Does Economic Inequality Cause Financial Crises?

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Abstract

Inequality rose rapidly in the run up to the 1929 stock market crash and the 2007 financial crisis. Both crises precipitated long and deep recessions. This paper seeks to determine if there is any deeper relationship between inequality and financial stability. The work presents an empirical investigation of the topic and theoretical model of how such a relationship could exist. My original contribution to the literature is threefold: (1) the empirical detection of a small interaction between economic inequality and propensity to financial crises, (2) the presentation of a novel measure of financial stability using principal component analysis and its interaction with economic inequality, and (3) the presentation of a novel theoretical model that demonstrates a possible mechanism by which inequality may reduce financial stability.

Declaration

The work in this thesis is based on research carried out at the Department of Economics and Finance, of the School of Economics, Finance and Business at the Durham University, England. No part of this thesis has been submitted elsewhere for any other degree or qualification and it is all my own work unless referenced to the contrary in the text.

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I would also like to thank my supervisor, Prof. Parantap Basu, whose advice always challenged me to be rigorous in my thinking and clearer in my exposition despite my instincts to the contrary. I would also like to thank my second supervisor Dr Thomas Renstrom for his input and support.

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Chapter 1

Introduction

So distribution should undo excess, and each man have enough.

[King Lear, Act 4, Scene 1]

–William Shakespeare

Within the last five years the study of financial crises has once again gained significant popularity in the mainstream media and academic circles. The near collapse of the banking system has raised familiar, but fundamental questions for economists about the way in which the financial system is organised. Although this is not the fundamental question raised by this research, it has a strong bearing on the motivation behind it. While financial crises are certainly inconvenient to those affected by them, there is a mounting body of work to suggest that there are real and significant costs involved. The cost of a crisis is found to be of the order of 63% of real per capita GDP (Boyd et al., 2005). This work aims to answer the simple question: *How does economic inequality affect the stability of the financial system?*

Much attention has already been paid to the effect of inequality on economic growth with a general consensus that inequality negatively impacts growth (Aghion and Williamson, 1999). However, this is not a universal finding and some research finds the converse to be true. Forbes (2000) argues that the differing results are due to improved data and methods of analysis, but this is still a contentious position.

Work has been undertaken to identify the effects of financial crises on inequality. Papers such as Halac et al. (2004) and Baldacci et al. (2002) demonstrate that economic inequality is usually increased by financial crises. The reverse relationship has also been posited, with no lesser proponents than Rajan (2011) and Krugman (2010), suggesting that there seems to be some correlation between large financial crises and levels of inequality. The core aim of this research is to determine if this is a mere coincidence or a more structural relationship. The observation behind this hypothesis is shown in figure 1.1, which shows the proportion of income taken by the top 1% of earners in the USA over the last century. We can see two distinct peaks, just before the Great Depression and the 2007 financial crisis.

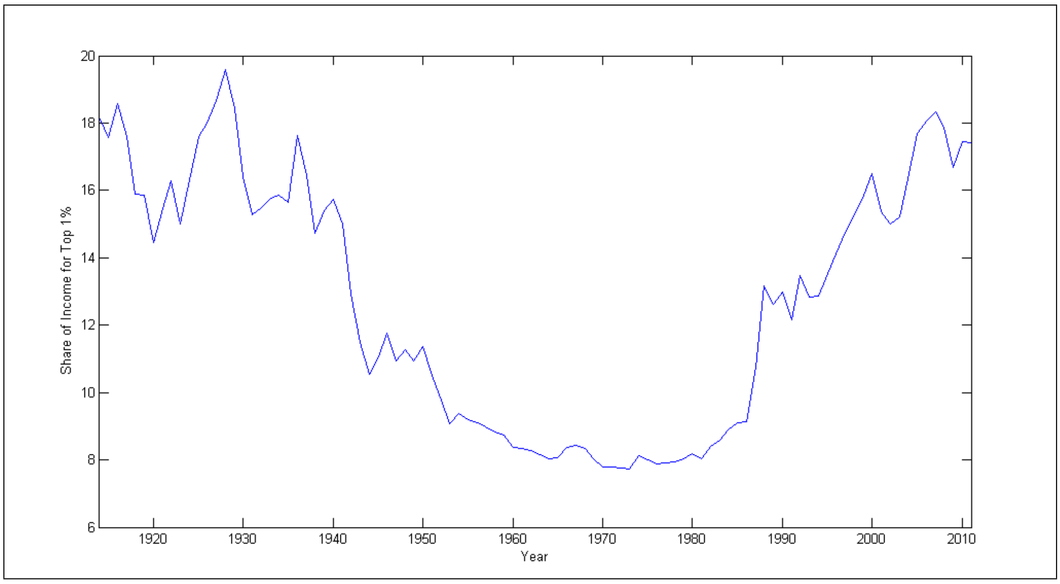

Figure 1.1: US top 1% income share over the twentieth century, (Facundo et al., 2012)

The link between economic inequality and financial stability is not anecdotally obvious. Comparing countries regarded as having high inequality [1] and those with more equality [2] it is not immediately obvious which has the greater propensity to financial crises. This is especially true when one turns to banking crises; most of the world has experienced a banking crisis of some sort over the last 25 years. Moreover, differences in governmental responses to unravelling crises can mask the true depth of the crisis making it difficult to measure the underlying severity.

Chapter 3 will look at this relationship in more detail and show that high inequality does appear to increase the propensity towards financial crisis. I first use a logit model to show that the probability of a country experiencing a financial crisis in 2007 was higher if that country had a higher Gini coefficient. I then show that this result can be generalised to other crises, however, these results are not statistically robust. Finally, in chapter 3, I use a principal

[1] For example US, UK and Portugal
[2] For example Japan and Scandinavia

component decomposition to look more closely at the role inequality played in the 2007 financial crisis.

My empirical results are contrary to the findings of Bordo and Meissner (2012), who did not find such a relationship. I argue that we cannot rule out a weak relationship despite the dominance of other factors, because of the sparse data on inequality. My work also contributes a relatively novel approach to measuring financial stability, and uses this measure to analyse possible causes of crises.

It is appropriate at this point to ask how inequality may affect the probability of a crisis and to propose some transmission mechanism whereby greater inequality reduces stability. Krugman (2010) suggests that rising inequality is related to political polarisation. The result is that the wealthy have undue political influence, which causes suboptimal policy and financial fragility. In this vein, Landier and Plantin (2011) present a theoretical model of how such political interaction could occur. However, this thesis will propose a purer microeconomic mechanism model of how inequality reduces stability.

Economic inequality could have several effects on the stability of the financial system. One scenario is where inequality does not directly cause a financial crisis but serves to deepen what could be small shocks.

Consider an extreme example in which a large proportion of the population are engaged in menial and low paid work while the remainder are disproportionately wealthy. In this extreme country, the poor are all but unable to participate in any financial markets:

- They have bank accounts but little stock holding or insurance,
- They will be limited in their ability to obtain credit, and
- They will own a very small fraction of assets.

Conversely the rich would be responsible for investing nearly all of the country's wealth; they are responsible for diversification, choice of assets and obtaining credit.

The poor are then wholly dependent on the rich making good choices so that the country becomes wealthier. This may be at odds with a purely self-interested agenda e.g. extracting rents may be more profitable than investing in productive enterprise. In the event that the rich take undue risks, fail to diversify their investments or borrow money irresponsibly then the entire country could suffer. A small shock in financial markets may then be amplified.

This economy does not look familiar from a developed western perspective, where there is much greater engagement with the financial system. It is true that in the USA the wealthy do account for a large proportion of wealth, the top 5% of earners account for 50% of output (Wolff, 2010). However, over one quarter of US citizens directly own some stock (Haliassos and Bertaut, 1995), nearly three quarters participate in some sort of insurance (Chen et al., 2001) and less than 10% of the population is unbanked (Rhine et al., 2006). Within this context the extreme narrative presented above does not seem to work and instead we must look for a subtler story.

Consider an economy where financial market participation is relatively high, however, there is significant disparity between high and low incomes. The young generally have lower incomes which increase over time, but the final level of income could be very much higher (in the event one becomes CEO), or relatively low, (one remains a teaching assistant).

Each agent in the economy engages in lifetime consumption smoothing. They borrow when they have low incomes and save when they have high incomes. In general, an agent starting out with a low income will borrow based on the expectation that their income will rise. The level of borrowing will be determined by their preference structure: rate of time preference, marginal utility of consumption and risk aversion. An increase at the top end of the income distribution (the density or level) will not necessarily cause an increase in the expected level of future income and risk aversion is likely to cancel out any desire for increased borrowing.

In the case where defaults are permitted, including bankruptcy provisions in the UK and the USA, higher income inequality will lead to an increased propensity to borrowing and therefore greater financial instability. If defaults are permitted at non zero consumption then the agent will be inclined to borrow more because they do not lose as much consumption in the lowest realisations of income. That is to say agents form expectations about future consumption levels which discount some future low income states.

Chapter 4 will show how this defaults based mechanism propagates through an economy to increase financial instability. The mode of transmission proposed has the same result to that of Kumhof and Rancière (2010), however, here the mechanism is somewhat simpler. While the mechanism may be more intuitive I am forced to use models of non-commitment, which are somewhat less analytically tractable. This model is novel and represents an original contribution to the literature.

Chapter 2

Literature Review

At this juncture, however, the impact on the broader economy and financial markets of the problems in the subprime market seems likely to be contained.

– Fed chairman, Ben Bernanke, Congressional testimony, March, 2007

Chapters 3 to 4 will look at the relationship between economic inequality and financial crises. However, before establishing an empirical relationship or theoretical mechanism it is important to establish what is meant by: (1) financial crisis and (2) economic inequality. In this chapter, the literature will be reviewed around these two concepts and the terms defined for the purposes of this research.

2.1 Financial Crises

There exists a great wealth of literature on financial crises, principally because no two crises are the same and so are studied under various categorisations. Here, I will use two taxonomies to discuss crisis: (1) by the manifestation, whether a crisis affects banks, currencies and so on; and (2) how the crisis originates, the three generations of crisis.

It is prudent to note that very few crises happen in isolation and no cause is pure. Quite often crises will occur simultaneously in several countries and markets, including banking, currency and asset markets. Furthermore, an individual crisis can reflect components of all three generations. It is therefore important to think about these events in broad terms.

2.1.1 Manifestations of crises

The term 'financial crisis' is broadly taken to mean the sudden and unexpected change in the value of one or more assets that results in some distribution to the normal business of finance. This is by no means a new phenomenon, in the words of Reinhart and Rogoff (2009) there have been *"Eight Centuries of Financial Foll"*. However, Reinhart and Rogoff (2009) also note that even as early as the Byzantine Empire currencies were rapidly debased to reduce levels of sovereign debt.

Even in its early development, the complex nature of any financial system results in inherent instability. Banks are by definition unstable, they lend money over the long term in mortgages and promise depositors repayment on demand. Rulers and then governments find themselves conflicted between sovereign, or even divine independence, and the fixed nature of debt contracts. Herding behaviour has been seen in every asset market from Dutch Tuplip bulbs (Garber, 2001) to Collateralized Debt Obligations (Mason and Rosner, 2007).

The financial system has evolved a mind boggling array of assets and interactions. However, the base of the financial system is still the currency; banks are the principle intermediaries and the majority of assets are still held in stocks of companies and property. This section will look at various events which can be considered financial crises. However, the remainder of this paper will be primarily concerned with banking crises, and so this will be the focus of discussion.

Currency Crises

The academic literature on financial crises is historically centred around the study of the sudden changes in exchange rate positions. The unwinding of currency regimens in the 1990s provided the first case where detailed data available for robust empirical study.

These crises are typically defined as the collapse of a fixed peg exchange rate; for example the collapse of the European Monetary System (EMS), the Mexican (Tequila) Crisis, and the Asian Flu after the collapse of the Bhat. The primary reason for this identification is the fact that such events are often dramatic and as such the crisis is easier to identify.

However, the moment the revaluation occurs is not the beginning of the crisis. Currency crises are often defined in terms of speculative pressure; several numerical definitions have been proposed based around the level of international reserves of a country, changes in the exchange rates and interest rates, see for example Eichengreen et al. (1996) or Kaminsky et al. (1998).

Defining a crisis robustly will be further discussed in Section 2.1.3.

Currency crises are often the result of international asymmetry, e.g. balance of payments or sovereign debt (Krugman, 1979). However, crises can also ensue because of an inappropriate peg, due to misjudged competitiveness or political desire to portray an economy as stronger than it really is. This is particularly pertinent to Greece's position in the Euro crisis, [1] Pisani-Ferry (2012).

Banking Crises

The business of banking is vital to modern financial systems; fundamentally they are vehicles of risk sharing and maturity transformation, taking deposits and making loans. In most developed economies banks provide payment clearing systems and therefore, have a pivotal role in promoting financial stability. There is a fundamental problem in the business model of banks in that they take short term deposits and issue long term loans, so clearly care must be taken to ensure that provision is made to meet depositors demands for liquidity.

Several models have been proposed to demonstrate why banking crises can occur. In isolation a bank can experience a 'run i.e. the point where depositors demand for liquidity is greater than a banks' liquid assets. In the event that the bank is able to meet demand by selling illiquid assets, albeit at a loss, they are said to be insolvent; in the event that claims on a bank outweigh the proceeds of the sale of assets, the bank is said to be bankrupt.

The seminal work in this area was by Diamond and Dybvig (1983) who demonstrated how bank runs occur and under certain circumstances may be useful. However, they also noted that when deposit insurance is introduced a more optimal solution can be reached. Allen and Gale (1998) were the next protagonists who demonstrated, in 1998 and 2000, that when banks hold claims against each other a systematic failure can occur, dependent on the topology of said claims. These models could easily be adapted to the situations where a significant proportion of bank deposits are owned by a small proportion of the population. Hence being an obvious place to begin a theoretical analysis of the crises involving wealth disparity.

It can be argued that bank runs pose a real and present danger to any economy (Diamond and Dybvig, 1983). In their model, Diamond and Dybvig (1983) consider two groups of consumers who maximise their utility withdrawing funds at two different times. The bank invests in a produc-

[1] Greece is maintaining parity between its currency and the rest of the Eurozone, which is, to all intents and purposes, a currency peg.

tive asset that returns $R > 1$ units of consumption at the later time or, if liquidated early, will return 1 consumption unit. They consider several possible outcomes of risk sharing problems under which depositors are repaid on a first come first served basis. They conclude that under the provision of deposit insurance a more optimal solution can be reached.

Allen and Gale (1998) go on to expand this basic model by considering the same two groups with early and late consumer preference, giving banks the option to hold either liquid short term assets returning 1 unit of consumption at the early date (cash), or an illiquid asset returning R units of consumption at the later date. In the event that a demand is made against the bank for withdrawals greater than the amount of liquid asset holdings, the available funds are divided equally among all depositors. By considering possible ratios of consumers being early or late they discuss possible optimal allocations for the bank, and use this equilibrium model to demonstrate circumstances under which external agencies should intervene to support a failing bank.

Further to considering individual banks, the model proposed by Allen and Gale can be used to analyse several connected banks (Allen and Gale, 2000). In this case, banks are able to make deposits with each other, this allows for excess liquidity demands in a single bank to be met by those banks that have excessive liquidity. Given what is described as a 'complete market', the result is that the banks independently are more stable, however, when not all banks are connected, an excess demand for liquidity in one region precipitates through the banking system and can result in a more profound systemic failure.

This represents one of the first examples of contagious behaviour, which is to say that the same model can be applied to banks in different regions or even countries. Allen and Gale (2000) not only demonstrate the potential for contagion, but also that within a finite incomplete network of banks the bankruptcy of one bank will result in all other banks suffering a similar fate.

Demirg-Kunt and Detragiache (2011) test several indicators of banking crises, including a broad range of macroeconomic fundamentals and features of the regulatory environment. They find countries with weak fundamentals are more likely to experience banking failures; in particular, low growth combined with high inflation or high real interest rates cause problems. They also find that the provision of explicit deposit insurance creates moral hazard; rewarding banks for taking irresponsible risks.

Individual banks fail on a fairly regular basis, often to be bought out by a stronger competitor. The more interesting type of banking crisis is systematic banking failure, in which many banks in the same country face similar difficulties (Demirg-Kunt and Detragiache, 2011). Examples include the American Savings and Loan Crisis, the Argentine and Chilean Crises of the mid Eighties, the Japanese Crisis, the Scandinavian Crises and more recently the US Sub-Prime Crisis (Gourinchas and Obstfeld, 2012).

It is interesting to think of the recent events in the UK in the context of individual versus systematic banking failure. The collapse of Northern Rock was probably the result of panic, and certainly changed the way markets perceived risk, but the collapse of Lehman Brothers brought the entire banking system to the brink of failure. Were it not for capital injections from governments it is difficult to see how any bank could have survived.

Asset Prices

There are many examples of financial crises caused by the rapid depreciation of asset prices, for example (i) in stock prices the 1929 Wall Street Crash, the 1987 Crash, the Dot-Com bubble or (ii) in the house prices the US in 2007, the UK in the early 1990s or Spain in the late 1990s. Once again such crises are considered in three generations: (i) the idea that assets are fundamentally over priced; (ii) investors believe assets are over priced and sell en masse even if it is a fundamentally ex ante irrational action; (iii) prices in one market falling cause mass liquidation across other markets.

Much has been postulated about the presence of so-called 'asset price bubbles'. A bubble is said to occur when the price of an asset departs significantly from its underlying value. For example, in the run up to the 2000 Dot-Com crash the S&P500 was trading with a price to earnings ratio of over 40 (Datastream, 2012) compared to historic averages of between 10 and 20 (Shiller, 1987). While Shiller (1987) may describe this as 'irrational exuberance', there is also an argument that bubbles may be wholly rational (Diba and Grossman, 1988).

The basic model for stock market crashes begins with the idea of over-optimistic investors inflating prices beyond their fundamental value; the logic suggests that eventually investors will realise their mistake and prices will adjust. Shiller (1987) comments on surveys of investors undertaken in the wake of the 1987 Wall Street and Japanese Crashes, he notes that most investors felt that assets were over priced but could not offer any explanation as to why this might have been the case.

There are as many theories of stock price collapse as there are securities traded on those markets. For example, Cass and Shell (1983) suggest that belief in sun spots can result in the unstable equilibrium.

It also proves interesting to look at house price movements, especially in the context of the current crisis and the recession of the early 1990s. There is an increasing literature developing around house prices, and especially the role of monetary policy on residential house prices. For example, Aoki

(2003) looks at the role of such policy and house prices in the consumer credit market.

2.1.2 Generations of Crises

The crisis literature also breaks down the causes of a crisis into three generations. These are not exclusive categorisations but provide a taxonomy under which to consider the events running up to a crisis.

1^{st} *Generation: Fundamentals Crisis*

A 'Fundamentals' crisis is the simplest form of financial crisis. For example, a government with a fixed exchange rate peg pursues an expansionary fiscal policy resulting in downward pressure on the currency and drain on the central bank's foreign reserves (Krugman, 1979). Fundamental indicators will predominantly include macroeconomic fundamentals e.g. international reserves, real interest rates, current account, imports, exports, money supply and unemployment. The crisis occurs as a result of the crossing of a threshold where fiscal/monetary policy becomes clearly unsustainable.

More generally, this is a crisis that *should* happen; that is to say that bad or misguided management, external shocks or malice result in unsustainable fundamentals and so the crisis unfolds. This type of crisis seems to explain the propagation of crises in Latin America in the 1970's (Flood and Marion, 1998; Dornbusch et al., 1995). However, there is no obvious reason for expectations to change rapidly in the cases of the collapse of the ERM or in the Asian Crisis. Also, in the case of the ERM collapse there was not a significant reduction in output that would be expected after a fundamentals based crisis (Dornbusch et al., 1995). Resolutions for these inconsistencies are provided by the remaining two generations.

2^{nd} *Generation: Self-fulfilling Prophecy*

Second generation crises are predominantly the result of the literature on multiple equilibria by Obstfeld (1986). Such crises are referred to as self fulfilling prophecies. If speculators believe a peg is going to change imminently they may take positions to protect against, or benefit from that change. This position can then drive collapse, even if the country has good macroeconomic fundamentals (Eichengreen et al., 1996).

3^{rd} *Generation: Contagion*

This is defined as the spread of a crisis from a different country or market. This type of crisis is widely cited in the media (Economist, 2011), but less clearly defined academically. There is not a well-defined academic consensus on the role of contagion in financial crises, however, discussion of crises often involve the semantics of contagion:

> *The Asian Flu began with continued attacks on Thailand ... Within days, speculators had attacked Malaysia, the Philippines and Indonesia... then the crises spread across the Pacific to Chile and Brazil.* (**?**, 3)

Several modes of transmission have been suggested, for example Kyle and Xiong (2007) suggest that liquidity constraints on certain traders may cause spill over from different markets.

Contagion is often defined simply as a clustering effect (Eichengreen et al., 1996); that is, a crisis is more likely to occur if a crisis has already occurred elsewhere in the world. They find an increase in the probability of a crisis to occur after another crisis when macroeconomic fundamentals have been controlled for, however, this study only considers developed markets. It is also noteworthy that a year earlier Eichengreen et al. (1995) used the same data set to demonstrate the significance of the effects of several of the variables used as control variables in predicting currency crises.

In recent years, the definition of contagion that was adopted by Eichengreen et al. (1996) has dropped slightly out of favour, and to be replaced by the stricter definition of Forbes and Rigobon (2002). While the idea of using increased correlation after a major event as an indication of contagion is not new, Forbes and Rigobon (2002) demonstrate how heteroskedasticity in market returns can cause biases in such measurements. For example, periods of high volatility may precipitate more than one crisis and this may look like a correlation between crises, when in reality it is a common cause.

Forbes and Rigobon (2002) then consider three stock market crises, many related to, or associated with a currency crisis: The 1997 East Asian Crisis, the 1994 Mexican Peso Crisis and the 1987 U.S. Stock Market Crash. In these three cases, they demonstrated how conventional, biased, correlation measures would seem to suggest the presence of contagion and how using the heteroskedasticity adjustment eliminates this evidence. They do however take care to note that they do find evidence of interdependence between markets and that this is present in both crisis and tranquil periods.

This work was not met with magnanimous support, for example Favero and Giavazzi (2002) note that while the adjustment for heteroskedasticity is worthwhile Forbes and Rigobon (2002) did not attempt to control for common shocks or in fact try to model for a normal interaction. As such they criticise the definition of contagion used by Forbes and Rigobon (2002), suggesting that contagion is in the change in co-movements during a crisis due to the creation of new mechanisms to transmit interconnections. This can of course be compared directly to Eichengreen et al. (1996) compensating for macroeconomic similarities.

Furthermore, Chiang et al. (2007) dig deeper into the ideas proposed by Forbes and Rigobon (2002), noting that their method ignores the problems of missing variables. Forbes and Rigobon (2002) do in fact provide an appendix on this topic and conclude that the heteroskedasticity correction is not as accurate under the presence of missing variables. Chiang et al. (2007) also consider problems involved in modelling the dynamic nature of the cross market correlation by using a multivariate GARCH Model in order to capture the time varying nature of the correlation matrix and to avoid the arbitrary allocation of crisis and tranquil periods.

This discussion serves to highlight the importance of controlling for other effects aside from that which is being studied. Chapter 3 will make extensive use of the methods outlined by Eichengreen et al. (1996).

2.1.3 Defining a Crisis

So far we have discussed types of crisis in terms of the markets that they affect and the three generations of crisis. However, 'crisis' is much more of a human word than a technical word, as it suggests something both scary and out of the ordinary. Hence why it is often used in the press to evoke a strong emotional response. Therefore, in each case, one must be careful to note that the definition will contain a certain element of subjectivity. To have any rigour, the word 'crisis' must be interpreted to mean a significant statistical departure from 'normal' market conditions. This will be important when interpreting empirical results to test the sensitivity to these assumptions by varying any numerical definitions.

Therefore, the first step in defining a crisis must be to measure some variables related to the crisis, then to ascertain its 'normal' behaviour, and to identify instances in history where it is not behaving normally. This measurement will depend very much on the type of crisis being observed, and we must appeal to theory to give a guide as to which variables may be affected. Thus, for example, in the case of currency crises, Eichengreen et al. (1996) use the following definitions:

$$
\begin{aligned}
EMP_{i,t} &= [(\alpha\Delta e_{i,t}) + (\beta\Delta(i_{i,t} - i_{G,t})) - (\gamma(\Delta r_{i,t} - \Delta r_{G,t}))] \qquad (2.1)\\
EMP_{i,t} &= \text{Exchange Market Pressure (EMP) in country i at time t}\\
e_{i,t} &= \text{Price of a DM in i's currency at time t}\\
i_{i,t} &= \text{Interest rate in country i at time t}\\
r_{i,t} &= \text{Foreign reserves held by country i at time t}\\
\alpha, \beta, \gamma &= \text{Weights}
\end{aligned}
$$

This definition is used to study the collapse of the ERM and therefore the variables are taken relative to the Deutsch Mark. The rationale behind the proposed measurement is that the crisis is the result of speculative pressure, with the markets betting against the currency. The result of this pressure, in the form of selling off assets denominated in the given currency, will be to reduce the value of that currency. Moreover, if the monetary authority attempts to protect the currency, by either raising rates or selling reserves, this will also be detected even if the currency does not change in value.

A variable crisis is then defined such that;

$$
Crisis_{i,t} = \begin{cases} 1 & EMP_{i,t} > 1.5\sigma_{EMP} + \mu_{EMP}, \\ 0 & \text{otherwise.} \end{cases} \qquad (2.2)
$$

We may be tempted to think of this as over complicating the question, a currency crisis is surely easy to identify as it results in devaluation of the currency. However, this is not always the case, in the event that a government or monetary authority successfully defends the currency we cannot say that a crisis did not occur. Eichengreen et al. (1996) use this definition to identify various crisis episodes on which to do analysis. They then go on to test the sensitivity of their results to a change in the weights within the EMP and the condition that a crisis occurs when the EMP deviates 1.5 standard deviations from the long-term mean. They find it does not affect the conclusions of the paper.

Kaminsky et al. (1998) provides a broad overview of 25 empirical studies considering possible indicators of currency crises/speculative pressure and agrees with Eichengreen et al. (1996), in all but the inclusion of interest rates as a measure of speculative pressure. The motivation for this is that in considering developing countries, one must bear in mind restrictions on the market defining interest rates.

In the case of banking crises, it is difficult to be as precise. A well capitalised system may experience large defaults but still survive. Conversely, panic may cause the system to fail. High interest rates may increase profit or cause instability, or a broadly capitalised system may suffer significant external shocks and fail despite the appearance of ex ante stability. For each variable that could define a banking crisis measure there is an example of a crisis that does not fit the measure. Generally, one must turn to case study evidence. Reinhart and Rogoff (2011) make the following definition:

> *We mark a banking crisis by two types of events: (1) bank runs that lead to the closure, merging, or takeover by the public sector of one or more financial institutions; and (2) if there are no runs, the closure, merging, takeover, or large-scale government assistance of an important financial institution (or group of institutions) that marks the start of a string of similar outcomes for other financial institutions.* Reinhart and Rogoff (2011, p. 1680)

Reinhart and Rogoff (2011) also emphasise the distinction between systemic failure and financial instability. For example, they would not take the collapse of Northern Rock in the UK to be the start of the 'banking crisis' as it only affected one institution. This example serves to demonstrate the key drawback in this events-driven definition, namely that the timing of events may make the timing of the 'crisis' difficult to pinpoint. It is easy to see how in the case of the UK recently, the crisis began in mid 2007 but the 'crisis event', using the above definition, would be the capital injection into Lloyds Banking Group and RBS in 2008. Equally, events can make crises seem to start too early when the worst is much later; one could look to Ireland in the recent past as an example (Gourinchas and Obstfeld, 2012).

In this paper the overall stability of the financial system is the primary concern and therefore we must restrict the study to systemic crises. As defined:

> *under our definition, in a systemic banking crisis, a country's corporate and financial sectors experience a large number of defaults and financial institutions and corporations face great difficulties repaying contracts on time. As a result, non-performing loans increase sharply and all or most of the aggregate banking system capital is exhausted. This situation may be accompanied by depressed asset prices (such as equity and real estate prices) on the heels of run-ups before the crisis, sharp increases in real interest rates, and a slowdown or reversal in capital flows. In some cases, the crisis is triggered by depositor runs on banks, though in most cases it is a general realization that systemically important financial institutions are in distress* Laeven and Valencia (2008, p. 5)

The above mentioned papers, Laeven and Valencia (2008), Reinhart and Rogoff (2011), and Bordo et al. (2001), present comprehensive databases of banking crises over the last century. Each of these databases are based on assessing evidence case by case and therefore, there is some level of subjectivity involved in the definition. Atkinson and Morelli (2011a) suggests taking a majoritarian view of the above three databases, defining a crises to have happened/begun if at least two of the studies agree. While this method is at least conclusive, given caveats for the various periods studied, it would appear somewhat *ad hoc*.

The first two sections of empirical work presented below will use this majoritarian definition of a crisis. However, the final section will present a different method drawing from studies of dynamic factor analysis.

2.2 Inequality

Inequality is a much discussed, little understood and highly contested concept in modern economics, even before one brings into question fairness, rights and politics. However, there is a growing consensus that inequality matters. In the pop-economics field, the Spirit Level (Wilkinson and Pickett, 2010) opened the recent debate claiming inequality damages many social outcomes. The popularist political debate rumbles on, but the academic questions are no closer to a solution.

It is important to first decide what inequality actually means. Intuitively, inequality is the difference between the rich and poor in a given society. However we are immediately confronted by the question of wealth vs. income. The former would perhaps seem more appropriate for a discussion on financial stability. It is, after all, wealth rather than income that makes up the majority of bank deposits. However, data on wealth inequality is even more sparse than that on income; moreover, wealth may not be liquid and therefore hard to value, for example the ‘cash poor’ land owner. It is also noted that one may wish to think in terms of family or households and, in the ideal case, total lifetime economic resources.

The question is then how to incorporate inequality into a well defined econometric model, ideally we would like a single numerical measure of inequality. The most common numerical measure of inequality is the Gini coefficient, which takes the values in the interval $[0, 1]$ with 0 being perfect equality and 1 being perfect inequality. The numerical value is usually calculated with reference to the Lorenz curve. Hence, for the distribution Ψ_y the function $L(x)$ is the proportion of households with income less than or equal to $\Psi^{-1}(x)$ where $x \in [0, 1]$:

$$L(x) = \frac{\int\limits_{-\infty}^{\Psi^{-1}(x)} yd\Psi_y}{\int\limits_{-\infty}^{\infty} yd\Psi_y} \tag{2.3}$$

It is useful to note that $L(\Psi(y_j))$ is increasing and convex.

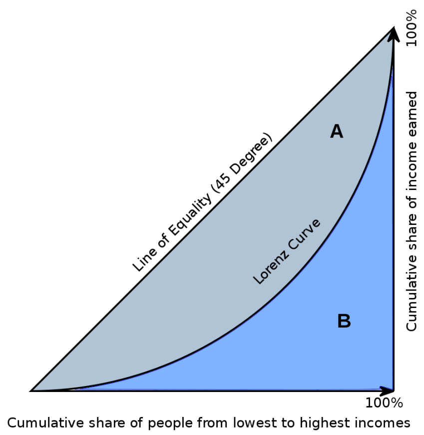

Figure 2.1: Example of Gini coefficient calculation from Lorenz Curve (Reidpath, 2009)

If income was distributed with perfect equality, the curve L would be a straight line at 45^o. Then intuitively the Gini coefficient is the ratio of the areas under the 45^o line and the area between the 45^o and the Lorenz curve. In Figure 2.1 this would the ratio $A/(A+B)$ or analytically it is given by:

$$G = 1 - 2\int\limits_{0}^{1} L(X)dX \tag{2.4}$$

$$= 1 - \frac{1}{\mu}\int\limits_{y_{min}}^{y_{max}} (1-\Psi(y))^2 dy \tag{2.5}$$

However, as Atkinson and Morelli (2011a) ask, 'Which part of the parade should we be watching?' Gini is a measure of overall dispersion, but there is some evidence that this value does not capture all of the changes that are occurring.

