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Procure Financing for Shipping by Auctions

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by

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Abstract

We study a risk management problem in the scenario of ship procurement. A shipping firm faces a certain financing pressure for the procurement of a new ship. On the other hand, the capacity of the ship excesses the demand requirement of the firm. The firm wants to reduce the payment and control the risk by selling a percentage of capacity to another shipping company. We introduce an auction mechanism for the firm to select the partner and determine the sharing percentage. Acting as the auctioneer, the firm announces a certain percentage of capacity to a set of buyers. The payment from the buyer is determined as the highest bid level except the winning price in a second-price auction. The bidding strategy depends on two signals: the demand and financial fiction. For both the risk-neutral and risk-averse utility functions, we find the unique equilibrium for buyers, and the unique percentage of sharing capacity for the auctioneer. Our new policy not only reduces the cost of financial fiction, but also increases the overall utilization of ship capacity. The numerical experiments illustrate that the percentage of the payments from auction is usually higher than the percentage of the capacity shared to the partner. The firm improves the performance significantly through our auction mechanism.

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To my parents

Wu Zhengyi and Ma Youxia

To my wife

Wu Yanling

Chapter 1

Introduction

1.1 Problem description

Investments and financing are among the most important decisions in a firm's operations. Especially for those long-term capacity investments involve hudge expenditures, the integrated optimal capacity planning and the financing strategies for a risk-averse finacially constrained decision maker are critical in the presence of uncertainty. Financial resources are not free for firms. They need external funds to support the investments. As a cost the financing cost will be imposed in the form of interests, etc. Usually the marginal financing cost is increasing in scale of financing due to the increasing risk premium and other friction factors. Even the firm only use its internal funds the expected marginal revenue will decrease in the amount of money be invested. This have the same effect of the increasing financing cost. Under this scenario the firm need to find an optimal strategy to balance the increasing financing cost and the benefits from the investment in capacity decisions.

This paper is motivated by the real problem arises from the shipping business. We study a risk management problem in the scenario of ship procurement. A shipping firm faces a certain financing pressure for the procurement of a new ship. On the other hand, the capacity of the ship excesses the demand requirement of the firm. The firm wants to reduce the payment and control the risk by selling a percentage of capacity to another shipping company. We introduce an auction mechanism for the firm to select the partner and determine the sharing percentage. Acting as the auctioneer, the firm announces a certain percentage $x \in [0, 1]$ of capacity to a set of buyers. The buyers compete for the sharing percentage by submitting their bids that indicate their willingness to pay. The payment from the buyer is determined as the highest bid level except the winning price in a second-price auction.

Both the seller (the shipping firm) and buyers (potential partners) can benefit from the capacity-cost sharing scheme. From the buyer's perspective, the capacity-cost sharing scheme offers an opportunity to expand its capacity to better meet its demand for a reasonable price. If not sharing the capacity and cost with the seller, the buyer may not be able to procure an appropriate vessel by its own. So that it could lose the opportunity to make money. So the buyers have incentive to participate in the capacity-cost sharing scheme.

The seller also benefit from the capacity-cost sharing scheme. First, by adopting the capacity-cost sharing scheme the seller can reduce its demand risk. The shipping company's furture demand is random instead of deterministic. The stochastic fluctuation of the demands is a risk factor in the company's operation. For a risk-averse decision maker it is reasonable to share parts of its capacity to cover its loss in case of extreme low demand realization. Second, the seller can reduce its total cost. For a financially constrained shipping company facing increasing marginal cost on expenditure, the financing cost will increase dramatically. By sharing parts of its capacity, the seller can acquire the buyer's payment as a source of financing which is free of interest. This will reduce both purchasing cost and financing cost for the seller. Third, the seller can make use of extra capacity. The shipping company's demand may be lower than the total capacity of the ship with very high probability. In this case the extra capacity is totally worthless for the company. The company can benefit from selling the capacity without any possible loss.

Besides the above benefits for buyers and seller, the utilization of the vessel will increase by sharing the capacity. This is the benefit for the society.

We propose auction as the mechanism to select the partner as well as determine the selling price in the capacity-cost sharing scheme. Auction is a kind of selling mechanism which is broadly used in selling artworks, antiques, natural resources, financial instruments and etc. A of the good features of auction is the price discovering function by increasing the competition among bidders to force them reveal their private valuation of the item. Auction is an ideal mechanism for our capacity-cost sharing scheme. The buyer who need the capacity share most can win the capacity share by submitting the highest bid. And the seller of the capacity share can benefit from the competition imposed by auction. Even if the shipping company doesn't adopt auction as the selling mechanism, as an analysis framework auction will offer useful insights for the firm's pricing decision.

In this paper we assume that the seller will announce the capacity share to sell in advance, then the buyers will compete by submitting bids for the capacity. We will focus on the implementation of second-price sealed auction. However we build a general model that is consistent with other forms of auctions in Chapter 2. We expect to (i) study the optimal capacity decision for the seller and the bidding strategy for buyers; and (ii) try to understand the role of demand and financial friction in all participants' decision making.

1.2 Related literature

Three streams of literatures are related to our work. They are (i) researches on capacity reservation studied by Wu et al. (2002) and Kleindorfer and Wu (2003); (2) researches on auctions with constraints studied by Che and Gale (1998) and Malakhov and Vohra (2009); and (iii) researched on auction application in procurement problem studied by Dasgupta et al (1990), Che (1993), Branco (1997) and Chen (2007).

Table 1.1: Related literatures

Literature	Mechanism	Signal Space*		Risk Attitude
Wu et al. (2002)	Not Specified	-	-	Risk Neutral
Kleindorfer and Wu (2003)	Not Specified	-	-	Risk Neutral
Che and Gale (1998)	Auction	M	C	Risk Neutral
Malakhov and Vohra (2009)	Auction	M	D	Risk Neutral
Dasgupta et al (1990)	Auction	U	C	Risk Neutral
Che (1993)	Auction	U	C	Risk Neutral
Branco (1997)	Auction	U	C	Risk Neutral
Chen (2007)	Auction	U	C	Risk Neutral
Our Paper	Auction	M	C	Risk Averse

* M is for multi-dimensional. U is for uni-dimensional. C is for continuous. D is for discrete.

Wu et al. (2002) studied the seller's optimal bidding strategy and the buyers' contracting strategies for capital-intensive goods. In their model, the seller acts as a Stackelberg leader who offered the capacity reserve cost and executive cost as the bid, then the buyer determine the capacity reserve level according to its own interest. Adopting this framework as the base case, Kleindorfer and Wu (2003) reviewed a lot of literatures involves with the problem of optimal capacity reservation by different contracting linked with B2B exchanges in capital-intensive industries. Our research is distinguished with the above works. Wu et al. (2002) and Kleindorfer and Wu (2003) presume relationship and changes have already determined by some price discovery mechanism. They focused on the efficient integration of long-term and short-term contracting. In our paper we use proposed auction as the selling mechanism for capacity-cost sharing. And try to address the benefits of this scheme.

Che and Gale (1998) studied the performance of first-price and second-price auctions when bidders' financial resources are costly. In their paper they presumed the existence of the buyer's equilibrium for the dual-dimensional signal setting. However they did not offer a methodology to characterize the equilibrium. In our paper we derive the buyer's equilibrium strategy for the dual-dimensional signal setting under second-price auction. This distinguishes our paper with theirs. Malakhov and Vohra (2009) studied the optimal auction design problem for a seller facing a group of buyers with two-dimensional private signal about the marginal valuation of the goods and its capacity constraints. They presume the signals to be finite discrete. Under this critical assumption they can implement linear programming for their problem. In our paper we assume that the signals are continuous, thus their methodology will not work.

Dasgupta et al (1990) is one of the earliest paper studied auction mechanism in procurement problems. They proved that under certain conditions quantity auction is the optimal mechanism for the buyer in the procurement problem. Our paper is distinct with Dasgupta et al (1990). In Dasgupta et al (1990) signals are uni-dimensional. But in our paper signals have two dimensions. Che (1993) and Branco (1997) studied the optimal auction design problems for which the bids have multi-dimensions. The auctioneer evaluate the multidimensional bids by some scoring function designed for its best interest. It's different from our paper. In our paper the buyer's signals are dual-dimensional while the bids only have one dimension. One of our problem is to derive a bid function for buyers that constitute an equilibrium. Chen (2007) studied the supply contract auction.

Miller et al (2007) studied an auction mechanism in which all bidders' types are multidimensional. In mechanism design problems, type (or signal) refers to the characters of the participators. In an auction, every bidder is privately informed with its type, and other bidders' types are assumed to follow some distributions. Bidders make decisions on bid according to its own type and the distribution of others' types as the available information. In private value auction, types are assumed to be unidimensional as the valuation of the product. But in some applications, it is appropriate to assume the type to be a vector. In this paper, the authors proof the existence of a revelation mechanism for multidimensional continuous type auctions with interdependent valuations.

1.3 Organization of the paper

The rest of the paper is organized as follows. Chapter 2 present the framework of the model for the seller's optimal capacity sharing decision and the buyer's incentive compatible bidding strategy. Chapter 3 and follows with the model with second-price auction implementation. The seller's decision and utility are compared with the risk-neutral assumption and risk-averse assumption. Chapter 5 is the conclusion and point the future research.

Chapter 2

The Model

In this chapter we construct models for buyer's problem and seller's problem. Some assumptions are proposed and justified in Section 2.1. In Section 2.2 we develop a model framework for general case which do not specify the rule of auction. Models for second-price auction and first-price auction are constructed in Section 2.3 and Section 2.4 respectively.

