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UNIVERSITY OF MIAMI

APPLICATIONS OF MATCHING AND SORTING THEORY TO TOPICS IN
OPERATIONS MANAGEMENT

By

Hewen Liu

A DISSERTATION

Submitted to the Faculty
of the University of Miami
in partial fulfillment of the requirements for
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APPLICATIONS OF MATCHING AND SORTING THEORY TO TOPICS IN
OPERATIONS MANAGEMENT

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Applications of Matching and Sorting Theory (August 2019)
to Topics in Operations Management

Abstract of a dissertation at the University of Miami.

Matching and sorting theory are widely applied to operations management. Our analysis focuses on the matching in supply chains and sorting between projects and heterogeneous firms. In chapter 1, we investigate the decentralized allocation in pharmaceutical research and development (R&D) projects. We construct a theoretical framework of R&D competition between heterogeneous firms for differentiated drugs. The decentralized allocation features a mismatch between firms and R&D projects. Efficient outcome prescribes that each firm works on a separate R&D project with the more valuable R&D project being allocated to the established firm. The impact of transparency on efficiency is contingent on the disclosure rules and may not improve the welfare. We also demonstrate the trade-off between the firm revenue and positive externality of the development of new drugs. Our research extends the R&D competition to a two-sided heterogeneous case and enriches the transparency analysis. In chapter two, we analyze a decentralized assortative matching model for bilateral supply chains. The risk-averse suppliers and retailers match mutually to form as supply chains, in which each supply chain optimally determines production plans and revenue sharing contracts. We show that in equilibrium, the less risk averse firm is not always matched with higher risk averse firm to share risk and revenue. We find that both positive assortative matching (PAM) and negative assortative matching

(NAM) can occur in equilibrium. Equilibrium sorting pattern depends on the trade-off between the expected revenue and demand uncertainty. The main results extend the supply chain matching research by endogenizing both production choice and risk sharing contract.

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Chapter 1

Decentralized Pharmaceutical R&D Competition for New Drug Development: Transparency and Externality

1.1 Overview

The successful research and development (R&D) is critical for pharmaceutical firms to achieve a superior performance. Therefore, the spending on R&D has become the major component of pharmaceutical firm expenditures. According to the study of Gagnon and Lexchin (2008), pharmaceutical firms spent billions of US dollars on R&D each year. Paul et al. (2010) assessed that bringing each single new drug to market costs about US $1.8 billion in 2010. More specifically, instead of increasing the value of drug, the majority of expenditures are devoted to clear the approval process (DiMasi et al., 2016). Unfortunately, R&D project could fail and the expenditure would be wasted. In the pharmaceutical industry, new drug development becomes even harder as the number of new drugs approved per billion US dollars R&D spending has halved roughly every 9 years since 1950 and the reason behind the R&D failures largely comes

from the intense competition among heterogeneous firms (Scannell et al., 2012)[1].In the pharmaceutical R&D competition, only the first innovator could secure the profitable intellectual property. After a new drug is successfully developed by the first firm, the patent law ensures the first firm produces and sells the new drug exclusively. In the meantime, the firm endeavors less R&D investment may fail in this R&D competition and incur financial loss if two competing pharmaceutical firms target the drug with equivalent efficacy. The other fact in pharmaceutical industry is that many pharmaceutical firms only develop single project in a long term. Most of the active pharmaceutical firms only successfully develop one single drug in past decade Herper (2013) and there were many pharmaceutical firms quit the business due the failure of R&D competition. The failure of ImClone Systems is not uncommon in R&D competition. ImClone Systems was one of the most promising company which was developing a colourectal cancer drug in late 1990s to early 2000s. However, its product 'Erbitux' got rejected by FDA in 2001 due to the insufficient clinical trial evidence on efficacy. When ImClone resubmitted 'Erbitux''s application to FDA 3 years later, the other colourectal cancer drug 'Avastin' already took the most market share and the whole case directly led to the failure of ImClone. Therefore, the R&D project choice and the investment sorting to R&D project is critical to pharmaceutical firms' strategy.

Inspired by the above facts in pharmaceutical industry, we aim at developing a R&D competition model to analyze the sorting between firms and projects by incorporating the expenditure on approval clearing stage. More specifically, our analysis consists of a set of firms' R&D strategies into the model of competition between heterogeneous firms. Both firms choose firms' R&D project with respect to the drug

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[1]The number is adjusted by inflation.

value. It is intuitive that the high revenue R&D project draws more intense competition than the low revenue R&D project. To avoid the R&D failure, both established and secondary firm must deliberate over other firm's R&D project choice. Meanwhile, firms could decide the amount of R&D investment to control the speed of R&D.

Our first model considers confidential competition in which firms make their project choice without observing each others' decisions. In the first stage of the competition, firms decide which project to develop and how much investment the firm is willing to commit to accelerate the R&D blindly. The pharmaceutical companies can allocate the investment to facilitate the approval stage and those resources that are allocated to the approval stage determines how firms values the R&D competition. Our analysis incorporates the approval investment and treats it as an essential element for the success of pharmaceutical R&D. Therefore, in the next stage, the patent law will secure the profit for first innovator which is the firm that invests more in each project while the firm that invests less will lose the competition and the investment will be wasted. The investment itself doesn't increase the value of the project but it is the only factor that determines which firm can sell the drug legitimately and make profit. The more the company invests in clinical trials, the faster the drug can be patented. For the firms, the high value project is more profitable if successfully developed but it also comes with higher risk of losing the competition. The ability of R&D critically affects the competition. The established company commonly has more capable manufacturing and marketing departments, thus, any project developed by the established firm would come with more profit than the same project finished by the secondary firm.

In competition, the secondary firm can optimize its expected payoff in two ways. First, the secondary firm can aggressively choose the high-profit project with a sufficient investment plan. Once the secondary firm decides to invest in a high-profit project, they would be prone to investing more than their rival to seize the opportunity of winning the competition. Second, the secondary firm can also execute a secure strategy to work on a low-profit project, such as an orphan drug targeting rare diseases, coming with a lower chance to encounter competition with other firms. The trade-off for the established firm also arises as the firm can either invest adequately to ensure the successful development of a high-profit project, or they can expect the secondary firm to choose a low-profit project and economize the expenditure. In particular, if both of the firms coincidentally compete in the high-profit project, the secondary firm has stronger incentive to invest.

In equilibrium, by weighting the capability of each firm and the potential revenue of drugs, the secondary firm has a positive probability of selecting either the high-profit or the low-profit R&D projects, whereas the established firm only chooses the high-profit project in the confidential competition.

Decentralized competition induces a intense competition in high value project and the efficiency loss arises through three channels:

1. The investment is wasteful in terms of increasing the value of the drug. The inherent value of pharmaceutical R&D project is not affected by the investment as the investment only facilitates the approval stage. In our paper, we also discuss the case when investment is not wasteful in section 7.

2. It is possible that the secondary firm wins the high value project and negative assortative sorting causes efficiency loss from mismatch.

3. The low value project can be neglected in decentralized sorting when both firms choose to develop the high value project.

The efficiency loss is largely due to simultaneous project choice. We also characterize the sorting equilibrium when the heterogeneous firms are randomly selected to be the first firm to register the project. When the simultaneous project choice is infeasible in R&D competition, all the firms weakly prefer to register the project first by considering the first mover advantage. The possible rushing registration behavior increases the necessity of studying the random first mover sequential competition. Depending on the difference in project value and ability, low ability firm has a chance to successfully finish the high value project when it is chosen to be the first one to register the project. Suppose the difference in firms' ability and the project value are relatively small, the high ability firm is better off picking the low value project when it becomes the second firm to register the project and the low ability firm already chose the high value project. The trade off between competing the high value project and being the only firm choosing the high value project arises because the high value project comes with higher revenue while the low value project does not require investment. When the difference in firm's ability and project value are large enough, high ability firm always chooses the high value project regardless of low ability firm's project choice. Based on Baye et al. (1996), if low ability firm chooses the same project as high ability firm does in the complete information contest, its payoff in the competition is always zero. Therefore low ability firm's strategy is to evade competition with high ability firm. The equilibrium in random first mover R&D competition indicates that the firms always choose different project and the investment in approval stage is zero for both firms. By adding the registration stage, the efficiency loss from

unselected project is eliminated and both positive and negative sorting can become the equilibrium sorting depending on the exogenous project value. The efficiency in random first mover registration is also higher than the confidential competition.

Finally, we also consider the implication of non-wasteful investment. The investment might not be valueless when we consider the more investment in the project can facilitate the drug approval process and the promptness of R&D can convert to social benefit. The confidential development plan may lead to a fierce competition in the more valuable project and generate a grand investment in high value project to accelerates the development, but it also has a pronounced effect on leaving the low revenue project undeveloped. Empirically, the "orphan drug" is less likely to be chosen by the firms' R&D plan, and the insufficiency of "orphan drug" significantly jeopardize the social welfare. Otherwise, the sequential information disclosures process between the competitors leads to an assortative sorting with the possibility of a mismatch. The assortative sorting dampens the incentive to invest, but both drugs will be developed. Consequently, we propose a social welfare function that weights the value of acceleration investment and the efficiency loss from undeveloped drug. The curvature of the welfare function crucially determines the optimal pharmaceutical R&D mechanism.

1.2   Literature Review

Several recent studies have focused on the phenomenon that pharmaceutical innovation has encountered an efficiency decline in the past century. Scannell et al. (2012) show that the rate of decline in the approval of new drugs per billion US dollars spent has stayed fairly constant over every 10 years since 1950. Kaitin and DiMasi (2011) also state that the total drugs approvals was at a 25-year low in 2009. Pammolli

et al. (2011) demonstrate the empirical evidence that the increase in pharmaceutical R&D investment hasn't successfully led to an increase in the output in terms of new drugs being approved. Their study also indicates that the investment of pharmaceutical R&D is becoming more concentrated on the drugs that have a relatively little chance to be developed. The confidential competition model in our study also find concentrated competition in the development of high return drug may happen in the equilibrium. Yin (2008) examines the impact of the Orphan Drug Act (ODA). The ODA is aimed at stimulating the R&D which targets rare diseases by the tax credit. The study finds that the limitation of the ODA may leave revenue margins unaffected. Therefore, firms may be unresponsive to tax credits. In our study, we show that the regulation on the project claim may motivate the firms to choose to develop the orphan drug.

The product development tournaments and contests have long received wide interest from operations management and economics. Taylor (1995) addresses that the free entry competition is not the optimal mechanism to stimulate the efforts from contestants. Moldovanu and Sela (2001) also consider the optimal-prize structure in the centralized allocation contest. They suggest that if the cost function of contestants is concave or linear, all the prizes should be allocated to the most efficient agent, and if the cost function is convex, the whole prize should be separated into several fixed prizes. Che and Gale (2003) compare multiple models to maximize the efforts from the researchers and suggest if the ability is asymmetric, the most efficient researcher should be blocked from the contest. Fullerton and McAfee (1999) also discuss the efficient contestant number in the centralized one prize contest is two. Terwiesch and Xu (2008) characterized a stochastic model to analyze how the optimal innovation contest mechanism, i.e., the number of contestants and the reward scheme, can

vary by the principles' value function. Deng and Elmaghraby (2005) study a dynamic model to assess the optimal duration of contest. In our analysis, we not only value the aggregated effort that the competition can invite, but we also discuss the efficiency loss from the unselected prize(project).

The efficiency analysis in innovation contest also have studied by empirical and experimental researches. Liu et al. (2014) have an experimental research on the efficiency of all pay auction. Their experimental evidence illustrates that a higher-value reward induces more overall effort and an outstanding competitor reduces the rest of innovation efforts. Boudreau et al. (2016) test correlation between the number of contestants and the innovation effort. Their work shows that the high ability contestant would exert more effort whereas the low ability contestant act oppositely by increasing the total number of contestants. Boudreau et al. (2011) analyze how the uncertainty and competitiveness affect the innovation efforts. The innovation contest also tested to be more efficient by reducing the complexity and uncertainty (Ulrich and Ellison, 1999).

A number of researches consider the interaction between uncertainty and the R&D competition. Weeds (2002) studies a model in which firms face the uncertain returns in the R&D process. The uncertainty in the model comes from both technological success and the stochastic project value. Murto (2004) considers a exit strategy of duopoly R&D competition with uncertain revenue in the stochastic model. Riedel (2009) also analyzes an optimal stopping strategy for the firms which have multiple priors in the first stage. Unlike our analysis, the above research focuses on the uncertainty from the project revenue instead of other competitors' investments. Ishida et al. (2011) consider the Cournot model that prescribes that increasing in the number of low ability firms can stimulate R&D by the low-cost firms and the high ability

firms' profit may increase with a larger number of the low ability firms. In those cases, however, the project is not differentiated, and the competition is not separable into different sub-competitions.

1.3   Confidential Competition Model

We model the competition between two heterogeneous firms for two heterogeneous projects. A high value project (H) represents a drug with a large market margin, and a low value project (L) represents a drug with a lower market margin. The pharmaceutical firms are also heterogeneous with respect to their research and development ability. Suppose that the ability of firms can be classified into high and low, noted as H and L. The ability reflects their scale and marketing ability. In each period, the pharmaceutical firms compose their R&D plan by choosing a desirable project and investment level, simultaneously and independently. The available investment amount for agents is $x \in_+$. If firms choose to work on the same project, the firm with the higher investment amount is able to finish the development first and file the patent with legitimate monopoly power. In turn, the firm that invest less in R&D incurs a loss by the failure in the research competition. The cost function is $c(x_i) = x_i$. The cost function is captured by the amount of firms' investments in the R&D development. Each R&D department is only capable to work on one project in each period.

A firm's ability affects the revenue of drug development in a positive way, i.e., a higher ability firm owns a larger manufacturing factory, a more capable lab, and it might also be able to promote their products more successfully. Therefore we set up the assumption of revenues as follows

Assumption 1. *The revenue from successfully developing project i is $a\pi_i$ for firm H; the revenue from successfully developing project i is π_i for firm L with $i \in [H, L]$ and $a > 1$.*

Timing in confidential competition. When the R&D competition is confidential, the timing of the game is follow:

1. *Decision stage:* In the initiation stage, the firms endogenously compose the production development strategy in two dimensions:

   - which drug the firm prefers to develop;

   - how much investment they would exert to accelerate the development process–x_i, $i \in [H, L]$.

2. *Development stage:* Each firm works on its R&D strategy that it has committed to in the decision stage.

3. *Final stage:* Whichever firm invests more in the project they choose wins the competition with the project value. The firm that invests less in the development stage gains zero revenue.

1.4 Decentralized Competition Equilibrium in R&D Competition

In this section we describe the equilibrium of 2×2 R&D competition in which each firm's project choice is confidential. By allowing the heterogeneous firms to simultaneously choose their R&D strategy, we found that competition would occur only in the high value project and the low value project may be left undeveloped.

We detail the equilibrium in the appendix, and address the intuition and the equilibrium strategies here.

Lemma 1.4.1. ***Equilibrium structure.*** *The structure of the unique mixed strategy equilibrium is as follow: only firm L has positive probability to choose both projects, and firm H only choose the high value project.*

In the decentralized competition, the efficient allocation is not a Nash Equilibrium[2]. In one time decentralized competition, if firm H anticipates firm L will not select high value project, the best response of firm H is to invest 0, but then firm L has incentive to invest a small amount ϵ to win the high value project in this case. Nor is it a Nash equilibrium if firm H chooses the low value project and firm L gets the high value project for a similar reason. The other possible allocation is one in which both firms choose both projects with positive possibility. However, the ex-ante expected profit from competing for the low value project is strictly lower than that from the competition of high value project for firm H. On the other hand, the low value project acts as a outside option for low ability firm. Therefore the larger value of π_L lessens the investment x_L and also the probability of choosing the high value project. Anticipating the expected revenue for firm L is π_L, the firm H is better off to either bid 0 at high value project or compete with firm L instead of participating in the competition in the low value project rather than choose the low value project[3].