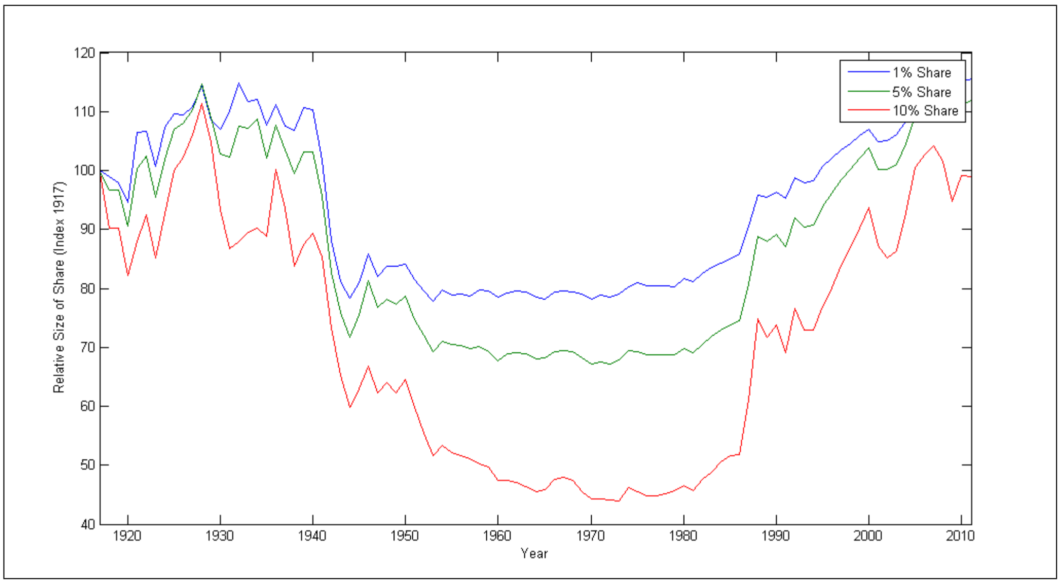

Figure 2.2: Relative size of top income shares indexed at 100 in 1917

Figure 2.2 shows that significant changes are occurring in different parts of the distribution and not all of these can be accounted for by a single measure. Specifically, we see that in the latter half of the twentieth century the share taken by the top 1% rose faster than that taken by the top 5% or 10%. It will therefore be useful to consider other measures if they are available. Two principal sources will be used:

1. The World Incomes Inequality Database, which systematically pools research on income distribution (WIDER, 2008). The database covers 159 countries with time series varying over the last century and ranks observations by the quality of the source.

2. The World Top Incomes Database is a relatively new source created for the purposes of researching the effect of income distribution (Facundo et al., 2012). The creators set out to create a source which is comparable across time and countries based on the best available data. However, the data is still predominantly based on tax statistics and therefore subject to the below mentioned issues. This database focuses on actual shares of income rather than simply Gini coefficients, although it is limited to 23 countries.

Both data sources rely heavily on information based on the analysis of individual studies of government tax receipts. Such sources are often criticised as they are collected for an administrative rather than scientific purpose and are subject to bias because of tax evasion or avoidance (Atkinson and Brandolini, 2001). However, given the sparsity of data, it is not prudent to discard too much data and instead, we must be careful about over stating the strength of any results.

There are further problems in using Gini type measures of inequality, the first obvious question is 'Inequality of what?' Generally speaking this thesis discusses inequality of income, however, some would argue that inequality of consumption is, in many ways, more important. These quantities do not necessarily move together, as Krueger and Perri (2006) demonstrate, rising income inequality does not necessarily imply rising consumption inequality. This is in part due to changes in patterns of borrowing and saving, which provides an interesting narrative for the theoretical aspect of this work. Moving beyond mere measurement, Sen (1973) began a strand of literature that goes further to question what inequality really means. That is to say that a discussion based purely on measurements of money missed out the social and political disenfranchisements that may be more important than spending power. While such issues are very important, this study requires a quantitative measurement and therefore we will use the most widely available measure, that is the Gini coefficient.

Chapter 3

Empirical Investigation

God does not care about our mathematical difficulties. He integrates empirically.

–Albert Einstein quoted in Ishaq (2005)

3.1 Introduction

The literature review has provided an overview of current discussions in the area of inequality and financial stability. It is clear that some relationship may exist, but the nature of this relationship remains hard to pin down. This chapter will aim to look at the best available data and answer two simple empirical questions:

- Are the financial systems of countries with greater inequality less structurally sound than those with more equality?
- Have countries with higher inequality suffered more financial crises than those with higher equality?

The exposition that follows will demonstrate that these questions are not straightforward. To begin with, the term 'crisis' is vague and inequality is badly measured, which is compounded by the fact that the data is sparse in some areas; therefore careful treatment is required. For these reasons, among others, relatively novel approaches are used. These methods are not new but they are not often applied to economics in this way.

Two methods are presented that demonstrate that we cannot rule out inequality as a risk factor for financial crises. We begin by looking at the most recent crisis and try to explain the cross-sectional experience in terms of macroeconomic variables using a logistic regression. We find that neither conventional predictors nor inequality provided significant *ex ante* warning of the crisis. The next section looks at a broader selection of crises, again using logistic regression, but now in a panel data setting. Here we find some evidence that high inequality may be a risk factor for some types of crises.

The logistic regressions both rely on macroeconomic data, as control variables, and the Gini coefficient, as a measure of inequality. Both sets of data are incomplete. To make the best use of data I employ a Markov Chain Monte Carlo imputation on the macroeconomic dataset. However, for the Gini coefficient data set I show that it is sufficient to employ a simpler interpolation strategy.

The second method looks at how we define crises and financial stability. Here we demonstrate that a principal component decomposition of various indicators provides a good measure of financial stability. We show that the factors associated with financial fragility are well predicted by measures of inequality. Moreover, we show that by using this decomposition we are able to attribute a channel through which inequality may affect financial stability.

Neither method presented establishes, with certainty, that economic inequality causes financial crises, however, it is shown that there does appear to be some relationship and that higher inequality seems to increase the likelihood of a crisis.

This chapter will proceed as follows:

3.2 Data Used

3.4 A Logistic regression approach

- The 2007 Crisis: A brief introductory study
- Panel Data Study: Looking at a wide range of indicators over time

3.5 Measuring financial stability: PCA Analysis

3.2 Data

This section will address issues around choice of data for empirical study. The first problem to address is that of missing data; we will present a robust imputation method using Markov Chain Monte Carlo (Geyer, 1992). This method will be applied to a data set based on the International Monetary

Fund's (IMF) International Financial Statistics (IFS) (International Monetary Fund, 2012) and the Penn World Tables (Alan Heston and Aten, 2012).

The next problem to deal with is that of defining what exactly is meant by the word 'crisis', here we will propose two options: (1) taking the definitions from the literature (2) applying a principal component analysis to the IMF Financial Soundness Indicators dataset (Fund, 2012). The former will be discussed in this section while the latter can be found in Section 3.5.

Finally this section will further discuss the issues around measuring inequality and will propose that, while not ideal, the Gini coefficient is the best measure of inequality. Moreover, it will be proposed that we can make best use of the available data by treating Gini as constant over time.

3.2.1 Macroeconomic Data and Justification of Imputation

The methods proposed in this work will both make extensive use of macroeconomic data. Data will be taken from the International Monetary Fund's (IMF) International Financial Statistics (IFS) (International Monetary Fund, 2012) and Penn World Tables (Alan Heston and Aten, 2012). In each case, the source that has the greatest amount of available data is used for each series. The maximum possible sample from 1949 to 2012 across all 216 countries in the IFS/Penn World Tables gives a total maximum of 13,608 observations. Table 3.1 shows the overall availability of the relevant data. Data sources for the Gini coefficient are discussed in the previous section.

However, of the 2064 observations of the Gini coefficient only 331 points also have the other variables available; details of the exact sample are given in Appendix A.3. However, there are 1760 observations for which at most 3 points are missing. This is an ideal situation in which to employ Multiple Imputation (MI), a process by which we systematically account for missing data.

As discussed in Section 2.1.3, data on crises will be taken from Laeven & Valencia (2008), Reinhart and Rogoff (2012) and Bordo et al (2001). For the purposes of quantification of crises, we will use information from the IMF's Global Financial Stability Report. For this reason, countries are limited to those covered by the Stability Report. This is a total of 97 countries broken down as per Table A.14. It is also worth noting that this selection also accounts for more than 85% of Global Output and 70% of global population in 2006[1].

[1] Data Taken from the IMF IFS.

Table 3.1: Availability of Data

Variable	Missing Values	Observations
Gini	11,544	2,064
Top 5% Share	12,836	772
Top 0.5% Share	12,688	920
Top 0.1% Share	12,710	898
Top 1% Share	12,680	928
Pareto-Lorenz Coefficient	12,609	999
GDP	5,541	8,067
CPI	5,541	8,067
REER	12,932	676
Fx	4,348	9,260
Long Rates	11,565	2,043
Short Rates	10,926	2,682
Unemployment	11,048	2,560
Gov Cons	5,541	8,067
Current Account	8,459	5,149
Fiscal Pos	10,268	3,340
Equities	11,845	1,763
population	3,108	10,500
Trade Balance	3,353	10,255
Reserves/M2	10,583	3,025

Imputation

The process of imputation is often seen as controversial, but is a natural extension of the principle that where possible, we should not discard data unnecessarily. This method should not be seen as interpolation, instead it should be seen as a statistically robust aid to estimation when dealing with missing data. The basic principle states that given a set of incomplete data it may be possible to generate several sets of complete data. If the new data generated has the correct characteristics we may be able to draw inference about the analysis performed across all sets of data.

It was established by Rubin (1987) that Multiple Imputation (MI) yields valid inference if:

1. The Imputation is valid, specifically drawn from a Bayesian Posterior predictive distribution

2. The complete-data analysis is statistically valid in the absence of miss-

ing data

The second point (2) must be assumed in order to proceed, but this is a general assumption about the validity of econometrics. However, one must be careful to ensure the first condition is satisfied. For example, it is not sufficient to simply generate random points that match moments of the distribution of observed data. In this work I use a generalised Markov Chain Monte Carlo (MCMC) method (Schafer, 1997), details of the algorithm are given in the following section (3.2.1).

The series for GDP, CPI, Foreign Exchange Rate, Imports and Reserves are complete and population, while not analysed in the model, is also complete. The other series are given a multivariate normal prior, then conditioned by MCMC on observed data and regressions against other complete and incomplete variables using a pseudo Estimation Maximisation (EM)Algorithm. The result should be sets of data consistent with both the statistical properties of the imputed variables and the structural properties of the observed data.

Generally a small number of imputations are required, usually less than 3-6 (Schafer, 1997). However, given the relatively high proportions of some missing variables, 50 imputations are generated and convergence of results is shown. It is interesting to note that the MI method is effectively a simulation based maximum likelihood (ML) method (Enders and Bandalos, 2001). It should, in principle, be possible to devise an appropriate ML estimator for the model parameters given the missing data. However, this would be both time consuming and unnecessary, given the ease at which MI can be used.

Implementation and Markov Chain Monte Carlo

We begin by defining that a matrix, X, contains some observed data, X_{Obs}, and some missing data, X_{Mis}. We assume that the data is missing at random in the sense of Rubin (1987). To understand what this means, we define a function I_i as an indicator which is 0 for missing values and 1 for observed values. Following Little and Rubin (2002), missing completely at random can then be defined in terms of the conditional distributions of the indicator function, that is to say:

$$\begin{aligned} P(I|X) &= P(I|X_{Obs}, X_{Mis}) && (3.1) \\ &= P(I) && (3.2) \end{aligned}$$

However, this condition is stronger than we strictly require for the MCMC algorithm to give acceptable results (Rubin, 1987). Instead it is enough to require that $P(I|X) = P(I|X_{Obs})$ equivalent to Rubin's (1987) missing at random condition. We assume that the missing data is drawn from a multivariate normal distribution $N(\mu_i, \Sigma_i)$, we have a normal prior in a Bayesian sense. The aim is then to estimate the missing values of variables such that the imputed variables are consistent with the statistical distribution of the observed data; given our prior. That is to say, we know the observed parameter estimates $\hat{\theta}_i = (\hat{\mu}_i, \hat{\Sigma}_i)$, and this should form our initial 'guess' of the true complete data distribution[2].

We can then define the MCMC algorithm as follows:

1. Imputation Step: Draw a random set of X_{Mis} from the distribution $X^{(t)}_{Mis} \sim P(X_{Mis}|X_{Obs}, \theta^{(t-1)})$
2. Update Step: Update the distribution $\theta^{(t)} \sim P(X_{Mis}|X_{Obs}, X^{(t)})$

The stream of $\theta^{(t)}$ then forms a Markov Chain, which converges to a posterior distribution suitable for the imputation. The distribution will then be proper in the sense of Rubin(1987), an appropriate Bayesian representation of the space of parameters.

The implementation used in this work, from the MI routine in STATA 11, accepts additional parameters in θ; in this way we use other readily available data to help condition the distribution of missing data. For our purposes, we use GDP and Population in addition to the main macroeconomic variables.

Combining Imputations

The imputations can be combined as follows, if the parameter $\hat{Q}_i$ is estimated for $i \in N$ imputations the best estimate of $\bar{Q}$ is given by the simple mean:

$$\bar{Q} = \frac{1}{N}\sum_{i=1}^{N}\hat{Q}_i \tag{3.3}$$

The variance, V of this estimate must then be taken as the sum of the between-imputations variance, B, and the within-imputation variance U:

$$V = \bar{U} + (1 + \frac{1}{N})B \tag{3.4}$$

[2] It is important to note that the complete data parameters, θ would be those given all observed and unobserved data. This is of course our best estimate of the 'true' population parameters

Where:

$$\bar{U} = \frac{1}{N}\sum_{i=1}^{N}\hat{U}_i \tag{3.5}$$

$$B = \frac{1}{N-1}\sum_{i=1}^{N}(\hat{Q}_i - \bar{Q})^2 \tag{3.6}$$

Where $\hat{U}_i$ are the set of parameter variance estimates obtained from the individual imputation regressions. This variance can then be used in place of the variance estimate from the individual regression equations for t-statistics and other statistical tests.

3.2.2 Measuring Inequality

The literature review (Section 2.2) highlighted problems associated with measuring inequality and it is sensible, at this point, to look more closely at the statistical properties of the Gini coefficient. The countries covered by the databases can be found in Appendix A.3, along with descriptive statistics by country. It is useful to note that for Gini coefficients, using the best available data from both sources, the cross-sectional variation between countries is higher than the variation over time, summarised in Table 3.2.

Table 3.2: Statistical Longitudinal Properties of Gini Coefficients

	Mean	Std. Dev.	Min	Max	Observations
All	38.6	10.9	15.9	79.5	N = 1151
Between		9.98	19.1	63.7	n = 128
Within		5.48	19.1	63.8	T = 8.99

Table 3.2 gives the statistical properties of the sample of Gini coefficients quoted as a percentage (out of 100 rather than out of 1). The number of points (N), countries (n) and average length of time series (T) is also shown. We can see that the standard deviation between countries is higher than that within each country. This does not conclusively show that variation across time is less significant than variation between countries as it does not prove it statistically. Simple ANOVA analysis reveals that means of the countries are significantly different from each other, which also implies it is the variation between countries rather than over time that matters.

One would ideally form a panel of data with time series for each country, however, the average period covered is 1967-2000, but the average length of a continuous time series is only 9 years. Moreover, 47 countries have no continuous time points and only 10 have more than 25 years. A more detailed break down of the lengths of time series available is given in Appendix A.2.

To combat the problem of short time series we will make use of the fact that Gini is a relatively stable measure. While this makes it difficult to analyse dynamic effects it also means I am able to extrapolate data easily. I test the hypothesis in two parts: (1) that there is no time trend and (2) that the value of the Gini coefficient is highly persistent. To test a time trend we take the regression equations as:

$$G_{it} = \beta_{0i} + \beta_{1i}t + \epsilon_{ti} \tag{3.7}$$

In equation (3.7) I write the Gini coefficient, G_{it}, of country i at time t as the sum of country specific constant, time trend β_{1i} and stochastic error term ϵ_{ti}. Appendix A.4 gives the results of these regressions for each country in the sample. In this table we see that in all cases the time trend is not significant at the 5 % level, with a critical t-value of 1.96.

We can then jointly test the hypothesis that the time trend is zero over all regressions.

I also run an ancillary pooled regression holding β_{0i}, β_{1i} and both constant over countries. I find that it is not possible to assume the intercept terms, β_{0i}, are the same over countries but it is also not possible to reject the hypothesis that all of the slope coefficients are equal to zero.

The absence of a time trend does not by itself suggest that the values are stable, for that we need to determine if the best predictor of a future Gini coefficient is the current level. To show this we use the following specification for the regression equation:

$$G_{it} = \beta_{0i} + \beta_{1i}G_{it-1} + \epsilon_{ti} \tag{3.8}$$

Equation (3.8) will allow us to test if the process for the evolution of a Gini coefficient follows a random walk. Furthermore, Appendix A.5 shows the result of this regression. We find that there is insufficient evidence to suggest that the coefficient of any G_{it-1} is not equal to 1.

3.3 Relationship between borrowing spread and Gini Coefficient

In the core model presented in the following chapter I establish a mechanism by which inequality can produce a higher borrowing spread. It is therefore useful to establish a link, albeit weak, between the Gini coefficient and borrowing spread. I first note that across my full panel of data there is a small, 5%, correlation between these two values. This suggests that if any theoretical relationship is found it is unlikely to be strong.

3.4 Logistic regression in predicting crises

This section will provide two methods to analyse the ability of inequality to predict the probability of a crisis. Both methods will use the logistic regression, a standard in the literature on banking crisis (Laeven & Valencia, 2012). Following the literature of Eichengreen et al. (1995), we define a regression equation:

$$Crisis_{i,t} = \lambda I(L)_{i,t} + \epsilon_{i,t} \tag{3.9}$$

where λ are the coefficients of the regression to the estimated and $\epsilon_{i,t}$ are appropriately defined shocks. The questions are then: how to estimate this regression; the functional form of I and; what components should be present in the state vector $L_{i,t}$? Following Demirg-Kunt and Detragiache (2000), this regression is estimated using a logistic form often used in studies of banking problems. The logistic regression assumes that the probability of a crisis given the extant conditions, $\pi(L_{i,t})$ is given by the identity in equation (3.10).

$$\pi(L_{i,t}) = \frac{1}{e^{-\beta \mathbf{L_{it}}} + 1} \tag{3.10}$$

It is important to note that because some countries do not suffer crises, and therefore record all zeros in the crisis variable, it is not possible to use the between groups estimator in a panel data logit context. Therefore, we must use a Random Effects model, but this presents some problems in analysis and must be treated carefully. In this case, we change our specification to decompose the error, $\epsilon_{i,t} = w_i + \mu_{it}$ such that:

$$\begin{aligned} w_i &\sim N(0, \sigma_w^2) \\ \mu_i &\sim N(0, \sigma_\mu^2) \end{aligned} \tag{3.11}$$

Using the Random Effect model with a logistic form produces a consistent estimate for coefficients, however, estimates are not necessarily efficient

(Maddala, 1987). Specifically, in the multinomial logistic distribution all correlations between groups are constrained to be one half. This problem is dealt with using robust standard errors (Petersen, 2009); in the procedure outlined below I use the Huber Sandwich Estimator (Freedman, 2006).

3.4.1 Inequality and the 2007 Financial Crisis

The most obvious starting place for analysis of crises in financial markets is in the study of the recent past. While it could be argued that results are only applicable to a very specific event, one can also argue that the structure of the current banking market is more similar to the 2007 Crisis than, say, to the Scandinavian or Japanese Crises of the 1990s. For example the value of the Credit Default Swap market has increased nearly 100 fold in 10 years (ISDA, 2012). This sort of innovation has significantly impacted on the overall stability of the financial system and mechanisms by which shocks are transmitted and macroeconomic variables interact. It is therefore pertinent, when asking questions about what may happen in the future, to look closely at more recent events.

I will therefore perform the following regression:

$$Crisis_i = \lambda I(L)_i + \epsilon_i \tag{3.12}$$

Where $Crisis_i$ takes a value of 1 if a banking crisis is reported after 2007 and 0 otherwise. In this case only Laeven & Valencia (2012) report the most recent crises and so I use their definition. The vector $I(L)_i$ contains macroeconomic variables along with the Gini coefficient. Moreover, the relative stability of Gini coefficients means that we do not necessarily need to have the most recent Gini coefficient data in order to estimate the current value. Instead, the most recent observation of the specific Gini is used for all countries.

The other macroeconomic variables used in $I(L)_i$ are discussed in more detail in the following section. However, they are chosen to be broadly consistent with the extant literature.

Method

To simplify the analysis for this section I rely on countries with complete data for 2006. this covers all developed and many emerging markets. These countries are listed in Appendix A.3. I then use the macroeconomic variables listed in Table 3.6 as predictors of the the variable $Crisis_i$, which takes the value of 1 for any country which experienced a crisis after 2007. I then use a very simple logistic regression based on maximum likelihood estimation. I use the R package logit for this purpose.

Results

From Table 3.3 we can see that in the 2007 crisis countries with a higher Gini coefficient were more likely to suffer financial crises than those with lower. We must also note that none of the predictors are significant, however, Gini seems to be closer to being significant than the other factors. This in itself is interesting as it suggests that the likelihood of being caught up in the recent crisis was not dependent on macroeconomic conditions prior to the crisis.

Table 3.3: Logit Regression of Crisis

	Coef.	Std. Err.	t	$P > t$	[95% Conf.	Interval]
gini	0.289	0.188	1.530	0.126	-0.081	0.659
dgdp	52.419	43.800	1.200	0.232	-33.780	138.617
fx	-6.683	38.717	-0.170	0.863	-82.847	69.480
cpi	-16.885	37.873	-0.450	0.656	-91.396	57.627
reer	0.019	0.111	0.170	0.866	-0.199	0.237
shortr	14.069	96.679	0.150	0.885	-177.161	205.298
curr	0.000	0.000	-0.020	0.986	-0.001	0.001
imp	0.000	0.000	-0.660	0.506	0.000	0.000
exports	0.000	0.000	0.460	0.644	0.000	0.000
shares	-0.007	0.065	-0.100	0.917	-0.135	0.122
dm2	-0.006	0.204	-0.030	0.975	-0.410	0.397
cons	1.953	14.488	0.130	0.893	-26.587	30.493

Strictly speaking the result presented above does not show significance of the Gini coefficient at the accepted 5% level or even 10% level. However, given that it is the most significant predictor I argue that we cannot simply discard this factor. Moreover, recent research in statistics has suggested that the t-test may not be the most appropriate way of measuring the impact of a specific variable. [3]

The 2007 crisis will be considered in more detail in Section 3.4.

[3] Instead a Bayesian formulation is currently proposed as more appropriate (Kruschke, 2012), which emphasises the importance of priors in conditioning statistical tests. In this case selecting a gamma prior on the distribution of errors yields significant coefficients, but this method is both contentious and open to abuse so is not presented rigorously.

3.4.2 A Panel Data Study

Given that a relationship seems to exist for the 2007 financial crisis, the question remains as to the generality of the result. Looking at a wider sample of data, taking all countries in the WIIDR/World Top Income databases an obvious first step is look at the correlations between crises and inequality (WIDER, 2008; Facundo et al., 2012).

To give the broadest possible base, the crises determined in Laeven and Valencia (2008) will be used to define a crisis variable $Crisis_{i,t}^{j}$ for country i, in time t and variety j. Once again using the Laeven and Valencia (2008), categorisations Banking ($j = B$), Currency ($j = C$) and Sovereign Debt ($j = S$) crises will be considered along with Any Crisis ($j = A$), a summary definition is given in Table 3.4. Then, the first sense check is to look at simple correlations between the variables. This is presented in Table 3.5. The Gini coefficient is taken from the WIIDR data set and the other measures of inequality are taken from the World Top Income Database.

Table 3.4: Definitions of crises

Type	Definition
Banking	A systemic banking crises as defined in Section 2.1.3 i.e. a significant number of banks experience runs and/or the government has intervened to prevent such runs
Currency	Nominal depreciation of at least 30% which is at least 10% faster than previous year
Sovereign	Sovereign default or restructure
Any	At least one of the above crises

Table 3.5: Correlation of Crisis with Measures of Inequality

	Banking	Currency	Soverign Debt	Any
Gini	2.98%	8.59%	4.95%	9.36%
Top 5% Share	2.03%	-0.99%	4.24%	2.22%
Top 1% Share	4.98%	1.58%	5.03%	5.95%
Top 0.5% Share	6.34%	0.21%	4.93%	6.15%
Top 0.1% Share	6.47%	0.15%	3.30%	5.54%
Pareto Lorenz	-8.45%	-2.34%	-1.98%	-8.03%

It is interesting to note that the correlation with banking crises is increasing in the marginality of the income share. The correlation between banking crises and income share is higher for the Top 0.1 % of incomes than the Top 5%. While the Gini coefficient and Pareto Lorenz (P-L) coefficient apply to different samples and are not wholly comparable, it is still interesting to note that Gini seems to be more strongly correlated with Currency Crises and P-L with Banking. The possible explanation of this could be that Gini measures the broad income dispersion, whereas P-L is more of a measure of concentration in the high shares. This is encouraging given the hypothesis of this research.

Empirical Method

These simple correlations may not be enough to demonstrate the link that we are interested in and so, as per the literature of Eichengreen et al (1996), we define a regression equation:

$$Crisis_{i,t} = \lambda I(L)_{i,t} + \epsilon_{i,t} \tag{3.13}$$

In equation (3.13) we see that observations are not only identified by a country index, i, but also by a time index, t. This panel of data is not balanced and so we must be careful to apply robust standard errors to the estimates. The macroeconomic control variables to be included can be found in Table 3.6. Indicators found to be significant by Demirg-Kunt and Detragiache (2000) are noted.

Table 3.6: Determinants of Banking Crises and Expected Result

Variable	Significant	Explanation	Expected Relationship
GDP	Yes	State of Macro Economy	Negative
Short Rate	Yes	Financial Liberalisation	Positive
Inflation	Yes	Economics Management	Positive
M2/Reserves	Yes	Financial Stability	Positive
Fiscal Surplus/Deficit	No	Ability of Government to Intervene	Negative
Long Rates	NA	Exceptions/Refinancing	Negative
REER	NA	International Competitiveness	Positive
XS Equity Returns	NA	Expectations	Negative
Unemployment	NA	Various	Positive
Trade Balance	NA	International Stability	Negative

A change in GDP is generally taken to be an indicator of macroeconomic stability and confidence in the economy; a growing economy would be less likely to experience a crisis. The nominal short rate, could have many effects on the stability of the financial system, but in this case it is being used as a proxy for refinancing costs on banks' balance sheets. The higher the short rate the more difficult it is for banks to obtain refinancing. Inflation gives a general indication on the stability of the economy and how well it is being managed. Finally, the Government Budget Position encompasses the ability of a government to assist, if needed, along with effective governance of government finances.

The value of M2/Reserves indicates the relative strength of the government in international money markets; how well able they are to defend the currency if the need arises.

In addition I consider variables not included by Demirg-Kunt and Detragiache (2000), but consistent with the literature on financial crises. Spreading the possible net as wide as possible strengthens the analysis given that including variables does not reduce power but missing out variables can result in bias. Long rates are included to capture expectations about the future economy along with long-term effects on banks' balance sheets. Excess Equity Returns also capture expectations about the future whereas Trade Balance and Real Effective Exchange Rate (REER) should capture relative international competitiveness.

Given that we are looking for *ex ante* predictors of crises and that the macroeconomic situation will be altered after the onset of a crisis I use the lagged values of the explanatory variables. The results are, therefore, indicators of which factors determined the probability of a crisis in the following year.

Results

Each type of crisis is estimated separately along with the combined variable. All models apart from the Sovereign Debt, are found to be significant and the Random Effects specification is found to be appropriate. [4] Gini appears to be a good predictor of Currency Crises and also, the overall crisis variable but does not predict Banking Crises well. In each case, the sign of the Gini Coefficient in the model is positive and therefore, it would seem that inequality does increase the probability of a crisis, although this is not statistically robust.

[4] This is a not a trivial test, the models are estimated on the sample of countries who experience at least one crisis using both fixed and random effects and the result is compared using a Hausman Test (Hausman and McFadden, 1984). This does not in fact lead us to the conclusion that the model specified is appropriate for the entire sample but does suggest that proceeding in this way should be valid.

Table 3.7: Random Effects Model of Various Crises With Gini Measure (Standard Errors in italics

	Banking	Currency	Sovereign	Any
Gini coefficient	0.0079	0.0156	0.0158	0.0106
	0.0095	*0.0079*	*0.0127*	*0.0064*
L. GDP	-3.3713	-3.1793	-2.1036	-2.9981
	1.5059	*1.2553*	*1.9832*	*0.9809*
L. D. GDP	1.3334	2.5551	0.4856	1.7822
	1.0820	*0.8213*	*1.4269*	*0.6901*
L CPI	-0.2457	0.2068	-0.1989	0.2269
	0.3660	*0.3791*	*0.4627*	*0.3127*
L. Short Rate	-0.1306	0.6071	1.0837	0.4560
	3.0178	*2.1762*	*2.9237*	*1.8648*
L. FX	0.4567	0.4824	0.6552	0.5883
	0.1784	*0.1609*	*0.1943*	*0.1273*
L. Unemployment	-0.0034	0.0029	0.0121	0.0021
	0.0218	*0.0190*	*0.0272*	*0.0137*
L. Government Cons	-1.0747	0.6047	-1.1532	-0.3719
	0.8264	*0.6454*	*0.9469*	*0.5388*
L. Equity Returns	-7.38E-04	-1.06E-03	7.28E-04	-6.23E-04
	3.00E-03	*2.32E-03*	*3.03E-03*	*1.93E-03*
L. Fiscal Position	-2.53E-08	-1.79E-10	3.30E-08	-1.23E-09
	4.42E-08	*3.54E-08*	*5.17E-08*	*2.99E-08*
L. Trade Balance	-3.15E-06	4.65E-07	1.86E-06	-2.06E-06
	1.16E-06	*2.48E-06*	*4.36E-06*	*1.23E-06*
L. Cash	-2.05E-06	-3.17E-06	3.74E-06	-1.03E-06
	1.06E-05	*8.71E-06*	*1.12E-05*	*6.77E-06*
L. Spread	0.0027	0.0035	-0.0028	0.0022
	0.0113	*0.0087*	*0.0112*	*0.0072*
F-Test	2.21	2.35	0.87	3.54

Looking at the Banking Crisis regression first, only lagged Inflation, the lagged Exchange Rate and lagged change in GDP are significant. High inflation will reduce incentives to hold deposits and so can reduce capital available to banks, hence appearing with a positive sign. Whereas, falling GDP can result in an increased rate of loan delinquency, hence, change in GDP appears in the regression with a negative sign. The positive sign on the exchange rate coefficient can be explained in terms of the Asian Crisis, where foreign denominated liabilities became unsustainably expensive due to currency movements.

Currency crises seem to be well predicted by a similar set of variables, although Gini and contemporaneous inflation, also become important. Price instability will certainly have a negative impact on the valuations of a currency and therefore, it is unsurprising that CPI plays a significant role in currency crises.

While the other variables are not significant in either regression, it is useful to look at the sign anyway. Given the imputation strategy, it is not unusual to see low t-values. High unemployment seems to be associated with crises and again this is intuitive. High unemployment may result in increased loan delinquency and may be the result of an overvalued currency and a lack of international competitiveness.

