12-2016

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Recommended Citation

Pothabathula, Swapna, "An integrated economic equilibrium model for electricity markets." (2016). *Electronic Theses and Dissertations.* Paper 2594.
https://doi.org/10.18297/etd/2594

AN INTEGRATED ECONOMIC EQUILIBRIUM MODEL FOR ELECTRICITY MARKETS

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A Thesis
Submitted to the Faculty of the
J.B. Speed School of Engineering of the University of Louisville
in Partial Fulfillment of the Requirements
for the Degree of

Master of Science
in Industrial Engineering

Department of Industrial Egnineering
University of Louisville
Louisville, Kentucky

December 2016

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ACKNOWLEDGEMENTS

I would like to express my deepest gratitude to my thesis advisor, Dr. Lihui Bai, for her never ending support and motivation. Her guidance along the way was really valuable and her patience and encouragements never failed to make me feel confident again in the research that I am doing. My sincere thanks to Dr. Thomas Riedel and Dr. Ki-Hwan Gabriel Bae for reviewing and providing some comments to improve this thesis. My appreciation also goes to Dr. John Kielkopf and my friend Guangyang Xu for their valuable LaTeX template of the thesis.

My greatest appreciation goes to my beloved parents, Srinivasa Rao and Venkata Ramanamma, my husband Prashant and my brothers for their love and support. Without their trust, I would have never reached this far.

Last but not least, I would like to thank my professors and fellow friends for their support, helpful comments and encouragement. Their help is deeply appreciated.

ABSTRACT

AN INTEGRATED ECONOMIC EQUILIBRIUM MODEL FOR ELECTRICITY MARKETS

Swapna Pothabathula

November 28th, 2016

The increasing energy challenges worldwide are forcing researchers to explore ways for energy systems that work more efficiently on their own and with each other. This thesis develops an integrated electricity market equilibrium model (EMEM) as a mixed complementarity problem (MCP). In particular, we first develop an equilibrium model to study an electricity market consisting of coal producers, electricity generation firms, natural gas producers, natural gas marketers, natural gas pipeline owners, natural gas consumers and electricity consumers. The equilibrium model not only captures a decentralized optimization for each player but considers the interaction amongst players. Second, we formulate the equilibrium model as a mixed complementarity problem (MCP) and conduct computational studies and sensitivity analyses to shed lights on energy policy making. Numerical results show that the proposed integrated equilibrium model can effectively govern the supply and demand among all players.

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CHAPTER 1

INTRODUCTION

With the changing climate and resource scarcity today, the world's energy system is on the verge of a major transformation. Energy systems can refer to Electric power system (supply, transmission and consumption of electric power), thermodynamic system (a physics concept for analysis of thermal energy exchange) and bioenergetics system (metabolic processes for converting energy in living organisms). This thesis deals with electricity markets and components involved in it.

Various technologies are building blocks for the transition to a sustainable energy future. Sustainable energy not only focuses on renewable energy sources such as hydroelectricity, solar energy, wind energy, wave power, geothermal energy, bio-energy, tidal power but also technologies designed to improve energy efficiency. There are various ways in which energy efficiency can be improved; one such way is to integrate various forms of energy in order to achieve the maximum utilization of all the sources involved in integration.

In this thesis, our focus on electric energy market is to improve the efficiency of the overall system. The latter is multi-faceted in nature. For example, electricity can be generated by multiple sources such as coal, natural gas, wind, solar, etc. Within each generation type, there are multiple market players such as coal producers, coal-fired power generation companies and electric power grid owners for the coal-firing generation. Similarly, nature gas producers, pipeline owners, pipeline operators and power grid owners are key players in the gas-firing generation. In the literature, many researchers have studied optimization models for one player or one subsystem. For example, Kolstad and Abbey [3] examine the effects of market behavior on international steam coal trade while Thompson et al. [4] study natural gas storage optimization problems. However, few papers deal with integration of coal and natural subsystems. In our view, only when these subsystems are studied jointly can we accurately assess the overall electricity market and its behavior. Therefore, the research in this thesis aims to fill this gap in the literature by integrating all the components (i.e., subsystems) involved in the electricity market and study the economic equilibrium for this integrated electricity market system.

To further motivate the development of our proposed economic equilibrium model for the electricity market, in the last decade, the electricity industry has experienced significant changes towards deregulation and competition with the aim of improving economic efficiency (see e.g. [1]). When markets are deregulated, pricing of commodities and services should be purely based on market supply and demand. Therefore, an economic equilibrium model and its resulting market-clearing prices can be useful for policy makers to assess the overall system efficiency. Indeed, along the same line of motivation, many have studied equilibrium models in the energy research. For example, Kazempour and Hopkins [5] analyze the impact of large-scale wind power integration on the electricity market equilibrium. As another example, Fuller et al. [6] propose a mathematical model to determine the optimal energy storage systems (ESS) operation as well as the market clearing prices. Our proposed economic equilibrium model is guided by the same principle aiming to offer insights for a deregulated electricity market.

In particular, the proposed economic equilibrium model for electricity markets considers seven types of market players: electricity generation firms that own both coal and gas fired generators, natural gas producers, natural gas pipeline operators, natural gas marketers, and coal mine owner, and finally electricity consumers and natural gas consumers. All generation and services are subject to fixed and deterministic capacities, while consumer demands for electricity are elastic. We develop mathematical models that each player (or subsystem) wishes to optimize and then integrate these optimized subsystems into an equilibrium model by using market-clearing conditions to represent the interaction between pairs of players. In other words, the market equilibrium model optimizes each of the individual components (i.e. electricity sector, natural gas sector and coal sector) and ensures that the market clearing conditions are satisfied whenever the individual components interact with each other. It is important to note that, as by products, the market-clearing prices for the equilibrium model offer guidelines for commodity pricing such as electricity price at demand region and coal prices for different regions in the electricity market.

The contribution of this thesis is two-fold. First, we develop an equilibrium model to study an electricity market consisting of coal producers, electricity generation firms, natural gas producers, natural gas marketers, natural gas pipeline owners, natural gas consumers and electricity consumers. The equilibrium model not only captures a decentralized optimization for each player but considers the interaction amongst players. Second, we formulate the equilibrium model as a mixed complementarity problem (MCP) and conduct various sensitivity analyses to shed lights on energy policy making.

The rest of this thesis is organized as follows. Chapter 2 reviews the literature on the electricity market, coal market, natural gas market and equilibrium models in electricity market. Chapter 3 formulates the mathematical models for each of the components involved in the electricity market. Chapter 4 presents the computational experiments and their results and conclusions and future research are presented in Chapter 5.

CHAPTER 2

LITERATURE REVIEW

In this chapter, we review the body of research in the operations research literature that is closely related to our research in this thesis. This review is divided into five parts on: electricity market optimization, natural gas market optimization, coal market optimization, integrated energy systems and finally the applications of mixed complementarity problems, respectively.

2.1 Optimization Models for Electricity Markets

Optimization of electricity markets is widely studied by many researchers with various objectives or constraints. For example, Hobbs [7] considers imperfect competition among electricity producers. This paper focuses on developing a linear complementarity model of the Nash-Cournot competition in bilateral and POOLCO power markets (A POOLCO is a privatized power exchange that operates auctions, hosts spot market sales and generally functions as a privately owned market place for energy sales in the wholesale marketplace). The model is formulated as mixed linear complementarity problem. The model considers two types of players, i.e., electricity producers and grid owners, and its solutions are essentially the imperfectly competitive equilibria. Similarly, Cardell et al. [15] present a work on market power and strategic interaction in electricity markets. This paper presents a model where a firm no longer exercises market power by restricting its own production. This model constrains the electrical network where generator exercises market power by increasing its production in order to block transmission. The players in the model are Cournot firms and a collection of competitive fringe participants. This model illustrates the possible strategic interactions between the players. Furthermore, Ramos et al. [16] model the competitive behavior of electric firms by incorporating equilibrium constraints. These constraints provide characterization of the first order optimality conditions of strategic companies. The objective of this approach is to maximize profit while meeting the equilibrium constraints.

One other stream of research in the electricity market optimization studies the market power of various players. For example, Kahn et al. [17] model the electric markets for estimating the price or cost margins; however, they conclude that the model may not be able to tell whether a firm or set of firms can succeed in manipulating market prices. Borenstein et al. [19] provide an an empirical analysis of the potential for market power in Californias electricity industry. The model in this paper shows the two most important factors in determining the extent and severity of market power. The factors are available hydroelectric production and the elasticity of demand. The model indicates that there is potential for significant market power during high demand hours (where elastic demand comes into picture). Finally, Helman et al. [18] present a strategic pricing model in bilateral and Poolco electricity markets by using Nash-Cournot approach and use the U.S. eastern interconnection as a case study. This paper has two specific models a Nash-Cournot framework and represents transmission constraints by a linearized DC network. The formulation in this model helps in computation for larger markets while guaranteeing the existence of unique price equilibria. As mentioned previously, the above literature mainly focuses on electricity markets optimization, but not-integrated energy system as a whole. The major difference between these literatures and the current thesis is that we study an integrated energy equilibrium model that includes coal market, electricity market, natural gas market and elastic demand.

2.2 Optimization Models for Natural Gas Markets

Natural gas markets optimization is not new in the literature but they have generally focused on either optimization of gas operations or computation of market equilibrium prices, flows and quantities. We review several papers in this area.

Some works present natural gas market equilibrium model that involves natural gas market alone. For instance, Gabriel et al. [8] discuss mixed complementarity based equilibrium model of natural gas markets. This paper presents a natural gas market equilibrium model that has producers, storage reservoir operators, peak gas operators, pipeline operators, marketers and consumers. The equilibrium model is an instance of a mixed nonlinear complementarity problem for natural gas market (NCP). The NCP formulation is derived from considering the Karush-Kuhn- Tucker optimality conditions of the optimization problems faced by these participants. The natural gas equilibrium model is validated by considering nine market participants, three seasons and using four scenarios. Similarly, Zhuang et al. [20] discuss a large-scale complementarity model of the North American natural gas market. This paper analyses the natural gas market using a linear complementarity equilibrium model. The players considered are producers, storage and peak gas operators, third party makers and end user sectors. The model is validated based on National Petroleum Council scenarios. Few researches also study natural gas optimization models with main focus on cost minimization. Avery et al. [21] develop an optimization model for purchase, storage and transmission contracts for natural gas utilities. The model minimizes cost while satisfying regulatory agencies and presents a decision support system for natural gas utilities to plan operations. The model considers wellhead, consumers, transport and storage as its players.

Additionally, there are few works that consider optimization techniques to meet natural gas demand. Zheng et al. [22] model the natural gas markets for meeting the demand. The players considered are natural gas producers, transportation network, and market (consumers). Optimization techniques are widely used to meet the demand for natural gas and have yielded a lot of promising results. The pipeline network for natural gas markets is also widely studied. There are optimization models for dimensioning of pipeline networks. De Wolf et al. [23] present optimal dimensioning of pipe networks with applications to gas transmission networks. While all these works deal either with natural gas market in a decentralized fashion or develop an equilibrium model for natural gas market alone, our thesis presents an equilibrium model for electricity markets and its players as an integrated system.

2.3 Optimization Models for Coal Markets

Coal was the first fossil fuel used for heat and power. Coal consumption declined as oil and natural gas replaced coal in heating, electric engines, steam engines and motors. Throughout the long period in decline of coal consumption, electric utilities expanded their use of coal enormously.

There are quite a lot of models that discuss minimization of costs for fuel inputs, forecasts of coal prices, coal production and consumption. There are programming models for coal markets that analyze the demand for coal. Labys et al. [25] present a quadratic programming model of the Appalachian steam coal market. The authors attempt to analyze the Appalachian steam coal market using a programming approach. The destinations of Appalachian coals have been determined on the basis for the demand for steam coal created by major utility companies. This model determines the extent to which Appalachian coal can meet eastern US steam coal demands by minimizing costs of fuel inputs. Energy Information Administration [EIA] developed a mathematical model for computer implementation of the National Coal Model [26]. This report contains the objectives used in the development of the National Energy Modeling System (NEMS) Coal market Module. The conceptual and methodological approach is used for the development of this system. The CMM provides annual forecast of prices, production, and consumption of coal for the NEMS. The CMM has two submodules Coal Production Submodule (CPS) and Coal Distribution Submodule (CDS). The CPS provides supply inputs that are integrated by the CDS to satisfy demands for coal received from demand models. The CDS forecasts annual world coal trade flows from major supply to major demand regions and provides annual forecasts of U.S coal exports for input into NEMS. This work completely focuses on forecasts of prices, production and consumption of COAL only.

There are few works that formulates an equilibrium model for coal markets.