In equilibrium, firm H invests "0" with a strictly positive probability and has the remaining probability to invest positively to compete for the high value project; the

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[2]The efficient allocation in this study represents the case when firm H chooses the high value project with probability 1 and firm L chooses the low value project with probability 1. The positive sorting increases the principal's (social) benefit by both of the drugs can be developed and the establish firm can produce more efficiently of the more valuable medical.

[3]There is no solution satisfy both $h > 0$ and $a > 1$ if we allow firm H to invest in the low value project. Also see proof in appendix.

secondary firm is indifferent between investing in high value project and choosing the low value project with no investment. The equilibrium exhibits upward competition i.e., only equilibrium strategy of low ability firm has positive probability to select both of the projects, and the high ability firm has no incentive to select to the low value project. Also, the equilibrium is unique when there are two firms participate this competition[4].

In the competition of high revenue drug, the firms' expected utilities are depending on the other firms' choice and we form it as:

$$u_H = a\pi_H[hF_L(x_L) + (1 - h)] - x_H \tag{1.1}$$

$$u_L = \pi_H F_H(x_H) - x_L \tag{1.2}$$

u_i represents the profit of firm i, and the cost for the high value project is x_i for firm i, $i \in [H, L]$. Firm L has probability h to choose the high value project and probability $1 - h$ to choose the low value project. The investment that firm H exerts in the high value project follows the distribution $F_H(x)$, and the investment exerted by firm L in the high value project follows $F_L(x)$.

The unique equilibrium must satisfy both Lemma 1.4.1 and the functions (1.1) and (1.2).

Proposition 1. *The unique mixed strategy equilibrium satisfy both Lemma 1.4.1 and the utility functions (1.1) and (1.2). The R&D strategies have the following properties:*

---

[4]When we extend the model to N firms, the upward competition equilibrium is still consistent by holding certain exogenous constraints.
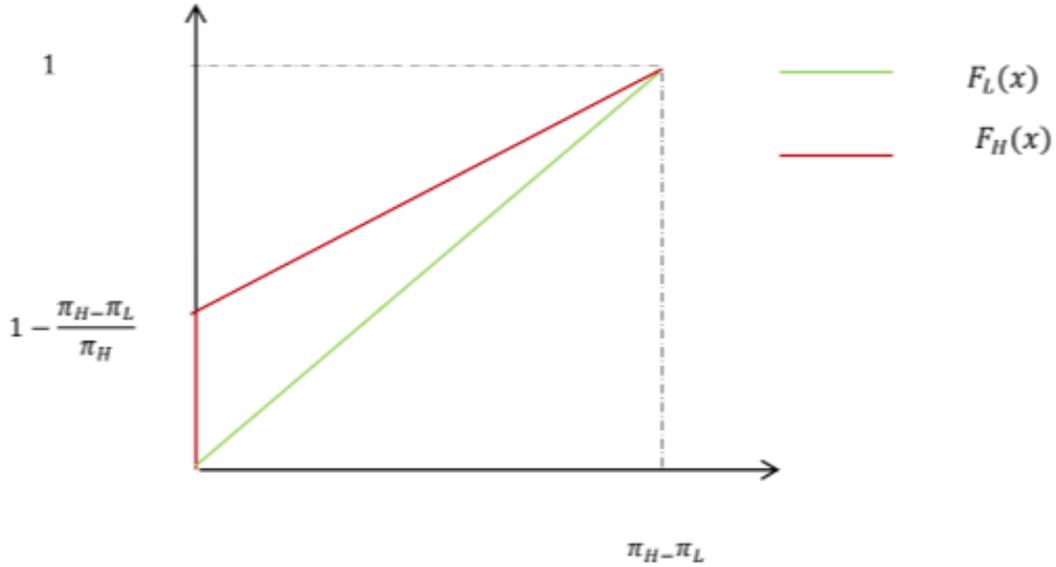
Figure 1.1: $F_L(x_L)$ and $F_H(x_H)$ in high value project competition

- Firm H's investment in high value project follows CDF $F_H(x_H) = 1 - \frac{\pi_H - \pi_L}{\pi_H} + \frac{x_H}{\pi_H}$ where $x_H \in [0, \pi_H - \pi_L]$. The expected profit for firm H is $u_H = \pi_H(a-1) + \pi_L$.

- Firm L has probability $h = \frac{\pi_H - \pi_L}{a\pi_H}$ to select project H and randomizes investment with $F_L(x_L) = \frac{x_L}{\pi_H - \pi_L}$ where $x_L \in [0, \pi_H - \pi_L]$. Firm L has the remaining probability $\frac{(a-1)\pi_H + \pi_L}{a\pi_H}$ to select low value project. The expected profit for firm L is $u_L = \pi_L$

Proof See appendix.

When both firm H and L enter the competition of high value project, $F_L(x_L)$ first-order stochastically dominance $F_H(x_H)$. It addresses the fact that firm L would exert more investment to accelerate the high value drug development if both firms enter the high value project competition. The strategic force leads to this result is that the established firm can anticipate the secondary firm has $h = \frac{\pi_H - \pi_L}{a\pi_H}$ probability
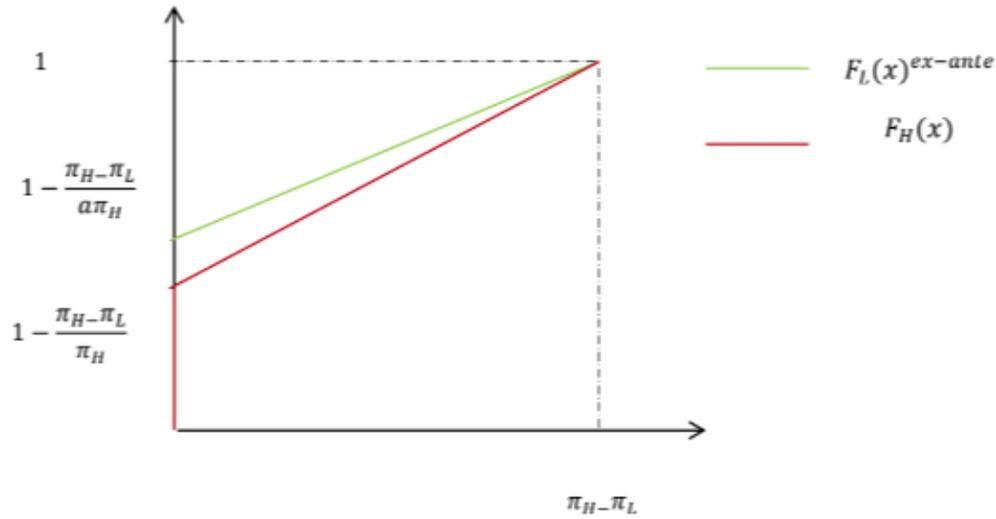
Figure 1.2: Ex-ante CDFs in high profit project competition

to select the low value project. Therefore, the established firm copes with $1 - \frac{\pi_H - \pi_L}{\pi_H}$ probability to invest 0 in the competition.

We also analyze the ex-ante investment functions in the high value project competition and graph the CDFs in figure 1.2. In the comparison of the probability that high ability firm invest 0 ,i.e.,$1 - \frac{\pi_H - \pi_L}{\pi_H}$ and the probability that low ability firm select low value project i.e., $1 - \frac{\pi_H - \pi_L}{a\pi_H}$, we find that the low ability firm has less probability to engage in the high value project competition.

Remark 1 *Firm H's ex-ante investment function in high value project first order stochastically dominates firm L's.*

1.5 Efficiency Analysis in R&D Competition

We raise a discussion about the efficiency in the decentralized competition in the following section. The efficient outcome in the R&D competition describes the situa-

tion when firm H develop the high profit project and firm L develops the low profit project. Under the efficient outcome, each firm work on a separate project with the high value project being allocated to the established firm. However, based on the above analysis, there is a significant probability h that the efficient outcome cannot become the equilibrium outcome in a decentralized competition. More specially, the efficiency loss comes from:

1. The high value project might be won by low ability firm.

2. The low value project might be neglected.

In order to address the significant efficiency loss in decentralized competition, we compare the efficiency of and decentralized competition and random centralized sorting, i.e., the projects can be allocated to firms by industry coordinator. In the most crude case, if the coordinator doesn't have any information on firms' ability and sort the projects randomly, the efficient outcome could occur with probability $\frac{1}{2}$. If the coordinator receives any information of firms' ability, the total expected profit in the industry will be lifted with a higher probability of the positive sorting.

Proposition 2. *Compared to centralized allocation, the decentralized simultaneous competition equilibrium is associated with significant efficiency loss.*

The efficiency generated in decentralized simultaneous competition is even less than the case when the centralized allocator randomly assign the high value project and low value project to firms. More specifically, the sum of expected revenues for both firms under random centralized allocation, denoted U^{random}, is larger than which under decentralized allocation equilibrium, denoted $U^{decentralized}$.

$$U^{random} > U^{decentralized} \tag{1.3}$$

Proof See appendix.

The decentralized R&D competition equilibrium reduces the efficiency loss by ruling out the possibility that high ability firm executes the low value project development, but it dampens the efficiency by inducing the possibility of leaving the low profit drug undeveloped. Our result shows that the efficiency loss from mismatch is greater than that from leaving one project undeveloped.

1.6 Random First Mover in Sequential Registration

The above analysis is based on the case which firms choose the project simultaneously. When decentralized simultaneous competition is infeasible, both of the firms are motivated to register ahead of their rival. Thus, the random first mover could be a important mechanism in practice.

In the following analysis, we analysis the sequential model when the players are randomly becomes the first mover, and we also discuss more possible sequential games in appendix. In the random first mover analysis, each firm has probability $\frac{1}{2}$ to be able to register the project ahead of its rival.

The timing in sequential claims model:

1. *Random stage:* Nature randomly chooses the first firm to register the project. The probability that each firm becomes the first mover is $\frac{1}{2}$.

2. *Decision stage 1:* The firm H or L chosen to firstly register the project claims the project choice.

3. *Decision stage 2:* After the acknowledgement of rival's project choice, firm L or H registers the project choice.

4. *Development stage:* Each firm works on its R&D strategy that it has committed to in the decision stage, investment is private information to each firm.

5. *Final stage:* Investment revealed. Whichever firm invests more in each project wins and receives the revenue. The firm losses gains zero.

Proposition 3. *In random first mover model, the equilibrium outcome is depending on the firms' ability and project value, where:*

- *When $(a-1)\pi_H < a\pi_L$ and low(high) ability firm chosen to be the first firm to register the project, low(high) ability firm chooses the high value project; high(low) ability firm chooses the low value project.*

- *When $(a-1)\pi_H > a\pi_L$, high ability firm always chooses the high value project and low ability firm always chooses the low value project.*

- *In all cases, neither of firms would invest in approval stage.*

Proof See appendix.

We can derive the equilibrium based on backward induction. When low ability firm selected to be the first mover and choose the high value project, high ability firm is deciding between competing with low ability firm for the high value project and choosing the low value project with no cost on investment. In sequential competition,

the project choice has no uncertainty in development stage and we can apply Baye et al. (1996) to calculate the payoffs. In the sub-game where both firm compete in high value project, the low ability firm would be anticipated to gain zero expected payoff and the high ability firm's expected payoff is $(a - 1)\pi_H$. Otherwise, the high ability firm can choose low value project to avoid the competition with payoff: $a\pi_L$. Thus, when $(a-1)\pi_H < a\pi_L$ holds, high ability firm values the high value project less than choosing the low value project with no investment as a second mover. The low ability firm would choose high value project in equilibrium by anticipating that high ability firm would choose a different project when low ability firm is the first mover. On the other hand, when high ability firm becomes the first mover, it always chooses the high value project in equilibrium and the sorting is positive assortative. However, in the case when $(a - 1)\pi_H > a\pi_L$, high ability firm has pure strategy to choose the high value project in equilibrium because the high value project is valuable enough and always worth to compete for. Back to the low ability firm's registration stage, instead of competing in high value project with zero payoff the low ability firm is better off to choose the low value project with payoff π_L. The comparison between $(a - 1)\pi_H$ and $a\pi_L$ represents the difference in inherent project value and it critically affects the high ability firm's choice.

Based on the above analysis, we can conclude that in any case of random first mover sequential competition, two firms are always separated on project choice. Thence, by knowing the other company is always choose the different project as itself, neither of firms would invest in approval stage to facilitate the process in equilibrium.

We already illustrate that the centralized random assignment generates more the efficiency than the decentralized simultaneous sorting equilibrium in Proposition 2. In the sequential registration competition, the equilibrium generates at least as same

efficiency as the random centralized assignment. More specifically, when $(a-1)\pi_H < a\pi_L$, the equilibrium sorting is positive sorting and no investment wastes on the approval stage, the total payoff is $a\pi_H + \pi_L$ which is higher than random centralized assignment; when $(a-1)\pi_H < a\pi_L$, equilibrium sorting is either positive assortative sorting or negative assortative sorting and each outcome comes with probability $\frac{1}{2}$ and the expected total payoff is $\frac{1}{2}(a+1)(\pi_H + \pi_L)$.

Overall, the sequential announcement reduces the competition within the firms and guarantees the development for both of the drugs. However, depending on the inherent value of the projects, both negative assortative sorting and positive assortative sorting can become equilibrium outcome. We also consider the case when the regulation requires only one of the firms to claim the project. The result is briefly listed in discussion session.

1.7 Acceleration and the Externality: The Value of Investment

In this section we discuss the value of the R&D competition. The inherent feature of the pharmaceutical drugs values the development speed as a critical factor. We derive implications of our result for the regulation of the pharmaceutical R&D competition. In fact, the investment on the R&D progress not only acts in the R&D competition, but it also generates positive externality on social welfare. Intuitively, the faster the drugs get developed, the more people can benefit from the medical improvement. By taking the spillover effect of medical breakthrough into account, we construct a simple social welfare function to illustrate the total benefit generated from the R&D

competition and we further discuss the mechanism selection in pharmaceutical R&D competition.

We construct a linear social welfare function that consists of the benefit of the medical innovation and the medical firms' revenue. The total revenue of the two firms firms $u_L + u_H$ is a component of social welfare. On the other hand, a shorter waiting time for new drugs is valuable to the society. The larger size of investment speeds up drug development and the increased total investment due to competition may be welfare-improving. Not only the winning company's investment generates the positive externality on social welfare, the losing firm's investment also stimulates the improvement of biological technology and medical education. Thence, we capture the total investment from two firms $x_H + x_L$ to represent the positive externality associated with pharmaceutical R&D competition.

Assumption 2. *Social welfare is given by:*

$$W(x_L, x_H) = E[u_L + u_H] + E[c(x_L + x_H)] \tag{1.4}$$

The welfare function has significant normative implications on market regulation. Linearity assumption in investment facilitates a clean analysis on qualitatively important aspect that public health regulator may have preferences over the total investment. When the society is relatively patient to wait for the new drug and the investment in R&D can be only seen as "burning money", the competition in one drug can cause a negative effect in utilization of resource. In this circumstance, c is negative and the more investment put in the competition would only lessen the social welfare. Under this scenario, centralized sorting and random first mover mechanisms

are preferable. This is because under these mechanisms, both projects are finished and total investment is zero.

In the following cases, we limit our analysis to a comparison of random first mover sequential model and decentralized simultaneous project choice model. In addition, we only consider the efficient sorting in the random first mover model which is the case that the high ability firm always choose the high value project and the low ability firm always choose the low value project.

The following proposition specifies conditions under which either mechanism does better in maximizing social welfare.

> **Proposition 4.** *When $c > \frac{2a\pi_H}{(\pi_H - \pi_L)(1+a)}$, the simultaneous project choice competition generates more social welfare than the random first mover competition (positive assortative sorting case). When $c < \frac{2a\pi_H}{(\pi_H - \pi_L)(1+a)}$, the random first mover competition generates more social welfare than the simultaneous project choice competition.*
>
> *Proof See appendix.*

The welfare function can be decomposed into two parts, the firm's revenue $E[u_L + u_H]$ and externality from investment $E[c(x_L + x_H)]$. The total expected revenues in confidential competition is $(a-1)\pi_H + 2\pi_L$ and is $a + \pi_H + \pi_L$ in random first mover competition. It is clear that the confidential competition is associated with efficiency loss because low ability firm will choose the high value project with probability $\frac{\pi_H - \pi_L}{a\pi_H}$. In the meantime, random first mover competition can positively assort the projects and firms and it apparently is better mechanism to improve firms revenues. However, the random first mover competition induces zero investment while confidential competition induces $E[c(x_L + x_H)]$ investment to facilitate the R&D development.

