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# Fertility, Mortality and the Macroeconomy in an Altruistic, Overlapping Generations Model

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UNIVERSITY OF MIAMI

FERTILITY, MORTALITY AND THE MACROECONOMY IN AN ALTRUISTIC,  
OVERLAPPING GENERATIONS MODEL

By

Isaac Petit-Frère

A DISSERTATION

Submitted to the Faculty  
of the University of Miami  
in partial fulfillment of the requirements for  
the degree of Doctor of Philosophy

Coral Gables, Florida

December 2009

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Fertility, Mortality, and the Macroeconomy in  
an Altruistic, Overlapping Generations Model

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The economic literature has found difficulty linking fertility and mortality rates. Previous versions of the dynastic (parental altruism) model have failed to predict the negative relationship between fertility and infant survival, since it was postulated that parents view children as normal goods and increases in childhood survival would result in a decrease in unit-child costs. In this work, I find that a simple reformulation of the Becker-Barro altruism hypothesis successfully predicts the observed demographic transition in the past century, as well as explaining fertility differences across countries. I contest that fertility decision is dependent on the number of surviving children and not the number of children born. Child bearing is therefore perceived as risk-taking behavior given the stochastic nature of childhood survival. Essentially, higher childhood survival requires fewer children (i.e., less "hoarding") in order to ensure the desired family size. The model predicts that higher childhood survival rates will lead to a decrease in fertility.

I calibrate an infinitely-lived overlapping-generations dynastic utility model and compare the fertility predictions of the baseline model with the data for the year 2000. In doing so, I have relaxed the dual normalization of the utility of death and the overall level of utility. This is necessary given that the value of children's lives are important in the parent's fertility decision. Parents jointly care for the number and utility levels of their

children. I will calibrate this number and estimate this implied value of life. I find that the consumption level an agent is indifferent between life and death to be less than 1% of current consumption. I also find that parents care for their children future 47% more than that of their own. All in all, this experiment finds that fertility differences cannot be explain by differences in mortality rates alone and that incorporating human capital investment in the household production function will yield desirable results. Simply stated, lower income countries have lower opportunity costs of birthing children and will choose quantity over quality.

I find that the steady state analysis of this model can explain over 62% of the cross-country variation, while mortality rates alone can explain 10%-25%. The model generally performs better for low survival, high fertility countries and vice versa. While the model tends to perform well for these economies, more needs to be done to explain fertility in the transition economies.

*To my family and future wife for their support*

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## CHAPTER 1

### INTRODUCTION

*The power of population is indefinitely greater than the power in the earth to produce subsistence for man.- Thomas Robert Malthus 1798. An Essay on the Principle of Population. Chapter 1, p. 13*

In his controversial *An Essay on the Principle of Population*, Thomas Malthus introduced the importance of an analytical, economic approach to demographic analysis. His idea that population growth was limited by a series of positive and negative checks remained fairly consistent with the observations of the late 18<sup>th</sup> and early 19<sup>th</sup> centuries. The earth's limited capacity to sustain life constrained what seemed to be the limitless potential of population growth. In essence, population, which unchecked can increase in a geometrical ratio, is constrained by the production abilities of man, which can only increase in an arithmetical ratio. This pattern is commonly referred to as the *Malthusian Trap*. Population would be held constant by *positive checks* on the death rate (i.e., disease, starvation, pestilence, etc.) or by negative, *preventive checks* on the birth rate (prevention of birth due to fear of increases in death rate). This would inevitably lead to an equilibrium of low wage<sup>1</sup>, high fertility, high mortality and stagnant population growth. Beginning in the 19<sup>th</sup> century, the triumphs of his theories would quickly wane as populations around the globe began to experience persistent declines in mortality rates followed by observed declines in fertility rates, which together came to be known as the *demographic transition*.

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<sup>1</sup>Income level will be equal to that of mere subsistence.

## HISTORICAL OBSERVATIONS

In the field of demography, *demographic transition* can be defined as the theory of explaining the transition from high mortality and fertility rates to low ones<sup>2</sup>. The transition began in most European countries and was later followed by China and the United States<sup>3</sup>. The four stage demographic transition model credited to demographer Warren Thompson (1929) and developed by Notestien (1945) places transition in the following stages:

- **Stage 1** represents the Malthusian, Pre-Industrial phase in which a country experiences high mortality and high fertility and fairly stagnant population growth.
- **Stage 2** signifies the beginning of demographic change. Health improvements, which, can be attributed to sanitation improvements, technological advance and health care, among others, lead to decreases in mortality rates. The corresponding chain of events lead to a subsequent increase in the population growth rate.
- **Stage 3** shows the beginning of a significant economic change. Countries begin to industrialize and show signs of economic growth<sup>4</sup>. During this period an increase in wages leads to higher investment in education and higher cost of raising children. The end result is a convergence of population growth to a stagnant rate.
- In **Stage 4**, increases in factor productivity and human capital stock will lead to low mortality and low fertility. At this stage countries may experience a decline in population due to below-replacement birth rates<sup>5</sup>.

---

<sup>2</sup>The information on demographic transition, presented in this paper, must be credited to Boldrin and Jones (2002), who beautifully present this empirical information in a comprehensive and concise fashion. I have provided additional information that I have acquired on demographic modeling. For more detailed information, please read Boldrin, Jones (2002) pg.

<sup>3</sup>see Livi-Bacci (1989) and Chesnais (1992)

<sup>4</sup>Ehrlich and Lui have developed a theory in an attempt to explain this phenomenon.

<sup>5</sup>Replacement fertility rate is the fertility rate required for woman to have enough children to replace herself and her partner

Today, the proportion of the population that is in Stage 1 is negligible. Recently, significant health improvements in the poorest, developing countries, such as Afghanistan, Laos and most of Sub-Saharan Africa, have elevated Stage 1 to Stage 2 status. These countries are expected to experience a transitory period of *population explosion*, where high fertility rates are accompanied by low mortality rates. Most of the world, including many developing countries, have begun, or are well into, the Stage 3 transition phase experiencing fertility declines of 10-40%. The richer, developed countries are now in Stage 4 and have maintained consistent below-replacement fertility rates resulting in a negative population growth. These observations have led to the general consensus that a fall in mortality inevitably results in a fall in fertility<sup>6</sup>.

Observing a statistically significant, cross-country correlation<sup>7</sup> between the Total Fertility Rate ( $TFR$ <sup>8</sup>) and the Infant Mortality Rate ( $IMR$ <sup>9</sup>), I emphasize the importance of considering infant mortality in analyzing fertility differences. This is contrary to the conventional thought, which attributes demographic change and fertility decline to technological advance and the industrial revolution.<sup>10</sup> This study develops a variant of the Becker-Barro (BB) dynastic utility model with the intention of explaining time-series and cross-country empirical observations of fertility, survival, and population growth, as it relates to the macroeconomy. Here it is assumed that parents care about the number

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<sup>6</sup>Please refer to Preston (1978) which provides us with an extensive review of literature pertaining to the effects of child mortality. Galloway, Lee and Hummel (1998) and Coale (1986) observed significant empirical evidence supporting the possible causal relationship between infant mortality and fertility.

<sup>7</sup>The cross sectional correlation coefficient found was approximately  $-0.814$ .

<sup>8</sup>the average number of children born to a woman over her lifetime, given she would experience the current fertility rate through her lifetime

<sup>9</sup>The Infant Mortality Rate (or  $IMR$ ) is defined as the number of infant deaths per 1000 live births. The difficulties in evaluating cross-country comparisons will be the difference in the definition of *live births*, which may or may not include perinatal and neonatal mortalities.

<sup>10</sup>Read Becker, Murphy, and Tamura (1990), Ehrlich and Lui (1991), Galor and Weil (1998), Lucas (1998), and Hansen and Prescott (1999) Ehrlich and Lui (1991), Doepke (2000), and Fernandez-Villaverde (2001) on this topic



of surviving children (i.e., the resulting family size). This is distinctly different from the hypothesis that parents desire directly the birth of children. Since per child costs decline as more children survive, under the assumption that people care only about the number of children born, declining child mortality will induce individuals to have more children. This is contrary to observation. Under the assumption that people care about surviving children, declines in child mortality lead to lower fertility as fewer births are needed to achieve a given expected number of surviving children. In addition, as the likelihood of child survival increases, risk-averse parents have less desire to produce and *hoard* children as *insurance*.

This work develops a variant of the BB model and evaluates its performance by estimating its parameters and comparing the model's predictions of observable variables to the actual values across countries.. The model successfully replicates historical patterns, but finds that cross-country fertility differences cannot be solely explained by mortality differences. This suggests the importance of integrating various macroeconomic factors (i.e., interest rates, income, exogenous GDP growth, etc.) to further our understanding. To compensate for these shortcomings, this thesis introduces human capital investments as it relates to the opportunity cost of raising children, in efforts to improve the results.

## THEORIES OF FERTILITY

One of the difficulties in modeling household decision behavior is the inherent complexity of the representative agents' preference structure. The difficulty arises in defining what motivates one to birth and raise children. The initial economic theory of fertility, introduced by Harvey Leibenstein, states that the household obtains utility from child services at a cost which is included in its household production function. Therefore parental

preferences for child services are manifested in three forms: the consumption services of children, the labor contribution to the family, and old-age security. The benefits of these of child services occur during different periods in the agents life with consumption utility occurring during young adulthood, labor productivity occurring during mature adulthood and old-age security during retirement. The most accurate theory of fertility would successfully incorporate these three effects.

Leibenstein's contribution led to the rise of two schools of thought concerning the economic theory of fertility<sup>11</sup>. The first was developed by Gary S. Becker<sup>12</sup> which defined parental altruism as the primary motivation for fertility (i.e., the consumption services of children) and the second was developed by John Caldwell<sup>13</sup> which postulated that utility was dominated by old-age security (i.e., *reverse altruism*). The evolution of the theory of parental altruism as the primary motivation for fertility (i.e., the consumption services of children) lead to the development of the Becker and Barro (BB) dynastic utility function, which outlined the preference of the agent for dynasty building (i.e., family creation). The theory states that parents derive utility from selfish, dependent children in two parts: first, the number of children (i.e., family size); second, the consumption utility of each child. The model was successful in linking fertility to the various macroeconomic variables. Given their constrained resources, parents must choose an equilibrium quantity-quality bundle which maximizes total utility. To contribute to the quality of children the parent must invest time, education, income, etc., at market costs. Linking

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<sup>11</sup>The quality/quantity tradeoff (ie. labor productivity utility) explains, in part, a proportion of the income effect on the demographic transition but currently fails to provide us with further understanding of the health effects on fertility.

<sup>12</sup>and was later corroborated by Nambroodiri (1972), Easterlin (1975), and Easterlin (1986)

<sup>13</sup>Caldwell (1978) concluded that cross-country differences in fertility can be explained by differences in contributions from children. These ideas were further developed by Lillard and Willis (1997) and Kirk (1996)

household production to fertility allowed the possibility explaining the effects of changes in macroeconomic variables on family-size decision-making behavior. Unfortunately, as mentioned early on by the authors of the theory, the model failed to predict the aforementioned demographic transition. Instead the model predicted, counterfactually, a negative relationship between fertility and mortality since children were predominantly viewed as consumption goods and mortality declines implied corresponding fall in per-child costs. The BB model also failed to explain the income/wealth effects on fertility. This has lead many to question and some to abandon the use of the dynastic utility function as a tool in modeling demographics.

In response many have supported and continue to support the opposing theory, which suggests that the primary fertility motive is old-age security. Proponents of the old-age security hypothesis argue that preferences are primarily dominated by *reverse altruism*; children care for selfish parents and therefore parents birth children as investment (retirement) goods. The old-age security hypothesis, although useful in overlapping generations models incorporating social security and old-age pensions, is incomplete and limited in explaining demographic transitions and cross-country demographic patterns. To state that fertility is solely motivated by old age security alone is incorrect and requires careful reconsideration.

Chapter 2 summarizes previous contributions to the economic theory of fertility. The economic literature reveals difficulties linking fertility with infant mortality rates (IMR). Essentially the limitation of the BB model is not its assumption of parental altruism, but the formulation of future preferences and child costs. The BB dynastic utility formulation states that the cost per child depends on the number of surviving children. As more children survive, relative cost per child falls leading to an increase in fertility. I address

this problem in Chapter 3 with a simple reformulation of the BB model. I contest that it is more accurate to assume that economic agents have a preference for the resulting family size (i.e., the number of surviving children, **not** the number of children born). Therefore child-rearing is perceived as risk-taking behavior since parents do not know the exact number of surviving children. Instead parents will make their decisions based on expected utilities. The inherent precautionary motive of rearing children may cause what has been referred to as the *hoarding effect*<sup>14</sup>. High mortality families may overproduce to ensure they achieve the desired family size. As mortality rates decline, the necessary number of children born will consequently fall. I will also examine the life cycle consumption and fertility behavior of households who face the possibility of intermediate death during both young adulthood and mature adulthood.

#### UTILITY AT DEATH

In uses of additive intertemporal utility functions where births and lifespan are exogenous results are usually invariant to the addition of a constant to the utility function. In other words, only the utility function's property of intertemporal substitution matters, not its level. The modeler is thus free to normalize the utility of a specific level of consumption to any level. This is not the case where births or lifespan are endogenous, as in this work.

In much of the literature where number of births or lifespan are endogenous, the utility of being dead (or unborn) is normalized to zero, but so is the utility of a given level of consumption (usually zero consumption) given that the individual is alive. This double normalization, which is normally made implicitly, is not innocuous as it affects

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<sup>14</sup>Taylor et. al. (1976) and more recently, Sah (1991) and Kalemli-Ozcan (2003)

the relative valuation of children versus own consumption (or of consumption versus extending life)<sup>15</sup>. I will calibrate this parameter using data on mortality risk and wage differentials.

## HOUSEHOLD PRODUCTION

As we will observe in Chapter 5, cross-country fertility differences cannot be explained through differences in mortality alone. There are notable structural macroeconomic differences that must be addressed in order to compensate for the inaccuracies in the model. In Chapter 6, I consider various important economic factors which contribute to the fertility decision behavior of the representative agent. As we will observe in the model, the differentials between the model and the data are much larger for the poorer, lower health countries. A proportion of this difference may be explained through the income effects of the quantity-quality trade-off proposed in earlier research<sup>16</sup>. The empirical evidence<sup>17</sup> suggests a negative relationship between family size and the educational level of children. To incorporate the quantity-quality trade-off, I consider the child birthing behavior in the household production process. Household production theory posits the following; agents allocate time and resources in order to transform intermediate goods into final goods later to be used for consumption. Therefore, in a model incorporating household production, child creation will require both time and goods but will be driven by the desire of the household to replicate itself. The market value of wages, income and

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<sup>15</sup>Becker and Stout (1992) found it intrinsically wrong to assume the utility of death to be equal to the utility of zero consumption, as most of the literature has.

<sup>16</sup>Initially formulated by Becker (1960) and developed by many in the subsequent years including Becker and Lewis (1973) and Becker and Tomes (1976), a unified analytical approach is generally non-existent.

<sup>17</sup>Rosenzweig and Wolpin (1980), Berhman et. al. (1989) and Stafford (1987), Conley (2004), Goux and Maurin (2004) are a few examples.

savings will inevitably affect the household's child production decision. It can be expected that as income increases the time costs of birthing children will also increase, resulting in a decline in the time intensive production of children. The necessary quantity/quality trade-off households face originates from the choice of either investing in the human capital of each child (i.e., education, life skills, etc.) or birthing more children.

This section yields a few interesting conclusions. First, as child creation becomes more labor intensive less children are born. Second, as income increases the production of children transfers from labor to goods. Third, as income increases human capital transfers to children also increase. And lastly, the income effect on the number of children born is negative.

This thesis is organized in the following manner: Chapter 2 reviews the previous contributions pertaining to topics discussed; Chapter 3 develops a reformulation of the Becker-Barro model addressing the utility at death, incorporating a revised approach to household production and, most importantly, expected altruism; I calibrate the model in Chapter 4. An important contribution of this thesis is the estimation of the utility level that corresponds with the value of life which, to the best of my knowledge, is yet to be done. This parameter is estimated by incorporating the given information on mortality risk and wage differentials to the hedonic wage methodology. I then discuss the results in Chapter 5; To improve these results, I discuss possible extensions in Chapter 6. I will conclude and discuss certain limitations in Chapter 7.

## CHAPTER 2

### PREVIOUS LITERATURE

#### 2.1 Foundations of an Economic Approach to Fertility Theory

Between the late 50's and early 60's, almost two centuries following the work of Malthus, a surge of literature introduced a pragmatic, theoretical approach to demographic modeling and analysis. In response to the previous failures of arcane time-series methods, Harvey Leibenstein (1957) incorporated consumer demand theory in the creation of children. Essentially households would decide whether or not to have an additional child based on the corresponding marginal costs and benefits. More importantly, his theory that parents acquire utility from *child-services* served as the theoretical foundation for linking economics and demography in the many years that followed. He argued that child-services come in three forms and are actualized during different periods of a parent's life. Young adult parents acquire utility from the *consumption-services* of newborn, unproductive children<sup>18</sup>. Second, mature adult parents realize the *labor-productivity-services* of their young adult children<sup>19</sup>. The third and final service occurs in old-age retirement when middle-aged children provide *old-age security* to adult parents. The current philosophical debate is a question of whether the *consumption-services* (i.e., parental altruism) or *old-age security* (i.e., reverse altruism) dominates the household fertility decision.

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<sup>18</sup>Children are essentially perceived as consumption goods, in which utility for children is concave and obeys the inada conditions.

<sup>19</sup>Mature adult parents invest in the capital required for improving the labor productivity of children.

## 2.2 Two Schools of Thought: Parental Altruism versus Old-Age Security

### PARENTAL ALTRUISM

Following Leibenstein's work, Gary Becker (1960) introduced the household production process linking various macroeconomic variables to fertility. The following three decades lead to the culmination of ideas contributing to the notion of *parental altruism*<sup>20</sup>. The influential work of Gary Becker and Robert Barro (1988) elegantly reformulated the propositions of original Becker paper and served as a catalyst in the economic modeling of the demand theory of fertility. The Becker-Barro (BB) dynasty utility model hypothesizes that parents are altruistic towards their selfish children or, simply stated, derive utility from their consumption as well as their children'. Households also have a preference for continuing their *dynasty* in which the limited life of the parent is continued through the children created. Concave utility is obtained from the number of children born suggesting that children are viewed as normal goods constrained by the inherent costs of raising them. But the shortcomings of the model, as suggested by the authors, was the inability to predict a permanent fertility decline given a permanent decline in IMR or permanent increase in income and wealth.

