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Rolling ADF Tests: Detecting Rational Bubbles in Greater China Stock Markets

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Rolling ADF Tests: Detecting Rational Bubbles in Greater China Stock Markets



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Abstract

Following Phillips, Wu and Yu (2007), this paper extends their bubble detecting work to several Greater China stock markets. Two alternative bubble detecting methods, the forward recursive ADF tests raised by Phillips et al. (2007) and the modified version, forward rolling ADF tests, are implemented and compared. Monte Carlo simulations are performed to determine the critical values of the ADF statistic under different sample size. Empirical results demonstrate that only rolling ADF tests are successful in detecting rational bubbles by overcoming the problem of periodically collapsing bubble. As we have expected, bubbles in China Mainland stock market are detected. Out of our expectation, significant and long standing bubbles are also found in Hong Kong, Taiwan and Singapore stock markets. However, the styles of rational bubbles in different stock markets are different. Differences between the transition stage of China Mainland and the mature stage of other Greater China economies should be one important reason that leads to the different stock market speculative behaviors during the same period. At last, the potential time when bubble begins to collapse is investigated.

Keywords: Rational bubbles, forward recursive ADF tests, forward rolling ADF tests, bubble size, speculative behavior, bubble collapse

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1 Introduction

1.1 Economic Background

During the last season of 2007, stock market price of China Mainland reached its highest level in history from its ten-year lowest level in Dec 2005. In less than two years, the index doubled twice or more. Many informal commentators attributed this steep rise in stock prices to the presence of a bubble. However, price level and dividend level in the stock market of China Mainland also changed greatly during this period. As a result, whether there was really a bubble or not is an issue that should be substantiated using economic theories and econometric methods. What's more interesting, some other Greater China economies which are closely related with China Mainland also experienced prosperities in their stock markets during the same period, although the sign of bubble in these markets is not as obvious as in China Mainland. Do bubbles also exist in these economies? If the answer is "yes", what are the differences among these bubbles? Further more, we should find and provide possible economic explanations or mechanisms for the phenomenon.

1.2 Literature Review

A large and growing number of papers in the literature are focusing on financial bubbles. Some of them have similar starting point of defining financial bubbles as rational bubbles based on rational expectation models. In this paper, we also define bubble as rational bubble. By solving consumers' optimization problem and assuming no-arbitrage and no rational bubble, we can get the standard present value model for asset price (Gurkaynak, 2005):

$$P_t = \frac{1}{1+R} E_t(P_{t+1} + D_{t+1}) \quad (1)$$

$$= \sum_{i=1}^{\infty} \left(\frac{1}{1+R} \right)^i E_t(D_{t+i}) \quad (2)$$

where P_t is the real stock price at time t and D_t is the real dividend received from the equity between $t-1$ and t , and R is the discount rate ($R > 0$).

Equity prices contain a rational bubble if investors are willing to pay more for the stock than they know is justified by the value of the discounted dividend stream because they expect to be able to sell it at an even higher price in the future, making the current high price an equilibrium price. The pricing of the asset is still rational (Gurkaynak, 2005). This can be explained by the following equations:

$$P_t = \sum_{i=1}^{\infty} \left(\frac{1}{1+R} \right)^i E_t(D_{t+i}) + B_t = P_t^f + B_t \quad (3)$$

$$st. E_t(B_{t+1}) = (1+g)B_t \quad (4)$$

where the first part in equation (3) is the fundamental part and the second is the bubble part. Equation (4) reflects the self-confirming belief. The prices are rational because they are consistent with investors' expectation or belief.

In accordance with the idea of rational bubble, economists were trying and are trying to detect bubbles using many different methods, but till now almost none of them are enough satisfying (Gurkaynak, 2005). We will first briefly review the bubble testing methods that have little structure on bubbles. Two of them are famous.

Variance bounds test for equity prices are initiated by Shiller (1981) and LeRoy and Porter (1981). This method is first initiated for testing random walk hypothesis in stock price but then used by some economists as a criterion for the existence of bubble. It proposes an upper bound on the variance of the observed price series based on dividend series. If the variance bound is violated in data, this may be an evidence of the presence of a bubble. Variance bounds test have sharp drawbacks: theoretically, it is not originally designed as a test for bubble but as a test for the present value model and the rejection may be due to other model failings but not a bubble; practically, Cochrane (1992) finds a striking counterexample that violates the variance bound without requiring a bubble.

Another one is the duration dependence test used by Chan et al. (1998), Harman and Zuehlke (2001, 2004), among many others. It is built on the rational expectation model and the statistical theory of duration dependence. Similar to the theory of rational bubble, in an asset market experiencing a bubble, investors are aware that prices of securities exceed their fundamental values but may still want to purchase securities because they believe that prices will appreciate further. Rational bubble exists only if the probability of a higher return rises to compensate for the increased probability of crash. Thus according to the duration dependence method, the probability that a run of positive abnormal return ends should decrease with the length of the run (sequence of the returns with the same sign) if bubble exists in the security market (Mokhtar et al., 2006). This method is good at detecting bubble in some cases: Mokhtar et al. detect bubble in Malaysia stock market and Rangel et al. (2007) do so in Singapore stock market. However, the definition for the sign in a run is arbitrary and different from paper to paper, which can change the empirical conclusions for whether bubble exists (Rangel et al., 2007).

According to the above two methods, it may be argued that empirical tests for bubbles are uninteresting because they can be ruled out by other reasonable factors. Other economists, however, alleviate this problem by trying to add structures on bubbles based on other specific definition or theory for bubble. The rest of this paper looks into the bubble model we choose.

There is an insightful description for rational speculative bubble in the stock market made by Koustas and Serletis (2005) that rational speculative bubbles must be continually expanding since stock buyers must pay a price higher than that suggested by the fundamentals if they believe that someone else will subsequently pay an even higher price. As long as the belief is persistent, bubble will be growing explosively. This description implies that bubble may appear in a form of explosive time series. Different from the above two models, Diba and Grossman (1988) propose a test for bubble which allows for unobserved fundamentals, and imposes such structure on

which deviations from fundamentals in data may be blamed on the presence of bubble. Two famous tests, Dickey-Fuller tests and cointegration tests are used to detect bubble. However, both of them fail in finding evidence of bubble in a prosperous market (Gurkaynak, 2005).

Evans (1991) argues that it is possible that bubble will collapse to a small nonzero value and then continue increasing. This leads to the result that the unit root based tests have difficulty in detecting collapsing bubbles because they behave more like stationary processes than explosive processes as a result of the periodic collapses involved. Evans' criticism of unit root tests for rational bubbles lead to a number of papers trying to overcome the difficulty of detecting collapsing bubbles, such as Hall, Psaradakis, and Sola (1999), Norden and Vigfusson (1998) and Driffill and Sola (1998). Recently, Phillips, Wu and Yu (2007) develop a new method that ameliorated the periodically collapsing bubble problem and obtained satisfactory results using forward recursive ADF test. They not only succeed in detecting significant bubble during the expected NASDAQ bubble period, but also in data-stamping the start and the end of the bubble.

In this paper, we follow their work and modify the recursive ADF test to rolling ADF test and apply the new method to four Greater China stock markets. The results show some interesting things that we have expected and also something that we have not expected. Unlike previous studies, this paper takes the problem of bubble collapse into consideration for the first time. However, due to the limit of sample size, no formal empirical work is done for bubble collapse. Note that we will only consider positive bubble in this paper. That is, bubble exists on the first day of the market and will not burst or become negative. Arguments about positive bubble can be found in Diba and Grossman (1988) and Evans (1991).

The content of this paper is arranged as follows. In section 2, the models which represent bubble are presented. Two different bubble testing procedures, forward re-

cursive ADF tests and its modified version, forward rolling ADF tests are introduced. Section 3 discusses the data we use in this paper. Section 4 applies the two models introduced in section 2. Important empirical results are shown and analyzed. The last section gives several explanations for the empirical findings and concludes.

2 Models and Methods

2.1 Theoretical Framework for Bubble

For the derivation of the core of the bubble detecting model, this paper follows the setup in Phillips et al. (2007). In the rational bubble literature that bubbles, if they are present, should manifest explosive characteristics in prices. This statistical property motivates an expression of bubble in terms of explosive autoregressive behavior propagated by a process of the autoregressive (AR) form

$$x_t = \mu_x + \rho x_{t-1} + \varepsilon_{x,t} \quad (5)$$

where for certain subperiods of the data, $\rho > 1$. Figure 1 in the appendices shows typical time series plots for stationary ($\rho = 0.8$), random walk ($\rho = 1$) and explosive ($\rho = 1.03$) processes with intercept $\mu_x = 0$ and error term $\varepsilon_{x,t} \sim i.i.d.N(0,1)$. The differences in the trajectories are apparent and useful to help us understand what is different between a stationary process and a bubble process.

In Phillips et al. (2007), similar to many researches, the concept of rational bubble is illustrated using the present value theory of finance whereby fundamental asset prices are determined by the sum of the present discounted values of expected future dividend sequence. We follow this idea and begin with equation (1), the standard present value model for asset price.

Following Campbell and Shiller (1989), Phillips et al. obtain a log-linear bubble

expression:

$$p_t = p_t^f + b_t \quad (6)$$

where

$$p_t^f = \frac{\kappa - \gamma}{1 - \rho} + \sum_{i=0}^{\infty} \rho^i E_t d_{t+1+i} \quad (7)$$

$$b_t = \lim_{i \rightarrow \infty} \rho^i E_t p_{t+i}$$

$$E_t(b_{t+1}) = \frac{1}{\rho} b_t = (1 + \exp(\overline{d - p})) b_t \quad (8)$$

with $p_t = \log(P_t)$, $d_t = \log(D_t)$, $\gamma = \log(1 + R)$, $\rho = 1 / (1 + \exp(\overline{d - p}))$, and $\overline{d - p}$ being the average log dividend-price ratio, and κ is a variable that related to ρ .

Following convention, p_t^f , which is exclusively determined by expected dividends, is called the fundamental component of the stock price, and b_t which satisfies the difference equation (9) below, is called the rational bubble component. Both components are expressed in natural logarithms.

$$b_t = \frac{1}{\rho} b_{t-1} + \varepsilon_{b,t} \equiv (1 + g) b_{t-1} + \varepsilon_{b,t} \quad (9)$$

$$E_{t-1}(\varepsilon_{b,t}) = 0$$

where $g = \frac{1}{\rho} - 1 = \exp(\overline{d - p}) > 0$ is the growth rate of the natural logarithm of the

bubble and $\varepsilon_{b,t}$ is a martingale difference.