Haftendorn et al. [10] models to assess international coal markets until 2030. This paper presents an equilibrium model of international market for steam coal. The players considered in this model are producers and exporters. This equilibrium model is formulated in the complementarity format. This model is tested by using a base case scenario and suggestions for alternative scenarios. Franziska et al [28] model and analyse the International Steam Coal trade. This paper presents the analysis of prices and trade flows for steam coal in the international market. This is done by simulating the market for a couple of years using the complementarity modeling technique. The paper presents two models 1) quality based model for coal and 2) a model that incorporates energy values. The conclusion of this paper is that an energy-based model is more superior than a quality based model. Complementarity format discussed in these two works Haftendorn et al. [10] and Franziska Holz et al [28] exactly fits into a part of the work done for this thesis.

Furthermore, there are transportation models in the literature review for coal markets. LeBlanc [27] formulates a transportation Model for the US Coal Industry. This work presents a general economic model that minimizes the cost of coal shipments in the United States. The economic model described is a linear programming model. This model assumes coal demand and sulfur dioxide emission regulations. The results of the model indicate significant differences in flows and shadow prices under varied assumptions. As Demand plays a vital role in any supply chain, there are literature reviews that model for coal demand. Labys et al. [25] and Maggi et al. [29] has their work on development and perspectives on supply and demand in the global hard coal market. They discuss the root causes for the extreme price developments of coal and provides insights on changing supply and demand structure within the seaborne hard coal market. Maggi et al. [29] build analytical methods for the development and perspectives on supply and demand in the global hard core market. These are few literature reviews that model coal market by itself for various reasons mentioned but none of them consider modelling two or more different systems/entities and solve for equilibrium.

2.4 Optimization Models for Integrated Energy Systems

Integrated energy systems play a vital role in efficient energy planning and sustainable development. There are very few works in the literature review that consider modelling the energy system as a whole. Quelhas et al. [9] present a multi-period generalized network model of the U.S. integrated energy system. It is a multi-period generalized network flow model of the integrated energy system in the United States. The players considered in the model are coal and natural gas suppliers to the electric load centers. The mathematical model is developed by connecting the electricity demand nodes and fuel supply via a transportation network. The model incorporates production, storage and transportation of coal, natural gas and electricity in one mathematical framework. The model is solved for the most efficient allocation of quantities and corresponding prices. The objective of the proposed model is to minimize the total costs which include fuel production cost, fuel transportation costs, fuel storage costs, electricity generation costs and electricity transmission costs. It also provides numerical results which describes the application of proposed model. There are few literature reviews that consider modelling the integrated energy systems for economic studies. Gil et al. [30] develop an integrated energy transportation networks for analysis of economic efficiency and network interdependencies. This paper presents an integrated mathematical framework for coal, gas, water, and electricity production and transportation. The model named as the National Electric Energy System (NEES) is formulated using a network flow optimization model. It is modelled fundamentally by balancing energy at various nodes such as production nodes, storage nodes, generation nodes and electric transmission nodes. The objective function is set to minimize the total cost in the entire framework. The solution for this model is an algorithm anchored in the network simplex method. Also, McCalley et al. [31] model an integrated energy system for determining nodal prices. This is a generalized network flow model for integrated energy systems. This model is used to analyze the economic interdependencies of energy systems. This model comprises multiple entities such as electric network and fuel supply and delivery systems. The model is solved using an optimization algorithm that also provides nodal prices as a byproduct. These nodal prices provide a way to analyze the economic interdependencies between the various fuel networks and the electric network. (Nodal pricing is defined as a method of determining prices in which market clearing prices are calculated for a number of locations on the transmission grid called as nodes).

However, all the above researches are trying to model the integrated energy systems but none of them considers the competition among participants. Hence, in this thesis we intend to develop an integrated economic equilibrium model for electricity markets that consider electricity market (production and transmission), elastic demand of electricity by consumers, natural gas market (upstream natural gas producers, midstream natural gas pipeline owners, downstream natural gas marketers) and coal market (coal producers) along with market clearing conditions that serve as binding bridge among these players.

2.5 Formulation of Mixed Complementarity Problems

In our thesis, optimization models are developed for each of the players involved in the integrated energy system. All these players are interconnected to each other by market clearing conditions. These individual optimization models are modelled into a single equilibrium model by formulating into a mixed complementarity problem. There are few works in the literature review that use a similar approach for modelling. Few such literatures are Cottle et al. [33] explain the formulation of linear complementarity problem. Similarly, Gabriel et al. [37] present complementarity modeling in Energy markets. This book presents clear picture of the modeling advantages of complementarity problems vs optimization and standard models. Modelling for equilibrium constraints is seen in some literature reviews, Luo et al. [35] model mathematical programs with equilibrium constraints. This paper mentions the method for binding various players using equilibrium constraints which is used as an example in our thesis. Dirks et al. [34] has a collection of nonlinear mixed complementarity problems. This paper explains the formulation for nonlinear mixed complementarity problems which is used for nonlinear cost functions in my thesis.

After developing the mathematical model for the integrated energy system as a whole, it is tested for numerical results and validation. General Algebraic Modeling System (GAMS) is one such mathematical modelling language used by many researchers. Ferris et al. [32] describe complementarity problems in GAMS and the PATH solver. This paper presents the method to find a solution for a square system of nonlinear equations by converting them into complementarity problem. This paper explains the methodology for solving such problems in GAMS and provides details about the PATH solver for finding the solution. There are also works on extensions of GAMS for complementarity problems arising in applied economic analysis. All of the above cited literature has been used to implement the developed integrated energy equilibrium model into GAMS and obtain numerical results.

2.6 The choice of production cost functions for electricity, coal, natural gas and elastic demand

The production cost functions for coal, natural gas and electricity have been widely cited in many literature reviews. Few such researches have been studied during the course of our thesis. Bhagwat et al. [38] develop a report on cost of underground coal mining in Illinois. This report has been used as reference for coal production cost function. They presents a linear production cost function for coal that depend on various factors such as annual production, age of mines, labor productivity, mine development cost, coal cleaning level. On considering the typical values for all these factors, a constant value has been determined. The final coal transportation cost used in our thesis has turned out to be a linear cost function.

Gabriel et al. [8] develop a mixed complementarity-based equilibrium model of natural gas markets. This paper has been used a reference for production cost function of electricity and natural gas. The electricity production cost function is a linear function and is used as such. The natural gas cost function is modified according to the units considered in my thesis.

For elastic demand, there are quite a lot of literature reviews major them being traffic models. Dafermos [39] presents a multimodal network equilibrium problem with elastic demand. It defines the concepts of user-optimality and equilibrium. The algorithm proceeds by iteration, each step of which amounts to computing the equilibrium pattern for a single modal linear traffic equilibrium problem with elastic demands. Arnott et al. [40] develop a structural model of peak-period congestion. This paper considers the modeling of road congestion subject to peak-load demand. The model treats elastic (i.e., price-sensitive) demand and examines some economic implications of the structural approach. There are very few works that consider elastic demand in energy markets and are mainly seen in bidding strategies for electricity markets Wang et al. [41]. In this paper they propose an evolutionary imperfect information game approach to analyze bidding strategies in electricity markets with price-elastic demand. All of the above cited literature have been used as a basis for elastic demand function used in our thesis.

CHAPTER 3

THE ELECTRICITY MARKET EQUILIBRIUM MODEL

In this chapter, we discuss the mathematical model behind EMEM. This chapter is divided into two parts on: notations and assumptions, and mathematical model for integrated energy system.

3.1 Notations and Assumptions

As mentioned previously, we consider an electricity market consisting of seven elements: power generation firms that own both coal-firing and gas-firing generators, coal producers, natural gas producers, natural gas pipeline owners, natural gas marketers, natural gas consumers and electricity consumers. The relationships between these elements are as follows. First, electricity and natural gas consumers are represented by electricity demand nodes (i) and natural gas demand regions (j), respectively. Second, each natural gas producer n belongs to a natural gas supply region w through a mapping function $\tau(n) = w$, and each coal producer m belongs to a coal supply region v through a mapping function $\tau(m) = v$. Third, each generation unit u belongs to a generation firm f through a mapping function $o(u) = f$. Additionally, unit u has a designated generation technology $h(u)$ to indicate the coal type if coal-firing or if it is gas-firing and a designated transmission network through a mapping function $\theta(u)$. Finally, we consider two uses $k = 1, 2$ of natural gas: generating electricity or others including industrial, commercial and residential. They have separate demands at each natural gas demand region j. These elements each has associated attributes. A generation unit u is associated with its unique heat rate (MMBTU/MWh), heat content (MMBTU/short ton), non-fuel related marginal costs (\$/MWh), production cost (\$/MWh), and capacity

(MW). Furthermore, natural gas producer and coal producer each has production capacity. For natural gas pipeline owner, each pipeline l has its origin, destination regions as well as capacity (MCF). Figure 1 provides a schematic illustration for a small integrated system and Table 1 summarizes the sets, indices and parameters for these components used in our model.

$i = 1, \ldots, I$	electricity demand nodes
$j = 1, \ldots, J$	natural gas demand regions
$n = 1, \ldots, N$	natural gas producers
$w = 1, \ldots, W$	natural gas producer locations
$\tau(n)$	mapping of natural gas producer n to its location (i.e., $\tau(n) = w$, $w = 1, \ldots, W$)
$\omega(n)$	mapping of natural gas producer n to its supply region w (i.e., $\omega(n) = w$, $w = 1, \ldots, W$)
$m = 1, \ldots, M$	coal mine owners
$v = 1, \ldots, V$	coal mine locations
$\tau(m)$	mapping of coal mine owner m to its location (i.e., $\tau(m) = v$, $v = 1, \ldots, V$)
$\tau(u)$	mapping of coal unit u to its coal supply region (i.e., $\tau(u) = v$, $v = 1, \ldots, V$)
$f = 1, \ldots, F$	power generation firms, with each firm possibly owning several units
$u = 1, \ldots, U$	generation units
$o(u)$	mapping of a generation unit u to its owner f (i.e., $o(u) = f$)
$h(u)$	electricity generation technology of unit u (coal, natural gas);
$k = 1, \ldots, K$	fuel consumption sectors (electricity, industrial, commercial, residential)
$o(k)$	mapping of sector k (eg. $o(k) = 1$ means that k is electricity sector)
$a = 1, \ldots, A$	natural gas marketers
$l = 1, \ldots, L$	natural gas pipelines
$o(l), d(l)$	the origin and destination of pipeline l
HR_u	heat rate of generation unit u [MMBTU/MWh]
HC_c	heat content of coal type c [MMBTU/short ton]
$C_u(\cdot)$	non-fuel related marginal costs for generation unit u [\$/MWh]
$C_n(\cdot)$	production cost for natural gas producer n [\$/MCF]
$\bar{X}_u$	capacity of generation unit u [MW]
$\bar{G}_n$	natural gas production upper bound for producer n [MCF]
$\bar{L}_l$	pipeline l capacity [MCF]
$\bar{Z}_m$	capacity of coal mine m [short ton]
t	time periods
H_t	number of hours in period t
D_t	number of days in period t

TABLE 1

Description of parameters in EMEM

The equilibrium model is driven by the maximization of the total utility of electricity and natural gas consumers. It will determine, while respecting capacity constraints at various player sites, the production levels for electricity generation units, natural gas producers and coal producers, and the flows from coal producers to generation units, from natural gas producers to pipelines then to generation units, from natural gas producers to pipelines then to natural gas demand regions, and from generation units to electricity demand regions. Table 2 summarizes the variables representing these quantities and some associated function used in the mathematical model.

x_{ut}	electricity generated by unit u in time t [MW]
s_{uit}	electricity sales by unit u to demand region i in time t [MW]
g_{nt}	natural gas produced by producer n in time t [MCF/period]
z_{mt}	coal produced by owner m in time t [short ton/period]
q_{awjkt}	natural gas bought by marketer a from supply region w to ship to region j, sector k in time t [MCF/period]
f_{lt}	natural gas flow through pipeline l
p^g_{wt}	upstream (wellhead) natural gas prices at region w in time t [\$ / MCF]
p^g_{jkt}	downstream natural gas prices paid by consumers in region j, sector k in time t [\$/MMBTU]
$P^g_{jkt}(\cdot)$	inverse demand function of natural gas in region j, sector k in time t
p^l_t	natural gas transportation rates for pipeline l in time t [\$/MCF]
p^c_{vt}	coal prices for region v coal in time t [\$/short ton]
$p^s_{vo(u)}$	shipping cost of transporting coal from coal mine region v to the region of generation unit u [\$/short ton]
p^e_{it}	electricity price at demand region i [\$/MWh]
$P^e_{it}(\cdot)$	inverse demand function of electricity at node i in time t [\$/MWh]
d^e_{it}	electricity demand at region i in time t [MW]

TABLE 2

Variables and Functions in EMEM

3.2 Mathematical model for Integrated Energy System

In this section, we examine each players optimization problem first, and then present the integrated equilibrium model. First, optimization problem for generation firm $f's$ is discussed. Firm f objective function is defined as maximization of its profit. The first term $\sum_i H_t p^e_{it} s_{uit}$ represents the revenue of f generated by total sales of electricity by u to node i at t. $C_u(H_t x_{ut})$ is the cost for generation of electricity for u. These two terms are summed over those generation units u that belong to firm f. Cost incurred due to natural gas is represented by

$p^g_{\tau(u)Et}H_tHR_ux_{ut}$ and is summed over natural gas fired generation units. Cost for purchasing coal is defined as $p^c_{h(u)t}(HR_u/HC_{h(u)})x_{ut}H_t$ and cost for shipping coal is defined as $p^s_{vo(u)}(HR_u/HC_{h(u)})x_{ut}H_t$. Both of these terms are summed over coal fired generation units u that belong to firm f. Hence, profit for firm f is defined as revenue minus sum of all costs. The constraint $s_{uit} = x_{ut}$ means that electricity generated by u at t is equal to sales of electricity by u to i at t summed over all demand nodes i.

