Management of Foreign Reserves: An Approach Based on Vine-Copula, Regime-Switching Dependence and Bayesian Opinion Pooling

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Management of Foreign Reserves: An Approach Based on Vine-Copula, Regime-Switching Dependence and Bayesian Opinion Pooling

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ABSTRACT

This research is constructed to address the issue of structure management for colossal foreign exchange reserves holders, such as China and other emerging economies. Contrary to the discussion of optimal quantity on the reserve level, structure management considers the ideal applications of the national wealth, specifically the compositions in the reserves' financial investments. Two perspectives are considered for the safety and liquidity tranche of the foreign reserves, and another one for the return tranche. The thwo perspectives are further developed into three chapters of this thesis and they form a comprehensive set of analyses for the structure management.

First, the optimal currency composition for huge foreign reserves in the safety and liquidity tranche is investigated. The asymmetry fat-tails and complex dependence structure in distributions of currency returns are examined for their vital role in the portfolio risk appraisal. In a D-vine copula approach, it is shown that under the disappointment aversion effect, the central bank in our model can achieve sizeable gains in economic value by switching from the mean-variance to copula modelling. It is also found that this approach will lead to an optimal currency composition that allows China to have more space for international currency diversification, while maintaining the leading position of the US dollar in the currency shares of China’s reserves.

Next, the strategic asset allocation for China’s foreign reserves in the same safety tranche is studied using a risk-based approach. Four aspects of the risk management

are investigated: investment universe, dependence structure, allocation strategies under risk minimization and trade-off between risks and returns. A regime-switching copula model is developed to investigate the dynamic dependence between assets. The optimal allocation is derived following two strategies: risk minimization and trade-off between risk and returns in utility maximization with disappointment avoidance. If the central bank focuses solely on risk minimization, the asymmetries in the asset return dependence encourage the flight to safety. However, if higher risks are allowed in exchange for higher returns, even if the exchange is very conservative, the asymmetries would discourage the flight to safety. Therefore, we suggest that China should mitigate its flight to safety after 2008 and increase holdings of short-term bank deposits, long-term treasury bonds and euro bonds.

Finally, the strategic asset allocation problem for China's Sovereign Wealth Fund, the China Investment Corporation, is examined. This is considered to be the return tranche of China's foreign reserves. Bearing the responsibility to pursue higher returns for China's huge volume of foreign exchange reserves, the China Investment Corporation is endowed with a capable funding position. However, its emphasis on safety is still considered more serious than that of other institutional investors. A new method combining the merits of the shrinkage estimation, vine-copula structure, and Black-Litterman model, is proposed and tested to satisfy the revealed investment objectives. Empirical analysis suggests that there is more emphasis on emerging market economies rather than advanced economies when diversifying in fixed-income securities; whereas that emphasis is reversed on the equities side. In addition, using the commodity ETFs to represent the significance of gold in the portfolio, it is discovered that gold is a formidable competitor to the investment in equities.

Declaration

The content of this doctoral dissertation is based on the research work completed at Durham University Business School, UK. No material contained in the thesis has previously been submitted for a degree in this or any other university.

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CHAPTER ONE

This chapter begins by giving the background of the study. The importance of the topic and the objective of the thesis are explained. Next, motivations for each section of the thesis are broken down into detailed research questions. Key conclusions reached in exploring these questions, and the main contributions of the thesis, are outlined. Finally, the organization and structure of the entire thesis are outlined.

CHAPTER 1

INTRODUCTION

China, the second largest economy, has impressed the world with its own way of economic development: fast, full of character, but also puzzling to many. One of the most outstanding features of the nation's rapid wealth accumulation is the simultaneously rapidly increasing volume of China's foreign exchange reserves. According to data from the State Administration of Foreign Reserves (SAFE), the reserves level grew from 11.093 billion US dollars to 3.497 trillion US dollars over the period from 1990 to June 2013. Likewise, there are many other emerging economies with similarly huge accumulation of foreign reserves. How can this huge volume of wealth be managed? The foreign reserves are considered to function as a national source of security for economic development and financial stability. Can this vital purpose be satisfactorily served? Would the large sum of foreign exchange reserves bring a heavy burden of opportunity costs or huge benefits from investments? Meanwhile, China's mode of economic development and reserves accumulation is viewed by many other developing countries as a significant alternative for improving living standards in comparison with the existing way of western developed countries. The investment recipients, usually the developed countries, also pay great attention to the largest foreign reserves in the world. Therefore, management of this vast amount of national wealth is of great interest not only to China, but also to the world.

The management of the foreign reserves can be categorized into quantity optimisation and structure analysis. This research looks at the management issue from the structure perspective, which means the allocation of the foreign reserves is studied in order to match the management objectives with the market conditions and developments. More specifically, the research views the allocation of the reserves in three tiers. The first is the currency allocation, where the proportion of each foreign currency composing the foreign exchange reserves is analysed. In the second tier, strategic asset allocation decisions are investigated. This is to uncover the composition of the asset classes, e.g. bonds, equities and other securities, for the reserves. The second tier is based on the result of the first. The result of the investigation into currency composition shows that the US dollar denominated assets take the dominant position. It is a natural continuum that in the second tier, compositions of mainly dollar assets are given priority for analysis. In the third tier of this research, strategic asset allocation decisions for the Sovereign Wealth Fund (hereafter SWF), whose investment objective is mainly the demand of higher returns, become the research target, and the SWF of China, the China Investment Corporation (hereafter CIC) is used as an illustration for our method.. The important difference between this tier and the previous two lies in a major shift in terms of investment objectives. The first two layers are usually categorized under the name of ‘safety tranche’ of the investment of foreign reserves, where the safety are of paramount importance. However, for reserve holders with more than adequate quantity levels, like China, the other important aspect in investment is the demand for higher returns. In the literature, this is usually denoted as the management for the ‘return tranche’ of the foreign

exchange reserves. In practice, SWFs are usually set up for the higher return investment objective. By covering these three tiers, the thesis aims at a relatively comprehensive set for the management of China's huge amount of foreign reserves.

The rest of this chapter is organized as follows: First, the research motivations for various problems proposed are explained. Then, the main findings and contributions of this research are summarized. Finally, the organization of the whole thesis is set out.

1.1 Research Motivations and Objectives

The development of foreign exchange reserves has gone through different phases in history, and their management has always been a concern for central banks. Especially since the Asian crisis in the late 1990s, the world has witnessed the rapid build-up of an unprecedented amount of foreign reserves, so that very many countries now dismiss the threat of reserves deficiency, whereas there are acute challenges for managing the reserves structure. Since the global financial crisis in 2008, the international financial landscape has changed significantly. The main reserves assets are rendering near zero interest rates, posing serious challenges for central banks' international investments. Meanwhile, central banks of countries such as China typically sterilize the accumulation of foreign reserves by issuing domestic debt. The ensuing cost of domestic interest rate with the huge amount of foreign reserves compounds the challenge for the reserves management. Dominguez *et al.* (2012) propose the concept of quasi-fiscal costs for holding reserves, which would be incurred if the interest rate on reserve assets is lower

than the domestic interest rate. Walther (2012) estimates that such social cost could be substantial in an environment of low international investment yield with rising levels of reserves.

This research aims to provide a relatively comprehensive solution for the management of the foreign reserves from the structure perspective. The sheer scale of the management and the complexity in the nature of the reserves as the secure storage of national wealth require a systematic methodology and multi-purposed investment strategies. In this research, the concept of 'tranche management' suggested by the International Monetary Fund (IMF hereafter) (IMF, 2001) is therefore utilised and improved, and the issue of structure management is studied in the vertical direction as well as in the horizontal direction.

The management or investment demands of foreign reserves need to be covered in three aspects: liquidity, risk and return. Based on his review of historical literature on reserves management, Roger (1993) emphasizes that the important special function of foreign reserves is to fund the everyday international trading and financing activities. These transactional needs determine the necessity for the liquidity management of the reserves. With respect to the second aspect mentioned above, Beck and Rahhari (2011) give an example for the importance of risk management of the reserves. They propose a theoretical model on the structure management of reserves in the presence of sudden stops, i.e. the unexpected reversal of capital flows, and provide empirical evidence to show the importance of such attention to the sudden stop risks. The other aspect for

investment is of course return. Since the 1990 Asia financial crisis, the emerging economies, especially China, have built up foreign exchange reserves beyond the level that is adequate for trading and financing activities. The quasi-fiscal costs or the opportunity costs of holding reserves suggested by Dominguez *et al.* (2012) are difficult burdens, and place immense pressure on the return aspect of the reserves investment. For all these reasons, the tranche management studies suggest that these multiple investment needs should be catered for by dividing the foreign reserves into two sections, or two tranches. According to basic financial asset pricing theories, the financial assets with high liquidity and low risk would not usually offer high return. The objectives of safety and liquidity can be achieved simultaneously, and therefore the first tranche of foreign reserves wealth should be restricted to these safe and liquid assets. On the other side, the second tranche of foreign reserves wealth focuses on high return and gives rise to the SWFs, which emphasize long-term investments with higher return and accordingly higher risk characteristics.

In order to cover these multiple needs comprehensively, this research deals with the structure management of foreign reserves both in the vertical direction and in the horizontal direction. Chapter 2 and Chapter 3 are devoted to the management issues in the first tranche of foreign reserves. These can be considered as approaches in a vertical direction to the problem of structure management, because focusing on the same demand of safety, they offer solutions at different depths. Chapter 2 is the first step to decide the optimal currency composition. Before any international investments can be more specifically allocated, the currency in which these investments are denominated must be identified. This

consideration is also helpful for answering the intriguing question as to whether the leading position of the US dollar is challenged by other competing international currencies. Based on the analysis in Chapter 2, the subsequent chapter looks further into the structure management problem in the safety tranche by analysing the financial asset class composition mainly denominated in the most popular currency. These two chapters follow a natural order to solve the structure management issue focusing on the need for safety. In considering the parallel reserves tranche, with higher return and risk, Chapter 4 can be viewed as being in the horizontal direction of the general structure management problem. The investment objective for higher return is incompatible with the safety demands. The source of higher returns determined by an efficient financial market should come from the risk or liqudity premium. Thus, in the second tranche of foreign exchange reserves structure management, the part of wealth in the reserves that is considered to be beyond what is adequate for meeting the safety needs is set aside to form the SWFs. This is a common practice in many countries, and in China the SWF is the CIC. The strategic asset allocation decisions by the CIC are investigated in Chapter 4.

In addition to the above general analyses on motivations of the whole thesis, there are more specific research questions for the study of each chapter, as detailed below:

In Chapter 2, the first step in the vertical direction, the investigation focuses on the safety demands.

The first research question in this chapter is what the currency composition of the foreign reserves of China should be like. Since transactional activities are identified as important sources for the need for safety (Dellas and Yoo, 1991; Petursson, 1995 and Papaioannou *et al.*, 2006), the currency composition question should be answered taking the international trading and financing constraints into account. The suggestions on the optimal currency composition based on these factors comprise one of the core motivations of this chapter.

The second research question arises in the process of answering the first. In the turbulent times during the 2008 financial crisis and its aftermath, the central bank was reluctant to be involved in any risk-taking activities, and its emphasis on risk control should be properly reflected in its actions in investments. With careful regard to this subjective preference of the central bank, and considering objectively the unconventional features in financial markets under the crisis, it is important to ask which methodology should be utilised for accurate and responsive management of the foreign reserves.

The third research question concerns the risk features of the financial market. Can the existing currency returns distribution assumption, i.e. the Gaussian distribution assumed by common methods such as the mean-variance analysis, accurately reflect the market risks? The drawbacks of the Gaussian distributed returns and the mean-variance method lie in the discovery of the prevalent non-normal features, such as fat-tails and asymmetries in both univariate returns and the dependence among currency returns (Ang and Chen, 2002; Bae *et al.*, 2003; Hong *et al.*, 2007; Ammann and Suss, 2009). While there are plenty of papers

proving the prevalence of the fat-tails and asymmetries in the equity market, are these features presented in the more actively traded foreign exchange market? If their existence can be confirmed, would they affect the risk appraisal for the currency composition decisions of foreign reserves, and what are the effects?

The next research question is more technical. Copula model is a good candidate method for the second research question raised above, i.e. to reflect the conservative investment preference of the central bank. However, it is important to know whether there are other available methods, which can capture risk well under the same multivariate situation. As it turns out, the group-t distribution is flexible under the multivariate portfolio management problem and the Archimedean copulas are good at describing the asymmetric risks. The question is whether our proposed vine-copula model structure is in any way superior to the above two and whether the advantages can be confirmed in empirical examinations. In terms of methodology, is the proposed model able to advance the existing literature?

The final research question in Chapter 2 asks whether more currencies, especially the currencies of emerging economies, should be included in China's portfolio. Should they play more important roles? Also, should China engage in more diversification away from the US dollar? An important topic in international economics is the debate on the globally dominant status of a currency, and whether there are formidable competitors to the US dollar. China has foreign exchange reserves of sufficient size that its currency composition decisions can

shed light on international currencies' diversifications. This is the last, but not the least, consideration motivating Chapter 2.

The research topic in Chapter 3 is a close sequel to that in Chapter 2. It is the second step in the vertical direction towards the general question of the structure management of China's foreign exchange reserves.

Following the optimal currency composition decisions explored in Chapter 2, this chapter extends into a deeper level and studies the decisions for financial assets allocation with the same emphasis on safety. Such decisions are called the strategic asset allocation. In investment, the strategic asset allocation often refers to the strategy that calls for setting target allocations and periodically rebalancing the allocation back to the targets if, as time goes on, the investment deviates from the original percentages. The key to the strategy is the allocation target. It is interpreted in terms of holding percentages of different asset classes, which are portfolios containing financial assets with similar traits. The asset classes are more general than specific financial securities, so that the strategic asset allocation decisions can accommodate investments with larger volume of wealth, such as in our case of China's foreign reserves.

The most direct research question in this chapter is what the optimal composition of asset classes looks like. Similar to the previous chapter, the composition and the methods used in the optimisation can be helpful for China's and other nations' foreign reserves management.

The second research question is to identify the investment universe. There are many US dollar denominated asset classes with different financial characteristics. Which ones should qualify as the investment candidates for China's foreign reserves? One way of classifying the financial assets is to look at their features in three aspects: return, risk/safety, and liquidity. The strategic asset allocation decisions explored in this chapter bear the responsibility for controlling safety. They belong to the first tranche of foreign reserves management. However, in the literature the criterion of liquidity is perceived as being difficult to quantify compared with the other two criteria of safety and return. Therefore, it is important to check the criterion of liquidity from a qualitative perspective rather than from a quantitative one. A permissive investment universe needs to be established according to these qualitative criteria, before the pursuit of the optimal asset allocation. For the central banks, it is interesting to ask what their investment profiles, i.e. the criteria including the above-mentioned liquidity, are like, and how such profiles can help determine the investment universe.

The third research question of Chapter 3 concerns the recognition of non-Gaussian risks in the market. In order to reflect the conservative attitude of the central bank in managing the safety tranche of the foreign reserves, the market conditions must be accurately captured. Similar to the previous chapter, the non-Gaussian features in the asset classes need to be examined. Their existence must be confirmed and their effects on the risk appraisal and asset allocation decisions should also be investigated. Once again the advantage of the vine-copula models can be utilised in this regard. However, there are also obvious differences in application compared to the previous chapter. This is due to the new feature in this chapter's

question, the debate regarding 'flight to safety', which is another research motive discussed later.

The next research question is about the measurement of risk. Since the liquidity requirement has already been considered in the process of selection for the permissive investment universe, the remaining consideration is the safety criterion, the other element in the name of this tranche of foreign reserves, the safety tranche. Unlike the qualitative criterion of liquidity, there are many quantitative methods to represent risk in financial investment decisions. However, appropriate measurements for risk have long been intensely debated. The most commonly used risk measure is variance, but this is criticized for treating the deviations above and below the expected value equally. Intuitively, it is difficult to rationalize that an investor would have no preferences over a certain amount of volatility no matter whether it meant profits or losses. Instead, he or she should favour extreme high return, but shun the volatility when it leads to losses. The feature whereby an investor would prefer returns rather than losses in risk measurement is called asymmetric preference. Potential solutions have been developed, such as Value-at-Risk (VaR), Conditional Value-at-Risk (CVaR)/Expected Shortfall (ES), and some modifications in the utility functions to appreciate asymmetric preference. It is interesting to ask which risk measure should be chosen and what viewpoint the reserve manager expresses if he or she chooses a particular measure.

The fifth research question in Chapter 3 revolves around the decision of 'flight to safety', i.e., whether the investment should be transferred from higher return

assets to safer ones. Studies have found that such behaviours are popular under stressful financial periods (Caballero and Krishnamurthy, 2008; Beber *et al.*, 2009). The IMF has issued a report (IMF, 2012) stressing the pressure exerted by the 'flight to safety' movement around the globe and highlighting the challenge of a global safety assets shortage after the financial crisis. Therefore, it is interesting, and bears worldwide significance, to ask whether such behaviours have been seen in China's safety tranche of foreign reserves investments. What should China's strategic asset allocation strategy be under this situation? During the current recovering period after the climax of the 2008 global financial crisis, should the flight to safety strategy continue or is it time to diversify away from safety into risky but more highly rewarded sections? The optimal asset allocation result would be helpful in this regard. However, bearing these questions in mind, more technical questions regarding a proper risk appraisal model should be raised, since there are multiple safety assets to be highlighted. This is also the reason why the vine-copula method in Chapter 2 can no longer suffice.

A new method of regime-switching vine-copulas emphasizing multiple safety assets is proposed for the flight to safety decision. The multiple regimes mean there are multiple vine-copulas in response to the multiple safety assets. Compared to other methods, the Gaussian distribution based models cannot reflect the fat-tails and asymmetries in the financial market, as mentioned in the previous chapter. The vine-copula model proposed in the previous chapter, within which various types of bivariate Archimedean copulas connect the multiple variables, successfully eliminates the drawbacks of the group-t copula model and

the expanded Archimedean copulas in asymmetry modelling and multivariate flexibility. However, it fails to reflect the importance of the multiple safety assets.

These vine-structured copulas have a drawback in design, which becomes apparent in the decision of 'flight to safety'. The connecting structure of vine-copulas does not treat every component equally. It must organize all the variables into different tiers in order to weave them together using conditional bivariate probability distributions. In tiers other than the first, the variables need to go through transformations of conditional probability functions. Therefore, only the variables in the first tier retain their original form and their empirical features can be captured most accurately.

In the strategic asset allocation problem of Chapter 3, the investigation against the liquidity criterion left two equally important safety assets in the investment universe. For the flight to safety decision, the investment wants to come back from other risky assets towards either or both of the two safety assets. The question of how each of these safety assets relates to the other risky assets in the investment universe is critical. The vine-copula utilised in the previous chapter can only feature the relationship of one safety asset with other risky assets in its first tier. Two safety assets would need two vine-copulas simultaneously. How to solve this problem is not only an intrinsic part of answering our research question regarding the decision of 'flight to safety', but also provides illumination for future research in dealing with the dependence structure emphasizing multiple variables.

In Chapter 4, the strategic asset allocation decision for the SWF of China is investigated. In contrast to the previous two chapters, both of which focus on the

safety needs of investment and can be viewed as in the vertical direction of the structure management, Chapter 4 explores the thesis topic in the horizontal direction, the pursuit of higher returns. This pursuit is to a large extent incompatible with the goals of controlling safety and liquidity. Determined by the basic financial market efficiency, any arbitrage opportunity, such as existence of assets with high return which are at the same time safe and liquid, should be exploited and thus eliminated. Therefore, the strategic asset allocation decisions emphasizing higher returns in this chapter are in parallel with the topics in the previous chapters. There must be two different tranches of foreign reserves, focusing on returns and safety separately. Together, the researches across both tranches form a relatively comprehensive analysis of the structure management problem.

Moving from the liquidity tranche to the return tranche, SWFs are set up by many countries around the globe with sufficient foreign exchange reserves. China has several SWFs, but technically only the CIC is considered as being for the pure pursuit of investment returns. In this chapter, similar to the previous two chapters, the most direct research question is what the optimal asset allocation for the return tranche of investment should be like. In answering this general question, the following specific questions provide motivations for the study: What is the CIC's internal identity, or which category does it belong to among the various SWFs around the world? What are the external investment situations like? What are the investment objectives of the CIC under such internal and external circumstances and by what method can these be fulfilled?

The CIC's internal identity is intriguing, and its profile largely determines the investment mode and objectives. SWFs are not rare, and have existed for more than a century. Today, the scope of SWFs is clearly defined by the IMF, and they are usually categorized into commodity and non-commodity funds (Kunzel, *et al.*, 2011; Mihai, 2013). The investment management philosophies between the two types are largely different, because of the difference in their funding sources and future usages. Lyons (2007) and Santiso (2008) point out another way to categorize SWFs, according to whether the investments are for strategic reasons. Here, strategic reasons are objectives other than pure pursuit of financial benefits. These might include the ability to control the firms from a foreign country, with the intention of giving competitive advantage to a domestic competitor. Many countries are averse to such strategic SWFs. The characteristics analysis on the CIC is of interest to provide knowledge of its investment objectives.

Regarding the internal identity of the CIC, its history since its establishment in 2007 is another important research question in this chapter. This can help in understanding the CIC's investment objectives and obligations. The management structure and the question of who controls the SWF gives insights on the fund's objectives and efficiency. Although the CIC is young and does not have a lengthy transaction record from which to derive its investment profile, the historical attitude of the governing body and the arrangement of the managing team can offer some clues. Moreover, the CIC was established just before the dawn of the financial crisis. Its performance and strategy history are important lessons for its growth, and great indicators for its future investment strategies.

In the investigation of the internal identity of the CIC, one of the most important aspects is its funding position. In investment textbooks, for individual and institutional investors the funding position largely determines the investing ability and the risk preference, which are intrinsic components forming the overall investment objective. For example, whether the CIC is funded mostly by liabilities, which require regular interest payments, or by equity shares, where long-term benefits are taken into consideration, makes a huge difference to its investment horizon, i.e. the holding terms of the assets. Through these research questions on the internal identity, a clearer picture of the CIC's investment preference and ability can hence be expected.

In addition to the internal identity, it is also important to understand the external investment environment faced by the CIC. In particular, it is interesting to explore the openness of the fund. This is critical, because the international financial investment recipients, usually the developed countries, tend to be very cautious with regard to SWF investment for strategic purposes, e.g. taking control of some industries or affecting their operational decisions in order to give the SWF's origin country an advantage in competition. The openness of the managing team, i.e. the percentage and the level of international expertise in the team, often indicates the absence of such strategic drives. Selfish conduct in the nation's interests is more likely on the part of that nation's citizens.

Another external aspect is the influence of the financial crisis on the attitudes of the developed countries towards the outside SWF investments. Both this and the previously mentioned openness of the management are important questions

regarding the external conditions for the CIC to pursue a return emphasized, risk balanced investment objective.

In answering the above-mentioned research questions on both the internal and external aspects, it seems that a suitable investment strategy for the CIC should still be multi-purposed, with the need for higher returns highlighted, but also with caution regarding risks. In contrast to the management of reserves in the liquidity tranche, the CIC focuses more on returns rather than on the safety requirements. However, because the foreign reserves are nonetheless a significant part of national wealth, the management should still direct great attention to the risk management. This brings a new requirement for a strategic asset allocation method that can fulfil such investment objectives. The next research question in Chapter 4 regards methodology, and whether a new way can be proposed in order to combine the abilities in financial performance, risk management and allocation efficacy. Good financial performance means higher return. Accurate risk appraisal is for safety control, and allocation efficacy stands for stable and diversified allocations.

The frequently applied method of mean-variance analysis, as proposed by Markowitz (1952), formed the foundation for modern portfolio theory and has been a cornerstone of many other financial theories. However, this method is not suitable in our case. Due to its simplicity in incorporating covariance into the optimisation of portfolios, the intuitive idea that diversification can reduce risks became concrete in practice. However, what is not so intuitive is that slight changes in the input parameters of the model can often lead to unexpected swings

in the final optimal weight result. Sometimes a rise in the expected return of an asset causes a drop in its holding percentage. Such counter-intuitive consequence is due to the complex covariance matrix, which gives rise to the model's success, especially under multiple assets situations, where portfolio optimisation and diversification are most needed. As such, this classic model has long been criticized for lack of stability.

As pointed out earlier, the mean-variance method, which builds upon Gaussian distributed asset returns, also fails to capture the extreme risks involved in distributional features such as fat-tails and asymmetries. Therefore, with regard to cautious risk appraisal, this method also fails to satisfy the CIC's investment objectives. In addition, since pursuing higher return is the number one priority for China's SWF, the CIC, the more advanced ability to forecast return is of importance.

Overall, our pursuit of a new method that can perform in the above three aspects of allocation stability, risk appraisal and financial performance is well motivated. In order to move towards the solution of this problem, a more specific question is to identify the best existing methods in these three aspects. Black-Litterman has strength in offering stable and diversified allocations by incorporating the market equilibrium. The copula model used in the previous two chapters can offer good appreciation of the fat-tails and asymmetries in return risks. The shrinkage method to reduce estimation error can enhance the overall financial performance of an investment strategy. However, there is no method that can do all three at the same time. The prospect of finding a way to combine the merits of these three is

an attractive one, and such a method would be perfect for the CIC's investment objectives. The final question of this chapter is whether it is possible to combine the three best methods and whether an overall improvement can be achieved. Robustness tests are needed in order to provide a reliable answer to these questions.

1.2 Contributions and Main Findings

The structure management of foreign exchange reserves is comprehensively studied in this thesis, and China is used as an example to demonstrate the empirical results. The overall contribution of this research can be summarized as the decomposition of the structure management problem into three concrete models under the framework of tranches management for foreign reserves. The framework suggests that the adequate level of foreign reserves should be divided into a safety tranche, where the liquidity and risk requirement should be emphasized, and a return tranche, where higher risks are allowed and correspondingly higher returns can be pursued. More specific contributions can be found in the following paragraphs. They correspond to the research questions identified earlier.

In Chapter 2, the first finding is the optimal currency composition for China's foreign reserves in the safety tranche. The result provides insight on the optimal currency structure from multiple perspectives. The safety tranche of the foreign reserves is mainly proposed for the purpose of keeping the everyday functions of foreign reserves, to maintain the trading and financing activities of China with foreign countries. Therefore, the relative importance of each currency with respect

to its role in the international trading and financing activities is taken into consideration. In addition to the perspective of everyday functions, other scenarios with Gaussian and non-Gaussian assumptions on currency returns are explored and discussed. The overall results confirm the importance of the US dollar in the optimal currency structure. It takes the largest share among the 12 potential currency candidates.

The second empirical contribution of Chapter 2 is the expansion of the number of potential currencies included in the investment of China's huge volume of foreign reserves. Not only are the major international 'hard currencies' covered, such as the US dollar, euro, Japanese yen, UK pound sterling and Swiss franc, but the currencies in the emerging economies are also accounted for. In addition, currencies of China's surrounding countries are considered as promising candidates. The larger number of currencies also allows for greater room for portfolio diversification.

The third contribution lies in the enlightenment provided by the optimal composition results on the question of whether the status of the US dollar is challenged. In the rivalry to be the dominant international currency, other currencies such as the euro and the yen cannot pose a serious threat to the US dollar, from the perspective of China's composition. The optimal currency results with binding international trading or financing restrictions still pronounce the US dollar as the winner. However, if risks are watched more closely, by using the proposed copula method, there is potential for more diversification from the US

dollar to emerging currencies. This change is suggested by a comparison between a copula dependence model and a Gaussian dependence model.

Also with regard to empirical results, the fourth contribution in this chapter lies in the confirmation of non-Gaussian distributional features, such as fat-tails and dependence asymmetries, in currency returns. Their existence is discovered by the vine-copula model and the influence in optimal currency compositions is shown by comparison studies. In this safety tranche of foreign reserves management it is required that the risk appraisal must be accurate. The effect of incorporation of such features in risk appraisal is important for the currency composition decision.

The fifth contribution in Chapter 2 is in terms of methodology. The proposed method of vine-copula combined with ARMA-GARCH (Autoregressive Moving Average Autocorrelation - General Autoregressive Conditional Heteroskedasticity) model can accurately reflect the time-dynamics as well as the non-Gaussian features in a multivariate situation in order to achieve a well-diversified portfolio. Compared to the existing models in the literature, the proposed model possesses unparalleled advantages in terms of flexibility in multivariate scenarios and capacity to describe dependence asymmetries. Specifically, the existing methods refer to the group t model suggested in Demarta and McNeil (2005) and the multivariate Archimedean copulas expanded from bivariate copulas described in Nelson (2006). The difference lies mainly in the dependence structures. The merit of the group t model is that it can allow for more dependence parameters when the number of variables connected by the dependence is increased. In a multivariate situation, this is important because the higher complexity in dependence induced

by the inclusion of more input variables can be captured. However, only Student t distribution is allowed in the group t model, and the t distribution cannot reflect dependence asymmetry. Conversely, the expanded Archimedean copula is capable of modelling the asymmetries. However, the number of dependence parameters cannot increase accordingly as the complexity rises. These existing models cannot achieve flexibility and asymmetry capability at the same time. The vine-copula model proposed in this chapter uses bivariate Archimedean copulas as elements and the vine-structured conditional copulas to weave the elements together for the multivariate scenario. Therefore, it possesses the ability of Archimedean copulas in capturing asymmetries. The number of bivariate copulas can increase together with the input variables, and thus overcome the flexibility issue.

In Chapter 3, the first contribution is the optimal strategic asset allocation. It can be offered as policy guidance for the management of foreign reserves in the safety tranche. Following the currency allocation decision from the previous chapter, US dollar denominated investments are assigned with the largest share. Furthermore, the findings from Chapter 3 on the optimal asset class composition assist the structure management of foreign reserves. As in Chapter 2, different perspectives are pursued, which lead to different optimal allocations.

In the pursuit of the optimal strategic asset allocation structure, the second contribution in this chapter lies in the first step, the discovery of the investment universe according to the safety and liquidity demands. The asset classes are divided into the short-term and long-term horizons, where in the short-term section there are two asset classes with qualified liquidity requirement: bank deposits and

treasury bills; and in the long-term section there are five: long-term treasury bonds, US government agency debts, corporation bonds, equities and European government bonds.

The third contribution is the finding of the different risk measures corresponding to the different investment attitudes of reserves managers. Conditional Value-at-Risk and Disappointment Avoidance are applied to measure the risk level and allocation performance, reflecting respectively a risk-only preference and a risk emphasized but return balanced viewpoint. The optimal solutions are obtained conditional on which perspective is taken. It is likely in the current financial environment that either perspective can be taken by the central bank of China. Therefore, both risk measurements are attempted for a conservative foreign reserves manager.

