2009

Expected Equity Option Returns

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SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE IN FINANCE

SINGAPORE MANAGEMENT UNIVERSITY

2009

ABSTRACT

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Substantial progress has been made in investigating ‘Overpriced Puts Puzzle’ which exists in index futures options. However, scarce studies focus on whether single-stock options also have similar problems. This thesis analyzes the returns of individual stocks’ calls, puts, and their portfolios, both theoretically and empirically. Adopting the methodology of Broadie, Chernov, and Johannes (2008), I find that (1) calls have positive expected returns while puts have negative expected returns. The expected returns of both calls and puts are increasing in the strike price. (2) CAPM alphas and Sharpe ratios are reasonable for calls options, but they are too negative for OTM puts. (3) The finite sample distributions simulated by SV and SVJ models do not provide much information on the mispricing of sole calls or sole puts, while the examination of option portfolios show that only the most actively traded options exhibit similar volatility risk premiums in their straddle prices.

TABLE OF CONTENTS

LIST OF TABLES AND FIGURES

LIST OF TABLES

LIST OF FIGURES

ACKNOWLEDGMENT

First of all, I am honored to express my deepest gratitude to my supervisor Prof. Lim Kian Guan whose help, stimulating suggestions and encouragement helped me in all the time of research for and writing of this thesis. Although he is especially busy with his administrative work in the school, he still kept instructing me during the whole process and he amended my thesis in earnest, even for those inconspicuous grammar mistakes. All the remaining mistakes are mine. He has also given me the largest technology support to the completion of my thesis. Despite that it may bring him inconvenience, he generously provided me the computers in his office to help me enhanced the speed of the calculation greatly. Moreover, I am indebted to Prof. Lim's kindly understanding of my repeating leaves for China because of the misfortune of my family.

I want to thank Prof. Jeremy Goh, Prof. Wu Chunchi, Prof. Christopher Ting, and Prof. Tu Jun for instructing me courses during my graduate study. I also want would like to thank all the classmates in our program for those happy and memorable days we spend together in Singapore Management University.

At last, I would like to give my special thanks to my parents. All the time, they try their best to protect me against any possible hurts in my life. It is their support and encouragement that let me come through the misery

Chapter One: Introduction

1.1 Introduction

The rationality of option pricing has become the focus of much attention in academia, especially for index futures options. It is a common perception that index options are mispriced. One piece of evidence is that since the 1987 stock market crash, the Black-Scholes (BS) formula has been producing systematic biases across moneyness and maturity of index options. In particular, the BS formula has been significantly underpricing short maturity, deep out-of-the-money (OTM) puts. This property has been referred to as a ‘volatility smile’ (see Rubinstein (1994), Jackwerth and Rubinstein (1996), and Bates (1996)). Given the empirical failures of the BS model, much research has gone into identifying models that relax some of the restrictive BS assumptions (e.g. stochastic volatility models and jump diffusion models). These extended models have been tested empirically. However, Bakshi, Cao and Chen (1997) find that the ‘smile’ still exist, although the stochastic volatility and jump features can providea significant improvement. Another piece of evidence is from the investigation on option returns. The first paper to focus on the theoretical and empirical nature of option returns is Coval and Shumway (2001). Using zero-beta (BS betas) straddles, they find both call and put contracts earn exceedingly low returns, and argue that systematic stochastic volatility may be an important factor for pricing. Bondarenko (2003) also reports that the historical S&P 500 put options excess returns are significantly negative, as well as the

CAPM alphas. Moreover, other popular measures, such as the Sharpe ratios, also indicate that index put prices have been very high. Noting that option returns are highly non-normal and those metrics all assume normality, Broadie, Chernov, and Johannes (2008) claim that simply using CAPM alphas or Sharpe ratios is not appropriate, and provide an alternative method that let returns anchor null hypothesis values when testing whether they are significantly different from those generated by a given null model. Unfortunately, because of the statistical difficulties present when analyzing options returns, they do not find that index put returns are inconsistent with BS and SV models. Similar tests using option portfolio strategies, such as straddles, show that options are not mispriced when incorporated with certain volatility and estimation risks.

However, these research results are based on index options, and there is scarce literature that studies whether single-stock options also have the ‘mispriced’ problems like index options. Bakshi, Kapadia, and Madan (2003) claim that individual risk-neutral distributions differ from that of the market index by being far less negatively skewed. Garleanu, Pedersen, and Poteshman (2005) find that single-stock options appear cheaper than index options, and their ‘smile’s are flatter. Since few researches have tried to explore this by testing option returns, I explore this topic in my thesis using the methodology of Broadie, Chernov, and Johannes (2008).

This thesis focuses on the returns of individual stocks’ calls, puts, and their portfolios both theoretically and empirically. Consistently, I find that calls have positive expected returns and puts have negative expected returns. Moreover, the expected returns of both calls and puts are increasing in the strike price. CAPM alphas and Sharpe ratios seem reasonable for call options, but they are too negative for OTM put options. Nevertheless, considering the shortcomings of these metrics to measure non-normal option returns, it is not sufficient evidence to prove that OTM puts for individual stocks are overpriced. Moreover, the finite sample distributions simulated by SV and SVJ models do not provide much information on the mispricing of sole calls or sole puts, while the examinations of option portfolios show that only the most actively traded options exhibit similar volatility risk premiums in their straddle prices.

1.2 Organization of the study

The rest of the thesis will be organized in this way:

Chapter Two provides a review of the existing literature analyzing the mispricing problems of index options.

Chapter Three describes the theoretical characters of expected option returns, CAPM alphas, and Sharpe ratios.

Chapter Four describes the data used. The methodology of how to estimate parameters under $\mathbb{P}$-measure and how to construct finite sample distribution is discussed in this chapter.

Chapter Five reports the findings of the empirical tests, and where possible, discussion and explanations are given to provide insights on the results.

Chapter Six summarizes the key results, points out some limitations of the study and also directions for future research.

Chapter Two: Literature Review

2.1 Introduction

It is well known that the Black-Scholes implied volatility smile indicates that OTM put index options are expensive relative to the ATM index puts, and the issue is to then determine if these put options are in fact mispriced. (see Jackwerth (2000), Coval and Shumway (2001), Bondarenko (2003), and etc.) Generally speaking, put options, which deliver payoffs in bad states of the world, indeed will earn lower returns than call options, which deliver their payoffs in good states. As mentioned in Coval and Shumway (2001), option returns can be thought of as pricing two kinds of risks. The first one is a leverage effect. Because an option allows investors to assume much of the risk of the option's underlying asset with a relatively small investment, options have characteristics similar to levered positions in the underlying asset. Therefore, call options written on securities with expected returns above the risk-free rate should earn expected returns that exceed those of the underlying security, while put options should earn expected returns below that of the underlying security. Coval and Shumway (2001) show that the Black-Scholes model has priced this implicit leverage. Secondly, another risk of options comes from the curvature of option payoffs, which results in the skewness of option returns' distribution (i.e. long call returns can be as high as infinity, but no lower the -100%) and the sensitivity of option returns to the higher moments of the underlying asset's returns. Under the assumption of Black-Scholes model, that the market is dynamically complete and perfect, options should earn no such risk premium, since they are redundant assets. However, much evidence (see Coval and Shumway (2001)) seems to show that the financial market does price certain risk into option prices. It is known as "overpriced puts" puzzle. Researches dedicated to this problem mainly focus on three explanations (see Bondarenko (2003) and Isaenko (2007)):

- Risk premium. As mentioned above, because of the high risks of asymmetric payoff and the leverage effect, high prices of puts may be expected and reflect normal risk premiums under some equilibrium model. It is possible that the standard models fail to explain the data, but another "true" model can. In this "true" model, investors strongly dislike negative returns of the underlying securities and are willing to pay considerable premiums for portfolio insurance offered by puts. It is noted that researchers who are concerned with this explanation often focus on two aspects. One is to look for new factors that should be priced in option prices. The other is to discuss investors' particular preference to risks.
- The Peso problem. If this problem exists, it is to say that the puts are not mispriced, and the "overpriced puts" phenomenon is due to the small sample under investigation, which is affected by the Peso problem. This refers to a situation when an influential event could have reasonably happened but did not happen in the sample yet. According to this explanation, the mispricing would have disappeared if data for a much longer period were available.
- Biased belief. This explanation assumes that investors' subjective beliefs are

mistaken, that is the underlying securities realized returns have not been anticipated by investors. The biased beliefs of investors can result in the mispricing problem of options, since they may overstate empirical as well as risk-neutral probabilities of negative returns.

2.2 Overpriced Puts

The market for index options developed in the mid to late 1980s. The first evidence of overpriced puts comes from the well known Black-Scholes implied volatility smile (see Jackwerth and Rubinstein (1996)), with much steeper slope in the OTM puts. Later, Jackwerth (2000) uses prices to characterize the shape of the risk-neutral density, and found that this risk-neutral distribution computed from S&P 500 index put options exhibits a pronounced negative skew after the crash of 1987. Based on a single factor model, he shows that utility over wealth has convex portions, interpreted as evidence of option mispricing. Jackwerth (2000) has also simulated special trading strategies to exploit this mispricing, and shows that put writing strategies gain excess returns, even after accounting for the possibility of further crashes, transaction costs, and hedges against the downside risk, which is another evidence of mispricing.

Aït-Sahalia, Wang, and Yared (2001) compare the risk-neutral density estimated in complete markets from a cross-section of S&P 500 option prices to the risk-neutral density inferred from the time series density of the S&P 500 index, and find that the market prices options with an overly skewed and leptokurtic risk-neutral density.

Aït-Sahalia, Wang, and Yared (2001) reject the joint hypothesis that the S&P 500 options are efficiently priced and that the S&P 500 index follows one-factor diffusion. Moreover, Sharpe ratios achieved by special trading strategies are much larger than those of the market, which further support the mispricing of options.

Coval and Shumway (2001) is the first paper to focus on the theoretical and empirical nature of option returns. By analyzing weekly S&P100 option returns with Black-Scholes betas of calls and puts from 1986 to 1995, they find that both calls and puts earn returns that are too low to be consistent with the Black-Scholes/CAPM model.

Bondarenko (2003) estimates the monthly hold-to-maturity returns of S&P 500 index options from August 1987 through December 2000. He also finds that the average put option returns monotonically increase with strike price. Moreover, the put returns are highly negative and statistically significant, with average excess return -39% per month for ATM puts and -95% per month for deep OTM puts. Other evidence provided by Bondarenko (2003) includes the highly negative Jensen’s alpha for ATM puts which is -23% per month. And it is estimated that the cumulative wealth transfer from buyers to sellers of the S&P 500 futures options is as much as $18 billion over the studied period.

A more recent paper, Broadie, Chernov, and Johannes (2008), studies the monthly hold-to-maturity returns of S&P 500 index options using a longer period from August 1987 to June 2005, and finds similar result that the put options are mispriced. The average monthly returns are -57% for OTM puts and -30% for ATM puts, and are statistically different from zero using *t*-statistics, as *p*-value are close to zero. Broadie, Chernov, and Johannes (2008) compare their statistics in sub-sample from 1987 to 2000 to the ones in the Bondarenko (2003), and finds that the returns are very close, in spite that results of Broadie, Chernov, and Johannes (2008) studies are slightly more negative for every moneyness category except the deepest OTM category. Besides, average put returns are unstable over time. Put returns were extremely negative in the late 1990s during the dot-com "bubble", but were positive and large from late 2000 to early 2003. This may demonstrate a problem with tests using short sample periods.

2.3 Investigating the Explanation of "Overpriced Puts Puzzle"

2.3.1 Risk premium

Most of the existing papers believe that it is the failure of standard models that brings on the overpriced puts puzzle. They are trying to find a "true" model to better fit this fact of extremely negative put returns. Some of them make attempts to investigate newly priced factors such as volatility risk premium, jump risk premium, etc. (e.g. Jones (2006) and Cao and Huang (2007)), and others focus on the changing preferences of investors (e.g. Benzoni, Collin-Dufresne, and Goldstein (2005)).

Coval and Shumway (2001) look at the average returns of zero-beta straddles formed with futures options on the U.S. Treasury bond, Eurodollar, Nikkei 225 Index, and Deutsche Mark. The assumption is that if the only systematic volatility in the economy is market volatility, then only assets with volatilities that are positively correlated with that of the market should earn a risk premium. They find that straddles on assets with volatilities that are positively correlated with market volatility tend to earn negative returns, which is interpreted as very tentative evidence that market volatility risk is priced. Furthermore, Coval and Shumway (2001) regress time series excess returns of CRSP's size-decile portfolios on excess returns of the market and the excess returns of their zero-beta straddles, assuming that if straddle returns are highly sensitive to innovations in volatility, the excess returns of straddle should capture any ability of volatility risk to account for cross-sectional variation in excess returns. Again, they find a distinct pattern in the sensitivities of the size portfolios to the straddle factor. Coval and Shumway (2001) regard all of these results as the evidence that volatility risk is priced in the options market.

Bondarenko (2003) uses a novel test based on equilibrium models in his study. He claims that under fairly general conditions, securities prices must satisfy a new martingale restriction, $E_t^v\left[\frac{Z_s}{h_s(v)}\right] = \frac{Z_t}{h_t(v)}$, $t<s<T$, where v_t is the asset's price, $h_t(v_T)$ is the conditional risk-neutral density of the asset's final price, $E_t^v[\cdot] \coloneqq E_t[\tilde{v}_T = v]$ is the expectation conditional on the final price being v, and Z_t is the value of a general derivative security with a single payoff Z_T at time T. Bondarenko (2003) estimate the conditional risk-neutral density $h_t(v_T)$ from prices of traded options, and find that no equilibrium model for which the pricing kernel $m_T = m(v_T)$ is a flexible and unspecified function of v_T can possibly explain the put anomaly (including the Black-Scholes model, jump-diffusion model and stochastic volatility model), even when

allowing for the possibility of incorrect beliefs and a biased sample. Bondarenko (2003) asserts that a candidate equilibrium model must produce a projected kernel which is strongly path-dependent with respect to the market portfolio.

Jones (2006) estimates a flexible class of nonlinear models using all S&P 500 Index futures options traded between 1986 and 2000. He finds that two- or three-factor models are most successful in explaining both expected and realized option returns, and volatility risk and possibly jump risk are priced in the cross section of index options. However, these additional risk premiums are insufficient to explain average option returns, and deep OTM puts still exhibit overpricing. To explain the failures of all the specifications, Jones (2006) proposes that it is possible that the addition of some unknown state variable may resolve these puzzles, although it is difficult to speculate on what those state variables might be.

Cao and Huang (2007) analyze common factors that affect returns on S&P 500 index options, using daily returns of S&P index options from 1988 to 1994. They find that 93% of the variation in option returns can be explained by three factors, which respectively account for 87%, 4%, and 2% of the variation in option returns. The first factor is interpreted as the underlying asset, denoted by the underlying S&P 500 index returns. The last two factors are both regarded as volatility factors: one is the equally weighted option index, and the other is the option-implied volatility. Since the former offers significant incremental explanatory power for option returns, especially for the OTM options (4% VS 2%), Cao and Huang (2007) believe that the equally weighted option index is a better proxy for the volatility factor. Furthermore, Cao and Huang (2007) perform mean-variance spanning tests, using the underlying and an equally weighted option index as benchmark assets to span OTM, ATM, or ITM option returns, both individually and jointly. Although their results fail to reject that the underlying asset and an ATM option can span OTM options, they reject the notion that they can span ITM options. It indicates that one or more other factors also play a role in determining S&P 500 index option returns.

Garleanu, Pedersen and Poteshman (2005) suggest a new factor to explain the option-pricing puzzle. Empirically, they find that end users, defined as proprietary traders and customers of brokers, have a net long position in S&P 500 index options with large net positions in OTM puts, while dealers, such as market makers are shorting index options. Further, the steepness of the smirk, measured by the difference between the implied volatility of low-moneyness options and ATM options, is positively related to the skew of option demand, measured by the demand for low-moneyness options minus the demand for high-moneyness options. Theoretically, they model the demand-pressure effect on prices, assuming preference of constant absolute risk aversion. To test their demand-pressure model, Garleanu, Pedersen and Poteshman (2005) compute net end-user demand for an option by the sum of the end users long open interest minus the sum of the end users short open interest, and find that options are overall more expensive when there is more end-user demand for options and that the expensiveness skew across moneyness is positively related to skew in end-user demand across moneyness.

Han (2008) studies whether investor sentiment affects S&P 500 option prices. Investor sentiment is the aggregate error in investor beliefs. He uses three investor sentiment proxies in the empirical test. The first proxy is a popular sentiment index based on Investors Intelligence's weekly surveys of approximately 150 investment newsletter writers. The second sentiment proxy is the net position of large speculators in S&P 500 futures, which is calculated as the number of long noncommercial contracts minus the number of short noncommercial contracts, scaled by the total open interest in S&P 500 futures. The last proxy is the residuals of the log price-earnings ratio of the S&P 500 index regressed on earnings growth expectations, log dividend payout, and several other variables such as expected inflation and real 30-year treasury-bond yield. The empirical tests focus on the time-series relation between sentiment proxies and the skewness of the risk-neutral density of monthly S&P 500 index returns. Han (2008) finds that when investors are more bearish, they would have a stronger demand and be willing to pay more for state contingent claims that pay off when the index level is low. This leads to a more negatively sloped pricing kernel, and thus a more negative index risk-neutral skewness. Moreover, these results still hold after controlling for a set of rational factors that may be related to the sentiment proxies, and after controlling for variables related to index risk-neutral skewness. Han (2008) concludes that investor sentiment is an important determinant of index option prices.

Benzoni, Collin-Dufresne and Goldstein (2005) explore whether the standard preferences can explain the prices of OTM S&P 500 put options. While many researchers have argued that overpriced put puzzle cannot be justified in a general equilibrium setting if the representative agent has standard preferences, Benzoni, Collin-Dufresne and Goldstein (2005) demonstrate that the volatility smirk can be rationalized if the agent is endowed with Epstein-Zin preferences and if the aggregate dividend and consumption processes are driven by a persistent stochastic growth variable that can exhibit jump. In their framework, the risky asset performs poorly in a bad state and investors are willing to pay a high price for a security that provides insurance against this state. Benzoni, Collin-Dufresne and Goldstein (2005) further extend the model to a Bayesian setting in which the agent formulates a prior on the average value of the jump size, and then updates her prior when she observes an extreme event such as the 1987 crash. They find that their model can capture the implied volatility pattern of option prices both before and after the 1987 crash. However, for the case with Bayesian updating, the model consistent with pre- and post-crash data seems to predict a crash on the day of the event larger than what was observed in 1987. Benzoni, Collin-Dufresne and Goldstein (2005) assert that allowing for Bayesian updating not only on the size of the jump, but also on its intensity, or modeling volatility as stochastic with jump may improve the fit on the real data.

2.3.2 The peso problem & biased belief

In empirical tests, the effect caused by peso problem and biased belief are hard to isolate. When a rare event that one believes will happen do not actually happen, people cannot ascribe all to the person's biased belief. It is possible that over a considerably long period, evidence will show the correctness of his belief. For this reason, the peso problem and biased belief can only be tested jointly in studies.