As evident from (6), the stochastic properties of p_t are determined by those of p_t^f and b_t . When bubble is not expanding i.e., $g = 0, \forall t$, we will have $p_t = p_t^f + b_0$, where b_0 is the constant initial bubble which we do not know and p_t is determined solely by p_t^f and hence by d_t . In this case, from (7), we obtain

$$d_t - p_t = -\frac{\kappa - \gamma}{1 - \rho} - \sum_{i=0}^{\infty} \rho^i E_t(\Delta d_{t+1+i}) - b_0 \quad (10)$$

However, when bubble is expanding, i.e., $g > 0$, since equation (9) implies explosive behavior in b_t , p_t will also be explosive by equation (6). In this case there will be two situations for dividend series. One is that d_t is an integrated process I(1) or a stationary process I(0). Another case is that d_t is also explosive but this is hard to be explained. In the empirical work, it will be easier to judge whether bubble exists when the first situation is satisfied.

Based on equation (10), Phillips et al. look for explosive behavior in p_t and non-explosive behavior in d_t via right-tailed unit root tests. If p_t is explosive while d_t is non-explosive, there is evidence of bubble according to this special framework. Bubble may also exist in other situations, but in this paper we will only follow this idea to find evidence of bubble.

2.2 Forward Recursive and Forward Rolling ADF Tests

After the model for bubble is determined, the tests are implemented as follows. For time series x_t (log stock price divided by log dividend), we apply the augmented Dickey-Fuller (ADF) test for a unit root against the alternative of an explosive root (the right-tailed). That is, we estimate the following autoregressive specification by least squares

$$x_t = \mu_x + \delta x_{t-1} + \sum_{j=1}^J \phi_j \Delta x_{t-j} + \varepsilon_{x,t} \quad (11)$$

$$\varepsilon_{x,t} \sim NID(0, \delta_x^2)$$

for some given value of the lag parameter J , where NID denotes independent and normal distribution. The unit root null hypothesis is $H_0 : \delta = 1$ and the right-tailed

alternative hypothesis is $H_1 : \delta > 1$.

The optimal lag order J_{op} in equation (11) is determined by using significance tests suggested in Campbell and Perron (1991). The significance tests can be implemented like follows. Starting with J_{max} , the upper bound of the lag length, if the last included lag is significant (under the 5% significant level using the standard normal asymptotic distribution), we select $J_{op} = J_{max}$. If not, we reduce the order of the estimated autoregression by one until the coefficient on the last included lag is significant. If none is significant, we will suppose that $k = 1$. In the recursive and rolling procedures, such selection procedure will be used for every subsample repeatedly. Other lag length selection criteria such as Bayesian information criterion (BIC) and Akaike information criterion (AIC) are more common and also applicable here. However, simulation evidence presented in Hall (1990, 1994) suggests that data-based method induces little size distortion in finite samples and the performance of the ADF test is considerably improved when the lag length is selected from the data. Thus, our setup in this paper is that J_{op} should be determined by the data.

Further more, Said and Dickey (1984) argue that to get consistent estimates of the coefficients in equation (11) it is necessary to let J_{max} , the starting lag length in our significant test, be a function of sample size T . Schwert (1989) follows the intuition of Said and Dickey and suggested that $J_{max} = \text{int}\{12 \cdot (T/100)^{1/4}\}$. This is also consistent with the suggestion of Said and Dickey to use a high-order autoregressive process to approximate an unknown ARIMA process. In the empirical part, we will combine the significance test method and Schwert's setup for J_{max} to find the optimal lag order J_{op} .

In forward recursive ADF tests, ADF statistic is computed for each recursive sub-

sample. The subsamples are all from the data with size T . The first subsample includes the observations from the first observation $t_{rec}=1$ to the j th observation $t_{rec}=j$ and has a size of j . We extend each of the following subsample by adding one more observation than the previous one. Thus the last subsample is equivalent to the full sample and the total number of subsamples is $T-j$. The size of the initial subsample j can be selected arbitrarily as long as we only care about the result related with the later part of the data. In Phillips (2007), j is chosen to be 10% of the NASDAQ sample. We will follow this setting in the empirical part.

The rolling ADF procedure is a little bit different. The ADF statistic is computed for each rolling subsample. The subsamples are also from the data with size T . The first subsample includes the observations from the first observation $t_{rol}=1$ to the k th observation $t_{rol}=k$ and has a size of k . The second subsample includes the observations from the second observation $t_{rol}=1$ to the $(k+1)$ th observation $t_{rol}=k+1$ and also has a size of k . Similarly, for each of the following subsample we just move the subsample forward by one observation but keep the size fixed at k . Thus, every subsample has the same size and the total number of subsamples is $T-k$. In this paper, the conclusion of bubble is sensitive to the choice of k since ADF statistic changes as subsample size changes. Hence, how to determine the best subsample size is an important work before rolling ADF tests are implemented.

3 Data

We investigate the stock markets from four Greater China economies: China Mainland, Hong Kong, Taiwan and Singapore. Monthly observations on the composite

stock price index and composite dividend yield are obtained from DataStream and compute the composite dividend series from these two series. Then we take the monthly Consumer Price Index (CPI) from CEIC dataset to convert the above nominal series to real series. Our sample covers the period from May 1994 to Jun 2008 and comprises 170 monthly observations in each market.

Figure 2 in the appendices plots the time series trajectories of real price and real dividend for the four markets. These time series are all normalized to 100 at the beginning of the sample and then transformed to log form. The index series of China Mainland, Hong Kong and Singapore show some similarity that they climb up rapidly together in the last several periods. However, their dividend series look quite different from each other. The dividend series of China Mainland does not climb up with index during the last periods while Hong Kong and Singapore dividend series do so. We may guess that there must be some bubbles in China Mainland stock market but we cannot judge whether bubbles exist in Hong Kong and Singapore markets at this stage. For the stock market of Taiwan, there seems no sign of bubble according to the figure since the index series grows more slowly and steadily than the dividend series. However, it is too early to make judgment until empirical tests are performed.

4 Empirical Results

In this paper, R (time discount rate) is assumed to be time invariant since the change of R is not significant relative to the stock price and dividend during the whole sample. Another reason that the change of R is ignored is: fewer variables lead to fewer measure and estimation errors.

4.1 Subsample Size

For recursive ADF test different people may set different k_{start} but this has nothing to do with other subsamples except for the earlier ones. Thus, there is no problem with the selection of subsample size for recursive ADF tests. Different from recursive ADF tests, subsample size for rolling ADF tests is fixed in the rolling process. When sample size is too small, statistical characteristics of the estimators is poor, problem such as bias of coefficients will become serious. However, when sample size is too large, the problem of periodically collapsing bubbles demonstrated by Evan will become apparent. This is because some observations that do not belong to an explosive process may be included into the test. Conventionally, a better subsample size should be figured out first before further analysis can be carried out. In this paper, however, the best choice is that there is no single best subsample size but several good subsample size.

We consider nine different subsample size: 25, 30, 35, 40, 45, 50, 55, 60, and 100. 25 is a relatively small subsample size and 100 is a relatively large one. For each subsample size, forward rolling ADF tests are implemented. However, it is difficult and not rigorous to say which subsample size is better while the empirical results are similar. In this paper, no benchmark is used to determine the best subsample size. Instead, there are many good subsample size that can be used together to do analysis. Furthermore, good subsample size is different among different markets. Our judgments can be improved by considering various conditions. Power analysis may be useful to figure out an optimal subsample size, but in this paper the importance of this is small. This is because the main conclusion will not change when subsample size changes no too much.

We choose the subsample size under which the total number of subsamples with significant ADF statistic in the index series is large enough for us to find evidence of bubbles. This procedure seems to be intended but is not. There are three reasons.

Firstly, such criteria is in line with the condition under which evidence of bubble can be found by using equation (10), as we have introduced in section 1 of part 2. Secondly, empirical tests find out different good sample size for different stock markets, which is enlightening and can be interpreted like follows. Different markets have different economic behaviors and the cycles of a bubble process may also be different. If we use a fixed subsample size in every market, we may make wrong judgment due to incorrect selection of subsample size or to periodically collapsing bubble. Thus, the third reason is that this procedure can further reduce periodically collapsing bubble problem.

Here we jump the details of rolling ADF tests that will be discussed in the next section and directly obtain our selection for good subsample size. Table 2 in the appendices shows the number of subsamples with significant ADF statistics under different subsample size for each market. As we have expected, different subsample size can lead to different results in bubble detecting in the same market. And for different markets, there is also some difference in good subsample size. For China Mainland market, the number of significant ADF statistics of the stock index series becomes very small when subsample size is 60 and diminishes when subsample size is 100, which tells us that if there is bubble in China Mainland stock market, the bubble cycle should be less than 60 month. Similar to that of China Mainland, the number of significant ADF statistics of Taiwan stock index series becomes very small when subsample size is 55 and diminishes when subsample size is larger than 60. Hong Kong and Singapore, however, may have longer bubble cycle as significant ADF statistics of their stock index series can be found when subsample size is 60 or even 100.

The above findings demonstrate that, in Hong Kong and Singapore stock markets, bubble generation cycle can be longer than in China Mainland and Taiwan. Shorter cycle may be resulted in more serious speculative behavior in a stock market. What's more, it tells us that bubbles may be more likely to be formed and to collapse in China and Taiwan stock markets than others.

4.2 Recursive and Rolling ADF Statistics

Fuller (1996) gives a table for ADF critical values under several sample size. However, more different subsample size is involved in this paper and their critical values are not available. Thus, Monte Carlo simulations for critical values under the above sample size are implemented first. Critical values for each recursive subsample size are obtained approximately by linear interpolation using the simulated ADF statistics in Table 1.

Both the recursive and rolling ADF tests are applied to the datasets. If the ADF statistic for index lies above the critical value (5% confidence level, see Table 1) and ADF statistic for dividend lies below it, there is evidence of bubble according to the argument by Phillips et al. (2007). Both the statistics of recursive ADF tests and of rolling ADF tests are shown in the appendices (See Figure 3 and Figure 4). Figure 3 plots the recursive ADF statistics of log real index and dividend in the four stock markets. The interpolated critical values are plotted in the figures too.

Forward recursive ADF tests are quite capable in detecting NASDAQ bubble in the paper by Phillips et al. (2007). That may be because of the relatively large sample size and smooth track of NASDAQ time series from the starting point to the highest level. In this paper, however, the samples we choose are not smooth at all but very flexuous. When forward recursive ADF tests are implemented the lines of ADF statistics are almost all below the lines of the critical value for all of these four economies especially for the period of latest two years. No matter whether we follow the convention and apply the ADF test to the full sample (from May 1990 to January 2008) or just to the recursive subsamples, the empirical results can not reject the null hypothesis $H_0 : \delta = 1$ in favor of the right-tailed alternative hypothesis $H_1 : \delta > 1$ at the 5 percent significance level for the time series, and therefore there is no evidence of bubble in these stock markets. However, this is not consistent with our intuition at all. At least for the stock market of China Mainland, the result should have been different.