Table 3.8: Odds Ratio for the Gini Regressions

	Banking	Currency	Sovereign	Any Crisis
gini1	1.003	1.045	1.023	1.023
ldgdp	0.107	0.913	0.485	0.213
lcpi	4.287	12.322	1.039	5.809
lshortr	0.416	0.363	1.638	1.533
dgdp	0.000	0.000	0.059	0.000
cpi	3.457	0.009	1.212	0.096
fx	4.467	7.030	2.002	3.873
unemp	1.044	1.007	0.990	1.010
govcons	2.260	0.182	0.207	0.714
curr	1.000	1.000	1.000	1.000
shares	0.998	1.000	1.000	0.999
fiscpos	1.000	1.000	1.000	1.000
trade	1.000	1.000	1.000	1.000
cash	0.992	1.007	0.999	0.997

Table 3.8 shows the Odds Ratios for the various models. This measure tells us the relative importance of given values in predicting the probability of

a crisis. Due to the nonlinearity of this model this information is not easily inferred from the raw coefficients. Looking again at only the significant coefficients we see that lag of CPI and the exchange rate have the greatest impact on the probability of a crisis. The odds ratio of the Gini coefficient is only slightly greater than 1, suggesting a change in its value has only a slight change on the probability of a crisis. The effect is certainly less than the other macro variables, however, it was still significant in the Currency and All Crisis regressions and therefore we cannot discount the idea that increased inequality increases the likelihood of experiencing a crisis.

The tables in Appendix A.7 are included for completeness. It uses the Pareto measure rather than the Gini measure of inequality for the same set of regressions as above. In this case, it can be seen that the Pareto measure is not a good predictor of any of the crises and instead, we see Unemployment and Government consumption becoming significant. This result may suggest that the overall distribution of incomes is important in determining crises but not the higher incomes. It would also seem that Government Consumption is redistributive and a significant driver of inequality is unemployment.

3.5 A more robust measure of stability

So far we have discussed relatively subjective measures of financial crises and it would seem desirable to have a more robust approach. We are more interested in financial stability and this is a topic the IMF regularly reports on in the International Financial Stability Report (FSR) (Datastream, 2012). This report has only collated data since 2005, but includes measures of delinquent loans, bank equity/capital and certain types of exposure. These measures are very closely aligned with the measures used to define 'crises' in case study approaches. Therefore, the question is, which of these measures is truly important to financial stability?

It is likely that justification for inclusion of all indicators would be found. However the aim is then to regress inequality against a variable defining financial stability. With more than 20 indicators to choose from picking those that are most important is not obvious. A list of variables can be found in table 3.11. This section will present a method of reducing the dimensionality of the problem and will then look at how the Gini coefficient interacts with these factors.

Reducing the dimensionality of the problem has clear advantages and so the obvious method is Principal Component Analysis (PCA). PCA is a well established method of looking at high dimension data and picking out factors that account for the greatest variation across the data set. In this context each 'factor' or component is a linear sum of the variables and we must determine what weights to assign in this sum to maximise the variance of the factor.

3.5.1 Calculating Principal Components

The usual definition of PCA would take a normalised, mean zero, matrix of data $\mathbf{X}$ with rows representing observations and columns representing variables and form the covariance matrix $\mathbf{X}\mathbf{X}^{\mathbf{T}}$. The weights are then the eigenvectors of the covariance matrix, usually found by single value decomposition (SVD). There will be as many principal components as there are columns in $\mathbf{X}$ (variables) and it is usual to label them in decreasing order of variance, that is PCA-1 will have the highest variance. At this stage, the actual loadings can be considered and a real-world interpretation of the factors given.

However, within the FSR, there are missing data points which means that we are unable to directly apply the SVD approach, instead we will combine the PCA specification with an Expectation Maximisation (EM) algorithm. The EM algorithm is often used when data is incomplete to find the Maximum Likelihood[5] estimation of model parameters. In this way partial observations are not wholly discounted and so we are able to take advantage of all available data.

To facilitate this analysis, we must first adopt a slightly different specification of the principal component problem, taken from Grung and Manneo (1998). Instead of posing the problem in terms of decomposition, we pose the problem in terms of the re-composition. In other words defining the components to give the data when summed appropriately. Consider a $M \times N$ matrix $\mathbf{X}$, after performing the PCA decomposition, we wish to find the set of coefficients $\mathbf{t}$ and principal components $\mathbf{p}$ (scores), such that the remainder $\sum r_i^T r_i$ is minimised in:

$$\mathbf{x}_i^T = \sum t_{ik}\mathbf{p}_k^T + \mathbf{r}_i^T \tag{3.14}$$

As discussed the widely known method used to find these $\mathbf{p}$, is to choose a set of orthogonal $\mathbf{p_k}$ in order of variance. However, in the case where there are missing data we cannot simply compute the product $\mathbf{XX^T}$ and therefore we cannot perform a single value decomposition. Instead, I pose the problem in terms of a least squares minimisation. That is we aim to find the set of weights and components that minimises F where:

$$F = \sum_{ij}(x_{ij}T - \sum_k t_{ik}p_{jk})^2 \tag{3.15}$$

Here, F, is the sum of squared errors between the data x_{ij} and the implied data $\sum_k t_{ik}p_{jk}$. We also note that both $\mathbf{t_i}$ and $\mathbf{p_j}$ are chosen to be orthogonal. The method of calculating the principal components is given in Appendix A.1 summarising the exposition by Grung and Manneo (1998). The stages of calculation are, at a high level:

1. Define the problem in terms of known data
2. Calculate the first component
3. Use iteratively to find the further components

[5] Strictly speaking this is a pseudo-maximum likelihood estimation as no likelihood function is given explicitly.

The outlined procedure adopts a least squares approach to the problem. For complete data this process and an appropriate orthogonality procedure guarantees a unique representation. However, in the case where some data are missing, this is not guaranteed. In fact, there are likely to be many local minima of the function. The implementation I use here applies a robust Bayesian formulation as per Ilin and Raiko (2008). See Appendix A.1 for further details. I used the implementation from the R-Package pca-methods.

3.5.2 Discussing principal components

Table 3.9 gives the basic statistical properties of the first eight principal components calculated using the least squares with Bayesian updating.

Table 3.9: Summary of Principal Components

		PC_1	PC_2	PC_3	PC_4	PC_5	PC_6	PC_7	PC_8
Average		-0.016	0.011	0.035	0.010	-0.015	0.013	0.000	-0.013
St. Dev.		0.859	0.840	0.733	0.767	0.632	0.603	0.605	0.584
Correl w/ Crisis		-0.064	0.089	-0.219	-0.014	0.041	0.038	-0.084	0.119
Average	2005	-0.084	-0.060	0.208	-0.093	-0.070	0.002	0.099	-0.098
	2006	-0.097	-0.052	0.237	-0.121	-0.097	-0.042	0.056	-0.073
	2007	-0.045	-0.136	0.198	-0.019	-0.004	-0.040	0.079	-0.069
	2008	0.057	0.012	0.007	0.032	0.104	0.040	-0.027	0.092
	2009	-0.005	0.159	-0.017	0.145	-0.090	0.026	-0.041	0.026
	2010	0.027	0.152	-0.091	0.073	-0.040	0.101	-0.067	0.010
	2011	0.110	-0.012	-0.532	0.142	0.153	0.062	-0.194	0.072
Correlation	PC_1	1.000	0.027	0.056	-0.038	0.011	-0.007	-0.098	-0.032
	PC_2	0.027	1.000	-0.053	-0.019	-0.048	0.155	-0.031	0.025
	PC_3	0.056	-0.053	1.000	0.011	-0.075	0.007	0.064	-0.293
	PC_4	-0.038	-0.019	0.011	1.000	-0.031	0.030	-0.013	0.000
	PC_5	0.011	-0.048	-0.075	-0.031	1.000	-0.038	-0.026	0.101
	PC_6	-0.007	0.155	0.007	0.030	-0.038	1.000	0.005	-0.042
	PC_7	-0.098	-0.031	0.064	-0.013	-0.026	0.005	1.000	-0.077
	PC_8	-0.032	0.025	-0.293	0.000	0.101	-0.042	-0.077	1.000

3.5.3 Interpreting the PCs

The principal component decomposition of the Financial Stability indicators yields some interesting results. Of the 17 variables reported in the IMF IFS, we are only able to reduce the number of variables by half. Table 3.10 gives the contribution to variance of each factor. We can see that the first two components are similarly weighted (20%) and the following five components have approximately half this weight. This means there are potentially eight factors of interest, that is eight factors that explain the variation in the 17 variables.

Table 3.10: Proportion of Variance Accounted For by Each Principal Component

	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9	PC10
Contribution to Variance	0.236	0.191	0.113	0.117	0.108	0.089	0.091	0.055	0	0
Cumulative	0.236	0.426	0.540	0.657	0.765	0.854	0.945	1.000	1.00	1.00

The results of this analysis are shown in Table 3.11. For each principal component, there is an equivalent set of weights applied to each of the variables. Interpreting the first eight components is not a trivial task and we must be careful not to introduce too much subjectivity. However, it is useful to develop a taxonomy for dealing with the factors. Therefore, we will describe the first three factors as follows:

1. Capital adequacy: Increasing in capital to assets and decreasing in exposure to financial derivative, this factor should be increasing in financial stability

2. Income factors: Decreasing in cost of capital and increasing in Non-Performing Loans, also accounts for sensitivity to foreign liabilities, this factor has an unclear relationship to stability

3. Financial Risk: Increasing in Capital and Return on Assets, decreasing in Deposits to Loans, Interest Margin and Capital to Risk Weighted Assets, this factor should be increasing in financial stability

4. Large Risks: PC8 is also interesting as it is increasing the proportion of capital taken with large exposures (large loans to a single entity) and negative with the return on assets. This would seem to be a measure of the extent to which the banks' risk is concentrated.

Tables 3.12 helps us further pin down the meaning of the factors by looking at the correlation between the factors and various macroeconomic variables. The variables presented are chosen to be broadly consistent with other work presented earlier in this chapter; not all variables will be directly referenced but the set is chosen to provide a baseline for comparison. We can then look at the factors in macroeconomic terms:

1. Capital adequacy: Positively related to a strong macro position, for example, decreasing in CPI and increasing in Imports, GDP and Fiscal position. However, Unemployment and Exports are anomalous.

2. Income factors: Decreasing in the Long Rate, Fiscal Position and Changes in Money Supply suggesting this factor is negatively correlated with macroeconomic stability.

3. Financial Risk: Increasing in GDP, CPI, REER, Current Account, and Share price and decreasing in Trade, this factor seems predominantly concerned with the strength of the domestic economy.

Table 3.11: Principal Component Weightings

	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8
Change in Capital	0.07	0.07	0.26	0.06	-0.01	-0.26	0.09	0.58
Capital to Assets	0.72	0.28	0.14	0.12	-0.05	0.07	-0.13	-0.07
Deposit To Loans	0.17	-0.03	-0.43	0.08	0.22	-0.02	0.48	-0.06
Proportion of Foreign Liabilities	0.34	0.5	0.11	0.25	-0.09	-0.36	0.33	0.03
Foreign Loans to Total Loans	0.31	0.5	0.18	0.25	-0.13	-0.3	0.34	0.03
Financial Derivatives to Capital	-0.54	-0.4	-0.09	0.23	0.04	-0.45	0.7	-0.14
LiabFinDerivtoCap	-0.55	-0.41	-0.08	0.23	0.03	-0.43	0.71	-0.15
Interest Margin to Gross Income	0.06	-0.11	-0.28	-0.11	-0.75	-0.1	-0.04	-0.02
Large Exposure to Capital	0.14	-0.06	0	-0.02	-0.02	0.37	0.16	0.55
Liquid Assets to Short Term Liab	0.07	-0.29	0.04	-0.04	-0.27	0.59	0.26	-0.07
Liquid Assets Ratio	0.17	-0.18	-0.09	0.09	-0.02	0.46	0.57	0.06
Non Interest Expend to Gross Income	0.07	-0.1	-0.26	-0.07	-0.78	-0.09	0	0.03
Nonperforming Loans to Capital	-0.01	0.6	-0.04	-0.45	0.07	0.12	0	0.07
Nonperforming to Total Loans	-0.16	0.71	-0.04	-0.26	-0.08	0.16	0.08	-0.05
Regulatory Capital to RWA[6]	0.78	0.11	-0.25	0.13	0.1	0.04	-0.02	-0.01
T1 Capital to RWA	0.75	0.1	-0.29	0.16	0.13	0.03	0.05	-0.02
Return on Assets	0.12	-0.21	0.63	-0.02	-0.15	0.01	0.13	-0.34
Return on Equity	0.43	-0.29	0.04	-0.58	0.1	-0.17	0.02	0

3.5.4 Relationships to Crises

We must now ask how well these factors are able to identify a 'crisis'. For this we use the standard Laeven & Valencia (2012) database of crises to define

Table 3.12: Correlations between the first eight principal components of financial stability and macroeconomic variables

	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8
Change in GDP	1.77%	-3.15%	9.54%	-0.03%	4.94%	-4.55%	-0.80%	0.27%
Lag Change in GDP	-1.82%	-4.64%	7.30%	-0.40%	-2.06%	-2.66%	-2.62%	-3.13%
CPI	-6.25%	-6.49%	1.90%	-2.75%	-7.82%	-2.57%	-0.88%	4.28%
Lag Change in GDP	-3.10%	-3.49%	6.96%	0.57%	0.97%	5.10%	-5.93%	3.18%
REER	5.25%	2.57%	13.17%	3.94%	-10.29%	-1.89%	5.32%	-10.91%
Imports	-0.40%	9.39%	-9.31%	-7.02%	-3.25%	14.85%	3.35%	1.92%
FX	6.31%	2.09%	2.05%	3.76%	6.65%	1.50%	1.66%	-6.61%
Lag Change in GDP	4.27%	-1.68%	0.93%	-1.92%	-2.51%	-6.28%	2.05%	0.65%
International Reserve	3.53%	3.01%	-6.31%	-0.21%	1.78%	11.54%	1.44%	-4.66%
Change in Int Reserve	2.68%	-4.90%	5.47%	0.44%	-2.23%	1.13%	-0.21%	2.91%
Long Rate	0.73%	-16.73%	-4.47%	-4.98%	9.67%	7.74%	-2.07%	-6.62%
Short Rate	5.00%	-7.22%	6.16%	-7.75%	1.86%	-3.48%	-3.76%	-1.30%
Unemployment	6.65%	-5.69%	-3.09%	-3.44%	9.60%	0.02%	-3.28%	5.48%
Government Consumption	-0.49%	-2.81%	-1.09%	0.27%	-0.80%	-1.96%	-0.63%	-1.23%
Current Account	0.52%	1.99%	9.81%	-5.26%	-0.69%	-7.18%	4.79%	-0.04%
Fiscal Deficit (Surplus)	3.21%	-14.66%	3.42%	8.75%	2.04%	-5.72%	-0.93%	-2.55%
Exports	-4.99%	7.17%	-7.68%	-5.06%	-3.75%	9.37%	5.88%	3.78%
Shares	0.93%	0.10%	16.95%	7.31%	4.81%	-5.03%	-0.72%	-1.66%
Change in Money Supply	-2.86%	-15.19%	-0.99%	-8.54%	9.93%	-11.21%	-0.02%	8.87%

a logical variable where 1 means a country is thought to have experienced a crisis and 0 means they did not. I then use the logistic regression discussed in the earlier sections of this empirical work to estimate the crisis variable based on the factors PC1 to PC8.

The results of this regression are given in Table 3.13. It can be seen that all but three of the components are significant at the 5 % level. Moreover, a Wald test confirmed that these values are not jointly significant.[7] As predicted PC1, PC3 and PC7 are decreasing in the probability of a crisis whereas PC2 and PC8 are increasing.[8]

Finally, it is also reasonable to ask to what extent the principal component factors explain the severity of a crisis, that is to say, if two countries are experiencing a financial crisis and one has a lower PC1 is that crisis more

[7] A test statistic of 1.4 along with three degrees of freedom gives a probability of 0.71 that the values are all equal to zero (Buse, 1982).

[8] In this regression I use pooled data, alternative specifications of the regression applying country and time dummies was rejected using Wald Tests.

Table 3.13: Logistic Regression of Principal Components on Crisis Dummy

	Estimate	Std. Error	z value	Pr(>—z—)
(Intercept)	-2.217	0.158	-14.064	0.000
PC1	-0.253	0.130	-1.949	0.051
PC2	0.421	0.135	3.121	0.002
PC3	-1.506	0.241	-6.243	0.000
PC4	0.043	0.161	0.269	0.788
PC5	0.263	0.228	1.152	0.249
PC6	0.057	0.182	0.313	0.755
PC7	-0.547	0.178	-3.073	0.002
PC8	0.749	0.206	3.641	0.000

severe? To answer this question we took the Laeven & Valencia (2012) data for GDP loss in a three year window after the crisis and regressed it against the principal components in the first year of the crisis. I find that PC1 and 8 are both good predictors of the cost of crises. See Appendix A.8 for details of regression coefficients.

3.5.5 Categorising Crises

I will now consider what other information the principal components can give us about specific crises. This is done by looking at the principal component which is most extreme for a given observation. The aim is to find which principal component characterises each crisis. We begin by calculating a standardised measure of how extreme a given observation is, in this case we calculate $(x_i - \mu_i)\sigma_i^{-2}$. Table 3.14 details which PC was largest and smallest for each country experiencing a crisis in 2007-2011.

We can see that in 2008 nearly one third of crises were characterised by low values of PC 1, 3 and 7 each. It is also interesting to note that the proportion of countries with low PC 1 and 7 shrinks by a factor of one fifth by 2012, factor 3 is still the most common low factor. This seems to suggest that, while capital adequacy has improved, financial risks are still prevalent. Finally we can see that high values of PC 6 and 8 seem to be associated with crisis conditions. This analysis concurs with our previous interpretation of the principal components.

I now consider how the principal components differ between countries that have experienced crises and those that have not; this table is presented in appendix A.8. It can be seen that PC1 and 3 are lower for countries that experience crises and 8 higher.

Table 3.14: Categorising crises using PCA

	Low Tail Extreme					High Tail Extreme				
	2007	**2008**	**2009**	**2010**	**2011**	**2007**	**2008**	**2009**	**2010**	**2011**
Austria		3	7	7	6		6	6	6	5
Belgium		6	3	7			8	2	4	
Denmark		7	1	6	3		5	8	8	7
France		1	7	7			8	6	8	
Germany		7	7	7	2		4	2	6	6
Greece		7	5	2	8		6	6	8	6
Hungary		5	5	3	5		6	6	6	6
Iceland			4					2		
Ireland		4	7	5	5		2	8	2	1
Italy		6	7	4	7		8	2	2	8
Kazakhstan		7	3	3	7		8	5	7	5
Latvia		3	7	1	3		5	1	6	5
Luxembourg		3	5	5	3		6	2	2	6
Netherlands		3	6	4	3		2	8	5	5
Nigeria			7					1		
Portugal		6	3	6	3		7	7	2	6
Russia		1	6	6	6		7	8	7	7
Slovenia		1	3	1	3		3	4	7	6
Spain		1	4	4	4		6	2	2	2
Sweden		7	6	1	1		5	3	6	8
Switzerland		1	1	7	7		8	5	5	5
UK	2	3	6	6	6	4	1	5	5	5
Ukraine		6	1	3	3		8	5	7	1
USA	2	2	6	6	3	4	4	4	5	5

3.5.6 Borrowing Spread

Borrowing spread[9] is an important component of financial stability. This measure will be used extensively in the theoretical chapter. The wedge between borrowing and lending costs represents a market failure. This results in a limit to risk sharing. Borrowing spread is not directly measured within the IMF FSR and therefore, it is useful to ascertain how well the principal component decomposition explains this spread.

Table 3.15: PCA a predictor of borrowing spread

	Estimate	Std. Error	t value	Pr(>—t—)
(Intercept)	6.5688	0.2821	23.28	0.0000
PC1	-0.4166	0.3172	-1.31	0.1898
PC2	0.5410	0.3488	1.55	0.1217
PC3	0.9170	0.4790	1.91	0.0562
PC4	-0.9415	0.3463	-2.72	0.0068
PC5	-0.4380	0.3822	-1.15	0.2525
PC6	0.2295	0.5204	0.44	0.6595
PC7	-0.5843	0.6236	-0.94	0.3494
PC8	1.0194	0.6763	1.51	0.1325

Only PC4 is significant at the 5% level and negatively correlated with borrowing spread, although PC3 is very close but with a positive correlation. This would suggest that in the 2007 crisis a high borrowing premium was not associated with an increased chance of a crisis. This is contrary to the literature which suggests that borrowing premium is a good indicator of financial fragility (Gertler et al., 2007; Haramillo et al., 1996).

3.5.7 Inequality and the Principal Components

Table 3.16 shows regressions of the Gini coefficient against each principal component. We see that Gini is a good predictor of PC1, PC3 and PC8. The coefficients are all less than or equal to 0.01 but still positive and significant. We have already established that PC1 and PC3 are negatively correlated with the probability of a crisis and PC8 positively.

The PC8 relationship would seem to suggest that a crisis due to the concentration of banking risks may be exacerbated by high inequality. Whereas, the PC1 and PC3 relationships would seem to suggest a crisis due to more broad capital depletion is less likely to be associated with high inequality.

[9] Taken from Bank (2012)

Table 3.16: Gini as a predictors of the first eight principal components

	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7	PC 8
(Intercept)	-0.07^{*}	0.04	-0.05	0.01	-0.03	0.02	0.00	0.02
	(0.04)	(0.04)	(0.03)	(0.04)	(0.03)	(0.03)	(0.03)	(0.03)
gini	0.00^{**}	0.00	0.01^{***}	0.00	0.00	0.00	0.00	0.00^{***}
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
R^2	0.01	0.00	0.04	0.00	0.00	0.00	0.00	0.08

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

3.6 Conclusion

This chapter has demonstrated that we cannot rule out the possibility that inequality is related to financial crises. I have shown that there may exist a relationship in which increased levels of inequality increase the likelihood of a crisis. Countries with high inequality before 2007 seem to have been more likely to suffer crises after the Credit Crunch.

I also show that inequality seems to be associated with a concentration of risks in the financial sector. We have not tested the reasons for this but it is possible that the concentration of wealth motivates banks to take larger single risks.

Appendices

Chapter A

Empirical Chapter Appendices

A.1 PCA with Missing Values

As in the earlier discussion we begin with the definition:

$$F = \sum_{ij}(x_{ij}T - \sum_{k} t_{ik}p_{jk})^2 \tag{A.1}$$

Here F is the sum of square errors between the data x_{ij} and the implied data $\sum_k t_{ik}p_{jk}$, we also note that both $\mathbf{t_i}$ and $\mathbf{p_j}$ are chosen to be orthogonal. The missing components of the $\mathbf{X}$ must be removed from the objective function and, following (Grung and Manneo, 1998), we change notation such that $\mathbf{Y}$ is the full matrix and $\mathbf{X}$ is the known part. We then define the transform $\mathbf{C}$ such that $x_{ij} = c_{ij}y_{ij}$ implying that $c_{ij} = 0$ if y_{ij} is unknown and 1 otherwise. This leads to a specification in terms of the known matrix:

$$F = \sum_{ij} c_{ij}(y_{ij} - \sum_{k} t_{ik}p_{jk})^2 \tag{A.2}$$

$$= \sum_{ij}(X_{ij} - \sum_{k} t_{ik}c_{ij}p_{jk})^2 \tag{A.3}$$

The PC decomposition is then found by minimising the function F over the space of $\{p_{jk}\} \otimes \{t_{ik}\}$. For the first principal component Christofferson (1970) derives the minima of this function by differentiating with respect to t_i and p_j, this gives closed form solutions for the first principal component and weights p_1, t_1.

Then, if say p_{jk} were known then we could define $A^i_{jk} = c_{ij}p_{jk}$ and thus we would have:

$$F_{(i)} = \sum_j (X_{ij} - \sum_k t_{ik}A^{(i)}_{jk})^2 \tag{A.4}$$

Therefore we could write the solution for $t^{(i)}$ as:

$$t^{(i)} = x^{(i)}A^{(i)}({A^{(i)}}^T A^{(i)})^{-1} \tag{A.5}$$

Now if we instead knew $\{t_{ik}\}$ we could write $B^j_{ik} = t_{ik}c_{ij}$ then:

$$F_{(j)} = \sum_j (X_{ij} - \sum_k B^{(j)}_{ik} p_{jk})^2 \tag{A.6}$$

Then once again we define:

$${p^{(j)}}^T = ({B^{(j)}}^T B^{(j)})^{-1} B^{(j)} x^{(j)} \tag{A.7}$$

This process can be applied iteratively to find the complete set of components. In itself this is not sufficient information to uniquely define the components and in application we create a distribution of components which is bootstrapped to produce a Markov Chain. This chain is then used to find the mean value of the missing values given the available data.

A.2 Length of Time Series for Gini Coefficients

Table A.1: Length of Time Series for Gini

Length	Countries<=	Countries>=
1	47	
5	96	66
10	112	47
15	128	32
20	139	19
25	143	10

A.3 Countries in Sample Gini Values

Table A.2: Gini Coefficient Summary

	Obs	Start	End	Mean Gini	StdDev G	Mean Pareto	StdDev P
Albania	3	1996	2004	29.49	1.52		
Algeria	2	1988	1995	37.65	3.18		
Argentina	15	1992	2006	48.69	2.94	1.68	0.12
Armenia	10	1994	2006	45.67	9.74		
Australia	53	1951	2004	32.64	4.70	2.58	0.44
Austria	20	1970	2006	27.10	3.27		
Bangladesh	12	1973	2005	36.24	3.56		
Barbados	9	1970	1981	35.08	5.71		
Belarus	12	1995	2006	30.27	3.95		
Belgium	22	1969	2006	30.73	6.12		
Benin	1	2003	2003	36.48			
Bolivia	13	1968	2004	54.29	4.77		
Botswana	2	1986	1994	44.55	4.45		
Brazil	31	1958	2005	58.58	3.04		
Bulgaria	17	1990	2006	33.02	6.55		
Burkina Faso	4	1994	2003	58.10	21.76		
Burundi	2	1992	1998	37.56	6.02		

Table A.3: Gini Coefficient Summary Part 2

	Obs	Start	End	Mean Gini	StdDev G	Mean Pareto	StdDev P
Cambodia	4	1994	2004	45.13	2.76		
Cameroon	3	1983	2001	47.92	3.55		
Canada	33	1951	2000	30.80	2.97	2.28	0.33
Chile	33	1964	2003	52.05	4.47		
Colombia	29	1960	2004	53.49	5.73		
Costa Rica	27	1961	2006	47.70	2.26		
Croatia	8	1991	2005	29.03	4.07		
Cyprus	5	1966	2006	29.18	1.75		
Czech Rep.	14	1993	2006	25.90	1.44		
Denmark	38	1952	2006	32.82	7.98	3.32	0.20
Djibouti	2	1996	2002	44.70	5.37		
Dominican Rep.	17	1969	2006	49.51	2.50		
Ecuador	14	1965	2006	55.14	9.58		
Egypt	11	1958	2004	38.04	6.87		
El Salvador	14	1977	2004	50.86	4.14		
Estonia	15	1992	2006	35.42	1.81		
Ethiopia	4	1981	2000	40.65	11.52		
Fiji	4	1968	1991	42.94	0.79		
Finland	28	1952	2006	26.11	6.30		
France	23	1956	2006	33.26	7.27	2.40	0.15
Gabon	4	1968	1994	56.90	8.85		
Georgia	5	1997	2005	47.24	3.86		
Germany	37	1960	2006	30.13	6.06	1.67	0.09
Ghana	7	1987	1999	44.00	8.20		
Greece	30	1957	2006	38.70	4.95		
Guatemala	9	1966	2004	51.72	8.64		
Guinea	2	1991	1994	62.95	11.10		
Guinea-Bissau	2	1991	1994	50.00	8.06		
Guyana	3	1956	1999	46.70	6.43		
Haiti	1	2000	2000	50.90			
Honduras	17	1968	2006	55.35	2.70		
Hungary	22	1984	2006	26.03	3.52		
Iceland	3	2004	2006	25.00	1.00		
India	38	1951	2004	34.99	5.89	1.90	0.25
Indonesia	17	1964	2005	36.23	6.90	1.37	0.07
Ireland	15	1973	2006	33.32	3.34	2.23	0.41
Israel	14	1954	2001	37.39	6.36		
Italy	32	1967	2006	35.84	3.49	2.45	0.20

Table A.4: Gini Coefficient Summary Part 3

	Obs	Start	End	Mean Gini	StdDev G	Mean Pareto	StdDev P
Jamaica	19	1958	2004	47.96	9.10		
Japan	34	1954	1998	33.27	4.67	2.69	0.27
Jordan	8	1973	2003	38.22	4.02		
Kazakhstan	6	1996	2006	40.10	7.44		
Kenya	10	1967	1999	59.97	10.26		
Korea, Republic of	22	1961	2004	34.46	4.29		
Kyrgyz Republic	13	1994	2006	46.07	6.42		
Latvia	13	1994	2006	33.84	3.18		
Lesotho	5	1986	1999	60.58	5.70		
Liberia	1	1974	1974	43.00			
Lithuania	13	1994	2006	33.95	1.63		
Luxembourg	15	1985	2006	26.12	2.87		
Macedonia, FYR	14	1993	2006	29.44	5.30		
Madagascar	6	1962	2001	44.73	3.43		
Malawi	7	1969	2004	52.39	8.18		
Malaysia	15	1958	2004	48.36	4.26		
Mali	3	1989	2001	49.90	20.17		
Malta	3	2000	2006	28.67	1.15		
Mauritania	7	1987	2000	55.83	17.11		
Mauritius	6	1975	2001	38.43	2.30	2.27	0.38
Mexico	19	1956	2005	52.77	4.24		
Moldova	14	1993	2006	38.77	3.19		
Mongolia	4	1995	2002	33.62	8.01		
Morocco	11	1955	1999	48.62	7.81		
Mozambique	2	1996	2002	43.35	5.58		
Namibia	1	1993	1993	73.90			
Nepal	5	1976	2004	45.81	9.29		
Netherlands	27	1952	2006	30.98	5.40	3.22	0.44
New Zealand	41	1954	2004	47.05	13.14	2.78	0.38
Nicaragua	4	1993	2005	54.47	1.57		
Niger	3	1992	1995	46.20	8.95		
Nigeria	13	1959	2003	45.96	9.43		
Norway	30	1957	2006	27.39	4.27	2.34	0.66
Pakistan	24	1963	2005	34.42	4.03		
Panama	20	1960	2004	52.63	6.69		
Papua New Guinea	1	1996	1996	50.40			
Paraguay	11	1983	2005	53.52	6.44		
Peru	19	1961	2005	51.29	7.11		
Philippines	12	1957	2003	48.06	2.91		
Poland	27	1980	2006	28.66	5.36		
Portugal	14	1973	2006	36.36	2.55	2.92	0.32

Table A.5: Gini Coefficient Summary Part 4

	Obs	Start	End	Mean Gini	StdDev G	Mean Pareto	StdDev P
Romania	18	1989	2006	30.75	4.73		
Russian Federation	13	1992	2006	43.61	4.67		
Rwanda	2	1984	2000	37.16	11.69		
Senegal	5	1970	2001	46.27	14.48		
Seychelles	1	1978	1978	46.00			
Sierra Leone	5	1967	2003	52.62	9.51		
Singapore	30	1966	2000	44.50	3.04	2.32	0.15
Slovak Republic	12	1993	2006	24.94	2.15		
Slovenia	16	1991	2006	23.81	2.33		
South Africa	13	1959	2000	53.95	5.91	2.85	0.26
Spain	22	1965	2006	32.22	3.69	2.33	0.16
Sri Lanka	13	1953	2002	44.28	7.06		
Suriname	1	1999	1999	52.81			
Swaziland	3	1974	2001	57.76	6.76		
Sweden	56	1951	2006	34.83	14.24	2.84	0.54
Switzerland	8	1978	2002	32.97	2.23	1.92	
Tajikistan	2	2003	2004	33.03	0.79		
Tanzania	9	1967	2001	51.03	11.09	2.71	0.04
Thailand	19	1962	2002	49.79	7.15		
Tunisia	7	1965	2000	44.27	4.70		
Turkey	13	1952	2003	49.48	6.71		
Uganda	5	1970	2002	45.76	7.77		
Ukraine	12	1995	2006	40.87	5.47		
United Kingdom	48	1954	2006	29.28	4.49	2.37	0.39
United States	54	1950	2004	41.36	3.56	2.15	0.36
Uruguay	24	1961	2005	42.40	2.36		
Vietnam	2	2002	2004	35.98	2.23		
Zambia	10	1970	2004	60.04	9.67		
Zimbabwe	4	1968	1995	64.07	6.93		
Grand Total	1761	1950	2006	39.66	11.43	2.47	0.53