Coval and Shumway (2001) test the returns to the "crash-neutral" straddle, a straddle position achieved by purchasing a straddle position and selling a deeply OTM put option. This crash-neutral straddle's return during a market crash is limited to some level that is specified when the position is created. So the measures of the position's expected returns are not downward-biased by infrequent crash observations or high-priced crash risk. However, Coval and Shumway (2001) find that this strategy still generates average losses of nearly 3 percent per week. The authors regard these results as rejection of the peso problem and claim that options are earning low returns for reasons that extend beyond their ability to provide insurance against crashes.

As mentioned above, Bondarenko (2003) uses a new restriction to test the overpriced put puzzle. It is said that this restriction will not be affected by the selection bias and also the belief bias, because it involves conditioning on the final price. Likewise, the restriction is not affected by the peso problem. After ruling out peso problem's influence, since the put anomaly still cannot be well explained, Bondarenko (2003) claims that the peso problem and bias belief are not the necessary explanations for overpriced put puzzle.

Broadie, Chernov, and Johannes (2008) adopt a different approach. Firstly, they estimate the stochastic-volatility random-jump model using daily S&P 500 index returns spanning the same time period as their options data, from 1987 to 2005. Then they use MCMC methods to simulate the posterior distribution of the parameter and state variables. Assuming these parameters are $\mathbb{P}$-parameters, generating the observed S&P 500 index returns, they claim that investors priced options taking into account estimation risk by increasing/decreasing the $\mathbb{Q}$-measure parameters by one standard deviation from the $\mathbb{P}$-parameters. Broadie, Chernov, and Johannes (2008) compute the difference between expected variance under $\mathbb{Q}$- and $\mathbb{P}$-measures, and find that estimation risk appears to be priced in the option market. Nonetheless, estimation risk is not totally equal to the peso problem while biased belief can also be a cause of estimation risk.

2.4 Conclusion

It is recognized that the put index options, especially OTM puts, have been overpriced. Many studies have made attempts to explain this puzzle. Most researchers believe that it is the failure of standard models that brings on the overpriced puts puzzle. Other factors are being tested. Stochastic volatility and jump risk are two important factors that first been used to explain the option returns, and they do have significant effect on improving the models. However, it has been shown that they are still insufficient to provide a full explanation. Other new factors, such as demand pressure and investor sentiment, also shed certain light on the discussion. Peso problem and investors' biased beliefs may also cause the overpriced put puzzle. Nevertheless, studies that reject these explanations and those that support them both exist.

Chapter Three: Theoretical Foundations

3.1 Expected Option Returns

In this thesis, I focus on one-month hold-to-expiration option returns (see Broadie, Chernov and Johannes (2008)), which are defined as

$$\begin{aligned} Call\ Returns: r_{t,T}^{c} &= \frac{(S_{t+T} - K)^{+}}{C_{t,T}(K, S_t)} - 1 \\ Put\ Returns: r_{t,T}^{p} &= \frac{(K - S_{t+T})^{+}}{P_{t,T}(K, S_t)} - 1 \end{aligned} \tag{3.1}$$

where $x^{+} \equiv \max(x, 0)$, $C_{t,T}(K, S_t)$ and $P_{t,T}(K, S_t)$ are the time-t price of a call and a put written on S_t, struck at K, and expiring at time t+T. Hold-to-expiration returns are typically analyzed in both academic studies and in practice for two reasons. First, option trading involves significant costs while strategies that hold until expiration incur these costs only at initiation. Secondly, higher frequency option returns (for example, weekly returns) generate a number of theoretical and statistical issues which can be avoided using monthly returns. Specifically, many studies compute weekly returns by holding a longer-dated option for one week, but it presents an important theoretical complication since weekly return characteristics vary by maturity: a one-week return on a five-week option is theoretically different from a one-week return on a one-week option. Besides, OTM options are usually less traded, especially for individual stock options. This implies that weekly option returns will be generated by allowing for moneyness and maturity windows.

Given the hold-to-maturity option returns, expected option returns are as below

$$
\begin{aligned}
Call&: E_t^P\left(r_{t,T}^c\right) = E_t^P\left(\frac{(S_{t+T}-K)^+}{C_{t,T}(K,S_t)}\right) - 1 = \frac{E_t^P((S_{t+T}-K)^+)}{C_{t,T}(K,S_t)} - 1 \\
&= \frac{E_t^P((S_{t+T}-K)^+)}{E_t^Q(e^{-rT}(S_{t+T}-K)^+)} - 1 \\
Put&: E_t^P\left(r_{t,T}^p\right) = E_t^P\left(\frac{(K-S_{t+T})^+}{P_{t,T}(K,S_t)}\right) - 1 = \frac{E_t^P((K-S_{t+T})^+)}{P_{t,T}(K,S_t)} - 1 \\
&= \frac{E_t^P((K-S_{t+T})^+)}{E_t^Q(e^{-rT}(K-S_{t+T})^+)} - 1
\end{aligned}
\tag{3.2}
$$

Equation (3.2) implies that the gap between the $\mathbb{P}$ and $\mathbb{Q}$ probability measures determines expected option returns, and the magnitude of the returns is determined by the relative shape and location of the two probability measures.

Coval and Shumway (2001) have shown that since no existing asset-pricing theory permits a stochastic discount factor that is positively correlated with the market level and most individual security prices and all call options written on that security will have positive expected returns and increasing in their strike price, while all put options should have expected returns below the risk-free rate that is increasing in the strike price.

PROPOSITION 1 (from Coval and Shumway (2001)): *If the stochastic discount factor is negatively correlated with the price of a given security over all ranges of the security price, any call option on that security will have a positive expected return that is increasing in the strike price.*

Proof: As Equation (3.2),

$$E_t^P\left(r_{t,T}^c\right) = \frac{E_t^P((S_{t+T} - K)^+)}{E_t^Q(e^{-rT}(S_{t+T} - K)^+)} - 1$$

Thus, the derivative of expected call returns with respect to strike price can be expressed as (assuming *t=0*)

$$\frac{\partial E^P(r_T^c)}{\partial K} = e^{-rT}\frac{\frac{\partial}{\partial K}E^P[(S_T - K)^+]\cdot E^Q[(S_T - K)^+] - E^P[(S_T - K)^+]\cdot\frac{\partial}{\partial K}E^Q[(S_T - K)^+]}{\left[E^Q[e^{-rT}(S_T - K)^+]\right]^2}$$

The numerator of last equation is

$$\begin{aligned}
&\frac{\partial}{\partial K}E^P[(S_T - K)^+]\cdot E^Q[(S_T - K)^+] - E^P[(S_T - K)^+]\cdot\frac{\partial}{\partial K}E^Q[(S_T - K)^+] \\
&= \frac{\partial}{\partial K}\int_K^\infty (S_T - K)dP\cdot\int_K^\infty (S_T - K)dQ - \int_K^\infty (S_T - K)dP\cdot\frac{\partial}{\partial K}\int_K^\infty (S_T - K)dQ \\
&= -\big(1 - P(K)\big)\cdot\int_K^\infty (S_T - K)\frac{dQ}{dP}dP + \int_K^\infty (S_T - K)dP\cdot\int_K^\infty \frac{dQ}{dP}dP \\
&= -\big(1 - P(K)\big)^2\left[\int_K^\infty (S_T - K)\frac{dQ}{dP}\frac{dP}{1 - P(K)} - \int_K^\infty (S_T - K)\frac{dP}{1 - P(K)}\int_K^\infty \frac{dQ}{dP}\frac{dP}{1 - P(K)}\right] \\
&= -\big(1 - P(K)\big)^2\left[E^P\left[(S_T - K)\cdot\frac{dQ}{dP}\ |S_T \geq K\right] - E^P[(S_T - K)|S_T \geq K].E^P\left[\frac{dQ}{dP}|S_T \geq K\right]\right]
\end{aligned}$$

Note that the second part of the last equality is rightly the covariance of $(S_T - K)$ and $\frac{dQ}{dP}$, conditional on the option being in the money ($S_T \geq K$):

$$\begin{aligned}
&\operatorname{cov}\left[(S_T - K), \frac{dQ}{dP}\ |S_T \geq K\right] \\
&= E^P\left[(S_T - K)\cdot\frac{dQ}{dP}\ |S_T \geq K\right] - E^P[(S_T - K)|S_T \geq K].E^P\left[\frac{dQ}{dP}|S_T \geq K\right] < 0
\end{aligned}$$

where $\frac{dQ}{dP}$ is the stochastic discount factor. Then, we have

$$\frac{\partial E_t^P\left(r_{t,T}^c\right)}{\partial K} > 0.$$

This implies that expected call returns is increasing in the strike price. Because a call option with a zero strike price has the same positive expected return as the underlying asset, all the calls should also have positive expected returns above that of the underlying. Q.E.D.

PROPOSITION 2 (from Coval and Shumway (2001)): *If the stochastic discount factor is negatively correlated with the price of a given security over all ranges of the security price, any put option on that security will have an expected return below the risk-free rate that is increasing in the strike price.*

Proof: As Equation (3.2),

$$E_t^P\left(r_{t,T}^p\right) = \frac{E_t^P\left((K - S_{t+T})^+\right)}{E_t^Q\left(e^{-rT}(K - S_{t+T})^+\right)} - 1$$

Thus, the derivative of expected call returns with respect to strike price can be expressed as (assuming *t=0*)

$$\frac{\partial E^P\left(r_T^p\right)}{\partial K} = e^{-rT}\frac{\frac{\partial}{\partial K}E^P[(K - S_T)^+]\cdot E^Q[(K - S_T)^+] - E^P[(K - S_T)^+]\cdot\frac{\partial}{\partial K}E^Q[(K - S_T)^+]}{\left[E^Q[e^{-rT}(K - S_T)^+]\right]^2}$$

The numerator of last equation is

$$\begin{aligned}
&\frac{\partial}{\partial K}E^P[(K - S_T)^+]\cdot E^Q[(K - S_T)^+] - E^P[(K - S_T)^+]\cdot\frac{\partial}{\partial K}E^Q[(K - S_T)^+] \\
&= \frac{\partial}{\partial K}\int_0^K (K - S_T)dP\cdot\int_0^K (K - S_T)dQ - \int_0^K (K - S_T)dP\cdot\frac{\partial}{\partial K}\int_0^K (K - S_T)dQ \\
&= P(K)\cdot\int_0^K (K - S_T)\frac{dQ}{dP}dP + \int_0^K (K - S_T)dP\cdot\int_0^K\frac{dQ}{dP}dP \\
&= P(K)^2\left[E^P\left[(K - S_T)\cdot\frac{dQ}{dP}\;|S_T < K\right] - E^P[(K - S_T)|S_T < K].\,E^P\left[\frac{dQ}{dP}|S_T < K\right]\right] \\
&= \mathrm{cov}\left[(K - S_T), \frac{dQ}{dP}\;|S_T < K\right] > 0
\end{aligned}$$

Thus, we have

$$\frac{\partial E^P\left(r_T^p\right)}{\partial K} > 0.$$

With the fact that put returns are increasing in the option strike price and that a put option with infinite strike price has an expected return equal to the risk-free rate, we know that all put options will have expected returns below the risk-free rate. Q.E.D.

Propositions 1 and 2 actually describe the so-called leverage effect in options returns, and this implicit leverage should be priced no matter which model is used.

3.2 CAPM Alpha and Sharpe Ratio

It is common that the literature uses CAPM alphas to test if an asset is mispriced. For example, CAPM model is written as

$$E_t[r_i] - r_f = \beta_i \cdot E_t\left[r_m - r_f\right] + \varepsilon_{\mathrm{i}}$$

Thus a non-zero α in $E[r_i] - r_f = \alpha + \beta_i \cdot E\left[r_m - r_f\right] + \varepsilon_{\mathrm{i}}$ is interpreted as evidence of either mispricing or a risk premium not captured by CAPM.

Thus, we can also write expected excess option returns into this CAPM form. Take the Black-Scholes model as an example. The link between instantaneous derivative returns and excess underlying security returns is

$$\frac{df(S_t)}{f(S_t)} = rdt + \frac{S_t}{f(S_t)}\frac{\partial f(S_t)}{\partial S_t}\left(\frac{dS_t}{S_t} - rdt\right). \tag{3.3}$$

proof: In Black-Scholes, the underlying security satisfies geometric Brownian motion:

$$dS_{t=}r_f S_t dt + \sigma S_t dB_t$$

where S_t is the price of underlying security at time *t*, r_f is the risk-free rate, and B_t is standard Brownian motion. According to Ito-lemma, the dynamics of derivative's price is given by

$$df(S_t) = \frac{\partial f(S_t)}{\partial t}dt + \frac{\partial f(S_t)}{\partial S_t}dS_t + \frac{1}{2}\frac{\partial^2 f}{\partial S_t^2}\sigma^2 S_t^2 dt \tag{3.4}$$

On the other hand, the Black–Scholes PDE shows that

$$\frac{\partial f(S_t)}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S_t^2}\sigma^2 S_t^2 + rS_t\frac{\partial f(S_t)}{\partial S_t} - r_f f(S_t) = 0 \qquad (3.5)$$

Substituting PDE (3.4) into Equation (3.3), we see that

$$\begin{aligned} &df(S_t) \\ &= \left(-\frac{1}{2}\frac{\partial^2 f}{\partial S_t^2}\sigma^2 S_t^2 - r_f S_t\frac{\partial f(S_t)}{\partial S_t} + r_f f(S_t)\right)dt + \frac{\partial f(S_t)}{\partial S_t}dS_t + \frac{1}{2}\frac{\partial^2 f}{\partial S_t^2}\sigma^2 S_t^2 dt \qquad (3.6) \\ &= r_f f(S_t)dt + S_t\frac{\partial f(S_t)}{\partial S_t}\left(\frac{dS_t}{S_t} - r_f dt\right) \end{aligned}$$

Q.E.D.

If we consider Equation (3.3) as the instantaneous Black-Scholes CAPM for derivatives, an approximate CAPM model for finite holding period returns is

$$\begin{aligned} E_t^P\left[\frac{f(S_{t+T}) - f(S_t)}{f(S_t)} - r_f T\right] &\approx \frac{S_t}{f(S_t)}\frac{\partial f(S_t)}{\partial S_t}\cdot E_t^P\left[\frac{S_{t+T} - S_t}{S_t} - r_f T\right] \\ &= \beta_t \cdot E_t^P\left[\frac{S_{t+T} - S_t}{S_t} - r_f T\right]. \end{aligned}$$

Thus, by testing whether $\alpha_T = 0$ via regression

$$\frac{f(S_{t+T}) - f(S_t)}{f(S_t)} - r_f T = \alpha_T + \beta_t\left(\frac{S_{t+T} - S_t}{S_t} - r_f T\right) + \varepsilon_{t,T},$$

we can tell whether options are mispriced or there is risk premium.

However, this strategy has a serious problem: in discrete time, this CAPM model for options can only be derived approximately, although it holds in continuous time. The degree of bias depends on the length of the holding period. Moreover, since option returns are highly skewed, the errors $\varepsilon_{t,T}$ are also highly skewed, which does not agree with standard tests of parameter significance assuming normal distributions.

When it comes to more complicated models, such as SVJ model, α_Ts are theoretically notzero (see Broadie, Chernov, Johannes (2008)), because of the volatility risk and jump risk premiums:

$$\frac{1}{dt}E_t^P\left[\frac{df(S_t)}{f(S_t)}-r_f dt\right]$$

$$=\beta_t^s\left[dS_t/S_t-\left(r_f-\lambda^Q\bar{\mu}^Q\right)dt\right] \tag{3.7}$$

$$+\beta_t^v\kappa_v^P\left(\theta_v^P-\theta_v^Q\right) \tag{3.8}$$

$$+\left[\lambda^P E_t^P[f(S_t e^Z)-f(S_t)]-\lambda^Q E_t^Q[f(S_t e^Z)-f(S_t)]\right]/f(S_t) \tag{3.9}$$

where, $\beta_t^s=\partial log[f(S_t,V_t)]/\partial log S_t$ and $\beta_t^v=\partial log[f(S_t,V_t)]/\partial V_t$. Note that (3.8) and (3.9) represent volatility risk and jump risk premiums respectively. Generally speaking, for OTM puts, pricing volatility and jump risks implies that $\theta_v^P<\theta_v^Q$ and $E_t^P[f(S_t e^Z)]<E_t^Q[f(S_t e^Z)]$. Thus, negative alphas are fully consistent with volatility and jump risk premium, and are not indicative of mispricing.

The Sharpe ratio is another measure of the excess return per unit of risk in an investment asset or a trading strategy. It is defined as

$$SR=\frac{E\left[r-r_f\right]}{\sqrt{Var\left[r-r_f\right]}}. \tag{3.10}$$

It is said that the asset with the higher Sharpe ratio gives more return for the same risk. In fact, Sharpe ratio does provide an appropriate metric when returns are normally distributed, but it is problematic to measure the mispricing of options, whose returns are highly skewed.

3.3 Option Pricing Models

Since the classical Black-Scholes model proposed by Black and Scholes (1973), option pricing has witnessed an explosion of new models that relax some of the restrictive Black-Scholes assumptions. Examples include (a) the jump-diffusion model with constant volatility of Merton (1976); (b) the stochastic-volatility model of Heston (1993); and (c) the stochastic-volatility jump-diffusion models of Bates (1996) and Scott (1997). These models can be nested to a general model with mean-reverting stochastic volatility and lognormal distributed Poisson driven jumps in prices:

$$dS_t = (r_f + \mu) S_t dt + S_t \sqrt{V_t} dW_t^s(P) + d\left(\sum_{j=1}^{N_t(P)} S_{\tau_{j-}} \left(e^{Z_j^s(P)} - 1\right)\right) - \lambda^P \bar{\mu}^P S_t dt \qquad (3.11)$$

$$dV_t = \kappa_v^P (\theta_v^P - V_t) dt + \sigma_v \sqrt{V_t} dW_t^v(P) \qquad (3.12)$$

where r_f is the risk-free rate, μ is the equity premium, W_t^s and W_t^v are two correlated standard Brownian motions ($E[W_t^s W_t^v] = \rho t$), $N_t(P) \sim \text{Poisson}(\lambda^P t)$, $Z_j^s(P) \sim \mathcal{N}(\mu_z^P,\ (\sigma_z^P)^2)$, and $\bar{\mu}^P = \exp(\mu_z^P + (\sigma_z^P)^2/2) - 1$.

This general form is actually the SVJ model under the real-world measure $\mathbb{P}$. For Black-Scholes model, there is no jump ($\lambda^P = 0$) and the volatility is constant ($V_0 = \theta_v^P = \sigma^2, \sigma_v = 0$); Merton's model is a special case with jumps but constant volatility ($\lambda^P \neq 0, V_0 = \theta_v^P = \sigma^2, \sigma_v = 0$); and Heston's SV model is another special case with no jump but stochastic volatility ($\lambda^P = 0$). When volatility is constant, we use the notation $\sqrt{V_t} = \sigma$.

