ABSTRACT

Title of Dissertation: ESSAYS IN BEHAVIORAL ECONOMICS: APPLYING PROSPECT THEORY TO AUCTIONS

Anmol Ratan, Doctor of Philosophy, 2010

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I explore the implications of reference-dependent preferences in sealed-bid auctions. In the first part, I develop a Prospect theory based model to explain bidding in first-price auctions. I show that bidding in *induced-value* first-price sealed-bid auctions can be rationalized as a combination of reactions to underlying ambiguity and anticipated loss aversion. Using data from experimental auctions, I provide evidence that in induced-value auctions with human bidders, this approach works well. In auctions with prior experience and /or against risk-neutral Nash rivals where ambiguity effects could be altogether irrelevant, anticipated loss aversion by itself can explain aggressive bidding. This is a novel result in the literature. Using data from experiments, I find that ambiguity effects become negligible in auctions with experienced human bidders against (i) experienced human rivals and (ii) Nash computer rivals, when loss aversion is taken in consideration. The estimates for loss aversion are similar in auctions with human bidders (with or without experience).

Next, I extend my approach of anticipated loss aversion to address bidding outcomes in first- and second-price sealed-bid auctions. As shown in first part, the model predicts overbidding in first-price induced-value auctions consistent with evidence from most laboratory experiments. However, substantially different bidding behavior could result in commodity auctions where money and auction item are consumed along different dimensions of the consumption space. Differences also result in second-price auctions. The study thereby indicates that transferring qualitative behavioral findings from induced-value laboratory experiments to the field may be problematic if subjects are loss averse and anticipate such losses at the time of bidding.

Finally, I explore the effect of resale or procurement opportunities, to which bidders have heterogeneous market access, on bidding in first- price sealed-bid auctions. My models suggest that in auctions with resale, loss aversion causes underbidding with respect to the risk-neutral-Nash prediction. Bidders with greatest level of market access are least affected by loss aversion and therefore bid closer to the risk-neutral-Nash than bidders with smaller market access. In auctions with procurement, the effect of loss aversion is such that it causes overbidding (underbidding) for bidders with respect to the risk-neutral-Nash. Bidders with greatest level of market access are again least affected by loss aversion and therefore bid much conservatively and closer to the risk-neutral-Nash than bidders with very low market access. If market access is interpreted as a proxy for experience, the predictions of my model are qualitatively similar to the findings in List (2003, 2004). Since these indirect effects are obtained without altering reference-dependent preferences, it raises the possibility that the effects obtained in List (2003, 2004) in *field* settings may not arise entirely due to the direct effect of experience on reference-dependent preferences. This calls for a more careful reexamination of the underlying issues.

ESSAYS IN BEHAVIORAL ECONOMICS: APPLYING PROSPECT THEORY TO AUCTIONS

By

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Dissertation submitted to the Faculty of the Graduate School of the
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Dedication

To My Parents, Avtar, Dada, Nani, Shri Ram Swarup Arya, Prof. Birbal Singh, Mr. Roshan Lal,

Mr. Yudhvir Singh

and

Teachers and Friends

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Chapter 1: Reference-Dependent Preferences in Auctions

In this dissertation, I explore the effect of reference-dependent preferences on bidding in auctions. Loss aversion is widely suspected as the primary influence that manifests in trading of various commodities – from chocolate bars to coffee mugs, coins, or sportscards – for money or other goods (Knetsch 1989; Tversky and Kahneman 1991; Kahneman, Knetsch, and Thaler 1990; Benartzi and Thaler 1995, Camerer 1995, List 2003). In their formulation of the Prospect theory, Tversky and Kahneman (1991) assume the *current endowment level* as a fixed reference point. Any change from the reference point is interpreted as "gains" or "losses" by individuals and the disutility induced by "losses" are larger than the utility induced by "gains" producing a kink at the reference point. Such preferences reconcile the "endowment effect" obtained in the literature above.

The literature on consumer psychology has discussed how consumers get affected by reference prices in everyday transactions (Urbany, Bearden and Weilbaker 1988, Kalyanaram and Winer 1995, Mazumdar et al. 2005). Various experiments on online auctions suggest that bidders get influenced by auctions' reserve prices (Ariely and Simonson 2003, H¨aubl and Popkowski Leszczyc 2003), Kamins et al. 2004, and Suter and Hardest, 2005). Such effects have also been observed in e-Bay "buy now" auctions where buy prices are believed to affect bidding decisions (Dodonova and Khoroshilov, 2004, Popkowski Leszczyc et al. 2007, Shunda 2009). In such uncertain circumstances with anticipatory (forward-looking) attitudes a fixed reference point based approach seems less plausible.

In such circumstances when bidders face uncertain payoff consequences, and "anticipate" uncertain outcomes, Kahnemann and Tversky (1979), Gul (1991) Sugden (2003) and Köszegi and Rabin (2006) suggested alternative formulation of reference points. While Kahnemann and Tversky (1979) and Gul (1991), propose the endogenously determined expected value of the prospect, Sugden (2003) assumes the reference to be given by the *current endowment* which might adjust over the time as a reference point. One another model with endogenous definition of reference points was proposed by Köszegi and Rabin (2006). The Köszegi and Rabin's approach is more general and bears similarities with previous approaches in simpler environments[1] and allows heterogeneous loss aversion for commodities that make the consumption bundle. This brings certain advantages that are apparent in commodity transactions where consumption space in multidimensional.

I base the analysis in this dissertation on a model of loss aversion with endogenous reference points similar to Köszegi and Rabin (2006) to explore the effect on bidding in auctions. This is different from an approach with an exogenous fixed reference point in which winning the auction is interpreted as a "gain" while losing leaves the initial wealth unaffected. I argue that the reference point may get influenced by expected gains and therefore auction outcomes could be interpreted as "gains" or "losses." It is plausible that a bidder who draws a high value and expects to win the auction interprets "not winning" as a "loss" and likewise that a bidder with low induced-value interprets winning the auction as a "gain." In a sealed-bid auction, after placing a bid, bidders face a lottery of winning or losing the auction. The

[1] For example, with forward looking rational expectations based reference, it is equivalent to Kahnemann and Tversky (1979) and Gul (1991) in induced value first-price auction.

probabilities and potential payoffs depend on their own and other bidders' bids. The final outcome is then evaluated with respect to any possible outcome from this lottery as a reference point.

It is noteworthy that my approach though similar to Köszegi and Rabin (2006) is slightly different. In an auction where bidders have rational forward-looking expectations, the probability of various auction outcomes could be derived upon placing the bid. It is therefore natural to assume that the bidders anticipate the effects of bidding on both the reference as well as payoff distribution, i.e. both are chosen at the *same* time. This makes my approach slightly different from the Köszegi and Rabin (2006).[2] Because of its *anticipatory* nature, my approach becomes equivalent to Kahnemann and Tversky (1979) and Gul (1991) in induced-value auctions and it is well suited to capture the effect of heterogeneous loss aversion in commodity auctions.

I apply a more general Prospect theory approach which allows non-linear probability weighting and anticipated loss aversion to explore bidding in induced-value auctions (Chapter 2). In chapter 3, I apply a reference-dependent approach to derive potential differences in induced-value and commodity auctions. More specifically, I show that simultaneous presence of money and commodity loss aversion could influence bidding differently than money loss aversion alone. Finally, in chapter 4, I explore the effect of heterogeneous resale or procurement access on bidding.

The dissertation extends the domain of loss averse preferences as applied in other contexts to auctions. This rationalizes aggressive bidding in first-price auctions and

[2] This is discussed in greater detail in the following chapters.

yields some interesting implications for bidding in auctions in general. Highlighting potential differences between induced-value and commodity auctions adds to the current debate on the link between lab and field settings (e.g., Harrison and List 2004; Levitt and List 2007a, 2007b; List 2003). The findings more generally raise some concerns for transferring qualitative behavioral findings from the lab to the field.

Finally in chapter 4, I explore the effect of differences in trading intentions that arise due to heterogeneous access on bidding in auctions. While a formal treatment of how experience effects bidding directly could be more challenging, if market access is interpreted as a proxy for bidder experience, it becomes possible to analyze the indirect effects of experience on bidding in an auction with outside alternatives. In auctions with resale, loss aversion causes underbidding with respect to the risk-neutral-Nash predictions. Bidders with highest access over favorable prices are least affected by loss aversion and therefore bid closer to the risk-neutral-Nash than bidders with smaller access to favorable prices. In auctions with procurement, the attachment effect is such that it causes overbidding (underbidding) for bidders with respect to the risk-neutral-Nash. Bidders with greatest level of market access are again least affected by loss aversion and therefore bid much conservatively and closer to the risk-neutral-Nash than bidders with less favorable access to procurement prices. Thus, the predictions of my model are qualitatively similar to the findings in List (2003, 2004) which suggest that market experience attenuates the endowment effect. Since these indirect effects are observed without altering reference-dependent preferences, it raises the possibility that the effects obtained in List (2003, 2004) may not arise entirely due to direct effect of experience on such preferences.

Chapter 2: Reference-Dependent Preferences in First-Price Auctions

1. Introduction

Auctions have become extremely popular for transferring goods and services. Their use can be traced back to 500 B.C. in ancient Babylon. Since Vickrey (1961)[3] economists have tried to explore bidding and auction outcomes under various experimental settings.[4] In induced-value first-price auctions, subjects bid in excess of the risk-neutral-Nash predictions in laboratory conditions ("Overbidding" Anomaly: Cox et al 1982, 1988, 1996; Harrison 1989). Although risk aversion can explain such aggressive behavior, skepticism surrounding risk aversion as the sole explanation has prompted scholars to explore other behavioral alternatives[5] (Salo and Weber 1995, Goeree et al. 2002, Dorsey and Razzolini 2003, Morgan, Steiglitz and Reis 2003, Kagel 1995, Filiz-Ozbay and Ozbay 2007). In this paper I propose a different alternative which combines elements of Prospect theory: loss aversion and non-linear probability weighting.

In first-price sealed-bid auctions, the probability of winning for a given bid depends on the joint distribution of induced-values, risk attitudes, and the unknown strategies of rival bidders. Thus, missing information about other bidders' induced-values, risk posture, and/or bidding strategies exposes bidders to submit bids

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[3] Vickrey (1961) provides the theoretical foundations of various auction mechanisms.

[4] There is a rich variation of laboratory and field experiments that employ various types of subjects and auctioned objects.

[5] Some other behavioral explanations include-nonlinear probability weighting (ambiguity aversion), spiteful preferences, regret aversion, etc.

in an inherently "ambiguous"[6] environment. Ambiguity effects as captured in Ellsberg paradox (1961) have been observed in market experiments (Camerer and Kunreuther 1989, Sarin and Weber 1993)[7] and could influence bidding in auctions as well (Salo and Weber 1995, Chen et al 2007)[8]. In auctions against human bidders, prior bidding experience[9] could make it easier to derive missing information thereby reducing the level of ambiguity. Moreover, additional controls for missing information have been applied which present even smaller levels of ambiguity in these auctions. For example, when bidding against risk-neutral Nash computer bidders, there is no uncertainty about bidders' risk attitudes and bidding strategies. Therefore, ambiguity effects should become smaller in these auction environments. While efforts have been made to explore the effect of ambiguity on bidding in first-price auctions (Chen et al. 2007) some other behavioral explanations can't explain overbidding in auctions against Nash computer bidders'.[10] In this paper, I exploit the difference between bidding against human bidders versus computer bidders to demonstrate the existence of ambiguity effects as well as another determinant of behavior: loss aversion.

[6] Thus, ambiguity reflects a scenario where missing probabilistic information must be derived.

[7] In Sarin and Weber (1993) the market prices for ambiguous assets were consistently below the corresponding prices for equivalent unambiguous assets. An asset is a two-stage lottery with risk (well-defined probabilities) and ambiguity (probabilities not well-defined). This effect was stronger when these assets were traded simultaneously. However there is weaker evidence that ambiguity affects insurance markets in Camerer and Kunreuther (1989).

[8] Ambiguity (unlike risk) better characterizes decision making in many real-world situations. E.g., the success rate of new drugs, insurance against previously unknown environmental hazards, terrorist activities, outcomes of R&D and success of new products in consumer goods markets (see references in Chen et al. 2007).

[9] In auctions where all bidders have prior experience and act similarly.

[10] Spiteful preferences or ambiguity aversion cannot explain why humans bid aggressively against computers whose bidding strategies are known, and therefore the objective probability of winning the auction conditional on bid can be derived fairly easily or conveyed to human bidders.

I base the analysis in this paper on a model of loss aversion with endogenous reference points similar to Köszegi and Rabin (2006). This is different from an approach with an exogenous fixed reference point in which winning the auction is interpreted as a "gain" while losing leaves the initial wealth unaffected. I argue that the reference point may get influenced by expected gains and therefore auction outcomes could be interpreted as "gains" or "losses." It is plausible that a bidder who draws a high value and expects to win the auction interprets "not winning" as a "loss" and likewise a bidder with low induced-value interprets winning the auction as a "gain." This has been observed in other contexts. For example, loss aversion has been observed in trading of various commodities – from chocolate bars to coffee mugs, coins, or sportscards – for money or other goods (Knetsch 1989; Tversky and Kahneman 1991; Kahneman, Knetsch, and Thaler 1990; Benartzi and Thaler 1995, List 2003). I show that *anticipated* loss aversion by itself (irrespective of other behavioral explanations) can explain aggressive bidding in first-price auctions and captures an important behavioral influence on bidding. Thus, my approach provides a justification for aggressive bidding in auctions where ambiguity effects could be minimal or altogether absent. Other behavioral explanations-spiteful preferences, non-linear probability weighting, anticipated regret aversion, disappointment aversion - could also explain aggressive bidding just like *anticipated* loss aversion. Unlike a regret-based explanation (Filiz-Ozbay and Ozbay 2007), my approach does not rely on ex-post information to explain aggressive bidding; spiteful preferences (Morgan, Steiglitz and Reis 2003) can't explain why human bidders bid aggressively against computer bidders. And finally, when the auction winner earns only the monetary profit as in laboratory experiments,[11] my approach is equivalent to the disappointment aversion model as in Gul (1991).[12]

Two prominent approaches to address ambiguity attitudes in the literature are the maximin expected utility (MMEU model) (Gilboa and Schmeidler 1989) and Choquet expected utility (CEU) model (Gilboa 1987, Schmeidler 1989). I take the CEU approach, which allows subjective distortion of objective probability measures to capture attitudes towards ambiguity, exactly as in Salo and Weber (1995) and Goeree et al (2002). I propose a model of endogenous expectations, similar to Köszegi and Rabin (2006), to accommodate reference-dependent preferences and attitudes towards ambiguity. This is consistent in the spirit of Prospect theory, which allows both non-linear probability weighting and loss aversion.

Theoretically, as special cases of my approach, either *non-linear probability weighting* or *loss aversion* can explain observed bidding outcomes. I show that when bidders are loss averse and fully anticipate potential "losses", overbidding is justified even without non-linear probability weighting. Thus, I suggest loss aversion as an alternative explanation for aggressive bidding in auctions. When I rely on non-linear probability weighting alone, my approach is behaviorally equivalent to previous explanations that explain overbidding in terms of risk aversion or ambiguity-aversion (Salo and Weber 1995; Goeree et al 2002).

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[11] This is different in *field* where auction object is exchanged for a monetary price (bid). In Lange and Ratan (2009) we discuss the differences that could arise between the auctions conducted in *induced-value (laboratory)* settings and *field* in the context of the model, offered here.

[12] Since I allow nonlinear probability weighting, my approach differs from Gul's approach; in the special case of linear probability weighting, the two approaches are similar. This equivalence breaks down in field auctions where the auction object is exchanged for the bid. The implications of a model based on loss aversion for various auction settings are further explored in Lange and Ratan (2009).

Using data from First-price auction lab experiments, I provide evidence that the general approach that combines loss aversion and non-linear probability weighting provides a good fit for observed bids. This approach is capable of addressing the differences in ambiguity across auction environments and explains aggressive bidding in auctions with prior experience (with loss aversion) against (i) experienced human bidders and (ii) risk-neutral Nash-computer bidders. In these auctions, drawing probabilistic inferences (conditional on bids) is relatively easier, and ambiguity effects could be irrelevant,[13] and therefore smaller deviations between subjective and objective probabilities are expected.[14]

I estimate the behavioral parameters in my models using experimental data (Cox et al 1982, Harrison 1989) and test the hypothesis for probability weighting under less ambiguous circumstances. I provide evidence that in auctions against human bidders aggressive bidding can be rationalized as a combination of *loss aversion* and *ambiguity-aversion*; the estimates for loss aversion in auctions with human bidders (irrespective of prior experience) are similar, whereas probability weighting becomes less convex in auctions that present successively reduced levels of ambiguity. This results in smaller deviations between subjective and objective probabilities. When loss aversion is allowed, this yields an almost linear probability weighting in auctions with prior experience against (i) experienced human and (ii) risk-neutral Nash bidders.

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[13] In an experiment reported in Dorsey and Razzolini (2003), the probability of winning conditional on bids is conveyed to the subjects.

[14] The evidence on ambiguity attitudes suggests that ambiguity aversion is more prevalent. In addition to the experiments that are replications of the Ellsberg paradox (Fox and Tversky 1998), Sarin and Weber (1993) find that the price of ambiguous two-stage lotteries is lower than equivalent unambiguous lotteries obtained through double-market auctions.

In the following sections I motivate the general Prospect theory model for bidding in auctions (sections 2 and 3). I apply the model to auctions with risk-neutral Nash bidders (section 4), and analyze the experimental data in sections 5 and 6. Finally, I discuss my results and conclude (sections 7 and 8).

2. Prospect Theory: Reference-Dependence and Non-linear Probability Weighting

In this section I describe the behavioral assumptions in my approach to address bidding in laboratory first-price auctions. In laboratory auctions, values are induced and profits are paid in monetary units. Thus, consumption occurs in a single dimension.[15] Following Köszegi and Rabin (2006), an individual's utility $u(c \mid r)$ depends both on her consumption $c \in \mathbb{R}$ and her reference level $r \in \mathbb{R}$. The "direct" consumption utility $v(c)$ is obtained when realized consumption is the same as the reference level, i.e., $v(c) = u(c \mid c)$, and the individual utility when her consumption differs from her reference is defined as

$$u(c \mid r) = v(c) - k_l \max[0, v(r) - v(c)] \qquad (1)$$

with $0 \le k_l$. k_l is the scalar gradient which captures the sensation of "loss" when less favorable outcomes are realized.[16]

Ex ante, both reference levels and consumption could be stochastic. Following Köszegi and Rabin (2006), the reference level is a probability measure G over $\mathbb{R}$

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[15] Unlike laboratory auctions where induced-values are induced in money and profits are paid in monetary units, in real auctions the object is awarded to the winner in return for money. In Lange and Ratan (2009), we discuss the implications arising from this difference when loss aversion associated with the object and money may differ.

[16] I normalize psychological "gains" to zero.

and consumption is drawn according to the probability measure H over . Then, the individual's overall expected utility over risky outcomes is given by

$$U(H \mid G) = \int \int u(c \mid r) dG(r) dH(c) \qquad (2)$$

In an equilibrium (for a first-price auction) captured by a strictly increasing symmetric bidding function, the bid determines the probability of winning and the consequent profits for a bidder. Since no further action is possible after placing the bid, the bid not only defines the probability of consumption outcomes (H) but also defines the probability of reference outcomes (G). Thus, for a bidder with rational expectations $H = G$, and the reference point G is endogenously determined.[17]

The other important feature of prospect theory is non-linear probability weighting (Kahneman and Tversky 1979). As discussed earlier, auction environments could vary in terms of underlying ambiguity. Two prominent approaches to address ambiguity attitudes in the literature are maximin expected utility (MMEU) (Gilboa and Schmeidler 1989) and Choquet expected utility (CEU) (Gilboa 1987, Schmeidler 1989). In the MMEU model, decision makers have a set of priors over outcomes and choose the actions that maximize the minimum expected utility over the set of priors. In the CEU model, decision makers' beliefs are represented by a set of non-additive probability measure (capacities).[18] I take the CEU approach, which allows subjective

[17] Alternative reference-dependent models with endogenous definition of reference points are given by Sugden (2003) and Munro and Sugden (2003) who assume the reference to be given by the *current endowment* which might adjust over the time. One other fixed reference could be the weighted expected value of the prospect, which is also determined endogenously in one-shot games (Kahnemann and Tversky 1979).

[18] Some recent contributions aim at characterizing ambiguity without restricting attention to specific decision models, or functional-form considerations. E.g., Klibanoff, Marinacci and Mukerji (2005).

distortion of objective probability measures to capture attitudes towards ambiguity.[19] Ambiguity effects should become smaller in auctions with prior bidding experience and/or against risk-neutral Nash bidders[20], thereby producing smaller distortions of objective probabilities. I therefore assume that each bidder distorts the objective probability measure P through the following probability weighting function as in Salo and Weber (1995) and Goeree et al (2002):

$$\omega(P) = P^{\beta} \text{ where } \beta > 0 \quad (3)$$
[21]

Under this assumption H and G in (2) could be non-linearly weighted measures of probability as defined in (3).[22] Thus an individual solves the following program:

$$\max U(H \mid H) \quad (4)$$

This specification is however slightly different from the general setting discussed by Köszegi and Rabin (2006). In their approach, action takes place *after* a reference distribution has been formed. Given a reference distribution G, the individual therefore chooses $H(G)$ to maximize $U(H|G)$. In equilibrium, rational expectations then require that the consumption distribution is chosen such that it is consistent with

[19] Thus, I assume that probability distortions arise entirely as a response to ambiguity. This approach is similar to Salo and Weber (1995) and Goeree et al (2002).
[20] Since deriving missing information about rivals' risk attitudes and equilibrium bidding strategies could become easier.
[21] β governs the elevation of the probability weighting function with respect to the 45-degree line. The 45-degree line describes linear probability weighting. $\beta < (>) 1$ implies overweighting (underweighting) of probability. This functional specification is a special case of the probability weighting function described in Prelec (1998): $\omega(P) = \exp(-\beta(-\log P)^{\alpha})$, in which $\alpha = 1$; thus, my approach is less general. Moreover, in previous attempts to fit the more general form for bidding in first-price auctions, I found that $\alpha \to 1$. Later I discuss other evidence in the literature that supports this functional form for uncertain circumstances where probabilities are derived and not known exclusively.
[22] Later, I show how the auction outcomes are weighted in my model.

the formulation of the reference point, i.e. $H(G)=G$. In sealed-bid auction equilibrium, given the beliefs of bidders' bidding strategies, a bid uniquely determines the probability of various auction outcomes for each bidder. This allows the formulation of a probability distribution over consumption and reference outcomes simultaneously. A rational bidder applies the same weighting to the objective probability measure associated with reference and consumption levels. This allows a complete specification of overall expected utility for a bidder who fully anticipates ensuing "losses" as defined in (4).

3. The First-Price Auction Environment

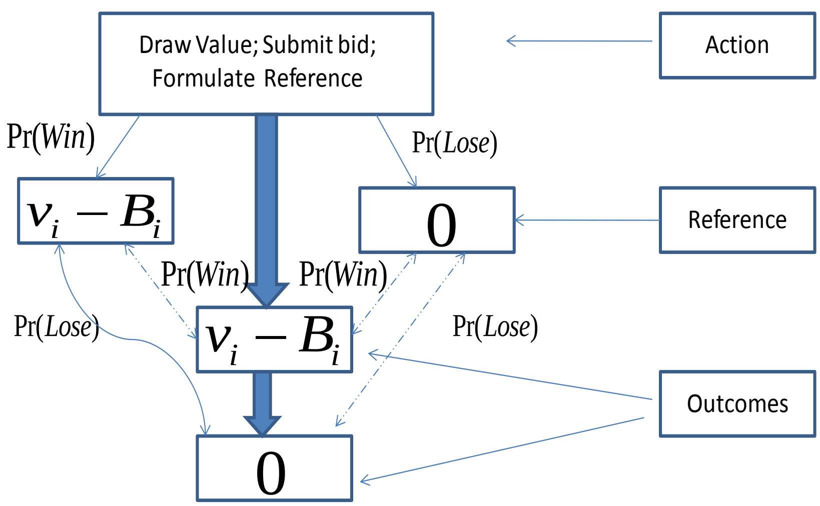
In this section I discuss the bidding problem in a first-price auction for a bidder with behavioral characteristics as described in the previous section.

I consider n bidders $i=1,...,n$ and assume symmetric behavioral preferences, i.e. that bidders share the same characteristics for loss aversion and probability weighting; this is common knowledge. In my framework, unique identification of risk preferences and non-linear probability weighting is not possible. Therefore, bidders are assumed to be risk-neutral in the numeraire consumption, i.e. $v_i(c)=c$. In the laboratory auction, v_i is directly induced in monetary units. Each bidder draws her induced-value v_i from a probability distribution defined by the distribution function F defined over $[\underline{v},\bar{v}]$ ($\bar{v}\geq\underline{v}\geq 0$); each bidder knows his induced-value, and knows that other bidders' induced-values are also drawn independently from distribution F.[23]

[23] In laboratory auctions, overbidding beyond induced-value entails negative payoff and is always suboptimal.However, in Harrison (1989) this restriction is not imposed explicitly.

The bidding problem for a typical bidder in a laboratory first-price auction is described in figure 1. In equilibrium for symmetric bidders, which can be depicted through a strictly increasing bid function $B_j(v_j) = B(v_j)$ where $j \neq i$ for all other bidders, a bid B_i for bidder i defines her objective probability of winning the auction. This is weighted non-linearly by the bidder. Thus, a bidder can formulate an endogenous reference lottery for each feasible bid that captures his expectations (beliefs) of various auction outcomes. The auction outcome follows. Ex-ante, losing the auction could be interpreted as loss and weighted with respect to the endogenous reference formulated at the time of bidding.

Figure 1: Bidding Problem in a First-Price Auction



Note that a bidder's reference is defined by her beliefs about the relevant outcomes held between the time she formulates her bid and shortly before the auction outcome is observed. The degenerate utility in a first -price sealed-bid auction that captures the gain-loss utility as described in (1) takes the following values:

$$u_{PT}(c|r)=\begin{cases} v_i - B_i & when\ \ c=r=v_i - B_i \\ v_i - B_i & when\ \ c=v_i - B_i, r=0 \\ -k_l(v_i - B_i) & when\ \ c=0, r=v_i - B_i \\ 0 & when\ \ c=r=0 \end{cases}$$

The overall expected utility for a bidder with preferences as given in section 2 (based on conditions (1)-(4)) is given by:

$$\pi_{PT}(v_i,B_i)=\omega(f(B_i))(v_i-B_i)-k_l\omega(f(B_i))(1-\omega(f(B_i)))(v_i-B_i) \qquad (5)$$

where $f(B_i)$ and $\omega(f(B_i))$ are the objective and weighted probability of winning for a given bid. The first term is the weighted expected direct consumption utility (value) and the second captures the psychological "losses" when the bidder unexpectedly loses the auction.[24] Note that bidding yields nonnegative payoff for moderate levels of loss aversion; for high levels of loss aversion bidding $B_i = v_i$ maximizes overall payoff.[25] Also note that weighted expected value is also determined endogenously for an equilibrium bid and could be used as a fixed reference to evaluate the reference-dependent utility of various outcomes (Kahnemann and Tversky 1979). This is equivalent to the lottery (Köszegi-Rabin) approach as discussed in the previous section and yields the same overall expected utility as in (5). [26]As mentioned before, with linear probability weighting and induced-value (laboratory) settings where

[24] Note when there is no loss aversion, $k_l = 0$, this becomes a probability weighted model alone. In addition, when there is no probability weighting, this becomes a risk-neutral Nash model.
[25] The maximum payoff in this case is zero.
[26] In the fixed reference approach, only the auction outcome of not winning yields psychological loss with respect to the reference of the expected value for a given bid.

auction winner earns the monetary profit, my approach is equivalent to the disappointment aversion model as in Gul (1991).[27]

It should be noted that (5) implies that a non-negative expected utility gain $\pi_{PT}(v_i, B_i)$ from participating in the auction can only result if $1 > k_l(1-\omega(f(B_i)))$. That is, auction yields positive utility only for bidders with $\omega(f(B_i)) > 1-1/k_l$. If $k_l < 1$, this condition holds for all bidders. If $k_l \geq 1$, the condition implies only bidders with a sufficiently large probability to win derive positive payoff from placing positive bids.

I restrict attention to symmetric monotonically increasing equilibria in pure strategies. In equilibrium, the chances of player i to win are given by $H^{n-1}(v_i)$. With the above argument, auction yields positive utility only if $\omega(H^{n-1}(v_i)) > 1-1/k_l$. Given $\omega(.)$, the threshold value $\hat{v}(k_l)$ beyond which positive utility is realized is defined by

$$\omega(H^{n-1}(\hat{v}(k_l))) = \max[0, 1-1/k_l] \tag{5a}$$

Note that $\hat{v}(k_l) = \underline{v}$ if $k_l \leq 1$. Bidders with $v_j \in [\hat{v}(k_l), \overline{v}]$ shall place positive equilibrium bids that yield positive overall payoff. Maximizing (5) with respect to B_i yields a strictly increasing (optimal) bid function.

[27] In Gul's model, disappointment could arise from paying a higher than expected price and/or losing the profit (based on higher price) due to losing the lottery. In a first-price auction, the price paid equals the bid in case of winning; so the only source of disappointment arises from not realizing the expected profit (certainty equivalent- $f(B_i)(v_i - B_i)$) when the auction is lost which occurs with probability $(1-f(B_i))$. The last term therefore fully captures the disappointment as discussed in Gul (1991).

Proposition 1: **(-First Price Auction against Human bidders-)** *The unique monotonically increasing symmetric Bayesian Nash equilibrium (BNE) bid function for loss averse bidders who weigh probabilities non-linearly is*

$$B(v_i)_{PT} = \begin{cases} \dfrac{\int_{\hat{v}(k_l)}^{v_i} x[1-k_l(1-2\omega(F^{n-1}(x))]d\omega(F^{n-1}(x))}{\omega(F^{n-1}(v_i))[1-k_l(1-\omega(F^{n-1}(v_i)))]} & \text{for} \quad v_i \geq \hat{v}(k_l) \\ v_i & \text{for} \quad v_i < \hat{v}(k_l) \end{cases}$$

Proof: See Appendix.