2.1 Notation and Assumptions

We consider the problem faced by seller that must determine the portion $x \in [0, 1]$ of capacity share to sell in the auction which refers to the seller's problem. And the problem faced by buyers that must propose appropriate bids Y that best fit their private signals which refers to the buyer's problem. The seller act as an Stackelberg leader who posts the capacity share to be sold first, then the buyers bid for the posted capacity share x.

Let N be the number of buyers and let the subscript $i = 1, 2, \ldots, N$ denote different buyers. The subscript $i = 0$ will used to denote the seller especially. We use

Table 2.1: Notation

$U(\cdot)$	utility function
r	revenue for the total capacity
x	capacity share to be sold in auction
$Y(x, d, \alpha)$	bidding function or bidding level
C	total cost for the vessel
d_i	expected demand for buyer i (seller if $i = 0$)
α_i	financing friction for buyer i (seller if $i = 0$)
ϵ_i	stochastic noise with $E[\epsilon_i] = 0$

the subscript $A = G, F, S$ to denote the types of auctions. For every buyer and seller, its signal is assumed to be a dual-dimensional vector $(d, \alpha)^T$. Here α is the financial friction which represent the participator's financial condition by affect its financing cost. And d represent the participator's expected demand. Here we propose some assumptions for our model.

Assumption 2.1. *Signal is private informed for $(d_i, \alpha_i)^T, i = 0, 1, \ldots, N$. α and d are independent with each other.*

Assumption 2.1 is standard in private value auction literatures. Under this assumption every buyer is privately informed with its own signal $(d, \alpha)^T$. But other buyers' signal $(d', \alpha')^T$ are unknown random variables drawn from the same distribution. The private information assumption is a reasonable generalization of the real world. Usually it is not possible for a company to know its competitors' information exactly. But a distribution can be assigned to the signals due to some public apriori information. We assume that the expected demand d and financial friction α are independent for computational simplicity. But in fact most results in this paper do not rely on the independent assumption. Assume that $d_i \sim iid\ F_d$ and $\alpha_i \sim iid\ F_\alpha$. And $\text{supp}(f_d) = [\underline{d}, \bar{d}]$, $\text{supp}(f_\alpha) = [\underline{\alpha}, \bar{\alpha}]$.

Assumption 2.2. *The demand of buyer i (the seller if $i = 0$) is $D_i = d_i + \epsilon_i$, $i = 0, 1, \ldots, N$. $\epsilon_i \sim iid\ F_\epsilon$ are random noise with zero mean.*

We can view D_i as the total demand for i during the entire operational periods of the vessel. So it is reasonable to assume ϵ follows normal distribution by applying the Law of Large Number. We make the iid assumption on the random noise for simplicity. However in more general cases buyers' demand can be correlated. The zero mean assumption for ϵ_i is not necessary. But for simplicity we can add the nonzero part of $E[\epsilon_i]$ to d_i.

Assumption 2.3. *If the payment size is z the financing cost function is assumed to be $c(\alpha, z) = \frac{\alpha}{2} z^2$. And the payment function is a common knowledge among buyers and the seller.*

There are some reasons to adopt the financing cost function in Assumption 2.3. First, the partial derivative on z is positive increasing which represent the increasing marginal property of the financing cost. Second, the increasing rate of the marginal financing cost is constant which equals to the financial friction α. And lastly, the marginal financing cost equals to zero while it is evaluated at $z = 0$ which is consistent with our heuristics. The assumed financing cost function is the unique continuous function that satisfy the above three conditions. By assuming the above financing cost function we can characterize the participants' financial condition by a single parameter α. This will facilitate our analysis and give us useful insight on the role of financial friction.

To model the buyer's risk preferences we propose the following assumptions concerning on the concavity of utility function.

Assumption 2.4. *Both buyers and the seller are assumed to be risk-averse with the*

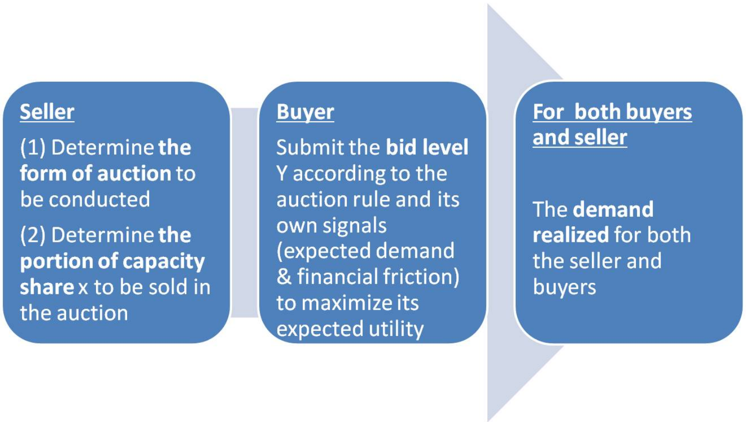

Figure 2.1: The Framework for Model

same utility function if not further explained. Thus we have $U'(\cdot) \geq 0$ and $U''(\cdot) \leq 0$. Further we assume $U(\cdot) \geq 0$.

We assume the same utility function for both buyers and seller to simplify the model. We care about the effects of risk-aversion in general form so we do not assume the specific utility function. Only the up-to the second order analytic properties are assumed for the utility function.

2.2 Model Framework

Figure 2.1 illustrates the framework for our model. The seller acts as the Stackelberger leader and buyers make their decisions based on the seller's posted capacity share x. For any predetermined posted capacity share x every buyer need to submits a bid Y that indicates how much he is willing to pay for x. This bid should best fit the buyer's interests that can maximizes his expected utility. The buyer's problem (BP) is to find out the optimal bid level Y according to its own signal $(d, \alpha)^T$ and the seller's posted capacity share x. Bayesian-Nash equilibrium is an appropriate concept for the solution of BP as a game. A Bayesian-Nash Equilibrium is the state that everyone in the game is not incent to change its decision as long as other players keep their decisions unchanged. For a buyer the Bayesian-Nash Equilibrium bidding function Y is a function which maps the buyer's own signal $(d, \alpha)^T$ to the deterministic bid level $Y(x, d, \alpha)$ for any predetermined capacity share x. But from other buyers' and the seller's point of view, a given buyer's bid level is a random function of x due to the randomness of the buyer's signal $(d, \alpha)^T$. The seller's problem (SP) is to select an optimal capacity share x for the auction according to the equilibrium among buyers, which will maximize its expected utility. Here we will first model the Buyer's Problem and then the Seller's Problem assuming the existence of the equilibrium. However it will be proved in section 3.3 that the equilibrium exist in the second-price auction for both risk-neutral and risk-averse cases.

To analyze the buyer's problem we assume that the posted capacity share x to be sold and the type of auction to be conducted were given. Here we first formulate the model for the general auction type $A = G$. The buyer's objective is to maximize its expected utility by choosing an appropriate bid level Y. Without loss of generality let's assume the buyer under our consideration is buyer $i,\ i = 1, 2, \ldots, N$. A characteristic function $1_G(y_i, y_{-i})$ is defined to tell whether buyer i win the auction or not if his bid is y_i and other bidder's bids are y_{-i}. $1_G(y_i, y_{-i})$ equals to 1 if i wins according to the rule of auction G, and it equals to 0 if i lost in the auction. The functional form of $1_G(y_i, y_{-i})$ is determined by the type of auction to be conducted. Examples of the characteristic function for second-price and first-price auction will be provided in the later part of this section. Assume that i's bid is y_i and others' bid are y_{-i}, if i win the auction in the end his profit is $\pi_i = r\min(x, d_i + \epsilon_i) - \frac{\alpha_i}{2}P_G^2(y_i, y_{-i}) - P_G(y_i, y_{-i})$ and the utility for π_i is $U(\pi_i^B) = U\left(r\min(x, d_i + \epsilon_i) - \frac{\alpha_i}{2}P_G^2(y_i, y_{-i}) - P_G(y_i, y_{-i})\right)$. Here $r\min(x, d_i+\epsilon_i)$ is the operations revenue of the share capacity x if the realized demand for i is $d_i + \epsilon_i$, and the second term $\frac{\alpha_i}{2}P_G^2(y_i, y_{-i})$ is the financing cost for i if his bid level is y_i. $P_G(y_i, y_{-i})$ is the payment function which indicate the amount of payment under the rule of auction G for the bidder if the outcome of the auction is (y_i, y_{-i}) and he wins the auction. In case that i lost the auction his profit is $\pi_i^B = 0$ and the utility is $U(0)$. So if given the outcome of the auction as (y_i, y_{-i}) the buyer i's utility is $U\left(r \min(x, d_i + \epsilon_i) - \frac{\alpha_i}{2} P_G^2(y_i, y_{-i}) - P_G(y_i, y_{-i})\right) 1_G(y_i, y_{-i}) + U(0)\,(1 - 1_G(y_i, y_{-i}))$.