The pricing of options need to use the dynamics under the risk-neutral measure $\mathbb{Q}$:

$$dS_t = r_f S_t dt + S_t\sqrt{V_t}dW_t^s(Q) + d\left(\sum_{j=1}^{N_t(Q)} S_{\tau_{j-}}\left(e^{Z_j^s(Q)} - 1\right)\right) - \lambda^Q \bar{\mu}^Q S_t dt \tag{3.13}$$

$$dV_t = \kappa_v^Q\left(\theta_v^Q - V_t\right)dt + \sigma_v\sqrt{V_t}dW_t^v(Q) \tag{3.14}$$

where$N_t(Q)$~Poisson$(\lambda^Q t)$,$Z_j^s(Q) \sim \mathcal{N}\left(\mu_z^Q,\ \left(\sigma_z^Q\right)^2\right)$, and$\bar{\mu}^Q = \exp\left(\mu_z^Q + \left(\sigma_z^Q\right)^2/2\right) - 1$. Thus, the diffusive equity premium is represented by $\mu' = \mu - \lambda^P\bar{\mu}^P + \lambda^Q\bar{\mu}^Q$, while differences between the risk-neutral and real-world jump and stochastic volatility parameters are referred to as jump or stochastic volatility risk premium. The difference between expected variance under $\mathbb{Q}$- and $\mathbb{P}$-measures in the model is

$$\begin{aligned} &E\left[V_{t,T}^Q\right] - E\left[V_{t,T}^P\right] \\ &= \left(\theta_v^Q - \theta_v^P\right)\left(1 + \frac{e^{-\kappa_v^P T} - 1}{\kappa_v^P T}\right) + \lambda^Q\left(\left(\mu_z^Q\right)^2 + \left(\sigma_z^Q\right)^2\right) - \lambda^P((\mu_z^P)^2 + (\sigma_z^P)^2). \end{aligned} \tag{3.15}$$

Parameters θ_v and κ_v can both characterize the stochastic volatility risk, and potentially change under the risk-neutral measure (Cheredito, Filipovic, and Kimmel (2003)). However, since Broadie, Chernov, and Johannes (2008) show that average option returns are not sensitive to empirically plausible changes in κ_v^P, one can make changes in θ_v^Q from θ_v^P but constrain $\kappa_v^Q = \kappa_v^P$ to explore the stochastic volatility risk premium for SV models. Because volatility is highly persistent (i.e. κ_v^P is small), when T is short (i.e. one-month options in my data sample), θ_v^Q needs to be comparatively larger than θ_v^P to generate the gap between $\mathbb{Q}$ and $\mathbb{P}$. On the other hand, changes of measure for jump processes are more flexible than those for diffusion processes: parameters λ, μ_z ,and σ_z^2 have impact on expected variance for all maturities and do not depend on slow rates of mean-reversion.

Chapter Four: Data and Methodology

4.1 Data

Unlike existing literature, which usually focus on understanding index options, this study is examines individual equity options. The primary data used in this thesis are a triple panel (in the three dimensions of strike price, maturity, and underlying ticker) of bid-ask option quotes written on 5 stocks, obtained from the OptionMetrics Database, and the corresponding underlying stock returns, obtained from the CRSP Database. The sample contains options on the 5 most actively traded (measured by the total trading volume in the test period) and familiar stock options: Microsoft, CISCO Systems, IBM, General Motors, and General Electric. I collected data on these options from January 1996 to April 2006, a total of 124 months. Knowing contracts expire on the third Friday of each month, which implies there are 28 or 35 calendar days to maturity depending on whether it was a four- or five-week month, one month options are thus selected.

However, given that these stock options are all American, it complicated the estimation procedure of hold-to-expiration option returns because of the considerable probability of early exercise. One method to circumvent this problem is transforming American option prices to European option prices (see Broadie, Chernov, and Johannes (2008), Broadie, Chernov, and Johannes (2007), and Bondarenko (2003)); details of this procedure are given in Appendix A.

Since options exist only for specific strike prices, prices for standard moneyness (ranging from 0.75 to 1.15 with 0.05 increments) cannot be directly observed. However, theoretical distributions and no-arbitrage conditions imply that options prices are continuous, monotone, and convex functions of the strike price. Following Bates (1991), I adopt the strategy of interpolating options prices for desired strike prices from a constrained cubic spline fitted through the options prices as a function of the moneyness (strike price/underlying price).

4.2 Methodology

This thesis follows mainly the methodology of Broadie, Chernov, and Johannes (2008). That is to compare the observed values of the common statistics (average returns, CAPM alphas, and Sharpe ratios) in the data to those generated by option pricing models (3.7) and (3.8). Here, formal models provide appropriate null values for anchoring hypothesis tests, and a mechanism for dealing with the severe statistical problems associated with option returns.

4.2.1 Parameter Estimation

To obtain the statistics generated by option pricing models, I need to know all the parameters of these models under the real-world $\mathbb{P}$-measure. Following Broadie, Chernov, and Johannes (2008), I also calibrate the models to fit the realized historical behavior of the underlying stock returns over my observed sample. Markov Chain Monte Carlo (MCMC) methods are utilized here to achieve this goal (see Erake, Johannes, and Polson (2003), Jacquier, Polson, Rossi (2004), and Johannes and Polson (2003)). Erake, Johannes, and Polson (2003) show that this approach has four advantages: (a) MCMC provides estimates of the latent volatility, jump times, and jump sizes; (b) MCMC accounts for estimation risk; (c) MCMC methods have been shown in related settings to have superior sampling properties to competing methods (i.e. GMM and EMM); and (d) MCMC methods are computationally efficient for researchers to check its accuracy using simulations.

Take SVJ model as an example. The basis of the MCMC estimation is a time-discretization of (3.7) and (3.8)

$$Y_t = \tilde{\mu} + J_t Z_t^s + \sqrt{V_{t-1}}\varepsilon_t^s \tag{4.1}$$

$$V_t = \kappa_v \theta_v + (1 - \kappa_v)V_{t-1} + \sqrt{V_{t-1}}\sigma_v \varepsilon_t^v \tag{4.2}$$

where $Y_t = S_t/S_{t-1} - 1$ is the stock return, $\tilde{\mu} = r_f + \mu$, $J_t = 1$ indicates a jump arrival, and ε_t^s and ε_t^v are standard normal random variables with correlation ρ.

The estimate of parameter $\tilde{\mu}$ can be directly substituted by the mean of historical stock returns. For other parameters, according to Bayesian Rule, the posterior distribution summarizes the sample information regarding the parameters, Θ, and the latent volatility, jump times, and jump sizes:

$$p(\Theta, J, Z^s, V|Y) \propto p(Y|\Theta, J, Z^s, V)p(\Theta, J, Z^s, V)$$

where J, Z^s, V, and Y are vectors containing the time series of the relevant variables. The posterior combines the likelihood, $p(Y|\Theta, J, Z^s, V)$, and the prior, $p(\Theta, J, Z^s, V)$. As the posterior distribution is not known in closed form, the MCMC algorithm generates samples by iteratively drawing from the following conditional posteriors (supported by Gibbs Sampler and Metropolis-Hastings):

Parameters: $p(\Theta_i|\Theta_{-i}, J, Z^s, V, Y), i = 1, \ldots, k$

Jump times: $p(J_t = 1|\Theta, Z^s, V, Y), t = 1, \ldots, T$

Jump sizes: $p(Z^s|\Theta, J_t = 1, V, Y), t = 1, \ldots, T$

Volatility: $p(V_t|\Theta, J_t = 1, V_{t-1}, V_{t+1}, Z^s, Y), t = 1, \ldots, T$

where Θ_{-i} denotes the elements of the parameter vector except Θ_i. The details of this MCMC algorithm are explained in Appendix B.

The parameter estimates (posterior means) and posterior standard deviations are reported in Table I. Compared to those parameter estimates of S&P 500 index futures, we can see there are some difference between individual stocks and index. For example, the average estimate λ for individual stocks is about 0.02, implying that jumps arrive at a rate of about 5.6 per year, much more frequently than the rate of index jumps estimated by Broadie, Chernov, and Johannes (2008), 0.9 per year. Secondly, the θ_vs of individual stocks, which measures the volatility level of their returns, are also much larger than that of index, viz. about 4 versus 0.9. Both of these two evidences show that returns of individual stocks are much noisier than index. Besides, unlike index, which has average negative jumps in the returns series, individual stocks (except GE) mainly have jumps with positive means, but the jumps in individual stock returns are not significantly different from zero. It implies that individual stocks have approximately equal upward and downward jump risks.

4.2.2 Finite Sample Distribution via Monte Carlo Simulation

As that discussed in Section 3.1, expected call (put) option returns can be written in Equation (3.2). Note that expected call (put) returns are actually independent of S_t:

$$\begin{aligned} Call: E_t^P\left(r_{t,T}^c\right) &= \frac{E_t^P\left(\left(R_{t,T}-k\right)^+\right)}{E_t^Q\left(e^{-r_f T}\left(R_{t,T}-k\right)^+\right)} - 1 \\ Put: E_t^P\left(r_{t,T}^p\right) &= \frac{E_t^P\left(\left(k-R_{t,T}\right)^+\right)}{E_t^Q\left(e^{-r_f T}\left(k-R_{t,T}\right)^+\right)} - 1 \end{aligned} \tag{4.3}$$

where $k = K/S_t$ is the initial moneyness of the option, and $R_{t,T} = S_{t+T}/S_t$ is the stock returns. It implies that expected option returns depend only on the moneyness, maturity, interest rate, and the distribution of stock returns. This fact makes possible the analysis of option returns via Monte Carlo simulation.

- The estimation of $E_t^Q\left(e^{-r_f T}\left(R_{t,T}-k\right)^+\right)$ and $E_t^Q\left(e^{-r_f T}\left(k-R_{t,T}\right)^+\right)$:

Given risk-neutral parameters, $\theta_v^Q, \kappa_v^Q, \lambda^Q, \mu_z^Q$, and σ_z^Q, for each k and time t, one can simulate G (i.e. G=25,000) stock returns $R_{t,t+1}^{Q,(g)}$. The prices of options of moneyness k under $\mathbb{Q}$-measure at time t are then:

$$\begin{aligned} Call: E_t^Q\left(e^{-r_f T}\left(R_{t,t+1}-k\right)^+\right) &= \frac{1}{G}\sum\nolimits_{g=1}^{G} e^{-r_f T}\left(R_{t,t+1}^{Q,(g)}-k\right)^+ ; \\ Put: E_t^Q\left(e^{-r_f T}\left(k-R_{t,t+1}\right)^+\right) &= \frac{1}{G}\sum\nolimits_{g=1}^{G} e^{-r_f T}\left(k-R_{t,t+1}^{Q,(g)}\right)^+ . \end{aligned} \tag{4.4}$$

The choice of risk-neutral parameters will be discussed later.

- Average option returns: Following Broadie, Chernov, and Johannes (2008), to compute the finite sample distribution of various option return statistics, I simulate N=124 months (the sample length in the data) of index levels G=25,000 times using SV and SVJ models separately by standard simulation techniques. For each simulation path g and each month t, call (put) returns for a fixed moneyness are

$$Call: r_{t,t+1}^{c,(g)} = \frac{\left(R_{t,t+1}^{P,(g)} - k\right)^{+}}{E_t^Q\left(e^{-r_f}\left(R_{t,t+1} - k\right)^{+}\right)} - 1$$

$$Put: r_{t,t+1}^{p,(g)} = \frac{\left(k - R_{t,t+1}^{P,(g)}\right)^{+}}{E_t^Q\left(e^{-r_f}\left(k - R_{t,t+1}\right)^{+}\right)} - 1 \tag{4.5}$$

where $t = 1, \cdots, N$ and $g = 1, \cdots, G$. Average option returns for each simulation g using N months of simulation data are

$$Call: \bar{r}^{c,(g)} = \frac{1}{N}\sum_{t=1}^{N} r_{t,t+1}^{c,(g)}$$

$$Put: \bar{r}^{p,(g)} = \frac{1}{N}\sum_{t=1}^{N} r_{t,t+1}^{p,(g)} \tag{4.6}$$

Now we have got a set of G average returns, which forms the finite sample distribution. This is a parameter bootstrapping approach, providing exact finite sample inference under the null hypothesis that a given model holds.

Similarly, finite sample distributions for other statistics, such as CAPM alphas and Sharpe ratios can be constructed.

- CAPM alphas: Take call options as an example. For each simulation trial g, do the time series OLS regression:

$$Call: r_{t,t+1}^{c,(g)} - r_f = \alpha^{(g)} + \beta^{(g)}(R_{t,t+1}^{(g)} - r_f) \tag{4.7}$$

where $t = 1, \cdots, N$ and $g = 1, \cdots, G$. $\alpha^{(g)}$s compose a set of G CAPM alphas, forming the finite sample distribution.

- Sharpe ratios: Also take call options as an example. For each simulation trial g, there are time series of N month simulation call returns $r_{t,t+1}^{c,(g)}$. Sharpe rations for simulation g using N month of data are

$$SR^{(g)} = \frac{E\left[r_{t,t+1}^{c,(g)} - r_f\right]}{\sqrt{Var\left[r_{t,t+1}^{c,(g)} - r_f\right]}}, \tag{4.8}$$

The set of N $SR^{(g)}$ forms the finite sample distribution of Sharpe ratios.

Chapter Five: Results Analysis

5.1 Observed Average Option Returns

Table II and III record a variety of statistics for monthly call and put returns of five individual stocks, i.e. Microsoft (MSFT), CISCO Systems (CSCO), IBM, General Motors (GM), and General Electric (GE) over a 124-month period from January 1996 to April 2006. I record the mean, median, minimum, and maximum monthly hold-to-maturity returns for each of the nine groups of moneyness. The groups range from options with moneyness 0.75 to 1.15. The *t*-statistics and *p*-values associate with a null hypothesis of zero mean return is recorded in the second and third rows. I also report the skewness and kurtosis for each of the stock options and moneyness.

Looking at mean returns of call options in Table II, we see that most of them do earn positive average returns, except two OTM calls of CSCO, two ITM calls of GM, and two ITM calls of GE. Moreover, average returns are strictly increasing with the moneyness for calls of MSFT and IBM, while average call returns of CSCO, GM, and GE are almost monotonically increasing in moneyness. The ATM calls earn average returns of about 9 percent for MSFT, 10 percent for CSCO, 12 percent for IBM, 4 percent for GM, and 20 percent for GE. Call options that are 10% out of the money earn 11 percent more per month than those equivalently in the money for MSFT, 10% OTM calls of IBM earn 58 percent more per month than those 10% ITM;   and 10% OTM calls of GM earn 41 percent more per month than those 10% ITM;. CSCO and GE show exceptions. They report negative differences between their 10% OTM and 10% ITM calls respectively. Positive average call returns are not significantly different from zero for all five stocks, according to the *t*-statistics and *p*-values, while negative average call returns are almost yet significant. Nevertheless, we still acknowledge that call option returns are, as a whole, appear to be qualitatively consistent with Proposition 1. Not surprisingly, the median, minimum, maximum, skewness statistics demonstrate a substantial degree of positive skewness in the call returns, which is increasing in their moneyness.

Turning to put option returns in Table III, we see the results (except for GM) that are consistent with Proposition 2. Put options have returns that are almost statistically negative, and monotonically increasing in moneyness for MSFT, CSCO, IBM, and GE. Since the highest returns should be obtained by the deepest in-the-money puts, GE has a positive average put return at moneyness equal to 1.15, but it is not significantly different from zero. The exception occurs in the results of GM. All the average put returns are positive, and they do not show an increasing pattern with respect to moneyness. This phenomenon may be due to both mispricing and extreme shocks in the test period may account for this. Looking at the results of other four stocks, the ATM puts earn average return of about -26 percent for MSFT, -29 percent for CSCO, -10 percent for IBM, and -19 percent for GE. Put options that are 10% out of the money lose 57 percent more per month than those in the money for MSFT; 10% OTM puts of CSCO lose 46 percent more per month than those 10% ITM; 10% OTM puts of IBM lose 24 percent more per month than those 10% ITM; and 10% OTM puts of GE lose 59 percent more per month than those 10% ITM. Again, as expected, the median, minimum, maximum, and skewness statistics indicate that put returns exhibit substantial positive skewness.

In order to have a clearer perspective, Figure I and II shows the time series for 10% OTM, ATM, and 10% ITM call and put returns of MSFT, highlighting some of the issues present when evaluating the statistics generated by option returns in Table II and III. OTM call returns have infrequent but very large values and many repeated values which are -100%, exhibiting a highly positive skewness pattern. Along with increases in moneyness, positive returns in time series become more frequent and the magnitudes are much smaller. Compared to OTM calls, OTM puts are even more extreme, with much less positive values. Besides, the magnitudes of positive returns of OTM puts are half the size of OTM calls. As moneyness is increasing, put returns also experience more positive values. However, like OTM options, puts are always more positively skewed than calls at the same level of moneyness. This is not a surprising fact, because Proposition 1 and 2 guarantee that calls should have positive expected returns, while puts should have negative ones.

The CAPM alphas for calls and puts are also reported in Table II and III. Beginning with Table II, only the 25% ITM calls of IBM and GM have significant positive CAPM alphas. Except for this, the results for calls are all insignificantly from zero. It seems to be consistent with the assumption of CAPM model, indicating that volatility and jump risks may be not priced in individual stock calls. On the other hand, puts exhibit a different picture (see Table III). MSFT, CSCO, IBM, and GE all have significant negative CAPM alphas in their OTM puts (GM is still an exception). Considering that OTM puts are the options that most likely to be priced with volatility and jump risks, this result is not inconsistent. Puzzling CAPM alphas are those for ITM puts, which are significantly positive for the 10% and 15% ITM puts of IBM and GE. Explanation may be that they carry positive volatility or jump risk premium.

Sharpe ratios for calls are positive but small, about 0.04 ~ 0.10. Puts, on the contrary, have negative Sharpe ratios, and the magnitude is very large, about -1.5 ~ -0.15. The literature mainly concludes that put returns are puzzling and likely to be mispriced. However, since put option returns are highly skewed, I argue that Sharpe ratio may not be an appropriate metric for mispricing.

5.2 Finite Sample Distribution of Individual Options

Broadie, Chernov, and Johannes (2008) perform the finite sample distribution analysis using the simplest option pricing models, the Black-Scholes and stochastic-volatility models. However, since it is well known that Black-Scholes model is too simplistic characterize the dynamics of option prices, I omit the analysis by Black-Scholes model, but focus on SV and SVJ models.

5.2.1 Stochastic-volatility Model

Following the steps described in 4.2.2, I assume there is only equity premium priced in options. That is $\theta_v^Q = \theta_v^P$, implying that there is not a diffusive volatility premium. Table IV and V provides population average returns, CAPM alphas, and Sharpe ratios for SV model, as well as *p*-values. Here, the *p*-values are different from those in Table II and III. They do not support the null hypothesis that option returns equal zero, but the null hypothesis at values generated by the option pricing models.

Firstly, look at the finite sample distribution of call option returns in Table IV. The means of the simulation returns can be thought as the theoretical returns by SV model. Note that these returns increase with moneyness and are all positive values, consistent with Proposition 1. Another phenomenon is that for four out of the five stocks, the observed returns of calls whose moneynesses are close to being at-the-money, are significantly smaller than the average simulated returns (i.e. 5% OTM, ATM, and 5% ITM calls of MSFT and CSCO, 5% OTM and ATM calls of IBM and GE). Other significant results occur in deep ITM calls (i.e. 25% ITM calls of MSFT, CSCO and IBM, 25% and 20% ITM calls of GE). This result seems to be puzzling, because the common perception is that OTM options, either calls or puts, are most easily to be mispriced. However, Broadie, Chernov, and Johannes (2008) shows that it does not necessary to claim that OTM options are not mispriced. They provide simulation results to explain that option returns, especially OTM options, are very sensitive to equity premium μ and volatilities. With regard to this fact, the insignificance of OTM calls may result from the large sampling uncertainty in the distribution of average option returns. Nevertheless, through my results, there is no evidence that OTM call returns are inconsistent with SV model.