It is clear from the above that (i) $\hat{v}(k_l)$ varies with k_l and β and (ii) for bidders with $v_i \geq \hat{v}(k_l)$ the equilibrium bid depends on k_l and β.[28] Thus, for $v_i \geq \hat{v}(k_l)$ we can explore the marginal effects of changes in k_l and β on equilibrium bids.

Proposition 2 (i) ***(-Effect of loss aversion-)*** *Greater loss aversion yields aggressive bidding, i.e.,* $\frac{\partial B_{PT}}{\partial k_l} > 0$ **(ii)** ***(-Effect of probability weighting-)*** *Greater* β *(more convex probability weighting) yields more aggressive bidding, i.e.* , $\frac{\partial B_{PT}}{\partial \beta} \geq 0$ *except when* $0.9951 < k_l < 1$ *and bidders with very small induced-values such that* $Z(y_i) = 2k_l^2 y_i^2 + 3k_l(1-k_l)y_i + (1-k_l)^2 + k_l(1-k_l)y_i \ln y_i < 0$ *(where* $y_i = F(v_i)^{\beta(n-1)}$ *) bid less aggressively i.e.* $\frac{\partial B_{PT}}{\partial \beta} < 0$.

Proof: See Appendix.

[28] For $v_i < \hat{v}(k_l)$ equilibrium bid $B(v_i) = v_i$ does not depend on k_l and β.

Intuitively, the tradeoff that determines the optimal bid for loss averse bidders differs from the tradeoff without loss aversion. Loss averse bidders are willing to pay a higher price to avoid the "losses" from not realizing the profits upon winning. This induces more aggressive bidding for any monotonic probability weighting. Thus, anticipated loss aversion by itself explains overbidding with respect to risk-neutral Nash equilibrium.[29] For example, if the ambiguity confronting the bidder is smaller, (such that ambiguity effects could be smaller or altogether irrelevant[30]) then anticipated loss aversion would suffice to rationalize aggressive bidding.

Before I explore the effect of probability weighting on equilibrium bidding it is noteworthy that bidders could avoid "losses" in the following ways: (a) if the value draw is not high enough then bid upto their value to avoid "losses", (b) and if the value draw is high enough they could either bid (i) more aggressively or (ii) less aggressively, in response to more convex probability weighting. In the latter scenario, when the value draw is high enough less aggressive bidding could happen because bid also affects the expectation of auction outcomes simultaneously. Higher β means lower elevation of the probability weighting curve and causes more aggressive bidding which suggests ambiguity-aversion (or bidder pessimism) in most circumstances except the following: when $0.9951 < k_l < 1$ some bidders with very small

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[29] In addition to other behavioral influences that could justify aggressive bidding with respect to the RNNE bid.

[30] For example, in auctions with experienced bidders and/or against risk-neutral Nash bidding strategies, deriving missing information regarding the probability of winning for a bid could be easier. Such auctions therefore present smaller levels of ambiguity for a bidder.

induced-values could bid less aggressively. [31] Therefore, as a special case of Proposition 1, one can justify aggressive bidding entirely as a response to underlying ambiguity with non-linear probability weighting (without loss aversion $k_l = 0$). Aggressive bidding with respect to the RNNE would then suggest that "ambiguity-aversion" or "bidder pessimism" causes underweighting the probability of winning for given bids (Salo and Weber 1995, Goeree. et al 2002).

Proposition 3: G*reater competition (more bidders) yields more aggressive bidding, i.e.,* $\frac{\partial B_{PT}(v_i)}{\partial n} \geq 0$ *except when* $0.9951 < k_l < 1$ *and bidders with very small induced-values such that* $Z(y_i) = 2k_l^2 y_i^2 + 3k_l(1-k_l)y_i + (1-k_l)^2 + k_l(1-k_l)y_i \ln y_i < 0$ *(where* $y_i = F(v_i)^{\beta(n-1)}$ *) bid less aggressively i.e.* $\frac{\partial B_{PT}(v_i)}{\partial n} < 0$.

Proof: See Appendix.

The marginal response to greater competition is similar to the effect of probability weighting; as before, when value draw is high enough, bidders could avoid "losses" by bidding more or less aggressively, in response to more competition; this happens because their bid affects their expectation of auction outcomes simultaneously. The effect of greater competition is analogous to more convex probability weighting and causes aggressive bidding in most circumstances except the following: when

[31] For any bidder, more convex probability weighting, affects overall payoffs by affecting the weighted probability of winning, direct expected payoff and anticipated "losses"; for most bidders the net effect of more convex probability is such that it yields more aggressive bidding; however when $0.9951 < k_l < 1$ for some bidders with low induced-values the net effect could yields less aggressive bidding.

$0.9951 < k_l < 1$ some bidders with very small induced-values could bid less aggressively. Thus, despite behavioral preferences, in most circumstances bidders respond to greater competition along conventional lines by bidding more aggressively.[32]

In the following sections, I provide evidence that my approach which allows loss aversion performs quite well in induced-value auctions, but identifying suitable reference points[33]presents a major challenge in applying Prospect theory based approaches to other contexts, e.g., in common value auctions.

As discussed earlier, the general model is capable of addressing the differences in ambiguity across auction environments. Intuitively, ambiguity effects should become smaller in auctions where bidders have prior bidding experience against (i) experienced human bidders and (ii) risk-neutral Nash bidders, thereby producing smaller deviations between weighted and objective probabilities. I shall explore this hypothesis in the following section. It should be noted, however, that bidding against Nash risk-neutral bidders is not a special case of the equilibrium bid as discussed so far. Instead, it merely represents the best response of the player. In the following I derive the best response bid under given behavioral assumptions in these auctions.

4. Auctions against Nash (risk-neutral) bidders

In this section I address bidding in induced-value auctions against Nash risk-neutral computer bidders. In these auctions, bidders are informed ex-ante that other bidders

[32] Except when $0.9951 < k_l < 1$ some bidders with very small induced-values the net effect of greater competition yields less aggressive bidding, as in the case of probability weighting before.

[33] How people develop reference points could be contextual and plausible reference points could differ under different circumstances.

always bid a certain fraction (the risk-neutral Nash bid) of their induced-values.[34] The auction environment is the same except that bidders face Nash risk-neutral computer bidders. There is no uncertainty in these auctions about risk attitudes and equilibrium strategies that rival bidders employ. Thus, the ambiguity confronting the bidder becomes smaller in these auctions. Some other behavioral explanations for overbidding (considered in isolation) do not apply in these auctions. E.g., it is unlikely that humans will harbor spite against computer bidders; thus, spiteful preferences cannot explain aggressive bidding in these auctions. Similarly, the estimates of risk aversion obtained in these auctions are not similar to those observed in auctions against human bidders.[35] Although combining risk aversion with spite could explain overbidding against risk-neutral Nash computerized bidders, such a model by itself is not capable of addressing the changes in ambiguity on bidding behavior in these auctions.[36] The Prospect theoretic framework that I motivated earlier is capable of addressing changes in ambiguity on bidding in these auctions.

Each bidder relies only on her induced characteristics, as described in the preference structure defined in (1)-(4).[37] Consistent with the experimental setup, I assume that induced-values are drawn from a uniform distribution over the support $[0,1]$. Since it is known that rival bidders' bids are Nash (risk-neutral) best responses,[38] the bidder

[34] In some variants of these experiments (Dorsey and Razzolini 2003), probability of winning, conditional upon bids was also shown to bidders.

[35] This is obvious by looking at the estimates of the probability-weighted model (no loss aversion) in auctions against risk- neutral Nash bidders (Table 5). Variations in probability weighting would therefore suggest variation in risk attitudes.

[36] Among other explanations, ambiguity aversion and risk aversion could also rationalize bidding outcomes in these auctions. However, uniquely identifying risk and ambiguity attitudes could be extremely difficult when they are modeled together.

[37] We don't need to assume symmetric behavioral characteristics to derive the optimal bid response.

[38] For example, in a first-price auction with 4 bidders, computers always bid three-quarters of their induced-value.

need not take into account the strategic consequences of his bids. This yields the following overall expected utility for the bidder who maximizes expected payoffs:

$$\max_{\underline{v} \le B_i \le \overline{v}} \quad \pi_{PT}(v_i, B_i) = \left[\omega((\theta B_i)^{n-1}) - k_l \omega((\theta B_i)^{n-1})(1 - \omega((\theta B_i)^{n-1}) \right] (v_i - B_i) \qquad (7)$$

where $\theta = n/(n-1)$, $(\theta B_i)^{n-1}$, and $\omega((\theta B_i)^{n-1})$ are the objective and weighted probability of winning conditional on bid. The first term is the weighted direct consumption utility (value) and the second captures the psychological "losses" when the bidder loses but had expected to win the auction. Given the risk-neutral-Nash opponent bidders, bidders can ensure winning by placing a bid- $(n-1)\overline{v}/n$.

As before, (7) implies that a non-negative expected utility gain $\pi_{PT}(v_i, B_i)$ from participating in the auction can only result if $1 > k_l(1 - \omega(f(B_i)))$. That is, auction yields positive utility only for bidders with $\omega(f(B_i)) > 1 - 1/k_l$. If $k_l < 1$, this condition holds for all bidders. If $k_l \ge 1$, the condition implies only bidders with a sufficiently large probability of winning shall derive positive payoff from the auction by placing positive bids.

I restrict attention to symmetric monotonically increasing equilibria. In equilibrium, the chances of player i to win, are given by $(\theta B_i)^{n-1} = H^{n-1}(v_i)$. With the above argument, auction yields positive utility only if $\omega(H^{n-1}(v_i)) > 1 - 1/k_l$. Given $\omega(.)$, the threshold value $\hat{v}(k_l)$ beyond which positive utility is realized is defined by

$$\omega(H^{n-1}(\hat{v})) = \max[0, 1 - 1/k_l] \qquad (7a)$$

Note that $\hat{v}(k_l) = \underline{v}$ if $k_l \le 1$. For bidders with $v_i \le \hat{v}(k_l)$ bidding their induced-value ensures maximizes overall payoff. Bidders with $v_j \in [\hat{v}(k_l), \overline{v}]$ shall place positive equilibrium bids that yield positive overall payoff. Maximizing (7) with respect to B_i yields a strictly increasing (optimal) bid response function.[39]

Proposition 4: **(-First-price auction against Nash bidders-)** *The unique optimal bid for loss averse bidders who weigh probabilities non-linearly (against Nash risk-neutral bidders) is captured through the following monotonic relationship:*

$$v_i = \begin{cases} \min\left\{ B_i + \dfrac{B_i}{(n-1)\beta}\left(\dfrac{1\text{-}k_l + k_l(\theta B_i)^{(n-1)\beta}}{1\text{-}k_l + 2k_l(\theta B_i)^{(n-1)\beta}}\right), \dfrac{n\text{-}1}{n}\overline{v} \right\} & \text{for} \quad v_i \geq \hat{v}(k_l) \\ B_i & \text{for} \quad v_i < \hat{v}(k_l) \end{cases}$$

Proof: See Appendix.

It is clear from the above that (i) $\hat{v}(k_l)$ varies with k_l and β and (ii) for bidders with $v_i \geq \hat{v}(k_l)$ the equilibrium bid depends on k_l and β. For $v_i \geq \tilde{v}$, the optimal bid attains a corner solution i.e. $B_i + \dfrac{B_i}{(n-1)\beta}\left(\dfrac{1\text{-}k_l + k_l(\theta B_i)^{(n-1)\beta}}{1\text{-}k_l + 2k_l(\theta B_i)^{(n-1)\beta}}\right) = \dfrac{n\text{-}1}{n}\overline{v}$. This suggests that beyond the threshold induced-value $\tilde{v}$ it is optimal for bidders to bid $(n-1)\overline{v}/n$ that ensures winning the auction. If a bidder chooses a bid below $(n-1)\overline{v}/n$ and anticipates "losses", then her bid is adjusted against loss aversion. For bidders with $v_i < \hat{v}(k_l)$ equilibrium bid $B(v_i) = v_i$ does not depend on k_l and β. As a special case, when bidders are not loss averse and do not weigh probabilities non-linearly, this yields a best response in a Nash equilibrium. This allows characterizing the effect of loss aversion on bidding.

[39] For all plausible parameters β and k_l the payoff function has a unique interior or corner optimum.

Proposition 5: **(i) (-Effect of Loss Aversion-)** *In auctions with induced-values (against Nash risk-neutral bidders), for* $\hat{v}(k_l) \le v_i < \tilde{v}$ *loss aversion induces more aggressive bidding, i.e.* $\frac{\partial B_{PT}}{\partial k_l} > 0$ **(ii) (-Effect of probability weighting-)** *Greater* β *(more convex probability weighting) yields more aggressive bidding i.e.* $\frac{\partial B_{PT}}{\partial \beta} \ge 0$ *except when* $k^* < k_l < 1$ *and bidders such that* $[1-k_l(1-2\omega(\theta B_i))] < \frac{n\beta(k_l-1)\ln(\theta B_i)\omega(\theta B_i)k_l}{[1-k_l(1-\omega(\theta B_i))]}$*, bid less aggressively i.e.* $\frac{\partial B_{PT}}{\partial \beta} < 0$.

Proof: See appendix.

This suggests that loss aversion has no effect on bidding when bidders either have very high or low induced-values. Beyond a certain threshold induced-value $\tilde{v}$ it is optimal to bid $(n-1)\bar{v}/n$ and ensure winning the auction against risk-neutral Nash computer bidders.[40] Bidders with very low induced-values, avoid "losses" by bidding their upto their value. However, for most bidders with intermediate range of induced-values, anticipated loss aversion justifies aggressive bidding, with or without non-linear probability weighting. Since the role of probability weighting is limited in these auctions, loss aversion by itself provides a sufficient justification for aggressive bidding, as evident in auction outcomes obtained through laboratory experimentation.

While discussing the effect of probability weighting on bidding, it is important to understand that the effect of probability weighting in such auctions could

[40] Note that, $(n-1)\bar{v}/n$ is the highest possible bid in a risk-neutral Nash model.

be limited. Nevertheless, just like in auction against human bidders, bidders could avoid "losses" in the following ways: (a) if the value draw is not high enough then bid upto their value to avoid "losses", (b) and if the value draw is high enough they could either bid (i) more aggressively or (ii) less aggressively, in response to more convex probability weighting; this happens because bid affects the expectation of auction outcomes simultaneously. Higher β means lower elevation of the probability weighting curve and in most circumstances causes more aggressive bidding which suggests ambiguity-aversion (or bidder pessimism); except when $k^* < k_l < 1$ and for v_i such that $[1-k_l(1-2\omega(\theta B_i))] < \frac{n\beta(k_l-1)\ln(\theta B_i)\omega(\theta B_i)k_l}{[1-k_l(1-\omega(\theta B_i))]}$, more convex probability weighting causes less aggressive bidding.

Proposition 6: (-Effect of greater competition-) *For most human bidders (in most circumstances) greater competition yields more aggressive bidding i.e.* $\frac{\partial B_{PT}}{\partial n} > 0$ *except when* $\hat{k} < k_l < 1$ *and bidders such that* $[1-k_l(1-2\omega(\theta B_i))] < \frac{(n-1)[\beta(1-\theta B_i) - B_i \ln(\theta B_i)]\omega(\theta B_i)k_l}{[1-k_l(1-\omega(\theta B_i))]}$, *bid less aggressively i.e.* $\frac{\partial B_{PT}}{\partial n} < 0$.

Proof: See Appendix

The marginal response to greater competition (more bidders) is similar to the marginal effect of probability weighting; as before, bidders could bid more or less aggressively, in response to greater competition; this happens because their bid also affects their expectation of auction outcomes simultaneously. The effect of greater competition is analogous to more convex probability weighting and in most circumstances causes more aggressive bidding; for $\hat{k} < k_l < 1$ and for v_i such that $[1-k_l(1-2\omega(\theta B_i))] < \frac{(n-1)[\beta(1-\theta B_i) - B_i \ln(\theta B_i)]\omega(\theta B_i)k_l}{[1-k_l(1-\omega(\theta B_i))]}$, bidders bid less aggressively in response to greater competition. Thus, despite behavioral preferences, in most circumstances bidders respond to greater competition along conventional lines by bidding more aggressively.

In the following section I fit the general model with probability weighting and loss aversion and its restricted versions which take into account loss aversion and non-linear probability weighting in isolation to explain bidding using data from auctions with (i) human bidders and (ii) risk-neutral Nash computer bidders. Note that the equilibrium bidding behavior as specified in Propositions 1 and 3 differs across these auctions.

5. Empirical Analysis

Data

I use data from induced-value first-price auctions reported in Cox et al. (1982) and Harrison (1989). Cox et al. (1982) reports 210 auctions with different number of bidders, totaling 1170 bids in first-price auctions.[41] A description of the data in Cox et al. (1982) is provided in Table 1.

[41] I ignore auctions with 3 bidders in these experiments. The results for these auctions are considered anomalous, and breakdown of non-cooperative bidding is suspected (Cox et al. 1982)

[Table 1 here]

The experiments in Cox et al. (1982) employed undergraduate students enrolled in introductory economics classes at the University of Arizona and Indiana University and were conducted over a number of years in the 1980s. The results based on this data have formed a benchmark for investigation of bidding outcomes in first-price auctions experiments (see Harrison 1989, Salo and Weber 1995, Goeree et al. 2002). The first-price auctions were conducted in sessions along with Dutch and second-price auctions for single (hypothetical) objects. All sessions consisted of 30 sequential auctions (e.g., 10 Dutch, 10 first-price, and 10 Dutch). These auctions had the following features: Identifying variables include auction series, type of auction, observed bid/price, number of bidders, period, subject, and the support of the uniform distribution from which induced-values were drawn and induced. Bidders were paid $3.00 for participation and a series of 30 auctions had an expected profit of $12. Thus, the total expected earnings were about $15 per subject. A session lasted for about one hour. Induced-values (in discrete multiples of 10 cents) were induced from uniform distributions with support over 0 and an upper limit that varied across different sets of auctions (see Table 2 for description). The number of bidders and the support from which induced-values were drawn (with replacement) were varied such that expected gains were similar across auctions. Overbidding beyond induced-values was not allowed, the object was awarded to the highest bidder at his bid, and the winning bid was displayed after the auction was concluded. The winner's identity and bid were not conveyed to the other bidders.[42] The summary statistics of the data reported in Cox et al (1982) is provided in table 2.

[Table 2 here]

The series of auctions where bidders have prior experience of bidding in first-price auctions have a suffix “x” in the name (see Table 1). Thus, for auctions with 4 and 5 bidders, we can explore the effect of “experience” on behavioral parameters.

I also use data from Harrison (1989) in addition to Cox et al. (1982). Six experimental sessions were conducted using the design indicated in Table 3. The general procedures follow those introduced by Cox, Smith and Walker (1985b) and Cox et al. (1988), and are broadly similar to Cox et al. (1982). All subjects were economics undergraduates at the University of Western Ontario and received \$3 just for showing up at the experimental session. The expected profit for a session of 20 auctions was roughly \$10. Therefore, total expected earnings were \$13 for each subject. All experimental sessions had 4 bidders whose induced-values were drawn from a uniform distribution with lower and upper valuations of \$0.01 (or 1 point) and \$10.00 (or 1000 points). A description of the data reported in Harrison (1989) is provided in Table 3.

[Table 3 here]

I restrict my analysis to auctions with dollar payoff and compare the auctions with auctions involving inexperienced human rival in the following treatments: (i) subject experience and (ii) use of computer-simulated “Nash risk-neutral bidders.” Subjects

[42] This is quite unlike in real first-price auctions where such information can be public. The non-availability of ex-post information that becomes the basis of “regret” therefore weakens anticipated “regret” as an explanation for overbidding in these auctions (Filiz-Ozbay and Ozbay 2007). Note that my explanation is invariant to ex-post information structure in these auctions.

have a similar level of experience in series 1, 2, and 3, respectively. In auctions against risk-neutral Nash bidders, a computer entered risk-neutral Nash equilibrium bids for the 3 bidders that each human bidder faces in an auction. Subjects were informed ex-ante that the computer would bid 75% of the valuation that it drew for each of the 3 simulated bidders. The auctions in Harrison (1989) are different from the auctions in Cox et al. (1982) in the following ways: Bidding beyond induced-value is allowed in Harrison (1989). Bidders (human or simulated) were assigned randomly in each period. This controls for the use of multi-period strategies that can be employed when this randomization procedure is not in use. Valuations vary across bidders in a given replication and across periods. Each replication in a given period also employs the same N valuations, since replications occur simultaneously in a given experiment. The summary statistics of the auctions in Harrison (1989) is provided in Table 4.

[Table 4 Here]

Pooling of data

1. Induced-value distributions were varied across auctions with varying numbers of bidders in Cox et al. (1982) such that expected gains from participation in auctions were roughly similar. In my framework this design may not have the desired effect. Also, auctions with different numbers of bidders may present unique levels of ambiguity. Therefore, I do not pool the data from all the auctions together.
2. In Cox et al. (1982) there are two series of auctions, “fdf” and “dfd” each composed of 10 consecutive auctions of a type. For example, “fdf” represents

10 first-price, 10 dutch, and 10 first-price auctions, and "dfd" represents 10 dutch, 10 first-price, and 10 dutch auctions. I pool data from 20 first-price auctions from the series "fdf" and 10 first-price auctions from the series "dfd".

Similarly, data from 20 sequential first-price auctions are pooled together from Harrison (1989). As observed earlier, randomization procedures adopted in Harrison (1989) control for the use of multi-period strategies that can be employed when this randomization procedure is not in use. (1989).

An overview of bidding behavior

An overview of bidding across auctions in Cox et al. (1982) and Harrison (1989) (in tables 2 and 4) reveals the following: (a) in auctions with 4 bidders, the number of bids above the risk-neutral Nash (henceforth overbids) ranges between 81-91% in Harrison (1989), as compared to 77.5-82.5% in Cox et al. (1982); (b) and in auctions with 5 or more bidders in Cox et al. (1982), the number of overbids ranges between 66-86%. For all auctions (a) the amount by which bids exceed the risk-neutral Nash bids (overbid[43]) in Harrison (1989) is also higher (around 22%) than in Cox et al. (1982) (around 16%) and (b) the percentage absolute deviation[44] around RNNE is also higher in Harrison (19-24%) than in Cox et al. (1982) (12-20%).

In Cox et al.(1982)(a) in the second set of auctions with 6 bidders (series b), the number of overbids is substantially lower (66.7%) than in any other auctions; the average percentage overbid is also the lowest among all auctions, whereas the average percentage bid below the risk neutral Nash (henceforth underbid) is similar to

[43] Overbid=(bid-RNNE)/RNNE; Underbid=(RNNE-bid)/RNNE.
[44] Absolute deviation=abs(bid-RNNE)/RNNE.

other auctions; (b) in the other set of auctions with 6 bidders (series a) the number of overbids is 78.3%, which is similar to other auctions, but the average percentage underbid is around 23%, which is somewhat high; (c) in both series of auctions with 6 bidders, 4 out of 10 bidders bid below RNNE in 50% of the auctions; and (d) in auctions with 9 bidders, low valuation bidders tended to bid close to zero, which yields an unusually high average underbid of around 27% below RNNE; 4 out of 10 bidders bid below RNNE 50% of the time. Clearly, observed bids reflect differences in auction procedures, payoffs, and bidder characteristics.

In Harrison (1989) prior experience seems to affect bidding in against human bidders and against Risk-neutral Nash bidders. The number of bids above RNNE declines from 91% in auctions with inexperienced bidders to 89% in auctions with experienced bidders. This further declines to 81% in auctions with experienced bidders who face Nash bidders (see Table 4). The average percentage overbid above the RNNE declines from 23% to 21% in auctions against human bidders. This declines further to 18% in auctions with experienced bidders against Nash bidders. The average percentage absolute deviation around RNNE declines from 24% to 21% in auctions against human bidders. This further declines to 19% in auctions with experienced bidders against Nash bidders.

Such effects are not obvious in auctions in Cox et al. (1982). In auctions with 4 bidders, number of overbids increase from 77.5% with inexperienced bidders to 82.5% with experienced bidders. However, average overbid (underbid) declines from 16.3% (34.2%) to 15.5% (20.9%). This yields a decline in average absolute deviation around RNNE from 20% to 16.3%. Thus, prior experience lowers absolute deviation around RNNE. However, an opposite effect is observed in auctions with 5 bidders. Although the number of bids with prior experience above RNNE declines from 86.7% to 80%, the average percentage overbid declines from 14.2% to 13.8%; the average percentage underbid however increases from 17.6 to 20.5%. The average percentage absolute deviation around RNNE increases from 14.6% to 15.1%. Clearly, the effect of experience in auctions with 5 bidders, in terms of average percentage absolute deviations around RNNE, is different from that observed in other auctions.

Omitted Observations

In Cox et al. (1982), I estimate the parameters for different levels of competition without pooling the data. In auctions with 9 bidders, bidders with low induced-values tend to bid close to zero, clearly suggesting that cognitive costs of bidding exceed potential gains from optimal bidding. All bids that suggest more than 20% absolute deviation around RNNE (most of these bids are underbids close to zero) are therefore ignored for estimation purposes. I ignore bids that exceed induced-values in Harrison (1989). In auctions against risk-neutral Nash bidders, only those bids that do not exceed the highest possible bid of 750 have been considered. Thus, the number of bids considered for estimation purposes are less than the number of bids reported in Harrison (1989). Outliers have been removed throughout.

Estimation Procedure

I use non-linear least squares estimation to minimize the squared errors between the observed and predicted bids to identify the behavioral parameters for the bidding function in a symmetric Bayesian Nash equilibrium.[45] This estimation has been done for the general model (outlined in Proposition 1) and the restricted versions of the general model which allow loss aversion and non-linear probability weighting in isolation. I have used MATLAB to implement a "Trust-region reflective Newton" search for the best-fitting parameters.[46]

Estimates

The combined results for all the auctions are listed in Table 5; the table lists estimated behavioral parameters for auctions with varying levels of experience, number of bidders, and nature of opponent bidders (humans or risk-neutral Nash bidders). The estimates for auctions against risk-neutral Nash bidders are reported in the last set of rows in Table 5.

[Table 5 here]

i. Probability weighting and loss aversion in the general model

The estimates of β are greater than 1 (and significantly different from zero in most cases[47]) in auctions against human bidders in both Cox et al. (1982) and Harrison (1989). Except for the auctions with 6 bidders in Cox et al. (1982), the estimates of β are greater than 1.[48] This yields convex probability weighting and therefore suggests

[45] If the errors between the predicted and observed bids are assumed independent identical normal random variables i.e. $\varepsilon_i \sim NID(0, \sigma^2)$, then maximum likelihood and nonlinear least squares estimation are equivalent. ML estimates are consistent, asymptotically efficient and asymptotically normal; however, if this does not hold nonlinear least squares though not efficient remain consistent and asymptotically normal.
[46] The programming code underlying all the ensuing results is available upon request.
[47] Based on t-ratio.
[48] In auctions with 6 bidders (series B), the number of overbids is substantially lower (66.7%) than for any other auctions; the average overbid is also the lowest among all auctions, whereas the average underbid is similar to other auctions. In the other set of auctions with 6 bidders (series a) the number of overbids is 78.3%, which is similar to other auctions, but the average underbid is around 23%, which is

"ambiguity-aversion" along the lines of Salo and Weber (1995) and Goeree et al. (2002). In Harrison (1989), the estimates of β successively decline from 1.51 in auctions with inexperienced human bidders to 1.16 in auctions against human bidders and prior experience; this further declines to 1.01 in auctions against risk-neutral Nash bidders and prior experience. Note that a model based on risk-aversion alone cannot explain these changes.[49]

The estimates of k_l are approximately close to 1 and significantly different from zero in most auctions against human bidders in Cox et al. (1982) and Harrison (1989). Except for auctions against risk-neutral Nash bidders in Harrison (1989), where the estimate for k_l is smaller but not significantly different from zero, the estimates are approximately close to 1, which supports loss aversion based on my model.

ii. Probability weighting without loss aversion

Although the estimates of β are greater than 1 and significantly different from zero in all auctions against human bidders for β in both Cox et al. (1982) and Harrison (1989), their magnitude is much larger. This yields more convex probability weighting and suggests larger deviations between the objective and weighted probabilities of auction outcomes.[50] The estimates for auctions with 6 bidders in Cox et al. (1982) are much lower than the estimates for all other auctions. In Harrison

somewhat high. In both series of auctions with 6 bidders, 4 out of 10 bidders bid below RNNE in 50% of the auctions. These auctions are therefore unusual and the estimates of β which suggest overweighting (concave probability weighting), are somewhat out of order.

[49] Another aspect of the estimates for β relate to the deviation from 1 in the expected utility based models. In most auctions, when the estimates are greater than 1 in more than 50% cases (more than half of the auctions) they significantly improve the explanatory power of the model based on sum of squared errors and F-test.

[50] Also note than when loss aversion was considered the estimates for probability weighting were quite similar to each other which is not true when loss aversion is ignored.

(1989) the estimates of β decline from 3.02 in auctions with inexperienced bidders to 2.32 in auctions with human bidders and prior bidding experience; this further declines to 1.70 in auctions against risk-neutral Nash bidders and prior experience. As before, a model based on risk-aversion alone cannot explain these changes.

iii. Loss aversion without probability weighting

The estimates of k_l for most auctions in Cox et al (1982), except for auctions with 6 bidders (series B), are approximately close to 1 and significantly different from zero. The estimates of k_l in auctions in Harrison (1989) are 1.00, 1.01, and 0.91 respectively and significantly different from zero. Thus, even when probability weighting is ignored, based on the estimates obtained for auctions in Harrison (1989), these estimates become smaller in auctions with smaller ambiguity levels (with human bidders and prior experience or Nash bidders).