The fourth contribution concerns the ‘flight to safety’ decision under and shortly after the 2008 global financial crisis. The findings answer whether the safety focused investment of foreign reserves should transfer from the risky section of securities to the safety section. This chapter offers insights on this by considering the US treasury bonds in short and long horizons as the safe assets. The delicate balance between safety and profitability under dynamic market conditions due to the financial crisis is analysed. The result recommends the flight to safety under the circumstance of the climax of financial crisis. However, an important policy suggestion is that in the current period shortly after the height of the crisis, although the signs of recovery are still feeble, diversification away from the safety assets should begin. The reversing trend from 'flight to safety' to 'cautious move to

risky' is achieved from the Disappointment Avoidance (risk-return balanced) perspective, and is encouraged even more if the asymmetries in asset returns are incorporated.

The fifth contribution is in terms of methodology. A regime-switching vine-copula method is proposed for more accurately measuring the risks. As discussed previously in the section on research questions, despite the advantages of the vine-copula model in terms of multivariate flexibility and the capability for capturing asymmetries, a serious drawback made apparent in the 'flight to safety' problem of Chapter 3 is that it cannot emphasize multiple variables at the same time. There are two safety assets in Chapter 3. Therefore, two vine-structures are proposed and they are governed by a Markov chain. The two pivot assets in the two regimes are, respectively, the two safety assets, short-term and long-term treasury bonds. This multiple-regimes methodology can also be extended to any other vine-copula situation where multiple variables need to be highlighted at the same time.

Attending only to the safety needs for managing China's foreign exchange reserves is not sufficient, due to the high opportunity costs for carrying such a huge volume of wealth. Chapter 4 gives suggestions on the strategic asset allocation decisions for China's SWF, the CIC. These suggestions, combined with the topics in the previous two chapters on the structure management for the safety tranche of the foreign reserves, comprise a relatively comprehensive policy suggestion set. Through analyses on the SWF's identity, i.e. funding position and performance history, and both internal and external investment environments, the chapter sheds light on the investment objectives of the CIC. A strategy of

diversified asset allocation with long investment horizon and the priority of higher returns rather than safety should be pursued. Therefore, this chapter includes various investment candidates across different types of assets and geographical locations.

In order to solve the above investment problem, a new method is developed combining three strands of well-established researches in asset allocation studies. Since there are plenty of financial institutions with similar investment objectives, i.e. high return requirement with serious emphasis on risk appraisal, the proposed method can be widely applicable with small modifications. The three strands of research are the Black-Litterman model for incorporating market equilibrium, Jorion's (1991) shrinkage estimation for reducing estimation error, and the vine-copula method for capturing risks in asymmetric returns. In this chapter, it is proven using robustness tests that the proposed method combining the three components can integrate their merits and improve on allocation efficacy, financial performance and risk appraisal ability, respectively.

In summary, the first contribution in Chapter 4 is the policy suggestion on the strategic asset allocation decisions for China's foreign reserves management in the high return tranche. The allocation is well diversified in 15 asset classes covering a wide range of financial assets and commodity representatives. These candidates for investment also include both developed and emerging countries across various geographic locations. The second contribution comprises the analyses on the internal identity and external environment of the CIC, China's SWF responsible for managing the return tranche of foreign reserves. Deriving from the CIC's

categorical identity, its history and funding position, as well as the openness of the management team and the attitudes of the investment receiving countries, these analyses provide the basis for the CIC's investment objectives and obligations. The third contribution is in terms of methodology. A new technique, which combines three famous methods, each with its own specialities in asset allocation studies, is proposed and proven to be effective. The method should be ideal for SWFs like the CIC with higher return as the main pursuit, while also emphasizing risks.

1.3 Organization of the Thesis

Chapter 2: Currency Composition

Chapter 2 investigates the optimal currency composition for China's foreign reserves. The asymmetry fat-tails and complex dependence structure in distributions of currency returns are examined. A skewed, fat-tailed, and pair-copula construction is then built to capture features of higher moments. In a D-vine copula approach, it is shown that under the disappointment aversion effect, the central bank in our model can achieve sizeable gains in economic value by switching from the mean-variance to copula modelling. It is found that this approach will lead to an optimal currency composition that allows China to have more space for international currency diversification while maintaining the leading position of the US dollar in the currency shares of China's reserves.

Chapter 3: Strategic Asset Allocation for Foreign Reserves

In a risk-based approach, Chapter 3 studies the strategic asset allocation for the safety tranche of China's foreign reserves. Four aspects of the risk management

are investigated: investment universe; dependence structure; allocation strategies under risk minimization or under the trade-off between risk and return; and central banks' flight to safety. A regime-switching copula model is developed to investigate the dynamic dependence between the assets. The model contains two regimes, and the interchange between them is governed by a Markov chain. Each safety asset forms the core variable of one of the two vine-copulas, and identifies the copula regime. As such, this design has the advantage of highlighting the relationship between the two safety assets and other asset classes. The optimal allocation is derived by conducting two strategies, i.e. risk minimization and trade-off between risk and returns in utility maximization with Disappointment Avoidance. If the central bank is focused solely on risk minimization, the asymmetries in dependence encourage the flight to safety. However, if higher risks are allowed in exchange for higher returns, even if the exchange is very conservative, the asymmetries would discourage the flight to safety.

Chapter 4: Strategic Asset Allocation for Sovereign Wealth Funds

Chapter 4 examines the strategic asset allocation problem for China's Sovereign Wealth Fund, the CIC. Through investigation of the CIC's investment identity and performance history, its investment objectives are revealed. Bearing the responsibility to pursue higher returns for China's huge volume of foreign exchange reserves, the CIC is endowed with a capable funding position and only long-term performance assessment requirements. However, its emphasis on safety is still considered more serious than that of other institutional investors. A new method combining the merits of the shrinkage estimation (Jorion, 1985, 1986 and

1991), vine-copula structure (Aas and Berg, 2009), and the Black-Litterman model (Black and Litterman, 1991 and 1992), is proposed to satisfy the revealed investment objectives. Robustness tests for the method's advantages in terms of financial performance, risk appraisal and allocation efficacy show positive feedback on its overall effectiveness. Empirical analysis suggests that there is more emphasis on emerging market economies rather than advanced economies when diversifying in fixed-income securities; whereas that emphasis is reversed on the equities side. In addition, using the commodity ETFs to represent the significance of gold in the portfolio, it is discovered that gold is a formidable competitor to the investment in equities.

Chapter 5: Conclusions

The overall conclusion is given with respect to the previous chapters. The limitations of the research are outlined and future improvements are proposed. In the management of the safety tranche of foreign reserves, there should be further investigation of more specific considerations on the reasons for the safety. Two directions are proposed, namely asset-liability management and incorporation of transaction costs. In the management of the return tranche of the foreign reserves, further contributions can be made in the aspects of both data and methodology. Better asset indices can better reflect the investment preference. Also, wider applications of the proposed method in other areas of portfolio management can be achieved under the condition of more robustness tests in other asset allocation markets and situations.

CHAPTER TWO

In this chapter, the optimal currency composition for China's foreign reserves is investigated. First, literature on the topic is reviewed highlighting the importance of risk evaluation in the currency management. Then, the asymmetry, fat-tails and complex dependence structure in distributions of currency returns are examined. Next, in a D-vine copula approach, it is shown that the central bank in our model can achieve sizeable gains in economic value from switching from the mean-variance to copula modelling, and finally this approach leads to an optimal currency composition that allows China to have more space for international currency diversification while maintaining the leading position of the US dollar in the currency shares of China's reserves.

CHAPTER 2

OPTIMAL CURRENCY COMPOSITION OF CHINA'S FOREIGN RESERVES IN SAFETY TRANCHE

2.1 Introduction

Management of foreign reserves has been a constant concern for central banks (Nugee, 2000). In the wake of the rapid accumulation of reserves that has taken place since the start of the Asian crisis in the late 1990s, the challenge has become even more acute. According to International Monetary Fund, the amount of global foreign reserves grew from around 2 trillion US dollars in 1999 to more than 10 trillion dollars by the end of 2012, while during the same period, international monetary relations underwent fundamental changes. In a time of the global financial crisis, interest rates of main reserve assets are approaching zero, resulting in a low yield environment for central banks' investment of their foreign reserves. On the domestic front, central banks typically sterilize the accumulation of foreign reserves by issuing domestic debt. The difference between the returns on investment of external assets and the cost of issuing domestic debt represents the social cost of holding reserves, which increases with interest spreads and the size of reserve holdings. If the interest rate on reserve assets is lower than the domestic interest rate, holding reserves incurs quasi-fiscal costs (Dominguez *et al.*, 2012). In an environment of low international yield and with rising levels of reserves, this social cost could be substantial (Walther, 2012).

To compound the situation, the value of the dollar fluctuated widely during the period, with a largely downward trend, so eroding the purchasing power of nations' reserve stocks. The euro, once a promising contender to the dollar (Chinn and Frankel, 2006, 2008), had to fight for its survival in the shadow of the eurozone crisis. The crisis also plunged the world economy into its worst recession since the Great Depression. In the circumstances, sound and prudent management of foreign reserves has become all the more critical, especially for large reserve holders such as China (Ryan, 2009).

Reserve management involves determination of two essential aspects, i.e. the desired amount and the form of reserve assets a country should hold (Roger, 1993). For larger reserve holders, recent research indicates that the appropriate reserve composition is more critical than the reserve level (Beck and Weber, 2011). Following this insight, the current study concentrates on how to derive the optimal currency composition for China while taking the reserve level as exogenously given. As the world's largest reserve holder, China reportedly holds as much as 70% of its total reserves in US dollars. This exposes China to great currency risk. Consequently, it is desirable and necessary for China to hedge against the currency exposure by diversifying the currencies denominating the reserve assets.

Existing literature of reserve management offers two conventional approaches to analysing currency composition, i.e. the mean-variance approach and the transactions approach (Roger, 1993). In the mean-variance approach, the central bank is treated as an investor who is concerned only about the risk and returns on investment of reserves, and the returns are measured in terms of a basket of

currencies or commodities. The analyst has to find the currency share (weight) that can maximize the value of the investment portfolio for any given level of risk. The transaction approach argues that the central bank should seek to optimise the currency composition of the net foreign assets rather than of gross foreign reserves, which can be achieved by manipulating the structure of gross assets, gross liabilities or both (Dooley, 1986). While this means that the currency composition can be optimised on the side of either assets or liabilities, Dooley suggests that more considerations should be given to transaction cost on the assets side and to mean-variance on the liabilities side. In a subsequent empirical investigation, Dooley *et al.* (1989) identify some key determinants of the transaction considerations, such as a currency's usage in international trade and financial transactions, the exchange rate regime, and country size.

While it certainly makes sense to optimise reserves on the assets side while taking into account the known foreign exchange liabilities, as suggested by the transactions approach, it is difficult for academic researchers to have access to detailed data on central banks' foreign assets and liabilities, which makes meaningful research in this approach virtually impossible. In contrast, the mean-variance analysis can be conducted using data in the public domain and computationally it is rather tractable. This may partly explain the ready application of the mean-variance approach to analysing optimal currency composition of reserves (Ben-Bassat, 1980; Rikkonen, 1989; Dellas and Yoo, 1991; Murray *et al.*, 1991; Petursson, 1995; Levy and Levy, 1998; Papaioannou *et al.*, 2006).

However, the mean-variance approach has its weaknesses as a tool for analysing wealth diversification. The essence of the approach assumes that investors maximize the expected returns for a given level of risk. But for asset returns, they are usually fat-tailed and, for variance as the measure of risk in the mean-variance, it implies the world is Gaussian (Bouye, *et al.*, 2000). Furthermore, it is well known that financial risks are often correlated in a non-Gaussian way (Clemen and Reilly, 1999; Embrechts *et al.*, 1999; Ane and Kharoubi, 2003).

Recent research has highlighted in particular the inadequacy of this approach to take account of influences of asymmetries in individual distributions and in dependence, occurrence of extreme events and the complexity in the dependence structure of asset returns as documented in papers such as Ait-Sahalia and Brandt (2001), Longin and Solnik (2001), Ang and Chen (2002), Bae *et al.* (2003), Hong *et al.* (2007) and Ammann and Suss (2009). These effects can fundamentally affect portfolio performance and the corresponding investment decision. Campbell *et al.* (2001) show that the portfolio efficient frontier is altered by the non-normal marginal distribution.

It turns out that the fundamental difficulties with the mean-variance approach, i.e. the Gaussian assumption and the joint distribution modelling, can be treated as a copula problem. A copula is a function that links univariate marginals to their multivariate distribution. Since the seminal work of Embrechts *et al.* (1999), copulas have found increasing applications in financial research. In the field of portfolio management, copulas have also been applied to modelling multivariate distributions in problems of portfolio optimisation (Hennessy and Lapan, 2002; Thorp and

Milunovich, 2005; Natale, 2008; Christoffersen and Langlois, 2011; Garcia and Tsafack, 2011).

Patton (2006) applies the copula function to highlight construction of foreign currency portfolios. Hurd *et al.* (2007) provide a copula-based study of the bilateral exchange rate between the euro-sterling and the dollar-sterling exchange rates. Dias and Embrechts (2010) model exchange rate dependence dynamics at different time horizons in a time-varying setting. Wang *et al.* (2010) estimate risk of foreign exchange portfolio using models including the copula framework. Kitamura (2011) applies the copula approach to investigate the impact of order flow on foreign exchange market.

Despite the fact that the copula literature is large and growing, the great part of the research involves only bivariate modelling and construction of higher dimensional copulas is rather limited (Genest *et al.*, 2009). To extend bivariate copulas to higher dimensions, Joe (1997), Bedford and Cooke (2001, 2002), and Kurowicka and Cooke (2006) have proposed the pair-copula decomposition approach. Aas *et al.* (2009) illustrate how multivariate data with complex patterns of dependence in the tails can be modelled using a cascade of pair-copulas acting on two variables at a time and show that the pair-copula approach is a flexible and intuitive way of extending bivariate copulas to higher dimensions.

This study contributes to the reserve management literature by applying the copular approach that models asymmetric, fat-tail, and multiple dependence to the currency composition of foreign reserves in the context of China. The pair-copula construction method is applied for modelling the dependence structure among

international currencies. Specializing in modelling multivariate cases, the pair-copulas are based on a decomposition of higher-dimensional copula into bivariate ones, of which some are conditional and unconditional functions of modelled variables (Aas and Berg, 2011).

In conventional extension of a bivariate Archimedean copula to a multivariate case, the dependence parameters will not increase with the number of variables, hence one would end up with an over-simplified dependence structure. As suggested in Demarta and McNeil (2005) the group t copula does not suffer from this inability to increase parameters, however, it lacks the ability of an Archimedean copula to model asymmetric dependence. This is particularly problematic for currency returns since their modelling requires flexibility in both the high dimensional situations and in complex dependence features such as asymmetries. The pair copula construction method overcomes this problem by composing multiple variables through layers of bivariate copulas, each with its own different dependence parameters. As such, the pair copula construction represents an efficient technique that allows the construction of flexible and accessible multivariate copula extensions for optimal portfolio formation and quantitative risk management.

Based on their importance in China's trade and financial transactions, twelve currencies are chosen in this research as the possible candidates for the optimal currency composition of China's foreign exchange reserves. With this selection, we form the optimal portfolio based on the pair copula construction, the performance of which is then compared with the outcome obtained under a Gaussian copula approach. Using the performance measure of economic value of switching to the

vine copula, the pair copula method shows clear advantages. The dominance of the copula method is also manifested under *ad hoc* weight constraints to reflect some common transaction motives, i.e. the international trade needs and foreign financing needs. Taking into account asymmetry, fat-tail and complex dependence, the pair copula approach suggests that China should hold a smaller proportion of US dollars than conventionally thought, around 40% of the total reserves for 2001-2009, the period under examination. The remainder of the chapter is arranged as follows. Section 1.2 summarises related literature. Section 1.3 discusses the methodology of how to build asymmetry marginals and the fat-tailed dependence structure. In addition, we specify a utility function that incorporates disappointment aversion as in Gul (1991), Ang *et al.* (2005) and Hong *et al.* (2007), which enables the portfolio optimisation on non-Gaussian distribution. Data analysis and model results are presented in section 1.4, and we conclude in section 1.5.

2.2 Related Literature

2.2.1 Currency composition of foreign reserves

The problem of foreign reserves management can be viewed as the optimal quantity problem and the optimal structure problem (Roger, 1993). After the World War II, in the context of the 'lack of US dollar', the former problem is the research focus. After the Bretton Woods system broken down, multiple currencies gradually challenge the absolute dominance of US dollar, and the international capital flows increase by multi-folds. As a result, the quantity of the foreign reserves of many countries has improved after the 1970s. Noticeably, the Asian financial crisis in the 1990s teaches emerging economies to accumulate large amount of foreign reserves. China, in

particular, has become the No.1 reserves holder in the world. Under such background, the emphasis on optimal quantity abdicates to the optimal currency structure problem (Bird and Rajan, 2003; Borio, *et al.*, 2008).

The focus of the present chapter is on this problem of optimal currency composition for China's foreign reserves. There are two schools of studies with reference to this question. In the first category, regression analyses are utilised to look for factors influencing the currency composition. The other direction of research takes individual countries as representative investors in the foreign exchange market, and attempts to figure out the ideal currency composition for the central bank to hold. Our present research belongs to this second category, which is to find the optimal currency structure for the foreign reserves of China. In this part of background introduction, the literature concerning currency composition in both categories is reviewed. By reviewing the first category, we intend to find out what are the commonly agreed factors affecting the composition. For the second category, we reveal how these factors identified in the first category can be integrated in the optimisation.

The earliest study in the first category, Heller and Knight (1978), argues that the two major determinants of the currency composition of international reserves are the exchange rate arrangement of a country and the cost-benefit characteristics for holding reserves. The seemingly abstract categorization inspires later research in identifying more specific factors. Dooley (1986) then argues that a country’s foreign currency composition is determined by its transaction and precautious needs. This is yet another rough idea, before Dooley, *et al.* (1989) further develop a regression

model, and point out specifically three factors, i.e. volume of foreign trade, foreign debt and the country's currency exchange regime. The model uses the International Monetary Fund's (IMF) exclusive data and due to its success in explaining the currency structure, Eichengreen and Mathieson (2000) update it with new dataset from the IMF. They establish the common recognition that the currency composition is largely determined by the reserves usage, i.e. in facilitating international trading and financing activities. Chin and Frannkel (2008) later use panel data analysis to find out that GDP, inflation, appreciation or devaluation of the currency, extent of fluctuation of the exchange rate, and the change volume of currency to be the important determinants. However, three-factor model by Dooley, *et al.* (1989) is more classic and more influential. In the current chapter to optimise the foreign reserves with the emphasis on the safety demands, the importance of each currency in China's international trading and financing activities should be taken into consideration, since they are identified by Dooley, *et al.* as the main reasons for liquidity and safety.

Dooley, *et al.* (1989) propose that the currency composition of international reserves should be affected by the ratio of transaction in a given currency to total transactions, the arrangement of exchange rate of a country's currency and the scale of debt denominated in a particular country relative to total foreign debts. To be more specific, here quotes the econometrical formulation of the model:

$$\frac{A_{i,k,t}}{\overline{A}_{i,t}} = \beta_0 + \sum_{\substack{v=1 \\ v \neq i}}^{5} \beta_{1,v} (TR_{i,v,t} / TT_{i,t}) + \sum_{\substack{v=1 \\ v \neq i}}^{5} \beta_{2,v} (D_{i,v,t} / TT_{i,t}) + \sum_{s=1}^{5} \beta_{3,s} E_{i,s,t} + \mu_{i,t},$$

(2.1)

where,

$A_{i,k,t}$ = reserves of country i held as assets denominated in the currency of reserve country k at time t

$D_{i,v,t}$ = debt service payments of country i denominated in the currency of reserve country k at time t

$E_{i,s,t}$ = exchange rate arrangement of type s adopted by country i at time t

$\bar{A}_{i,t}$ = total end-of-period foreign exchange reserves for country i at time t

$TT_{i,t}$ = sum of exports, imports, and debt-servicing payments

$TR_{i,v,t}$ = trade flows at time t

The ratios of $TR_{i,v,t}/TT_{i,t}$ and $D_{i,v,t}/TT_{i,t}$, i.e. the factors in the regression, are used to obtain the *ad hoc* international trading and financing constraints when making the optimisations. Similar applications can be seen in Papaioannou, *et al.* (2006).

The objective of this chapter is more consistent with the literature in the second category, which is to find the optimal currency structure of foreign reserves. The current in this regard is reveal in the chronicle review of these studies.

Ben-Bassat (1980) pioneers in optimal currency structure of foreign reserves at the beginning of a new international monetary system. The managed floating exchange rates system gradually replaces the Bretton Woods system. There are previous studies on the choice of international portfolio. However, Ben-Bassat is the first to

find an optimal solution for a nation with multiple potential currencies in the portfolio. Officer and Willett (1969), Hagemann (1969), Steckler and Pickarz (1970) and Makin (1971) study the currency composition of foreign reserves for a country, but only restrict themselves in the Bretton Woods system mindset, i.e. a two-option choice between gold and dollar.

The simplicity of Mean-Variance analysis and its suitability for multiple currencies management gains popularity among various central banks in their management of foreign reserves. Many central banks conduct their own researches in this manner, such as the Finland central bank (Rikkonen, 1989), the Korea central bank (Dellas and Yoo, 1991), the Canada central bank (Murray, *et al.*, 1991), the Iceland central bank (Petursson, 1995), and the Israel central bank (Levy and Levy, 1998). Australia and New Zealand central banks report the use of the Mean-Variance method in their official website. In addition, Dellas and Yoo (1991) use real data of Korea central bank and report resemblance between the actual currency composition and their Mean-Variance result. This is an important evidence for the popularity of the Mean-Variance method in central banks, because the data on central banks' currency composition is usually exclusive to insiders.

With regard to how the Mean-Variance method is applied specifically, the paper by Papaioannou, *et al.* (2006) is reviewed for demonstration, because this paper synthesizes most features in the previous works and provides simple and elegant treatment on both the 'mean' and the 'variance' side. They propose four assumptions on currency returns, i.e. the mean side, and utilise the Dynamic Conditional Correlation - General Autoregressive Conditional Heteroskedasticity (DCC-GARCH)

model to capture the dynamics in the currency risks, i.e. the variance side. The optimisation risk constraint is gained by a Value-at-Risk analysis.

The methodology in this paper can be decomposed into two parts, the currency return assumption, and the variance-covariance matrix estimation. With respect to the currency return assumptions, there are differences between the foreign exchange market and the equity or bond markets. In the foreign exchange market, various theories dictate the relationship between currencies. However, in reality none of them is absolutely obliged. Therefore, Papaioannou, *et al.* summarize these currency return assumptions appeared in previous papers (Rikkonen, 1989; Dellas and Yoo, 1991; and Petursson, 1995). These include random walk, perfect foresight and the uncovered interest parity assumptions for the expect returns. In addition, they propose the fourth assumption to incorporate the transaction costs of currencies by imposing the bid-ask spread on the uncovered interest parity. The rationale is that if the uncovered interest parity is assumed to be true, each currency return should be determined by its liquidity premium, which is denoted by the transaction cost. With respect to the second part of variance-covariance estimation, the variance-covariance matrix is estimated by the Constant Conditional Correlation - General Autoregressive Conditional Heteroskedasticity (CCC-GARCH) model and the DCC-GARCH model, so that the dynamics in currency risks can be captured.

The portfolio optimisation that follows is the classic Mean-Variance method problem based on the forecasted “mean” part and “variance” part from the above specifications. The application of the Mean-Variance method in this paper is more advanced than any paper before. The assumption on currency returns and the DCC-

GARCH model represents apparent innovation in capturing the market characteristics, such as risk and return, which are important for making investment decisions. However, the Mean-Variance analysis has its innate deficiencies. These are discussed later and this chapter of the thesis intends to make improvements upon them.

2.2.2 Deficiencies of the Mean-Variance analysis

The key techniques of portfolio management start with Markowitz' Mean-Variance paradigm and the further development can be roughly grouped into two sections. The basic idea of the portfolio optimisation is that the expected utility of certain combinations of asset returns needs to be maximized. In mathematical terms, the asset returns are assumed with some stochastic distributions, and the utility is obtained by building a utility function based on the random returns. Therefore, the first direction of the development of the Mean-Variance method concerns its simple assumption of the distributions of asset returns and forms of utility functions. The Markowitz method belongs in this category and uses only the first two moments of the utility function. It is only exact, not an approximation, under the assumption Gaussian distributed returns or when the utility function is quadratic. The second direction is about inter-temporal optimisation of the final utility in a multi-period or continuous setting with intermediate portfolio rebalancing (Merton, 1969, 1971; Samuelson, 1969; and Fama, 1970). These developments reflect the drawbacks of the mean-variance analysis. Our focus is on the first direction of the single-period problem. The Mean-Variance method is built on that the asset returns follow

Gaussian distribution. There is abundant discovery of the deviation from Gaussian distribution of the series in various financial markets.

The Mean-Variance analysis cannot deal with the asymmetry and the fat-tail features discovered in financial data (Kraus and Litzenberger, 1976; Lim, 1989; Harvey and Siddique, 1999 and 2000; Ait-Sahalia and Brandt, 2001; Longin and Solnik, 2001; Ang and Chen, 2002; Bae *et al.*, 2003; Hong and *et al.*, 2007; Ammann and Suss, 2009).

The copula, originally a method in mathematics developed by Sklar (1959) (in Nelson, 2006), is thus introduced to the finance studies. Copula theory starts with the description of bivariate dependence relationship. When the number of variables increases, the concept of multivariate copulas can be obtained by simply extending from their bivariate origin. However, such expansion would lose much flexibility in capturing asymmetries in multiple pairs of variables, because the number of parameters for dependence cannot increase with the number of relationships. Therefore, more advanced copula construction methods, such as Nested-Archimedean Copula and Pair-Copula Construction methods, are developed. Compared to the expansion from a bivariate copula to a multivariate one, these construction methods attempt to connect multiple bivaraite copulas together to reflect the multivariate situation. Therefore the number of dependence parameters increase with the number of bivariate copulas, and as a result the flexibility problem in multivariate situation can be solved. In implementation the key lies in the connection method for the multiple bivaraite copulas. The decomposition of a multivariate probability distribution function into multiple conditional bivariate probability

distribution function is utilised for this purpose. More detailed introduction is given in the later part of this literature review, and an appropriate construction method is chosen to replace the Mean-Variance method.

Another limitation of the single period Mean-Variance method is its inability to model the time-varying dynamics in financial time-series data. The time-varying dynamics is an important feature in portfolio management theories, and it has motivated many famous models to be developed, such as the Autoregressive Moving Average autocorrelation (ARMA) model and various types of the GARCH model. There are time-dynamic models in the multivariate case, such as the CCC- and DCC-GARCH model, but built on the multivariate Gaussian distribution. Clearly the strength in modelling such time-varying features lies in the univariate situation. This is another advantage of the copula method. It treats a multivariate distribution separately in terms of multiple univariate variables and their dependence structure. In addition to the ability to capturing the asymmetries in dependence, the copula model makes it easy to exploit our strength in univariate time-varying models.

More details of these merits of copula are introduced in the next section. Its general merits for gaining wide applications in many areas of finance are review, and also its advantages in portfolio management and currency composition area are discussed.

2.2.3 Merits of the Copula

What is a copula function, and how can it be estimated and evaluated? Why copula is advantageous in describing the dependence between variables and why the application of copula can improve the portfolio management? These questions are

answered in this part. Copula gets wide applications in risk management, asset pricing, risk measurement, and portfolio management because in general it provides better representation of the dependence supported by the financial data than the existing Pearson's correlation in terms of features like asymmetry, kurtosis, and tail dependence. With special reference to portfolio management, the capture of these characteristics, which the Gaussian distribution prevents, does make a difference regarding the portfolio choices. These merits of copula in general terms and in special, will be reviewed in this part, but let us first look at some essential abstracts of the definition, estimation and evaluation of the copula model.

2.2.3.1 Definition, estimation and evaluation of copula

A bivariate-copula function is a "2-increasing" function, a bivariate analog of a non-decreasing one variable function, which represents a bivariate joint distribution's dependence structure (Nelson, 2006). The idea behind the inception of copula is that it is a transformation of any continuous joint distribution function into a standardised joint distribution function, and reversely, the joint distribution can be composed back by its copula and its marginal distributions. In order to appreciate the idea, the distinction between a joint distribution and its marginals needs to be elaborated. A bivariate joint distribution function tells us the probability of random events defined by two variables. Its marginal distribution functions describe random events defined by only one of the variables and the marginals (aka the marginal distribution functions) can be derived from the joint distribution. Therefore, copula is a standardised joint distribution, which means its marginals are uniformly distributed on [0,1], that can be transformed from and to any joint distribution with the help of the chosen joint distribution's marginals (Embrechts, 2009).

Let $H(x,y)$ be a joint distribution with marginals $X\sim F(x), Y\sim G(y)$. Use the "probability integral transforms" denoted by $U_1 = F(X), U_2 = F(Y)$. It means that through the transformation by imposing the random variables' own marginal distribution functions on themselves, the new variable would follow a uniform distribution on [0,1]. We have the following:

$$\begin{aligned} H(x,y) &= P(X \le x, Y \le y) \\ &= P(F(X) \le F(x), G(Y) \le G(y)) \\ &= P(U_1 \le F(x), U_2 \le G(y)) \\ &= C(u_1, u_2) \end{aligned} \tag{2.2}$$

According to the Sklar's theorem, if the margin distribution functions and the joint distribution functions are continuous, the copula C will be unique. However, the uniqueness of copula cannot be assured under discrete distribution functions. Genest and Neslehova (2007) provide a good introduction for problem in the discrete situation.

The bounds of the dependence can be easily denoted if copula is used to represent the dependence among random variables. The Frechet-Hoeffding bounds: for any copula C and for any u_1, u_2 in [0,1] there is

$$W(u_1,u_2) = \max(u_1 + u_2 - 1, 0) \le C(u_1,u_2) \le \min(u_1,u_2) = M(u_1,u_2) \tag{2.3}$$

When the copula is $M(u_1, u_2)$, $P(u_1 = u_2) = 1$ and X and Y are positively related; when the copula is $W(u_1, u_2)$, $P(u_1 = 1 - u_2) = 1$ and X and Y are negatively related. For more detailed introduction to copula, Nelson (2006), Joe (1997) and Cherubini, *et al.* (2004) can be referred to. Patton (2012) reviews the application of copulas in financial time series modelling.