Next, consider CAPM alphas and Sharpe ratios for call options of each stock, which are also reported in Table IV. Nonetheless, they do not shine much light on the analysis, still showing insignificant results for ATM calls. Except for that, we do not see much orderlyness of CAPM alphas of the finite sample distributions along with moneynesses: CAPM alphas are positive or negative at random moneyness levels. As shown in Broadie, Chernov, and Johannes (2008), when applying linear factor models (CAPM model) to nonlinear option returns, even one single extreme observation will impact the estimation of CAPM alphas a lot[1], resulting in problems when regarding CAPM alphas as metrics. On a whole, the magnitudes of Sharpe ratios of the finite sample distributions are larger than that observed. And also, the magnitudes of Sharpe ratios for SVJ models are larger than those for SV models.

Turn to put options in Table V. Consistent with Proposition 2, the mean returns simulated by SV model are all negative and tend to increase with moneyness. The significant results only happen to 10%, 5% OTM and ATM puts of CSCO, and 25% OTM puts of GE (25%, 20%, 15% OTM puts of GM are significant because the observed returns are all positive, as discussed in section 5.1). It may imply that put returns for individual stocks are consistent with SV model, or that the sampling

[1] It means that even one single large observation can substantially shifts the intercept from negative to positive, vice versa.

uncertainty generated by changing volatility significantly increases p-value, resulting in the indiscernible mispricing puzzle. When looking at CAPM alphas and Sharpe ratios, we see that the conclusions are similar, providing no more information over average returns. However, one thing should be noticed is that the magnitude of Sharpe ratios is extremely high for the deepest OTM (25% OTM) puts, at around 600. This happens because the returns of deep OTM puts are highly skewed: almost all the values are -100%. Hence, the variance of returns is usually very small, resulting in such a large Sharpe ratio. Nonetheless, the very small variance is also so sensitive to even one large positive shock that once it occurs, the magnitude of Sharpe ratio will accordingly experience a considerable drop. For this reason, Sharpe ratio is not a good metric for deep OTM puts.

5.2.2 Stochastic-volatility Jump-diffusion Model

Next, consider the SVJ model. I assume that there is neither a diffusive volatility premium nor a jump risk premium: $\theta_v^Q = \theta_v^P$, $\lambda_z^Q = \lambda_z^P$, $\mu_z^Q = \mu_z^P$, and $\sigma_z^Q = \sigma_z^P$. Population average returns, CAPM alphas, and Sharpe ratios for SVJ model, as well as p-values are shown in Table IV and V.

Begin with call options. Expected call returns are lower (almost) in the SVJ model than in the SV model, especially for OTM options. This is due to the fact that expected returns are a concave function of volatility, which implies that the more the volatility fluctuates, the lower the expected returns. Considering that jumps can cause fluctuations in stochastic volatilities, the expected returns of SVJ model are, theoretically, lower than that of SV model. Nonetheless, we can see from Table IV that this difference is not very significant. It may be because returns of individual stocks are much noisier, compared with index futures. As we see in Table I, the jump frequencies λs are considerably high, and the means of jumps sizes are not significantly from zero, indicating that SVJ model doesn't improve SV model very much in characterizing the dynamics of individual stocks. The *p*-values in SVJ model show that significant results happen in 5% OTM, ATM, and 25% ITM calls of MSFT, nearly all calls of CSCO, 5% OTM, 25%, 20%, and 15% ITM calls of IBM, 15% and 10% ITM calls of GM, and 25%, 20% ITM calls of GE. This is a similar result with that of SV model.

We turn to put options, next. More obviously than calls, expected put returns for SVJ model are more negative than for SV model at all moneyness levels. *p*-values shows that 25% OTM puts of MSFT, 25% and 20% OTM, ATM, and 5% and 10% ITM puts of IBM, and 25% OTM puts of GE (ignoring results of GM, because the observed abnormal returns, as discussed in section 5.1) have significant results. Among these results, IBM's tell us that the expected put returns of SVJ model may be so negative that observed average returns are actually significantly larger than the expected ones by SVJ.

As discussed in section 5.2.1, CAPM alphas and Sharpe ratios for both calls and puts do not provide more information about option mispricing.

In summary, the analysis in section 5.2 presents three results. Firstly, CAPM alphas and Sharpe ratios are generally noisier than mean returns, indicating that they are not more informative statistics than mean returns. Secondly, SVJ model does not improve SV model a lot for individual stocks, although it introduces more flexible volatilities in dynamics. Third, sampling uncertainty is substantial for both call and put returns, since the returns for many of the strikes are statistically insignificant. While it is well known that SV and SVJ models are not perfect specifications for stocks, it makes us think that average raw option returns are so noisy that little can be said about option mispricing. Broadie, Chernov, and Johannes (2008) and Coval and Shumway (2003) suggest that tests using option portfolio may be much more informative.

5.3 Finite Sample Distribution of Option Portfolios

This section explores the performance of option portfolios in finite sample distribution. I consider a variety of portfolios including covered puts, which consist of a long put position combined with a long position in the underlying index; ATM straddles, which consist of a long position in an ATM put and an ATM call; crash-neutral straddles, which consist of a long position in ATM straddle combined with a short position in one unit of 10% OTM put; and put spreads (also known as a crash-neutral puts), which consist of a long position in an ATM put and a short position in a 10% OTM put.[2] Since a large part of the variation in average option returns is driven by the underlying assets,

[2] In the discussion in Broadie, Chernov, and Johannes (2008), they also include delta-hedged puts, which consist of a long put position combined with a position in Black-Scholes delta unites of the underlying asset. However, they also point out the shortcomings of analyzing delta-hedged returns, so I omit this analysis in my thesis.

the above mentioned portfolios mitigate the impact of the level of the stocks or the tail behavior of the stocks. Table VI reports the population average returns as well as *p*-values for both SV model and SVJ model. For each option portfolio, I only focus on the returns to the long side, in order to consistent with earlier results. As shown in section 5.3, CAPM alphas and Sharpe ratios do not add new information, so they are not reported.

Table VI shows that the average returns on the covered put positions and put spread positions are not significant for all the stocks and moneyness levels. Note that the *p*-values of the ATM straddle returns for MSFT and CSCO is quite small. Thus MSFT and CSCO both have significantly different ATM straddle returns from those simulated by SV and SVJ models. The observed average ATM straddle return of MSFT is -6.05% per month, while finite sample distributions report returns around 11% per month. For CSCO, the observed average ATM straddle return is -7.79% per month, while finite sample distributions report returns around 33% per month (SV) or 18% per month (SVJ). Although SVJ model estimates show expected ATM straddle returns to be much lower than SV model, the difference between 18% and -7.79% still indicates a significant result (*p*-value is 0.27%). Unlike MSFT and CSCO, the ATM straddle of GM has its observed average returns significantly higher than those in finite sample distributions. Again, we say the GM case has shown abnormal in the test data sample that it is difficult to interpret the GM results. The observed average ATM straddle returns for IBM and GE are not significantly different from those in finite sample distributions, and comparatively much closer to those of SVJ model than SV model.

The crash-neutral straddles are considered in the analysis because their return during a market crash is limited to some level (10% loss in my thesis) that is specified when the position is created. In this way, measures of the position's expected returns are not downward-biased by infrequent crash observations or high-priced crash risk. Looking at the results in Table VI, MSFT still earns a negative return -1.26%, CSCO earns 0.91%, IBM earns 7.88%, and GE earns 10.31%. Like ATM straddles, the significant results also happen to MSFT and CSCO, which implies that volatility risks are priced in the options of these two stocks.

Recalling the analysis of S&P 500 index options by Broadie, Chernov, and Johannes (2008), we see both ATM straddles and crash-neutral straddles lost on average significantly compared with the positive mean returns of finite sample distributions. Therefore, it seems that only the options of MSFT and CSCO have similar characteristics with the index's. Noting that MSFT and CSCO are two individual stocks, whose options are trading the most actively, we may explain this phenomenon by Garleanu, Pedersen, and Poteshman (2007). They argue that in the real world options cannot be perfectly hedged. Consequently, if intermediaries such as market makers and proprietary traders who take the other side of end-user option demand[3] are risk-averse, end-user demand for options will impact option prices. In particular, options are overall

[3] They compute net end-user demand for an option in this way. They assume that firm proprietary traders are end users and compute the net demand for an option as the sum of the public customer and firm proprietary trader short open interest. Net demand computed in this way is referred to as non-market-maker net demand.

more expensive when there are more end-user demands for options and that the expensiveness skew across moneyness is positively related to skewness in end-user demand across moneyness. Empirical evidence is that on average index options are quite expensive by the measure of implied volatilities, and that they have high positive end-user demand. On the contrary, equity options are on average slightly inexpensive and have a small negative end-user demand. As MSFT and CSCO's options are traded most actively in the market compared with other equity options, it makes us think whether they have possessed similar characters with index options, such as positive end-user demand. Further research should analyze more equity stocks, and to test whether this guess is true.

Chapter Six: Conclusions

6.1 Summary of the Results

In this thesis, I mainly use the methodology of Broadie, Chernov, and Johannes (2008) to explore whether 'mispricing' problems exist in individual stock options. Recognizing that simply looking at average option returns, CAPM alphas, or Sharpe ratios is problematic for the analysis of highly skewed option returns, I rely on standard option-pricing models (i.e. SV and SVJ models) to compute analytical expected option returns and to construct finite sample distributions of average option returns using Monte Carlo simulation. By investigating whether historical statistics are significant in the finite sample distributions, I can find out if these models are too simple to provide accurate descriptions of option prices.

Theoretically, I cite and verify the propositions in Coval and Shumway (2001): calls earn positive returns while puts' returns are mainly negative. Meanwhile, the returns of both calls and puts tend to increase with moneyness levels. As to non-zero CAPM alphas, they can only hold in instantaneous Black-Scholes model, but may be inconsistent for discrete-time BS models, as well as in complicated models such as SV and SVJ. In particular, OTM puts, which are usually priced with volatility risk premium, will consequently see negative CAPM alphas. Moreover, general standard of Sharpe ratios is also inapplicable for option returns, for the option returns show a highly non-normal pattern.

Empirically, I present a number of interesting findings. First, I find that calls do have positive expected returns and puts have negative expected returns. Moreover, the expected returns of both calls and puts are increasing as the strike price increases. These results are consistent with proposition 1 and 2. Second, although CAPM alphas and Sharpe ratios seem reasonable for call options, they are too negative for OTM put options. Nevertheless, as the analysis in the theory shows there is no convincing evidence to prove OTM puts for individual stocks are overpriced. The third finding is that the finite sample distributions simulated by SV and SVJ models do not likely provide much information on mispricing of sole calls and sole puts. Specifically, average returns, CAPM alphas, and Sharpe ratios for options are statistically insignificant when compared to the SV and SVJ models. However, Broadie, Chernov, and Johannes (2008) claim that these finding should not be interpret as evidence that SV or SVJ models are correct, but as highlighting the statistical difficulties present when analyzing option returns. With regard to this, fourthly, I use the similar method to test option portfolios. I find that only the most actively traded options (MSFT and CSCO) exhibit similar volatility risk premiums in their straddle prices. One explanation may be that actively traded equity options are more demanded by end users while other equity options face negative end user demands. According to the findings of Garleanu, Pedersen, and Poteshman (2007), positive demands cause the option prices to look expensive.

6.2 Limitation of the Study

Limitations of the thesis are listed below:

1. Because of the limitation of time and computer speed, this thesis only focuses on five stocks' options. As this thesis is discussing individual stock options, the results of five options are apparently just exploratory.

2. In the thesis, the time series is from January 1996 to April 2006, total 124 months. Large crashes, such as that in October 1987, are not included in the sample. Moreover, considering missing values, sometimes less than 124 data are analyzed. Therefore, the investigation may be affected by the Peso problem.

6.3 Direction for Future Research

The future study can be extended in the following areas:

1. Do similar tests on more individual stock options, to see whether the results are consistent with those found in this thesis.

2. In this thesis, I assume the evolution of volatility under the real-world $\mathbb{P}$ and the risk-neutral $\mathbb{Q}$ measures are the same, which means there is no diffusive stochastic volatility risk premium. Thus, to further explore the risk premiums, one can correct parameters of $\mathbb{Q}$ measures to make the finite sample distributions fit the observed statistics, and investigate the gap between $\mathbb{P}$ and $\mathbb{Q}$.

3. As long as the trading volume data of options are available, the empirical study of the relation between option demands and option prices should be conducted.

Table I

$\mathbb{P}$-measure Parameters of Stocks

This table reports parameter values that I will use in constructing the finite sample distributions. All the parameters are estimated by MCMC method. Standard errors for the estimation are also reported.

	μ	λ	μ_z	σ_z	θ_v	κ_v	σ_v	ρ
Panel A: MICROSOFT COR. (MSFT)								
SV	0.0879	.	.	.	4.1218	0.0124	0.2695	-0.0418
(std)	.	.	.	.	(28.866)	(0.0042)	(0.0381)	(0.0697)
SVJ	0.0879	0.0209	0.2956	6.0819	3.8493	0.0032	0.1325	-0.2263
(std)	.	(0.0063)	(1.0155)	(0.9797)	(51.4495)	(0.0019)	(0.0145)	(0.1101)
Panel B: CISCO SYSTEMS INC. (CSCO)								
SV	0.1119	.	.	.	10.773	0.0061	0.2550	-0.4751
(std)	.	.	.	.	(62.180)	(0.0042)	(0.0267)	(0.0772)
SVJ	0.1119	0.0161	1.2293	8.0194	7.2941	0.0037	0.1813	-0.4724
(std)	.	(0.0052)	(1.6277)	(1.3745)	(94.411)	(0.0017)	(0.0188)	(0.0786)
Panel C: IBM COR. (IBM)								
SV	0.0750	.	.	.	3.9678	0.0099	0.2379	-0.4475
(std)	.	.	.	.	(4.4781)	(0.0036)	(0.0267)	(0.0620)
SVJ	0.0750	0.0300	0.7730	5.8477	2.7144	0.0045	0.1347	-0.5140
(std)	.	(0.0062)	(0.7774)	(0.6541)	(67.014)	(0.0021)	(0.0159)	(0.0836)
Panel D: GENERAL MOTORS COR. (GM)								
SV	0.0161	.	.	.	3.8227	0.0215	0.2839	-0.2892
(std)	.	.	.	.	(0.5469)	(0.0055)	(0.0304)	(0.0752)
SVJ	0.0161	0.0318	1.1263	4.8599	3.2117	0.0125	0.1809	-0.2761
(std)	.	(0.0097)	(0.7233)	(0.6814)	(0.6828)	(0.0036)	(0.0166)	(0.0993)
Panel E: GENERAL ELECTRIC COR. (GE)								
SV	0.0654	.	.	.	3.2209	0.0129	0.2079	-0.5897
(std)	.	.	.	.	(0.6043)	(0.0034)	(0.0194)	(0.0525)
SVJ	0.0654	0.0123	-0.2327	4.7237	2.8930	0.0063	0.1449	-0.6058
(std)	.	(0.0052)	(1.3815)	(1.1826)	(7.0523)	(0.0025)	(0.0196)	(0.0672)

Table II

Call Option Returns

This table reports summary statistics for call option returns of the five stocks discussed in this thesis. The sample period is from January 1996 to April 2006 (124 months). Mean return denotes the average of one-month hold-to-maturity return. CAPM alpha and Sharpe ratio are statistics described in section 3.2.
***, **, * denote significance level of 1%, 5%, and 10% respectively.

Moneyness	0.75	0.80	0.85	0.90	0.95	1.00	1.05	1.10	1.15
Panel A: MICROSOFT COR. (MSFT)									
Mean Return	0.02	0.07	0.08	0.09	0.09	0.09	0.13	0.20	0.88
t-Statistic	0.48	1.57	1.54	1.32	0.96	0.66	0.57	0.53	0.85
p-value,%	62.92	11.92	12.58	18.83	33.71	51.33	56.71	59.45	39.66
Median	-0.00	0.03	0.03	0.02	-0.15	-0.71	-1.00	-1.00	-1.00
Minimum	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00
Maximum	0.95	1.81	2.09	2.90	4.28	6.88	12.47	24.10	85.42
Skew	-0.26	0.27	0.44	0.73	1.13	1.83	2.99	4.49	7.08
Kurt	0.80	0.92	0.48	0.72	1.42	3.71	9.90	21.10	54.33
CAPM α	0.00	0.01	0.00	-0.02	-0.06	-0.12	-0.18	-0.20	0.20
t-Statistic	0.05	0.63	0.09	-0.80	-1.55	-1.61	-1.12	-0.65	0.20
p-value,%	95.75	52.92	93.01	42.45	12.39	11.02	26.61	51.80	84.19
Sharpe ratio	0.04	0.14	0.14	0.11	0.08	0.06	0.05	0.05	0.09
Panel B: CISCO SYSTEMS INC. (CSCO)									
Mean Return	0.00	0.03	0.05	0.06	0.09	0.10	0.08	-0.06	-0.19***
t-Statistic	0.09	0.48	0.83	0.76	0.87	0.76	0.44	-0.26	-0.67
p-value,%	92.62	63.16	40.56	44.90	38.55	44.66	65.87	79.15	0.50
Median	-0.01	-0.02	-0.04	-0.04	-0.15	-0.79	-1.00	-1.00	-1.00
Minimum	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00
Maximum	1.22	1.75	2.53	2.29	3.31	5.07	9.39	13.00	18.46
Skew	-0.02	0.31	0.59	0.63	0.91	1.42	2.39	3.31	4.68
Kurt	-0.11	0.16	0.38	-0.30	-0.03	1.25	5.81	11.02	23.11
CAPM α	-0.01	-0.01	-0.01	-0.03	-0.03	-0.04	-0.10	-0.24	-0.34
t-Statistic	-0.94	-1.35	-0.85	-1.04	-0.60	-0.56	-0.76	-1.27	-1.36
p-value,%	34.90	17.89	39.60	30.02	55.00	57.89	44.70	20.53	17.67
Sharpe ratio	0.00	0.04	0.07	0.07	0.08	0.07	0.04	-0.03	-0.06

Table II (continued)

Moneyness	0.75	0.80	0.85	0.90	0.95	1.00	1.05	1.10	1.15
Panel C: IBM COR. (IBM)									
Mean Return	0.02	0.03	0.03	0.06	0.07	0.12	0.19	0.64	1.09
t-Statistic	0.42	0.65	0.64	0.91	0.81	0.80	0.76	1.09	0.66
p-value,%	67.48	51.99	52.59	36.25	42.09	42.63	45.11	27.98	51.35
Median	0.04	0.05	0.03	0.04	-0.08	-0.72	-1.00	-1.00	-1.00
Minimum	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00
Maximum	0.95	2.02	2.20	2.94	4.31	7.69	16.65	50.04	149.23
Skew	-0.44	0.30	0.33	0.69	1.09	1.98	3.42	5.71	9.36
Kurt	0.38	1.57	0.70	0.85	1.49	4.75	13.51	37.93	88.61
CAPM α	0.01**	0.00	-0.00	0.00	-0.01	-0.02	-0.01	0.31	0.57
t-Statistic	1.99	0.55	-0.24	0.24	-0.34	-0.21	-0.07	0.61	0.36
p-value,%	4.91	58.59	80.73	81.31	73.12	83.46	94.19	54.49	71.80
Sharpe ratio	0.04	0.06	0.05	0.08	0.07	0.07	0.07	0.10	0.07
Panel D: GENERAL MOTORS COR. (GM)									
Mean Return	0.02	0.01	-0.03***	-0.05***	0.01	0.04	0.25	0.36	0.64
t-Statistic	0.28	0.21	-0.51	-0.64	0.14	0.29	0.81	0.68	0.57
p-value,%	78.37	83.11	0.61	0.53	88.85	77.59	41.69	49.96	57.05
Median	0.09	0.05	0.03	-0.11	-0.18	-0.97	-1.00	-1.00	-1.00
Minimum	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00
Maximum	1.01	1.17	1.38	1.94	3.23	7.36	27.16	51.11	99.34
Skew	-0.24	0.11	0.23	0.62	1.09	2.03	5.09	6.68	8.62
Kurt	0.16	-0.19	-0.22	-0.23	0.66	4.49	34.14	51.99	78.19
CAPM α	0.03***	0.03*	0.01	0.00	0.04	0.07	0.28	0.41	0.58
t-Statistic	2.71	1.92	0.40	0.15	0.77	0.74	1.12	0.84	0.53
p-value,%	0.87	5.81	69.02	87.72	44.06	46.03	26.69	40.29	59.76
Sharpe ratio	0.03	0.02	-0.05	-0.06	0.01	0.02	0.07	0.06	0.06