Electricity generation firm f:

$$\begin{aligned}
\max \quad & \sum_t \{ \sum_{u:o(u)=f} [\sum_i H_t p^e_{it} s_{uit} - C_u(H_t x_{ut})] - \sum_{u:h(u)=\text{gas}} p^g_{\tau(u)Et} H_t HR_u x_{ut} - \\
& \sum_{u:h(u)=\text{coal}} [p^c_{h(u)t}(HR_u/HC_{h(u)})x_{ut}H_t + p^s_{vo(u)}(HR_u/HC_{h(u)})x_{ut}H_t]\} \qquad (1) \\
\text{s.t.} \quad & \sum_i s_{uit} = x_{ut}, \quad \forall t, \forall u : O(u) = f \quad (\lambda_{ut}) \qquad (2) \\
& s_{uit}, x_{ut} \geq 0,
\end{aligned}$$

Consumer's utility optimization problem is defined as follows. The objective function tries to maximize their amount of electricity for least amount of money. The first term $\sum_t \sum_i \int_0^{d^e_{it}} P^e(\mu_{it})d\mu_{it}$ is the inverse demand function integrated over electricity demand d^e_{it}. It is the price at which customers are willing to pay for electricity. The second term $\sum_t \sum_i p^e_{it}d^e_{it}$ is the actual price for electricity demanded.

Electricity consumers utility maximization:

$$\begin{aligned}
\max \quad & \sum_t \sum_i \int_0^{d^e_{it}} P^e(\mu_{it})d\mu_{it} - \sum_t \sum_i p^e_{it}d^e_{it} \qquad (3) \\
\text{s.t.} \quad & d^e_{it} \geq 0,
\end{aligned}$$

Supply must always be equal to demand and following equation represents it. It is the market clearing condition between electricity generators and consumers.

Electricity market clearing condition:

$$d^e_{it} = \sum_u s_{uit}, \; \forall i, t \qquad \cdots p^e_{it} \qquad (4)$$

Next, optimization problem for coal producers is discussed. The term $\sum_t p^c_{\tau(m)t} z_{mt}$ represents revenue generated by coal producer m by selling z_{mt} shortton of coal at price $p^c_{\tau(m)t}$. Cost for production of coal is represented as $C_m(z_{mt})$. The constraint z_{mt} less than or equal to $\bar{Z}_m$ says that coal production is limited to a certain capacity.

Coal mine owner m:

$$\begin{aligned} \max \quad & \sum_t p^c_{\tau(m)t} z_{mt} - \sum_t C_m(z_{mt}) & (5) \\ \text{s.t.} \quad & z_{mt} \leq \bar{Z}_m, \qquad (\gamma_{mt}) & (6) \\ & z_{mt} \geq 0, & \end{aligned}$$

Total coal produced by all producers m must be utilized for generation of electricity. The mathematical form for this market clearing condition is defined as follows.

Coal market clearing condition:

$$\sum_{m:\tau(m)=v} z_{mt} = \sum_{u:h(u)=\text{coal},\tau(u)=v} (HR_u/HC_{h(u)}) x_{ut} H_t, \quad \forall v, t \qquad (p^c_{vt}) \tag{7}$$

Natural gas producer optimization problem is discussed as follows. Like coal and electricity players, natural gas also has similar objective function. The term $\sum_t D_t p^g_{w(n)t} g_{nt}$ represents the revenue for producer n generated by selling natural gas at a price of g_{nt}. The production cost is defined as $\sum_t D_t C_n(g_{nt})$. Natural gas producer n is allowed to produce up to certain capacity $\bar{G}_n$.

Natural gas producer n:

$$\begin{aligned} \max \quad & \sum_t D_t p^g_{w(n)t} g_{nt} - \sum_t D_t C_n(g_{nt}) & (8) \\ \text{s.t.} \quad & g_{nt} \leq \bar{G}_n, \qquad (\delta_{nt}) & (9) \\ & g_{nt} \geq 0, & \end{aligned}$$

Natural gas produced by all the producers n that belong to region w must be equal to the amount of natural gas bought by marketers a from supply region w to ship to regions j, sectors k in time t. This market clearing condition is mathematically defined as follows.

Upstream Natural gas market clearing condition:

$$\sum_{n:w(n)=w} g_{nt} = \sum_a \sum_j \sum_k q_{awjkt}, \quad \forall w, t \qquad (p^g_{wt}) \tag{10}$$

Natural gas pipeline operator l maximization problem depends on flow through pipeline l and natural gas transportation rates for pipeline l in time t. The flow through pipeline l (f_{lt}) is restricted with an upper bound capacity of L_l.

Natural gas pipeline operator l:

$$\max \quad \sum_t D_t p^l_t f_{lt} \tag{11}$$

$$\text{s.t.} \quad f_{lt} \leq L_l, \ \forall t \qquad (\alpha_{lt}) \tag{12}$$

$$f_{lt} \geq 0,$$

Market clearing condition for midstream pipeline market is merely flow conservation principle. Flow through pipeline l at time t must be equal to natural gas bought by marketers a from supply regions w to ship to regions j, sectors k in time t. Mathematically, it can be represented as follows.

Midstream pipeline market clearing condition:

$$f_{lt} = \sum_a \sum_k q_{ao(l)d(l)kt} \ \forall l, t \qquad (p^l_t) \tag{13}$$

For natural gas marketer a, $\sum_t \sum_j \sum_k D_t p^g_{jkt} \cdot (\sum_w q_{awjkt})$ represents revenue generated by selling natural gas to consumers. Cost for buying natural gas from producers is shown as $\sum_t \sum_w D_t p^g_{wt} \cdot (\sum_j \sum_k q_{awjkt})$ and cost for natural gas transforation is $\sum_t D_t p^l_t \cdot (\sum_l \sum_k \ q_{ao(l)d(l)kt})$. Hence, maximization problem for marketer a is defined as follows.

Natural gas marketer a:

$$\begin{aligned}
\max \quad & \sum_t \sum_j \sum_k D_t p^g_{jkt} \cdot (\sum_w q_{awjkt}) - \sum_t \sum_w D_t p^g_{wt} \cdot (\sum_j \sum_k q_{awjkt}) - \\
& \sum_t D_t p^l_t \cdot (\sum_l \sum_k \; q_{ao(l)d(l)kt}) \qquad (14) \\
\text{s.t.} \quad & q_{awjkt} \geq 0,
\end{aligned}$$

Market clearing condition for downstream natural gas is similar to coal market clearing condition, i.e. Total natural gas produced must be utilised by both electric and non-electric sectors. For k = electric sector, 30% of natural gas produced must be fully utilized for production of electricity. For k = non - electric sector, 70% of natural gas must be utilized by non-electric sector. Mathematical form for this is defined as follows.

Downstream natural gas consumption market clearing condition:

$$\begin{aligned}
\sum_a \sum_w 1.028 \cdot D_t q_{awjkt} = & \sum_{u:\ o(u)=j,\ h(u)=\text{gas}} H_t HR_u x_{ut},\ \forall j, t, k = \text{elec sector} \quad (p^g_{jkt}) \\
\sum_a \sum_w q_{awjkt} = & \quad (7/3) \sum_a \sum_w \sum_{k:k=\text{elec sector}} q_{awjkt},\ \forall j, t, k = \text{non} - \text{elec sector} \quad (p^g_{jkt})
\end{aligned} \qquad (15)$$

Individual optimization problems for coal, electricity and natural gas markets are integrated into equilibrium model by using market clearing conditions. This model is presented as mixed complementarity problem (MCP) by using Karush Kuhn Tucker (KKT) conditions and Lagrangian Equations. $L^f(x, s; \lambda)$ represents Lagrangian Equation for electricity markets. $\frac{\partial L^f}{\partial s_{uit}}$ is the first order derivative of Lagrangian Equation w.r.t positive variable s_{uit}. Similarly, $\frac{\partial L^f}{\partial x_{ut}}$ is the first order derivative w.r.t positive variable x_{ut}. Likewise, $L^c(d)$, $L^n(g; \delta)$, $L^l(f; \alpha)$, $L^a(q)$ and $L^m(z; \gamma)$ are Lagrangian Equations for consumer's utility, natural gas producers, natural gas pipeline owners, natural gas marketers and coal producers respectively. The first order derivatives of all Lagrangian Equations w.r.t positive variables form MCP and are shown under Lagrangian Equations.

Parameters	H_t
	HR_u
	$HC_{h(u)}$
	$p^s_{vo(u)}$
	$\bar{Z}_m$
	D_t
Positive Variables	x_{ut}
	s_{uit}
	d^e_{it}
	z_{mt}
	g_{nt}
	f_{lt}
	q_{awjkt}
Functions	$C_u(H_t x_{ut})$
	$C_m(z_{mt})$
	$P^e(\cdot)$
	$C_n(g_{nt})$

<u>Lagrangian Equations:</u>

$$L^f(x,s;\lambda) = -\sum_t \{ \sum_{u:o(u)=f} [\sum_i H_t p^e_{it} s_{uit} - C_u(H_t x_{ut})] - \sum_{u:h(u)=\text{gas}} p^g_{\tau(u)kt} H_t HR_u x_{ut}$$

$$- \sum_{u:h(u)=\text{coal}} [p^c_{vt}(HR_u/HC_{h(u)}) x_{ut} H_t + p^s_{vo(u)}(HR_u/HC_{h(u)}) x_{ut} H_t]\}$$

$$+ \sum_t \sum_{u:o(u)=f} [\lambda_{ut} \cdot (x_{ut} - \sum_i s_{uit})]$$

$$\frac{\partial L^f}{\partial s_{uit}}|_{u:o(u)=f} = 0 \Longrightarrow -H_t p^e_{it} - \lambda_{ut} = (\geq)0 \tag{16}$$

$$\frac{\partial L^f}{\partial x_{ut}}|_{u:o(u)=f,h(u)=\text{coal},h(u)\neq\text{gas}} = 0 \Longrightarrow \frac{\partial C_u(H_t \cdot x_{ut})}{\partial(H_t \cdot x_{ut})} \cdot H_t$$

$$+ p^c_{\tau(u)t} \cdot (HR_u/HC_{h(u)}) \cdot H_t + p^s_{\tau(u)o(u)} \cdot (HR_u/HC_{h(u)}) \cdot H_t + \lambda_{ut} = (\geq)0 \tag{17}$$

$$\frac{\partial L^f}{\partial x_{ut}}|_{u:o(u)=f,h(u)\neq\text{coal},h(u)=\text{gas}} = 0 \Longrightarrow \frac{\partial C_u(H_t \cdot x_{ut})}{\partial(H_t \cdot x_{ut})} \cdot H_t + p^g_{\tau(u)kt} \cdot H_t HR_u + \lambda_{ut} = (\geq)0 \tag{18}$$

$$\begin{aligned} L^c(d) &= -\sum_t \sum_j \int_0^{d^e_{it}} P^e(\mu_{it}) d\mu_{it} + \sum_t \sum_i p^e_{it} d^e_{it} \\ \frac{\partial L^c}{\partial d^e_{it}}|_{u:o(u)=f} &= 0 \Longrightarrow -P^e(d^e_{it}) + p^e_{it} = (\geq)0 \end{aligned} \tag{19}$$

$$\begin{aligned} L^n(g;\delta) &= -\sum_t D_t p^g_{wt} g_{nt} + \sum_t D_t C_n(g_{nt}) + \sum_t \delta_{nt}(g_{nt} - \bar{G}_n) \\ \frac{\partial L^n}{\partial g_{nt}} &= 0 \Longrightarrow -D_t p^g_{wt} + D_t \frac{\partial C_n(g_{nt})}{\partial g_{nt}} + \delta_{nt} = (\geq)0 \end{aligned} \tag{20}$$

$$\begin{aligned} L^l(f;\alpha) &= -\sum_t D_t p^l_t f_{lt} + \sum_t \alpha_{lt}(f_{lt} - \bar{L}_l) \\ \frac{\partial L^l}{\partial f_{lt}} &= 0 \Longrightarrow -D_t p^l_t + \alpha_{lt} = (\geq)0 \end{aligned} \tag{21}$$

$$\begin{aligned} L^a(q) = -\sum_t \sum_j \sum_k 1.028 D_t p^g_{jkt} \cdot (\sum_w q_{awjkt}) &+ \sum_t \sum_w D_t p^g_{wt} \cdot (\sum_j \sum_k q_{awjkt}) \\ &+ \sum_t D_t p^l_t \cdot (\sum_l \sum_k \; q_{ao(l)d(l)kt}) \\ \frac{\partial L^a}{\partial q_{awjkt}} = 0 \Longrightarrow -D_t p^g_{jkt} &+ D_t p^g_{wt} + D_t p^l_t = (\geq)0 \end{aligned} \tag{22}$$

$$\begin{aligned} L^m(z;\gamma) &= -\sum_t p^c_{\tau(m)t} z_{mt} + \sum_t C_m(z_{mt}) + \sum_t \gamma_t(Z_{mt} - \bar{Z}_m) \\ \frac{\partial L^m}{\partial z_{mt}} &= 0 \Longrightarrow -p^c_{\tau(m)t} + \frac{\partial C_m(z_{mt})}{\partial z_{mt}} + \gamma_{mt} = (\geq)0 \end{aligned} \tag{23}$$