A great deal of literature further corroborates this claim. Lillard and Willis (1997), using data on intergenerational transfers, found little evidence to support altruism as a primary fertility motive. Alvarez (1999) computed the counterfactual results of the dynastic utility function finding the model's prediction that wealthier parents will have more children<sup>21</sup>. Fernandez-Villaverde (2001) provided quantitative results by calibrating

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<sup>20</sup>Easterlin (1975 ), Easterlin (1986), Landes and Posner (1978), Namboodiri (1972)

<sup>21</sup>In this paper, Alvarez combines the Becker-Barro model with the Laitner-Loury model to incorporate the uninsurable risk, intergenerational transfers, and fertility.



the Becker and Barro model. He found that the dynastic utility model provided little empirical evidence to prove a relationship between the IMR, fertility, and population growth. As expected the model failed to predict the observed demographic statistics. Boldrin, De Nardi and Jones (2002) exposed the inability of the Becker-Barro model in explaining the effects of social security on fertility<sup>22</sup>. The majority of the literature on this topic concludes that the dynastic framework predicts counterfactual results.

### OLD-AGE SECURITY

In an attempt to link fertility with the observed economic variables, many have abandoned the use of the dynastic utility function and the altruism hypothesis, using old-age security as the primary motive for rearing children. This set of propositions stems from the early, pioneering work of Caldwell (1978 and 1982)<sup>23</sup>, Willis (1980), Nugent (1985), and Srinivasan (1988). Old-age security or *reverse altruism* postulates that children care for parents and provide for them during old-age. Children are essentially perceived by parents as investment goods and parents have information as to how much they can expect to receive from their children when old. The Boldrin and Jones (2002) paper developed a model which links infant mortality, fertility and capital savings behavior. Boldrin, DeNardi and Jones (2005) performed an experiment comparing the results of *altruism* verses *old-age security*. After calibrating and simulating both the Caldwell and BB models, they found that, in the BB model, increases in government old-age pensions yielded a very small, insignificant change in the household fertility. These ideas have

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<sup>22</sup>Boldrin, DeNardi, and Jones found that the effect of increases in old-age pensions in the Becker-Barro framework was inconsistent with empirical findings in Europe, while finding that the Caldwell model account for 55% of the variation.

<sup>23</sup>Caldwell (1978 and 1982) concluded that transfers from children alone can explain high fertility rates

perpetuated into the resulting, modern theory supporting this claim preceded by the influential Boldrin and Jones (2002) article.

Although the existence of the old-age security motive is undeniable, we must examine the plausibility of the assumption that children are solely investment goods. Under these assumptions, it is difficult to justify the choice to bear children as rational behavior given the relatively high costs and low monetary returns of birthing children. Essentially parents would choose cheaper forms of investment yielding higher returns and would choose not to have children.

In a general equilibrium model, the rates of return, in children and capital, must equate implying a defined relationship between capital investment and number of children. This relationship may likely lead to a corner solution. Zhang and Nishimura (1993) addressed the question of the existence of an interior solution. They found the possibility of arriving to corner solutions where households may choose to have no children. Lagerloff (1996) also supported the notion that it was incorrect to assume the existence of an interior solution. He also found the corner solutions of Nishimura and Zhang (1992) and Zhang and Nishimira (1993). Finding levels of fertility other than the common equilibrium solutions of the Caldwell model lead to higher utility levels, questioning the plausibility of an interior solution<sup>24</sup>. These observations, in comparison to empirical findings, suggest that there must be another motivating force behind the fertility and that it is important to emphasize the validity of altruism in modeling demographic transition without contesting the relevance of the old-age security hypothesis.

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<sup>24</sup>Lagerloff concluded that household will not have children and will only invest capital.

### 2.3 Family-Making Decisions under Uncertainty

The initial questions concerning the effects of IMR on fertility can be explained through the stochastic nature of fertility itself. The precautionary fertility motive is derived from the uncertainty of childhood survival. As suggested by Schultz<sup>25</sup> (1997), the fertility decision, in response to the observed infant mortality rate, is primarily dominated by the insurance strategy<sup>26</sup>. Based on a Ben-Porath (1976) study<sup>27</sup>, Sah (1991) introduced the first theoretical paper to examine fertility and population growth in a stochastic environment. Assuming parental altruism, risk-averse parents are inclined to *hoard* or overproduce offspring to compensate for the possibility of death. His proposition lead to the prediction of a positive relationship between the production of children and infant mortality. He went further to estimate the welfare effects of declines in child mortality on parents. This thesis contributes to the burgeoning body of literature supporting this claim.

Closely related to my work is a recent study by Doepke (2005) which studies the performances of three extensions to the Becker-Barro model<sup>28</sup>. The model which incorporates continuous fertility<sup>29</sup> performed fairly well in predicting the observed historical declines in fertility in various countries. In this dissertation, I will calibrate the model to match the observed moments in the United States in the year 2000. I will then utilize the model to predict the cross country fertility rates comparing the model predictions to

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<sup>25</sup>According to Schultz, fertility is dominated by the replacement strategy and the insurance strategy. His study find the replacement effect explains less than 50% and generally falls around 20%.

<sup>26</sup>The insurance strategy leads to the *precautionary demand for children* or the *hoarding mechanism*.

<sup>27</sup>Ben-Porath postulated that the expected utility depended on the expected number of surviving children.

<sup>28</sup>Incorporating the uncertainty of family size, his study included a model with continuous fertility, one with discrete fertility and another with discrete and sequential fertility. In all three model he found a positive relationship between fertility and infant mortality.

<sup>29</sup>This assumption ignores Sah's claim that the discrete fertility choice provides more realistic results. I will assume continuous fertility in order to avoid possible computational hurdles.

empirical observation. I will also consider the effects of other macroeconomic variables (i.e., income, interest rates, exogenous growth rates) on fertility.

## CHAPTER 3

### MODEL

Consider an economy with discrete time  $t = 1, 2, \dots$ , where the representative agent may live for a maximum of 4 periods (childhood, young adulthood, mature adulthood, and retirement). Childhood is considered to be a period of parasitic, unproductive living. Children are partially dependant on parents for future income through the bequests which will be received during young and mature adulthood. Agents are endowed with two periods of inelastic labor supply, during young and mature adulthood, valued at  $w_t$  and  $w_{t+1}$ , respectively. Given perfect foresight, young adults (ages 21-40) will also choose an optimal lifetime saving and consumption bundle  $\{(s_t, s_{t+1}), (C_{1t}, C_{2,t+1}, C_{3,t+2})\}$  that maximizes expected lifetime utility,  $v_t$ .

The representative agent also considers the perpetuation of her life through the lives of future children. Unreciprocated altruism towards children is manifested in two ways, by the number of children born and the bequests entrusted to them. Young adulthood will be assumed to be the fertile period where the fertility decision,  $\eta_t$ , will be made and realized. This altruistic pattern continues through temporal bequest decisions occurring during both mature adulthood (ages 41-60) and retirement (ages 61-80). As mature adults, agents will transfer total bequest,  $B_{t+1}^A$ , to  $\pi_1 \eta_t$  surviving young adult children. Older retired agents give total bequest,  $B_{t+2}^O$ , to  $\pi_2 \pi_1 \eta_t$  surviving, mature adult children. Therefore agents are also bequeathed with from their parents.

Observe that households are faced with the possibility of death in each period. Agents will make their decisions according to the present value of future trading decisions and will also factor the possibility of death in their valuation process via actuarially fair intrafamily insurance policies. The agent has full information about its survival probabilities.  $\pi_1$  is the probability of surviving from childhood to young adulthood.  $\pi_2$  and  $\pi_3$  are the probabilities of surviving from young adulthood to mature adulthood and from mature adulthood to retirement,  $b_t^A$  and  $b_{t+1}^O$ , respectively.

### 3.1 Fertility Cost

The unit cost of raising children is constant and linearly proportional to income.

$$\theta_t = f_\theta w_t + g \tag{3.1}$$

such that,  $f_\theta, g \in (0, 1)$ .

Under the assumption that affective labor is inelastically supplied,  $f_\theta$ , represents the proportion of time spent raising each child, while  $g$  characterizes the consumption cost of child rearing. The constant unit cost assumption is introduced here contrary to the evidence which suggests existent scale effects, which cause the cost per child to decline as the number of children rises, vary across countries such that  $\theta_t = f_\theta(w_t, \eta_t)$  and  $\partial f_\theta / \partial \eta_t < 0$  (i.e., the cost per child declines as the number of child rises). The lack of consideration to the prevailing scale effects will not abstract from the important conclusions and implications of the model. Of significant importance is the cost of administering bequests to children which will be discussed in further detail in the following section.

### 3.2 Budget Constraint

Given the ergodicity of corresponding survival rates and the homogeneous properties of households<sup>30</sup>, all decisions will be made at time  $t$  and agents face the following budget constraints

$$C_{1t} + \theta_t \eta_t + s_t \leq w_t + b_t^A \quad (3.2)$$

$$\pi_2 C_{2,t+1} + \pi_1 \delta_{t+1} \eta_t b_{t+1}^A + s_{t+1} \leq \pi_2 (w_{t+1} + b_{t+1}^O) + (r_t + 1) s_t \quad (3.3)$$

$$\pi_3 C_{3,t+2} + \pi_2 \pi_1 \delta_{t+1} \eta_t b_{t+2}^O \leq (r_{t+1} + 1) s_{t+1} \quad (3.4)$$

which have been derived through a series of intrafamily insurance policies<sup>31</sup>. Equation (3.2) denotes the budget constraint of young adults and subsequently equations (3.3) and (3.4) characterize the budget constraints of mature adults and retired adults, consecutively. Parents insure gifts given to children in the event that they fail to survive in the next period. Introducing insurance policies will eliminate the possibility of heterogeneity and income distribution problems.

At time  $t$ , the household is faced with the following present value intertemporal budget constraint:  $C_{1t} + \theta_t \eta_t + \frac{\pi_2 C_{2,t+1} + \pi_1 \delta_{t+1} \eta_t b_{t+1}^A}{(r_t + 1)} + \frac{\pi_3 C_{3,t+2} + \pi_2 \pi_1 \delta_{t+1} \eta_t b_{t+2}^O}{(r_t + 1)(r_{t+1} + 1)} \leq w_t + b_t^A + \frac{\pi_2 (w_{t+1} + b_{t+1}^O)}{(r_t + 1)}$ , where  $\theta_t$  represents the unit cost of rearing children,  $\delta_{t+1}$  represents the cost of transferring inter vivos bequests and  $r_t$  and  $r_{t+1}$  represent the intertemporal rates of return on investment.

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<sup>30</sup>All households will experience the same survival shocks.

<sup>31</sup>Please refer to the appendix for more detail.

### 3.3 Households

#### 3.3.1 Formulation of Periodic Utility Function

We are faced with the following conundrum; in order to accurately state the valuation of life, an understanding of the value of death is require. More specifically the value of life is a measurement relative to the value of death. The present value of lifetime future decisions is directly affected by the agents' preferences over an uncertain length of life. Each period the household faces two possibilities, life and death, and it values each state differently. At each period,  $t$ , the utility of being alive in the next period is dependant upon both the utility of being alive which, assuming an optimal decision has been made is  $U(C_{t+1})$ , and the utility at death which I will define as,  $U^*$ . If  $U^*$  is greater than the maximum utility,  $U(C_{t+1})$ , the agent will choose death over life and borrowing against future earning for consumption in period,  $t$ . Therefore the agents total maximum lifetime utility will be  $U(C_t) + U^*$ . Present value decisions are definitively dependent on the household's value of death. The agent faces death each period with probability  $(1 - \pi_i)$  how she values death will be important on how she values the present versus the future.

In a simple two-period model where the household faces probability,  $\pi$ , of surviving to period 2 and utility of death,  $U^*$ , the von Neumann-Morgentstern utility function is characterized by

$$v_t = U(C_t) + \pi U(C_{t+1}) + (1 - \pi) U^*$$

As suggested by Becker and Stout (1992), it is intrinsically wrong to assume values for the utility of death and the utility of zero consumption. As previously mentioned, the utility levels are important in analyzing a model with uncertain life and endogenous fertility. A principle of equivalence exists when comparing the decision making behavior of an



agent whose utility of zero consumption,  $U(0) = 0$  and the utility of death,  $U^* > 0$ , and conversely choosing  $U(0) > 0$  and  $U^* = 0$ . When normalizing the linear hedonic lifetime utility function, it makes no difference with method is chosen, as long as the utility of zero consumption is not forced to be the utility at death. It is a matter of preference which will be normalized to be zero.

Generally speaking, the economic literature has failed to address this problem by assuming both the level utility and the utility of death to be zero. For standard functional forms this poses serious problem in expected utility theory. For example, if we assume

$$U(C) = \frac{C^{1-\sigma}}{1-\sigma}$$

and  $U(C) < 0$  for all  $C$ . This implies that the utility at death is greater than the utility of being alive for all  $C \in \mathbb{R}^+$ . Consumption,  $C$ , must approach infinity in order for the household to be indifferent between life and death, which is an impossible realization under a bounded budget constraint. Also, given that one chooses the utility at death to be zero, the a priori presupposition of the utility of zero consumption limits our understanding of the agents' preferences over time. For example, the lack of generalization for log utility in the life cycle (i.e.,  $\sigma = 1$ ) presumes a fixed level of consumption for which the household is indifferent between life and death (i.e.,  $C^* = 1$ ) A generalization of the utility function provides us with the possibility of accurately observing the value of life and its sensitivity to the coefficient of relative risk aversion.

Therefore I will assume

$$U(C) = \frac{C^{1-\sigma}}{1-\sigma} + \zeta \quad (3.5)$$

The level of consumption,  $C^*$ , which the individual is indifferent between being alive or dead is

$$C^* = [\zeta (\sigma - 1)]^{\frac{1}{1-\sigma}}$$

for  $\sigma > 1$ . Lower values of  $\zeta$ , requires  $C^*$  to be very high and possibly infeasible. In order to avoid efficiency problems,  $\zeta$  must be sufficiently high for  $C_{i,t} \geq C^*$ , for all  $i = 1, 2, 3$  and  $t = 0, 1, 2, \dots$ . For  $\sigma = 1$  (i.e., log utility)

$$C^* = e^{-\zeta}$$

Accurately assigning a value to  $\zeta$  requires some understanding of the household's preferences and should be treated carefully. Contrary to the Becker and Stout statement that the utility of death is undefinable, the utility of zero may be acquired from the agent's indifference set between wage and mortality risk. I have yet to find literature attempting to estimate this parameter. I will estimate this parameter using information on mortality risk and wage differentials.

### ***3.3.2 Expected Lifetime Utility***

Given that the household has the possibility of living 3 additional periods and  $U^*$  is normalized to 0, the lifetime expected utility function at period  $t$  is

$$\nu_t = U(C_{1t}) + \beta\pi_2 [U(C_{2,t+1}) + \beta\pi_3 U(C_{3,t+2})] \quad (3.6)$$

$\beta$  is the time preference discount factor.  $C_1$ ,  $C_2$  and  $C_3$  are consumption during young adulthood, mature adulthood and retirement, respectively.

### 3.3.3 *Dynastic Preferences*

Although the agent can only live for a maximum of 4 periods, she is survived through future generations. Her preference for perpetuating future generations is represented by the dynastic utility function introduced by Becker and Barro (BB). Each period a new generation is born and the household values each child at a utility rate discounted by a certain number of children. The household not only cares for its lifetime consumption but also the number of children and the consumption of its future generations. The lifetime utility of the dynasty will be characterized as

$$E_Q \left[ \sum_{t=1}^{\infty} (\alpha(\eta_t)^{1-\varepsilon})^t [U(C_{1t}) + \beta\pi_2 U(C_{2,t+1}) + \beta^2\pi_3 U(C_{3,t+2})] \right]$$

$C_1$  : young age consumption

such that,  $C_2$  : adult consumption

$C_3$  : old age consumption

The household will maximize the following utility function.

$$V_t = \nu_t + E_Q [\alpha(\eta_t^*)^{1-\varepsilon}] V_{t+1} \quad (3.7)$$

where  $\eta_t^* \sim Q(\eta_t)$ .

Unlike BB, the household obtains utility from the number of surviving children and **not** the number of children born. Observe that the model differs from BB in the sense that households view fertility as a risky endeavor. Olsen (1994) suggested that "there

is substantial uncertainty about the rate of child survival". He goes on to exploit the possibility of family sizes becoming too large to ensure the desired family size, sometimes referred to as a hoarding or precautionary effect. The number of surviving children,  $\eta_t^*$ , depends on the number of children born,  $\eta_t$ , and the probability distribution,  $Q(\eta_t)$ , of surviving children. As we will see, this reformulation will provide significantly different predictions concerning fertility and infant mortality rates. Another aspect worth considering, but not explored in this paper, is the effect of the death of children on the household. Although there is evidence to suggest that disutility from children dying exists, I will not consider it in this paper and will discuss its consequences later. The household's value function will be

$$V_t = \max \left\{ \nu_t + \left[ \alpha(\pi_1 \eta_t)^{1-\varepsilon} + \frac{\alpha \pi_1 (1-\varepsilon) (\pi_1 \eta_t)^{-\varepsilon}}{2} \sigma_\eta \right] V_{t+1} \right\} \quad (3.8)$$

where  $\sigma_\eta$  represents the variance of the number of surviving children<sup>32</sup>.

I will simplify the model in the following functional form:

$$V_{t+1} = \max \{ \nu_t + \rho(\eta_t) V_{t+1} \} \quad (3.9)$$

such that,  $\rho(\eta_t) = \alpha(\pi_1 \eta_t)^{1-\varepsilon} + \frac{1}{2} \alpha \pi_1 (1-\varepsilon) (\pi_1 \eta_t)^{-\varepsilon} \sigma_\eta$ <sup>33</sup>.

For simplicity's sake I have obtained the second-order approximation of the survival distribution. The household's fertility decision will depend on the expected number of surviving children and the variance of the survival distribution. Two households with the same infant mortality rate may have the different optimal fertility decision rules.

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<sup>32</sup>The notion of expected altruism is similar to a study by Ehrlich and Kim in which the utility parents derive from children is characterized by the altruism function which includes the child survival rate.

<sup>33</sup> $\frac{\partial \rho(\eta_t)}{\partial \eta_t} = \alpha \pi_1 (1-\varepsilon) (\pi_1 \eta_t)^{-\varepsilon} - \frac{1}{2} \alpha \varepsilon \pi_1^2 (1-\varepsilon) (\pi_1 \eta_t)^{-\varepsilon-1} \sigma_\eta$

Households with higher variance must birth more children to insure that they achieve the desired family size.