Table II (continued)

Moneyness	0.75	0.80	0.85	0.90	0.95	1.00	1.05	1.10	1.15
				Panel E: GENERAL ELECTRIC COR. (GE)					
Mean Return	-0.01***	-0.01***	0.03	0.05	0.06	0.20	0.70	-0.11	0.08
t-Statistic	-0.12	-0.33	0.62	0.81	0.72	1.35	1.41	-0.32	0.12
p-value,%	0.90	0.74	53.51	42.21	47.27	18.02	16.03	74.89	90.27
Median	-0.01	-0.03	0.01	-0.01	-0.08	-0.70	-1.00	-1.00	-1.00
Minimum	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00
Maximum	0.73	0.87	1.34	1.84	2.82	8.62	51.95	27.38	36.62
Skew	-0.62	-0.33	-0.01	0.25	0.64	1.84	7.01	5.75	5.94
Kurt	0.55	-0.01	-0.34	-0.49	-0.21	4.80	60.50	38.47	37.78
CAPM α	0.01	0.00	0.02	-0.00	-0.02	0.08	0.49	-0.29	-0.05
t-Statistic	0.97	0.21	1.50	-0.02	-0.64	0.77	1.03	-0.88	-0.09
p-value,%	33.67	83.69	13.74	98.51	52.47	44.50	30.69	37.95	93.13
Sharpe ratio	-0.02	-0.04	0.05	0.07	0.06	0.12	0.13	-0.03	0.01

Table III

Put Option Returns

This table reports summary statistics for call option returns of the five stocks discussed in this thesis. The sample period is from January 1996 to April 2006 (124 months). Mean return denotes the average of one-month hold-to-maturity return. CAPM alpha and Sharpe ratio are statistics described in section 3.2.
***, **, * denote significance level of 1%, 5%, and 10% respectively.

Moneyness	0.75	0.80	0.85	0.90	0.95	1.00	1.05	1.10	1.15
Panel A: MICROSOFT COR. (MSFT)									
Mean Return	-0.93***	-0.91***	-0.82***	-0.76***	-0.62***	-0.26**	-0.19**	-0.12*	-0.09
t-Statistic	-14.16	-10.10	-7.60	-6.83	-5.91	-2.29	-2.37	-1.87	-1.54
p-value,%	0.00	0.00	0.00	0.00	0.00	2.38	1.94	6.36	12.75
Median	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-0.51	-1.17	-0.09
Minimum	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00
Maximum	4.74	8.19	9.56	9.18	6.70	7.67	3.31	2.42	1.85
Skew	9.33	10.10	7.75	6.10	3.94	2.90	1.33	0.69	0.43
Kurt	81.00	102.00	64.57	41.15	17.32	11.71	1.95	0.52	0.38
CAPM α	-0.90***	-0.85***	-0.73***	-0.64***	-0.47***	-0.08	-0.03	0.01	0.02
t-Statistic	-13.83	-9.47	-7.15	-6.27	-5.56	-0.98	-0.82	0.39	1.41
p-value,%	0.00	0.00	0.00	0.00	0.15	32.95	41.42	70.05	16.03
Sharpe ratio	-1.52	-1.00	-0.72	-0.62	-0.54	-0.21	-0.22	-0.17	-0.15
Panel B: CISCO SYSTEMS INC. (CSCO)									
Mean Return	-0.89***	-0.83***	-0.67***	-0.59***	-0.44***	-0.29***	-0.20**	-0.13*	-0.09
t-Statistic	-14.37	-8.98	-5.17	-4.72	-3.69	-2.82	-2.28	-1.81	-1.51
p-value,%	0.00	0.00	0.00	0.00	0.03	0.56	2.45	7.20	13.43
Median	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-0.47	-0.18	-0.07
Minimum	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00
Maximum	3.66	5.83	8.89	7.83	6.25	4.24	2.73	1.96	1.61
Skew	6.13	6.08	4.85	4.13	2.81	1.84	1.16	0.73	0.46
Kurt	38.87	37.35	24.28	17.95	8.03	2.80	0.60	-0.20	-0.34
CAPM α	-0.88***	-0.77***	-0.57***	-0.48***	-0.31***	-0.16**	-0.07	-0.02	-0.01
t-Statistic	-15.06	-8.98	-4.86	-4.46	-3.32	-2.27	-1.46	-0.72	-0.44
p-value,%	0.00	0.00	0.00	0.00	0.12	2.52	14.69	47.00	66.11
Sharpe ratio	-1.46	-0.85	-0.47	-0.43	-0.33	-0.26	-0.21	-0.17	-0.15

Table III (continued)

Moneyness	0.75	0.80	0.85	0.90	0.95	1.00	1.05	1.10	1.15
Panel C: IBM COR. (IBM)									
Mean Return	-1.00***	-0.83***	-0.53***	-0.27	-0.17	-0.10	-0.08***	-0.03	-0.02
t-Statistic	.	-6.31	-2.63	-1.20	-0.79	-0.68	-0.81	-0.50	-0.43
p-value,%	.	0.00	0.96	23.35	43.09	49.75	0.42	61.52	67.00
Median	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-0.42	-0.11	-0.06
Minimum	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00
Maximum	-1.00	11.72	12.64	13.93	15.35	5.62	3.02	2.17	1.61
Skew	.	9.00	4.91	3.82	3.70	1.90	1.24	0.78	0.41
Kurt	.	84.36	23.96	14.96	16.62	2.74	0.82	0.15	0.09
CAPM α	-1.00***	-0.78***	-0.40**	-0.06	0.02	0.07	0.05	0.05**	0.05***
t-Statistic	-6359.1	-5.95	-2.17	-0.34	0.13	0.74	1.12	2.02	2.93
p-value,%	0.00	0.00	3.25	73.52	89.41	45.79	26.39	4.57	0.42
Sharpe ratio	-680.72	-0.63	-0.24	-0.11	-0.07	-0.06	-0.08	-0.05	-0.04
Panel D: GENERAL MOTORS COR. (GM)									
Mean Return	0.04	0.17	0.25	0.04	0.05	0.03	0.01	0.03	0.02
t-Statistic	0.04	0.23	0.33	0.08	0.18	0.19	0.13	0.40	0.23
p-value,%	96.55	81.87	74.43	93.37	85.81	85.11	89.97	69.01	0.82
Median	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-0.33	-0.09	-0.11
Minimum	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00
Maximum	57.47	48.17	63.34	39.67	27.72	11.80	4.97	3.05	2.13
Skew	7.80	6.26	7.19	6.63	6.11	3.15	1.63	0.81	0.69
Kurt	62.08	39.35	53.21	49.28	47.00	13.74	3.37	0.43	0.36
CAPM α	0.19	0.27	0.22	-0.03	0.08	0.05	0.03	0.05*	0.04**
t-Statistic	0.22	0.39	0.31	-0.08	0.32	0.45	0.67	1.97	2.05
p-value,%	82.27	69.59	75.34	93.83	74.89	65.21	50.60	5.16	4.34
Sharpe ratio	0.01	0.02	0.03	0.01	0.02	0.02	0.01	0.03	0.02

Table III (continued)

Moneyness	0.75	0.80	0.85	0.90	0.95	1.00	1.05	1.10	1.15
Panel E: GENERAL ELECTRIC COR. (GE)									
Mean Return	-1.00***	-0.85***	-0.83***	-0.64***	-0.37**	-0.19	-0.08	-0.05	0.04
t-Statistic	.	-5.91	-5.76	-4.75	-2.51	-1.60	-0.85	-0.74	0.62
p-value,%	.	0.00	0.00	0.00	1.35	11.26	39.66	46.12	53.44
Median	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-0.30	-0.04	0.06
Minimum	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00
Maximum	-1.00	11.89	14.51	10.91	7.63	4.80	3.13	2.17	1.57
Skew	.	9.43	9.82	5.59	2.75	1.62	0.95	0.50	0.11
Kurt	.	89.00	99.51	26.49	7.14	1.78	0.04	-0.28	-0.33
CAPM α	-1.00***	-0.83***	-0.78***	-0.54***	-0.22**	-0.04	0.05	0.06***	0.08***
t-Statistic	-5168.6	-5.83	-5.59	-4.61	-1.98	-0.60	1.35	2.95	5.35
p-value,%	0.00	0.00	0.00	0.00	4.97	55.08	17.93	0.39	0.00
Sharpe ratio	-635.39	-0.63	-0.55	-0.44	-0.23	-0.15	-0.08	-0.07	0.06

Table IV

Finite Sample Distribution of Call Option Returns

This table reports population expected option returns, CAPM alphas, and Sharpe ratios, as well as finite sample distribution p-values for the stochastic volatility and stochastic volatility jump diffusion models. I assume that all risk premium (expect for the equity premium) are equal to zero.
***, **, * denote significance level of 1%, 5%, and 10% respectively.

		Moneyness	0.75	0.80	0.85	0.90	0.95	1.00	1.05	1.10	1.15
Panel A: MICROSOFT COR. (MSFT)											
Mean Return			0.02	0.07	0.08	0.09	0.09	0.09	0.13	0.20	0.88
	SV	E^P	0.09*	0.10	0.13	0.16	0.22**	0.41*	0.90*	1.63	1.66
		p-value, %	8.14	49.75	45.25	36.86	2.19	8.16	6.43	16.81	70.36
	SVJ	E^P	0.09*	0.11	0.13	0.17	0.23	0.42*	0.90*	1.54	1.41
		p-value, %	5.65	40.48	37.32	29.42	17.34	6.53	5.80	16.98	77.94
CAPM α			0.00	0.01	0.00	-0.02	-0.06	-0.12	-0.18	-0.20	0.20
	SV	E^P	-0.00	-0.00**	-0.01	-0.02	-0.03	0.03*	0.24*	0.47	0.21
		p-value, %	35.00	3.07	12.13	69.35	37.57	9.32	8.47	24.11	98.99
	SVJ	E^P	-0.00	-0.00**	-0.01*	-0.02	-0.03	0.02	0.21*	0.39	0.53
		p-value, %	27.95	1.62	6.80	58.65	42.42	10.72	9.25	27.19	86.80
Sharpe ratio			0.04	0.14	0.14	0.11	0.08	0.06	0.05	0.05	0.09
	SV	E^P	0.04*	0.14	0.14	0.11	0.08	0.06**	0.05**	0.05*	0.09
		p-value, %	7.66	45.12	49.00	40.98	22.96	6.38	2.81	9.57	65.90
	SVJ	E^P	0.23*	0.23	0.21	0.20	0.19	0.20**	0.19**	0.14	0.03
		p-value, %	5.28	34.98	38.86	32.16	17.44	4.70	2.27	10.10	66.78
Panel B: CISCO SYSTEMS INC. (CSCO)											
Mean Return			0.00	0.03	0.05	0.06	0.09	0.10	0.08	-0.06	-0.19
	SV	E^P	0.13**	0.15*	0.19*	0.27**	0.43**	0.75***	1.27***	2.06***	4.82
		p-value, %	3.90	5.98	8.37	4.19	1.50	0.42	0.24	0.71	10.47
	SVJ	E^P	0.13**	0.15**	0.17*	0.22*	0.33**	0.56**	0.87**	1.29**	2.81
		p-value, %	1.64	3.77	8.27	6.68	4.5	1.82	1.27	2.18	12.27
CAPM α			-0.01	-0.01	-0.01	-0.03	-0.03	-0.04	-0.10	-0.24	-0.34
	SV	E^P	-0.01	-0.01	0.00	0.03	0.11**	0.29***	0.57***	0.97***	2.44
		p-value, %	99.65	56.62	46.17	11.71	2.12	0.36	0.27	0.86	11.28

Table IV (continued)

	Moneyness	0.75	0.80	0.85	0.90	0.95	1.00	1.05	1.10	1.15
					Panel B: CISCO SYSTEMS INC. (CSCO) (continued)					
SVJ	E^P	-0.01*	-0.02	-0.03	-0.03	-0.01	0.07	0.17	0.27	0.84
	p-value, %	8.96	32.10	24.31	72.62	69.96	22.83	12.06	10.49	22.34
Sharpe ratio		0.00	0.04	0.07	0.07	0.08	0.07	0.04	-0.03	-0.06
SV	E^P	0.21**	0.21*	0.21	0.23**	0.26**	0.29***	0.29***	0.25***	0.19***
	p-value, %	3.72	6.60	10.11	4.71	1.42	0.19	0.02	0.01	0.02
SVJ	E^P	0.24**	0.23**	0.22*	0.22*	0.23**	0.25**	0.24***	0.20***	0.15***
	p-value, %	1.63	3.70	8.25	6.64	4.17	1.02	0.27	0.16	0.53
					Panel C: IBM COR. (IBM)					
Mean Return		0.02	0.03	0.03	0.06	0.07	0.12	0.19	0.64	1.09
SV	E^P	0.09*	0.10	0.11	0.14	0.22	0.44*	1.22**	3.90	2.90
	p-value, %	6.91	14.50	17.00	24.94	15.69	5.68	3.17	18.32	60.09
SVJ	E^P	0.10**	0.11**	0.13*	0.16	0.21	0.37	0.93*	2.69	1.45
	p-value, %	1.02	3.08	5.01	13.46	14.67	10.94	7.49	31.08	89.85
CAPM α		0.01	0.00	-0.00	0.00	-0.01	-0.02	-0.01	0.31	0.57
SV	E^P	10.01	0.00	-0.01	-0.01	0.01	0.12	0.58**	2.09	1.11
	p-value, %	25.59	95.94	52.41	51.05	62.81	13.71	4.86	24.86	78.44
SVJ	E^P	0.01*	0.00	-0.02**	-0.03***	-0.06	-0.05	0.14	0.72	-0.19
	p-value, %	5.82	24.02	1.87	0.81	13.80	70.97	50.73	71.12	26.26
Sharpe ratio		0.04	0.06	0.05	0.08	0.07	0.07	0.07	0.10	0.07
SV	E^P	0.22*	0.20	0.18	0.18	0.19	0.22**	0.22***	0.16	-0.19
	p-value, %	6.60	12.87	16.31	25.67	16.18	4.23	0.77	19.18	71.61
SVJ	E^P	0.30	0.28	0.24	0.22	0.20	0.20	0.19	0.13	-2.22
	p-value, %	0.90	2.18	3.82	11.21	13.29	8.36	3.65	55.88	45.41
					Panel D: GENERAL MOTORS COR. (GM)					
Mean Return		0.02	0.01	-0.03	-0.05	0.01	0.04	0.25	0.36	0.64
SV	E^P	0.04	0.04	0.03	0.02	0.04	0.10	0.34	0.79	0.66
	p-value, %	53.38	59.76	27.84	32.50	73.96	67.10	75.03	62.27	99.04

Table IV (continued)

	Moneyness	0.75	0.80	0.85	0.90	0.95	1.00	1.05	1.10	1.15
Panel D: GENERAL MOTORS COR. (GM) (continued)										
SVJ	E^P	0.07	0.07	0.08^{*}	0.07^{*}	0.10	0.17	0.43	0.90	0.70
	p-value, %	18.43	19.01	5.68	8.34	34.85	38.46	55.72	54.32	96.68
CAPM α		0.03	0.03	0.01	0.00	0.04	0.07	0.28	0.41	0.58
SV	E^P	0.02^{**}	0.01^{***}	-0.00	-0.02	-0.01	0.02	0.21	0.55	0.35
	p-value, %	4.14	0.38	35.49	15.17	18.74	53.86	72.04	84.21	85.67
SVJ	E^P	0.02^{**}	0.01^{***}	-0.01^{*}	-0.03^{**}	-0.04^{**}	-0.04	0.09	0.31	0.02
	p-value, %	1.13	0.06	7.92	1.95	2.72	16.64	31.01	88.46	51.12
Sharpe ratio		0.03	0.02	-0.05	-0.06	0.01	0.02	0.07	0.06	0.06
SV	E^P	0.11	0.08	0.05	0.02	0.03	0.05	0.09	0.06	-2.75
	p-value, %	50.09	58.03	28.27	34.20	77.56	71.98	79.57	99.22	32.68
SVJ	E^P	0.20	0.16	0.13^{*}	0.09^{*}	0.09	0.09	0.12	0.08	-2.01
	p-value, %	16.08	17.89	5.66	8.49	34.95	38.73	54.42	83.38	34.84
Panel E: GENERAL ELECTRIC COR. (GE)										
Mean Return		-0.01	-0.01	0.03	0.05	0.06	0.20	0.70	-0.11	0.08
SV	E^P,	0.07^{**}	0.08^{**}	0.11	0.13	0.20	0.57^{**}	2.18^{**}	28.37	2.45
	p-value, %	3.95	2.37	10.38	21.09	13.06	4.84	9.27	13.5	52.04
SVJ	E^P,	0.07^{**}	0.08^{**}	0.11	0.12	0.17	0.50	1.96	25.04	1.98
	p-value, %	4.36	2.78	13.22	30.20	22.45	10.22	12.82	14.59	58.72
CAPM α		0.01	0.00	0.02	-0.00	-0.02	0.08	0.49	-0.29	-0.05
SV	E^P, %	0.01	0.00	0.01	0.00	0.02	0.25^{*}	1.33	18.30	0.82
	p-value, %	24.02	30.67	72.90	98.39	31.42	9.77	13.84	15.87	72.54
SVJ	E^P, %	0.01	0.00	0.01	-0.01	-0.00	0.20	1.17	16.04	0.57
	p-value, %	22.54	32.43	35.10	56.88	64.72	22.02	21.90	17.56	80.04
Sharpe ratio		-0.02	-0.04	0.05	0.07	0.06	0.12	0.13	-0.03	0.01
SV	E^P, %	0.21	0.19	0.20	0.17	0.19	0.26^{**}	0.24^{**}	0.14^{**}	-13.81
	p-value, %	4.60	2.66	10.66	22.84	13.89	4.70	3.30	2.18	61.94
SVJ	E^P, %	0.20^{**}	0.18^{**}	0.19	0.16	0.16	0.23	0.22^{*}	0.11^{*}	-18.05
	p-value, %	4.99	3.05	12.89	31.52	24.30	11.04	6.32	3.82	55.69

Table V

Finite Sample Distribution of Put Option Returns

This table reports population expected option returns, CAPM alphas, and Sharpe ratios, as well as finite sample distribution p-values for the stochastic volatility and stochastic volatility jump diffusion models. I assume that all risk premium (expect for the equity premium) are equal to zero.
***, **, * denote significance level of 1%, 5%, and 10% respectively.