According to the above analysis we can formulate the buyer's problem as bellow.

$$
\begin{aligned}
\text{(BP-G)} \quad & \max \quad E\left[U\left(\pi_i^B\right) 1_G(y_i, y_{-i})\right] + U(0) E\left[(1 - 1_G(y_i, y_{-i}))\right] \\
& s.t. \quad \pi_i^B = r \min(x, d_i + \epsilon_i) - \tfrac{\alpha_i}{2} P_G^2(y_i, y_{-i}) - P_G(y_i, y_{-i}) \\
& \qquad\;\; y_i \geq 0
\end{aligned}
\tag{2.1}
$$

Notice that in (BP-G), we can calculate the expectations in the objective function by taking conditions on the value of the random function $1_G(y_i, y_{-i})$. Since $1_G(y_i, y_{-i})$ is a characteristic function, its expectation is the winning probability $G_G(y_i)$ for the seller. So the buyer's problem (BP) can be transformed to the formulation bellow.

$$
\begin{aligned}
\text{(BP-G}'\text{)} \quad & \max \quad E\left[U\left(\pi_i^B\right)\right] G_G(y_i) + U(0)\left[1 - G_G(y_i)\right] \\
& s.t. \quad \pi_i^B = r \min(x, d_i + \epsilon_i) - \tfrac{\alpha_i}{2} P_G^2(y_i, y_{-i}) - P_G(y_i, y_{-i}) \\
& \qquad\;\; y_i \geq 0
\end{aligned}
\tag{2.2}
$$

Here we assumed the existence of equilibrium bidding strategy Y for the buyers in advance. The equilibrium bidding strategy Y will map all the buyers' signals into bid level y. So if the equilibrium bidding strategy exist there will be a CDF $F_Y(y)$ for bid level y which is consistent with the equilibrium. The winning probability G_G is determined by F_Y. Our approach is to first assume the winning probability G_G so the buyer's problem can be formulated as the individual optimization problem (BP-G$'$).Then we will verify the existence of the equilibrium and further characterize the winning probability G_G.

The solution of (BP-G$'$) is an incentive compatible bidding strategy $Y^{EQ}(x,d,\alpha)$ which is considered to be common knowledge among buyers and seller. For the seller, there're two sources of revenue. They're revenue from its operations of the reserved capacity $1-x$ and the revenue from the capacity share x that is sold in auction. The seller's operational revenue $\mathrm{OR}(x,\epsilon_0)$ is determined by its reserved capacity x and its demand while the auction revenue $\mathrm{AR}(x,d,\alpha)$ depends on the buyer's equilibrium and the distribution of their signals. For the cost side, there're two terms which are financing cost and purchasing cost. The purchasing cost is the total price for the vessel C. And the financing cost $\mathrm{FC}(x,d,\alpha)$ depends on the buyer's payments. Notice that the financing cost can be 0 if the buyer's payments is equal or greater than the purchasing cost C in which case the seller need not seek for the costly funding.

The seller's problem (SP-G) for general auctions is formulated bellow.

$$
\begin{array}{lll}
\text{(SP-G)} & \max & E\left[U\left(\mathrm{OR}(x,\epsilon_0)+\mathrm{AR}(x,d,\alpha)-\mathrm{FC}(x,d,\alpha)-C\right)\right] \\
 & s.t. & \mathrm{OR}(x,\epsilon_0)=r\min(1-x,d+\epsilon_0) \\
 & & \mathrm{AR}(x,d,\alpha)=H_G\left(Y_1^{EQ},Y_2^{EQ},\dots,Y_N^{EQ}\right) \\
 & & \mathrm{FC}(x,d,\alpha)=\frac{\alpha_0}{2}E\left[\left(C-H_G\left(Y_1^{EQ},Y_2^{EQ},\dots,Y_N^{EQ}\right)\right)^+\right]^2 \\
 & & x\in[0,1]
\end{array}
\tag{2.3}
$$

Here $H_G\left(Y_1^{EQ},Y_2^{EQ},\dots,Y_N^{EQ}\right)$ is a function determined by the type of auction to be conducted which indicate the revenue for the auctioneer if the outcome of the auction is $\left(Y_1^{EQ},Y_2^{EQ},\dots,Y_N^{EQ}\right)$.

2.3 Second-Price Sealed Auction

If second-price auction is conducted, the buyer with the highest bid will win but only need to pay the second highest bid. The payment function for the second-price auction is defined as $P_S(y_i, y_{-i}) = \max\{y_j : \forall j \neq i\}$ and $H_S\left(Y_1^{EQ}, Y_2^{EQ}, \ldots, Y_N^{EQ}\right) = Y_{(N-1)}^{EQ}(x, d, \alpha)$. Suppose the winning probability for any buyer is $G_S(y)$ which equals to $F_Y^{N-1}(y)$ in this case. By conditioning we can simplify the expected utility in (BP-G) which results in the following formulation for the buyer's problem for second-price sealed auction implemented case (BP-S).

$$\begin{aligned}
\text{(BP-S)} \quad \max \quad & E\left[U\left(\pi_i^B\right)|Z < y\right] G_S(y) + U(0)\left(1 - G_S(y)\right) \\
s.t. \quad & \pi_i^B = r\min(x, d_i + \epsilon_i) - \tfrac{\alpha_i}{2} Z^2 - Z \\
& Z = \max\{y_j : \forall j \neq i\} \\
& y_i \geq 0
\end{aligned} \tag{2.4}$$

Suppose the equilibrium bidding function for buyers is Y^{EQ} then the seller's problem (SP-S) can be formulated as follows.

$$\begin{aligned}
\text{(SP-S)} \quad \max \quad & E\left[U\left(\text{OR}(x, \epsilon_0) + \text{AR}(x, d, \alpha) - \text{FC}(x, d, \alpha) - C\right)\right] \\
s.t. \quad & \text{OR}(x, \epsilon_0) = r\min(1 - x, d + \epsilon_0) \\
& \text{AR}(x, d, \alpha) = Y_{(N-1)}^{EQ}(x, d, \alpha) \\
& \text{FC}(x, d, \alpha) = \tfrac{\alpha_0}{2} E\left[\left(C - Y_{(N-1)}^{EQ}(x, d, \alpha)\right)^+\right]^2 \\
& x \in [0, 1]
\end{aligned} \tag{2.5}$$

2.4 First-Price Sealed Auction

If first-price auction is conducted, the buyer with the highest bid will win and pay what her bid. The payment function for the second-price auction is defined as $P_S(y_i, y_{-i}) = y_i$ and $H_S\left(Y_1^{EQ}, Y_2^{EQ}, \ldots, Y_N^{EQ}\right) = Y_{(N)}^{EQ}(x, d, \alpha)$. Suppose the winning probability for any buyer is $G_F(y)$ which equals to $F_Y^N(y)$ in this case. By conditioning we can simplify the expected utility in (BP-F) which results in the following formulation for the buyer's problem for second-price sealed auction implemented case (BP-F).

$$
\begin{aligned}
\text{(BP-F)} \quad \max \quad & E\left[U\left(\pi_i^B\right)\right] G_F(y) + U(0)\left(1 - G_F(y)\right) \\
s.t. \quad & \pi_i^B = r\min(x, d_i + \epsilon_i) - \tfrac{\alpha_i}{2} y_i^2 - y_i \\
& y_i \geq 0
\end{aligned} \tag{2.6}
$$

Suppose the equilibrium bidding function for buyers is Y^{EQ} then the seller's problem (SP-F) is as follows.

$$
\begin{aligned}
\text{(SP-F)} \quad \max \quad & E\left[U\left(\text{OR}(x, \epsilon_0) + \text{AR}(x, d, \alpha) - \text{FC}(x, d, \alpha) - C\right)\right] \\
s.t. \quad & \text{OR}(x, \epsilon_0) = r\min(1 - x, d + \epsilon_0) \\
& \text{AR}(x, d, \alpha) = Y_{(N)}^{EQ}(x, d, \alpha) \\
& \text{FC}(x, d, \alpha) = \tfrac{\alpha_0}{2} E\left[\left(\left(C - Y_{(N)}^{EQ}(x, d, \alpha)\right)^+\right]^2 \\
& x \in [0, 1]
\end{aligned} \tag{2.7}
$$

Chapter 3

Implement with Second Price Sealed Auction

In this chapter we will focus on the case that the second-price sealed auction is implemented. We'll first solve both buyer's problem and seller's problem for the risk-neutral case for which both buyers and the seller are assumed to be risk-neutral. And then all results will be extend to the risk-averse case. The proofs of statements are attached in Section 3.3.

3.1 Risk-Neutral Case

By replacing the concave utility function with a homogeneous linear function we get the modified objective function for the risk-neutral buyer's problem for the second-price auction (RN-BP-S).

$$\begin{aligned} \text{(RN-BP-S)} \quad & \max \quad E\left[rE(x,d_i) - \tfrac{\alpha_i}{2}Z^2 - Z|Z<y\right]G_S(y) \\ & s.t. \quad Z = \max\{y_j : \forall j \neq i\} \\ & \qquad\ \ y_i \geq 0 \end{aligned} \tag{3.1}$$

An observation of the buyer's objective function is that by increasing the bid level y the buyer's conditional expected profit will decrease but at the same time the winning probability $G_S(y)$ will increase. So The buyer need to find a bid level that can balance these two mutual contradict effects. Proposition 3.1 gives the Bayesian-Nash Equilibrium for the buyer's problem (RN-BP-S).

Proposition 3.1. *The Bayesian-Nash Equilibrium bidding function $Y_S^{EQ}(x,\alpha,d)$ for (RN-BP-S) is the solution of the equation bellow.*

$$rE(x,d) - \frac{\alpha}{2}y^2 - y = 0 \tag{3.2}$$

Thus we have,

$$Y_S^{EQ}(x,\alpha,d) = \frac{\sqrt{1+2\alpha rE(x,d)}-1}{\alpha} \tag{3.3}$$

From Proposition 3.1 we know that if the second-price auction is conducted the equilibrium strategy for buyers is bid to the maximum level that makes their expected profit non-negative. This proposition is consist with the classic results about second-price auction. To understand the equilibrium we consider the example bellow. Assume outcome suggested by the equilibrium is $(y_1^{EQ}, y_2^{EQ}, \ldots, y_N^{EQ})$ we claim that for a bidder (let's say bidder 1) it's optimal to keep its bid level unchanged if other bidders did not change their bids. Let's assume $y_1^{EQ} = \max(y_1^{EQ}, y_2^{EQ}, \ldots, y_N^{EQ})$. In this case bidder 1 will win the auction and his expected profit will be $y_1^{EQ} - \max(y_i^{EQ}, i \neq 1) > 0$. It is not optimal for bidder 1 to increase the bid because no incremental expected profit is induced by doing so. Decreasing the bid level is not optimal either. Because the winning probability will decrease while the expected profit will not increase if bidder 1 win the auction in the end. For the case that $y_1^{EQ} < \max(y_1^{EQ}, y_2^{EQ}, \ldots, y_N^{EQ})$ it is not optimal to decrease the bid level because the bidder will still lost the auction and the expected profit remains 0. Increasing the bid level is not optimal either. This is because nothing changed if the increased bid level is still lower than the winning bid, but if the increased bid level exceeded the former winning bid the bidder's expected profit will become negative which is even worse than before.

