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Essays on Household Debt, Macroprudential Policy and Monetary Policy in South Korea

Hee Chang Jang

A thesis presented for the degree of
Doctor of Philosophy in economics at Durham University



October 2017

Dedicated to

My wife Ha Jung and my children
Jaehoo and Hyunseo

Essays on Household Debt, Macroprudential Policy and Monetary Policy in South Korea

Hee Chang Jang

Submitted for the degree of Doctor of Philosophy

October 2017

Abstract: Household debt in South Korea is high and still rising. Household debt to GDP ratio had risen at the similar pace with that in the US until 2007 but it has still been rising whereas it has been falling since 2017 in the US. As a result, it is now higher in South Korea than in the US. There was a dramatic growth in household debt in the US preceding the recent Great Recession and high level of household debt was viewed to amplify the severity of economic recession in the US constraining consumer spending. In this context, high and continuously rising household debt could be a potential risk factor for the South Korean economy. Macroprudential policy, which indicates policy aims to reduce financial systemic risk pre-emptively, is a crucial measure to slow down the pace of household debt growth in South Korea. However, there is no established tool to analyse or evaluate its effects and relationship to monetary policy.

The second chapter presents the trend and distribution of household debt in South Korea, and brief history of policy responses to continuously increasing household debt.

The third chapter shows how macroprudential policy works by using a simple heterogeneous DSGE model with collateral constraint. The model is based on so-called borrower-saver model. Despite of its simplicity, the model can clearly explain how macroprudential policy affects household debt and related variables in South Korea. In addition, dynamics of this model imply increasing amortisation rate is superior

measure to decreasing LTV ratio because it induces less volatility in economy. The collateral constraint in this thesis is designed to distinguish household debt (stock) and borrowing (flow). As a result, it is more realistic than the one mostly used in literature. This collateral constraint setting contributes to the better results especially when we analyse the phase of tightening household credit conditions. Furthermore, it enables us to see how amortization rate affects the South Korean economy.

The fourth chapter extends the model mainly to see how credit tightening and monetary policy work differently and how they interact. Habit formation in non-durable good consumption, price rigidity in non-durable good producers, fixed cost in intermediate good production and monetary policy are added in the model. Not only the newly added elements themselves but also inflation make model's responses different from those in the previous chapter. Nominal and real rigidities make dynamics last longer and more realistic. Due to the structure of collateral constraint, a rise in inflation can reduce the level of real household debt whereas there is no inflation effect on real household debt with the common type of collateral constraint. This also influences responses to monetary policy shock. The results demonstrate credit tightening is better than monetary policy in slowing down the growth rate of household debt. Among all policy measures considered, decreasing amortization rate is the most effective and increasing LTV ratio is the second. These implies that ongoing policy efforts to slow down the growth rate of household debt in South Korea is on the right track.

The fifth chapter shows welfare effects of macroprudential policy. The results illustrate it is impossible to get social welfare gains in a situation given in South Korea when discretionary macroprudential policy comes into effect. If government adopts countercyclical macroprudential rule, it is possible to improve social welfare but it requires welfare loss either of borrower or saver.

Declaration

The work in this thesis has been submitted elsewhere for any other degree or qualification and it is all my own work unless referenced to the contrary in the text.

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Chapter 1

Introduction

1.1 Introduction

Household debt in South Korea is high and still rising. Household debt to GDP ratio had risen at the similar pace with that in the US until 2007 but it has still been rising whereas it has been falling since 2007 in the US. As a result, it is now higher in South Korea than in the US. There was a dramatic growth in household debt in the US preceding the recent Great Recession. The high level of household debt was viewed to amplify the severity of economic recession in the US constraining consumer spending. In this context, high and continuously rising household debt is considered as a potential factor to threaten the stability of South Korean economy. Macroprudential policy, which indicates policy aims to reduce financial systemic risk pre-emptively, is a crucial measure to slow down the pace of household debt increase in South Korea. So, South Korean government is trying to slow down the growth rate of household debt by using macroprudential policy as we see in chapter 2, in spite of the lack of proper tools to estimate overall effects of its policies. This is because household debt is not usually incorporated into the macroeconomic models for either policymaking or academic analysis.¹ So we need new macroeconomic models which clearly consider household debt to cope with the ongoing economic developments in South Korea.

¹Eggertsson and Krugman (2012) point out that mainstream macroeconomic models usually do not have debt in them although debt is popular issue in economic discussion.

It is not surprising that the mainstream macroeconomic theories and models cannot explain clearly the relationship between the recent economic developments and household debt or suggests how to conduct macroprudential or monetary policy considering household debt because household debt is not included in them.² Because those theories and models depend on the Modigliani and Miller (1958) framework in which real sectors are not affected by financial sectors and, furthermore, debt is always net zero from a macroeconomic perspective: the liabilities of all borrowers always exactly match the assets of all lenders (Cecchetti et al., 2011). Therefore, representative household models cannot or do not have to consider household debt. However, there were some economists who clearly recognized the role of debt from a macroeconomic perspective. The first one who documented it is Fisher (1933) who had developed debt-deflation theory of depression. The concept of Fisher's debt-deflation is that the depression can be caused by a vicious circle of deflation which means that deflation increases the real burden of debt and then causes further deflation. Mishkin (1978) argues that the balance-sheet approach, which is based on the Fisher's debt-deflation theory, can provide an explanation for the reason why the aggregate demand dropped so severely in 1930 and it can explain the contraction of 1929-1933 and the severity of the 1937-1938 recession. Minsky and Kaufman (2008) shows a recurring cycle of instability, in which high leverage caused by complacency about debt during the calm periods for the economy leads to crisis. King (1994) presented his view on household debt in his 1994 European Financial Association Presidential Address based on Fisher's debt-deflation theory. He suggests that the real business cycle model is required to incorporate household debt into it.

The recent economic developments following the Global Financial Crisis of 2007-2008 has led a wide range of analytical research investigating the role of household debt in business cycle fluctuation.³ Moreover, the theoretical literature, developed by Guerrieri and Lorenzoni (2017), Hall (2011), Midrigan and Philippon (2011), Eggertsson and Krugman (2012), Justiniano et al. (2015), and Korinek and Simsek (2016)

²Even the most recent and sophisticated DSGE models based on Smets and Wouters (2007) and Christiano et al. (2005) do not include a financial system. Moreover, according to Kocherlakota et al. (2009), "Macro models with financial market frictions, such as borrowing constraints or limited insurance, were not used widely for macro policy analysis before the recent financial crisis."

³For example, Glick, Lansing, et al. (2009), Isaksen et al. (2014), and Chmelar et al. (2012).

among others, not only looks at possible logical relationships between household debt and recession, but also develop macroeconomic models which incorporate household debt or borrowing into. It needs to be mentioned that in this literature the level of household debt (or borrowing) plays roles to determine the length of recession and strength of the following economic recovery. In other words, the focus of research is rather limited on the periods of a recession and the following recovery. There is also empirical literature which investigates household debt and its macroeconomic effects. Cecchetti et al. (2011) find that when household debt goes beyond 85% of GDP, it becomes a drag on growth while for corporate debt they report a threshold around 90% of GDP. Mian and Sufi (2012) show a disproportionately larger decline in consumption and employment in counties that had higher household debt-to-income ratio by 2006 in the US. Martin and Philippon (2014) demonstrate consistent results with Mian and Sufi (2012) analysing euro area countries. Jordà et al. (2011) suggest that a credit build-up in the boom generally may heighten the vulnerability of economies based on a study of over 200 recession episodes in 14 advanced countries. Baker (2014) show that the drop in consumption during the 2007-2009 recession in the US was approximately 20% greater than what would have been seen with the household balance sheet position in 1983.

The main feature of models, which differentiates this thesis from other existing literature, is that debt (stock) and borrowing (flow) can be clearly distinguished in impatient household's collateral constraint. Especially when the value of collateral goes down, which means a reduction in loan-to-value (LTV) ratio and/or house prices, borrowers do not need to renew all the existing debt contracts under the less favourable conditions because lenders cannot force borrowers to repay the outstanding debt. Under this collateral constraint, household borrowing in each period is just a small portion of household debt. This clear distinction between household debt and borrowing is the key ingredient of this thesis. Household debt is usually assumed to be entirely renewed every period. But, in reality, borrowers do not need to renew all the outstanding debt especially under the less favourable situation. For South Korea, this clear distinction between household debt and borrowing has never been adopted before and can provide more realistic policy analysis when policymakers try to tighten credit conditions such as lowering LTV ratio or increase amortisation rate to slow down the pace of rise in

record-high household debt. Recently in South Korea credit tightening policies are introduced and expected to be introduced further in the near future.

This thesis aims to contribute to the macroeconomic policy analysis by focusing on how the macroeconomic variables in South Korea are influenced by household leveraging and deleveraging with the collateral constraints which can tell the difference between debt and borrowing. In the following chapters, I construct Dynamic Stochastic General Equilibrium (DSGE) models with household debt for South Korea to provide tools to analyse policy effects of macroprudential and monetary policies. This study is expected to provide useful tools for the policy makers in South Korea but these tools can be utilised by the policy makers of other countries with similar economic developments.

Highlighting this research gap, the following research questions are addressed in this thesis:

1. How would macroprudential policy affect the South Korean economy?
2. How would the effects of macroprudential policy be different from those of monetary policy in South Korea?
3. Is macroprudential policy more effective than monetary policy in slowing down the pace of increasing in household debt in South Korea?
4. What are the welfare implications of macroprudential policy in South Korea?

1.2 Contribution of this thesis

I develop Dynamic Stochastic General Equilibrium (DSGE) models which incorporate into household debt with more realistic borrowing constraints for the South Korean economy (Methodological Contribution). Then, I suggest some policy implications for South Korean policy makers to deal with high level of household debt considering overall macroeconomic effects and further provide household welfare implications of macroprudential policies (Policy Contribution).

1.3 Organisation of this thesis

Chapter 2 shows the recent trend and distribution of household debt and related policy responses in South Korea. Chapter 3 presents a model to analyse the effects of macroprudential policy in South Korea. This model is based on a simple standard Real Business Cycle (RBC) model which focuses on the real variables. It does not consider either nominal variables or monetary policy to see the effects of macroprudential policy with minimum scale. Before elaborating model specifications, key existing literature, which provides basic structure of models in this thesis, is discussed in detail. In calibration, we show how this model fits the South Korean economy considering household debt. And results from stochastic simulations are presented to show how the South Korean economy reacts dynamically to the exogenous shocks. Results show reasonable dynamics of variables considering the degree of simplicity of a RBC model leaving more detailed outcomes from the following DSGE model. In chapter 4, we add the monetary side to the basic model presented in the previous chapter and introduce a nominal rigidity in firm's pricing. In other words, a medium-scale New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model for South Korea, which incorporates nominal household debt, is presented to analyse a more comprehensive effects of macroprudential policy and monetary policy on macroeconomy in South Korea. Chapter 5 shows welfare effects of discretionary macroprudential policy using the model introduced in the previous chapter and analyses further to find optimal countercyclical macroprudential policy rule. Chapter 6 concludes with a summary of all results, limitations of this thesis, and future directions for research.

Chapter 2

Household Indebtedness in South Korea

2.1 Introduction

Household debt in South Korea is high and still rising. In this chapter, the trend and distribution (by income and age) of household debt in South Korea are illustrated and then the brief history of policy responses related to household debt is described.

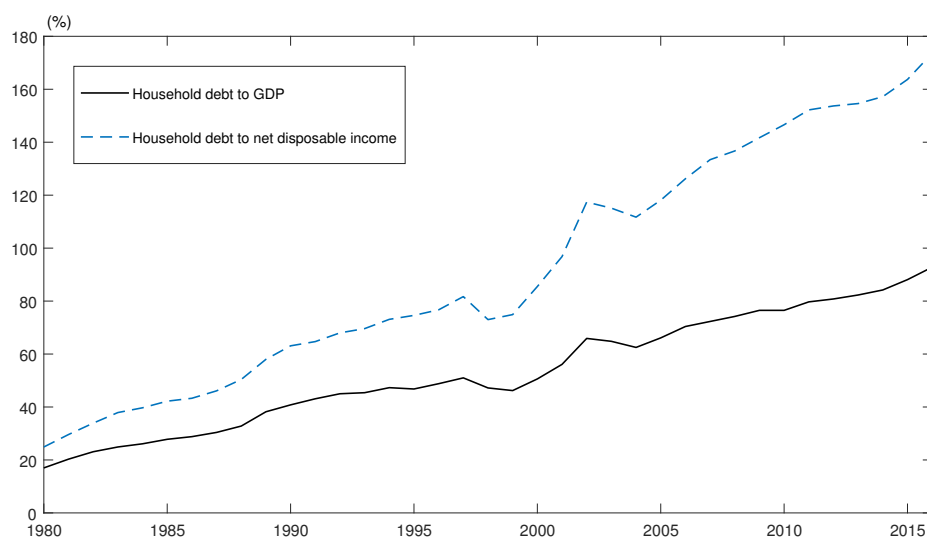
2.2 Trend and distribution

2.2.1 Trend

In South Korea, financial liberalisation and deregulation started in the early 1990s but mortgage was not liberalised before the 1997 Foreign Currency Crisis. As shown in Figure 2.1, household debt started to increase significantly since early 2000s following the substantial liberalisation of mortgage late 1990s. After recording 20-30% growth in household debt during the period of 2001-2002, the South Korean government reacted to the rapid credit growth by tightening financial regulation and supervision in 2003. Household debt increased until the end of 2003 when so-called credit card crisis happened and it shrank until 2004. However, it rebounded rapidly since 2005 and has

not been affected significantly by the Global Financial Crisis in 2007-2008. As a result, household debt has been rising at a steady pace without any significant adjustment since 2005 and, as of the end of 2016, household debt level in South Korea was over 90% of GDP and over 170% of net disposable income.

Figure 2.1: Trend of household debt in South Korea



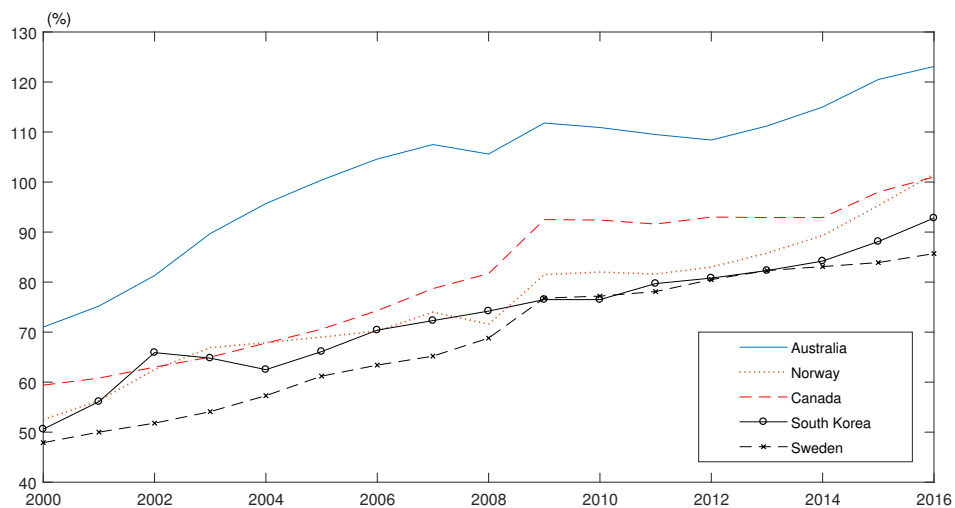
Sources: BIS, Bank of Korea

In Figure 2.2 and Figure 2.3, household debt to GDP ratios in selected advanced economies are compared. Figure 2.2 shows countries in which household debt is increasing like in South Korea. All these countries including South Korea have experienced very similar increasing trends in household debt to GDP ratios. Figure 2.3 describes countries in which household debt is decreasing after reaching its peak between 2007-2009. Household debt reached its peak in 2007 in the US and in 2009 in the rest of countries.

2.2.2 Distribution

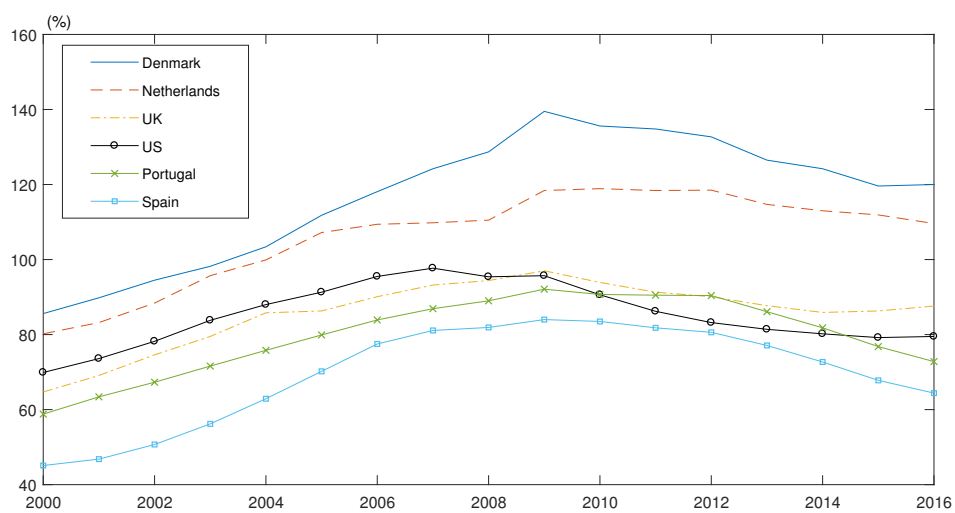
Table 2.1 shows household debt share by income quantile as of the end of March 2015. High income (4th and 5th quantile) households have about 70% of the total household debt and about 75% of the total household income. Low income (1st and 2nd quantile) households have only around 15% of household debt, which is quite higher than their income share (around 10%). Table 2.2 shows household debt share by age group as

Figure 2.2: Increasing household debt in selected countries (% of GDP)



Source: BIS

Figure 2.3: Decreasing household debt in selected countries (% of GDP)



Source: BIS

of the end of March 2015. Only the oldest age group (older than 60) households have higher share in household debt than in income. More than 80% of household debt is held by three older age groups (40-49, 50-59 and 60-).

Table 2.1: Household debt by income quintile (2015)

| | 1st (lowest) | 2nd | 3rd | 4th | 5th (highest) |
|------------------------|--------------|------|------|------|---------------|
| Household income share | 1.8 | 8.0 | 15.5 | 25.1 | 49.7 |
| Household debt share | 4.1 | 11.0 | 15.2 | 23.7 | 46.0 |

Source: Survey of Household Finances, Statistics Korea

Table 2.2: Household debt by age group (2015)

| | ~ 29 | 30 ~ 39 | 40 ~ 49 | 50 ~ 59 | 60 ~ |
|------------------------|------|---------|---------|---------|------|
| Household income share | 1.0 | 16.4 | 32.2 | 32.9 | 17.4 |
| Household debt share | 0.5 | 14.1 | 30.1 | 32.0 | 23.2 |

Source: Survey of Household Finances, Statistics Korea

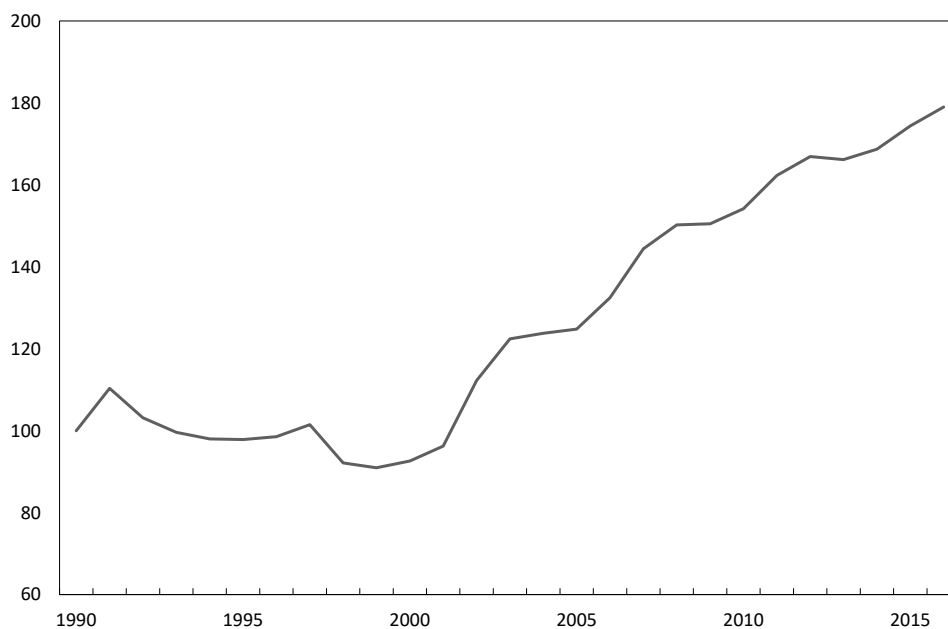
2.3 Policy responses

In South Korea, policy responses related to increasing household debt have been actually more closely related to increasing house prices. As Figure 2.4 shows, house price in South Korea has kept rising since 2000. Policies have been mainly focused on housing demand rather than housing supply. Housing demand can be affected by the availability of mortgage loan. Thus, policy responses to increasing household debt and house prices were conducted mainly by changing LTV ratios or DTI (debt to income) ratios.¹ Table 2.3 summarises the brief history of LTV regulation. It was first introduced in late 2002. Except in 2004 and 2014, LTV ratio were lowered to tighten household credit conditions because household debt and house prices have never decreased since the introduction of LTV regulation in 2002. In addition, interest-only mortgage was prohibited since 2016. Before that, there was no amortisation requirement. The share of interest-only mortgage was 93.6% as of the end of 2010 and it dropped to 61.1% as

¹See Igan and Kang (2011) for the brief history of changing DTI ratios in South Korea

of the end of 2015. Although more recent mortgage borrowers chose to amortise their debt, they were not forced by regulation.

Figure 2.4: House price index (1990=100)



Source: Kookmin Bank

Table 2.3: Brief history of LTV regulation

| | | Target (maturity, area and collateral value) | Change in LTV ratio |
|-----------|-----------------------|--|---------------------|
| Sep. 2002 | Introduction | Speculation-prone zone | 60% |
| Nov. 2002 | Expanding target area | All area | 60% |
| May. 2003 | Tightening | Under 3 years, Speculative and speculation-prone zone | 60% → 50% |
| Oct. 2003 | Tightening | Under 10 years, Speculative zone | 50 ~ 60% → 40% |
| Mar. 2004 | Loosening | Amortised over 10 years, All area | 60% → 70% |
| Jul. 2005 | Tightening | Non-amortised over 10 years, All area, Over 600mil won | 60% → 40% |
| Jul. 2009 | Tightening | Seoul Metropolitan Area, Over 600mil won | 60% → 50% |
| Jul. 2014 | Loosening | All maturity, All area, All collateral | 40 ~ 70% → 70% |
| Jul. 2014 | Loosening | All maturity, All area, All collateral | 40 ~ 70% → 70% |
| Jun. 2017 | Tightening | All maturity, Seoul and some other cities, Over 500mil won | 70% → 60% |

Source: Igan and Kang (2011) (recent three measures added by author)

Chapter 3

Household Debt and Macroprudential Policy in South Korea: a Simple DSGE Model

3.1 Introduction

Household debt in South Korea is high and still rising. Household debt to GDP ratio had risen at the similar pace with that in the US until 2007 but it has still been rising whereas it has been falling since 2007 in the US. As a result, it is now higher in South Korea than in the US as in Figure 3.1.¹ There was a dramatic growth in household debt in the US preceding the recent Great Recession. The high level of household debt was viewed to amplify the severity of economic recession in the US constraining consumer spending. In this context, high and continuously rising household debt is considered as a potential factor to threaten the stability of South Korean economy. In this context, macroprudential policy, which indicates policy aims to reduce financial systemic risk pre-emptively, is a crucial measure to slow down the pace of household debt increase in South Korea. So, South Korean government is trying to slow down the growth rate of household debt by using macroprudential policy as we see in chapter 2, in spite

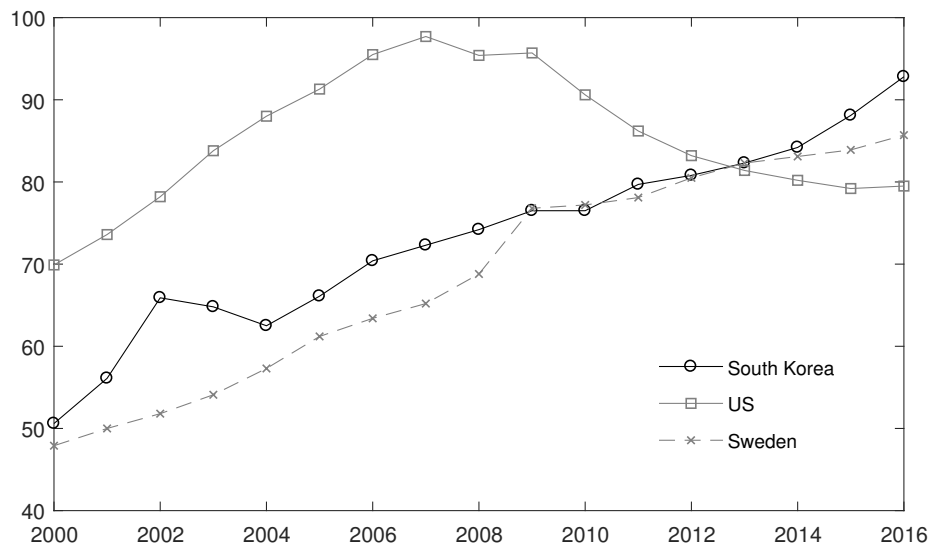
¹Household debt to GDP ratio in Sweden shows very similar increasing pace with that in South Korea.

of the lack of proper tools to estimate overall effects of its policies. This is because household debt is not usually incorporated into the macroeconomic models for either policymaking or academic analysis.² So I begin to construct a new macroeconomic model which clearly shows the role of household debt in South Korean macroeconomy and with which policymakers can simulate their policy measures.

It would be good enough to start from a simple RBC model without monetary policy rather than a larger scale general equilibrium model so as to focus on the basic mechanism of household debt and macroprudential policy in the economy. The results from this model show a fairly good performance in matching steady-state ratios and volatility in South Korea, especially regarding household debt related variables.

The rest of this chapter is structured as follows. Section 3.2 highlights key literature. Section 3.3 describes the model. Section 3.4 discusses main characteristics of equilibrium and steady-state. Calibration of parameter values is presented in section 3.5. Quantitative results are illustrated in section 3.6. Finally, conclusion is presented in section 3.7.

Figure 3.1: Household debt to GDP ratio



Source: BIS

²Eggertsson and Krugman (2012) point out that mainstream macroeconomic models usually do not have debt in them although debt is popular issue in economic discussion.

3.2 Literature Review

It is not surprising that the mainstream macroeconomic theories and models cannot explain clearly the relationship between the recent economic developments and household debt or suggests how to conduct macroprudential or monetary policy considering household debt because household debt is not included in them.³ Economic theories and models basically depend on the Modigliani and Miller (1958) framework in which real economy is not affected by financial sectors. Furthermore, debt is always net zero from a macroeconomic perspective: the liabilities of all borrower always exactly match the assets of all lenders (Cecchetti et al., 2011). If a macroeconomic model does not consider household debt, households do not need to be heterogeneous. However, to incorporate household debt into a model, households need to be assumed heterogeneous as King (1994) suggested.⁴ There are two ways to put heterogeneous households in a macroeconomic model in terms of the source of their heterogeneity. The first one is to assume that heterogeneity between households comes from uninsurable idiosyncratic shocks⁵ even though they are initially homogeneous. The second one is to assume that households have different time preferences from the beginning: patient household is a saver and impatient one is a borrower. In this thesis, the latter is used to model household's heterogeneity as in Campbell and Hercowitz (2005), Iacoviello (2005), Eggertsson and Krugman (2012) and Justiniano et al. (2015).

As Becker (1980)⁶ shows, there is no steady-state with positive consumption by all households if we set the economy with heterogeneous households in terms of different time preference. There are two different approaches to overcome this unrealistic result. First, setting borrowing limit for impatient household. The simplest way is to set this limit as exogenous as in Eggertsson and Krugman (2012). The more sophis-

³Even the most recent and sophisticated DSGE models based on Smets and Wouters (2007) and Christiano et al. (2005) do not include a financial system. Moreover, according to Kocherlakota et al. (2009), "Macro models with financial market frictions, such as borrowing constraints or limited insurance, were not used widely for macro policy analysis before the recent financial crisis."

⁴Household heterogeneity in King (1994) comes from different marginal propensity to spend between debtors and creditors.

⁵It is called Bewley-Aiyagari-Hugget model. For example, Guerrieri and Lorenzoni (2017) study this kind of model. The details of this model can be found in Heathcote et al. (2009).

⁶He assumes that a household's utility function is time-additive and stationary with a constant rate of time preference.

ticated way is to tie borrowing limit to the value of collateral. This kind of collateral constraint is spawned by Kiyotaki and Moore (1997)'s seminal paper. Second, making time preference time-variant. This type of time preference is called endogenous time preference. Shi and Epstein (1993) show that all heterogeneous households can have positive wealth in the long run under this specification. But this approach is technically hard to deal with macroeconomic models. So many recent researchers adopt the first approach which sets collateral constraint on impatient household. It is also used in this thesis.

Following Kiyotaki and Moore (1997), large literature use a collateral constraint in a macroeconomic model not only for firms but also for households. Among others, Iacoviello (2005), Campbell and Hercowitz (2005, 2009), Iacoviello and Neri (2010), Guerrieri and Iacoviello (2017), and Justiniano et al. (2015) adopt this framework for households. Kiyotaki and Moore (1997) set a collateral constraint for firms and collateral is the land. Iacoviello (2005) develops it as a collateral constraint for impatient household and housing stock is used as collateral. Campbell and Hercowitz (2005) use a similar setting but there is an interesting difference between Iacoviello (2005) and Campbell and Hercowitz (2005). Iacoviello (2005) uses the same collateral constraint as in Kiyotaki and Moore (1997), which has only loan-to value (LTV) ratio and in which household debt is entirely renewed every time. A collateral constraint in Campbell and Hercowitz (2005) is slightly different. It explicitly incorporates amortisation rate as well as LTV ratio and household debt is not entirely renewed every time under debt contract. This mechanism enables us to look at the behaviour in borrowing and accumulated debt separately. The model in this chapter is an extension of Campbell and Hercowitz (2005) by making LTV ratio and amortisation rate time-variant.⁷ As macroprudential policy to secure financial stability is a recent topic in academia as well as in policymakers, literature which focuses specifically on it has relatively short history. Gelain et al. (2013) find that macroprudential tools such as change in LTV ratio is effective for dampening excess volatility in the economy. Rubio and Carrasco-Gallego (2014) analyse that rule based LTV ratio can improve the stability of economy.

⁷Justiniano et al. (2015) set a similar collateral constraint, but only LTV ratio is time-variant. Chen and Columba (2016) illustrate time-variant amortisation requirement model but their analysis is done only in terms of permanent changes in amortisation requirement.

Justiniano et al. (2015) show that decrease in LTV ratio leads to decline in debt to GDP ratio.

There is empirical literature investigates household debt and its macroeconomic effects. Cecchetti et al. (2011) find that when household debt goes beyond 85% of GDP, it becomes a drag on growth while they report a threshold around 90% of GDP for corporate debt. Mian and Sufi (2012) show a disproportionately larger decline in consumption and employment in counties that had higher household debt-to-income ratio by 2006 in the US. Martin and Philippon (2014) demonstrate consistent results with Mian and Sufi (2012) analysing euro area countries. Jordà et al. (2011) suggest that a credit build-up in the boom generally may heighten the vulnerability of economies based on a study of over 200 recession episodes in 14 advanced countries. Baker (2014) show that the drop in consumption during the 2007-2009 recession in the US was approximately 20% greater than what would have been seen with the household balance sheet position in 1983.

Many researchers have studied how household debt can affect macroeconomy following the Global Financial Crisis of 2007-2008. However, a vast majority of research focuses only on the US and European countries where the recent financial crisis happened. Comparing South Korean economy with that of the US from the perspective of household debt and its macroeconomic influences, there must be not only similarities but also differences. In South Korea, household debt has not apparently harmed the economy yet, whereas it affected the economy negatively in the US through the recent episodes of financial crisis and following slow recovery. In this context, many researchers in South Korea just focus on sustainability of household debt as in Kim et al. (2014) rather than building macroeconomic models which include household debt. Some recent studies (Jung, 2015; Lee, 2011; Lee and Song, 2015) for the South Korean economy have considered the role of household debt by using structural DSGE models but incorporated only constant LTV ratio as a parameter in borrower's collateral constraint. As for macroprudential policy, Igan and Kang (2011) find the impact of change in LTV ratio on house price rather than overall economy.

3.3 Model

In this chapter, I try to build a simple DSGE model based on a Kydland and Prescott (1982)' RBC model. A collateral constraint is incorporated as in Campbell and Hercowitz (2005) to understand how household borrowing and debt affect major macro variables and vice versa. Shocks come from changes not only in productivity but also in LTV ratio and amortisation rate. In addition, we add housing preference shock to produce house price change which is not related to credit conditions.⁸ To make the model as simple as possible, price stickiness and monetary policy are ruled out.⁹ So we can see the simple and basic role of household debt in a simplified economy. The more complex model with inflation, price rigidity and monetary policy will be introduced in the next chapter mainly for the monetary policy analysis.

Time is discrete and its horizon is infinite in this economy. There are impatient and patient households, non-durable good producer, house producer and government. Households are heterogeneous in terms of different time preference. Impatient household has higher discount rate (lower discount factor) than patient household, so that impatient household borrows from patient household against collateral (houses). Impatient household's borrowing is limited to the certain ratio of collateral value. Both type of household consumes non-durable goods, own houses, and provide labours. Patient household owns the entire capital stock because impatient household never owns capital with perpetually binding borrowing constraint. Non-durable good producer produces non-durable goods, hiring labour from both households and combining them with capital according to a constant return to scale (CRS) production function. House producer purchases a certain amount of non-durable goods to transform them into houses, which it sells to households. Government balances its budget. There are four key assumptions. First, households are heterogeneous in terms of different time preference, which induces lending and borrowing among them. Second, households own houses which serve as collateral for impatient household to finance. Third, there are two kinds of producers in the supply side: one is a good producer and the other is a

⁸We follow this setting as in Iacoviello (2005), Iacoviello and Neri (2010) and Justiniano et al. (2015).