Variables	s_{uit}	x_{uit}	d^e_{it}	g_{nt}	f_{lt}	q_{awjkt}	z_{mt}	λ_{ut}	γ_{mt}	δ_{nt}	α_{lt}
Constraints	(16)	(17)(18)	(19)	(20)	(21)	(22)	(23)	(2)	(6)	(9)	(12)
Variables	p^e_{it}	p^c_{vt}	p^g_{wt}	p^l_t	p^g_{jkt}						
Constraints	(4)	(7)	(10)	(13)	(15)						
Complementarity	positive variables: $x_{ut} \geq 0$, $s_{uit} \geq 0$, $d^e_{it} \geq 0$,$g_{nt} \geq 0$,$f_{lt} \geq 0$										
	$q_{awjkt} \geq 0$, $z_{mt} \geq 0$, $\gamma_{mt} \geq 0$, $\delta_{nt} \geq 0$, $\alpha_{lt} \geq 0$										
	free variable(λ_{ut}) matches with equality constraints										

Table shown above lists the relationship between constraints and variables in MCP. For example, s_{uit} is tied to Equation 16, p^e_{it} shadow price for electricity and is tied to Equation 4. Likewise all the variables that are related to their respective constraints are briefed in this table.

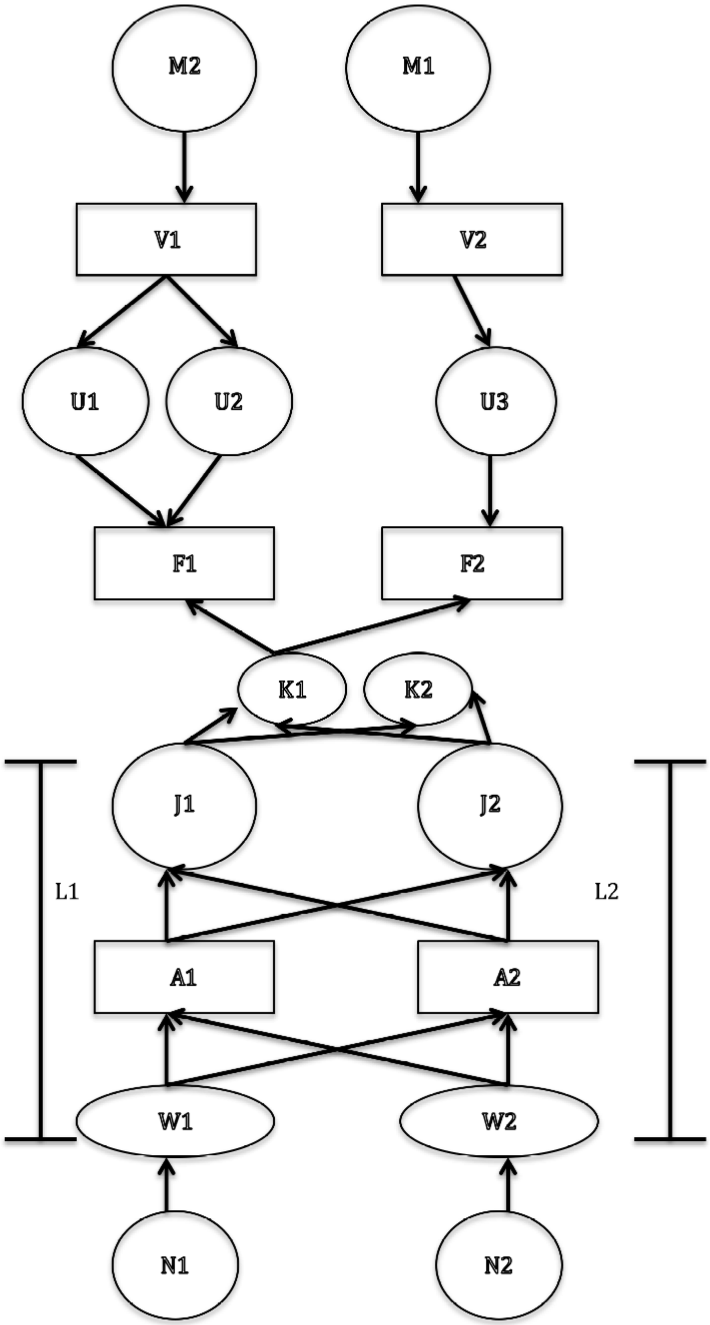

Figure 1. The network for Electricity Market with Seven Players

CHAPTER 4

COMPUTATIONAL RESULTS

In this chapter we present the numerical results for EMEM model. This chapter is divided into five parts on: parameters and functions setup, EMEM without natural gas, EMEM with natural gas,EMEM with natural gas and non-electric sector, sensitivity analysis on EMEM.

4.1 Discussions on Parameter and Functions setup

We evaluate the proposed equilibrium model in the MCP format through simulation using the PATH solver in GAMS (General Algebraic Modeling System). In this section we discuss parameters and functions used in the model. Table 3 shows heat rate and heat content of coal for generation units u. Heat content for generation units u_4 and u_5 is not specified since they are natural gas fired units. Table 4 shows the shipping cost of transporting coal from coal mine region v to the region of generation unit u. These values are based on those published by the EIA [13].

u	HR_u(MMBTU/MWh)	$HC_{h(u)}$(MMBTU/shortton)
1	10.089	19.21
2	9.86	21.28
3	11.91	18.96
4	7.0	
5	7.5	

TABLE 3

Heat rate of u and heat content of coal

Coal producers are limited to certain capacity of producing coal at any given time period t. Bhagwat et al. [38] generate a report on coal production capacities in

v	u	$p^s_{vo(u)t}$(\$/shortton)
1	1	5
1	2	3
2	3	6

TABLE 4

Shipping prices of coal

various parts of the U.S. Using this report, coal producer m_1 is limited to a capacity of 100,000 short ton/t and coal producer m_2 is limited to a capacity of 120,000 short ton/t. Similarly, capacities for pipelines l_1 and l_2 are set to be 2,740 MCF/day and 1,860 MCF/day respectively [14]. The Number of hours in a given time period t is set around a month. Table 5 shows number of hours and number of days in time t.

t	H_t(Number of hours)	D_t(Number of days)
1	700	30
2	660	28
3	600	25

TABLE 5

Number of hours and days in period t

There are several important functions in the proposed model. First, $P^e_{jt}(\cdot)$ is inverse demand function of electricity at node j in time t [\$/MWh] and is set as $P^e_{jt}(\cdot) = c_o - c_1 d^e_{it}$ $(c_o = 60,\ c_1 = 0.1)$. Second, $C_u(\cdot)$ is non-fuel related marginal costs for generation unit u [\$/MWh] and is set as $C_u(\cdot) = 0.0002x_{ut}$ in reference with Gabriel et al. [8]. Third, $C_m(\cdot)$ is production cost for coal producer m [\$/shorttons] and is set as $C_m(\cdot) = 31z_{mt}$ in reference with Bhagwat et al. [38]. The cost per ton of clean coal as defined in the report is $C_m(\cdot) = 44.22 - (1.9085 \times 10^{-6})x_1$ $-0.19906x_2 - (6.3166 \times 10^{-3})x_3 + (6.4903 \times 10^{-2})x_4 + 0.75955x_5$ where x_1 = annual production (tons per year), x_2 = age of mine (years), x_3 = labor productivity (tons per worker per year), x_4 = mine development cost (dollars per ton annual production), x_5 = coal cleaning level. On considering the typical values for x_2, x_3, x_4 and x_5 from the graphs as 12, 2500, 30 and 3 respectively, the final constant value turned out to be 31.02. This results in the final production cost per unit to be

$C_m(\cdot) = 31 - (1.9085 \times 10^{-6})$(annual production). Hence, the final total monthly cost is set to be $C_m(\cdot) = 31$(monthly production) $-(1.9085 \times 10^{-6})(12)$(monthly production)^2. In order to ignore non-convexity nature of the production cost function, it is considered as linear function, $C_m(\cdot) = 31$(monthly production). Fourth, $C_n(\cdot)$ is the production cost for natural gas producer n [\$/MCF] is set to be $C_n(\cdot) = 0.005g_{nt}$ $+0.000003g_{nt}^2$ in reference with Gabriel et al. [8]. Although our numerical experiments use MCF/day in the natural gas production, this production cost is validated and produced similar results as in Gabriel et al. [8].

4.2 Results for Energy Market Equilibrium Model (EMEM) without Natural Gas

The mixed complementarity problem (MCP) that has been developed using KKT conditions is numerically tested using parameters and functions as described in Section 4.1. This section deals with the equilibrium model that has coal and electricity as players. The numerical example that is considered for testing this model has two electricity generation firms f_1, f_2, three electricity generation units u_1, u_2, two coal mine regions v_1, v_2 and two coal producers m_1, m_2. Figure 2 shows the schematic diagram that describes the interconnection between players. For instance, generation unit u_1 and generation unit u_2 belong to electricity generation firm f_1. Similarly, u_1 and u_2 are served by coal producer m_2 through coal mine region v_1. In these tables, it has to be noted that suffix CE means the model with coal and electricity at equilibrium. There are other suffixes to be introduced in later sections for other modelling scenarios.

Table 6 shows electricity sales by generation unit u to electricity demand region i in time t. Generation units u_1 and u_2 belong to generation firm f_1 and receives its coal from coal producer m_2. The model chooses u_2 and not u_1 for generation of electricity for firm f_1. This is because u_2 is more efficient than u_1 (efficiency of generation unit is inversely proportional to the ratio of heat rates and heat contents). From Table 3 it can be seen that the ratio of HR_u(MMBTU/MWh) to $HC_{h(u)}$(MMBTU/shortton) for generation unit u_1 is 0.4634 and for generation unit u_2 is 0.5251. Generation unit u_3 produces electricity irrespective of its

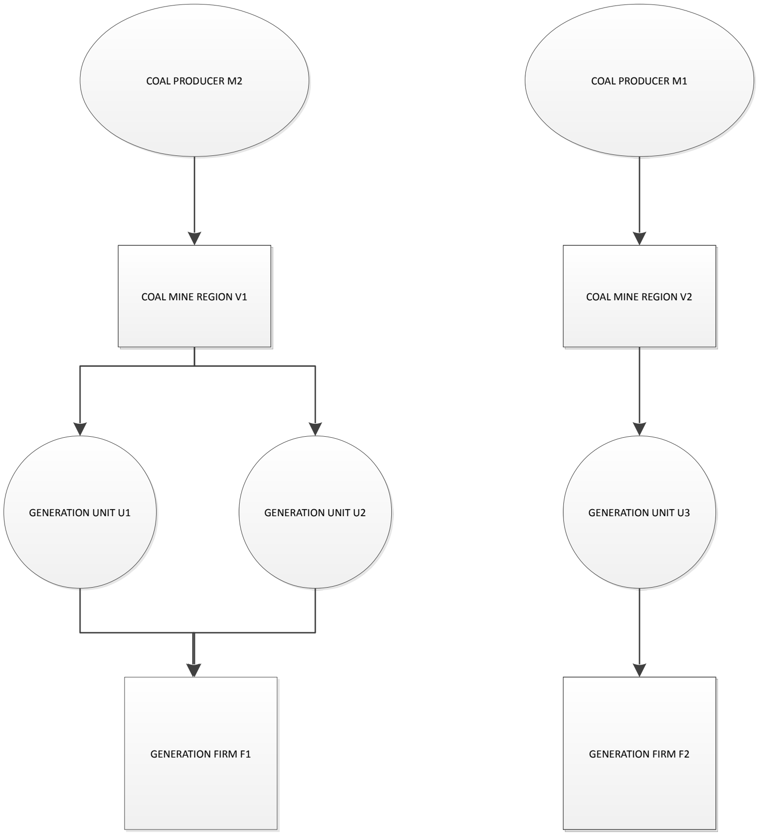

Figure 2. Coal generation units and electricity generation firms

efficiency (since it is the only generation unit for firm f_2). Also, it can be seen that sales of electricity by generation unit u_1 to demand node i_1 at time t_1 is 185 Megawatts and that of unit u_1 to node i at time t_2 is 196 Megawatts. It varies with time because generation of electricity is inversely proportional to number of hours in each time period. Table 5 shows that there are 700 hours in t_1 and 660 hours in t_2. The demand nodes i_1, i_2 and i_3 are all identical and hence receive same amount of electricity at give time time t from generation unit u.