We can characterize the iso-bid curve since we know the equilibrium bidding function. Iso-bid curve is a curve that divide the entire signal plane into two parts. The signals on the same iso-bid curve share the same bid level. The signals laid on the upper-left area of an iso-bid curve are associated with lower bid levels, while the signals laid on the lower-right area of the iso-bid curve are associated with higher bid levels. The next corollary gives the formulation of the iso-bid function for the second-price auction case.

Corollary 3.1. *The iso-bid function for any bidder is given by the equation bellow.*

$$\mathcal{A}(d, x, y) = \frac{2r}{y^2} E(x, d) - \frac{1}{y} \tag{3.4}$$

Figure 3.1 illustrates the shape of buyer's equilibrium bid function $Y_S^{EQ}(x, \alpha, d)$ and the corresponding iso-bid curves. The sharp of $Y_S^{EQ}(x, \alpha, d)$ are displayed in subfigure (a) and subfigure (b) while the shape of $\mathcal{A}(d, x, y)$ are displayed in subfigure

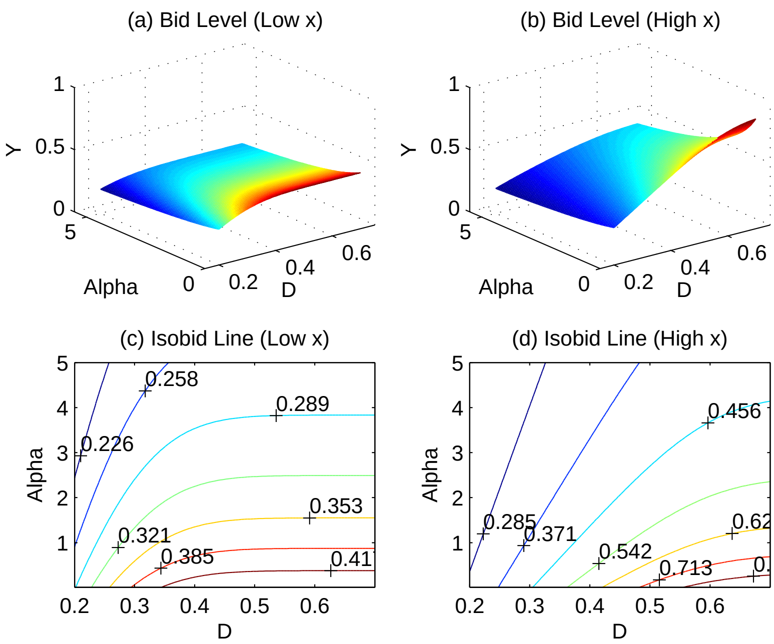

Figure 3.1: The Equilibrium bid function

(c) and subfigure (d). We performed two numerical experiments with different posted capacity share x to see the effects of x on the bid level and the iso-bid curve. For the low capacity share case x is set to be 0.3, and it is set to be 0.6 for the high capacity share case. Other parameters for this numerical experiment are $\underline{\alpha} = 0.01$, $\bar{\alpha} = 5$, $\underline{d} = 0.2$, $\bar{d} = 0.7$, $r = 1.5$ and $C = 1$. All these parameters are used in the followed numerical experiments if not further explained.

We can observe from Figure 3.1 (a) (b) that the equilibrium bid function Y_S^{EQ} is increasing in the buyer's expected demand while decreasing in the financial friction α. Buyers need to consider both demand factor and financial factor in their bidding decisions. With the same demand the buyer can bid more aggressively if his financial condition is better. While if the financial conditions are similar, the buyer will bid more if his expected demand is higher. These trends are more obvious when the capacity share x is high.

As observed in subfigure (c) and subfigure (d) the iso-bid curve is concave. Every iso-bid curve is attached with a bid level. All signals laid on the same iso-bid curve are assigned to the same equilibrium bid level. The iso-bid curves on the northwest corner are associated with lower equilibrium bid level while the iso-bid curves on the southeast corner are associated with higher equilibrium bid level. To attain a bid level y the buyer's expected demand must increase if his financial condition become worse. When the bid level y is low the financial friction α and expected demand d will move in a linear manner. However when the bid level y become higher and higher a small shift of financial friction α will cause a large change in expected demand to retain the bid level.

We can use iso-bid curves to derive the winning probability that is consistent with the equilibrium bidding function in Proposition 3.1. The next corollary characterizes the winning probability.

Corollary 3.2. *Suppose N buyers are participating in the auction. Buyers' signals are independent identical distributed with probability density function $f_\alpha(a)$ and $f_d(d)$ then the winning probability is,*

$$G_S(y) = F_S^{N-1}(y) \tag{3.5}$$

Where $F_S(y)$ is the cumulative distribute function for bid level Y_S^{EQ}.

$$F_S(y) = \int\int_\Omega f_\alpha(a) f_d(\xi) da d\xi \tag{3.6}$$

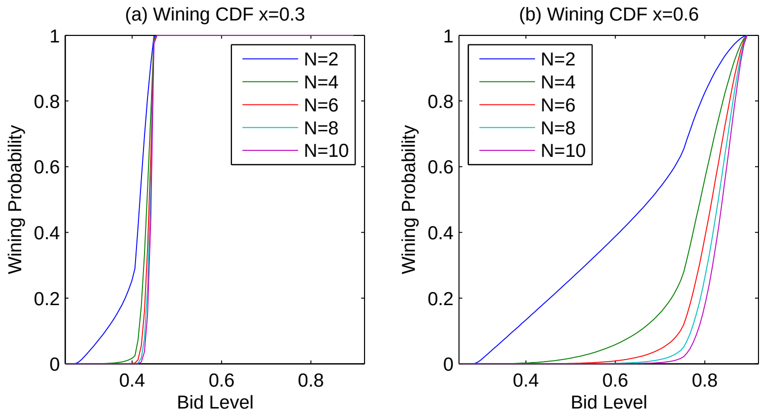

Figure 3.2: Winning Probability

$$\Omega_y = \left\{(a, \xi) \in [\underline{\alpha}, \bar{\alpha}] \times [\underline{d}, \bar{d}] : \alpha \geq \mathcal{A}(d, x, y)\right\} \tag{3.7}$$

Ω_y is the area with signals which are associated with bid levels lower than y. By taking integral on this area we obtain the CDF F_Y for the equilibrium bid level which is consistent with Proposition 3.1.

Figure 3.2 plots the buyer's winning probability. Subfigure (a) and subfigure (b) are the buyer's winning probability for Low capacity share case and high capacity share case. In each case we plot the winning probability for $N = 2$, $N = 4$, $N = 6$, $N = 8$ and $N = 10$. When the number of buyer is small the winning bid is more evenly distributed. When the number of buyers increased the density of winning bid will convergent to a higher level very fast. So if there're enough buyers the seller get very good chance to sell the capacity share for a high price. This is a benefit of auction for the seller. Another observation between (a) and (b) in Figure 3.2 is that by increasing x the upper bound for the winning bid will increase. This is the benefit from the increasing competition.

Next we will move to the solution of the seller's problem (RN-SP-S) for second-price auction. For the risk-neutral seller the objective is to maximize the expected profit which is consist of the expected operational revenue (EOR), the expected auction revenue (EAR), the expected financing cost (EFC) and the purchasing cost C. The decision variable for the seller is the capacity share x.

$$\begin{aligned} \text{(RN-SP-S)} \quad \max \quad & \text{EOR}(x) + \text{EAR}(x) - \text{EFC}(x) - C \\ s.t. \quad & \text{EOR}(x) = rE(1-x, d_0) \\ & \text{EAR}(x) = E\left[Y^{EQ}_{(N-1)}(x, \alpha, d)\right] \\ & \text{EFC}(x) = \tfrac{\alpha_0}{2} E\left[\left(C - Y^{EQ}_{(N-1)}(x, \alpha, d)\right)^+\right]^2 \\ & x \in [0, 1] \end{aligned} \tag{3.8}$$

We will study the seller's expected profit term by term. The next proposition is about the concavity of the seller's expected operational revenue EOR(x) as a functions of the capacity share x.