There are several reasons that we do not believe in forward recursive ADF tests in this paper.

The failure of forward recursive ADF tests to detect bubbles in this paper may be still due to the problem of periodically collapsing bubbles. Since the subsample size is increasing in forward recursive ADF tests, as the subsample size approaches the total sample size, the forward recursive ADF test approaches the conventional pure ADF test. Simultaneously, the problem with periodically collapsing bubbles becomes apparent asymptotically. Finally, the subsample equals the total sample itself and the forward recursive ADF test becomes the conventional pure ADF test.

What's more important, different from NASDAQ stock market, the stock markets of the four economies in our paper do not have satisfying explosive form during the whole sample period. Thus they show more chance to suffer periodically collapsing bubbles than NASDAQ stock market. Other methods are required to detect bubble for such more general conditions.

Forward recursive ADF tests also suffer some extent of theoretical flaws. 1) They are self-contradicted intrinsically. In the recursive tests procedure, bubbles are probably not to be detected for the first fewer subsamples and we may conclude that these subsamples are not explosive. However, as long as we can detect bubbles later for longer subsamples, we may conclude that these longer subsamples are explosive. Longer subsamples contain the first fewer subsamples, so these two conclusions seem self-contradicted. 2) The subsample size is changing and different in the recursive procedure especially when a dataset is large. Due to difficulty in simulating all critical values for different sample size, however, they are fixed for convenience. This may lead to incorrect judgment on whether bubble exists.

When rolling ADF tests are applied, however, the results improve greatly. For each market, many ADF statistics of stock index series where there may be bubbles are above the line of critical value as we have expected. In the periods of the latest

two years, there are successive significant ADF statistics under most of the different subsample size in each market. The empirical results for rolling ADF tests show that there is apparent evidence of long-standing bubbles for the stock market of China Mainland. What is out of our previous expectation, apparent and long-standing bubbles are also detected in Hong Kong, Taiwan and Singapore stock markets.

Before we investigate the exact periods of bubbles, we would like to first define or to further clarify the definition of bubble. ADF tests are implemented for every subsample and the outcome is an ADF statistic for each subsample. The months where successive significant ADF statistics are found should not be considered as bubble period. Instead, each significant ADF statistic is related to a subsample which is experiencing bubble. Thus, when we observe a significant ADF statistic, its corresponding subsample is a bubble process. In other words, a bubble is an explosive process, not points with significant ADF statistics. In the paper by Phillips et al. (2007), bubble is successfully dated but the definition is confusing that bubble period is defined to be the points where ADF statistics are significant. This is not proper. A point where ADF statistic is significant is just the last month of its corresponding subsample where ADF test is implemented. Thus, the points before the one where ADF test is significant should be included to analyze the bubble as a process. That is to say, a subsample should be considered as a whole. The first observation in a subsample is the start of the bubble expanding process.

Significant ADF statistics are found through the whole sample period, but we will not consider all of them since our focus is for the latest two years. Table 4 lists all the periods from Jul 2006 to Jun 2008 which are the end of a subsample with significant ADF statistics. We cannot conclude which is better than others but we make a general analysis using all of the findings in the table. There are three important findings. 1) Evidence of bubbles can be found in China Mainland market mainly from Jan 2007 to Feb 2008, in Hong Kong mainly from early 2007 to Jun 2008, in Taiwan mainly from early 2007 to Apr 2008 and in Singapore mainly from Feb 2007 to Feb 2008. 2) Suc-

cessive subsample size can lead to similar empirical results but for subsamples with large difference the conclusions are very different. For China Mainland market, results under subsample size 25 to 50 have much overlapping but there is no evidence of bubbles for subsample size 100. Such kind of conclusions can also be found in other three markets. 3) No matter which subsample size is used, we cannot find a bubble process that can last as late as Mar 2008 in China Mainland and Singapore. That is to say, when observations after Mar 2008 are included in an ADF test, the whole subsample cannot form an explosive process. However, we find that bubble process can last later in Hong Kong and Taiwan. This may tell us that bubbles collapse earlier in China Mainland and Singapore than in Hong Kong and Taiwan.

4.3 Explosive Speed and Bubble Size

The definition of bubble is the logarithm of the ratio of index to its fundamental.

$$b_t = \log\left(\frac{P_t}{P_t^f}\right) = p_t - p_t^f \quad (12)$$

where P_t , P_t^f are real prices. There are so many successive subsamples with significant ADF statistics that we cannot consider them all. In order to make the arguments more reliable, however, we interpret the explosive speed and bubble size in a stock market based on the subsample with the most significant ADF statistics only. This subsample has a form that is closer to an explosive process as in Figure 1 than other subsamples. We select that very subsample from all the subsamples under different subsample size. Figure 5 plots these four subsamples where price series and dividend series are separated in order to make us clear about the different patterns. From the figures, we can find that all the four price series are more likely to be in an explosive process than the dividend series.

In Phillips et al. (2007), the initial size of bubble is not computed by regressions but is supposed to be 10 percent of the stock price. This kind of setup is very useful as a tool of analysis when we do not know what the initial bubble size is for the subsam-

ple we concern. In this paper, we do not assume initial bubble size or compute the exact number of bubble size since that is quite arbitrary and can lead to various conclusions. Instead, we only look at how much the bubble grows in percentage from the first period to the last period in the subsample we use.

Here, we compute the coefficients of AR (1) process for the subsamples we mentioned above in the hope to capture some rough but helpful information in the explosive process in the Greater China stock markets (Since least squares regression will produce downward biased coefficient estimates in the first order autoregression, it is useful to take account of it and conduct inference on autoregressive coefficients if we need exact information on bubble size). Table 5 in the appendices lists the autoregressive coefficients for the subsample with the most significant ADF statistics in China Mainland, Hong Kong, Taiwan and Singapore.

First order autoregressive coefficients show the average growing speed of index in the above markets. The subsamples we choose have different size of 25, 55, 40 and 50 for each market. This is in line with our previous finding that China Mainland and Taiwan may suffer shorter bubble cycle than Hong Kong and Singapore. For the subsample most likely to be explosive, China Mainland has the shortest bubble expanding speed and fastest price explosive speed of 0.92% per month. For Hong Kong, Taiwan and Singapore, price explosive speeds are 0.38%, 0.46% and 0.44% per month separately. As shown in past researches, least squares regression will produce downward biased coefficient estimates in the first order autoregression especially when sample size is small. Among several statistical methods initiated to deal with this problem, we use Gouriéroux's indirect inference (1993) to adjust the simple least square estimates. Our indirect inference estimates are obtained via simulation with 30000 replications. The corresponding adjusted AR(1) coefficient estimates are 1.8%, 1.2%, 1.35% and 1.25%.

Suppose the initial size of bubble in our subsample is $b_0 = p_0 - p_0^f$, we can get

the size of bubble at the end of the subsample by using equation (12):

$$b_T = p_T - p_T^f = (1+c)^T p_0 - p_T^f \quad (13)$$

where b_T is the bubble size of the subsample with sample size T, c is the AR(1) coefficient and price growing speed, p_T^f is the fundamental price at the end of the subsample. Thus, the percentage the bubble expands during the subsample is:

$$\frac{b_T}{b_0} = \frac{p_T - p_T^f}{p_0 - p_0^f} = \frac{(1+c)^T p_0 - p_T^f}{p_0 - p_0^f} \quad (14)$$

Notice that equation (14) is difficult to implement because we do not know what p_0^f is. In the four subsamples we concern, no evidence of explosive process can be detected in corresponding dividend series, which lead to the conclusion that fundamental prices should not be explosive. However, dividend series seem to move in other type of form and hence p_T^f is also difficult to be determined. To make things easier, we suppose that fundamental price grows at the same speed with actual price. Then the percentage the bubble expands during the subsample now become:

$$\frac{b_T}{b_0} = \frac{p_T - p_T^f}{p_0 - p_0^f} > \frac{(1+c)^T (p_0 - p_0^f)}{p_0 - p_0^f} > (1+c)^T \quad (15)$$

which can be figured out by using our empirical results. If dividend series or the fundamental price does not increase explosively, we can interpret equation (15) as a conservative approximation or a lower bound to the true percentage that the bubble expands from the initial point.

China suffers a serious bubble expanding of 156% within only 25 months. The percentages that bubble expand in Hong Kong, Taiwan and Singapore is separately 192% after 55 months, 171% after 40 months and 186% after 50 months. According to our derivations above, the true expanding percentages should be larger than these figures, since there is no evidence of explosive process in their dividend series. Since we do not know the initial bubble in each market, it is hard to conclude which market

suffers the most serious bubble. However, it is easy to see that China suffers the fiercest bubble expanding in much shorter period than others.

For the above four subsamples with the highest possibility of suffering explosive bubbles, subsample size are different. China Mainland has the shortest explosive process while the other three markets have much longer explosive process. What's more, the explosive speed in China Mainland is the largest. These may be because that speculative behavior in China Mainland is much stronger than in the other three markets, which leads to quick expanding in the bubble size and shorter explosive process. The peaks of bubbles for above four subsamples are different. The largest bubble in China and Singapore are during Jun, 07 while in Hong Kong it's Jan, 08 and in Taiwan Nov, 07.

4.4 When Bubble Begins to Collapse

It will be interesting to investigate the bubble collapse in a bubble stock market. Previous researches always focus on bubble testing, which is only about whether bubbles exist or in another word, bubble generation process. Scare research focuses on the topic of bubble collapsing process.

Intuitively, bubble collapsing speed is always much faster than bubble generation. Sudden collapsing bubble can lead to very serious economic and social problems. And such process is relatively easy to transmit from country to country. When bubble is collapsing, it has similar mechanism with bubble generation but perform in the other way round. Theoretically, bubble collapsing can also have an exponential form because everyone believes that nobody will buy the stock and they must sell their stock in much lower price than it should be.

In all of these four bubble stock markets, there are obvious quick drops during the last periods. Back recursive or back rolling ADF tests which can be performed adversely to forward ADF tests seem quite capable if we want to detect bubble collaps-

ing. Unfortunately, the sample size we can use is too short at the present time. We are not able to get deeper findings until we can find larger sample. Further research on this topic is worthwhile to do so that we can further understand what bubble is.