⁹Without fiscal policy, housing stock and housing investment cannot help but being estimated higher than the actual data. Thus, fiscal policy is included in this model.

house producer. Fourth, there is no central bank, that is, no monetary policy.

Impatient Household

Impatient and patient households are denoted by b (borrower) and s (saver), respectively. Impatient household shares ψ of the population ($0 < \psi < 1$). Utility function for impatient household is as follows.¹⁰

$$U_{b,t} = \ln C_{b,t} + \phi_t \ln H_{b,t} - \frac{L_{b,t}^{1+\eta}}{1+\eta} \quad (3.3.1)$$

where $C_{b,t}$ is consumption of non-durable goods, $H_{b,t}$ is stock of houses (durable goods), $L_{b,t}$ is hours worked, η is inverse Frisch elasticity of labour supply and ϕ_t is preference for housing services. The price of non-durable goods is assumed to be one. Fluctuations in ϕ_t can be interpreted as random changes in marginal utility of housing stock. A cycle in house prices can be mimicked by changing ϕ_t . ϕ_t is exogenous and its log follows AR(1) process and ϕ is the steady-state value of ϕ_t .

$$\ln \phi_t = \rho_\phi \ln \phi_{t-1} + (1 - \rho_\phi) \ln \phi + \varepsilon_{\phi,t} \quad (3.3.2)$$

where $0 < \rho_\phi < 1$ and $\varepsilon_{\phi,t}$ is an *i.i.d.* zero mean normal random disturbance with constant variance σ_ϕ^2 .

Impatient household maximises its lifetime expected utility at time 0.

$$E_0 \sum_{t=0}^{\infty} \beta_b^t U_{b,t}$$

where β_b is impatient household's discount factor and $\beta_b < \beta_s$.

Budget constraint is

$$C_{b,t} + P_{h,t} N_{b,t} + (R_{t-1} - 1 + \varrho_{t-1}) D_{b,t} \leq W_{b,t} L_{b,t} + D_{b,t+1} - (1 - \varrho_{t-1}) D_{b,t} - T_{b,t} \quad (3.3.3)$$

where $N_{b,t} = H_{b,t+1} - (1 - \delta_h) H_{b,t}$, $N_{b,t}$ is residential investment (new houses), $P_{h,t}$

¹⁰We assume separability in household utility function of durables (houses) and non-durables as in Iacoviello (2005), Campbell and Hercowitz (2005), and many other previous work. Bernanke (1985) shows that separability in household utility of durables and non-durables is not rejected empirically, so that separability in household utility function across goods does not harm the plausibility of results from it.

is house price, $W_{b,t}$ is the (real) wage, $T_{b,t}$ is lump-sum tax and transfer from the government and $D_{b,t}$ is the amount of (real) debt at the end of time $t-1$ and at the beginning of time t . The interest paid for existing debt at time t is $(R_{t-1} - 1)D_{b,t}$ where R_{t-1} is gross (real) interest rate. Borrowing is different from debt in our model because debt is not fully paid back each period.¹¹ A ratio of debt paid back at time t is amortisation rate, ϱ_{t-1} . New borrowing at time t is $D_{b,t+1} - (1 - \varrho_{t-1})D_{b,t}$.

Impatient household can only borrow up to a certain fraction of newly purchased houses which serve as collateral at time t , as in Campbell and Hercowitz (2005) and Chen and Columba (2016). Its collateral constraint is as follows.

$$D_{b,t+1} - (1 - \varrho_{t-1})D_{b,t} \leq \theta_t P_{h,t} N_{b,t}^{12} \quad (3.3.4)$$

where ϱ_{t-1} is the stochastic amortisation rate. ϱ_{t-1} can be different from depreciation rate, δ_h . ϱ_t is exogenous and its log follows AR(1) process and ϱ is the steady-state value of ϱ_t .

$$\ln \varrho_t = \rho_\varrho \ln \varrho_{t-1} + (1 - \rho_\varrho) \ln \varrho + \varepsilon_{\varrho,t} \quad (3.3.5)$$

where $0 < \rho_\varrho < 1$ and $\varepsilon_{\varrho,t}$ is an *i.i.d.* zero mean normal random disturbance with constant variance σ_ϱ^2 .

Similarly, θ_t is the stochastic loan-to-value (LTV) ratio and θ is the steady-state value of θ_t .

$$\ln \theta_t = \rho_\theta \ln \theta_{t-1} + (1 - \rho_\theta) \ln \theta + \varepsilon_{\theta,t} \quad (3.3.6)$$

where $0 < \rho_\theta < 1$ and $\varepsilon_{\theta,t}$ is an *i.i.d.* zero mean normal random disturbance with constant variance σ_θ^2 .

Lagrangian for impatient household can be defined as follows.

$$\begin{aligned} \mathcal{L} = & E_0 \sum_{t=0}^{\infty} \beta_b^t \left[\ln C_{b,t} + \phi_t \ln H_{b,t} - \frac{L_{b,t}^{1+\eta}}{1+\eta} + \lambda_{b,t} [W_{b,t} L_{b,t} + D_{b,t+1} - T_{b,t} - C_{b,t} \right. \\ & \left. - P_{h,t} \{H_{b,t+1} - (1 - \delta_h) H_{b,t}\} - R_{t-1} D_{b,t}] + \lambda_{b,t} \mu_{b,t} [(1 - \varrho_t) D_{b,t} \right] \end{aligned}$$

¹¹In most of literature (Iacoviello, 2005; Kiyotaki and Moore, 1997), debt is assumed to be fully paid back at the beginning of each period and get a new borrowing at the end of period. So borrowing is always equal to debt.

¹²Campbell and Hercowitz (2005) describes borrower's collateral constraint as $D_{b,t} = \sum_{j=0}^{\infty} (1 - \varrho)^j \theta P_{h,t-j} N_{b,t-j}$. This is almost same with Equation 3.3.4, if written recursively. It is different from Equation 3.3.4 in that ϱ and θ is set to be constant.

$$+ \theta_t P_{h,t} \{H_{b,t+1} - (1 - \delta_h)H_{b,t}\} - D_{b,t+1} \} \Big] \quad (3.3.7)$$

where $\lambda_{b,t}$ is the current-value Lagrangian multiplier on budget constraint and $\lambda_{b,t}\mu_{b,t}$ is the current-value Lagrangian multiplier on borrowing constraint. $\lambda_{b,t}$ is the shadow value of impatient household's budget constraint and measures the marginal value in units of non-durable good of relaxing the budget constraint and $\mu_{b,t}$ measures the marginal value in units of non-durable good of relaxing the borrowing constraint (Campbell and Hercowitz, 2005). This function is maximised with respect to $C_{b,t}$, $H_{b,t+1}$, $L_{b,t}$, and $D_{b,t+1}$.

Patient Household

Patient household also maximises its lifetime expected utility at time 0.

$$E_0 \sum_{j=0}^{\infty} \beta_s^j \left[\ln C_{s,t} + \phi_t \ln H_{s,t} - \frac{L_{s,t}^{1+\eta}}{1+\eta} \right] \quad (3.3.8)$$

Budget constraint is

$$C_{s,t} + P_{h,t}N_{s,t} + I_{s,t} + R_{t-1}D_{s,t} \leq W_{s,t}L_{s,t} + R_{k,t}K_{s,t} + D_{s,t+1} - T_{s,t} \quad (3.3.9)$$

where $N_{s,t} = H_{s,t+1} - (1 - \delta_h)H_{s,t}$, $I_{s,t} = \frac{I_t}{(1-\psi)}$, and $K_{s,t} = \frac{K_t}{(1-\psi)}$. $I_{s,t}$ is patient household's investment in production capital, $K_{s,t}$ is the stock of capital owned by patient household, $R_{k,t}$ is capital rental rate and $T_{s,t}$ is lump-sum tax and transfer from the government.

The stock of capital is determined by the following equation.

$$K_{t+1} = (1 - \delta_k)K_t + F_t \quad (3.3.10)$$

where δ_k is the physical depreciation rates of capital stock and the function $F_t(I_t, I_{t-1})$ summarizes the technology that transforms I_t and I_{t-1} into installed capital for use at time t, as in Christiano et al. (2005). $F_t(I_t, I_{t-1})$ is given by

$$F_t(I_t, I_{t-1}) = \left[1 - S_k \left(\frac{I_t}{I_{t-1}} \right) \right] I_t \quad (3.3.11)$$

where $S_k(\frac{I_t}{I_{t-1}}) = \zeta_k \frac{1}{2} (\frac{I_t}{I_{t-1}} - 1)^2$ is investment adjustment cost.¹³ In steady-state, $S_k = S_k' = 0$ and $S_k'' = \zeta_k > 0$.

Lagrangian for patient household can be defined as follows.

$$\begin{aligned} \mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta_s^t & \left[\ln C_{s,t} + \phi_t \ln H_{s,t} - \frac{L_{s,t}^{1+\eta}}{1+\eta} + \lambda_{s,t} \left[W_{s,t} L_{s,t} + \frac{R_{k,t} K_t}{1-\psi} + D_{s,t+1} - T_{s,t} \right. \right. \\ & \left. \left. - C_{s,t} - P_{h,t} \{ H_{s,t+1} - (1-\delta_h) H_{s,t} \} - \frac{I_t}{1-\psi} - R_{t-1} D_{s,t} \right] \right. \\ & \left. + \frac{\lambda_{s,t} \mu_{s,t}}{1-\psi} \left[\left\{ 1 - \zeta_k \frac{1}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right\} I_t - K_{t+1} + (1-\delta_k) K_t \right] \right] \end{aligned} \quad (3.3.12)$$

where $\lambda_{s,t}$ is the current-value Lagrangian multiplier on budget constraint. This function is maximised with respect to $C_{s,t}$, $H_{s,t+1}$, $L_{s,t}$, K_{t+1} , $D_{s,t+1}$ and I_t .

Non-durable Good Producer

The production function of non-durable good producer is described by a Cobb-Douglas function with constant return to scale (CRS) by combining labours of both types of households and capital. The imperfect elasticity of substitution between the labour supplied by savers and by borrowers is assumed as in Iacoviello and Neri (2010).¹⁴

$$Y_t = A_t^{1-\alpha} K_t^\alpha \left[\{\psi L_{b,t}\}^\nu \{(1-\psi) L_{s,t}\}^{1-\nu} \right]^{1-\alpha}, \quad 0 < \alpha < 1 \quad \text{and} \quad 0 < \nu < 1 \quad (3.3.13)$$

where A_t is exogenous labour-augmenting (or, equivalently, Harrod-neutral) technological progress. The level of technology is non-stationary and it follows a stationary AR(1) process in the log. We abstract from growth.

$$\ln A_t = \rho_a \ln A_{t-1} + \varepsilon_{a,t}, \quad \varepsilon_{a,t} \sim i.i.d.N(0, \sigma_a^2) \quad (3.3.14)$$

Non-durable good producer maximises profits subject to the production function above.

$$\max_{L_{b,t}, L_{s,t}, K_t} E_0 \sum_{t=0}^{\infty} \beta_s^t \lambda_{s,t} \left[A_t^{1-\alpha} K_t^\alpha \left[\{\psi L_{b,t}\}^\nu \{(1-\psi) L_{s,t}\}^{1-\nu} \right]^{1-\alpha} - W_{b,t} \psi L_{b,t} \right]$$

¹³Without investment adjustment cost, capital stock will increase sharply in response to a technology shock, which is counterfactual. Campbell and Hercowitz (2005) assumes fixed capital as an extreme case of investment adjustment cost.

¹⁴As Iacoviello and Neri (2010) point out, this assumption is for analytical simplicity because perfect substitution in production function makes a complex interplay between borrowing constraints and labour supply decisions.

$$- W_{s,t}(1 - \psi)L_{s,t} - R_{k,t}K_t \Big] \quad (3.3.15)$$

House Producer

The production functions of house producer are as in Justiniano et al. (2015).

$$N_t = \left\{ 1 - S_{h,t} \left(\frac{I_{h,t}}{I_{h,t-1}} \right) \right\} I_{h,t} \quad (3.3.16)$$

They purchase an amount of $I_{h,t}$ of final (non-durable) goods. $S_{h,t}(x) = \zeta_h \frac{1}{2}(x - 1)^2$. In steady-state, $S_h = S'_h = 0$ and $S''_h = \zeta_h > 0$. As ζ_h increases, the supply of houses becomes less elastic.

House producer maximises profits subject to the production function above.

$$\max_{I_{h,t}} E_0 \sum_{t=0}^{\infty} \beta_s^t \lambda_{s,t} \left[P_{h,t} \left\{ 1 - \zeta_h \frac{1}{2} \left(\frac{I_{h,t}}{I_{h,t-1}} - 1 \right)^2 \right\} I_{h,t} - I_{h,t} \right] \quad (3.3.17)$$

where $\lambda_{s,t}$ is the marginal utility of income of saver, that is to say, the current-value Lagrangian multiplier on budget constraint of patient household.

Fiscal Policy

The government spending follows AR(1) process in its log:

$$\ln G_t = \rho_G \ln G_{t-1} + (1 - \rho_G) \ln G + \varepsilon_{G,t} \quad (3.3.18)$$

where $0 < \rho_G < 1$ and $\varepsilon_{G,t}$ is an *i.i.d.* zero mean normal random disturbance with constant variance σ_G^2 .

The government raises revenue via lump sum taxes and balances its budget:

$$G_t = gY_t = \psi T_{b,t} + (1 - \psi)T_{s,t} \quad (3.3.19)$$

Thus, patient household can only lend to impatient household, and the net supply of borrowing is zero.

The share of taxes borrower pays is set to be γ . This indicates the degree of

government redistribution. As γ decreases, government redistributes more to borrower.

$$\psi T_{b,t} = \gamma G_t = \gamma g Y_t \quad (3.3.20)$$

Market Clearing Conditions

Labour, house and debt markets are cleared as follows.

$$L_t = (1 - \psi)L_{s,t} + \psi L_{b,t} \quad (3.3.21)$$

$$N_t = (1 - \psi)N_{s,t} + \psi N_{b,t} \quad (3.3.22)$$

$$\psi D_{b,t+1} + (1 - \psi)D_{s,t+1} = 0 \quad (3.3.23)$$

The government balances its budget.

To get goods market clearing condition, we need to aggregate budget constraints of both type of household as follows.

$$\begin{aligned} & (1 - \psi)C_{s,t} + \psi C_{b,t} + P_{h,t}[(1 - \psi)N_{s,t} + \psi N_{b,t}] + I_t + R_{t-1}[(1 - \psi)D_{s,t} + \psi D_{b,t}] \\ & = W_{b,t}\psi L_{b,t} + W_{s,t}(1 - \psi)L_{s,t} + R_{k,t}K_t + (1 - \psi)D_{s,t+1} + \psi D_{b,t+1} - [(1 - \psi)T_{s,t} + \psi T_{b,t}] \end{aligned}$$

$Y_t = W_{b,t}\psi L_{b,t} + W_{s,t}(1 - \psi)L_{s,t} + R_{k,t}K_t$ and $P_{h,t}N_t = I_{h,t}$ because the profits of non-durable good producer and house producer are assumed to be zero. Therefore, goods market clearing condition is

$$Y_t = C_t + I_{h,t} + I_t + G_t \quad (3.3.24)$$

where $C_t = (1 - \psi)C_{s,t} + \psi C_{b,t}$ and $I_t = (1 - \psi)I_{s,t}$.

3.4 Equilibrium and Steady State

3.4.1 Equilibrium

As mentioned above, household debt has not been commonly incorporated into macroeconomic models. Because of this lack, our understanding of economy regarding household debt may stay in the area of conjecture without models. Even recent literature

has an emphasis on resulting responses of economy mainly based on impulse response function rather than provide mechanism behind in detail. This may be partly because of the complex structure of medium-scale DSGE models which are mostly used. Although we can conjecture the dynamics of economy regarding household debt without a model, verifying it based on a model will make our understanding much more clear.

The first order conditions of borrower's utility maximisation are as follows.

$$\lambda_{b,t} = \frac{1}{C_{b,t}} \quad (3.4.1)$$

$$P_{h,t}(1 - \mu_{b,t}\theta_t) = \beta_b E_t \left[\frac{\lambda_{b,t+1}}{\lambda_{b,t}} \left\{ \phi_{t+1} \frac{C_{b,t+1}}{H_{b,t+1}} + (1 - \delta_h) P_{h,t+1}(1 - \mu_{b,t+1}\theta_{t+1}) \right\} \right] \quad (3.4.2)$$

$$W_{b,t} = C_{b,t} L_{b,t}^\eta \quad (3.4.3)$$

$$\lambda_{b,t}(1 - \mu_{b,t}) = \beta_b E_t \lambda_{b,t+1} \{ R_t - (1 - \varrho_t) \mu_{b,t+1} \} \quad (3.4.4)$$

Equation 3.4.1 shows the marginal value of additional current resources of borrower. As Campbell and Hercowitz (2005) explains, the borrower does not have a standard linear inter-temporal budget constraint due to the borrowing constraint. Therefore, $\lambda_{b,t}$ cannot be interpreted as the value of relaxing the inter-temporal budget constraint. In Equation 3.4.2, $P_{h,t}(1 - \mu_{b,t}\theta_t)$ can be interpreted as the effective relative house price. The effective relative house price is less than the actual relative house price ($P_{h,t}$) because collateral constraint relaxes when borrower purchase houses. Equation 3.4.3 describes borrower's optimal labour supply condition. When borrower does not have a debt at all, which means borrower does not have any housing stock, $C_{b,t}$ is equal to $W_{b,t}L_{b,t}$ and then $L_{b,t} = 1$. Therefore, borrower works more only in order to purchase houses and get borrowing in this case. However, when borrower has any debt, which is more realistic setting such as in steady-state, borrower's labour supply is not constant. Equation 3.4.4 is different from the standard consumption Euler equation because impatient household has a borrowing constraint. This can be rewritten as

$$U'(C_{b,t})(1 - \mu_{b,t}) = \beta_b E_t U'(C_{b,t+1}) \{ R_t - (1 - \varrho_t) \mu_{b,t+1} \}$$

If there is no borrowing constraint ($\mu_{b,t} = \mu_{b,t+1} = 0$), this reduces to the standard consumption Euler equation.

The rest of equilibrium conditions are listed in appendix 3.A as a full set.

3.4.2 Steady State

In steady-state, we can see borrowing constraint binds as follows.

$$\mu_b = \frac{1 - \frac{\beta_b}{\beta_s}}{1 - \beta_b(1 - \varrho)} > 0 \quad (3.4.5)$$

where real gross interest rate R is equal with $\frac{1}{\beta_s}$. From Equation 3.4.2, borrower's housing stock to non-durable consumption ratio is

$$\frac{H_b}{C_b} = \frac{\beta_b \phi}{(1 - \mu_b \theta)[1 - (1 - \delta_h)\beta_b]} \quad (3.4.6)$$

When the effective relative house price $(1 - \mu_b \theta)$ rises (steady-state value of $P_{h,t}$ is 1), $\frac{H_b}{C_b}$ falls. The effective relative house price increases when borrower's credit conditions get worse (θ lowers). In other words, tightening credit conditions makes borrower's housing stock to non-durable consumption ratio reduce.

From Equation 3.3.4, borrower's debt to housing stock ratio is

$$\frac{D_b}{H_b} = \frac{\theta P_h \delta_h}{\varrho} \quad (3.4.7)$$

When borrower's credit conditions get worse (θ lowers or ϱ rises), borrower's debt to housing stock ratio decreases.

The rest of deterministic steady-state values are listed in appendix 3.B as a full set.

3.5 Calibration

In South Korea, key statistics show relative stability after the Korean Financial Crisis 1997-1998. Meanwhile, household debt to GDP ratio shows its relative stability since 2003. So we choose the value of parameters using statistics for the period of 2003-2015.

During that period, annual inflation is 2.65% and nominal annual interest rate is 3.20%.¹⁵ Therefore the steady-state gross real interest rate (R) is set to be 1.001. In steady-state, $R = \frac{1}{\beta_s}$. Thus, saver's discount factor (β_s) is set to be 0.998. Calibration

¹⁵Calculated by using average CPI and Call Rate.

for the borrower's discount factor (β_b) is difficult because it does not affect the interest rate. We set $\beta_b = 0.995$ so that it is smaller than β_s but needs to be large enough to guarantee an equilibrium in which borrowing constraint will hold. These values are very close to those in Campbell and Hercowitz (2005) and Iacoviello (2005).¹⁶ η is calibrated to be one so that a Frisch elasticity of labour supply ($\frac{1}{\eta}$) is one, as in Justiniano et al. (2015).¹⁷ The population share of borrower (ψ) is set to 0.657, which is average share of borrower in household during 2012-2015.¹⁸ The loan-to-value ratio (θ) is chosen to be 0.65 to match the household debt-to-GDP ratio (77.1%). The depreciation of houses (δ_h), amortisation rate (ρ)¹⁹ and housing preference parameters (ϕ) are set to 0.005, 0.007 and 0.135 to match three targets. The first target is the house-to-GDP ratio, which we estimate from National Balance Sheet and National Accounts data as the average ratio between the market value of houses owned by household and non-profit organisations and nominal GDP (214.2%). The second target is the household debt to GDP ratio (77.1%). The last target is the ratio of residential investment to GDP (4.5%). On the production side, we follow standard practice and set the elasticity of the production function (α) equal to 0.33, and the depreciation of productive capital (δ_k) to 0.025, which match the investment-to-GDP ratio (27.5%). The wage share of patient household (ν) is set to be 0.8 to match the regular income share of indebted household (62.9%) in Survey of Household Finances and Living Condition. Investment adjustment cost parameter in capital production (ζ_k) is set at 0.5. Adjustment cost parameter in house production (ζ_h) is calibrated at 1.0.²⁰ These values of adjustment

¹⁶In Justiniano et al. (2015), discount factors of saver and borrower are 0.998 and 0.99, respectively.

¹⁷Hansen (1985) and Campbell and Hercowitz (2005) set household's utility function to be linear in leisure, which means $\eta = 0$. Iacoviello (2005) set $\eta = 0.01$, which is very close to be linear. Meanwhile, for South Korea, Elekdag et al. (2006) estimated η using Bayesian method and it was 0.889.

¹⁸Share of borrower in South Korea can be found in Survey of Household Finances and Living Condition which is available only since 2012.

¹⁹In South Korea, more than 90% of mortgage was interest-only type before 2010. According to Financial Services Commission, 93.6% of mortgage was interest-only type as of the end of 2010. Therefore, amortisation rate should be set very low. However, it is set to be higher than house depreciation rate to cancel out over-estimation of debt level in this model. Without inflation in the model, debt to GDP ratio is estimated far above than the actual data. In chapter 4, it can be set much lower because there is a positive inflation in that model.

²⁰Justiniano et al. (2015) assumes an extreme case of nearly fixed supply of house ($\zeta_h = 600$) and it is for 'credit liberalisation' experiment. Chen and Columba (2016) also sets fixed supply of houses considering unchanged housing stock per capita since 1990s in Sweden. However, in South Korea, the housing stock per capita has been growing from 1995 to 2015 according to Ministry of Land, Infrastructure and Transport.

cost parameters are calibrated to match volatility in capital investment and housing investment. The autocorrelations of shocks ($\rho_a = 0.95$, $\rho_\varrho = \rho_\theta = \rho_\phi = \rho_g = 0.85$) are set as in Iacoviello (2005) and Justiniano et al. (2015).

Table 3.1: Parameter values

| Description | Parameter | Value |
|---|------------------|-------|
| Discount factor of patient household | β_s | 0.998 |
| Discount factor of impatient household | β_b | 0.995 |
| Frisch elasticity of labour supply | $\frac{1}{\eta}$ | 1.000 |
| Population share of borrower | ψ | 0.657 |
| Capital share of the production function | α | 0.330 |
| Depreciation of productive capital | δ_k | 0.025 |
| Depreciation of houses | δ_h | 0.005 |
| Amortisation | ϱ | 0.007 |
| Adjustment cost parameter in capital investment | ζ_k | 0.500 |
| Adjustment cost parameter in house production | ζ_h | 1.000 |
| Wage share of patient household | ν | 0.800 |
| Loan-to-value ratio | θ | 0.650 |
| Housing preference | ϕ | 0.135 |
| Autocorrelation of technology shock | ρ_a | 0.950 |
| Autocorrelation of amortisation shock | ρ_ϱ | 0.850 |
| Autocorrelation of credit shock | ρ_θ | 0.850 |
| Autocorrelation of housing preference shock | ρ_ϕ | 0.850 |
| Autocorrelation of fiscal shock | ρ_g | 0.850 |

3.6 Quantitative Analysis

Based on the calibrated parameter values, the model performance is assessed by comparing the model's steady-state ratios and second moments with the actual data.

As shown in Table 3.2, the model's steady-state ratios are close to the actual data. Consumption to output ratio (51.0%), capital investment to output ratio (30.6%), residential investment to output ratio (4.3%), housing stock to output ratio (212.5%), and household debt to output ratio (78.1%) are close to the ratio from the data (50.9%, 27.5%, 4.5%, 214.2%, and 77.1%²¹, respectively). The fact that household debt related

²¹Household debt to GDP ratio data is available from the Bank of Korea and Bank for International Settlement (BIS). We use it from the former (77.1%). The ratio from the latter is 74.2%. For the purpose of international comparison, the ratio from the latter is usually chosen.

ratios such as housing stock to output ratio and household debt to output ratio show fairly close values to the actual data without hampering relevance of the rest of ratios can be a contribution of this model.

Table 3.2: Steady-state ratios

| | Ratio | Data | Model |
|-------------------------------|---|--------|--------|
| $\frac{C}{Y}$ | $\frac{\text{Consumption}}{\text{Output}}$ | 50.9% | 51.0% |
| $\frac{I}{Y}$ | $\frac{\text{Capital investment}}{\text{Output}}$ | 27.5% | 30.6% |
| $\frac{I_h}{Y}$ | $\frac{\text{Residential investment}}{\text{Output}}$ | 4.5% | 4.3% |
| $\frac{H}{4 \times Y}$ | $\frac{\text{Housing stock}}{\text{Output}}$ | 214.2% | 212.5% |
| $\frac{\psi D_b}{4 \times Y}$ | $\frac{\text{Household debt}}{\text{Output}}$ | 77.1% | 78.1% |

Table 3.3 shows results of matching second moments from simulation (periods=1000) of the model and the actual data. This is crucial for the evaluation of model performance especially regarding business cycle analysis. In the table, model 2 indicates the model with the typical collateral constraint which assumes debt is fully paid back at the beginning of each period and get a new borrowing (debt) as in Kiyotaki and Moore (1997) and Iacoviello (2005). Model 1 and Model 2 are identical except the borrower's collateral constraint. Second moments for Model 1 are closer to the household debt data than those for Model 2 while maintaining similar performance for the rest of variables. The standard deviation of household debt ($D_{b,t}$) is much closer to the data in Model 1 than Model 2. Correlations with output are also closer to the data in Model 1 than Model 2. The collateral constraint setting (Equation 3.3.4) contributes for producing volatility and correlation closer to the data especially in household debt because it assumes only a fraction of household debt is renewed or added as in the actual data.

The model dynamics responding to the shocks will be shown next. To show the effects of our collateral constraint clearly, dynamics of Model 1 and Model 2 are

Table 3.3: Second moments

| Standard deviation | Data | Model 1 | Model 2 |
|-------------------------|--------|---------|---------|
| C_t | 0.0108 | 0.0105 | 0.0127 |
| I_t | 0.0296 | 0.0237 | 0.0324 |
| $I_{h,t}$ | 0.0617 | 0.0588 | 0.0374 |
| Y_t | 0.0113 | 0.0118 | 0.0120 |
| $D_{b,t}$ | 0.0146 | 0.0180 | 0.0928 |
| Correlation with output | Data | Model 1 | Model 2 |
| C_t | 0.69 | 0.93 | 0.52 |
| I_t | 0.70 | 0.92 | 0.77 |
| $I_{h,t}$ | 0.05 | 0.44 | 0.31 |
| $D_{b,t}$ | 0.35 | 0.47 | -0.19 |

Notes: 1. All variables are logged and HP-filtered for the period 2003-2015.
2. I_t is non-residential investment data. It is calculated as follows. $I_t =$ Gross fixed capital formation - residential investment.

compared. Responses to four shocks will be illustrated in this chapter: a technology shock, a LTV ratio shock, a amortisation rate shock, and a housing preference (house price) shock. First, responses to a technology shock show general characteristics of this model. Second, responses to the decrease in LTV ratio and the rise in amortisation rate briefly describe how the South Korean economy with high level of household debt reacts to the demand-side macroprudential (credit tightening) policies. South Korean government is trying to slow down the growth rate of household debt by tightening household credit conditions. These two cases can provide right quantitative dynamics of key variables in South Korean economy in this situation. Third, we simulate responses to a negative shock in households' taste for housing services in order to generate a drop in house price.

3.6.1 Technology shock

We start with households' dynamics. Figure 3.2 plots households' responses to a positive technology shock of 1%. All the variables are expressed as percent deviations from their steady-state values. The technology shock raises the wage for both households because it makes non-durable good producer demand more labour. Thanks to this wage rise, both households can increase their consumptions either in non-durable

good or durable good (house). Saver can get additional benefit from the rise in capital rental rate. So they can both increase non-durable good consumption. Borrower also purchases more houses because they do not have any other choices. However, saver's choice is different from that of borrower. Saver has to lend more money to borrower to maintain debt market equilibrium. So saver has to reduce house purchases because house purchase is a kind of saving which can be an alternative to lending. Two models show different dynamics in house purchase and borrower's hours worked. The model 1 we build reacts more gradually in household debt related variables due to its structure of collateral constraint. For example, borrower's housing stock cannot jump up in period 1 so it shows gradually increasing shape.

Responses of aggregate variables provide similar picture of dynamics in Figure 3.3.²² Due to the positive technology shock, all the variables except interest rate increase.²³ Interest rate is expressed as level deviation from its steady-state value. Two models show very similar dynamics other than in debt-related variables. Household debt cannot jump up in Model 1 so that household debt to output ratio drops at once whereas it gradually decreases in Model 2. Meanwhile, we can see that temporary increase in wage income leads borrower to have more debt to finance more consumption, in contrast with the standard representative household model. This is because of the existence of borrowing constraint which is assumed to always bind in this model as Campbell and Hercowitz (2005) points out. As we see in households' dynamics, Model 1 shows more gradual response in household debt than Model 2.

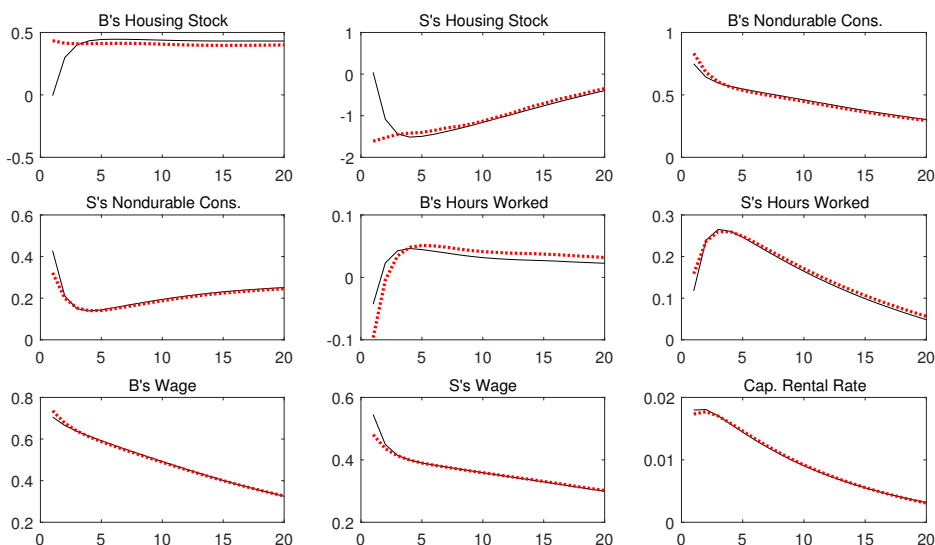
3.6.2 Decreasing LTV ratio

Decrease in LTV ratio immediately affects the amount of impatient household's borrowing. However, it cannot affect the existing household debt because a lender cannot force a borrower to accept worsened condition for the existing debt. So decrease in LTV ratio reduces only new borrowing rather than whole household debt. Responses

²²Interest rate is expressed as level deviation from its steady-state value.

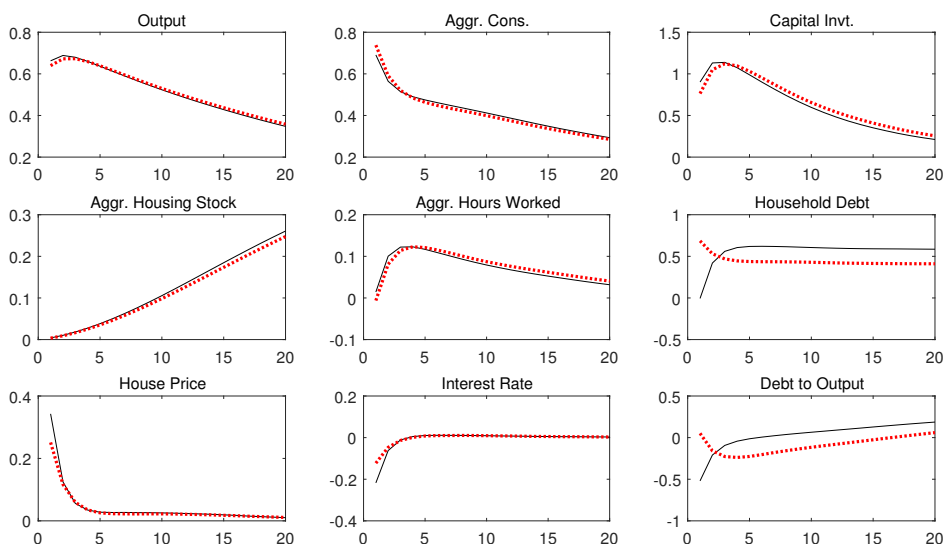
²³Without investment adjustment cost, interest rate also increases as in a typical RBC model. However, in this thesis, interest rate falls in response to the technology shock due to the investment adjustment cost.

Figure 3.2: Household responses to technology shock



Notes: 1. All variables except capital rental rate are expressed as percent deviation from their initial steady-state values. Capital rental rate is percentage point deviation. 2. Solid lines: Model 1, Dotted lines: Model 2.