Proposition 3.2. *Then seller's expected operational revenue EOR(x) is concave decreasing in the capacity share x.*

The marginal expected operational revenue for the seller equals to $-r\Pr(\epsilon_0 > 1 - x - d_0)$ which is the production of the revenue for the total capacity share $-r$ and the probability that the realized demand is greater than the reserved capacity $1 - x$. It is clear that the probability of the even that the realized demand is greater than $1 - x$ is increasing in the posted capacity share x. So the marginal expected operational revenue is decreasing concave in x. Following the same logic we know that the expected operational revenue is increasing concave in the seller's expected demand d_0. Figure 3.3 illustrates our observations in a more intuitive way. The

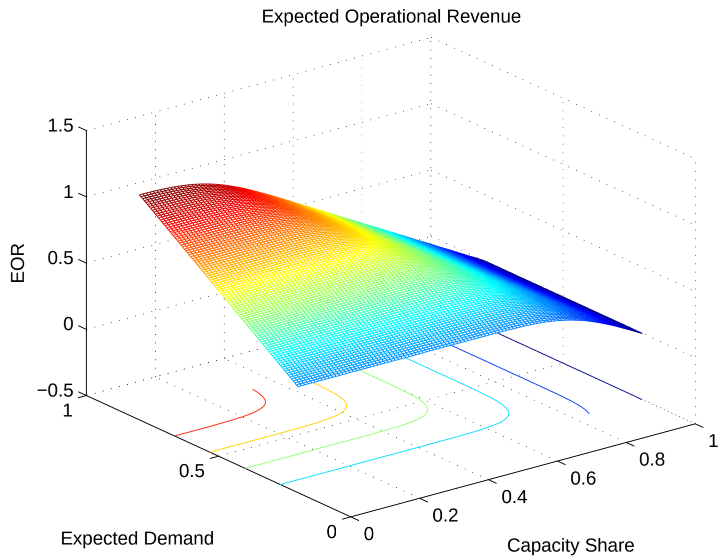

Figure 3.3: Expected Operational Revenue

surface in the figure is the seller's operational revenue. We can observe that for any capacity share x, the operational revenue is increasing in the d when d is less than $1-x$. But when b become greater than $1-x$ there is only very little increases in revenue if the d is increasing. So without considering the financial friction select the reservation capacity $1-x$ to fulfill the expected demand is a good strategy for the seller's capacity decision. However in the presence of financial friction this strategy is not optimal. We will illustrate this point in Chapter 4.

The seller's expected auction revenue $\mathrm{EAR}(x)$ and expected financing cost $\mathrm{EFC}(x)$ depend on the winning buyer's payment which equals to the second highest bid among all buyers. Next we first establish the concavity of the equilibrium bid function as a function of the capacity share x. Later we will find that the concavity of the bid function is a sufficient condition for the concavity of $\mathrm{EAR}(x)$ and $-\mathrm{EFC}(x)$.

Proposition 3.3. *If second-price sealed auction is conducted, the buyer's equilibrium bid function $Y_S^{EQ}(x, \alpha, d)$ is concave increasing in x for all signals.*

However Proposition 3.3 can not guarantee the path-wise concavity of the $(N-1)-$th order statistic $Y^{EQ}_{(N-1)}(x, \alpha, d)$. In fact it is not difficult to construct counter examples to prove that the $(N-1)-$th order statistic is not concave in general case. But we can prove expectation of $Y^{EQ}_{(N-1)}$ is concave in x. The idea is that for any x we can order $Y^{EQ}_i, i = 1, 2, \dots, N$ strictly with probability 1 in a neighborhood of x. The order is disrupted with probability 0 which can be omitted while taking expectation. First we present two lemmas which deal with the probabilistic property of the equilibrium bid function $Y^{EQ}_{(N-1)}(x, \alpha, d)$ and the continuity of the iso-bid function $\mathcal{A}(d, x, y)$ respectively.

Lemma 3.1. *For any given capacity share x,* $\Pr(Y_i = Y_j, \forall i \neq j) = 0$.

Lemma 3.1 claims that fix x the probability that any two bids from different buyers equals is zero. Since the d and α are continuous random variables, the even that any two bids equals is a countable subset of the probability space. Thus the probability is zero.

Lemma 3.2. *Fix the expected demand d, as a function of (x, y) the iso-bid function $\mathcal{A}(d; x, y)$ is continuous.*

With the continuity, there exist a neighborhood for every x in which the order of bids reserved. Together with Lemma 3.1 we can prove the concavity of the winning buyer's expected payment for second price auction which is the key to prove the concavity of EAR(x) and $-$EFC(x).

Proposition 3.4. *Defined the Expected Auction Revenue $EAR(x)$ and the Expected Financing Cost $EFC(x)$ by the following equations.*

$$EAR(x) = E\left[Y^{EQ}_{(N-1)}(x,\alpha,d)\right] \quad (3.9)$$

$$EFC(x) = \frac{\alpha_0}{2} E\left[\left(C - Y^{EQ}_{(N-1)}(x,\alpha,d)\right)^+\right]^2 \quad (3.10)$$

For any $x_0 \in [0,1]$ *there exist a* $\Delta > 0$ *that*

1. *EAR*(x) *is concave increasing in* x, $\forall x \in (x_0, x_0 + \Delta)$;
2. *EFC*(x) *is convex decreasing in* x, $\forall x \in (x_0, x_0 + \Delta)$.

The idea of Proposition 3.4 is that we can prove EAR(x) (EFC(x)) is pointwise concave (convex) in the interval $x \in [0,1]$. This implies that EAR(x) (EFC(x)) is concave (convex) in $[0,1]$. We take the advantage of the continuity of $\mathcal{A}(d,x,y)$ in Lemma 3.2. For a sample path we assume that all bids at x can be ranked strictly. Due to the continuity of $\mathcal{A}(d,x,y)$ this rank will not be destroyed in a neighborhood of x. Since we know that the equilibrium bid function is concave increasing in x from Proposition 3.3 EAR(x) (EFC(x)) is concave (convex) in this neighborhood. This local concavity (convexity) is valid for any sample with positive probability because of Lemma 3.1. So the concavity of EAR(x) and the convexity of EFC(x) are proved.

Figure 3.4 illustrate the shape of the seller's expected auction revenue against the capacity share x. When capacity is sold the seller can get more revenue from the auction. If there are more buyers participate in the auction the auction revenue for the seller will be higher. This phenomenon is shaper if the capacity share x is big. Figure 3.5 displays the buyer's expected financing cost against his financial friction α and the decision on capacity share x. Generally the expected financing cost is increasing with the financial friction. If the financial condition for the seller is bad he may need to pay plenty of financing cost to support his purchasing plan. However

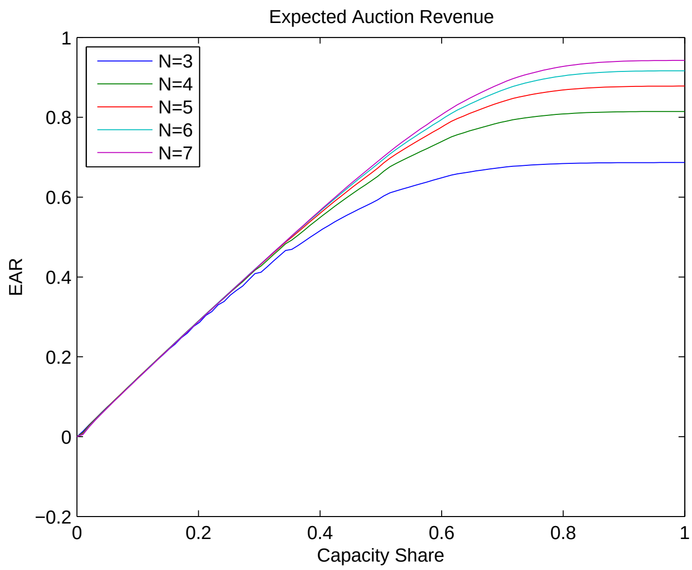

Figure 3.4: Expected Auction Revenue

as it is illustrated in Figure 3.5 the seller can ease his financial pressure by selling more capacity. As x increased, the seller can collect more money from the winning buyer's payment as a interest-free fund. So he doesn't need to procure as much fund as before. The expected financing cost will be reduced dramatically especially for the seller with poor financial condition.

Proposition 3.4 together with Proposition 3.2 imply the existence and uniqueness of the optimal posted capacity share to be sold x^{opt} for the seller. The following theorem gives the solution for this optimal capacity decision.

Theorem 3.1. *The optimal posted capacity share decision* x^{opt} *is uniquely exist for the risk-neutral seller if the buyer's equilibrium bid function* $Y_S^{EQ}(x, \alpha, d)$ *is concave increasing in* x*.* x^{opt} *is the solution of the equation bellow.*

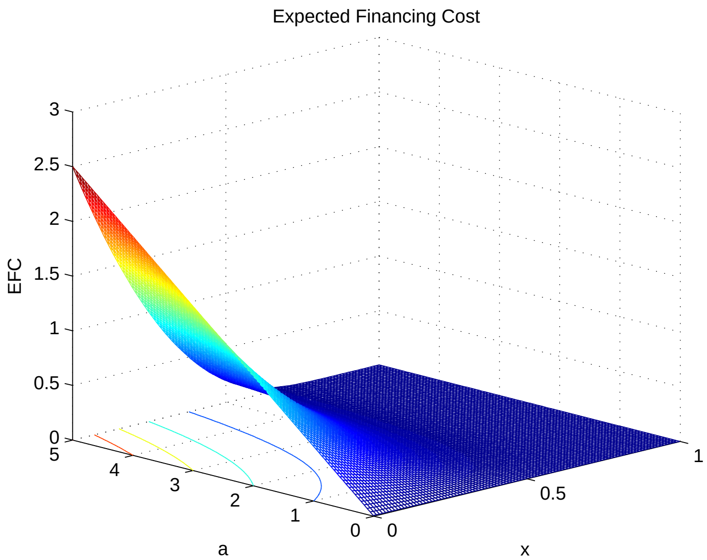

Figure 3.5: Expected Financing Cost

$$r\Pr\left(\epsilon_0 > 1 - x - d_0\right) = \frac{d\left(E\left[Y^{EQ}_{(N-1)}(x)\right]\right)}{dx} - \frac{\alpha_0}{2}\frac{d\left(E\left[\left(C - Y^{EQ}_{(N-1)}(x)\right)^{+}\right]^2\right)}{dx} \tag{3.11}$$

The next proposition provides the relationship between the optimal capacity share x^{opt} and the seller's signal (α_0, d_0).