According to our empirical findings, we can guess when a bubble begins to collapse. When this happens, there is still bubble in the market but the explosive process does not continue anymore. Thus, rolling ADF statistic should be still significant but the statistic should become less significant than previous ones. Since we have already found the subsamples with most significant ADF statistics in the previous section, it is very likely that bubbles begin to collapse after the end of these subperiods. The potential month when the bubble begins to collapse in China Mainland and Singapore is after Jun 07, in Hong Kong after Jan 08 and in Taiwan after Nov 07.

5 Conclusion

In order to find evidence of rational bubbles in four Greater China stock markets, two different methods (forward recursive ADF tests and forward rolling ADF tests) are used and compared. Forward recursive ADF tests fail in detecting bubbles in our samples while rolling ADF tests do so. The same to the informal comments, we find strong evidence of long-standing bubbles in China Mainland stock market. We also detect bubbles in the rest three Greater China stock markets (Hong Kong, Taiwan and Singapore). This is an evidence of strong interaction in today's international financial markets. China Mainland stock market may have suffered the fastest bubble expanding, and within a very short time its bubble expands as much as other markets whose bubble expanding duration is much longer. What's more, the bubble in China Mainland stock market collapses in the earliest time. The empirical results for rolling ADF tests further show that the length of bubble cycles in different markets may be different.

It is a quite common economic phenomenon in China Mainland that to invest in stock market is much like a kind of gambling. Speculative behavior in China stock market is prevailing for various kinds of people. What's more, as an economy under a transition stage, financial regulations in China Mainland are very weak. Due to the close economics relationship between these three economies (China Mainland, Hong Kong and Singapore), the conclusions of bubbles are similar. Speculation may have spread among them and China Mainland is very likely to be the starting market. However, both investors' understanding about investment and financial regulations are more complete in Hong Kong, Taiwan and Singapore. This is why their bubble processes are slower, longer and less serious. It is important for China Mainland that through effective financial policies like those developed economies, speculative behavior is possible to be controlled or decreased.

If bubbles exist in asset markets, market prices of assets will differ from their fundamental values. Markets would not necessarily be allocating the savings of individuals to the best possible investment uses. As bubbles always lead to financial and social unrest, public policies might be designed to attempt to rid the markets of bubbles. It is more important that after continuing education, investor's in China Mainland can become really "rational".

There are still some shortcomings in this paper that may be improved. The model for bubble is only applicable for special cases when there is only explosive behavior in index but not in dividend, so refinement of model is needed for more general applications. The datasets need to be extended to capture more historical information so that deeper empirical research on the collapsing of bubble is available. At last, model comparison between our method and previous methods are also valuable and important. Finite sample performance should be examined among different methods so that we can find out the best one. Future research will cover these issues.

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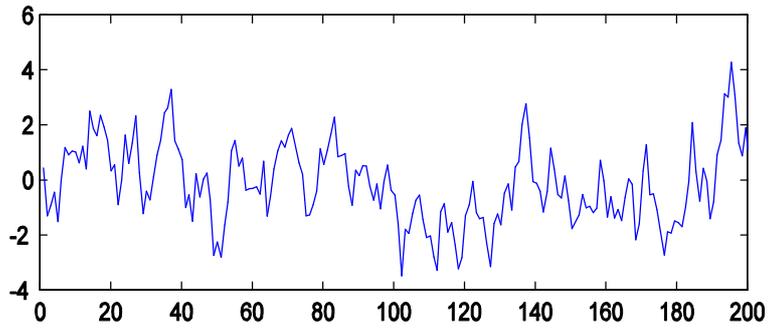
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Appendices

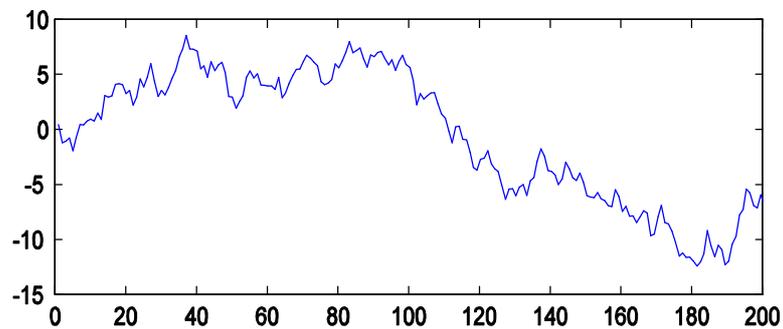
Figure 1 Typical time series

1. Simulated AR (1) model with $\rho = 0.8$ (sample size is 200)



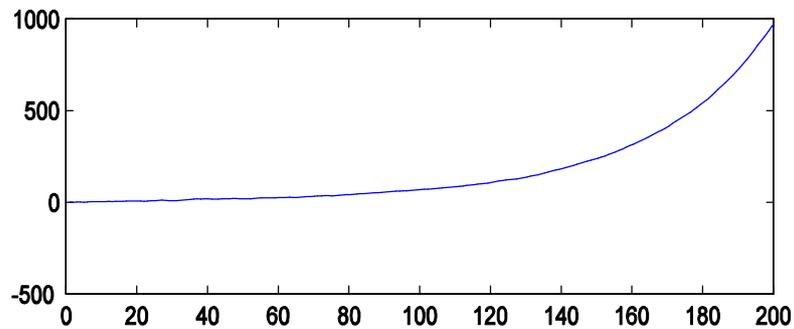
A stationary process with mean 0 and variance less than infinity

2. Simulated AR (1) model with $\rho = 1$ (sample size is 200)



A random walk process

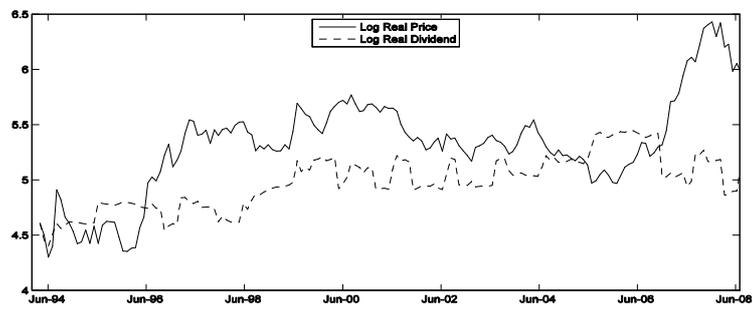
3. Simulated AR (1) with $\rho = 1.03$ (sample size is 200)



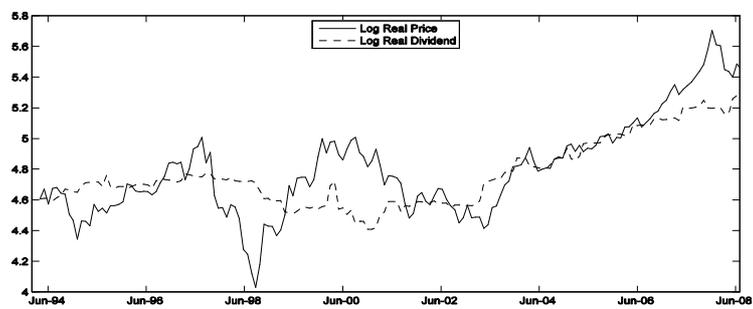
An explosive process

Figure 2 Log normalized real index and dividend (May 1994-Jun 2008)

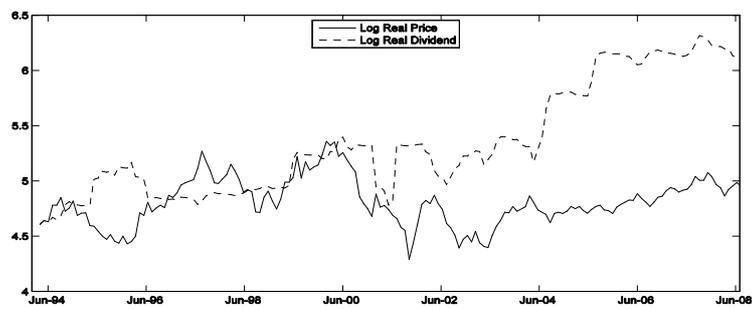
China Mainland



Hong Kong



Taiwan



Singapore

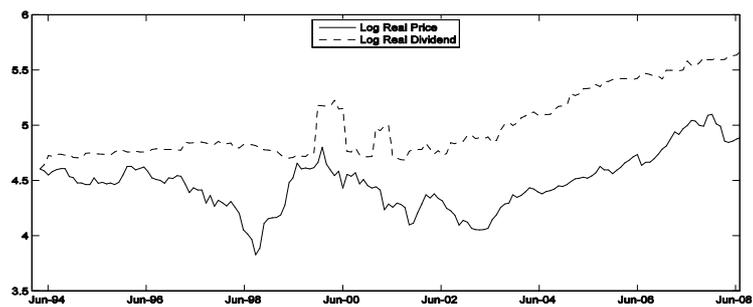
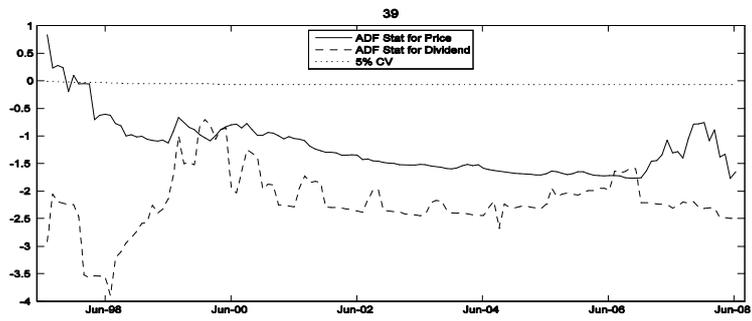
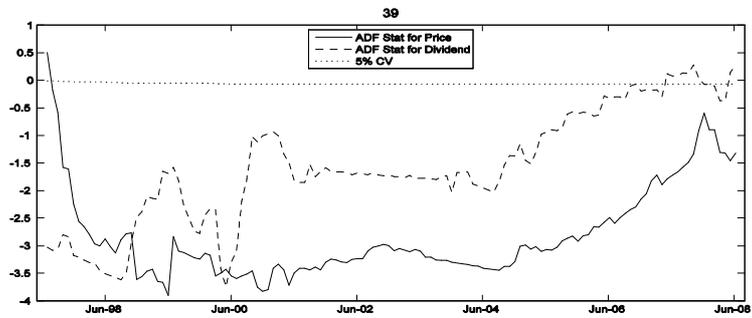