After the brief introduction of the concept of copula, the next question is the estimation of copula parameters and the evaluation of the estimated model. The estimation methods, or the data-fitting methods, include full parametric methods, non-parametric methods, semi-parametric methods and Bayesian methods. For the full parametric estimation, Joe (1997, Chapter 10) introduces a two-stage maximum likelihood method: if the parameters of margins can be separated from the copula, they are first estimated by univariate maximum likelihood, and then the inference of copula parameters is based on the estimated marginal parameters. This method is called the "inference functions for margins" (IFM) in the literature. With respect to the non-parametric estimation method, one can refer to Genest and Rivest (1993) and Caperaa, Fougeres and Genest. (1997). The most popular estimation, however, is the ranking-based estimation, and it is started by Genest, *et al.* (1995). This method allows for the univariate parameters to be estimated non-parametrically or semi-parametrically, whereas the copula parameters are estimated parametrically based on the rankings of the samples. Kim, *et al.* (2007) demonstrate the superiority of this estimation method over the full parametric estimation by a simulation method. The ranking-based method is widely applied in researches such as Shih and Louis (1995) and Chen and Fan (2006). The latter develops it further for time series analysis. With respect to the evaluation of the goodness-of-fit of the estimated copula models

Genest and Favre (2007) and Genest, *et al.* (2009) provide good reference on the topic.

The review on these main directions of copula shows the literature on its application in finance is rich and advancing quickly. It gives good foundation for studying the currency composition structure of China's foreign reserves in this chapter, and topics in the following chapters in this thesis, which are closely related to excellent risk appraisal abilities for the conservative central bank. The wide application of copula is due to its advantages generally in the area of finance, and particularly in the area of portfolio management.

2.2.3.2 The merits of copula in finance: the general case

Copula theory has gained explosive development in recent years (Genest, *et al.*, 2009). The concept of correlation is used in various aspects in actuarial science, finance and statistics, whereas its underlying assumption of multivariate Gaussian distribution is found to be violated in many areas. For example, literature that shows evidence of asymmetries in asset returns includes: Kraus and Litzenberger (1976), Lim (1989), Harvey and Siddique (1999, 2000) and Ait-Sahalia and Brandt (2001). Literature that shows asymmetries in the dependence of asset returns includes: Longin and Solnik (2001), Ang and Chen (2002), Bae *et al.* (2003) and Ammann and Suss (2009). As an alternative to the Pearson's correlation for modelling the dependence, the copula has the potential to replace it in wide areas of finance. With respect to the drawbacks of the Gaussian distribution assumption, not only does the copula allow for modelling the marginals separately in order to capture the asymmetries, but also can it capture anomalies in dependence, which can help

explain co-movement asymmetries, for example, why many stocks are falling together more often in a crash than rising together in a boom.

Embrechts, *et al.* (2002) discuss the advantage the copula by pointing out the deficiencies of the commonly used Pearson's correlation coefficient. The correlation is often called linear correlation. The definition is as following:

$$\rho(X,Y) = \frac{Cov[X,Y]}{\sqrt{\sigma^2[X]\sigma^2[Y]}} \tag{2.4}$$

The Pearson's correlation coefficient is called "linear" because only when in perfect linear dependence, $P(Y = aX + b) = 1$ for $a \in R\backslash\{0\}, b \in R$, the correlation $\rho(X,Y) = \pm 1$.

The drawbacks of the linear correlation include: first, the variance of each variable must be finite in order for the correlation to exist; second, zero correlation does not imply the independence of two random variables; and most importantly, linear correlation is not invariant under non-linear strictly increasing transformations. As a consequence, three fallacies are often made when in need for dependence description by using the correlation: First, it is thought that marginal distribution functions and correlation can determine the joint distribution function. Second, it is thought that all value of correlation in $[-1,1]$ are attainable, no matter what forms of the marginals. Third, for a linear portfolio, the worst VaR case coincides with the largest correlation case. These three seemingly true arguments cannot hold in general, especially for non-elliptical distributions where the necessity of copula is highlighted.

The authors then present the desired properties that a proper measure of dependence should have. These properties are built using copula:

"P1. $\delta(X,Y) = \delta(Y,X)$ (Symmetry).

P2. $-1 \leq \delta(X,Y) \leq 1$ (Normalisation).

P3. $\delta(X,Y) = 1 \leftrightarrow X,Y$ comonotonic; $\delta(X,Y) = -1 \leftrightarrow X,Y$ countermonotonic.

P4. For $T: R \to R$ strictly monotonic on the range of X : $\delta(T(X),Y) = \begin{cases} \delta(X,Y) \ T\ increasing \\ -\delta(X,Y)\ T\ decreasing \end{cases}$.

P5. $\delta(X,Y) = 0 \leftrightarrow X,Y$ are independent"

The definition of comonotonic and countermonotonic are based on the copula theory. (X,Y) are comonotonic or coutermonotonic if their copula is the upper Frechet-Hoeffding bounds or the lower Frechet-Hoeffding bounds.

It has been proven that no dependence measure can simultaneously fulfil P4 and P5. Therefore ranking-based correlations which are established upon copula, like Spearman's rho and Kentall's tau, are ideal dependence measures for being able to possess P1 to P4, the most possible properties for a measure. Linear correlation is not a perfect choice since it only satisfies P1 and P2, two out of the four desired properties.

In addition, the ranking-based correlation is just one example of the dependence measure extracted from the copula functions. Copula functions have the whole

information about the dependence. Other measures can also be derived depending on the analysts' interests, for example, tail dependence which focuses on dependence of extreme events.

As summarized by McNeil, *et al.* (2005, Chapter 5), the advantages of copula, because of which researches across many areas in finance have vigorously applied it, contain two aspects. The first is copula, as described above, provides deeper understanding of the dependency. It helps us to avoid pitfalls of the correlation and generalizes a foundation of the dependency, upon which satisfactory dependence measures can be established according to analysing purposes. The second merit of copula is the "bottom-up approach to multivariate model building". In finance it is often the case that the univariate behaviour is better understood than the dependence between series. The separation of margins and dependence of a joint distribution is an endowment by copula functions to enable us taking advantage of what we can do better. Nearly all the literature of copula's application in finance use copulas for reasons in these two aspects.

2.2.3.3 The merits of copula in portfolio choice

The application of copulas in finance lies in mainly four regions, i.e. risk management, risk measurement, derivatives pricing and portfolio management (Genest, *et al.*, 2009). Besides the general considerations reviewed above, in each particular area there are evidence of the effectiveness of copula application as well. Our interest lies in the area of portfolio management. One major feature copula offers is more realistic distribution functions of the asset returns, to incorporate characteristics like the skewness and kurtosis etc., both in marginals and in

dependence. This is what the ordinary Mean-Variance analysis cannot provide. Therefore, firstly, we would like to uncover the impact of this advantage of copula over the Gaussian distribution of Mean-Variance analysis on the choice of optimal portfolio. The studies in this regard try to prove that the incorporation of skewness in the distributions of asset returns does improve the portfolio selection.

Secondly, the comparison between copula method and other is shown. The problem of skewness and kurtosis incorporation in portfolio optimisation can also be solved by methods other than the copula. Literature like Kraus and Litzenberger (1976), de Athayde and Flores (2004), Jondeau and Rockinger (2006a) and Harvey, *et al.* (2010) attempt to deal with this sort of problem through optimisation based on higher moments of the portfolio distribution. It can be seen that the copula method is more flexible at the slight expense of losing some level of tractability. Such flexibility refers to both the modelling of parameter dynamics and the separation of margins and dependency.

At last, some out-of-sample tests of copula in portfolio management are reviewed. With the merits of copula demonstrated in theory, it is important to see whether the flexibility offered by copula is really effective in empirical studies.

Differences between copula and Mean-Variance method

As one of the main differences between the copula and the Gaussian distribution assumption lies in the capability to capture the asymmetry in return series, it is important to see whether such difference would lead to different portfolio choices. Patton (2002) demonstrates that the skewness in dependence among assets returns

affect the skewness of the portfolio. Other studies reveal that investors have different preferences over portfolios with different level of skewness. Arrow (1971) builds a model for this idea by setting up utility functions preferring positively skewed portfolios rather than the negatively skewed ones. Empirical evidence of such preference are found in Arditti (1967), Kraus and Litzenberger (1976), Simkowitz and Beedles (1978), Scott and Horvath (1980), Levy and Sarnat (1984), Sortino and Price (1994), Sortino and Forsey (1996), Harvey and Siddique (2000), and Dittmar (2002). Therefore, since the skewness or asymmetries in asset returns do matter to portfolio choices, the negligence of such features should not be encouraged, and the Gaussian distribution in the Mean-Variance analysis should not be suggested. More details in these studies are shown below, in order to demonstrate the importance of copula for being able to capture these abnormal features in currency returns.

In order to show the linkage between asymmetric currency returns and the asymmetric portfolio, the concept of asymmetry needs to be defined. A concept called radical symmetry from Nelson (2006) is borrowed to facilitate the establishment of the asymmetry measure.

"Definition (Bivariate radical symmetry) $(X, Y) \sim H = C(F, G)$ are said to be radical symmetric about (μ_x, μ_y) if the joint distribution of $(X - \mu_x, Y - \mu_y)$ is the same as $(\mu_x - X, \mu_y - Y)$."

It has been proved that the necessary and sufficient condition for a joint distribution being radical symmetry is that if the margins X and Y are *individually* symmetric to μ_x and μ_y, the copula of the joint distribution must be radical symmetric. The radical

symmetric distribution implies the concept of conditional symmetry, which is used later to build the measure of asymmetry. The conditional symmetry is as follows:

"Let $(X,Y)\sim H=C(F,G)$ be radial symmetric about (μ_x,μ_y). Then there are the conditional functions, $E\left[X\middle|Y=y+\mu_y\right]-\mu_x=\mu_x-E[X|Y=\mu_y-y]$, and $E[Y|X=x+\mu_x]-\mu_y=\mu_y-E[Y|X=\mu_x-x]$"

If the above the identity does not hold, it means that asymmetry exists in the joint distribution. Nelson (2006) controls the margins to be symmetric and violates at least one of the above two equalities, so that it is ensured that the asymmetry happens in dependence. Under this condition, it is proven that the linearly composed portfolio would be skewed because of the skewness in dependency, in his Proposition III.5 cited in below:

"Let $(X,Y)\sim H=C(F,G)$, and let X and Y be symmetric about μ_x and μ_y respectively. If $\mu_x-E\left[X\middle|Y=\mu_y-y\right]\geq E\left[X\middle|Y=\mu_y+y\right]-\mu_x$ for all $y\geq 0$, and $\mu_y-E[Y|X=\mu_x-x]\geq E[Y|X=\mu_x+x]-\mu_y$ for all $x\geq 0$, with at least one of the weak inequalities holding strictly for some x or y, then $Z\equiv\omega X+(1-\omega)Y$ for $\omega\in(0,1)$ will be negatively skewed.

Proof: Since X and Y are symmetric, it is only needed to look at the co-skewness terms. Consider $M_{12}[X,Y]$:

$$M_{12}[X,Y]\equiv E\left[(X-\mu_x)\left(Y-\mu_y\right)^2\right]=E[\left(Y-\mu_y\right)^2(E[X|Y=y]-\mu_x)]$$

$$\equiv\int_{-\infty}^{+\infty}\left(y-\mu_y\right)^2(E[X|Y=y]-\mu_x)g(y)dy$$

$$= \int_{\mu_y}^{+\infty} \left(y - \mu_y\right)^2 (E[X|Y = y] - \mu_x) g(y) dy$$

$$+ \int_{-\infty}^{\mu_y} \left(y - \mu_y\right)^2 (E[X|Y = y] - \mu_x) g(y) dy$$

$$= \int_{\mu_y}^{+\infty} \left(y - \mu_y\right)^2 (E[X|Y = y] - \mu_x) g(y) dy$$

$$+ \int_{\mu_y}^{+\infty} \left(y - \mu_y\right)^2 \left(E\left[X\middle|Y = 2\mu_y - y\right] - \mu_x\right) g\left(2\mu_y - y\right) dy$$

$$= \int_{\mu_y}^{+\infty} \left(y - \mu_y\right)^2 \left(E[X|Y = y] - 2\mu_x + E\left[X\middle|Y = 2\mu_y - y\right]\right) g(y) dy$$

$$\leq 0 \qquad (2.5)$$

since $\left(y - \mu_y\right)^2$ and $g(y)$, which is the marginal density of y, are positive for all y, and $E[X|Y = y] - 2\mu_x + E\left[X\middle|Y = 2\mu_y - y\right]$ is negative for all $y \geq \mu_y$. The $M_{12}[X, Y] \leq 0$ can be similarly shown, thus that $Skew[Z] < 0 \; for \; \omega \in (0,1)$."

Since the asymmetry in currency returns will definitely trigger asymmetries in the currency portfolio's asymmetry, and it is shown in various empirical papers that investors have different preferences over different asymmetric portfolios, the application of copula model is necessary to capture such features, compared to the negligence of the Gaussian distribution by the Mean-Variance analysis.

Differences between copula and the higher-moments method

To incorporate the asymmetry and other higher moments and co-moments of the return distributions, another branch of portfolio management research aims at this same target as the copula methods, which is called higher-moments portfolio selection problems. Here we distinguish the difference between these two and claim that the copula method is more flexible in terms of modelling the financial data dynamics. As in Jondeau and Rockinger (2006a) and many other researches in the same branch of the higher moments problem (e.g. Kraus and Litzenberger, 1976; de Athayde and Flores, 2004; Harvey, *et al.*, 2010), the optimal portfolio allocation problem under higher moments can be described as follows:

The objective function of portfolio optimisation can be approximated by Taylor's series expansion as:

$$E[U(W)] = U(\bar{W}) + U^{(1)}(\bar{W})E[W - \bar{W}] + \frac{1}{2}U^{(2)}(\bar{W})E[(W - \bar{W})^2] + \frac{1}{3!}U^{(3)}(\bar{W})E[(W - \bar{W})^3] + \frac{1}{4!}U^{(4)}(\bar{W})E[(W - \bar{W})^4] + O(W^4) \quad (2.6)$$

In Equation 2.6, the expected value of investor's utility, $U(W)$, is approximated by finding up to the fourth centre moments of the portfolio wealth. These moments are defined as:

$$\mu_p = E[r_p],$$

$$\sigma_p^2 = E\left[(r_p - \mu_p)^2\right] = E[(W - \bar{W})^2],$$

$$s_p^3 = E\left[\left(r_p - \mu_p\right)^3\right] = E[(W - \bar{W})^3],$$

$$\kappa_p^4 = E\left[\left(r_p - \mu_p\right)^4\right] = E[(W - \bar{W})^4].$$

(2.7)

These moments of the portfolio can also be expressed in a tractable manner in terms of the weight of each asset composing the portfolio and the asset returns' moments and co-moments (portfolios moments expressed by components moments):

$$M_1 = \alpha' \mu,$$

$$M_2 = E[(R - \mu)(R - \mu)'],$$

$$M_3 = E[(R - \mu)(R - \mu)' \otimes (R - \mu)'],$$

$$M_4 = E[(R - \mu)(R - \mu)' \otimes (R - \mu)' \otimes (R - \mu)'].$$

(2.8)

The problem of optimising the expected utility is then turned into the estimation of the moments of portfolio, and further into the estimation of moments of each assets composing the portfolio. It is often assumed asset returns are time-invariant in such literature, thus the estimation can be completed by using simple sample estimators of the moments.

However, in comparison with copula models, the latter is preferable for two reasons. The first is the separation of the marginals and dependence structure of a joint

distribution gives more flexibility in modelling and helps to avoid pitfalls in the application of correlation coefficient, (McNeil, Frey and Embrechts, 2005, Chapter 5). Second, suggested by literature on time-varying dependence (Patton, 2006; Jondeau and Rockinger, 2006b; Rodriguez, 2007; Cailault and Guegan 2009), the dependence parameter can be easily rendered with time-varying feature.

Out-of-sample performance of copula in portfolio management

With respect to the out-of-sample experiment of copula application to incorporate asymmetry in data for portfolio optimisation, the results are mixed depending on properties of each dataset. Patton (2004) analyse the importance of skewness for asset allocation based on the data of a small cap stock portfolio, a large cap stock portfolio and a risk free asset, and the significance of asymmetry modelled by copula is found in case of no short-sale constraints. Hatherley and Alcock (2007) use Australian equities as the study objects and get the optimal portfolio by minimising CVaR (the conditional value-at-risk) of the portfolio. The effect of copula application in this case is confirmed in their research. There are also some other working papers about the out-of-sample importance of the copula. Riccetti (2010) argues that the use of copula is effective when the portfolio is composed by one bond index and some stock indices. Xu (2005) also find the copula assumption does make a change to the optimal weights of portfolio. These results encourage the usage of copula in this chapter, but at the same time suggest the importance of the examination for the existence of asymmetries in our data and the effectiveness in the optimal currency composition result.

2.3 Methodology

2.3.1 Distribution building

Two steps are involved in building the multivariate distribution using copulas. The first is to build the single variable distribution for each return series and the second is to build the dependence by copula for joining the separate return distributions together. A bivariate copula function $C(u_1, u_2)$ is defined by Equation 2.2. The derivation of a copula starts originally from a multivariate distribution function. However, by reversing the process, copulas can be combined with other marginal distribution function to form new varieties of multivariate distributions. The single return distributions and the copula for dependence are selected in the following manner.

Distribution of each return series

For univariate return series, Hansen's skewed Student-t distribution is considered as an option for modelling the residuals from some conditional mean and conditional variance models. This is to reflect the asymmetry features of each currency's returns. The density function of the skewed Student-t distribution is defined by:

$$d(z;\eta,\lambda) = \begin{cases} bc\left(1+\frac{1}{\eta-2}\left(\frac{bz+a}{1-\lambda}\right)^2\right)^{-[(\eta+1)/2]} & if\ z < -a/b \\ bc\left(1+\frac{1}{\eta-2}\left(\frac{bz+a}{1+\lambda}\right)^2\right)^{-[(\eta+1)/2]} & if\ z \geq -a/b \end{cases} \tag{2.9}$$

where

$$a \equiv 4\lambda c\frac{\eta-2}{\eta-1}, b^2 \equiv 1+3\lambda^2-a^2, c \equiv \frac{\Gamma\left(\frac{\eta+1}{2}\right)}{\sqrt{\pi(\eta-2)}\Gamma\left(\frac{\eta}{2}\right)} \tag{2.10}$$

and η and λ denote the degree-of-freedom parameter and the asymmetry parameter of the distribution. We write $Z \sim ST\ (\eta, \lambda)$, if a random variable Z has the density $d(z; \eta, \lambda)$. Similarly $Z \sim T(\lambda)$ denotes a random variable following a standardized t distribution and $Z \sim N$ means that it follows a standardized normal distribution. The Student t distribution and Gaussian distribution are also deployed to model the residuals.

The conditional mean model of ARMA (u, v) is employed with (u, v) ranging from 0 up to 3 lags. For modelling the conditional volatility, GARCH (p, q) and APARCH (p, q) are used with (p, q) ranging from 0 to 3 are to fit the currency data.

The Akaike information criterion (AIC) is used to determine the lag length, the choice between the GARCH and APARCH volatility model, and the type of residual distribution for the best fit. We have 12 currencies for 9 years' horizon and this method provides a wide range to find the best fit model for each individual currency return. Specifically, we have:

$$r_t = c_0 + \sum_{\mathrm{i}=1}^{\mathrm{u}} ar_{\mathrm{i}} r_{t-\mathrm{i}} + \sum_{\mathrm{j}=1}^{\mathrm{v}} ma_{\mathrm{j}} \varepsilon_{t-\mathrm{j}} + \varepsilon_t, \tag{2.11}$$

$$\varepsilon_t = \sigma_t z_t, \tag{2.12}$$

$$\sigma_t^2 = \omega_0 + \sum_{\mathrm{i}=1}^{\mathrm{p}} \alpha_{\mathrm{i}} {\varepsilon_{\mathrm{t-i}}}^2 + \sum_{\mathrm{j}=1}^{\mathrm{q}} \beta_{\mathrm{j}} \sigma_{t-\mathrm{j}}^{\delta}, \tag{2.13}$$

$$\sigma_t^{\delta} = \omega_0 + \sum_{\mathrm{i}=1}^{\mathrm{p}} \alpha_{\mathrm{i}} (|\varepsilon_{\mathrm{t-i}}| - \gamma_{\mathrm{i}} \varepsilon_{\mathrm{t-i}})^{\delta} + \sum_{\mathrm{j}=1}^{\mathrm{q}} \beta_{\mathrm{j}} \sigma_{t-\mathrm{j}}^{\delta}, \tag{2.14}$$

$$z_t \sim ST(\eta_t, \lambda_t) \tag{2.15}$$

$$z_t \sim T(\lambda_t) \tag{2.16}$$

$$z_t \sim N \tag{2.17}$$

where Equation 2.13 is the GARCH specification, 2.14 is the APARCH model, and Equations 2.15, 2.16 and 2.17 are three types of residual distribution, i.e. the Skewed t, t and Gaussian distribution, respectively.

After the initial estimation, we save the standard residual terms, z_t, which are to be plugged into the copula model in the next step for estimating parameters of the dependence structure.

Pair-copula construction for dependence structure

A brief introduction to the pair copula construction à la Bedford and Cooke (2002) is presented here. Consider a random vector $X = (X_1, \dots, X_n)$ with a joint density function of $f(x_1, \dots, x_n)$. The pair copula decomposition is a result of the combined application of conditional density equation and the density form of Sklar's theorem, as in the following:

$$f(a, b) = f(a|b) \cdot f(b) \tag{2.18}$$

$$f(a, b) = c(F(a), F(b)) \cdot f(a) \cdot f(b) \tag{2.19}$$

By applying the conditional density equation, the joint density function $f(x_1, \dots, x_n)$ can be expressed as:

$$f(x_1, \dots, x_n) = f(x_n) \cdot f(x_{n-1}|x_n) \cdot f(x_{n-2}|x_{n-1}, x_n) \cdots f(x_1|x_2, \dots, x_n)$$

(2.20)

The order of the variables is changeable. By applying the density form of Sklar's theorem, each factor on the right-hand side of the above equation can be decomposed into a product of several conditional pair-copulas and an unconditional marginal density function as shown below:

$$f(x_1|x_2, x_3) = \frac{c_{13|2}[F(x_1|x_2), F(x_3|x_2)] \cdot f(x_1|x_2) \cdot f(x_3|x_2)}{f(x_3|x_2)}$$

$$= c_{13|2}[F(x_1|x_2), F(x_3|x_2)] \cdot f(x_1|x_2) \quad (2.21)$$

where $f(x_1|x_2)$ can be further decomposed using the same method, so:

$$f(x_1|x_2, x_3) = c_{13|2}[F(x_1|x_2), F(x_3|x_2)] \cdot c_{12}[F(x_1), F(x_2)] \cdot f(x_1) \quad (2.22)$$

The choices of the pair variables of the copulas are also changeable. These various types are organised by the "vines" structure. Typical examples are the "C-vine" (canonical vine) and the "D-vine" (Kurowicka and Cooke, 2006). The main difference between them is that the C-vine places more emphasis on a pivotal variable as a root to connect other variables, whereas the D-vine states parallel relationship among variables. Fig. 2.1 demonstrates the comparison between the two structures in a 5-variables case. The n-dimensional density functions of the D-vine and C-vine decomposition are given by Equations 2.23 and 2.24, respectively:

$$\prod_{k=1}^{n} f(x_k) \prod_{j=1}^{n-1} \prod_{i=1}^{n-j} c_{i,i+j|i+1,\dots,i+j-1}\{F(x_i|x_{i+1},\dots,x_{i+j-1}), F(x_{i+j}|x_{i+1},\dots,x_{i+j-1})\}$$

(2.23)

$$\prod_{k=1}^{n} f(x_k) \prod_{j=1}^{n-1} \prod_{i=1}^{n-j} c_{j,j+i|1,\dots,j-1}\{F(x_j|x_1,\dots,x_{j-1}), F(x_{j+i}|x_1,\dots,x_{i+j-1})\}$$

(2.24)

The likelihood function can be calculated using the same formulae as above, after the sample for x_k is decided, i.e. the standardized residuals from the GARCH estimation and the type of pair-copulas are determined.

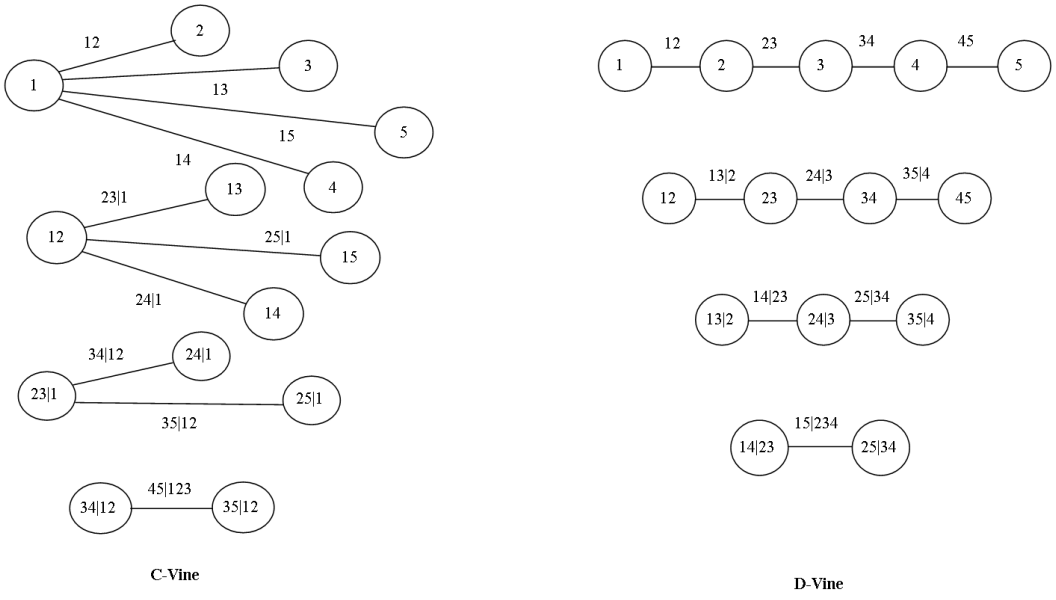


Fig. 2.1 C-Vine and D-Vine Copulas' Structure: Illustration with 5 Variables

In total, we have 12 currencies as candidates for the optimal currency portfolio. The sample time period spans for 9 years. To determine the best fit type of copula for each pair of variables on the vine nodes, we offer a range of 31 copulas which is wide enough to capture the complex dependence between the 12 currencies. For different layers of pair copula, we use 10 different copulas specifically the Gaussian, Student t, Clayton, Gumbel, Frank, Joe, Clayton-Gumbel, Joe-Gumbel, Joe-Clayton, and Joe-Frank copulas. Of these 10 copulas, 7 have their variants that are rotated 180 degrees, 90 degrees, and 270 degrees, making a total of 31 copulas. The copulas without variants are the Gaussian, Student-t and Frank. This setting allows the Archimedean copulas to capture any asymmetric dependence between upper and lower tails, and enables the rotated copulas to capture similar features in the second and third quarters of the dependence. This will be further illustrated later when analysing the currency returns data. The estimation is carried out by maximizing the pseudo-likelihood. The algorithms are based on modification of Aas *et al.* (2009) and the package 'CDVine' in R.

The distribution building is finalized by combining the univariate returns and the copula dependence model. Monte Carlo simulations are conducted to generate each distribution containing 500,000 observations.[1] In generating the return distribution, GARCH forecasts for the portfolio management period, assumed in this study to be 1 year until next adjustment of compositions, are required and the average of these forecasts is incorporated in the return distribution.

[1] 1-million-sample-distribution is tried at some time points, showing no significant differences.

To compare with the pair-copula model, a Gaussian copula model is also estimated using the same dataset from univariate currency returns. The estimation is straightforward, for only the covariance parameters are involved. It is found that the Gaussian copula cannot capture the asymmetric and complex dependence features in the data.

2.3.2 The investor's preference

The investor's preference is first described by Markowitz (1952), where the portfolio selection problem is formulated as a tradeoff between mean and variance of a portfolio of assets. Although from then on, vast developments have been made to capture the investment objectives, the rationale of tradeoff between return and risk largely remains intact. Especially in the area of risk measurement, in addition to variance used originally in Markowitz (1952), Philippatos and Wilson (1972) applied entropy; Price, *et al.* (1982) used lower partial moments; Gaivoronski and Pflug (2005) used Value-at-Risk; and Rockafeller and Uryasev (2002) used Conditional Value-at-Risk (CVaR).

The minimisation of CVaR risk measurement seems suitable to our currency composition management focusing mainly on safety. However, in this chapter safety is still not the only concern. As a management strategy, certain level of flexibility in the model, which allows adjustment as the investor's preference weight changes between return and risk should, still be maintained. Therefore a utility function that both generalises the choice between return and risk and in the meanwhile puts significant emphasis on the risks, especially the non-Gaussian risks, is needed for our analysis.

In our study, the portfolio optimisation problem can be summarized as maximization of appropriate expected utility while the utility function is based on the distributions from the above models:

$$\max_w E(U(W)) \tag{2.25}$$

$$W = 1 + w'R \tag{2.26}$$

where w is a vector representing the weights of currencies, R a vector of currency returns, and W is the wealth value of the portfolio.

The commonly used utility function is that of the power Constant Relative Risk Aversion (CRRA). However, this specification proves to be unable to capture the asymmetry and higher moments' effects of the distribution on portfolio choice. Following Gul (1991), Ang *et al.* (2005) and Hong *et al.* (2007), we use the Disappointment Aversion (DA) preference for our optimisation objective, on the ground that the commonly used CRRA utility function is a local mean-variance preference. The DA utility is defined by the following equation:

$$DA(W) = \frac{1}{K}\left(\int_{-\infty}^{\mu_w} u(W)dF(W) + A\int_{\mu_w}^{\infty} u(W)dF(W)\right) \tag{2.27}$$

where $u(\cdot)$ is the felicity function in the form of CRRA utility:

$$u(W) = \begin{cases} (1-\gamma)^{-1}\cdot(W)^{1-\gamma} & if\ \gamma \neq 1 \\ \ln(W) & if\ \gamma = 1 \end{cases}, \tag{2.28}$$

μ_w is the certainty equivalent according to the CRRA power utility; $F(\cdot)$ is the cumulative distribution function of the wealth; and K is a constant scalar given by:

$$K = P(W < \mu_w) + AP(W > \mu_w). \quad (2.29)$$

The DA preference is a transformation based on the chosen $u(\cdot)$, or the CRRA power utility function in this case, in which the risk aversion parameter (RA) stands for the risk preference of the representative investor. The transformation puts different weights upon utility above and below the reference point, μ_w. Usually parameter A is set to be smaller than 1 so that the utility below average (the loss) gives larger impacts than the utility above the average (the profit). For example, if A is set to be 0.5, then the lower part of the utility is given twice the weight given to the upper part utility. This emphasis on the loss rather than profit is in accordance with the management nature of the central banks, whose primary goal is to avoid negative shocks to foreign assets rather than to increase wealth. Parameter A stands for the asymmetry preference of the representative investor. Therefore the optimisation problem becomes:

$$\max_w DA(W) \quad (2.30)$$

$$W = 1 + w'R \quad (2.31)$$

In our analysis, we set up three levels of DA parameter, A, to be 0.25, 0.45 and 0.65, and four levels of relative risk aversion coefficient in the CRRA power utility function RA , to be 3, 7, 10 and 20. Similar range of risk aversion are used in Campbell and Viceira (1999), Ait-Sahalia and Brandt (2001) and Patton (2004).