**Household Problem** The household's problem is therefore:

$$V_t = \max_{\{\eta_t b_{t+1}^A, b_{t+2}^O \geq 0\}} U(C_{1t}) + \beta \pi_2 [U(C_{2,t+1}) + \beta \pi_3 U(C_{3,t+2})] + \rho(\eta_t) V_{t+1}$$

$$\text{such that: } C_{1t} + \theta_t \eta_t + \frac{\pi_2 C_{2,t+1} + \pi_1 \delta_{t+1} \eta_t b_{t+1}^A}{(r_t+1)} + \frac{\pi_3 C_{3,t+2} + \pi_2 \pi_1 \delta_{t+1}^* \eta_t b_{t+2}^O}{(r_t+1)(r_{t+1}+1)} \leq w_t + b_t^A + \frac{\pi_2(w_{t+1} + b_{t+1}^O)}{(r_t+1)}.$$

#### FIRST ORDER CONDITIONS

The household's problem yields the following optimally conditions:

$$C_{1t} : U'(C_{1t}) = \lambda_t \quad (3.10)$$

$$C_{2,t+1} : \beta U'(C_{2,t+1}) = \frac{\lambda_t}{r_t + 1} \quad (3.11)$$

$$C_{3,t+2} : \beta^2 \pi_2 U'(C_{3,t+2}) = \frac{\lambda_t}{(r_t + 1)(r_{t+1} + 1)} \quad (3.12)$$

$$\eta_t : \lambda_t \left[ \theta_t + \frac{\pi_1 \delta_{t+1} b_{t+1}^A}{r_t + 1} + \frac{\pi_2 \pi_1 \delta_{t+1}^* b_{t+2}^O}{(r_t + 1)(r_{t+1} + 1)} \right] = \rho'(\eta_t) V_{t+1} \quad (3.13)$$

$$b_{t+1}^A : \rho(\eta_t) \lambda_{t+1} - \frac{\pi_1 \delta_{t+1} \eta_t \lambda_t}{r_t + 1} \leq 0 \quad (3.14)$$

$$b_{t+2}^O : \frac{\rho(\eta_t) \lambda_{t+1}}{(r_{t+1} + 1)} - \frac{\pi_1 \delta_{t+1}^* \eta_t \lambda_t}{(r_t + 1)(r_{t+1} + 1)} \leq 0 \quad (3.15)$$

The following Kuhn-Tucker condition,  $\lambda_t \left[ w_t + b_t^A + \frac{\pi_2(w_{t+1} + b_{t+1}^O)}{(r_t+1)} \right] - \lambda_t \left[ C_{1t} + \theta_t \eta_t + \frac{\pi_2 C_{2,t+1} + \pi_1 \delta_{t+1} \eta_t b_{t+1}^A}{(r_t+1)} + \frac{\pi_3 C_{3,t+2} + \pi_2 \pi_1 \delta_{t+1}^* \eta_t b_{t+2}^O}{(r_t+1)(r_{t+1}+1)} \right] = 0$ , combined with equation (3.10) suggests that the household's budget constraint binds with equality. In equation (3.13), the marginal benefit of an additional child,  $\alpha \pi_1 (1 - \varepsilon) (\pi_1 \eta_t)^{-\varepsilon} V_{t+1}$ , equates with the lifetime cost of rearing and raising the child. Equation (3.13) captures the no-

tion that the marginal value of birthing a child is equal to the cost of raising the child which includes the initial time cost,  $\theta_t$ , and the cost of future bequest obligations<sup>34</sup>.

**Proposition 1** *Assuming an interior solution exists in the optimally conditions and*

1.  $U(\cdot)$  is twice differentiable,  $U'(\cdot) > 0$  and  $U''(\cdot) < 0$
2.  $d_t = \{C_{1t}, C_{2,t+1}, C_{3,t+2}, \eta_t, b_{t+1}^A, b_{t+2}^O\}$  such that  $d_{t,i} \in \mathbb{R}^+$  for all  $i = 1, \dots, 6$  and  $t = 0, 1, 2, \dots$
3.  $\delta_{t+1} = \delta_{t+1}^*$  for all  $w_t, w_{t+1}, b_{t+1}^A, b_{t+2}^O, \eta_t \geq 0$  and  $t = 0, 1, 2, \dots$

*The agent will choose the combination of  $b_{t+1}^A$  and  $b_{t+2}^O$ , such that if  $b$  solves equation (3.14) and  $b = jb_{t+1}^A + kb_{t+2}^O$ .*

The model predicts that the household is indifferent between choosing to give to children during adulthood and retirement if  $j = k$ . Assuming  $j = k$ , there are an infinite number of possible values for  $b_{t+1}^A$  and  $b_{t+2}^O$ . Now if  $j > k$ , the household will choose  $b_{t+2}^O > 0$  and  $b_{t+1}^A = 0$ . Essentially it requires less  $b_{t+2}^O$  to achieve the utility maximizing bequest level. And the converse is true. That is, if  $k > j$ ,  $b_{t+2}^O = 0$  and  $b_{t+1}^A > 0$ .

**Proposition 2** *Assuming the conditions of Proposition 1 and*

1.  $U(\cdot)$  is CRRA
2.  $r_t \geq 0\pi$

*The agent will choose  $b_{t+2}^O > 0$  and  $b_{t+1}^A = 0$  since  $j = 1$  and  $k = \frac{\pi_2}{r_{t+1}+1} \leq 1$ .*

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<sup>34</sup>At this point the uniqueness of the solution may be a concern due to the non-convexity of the budget constraint. The cost of raising a child is endogenously related to the household decision variables.

This result follows general economic intuition. Although parents have insured bequests against future loss of life, the present value of the cost of future bequest donations are greater during mature adulthood than retirement. Parents would prefer to give to their mature children during old-age retirement. It is limited in doing so by the following optimally condition

$$\rho(\eta_t) U'(C_{1,t+1}) = \frac{\pi_1 \delta_{t+1} \eta_t}{r_t + 1} U'(C_{1t}) \quad (3.16)$$

With this result I will arbitrarily choose the decision point to be during mature adulthood and not during retirement (i.e.,  $b_{t+1}^A > 0$  and  $b_{t+2}^O = 0$ ). Rearranged

$$U'(C_{1t}) = (r_t + 1) \frac{\rho(\eta_t)}{\delta_{t+1} \pi_1 \eta_t} U'(C_{1,t+1}) \quad (3.17)$$

which states that the average return on future investment per child is equal to the current marginal utility of consumption. Interestingly the fertility decision is determined by equation (3.17) given that the intergenerational marginal rates of substitution must equate to the returns. As a result the agent chooses bequests that adjust costs to equal to the given marginal value of an additional child. Given perfect foresight I assume markets are complete, which subjects the representative agent to the following condition.

A competitive equilibrium is the set of choices  $\{\eta_t, b_{t+1}^A, b_{t+2}^O\}$  such that, given prices  $\{w_t, w_{t+1}, r_t, r_{t+1}, \theta_t, \delta_{t+1}\}$  and parameters  $\{\mu, \sigma, \zeta, \beta, \pi_1, \pi_2, \pi_3, \sigma_\eta, \alpha, \varepsilon, f, g\}$  the representative agent's maximization problems are solved and the goods markets clear.

### 3.4 Demographics

There are 3 types of individuals; young adult, mature adult, and old. Populations evolve according to the fertility decision,  $\eta_t$ , and corresponding survival rates,  $\pi_1, \pi_2,$

and  $\pi_3$ .  $\pi_1$ , represents the percentage of children who survive into young adulthood.  $\pi_2$ , will represent the survival rate of young adults who survive into mature adulthood, conditional upon surviving childhood.  $\pi_3$ , will represent the rates mature adults surviving into retirement, conditional upon  $\pi_2$ . Their population dynamics are characterized by the following equations:

$$N_{t+1}^Y = \pi_1 \eta_t N_t^Y \quad (3.18)$$

$$N_{t+1}^A = \pi_2 N_t^Y \quad (3.19)$$

$$N_{t+1}^O = \pi_3 N_t^A \quad (3.20)$$

Equation (3.18) characterizes the law of motion for the young adult population. The mature adult population,  $N_t^A$ , and the older retired adult population,  $N_t^O$ , depend on the conditional survival rates of young and mature adults,  $\pi_2$  and  $\pi_3$ , which is represented by equations (3.19) and (3.20) respectively. The total population evolves under the following rule:

$$N_{t+1} = (\pi_1 \eta_t + \pi_2) N_t^Y + \pi_2 \pi_3 N_{t-1}^Y - \pi_2 \pi_3 N_{t-2}^Y \quad (3.21)$$



### 3.5 Steady State

Given the exogenous income growth rate, we have a balanced growth path for consumption and bequests resulting in the following steady state conditions.

$$U'(\bar{C}_1) \left[ \bar{\theta} + \frac{\bar{\delta}(1+\mu)\pi_1\bar{b}^A}{r+1} \right] = \frac{\partial \rho(\bar{\eta})}{\partial \bar{\eta}} \bar{V} \quad (3.22)$$

$$\rho(\bar{\eta})(1+\mu)^{-\sigma} = \frac{\bar{\delta}\pi_1\bar{\eta}}{\bar{r}+1} \quad (3.23)$$

such that  $\bar{\eta}, \bar{k}, \bar{b}^O > 0$ ,  $\bar{C}_1 + \bar{\theta}\bar{\eta} + \frac{\pi_2 C_2 + \bar{\delta}(1+\mu)\pi_1\bar{\eta}\bar{b}^A}{(r+1)} + \frac{\pi_3 C_3}{(r+1)^2} = \bar{w} + \bar{b}^A + \frac{\pi_2(1+\mu)\bar{w}}{(r+1)}$  and  $\bar{V} = \frac{U(\bar{C}_1) + \beta\pi_2 U(\bar{C}_2) + \beta^2\pi_3 U(\bar{C}_3)}{1-\rho(\bar{\eta})}$ .

A simplified first order approximation of  $\rho(\eta)$  provides us with the following steady state fertility equation

$$\eta = \left[ \frac{\alpha(r+1)}{(1+\mu)^\sigma} \right]^{1/\varepsilon} \frac{1}{\bar{\delta}\pi_1} \quad (3.24)$$

which can be referred to as a golden rule for fertility.

This equation provides us with a striking result. Fertility is independent of wealth, income and first period unit child time costs. Given perfect oversight and insured mortality risk, equation 3.17 posits that parents value each dollar spent on themselves to each dollar provided to each child. In the long run, the additive nature of the utility function assumes this fixed proportion independent of the wealth and income levels. From this equation I have observed the following facts:

1. As  $\pi_1$  increases,  $\eta$  falls for all  $\pi_1$

This is an important result of this paper given previous critical judgements of the altruism hypothesis. A simple reformulation in which an agents' preferences depends not on the number of children born but on the number of surviving children

leads to favorable results and consistent pattern predictions. An increase in the infant survival rate requires fewer births to obtain the desired family size.

2. As  $\delta$  increases,  $\eta$  decreases for all  $\delta$

As the adoption cost per child increases, it is expected that the number of children born will fall. Therefore according to the assumption that  $\delta$  rises with income agents with high income will have low fertility and vice versa.

3. As  $\alpha$  increases,  $\eta$  increases for all  $\alpha$

$\alpha$  represents the agents' time preference for future generations. As  $\alpha$  increases utility derived from having children increases (i.e., children become more valuable. If  $\alpha > \beta$  parents value their children's future more than their own and vice versa.

4. As  $r$  increases,  $\eta$  increases for all  $r$

The golden rule for fertility indicates a constant intertemporal rate of substitution between bequests given to children and the number of children born. The cost of giving to children is therefore equal to discounted return on future generation. As interest rates rise the return on future generations rises and therefore more children are born.

Essentially,  $s_t$ , is a mechanism by which the agent smooths consumption during her lifetime. The opportunity cost of consuming today is the interest rate,  $r$ , or the return on waiting for future consumption. Therefore  $r$  may be defined as the cost of smoothing consumption. Therefore, given the household has a preference for consumption smoothing, an increase in  $r$  discourages young consumption,  $C_1$ , which is consequently substituted by having more children.

## BALANCED POPULATION GROWTH PATH

Population growth will depend on the steady state fertility rate of young adults.

$$N_{t+1} = \pi_1 \bar{\eta} N_t \quad (3.25)$$

All three populations (young, adult, and old) will grow at steady state rate of  $\pi_1 \bar{\eta} - 1$ .

Observe that the model fails to predict differences in population growth given differences in infant survival.

$$\begin{aligned} g_\eta &= \pi_1 \bar{\eta} - 1 \\ &= A_\eta - 1 \end{aligned}$$

where  $A_\eta = \left[ \frac{\alpha(r+1)}{(1+\mu)^\sigma} \right]^{1/\varepsilon} \frac{1}{\delta}$ .

## STEADY STATE POPULATION DISTRIBUTIONS

Given a steady state fertility and population growth rate, steady state population ratios can be calculated as

$$\begin{aligned} N_t &= N_t^Y + N_t^A + N_t^O \\ 1 &= g_Y + g_A + g_O \end{aligned}$$

where  $g_i$  will be referred to as the population share of agent  $i$  or

$$g_i = \frac{N_t^i}{N_t}$$

and

$$N_{t-1}^i = \frac{N_t^i}{(\pi_1 \bar{\eta} - 1)}$$

$$g_i = \frac{N_t^i}{N_t}$$

It is found that given  $N_{t+1}^Y = \pi_1 \bar{\eta} N_t^Y$ ,  $N_{t+1}^A = \pi_2 N_t^Y$  and  $N_{t+1}^O = \pi_3 N_t^A$  for all  $t$ . And steady state growth  $N_{t+1}^i = \pi_1 \bar{\eta} N_t^i$  for all  $i, t$

$$g_Y = \frac{(\pi_1 \bar{\eta})^2}{(\pi_1 \bar{\eta})^2 + \pi_2 \pi_1 \bar{\eta} + \pi_3 \pi_2}$$

$$g_A = \frac{\pi_2 \pi_1 \bar{\eta}}{(\pi_1 \bar{\eta})^2 + \pi_2 \pi_1 \bar{\eta} + \pi_3 \pi_2}$$

$$g_O = \frac{\pi_3 \pi_2}{(\pi_1 \bar{\eta})^2 + \pi_2 \pi_1 \bar{\eta} + \pi_3 \pi_2}$$

and consequently  $\sum_i g_i = 1$ .

## CHAPTER 4

### CALIBRATION

Provided in this section is a general overview of the methodology for acquiring the necessary parameters in the model. Aside from following previous literature and fixing the standard parameters, the subsequent sections will match the important target moments (i.e., fertility, bequest, time costs, etc.) to the necessary model parameters. The choice for the model's functional forms are fairly standard with exception to the utility function which now includes the parameter characterizing the level of utility, which later implies a value of life. This value will be acquired through a series of demographic moments and prior research on mortality risk and wage differentials. This will be found to be a significant contribution in that I have considered a non-singular and non-zero value for this utility and acquiring an estimate using a general hedonic wage function. In the following subsections, I will assume that one period,  $t$ , is 20 years.

#### 4.1 Functional Forms

##### 4.1.1 *Expected Utility*

As previously mentioned in section (3.3.2), the household's lifetime utility is:

$$\nu_t = U(C_{1t}) + \beta\pi_2 [U(C_{2,t+1}) + \beta\pi_3 U(C_{3,t+2})]$$

where  $U(C) = \frac{C^{1-\sigma}}{1-\sigma} + \zeta$ .

### 4.1.2 Expected Altruism

Although the functional form of parental altruism is  $\alpha(\eta_t)^{1-\varepsilon}$ , we care to calibrate the altruism of the expected number of children,  $E_Q[\alpha(\eta_t)^{1-\varepsilon}]$  which has been approximated to be

$$\rho(\eta_t) = \alpha(\pi_1\eta_t)^{1-\varepsilon} + \frac{1}{2}\alpha\pi_1(1-\varepsilon)(\pi_1\eta_t)^{-\varepsilon-1}\sigma_\eta$$

where  $\sigma_\eta$  is the variance of the survival distribution of children.

## 4.2 Parameters and Estimation

The following table is a summary of initial calibrated parameters

Table 4.1 Summary of Initial Parameter Values

Category	Parameter	Model	Source
Health	$\pi_1$	0.9863	WHO (2000)
	$\pi_2$	0.9775	WHO (2000)
	$\pi_3$	0.9097	WHO (2000)
Real Growth Rate	$\mu$	0.24	BEA (1925-2000)
Fertility Cost	$(f, g)$	(0.03, 0)	Standard
Relative Risk Aversion	$\sigma$	1	Assigned Log-Utility
Discount Rate	$\beta$	0.54	Annual Rate of 0.97

The initial parameter values are fairly standard. The Bureau of Economic Analysis has reported that the average annual growth rate of the economy, from 1925-2000, is approximately 1.1%. Therefore  $\mu$  will be set to match the 20 year compounded growth rate. I have set  $\beta$  equal to 0.54 given the annual discount rate is assumed to be 0.97. I will also assume log utility in my preliminary experiments but will provide a summary of results for  $\sigma = 0.5, 0.75, 1.25, 1.5$  and 2.

**Mortality Tables** The World Health Organization has estimated age specific death rates for over 193 countries over 25 years. The age-specific death rate is calculated as

$$M_X^t = \frac{\text{Number of Deaths at age } x \text{ in year } t}{\text{Estimated Population Alive at age } x \text{ on June 30 in year } t}$$

Using this information a life (or mortality) table is constructed with various statistics including age specific survival rates  $(1 - M_X^t)$ . It is calculated that the model's survival rates,  $\pi_1$ ,  $\pi_2$  and  $\pi_3$ , are

$$\begin{aligned}\pi_1 &= \prod_{X=1}^{20} (1 - M_X^t) \\ \pi_2 &= \prod_{X=21}^{40} (1 - M_X^t) \\ \pi_3 &= \prod_{X=41}^{60} (1 - M_X^t)\end{aligned}$$

Therefore in the United States in 2000, the survival rates,  $(\pi_1, \pi_2, \pi_3) = (0.9863, 0.9775, 0.9097)$ .

The WHO has collected various health proxies which may be useful in future work to possibly explain the observed differences in mortality (i.e., population distribution, cause of death, education, health investment, etc.).

**Fertility Cost** The cost of each child takes two parts, the time to birth the child,  $\theta$ , and the cost of producing the child,  $\delta$ . I have borrowed the parameter values from Boldrin and Jones (2005) which estimates that the average mother utilizes 6% of her time per child to birth that child.

Producing a child requires bequest  $b^A$  at a time cost of  $\delta$  of adoption. Therefore total bequests are set to match

$$\text{Total Bequest} = \delta (1 + \mu) \pi_1 \eta b^A$$

I will assume the transfer cost to be proportional to time

$$\delta = f_\delta w$$

Therefore the bequest share will be

$$\phi = f_\delta (1 + \mu) \pi_1 \eta b^A$$

Inserting this equation to the steady state budget constraint yields the following equation<sup>35</sup>

$$\delta = \frac{\phi}{\pi_1 \eta} \left[ A_1 + \frac{\pi_2}{r+1} + \frac{A_3 \pi_3}{(r+1)^2} \right] \gamma \kappa + \phi - \left[ 1 + \pi_2 \frac{(1+\mu)}{r+1} \right]^{-1}$$

such that  $\gamma$  is the consumption to income ratio and  $\kappa = g_Y [\beta (r+1)]^{-\frac{1}{\sigma}} + g_A + g_O [\beta \pi_2 (r+1)]^{\frac{1}{\sigma}}$ .