The estimates for k_l are approximately close to 1 in models where loss aversion is allowed except for auctions against risk-neutral-Nash bidders in Harrison (1989) where the estimate is smaller than 1 and significantly different from zero.

The implied ratio of loss-gain utility is therefore close to 2. Tversky and Kahneman (1991)[51] suggest a ratio of 2:1 for the "gains" and "losses" based on acceptable lottery gambles.[52] The estimates I obtain suggest that the ratio of "gain-loss" utility is qualitatively similar to that observed in Tversky and Kahneman (1991) and reported

[51] "…these findings suggest that a loss aversion coefficient of about two may explain both risky and riskless choices involving monetary outcomes and consumption goods" (Tversky and Kahneman, 1991, p.1053)

[52] As mentioned earlier, not winning the auction does not result in monetary "losses"; thus a ratio of "losses" to gains would be $(1+k_l)/1$.

elsewhere (Ho, Lim and Camerer, 2006).[53] Note that my model with linear probability weighting and $k_l = 1$ is equivalent to a model with risk-aversion with Arrow-Pratt coefficient of 0.5. This similarity is supported by the estimates obtained for β and k_l, in auctions with least ambiguous circumstances. However, unlike the model based on risk-aversion (constant relative risk-aversion or CRRAM) alone, the general prospect theory model, can address changes in ambiguity levels; the estimates for probability weighting obtained across these auctions, are consistent with how individuals respond to changes in underlying circumstances.

In the following section, I state the results based on differences in estimates for β and k_l obtained in auctions with prior bidding experience and/or against Nash risk-neutral bidders; in section 7, I further discuss the implications of my results in the context of related literature.

6. The effect of bidding experience and type of opponent bidders

Ambiguity-aversion has attracted attention because individuals are typically not aware of precise probabilities in the real world. In auctions, the probability of winning for a given bid depends on bidders' bidding strategies, which is not readily known in most induced-value auctions. Clearly, deriving probabilities in these auctions is a complicated task, and therefore ambiguity could affect bidding as in other market experiments (Sarin and Weber 1993, Salo and Weber 1995). As people become familiar and gain experience of bidding, deriving probabilities of various

[53] The estimated coefficient for loss aversion makes my model equivalent to a model with risk-aversion coefficient of 0.5 without nonlinear probability weighting; the generality due to nonlinear probability weighting adds to the explanatory power of my model over a model with risk-aversion alone.

outcomes could become easier.[54] In my model, this could result in smaller deviations between subjective and objective probabilities under less ambiguous circumstances. The data for auctions where bidders have prior bidding experience and/or face risk-neutral Nash bidders present an opportunity to explore these effects. Since these induced-value auctions are similar, besides variations in experience level and the nature of opponent bidders, as a preliminary hypothesis one could argue that changes in the underlying circumstances (ambiguity) are not likely to affect the degree of loss aversion (the gradient for loss aversion)[55] [56] In this section I discuss the experimental evidence which supports my hypothesis and suggests minimal role for non-linear probability weighting in auctions characterized by less ambiguous circumstances.

Based on my discussion above, I propose the following hypothesis.

Hypothesis: (a) *The deviations between weighted and objective probabilities become smaller as auctions environments become less ambiguous, i.e,*

$$\beta_{inexperienced}^{humanrivals} > \beta_{experienced}^{humanrivals} > \beta_{experienced}^{RNNrivals}$$

whereas ***(b)*** *the coefficient of loss-gain utility* k_l *is similar across auction environments.*

[54] Such expertise is likely to develop faster in other contexts, e.g., in games of chance.

[55] Loss aversion may vary across commodities (Horowitz and McConnell 2002, Koszegi and Rabin 2006) and could potentially depend on availability of substitutes and trading intentions (Kahneman, Knetsch and Thaler 1990; List 2003).

[56] The assumption in Kahnemann and Tversky (1979), which suggests that probability weighting and loss aversion are independent, is too simplistic. There is some literature that suggests that probability weighting and loss aversion could be related. Intuitively it is plausible that loss aversion could become smaller in less ambiguous circumstances (Chambers and Melkonyan 2008, Plott and Zeiler 2005).

Since my hypothesis pertains to both loss aversion and probability weighting, I shall focus only on the results from the general model to explore the effect of prior bidding experience against experienced human and risk-neutral Nash bidders[57].

I first test the following hypothesis for (gradient of) loss aversion using a generalized likelihood ratio test:

$$H_0 : k_l^{i,g} = k_l^{j,h}, \quad H_1 : Not\ H_0$$

where *i,j*=level of experience and *g,h*=nature of bidders. Then I test the following hypothesis for probability weighting:

$$H_0 : \beta_i^g = \beta_j^h; \quad H_1 : Not\ H_0$$

If the first test does not reject the null hypothesis, I test the following hypothesis for probability weighting under the assumption that loss aversion remains the same for robustness:

$$H_0 : \beta_i^g = \beta_j^h \mid k_{li}^g = k_{lj}^h; \quad H_1 : Not\ H_0$$

The likelihood ratio has a χ_r^2 distribution where r is the number of restrictions imposed in the null hypothesis. On the basis of these tests (see Table 6), I obtain the following result (figures 1-5 in appendix for bidding functions and probability weighting functions, which are based on the estimates listed in Table 5, supplement the results below).

Result 1.A: (-Less convex probability weighting due to experience-) ***Prior bidding experience reduces the non-linearity of probability weighting in auctions (i) against***

[57] Going by the sum of squared residuals (SSE) alone, the restricted versions of the general model do not throw unambiguous evidence in favor of one approach over the other. As observed earlier, the similarity of estimates suffer, when either of these influences on behavior is ignored.

experienced human bidders and (ii) against risk-neutral Nash bidders. This yields smaller deviations between subjective and objective probabilities of equilibrium auction outcomes.

This result addresses the effect of prior experience on bidding in auctions which present successively smaller levels of ambiguity as opponents change from (i) experienced human bidders to (ii) risk-neutral Nash bidders.

First, I shall address the former auctions. The estimates for β are smaller in these auctions with 4 bidders and prior bidding experience (compared to auctions with bidders without experience) in Harrison (1989) and Cox et al. (1982). This decline is significant at the 1% level for auctions in Harrison (1989) and not significant for auctions with 4 bidders in Cox et al. (1982) (see Table 6). In auctions with 5 bidders, the increase in the estimate for β for experienced bidders in Cox et al. (1982) contradicts my hypothesis but is not significant. If prior experience is expected to reduce deviations with respect to the risk-neutral Nash bid then the deviations obtained in auctions with 5 bidders belies the expectation, which parallels the movements obtained for β .

Next, in auctions against risk neutral Nash bidders (Harrison 1989), bidders have prior bidding experience as well. Thus, of all auctions under consideration, bidding in these auctions occurs in least ambiguous circumstances. In these auctions, the decline in the estimate for β as compared to auctions without prior bidding experience is significant. This supports my primary hypothesis about the effect of ambiguity on bidding in these auctions.

Result 1.B: (-Less convex probability weighting due to fixed opponents' strategies-) ***In auctions with prior bidding experience against risk-neutral Nash bidders, fixing the opponents' bidding strategies reduces the non-linearity of probability weighting (with and without loss aversion). This yields smaller deviations between subjective and objective probabilities of equilibrium auction outcomes.***

While the previous result compares the estimates for β with prior bidding experience, the auctions against risk-neutral Nash rivals differ from the auctions with human opponent bidders (with same experience levels) since the opponents bidding strategies are fixed. The focus of previous attempts (Salo and Weber 1995) to explain aggressive bidding relates to the ambiguous circumstances arising due to uncertain behavior of opponent bidders. The extra control in bidding against risk-neutral Nash bidders allows us to examine the implications for β using my approach. As before, in auctions against risk-neutral Nash bidders (Harrison 1989), the decline in the estimate for β as compared to auctions against human bidders, is significant.

Thus, so far, as we move from auctions with inexperienced bidders to auctions with experienced bidders and risk-neutral Nash opponent bidders, the estimates of β display significant downward movement with successively smaller levels of ambiguity. It is therefore appropriate to reflect on the role of ambiguity attitudes in auctions with least ambiguous circumstances, based on the estimates obtained for behavioral parameters.

Result 1.C: (-Linear probability weighting in least ambiguous circumstances-) ***In auctions, with prior bidding experience, against risk-neutral Nash bidders, by allowing loss aversion, an almost linear probability weighting is obtained.***

Without loss aversion, although non-linearity of probability weighting declines with successively smaller levels of ambiguity, the deviations between subjective and objective probabilities remain. However, with loss aversion, I obtain almost linear probability weighting which suggests that aggressive bidding can be rationalized by loss aversion alone without invoking ambiguity effects.

I shall now turn to the estimates for loss aversion observed in various auctions.

Result 2.A: (-No effect on loss aversion due to experience-) ***Prior bidding experience has no effect on loss aversion in auctions against experienced human bidders.***

Result 2.B: (-Loss aversion declines in least ambiguous circumstances-) ***The degree of loss aversion obtained in auctions with prior bidding experience against risk-neutral Nash bidders is smaller than that obtained in auctions with human opponent bidders.***

The estimates for k_l are almost identical in all the auctions except in auctions against risk-neutral Nash bidders, where the estimated gradient for "losses" k_l is smaller.

This decline is significant when compared to the estimates obtained in auctions with human bidders in Harrison (1989). This allows a reflection of the possible shortcomings of my approach. In more general field settings, the degree of loss aversion may vary across commodities (Horowitz and McConnell 2002, Köszegi and Rabin 2006). It may be affected by the availability of market substitutes (Horowitz and McConnell 2002) or trading intentions (List 2003, 2004; Kahnemann, Knetsch and Thaler 1990). The difference in loss aversion obtained in induced-values settings (where the above do not apply) possibly suggests that behavioral influences, other than probability weighting and loss aversion, coexist. For example, if bidders display spite against human bidders and not against Nash bidders (computers), the differences in loss aversion as obtained are expected.[58]

7. Further discussion of the empirical findings

In this section I discuss the significance of my findings in the context of the experimental literature on auctions as well as the experimental literature in general. I compare my findings to previous literature that explores probability weighting and loss aversion in experiments.

Several studies on decision under risk show the tendency of subjects to overweigh small objective probabilities and underweight medium and large objective probabilities (Tversky and Kahneman 1992, Camerer and Ho 1994, Fox and Tversky 1998, Gonzalez and Wu 1999). This pattern yields an inverted S-shaped probability

[58] The changes in estimates for β and k_l (when considered in isolation) are also similar to the change in estimates obtained in the general model.

weighting function as in Kahneman and Tversky (1979).[59]In the real world actual probabilities may not be known precisely. Recent evidence (Barron and Erev 2003; Hertwig et al. 2004; Barron and Ursino 2007) suggests that the inverted S-shaped curve does not capture decision making under uncertainty where probabilities are typically derived through repeated sampling (experience)[60].This literature suggests that individuals underweigh small probabilities under uncertainty, which is different from what they do under risky circumstances as reflected in the inverted S-shaped probability weighting (Prelec 1998, Wu and Gonzalez 1999).[61] In an auction equilibrium, winning is a rare event for bidders with low induced-values. Thus, the estimated convex probability weighting in my models (with or without loss aversion) is consistent with this literature. As discussed earlier, this is also consistent with Salo and Weber (1995) and Goeree et al (2002) who suggest "ambiguity-aversion" in auctions.[62]

The literature suggests loss aversion in various settings and provides experimental evidence for choices over trade of mugs, pens, candy bars, subscription for electric

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[59] This function typically intersects the linear probability weighting function somewhere between 0.3 and 0.4.

[60] In these experiments subjects were asked to choose among two options; for example, when asked to choose between a sure $3 , and $4 with probability 0.8, and $0 with probability 0.2. In one treatment the probabilities are specified clearly (descriptive) and in the other the probabilities are derived by random sampling of the options (experience-based learning). The proportion of subjects who choose the risky ($4 with probability 0.8) option is significantly higher in the treatment with uncertainty (experience-induced learning).

[61] In experiments, underweighting of rare events could occur due to sampling errors. For example, people are likely to draw rare events less often than objective probability implies, especially if their samples are small. Barron and Ursino (2007) find that underweighting of rare events as observed in one-shot decisions is robust to removal of unrepresentative samples. This suggests that underweighting of rare events in experience-based decisions occurs due to overweighting of most recent outcomes.

[62] In Chen et al. (2007), ambiguity attitudes could get confounded with the pessimistic reasoning that applies to symmetric bidders. For example, when a rival's induced-value distribution is unknown , a bidder with low valuation might assume that the rival also makes a similar assumption about his values (symmetry). This could produce lower bids in equilibrium. Thus the experimental design in Chen et al. (2007) does not separates "ambiguity attitudes" from such ex ante pessimistic reasoning.

services, job attributes, sportscards, etc. (Knetsch 1989, Tversky and Kahneman 1991, Kahneman, Knetsch, and Thaler 1990, Benartzi and Thaler 1995, List 2003). The estimate for the ratio of the slopes of the value function in two domains, for small and moderate "gains" and "losses" of money, is about 2:1 (Tversky and Kahneman 1991). In a slightly different context, Kahneman, Knetsch, and Thaler (1990) investigate loss aversion in a purely deterministic environment. In an experiment, half of a group of Cornell students are given a Cornell insignia coffee mug, while the other half are not. When mug owners are given an opportunity to trade and nonowners are given an opportunity to buy, Kahneman, Knetsch, and Thaler (1990) found that the reservation prices for the two groups were significantly different. Specifically, the ratio of the median of the reservation price of the sellers to the buyers is roughly 2.5:1. My findings are broadly consistent with this literature (Tversky and Kahneman, 1991; Ho, Lim and Camerer, 2006).[63]

It is however important to emphasize that doubts have been raised in the literature about the robustness of loss aversion as a description of individual preferences. List (2003, 2004) provides evidence using field experiments that loss aversion attenuates with previous trading experience. Plott and Zeiler (2005) suggest that an endowment effect arises due to subject misconceptions (ambiguity) about experimental tasks. They suggest that when all known controls for subject misconceptions are employed

[63] Note however that because loss aversion is modeled slightly differently in my approach, this equivalence is not obvious. If $u(x) = \{x^{\alpha} \text{ for } x > 0; -\lambda(-x^{\beta}) \text{ for } x \leq 0$. Therefore, $k_l = \lambda - 1$. Clearly, these estimates suggest $k_l > 0$. My approach rules out very high levels of loss aversion so bidding remains acceptable.

the WTA-WTP disparity is not observed.[64] The lessons from this literature suggest the following possibilities: (i) ambiguity affects loss aversion; (ii) trading intentions could affect choices such that loss aversion disappears and (iii) market experience, which could affect both ambiguity and/or trading intentions and thereby loss aversion. My results that are obtained within the context of induced-value laboratory experiments add to this literature and provide support along the lines of List (2003, 2004) which suggest that loss aversion could become smaller in the field. However, unlike List (2003, 2004), my results do not suggest that loss aversion will disappear completely. This might be due to the complexity of the auction environment. If cognitive capital that attenuates loss aversion develops slowly, then such learning is likely to be slower in auctions than in other simpler choice/trading environments as in List (2003, 2004). My results also suggest that ambiguity could affect loss aversion since the estimates for loss aversion are slightly smaller in auctions against risk-neutral Nash bidders. However, in *field* auctions, even if ambiguity effects can be ruled out, trading intentions could still influence loss aversion. [65]

8. Conclusions

In this chapter, I provide a model of bidding in first-price auctions that combines loss aversion and non-linear probability weighting. This approach applies to a wider domain of auction environments which differ in terms of levels of ambiguity. In auctions against human bidders, missing information about bidders' risk postures and bidding strategies present greater levels of uncertainty (ambiguity) in comparison to

[64] Although, recent research raises doubts about the claims in Plott and Zeiler (2005) (see Isoni, Loomes and Sugden 2009)

[65] This is further explored in Ratan (2009).

bidding against risk-neutral Nash (computer) bidders. The analysis of experimental auction data suggests that aggressive bidding against inexperienced human bidders can be rationalized by anticipated loss aversion and ambiguity effects. Interestingly, my approach suggests that ambiguity effects become less relevant as levels of ambiguity decline with prior experience in auctions against (i) experienced human bidders and (ii) risk-neutral Nash bidders. When loss aversion is taken into account, the best-fitting parameters in auctions with smaller levels of ambiguity yield almost linear probability weighting.

However, other behavioral explanations that induce aggressive bidding in these auctions may coexist with the influences that are prominent in my approach. For example, theoretically, risk aversion could be combined with spiteful preferences and/or non-linear probability weighting (ambiguity-aversion) to create a bidding response that is observationally equivalent to my approach. However, using this approach, in auctions against risk-neutral Nash bidders where ambiguity effects and spitefulness could be altogether irrelevant, the obtained level of aggregate risk aversion is still very high.[66] This brings out the advantages of my approach over other approaches: it provides a reasonable account of aggregate bidding behavior, and addresses the role of ambiguity very well. The declining role of ambiguity effects in auctions that present successively smaller levels of ambiguity is consistent with the smaller levels of non-linear probability weighting obtained using my approach. This enhances the performance criteria for other behavioral approaches that can be applied

[66] For example, using constant-risk-aversion approach and linear probability weighting (similar to that in obtained using my approach), the Arrow-Pratt measure for auctions in Harrison (1989) with prior experience in auctions against (a) human bidders and (b) risk-neutral Nash bidders varies between 0.42-0.52.

in auction environments. Further research is required to disentangle the effects of various behavioral influences in auctions to attain this objective.

More investigation of the indirect effects of ambiguity on loss aversion could possibly help refine the Prospect theory based accounts of behavior under risk and/or uncertainty. However, attaining these objectives within the complexity of auction environments could be difficult.

Chapter 3: Multi-Dimensional Reference-Dependent Preferences in Sealed-bid Auctions: How (most) laboratory experiments differ from the field[⊗]

1. Introduction

In the previous chapter, I have shown that money loss aversion could explain aggressive bidding in induced-value auctions across auction environments in which bidders face human rivals to Nash computer rivals. However, commodity auctions also differ from induced-value auctions in the following respect: in commodity auctions the auction object is exchanged for monetary bid. This is important because transferring insights from laboratory experiments to inform structural models in the field can be problematic if individuals are loss averse in various dimensions of the consumption space in sealed-bid auctions. Moreover, the findings in the recent literature suggest that bidding in lab environments could differ from field settings (List 2003, 2004). We investigate the theoretical implications of reference-dependent preferences in commodity auctions (which are different from induced-value settings in the consumption space) and could therefore have altogether different effect on bidding in field settings as discussed in List (2003, 2004).

Differences in moral considerations, the nature and extent of scrutiny, (social) context, subject pool, and differences in stakes have recently received increasing interest in the literature (e.g., Harrison and List 2004; Karlan 2005; Levitt and List

[⊗] Coauthored with Andreas Lange: *Games and Economic Behavior, 2010,* Vol. 68(2), pp 634-645

2007a, 2007b) that could make transferring qualitative insights from the lab to field settings problematic. However, we argue that the nature of the traded commodities forms another significant difference between most laboratory and field settings:[67] in most laboratory auction experiments both the induced-value of the item as well as the bids are measured in monetary units and therefore along a single dimension from the perspective of bidders.[68] In almost all field settings, however, the auctioned item and payments form different dimensions of the consumption space.

In this paper, we demonstrate that qualitatively different bidding behavior may result from these different dimensionalities of the relevant consumption space. For this, we explore first- and second-price auctions when bidders are loss averse. The phenomenon of loss aversion is well established in the experimental literature (for summaries, see Knetsch et al. 1991; Camerer 1995; Horowitz and McConnell 2002). We use a reference-dependent utility model with loss aversion based on Köszegi and Rabin (2006):[69] after placing the bid, bidders expect to win the auction with some probability and therefore compare the outcome of the auction with this reference point. Our model predicts overbidding in induced-value experiments, but substantially different bidding behavior when a non-monetary item is auctioned. Our

[67] Harrison and List (2004) point out that this difference can be essential: “If the nature of the commodity itself affects behavior in a way that is not accounted for by the theory being applied, then the theory has at best a limited domain of applicability that we should be aware of, and at worse is simply false” (p.1012).

[68] The winning bidder receives the (induced) value which was randomly assigned to him *minus* the auction price (his bid in the first price auction; the highest bid of an opponent in the second-price auction).

[69] The reference dependence or status-quo bias of choice behavior has received considerable attention in the literature. It is usually discussed in a multi-dimensional setting, e.g. when describing differences in willingness-to-pay willingness-to-accept (see, e.g. Coursey et al. 1987; Knetsch 1989; Knetsch et al. 1991; Kahneman et al. 1990; Bateman et al. 1997). Köszegi and Rabin (2006) provide a model with an endogenous determination of the reference point.

results thereby have implications for the interpretation of laboratory data as well as for transferring qualitative insights on bidding behavior to the field.

The predicted overbidding in first-price sealed-bid auctions with induced-values is consistent with experimental findings which suggest that bidders in such auctions consistently bid in excess of the predictions of the risk-neutral-Nash (RNNE) model (Kagel 1995; Cox et al. 1982, 1988; Harrison 1989). This overbidding anomaly has drawn considerable attention because explaining overbidding by risk-aversion would require bidders to be excessively risk-averse (Kagel 1995). Therefore, many alternative behavioral models have been suggested (e.g., Salo and Weber 1995; Goeree et al. 2002; Morgan, Steiglitz and Reis 2003; Dorsey and Razzolini 2003; Filiz-Ozbay and Ozbay 2007).[70]

Our model of loss aversion provides one additional explanation for overbidding. Naturally, all the above behavioral motivations could exist simultaneously such that we do not intend to propose loss aversion as the single cause of overbidding. We focus the model on loss aversion only to demonstrate qualitative differences between induced-value and commodity auctions.

Besides in first-price auctions, our model also predicts qualitative differences in second-price auctions. Here, loss aversion leads to truthful revelation of the underlying induced-value, while overbidding or underbidding may result if the

[70] Goeree et al. (2002) study noisy bidding behavior in a quantal response equilibrium along with non-linear probability weighting and joy of winning as potential causes of overbidding. Dorsey and Razzolini (2003) compare auctions and lotteries and show that subjects' overbidding is consistent with a misperception of probabilities of winning in auctions. Salo and Weber (1995) consider ambiguity-aversion and Morgan et al. (2003) studies spiteful preferences as explanations for overbidding. Filiz-Ozbay and Ozbay (2007) suggest that anticipation of loser's regret could be the potential reason for overbidding.

auction item and payments form two different dimensions of the consumption space. It should be noted, however, that there is some experimental evidence for bid-shading even in induced-value second-price auctions (Kagel 1995; Kagel and Levin 1993). Our model is not able to predict such behavior.[71] Bidding above or, less often, below a subject's own value is typically explained by a lack of familiarity with the second-price format and weak learning feedback mechanisms (Kagel et al. 1987; Kagel and Levin 1993; Harstad 2000).[72] Similarly, the extent of over- and underbidding is reduced when bidders have time to introspect their actions (Aseff 2004). While not addressing such bid-shading, our theory predicts potential qualitative differences in bidding behavior in induced-value vs. commodity auctions.

Out theory thereby indicates that findings on bidding behavior obtained in induced-value experiments cannot necessarily be transferred to the field because of the multi-dimensionality of the product space.[73] Even though we demonstrate this dimensionality effect using a specific reference-dependent model in an auction setting, our findings more generally raise some concerns for transferring qualitative behavioral findings from the lab to the field. With this, we add to the current debate

[71] It should be noted that this is standard for most of the behavioral literature cited above: while models are explain overbidding in first price auctions, they generally do not affect the dominant strategy in second-price auctions. Exceptionally, a joy of winning hypothesis would be able to generate overbidding in second-price such auctions.

[72] Harstad (2000) shows that bidders with experience in English auctions may indeed bid closer to the Nash prediction than inexperienced bidders. In typical second-price lab experiments with a small number of bidders and uniform distribution of values, such information feedback is rather weak. Bidders are therefore less likely to be punished for overbidding, i.e. the chance of winning while paying a price which is still above the player's value is rather small.

[73] It should again be noted that we do not claim that our behavioral assumption is able to serve as the *single* explanation of bidding anomalies in first- and second-price auctions. However, many subjects probably combine risk-aversion and potentially loss aversion with other behavioral motivations as discussed above. In this sense, our reference-dependent framework is sufficient to point out potential differences in qualitative model predictions between lab and field environments.

on the link between lab and field settings (e.g., Harrison and List 2004; Levitt and List 2007a, 2007b; List 2003).

Our paper is structured as follows. We first lay out the underlying assumptions of our reference-dependent model in section 2. In section 3, we then examine the implications of loss aversion in auction settings where object and bids are valued along the same dimension or – alternatively – along separate dimensions. We study first-price auctions in section 3.1, second-price auctions in section 3.2, and then compare the revenues in section 3.3. We conclude by discussing the implications for the interpretation of laboratory data.

2. Loss aversion and reference-dependent preferences

Loss aversion as a cause for behavioral anomalies is widely discussed in the literature (e.g., Knetsch et al. 1991; Camerer 1995, Horowitz and McConnell 2002). Diverse studies show large disparities between willingness-to-pay (WTP) and willingness-to-accept (WTA). As a stylized fact, the discrepancy is largest for non-market goods, smaller for ordinary induced goods, and smallest for money-valued items in experiments (Horowitz and McConnell 2002, Camerer 1995). Consistent with this finding, List (2003) indicates that experience and therefore the frequency of trades of a specific commodity can reduce the endowment effect.

Endowment effects and the WTA/WTP gap can be explained by reference-dependent preferences, i.e. where the valuation of the final outcome depends on the reference point and subjects are averse to “losses”. In their prospect theory, Tversky and Kahneman (1991) describe preferences by indifference curves with a kink at the reference point. This reference point is often assumed to be the status quo consumption level, given by the endowment. However, if bidders are endowed with a lottery, i.e. face uncertain payoff consequences, “gains” and “losses” must be compared to this lottery as a reference point. Sugden (2003) and Köszegi and Rabin (2006) provide models allowing for such lotteries as reference points.

We apply Köszegi and Rabin’s (2006) framework to auction environments. After placing a bid, bidders basically face a lottery of winning or losing the auction. The probabilities and potential payoffs depend on their own and other bidders’ bids. The final outcome is then evaluated with respect to any possible outcome from this lottery as a reference point.

Formally, we consider $k+1$ commodities, including a numeraire commodity 0. An individual’s utility $u(c \mid r)$ depends both on her consumption $c=(c_0, c_1) \in \mathbb{R} \times \mathbb{R}^k$ and her reference level $r=(r_0, r_1) \in \mathbb{R} \times \mathbb{R}^k$. Consistent with Köszegi and Rabin (2006), we assume that utility is additively separable in the numeraire and the remaining dimensions:

$$u(c \mid r) = u_0(c_0 \mid r_0) + u_1(c_1 \mid r_1) \qquad (1)$$

Defining the consumption utility as $v_t(c_t) = u_t(c_t \mid c_t)$ ($t \in \{0,1\}$), we assume

$$u_t(c_t \mid r_t) = v_t(c_t) - \lambda_t \max[0, v_t(r_t) - v_t(c_t)] \qquad (2)$$

with $\lambda_t \geq 0$. That is, bidders are loss averse and loss aversion is linear in utility changes.[74]

The parameter $\lambda_t \geq 0$ hereby measures the degree of loss aversion in dimension t. It directly relates to the ratio of loss-gain-utility given by $(1+\lambda_t)/1$. As a perhaps more familiar measure for loss aversion, the WTA/WTP-ratio is – at the margin –given by $WTA/WTP = (1+\lambda_0)(1+\lambda_1)$.[75] The finding that the ratio varies widely and depends on good characteristics[76] implies the λ_1 will depend on the specific auction item.

Ex ante, both reference levels and consumption could be stochastic. Following Köszegi and Rabin (2006), the reference level is a probability measure G over $\mathbb{R}^{k+1}$ and consumption is drawn according to the probability measure F over $\mathbb{R}^{k+1}$. Then, the individual's expected utility over risky outcomes is given by

$$U(F \mid G) = \int\int u(c \mid r)\, dG(r)\, dF(c) \tag{3}$$

[74] For the case of two dimensional commodity space, this specification would lead to indifference curves with kinks at the reference level which reflects reference-dependence or status quo bias (see Knetsch et al. 1991).

[75] For a fixed reference point, the WTP for a small increase in consumption Δ is determined by $u(r \mid r) = u_0(r_0 - WTP(\Delta) \mid r_0) + u_1(r_1 + \Delta \mid r_1)$ such that $WTP'(0) = v_1'(r_1)/((1+\lambda_0)v_0'(r_0))$. The WTA is given by $u(r \mid r) = u_0(r_0 + WTA(\Delta) \mid r_0) + u_1(r_1 - \Delta \mid r_1)$ such that $WTA'(0) = v_1'(r_1)(1+\lambda_1)/v_1'(r_0)$. At the margin, the WTA/WTP ratio is therefore given by $WTA'(0)/WTP'(0) = (1+\lambda_0)(1+\lambda_1)$.

[76] The ratio tends to be smaller the closer the good comes to an ordinary induced consumption good (Horowitz and McConnell 2002). For money lotteries, they find a ratio close to 2. However, there is some debate about assigning the degree of the observed ratios fully to loss aversion (Plott and Zeiler 2005).

Bidders are assumed to have rational expectations in the formulation of reference and consumption probability. That is, the reference point is endogenously determined.[77] In our application to an auction setting, the bid affects a bidder's chances to win the auctioned item and – in the first-price auction – her consumption of the numeraire in case of winning. By placing a bid, the bidder therefore not only changes the distribution of consumption levels (F) but also generates the reference distribution (G). The individual undertakes no further action after placing the bid. Rational expectations therefore imply $F = G$ such that the bidders solves the following program:

$$\max_F U(F \mid F). \qquad (4)$$

Note that the temporal structure is slightly different from Köszegi and Rabin (2006). In their case, bidders correctly anticipate their actions which take place *after* new information is received. The ex ante payoff distribution which incorporates these anticipated actions then forms the reference distribution. As a consequence, actions take place *after* a reference distribution has been formed and new information is received. Given a reference distribution G the bidder therefore chooses $F(G)$ to maximize $u(F \mid G)$. Rational expectations then require consistency of the consumption distribution with the formulation of the reference distribution, i.e. $F(G) = G$.