	Moneyness	0.75	0.80	0.85	0.90	0.95	1.00	1.05	1.10	1.15
Panel A: MICROSOFT COR. (MSFT)										
Mean Return		-0.93	-0.91	-0.82	-0.76	-0.62	-0.26	-0.19	-0.12	-0.09
SV	E^P	-0.95	-0.88	-0.67	-0.19	0.63	-0.19	-0.19	-0.14	-0.10
	p-value, %	97.99	92.43	64.49	20.18	15.17	57.25	91.80	81.68	89.04
SVJ	E^P	-0.97*	-0.91	-0.71	-0.24	0.57	-0.22	-0.20	-0.15	-0.11
	p-value, %	4.74	99.72	72.48	23.04	15.85	68.32	91.33	63.73	72.84
CAPM α		-0.90	-0.85	-0.73	-0.64	-0.47	-0.08	-0.03	0.01	0.02
SV	E^P	-0.94	-0.82	-0.52	0.13	1.17	0.02	-0.01	0.01	0.02
	p-value, %	97.86	94.79	65.21	20.24	13.80	33.64	58.64	99.40	73.58
SVJ	E^P	-0.95	-0.86	-0.57	0.09	1.14	9.02	-0.02	0.01	0.02
	p-value, %	97.00	98.10	72.27	22.67	14.12	36.70	66.96	86.00	58.71
Sharpe ratio		-1.52	-1.00	-0.72	-0.62	-0.54	-0.21	-0.22	-0.17	-0.15
SV	E^P	-601.62*	-329.93	23.25	-0.19*	-0.08*	-0.15	-0.20	-0.19	-0.18
	p-value, %	7.19	76.05	66.15	7.49	1.39	59.14	87.37	86.19	83.58
SVJ	E^P	-624.01	-372.67	-31.84	-0.25	-0.10	-0.17	-0.22	-0.21	-0.20
	p-value, %	4.72	23.13	61.94	9.49	1.82	71.41	95.86	68.81	67.55
Panel B: CISCO SYSTEMS INC. (CSCO)										
Mean Return		-0.89	-0.83	-0.67	-0.59	-0.44	-0.29	-0.20	-0.13	-0.09
SV	E^P	-0.55	-0.12	0.47	0.35**	0.14**	0.02*	-0.07	-0.10	-0.13
	p-value, %	43.44	19.17	10.85	4.33	2.14	5.28	25.82	71.44	65.88
SVJ	E^P	-0.88	-0.67	-0.29	-0.20	-0.22	-0.21	-0.21	-0.18	-0.18
	p-value, %	94.54	64.2	41.40	20.57	23.76	52.55	88.72	46.35	18.32
CAPM α		-0.88	-0.77	-0.57	-0.48	-0.31	-0.16	-0.07	-0.02	-0.01
SV	E^P	-0.35	0.24	1.02*	0.81**	0.48***	0.29***	0.15***	0.08***	0.03
	p-value, %	39.05	16.95	9.24	3.10	0.40	0.11	0.18	0.80	12.50

Table V (continued)

	Moneyness	0.75	0.80	0.85	0.90	0.95	1.00	1.05	1.10	1.15
Panel B: CISCO SYSTEMS INC. (CSCO) (continued)										
SVJ	E^P	-0.80	-0.49	0.06	0.15	0.09*	0.06*	0.02	0.01	-0.01
	p-value, %	86.06	55.94	32.80	12.06	6.28	6.59	16.48	35.15	80.08
Sharpe ratio		-1.46	-0.85	-0.47	-0.43	-0.33	-0.26	-0.21	-0.17	-0.15
SV	E^P	-53.40	-1.05	0.09**	0.04***	0.03***	-0.00**	-0.06	-0.11	-0.17
	p-value, %	87.26	97.71	1.25	0.23	0.24	1.02	12.49	53.24	82.19
SVJ	E^P	-267.33	-28.41	-0.32	-0.13*	-0.13*	-0.16	-0.21	-0.23	-0.27
	p-value, %	77.63	70.75	73.42	6.14	9.39	38.91	99.88	55.44	22.83
Panel C: IBM COR. (IBM)										
Mean Return		-1.00	-0.83	-0.53	-0.27	-0.17	-0.10	-0.08	-0.03	-0.02
SV	E^P	-0.90*	-0.65	0.25	0.20	0.10	-0.09	-0.11	-0.11	-0.07
	p-value, %	9.43	80.24	60.35	52.48	42.92	98.92	71.35	27.88	40.26
SVJ	E^P	-0.99**	-0.93*	-0.60	-0.44	-0.31	-0.30*	-0.23*	-0.18**	-0.10*
	p-value, %	2.46	7.71	93.65	71.46	55.43	7.73	6.48	2.4	9.42
CAPM α		-1.00	-0.78	-0.40	-0.06	0.02	0.07	0.05	0.05	0.05
SV	E^P	-0.86	-0.49	0.76	0.63	0.44	0.13	0.06	0.02	0.04
	p-value, %	78.34	78.36	56.58	46.12	27.41	54.44	83.06	13.86	34.27
SVJ	E^P	-0.99	-0.89	-0.35	-0.12	0.01	-0.05	0.14*	0.72***	-0.19**
	p-value, %	79.74	86.30	97.48	93.15	95.67	24.96	9.58	0.07	3.78
Sharpe ratio		-680.72	-0.63	-0.24	-0.11	-0.07	-0.06	-0.08	-0.05	-0.04
SV	E^P	-557.02	-247.93	-14.55	-0.06	-0.00	-0.07	-0.11	-0.15	-0.13
	p-value, %	99.95	72.47	63.28	79.85	48.77	94.50	68.89	28.69	40.38
SVJ	E^P	-665.28	-526.58*	-132.85	-0.76	-0.19	-0.25	-0.26*	-0.27**	-0.21*
	p-value, %	99.99	8.66	41.70	39.38	50.66	12.86	8.72	3.16	9.34
Panel D: GENERAL MOTORS COR. (GM)										
Mean Return		0.04	0.17	0.25	0.04	0.05	0.03	0.01	0.03	0.02
SV	E^P	-0.95**	-0.85**	-0.39*	-0.15	-0.09	-0.03	-0.03	-0.00	-0.00
	p-value, %	1.08	1.28	6.38	63.35	52.32	65.18	64.12	63.6	74.98

Table V (continued)

	Moneyness	0.75	0.80	0.85	0.90	0.95	1.00	1.05	1.10	1.15
Panel D: GENERAL MOTORS COR. (GM) (continued)										
SVJ	E^P	-0.99^{***}	-0.94^{***}	-0.67^{**}	-0.44^{*}	-0.30^{**}	-0.18^{*}	-0.13	-0.07	-0.05
	p-value, %	0.22	0.21	2.92	8.34	4.94	9.34	10.28	12.35	22.72
CAPM α		0.19	0.27	0.22	-0.03	0.08	0.05	0.03	-0.05	0.04
SV	E^P	-0.05^{***}	-0.84^{***}	-0.34	-0.09	-0.03	0.02	0.01	0.03	0.03
	p-value, %	0.89	1.22	18.00	87.48	55.42	74.86	61.95	41.34	29.90
SVJ	E^P	-0.98^{***}	-0.92^{***}	-0.58^{**}	-0.30	-0.15	-0.42	-0.01	0.02	0.02
	p-value, %	0.28	0.50	3.93	42.43	18.90	31.21	23.93	16.33	14.10
Sharpe ratio		0.01	0.02	0.03	0.01	0.02	0.02	0.01	0.03	0.02
SV	E^P	-631.72^{**}	-286.01	-10.71^{*}	-0.11	-0.06	-0.03	-0.04	-0.01	-0.01
	p-value, %	1.07	40.78	8.29	45.05	47.71	60.94	61.48	63.62	76.38
SVJ	E^P	-701.12^{***}	-460.59^{***}	-49.45^{*}	-0.30	-0.18	-0.14	-0.14	-0.10	-0.10
	p-value, %	0.20	0.16	9.85	18.98	12.89	13.76	12.71	14.39	24.85
Panel E: GENERAL ELECTRIC COR. (GE)										
Mean Return		-1.00	-0.85	-0.83	-0.64	-0.37	-0.19	-0.08	-0.05	0.04
SV	E^P	-0.97^{*}	-0.85	-0.38	0.09	0.22	0.01	-0.07	-0.04	0.03
	p-value, %	6.66	98.69	45.61	22.70	13.07	24.04	92.99	88.88	28.39
SVJ	E^P	-0.99^{**}	-0.92	-0.56	-0.11	0.11	-0.02	-0.07	-0.04	-0.02
	p-value, %	3.81	87.96	61.18	31.15	18.13	31.21	97.85	84.34	30.42
CAPM α		-1.00	-0.83	-0.78	-0.54	-0.22	-0.04	0.05	0.06	0.08
SV	E^P	-0.96	-0.78	-0.12	-0.48	0.59^{*}	0.26^{**}	0.10	0.09	0.08
	p-value, %	89.69	94.49	42.06	18.83	7.40	2.60	23.75	16.30	47.42
SVJ	E^P	-0.99	-0.88	-0.39	0.20	0.43	0.20^{*}	0.09	0.09	0.07
	p-value, %	91.41	93.43	57.70	27.07	10.96	5.27	40.72	20.18	39.07
Sharpe ratio		-635.39	-0.63	-0.55	-0.44	-0.23	-0.15	-0.08	-0.07	0.06
SV	E^P	-585.33	-367.77	-36.12	-0.15^{*}	0.02^{**}	-0.01	-0.07	-0.06	-0.05
	p-value, %	99.98	24.20	72.85	6.91	2.54	12.29	91.27	85.84	28.05
SVJ	E^P	-612.24	-454.86	-69.31	-0.24	-0.01^{**}	-0.03	-0.08	-0.05	-0.05
	p-value, %	99.99	14.70	64.60	56.82	4.27	19.86	97.96	83.00	31.10

Table VI

Finite Sample Distribution of Option Portfolio Returns

This table reports the sample average returns on option portfolios. Population expected returns and finite sample *p*-values are computed from the stochastic volatility and stochastic volatility jump diffusion models. I assume that volatility and jump risk premium are equal to zero.
***, **, * denote significance level of 1%, 5%, and 10% respectively.

		Covered Puts									ATM	Crash-Neutral	Put
Moneyness		0.75	0.80	0.85	0.90	0.95	1.00	1.05	1.10	1.15	Straddle	Straddle	Spread
Panel A: MICROSOFT COR. (MSFT)													
Mean Return, %		1.47	1.92	1.54	1.42	1.00	0.77	0.68	0.73	0.78	-6.05	-1.26	-20.56*
SV	E^P, %	2.08	1.99	1.83	1.65	1.45	1.23	0.94	0.80	0.71	11.21**	12.93*	-9.62
	p-value,%	55.99	94.32	74.55	77.33	53.60	43.46	54.20	80.26	72.41	2.84	8.92	64.40
SVJ	E^P, %	2.23	2.12	1.94	1.76	1.53	1.28	0.94	0.76	0.66	11.57**	13.44*	-11.18
	p-value,%	45.74	82.87	63.74	66.84	45.03	37.26	52.06	89.03	51.08	2.40	7.42	71.76
Panel B: CISCO SYSTEMS INC. (CSCO)													
Mean Return, %		0.49	1.17	1.27	1.06	0.82	0.67	0.59	0.55	0.44	-7.79	0.91	-12.74
SV	E^P, %	3.22	3.14	3.15	3.16	3.11	2.97	2.60	2.12	1.40	33.34***	37.85***	19.45
	p-value,%	17.89	16.42	14.61	18.13	13.36	11.26	10.92	11.11	14.40	0.00	0.06	13.56
SVJ	E^P, %	3.19	3.00	2.83	2.62	2.36	2.07	1.68	1.29	0.74	17.97***	25.08***	-8.25
	p-value,%	14.04	12.89	16.23	13.49	19.98	17.48	18.27	12.54	40.70	0.27	0.86	81.26
Panel C: IBM COR. (IBM)													
Mean Return, %		1.12	1.31	1.25	1.35	1.09	0.98	0.94	0.87	0.96	3.62	7.88	-0.10
SV	E^P, %	1.88	1.78	1.71	1.63	1.55	1.41	1.16	0.79	0.82	14.95	16.18	15.92
	p-value,%	44.71	60.71	58.69	71.68	49.48	42.14	56.67	73.98	34.11	13.95	30.14	44.61
SVJ	E^P, %	2.27	2.15	2.01	1.79	1.49	1.17	0.86	0.57	0.71	8.73	10.95	-22.65
	p-value,%	18.32	29.27	31.49	53.48	51.28	69.90	81.40	11.70	1.97	47.06	68.66	63.04

Table VI (continued)

Moneyness		Covered Puts 0.75	0.80	0.85	0.90	0.95	1.00	1.05	1.10	1.15	ATM Straddle	Crash-Neutral Straddle	Put Spread
Panel D: GENERAL MOTORS COR. (GM)													
Mean Return, %		0.54	0.37	-0.13	-0.31	0.14	0.20	0.32	0.55	0.63	6.09	9.02	15.84
SV	E^P, %	0.40	0.32	0.23	0.14	0.15	0.27	0.35	0.52	0.53	-0.24	2.34	13.63
	p-value,%	90.50	95.44	67.42	55.24	98.63	87.84	94.12	91.76	40.16	36.24	38.86	93.84
SVJ	E^P, %	1.14	1.04	0.91	0.70	0.56	0.51	0.46	0.56	0.53	0.68	4.22	-9.25
	p-value,%	57.55	48.19	21.54	17.50	51.04	52.80	69.88	94.37	43.03	44.08	53.74	29.72
Panel E: GENERAL ELECTRIC COR. (GE)													
Mean Return		0.79	0.36	0.63	0.88	0.80	0.77	0.79	0.96	1.13	2.37	10.31	-6.36
SV	E^P, %	1.57	1.51	1.47	1.41	1.42	1.40	1.18	1.09	1.09	15.52*	16.72	49.86
	p-value,%	44.87	19.93	27.90	44.92	30.74	18.18	21.33	44.85	75.69	17.64	41.28	30.00
SVJ	E^P, %	1.47	1.41	1.35	1.25	1.23	1.22	1.05	1.03	1.07	11.65	13.17	39.37
	p-value,%	49.36	22.15	34.35	59.21	47.37	33.33	38.71	66.16	56.86	19.98	70.91	37.52

Figure I

Time Series of Call Returns, MSFT

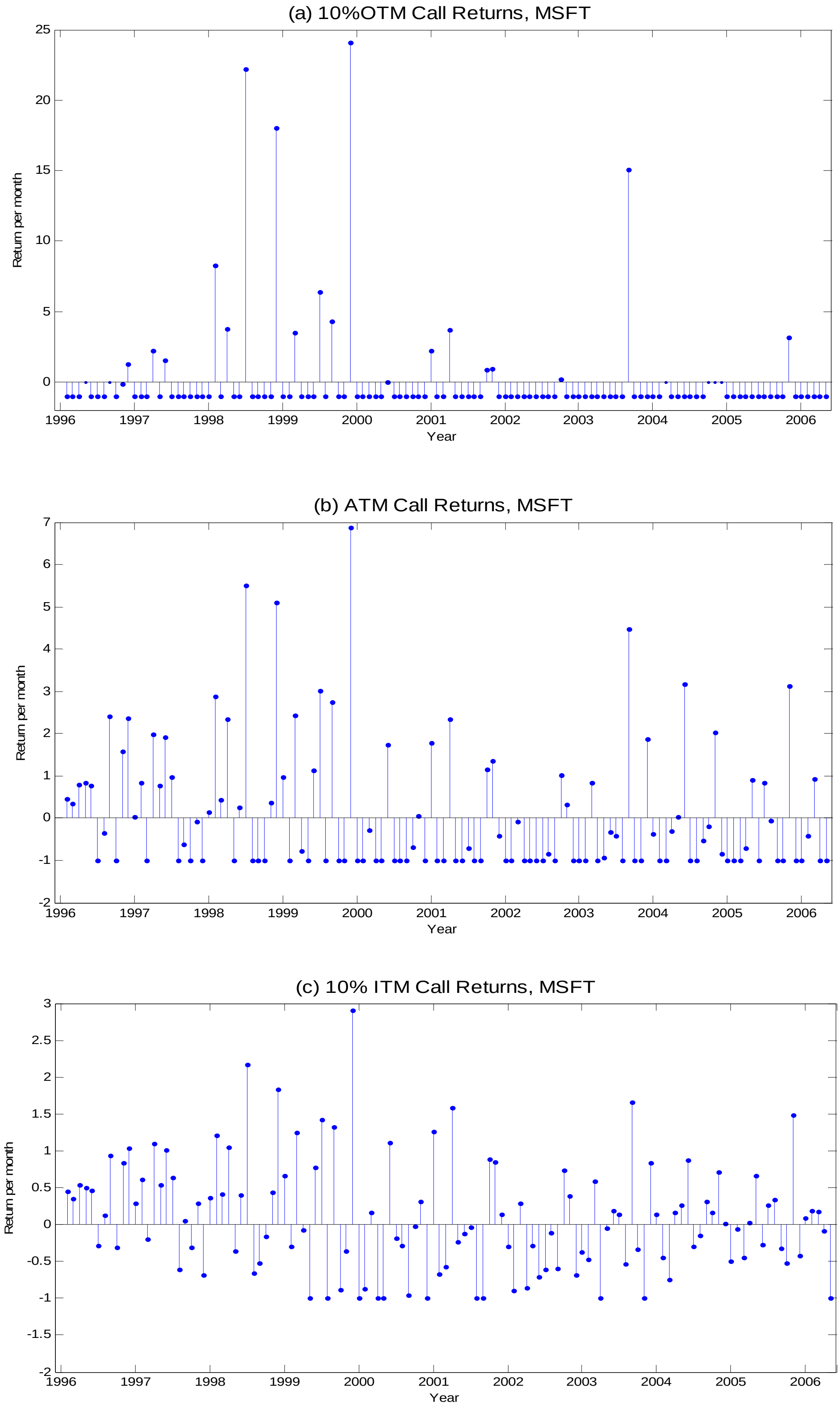
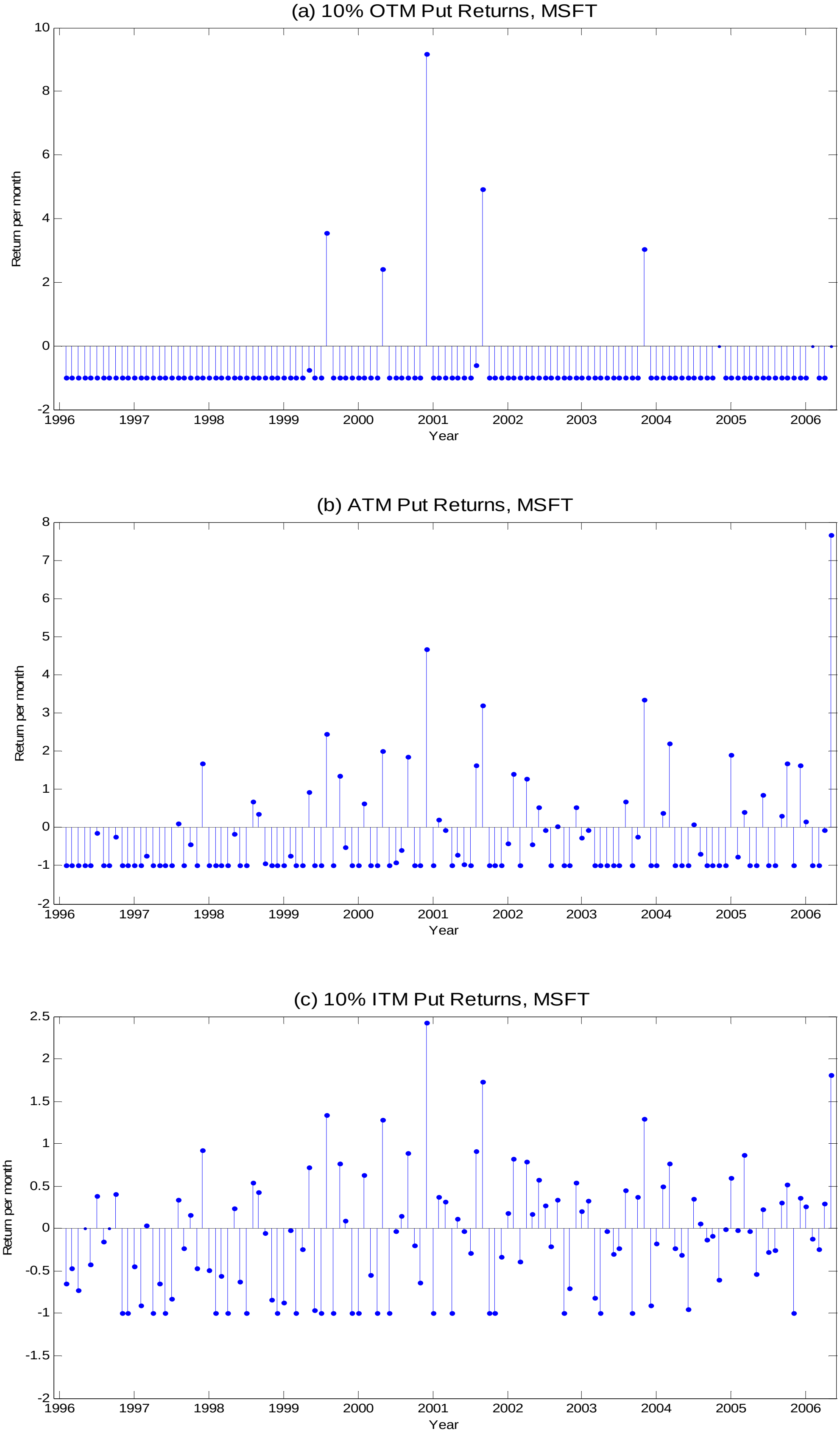