A.4 Summary of Trend Regressions For Gini Coefficients

Table A.6: Significance of time trend in predicting Gini Coefficient

	Obs	Mean	Time Trend	Const	SE	t
Afghanistan, Islamic Republic of	4	27.82				
Albania	12	31.18	1.02	-2013.31	6.19	0.16
Algeria	9	37.76	-0.72	1471.97	0.02	-43.89
American Samoa	2				0.00	
Andorra	2				0.00	
Angola	4	58.64				
Antigua and Barbuda	2				0.00	
Argentina	29	48.13	1.30	-2553.30	159.29	0.01
Armenia	17	35.40	-0.71	1458.46	5.04	-0.14
Aruba	2				0.00	
Australia	8	33.05	3.85	-7572.72	7519.44	0.00
Austria	4	29.15				
Azerbaijan	10	35.06	-1.51	3056.87	85.11	-0.02
Bahamas, The	2				0.00	
Bahrain	2				0.00	
Bangladesh	15	31.83	-0.93	1883.77	145.73	-0.01
Barbados	3	35.70				
Belarus	19	27.76	1.00	-1962.99	27.18	0.04
Belgium	4	32.97				
Belize	14	57.58	-0.60	1259.06	2.98	-0.20
Benin	4	38.62				
Bermuda	2				0.00	
Bhutan	9	42.45	-0.23	499.69	30.87	-0.01
Bolivia	18	56.45	0.53	-1006.82	19.44	0.03
Bosnia & Herzegovina	10	33.34	0.39	-740.15	13.06	0.03
Botswana	9	57.59	0.59	-1121.67	2.02	0.29
Brazil	33	58.67	-1.17	2393.37	99.19	-0.01
Brunei Darussalam	2				0.00	

Table A.7: Significance of time trend in predicting Gini Coefficient: 2

	Obs	Mean	Time Trend	Const	SE	t
Bulgaria	15	28.45	0.58	-1122.16	15.25	0.04
Burkina Faso	11	44.24	-0.82	1678.30	4.66	-0.18
Burundi	10	36.33	-0.08	193.07	27.34	0.00
Cambodia	11	40.59	0.61	-1187.35	15.36	0.04
Cameroon	10	40.00	-3.64	7332.52	367.44	-0.01
Canada	8	32.33	42.39	-83923.65	1365875.36	0.00
Cape Verde	4	50.52				
Cayman Islands	2				0.00	
Central African Republic	10	53.73	-0.28	616.74	69.20	0.00
Chad	4	39.78				
Channel Islands	2				0.00	
Chile	17	54.55	-3.61	7272.71	598.35	-0.01
China	16	34.95	1.26	-2485.99	25.64	0.05
Colombia	25	56.74	1.18	-2308.89	84.97	0.01
Comoros	4	64.30				
Congo, Dem. Rep.	4	44.43				
Congo, Rep.	4	47.32				
Costa Rica	30	47.18	0.90	-1746.94	80.79	0.01
Cote d'Ivoire	16	40.49	0.91	-1765.51	39.41	0.02
Croatia	14	28.91	1.34	-2645.33	27.92	0.05
Cuba	2				0.00	
Curacao	2				0.00	
Cyprus	2				0.00	
Czech Republic	10	23.94	0.61	-1181.69	4.44	0.14
Denmark	8	31.35	-1.35	2709.70	627.35	0.00
Djibouti	4	39.96				
Dominica	2				0.00	
Dominican Republic	23	49.95	0.12	-180.16	3.52	0.03
Ecuador	20	53.11	-0.31	679.80	12.25	-0.03
Egypt	12	31.56	-0.29	604.10	4.73	-0.06
El Salvador	21	50.41	-0.85	1758.37	95.90	-0.01
Equatorial Guinea	2				0.00	
Eritrea	2				0.00	
Estonia	16	34.74	0.60	-1159.29	15.10	0.04
Ethiopia	11	33.05	-0.43	899.59	29.66	-0.01
Faeroe Islands	2				0.00	
Fiji	9	44.82	-0.75	1556.88	0.15	-5.12
Finland	4	26.88				
France	4	32.74				
French Polynesia	2				0.00	
Gabon	4	41.45				
Gambia, The	9	48.76	-0.85	1744.09	0.83	-1.02

Table A.8: Significance of time trend in predicting Gini Coefficient: 3

	Obs	Mean	Time Trend	Const	SE	t
Georgia	19	40.25	0.88	-1729.72	10.92	0.08
Germany	4	28.31				
Ghana	12	38.60	1.86	-3676.60	117.77	0.02
Greece	8	38.95	-2.08	4160.59	2577.14	0.00
Greenland	2				0.00	
Grenada	2				0.00	
Guam	2				0.00	
Guatemala	15	56.70	-1.80	3657.06	132.29	-0.01
Guinea	11	42.84	-1.55	3146.37	65.48	-0.02
Guinea-Bissau	9	41.68	-0.37	771.29	40.79	-0.01
Guyana	9	48.05	-0.36	759.71	13.66	-0.03
Haiti	4	59.21				
Honduras	28	56.33	1.25	-2434.09	56.55	0.02
Hong Kong SAR	4	43.44				
Hungary	17	26.89	1.68	-3323.14	74.39	0.02
Iceland	2				0.00	
India	12	32.53	-1.59	3178.50	549.30	0.00
Indonesia	15	30.29	1.86	-3677.73	166.77	0.01
Iran, Islamic Rep.	12	43.28	-1.60	3239.63	79.90	-0.02
Iraq	4	30.86				
Ireland	4	34.28				
Isle of Man	2				0.00	
Israel	8	38.51	14.39	-28472.89	164914.46	0.00
Italy	4	36.03				
Jamaica	14	42.80	0.72	-1393.83	15.45	0.05
Japan	4	24.85				
Jordan	14	37.38	-1.04	2112.80	66.26	-0.02
Kazakhstan	18	32.35	0.05	-72.52	16.34	0.00
Kenya	11	47.70	-0.37	781.79	64.19	-0.01
Kiribati	2				0.00	
Korea, Republic of	3	32.00				
Korea	4	31.59				
Kosovo	2				0.00	
Kuwait	2				0.00	
Kyrgyz Republic	17	36.72	-0.01	53.92	50.64	0.00
Lao PDR	11	33.68	1.49	-2939.64	66.94	0.02
Latvia	18	32.68	1.13	-2220.63	9.95	0.11
Lebanon	2				0.00	
Lesotho	11	57.41	-0.44	938.57	18.00	-0.02
Liberia	4	38.16				
Libya	2				0.00	
Liechtenstein	2				0.00	
Lithuania	16	32.06	1.04	-2048.96	13.26	0.08
Luxembourg	4	30.76				

Table A.9: Significance of time trend in predicting Gini Coefficient: 4

	Obs	Mean	Time Trend	Const	SE	t
Macao SAR, China	2				0.00	
Macedonia,	16	38.71	0.60	-1164.27	9.29	0.06
Madagascar	14	44.68	-0.34	732.48	18.17	-0.02
Malawi	9	44.67	-0.27	576.37	47.00	-0.01
Malaysia	16	46.36	-1.07	2184.88	74.63	-0.01
Maldives	9	50.32	-0.12	282.10	317.65	0.00
Mali	11	40.65	-0.69	1417.67	8.63	-0.08
Malta	2				0.00	
Marshall Islands	2				0.00	
Mauritania	13	42.01	-0.63	1308.75	24.04	-0.03
Mauritius	2				0.00	
Mexico	17	49.07	-0.20	440.96	5.97	-0.03
Micronesia, Fed. Sts.	4	61.10				
Moldova	22	35.41	0.54	-1047.60	16.41	0.03
Monaco	2				0.00	
Mongolia	11	33.21	1.20	-2375.21	26.92	0.04
Montenegro	11	30.07	0.37	-706.40	0.47	0.79
Morocco	12	39.87	7.32	-14580.44	3921.77	0.00
Mozambique	10	45.75	1.62	-3199.84	83.60	0.02
Myanmar	2				0.00	
Namibia	9	69.12	-0.53	1122.97	10.72	-0.05
Nepal	11	35.49	0.57	-1104.22	41.67	0.01
Netherlands	4	30.90				
New Caledonia	2				0.00	
New Zealand	8	47.94	-0.76	1561.83	8.04	-0.10
Nicaragua	11	44.80	-0.89	1827.97	0.38	-2.38
Niger	11	39.02	-0.04	118.78	19.53	0.00
Nigeria	12	44.38	1.42	-2801.14	123.95	0.01
Northern Mariana Islands	2				0.00	
Norway	4	25.79				
Oman	2				0.00	
Pakistan	15	31.57	-1.62	3267.29	128.80	-0.01
Palau	2				0.00	
Panama	21	55.55	-0.13	311.74	9.69	-0.01
Papua New Guinea	4	50.88				
Paraguay	21	53.75	0.24	-421.34	18.32	0.01
Peru	23	50.85	-0.46	967.43	46.97	-0.01
Philippines	16	43.84	-0.51	1065.69	51.52	-0.01
Poland	24	31.90	1.83	-3628.11	115.79	0.02
Portugal	4	38.45				
Puerto Rico	2				0.00	
Qatar	4	41.10				

Table A.10: Significance of time trend in predicting Gini Coefficient: 5

	Obs	Mean	Time Trend	Const	SE	t
Romania	21	29.88	1.92	-3816.79	91.39	0.02
Russia	20	39.32	0.31	-576.68	32.41	0.01
Rwanda	11	46.08	0.67	-1286.38	32.17	0.02
Samoa	2				0.00	
San Marino	2				0.00	
Sao Tome and Principe	4	50.82				
Saudi Arabia	2				0.00	
Senegal	11	44.01	-0.55	1142.08	21.44	-0.03
Serbia	15	30.86	-0.82	1680.47	1.06	-0.78
Seychelles	9	54.25	0.15	-250.10	241.49	0.00
Sierra Leon	4	42.52				
Singapore	4	42.48				
Sint Maarten (Dutch part)	2				0.00	
Slovak Republic	16	25.82	1.50	-2981.41	73.40	0.02
Slovenia	13	28.72	1.82	-3605.65	96.08	0.02
Solomon Islands	2				0.00	
Somalia	2				0.00	
South Africa	12	60.85	0.94	-1821.87	18.19	0.05
South Sudan	4	45.53				
Spain	4	34.66				
Sri Lanka	12	36.34	1.56	-3076.83	97.26	0.02
St. Kitts and Nevis	2				0.00	
St. Lucia	4	42.58				
St. Martin (French part)	2				0.00	
St. Vincent and the Grenadines	2				0.00	
Sudan	4	35.29				
Suriname	4	52.88				
Swaziland	10	54.27	-0.69	1432.23	14.09	-0.05
Sweden	8	39.55	-0.67	1366.69	4.40	-0.15
Switzerland	4	33.68				
Syrian Arab Republic	4	35.78				
Tajikistan	12	31.72	0.65	-1265.21	6.04	0.11
Tanzania	10	35.34	2.37	-4699.98	254.06	0.01
Thailand	20	43.14	-2.78	5601.24	449.88	-0.01
Timor-Leste	9	35.73	-0.40	827.82	13.62	-0.03
Togo	4	34.41				
Tonga	2				0.00	
Trinidad and Tobago	9	41.44	-0.86	1749.59	0.61	-1.41

Table A.11: Significance of time trend in predicting Gini Coefficient: 6

	Obs	Mean	Time Trend	Const	SE	t
Tunisia	12	43.09	-2.95	5909.76	1789.17	0.00
Turkey	16	41.67	-1.98	4004.39	161.95	-0.01
Turkmenistan	10	34.18	0.45	-870.88	25.28	0.02
Turks and Caicos Islands	2				0.00	
Tuvalu	2				0.00	
Uganda	14	42.84	0.47	-904.81	15.47	0.03
Ukraine	20	29.09	-0.10	229.09	16.64	-0.01
UAE	2				0.00	
UK	8	30.74	1.81	-3562.39	1710.48	0.00
USA	8	42.40	-10.74	21245.14	56354.66	0.00
Uruguay	25	44.49	3.58	-7107.53	1571.27	0.00
Uzbekistan	11	35.38	0.39	-735.51	49.05	0.01
Vanuatu	2				0.00	
Venezuela	20	48.01	-0.78	1603.32	32.59	-0.02
Vietnam	13	36.15	0.71	-1383.43	15.23	0.05
Virgin Islands (U.S.)	2				0.00	
West Bank and Gaza	9	37.08	-0.32	674.54	3.16	-0.10
Yemen, Rep.	9	35.57	0.82	-1612.73	1.15	0.72
Zambia	13	50.55	-0.14	334.84	19.86	-0.01

A.5 Gini Persistence

Table A.12: Analysing Persistence in predicting Gini Coefficient

Country	Coefficient	P=1	Country	Coefficient	P=1
Albania	1.04	0.18	Lithuania	1.05	0.36
Algeria	0.88	1.00	Macedonia,	1.04	0.14
Argentina	1.00	0.96	Madagascar	0.99	0.73
Armenia	0.95	0.14	Malawi	0.78	1.00
Australia	1.14	1.00	Malaysia	0.99	0.77
Azerbaijan	0.98	0.78	Maldives	0.59	1.00
Bangladesh	0.92	0.31	Mali	0.86	0.08
Belarus	1.01	0.78	Mauritania	0.97	0.73
Belize	0.98	0.14	Mexico	1.00	0.89
Bhutan	0.81	1.00	Moldova	1.01	0.65
Bolivia	1.02	0.49	Mongolia	1.03	0.64
Bosnia & Herzegovina	1.11	0.47	Montenegro	1.00	0.94
Botswana	1.12	1.00	Morocco	1.01	0.17
Brazil	1.00	0.72	Mozambique	1.01	0.82
Bulgaria	1.00	1.00	Namibia	0.86	1.00
Burkina Faso	0.92	0.15	Nepal	0.99	0.97
Burundi	0.97	0.91	New Zealand	0.61	1.00
Cambodia	0.99	0.91	Nicaragua	0.93	0.02
Cameroon	0.98	0.29	Niger	0.98	0.85
Canada	1.01	1.00	Nigeria	1.05	0.39
Central African Republic	0.91	0.76	Pakistan	0.98	0.61
Chile	0.99	0.20	Panama	1.00	0.88
China	1.05	0.04	Paraguay	1.01	0.72
Colombia	1.00	0.70	Peru	0.97	0.48
Costa Rica	1.00	0.87	Philippines	0.99	0.61
Cote d'Ivoire	1.00	0.91	Poland	1.01	0.24
Croatia	1.06	0.21	Romania	1.01	0.20
Czech Republic	1.11	0.62	Russia	1.01	0.89

Table A.13: Analysing Persistence in predicting Gini Coefficient: 2

Denmark	0.65	1.00	Rwanda	1.10	0.64
Dominican Republic	1.00	0.89	Senegal	0.88	0.20
Ecuador	1.00	0.85	Serbia	0.98	0.23
Egypt	0.99	0.74	Seychelles	1.54	1.00
El Salvador	1.00	0.86	Slovak Republic	1.02	0.59
Estonia	1.02	0.83	Slovenia	1.05	0.30
Ethiopia	0.96	0.78	South Africa	1.01	0.83
Fiji	0.91	1.00	Sri Lanka	1.05	0.29
Gambia, The	0.94	1.00	Swaziland	0.91	0.42
Georgia	1.01	0.71	Sweden	0.46	1.00
Ghana	1.05	0.01	Tajikistan	1.01	0.78
Greece	0.79	1.00	Tanzania	1.06	0.22
Guatemala	0.99	0.73	Thailand	0.99	0.40
Guinea	0.94	0.09	Timor-Leste	0.81	1.00
Guinea-Bissau	0.74	1.00	Trinidad and Tobago	0.95	1.00
Guyana	0.86	1.00	Tunisia	0.95	0.21
Honduras	1.00	0.98	Turkey	0.99	0.20
Hungary	1.04	0.24	Turkmenistan	1.22	0.14
India	1.00	0.99	Uganda	1.00	0.92
Indonesia	1.02	0.58	Ukraine	1.00	0.92
Iran, Islamic Rep.	0.95	0.20	UK	1.41	1.00
Israel	1.04	1.00	USA	0.97	0.42
Jamaica	1.00	0.92	Uruguay	1.01	0.43
Jordan	0.99	0.86	Uzbekistan	1.02	0.93
Kazakhstan	1.00	0.96	Venezuela	0.98	0.32
Kenya	0.98	0.84	Vietnam	1.00	0.95
Kyrgyz Republic	0.98	0.85	West Bank and Gaza	0.92	1.00
Lao PDR	1.06	0.45	Yemen, Rep.	1.13	1.00
Latvia	1.04	0.10	Zambia	1.00	0.99
Lesotho	0.97	0.78			

A.6 Countries in the World Top Incomes Database

Table A.14: Break Down of GFSR Countries by Region/Type

Advanced Economies	30
Emerging and Developing Economies	67
C&E Europe	21
Developing Asia	8
Middle East North Africa	8
Sub Saharan Africa	13
Western Hemisphere	16

A.7 Ancillary Regression Results for Panel Data Study

Table A.15: Random Effects Model of Various Crises With Pareto Measure

	Banking	Currency	Soverign	Any Crisis
Pareto	-0.0824	0.0109	-0.0583	-0.0402
	0.42	*0.472*	*0.709*	*0.292*
GDP	-2.17	1.16	-0.282	-1.04
	2.78	*3.31*	*4.92*	*2.22*
CPI	1.46(*)	2.73(*)	0.104	1.85(**)
	0.793	*1.41*	*1.41*	*0.802*
Exchange Rate	-0.859	-0.437	0.685	0.628
	2.05	*2.77*	*3.72*	*1.64*
Short Rate	-9.07(***)	-9.83(***)	-2.76	-9.58(***)
	2.58	*2.98*	*4.29*	*2.1*
Unemployment	1.25	-4.95(***)	0.206	-2.46(**)
	1.02	*1.36*	*1.82*	*0.992*
Government Consumption	1.5(**)	1.82(*)	0.621	1.29(*)
	0.732	*1.01*	*1.26*	*0.641*
Current Account	0.0417	-0.00122	-0.0155	0.00512
	2.87E-02	*3.98E-02*	*6.51E-02*	*2.48E-02*
Equity	*8.16E-01*	*-1.84E+00*	*-1.69E+00*	*-3.97E-01*
	1.24E+00	*1.19E+00*	*2.17E+00*	*9.08E-01*
Fiscal Position	*-1.52E-05*	*-3.60E-06*	*9.80E-06*	*-8.04E-06*
	1.64E-05	*2.06E-05*	*3.03E-05*	*1.33E-05*
Trade Balance	*-1.58E-03*	*-1.21E-03*	*-2.49E-04*	*-1.65E-03*
	1.88E-03	*2.48E-03*	*2.86E-03*	*1.41E-03*
M2/Res	*9.98E-08*	*-5.58E-07*	*2.25E-06*	*1.21E-07*
	3.29E-06	*4.55E-06*	*8.34E-06*	*2.77E-06*
_cons	*-2.07E-06*	*2.26E-06*	*3.93E-06*	*-6.93E-07*
	2.97E-06	*5.12E-06*	*9.20E-06*	*2.60E-06*
F-Test	3.57	4.24	0.67	5.56

A.8 Ancillary regressions for PCA Decomposition

Table A.16: Odds Ratio for Pareto Measure Regressions

	Banking	Currency	Soverign	Any Crisis
pareto	0.921	1.011	0.943	0.961
ldgdp	0.114	3.196	0.754	0.355
lcpi	4.324	15.355	1.110	6.363
lshortr	0.424	0.646	1.984	1.874
dgdp	0.000	0.000	0.064	0.000
cpi	3.506	0.007	1.229	0.085
fx	4.469	6.180	1.862	3.616
unemp	1.043	0.999	0.985	1.005
govcons	2.261	0.158	0.184	0.673
curr	1.000	1.000	1.000	1.000
shares	0.998	0.999	1.000	0.998
fiscpos	1.000	1.000	1.000	1.000
trade	1.000	1.000	1.000	1.000
cash	0.992	1.005	0.998	0.996

Table A.17: Mean value of principal components by year and experience of a crisis.

		PC_1	PC_2	PC_3	PC_4	PC_5	PC_6	PC_7	PC_8
Crisis	2005	-0.244	-0.523	1.489	0.159	-0.386	0.066	0.382	-0.825
	2006	NA	NA	NA	NA	NA	NA	NA	NA
	2007	-0.28	-0.50	0.12	0.39	-0.01	-0.06	0.01	-0.03
	2008	-0.14	0.22	-0.38	-0.05	0.05	0.16	-0.11	0.23
	2009	-0.13	0.49	-0.28	0.11	-0.12	-0.01	-0.13	0.25
	2010	-0.33	0.28	-0.35	-0.18	0.11	0.00	-0.11	0.14
	2011	0.02	-0.16	-0.57	0.00	0.21	0.15	-0.20	0.06
No Crisis	2005	-0.08	-0.06	0.19	-0.10	-0.07	0.00	0.10	-0.09
	2006	-0.10	-0.05	0.24	-0.12	-0.10	-0.04	0.06	-0.07
	2007	-0.04	-0.13	0.20	-0.03	0.00	-0.04	0.08	-0.07
	2008	0.11	-0.05	0.12	0.06	0.12	0.00	0.00	0.05
	2009	0.03	0.06	0.07	0.15	-0.08	0.04	-0.01	-0.04
	2010	0.15	0.11	0.00	0.16	-0.09	0.13	-0.05	-0.03
	2011	0.16	0.07	-0.51	0.22	0.12	0.01	-0.19	0.08

Chapter 4

Two Period Model

The grand aim of all science is to cover the greatest number of empirical facts by logical deduction from the smallest number of hypotheses or axioms.

–Albert Einstein quoted in Nash (1963)

4.1 Introduction

The empirical section and literature review have established the following stylised facts:

- The income share of the high earners may affect financial stability
- Borrowing premium is a good proxy for financial stability

In this chapter, a simple two period model will be used to study the possible effects of inequality. Banks will intermediate between consumers with varying income levels. The banks will charge a premium for borrowing and this premium will be taken as indicative of the stability of the economy.

Section 4.2, Model Structure, will define the parameters of the model and highlight important structural features of the proposed economy. Then section 4.3, Inequality, will define what is meant by inequality in the context of this model. The Household and Bank problems will then be considered in detail in sections 4.4 and 4.5 respectively, including several lemmas demonstrating how borrowing changes with inequality. Section 4.6, Market Clearing, will seek to show that all markets clear and equilibrium is possible given the proposed structure. Finally, the Borrowing Premium will be further discussed in Section 4.8.

These results are then shown numerically; the Computational Strategy section (4.9) explains how the model is solved and the Data for Calibration (4.10) provides a set of parameters for the model. Finally, the results section (4.11) verifies the analytic results numerically.

4.2 Model Structure

In this model, a continuum of atomistic households receive endowments y_0 in Period 0. The households are heterogeneous in income. It will be assumed that the income is bounded such that $y_i \in [y_{min}, y_{max}]$. I will also assume that all incomes are positive and none zero. Income shocks are assumed to be independent, that is to say: $E[y_1 \mid y_0] = E[y_1]$. This assumption is clearly not born out in reality, there is clear empirical evidence of autocorrelation in income levels (Rey and Montouri, 1999). The assumption will be maintained for analytic tractability and possible extensions, relaxing the assumption discussed later in this chapter.

There will be a subsistence level of consumption $\bar{c}$ below which income cannot fall and it is assumed that autarky is always possible, thus $y_{min} > \bar{c}$. It will be assumed that all households have the same preference structure consisting of a concave utility function u and subjective discount factor β. Assuming strict concavity of u leads to the following identities for all positive c:

$$u'(c) > 0 \tag{4.1}$$
$$u''(c) < 0 \tag{4.2}$$
$$u'''(c) > 0 \tag{4.3}$$

Households take the income received in period 0 and solve an optimal choice problem given:

1. Their idiosyncratic first period income y_0
2. The Market Interest Rates for loans R_L and deposits R_D
3. Distribution of Incomes Ψ_y

We will take the distribution of Ψ_y to be uniform for increased analytical tractability. The result of the decision problem will be the optimal response functions for deposits (d_0) and loans(l_0), at least one of these must be zero for a given y_0 [1]:

$$d_0^* = d^*(R_D, R_L, y_0, \Psi_y) \tag{4.4}$$
$$l_0^* = l^*(R_D, R_L, y_0, \Psi_y) \tag{4.5}$$

The total loans issued by the banks must equal the total loans taken by consumers, Loan and Deposit Market clearing, then implies:

$$D_0 = \int d_0^*(R_D, R_L, y_0, \Psi_y) d\Psi_y \tag{4.6}$$
$$L_0 = \int l_0^*(R_D, R_L, y_0, \Psi_y) d\Psi_y \tag{4.7}$$

Call the set of households H, then Borrowers are H^- with $l_0^* > 0,\ d_0^* = 0$ and Savers are H^+ with $l_0^* = 0,\ d_0^* > 0$. Because of the borrowing premium, there exists a set of consumers who choose to neither borrow nor save and they will be defined as H^0. The sets H^+, H^- and H^0 are determined in period zero by the realisation of income. Therefore, we can define the threshold incomes y_B and y_S such that:

1. For $y_0 < y_B$ the consumer would be a borrower in H^-
2. For $y_0 > y_S$ the consumer would be a saver in H^+
3. For $y_B < y_0 < y_S$ the consumer would live in autarky in H^0

Figure 4.2 shows the relative positions of these incomes on a stylised scale. The ordering of these income levels is guaranteed if $R_L > R_D$, given that defaults are never negative this holds almost surely.

0	$\bar{c}$	y_{min}	y_D	y_B	$\bar{y}$	y_S	y_{max}
0	2	3	5	7	8	9	13

Figure 4.1: Relative scales of income levels

Debts are assumed to be legally enforceable unless paying off the debt would take consumption beneath $\bar{c}$, in this case the bank can only reclaim income in excess of $\bar{c}$. This is comparable with fair recovery practices in the

[1] This will be further discussed in the following sections.

UK and will be discussed in more detail in the Data section. Therefore, in Period 1, a household which has borrowed may be unable to repay all of their debts if $\bar{c} > y_1 - R_L l_0^*$ this set of defaulters will be called H_1^q. It then follows that $H = H^- \cup H^+ \cup H^0$ and $H_1^q \subset H^-$. H_1^q is only determined in Period 1.

To make the order of events clear, Figure 4.2 shows a time line. Before any incomes are realised, at time -1, a continuum of banks know the distribution of incomes the households will receive and the preferences of the households. Using this information, the banks are able to set their interest rates in advance of the realisation of incomes in period 0. The banks intermediate competitively between borrowers and lenders. The total loans and deposits supplied by all banks will be denoted L_0 and D_0. Then the banks have an aggregate Period 0 balance sheet constraint and by competitive markets a Period 1 zero profit condition.

$$L_0 = D_0 \tag{4.8}$$

$$\pi = R_L L_0 - R_D D_0 - Q = 0 \tag{4.9}$$

Where Q is the value of loans defaulted on and R_L and R_D are the gross, including capital, returns on loans and deposits. Then noting that $L_0 = D_0$ in equilibrium this yields:

$$R_L = R_D + \frac{Q}{L_0} \tag{4.10}$$

Therefore, the interest rate charged on loans is equal to the interest rate paid on deposits plus the default premium, which will be denoted $\varsigma = \frac{Q}{L_0}$.

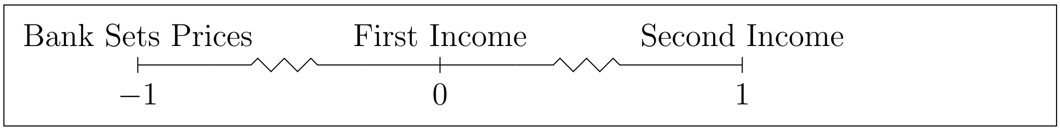

Figure 4.2: Time Line for Decision Making

The measure of financial stability is the premium charged for borrowing. This premium will then be determined by the response functions of the consumers. These functions and their aggregates are unlikely to be of a closed form. Even in the simplest case of quadratic utility the decision rules would be straightforward, but the interest rates would be transcendental. The following section will establish some properties of these functions. Specifically, it will be shown that the relative demand for borrowing is higher when inequality is higher, which then drives up the borrowing premium. This is shown in comparative statics but the general result is shown numerically in the following sections.

4.3 Inequality

The Gini coefficient, discussed in Section 2.2, is a measure of inequality for a given income distribution, a Gini of 0.0 means that income is evenly spread out across an entire population, perfect equality, and a Gini of 1.0 suggests income is concentrated with a single individual. This model will make great use of the uniform distribution, while incomes are very clearly not uniformly distributed this distribution provides a useful baseline. The simplicity of the formulation allows for better intuition for the results and I will expand on the effect of other distributions later in this chapter. it is therefore useful to establish the parametric form of the above definitions for this distribution:

$$\Psi(y) = \frac{y - y_{min}}{y_{max} - y_{min}} \tag{4.11}$$

$$y = y_{min} + (y_{max} - y_{min})\Psi(y) \tag{4.12}$$

$$= y_{min} + (y_{max} - y_{min})x \tag{4.13}$$

$$L(\Psi(y)) = \frac{y^2 - y_{min}^2}{y_{max}^2 - y_{min}^2} \tag{4.14}$$

$$= \frac{x^2(y_{max} - y_{min})^2 + 2y_{min}x(y_{max} - y_{min})}{y_{max}^2 - y_{min}^2} \tag{4.15}$$

$$= \frac{x(y_{max} - y_{min})(xy_{max} + (2 + x)y_{min})}{y_{max}^2 - y_{min}^2} \tag{4.16}$$

$$= \frac{x^2 y_{max} + x(1 + x)y_{min})}{y_{max} + y_{min}} \tag{4.17}$$

$$G = 1 - 2\int_0^1 L(x)dx \tag{4.18}$$

$$= 1 - \frac{2}{y_{max} + y_{min}}[\frac{x^3}{3}(y_{max} + y_{min}) + x^2 y_{min}]_0^1 \tag{4.19}$$

$$= \frac{1}{3} - \frac{y_{min}}{(y_{max} + y_{min})} \tag{4.20}$$

$$\frac{\partial G}{\partial y_{max}} = \frac{y_{min}}{(y_{max} + y_{min})^2} \tag{4.21}$$

$$\psi(y) = \frac{1}{y_{max} - y_{min}} \tag{4.22}$$

Therefore, for a given y_{min}, the Gini coefficient is increasing in y_{max} up to a limiting value of $\frac{1}{3}$. We can, in this case, take y_{max} as our measure of inequality for a given y_{min}. Raising y_{max} while keeping y_{min} constant, increases the mean value of income as well as the Gini coefficient. This is consistent with recent observations in the UK and the US economies; where rising average real income is significantly attributable to increases in high incomes, as noted in Section 2.2.

4.4 Household Choices

This section will establish the first order conditions for the consumer along with some useful partial derivatives. A possible mode by which inequality could affect the borrowing premium will be presented.