The equilibrium model shows that coal producers m_1 and m_2 produce coal to

u	i	t	s_{uit}CE(MW)	u	i	t	s_{uit}CE(MW)
1	1	1	0	2	2	1	185
1	1	2	0	2	2	2	196
1	1	3	0	2	2	3	215
1	2	1	0	3	1	1	113
1	2	2	0	3	1	2	121
1	2	3	0	3	1	3	132
2	1	1	185	3	2	1	113
2	1	2	196	3	2	2	120
2	1	3	215	3	2	3	132

TABLE 6

Electricity Sales for EMEM: with CE

Electricity demand		
i	t	CE[Megawatts]
1	1	298
1	2	316
1	3	348
2	1	298
2	2	316
2	3	348

TABLE 7

Electricity demand for EMEM: with CE

their fullest capacities i.e. m_1 produces 100,000 [short ton /t] and m_2 produces 120,000 [short ton /t], as discussed previously. Table 7 shows elastic demand [Megawatts] for electricity consumer nodes i_1 and i_2 at time periods t_1,t_2 and t_3. The total amount of electricity generated by both firms f_1 and f_2 at a given time t is equally distributed among consumer nodes i (since consumer nodes are all identical).

Table 8 shows results for shadow price of electricity p^e_{it}CE($/MWh). It can be seen that the price of electricity for consumer node i_1 at time t_1 is $30.13/MWh and for consumer node i_2 at t_1 is also $30.13/MWh. This is because both the consumer nodes are served with same amount of electricity at given time period as discussed earlier. The price of electricity at time period t_3 is least when compared to other time periods because there is more supply of electricity during time t_3 (supply of electricity at $t_3 > t_2 > t_1$ and so is the price). Table 9 shows results for shadow

Electricity price		
i	t	p^e_{it}CE($/MWh)
1	1	30.13
1	2	28.32
1	3	25.15
2	1	30.13
2	2	28.32
2	3	25.12

TABLE 8

Electricity price for EMEM: with CE

Coal price		
v	t	p^c_{vt}CE($/Shortton)
1	1	62.02
1	2	58.12
1	3	51.28
2	1	41.96
2	2	39.08
2	3	34.04

TABLE 9

Coal price for EMEM: with CE

price of coal as p^c_{vt}CE($/Shortton). The shadow prices of coal for coal mine region v_1 at all times is greater than shadow prices of coal mine region v_2. Recall from Figure 2 that coal producer m_1 belongs to region v_2 with a maximum capacity of 100,000(Shortton/t) and coal producer m_2 belongs to region v_1 with a maximum capacity of 120,000(Shortton/t). In other words, region with less capacity (v_1) has greater shadow price for coal when compared to region with more capacity. In this numerical example it varies with times t_1, t_2 and t_3 for region v_1 such as $62.02/Shortton, $58.12/Shortton and $51.28/Shorton respectively because of different number of hours during each time-period. Similar explanation holds for shadow price of coal for coal mine region v_2.

4.3 Results for Energy Market Equilibrium Model (EMEM) with Natural Gas

Figure 3 shows the schematic diagram of all the players involved when natural gas market is added to the existing model in Figure 2 . Natural gas players added to the baseline model are: two natural gas producers w_1, w_2 (identical), two pipelines l_1, l_2, two marketers a_1, a_2(identical) and two consumer nodes j_1, j_2 are added to the previous model. The results of this extended model are discussed in this section.

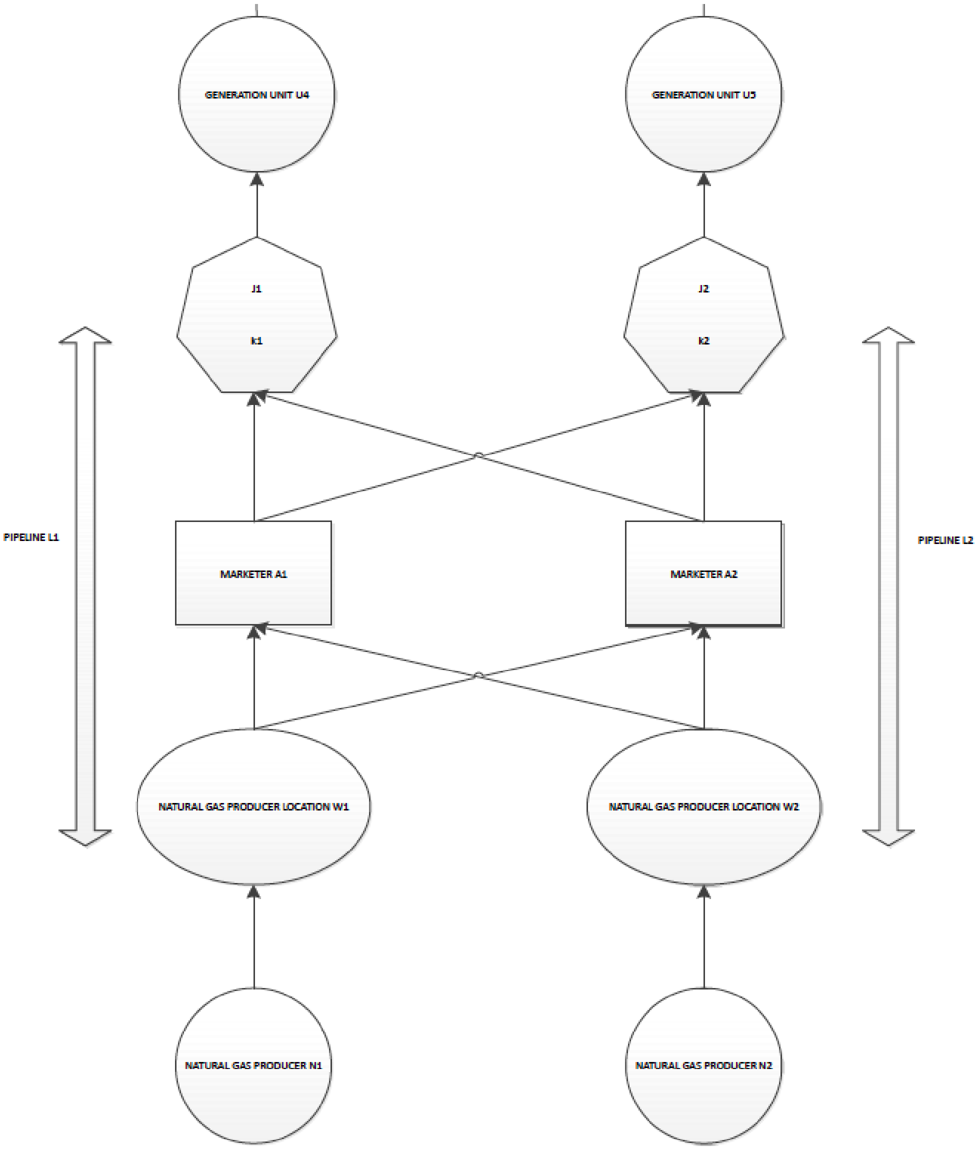

Figure 3. Electricity Market for EMEM: CNE

Before further examining the computational results, note that suffix "CE" means equilibrium model with coal and electricity while "CNE" means equilibrium model with coal, natural gas and electricity. The equilibrium model has used all the available capacity of coal for each of the coal producers, i.e. coal produced by coal producer m_1 is 100,0000 shortton/t at all the times and coal produced by coal producer m_2 is 120,0000 shortton/t at all times. Note that the additional natural gas fired units only causes total electricity demand to increase, but doesn't cause less coal consumption. The elastic demand for electricity nodes i_1 and i_2 is shown on Table 10. It can be seen that there is an increase in elastic demand after the introduction of natural gas into the equilibrium model. An increase of 13.69 Megawatts of electricity demand can be observed (generation unit u_4 produces 8.38 Megawatts and generation unit u_5 produces 5.31 Megawatts).

Electricity demand			
i	t	CE[Megawatts]	CNE[Megawatts]
1	1	298	311.69
1	2	316	329.69
1	3	348	361.69
2	1	298	311.69
2	2	316	329.69
2	3	348	361.69

TABLE 10

Electricity demand for EMEM: CNE

Table 11 displays the results for sales of electricity by generation unit u to demand node i during time t. As shown in the Table 11, there are electricity sales from generation units u_4 and u_5 since they are natural gas fired units. Also sales from generation unit u_4 is greater than unit u_5 . The efficiency of natural gas fired unit depends on heat rate of the respective generation unit. Here, heat rate of NG fired unit u_4 is 7.0 (MMBTU/MWh) and for u_5, it is 7.5 (MMBTU/MWh). In this case, unit u_5 is more efficient than unit u_4 but still produces less amount of electricity because of the fact that natural gas supplied to unit u_4 is greater than unit u_5 (2,740 MCF/day for generation unit u_4 and 1,860 MCF/day for generation units u_5). It can be observed from Table 11 that sales of electricity by generations units u_2 and u_3 are different during various time periods. This is because, electricity fired units depend on number of hours in that time period and coal received by the generation unit (Number of hours are set as different during each time period , u_4 receives 120,0000 shortton/t and u_5 receives 100,0000 shortton/t).

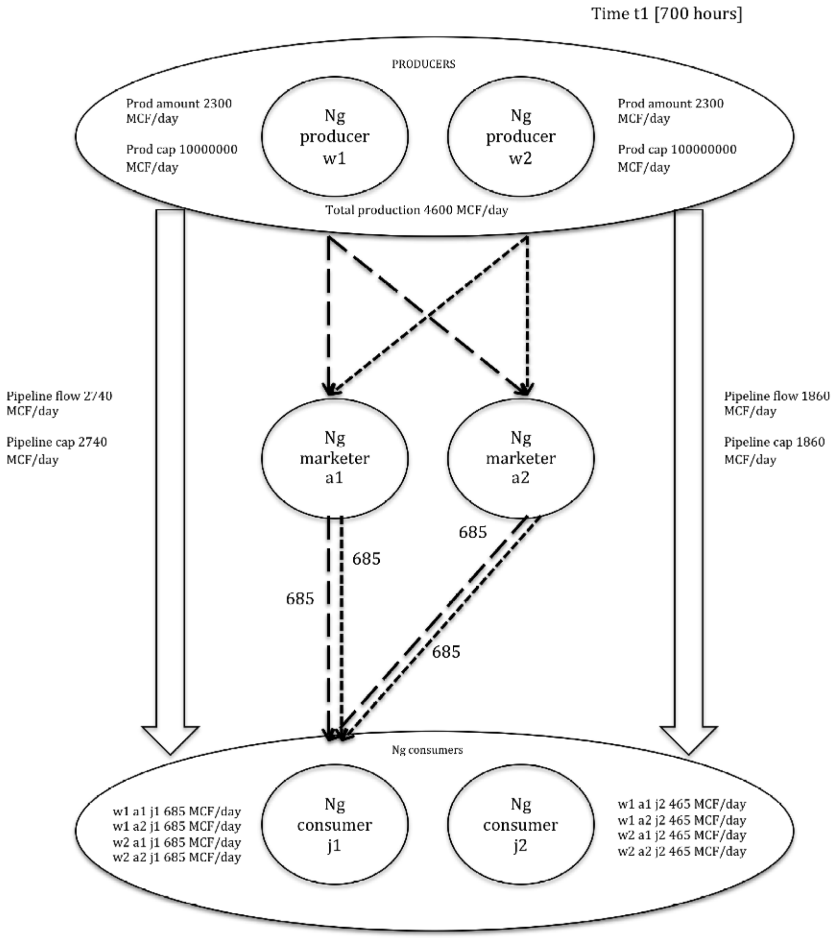

Figure 4. Flow of natural gas from producers to electric sector

Figure 4 shows natural gas flow from upstream (natural gas producers) to downstream (natural gas consumers) via midstream (natural gas pipelines). There are two identical natural gas producers considered in the model, thus both of them produce 2,300 MCF/day at all times. Also, the capacity of pipelines is fully utilized. Flow of natural gas through pipeline l_1 is 2,740 MCF/day and through pipeline l_2 is 1,860 MCF/day. It has to be noted that the flow of natural gas that leads to consumer j_1 is considered as pipeline l_1 and the pipeline that leads to consumer j_2 is considered as pipeline l_2. The amount of natural gas consumed by consumer j_1 is

685 MCF/day at all times and the amount of natural gas consumed by consumer j_2 is 465 MCF/day at all times. This is governed by pipelines and their capacities that connect consumers (685 times 4 = 2,740 (MCF/day) which is the pipeline capacity of $l = 1$ and 465 times 4 = 1,860 (MCF/day) which is the pipeline capacity of $l = 1$). This shows that the numerical results exactly match with the pipeline capacities and accept flow principle.