Figure II
Time Series of Put Returns, MSFT



Appendix

Appendix A: Adjusting for the Early Exercise Premium

Options in my base case are all the American type, their prices $P(S_0, T)$ and $C(S_0, T)$ are slightly higher than prices of the corresponding European options $p(S_0, T)$ and $c(S_0, T)$. However, since I am dealing with the one-month hold-to-expiration returns, American option prices will be no longer appropriate in this analysis. Thus, one problem for this study is to convert these American option prices into equivalent European option prices. Broadie, Chernov, and Johannes (2008) use the approach provided Broadie, Chernov, and Johannes (2007). Taking American Puts as an example, they use the observed price $P(S_0, T)$ to compute an American Black-Scholes implied volatility, that is, a value σ^{BS} such that $P(S_0, T) = \mathrm{BS}^A(\sigma^{BS}, K, T, S_0, r, \delta)$, where BS^{A} denotes the Black-Scholes American option price. They then estimate that an equivalent European option would trade in the market at a price $\mathrm{BS}^E(\sigma^{BS}, K, T, S_0, r, \delta)$, where BS^E denotes the Black-Scholes European option price. Broadie, Chernov, and Johannes (2008) compute American Black-Scholes implied volatilities using a binomial tree method. Broadie, Chernov, and Johannes (2007) show that this produces accurate early exercise adjustments in models with stochastic volatility and jumps in prices. However, binomial approximation methods for American options are usually cumbersome and expensive to use. Bondarenko (2003) recommends applying Barone-Adesi and Whaley (1987) approximation, which is efficient and inexpensive comparatively. Barone-Adesi and Whaley (1987) has shown the accuracy of their method to value American call and put options. To construct my dataset, I will mainly adopt the approach of Bondarenko (2003). Considering prices of OTM and ATM options are both more reliable and less affected by the early exercise feature. I will firstly compute the equivalent European option prices using put prices for moneyness $k \leq 1.00$ and call prices for $k \geq 1.00$, where $k = X/S_0$. Then for the ITM options, I compute the prices through the put-call parity relationship,

$$p_0 + S_0 = c_0 + Xe^{-r_f T} \quad \text{(A.1)}$$

where p_0 (c_0) is European put (call) price at $t = 0$, S_0 is spot price at $t = 0$, X is strike price, and r_f is risk-free rate. The correction of American option prices will be shown below in detail.

a. OTM and ATM Put Options

Like Black and Scholes (1973), Barone-Adesi and Whaley (1987) assume that the underlying commodity price-change movements follow the stochastic differential equation,

$$dS/S = (\mu - \delta)dt + \sigma dz \quad \text{(A.2)}$$

where μ is the drift rate of stock price, δ is the continuous dividend rate, σ is the volatility, and z is a Wiener process. If there is no arbitrage in the market, the partial differential equation governing the movements of the stock option (V) through time is

$$\frac{1}{2}\sigma^2 S^2 V_{SS} + (r_f - \delta)SV_S - r_f V + V_t = 0 \quad \text{(A.3)}$$

Knowing the terminal payoff for put option is $\max(0, S_T - X)$, the value of European put option is

$$p(S,T) = Xe^{-r_f T}N(-d_2) - Se^{-\delta T}N(-d_1), \quad \text{(A.4)}$$

where $d_1 = [ln\,(S\,/\,X) + (r_f - \delta + 0.5\sigma^2)]T/\sigma\sqrt{T}$, $d_2 = d_1 - \sigma\sqrt{T}$, and $N(\cdot)$ is the cumulative univariate normal distribution.

Barone-Adesi and Whaley (1987) further assume that American options, as well as European options, also satisfy the partial differential Equation (A.4), then (A.4) also applies to the early exercise premium of the American option. Denote the early exercise premium as $\varepsilon_C(S,T)$, that is

$$\varepsilon_C(S,T) = P(S,T) - p(S,T), \quad \text{(A.5)}$$

where *P(S,T)* is the American option value and *p(S,T)* is the European option value. The partial differential equation will become

$$\frac{1}{2}\sigma^2 S^2 \varepsilon_{SS} - r_f \varepsilon + bS_{\varepsilon S} + \varepsilon_t = 0 \tag{A.8}$$

Since T denotes the period from the option's expiration to the present, $\varepsilon_T = -\varepsilon_t$. Let $M = 2r_f/\sigma^2$ and $N = 2(r_f - \delta)/\sigma^2$, Equation (A.8) is changed to

$$S^2\varepsilon_{SS} - M\varepsilon + NS_{\varepsilon S} - \left(M/r_f\right)\varepsilon_T = 0 \tag{A.9}$$

Define the early exercise premium as $\varepsilon_C(S,K) = K(T)f(S,K)$, and substitute it into Equation (A.9) to get Equation (A.10),

$$S^2 f_{SS} + NSf_S - Mf\left[1 + \left(K_T/r_f K\right)(1 + Kf_K/f)\right] = 0. \tag{A.10}$$

Choosing $K(T) = 1 - e^{-r_f T}$, substituting into (A.10), and simplifying give

$$S^2 f_{SS} + NSf_S - (M/K)f - (1-K)Mf_k = 0 \tag{A.11}$$

The approximation will be made in Equation (A.11). For options with very short times to expiration, *T* approaches 0, f_K approaches 0, and the term, $(1-K)Mf_K$, disappears. Therefore, drop the term $(1-K)Mf_K$ in (A.11), and get Equation (A.12),

$$S^2 f_{SS} + NSf_S - (M/K)f = 0. \tag{A.12}$$

Assuming *f* is in the form aS^q, the general solution to (A.12) is

$$f(S) = a_1 S^{q_1} + a_2 S^{q_2}. \tag{A.14}$$

where $q_1 = \left[-(N-1) - \sqrt{(N-1)^2 + 4M/K}\right]/2 < 0$ and $q_2 = \left[-(N-1) + \sqrt{(N-1)^2 + 4M/K}\right]/2 > 0$.

Since the value of put option should be no more than $S - X$, and $a_2 S^{q_2}$ will approach ∞ when S approaches 0, it is required that $a_2 = 0$. Therefore, the approximate value of the American put option is written as

$$P(S,T) = p(S,T) + Ka_1 S^{q_1} \tag{A.15}$$

It is known that $P(S,T)$ is increasing as S increasing. There must be a critical price S^*, which has the relationship that

$$X - S^* = p(S^*,T) + Ka_1 S^{*q_1} \tag{A.16}$$

and the slope of the exercisable value of the put, -1, is set equal to the slope of $P(S^*,T)$, that is

$$-1 = -e^{-\delta T} N[-d_1(S^*)] + Kq_1 a_1 S^{*q_2 - 1}. \tag{A.17}$$

Thus $a_1 = -\{1 - e^{-\delta T} N[-d_1(S^*)]\}/Kq_1 S^{*q_1 - 1}$, and S^* should be the numerical solution of the following Equation (A.18),

$$X - S^* = p(S^*,T) - \{1 - e^{-\delta T} N[-d_1(S^*)]\} S^*/q_1 \tag{A.18}$$

With S^* known, the approximate value of an American put option becomes

$$\begin{aligned} &P(S,T) = p(S,T) + A_1 (S/S^*)^{q_1}, && \text{when } S > S^*, \text{and} \\ &P(S,T) = X - S && \text{when } S \le S^*, \end{aligned} \tag{A.19}$$

where $A_1 = -(S^*/q_1)\{1 - e^{-\delta T} N[-d_1(S^*)]\}$.

Since I am focusing on the OTM put options, their spot prices will not be below S^*. Therefore, I only need to use the first equation to compute the Black-Scholes Volatilities. Denote the observed American put price as $P^*(S_0, T, \sigma^{BS})$. It is given that

$$\begin{aligned} P^*(S_0, T, \sigma^{BS}) &= P(S_0, T, \sigma^{BS}) \\ &= p(S_0, T, \sigma^{BS}) + A_1 (S/S^*)^{q_1} \end{aligned} \tag{A.20}$$

In the end, the corrected European option price is determined by $p(S_0, T, \sigma^{BS})$.

Note that this approach to compute equivalent European option price from American option price is contrived for continuous-dividend stock option ($\delta \neq 0$) or non-dividend stock option ($\delta = 0$). However, for individual stocks, dividends are often paid in discrete time. To avoid this problem, I assume that dividend is paid out at pre-determined times, and calculate the equivalent spot price S_0' at $t = 0$, by subtracting the discounted dividends from S_0:

$$S_0' = S_0 - \sum_i D(t_i) e^{-r \cdot \frac{t_i}{360}} \tag{A.21}$$

where t_i is the time of the i^{th} dividend, and $D(t_i)$ is dividend amount at t_i. Then the equivalent European option prices can be estimated by $S_0{}'$, according to Equation (A.20) as if non-dividend stocks.

b. OTM and ATM Call Options

It is known that for non-dividend stocks, their American call options will be valued using the European formula,

$$C(S,T) = c(S,T) = SN(d_1) - Xe^{-r_f T} N(d_2), \tag{A.22}$$

where $d_1 = [ln\,(S\,/\,X) + (r_f - \delta + 0.5\sigma^2)]T/\sigma\sqrt{T}$, $d_2 = d_1 - \sigma\sqrt{T}$, and $N(\cdot)$ is the cumulative univariate normal distribution. On the other hand, for discrete-dividend stocks, American calls will be execute right before the first dividend. For this reason, the correction of OTM American call options will follow the below approach:

If the underlying stocks of American calls pay no dividend during the month when calls exist, keep the observed American call prices as the equivalent European call prices. Otherwise, if there are more than one time of dividend payments in the month, suppose the first payment occurs at t', and $T' = t' - t_0$ denotes the period from issue day to first dividend payment day. Therefore, the American call price can be expressed as

$$C(S_0, T', \sigma^{BS}) = S_0 N\big(d_1(S_0, T', \sigma^{BS})\big) - Xe^{-r_f T'} N\big(d_2(S_0, T', \sigma^{BS})\big), \tag{A.23}$$

where $C(S_0, T', \sigma^{BS})$ is the observed American call price.

Implied volatility σ^{BS} can thus be acquired through Equation (A.23). With the known σ^{BS}, I will further estimate the equivalent European call price by

$$c(S_0', T, \sigma^{BS}) = S_0' N\big(d_1(S_0', T, \sigma^{BS})\big) - Xe^{-rT} N\big(d_2(S_0', T, \sigma^{BS})\big) \qquad \text{(A.24)}$$

where S_0' is the equivalent spot price at $t = 0$, the same as that in the correction of American put options in the above section.

c. ITM Call and Put Options

Since ITM calls and puts are more possible to be executed before the maturity date, the correction of them is following the put-call parity (A.1), using corresponding OTM call or put prices.

Appendix B: MCMC Algorithm

a. Model

In Appendix B, I will discuss on the stochastic-volatility jump-diffusion (SVJ) model. Stochastic-volatility model of Heston (1993) can be considered as a special case of SVJ, and the estimation process of SV model is similar.

$$Y_t = \tilde{\mu} + J_t Z_t^s + \sqrt{V_{t-1}}\varepsilon_t^s \tag{B.1}$$

$$V_t = \alpha_v + \beta_v V_{t-1} + \sqrt{V_{t-1}}\sigma_v \varepsilon_t^v \tag{B.2}$$

where $\alpha_v = \kappa_v \theta_v$, and $\beta_v = 1 - \kappa_v$. Define $u_t^v \stackrel{\text{def}}{=} \sigma_v \varepsilon_t^v$. The covariance matrix of $\boldsymbol{r_t} \stackrel{\text{def}}{=} (\varepsilon_t^s \quad u_t^v)'$, is Σ^*:

$$\Sigma^* \stackrel{\text{def}}{=} \begin{pmatrix} 1 & \rho\sigma_v \\ \rho\sigma_v & \sigma_v^2 \end{pmatrix} \tag{B.3}$$

b. Priors

For the parameters, Clifford-Hammersley theorem (Hammersley and Clifford (1970) and Besag (1974)) implies that $p(\sigma_v, \rho | J, Z^s, V, Y)$, $p(\alpha_v, \beta_v | J, Z^s, V, Y)$, $p(\mu_Z, \sigma_Z^2 | J, Z^s)$, and $p(\lambda | J)$ characterize $p(\Theta | J, Z^s, V, Y)$. Assuming standard conjugate prior distributions for the parameters, $(\alpha, \beta)^T \sim \mathcal{N}$, $(\mu_Z, \sigma_Z^2)^{\mathrm{T}} \sim \mathcal{N}/\mathcal{IG}$ $\lambda \sim \mathcal{B}(\alpha_J, \beta_J)$, where $\mathcal{N}$ is the normal distribution, $\mathcal{N}/\mathcal{IG}$ is the normal-inverse gamma distribution, and $\mathcal{B}$ is the beta distribution. The major challenge here is to formulate a prior for (σ_v, ρ), since Σ^* has its $(1,1)$ element fixed to 1. This means that standard inverted Wishart priors cannot be used. Jacquier, Polson, Rossi (2004) suggest that (σ_v, ρ) can be transform to (ψ, Ω) as follows:

$$\Sigma^* = \begin{pmatrix} 1 & \psi \\ \psi & \Omega + \psi^2 \end{pmatrix} \tag{B.4}$$

where $\psi = \sigma_v \rho$, $\Omega = \sigma_v^2(1-\rho^2)$. As observed by McCulloch, Polson, and Rossi (2000), it is said that $(\psi, \Omega) \sim \mathcal{N}/\mathcal{IG}$.

In summary, the prior distributions for the parameters are:

$$\Omega \backsim \mathcal{IG}(v_0, v_0 t_0^2) \qquad \psi|\Omega \sim \mathcal{N}(\psi_0, \Omega/p_0) \qquad v_0 = 2.5,\ t_0 = 0.2,\ \psi_0 = 0,\ p_0 = 2$$

$$(\alpha, \beta)^T \sim \mathcal{N}\left((0,1)^T, \begin{pmatrix} \sigma_\alpha^2 & 0 \\ 0 & \sigma_\beta^2 \end{pmatrix}\right) \qquad \sigma_\alpha^2 = \sigma_\beta^2 = 1$$

$$\sigma_z^2 \backsim \mathcal{IG}(v_{z0}, v_{z0} t_{z0}^2) \qquad \mu_z|\sigma_z^2 \sim \mathcal{N}(\mu_{z0},\ \sigma_z^2/p_{z0}) \qquad v_{z0} = 2,\ t_{z0} = 5,\ \mu_{z0} = -3,\ p_{z0} = 1$$

$$\lambda \sim \mathcal{B}(\alpha_J, \beta_J) \qquad \alpha_J = 2,\ \beta_J = 400$$

c. Posterior Distributions

The joint distribution of data and volatilities is

$$p(\boldsymbol{Y}, \boldsymbol{V}|\boldsymbol{\Theta}, \boldsymbol{J}, \boldsymbol{Z^s}) \propto \left(\prod_{t=2}^{T} V_{t-1}^{-1}\right) |\Sigma^*|^{-\frac{T-1}{2}} exp\left(-\frac{1}{2} tr\left(\Sigma^{*-1} A\right)\right) \tag{B.5}$$

Here

$$A = \sum_t \boldsymbol{r}_t \boldsymbol{r}'_t$$

$$= \begin{pmatrix} \sum_t \frac{(Y_t - \tilde{\mu} - J_t Z_t^s)^2}{V_{t-1}} & \sum_t \frac{(Y_t - \tilde{\mu} - J_t Z_t^s)(V_t - \alpha_v - \beta_v V_{t-1})}{V_{t-1}} \\ \sum_t \frac{(Y_t - \tilde{\mu} - J_t Z_t^s)(V_t - \alpha_v - \beta_v V_{t-1})}{V_{t-1}} & \sum_t \frac{(V_t - \alpha_v - \beta_v V_{t-1})^2}{V_{t-1}} \end{pmatrix} \tag{B.6}$$

is the residual matrix.

For parameters:

- $p(\psi, \Omega|\, J, Z^s, V, Y) \propto p(Y, V|\Theta, J, Z^s) p(\psi, \Omega)$.