The households consume in both periods, taking utility from consumption. They may make deposits with the bank and receive a gross return R_D or take out consumption loans and repay at a gross rate of R_L. The households budget constraints are then:

$$c_0 + w_0 = y_0 \tag{4.23}$$
$$c_1 = max(\bar{c}, y_1 + R_1 w_0) \tag{4.24}$$
$$\tag{4.25}$$

Where:

$$R_1 = \begin{cases} R_D & \text{if } w_0 > 0, \\ R_L & \text{if } w_0 < 0, \end{cases} \tag{4.26}$$

Here, a general term $w_0 = y_0 - c_0$ has been used to simplify the exposition of the maximisation problem, which can be interpreted as 'bank balance'. Borrowing, $w_0 < 0$, will be denoted l_0 and saving, $w_0 > 0$, will be denoted d_0. The decision problem is to find $V = \max[u(c_0) + \beta u(c_1)]$ subject to the budget constraints:

$$c_0 \in [\bar{c}, y_0 + (y_{max} - \bar{c})/R_L] \tag{4.27}$$

Given the choice of consumption in period zero, c_0, consumption in period

1 in then given by:

$$c_1 = \begin{cases} y_1 + R_D(y_0 - c_0) & \text{if } c_0 < y_0, \\ y_1 - R_L(c_0 - y_0) & \text{if } c_0 > y_0 \text{ and } y_1 \geq R_L(c_0 - y_0) + \bar{c}, \\ \bar{c} & \text{if } c_0 > y_0 \text{ and } y_1 < R_L(c_0 - y_0) + \bar{c}. \end{cases} \tag{4.28}$$

Within this formulation is is clear that agents who save, $c_0 < y_0$, will always consume more than y_1 in period 1. Then agents who borrow, $c_0 > y_0$ may borrow so little that they will never default (group 2) or borrow sufficient that they risk defaulting (group 3). We can then define a set of corresponding lifetime value functions:

$$V(c_0; y_0) = \begin{cases} u(c_0) + \beta E_0[u(y_1 + R_D(y_0 - c_0))] & \text{if } c_0 < y_0, \\ u(c_0) + \beta E_0[u(max[\bar{c}, y_1 + R_L(y_0 - c_0)])] & \text{if } c_0 > y_0 \end{cases} \tag{4.29}$$

The first section of the piecewise representation of $V(c_0; y_0)$ is very familiar and we can trivially replace the expectation with an integral. However, the second component, relating to borrowers, is more tricky. Here there will be different consumption functions for different realisations of y_1. Therefore, I further break down this function in terms of y_1:

$$V(c_0; y_0) = \begin{cases} u(c_0) + \beta \int_{y_{min}}^{y_{max}} [u(y_1 + R_D(y_0 - c_0))] d\Psi_{y_1} \\ \quad \text{if } c_0 \in \{\bar{c}, y_0\}, \\ u(c_0) + \beta \int_{y_{min}}^{y_{max}} [u(y_1 + R_L(y_0 - c_0))] d\Psi_{y_1} \\ \quad \text{if } c_0 \in \{\bar{c}, y_0 + (y_{min} + \bar{c})/R_L\}, \\ u(c_0) + \beta[\int_{y_{min}}^{R_L(c_0 - y_0) + \bar{c}} [u(\bar{c})] d\Psi_{y_1} + \int_{R_L(c_0 - y_0) + \bar{c}}^{y_{max}} [u(y_1 + R_D(y_0 - c_0))] d\Psi_{y_1}] \\ \quad \text{if } c_0 \in \{y_0 + (y_{min} + \bar{c})/R_L, y_0 + (y_{max} + \bar{c})/R_L\} \end{cases} \tag{4.30}$$

Then writing:

$$\frac{dU(c)}{dc} = u(c) \tag{4.31}$$

And noting that, for a uniform distribution, $d\Psi_{y_1} = (y_{max} - y_{min})^{-1} dy_1$

We have:

$$V(c_0;y_0)=\begin{cases} u(c_0)+\beta(y_{max}-y_{min})^{-1}[U(y_{max}+R_D(y_0-c_0))-U(y_{min}+R_D(y_0-c_0))] \\ \text{ if } c_0\in\{\bar{c},y_0\}, \\ u(c_0)+\beta(y_{max}-y_{min})^{-1}[U(y_{max}+R_L(y_0-c_0))-U(y_{min}+R_L(y_0-c_0))] \\ \text{ if } c_0\in\{\bar{c},y_0+(y_{min}+\bar{c})/R_L\}, \\ u(c_0)+\beta(y_{max}-y_{min})^{-1}[(R_L(c_0-y_0)+\bar{c}-y_{min})u(\bar{c})]+ \\ U(y_{max}+R_L(y_0-c_0))-U(\bar{c})] \\ \text{ if } c_0\in\{y_0+(y_{min}+\bar{c})/R_L,y_0+(y_{max}+\bar{c})/R_L\} \end{cases} \tag{4.32}$$

Now differentiating with respect to consumption:

$$\frac{\partial V(c_0;y_0)}{\partial c_0}=\begin{cases} u'(c_0)-\beta R_D(y_{max}-y_{min})^{-1}[u(y_{max}+R_D(y_0-c_0))-u(y_{min}+R_D(y_0-c_0))] \\ \text{ if } c_0\in\{\bar{c},y_0\}, \\ u'(c_0)-\beta R_L(y_{max}-y_{min})^{-1}[u(y_{max}+R_L(y_0-c_0))-u(y_{min}+R_L(y_0-c_0))] \\ \text{ if } c_0\in\{\bar{c},y_0+(y_{min}+\bar{c})/R_L\}, \\ u'(c_0)-\beta R_L(y_{max}-y_{min})^{-1}[\\ u(y_{max}+R_L(y_0-c_0))-u(\bar{c})]] \\ \text{ if } c_0\in\{y_0+(y_{min}+\bar{c})/R_L,y_0+(y_{max}+\bar{c})/R_L\} \end{cases} \tag{4.33}$$

And setting these derivative equal to zero:

$$u'(c_0)=\begin{cases} \beta R_D(y_{max}-y_{min})^{-1}[u(y_{max}+R_D(y_0-c_0))-u(y_{min}+R_D(y_0-c_0))] \\ \text{ if } c_0\in\{\bar{c},y_0\}, \\ \beta R_L(y_{max}-y_{min})^{-1}[u(y_{max}+R_L(y_0-c_0))-u(y_{min}+R_L(y_0-c_0))] \\ \text{ if } c_0\in\{\bar{c},y_0+(y_{min}+\bar{c})/R_L\}, \\ \beta R_L(y_{max}-y_{min})^{-1}[\\ u(y_{max}+R_L(y_0-c_0))-u(\bar{c})]] \\ \text{ if } c_0\in\{y_0+(y_{min}+\bar{c})/R_L,y_0+(y_{max}+\bar{c})/R_L\} \end{cases} \tag{4.34}$$

The following two sections will deal with the two analytical problems (1) capital market imperfections and (2) default.

4.4.1 Borrow or Save?

The borrowing premium is the result of the ability of agents to default. However, the resultant capital market failure requires careful treatment even before introducing the possibility of default. Therefore, this section will ignore default and focus on the behaviour of agents due to the borrowing spread alone.

The question is then which consumers, H, will be borrowers, H^-, and which will be savers, H^+. Now, consider the two values of y_0 for which $w = 0$, denoted y_B and y_S, where y_B is the level of income at which it would be *just* optimal to not borrow or save respectively.

$$u'(y_B) = \beta R_L E_0 u'(y_1) \tag{4.35}$$
$$u'(y_S) = \beta R_D E_0 u'(y_1) \tag{4.36}$$

Lemma B.1.1 in Appendix B.1.1 states that if $R_L > R_D$ then $y_B < y_S$ and households whose income falls between theses limits will neither deposit with nor borrow from the bank. For these autarkic consumers in H^0, it follows that $c_0 = y_0$ and $c_1 = y_1$

The set of households, H, is then partitioned into borrowers, denoted H^-, and savers, H^+, by the above relations, note $H = H^- \cup H^+$. It is interesting to note that the threshold income y_B is increasing in y_{max} as established by Corollary B.1.2 in Appendix B.1.1.

The optimal response of a household in H^- is then:

$$u'(y_0 + l_0^*) = \beta R_L E_0 u'(y_1 - R_L l_0^*) \tag{4.37}$$

And for a household in H^+:

$$u'(y_0 - d_0^*) = \beta R_D E_0 u'(y_1 + R_D d_0^*) \tag{4.38}$$

Optimal consumption will be denoted c^*, optimal borrowing l_0^* and saving d_0^*. The remainder of this section will now aim to establish the properties of l_0^*, which will in turn be used to establish the properties of the borrowing premium. Two lemmas are now presented; the proofs can be found in Appendix B.1.1

Lemma 4.4.1. *Optimal borrowing for a given household is decreasing in* y_0

Corollary 4.4.2. *The derivative* $\frac{\partial l^*}{\partial y_0} < 1$.

Lemma 4.4.3. *Optimal borrowing for a given household is increasing in* y_{max}

Intuitively, Lemmas 4.4.1 and 4.4.3, result from the fact that the level of borrowing is proportional to the expected increase in income between period 0 and period 1. That is to say that the motivation for borrowing is that the income in period 0 is less than the expected level of income in period 1 and therefore, the consumption can be smoothed by moving some of the deficit into period 1.

4.4.2 Defaults

Although households are compelled by law to repay debts, following British law (OFT, 2011), creditors cannot demand repayment of debt that reduces consumption beneath a subsistence level $\bar{c}$. A consumer who cannot afford to repay their debts is allowed to consume $\bar{c}$ but is still compelled to repay everything they can afford. Henceforth, households who have chosen to borrow will default if:

$$\bar{c} > y_1 - R_L l_0^* \tag{4.39}$$

This means where defaults are possible, $Eu'(c_1) = Eu'(\max(\bar{c}, y_1 - R_L l^*))$. The first order condition must be adjusted to take into account the possibility of default. Note that this does not imply 'strategic default', where the borrower plans to default. Instead, this takes account of the possibility of 'bad' states of nature. Thus, the appropriate first order condition in the presence of default will be:

$$u'(y_0 - l^*) = \beta R_L E_0 u'(\max(\bar{c}, y_1 - R_L l^*)) \tag{4.40}$$

No consumer will be allowed to borrow so much that they default with certainty. The maximum that a consumer could possibly receive in period 1 is y_{max}, the maximum available to repay debts is then $y_{max} - \bar{c}$. Therefore, there is an implied borrowing limit of $\frac{y_{max}-\bar{c}}{R_L}$ i.e. $l^* \in [0, \frac{y_{max}-\bar{c}}{R_L}]$.

The ability of agents to default increases the amount borrowed. This is intuitively obvious. The proof requires a little investigation and is laid out as the proof of Lemma 4.4.4 found in Appendix B.1.1.

Lemma 4.4.4. *The amount borrowed by a consumer is higher when defaults are permitted.*

While this result is important, we need to establish the properties of the borrowing decision in the presence of default. The results are laid out in the following set of lemmas and the proofs can be found in Appendix B.1.1. It will be shown that borrowing is still increasing in y_{max}. Finally, it will be shown there exists a threshold period 0 income y_D above which an agent may borrow, but will never default. These results will be used to establish the properties of the default premium in the Banks Section.

Lemma 4.4.5. *The level of borrowing l_D^* is increasing in y_{max}.*

Lemma 4.4.6. *There exists a threshold $y_D \leq y_B$ below which households borrow allowing for the possibility of default and above which they will never default.*

From 4.40 we can see that the demand for loans is only a function of incomes greater than $R_L l_D^* + \bar{c}$. Agents with relatively low incomes thus borrow based only on higher income and in the event that they receive a low income in Period 1, they will simply default. It is important to note that there is nothing irrational about this behaviour and neither is it strategic default. Agents are choosing borrowing based on the relevant legal framework, however, the bank will consider this when setting interest rates.

4.4.3 Illustration of Decision Rules

Given the complexity of the decision rules, it is useful to consider a practical example of what these decision rules look like. Figure 4.3 shows stylised decision rules for agents over various y_0.

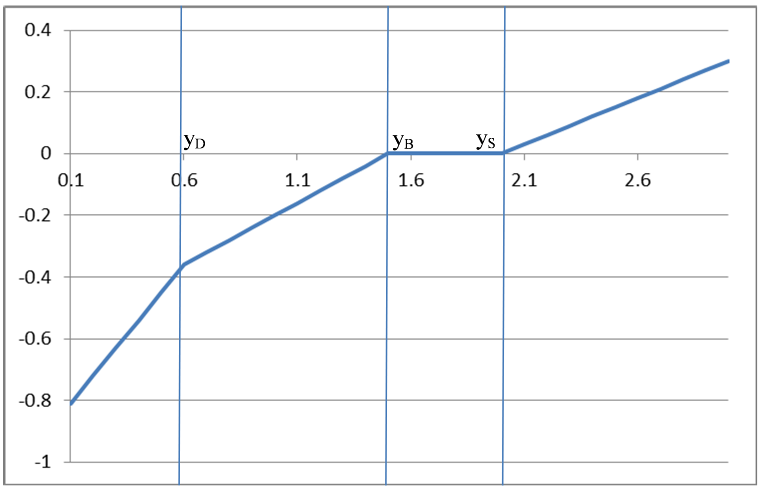

Figure 4.3: Stylised Decision Rule

In this example we can see four distinct regions discriminated by y_D, y_B and y_S. It is also possible to see clearly that, if this were a real decision rule, the economy would not be in equilibrium. The area between the line and the axis representing borrowing, $y_0 < y_B$, and deposits, $y_0 > y_s$, and these quantities are clearly not equal. Figure 4.6 shows a more realistic rule generated by the numerical methods outlined in later sections.

Figure 4.4: Optimal decision rules for log utility

In this diagram the actual decision rule is shown in purple. The other lines are shown over the full range of incomes to demonstrate more clearly the shapes of the curves for each of the three decision rules.

4.4.4 Concavity of the value function

The ability to default results in some possible breaches to the requirement for the value function to be concave in choice variable. That is to say that there may be a minima as well as a maxima in the value function, therefore solving the first order conditions may not result in utility maximisation. Figure 4.5 shows an example value function with a minima near -1 and a maxima just above zero. In numerical work it is important to ensure that we have not found a minima and therefore, at each calculation of the decision rule I test that the value function is decreasing at $w \pm \delta$. If I find that the minima has, in fact, been given then I resolve the problem by adjusting the limits of the available range to exclude the minima.

4.5 Banks

In Period 0, the banking sector takes deposits D_0 from households and lends L_0 to households. The Loan and Deposit markets must clear, that is to say

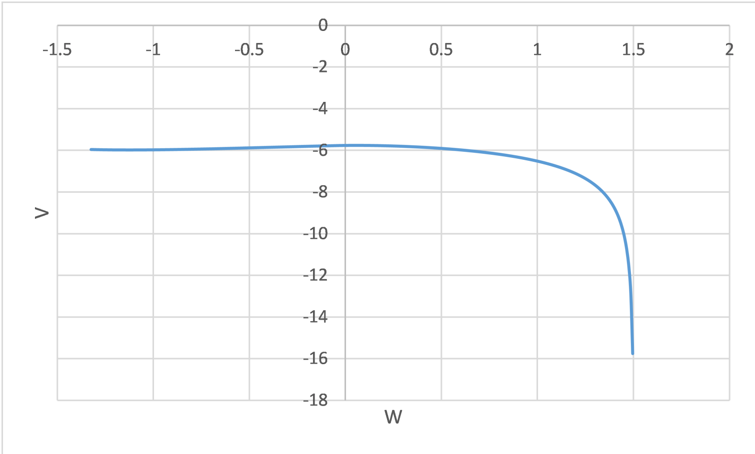

Figure 4.5: Value function for various choices of w in period zero with CRRA utility and $\gamma = 1.25$

that the supply of Loans/Deposits from the banks must equal the demand for Loans/Deposits from the Households:

$$L_0 = \int_{H^-} l^* d\Psi_y \tag{4.41}$$

$$D_0 = \int_{H^+} d^* d\Psi_y \tag{4.42}$$

The aggregate behaviour of the Loans and Deposits follow the individual choices. Lemma 4.5.1 establishes one of the key results; the proof of this lemma can be found in B.2.

Lemma 4.5.1. *For a given interest rate, R_L, the aggregate demand for borrowing is increasing in y_{max} if incomes are uniformly distributed .*

The only resources available to the banks are the deposits from households and the only investment opportunity is loans issued to households. Therefore, for each of the atomistic continuum of banks $i \in \{0, 1\}$, we have a first period balance sheet constrain of:

$$L_0^i = D_0^i \tag{4.43}$$

Banks are assumed to be identical, atomistic and symmetrical and therefore, it is reasonable to focus the analysis on the aggregates for the banking system. We must be careful to ensure that L_0^i and D_0^i are measurable on the set of banks. However, this is ensured by matching each pair of consumers $j \in H^-$ and $k \in H^+$ with a bank $i \in \{0,1\}$ which is equivalent of a mapping $\mathbb{R} \leftarrow \mathbb{R}$. Therefore, the banking sector has an aggregate balance sheet constraint in period 0 which is:

$$L_0 = D_0 \tag{4.44}$$

By the same argument, we are able to deal with the aggregate zero profit condition. In Period 1, the banks repay the depositors, plus interest and collect on debts where possible. The bank is only able to reclaim debts up to the income received by households and may suffer defaults if the income of the individual household falls below liabilities.

$$\pi = R_L L_0 - R_D D_0 - Q \tag{4.45}$$

This then yields:

$$R_L = R_D + \frac{Q}{L} \tag{4.46}$$
$$= R_D + \varsigma \tag{4.47}$$

The interest rate charged on loans is equal to the interest rate paid on deposits plus the default premium ς. The borrowing premium is driven by the level of defaults relative to the level of loans.

In Period 1 a household that has borrowing is unable to repay all of their debts if $\bar{c} > y_1 - R_L l_0^*$. Recall the set of Household is H, Borrowers are H^- with $l > 0,\ d = 0$, Savers are H^+ with $l = 0,\ d > 0$ and those who may default are H_0^q and those who do default H_1^q with $\bar{c} > y_1 - R_L l_0^*$. Note that $H = H^- \cup H^+$ and $H_1^q \in H_0^q \in H^-$.

The total liabilities of agents in H_1^q are $R_L l_0$. The loss suffered by a bank is then the difference between the payment they expect to receive, $R_L l_0$, and the funds available to repay the debt, income after subsistence expenditure $y_1 - \bar{c}$:

$$q(l_0) = R_L l_0 - (y_1 - \bar{c})$$

Taking expectations at period 0:

$$Eq(l_0) = E[R_L l_0 - (y_1 - \bar{c})]$$
$$= \int_{y_{min}}^{R_L l_0 + \bar{c}} [R_L l_0 - (y_1 - \bar{c})] d\Psi_{y_1} \tag{4.48}$$

Which for a uniform distribution gives:

$$Eq(l_0) = \frac{1}{y_{max} - y_{min}} \int_{y_{min}}^{R_L l_0 + \bar{c}} [R_L l_0 - (y_1 - \bar{c})] dy_1$$
$$= \frac{1}{y_{max} - y_{min}} [(R_L l_0 + \bar{c}) y_1 - \frac{y_1^2}{2}]_{y_{min}}^{R_L l_0 + \bar{c}}$$
$$= \frac{1}{y_{max} - y_{min}} [(R_L l_0 + \bar{c})(R_L l_0 + \bar{c}) - \frac{(R_L l_0 + \bar{c})^2}{2} - (R_L l_0 + \bar{c}) y_{min} + \frac{y_{min}^2}{2}]$$
$$= \frac{1}{y_{max} - y_{min}} [\frac{(R_L l_0 + \bar{c})^2}{2} - (R_L l_0 + \bar{c}) y_{min} + \frac{y_{min}^2}{2}]$$
$$= \frac{1}{2} \frac{1}{y_{max} - y_{min}} [R_L l_0 + \bar{c} - y_{min}]^2$$
$$= \frac{1}{2} \frac{1}{y_{max} - y_{min}} [R_L^2 l_0^2 + 2 R_L l_0 (\bar{c} - y_{min}) + (\bar{c} - y_{min})^2] \tag{4.49}$$

Then substituting back for Q:

$$Q = \int_{y_{min}}^{y_D} \frac{1}{2} \frac{1}{(y_{max} - y_{min})^2} [R_L^2 l_0^2 + 2 R_L l_0 (\bar{c} - y_{min}) + (\bar{c} - y_{min})^2] dy_0 \tag{4.50}$$
$$= \frac{1}{2} \frac{1}{(y_{max} - y_{min})^2} [R_L^2 L_2 + 2 R_L L_1 (\bar{c} - y_{min}) + (y_D - y_{min})(\bar{c} - y_{min})^2] \tag{4.51}$$

Where:

$$L_1 = \int_{y_{min}}^{y_{D_0}} l_0 dy_0 \tag{4.52}$$
$$L_2 = \int_{y_{min}}^{y_{D_0}} l_0^2 dy_0 \tag{4.53}$$

The borrowing premium is then:

$$\begin{aligned} \varsigma &= \frac{Q}{L} \\ &= \frac{1}{2}\frac{1}{L}\frac{1}{y_{max}-y_{min}}[R_L^2 L_2 + 2R_L L_1(\bar{c}-y_{min}) + (y_{D_0}-y_{min})(\bar{c}-y_{min})^2] \end{aligned} \tag{4.54}$$

The banks' problem then gives two conditions with the equilibrium defined as the R_L,R_D which solves:

$$D_0^*(R_D) = L^*(R_L, p)\ s.t. \tag{4.55}$$

$$R_L = R_D - \varsigma \tag{4.56}$$

$$\varsigma = \frac{1}{2}\frac{1}{L}\frac{1}{y_{max}-y_{min}}[R_L^2 L_2 - 2R_L L_1(\bar{c}-y_{min}) + (y_{D_0}-y_{min})(\bar{c}-y_{min})^2] \tag{4.57}$$

4.6 Market Clearing

It is now important to establish that there is enough information within the model to define all of the endogenous quantities. To demonstrate closure of the model, we begin with the period zero household budget constraints for borrowers, autarkic consumers and savers:

$$c_0^B = y_0 + l_0 \tag{4.58}$$

$$c_0^A = y_0 \tag{4.59}$$

$$c_0^S + d_0 = y_0 \tag{4.60}$$

Integrating over Period 0 incomes:

$$\int_{y_{min}}^{y_B} c_0^B d\Psi_{y_0} = \int_{y_{min}}^{y_B} l_0 d\Psi_{y_0} + \int_{y_{min}}^{y_B} y_0 d\Psi_{y_0} \tag{4.61}$$

$$\int_{y_B}^{y_S} c_0^A d\Psi_{y_0} = \int_{y_B}^{y_S} y_0 d\Psi_{y_0} \tag{4.62}$$

$$\int_{y_S}^{y_{max}} c_0^S d\Psi_{y_0} + \int_{y_S}^{y_{max}} d_0 d\Psi_{y_0} = \int_{y_S}^{y_{max}} y_0 d\Psi_{y_0} \tag{4.63}$$

Denoting the partial aggregates as upper-case letters with the relevant subscripts we can then write the resource constraints for each subgroup in the model:

$$C_0^B = L_0 + Y_0^B \tag{4.64}$$
$$C_0^A = Y_0^A \tag{4.65}$$
$$C_0^S + D_0 = Y_0^S \tag{4.66}$$

Where Y_0^B, Y_0^A and Y_0^S are the total period 0 income received by borrowers, autarkic consumers and savers respectively; this means $Y_0 = Y_0^B + Y_0^A + Y_0^A$. Summing these constraints and writing the total Period 0 income as Y_0 yields the period zero resource constraint:

$$C_0^B + C_0^A + C_0^S + D_0 = Y_0 + L_0 \tag{4.67}$$

And since we know $D_0 = L_0$ this simply reduces to a constraint that all income must be consumed in period 0 i.e.

$$C_0^B + C_0^A + C_0^S = Y_0 \tag{4.68}$$

For Period 1, we are able to perform the same accounting exercise, although we must be careful to properly account for the defaults:

$$c_1^B = max(y_1 - R_L l_0, \bar{c}) \tag{4.69}$$
$$c_1^A = y_1 \tag{4.70}$$
$$c_1^S = y_1 + R_D d_0 \tag{4.71}$$

We now note that in the case where $y_0 - R_L l_0 < \bar{c}$ the level of defaults q is given by $q = R_L l_0 - (y_0 - \bar{c})$. Intuitively, debts are reduced by q such that income does not fall below $\bar{c}$ can therefore write equation (4.69) as:

$$c_1^B = y_0 - R_L l_0 + q \tag{4.72}$$

Integrating over period 1 incomes requires that l_0 and d_0 are both y_0 and y_1 measurable. Put simply, those who borrow in period 0 are still borrowers in period 1. We can therefore write:[2]

$$\int_{y_{min}}^{y_{max}} c_1^B d\Psi_{y_1} = \int_{y_{min}}^{y_{max}} y_0 - R_L l_0 + q d\Psi_{y_1} \forall y_0 \in [y_{min}, y_B] \tag{4.73}$$

$$\int_{y_{min}}^{y_{max}} c_1^A d\Psi_{y_1} = \int_{y_{min}}^{y_{max}} y_1 d\Psi_{y_1} \forall y_0 \in [y_B, y_S] \tag{4.74}$$

$$\int_{y_{min}}^{y_{max}} c_1^S d\Psi_{y_1} = \int_{y_{min}}^{y_{max}} y_1 + R_D d_0 d\Psi_{y_1} \forall y_0 \in [y_S, y_{max}] \tag{4.75}$$

And writing in partial aggregates, where the state of the agent is determined by their period zero income:

$$C_1^B = Y_1^B - R_L L_0 + Q \tag{4.76}$$

$$C_1^A = Y_1^A \tag{4.77}$$

$$C_1^S = Y_1^S + R_D D_0 \tag{4.78}$$

Where Y_1^B, Y_1^A and Y_1^S are the total income received in period 1 by agents who were borrowers, autocratic consumers and savers in period 0; this means $Y_1 = Y_1^B + Y_1^A + Y_1^A$. And once again summing gives:

$$C_1^B + R_L L_0 + C_1^A + C_1^S = Y_1 + R_D D_0 + Q \tag{4.79}$$

Furthermore, all resources available must be consumed in each period and hence market clearing must ensure that:

$$C_0^B + C_0^A + C_0^S = Y_0 \tag{4.80}$$

$$C_1^B + C_1^A + C_1^S = Y_1 \tag{4.81}$$

Subtracting the market clearing (Equations 4.80 and 4.81) from the resource constraints (Equations 4.67 and 4.79) yields the banking sectors' balance sheet and zero profit conditions:

[2] This requirement simply means that there exists a mapping from the distribution of agents in H^- from period 0 to period 1. Strictly speaking, the notation here is an abuse of the term $d\Psi_{y_1}$, which refers to the distribution of both y and l_0 on y_1 which are not the same. However, under the uniform distribution there is a simple factor of $y_B - y_{min}$ difference and therefore, the aggregate relationship still holds.

$$L_0 = D_0 \tag{4.82}$$
$$R_D D_0 = R_L L_0 - Q \tag{4.83}$$

The equilibrium is then defined $C_0^B, C_0^A, C_0^S, C_1^B, C_1^A, C_1^S, L_0, D_0, R_D$ and R_L defined by the first period clearing conditions Equations 4.64 to 4.66, second period clearing Equations 4.76 to 4.77, the bank conditions Equations 4.82 and 4.83 and the resource constraints 4.67 and 4.79. This yields eight aggregates and two prices in 10 equations and therefore, the model is well defined.

However, it has been demonstrated above that the equilibrium can actually be defined as R_L,R_D which solves:

$$D_0^*(R_D) = L^*(R_L) \tag{4.84}$$
$$R_L = R_D + \varsigma \tag{4.85}$$
$$\varsigma = \frac{1}{2}\frac{1}{L}\frac{1}{y_{max} - y_{min}}[R_L^2 L_2 - 2R_L L_1(\bar{c} - y_{min}) + (y_{D_0} - y_{min})(\bar{c} - y_{min})^2] \tag{4.86}$$

This is effectively a statement of Walras Law, that is, the financial market clears then the goods market must also clear when there are only two markets.

4.7 Definition of Equilibrium

For the purpose of clarity, I bring together the equations that define the equilibrium of the system in this section. The exogenous state of the system is given as: (1) the income levels $y_{max} > y_{min} > \bar{c} > 0$, the discount rate $\beta \in (0, 1)$ and the concave utility function $u(.)$. The equilibrium is then defined by: (1) the pair of prices R_L and R_D; (2) the threshold incomes $y_d < y_B < y_s$; (3) the loan demand function for borrowers who may default $y_0 \in [y_min, y_D] \mapsto l_D^*(y_0, R_L; y_{max}$; (4) the loan demand function for borrowers who will never default $y_0 \in [y_D, y_B] \mapsto l_{ND}^*(y_0, R_L; y_{max}$; a deposit function for savers $y_0 \in [y_S, y_{max}] \mapsto d_D^*(y_0, R_D; y_{max}$; (5) the aggregate loan function $L(R_L; y_{max}$; (6) the aggregate deposit function $D(R_D; y_{max}$ and the aggregate defaults $Q(R_L; y_{max}$. These quantities are related by the set of equations:

1. First Order Conditions

   - for $y_0 \in [y_{min}, y_D]$, $u'(c_0) = \beta R_L (y_{max} - y_{min})^{-1}[u(y_{max} + R_L(y_0 - c_0)) - u(\bar{c})]]$

- for $y_0 \in [y_B, y_B]$, $u'(c_0) = \beta R_L(y_{max} - y_{min})^{-1}[u(y_{max} + R_L(y_0 - c_0)) - u(y_{min} + R_L(y_0 - c_0))]$
- for $y_0 \in [y_S, y_{max}]$, $u'(c_0) = \beta R_D(y_{max} - y_{min})^{-1}[u(y_{max} + R_D(y_0 - c_0)) - u(y_{min} + R_D(y_0 - c_0))]$

2. Income Thresholds

- y_D such that $u'(y_D + (y_{min} - \bar{c})/R_L) = \beta R_L(y_{max} - y_{min})^{-1}[u(y_{max} + (y_{min} - \bar{c}) - u(\bar{c})]$
- y_B such that, $u'(y_B) = \beta R_L(y_{max} - y_{min})^{-1}[u(y_{max}) - u(y_{min})]$
- for y_S such that, $u'(y_S) = \beta R_D(y_{max} - y_{min})^{-1}[u(y_{max}) - u(y_{min})]$

3. Aggregates

- $L(R_L, y_{max}) = \frac{1}{y_{max} - y_{min}} \int\limits_{y_{min}}^{y_D} l_D^* dy_0 + \frac{1}{y_{max} - y_{min}} \int\limits_{y_D}^{y_B} l_{ND}^* dy_0$
- $D(R_D, y_{max}) = \frac{1}{y_{max} - y_{min}} \int\limits_{y_S}^{y_{max}} d^* dy_0$
- $Q(R_L, y_{max}) = \frac{1}{2} \frac{1}{(y_{max} - y_{min})^2} \int\limits_{y_{min}}^{y_D} [R_L^2 l_0^2 + 2R_L l_0(\bar{c} - y_{min}) + (\bar{c} - y_{min})^2] dy_0$

4. Market Clearing

- $L(R_L, y_{max}) = D(R_D, y_{max})$
- $R_L L(R_L, y_{max}) - Q(R_L, y_{max}) = R_D D(R_D, y_{max})$

Consumers with $y_0 \in [y_B, y_S]$ have $c_0 = y_0$, that is to say they neither borrow nor save.

In choosing this definition I assume that the solution is interior for all consumers.

4.8 Defaults and Borrowing Premium

We have previously analysed the effect of an ability to default on individual consumers. It has already been shown that where defaults are possible, the level of borrowing for each y_0 is higher and the derivative of l_0 with respect to y_{max} is positive irrespective of the ability to default. This section will establish the effect on aggregates and borrowing premium.