2.4 Data Description and Investment Strategy

Unlike when calculating securities returns, to compute returns of each currency we need two types of datasets, i.e. the interest rate of the currency-issuing country and

the exchange rate of the foreign currency to the currency of the home country, which is China in our case. To concentrate on the currency effect, we assume that international reserves are solely invested in government bonds. To comprehend the effects of diversification, a sufficient number of currency assets are to be included in a foreign currency portfolio. We select 12 currencies for the central bank of China. Therefore, we need 12 corresponding interest rates of these countries and 12 foreign exchange rates to the Chinese yuan. The horizon of the data sample is from 1 January 1999 to 31 December 2009 and the data are in daily frequency.

The interest rate dataset consists of 8 interbank rates and 4 money market rates. Of the 8 interbank rates, 7 are from the London market, i.e. the London Interbank Offered Rate (LIBOR) and the remaining one is the interbank rate for the country to which the home currency belongs, in this case Singapore Sibor. All 8 interbank rates are from Thomson Reuters DataStream. Due to data availability, the other four rates are money market rates from the IMF International Financial Statistics. Table 2.1 presents a summary of the interest rates.

Table 2.1 Summary of Interest Rates Data

Interest Rates Data								
Country	**US**	**EURO**	**JAPAN**	**UK**	**SWITZERLAND**	**CANADA**	**AUSTRALIA**	**SINGAPORE**
Type	Interbank rates (12 Month)							
Market	LIBOR							SIBOR
Frequency	Daily	Daily	Daily	Daily	Daily	Daily	Daily	Daily
Source	Thomson Reuters DataStream							
Mnemnic Code	BBUSD12	BBEUR1Y	BBJPY12	BBGBP12	BBCHF12	BBCAD12	BBAUD12	SNGIB1Y

Interest Rates Data				
Country	**NEWZEALAND**	**SOUTH KOREA**	**RUSSIA**	**THAILAND**
Type	Money Market Rate			
Frequency	Daily	Daily	Daily	Daily
Source	IMF International Financial Statistics			

Source: Compiled by the author

As to the exchange rates, 8 of the total 12 are from Thomson Reuters DataStream. Historic data on exchange rates of the Korean won and Russian rouble against the Chinese yuan are from a foreign exchange service company.[2] Table 2.2 gives a summary of the data sources.

Table 2.2 Exchange Rate Data

Foreign Exchange Rate Data						
Currency	**USD**	**EURO**	**JPY**	**GBP**	**CHF**	**CAD**
Type	WM/Reuters Mid Price					
Frequency	Daily					
Source	Thomson Reuters DataStream					
Mnemnic Code	CHIYUA$	CHEURSP	CHJPYSP	CHIYUAN	CHCHFSP	CHCADSP

Foreign Exchange Rate Data Specification						
Currency	**AUD**	**SGD**	**NZD**	**THB**	**KRW**	**RUB**
Type	WM/Reuters Mid Price				Mid Price	
Frequency	Daily					
Source	Thomson Reuters DataStream				OANDA	
Mnemnic Code	CHAUDSP	CHSGDSP	CHNZDSP	CHTHBSP		

Source: Compiled by the author

Currency returns are derived by combining the interest rate and exchange rate returns:

$$r_{i,t} = s_{i,t} + b_{i,t} \tag{2.32}$$

[2] OANDA Corporation. www.oanda.com. Last accessed on May 5th, 2014.

where $b_{i,t}$ is the interest rate of currency i and $s_{i,t}$ is the exchange rate return of currency i against the Chinese yuan.

For tractability, we assume that it is desirable for reserve managers to adopt a buy-and-hold investment strategy with yearly rebalancing. We take previous three years' daily returns as the base for estimating coefficients on model parameters and use one-year-ahead values from the conditional mean and volatility models as the corresponding expected values. Economic values are used as the performance measure, following Ang *et al.* (2005) and Hong *et al.* (2007). This measure is based on portfolio distributions, and indicates how much certainty equivalent wealth is needed for the worse model to have the same amount of utility as the better distribution model.

2.5 Empirical Results

2.5.1 Currency returns and non-Gaussian features

Univariate currency returns

Descriptive analyses of the 12 currency returns during the sample period are carried out. Table 2.3 displays the results for 2005 as an example. It can be seen that, in 2005, the returns of only two currencies, the euro and pound sterling have small skewness and excess kurtosis. Normality of their returns is not rejected by the Jarque-Bera tests. The prevalent non-normal distribution prompts us to add t distribution and skewed t distribution to modelling the residuals. With respect to the autocorrelation in conditional mean and volatility clustering, the Ljung-Box tests on raw data and squared returns are performed with 5 and 10 lag lengths. The LM

ARCH test of Engle (1982) is also carried out. Of 12 currency returns, five have at least one test indicating autocorrelation or heteroskedastcity. This finding motivates us to apply the ARMA-GARCH/APARCH model. To demonstrate this furthermore, the same table for the whole sample from 1999 to 2009, instead of the three years before 2005 in Table 2.3, is presented with the same discoveries of autocorrelation, heteroskedasticity and non-normal distributions. See Table 2.4 for the result for the whole sample. Although in the portfolio optimisation process it is the three-year rolling window estimations that are utilised, the whole sample result suggests the prevalence of the empirical features that motivate our model.

Table 2.3 Descriptive Statistics for Currency Returns (2005 Sample)

	USD	EURO	JPY	GBP	CHF	CAD	AUD	SND	NZD	KRW	RUB	THB
Skewness	-27.140	-0.085	0.006	-0.150	0.106	-0.030	-0.308	-0.157	-0.492	0.097	-2.226	-0.368
Excess Kurtosis	748.830	0.054	1.219	0.232	0.920	1.480	1.361	1.489	1.375	2.882	27.503	5.984
Jarque-Bera	**1.84E+07**	1.0488	**48.454**	4.7135	**29.071**	**71.598**	**72.812**	**75.574**	**93.27**	**272.23**	**25325**	**1185.8**
p-value	0.000	0.592	0.000	0.095	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
LM ARCH	0.001	1.972	1.266	2.559	1.258	0.537	0.324	**3.814**	**2.310**	**51.157**	0.140	**4.377**
p-value	0.999	0.140	0.283	0.078	0.285	0.584	0.723	0.023	0.100	0.000	0.869	0.013
Ljung-Box 5	0.914	4.232	5.672	**11.747**	10.870	6.382	4.606	3.590	4.716	**112.583**	0.368	6.085
p-value	0.969	0.516	0.340	0.038	0.054	0.271	0.466	0.610	0.452	0.000	0.996	0.298
LB 10	0.914	9.231	12.458	13.456	12.896	11.344	10.980	5.947	10.052	**133.842**	10.212	15.589
p-value	0.969	0.510	0.256	0.199	0.230	0.331	0.359	0.820	0.436	0.000	0.422	0.112
LB Square5	0.005	8.971	7.445	10.867	4.967	2.269	1.625	7.491	6.123	**135.773**	0.322	**81.197**
p-value	1.000	0.110	0.190	0.054	0.420	0.811	0.898	0.187	0.294	0.000	0.997	0.000
LB Square10	0.011	**19.493**	11.083	**33.073**	7.933	13.481	26.533	11.034	**19.073**	**143.272**	2.116	**83.932**
p-value	1.000	0.034	0.351	0.000	0.635	0.198	0.003	0.355	0.039	0.000	0.995	0.000

Notes:
(i). LB is short for Ljung-Box test and LB 10 means the Ljung-Box test on raw data with lag length of 10. LB Squre5 means the Ljung-Box test on squared terms with lag length of 5.
(ii). Except for skewness and excess kurtosis, the rest tests in the table are presented with both statistics values and their probability values (p-values) to indicate the significance, and the significant statistics are highlighted by bold font

Table 2.4 Descriptive Statistics for Currency Returns (Whole Sample)

	USD	EURO	JPY	GBP	CHF	CAD	AUD	SND	NZD	KRW	RUB	THB
Skewness	-8.889	0.245	-0.116	-0.007	0.176	-0.163	-0.428	-0.014	-0.355	0.133	-2.339	-0.469
Excess Kurtosis	270.580	2.927	4.982	4.942	2.022	4.274	8.116	3.281	2.795	12.797	38.808	70.490
Jarque-Bera	**8.790E+06**	**1052.600**	**2973.900**	**2919.100**	**503.660**	**2196.400**	**7961.700**	**1286.600**	**993.710**	**19585.000**	**1.827E+05**	**5.941E+05**
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
LM ARCH	0.030	**41.791**	**36.542**	**38.095**	**14.852**	**206.530**	**291.630**	**31.107**	**155.030**	**83.285**	**119.700**	**306.770**
p-value	0.970	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Ljung-Box 5	**11.106**	3.577	**15.555**	**15.276**	**16.610**	**11.834**	**18.869**	**15.490**	3.170	**179.684**	**79.632**	**175.617**
p-value	0.049	0.612	0.008	0.009	0.005	0.037	0.002	0.008	0.674	0.000	0.000	0.000
LB 10	**33.744**	**23.590**	**22.798**	**26.513**	**21.893**	**38.255**	**38.029**	**24.541**	9.490	**235.190**	**101.496**	**264.868**
p-value	0.000	0.009	0.012	0.003	0.016	0.000	0.000	0.006	0.486	0.000	0.000	0.000
LB Square5	0.169	**144.383**	**125.566**	**442.611**	**52.145**	**1119.980**	**967.616**	**110.909**	**541.421**	**531.598**	**380.376**	**562.222**
p-value	0.999	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
LB Square10	**0.385**	**246.545**	**185.378**	**861.929**	**85.085**	**2192.640**	**2004.390**	**226.359**	**931.561**	**1165.690**	**491.188**	**797.854**
p-value	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Source: Compiled by the author.

The parameters for modelling each currency returns are presented in Table 2.5. The best model is determined by selecting the minimal AIC. The first two rows show the best fit type of conditional mean and conditional variance models. APARCH models explain asymmetries in some skewed currencies. The selection of residuals distribution type is also as expected from the descriptive statistics. Euro and pound sterling are fitted with normal distribution whereas the US dollar and the New Zealand dollar with high skewness are fitted with skewed Student-t distribution. Other currencies with high excess kurtosis are accounted for by t distributions. Most of the parameters are found to be significant, as indicated with bold typeface.

Table 2.6 reveals the effectiveness of ARMA-GARCH/APARCH models in removing the time-dynamics in currency returns. The Ljung-Box and LM ARCH tests show all currency returns' residuals are now white noise. Kolmogorov-Smirnov tests are performed to compare residuals with their fitted distribution. The result shows that no currency can reject its best fit distribution. These results provide solid foundations for copula modelling.

Table 2.5 Univariate Returns Model Estimation (2005 Sample)

	USD	EURO	JPY	GBP	CHF	CAD	AUD	SND	NZD	KRW	RUB	THB
mean type	Arma (3, 3)	Arma (3, 2)	Arma (3, 1)	Arma (3, 3)	Arma (2, 1)	Arma (3, 1)	Arma (2, 2)	Arma (2, 3)	Arma (3, 3)	Arma (3, 2)	Arma (3, 1)	Arma (3, 3)
variance type	Aparch (1, 1)	Garch (1, 1)	Garch (1, 1)	Garch (1, 1)	Garch (1, 1)	Garch (1, 1)	Aparch (1, 1)	Garch (1, 1)	Garch (1, 1)	Garch (1, 1)	Aparch (1, 1)	Aparch (1, 1)
Distribution	sstd	norm	std	norm	Std	Std	std	std	sstd	std	std	std
Mu	1.101E-07	5.080E-04	-2.470E-06	8.790E-04	-4.490E-06	**7.090E-06**	**-2.390E-05**	2.750E-04	1.740E-04	1.370E-04	5.610E-07	9.160E-06
p-value	4.536E-01	3.124E-01	5.979E-02	1.682E-01	9.860E-01	2.160E-06	2.000E-16	1.611E-01	1.247E-01	1.657E-01	NA	7.469E-01
ar1	**3.920E-01**	**-6.730E-01**	**8.940E-01**	**-9.580E-01**	-1.960E-01	**9.290E-01**	**4.840E-02**	-1.000E+00	**-4.530E-01**	-2.720E-01	9.620E-01	**2.030E-01**
p-value	2.000E-16	3.920E-04	2.000E-16	3.630E-05	4.770E-01	2.000E-16	2.000E-16	NA	1.660E-06	5.620E-02	NA	1.003E-02
ar2	**2.720E-01**	**-6.740E-01**	**1.080E-01**	**-6.960E-01**	-4.080E-03	**8.830E-02**	**9.510E-01**	**-5.120E-01**	**3.160E-01**	**5.580E-01**	7.940E-03	**-2.190E-01**
p-value	2.000E-16	6.450E-07	1.343E-02	4.660E-05	9.240E-01	3.960E-02	2.000E-16	1.970E-04	6.800E-04	2.180E-05	6.163E-01	3.620E-06
ar3	**3.400E-01**	-4.810E-02	-2.210E-03	**-6.850E-01**		-3.900E-02			**7.700E-01**	**2.170E-01**	1.760E-02	**6.890E-01**
p-value	2.000E-16	1.910E-01	9.522E-01	8.970E-04		2.322E-01			2.000E-16	2.790E-04	NA	2.000E-16
ma1	**-4.950E-01**	**6.840E-01**	**-1.000E+00**	**1.000E+00**	8.980E-02	**-1.000E+00**	**-9.090E-02**	9.390E-01	**4.320E-01**	-5.110E-02	-9.820E-01	-1.390E-01
p-value	2.000E-16	2.560E-04	2.000E-16	1.090E-04	7.450E-01	2.000E-16	2.000E-16	NA	5.770E-07	7.102E-01	NA	7.174E-02
ma2	**-1.890E-01**	**6.870E-01**		**7.340E-01**			**-9.410E-01**	**4.620E-01**	**-3.380E-01**	**-6.780E-01**		**2.660E-01**
p-value	2.000E-16	1.450E-07		6.560E-05			2.000E-16	9.600E-04	3.810E-05	1.880E-12		5.400E-08
ma3	**-1.890E-01**			**6.490E-01**				-4.200E-02	**-8.200E-01**			**-7.070E-01**
p-value	2.000E-16			5.820E-04				2.027E-01	2.000E-16			2.000E-16
Omega	**3.360E-05**	7.380E-07	7.300E-07	**9.050E-07**	1.090E-06	5.950E-07	8.140E-07	1.900E-07	7.440E-07	6.510E-07	5.680E-09	**1.290E-04**
p-value	2.430E-03	1.840E-01	1.236E-01	7.026E-02	2.120E-01	1.724E-01	1.840E-01	1.395E-01	1.276E-01	2.664E-01	1.000E+00	3.896E-02
alpha1	**2.500E-01**	1.350E-02	**5.000E-02**	**4.320E-02**	3.540E-03	**2.810E-02**	1.830E-02	**3.620E-02**	**1.620E-02**	**1.410E-01**	**1.000E+00**	**3.220E-01**
p-value	1.030E-07	1.169E-01	4.850E-03	5.818E-03	6.680E-01	2.120E-02	3.230E-01	1.358E-02	2.051E-02	1.487E-03	1.760E-02	3.170E-03
gamma1	8.880E-02						**3.310E-01**				1.070E-01	1.200E-01
p-value	5.160E-01						1.680E-05				3.909E-01	3.207E-01

beta1	**8.140E-01**	**9.660E-01**	**9.310E-01**	**9.260E-01**	**9.760E-01**	**9.540E-01**	**9.610E-01**	**9.380E-01**	**9.710E-01**	**8.710E-01**	**8.860E-01**	**6.380E-01**
p-value	2.000E-16	2.000E-16	2.000E-16	2.000E-16	2.000E-16	2.000E-16	2.000E-16	2.000E-16	2.000E-16	2.000E-16	2.000E-16	9.730E-09
Delta	**6.680E-01**						2.000E+00				**8.360E-01**	**1.240E+00**
p-value	2.240E-08						1.990E-01				7.850E-07	4.100E-03
Skew	**9.890E-01**								**8.740E-01**			
p-value	2.000E-16								2.000E-16			
Shape	**2.680E+00**		**5.030E+00**		**5.450E+00**	**6.040E+00**	**6.640E+00**	**6.210E+00**	**6.780E+00**	**4.320E+00**	**2.010E+00**	**2.870E+00**
p-value	4.440E-16		2.550E-06		4.090E-05	1.450E-05	4.140E-05	7.290E-06	8.640E-05	7.320E-09	2.000E-16	3.690E-14

Notes:

(i).The first two rows in the table indicate the type of mean and variance functions for each currency returns and their best fit lag lengths. The third row reports the best fit distribution forms for their residuals. Skewed Student-t, Student-t and Gaussian distributions are respectively denoted by ‘sstd’, ‘std’, and ‘norm’.

(ii).The rest of the table lists coefficient values and their p-values to indicate significance for corresponding models in the first three rows. Significance is highlighted with the bold fonts.

Table 2.6 Statistical Tests for Effectiveness of Univariate Models (2005 Sample)

	USD	EURO	JPY	GBP	CHF	CAD	AUD	SND	NZD	KRW	RUB	THB
Ljung-Box 10	0.027	3.253	7.919	4.143	6.027	5.697	6.633	4.113	9.764	10.670	0.004	4.908
p-value	1.000	0.975	0.637	0.941	0.813	0.840	0.760	0.942	0.461	0.384	1.000	0.897
Ljung-Box 15	0.043	7.915	9.752	6.079	12.138	6.673	17.349	7.794	18.370	12.990	0.004	17.094
p-value	1.000	0.927	0.835	0.978	0.669	0.966	0.298	0.932	0.244	0.603	1.000	0.313
LB Square10	0.013	16.025	7.202	4.169	8.473	8.265	12.441	9.547	5.005	11.034	0.004	3.423
p-value	1.000	0.099	0.706	0.939	0.583	0.603	0.257	0.481	0.891	0.355	1.000	0.970
LB Square 15	0.020	19.544	9.240	7.160	9.754	11.013	20.304	16.794	18.920	15.732	0.004	4.065
p-value	1.000	0.190	0.865	0.953	0.835	0.752	0.161	0.331	0.217	0.400	1.000	0.998
LM ARCH	0.016	18.649	7.214	4.534	8.852	9.762	11.552	11.575	6.107	12.429	0.753	3.870
p-value	1.000	0.097	0.843	0.972	0.716	0.637	0.482	0.480	0.911	0.412	1.000	0.986
KS test	0.030	0.028	0.043	0.026	0.042	0.033	0.045	0.037	0.033	0.020	0.046	0.028
p-value	0.489	0.640	0.137	0.708	0.153	0.426	0.111	0.228	0.362	0.920	0.080	0.572

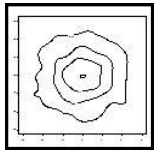
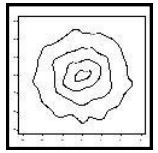
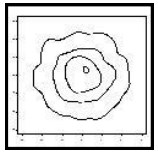
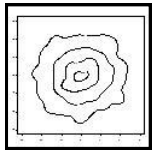
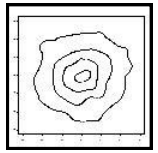
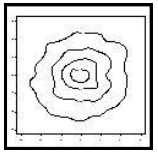
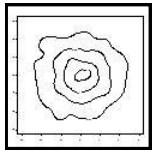
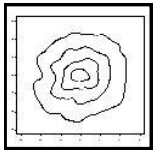
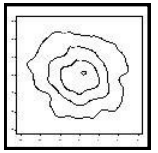
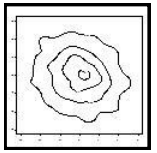
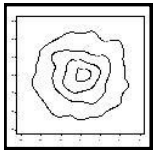
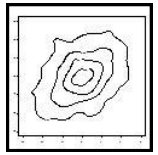
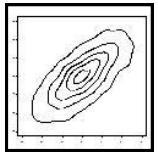
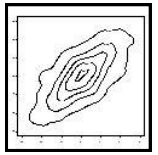
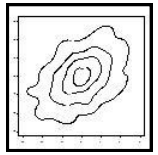
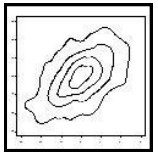
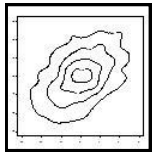
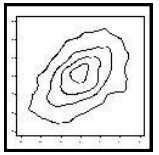
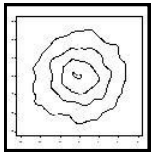
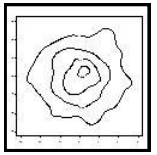
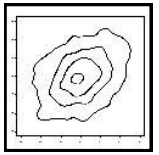
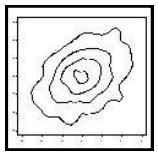
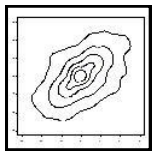
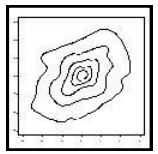
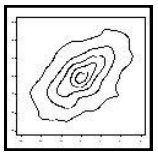
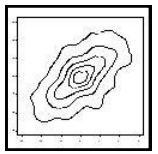
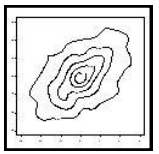
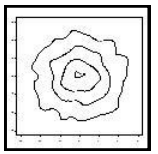
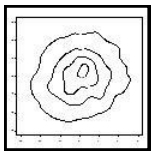
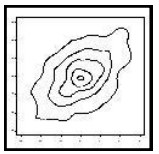
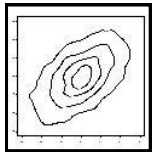
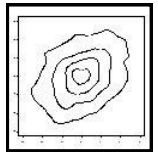
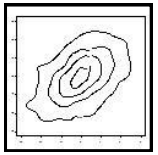
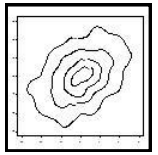
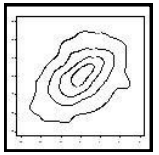
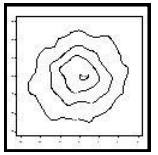
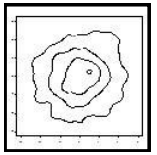
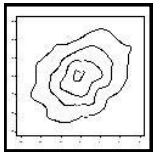
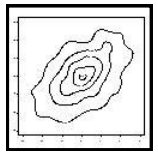
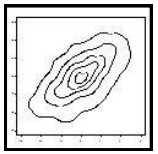
Notes:
(i). LB stands for the Ljung-Box test and LB 10 means the Ljung-Box test on raw data with 10 lags. LB Squre15 means the Ljung-Box test on squared terms with a lag length of 15.
(ii). All tests in the table are presented with both coefficient values and their probability values (p-values) to indicate the hypothesis rejection. None of the null hypothesis can be rejected.

Analysis of dependence

Descriptive analyses of the dependence are also carried out. Table 2.7 reports the results for 2005 as an example. The lower triangular lists three dependence measures, i.e. the upper tail dependence, lower tail dependence and Kendall's tau. For example, in the 7^{th} row and 2^{nd} column of the table, the three numbers 0.6148, 0.3734 and 0.3630 indicate that the relation between the 7^{th} currency AUD and the 2^{nd} currency euro has a Kendall's tau of 0.3630, and its upper tail is greater than the lower tail. This implies that it has a fat-tail with tail dependence greater than zero. It also suggests the existence of asymmetric dependence, which indicates that extreme losses occur less often than do extreme earnings. The upper triangular of Table 2.7, further illustrates dependence between two variables. The empirical meta contour graphs are fitted in their corresponding positions. For example, the dependence between AUD and the euro, in the 2^{nd} row and 7^{th} column, is shown to be clearly asymmetric.

Our vine copula structure allows a wide selection of copula functions. The flexibility of the approach manifests in two aspects. First, it can capture fat-tails and asymmetric dependence. Such dependence is complex, especially in high dimensional situations. As revealed in Table 2.7, many currency pairs have greater than zero tail dependence and uneven upper and lower tails. Conventional assumption of Gaussian and elliptical copulas are unable to capture these features, which may significantly affect portfolio optimisation. See Figs. 2.2 and 2.3 for further illustration.

Table 2.7 Descriptive Analysis of Dependence (2005 Sample)

USD											
0.0620 0.0000 0.0334	EURO										
0.0506 0.0000 0.0201	0.0995 0.0995 0.2497	JPY									
0.0434 0.0000 0.0088	0.1681 0.1681 0.5488	0.0057 0.0057 0.2227	GBP								
0.0168 0.0168 0.0521	0.6659 0.4983 0.5028	0.2583 0.2583 0.3856	0.0000 0.0000 0.3687	CHF							
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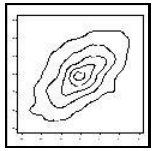
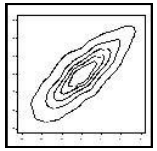
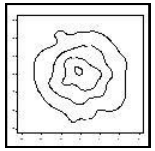
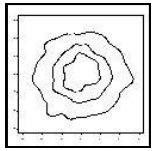
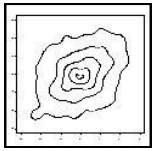
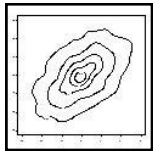
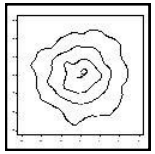
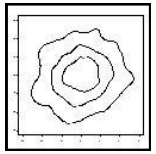
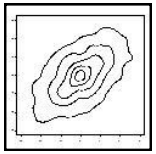
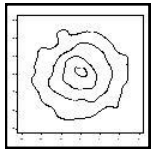
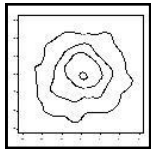
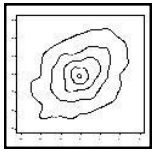
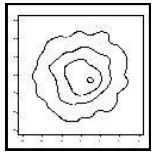
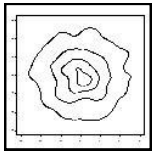
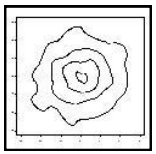
0.0686 0.0000 0.0490	0.6148 0.3734 0.3630	0.2170 0.2170 0.3952	0.0293 0.0293 0.3378	0.2858 0.2858 0.4585	0.0000 0.0000 0.4193	AUD					
0.0000 0.0000 -0.0118	0.0000 0.0000 0.3160	0.2437 0.2437 0.4642	0.0000 0.0000 0.2713	0.1367 0.1367 0.3728	0.0986 0.0986 0.2533	0.1331 0.1331 0.3966	SND				
0.0041 0.0041 0.0535	0.0000 0.0000 0.3299	0.2302 0.2302 0.3659	0.2095 0.2632 0.3283	0.2833 0.2833 0.4411	0.0706 0.0706 0.3673	0.6757 0.4250 0.6572	0.0810 0.0810 0.3768	NZD			
0.0000 0.0000 -0.0565	0.0000 0.0000 0.0078	0.0000 0.0000 -0.0145	0.0000 0.0000 0.0091	0.0000 0.0000 -0.0048	0.0000 0.0000 -0.0528	0.0000 0.0082 0.0019	0.0000 0.0000 0.0104	0.0000 0.0000 0.0240	KRW		
0.0000 0.0000 -0.1062	0.0751 0.0000 0.0166	0.0000 0.0000 0.0697	0.0032 0.0000 0.0600	0.0000 0.0000 0.0168	0.0000 0.0000 0.0120	0.0000 0.0000 0.0256	0.0000 0.0000 0.0640	0.0000 0.0012 0.0400	0.0543 0.0000 0.0398	RUB	
0.0000 0.0000 -0.0062	0.0000 0.0000 0.2107	0.5830 0.2848 0.3222	0.0000 0.0000 0.1981	0.0158 0.0158 0.2168	0.1606 0.0124 0.1730	0.1242 0.1242 0.2254	0.6049 0.3469 0.3853	0.2257 0.0037 0.2123	0.0000 0.0000 -0.0205	0.0000 0.0000 0.0417	THB

Notes:

The lower triangular lists three dependence measures: the upper and lower tail dependence and Kendall's tau, respectively. The upper triangular are empirical meta-contour graphs.

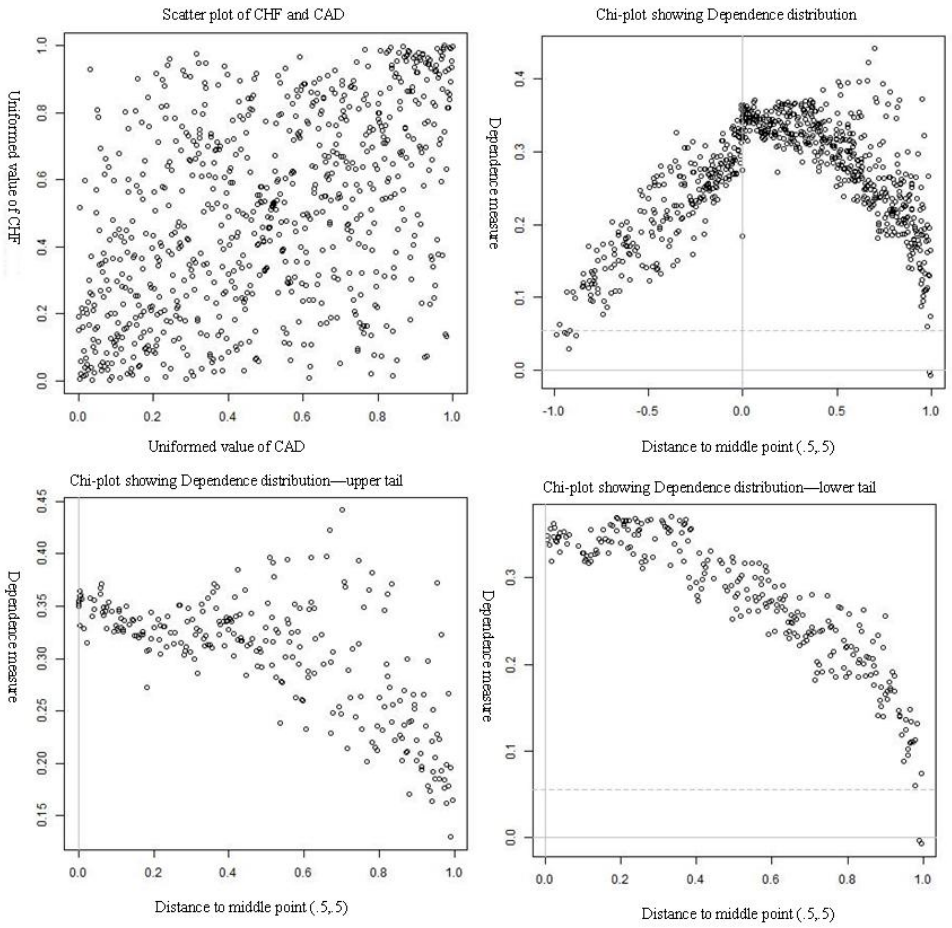


Fig. 2.2 Scatter Plot and Chi-Plots for Fat-Tail and Asymmetric Dependence Demonstration in 2005

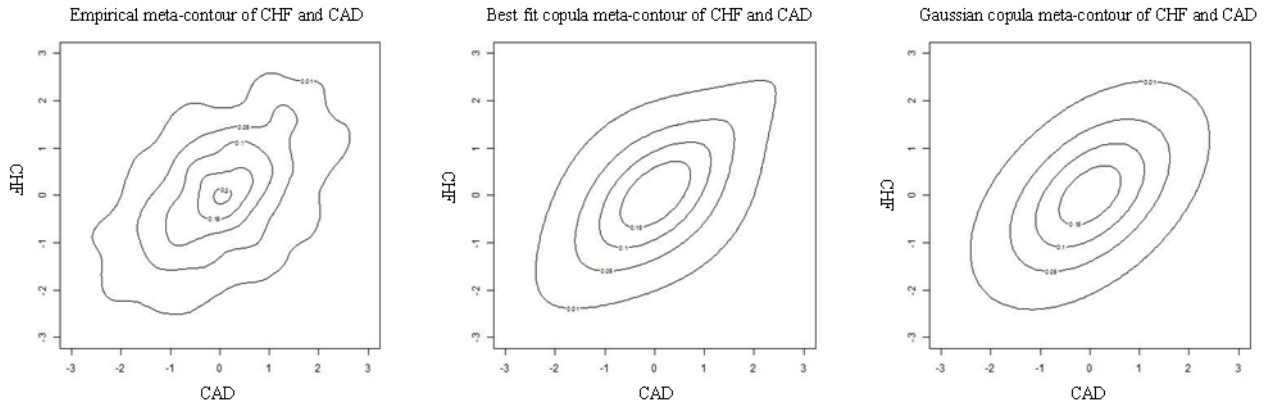


Fig. 2.3 Meta-Contours Showing Copula Model Captures Features in Empirical Data Better

Fig. 2.2 contains four graphs depicting the relation between the CHF and CAD in 2005. The scatter plot in the upper left, and the chi-plot in the upper right using the method of Fisher and Switzer (1985) are for the whole sample; the chi-plot in the lower left is for both variables increasing together above their averages (the upper tail dependence), and the one in the lower right is for their decreasing together (the lower tail dependence). The horizontal axis of a chi-plot is the distance between the data point (x, y) and the centre of the dataset, whereas the vertical axis is a correlation coefficient on dichotomized values of the two variables.