The investment from confidential competition can be represented as $c(1+\frac{1}{a}(\pi_H+\pi_L))$ explicitly. Therefore the trade off arises regarding the benefit of social welfare. When $c > \frac{2a\pi_H}{(\pi_H-\pi_L)(1+a)}$, the acceleration of R&D is more valuable to social welfare than the firm's revenue. In contrast, when $c < \frac{2a\pi_H}{(\pi_H-\pi_L)(1+a)}$, the efficiency loss from confidential competition is larger than the social welfare gain from inviting the investment to facilitate the R&D. When c is small, the investment has small contribution to social welfare but revenues drops by a large amount due to the possibility of both firms entering the high value project competition.

The value of equation $\frac{2a\pi_H}{(\pi_H-\pi_L)(1+a)}$ largely depends on the project value and the difference in firms' ability. The value of $\frac{2a\pi_H}{(\pi_H-\pi_L)(1+a)}$ is increasing in π_L and decreasing in π_H[5]. The intuition of the comparative statics is as follows: the better the low value project is, the more costly it is to leave the low value project unselected. On the other hand, when the high value project is more valuable, firms would invest more in the high value project and it is easier for confidential competition to become better mechanism. Also $\frac{2a\pi_H}{(\pi_H-\pi_L)(1+a)}$ is increasing in a. Because we only compare the positive sorting with the confidential competition, efficiency loss of mismatch in high value project is lifted because the high ability firm is more capable in manufacturing but the low ability firm sometimes can win the high value project.

As the prevailing fact that the enactments of legislation in many countries advocate the innovation on the rare disease medication, the rare disease drugs can be less profitable by targeting at the small group of patient. When there is no transparency in the competition, the research resources are concentrated on the drugs which have more potential users, whereas the drug for rare disease can be ignored with a significant probability. The sequential claim within two agents can eliminate

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[5]See Proof in appendix.

the potential social efficiency loss by assortatively allocating the projects to the R&D departments. Thus, if the innovation of both drugs is the social optimum, the sequential claims should be suggested to the industry. This can be implemented by a regulator by requiring the claims of projects before the investment stage when the development of both drugs is the priority of public health regulation. Conversely, if pestilence is prevalent, the social welfare could highly depend on the promptness of the approval of target medication. Since the random first mover game would drag the investment to zero, when c becomes large enough until $c > \frac{2a\pi_H}{(\pi_H - \pi_L)(1+a)}$ is satisfied, the efficiency loss from leaving low value project unselected becomes smaller than welfare gain with acceleration in high value project. In this case confidential competition is preferable since it generates more investments that speeds up innovation. A regulator would do well by not requiring disclosures of ongoing projects.

1.8 Discussion and Future Prospect

1.8.1 The Semi-Transparent Competition in 2×2 Model

In this session we consider the case that lies between the confidential and the sequential claim case where only one firm claims the project in the decision stage. For an example of a realistic case: according to SEC regulation[6], the publicly traded firm is obligated to reveal their R&D expenditure. However the regulation is not applicable to private firm. Thus, the project selection can be semi-transparent in competition.

When the established firm claim the project and the secondary firm doesn't, the equilibrium is identical to that in the confidential case. The established firm strictly

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[6] Regulation D (SEC) of United States Securities and Exchange Commission.

prefers the high value project and its investment is drawn from a continuous random variable supported on $[0, \pi_H - \pi_L]$ and a mass point on 0; the secondary firm is indifferent between competing high value project with the randomized investment on $[0, \pi_H - \pi_L]$ and choosing low value project with 0 investment. However, if the secondary firm is obligated to reveal their R&D project choice, the secondary firm will select the low value project with 0 investment, and the established firm chooses the pure strategy to select the high value project with 0 investment in equilibrium. The strategical force that reduces the secondary firm's willingness to compete for the high value project is that if the secondary firm picks the high value project, the established firm has incentive to increase the investment until it wins the project for sure, i.e, ensuring u_L is lower than π_L by investing more in the contest. By adding an extra stage of registration, the efficient outcome becomes the equilibrium outcome in most of the case.

1.8.2   Discussion - Cost Function

In order to construct an appropriate model to analyze pharmaceutical R&D competition, we have focused on the case in which the abilities of firms affect the value of the project instead of cost. If the cost function is linear and different between firms, the equilibrium structure should be same. The higher valuation in project for high ability firm is equivalent to less marginal cost in investing. The reasoning and intuition in the paper therefore also apply for case where cost functions are different between firms. When the cost function quadratic and different between firms, it is difficult to derive a simple closed form solution for heterogeneous projects competition. However, based on the result in Xiao (2016) the ex-ante investment function from high ability firm in

high value project is stochastic dominant the ex-ante investment function from low ability firm in high value project. The same intuition is captured by Remark 1.

1.8.3 Future Prospect - Investment on Information Provision

In our model, investment is a essential factor in R&D contest and the more the firm invest and the faster the firm would able to file the patent and secure profit. This assumption is can be useful when the licensing authority is not applicable in pharmaceutical industry. Suppose pharmaceutical firms required to propose a preferable project to authority and firm can invest to convince authority to approve the project to itself. The authority updates the belief on the quality of project by the realization of the investment, i.e., $c(x_H + x_L)$ in our model. In this way, the investment not valuable to social welfare but also critical to R&D's success. Based on the observation, the authority may sign approval for up to one firm in each project. Thus the investment is informative about the inherent quality of the project. In fact, it is common in pharmaceutical industry that the project competition based on the quality of trail, especially projects that government funded. The information provision investment can be constructed in a Bayesian persuasion model with different project and it would be a novel literature contribution in the field of R&D competition. There are handful Bayesian persuasion literature study the contest between different ability agents, such as Boleslavsky and Cotton (2015) and Boleslavsky and Cotton (2018). Based on the previous literature, our future work can focus on the case that heterogeneous firms simultaneously choose between projects with different potential revenue, along with strategic informative investments.

1.9 Conclusion

In this paper, we study the R&D competition in the pharmaceutical industry. We show that when project choice is confidential before the competition is finished, the uncertainty of the project choice creates the investment competition in the high value project, and the low value project can be left unfinished. In contrast, if the firms claim the project choice publicly ahead of the investment stage, the assortative matching is the equilibrium outcome, but the investment is "zero" from both of the firms.

This result implies that the blame of inefficient R&D activity in pharmaceutical industry may come from the following aspects. First, firms are motivated to crowd in the most profitable medical project. Without a authorized central allocator, firms can't clearly anticipate rival's choice and the high value project is worthy for even low ability firm to try its luck. Second, when a secondary firm participate the high value project competition, it would put even more investment in the contest and if it won, their capacity is small and the R&D activity generates less revenue than established firm does.

However, depending on the explicit form of the social welfare function, the decentralized simultaneous project choice competition and the random first mover competition can both become the social optimal outcome. If reduction of the development period is the first priority of the public health system, the regulation should restrict the information disclosure, and if the development of low revenue drugs is a crucial demand in the society, the regulation should encourage transparency. The approval investment induces the acceleration in the clinical trails, and when the time is more valuable for the public health system, the decentralized simultaneous project choice competition can benefit more the society than the complete information competition.

Chapter 2

The Role of Product Choice in Risk-Sharing Management of Stable Supply Chains

2.1 Motivation

The coordination supply chain management has drawn a plenitude of research interests (Sodhi et al., 2012). The supply chains commonly face various types of risk. Particularly, the uncertain demand can crucially affect the performance of supply chains (Tang, 2006). On the other hand, the business environment is becoming more competitive and the appearance of global supply chains broaden the supply chain matching market (Motwani et al., 1998). Inspired by the expanding supply chain matching activities globally, we study a matching model between suppliers and retailers to demonstrate how stable formation of partnership varies to maintain the superior profit and mitigate the supply chain risk.

The risk averse behavior is commonly reflected in supply chain's business activity. Retailers who sell short life cycle products, such as fashion apparel, beauty products, electronics, can sign buyback contracts to ease the demand uncertainty and return the

unsold merchandise to retailers (Wang and Webster, 2007), (Donohue, 2000), (Kratz, 2005), (Anupindi and Bassok, 1999). In fact, 30% to 35% of new books are returned to the publisher (Terwiesch and Cachon, 2012). On the other hand, risk averse retailers commonly use markdown money policy which includes quantity discount contract, consumer rebates contract, quantity flexibility contract, and backup agreements to push the order quantities up (Shen et al., 2013). Motivated by risk aversion on both sides supply chain, we study a coordination supply chain model with risk averse suppliers and retailers.

Supply chain contracting and risk sharing has drawn significant attention by previous researches. However, most of the existing literature study the contract formation only between certain fixed supply chains instead of considering the efficient matching among heterogeneous risk averse firms. In the era of technology revolution and globalization, the matching behavior in supply chain formation shall not be neglected in supply chain risk management. Also, the previous supply chain management studies demonstrate the contract and the production planning separately. However, the risk sharing between the supplier and retailer usually heavily depends on both demand risk for products and the risk sharing contract within the supply chain. Therefore, it is critical to characterize a theoretical risk sharing model, in which contract and production choice are jointly considered. In this paper, we analyze an equilibrium model that endogenizes supply chain formation as well as the product choice and risk sharing contract. Doing this allows us to derive insights on how demand uncertainty influences formation of supply chains.

Our analysis gives the following insights: risk sharing contracts as well as product choice are means that risk averse suppliers and retailers use to mitigate their risk exposure. Risk sharing contracts work best when one side is relatively risk averse

while the other is less so. Therefore , firms do best to match with differentially risk averse supply chain partner when the main concern is risk sharing. On the other hand, product choice of a supply chain requires a costly compromise between differentially risk averse partner. The cost of this compromise is minimized when the partners are similar in their risk attitude. We show how the fundamentals of the economic environment determines how this trade-off is balanced.

To understand the matching equilibrium of the risk-averse suppliers and retailers, we incorporate the optimal production choice into the supply chain risk-sharing contract. Risk-averse companies are matched into coordination supply chains on the basis of their risk preferences, product choice, and a risk-profit sharing contract which satisfies the interest of both supplier and retailer. Even though, in reality, the supply chain structure may vary in many forms, we focus on the simple bilateral balanced supply chains matching, i.e., each supply chain contains one supplier and one retailer. Our result provides novel perspective and motivation for future studies in supply chain matching and contracting research.

We develop a matching model that features the endogenous product choice with balanced suppliers and retailers to answers the following questions:

- What is the sorting pattern in equilibrium?

- What kind of products are optimal for the matched supply chains?

- What is the risk-profit sharing contract for the matched supply chains?

- How the general demand risk in the market affects equilibrium sorting?

Therefore, the intention of this study is to introduce a stable matching equilibrium associated with three dimensions of a strategy set: optimal coordinating contracts,

product choice and matching patterns [7].

Due to the long leading time in most of the manufacturing, first we introduce why the production choice is crucial for the supply chain risk management. For instance, if the supply chain has two available product profiles: 1, a less risky product which has stable demand but many substitutes; 2, a new product which has no competitor but unclear consumer demand. The optimal production choice for the supply chain is ambiguous in between the two portfolios. The product choice is contiguous on the risk preferences of supply chains. Thus, by providing the diverse product proposals to supply chains, the optimal product choice for each supply chain is determined by the uncertainty, expected profit, and the risk preference of entire supply chain.

Second, to guarantee the stability of the matched supply chains, the matching of partner crucially depends on the contract that both the supplier and the retailer must agree on. To see how contract may distribute risk differently between a supplier and a retailer we can consider the following examples: The classical buyback contract permits the supplier to acquire larger share of the profit but also to bear most of the risk i.e., the retailer will order the proper amount of existing product with approximately a 30% discount off of retail prices. If products are unpopular in the market, the extra stock will be returned to the supplier with only a small amount of restocking fee. In this case, the risk-averse supplier has a bigger chance to produce an insufficient amount, which means that the product might become out of stock more quickly and the supply chain may lose potential profit. To refrain from low production from a high risk-averse supplier, many retail companies sign the "exclusive products" contracts

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[7]As a result of the utilization of the Internet, the risk preference of companies becomes more transparent and the search cost to find suppliers and retailers has been heavily reduced. Taking these facts into account, this paper is focuses on the frictionless matching in the risk-averse coordination supply chain.

with manufacturers to produce goods that can only be sold in their stores and waive the right to return the stock, such as "Great Value" in Walmart or "Kirkland" in Costco. Under this type of contract, the supplier takes less of the share of profit and retailers are prohibited from returning excessive products. If the product get unpopular in the market, the excessive stock is burden on the retailers' side. Consequently, the risk and profit are both transferable within the coordination supply chains.

We illustrate the concept of assortative matching equilibrium and characterize the equilibrium matching in the balanced market. The critical forces that drive the equilibrium outcome can be described as: a less risk-averse firm can either match with an opposing type of partner to be allocated a larger proportion of uncertain profit and riskiness, or match with a firm with similar preference to achieve a more challenging project which is associated with higher expected profit and higher demand uncertainty. Similarly, a high risk-averse firm also maximize their expected return in two channels: matching with a low risk-averse agent but to take less profit and less risk or teaming up with an identical agent to choose a safer production plan with an equally split contract.

Our model also describes how general demand risk in the market affects the equilibrium sorting. The partner formation is likely to change rapidly to cope with the demand variation (Christopher and Towill, 2000) and (Miles and Snow, 1984). In fact, the general consumers in the market can be expected to vary from conservative to open-minded, therefore, the demand uncertainty for each production choice is also different accordingly. Intuitively, the more conservative the market is, the less likely that the innovative product can get success and vice versa. Therefore, when market becomes more conservative, matching with diverse partner to trade the risk and profit is more beneficial than matching with a homogeneous firm. Conversely, when

customers more easily accept the innovative products, matching with a homogeneous partner and choosing a desired risk- revenue bundle is more attractive. By including the the production choice within the bilateral supply chain, we found that the general demand condition in the market is a crucial factor for the matching between suppliers and retailers.

The rest of this paper is organized as follows: we review the literature in section 2. Section 3 describes our model. We characterize feasible payoffs of arbitrary supply chains in section 4 and characterize the stable matching in section 5. We discuss the result in section 6 and conclude our study in section 7.

2.2   Literature Review

This study closely follows two lines of literature: risk aversion in the coordination supply chain and the bilateral assortative matching model.

The coordination supply chain study has significant researches analyzing the risk sharing contract with risk-averse firms. Agrawal and Seshadri (2000) study a supply chain risk sharing contract question with multiple heterogeneous risk-averse retailers and single risk neutral distributor. By offering the multiple quantity-discount contract, the total efficiency for all agents can be improved. The risk-averse agents can choose an ideal quantity with positively correlated discount. Gan et al. (2004) illustrate the risk sharing contracts among many risk aversion retailers and one risk neutral supplier. They characterize the Pareto-optimal solutions in the news-vendor setting with single neutral supplier and multiple risk-averse retailers. Chen et al. (2014) study the stable matching of one risk-averse retailer and a group of competing suppliers. They allow the model to include more than one supplier and their model

indicates that the least risk-averse agents bear all risk, while the contract determines the inventory for maximum expected payoff. They also emphasize the importance of stability, which guarantees the coordination supply chain is maximizing both agents' individual and entire supply chain's profits simultaneously without deviation. Giannoccaro and Pontrandolfo (2004) also study a decentralized coordination model within the supply chains. They suggest the coordination contracts can improve the market efficiency in a three-tier model. He and Zhao (2012) propose a model to describe the efficiency-improving coordination contract to the supply chains which face the risk from both demand and supply side. Another related paper is from Chauhan and Proth (2005) studying a centralized revenue-risk sharing contract model.