**Steady State Population Distribution** The model concludes that the population growth rate is driven by the fertility and survival rate of the young,  $\pi_1 \eta - 1$ . Therefore given a steady state fertility and population growth rate, the model defines the steady

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<sup>35</sup>Derivation of this equation can be found in the appendix.



state population ratios as

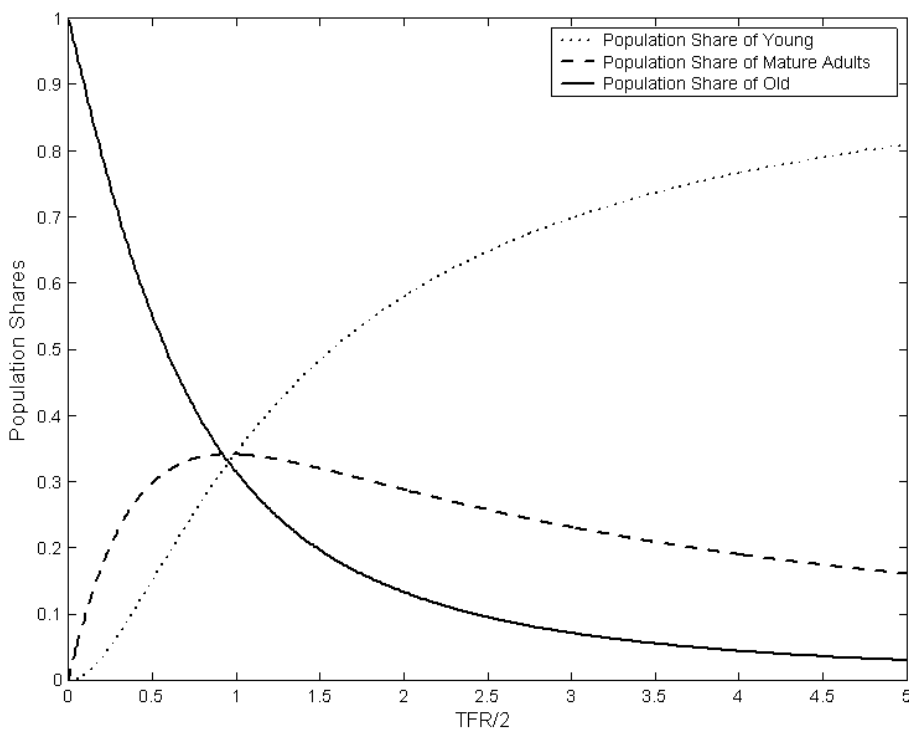
$$g_Y = \frac{(\pi_1 \bar{\eta})^2}{(\pi_1 \bar{\eta})^2 + \pi_2 \pi_1 \bar{\eta} + \pi_3 \pi_2}$$

$$g_A = \frac{\pi_2 \pi_1 \bar{\eta}}{(\pi_1 \bar{\eta})^2 + \pi_2 \pi_1 \bar{\eta} + \pi_3 \pi_2}$$

$$g_O = \frac{\pi_3 \pi_2}{(\pi_1 \bar{\eta})^2 + \pi_2 \pi_1 \bar{\eta} + \pi_3 \pi_2}$$

Assuming all else constant, population distributions vary with fertility and is represented by Figure 4.1.

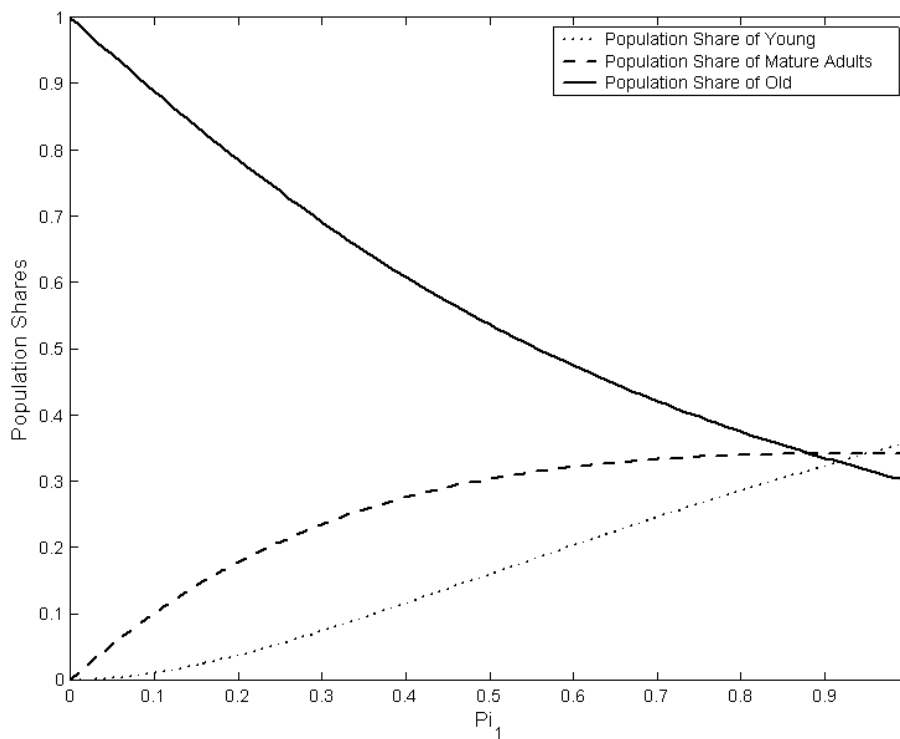
Figure 4.1 Population Shares vs. Fertility



High fertility countries will have significantly higher concentrations of young, whereas countries with below replacement fertility rates will have larger shares of old which is what we are experiencing today. Now assuming all else constant, including fertility, except for changes in the survival rate of the young Figure 4.2 below gives us the following

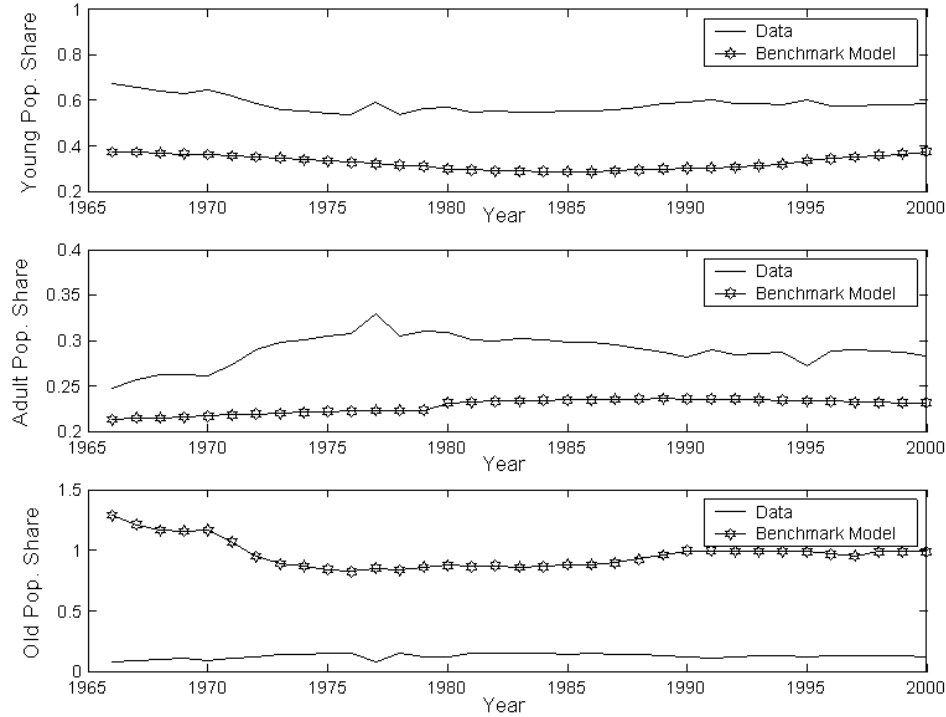
population shares.

Figure 4.2 Population Shares vs. Young Survival



As expected, the share of young in the population will increase with their survival rates. Observe the significant effect of childhood survival on population distribution holding fertility constant. As  $\pi_1$  approaches 1, the population shares are fairly equal. Figure (4.3) below compares the time series population and population shares between the model and the data.

Figure 4.3 Population Distributions: Model vs. Data



The time series data containing the mortality and fertility rates, from 1960 to 2000, was acquired from the CDC Life Tables and Hamilton and Ventura (2006) from the Division of Vital Statistics, National Center for Health Statistics. Observe that the model tends to understate the young and mature adult population shares. As we have observed in Figure (4.1), the population distribution is heavily dominated by the agent's fertility decision. It is expected that the fertility measure of the data will noticeably overstate the model due to the limited period of time which women are assumed to be fertile. The actual fertile period (approximately between ages 15 – 44) significantly exceeds our current assumption of the fertility decision ranging between ages 21 to 40. Aside from these inconsistencies the model predictions of population distributions for the United States perform reasonably well.

### 4.3 Target Variables

Below is a summary of parameter values for the steady state equations.

Table 4.2 Summary of Target Variables

Parameter		
Consumption to Income	$\gamma$	0.75
Bequest to Income Share	$\phi$	0.0549 or 1.561
Risk to Income Elasticity	$\varepsilon_m$	0.5
Population Ratios	$(g_Y, g_A, g_O)$	(0.3541, 0.3407, 0.3051)

**Consumption to Income Ratio** The Bureau of Labor and Statistics (BEA) provides us with a standard value for the consumption to income ratio is 0.75 acquired from NIPA data .

**Bequest to Income Ratio** The most common source of information used to acquire bequest information in United States is a 1983 Consumer Finance Survey found in Gale and Scholz (1994). Other sources of information include Kotlikoff and Summers (1981) which includes the accrued interest as part of intergenerational wealth accumulation.

Table 4.3 Transfers

Transfer (\$ billions)	
Net Worth	11,007
Intervivos Transfers	42
College Expenses	32.5
Trusts	308.9
Life Insurance	3,457
Inheritances	43.7

Based on this information, we have acquired the following percentage transfer flows

Table 4.4 Percentage Transfer Flows

Percentage of Net Worth (%)	
Intervivos Transfers	0.51
College Expenses	0.29
Bequest	0.43

The total transfer percent of net wealth is approximately 1.23%. Gale and Scholz claim this value to be a lower bound for a few reasons. First, inter vivos transfers will be higher for the higher income and wealth percentile of the population, which is also the proportion of the population that generally fails to report in these surveys. Second, the exclusion of intended bequests due to lack of information and third, data censoring for transfer less than \$3,000. If all bequest are included, transfer account for 35% of net wealth Therefore to estimate the relationship between bequest and income

$$\phi = \text{Percentage of Bequest Flows} \times \text{Wealth to Income Ratio}$$

Wealth to income ratio, reported by the Federal Reserve Bank, is referred to as the ratio of household net worth to disposable income and is estimated to be 4.46 around this time period.

**Risk to Income Elasticity** Various important studies have estimated worker's preferences over risk, wage compensating differentials and the value of statistical life. These observables have been acquired through a large set of surveys beginning in the 1960's. Commonly used is the information collected by the Bureau of Labor and Statistics (BLS) which was later enriched by the National Institute of Occupational Safety and Health (NOISH), through concerns about how fatality data was collected. A more detailed ap-

proach led to the BLS recording information on both fatal and non-fatal injury through the Census of Fatality Occupational Injuries (CFOI), beginning in 1992. Earlier research on estimating the value of statistic life included the Michigan Survey of Working Conditions and Quality Employment Survey which estimated the hedonic linear wage equation based on a survey of risk perceptions. Occupational risk has also been evaluated using worker's compensation records. The difficulty arises when estimating the equations on a national level given the information is acquired differently by state. According to Viscusi and Aldy (2003), the elasticity of risk to wage is reported to range from 0.5 to 0.6. I will assume the value of 0.5.

#### 4.4 Estimated Parameters

Given the aforementioned parameter estimations, the objective is to match the remaining steady state optimality conditions with observable fertility and bequests. I have obtained the following parameters

Table 4.5 Summary of Calibrated Parameters

Parameter	
$\alpha$	0.7857
$\varepsilon$	0.9692
$\zeta$	-4.1292

The utility parameter,  $\zeta$ , is estimated to be  $-4.1292$  which would suggest that the level of consumption in which households are indifferent between being alive and dead (i.e.,  $C_{dead} = e^{-\zeta}$ ) is approximately 0.31% of U.S. consumption. Although this may seem insignificant in high income countries, this value represents more than 10% of *GDP* for poorer developing economies. The household discounts the future utility of children by 0.7857, which is noticeably 45% greater than  $\beta$ , the parent's time preference discount.

Therefore parents value their children's future more than their own, especially for those with above replacement fertility rates. The estimated elasticity of parental altruism is approximately 0.9692 which is generally on par with previous a priori, qualitative conclusions found in the literature. These values change with various estimates of the coefficient of risk aversion. The agent's time discount of future generations is consequently

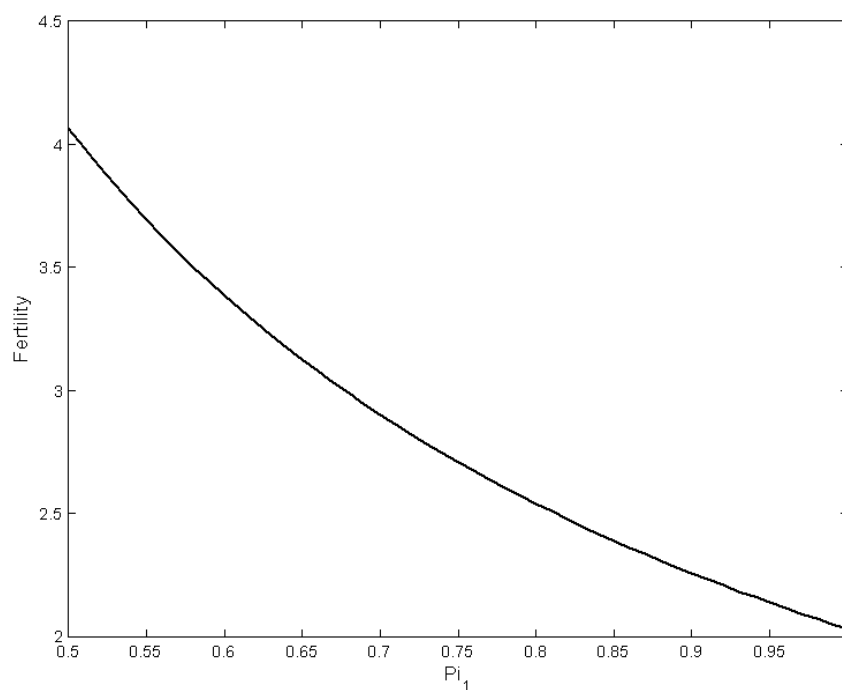
Table 4.6 Calibrated Parameters given Coefficient of Relative Risk Aversion

Parameter	$\sigma = 0.5$	$\sigma = 0.75$	$\sigma = 1.25$	$\sigma = 1$	$\sigma = 1.5$	$\sigma = 2$
given						
$\alpha$	0.7432	0.7596	0.8179	0.7857	0.8545	0.9397
$\varepsilon$	0.8455	0.8989	0.9991	0.9692	0.9999	1.0000
$\zeta$	-4.0819	-4.1218	-4.1295	-4.1292	-4.1280	-4.1242

affected by aversion to risk. As the agent becomes more risk averse the value of their children's utility increases. Without compromising the results, I have arbitrarily chosen log utility for the remaining experiments in this paper.

Assuming all macroeconomic variables (i.e., interest rate,  $r$ , growth rate,  $\mu$ , and income,  $w$ ) are constant, the only deciding factor on the agents' fertility decision is the survival rate of her children represented by Figure 4.4 below

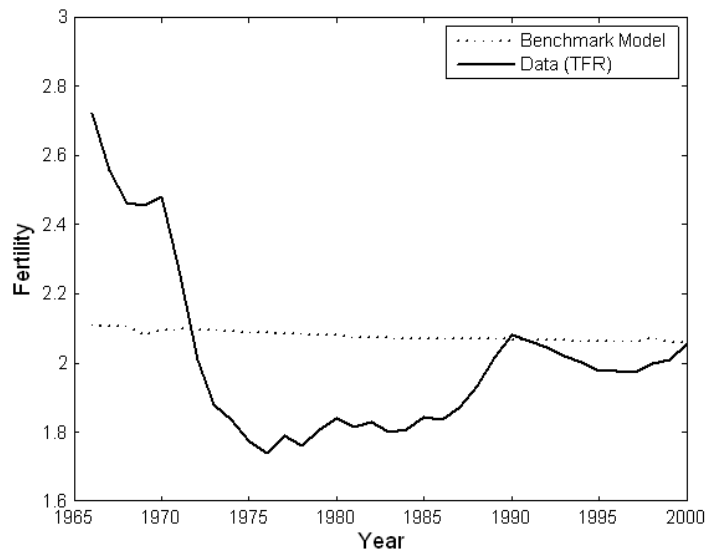
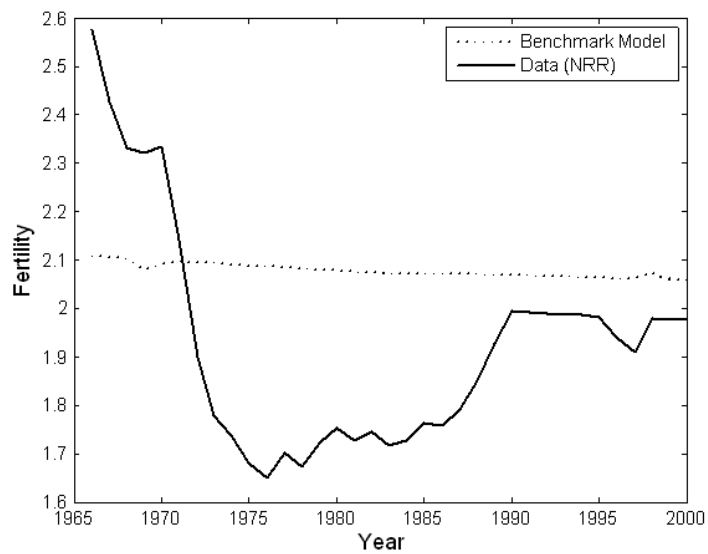
Figure 4.4 Fertility vs. Young Survival



Although the model successfully predicts the necessary negative relationship, we can observe that, at extremely low survival rates fertility remains relatively low compared to *TFR* for the various countries in Figure 5.1, suggesting that cross country fertility differences cannot be explained by infant mortality rates alone. Therefore we must also consider the cross-country macroeconomic differences in the analysis.