Proposition 3.5. *The seller's optimal capacity share to be sold x^{opt} is decreasing in the seller's expected demand d_0 while increasing in the seller's financial friction α_0.*

If the seller's expected demand d_0 is increasing it is more likely that he will have a higher realized demand. Thus he is incented to reserve more capacity to fulfill his future demands which will lead to more revenue. This will result in a shrinkage in the optimal capacity share x^{opt}. But if the seller's financing friction α_0 is increasing the seller's relative advantage in financing will decrease. It is better for the seller to

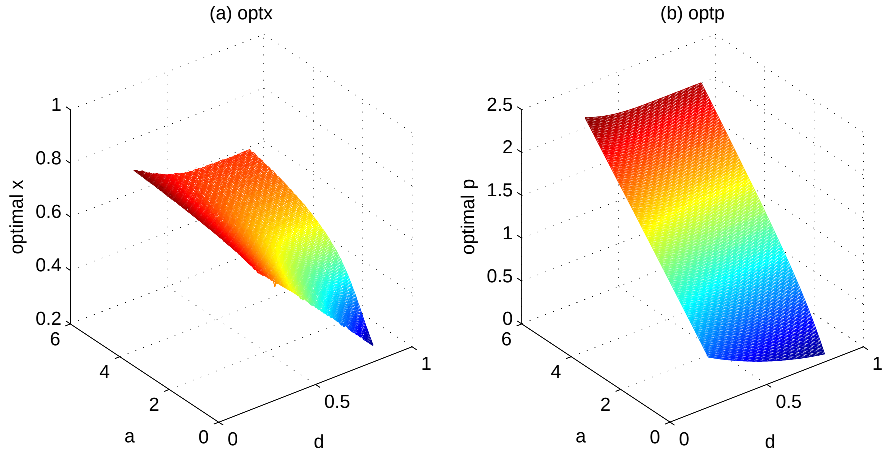

Figure 3.6: Seller's Optimal Capacity Decision

put more capacity share to sale as a way to release the financial pressure even his expected demand is relative high. The capacity-cost sharing scheme and the auction mechanism offer better chance for the seller to optimize its capacity and financing decisions.

3.2 Risk-Aversion

In this section we will focus our attentions on risk-averse cases. All the results in Section3.1 to risk-averse cases. First we develop a proposition about the buyer's problem (BP-S) in Equation (2.4).

Proposition 3.6. *$U(\cdot)$ is a concave increasing utility function. The buyer's Bayesian-Nash equilibrium bid function $Y_S^{EQ}(x, \alpha, d)$ is the solution of the equation bellow.*

$$E_\epsilon \left[U \left(r \min(x, d+\epsilon) - \frac{\alpha}{2} y^2 - y \right) \right] = U(0) \tag{3.12}$$

The result is similar to the risk-neutral case in Section 3.1. The equilibrium bidding strategy requires the expected utility equals to $U(0)$. Due to the concavity of $U(\cdot)$ it's easy to know that if the buyer is risk averse the bid level y will be less than the risk-neutral case.

The next proposition is about the concavity of $Y_S^{EQ}(x, \alpha, d)$ as a function of the capacity share x.

Proposition 3.7. *Fix any signal (α, d), the equilibrium bidding function $Y_S^{EQ}(x, \alpha, d)$ for (BP-S) is concave increasing in the capacity share x.*

The concavity is reserved for the equilibrium bid function $Y_S^{EQ}(x, \alpha, d)$ in the case that the buyers are risk-averse. This is result of monotonicity of the utility function $U(\cdot)$. In the next theorem we will find that the concavity of $Y_S^{EQ}(x, \alpha, d)$ is a sufficient condition for the existence and the uniqueness of optimal capacity share x^{opt} for risk-averse seller.

Theorem 3.2. *The optimal posted capacity share decision x^{opt} is uniquely exist for the risk-averse seller if the buyer's equilibrium bid function $Y_S^{EQ}(x, \alpha, d)$ is concave increasing in x. x^{opt} is the solution for the first order condition.*

Similar results about the relationship between the optimal capacity share x^{opt} and the buyer's signal (α_0, d_0) can be derived for risk-averse seller.

Proposition 3.8. *The seller's optimal capacity share to be sold x^{opt} is decreasing in the seller's expected demand d_0 while increasing in the seller's financial friction α_0.*

3.3 Proof of Statements

3.3.1 Proof of Proposition 3.1.

The expected profit for the buyer for bidding y is

$$\Pi^S(y) = \left(rE(x,d) - E\left[\frac{\alpha}{2}z^2 + z\middle|z < y\right]\right) G_S(y) \tag{3.13}$$

Set the first order derivative equals to zero. We get the equation bellow.

$$\left(rE(x,d) - \frac{\alpha}{2}y^2 - y\right) G'_S(y) = 0 \tag{3.14}$$

$\Longrightarrow$

$$rE(x,d) - \frac{\alpha}{2}y^2 - y = 0 \tag{3.15}$$

So the bid function is

$$Y_S^{EQ}(x,\alpha,d) = \frac{\sqrt{1+2\alpha rE(x,d)} - 1}{\alpha} \tag{3.16}$$

This proved Proposition 3.1.

3.3.2 Proof of Proposition 3.2.

$$\text{EOR}(x) = E_{\epsilon_0}\left[r\min(1-x, d_0+\epsilon_0)\right] \tag{3.17}$$

By taking derivative of EOR(x) we have.

$$\frac{d\text{EOR}(x)}{dx} = -r\Pr(\epsilon_0 > 1-x-d_0) \leq 0 \tag{3.18}$$

$$\frac{d^2\text{EOR}(x)}{dx^2} = -rf_{\epsilon_0}(1-x-d_0) \leq 0 \tag{3.19}$$

This proved Proposition 3.2.

3.3.3 Proof of Proposition 3.3.

Choose an arbitrary pair of signal from the sample space, keep it fixed. Without loss of generality let's assume the signal is (α, d). By taking derivatives we have the following equations which proved $E(x, d)$ is concave increasing in x.

$$\begin{cases} E_1'(x,d) = F_\epsilon(x-d) \\ E_1 1''(x,d) = -f_\epsilon(x-d) \end{cases} \tag{3.20}$$

Due to the monotonicity of $Y_S^{EQ}(x, \alpha, d)$ on $E(x, d)$, $Y_S^{EQ}(x, \alpha, d)$ is concave increasing in x. Since (α, d) is arbitrarily selected Proposition 3.3 is proved.

3.3.4 Proof of Lemma 3.1.

$$\Pr(Y_i = Y_j | Y_j = t) = \iint_{(a,\xi)\in\{(a,\xi):a=\mathcal{A}(\xi,x,t)\}} f_\alpha(a) f_d(\xi) da d\xi = 0 \tag{3.21}$$

$\Longrightarrow$

$$\Pr(Y_i = Y_j) = \int_{L(x)}^{U(x)} \Pr(Y_i = Y_j | Y_j = t) f_{Y_j}(t) dt = 0 \tag{3.22}$$

This proved Lemma 3.1.

3.3.5 Proof of Proposition 3.4.

$\forall x_0 \in [0,1]$, let $D = [\underline{d}, \bar{d}]$, $A = [\underline{\alpha}, \bar{\alpha}]$. Define

$$I = \Big\{ \prod_{i=1}^{N-1} (\alpha_i, d_i) \in \prod_{i=1}^{N-1} (A \times D) : \exists i \neq j, Y_i(\alpha_i, d_i; x) = Y_j(\alpha_j, d_j; x) \Big\} \tag{3.23}$$

1. $\forall \prod_{i=1}^{N-1} (\alpha_i, d_i) \in I^c$

   Without lose of generality assume that

$$Y_1 < Y_2 < \cdots < Y_{N-1} < Y_N \tag{3.24}$$

$\forall Y_i < Y_{i+k}$, $\exists Y$ that $Y_i < Y < Y_{i+k}$. And

$$\begin{cases} \mathcal{A}(d_i; x, Y) < \alpha_i \\ \mathcal{A}(d_{i+k}; x, Y) > \alpha_{i+k} \end{cases} \tag{3.25}$$

Denote $r_1 = \alpha_i - \mathcal{A}(d_i; x, Y)$ and $r_2 = \mathcal{A}(d_{i+k}; x, Y) - \alpha_{i+k}$. Due to the continuous of iso-bid function (refers to Lemma **??**). There exist δ_1 and δ_2 that

$\forall(\hat{x}_1, \hat{y}_1) \in \left\{(\hat{x}, \hat{y}) : \|(\hat{x}, \hat{y}) - (x, Y)\|_2 < \delta_1\right\}$ we have $\mathcal{A}(d_i; \hat{x}_1, \hat{y}_1) < \alpha_i$;

$\forall(\hat{x}_2, \hat{y}_2) \in \left\{(\hat{x}, \hat{y}) : \|(\hat{x}, \hat{y}) - (x, Y)\|_2 < \delta_2\right\}$ we have $\mathcal{A}(d_{i+k}; \hat{x}_2, \hat{y}_2) < \alpha_i$;

Let $\delta = \min(\delta_1, \delta_2)$, $\exists \Delta > 0$ that

$(x + \Delta, y) \in \left\{(\hat{x}, \hat{y}) : \|(\hat{x}, \hat{y}) - (x, Y)\|_2 < \delta_2\right\}$, which satisfy the following equations.

$$\begin{cases} \mathcal{A}(d_i; x + \Delta, y) < \alpha_i \\ \mathcal{A}(d_{i+k}; x + \Delta, y) < \alpha_{i+k} \end{cases} \tag{3.26}$$

This means for all $x \in (x_0, x_0 + \Delta)$, $Y_i(x) < Y_{i+k}(x)$ if $Y_i(x_0) < Y_{i+k}(x_0)$. So $Y_{(N-1)(x)} = Y_{N-1}(x)$, $\forall x \in (x_0, x_0 + \Delta)$.