In contrast to the above studies, our model describes the stable matching equilibrium that analyzes the matching of N retailers and N suppliers with heterogeneous degree of risk aversion. Moreover, we focus on how the supply chains balance the risk and profit by choosing different production plans instead of the inventory. Depending on the general demand risk in the market, either low risk-averse firm coordinates with high risk-averse firm or each firm coordinates with the other firm possessing similar risk-aversion type. Therefore, our research is novel to the supply chain management literature.

There is a considerable body of literature devoted to the study of assortative matching. If transfers are not allowed within the matched pair, the unique assortative matching equilibrium is negative assortative (Legros and Newman (2007),Chiappori and Reny (2016)). Chiappori and Reny (2016) intuitively introduce the model under the background of marriage matching market where the more risk-averse male is stably matched with the less risk-averse female to share the risk. However, the negative assortative equilibrium is inconsistent with the result of Di Cagno et al. (2012). Their

experiment finds the evidence that there are significant number of players willing to team with a partner with similar risk aversion types in order to play a lottery with a higher prize.

The recent paper Li et al. (2013) analyzes the role of endogenous effort in the assortative matching equilibrium. They find that when the effort can be exerted to increase the expectation and reduce the variance in the couple's income bundle, the positive assortative matching can be the stable matching outcome. The main issue of last paper is similar to ours. We study the agents' direct choice of the product instead of effort. Finally, Wang (2014) studies an assortative matching model on informal insurance markets. Her work also follows assortative matching literature and discusses the possibility of positive assortative matching in risk sharing market.

2.3 The Model

Consider a market with balanced groups of N suppliers and N retailers. Suppliers and retailers need to team up as bilateral supply chains to produce and sell the products during the sale season. Both suppliers and retailers are risk-averse. Suppliers and retailers possess CARA utility function over money outcomes, w:

$$u(w, r) = -e^{-rw}, \tag{2.1}$$

where $r \in [r, \bar{r}]$ is an Arrow-Pratt degree of risk aversion coefficient.

Agents are heterogeneous with respect to their risk aversion coefficients. Agent i's risk aversion coefficient is denoted by r_i and is publicly known.

We frame P_{ij} to represent the unit profit function of products ij:

Assumption 3.

$$P_{ij} = \alpha_{ij} - \frac{\alpha_{ij}}{k}Q_{ij} + \epsilon_{ij}$$

The unit profit is depicted by the endogenous production quantity chosen Q_{ij}; the total market size is α_{ij}; and the ratio of market size and price sensitivity as k. The consumers' sensitivity of price captured by $\frac{\alpha}{k}$. The price-demand uncertainty is represented by $\epsilon_{ij} \sim N(0, \sigma_{ij}^2)$.

The coordination supply chain jointly determines the quantity by maximizing the expectation of profit function π_{ij}:

$$E[\pi_{ij}] = E[P_{ij}Q_{ij}] \tag{2.2}$$

The optimal quantity for the supply chain (i, j) is $\frac{k}{2}$, and the profit function for production (i, j) π_{ij} follows $N(\frac{\alpha_{ij}k}{4}, \frac{k^2}{4}\sigma_{ij}^2)$.

2.3.1 Endogenous Product Choice

We make the following assumption about the frontier of the feasible production plans:

Assumption 4. *A production plan (α, σ^2) is feasible if and only if:*

$$\alpha \leqslant (\sigma^2)^\gamma \tag{2.3}$$

The above condition depicts the general demand in the market. The parameter γ denotes the growth rate of α and it describes how costly it is to increase expected profit by increasing the demand variance. When γ is relatively small, the customers

are more conservative due to the low risk associated with increasing the expected revenue in the low revenue product. Conversely, when γ is relatively big, the α function is more smooth which represents that the customers are easier to accept a novelty because the high risk production plan comes with less demand uncertainty. In this paper we mainly focus on the case when $0 < \gamma < 1$. By imposing an increasing function of expectation of π with $0 < \gamma < 1$, we capture the idea that higher expected profit is associated with higher uncertainty in demand. The concave growth rate of $\alpha(\sigma^2)$ implies decreasing marginal expected returns to riskiness in the market. We also discuss the assortative matching equilibrium under convex $\alpha(\sigma^2)$ in the discussion section.

Timing. When we consider the matching between supplier and retailers, the game takes place as follows:

1. Suppliers and retailers are matched into supply chains.

2. The supply chain that is formed with supplier i and retailer j commits to the production plan $(\alpha_{ij}, \sigma_{ij}^2)$ and a revenue sharing contract.

3. During the sale season, the supplier produces according to the production plan they agreed on and the retailer sells it according to the contract.

4. At the end of sale season, the shocks are realized and supplier and retailer split the revenue according to their contract.

2.3.2 Equilibrium Concept

A matching is a one-to-one function $f : N \to N$ mapping each supplier to a retailer.

Matching structure. A matching function f is positive (negative) assortative [8] if

$$r_i \geqslant r_j \tag{2.4}$$

implies

$$r_{f(i)} \geqslant (\leqslant) r_{f(j)} \tag{2.5}$$

Contracts. Fix a supply chain (i, j) and a production plan $\pi_{ij} \sim N(\frac{\alpha_{ij}k}{4}, \frac{k^2}{4}\sigma_{ij}^2)$. A contract $[m_i(\cdot), m_j(\cdot)] : \mathbb{R} \to \mathbb{R}^2$ is a pair of functions such that when realized profit is Π, the supplier's payoff is $m_i(\pi)$ and retailer's payoff is $m_j(\pi)$.

Let (u_i, u_j) stand for a payoff pair for the supplier and the retailer. (u_i, u_j) is feasible if there exist a production plan (α, σ^2) and a contract $(m_i(\pi), m_j(\pi))$ with

$$m_i(\pi) + m_j(\pi) \leqslant \pi \tag{2.6}$$

for all π, and

$$u_i \leqslant E[m_i(\pi)], u_j \leqslant E[m_j(\pi)] \tag{2.7}$$

---

[8] Suppose a supply chain matching market contains 100 suppliers and 100 retailers with heterogeneous risk aversion type. From the lowest Arrow- Pratt degree of risk-averse (the least risk-averse) $i = 1$ to the highest Arrow- Pratt degree of risk-averse (the most risk-averse) $i = 100$ we mark suppliers as $\{S_1, S_2...S_{100}\}$ retailers as $\{R_1, R_2...R_{100}\}$. Negative assortative matching is the matching pattern in which agents are paired as $(S_1, R_{100}), (S_2, R_{99}), (S_3, R_{98})...(S_{100}, R_1)$, which is the most risk-averse retailer willing to match with least risk-averse supplier in the market. In contrast, if positive assortative matching is the one where the most risk-averse retailers is matched with the most risk-averse supplier, and vice versa. More specifically, under PAM the supply chain will paired as $(S_1, R_1), (S_2, R_2), (S_3, R_3)...(S_{100}, R_{100})$.

An equilibrium consists of a payoff allocation $\{(u_i, u_j)\}_{i,j} \in N$ and a matching function f such that

1. For all i, j if $f(i) = j$, then (u_i, u_j) is feasible for (i, j);

2. There exists no (i, j) and a payoff pair $(\tilde{u}_i, \tilde{u}_j)$ such that:

   (a) $(\tilde{u}_i, \tilde{u}_j)$ is feasible for (i, j), and

   (b) $\tilde{u}_i > u_i$, $\tilde{u}_j > u_j$.

2.4 Feasible Profits

In this section we characterize the set of feasible payoffs for an arbitrary supply chain (i, j). We separate this into two steps: first, we characterize the achievable payoffs, taking as given the production plan. Then based on this, we characterize Pareto frontier of possible payoffs by varying the production plan.

2.4.1 Coordination Contract

Stability requires that the contract of a supply chain generates a Pareto optimal payoff vector, given the production plan $\pi_{ij} \sim N(\frac{\alpha_{ij}k}{4}, \frac{k^2}{4}\sigma_{ij}^2)$. Therefore any equilibrium contract must solve:

$$\max_{m_{ij}}\{Eu_i[\pi_{ij} - m_{ij}(\pi_{ij})]\} + \lambda\{Eu_j[m_{ij}(\pi_{ij})]\} \tag{2.8}$$

By varying λ over $_{++}$, the solution to this problem spans the Pareto frontier of possible payoff for the matched pair.

Proposition 1 characterizes the solution of this problem for arbitrary λ

Proposition 5. *Fix $\lambda > 0$, the contract that solves (8) is linear with respect to total revenue π_{ij} and is given by:*

$$\pi_{ij} - m_{ij}(\pi_{ij}) = \frac{r_j \pi_{ij} - log\lambda}{r_i + r_j} \tag{2.9}$$

$$m_{ij}(\pi_{ij}) = \frac{r_i \pi_{ij} + log\lambda}{r_i + r_j} \tag{2.10}$$

Proof See appendix.

Let $CE_i(m_i(\pi_{ij}))$ and $CE_j(m_i(\pi_{ij}))$ represent the certainty equivalents of the "lottery" induced by contracts $m_i(\pi_{ij})$ and $m_j(\pi_{ij})$ for agent i and j respectively. We define the compound Arrow-Pratt coefficient of risk aversion for supplier j and retailer i as $R_{ij} = \frac{r_i r_j}{r_i + r_j}$.

Lemma 2.4.1. *If (m_i, m_j) solves (2.8), then*

$$CE_i(m_i(\pi_{ij})) + CE_j(m_i(\pi_{ij})) = \alpha_{ij}(\sigma_{ij}^2)\frac{k}{4} - R_{ij}\frac{k^2}{8}\sigma_{ij}^2 \tag{2.11}$$

Proof See appendix.

Let $CE_{ij}(\alpha, \sigma)$ represent the total certainty equivalent for the team with production plan $(\alpha_{ij}, \sigma_{ij}^2)$, that is,

$$CE_{ij} = \alpha_{ij}(\sigma_{ij}^2)\frac{k}{4} - R_{ij}\frac{k^2}{8}\sigma_{ij}^2 \tag{2.12}$$

Note that the total certainty equivalent is independent of λ, which only affects how supply chain splits the revenue and not the business strategy. Under our setting, λ

represents the irrelevant transfer between the supplier and retailer during cooperation. Moreover, the total certainty equivalent is frictionlessly transferable within the supply chain. This transferability allows us to apply Becker (1973) approach directly when payoffs are expressed in certainty equivalent terms.

It suffices to modify the definition of **feasibility** as follow: A pair of certainty equivalents (CE_i, CE_j) is feasible if and only if

$$CE_i + CE_j \leqslant \max_{\alpha, \sigma^2} CE_{ij}(\alpha, \sigma^2) \tag{2.13}$$

subject to $\alpha \leqslant (\sigma^2)^\gamma$.

The solution to (13) is:

$$\hat{\sigma}_{ij}^2 = (\frac{k}{2\gamma} R_{ij})^{\frac{1}{\gamma-1}} \tag{2.14}$$

and

$$\hat{\alpha}_{ij} = (\frac{k}{2\gamma} R_{ij})^{\frac{\gamma}{\gamma-1}} \tag{2.15}$$

The result shows that σ_{ij}^2 is decreasing in R_{ij}. It is intuitive that the more risk-averse the supply chain is, the less risky production plan is optimal. From here, let $\overline{CE}_{ij}$ represents the total certainty equivalent value of (13).

Lemma 2.4.2. *(Becker (1973)) The equilibrium matchings are negative (positive) assortative if the team's certainty equivalent $\overline{CE}_{ij}$ is submodular (supermodular) in (r_i, r_j) $\frac{\partial^2 CE}{\partial r_i \partial r_j} \leqslant 0 (\geqslant 0)$.*

The above lemma suggests the modularity of transferable certainty equivalent determines the stable matching for supply chains.

2.5 Benchmark Model: Exogenous Product Choice

Before presenting the intuition of the matching model with the endogenous product choice, we solve the benchmark model when the product choice is exogenously given. It is a restricted setting in which supply chains can only work for an identical project. The equilibrium outcome is NAM and it is consistent with Chiappori and Reny (2016).

Similar to the analysis from assumption 1, the profit function of supply chain i, j follows a normal distribution $N(\frac{\alpha k}{4}, \frac{k^2}{4}\sigma^2)$ where α and σ are exogenous and $\alpha, \sigma > 0$. Following the same equilibrium concept, the certainty equivalent for each supply chain is

$$CE_{ij} = \alpha\frac{k}{4} - R_{ij}\frac{k^2}{8}\sigma^2 \tag{2.16}$$

For each pair (i, j), the Pareto frontier pairs (CE_i, CE_j) satisfy $CE_i + CE_j = \overline{CE}_{ij}$. Therefore, the sorting properties of equilibrium matching are determined by the modularity properties of $\overline{CE}_{ij}$. The following Lemma records this.

Proposition 6. *When the product choice is exogenously given, all equlibrium matchings are negative assortative (NAM).*

Proof See appendix

We list the result in the above proposition and keep the discussion in the next section.

2.6 Coordinating Supply Chain with Endogenous Product Choice

Consider a simple example of two suppliers and two retailers in matching market, each group (suppliers and retailers) has one high risk-averse agent and one low risk-averse agent, i.e. each type includes agents such that $i \in (H, L)$ where $r_H > r_L$. Assume CE_{ij} is the total certainty equivalent of the supply chain formed with agents type i, j where $i, j \in [H, L]$. Let $\Delta CE_i = CE_{iL} - CE_{iH}$. ΔCE_i describes how much total certainty equivalent decreases by providing agent i a higher risk-averse agent as his/her new partner. Note that if this decrease is larger for $i = H$ than $i = L$, equilibrium matchings are NAM. Conversely, if it is larger for $i = L$ the equilibrium matchings are PAM.

When the production plan is exogenous, CE_{ij} is linear decreasing with respect to the risk aversion coefficient R_{ij}, therefore, the equilibrium matching is NAM and the total certainty equivalent indicates suermodularity in the agents' risk aversion coefficient. The supermodularity of R_{ij} reveals the property that $\Delta CE_H > \Delta CE_L$. That is, the high risk-averse agent has a higher cost to match with another high risk-averse agent than a low risk-averse agent does. In terms of the increase of social surplus, the total certainty equivalent increase from replacing one high risk-averse agent in the pair (H, H) with the low risk-averse agent to form a new pair (H, L) is higher than replacing one high risk-averse agent in a pair (H, L) with a low risk-averse agent to form a new pair (L, L).

By allowing the endogenous production plan, the matched pair can pick the production plan that maximizes the total certainty equivalent on the production plan constraint function $\alpha \leqslant (\sigma^2)^\gamma$ and r_i, r_j are not always substituted. Both $\Delta CE_H < (>)$

ΔCE_L are possible and the result crucially depends on γ.

To understand this, it is convenient to analyze the indifference curve of a supply chain:

$$\alpha_{ij}(\sigma_{ij}^2(R_{ij})) = R_{ij}\frac{k}{2}\sigma_{ij}^2(R_{ij}) + \frac{4}{k}CE_{ij} \tag{2.17}$$

When one of the partners is replaced with a higher risk-aversion partner, the indifference curve rotates. When the product choice sis exogenous, it has to cross the risk return boundary the same point as before. When the product choice is endogenous the new supply chooses a new production plan so that the rotated indifference is now tangent to the risk-return frontier.

We can break down the impact of replacing each agent's L type partner with an H type into two parts:

- The indifference curve implement the rotation motion to express the risk preference. This motion is well studied by exogenous case, we record it as ΔCE_i^{ex}.