Figure 3 Recursive ADF tests

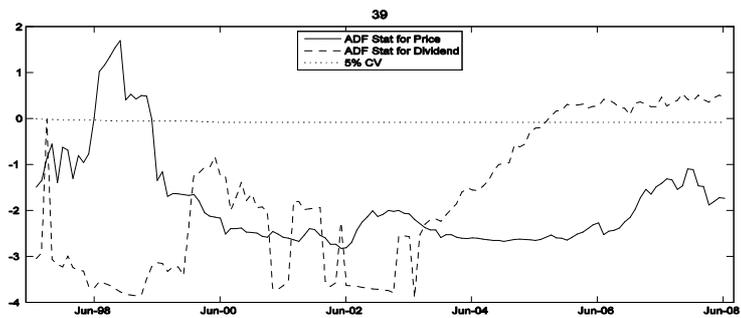
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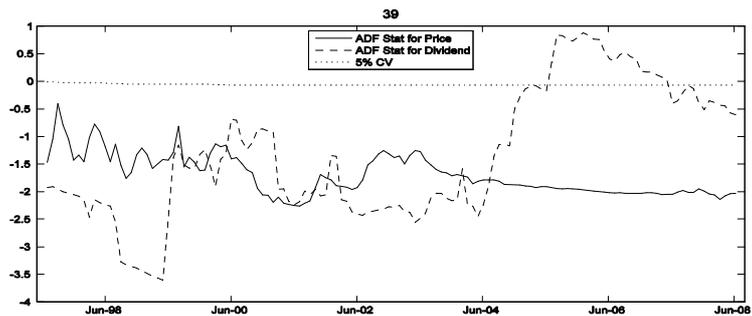
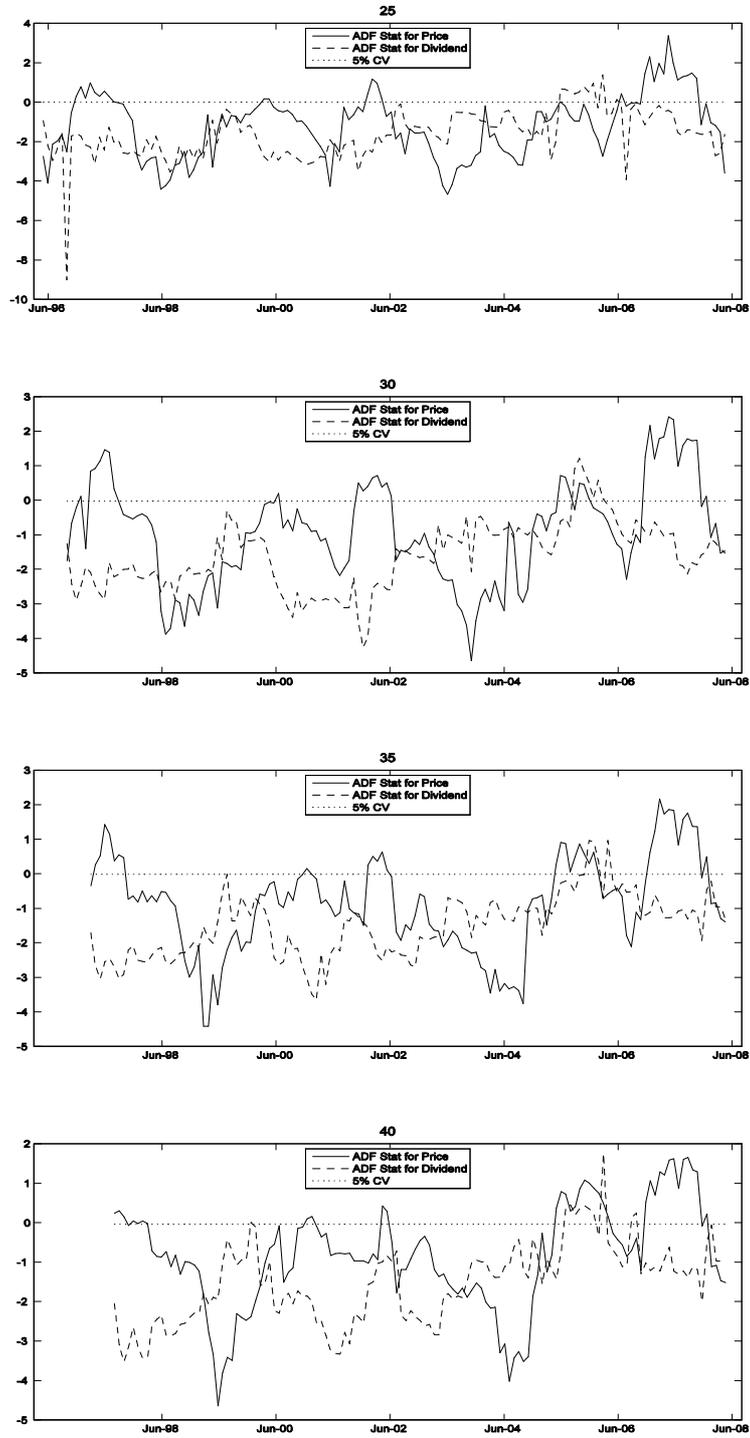
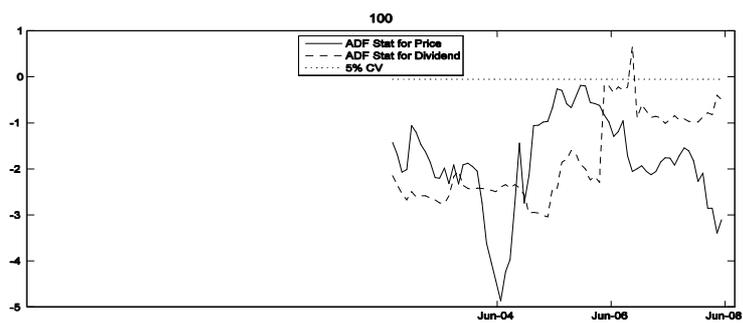
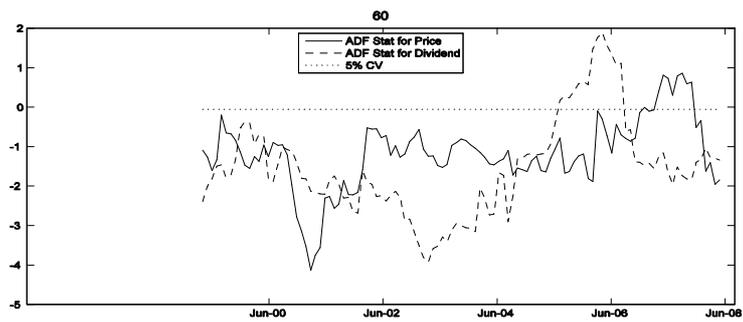
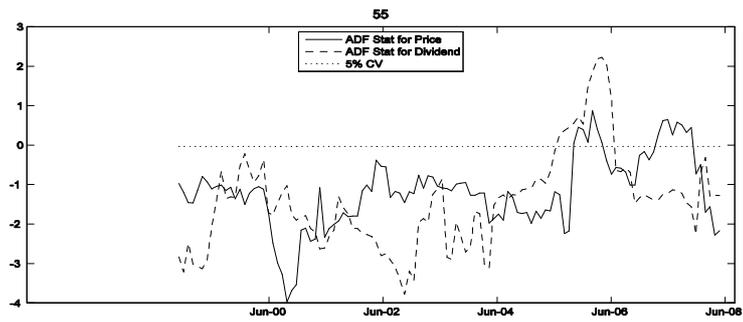
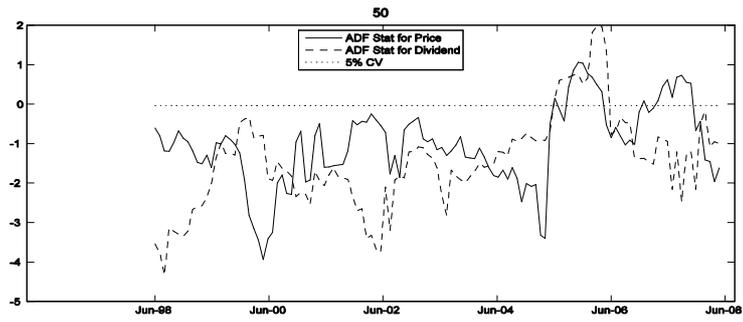
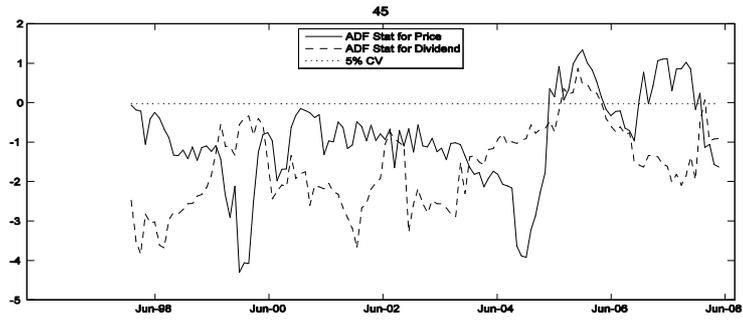


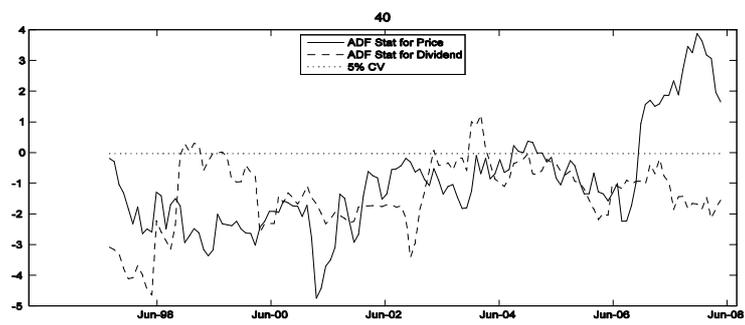
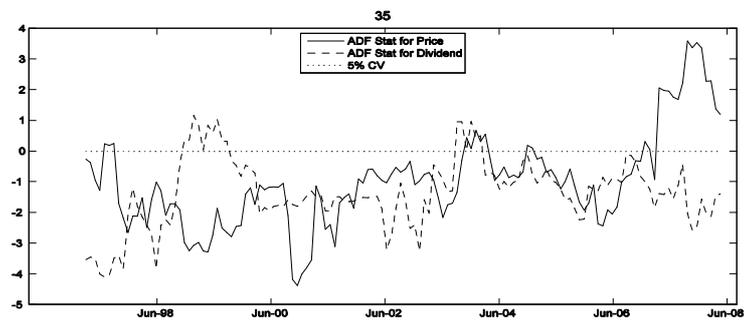
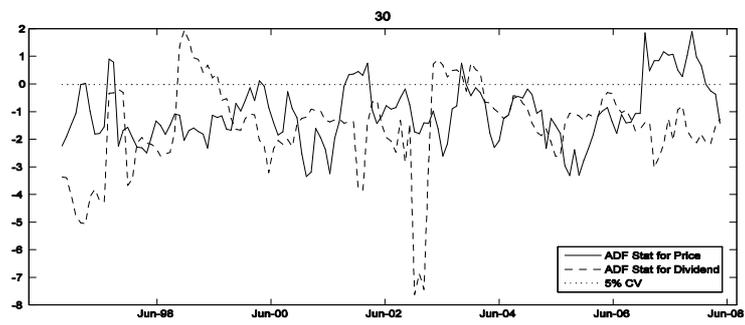
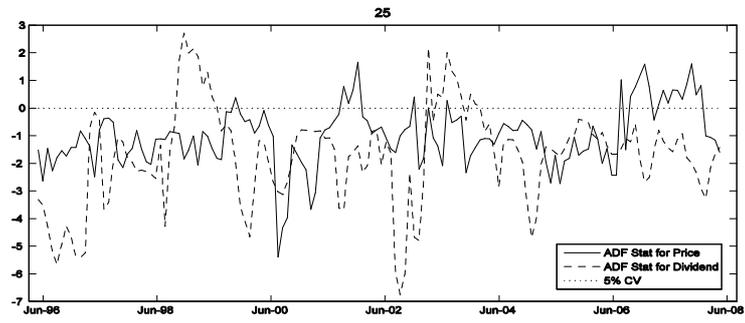
Figure 4 Rolling ADF tests