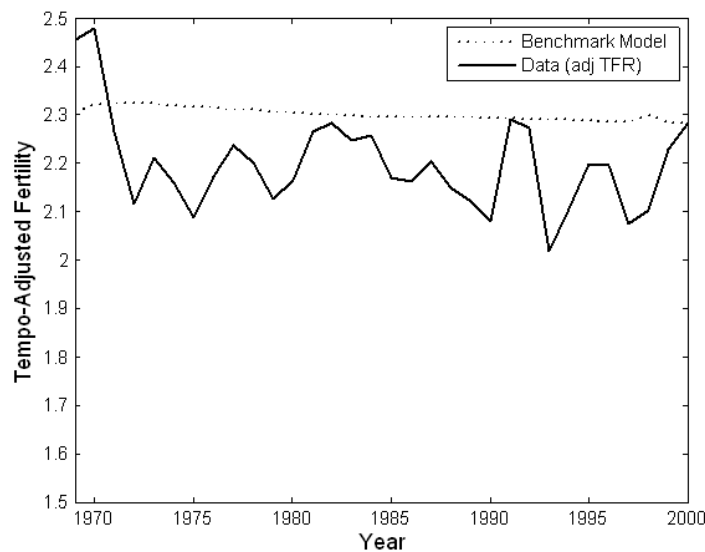


Figure 4.5 Timeseries Fertility: Model vs. Data

Figure 4.6 Time Series *NRR*: Model vs. Data

Figures (4.5) and (4.6) display a destabilization occurring between the years 1966 and 1990<sup>36</sup>. A significant amount of research directly addresses this worldwide phenomenon of the Baby Boom and the following Baby Bust. One can postulate that high degree of variation in  $TFR$  during these years may have been attributed to unexplained changes in timing of births. I address this problem by introducing and adjusted measurement of fertility controlling for the *tempo effects*<sup>37</sup>.

Figure 4.7 Time Series Time-Adj. Fertility: Model vs. Data



Observe that adjusting for the tempo effect, the fit of the data to the model is significantly improved. I can expect that the model's performance may be more accurately measured through the tempo adjusted fertility and reproduction rates. In the subsections following Chapter 5, the model I will evaluate the model's performance with respect to the various measurements of fertility.

<sup>36</sup>I have acquired the time series survival rate data through the United States Center for Disease Control (CDC) life tables from 1960 to 2004. To calculate the tempo adjusted fertility rates, I collected the Total Fertility Rate ( $TFR$ ) and the Mean Age at Childbirth ( $MAC$ ) for the U.S. from 1960 to 2002 from a study by Hamilton and Ventura (2005) from the Division of Vital Statistics, National Center for Health Statistics

<sup>37</sup>The tempo-adjusted fertility rates are discussed in more detail in Section 5.1.1.

#### 4.4.1 Mortality Risk and Wage Differentials $\zeta$

Given the additional parameter,  $\zeta$ , the model requires additional information. In this section I plan to exploit previous work on mortality risk and wage differentials to determine its value. Earlier work on this topic estimates the value of mortality risk based on wage compensation structures. Earlier papers have attempted to empirically estimate the supply and/or demand for risk. Through regression analysis they have obtained relationships between wage and income, controlled for wage related observables. Closely related to the method I have utilized in this paper, Viscusi and Aldy (2005) estimate a relationship between risk and wage based on a set of market equilibria.

Consider the following scenario. The household is concerned about the risk/occupation bundle it faces adulthood. For  $p \in (0, 1)$  and  $w_2 \in \mathbb{R}^+$ , the household is indifferent between choosing from the set of bundles  $(p_2, w_2)$ . It's important to assume that the household faces a subset of  $p$ 's such that the household's expected survival is  $\pi_2$ , in order to maintain the initial insurance policies do not change. If we assume the likelihood of obtaining a set,  $(p^i, w_2^i)$  is equal to  $(p^j, w_2^j)$ , for all  $i, j$ , then  $\pi_2$  must be equal to the average  $p$ 's. The set of  $p$ 's will therefore be such that  $\pi_2 = \int_a^1 p dp$ . Therefore  $p \in (2\pi_2 - 1, 1)$ .

Given the value function;  $V(p, C_2) = \bar{V}$  for all  $p, C_2$ . Where

$$V(p, C_2) = \frac{1}{1 - \rho(\eta)} \{U(C_1) + \beta p [U(C_2) + \beta \pi_3 U(C_3)]\}$$

In the steady state  $C_1 = [\beta(r_1 + 1 - \delta)]^{-\frac{1}{\sigma}} C_2$  and  $C_3 = [\beta \pi_2(r_2 + 1 - \delta)]^{\frac{1}{\sigma}} C_2$ . Using the hedonic wage methodology, a first-order Taylor-series approximation about  $(\bar{p}, \bar{C}_2)$  provides

$$V(p, C_2) = \bar{V} \simeq \bar{V} + \bar{V}_p(p - \bar{p}) + \bar{V}_c(C_2 - \bar{C}_2)$$

Therefore

$$\frac{1}{\kappa} \frac{dw}{dp} = -\frac{\bar{V}_p}{\bar{V}_c}$$

Given  $\frac{dC_2}{dp} = -\frac{\bar{V}_p}{\bar{V}_c}$ ,  $\frac{dc}{dp} = \frac{dw}{dp}$  and  $\frac{dc}{dp} = \kappa \frac{dC_2}{dp}$  since  $c = \kappa C_2$  and  $\gamma = \frac{c}{w}$ . I can analytically solve

$$\zeta = \frac{(\gamma w)^{1-\sigma}}{\gamma \kappa} \frac{\varepsilon_m}{T} \frac{\pi_2^{\frac{1-T}{T}}}{1 - \pi_2^{\frac{1}{T}}} \left[ \beta^{-\sigma} (r+1)^{\frac{\sigma-1}{\sigma}} + \pi_2 + \pi_3 (r+1)^{\frac{1-\sigma}{\sigma}} \left( \frac{1}{\beta \pi_2} \right)^{-\sigma} \right] - \frac{(\gamma w)^{1-\sigma}}{1-\sigma}$$

such that  $\varepsilon_m$  is the elasticity of risk to income and  $\kappa = g_Y [\beta (r+1)]^{-\frac{1}{\sigma}} + g_A + g_O [\beta \pi_2 (r+1)]^{\frac{1}{\sigma}}$ .

More detail is provided in the appendix.

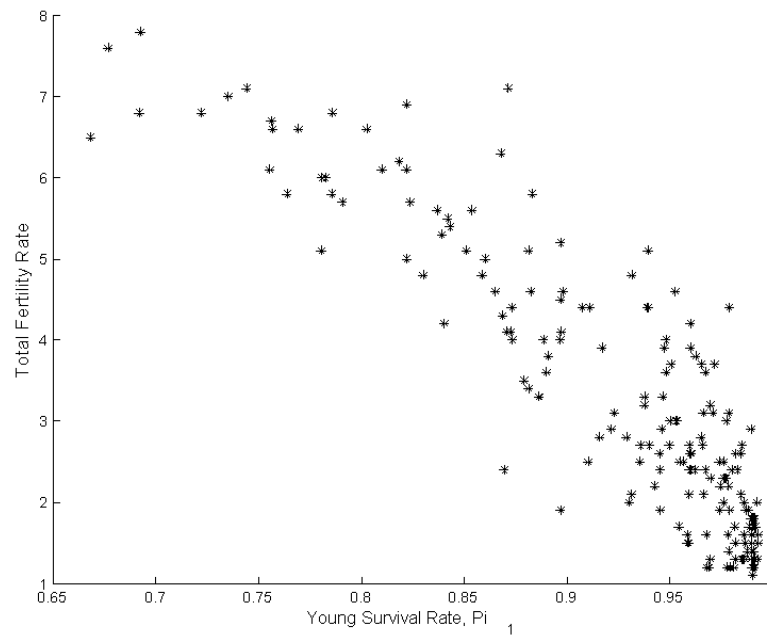
The model with the estimated parameter values performs pleasantly well with the time series data in the United States. The advantages of comparing the time series is the fairly constant interest rates and growth rates given I am observing only one country. But when transmitting the model from time series to cross sectional analysis careful consideration of the structural, macroeconomic factors will be required to accurately evaluate the predictions of the model.

## CHAPTER 5

### RESULTS

Once the model has been developed and calibrated, the experiment will be to compare the model's prediction to the data represented in Figure 5.1 below.

Figure 5.1 Data: Fertility ( $TFR$ ) vs Young Survival Rates



I will test to see how well a model, which only considers differences in mortality rates, performs in comparison to the data. Given the obvious shortfalls of the assumptions of different measurements of fertility, I will also compare the model's performance to alternative fertility measurements (i.e., net rate of reproduction, general fertility rate, and time-adjusted fertility rates)<sup>38</sup>.

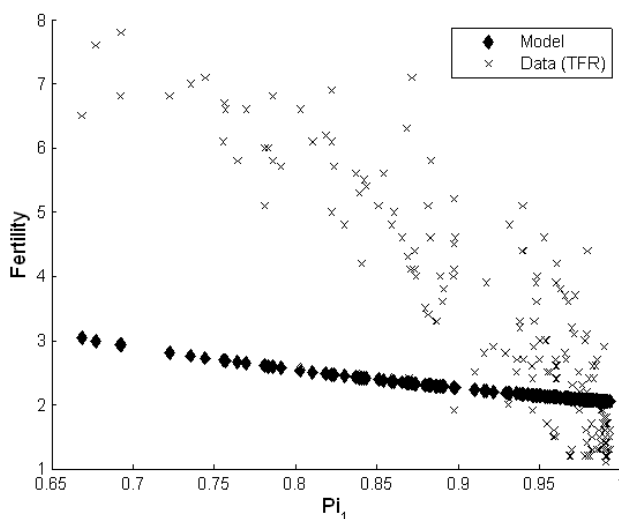
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<sup>38</sup>I discuss these shortfalls in the following sections.

## 5.1 TFR with respect to young survival rates

Below is the following graphical representation of the results comparing total fertility between the model and data.

Figure 5.2 Total Fertility vs.  $\pi_1$ : Model vs. Data



My initial experiment is to assume that countries differ only in the survival rates of the young. With this assumption the model generally underpredicts fertility across the board. The countries which overpredict, albeit the minority, include the Organization for Economic Cooperation and Development (OECD)<sup>39</sup>, the European Union (EU) and the Russian Federation as well as the Asian Tigers<sup>40</sup>. The model also underestimates fertility for all other Eastern Asian and Eastern European countries with exception to Albania<sup>41</sup>. In support of the *real interest rate parity hypothesis*<sup>42</sup>, in this section, I have assumed all countries are identical to the United States and only differ in the childhood

<sup>39</sup>with exception to Mexico, Italy and Turkey

<sup>40</sup>A group of southeast Asian countries with exceptionally long periods of high growth. They are Hong Kong, Singapore, South Korea and Taiwan.

<sup>41</sup>I have also failed to include Kosovo and Montenegro due to lack of data.

<sup>42</sup>The real interest rate parity hypothesis (RIPH) postulates that the real interest rates of different countries should be identical; provided that markets are frictionless and economic agents' expectations are rational.

survival rates. This experiment has suggested that fertility differences may be attributed to various cross-country macroeconomic factors including differences in interest rates across countries. Especially in the year 2000, for which I am conducting this experiment, evidence indicates real interest rates in the US are relatively higher in comparison to the European Union and consequently all OECD countries.

The model generally underpredicts fertility for countries with higher fertility rates than that of the United States. These countries are generally located in the regions of Sub-Saharan Africa<sup>43</sup>. The model, simplified with such particularly rigid assumptions, performs very well in Latin American countries, with differentials<sup>44</sup> less than 0.4. The observed exceptions are Guatemala, Bolivia, Honduras, Paraguay and Nicaragua, with differentials ranging from 1.1 to 2.55<sup>45</sup>. Barbados, Cuba and Trinidad<sup>46</sup> are the only low fertility countries which the model underpredicts. Monaco and San Marino are insignificantly small regions likely to be influenced and subjected to the fertility rates and survival rates of nearby, neighboring OECD country of Italy. San Marino is landlocked in Italy, while Monaco is located on the northwestern border.

Most Western Asian countries suffer from both high fertility and high mortality with exception to Armenia, Azerbaijan and Iran.

Countries with high fertility rates (i.e., above approximately 2) are generally higher than the model's predicted values. The differentials between the data and model increase as fertility rates increase. For fertility rates above 2.5, differentials can be as high

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<sup>43</sup>This is with exception to Tunisia, which ranks highest in economic competitiveness among African and Middle East countries. This also excludes Mauritius and Seychelles, both island systems located off the east coast of Africa.

<sup>44</sup>The differential, in this case, is defined as the distance between the predictions of the model and the data.

<sup>45</sup>Discuss Guatemala's anomalies with 2.55 as well as other reasons why these countries have higher fertility differences

<sup>46</sup>short for, Trinidad and Tobago

as 4.68. This suggests that there are significant structural factors, whether social or economic, which have yet to be considered. These countries are mostly located in the same geographic regions such as Sub-Saharan African, Western Asia and a few Caribbean Countries. In the Arab regions of Western Asia, families are highly valued and Muslim women value themselves based on the number of children they bare. These differences in preference structures are not discussed in this model. Many argue that preferences towards family size are what differentiates the African and Western Asian regions from the rest of the world<sup>47</sup>. Also many other socioeconomic factors such as the availability of contraceptives, cultural views on women and families, education and the cross country preferential differences, may distort optimal decisions compromising the model's results. But under these circumstances the model performs fairly well.

Most importantly significant economic changes may affect child bearing and family planning decisions. Observed increases in economic growth may influence expectations which may shift investment of time to education, consequently delaying marriage and child birthing. The timing of child-bearing is a factor that must be considered and may suggest alternative uses of measurements of fertility, such as net reproduction rates and tempo adjusted fertility rates. To use these various fertility measurements I will need detailed information on age specific fertility and birth rates. Further discussion on how these rates are measured can be found in the Appendix.

It is imperative that the definitions of the data coincide with the variables in the model. The variable of most importance and concern is fertility. In conducting this experiment, the question arises; is the Total Fertility Rate (*TFR*) an appropriate mea-

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<sup>47</sup>Easterlin, 1975; Pritchett, 1994; Becker, 1990; Caldwell and Caldwell, 1987; Makinwa-Adebusoye, 2001



surement of per capita family size decisions? Other than the common measurement obstacles, the *TFR* may be an inaccurate measure of the performance of the model, strictly due to its definition. The *TFR*, which is defined as the *expected number of children that would be born to a woman over her lifetime if she were to experience the exact age specific fertility rates through her lifetime and she were to survive from birth through the end of her reproductive life*, is not exactly equivalent to the definition of fertility, or  $\eta$ , in the model. The variable,  $\eta$ , is defined as the number of children each agent births. The first approach follows the standard procedure in the literature. I have assumed a unisex economy, the current calibration matches  $\eta$  to  $TFR/2$ . The model does not assume nor does it require that at time,  $t$ , a woman will experience the current age specific rates throughout her lifetime. Also, the model does not require that the agent survive the entire period of child bearing. In the model, agents choose  $\eta$  and observe the number of children who survive before facing the fate of whether or not they will survive to mature adulthood. Please refer to the appendix for more information. In the following sections I will compare the results using various definitions of fertility.

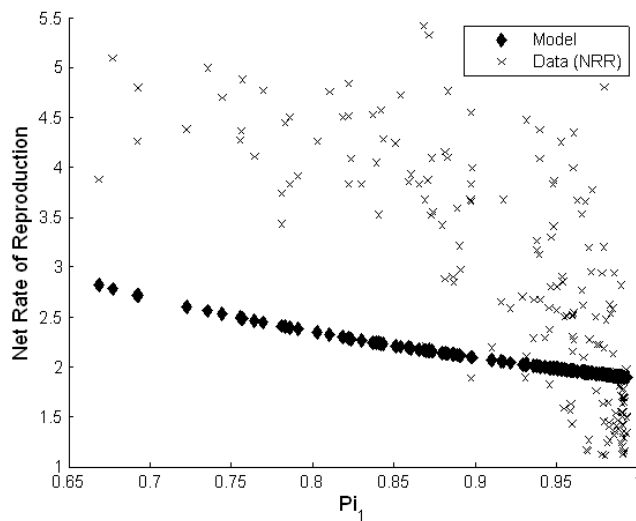
### ***5.1.1 Utilizing Various Measures of Fertility***

#### NET REPRODUCTION RATE

The *Net Reproduction Rate* or the *Net Rate of Reproduction* (*NRR*) measures the expected total number of daughters a woman is expected to have in her lifetime, given she experiences during her lifetime the current age specific fertility and mortality rates for that year. This measurement is different from the *Gross Reproduction Rate* or *GRR*, which ignores the mortality rates and estimates the total number of children born to a woman during her lifetime assuming she survives her child bearing years. Below are the

following results comparing  $NRR$  to fertility in the model,  $\eta$

Figure 5.3 Net Rate of Reproduction: Model vs Data



When comparing the model to  $NRR$ , we see significant improvement for the poor health countries. As expected the age specific mortality rates associated with the net rate of reproduction allows for a significant adjustment for the extreme low health cases. On the other hand I find no improvement for the high health countries given mortality rates are fairly close to one, consisted between the sexes and across countries. Adjusting the model to incorporate child costs which, in this case, are proportionally related to income, will improve the results significantly.

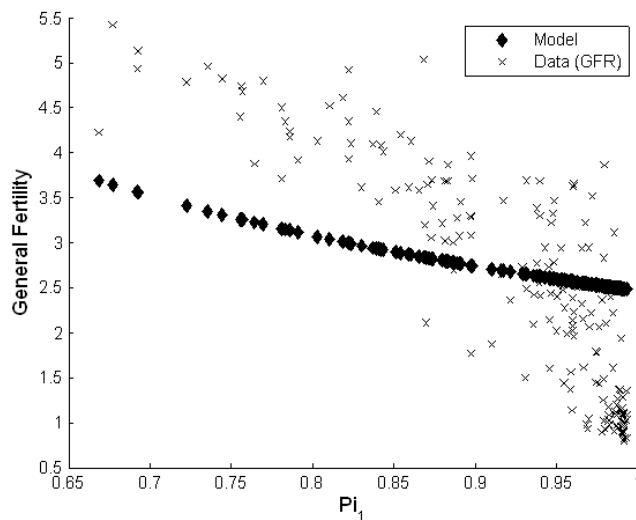
#### GENERAL FERTILITY RATE

The General Fertility Rate, or the GFR, is defined as the birth rate per woman of child bearing age. More specifically,

$$GFR_t = \frac{\text{total number of birth at time, } t}{\text{total number of woman aged 15-44}} \times 1000$$

In order to calculate  $GFR$ , details of population distributions are required, as well as, birthing information. The US Census Bureau provides data on age specific populations for 5 year age cohorts but is limited by the number of years available. Through the International Database (IDB) and the Population Prospectus (2006), I can acquire information from 1996 to the projected years up to 2050. Below are the following results comparing  $GFR$  to  $\eta$ .

Figure 5.4 General Fertility Rate: Model vs Data



The  $GFR$ , which represents the average birth rate per woman, performs better for countries with survival rates less than 0.9. For the high health and high developed countries the model overpredicts by a significant amount. This can be caused by many reasons. The general fertility rate is essentially a population-weighted average of the age-specific fertility rates. Therefore high health, high fertility agents will suffer in their measurement of TFR given the younger population (those with higher fertility) represents a smaller proportion of the fertile population. In the following sections I will introduce various adjustments which may improve the results and more accurately describe the model variables to the data moments.