In a sealed-bid auction, however, bidders' bids directly affect their payoff distribution. After placing the bid, a rational bidder's payoff expectations, i.e. her

[77] Alternative reference-dependent models with endogenous definition of reference points are given by Sugden (2003) and Munro and Sugden (2003) who assume the reference to be given by the *current endowment* which might adjust over the time.

reference distribution, directly depend on her action (bid) and her beliefs of rivals' bidding strategies in equilibrium. Compared to these payoff expectations, bidders then realize the auction outcome and potential loss sensations when the auction results are announced.[78] It is therefore natural to assume that the bidders anticipate the effects of bidding on both the reference as well as payoff distribution, i.e. that both are chosen at the *same* time. Rational expectations require that they coincide as in optimization program (4).

3. The auction environment

We consider n bidders $i = 1,...,n$. We assume symmetric preference structures as given in the previous section. For simplicity, bidders are assumed to be risk-neutral in the numeraire consumption, i.e. $v_0(c_0) = c_0$. We study two different auction environments: a commodity auction (CA) where a consumption bundle $\Delta \in \mathbb{R}^k$ is auctioned off, and an induced-value auction (IV) where values of the auction item are induced in the numeraire (money) dimension. While (CA) resembles a naturally-occurring auction setting with homegrown values, the (IV) auction characterizes most of laboratory auction environments.

In the commodity auction, the value of the auctioned item $\Delta \in \mathbb{R}^k$ for bidder i is measured in consumption utility gains $w^i = v_1(c_1^i + \Delta) - v_1(c_1^i) \geq 0$ where c_1^i denotes the initial endowment of bidder i. Differently in the induced-value auction, w^i is directly induced as gain in the money dimensions. In both cases we assume that each

[78] The model thereby rests on the assumption that between placing the bid (or planning to place the bid) and the final realization of the auction outcome, a reference distribution is established

bidder knows her induced-value w^i and is informed that others' values are drawn from a distribution $H(\bullet)$ defined over $[\underline{w}, \overline{w}]$ ($\overline{w} > \underline{w} \geq 0$).

Bidders can place bids $0 \leq b^i \leq c_0^i$ where c_0^i denotes the available income of bidder i. Throughout the paper, we assume that the budget constraint is not binding. We consider both first- and second-price independent induced-value auctions.

3.1 First price auctions

We first consider the commodity auction (CA) where the value of the auctioned item for bidder i is given by $w^i = v_1(c_1^i + \Delta) - v_1(c_1^i) \in [\underline{w}, \overline{w}]$. If bidder i places a bid $b^i \geq 0$, her consumption in case of winning the auction is given by $(c_0^i - b^i, c_1^i + \Delta)$, while the consumption in case of losing is given by the initial endowment (c_0^i, c_1^i). For a bidder with preferences as given in section 2 (conditions (1)-(4)), the expected utility gain from participating in the auction is therefore given by

$$\Pi^{1,\mathrm{CA}}(b^i, w^i) = f(b^i)(w^i - b^i) - f(b^i)(1 - f(b^i))[\lambda_0 b^i + \lambda_1 w^i] \qquad (5)$$

where $f^i = f(b^i)$ denotes the probability of winning of bidder i when placing a bid b^i. Besides the expected consumption utility $f(b^i)(w^i - b^i)$, equation (5) comprises expected "losses" in two dimensions: (i) $(1 - f(b^i)) f(b^i) w^i$: the bidder expects to win w^i with probability $f(b^i)$ such that she experiences not winning as a loss with probability $(1 - f(b^i))$. (ii) $f(b^i)(1 - f(b^i)) b^i$: the bidder expects to lose and therefore not to pay b^i with probability $(1 - f(b^i))$ such that she experiences the payment of the price in case of winning the auction a loss with probability $f(b^i)$.

It should be noted that (5) implies that a non-negative expected utility gain $\Pi^{1,\mathrm{CA}}(b^i, w^i)$ from participating in the auction can only result if $w^i[1-\lambda_1(1-f(b^i))] \geq b^i[1+\lambda_0(1-f(b^i))]$. That is, positive bids only will be placed by bidders with $f(b^i) > 1-1/\lambda_1$. If $\lambda_1 < 1$, this condition holds for all bidders. If $\lambda_1 > 1$, the condition implies only bidders with a sufficiently large probability to win place positive bids.

As usual, we restrict our attention to symmetric monotonically increasing equilibria in pure strategies. In equilibrium, the chances of player i to win, are therefore given by $H^{n-1}(w^i)$. With the above argument, positive bids may only result if $H^{n-1}(w^i) > 1-1/\lambda_1$. The corresponding threshold value w_L^{CA} we define by

$$H^{n-1}(w_L^{CA}) = \max[0, 1-1/\lambda_1] \qquad (6)$$

Note that $w_L^{CA} = \underline{w}$ if $\lambda_1 \leq 1$. Bidders with $w^j \in [w_L^{CA}, \overline{w}]$ place positive equilibrium bids.

We obtain the following proposition:

Proposition 1: (commodity first-price auction–CA) *The unique monotonically increasing symmetric Bayesian Nash equilibrium bidding function for commodity auctions is given by*

$$b^{1,\mathrm{CA}}(w) = \begin{cases} w\dfrac{1-\lambda_1(1-H^{n-1}(w))}{1+\lambda_0(1-H^{n-1}(w))} - \dfrac{\int_{w_L}^{w} H^{n-1}(z)[1-\lambda_1(1-H^{n-1}(z))]dz}{H^{n-1}(w)[1+\lambda_0(1-H^{n-1}(w))]} & \text{if } w \geq w_L^{CA} \\ 0 \qquad \text{if } w < w_L^{CA} \end{cases} \qquad (7)$$

Proof: see Appendix.

Without any loss sensation ($\lambda_0 = \lambda_1 = 0$), the bidding function $b^{1,CA}(\bullet)$ given in (7) reduces to the risk-neutral Nash bidding function. Loss aversion in money ($\lambda_0 > 0$) or in the commodity dimension ($\lambda_1 > 0$) will affect bids. For example, consider the minimum bid $b^{1,CA}(\underline{w}) = \max[0, \underline{w}(1-\lambda_1)/(1+\lambda_0)]$ which is generally smaller than the minimal value $\underline{w}$. More generally we obtain the following results:

Proposition 2 (effects of loss aversion–CA). *Equilibrium bids are decreasing in the degree of loss aversion* λ_0 *in the numeraire (money) dimension. There exists* $0 \le \hat{w} \le 1$ *with* $2H^{n-1}(\hat{w}) > 1$ *such that for increasing loss aversion* $\lambda_1 > 0$ *in the commodity dimension, bids are decreasing if* $w < \hat{w}$ *but increasing if* $w > \hat{w}$ *as long as* $w \ge w_L^{CA}$.

Proof: see Appendix.

The comparative statics shows that the impact of loss aversion depends on the dimension in the commodity space in which "losses" occur: while loss aversion in the numeraire (payment) dimension has an unambiguously decreasing effect on bids, the qualitative impacts of loss aversion in the commodity dimension differ across the signal range $w \in [\underline{w}, \overline{w}]$.

Intuitively, increasing the bid increases the potential loss of money in commodity auctions such that money loss aversion implies lower bids. Additionally, the potential loss of the commodity serves as a "bifurcating" force: If a bidder is likely to win to start with, he can decrease chance of disappointment by increasing probability of winning. As a consequence, loss aversion leads to higher bids. However, if a bidder is unlikely to win to start with, he can decrease expectations by further decreasing probability of winning, i.e. by bidding lower due to loss aversion. This bid reducing effect applies in particular for values with $H^{n-1}(w) < 1/2$ which holds for any given value w if n is sufficiently large.

We now turn to the induced-value (IV) auction, where individual values w^i are directly induced as gains in the money dimension. Here, a bidder with preferences as given in section 2 (conditions (1)-(4)), experiences the following expected utility gain from participating in the auction:

$$\Pi^{1,\text{IV}}(b^i, w^i) = f(b^i)(w^i - b^i) - \lambda_0 f(b^i)(1 - f(b^i))(w^i - b^i) \qquad (8)$$

We again see that positive expected utility can only result if $f(b^i) > 1 - 1/\lambda_0$. Similar to the commodity auctions, this defines a threshold w_L^{IV} for participation with positive bids: $H^{n-1}(w_L^{IV}) = \max[0, 1 - 1/\lambda_0]$. Bidders with $w_i < w_L^{IV}$ who exist if $\lambda_0 > 1$ can gain non-negative expected utility by bidding their own value, i.e. $b_i = w_i$.

Comparing conditions (5) and (8) we immediately see that the bidding behavior in the induced-value auction can be obtained from the preceding analysis of commodity auctions by setting $\lambda_1^{\text{CA}} = \lambda_0$ and $\lambda_0^{\text{CA}} = -\lambda_0$. In the induced-value auction, “losses” occur at a level $w^i - b^i$ if player i had expected to win but loses the auction. Correspondingly, bidders with a small chance of winning can obtain a non-negative utility in IV auctions not only by (i) bidding zero and ensuring zero probability of winning, but also by (ii) bidding their own value, i.e. $b_i = w_i$.

Adapting Proposition 1 we immediately obtain the bidding function for induced-value auctions:

Corollary 1 (induced-value first-price auction–IV). *The unique continuous monotonically increasing symmetric Bayesian Nash equilibrium bidding function for induced-value auctions is given by*

$$b^{1,\mathrm{IV}}(w) = \begin{cases} w - \dfrac{\int_{w_L^{IV}}^{w} H^{n-1}(z)[1-\lambda_0(1-H^{n-1}(z))]dz}{H^{n-1}(w)[1-\lambda_0(1-H^{n-1}(w))]} & \text{if } w \geq w_L^{IV} \\ w \quad \text{if } w < w_L^{IV} \end{cases} \tag{9}$$

The minimal bid is given by $b^{1,\mathrm{IV}}(\underline{w}) = \underline{w}$ and not affected by loss sensation. In general, however, we obtain the following result:

Corollary 2 (effects of loss aversion–IV). *Increases in the degree of loss aversion unambiguously increase bids in induced-value first-price auctions. Bids are therefore more aggressive than in the risk-neutral Nash equilibrium.*

Proof: see Appendix.

In induced-value settings, our model therefore provides one potential explanation for overbidding in first-price sealed-bid auctions when compared to the risk-neutral-Nash prediction (e.g., Kagel 1995). Loss aversion thereby may complement other behavioral driving forces such as: risk-aversion, probability weighting or ambiguity-aversion (Salo and Weber 1995), or loser's regret (Filiz-Ozbay and Ozbay 2007). All these different behavioral motivations could exist simultaneously such that we do not expect a single one to completely explain the data from laboratory experiments. As such, we do not neither anticipate nor intend to show that the described model perfectly fits observed data.[79] Instead, we take evidence for loss aversion from numerous choice experiments as a basis for the claim that it may very well also affect bidding behavior in auctions. Concentrating on loss aversion, we see our contribution in demonstrating potential qualitative differences in bidding between induced-value and commodity auctions.

As such, the effects of loss aversion in the money dimension in the induced-value auction are opposite from those in the commodity auction. While loss aversion in the money dimension would imply increased bids in the induced-value setting which applies in most lab experiments, bids could decrease if the consumption of the commodity occurs in a different dimension. The intuition behind this reversal of the impact of money loss aversion can be seen when considering how a subject might end up experiencing the money loss: in the induced-value setting, the loss (compared to expected payoff) occurs when *losing* the auction. Increasing the bid decreases the surplus ($w-b$) and therefore decreases the potential loss of money. As a result, money loss aversion induces higher bids. This is different in the commodity auction

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[79] Ratan (2008, Chapter 2 above) provides some evidence of the empirical validity of the reference-dependent approach for explaining data from induced-value laboratory experiments. Using existing data from Cox et al. (1982) and Harrison (1989), he finds that loss aversion parameter λ_0 between 0.9 and 1.0 provide the best fit of the reference-dependent model to the respective data sets. The results are obtained using separate nonlinear least squares estimations to identify the loss aversion parameter based on a symmetric equilibrium bidding function for the different data sets and different number of bidders ($n \in \{4,5,6,9\}$. The best fit loss aversion levels would correspond to a ratio of gain-loss-utility close to 2, similar to the levels suggested by Tversky and Kahneman (1991).

setting. Here, the money loss occurs when *winning* the auction. Therefore, increasing the bid increases the potential loss of money in commodity auctions and money loss aversion implies lower bids.

We summarize these results as follows:

Corollary 3 (lab vs. field environment) *Loss aversion has different qualitative implications on bidding behavior depending on whether or not payments are made in the same dimension of the commodity space as auction item is consumed. If being made in the same dimension (induced-value in laboratory setting), increasing loss aversion with respect to the numeraire induces bids to be more aggressive, while they become more conservative if consumption dimensions differ (commodity auctions).*

This result suggests overbidding in the field cannot necessarily be predicted on the basis of overbidding in induced-value experiments. Using a behavioral model of loss aversion, this substantiates the observation by Harrison and List (2004) that the applicability of theories may be limited if they do not explicitly account for the nature of traded commodities.

3.2 Second-price auctions

We next consider second-price sealed-bid auctions again first for commodity auction where the value of the auctioned item for bidder i is given by $w^i = v_1(c_1^i + \Delta) - v_1(c_1^i)$. Values are again distributed in $[\underline{w}, \overline{w}]$ according to $H(\bullet)$.

For given bidding strategies of the other bidders, bidder i's probability of winning with a bid of b^i is again denoted by $f(b^i)$. The payment which bidder i has to make in this case, is however given by the second largest bid, i.e. the largest bid of a competitor and therefore follows the distribution $f(b^i)$. Then, the expected utility including the loss sensation is given by

$$\begin{aligned}\Pi^{2,\mathrm{CA}}(b^i,w^i) = \int_{b(\underline{w})}^{b^i}[w^i - p]df(p) - \lambda_0\int_{b(\underline{w})}^{b^i}\int_{b(\underline{w})}^{p}(p-s)df(s)df(p) \\ - \lambda_0(1-f(b^i))\int_{b(\underline{w})}^{b^i}pdf(p) - \lambda_1 w^i f(b^i)(1-f(b^i))\end{aligned} \qquad (10)$$

Here the first term gives the standard expected consumption utility when winning the auction. The second term reflects money losses when bidder i wins the auction and has to pay p, while she expected to pay $s < p$. Note that when paying p experienced "losses" are given by $\max[p-s,0]$. Due to the consistency of the reference and outcome distribution (see condition (5)), s is also distributed with distribution $f(s)$. The third term reflects money losses from winning the auction and paying p when having expected to lose (with probability $1-f(b^i)$). Finally, the last term again describes the "losses" suffered from not obtaining the auction commodity.

Condition (10) implies that a positive bid only may result if $f(b^i) > 1-1/\lambda_1$ for positive utility gains to be generated. Therefore, we again obtain a threshold value w_L^{CA} as defined in (6), below which bidders place zero bids or do not participate in the auction. Note again that $w_L^{CA} = \underline{w}$ if $\lambda_1 \leq 1$, such that partial pooling at zero bids only occurs if $\lambda_1 > 1$.

Bidder i chooses b^i to maximize (11). Differentiating and simplifying (11) yields

$$w^i - b^i = \lambda_0 b^i - 2\lambda_0\int_{b(\underline{w})}^{b^i}pdf(p) + \lambda_1 w^i(1-2f(b^i)) \qquad (11)$$

While it is well known that without loss aversion ($\lambda_0 = \lambda_1 = 0$) truthful bidding will result, condition (11) shows immediately that bidding behavior changes in our setting due to the assumed loss aversion. In particular, underbidding results at the lowest valuation ($b(\underline{w}) = \max[0, \underline{w}(1-\lambda_1)/(1+\lambda_0)]$).

Proposition 3: (commodity second-price auction–CA) *The unique monotonic symmetric Bayesian Nash equilibrium in commodity second-price auctions with loss averse bidders is given by*

$$b^{2,\mathrm{CA}}(w) = w\frac{1-\lambda_1(1-2H^{n-1}(w))}{1+\lambda_0} + \frac{2\lambda_0}{(1+\lambda_0)^2}\int_{w_L^{CA}}^{w} z[1-\lambda_1(1-2H^{n-1}(z))]\exp\left(\frac{2\lambda_0}{1+\lambda_0}(H^{n-1}(w)-H^{n-1}(z))\right)dH^{n-1}(z) \quad \text{if } w \geq w_L^{CA} \tag{12}$$

and $b^{2,\mathrm{CA}}(w) = 0$ if $w < w_L^{CA}$.

Proof: See Appendix.

Bidders do not truthfully reveal their true value in order to reduce expected losses: the chance of experiencing "losses" from not obtaining the auction item can be decreased by lowering the reference probability of winning (low bid) or by increasing the probability of winning (high bid). From (11) it becomes obvious that the former is optimal for small value draws while the latter will result for high value draws. Along the money dimension, the effect of loss aversion is less straightforward. However, we can use condition (11) to generate the following comparative statics results.

Proposition 4: (effects of loss aversion–second-price CA) *If bidders are loss-neutral in the money dimension (* $\lambda_0 = 0$ *), bids are larger (smaller) than the value* w

if $H^{n-1}(w) > 1/2$ ($H^{n-1}(w) < 1/2$). *For money loss averse bidders (*$\lambda_0 = 1$*), bids decrease in commodity loss aversion* λ_1 *for small valuation (in particular if* $H^{n-1}(w) < 1/2$*). Equilibrium bids decrease in the degree of money loss aversion* λ_0 *for sufficiently small values* w*. In general, they can decrease or increase in* λ_0.

Proof: see Appendix.

Proposition 4 shows that loss aversion in the commodity dimension can both increase or decrease bids depending on the signal range. Loss aversion therefore breaks the standard result of truthful revelation of values in second-price sealed-bid auctions: for high value bidders, overbidding can result, while underbidding is predicted for low value draws.

We now turn again to the induced-value second-price auction. Here, the expected utility gain from participating in the auction is given by

$$\Pi^{2,\mathrm{IV}}(b^i, w^i) = \left[\int_{b(\underline{w})}^{b^i} [w^i - p - \lambda_0 \int_{b(\underline{w})}^{p} (p-s)\,df(s)]\,df(p)\right] - \lambda_0 (1 - f(b^i)) \int_{b(\underline{w})}^{b^i} (w^i - s)\,df(s) \tag{13}$$

The bracket has the same interpretation as the first two terms in (10) and gives the utility gain from winning the auction at price p. Here, "losses" might be experienced because the bidder might have expected to pay a smaller price. Similarly, the second term describes the "losses" experienced from not winning the auction: the bidder might have expected to win at some price $s < b^i$ and therefore suffers "losses" of size $w^i - s$.

Maximizing (13), we obtain

$$\frac{\partial}{\partial b^i}\Pi^{2,\mathrm{IV}}(b^i, w^i) = (w^i - b^i)\left[1-\lambda_0 + 2\lambda_0 f(b^i)\right] f'(b^i) = 0 \quad (14)$$

which shows that, if placing a positive bid, bidders reveal their valuation truthfully despite loss aversion. Note, however, that (14) implies that positive expected utility can only result if $f(b^i) > 1 - 1/\lambda_0$ such that bidders with $w_i < w_L^{IV}$ will place zero bids which eliminate their chances to win.

Proposition 5: (induced-value second-price auction–IV) *The unique monotonic symmetric Bayesian Nash equilibrium in induced-value second-price auctions with loss averse bidders is given by truthful revelation of valuations, i.e.*

$$b^{2,\mathrm{IV}}(w) = \begin{cases} w & \text{if } w \geq w_L^{IV} \\ 0 & \text{if } w < w_L^{IV} \end{cases} \quad (15)$$

Loss aversion at $\lambda_0 \leq 1$ (i.e. $w_L = \underline{w}$) therefore has no effects on bidding behavior in induced-value auctions. The different finding for commodity auctions (Proposition 4), again demonstrates that loss aversion gives one obstacle of transferring findings from induced-value settings to naturally-occurring auctions.

3.3 Revenue equivalence

To complete the analysis of equilibrium bidding, we finally discuss the potential revenue equivalence. For the induced-value auction (IV), Corollary 4 and Proposition 5 imply that loss aversion does not change second-price bidding but increases bids in first-price auctions compared with the risk-neutral prediction. Expected revenues in a first-price auction with loss aversion therefore exceed those in a second-price auction.

Proposition 6: (revenue comparison–IV) *Revenues in a first-price induced-value auction are larger than in a second-price auction if (symmetric) bidders are loss averse in the money dimension.*

This revenue-ranking is consistent with laboratory findings (e.g., Cox et al. 1982; Kagel and Levin 1993).[80] The effects of loss aversion are different in commodity auctions (CA). Here, loss aversion changes bids in both auction formats. We obtain the following result on revenues:

Proposition 7: (revenue comparison–CA) *If bidders are not averse to losses in the numeraire dimension ($\lambda_0 = 0$), first- and second-price auctions are revenue-equivalent in the commodity auction. For positive levels of λ_0, first-price auctions revenue-dominate second-price auctions.*

Proof: see Appendix.

The proof relies on the fact that expected utility gains are identical in both auctions for any player (equation (5) and (10)). For $\lambda_0 = 0$, this difference in expected utility gains between the two auction formats coincides with the difference between expected payments and is zero. For $\lambda_0 > 0$, revenue equivalence is not guaranteed. Here, expected payments are larger in the first-price than in the second-price auction for all values of w. Expected revenues in the first-price auction therefore dominate those in second-price auctions in both induced-value auctions and commodity-auctions.

[80] Note that risk-aversion generates the same qualitative revenue-ranking.

4 Conclusions

In this paper, we discussed the effects of loss aversion with endogenous reference points on bidding behavior in first- and second-price sealed-bid auctions. We demonstrated that it is important to consider the multi-dimensionality of the commodity space: The effects of loss aversion qualitatively differ depending on whether the auction item is money (induced-value) or some consumption good which generates a loss sensation when an individual unexpectedly loses the auction. In particular, we showed that loss aversion in induced-value first-price auctions leads to bids in excess of the risk-neutral prediction, while underbidding may result in commodity auctions.

The extent to which loss aversion affects bids in commodity auctions, may further depend on the characteristics of the auction item. Corresponding to varying ratios between willingness-to-pay and willingness-to-accept measures (Horowitz and McConnell 2002), we hypothesize that the effects of loss aversion in the commodity dimension are most prevalent in auctions of unique items. That is, if more opportunities exist to acquire close substitutes, the sensation of loss and therefore their impact on bidding behavior might be smaller.

Our findings put a word of caution on transferring qualitative behavioral findings from induced-value laboratory experiments to the field. Besides other differences the auction environments (e.g., subject pools, value of traded goods), we find that the one-dimensionality of the commodity space in most laboratory experiments in itself may be problematic. That is, auction experiments may need to include more than just a money dimension in order to better understand economic behavior in the field. The challenge will lie in designing such experiments while still keeping control over the underlying value distribution.

Chapter 4: Trading Intentions and Reference-Dependence in Auctions: Does Experience manifest through Heterogeneous Access to Outside Markets?

1. Introduction

In the previous chapters, I explored the effect of reference-dependent preferences on bidding in auctions. Given the relevance of commodity loss aversion, in this chapter, I explore the effect of reference-dependent preferences in the presence of resale or procurement opportunities on bidding in commodity auctions. Such alternatives influence trading intentions and therefore are likely to alter the effect of commodity loss aversion on bidding.

The main focus of this chapter is literature which suggests that experience has a direct effect on behavioral preferences. List (2003, 2004) finds that while prospect theory applies well to less experienced traders the behavior of more experienced traders comes closer to the predictions of neoclassical theory. I explore whether similar effects on individual decisions could arise due to other unobservable features (besides prior market experience) that mimic experience. I show that heterogeneous access to outside markets could alter trading intentions[81] thereby producing effects similar to those that could be attributed to the direct linkage between experience and

[81] This has been suggested in Kahneman, Knetsch, and Thaler (1990, p. 1328), who note "there are some cases in which no endowment effect would be expected, such as when goods are purchased for resale rather than for utilization."

preferences; such possibilities could arise in first-price sealed-bid auctions in the field, with anticipated loss aversion.

Previous literature has discussed the effect of resale (Bikhchandani and Huang 1989, Gupta and Lebrun 1999, Haile 2001, 2003, Garrat and Troger 2003 etc) and outside procurement prices on bidding in auctions. In Bikhchandani and Huang (1989) and Haile (2001, 2003) uncertainty over induced-values gets resolved overtime and creates opportunities for resale among the participants in the first-stage auction. [82] The existence of posted prices at which a commodity can be procured outside the auction on bidding with reference-dependent preferences has received contemporary interest (Shunda 2009, Reynolds and Wooders 2005, Durham, Roelofs and Standifird, 2004, Mathews 2003a and Mathews and Katzman 2004). The focus of this literature is an eBay type buy-price auction in which the seller offers the commodity to the bidder at a given price in the pre-auction stage. I explore a different but relevant scenario where access to outside markets is uncertain.

In contrast to previous literature on auctions with resale where secondary markets have same participants, in my framework, resale opportunities could arise due to absence of all interested bidders in the auction. In such a scenario, although participants in the primary auction are well aware of absent bidders but might have unequal access to them. Thus, opportunities for resale arise due to reasons different from induced-value uncertainty or asymmetry between bidders in the first-stage auctions. In my framework with procurement outside the auction, the effect of outside

[82] In Gupta and Lebrun (1999), inefficient outcomes could arise due to asymmetric distribution of bidders' valuation which creates opportunities for resale.

markets on bidding is different than in auctions with buy-price[83] due to the simultaneous existence of outside markets to which bidders may have unequal access. Such heterogeneous access to outside markets could arise due to differences in prior market experience, bargaining ability, transactions costs, time committed to resale/procurement effort or other unobservable characteristics.

While a formal treatment of how experience affects bidding directly could be more challenging, if uncertain market access is interpreted as a proxy for bidder experience, it becomes possible to analyze the indirect effects of experience on bidding in an auction with outside alternatives. In recent years, following rapid advances in internet based communication search and transactions costs have gone down drastically thereby expanding the reach of consumers for everyday commodities (Lee 1998, Ariely and Lynch 2000) and could potentially affect auction outcomes.

I analyze the effect of experience on bidding within the context of reference-dependent preferences to isolate the effect of trading intentions that arise due to heterogeneous market access. As shown in chapter 2, anticipated loss aversion can explain aggressive bidding in first-price auctions. As discussed in chapter 1, the literature on consumer psychology has discussed how consumers get affected by reference prices in everyday transactions. Such effects have been discussed in e-Bay "buy now" auctions where buy prices are believed to affect bidding decisions (Shunda 2009, Dodonova and Khoroshilov 2004, Popkowski Leszczyc et al. 2007).

[83] In these auctions the primary interest is: (i) how do bidders behave regarding accepting or rejecting a transaction at the buy price in the first stage and (ii) how does it affect bidding in the second stage auction?

In my approach with loss aversion, resale or procurement options affect equilibrium bidding such that there is a direct speculative effect on bidding (as in the expected utility framework) and forward-looking (anticipatory) behavior influences equilibrium bidding by affecting the “attachment” of a bidder to the commodity. Due to the additional behavioral effects, deviations arise with respect to the risk-neutral-Nash predictions. In auctions with resale, “loss aversion” causes underbidding with respect to the risk-neutral-Nash prediction. Bidders with highest access over favorable prices are least affected by “loss aversion” and therefore bid closer to the risk-neutral-Nash than the bidders with smaller access to favorable prices. In auctions with procurement, the attachment effect is such that it may cause overbidding (underbidding) with respect to the risk-neutral-Nash. Bidders with greatest level of market access are again least affected by “loss aversion” and therefore bid much conservatively and closer to the risk-neutral-Nash than the bidders with less favorable access to procurement prices. Thus, the predictions of my model are qualitatively similar to the findings in List (2003, 2004) which suggest that market experience attenuates the endowment effect. Since these indirect effects are observed without altering reference dependent preferences, it raises the possibility that the effects obtained in List (2003, 2004) in field settings may not arise entirely due to direct effect of experience on such preferences.

In the following sections I discuss the modern marketplace that makes outside markets relevant for auctions (section 2), describe a model of reference-dependent preferences (section 3) and discuss optimal bidding with resale or outside procurement for first-price auctions (section 4). Then I discuss the findings in the context of the literature (section 5) and conclude (sections 6).

2. The Modern marketplace: Multiple trading instruments, search and technological innovation

In contemporary markets, everyday goods and intangible assets are traded via a multiplicity of mechanisms like auctions, posted prices and/or decentralized bilateral bargaining (chapter 5, Handbook of Experimental Economics). In recent times, lower search and transaction costs present a much wider marketplace to an individual consumer. Automobiles, government property, electronics, pollution permits, spectrum licenses, exploration rights etc are sold via auctions and other instruments in many countries.

While the simultaneous existence of auctions and posted prices has been discussed in the context of eBay buy price auctions, the sale of used cars, electronics and real estate through auctions and posted prices are commonplace.[84] The buy price transactions account for a share of between 32% and 49% of the quarterly sales that range between \$10.6 and \$16.2 billion on eBay; sellers choose to augment their auction with a buy price in between 30% and 60% of online auctions, and, among those auctions with a buy price, between 10% and 40% end with a transaction at the buy price (Shunda 2009). A sizable number of SO_2 emissions permits worth millions of dollars have been sold both via auctions (spot and advance auctions) and induced

[84] Also notable among these are the sale of SO_2 emissions permits by EPA in the United States, simultaneously through auctions and permit markets (http://www.epa.gov/airmarkt/trading/buying.html, Schmalensee et al. 1998, Jaskow et al. 1998).

markets since 1993 by EPA (Jaskow et al. 1998) until now.[85] While subsequent resale of emissions permits is widely reported by EPA and related literature, the relevance of cost reducing technological advances that provide the same services is especially relevant for EPA advance auctions.