Figure 3.3: Aggregate responses to technology shock



Notes: 1. All variables except interest rate and debt to output are expressed as percent deviation from their initial steady-state values. Interest rate and debt to output are percentage point deviation. 2. Solid lines: Model 1, Dotted lines: Model 2.

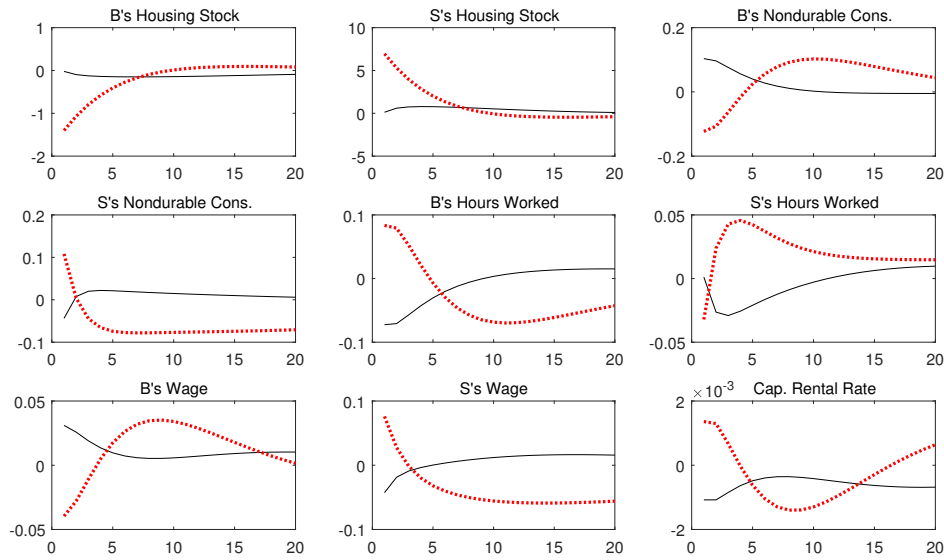
of Model 1 demonstrate this mechanism. However, in Model 2, a negative LTV ratio can affect an entire household debt due to its assumption.²⁴ Figure 3.5 clearly describes how household debt reacts depending on the assumption of debt and borrowing. Responses of household debt and household debt to output ratio in Model 2 is about 10 times larger than those in Model 1. Historically, household debt to output ratio has never changed as dramatically as in Model 2 in response to changes in LTV ratio. Response of Model 1 is much closer to the historical dynamics of household debt to output ratio. In case of positive LTV ratio shock, borrower may renew its existing debt contract to benefit better condition but in reality, the existing debt contracts are not entirely renewed even along with the increase in LTV ratio.

Figure 3.4 describes households' responses to a negative LTV ratio shock of 1%. Borrower is forced to reduce the purchase of houses as collateral. It induces saver to buy more houses and decrease non-durable good consumption. Borrower can increase non-durable good consumption thanks to the reduced house purchase, which also leads borrower to work less. Thus, saver has to work more. The model 1 shows more moderate responses in household debt related variables as well. For instance, borrower's housing stock does not drop suddenly in period 1.

Responses of aggregate variables show more differences between models in Figure 3.5. Considering the negative characteristics of this shock, we can expect a negative response of output and decline in house price. Model 1 shows appropriate results to meet this conjecture. However, Model 2 shows opposite responses in aggregate level. Therefore, it can be said that Model 1 does produce more realistic aggregate volatility than Model 2 when LTV ratio decreases temporarily and unexpectedly. This can be a main improvement of our model. Therefore household debt to output ratio dynamics in Model 1 is much more acceptable than in Model 2.

²⁴In model 2, borrowing is not distinguished from debt.

Figure 3.4: Household responses to decrease in LTV ratio



Notes: 1. All variables except capital rental rate are expressed as percent deviation from their initial steady-state values. Capital rental rate is percentage point deviation.
2. Solid lines: Model 1, Dotted lines: Model 2.

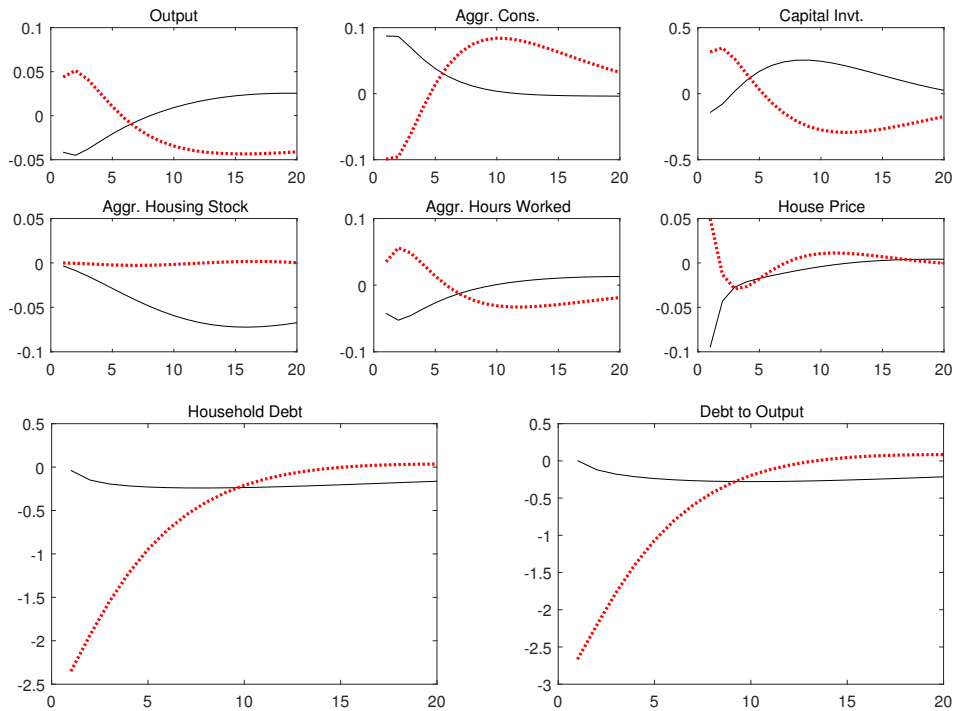
3.6.3 Increasing amortisation rate

Figure 3.6 plots the household impulse responses to 5.5% increase²⁵ in amortisation rate and compare them with those of negative LTV ratio shock with the same model (Model 1). For the purpose of comparison, the magnitude of shock is increased to 5.5%, so as to produce the similar reduction in household debt to output ratio with the case of negative 1% LTV ratio shock.

We can clearly see increase in amortisation rate produces less volatility than lowering LTV ratio. Figure 3.6 shows less fluctuation in all variables on household level. Interestingly, borrower's hours worked keep increasing for more than ten quarters whereas they decrease when LTV ratio lowers. When amortisation rate increases, borrower needs to pay back more fraction of its debt every period. So it needs to work more and reduce non-durable good consumption.

²⁵5.5% of amortisation rate is only 0.00039 because its given value is 0.007 as in Table 3.1. Although change in amortisation rate can apply only to new borrowing like change in LTV ratio, the model does not have this property to reduce computational complexity. During 2003-2015, average quarterly new borrowing is 2% of existing debt. So, change in amortisation rate in this model has to be interpreted as 50 times larger change in amortisation rate of new borrowing. Therefore, increase by 0.00039 can be interpreted as increase by 0.0195. Amortisation rate 0.007 means 143-year amortisation of debt and 0.0265 means 38-year amortisation. Although this is not a small change, 38-year amortisation of debt is still quite loose requirement.

Figure 3.5: Aggregate responses to decrease in LTV ratio



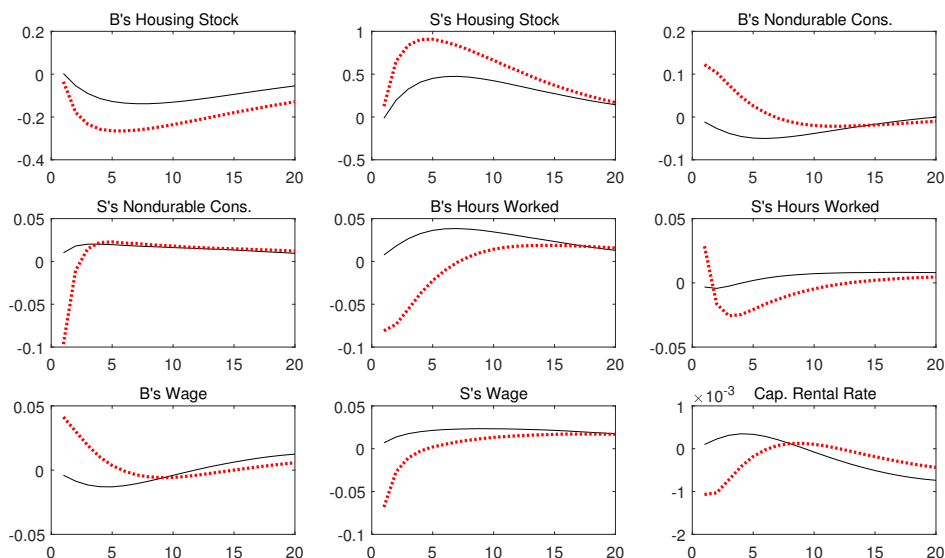
Notes: 1. All variables except debt to output are expressed as percent deviation from their initial steady-state values. Debt to output is percentage point deviation. 2. Solid lines: Model 1, Dotted lines: Model 2.

On aggregate level (Figure 3.7), output increases thanks to more labour supply whereas it decreases when LTV ratio lowers. Aggregate non-durable good consumption reduces as borrower decreases it.

3.6.4 Negative house price shock

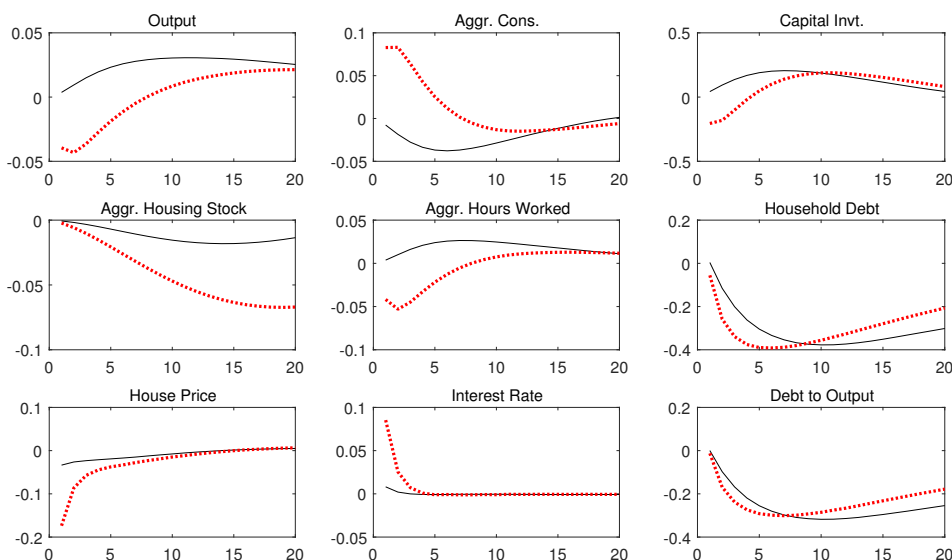
Figure 3.8 and Figure 3.9 plot the impulse responses to 3.7% decline in housing preference and compare them with those of Model 2 with same shock and negative 1% LTV ratio shock with the Model 1. For the purpose of comparison, the magnitude of shock is increased to 3.7%, so as to produce the similar magnitude of drop in house price as in the case of negative 1% LTV ratio shock. If we compare results from Model 1 with those from Model 2 with same house preference shock, there are interesting differences. As we can see in Figure 3.9, maximum reduction in output and house price in Model 1 is almost twice as much as in Model 2 despite maximum reduction in household debt is opposite. This is because in Model 2, capital rental rate increases while it decreases in Model 1. Increasing capital rental rate in Model 2 makes saver cancel out borrower's

Figure 3.6: Household responses to increase in amortisation rate



Notes: 1. All variables except capital rental rate are expressed as percent deviation from their initial steady-state values. Capital rental rate is percentage point deviation. 2. Solid lines: Amortisation rate shock, Dotted lines: LTV ratio shock.

Figure 3.7: Aggregate responses to increase in amortisation rate

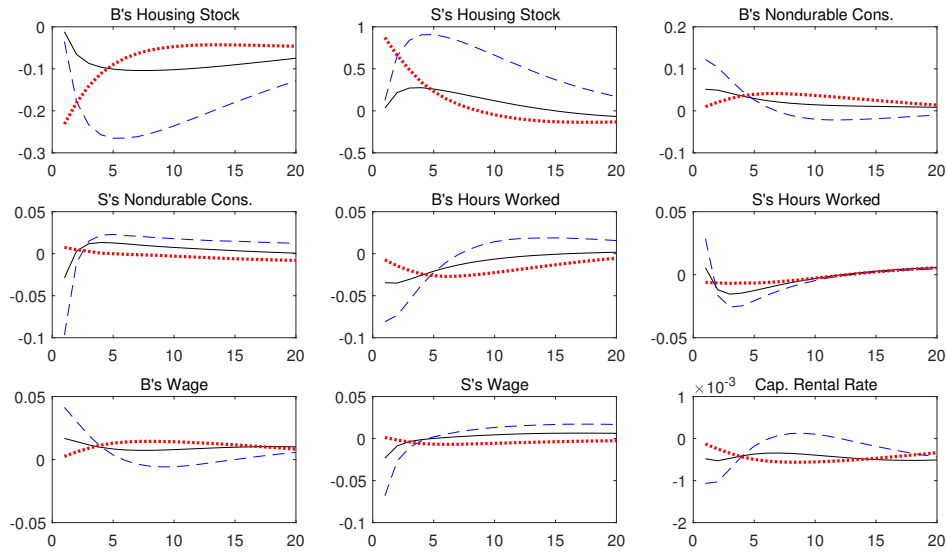


Notes: 1. All variables except interest rate and debt to output are expressed as percent deviation from their initial steady-state values. Interest rate and debt to output are percentage point deviation. 2. Solid lines: Amortisation rate shock, Dotted lines: LTV ratio shock.

negative responses more than in Model 1. In addition, borrower needs to work more in Model 2 than in Model 1 in order to compensate the reduced debt.

If we compare dynamics caused by house price drop (housing preference decrease) with those in response to decreasing LTV ratio, we can see decline in household debt is much smaller in house price drop than in LTV ratio shock. This is because impatient household's borrowing limit is determined by combination of house price and LTV ratio. Therefore overall effects are smaller both on household level and aggregate level when housing preference reduces than when LTV ratio decreases. This has an implication that impacts of house price drop on economy depend on where the shock comes from. Despite we see the same amount of house price drop, its effects could be quite different between in case of LTV ratio shock and in case of house price drop.

Figure 3.8: Household responses to decrease in house price

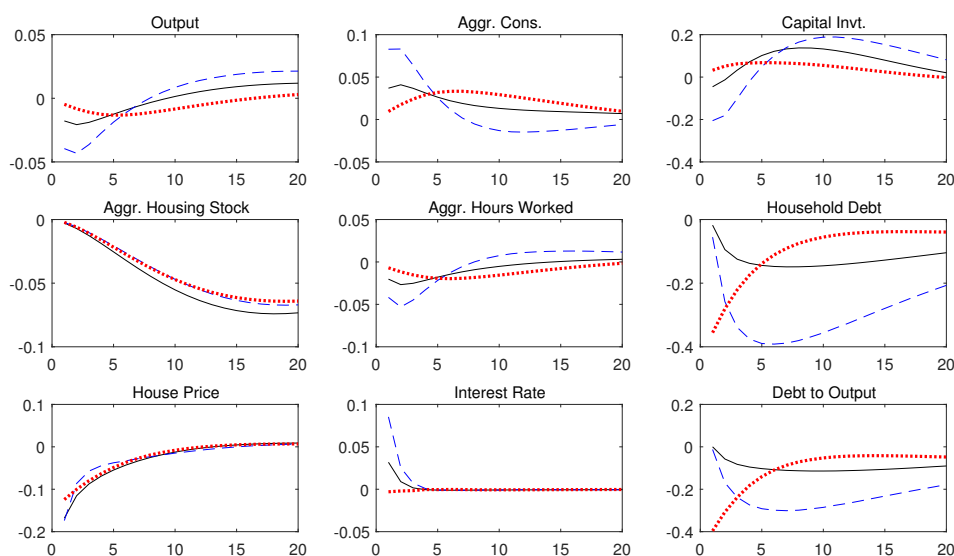


Notes: 1. All variables except capital rental rate are expressed as percent deviation from their initial steady-state values. Capital rental rate is percentage point deviation.
2. Solid lines: Model 1, Dotted lines: Model 2, Dashed lines: LTV ratio shock with Model 1

3.7 Conclusion

It is attempted to build a modified Real Business Cycle model which incorporates borrowing, household debt and house producer to analyse household debt more precisely. The model is calibrated using the data from South Korea and succeeds in matching the

Figure 3.9: Aggregate responses to decrease in house price



Notes: 1. All variables except interest rate and debt to output are expressed as percent deviation from their initial steady-state values. Interest rate and debt to output are percentage point deviation. 2. Solid lines: Model 1, Dotted lines: Model 2, Dashed lines: LTV ratio shock with Model 1

actual data from South Korea. It is found that macroprudential policies such as LTV ratio decrease and amortisation rate increase have different effects on households and whole economy. Increasing amortisation rate is more effective measure in slowing down the speed of household debt growth in that it produces less volatility in the economy. In addition, the source of house price drop is very important in estimating its effects on households and overall economy.

These evidences show that recent demand-side macroprudential policy in South Korea such as decrease in LTV ratio and prohibition of interest-only mortgage, which leads to amortisation rate increase, could effectively work in slowing down the growth rate of household debt. When government implements these macroprudential policies, appropriate quantitative results from this general equilibrium model can be helpful in calibrating or finding proper mixture of different policies.

Finally, this model can be extended to take into account monetary policy. In recent debate on macroprudential policy, it is very crucial how macroprudential policy works with monetary policy and how much their impacts are different from each other. In the next chapter, it will be attempted to extend this model for the analysis of monetary policy effects by including central bank. For this analysis, nominal rigidity

such as sticky price and real rigidity such as habit formation in consumption and fixed cost in non-durable good production need to be included because it is well known that monetary policy analysis is unrealistic without these rigidities.

Appendix

3.A Equilibrium

The model has a unique stationary equilibrium in which impatient household borrows up to the borrowing limit. The equilibrium conditions are as follows.

$$[1] \quad \lambda_{b,t} = \frac{1}{C_{b,t}}$$

This can be rewritten as $U'(C_{b,t}) = \lambda_{b,t}$.

$$[2] \quad P_{h,t}(1 - \mu_{b,t}\theta_t) = \beta_b E_t \left[\frac{\lambda_{b,t+1}}{\lambda_{b,t}} \left\{ \phi_{t+1} \frac{C_{b,t+1}}{H_{b,t+1}} + (1 - \delta_h) P_{h,t+1}(1 - \mu_{b,t+1}\theta_{t+1}) \right\} \right]$$

$$[3] \quad W_{b,t} = C_{b,t} L_{b,t}^\eta$$

This can be rewritten as $\frac{-U'(L_{b,t})}{U'(C_{b,t})} = W_{b,t}$.

$$[4] \quad \lambda_{b,t}(1 - \mu_{b,t}) = \beta_b E_t \lambda_{b,t+1} \left\{ R_t - (1 - \varrho_t) \mu_{b,t+1} \right\}$$

This is different from the standard consumption Euler equation because impatient households has a borrowing constraint. This can be rewritten as

$$U'(C_{b,t})(1 - \mu_{b,t}) = \beta_b E_t U'(C_{b,t+1}) \left\{ R_t - (1 - \varrho_t) \mu_{b,t+1} \right\}$$

If there is no borrowing constraint ($\mu_{b,t} = \mu_{b,t+1} = 0$), this reduces to the standard consumption Euler equation.

$$[5] \quad C_{b,t} + P_{h,t} N_{b,t} + R_{t-1} D_{b,t} = W_{b,t} L_{b,t} + D_{b,t+1} - T_{b,t}$$

$$[6] \quad D_{b,t+1} - (1 - \varrho_{t-1}) D_{b,t} = \theta_t P_{h,t} N_{b,t}$$

$$[7] \quad N_{b,t} = H_{b,t+1} - (1 - \delta_h)H_{b,t}$$

$$[8] \quad \lambda_{s,t} = \frac{1}{C_{s,t}}$$

This can be rewritten as $U'(C_{s,t}) = \lambda_{s,t}$.

$$[9] \quad P_{h,t} = \beta_s E_t \left[\frac{\lambda_{s,t+1}}{\lambda_{s,t}} \left\{ \phi_{t+1} \frac{C_{s,t+1}}{H_{s,t+1}} + (1 - \delta_h)P_{h,t+1} \right\} \right]$$

$$[10] \quad W_{s,t} = C_{s,t} L_{s,t}^\eta$$

This can be rewritten as $\frac{-U'(L_{s,t})}{U'(C_{s,t})} = W_{s,t}$.

$$[11] \quad \lambda_{s,t} = \beta_s R_t E_t \lambda_{s,t+1}$$

This is the standard consumption Euler equation for the patient household and can be rewritten as:

$$U'(C_{s,t}) = \beta_s R_t E_t U'(C_{s,t+1})$$

$$[12] \quad \mu_{s,t} = \beta_s E_t \frac{\lambda_{s,t+1}}{\lambda_{s,t}} \{ R_{k,t+1} + (1 - \delta_k)\mu_{s,t+1} \}$$

$$[13] \quad N_{s,t} = H_{s,t+1} - (1 - \delta_h)H_{s,t}$$

$$[14] \quad K_{t+1} = (1 - \delta_k)K_t + F_t(I_t, I_{t-1})$$

$$[15] \quad F_t(I_t, I_{t-1}) = \left\{ 1 - \zeta_k \frac{1}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right\} I_t$$

$$[16] \quad 1 = \mu_{s,t} \left[1 - \zeta_k \frac{1}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \zeta_k \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right] \\ + \beta_s E_t \frac{\lambda_{s,t+1}}{\lambda_{s,t}} \mu_{s,t+1} \zeta_k \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2$$

$$[17] \quad \nu(1 - \alpha)Y_t L_{b,t}^{-1} = \psi W_{b,t}$$

This can be rewritten as $MPL_{b,t} = \psi W_{b,t}$.

$$[18] \quad (1 - \nu)(1 - \alpha)Y_t L_{s,t}^{-1} = (1 - \psi)W_{s,t}$$

This can be rewritten as $MPL_{s,t} = (1 - \psi)W_{s,t}$.

$$[19] \quad \alpha Y_t K_t^{-1} = R_{k,t}$$

This can be rewritten as $MPK_t = R_{k,t}$.

$$[20] \quad Y_t = A_t^{1-\alpha} K_t^\alpha \left[\{\psi L_{b,t}\}^\nu \{(1 - \psi)L_{s,t}\}^{1-\nu} \right]^{1-\alpha}$$

$$[21] \quad 1 = P_{h,t} \left\{ 1 - \zeta_h \frac{1}{2} \left(\frac{I_{h,t}}{I_{h,t-1}} - 1 \right)^2 - \zeta_h \left(\frac{I_{h,t}}{I_{h,t-1}} - 1 \right) \right\} \frac{I_{h,t}}{I_{h,t-1}} \\ + \beta_s E_t \frac{\lambda_{s,t+1}}{\lambda_{s,t}} P_{h,t+1} \zeta_h \left(\frac{I_{h,t+1}}{I_{h,t}} - 1 \right) \left(\frac{I_{h,t+1}}{I_{h,t}} \right)^2$$

$$[22] \quad N_t = \left\{ 1 - \zeta_h \frac{1}{2} \left(\frac{I_{h,t}}{I_{h,t-1}} - 1 \right)^2 \right\} I_{h,t}$$

$$[23] \quad \psi T_{b,t} = \gamma G_t = \gamma g Y_t$$

$$[24] \quad \psi D_{b,t+1} + (1 - \psi)D_{s,t+1} = 0$$

$$[25] \quad N_t = (1 - \psi)N_{s,t} + \psi N_{b,t}$$

$$[26] \quad C_t = (1 - \psi)C_{s,t} + \psi C_{b,t}$$

$$[27] \quad L_t = (1 - \psi)L_{s,t} + \psi L_{b,t}$$

$$[28] \quad Y_t = C_t + I_{h,t} + I_t + G_t$$

$$[29] \quad \ln A_t = \rho_a \ln A_{t-1} + \varepsilon_{a,t}, \quad \varepsilon_{a,t} \sim i.i.d.N(0, \sigma_a^2)$$

$$[30] \quad \ln \varrho_t = \rho_\varrho \ln \varrho_{t-1} + (1 - \rho_\varrho) \ln \varrho + \varepsilon_{\varrho,t}, \quad \varepsilon_{\varrho,t} \sim i.i.d.N(0, \sigma_\varrho^2)$$

$$[31] \quad \ln \theta_t = \rho_\theta \ln \theta_{t-1} + (1 - \rho_\theta) \ln \theta - \varepsilon_{\theta,t}, \quad \varepsilon_{\theta,t} \sim i.i.d.N(0, \sigma_\theta^2)$$

$$[32] \quad \ln \phi_t = \rho_\phi \ln \phi_{t-1} + (1 - \rho_\phi) \ln \phi - \varepsilon_{\phi,t}, \quad \varepsilon_{\phi,t} \sim i.i.d.N(0, \sigma_\phi^2)$$

$$[33] \quad \ln G_t = \rho_G \ln G_{t-1} + (1 - \rho_G) \ln G + \varepsilon_{G,t}, \quad \varepsilon_{G,t} \sim i.i.d.N(0, \sigma_G^2)$$

3.B Steady State

The model can be solved when θ and ϕ are regarded as parameters. We abstract from growth ($A = 1$).

$$\begin{aligned}
P_h &= \mu_s = 1 & R &= \frac{1}{\beta_s} \\
\mu_b &= \frac{1 - \frac{\beta_b}{\beta_s}}{1 - \beta_b(1 - \varrho)} & R_k &= \frac{1}{\beta_s} - (1 - \delta_k) \\
aux_1 &= \frac{\beta_b \phi}{(1 - \mu_b \theta)[1 - (1 - \delta_h)\beta_b]} \\
aux_2 &= \frac{\beta_s \phi}{1 - (1 - \delta_h)\beta_s} \\
aux_3 &= \frac{\frac{r_k}{\psi \alpha} \{\nu(1 - \alpha) - \gamma g\}}{1 + aux_1 \delta_h \left[1 + \frac{\theta P_h}{\varrho} (R - 1)\right]} \\
aux_4 &= \frac{\frac{R_k}{\alpha} (1 - g) - \delta_k - \psi aux_3 (1 + \delta_h aux_1)}{(1 - \psi)(1 + \delta_h aux_2)} \\
L_b &= \left[\frac{\nu(1 - \alpha) R_k}{\alpha \psi aux_3} \right]^{\frac{1}{\eta+1}} & L_s &= \left[\frac{(1 - \nu)(1 - \alpha) R_k}{\alpha (1 - \psi) aux_4} \right]^{\frac{1}{\eta+1}} \\
K &= \left(\frac{\alpha}{R_k} \right)^{\frac{1}{1-\alpha}} \{\psi L_b\}^\nu \{(1 - \psi) L_s\}^{1-\nu} & Y &= \frac{R_k K}{\alpha} \\
C_b &= aux_3 K & C_s &= aux_4 K \\
\lambda_b &= \frac{1}{C_b} & \lambda_s &= \frac{1}{C_s} \\
H_b &= aux_1 C_b & H_s &= aux_2 C_s \\
N_b &= \delta_h H_b & N_s &= \delta_h H_s \\
N &= (1 - \psi) N_s + \psi N_b & I_h &= N \\
D_b &= \frac{\theta P_h N_b}{\varrho} & D_s &= \frac{\psi}{\psi - 1} D_b \\
C &= (1 - \psi) C_s + \psi C_b & L &= (1 - \psi) L_s + \psi L_b \\
F &= \delta_k K & I &= F \\
W_b &= \frac{\nu(1 - \alpha) Y}{\psi L_b} & W_s &= \frac{(1 - \nu)(1 - \alpha) Y}{(1 - \psi) L_s} \\
T_b &= \frac{\gamma g Y}{\psi} & G &= g Y
\end{aligned}$$

The steady state can be briefly described as follows.

$$\begin{aligned}\frac{C}{Y} &= \frac{\alpha[\psi aux_3 + (1 - \psi)aux_4]}{\delta_k - 1 + \frac{1}{\beta_s}} \\ \frac{I_h + I}{Y} &= \frac{\alpha\delta_k[(1 - \psi)aux_1aux_3 + \psi aux_2aux_4] + \alpha\delta_k}{\delta_k - 1 + \frac{1}{\beta_s}} \\ \frac{N}{Y} &= \frac{\alpha\delta_k[\psi aux_1aux_3 + (1 - \psi)aux_2aux_4]}{\delta_k - 1 + \frac{1}{\beta_s}} \\ \frac{H}{Y} &= \frac{\alpha[\psi aux_1aux_3 + (1 - \psi)aux_2aux_4]}{\delta_k - 1 + \frac{1}{\beta_s}} \\ \frac{D_b}{Y} &= \frac{\alpha\theta P_h\delta_h aux_1aux_3}{\varrho(\delta_k - 1 + \frac{1}{\beta_s})}\end{aligned}$$

3.C Dynare Code

```

#define bc=0
//if bc=0 Model 1
//if bc=1 Model 2

var
LAMBDA_B // Lagrangian multiplier on borrower budget constraint
LAMBDA_S // Lagrangian multiplier on saver budget constraint
CB // Consumption of borrower
CS // Consumption of saver
C // Aggregate Consumption
THETA // LTV ratio
PHI // Housing preference
Q // House price
HB // Housing stock of borrower
HS // Housing stock of saver
H // Aggregate housing stock
WB // Wage of borrower
WS // Wage of saver
LB // Labour supply of borrower
LS // Labour supply of saver
L // Aggregate Labour supply
MUB // Lagrangian multiplier on borrower borrowing constraint
MUS // Lagrangian multiplier on saver capital accumulation
R // Gross interest rate
AMO // Amortisation rate
DB // Debt of borrower
DS // Debt of saver
NB // Housing investment of borrower
NS // Housing investment of saver

```

```

N          // Aggregate Housing investment
RK         // Capital rental rate
K          // Capital stock
IK         // Capital investment
F          // Investment adjustment cost
Y          // Output
IH         // Housing investment
AT         // AR(1) technology process
G          // Government spending
TB         // Tax for borrower
TS         // Tax for saver
D2Y       // Debt to output
log_Y log_WB log_WS log_L log_LB log_LS log_DB log_CB log_CS log_C
log_HB log_HS log_H log_Q log_IK log_IH log_AT log_THETA log_PHI
;

varexo
EPS_AT     // technology shock
EPS_AMO    // amortisation rate shock
EPS_THETA  // LTV ratio shock
EPS_PHI    // housing preference shock
EPS_G      // fiscal shock
;

parameters
alpha      // capital share
niu        // wage share of borrower
bettab     // discount factor of borrower
bettas     // discount factor of saver
eta        // inverse Frisch elasticity of labour supply
deltak     // capital depreciation rate
deltah     // housing depreciation rate
psi        // population share of borrower
zetak      // investment adjustment cost parameter
zetah      // adjustment cost parameter in house production
rhoat      // autocorrelation technology shock
rhoamo     // autocorrelation amortisatino shock
rhotheta   // autocorrelation LTV shock
rhophi     // autocorrelation housing preference shock
rhog       // autocorrelation fiscal shock
sharetb    // tax share of borrower
shareg     // government spending share in output
;

bettas=.998;
bettab=.995;
eta=1;
psi=.657;
alppha=.33;
deltak=.025;
deltah=.005;

```

```

zetak=0.5;
zetaH=1;
niu=.8;
rhoat=.95;
rhoamo=.85;
rhotheta=.85;
rhophi=.85;
shareg=.142;
sharetb=.55;
rhog=.85;

model;
LAMBDA=1/CB;

@if bc==0
Q*(1-MUB*THETA)=bettab*(LAMBDA(+1)/LAMBDA)*(PHI(+1)*(CB(+1)/HB)+(1-deltah)
*Q(+1)*(1-MUB(+1)*THETA(+1)));
#else
Q*(1-MUB*THETA)=bettab*(LAMBDA(+1)/LAMBDA)*(PHI(+1)*(CB(+1)/HB)+(1-deltah)
*Q(+1));
#endif

WB=CB*LB^eta;

@if bc==0
LAMBDA*(1-MUB)=bettab*LAMBDA(+1)*(R-(1-AMO)*MUB(+1));
#else
LAMBDA*(1-MUB)=bettab*LAMBDA(+1)*R;
#endif

CB+Q*NB+R(-1)*DB(-1)=WB*LB+DB-TB;

@if bc==0
DB-(1-AMO(-1))*DB(-1)=THETA*Q*NB;
#else
DB=THETA*Q*HB;
#endif

NB=HB-(1-deltah)*HB(-1);
LAMBDA=1/CS;
Q=bettas*(LAMBDA(+1)/LAMBDA)*(PHI(+1)*(CS(+1)/HS)+(1-deltah)*Q(+1));
WS=CS*LS^eta;
LAMBDA=bettas*LAMBDA(+1)*R;
MUS=bettas*(LAMBDA(+1)/LAMBDA)*(RK(+1)+(1-deltak)*MUS(+1));
NS=HS-(1-deltah)*HS(-1);
K=(1-deltak)*K(-1)+F;
F=(1-zetak*(1/2)*(IK/IK(-1)-1)^2)*IK;
1=MUS*(1-zetak*(1/2)*(IK/IK(-1)-1)^2-zetak*(IK/IK(-1)-1)*(IK/IK(-1)))+bettas
*(LAMBDA(+1)/LAMBDA)*MUS(+1)*zetak*(IK(+1)/IK(-1))*(IK(+1)/IK)^2;
niu*(1-alpha)*Y*LB^(-1)=psi*WB;