Since $Y_{N-1}(x)$ is concave increasing in x for all $\prod_{i=1}^{N-1}(\alpha_i, d_i) \in I^c$. We have

$$\begin{aligned}
\mathrm{EAR}(x) &= E\big[Y_{(N-1)}\big] \\
&= E\Big[Y_{(N-1)}\Big|\prod_{i=1}^{N-1}(\alpha_i,d_i)\in I^c\Big]\Pr\Big(\prod_{i=1}^{N-1}(\alpha_i,d_i)\in I^c\Big) \\
&\quad +E\Big[Y_{(N-1)}\Big|\prod_{i=1}^{N-1}(\alpha_i,d_i)\in I\Big]\Pr\Big(\prod_{i=1}^{N-1}(\alpha_i,d_i)\in I\Big) \\
&= E\Big[Y_{(N-1)}\Big|\prod_{i=1}^{N-1}(\alpha_i,d_i)\in I^c\Big] \qquad (3.27)
\end{aligned}$$

Which is concave increasing in x, $\forall x \in [x_0, x_0 + \Delta]$.

2. Since $\Big[\big(C - Y_i(x)\big)^+\Big]^2$ is convex decreasing in x, $\forall x \in [0, 1]$, for the same reason presented in the prove of (1) we can prove (2).

This proved Proposition 3.4.

3.3.6 Proof of Theorem 3.1.

Since EOR(x) and EAR(x) are concave increasing in x and EFC(x) is convex decreasing in x, the seller's objective function is concave increasing in x. So the seller's problem has unique optimal solution. The solution is determined by the first order condition which is presented as bellow.

$$r\Pr(\epsilon_0 > 1 - x - d_0) = \frac{d\left(E\left[Y^{EQ}_{(N-1)}(x)\right]\right)}{dx} - \frac{\alpha_0}{2}\frac{d\left(E\left[\left(C - Y^{EQ}_{(N-1)}(x)\right)^+\right]^2\right)}{dx} \tag{3.28}$$

This proved Theorem 3.1.

3.3.7 Proof of Proposition 3.5.

Define $H(x, \alpha, d)$ as followed.

$$H(x, \alpha, d) = \frac{r\partial E(1-x, d_0)}{\partial x} + \frac{\partial\left(\text{EAR}(x)\right)}{\partial x} - \frac{\alpha_0}{2} \frac{\partial\left(E\left[\left(C - Y^{EQ}_{(N-1)}(x)\right)^{+}\right]^2\right)}{\partial x} \tag{3.29}$$

$$\frac{\partial H}{\partial x^{opt}} = \frac{d^2\text{EOR}(x)}{dx^2} + \frac{d^2\text{EAR}(x)}{dx^2} - \frac{d^2\text{EFC}(x)}{dx^2} \leq 0 \tag{3.30}$$

$$\frac{\partial H}{\partial \alpha_0} = -\frac{1}{\alpha_0}\frac{d\text{EFC}(x)}{dx} \geq 0 \tag{3.31}$$

$$\frac{\partial H}{\partial d_0} = -rf_{\epsilon_0}(1 - x - d_0) \leq 0 \tag{3.32}$$

So the partial derivatives of x^{opt} are as followed.

$$\frac{\partial x^{opt}}{\partial \alpha_0} = -\frac{\frac{\partial H}{\partial \alpha_0}}{\frac{\partial H}{\partial x^{opt}}} \geq 0 \tag{3.33}$$

$$\frac{\partial x^{opt}}{\partial d_0} = -\frac{\frac{\partial H}{\partial d_0}}{\frac{\partial H}{\partial x^{opt}}} \leq 0 \tag{3.34}$$

These proved Proposition 3.5.

3.3.8 Proof of Proposition 3.6.

Since ϵ is independent with Y_i, $\forall i$.

$$E\left[U\left(\pi(x,\alpha,d,Z,\epsilon)\right)|Z<y\right] = E_Z\left[E_\epsilon\left[U\left(\pi(x,\alpha,d,Z,\epsilon)\right)\right]|Z<y\right] \tag{3.35}$$

We have

$$E\left[U\left(\pi(x,\alpha,d,Z,\epsilon)\right)|Z<y\right]G_S(y) = \int_0^y E_\epsilon\left[U\left(\pi(x,\alpha,d,t,\epsilon)\right)\right]G'_S(t)dt \tag{3.36}$$

Taking the derivative of the buyer's expected profit refers to the bid level y results the following equation.

$$E_\epsilon\left[U\left(\pi(x,\alpha,d,y,\epsilon)\right)\right]G'_S(y) - U(0)G'_S(y) = 0 \tag{3.37}$$

So we have

$$E_\epsilon\left[U\left(\pi(x,\alpha,d,y,\epsilon)\right)\right] = U(0) \tag{3.38}$$

$\Longrightarrow$

$$E_\epsilon \left[U\left(r\min(x, d+\epsilon) - \frac{\alpha}{2}y^2 - y\right)\right] = U(0) \tag{3.39}$$

Which proved the proposition.

3.3.9 Proof of Proposition 3.7.

$$F(x,y) \stackrel{\text{def}}{=} E_\epsilon \left[U\left(r\min(x, d+\epsilon) - \frac{\alpha}{2}y^2 - y\right)\right] - U(0) \tag{3.40}$$

We know that by definition

$$F(x,y) \equiv 0 \tag{3.41}$$

So we have

$$\begin{cases} \frac{dF(x,y)}{dx} = 0 \\ \frac{d^2F(x,y)}{dx^2} = 0 \end{cases} \tag{3.42}$$

$\Longrightarrow$

$$\begin{cases} y'(x) = -\frac{F_1'}{F_2'} \\ y''(x) = -\frac{F_{11}''+F_{12}''y'+\left(F_{21}''+F_{22}''y'\right)y'}{F_2'} \end{cases} \tag{3.43}$$

It is obviously that there's no cross term in F so we know that

$$F''_{12} = F''_{21} = 0 \tag{3.44}$$

$\Longrightarrow$

$$y''(x) = \frac{F''_{11} + F''_{22}\left(\frac{F'_1}{F'_2}\right)^2}{F'_2} \tag{3.45}$$

Since

$$\begin{aligned} F'_1 &= \frac{\partial\left(E_\epsilon\left[U\left(r\min(x, d+\epsilon) - \frac{\alpha}{2}y^2 - y\right)\right]\right)}{\partial x} \\ &= \int_{\underline{\epsilon}}^{\bar{\epsilon}} U' \cdot \left(r\frac{d\left(\min(x, d+t)\right)}{dx}\right) f_\epsilon(t)dt \\ &= \int_{\underline{\epsilon}}^{\bar{\epsilon}} U' \cdot \left(rI_{\{t:x<d+t\}}(t)\right) f_\epsilon(t)dt \geq 0 \end{aligned} \tag{3.46}$$

$$\begin{aligned} F''_{11} &= \frac{\partial F'_1}{\partial x} \\ &= \int_{\underline{\epsilon}}^{\bar{\epsilon}} U'' \cdot \left(rI_{\{t:x<d+t\}}(t)\right)^2 f_\epsilon(t)dt \leq 0 \end{aligned} \tag{3.47}$$

$$\begin{aligned} F_2' &= \frac{\partial\left(E_\epsilon\left[U\left(r\min(x,d+\epsilon)-\frac{\alpha}{2}y^2-y\right)\right]\right)}{\partial y} \\ &= \int_{\underline{\epsilon}}^{\bar{\epsilon}} U'\cdot\left(\frac{d\left(-\frac{\alpha}{2}y^2-y\right)}{dy}\right) f_\epsilon(t)dt \\ &= -\int_{\underline{\epsilon}}^{\bar{\epsilon}} U'\cdot(\alpha y+1)f_\epsilon(t)dt \le 0 \end{aligned} \tag{3.48}$$

$$\begin{aligned} F_{22}'' &= \frac{\partial F_2'}{\partial y} \\ &= -\int_{\underline{\epsilon}}^{\bar{\epsilon}} \left(-U''(\alpha y+1)^2+\alpha U'\right) f_\epsilon(t)dt \le 0 \end{aligned} \tag{3.49}$$

So

$$y''(x) = -\frac{F_{11}''+F_{22}''\left(\frac{F_1'}{F_2'}\right)^2}{F_2'} \le 0 \tag{3.50}$$

This proved the proposition.

3.3.10 Proof of Theorem 3.2.

Since the concavity (convexity) of the operational revenue term and auction revenue term (financing cost term) are reserved, and the utility function is monotonically increasing the objective function is concave in x. So the first order condition gives the optimal solution.

3.3.11 Proof of Proposition 3.8.

The proof is similar to the proof of Proposition 3.5.

Chapter 4

A Numerical Example

In this chapter we use a numerical example to illustrate the benefits of our capacity-cost sharing scheme and the auction implementation. In our example there're 10 bidders participate in the auction. Buyer's expected demand d_i are randomly drawn from $U(0.2, 0.8)$. While their financial friction α_i are randomly drawn from $U(0.01, 5.00)$. The purchasing cost c is set to be 1 and the total revenue r is set to be 2.25. Second-price sealed auction is implemented in our example. All participants are risk-neutral. The benefits of our capacity-cost sharing scheme is studied in Section 4.1. In Section 4.2 we compare the seller's optimal strategy with an alternative strategy. The buyer's expected cost share is compared in Section 4.3.

4.1 The Benefits of Capacity-Cost Sharing Scheme

We study the benefits of our capacity-cost sharing scheme for the seller in this section. Second-price sealed auction is implemented to select a partner for our shipping firm. As the alternative scheme the seller will choose not to share the capacity. Under this scheme the seller will pay all the purchasing cost by its own. We compare the expected profits for the seller for different signals. The result is displayed in Table 4.1.