- On the other hand, when the production plan is endogenous, the indifference curve also comply the movement on the frontier of $\alpha(\sigma^2)$, we note it as ΔCE_i^{en}

ΔCE_i can be decomposed into two steps as we discussed above:

$$\Delta CE_i = \frac{k}{4} \left\{ \underbrace{[(\alpha_{iL} - \frac{\sigma_{iL}^2 k}{2}R_{iL}) - (\alpha_{iL} - \frac{\sigma_{iL}^2 k}{2}R_{iH})]}_{\text{Certainty equivalent loss in exogenous production plan } \Delta CE_i^{ex}} - \underbrace{[(\alpha_{iH} - \frac{\sigma_{iH}^2 k}{2}R_{iH}) - (\alpha_{iL} - \frac{\sigma_{iL}^2 k}{2}R_{iH})]}_{\text{Certainty equivalent adjustment by endogenous production plan adjustment } \Delta CE_i^{en}} \right\} \tag{2.18}$$

By decomposing the change in the certainty equivalent as (2.18), ΔCE_i^{ex} and ΔCE_i^{en} clarify how the endogeneity of production plan may lead to super modular-

ity. In Figure 1, we subtract the endogenous production adjustment from the total certainty equivalent and demonstrate how ΔCE_i^{ex} changes if only the exogenous production plan is permitted. The total certainty equivalent decrease is always more drastic for H type agents because of $\Delta CE_H^{ex} > \Delta CE_L^{ex}$

However, the high risk-averse supply chain (H, H) will have a larger certainty equivalent adjustment when they are able to choose production plan on the frontier, reflected by $\Delta CE_H^{en} > \Delta CE_L^{en}$. Specifically, when the more risk-averse pair is able to choose the production plan, they are capable of getting a higher certainty equivalent by choosing a low risk production plan than a lower risk-averse pair doing so.

In the supply chain matching market, high risk-averse agents are balancing between matching with H or with L. By matching with H, the supply chain (H, H) would acquire a low uncertainty production plan with low profit; by matching with L, the supply chain (H, L) would acquire a medium uncertainty production plan with medium profit.

The low risk-averse agents also facing the trade off between matching with H and L. Likewise, (H, L) pair would adopt a medium profit production plan. When the matching is negative, L can take larger share of revenue by providing informal insurance to the H. In contrast, (L, L) is able to take a high profit high uncertainty production plan with an equally share payment contract.

The modularity of total certainty equivalent is determined by γ and the condition is suggested by Proposition 7:

Proposition 7. *When $\gamma > \frac{1}{2}$ all equilibrium matchings are positive assortative(PAM), and when $\gamma < \frac{1}{2}$ all equilibrium matchings are negative assortative matching (NAM).*
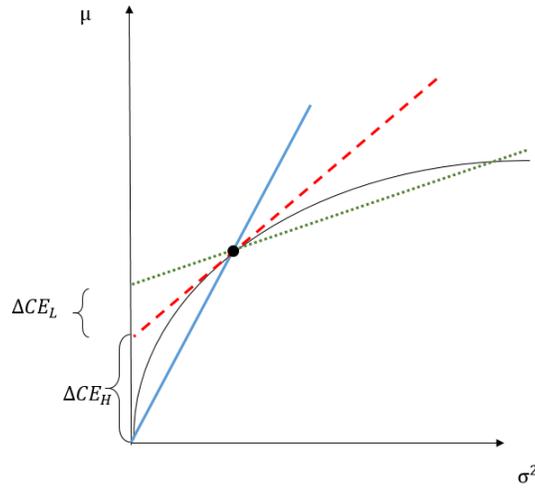
Figure 2.1: Optimal product choice in exogenous production plan
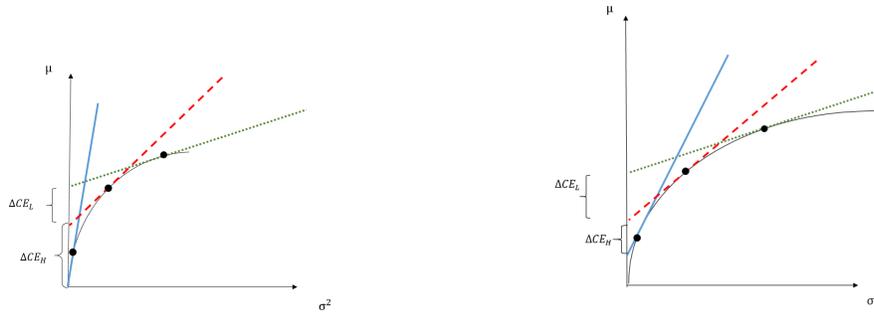
Proof See appendix.

Figure 2.2: Optimal product choice in en-
dogenous production plan when $\gamma < \frac{1}{2}$

Figure 2.3: Optimal product choice in en-
dogenous production plan when $\gamma > \frac{1}{2}$

In figure 2.1 to 2.3, we illustrate the matching equilibrium in 2×2 case. The
space in figure refers to all available of production choice and the slope of indifference
curve represents the marginal rate of substitution between risk and expected profit.
When product choice is exogenous, the only channel to increase the total certainty
equivalent is sharing risk between the supply chain. Thus, all the indifference curve

has to pass a certain product choice. As shown in figure 2.1, $\Delta CE_H^{ex} > \Delta CE_L^{ex}$ always holds and it represents that the equilibrium matching is NAM.

Figure 2.2 and 2.3 illustrates that the marginal growth rate of the expected profit with respect to risk γ. It is the crucial ingredient to determine the stable matching pattern with endogenous production plan. When the γ becomes larger, more specifically when $\gamma > \frac{1}{2}$, the market is less uncertain and product choice becomes the primary channel to optimize total certainty equivalent. Therefore, $\Delta CE_H < \Delta CE_L$ and PAM is equilibrium matching. If $< 0\gamma < \frac{1}{2}$, the market is relative uncertain, the main concern with respect to increase total certainty equivalent is still risk sharing and NAM is equilibrium matching.

We can now conclude that when the growth rate of $\alpha(\sigma^2)$ is smaller, matching with a heterogeneous type of agent generates a higher certainty equivalent for the entire industry. On the contrary, when the growth rate of $\alpha(\sigma^2)$ is larger, PAM is the equilibrium matching.

2.7   Discussion

This paper suggests an improved matching mechanism theory in the coordination supply chain analysis to predict a more accurate matching pattern. Suggested by Ackerberg and Botticini (2002), the production plan raised by endogenous matched partners can be the indicator of agents' risk taking types. By observing the endogenous production plan, assortative matching theory is able to reveal the risk sharing behavior and the degree of risk aversion between the partners. Empirically, the agents' degree of risk aversion considered to be difficult to observe and measure. Therefore, if the theory has flaws initially, the predicted agents' degree of risk aversion can be

questionable. In Example 1, we compose a comparison between random matching and assortative matching equilibrium to address the importance of accounting for the endogenous production plan.

Example

In Figure 2.4, we compare the optimal production plan suggested by this model. Recall that the cutoff of PAM and NAM in this model is $\gamma = 0.5$. If the environment changes slightly by υ, the matching pattern can be altered drastically.

According to (Babcock et al., 1993), individual's Arrow-Pratt risk aversion coefficient lays between [0,1], therefore I randomly draw the risk-averse heterogeneous retailers and suppliers in such interval[9]. Figure 2.4 exhibits how the optimal production plan would vary by slightly changing the business environment γ. When $\gamma = 0.5 + \upsilon$, PAM is equilibrium matching and the production plan selection is much more dispersed than the random matching; when $\gamma = 0.5 - \upsilon$, NAM is equilibrium matching and the production plan is highly concentrated compared to the random matching. [10].

Due to the unobservability of risk aversion, the empirical exercise uses current theory to reveal the risk aversion. If the endogenous production plan is not taken into account, using the matching behavior to predict the risk aversion of supply chains may lead to a flawed prediction.

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[9]Due to the widespread of risk coefficient cause the difficulty of scaling and emphasize the comparison, we picked the risk-averse coefficient $r \in [0.4, 0.6]$

[10]Due to the concavity of $\mu(\sigma)$ function, the low risk project ought to be chosen by more supply chains. However compared to the NAM, production choice under PAM is very evenly spread
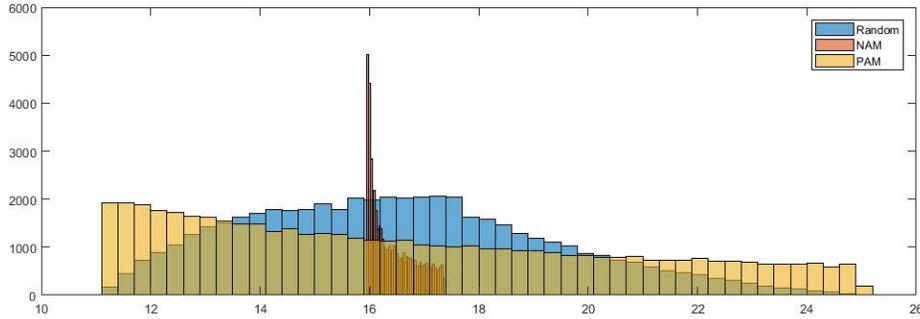
Figure 2.4: Optimal σ^2 choice of random matching vs endogenous matching under NAM where $\gamma = 0.3$

2.7.1 Convex Production Set

To generalize the model, we also consider the convex set of available production. Intuitively, if the uncertainty is relatively less in the market, and the companies can increase the market size with less uncertainty in demand, the matched pairs choose their production plan more aggressively.

Consider the case where a production plan (α, σ^2) is feasible if and only if:

$$\alpha = (\sigma^2)^\gamma \tag{2.19}$$

With $\gamma > 1$.

By replacing the concave function in Assumption 1 by (2.19), the supply chains can choose a project with higher expected market demand in return for a relative small demand risk. The increase of one unit of potential market associated with less than one unit uncertainty encourages the matched supply chains to choose a higher risk-profit product.

The convex curvature suggests that the corner solutions should be the optimal production choices. For convenience, we assume $(\bar{\alpha}^2, \bar{\sigma}^2)$ is the highest ranked bundle

that supply chain can choose due to regulation. Anticipating the marginal expected payoff associated with less marginal variance, supply chains optimally chooses the product with highest profit and risk $(\bar{\alpha}^2, \bar{\sigma}^2)$. When $(\bar{\alpha}^2, \bar{\sigma}^2)$ is the mass point of the product choice of supply chains, the stable equilibrium is consistent as the exogenous production case. If the production choice is limited at the upper bound of α by market regulation, supply chains have no incentive to move to a less risky project. Therefore, the stable matching equilibrium outcome is NAM. The other possible corner solution is to quit the market. The solution under the convex function is relatively trivial and therefore our study focuses on the concave $\alpha(\sigma^2)$ function. If upper limit of production choice doesn't exist, the matched supply can choose the product with infinite high expected profit. In this case, depending on the parameters, the supply chain either exits the market by choosing the lowest risk/expected profit portfolio or chooses infinite risk/expected profit portfolio. The lowest product choice leads to the stationary point and the equilibrium outcome is NAM and the infinite product value choice leads to no solution to the optimization problem.

2.8   Conclusion

The objective in this study is to analyze how the flexibility of product choice affects the formation of risk averse supply chains. We have considered the case which risk-averse suppliers and retailers can freely choose the product and determine the revenue sharing contract. This approach also permits more general assumptions with respect to risk sharing problem in supply chain management by considering $N \times N$ risk-averse suppliers and retailers in the matching market.

In the benchmark model, we demonstrate that if each supply is only able to work for a identical project, NAM is the only equilibrium outcome. When we take the flexibility of product choice into account, both NAM and PAM can become the equilibrium matching. The flexibility in product choice represents a more practical relationship in supply chains. When product choice interrelates to matching mechanism, NAM may become unstable because it is too costly for less risk averse firms to burden all the risk.

Throughout the analysis, our model introduces the distinct scope to examine risk averse agents in coordination supply chain by extending the strategy space to three dimensions: matching pattern, product selection, and revenue sharing contract.

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Appendix A

Proof of Proposition 1

Consider Lemma 4.1, if both firms simultaneously choose to participate the competition in the high value project, the utility functions for both firms are:

$$u_H = a\pi_H[hF_L(x_L) + (1-h)] - x_H \tag{A.1}$$

$$u_L = \pi_H F_H(x_H) - x_L \tag{A.2}$$

Where u_i is expected profit of firm i.

In order to apply Baye et al. (1996), we reorganize (1) (2) as:

$$u_H - a\pi_H(1-h) = a\pi_H hF_L(x_L) - x_H \tag{A.3}$$

and

$$u_L = \pi_H F_H(x_H) - x_L$$

When firm L chooses project H, the expected profit for firm H is $a\pi_H hF_L(x_L) - x$. Thus, in the asymmetrical all pay contest, firm H values the ex-post expected profit

as $a\pi_H h$, and if it wins the competition, the total expected profit for firm H is $u_H = a\pi_H h + a\pi_H(1-h)$, that is also $a\pi_H$. Firm H has 0 probability to deviate from high value project, the expected profit for firm L in high value project does not change in the ex-post competition.

Lemma A.0.1. *(Baye et al., 1996) If firms value the project differently where $v_1 > v_2$ the firm 2 will have expected payoff at 0. And firm 1 has expected payoff at $v_1 - v_2$ in the competition.*

In order to pin down the investment functions, we analyze which firm has higher ex-post value for the high value project. In this "auction", the profits we are comparing are the low ability firm expected profit when it choose the high value project and the high ability agent's expected profit condition on the low ability firm choose the high profit project for sure, which is $u_H - a\pi_H(1-h)$. So naturally, high ability firm has less value for the project when the high ability firm can possibly win this auction for free. In contrast, if firm H values the competition more than firm L then firm L's investment distribution will be first order dominated and firm L will has less incentive to choose high value project. If that is the case, we found in equilibrium condition that the profit of firm L $0 = u_L$ and the $h = 0$ and the equilibrium doesn't exist. So our following analysis is based on assumption that $aw_H h < w_H$, which indicates that firm H values the ex-post auction less than firm L does.

In such ex-post competition, firm H values $aw_H h$ less than firm L values π_H, therefore, for any x_H firm H exerts, its expected value for the ex-post competition should be "0". We can conclude that the best response from firm L's investment function must satisfy:

$$a\pi_H h F_L(x_L) - x = 0 \tag{A.4}$$

And also, the expected profit of firm H also satisfy:

$$u_H = a\pi_H(1-h) \tag{A.5}$$

The investment function of firm L denoted as:

$$F_L(x_L) = \frac{x_L}{a\pi_H h} \tag{A.6}$$

With $x \in [0, a\pi_H h]$. $a\pi_H h$ represents the highest amount that firm H willing to invest and firm L has no incentive to invest above that. In the other hand, firm L is indifferent with compete the high value project and choose the low value project. Therefore, the expected profit of firm L is $u_L = w_L$. Also since firm L values π_H more than firm H values $aw_H h$, the expected profit for firm L should also equals to the difference of the valuation, that is: $u_L = \pi_H - a\pi_H h$. Thus, the probability of firm L to select the high value project is,

$$h = \frac{\pi_H - \pi_L}{a\pi_H} \tag{A.7}$$

The expected profit of firm L can also pin down the investment function of firm H,

$$\pi_H - a\pi_H h = \pi_H F_H(x_H) - x_L \tag{A.8}$$

Therefore,

$$F_H(x) = 1 - ah + \frac{x_H}{\pi_H} \tag{A.9}$$

Where $x \in [0, a\pi_H h]$. It is clear that firm H's strategy has probability of $1 - ah$ to invest 0, and it has the complementary probability to random invest between

$[0, a\pi_H h]$.

The known h can directly solve the expected profit of firm H:

$$u_H = (a - 1)\pi_H + \pi_L \tag{A.10}$$

In sum, in equilibrium of 2 firms decentralized competition: firm H choose high value project with probability "1", and its investment mixes over $x_H \in [0, \pi_H - \pi_L]$ according to distribution $F_H(x_H) = 1 - \frac{\pi_H - \pi_L}{\pi_H} + \frac{x_H}{\pi_H}$, with mass point on "0". Firm L's investment in high value project mixes over $x_L \in [0, \pi_H - \pi_L]$ according to distribution $F_L(x_L) = \frac{x_L}{\pi_H - \pi_L}$. The probability that firm H select high value project listed as (14).

Moreover, in the two firms competition, the upward competition equilibrium holds in general. The condition $a\pi_H h < \pi_H$ in equilibrium is equivalent to $\pi_H - \pi_L < \pi_H$.