As has been stated, households that have chosen to borrow will default if:

$$\bar{c} > y_1 - R_L l_0^* \tag{4.87}$$

The funds available for repayment of debt in this period are: $y_1 - \bar{c}$ and in this case the amount of debt upon which each household defaults will be:

$$\begin{aligned} q &= R_L l_0^* - (y_1 - \bar{c}) \\ &= R_L l_0^* - y_1 + \bar{c} \end{aligned} \tag{4.88}$$

This value will always be positive, strictly speaking and the loss through default to the intermediating banks is $\max\{q, 0\}$.

Lemma 4.8.1. *Eq is increasing in l_0^*.*

Proof. Note that $\forall\ y_1 > R_L l_0^* + \bar{c},\ q = 0$ then from 4.88

$$Eq = \int_{y_{min}}^{R_L l_0^* + \bar{c}} R_L l_0^* - y_1 + \bar{c} d\Psi_{y_1} \tag{4.89}$$

$$= \frac{1}{y_{max} - y_{min}} [(R_L l_0^* + \bar{c})(R_L l_0^* + \bar{c} - y_{min}) - \frac{1}{2}(R_L l_0^* + \bar{c})^2 + \frac{1}{2} y_{min}^2] \tag{4.90}$$

Then taking derivatives:

$$\frac{\partial Eq}{\partial E_0 y_1} = \frac{1}{y_{max} - y_{min}} [R_L l_0^* (2 + 2\bar{c} - y_{min}) - R_L(R_L l_0^* + \bar{c})] \tag{4.91}$$

We know that $\bar{c} - y_{min} > 0$ and that therefore the derivative is positive if:

$$l_0^*(2 + \bar{c}) > R_L l_0^* + \bar{c} \tag{4.92}$$

$$l_0^*(2 - R_L) + \bar{c}) > 0 \tag{4.93}$$

This hold true as long as interest rates are less than 100% which seems like a reasonable assumption.

□

The total amount of borrowing by households in this economy, L_0, is then the sum of the borrowing which may result in default and that which will not $L_0 = L_0^D + L_0^{ND}$ where:

$$L_0^D = \int_{y_{min}}^{y_D} l_D^* d\Psi_y \tag{4.94}$$

$$L_0^{ND} = \int_{y_D}^{y_B} l_{ND}^* d\Psi_y \tag{4.95}$$

If default was not an option then $y_D = y_min$ and so the total amount of borrowing, L_0^{ND}, would simply be:

$$L_0^{ND} = \int_{y_{min}}^{y_B} l_{ND}^* d\Psi_y \tag{4.96}$$

The total demand for borrowing in an economy where defaults are possible is greater than one in which defaults are not possible. Disaggregating 4.96 yields:

$$L_0^{ND} = \int_{y_{min}}^{y_D} l_{ND}^* d\Psi_y + \int_{y_D}^{y_B} l_{ND}^* d\Psi_y \tag{4.97}$$

It is then clear by Lemma 4.4.4 that:

$$\int_{y_{min}}^{y_D} l_{ND}^* d\Psi_y < \int_{y_{min}}^{y_D} l_D^* d\Psi_y \tag{4.98}$$

It then follows directly that $L_0^D > L_0^{ND}$.

These results show that the propensity for borrowing increases when defaults are possible and, for a given y_{min}, increases with y_{max}. However, given the complex nature of the interaction in equilibrium it is not possible to proceed further analytically, therefore I present numerical results in the following sections.

4.9 Computational Strategy

The parametrisation of the model outlined above, results in a non-linear set of simultaneous equations. The model was shown to clear, if and only if the market clearing and zero profit conditions hold:

$$\int_{y_{min}}^{y_B} l_0^* dy_0 = \int_{y_S}^{y_{max}} d_0^* dy_0 \tag{4.99}$$

$$R_L = R_D + \varsigma \tag{4.100}$$

The challenge is to find the set of interest rates $\{R_L, R_D\}$ that satisfy this relationship.

Substituting for a Log Utility function, let:

$$u(c) = ln(c) \tag{4.101}$$

$$u'(c) = c^{-1} \tag{4.102}$$

Substituting for a Log Utility function, let:

$$u(c) = \frac{c^(1-\gamma)}{1-\gamma} \tag{4.103}$$

$$u'(c) = c^{-\gamma} \tag{4.104}$$

The choice of utility function is a balance between tractability and realism. Both Log and CRRA have some empirical support and are no more computationally difficult to implement than quadratic utility. The high level strategy is then:

1. For a given $\{R_L, R_D\}$ calculate the decision rules, l_0^*, d_0^* for each y_0, noting that this implies $c_0^{S*}, c_0^{B*}, c_0^{A*}$

2. Integrate to calculate L_0, D_0 and Q

3. Test if equilibrium is achieved and, if not, evolve $\{R_L, R_D\}$

These three steps are not necessarily trivial to implement, the decision rules do not have closed form solutions for realistic utility functions and the integral must therefore be calculated numerically. We then have multiple levels of numerical calculation and so the root finding algorithm is likely to be inhibited by numerical noise. Finding the solution of these equations must then require a robust numerical approach. To begin with define X and Y as deviations from the equilibrium:

$$X = \int_{y_{min}}^{y_B} l_0^* dy_0 - \int_{y_S}^{y_{max}} d_0^* dy_0 \tag{4.105}$$

$$Y = R_L - R_D - \varsigma \tag{4.106}$$

The numerical strategy is to use a Vector Newton-Raphson style approach to find the roots of X, Y. However, such algorithms may not converge, given the absence of established regularity conditions and therefore, it is useful to specify an alternative objective function:

$$F = X^2 + Y^2 \tag{4.107}$$

If the vector method is found to not converge to a root, we can instead use a Non-Linear Least Squares approach to find a root. While this may be seen as not reaching an equilibrium, one must remember that floating point operations using 32bit doubles are only accurate to 16 significant figures and therefore, minimising a function to within 10^{-26} is equivalent to finding a root. Moreover, in both cases above this objective function is not necessarily smooth. That is to say for a given set of model parameters the function may have multiple local minima for given values of $\{R_L, R_D\}$.

The algorithm for calculating the solution is then:

1. For a given parameter set and functional specification $\{\bar{c}, y_{min}, y_{max}, u, \beta\}$ evaluate X and Y

   (a) Calculate policy functions using Newton-Raphson if necessary

   (b) Calculate aggregates using an adaptive quadrature

(c) Calculate X and Y

2. Check the stopping criteria

3. Evolve the system and repeat

This process has been stated using a general evolution step. This is due to difficulties encountered minimising the functions in practice and so a combination of a Newtwon/Broyden type Vector Methods and a Shuffled Complex Evolution Procedure was used to ensure convergence. These methods are implemented in C++ and the code used can be found in Appendix B.3 along with detailed comments about platform specific implementation. An overview of the code is presented along with descriptions of the algorithms in Appendix B.4.

4.10 Data for Calibration

The parameters required as inputs of the model are summarised in Table 4.1:

Table 4.1: Parameter inputs for the two period model and sources

Parameter	Value	Source
y_{min}	0.3825	Office for National Statistics (2012, pp. 57)
y_{max}	1.0000	
$\bar{c}$	0.3187	Davis et al. (2011)
β	0.98	Epstein and Hynes (1983)

The value of β will be taken as 0.98 as is current in the literature. The other three parameters are novel to this model and require a little attention. There is also the problem of scale, the model is not homogeneous in the income level and we need to be careful to calculate a realistic level of income available for consumption.

The nominal value of y_{min} is relatively easy to find, taking 2010 UK data the minimum wage is £ 5.80 per hour (copyright, 2012) and the average number of hours worked is 33.4 hours per week. This gives rise to a weekly minimum wage of £ 193.72 or an annual £ 10,101. However, this does not take into account the effects of tax and benefits delivered by the state. The ONS estimates that the average income of the poorest fifth of households is £15,242 and £61,376 for the richest after taking into account direct and indirect taxes. Dividing by the GDP deflator (Office for National Statistics, 2012, 56) yields 148.28 and 597.04 in consumption units.

Using the uniform distribution the average income, $\bar{Y}$, between the upper and lower limits on income, H and L respectively, is given by:

$$\int_L^H y d\Psi_y = \bar{Y}$$
$$H + L = 2\bar{Y} \tag{4.108}$$

To maintain the levels of upper and lower quartiles to match the above level we require that:

$$2y_{min} + \frac{1}{5}(y_{max} - y_{min}) = 2 \times 148.28$$
$$y_{max} + y_{min} + \frac{4}{5}(y_{max} - y_{min}) = 2 \times 597.04 \tag{4.109}$$

So for the upper and lower quartiles to match the above level we require that:

$$y_{min} = 18.434$$
$$y_{max} = 130.63 \tag{4.110}$$

We note that the Gross National Income (GNI) in 2010 was £1,136,596 million at market prices (Office for National Statistics, 2012, pp. 57). The population will be taken to be 62,262,000, this does include those under the age of 16. This gives an income per capita of £18,255 or 177.58 in consumption units. The total per capita income implied by the values of y_{min} and y_{max} is only 74.531, less than half of that reflected in the data. However, this simply reflects the fact that the uniform distribution is not capturing the significant proportion of national income taken by the wealthy.

Rather than matching the average for the upper and lower quartiles, it is possible to match the minimum wage and total income. This would yield $y_{min} = 98.26$ and $y_{max} = 256.90$.

The minimum level of consumption, $\bar{c}$, is somewhat more subjective. The OFT (2011, pp. 20) states that in the process of debt collection companies must refrain from:

> i. Pressurising debtors to pay more than they can reasonably afford[32] without experiencing undue difficulty.[33] As and when the

debtor has been located/identified or to pay within an unreasonably short period.

[32] For example, pressurising a debtor to make unreasonably large repayments or to pay off his debts in full in a single (or very few) repayment(s), when to do so would have an adverse impact on the debtor's financial circumstances.

[33] The OFT would regard 'without undue difficulty' in this context as meaning the debtor being able to make repayments while also meeting other debt repayments and other normal/reasonable outgoings and without having to borrow further to meet these repayments.

What this means in practice, is that a company providing credit can only recover debts in excess of an individual's living expenses. This will clearly vary between individuals and depends critically on individual circumstances. Two base lines will be considered both based on the weekly consumption of households composed of a single working individual: (1) the national average of £290.30 taken from ONS (2011, pp. 127, Table A23) and (2) a minimum of £227.97 established by Davis et al. (2011, pp. 19). Removing 'discretionary spending' e.g. social, cultural, alcohol and non-work related travel, yields figures of £108.9 and £161.40, respectively. Converting to consumption units per year, 55.237 and 81.86, this would seem to suggest that the Minimum Wage and GDP methodology is more appropriate to calculate y_{min} and y_{max}.

There is, however, still a problem of scale. The traditional solution is to match ratios of the real economy to the calibrated model. For this purpose the values of y_{min}, $\bar{C}$ and y_{max} will be taken as 0.3825, 0.3187 and 1.000 respectively. This corresponds to the higher estimate of the $\bar{c}$ and the corresponding estimate of y_{min} and y_{max}.

4.11 Results

This section will present the results of the calibration along with a demonstration of how the model solution works.

4.11.1 Decision Rule

The values of y_{min}, $\bar{C}$ and y_{max} are taken from Table 4.1, I then use either log or CRRA Utility and explore the space of γ. To begin I look at the decision rule, in Figure 4.6, this graph shows the overall decision rule, W, and the sub curves which contributed to the decision rule. The decision rule function exists over the entire space of Y_0 but the consumer will only ever make a choice based on W.

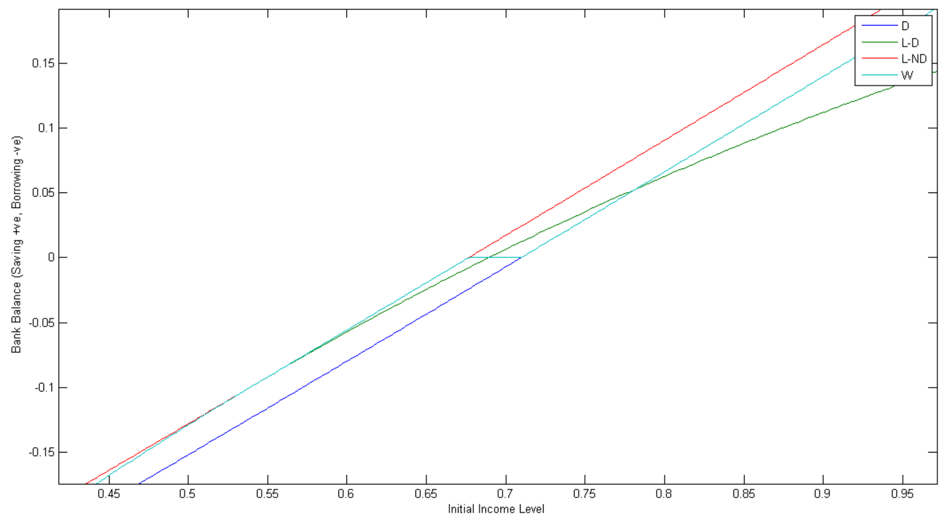

Figure 4.6: Optimal Choice when $\gamma = 2.1$ Looking at Non-Defaulter Transition

On Figure 4.1 we see the three transitions clearly, from defaulter to non-defaulter (0.5), borrower to autarkic consumer (0.675) and finally saver (0.7).

4.11.2 Objective Function and Solution

The solution to the model is given as the $\{R_L, R_D\}$, that solves the zero profit condition and the market clearing condition. Figure 4.7 shows the error in the zero profit condition for various combinations of $\{R_L, R_D\}$. It is useful, such that the solution is unique, for the function to have exactly one zero in R_L for each value of R_D. This is equivalent to saying the the function crosses the 0 plane only once; Figure 4.7 demonstrates that this is the case. [3]

The market clearing condition must also be satisfied in equilibrium. Once again I show that the error in this condition is zero along a single plane crossing the zero plane. Figure 4.8 shows the function evaluated over various values of $\{R_L, R_D\}$

The overall objective function is shown in Figure 4.9, however, it is difficult to see precisely the position this function is at zero. Therefore, it is

[3] More generally this was found to be true for CRRA utility with values of γ between 1 and 3.

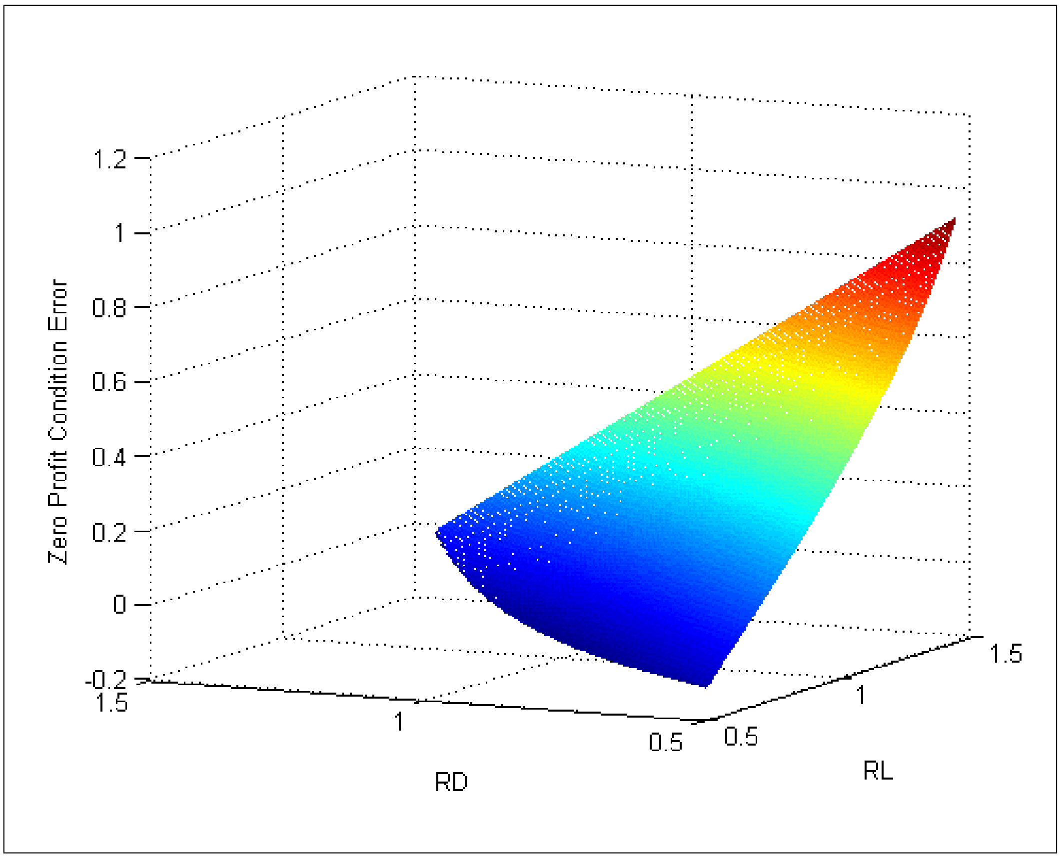

Figure 4.7: Error in the zero profit condition using Log Utility

easier to consider the slices where the two separate components are zero. Turning to Figure 4.10 we see that for the base case there is exactly one value of $\{R_L, R_D\}$ that satisfies both the zero profit condition and the market clearing condition. For this base economy, we find that the interest rates $\{R_L, R_D\} = \{1.01, 0.86\}$, that is a borrowing spread of 15%. This may seem high relative to the UK however, given the current very low level of deposit rates and high cost of unsecured credit, this number is not unreasonably high.

The SCE algorithm employed is also robust enough to conclude that if it is unable to find a zero within the bounds specified, no such zero exists. That is not to say that there may not be a pair (R_L, R_D) which solves the model but it does mean that the only solution is outside of the bounds $[0.5, 2.0]$. While there is some evidence of economies with negative real interest rates or indeed rates over 100%, neither of these cases would be considered economically stable.

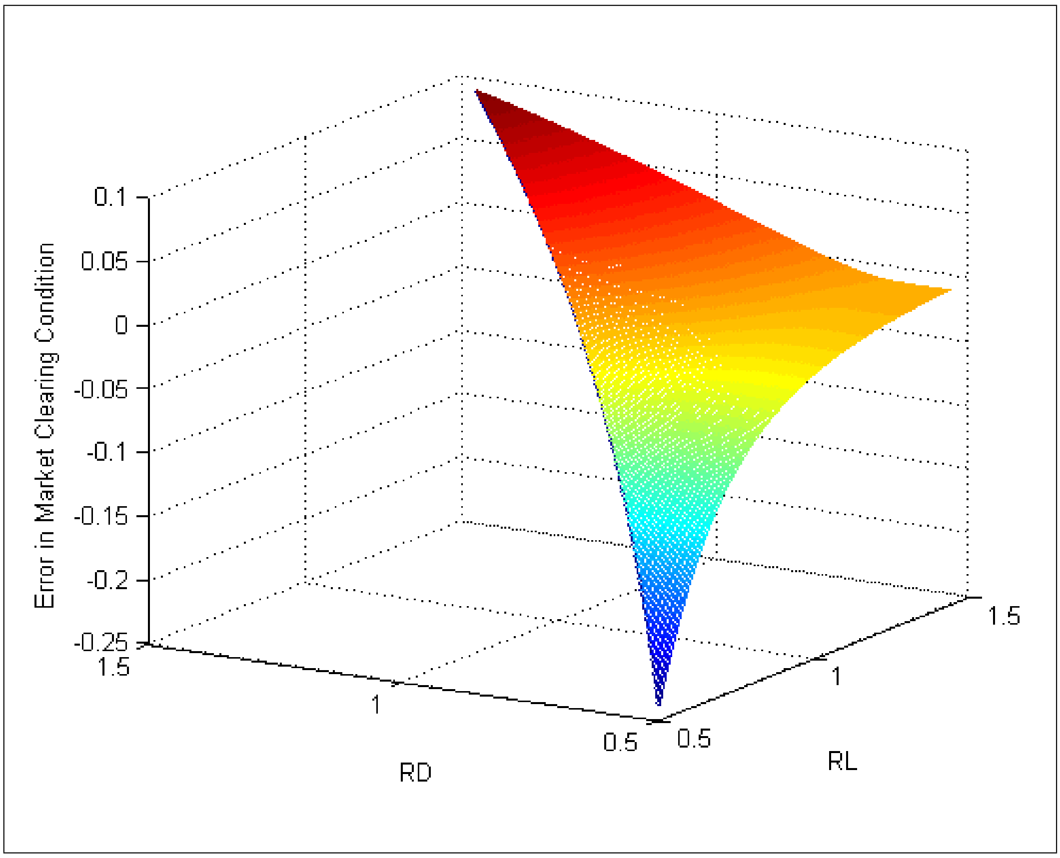

Figure 4.8: Error in the market clearing condition using Log Utility

4.11.3 Effect of Inequality

Inequality in this model is measured by y_{max} and so the question is: what happens to the borrowing premium when this upper limit is increased. Figure 4.11 shows how the borrowing premium changes for various values of y_{max} using both log utility and various CRRA functions. Beyond the range $1 \leq \gamma \leq 1.8$ the result did not hold and equilibrium were difficult to find.

Clearly, inequality serves to increase the borrowing premium in this case, moreover no feasible solution was found when y_{max} exceeded this level thus suggesting no reasonable equilibrium exists. Specifically, in the cases presented on the graph the residual value of F was less then 10^{-10} and this value increased rapidly for $y_{max} > 2.1$. The result was also found to be robust to changes in $\bar{c}$, γ, and y_{min}.

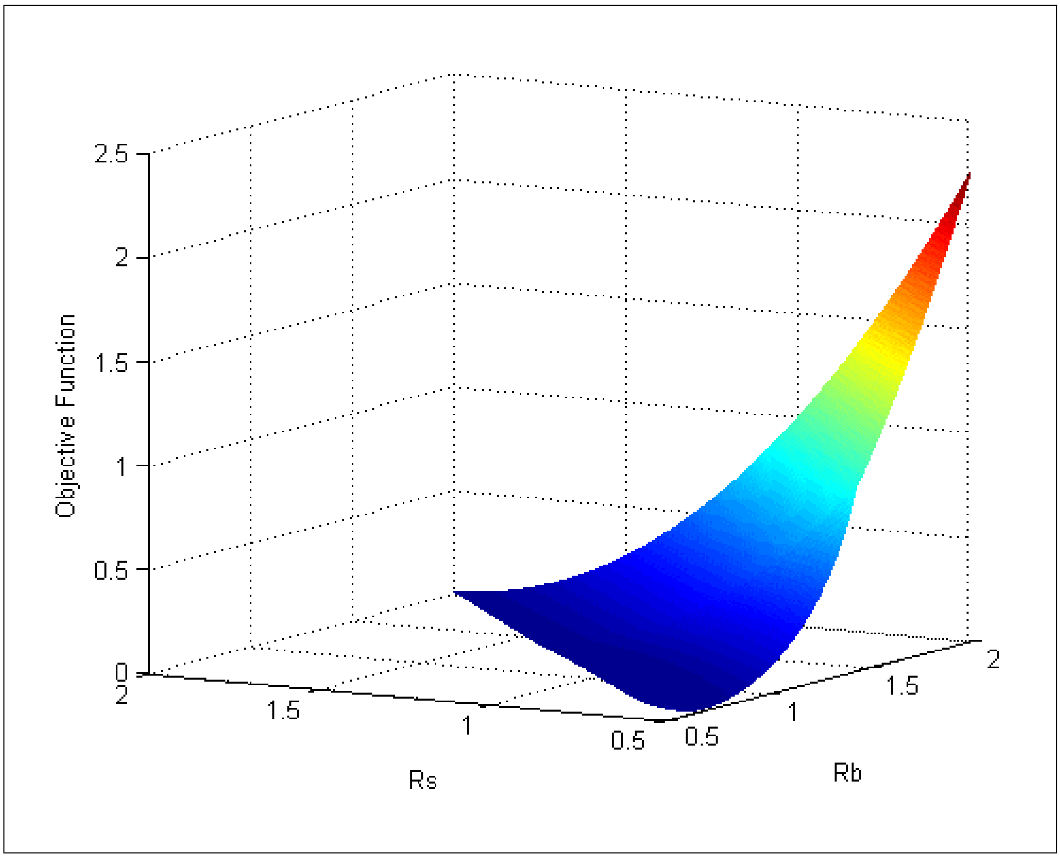

Figure 4.9: Objective function using Log Utility

4.12 Conclusion

This chapter has presented a model of how inequality can precipitate through an economy and result in a higher borrowing premium. Specifically we find that for a calibration of the model with log utility and data from the UK a borrowing premium of 15% is predicted. If the highest incomes were to rise by 20%, I would predict that the borrowing premium would rise to over 20%.

The key results are found numerically, however, they are supported by instructive analytical results. Specifically, I have shown that when defaults are permitted the desire for borrowing is increasing in y_{max}. I propose that this result is due to the fact that agents are unbound by the fear of debt repayments in low income states of the world, due to the ability to default. This increased propensity to borrow tells a very similar to story to Kumhof and Rancière (2010) in that inequality leads to a credit bubble.

The results of the theoretical work are not true universally, that is for certain levels of income borrowing premium does not rise. However, this

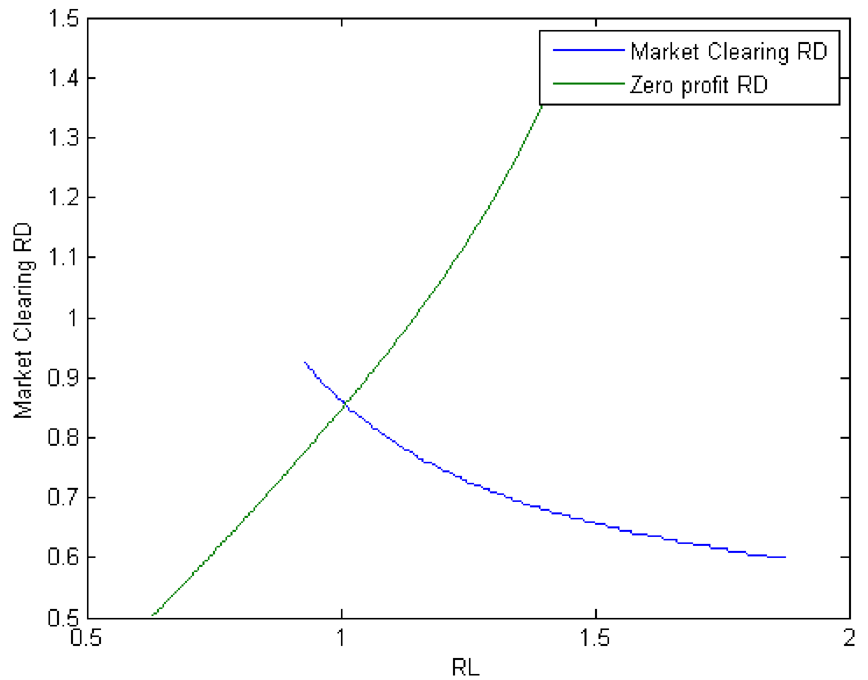

Figure 4.10: The values of $\{R_L, R_D\}$ that satisfy the zero profit and market clearing conditions

may not be as problematic to the hypothesis as it fist appears. If one were to consider Japan in the late 1980s or the USA in the 2005 one could argue that borrowing costs were too low promoting excessive risk taking. In this sense the model provides a rich framework for future analysis. It would be interesting to study these affects in more detail, ideally in the context of a dynamic generalisation of this model.

There are many additional extensions to this model which may provide a richer environment for study. For example, I did not consider the issue of collateral or securitisation; the ability of banks to reposes mortgaged houses will certainly alter the expected pay off of a distressed loan. This issue could be further investigated using this model if I were to include an asset market in both periods. Finally, the model specifies a capital market imperfection, namely a borrowing premium, it would be possible to alter this premium to be a function of the level of borrowing or the perceived riskiness of the loan. I do not expect any of these additions to significantly alter the core conclusion that inequality does interact with financial stability.

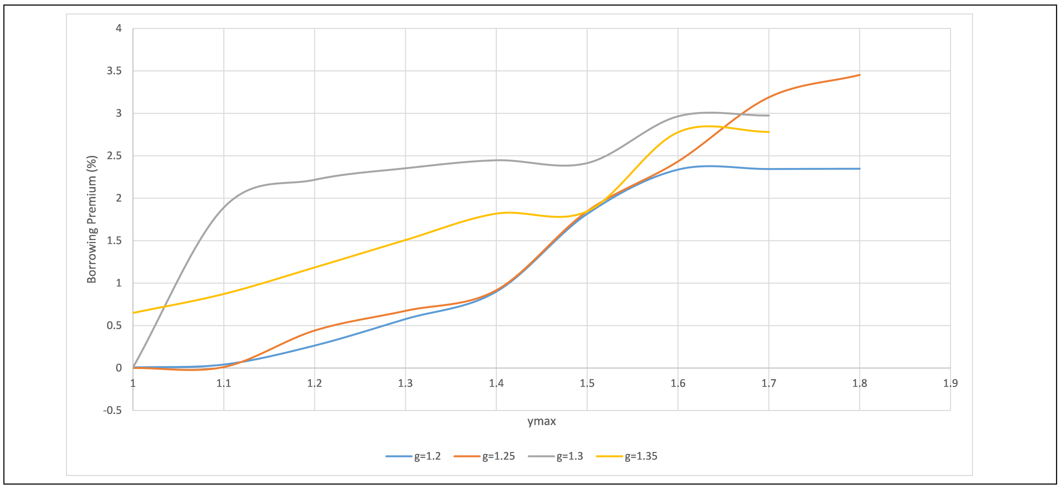

Figure 4.11: The effect of inequality on borrowing premium

Appendices

Chapter B

Two Period Model Appendices

B.1 Proof of Lemmas

B.1.1 Borrow or Save

Properties of the Threshold Incomes

Lemma B.1.1. *If $R_L > R_D$ then $y_B < y_S$ and households whose income falls between these limits will neither deposit with nor borrow from the bank.*

Proof. By concavity of u and independence of $E_0[y_1]$ the result follows directly. □

Corollary B.1.2. *The threshold income where borrowing becomes the optimal strategy is increasing in y_{max}*

Proof. Noting that for the uniform distribution:

$$u'(y_B) = \beta R_L E u'(y_1) \tag{B.1}$$

$$= \beta R_L \int_{y_{min}}^{y_{max}} u'(y_1) d\Psi_{y_1} \tag{B.2}$$

$$= \frac{\beta R_L}{y_{max} - y_{min}} \int_{y_{min}}^{y_{max}} u'(y_1) dy_1 \tag{B.3}$$

Then taking the derivative with respect to y_{max} using Leibniz Rule to differentiate under the integral sign:

$$\frac{\partial y_B}{\partial y_{max}} u''(y_B) = \frac{-\beta R_L}{(y_{max} - y_{min})^2} \int_{y_{min}}^{y_{max}} u'(y_1) dy_1 + \frac{\beta R_L}{y_{max} - y_{min}} u'(y_{max}) \quad \text{(B.4)}$$

Now, $u''(y_B) < 0 \forall y_B$ the problem is to prove that the LHS is negative which can be achieved by concavity of u:

$$\frac{\beta R_L}{(y_{max} - y_{min})^2} \int_{y_{min}}^{y_{max}} u'(y_1) dy_1 > \frac{\beta R_L}{y_{max} - y_{min}} u'(y_{max}) \quad \text{(B.5)}$$

$$u(y_{max}) - u(y_{min}) > [y_{max} - y_{min}] u'(y_{max}) \quad \text{(B.6)}$$

htb □

Properties of l_0

Lemma 4.4.1. *Optimal borrowing for a given household is decreasing in y_0*

Proof. Taking the derivative of equation (4.37) with respect to y_0 yields:

$$u''(y_0 + l^*)[1 + \frac{\partial l^*}{\partial y_0}] = -\beta R_L^2 E_0 u''(y_1 - R_L l^*) \frac{\partial l^*}{\partial y_0} \quad \text{(B.7)}$$

Then solving for $\frac{\partial l^*}{\partial y_0}$ yields:

$$\frac{\partial l^*}{\partial y_0} = -\frac{u''(y_0 + l^*)}{u''(y_0 + l^*) + \beta R_L^2 E_0 u''(y_1 - R_L l^*)} \quad \text{(B.8)}$$

Then noting $u''(c) < 0 \ \forall \ c$ the derivative is negative and so l^* is strictly decreasing in y_0.