The access of individual consumers to outside markets and potentially cost reducing technological advances is determined by characteristics like search and transactions costs, experience and consumption plans. It is therefore natural that access to outside markets affects bidding in auctions. In this chapter, I focus on bidding in auctions in the context of outside markets.

3. A Model of Reference-Dependent Preferences

As before, an individual's utility $u^i(c \mid r)$ depends both on her consumption $c = (c_0, c_1) \in \mathbb{R}^2$ and her reference level $r = (r_0, r_1) \in \mathbb{R}^2$.[86] Consistent with Köszegi and Rabin (2006), I assume that utility is additively separable in the numeraire and commodity dimensions: $u^i(c \mid r) = u_0^i(c_0 \mid r_0) + u_1^i(c_1 \mid r_1)$. The "direct" consumption utility $v_t^i(c_t), t \in \{0,1\}$ is obtained when realized consumption is the same as the reference level, i.e., $v_t^i(c_t) = u_t^i(c_t \mid c_t)$, and the individual utility when her consumption differs from her reference is defined as :

$$u_t^i(c_t \mid r_t) = v_t^i(c_t) - k_t \max[0, v_t^i(r_t) - v_t^i(c_t)] \tag{1}$$

[85] http://www.epa.gov/airmarkt/trading/2009/09summary.html
[86] The consumption bundle for a bidder comprises of the auction commodity indexed by 1 and monetary payments indexed by 0.

with $0 \leq k_t, t \in \{0,1\}$. k_t is the scalar which captures the sensation of "loss" when less favorable outcomes are realized.[87] I simplify by assuming that individuals experience "losses" only in the commodity dimension and not in money[88] i.e. $k_0 = 0$ and $0 \leq k_1$.

The Auction environment: I consider an auction that has n bidders with symmetric risk-neutral preferences. I assume that each bidder has the same consumption utility $\bar{v} > 0$ for the commodity. Each bidder knows that (i) rivals' have the same consumption utility and (ii) behavioral and/or risk preferences. There exists a resale or procurement market to which bidders have heterogeneous access (specified later). The auctioneer invites sealed-bids from bidders present at the auction and the commodity is awarded to the highest bidder in exchange for her (second highest) bid in the first (second) price auction.

3.1 Stochastic Reference in Sealed-bid Auction

Ex ante, auction outcomes are uncertain and depend on the rivals' bidding strategies and other characteristics (references). Bidders who anticipate these outcomes may develop expectations regarding winning or losing; to the extent winning the auction is possible, "not winning" the auction could induce psychological "losses". Following Köszegi and Rabin (2006), the reference level is a probability measure G over $\mathbb{R}^2$

[87] Thus, psychological "gains" are normalized to zero.
[88] This is a suitable description for real world where day to day exposure to monetary transactions yields no loss aversion in money dimension.

and consumption is drawn according to the probability measure F over $\mathbb{R}^2$. Then, the individual's overall expected utility over risky outcomes is given by

$$U(F \mid G) = \int\int u(c \mid r) dG(r) dF(c) \qquad (2)$$

As in earlier chapters, I deviate from the Köszegi-Rabin formulation of overall expected utility. In an equilibrium (for a sealed-bid auction) captured by a strictly increasing symmetric bidding function, the bid determines the probability of winning and the consequent auction outcomes for a bidder. Since no further action that affects auction outcomes is possible after placing the bid, the joint probability distribution over potential resale/trade and auction outcomes-F (which is determined by the equilibrium bid) defines the distribution over potential consumption outcomes for a bidder with rational expectations, and also generates an identical probability distribution -G over (reference) auction outcomes for a bidder with rational expectations. Therefore, in an auction with resale or procurement possibilities, an individual solves the following program:

$$\max_{F} U(F \mid F) \qquad (3)$$

This specification is similar to the (anticipatory) approach taken in earlier chapters and different from the general setting discussed by Köszegi and Rabin (2006).

3.2 Heterogeneous access to outside Markets

Individual bidders have heterogeneous access to prices in resale or procurement markets. Such differences may arise due to differences in individual characteristics like experience, bargaining ability, transactions costs etc. among otherwise similar bidders.

3.2.1 Resale outside the auction

There exists a resale market in which a price t is offered in exchange for the commodity. Let's consider the equilibrium behavior of an auction winner who pays a bid B with reference-dependent preferences when resale price is uncertain. Consider the utility from resale at price t and no resale under the most unfavorable circumstances for resale (when no resale at price t is the reference level)

$$u(\textit{resale at } t \mid \textit{No resale at } t) = t - B_i - k_1\bar{v}$$

$$u(\textit{No resale} \mid \textit{No resale at } t) = \bar{v} - B_i$$

Clearly if t is high enough then

$u(\textit{resale at } t \mid \textit{No resale at } t) \geq u(\textit{No resale} \mid \textit{No resale at } t)$; this happens when

$t - B_i - k_1\bar{v} \geq \bar{v} - B_i \Leftrightarrow t \geq (1+k_1)\bar{v}$. In such circumstances, trading is optimal. Now consider the utility from trading at price t and not trading under the most favorable circumstances for trading (when trading at price t is the reference level) (since there is no money loss aversion)

$$u(\textit{resale at } t \mid \textit{resale at } t) = t - B_i$$

$$u(\textit{No resale} \mid \textit{resale at } t) = \bar{v} - B_i$$

Clearly if $t < \bar{v}$, no resale is optimal.

I assume that there exists a resale market in which only two prices are offered - $0 \leq m \leq \bar{v}$ or $M > \bar{v}(1+k_1)$ in exchange for the commodity. Thus, the exchange prices are such that in equilibrium each bidder is willing to exchange the commodity only at the higher price M. However, bidders have heterogeneous access to prices. For a bidder, μ_i is the probability of drawing the higher resale price M which is randomly chosen for bidder i from a distribution H over the interval $[0,1]$ [89] and constitutes his private knowledge. It is common knowledge that rivals' access over resale prices is determined independently (randomly) from the same distribution. Thus, μ_i takes a higher value for greater access to price M and yields higher probability for resale.

3.2.2 Procurement outside the auction

There exists a procurement market in which a price t is offered in exchange for the commodity. Let's consider the equilibrium behavior of an auction winner with reference-dependent preferences when procurement price is uncertain. Consider the utility from procurement at t and no procurement under the most unfavorable circumstances for procurement (when no procurement at t is the reference level)

$$u(procurement\ at\ t \mid No\ procurement\ at\ t) = \bar{v} - t$$

$$u(No\ procurement \mid No\ procurement\ at\ t) = 0$$

Clearly if t is high enough then

$$u(procurement\ at\ t \mid No\ procurement\ at\ t) \geq u(No\ procurement \mid No\ procurement\ at\ t).$$

This happens when $\bar{v} > t$. In such circumstances, procurement is optimal. Now consider the utility from procurement at t and no procurement under the most

[89] The prices take only two values for simple exposition; alternatively it is likely that bidders with greater access draw higher prices with higher probability. This generalization though useful is not required for drawing the main results.

favorable circumstances for procurement (when procurement at t is the reference level) (since there is no money loss aversion)

$$u(procurement\ at\ t \mid procurement\ at\ t) = \bar{v} - t$$

$$u(No\ procurement \mid procurement\ at\ t) = -k_1\bar{v}$$

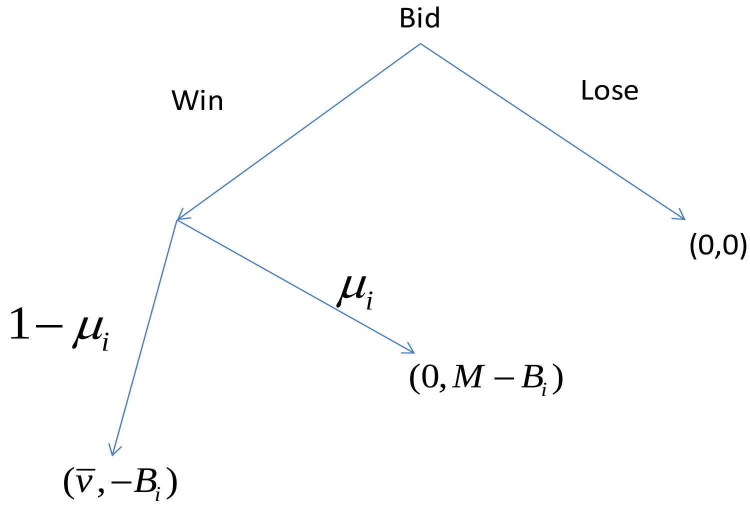
Clearly if $t > \bar{v}(1+k_1)$, no procurement is optimal. I assume that there exists a procurement market in which only two prices are offered - $0 \le r \le \bar{v}$ or $R > \bar{v}(1+k_1)$ in exchange for the commodity. Thus, the procurement prices are such that in equilibrium each bidder is willing to procure the commodity only at the lower procurement price r. However, bidders have heterogeneous access to prices. For a bidder μ_i is a probability of drawing the higher procurement price R which is randomly chosen for bidder i from a distribution H over the interval $[0,1]$. It is common knowledge that rivals' access over prices is also chosen independently (randomly) from the same distribution. *Thus, μ_i is defined such that it takes a lower value for more favorable access over procurement prices and the probability for outside procurement at price r* is $1-\mu_i$.

4. First-Price Auction

4.1 Auction with resale

The decision process for a first-price-auction with resale is described in figure 7.

Figure 7: Bidding Problem in First-price Auction with Resale Intentions



The ex-ante overall expected utility for a bidder is

$$\pi_{PT}(\mu_i, B_i) = f(B_i)(1-\mu_i)(\bar{v}-B_i) + f(B_i)\mu_i(M-B_i)$$
$$-k_1 f(B_i)(1-f(B_i))(1-\mu_i)\bar{v} - k_1 f(B_i)^2 \mu_i(1-\mu_i)\bar{v} \qquad (4)$$

where $f(B_i)$ denotes the probability of winning conditional on bid B_i. The first-term and second terms are the expected (direct) consumption utility (net of monetary payments) realized when the auction is won and the commodity is kept or successfully traded. The third and fourth terms capture commodity loss aversion; this is realized when the bidder loses the auction but had expected to win and consume the commodity or when the bidder trades away the commodity when she expected to consume it upon winning. Given the high resale price, the level of access over favorable prices which varies for bidders determines the probability of resale. Thus, loss aversion affects bidders heterogeneously.

Note that (4) implies that a non-negative expected utility gain $\pi_{PT}(\mu_i, B_i)$ from participating in the auction results only if

$$(1-\mu_i)\overline{v}+\mu_i M \geq B_i \text{ and } \frac{(1-\mu_i)\overline{v}+\mu_i M - B_i}{k_1\overline{v}(1-\mu_i)} \geq 1 - f(B_i)(1-\mu_i) \qquad (4a)$$

That is, positive bids will be placed only by bidders with sufficiently large access to the high resale price or high consumption utility.

As usual, I restrict my attention to symmetric monotonically increasing equilibria in pure strategies. Let us assume that all opponents of bidder i bid according to a strictly increasing bidding strategy $B(\mu_i)$ $\mu_i \in (0,1)$. When placing a bid B^i, bidder i's probability of winning is therefore given by $f(B^i) = H^{n-1}(B^{-1}(B^i))$. In equilibrium, the chances of player i to win, are therefore given by $H^{n-1}(\mu_i)$. From (4a) the following can be derived: when $k_1 \leq 1$, $\hat{\mu}_r^1 = 0$ [90]and when $k_1 > 1$, positive bids will result only if $H^{n-1}(\mu_i) \geq [\frac{1}{1-\mu_i} - \frac{(1-\mu_i)\overline{v}+\mu_i M - B(\mu_i)}{k_1\overline{v}(1-\mu_i)^2}]$ (this is straightforward from (4a)). Thus, the corresponding threshold value $\hat{\mu}_r^1$ is implicitly defined by

$$H^{n-1}(\hat{\mu}_r^1) = \frac{1}{1-\hat{\mu}_r^1} - \frac{(1-\hat{\mu}_r^1)\overline{v}+\hat{\mu}_r^1 M}{k_1\overline{v}(1-\hat{\mu}_r^1)^2} \qquad (4b)$$

Clearly, optimal bidding depends on the degree of loss aversion and access to resale prices. Bidders with $\mu_i \in [\hat{\mu}_r^1, 1]$ get non-negative expected utility gain $\pi_{PT}(\mu_i, B_i)$ and place positive bids.

---

[90] Note that nonnegative expected overall utility implies

$Z = \left(\frac{1}{k_1} - 1\right) + \left(\frac{1}{k_1}\frac{\mu_i M - B_i}{\overline{v}(1-\mu_i)} + H^{n-1}(\mu_i)(1-\mu_i)\right) \geq 0$; when $B(\mu_i) = 0$

$\frac{\partial}{\partial \mu_i}\left(\frac{1}{k_1}\frac{\mu_i M - B_i}{\overline{v}(1-\mu_i)} + H^{n-1}(\mu_i)(1-\mu_i)\right) > 0$; when $k_1 \leq 1$ and $\mu_i = 0$, therefore $Z \geq 0$.

Maximizing the expected payoff in (4) yields the following monotonic bid function:

Proposition 1: (-First-price auction with resale-) *For a uniform* $H(\bullet)$*, the monotonic Bayesian Nash equilibrium bid function for (symmetric) bidders with commodity loss aversion is given by*

$$B(\mu_i)_{PT} = \begin{cases} \dfrac{\int_{\hat{\mu}_r^1}^{\mu_i} [(1-x)\bar{v} + xM]\, dH^{n-1}(x)}{H^{n-1}(\mu_i)} - k_1 \bar{v} \dfrac{\int_{\hat{\mu}_r^1}^{\mu_i} [(1-2H^{n-1}(x))(1-x) + 2H^{n-1}(x)x(1-x)]\, dH^{n-1}(x)}{H^{n-1}(\mu_i)} & \text{if } \mu_i \geq \hat{\mu}_r^1 \\ 0 \quad \text{if } \mu_i < \hat{\mu}_r^1 \end{cases}$$

Proof: See Appendix

The optimal bidding under the expected utility scenario with risk-neutral preferences is $\dfrac{\int_0^{\mu_i} [(1-x)\bar{v} + xM]\, dH^{n-1}(x)}{H^{n-1}(\mu_i)}$ which is obtained by substituting $k_1 = 0$ in above; it reflects the effect of speculation on bidding. Clearly, greater access to high resale price yields greater expected overall payoff to the bidder and causes higher bidding. For $k_1 = 0$, as $\mu_i \to 0, B \to \bar{v}$ [91] i.e. the bidder with least access to the high resale price bids her consumption utility and derives zero expected payoff from the auction. With commodity loss aversion, we obtain underbidding compared to the risk-neutral-Nash bid function. Since, the high resale price more than compensates for loss aversion $M \geq \bar{v}(1+k_1)$ each bidder anticipates to exchange the commodity at the high resale price. Since she will lose the commodity as a result of this exchange she tries to avoid

[91] This is derived from the bid function using L' Hospital's rule.

the commodity "losses" by bidding less. Due to loss aversion, as $\mu_i \to 0, B_i \to \max[0, \bar{v}(1-k_1)]$ [92]; this reflects that only nonnegative bids are allowed. More generally the following is obtained:

Proposition 2 (-Effect of loss aversion in first-price auction with resale-)

For a uniform distribution $H(\bullet)$ *over resale prices, commodity loss aversion has a decreasing effect on bids i.e.* $\frac{\partial B}{\partial k_1} < 0$.

Proof: see Appendix.

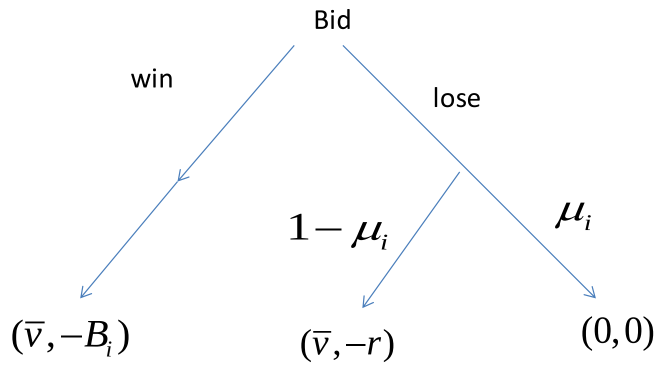
The effect of commodity loss-aversion on bids in this scenario is different from induced-value auctions without resale where money loss aversion yields overbidding (chapter 2) and from commodity auctions where the effect of commodity loss aversion on bids depends on induced consumption utility (chapter 3). Intuitively, for any given access over resale prices, the bidders are willing to exchange the commodity at the high resale price M and anticipate the ensuing commodity loss aversion; in the presence of sufficient competition, bidding less aggressively lowers their chances of winning the auction and experiencing ensuing commodity loss.

4.2 Auction with outside procurement

The decision process for a first-price-auction with procurement is described in figure 8.

[92] This is derived from the bid function using L' Hospital's rule.

Figure 8: Bidding Problem in First-price Auction with procurement intentions



The ex-ante overall expected utility for a bidder is

$$\pi_{PT}(\mu_i, B_i) = f(B_i)(\bar{v} - B_i) + (1 - f(B_i))(1-\mu_i)(\bar{v} - r) \\ - f(B_i)(1 - f(B_i))\mu_i k_1 \bar{v} - (1 - f(B_i))^2 \mu_i (1-\mu_i) k_1 \bar{v} \qquad (5)$$

Where $f(B_i)$ denotes the probability of winning conditional on bid B_i. The first-term and second terms are the expected (direct) consumption utility (net of monetary payments) realized when the commodity is won through the auction or the commodity is successfully accessed and exchanged for a price r. The third and fourth terms capture commodity loss aversion; this is realized when the bidder loses the auction and could not procure the commodity but had expected to win commodity through the auction or procure the commodity upon losing. Given a relatively low procurement price r, access over procurement prices which varies for bidders, determines the probability of procurement. Thus, loss aversion affects bidders heterogeneously.

Note that the reservation overall expected utility for the bidder equals $(1-\mu_i)(\overline{v}-r)-k_1\mu_i(1-\mu_i)\overline{v}$ due to outside procurement possibilities. Thus, (5) implies that participation in the auction exceeds the utility from just relying on outside procurement only if

$$\mu_i\overline{v}+r(1-\mu_i)-B_i \geq k_1\overline{v}\mu_i[2\mu_i - f(B_i)\mu_i - 1] \qquad (5a)$$

Clearly, auction participation matters depends on: k_1, distribution $H(\bullet)$, the number of bidders, among others. It is obvious that for moderate levels of loss-aversion, (5a) holds for bidders with any distribution $H(\bullet)$ over prices. Bidders derive expected utility greater than the reservation level of utility by bidding above r, for very large and small levels of access to prices for any $H(\bullet)$ and n. [93]However, (5a) may not hold for intermediate range of μ_i under certain $H(\bullet)$ and n.

As before, I restrict my attention to symmetric monotonically increasing equilibria in pure strategies such that there exists a strictly increasing bidding strategy $B(\mu^j)$ $\mu^j \in (0,1)$ and $f(B^i)=H^{n-1}(B^{-1}(B^i))$. [94]With the above argument, positive bids result only if $H^{n-1}(\mu_i) \geq 2-\frac{1}{\mu_i}-\frac{\mu_i\overline{v}+r(1-\mu_i)-B}{k_1\overline{v}\mu_i^2}$. [95] When $H(\bullet)$ is uniform and $n=2$, we can guarantee that (5a) holds for all possible bidder types and bidding above r is optimal for all $\mu_i > 0$. As noted earlier, this is also true when k_1 is sufficiently small as defined in (5a); even under such circumstances, optimal bidding

[93] This is easily shown by putting $B_i = r$ in (5a) and taking limits as $\mu_i \to 0,1$.

[94] In view of the above, therefore some level of generality is lost while deriving the main results.

[95] Note however that bidding r is as good as outside procurement without loss-aversion; however with loss-aversion the expectations associated with some positive chances of winning yields less expected overall utility than outside procurement. Therefore, it is optimal to bid zero with loss-aversion if the chances of winning are not high and outside procurement is possible.

depends on the degree of loss aversion and access to procurement prices. Maximizing the expected payoff in (5) yields the following monotonic bid function:

Proposition 3:(-First price auction with procurement-) *For a uniform* $H(\bullet)$ *and* $n=2$, *the monotonic Bayesian Nash equilibrium bid function for (symmetric) bidders with commodity loss aversion is given by*

$$B(\mu_i)_{PT} = \frac{\int_0^{\mu_i} [\bar{v} - (1-x)(\bar{v} - r)] dH^{n-1}(x)}{H^{n-1}(\mu_i)} - \frac{k_1 \bar{v} \int_0^{\mu_i} [(1-2H^{n-1}(x))x - 2(1-H^{n-1}(x))x(1-x)] dH^{n-1}(x)}{H^{n-1}(\mu_i)}$$

Proof: See Appendix

The optimal bidding under the expected utility scenario with risk-neutral preferences is $\frac{\int_0^{\mu_i} [\bar{v} - (1-x)(\bar{v} - r)] dH^{n-1}(x)}{H^{n-1}(\mu_i)}$ which is obtained by substituting $k_1 = 0$ in above and reflects the effect of (procurement) speculation on bidding. Higher access to the low procurement price r, makes winning the commodity in the auction less important and causes lower bidding. This is different from before where higher access to the favorable resale price causes higher bidding. As $\mu_i \to 0, B \to r$ i.e., the bidder with almost certain access to the low procurement price bids only the low procurement price r with or without loss aversion.

Under the circumstances, with loss aversion we obtain overbidding with respect to the risk-neutral-Nash bid function.

Proposition 4 (-Effect of loss aversion in first-price auction with procurement-) *For a uniform distribution* $H(\bullet)$ *over procurement prices and* $n = 2$, *bids are increasing in commodity loss aversion i.e.* $\frac{\partial B}{\partial k_1} > 0$.

Proof: *see Appendix*

The comparative static shows that the impact of loss aversion depends on (i) the distribution of access to prices and (ii) number of bidders in the auction. Commodity "losses" are realized when the bidder loses the auction and could not procure the commodity but had expected to (i) procure the commodity upon losing or (ii) win commodity through the auction. The first type of *losses* always induces higher bids. The second type of *losses* could either cause higher or lower bids. If a bidder is likely to win to start with, she can decrease chances of disappointment by increasing probability of winning. As a consequence, loss aversion leads to higher bids. However, if a bidder is unlikely to win to start with, he can decrease expectations by further decreasing the probability of winning, i.e. by bidding lower due to loss aversion. The overall effect on bidding depends on the net of these effects (this is discussed in the proof for proposition 4).

Nevertheless, for a uniform $H(\bullet)$ over procurement prices and $n = 2$, commodity loss aversion always induces higher bidding.

5. The Role of Market Access

The heterogeneous access to prices influences bidding in auction with resale or procurement possibilities. The risk-neutral-Nash equilibrium bids capture the direct speculative effect of prices in outside markets. The deviations from risk-neutral-Nash bids arise due to loss aversion and depend on (i) access to prices in outside markets and (ii) the level of competition. The bidders with favorable access to prices bid closer to the predictions of the risk-neutral-Nash bids. In general, we obtain the following:

Proposition 5: For a uniform distribution $H(\bullet)$ of market access over resale prices, in (first price) auctions, deviations from risk-neutral-Nash equilibrium due to commodity loss aversion decline with greater access to high resale prices i.e. $\frac{\partial(B_{RN}(\mu_i) - B_{PT}(\mu_i))}{\partial \mu_i} < 0$

Proof: See Appendix

This applies to first- price auctions. Greater access to high resale price induces lesser expectation of retaining the commodity for consumption, which yields a smaller effect of commodity loss aversion on bidding in auction. Thus, deviations from the risk-neutral-Nash prediction become smaller with greater access to high resale prices. Thus, greater access to resale prices affects trading intentions such that the effects are similar to those attributed to direct effect of experience on preferences (List 2003, 2004).

In auctions with outside procurement, risk-neutral-Nash yields less aggressive bidding. Due to commodity loss aversion, conditional on the (i) number of rivals, and (ii) access to prices, bidders could overbid or underbid below the Risk-neutral-Nash bids. Thus, unlike in auctions with resale a consistent decline in deviations from the Risk-neutral-Nash bidding with access to procurement price may not be observed. However, given that bidders with low levels of market access over low procurement prices always overbid and bidders above a high level of access could either underbid or overbid larger deviations with respect to the risk-neutral-Nash are obtained for bidders with very small access to low procurement price. In general we obtain:

Proposition 6: For a uniform distribution $H(\bullet)$ over procurement prices and $n = 2$, in (first-price) auctions, deviations from risk-neutral-Nash equilibrium for bidders with high levels of market access are smaller than the bidders with very low levels of market access i.e.

$$\lim_{\mu_i \to 0} | B_{PT}(\mu_i) - B_{RN}(\mu_i) | \quad < \quad \lim_{\mu_i \to 1} | B_{PT}(\mu_i) - B_{RN}(\mu_i) |$$

Proof: See Appendix

This applies to first-price auctions. I simulate bidding functions for a uniform access over procurement prices, and (i) various levels of loss aversion. (See figure 9 in the appendix). This proposition is addressed to compare the behavior of bidders with relatively high and low access to the low procurement price just like the behavior of inexperienced traders with highly experienced traders in List (2003, 2004). Thus, greater access to procurement prices affects trading intentions such that the effects are similar to those attributed to direct effect of experience on preferences (List 2003, 2004).

6. Conclusion and Policy Implications

Auctions are the preferred mode of transfer of *unique* goods and services and maximization of seller's revenue. In recent years, following rapid advances in internet based communication, search and transactions costs have gone down drastically thereby expanding the reach of consumers for everyday commodities. This is one way in which outside markets are more relevant than before. This is especially relevant in the context of auction commodities when technological changes could make low cost alternatives a possibility in future. Therefore, the framework is relevant for a scenario where the auction commodity could potentially become available in future at relatively low cost (Schmalensee et al. 1998). Unobservable effort and/or ability to innovate or access to such alternatives are likely to be important influences on bidding in the presence or absence of other distinguishing characteristics of bidders. Such influences matter when the auction commodity is not required for immediate consumption and therefore search and/or technological advances are possible. Among commodities sold through auctions- real estate, environmental permits, automobiles, art, collectibles, spectrum licenses, electronics, debt instruments, exploration and extraction rights, are subject to influences that could arise due to outside procurement either because they are (i) non-unique and have outside markets (therefore searchable) and/or (ii) likely to become available at cheaper prices due to technological changes.[96] The framework I have presented in this paper addresses the effect of such circumstances in auctions.

I have explored the effect of resale or procurement outside the auction on bidding in a sealed-bid first-price auction. Individuals who participate in auctions possess unobservable characteristics like experience levels, bargaining skills, ability to innovate etc. that could yield heterogeneous access over prices in resale or procurement markets. I explore the effect of differences in trading intentions that arise due to heterogeneous access over outside market prices on bidding in auctions. I show that bidders with greater access to favorable prices are in the most advantageous position such that they bid (i) aggressively in auction with profitable resale opportunities and (ii) conservatively in auctions where low cost procurement is possible.

If bidders are loss averse in commodity, there is an additional "attachment" effect on bidding. In auctions with resale, loss aversion causes underbidding with respect to the risk-neutral-Nash prediction. Bidders with favorable access over prices are again least affected by loss aversion and therefore bid closer to the risk-neutral-Nash than the bidders with less favorable access. In auctions with procurement, the attachment effect is such that it may cause overbidding (underbidding) for bidders with respect to the risk-neutral-Nash. Nevertheless, bidders with favorable access over prices are again least affected by loss aversion and therefore bid much conservatively and closer to the risk-neutral-Nash than the bidders with less favorable access over prices.

[96] The demand for electronic storage of data is one such example: within last few years the modes of storage has changed rapidly.

If access to favorable prices is interpreted as a proxy for experience levels, the predictions of my model are qualitatively similar to the findings that suggest a direct linkage between experience and preferences (List 2003, 2004). If such heterogeneous access is interpreted as proxy for experience then my model captures the indirect relationship between experience and bidding. Since these indirect effects are observed without altering loss averse preferences, it raises the possibility that the effects obtained in List (2003, 2004) may not arise entirely due to direct linkage between experience and preferences.

Appendices

A.1 Proofs

Chapter 2

Proof for Proposition 1

For $v_i \geq \hat{v}(k_l), [1-k_l(1-f(B(v_i)))] \geq 0$ maximizing (5), bidder i chooses B_i according to

$$\omega' f'(B_i)(v_i - B_i) - \omega f^i - \omega' f'(B_i)[1-2\omega(f^i)k_l](v_i - B_i) + \omega(f^i)(1-\omega(f^i))k_l = 0$$

(A.1)

Here $f^i = f(B_i) = F^{n-1}(B^{-1}(B_i))$ and therefore $f'(B_i) = (F^{n-1})'(B^{-1}(B_i))(B^{-1})'(B_i)$. In equilibrium, we have $B^{-1}(B_i) = v_i$, $(B^{-1})'(B_i) = 1/B'(v_i)$, and $f^i = F^{n-1}(v_i)$.

Rearranging (A.1) gives

$$\omega'(F^{n-1})'(v_i)v_i[1-(1-2\omega(F^{n-1}(v_i)))k_l] = [\omega(F^{n-1}(v_i))(1-k_l(1-\omega(F^{n-1}(v_i))))B(v_i)]'$$

(A.2)

Integrating yields

$$B(v_i)_{PT} = \frac{\int_{\hat{v}(k_l)}^{v_i} x[1-k_l(1-2\omega(F^{n-1}(x))]d\omega(F^{n-1}(x))}{\omega(F^{n-1}(v_i))[1-k_l(1-\omega(F^{n-1}(v_i)))]}$$

(A.3)

as the unique candidate for a symmetric monotonic bidding equilibrium.

Monotonicity of $B(v_i)_{PT}$ can easily be established by differentiating (A.3) and using the following:

for $v_i \geq \hat{v}(k_l), [1-k_l(1-f(B(v_i)))] \geq 0 \Rightarrow [1-k_l(1-2f(B(v_i)))] \geq 0$.