Proof of Proposition 2

By proposition 1, F_L is stochastic dominance over F_H, therefore, the expected probability that the low ability firm wins the high value project is greater than high ability firm's $\mathrm{E}[P(x_L \leqslant x_H)] \leqslant \mathrm{E}[P(x_L \geqslant x_H)]$. The sum of expected revenue is:

- $\mathrm{E}[P(x_L \leqslant x_H)]a\pi_H + \mathrm{E}[P(x_L \geqslant x_H)]\pi_H$ when both firm choose the high value project.

- $a\pi_H + \pi_L$ when the sorting is positive assortative.

In equilibrium, both firm choose the high value project with probability h, and positive assortative happens with probability $1 - h$.

Therefore, the total expected revenue for the industry in the decentralized competition is:

$$U^{decentralized} = h[\mathrm{E}[P(x_L \leqslant x_H)]a\pi_H + \mathrm{E}[P(x_L \geqslant x_H)]\pi_H] + (1-h)(a\pi_H + \pi_L) \quad (A.11)$$

The centralized random allocation creates the following expected revenue in the industry:

$$U^{random} = \frac{1}{2}(a+1)(\pi_H + \pi_L) \quad (A.12)$$

The random centralized sorting prescribes the probabilities of both negative and positive sorting are equal to $\frac{1}{2}$.

The **sufficient condition** for proposition 2 is that the expected probability of each firm wins the high value project is same, $\mathrm{E}[P(x_L \leqslant x_H)] = \mathrm{E}[P(x_L \geqslant x_H)] = \frac{1}{2}$, and $U^{decentralized}$ is still less than U^{random}. The total revenue for the industry is $a\pi_H$ when both of the firms choose the high revenue project and high type firm develops it first and it is larger than π_H when the low ability firm develops it first. Therefore, if the industrial revenue in decentralized competition is less than the random allocation even we take the upper bond of $\mathrm{E}[P(x_L \leqslant x_H)]$, we can conclude the decentralized competition is less efficient than a centralized sorting. Thus, we consider the comparison between $U^{decentralized'}$, which represents $\mathrm{E}[P(x_L \leqslant x_H)] = \mathrm{E}[P(x_L \geqslant x_H)] = \frac{1}{2}$ in $U^{decentralized}$, and U^{random}.

$$U^{decentralized'} = h(\frac{1}{2}\pi_H + \frac{1}{2}a\pi_H) + (1-h)(a\pi_H + \pi_L) \quad (A.13)$$

The revenue difference between the random allocation and decentralized allocation when $\mathrm{E}[P(x_L \leqslant x_H)] = \mathrm{E}[P(x_L \geqslant x_H)] = \frac{1}{2}$ is:

$$U^{decentralized'} - U^{random} = h(\frac{1}{2}\pi_H + \frac{1}{2}a\pi_H) + (1-h)(a\pi_H + \pi_L) - \frac{1}{2}(a+1)(\pi_H + \pi_L)$$

$$= \frac{1}{2}h(a+1)\pi_H + (1-h)a\pi_H - h\pi_L - \frac{1}{2}(a+1)\pi_H - \frac{1}{2}\pi_L$$

$$= \pi_H[\frac{1-a}{2}(1-h)] - \pi_L(h + \frac{1}{2}) < 0$$

By assumption 1, $a > 1$, it is clear to claim $U^{decentralized'} - U^{random} < 0$, and $U^{decentralized'} - U^{random} < 0$ is a sufficient condition to $U^{decentralized} - U^{random} < 0$. Thus far, we prove that the revenue from decentralized allocation is strictly less than the random centralized allocation.

Proof of Proposition 3.

In random first mover registration game, there are probability $\frac{1}{2}$ that each player register first, therefore, we characterize the equilibrium under both scenarios.

First, the established firm is the first mover to reveal its proposal of R&D. Second, the subordinate firm is the first one to announce the R&D plan. Because of the first mover advantage, in equilibrium, firms have incentive to register the high value project ahead of their rival. Furthermore, in the extreme case when registration is simultaneous, the unique pure strategy equilibrium is positive assortative matching.

The Established Firm Registers the Project First

First, we consider that the queue of registration requires the leading firm in the industry to announce the project first, and then the secondary can make its decision accordingly. In the unique equilibrium, the established firm chooses a pure strategy to work on the high value project, and the secondary chooses a pure strategy to work on the low value project, as the secondary firm anticipates its expected profit is weakly

higher to choose the low value project.

If the decision stage is transparent, the sub-games of project selection has 4 possible results: both firms choose the high value project, both firms choose the low value project; positive sorting, and negative sorting. The extensive form game shows in figure 4. By lemma 9.1, if both firms choose the same project the firm who values the projects lower get the expected profit 0. In the contrast of the confidential game, firms first realize the rival's selection and then invest accordingly. In this case, the high ability firm always values the projects more, therefore, the low ability firm gets 0 expected profit when they select the same project by Lemma9.1.

When the established firm selects the high value project, the best response from firm L is to choose the low value project to get the expected π_L instead of competing with firm H. Conversely, if the firm L first choose low value project, firm L's best response is to choose the high value project. Firm H anticipates the low ability firm will avoid its selection for sure, so firm H chooses the high value project in equilibrium. In turn, firm L chooses the pure strategy to select low value project in equilibrium.

Remark When the established firm is the first one to claim the project, the dominant strategy of the established firm is to claim the high value project. The secondary firm chooses a pure strategy to select the low value project. With the least investment on accelerating the research process $x = 0$ for both firms, the profit for the established firm is $a\pi_H$ and it is π_L for the secondary firm.

As the established firm can fully notice if it is the only one in the high value project contest, it is not optimal for secondary firm to compete the high value project. As a consequence, the established firm would invest more on the contest to lessen the wining probability for the secondary firm. If both of the firms compete for the
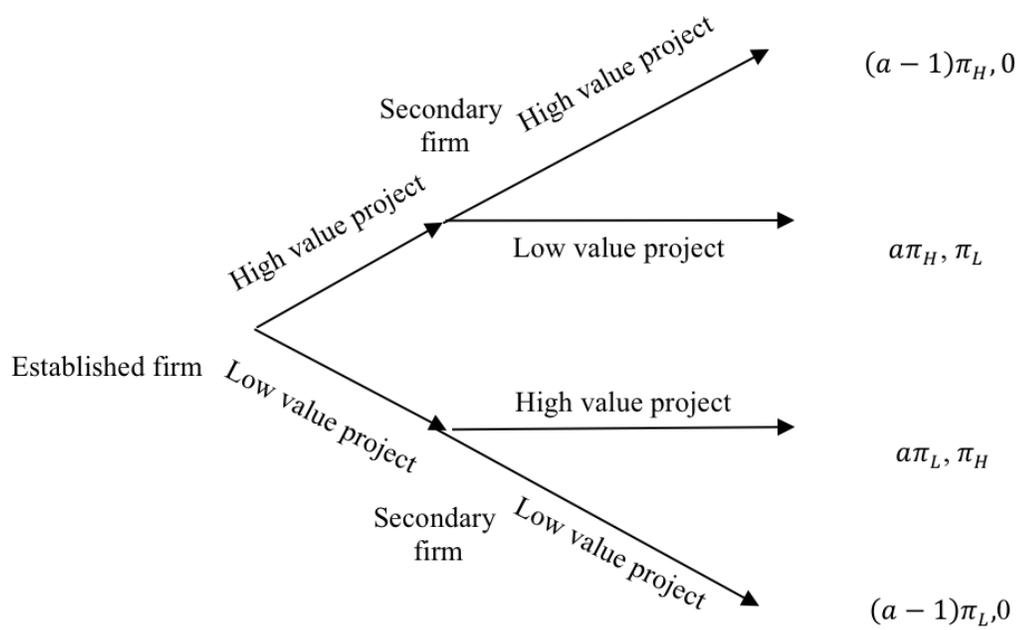
Figure A.1: The established firm is the first mover to claim the project

same project, the high ability firm would be willing to invest sufficiently in the R&D department to induce a zero expected utility of the secondary firm. Therefore, the sequential claim assortatively sorts the projects to firms and the positive sorting is the equilibrium outcome when the established firm is the first mover.

Remark *When both firms claim the projects simultaneously ahead of the development stage, the unique pure strategy equilibrium is positive assortative matching too.*

The Secondary Firm Registers the Project First

When the order of claims is reversed, the result is ambiguous. Depending on the exogenous conditions, both positive sorting and negative sorting can be the equilibrium outcome. To derive the equilibrium R&D strategies, we first focus on the case where

$$(a - 1)\pi_H < a\pi_L \tag{A.14}$$

This condition can be viewed as: 1, the difference in ability between the agents is relatively small; 2, the difference between the project value is relatively small.

When the secondary firm is able to claim first and the exogenous parameters satisfy (A.14):

- A unique pure strategy equilibrium of the sequential project competition exists.

- The secondary firm chooses pure strategy to select project H and the established firm chooses pure strategy to select project L.

- The expected profit for the established firm is $u_H = a\pi_L$, and it is $u_L = \pi_H$ for the secondary firm.

- There is no investment on acceleration where $x = 0$ for all firms.

The proof contains similar intuition as the case when the established firm is the first mover. The only difference here is that when firm L has the first mover advantage, it can lead the game to different sub-games. when $(a-1)\pi_H < \pi_L$ and firm L chooses the high value project, it is more profitable for firm H just avoid the competition in high value project and choose low value project. when $(a-1)\pi_H > \pi_L$, firm H has a dominant strategy to choose high value project regardless of firm L's choice. Thus, when $(a-1)\pi_H > \pi_L$, positive sorting is equilibrium outcome; when $(a-1)\pi_H < \pi_L$, negative sorting is equilibrium outcome.

In the sequential claim stage, the established firm has no incentive to compete in the high value project when $(a-1)\pi_H < a\pi_L$. If both firms enter the high value project competition, the expected value for established firm is $(a-1)\pi_H$, and if the established firm strategically select the low value project in order to avoid the competition, the expected profit is $a\pi_L$. Therefore, when (A.14) is satisfied, the secondary firm is able to develop the high value project and the low value project will be developed by the established firm.

The second case considers the condition:

$$(a - 1)\pi_H > a\pi_L \tag{A.15}$$

The condition in (A.15) can be interpreted as the differences between firms' abilities and the project values are both relatively large.

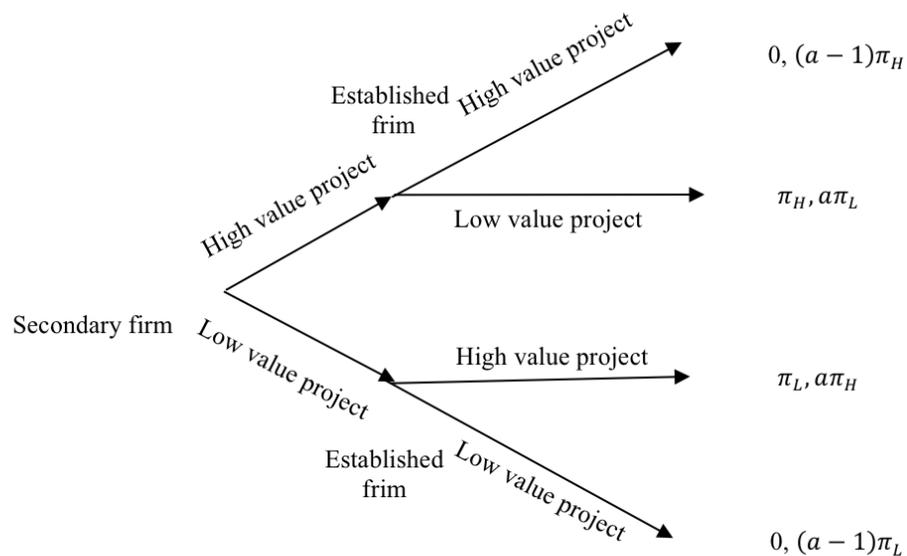When the secondary firm is able to claim first and the exogenous parameter satisfy (19):

Figure A.2: The secondary firm is the first to claim the project

- A unique pure strategy equilibrium of the sequential project competition exists.

- The secondary firm chooses pure strategy to select project L and the established firm chooses pure strategy to select project H.

- The expected profit for the established firm is $u_H = a\pi_H$ and it is $u_L = \pi_L$ for the secondary firm.

- There is no investment on acceleration where $x = 0$ for all firms.

When the secondary firm possesses the first mover advantage, the equilibrium sorting depends on the established firm's valuation. When the high value project competition bring more costs than the increase in revenue, the established firm will

strategically select the low value project as a second mover. Conversely, if the difference in ability is relative large and the difference in the project is relative big, the established firm would invest adequately in the development of high value drug to push the low ability R&D department out of the competition.

Proof of Proposition 4

From Proposition 1, the utility (revenue) of H and L in decentralized R&D competition are $(a-1)\pi_H + \pi_L$ and π_L accordingly. Therefore

$$E[u_H + u_L] = (a-1)\pi_H + 2\pi_L$$

in decentralized R&D competition. In random first mover competition, if $(a-1)\pi_H > a\pi_L$ is satisfied, the total utility for both firm are $a\pi_H + \pi_L$. Based on $W(x_L, x_H) = E[u_L + u_H] + E[c(x_L + x_H)]$, we need to calculate the expected externality in confidential model.

$$E[c(x_L + x_H)] = c[E(x_L) + E(x_H)]$$

In the confidential competition model, the expected value for $E(x_L) = \frac{\pi_H - \pi_L}{a\pi_H} \frac{\pi_H - \pi_L}{2}$, and $E(x_H) = \frac{\pi_H - \pi_L}{\pi_H} \frac{\pi_H - \pi_L}{2}$. The expected investment is derived from the CDFs of firms investment in the high value project in Proposition 1. Therefore, $E[c(x_L + x_H)] = c(1+a)\frac{\pi_H - \pi_L}{2a\pi_H}$ and the total social welfare from confidential competition is:

$$(a-1)\pi_H + 2\pi_L + c(1+a)\frac{\pi_H - \pi_L}{2a\pi_H} \tag{A.16}$$

For the random first mover model, we proved that the investment is always zero from both firms. And as we stated, we only consider the total social welfare when the positive assortative sorting is the equilibrium outcome. Therefore, the total social welfare from random first mover model is:

$$a\pi_H + \pi_L \tag{A.17}$$

- When $c > \frac{2a\pi_H}{(\pi_H - \pi_L)(1+a)}$, (A.16) is larger than (A.17).

- When $c < \frac{2a\pi_H}{(\pi_H - \pi_L)(1+a)}$, (A.16) is smaller than (A.17).

So we can prove proposition 4.

For the discussion in Chapter 1, Section 7 we also list the partial derivatives here to support my argument:

$$\frac{\partial \pi_L}{\partial \frac{2a\pi_H}{(\pi_H - \pi_L)(1+a)}} = \frac{2a\pi_H}{(a+1)(\pi_H - \pi_L)} > 0 \tag{A.18}$$

$$\frac{\partial \pi_H}{\partial \frac{2a\pi_H}{(\pi_H - \pi_L)(1+a)}} = -\frac{2a\pi_L}{(a+1)(\pi_H - \pi_L)} < 0 \tag{A.19}$$

$$\frac{\partial a}{\partial \frac{2a\pi_H}{(\pi_H - \pi_L)(1+a)}} = \frac{2\pi_H}{(a+1)^2(\pi_H - \pi_L)} > 0 \tag{A.20}$$

(A.18)-(A.20) indicate that a and π_L is increasing in $\frac{2a\pi_H}{(\pi_H - \pi_L)(1+a)}$ and π_H is decreasing in $\frac{2a\pi_H}{(\pi_H - \pi_L)(1+a)}$.

Table A.1: Equilibrium Outcomes with Registrations

Registration Queue	Contest Equilibrium
Simultaneous Registration	Positive Assortative Sorting (Efficient outcome)
Established Firm Register First	Positive Assortative Sorting (Efficient outcome)
Secondary Firm Register First and $(a-1)\pi_H < a\pi_L$	Negative Assortative Sorting
Secondary Firm Register First and $(a-1)\pi_H > a\pi_L$	Positive Assortative Sorting (Efficient outcome)
Only Established Firm Release the Project Choice	Same as Confidential Contest
Only Secondary Firm Release the Project Choice	Positive Assortative Sorting (Efficient outcome)

Supplement

The Equilibrium Sorting of Different Registrations

In table 1 we list some of the possible information revelation scenarios and the equilibrium outcomes accordingly.