```

```

(1-niu)*(1-alfpha)*Y*LS^(-1)=(1-psi)*WS;
alpha*Y*K(-1)^(-1)=RK;
Y=AT^(1-alfpha)*K(-1)^alpha*((psi*LB)^niu*((1-psi)*LS)^(1-niu))^(-1-alfpha);
1=Q*(1-zetah*(1/2)*(IH/IH(-1)-1)^2-zetah*(IH/IH(-1)-1)*(IH/IH(-1)))+bettas
*(LAMBDA(+1)/LAMBDA)*Q(+1)*zetah*(IH(+1)/IH-1)*(IH(+1)/IH)^2;
N=(1-zetah*(1/2)*(IH/IH(-1)-1)^2)*IH;
psi*TB=sharetb*shareg*Y;
G=psi*TB+(1-psi)*TS;
psi*DB+(1-psi)*DS=0;
N=(1-psi)*NS+psi*NB;
C=(1-psi)*CS+psi*CB;
L=(1-psi)*LS+psi*LB;
Y=C+IH+IK+G;
H=psi*HB+(1-psi)*HS;
D2Y=psi*DB/Y/4;
ln(AT)=rhoat*ln(AT(-1))+EPS_AT;
ln(AMO)=rhoamo*ln(AMO(-1))+(1-rhoamo)*ln(steady_state(AMO))+EPS_AMO;
ln(THETA)=rhoteta*ln(THETA(-1))+(1-rhoteta)*ln(steady_state(THETA))-EPS_THETA;
ln(PHI)=rhophi*ln(PHI(-1))+(1-rhophi)*ln(steady_state(PHI))-EPS_PHI;
ln(G)=rhog*ln(G(-1))+(1-rhog)*ln(steady_state(G))+EPS_G;
log_Y=log(Y);
log_WB=log(WB);
log_WS=log(WS);
log_L=log(L);
log_LB=log(LB);
log_LS=log(LS);
log_DB=log(DB);
log_CB=log(CB);
log_CS=log(CS);
log_C=log(C);
log_HB=log(HB);
log_HS=log(HS);
log_H=log(H);
log_Q=log(Q);
log_IK=log(IK);
log_IH=log(IH);
log_AT=0;
log_THETA=log(THETA);
log_PHI=log(PHI);
end;

steady_state_model;
THETA=.65;
PHI=.135;
AMO=.007;
AT=1;
Q=1;
MUS=1;
R=1/bettas;

@if bc==0

```

```

MUB=(1-bettab/bettas)/(1-bettab*(1-AMO));
#else
MUB=1-bettab/bettas;
#endif

RK=1/bettas-(1-deltak);

#ifdef bc==0
AUX1=(bettab*PHI)/((1-MUB*THETA)*(1-(1-deltah)*bettab));
#else
AUX1=(bettab*PHI)/((1-MUB*THETA)-(1-deltah)*bettab);
#endif

AUX2=(bettas*PHI)/(1-(1-deltah)*bettas);

#ifdef bc==0
AUX3=((RK*(niu*(1-alpha)-sharetb*shareg))/(psi*alpha))/(1+AUX1*deltah
*(1+((THETA*Q)/AMO)*(R-1)));
#else
AUX3=((RK*(niu*(1-alpha)-sharetb*shareg))/(psi*alpha))/(1+AUX1*Q*(THETA
*(R-1)+deltah));
#endif

AUX4=(RK*(1-shareg)/alpha-deltak-psi*AUX3*(1+deltah*AUX1))/((1-psi)
*(1+deltah*AUX2));
LB=((niu*(1-alpha)*RK)/(alpha*psi*AUX3))^(1/(eta+1));
LS=((1-niu)*(1-alpha)*RK)/(alpha*(1-psi)*AUX4))^(1/(eta+1));
K=(alpha/RK)^(1/(1-alpha))*(psi*LB)^niu*((1-psi)*LS)^(1-niu);
Y=(RK*K)/alpha;
CB=AUX3*K;
CS=AUX4*K;
LAMBDA B=1/CB;
LAMBDA S=1/CS;
HB=AUX1*CB;
HS=AUX2*CS;
NB=deltah*HB;
NS=deltah*HS;
N=(1-psi)*NS+psi*NB;
IH=N;
#ifdef bc==0
DB=(THETA*Q*NB)/AMO;
#else
DB=THETA*Q*HB;
#endif

DS=(psi/(psi-1))*DB;
C=(1-psi)*CS+psi*CB;
L=(1-psi)*LS+psi*LB;
F=deltak*K;
IK=F;
WB=(niu*(1-alpha)*Y)/(psi*LB);

```

```
WS=((1-niu)*(1-alpha)*Y)/((1-psi)*LS);
TB=sharetb*shareg*Y/psi;
G=shareg*Y;
TS=(G-psi*TB)/(1-psi);
H=psi*HB+(1-psi)*HS;
D2Y=psi*DB/Y/4;
log_Y=log(Y);
log_WB=log(WB);
log_WS=log(WS);
log_L=log(L);
log_LB=log(LB);
log_LS=log(LS);
log_DB=log(DB);
log_CB=log(CB);
log_CS=log(CS);
log_C=log(C);
log_HB=log(HB);
log_HS=log(HS);
log_H=log(H);
log_Q=log(Q);
log_IK=log(IK);
log_IH=log(IH);
log_AT=0;
log_THETA=log(THETA);
log_PHI=log(PHI);
end;

resid(1);
steady;
check;

shocks;
var EPS_AT; stderr 0.01;
var EPS_THETA; stderr 0.01;
var EPS_AMO; stderr 0.055;
var EPS_PHI; stderr 0.037;
var EPS_G; stderr 0.01;
end;

options_.pruning=1;
stoch_simul(periods=1000, hp_filter=1600, order = 2, irf=100);
```

Chapter 4

Household Debt, Credit Tightening and Monetary Policy in South Korea: a medium-scale DSGE Model

4.1 Introduction

Elevated concern on the financial stability after the recent Great Recession turns many policymakers' and researchers' attention to macroprudential policy (credit tightening) and its relationship with monetary policy. Suh (2012) and Svensson (2016) argue that monetary policy and macroprudential policy should be separated because they are efficient for different target variables whereas Woodford (2012) argues that financial stability should be an objective of monetary policy.

High and rising household debt has been a main concern regarding financial stability in South Korea for more than ten years. To slow down the growth rate of household debt, various policy efforts has been implemented by policymakers. In this chapter, I investigate which policy measures are more effective in reducing household debt to output ratio using medium-scale New Keynesian DSGE model with the collateral constraint which is introduced in the previous chapter. Literature using DSGE model with

a collateral constraint usually does not distinguish household debt from borrowing by assuming that household debt is fully paid back every period. This assumption may be good enough to analyse effects of credit liberalisation but it could be too strong in investigating effects of credit tightening. Moreover, when household debt is a concrete policy target, it needs to be defined clearly in a model separating from borrowing. Clear separation of household debt (stock) from borrowing (flow) makes analysis more realistic and precise at least in household debt related variables. Available credit tightening measures are LTV ratio and amortisation rate which are all demand-side credit tightening measures.

The rest of this chapter is structured as follows. Section 4.2 highlights key literature. Section 4.3 describes the model. Section 4.4 presents calibration of parameter values. Quantitative Analysis is illustrated in section 4.5. Finally, conclusion is presented in section 4.6.

4.2 Literature Review

There are recent literature that compares effects of macroprudential policy and monetary policy in general equilibrium models. This literature can be grouped by either the type of macroprudential policy measures or the types of the model used for analysis. Type of macroprudential policy measures can be classified into two categories. One is supply-side measures such as bank capital requirement and maximum leverage ratio. The other is demand-side measures such as LTV ratio, amortisation rate and tax. When supply-side measures are analysed, models commonly incorporate a banking sector because the policy target is a banking sector. In case of analysis of demand-side measures, collateral constraint is added into models as a key feature because financial demand works through collateral constraint. The focus of this chapter is limited to demand-side macroprudential policy, so that the model used in this chapter is in line with models with collateral constraint as in Campbell and Hercowitz (2005) and Iacoviello (2005).

Literature on supply-side macroprudential policy focus mainly on capital requirements. Angeloni and Faia (2013) find the optimal combination of capital requirements

and monetary policy, which is mildly anti-cyclical capital requirements and a monetary policy which reacts to inflation and asset prices by using a DSGE model with a banking sector. Angelini et al. (2014) show time-varying capital requirements can be a useful complement to monetary policy by using a DSGE model with a banking sector as well as collateral constraint. Collard et al. (2017) argue that macroprudential policy such as capital requirements is appropriate for accommodating risk-taking incentives and monetary policy is appropriate for alleviating macroeconomic effects of macroprudential policy by using a DSGE model with a banking sector. The main topic among demand-side macroprudential policy measures is LTV ratio.

4.3 Model

The model in this chapter is a medium-scale DSGE model with a financial friction (collateral constraint). This builds on a Campbell and Hercowitz (2005) and Justiniano et al. (2015). Based on the model in chapter 3, habit formation in non-durable good consumption, price rigidity and fixed cost in intermediate-good production and monetary authority are added mainly for monetary policy analysis. Price rigidity basically follows Calvo price-setting and the way of price-setting by non-optimal setters is based on Christiano et al. (2005). Fixed cost of production is also based on Christiano et al. (2005). Monetary policy follows a Taylor-type interest rate rule. Credit tightening is assumed to be implemented as exogenous shocks. The rest of structure and assumptions of the model are the same as in chapter 3.

Impatient Household

Impatient household's utility function is the same as in chapter 3 except habit formation.

$$U_{b,t} = \ln(C_{b,t} - hC_{b,t-1}) + \phi_t \ln H_{b,t} - \frac{L_{b,t}^{1+\eta}}{1+\eta} \quad (4.3.1)$$

where h is the consumption habit formation parameter.

The log of housing preference ϕ_t follows AR(1) process as in chapter 3.

$$\ln \phi_t = \rho_\phi \ln \phi_{t-1} + (1 - \rho_\phi) \ln \phi + \varepsilon_{\phi,t} \quad (4.3.2)$$

Impatient household maximises its lifetime expected utility at time 0 as in chapter 3.

$$E_0 \sum_{t=0}^{\infty} \beta_b^t U_{b,t}$$

Budget constraint is different from that in chapter 3 because of an inflation. In this economy, we need to introduce an inflation for monetary policy analysis. As a result, we need to distinguish nominal variables from real variables.

$$C_{b,t} + q_t N_{b,t} + (R_{t-1} - 1 + \varrho_{t-1}) \frac{d_{b,t}}{\pi_t} \leq w_{b,t} L_{b,t} - T_{b,t} + d_{b,t+1} - (1 - \varrho_{t-1}) \frac{d_{b,t}}{\pi_t} \quad (4.3.3)$$

where $\pi_t (= \frac{P_t}{P_{t-1}})$ is an inflation at time t, P_t is the nominal price of non-durable good, $q_t (= \frac{P_{h,t}}{P_t})$ is real house price, $w_{b,t} (= \frac{W_{b,t}}{P_t})$ is the real wage, $T_{b,t}$ is real lump-sum tax and transfer from the government and $d_{b,t+1} (= \frac{D_{b,t+1}}{P_t})$ is the amount of real debt at the end of time t and at the beginning of time t+1. The interest paid for existing real debt at time t is $(R_{t-1} - 1) \frac{d_{b,t}}{\pi_t}$ where R_{t-1} is gross nominal interest rate.

Impatient household's nominal collateral constraint is the same as in chapter 3. However, real debt is affected by inflation.

$$d_{b,t+1} - (1 - \varrho_{t-1}) \frac{d_{b,t}}{\pi_t} \leq \theta_t q_t N_{b,t} \quad (4.3.4)$$

ϱ_t is exogenous and its log follows AR(1) process.

$$\ln \varrho_t = \rho_\varrho \ln \varrho_{t-1} + (1 - \rho_\varrho) \ln \varrho + \varepsilon_{\varrho,t} \quad (4.3.5)$$

θ_t is exogenous and its log follows AR(1) process.

$$\ln \theta_t = \rho_\theta \ln \theta_{t-1} + (1 - \rho_\theta) \ln \theta + \varepsilon_{\theta,t} \quad (4.3.6)$$

Lagrangian for impatient household can be defined as follows.

$$\begin{aligned} \mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta_b^t & \left[\ln (C_{b,t} - h C_{b,t-1}) + \phi_t \ln H_{b,t} - \frac{L_{b,t}^{1+\eta}}{1+\eta} + \lambda_{b,t} [w_{b,t} L_{b,t} - T_{b,t} + d_{b,t+1} \right. \\ & - C_{b,t} - q_t \{H_{b,t+1} - (1 - \delta_h) H_{b,t}\} - R_{t-1} \frac{d_{b,t}}{\pi_t}] + \lambda_{b,t} \mu_{b,t} \left[(1 - \varrho_{t-1}) \frac{d_{b,t}}{\pi_t} \right. \\ & \left. \left. + \theta_t q_t \{H_{b,t+1} - (1 - \delta_h) H_{b,t}\} - d_{b,t+1} \right] \right] \quad (4.3.7) \end{aligned}$$

This function is maximised with respect to $C_{b,t}$, $H_{b,t+1}$, $L_{b,t}$ and $d_{b,t+1}$.

Patient Household

Patient household also maximises its lifetime expected utility at time 0 as in chapter 3.

$$E_0 \sum_{t=0}^{\infty} \beta_s^t \left[\ln(C_{s,t} - hC_{s,t-1}) + \phi_t \ln H_{s,t} - \frac{L_{s,t}^{1+\eta}}{1+\eta} \right] \quad (4.3.8)$$

Budget constraint is

$$C_{s,t} + q_t N_{s,t} + \frac{I_t}{1-\psi} + R_{t-1} \frac{d_{s,t}}{\pi_t} \leq w_{s,t} L_{s,t} + r_{k,t} \frac{K_t}{1-\psi} + \frac{\Pi_{s,t}}{P_t} + d_{s,t+1} - T_{s,t} \quad (4.3.9)$$

where $I_{s,t}(= \frac{I_t}{1-\psi})$ is patient household' investment in production capital, $K_{s,t}(= \frac{K_t}{1-\psi})$ is the stock of capital owned by patient household, and $r_{k,t}(= \frac{R_{k,t}}{P_t})$ is real capital rental rate, and $\Pi_{s,t}$ is the share of nominal profits of the intermediate-good producers owned by the savers.

The stock of capital is determined by the following equation as in chapter 3.

$$K_{t+1} = (1 - \delta_k)K_t + F_t \quad (4.3.10)$$

$F_t(I_t, I_{t-1})$ is given by

$$F_t(I_t, I_{t-1}) = A_{I,t} \left[1 - S_k \left(\frac{I_t}{I_{t-1}} \right) \right] I_t \quad (4.3.11)$$

where $A_{I,t}$ is investment-specific technology shock and its log follows AR(1) process.

$$\ln A_{I,t} = \rho_I \ln A_{I,t-1} + \varepsilon_{I,t} \quad (4.3.12)$$

Lagrangian for patient household can be defined as follows.

$$\begin{aligned} \mathcal{L} = & E_0 \sum_{t=0}^{\infty} \beta_s^t \left[\ln(C_{s,t} - hC_{s,t-1}) + \phi_t \ln H_{s,t} - \frac{L_{s,t}^{1+\eta}}{1+\eta} + \lambda_{s,t} \left[w_{s,t} L_{s,t} + r_{k,t} \frac{K_t}{1-\psi} \right. \right. \\ & + \left. \frac{\Pi_{s,t}}{P_t} + d_{s,t+1} - T_{s,t} - C_{s,t} - q_t \left\{ H_{s,t+1} - (1 - \delta_h) H_{s,t} \right\} - \frac{I_t}{1-\psi} - R_{t-1} \frac{d_{s,t}}{\pi_t} \right] \\ & + \left. \frac{\lambda_{s,t} \mu_{s,t}}{1-\psi} \left[A_{I,t} \left\{ 1 - \zeta_k \frac{1}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right\} I_t - K_{t+1} + (1 - \delta_k) K_t \right] \right] \quad (4.3.13) \end{aligned}$$

where $\lambda_{s,t}$ is the current-value Lagrangian multiplier on budget constraint. This function is maximised with respect to $C_{s,t}$, $H_{s,t+1}$, $L_{s,t}$, $d_{s,t+1}$ and K_{t+1} .

Non-durable Good Producers

There is one final non-durable good, Y_t , which is produced by the following technology.

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (4.3.14)$$

where $i \in [0, 1]$, $\varepsilon > 1$. ε is the elasticity of substitution among goods and $Y_t(i)$ is an i th intermediate-good. Final non-durable good producer maximises profits subject to the production function above and demands for intermediate-goods as follows.

$$\max_{Y_t(i)} P_t Y_t - \int_0^1 P_t(i) Y_t(i) di \quad (4.3.15)$$

Therefore,

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t \quad (4.3.16)$$

where $P_t = \left[\int_0^1 P_t(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}$

There are a continuum of intermediate-good producers which produce a differentiated good $Y_t(i)$ at a price $P_t(i)$. The production function of intermediate-good producers is described by a Cobb-Douglas function with constant return to scale (CRS). Intermediate-good producers combine labours of both types of households and capital.

$$Y_t(i) = A_t^{1-\alpha} K_t(i)^\alpha \left[\{\psi L_{b,t}(i)\}^\nu \{(1-\psi)L_{s,t}(i)\}^{1-\nu} \right]^{1-\alpha} - A_t \Gamma \quad (4.3.17)$$

where $0 < \alpha < 1$ and $0 < \nu < 1$. Γ is the fixed cost of production which is chosen to ensure that steady-state profits are zero as in Christiano et al. (2005) and A_t is exogenous labour-augmenting (or, equivalently, Harrod-neutral) technological progress. The level of technology is non-stationary and it follows a stationary AR(1) process in the log. We abstract from growth.

$$\ln A_t = \rho_a \ln A_{t-1} + \varepsilon_{a,t}, \quad \varepsilon_{a,t} \sim i.i.d.N(0, \sigma_a^2) \quad (4.3.18)$$

Intermediate-good producers maximise profits subject to the production function above. Intermediate-good producers solve two-stages problems. Firstly, they minimise their real costs.

$$\min_{L_{b,t}(i), L_{s,t}(i), K_t(i)} w_{b,t} \psi L_{b,t}(i) + w_{s,t} (1-\psi) L_{s,t}(i) + r_{k,t} K_t(i) \quad (4.3.19)$$

subject to their supply curve:

$$Y_t(i) = A_t^{1-\alpha} K_t(i)^\alpha \left[\{\psi L_{b,t}(i)\}^\nu \{(1-\psi)L_{s,t}(i)\}^{1-\nu} \right]^{1-\alpha} - A_t \Gamma$$

The Lagrangian is

$$\begin{aligned} \mathcal{L} = & w_{b,t} \psi L_{b,t}(i) + w_{s,t} (1-\psi) L_{s,t}(i) + r_{k,t} K_t(i) + \varsigma_t(i) \left[Y_t(i) \right. \\ & \left. - A_t^{1-\alpha} K_t(i)^\alpha \left[\{\psi L_{b,t}(i)\}^\nu \{(1-\psi)L_{s,t}(i)\}^{1-\nu} \right]^{1-\alpha} + A_t \Gamma \right] \end{aligned} \quad (4.3.20)$$

where $\varsigma_t(i)$ is the current-value Lagrangian multiplier on supply curve. The real marginal cost is identical across producers. Therefore $\varsigma_t(i)$ is equal to mc_t .

Secondly, intermediate-good producers maximise their discounted real profits. Following Calvo (1983), a fraction $1 - \xi_p$ of them can set their nominal prices optimally in each period. The rest of them simply index their nominal prices to lagged aggregate inflation as in Christiano et al. (2005). The intermediate-good price in period t is:

$$P_t(i) = \begin{cases} P_t^*(i) & \text{if } P_t(i) \text{ chosen optimally} \\ \pi_{t-1} P_{t-1}(i) & \text{otherwise} \end{cases}$$

where $\pi_t = \frac{P_t}{P_{t-1}}$.

The price of an intermediate-good producer which can set its price optimally only in period t is as follows in subsequent period:

$$\begin{aligned} P_{t+1}(i) &= \pi_t P_t^*(i) \\ P_{t+2}(i) &= \pi_{t+1} \pi_t P_t^*(i) \\ &\vdots \\ P_{t+\tau}(i) &= \prod_{s=1}^{\tau} \pi_{t+s-1} P_t^*(i) \end{aligned}$$

Then, the real price of an intermediate-good producer which can set its price optimally only in period t is as follows at time $t + \tau$.

$$\frac{P_{t+\tau}(i)}{P_{t+\tau}} = \prod_{s=1}^{\tau} \pi_{t+s-1} \frac{P_t^*(i)}{P_{t+\tau}}$$

Intermediate-good producers discount profits by $\beta_s \frac{\lambda_{s,t+\tau}}{\lambda_{s,t}}$, which is their discount factor,

as well as a fraction of non-optimal price-setting producers, ξ_p . Then, intermediate-good producers maximise their real profits as follows:

$$\max_{P_t(i)} E_t \sum_{\tau=0}^{\infty} (\beta_s \xi_p)^\tau \frac{\lambda_{s,t+\tau}}{\lambda_{s,t}} \left[\left(\prod_{s=1}^{\tau} \pi_{t+s-1} \frac{P_t(i)}{P_{t+\tau}} - mc_{t+\tau} \right) Y_{t+\tau}(i) \right] \quad (4.3.21)$$

subject to

$$Y_{t+\tau}(i) = \left(\prod_{s=1}^{\tau} \pi_{t+s-1} \frac{P_t(i)}{P_{t+\tau}} \right)^{-\varepsilon} Y_{t+\tau}$$

where $mc_{t+\tau}$ is the real marginal cost at time $t + \tau$.

Substituting the demand curve in the objective function, we get:

$$\max_{P_t(i)} E_t \sum_{\tau=0}^{\infty} (\beta_s \xi_p)^\tau \frac{\lambda_{s,t+\tau}}{\lambda_{s,t}} \left[\left\{ \left(\prod_{s=1}^{\tau} \pi_{t+s-1} \frac{P_t(i)}{P_{t+\tau}} \right)^{1-\varepsilon} - \left(\prod_{s=1}^{\tau} \pi_{t+s-1} \frac{P_t(i)}{P_{t+\tau}} \right)^{-\varepsilon} mc_{t+\tau} \right\} Y_{t+\tau} \right]$$

This can be transformed as follows.

$$\max_{P_t(i)} E_t \sum_{\tau=0}^{\infty} (\beta_s \xi_p)^\tau \frac{\lambda_{s,t+\tau}}{\lambda_{s,t}} \left[\left\{ \left(\prod_{s=1}^{\tau} \frac{\pi_{t+s-1} P_t(i)}{\pi_{t+s} P_t} \right)^{1-\varepsilon} - \left(\prod_{s=1}^{\tau} \frac{\pi_{t+s-1} P_t(i)}{\pi_{t+s} P_t} \right)^{-\varepsilon} mc_{t+\tau} \right\} Y_{t+\tau} \right]$$

The first order condition is:

$$E_t \sum_{\tau=0}^{\infty} (\beta_s \xi_p)^\tau \lambda_{s,t+\tau} \left[\left\{ (1 - \varepsilon) \left(\prod_{s=1}^{\tau} \frac{\pi_{t+s-1}}{\pi_{t+s}} \right)^{1-\varepsilon} \frac{P_t(i)}{P_t} + \varepsilon \left(\prod_{s=1}^{\tau} \frac{\pi_{t+s-1}}{\pi_{t+s}} \right)^{-\varepsilon} mc_{t+\tau} \right\} Y_{t+\tau} \right] = 0$$

$P_t^*(i)$ is equal to P_t^* because we only consider a symmetric equilibrium. Then, this condition yields the optimal price as follows:

$$\frac{P_t^*}{P_t} = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{\tau=0}^{\infty} (\beta_s \xi_p)^\tau \lambda_{s,t+\tau} \left(\prod_{s=1}^{\tau} \frac{\pi_{t+s-1}}{\pi_{t+s}} \right)^{1-\varepsilon} mc_{t+\tau} Y_{t+\tau}}{E_t \sum_{\tau=0}^{\infty} (\beta_s \xi_p)^\tau \lambda_{s,t+\tau} \left(\prod_{s=1}^{\tau} \frac{\pi_{t+s-1}}{\pi_{t+s}} \right)^{-\varepsilon} Y_{t+\tau}}$$

Let

$$Z_t = \frac{\varepsilon}{\varepsilon - 1} E_t \sum_{\tau=0}^{\infty} (\beta_s \xi_p)^\tau \lambda_{s,t+\tau} \left(\prod_{s=1}^{\tau} \frac{\pi_{t+s-1}}{\pi_{t+s}} \right)^{1-\varepsilon} mc_{t+\tau} Y_{t+\tau}$$

$$M_t = E_t \sum_{\tau=0}^{\infty} (\beta_s \xi_p)^\tau \lambda_{s,t+\tau} \left(\prod_{s=1}^{\tau} \frac{\pi_{t+s-1}}{\pi_{t+s}} \right)^{-\varepsilon} Y_{t+\tau}$$

and then the first order condition can be briefly expressed as

$$\pi_t^* = \frac{Z_t}{M_t} \quad (4.3.22)$$

where $\pi_t^* = \frac{P_t^*}{P_t}$.

The price index evolves as follows.

$$P_t^{1-\varepsilon} = \xi_p (\pi_{t-1} P_{t-1})^{1-\varepsilon} + (1 - \xi_p) P_t^{*1-\varepsilon}$$

Dividing by $P_t^{1-\varepsilon}$, we get:

$$1 = \xi_p \left(\frac{\pi_{t-1}}{\pi_t} \right)^{1-\varepsilon} + (1 - \xi_p) \pi_t^{*1-\varepsilon} \quad (4.3.23)$$

House Producer

The production functions of house producer is the same as in chapter 3.

$$N_t = A_{H,t} \left\{ 1 - S_{h,t} \left(\frac{I_{h,t}}{I_{h,t-1}} \right) \right\} I_{h,t} \quad (4.3.24)$$

where $A_{H,t}$ is housing investment-specific technology shock and its log follows AR(1) process.

$$\ln A_{H,t} = \rho_H \ln A_{H,t-1} + \varepsilon_{H,t} \quad (4.3.25)$$

House producer maximises its real profits as follows.

$$\max_{I_{h,t}} E_0 \sum_{t=0}^{\infty} \beta_s^t \lambda_{s,t} \left[q_t \left\{ 1 - \zeta_h \frac{1}{2} \left(\frac{I_{h,t}}{I_{h,t-1}} - 1 \right)^2 \right\} I_{h,t} - I_{h,t} \right] \quad (4.3.26)$$

where $\lambda_{s,t}$ is the marginal utility of income of saver, that is to say, the current-value Lagrangian multiplier on budget constraint of patient household.

Fiscal and Monetary Policy

The government spending follows AR(1) process in its log:

$$\ln G_t = \rho_G \ln G_{t-1} + (1 - \rho_G) \ln G + \varepsilon_{G,t} \quad (4.3.27)$$

where $0 < \rho_G < 1$ and $\varepsilon_{G,t}$ is an *i.i.d.* zero mean normal random disturbance with constant variance σ_G^2 .

The government raises revenue via lump sum taxes and balances its budget:

$$G_t = gY_t = \psi T_{b,t} + (1 - \psi) T_{s,t} \quad (4.3.28)$$

Thus, patient household can only lend to impatient household, and the net supply of borrowing is zero.

The share of taxes borrower pays is set to be γ . This indicates the degree of government redistribution. As γ decreases, government redistributes more to borrower.

$$\psi T_{b,t} = \gamma G_t = \gamma g Y_t \quad (4.3.29)$$

Monetary policy follows a Taylor-type interest rate rule (Taylor, 1993) as follows.

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\omega_R} \left[\left(\frac{\pi_t}{\pi} \right)^{\omega_\pi} \left(\frac{Y_t}{Y} \right)^{\omega_Y} \right] e^{\xi_{r,t}} \quad (4.3.30)$$

where π is the central bank's inflation target. The parameters ω_R , ω_π , and ω_Y capture the degree of smoothing in monetary policy, and the policy weight on inflation, and output, respectively. $e^{\xi_{r,t}}$ is a monetary policy shock with an expected value of one.

Aggregation and Market Clearing Conditions

Total nominal profits in the economy is the integral of profits across intermediate-good producers:

$$\begin{aligned} \Pi_t &= \int_0^1 \Pi_t(i) di \\ &= \int_0^1 \{P_t(i)Y_t(i) - [W_{b,t}\psi L_{b,t}(i) + W_{s,t}(1-\psi)L_{s,t}(i) + R_{k,t}K_t(i)]\} di \\ &= \int_0^1 P_t(i)Y_t(i) di - W_{b,t}\psi \int_0^1 L_{b,t}(i) di - W_{s,t}(1-\psi) \int_0^1 L_{s,t}(i) di - R_{k,t} \int_0^1 K_t(i) di \end{aligned}$$

Since $\int_0^1 L_{b,t}(i) di = L_{b,t}$, $\int_0^1 L_{s,t}(i) di = L_{s,t}$ and $\int_0^1 K_t(i) di = K_t$, we get:

$$\Pi_t = \int_0^1 P_t(i)Y_t(i) di - W_{b,t}\psi L_{b,t} - W_{s,t}(1-\psi)L_{s,t} - R_{k,t}K_t$$

Using the demand function for goods:

$$\Pi_t = P_t^\varepsilon Y_t \int_0^1 P_t(i)^{1-\varepsilon} di - W_{b,t}\psi L_{b,t} - W_{s,t}(1-\psi)L_{s,t} - R_{k,t}K_t$$

Since $\int_0^1 P_t(i)^{1-\varepsilon} di = P_t^{1-\varepsilon}$, we get:

$$\Pi_t = P_t Y_t - W_{b,t}\psi L_{b,t} - W_{s,t}(1-\psi)L_{s,t} - R_{k,t}K_t$$

Then, total real profits in the economy are as follows:

$$\frac{\Pi_t}{P_t} = Y_t - w_{b,t}\psi L_{b,t} - w_{s,t}(1 - \psi)L_{s,t} - r_{k,t}K_t$$

$q_t N_t = I_{h,t}$ because profits of house producer are assumed to be zero.

Debt market is cleared as follows.

$$\psi d_{b,t+1} + (1 - \psi)d_{s,t+1} = 0 \quad (4.3.31)$$

The government balances its budget.

Plugging all these conditions to the aggregated budget constraints of both type of households below,

$$\begin{aligned} & (1 - \psi)C_{s,t} + \psi C_{b,t} + q_t[(1 - \psi)N_{s,t} + \psi N_{b,t}] + I_t + \frac{R_{t-1}}{\pi_t}[(1 - \psi)d_{s,t} + \psi d_{b,t}] \\ & = w_{b,t}\psi L_{b,t} + w_{s,t}(1 - \psi)L_{s,t} + r_{k,t}K_t + (1 - \psi)\frac{\Pi_{s,t}}{P_t} - [(1 - \psi)T_{s,t} + \psi T_{b,t}] \\ & \quad + (1 - \psi)d_{s,t+1} + \psi d_{b,t+1} \end{aligned}$$

where $(1 - \psi)\Pi_{s,t} = \Pi_t$ we can get the economy's resource constraint:

$$Y_t = C_t + I_{h,t} + I_t + G_t \quad (4.3.32)$$

where $C_t = (1 - \psi)C_{s,t} + \psi C_{b,t}$.

4.4 Parameter Estimates

4.4.1 Methods and Data

I linearise the equations describing the equilibrium around the steady state (See the appendix 4.A, appendix 4.B, appendix 4.C for the equilibrium equations, the steady state and the log-linearised equations). The Bayesian estimation is used to set up parameter values and some parameters are calibrated before the Bayesian estimation. Prior distributions for the parameters are chosen and posterior distributions are estimated by using the Metropolis-Hastings algorithm with 500,000 draws.

The sample period for the estimation is 2003:Q1 to 2015:Q4 as in chapter 3. Eight series of quarterly data are employed as observable data: real GDP, real private consumption, residential investment, non-residential investment, employment, CPI inflation, uncollateralised overnight call rate, real house prices.¹

$$Obs_t = [\Delta \ln Y_t^{obs}, \Delta \ln C_t^{obs}, \Delta \ln I_{h,t}^{obs}, \Delta \ln I_t^{obs}, \Delta \ln L_t^{obs}, \pi_t^{obs}, R_t^{obs}, \Delta \ln q_t^{obs}]$$

Real GDP, real private consumption, residential investment, non-residential investment and employment are divided by population and real house prices are the nominal house prices divided by CPI inflation.

4.4.2 Calibrated Parameters

Table 4.1: Calibrated Parameters

| Description | Parameter | Value |
|--|---------------------------|-------|
| Discount factor of patient household | β_s | 0.998 |
| Discount factor of impatient household | β_b | 0.995 |
| Population share of borrowers | ψ | 0.657 |
| Average net mark-up of intermediate-good producers | $\frac{1}{\varepsilon-1}$ | 0.15 |
| Depreciation of productive capital | δ_k | 0.025 |
| Depreciation of houses | δ_h | 0.005 |
| Amortisation | ϱ | 0.001 |

The discount factors of saver and borrower (β_s and β_b), depreciation rates of capital and housing stock (δ_k and δ_h), population share of borrowers (ψ), average net mark-up of intermediate-good producers (ψ) and amortisation rate (ϱ) are calibrated.

During that period, annual inflation is 2.65%, so that π is set to be 1.006. The steady-state gross interest rate (R) is chosen to be 1.008, which matches the average annual Call Rate 3.20%. In steady-state, $R = \frac{\pi}{\beta_s}$. Therefore, saver's discount factor (β_s) is calibrated at 0.998 as in chapter 3. Amortisation rate is set very low (0.001) as mentioned in chapter 3. Before 2010, it used to be very low but it is increasing very fast recently. Interest-only mortgage ratio was 93.6% as of the end of 2010 but

¹All data are obtained from the Bank of Korea Economic Statistics System (<http://ecos.bok.or.kr>).

reduced to 61.1% as of the end of 2015. This dramatic change is a motivation to build a model which can simulate responses to amortisation rate shock. ε is calibrated at 7.67 so that average net mark-up of intermediate-good producers is calibrated at 0.15. Depreciation rates of capital and housing stock (δ_k and δ_h) and population share of borrowers (ψ) are the same as in chapter 3.

The model's steady-state ratios² are very close to the actual data as shown in Table 4.2, but housing stock to output ratio from the model (229.1%) is quite higher than those from the data (214.2%).