From the first chi-plot we can see that since the right half of this graph describes data moving in the same direction (rising or falling at the same time) and the left half describes data moving in different directions (one rises/falls, while the other falls/rises), the fact that dependence on the right is greater than that on the left means these two currencies are more correlated when increasing or decreasing simultaneously. Further, on reading the points towards the right of the plot (the furthest distance from the centre) the tail dependence is found to be above zero. This shows the fat-tail. Comparison between the second and third chi-plots shows that the upper tail has greater dependence than the lower tail, since the higher correlation points are from the upper tail in the lower left graph, rather than the lower tail in the lower right graph, and this pattern reveals asymmetry.

Fig. 2.3 shows that the relation described in Fig. 2.2 can be captured exactly by a D-vine structure. The figure includes three meta-contour plots. The first is the empirical contour, the second is taken from the estimated best fit copula in the D-vine structure, whereas the third is a comparison with the Gaussian copula if no

selection is permitted. It can be seen that the Clayton-Gumbel copula in the second plot better captures the essence of the empirical dependence.

To facilitate the demonstration of this point, Fig. 2.4 gives the same scatter plot and chi-plots as in Fig. 2.2 for the whole sample again from 1999 to 2009 for the purpose of showing such feature is universal. From the whole sample case in Fig. 2.4, it is also discovered from the chi-plots that the dependence is actually distributed unevenly. The non-zero dependence in the upper and lower ends means fat-tails, and the different patterns in the lower half two chi-plots indicate dependence asymmetry. Such features are typical and universal in all the individual years' cases.

The second aspect of our copula model's flexibility lies in the rotated copulas included in the fitting range, especially those Archimedean copulas being rotated 90 and 270 degrees. This makes it possible for our approach to capture dependence between variables that are correlated when moving in different directions. In the vine structures only part of the nodes are fed with the original residuals data. Many nodes need to be changed according to the conditional distribution functions. As such, there is a good chance that the dependence between changed variables is fit best by a rotated copula. Fig. 2.5 shows a meta-contour of the second copula in the sixth tier in the D-vine structure for the dependence of currency returns in 2005. It can be seen that the correlation in the upper left corner is greater than in the lower right corner. This best fit copula is a 270 degree rotated Clayton copula.

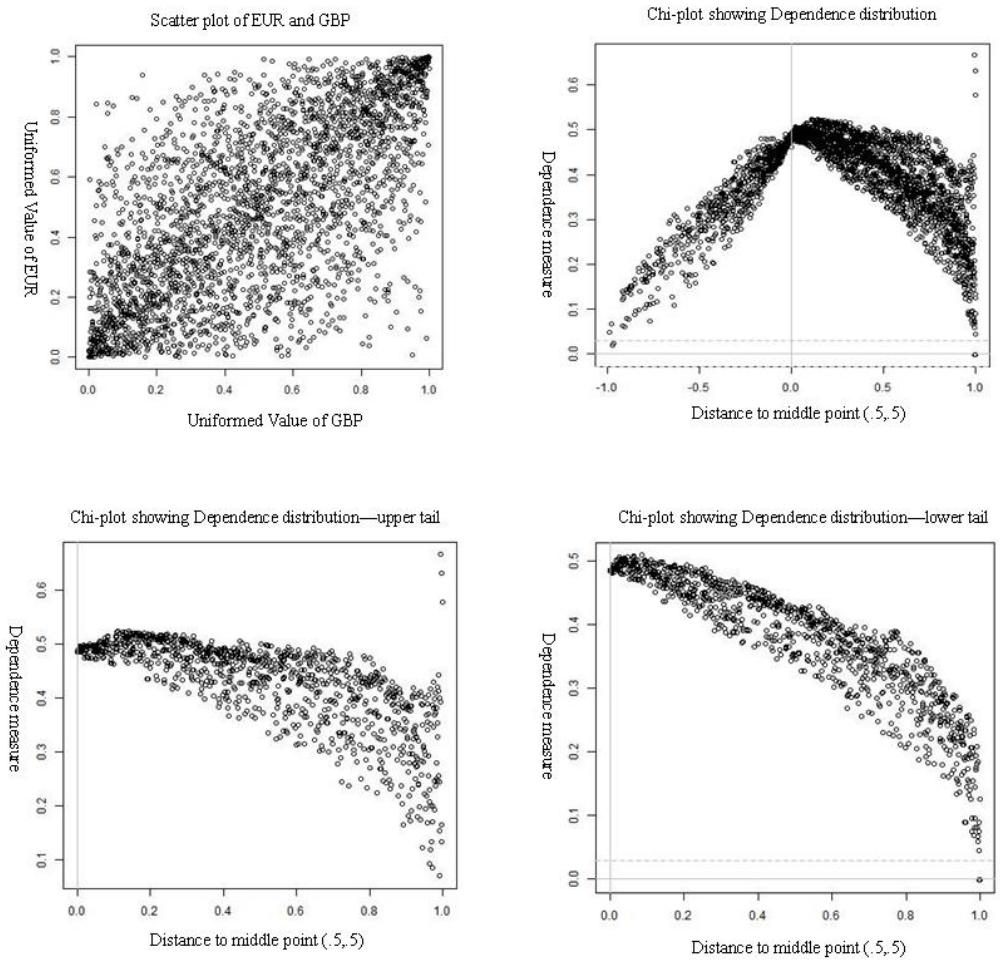


Fig. 2.4 Scatter Plot and Chi-Plots for Fat-Tail and Asymmetric Dependence Demonstration Whole Sample

In Fig. 2.6 similar discovery of rotated copulas capturing the relationship of currencies moving in different directions is shown again using the whole sample from 1999 to 2009. It is a plot of meta-contour of the second copula in the eighth tier in the D-vine structure, with the best fit copula to be a 90 degree rotated BB8 copula.

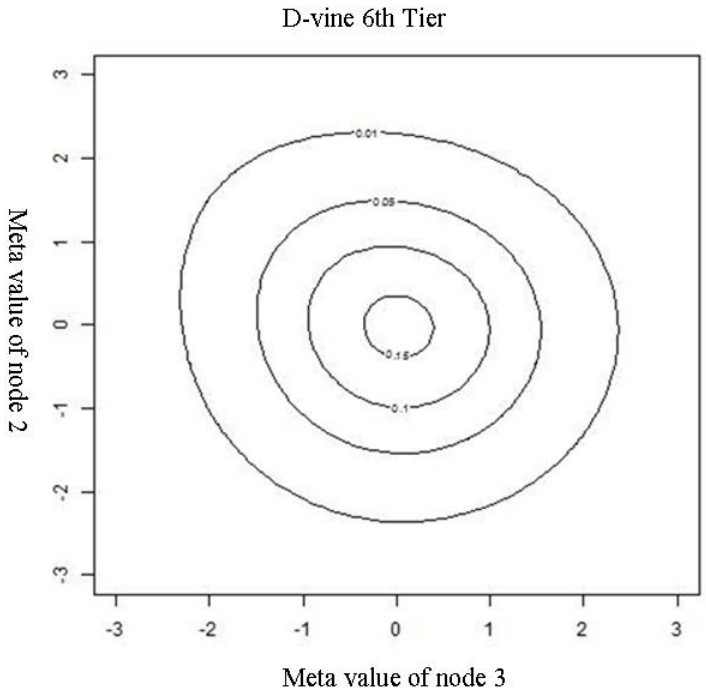


Fig. 2.5 Meta-Contour Illustration for 270 Degree Rotated Copula in 2005

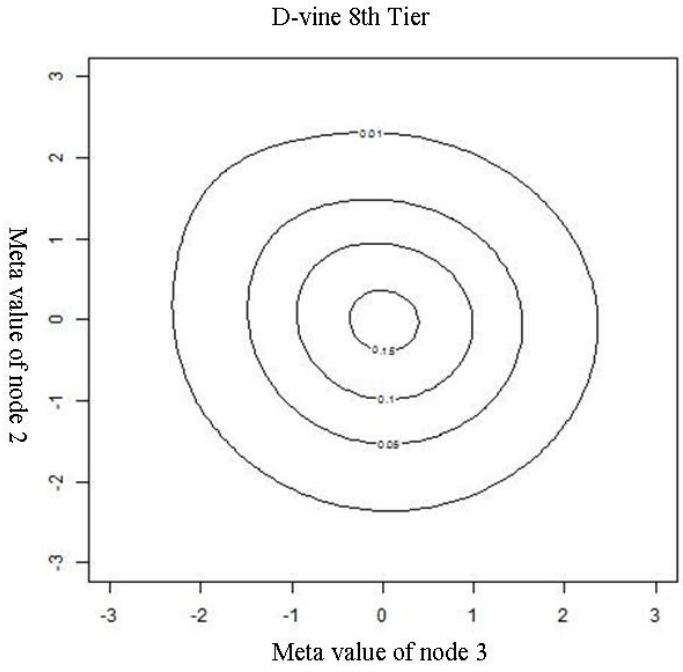


Fig. 2.6 Meta-Contour Illustration for 90 Degree Rotated Copula in the Whole Sample

To formally test the overall fit of the pair copula models, we conduct the Vuong ratio test (Vuong, 1989) by comparing the C-vine and D-vine copulas with a Gaussian copula and by comparing between the two vine structures. The Vuong test is a likelihood-ratio based test often used for comparing different non-nested models.

Table 2.8 presents the Vuong test statistics and p-values for three sets of comparisons. The test results are interpreted in terms of the p-values. If the p-value of a test is smaller than 5%, we prefer the first model at the 5% significance level. If it is greater than 95%, the second model is preferred. Thus we can see from the tests that both C-vine and D-vine copulas are to be preferred over the Gaussian copula. The flexibility provided by the vine-structures and inspected individually in above examples are highly effective in the overall 12-dimensional joint dependence in the sample years. However, the comparison between the C- and D-vines, is less conclusive. A winner can be selected if we raise the significance level from 5% to 10%. Below the 10% significance level, the D-vine is preferred for 2002 and 2008, whereas ethe C-vine is desired only for 2005. For all other the years the difference is hardly significant. The fact that the D-vine has a slight edge over the C-vine is probably due to the fact that in the first tiers of C-and D-vines, the latter contains more highly correlated pairs.

Table 2.8 Vuong Test for Three Pairs of Comparisons

	2001	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**
C-Gaussian	5.975	5.811	6.446	5.573	4.283	5.209	5.446	6.252	4.893
p-value	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000
D-Gaussian	5.634	5.964	6.321	5.332	4.528	4.995	6.205	6.253	6.400
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
C-D	0.695	0.116	-0.394	0.739	-1.692	0.173	-1.208	-0.101	-1.491
p-value	0.487	0.908	0.693	0.460	0.091	0.863	0.227	0.920	0.136

Notes:
(i). C-Gaussian means comparison between C-vine copula and Gaussian copula.
(ii). The Vuong tests are interpreted by inspecting p-values. If it is smaller than the significance level, the former model in the comparing pair is preferred. If larger than one minus the significance level the latter is preferred. No decision can be made if in the middle.

2.5.2 Analysis of optimal portfolios

Influences of risk aversion and disappointment aversion

Tables 2.9 and 2.10 show seven statistics that describe the optimal portfolio under different constructions. In addition to conventional measures such as portfolio mean, standard deviation, and the Sharpe ratio, we also look for skewness, kurtosis, VaR (value at risk) and CVaR (the conditional value at risk). Table 2.9 provides an overview of copula model estimates when the risk aversion variable (RA) takes different values; the disappointment avoidance variable, A, is set for 2005 at $A = 0.25$, which is the least of the three commonly adopted disappointment avoidance values.

Table 2.10 is a comparison under three values of A when R$A = 20$ for the same year of 2005. Generally speaking, for 2005, the average daily returns of the optimal portfolios across the models are all positive. The distinction between the three models of Gaussian copula, D-vine and C-vine methods is clear in terms of skewness and kurtosis. For the rest of the measures, the differences are not as apparent, which lends the support for our use of DA preference. With the DA utility function, the portfolio optimisation can take into consideration the higher moments like skewness and kurtosis, which is the distinction between vine and Gaussian models.

Table 2.9 Descriptive Statistics for Different Degrees of Risk Aversion

when A=0.25 for 2005							
RA=3							
Model	Mean	s.d.	Sharpe ratio	skewness	kurtosis	VaR	CVaR
Gaussian	0.000451	0.005678	0.079404	-0.24963	4.856321	-0.00895	-0.24946
D-vine	0.000452	0.005992	0.075445	-0.23347	5.728709	-0.0094	-0.26427
C-vine	0.000449	0.005454	0.082337	-0.38095	14.861	-0.00844	-0.23346
RA=7							
Model	Mean	s.d.	Sharpe ratio	skewness	kurtosis	VaR	CVaR
Gaussian	0.000434	0.005085	0.085346	-0.10572	4.692941	-0.00785	-0.21691
D-vine	0.000433	0.005309	0.081606	-0.26512	11.21563	-0.00815	-0.227
C-vine	0.000439	0.00513	0.08565	-0.46894	23.37002	-0.00783	-0.2161
RA=10							
Model	Mean	s.d.	Sharpe ratio	skewness	kurtosis	VaR	CVaR
Gaussian	0.00042	0.004794	0.087557	-0.08073	4.685982	-0.00736	-0.20315
D-vine	0.000424	0.005117	0.082878	-0.22989	14.21159	-0.00781	-0.21689
C-vine	0.000433	0.005005	0.086599	-0.45262	23.90282	-0.0076	-0.20958
RA=20							
Model	Mean	s.d.	Sharpe ratio	skewness	kurtosis	VaR	CVaR
Gaussian	0.000307	0.002536	0.12104	-0.06807	5.009664	-0.0038	-0.10557
D-vine	0.000322	0.003789	0.085092	-0.01971	20.90247	-0.00575	-0.15901
C-vine	0.000333	0.003756	0.08871	-0.42971	23.38223	-0.00568	-0.15653

Table 2.10 Descriptive Statistics for Different Values of Asymmetry Preference

Portfolio Descriptive Statistics when RA=20 for 2005							
A=0.25							
Model	Mean	s.d.	Sharpe ratio	skewness	kurtosis	VaR	CVaR
MV	0.000307	0.002536	0.12104	-0.06807	5.009664	-0.0038	-0.10557
D-vine	0.000322	0.003789	0.085092	-0.01971	20.90247	-0.00575	-0.15901
C-vine	0.000333	0.003756	0.08871	-0.42971	23.38223	-0.00568	-0.15653
A=0.45							
Model	Mean	s.d.	Sharpe ratio	skewness	kurtosis	VaR	CVaR
MV	0.000307	0.002538	0.120982	-0.06858	5.007607	-0.00381	-0.10566
D-vine	0.000322	0.003788	0.085094	-0.01976	20.91866	-0.00575	-0.15898
C-vine	0.000333	0.003756	0.08871	-0.42983	23.38422	-0.00568	-0.15654
A=0.65							
Model	Mean	s.d.	Sharpe ratio	skewness	kurtosis	VaR	CVaR
MV	0.000307	0.002534	0.121074	-0.06889	5.011703	-0.0038	-0.10551
D-vine	0.000322	0.003789	0.085093	-0.01991	20.91404	-0.00575	-0.15899
C-vine	0.000333	0.003757	0.088708	-0.43131	23.37734	-0.00568	-0.15655

Notes:

(i).A is the disappointment avoidance parameter with its values ranging in $[0,1]$. With the disappointment avoidance utility, the investor treats the earnings above the expectation only as A times of the losses below the expectation. The smaller the value of A, the more emphases the investor puts on losses below expectation than on earnings above.

(ii).RA is the risk aversion parameter. The higher the value of RA, the more risk averse the investor is.

(iii).s.d. is short for standard deviations. The Sharpe ratio is calculated as the ratio between mean and s.d. representing return per unit of risk. VaR is short for Value at Risk. CVaR is short for Conditional Value at Risk.

In Table 2.9, one can see the effects of a change in risk aversion in any of the three models, especially in terms of the conventional risk measure, i.e. standard deviations. As the degree of risk aversion of the central bank increases, the portfolio with highest DA influence has less standard deviations and lower average returns. Table 2.10 shows the influence of the disappointment aversion effects. The smaller the value taken by A, the less tolerance of a negatively skewed distribution, implying that the possibility of negative extreme events is more stringently excluded. As expected, in all copula models skewness increases with the value of A. In what follows, we shall choose a pair of RA and A whose values are assumed to be the most likely representation of the central bank's preference. Given that the central bank is a very conservative institution in managing investment of its foreign reserves, we set A to take the smallest value from the range, i.e. 0.25, while RA equals to 20, the largest out of the four values to represent the central bank of China's behaviour.

Economic value of switching from mean-variance to pair-copula method

The notion of economic values can be traced back to Ang *et al.* (2005) and Hong *et al.* (2007). It calculates the certainty equivalent wealth gains based on the better fitted distribution model as compared to the coarser model. In this study, we use economic value to represent how much is earned by the pair-copula model compared to the mean-variance model. In so doing, we assume DA utility for the Chinese central bank and take into account the asymmetries, fat-tails and dependence complexities in the returns distribution. Hence, this performance

measure is built on a comprehensive base that incorporates the conservative property of the central bank and the advantages offered by copula modelling.

Let us denote the certainty equivalent wealth of a mean-variance model as W^{nor} and the certainty equivalent wealth of the D-vine model as W^{copu}. The certainty equivalent wealth is a scalar which will give the same amount of DA utility if the distribution of the wealth is plugged into the utility function. The notion of the economic values is that if the D-vine distribution is believed to be true, how much percentage of returns that the investor needs giving up in order to have the same DA utility as can be obtained from the traditional mean-variance method. This can also be regarded as the economic value of switching from a mean-variance to a pair-copula model. Denoting this amount as CE, it can be solved through the following equations:

$$DA(W^{nor}) = \frac{1}{K}\left(\int_{U(w^*)<E(\mathrm{U}(w^*))} U(w^*)p(R^{copu})\,dR^{copu}\right.$$

$$\left.+A\int_{\mathrm{U}(w^*)>E(\mathrm{U}(w^*))} U(w^*)p(R^{copu})\,dR^{copu}\right) \tag{2.33}$$

where

$$w^* = 1 + R^{copu} - CE \tag{2.34}$$

Table 2.11 displays the economic value of switching from mean-variance to the D-vine model when the disappointment avoidance parameter is taken to be 0.25 with five different risk aversion preferences. Across all risk preferences, Table 2.11

records that the annualized gain ranges from 0.563 basis points to 15.5%and the average is 0.962%. The annualized gains are calculated from the result from daily data assuming that there are 250 working days in a year. When the central bank of China takes the most conservative stance so that $RA = 20$, the average annual gain is even higher, at 1.05% for the period from 2001 to 2009.

Table 2.11
Economic Value of Switching from Gaussian Copula to D-Vine Copula Modelling

Economic value of Gaussian copula to D-vine when A=0.25				
	RA=3	RA=7	RA=10	RA=20
2001	8.68E-04	6.15E-04	2.04E-02	2.33E-02
2002	1.86E-04	3.13E-04	4.18E-04	4.63E-04
2003	5.63E-05	2.70E-04	2.93E-04	8.60E-03
2004	1.06E-02	6.53E-03	2.04E-03	3.05E-03
2005	2.53E-04	3.00E-04	4.55E-03	2.14E-02
2006	3.88E-03	1.11E-02	4.93E-03	7.95E-03
2007	1.92E-04	0.1515	4.78E-03	7.80E-03
2008	7.15E-03	2.70E-03	4.78E-03	1.41E-02
2009	4.75E-03	3.50E-03	4.78E-03	8.28E-03

Notes:
(i).The table shows the annualized economic value for attending features of asymmetries and fat-tails by switching from the Gaussian copula to the D-vine copula modelling. The value is calculated as how much earnings can be deducted to lower the D-vine copula model's utility down to the same level as the mean-variance model's utility.
(ii).A is the disappointment avoidance parameter with its values ranging in [0,1]. Under the disappointment avoidance utility, the investor treats the earnings above the expectation only as A times of the losses below the expectation. The smaller the value of A means that the more emphases the investor puts on losses below the expectation than earnings.
(iii).RA is the risk aversion parameter. The higher the value of RA, the more risk averse the investor is.

Comparison with foreign debt and trade constraints

In this sub-section, we analyse influences of two *ad hoc* weight constraints on the choice of currency portfolio. These two sets of constraints are in correspondence to the currency shares of China's external debt and shares of bilateral trade between China and a particular partner in China's total foreign trade. We have shown that the pair-copula method is beneficial, but the gains are obtained when no constraints are imposed on currency weights.

Taking foreign trade and debt into consideration will make our model resemble the reality more closely. One major function of a country's foreign reserves is to fulfil the payment needs of international trade and debt. These two constraints of minimal weights are set up following Papaioannou *et al.* (2006). Further application of this set up can be found in Wu (2007).

Table 2.12 presents trade shares of Chinese partners according to the IMF's Direction of Trade. We take 50% of these shares as the minimal weight in the optimal currency structure for China's foreign reserves. For example, in China's total international trade in 2009, trade with the US accounts for 13.55% of China's total trade in value terms and so we assume that in China's currency structure of foreign reserves, at least 6.775% should be kept in the USD.

The second constraint involves China's international financial activity. The currency shares of China's external debt are obtained from the Global Development Finance Database of the World Bank, and are listed in Table 2.13. A threshold of 50% of

these currency shares are taken for the minimal weight of the corresponding currency in China's currency composition of foreign reserves.

Table 2.14 shows annual gains of the economic value with foreign debt and international trade constraints. The average annualized economic value under the debt constraints is 4.12% and under the trade constraints it is 13.4%. These are greater than that in the case without weight constraints.

Optimal currency composition for China's reserves

We report estimates of the optimal currency composition for China's foreign reserves in Tables 2.15 and 2.16. The estimation is based on the generally preferred D-vine copula construction for the sample period of 2001 to 2009. Results in Table 2.15 are those obtained under the trade constraints, while outcome in Table 2.16 are derived with the external debt constraints. Across the sample years, we see a clear pattern of currency distributions, i.e. the US dollar, euro and Japanese yen are the three main currencies that consistently dominate the currency structure of China's reserves. Of these first tier currencies, the US dollar maintains the leading position despite occasionally being challenged in the early 2000s by the Japanese yen (in 2001) and the euro (in 2003). However, although the dollar's primary standing is solid, its edge over other currencies is not as great as conventionally thought. Generally, in China's case, the optimal proportion for the dollar in the reserves is around 40-45%. The big-three currencies are followed by a large group of second-tier currencies. This research has derived optimal shares for each of these currencies in China's reserves. They provide ample rooms for China to diversify its reserve holdings into non-dollar assets.

Table 2.12 Trade Shares of China's Partners

	2001	2002	2003	2004	2005	2006	2007	2008	2009
USD	15.80%	15.67%	14.87%	14.72%	14.92%	14.94%	13.94%	13.06%	13.55%
EURO	12.26%	11.61%	12.39%	12.24%	12.26%	12.35%	12.79%	12.93%	12.79%
JPY	17.22%	16.41%	15.69%	14.53%	12.97%	11.78%	10.85%	10.42%	10.37%
GBP	2.02%	1.83%	1.69%	1.71%	1.72%	1.74%	1.81%	1.78%	1.77%
CHF	0.47%	0.43%	0.42%	0.45%	0.41%	0.39%	0.44%	0.44%	0.44%
CAD	1.45%	1.28%	1.18%	1.34%	1.35%	1.32%	1.39%	1.35%	1.34%
AUD	1.76%	1.68%	1.59%	1.76%	1.91%	1.86%	2.01%	2.29%	2.71%
SND	2.14%	2.26%	2.27%	2.31%	2.34%	2.32%	2.17%	2.05%	2.17%
NZD	0.23%	0.23%	0.21%	0.22%	0.19%	0.17%	0.17%	0.17%	0.21%
KRW	7.04%	7.10%	7.43%	7.79%	7.87%	7.63%	7.36%	7.27%	7.07%
RUB	2.09%	1.92%	1.85%	1.83%	2.04%	1.89%	2.21%	2.22%	1.75%
THB	1.41%	1.38%	1.49%	1.50%	1.53%	1.57%	1.59%	1.61%	1.73%

Source: International Monetary Fund: Direction of Trade, various issues.

Table 2.13 Currency Shares of China's External Debt

	2001	2002	2003	2004	2005	2006	2007	2008	2009
USD	74.08%	72.45%	71.27%	70.77%	74.69%	76.27%	80.62%	81.68%	83.83%
EURO	4.74%	5.69%	7.16%	9.02%	8.00%	8.39%	8.07%	6.62%	6.21%
JPY	14.54%	15.39%	16.73%	15.92%	13.47%	12.02%	8.38%	9.14%	7.86%
GBP	0.10%	0.11%	0.11%	0.10%	0.09%	0.08%	0.07%	0.04%	0.03%
CHF	0.10%	0.11%	0.11%	0.10%	0.07%	0.06%	0.04%	0.03%	0.01%

Source: World Bank: Global Development Finance Database

Table 2.14
Economic Value of Switching from Mean-Variance to D-Vine Copula Modelling

Economic Values Constrained when A=0.25 and RA=20									
	2001	2002	2003	2004	2005	2006	2007	2008	2009
Debt Cons	6.33E-03	3.33E-04	2.47E-04	6.10E-03	0.142	1.11E-03	8.38E-03	0.1238	0.083
Trade Cons	0.552	4.70E-15	1.58E-15	0.0965	0.223	2.68E-03	0.23525	0.09075	1.51E-03

Notes:
(i).The table shows the annualized economic value for attending features of asymmetries and fat-tails by switching from mean-variance to D-vine copula Modelling. The value is calculated as how much earnings can be deducted to lower the D-vine copula model's utility down to the same level as the mean-variance model's utility.
(ii).The optimal currency compositions based on which the economic value is obtained are calculated with debt or trade constraints. These constraints are set as minimal weights of currencies for China's debt or transactions with its trading partners, and the weights are taken as 50% of each partner' share in China's debt or trade relation.

Table 2.17 shows the optimal currency composition for China if the Gaussian copula model with international trade constraints is used. The results are generally similar to those of the previous exercises, in that if we attend to both the trade and debt constraints in the copula model we derive an average proportion of 41.75% for the USD, whereas the conventional estimate of China's USD reserves is above 60%. However in comparison with the D-vine copula results (in Table 2.15), allocations under the Gaussian copula show heavier concentration on several currencies. This means that the Gaussian copula approach may squeeze the space for efficient currency diversification.