Electricity Sales									
u	i	t	s_{uit}CE(MW)	s_{uit}CNE(MW)	u	i	t	s_{uit}CE(MW)	s_{uit}CNE(MW)
1	1	1	0	0	3	2	1	113	113
1	1	2	0	0	3	2	2	120	120
1	1	3	0	0	3	2	3	132	132
1	2	1	0	0	4	1	1	0	8.38
1	2	2	0	0	4	1	2	0	8.38
1	2	3	0	0	4	1	3	0	8.38
2	1	1	185	185	4	2	3	0	8.38
2	1	2	196	196	4	2	3	0	8.38
2	1	3	215	215	4	2	3	0	8.38
2	2	1	185	185	5	1	1	0	5.31
2	2	2	196	196	5	1	2	0	5.31
2	2	3	215	215	5	1	3	0	5.31
3	1	1	113	113	5	2	1	0	5.31
3	1	2	120	120	5	2	2	0	5.31
3	1	3	132	132	5	2	3	0	5.31

TABLE 11

Electricity sales for EMEM: CNE

Tables 12 and 13 display the results of shadow or dual prices of coal and electricity respectively. Table 12 shows the price of electricity at equilibrium with and without the addition of natural gas to the model. It can be seen that the price of electricity is less when compared to the equilibrium model without natural gas. This is because more electricity is produced than previous model due to the presence of natural gas. In both cases, the price of electricity falls in the range of [35,40] as reported by EIA 2009 [12]. Similarly, table 13 compares the price of coal between the two models. Once again, the range of [38,40] is consistent with those reported by EIA 2009 [12].

Table 14 shows the upstream (natural gas producer), midstream (pipeline owner) and downstream (natural gas consumer) prices. The production price of

Electricity price			
i	t	p^e_{it}CE($/MWh)	p^e_{it}CNE($/MWh)
1	1	30.13	28.76
1	2	28.32	26.95
1	3	25.15	23.78
2	1	30.13	28.76
2	2	28.32	26.95
2	3	25.12	23.78

TABLE 12

Electricity price for EMEM: CNE

natural gas is considerably very less due to enormous amount of natural gas production capacity (Natural gas capacity of both the producers is 100,000000 MCF/day). It can also be seen that the shadow price of natural gas transportation cost and is fairly high (almost $4/MCF at all time periods) when compared to natural gas production price because production is governed by natural gas pipeline capacity in this equilibrium model. Also, the natural gas pipeline capacity is fully utilized. The downstream natural gas price is the sum of natural gas production price and natural gas transportation price at respective time periods (0.019 + 4.09 = $4.10/MCF). Interestingly, the range of [2.5,5.0] is consistent with those reported by EIA 2009 [12].

Coal price			
v	t	p^c_{vt}CE($/Shortton)	p^c_{vt}CNE($/Shortton)
1	1	62.02	59.07
1	2	58.12	55.16
1	3	51.28	48.32
2	1	41.96	39.78
2	2	39.08	36.90
2	3	34.04	21.86

TABLE 13

Coal price for EMEM: CNE

NG Region		NG upstream price	NG midstream price	NG downstream price
w	t	p_{wt}CNE($/MCF)	p_{lt}CNE($/MCF)	p_{wt}CNE($/MCF)
1	1	0.019	4.09	4.1
1	2	0.019	3.83	3.85
1	3	0.019	3.37	3.39
2	1	0.019	3.81	3.83
2	2	0.019	3.57	3.59
2	3	0.019	3.15	3.17

TABLE 14

Natural gas prices for EMEM: CNE

4.4 Results for EMEM with non-electricity natural gas usage

The model discussed in this section is different from the one that has been discussed in previous section 4.5 due to the presence of non-electric sector that consumes natural gas. In addition to two natural gas producers w_1, w_2 (identical), two pipelines l_1, l_2, two marketers a_1, a_2 (identical) and two consumer nodes j_1, j_2, now there are two sectors added to the model k_1, k_2 where sector k_1 belongs to electric sector (meaning - natural gas that is supplied to sector k_1 is exclusively used for the production of electricity), sector k_2 belongs to non-electric sector (e.g., consumers and businesses who use natural gas for heating). Figure 5 shows the schematic representation of all the players with their interactions. We have conducted preliminary testing for this framework using GAMS/PATH, which is not discussed in this thesis.

Like previous section, “CE” refers to the results of model in section 4.c.1, “CNE” refers to the results of model in Section 4.5 and “CNE-NonE” which means coal, natural gas, electricity and non electric sector for natural gas refers to the results of model in this section. Here, the results of all the three models are compared with each other.

Table 15 displays the results for sales of electricity by generation unit u to demand node i during time t. As shown in the Table 15, there are electricity sales from generation units u_2, u_3, u_4 and u_5. The sales from generation unit u_4 and generation unit u_5 are dropped (from 8.38 MW to 2.54 MW for unit 4 and 5.31 MW

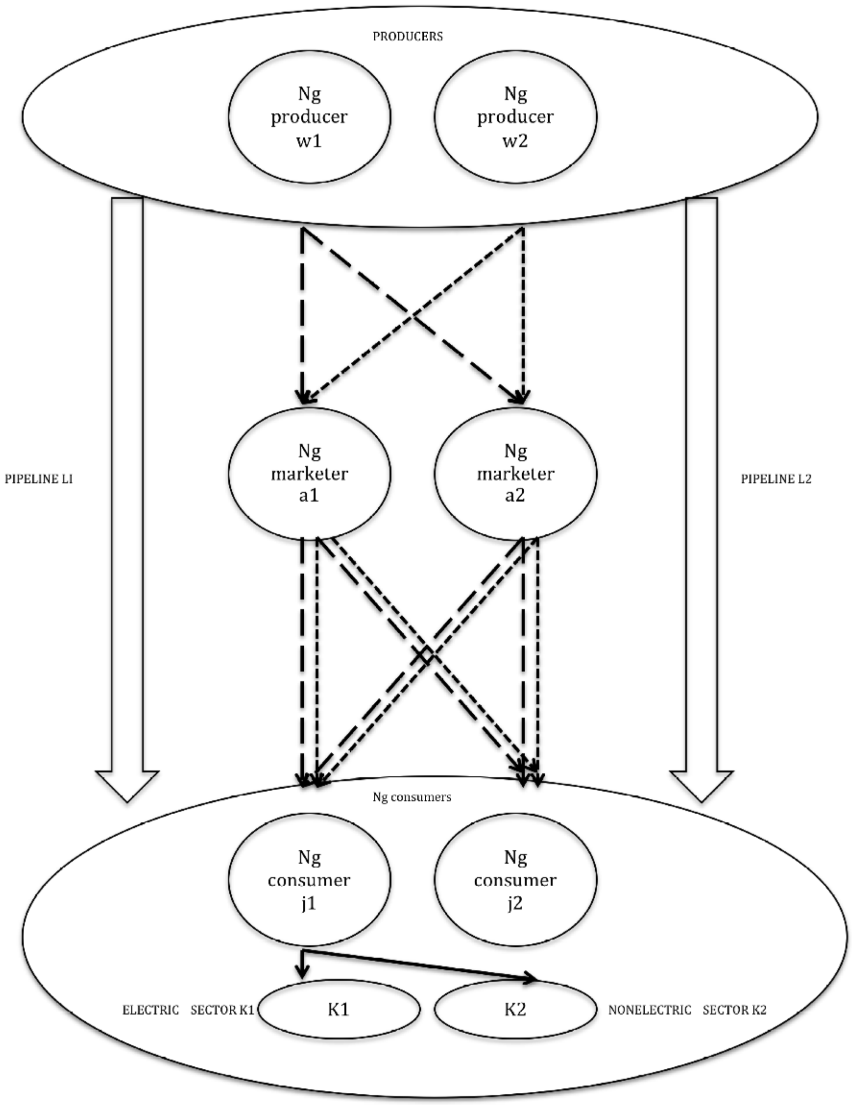

Figure 5. Natural gas market after the introduction of NonE sector

to 1.60 MW) in this case due to the addition of non-electric sector within natural gas market. The quantity of natural gas that has to be supplied to non-electric sector is governed by an additional constraint added in this model. This constraint says that almost 70% of the total natural gas produced must be supplied to non-electric sector. As a result, the amount of electricity generated by units u_4 and u_5 is less than previous model and can be between two columns headed as s_{uit}CNE(MW) and s_{uit}CNE-NonE(MW).

Like previous model described in Section 4.5 , this equilibrium model has also used all the available capacity of coal for each of the coal producers i.e. coal produced by coal producer m_1 is 100,0000 shortton/t at all the times and coal produced by coal producer m_2 is 120,0000 shortton/t at all times. Note that the additional non-electric sector doesn't cause less coal consumption but only produces less electricity.

Electricity Sales (MW)											
u	i	t	s_{uit}CE	s_{uit}CNE	s_{uit}CNE-NonE	u	i	t	s_{uit}CE	s_{uit}CNE	s_{uit}CNE-NonE
1	1	1	0	0	0	3	2	1	113	113	113
1	1	2	0	0	0	3	2	2	120	120	120
1	1	3	0	0	0	3	2	3	132	132	132
1	2	1	0	0	0	4	1	1	0	8.38	2.54
1	2	2	0	0	0	4	1	2	0	8.38	2.54
1	2	3	0	0	0	4	1	3	0	8.38	2.54
2	1	1	185	185	185	4	2	3	0	8.38	2.54
2	1	2	196	196	196	4	2	3	0	8.38	2.54
2	1	3	215	215	215	4	2	3	0	8.38	2.54
2	2	1	185	185	185	5	1	1	0	5.31	1.60
2	2	2	196	196	196	5	1	2	0	5.31	1.60
2	2	3	215	215	215	5	1	3	0	5.31	1.60
3	1	1	113	113	113	5	2	1	0	5.31	1.60
3	1	2	120	120	120	5	2	2	0	5.31	1.60
3	1	3	132	132	132	5	2	3	0	5.31	1.60

TABLE 15

Electricity sales for EMEM: CNE-NonE

Electricity demand (MW)				
i	t	CE[Megawatts]	CNE[Megawatts]	CNE-NonE[Megawatts]
1	1	298	311.69	302.85
1	2	316	329.69	320.95
1	3	348	361.69	352.63
2	1	298	311.69	302.85
2	2	316	329.69	320.95
2	3	348	361.69	352.63

TABLE 16

Electricity demand for EMEM: CNE-NonE

The elastic demand for electricity nodes i_1 and i_2 is shown on Table 17. It can be seen that there is decrease in elastic demand after the introduction of non-electric sector for consumption of natural gas into the equilibrium model. Decrease of around 10 Megawatts of electricity demand can be observed for

Electricity price				
i	t	p^e_{it}CE(\$/MWh)	p^e_{it}CNE(\$/MWh)	p^e_{it}CNE-NonE(\$/MWh)
1	1	30.13	28.76	29.71
1	2	28.32	26.95	27.9
1	3	25.15	23.78	24.73
2	1	30.13	28.76	29.71
2	2	28.32	26.95	27.90
2	3	25.12	23.78	24.73

TABLE 17

Electricity price for EMEM: CNE-NonE

Coal price				
v	t	p^c_{vt}CE(\$/Shortton)	p^c_{vt}CNE(\$/Shortton)	p^c_{vt}CNE-NonE(\$/Shortton)
1	1	62.02	59.07	61.13
1	2	58.12	55.16	57.22
1	3	51.28	48.32	50.38
2	1	41.96	39.78	41.30
2	2	39.08	36.90	38.42
2	3	34.04	21.86	33.37

TABLE 18

Coal price for EMEM: CNE-NonE

electricity demand node i_1 at time t_1 (generation unit u_4 produces 2.54 Megawatts (8.38 Megawatts earlier) and generation unit u_5 produces 1.60 Megawatts (5.31 Megawatts earlier). This decrease in production of electricity due to consumption of natural gas by non-electric sector is reflected in elastic demand.