Table 4.2: Steady-state ratios

| | Ratio | Data | Model |
|-------------------------------|---|--------|--------|
| $\frac{C}{Y}$ | $\frac{\text{Consumption}}{\text{Output}}$ | 50.9% | 50.9% |
| $\frac{I}{Y}$ | $\frac{\text{Non-residential investment}}{\text{Output}}$ | 27.5% | 30.3% |
| $\frac{I_h}{Y}$ | $\frac{\text{Residential investment}}{\text{Output}}$ | 4.5% | 4.6% |
| $\frac{H}{4 \times Y}$ | $\frac{\text{Housing stock}}{\text{Output}}$ | 214.2% | 229.1% |
| $\frac{\psi D_b}{4 \times Y}$ | $\frac{\text{Household debt}}{\text{Output}}$ | 77.1% | 81.0% |

4.4.3 Prior Distributions

Priors are illustrated in Table 4.3 and Table 4.4. They are chosen based on previous studies (Iacoviello and Neri, 2010; Lee and Song, 2015). For the adjustment costs in capital investment and house production (ζ_k and ζ_h), I use a gamma distribution with a prior mean of 4.0 and standard deviation of 1.5. I set a prior mean on habit formation parameter in consumption (h) at 0.5. I choose a beta prior for the Calvo price parameter (ξ_p) with a mean of 0.75 and standard deviation of 0.1. For the capital

²These ratios are results from the model using the calibrated parameters and the mean estimates shown in Table 4.3 and Table 4.4.

Table 4.3: Priors and posteriors of the structural parameters

| | Prior | | | Posterior | | | |
|--------------|--------|------|--------|-----------|--------|--------|--------|
| | Dist. | Mean | Stdev. | Mean | Stdev. | 5% | 95% |
| ζ_k | Gamma | 4.00 | 1.50 | 3.086 | 1.704 | 0.0769 | 5.2096 |
| ζ_h | Gamma | 4.00 | 1.50 | 6.467 | 1.924 | 3.504 | 9.712 |
| h | Beta | 0.50 | 0.10 | 0.354 | 0.096 | 0.195 | 0.507 |
| ξ_p | Beta | 0.75 | 0.10 | 0.447 | 0.031 | 0.394 | 0.491 |
| α | Beta | 0.33 | 0.02 | 0.327 | 0.011 | 0.309 | 0.343 |
| ν | Beta | 0.80 | 0.10 | 0.797 | 0.010 | 0.782 | 0.813 |
| ω_R | Beta | 0.80 | 0.10 | 0.882 | 0.023 | 0.848 | 0.918 |
| ω_Y | Beta | 0.20 | 0.10 | 0.103 | 0.060 | 0.014 | 0.189 |
| ω_π | Normal | 3.00 | 0.20 | 3.005 | 0.197 | 2.669 | 3.312 |
| θ | Beta | 0.65 | 0.10 | 0.626 | 0.046 | 0.552 | 0.703 |
| ϕ | Normal | 0.20 | 0.10 | 0.147 | 0.020 | 0.114 | 0.180 |
| η | Gamma | 1.00 | 0.20 | 1.316 | 0.260 | 0.898 | 1.73 |

share of the production function (α), I use a beta distribution with a prior mean of 0.33 and standard deviation of 0.02. I select the prior mean of 0.80 for the wage share of impatient household (ν) with a standard deviation of 0.1. I set the prior mean for the LTV ratio (θ) to be 0.65 with a standard deviation of 0.1. For the housing preference parameter (ϕ), I use a normal distribution with a mean of 0.20 and standard deviation of 0.1. I set a prior mean on inverse Frisch elasticity of labour supply³ at 1.0 with a standard deviation of 0.2. I choose a prior mean of 0.8 and standard deviation of 0.1 for the monetary policy smoothing parameter (ω_R). For the monetary policy weight on output (ω_Y), a beta distribution with a mean of 0.2 and standard deviation of 0.1. I use a normal distribution with a mean of 3.0 as in Lee and Song (2015) to match the volatility in inflation. Inverse gamma priors are used for the standard errors of the shocks and beta priors for the persistence parameters.

³Strictly speaking, $1/\eta$ has to be interpreted as the elasticity with which marginal people substitute in and out of employment with respect to a change in the wage because employment data are used for its estimation. If hours worked data is used for its estimation, it can be interpreted as the elasticity of labour supply (Christiano et al., 2010).

Table 4.4: Priors and posteriors of the shock processes

| | Prior | | | Posterior | | | |
|------------------|------------|------|--------|-----------|--------|-------|-------|
| | Dist. | Mean | Stdev. | Mean | Stdev. | 5% | 95% |
| ρ_a | Beta | 0.80 | 0.10 | 0.809 | 0.087 | 0.677 | 0.953 |
| ρ_G | Beta | 0.80 | 0.10 | 0.515 | 0.122 | 0.312 | 0.705 |
| ρ_ϱ | Beta | 0.80 | 0.10 | 0.800 | 0.099 | 0.651 | 0.957 |
| ρ_θ | Beta | 0.80 | 0.10 | 0.722 | 0.094 | 0.568 | 0.823 |
| ρ_ϕ | Beta | 0.80 | 0.10 | 0.867 | 0.068 | 0.782 | 0.961 |
| ρ_I | Beta | 0.80 | 0.10 | 0.776 | 0.105 | 0.615 | 0.947 |
| ρ_H | Beta | 0.80 | 0.10 | 0.858 | 0.089 | 0.727 | 0.991 |
| σ_a | Inv. gamma | 0.01 | 2.00 | 0.013 | 0.002 | 0.010 | 0.016 |
| σ_G | Inv. gamma | 0.01 | 2.00 | 0.049 | 0.012 | 0.036 | 0.071 |
| σ_r | Inv. gamma | 0.01 | 2.00 | 0.002 | 0.0003 | 0.002 | 0.003 |
| σ_ϱ | Inv. gamma | 0.01 | 2.00 | 0.008 | 0.006 | 0.002 | 0.014 |
| σ_θ | Inv. gamma | 0.01 | 2.00 | 0.048 | 0.013 | 0.027 | 0.068 |
| σ_ϕ | Inv. gamma | 0.01 | 2.00 | 0.109 | 0.070 | 0.002 | 0.020 |
| σ_I | Inv. gamma | 0.01 | 2.00 | 0.006 | 0.002 | 0.003 | 0.009 |
| σ_H | Inv. gamma | 0.01 | 2.00 | 0.007 | 0.003 | 0.003 | 0.012 |

4.4.4 Posterior Distributions

Table 4.3 and Table 4.4 describe the posterior mean and 95 probability intervals for the estimated parameters. The posterior mean of adjustment cost parameter of capital investment ζ_h is 6.47 which is higher than the mean of ζ_k , 3.09. The degree of habit formation is moderate ($h=0.354$). The price stickiness in non-durable good production is 0.447 which is also moderate. This implies that prices are re-optimised about every six or seven months. Capital share is 0.327 which is very close to the prior mean, 0.33. Wage share of impatient household (ν) is 0.797, which implies that indebted households in South Korea are usually in higher income group. This result is totally opposite to the result from Iacoviello and Neri (2010). The posterior mean of LTV ratio is 0.626, which is fairly close to the prior mean and quite lower than 0.925 in Lee and Song (2015). The estimated value of housing preference parameter ϕ (0.147) is close to the calibrated value 0.12 in Iacoviello and Neri (2010) and lower than 2.0 in Lee and Song (2015). The posterior mean of η (1.32) is close to the conventionally calibrated value of 1. Estimated values of monetary policy rule related parameters are in line with previous studies. The persistence of shocks except government spending is higher than 0.7 as shown in Table 4.4.

4.5 Properties of the Estimated Model

4.5.1 Cyclical properties

Table 4.5 shows results of matching second moments from the model and data. In the table, Model 2 indicates the model with the typical collateral constraint as in the previous chapter. Model 1 and Model 2 are identical except the borrower's collateral constraint. In terms of standard deviation, two models show quite different performance in household debt. In household debt ($D_{b,t}$), Model 2 shows too high volatility but standard deviation of Model 1 is much closer to the data. Both models fail to show large enough volatility in residential investment. In correlation with output, Model 2 fails to match the correlation in household debt but Model 1 is closer to the data. Both models fail to show proper correlation in residential investment. Results imply that at least regarding household debt, Model 1 can show relatively realistic second moments. This implication will be confirmed in impulse responses as well.

Table 4.5: Second moments

| Standard deviation | Data | Model 1 | Model 2 |
|-------------------------|--------|---------|---------|
| C_t | 0.0108 | 0.0127 | 0.0132 |
| I_t | 0.0296 | 0.0203 | 0.0230 |
| $I_{h,t}$ | 0.0617 | 0.0287 | 0.0245 |
| Y_t | 0.0113 | 0.0110 | 0.0110 |
| $D_{b,t}$ | 0.0146 | 0.0303 | 0.1615 |
| Correlation with output | Data | Model 1 | Model 2 |
| C_t | 0.69 | 0.65 | 0.52 |
| I_t | 0.70 | 0.68 | 0.70 |
| $I_{h,t}$ | 0.05 | 0.33 | 0.25 |
| $D_{b,t}$ | 0.35 | 0.55 | -0.09 |

Note: All variables are logged and HP-filtered for the period 2003-2015.
 2. I_t is non-residential investment data. It is calculated as follows. $I_t =$ Gross fixed capital formation - residential investment.

4.5.2 Impulse Responses

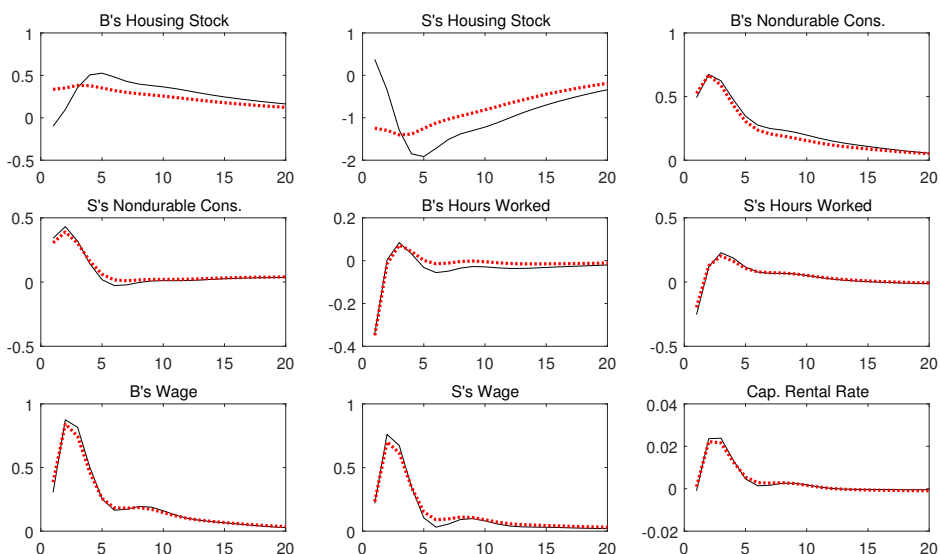
Technology shock

Although the models in this chapter and chapter 3 share common structure, there are some differences in responses to shocks. In addition, difference in collateral constraint also affect more than in chapter 3, that is, gap between Model 1 and Model 2 becomes slightly wider.

Overall responses are far more gradual and last longer than in chapter 3 because of the price stickiness and habit formation in this model. The most notable difference can be found in household debt and household debt to output ratio. As shown in Figure 4.2, household debt keeps increasing after the shock and the amount of its increase is much larger than in the previous chapter model. In addition, household debt to output ratio quickly recovers its initial level and gradually increases thereafter to above its initial level in response to technology shock whereas it suddenly drops and takes long time to recover its initial level in the previous chapter model. There are two factors make these happen. One is relative house price. In this model, technology shock induces higher house price for longer time than in the previous chapter model. Higher house price means higher value in collateral. The other is inflation. When inflation falls, real debt grows. If these two factors work rapidly and strongly enough, household debt to output ratio can increase in spite of rise in output. Model 2 shows little differences from Model 1 in most of variables except household debt and household debt to output ratio. In Model 2, responses of those two variables are too volatile to be regarded as realistic responses to 1% technology shock. Model 1 shows relatively realistic volatility regarding household debt.

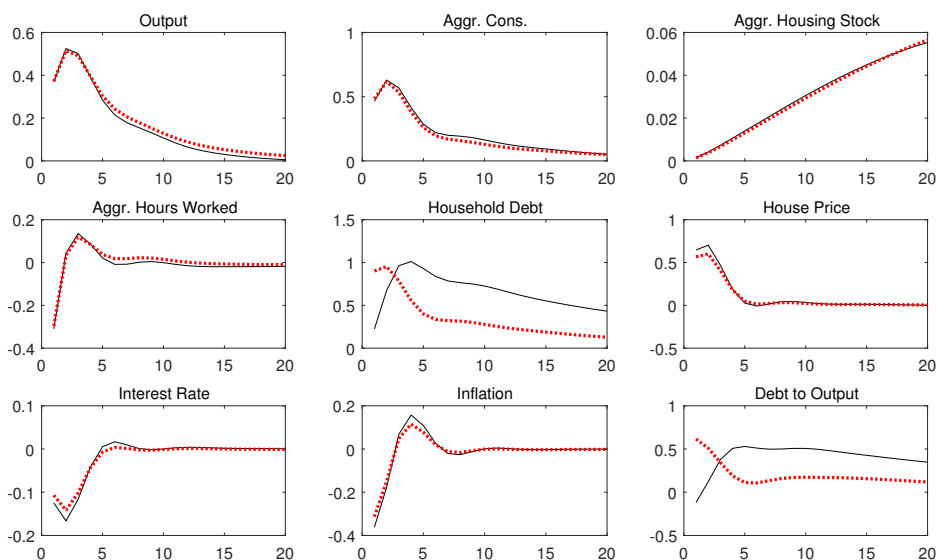
Figure 4.1 plots households' responses. Borrower's housing stock in Model 1 recovers its steady-state level within two quarters and increase thereafter for a long period but in Model 2, it jumps up and keeps above its steady-state level thereafter. This affects saver's responses in opposite direction. Increasing household debt boosts borrower's non-durable good consumption and reduces its hours worked.

Figure 4.1: Household responses to technology shock



Notes: 1. All variables except capital rental rate are expressed as percent deviation from their initial steady-state values. Capital rental rate is percentage point deviation. 2. Solid lines: Model 1, Dotted lines: Model 2.

Figure 4.2: Aggregate responses to technology shock



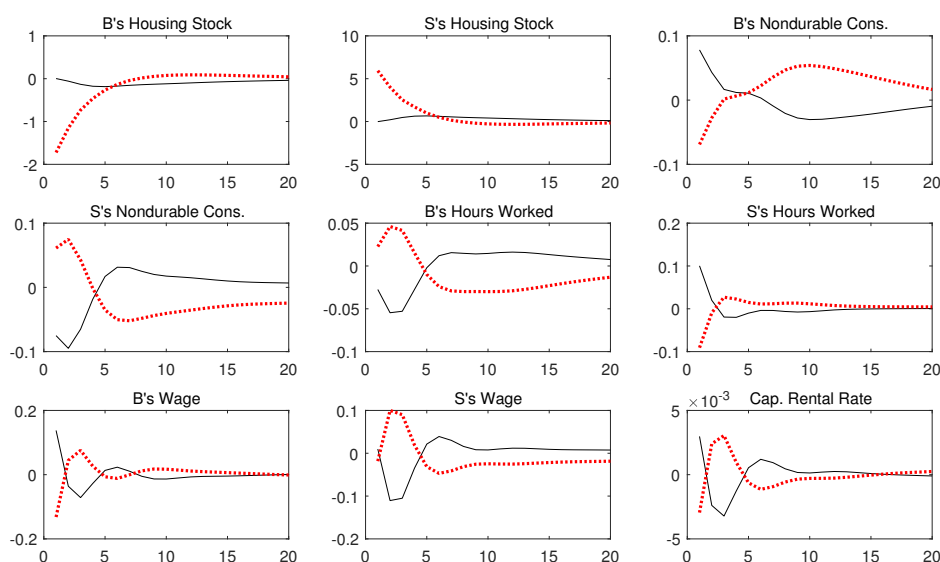
Notes: 1. All variables except interest rate, inflation and debt to output are expressed as percent deviation from their initial steady-state values. Interest rate, inflation and debt to output are percentage point deviation. 2. Solid lines: Model 1, Dotted lines: Model 2.

Decreasing LTV ratio

Figure 4.3 and Figure 4.4 plot responses to a negative LTV ratio shock of 1%. Overall results are not very different from those in the previous chapter. Only notable difference can be found in magnitude of responses of house prices between Model 1 and 2. They are almost twice as much as in chapter 3. In this context, Model 1 looks more realistic than Model 2 because Model 2's responses are too much, especially related to household debt.

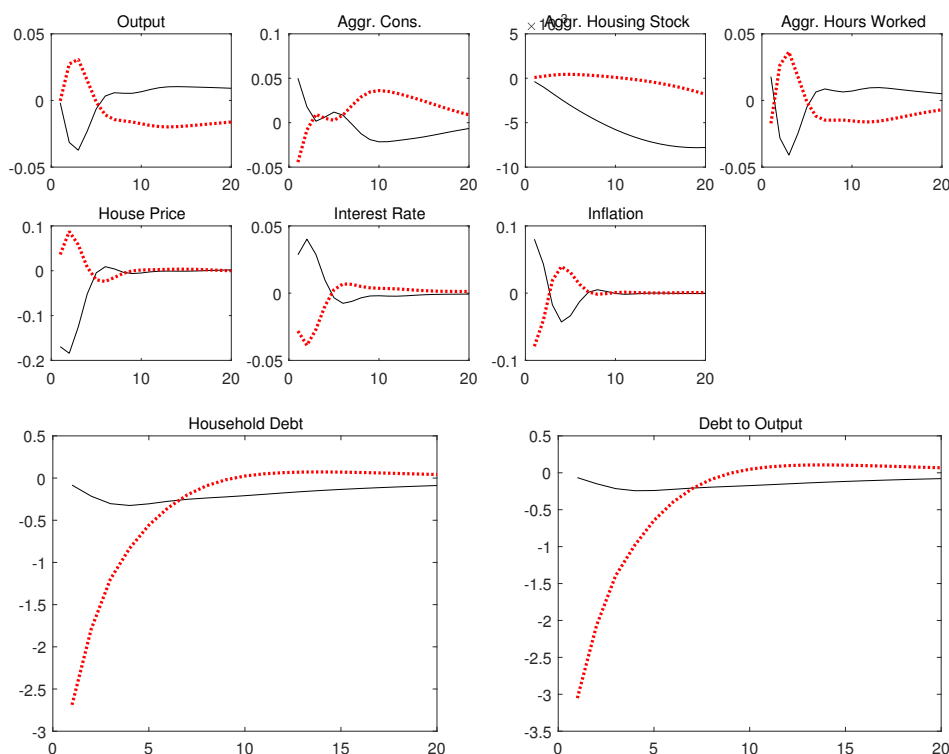
As mentioned in the previous chapter, we can intuitively expect negative responses of output and house price. Model 1 shows appropriate results to meet this conjecture. However, Model 2 shows opposite responses not only in these two variables but also in almost every variable. Therefore, it can be said that Model 1 does produce more realistic responses than Model 2 when LTV ratio decreases temporarily and unexpectedly. Therefore household debt to output ratio dynamics in Model 1 is much more acceptable than in Model 2.

Figure 4.3: Household responses to decrease in LTV ratio



Notes: 1. All variables except capital rental rate are expressed as percent deviation from their initial steady-state values. Capital rental rate is percentage point deviation.
2. Solid lines: Model 1, Dotted lines: Model 2.

Figure 4.4: Aggregate responses to decrease in LTV ratio



Notes: 1. All variables except interest rate, inflation and debt to output are expressed as percent deviation from their initial steady-state values. Interest rate, inflation and debt to output are percentage point deviation. 2. Solid lines: Model 1, Dotted lines: Model 2.

Increasing amortisation rate

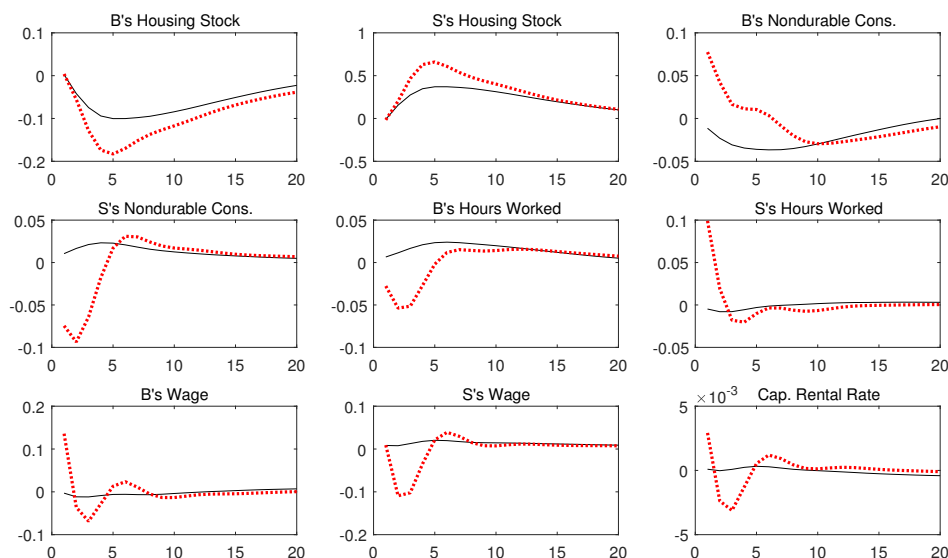
Figure 4.5 plots the household impulse responses to 34% increase⁴ in amortisation rate and compare them with those to negative 1% LTV ratio shock with the same model (Model 1). For the purpose of comparison, the magnitude of shock is increased to 34%, so as to produce almost the same reduction in household debt to output ratio with the case of negative 1% LTV ratio shock.

Overall, responses of variables both in household and aggregate levels are not different from those in the previous chapter. As mentioned in chapter 3, increasing

⁴34% of amortisation rate is only 0.00034 because its given value is 0.001 as in Table 4.1. Although change in amortisation rate can apply only to new borrowing like change in LTV ratio, the model does not have this property to reduce computational complexity. During 2003-2015, average quarterly new borrowing is 2% of existing debt. So, change in amortisation rate in this model can be interpreted as 50 times larger change in amortisation rate of new borrowing. Therefore, increase by 0.00034 can be interpreted as increase by 0.017 in amortisation rate of new borrowing. Therefore this change in amortisation rate is actually from 0.001 (1000-year amortisation of debt) to 0.018 (56-year) in amortisation rate of new borrowing. Although this is a big change in amortisation rate of new borrowing, 56-year amortisation of debt is still quite generous.

amortisation rate produces less volatility in the economy given the same amount of reduction in household debt to output ratio.

Figure 4.5: Household responses to increase in amortisation rate



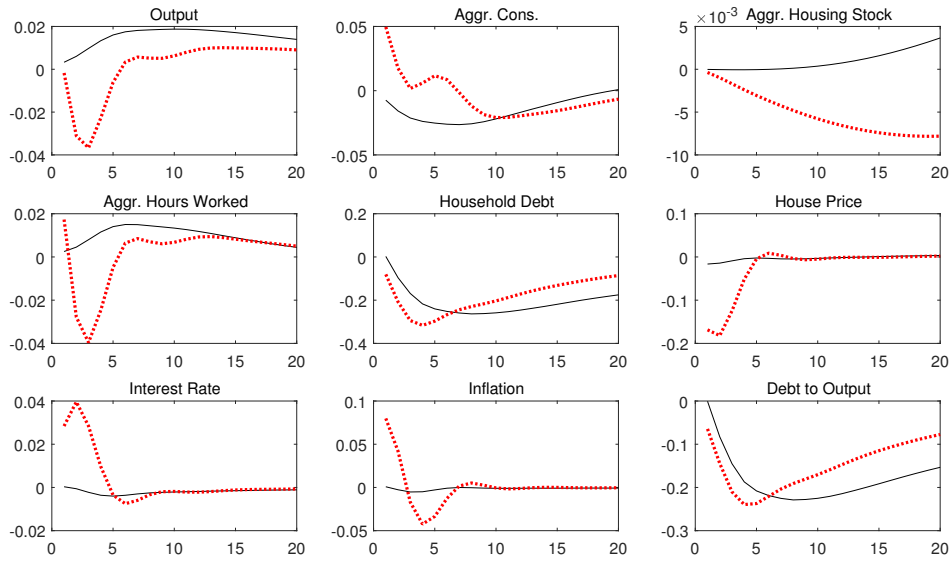
Notes: 1. All variables except capital rental rate are expressed as percent deviation from their initial steady-state values. Capital rental rate is percentage point deviation.
2. Solid lines: Amortisation rate shock, Dotted lines: LTV ratio shock.

Negative house price shock

Figure 4.7 and Figure 4.8 plot the impulse responses to 3.5% decline in housing preference and compare them with those of Model 2 with same shock and Model 1's responses to negative 1% LTV ratio shock. For the purpose of comparison, the magnitude of shock is increased to 3.5%, so as to produce the similar magnitude of drop in house price as in the case of negative 1% LTV ratio shock.

As shown in chapter 3, household debt declines much smaller than in LTV ratio shock. Output decreases to some degree in Model 1 while it shows little response in Model 2. Aggregate consumption slightly goes up in response to drop in house price. This result does not conform with literature (Campbell and Cocco, 2007; Iacoviello and Neri, 2010) which find a positive relationship between on house price and consumption (housing wealth effect). Iacoviello and Neri (2010) uses the same collateral constraint with Model 2. According to Iacoviello and Neri (2010), borrower's consumption can

Figure 4.6: Aggregate responses to increase in amortisation rate



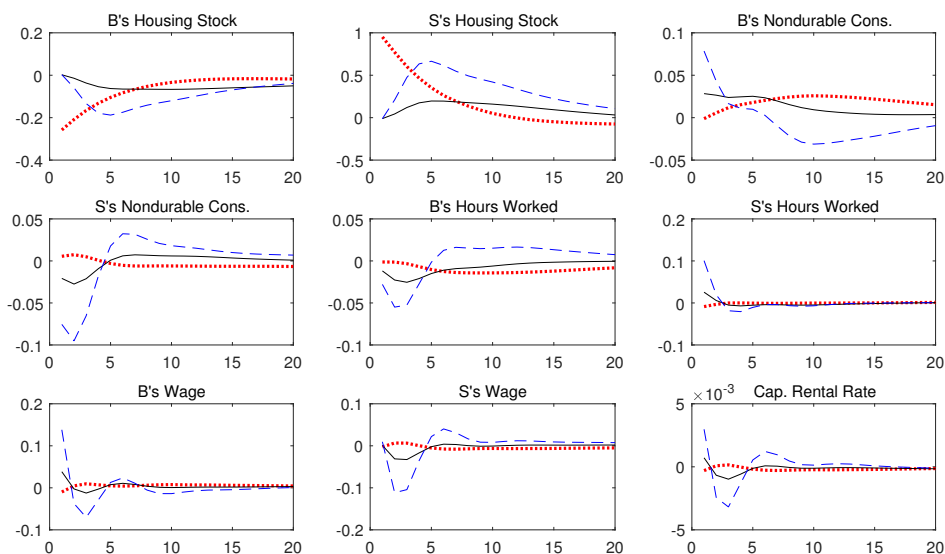
Notes: 1. All variables except interest rate, inflation and debt to output are expressed as percent deviation from their initial steady-state values. Interest rate, inflation and debt to output are percentage point deviation. 2. Solid lines: Amortisation rate shock, Dotted lines: LTV ratio shock.

increase following the rise in house price thanks to more borrowing as Model 2 shows considerable change in household debt in response to house price shock. However, increase (decrease) in borrowing may not be as large as Model 2 shows unless existing mortgage contracts are renewed to utilise a better (worse) condition. If a house price rise affects only new borrowing, household debt cannot increase (decrease) sharply. Although Model 1's consumption response does not conform with literature, its output response may be closer to the data and literature than Model 2's output response.

Monetary policy shock

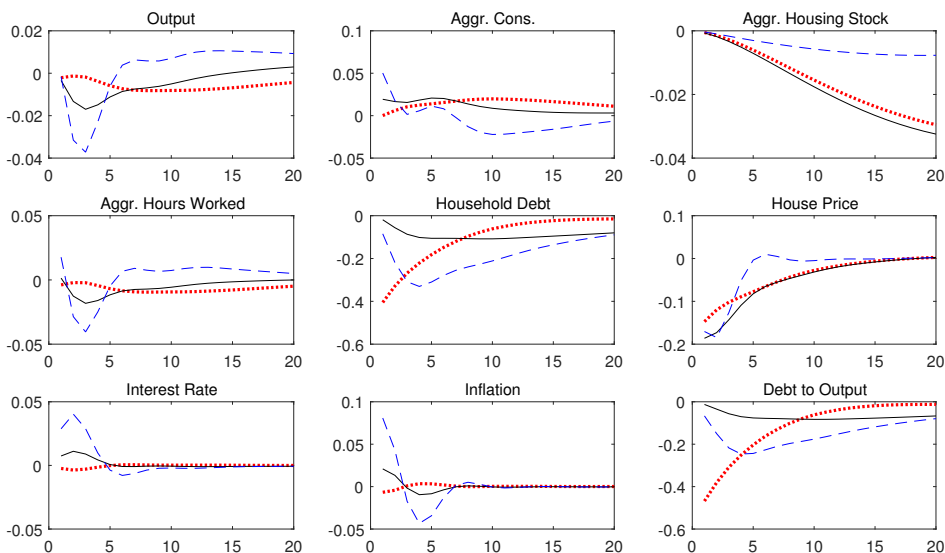
Model 1 shows less volatility in response to monetary policy shock (25bp increase in annual policy rate) than Model 2 as shown in Figure 4.9 and Figure 4.10. Notable differences can be found in household debt related variables. In Model 2, household debt reduces immediately as house price declines in response to monetary tightening. In contrast, household debt rather increases in Model 1 because lower house price negatively affects only on new borrowing and the existing (real) household debt increases due to the lowered inflation. If price-stickiness is high enough (when ξ_p is higher than 0.6), household debt can slightly decrease in response to monetary policy tightening. But with the estimated value of ξ_p (0.447), household debt cannot be reduced by rais-

Figure 4.7: Household responses to decrease in house price



Notes: 1. All variables except capital rental rate are expressed as percent deviation from their initial steady-state values. Capital rental rate is percentage point deviation. 2. Solid lines: Model 1, Dotted lines: Model 2, Dashed lines: LTV ratio shock with Model 1

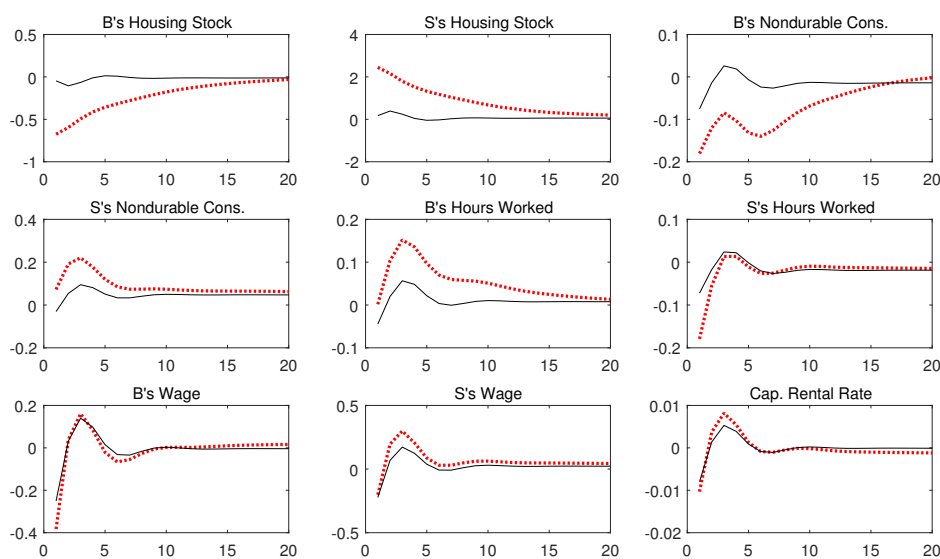
Figure 4.8: Aggregate responses to decrease in house price



Notes: 1. All variables except interest rate, inflation and debt to output are expressed as percent deviation from their initial steady-state values. Interest rate, inflation and debt to output are percentage point deviation. 2. Solid lines: Model 1, Dotted lines: Model 2, Dashed lines: LTV ratio shock with Model 1

ing policy rate. In model 2, household debt decreases much more than output, so as to make household debt to output ratio go down. In model 1, household debt increases while output declines, so that household debt to output ratio rises. These results imply that effects of monetary policy tightening on real household debt level depend on price-stickiness and with the estimated degree of price-stickiness, household debt level slightly increases when policy rate rises.

Figure 4.9: Household responses to monetary policy shock (25bp \uparrow)

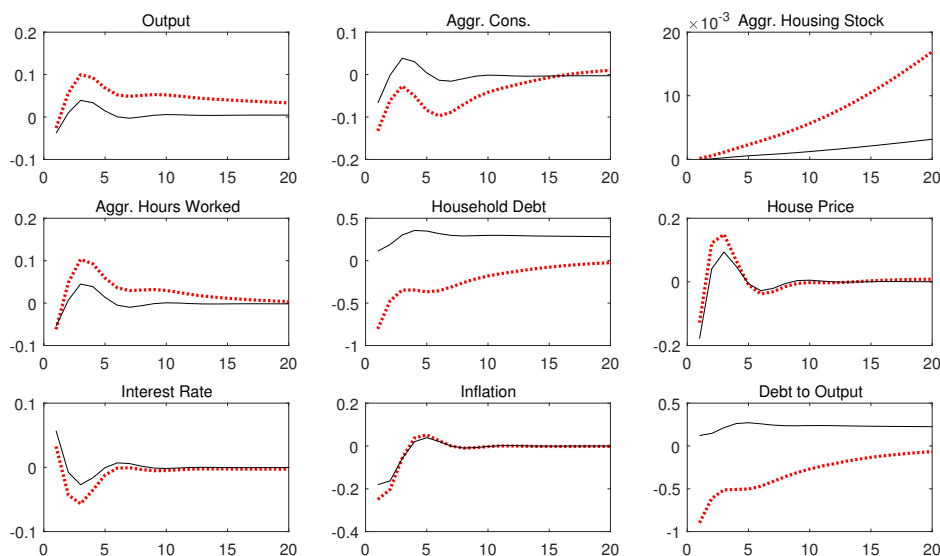


Notes: 1. All variables except capital rental rate are expressed as percent deviation from their initial steady-state values. Capital rental rate is percentage point deviation.
2. Solid lines: Model 1, Dotted lines: Model 2.