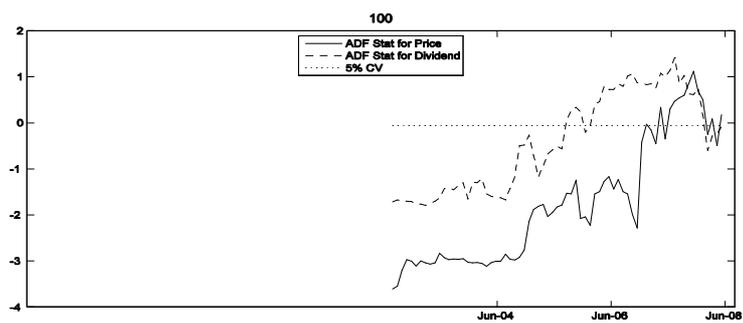
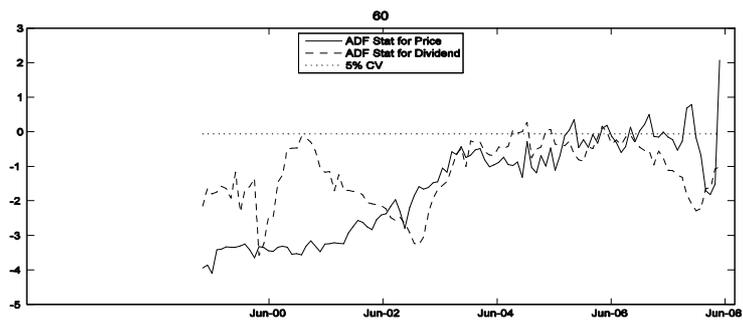
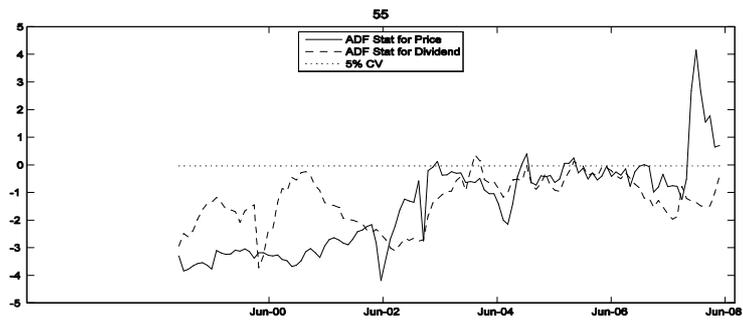
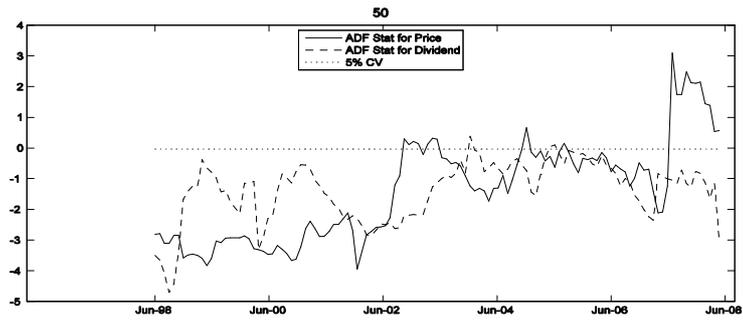
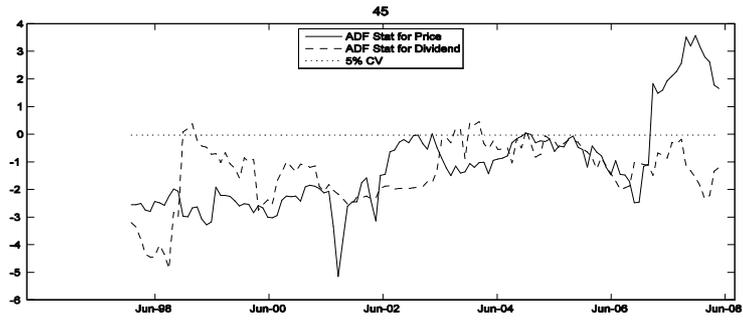
China Mainland



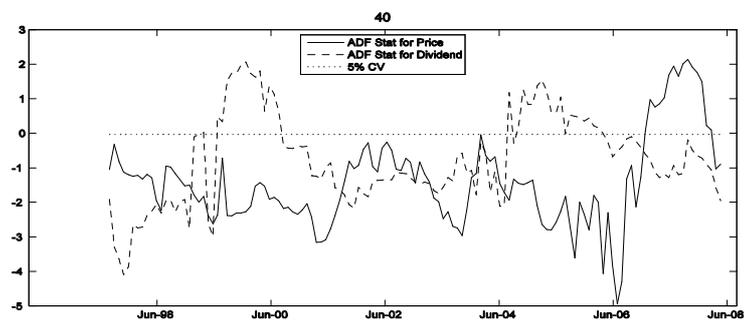
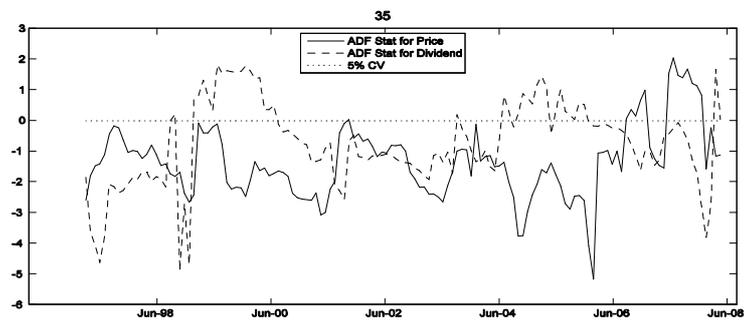
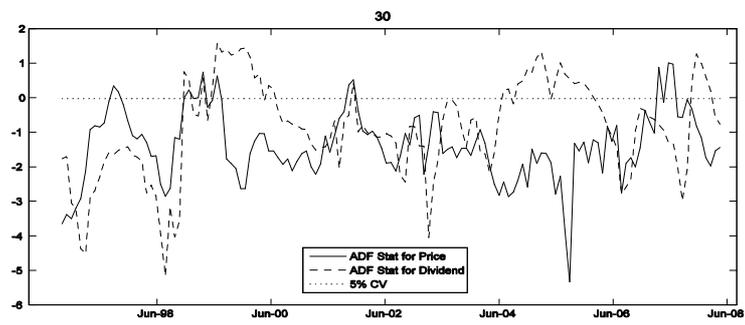
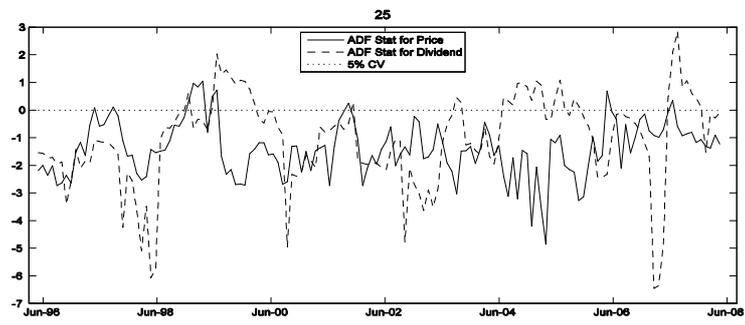


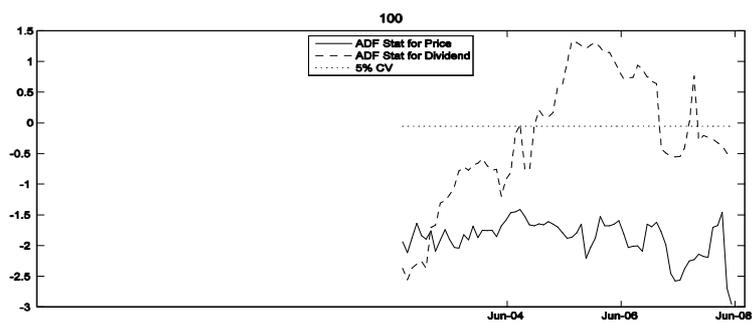
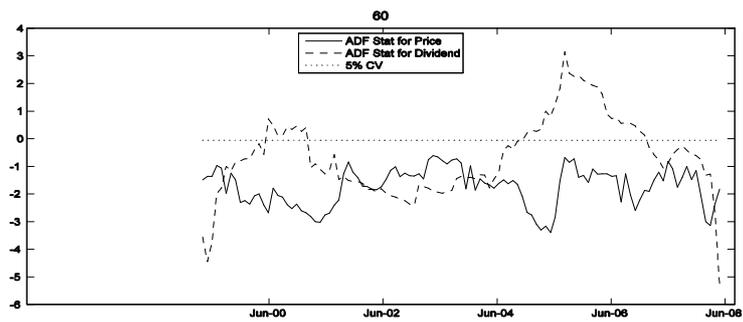
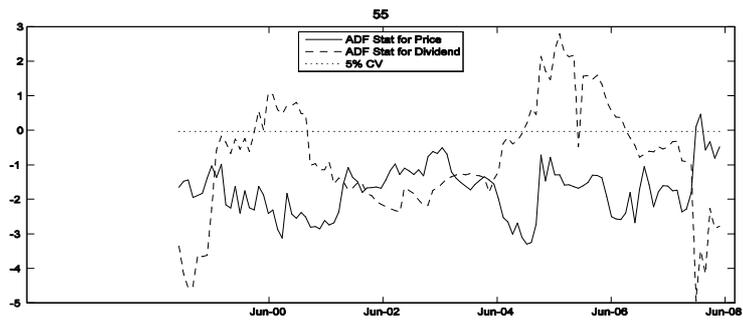
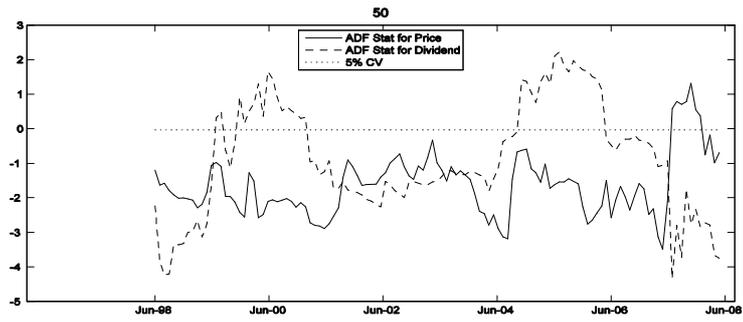
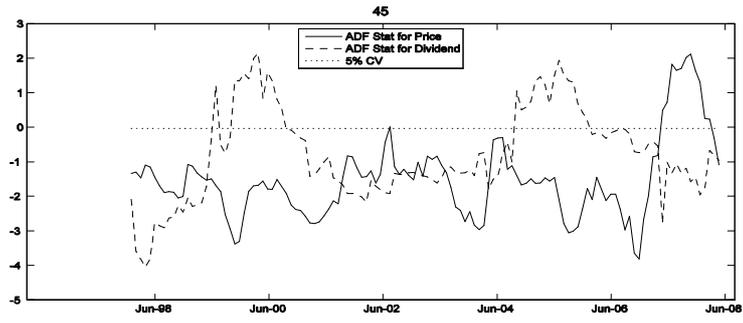
Hong Kong