Note that $|\Sigma^*| = \Omega$, and rewrite Σ^{*-1} as

$$\Sigma^{*-1} = \frac{1}{\Omega}\begin{pmatrix} \Omega + \psi^2 & -\psi \\ -\psi & 1 \end{pmatrix}$$

With the prior distribution in section b, we have:

$$\Omega \backsim \mathcal{IG}\left(v_0 + T - 2, v_0 t_0^2 + a_{22} - \frac{(a_{12} + p_0\psi_0)^2}{p_0 + a_{11}} + p_0 \psi_0^2\right) \tag{B.7}$$

$$\psi|\Omega \sim \mathcal{N}\left(\frac{a_{12}+p_0\psi_0}{a_{11}+p_0}, \frac{\Omega}{a_{11}+p_0}\right)$$

- $p(\alpha_v, \beta_v | J, Z^s, V, Y) \propto p(Y, V|\Theta, J, Z^s)p(\alpha_v, \beta_v)$

$$(\alpha_v, \beta_v) \backsim \mathcal{N}\left(\mu_{\alpha\beta}, \Sigma_{\alpha\beta}\right) \tag{B.8}$$

where

$$\mu_{\alpha\beta} = \Sigma_{\alpha\beta}\begin{pmatrix} \frac{1}{\Omega}\sum_t\left(\frac{V_t}{V_{t-1}} - \psi\frac{Y_t-\mu-J_tZ_t^s}{V_{t-1}}\right) \\ \frac{1}{\Omega}\sum_t\left(V_t-\psi(Y_t-\tilde{\mu}-J_tZ_t^s)\right)+\frac{1}{\sigma_\beta^2} \end{pmatrix}$$

$$\Sigma_{\alpha\beta} = \begin{pmatrix} \frac{1}{\Omega}\sum_t V_{t-1}^{-1}+\frac{1}{\sigma_\alpha^2} & \frac{T-1}{\Omega} \\ \frac{T-1}{\Omega} & \frac{1}{\Omega}\sum_t V_{t-1}+\frac{1}{\sigma_\beta^2} \end{pmatrix}^{-1}$$

- $p(\mu_z, \sigma_z^2 | Z^s) \propto p(Z^s|\mu_z, \sigma_z^2)p(\mu_z, \sigma_z^2)$

$$\sigma_z^2 \backsim \mathcal{IG}\left(v_{z0}+T-1, v_{z0}t_{z0}^2 - \frac{(\sum_t Z_t + p_{z0}\mu_{z0})^2}{T+p_{z0}} + \sum_t Z_t^2 + p_{z0}\mu_{z0}^2\right)$$

$$\mu_z|\sigma_z^2 \sim \mathcal{N}\left(\frac{\sum_t Z_t + p_{z0}\mu_{z0}}{T+p_{z0}}, \frac{\sigma_z^2}{T+p_{z0}}\right) \tag{B.9}$$

- $p(\lambda | J) \propto p(J|\lambda)p(\lambda)$

$$\lambda \sim \mathcal{B}\left(\sum_t J_t + \alpha_J, \beta_J + T - \sum_t J_t\right) \tag{B.10}$$

For the state variables:

- $p({Z_t}^s|\Theta, J_t, V_{t-1}, V_t, Y_t) \propto p(Y, V|\Theta, J, Z^s)p({Z_t}^s|\mu_z, \sigma_z^2)$

$$Z_t^s \backsim \mathcal{N}\left(-\frac{B}{A}, \frac{1}{A}\right) \tag{B.11}$$

where

$$A = \frac{1}{\sigma_z^2} + \frac{J_t}{V_{t-1}}\left(\frac{\psi^2}{\Omega}+1\right)$$

$$B = -\frac{\mu_z}{\sigma_z^2} + \frac{\psi J_t(V_t-\alpha_v-\beta_v V_{t-1})}{\Omega V_{t-1}} - \left(\frac{\psi^2}{\Omega}+1\right)\frac{J_t(Y_t-\tilde{\mu})}{V_{t-1}}$$

- $p(J_t = 1|\Theta, V_{t-1}, V_t, Y_t, {Z_t}^s) \propto p(Y, V|\Theta, J, Z^s)p(J_t = 1|\lambda)$

$$p(J_t = 0|\Theta, V_{t-1}, V_t, Y_t, {Z_t}^s) \propto p(Y, V|\Theta, J, Z^s)p(J_t = 0|\lambda)$$

$$\begin{aligned} &p(J_t = 1) \\ &\quad \propto \lambda exp\left(-\frac{1}{2}\left(1+\frac{\psi^2}{\Omega}\right)\left(\frac{Y_t - \tilde{\mu} - Z_t^s}{\sqrt{V_{t-1}}}\right)^2 + \frac{\psi}{\Omega}\frac{Y_t - \tilde{\mu} - Z_t^s}{\sqrt{V_{t-1}}}\frac{V_t - \alpha_v - \beta_v V_{t-1}}{\sqrt{V_{t-1}}}\right) \\ &p(J_t = 0) \\ &\quad \propto (1-\lambda)exp\left(-\frac{1}{2}\left(1+\frac{\psi^2}{\Omega}\right)\left(\frac{Y_t - \tilde{\mu}}{\sqrt{V_{t-1}}}\right)^2 + \frac{\psi}{\Omega}\frac{Y_t - \tilde{\mu}}{\sqrt{V_{t-1}}}\frac{V_t - \alpha_v - \beta_v V_{t-1}}{\sqrt{V_{t-1}}}\right) \end{aligned} \tag{B.12}$$

According to (B.12), J_t satisfies the Bernoulli probability.

- $p(\boldsymbol{V}|\boldsymbol{\Theta}, \boldsymbol{J}, \boldsymbol{Z^s}, \boldsymbol{Y})$: Break $\boldsymbol{V}$ into T components $V_t|V_{t-1}, V_{t+1}$.

$$\begin{aligned} &p(V_t|V_{t-1}, V_{t+1}, \boldsymbol{\Theta}, \boldsymbol{Y}) \\ &\quad \propto {V_t}^{-1} exp\left[\left(-\frac{(Y_{t+1} - \tilde{\mu} - J_{t+1}Z_{t+1}^s)^2}{2}\left(1+\frac{\psi^2}{\Omega}\right)\right.\right. \\ &\quad \left.\left. + \frac{\psi}{\Omega}(Y_{t+1} - \tilde{\mu} - J_{t+1}Z_{t+1}^s)(V_{t+1} - \alpha_v) - \frac{1}{2\Omega}(V_{t+1} - \alpha_v)^2\right)\frac{1}{V_t} - \frac{(V_t - \eta_t)^2}{2\Omega \mathrm{V}_{\mathrm{t-1}}}\right] \end{aligned} \tag{B.13}$$

where

$$\eta_t = \alpha_v + \beta_v V_{t-1} - \frac{1}{2}{\beta_v}^2 V_{t-1} + \psi(Y_t - \tilde{\mu} - J_t Z_t)$$

Let

$$\gamma = -\frac{1}{2}(Y_{t+1} - \tilde{\mu} - J_{t+1}Z_{t+1}^s)^2 - \frac{1}{2\Omega}(\psi(Y_{t+1} - \tilde{\mu} - J_{t+1}Z_{t+1}^s) - V_{t+1} + \alpha_v)^2$$

$$\delta = -\frac{1}{2\Omega \mathrm{V}_{\mathrm{t-1}}}$$

then

$$p(V_t|V_{t-1}, V_{t+1}, \boldsymbol{\Theta}, \boldsymbol{Y}) \propto exp\left(-logV_t + \gamma\frac{1}{V_t} + \delta(V_t - \eta_t)^2\right) \tag{B.14}$$

Note that the density $p(V_t|V_{t-1}, V_{t+1}, \boldsymbol{\Theta}, \boldsymbol{Y})$ has both inverse-gamma kernel and normal kernel, but it is not a standard distribution. Thus simple Gibbs sampler does not apply for the simulation of V_t. The Metropolis-Hastings algorithm had to be used in this case.

d. Metropolis-Hastings algorithm for V_t

I use an independence Metropolis algorithm here. Noting that the density has both inverse-gamma kernel and normal kernel in it, two methods for constructing the approximate standard distributions can be considered.

- Normal distribution

To approximate the first two terms of (B.14) by a normal kernel, firstly, we need to find ϖ_t, at which point, $p(V_t|V_{t-1}, V_{t+1}, \mathbf{\Theta}, \boldsymbol{Y})$ achieves its maximum value.

$$p(\varpi_t) = \max\left(p(V_t|V_{t-1}, V_{t+1}, \mathbf{\Theta}, \boldsymbol{Y})\right)$$

Given the Taylor expansion of the first two terms of (B.14) at ϖ_t:

$$-logV_t = -\frac{2V_t}{\varpi_t} + \frac{V_t^2}{2\varpi_t^2} + \cdots$$

$$\gamma\frac{1}{V_t} = -\gamma\frac{3V_t}{\varpi_t^2} + \gamma\frac{V_t^2}{\varpi_t^3} + \cdots$$

approximate normal distribution q is:

$$q_t \sim \mathcal{N}(\varpi_t, 1/\Xi) \tag{B.15}$$

where

$$\Xi = -\frac{1}{\varpi_t^2} - \gamma\frac{2}{\varpi_t^3} - 2\delta$$

- Inverse gamma distribution

Rewrite (B.14) as

$$p(V_t|V_{t-1}, V_{t+1}, \mathbf{\Theta}, \boldsymbol{Y}) \propto \frac{1}{V_t}exp\left(-\frac{-2\gamma}{2V_t}\right)\cdot\frac{1}{\sqrt{2\pi\cdot\left(-\frac{1}{2\delta}\right)}}exp\left(-\frac{1}{2}\frac{(V_t-\eta_t)^2}{-\frac{1}{2\delta}}\right) \tag{B.16}$$

I approximate the normal kernel by an inverse-gamma with the same mean and variance $\mathcal{IG}(\alpha_N, \beta_N)$:

$$\alpha_N = \frac{2\eta_t^2}{-\frac{1}{2\delta}} + 4 = -4\eta_t^2\delta + 4$$
$$\beta_N = (\alpha_N - 2)\eta_t \tag{B.17}$$

Combine the approximate new inverse gamma kernel with the original one, the density of $p(V_t|V_{t-1}, V_{t+1}, \mathbf{\Theta}, \boldsymbol{Y})$ is transformed to standard inverse gamma distribution:

$$q_t \sim \mathcal{IG}\left(\alpha_q, \beta_q\right) \tag{B.18}$$

where

$$\alpha_q = \alpha_N + 2$$

$$\beta_q = -2\gamma + \beta_N = -2\gamma + \left(\alpha_q - 4\right)\eta_t$$

■ Independent Metropolis-Hastings algorithm

Step 1: Draw $V_t^{(g+1)}$ from the proposal density q_t

Step 2: Accept probability

$$\alpha\left(V_t^{(g)}, V_t^{(g+1)}\right) = min\left(\frac{p\left(V_t^{(g+1)}\right) q\left(V_t^{(g)}\right)}{p\left(V_t^{(g)}\right) q\left(V_t^{(g+1)}\right)}, 1\right)$$

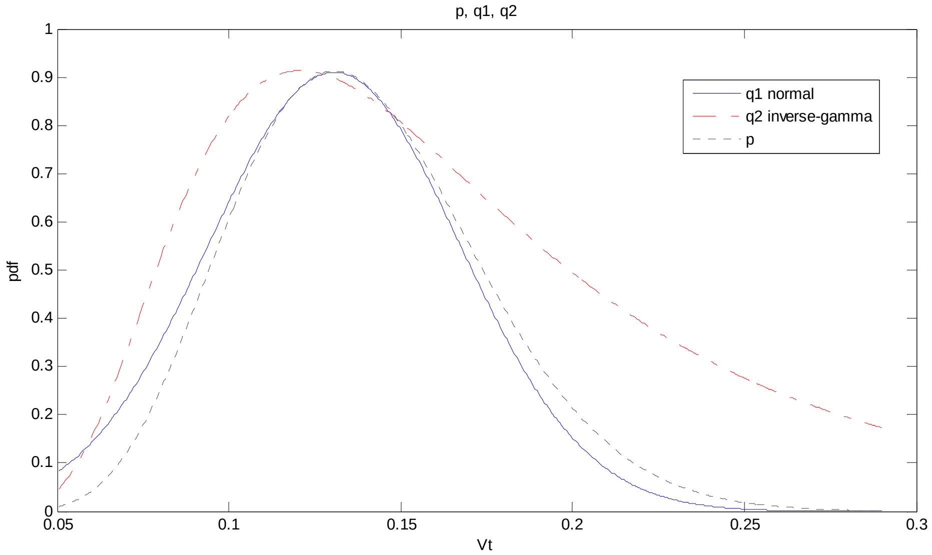
That is, if $\alpha\left(V_t^{(g)}, V_t^{(g+1)}\right) = 1$, $V_t^{(g+1)} = V_t^{(g+1)}$; else if $\alpha\left(V_t^{(g)}, V_t^{(g+1)}\right) < 1$, $V_t^{(g+1)} = V_t^{(g+1)}$ with probability $\frac{p\left(V_t^{(g+1)}\right)q\left(V_t^{(g)}\right)}{p\left(V_t^{(g)}\right)q\left(V_t^{(g+1)}\right)}$, and $V_t^{(g+1)} = V_t^{(g)}$ with probability $1 - \frac{p\left(V_t^{(g+1)}\right)q\left(V_t^{(g)}\right)}{p\left(V_t^{(g)}\right)q\left(V_t^{(g+1)}\right)}$

■ The choice between normal and inverse-gamma approximations

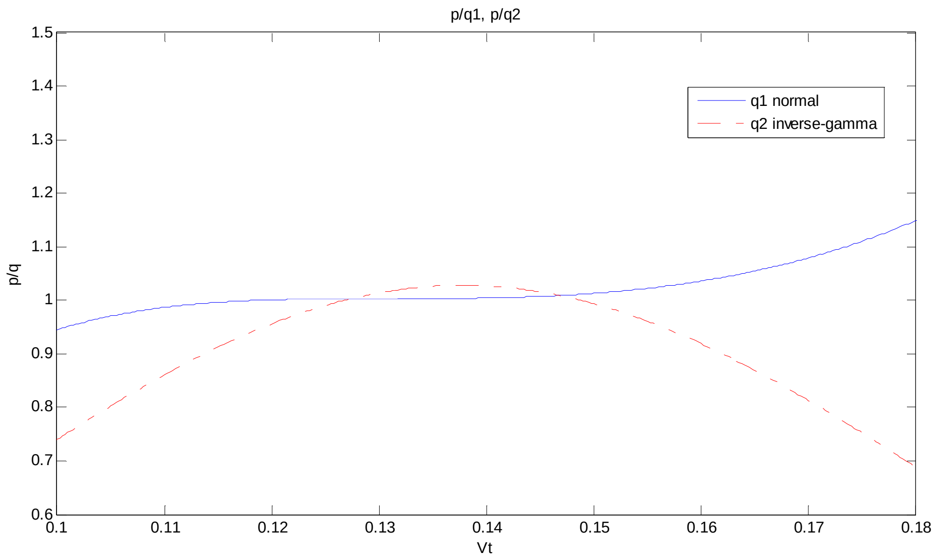
One key to measure the performance of distribution approximations is the ratio $p(V_t)/q(V_t)$, which drives acceptance and repeat probabilities, as shown in independent Metropolis algorithm. The flatter $p(V_t)/q(V_t)$ is, the more efficient the blanket. Empirical tests shows that in most cases, $p(V_t)$ is much more like a normal distribution (see Figure B1(a)). And a normal approximation $q(V_t)$ does perfectly fit the original distribution $p(V_t)$ and better performed than does the inverse-gamma approximation, as shown in Figure B1(b). The result of simulation distribution is

Figure B1

Choosing the Blanket Density for Stochastic Volatility



(a)



(b)

Figure B2
Simulation by Metropolis Algorithm

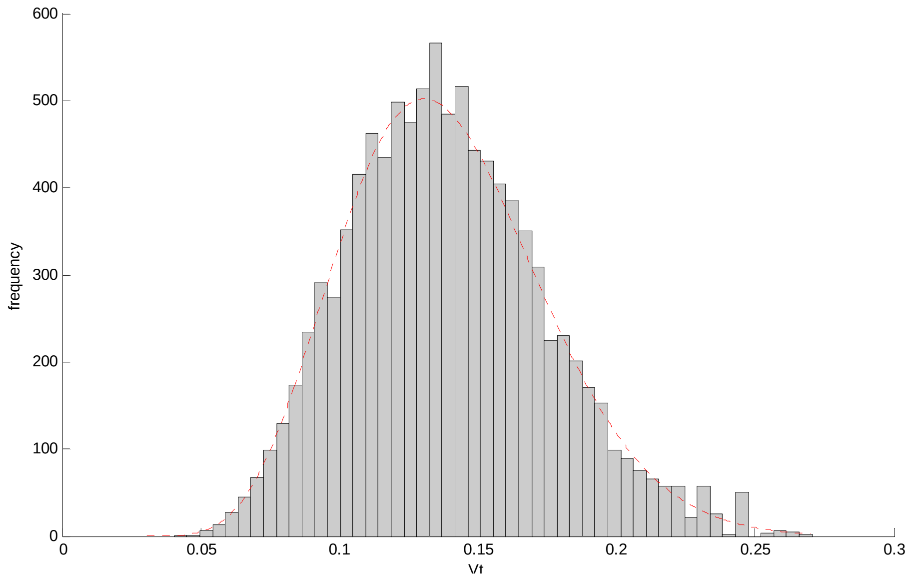
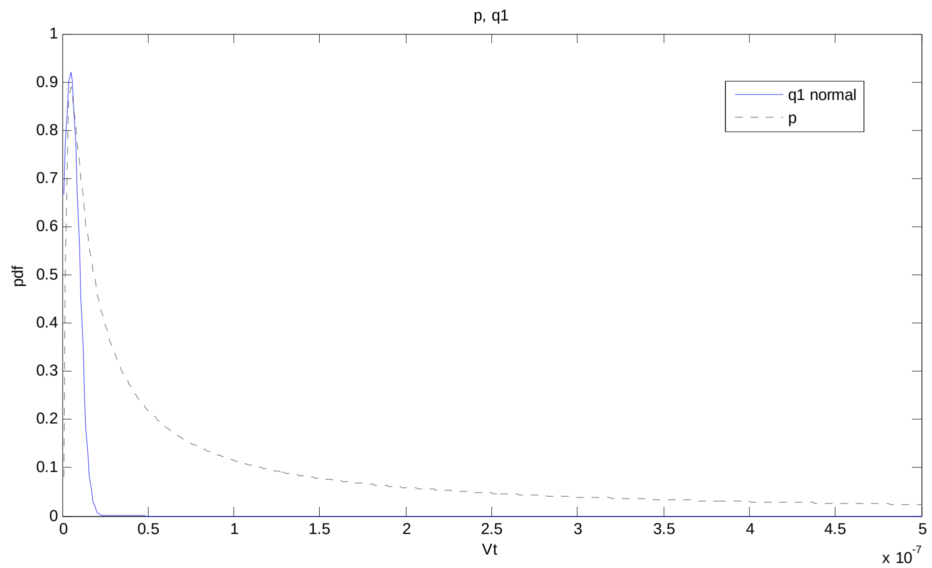


Figure B3
Special Case for Normal Approximation



reported in Figure B2, which is quite desirable.

However, empirical practices tell us, normal approximation cannot be applied in some special cases. For example, when γ in (B.14) is especially smaller than δ, the inverse-gamma kernel will quite dominate the normal kernel, making $p(V_t)$ has an extremely right fat-tail problem (see Figure B3). Unfortunately, an inverse-gamma approximation cannot be applied either, because the large difference between their peaks. To solve this problem, I suggest that instead of simulating V_t by Metropolis, one can assign the mean of the inverse-gamma approximation to V_t. That is

$$V_t = \frac{\beta_q}{\alpha_q - 2}.$$

Since this special case does not happen frequently, this method is tested to be practicable.

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