It remains to show the second-order condition for the maximization problem. Using the envelope theorem and (A.1), this is equivalent to $\partial^2 \pi_{PT}(B(v_i), v_i)/\partial B \partial v_i \geq 0$

which holds true since $\partial^2 \pi_{PT}(B(v_i), v_i)/\partial B \partial v_i = \omega' f'(B(v_i))[1-k_l(1-2f(B(v_i)))] \geq 0$

since $[1-k_l(1-f(B(v_i)))] \geq 0$

Applying L'hospital's rule to (A.3) yields the bid for lowest induced-value.

For $v_i < \hat{v}(k_l), B(v_i) = v_i$ maximizes payoff (yields zero payoff).

▪

Proof for Proposition 2 (i)

Note first that by definition of $\hat{v}(k_l)$, $\frac{\partial \hat{v}(k_l)}{\partial k_l} > 0$; for $v_i \geq \hat{v}(k_l)$ rewrite bid function

(A.3) as $$B(v_i)_{PT} = v_i - \frac{\int_{\underline{v}}^{v_i} \omega(F^{n-1}(x))[1-k_l(1-\omega(F^{n-1}(x))]dx}{\omega(F^{n-1}(v_i))[1-k_l(1-\omega(F^{n-1}(v_i)))]} \qquad (2.1)$$

From above

$$den^2 \frac{\partial B_{PT}}{\partial k_l} = -[den \int_{\underline{v}}^{v_i} -\omega(x)(1-\omega(x))dx + \omega(v_i)(1-\omega(v_i)) \int_{\underline{v}}^{v_i} (\omega(x) - k_l \omega(x)(1-\omega(x)))dx]$$

where $den = \omega(v_i) - \omega(v_i)(1-\omega(v_i)k_l; \omega(v_i) = \omega(F(v_i)^{n-1}), \omega(x) = \omega(F(x)^{n-1})$. Upon expansion and cancellation this reduces to $den^2 \frac{\partial B_{PT}}{\partial k_l} = \int_{\underline{v}}^{v_i} (1-\omega(x))dx - \int_{\underline{v}}^{v_i} (1-\omega(v_i))dx$

. For all $x < v_i$ and monotonic probability weighting $\omega(v_i) > \omega(x)$. Thus, $\frac{\partial B_{PT}}{\partial k_l} > 0$.

Proof for Proposition 2 (ii)

Note first that by definition of $\hat{v}(k_l)$, $\frac{\partial \hat{v}(k_l)}{\partial \beta} > 0$; for $v_i \geq \hat{v}(k_l)$ I show that (i)

$\frac{\partial}{\partial \beta} B(v_i) \geq 0$ for $k_l \geq 1$ (ii) and $\frac{\partial}{\partial \beta} B(v_i) \geq 0$ is guaranteed for $0 \leq k_l \leq 0.995066$

Let $P(x) = F(x)^{n-1}$ and $P(v) = F(v)^{n-1}$ and drop subscript i for simplicity. From (2.1)

$$\frac{\partial}{\partial \beta} B(v_i) \geq 0 \Leftrightarrow \frac{\frac{\partial}{\partial \beta}[\omega(P(v))\left[1-k_l(1-\omega(P(v)))\right]]}{\omega(P(v))\left[1-k_l(1-\omega(P(v)))\right]} \geq \frac{\int_{\underline{v}}^{v_i} \frac{\partial}{\partial \beta}\left[\omega(P(x))\left[1-k_l(1-\omega(P(x)))\right]\right]dx}{\int_{\underline{v}}^{v_i} \omega(P(x))\left[1-k_l(1-\omega(P(x)))\right]dx}$$

(3.1)

Now

$$\frac{\partial}{\partial \beta}\left[\omega(P(v))\left[1-k_l(1-\omega(P(v)))\right]\right] = \left[1-k_l(1-2\omega(P(v)))\right]\frac{\partial \omega(P(v))}{\partial \beta}$$

where

$$\frac{\partial \omega(P(v))}{\partial \beta} = \frac{\partial (P(v)^{\beta})}{\partial \beta} = \omega(P(v)) \ln P(v)$$

Since $\ln P(v) \leq 0$ (3.1) is equivalent to

$$\frac{\int_{\underline{v}}^{v_i}\left[\omega(P(x))\left[1-k_l(1-\omega(P(x)))\right]\right]dx}{\omega(P(v))\left[1-k_l(1-\omega(P(v)))\right]} \le \frac{\int_{\underline{v}}^{v_i}\ln P(x)\omega(P(x))\left[1-k_l(1-2\omega(P(x)))\right]dx}{\ln P(x)\omega(P(v))\left[1-k_l(1-2\omega(P(v)))\right]}$$

To how this, it is sufficient to show that for

$x \le v$

$$\frac{\omega(P(x))\left[1-k_l(1-\omega(P(x)))\right]}{\omega(P(v))\left[1-k_l(1-\omega(P(v)))\right]} \le \frac{\ln P(x)\omega(P(x))\left[1-k_l(1-2\omega(P(x)))\right]}{\ln P(v)\omega(P(v))\left[1-k_l(1-2\omega(P(v)))\right]}$$
$$\Leftrightarrow \frac{\left[1-k_l(1-\omega(P(x)))\right]}{\left[1-k_l(1-\omega(P(v)))\right]} \le \frac{\ln P(x)\left[1-k_l(1-2\omega(P(x)))\right]}{\ln P(v)\left[1-k_l(1-2\omega(P(v)))\right]} \tag{3.1a}$$

which is equivalent to

$$\frac{\ln P(x)\left[1-k_l(1-2\omega(P(x)))\right]}{\left[1-k_l(1-\omega(P(x)))\right]} = \frac{1}{\beta}\frac{\ln P(x)^{\beta}\left[1-k_l(1-2\omega(P(x)))\right]}{\left[1-k_l(1-\omega(P(x)))\right]}$$
$$= \frac{1}{\beta}\frac{\ln \omega(P(x)\left[1-k_l(1-2\omega(P(x)))\right]}{\left[1-k_l(1-\omega(P(x)))\right]}$$

being increasing in x ; Or equivalently

$T(y) = \dfrac{\ln y\left[1-k_l(1-2y)\right]}{\left[1-k_l(1-y)\right]}$ being increasing in y when $0 \le y \le 1$;

i.e. $\dfrac{\partial T}{\partial y} = \dfrac{1}{y}[1-k_l(1-y)][1-k_l(1-2y)] + k_l(1-k_l)\log y \ge 0$

(3.2)

Case 1: For $k_l = 1$, $\dfrac{\partial T}{\partial y} \ge 0 \Rightarrow \dfrac{\partial B}{\partial \beta} \ge 0$ for $0 \le y \le 1$.

For $k_l > 1$, $\hat{v}(k_l) > \underline{v}$. (i) Since $\frac{\partial \hat{v}}{\partial \beta} > 0$ more bidders bid $B = v$ (bid more aggressively in response to greater ambiguity). (ii) For $v_i \geq \hat{v}(k_l)$,

$[1 - k_l(1 - f(B(v_i)))] \geq 0 \Rightarrow [1 - k_l(1 - 2f(B(v_i)))] \geq 0$

$\frac{\partial T}{\partial y} \geq 0 \Rightarrow \frac{\partial B}{\partial \beta} \geq 0$ for $0 \leq y \leq 1$.

Case 2: For $k_l < 1$, all $v_i \geq \hat{v}(k_l)$ when $k_l \to 0$ or $k_l \to 1$, $\frac{\partial T}{\partial y} \geq 0 \Rightarrow \frac{\partial B}{\partial \beta} \geq 0$

Let $Z(y) = 2k_l^2 y^2 + 3k_l(1-k_l)y + (1-k_l)^2 + k_l(1-k_l)y \ln y$ (3.3)

Then we need to show $Z(y) \geq 0$ for $0 \leq y \leq 1$

$Z(y)$ is strictly convex with a strict minimum attained at y^* such that

$$\frac{\partial}{\partial y} Z(y)\bigg|_{y=y^*} = 0 \qquad (3.4)$$

i.e. $\frac{4k_l y^*}{1-k_l} + \ln y^* = -4$ (3.5)

The function $[\frac{4k_l y}{1-k_l} + \ln y]$ is a strictly monotonically increasing continuous function of y which increases from $-\infty$ at $y = 0$ to $\frac{4k_l}{1-k_l}$ at $y = 1$. Hence there exists a unique y^* at which (3.4) holds. Using (3.3) it can be shown that

$Z(y^*) = (1-k_l)^2 - 2k_l^2 y^{*2} - k_l(1-k_l)y^*$; rearranging (3.5) yields

$$k_l = \frac{4 + \ln y^*}{4 + \ln y^* - 4y^*} \qquad (3.5)$$

Again using (3.4) it can be shown that $Z(y^*) = (1-k_l)\left[1-k_l(1-\frac{y^*\ln y^*}{2}-y^*)\right]$.

Thus $Z(y^*) \geq 0$ iff $k_l[1-\frac{y^*\ln y^*}{2}-y^*] \leq 1$

i.e. $\frac{4+\ln y^*}{4+\ln y^* - 4y^*}[1-\frac{y^*\ln y^*}{2}-y^*] \leq 1$. Suppose $\ln y = -4$ then $y = e^{-4}$; and

since $\frac{4k_l y}{1-k_l} + \ln y \geq \ln y \Rightarrow y^* \leq e^{-4} \Rightarrow 4 + \ln y^* - 4y^* \leq 0$.

Hence $Z(y^*) \geq 0$ iff $(4+\ln y^*)(2-y^*\ln y^* - 2y^*) \geq 2(4+\ln y^* - 4y^*)$ i.e. , $y^* \geq e^{-6}$

(3.6)

From (3.5) k_l increases as y^* decreases. Thus $y^* \geq e^{-6}$ iff

$k_l \leq \frac{4+\ln e^{-6}}{4+\ln e^{-6} - 4e^{-6}} = 0.995066$. Thus, when $k_l \leq \frac{4+\ln e^{-6}}{4+\ln e^{-6} - 4e^{-6}} = 0.995066$

$\frac{\partial T}{\partial y} \geq 0 \Rightarrow \frac{\partial B}{\partial \beta} \geq 0$ is guaranteed.

From (3.6) when $k_l > \frac{4+\ln e^{-6}}{4+\ln e^{-6} - 4e^{-6}}$, if there exists $y^* = F(v)^{\beta(n-1)} < e^{-6} = 0.0025$;

then for very small induced-values such that $Z(y) < 0$ as specified in (3.3),

$\frac{\partial T}{\partial y} < 0 \Rightarrow \frac{\partial B}{\partial \beta} < 0$.

Proof for Proposition 3

Note first that by definition of $\hat{v}(k_l)$, $\frac{\partial \hat{v}(k_l)}{\partial n} > 0$; for $v_i > \hat{v}(k_l)$ I show that (i)

$\frac{\partial}{\partial n} B(v_i) \geq 0$ for $k_l \geq 1$ (ii) and $\frac{\partial}{\partial n} B(v_i) \geq 0$ is guaranteed for $0 \leq k_l \leq 0.995066$.

Let $P(x) = F(x)^{n-1}$ and $P(v) = F(v)^{n-1}$ and drop subscript i for simplicity

$$\frac{\partial}{\partial n}B(v_i)\geq 0 \Leftrightarrow \frac{\frac{\partial}{\partial n}[\omega(P(v))\left[1-k_l(1-\omega(P(v)))\right]]}{\omega(P(v))\left[1-k_l(1-\omega(P(v)))\right]} \geq \frac{\int_{\hat{v}}^{v_i}\frac{\partial}{\partial n}\left[\omega(P(x))\left[1-k_l(1-\omega(P(x)))\right]\right]dx}{\int_{\hat{v}}^{v_i}\omega(P(x))\left[1-k_l(1-\omega(P(x)))\right]dx}$$

(4.1)

Also

$$\frac{\partial}{\partial n}\left[\omega(P(v))\left[1-k_l(1-\omega(P(v)))\right]\right]=\left[1-k_l(1-2\omega(P(v)))\right]\frac{\partial\omega(P(v))}{\partial n}$$

Where

$$\frac{\partial\omega(P(v))}{\partial n}=\frac{\partial\omega(F(v)^{\beta(n-1)})}{\partial n}=\beta\ln F(v)F(v)^{\beta(n-1)}=\beta\ln F(v)\omega(P(v))<0$$

Since $\ln P(v)\leq 0$ (4.1) is equivalent to

$$\frac{\int_{\hat{v}}^{v_i}\left[\omega(P(x))\left[1-k_l(1-\omega(P(x)))\right]\right]dx}{\omega(P(v))\left[1-k_l(1-\omega(P(v)))\right]}\leq\frac{\int_{\hat{v}}^{v_i}\ln F(x)\omega(P(x))\left[1-k_l(1-2\omega(P(x)))\right]dx}{\ln F(x)\omega(P(v))\left[1-k_l(1-2\omega(P(v)))\right]}$$

Upon multiplying both sides by β we get the same inequality (3.1a). Thus, the rest of the proof is the same as outlined above for proposition 2(ii) for various range of value for k_l. The same conclusions follow.

■

Proof for Proposition 4

I shall first characterize the interior solution underlying the first-order condition for the objective function, assuming a monotonic bid-value relationship exists. Then show that (i) the best-response bid-value relationship is strictly increasing for

$\hat{v}(k_l) \le v_i \le \tilde{v}$ and (ii) the expected payoff is local and global maximum at the optimal bid.

(i) $\max_{\underline{v} \le B_i \le \bar{v}} \pi_{PT}(v_i, B_i) = \left[\omega((\theta B_i)^{n-1}) - k_l \omega((\theta B_i)^{n-1})(1-\omega((\theta B_i)^{n-1}))\right](v_i - B_i)$

(2.1)

For $v_i \ge \hat{v}(k_l), [1-k_l(1-f(B(v_i)))] \ge 0$ and $v_i \le \tilde{v}$,

$$\frac{\partial \pi_{PT}}{\partial B_i} = 0 \Rightarrow v_i = B_i + \frac{\omega((\theta B_i)^{n-1})\left[1-(1-\omega((\theta B_i)^{n-1}))k_l\right]}{\frac{\partial \omega((\theta B_i)^{n-1})}{\partial B_i}\left[1-(1-2\omega((\theta B_i)^{n-1}))k_l\right]} \qquad (2.2)$$

This defines a unique bid for each value.

For $v_i < \hat{v}(k_l), B(v_i) = v_i$ maximizes payoff (yields zero payoff).

For $v_i > \tilde{v}$ the following holds: $B(v_i) = \left(\frac{n\text{-}1}{n}\right)\bar{v}$

(ii) For $\beta > 0,\ v_i \ge \hat{v}(k_l), [1-k_l(1-f(B(v_i)))] \ge 0$ and $v_i \le \tilde{v}$ using (2.2) we

obtained the following above: $v_i = B_i + \frac{B_i}{(n-1)\beta}\left(\frac{1-k_l+k_l(\theta B_i)^{\beta(n-1)}}{1-k_l+2k_l(\theta B_i)^{\beta(n-1)}}\right)$

$\frac{\partial v}{\partial B_i} = 1 + \frac{B_i}{(n-1)\beta}\left(\frac{\partial Z}{\partial B_i}\right) + \frac{Z}{(n-1)\beta}$ where $Z = \left(\frac{1-k_l+k_l(\theta B_i)^{\beta(n-1)}}{1-k_l+2k_l(\theta B_i)^{\beta(n-1)}}\right) \in [0,1]$; and

$$\frac{\partial Z}{\partial B_i} = \frac{-(1-k_l)k_l\theta^{\beta(n-1)}B_i^{\beta(n-1)-1}}{(1-k_l+2k_l(\theta B_i)^{\beta(n-1)})^2} \Rightarrow \frac{B_i}{(n-1)\beta}\left(\frac{\partial Z}{\partial B_i}\right) = \frac{B_i}{(n-1)\beta}\left(\frac{-(1-k_l)k_l\theta^{\beta(n-1)}B_i^{\beta(n-1)-1}}{(1-k_l+2k_l(\theta B_i)^{\beta(n-1)})^2}\right)$$

For $0 < k_l < 1$, $-1 < \frac{B_i}{(n-1)\beta}\left(\frac{\partial Z}{\partial B_i}\right) < 0 \Rightarrow \frac{\partial v}{\partial B_i} > 0$; For $k_l \ge 1$, the numerator and denominator are such that $\frac{B_i}{(n-1)\beta}\left(\frac{\partial Z}{\partial B_i}\right) > 0 \Rightarrow \frac{\partial v}{\partial B_i} > 0$.

Thus the bid-value relationship is strictly increasing for $\hat{v}(k_l) \le v_i \le \tilde{v}$.

(iii) For $v_i \geq \hat{v}(k_l), [1-k_l(1-f(B(v_i)))] \geq 0$ and $v_i \leq \tilde{v}$ at the optimal bid $\frac{\partial \pi_{PT}}{\partial B_i} = 0$.

To show $\frac{\partial^2 \pi_{PT}}{\partial B_i^2} \leq 0$. Differentiate the first order condition $\frac{\partial \pi_{PT}}{\partial B_i} = 0$ with respect to v_i yields $\frac{\partial^2 \pi_{PT}}{\partial B_i^2}\frac{\partial B_i}{\partial v_i} + \frac{\partial^2 \pi_{PT}}{\partial B_i v_i} = 0$. Then we need to show that $\frac{\partial^2 \pi_{PT}}{\partial B_i \partial v_i} \geq 0$ for the proof to work since $\frac{\partial B}{\partial v_i} > 0$. Differentiating (2.1) with respect to v_i yields $\frac{\partial^2 \pi_{PT}}{\partial B_i \partial v_i} = \frac{\partial \omega((\theta B_i)^{n-1})}{\partial B_i}[1-k_l(1-2\omega(f(B_i)))]$. Since $\frac{\partial \omega((\theta B_i)^{n-1})}{\partial B_i} > 0$ and $[1-k_l(1-\omega(f(B_i))] \geq 0 \Rightarrow [1-k_l(1-2\omega(f(B_i))] \geq 0$; therefore $\frac{\partial^2 \pi_{PT}}{\partial B_i \partial v_i} \geq 0$. Thus, the first order condition describes a global optimum. Bidding $\frac{n-1}{n}\bar{v}$ ensures that the auction is won. Therefore for $v_i \geq \tilde{v}$, the global optimum is given by $B = \frac{n-1}{n}\bar{v}$

■

Proof for Proposition 5

For $\hat{v}(k_l) \leq v_i \leq \tilde{v}$ we need to show that $\frac{\partial B_i}{\partial k_l} > 0$. Differentiate $\frac{\partial \pi_{PT}}{\partial B_i} = 0$ with respect to k_l yields $\frac{\partial^2 \pi_{PT}}{\partial B_i^2}\frac{\partial B_i}{\partial k_l} + \frac{\partial^2 \pi_{PT}}{\partial B_i \partial k_l} = 0$. Then we need to show that $\frac{\partial^2 \pi_{PT}}{\partial B_i \partial k_l} \geq 0$ for the proof to work since it has been shown (above for proposition 4) that $\frac{\partial^2 \pi_{PT}}{\partial B_i^2} \leq 0$. Differentiating (2.1) with respect to k_l yields

$$\frac{\partial^2 \pi_{PT}}{\partial B_i \partial k_l} = \frac{-B_i}{(n-1)\beta}\left(\frac{(1-k_l+2k_lY)(Y-1)-(1-k_l+k_lY)(2Y-1)}{(1-k_l+2k_lY)^2}\right) = \frac{-B_i}{(n-1)\beta}\left(\frac{Y-2Y}{(1-k_l+2k_lY)^2}\right) \geq 0$$

where $Y = (\theta B_i)^{\beta(n-1)} > 0$; thus from above, $\frac{\partial^2 \pi_{PT}}{\partial B_i \partial k_l} > 0 \Rightarrow \frac{\partial B_i}{\partial k_l} > 0$

▪

Proof: for Proposition 5(ii)

If $v_i - B_i - \frac{B_i}{(n-1)\beta}\left(\frac{1-k_l+k_l(\theta B_i)^{(n-1)\beta}}{1-k_l+2k_l(\theta B_i)^{(n-1)\beta}}\right) = 0$ is equivalent to $F(\beta, B(\beta)) = 0$ where

subscript i is dropped for simplicity. By implicit function theorem, if $\frac{\partial F}{\partial B} \neq 0$, then

$$\frac{\partial B}{\partial \beta} = \frac{\partial F}{\partial \beta} / \frac{\partial F}{\partial B}$$

$$\frac{\partial B}{\partial \beta} = \frac{\frac{B\Gamma(.)}{\beta^2(n-1)} - \frac{A(.)X_2(.)(k_l-1)}{D(.)^2}}{1+\frac{\Gamma(.)}{\beta(n-1)}+\frac{A(.)X_1(.)(k_l-1)}{D(.)^2}}$$ where

$$\Gamma(.) = \left(\frac{1-k_l+k_l(\theta B_i)^{(n-1)\beta}}{1-k_l+2k_l(\theta B_i)^{(n-1)\beta}}\right),\ A(.) = \frac{B_i}{(n-1)\beta},\ X_2(.) = k_l\omega(.)n\ln(\theta B) < 0,\ X_1(.) = k_l\beta n\frac{(\theta B)^{\beta(n-1)}}{(\theta B)}$$

and $D(.) = 1-k_l+2k_l(\theta B_i)^{(n-1)\beta}$

It is relatively straightforward to show that for $v > \hat{v}(k_l), \Gamma(.) > 0, D(.) > 0$

$$1+\frac{\Gamma(.)}{\beta(n-1)}+\frac{A(.)X_1(.)(k_l-1)}{D(.)^2} > 0$$

$$\Leftrightarrow \beta(n-1)D(.)^2 + \Gamma(.)D(.)^2 > A(.)X_1(.)(k_l-1)\beta(n-1)$$

Since $\Gamma(.)D(.)^2 > 0$ the above holds if $\beta(n-1)D(.)^2 > (n-1)\beta\omega(.)(1-k_l)k_l$. Note that this holds when $k_l \geq 1$. When $k_l < 1$, the above is equivalent to

$$D(.)^2 > \omega(.)(1-k_l) \Leftrightarrow [(1-k_l)+4\omega(.)k_l]+4\omega(.)^2 k_l^2/(1-k_l) > \omega(.)$$
$$\Leftrightarrow (1-k_l)_l > \omega(.)(1-4k)$$

which holds for all $k_l < 1$. Therefore $1+\frac{\Gamma(.)}{\beta(n-1)}+\frac{A(.)X_1(.)(k_l-1)}{D(.)^2} > 0$.

Thus $\frac{\partial F}{\partial B} > (<)0$ iff $\frac{B\Gamma(.)}{\beta^2(n-1)}-\frac{A(.)X_2(.)(k_l-1)}{D(.)^2} > (<)0$

i.e.

$$\frac{B}{\beta(n-1)}\left[\frac{\Gamma(.)}{\beta}-\frac{X_2(.)(k_l-1)}{D(.)^2}\right] > (<)0 \Leftrightarrow \Gamma(.)D(.)^2 > (<)n\beta\ln(\theta B)(k_l-1)[\omega(\theta B)k_l]$$

when $k_l \geq 1$, the LHS exceeds the RHS since $\ln(\theta B) \leq 0$. Therefore $\frac{\partial B_{PT}}{\partial\beta} \geq 0$.

When $k_l < 1$, as $B_i \to 0$ or $B_i \to \theta$, the LHS exceeds the RHS ; given the bids and values are monotonically increasing therefore for the extreme induced-values $\frac{\partial B_{PT}}{\partial\beta} \geq 0$; for some $k^* < k_l < 1$ for v_i it follows from above, if

$$[1-k_l(1-2\omega(\theta B_i))] < \frac{n\beta(k_l-1)\ln(\theta B_i)\omega(\theta B_i)k_l}{[1-k_l(1-\omega(\theta B_i))]}$$, then $\frac{\partial B_{PT}}{\partial\beta} < 0$.

Proof: for Proposition 6

As before, $v_i - B_i - \frac{B_i}{(n-1)\beta}\left(\frac{1\text{-}k_l + k_l(\theta B_i)^{(n-1)\beta}}{1\text{-}k_l + 2k_l(\theta B_i)^{(n-1)\beta}}\right) = 0$ is equivalent to

$F(n, B(n)) = 0$ where subscript i is dropped for simplicity. By implicit function theorem, if $\frac{\partial F}{\partial B} \neq 0$, then $\frac{\partial B}{\partial n} = \frac{\partial F}{\partial n} / \frac{\partial F}{\partial B}$ i.e. $\frac{\partial B}{\partial n} = \dfrac{\frac{B\beta\Gamma(.)}{\beta^2(n-1)^2} - \frac{A(.)X_2(.)(k_l-1)}{D(.)^2}}{1+\frac{\Gamma(.)}{\beta(n-1)} + \frac{A(.)X_1(.)(k_l-1)}{D(.)^2}}$

where

$$\Gamma(.) = \left(\frac{1 - k_l + k_l(\theta B_i)^{(n-1)\beta}}{1 - k_l + 2k_l(\theta B_i)^{(n-1)\beta}}\right),\ A(.) = \frac{B_i}{(n-1)\beta},\ X_2(.) = k_l[\omega(.)(B\ln(\theta B) + \beta\frac{B(n-1)-n}{n})] < 0,$$

$$X_1(.) = k_l\beta n\frac{(\theta B)^{\beta(n-1)}}{(\theta B)} \text{ and } \quad D(.) = 1 - k_l + 2k_l(\theta B_i)^{(n-1)\beta}$$

It is relatively straightforward to show (as shown before in the proof for Prop.5(ii)) that for $v > \hat{v}(k_l)$ and for all k_l,

$$1+\frac{\Gamma(.)}{\beta(n-1)} + \frac{A(.)X_1(.)(k_l-1)}{D(.)^2} > 0 .$$

Thus $\frac{\partial F}{\partial B} > (<)0$ iff $\frac{B\beta\Gamma(.)}{\beta^2(n-1)^2} - \frac{A(.)X_2(.)(k_l-1)}{D(.)^2} > (<)0$

i.e. $\frac{B}{\beta(n-1)}\left[\frac{\Gamma(.)}{n-1} - \frac{X_2(.)(k_l-1)}{D(.)^2}\right] > (<)0$ (6.1)

which can be shown to be equivalent to

$$(1+\frac{k_l\omega(.)}{1-k_l})(1-k_l+2k_l\omega(.)) > (<) -(n-1)k_l\omega(.)[B\ln(\theta B) + \beta(B(1-1/n)-1)]$$

When $k_l \geq 1$, the LHS exceeds the RHS in equation (6.1) since $X_2 < 0$. Therefore $\frac{\partial B_{PT}}{\partial n} > 0$. When $k_l < 1$, as $B_i \to 0$ or $B_i \to \theta$, the LHS exceeds the RHS ; given the bids and values are monotonically increasing therefore for the extreme induced-values $\frac{\partial B_{PT}}{\partial n} > 0$; for some $\hat{k} < k_l < 1$ for v_i it follows from above that if

$[1-k_l(1-2\omega(\theta B_i))] < \frac{(n-1)[\beta(1-\theta B_i) - B_i \ln(\theta B_i)]\omega(\theta B_i)k_l}{[1-k_l(1-\omega(\theta B_i))]}$, then $\frac{\partial B_{PT}}{\partial n} < 0$.

▪

Chapter 3

Proof of Proposition 1:

Bidders with $w^j \in [\underline{w}, w_L^{CA}]$ can only obtain non-negative expected utility by having zero chances of winning. This is guaranteed when placing a zero bid. For $w^j \in [w_L^{CA}, \overline{w}]$, we assume that all opponents of bidder i bid according to a strictly increasing bidding strategy $b(w^j)$.

Maximizing (5), bidder i chooses b^i according to

$$f'(b^i)[w^i - b^i - (1-2f^i)(\lambda_0 b^i + \lambda_1 w^i)] = f^i[1+(1-f^i)\lambda_0] \tag{A.1}$$

When placing a positive bid b^i, bidder i's probability of winning is therefore given by $f(b^i) = H^{n-1}(b^{-1}(b^i))$ and therefore $f'(b^i) = (H^{n-1})'(b^{-1}(b^i))(b^{-1})'(b^i)$. In equilibrium, we have $b^{-1}(b^i) = w^i$, $(b^{-1})'(b^i) = 1/b'(w^i)$, and $f^i = H^{n-1}(w^i)$.