Figure 6 shows natural gas flow from upstream (natural gas producers) to downstream (natural gas consumers) via midstream (natural gas pipelines). There

Time	W/O Non-Electric Sector			With Non-Electric Sector		
t	w	p_{wt}CNE(\$/MCF)	p_{wt}CNE-NonE(\$/MCF)	l	p_{lt}CNE(\$/MCF)	p_{lt}CNE-NonE(\$/MCF)
1	1	0.019	0.019	1	4.09	4.22
2	1	0.019	0.019	1	3.83	3.96
3	1	0.019	0.019	1	3.37	3.51
1	2	0.019	0.019	2	3.81	3.94
2	2	0.019	0.019	2	3.57	3.70
3	2	0.019	0.019	2	3.15	3.27

TABLE 19

Natural gas upstream and midstream prices for EMEM: CNE-NonE

			W/O Non-Electric Sector	With Non-Electric Sector
j	k	t	p_{jkt}CNE (\$/MCF)	p_{jkt}CNE-NonE(\$/MCF)
1	1	1	4.1	4.24
1	1	2	3.85	3.98
1	1	3	3.39	3.53
1	2	1	0	4.24
1	2	2	0	3.98
1	2	3	0	3.53
2	1	1	3.83	3.96
2	1	2	3.59	3.72
2	1	3	3.17	3.29
2	2	1	0	3.96
2	2	2	0	3.72
2	2	3	0	3.29

TABLE 20

Natural gas downstream prices for EMEM: CNE-NonE

are two identical natural gas producers considered in the model and can be seen that both of them produce 2,300 MCF/day at all times. Also, the capacity of pipelines is fully utilized. Flow of natural gas through pipeline l_1 is 2,740 MCF/day and through pipeline l_2 is 1,860 MCF/day. Similar to previous Section 4.5, it has to be noted that the flow of natural gas that leads to consumer j_1 is considered as pipeline l_1 and the pipeline that leads to consumer j_2 is considered as pipeline l_2. The amount of natural gas consumed by consumer j_1 is 685 MCF/day at all times and the amount of natural gas consumed by consumer j_2 is 465 MCF/day at all times. This is governed by pipelines and their capacities that connect consumers ($685 \times 4 = 2740$ (MCF/day) which is the pipeline capacity of l_1 and $465 \times 4 = 1860$ (MCF/day) which is the pipeline capacity of l_1). This shows that the numerical results exactly match with the pipeline capacities and accept flow principle. As there are two sectors introduced - k_1 electric and k_2 non-electric, amount5 of natural gas received by consumer nodes j_1 and j_2 are distributed between k_1 and k_2. This distribution is governed by a constraint that says almost 70% of natural gas has to be sent to non-electric sector. Hence, amount of natural gas received by electric sector k_1 via natural gas consumer node j_1 at time t_1 is 207.5 (MCF/day) instead of 685 MCF/day. Remaining amount of natural gas is sent to non-electric sector k_2, i.e. 477.5(MCF/day). Figure 6 and Table 21 show the distribution of natural gas natural gas producer w to market a to consumer node j to sector k in time t. Note that Figure 6 shows values only for w_1, a_1, j_1, k_1 and w_1 ,a_1, j_1, k_2 at time t_1. Also Table 21 shows values for marketer a_1, marketer a_2 is ignored as both the marketers are identical and have exactly the same values.

					W/O NE	With Non-Electric						W/O NE	With Non-Electric
a	w	j	k	t	q_{awjkt}CNE	q_{awjkt}CNE with NE	a	w	j	k	t	q_{awjkt}CNE	q_{awjkt}CNE with NE
1	1	1	1	1	685	207	1	2	1	1	1	685	207
1	1	1	1	2	685	207	1	2	1	1	2	685	207
1	1	1	1	3	685	207	1	2	1	1	3	685	207
1	1	1	2	1	0	477	1	2	1	2	1	0	477
1	1	1	2	2	0	477	1	2	1	2	2	0	477
1	1	1	2	3	0	477	1	2	1	2	3	0	477
1	1	2	1	1	465	140	1	2	2	1	1	465	140
1	1	2	1	2	465	140	1	2	2	1	2	465	140
1	1	2	1	3	465	140	1	2	2	1	3	465	140
1	1	2	2	1	0	324	1	2	2	2	1	0	324
1	1	2	2	2	0	324	1	2	2	2	2	0	324
1	1	2	2	3	0	324	1	2	2	2	3	0	324

TABLE 21

Natural gas sales by marketer for EMEM: CNE-NonE

Tables 17 and 18 display the results of shadow or dual prices of electricity and coal respectively. Table 18 compares the price of coal at equilibrium with and without the addition of non-electric sector to the model. It can be seen that the price of coal is greater (\$61.13/shortton for v_1 at t_1) when compared to the equilibrium model without non-electric sector (\$59.07/shortton) for v_1 at t_1) but less than the model that has only coal and electricity at equilibrium (\$62.02/shortton) for v_1 at t_1). This is because shadow price directly depends on amount of electricity demand that is being fulfilled. For example, Table 16 shows that the CE scenario yields the least electricity demand for consumer node i_1 at time t_1 298 MW, followed by 302 MW in the CNE-NonE scenario and then by 312 MW in the CNE scenario. This is perfectly aligned with the order of the shadow price of coal, which is \$59.07/shortton in the CNE scenario followed by \$61.13/shortton in the CNE-NonE scenario and then by \$62.02/shortton in the CE scenario. In all the three cases, the price of coal is close to the range of [38,40] as reported by EIA 2009 [12]. Similarly, table 17 compares the price of electricity between the three models and it can be seen that for consumer node i_1 at time t_1 the price is \$30.13/MWh in CE scenario followed by \$28.76/MWh in the CNE scenario and \$29.71/MWh in the CNE-NonE scenario.It follows exactly same reasons as described for coal shadow price. Once again, the range of [35,40] is consistent with those reported by EIA 2009 [12].

Table 19 shows the upstream (natural gas producer) and midstream (pipeline owner). The production price of natural gas doesn't change when compared to previous model i.e. 0.0019 (\$/MWh)) due to enormous amount of natural gas production capacity (Natural gas capacity of both the producers is 100,000000 MCF/day). Table 20 shows the downstream price (natural gas marketer) of natural gas at which marketer a sells to consumer node j for sect$k$ during time t. This price slightly increases (\$4.224/MCF at j_1, k_1, t_1) when compared to previous model (\$4.1/MCF at j_1, k_1, t_1) without non-electric sector because of change in amount of natural gas that is being sent to non-electric sector. Interestingly, the range of [2.5,5.0] is also consistent with those reported by EIA 2009 [12] in this model with non-electric sector.

4.5 Sensitivity analysis for EMEM

In this section, sensitivity analysis is performed on EMEM model including natural gas market and further its non-electric users. This analysis helps in better understanding of affect of parameters on the developed model. We first vary natural gas pipeline capacity and change it form 2,740 MCF/day and 1,860 MCF/day to 100,00000 MCF/day for both.

Efficiency of generation units depend on their heat rate and heat content. Generation unit u_4 costs \$18.45/MWh and generation unit u_5 costs \$19.77/MWh. Since u_4 is more efficient than u_5, natural gas is sent to unit u_4. This results in the production of electricity and sales by unit u_4 as shown in Table 22. It has to be recalled that generation units $u1, u2$ and u_4 belong to firm f_1 and $u3, u5$ belongs to firm f_2. Unit u_2 doesn't produce electricity in this case because unit u_4 that belongs to the same firm as unit u_2 produces electricity using natural gas. Coal producer m_1 uses its full capacity and produces 100,000 shortton/t as it supplies to generation

Electricity Sales (MW)									
u	i	t	s_{uit}CNE-NonE	s_{uit}CNE(SA pipeline cap)	u	i	t	s_{uit}CNE-NonE	s_{uit}CNE(SA pipeline cap)
1	1	1	0	0	3	2	1	113	113
1	1	2	0	0	3	2	2	120	120
1	1	3	0	0	3	2	3	132	132
1	2	1	0	0	4	1	1	2.54	396
1	2	2	0	0	4	1	2	2.54	390
1	2	3	0	0	4	1	3	2.54	380
2	1	1	185	0	4	2	3	2.54	396
2	1	2	196	0	4	2	3	2.54	396
2	1	3	215	0	4	2	3	2.54	396
2	2	1	185	0	5	1	1	1.60	0
2	2	2	196	0	5	1	2	1.60	0
2	2	3	215	0	5	1	3	1.60	0
3	1	1	113	113	5	2	1	1.60	0
3	1	2	120	120	5	2	2	1.60	0
3	1	3	132	132	5	2	3	1.60	0

TABLE 22

Electricity sales for EMEM: Sensitivity Analysis

unit u_3. Coal producer m_2 is not allowed to produce coal as units u_1 and u_2 do not produce electricity in this case. Natural gas producers almost produce 200,000 MCF/day rather than 2000 - 3000 MCF/day due to increase in pipeline capacity enormously.

Electricity demand			
i	t	CNE-NonE[Megawatts]	CNE-NonE(SA pipeline cap)
1	1	302	509
1	2	320	511
1	3	352	513
2	1	302	509
2	2	320	511
2	3	352	513

TABLE 23

Electricity demand for EMEM: Sensitivity Analysis

Table 23 shows the increase in electricity demand after increase in pipeline capacity of natural gas. Almost 50% of electricity demand is satisfied by unit u_4 which is natural gas fired. Flow of natural gas through pipeline l_1 is around 400,000 MCF/day and there is no flow through pipeline l_2 as it leads to generation unit u_5 (unit u_5 doesn't produce electricity as discussed earlier).

The prices of coal and electricity becomes interesting in this sensitive analysis. Table 27 compares the shadow price of coal. Price of coal for coal producer m_2 is only 8.34 \$/shortton at time t_1, which was 41.30 \$/shortton. This is because of increase in use of natural gas for production of electricity. Table 25 shows the price of electricity, which is much lower at \$9.009/MWh for $i1, t1$, almost one-third of the price as it was without pipeline capacity expansion (\$29.71/MWh for $i1, t1$). This is because of increase in elastic demand and excess availability of resources for production of electricity.

		Coal price	
v	t	p^c_{vt}CNE(\$/Shortton)	p^c_{vt}CNE (SA pipeline cap)(\$/Shortton)
1	1	61.13	52.72
1	2	57.22	52.73
1	3	50.38	52.59
2	1	41.30	8.34
2	2	38.4	8.13
2	3	33.37	7.78

TABLE 24

Coal price for for EMEM: Sensitivity Analysis

		Electricity price	
i	t	p^e_{it}CNE(\$/MWh)	p^e_{it}CNE(SA pipeline cap)(\$/MWh)
1	1	29.71	9.009
1	2	27.90	8.882
1	3	24.73	8.659
2	1	29.71	9.009
2	2	27.90	8.882
2	3	24.73	8.659

TABLE 25

Electricity price for EMEM: Sensitivity Analysis

Natural gas price of upstream is also one interesting thing as the production price increased from \$0.019/MCF to almost \$1.2/MCF because of increase in production and relatively less gap between production and available capacity. In previous model, production was 2,300 MCF/day while available capacity is 100,000000 MCF/day. But, in this model production is almost 2,000,000 MCF/day with same available capacity. Where as, midstream price of natural gas is nearly zero due to enormous amount of available capacity. This resulted in natural gas downstream price to drop from \$4.0/MCF - \$5.0/MCF to nearly \$1.2/MCF - \$1.3/MCF.

Secondly, we vary coefficients of inverse demand function of electricity. The present function is $P^e_{jt}(\cdot) = c_o - c_1 d^e_{it}$ $(c_o = 60,\ c_1 = 0.1)$. Initially, co-efficient c_o is increased by 40 till c_o reaches 200 keeping c_1 constant. In all the cases, change in inverse demand function doesn't affect sales of electricity by generation unit u to demand node i at time t (s_{uit}). Sales of electricity remain exactly the same as in Table 11. It doesn't affect the amount of coal produced either. Coal producers m_1 and m_2 continue to produce 100,000 shortton/t and 120,000 shortton/t in all cases. Also, same holds for electricity demand d^e_{it} and can be seen in Table 10. Flow of natural gas through pipelines l_1 and l_2 is 2,740 MCF/day and 1,860 MCF/day in all cases in this analysis.

Since, values of s_{uit}, z_{mt}, d^e_{it}, g_{nt}, f_{lt} and q_{awjkt} do not change with change in coefficient c_o of inverse demand function, their tables showing values are ignored. Interesting part of this sensitivity analysis lies in shadow prices of coal (p^c_{vt}), electricity (p^e_{it}), natural gas pipeline $(p_{lt})$and natural gas marketer (pg_{jkt}). Inverse demand function is a function that maps the quantity of output demanded to the market price for that output. As demand satisfied depends on availability of resources (coal and natural gas in this case), it remains unchanged. Therefore, the price of electricity directly depends on coefficient c_o.

Electricity price p^e_{it} (\$/MWh)						
i	t	$c_o = 60$	$c_o = 80$	$c_o = 120$	$c_o = 160$	$c_o = 200$
1	1	28.76	48	88	128	168
1	2	26.95	46	86	126	166
1	3	23.78	43	83	123	163
2	1	28.76	48	88	128	168
2	2	26.95	46	86	126	166
2	3	23.78	43	83	123	163

TABLE 26

Electricity price for EMEM: different coefficients of inverse demand function (c_o)

Table 26 shows the price of electricity for node i at equilibrium when coefficient c_o of inverse demand function is changed. It can be seen that increase in co-efficient of c_o results in increase of electricity price. When observed closely, electricity price for node i_1, time t_1, $c_o = 120$ is 128 [\$/MWh] and for $c_o = 200$ is 168 [\$/MWh]. This is just an example for one case but similar pattern can be observed in all the cases. An addition of 40 to coefficient c_o results in an increase of 40 \$/MWh of electricity price and follows a linear relationship. We'd like to investigate the theoretical aspect of this observation in a future study.