## TEMPO-ADJUSTED RATES

**Tempo-Adjusted Fertility Rate** The demographic literature discusses the issues and differences between the period measurement (i.e., total fertility) and the cohort measurement (i.e., completed total fertility). The primary concern with the conventional measurement of total fertility is its susceptibility to the timing of births. This is commonly referred to as the *tempo effect*. The total fertility rate estimates the expected number of children given current age specific fertility rates. For women who choose to delay child bearing the current *TFR* will be understated and compromised by this tempo effect. As stated by Bongaarts and Feeney (1998), the *TFR* is primarily dominated by two effects, the *quantum effect* and the *tempo effect*. The model predicts the quantum, or level, measurement of fertility and therefore requires a tempo-adjusted, quantum measurement of the *TFR*. *TFR* is understated when mothers delay childbearing and overstated when first time mothers birth at a younger age. The recent shift towards the postponement of childbearing, in developed countries, is the likely for cause for the current overstatement of *TFR* measurements.

The most accurate measurement would be, given the cohort of women, the average number of births over their life cycle, which is referred to as the *completed fertility rate* or *CFR*. The nearest estimate to the CFR is the tempo-adjusted, quantum measurement of the total fertility,  $TFR^*$ , which according to Bongaarts and Feeney is

$$TFR_t^* = \sum_i TFR_{i,t}^*$$

such that  $TFR_i^*$  is the tempo-adjusted fertility rate for the  $i^{th}$  age order. More specifically,

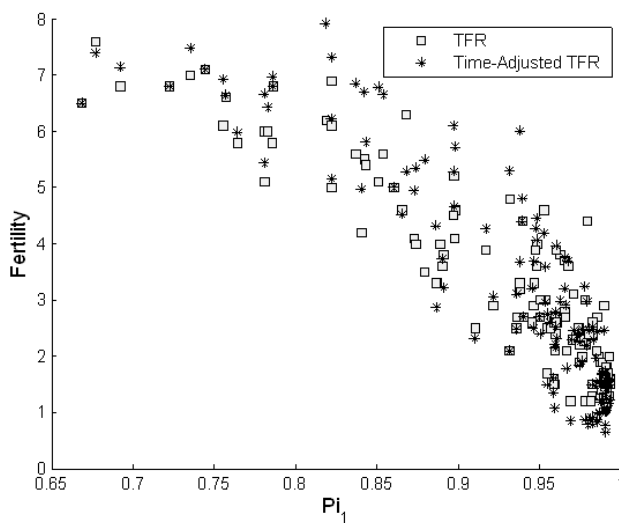
$$TFR_{i,t}^* = \frac{TFR_{i,t}}{(1 - r_{i,t})}$$

where  $TFR_i$  is the  $i^{th}$  age specific fertility rate and  $r_i$  is rate of change in the mean age at childbearing,  $MAC^i$ , for age order  $i$  at the beginning and end of the period

$$r_{i,t} = \frac{MAC_{t+1}^i - MAC_{t-1}^i}{2}$$

The results are depicted in Figures 5.5 and 5.6

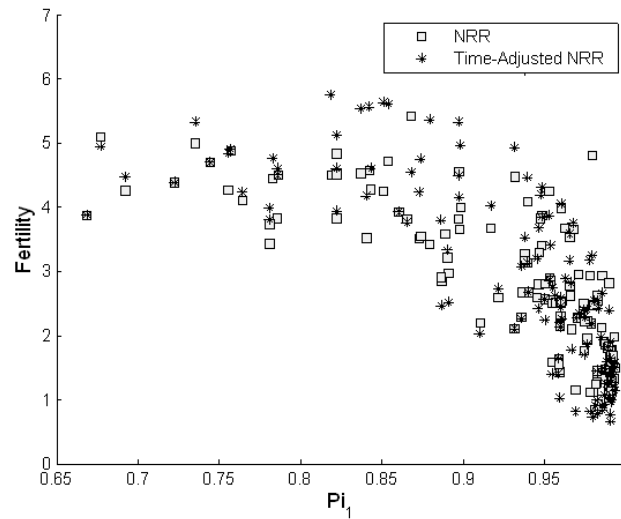
Figure 5.5 Data: Time-Adjusted  $TFR$  vs  $TFR$  across countries



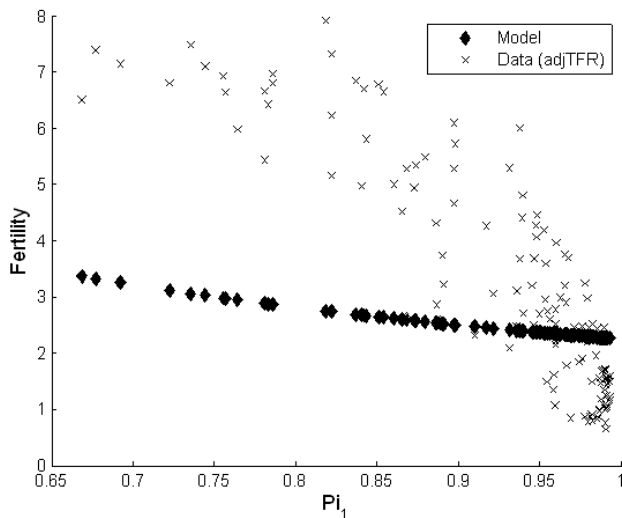
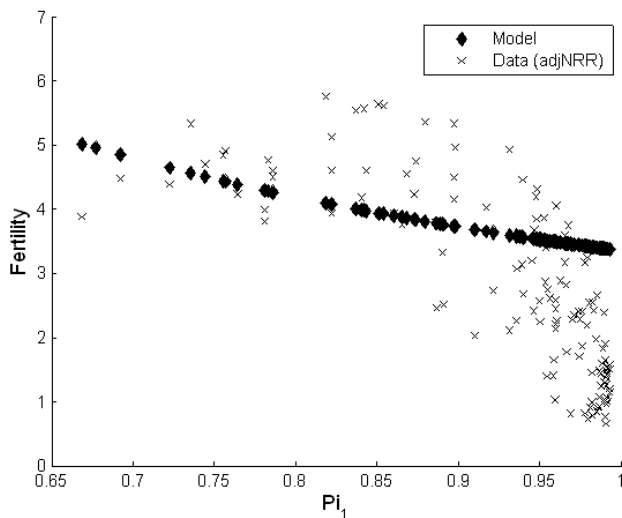
The evidence has led many to question the effect of the mother's age on fertility but fails to address the causal links leading to this relationship. Although we have abstracted from this fact, the evidence suggests that woman who birth children later in life tend to have more children.

**Tempo-Adjusted Net Reproduction Rate** The NRR, which factors age specific fertility and mortality rates, is adjusted with the same methodology discussed in the previous section and yields the following graphical result

Figure 5.6 Data: *NRR* vs. Time-Adjusted *NRR* across countries



Therefore it is safe to assume that the tempo effect play a negligibly small role in explaining cross country variations in fertility. The tempo adjustment for the total fertility rate and the net reproduction rates change very little in comparison to their standard rates. I also find that Figures 5.7 and 5.8 show a small change in comparison to Figures 5.2 and 5.4.

Figure 5.7 Time-Adjusted Total vs.  $\pi_1$ : Model vs. DataFigure 5.8 Time-Adjusted *NRR* vs.  $\pi_1$ : Model vs Data

#### SUMMARY OF RESULTS WITH RESPECT TO INFANT SURVIVAL, $\pi_1$

The TFR or the Total Fertility Rate, whether or not it is adjusted for time, generally performs better for high survival countries. Given these economies have generally experienced the latter part of the demographic transition (i.e., late Stage 3 and Stage

4), the distribution of age specific birth rates is fairly constant among them and more importantly the survival rates are very close to one. It is expected that TFR is a more representative measure of fertility in these countries. But in lower income low survival countries, the TFR rate would not accurately represent fertility in the model given it fails to consider the age specific mortality rates. In these cases, the NRR or the net rate of reproduction seems more appropriate is reflected by the observations in Figures 5.3 and 5.8. The GFR or the General Fertility Rate provides us with a population weighted average, which is important in evaluating a set of countries with significantly different population distributions<sup>48</sup>. The model ignores important market effects to the opportunity cost of birthing children which will be further considered in Chapter 6.

**Calibration** A calibration of altruism parameters,  $(\alpha, \varepsilon)$ , are required given the various population measurements of fertility.

Table 5.1 Calibration Summary

Fertility Measure to Match	Parameters		
	$\alpha$	$\varepsilon$	$\zeta$
<i>TFR</i>	0.7857	0.9692	-4.0935
<i>NRR</i>	0.7812	0.9679	-4.1292
<i>GFR</i>	0.7978	0.9723	-4.2219
<i>time – adjusted</i>			
<i>adjTFR</i>	0.7921	0.9708	-4.1786
<i>adjNRR</i>	0.8181	0.9767	-4.3660.
<i>GFR*</i>	0.7569	0.9588	-3.8598

<sup>48</sup>Historically high fertility countries will likely have higher expected fertility measurements given a large proportion of the population (i.e. the young adults) comprise, in some cases, more than half of the population.



### 5.1.2 *Reliability of Mortality Data*

As suggested by the World Health Organization (WHO), for which the mortality data in this experiment has been acquired, the mortality rates, more often than not, are understated due to the under-reporting of deaths, especially in low income countries. WHO cites the unreliability of due to the "indifference to the need for reporting, recall laps and dysfunctional vital registrations systems. Various methods above age 5 are utilized to make up for the data's shortcomings<sup>49</sup>. But for ages below 5, there is no compensating method. This age region, which proves to be the most important in accurately measuring the survival of the young is often subject to misreporting.

Several factors contribute to the distortion of infant mortality measurements including perinatal, neonatal and post natal death rates<sup>50</sup>. Vital statistics often ignore deaths before birth which may be of significant importance in the poorer regions of the world. The debate of what the accurate measurement of infant mortality should be has led to noticeable cross country inconsistencies<sup>51</sup>. The infant mortality rate can be broken up into two prominent categories, the *neonatal mortality rate* and the *post-neonatal mortality rate*. The neonatal mortality rate, defined as the number of deaths, per 1000 births, of children less than 28 days, is often ignored, inconsistent or miscalculated in cross country experiments. It is likely found that the infant mortality rate (*IMR*) is dominated by the reporting of the death rate of children between the ages of 28 days and one year. Given the relatively small number of deaths inaccurate measurements of neonatal mortality

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<sup>49</sup>WHO uses the Brass Growth-Balance Method, the Bennett-Horiuchi Method and the Generalized Growth Balance Method. These methods are discussed in intricate detail in the WHO handbook.

<sup>50</sup>Perinatal mortality only includes deaths between fetal viability (ie. 22 weeks of gestation) and the end of the 7th day of delivery. Neonatal mortality only includes deaths in the first 28 days of life. Post-neonatal death only includes deaths after 28 days of life but before one year. Child mortality includes death within the first five years after birth.

<sup>51</sup>Hartford, 1992; Howell, 1991

may be detrimental.

Most infant mortality measurements ignore or fail to account for perinatal death which is the period between 22 weeks after gestation and 7 days after birth. In essence, in order for a child to die it must be born. Although the WHO defines a live birth as, *a human being who demonstrates independent signs of life including breathing, voluntary muscle movement or heartbeat*, the accuracy of the data is unclear. Many countries particularly those with higher mortality rates, fail to follow those standards.

Therefore children who die in perinatal or near perinatal periods are often unregistered. For some countries the child must be alive at the time of registration in order to qualify as a live birth, which in certain cases may take a few days. Therefore neither the birth nor the death is registered. This significantly distorts the data and fails to account for what may be the periods in which most infant deaths occur. It can be expected that the credibility of information disintegrates in lesser developed and consequently high fertility, high mortality countries; especially the areas where the model fails.

## CHAPTER 6

### FERTILITY AND HUMAN CAPITAL INVESTMENT

The tempo-effects and measurement errors cannot account for the observed differentials between the predictions of the model and the data. Naturally, due to the previous formulation of the Becker-Barro framework, income effects are non-existent. On the other hand, wages assuredly effect the parents perception on birthing children, as well as, their sensitivity to changes in mortality. The evidence alludes us to the notion that the effects of mortality on fertility are further exacerbated by income changes. When comparing fertility, mortality and real wages, Eckstein et. al. (1999) found substantial evidence to support the claim that the fertility decline could be explained by mortality<sup>52</sup>. But the indirect effects of real wages on fertility can be manifested in various forms. Wealthier parents delay marriage and the time of first pregnancy which may consequently cause a decline in family size<sup>53</sup>. In the economics of the family, increased wages encourage female participation in the workforce leading to a decline in family sizes. And most importantly when choosing family size parents are faced with the quantity-quality trade-off initially proposed by Becker (1960).

In this chapter I will focus on the quality-quantity trade-off. Originally proposed by Becker (1960) and further developed by Becker and Lewis (1973) and Becker and Tomes (1976), the quantity-quality theory attempts to explain the negative correlation between family size and child quality. The extensive empirical literature on this topic fails to agree

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<sup>52</sup>Eckstein et. al. successfully fitted the model to match two centuries of time series data in Sweden.

<sup>53</sup>Bailey and Chambers (1999) analyze the effect of wages and mortality on fertility and nuptuality.

on causality and adheres more to the notion that of the jointness of parental preferences. Empirical analysis has become the standard in solving these questions due to the lack of an analytic, theoretical approach.

This chapter develops a model where parents choose between the number of children they will raise,  $\eta$ , and investing in the quality of each child,  $e$ . The quality of the child is naturally projected into their income. Higher quality children are more productive and will earn more money. As we observed in Chapter 3 The parent's old age retirement decisions do not play into the parents' fertility decisions early on in life. Therefore I will consider a two period model in which investment in both quality and quantity require time.

## 6.1 A Model Incorporating Human Capital Investment

Consider the following two period model

$$V_t = \max_{\eta_t, e_{t+1}} \{u(c_{1t}) + \beta\pi_2 u(c_{2,t+1}) + \rho(\eta_t) V_{t+1}\}$$

such that  $\rho(\eta_t) = E[\alpha(\eta_t)^{1-\varepsilon}]$  and

$$w_t(1 - \theta\eta_t) + \frac{\pi_2 w_t(1 - \pi_1\eta_t e_{t+1})}{(r_t + 1)} - c_{1t} - \frac{\pi_2 c_{2,t+1}}{(r_t + 1)} \geq 0$$

$c_{1t}$  and  $c_{2,t+1}$ , represent young adult and mature adult consumption respectively. Young adults birth  $\eta_t$  children at a time cost of  $\theta$ . Mature adult parents provide human capital investment,  $e_{t+1}$  to  $\pi_1\eta_t$  children. Parents are motivated to invest in human capital given the positive effects on children

$$w_t = \bar{w}_t + h(e_t)$$

such that  $h(e_t) = A_h e_t^\delta$  and  $\delta \in (0, 1)$ . To avoid possible income distribution problems, I have incorporated the intergenerational insurance mechanisms from chapter 3.

#### OPTIMALITY CONDITIONS

Maximizing the Lagrangian leads to the following optimality conditions.

$$\begin{aligned}
 c_{1t} & : u'(c_{1t}) = \lambda_t \\
 c_{2,t+1} & : \beta u'(c_{2,t+1}) = \frac{\lambda_t}{(r_t + 1)} \\
 \eta_t & : -\lambda_t w_t \left[ \theta + \frac{\pi_2 \pi_1 e_{t+1}}{(r_t + 1)} \right] + \rho'(\cdot) V_{t+1} = 0 \\
 e_{t+1} & : -\lambda_t w_t \frac{\pi_2 \pi_1 \eta_t}{(r_t + 1)} + \lambda_{t+1} \rho(\cdot) h'(e_{t+1}) \left[ 1 - \theta \eta_{t+1} + \frac{\pi_2 (1 - \pi_1 \eta_{t+1} e_{t+2})}{(r_{t+1} + 1)} \right] = 0
 \end{aligned}$$

and the following Kuhn-Tucker condition

$$\lambda_t \left[ w_t (1 - \theta \eta_t) + \frac{\pi_2 w_t (1 - \pi_1 \eta_t e_{t+1})}{(r_t + 1)} - c_{1t} - \frac{\pi_2 c_{2,t+1}}{(r_t + 1)} \right] = 0$$

The lifetime budget constraint binds with equality. The first-order condition for fertility equates the marginal utility of birthing an additional child to the total lifetime costs of raising them. The cost of raising the child includes both the time cost of rearing the child in young age as well as the cost of investing human capital in the child during mature adulthood.

The human capital investment condition equates the marginal cost to the marginal benefit of providing children with human capital. The non-convexity of the budget constraint indicates that, with respect to human capital, the marginal cost rises and the marginal benefit falls with the number of surviving children.

## STEADY STATE CONDITIONS

I will assume the following steady states conditions:  $\eta_t = \eta_{t+1} = \eta$ ,  $e_t = e_{t+1} = e$ ,  $\bar{w}_t = \bar{w}_{t+1} = \bar{w}$  for all  $t = 1, 2, 3, \dots$

$$\begin{aligned} c_1, c_2 & : u'(c_1) = \beta(r+1)u'(c_2) \\ \eta & : -\lambda w \left[ \theta + \frac{\pi_2 \pi_1 e}{(r+1)} \right] + \rho'(\cdot) V = 0 \\ e & : -w \frac{\pi_2 \pi_1 \eta}{(r+1)} + \rho(\cdot) h'(\cdot) \left[ 1 - \theta \eta + \frac{\pi_2 (1 - \pi_1 \eta e)}{(r+1)} \right] = 0 \end{aligned}$$

where  $c_1 = w(1 - \theta \eta) + \frac{\pi_2 w(1 - \pi_1 \eta e)}{(r+1)} - \frac{\pi_2}{(r+1)} c_2 = f_c(s)$ , where  $s = (\eta, e, c_2, \bar{w})$ .

It is expected that  $\eta$  is decreasing in  $\bar{w}$ .  $\bar{w}$  represents the opportunity cost of quantity over quality. As  $\bar{w}$  increases it is more desirable to invest in the human capital of the child. Essentially as  $\bar{w}$  increases the marginal cost per child will increase and parents will birth less children and devote more time to human capital production.

## 6.2 Calibration

Assuming the following functional forms

$$\begin{aligned} \nu_t & = U(C_{1t}) + \beta \pi_2 [U(C_{2,t+1}) + \beta \pi_3 U(C_{3,t+2})] \\ \rho(\eta_t) & = \alpha (\pi_1 \eta_t)^{1-\varepsilon} + \frac{1}{2} \alpha \pi_1 (1-\varepsilon) (\pi_1 \eta_t)^{-\varepsilon-1} \sigma_\eta \\ h(e_t) & = A_h e_t^\delta \end{aligned}$$

where  $U(C) = \frac{C^{1-\sigma}}{1-\sigma} + \zeta$  and  $\rho(\eta_t)$  is a second order estimation of  $E_Q[\alpha(\eta_t)^{1-\varepsilon}]$ . The following table provides us with a summary of the parameter values.