Rearranging (A.1) therefore immediately gives

$$(H^{n-1})'(w^i)w^i[1-(1-2H^{n-1}(w^i))\lambda_1] = [H^{n-1}(w^i)(1+(1-H^{n-1}(w^i))\lambda_0)b(w^i)]' \tag{A.2}$$

Integrating yields

$$b(w) = \frac{\int_{w_L^{CA}}^{w} z[1-\lambda_1(1-2H^{n-1}(z))]dH^{n-1}(z)}{H^{n-1}(w)(1+\lambda_0(1-H^{n-1}(w)))} \quad \text{for } w \in [w_L^{CA}, \bar{w}] \qquad \text{(A.3)}$$

as the unique candidate for a symmetric monotonic bidding equilibrium. Partial integration leads to (7). Strict monotonicity of $b(w)$ in $w \in [w_L^{CA}, \bar{w}]$ can easily be established by differentiating (A.3). It remains to show the second-order condition for the maximization problem for $w \in [w_L^{CA}, \bar{w}]$. Using the envelope theorem and (A.1), this is equivalent to $\partial^2 \Pi^{1,\mathrm{CA}}(b^{1,\mathrm{CA}}(w), w)/\partial b \partial w > 0$ which holds true since $\partial^2 \Pi^{1,\mathrm{CA}}(b^{1,\mathrm{CA}}(w), w)/\partial b \partial w = f'(b^{1,\mathrm{CA}}(w))[1-(1-2f(b^{1,\mathrm{CA}}(w)))\lambda_1] > 0$.□

Proof of Proposition 2

Differentiating $b^{1,\mathrm{CA}}(w)$ with respect to λ_0 for $w \in [w_L^{CA}, \bar{w}]$ gives

$$\frac{\partial b^{1,\mathrm{CA}}(w)}{\partial \lambda_0} = -\frac{b^{1,\mathrm{CA}}(w)}{1+(1-H^{n-1}(w))\lambda_0}(1-H^{n-1}(w)) < 0 \qquad \text{(A.4)}$$

while

$$\frac{\partial b^{1,\mathrm{CA}}(w)}{\partial \lambda_1}[H^{n-1}(w)(1+(1-H^{n-1}(w))\lambda_0)] = -\int_{w_L^{CA}}^{w} z(1-2H^{n-1}(z))dH^{n-1}(z)$$

(A.5)

which immediately implies that $\frac{\partial b^{1,\mathrm{CA}}(w)}{\partial \lambda_1} < 0$ for small w, in particular those with $H^{n-1}(w) \leq 1/2$. At $w = \bar{w}$, however, we obtain (with partial integration)

$$-\int_{w_L^{CA}}^{\bar{w}} z(1-2H^{n-1}(z))dH^{n-1}(z) = \int_{w_L^{CA}}^{\bar{w}} H^{n-1}(z)(1-H^{n-1}(z))dz > 0 \qquad \text{(A.6)}$$

and therefore $\frac{\partial b^{1,\mathrm{CA}}(\overline{w})}{\partial \lambda_1} > 0$. Combined with $-\frac{\partial}{\partial w}\int_{w_L^{CA}}^{w} z(1-2H^{n-1}(z))dH^{n-1}(z) > 0$ for $H^{n-1}(w) > 1/2$ this implies the existence of an $0 < \hat{w} < 1$ with $H^{n-1}(\hat{w}) > 1/2$ such that $\frac{\partial b^{1,\mathrm{CA}}(w)}{\partial \lambda_1} < 0$ for $w < \hat{w}$ and $\frac{\partial b^{1,\mathrm{CA}}(w)}{\partial \lambda_1} > 0$ for $w > \hat{w}$. With (A.5), this threshold value is given by $\int_{\underline{w}}^{\hat{w}} z(1-2H^{n-1}(z))dH^{n-1}(z) = 0$. Note that this implies $H^{n-1}(\hat{w}) > 1/2$. □

Proof of Corollary 2.

Differentiating (9) with respect to λ_0:

$$\begin{aligned}
&\frac{\partial b^{1,\mathrm{IV}}(w)}{\partial \lambda_0} H^{n-1}(w)[1-\lambda_0(1-H^{n-1}(w))]^2 \\
&= [1-\lambda_0(1-H^{n-1}(w))]\int_{w_L^{IV}}^{w} H^{n-1}(z)(1-H^{n-1}(z))dz \\
&\quad -(1-H^{n-1}(w))\int_{w_L^{IV}}^{w} H^{n-1}(z)[1-\lambda_0(1-H^{n-1}(z))]dz \\
&= \int_{w_L^{IV}}^{w} H^{n-1}(z)(1-H^{n-1}(z))dz - (1-H^{n-1}(w))\int_{w_L^{IV}}^{w} H^{n-1}(z)dz \\
&= \int_{w_L^{IV}}^{w} H^{n-1}(z)(H^{n-1}(w)-H^{n-1}(z))dz \\
&> 0
\end{aligned} \tag{A.7}$$

□

Proof of Proposition 3:

Bidders with $w^j \in [\underline{w}, w_L^{CA}]$ can only obtain non-negative expected utility by having zero chances of winning. This is guaranteed when placing a zero bid. For $w \in [w_L^{CA}, \overline{w}]$, we differentiate condition (11) with respect to w ($b^i = b(w^i) = b(w)$):

$$1 - b'(w)[1 + \lambda_0 - 2b(w) f'(b(w))] = \lambda_1 (1 - 2f(b(w))) - 2\lambda_1 w f'(b(w)) b'(w)$$
$$b'(w)[1 + \lambda_0] - 2b(w)(H^{n-1}(w))' = (1 - \lambda_1) + 2\lambda_1 [wH^{n-1}(w)]'$$

(A.8)

Solving the affine linear differential equation (A.8), we obtain

$$b^{2,\mathrm{CA}}(w) = \int_{w_L^{CA}}^{w} \frac{1 - \lambda_1 + 2\lambda_1 (zH^{n-1}(z))'}{1 + \lambda_0} \exp\left(\frac{2\lambda_0}{1 + \lambda_0} (H^{n-1}(w) - H^{n-1}(z)) \right) dz$$
$$+ b^{2,\mathrm{CA}}(w_L^{CA}) \exp\left(\frac{2\lambda_0}{1 + \lambda_0} (H^{n-1}(w) - H^{n-1}(w_L^{CA})) \right)$$

(A.9)

as the unique candidate for a symmetric monotonic bidding equilibrium for $w \in [w_L^{CA}, \bar{w}]$. Noting that (11) implies $b^{2,\mathrm{CA}}(w_L^{CA}) = \max[0, \underline{w}(1 - \lambda_1)/(1 - \lambda_0)]$, partial integration yields (13). Monotonicity of $b^{2,\mathrm{CA}}(w)$ for $w \in [w_L^{CA}, \bar{w}]$ is easily be established by differentiating (A.9). It remains to show the second-order condition for the maximization problem for $w \in [w_L^{CA}, \bar{w}]$. Using the envelope theorem, this is equivalent to $\frac{\partial^2}{\partial b^i \partial w^i} \Pi^{2,\mathrm{CA}}(b^i, w^i) > 0$ or – using condition (10) – equivalently $f'(b^i)\left[1 - \lambda_1 (1 - 2f(b^i))\right] > 0$ which obviously holds true in the relevant range.□

Proof for Proposition 4

We first rewrite (11) as:

$$w = (1 + \lambda_0) b^{2,\mathrm{CA}}(w) - 2\lambda_0 \int_{w_L^{CA}}^{w} b^{2,\mathrm{CA}}(z) dH^{n-1}(z) + \lambda_1 w (1 - 2H^{n-1}(w))$$

(A.10)

At $\lambda_0 = 0$, we immediately obtain

$$b^{2,\mathrm{CA}}(w) - w = -\lambda_1 w[1 - 2H^{n-1}(w)]$$

which implies bids above the valuation for $H^{n-1}(w) > 1/2$ and below value for $H^{n-1}(w) < 1/2$ when $\lambda_0 = 0$.

For general λ_0, differentiating (A.10) with respect to λ_1 yields

$$(1+\lambda_0)\frac{\partial}{\partial \lambda_1} b^{2,\mathrm{CA}}(w) = 2\lambda_0 \int_{w_L^{CA}}^{w} \frac{\partial}{\partial \lambda_1} b^{2,\mathrm{CA}}(z)\, dH^{n-1}(z) - w(1 - 2H^{n-1}(w))$$

which immediately proves $\frac{\partial}{\partial \lambda_1} b^{2,\mathrm{CA}}(w) < 0$ at $w = w_L^{CA}$ as long as $H^{n-1}(w_L^{CA}) < 1/2$. This also implies $\frac{\partial}{\partial \lambda_1} b^{2,\mathrm{CA}}(w) < 0$ for all $w > w_L^{CA}$ with $H^{n-1}(w) < 1/2$. For $w < w_L^{CA}$, we naturally have $\frac{\partial}{\partial \lambda_1} b^{2,\mathrm{CA}}(w) = 0$.

Furthermore, differentiation with respect to λ_0 gives

$$(1+\lambda_0)\frac{\partial}{\partial \lambda_0} b^{2,\mathrm{CA}}(w) = -b^{2,\mathrm{CA}}(w) + 2\lambda_0 \int_{w_L^{CA}}^{w} \frac{\partial}{\partial \lambda_0} b^{2,\mathrm{CA}}(z)\, dH^{n-1}(z) + 2\int_{w_L^{CA}}^{w} b^{2,\mathrm{CA}}(z)\, dH^{n-1}(z) \tag{A.11}$$

which proves $\frac{\partial}{\partial \lambda_0} b^{2,\mathrm{CA}}(w) < 0$ at $w = w_L^{CA}$. For $w < w_L^{CA}$, again $\frac{\partial}{\partial \lambda_0} b^{2,\mathrm{CA}}(w) = 0$.

In order to see that money-loss aversion can increase bids for large values, consider the derivative at $\lambda_0 = \lambda_1 = 0$:

$$\frac{\partial}{\partial \lambda_0} b^{2,\mathrm{CA}}(w) = -b^{2,\mathrm{CA}}(w) + 2\int_{w_L^{CA}}^{w} b^{2,\mathrm{CA}}(z)\, dH^{n-1}(z) = -w + 2\int_{w_L^{CA}}^{w} z\, dH^{n-1}(z) \tag{A.12}$$

which immediately shows that the sign of the derivative depends on the distribution $H^{n-1}(\bullet)$, thereby completing the proof.

□

Proof of Proposition 7:

It is sufficient to show that expected payments made by a player with value w coincide in both auctions. They are given by $H^{n-1}(w)b(w)$ in the first-price auction, and by $\int_{\underline{w}}^{w} b^{2,\mathrm{CA}}(z)dH^{n-1}(z)$ in the second-price auction. Reconsidering the expected utility gains in (5) and (10), we obtain:

$$\begin{aligned}
&\Pi^{1,\mathrm{CA}}(b^{1,\mathrm{CA}}(w),w)-\Pi^{2,\mathrm{CA}}(b^{2,\mathrm{CA}}(w),w)+\\
&[1+\lambda_0(1-H^{n-1}(w))]\left[H^{n-1}(w)b^{1,\mathrm{CA}}(w)-\int_{\underline{w}}^{w} b^{2,\mathrm{CA}}(z)dH^{n-1}(z)\right]\\
&=\lambda_0\int_{\underline{w}}^{w}\int_{\underline{w}}^{z}[b^{2,\mathrm{CA}}(z)-b^{2,\mathrm{CA}}(z')]dH^{n-1}(z')dH^{n-1}(z) \qquad \text{(A.13)}\\
&=\lambda_0\int_{\underline{w}}^{w}\int_{\underline{w}}^{z} b^{2,\mathrm{CA}}{}'(z')H^{n-1}(z')dz'dH^{n-1}(z)\\
&=\lambda_0 H^{n-1}(w)\int_{\underline{w}}^{w} b^{2,\mathrm{CA}}{}'(z)H^{n-1}(z)dz-\lambda_0\int_{\underline{w}}^{w} b^{2,\mathrm{CA}}{}'(z)H^{2n-2}(z)dz
\end{aligned}$$

The probability of winning with a given type w is $H^{n-1}(w)$ in both auctions. Therefore, the first-order conditions (6) and (11) combined with the envelope theorem imply that

$$\begin{aligned}
\Pi^{1,\mathrm{CA}}(b(w),w)&=\int_{\underline{w}}^{w}\frac{\partial}{\partial z}\Pi^{1,\mathrm{CA}}(b(z),z)dz=\int_{\underline{w}}^{w} H^{n-1}(z)-\lambda_1 H^{n-1}(z)(1-H^{n-1}(z))dz\\
&=\int_{\underline{w}}^{w}\frac{\partial}{\partial z}\Pi^{2,\mathrm{CA}}(b(z),z)dz=\Pi^{2,\mathrm{CA}}(b(w),w)
\end{aligned}$$

(A.14)

Combining (A.13) and (A.14), we obtain

$$b^{1,\mathrm{CA}}(w)H^{n-1}(w)-\int_{\underline{w}}^{w} b^{2,\mathrm{CA}}(z)dH^{n-1}(z)$$
$$=\lambda_0 \frac{\int_{\underline{w}}^{w} b^{2,\mathrm{CA}}\,'(z)H^{n-1}(z)[H^{n-1}(w)-H^{n-1}(z)]dz}{1+\lambda_0(1-H^{n-1}(w))} \geq 0 \tag{A.15}$$

which proves that first-price auctions revenue-dominate second-price auctions if $\lambda_0 > 0$, while both auction formats lead to the identical expected revenue if $\lambda_0 = 0$. □

Chapter 4

Proof for Proposition 1

Bidders with $\mu_i \in [0, \hat{\mu}_r^1]$ can only obtain non-negative expected utility by having zero chances of winning. This is guaranteed when placing a zero bid. For $\mu_i \in (\hat{\mu}_r^1, 1]$, I assume that all opponents of bidder i bid according to a strictly increasing bidding strategy $B(\mu_i)$. Maximizing (4), bidder i chooses B^i according to

$$f'(B^i)[(1-\mu_i)\bar{v} + \mu_i M - (1-2f^i)(1-\mu_i)k_1\bar{v} - 2f^i\mu_i(1-\mu_i)k_1\mu_i - B^i] = f(B^i) \tag{A.1}$$

Here $f(B^i) = H^{n-1}(B^{-1}(B^i))$ and therefore $f'(B^i) = (H^{n-1})'(B^{-1}(B^i))(B^{-1})'(B^i)$. In equilibrium, we have $B^{-1}(B^i) = \mu_i$, $(B^{-1})'(B^i) = 1/B'(\mu_i)$, and $f^i = H^{n-1}(\mu_i)$. Rearranging (A.1) gives

$$(H^{n-1})'(\mu_i)[(1-\mu_i)\bar{v} + \mu_i M - (1-2f^i)(1-\mu_i)k_1\bar{v} - 2f^i\mu_i(1-\mu_i)k_1\mu_i] = [H^{n-1}(\mu_i)B(\mu_i)]' \tag{A.2}$$

Integrating yields

$$B(\mu_i)_{PT} = \frac{\int_{\hat{\mu}_r^1}^{\mu_i} [(1-x)\bar{v} + xM - (1-2H^{n-1}(x))(1-x)k_1\bar{v} - 2H^{n-1}(x)x(1-x)k_1\bar{v}]dH^{n-1}(x)}{H^{n-1}(\mu_i)} \tag{A.3}$$

as the unique candidate for a symmetric monotonic bidding equilibrium.

(Monotonicity) The bid function can be written as $B(\mu_i)_{PT} = \dfrac{\int_{\hat{\mu}_r^1}^{\mu_i} [g(x)] dH^{n-1}(x)}{H^{n-1}(\mu_i)}$ where

$g(x) = (1-x)\bar{v} + xM - (1-2H^{n-1}(x))(1-x)k_1\bar{v} - 2H^{n-1}(x)x(1-x)k_1\bar{v}$; thus for

$\mu_i \in (\hat{\mu}_r^1, 1]$

$$\frac{\partial B(\mu_i)}{\partial \mu_i} = \frac{H^{n-1}(\mu_i)(H)'^{n-1}(\mu_i)g(\mu_i) - (H)'^{n-1}(\mu_i)\int_{\hat{\mu}_r^1}^{\mu_i}[g(x)](H)'^{n-1}(x)}{(H^{n-1}(\mu_i))^2} > 0$$

Iff $H^{n-1}(\mu_i)g(\mu_i) - \int_{\hat{\mu}_r^1}^{\mu_i}[g(x)]dH^{n-1}(x) > 0$ i.e.

$$H^{n-1}(\mu_i)g(\mu_i) - [\int_{\hat{\mu}_r^1}^{\mu_i} g(x)(H)'^{n-1}(x) + \int_{\hat{\mu}_r^1}^{\mu_i} g'(x)(H)^{n-1}(x)dx - \int_{\hat{\mu}_r^1}^{\mu_i} g'(x)(H)^{n-1}(x)]dx > 0$$ i.e.

$$H^{n-1}(\mu_i)g(\mu_i) - \int_{\hat{\mu}_r^1}^{\mu_i}[g(x)H^{n-1}(x)]'dx + \int_{\hat{\mu}_r^1}^{\mu_i} g'(x)H^{n-1}(x)dx > 0$$ i.e., $\int_{\hat{\mu}_r^1}^{\mu_i} g'(x)H^{n-1}(x)dx > 0$

$$g'(x) = \underbrace{M - \bar{v}}_{\geq k_1\bar{v}} + 2k_1(H)'^{n-1}(x)(1-x)^2\bar{v} - 4k_1H^{n-1}(x)(1-x)\bar{v} + k_1\bar{v}$$
$$\geq 2k_1\bar{v}[1 + (H)'^{n-1}(x)(1-x)^2 - 2H^{n-1}(x)(1-x)]$$

Therefore it suffices to show that

$\int [1 + (H)'^{n-1}(x)(1-x)^2 - 2H^{n-1}(x)(1-x)]H^{n-1}(x)dx > 0$. This can be shown to be

equivalent to $\int H^{n-1}(x)(1-(1-x)H^{n-1}(x))dx + \frac{1}{2}(1-\mu_i)^2(H^{n-1}(\mu_i))^2 dx > 0$ which is

always true. Thus the bid function is strictly monotonic.

(Sufficiency) It remains to show the second-order condition for the maximization problem. Using the envelope theorem and (A.1), this is equivalent to $\partial^2\Pi(B(\mu_i),\mu_i)/\partial B\partial\mu_i > 0$.

$\partial^2\Pi(B(\mu_i),\mu_i)/\partial B\partial\mu_i = f'(B)[M-\bar{v}+k_1(1-2f)\bar{v}-k_1 2f(1-2\mu_i)\bar{v}]$ i.e.

$$\partial^2\Pi(B(\mu_i),\mu_i)/\partial B\partial\mu_i = (H^{n-1})'(\mu_i)[\underbrace{M-\bar{v}}_{\geq k_1\bar{v}}+k_1\bar{v}(1-4H^{n-1}(\mu_i)(1-\mu_i))]$$
$$\geq k_1\bar{v}[2(1-2H^{n-1}(\mu_i)(1-\mu_i))]$$

$\partial^2\Pi(B(\mu_i),\mu_i)/\partial B\partial\mu_i > 0$ iff $\frac{1}{2} > H^{n-1}(\mu_i)(1-\mu_i)) = Z(\mu_i)$. For a uniform distribution $H(\bullet)$, the maximum value $Z(\mu_i)^* = \left(\frac{n-1}{n}\right)\left(\frac{1}{n}\right) < \left(\frac{1}{2}\right)$ for $n \geq 2$.

Therefore $\partial^2\Pi(B(\mu_i),\mu_i)/\partial B\partial\mu_i > 0$

□

Proof for Proposition 2

Differentiating A.3 yields

$$\frac{\partial B(\mu_i)}{\partial k_1} = \frac{\int_{\hat{\mu}_r^1}^{\mu_i}[-(1-2H^{n-1}(x))(1-x)\bar{v}-2H^{n-1}(x)x(1-x)\bar{v}]dH^{n-1}(x)}{H^{n-1}(\mu_i)} = \frac{-\int_{\hat{\mu}_r^1}^{\mu_i}[g(x)]dH^{n-1}(x)}{H^{n-1}(\mu_i)} \quad ;$$

$$\int_{\hat{\mu}_r^1}^{\mu_i}[g(x)]dH^{n-1}(x) = \int(1-2H^{n-1}(x))(1-x)\bar{v}+2H^{n-1}(x)x(1-x)\bar{v}dH^{n-1}(x)$$
$$= \bar{v}\int(1-2H^{n-1}(x)(1-x))(1-x)dH^{n-1}(x) \quad .$$

Which can be shown to be equal to

$$\bar{v}[(1-\mu_i)H^{n-1}(\mu_i)+\int H^{n-1}(x)dx-(H^{n-1})^2(1-\mu_i)^2+\int 2(H^{n-1})^2(x)(1-x)dx>0$$

Therefore, it follows for any general distribution $H(\bullet)$ $\frac{\partial B(\mu_i)}{\partial k_1}<0$.

Proof of Proposition 3

For $\mu_i \in [0,1]$, and uniform $H(\bullet)$ and $n=2$ bidders derive expected overall utility greater than the reservation expected utility by bidding greater than r. Let us assume that all opponents of bidder i bid according to a strictly increasing bidding strategy $B(\mu_i)$. Maximizing (5), bidder i chooses B^i according to

$$f\,'(B^i)[\bar{v}-B^i-(1-\mu_i)(\bar{v}-r)-(1-2f^{\,i})k_1\mu_i\bar{v}+2(1-f^{\,i})\mu_i(1-\mu_i)k_1\mu_i = f(B^i) \tag{B.1}$$

Here $f(B^i)=H^{n-1}(B^{-1}(B^i))$ and therefore $f\,'(B^i)=(H^{n-1})'(B^{-1}(B^i))(B^{-1})'(B^i)$. In equilibrium, we have $B^{-1}(B^i)=\mu_i$, $(B^{-1})'(B^i)=1/B'(\mu_i)$, and $f^{\,i}=H^{n-1}(\mu_i)$. Rearranging (B.1) gives

$$(H^{n-1})'(\mu_i)[\bar{v}-(1-\mu_i)(\bar{v}-r)-(1-2H^{n-1}(\mu_i))k_1\mu_i\bar{v}+2(1-H^{n-1}(\mu_i))\mu_i(1-\mu_i)k_1\bar{v}]=[H^{n-1}(\mu_i)B(\mu_i)]' \tag{B.2}$$

Integrating yields

$$B(\mu_i)_{PT}=\frac{\int_{\hat{\mu}_p^1}^{\mu_i}[\bar{v}-(1-x)(\bar{v}-r)-(1-2H^{n-1}(x))xk_1\bar{v}+2(1-H^{n-1}(x))x(1-x)k_1\bar{v}]dH^{n-1}(x)}{H^{n-1}(\mu_i)} \tag{B.3}$$

as the unique candidate for a symmetric monotonic bidding equilibrium.

(Monotonicity) The bid function can be written as $B(\mu_i)_{PT} = \frac{\int_{\hat{\mu}_p^1}^{\mu_i} [g(x)] dH^{n-1}(x)}{H^{n-1}(\mu_i)}$; as before

in the proof for proposition 1 $\frac{\partial B(\mu_i)}{\partial \mu_i} > 0$ if $g'(x) > 0$.

Since $g(x) = \bar{v} - (1-x)(\bar{v} - r) - (1 - 2H^{n-1}(x))k_1 x\bar{v} + 2(1 - H^{n-1}(x))x(1-x)k_1\bar{v}$

$g'(x) = \bar{v} - r + k_1\bar{v}[1 + 2(H)'^{n-1}(x)x^2 - 4x(1 - H^{n-1}(x)))]$. For uniform $n = 2$ and

uniform $H(\bullet)$ it follows that $g'(x) > 0$. Thus the bid function is strictly monotonic.

(Sufficiency) It remains to show the second-order condition for the maximization

problem. Using the envelope theorem and the FOC, this is equivalent to

$\partial^2 \Pi(B(\mu_i), \mu_i) / \partial B \partial \mu_i > 0$. From the FOC,

$$\partial^2 \Pi(B(\mu_i), \mu_i) / \partial B \partial \mu_i = f'(B)[\bar{v} - r - (1 - 2f)k_1\bar{v} + 2(1 - f)(1 - 2\mu_i)k_1\bar{v}]$$

$$\partial^2 \Pi(B(\mu_i), \mu_i) / \partial B \partial \mu_i = f'(B)[\bar{v} - r + k_1\bar{v}(1 - 4\mu_i(1 - f))]$$

For $n = 2$ and uniform $H(\bullet)$, since $\bar{v} > r$, $\partial^2 \Pi(B(\mu_i), \mu_i) / \partial B \partial \mu_i > 0$ follows.

▪

Proof for Proposition 4

Differentiating the bid function in B.3 yields

$$\frac{\partial B}{\partial k_1} = \frac{\int_{\hat{\mu}_p^1}^{\mu_i} [-(1 - 2H^{n-1}(x)) + 2(1 - H^{n-1}(x))(1-x)] x\bar{v} dH^{n-1}(x)}{H^{n-1}(\mu_i)}$$

which gets simplified to

$$\frac{\partial B}{\partial k_1}=\frac{\int_{\hat{\mu}_p^1}^{\mu_i}[1-2x+2H^{n-1}(x)x]x\bar{v}dH^{n-1}(x)}{H^{n-1}(\mu_i)}$$

For uniform $H(\bullet)$, and $n=2$, $\frac{1}{2(1-H^{n-1}(x))}\geq x$, i.e. $\frac{\partial B}{\partial k_1}>0$. For moderate levels of loss-aversion participation in the auction is guaranteed and it can be shown that for a uniform distribution $H(\bullet)$, $n\leq 4$, $\frac{\partial B}{\partial k_1}>0$; for $n>4$, $\frac{\partial B}{\partial k_1}>(<)0$ thereby the claim at certain places that loss-aversion could cause overbidding or underbidding.

Proof for Proposition 5

In auctions with resale, for first-price auctions, the difference $B_{RN}-B_{PT}$ is given by

$$\Delta=\frac{\int_{\hat{\mu}_r^1}^{\mu_i}[(1-2H^{n-1}(x))(1-x)k_1\bar{v}+2H^{n-1}(x)x(1-x)k_1\bar{v}]dH^{n-1}(x)}{H^{n-1}(\mu_i)}$$

This can be written as $\Delta=\frac{\int_{\hat{\mu}_r^1}^{\mu_i}[g(x)]dH^{n-1}(x)}{H^{n-1}(\mu_i)}$; thus

$$\frac{\partial\Delta}{\partial\mu_i}=\frac{H^{n-1}(\mu_i)(H)'^{n-1}(\mu_i)g(\mu_i)-(H)'^{n-1}(\mu_i)\int_{\hat{\mu}_r^1}^{\mu_i}[g(x)](H)'^{n-1}(x)}{(H^{n-1}(\mu_i))^2}<0$$

Iff $H^{n-1}(\mu_i)g(\mu_i)-\int_0^{\mu_i}[g(x)]dH^{n-1}(x)<0$ i.e.

$$H^{n-1}(\mu_i)g(\mu_i)-[\int_{\hat{\mu}_r^1}^{\mu_i} g(x)(H)^{'n-1}(x)+\int_{\hat{\mu}_r^1}^{\mu_i} g'(x)(H)^{n-1}(x)dx-\int_{\hat{\mu}_r^1}^{\mu_i} g'(x)(H)^{n-1}(x)dx]<0 \quad \text{i.e.}$$

$$H^{n-1}(\mu_i)g(\mu_i)-\int_{\hat{\mu}_r^1}^{\mu_i}[g(x)H^{n-1}(x)]'+\int_{\hat{\mu}_r^1}^{\mu_i} g'(x)H^{n-1}(x)dx<0 \text{ i.e., } \int_{\hat{\mu}_r^1}^{\mu_i} g'(x)H^{n-1}(x)<0$$

.Since $g(x)=(1-2H^{n-1}(x))(1-x)k_1\overline{v}+2H^{n-1}(x)x(1-x)k_1\overline{v}$.

$$\int_{\hat{\mu}_r^1}^{\mu_i} g'(x)H^{n-1}(x)=-\int k_1\overline{v}[(1-4H^{n-1}(x)(1-x))+2(H^{n-1})'(x)(1-x)^2]H^{n-1}(x)dx\le 0$$

i.f.f.

$$-\int 2k_1\overline{v}[\frac{H^{n-1}(x)}{2}-(H^{n-1}(x))^2(1-x))-(H^{n-1}(x))^2(1-x))+(H^{n-1})'(x)H^{n-1}(x)(1-x)^2]dx\le 0$$

i.f.f. $\int[\frac{H^{n-1}(x)}{2}-(H^{n-1}(x))^2(1-x))]dx+\underbrace{\frac{1}{2}H^{n-1}(\mu_i)(1-\mu_i)^2}_{\ge 0}\ge 0$. For a uniform distribution $H(\bullet)$, $\int H^{n-1}(x)[\frac{1}{2}-(H^{n-1}(x))(1-x))]dx\ge 0$ since $\frac{1}{2}>(H^{n-1}(x))(1-x))$ for all $x>0$, as shown in the proof for the monotonicity of the bid function in proposition 1. Therefore $\frac{\partial\Delta}{\partial\mu_i}<0$.

Proof for Proposition 6

In auctions with outside procurement, in a first-price auction with sufficiently small number of bidders, the difference in bid functions is

$$\Delta = B_{PT} - B_{RN} = \frac{\int_{\hat{\mu}_p^1}^{\mu_i} [-(1-2H^{n-1}(x))xk_1\bar{v} + 2(1-H^{n-1}(x))x(1-x)k_1\bar{v}]dH^{n-1}(x)}{H^{n-1}(\mu_i)}$$

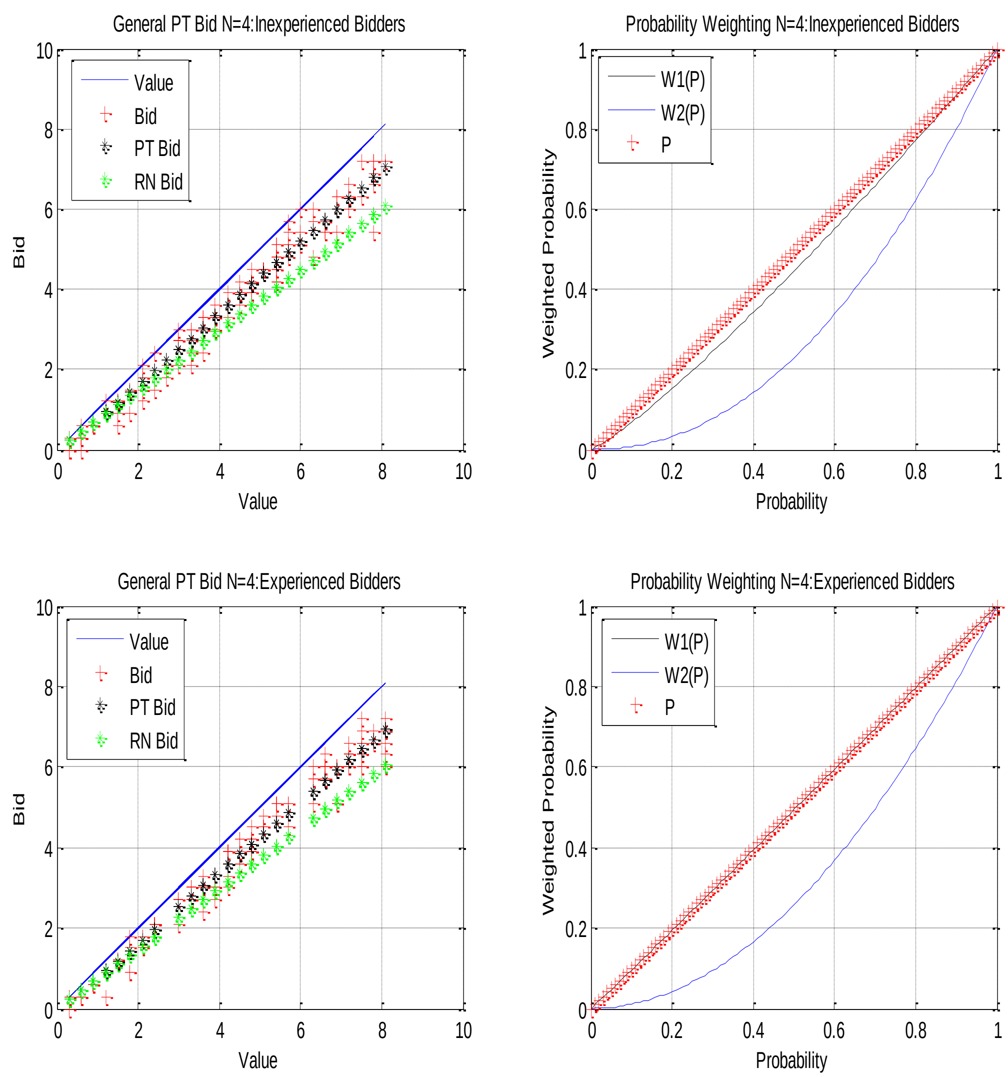
Applying L Hospitals' rule

$$\lim_{\mu_i \to 0} \Delta = (1-2H^{n-1}(\mu_i))\mu_i k_1\bar{v} - 2(1-H^{n-1}(\mu_i))\mu_i(1-\mu_i)k_1\bar{v}] = 0$$

As $\mu_i \to 1, H^{n-1}(\mu_i) > 1/2$, $\Delta = \frac{\int_{0}^{\mu_i} [-(1-2H^{n-1}(x))xk_1\bar{v} + 2(1-H^{n-1}(x))x(1-x)k_1\bar{v}]dH^{n-1}(x)}{H^{n-1}(\mu_i)} > 0$.