□

Corollary 4.4.2. $|\frac{\partial l^*}{\partial y_0}| < 1$

Proof. Noting that $\beta R_L > 1$ the result follows directly from equation. B.8. □

Lemma 4.4.3. *Optimal borrowing for a given household is increasing in y_{max}*

Proof. Follows directly as per Proof of Corollary B.1.2. Nothing that the derivative of l_0 with respect to y_{max} is:

$$\frac{\partial l^*_{ND}}{\partial y_{max}} = \frac{\beta R_L[u(y_{min} - R_L l^*) - u(y_{max} - R_L l^*) + ((y_{max} - y_{min}))u'(y_{max} - R_L l^*)]}{(y_{max} - y_{min})\{(y_{max} - y_{min})u''(y_0 + l^*) + \beta R_L^2[u'(y_{max} - R_L l^*) - u'(y_{min} - R_L l^*)]\}} \tag{B.9}$$

The denominator is clearly negative as $u'' < 0$ and $u'(y_{max} - R_L l^*) < u'(y_{min} - R_L l^*)$ and so the proof requires that the square brackets on the numerator is also negative, that is:

$$u(y_{min} - R_L l^*) + ((y_{max} - y_{min}))u'(y_{max} - R_L l^*) < u(y_{max} - R_L l^*) \tag{B.10}$$

This result must then be true by concavity of u. □

Default

Lemma 4.4.4. *The amount borrowed by a consumer is higher when defaults are permitted.*

Proof. The general first order condition is then:

$$u'(y_0 + l^*) = \beta R_L \int_{y_{min}}^{y_{max}} [u'(y_1 - R_L l^*)] d\Psi_{y_1} \tag{B.11}$$

The two cases are then (i) no defaults with loan size l^*_{ND} iff $\forall\ y_1 \in [y_{min}, y_{max}],\ y_1 - R_L l^*_{ND} \geq \bar{c}$ and (ii) possibility of defaults with loan size l^*_D iff $\exists\ y_1 \in [y_{min}, y_{max}]\ s.t.\ y_1 - R_L l^*_D < \bar{c}$ with first order conditions:

$$u'(y_0 + l_{ND}^*) = \beta R_L \int_{y_{min}}^{y_{max}} [u'(y_1 - R_L l_{ND}^*)] d\Psi_y \tag{B.12}$$

$$= \frac{\beta R_L}{y_{max} - y_{min}} [u(y_{max} - R_L l_{ND}^*) - u(y_{ymin} - R_L l_{ND}^*)] \tag{B.13}$$

$$u'(y_0 + l_D^*) = \beta R_L [u'(\bar{c}) \int_{y_{min}}^{R_L l_D^* + \bar{c}} d\Psi_{y_1} + \int_{R_L l_D^* + \bar{c}}^{y_{max}} [u'(y_1 - R_L l_D^*)] d\Psi_y] \tag{B.14}$$

$$= \frac{\beta R_L}{y_{max} - y_{min}} [u'(\bar{c})(R_L l_D^* + \bar{c} - y_{min}) + u(y_{max} - R_L l_D^*) - u(\bar{c})] \tag{B.15}$$

Now if the lemma is true and $l_D^* > l_{ND}^* \forall y_0 < y_D$ then by strict concavity of u and assuming $y_{min} - R_L l_D^* \neq \bar{c}$ we have :

$$u(\bar{c}) + u'(\bar{c}).(y_{min} - R_L l_D^* - \bar{c} > u(y_{min} - R_L l_D^*) \tag{B.16}$$

Then by B.15 we have:

$$u'(y_0 + l_D^*) < \frac{\beta R_L}{y_{max} - y_{min}} [u(y_{max} - R_L l_D^*) - u(y_{min} - R_L l_D^*)] \tag{B.17}$$

Subtracting from B.14 and rearranging yields:

$$u'(y_0 + l_{ND}^*) - \frac{\beta R_L}{y_{max} - y_{min}} [u(y_{max} - R_L l_{ND}^*) - u(y_{min} - R_L l_{ND}^*)] < \tag{B.18}$$

$$u'(y_0 + l_D^*) - \frac{\beta R_L}{y_{max} - y_{min}} [u(y_{max} - R_L l_D^*) - u(y_{min} - R_L l_D^*)] \tag{B.19}$$

Now define $g(x)$ as: Subtracting from B.14 and rearranging yields:

$$g(x) \equiv u'(y_0 + x) - \frac{\beta R_L}{y_{max} - y_{min}} [u(y_{max} - R_L x) - u(y_{min} - R_L x)] \tag{B.20}$$

Then by B.19 we have $g(l_{ND}^*) > g(l_D^*)$ which implies $l_D^* < l_{ND}^*$ because

$g(x)$ is decreasing in x, not the derivative:

$$g'(x) = u''(y_0 + x) - \frac{\beta R_L^2}{y_{max} - y_{min}} \left[u'(y_{max} - R_L x) - u'(y_{min} - R_L x)\right] \tag{B.21}$$
$$< 0 \tag{B.22}$$

□

Lemma 4.4.5. *The level of borrowing l_D^* is increasing in y_{max}.*

Proof. Taking the derivative of 4.40 and applying Leibniz Rule to differentiate under the integral taking into account the limits:

$$\begin{aligned}
u''(y_0 + l_D^*)\frac{\partial l_D^*}{\partial y_{max}} &= \beta R_L[\frac{R_L}{y_{max} - y_{min}}\frac{\partial l_D^*}{\partial y_{max}}u'(\bar{c}) - \frac{(R_L l_D^* + \bar{c} - y_{min})}{(y_{max} - y_{min})^2}u'(\bar{c}) \\
&\quad - \frac{R_L}{y_{max} - y_{min}}\int_{R_L l_D^* + \bar{c}}^{y_{max}} u''(y_1 - R_L l_D^*)\frac{\partial l_D^*}{\partial y_{max}}dy_1 \\
&\quad + \frac{1}{y_{max} - y_{min}}u'(y_{max} - R_L l_D^*) - \frac{R_L}{y_{max} - y_{min}}\frac{\partial l_D^*}{\partial y_{max}}u'(\bar{c}) \\
&\quad - \frac{1}{(y_{max} - y_{min})^2}\int_{R_L l_D^* + \bar{c}}^{y_{max}} [u'(y_1 - R_L l_D^*)]dy_1] \\
u''(y_0 + l_D^*)\frac{\partial l_D^*}{\partial y_{max}} &= \frac{\beta R_L}{(y_{max} - y_{min})^2}[-(R_L l_D^* + \bar{c} - y_{min})u'(\bar{c}) \\
&\quad - (y_{max} - y_{min})R_L \int_{R_L l_D^* + \bar{c}}^{y_{max}} u''(y_1 - R_L l_D^*)\frac{\partial l_D^*}{\partial y_{max}}dy_1 \\
&\quad + (y_{max} - y_{min})u'(y_{max} + R_L l_D^*) - \int_{R_L l_D^* + \bar{c}}^{y_{max}} [u'(y_1 - R_L l_D^*)]dy_1]
\end{aligned}$$

$$(y_{max} - y_{min})\frac{\partial l_D^*}{\partial y_{max}} =$$

$$\frac{\beta R_L[-(R_L l_D^* + \bar{c} - y_{min})u'(\bar{c}) + (y_{max} - y_{min})u'(y_{max} - R_L l_D^*) - \int_{R_L l_D^* + \bar{c}}^{y_{max}} [u'(y_1 - R_L l_D^*)]dy_1]}{(y_{max} - y_{min})u''(y_0 + l_D^*) + \beta R_L^2 \int_{R_L l_D^* + \bar{c}}^{y_{max}} u''(y_1 - R_L l_D^*)dy_1} \tag{B.23}$$

Clearly the denominator of this fraction is negative and therefore in order fro the derivative to be positive the numerator must also be negative. If this is the case then it must follow that:

$$-(R_L l_D^* + \bar{c} - y_{min})u'(\bar{c}) + (y_{max} - y_{min})u'(y_{max} - R_L l_D^*) < \int_{R_L l_D^* + \bar{c}}^{y_{max}} [u'(y_1 - R_L l_D^*)] dy_1 \quad \text{(B.24)}$$

Integrating:

$$u(\bar{c}) + (y_{max} - y_{min})u'(y_{max} - R_L l_D^*) < u(y_{max} - R_L l_D^*) + (R_L l_D^* + \bar{c} - y_{min})u'(\bar{c}) \quad \text{(B.25)}$$

Now noting that $u(\bar{c}) < u(y_{max} - R_L l_D^{max}) \ \forall \ l_D^*$ the inequality must hold for maximum borrowing given that $R_L l_D^* + \bar{c} = y_{max}$ because::

$$(y_{max} - y_{min})u'(y_{max} - R_L l_D^*) < (R_L l_D^* + \bar{c} - y_{min})u'(\bar{c}) \quad \text{(B.26)}$$

However for the minimum borrowing potential default state we have that $R_L l_D^{min} + \bar{c} = y_{min}$ which gives:

$$u(\bar{c}) + (y_{max} - y_{min})u'(y_{max} - R_L l_D^*) < u(y_{max} - R_L l_D^*) \quad \text{(B.27)}$$

This result must then hold by concavity of u.

□

Lemma 4.4.6. *There exists a threshold $y_D \leq y_B$ below which households borrow allowing for the possibility of default and above which they will never default.*

Proof. A consumer will never default if their debts are such that they can repay them even if they receive the minimum income in period 1. That is if $\bar{c} \leq y_1 - R_L l^* \ \forall \ y_1$. This would imply that:

$$l^D \leq \frac{y_{min} - \bar{c}}{R_L} \quad \text{(B.28)}$$

The level of income for which this is the optimal response is then given by:

$$u'(y_D + l^D) = \beta R_L E_0 u'(y_1 - R_L l^D)$$
$$u'(y_D + \frac{y_{min} - \bar{c}}{R_L}) = \beta R_L E_0 u'(y_1 - y_{min} + \bar{c}) \tag{B.29}$$

The value of y_D is therefore uniquely defined. Then noting that $y_{min} \leq \bar{c}$ we have:

$$E_0 u'(y_1 - y_{min} + \bar{c}) \geq E_0 u'(y_1) \tag{B.30}$$

And so:

$$u'(y_D + \frac{y_{min} - \bar{c}}{R_L}) \geq u'(y_B) \tag{B.31}$$

Then by convexity of u':

$$y_D + \frac{y_{min} - \bar{c}}{R_L} \leq y_B \tag{B.32}$$
$$y_D \leq y_B - \frac{y_{min} - \bar{c}}{R_L} \tag{B.33}$$

And again noting $y_{min} \leq \bar{c}$ it follows that $y_D \leq y_B$. Therefore $\forall y_0 \leq y_D$ a consumer may default and $\forall y_D < y_0 \leq y_B$ a consumer will borrow but never default. □

B.2 Banks

Lemma 4.5.1. *For a given interest rate R_L the aggregate demand for borrowing is increasing in y_{max} if incomes are uniformly distributed .*

Proof. Clearly for a given distribution $[y_{min}, y_{max}]$ an increase in y_{max} represents an increase in $E(y_1)$. Therefore by Corollary 4.4.1 we note l_0^* is increasing in y_{max}. Also noting that by Corollary B.1.2 H^- is expanding, in other words the upper bound on the integral is increasing. Therefore the integrand and limits of the integral in equation (4.41) are increasing in y_{max} and so L_0 is increasing in y_{max}.

□

B.3 Two Period Algorithms

B.3.1 Solving the Policy Function

Each of the three first order conditions must be solved for each value of y_0:

$$u'(y_0 + l_{ND}) = \beta R_L \int_{y_{min}}^{y_{max}} u'(y_1 - R_L l_{ND}) d\Psi(y_1) \tag{B.34}$$

$$u'(y_0 + l_D) = \beta R_L \int_{y_{min}}^{y_{max}} u'(\max(\bar{c}, y_1 - R_L l_D)) d\Psi(y_1) \tag{B.35}$$

$$u'(y_0 - d_0) = \beta R_D \int_{y_{min}}^{y_{max}} u'(y_1 + R_D d_0) d\Psi(y_1) \tag{B.36}$$

Then calculating the integrals yields:

$$u'(y_0 + l_{ND}) = \beta R_L \frac{1}{y_{max} - y_{min}} [u(y_{max} - R_L l_{ND}) - u(y_{min} - R_L l_{ND})] \tag{B.37}$$

$$\begin{aligned} u'(y_0 + l_D) &= \beta R_L \frac{1}{y_{max} - y_{min}} [\int_{y_{min}}^{R_L l_D + \bar{c}} u'(\bar{c}) dy_1 + \int_{R_L l_D + \bar{c}}^{y_{max}} u'(y_1 - R_L l_D) dy_1] \\ &= \beta R_L \frac{1}{y_{max} - y_{min}} [u'(\bar{c})(R_L l_D + \bar{c} - y_{min}) + u(y_{max}) - u(\bar{c})] \end{aligned} \tag{B.38}$$

$$u'(y_0 - d_0) = \beta R_D \frac{1}{y_{max} - y_{min}} [u(y_{max} + R_D d_0) - u(y_{min} + R_D d_0)] \tag{B.39}$$

Clearly, in all three cases the solutions are likely to be transcendental, as shown below even in the case of Quadratic Utility the solution to the possible default equation is very complicated and requires a numerical solution. Then define three new functions as the deviation from the solutions of these equations:

$$F_{LND} = u'(y_0 + l_{ND}) - \beta R_L \frac{1}{y_{max} - y_{min}}[u(y_{max} - R_L l_{ND}) - u(y_{min} - R_L l_{ND})] \quad \text{(B.40)}$$

$$F_{LD} = u'(y_0 + l_D) - \beta R_L \frac{1}{y_{max} - y_{min}}[u'(\bar{c})(R_L l_D + \bar{c} - y_{min}) + u(y_{max}) - u(\bar{c})] \quad \text{(B.41)}$$

$$F_D = u'(y_0 - d_0) - \beta R_D \frac{1}{y_{max} - y_{min}}[u(y_{max} + R_D d_0) - u(y_{min} + R_D d_0)] \quad \text{(B.42)}$$

The solutions are then the zeros of the equations B.40 to B.42, applying the Newton-Raphson method the general iteration in all cases is:

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} \quad \text{(B.43)}$$

It is also useful to note that in each case we can specify the range of the decision variable robustly, you cannot borrow more than the implied limit and you cannot save more than your current income. In the implimention fuctions are created which return the decision variable for a given y_0 to allow for exact integration. For the sake of robustness against failure of N-R method a secondary Secant Method is applied if N-R converges to value which is not a root. Thus each value of y_0 gives a rise to a value of l_{ND}, l_D and d_0.

Integrals

For the aggregate values we require the integrals of l_D, l_D l_{ND}^2, l_D^2 and d_0. Within the literature it is common to discretise the space and apply Simpson's Rule to calculate the integral, however, this method is only accurate under very restrictive regularity conditions. The most robust method would be using an Adaptive Simpsons (quadrature) method, in this method the space is discretised as in Simpsons Rule and then each dicretised unit is subdivided repeatedly until an exact area is calculated. In general, the reduction of the size of the discretised units is done linearly and this is potentially very computationally expensive, one can specify a maximum recurrence depth but in general the number of computations required for a desired level of accuracy in unknown. Therefore, we will employ a Double Exponential Method, in principle this method exploits knowledge about the function to significantly increase convergence.

First, the variable to be integrated is transformed such that we have an integral over the entire real line, using:

$$x = \tanh(\frac{1}{2}\pi \sinh t) \tag{B.44}$$

Then the integral is given by:

$$\int_{-1}^{1} f(x)dx \approx \sum_{k=-\infty}^{\infty} w_k f(x_k) \tag{B.45}$$

Where the abscissas and weights are given by:

$$x_k = tanh(\frac{1}{2}\pi \sinh(kh)) \tag{B.46}$$

$$w_k = \frac{\frac{1}{2}h\pi \cosh(kh)}{\cosh^2(\frac{1}{2}\pi sinh(kh))} \tag{B.47}$$

These abscissas functions are not calculated in the process but are rather stored as constants within the code to maximise computational efficiency. So, where as a general quadrature reduces the height of sections, linearly this method reduces as a double exponential function. It is important to note that this method only converges if the function being integrated is analytic. This is true almost surely between the appropriate limits however within the calculations it is important to check for any possible discontinuities.

Multidimensional Newtons Method

The first attempt at a solution to the overall problem will use a Vector Newton method, effectively path of steepest decent from a given starting point. For a given vector valued function $\mathbf{Z} = f(\mathbf{P}) = f(x, y)$ we define the Jacobian as:

$$\mathbf{J}(\mathbf{X}) = \begin{pmatrix} \frac{\partial f_1(x,y)}{\partial x} & \frac{\partial f_1(x,y)}{\partial y} \\ \frac{\partial f_2(x,y)}{\partial x} & \frac{\partial f_2(x,y)}{\partial y} \end{pmatrix} \tag{B.48}$$

The root of the system is then given by iterating:

$$\mathbf{P}_{i+1} = \mathbf{P}_i - (\mathbf{J}(\mathbf{P}_i))^{-1}\mathbf{f}(\mathbf{P}_i) \tag{B.49}$$

As the system is non-linear a smoothing technique is used, such that:

$$\mathbf{P}_{i+1} = \mathbf{P}_i - \frac{0.75}{\sqrt{i+1}}(\mathbf{J_i}(\mathbf{P}_i))^{-1}\mathbf{f}(\mathbf{P}_i) \tag{B.50}$$

$$\mathbf{J}_{i+1} = \mathbf{J}_i + \frac{0.999}{\sqrt{i+1}}\bar{\mathbf{J}}_{i+1} \tag{B.51}$$

Where $\bar{\mathbf{J}}_{i+1}$ is the finite element approximation of the gradient at $\mathbf{P}_i$. This smoothing improves the convergence behaviour of the algorithm at the cost of rate of convergence. However, the more robust behaviour far out ways the computational cost Press et al. (2007, pp. 477).

So to solve the model we are looking for the point $\mathbf{P} = \{R_D, R_L\}$ for which $\mathbf{Z} = \{0, 0\}$ in the code presented the two components of $\mathbf{Z}$ are tabled P and Q.

Shuffled Complex Evolution

Shuffled Complex Evolution is a hybrid genetic algorithm Duan et al. (1993), the method combines the following concepts:

- Probabilistic and deterministic approach
- Clustering
- Systematic Evolution of complex of points spanning the space
- Competitive Evolution

The principle of the algorithm is to begin with a set of points spanning the feasible space, the low points of this set are used to generate clusters of new points until a global minimum is found. The algorithm is outlines below, adapted from Duan et al. (1993):

1. Create Population:Using a uniform distribution and for each point R_D, R_L calculate the value of F for each point.
2. Sort the points in increasing order of F
3. Partition the population into complexes

4. Evolve Complexes

5. Shuffle complexes

6. Check fo Convergence

The Competitive Complex Evolution step involves the random selection of a subset of points, weighted such that lowest F values are more likely, and a series of transformations of these points which attempts to find a new minimum. The transformations attempted are as follows:

1. Reflection across the high point (By a factor of -1)

2. Further Reflection (By a factor of -2)

3. One Dimensional Contraction (By a factor of -0.5)

4. Contraction about the lowest point

The transformations are attempted in order and the best result transferred back to the complex, the complex is evolved several times until being replaced into the population. In this way, the space is explored as fully as possible. A detailed set of code is given in the appendix along with comments explaining each step. Convergence is deemed to have occurred if either of following are met:

1. The difference between the best and worst function is less than a given tolerance

2. The change in points i less than a given tolerance

B.4 Two Period Code

B.4.1 Utility

The Utility base class holds up to two parameters, a and b, and virtual functions for Utility, First and Second Derivatives and the inverse first derivative.

```
class Utility{
public:
  Utility(double A, double B){
    a=A;
    b=B;
  }
  Utility(){  }
  virtual double U(double c){ return 0.0; }
```

```
  virtual double Udiffinv(double c){return 0.0;}
  virtual double Udiff(double c) {return 0.0;}
  virtual double Udiff2(double c){return 0.0;}

  double a;
  double b;
};
```

There are then two versions of the derived class for Log and CRRA:

```
class CRRA: public Utility{
public:
  CRRA(double A, double B):Utility(A,B){
  }
  double U(double c)
  {
    double u;
    if (c <=0)
    { u = -99999999999.999+c*999999;
    }else{
      u=pow(c,(1-a))/(1-a);
    }
    return u;
  }
  double Udiffinv(double c)
  {
    double udiffinv;
    if (c<=0){
      udiffinv = -99999999999.999+c*999999;
    }else{
      udiffinv=pow(c,(1/(-1.0*a)));
    }
    return udiffinv;
  }
  double Udiff(double c)
  {
    double udiff;
    if (c<=0){
      udiff = 99999999999.999-c*999999;
    }else{
      udiff=pow(c,(-1.0*a));
    }
    return udiff;
  }
  double Udiff2(double c)
  {
    double udiff2;
    if (c<=0){
      udiff2 = -99999999999.999+c*999999;
    }else{
      udiff2=-1.0*a*(pow(c,(-1.0*a-1.0)));
    }
    return udiff2;
  }
};
```

```

class LOG: public Utility{
public:
  LOG(double A, double B):Utility(A,B){
  }
  double U(double c)
    {
      double u;
      if (c <=0)
      {
        u = -9999999.99999999;
      }else{
          u=log(c);
      }
      return u;
    }
  double Udiffinv(double c)
  {
    double udiffinv;
    if (c<=0){
      udiffinv = 99999999999.999+c*999999;
    }else{
      udiffinv=1.0/c;
    }
    return udiffinv;
  }
  double Udiff(double c)
  {
    double udiff;
    if (c<=0){
      udiff = 99999999999.999-c*999999;
    }else{
      udiff=1.0/c;
    }
    return udiff;
  }
  double Udiff2(double c)
  {
    return -1.0/(c*c);
  }
};
```

B.4.2 First Order Conditions

First define the base class for the generic first order condition containing appropriate data members:

```

class FOC{
protected:

  //
  //Parameters
```

```
  //

  double y0;
  double RF;
  double RL;
  double RD;
  double ymax;
  double ymin;
  double c;
  double beta;
  Utility &Ut;

  //
  //Transform result to get rid of Infinity and NANs
  //
  double NotNAN(double W)
{
  bool Val=1;
  double adj=0.01;
  double R[2];
  int x=0;
  double RET;
  if (y0<W){
    R[0]=this->value(y0*0.999);
    R[1]=this->value(y0*0.998);
    //cout <<``1:`` << R[0] <<``\t" << R[1] << endl;
    RET= R[0]+((R[1]-R[0])/0.001)*(y0-W);
  }else if(ymax+RF*W<0){
    R[0]=this->value(-0.999*ymin/RF);
    R[1]=this->value(-0.998*ymin/RF);
    //cout <<``2:`` << R[0] <<``\t" << R[1] << endl;
    RET= R[0]-((R[1]-R[0])/0.001)*(y0-W);
  }else{
    R[0]=this->value(-0.999*ymin/RF);
    R[1]=this->value(-0.998*ymin/RF);
    //cout <<``3:`` << R[0] <<``\t" << R[1] << endl;
    RET= R[0]-((R[1]-R[0])/0.001)*(y0-W);
  }
  if (RET!=RET){
    //cout <<``faaaiiilllll" << endl;
    RET = 999999999999.99;
  }else if(RET>DBL_MAX){
    RET = DBL_MAX-1.0;
  }else if(RET<-DBL_MAX){
    RET = -DBL_MAX+1.0;
  }
  //cout <<``RET NAN:``<< RET <<``  at:`` << this->id() << endl;
  return RET;

}
  double NotNANDF(double W)
{
  bool Val=1;
  double adj=0.01;
```

```
  double R[2];
  int x=0;
  double RET;
  if (y0<W){
    R[0]=this->value_df(y0*0.999);
    R[1]=this->value_df(y0*0.998);
    RET = R[0]+((R[1]-R[0])/0.001)*(y0-W);
  }else if(ymax+RF*W<0){
    R[0]=this->value_df(-0.999*ymin/RF);
    R[1]=this->value_df(-0.998*ymin/RF);
    RET = R[0]+((R[1]-R[0])/0.001)*(y0-W);
  }else{
    R[0]=this->value_df(-0.999*ymin/RF);
    R[1]=this->value_df(-0.998*ymin/RF);
    RET = R[0]+((R[1]-R[0])/0.001)*(y0-W);
  }

  if (RET!=RET){
    //cout <<"death" << endl;
    RET = -999999999999.99;
  }else if(RET>DBL_MAX){
    RET = DBL_MAX-1.0;
  }else if(RET<-DBL_MAX){
    RET = -DBL_MAX+1.0;
  }
  return RET;
}
public:
  //
  //Initialise routines
  //
  FOC(Utility &u):Ut(u){}
  FOC(double Y0, double rl, double rd, double Ymax, double Ymin,
      double C, double Beta, Utility &u):Ut(u)
  {
     y0=Y0;
     RL=rl;
     RD=rd;
     ymax=Ymax;
     ymin=Ymin;
     c=C;
     beta=Beta;

  }

  //
  // Virtual Functions to be overridden by specific implementati
     ons
  //
  virtual double dY(){ return (0);  }

  virtual double value(double W){ return (0); }

  virtual double value_df(double W){return (0); }
```

```
  virtual int id(){return (0);  }

  //
  //Operators
  //

  double df(double W){
    double RET=value_df(W);
    if ((RET!=RET)||(RET>DBL_MAX)||(RET<-DBL_MAX)){
      return NotNANDF(W);
    }else{
      return RET;
    }
  }

  virtual double operator()(double W){
    double RET=this->value(W);
    //cout << this->id() <<"\t" << W <<"\t" << RET << endl;
    if ((RET!=RET)||(RET>DBL_MAX)||(RET<-DBL_MAX)){
      return NotNAN(W);
    }else{
      return RET;
    }
  }

  //Ensure Destructor is correctly called
  virtual ~FOC(){}
};
```

Each First Order Condition contains a parameter set and reference to a utility function, the function id is used for robustness testing. The three version of the function then return the evaluation [operator()] and derivative [df] of the function at W for the three equations B.40 to B.42.The operator () actually calls the routine *value* which is overloaded depending on the specific type of FOC in use. If the result of this operator is infinite or undefined the routine calls NotNAN (or NotNANDF) which extrapolates continuously to ensure Newton-Raphson convergence.

The three specific first order conditions are then:

```
class FlD: public FOC
{
public:
  FlD (double Y0, double rl, double rd, double Ymax, double Ymi
      n, double C, double Beta, Utility &u):FOC( Y0,  rl, rd, Yma
      x, Ymin, C, Beta, u){RF=rl;}

  double value(double W){
    return Ut.Udiff(y0-W)-beta*RL*(1/(ymax-ymin))*((Ut.Udiff(c))
        *(-1.0*RL*W+c-ymin)+(Ut.U(ymax+RL*W))-(Ut.U(c)));
  }

  double value_df(double W){
```

```
    return -1.0*(Ut.Udiff2((y0-W))-beta*RL*(1/(ymax-ymin))*(-1.0*RL*(Ut.Udiff(c))+RL*(Ut.Udiff(ymax+RL*W)));
  }

  int id(){
    return -1;
  }
};

class FlND: public FOC
{
public:
  FlND (double Y0, double rl, double rd, double Ymax, double Ymin, double C, double Beta,Utility &u):FOC( Y0,  rl, rd, Ymax, Ymin, C, Beta, u){RF=rl;}

  double value(double W){
    return (Ut.Udiff((y0-W))-beta*RL*(1/(ymax-ymin))*((Ut.U(ymax+RL*W))-(Ut.U(ymin+RL*W)));
  }

  double value_df(double W){
    return -1*(Ut.Udiff2((y0-W))-beta*RL*(1/(ymax-ymin))*RL*((Ut.Udiff(ymax+RL*W))-(Ut.Udiff(ymin+RL*W)));
  }
  int id(){
    return -2;
  }
};

class Fd: public FOC
{
public:
  Fd (double Y0, double rl, double rd, double Ymax, double Ymin, double C, double Beta, Utility &u):FOC( Y0,  rl, rd, Ymax, Ymin, C, Beta, u){RF=rd;}

  double value (double W){
    return (Ut.Udiff((y0-W))-beta*RD*(1/(ymax-ymin))*((Ut.U(ymax+RD*W))-(Ut.U(ymin+RD*W)));
  }

  double value_df(double W){
    return -1*(Ut.Udiff2((y0-W))-beta*RD*(1/(ymax-ymin))*RD*((Ut.Udiff(ymax+RD*W))-(Ut.Udiff(ymin+RD*W)));
  }
  int id(){
    return -3;
  }
};
```

Each FOC overloads the pure virtual functions *value* and $value_d f$.