Table 2.15
Currency Composition by D-vine Copula with Trade Constraints

	2001	2002	2003	2004	2005	2006	2007	2008	2009
USD	7.97%	38.07%	7.65%	35.45%	12.46%	19.92%	7.46%	50.15%	31.14%
EURO	6.21%	7.25%	21.10%	9.48%	11.13%	6.48%	6.77%	6.70%	6.80%
JPY	75.41%	8.29%	7.96%	7.37%	11.48%	5.97%	5.53%	22.78%	24.64%
GBP	1.39%	8.29%	18.58%	7.29%	5.89%	11.76%	1.30%	1.15%	1.00%
CHF	0.34%	13.58%	0.49%	0.25%	5.22%	0.33%	0.37%	1.23%	0.75%
CAD	0.72%	1.30%	0.90%	0.99%	12.03%	9.83%	16.19%	1.24%	2.28%
AUD	1.01%	1.00%	1.09%	0.98%	5.48%	2.24%	1.67%	2.06%	5.52%
SND	1.33%	1.45%	1.29%	1.30%	6.20%	28.94%	2.44%	5.41%	2.46%
NZD	0.22%	2.54%	34.78%	8.91%	4.59%	1.48%	0.64%	0.30%	0.63%
KRW	3.53%	4.50%	4.03%	5.46%	8.90%	8.22%	3.68%	3.84%	3.55%
RUB	1.05%	12.96%	1.12%	21.76%	10.90%	4.03%	46.93%	3.20%	1.25%
THB	0.82%	0.76%	1.02%	0.75%	5.73%	0.79%	7.02%	1.94%	20.00%

Table 2.16 Currency Composition by D-vine Copula with Debt Constraints

	2001	2002	2003	2004	2005	2006	2007	2008	2009
USD	46.32%	45.99%	35.77%	35.52%	37.68%	38.48%	40.69%	49.46%	45.86%
EURO	2.48%	4.97%	19.05%	15.41%	4.21%	4.49%	4.41%	3.94%	7.91%
JPY	7.40%	7.78%	8.46%	8.08%	6.82%	6.09%	4.29%	14.36%	13.72%
GBP	0.99%	9.74%	3.41%	0.48%	0.33%	8.23%	0.43%	0.17%	2.58%
CHF	0.20%	14.75%	0.33%	0.25%	0.12%	0.17%	0.18%	2.08%	5.95%
CAD	41.11%	0.70%	0.23%	0.24%	27.24%	6.72%	17.37%	4.22%	0.02%
AUD	0.18%	0.17%	0.31%	0.17%	12.58%	0.93%	0.85%	4.73%	5.05%
SND	0.30%	0.39%	0.12%	0.13%	0.20%	22.78%	1.02%	7.56%	1.60%
NZD	0.13%	2.94%	31.51%	38.77%	6.52%	1.25%	0.63%	4.18%	6.27%
KRW	0.70%	2.49%	0.28%	0.89%	2.81%	7.48%	1.08%	2.82%	1.89%
RUB	0.07%	9.96%	0.28%	0.05%	0.88%	3.32%	23.53%	4.54%	5.14%
THB	0.12%	0.12%	0.24%	0.02%	0.60%	0.05%	5.51%	1.94%	4.02%

Table 2.17 Currency Composition by Gaussian Copula with Trade Constraints

	2001	2002	2003	2004	2005	2006	2007	2008	2009
USD	32.97%	32.71%	7.67%	7.60%	7.72%	9.12%	7.46%	43.90%	35.22%
EURO	6.25%	7.17%	21.46%	27.59%	6.34%	6.47%	6.77%	6.51%	6.87%
JPY	8.75%	8.32%	7.98%	7.47%	6.59%	5.97%	5.52%	21.84%	25.15%
GBP	1.57%	7.18%	13.51%	1.48%	1.11%	8.22%	1.30%	0.94%	1.01%
CHF	0.39%	16.89%	0.58%	0.55%	0.31%	0.33%	0.37%	0.29%	0.73%
CAD	39.95%	1.68%	0.99%	1.11%	26.96%	13.11%	19.56%	0.85%	2.88%
AUD	1.09%	1.05%	1.38%	1.10%	5.99%	1.61%	1.67%	1.31%	5.98%
SND	1.42%	1.70%	1.32%	1.36%	1.37%	42.38%	2.45%	1.57%	2.63%
NZD	0.27%	3.20%	37.85%	43.74%	5.76%	0.78%	0.63%	0.24%	0.54%
KRW	5.24%	4.91%	4.10%	4.16%	4.32%	6.60%	4.16%	3.69%	3.62%
RUB	1.23%	14.30%	2.06%	2.87%	32.50%	4.56%	42.60%	1.78%	1.24%
THB	0.86%	0.88%	1.10%	0.96%	1.02%	0.85%	7.49%	17.07%	14.11%

2.6 Conclusions

An appropriate currency structure is an essential aspect of sound management of foreign reserves. In this chapter, we set up a flexible framework based on pair-copula construction. This approach allows us to model critical features of currency returns, including the asymmetry, fat-tails and complex dependence structure. In the context of China, we apply the copula model to analyse how these features affect the currency returns and to derive an optimal currency structure for China's reserves management.

Each currency return is first modelled using a variety of ARMA-GARCH filters with different residual distributions to best suit dynamics in univariate returns series. The dependency structure to connect each currency returns are then modelled by pair-copula construction with two different vine structures. Based on the established distribution we use the preference under the disappointment aversion effect as the optimising objective to obtain the optimal currency composition. Our comparison shows that the mean-variance method cannot reflect the skewness whereas the pair-copula method can capture the features of higher moments such as skewness and kurtosis. Our further comparison shows the economic value of switching to the pair-copula models from the mean-variance framework. Considering the enormous amount of the international reserves held by emerging economies such as China, the central bank in our model can achieve sizable gains.

To analyse the Chinese case, we mimic China's currency shares of external payments by imposing *ad hoc* weight restrictions according to China's foreign trade and debt relations. Evidence shows that the pair-copula model with the D-vine structure has

advantages over other methods. In this approach, the US dollar consistently takes the largest share in China's reserve currency composition. However, incorporation of the features of asymmetry, fat tails and complex dependence structure would allow more rooms for other currencies to be chosen for currency diversification of China's reserves. It is therefore desirable and feasible for China to adopt the copula approach the currency composition of its reserves and diversification is important for countering dependence complexities to manage currency composition of its huge and growing reserves.

CHAPTER THREE

This chapter studies strategic asset allocation for China's foreign reserves using a risk-based approach. First, the background and motivation behind the analysis is presented. Then, a regime-switching copula model is developed to investigate the dynamic dependence between assets. Next, the optimal allocation is derived following two strategies: risk minimization and trade-off between risk and returns in utility maximization with disappointment avoidance. Finally, it is suggested according to analysis that China should mitigate its 'flight to safety' actions after 2008 and increase holdings of short-term bank deposits, long-term treasury bonds and euro bonds.

CHAPTER 3

STRATEGIC ASSET ALLOCATION FOR CHINA'S FOREIGN RESERVES IN SAFETY TRANCHE

3.1 Introduction

Sound management of foreign reserves has been a constant concern for central banks (Nugee, 2000). In recent years, this has acquired a new dimension due to the fallout of the global financial crisis. Yu (2011) maintains that the real value of China's foreign reserves is whipsawed by the price drop of the US treasuries and devaluation of the dollar. Dominguez *et al.* (2012) and Walther (2012) point out that, with the global financial crisis, countries are faced with an environment of low international yield with rising levels of reserves, whereby the social costs incurred for a large reserve holder can be substantial. According to the estimation of Zhang and Zhang (2007), relative to the mean capital returns of Chinese industries, the opportunity cost of China's reserves amounted to about 168 billion dollars in 2006. Wang (2012) shows that, for 2001 through 2011, the average yearly opportunity cost of China's reserve holdings is 114 billion dollars, or 2.6% of GDP.

Literature has underscored the contribution of strategic asset allocation to yield performance. Brinson *et al.* (1986) show that, in the case of US defined benefit pension plans over the period 1974 to 1983, 93.6% of the return variation for 91 such pension plans can be explained by asset allocation. Blake *et al.* (1999) suggest that

99.5% of the returns for more than 300 UK pension plans from 1986 to 1994 can be accounted for by strategic asset allocation. In terms of cross-sectional variation explained by the strategic asset allocation, i.e. performance difference among various funds due to their asset allocation, Ibbotson and Kaplan (2000) use 94 US mutual funds across the period 1988-1998, and find that about 40% of the variation of returns among the funds are due to asset allocation, while 60% are due to asset-class timing and security selection.

In this chapter, we consider strategic asset allocations for China's reserves in an approach that is based on risk management. The notion of 'risk management' implies that, among the tradition policy goals of reserve management that is liquidity, safety and returns, this study focuses on the safety objectives and leave the returns objectives to be fulfilled by special investment vehicles such as the sovereign wealth funds, which is common practice in reserve-abundant nations.

Risk management of foreign reserves can have many facets, but what we intend to explore are the following four areas: investment universe; dependence structure; risk measures and optimal allocation; and the decision of 'flight to safety'. Given the particular importance of the US market to China's reserve allocation, we use the data from that market as representative of China's foreign asset allocation policy.

Using these data, we build our investment universe for possible investment of Chinese reserves. We then consider the impact of the dependence structure on the management of the Chinese central bank's investment portfolio. In the process, we

apply the copula approach to model the dependence between assets. Then, the regime-switching dependence is estimated using the Hamilton (1989) filter.

The Conditional Value-at-Risk (CVaR) is taken as the risk measure in our study. Two strategies are adopted to derive the optimal asset allocation for China: one is based on CVaR and the other on Disappointment Avoidance utility maximisation. Finally, we examine the influences of dependence asymmetry and the fat-tails on the flight to safety, which is a widespread phenomenon in the time of the recent global financial crisis.

This chapter intends to make a number of contributions to the literature. It incorporates asymmetries and fat-tails into the decision on foreign reserve asset allocation. According to the market data, it tests whether central banks should engage in the flight to safety in response to the global financial crisis. A new copula structure is proposed in a multivariate dependence modelling environment. While in a common vine-copula structure, only some of the variable pairs can be directly described as copulas and their asymmetric dependence is accurately reflected, we devise a regime-switching model which can enlarge the describable range to all the variables we are interested in. This is particularly useful in our analysis of the 'flight to safety'.

The rest of the chapter is arranged as follows. In section 3.2, we briefly present the related literature. In sections 3.3 to 3.6, we discuss the four main aspects of the risk-based foreign reserves management in the context of China. Specifically, section 3.3 presents a discussion on the investment universe for China's reserve asset

management. In section 3.4 we discuss the importance of dependence structure in asset allocation. To properly capture the dependence structure in the data, we argue that it is desirable and necessary to apply the copula approach. Section 3.5 is devoted to the optimal asset allocation under two different strategies: the optimisation based on CVaR risk measurement and the Disappointment Avoidance utility maximisation. Section 3.6 furthers our study to consider the influences of dependence asymmetry and the fat-tails on the flight to safety. Section 3.7 concludes.

3.2. Related Literature

For asset allocation by central banks, Cardon and Coche (2004) propose a three-tier organisational establishment consisting of an oversight committee in charge of currency allocation, an investment committee in charge of asset allocation benchmark and a portfolio management team to carry out portfolio mandates. The purpose of such a structure is to ensure a central bank's requirements for liquidity, safety and returns. Putnam (2004) proposes a double-tranche management strategy, comprising a liquidity-challenged tranche and a volatility-premium tranche, to answer the reserve management requirement on liquidity and the desire for returns. Claessens and Kreuser (2004) also recognise the three investment objectives of central banks, i.e. liquidity, safety and returns, and propose a method that can incorporate these multiple objectives with macroeconomic and microeconomic factors and market conditions within a stochastic optimisation framework. Gintschel and Scherer (2004) propose a dual benchmark optimisation framework to consolidate the two requirements of liquidity and capital preservation simultaneously. Borio *et al.* (2008) detail the specifics of these requirements. Also, as proposed by León and

Vela (2011), foreign asset portfolio's construction departs from the conventional Modern Portfolio Theory (MPT) for two reasons. First, foreign reserve management is highly restrictive under the authority's control which has to meet a critical range of objectives as mentioned above such as liquidity, safety and profitability. Second, central banks show severe risk aversion towards financial losses. As such, there is a trend in recent research that captures the risk-return trade-off problems of different assets class to emphasize different motivations for assets allocation. In the latest literature, Rivadeneyra, *et al.* (2013) introduce a model on Canada's foreign exchange reserves asset allocation, claiming that the best allocation of foreign assets is by balancing the preferences of liquidity and returns. Overall, these analyses highlight foreign reserve allocation as governed by the requirements for liquidity, safety and profitability, in decreasing order of importance.

Development of the relevant studies in this field is also shaped by the international environment. Since the late 1990s, rising levels of reserves have placed the focus on diversifying investment using the asset classes with ample safety, to seek for higher returns. This often means expanding of the conventional investment universe. Fisher and Lie (2004) promote five improvement principles: broader investment universe with non-government bonds, e.g. mortgage-backed securities (MBS), asset-backed securities (ABS), corporate bonds, etc.; differing currency and country allocation; using transaction cost constraint to control liquidity; risk control at total portfolio level instead of at individual country level; and control default risk at total level. Remolona and Schrijvers (2004) explore the opportunities for investing in high yield securities in three ways: longer duration bonds, corporate bonds, and high-yielding

currencies. Ferket and Zwanenburg (2004) give two recommendations. The first is to broaden the investment universe by hedging currency risks, while the second is to perform more active duration management.

The current global financial crisis has seen intense debate about central banks' flight to safety and related operations regarding the safe assets (Caballero and Krishnamurthy, 2008; Beber *et al.*, 2009). McCauley and Rigaudy (2010) analyse data from the Bank of International Settlement (BIS) and find three characteristics in the movements of official holdings in the US dollar. First, there is no apparent duration shortening in the official reserve investment. Second, however, if the portfolios are divided into short-term and long-term sections, within each section there can be seen changes in asset classes featuring the flight to quality. Third, looking into the future, motive or pressure to diversify into high yielding assets still persists and the interrupted trend should resume, but with more caution. In their study, the failure of Lehman Brothers is an important event, which was followed by rapid flights to quality assets such as the short- and long-term treasury bonds.

Research carried out by the International Monetary Fund (IMF) suggests that the flight to safety by central banks should be discouraged. Pihlman and van der Hoorn (2010) argue that the withdrawal of deposits from banks in an apparent flight to safety would cause funding problems for the banking sector. Such behaviours would lead to other central banks' offsetting measures and would destabilise the market. An IMF (2012, Chap. 3) study discusses the post-crisis demand and supply of safe assets. It makes the similar point that flight to safety on the part of central banks would

worsen the shortage of safe assets in the market, which is contrary to the objectives of central banks.

Dependence structure of asset returns is very important in the risk management of portfolios (Patton, 2004; Fortin and Hlouskova, 2011). If the dependence between assets is not correctly specified, identification of risk might be misplaced. According to Poon *et al.* (2004) and Tastan (2006), a linear dependence measure, or the conventional Pearson's correlation, can lead to underestimation of risk. Extreme value theory (Longin and Solnik, 2001; Bae *et al.*, 2003), on the other hand, tends to overestimate the risks (Poon *et al.*, 2004) and fails to capture the tail dependence in the limits (Garcia and Tsafack, 2011). Using the copula method to model the dependence between assets proves to be a promising avenue for research in this area as the literature shows that the copula approach, especially the pair-copula construction has the right property to present the dependence structure (Joe, 1997; Bedford and Cooke, 2002; Aas *et al.*, 2009).

Dependence states may change with time. Van den Goorbergh *et al.* (2005) and Patton (2006) suggest time-varying parameters in a copula function, whereas Rodriguez (2007), Garcia and Tsafack (2011) and Wang *et al.* (2013) propose dynamic copula functions over time. Garcia and Tsafack (2011) identify one symmetric dependence structure and one asymmetric. The interchange between the two structures is governed by a Markov chain. Wang *et al.* (2013) also utilise a two-state regime-switching model, with which they investigate the possibility of changes between positive dependence and negative dependence.

Central banks can be affect by dependence asymmetry, especially in its decision on 'flight to safety'. Asymmetries in dependence are well documented in the literature (A ït-Sahalia and Brandt, 2001; Longin and Solnik, 2001; Ang and Chen, 2002; Bae *et al.*, 2003; Hong *et al.*, 2007; and Ammann and Suss, 2009). Safe assets are those which are deemed to be relatively low risk while at the same time offering high liquidity, in that there is a ready market to buy and sell them. For the 'flight to safety', the short- and long-term treasury bonds are very important (McCauley and Rigaudy, 2010).

Properly defining risk is the foundation of risk management. Value-at-Risk (Var) and Conditional Value-at-Risk (CVaR) are two most popular way to define risky in current risk management practic. As propose by Gordon and Baptista (2004), CVaR serves as a more coherent risk measure. Also they argue that CVaR has superior properties not only mathematically but also statistically compared with VaR. Therefore we define risk on the basis of (CVaR) introduced by Rockafellar and Uryasev (2000) as a popular tool for risk management.

3.3 Investment Universe

In the risk-based management of foreign reserves, selection of the permissible asset classes as the investment universe is of primary importance. As a first approximation, we use the actual allocation of Chinese reserve investment in the US market as the possible investment universe for China. We first look at the component assets and then use representative indices for each asset class to provide some descriptive analyses.

3.3.1 Reported Chinese asset allocation

The information for China's official asset class distribution is confidential. However, the U.S. Treasuary International Captial System (TIC) published annual survey report on the foreign holdings of U.S. securities provides researchers insights on China's foreign reserves dollar asset allocation. We also use these data as summarized in Table 3.1 as a starting point to gather information about China's foreign reserves investment objectives and the permissive universe for our analysis. It is worth noting that the TIC information cannot accurately reflect China's official holdings of U.S. securities. Setser and Pandey (2009) maintain that the TIC data fails to capture part of China's investment, if it is channelled into the U.S. from non-U.S. investment institutions. Zhang *et al.* (2010) argue that there is another factor leading to the TIC data as an inaccurate estimate. In the report, investments from the official and private investors are not distinguished. Taken both viewpoints into consideration, Zhang *et al.* (2010) estimate that the U.S. dollar asset should take around 59.5% ~ 62.6% in China's official reverses investment. We still use the TIC data in the following analysis because it has wider coverage, and we do not need very accurate estimates for our above mentioned the information.

We assume that China's investment in all-maturity euro government bonds accounts for 20% of its overall foreign reserve investment. All other US asset classes in Table 3.1 are based on the data of actual compositions, with their sum being scaled down to 80% of the total Chinese reserve investment.

Table 3.1 Allocation of China's Reserve Assets in the US Market

		2005	2006	2007	2008	2009	2010	2011
Short-Term	**Treasury**	5.86%	1.92%	1.95%	1.97%	8.56%	0.24%	0.22%
	Deposit	2.21%	1.86%	1.99%	1.67%	1.62%	1.10%	1.69%
Long-Term	**Treasury**	40.88%	40.70%	39.47%	33.92%	40.54%	54.28%	59.07%
	Agency	25.38%	28.55%	31.84%	34.26%	24.31%	17.64%	11.10%
	Corp	5.30%	6.54%	2.34%	1.71%	0.81%	0.54%	0.72%
	Equity	0.38%	0.43%	2.41%	6.47%	4.16%	6.20%	7.20%
	Euro	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%

Source: Wang (2011) and US Treasury International Capital System (TIC) data.

The two safe assets in the US market are the short- and long-term treasury debts. Throughout the period 2005-2011, there is a general trend for China to diversify its reserve investment into long-term asset classes in order to reap higher returns and offset the high carrying costs that accompany the high volume of China's foreign reserves. However, the central bank of China is in any case a conservative investor, and therefore the US long-term treasury bonds are the most favourable choice. These are followed by the investment in US agency debts as the second favourite selection.

From 2008 to 2009 we see a sudden rise in holdings of the short-term treasury bonds, reversing a long-term declining trend. Meanwhile, holdings of the long-term treasury bonds also increased, while holdings of long-term agency debts and equities decreased. We therefore see a reversal of an earlier trend of pursuing returns, and a general 'flight to safety' in response to the crisis. Subsequently, the returns-pursuing trend seems to have recovered, since the share of short-term treasury in the total investment dropped again and the long-term investments in treasury bonds and private equities rose. However, this time we can observe that the reserve managers are more cautious than before, because investments are more concentrated around the

long-term treasury bonds, and the share of agency debts in the overall investment never really picked up.

3.3.2 Investment universe and descriptive analysis

From historic holdings data, one may find what assets are commonly held by reserve managers. In the case of China, Chinese reserve assets are held in six US asset classes: short-term treasury bills， bank deposits, long-term treasury bonds, agency debts, corporate bonds, and equities (Wang, 2011). In addition, to account more closely for the reality, we assume that Chinese fund managers also hold some government bonds in euros. We then choose seven representative indices for each asset class as shown in Table 3.2.

Table 3.2 Data Sources

		Index	Frequency	Source	Mnemnic Code
Short-term	Treasury	JPM GBI US 1-3Y (US$)	Weekly	Thomson Reuters DataStream	JGUSBU$
	Deposit	JPM US CASH 12M			JPUS12L
Long-term	Treasury	US BENCHMARK 10 YEAR DS GOVT. INDEX			BMUS10Y
	Agency	BARCLAYS US AGGATES AGENCIES			LHUSAGN
	Corp	TR US CORP BMK AAA 10Y YIELD(US$)			TRUCCYJ
	Equity	MSCI USA			MSUSAML
	Euro	JPM EMU GOVERNMENT ALL MATS. (US$)			JEAGAU$

Notes:
MSCI stands for Morgan Stanley Capital International. TR US CORP BMK AAA stands for Thomson Reuters US corporate benchmark AAA graded bond index. JPM stands for JP Morgan. GBI and EMU stand for Global Bond Index and Economic and Monetary Union.

The short-term treasury bills and bank deposits are represented by the JP Morgan Global Bond Index 1-3-year and the US Cash index 12-month, respectively. The long-term treasury bonds are indicated by DataStream's benchmark US 10-year

government bond index. The agency debts are according to Barclays' US Aggregate Agency Debts. For corporate debts we use Thomson Reuters' US corporate benchmark AAA 10-year index. The equities are represented by the MSCI USA index.

Table 3.3 reports the descriptive statistics for the selected asset classes. Average returns, the covariance matrix and an empirical copula are calculated using the previous three years' weekly data. For the covariance matrix we show only the year 2010 for the economy of space, and with respect to the empirical copula we choose the relationship between the short- and long-term treasury bonds in 2010 as representative.

As expected, the returns of the short-term assets are lower than those for the long-term assets. Also, among all assets, both short- and long-term, the treasury bonds are the safe assets with low returns as well as low variance. It can be seen from the covariance matrix that many have negative values. This shows great potential for diversification. In the empirical copula section of the table, one may inspect the dependence structure (Wang *et al.*, 2013). Take the relation between the short- and long-term treasury bonds in 2010 as an example. Six quantile bins are applied for each series to create a 6 by 6 ranking table. The values are smallest at the top-left corner and increase towards the bottom-right corner. The frequency of the pair value appearing in the quartile is recorded in each respective bin. As can be seen from the numbers in bold in the top-left and bottom-right bins, they are greater than other values. This indicates the possibility of tail dependence between the series.

Table 3.3 Descriptive Statistics for Investment Universe

Average Returns								
		2006	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**
Short-Term	**Treasury**	-0.000295	8.77E-05	3.51E-04	2.61E-04	7.08E-05	-6.61E-05	-3.29E-05
	Deposit	0.0005453	8.56E-04	1.01E-03	9.28E-04	0.0006741	4.21E-04	3.21E-04
Long-Term	**Treasury**	-0.000299	7.82E-05	1.02E-03	0.0004788	0.0004052	0.0004124	0.0013924
	Agency	-0.000273	8.52E-05	3.34E-04	2.09E-04	5.882E-05	8.78E-05	2.67E-04
	Corp	-0.000236	-2.48E-04	-1.78E-03	-0.000561	-0.000695	-0.001306	-0.004109
	Equity	0.0015637	1.03E-03	-1.86E-03	-0.001469	-0.000688	0.0019764	0.0016277
	Euro	0.0010692	1.02E-03	1.74E-03	1.49E-03	0.0002337	-1.10E-05	0.0001953
Covariance Matrix for returns in 2010								
		Short-term		**Long-term**				
		Treasury	**Deposit**	**Treasury**	**Agency**	**Corp**	**Equity**	**Euro**
Short-Term	**Treasury**	5.38E-06	1.28E-06	2.13E-05	8.27E-06	-4.82E-05	-3.12E-05	1.04E-05
	Deposit	1.28E-06	1.52E-06	5.65E-06	2.77E-06	-1.67E-05	-1.55E-06	4.69E-06
Long-Term	**Treasury**	2.13E-05	5.65E-06	0.0001712	5.46E-05	-0.000422	-0.00018	7.02E-05
	Agency	8.27E-06	2.77E-06	5.46E-05	2.35E-05	-0.000146	-3.56E-05	3.18E-05
	Corp	-4.82E-05	-1.67E-05	-0.000422	-0.000146	0.0015279	0.0004399	-0.000193
	Equity	-3.12E-05	-1.55E-06	-0.00018	-3.56E-05	0.0004399	0.0013935	0.000152
	Euro	1.04E-05	4.69E-06	7.02E-05	3.18E-05	-0.000193	0.000152	0.0003496
Empirical copula between short and long term Treasury in 2010								

	15	6	3	1	1	0		
	7	6	9	1	1	2		
	2	9	4	6	4	1		
	2	3	7	7	3	4		
	0	1	2	7	11	5		
	0	1	1	4	6	**14**		

Notes:
The ranking table of empirical copula starts from the top-left corner, and increasing till the bottom-right corner. For example, the highlighted 15 in the top-left corner means the high frequency of both variables at the smallest quantile boxes.

3.4. Regime-Switching Copula Dependence

In order to assess investment risks, it is essential to capture the dependence characteristics correctly. The copula method can separate the univariate modelling of each asset return variable from its dependence modelling. For each return variable, the ARMA-GARCH model can be applied to capture its dynamics. In modelling multivariate dependence, the pair-copula construction (or the vine-copula construction) method avoids the over-simplification of the common multivariate copula.

3.4.1 Univariate return models

To account for the volatility clustering and autocorrelation in the individual returns, a traditional ARMA-GARCH model is used. We find that a simple ARMA (1,1) – GARCH (1,1) parameter can return most of the individual series back to the independently identically distributed state. We use the Glosten-Jagannathan-Runkle-GARCH (GJR-GARCH) to capture the leverage effect (where a negative shock exerts larger impact than a positive one), and a skew-t distributed residual term is used to replace the normally distributed residuals for the purpose of clearing the skewness and fat-tails in the individual series. The specifications are shown in the following equations:

$$r_t = c_0 + a_0 r_{t-1} + b_0 \varepsilon_{t-1} + \varepsilon_t, \tag{3.1}$$

$$\varepsilon_t = \sigma_t z_t, \tag{3.2}$$

$$\sigma_t^2 = c_0 + a_1\sigma_{t-1}^2 + b_1\varepsilon_{t-1}^2 + d_1\varepsilon_{t-1}^2 I_{t-1} \tag{3.3}$$

$$z_t \sim ST(\eta_t, \lambda_t) \tag{3.4}$$

where $I_{t-1} = 0$ if $\varepsilon_{t-1} \geq 0$, and $I_{t-1} = 1$ if $\varepsilon_{t-1} < 0$.

3.4.2 Pair-copula construction

We apply a pair-copula construction method to model the multivariate dependence. It allows for a multivariate density function to be decomposed into the product of several conditional bivariate copulas and the density functions of each individual variable. Different combinations of the decomposition elements can be selected; these are called 'vines'. Among the most frequently used vines are the D-Vine and C-Vine; they underscore the parallel relationships among variables and the strength of a pivotal variable in the multiple relationships respectively. We consider the C-Vine to be more suitable for our situation because of the central bank's emphasis on the safe asset class.

A multivariate density function can be decomposed into the products of multiple conditional density functions in the following manner:

$$f(x_1, \dots, x_n) = f(x_n) \cdot f(x_{n-1}|x_n) \cdot f(x_{n-2}|x_{n-1}, x_n) \cdots f(x_1|x_2, \dots, x_n) \tag{3.5}$$

For an element of the product on the right-hand-side of the equation, a conditional density function can be decomposed into the conditional bivariate copula as in the following example:

$$f(x_1|x_2,x_3) = \frac{c_{13|2}[F(x_1|x_2),F(x_3|x_2)]\cdot f(x_1|x_2)\cdot f(x_3|x_2)}{f(x_3|x_2)} \quad (3.6)$$

$$f(x_1|x_2,x_3) = c_{13|2}[F(x_1|x_2),F(x_3|x_2)]\cdot f(x_1|x_2) \quad (3.7)$$

where $f(x_1|x_2)$ can be further decomposed using the same method, so:

$$f(x_1|x_2,x_3) = c_{13|2}[F(x_1|x_2),F(x_3|x_2)]\cdot c_{12}[F(x_1),F(x_2)]\cdot f(x_1) \quad (3.8)$$

At the end of this process each conditional density function can be in the format as in Equation 3.8, as a product of the copulas and a single variable density function. However, the arrangements of the copula, and which two variables to include, can be determined by the researcher. The C-Vine structure can be illustrated in the following figure:

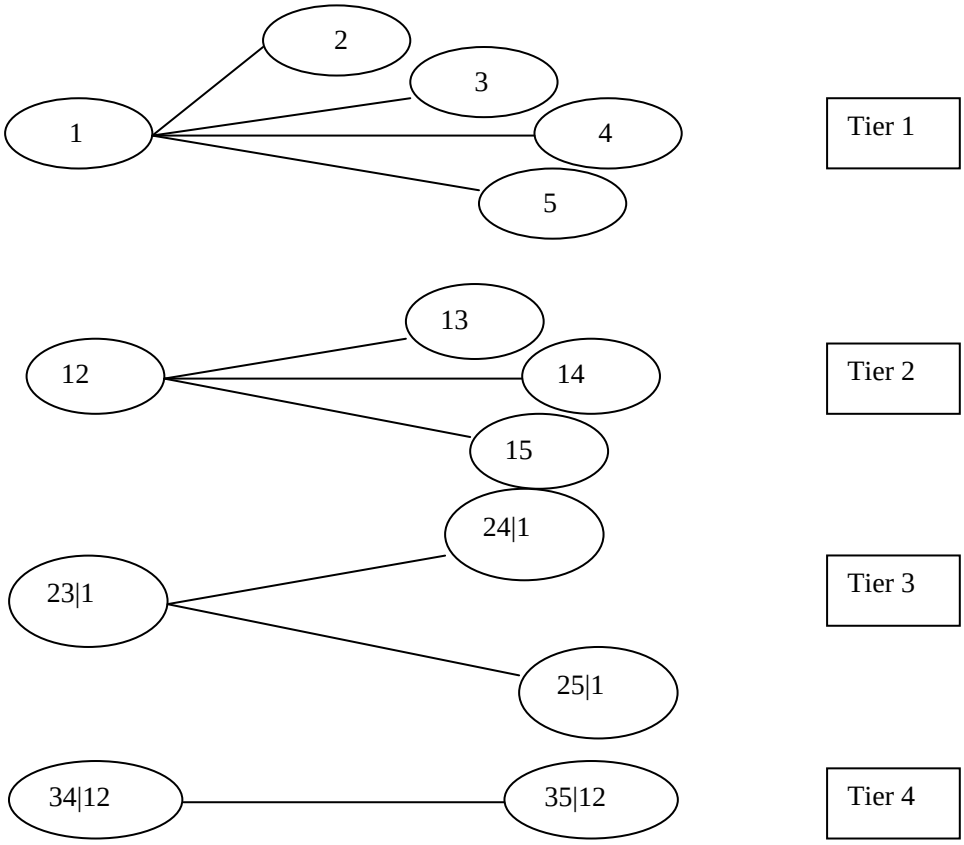


Fig. 3.1 Demonstration of A 5-Variable C-Vine Structure

The n-dimensional density for the C-Vine copula for the later maximal likelihood estimation is as in Equation 3.9:

$$\prod_{k=1}^{n} f(x_k) \prod_{j=1}^{n-1} \prod_{i=1}^{n-j} c_{j,j+i|1,\dots,j-1}\{F(x_j|x_1,\dots,x_{j-1}), F(x_{j+i}|x_1,\dots,x_{i+j-1})\} \quad (3.9)$$

In addition to the vine structure of the copula for multivariate dependence modelling, it is perhaps even more important to select the type of bivariate copulas to be fitted in the vine structure. Common selections include the normal copula, the student-t copula, and two types of Archimedean copula: the Clayton and the Gumbel copulas. The normal copula is the dependence structure of a multivariate normal distribution. It can reflect neither asymmetric dependence nor fat-tails. The student-t copula can capture fat-tails, and the higher possibility of extreme events. However, its dependence structure must be symmetric. The Clayton copula reflects negative tail dependence, whereas the Gumbel copula describes positive tail dependence. Given that central banks as an investor group behave conservatively, and are averse to their investment risks, especially the possible losses, the Clayton copula is most suitable to reflect that stance.