Coal price p^c_{vt} (\$/Shortton)						
v	t	$c_o = 60$	$c_o = 80$	$c_o = 120$	$c_o = 160$	$c_o = 200$
1	1	59.07	102	188	274	361
1	2	55.16	98	184	270	357
1	3	48.32	91	177	264	350
2	1	39.78	71	135	198	262
2	2	36.90	68	132	196	259
2	3	21.86	63	127	191	254

TABLE 27

Coal price for EMEM: different coefficients of inverse demand function (c_o)

The increase in coefficient c_o demands for more electricity for the given price. As mentioned earlier, amount of resources are limited in this model and are not allowed to produce more electricity. Table 27 shows coal price for region v at equilibrium. For region v_1, time t_1, coefficient $c_o = 80$, coal price is 102 \$/shortton and for $c_o = 120$ it is 188 \$/shortton. Increase in positive coefficient results in increase in coal price due to more demand. Interestingly, coal price also follows a specific pattern i.e. an increase in 40 to coefficient c_o results in an increase of around 86 \$/shortton of coal price.

Tables 28 and 29 shows the shadow price for natural gas transportation pipelines and downstream. Increase in coefficient c_o increase these prices. For instance, transportation price of pipeline l_1, at time t_1 for $c_o = 80$ is 6.94 [\$/MCF] and for $c_o = 120$ is 12.66 [\$/MCF]. Like coal and electricity prices, natural gas transportation price also follows a pattern with increase in positive coefficient of inverse demand function. An increase in 40 to coefficient c_o results in an increase of around 6 \$/MCF. Natural gas production price remains unchanged (0.019 \$/MCF)

Natural gas transportation price p_{lt} ($/MCF)						
l	t	$c_o = 60$	$c_o = 80$	$c_o = 120$	$c_o = 160$	$c_o = 200$
1	1	4.09	6.94	12.66	18.37	24.09
1	2	3.83	6.68	12.4	18.11	24.83
1	3	3.37	6.23	11.95	17.66	23.37
2	1	3.81	6.48	11.81	17.44	22.48
2	2	3.57	6.24	11.57	16.9	22.24
2	3	3.15	5.81	11.15	16.48	21.81

TABLE 28

Natural gas transportation rates for EMEM: different coefficients of inverse demand function (c_o)

because of its huge capacity. Natural gas downstream price is the sum of transportation price and production price. Table 29 shows that it follows similar results as in transportation price and adds 0.019 $/MCF for each case.

Ng downstream price p^g_{jt} ($/MCF)						
j	t	$c_o = 60$	$c_o = 80$	$c_o = 120$	$c_o = 160$	$c_o = 200$
1	1	4.1	6.96	12.68	18.39	24.1
1	2	3.85	6.7	12.42	18.13	24.85
1	3	3.39	6.25	11.96	17.68	23.39
2	1	3.83	6.5	11.83	17.16	22.5
2	2	3.59	6.26	11.59	16.92	22.26
2	3	3.17	5.83	11.17	16.5	21.83

TABLE 29

Downstream natural gas prices for EMEM: different coefficients of inverse demand function (c_o)

Next, we vary c_1 of inverse demand function of electricity. The original function is $P^e_{jt}(\cdot) = c_o - c_1 d^e_{it}$ $(c_o = 60,\ c_1 = 0.1)$. Co-efficient c_1 is changed keeping c_0 constant. Like in previous analysis, values of s_{uit}, z_{mt}, d^e_{it}, g_{nt}, f_{lt} and q_{awjkt} do not change with change in coefficient c_1. Decrease in co-efficient c_1 also causes increase in price because c_1 is a negative co-efficient of inverse demand function. Table 30 shows coal price for region v at equilibrium for different coefficients (c_1). Coal price for region v_1, time t_1 for $c_1 = 0.1$ is 59.07 $/Shortton and for $c_1 = 0.09$ is 65 $/Shortton.

Table 31 shows electricity price for demand node i. Negative co-efficient c_1 is

Coal price p^c_{vt} ($/Shortton)					
v	t	$c_1 = 0.1$	$c_1 = 0.09$	$c_1 = 0.06$	$c_1 = 0.03$
1	1	59.07	65	86	106
1	2	55.16	62	83	105
1	3	48.32	56	79	103
2	1	39.78	44	59	74
2	2	36.90	42	57	73
2	3	21.86	37	54	72

TABLE 30

Coal price for EMEM: different coefficients of inverse demand function(c_1)

Electricity price p^e_{it} ($/MWh)					
i	t	$c_1 = 0.1$	$c_1 = 0.09$	$c_1 = 0.06$	$c_1 = 0.03$
1	1	28.76	31	41	50
1	2	26.95	30	40	50
1	3	23.78	27	38	49
2	1	28.76	31	41	50
2	2	26.95	30	40	50
2	3	23.78	27	38	49

TABLE 31

Electricity price for EMEM: different coefficients of inverse demand function(c_1)

decreased from 0.1 to 0.03. Electricity price for node i_1, time t_1, $c_1 = 0.06$ is 41 $/MWh and $c_1 = 0.03$ is 50 $/MWh. Decrease in 0.03 to coefficient c_1 results in an increase of around 9 $/MWh. Tables 32 and 33 show natural gas transportation and downstream prices. As mentioned previously, due to enormous availability of natural gas capacity production price remains 0.019 $/MCF in all cases. The main reason for the increase in shadow price is the increased electricity demand due to decreased value of c_1.

Ng transportation price p_{lt} (\$/MCF)					
l	t	$c_1 = 0.1$	$c_1 = 0.09$	$c_1 = 0.06$	$c_1 = 0.03$
1	1	4.09	4.53	5.87	7.23
1	2	3.83	4.3	5.72	7.15
1	3	3.37	3.89	5.44	7.01
2	1	3.81	4.23	5.48	6.75
2	2	3.57	4.01	5.33	6.67
2	3	3.15	3.63	5.08	6.55

TABLE 32

Natural gas transportation rates for EMEM: different coefficients of inverse demand function(c_1)

Ng downstream price p^g_{jt} (\$/MCF)					
j	t	$c_1 = 0.1$	$c_1 = 0.09$	$c_1 = 0.06$	$c_1 = 0.03$
1	1	4.1	4.55	5.89	7.23
1	2	3.85	4.32	5.73	7.15
1	3	3.39	3.91	5.46	7.01
2	1	3.83	4.25	5.5	6.75
2	2	3.59	4.03	5.35	6.67
2	3	3.17	3.65	5.103	6.55

TABLE 33

Downstream natural gas prices for EMEM: different coefficients of inverse demand function(c_1)

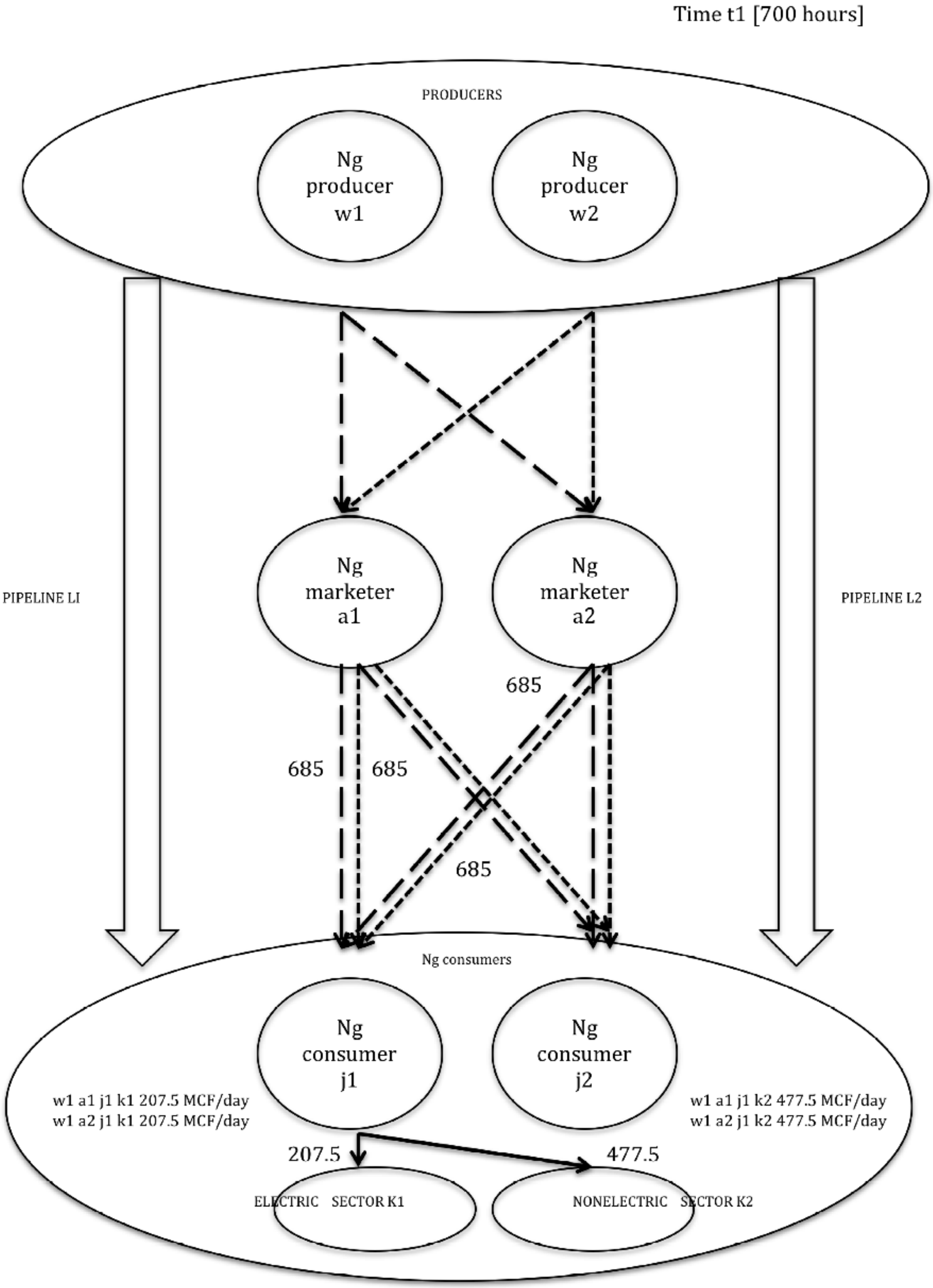

Figure 6. Flow of natural gas from producers to electric and NonE sector

CHAPTER 5

CONCLUSION AND FUTURE RESEARCH

5.1 Conclusion

This thesis makes an attempt to explore a way for energy systems that work more efficiently with each other in an integrated manner. The contribution of the thesis is two-fold. First, an equilibrium model is developed using individual optimization problems of players and their market clearing conditions whenever players interact with each other. Second, the model is converted into an MCP for numerical testing. Subsequently, we analyze these numerical results for three models: the model with coal and electricity markets; the model with coal, electricity and natural gas markets; the model with coal, electricity, natural gas markets along with non-electric sector.

Particularly, we solve the equilibrium model as a mixed complementarity problem. The objective is to: 1) create an equilibrium among the players involved in integrated energy system which efficiently governs supply and demand 2) help in decision making and evaluation for a potential energy policy.

Numerical results show that the shadow price of coal, electricity and natural gas perfectly align with those prices reported by EIA [13] and makes model valid for practical use in decision making. The equilibrium model is driven in the direction of maximization of electricity consumers surplus. We analyze the electricity demand and shadow prices for all the three models. It shows that price changes according to demand and is inversely proportional to it. For example, at demand node i_1 and time t_1 the elastic demand for electricity is 298 Megawatts when only considering coal as the generation source and the corresponding shadow price of electricity is \$30/MWh. When considering both coal and natural gas as generation sources (but without natural gas' non-electric usage) the elastic demand for electricity is 311.69 Megawatts and corresponding shadow price of electricity is \$28.70/MWh. This shows that more electricity is produced when natural gas is made available cause the shadow price of the electricity to drop. It exactly holds true for coal and natural gas markets as well. When non-electric sector is added to the model, the shadow prices slightly increase due to the decrease in production of the electricity (part of the natural gas is used by the non-electric sector) but is less than the shadow price with model that has only the coal and electricity markets. For a cleaner energy policy, natural gas is more environmental friendly than coal. So, this model helps decision makers to decide the quantities of coal and natural gas to be used in their energy system based on shadow prices (output) of the developed EMEM model.

5.2 Future Research

There are several future research directions. First, we would like to investigate the theoretical aspect of sensitivity analysis performed on EMEM model by changing various parameters like natural gas pipeline capacity, inverse demand function of electricity, time horizons and the efficiency of generations units in a future study. Second, we would like to extend the EMEM model to imperfect competition environment where electricity generators play Nash-Cournot games among each other, but are price takers of natural gas and coal delivered prices. Third, we would like to discover if coal mine owners have the market power.

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