The initial parameter values remain consistent with the findings in Table 4.1 with ex-

Table 6.1 Summary of Calibrated Parameters

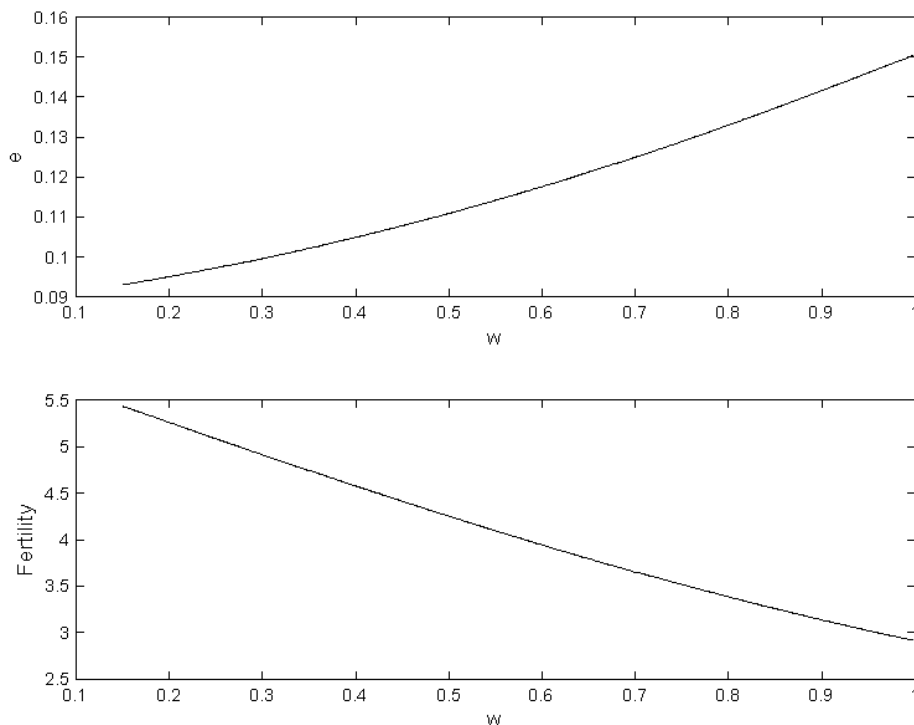
Category	Param.	Model	Source
Health	$\pi_1$	0.9863	WHO (2000)
	$\pi_2$	0.9775	WHO (2000)
	$\pi_3$	0.9097	WHO (2000)
Rel. Risk Aversion	$\sigma$	1	Assigned Log-Utility
Discount Rate	$\beta$	0.54	Assigned
	$\delta$	0.9	Edu. Share of Human Cap.
Altruism	$\alpha$	0.8176	Match Fertility Targets
	$\varepsilon$	0.9852	Match Fertility Targets
Zero Cons. Utility	$\zeta$	-4.1292	Risk-to-Wage Elasticity
Time Cost	$\theta$	0.03	Standard

ception to  $\mu$ ,  $A_h$ , and  $\delta$ .  $A_h$  has been set so that the proportion of time devoted to children does not exceed 15%. As studies have suggested to proportion of time mother's devote to children is estimated to be approximately 6%. Therefore it is reasonable to assume per capita proportion of time to child rearing,  $\theta$ , is approximately 3%. The remaining three parameters,  $(\alpha, \beta, \zeta)$ , are set to match the steady state optimality conditions combined with the derived wage differential equation

$$\zeta = \frac{1}{\gamma \kappa} \frac{\varepsilon_m}{T} \frac{1}{\pi_2^{\frac{T-1}{T}} - 1} \left[ \frac{1}{\beta} + \pi_2 \right] - \log \gamma$$

to the fertility target of 2.06, or  $\eta = 1.03$ , mortality risk to income elasticity,  $\varepsilon_m$ , of approximately 0.5, and the education share of GDP to 12.67%.  $\gamma$  represents the average consumption share in the US for the past 3 decades and  $\kappa$  represents the estimated mature adult share of lifetime consumption. The calibrated parameters provide us with the following

Figure 6.1 Human Capital Investment, Fertility, and Income



Given the estimated parameter values, the model predicts the following: human capital investment (i.e., education) rises with income and fertility falls as income rises. These observations are consistent with empirical evidence.

### 6.3 Results

This section combines the effects of fertility and human capital investment. I compare the model's predictions to the actual data. I use the same mortality data in Chapter 5 from the World Health Organization (WHO). I extrapolate income earned from variables other than labor from the World Development Indicators (WDI) Database for the year 2000. Figures 6.2 and 6.3 provide us with a visual representation of the results found.



Figure 6.2 Fertility and Income: Model vs. Data

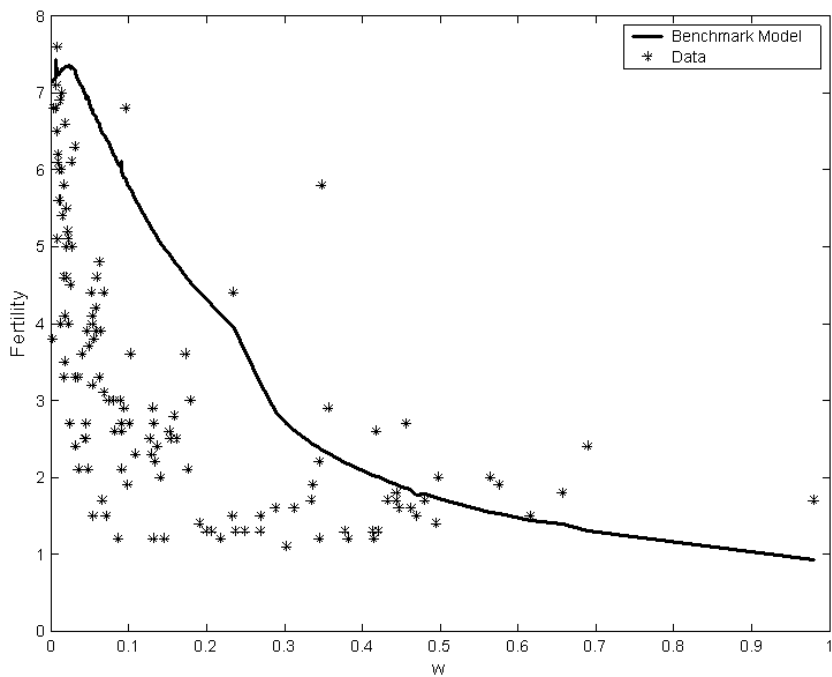


Figure 6.3 Fertility and Infant Survival: Model vs. Data

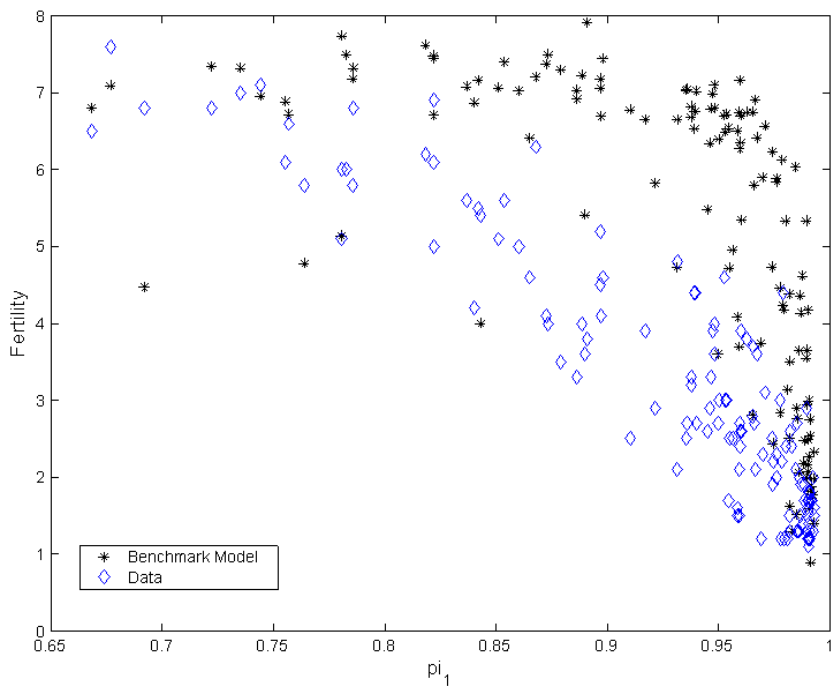
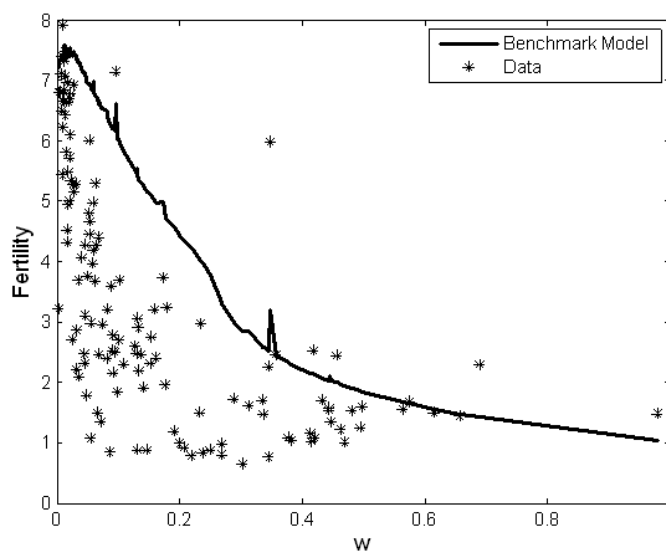


Figure 6.3 compares the predictions of the model to the actual values in the data. An R-squared analysis comparing the model's prediction to the data indicates that approximately 62% of the cross-country variation can be explained by the model. Observe that the model performs better for high fertility, low survival (Stage 1 and early Stage 2) countries and vice versa (late Stage 3 and Stage 4). The model explains over 78% of the cross countries variation for these countries<sup>54</sup>. Combining these observations with Figure 6.2, it can be concluded that the model over predicts fertility for low-income, high-survival countries (Stage 2 and Stage 3).

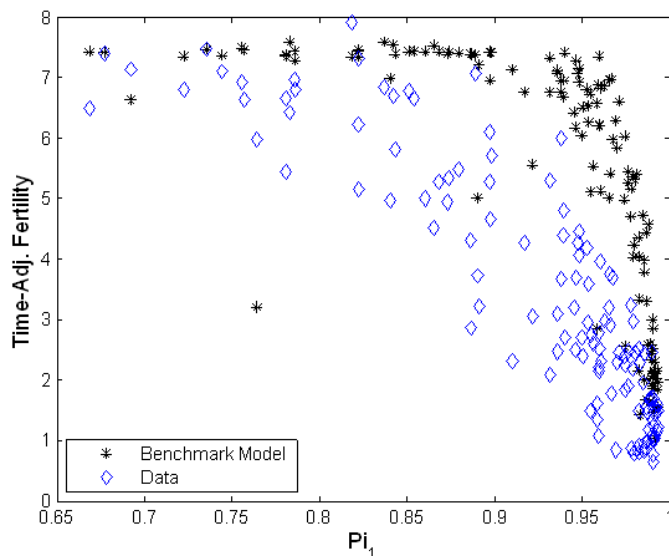
Figure 6.4 Time-Adjusted Fertility and Income: Model vs. Data



As I mentioned in Chapter 1, these economies are experiencing the crucial phases of development and demographic transition. The model overestimates the price effects child costs due to income. This indicates that a steady state analysis is insufficient in explaining the magnitude of the observed fertility differences.

<sup>54</sup>That is after eliminating the higher error countries, which incidently are countries with survival rates between 0.85 and 0.90, with errors greater than 1.

Figure 6.5 Time-Adjusted Fertility and Infant Survival: Model vs. Data



One may argue that the inaccuracies may be due to the measurement error inherent in the tempo-effects of the *TFR* measurement. I find, in Figures 6.4 and 6.5, the improvement in these transition countries to be negligible. Measurement errors cannot account for the significantly large differentials.

The transitory period involves many variables which have been omitted in this analysis. It can be expected that transition economies are more sensitive to changes in infant survival. The parents decision making behavior is based on current observations and fails to include expectations for the future. Higher growth and interest rates in transition economies will likely play an important role in the fertility decision. Higher expected income in the next generation will increase the expected returns on human capital investment. Therefore parents with higher future expectations will invest more in human capital and birth less children. Removing the assumption of intergenerational insurance will amplify the effects of infant survival and the survival rates of the parents themselves. Uninsured parents will invest more in quality and consequently less in quantity. Also,

fertility is more likely to be a sequential decision. Parents may suspend the birthing of children once the desired family size is achieved. Including these components will likely tame the overpredictions of the model for low income, high survival families.

Adding market structures and old-age pensions may also help explain the variations found in wealthier Stage 3 countries, given parents also receive old-age security from their children. The availability of capital markets and the productivity of human and physical capital provides an additional investment stream for wealthier families. The parents' optimal lifetime decision bundle will include the parents' expected returns on savings, investment, and future donations of children. The most complete theory is one which includes all three components of the child-services initially proposed by Harvey Leibenstein.

## CHAPTER 7

### CONCLUSION

The Becker-Barro model incorporating the precautionary child rearing motive successfully predicts a negative relationship between the infant survival rates and fertility. I calibrated the initial model and found that parents value their children's future 46% more than their own. Up to this point the research ignores the importance of the magnitude of utility and considers only the marginal change. In a model where the representative agent is uncertain of the length of life, the level of utility is as crucial as the rate of change in evaluated, agent's decision behavior. I have assumed the utility of death to be zero and estimate a utility level that corresponds with the value of life. Given previous findings on occupational mortality risk and wage differentials, I find it to be approximately  $-4.1292$  implying that the level of consumption where an agent is indifferent between life and death is estimated to be less than 1% of current consumption in developed economies but may be as high as 10% of consumption in poorer developing economies. I also find utility to be fairly unresponsive to changes in fertility given the altruism elasticity is close to 1.

This experiment also finds that infant mortality alone cannot explain the observed cross-country differences in fertility. When comparing the model's predictions to the data, childhood survival explains between 10% to 20% of the observed cross-country variation. Certain measures of fertility generally perform better than others. More specifically The Total Fertility Rate ( $TFR$ ) measure performs well for low fertility, high survival countries, whereas the General Fertility Rate ( $GFR$ ) measure performs better for high fertility low

income countries. The *GFR* is essentially a population weighted average of the *TFR*. This gives more consideration to the birth rate of the younger population which low income, high fertility are mostly comprised of. Accounting for tempo-adjusted fertility rates provides little improvement in this experiment. This analysis indicates that other variables must be considered.

In this work, I also consider human capital investment and find significant improvement in the results. Incorporating human capital investment enables the model to predict approximately 62% of the variation. This formulation provides an important results. Human capital investment increases and fertility falls with income. Successfully capturing a quantity-quality trade-off is a notable contribution in this area of economic research. While the model better explain the cross-country variation as a whole, it fails for Stage 2 and Stage 3 countries<sup>55</sup>. When excluding these transition countries, the model explains over 75% of the cross country variation. It is important to indicate that the inconsistencies of the model's predictions are likely due to steady state analysis. It is expected that the model will overpredict fertility levels in transition economies, given we have ignored important components of the economy affecting fertility. Countries experiencing the crucial phases of the demographic transition face economic constraints and uncertainties which, to this point, have been disregarded. Relaxing the assumptions of steady state conditions and intrafamily insurance and including these externalities will improve results.

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<sup>55</sup>These countries, as defined in Chapter 1, are experiencing the crucial phases of demographic transition, which generally include low income, high survival.

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## APPENDIX A

### PROOFS OF PROPOSITIONS

#### A.1 Proof of Proposition 1

Given the following first-order conditions and assuming utility is *CRRA*, non-borrowing inter-temporal budget constraints (i.e.,  $b_{t+1}^A, b_{t+2}^O \geq 0$ ) and equal transfer costs ( $\delta_{t+1} = \delta_{t+1}^*$ ), we find that

$$\begin{aligned} b_{t+1}^A &: \rho(\eta_t) \lambda_{t+1} - \frac{\pi_1 \delta_{t+1} \eta_t \lambda_t}{r_t + 1} = 0 \\ b_{t+2}^O &: \frac{\rho(\eta_t) \lambda_{t+1}}{(r_{t+1} + 1)} - \frac{\pi_1 \delta_{t+1} \eta_t \lambda_t}{(r_t + 1)(r_{t+1} + 1)} = 0 \end{aligned}$$

To further simplify, observe that the two equations are identical indicating a non-unique solution. Therefore combining equation (3.10)

$$\rho(\eta_t) U'(C_{1,t+1}) = \frac{\pi_1 \delta_{t+1} \eta_t}{r_t + 1} U'(C_{1t}) \quad (\text{A.1})$$

where  $C_{1t} = w_t + \frac{\pi_2}{(r_t+1)} w_{t+1} + b_t^A + \frac{\pi_2}{(r_t+1)} b_{t+1}^O - \theta_t \eta_t - \frac{\pi_2 C_{2,t+1} + \pi_1 \delta_{t+1} \eta_t b_{t+1}^A}{(r_t+1)} - \frac{\pi_3 C_{3,t+2} + \pi_2 \pi_1 \delta_{t+1} \eta_t b_{t+2}^O}{(r_t+1)(r_{t+1}+1)}$ . Therefore given  $s_t = \{w_t, w_{t+1}, r_t, r_{t+1}, \theta_t, \delta_{t+1}\}$ ,  $\psi_t = \{C_{2,t+1}, C_{3,t+2}, \eta_t\}$ ,  $d_t = \{b_{t+1}^A, b_{t+2}^O\}$ ,  $C_{1t} = f_C(\psi_t, d_t, s_t | d_{t-1})$ . Define the history of events for a set as  $\xi^t = \{\xi_t\}_{t=3}^\infty$  such that  $\xi_t = (\psi_t, s_t)$ . Assuming *CRRA* utility, equation (A.1) can be simplified such that

$$\Delta_1(d_t) = \varrho(\psi_t | \xi^t) \Delta_2(d^t)$$

such that  $\Delta_i(d_t)$  represents an additive sum of  $b_{t+1}^A$  and  $b_{t+2}^O$  where  $b = \Delta_i(d_t) = j_i b_{t+1}^A + k_i b_{t+2}^O$ . Therefore, there exists an infinite number of solutions which satisfy the optimality condition if  $j_i = k_i$ . For  $j_i > k_i$ ,  $b_{t+1}^A = 0$  and  $b_{t+2}^O > 0$ ; and vice versa.