Monetary policy shock with different LTV ratios

Figure 4.11 and Figure 4.12 plot responses to a monetary policy shock (25bp increase in annual policy rate) under three different LTV ratios; 0.75, 0.623 and 0.5. Under lower LTV ratio which means tighter credit conditions, household debt fluctuates by smaller amount as expected. Other aggregate variables seem to show no significant differences. If we look at the responses of output, aggregate consumption and aggregate hours worked closely, we can find slight differences. The higher LTV ratio, the higher the volatility. However, these differences are not big enough to conclude that LTV ratio significantly changes effects of monetary policy.

These results have the implication that the level of household debt does not affect the magnitude and transmission of monetary policy effects significantly because higher

Figure 4.10: Aggregate responses to monetary policy shock (25bp \uparrow)

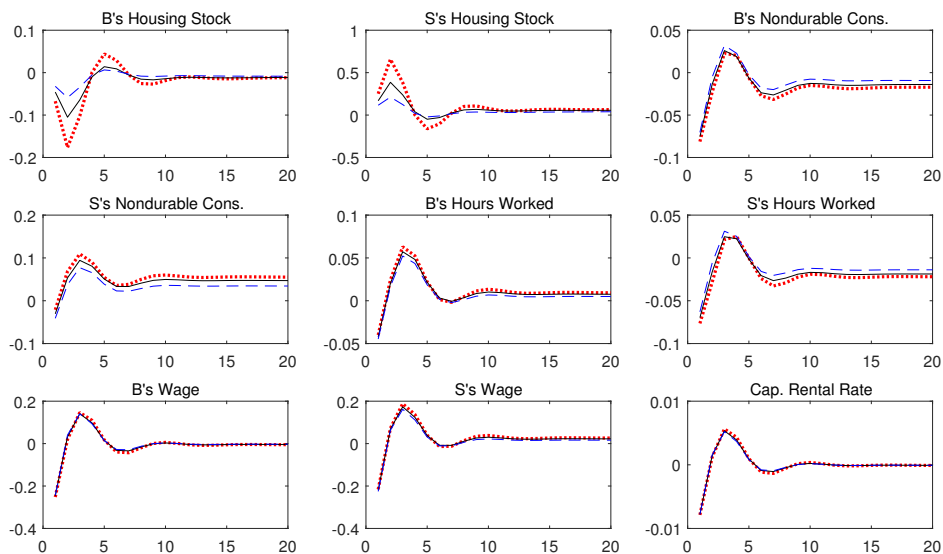
Notes: 1. All variables except interest rate, inflation and debt to output are expressed as percent deviation from their initial steady-state values. Interest rate, inflation and debt to output are percentage point deviation. 2. Solid lines: Model 1, Dotted lines: Model 2.

LTV ratios can be regarded as a proxy of higher level of household debt. If we interpret different LTV ratios as different macroprudential policy regimes, these results can imply that macroprudential policy regime does not influence monetary policy significantly.

Monetary policy shock with different amortisation rates

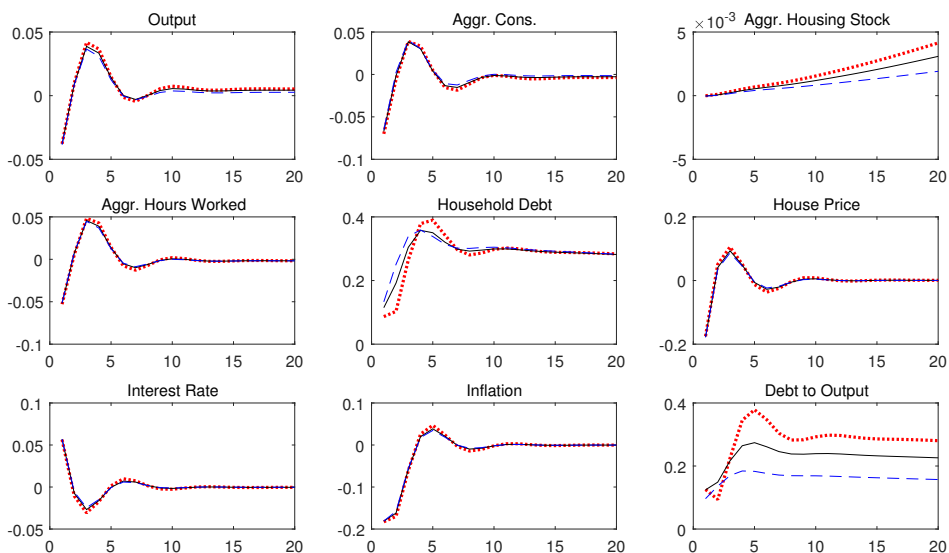
Figure 4.13 and Figure 4.14 plot responses to a monetary policy shock (25bp increase in annual policy rate) under three different amortisation rates; 0.001, 0.007 and 0.014. This experiment also designed to see the relationship between monetary policy and macroprudential policy regime. Once again we can see that monetary policy does not seem to be significantly influenced by amortisation rate (macroprudential regime). So, this result does not seem to be different from that in the previous experiment with different LTV ratios.

Figure 4.11: Household responses to monetary policy shock with different LTV ratios



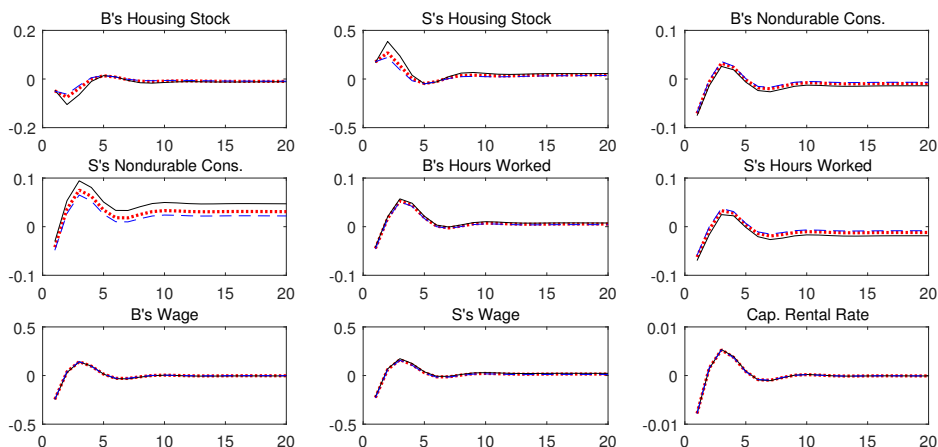
Notes: 1. All variables except capital rental rate are expressed as percent deviation from their initial steady-state values. Capital rental rate is percentage point deviation. 2. Solid lines: LTV ratio 0.623, Dotted lines: LTV ratio 0.75, Dashed lines: LTV ratio 0.45

Figure 4.12: Aggregate responses to monetary policy shock with different LTV ratios



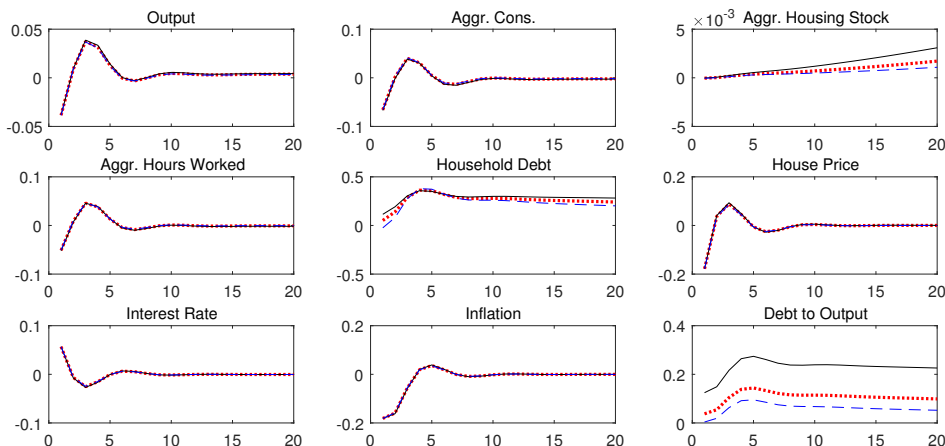
Notes: 1. All variables except interest rate, inflation and debt to output are expressed as percent deviation from their initial steady-state values. Interest rate, inflation and debt to output are percentage point deviation. 2. Solid lines: LTV ratio 0.623, Dotted lines: LTV ratio 0.75, Dashed lines: LTV ratio 0.45

Figure 4.13: Household responses to monetary policy shock with different amortisation rates



Notes: 1. All variables except capital rental rate are expressed as percent deviation from their initial steady-state values. Capital rental rate is percentage point deviation. 2. Solid lines: amortisation rate 0.001, Dotted lines: amortisation rate 0.007, Dashed lines: amortisation rate 0.014

Figure 4.14: Aggregate responses to monetary policy shock with different amortisation rates



Notes: 1. All variables except interest rate, inflation and debt to output are expressed as percent deviation from their initial steady-state values. Interest rate, inflation and debt to output are percentage point deviation. 2. Solid lines: amortisation rate 0.001, Dotted lines: amortisation rate 0.007, Dashed lines: amortisation rate 0.014

4.6 Conclusions

As household debt level in South Korea continues to grow, policymakers in South Korea have kept on eye on the level of household debt for more than ten years since mid-2000s and are trying to slow down the speed of its growth. This chapter tries to incorporate

both borrowing and household debt in a medium scale DSGE model with a proper collateral constraint in order to provide a proper policy analysis tool to address household debt issue. Results have at least three implications. First, only macroprudential policies such as lowering LTV ratio and increasing amortisation rate are effective in slowing down the speed of household debt growth. Tightening monetary policy even increases household debt level in this model economy. Among credit tightening measures, increase in amortisation rate is more effective than lowering LTV ratio as in the previous chapter because it makes less volatility in the economy. Second, different macroprudential regimes such as different LTV ratios and different amortisation rates do not change the magnitude and transmission of monetary policy effects. It also can be said that monetary policy is not significantly affected by the level of household debt. Third, as in the previous chapter, effects of house price change rely on its source of change. If house price is changed by housing demand, in other words, the source of change in house price is independent from other macroeconomic variables, its effects could be relatively limited.

These implications can be interpreted from the perspective of policymakers. When the government tries to address household debt problem with minimum disturbance in macroeconomy, raising amortisation rate could be the most effective than any other policies. When monetary policy authority makes a decision, the level of household debt may not be a critical factor to consider at least in that it does not change effects of monetary policy.

Appendix

4.A Equilibrium

The model has a unique stationary equilibrium in which impatient household borrows up to the borrowing limit. The equilibrium conditions are as follows.

$$[1] \quad \lambda_{b,t} = \frac{1}{C_{b,t} - hC_{b,t-1}} - h\beta_b E_t \frac{1}{C_{b,t+1} - hC_{b,t}}$$

This can be rewritten as $U'(C_{b,t}) = \lambda_{b,t}$.

$$[2] \quad q_t(1 - \mu_{b,t}\theta_t) = \beta_b E_t \left[\frac{\lambda_{b,t+1}}{\lambda_{b,t}} \left\{ \frac{\phi_{t+1}}{\lambda_{b,t+1} H_{b,t+1}} + (1 - \delta_h)q_{t+1}(1 - \mu_{b,t+1}\theta_{t+1}) \right\} \right]$$

$$[3] \quad L_{b,t}^n = \lambda_{b,t} w_{b,t}$$

This can be rewritten as $\frac{-U'(L_{b,t})}{U'(C_{b,t})} = w_{b,t}$.

$$[4] \quad \lambda_{b,t}(1 - \mu_{b,t}) = \beta_b E_t \frac{\lambda_{b,t+1}}{\pi_{t+1}} \left\{ R_t - (1 - \varrho_t)\mu_{b,t+1} \right\}$$

This is different from the standard consumption Euler equation because impatient households has a borrowing constraint. This can be rewritten as

$$U'(C_{b,t})(1 - \mu_{b,t}) = \beta_b E_t \frac{U'(C_{b,t+1})}{\pi_{t+1}} \left\{ R_t - (1 - \varrho_t)\mu_{b,t+1} \right\}$$

. If there is no borrowing constraint ($\mu_{b,t} = \mu_{b,t+1} = 0$), this reduces to the standard consumption Euler equation.

$$[5] \quad C_{b,t} + q_t N_{b,t} + R_{t-1} \frac{d_{b,t}}{\pi_t} = w_{b,t} L_{b,t} - T_{b,t} + d_{b,t+1}$$

$$[6] \quad d_{b,t+1} = (1 - \varrho_{t-1}) \frac{d_{b,t}}{\pi_t} + \theta_t q_t N_{b,t}$$

$$[7] \quad N_{b,t} = H_{b,t+1} - (1 - \delta_h) H_{b,t}$$

$$[8] \quad \lambda_{s,t} = \frac{1}{C_{s,t} - hC_{s,t-1}} - h\beta_s E_t \frac{1}{C_{s,t+1} - hC_{s,t}}$$

This can be rewritten as $U'(C_{s,t}) = \lambda_{s,t}$.

$$[9] \quad q_t = \beta_s E_t \left[\frac{\lambda_{s,t+1}}{\lambda_{s,t}} \left\{ \frac{\phi_{t+1}}{\lambda_{s,t+1} H_{s,t+1}} + (1 - \delta_h) q_{t+1} \right\} \right]$$

$$[10] \quad L_{s,t}^n = \lambda_{s,t} w_{s,t}$$

This can be rewritten as $\frac{-U'(L_{s,t})}{U'(C_{s,t})} = w_{s,t}$.

$$[11] \quad \lambda_{s,t} = \beta_s R_t E_t \frac{\lambda_{s,t+1}}{\pi_{t+1}}$$

This is the standard consumption Euler equation for the patient household and can be rewritten as:

$$U'(C_{s,t}) = \beta_s R_t E_t \frac{U'(C_{s,t+1})}{\pi_{t+1}}$$

$$[12] \quad \mu_{s,t} = \beta_s E_t \frac{\lambda_{s,t+1}}{\lambda_{s,t}} \{ r_{k,t+1} + (1 - \delta_k) \mu_{s,t+1} \}$$

$$[13] \quad N_{s,t} = H_{s,t+1} - (1 - \delta_h) H_{s,t}$$

$$[14] \quad K_{t+1} = (1 - \delta_k) K_t + F_t(I_t, I_{t-1})$$

$$[15] \quad F_t(I_t, I_{t-1}) = A_{I,t} \left\{ 1 - \zeta_k \frac{1}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right\} I_t$$

$$[16] \quad 1 = \mu_{s,t} A_{I,t} \left[1 - \zeta_k \frac{1}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \zeta_k \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right] \\ + \beta_s E_t A_{I,t+1} \frac{\lambda_{s,t+1}}{\lambda_{s,t}} \mu_{s,t+1} \zeta_k \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2$$

$$[17] \quad \nu_t^p = \xi_p \left(\frac{\pi_{t-1}}{\pi_t} \right)^{-\varepsilon} \nu_{t-1}^p + (1 - \xi_p) \pi_t^{*-\varepsilon}$$

where $\nu_t^p = \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} di$.

$$[18] \quad Y_t = \frac{A_t^{1-\alpha} K_t^\alpha \left[\{\psi L_{b,t}\}^\nu \{(1-\psi)L_{s,t}\}^{1-\nu} \right]^{1-\alpha} - A_t \Gamma}{\nu_t^p}$$

$$[19] \quad mc_t \nu (1 - \alpha) (\nu_t^p Y_t + A_t \Gamma) L_{b,t}^{-1} = \psi w_{b,t}$$

$$[20] \quad mc_t (1 - \nu) (1 - \alpha) (\nu_t^p Y_t + A_t \Gamma) L_{s,t}^{-1} = (1 - \psi) w_{s,t}$$

$$[21] \quad mc_t = \left(\frac{1}{1 - \alpha} \right)^{1-\alpha} \left(\frac{1}{\alpha} \right)^\alpha \left(\frac{1}{A_t} \right)^{1-\alpha} r_{k,t}^\alpha \left[\left(\frac{1}{\nu} w_{b,t} \right)^\nu \left(\frac{1}{1 - \nu} w_{s,t} \right)^{1-\nu} \right]^{1-\alpha}$$

$$[22] \quad \pi_t^* M_t = Z_t$$

$$[23] \quad Z_t = \frac{\varepsilon}{\varepsilon - 1} \lambda_{s,t} mc_t Y_t + \beta_s \xi_p E_t \left(\frac{\pi_t}{\pi_{t+1}} \right)^{1-\varepsilon} Z_{t+1}$$

$$[24] \quad M_t = \lambda_{s,t} Y_t + \beta_s \xi_p E_t \left(\frac{\pi_t}{\pi_{t+1}} \right)^{-\varepsilon} M_{t+1}$$

$$[25] \quad 1 = \xi_p \left(\frac{\pi_{t-1}}{\pi_t} \right)^{1-\varepsilon} + (1 - \xi_p) \pi_t^{*1-\varepsilon}$$

$$[26] \quad 1 = q_t A_{H,t} \left\{ 1 - \zeta_h \frac{1}{2} \left(\frac{I_{h,t}}{I_{h,t-1}} - 1 \right)^2 - \zeta_h \left(\frac{I_{h,t}}{I_{h,t-1}} - 1 \right) \right\} \frac{I_{h,t}}{I_{h,t-1}} \\ + \beta_s E_t A_{H,t+1} \frac{\lambda_{s,t+1}}{\lambda_{s,t}} q_{t+1} \zeta_h \left(\frac{I_{h,t+1}}{I_{h,t}} - 1 \right) \left(\frac{I_{h,t+1}}{I_{h,t}} \right)^2$$

$$[27] \quad N_t = A_{H,t} \left\{ 1 - \zeta_h \frac{1}{2} \left(\frac{I_{h,t}}{I_{h,t-1}} - 1 \right)^2 \right\} I_{h,t}$$

$$[28] \quad \psi T_{b,t} = \gamma g Y_t$$

$$[29] \quad G_t = (1 - \psi) T_{s,t} + \psi T_{b,t}$$

$$[30] \quad \frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\omega_R} \left[\left(\frac{\pi_t}{\pi} \right)^{\omega_\pi} \left(\frac{Y_t}{Y} \right)^{\omega_Y} \right]^{1-\omega_R} \xi_{r,t}$$

$$[31] \quad \psi d_{b,t+1} + (1 - \psi) d_{s,t+1} = 0$$

$$[32] \quad N_t = (1 - \psi) N_{s,t} + \psi N_{b,t}$$

$$[33] \quad C_t = (1 - \psi) C_{s,t} + \psi C_{b,t}$$

$$[34] \quad L_t = (1 - \psi) L_{s,t} + \psi L_{b,t}$$

$$[35] \quad Y_t = C_t + I_{h,t} + I_t + G_t$$

$$[36] \quad \ln A_t = \rho_a \ln A_{t-1} + \varepsilon_{a,t}, \quad \varepsilon_{a,t} \sim i.i.d.N(0, \sigma_a^2)$$

$$[37] \quad \ln \varrho_t = \rho_\varrho \ln \varrho_{t-1} + (1 - \rho_\varrho) \ln \varrho + \varepsilon_{\varrho,t}, \quad \varepsilon_{\varrho,t} \sim i.i.d.N(0, \sigma_\varrho^2)$$

$$[38] \quad \ln \theta_t = \rho_\theta \ln \theta_{t-1} + (1 - \rho_\theta) \ln \theta - \varepsilon_{\theta,t}, \quad \varepsilon_{\theta,t} \sim i.i.d.N(0, \sigma_\theta^2)$$

$$[39] \quad \ln \phi_t = \rho_\phi \ln \phi_{t-1} + (1 - \rho_\phi) \ln \phi - \varepsilon_{\phi,t}, \quad \varepsilon_{\phi,t} \sim i.i.d.N(0, \sigma_\phi^2)$$

$$[40] \quad \ln G_t = \rho_G \ln G_{t-1} + (1 - \rho_G) \ln G + \varepsilon_{G,t}, \quad \varepsilon_{G,t} \sim i.i.d.N(0, \sigma_G^2)$$

$$[41] \quad \ln A_{I,t} = \rho_I \ln A_{I,t-1} + \varepsilon_{I,t}, \quad \varepsilon_{I,t} \sim i.i.d.N(0, \sigma_I^2)$$

$$[42] \quad \ln A_{H,t} = \rho_H \ln A_{H,t-1} + \varepsilon_{H,t}, \quad \varepsilon_{H,t} \sim i.i.d.N(0, \sigma_H^2)$$

4.B Steady State

The model can be solved when θ , ϕ and ξ_r are regarded as parameters. We abstract from growth ($A = 1$).

$$\begin{aligned}
q &= \mu_s = \pi^* = \nu^p = 1 & R &= \frac{\pi}{\beta_s} \\
\mu_b &= \frac{\pi \left(1 - \frac{\beta_b}{\beta_s}\right)}{\pi - \beta_b(1 - \varrho)} & r_k &= \frac{1}{\beta_s} - (1 - \delta_k) \\
aux_1 &= \frac{\beta_b \phi(1 - h)}{(1 - \mu_b \theta)[1 - (1 - \delta_h)\beta_b](1 - \beta_b h)} \\
aux_2 &= \frac{\beta_s \phi(1 - h)}{[1 - (1 - \delta_h)\beta_s](1 - \beta_s h)} \\
aux_3 &= \frac{\frac{r_k}{\psi \alpha} \{\nu(1 - \alpha) - \gamma g\}}{1 + aux_1 \delta_h \left[1 + \frac{\theta q}{1 - (1 - \varrho)\frac{1}{\pi}} \left(\frac{R}{\pi} - 1\right)\right]} \\
aux_4 &= \frac{\frac{r_k}{\alpha}(1 - g) - \delta_k - \psi aux_3(1 + \delta_h aux_1)}{(1 - \psi)(1 + \delta_h aux_2)} \\
L_b &= \left[\frac{(1 - h\beta_b)\nu(1 - \alpha)r_k}{\alpha \psi aux_3(1 - h)} \right]^{\frac{1}{\eta+1}} & L_s &= \left[\frac{(1 - h\beta_s)(1 - \nu)(1 - \alpha)r_k}{\alpha(1 - \psi)aux_4(1 - h)} \right]^{\frac{1}{\eta+1}} \\
mc &= \frac{\varepsilon - 1}{\varepsilon} & K &= \left(\frac{mc\alpha}{r_k} \right)^{\frac{1}{1-\alpha}} \{\psi L_b\}^\nu \{(1 - \psi)L_s\}^{1-\nu} \\
Y &= \frac{r_k K}{\alpha} & C_b &= aux_3 K \\
C_s &= aux_4 K & \lambda_b &= \frac{1 - h\beta_b}{C_b(1 - h)} \\
\lambda_s &= \frac{1 - h\beta_s}{C_s(1 - h)} & H_b &= aux_1 C_b \\
H_s &= aux_2 C_s & N_b &= \delta_h H_b \\
N_s &= \delta_h H_s & N &= (1 - \psi)N_s + \psi N_b \\
I_h &= N & L &= (1 - \psi)L_s + \psi L_b \\
d_b &= \frac{\theta q N_b}{1 - (1 - \varrho)\frac{1}{\pi}} & d_s &= \frac{\psi}{\psi - 1} d_b \\
C &= (1 - \psi)C_s + \psi C_b & F &= \delta_k K \\
I &= F & \Gamma &= \frac{Y}{\varepsilon - 1} \\
w_b &= \frac{\nu(1 - \alpha)Y}{\psi L_b} & w_s &= \frac{(1 - \nu)(1 - \alpha)Y}{(1 - \psi)L_b} \\
M &= \frac{\lambda_s Y}{1 - \beta_s \xi_p} & Z &= \frac{\lambda_s Y}{1 - \beta_s \xi_p} \\
T_b &= \frac{\gamma g Y}{\psi} & G &= gY \\
T_s &= \frac{G - \psi T_b}{1 - \psi}
\end{aligned}$$

The steady state can be briefly described as follows.

$$\begin{aligned}\frac{C}{Y} &= \frac{\alpha[\psi aux_3 + (1 - \psi)aux_4]}{\delta_k - 1 + \frac{1}{\beta_s}} \\ \frac{I_h + I}{Y} &= \frac{\alpha\delta_k[(1 - \psi)aux_1aux_3 + \psi aux_2aux_4] + \alpha\delta_k}{\delta_k - 1 + \frac{1}{\beta_s}} \\ \frac{N}{Y} &= \frac{\alpha\delta_k[\psi aux_1aux_3 + (1 - \psi)aux_2aux_4]}{\delta_k - 1 + \frac{1}{\beta_s}} \\ \frac{H}{Y} &= \frac{\alpha[\psi aux_1aux_3 + (1 - \psi)aux_2aux_4]}{\delta_k - 1 + \frac{1}{\beta_s}} \\ \frac{D_b}{Y} &= \frac{\alpha\theta P_h\delta_h aux_1aux_3}{\varrho(\delta_k - 1 + \frac{1}{\beta_s})}\end{aligned}$$

4.C Log-linearized Equations

$$[1] \quad \hat{\lambda}_{b,t} = \frac{-1}{(1-h)(1-h\beta_b)} \{ \hat{C}_{b,t} - h\hat{C}_{b,t-1} - h\beta_b E_t(\hat{C}_{b,t+1} - h\hat{C}_{b,t}) \}$$

$$[2] \quad \hat{q}_t = \frac{\mu_b \theta (\hat{\mu}_{b,t} + \hat{\theta}_t)}{1 - \mu_b \theta} + E_t(\hat{\phi}_{t+1} - \hat{\lambda}_{b,t} - \hat{H}_{b,t+1}) \\ - \beta_b (1 - \delta_h) E_t \left[\hat{\phi}_{t+1} - \hat{\lambda}_{b,t+1} - \hat{H}_{b,t+1} - \hat{q}_{t+1} + \frac{\mu_b \theta (\hat{\mu}_{b,t+1} + \hat{\theta}_{t+1})}{1 - \mu_b \theta} \right]$$

$$[3] \quad \hat{w}_{b,t} = \hat{C}_{b,t} + \eta \hat{L}_{b,t}$$

$$[4] \quad \hat{\lambda}_{b,t} = \frac{\mu_{b,t}}{1 - \mu_b} \hat{\mu}_{b,t} + \frac{R}{R - (1 - \varrho)\mu_b} E_t(\hat{\lambda}_{b,t+1} + \hat{R}_t - \hat{\pi}_{t+1}) \\ - \frac{(1 - \varrho)\mu_b}{R - (1 - \varrho)\mu_b} (\hat{\lambda}_{b,t+1} - \hat{\pi}_{t+1} + \hat{\mu}_{b,t+1} - \frac{\varrho}{1 - \varrho} \hat{q}_t)$$

$$[5] \quad C_b \hat{C}_{b,t} + N_b(\hat{q}_t + \hat{N}_{b,t}) + \frac{R d_b}{\pi} (\hat{R}_{t-1} + \hat{d}_{b,t} - \hat{\pi}_t) = w_b L_b(\hat{w}_{b,t} + \hat{L}_{b,t}) - T_b \hat{T}_{b,t} + d_b \hat{d}_{b,t+1}$$

$$[6] \quad \hat{d}_{b,t+1} = \frac{1 - \varrho}{\pi} (\hat{d}_{b,t} - \hat{\pi}_t) - \frac{\varrho}{\pi} \hat{q}_{t-1} + \frac{\theta N_b}{d_b} (\hat{\theta}_t + \hat{q}_t + \hat{N}_{b,t})$$

$$[7] \quad \hat{N}_{b,t} = \frac{1}{\delta_h} \hat{H}_{b,t+1} - \frac{1 - \delta_h}{\delta_h} \hat{H}_{b,t}$$

$$[8] \quad \hat{\lambda}_{s,t} = \frac{-1}{(1-h)(1-h\beta_s)} \{ \hat{C}_{s,t} - h\hat{C}_{s,t-1} - h\beta_s E_t(\hat{C}_{s,t+1} - h\hat{C}_{s,t}) \}$$

$$[9] \quad \hat{q}_t = E_t(\hat{\phi}_{t+1} - \hat{\lambda}_{s,t} - \hat{H}_{s,t+1}) - \beta_s (1 - \delta_h) E_t(\hat{\phi}_{t+1} - \hat{\lambda}_{s,t+1} - \hat{H}_{s,t+1} - \hat{q}_{t+1})$$

$$[10] \quad \hat{w}_{s,t} = \hat{C}_{s,t} + \eta \hat{L}_{s,t}$$

$$[11] \quad \hat{\lambda}_{s,t} = \frac{\beta_s R}{\pi} E_t(\hat{R}_t + \hat{\lambda}_{s,t+1} - \hat{\pi}_{t+1})$$

$$[12] \quad \hat{\mu}_{s,t} = E_t(\hat{\lambda}_{s,t+1} - \hat{\lambda}_{s,t} + \hat{r}_{k,t+1}) - \beta_s (1 - \delta_k) E_t(\hat{r}_{k,t+1} - \hat{\mu}_{s,t+1})$$

$$[13] \quad \hat{N}_{s,t} = \frac{1}{\delta_h} \hat{H}_{s,t+1} - \frac{1 - \delta_h}{\delta_h} \hat{H}_{s,t}$$

$$[14] \quad \hat{K}_{t+1} = (1 - \delta_k) \hat{K}_t + \frac{F}{K} \hat{F}_t$$

$$[15] \quad \hat{F}_t = \hat{I}_t + \hat{A}_{I,t}$$

$$[16] \quad \hat{\mu}_{s,t} + \hat{A}_{I,t} = \zeta_k (\hat{I}_t - \hat{I}_{t-1}) - \beta_s \zeta_k E_t (\hat{I}_{t+1} - \hat{I}_t)$$

$$[17] \quad \hat{\nu}_t^p = \xi_p \left\{ \varepsilon (\hat{\pi}_t - \hat{\pi}_{t-1}) + \hat{\nu}_{t-1}^p \right\} + (1 - \xi_p) \varepsilon \hat{\pi}_t^*$$

$$[18] \quad \frac{Y \nu^p (\hat{Y}_t + \hat{\nu}_t^p)}{Y \nu^p + A \Gamma} = \left(1 - \alpha - \frac{A \Gamma}{Y \nu^p + A \Gamma} \right) \hat{A}_t + \alpha \hat{K}_t + (1 - \alpha) \{ \nu \hat{L}_{b,t} + (1 - \nu) \hat{L}_{s,t} \}$$

$$[19] \quad \hat{w}_{b,t} = \frac{\nu^p Y (\hat{m}c_t - \hat{L}_{b,t} + \hat{\nu}_t^p + \hat{Y}_t) + A \Gamma (\hat{m}c_t - \hat{L}_{b,t} + \hat{A}_t)}{\nu^p Y + A \Gamma}$$

$$[20] \quad \hat{w}_{s,t} = \frac{\nu^p Y (\hat{m}c_t - \hat{L}_{s,t} + \hat{\nu}_t^p + \hat{Y}_t) + A \Gamma (\hat{m}c_t - \hat{L}_{s,t} + \hat{A}_t)}{\nu^p Y + A \Gamma}$$

$$[21] \quad \hat{m}c_t = -(1 - \alpha) \hat{A}_t + \alpha \hat{r}_{k,t} + (1 - \alpha) \{ \nu \hat{w}_{b,t} + (1 - \nu) \hat{w}_{s,t} \}$$

$$[22] \quad \hat{\pi}_t^* + \hat{M}_t = \hat{Z}_t$$

$$[23] \quad \hat{Z}_t = (1 - \beta_s \xi_p) (\hat{\lambda}_{s,t} + \hat{m}c_t + \hat{Y}_t) + \beta_s \xi_p E_t \left\{ (1 - \varepsilon) (\hat{\pi}_t - \hat{\pi}_{t+1}) + \hat{Z}_{t+1} \right\}$$

$$[24] \quad \hat{M}_t = (1 - \beta_s \xi_p) (\hat{\lambda}_{s,t} + \hat{Y}_t) + \beta_s \xi_p E_t \left\{ \varepsilon (\hat{\pi}_{t+1} - \hat{\pi}_t) + \hat{M}_{t+1} \right\}$$

$$[25] \quad \hat{\pi}_t^* = \frac{\xi_p}{(1 - \xi_p) \pi^{*1-\varepsilon}} (\hat{\pi}_t - \hat{\pi}_{t-1})$$

$$[26] \quad \hat{q}_t + \hat{A}_{H,t} = \zeta_h (\hat{I}_{h,t} - \hat{I}_{h,t-1}) - \beta_s \zeta_h E_t (\hat{I}_{h,t+1} - \hat{I}_{h,t})$$

$$[27] \quad \hat{N}_t = \hat{I}_{h,t} + \hat{A}_{H,t}$$

$$[28] \quad \hat{T}_{b,t} = \hat{Y}_t$$

$$[29] \quad \hat{G}_t = \frac{1}{G}[(1 - \psi)T_s\hat{T}_{s,t} + \psi T_b\hat{T}_{b,t}]$$

$$[30] \quad \hat{R}_t = \omega_R\hat{R}_{t-1} + (1 - \omega_R)(\omega_\pi\hat{\pi}_t + \omega_Y\hat{Y}_t) + \xi_{r,t}$$

$$[31] \quad \hat{d}_{s,t+1} = -\hat{d}_{b,t+1}$$

$$[32] \quad \hat{N}_t = \frac{1}{N}[(1 - \psi)N_s\hat{N}_{s,t} + \psi N_b\hat{N}_{b,t}]$$

$$[33] \quad \hat{C}_t = \frac{1}{C}[(1 - \psi)C_s\hat{C}_{s,t} + \psi C_b\hat{C}_{b,t}]$$

$$[34] \quad \hat{L}_t = \frac{1}{L}[(1 - \psi)L_s\hat{L}_{s,t} + \psi L_b\hat{L}_{b,t}]$$

$$[35] \quad \hat{Y}_t = \hat{C}_t + \hat{I}_{h,t} + \hat{I}_t + \hat{G}_t$$

$$[36] \quad \hat{A}_t = \rho_a\hat{A}_{t-1} + \varepsilon_{a,t}$$

$$[37] \quad \hat{\varrho}_t = \rho_\varrho\hat{\varrho}_{t-1} + \varepsilon_{\varrho,t}$$

$$[38] \quad \hat{\theta}_t = \rho_\theta\hat{\theta}_{t-1} - \varepsilon_{\theta,t}$$