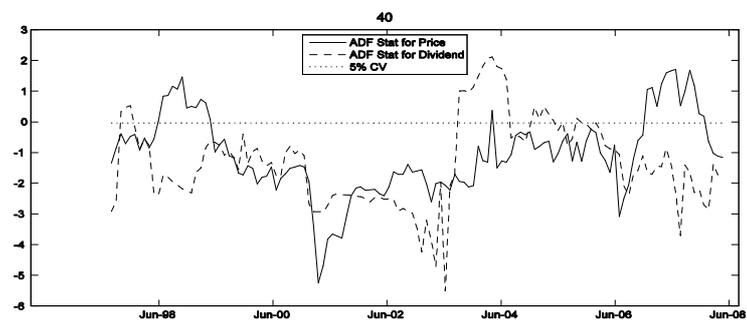
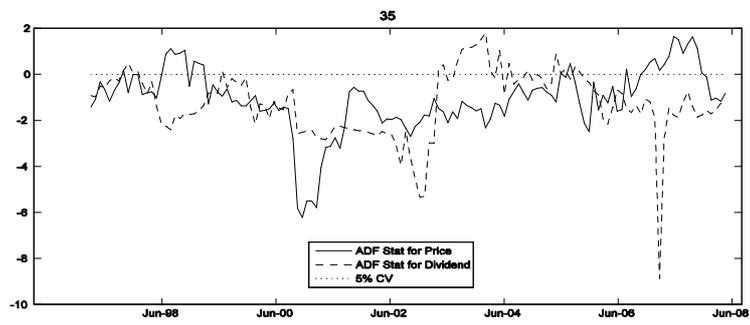
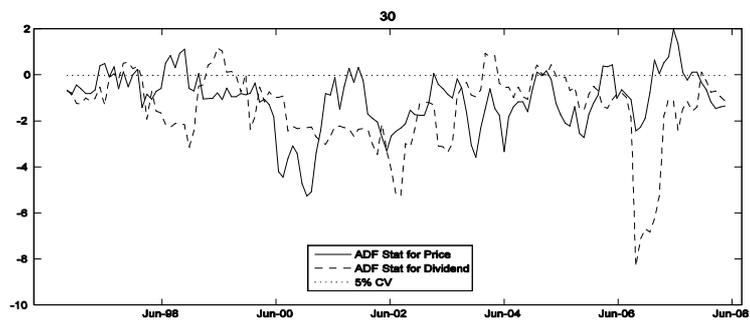
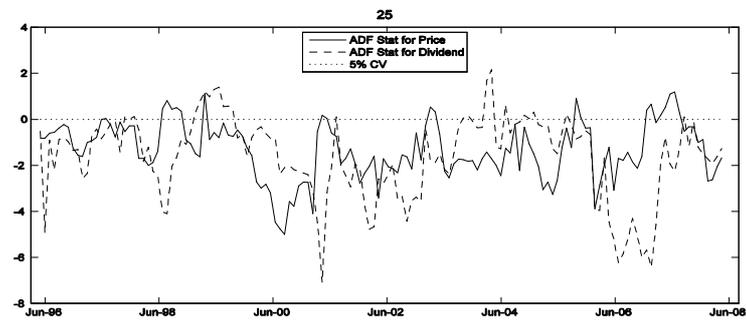


Taiwan





Singapore



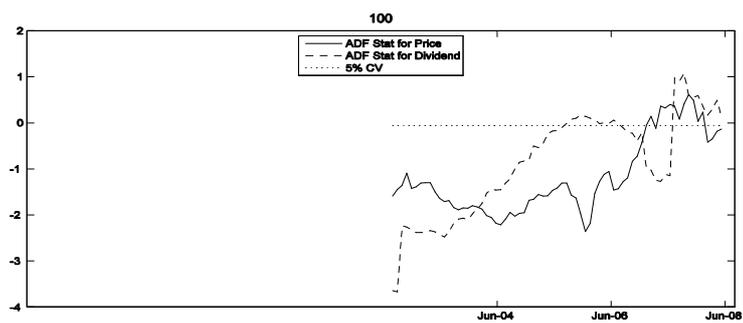
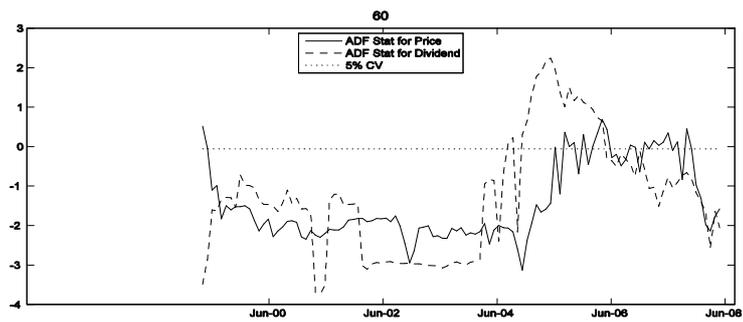
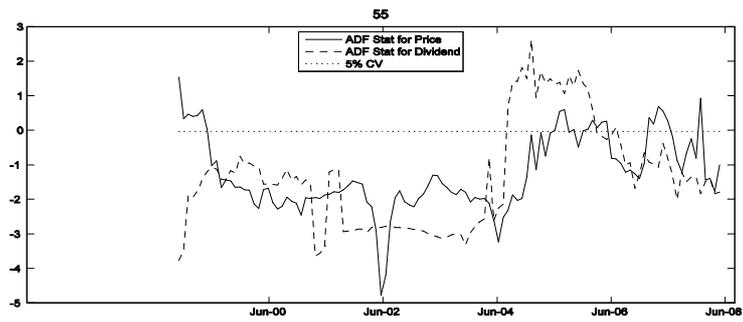
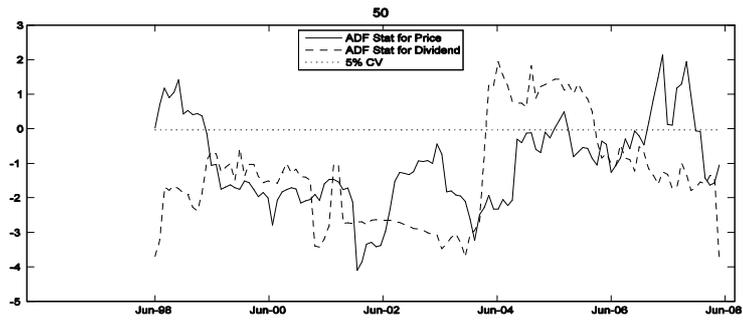
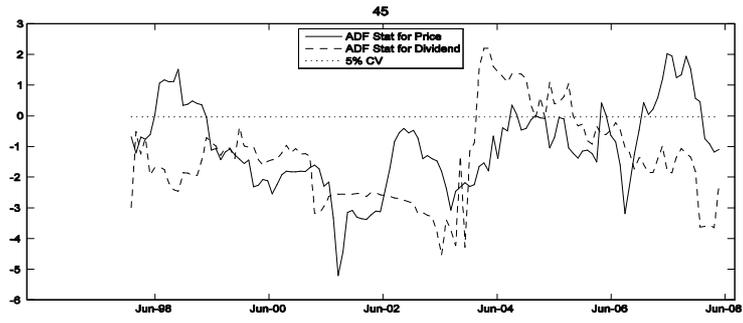
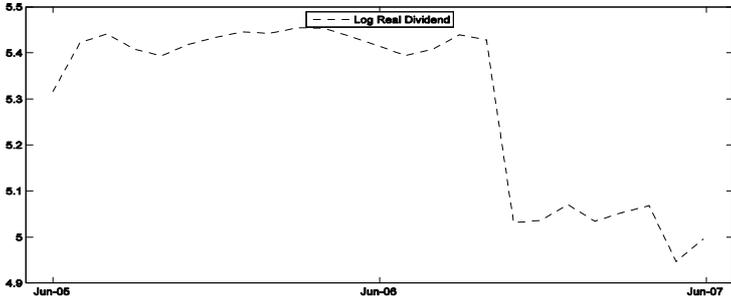
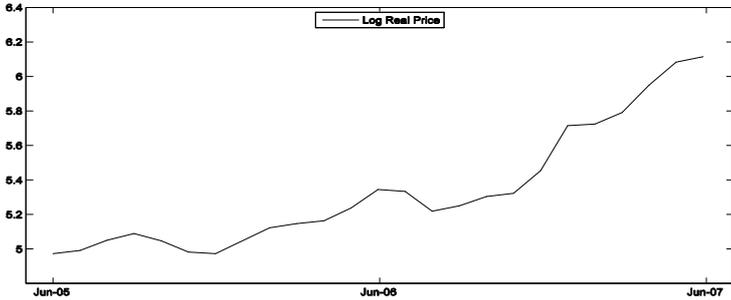
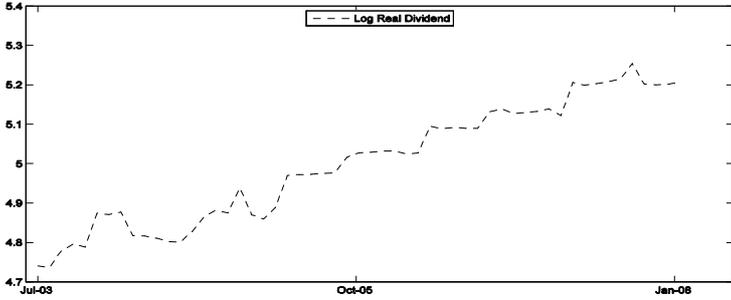
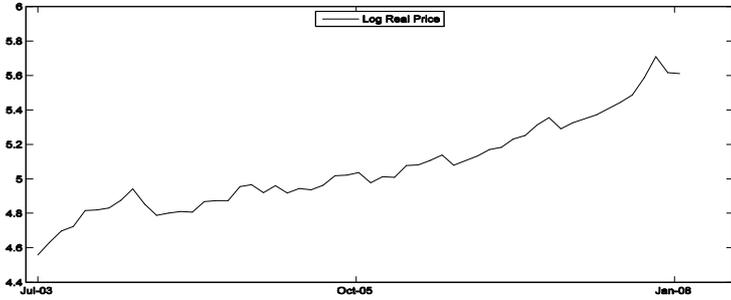