More Firms in the Confidential Competition

Thus far, the analysis has been focused on the condition that the number of the types and the firms ability are constrained to two. The 2×2 model may represent the general competition within two types of agents on two types of projects in the pharmaceutical industry. However, the upward competition in confidential competition can be consistent in more firms and projects case with exogenous conditions hold on the firms' ability and project value.

Three projects and three firms model. Assume three firms ranked by their abilities which are high H, medium M, and low L. The abilities of from H to L recorded as $a > b > 1$. The available projects are different with the revenue π and

$\pi_L < \pi_M < \pi_H$.

The upward competition equilibrium is constructed as that the high ability firm has probability 1 to choose the high value project and medium ability firm has probability h to choose the high value project, and $1-h$ to choose medium project. The low ability firm has probability m to choose the medium type project and $1-m$ to choose low type project. We only consider the cognate equilibrium structure to 2×2 model, and also the situation that the low ranked firm only "climb up one more step of the ladder", more specifically, firm L only decide between selecting medium value project and low value project. Medium ability firm meanwhile randomizes between high ability project and medium value project sometimes competing high value project with the high ability firm, and sometimes invest relative less in medium project. Also, the High ability agent has a unique preference on high value project.

For the convenience we assume F_k^i represents the investment function of firm i in project k, $i \in [H, M, L]$ and $k \in [H, M, L]$.

Proposition 8. *When $\frac{(a-b)\pi_H + (b-1)\pi_M + \pi_L}{a\pi_H} > \frac{\pi_M - \pi_L}{\pi_M}$ hold, the equilibrium of R&D competition is in mixed strategies.*

- *Firm H's investment in high value project follows $F_H^H(x_H) = 1 - \frac{\pi_H - \pi_L}{\pi_H} + \frac{x_H}{b\pi_H}$*

  where $x_H \in [0, b\pi_H - (b-1)\pi_M - \pi_L]$. The expected profit for firm H is $u_H = \pi_H(a-b) + (b-1)\pi_M + \pi_L$.

- *Firm M randomizes over selecting the high value project with probability $h = \frac{b\pi_H - (b-1)\pi_M - \pi_L}{a\pi_H}$ and selecting the medium value project with probability $1 - h = 1 - \frac{b\pi_H - (b-1)\pi_M - \pi_L}{a\pi_H}$. The investment functions for firm M are:*

  - *$F_H^M(x_M) = \frac{x_M}{b\pi_H - (b-1)\pi_m - \pi_L}$ where $x_M \in [0, b\pi_H - (b-1)\pi_M - \pi_L]$;*

$- F_M^M(x_M) = 1 - \frac{(\pi_M - \pi_L + x_M)a\pi_H}{[(a-b)\pi_H + (b-1)\pi_M + \pi_L]\pi_M}$ *where* $x_M \in [0, \pi_M - \pi_L]$

The expected profit for firm M *is* $u_M = (b-1)\pi_M + \pi_L$

- *Firm L randomizes over selecting the medium value project with probability* $m = \frac{\pi_M - \pi_L}{b\pi_M}$ *and selecting the low value project with probability* $1 - m = 1 - \frac{\pi_M - \pi_L}{b\pi_M}$. *Firm L's investment in medium value project follows* $F_M^L(x_L) = \frac{x_L}{\pi_M - \pi_L}$ *where* $x \in [0, \pi_M - \pi_L]$. *Firm L invest 0 in low value project. The expected profit for firm L is* $u_L = \pi_L$.

Condition $\frac{(a-b)\pi_H + (b-1)\pi_M + \pi_L}{a\pi_H} > \frac{\pi_M - \pi_L}{\pi_M}$ assures firm M invest less than firm L do in the medium value project. Distinct from the firm in the top or bottom rank, firm M not only confronts the competition from firm H in high value project for sure but also the competition with firm L in the medium value project. In the other words, the type in the middle has no "safe choice". In the three players model, when the above condition is not hold, the equilibrium will change accordingly[11].

n firms n projects model. When we extend the model to $n \times n$, the stronger assumptions is necessary for the upward competition equilibrium to exist. Assume n firms ranked from 1 to N with heterogeneous ability a_n that $a_i < a_{i+1}$, $i = \{1, 2, ..., n\}$ and $a_1 = 1$ for simplification. The projects' revenue ranked as $\pi_i < \pi_{i+1}$ and $i = \{1, 2, ..., n\}$.

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[11]We also construct one possible equilibrium when $\frac{(a-b)\pi_H + (b-1)\pi_M + \pi_L}{a\pi_H} > \frac{\pi_M - \pi_L}{\pi_M}$ is not satisfied, and we record it as two side competition equilibrium. This equilibrium structure is: firm H will only invest in high value project; firm M randomizes with all high, medium, and low value project. Low ability firm is indifferent medium and low value project. The extra citation l denotes the probability that firm M chooses the low value project. In equilibrium, unknowns m, l, h can be pinned down as: $l = \frac{(b+1)\pi_L - \pi_M}{2\pi_L}$, $m = 1 - \frac{\pi_L}{\pi_M} + \frac{(b+1)\pi_L - \pi_M}{b\pi_M}$, and $h = \frac{\pi_H(a-b) + \pi_M - \pi_L}{a\pi_H}$. The conditions that required for this unique mixed strategy equilibrium are: $\pi_m - \frac{1}{2}b\pi_l - \frac{3}{2}\pi_l > 0$ and $(b-1)\pi_M + \pi_L < (1 - \frac{\pi_H(a-b) + \pi_M - \pi_l}{a\pi_h} - \frac{(b+1)\pi_l - \pi_m}{2\pi_l})\pi_m$ and $\pi_H(a - 2b) + \pi_M - \pi_L < 0$. The cumulative distribution functions and expected utilities are explicitly characterized in the appendix

The upward competition in $n \times n$ model represents firm i either select project i or $i+1$. The firms face the competition with their adjacent ranking firms in the $n \times n$ model. We assume F_k^m represents the investment function of firm m in project k and p_z^v represents the probability that firm v select project z in equilibrium, k, m, v, z $\in \mathcal{N} = \{1, 2, ..., n\}$.

The upward competition exist if and only if the following conditions are satisfied:

1.
$$\frac{a_{n-1}\pi_i - [\sum_3^n (a_{k-1} - a_{k-2})\pi_{n-1}] - (a_2 - 1)\pi_2 - \pi_1}{\pi_n} < a_{n-1} \qquad (A.21)$$

2.
$$\frac{a_{i-1}\pi_i - [\sum_3^i (a_{k-1} - a_{k-2})\pi_{i-1}] - (a_2 - 1)\pi_2 - \pi_1}{\pi} \qquad (A.22)$$
$$< a_{i-1}\left(1 - \frac{a_i\pi_{i+1} - [\sum_3^{i+1}(a_{k-1} - a_{k-2})\pi_i] - (a_2 - 1)\pi_2 - \pi_1}{a_{i+1}\pi_{i+1}}\right)$$

for all $i = \{2, 3, ..., n-1\}$

The conditions indicate the difference in the projects is relative low, and the higher ability firm always invest less than the lower ability firm in the equilibrium. We summarize the equilibrium in Proposition 7:

Proposition 9. *When (6) and (7) hold, each firm employs mixed strategy in equilibrium.*

- *Firm N select project N with probability 1 and mixes the investment over*

$$[0, a_{n-1}\pi_n - [\sum_{k=3}^n (a_{k-1} - a_{k-2})\pi_{n-1}] - (a_2 - 1)\pi_2 - \pi_1]$$

  $k = \{3, 4, ..., n\}$ *and has mass point on 0. The expected profit for firm n is* $u_n = \sum_{k=3}^n \pi_k(a_k - a_{k-1}) + (a_2 - 1)\pi_2 + \pi_1$

- *For the firms in the middle range i.e., firm 2 to n-1, firm $i-1$ has probability $p_i^{i-1} = \frac{a_{i-1}\pi_i - [\sum_3^i (a_{k-1}-a_{k-2})\pi_{i-1}] - (a_2-1)\pi_2 - \pi_1}{a_i \pi_i}$ to invest in project i and probability $1 - p_i^{i-1}$ to select project $i-1$.*

  - *When firm $i-1$ select project i, it mixes the investment over*
    $$[0, a_{i-1}\pi_i - [\textstyle\sum_{k=3}^i (a_{k-1} - a_{k-2})\pi_{i-1}] - (a_2 - 1)\pi_2 - \pi_1]$$

  - *When firm $i-1$ select project $i-1$, it mixes the investment over*
    $$[0, a_{i-2}\pi_{i-1} - [\textstyle\sum_{k=3}^{i-1} (a_{k-1} - a_{k-2})\pi_{i-2}] - (a_2 - 1)\pi_2 - \pi_1] \text{ with mass point}$$
    on 0.

  The expected profit for firm $i-1$ is $u_{i-1} = \sum_{k=3}^{i-1} \pi_k (a_k - a_{k-1}) + (a_2 - 1)\pi_2 + \pi_1$

- *Firm 1 select project 2 with probability $p_2^1 = \frac{\pi_2 - \pi_1}{a_2 \pi_2}$ and select project 2 with probability $1 - p_2^1$.*

  - *When firm 1 select project 2, it mixes the investment over $[0, \pi_H - \pi_L]$.*

  - *When firm 1 select project 1, it invest 0.*

  The expected profit for firm 1 is $u_1 = \pi_1$

Proof see appendix.

In the equilibrium, the expected profit for higher ability firm is greater than the lower ability firm, but if both firms are competing for the same project, the low ability firm invests more aggressively than the high ability firm. As to the allocation of the projects, the upward competition limits the competition between the neighbours and reduce the volatile mismatch.

Proof of Proposition 8

Proof. In a three firms competition, the upward competition equilibrium requires more conditions to hold.

If both firm H and firm L choose the high value project, the expected profits are:

$$u_H = a\pi_H(1 - h) + hF_H^M(x_M) - x_H \tag{A.23}$$

$$u_M = b\pi_H F_H^H(x_H) - x_M \tag{A.24}$$

Reorganize those as:

$$u_H - a\pi_H(1 - h) = a\pi_H hF_H^M(x_M) - x_H \tag{A.25}$$

$$u_M = b\pi_H F_H^H(x_H) - x_M \tag{A.26}$$

We assume $a\pi_H h < b\pi_H$, that is, firm H values the auction more and because firm H knows that firm M has less than 1 probability to select the high value project. Thus, $u_H = a\pi_H(1 - h) = 0$ and $a\pi_H hF_H^M(x_M) - x = 0$. Also, the firm M's valuation for high value project is $u_M = b\pi_H - a\pi_H h$

The investment functions are:

$$F_H^M(x_N) = \frac{x_M}{a\pi_H h} \tag{A.27}$$

with $x \in [0, a\pi_H h]$.

$$F_H^H(x_H) = 1 - ah + \frac{x_H}{b\pi_H} \tag{A.28}$$

with $x \in [0, a\pi_H h]$

In the medium value project competition, the possible competitors are medium ability agent and low ability agent. The expected profit functions are:

$$u_M = b_M(1 - m) + b\pi_M m F_M^L(x_L) - x_M \tag{A.29}$$

$$u_L = h\pi_M + (1 - h)\pi_m F_M^M(x_M) - x_L \tag{A.30}$$

Reorganize those functions as:

$$u_M - b\pi_M(1 - m) = b\pi_M m F_M^L(x_L) - x_M \tag{A.31}$$

$$u_L - h\pi_M = (1 - h)\pi_M F_M^M(x_M) - x_L \tag{A.32}$$

Assume $(1 - h)\pi_M < bm\pi_M$ holds. The investment functions are:

$$F_M^M(x_M) = 1 - \frac{b}{1 - h} + \frac{x_M}{(1 - h)\pi_M} \tag{A.33}$$

with $x_M \in [0, (1 - h)\pi_M]$.

$$F_M^L(x_L) = \frac{L}{b\pi_M m} \tag{A.34}$$

with $x_L \in [0, (1 - h)\pi_M]$.

(A.31), (A.32), (A.33), (A.34) and $u_L = \pi_L$ suggest that $h = \frac{b\pi_H - (b-1)\pi_M - \pi_L}{a\pi_H}$ and $m = \frac{\pi_M - \pi_L}{b\pi_M}$ in equilibrium. Also the assumption $(1 - h)\pi_M > bm\pi_M$ is equivalent to $\frac{(a-b)\pi_H + (b-1)\pi_M + \pi_L}{a\pi_H} > \frac{\pi_M - \pi_L}{\pi_M}$ in equilibrium.

Thus, the expected profits of firms are: $u_H = \pi_H(a - b) + (b - 1)\pi_M + \pi_L$, $u_M = (b - 1)\pi_M + \pi_L$, and $u_L = \pi_L$ in equilibrium.

And the firms' investment follow:

$$F_M^L(x_L) = \frac{x_L}{\pi_M - \pi_L} \tag{A.35}$$

$$F_M^M(x_M) = 1 - \frac{ab\pi_H}{(a-b)\pi_H + (b-1)\pi_M + \pi_L} + \frac{x_M}{\frac{(a-b)\pi_H + (b-1)\pi_M + \pi_L}{a\pi_H}\pi_m} \tag{A.36}$$

with $x_L, x_M \in [0, \frac{(a-b)\pi_H + (b-1)\pi_M + \pi_L}{a\pi_H}\pi_M]$.

$$F_H^M(x_M) = \frac{x_M}{b\pi_H - (b-1)\pi_M - \pi_L} \tag{A.37}$$

$$F_H^H(x_H) = 1 - \frac{b\pi_H - (b-1)\pi_M - \pi_L}{\pi_H} + \frac{x_H}{b\pi_H} \tag{A.38}$$

with $x_H, x_M \in [0, b\pi_H - (b-1)\pi_M - \pi_L]$.

$\square$

Two-side competition equilibrium in three agents model

The condition $\frac{(a-b)\pi_H + (b-1)\pi_M + \pi_L}{a\pi_H} > \frac{\pi_M - \pi_L}{\pi_M}$ doesn't hold in general. We briefly demonstrate another equilibrium that firm M has possibility to select all projects in equilibrium.

In two-side competition equilibrium, firm H only choose high value project, firm M choose the mixed strategy that randomize in all projects, and firm L randomize with medium value project and low value project. The additional citation l denotes the probability that firm M choose low value project in equilibrium.

When $\pi_L l < b\pi_L(1 - m)$ holds, the expected profits in low value project competition are:

$$u_M = [(1-m)F_L^L(x_L) + m]b\pi_L - x_M$$

$$u_L = \pi_L[lF_L^M(x_M) + (1-l)] - x_L$$

Reorganize those as:

$$u_M - mb\pi_L = b\pi_L(1-m)F_L^L(x_L) - x_M$$

$$u_L - (1-l)\pi_L = \pi_L lF_L^M(x_M) - x_L$$

Thus,

$$F_L^L(x_L) = 1 - \frac{l}{b(1-m)} + \frac{x_L}{b\pi_L(1-m)}$$

$$F_L^M(x_M) = \frac{x_M}{\pi_L l}$$

When $b\pi_M m < (1-h-l)\pi_m$ holds, the expected profits for firm M and firm L in the medium value project competition can be organized as:

$$u_M - b\pi_M(1-m) = b\pi_M mF_M^L(x_L) - x_M$$

$$u_L - (l+h)\pi_M = (1-h-l)\pi_M F_M^M(x_M) - x_L$$

Thus,

$$F_M^L = \frac{x_L}{b\pi_M m}$$

$$F_M^M = 1 - \frac{bm}{1-h-l} + \frac{x_M}{\pi_m(1-h-l)}$$

When $aw_h h < bw_h$ holds, the expected profits for firm M and firm H in the high value project competition can be organized as:

$$u_H - a\pi_h(1 - h) = a\pi_H h F_M^H(x_M) - x_H$$

$$u_M = b\pi_h F_H^H(x_H) - x_H$$

Thus,

$$F_M^H(x_M) = \frac{x_M}{a\pi_H h}$$

$$F_H^H(x_H) = 1 - \frac{ah}{b} + \frac{x_H}{b\pi_H}$$

Therefore, the probability that firm$M$ select the low value project is

$$l = \frac{(b + 1)\pi_l - \pi_M}{2\pi_L}$$

; the probability that firm$L$ select the value project is

$$m = 1 - \frac{w_l}{w_m} + \frac{(b + 1)w_l - w_m}{bw_m}$$

; the probability that firm$M$ select the high value project is

$$h = \frac{\pi_H(a - b) + \pi_M - w_L}{a\pi_H}$$

.