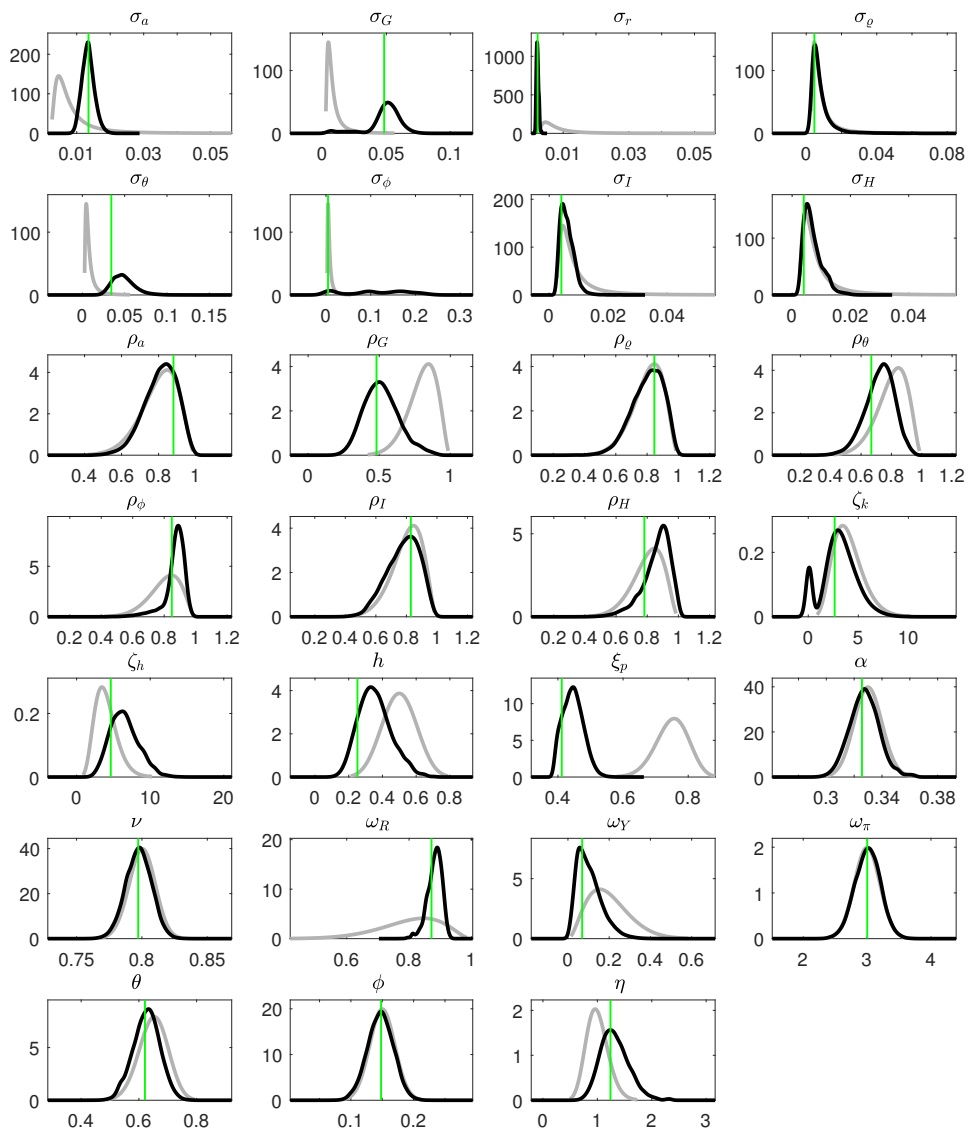
$$[39] \quad \hat{\phi}_t = \rho_\phi\hat{\phi}_{t-1} - \varepsilon_{\phi,t}$$

$$[40] \quad \hat{G}_t = \rho_G\hat{G}_{t-1} + \varepsilon_{G,t}$$

$$[41] \quad \hat{A}_{I,t} = \rho_I\hat{A}_{I,t-1} + \varepsilon_{I,t}$$

$$[42] \quad \hat{A}_{H,t} = \rho_H\hat{A}_{H,t-1} + \varepsilon_{H,t}$$

4.D Priors and Posteriors of Estimated Parameters



4.E Dynare Code

```

#define bc=0
// if bc=0 Model 1
// if bc=1 Model 2

var
LAMBDA_B // Lagrangian multiplier on borrower budget constraint
LAMBDA_S // Lagrangian multiplier on saver budget constraint
CB // Consumption of borrower
CS // Consumption of saver
C // Aggregate Consumption
THETA // LTV ratio
PHI // Housing preference
Q // House price
HB // Housing stock of borrower
HS // Housing stock of saver
WB // Wage of borrower
WS // Wage of saver
LB // Labour supply of borrower
LS // Labour supply of saver
L // Aggregate Labour supply
MUB // Lagrangian multiplier on borrower borrowing constraint
MUS // Lagrangian multiplier on saver capital accumulation
R // Nominal gross interest rate
AMO // Amortisation rate
DB // Debt of borrower
DS // Debt of saver
NB // Housing investment of borrower
NS // Housing investment of saver
N // Aggregate Housing investment
RK // Capital rental rate
K // Capital stock
IK // Capital investment
F // Investment adjustment cost
Y // Output
IH // Housing investment
PI // Inflation
VP // VP
PS //  $\pi^*$ 
MC // Marginal cost
M // M
Z // Z
TB // Tax for borrower
TS // Tax for saver
G // Government spending
AT // AR(1) technology process
IKS // AR(1) capital investment-specific technology process
IHS // AR(1) housing investment-specific technology process
;

var
Lobs Cobs Plobs Robs Ilobs Kobs Qobs Yobs;

varexo
EPS_AT // technology shock
EPS_AMO // amortisation rate shock
EPS_THETA // LTV ratio shock
EPS_PHI // housing preference shock
EPS_G // fiscal shock
EPS_MS // monetary policy shock
EPS_IK // capital investment-specific technology shock
EPS_IH // housing investment-specific technology shock
;

parameters
alpha // capital share
niu // wage share of borrower
bettab // discount factor of borrower
bettas // discount factor of saver
eta // inverse Frisch elasticity of labour supply
deltak // capital depreciation rate

```

```

deltah          // housing depreciation rate
psi             // population share of borrower
zetak          // capital investment adjustment cost parameter
zetah          // adjustment cost parameter in house production
epsil          // epsilon
xip            // xip
sharetb        // tax share of borrower
shareg         // government spending share in output
or             // degree of smoothing in monetary policy
op            // policy weight on inflation
oy            // policy weight on output
rhoat          // autocorrelation technology shock
rhoamo         // autocorrelation amortisatino shock
rhottheta     // autocorrelation LTV shock
rhophi        // autocorrelation housing preference shock
rhog          // autocorrelation fiscal shock
hf            // habit formation
rhoik         // autocorrelation fiscal shock
rhoih        // autocorrelation fiscal shock
PHIss PIss ATss Qss MUSss Rss PSss VPss THETAss AMOss //steady state values
;

bettas=.998;
bettab=.995;
eta=1;
psi=.657;
alpha=.3;
deltak=.025;
deltah=.005;
zetak=0.2;
zetah=1.3;

niu=.82;
rhoat=.95;
rhoamo=.85;
rhottheta=.85;
rhophi=.85;
rhoik=.85;
rhoih=.85;

or=.7;
op=3.0;
oy=.5;
xip=.62;
epsil=7.67;
rhog=.85;
shareg=.142;
sharetb=.55;
hf=0.42;

// steady state
AMOss=0.001;
PHIss=.12;
PIss=1.006;
ATss=1;
Qss=1;
MUSss=1;
Rss=PIss/bettas;
PSss=1;
VPss=1;
THETAss=0.65;
MSss=0;

model(linear);

%%%%%% STEADY STATE RELATIONSHIPS %%%%%%
@if bc==0
#MUBss=PIss*(1-bettab/bettas)/(PIss-bettab*(1-AMOss));
#else
#MUBss=1-bettab/bettas;
#endif

#RKss=1/bettas-(1-deltak);

@if bc==0
#AUX1=(bettab*PHIss)*(1-hf)/((1-MUBss*THETAss)*(1-(1-deltah)*bettab)*
(1-bettab*hf));
#else
#AUX1=(bettab*PHIss)*(1-hf)/((1-MUBss*THETAss)-(1-deltah)*bettab*(1-bettab*hf));

```

```

@#endif
#AUX2=(bettas*PHIss)*(1-hf)/((1-(1-deltah)*bettas)*(1-bettas*hf));
@#if bc==0
#AUX3=((RKss*(niu*(1-alpha)-sharetb*shareg))/(psi*alppha))/(1+AUX1*deltah
*(1+((THETAAss*Qss)/(1-(1-AMOSS)/PIss))*(Rss/PIss-1)));
@#else
#AUX3=((RKss*(niu*(1-alpha)-sharetb*shareg))/(psi*alppha))/(1+AUX1*Qss*(THETAAss
*(Rss/PIss-1)+deltah));
@#endif

#AUX4=(RKss*(1-shareg)/alpha-deltak-psi*AUX3*(1+deltah*AUX1))/((1-psi)
*(1+deltah*AUX2));
#LBss=((1-hf*bettab)*niu*(1-alpha)*RKss)/(alpha*psi*AUX3*(1-hf))^(1/(eta+1));
#LSss=((1-hf*bettas)*(1-niu)*(1-alpha)*RKss)/(alpha*(1-psi)*AUX4*(1-hf))
^(1/(eta+1));
#MCss=(epsil-1)/epsil;
#Kss=(MCss*alpha/RKss)^(1/(1-alpha))*(psi*LBss)^niu*((1-psi)*LSss)^(1-niu);
#Yss=(RKss*Kss)/alpha;
#CBss=AUX3*Kss;
#CSss=AUX4*Kss;
#LAMBDAAss=(1-bettab*hf)/(CBss*(1-hf));
#LAMBDAAsss=(1-bettas*hf)/(CSss*(1-hf));
#HBss=AUX1*CBss;
#HSss=AUX2*CSss;
#NBss=deltah*HBss;
#NSss=deltah*HSss;
#Nss=(1-psi)*NSss+psi*NBss;
#IHss=Nss;
@#if bc==0
#DBss=(THETAAss*Qss*NBss)/(1-(1-AMOSS)/PIss);
@#else
#DBss=THETAAss*Qss*HBss;
@#endif

#DSss=(psi/(psi-1))*DBss;
#Css=(1-psi)*CSss+psi*CBss;
#Lss=(1-psi)*LSss+psi*LBss;
#Fss=deltak*Kss;
#IKss=Fss;

#FCss=Yss/(epsil-1);
#WBss=(niu*(1-alpha)*Yss)/(psi*LBss);
#WSss=((1-niu)*(1-alpha)*Yss)/((1-psi)*LSss);

#Mss=LAMBDAAsss*Yss/(1-bettas*xip);
#Zss=LAMBDAAsss*Yss/(1-bettas*xip);
#TBss=sharetb*shareg*Yss/psi;
#Gss=shareg*Yss;
#TSss=(Gss-psi*TBss)/(1-psi);

%%%%%% MODEL %%%%%%%%%
LAMBDAAsss=(-1/((1-hf)*(1-hf*bettab)))*(CB-hf*CB(-1)-hf*bettab*(CB(+1)-hf*CB));

@#if bc==0
Q=MUBss*THETAAss*(MUB+THETA)/(1-MUBss*THETAAss)+(PHI(+1)-LAMBDAAsss*HB)-bettab*(1-deltah)
*(PHI(+1)-LAMBDAAsss(+1)-HB-Q(+1)+MUBss*THETAAss*(MUB(+1)+THETA(+1))/(1-MUBss*THETAAss));
@#else
Q*(1-MUB*THETA)=bettab*(LAMBDAAsss(+1)/LAMBDAAsss)*(PHI(+1)/(LAMBDAAsss(+1)*HB)+(1-deltah)
*Q(+1));
@#endif

eta*LB=WB+LAMBDAAsss;

@#if bc==0
LAMBDAAsss=MUBss*MUB/(1-MUBss)+(Rss/(Rss-(1-AMOSS)*MUBss))*(LAMBDAAsss(+1)+R-PI(+1))
-(((1-AMOSS)*MUBss)/(Rss-(1-AMOSS)*MUBss))*(LAMBDAAsss(+1)-PI(+1)+MUB(+1)-AMOSS
*AMO/(1-AMOSS));
@#else
LAMBDAAsss*(1-MUB)=bettab*LAMBDAAsss(+1)*R/PI(+1);
@#endif

CBss*CB+NBss*(Q+NB)+(Rss*DBss/PIss)*(R(-1)+DB(-1)-PI)=WBss*LBss*(WB+LB)-TBss*TB
+DBss*DB;

@#if bc==0
DB=((1-AMOSS)/PIss)*(DB(-1)-PI)-(AMOSS/PIss)*AMO(-1)+(THETAAss*NBss/DBss)
*(THETA+Q+NB);

```



```

@#else
DB=THETA*Q*HB;
@#endif

NB=HB/deltah-HB(-1)*(1-deltah)/deltah;
LAMBDA_S=(-1/((1-hf)*(1-hf*bettas)))*(CS-hf*CS(-1)-hf*bettas*(CS(+1)-hf*CS));
Q=PHI(+1)-LAMBDA_S-HS - bettas*(1-deltah)*(PHI(+1)-LAMBDA_S(+1)-HS-Q(+1));

eta*LS=WS+LAMBDA_S;
LAMBDA_S=(bettas*Rss/PIss)*(R+LAMBDA_S(+1)-PI(+1));
MUS=LAMBDA_S(+1)-LAMBDA_S+RK(+1)-bettas*(1-deltah)*(RK(+1)-MUS(+1));
NS=HS/deltah-HS(-1)*(1-deltah)/deltah;
K=(1-deltah)*K(-1)+Fss*F/Kss;
F=IK+IKS;
MUS+IKS =zetak*(IK-IK(-1))-bettas*zetak*(IK(+1)-IK);
VP=xip*(epsil*(PI-PI(-1))+VP(-1))-(1-xip)*epsil*PS;
Yss*VPss*(Y+VP)/(Yss*VPss+ATss*FCss)=(1-alpha-ATss*FCss/(Yss*VPss+ATss*FCss))*AT
+alpha*K(-1)+(1-alpha)*(niu*LB+(1-niu)*LS);
WB=(VPss*Yss*(MC-LB+VP+Y)+ATss*FCss*(MC-LB+AT))/(VPss*Yss+ATss*FCss);
WS=(VPss*Yss*(MC-LS+VP+Y)+ATss*FCss*(MC-LS+AT))/(VPss*Yss+ATss*FCss);
MC=-(1-alpha)*AT+alpha*RK+(1-alpha)*(niu*WB+(1-niu)*WS);
PS+M=Z;
Z=(1-bettas*xip)*(LAMBDA_S+MC+Y)+bettas*xip*((1-epsil)*(PI-PI(+1))+Z(+1));
M=(1-bettas*xip)*(LAMBDA_S+Y)+bettas*xip*(epsil*(PI(+1)-PI)+M(+1));
PS=xip*(PI-PI(-1))/((1-xip)*PSss^(1-epsil));
Q+IHS=zetak*(IH-IH(-1))-bettas*zetak*(IH(+1)-IH);
N=IH+IHS;
TB=Y;
Gss=G=psi*TBss*TB+(1-psi)*TSss*TS;
R=or*R(-1)+(1-or)*(op*PI+oy*Y)+EPS_MS;
DB=DS;
Nss*N=(1-psi)*NSss*NS+psi*NBss*NB;
Css*C=(1-psi)*CSss*CS+psi*CBss*CB;
Lss*L=(1-psi)*LSss*LS+psi*LBss*LB;
Y*Yss=C*Css+IH*IHss+IK*IKss+G*Gss;
AT=rhoat*AT(-1)+EPS_AT;
AMO=rhoamo*AMO(-1)+EPS_AMO;
THETA=rhotheta*THETA(-1)-EPS_THETA;
PHI=rhophi*PHI(-1)-EPS_PHI;
G=rhogog*G(-1)+EPS_G;
IKS=rhoik*IKS(-1)+EPS_IK;
IHS=rhoih*IHS(-1)+EPS_IH;

%%%%%% Measurment equations %%%%%%
Cobs=C;
Yobs=Y;
Ihobs=IH;
Ikobs=IK;
Piobs=PI;
Robs=R;
Qobs=Q;
Lobs=L;

end;

resid(1);
steady;
check;

%%%%%% Bayesian Estimation and Simulation %%%%%%
estimated_params;

stderr EPS_AT, INV_GAMMA_PDF, 0.01, 2.00;
stderr EPS_G, INV_GAMMA_PDF, 0.01, 2.00;
stderr EPS_MS, INV_GAMMA_PDF, 0.01, 2.00;
stderr EPS_AMO, INV_GAMMA_PDF, 0.01, 2.00;
stderr EPS_THETA, INV_GAMMA_PDF, 0.01, 2.00;
stderr EPS_PHI, INV_GAMMA_PDF, 0.01, 2.00;
stderr EPS_IK, INV_GAMMA_PDF, 0.01, 2.00;
stderr EPS_IH, INV_GAMMA_PDF, 0.01, 2.00;
stderr Qobs, INV_GAMMA_PDF, 0.01, 1;
stderr Ihobs, INV_GAMMA_PDF, 0.01, 1;
stderr Ikobs, INV_GAMMA_PDF, 0.01, 1;
stderr Cobs, INV_GAMMA_PDF, 0.01, 0.1;
stderr Lobs, INV_GAMMA_PDF, 0.01, 0.1;

rhoat, BETA_PDF, 0.80, 0.10;
rhog, BETA_PDF, 0.80, 0.10;

```

```
rhoamo,          BETA_PDF,      0.80, 0.10;
rhotheta,        BETA_PDF,      0.80, 0.10;
rhophi,          BETA_PDF,      0.80, 0.10;
rhoik,           BETA_PDF,      0.80, 0.10;
rhoih,           BETA_PDF,      0.80, 0.10;
zetak,           GAMMA_PDF,     4.00, 1.50;
zetah,           GAMMA_PDF,     4.00, 1.50;
hf,              BETA_PDF,      0.50, 0.10;
xip,             BETA_PDF,      0.75, 0.05;
alpha,           BETA_PDF,      0.33, 0.01;
niu,             BETA_PDF,      0.80, 0.01;
or,              BETA_PDF,      0.80, 0.10;
oy,              BETA_PDF,      0.20, 0.10;
op,              NORMAL_PDF,    3.00, 0.20;
THETAss,         BETA_PDF,      0.65, 0.05;
PHIss,           NORMAL_PDF,    0.15, 0.02;
eta,             GAMMA_PDF,     1.00, 0.20;

end;

%%%%%% Declaration of observable variables %%%%%%
varobs Lobs Cobs Piobs Robs Ihobs Ikobs Qobs Yobs;

options_.plot_priors=0;
estimation(datafile=kordata,mode_compute=4,prefilter=0,mh_replic=500000
,presample=4,mh_nblocks=2,mh_jscale=0.40,mh_drop=0.2,tex);
shock_decomposition (parameter_set=posterior_mode) Lobs Cobs Piobs Robs
Ihobs Ikobs Qobs Yobs;

options_.pruning=1;
stoch_simul(periods=1000,hp_filter=1600, order = 2,irf=100);
```

Chapter 5

Welfare Effects of Macroprudential Policy in South Korea

5.1 Introduction

The recent experience of the US Great Recession and fast growing household debt made South Korean government pay more attention to preventing financial instability by using macroprudential policy. As it is widely suggested that borrowers with collateral constraint lost more of their welfare than savers by the US Great Recession (Hur, 2016; Menno and Oliviero, 2016), pre-emptive macroprudential policies such as lowering LTV ratio and increasing amortisation rate may not equally affect the welfare of borrower and saver. If borrowers and savers are not equally affected by macroprudential policy, policymakers should estimate how and how much they are affected differently before conducting macroprudential policy and understand its distributional effects between households. However, it is not easy to quantify welfare effects of macroprudential policy because its transmission mechanism has not been clearly analysed on the basis of general equilibrium models and findings by some literature do not conform with our intuition. For example, Campbell and Hercowitz (2009) show increasing LTV ratio and lowering amortisation rate could worsen borrower's welfare while saver's welfare always improves, which is opposite to our intuition. Furthermore, once welfare effects are estimated, it needs to be checked whether there is room for Pareto-improving policy. Although some literature (Lambertini et al., 2013; Rubio and Carrasco-Gallego, 2014)

find room for social welfare gains or Pareto-superior outcomes by adopting optimal macroprudential policy rule, it still needs to be checked if the same results can be obtained for the South Korean economy.

Firstly, effects of discretionary countercyclical macroprudential policy are analysed. In South Korea, macroprudential policy has been conducted by government in discretionary way rather than in rule-based way as in many other countries. Thus, estimating effects of discretionary policy is more crucial and practical than finding optimal macroprudential rule. Results suggest that social welfare cannot help but decrease by discretionary credit tightening policy with given parameter values in South Korea. Only borrower can get welfare gain. Thus, Pareto-improving credit tightening is not possible. However, it is also found that credit loosening policy, which can be conducted when the financial conditions need to be improved, can increase social welfare considerably. Increasing amortisation rate could be better in credit tightening and increasing LTV ratio could be better in credit loosening. Next, optimal countercyclical macroprudential rules are found when there is only LTV ratio rule, when there is only amortisation rate rule and when there is a mixture of two rules. Results suggest that the most effective rule is the mixture of LTV ratio and amortisation rate rules.

The rest of this chapter is structural as follows. Section 5.2 introduces key literature. Section 5.3 explains the welfare measures used for analysis. Section 5.4 checks necessary conditions for welfare gains by discretionary macroprudential policy. Welfare effects of discretionary macroprudential policy are estimated in section 5.5. Optimal macroprudential rule is shown in section 5.6. Finally, conclusion is presented in section 5.7.

5.2 Literature Review

Since Kim and Kim (2003) show that welfare analysis cannot be performed correctly by the first-order approximation, many studies perform welfare analysis by using the proposed second-order approximation. Schmitt-Grohé and Uribe (2004) derive a practical solution method for the second-order approximation to the policy function of dynamic models. Schmitt-Grohé and Uribe (2007) use the same second-order approximation

to find out optimal monetary and fiscal policy rules which maximise welfare. In estimating welfare measures, this chapter follows Schmitt-Grohé and Uribe (2007). As Chen and Columba (2016) explain, model's equilibrium conditions are obtained by the second-order approximation and then welfare measures are estimated by simulating the model. The estimated parameter values in chapter 4 are used in this work.

Welfare effects of discretionary demand-side macroprudential policy in borrower-saver model are first documented by Campbell and Hercowitz (2009). Campbell and Hercowitz (2009) find that relaxing borrower's collateral constraint makes borrower's welfare fall by the dominant indirect effects of endogenous interest rate and other relative price changes despite of positive direct effect of credit loosening. Although this result is counter-intuitive, following research such as Chen and Columba (2016) and Rubio and Carrasco-Gallego (2014) also shows the similar results. In contrast, Mendicino et al. (2012) find that higher LTV ratio increases social welfare and lower LTV ratio decreases social welfare.

There are recent studies that find optimal demand-side macroprudential rule. Lambertini et al. (2013) shows optimal countercyclical LTV ratio rule against credit growth can improve social welfare. Rubio and Carrasco-Gallego (2014) also find the optimal parameter value of the LTV ratio rule which can improve social welfare.

5.3 Welfare Measure

As pointed out in Kim and Kim (2003), a second-order approximation has to be used solving the model in order to get correct results for welfare analysis. As in Rubio and Carrasco-Gallego (2014) and Schmitt-Grohé and Uribe (2007), the welfare of two types of households as well as social welfare is evaluated using a second-order approximation. The welfare for borrower and saver are as follows:

$$W_{b,t} = E_0 \sum_{j=0}^{\infty} \beta_b^j U_{b,t+j} \quad (5.3.1)$$

$$W_{s,t} = E_0 \sum_{j=0}^{\infty} \beta_s^j U_{s,t+j} \quad (5.3.2)$$

where

$$U_{b,t+j} = \ln (C_{b,t+j} - hC_{b,t+j-1}) + \phi_{t+j} \ln H_{b,t+j-1} - \frac{L_{b,t+j}^{1+\eta}}{1+\eta} \quad (5.3.3)$$

$$U_{s,t+j} = \ln (C_{s,t+j} - hC_{s,t+j-1}) + \phi_{t+j} \ln H_{s,t+j-1} - \frac{L_{s,t+j}^{1+\eta}}{1+\eta} \quad (5.3.4)$$

Equation 5.3.1 and Equation 5.3.2 can be written recursively as follows.

$$W_{b,t} = U_{b,t} + \beta_b W_{b,t+1} \quad (5.3.5)$$

$$W_{s,t} = U_{s,t} + \beta_s W_{s,t+1} \quad (5.3.6)$$

The value of welfare is based on the utility function which is not cardinal. To make this value intuitive, it needs to be converted in consumption equivalent unit as in Ascari and Ropele (2012). Specifically the difference between new steady-state welfare and old steady-state welfare is converted as follows.

$$W_b^{old} = \frac{1}{\beta_b} \left[\ln \{(1-h)C_b^{old}\} + \phi^{old} \ln H_b^{old} - \frac{L_b^{old^{1+\eta}}}{1+\eta} \right]$$

$$W_s^{old} = \frac{1}{\beta_s} \left[\ln \{(1-h)C_s^{old}\} + \phi^{old} \ln H_s^{old} - \frac{L_s^{old^{1+\eta}}}{1+\eta} \right]$$

where superscript *old* means value in the initial steady-state. If the welfare in new steady-state is higher than in initial steady-state, households should consume more of the constant fraction of non-durable good consumption in initial steady-state, CE_b and CE_s , respectively, in order to obtain the level of welfare in new steady-state. So positive values of CE_b and CE_s mean welfare gains from initial steady-state to new steady-state.¹ The welfare in new steady-state can be written as

$$W_b^{new} = \frac{1}{\beta_b} \left[\ln \{(1-h)(1+CE_b)C_b^{old}\} + \phi^{old} \ln H_b^{old} - \frac{L_b^{old^{1+\eta}}}{1+\eta} \right]$$

$$W_s^{new} = \frac{1}{\beta_s} \left[\ln \{(1-h)(1+CE_s)C_s^{old}\} + \phi^{old} \ln H_s^{old} - \frac{L_s^{old^{1+\eta}}}{1+\eta} \right]$$

Then, we get

$$CE_b = \exp[(1-\beta_b)(W_b^{new} - W_b^{old})] - 1 \quad (5.3.7)$$

$$CE_s = \exp[(1-\beta_s)(W_s^{new} - W_s^{old})] - 1 \quad (5.3.8)$$

¹ Thus, welfare analysis in this chapter is limited to welfare at steady-state.

As we define, CE_b and CE_s indicate the fraction of each household's own consumption in initial steady state. Thus, if we convert the sum of CE_b and CE_s to the fraction of aggregate consumption in old steady state, we get aggregate (social) welfare gains, CE , as follows.

$$\begin{aligned} (1 + CE)C^{old} &= \psi(1 + CE_b)C_b^{old} + (1 - \psi)(1 + CE_s)C_s^{old} \\ CE &= \frac{\psi CE_b C_b^{old} + (1 - \psi) CE_s C_s^{old}}{C^{old}} \end{aligned} \quad (5.3.9)$$

where aggregate consumption is a population weighted sum of each household's consumption as we define in the previous chapters. We can interpret CE as aggregate (social) welfare gains in terms of aggregate non-durable good consumption equivalent.²

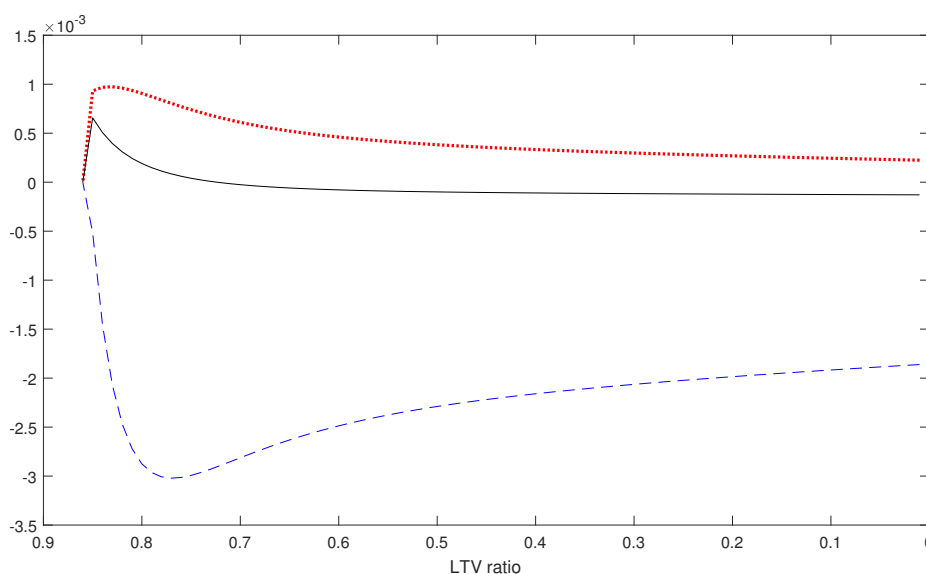
5.4 Necessary Conditions for Welfare Gains by Discretionary Macroprudential Policy

Although recent literature on macroprudential policy try to find an optimal rule under some assumptions, macroprudential policy in South Korea is not performed by rules but by government's discretion. However, their welfare effects have not been analysed based on general equilibrium models. In this section, possibility and necessary conditions for welfare gains by discretionary macroprudential policies are shown.

Figure 5.1 illustrates welfare gains along with decremental LTV ratios in consumption equivalent units. When LTV ratio decreases, saver's welfare gain is always negative and borrower's welfare gain is always positive. Social welfare gains start from positive and turn into negative when LTV ratio reaches around 0.72. Therefore, only when the initial LTV ratio is higher than 0.72, pre-emptive macroprudential policy of lowering LTV ratio can attain social welfare gains. If the initial LTV ratio is 0.626 as calibrated in chapter 4, there is no room for social welfare gains or Pareto-superior outcomes by lowering LTV ratio and only borrower can get welfare gain.

²Social welfare is usually defined as a weighted sum of each household's welfare (Pescatori, Mendicino, et al., 2005; Rubio and Carrasco-Gallego, 2014). However, as Mendicino et al. (2015) point out, there is no commonly accepted weights assigned to each household. Using aggregate consumption equivalent CE as social welfare gains is in line with a definition of aggregate consumption in the model and does not require any additional assumption for social welfare gains.

Figure 5.1: Welfare gains by decreasing LTV ratio



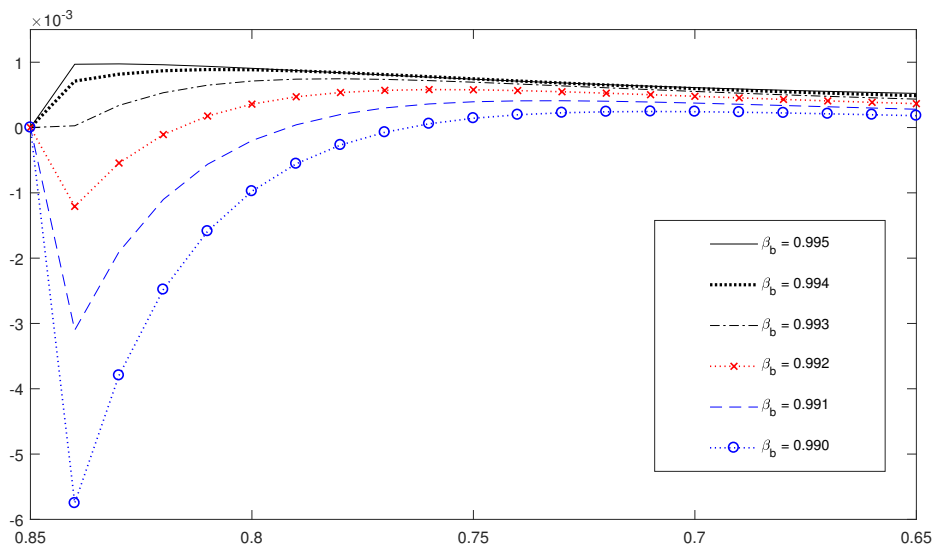
Notes: 1. LTV ratio decreases by 0.1 starting from 0.85. 2. Welfare gain (loss) = welfare with the new LTV ratio – welfare with the previous LTV ratio. 3. Solid lines: Social welfare gains, Dotted lines: Borrower's welfare gain, Dashed lines: Saver's welfare gain

Furthermore, an additional experiment is performed to check robustness of borrower's welfare gain when LTV ratio lowers. By lowering borrower's discount factor β_b starting from the calibrated value 0.995 while the value of β_s is fixed at 0.998, borrower's welfare gains are estimated when LTV ratio decreases.³ Because the result contradicts our intuition that tightening credit conditions reduces borrower's welfare as Campbell and Hercowitz (2009) mention. As we see in Figure 5.2 borrower's welfare gains are not always positive when LTV ratio decreases. If β_b is less than 0.993, borrower's welfare gain could be negative when LTV ratio reduces. So, the value of β_b seems to affect the dynamics of borrower's welfare gain. However, even when β_b is below 0.990, for example, 0.975, borrower's welfare gain can be always positive if the value of β_s is low enough, for example, 0.985. Therefore, it can be said that the dynamics of borrower's welfare gain rely on the gap between β_b and β_s rather than the value of β_b . When the gap is narrow enough, borrower's welfare gain is always positive when LTV ratio lowers as in Figure 5.1. When the gap is too wide, borrower's welfare gain may turn to be negative when LTV ratio lowers.

³Campbell and Hercowitz (2009) conclude welfare effects in their analysis do not rely on the value of β_b but their conclusion is based on results from just two alternative values.

This result has two implications. One is that when real interest rate is very low, which means β_s is very high, but β_b is very low, credit tightening may harm borrower's welfare much more than in any other times. On the contrary, credit loosening at that time can benefit borrower more than in any other times. The other implication is that when real interest rate is relatively high, which means low β_s , but β_b is very high, pre-emptive macroprudential policy such as lowering LTV ratio can improve at least borrower's welfare. The best timing of pre-emptive macroprudential policy in terms of minimum social welfare loss could be when the discount rate gap between two types of household is the lowest.