The density function of the bivariate Clayton copula is:

$$c(u_1, u_2) = (1+\theta)(u_1 \cdot u_2)^{-1-\theta} \times (u_1^{-\theta} + u_2^{-\theta} - 1)^{-\frac{1}{\theta}-2} \quad (3.10)$$

where $\theta > 0$ represents the dependence. The greater the parameter value gets, the more dependent the two variables are: $0 \leq u_1, u_2 \leq 1$.

The parameter can be easily transformed into the tail dependence for the dependence of their extreme losses. We denote the cumulative distribution functions of random

variables X and Y as F_X and F_Y. As in Nelson (2006), the lower tail dependence can be described as the limit of a conditional probability such that $\lim_{\alpha \to 0} \varphi^L = \Pr[X \leq F_X^{-1}(\alpha) | Y \leq F_Y^{-1}(\alpha)]$ for $\alpha \in (0,1)$, whereas the upper tail dependence is $\lim_{\alpha \to 0} \varphi^L = \Pr[X \geq F_X^{-1}(1-\alpha) | Y \geq F_Y^{-1}(1-\alpha)]$. The Clayton copula's parameter can be turned into the lower tail dependence, by:

$$\varphi = 2^{-\theta} \tag{3.11}$$

3.4.3 Regime-switching pair-copula

Both the D-Vine and the C-Vine copula vine structures have different tiers. In the case of a five-variable C-Vine, there are four tiers. Only the first tier of this copula construction directly models the data series. For the deeper tiers, the copula captures the dependence between the conditional function transformed data. Although these processed data can help flexibly model the multivariate dependence structure, they lose their intuitive interpretation since they are no longer the original returns of the asset classes. Fortunately, the first tier contains the most nodes, and in the C-Vine structure a pivotal asset can be selected. Its relationship with all the other assets can be displayed in the first tier.

Furthermore, we build a two-state regime-switching model with two different C-Vines:

$$\begin{aligned} &c(F(x_1), \dots, F(x_7); \boldsymbol{\theta_1}, \boldsymbol{\theta_2} | s_t) \\ &\quad = s_t c_1(F(x_1), \dots, F(x_7); \boldsymbol{\theta_1}) + (1 - s_t) c_2(F(y_1), \dots, F(y_7); \boldsymbol{\theta_2}) \end{aligned} \tag{3.12}$$

$$c_1(F(x_1), \dots, F(x_7); \boldsymbol{\theta_1})$$

$$= \prod_{j=1}^{6} \prod_{i=1}^{7-j} c_{j,j+i|1,\dots,j-1}\{F(x_j|x_1, \dots, x_{j-1}), F(x_{j+i}|x_1, \dots, x_{i+j-1})\}$$

(3.13)

$$c_2(F(y_1), \dots, F(y_7); \boldsymbol{\theta_2})$$

$$= \prod_{j=1}^{6} \prod_{i=1}^{7-j} c_{j,j+i|1,\dots,j-1}\{F(y_j|y_1, \dots, y_{j-1}), F(y_{j+i}|y_1, \dots, y_{i+j-1})\}$$

(3.14)

where $y_1, \dots, y_7$ are $x_3, x_1, x_2, x_4, \dots, x_7$, the original sequence $x_1, \dots, x_7$ with the third element, x_3, extracted and put in the first place, which is the position for the pivotal variable in a C-Vine structure. In a seven-asset-class dependence model, the two C-Vines have different pivotal assets, x_1 and x_3, representing the short- and long-term safe assets respectively. $s_t = \{0,1\}$ follows a Markov chain with the following transitional matrix:

$$M = \begin{pmatrix} P & 1-P \\ 1-Q & Q \end{pmatrix} \quad (3.15)$$

where $P = \Pr(s_t = 1|s_{t-1} = 1)$ and $Q = \Pr(s_t = 0|s_{t-1} = 0)$.

Estimation of the regime-switching dependence is via the Hamilton (1989) filter. Let the T observations be denoted as $\boldsymbol{U_T} \equiv \{U_1, \dots, U_T\}$, where $U_t \equiv \{u_{1,t}, \dots, u_{7,t}\}$. The conditional probabilities of the states are denoted as:

$$\hat{\xi}_{t|t-1} \equiv \begin{pmatrix} \Pr(s_t=1|\boldsymbol{U_{t-1}};\boldsymbol{\theta_1},\boldsymbol{\theta_2}) \\ \Pr(s_t=0|\boldsymbol{U_{t-1}};\boldsymbol{\theta_1},\boldsymbol{\theta_2}) \end{pmatrix} \tag{3.16}$$

The log-likelihood function is:

$$L(\boldsymbol{\theta_1}, \boldsymbol{\theta_2}; \boldsymbol{U_T}) = \sum_{t=1}^{T} \log(\hat{\xi}_{t|t-1}\eta_t) \tag{3.17}$$

$$\hat{\xi}_{t|t} = \left(\hat{\xi}_{t|t-1}\eta_t\right)^{-1}\left(\hat{\xi}'_{t|t-1} \circ \eta_t\right), \tag{3.18}$$

$$\hat{\xi}_{t+1|t} = M'\hat{\xi}_{t|t}, \tag{3.19}$$

$$\eta_t = \begin{pmatrix} c_1(u_{1,t},\dots,u_{7,t};\boldsymbol{\theta_1}) \\ c_2(u_{1,t},\dots,u_{7,t};\boldsymbol{\theta_2}) \end{pmatrix} \tag{3.20}$$

where $\circ$ denotes the Hadamard product. The parameters to be estimated include

$\theta \equiv \{\boldsymbol{\theta_1}, \boldsymbol{\theta_2}, P, Q, \hat{\xi}_{1|0}\}$, where $\hat{\xi}_{1|0}$ is the initial value, and is obtained by maximising the log-likelihood function.

Estimation of the individual series ARMA-GARCH model actually comes before the above dependence estimation. Their standardized residuals $z_{i,t}$ are used to compute the inputs of the copulas, $u_{i,t}$. The Canonical Maximum Likelihood (CML) approach suggests using empirical cumulative distribution functions (CDF) to mitigate the univariate marginal model misspecification. The empirical CDF is:

$$\hat{F}(\omega) = \frac{1}{1+T}\sum_{t=1}^{T} I(z_{i,t} \leq \omega) \quad (3.21)$$

where $I(\cdot)$ is an indicator function taking the value one when $z_{i,t} \leq \omega,$ and otherwise zero. After the marginal CDFs are gained, $u_{i,t} = \hat{F}(z_{i,t})$.

After the estimation, the stationary Markov states are derived. They are used in the dependence forecast simulation. The simulated distributions of asset classes returns contain 500,000 samples.

3.4.4 Dynamics in dependence structure

For multivariate copula modelling, the pair-copula is flexible but the drawback is that only bivariate copulas in the first tier contain the naked original variables. In the remaining tiers, the elements in the copulas are variables that have gone through conditional functional transformations. Therefore, only the variables in the first tier can directly use the features provided by a specific copula. In our case, we choose to use the bivariate Clayton copulas as the elements in the pair-copula construction because they reflect the negative tail dependence, a feature of utmost importance to the conservative central banks. A Clayton copula allows for the lower tail dependence but with zero upper tail dependence.

The asset class returns in the first tier of the C-Vine structure can be directly modelled by the Clayton copula, while for the remaining variables in the subsequent tiers, only their conditional transformation would be captured by the bivariate Clayton copulas. In a C-Vine multivariate dependence structure, all the copulas in the first tier must contain the same single variable, known as the pivotal variable.

Therefore, all the relationships between this pivotal variable and each of the other variables are described in the first tier.

In conventional beliefs, the treasury debts are deemed the safe assets (McCauley and Rigaudy, 2010). In our portfolio, we have both short- and long-term treasury bonds. When making the decision of 'flight to safety' in response to the crisis, the dependence structure between the safe assets and other asset classes is especially important. That is the main reason behind our selection of the C-Vine and the Clayton copula for our modeling. The safe assets should be considered as the pivotal assets. When the market shows simultaneous losses of the short-term treasury bonds and the rest of the asset classes in the portfolio, we model it as the first state in a regime-switching model. When the negative tail dependence between the long-term treasury bonds and the other asset classes is predominant, we have the second market state. The interchange between states is assumed to follow a Markov chain.

Table 3.4 presents the comparison of the Akaike and Bayes information criteria, or AIC and BIC respectively, between a time-invariant vine copula model with Clayton copulas as elements and our regime-switching model. It shows that the dynamic copula dependence rather than the time invariant dependence modeling, is to be preferred across all the years.

In Table 3.5, the estimated parameters are presented, underneath which are their standard errors in brackets. The parameters are estimated using maximal likelihood, with the standard errors calculated as the inverse of the information matrix. It is not possible to get the inverse of the matrices for the numerical Hessians of the likelihood function in 2009, 2011 and 2012. For the other years, the bold parameters

in the table indicate their significance and we can see that most of the parameters are significant. The parameters include those of the Clayton bivariate copulas for all the six pairs in the first tier of the C-Vine copula in both regimes, with the pivots being the short-term treasury bonds and the long-term treasury bonds, respectively. The tail dependence parameter is also calculated and presented.

Table 3.4 Comparison between Time-Invariant and Regime-Switching Dependence

		2006	2007	2008	2009	2010	2011	2012
Time-invariant dependence	AIC	-340.38	-529.451	-445.182	-450.776	-401.384	-446.991	-396.06
	BIC	-276.333	-465.405	-381.135	-386.729	-337.472	-383.079	-332.421
Regime-switching dependence	AIC	-22668.4	-47190.9	-187438	-2.50E+09	-34931.1	-39900.4	-33497.1
	BIC	-22531.2	-47053.6	-187300	-2.50E+09	-34794.2	-39763.5	-33360.8

Notes: AIC, Akaike information criterion, BIC, Bayes information criterion
Source: Calculated by the author.

Table 3.5 Estimated Parameters for Regime-Switching Dependence

Pivot Short-Term Treasury								
		2006	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**
Short-Term	**Deposit**	**2.72792**	**2.824768**	**2.270647**	2.012722	**2.40006**	5.517345	1.749994
		(0.0153)	(0.01845)	(0.0002)	(34.8052)	(0.0003)	(36751)	(991.032)
	Tail Dependence	0.150944	0.141143	0.207237	0.247805	0.189456	0.021833	0.297303
Long-Term	**Treasury**	**0.12184**	**0.079712**	**0.527286**	0.567634	**0.08593**	0.324128	0.044155
		(0.0008)	(0.00518)	(1.40E-05)	(51.5492)	(0.0012)	(6403.1)	(1597.2)
	Tail Dependence	0.919017	0.946246	0.693859	0.674722	0.942177	0.798781	0.969858
	Agency	**0.07643**	**0.390573**	**0.077316**	0.261836	**0.19035**	0.336606	0.755716
		(0.0003)	(0.00744)	(4.90E-08)	(68.1093)	(0.0042)	(15348)	(1078.41)
	Tail Dependence	0.9484	0.762827	0.947819	0.834026	0.876392	0.791902	0.592252
	Corp	6.86E-06	**0.784725**	5.86E-05	3.80E-07	2.15E-06	0.35082	0.102949
		(0.0005)	(0.00605)	(8.50E-05)	(0.94718)	(0.0001)	(7170.5)	(3151.01)
	Tail Dependence	0.999995	0.580463	0.999959	1	0.999999	0.784138	0.931127
	Equity	**0.40263**	**3.315903**	**0.310796**	0.637747	**0.35627**	1.884442	0.785948
		(8.00E-05)	(6.92E-05)	(0.00012)	(455.411)	(0.0005)	(14432)	(2374.5)
	Tail Dependence	0.756479	0.100419	0.806197	0.642716	0.781181	0.270848	0.579971
	Euro	**0.26342**	**0.657114**	**0.37406**	0.213288	**0.38385**	0.000838	0.30465
		(0.0515)	(0.0014)	(5.60E-05)	(122.789)	(0.0009)	(18866)	(5189.65)
	Tail Dependence	0.83311	0.634146	0.771608	0.862569	0.766392	0.999419	0.809639

Pivot Long-Term Treasury								
		2006	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**
Short-Term	**Treasury**	**0.6889**	**1.242092**	**0.080163**	1.098998	**0.35766**	0.510623	0.398049
		(0.009)	(0.0161)	(0.00019)	(8741.32)	(1.00E-05)	(55.137)	(3.5709)
	Tail Dependence	0.620327	0.422759	0.945951	0.466841	0.780428	0.701919	0.758884
	Deposit	**0.15606**	0.000433	**0.007629**	0.018521	**0.00096**	3.30E-07	3.93E-07
		(0.0005)	(0.02087)	(1.50E-05)	(10234.6)	(2.00E-05)	(2.0081)	(0.86161)
	Tail Dependence	0.897472	0.9997	0.994726	0.987244	0.999333	1	1
Long-Term	**Agency**	**1.08737**	**2.753499**	**3.082621**	1.919328	**0.71665**	0.949675	1.126658
		(7.00E-05)	(0.00947)	(1.60E-05)	(3981.34)	(0.0002)	(16.891)	(166.723)
	Tail Dependence	0.470618	0.148291	0.118043	0.264378	0.60851	0.517749	0.457975
	Corp	5.45E-06	1.04E-05	**0.028376**	0.004777	1.93E-06	1.88E-07	2.13E-07
		(1.00E-05)	(0.01308)	(3.70E-05)	(3529.17)	(2.00E-05)	(1.3953)	(0.99131)
	Tail Dependence	0.999996	0.999993	0.980523	0.996694	0.999999	1	1
	Equity	**0.29228**	**0.378192**	**0.085215**	0.960752	**0.54414**	0.359261	0.471872
		(0.0021)	(0.01312)	(2.50E-05)	(8214.43)	(0.0073)	(98.612)	(38.2501)
	Tail Dependence	0.816609	0.769401	0.942644	0.513789	0.685799	0.779564	0.721028
	Euro	**0.27171**	**0.24256**	**0.121213**	0.191197	**0.26364**	0.602685	0.489114
		(0.0014)	(0.01036)	(1.60E-06)	(213.724)	(2.00E-05)	(188.11)	(22.0261)
	Tail Dependence	0.828337	0.845244	0.919415	0.875878	0.832986	0.658527	0.712463
Transitional Possibilities								

		2006	2007	2008	2009	2010	2011	2012
	Initial	0.501181	**0.026516**	**0.452939**	0.974938	0.489935	6.89E-07	0.004017
		(0.9996)	(0.00218)	(5.30E-05)	(9459.85)	(0.9988)	(21.059)	(24.2597)
	Trans	**0.71797**	**0.133167**	**0.999996**	0.999999	**0.38468**	0.002863	0.065318
		(0.0004)	(3.31E-05)	(0.00056)	(3.69943)	(3.00E-05)	(310.44)	(1972.92)
		0.83075	**0.999999**	**0.969516**	0.135321	**0.25413**	0.9999	1
		(6.00E-06)	(2.16E-05)	(0.03473)	(707.816)	(0.0003)	(0.0426)	(0.68401)

Notes：The standard errors are in brackets.
Source: Calculated by the author.

In our study that emphasize the ‘flight to safety’ activities in response to the crisis, each of the regimes represents relationships among asset classes centred on one particular safe asset. As we can see, in both regimes the tail dependence parameter, reflecting the asymmetries in the dependence structure, remains relatively stable. There is no obvious pattern of change due to the financial crisis. Assets with high tail dependence remain at a high level of being held, and those that were low remain low. Before the crisis, some of the asset classes had a characteristic of changing tail dependence from year to year, for example equities in the first regime and agency debts in the second regime. These continue to fluctuate during the crisis.

In the first regime, when the short-term treasury bonds are the pivot, the assets with which the short-term treasury bonds have the highest tail dependence are the corporate bonds. There is a similar situation in the second regime, where the long-term treasury bonds being the pivot. In the first regime the short-term bank deposits have the smallest tail dependence with the short-term treasury bonds, while in the second regime it is with the long-term agency debts that the short-term treasury bonds have the smallest tail dependence.

Fig. 3.2 shows the possibilities of regime one in each of the seven years under examination. These are calculated using the estimated initial state possibilities and the transitional possibilities in Table 3.5. All the lines become stationary eventually. This means that at the analysis point of each year, the Markov chain obtained using the previous three years’ history has settled down. The crisis does not affect the stationary property of the regime states. However, this does not mean that the financial crisis has no influence on the dynamics of the dependence structure. In Fig.

3.2, one may not be able to see all the seven lines. This is due to the fact that in most of the years one regime will dominate the other. There are some lines overlapping with each other near probabilities 1 and 0. This is the case for the years 2007, 2009, 2011 and 2012. In these relatively calm years, mostly before the crisis or towards the late time of the crisis, there is little interchange between the two regimes. For the other years, the two regimes compete with each other, hence the changes in the asymmetric dependence. This is especially true for the cases in 2008 and in 2010 when the crisis was at its high time. Changes in these times may be viewed as the influence of the crisis.

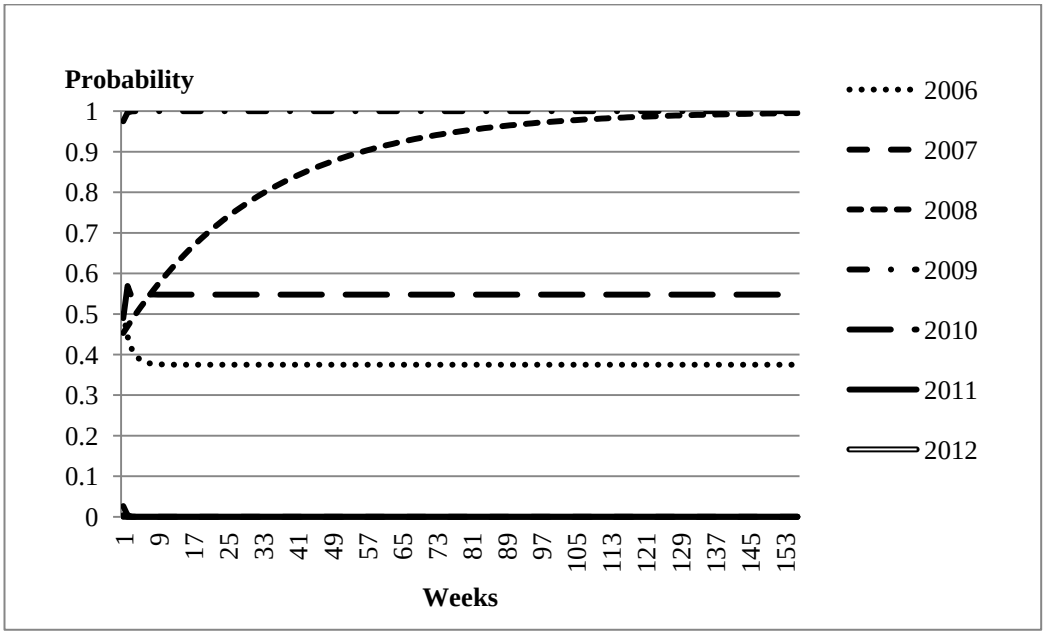


Fig. 3.2 Markov State Changes, 2006-2012

3.5 Risk Measurement and Optimal Asset Allocation

In strategic asset allocation for foreign reserves, the optimal allocation ultimately depends on the information obtained from modelling the distributions of returns and the reserve manager's preferences. Value-at-Risk (VaR) is a popular risk measure, due to its simplicity in presenting and conveying risk. However, it also suffers from some weaknesses, especially during a crisis. For example, it is only an indicator of threshold loss and cannot measure the worst case losses. In this light, we choose to use Conditional Value-at-Risk (CVaR) as a better base for a risk minimisation strategy for asset allocation. In addition, to account for the trade-off between higher returns and higher risks we adopt the DA (Disappointment Avoidance) utility in Ang, Bekaert and Liu (2005) for the risk minimisation. This preference would allow us to have sufficient appreciation of the central bank's conservatism in investing.

3.5.1 Allocation weight and ad hoc constraints

For sound financial management, fluctuations allowed for each asset class in each year must have a limit. In our study, we assume this to be $\pm 5\%$ over time, meaning that the allocation on one particular asset class can increase or decrease by only 5% in each year. Since China is a major holder of many classes of international assets, it is also in the nation's interest not adjusting the allocation of asset classes too much in a short period as otherwise the adjustment will cause large fluctuations. From the risk management point of view, drastic changes in asset allocation in a short period are often unattainable and often can have detrimental effects on stability of the market.

3.5.2 CVaR minimisation

CVaR minimisation reflects the fact that central banks are more concerned about risk than about returns. However, from the returns side, central banks are still constrained by the capital preservation consideration, implying that their investment returns must be at par with or above inflation. We use the US inflation as the benchmark, calculated using the Consumer Price Index (CPI-U) data from the US Bureau of Labor Statistics.

With the capital preservation constraint, the optimal combination of asset classes obtained by CVaR minimisation shows a pattern of 'flight to safety' in recent years. From the early times of the financial crisis in 2007 and 2008, there was a speedy conversion from the long-term to the short-term investment. The proportion of investment in short-term Treasuries rose at the maximal allowance of 5% per year from 2008, and the short-term deposits were preferred even one year before that time. In addition, this trend of safety flight shows no reversal. Investment in short-term financial instruments continues to grow until the end of the period. Only the investment in equities seems to pick up in 2011 and 2012, but its importance is largely insignificant, with a share of less than 1% in total reserve investment. Another difference between the actual weight changes and the results of our theoretical exercise is the importance of long-term treasury bonds. Compared with the long-term agency debts, investment in long-term treasury bonds is less favourable according to the copula approach than in the actual data. This might be due to the fact that China's perception of the credit risk of the agency debts is not in

total agreement with other market participants. The market assessment of risk did not suggest lowering the proportion of agency debts in total investment until 2012.

Table 3.6 Asset Allocation Based on Copula with CVaR Minimisation

COPULA with CVaR Minimisation								
		2006	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**
Short-Term	**Treasury**	0.86%	0.71%	5.71%	10.71%	15.70%	20.70%	25.69%
	Deposit	7.21%	12.20%	17.20%	22.20%	27.20%	32.20%	37.20%
Long-Term	**Treasury**	35.88%	30.88%	25.88%	20.88%	15.89%	10.89%	10.03%
	Agency	30.38%	25.38%	29.61%	30.75%	29.57%	30.23%	26.38%
	Corp	0.30%	0.00%	0.77%	0.00%	1.19%	0.00%	0.00%
	Equity	0.38%	5.37%	0.37%	0.00%	0.00%	0.53%	0.24%
	Euro	25.00%	25.45%	20.45%	15.45%	10.45%	5.45%	0.45%
	Mean	-0.00028	0.000544	0.001285	0.001526	0.000317	0.000607	0.000398
	CVaR	-0.15193	-0.17216	-0.3135	-0.17405	-0.19208	-0.09707	-0.04335

Source: Calculated by the author.

3.5.3 DA utility maximization

Our second optimisation strategy attempts to strike a balance between the emphasis on risk brought about by the central bank's conservative stance and the pressure for returns towards covering the growing costs of carrying foreign reserves. In this strategy, we apply the Disappointment Avoidance (DA) utility function as in Gul (1991), Ang *et al.* (2005) and Hong *et al.* (2007). The DA utility is defined as follows:

$$DA(W) = \frac{1}{K}\left(\int_{-\infty}^{\mu_w} u(W)dF(W) + A\int_{\mu_w}^{\infty} u(W)dF(W)\right) \qquad (3.22)$$

where $u(\cdot)$ is the felicity function in the form of CRRA utility, i.e.:

$$u(W) = \begin{cases} (1-\gamma)^{-1} \cdot (W)^{1-\gamma} \ \ if\ \gamma \neq 1 \\ \ln(W) \ \ if\ \gamma = 1 \end{cases}. \tag{3.23}$$

μ_w is the certainty equivalent according to the CRRA power utility; $F(\cdot)$ is the cumulative distribution function of the wealth; and K is a constant scalar given by:

$$K = P(W < \mu_w) + AP(W > \mu_w). \tag{3.24}$$

The DA preference is a transformation based on the chosen $u(\cdot)$, or in our case the CRRA power utility function, in which the risk aversion parameter (RA) stands for the risk preference of the representative investor. Usually parameter A is set to be smaller than 1 so that the utility below average (the loss) gives larger impacts than the utility above the average (the profit).

We set the values of the two parameters A and RA to be 0.25 and 20, respectively, to reflect the conservatism of central banks. For the Chinese central bank, the parameter A with a value of 0.25 means that earnings above the expectation are treated as having only a quarter of the importance of losses, whereas the very high value of the risk aversion parameter RA signifies the very high degree of risk aversion in the CRRA utility function. However, compared with the previous CVaR minimisation, the DA utility still allows the trade-off between returns and risk. Higher returns and lower risks both mean higher utilities to the investor. As can be seen from comparing Tables 3.6 and 3.7, in the pre-crisis period of 2006 and 2007 there was little difference in terms of the expected returns and risk. But from 2008 when the crisis started, the asset allocation according to DA utility optimisation has apparently

higher CVaRs as well as much higher expected returns. From the weights of each asset class in allocation, the DA utility seems mimic the actual data better.

We see a pattern similar to what we have discovered in the previous section. In the asset allocation based on DA utility maximization, there are increased holdings of safe assets, e.g. the short-term and long-term treasury bonds, in 2008 and 2009 when the financial crisis began to bite.

Also, the share of equities in the total reserve investment dropped in 2008 and 2009. Further, there is a familiar rise in the investment in higher risk assets from 2010 to 2012, implying a new shift in investment to returns. The percentages of short-term assets decrease for both the treasury bonds and bank deposits. Investment in equities picks up again. The long-term treasury bonds continue to grow regardless the growth of agency debts.

All these developments suggest that, in the post-crisis period, the Chinese central bank has started to pursue returns, but is more cautious than before. The share of the euro bonds in the DA analysis falls, but is less drastically than the drop in the CVaR analysis. Since DA utility maximization offers the possibility of allowing a sensible trade-off between higher returns and higher risk, it is likely that the Chinese fund managers may have started to opt for more euro government bonds, though slightly.

Table 3.7 Asset Allocation in Copula Model with DA Maximization

COPULA with DA Utility Maximization								
		2006	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**
Short-Term	**Treasury**	1.30%	0.04%	4.97%	0.15%	0.80%	0.06%	0.08%
	Deposit	7.13%	11.96%	16.92%	17.64%	19.46%	14.55%	14.16%
Long-Term	**Treasury**	35.91%	30.93%	26.49%	31.39%	36.31%	41.29%	46.26%
	Agency	30.22%	25.25%	21.04%	16.13%	12.20%	12.98%	8.33%
	Corp	0.32%	0.01%	0.72%	0.02%	0.11%	0.00%	0.01%
	Equity	0.15%	5.14%	2.33%	2.17%	3.61%	8.59%	13.58%
	Euro	24.97%	26.67%	27.53%	32.51%	27.51%	22.53%	17.58%
	Mean	-0.0003	0.0005	0.0017	0.0025	0.0005	0.002	0.0022
	CVaR	-0.1515	-0.1737	-0.3983	-0.2864	-0.4601	-0.3133	-0.2314

Source: Calculated by the author.

3.6 Decision on 'Flight to Safety'

Safe assets play a pivotal role in risk management of foreign reserves. Its role becomes especially important when there is ‘flight to safety’ in a time of financial turmoil. There has been heated discussion regarding the “flight to safety” in recent years. We contribute to this debate by offering the insights obtained from our analysis.

3.6.1 Flight to safety under CVaR

While we have discussed the general implications of the copula modelling for central banks’ decision to ‘flight to safety’, it is desirable and important to further pinpoint the influences of dependence asymmetry and the fat-tails on the flight. To highlight such effects, we make a comparison analysis of optimisations based on the variables with normal distribution and on the variables with the same mean and correlation matrix but whose distribution is modelled by copula. The differences in weights of each asset class in the allocation are the effects of the higher moments captured by the copula model.

Table 3.8 reports the optimal asset allocation under normal distribution that is calculated by minimising the CVaR of the portfolio while excluding asymmetries and fat-tails. The weights are then compared with those obtained from the copula model as reported in Table 3.6. The weight differences are reported in Table 3.9 by comparing the naïve model with the copula model in terms of pure risk minimisation. With the effects of the copula dependence, we see acceleration of the 'flight to safety', through the continuous increase in allocation to the short-term treasury bonds from the beginning of the crisis. Nonetheless, there are still movements of reserve investment shifting to Corporate bonds in 2008 and 2010 under the copula dependence structure.

Table 3.8 Asset Allocation in Naïve Model with CVaR Minimisation

Asymmetries and Fat-Tails Excluded with CVaR Minimisation								
		2006	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**
Short-Term	**Treasury**	0.86%	0.09%	5.09%	10.09%	15.09%	20.08%	25.08%
	Deposit	7.21%	12.20%	17.20%	22.20%	27.20%	32.20%	37.20%
Long-Term	**Treasury**	35.88%	30.88%	25.88%	20.88%	15.89%	11.46%	10.56%
	Agency	30.38%	25.38%	30.38%	30.75%	30.72%	30.18%	26.08%
	Corp	0.30%	0.00%	0.00%	0.00%	0.03%	0.00%	0.00%
	Equity	0.38%	5.37%	0.38%	0.00%	0.00%	0.00%	0.00%
	Euro	25.00%	26.07%	21.07%	16.07%	11.07%	6.07%	1.07%
	Mean	-0.00029	0.000544	0.001281	0.001554	0.000315	0.000607	0.000398
	CVaR	-0.15286	-0.17002	-0.29249	-0.16115	-0.16798	-0.08841	-0.03888

Source: Calculated by the author.

Table 3.9 Comparison between Naïve and Copula Models with CVaR

Effects of Asymmetries and Fat-tail with CVaR Minimisation								
		2006	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**
Short-Term	**Treasury**	0.00%	0.62%	0.62%	0.62%	0.61%	0.61%	0.61%
	Deposit	0.00%	0.00%	0.00%	0.00%	-0.01%	-0.01%	-0.01%
Long-Term	**Treasury**	0.00%	0.00%	0.00%	0.00%	0.00%	-0.57%	-0.52%
	Agency	0.00%	0.00%	-0.77%	0.00%	-1.14%	0.05%	0.30%
	Corp	0.00%	0.00%	0.77%	0.00%	1.16%	0.00%	0.00%
	Equity	0.00%	0.00%	0.00%	0.00%	0.00%	0.53%	0.24%
	Euro	0.00%	-0.62%	-0.62%	-0.62%	-0.62%	-0.62%	-0.62%

Source: Calculated by the author.

3.6.2 Flight to safety under DA utility

In Tables 3.10 and 3.11, the same procedure is carried out as for Tables 3.8 and 3.9, but this time with the optimisation objective being the maximising of the DA utility. Table 3.10 reveals the naïve asset allocation and Table 3.11 shows the differences between the weights in naïve asset allocation and in the copula model with DA maximization (comparison of results in Table 3.10 and Table 3.7).