Figure 5 Subsamples with the most significant ADF statistics in each market

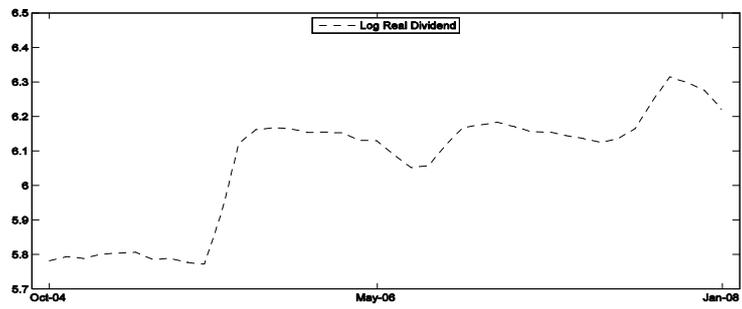
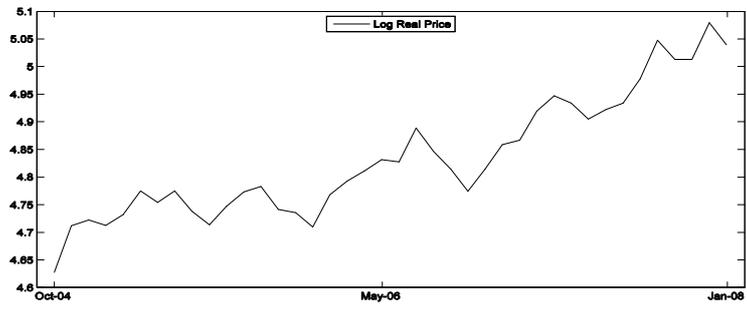
China



Hong Kong



Taiwan



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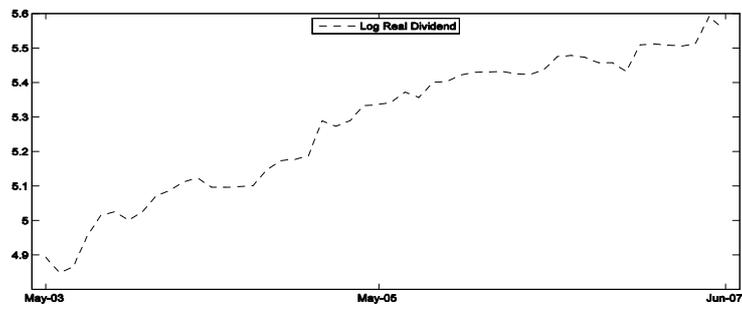
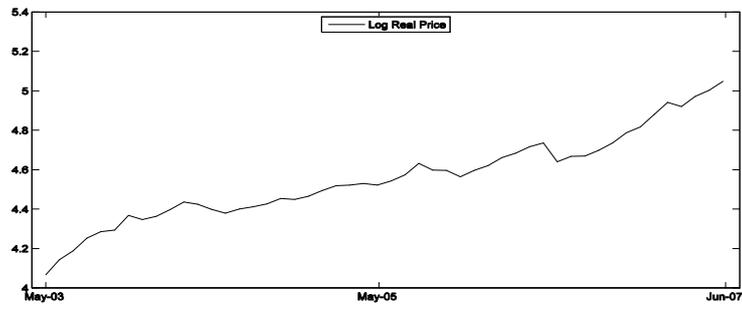


Table 1 Monte Carlo simulated critical values for ADF statistics

	0.01	0.025	0.05	0.1	0.9	0.95	0.975	0.99
25	-3.7417	-3.3249	-2.9947	-2.6415	-0.3763	-0.0079	0.32	0.7137
30	-3.6743	-3.2889	-2.9593	-2.6155	-0.3912	-0.0258	0.3057	0.6873
35	-3.6442	-3.2662	-2.9525	-2.6169	-0.3948	-0.0211	0.2984	0.6704
40	-3.6172	-3.2383	-2.9385	-2.613	-0.3987	-0.0376	0.2865	0.6692
45	-3.594	-3.2332	-2.93	-2.6067	-0.4075	-0.0346	0.2898	0.6848
50	-3.5694	-3.2283	-2.9326	-2.601	-0.4033	-0.0383	0.2849	0.6669
55	-3.5641	-3.2008	-2.9145	-2.5977	-0.4062	-0.0367	0.2775	0.6544
60	-3.5484	-3.1972	-2.9127	-2.5952	-0.4146	-0.0553	0.2738	0.6329
100	-3.515	-3.1668	-2.8862	-2.5784	-0.4274	-0.0563	0.2595	0.6348
150	-3.4582	-3.1369	-2.8758	-2.5763	-0.4313	-0.073	0.2483	0.6264
500	-3.4254	-3.1214	-2.8609	-2.5598	-0.4334	-0.0736	0.2513	0.6072
2000	-3.4398	-3.1316	-2.8724	-2.5689	-0.4442	-0.0741	0.241	0.5979

This table reports the simulated critical values for the ADF statistics in this paper. The first column shows the sample size. The first row shows the percentiles. The critical values are obtained by Monte Carlo simulation with 100000 replications. The detailed procedure is like this: 1) for sample size of 25, we simulate 100000 AR (1) unit root time series each with a fixed constant term; 2) since ADF statistic has the same sampling distribution as the DF statistic, we then use the DF test to compute the corresponding ADF statistic for each simulated sample, where the null hypothesis for the DF test is that there is a constant term but no time trend; 3) at last, we find the percentiles listed in the table for the 100000 ADF statistics; 4) procedures 1)-3) are repeated for different sample size.

In the paper, we conventionally use the 95 percentile critical values to evaluate explosive evidence in recursive and rolling ADF tests.

Table 2 Number of significant ADF statistics under different subsample size

	China Mainland		Hong Kong		Taiwan		Singapore	
25	30	10	23	19	11	36	21	23
30	35	7	24	19	12	40	29	20
35	37	4	26	16	14	39	28	24
40	36	11	23	13	15	34	26	22
45	25	10	22	9	12	28	28	19
50	18	12	20	3	7	35	24	23
55	15	12	14	4	2	33	23	20
60	8	14	12	7	0	35	21	20
100	0	1	12	28	0	30	12	23

This table reports the number of significant ADF statistics under different subsample size. The first column shows the sample size. The left column under each market shows the number of significant ADF statistics of stock index series while the right column shows that of dividend series.

Table 3 Most significant ADF statistics for stock price under different subsample size

	China Mainland	Hong Kong	Taiwan	Singapore
25	3.411	1.627	1.040	1.250
30	2.407	1.889	0.965	1.977
35	2.171	3.555	2.067	1.651
40	1.616	3.818	2.131	1.720
45	1.370	3.647	2.124	1.971
50	1.037	3.140	1.308	2.126
55	0.910	4.131	0.471	1.536
60	0.800	2.086	-0.651	0.697
100	-0.176	1.127	-1.410	0.618

This table reports the most significant ADF statistics of stock index series under different subsample size for each market. The first column shows the sample size. Significance of ADF statistic is the benchmark of this paper to select the subsamples which are most likely to suffer bubbles. The subsample size under which there exists the most significant ADF statistic (bold numbers) is different from market to market.

Table 4 Periods in the latest two years when significant ADF statistics are found under different subsample size

	China Mainland	Hong Kong	Taiwan	Singapore
25	Jan-Dec 07	Dec 06-Mar 07, May 07-Feb 08	none	Feb, Mar, May-Sep 07
30	Jan-Dec 07	Feb 07-Feb 08	May, July-Aug 07	Mar-Sep 07
35	Feb-Dec 07, Feb 08	May 07-Jun 08	Oct 06-Feb 07, Jul 07-Feb 08	Jan 07-Jan 08
40	Jan-Dec 07, Feb 08	Jan 07-Jun 08	Feb 07-Apr 08	Feb 07-Feb 08
45	Jan-Dec 07, Feb 08	Apr 07-Jun 08	Jun 07-Apr 08	Feb 07-Feb 08
50	Feb, May-Dec 07	Aug 07-Jun 08	Aug 07-Feb 08	Mar-Dec 07
55	May-Dec 07	Dec 07-Jun 08	Jan, Feb 08	Mar-Jul, Feb 07
60	May-Dec 07	Nov-Dec 07, Jun 08	none	Feb-Jul, Sep, Nov 07
100	none	none	none	none

This table reports the subsamples of which the ADF statistics of stock index series are significant. The first column shows the sample size. The dates in the table are the last days of these subsamples. We only report the dates for the latest two years (Jul 06-Jun 08) that we concern.

Table 5 Autoregressive Coefficients (Bubble Growing Speed) and Corresponding subsample ADF statistics

	period	size	ADF(price)	ADF(dividend)	cv	AR(1)	Adjusted
China Mainland	Jun 05-Jun 07	25	3.411	-0.5024	-0.0079	1.0092	1.018
Hong Kong	Oct 04-Jan 08	55	4.131	-1.3925	-0.0367	1.0038	1.012
Taiwan	Jul 04-Nov 07	40	2.131	-0.2349	-0.0376	1.0046	1.0135
Singapore	May 03-Jun 07	50	2.126	-1.2123	-0.0383	1.0044	1.0125