B.4.3 General W Function

The General W function takes the same parameters as the First Order Condition functions but using the input Y0 generates the optimal response W taking in to account which of the three FOC's applies. Note that when a Wopt object is created using the FOC parameter set it generates the three Y_0 cut off points: Y_B, Y_S, and Y_D0 using the function FindZeros. When the Wopt object is passed the value Y_0 it returns to the solution for the relevant FOC, for each evaluation the function checks for the feasibility and returns an extreme value.

```
class Wopt: public FOC
{
public:
   //
   //Place Holders for Y0 Limits etc
   //
   double YB;
   double YS;
   double YD0;
   double L1;

   //
   //Constructor calculates Y0 Limits
   //
  Wopt (double rl, double rd, double Ymax, double Ymin, double C, double Beta, Utility &u):FOC( 0.0,  rl, rd, Ymax, Ymin, C, Beta, u){
        FindZeros();
      L1=-9999999;
   }

   //
   //Takes an Input Y0 and gives the optimal Output W
   //
    double operator()(double Y0){
     double RES;
     double w=-1;
     double lb, ub;
     double xacc = FUNTOL;
     FOC* f;
     double DY = 1.0/(ymax-ymin);
     int jic=0;

     //decide what type of consumer this is

     if (Y0 < YB){
              lb=-((ymax-c)/RL);  //Max Borrowing equals max consumer could repay
              ub=0.0;          //As Y0 < YB we know W<0;
              if (Y0<YD0){     // Check is consumer may default
        f= new F1D(Y0, RL, RD, ymax, ymin, c, beta,Ut);  //Assign Default FOC
```

```
        jic=1;
      }else{
        f= new FlND(Y0, RL, RD, ymax, ymin, c, beta, Ut);    //Assign NO Default FOC
        jic=2;
      }
    }else if (Y0 > YS){ //Is Consumer is Saver?
            lb=0.0;         //As Y0 ?> YS we know W>0;
            ub=Y0*0.9999999;  //Assign max close to all endowment, not equal as this would cause c=0
      f= new Fd(Y0, RL, RD, ymax, ymin, c, beta, Ut);  //Assign Depositor FOC
      jic=3;
    }else{
           return 0.0;  //Not borrower or saver then do nothing
    }

    double testh, testl;
    double testdf;

    //find decision rule
    if (jic>0){
      w=rtsafe(f,lb,ub,xacc); //Call the robust Newton-Raphson Method
      RES=DY*w;
      //see if we have a corner solution
      if (w==999.99) //If N-R Fails we may have a corner solution
      {
        testh = f->operator()(ub);
        testl = f->operator()(lb);
        testdf= f->df(lb);
        switch (jic)
        {
        case 1:
          if (((testh<0)&&(testdf<0))||((testh>0)&&(testdf>0))){
            RES= DY*lb;
            break;
          }else{
            delete f;
            f= new FlND(Y0, RL, RD, ymax, ymin, c, beta, Ut);
            w=rtsafe(f,lb,ub,xacc);
            RES=w;
            break;
          }

        case 2:
          if (((testl<0)&&(testdf<0))||((testl>0)&&(testdf>0))){
            RES= DY*lb;
            break;
          }else{
            RES= 0.0;
            break;
          }
```

```

        case 3:
          if (((testh>0)&&(testdf<0))||((testh<0)&&(testdf>0))){
            RES= DY*Y0;
            break;
          }else{
            RES= 0.0;
            break;
          }
        default:
          cout <<"somthing very wrong!" << endl;
          RES= 0.0;
          break;
        }
      }
    }else{
      RES= 0.0;
    }
    //Make Sure Res exists
    if (RES!=RES){
      cout <<"faaaiiillllll" << endl;
    }else if(RES>DBL_MAX){
      RES = DBL_MAX*0.9;
    }else if(RES<-DBL_MAX){
      RES = -DBL_MAX*0.9;
    }
    delete f;
    return RES;
  }

   double dY()
   {
     return ymax-ymin;
   }

   double TEST() {return YB;}

  //
  // Calculate Aggregate Integrals
  //

   virtual double D(){
     double d=DEIntegrator<Wopt>::Integrate(*this,YS,ymax,QuadTol);
     return d;
   }

   virtual double L(){
     double l;
     double lb;
     if (L1==-9999999){
      L1= DEIntegrator<Wopt>::Integrate(*this,ymin,YD0,QuadTol);
     }

```

```
      if (YB>YD0){
       lb=DEIntegrator<Wopt>::Integrate(*this,YD0, YB,QuadTol);
       l = L1+lb;
      }else{
        l = L1;
      }
      return l;
    }

    virtual double L1_(){
      if (L1==-9999999){
       L1= DEIntegrator<Wopt>::Integrate(*this,ymin,YD0,QuadTol);
      }
      return L1;
    }

   virtual double L2_(){
     Wopt2 W2(*this);
      double l2=DEIntegrator<Wopt2>::Integrate(W2,ymin, YD0,QuadTol);
      return l2;
    }

private:
   int FindZeros()
   {
     if ((RD<1.0)||(RL<1.0)||(RL!=RL)||(RD!=RD))
       cout << RD <<``\t"␣<<␣RL␣<<␣endl;
␣␣␣␣double␣EU1=(1/(ymax-ymin))*(Ut.U(ymax)-Ut.U(ymin));
␣␣␣␣YB=Ut.Udiffinv(beta*RL*EU1);
␣␣␣␣YS=Ut.Udiffinv(beta*RD*EU1);
␣␣␣␣double␣lD=(ymin-c)/RL;
␣␣␣␣double␣EU2=(1/(ymax-ymin))*(Ut.U(ymax-RL*lD)-Ut.U(ymin-RL*lD));
␣␣␣␣YD0␣=␣Ut.Udiffinv(beta*RL*EU2)-lD;
␣␣␣␣if␣(YD0<ymin){␣YD0=ymin;}
␣␣␣␣if␣(ymax<YB){
␣␣␣␣␣␣YB=ymax;
␣␣␣␣␣␣YS=ymax;
␣␣␣␣␣␣YD0=ymax;
␣␣␣␣}
␣␣␣␣if␣(YD0>YB){
␣␣␣␣␣␣YD0=YB;
␣␣␣␣}
␣␣␣␣return␣0;
␣␣};

};
```

B.4.4 Newtons Method Implementation

A Robust Newtons method is implemented, it checks the root is bounded and then applies the method discussed in the main text. However, as the method does not necessarily converge a backup Secant method is implemented.

```
template <class T>
double rtsafe( T &funcd, const double x1, const double x2, const
    double xacc){
  double xh, xl;
  double fl=funcd->operator()(x1);
  double fh=funcd->operator()(x2);
  double fmid, xmid,fmido, xmido;
  fmido=999999999;
  xmido=999999999;
  if ((fl>0.0 && fh>0.0)||(fl<0.0 && fh<0.0)){
    return 999.99;
  }
  if(fl==0.0) return x1;
  if(fh==0.0) return x2;
  if (fl<0.0){
    xl=x1;
    xh=x2;
  }else{
    xh=x1;
    xl=x2;
  }

  double rts=0.5*(x1+x2);
  double dxold=abs(x2-x1);
  double dx=dxold;
  double f=funcd->operator()(rts);
  double df=funcd->df(rts);

  for (int j=0; j< MAXIT;j++){
    if ((((rts-xh)*df-f)*((rts-xl)*df-f) >0.0)||(abs(2.0*f)>abs(d
        xold*df))){
      dxold=dx;
      dx=0.5*(xh-xl);
      rts=xl+dx;
      if (xl==rts) j=MAXIT;
    }else{
      dxold=dx;
      dx=f/df;
      double temp=rts;
      rts -= dx;
      if (temp==rts) j=MAXIT;
    }

    if (abs(dx)<xacc) j=MAXIT;
    f=funcd->operator()(rts);
    df=funcd->df(rts);
    if(f<0.0){
      xl=rts;
    }else{
      xh=rts;
    }
  }

```

```

  if (abs(f)<=xacc){
    return rts;
  }else{

  //Newton fail attempting secant

  for (int j=0; j< 25;j++){
    xmid=(xh+xl)/2.0;
    fl=funcd->operator()(xl);
    fh=funcd->operator()(xh);
    fmid = funcd->operator()(xmid);
    //cout << fl <<"\t" << fmid <<"\t" << fh << endl;
    if (fh>0){
      if (fmid>0){
        xh=xmid;
      }else if(fmid<0){
        xl=xmid;
      }else{
        return xmid;
      }
    }else if (fh<0){
      if (fmid<0){
        xh=xmid;
      }else if(fmid>0){
        xl=xmid;
      }else{
        return xmid;
      }
    }else{
      return xh;
    }
    if ((abs(fmid-fmido)<xacc*xacc)||(abs(xmid-xmido)<xacc*xacc)){
      if (abs(fmid)<1e-10){
        return xmid;
      }
    }else{
      xmido=xmid;
      fmido=fmid;
    }

  }
  }
  //cout <<"Max Itter reached in rtsafe" << endl;
  return xmid;
}
```

B.4.5 Multidimensional Newtons Method

A hybrid Newtons Method/pseudo-Broydens method is implemented, for a given X0 the Jacobian is calculated and a new point developed. The the estimate of both the point and Jacobian is smoothed with a factor of $\frac{e}{\sqrt{k+1}}$

where k is the iteration number.

```
template <class T>
class Broyden
{
public:
    int iter;
    Broyden(T &FUNK, vector<double> X0, vector<double> LB, vector<double> UB){
        double d;
        int i,k;
        int n = (int) X0.size();
        f0=new vector<double>(n);
        x0=new vector<double>(n);
        *x0=X0;
        Matrix<double> A(n,n);
        Matrix<double> B(n,n);
        vector<double> x1, f1, s, s1, fd, ft;
        double e=0.999;
        vector<double> xd = X0;
        // Allocate temporary memory
        x1.resize(n);
        f1.resize(n);
        s.resize(n);
        int viable;
        double fact;
        const unsigned PSMAX=5;
    // Create identity matrix for startup (consider Jacobian alternative)
        if (n==2){
          A=Jac(FUNK,*x0);
        }else{
          A.MakeI();
        }

    // Main loop
        for (k=0;k< MAX_ITER;k++){
          ft =FUNK->operator[](*x0);
          *f0=ft;
          //cout <<"ft:`` << ft[0] << endl;
          if (A.det()>0){
            B=A.Inverse()*(0.75/sqrt((double)k+1.0));
          }else{
            B.MakeI();
            B=B*(0.75/sqrt((double)k+1.0));
          }
          s1=MultVe((*f0),-1.0);
          s=B*s1;
          d = 0.0;
          for (i=0;i<n;i++) {
            x1[i] = (*x0)[i] - s[i];
            d += s[i]*s[i];
          }
          if (d <= fabs(eps)) break;
          viable =10;
```

```
        while (viable!=0){
          viable=0;
          for (int q=0;q<n;q++){
            fact= ((double)rand())/((double) RAND_MAX);
            if(x1[q]<LB[q]){
              x1[q]=(fact*x1[q]+(1-fact)*UB[q]);
              viable++;
            }else if(x1[q]>UB[q]){
              x1[q]=(fact*x1[q]+(1-fact)*LB[q]);
              viable++;
            }
            if (x1[q]!=x1[q]) x1[q]=(fact*LB[q]+(1-fact)*UB[q]);
          }
        }
        f1=FUNK->operator[](x1);
    // Update A
        B=Jac(FUNK,x1)*(1-(e/sqrt((double)k+1.0)));
        A=A*(e/sqrt((double)k+1.0));
        A=A+B;

        //cout << k << endl;
        *f0 = f1;
        *x0 = x1;
      }
      iter = k;

    }

    vector<double> operator()(){
      return *x0;
    }
    vector<double> RES(){
      return *f0;
    }
  private:
    vector<double>* x0;
    vector<double>* f0;
    vector<double> MultVe(const vector<double> V, double D){
      int n = (int) V.size();
      vector<double> VR(n);
      for (int i=0;i<n;i++){
        VR[i]=D*V[i];
      }
      if (VR!=VR) cout <<"more pain..."␣<<␣endl;
␣␣␣␣return␣VR;
␣␣}

␣␣Matrix<double>␣Jac(T␣&FUNK,␣vector<double>␣X0){
␣␣␣␣int␣n␣=␣(int)␣X0.size();
␣␣␣␣Matrix<double>␣A(n,n);
␣␣␣␣vector<double>␣f0a(2);
␣␣␣␣vector<double>␣fd;
␣␣␣␣double␣e=1e-6;
␣␣␣␣vector<double>␣xd␣=␣X0;
```

```
␣␣␣␣f0a=FUNK->operator[](X0);
␣␣␣␣xd[0]+=e;
␣␣␣␣fd=FUNK->operator[](xd);
␣␣␣␣A.setValue(0,0,(f0a[0]-fd[0])/e);
␣␣␣␣A.setValue(0,1,(f0a[1]-fd[1])/e);
␣␣␣␣xd=X0;
␣␣␣␣xd[1]+=e;
␣␣␣␣fd=FUNK->operator[](xd);
␣␣␣␣A.setValue(1,0,(f0a[0]-fd[0])/e);
␣␣␣␣A.setValue(1,1,(f0a[1]-fd[1])/e);
␣␣␣␣double␣check␣=A.det();
␣␣␣␣if␣((check==0.0)||check!=check){␣A.MakeI();}
␣␣␣␣return␣A;
␣␣}
};
```

B.4.6 SCE Algorithm

Shuffled Complex Evolution is a hybrid genetic algorithm, the code used is presented below.

```
template <class T>
class POPULATION
{
  public:
    POPULATION(T &F, vector<double> X0, vector<double> lb, vector<double> ub, int n):f(F){
      //trasnfer to class vals
      double val;
      vector<vector<vector<double> > > POINTS2;
      vector<int> ordered;
      nPOINTS=n;
      x0=X0;
      LB=lb;
      UB=ub;
      int NDIM=(int) x0.size();

      //Creater Holders for points and fitness
      POINTS.resize(MAX_ITER+1);
      FITNESS.resize(MAX_ITER+1);
      for (int i = 0; i<MAX_ITER+1; i++){
        POINTS[i].resize(nPOINTS);
        FITNESS[i].resize(nPOINTS);
        for (int j = 0; j<nPOINTS; j++){
          POINTS[i][j].resize(NDIM);
        }
      }
          //create initial population
      for(int i=0;i<nPOINTS;i++){
        if (i == 0){
          POINTS[0][i] = x0;
        }else{
          for(int j=0;j<NDIM;j++){
```

```
          val=((double) rand())/((double)RAND_MAX);
          POINTS[0][i][j] = LB[j]+val*(UB[j]-LB[j]);
          }
        }
      }

      // calculate cost for each point in initial population
      FITNESS[0] = makeFIT(0);

      //sort points
      SortPoints(0);

      //Copy 0th to 1st itt
      FITNESS[1]=FITNESS[0];
      POINTS[1]=POINTS[0];

    }

    vector<double>GetPoint(int itt, int pt){ return POINTS[itt][pt];}
    double GetFit(int itt, int pt){ return FITNESS[itt][pt];}

    int NewItteration(int itt){
      POINTS[itt+1]=POINTS[itt];
      FITNESS[itt+1]=FITNESS[itt];
      return 0;
    }

    int ReplacePoint(int itt, int pt, vector<double> point, double fit){
      POINTS[itt][pt]=point;
      FITNESS[itt][pt]=fit;
      return 0;
    }

    int SortPoints(int itt)
    {
      places ordered(FITNESS[itt]);
      vector<vector<vector<double> > > POINTS2;
      vector<vector<double> > FITNESS2;
        //Sort the population into
      ordered=places(FITNESS[itt]);

      POINTS2=POINTS;
      FITNESS2=FITNESS;
      for(int i=0;i<nPOINTS;i++){
        POINTS2[itt][i] =  POINTS[itt][ordered[i]];
        FITNESS2[itt][i] = FITNESS[itt][ordered[i]];
      }
      FITNESS=FITNESS2;
      POINTS=POINTS2;
      return 0;
    }

```

```
    double TolFun(int itt){
      double A;
      double B;
      int z;
      z=(int)FITNESS[itt].size()-1;
      A=FITNESS[itt][0];
      B=FITNESS[itt][z];

      return B-A;
    }

    double Diff(int itt){
      int nPOINTS=(int)POINTS[0].size();
      int NDIM=(int)POINTS[0][0].size();
      vector<double> DIFF(nPOINTS*NDIM);
      double val;
      for(int i=0;i<nPOINTS;i++){
        for(int m=0;m<NDIM;m++){
          val=POINTS[itt-1][i][m]-POINTS[itt][i][m];
          DIFF[m*nPOINTS+i] =val*val;
        }
      }

      sort(DIFF.begin(),DIFF.end());

      return sqrt(DIFF[nPOINTS*NDIM-1]);
    }



  private:
    vector<vector<vector<double> > > POINTS;
    vector<vector<double> > FITNESS;
    T &f;
    vector<double> x0;
    vector<double> LB;
    vector<double> UB;
    int nPOINTS;

    vector<double>  makeFIT(T &F, vector<vector<double> > PTRY,
        vector<double> lb, vector<double> ub)
    {
      int pdim = (int) PTRY.size();
      vector<double> fit(pdim);
      for(int i = 0; i<nPOINTS; i++)
      {
        fit[i] = CALCULATE_COST(F,PTRY[i],lb,ub);
      }

      return fit;
    }

    vector<double>  makeFIT(int itt)
    {
```

```
        int pdim = (int) POINTS[0].size();
        vector<double> fit(pdim);
        for(int i = 0; i<pdim; i++)
        {
          fit[i] = CALCULATE_COST(f,POINTS[itt][i],LB,UB);
        }
        return fit;
      }

      // =========================================
      // COST FUNCTION EVALUATION
      // -----------------------------------------

      double CALCULATE_COST(T &FUN, vector<double> PTRY, vector<double> LB, vector<double> UB)
      {
        double big = 1000000000000;
        double YTRY;
        int NDIM = (int) PTRY.size();
        for (int i = 0; i<NDIM; i++)
        {
          // check lower bounds
          if (PTRY[i] < LB[i])
          {
            YTRY = big+(LB[i]-PTRY[i])*big/100;
            return YTRY;
          }
          // check upper bounds
          if (PTRY[i] > UB[i])
          {
            YTRY = big+(PTRY[i]-UB[i])*big/100;
            return YTRY;
          }
        }
        YTRY=FUN->operator()(PTRY);
        // calculate cost associated with PTRY
        return YTRY;
      }
};

template <class T>
class SCEMain{
public:
  SCEMain(){}

vector<double> operator() (T &f, vector<double> x0, vector<double> LB, vector<double> UB)
{
  if (x0.size() != LB.size()){cout <<‘‘LB and X0 have incompatible dimensions in SCE”␣<<␣endl;}
␣␣if␣(x0.size()␣!=␣UB.size()){cout␣<<‘‘UB␣and␣X0␣have␣incompatible␣dimensions␣in␣SCE” << endl;}

  /* initialize random seed: */
```

```
srand((int) time(NULL) );

// set EXITFLAG to default value
int EXITFLAG = -2;
int NDIM = (int) x0.size();
int nPOINTS_COMPLEX = 2*NDIM+2;
int nPOINTS_SIMPLEX = NDIM+2;
int nPOINTS = nCOMPLEXES * nPOINTS_COMPLEX;

POPULATION<T> POP(f, x0, LB,UB,nPOINTS);

vector<double> X(NDIM);

vector<double> temp(NDIM);
vector<double> temp2(nPOINTS);
vector<double> temp3(nPOINTS_COMPLEX);
vector<int> ordered(nPOINTS);
vector<vector<double> > COMPLEX(nPOINTS_COMPLEX, vector<double>(NDIM));
vector<double> COMPLEX_FITNESS(nPOINTS_COMPLEX);
vector<vector<double> > COMPLEX2(nPOINTS_COMPLEX, vector<double>(NDIM));
vector<double> COMPLEX_FITNESS2(nPOINTS_COMPLEX);
vector<vector<double> > SIMPLEX(nPOINTS_SIMPLEX, vector<double>(NDIM));
vector<double> SIMPLEX_FITNESS(nPOINTS_SIMPLEX);
vector<vector<vector<double> > > RES_REF;

//variable for for itterations
vector<int> k1(nPOINTS_COMPLEX);
vector<int> k2(nPOINTS_COMPLEX);
vector<int> LOCATION(nPOINTS_SIMPLEX);

// initialize counters
int nITERATIONS = 0;
int nFUN_EVALS = 0;

int itt_v=0;

for (int itt=1; itt<MAX_ITER-2; itt++){
  itt_v++;
  // The population matrix POPULATION will now be rearranged into so-called complexes.
  // For each complex ...

  for (int j = 0; j<nCOMPLEXES; j++)
  {
        // construct j-th complex from POPULATION
    for(int k = 0; k<nPOINTS_COMPLEX; k++){
        k1[k]=k;
        k2[k] = k1[k]*nCOMPLEXES + j;
        COMPLEX[k1[k]] = POP.GetPoint(itt,k2[k]);
        COMPLEX_FITNESS[k1[k]] = POP.GetFit(itt,k2[k]);
      }
```

```
    }

      // Each complex evolves a number of steps according to the competitive
      // complex evolution (CCE) algorithm as described in Duan et al. (1992).
      // Therefore, a number of 'parents' are selected from each complex which
      // form a simplex. The selection of the parents is done so that the better
      // points in the complex have a higher probability to be selected as a
      // parent. The paper of Duan et al. (1992) describes how a trapezoidal
      // probability distribution can be used for this purpose. The implementation
      // found on the internet (implemented by Duan himself) however used a
      // somewhat different probability distribution which is used here as well.

      for (int k = 0;k < nITER_INNER_LOOP;k++){
    //cout <<"1..." << endl;
    // select simplex by sampling the complex
    LOCATION=SimpSamp(nPOINTS_COMPLEX,nPOINTS_SIMPLEX);
    //cout <<"2..." << endl;
          // construct the simplex
    for (int m=0;m<nPOINTS_SIMPLEX;m++)
    {
      SIMPLEX[m] = COMPLEX[LOCATION[m]];
      SIMPLEX_FITNESS[m] = COMPLEX_FITNESS[LOCATION[m]];
    }

          // generate new point for simplex
          // first extrapolate by a factor -1 through the face of the simplex
          // across from the high point,i.e.,reflect the simplex from the high point


    RES_REF=EVOLVE(f, SIMPLEX, SIMPLEX_FITNESS, LB, UB);
    SIMPLEX_FITNESS=RES_REF[0][0];
    for (int i=0;i<nPOINTS_SIMPLEX;i++){
      SIMPLEX[i]=RES_REF[1][i];
    }

    // replace the simplex into the complex
          for (int i=1;i<nPOINTS_SIMPLEX;i++){
        COMPLEX[LOCATION[i]] = SIMPLEX[i];
        COMPLEX_FITNESS[LOCATION[i]] = SIMPLEX_FITNESS[i];
    }

    // sort the complex;
    places ordered(COMPLEX_FITNESS);
```

```
          COMPLEX2=COMPLEX;
                COMPLEX_FITNESS2=COMPLEX_FITNESS;
          for (int m=0; m<nPOINTS_COMPLEX;m++){
            COMPLEX[m]=COMPLEX2[ordered[m]];
            COMPLEX_FITNESS[m]=COMPLEX_FITNESS2[ordered[m]];
          }

        }//inner loop

            // replace the complex back into the population

        for (int k=0;k<nPOINTS_COMPLEX;k++){
          POP.ReplacePoint(itt,k2[k],COMPLEX[k1[k]],COMPLEX_FITNESS[k1[k]]);
        }


        // At this point, the population was divided in several complexes, each of which
        // underwent a number of iteration of the simplex (Metropolis) algorithm. Now,
        // the points in the population are sorted, the termination criteria are checked
        // and output is given on the screen if requested.

        // sort the population
        POP.SortPoints(itt);


        // end the optimization if one of the stopping criteria is met
        // 1. difference between best and worst function evaluation in population is smaller than TOLFUN
        // 2. maximum difference between the coordinates of the vertices in simplex is less than TOLX
        // 3. no convergence,but maximum number of iterations has been reached
        // 4. no convergence,but maximum time has been reached

        if (POP.TolFun(itt) < TOLFUN){
          EXITFLAG = 1;
        }else{
          if (POP.Diff(itt)< TOLX){
                    EXITFLAG = 2;
          }else{
            if(itt>MAX_ITER){
              EXITFLAG=0;
            }
          }
        }

        // if a termination criterium was met, the value of EXITFLAG should have changed
        // from its default value of -2 to -1, 0, 1 or 2
```

```
      //cout << itt <<"\t" << POP.GetFit(itt,0) <<"\t" << POP.GetPoint(itt,0)[0]<<"\t" << POP.GetPoint(itt,0)[1] <<"\t" << POP.GetFit(itt,nPOINTS-1) << endl;
      X=POP.GetPoint(itt,0);

      if (EXITFLAG != -2) {
        itt = MAX_ITER;
      }else{
        POP.NewItteration(itt);
      }
    }

    //cout << EXITFLAG << endl;
    X.resize(NDIM+1);
    X[NDIM]=POP.GetFit(itt_v,0);
    return X;
}

private:

// ========================================
// COST FUNCTION EVALUATION
// ----------------------------------------

double CALCULATE_COST(T &FUN, vector<double> PTRY, vector<double> LB, vector<double> UB)
{
    double big = 1000000000000.0;
    double big2 = 1000000.0;
    double YTRY=0.0;
    int NDIM = (int) PTRY.size();
    for (int i = 0; i<NDIM; i++)
    {
      // check lower bounds
      if (PTRY[i] < LB[i])
      {
        YTRY = big+(LB[i]-PTRY[i])*big2;
      }
      // check upper bounds
      if (PTRY[i] > UB[i])
      {
        YTRY = big+(PTRY[i]-UB[i])*big2;
      }
    }
    if (YTRY==0.0){
      YTRY=FUN->operator()(PTRY);
    }
    // calculate cost associated with PTRY

    return YTRY;
}

static vector<int> SimpSamp(int nPOINTS_COMPLEX,int nPOINTS_SIMPLEX)
```

```
{
  vector<int> LOCATION(nPOINTS_SIMPLEX);
  bool ALREADY_PARENT;
  int DUMMY;
  double val;

  LOCATION[0] = 0; // the LOCATION of the selected point in the complex

    for(int l = 1;l<nPOINTS_SIMPLEX;l++){
        ALREADY_PARENT = 0;
        while (ALREADY_PARENT==0)
      {
        val = ( (double)nPOINTS_COMPLEX+0.5)*( (double)nPOINTS_COMPLEX+0.5) - (double)nPOINTS_COMPLEX *( (double) nPOINTS_COMPLEX + 1.0 )*((double) rand()/((double)RAND_MAX));
        val = (double) nPOINTS_COMPLEX + 0.5 - sqrt(val);
            DUMMY = (int)floor(val);
        //val=(1.0/3.0)*((double)nPOINTS_COMPLEX)*(((double) rand()/((double)RAND_MAX))+((double) rand()/((double)RAND_MAX))+((double) rand()/((double)RAND_MAX)));
        //DUMMY=(int)floor(val);
        if (DUMMY<nPOINTS_COMPLEX){
          for (int m=0;m<nPOINTS_SIMPLEX;m++)
          {
            ALREADY_PARENT=1;
            if (LOCATION[m] == DUMMY){
              ALREADY_PARENT = 0;
              m=nPOINTS_SIMPLEX;
            }
          }
        }
      }
            LOCATION[l] = DUMMY;
  }

    sort(LOCATION.begin(),LOCATION.end());
  return LOCATION;
}

vector<vector<vector<double> > > EVOLVE(T &f, vector<vector<double> > SIMPLEX, vector<double> SIMPLEX_FITNESS,vector<double> LB,vector<double> UB)
{

  int PDIM =(int) SIMPLEX.size();
  int nPOINTS_SIMPLEX = PDIM;
  int NDIM =(int) SIMPLEX[0].size();
  //Result
  vector<vector<vector<double> > >RES(2);
  RES[0].resize(1);
  RES[1].resize(PDIM);

```

```
  RES[0][0].resize(PDIM);
  RES[0][0]=SIMPLEX_FITNESS;
  for (int i=0; i<PDIM; i++){
    RES[1][i].resize(NDIM);
    RES[1][i]=SIMPLEX[i];
  }
  vector<int> ordered;
  vector<vector<double> > RES_REF;
  vector<double> SORTED_FITNESS;
  SORTED_FITNESS=SIMPLEX_FITNESS;
  sort(SORTED_FITNESS.begin(),SORTED_FITNESS.end());
  double  SFTRY;
  vector<double> SXTRY(NDIM);
  double  SFTRYEXP;
  vector<double>  SXTRYEXP(NDIM);
  double  SFTRYCONTR;
  vector<double>  SXTRYCONTR(NDIM);
  string ALGOSTEP;

  RES_REF = AMOTRY(f,SIMPLEX,-1.0,LB,UB);

  //transfer result to vectors
  SFTRY=RES_REF[0][0];
  SXTRY=RES_REF[1];
  if (SFTRY <= SORTED_FITNESS[0]){
    // gives a result better than the best point,so try an additional
       // extrapolation by a factor 2
    RES_REF = AMOTRY(f,SIMPLEX,-2.0,LB,UB);

    //transfer result to vectors
    SFTRYEXP=RES_REF[0][0];
    SXTRYEXP=RES_REF[1];
    //check result
    if (SFTRYEXP <= SFTRY){
      RES[1][PDIM-1] = SXTRYEXP;
      RES[0][0][PDIM-1] = SFTRYEXP;
      ALGOSTEP ="reflection and expansion";
    }else{
      RES[1][PDIM-1] = SXTRY;
      RES[0][0][PDIM-1] = SFTRY;
      ALGOSTEP = "reflection";
    }
  }else if(SFTRY>=SORTED_FITNESS[PDIM-1]){
        // the reflected point is worse than the second-highest, so look
        // for an intermediate lower point, i.e., do a one-dimensional
        // contraction
        RES_REF = AMOTRY(f,SIMPLEX,-0.5,LB,UB);

    SFTRYCONTR=RES_REF[0][0];
    SXTRYCONTR=RES_REF[1];
    //check result
```

```
    if (SFTRYCONTR < SORTED_FITNESS[0]){
      RES[1][PDIM-1] = SXTRYCONTR;
      RES[0][0][PDIM-1] = SFTRYCONTR;
      ALGOSTEP =''one dimensional contraction";
    }else{
            // can't seem to get rid of that high point, so better contract
            // around the lowest (best) point
      for (int i=1;i<nPOINTS_SIMPLEX;i++){
        for (int m=0;m<NDIM;m++){
          RES[1][i][m] = 0.5*(SIMPLEX[i][m]+SIMPLEX[0][m]);
        }
        RES[0][0][i] = CALCULATE_COST(f,RES[1][i],LB,UB);
      }
      ALGOSTEP = ''multiple contraction";
    }
  }else{
        // if ytry better than second-highest point, use this point
    for (int i=1;i<nPOINTS_SIMPLEX;i++){
      RES[1][PDIM-1] = SXTRY;
      RES[0][0][PDIM-1] = SFTRY;
    }
    ALGOSTEP =''reflection";
  }
  //cout << ALGOSTEP.c_str() << endl;
  return RES;

}

// ====================================
// AMOTRY FUNCTION
// ------------------

vector<vector<double> > AMOTRY(T &FUN, vector<vector<double> > P,double FAC,vector<double> LB,vector<double> UB)
{
  //Extrapolates by a factor FAC through the face of the simplex across from
  // the high point, tries it, and replaces the high point if the new point is
  // better.

  int NDIM = (int) P[0].size();
  int PDIM = (int) P.size();
  vector<double> PSUM(NDIM);
  vector<double> PTRY(NDIM);

  //Results Vector
  vector<vector<double> > RES(2);
  RES[0].resize(1);
  RES[1].resize(NDIM);

  // calculate coordinates of new vertex
```

```
  for(int i=0; i<NDIM; i++)
  {
    PSUM[i]=0.0;
    for (int j=0; j<NDIM; j++){
      PSUM[i] += P[i][j]/((double) NDIM);
    }

    PTRY[i] = PSUM[i]*(1.0-FAC)+P[PDIM-1][i]*FAC;
  }

  // evaluate the function at the trial point.
  RES[0][0]= CALCULATE_COST(FUN,PTRY,LB,UB);
  RES[1]= PTRY;

  return RES;
}
};
```

Chapter 5

Conclusion

If all economists were laid end to end, they would not reach a conclusion

–Gorge Bernard Shaw quoted in Mankiw (2011)

This research has presented empirical and theoretical arguments that inequality causes increased financial instability.

The empirical aspects of this research, discussed in Chapter 3, applied standard logistic regression methods to ascertain in the probability of a crisis is increased by higher levels of inequality. I find that the relationship is consistent with the hypothesis. However, the coefficient in the regression model is not significant at the 5% level and so would be discarded by standard econometric techniques. I argue that this is not a fair test of the model given the available data. Specifically, the lack of accurate and lengthy time series on inequality makes inference more difficult.

Looking more closely at the interaction between inequality and financial stability before and after the 2007 Crisis. I find that inequality is a good predictor of and positively correlated with the level of large financial risks in which the banking system is engaged. This result is found using a principal component decomposition of financial stability indicators on the IMF's Financial Stability Indicators.

I conclude that these two empirical results combined suggest that even if inequality is not a first order driver of financial crises it is an important factor to consider. This is especially true given current trends of rising intra-country inequality.

A theoretical model was presented, in Chapter 4, in which inequality causes an increased propensity to borrow due to the ability of consumers to default. This excess demand for loans increases the wedge between borrowing and deposit rates that results from defaults on loans. I find that for a calibration to the UK economy in 2010, a borrowing spread of 15% is predicted. Moreover, the model is solved numerically and I demonstrate that if the level of high incomes rise the borrowing premium also rises.

The results of the theoretical work are not true universally, that is for certain levels of income borrowing premium does not rise. However, this may not be as problematic to the hypothesis as it fist appears. If one were to consider Japan in the late 1980s or the USA in the 2005 one could argue that borrowing costs were too low, thus promoting excessive risk taking. In this sense the model provides a rich framework for future analysis. I also did not consider the issue of collateral or securitisation, the ability of banks to reposes mortgaged houses will certainly alter the expected pay off of a distressed loan. This issue could be further investigated using this model if I were to include an asset market in both periods. Finally, the model specifies a capital market imperfection, namely a borrowing premium, it would be possible to alter this premium to be a function of the level of borrowing or the perceived riskiness of the loan. I do not expect any of these additions to significantly alter the core conclusion that inequality does interact with financial stability.

The culmination of this work is to suggest that inequality may pose risks to future financial stability. These results support the hypothesis of Rajan (2011). However, I find that the results are not conclusive in a robust statistical sense, and this is consistent with the work by Atkinson and Morelli (2011b). The lack of firm results is always disappointing but must be seen within the context of available data. Moreover, further work refining measures of inequality may help to better understand the roles inequality plays within the financial system.

Chapter 6

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