And the sufficient conditions support the two-side competition equilibrium are:

$$\pi_M - \frac{1}{2}b\pi_l - \frac{3}{2}\pi_L > 0$$

$$(b-1)\pi_M + \pi_L < (1 - \frac{\pi_H(a-b) + \pi_M - \pi_L}{a\pi_H} - \frac{(b+1)\pi_L - \pi_M}{2\pi_L})\pi_M$$

$$\pi_H(a - 2b) + \pi_M - \pi_L < 0$$

Proof of Proposition 9

In the N firms N projects model, the critical argument is very similar to the 3×3 model. However, the upward competition equilibrium needs $n - 1$ conditions to support the existence of it.

For example, only firm$n$ and firm $n - 1$ participate the highest value project competition, and their expected profits are:

$$u_n = a_n \pi_n((1 - p_n^{n-1}) + p_n^{n-1} F_n^{n-1}(x_{n-1})) - x_n \qquad (A.39)$$

$$u_{n-1} = a_{n-1} \pi_n F_n^n(x_n) - x_{n-1} \qquad (A.40)$$

Competition for project $n - 1$ indicates the expected profits for firm $n - 1$ and $n - 2$ are:

$$u_{n-1} = a_{n-1}\pi_{n-1}((1 - p_{n-1}^{n-2}) + p_{n-1}^{n-2}F_{n-2}^{n-1}(x_{n-2})) - x_{n-1},$$

$$u_{n-2} = a_{n-2}\pi_{n-1}((1 - p_n^{n-1})F_{n-1}^{n-1}(x_{n-1}) + p_n^{n-1}) - x_{n-2},$$

$$\vdots$$

We omit the competition for project 3 to $n - 2$.

Competition for project 2 indicates the expected profits for firm 1 and 2 are:

$$u_2 = a_2\pi_2((1 - p_2^1) + p_2^1 F_2^1(x_1)) - x_2$$

$$u_1 = \pi_2((1 - p_2^2)F_2^2(x_2) + p_2^2) - x_1$$

Also, $u_1 = \pi_1$.

The sufficient conditions for the upward competition are:

$$a_n p_n^{n-1} < a_{n-1},$$

$$a_{n-1}p_{n-1}^{n-2} < a_{n-2}(1 - p_n^{n-1}),$$

$$a_{n-2}p_{n-2}^{n-3} < a_{n-3}(1 - p_{n-1}^{n-2}),$$

$$\vdots$$

$$a_1 p_2^1 < (1 - p_2^2)$$

In equilibrium the probability that firms choose the different projects in their

mixed strategies are:

$$p_2^1 = \frac{\pi_2 - \pi_1}{a_2 \pi_2},$$

$$p_3^2 = \frac{a_2 \pi_3 - (a_2 - 1)\pi_2 - \pi_1}{a_3 \pi_3},$$

$$\vdots$$

$$p_i^{i-1} = \frac{a_{i-1}\pi_i - [\sum_3^i (a_{k-1} - a_{k-2})\pi_{k-1}] - (a_2 - 1)\pi_2 - \pi_1}{a_i \pi_i}$$

$$\vdots$$

$$p_n^{n-1} = \frac{a_{n-1}\pi_n - [\sum_3^n (a_{k-1} - a_{k-2})\pi_{k-1}] - (a_2 - 1)\pi_2 - \pi_1}{a_i \pi_i}$$

Appendix B

Proof of Proposition 5

Proof. We characterize the solution of:

$$\max_{m_{ij}}\{Eu_i[\pi_{ij} - m_{ij}(\pi_{ij})]\} + \lambda\{Eu_j[m_{ij}(\pi_{ij})]\}$$

In this model, we adopt the point-wise maximizing method to solve the above equation. Given any realization of the profit, the matched pair (i, j) can cope with an optimal contract. Therefore, we are able to shift the maximization problem to the following equation with each possible project realization instead of the expectation of the utility:

$$\max_{m_{ij}}\{u_i[\pi_{ij} - m_{ij}(\pi_{ij})]\} + \lambda\{u_j[m_{ij}(\pi_{ij})]\}$$

The first order condition delivers:

$$u_i'(\pi_{ij} - m_{ij}(\pi_{ij})) = \lambda u_j'(m_{ij}(\pi_{ij}))$$

The CARA utility function u is a negative exponential function so we naturally take logarithms on both side of the function:

$$r_i[\pi_{ij} - m_{ij}(\pi_{ij})] = r_j m_{ij}(\pi_{ij}) - \log\lambda$$

Subtract $m_{ij}(\pi_{ij})$:

$$m_{ij}(\pi_{ij}) = \frac{r_i \pi_{ij} + \log\lambda}{r_i + r_j}$$

Therefore we can address that, for supplier i, the optimal contract is:

$$\pi_{ij} - m_{ij}(\pi_{ij}) = \frac{r_j \pi_{ij} - \log\lambda}{r_i + r_j}$$

For retailer j, the optimal contract is:

$$m_{ij}(\pi_{ij}) = \frac{r_i \pi_{ij} + \log\lambda}{r_i + r_j}$$

The concavity of CARA utility guarantees the point-wise maximization is sufficient to provide the optimal sharing contract m_{ij}

$\square$

Proof of Lemma 2.4.1

Proof. From (9) and (10), we can conclude the optimal contract is linear with respect to total profit π_{ij} under CARA utility. Due to the fact that exponential utility is non-additive, we characterize the total certainty equivalent for pair matched (i, j). The Pareto frontier total certainty equivalent is additive and able to describe the stable matching equilibrium:

$$Eu_i\left(\frac{r_j \pi_{ij} - \log\lambda}{r_i + r_j}\right) + Eu_j\left(\frac{r_i \pi_{ij} + \log\lambda}{r_i + r_j}\right)$$

By the definition of certainty equivalent and normal distribution we can derive the certainty equivalent for each agent:

$$u_i(CE_i) = Eu_i[\pi_{ij} - m_{ij}(\pi_{ij})] = Eu_i(\frac{r_j\pi_{ij} - \log\lambda}{r_i + r_j})$$

and

$$u_j(CE_j) = Eu_j[m_{ij}(\pi_{ij})] = Eu_j(\frac{r_i\pi_{ij} + \log\lambda}{r_i + r_j})$$

Incorporate the available contract $\pi_{ij} \sim N(\frac{\alpha_{ij}k}{4}, \frac{k^2}{4}\sigma_{ij}^2)$, the expectation of CARA utilities for the firms (i, j) can be derived by:

$$Eu_i(\frac{r_j\pi_{ij} - \log\lambda}{r_i + r_j}) = \int_{-\infty}^{\infty} \frac{\exp\left(-\frac{(\pi_{ij} - \frac{r_j}{r_i+r_j}\frac{\alpha_{ij}k}{4})^2 - \frac{\log\lambda}{r_i+r_j}}{2(\frac{r_j}{r_i+r_j})^2\frac{k^2}{4}\sigma_{ij}^2}\right)}{\sqrt{2(\frac{r_j}{r_i+r_j})^2\frac{k^2}{4}\sigma_{ij}^2\pi}} u_i(\pi_{ij}) \, d\pi_{ij}$$

$$Eu_j(\frac{r_i\pi_{ij} + \log\lambda}{r_i + r_j}) = \int_{-\infty}^{\infty} \frac{\exp\left(-\frac{(\pi_{ij} - \frac{r_i}{r_i+r_j}\frac{\alpha_{ij}k}{4})^2 + \frac{\log\lambda}{r_i+r_j}}{2(\frac{r_i}{r_i+r_j})^2\frac{k^2}{4}\sigma_{ij}^2}\right)}{\sqrt{2(\frac{r_i}{r_i+r_j})^2\frac{k^2}{4}\sigma_{ij}^2\pi}} u_j(\pi_{ij}) \, d\pi_{ij}$$

The utility function of firms defined by (1), therefore, the above function can be represented as:

$$\exp(-r_i(\frac{r_j\pi_{ij} - \log\lambda}{r_i + r_j})^{CE_i})$$

$$= \frac{1}{\sqrt{2(\frac{r_j}{r_i+r_j})^2\frac{k^2}{4}\sigma_{ij}^2\pi}} \int_{-\infty}^{\infty} \exp\left(-\frac{(\pi_{ij} - \frac{r_j}{r_i+r_j}\frac{\alpha_{ij}k}{4})^2 - \frac{\log\lambda}{r_i+r_j}}{2(\frac{r_j}{r_i+r_j})^2\frac{k^2}{4}\sigma_{ij}^2} - r_i\pi_{ij}\right) d\pi_{ij}$$

$$\exp(-r_j(\frac{r_i\pi_{ij} - \log\lambda}{r_i + r_j})^{CE_j})$$

$$= \frac{1}{\sqrt{2(\frac{r_i}{r_i+r_j})^2\frac{k^2}{4}\sigma_{ij}^2\pi}} \int_{-\infty}^{\infty} \exp\left(-\frac{(\pi_{ij} - \frac{r_i}{r_i+r_j}\frac{\alpha_{ij}k}{4})^2 + \frac{\log\lambda}{r_i+r_j}}{2(\frac{r_i}{r_i+r_j})^2\frac{k^2}{4}\sigma_{ij}^2} - r_j\pi_{ij}\right) d\pi_{ij}$$

By the properties of the exponential utility and normal distribution we can simplify the equations as:

$$\exp(-r_i(\frac{r_j\pi_{ij} - \log\lambda}{r_i + r_j})^{CE_i}) = -exp(\frac{r_j}{r_i + r_j}\frac{k}{4}\alpha_{ij} - \frac{r_i}{2}(\frac{r_j}{r_i + r_j})^2\frac{k^2}{4}\sigma_{ij}^2 - \frac{\log\lambda}{r_i + r_j})$$

and

$$\exp(-r_j(\frac{r_i\pi_{ij} - \log\lambda}{r_i + r_j})^{CE_j}) = -exp(\frac{r_i}{r_i + r_j}\frac{k}{4}\alpha_{ij} + \frac{r_j}{2}(\frac{r_i}{r_i + r_j})^2\frac{k^2}{4}\sigma_{ij}^2 + \frac{\log\lambda}{r_i + r_j})$$

The certainty equivalent for firms can be summarized as:

$$CE_i = \frac{r_j}{r_i + r_j}\frac{k}{4}\alpha_{ij} - \frac{r_i}{2}(\frac{r_j}{r_i + r_j})^2\frac{k^2}{4}\sigma_{ij}^2 - \frac{\log\lambda}{r_i + r_j} \tag{B.1}$$

and

$$CE_j = \frac{r_i}{r_i + r_j}\frac{k}{4}\alpha_{ij} + \frac{r_j}{2}(\frac{r_i}{r_i + r_j})^2\frac{k^2}{4}\sigma_{ij}^2 + \frac{log\lambda}{r_i + r_j} \tag{B.2}$$

The sum of (B.1) and (B.2) is:

$$CE_{ij} = \frac{k}{4}\alpha_{ij}(\sigma_{ij}^2) - R_{ij}\frac{k^2}{8}\sigma_{ij}^2$$

Where $R_{ij} = \frac{r_i r_j}{r_i + r_j}$. □

Proof of Proposition 6

The certainty equivalent for supply chain ij is $CE_{ij} = \frac{k}{4}\alpha - R_{ij}\frac{k^2}{8}\sigma^2$ when the product choice is exogenous. By Lemma 5.1, the stable matching equilibrium is determined by $\frac{\partial^2 CE}{\partial r_i \partial r_j}$. It is obviously that

$$\frac{\partial^2 CE}{\partial r_i \partial r_j} = -\frac{k^2\sigma^2}{8}\frac{2r_i r_j}{(r_i + r_j)^3} < 0 \tag{B.3}$$

with $r_i, r_j, > 0$

Therefore, $\frac{\partial^2 CE}{\partial r_i \partial r_j} < 0$ indicates the stable matching is NAM when the supply chains cannot choose the product σ^2.

Proof of Proposition 7

Proof. From Lemma 2.4.1

$$CE_{ij} = \frac{k}{4}\alpha_{ij}(\sigma_{ij}^2) - R_{ij}\frac{k^2}{8}\sigma_{ij}^2$$

And assumption 1

$$\alpha \leqslant (\sigma^2)^\gamma, 0 < \gamma < 1$$

We can re-express the total certainty equivalent in equilibrium as:

$$CE_{ij} = \frac{k}{4}(\sigma_{ij}^2)^\gamma - R_{ij}\frac{k^2}{8}\sigma_{ij}^2$$

Take F.O.C with respect to σ^2

$$\gamma\frac{k}{4}\sigma_{ij}^{2(\gamma-1)} - \frac{R_{ij}k^2}{8} = 0$$

Therefore

$$\sigma_{ij}^2 = (\frac{k}{2\gamma}R_{ij})^{\frac{1}{\gamma-1}}$$

Plug the optimal σ_{ij} back to CE

$$CE_{ij} = (\frac{R}{2\gamma})^{\frac{\gamma}{\gamma-1}} - \frac{R}{2}(\frac{R}{2\gamma})^{\frac{1}{\gamma-1}}$$

$$CE_{ij} = R_{ij}^{\frac{\gamma}{\gamma-1}}[(\frac{k}{2\gamma})^{\frac{\gamma}{\gamma-1}} - \frac{1}{2}(\frac{k}{2\gamma})^{\frac{1}{\gamma-1}}]$$

$$CE_{ij} = \frac{r_i r_j}{r_i + r_j}^{\frac{\gamma}{\gamma-1}}[(\frac{k}{2\gamma})^{\frac{\gamma}{\gamma-1}} - \frac{1}{2}(\frac{k}{2\gamma})^{\frac{1}{\gamma-1}}]$$

First we take cross partial derivative with respect to r_i

$$\frac{\partial CE}{\partial r_i} = (\frac{\gamma}{1-\gamma})(\frac{1}{r_i} + \frac{1}{r_j})^{\frac{\gamma}{1-\gamma}-1}(-\frac{1}{r_i^2})[(\frac{k}{2\gamma})^{\frac{\gamma}{\gamma-1}} - \frac{1}{2}(\frac{k}{2\gamma})^{\frac{1}{\gamma-1}}]$$

Then we take cross partial derivative with respect to r_j

$$\frac{\partial^2 CE}{\partial r_i \partial r_j} = (\frac{\gamma}{1-\gamma})(\frac{\gamma}{1-\gamma} - 1)(\frac{1}{r_i} + \frac{1}{r_j})^{\frac{\gamma}{1-\gamma}-2}(-\frac{1}{r_i^2})(-\frac{1}{r_j^2})[(\frac{k}{2\gamma})^{\frac{\gamma}{\gamma-1}} - \frac{1}{2}(\frac{k}{2\gamma})^{\frac{1}{\gamma-1}}]$$

Since $0 < \gamma < 1$, if $k > \gamma$

$$\gamma > \frac{1}{2}, \frac{\partial^2 CE}{\partial r_i \partial r_j} > 0$$

PAM is the equilibrium outcome. And when

$$\gamma < \frac{1}{2}, \frac{\partial^2 CE}{\partial r_i \partial r_j} < 0$$

NAM is the equilibrium outcome.

If $k < \gamma$, the result is reversed. However, when $k < \gamma$, $CE_{ij} < 0$ and the supply chain will choose the $\sigma^2 = 0$ and quit the market. Therefore we will only consider $k > \gamma$.

□