#### A.2 Proof of Proposition 2

In this model, utility is *CRRA* and  $\sigma \geq 1$

$$C_{1,t+1} = \left[ \frac{\pi_1 \delta_{t+1} \eta_t}{\rho(\eta_t)(r_t + 1)} \right]^{\frac{1}{\sigma}} C_{1t} \quad (\text{A.2})$$

where  $f_C(\psi_t, d_t, s_t | d_{t-1}) = \phi_C(\psi_t, s_t) + \gamma_t(d_{t-1}) + \omega_t(d_t)$  and  $\gamma_t(\cdot), \omega_t(\cdot)$  are additive functions<sup>56</sup>. Therefore  $\phi_C(\psi_{t+1}, s_{t+1}) + \gamma_{t+1}(d_t) + \omega_t(d_{t+1})$

$$= \left[ \frac{\pi_1 \delta_{t+1} \eta_t}{\rho(\eta_t)(r_t+1)} \right]^{\frac{1}{\sigma}} [\phi_C(\psi_t, s_t) + \gamma_t(d_{t-1}) + \omega_t(d_t)].$$


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<sup>56</sup>Where  $\gamma_t(d_{t-1}) = b_t^A + \frac{\pi_2}{(r_t+1)} b_{t+1}^O$  and  $\omega_t(d_t) = \frac{\pi_2 \pi_1 \delta_{t+1} \eta_t}{(r_t+1)} \left[ b_{t+1}^A + \frac{\pi_2}{(r_{t+1}+1)} b_{t+2}^O \right] = \frac{\pi_2 \pi_1 \delta_{t+1} \eta_t}{(r_t+1)} \gamma_{t+1}(d_t)$

This would indicate that the decision vector<sup>57</sup>  $d_t$

$$\gamma_{t+1}(d_t) - \left[ \frac{\pi_1 \delta_{t+1} \eta_t}{\rho(\eta_t)(r_t + 1)} \right]^{\frac{1}{\sigma}} \omega_t(d_t) = \varrho(\psi_t | \xi^t) \Delta_2(d^t)$$

Therefore  $\Delta_1(d_t)$  would be defined as

$$\begin{aligned} \Delta_1(d_t) &= \gamma_{t+1}(d_t) - \left[ \frac{\pi_1 \delta_{t+1} \eta_t}{\rho(\eta_t)(r_t + 1)} \right]^{\frac{1}{\sigma}} \omega_t(d_t) \\ &= \gamma_{t+1}(d_t) - \pi_2 \rho(\eta_t)^{-\frac{1}{\sigma}} \left[ \frac{\pi_1 \delta_{t+1} \eta_t}{(r_t + 1)} \right]^{\frac{1}{\sigma} + 1} \gamma_{t+1}(d_t) \\ &= \gamma_{t+1}(d_t) \left[ 1 - \pi_2 \rho(\eta_t)^{-\frac{1}{\sigma}} \left[ \frac{\pi_1 \delta_{t+1} \eta_t}{(r_t + 1)} \right]^{\frac{1+\sigma}{\sigma}} \right] \end{aligned}$$

Therefore  $b$  solves the following

$$\gamma_{t+1}(d_t) = \frac{\varrho(\psi_t | \xi^t) \Delta_2(d^t)}{\left[ 1 - \pi_2 \rho(\eta_t)^{-\frac{1}{\sigma}} \left[ \frac{\pi_1 \delta_{t+1} \eta_t}{(r_t + 1)} \right]^{\frac{1+\sigma}{\sigma}} \right]} \quad (\text{A.3})$$

where  $b = \gamma_{t+1}(d_t) = b_{t+1}^A + \frac{\pi_2}{(r_{t+1} + 1)} b_{t+2}^O$ .

The household will choose the  $b$  that solves equation A.3 and is indifferent between how it is distributed between  $b_{t+1}^A$  and  $b_{t+2}^O$

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<sup>57</sup>Where  $\varrho(\psi_t | \xi^t) \Delta_2(d^t) = \left[ \frac{\pi_1 \delta_{t+1} \eta_t}{\rho(\eta_t)(r_t + 1)} \right]^{\frac{1}{\sigma}} [\phi_C(\psi_t, s_t) + \gamma_t(d_{t-1})] - [\phi_C(\psi_{t+1}, s_{t+1}) + \omega_t(d_{t+1})]$

**APPENDIX B**  
**INTRAFAMILY INSURANCE POLICIES**

Young adults insure  $b_{t+1}^A$  in the following manner: young adults pay  $x$  to insure that their children will receive bequests in the event of death.

$$\begin{aligned}\pi_2 & : (w_{t+1} + b_{t+1}^O) + (r_t + 1 - \delta) s_t - C_{2,t+1} - \pi_1 \eta_t b_{t+1}^A - s_{t+1} - px \\ 1 - \pi_2 & : (r_t + 1 - \delta) s_t - \pi_1 \eta_t b_{t+1}^A - s_{t+1} + (1 - p) x\end{aligned}$$

Assuming actuarially fair insurance policies,  $p = 1 - \pi_2$ . Given the strictly increasing nature of the marginal utilities, households will choose  $x$  such that:

$$\begin{aligned}(w_{t+1} + b_{t+1}^O) + (r_t + 1 - \delta) s_t - C_{2,t+1} - \pi_1 \eta_t b_{t+1}^A - s_{t+1} - (1 - \pi_2) x \\ = (r_t + 1 - \delta) s_t - \pi_1 \eta_t b_{t+1}^A - s_{t+1} + \pi_2 x\end{aligned}$$

Household's budget constraint is therefore:

$$\pi_2 (w_{t+1} + b_{t+1}^O) + (r_t + 1 - \delta) s_t = \pi_2 C_{2,t+1} + \pi_1 \eta_t b_{t+1}^A + s_{t+1}$$

Apply the same methodology for the mature adult budget constraint to obtain equation (3.4).

$$\begin{aligned}\pi_3 & : (r_{t+1} + 1 - \delta) s_{t+1} - C_{3,t+2} - \pi_2 \pi_1 \eta_t b_{t+2}^O - s_{t+1} - px \\ 1 - \pi_3 & : (r_{t+1} + 1 - \delta) s_{t+1} - \pi_2 \pi_1 \eta_t b_{t+2}^O - s_{t+1} + (1 - p) x\end{aligned}$$

**APPENDIX C**  
**APPROXIMATION OF EXPECTED ALTRUISM**

To obtain a more accurate estimate of  $E_Q [\alpha(\eta_t^*)^{(1-\varepsilon)}]$ , a Taylor approximation of  $\alpha(\eta_t^*)^{(1-\varepsilon)}$  about the mean is in order.

$$\alpha(\eta_t^*)^{1-\varepsilon} = \sum (\alpha(\pi_1\eta_t)^{1-\varepsilon})^{(j)} \frac{(\eta_t^* - \pi_1\eta_t)^j}{j!}$$

A second order approximation would yield

$$E_Q [\alpha(\eta_t^*)^{(1-\varepsilon)}] = E_Q \left[ \alpha(\pi_1\eta_t)^{1-\varepsilon} + \frac{\alpha\pi_1(1-\varepsilon)(\pi_1\eta_t)^{-\varepsilon}}{2} (\eta_t^* - \pi_1\eta_t)^2 \right]$$

$$E_Q [\alpha(\eta_t^*)^{(1-\varepsilon)}] = \alpha(\pi_1\eta_t)^{1-\varepsilon} + \frac{\alpha\pi_1(1-\varepsilon)(\pi_1\eta_t)^{-\varepsilon}}{2} Var(\eta_t^*)$$

## APPENDIX D

### MORALITY RISK AND WAGE DIFFERENTIAL CALIBRATION EQUATION

Data on wage differentials and mortality risk is required in order to acquired a value for  $\zeta$ . Consider the following:  $p^i$  is defined as the probability of surviving work Assume the household must choose between the following two bundles:  $(p^1, w^1)$  and  $(p^2, w^2)$ . We will also assume that  $w^2 > w^1$  and  $p^1 > p^2$ . The agent has the choose between the "risky", high wage bundle,  $(p^2, w^2)$  and the "safe", low wage bundle,  $(p^1, w^1)$ . In equilibrium the household will be indifferent between the two bundles, therefore

$$V(p^1, w^1) = V(p^2, w^2)$$

where  $V(p^i, w^i) = U(C_1^i) + p^i U(C_2^i)$  and  $U(C^i) = \frac{(C^i)^{1-\sigma}}{1-\sigma} + \zeta$ .

In general terms, given  $(p^i, w^i) \in \mathbb{R}^N$ ,  $p^i > 0$ ,  $w^i > 0$  and  $N$  defined as the number of bundles to choose from,  $V(p^i, w^i) = V(p^j, w^j)$  for all  $i, j = 1, 2, \dots, N$ . Therefore

$$\begin{aligned} \frac{(C_1^i)^{1-\sigma}}{1-\sigma} + \zeta + p^i \left( \frac{(C_2^i)^{1-\sigma}}{1-\sigma} + \zeta \right) &= \frac{(C_1^j)^{1-\sigma}}{1-\sigma} + \zeta + p^j \left( \frac{(C_2^j)^{1-\sigma}}{1-\sigma} + \zeta \right) \\ \frac{(C_1^i)^{1-\sigma}}{1-\sigma} + p^i \frac{(C_2^i)^{1-\sigma}}{1-\sigma} + p^i \zeta &= \frac{(C_1^j)^{1-\sigma}}{1-\sigma} + p^j \frac{(C_2^j)^{1-\sigma}}{1-\sigma} + p^j \zeta \\ r(C_1^i, C_2^i, p^i) + p^i \zeta &= r(C_1^j, C_2^j, p^j) + p^j \zeta \end{aligned}$$

So for all  $i, j = 1, 2, \dots, N$ ,

$$(p^i - p^j) \zeta = r(C_1^j, C_2^j, p^j) - r(C_1^i, C_2^i, p^i)$$

$$\zeta = \frac{r(C_1^j, C_2^j, p^j) - r(C_1^i, C_2^i, p^i)}{p^i - p^j}$$

My aim is to find a value  $\hat{\zeta}$  which would best fit the equation given all values  $i, j = 1, 2, \dots, N$ .

$$\hat{\zeta} = \frac{r(\tilde{C}_1^j, \tilde{C}_2^j, p^j) - r(\tilde{C}_1^i, \tilde{C}_2^i, p^i)}{p^i - p^j}$$

where  $\tilde{C}_i^i$  is the consumption level that maximizes total expected utility. Therefore



$\tilde{C}_k^i = f_k(w^i)$  for  $k = 0, 1, 2, \dots$ <sup>58</sup>Therefore it is found that

$$\begin{aligned}
 p^i \zeta - p^j \zeta &= \frac{(C_1^j)^{1-\sigma}}{1-\sigma} - \frac{(C_1^i)^{1-\sigma}}{1-\sigma} + p^j \frac{(C_2^j)^{1-\sigma}}{1-\sigma} - p^i \frac{(C_2^i)^{1-\sigma}}{1-\sigma} \\
 \Delta p \zeta &= \frac{(C_1^j)^{1-\sigma}}{1-\sigma} - \frac{(C_1^i)^{1-\sigma}}{1-\sigma} + \Delta p \left[ \frac{(C_2^j)^{1-\sigma}}{1-\sigma} - \frac{(C_2^i)^{1-\sigma}}{1-\sigma} \right] \\
 \zeta &= \frac{G(C_1^i, C_1^j) + \Delta p G(C_2^i, C_2^j)}{\Delta p} \\
 \zeta &= \frac{(1 + \Delta p) H(w^i)}{\Delta p}
 \end{aligned}$$

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<sup>58</sup>Since  $\tilde{C}_1^i = \frac{(2+r)(\beta p^i)^{-\frac{1}{\sigma}}}{(1+r)(\beta p^i)^{-\frac{1}{\sigma}+1}} w^i$  and  $\tilde{C}_2^i = \frac{(2+r)}{(1+r)(\beta p^i)^{-\frac{1}{\sigma}+1}} w^i$ .

**APPENDIX E**  
CALIBRATION NOTES

Using the steady state budget constraint

$$\bar{C}_1 + \bar{\theta}\bar{\eta} + \frac{\pi_2 C_2 + \pi_1 \bar{\delta}\bar{\eta}(1+\mu)\bar{b}^A}{(r+1)} + \frac{\pi_3 C_3}{(r+1)^2} = \bar{w} + \bar{b}^A + \frac{\pi_2 [(1+\mu)\bar{w}]}{(r+1)}$$

I can conclude that

$$\begin{aligned} \bar{C}_1 + \frac{\pi_2 C_2}{(r+1)} + \frac{\pi_3 C_3}{(r+1)^2} + \bar{\theta}\bar{\eta} + \frac{\pi_1 \bar{\delta}\bar{\eta}(1+\mu)\bar{b}^A}{(r+1)} &= \left[1 + \frac{\pi_2(1+\mu)}{(r+1)}\right] \bar{w} + \bar{b}^A \\ \left[A_1 + \frac{\pi_2}{(r+1)} + \frac{\pi_3 A_3}{(r+1)^2}\right] \kappa c + \bar{\theta}\bar{\eta} + \frac{\pi_1 \bar{\delta}\bar{\eta}(1+\mu)\bar{b}^A}{(r+1)} &= \left[1 + \frac{\pi_2(1+\mu)}{(r+1)}\right] \bar{w} + \bar{b}^A \end{aligned}$$

Now given that  $\phi = \frac{\pi_1 \bar{\delta}\bar{\eta}(1+\mu)\bar{b}^A}{(r+1)w}$  and  $\frac{c}{w} = \gamma$

$$\left[A_1 + \frac{\pi_2}{(r+1)} + \frac{\pi_3 A_3}{(r+1)^2}\right] \kappa\gamma + \phi = \left[1 + \frac{\pi_2(1+\mu)}{(r+1)}\right] + \frac{\bar{b}^A}{\bar{w}}$$

Therefore

$$\frac{\bar{b}^A}{\bar{w}} = \left[A_1 + \frac{\pi_2}{(r+1)} + \frac{\pi_3 A_3}{(r+1)^2}\right] \kappa\gamma + \phi - \left[1 + \frac{\pi_2(1+\mu)}{(r+1)}\right]$$

Now given  $\frac{\bar{b}^A}{\bar{w}}$  and  $\phi$

$$\begin{aligned} \phi &= \frac{\pi_1 \bar{\delta}\bar{\eta}(1+\mu)\bar{b}^A}{(r+1)w} \\ \bar{\delta} &= \frac{\phi(r+1)}{\pi_1 \bar{\eta}(1+\mu)} \left[\frac{\bar{b}^A}{w}\right]^{-1} \end{aligned}$$

## APPENDIX F

### VARIOUS MEASUREMENTS OF FERTILITY

Estimates of the various measurements of fertility can be obtained through age specific Crude Birth Rate, or  $CBR$  which is defined as

$$CBR_t = \frac{B_t}{P_t}$$

where  $P_t$  is the total population at time,  $t$  and  $B_t$  is the number of live births. Specifically, given there exists  $k$ ,  $i^{th}$  cohorts in a given population,  $P_t$ ,

$$B_t = \sum_{i=1}^k B_{i,t}$$

#### F.1 Total Fertility Rate, $TFR$

To calculate the  $TFR$ , the age specific fertility rate must be acquired and summed for  $k$  cohorts in a population

$$ASFR_{i,t} = \frac{B_{i,t}}{W_{i,t}} \times 1000$$

where  $ASFR_{i,t}$  is the age-specific fertility rate of women in the  $i^{th}$  age cohort,  $W_{i,t}$  is the population of in cohort  $i$  at time,  $t$  and  $h$  is the number of year in an age cohort. Given the age specific fertility rates

$$TFR_t = h \sum_{i=1}^k ASFR_{i,t}$$

This measurement is provided under the assumption the woman survives through her fertile years of 15-44.

#### F.2 General Fertility Rate, $GFR$

It can be observed that the General Fertility Rate is a weighted average of the age specific fertility rates. To be more specific, it is defined that the birth rate  $B_{i,t}$

$$B_{i,t} = ASFR_{i,t} \times W_{i,t}$$

the measure of  $GFR$  would be the total number of births per fertile woman. Therefore

$$\begin{aligned} GFR_t &= \frac{\sum_{i=1}^k B_{i,t}}{\sum_{i=1}^k W_{i,t}} = \sum_{i=1}^k \frac{ASFR_{i,t} \times W_{i,t}}{\sum_{i=1}^k W_{i,t}} \\ &= \sum_{i=1}^k \theta_{i,t} \times ASFR_{i,t} \end{aligned}$$

the  $GFR$  is in essence a population weighted average of age specific fertility rates (ie.  $\theta_{i,t} = W_{i,t} / \sum_{i=1}^k W_{i,t}$  and  $\sum_{i=1}^k \theta_{i,t} = 1$ ). This measurement of fertility possesses inaccuracies with the model discussed here. Therefore I will also utilize a population and time weighted measurement of fertility.

### F.3 Time-Weighted Adjustment of $GFR$

Therefore given period  $T$ , I will use the following measurement

$$\begin{aligned} \hat{G} &= \frac{\sum_{t=0}^T B_{i,t}}{\frac{1}{T} \sum_{t=0}^T \sum_{i=1}^k W_{i,t}} \\ &= \frac{\sum_{t=0}^T \sum_{i=1}^k ASFR_{i,t} \times W_{i,t}}{\frac{1}{T} \sum_{t=0}^T \sum_{i=1}^k W_{i,t}} \end{aligned}$$

To simplify even further

$$\begin{aligned} \hat{G} &= \frac{\sum_{t=0}^T \sum_{i=1}^k ASFR_{i,t} \times W_{i,t}}{\frac{1}{T} \sum_{t=0}^T \sum_{i=1}^k W_{i,t}} \\ &= \frac{\sum_{t=0}^T \sum_{i=1}^k ASFR_{i,t} \times W_{i,t} \times \sum_{i=1}^k W_{i,t}}{\frac{1}{T} \sum_{t=0}^T \sum_{i=1}^k W_{i,t} \times \sum_{i=1}^k W_{i,t}} \\ &= \sum_{t=0}^T \frac{\sum_{i=1}^k ASFR_{i,t} \times W_{i,t}}{\sum_{i=1}^k W_{i,t}} \frac{\sum_{i=1}^k W_{i,t}}{\frac{1}{T} \sum_{t=0}^T \sum_{i=1}^k W_{i,t}} \\ &= \sum_{t=0}^T \left[ \sum_{i=1}^k ASFR_{i,t} \times \theta_{i,t} \right] \frac{W_t}{\frac{1}{T} \sum_{t=0}^T W_t} \end{aligned}$$

Therefore

$$\hat{G} = \sum_{t=0}^T \left[ \sum_{i=1}^k ASFR_{i,t} \times \theta_{i,t} \right] \gamma_t$$

where  $\gamma_t = \frac{\sum_{i=1}^k W_{i,t}}{\frac{1}{T} \sum_{t=0}^T \sum_{i=1}^k W_{i,t}}$ . It can be concluded that this a population and time adjusted measure of fertility across the defined period,  $T$ .