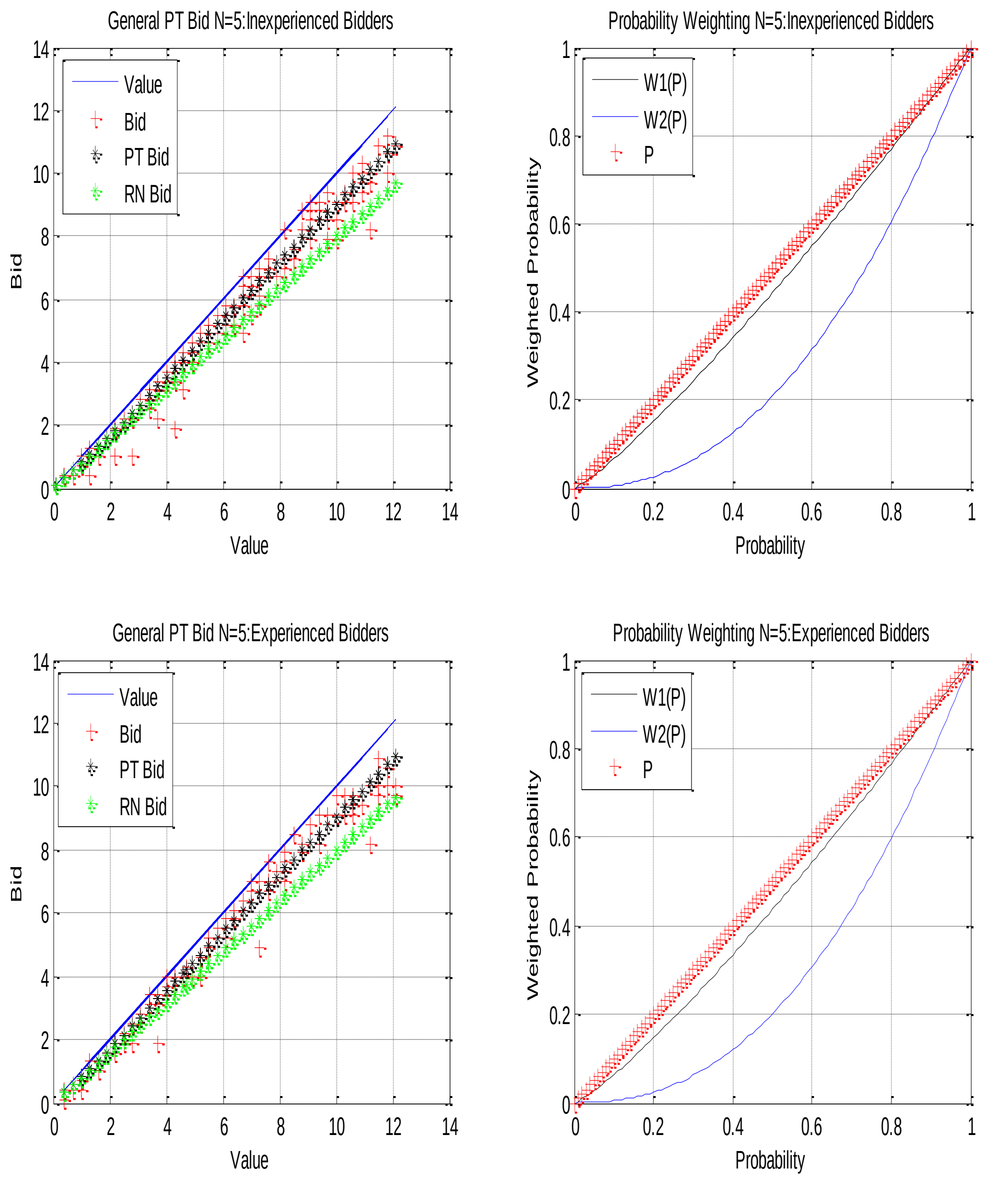
A.2 Figures

Figure 2: General PT Bid and Probability Function; CRS(1982); n=4 (Inexperienced and experienced bidders)



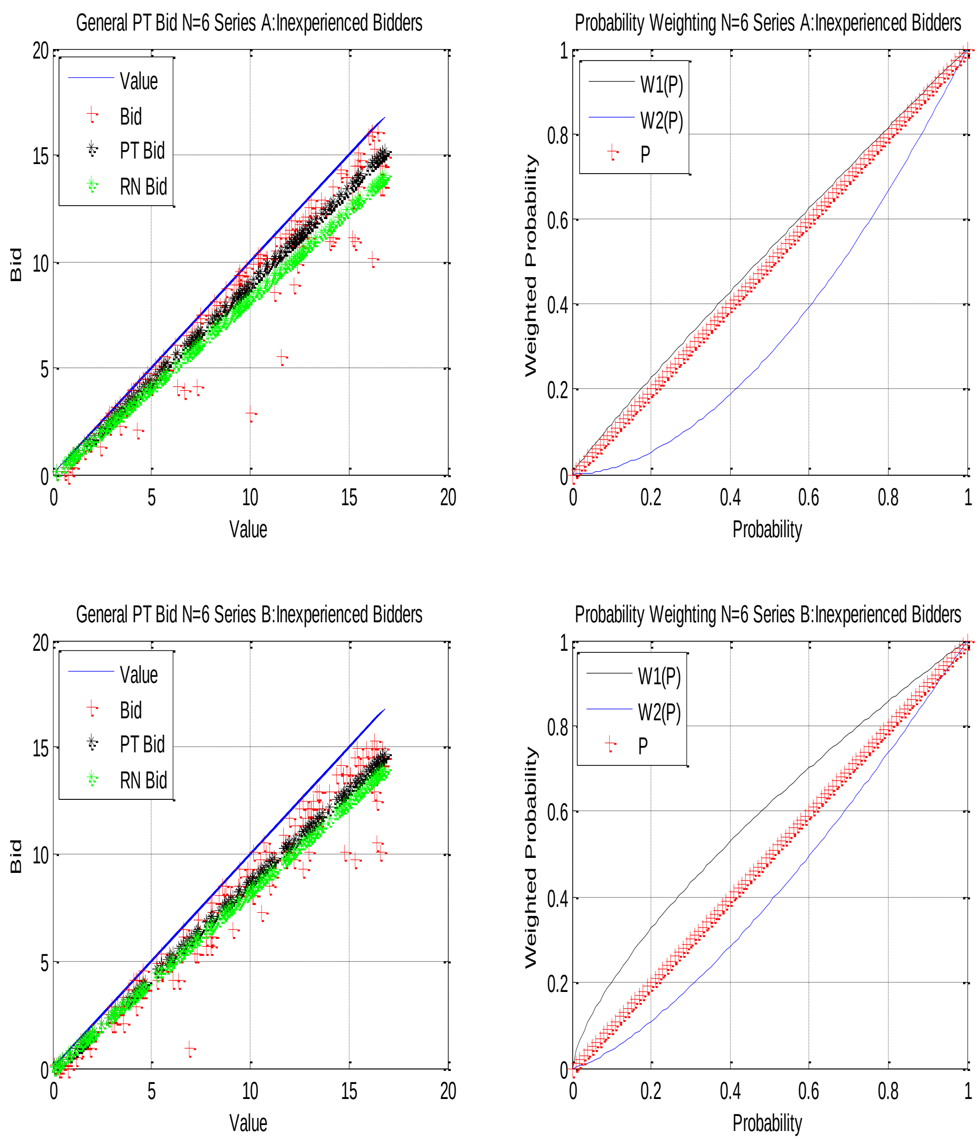
Notes: (1) The right column is a plot of the probability weighting function with and without loss aversion.

Figure 3: General PT Bid and Probability Function; CRS(1982); n=5 (Inexperienced and experienced bidders)



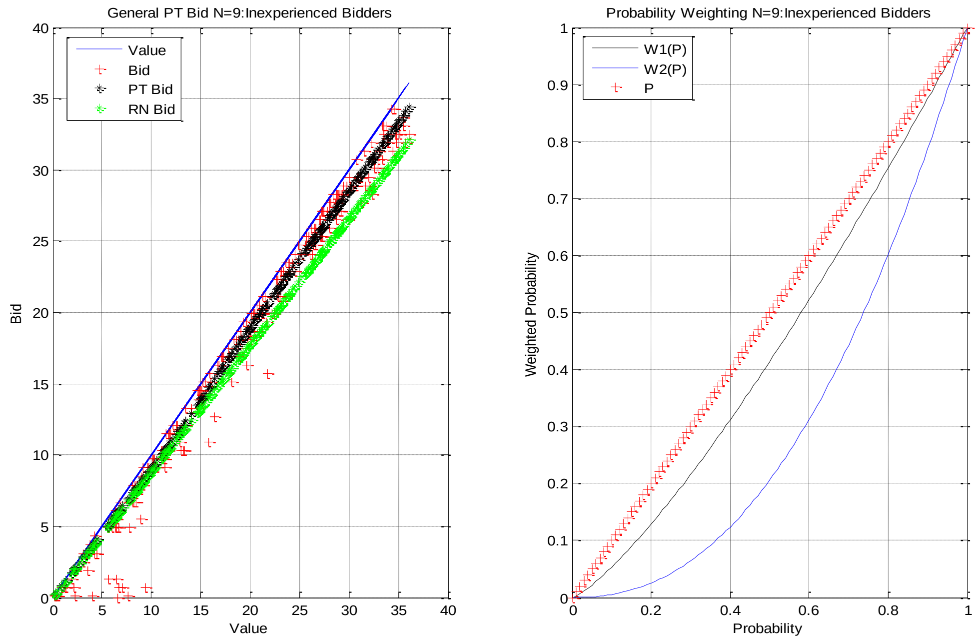
Notes: (1) The right column is a plot of the probability weighting function with and without loss aversion.

Figure 4: General PT Bid and Probability Function; CRS(1982); n=6 (Inexperienced bidders)



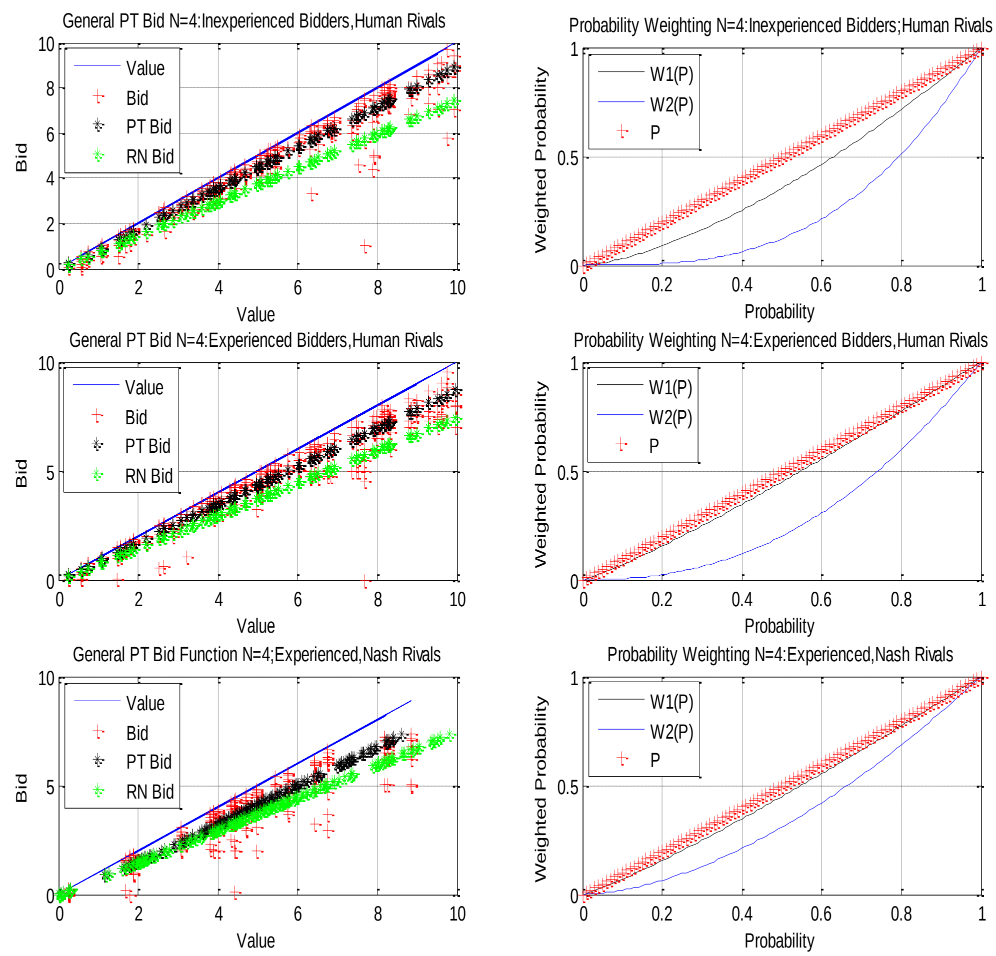
Notes: (1) The right column is a plot of the probability weighting function with and without loss aversion.

Figure 5: General PT Bid and Probability Function; CRS(1982); n=9 (Inexperienced bidders)



Notes: (1) The right column is a plot of the probability weighting function with and without loss aversion.

Figure 6: General PT Bid and Probability Function; Harrison(1989); n=4 (Inexperienced and experienced bidders); against Human and Risk-neutral Nash rivals

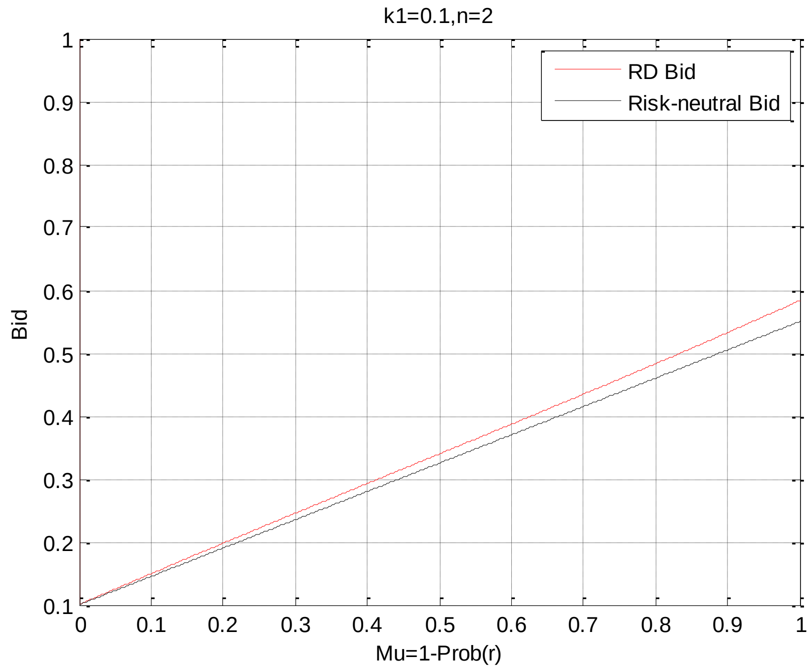


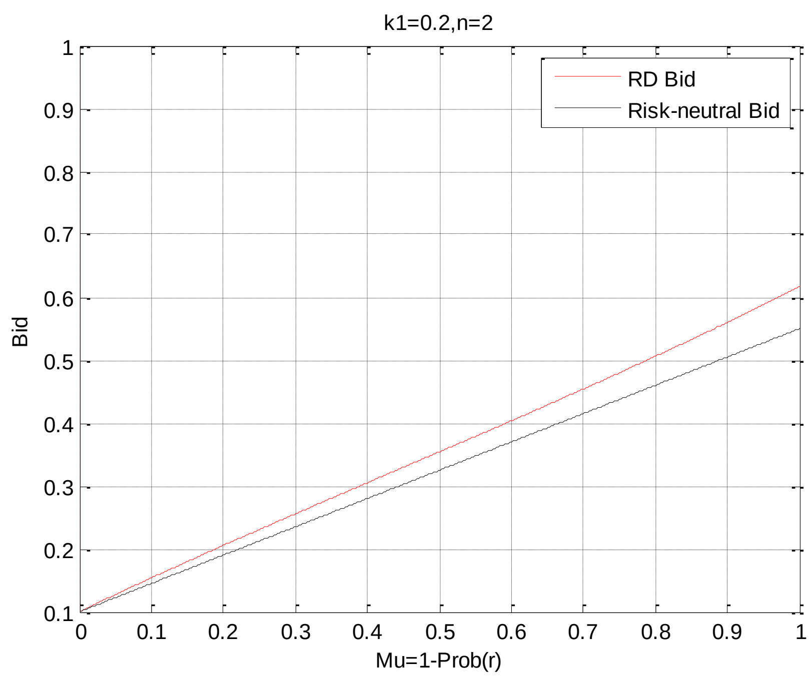
Notes: (1) The right column is a plot of the probability weighting function with and without loss aversion.

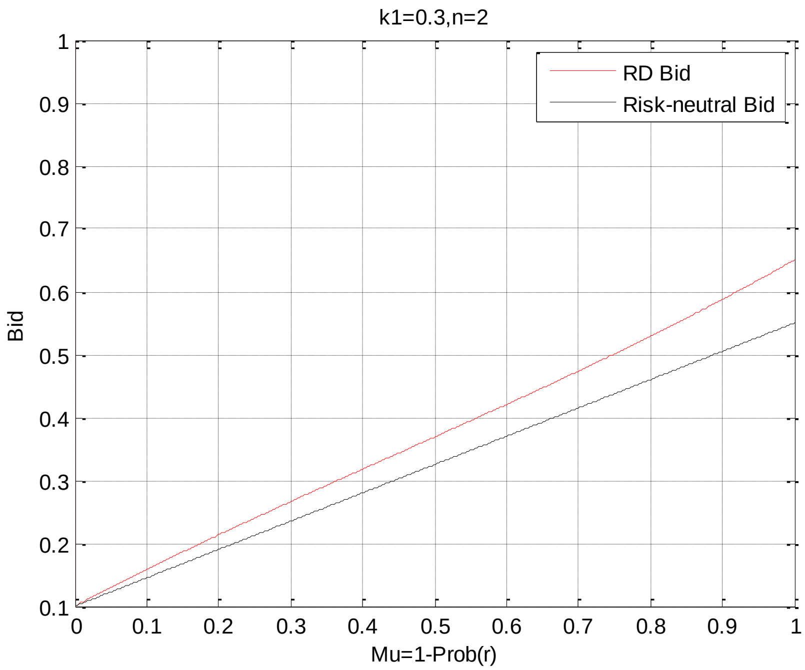
Figure 9: First-Price Auction with outside Procurement

Uniform distribution over procurement price r

$\overline{v} = 1, r = 0.1, n = 2$






k1=0.3,n=2
RD Bid
Risk-neutral Bid
Bid
Mu=1-Prob(r)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

A.3 Tables

Table 1: Auctions in Cox, Roberson and Smith (1982)				
	n=4 (No. of Auctions)	n=5 (No. of Auctions)	n=6 (No. of Auctions)	n=9 (No. of Auctions)
Inexperienced Bidders	fdf8 (20) dfd8 (10)	fdf9 (20) dfd9 (10)	fdf2(20) , fdf4 (20) dfd2 (10), dfd2 (10)	fdf5 (20) dfd5 (10)
Experienced Bidders	fdf8x (20) dfd8x (10)	fdf9x (20) dfd9x (10)		
(1) "n" denotes the number of bidders in a first-price auction				

Table 2: Descriptive Statistics for Auctions in Cox, Roberson and Smith (1982)									
Observations	No. of Bidders (Experience)		Highest Value	Value	Bid	No of Overbids (%)	Average Overbid (%)	Average Underbid (%)	Average Deviation (%)
120	4 (Inexperienced)	Mean (Std)	8.1 (-)	4.0 (2.3)	3.4 (2.1)	77.5 (-)	16.3 (-)	34.2 (-)	20.0 (-)
120	4 (Experienced)	Mean (Std)	8.1 (-)	4.5 (2.3)	3.8 (2.0)	82.5 (-)	15.5 (-)	20.9 (-)	16.3 (-)
150	5 (Inexperienced)	Mean (Std)	12.1 (-)	6.5 (3.4)	5.8 (3.1)	86.7 (-)	14.2 (-)	17.6 (-)	14.6 (-)
150	5 (Experienced)	Mean (Std)	12.1 (-)	5.6 (3.5)	5.1 (3.2)	80.0 (-)	13.8 (-)	20.5 (-)	15.1 (-)
180	6-seriesA (Inexperienced)	Mean (Std)	16.9 (-)	8.6 (4.9)	7.7 (4.5)	78.3 (-)	12.2 (-)	22.9 (-)	14.3 (-)
180	6-series B (Inexperienced)	Mean (Std)	16.9 (-)	8.8 (5.0)	7.6 (4.5)	66.7 (-)	9.5 (-)	21.0 (-)	13.1 (-)
270	9 (Inexperienced)	Mean (Std)	36.1 (-)	19.2 (10.0)	17.9 (10.0)	77.4 (-)	7.4 (-)	26.8 (-)	11.8 (-)
Note: (i) Overbid % defined with respect to RNNE i.e. no. of bids above the RNNE (ii) Overbid is 100*(bid-RNNE)/RNNE for each bid above RNNE (iii) Underbid is 100*(RNNE-bid)/RNNE for each bid below RNNE (iv) Deviation is 100*|(bid-RNNE)|/RNNE									

Table 3: First-Price Auctions in Harrison (1989)					
Common Design Features: $N = 4$, $\underline{v} = \$0.01$ or 1 Point, $\overline{v} = \$10.00$ or 1000 Points, 20 Periods					
Experiment	Level of Experience	Payoff in Dollars or Lottery Points	Simulated Nash Opponent?	Number of Replications per period?	Total Number of Human Bids?
1	Inexperienced	Dollars	No	4	320
1P	Inexperienced	Points	No	4	320
2	Experienced	Dollars	No	5	400
2P	Experienced	Points	No	4	320
3	Experienced	Dollars	Yes	14	280
3P	Experienced	Points	Yes	16	320

Table 4: Descriptive Statistics for Auctions in Harrison (1989)

Observations	No. of Bidders (Experience) Rivals		Highest Value	Value	Bid	No. of Overbids (%)	Average Overbid (%)	Average Underbid (%)	Average Deviation (%)
320	4 (Inexperienced) Human	Mean (Std)	10 (-)	5.09 (2.64)	4.56 (2.40)	91 (-)	23 (-)	26 (-)	24 (-)
400	4 (Experienced) Human	Mean (Std)	10 (-)	5.09 (2.64)	4.42 (2.31)	89 (-)	21 (-)	25 (-)	21 (-)
280	4 (Experienced) Nash	Mean (Std)	10 (-)	4.65 (2.26)	3.85 (1.98)	81 (-)	18 (-)	27 (-)	19 (-)

Note: (i) Overbid % defined with respect to RNNE i.e. no. of bids above the RNNE (ii) Overbid is 100*(bid-RNNE)/RNNE for each bid above RNNE (iii) Underbid is 100*(RNNE-bid)/RNNE for each bid below RNNE (iv) Deviation is 100*|(bid-RNNE)|/RNNE

Table 5: Prospect Theory Models of Bidding

Cox, Roberson and Smith(1982)

No. of Bidders (Experience) (Rivals)	No. of Observations (Periods × Bidders-outliers)	Model	$\hat{\beta}(S.E.)$	$\hat{k}_l(S.E.)$	Residual Sum of Squares (SSE)
4 (Inexperienced) Human	115	General	1.17(0.78)	0.98(0.16)**	12.28
		PW	2.13(0.76)**	-	13.26
		RD	-	0.99(0.06)**	12.85
		RNN	-	-	46.15
4 (Experienced) Human	118	General	1.02(0.48)*	0.99(0.11)**	12.43
		PW	1.96(0.59)**	-	12.81
		RD	-	0.99(0.07)**	12.44
		RNN	-	-	46.59
5 (Inexperienced) Human	146	General	1.17(0.50)**	1.00(0.02)**	26.62
		PW	2.26(0.74)*	-	27.22
		RD	-	1.00(0.004)**	28.10
		RNN	-	-	107.21
5 (Experienced) Human	146	General	1.20(0.52)**	1.00(0.01)**	24.40
		PW	2.31(0.82)**	-	25.50
		RD	-	1.00(0.003)**	26.09
		RNN	-	-	94.64
6-series A (Inexperienced) Human	175	General	0.92(0.48)	1.00(0.003)**	142.55
		PW	1.89(0.83)**	-	142.76
		RD	-	1.00(0.002)**	143.63
		RNN	-	-	223.84
6-series B (Inexperienced) Human	174	General	0.70(0.28)**	1.00(0.02)**	130.24
		PW	1.37(0.47)**	-	131.08
		RD	-	0.85(0.35)**	139.64
		RNN	-	-	159.91
9 (Inexperienced) Human	248	General	1.28(0.45)**	1.00(0.001)**	196.03
		PW	2.28(0.66)**	-	203.31
		RD	-	1.00(0.001)**	204.55
		RNN	-	-	644.96
4 (Inexperienced) Human	306#	General	1.51(1.00)	1.00(0.01)**	67.10
		PW	3.03(1.61)	-	67.27
		RD	-	1.00(0.01)**	85.32
		RNN	-	-	293.89
4 (Experienced) Human	371#	General	1.16(0.09)**	1.00(0.01)**	65.81
		PW	2.32(0.90)**	-	66.00
		RD	-	1.01(0.03)**	68.31
		RNN	-	-	253.01
4 (Experienced) Nash	268~	General	1.02(1.61)	0.91(0.89)	156.89
		PW	1.70(0.95)	-	162.98
		RD	-	0.91(0.37)*	156.91
		RNN	-	-	248.01

Notes: (1) The General model is based on Proposition 1;allows Nonlinear Probability Weighting and Loss-aversion (2) The PW model allows for Nonlinear Probability Weighting (no Loss-aversion) (3) The LA model allows loss-aversion defined in assumption B (linear Probability Weighting) only (4) The RNNE model is based on linear probability weighting where $\beta = 1$ and no loss-aversion (5) Asymptotic Standard Errors in brackets (6) SSE: Sum of squared errors based on the difference between actual and predicted bid (7) The estimates are based on search algorithms developed using MATLAB for the data described in Cox, Roberson and Smith (1982) (8)#Overbids beyond Private Values removed; ~Overbids beyond (3/4)*1000=Highest possible RNN bid removed (9) ** denotes significance at 1% level and * denotes significance at 5% level

Table 6: Hypothesis Tests			
Cox, Roberson and Smith(1982)			
No. of bidders (Observations) Experience Levels (Bidders)	Test	Estimated Log- likelihood ratio	p-value
4 (240) Inexperienced and Experienced (Human)	$k_l^{in\exp} = k_l^{\exp}$ $\beta^{in\exp} = \beta^{\exp}$ $\beta^{in\exp} = \beta^{\exp} \mid k_l^{in\exp} = k_l^{\exp}$	0.2073 2.0052 1.9244	0.9015 0.1568 0.1654
5 (300) Inexperienced and Experienced (Human)	$k_l^{in\exp} = k_l^{\exp}$ $\beta^{in\exp} = \beta^{\exp}$ $\beta^{in\exp} = \beta^{\exp} \mid k_l^{in\exp} = k_l^{\exp}$	1.6639 0.1457 0.0328	0.4352 0.7026 0.8542
Harrison (1989)			
4 (708) Inexperienced and Experienced (Human)	$k_l^{in\exp} = k_l^{\exp}$ $\beta^{in\exp} = \beta^{\exp}$ $\beta^{in\exp} = \beta^{\exp} \mid k_l^{in\exp} = k_l^{\exp}$	0.0465 16.2169** 14.5204**	0.9770 0.0010 0.0010
4 (584) Inexperienced against Human bidders and Experienced against Nash bidders	$k_l^{in\exp} = k_l^{\exp}$ $\beta^{in\exp} = \beta^{\exp}$	15.9527** 30.7884**	0.0030 0.0000
4 (660) Experienced against	$k_l^{in\exp} = k_l^{\exp}$ $\beta^{in\exp} = \beta^{\exp}$	23.1346** 9.2458**	0.0000 0.0024

Human bidders and Experienced against Nash bidders			
Note: (1) ** denotes significance at 1% level.			

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