Figure 5.2: Borrower's welfare gains by decreasing LTV ratio with different discount factors

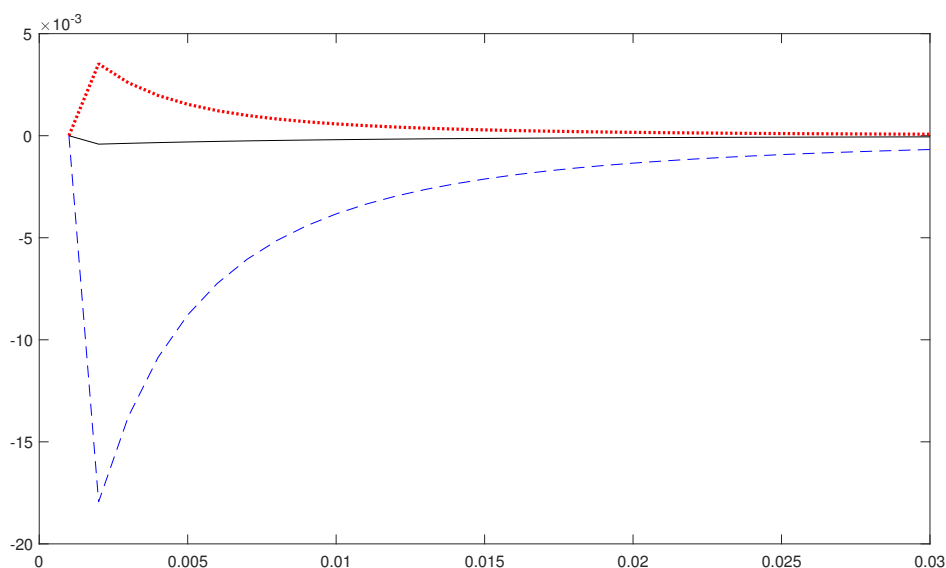


Notes: 1. LTV ratio decreases by 0.1 starting from 0.85. 2. Welfare gain (loss) = welfare with the new LTV ratio – welfare with the previous LTV ratio.

When amortisation rate increases, it is also impossible to get social welfare gains as shown in Figure 5.3. Only borrower can get welfare gain. As we expect, the dynamics of borrower's welfare gain rely on β_b as shown in Figure 5.4. If β_b is less than 0.975, borrower's welfare gain is always negative when amortisation rate rises. More precisely speaking, the dynamics of borrower's welfare gain depends on the gap between β_b and β_s as in LTV ratio case. Even when β_b is 0.970, borrower's welfare gain can be always positive if the value of β_s is low enough, for example, 0.985.

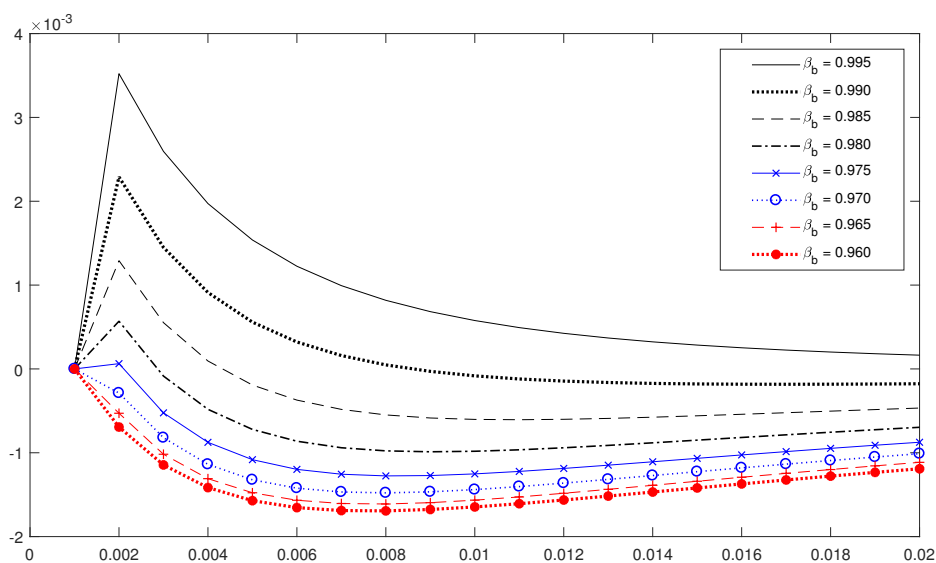
Given the calibrated values of β_b , β_s and ϱ , and estimated value of θ in chapter 4, we cannot attain social welfare gains but can attain only borrower's welfare gain when

Figure 5.3: Welfare gains by increasing amortisation rate



Notes: 1. Amortisation rate increases by 0.001 starting from 0.001. 2. Welfare gain (loss) = welfare with the new amortisation rate – welfare with the previous amortisation rate. 3. Solid lines: Social welfare gains, Dotted lines: Borrower's welfare gain, Dashed lines: Saver's welfare gain

Figure 5.4: Borrower's welfare gains by increasing amortisation rate with different discount factors



Notes: 1. Amortisation rate increases by 0.001 starting from 0.001. 2. Welfare gain (loss) = welfare with the new amortisation rate – welfare with the previous amortisation rate.

LTV ratio lowers to less than 0.626 or amortisation rate increase to higher than 0.001. Therefore, what we can do is finding a measure which can minimise social welfare loss when macroprudential policy is based on policymakers' discretion.

5.5 Welfare Effects of Discretionary Macroprudential Policy

In this section, welfare effects of different macroprudential policies are compared when they are conducted by discretion. Welfare effects of two macroprudential measures (changes in LTV ratio and amortisation rate) and a mixture of two measure (changes in LTV ratio and amortisation rate at the same time) are compared when these three measures attain the same goal in two different cases. The first case is when discount rate gap between borrower and saver is narrow as calibrated in the previous chapters. In this case, the goal of policy measures is set to be the same amount of reduction in household debt to output ratio (-9.7521%p).⁴ The second case is when discount rate gap between borrower and saver is wide. The second case is set by lowering β_b to 0.975 with the same value of β_s . In this case, the goal of two measures is set to be the same amount of increase in household debt to output ratio (10.87%p).⁵

5.5.1 Credit tightening when discount rate gap is narrow

When discount rate gap is narrow, three measures have very similar welfare effects as we see in Table 5.1. Their effects on social welfare show little differences. Increasing amortisation rate has marginally more negative effect on saver's welfare and more

⁴This value comes from the amount of reduction in household debt to output ratio when LTV ratio is lowered by 0.5 from 0.65 (initial steady-state) to 0.60 (new steady-state). The same amount of reduction in household debt to output ratio is attained by increasing amortisation by 0.00067935 from 0.001 to 0.00167935. The policy mix is a change in LTV ratio from 0.65 to 0.6256027 and a change in amortisation rate from 0.001 to 0.001343755.

⁵This value comes from the amount of increase in household debt to output ratio when LTV ratio is increased by 0.5 from 0.60 (initial steady-state) to 0.65 (new steady-state) while amortisation rate is set to be 0.002. The same amount of increase in household debt to output ratio is attained by decreasing amortisation rate by 0.00091726 from 0.002 to 0.00108274. The policy mix is a change in LTV ratio from 0.60 to 0.6257613 and a change in amortisation rate from 0.002 to 0.00154901

positive effect on borrower and social welfare although the overall differences are very small. In terms of gap between borrower's welfare gain and saver's welfare loss, lowering LTV ratio makes the least gap between two types of households' welfare. Decomposed effects in Table 5.2 show increasing amortisation rate reduces borrower's welfare less in housing and increases it less in hours worked while welfare gains in consumption have little differences among three measures. Therefore, if macroprudential policy has more weight on borrower's welfare gain or minimum social welfare loss, increasing amortisation rate is the most effective among three. However, if macroprudential policy has more weight on balanced welfare effects, lowering LTV ratio is the most effective.

Table 5.1: Comparing welfare effects of two macroprudential policies (Case 1)

| Measure | Welfare gains | | |
|------------------------------|---------------|---------|---------|
| | Borrower | Saver | Social |
| Lowering LTV ratio | 0.024 | -0.0126 | -0.0004 |
| Increasing amortisation rate | 0.027 | -0.0135 | -0.0003 |
| Policy mix | 0.026 | -0.0130 | -0.0003 |

Table 5.2: Decomposition of household's welfare gains (Case 1)

| | Welfare gains | | | |
|------------------------|---------------|---------|--------------|---------|
| | Consumption | Housing | Hours worked | Total |
| <Lowering LTV ratio> | | | | |
| Borrower | 0.0026 | -0.0020 | 0.0018 | 0.0024 |
| Saver | -0.0071 | -0.0015 | -0.0041 | -0.0126 |
| <Increasing amo. rate> | | | | |
| Borrower | 0.0025 | -0.0013 | 0.0015 | 0.0027 |
| Saver | -0.0076 | -0.0012 | -0.0047 | -0.0135 |
| <Policy mix> | | | | |
| Borrower | 0.0025 | -0.0017 | 0.0017 | 0.0026 |
| Saver | -0.0073 | -0.0014 | -0.0044 | -0.0130 |

5.5.2 Credit loosening when discount rate gap is wide

When discount rate gap is wide, three measures have clearly different welfare effects as we see in Table 5.3. When LTV ratio increases, social and borrower's welfare gains are the largest but saver's welfare loss is the biggest. Decomposed effects in Table 5.4 show increasing LTV ratio raises borrower's welfare mainly in housing and reduces saver's welfare both in consumption and hours worked. When amortisation rate lowers, borrower's welfare show little change but saver's welfare gain is the biggest. Decomposed effects show decreasing amortisation raises saver's welfare mainly in consumption and hours worked. Borrower's tiny welfare loss is the result of trade-off between welfare loss in housing and welfare gains in consumption and hours worked. When two policies are mixed, both type of households attain welfare gains only in housing. Therefore if macroprudential policy has more weight on borrower's and social welfare gains, increasing LTV ratio is the most effective among three. However, if macroprudential policy has more weight on balanced welfare effects, policy mix can be preferred. Although lowering amortisation rate can get the lowest social welfare gains, we can see that it is very close to the Pareto-improving policy because saver can get welfare gain while borrower's welfare loss is negligible.

Table 5.3: Comparing welfare effects of two macroprudential policies (Case 2)

| Measure | Welfare gains | | |
|----------------------------|---------------|---------|--------|
| | Borrower | Saver | Social |
| Increasing LTV ratio | 0.0083 | -0.0208 | 0.0034 |
| Lowering amortisation rate | -0.0001 | 0.0126 | 0.0020 |
| Policy mix | 0.0042 | -0.0033 | 0.0029 |

5.6 Optimal Macroprudential Policy Rule

In this section, it is shown that how optimal countercyclical macroprudential rule can improve social welfare. It is assumed that government follows the following rules which are based on the processes introduced in the previous chapter and similar to a Taylor-

Table 5.4: Decomposition of household's welfare gain (Case 2)

| | Welfare gains | | | |
|------------------------|---------------|---------|--------------|---------|
| | Consumption | Housing | Hours worked | Total |
| <Increasing LTV ratio> | | | | |
| Borrower | 0.0003 | 0.0109 | -0.0029 | 0.0083 |
| Saver | -0.0108 | 0.0010 | -0.0110 | -0.0208 |
| <Lowering amo. rate> | | | | |
| Borrower | 0.0006 | -0.0011 | 0.0004 | -0.0001 |
| Saver | 0.0069 | 0.0016 | 0.0041 | 0.0126 |
| <Policy mix> | | | | |
| Borrower | -0.0011 | 0.0076 | -0.0022 | 0.0042 |
| Saver | -0.0016 | 0.0012 | -0.0029 | -0.0033 |

type monetary policy rule.

$$\theta_t = \theta_{t-1}^{\rho_\theta} \left[\theta \left(\frac{x_t}{x_{t-1}} \right)^{-\chi_\theta} \right]^{1-\rho_\theta} \exp(-\varepsilon_{\theta,t}) \quad (5.6.1)$$

where $x_t = \{d_{b,t}, D2Y_t, q_t\}$, $D2Y_t$ is household debt to output ratio at time t and χ_θ is the parameter which measures the response to household debt related indicator x_t .

$$\varrho_t = \varrho_{t-1}^{\rho_\varrho} \left[\varrho \left(\frac{x_t}{x_{t-1}} \right)^{\chi_\varrho} \right]^{1-\rho_\varrho} \exp(\varepsilon_{\varrho,t}) \quad (5.6.2)$$

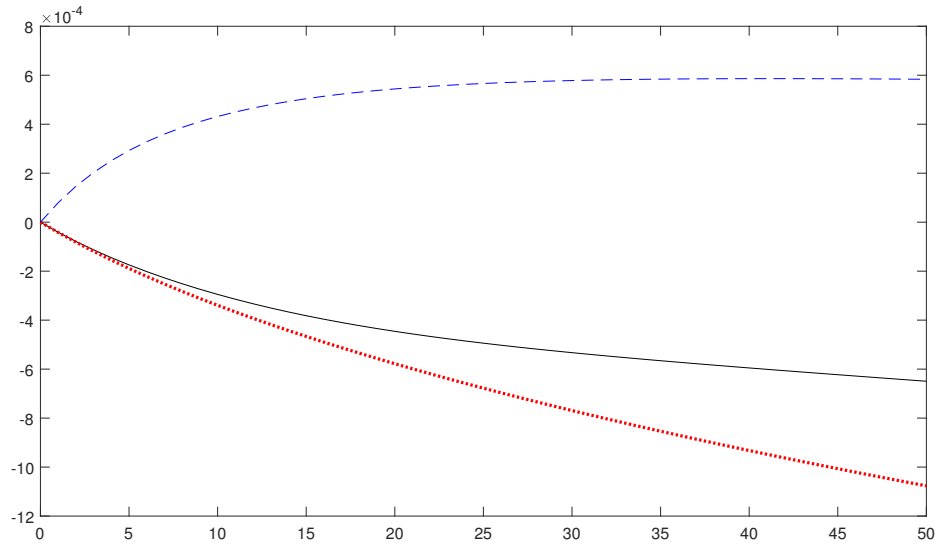
where $x_t = \{d_{b,t}, D2Y_t, q_t\}$, and χ_ϱ is the parameter which measures the response to household debt related indicator x_t .

For this experiment, the steady-state value of amortisation is set at 0.005 instead of 0.001. The calibrated value of 0.001 in chapter 4 is based on the fact that more than 90% of mortgage was interest-only type before 2010 in South Korea. However, interest-only type mortgage is prohibited since 2016 so that current average amortisation rate should be higher than 0.001. The rest of parameter values are the same as in chapter 4.

Figure 5.5 shows social welfare gains under the three alternative LTV ratio rules for the value of χ_θ from zero to 50. Only the LTV ratio rule which responds to the house price (q_t) attains social welfare gains. Social welfare gains are maximised when χ_θ is 41. However, this rule cannot make Pareto-superior outcomes as shown in Table 5.5.

Figure 5.6 describes social welfare gains under the three alternative amortisation rate rules for the value of χ_ϱ from zero to 50. All three amortisation rate rules attains

Figure 5.5: Welfare gains with LTV ratio rule



Notes: 1. χ_θ increases by 1 starting from zero. 2. Welfare gain (loss) = welfare with the new χ_θ – welfare with the initial $\chi_\theta (= 0)$. 3. Solid lines: $x_t = d_{b,t}$, Dotted lines: $x_t = D2Y_t$, Dashed lines: $x_t = q_t$

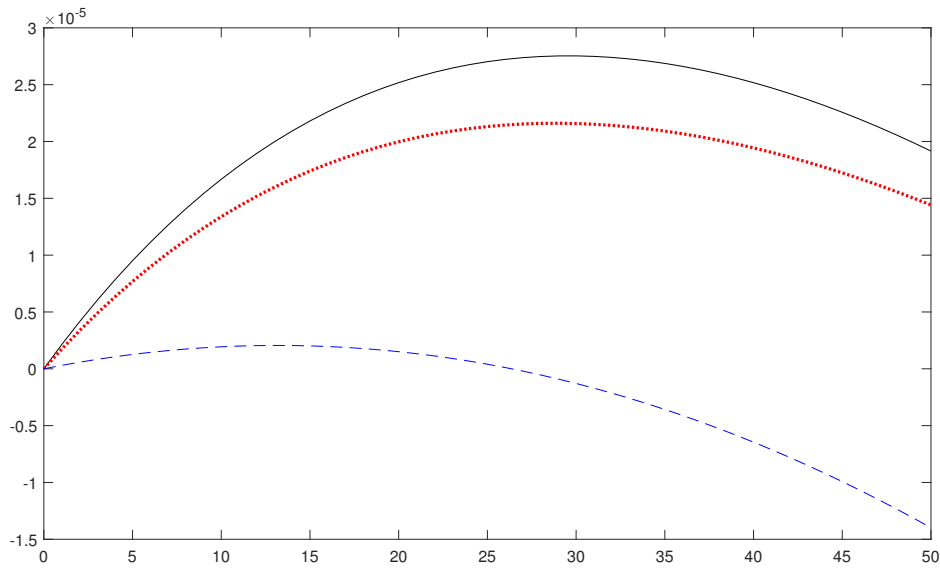
social welfare gains but the one which responds to household debt ($d_{b,t}$) can attain the highest social welfare gain when χ_θ is 30. However, this rule cannot make Pareto-superior outcomes as shown in Table 5.5. The other two rules also cannot attain Pareto improvement.

When LTV ratio rule and amortisation rate rule are mixed, the maximum social welfare gains are slightly higher than when there is only LTV ratio rule as shown in Table 5.5. The optimal parameter values are $\chi_\theta = 40$ when LTV ratio rule is against house price and $\chi_\theta = 11$ when amortisation rate rule is against household debt. There is no Pareto improving rule.

Table 5.5: Welfare gains by optimal macroprudential rules

| Rule | Welfare gains | | |
|--|---------------|---------|----------|
| | Borrower | Saver | Social |
| LTV ratio rule ($x_t = q_t$ and $\chi_\theta = 40$) | | | |
| + amortisation rule ($x_t = d_{b,t}$ and $\chi_\theta = 11$) | -0.00088 | 0.0074 | 0.000589 |
| LTV ratio rule only ($x_t = q_t$ and $\chi_\theta = 41$) | -0.00094 | 0.0076 | 0.000586 |
| Amortisation rate rule only ($x_t = d_{b,t}$ and $\chi_\theta = 30$) | 0.00027 | -0.0011 | 0.000028 |

Figure 5.6: Welfare gains with amortisation rate rule



Notes: 1. χ_θ increases by 1 starting from zero. 2. Welfare gain (loss) = welfare with the new χ_θ - welfare with the initial $\chi_\theta (= 0)$. 3. Solid lines: $x_t = d_{b,t}$, Dotted lines: $x_t = D2Y_t$, Dashed lines: $x_t = q_t$

5.7 Conclusion

In this chapter, welfare effects of macroprudential policy are examined on the basis of the model built in the previous chapter. It is found that discretionary macroprudential policies in South Korea cannot make social welfare gains because social welfare gains are possible only when the initial LTV ratio is higher than 0.72. guerrieri2017collateral $\chi_\theta = 41$ and $\chi_\theta = 30$.

These results imply that discretionary credit tightening policy cannot attain social welfare gains in a situation given in South Korea. So what government can do with discretionary credit tightening policy is to minimise social welfare losses and the best measure for that is increasing amortisation rate. If government adopts countercyclical macroprudential rule, it is possible to improve social welfare but it requires welfare loss either of borrower or saver. When the best rule for maximum social welfare gains is adopted, borrower's welfare loss is inevitable.

Major limitation of this analysis is that welfare measure does not provide information on the transition path. Welfare measure in this chapter can only provide information on the change in welfare from the initial steady-state to the new steady-state. More general welfare analysis which can include welfare changes over the trajectory is

left for the further future research.

Appendix

5.A Dynare Code

```
var
LAMBDA_B // Lagrangian multiplier on borrower budget constraint
LAMBDA_S // Lagrangian multiplier on saver budget constraint
CB // Consumption of borrower
CS // Consumption of saver
C // Aggregate Consumption
THETA // LTV ratio
PHI // Housing preference
Q // House price
HB // Housing stock of borrower
HS // Housing stock of saver
WB // Wage of borrower
WS // Wage of saver
LB // Labour supply of borrower
LS // Labour supply of saver
L // Aggregate Labour supply
MUB // Lagrangian multiplier on borrower borrowing constraint
MUS // Lagrangian multiplier on saver capital accumulation
R // Nominal gross interest rate
AMO // Amortisation rate
DB // Debt of borrower
DS // Debt of saver
NB // Housing investment of borrower
NS // Housing investment of saver
N // Aggregate Housing investment
RK // Capital rental rate
K // Capital stock
IK // Capital investment
F // Investment adjustment cost
Y // Output
IH // Housing investment
PI // Inflation
VP // VP
PS //  $\pi^*$ 
MC // Marginal cost
M // M
Z // Z
TB // Tax for borrower
TS // Tax for saver
```

```

G          // Government spending
FC          // Fixed Cost
AT          // AR(1) technology process
IKS        // AR(1) capital investment-specific technology process
IHS        // AR(1) housing investment-specific technology process
UTB        // Borrower Utility
UTS        // Saver Utility
VB         // Borrower Expected Utility
VS         // Saver Expected Utility
;

varexo
EPS_AT     // technology shock
EPS_AMO    // amortisation rate shock
EPS_THETA  // LTV ratio shock
EPS_PHI    // housing preference shock
EPS_G      // fiscal shock
EPS_MS     // monetary policy shock
EPS_IK     // capital investment-specific technology shock
EPS_IH     // housing investment-specific technology shock
;

parameters
alpha      // capital share
niu        // wage share of borrower
bettab    // discount factor of borrower
bettas    // discount factor of saver
eta       // inverse Frisch elasticity of labour supply
deltak    // capital depreciation rate
deltah    // housing depreciation rate
psi       // population share of borrower
zetak     // capital investment adjustment cost parameter
zeta      // adjustment cost parameter in house production
epsil     // epsilon
xip       // xip
sharetb   // tax share of borrower
shareg    // government spending share in output
or        // degree of smoothing in monetary policy
op        // policy weight on inflation
oy        // policy weight on output
rhoat     // autocorrelation technology shock
rhoamo    // autocorrelation amortisatino shock
rhotheta  // autocorrelation LTV shock
rhophi    // autocorrelation housing preference shock
rhog      // autocorrelation fiscal shock
hf        // habit formation
rhoik     // autocorrelation fiscal shock
rhoih     // autocorrelation fiscal shock
;

bettas=.998;
bettab=.995;
psi=.657;
deltak=.025;
deltah=.005;
epsil=7.67;
shareg=.142;

```

```

sharetb=.55;
rhoat=.8093;
rhog=.5148;
rhoamo=.8000;
rhotheta=.7221;
rhophi=.8674;
rhoik=.7762;
rhoih=.8582;
zetak=3.0859;
zetah=6.4665;
hf=0.3543;
xip=.4465;
alppha=.3273;
niu=.7974;
or=.8817;
oy=.1034;
op=3.0049;
eta=1.3158;

model;
LAMBDA=1/(CB-hf*CB(-1))-bettab*hf/(CB(+1)-hf*CB);
Q*(1-MUB*THETA)=bettab*(LAMBDA(+1)/LAMBDA)*(PHI(+1)/(LAMBDA(+1)*HB)+(1-deltah)
*Q(+1)*(1-MUB(+1)*THETA(+1)));
WB=LB^eta/LAMBDA;
LAMBDA*(1-MUB)=bettab*LAMBDA(+1)*(R-(1-AMO)*MUB(+1))/PI(+1);
CB+Q*NB+R(-1)*DB(-1)/PI=WB*LB+DB-TB;
DB=(1-AMO(-1))*DB(-1)/PI+THETA*Q*NB;
NB=HB-(1-deltah)*HB(-1);
LAMBDA=1/(CS-hf*CS(-1))-bettas*hf/(CS(+1)-hf*CS);
Q=bettas*(LAMBDA(+1)/LAMBDA)*(PHI(+1)/(LAMBDA(+1)*HS)+(1-deltah)*Q(+1));
WS=LS^eta/LAMBDA;
LAMBDA=bettas*LAMBDA(+1)*R/PI(+1);
MUS=bettas*(LAMBDA(+1)/LAMBDA)*(RK(+1)+(1-deltak)*MUS(+1));
NS=HS-(1-deltah)*HS(-1);
K=(1-deltak)*K(-1)+F;
F=IKS*(1-zetak*(1/2)*(IK/IK(-1)-1)^2)*IK;
1=MUS*IKS*(1-zetak*(1/2)*(IK/IK(-1)-1)^2-zetak*(IK/IK(-1)-1)*(IK/IK(-1)))+bettas
*IKS(+1)*(LAMBDA(+1)/LAMBDA)*MUS(+1)*zetak*(IK(+1)/IK-1)*(IK(+1)/IK)^2;
VP=xip*(PI(-1)/PI)^(-epsil)*VP(-1)+(1-xip)*PS^(-epsil);
FC=steady_state(FC);
Y*VP+AT*FC=AT^(1-alppha)*K(-1)^alppha*((psi*LB)^niu*((1-psi)*LS)^(1-niu))^
(1-alppha);MC*niu*(1-alppha)*(VP*Y+AT*FC)*LB^(-1)=psi*WB;
MC*(1-niu)*(1-alppha)*(VP*Y+AT*FC)*LS^(-1)=(1-psi)*WS;
MC=(1/(1-alppha))^(1-alppha)*(1/alppha)^alppha*(1/AT)^(1-alppha)*RK^alppha*(((1/niu)
*WB)^niu*((1/(1-niu))*WS)^(1-niu))^(1-alppha);
PS*M=Z;
Z=(epsil/(epsil-1))*LAMBDA*MC*Y+bettas*xip*(PI/PI(+1))^(1-epsil)*Z(+1);
M=LAMBDA*Y+bettas*xip*(PI/PI(+1))^(1-epsil)*M(+1);
1=xip*(PI(-1)/PI)^(1-epsil)+(1-xip)*PS^(1-epsil);
1=Q*IHS*(1-zetah*(1/2)*(IH/IH(-1)-1)^2-zetah*(IH/IH(-1)-1)*(IH/IH(-1)))+bettas
*IHS(+1)*(LAMBDA(+1)/LAMBDA)*Q(+1)*zetah*(IH(+1)/IH-1)*(IH(+1)/IH)^2;
N=IHS*(1-zetah*(1/2)*(IH/IH(-1)-1)^2)*IH;
psi*TB=sharetb*shareg*Y;
G=psi*TB+(1-psi)*TS;
R/steady_state(R)=(R(-1)/steady_state(R))^or*((PI/steady_state(PI))^op
*(Y/steady_state(Y))^oy)^(1-or)*exp(EPS_MS);
psi*DB+(1-psi)*DS=0;

```

```

N=(1-psi)*NS+psi*NB;
C=(1-psi)*CS+psi*CB;
L=(1-psi)*LS+psi*LB;
Y=C+IH+IK+G;
ln(AT)=rhoat*ln(AT(-1))+EPS_AT;
ln(AMO)=rhoamo*ln(AMO(-1))+(1-rhoamo)*ln(steady_state(AMO))+EPS_AMO;
ln(THETA)=rhotheta*ln(THETA(-1))+(1-rhotheta)*ln(steady_state(THETA))-EPS_THETA;
ln(PHI)=rhophi*ln(PHI(-1))+(1-rhophi)*ln(steady_state(PHI))-EPS_PHI;
ln(G)=rhog*ln(G(-1))+(1-rhog)*ln(steady_state(G))+EPS_G;
ln(IKS)=rhoik*ln(IKS(-1))+EPS_IK;
ln(IHS)=rhoih*ln(IHS(-1))+EPS_IH;
UTB=log(CB-hf*CB(-1))+PHI*LOG(HB(-1))-LB^(1+eta)/(1+eta);
UTS=log(CS-hf*CS(-1))+PHI*LOG(HS(-1))-LS^(1+eta)/(1+eta);
VB=UTB+bettab*VB(+1);
VS=UTS+bettas*VS(+1);
end;

steady_state_model;
THETA=0.626;
PHI=.1468;
AMO=0.001;
PI=1.006;
AT=1;
IKS=1;
IHS=1;
Q=1;
MUS=1;
R=PI/bettas;
MUB=PI*(1-bettab/bettas)/(PI-bettab*(1-AMO));
RK=1/bettas-(1-deltak);
AUX1=(bettab*PHI)*(1-hf)/((1-MUB*THETA)*(1-(1-deltah)*bettab)*(1-bettab*hf));
AUX2=(bettas*PHI)/(1-(1-deltah)*bettas)/(1-bettas*hf)*(1-hf);
AUX3=((RK*(niu*(1-alpha)-sharetb*shareg))/(psi*alpha))/(1+AUX1*deltah*(1
+((THETA*Q)/(1-(1-AMO)/PI))*(R/PI-1)));
AUX4=(RK*(1-shareg)/alpha-deltak-psi*AUX3*(1+deltah*AUX1))/((1-psi)*(1+deltah
*AUX2));
LB=((niu*(1-alpha)*RK*(1-bettab*hf))/(alpha*psi*AUX3*(1-hf)))^(1/(eta+1));
LS((((1-niu)*(1-alpha)*RK*(1-bettas*hf))/(alpha*(1-psi)*AUX4*(1-hf)))^(1/(eta+1));
PS=1;
MC=(epsil-1)/epsil;
K=(MC*alpha/RK)^(1/(1-alpha))*(psi*LB)^niu*((1-psi)*LS)^(1-niu);
Y=(RK*K)/alpha;
CB=AUX3*K;
CS=AUX4*K;
LAMBDA=(1-bettab*hf)/(CB*(1-hf));
LAMBDA=(1-bettas*hf)/(CS*(1-hf));
HB=AUX1*CB;
HS=AUX2*CS;
NB=deltah*HB;
NS=deltah*HS;
N=(1-psi)*NS+psi*NB;
IH=N;
DB=(THETA*Q*NB)/(1-(1-AMO)/PI);
DS=(psi/(psi-1))*DB;
C=(1-psi)*CS+psi*CB;
L=(1-psi)*LS+psi*LB;
F=deltak*K;

```

```
IK=F;
VP=1;
FC=Y/(epsil-1);
WB=(niu*(1-alpha)*Y)/(psi*LB);
WS=((1-niu)*(1-alpha)*Y)/((1-psi)*LS);
M=LAMBDA*Y/(1-bettas*xip);
Z=LAMBDA*Y/(1-bettas*xip);
TB=sharetb*shareg*Y/psi;
G=shareg*Y;
TS=(G-psi*TB)/(1-psi);
UTB=log(CB*(1-hf))+PHI*log(HB)-LB^(1+eta)/(1+eta);
UTS=log(CS*(1-hf))+PHI*log(HS)-LS^(1+eta)/(1+eta);
VB=UTB/(1-bettab);
VS=UTS/(1-bettas);
end;

resid(1);
steady;
check;

options_.pruning=1;
stoch_simul(periods=1000, hp_filter=1600, order = 2, noprint, irf=0) VB VS;
```

Chapter 6

Conclusions

South Korean government are trying to curb rapidly rising household debt mainly by using demand-side credit tightening policy. This study tries to provide appropriate models to analyse effects of demand-side credit tightening policy. These models also suggest how monetary policy affects not only households and the economy but also household debt, and how macroprudential (credit tightening) policy and monetary policy are different from each other as a measure controlling the level of household debt. Furthermore, effects of macroprudential policy are analysed in terms of social welfare as well as households' welfare.

To make the level of household debt policy target, the level of household debt needs to be defined in a model as realistic as possible. The third chapter shows how the collateral constraint, which clearly distinguishes household debt (stock) from borrowing (flow), works well or better than the collateral constraint mostly used in the previous literature in a simple DSGE model. Although, in chapter 3, the model is relatively simple and only calibration is used to set parameter values, the model succeeds in matching the actual data from South Korea and proves that it can be a better model to evaluate effects of macroprudential policy in South Korea. The collateral constraint contributes to the better results especially when we analyse the phase of tightening household credit conditions by using macroprudential policy. Furthermore, it enables us to see how amortization rate affects the Korean economy. Results from this model suggest that increasing amortisation rate is a superior measure to decreasing LTV ratio because it induces less volatility in the economy. In addition, they imply that

the source of house price decline needs to be checked before estimating its effects on households and entire economy because its effects are smaller when the house price declines independently.

Based on the successful results from the third chapter, the fourth chapter attempts to extend the model to see how macroprudential (credit tightening) policy and monetary policy work differently and they interact. Habit formation in non-durable good consumption, price rigidity in non-durable good producers, fixed cost in intermediate good production and monetary policy are added in the model. Not only the newly added elements themselves but also inflation make model's responses different from those in previous chapter. Nominal and real rigidities make dynamics last longer and more realistic. In this model, inflation can reduce the level of real household debt whereas there is no inflation effect on real household debt with the common type of collateral constraint in the previous literature. This also influences responses to monetary policy shock. The results have three implications. When it comes to slowing down the speed of household debt, monetary policy is not effective and may even bring opposite effects. Only credit tightening is effective. Among all policy measures considered, decreasing amortization rate is the most effective and increasing LTV ratio is the second. These implies that ongoing policy efforts to slow down the growth rate of household debt in South Korea is on the right track. Next, the magnitude and transmission of monetary policy are not significantly affected by macroprudential regime or the level of household debt. Last, the effect of house price drop may be relatively limited if the source of change in house price is independent from other macroeconomic variables.

The fifth chapter analyses welfare effects of macroprudential policy in South Korea. The results suggest that discretionary credit tightening cannot increase social welfare in a situation given in South Korea. Thus, its goal could be to minimise social welfare losses, if possible, by increasing amortisation rate. When discount rate gap between patient and impatient households is narrow as calibrated in chapter 4, increasing amortisation rate as a measure of discretionary credit tightening is the most effective in terms of minimum social welfare losses or maximum borrower's welfare gain. When discount rate gap is wide, increasing LTV ratio as a measure of discretionary credit loosening is the most effective in terms of maximum social welfare gains. Next,

adopting countercyclical macroprudential rule can improve social welfare but patient or impatient household cannot help but to lose its welfare. It is shown that the rule which mixes LTV ratio rule against house price and amortisation rate rule against household debt is the best among all rules.

There are limitations of this thesis. First, the models in this study assume the closed economy to simplify the analysis. To get more realistic results, the models can be extended to the open economy if we consider that South Korean economy is closer to the open economy. Second, welfare analysis does not provide information on the transition path. Welfare analysis in this study can only provide information on the change in welfare from the initial steady-state to the new steady-state. Third, it would be more rich analysis if effects of DTI (deb to income) regulation are included. In South Korea, DTI regulation is also one of major demand-side credit tightening measures. Fourth, it should be noted that the models used in this thesis are not the only ones for the analysis of housing market and business cycle. There could be many different point of views on this topic. In modelling household's time preference, a different approach such as present-bias¹ could be incorporated. Although houses are assumed to be only owned by households in this thesis, rented houses could be also considered as Shiller (2007) points out. House prices can be viewed from spatial aspects.² Although, in this thesis, impatient households are borrowers and patient households are savers, borrowers and savers could be younger generation and older generation as in Blanchard (1985).

¹See Laibson (2015) for the details of present biased discounting.

²See Ahlfeldt and Pietrostefani (2017) for the details of spatial aspects.

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