Table 4.1: The Benefits of the Scheme

Signals		Expected Profits		Benefits
d_0	α_0	Share	Not Share	for Seller
0.20	0.01	0.3028	-0.5550	0.2533
0.20	1.70	0.2737	-1.4000	0.2737
0.20	3.30	0.2465	-2.2000	0.2465
0.20	5.00	0.2180	-3.0500	0.2180
0.40	0.01	0.6486	-1.0500	0.6486
0.40	1.70	0.6031	-0.9500	0.6031
0.40	3.30	0.5628	-1.7500	0.5628
0.40	5.00	0.5219	-2.6000	0.5219
0.60	0.01	0.9142	0.3450	0.5692
0.60	1.70	0.8205	-0.5000	0.8205
0.60	3.30	0.7475	-1.3000	0.7475
0.60	5.00	0.6801	-2.1500	0.6801
0.80	0.01	1.1002	0.7931	0.3071
0.80	1.70	0.9086	-0.5190	0.9086
0.80	3.30	0.7959	-0.8519	0.7959
0.80	5.00	0.7085	-1.7019	0.7085

As observed from Table 4.1 the capacity-cost sharing scheme performs much better than the alternative non-sharing scheme. Under the non-sharing scheme the shipping firm's expected profits are negative for most cases. Even the buyer's expected demand is high (for example $d_0 = 0.8$) its profit could be negative if the financial friction is high. This is partially due to the high financing cost. The rational decision for the shipping firm is to abort the purchasing plan in this case. However if the shipping firm adopt our capacity-cost sharing scheme and implement our optimal strategy its expected profits will be positive for the same signals. The capacity-cost sharing scheme offers the shipping firm the opportunity to make extra profits by expending its shipping capacity which it can not afford alone. The last column of Table 4.1 represents the benefits of the sharing scheme compared to the non-sharing scheme.

The benefit is significant for the seller. Buyers can also benefit from the sharing scheme. Since the winning buyer only need to pay the second highest bid level instead of his winning bid, he will have a positive expected profit from the shared capacity.

4.2 The Benefits of Our Optimal Strategy

In this section we will compare the performance of the seller's optimal capacity decision (Strategy A) with an reasonable alternative strategy (Strategy B) under the capacity-cost sharing scheme.

Strategy A: The percentage x_A^* of capacity to share is determined by Theorem 3.1.

Strategy B: The percentage x_B^* of capacity to share is set to be $1 - d_0$.

Table 4.2: Expected Profits Comparisions

Signals		*A*		*B*		$(\pi_A^* - \pi_B^*)/\pi_B^*$
d_0	α_0	x_A^*	π_A^*	x_B^*	π_B^*	(%)
0.20	0.01	0.6967	0.3028	0.80	0.2533	19.54%
0.20	1.70	0.7023	0.2737	0.80	0.2279	20.10%
0.20	3.30	0.7080	0.2465	0.80	0.2019	20.92%
0.20	5.00	0.7131	0.2180	0.80	0.1783	22.24%
0.40	0.01	0.5527	0.6486	0.60	0.6362	1.94%
0.40	1.70	0.5658	0.6031	0.60	0.5973	0.96%
0.40	3.30	0.5830	0.5628	0.60	0.5606	0.40%
0.40	5.00	0.5921	0.5219	0.60	0.5215	0.08%
0.60	0.01	0.4010	0.9142	0.40	0.9141	0.01%
0.60	1.70	0.4421	0.8205	0.40	0.8079	1.57%
0.60	3.30	0.4712	0.7475	0.40	0.7073	5.69%
0.60	5.00	0.4940	0.6801	0.40	0.6004	13.28%
0.80	0.01	0.2454	1.1002	0.20	1.0889	1.04%
0.80	1.70	0.3451	0.9086	0.20	0.7642	18.89%
0.80	3.30	0.4134	0.7959	0.20	0.4569	74.21%
0.80	5.00	0.4575	0.7085	0.20	0.1303	443.86%

Strategy A adopt our result with the optimal capacity share while Strategy B select to sell its expected extra capacity. Notice that expectation is the best estimation for the future demand from statistical perspective. So if the shipping firm considers the capacity decision and financial friction separately it is quite reasonable to adopt strategy B.

Observed from Table 4.2 strategy A performs much better that strategy B. Especially when the shipping firm's expected demand is high ($d_0 = 0.80$) or low ($d_0 = 0.20$) the expected profit of strategy A is 20% higher than strategy B. We can see that when the the expected demand is low ($d_0 = 0.2$ or 0.4) the seller tends to sell less capacity than its expected extra capacity. The reason is when x increases buyers will have more financial pressure on bidding higher. So the marginal benefit from increasing x decreases. For the opposite case, if the expected demand is high ($d_0 = 0.60$ or 0.80) the seller tends to sell more than its expected extra capacity. By selling more the shipping firm can take advantage of buyers' financial condition and competition.

4.3 Cost Sharing Comparisons

In this section study the expected buyer's cost as a percentage of the total purchasing cost. We compare the buyer's cost share with its capacity share x. We find in Table 4.3 that the buyer's share on purchasing cost is larger than its share on the capacity for all signal combinations in our numerical examples. The difference between buyer's cost share and capacity share is significant. For most cases in our example the gap is more than 20%. This is a benefit of auction. Usually there is sufficient negotiation space between buyer and seller. The final price is determined by player's power and information in a bilateral negotiation. But under the scenario of auction buyers are forced to compete with each other. Auction will push the final price towards the winning buyer's bottom line.

Table 4.3: Cost Share Comparison

d_0	α_0	Cost Share (%)	x^* (%)	Difference (%)
0.20	0.01	87.06%	69.67%	17.38%
0.20	1.70	87.26%	70.23%	17.01%
0.20	3.30	87.42%	70.80%	16.62%
0.20	5.00	87.57%	71.31%	16.26%
0.40	0.01	79.53%	55.27%	24.26%
0.40	1.70	80.44%	56.58%	23.86%
0.40	3.30	81.58%	58.30%	23.28%
0.40	5.00	82.15%	59.21%	22.94%
0.60	0.01	65.57%	40.10%	25.47%
0.60	1.70	69.94%	44.21%	25.73%
0.60	3.30	72.78%	47.12%	25.66%
0.60	5.00	74.84%	49.40%	25.44%
0.80	0.01	45.17%	24.54%	20.63%
0.80	1.70	58.94%	34.51%	24.43%
0.80	3.30	66.94%	41.34%	25.60%
0.80	5.00	71.47%	45.75%	25.73%

Chapter 5

Conclusions

In this thesis we studied the risk management problem under the scenario of ship procurement. We introduced the capacity-cost sharing scheme for this problem. A general model was established to use auction as the mechanism to select a partner. We derived the optimal capacity decision for the seller and the equilibrium bidding strategy for buyers for second-price sealed auction implemented case. All results are extended to the case that the buyers and seller are risk-averse.

Several assumptions are made in this paper. The private information assumption we made in the paper is common in auction literatures. And it is an appropriate generalization. To model the participators' financial situation by assuming the financing cost function $c(\alpha, z)$ as the product of the financial friction α and $\frac{z^2}{2}$. This financing function is the only continuous function that satisfies the conditions which is reasonable for the financing cost. And this kind of financing cost function is consist with Che and Gale (1998). Participators' demands are assumed to be the summation of the expected demands and noises. We have assumed that the noises are independent identical distributed, this may be different from real world. By our results will not affect by this assumption. Another critical assumption is that both buyers and the seller share the same utility function. Since all participators are playing in the same industry it is reasonable to assume they have similar opinion on risk issues which is under our consideration.

The numerical example illustrated the benefits of the capacity-cost sharing scheme. Compared to the non-sharing scheme, the seller will have better chance to make more profits. For many cases in our example the buyer even can not afford the vessel purchasing plan without the sharing scheme. Buyers also benefit from the sharing scheme. It offers them an opportunity to expend their shipping capacity for a reasonable price. Besides, the utilization of the capacity is increased under the sharing scheme which is considered to be the benefit for the society. The implementation of auction increases the competition among buyers. According to our numerical example, the expected percentage of cost shared by the winning buyer is significant higher that the percentage of capacity it bid for. We also compared the seller's optimal capacity decision strategy with a reasonable alternative strategy, for which the seller considers the capacity decision separately with the financial friction. The result demonstrate the seller's expected profit will increase significantly if it adopts the optimal strategy suggested by this thesis.

There're still a lot of extensions for this project.

(i) The solutions for First-Price Sealed Auction. The uniqueness of the equilibrium bid function and the concavity of the equilibrium bid level on the capacity share x are not proved. The equilibrium bid function for multi-dimensional auctions was considered to be a hard problem in auction research for a long time. The main difficulty is that it is difficult to define an appropriate order on the multi-dimensional signals. However for our particular problem structure we can take advantage of isobid function. It is hopeful that the equilibrium bid function can be developed later.

(ii) Comparisons of the differences between risk-neutral case and risk-averse case for both first- and second-price auctions are another extension. This is not difficult for buyer's problem in techniques. We can make some discoveries on the buyer's decision by comparing the derivatives of their bid functions. However the seller's problem may be more complicated but our method will still work for that case.

(iii) Comparisons of the differences between First-Price Sealed Auction and Second-Price Sealed Auction for both risk-neutral and risk-averse case. The results on these comparisons are not clear now. Che and Gale (1998) developed a method to do such kind comparisons. We'll try to compare our results to theirs if the First-Price Sealed Auction is solved.

(iv) Our capacity-cost sharing scheme can be applied to many other problem concerning with manufacturing and service capacity decision problems.

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