The analysis with DA utility maximization sheds additional lights on China's reserve investment policy. We already know that asset allocation under DA utility is shaped by the concern with striking a balance between returns and risk. The CVaR minimisation assumes away the pressure that China is faced with to pursue returns. From Table 3.11, it can be seen that, in the pre-crisis period, the effects of asymmetries and fat-tails on optimal asset allocation are small. In 2006 and 2007 the optimal allocations under the two schemes show virtually no difference between the copula and naïve model. However, from the beginning of the crisis in 2008, the impacts become apparent. Under the *ad hoc* constraints we impose for avoiding

disruption to markets, the yearly allowance for allocation adjustment of each asset class is restricted to be a positive or negative 5% year on year. The difference in the weight between the naïve and the copula model is around 1% to 20% of the total allowed changes for the year. Furthermore, from the comparison between the copula modelling with DA maximization in Table 3.7 and the naïve modelling with DA maximization in Table 3.10, we can see that risk would be underestimated if asymmetries and fat-tails are ignored.

With respect to the decision of 'flight to safety', the results are instrumental. Sticking strictly to the rule of risk minimisation, the effects of asymmetries and fat-tails suggest more investment in conventional safe assets in the form of short-term treasury bonds, as shown in the case of CVaR minimisation. However, if taking a more balanced perspective between returns and risk, the result instead suggests avoidance of traditional safe assets such as the treasury bonds. Contrary to the 'flight to safety', the short-term deposits and the euro bonds seem appealing if the copula model is followed.

Table 3.10 Asset Allocation in Naïve Model with DA Maximization

Asymmetries and Fat-Tail Excluded with DA Utility Maximization								
		2006	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**
Short-Term	**Treasury**	1.32%	0.04%	4.97%	0.16%	0.79%	0.06%	0.08%
	Deposit	7.13%	11.98%	16.94%	18.12%	18.34%	13.44%	13.04%
Long-Term	**Treasury**	35.91%	30.93%	26.59%	31.49%	36.42%	41.39%	46.37%
	Agency	30.22%	25.25%	21.02%	16.11%	13.15%	13.94%	9.30%
	Corp	0.32%	0.01%	0.62%	0.02%	0.12%	0.00%	0.01%
	Equity	0.14%	5.13%	2.84%	2.10%	4.16%	9.14%	14.13%
	Euro	24.97%	26.67%	27.03%	32.01%	27.01%	22.03%	17.08%
	Mean	-0.0003	0.0005	0.0017	0.0025	0.0006	0.0021	0.0022
	CVaR	-0.1524	-0.1707	-0.3716	-0.2657	-0.3998	-0.2934	-0.2224

Source: Calculated by the author.

Table 3.11 Comparison between Naïve and Copula Models with DA

Effects of Asymmetries and Fat-tails with DA Utility Maximization								
		2006	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**
Short-Term	**Treasury**	-0.02%	0.00%	0.00%	-0.01%	0.01%	0.00%	0.00%
	Deposit	0.00%	-0.02%	-0.02%	-0.48%	1.12%	1.11%	1.12%
Long-Term	**Treasury**	0.00%	0.00%	-0.10%	-0.10%	-0.11%	-0.10%	-0.11%
	Agency	0.00%	0.00%	0.02%	0.02%	-0.95%	-0.96%	-0.97%
	Corp	0.00%	0.00%	0.10%	0.00%	-0.01%	0.00%	0.00%
	Equity	0.01%	0.01%	-0.51%	0.07%	-0.55%	-0.55%	-0.55%
	Euro	0.00%	0.00%	0.50%	0.50%	0.50%	0.50%	0.50%

Source: Calculated by the author.

3.7 Conclusions

Strategic asset allocation is an essential part of foreign reserve management. In a time of financial turmoil, it is of paramount importance to base the strategic asset allocation on the robust risk management. In this chapter we look at four aspects of this management: investment universe; the dependence structure; risk measure and asset allocation optimisation; and the decision on flight to safety. We apply the copula approach to the risk-based management of foreign reserves in terms of

strategic asset allocation. Special emphasis is paid on the impacts of asymmetries and fat tails on the asset allocation decisions.

In examining the dependence structure of the returns on the selected asset classes, we first analyse the univariate returns using an ARMA-GJR-GARCH model. A two-state regime-switching copula model for multiple asset classes is then developed to further analyse the dependence. A C-Vine copula is used to connect the seven representative asset classes that form China's investment universe. Twenty-one bivariate Clayton copulas are used as elements to form the joint dependence. The difference between the two regimes is that they have different pivotal variables in the first tier of the C-Vine structure. Each regime uses one safe asset as the protagonist, so that its asymmetric dependence with other assets can be better manifested.

Taking CVaR as the risk measure, two optimal asset allocation strategies are performed: the CVaR minimisation and the DA utility maximization. They represent respectively the situations where the central bank is concerned only with the risk for the level of returns that can only counter inflation, and the situation where the stance of the central bank is still conservative, but trade-off is allowed between higher returns and higher risk.

We deploy a regime-switching pair-copula multivariate model to highlight the features of safe assets. The two dependence regimes in our model allow focusing on two safe assets, short- and long-term treasury bonds, respectively. The interchange between the two regimes is governed by a Markov chain. We find that if the central bank is focused solely on risk, the asymmetries would encourage the flight to safety.

However, if higher risks are allowed in trading for higher returns, even the exchange is very conservative, the asymmetries would discourage the flight to safety.

Based on these analyses, we would like to present some insights with regard to the strategic asset allocation policy for China's foreign reserves. We first want to point out the importance of a proper asset allocation objective. As a central bank, the paramount investment emphasis is on liquidity and risk. This can restrict the range of investment universe. As discussed in Section 3, only limited number of asset classes qualifies. However, our study shows that the difference in asset allocations can still be evident if the investor prioritises her preference differently. In Section 3, the CVaR minimization represents a pure risk reduction stance whereas the DA utility maximization takes into account some moderate return requirements. The resulting allocations are very different in a way that wealth concentrates in short-term assets from the pure risk stance comparing to concentrates in long-term investment from the return stance. Both stances are under the prerequisite of liquidity and risk, but this shows the central bank still have plenty room to adjust its investment strategy. Secondly, this chapter confirms the importance of the copula structure, i.e. the non-normal dependence, for China's foreign reserves risk management. We suggest the central bank to pay more attention to the revealed tail dependence by the copula structure in Section 3. It is also shown in Section 6 that if these features are neglected, the allocation can be evidently different and can induce more risks. The third recommendation is about the decision of 'flight to safety' during turbulent financial periods. Our analysis incorporates information from both the investor side and the market side. The CVaR and DA utility objectives reflect the investor's preference, and the regime-switching copula structure offers accurate description for the market

risk appraisal. With the gradual passing of the recent global financial crisis, Chinese reserve manager may start to moderately increase their pursuant of returns by way of bolder investment in the classes of assets that are beyond traditionally believed safe assets. Given the nation's massive size of reserve assets, this may bring about a new era of international investment.

CHAPTER FOUR

This chapter examines the Strategic Asset Allocation problem for China's Sovereign Wealth Fund, China Investment Corporation. First, through investigation of China Investment Corporation's investment identity and performance history, its investment objectives are revealed. Next, a new method combining the merits of the shrinkage estimation, vine-copula structure, and Black-Litterman model, is proposed to satisfy the revealed investment objectives. Then, robustness tests for the method's advantages are conducted. Finally, the empirical analysis shed lights on the strategic asset allocation decisions for China's foreign reserves in return tranche.

CHAPTER 4

STRATEGIC ASSET ALLOCATION FOR CHINA'S FOREIGN RESERVES IN RETURN TRANCHE

4.1 Introduction

CIC, the relatively young SWF of China, has attracted much attention since its inception on 19 September, 2007. Due to the huge amount of China's foreign exchange reserves it can tap into, many are curious about its identity as an international investor, its investment objective and its strategic asset allocation. As CIC reports directly to the government of the State Council of the People's Republic of China, the political motivation of the young SWF also comes under scrutiny from international spectators. Meanwhile, in a new attempt to improve the situation whereby China's huge official foreign exchange reserves are heavily concentrated in low-yielding US Treasuries and bonds, CIC is also learning to fulfil its objective of seeking for higher returns for the foreign reserves and to adapt itself quickly to the global investment environment. In this paper, through reviewing the history and literature, and providing our own analyses, we discuss these three closely related questions, i.e. the identity of CIC, its investment situation, and its investment objectives.

In addition, in reflecting upon the existing literature on CIC strategic asset allocation, we discover that there are relatively few papers providing quantitative analysis or a proper portfolio management method suitable specifically to the investment

objectives of the CIC type of SWF. Since the development of the mean-variance approach of Markowitz (1952), the portfolio optimisation method has been widely recognized for its central role in helping strategic asset allocation. For SWFs like CIC, it is desirable to have asset allocations that are financially efficient, stable and diversified, and with good risk appraisal. A synthesized method is proposed combining the three features demanded by the SWFs for their strategic asset allocation.

The three features of financial efficiency, good risk appraisal and allocation efficacy have intuitive importance to portfolio management, and therefore each of these aspects has been well developed. With respect to allocation efficacy, by which we mean stability and level of diversification, the mean-variance analysis has been criticized. The most frequently applied solution is that proposed by Black and Litterman (1991, 1992) and further developed by He and Litterman (1999), and Satchell and Scowcroft (2000). They utilise the Bayesian rule to combine analysts' forecasts with the market equilibrium. This differs from the mean-variance method where the forecasts for every asset return are derived from the historic data. Based on the efficient market hypothesis, this method incorporates the market view as the basis for forecasting the future returns.

With respect to good risk appraisal for strategic asset allocation, many papers discover the asymmetric dependence feature in asset returns (Longin and Solnik, 2001; Ang and Chen, 2002; Bae *et al.*, 2003; Hong *et al.*, 2007). Some assets are more likely to go down together, thus diminishing the effect of diversification. In addition, the fat-tail feature means that extreme losses would be underestimated if the

common Gaussian distribution were assumed, as in the mean-variance analysis. Therefore, the copula method is important for risk management in asset allocation. The application of copula method in financial series estimation is developing rapidly. In particular, the vine-copula offers flexible tools to handle risk management in multivariate portfolio problems.

Another important issue in portfolio management is the 'estimation error'. Many papers attempt to deal with the estimation problem (Barry, 1974; Jorion, 1986, 1991; Pastor, 2000; Pastor and Stambaugh, 2000). This problem is closely related to the robustness of the optimal asset allocation and the accuracy of the model's predictability. Hence, proper treatment in this regard is expected to improve the overall financial performance of the portfolio management process. Estimation is the first stage in almost every portfolio optimisation model. However, if the possibility of estimation error is neglected, using different sets of observations from the same distribution can often lead to different results as to the underlying distribution. In response to this issue, Jorion's (1991) shrinkage method is widely applied, and has been proved to be effective in many cases. We intend to incorporate this into our method and expect it to be able to improve the overall financial performance (profitability) in our case.

In the following sections, we first present literature analysing the CIC investment objectives and the three aspects of our proposed method. Then, in Section 3, the methodology is proposed and elaborated. In Section 4, we provide empirical analysis on the case of CIC, targeting the effectiveness of the method as well as the

implications for CIC of our optimal strategic asset allocation result. In the final section, we conclude and point out limitations of this research.

4.2 Literature Review

4.2.1 Identity analysis

The definition of SWF according to the 'Santiago Principles' of the International Working Group of Sovereign Wealth Funds (IWG) and the International Monetary Fund (IMF) is as follows: 'SWFs are special purpose investment funds or arrangements, owned by the general government. Created by the general government for macroeconomic purposes, SWFs hold, manage, or administer assets to achieve financial objectives, and employ a set of investment strategies which include investing in foreign financial assets. The SWFs are commonly established out of balance of payments surpluses, official foreign currency operations, the proceeds of privatizations, fiscal surpluses, and/or receipts resulting from commodity exports.'

Based on this definition, Kunzel *et al.* (2011) and Mihai (2013) summarize the characteristics of most SWFs and categorize them broadly into commodity and non-commodity funds according to their funding sources. The commodity funds are those SWFs accumulated due to some particular type of commodity. According to the IWG, commodity funds can be further characterized by their functions related to fiscal stabilization, saving, or support for pension plans and social priorities (IWG, 2008). When planning the strategic asset allocation of the commodity funds, it is important to consider the price fluctuation of the particular commodity. Therefore, Gintschel and Scherer (2008) and Brown *et al.* (2010) develop investment frameworks to

incorporate commodity price changes. The non-commodity funds mainly comprise the Singaporean Temasek and Government of Singapore Investment Corporation, the Korea Investment Corporation of the Republic of Korea, and the China Investment Corporation of the People's Republic of China. The funding of these SWFs is from either fiscal surpluses or foreign exchange reserves. In the case of China the investment objectives are long-term, and to earn higher returns than the liquidity-emphasized part of foreign reserves.

The categorization of SWFs can be extended according to the presence or absence of a strategic purpose. Lyons (2007) and Santiso (2008) point out that strategic funds are not investing for macroeconomic or financial objectives, but rather to promote national economic development in particular firms or projects. Haberly (2011) provides some case studies on the strategic usage of SWFs. China's SWF, CIC, has high pressure for returns, and its conduct provides evidence to prove that it belongs to the class of non-strategic funds.

The funding position of a SWF is the most important information about its identity profile. It has a great impact on the investment objectives of the fund and the strategic asset allocation decisions. At the creation of CIC, there were two major factors influencing its funding situation and its initial investment performance.

The first was the high pressure for returns. Cognato (2008) and Zhang and He (2009) summarize three aspects of this high pressure. The opportunity cost for China of holding her huge amount of foreign reverses can be represented as the return gap between the investment in highly liquid US treasury and agency bonds and the foreign direct investment (FDI) China receives. As an example, the interest rate of

10-year US treasury bonds was around 3-6 percent from 2001 to 2007, whereas in 2005 the average FDI return in China was 22 percent, and thus the opportunity cost was 16-19 percent. In addition, there could be heavy losses due to the expected US dollar depreciation against the Chinese yuan. Zhang and He (2009) estimate that because in 2007 China held 1.68 trillion dollars of foreign reserves, a 10 percent depreciation of the dollar would have meant a loss equivalent to 5 percent of the Chinese GDP. Finally, the sterilization bonds issued by the central bank of China were at an annual rate of more than 4 percent. The low return from investing in the liquid and safe bonds might mean a net loss for holding foreign reserves.

In addition to the pressure for high returns, the second factor determining the initial funding situation of CIC was the competition for control between the central bank, the People's Bank of China (PBoC), and the Ministry of Finance (MoF). In other countries the MoF usually controls the SWF, whereas in China the State Administration of Foreign Exchange (SAFE), a subsidiary of the PBoC, had been the sole manager of the country's foreign exchange reserves (Eaton and Zhang, 2008). As a result of the rivalry between the PBoC and MoF, and the return pressure for China's SWF, the funding of the CIC at the time of creation was complicated. In 2007, the MoF raised 1.55 trillion Chinese yuan by issuing special bonds with an annual yield of around 4.5%, and then purchased 200 trillion US dollars' worth of assets from PBoC and injected those assets into CIC. Since the MoF is not a shareholder of CIC, the initial funds of 200 trillion dollars are recorded as liability, and CIC must pay interest on the special bonds. This funding position translates into a requirement for heavy returns as CIC's investment objective. Liao (2007) and Marin (2009) estimate that, taking into consideration future appreciation of the

Chinese yuan against major currencies, CIC would need to earn a 10% annual return. This is an unrealistic target given that Singapore's long-established and widely regarded as highly professional SWF, GIC, averaged only a 9.5% annual return over the 25 years to March 2006 (GIC annual report, 2008).

There were two immediate consequences of the funding arrangement (Eaton and Zhang, 2008; Marin, 2009). The first was the purchase of Huijin from PBoC, and the second was the pursuit of an over-concentrated high return, high risk investment strategy. The first caused concerns among foreign investment recipients that CIC would become a strategic investor acting in China's national interests by taking control of some core industries, which would limit CIC's opportunities. The second meant that the CIC was unable to afford an appropriate level of prudence, which was especially important at the dawn of the 2008 financial crisis. Both played important roles in the later transformation of CIC's funding position and investment objectives.

Huijin held large percentage shares in three national commercial banks: China Construction Bank, Bank of China and Industrial and Commercial Bank of China. After the listing of these banks in the Shanghai and Hong Kong stock markets, Huijin's 60 billion US dollar investment increased in value to 160 billion US dollars (Li, 2008). The price CIC paid for making Huijin its own subsidiary was only 67 billion US dollars. The transaction had effects in two aspects. It was a necessary move to enable CIC to repay the interest generated by its funding liability. However, it created a difficulty in terms of convincing the international environment that CIC is not a strategic investor (Wu and Seah, 2008a and 2008b). This is because the

national commercial banks are in many regards competitors to CIC's potential investment subjects.

The funding arrangement was also one factor driving CIC's choice of a risky investment strategy, which resulted in heavy losses. At the creation of CIC, there was a lack of expertise in investing. This fact, combined with the scheduled requirement for high interest, pushed CIC to choose a concentrated investment policy in high risk financial products, mostly in the US. Other reasons for this decision, as summarized by Wu *et al.* (2012), were that in 2006 oil and resource company asset prices were at record highs, and the US financial market was well developed. Wu and Seah (2008a) categorize the actions into three groups: (a) participation in some Initial Public Offerings (IPO); (b) utilisation of some external managers; and (c) investment in some firms hit by the crisis. Wu *et al.* (2011), Marin (2009) and Cognato (2008) reveal that the bad results attracted hostile criticism and media exposure. The disclosed initial investments included 3 billion US dollars in Blackstone private equity in May 2007, and 5 billion US dollars in Morgan Stanley convertible securities with 9% return in December 2007. In March 2008, CIC also invested 100 million US dollars in Visa Inc's IPO, and in September in a money market fund, Reserve Primary Fund. However, these investments were ill-timed, and at their lowest prices the investments in Blackstone and Morgan Stanley were 82% below their purchase price; the investment in Visa was 24% below its purchase price; and Reserve Primary Fund was the first money market fund to break the value of 1 dollar per share.

As a consequence of these initial failures, CIC managed to change its funding position, marking the start of a new identity for the fund. Agreement was reached with the MoF that the initial funding could be turned from liability into assets. Therefore, CIC was no longer required to pay MoF interest at regular intervals (Tong and Chong, 2010 in Wu *et al.* 2012). The SWF was permitted to change its investment strategy into 'seeking high, long-term and sustainable financial returns … within acceptable tolerance for risks' (CIC, 2012). The performance measure for the fund was also adjusted in line with its investment objective. In early 2011, it was decided that 'a rolling 10-year annualized return would be a major measure of performance' (CIC, 2012). Its strong position as a long-term fund, and the potential for growth, are evidenced by its small percentage in GDP and in foreign reserves (Wu *et al.*, 2012). Such objectives lead to an investment strategy of diversified portfolios across asset classes, geography and sectors. Also, as envisioned by CIC Chairman Lou Jiwei, the future recovery of the global economy will be fragile, and CIC's investment approaches will be prudent, taking these risks into considerations (CIC, 2012).

4.2.2 Situational analysis

The importance of the international environment includes the political attitudes towards the SWF among investment receiving nations. These attitudes affect the CIC's investment strategy, especially in terms of the available investment universe. Initially, at the creation of the fund, CIC's connections with government, for example through the acquisition of Huijin as its subsidiary, as well as transparency issues, meant that the international environment, especially the developed countries, did not

believe that the CIC was a non-strategic investor. Wu and Seah (2008a) note that in response to the perceived threats posed by SWFs, the US has set up the Committee on Foreign Investment in the United States to oversee SWFs' actions; Australia has the Foreign Investment Review Board; Germany implements controls on SWF investments in order to secure local jobs and strategic sectors; and France has promised to protect companies from takeovers by SWFs. These controls have resulted in the CIC's reluctance to invest in sensitive sectors such as airlines, telecommunications and oil (Bradsher, 2007). Hence the available investment universe is limited and mainly focused on financial assets (Wu *et al.*, 2012).

However, since the crisis the investment universe has broadened, and CIC's investments now include the energy and real estate sectors. There are two reasons for this. First, CIC continues to promote its commercial basis for investments. These efforts include the employment of professionals with international background, improvement in organizational structure and transparency, and giving up management board positions for several investments. A CIC annual report (CIC, 2012) shows that out of the total staff number of 405, there are 44 with overseas citizenship, 165 with overseas working experience and 224 with overseas education. The level of transparency is indicated by the Linaburg-Maduell Transparency Index (SWF Institute, 2013). CIC scores 7 out of 10, well above average. Wu *et al.* (2012) show that CIC has declined a seat on the management board for both the Blackstone and the Morgan Stanley investments. The second reason is the change in attitudes among developed nations. Hit by the crisis, the US and European Union now consider SWFs as necessary investments and liquidity providers for economic recovery (Mihai, 2013; Park and Estrada, 2009, p.78).

4.2.3 Investment objectives

Based on the above identity and situational analysis for CIC, its investment objectives can be derived. As a non-commodity SWF, CIC's investment is not under restraint from the price of any particular commodity. Also, the internal and external situations, i.e. high return pressure and international attitudes toward SWFs, prevent CIC from being a strategic investor and from pursuing control of some sensitive sectors through investments in other countries' economies. The common financial objectives, such as profitability, risk and liquidity, remain primary concerns for CIC. According to its funding position and the lessons from its own history of initial investment failure, CIC is now better prepared for investments with longer investment horizons. Although investment return is the primary objective, proper risk management is also crucial in order to avoid another round of public criticism over its asset managing capability, as happened with its initial investment failure. Wu *et al.* (2011) and the CIC annual report (CIC, 2012) show more diversified asset allocation across different asset classes, geography, and industry sectors. They also indicate better skills among the management team, including expertise gained both domestically and overseas. These signs prove CIC's emphasis on risk management. In addition, the international environment has improved since the 2008 financial crisis, and consequently CIC has been able to expand its investment universe to include non-financial assets such as real estate, natural resources and big commodities in many countries. New opportunities are ideally matched to China's strength in high volume of foreign reserves, but also present new challenges to a sound management and strategic asset allocation strategy. Therefore, we would like

to present a quantitative strategic asset allocation analysis in response to these investment objectives for CIC and other similar SWFs.

4.2.4 Incorporating estimation error, market equilibrium and vine-copula

We propose a new method for combining the market equilibrium, Jorion's (1991) shrinkage method for estimation error, and a vine-copula risk appraisal technique, using the Bayesian rule as in the Black-Litterman model. This methodology synthesizes the merits of its three components and suits the SWF's investment objectives of seeking returns with proper management of risks and stress testing.

The incorporation of the market equilibrium originated in the Black-Litterman model (Black and Litterman, 1991 and 1992; He and Litterman, 1992, Satchell and Scowcroft, 2000). It should lead to more diversified and more stable asset allocations.

The estimation error is an important topic in portfolio management and asset pricing. It refers to the inaccuracy of estimation of distribution parameters when using conventional estimation methods such as the sample mean and some maximal likelihood estimators. In order to improve the estimation Bayesian methods are usually applied; the seminal literature includes Barry (1974), Jorion (1985, 1986, 1991), Pastor (2000), Pastor and Stambaugh (2000), and DeMigual, *et al.* (2009). There are also non-Bayesian methods, such as the use of re-sampling as in Michaud (1998), and the restricting of asset weights, as in Jagannathan and Ma (2003). Generally speaking, all estimation error reduction methods are supposed to enhance the forecasting accuracy and provide robust asset allocation.

With regard to the vine-copula structure, this technique breaks the conventional paradigm of dependence between asset returns, i.e. the linear dependence measure of Pearson's correlation implied by a Gaussian returns distribution. Abnormal features such as asymmetry and fat-tail have been well documented by Ait-Sahalia and Brandt (2001), Longin and Solnik (2001), Ang and Chen (2002), Bae *et al.* (2003), Hong *et al.* (2007), and Ammann and Suss (2009), and the copula method is devised for just such a problem. Zhang *et al.* (2013) maintain that the vine-copula structure is necessary for the risk appraisal in China's foreign reserves investment.

Our method for combining these three components suits the CIC's investment objectives of pursuing high return while maintaining sound risk management. The incorporation of market equilibrium and the reduction of estimation error account for the profitability requirement, and these can help to build robust and diversified asset allocations. At the same time, risk management objectives are important because of CIC's identity as a foreign reserves manager, and because post-crisis, financial markets remain turbulent. Copula methods are often used for investment stress testing (Boss *et al.*, 2006; Sorge and Virolainen, 2006; Brechmann *et al.*, 2013). They provide the ideal tool to flexibly model asymmetries and fat-tails in the portfolio distribution. The following paragraphs provide brief literature reviews on the development of the three components respectively.

The Black-Litterman model, developed and later explained by Black and Litterman (1991, 1992), He and Litterman (1992), and Satchell and Scowcroft (2000), bridges the different views on asset distributions, i.e. the investor's personal views and the market view inferred from the Capital Asset Pricing Model (CAPM), otherwise

called the market equilibrium model. The method enlightens us in two respects: first by the incorporation of the market equilibrium to enhance portfolio stability, and second by its application of the Bayesian rule to combine distributions. The market equilibrium is expressed as:

$$\Pi = \delta \Sigma w_m \tag{4.1}$$

where Π is the vector for market equilibrium returns; $\delta = (\mu_m - r_f)/\sigma_m^2$, a coefficient equal to the excess return of the market portfolio over the variance of the market portfolio; Σ is the covariance matrix for asset returns; and w_m is the vector for market weights. The equation results from a reverse portfolio optimisation process assuming the validity of the CAPM.

The Bayesian theorem used for combining distributions simply states that:

$$f_3(\mu|\Pi) = \frac{f_1(\Pi|\mu) f_2(\mu)}{f(\Pi)} \tag{4.2}$$

where $f_1(), f_2(), f_3()$ are all probability density functions and f_2 is called the prior distribution and f_3 is called the posterior.

The Black-Litterman model uses the prior distribution to express the investor's views, and the market equilibrium is assumed to be attained conditional on the prior. The posterior distribution is the combination of investor's view and market equilibrium. Since both $f_1()$ and $f_2()$ are assumed to be Gaussian, a famous conjugate pair, the analytical solution exists and follows another Gaussian distribution. However, in our analysis we want to incorporate the copula function, which is generally non-Gaussian.

Therefore it is not possible to obtain an analytical solution for the posterior. This is the first obstacle we need to overcome.

The problem of estimation error or estimation risk can be well defined using the Bayesian theorem. First described by Zellner and Chetty (1965), this refers to the inaccuracy of parameter estimation by using some sample estimator. For a general asset allocation optimisation, the expected utility should be maximized:

$$EU = \int U(z)p(z|\theta)dz \tag{4.3}$$

where $U()$ represents the utility function with portfolio wealth of z; $p(z|\theta)$ is the probability density function of z conditional on the true probability function parameters θ.

However, the true value of θ can only be estimated based on observation, and therefore there is an optimisation problem based on sample estimators of the probability parameters:

$$\max\ E[U(z)|\theta = \hat{\theta}(z_{-t})] \tag{4.4}$$

where $\hat{\theta}(z_{-t})$ is the estimator based on previous observations. However, uncertainty exists when applying the sample estimator. The possibility exists that $\hat{\theta}(z_{-t})$ cannot give the accurate value of θ. If this uncertainty is ignored, we define the consequent utility optimisation error as the estimation error.

Assuming that the uncertainty is accounted for, the portfolio optimisation objective should be:

$$\max \mathrm{E}_{\theta}(E_{z|\hat{\theta}}[U(z)|\theta]) \tag{4.5}$$

with

$$\mathrm{E}_{\theta}\left(E_{z|\hat{\theta}}[U(z)|\theta]\right) = \iint U(z)p(z|\theta)dz\, p(\theta|z_{-t}, I_{-t})d\theta$$

$$= \int U(z) \int p(z|\theta)\, p(\theta|z_{-t}, I_{-t})d\theta dz$$

(4.6)

where $p(\theta|z_{-t}, I_{-t})$ is the probability function of applying $\hat{\theta}(z_{-t})$ to estimate θ. Then if the Bayesian rule is applied:

$$p(\theta|z_{-t}, I_{-t}) \propto p(z_{-t}|\theta)p(\theta|I_{-t}) \tag{4.7}$$

Different Bayesian based estimation error methods are established on different assumptions on the prior, $p(\theta|I_{-t})$, and the conditional function $p(z_{-t}|\theta)$. Barry (1974) chose the prior to be diffuse and asserted that the estimation error would become larger if the number of samples got smaller. Jorion (1986) developed that idea further, and assumed that the prior would follow a Gaussian distribution. They found that the estimator was in the format of the Stein Estimation (Stein, 1955 and 1962) and the returns would shrink towards the return of a minimal variance portfolio. Pastor (2000) and Pastor and Stambaugh (2000) alter the conditional function $p(z_{-t}|\theta)$ by utilising various asset pricing models to introduce the help of market models.

The application of the shrinkage estimation to reduce estimation error is popular. Jorion (1986 and 1991) has demonstrated its effectiveness using simulation studies and portfolio analyses. Since we have our own way to incorporate market equilibrium, we consider the Bayesian-Stein shrinkage method is ideal for us. As mentioned above, Jorion (1986) assumed both the conjugate pair $p(z_{-t}|\theta)p(\theta|I_{-t})$ to be Gaussian, and the prior is like:

$$p(\mu|\eta,\lambda) \propto \exp\left[-\left(\frac{1}{2}\right)(\mu - \mathbf{1}\eta)'(\lambda\Sigma^{-1})(\mu - \mathbf{1}\eta)\right] \tag{4.8}$$

where η is considered to be the expected value of the target parameter μ, and λ to be the confidence level of the prior. The estimation of the return would be like:

$$E(r) = (1-w)\bar{Y} + w\bar{Y_0} \tag{4.9}$$

with

$$w = \frac{\lambda}{T+\lambda} \quad \bar{Y_0} = \frac{\mathbf{1}'\Sigma^{-1}\bar{Y}}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} \tag{4.10}$$

where T is the sample number and Σ is the covariance matrix of returns.

The signicant meaning of the shirnkage factor, w, lies in that for large sample size, the correction for estimation risk disappears. The value of w dwindles and the expected value, $E(r)$, tend to the usual value $\bar{Y}$. To be more specific on the mechanism for obtaining the shrinkage factor, we show that when corrections are needed, the value of λ is estimated from the data directly. Assuming the probability

density function of $p(\lambda|\mu,\eta,\Sigma)$ to be a gamma distribution with mean at $(N+2)/d$, where d is defined as $(\mu-\mathbf{1}\eta)'\Sigma^{-1}(\mu-\mathbf{1}\eta)$ and is replaced in estimation by $(\bar{Y}-\mathbf{1}Y_0)'\Sigma^{-1}(\bar{Y}-\mathbf{1}Y_0)$. The shrinkage factor can be estimated in this way.

In the area of portfolio management, there is another popular method for the purpose of reducing estimation risk. It is the resampling or bootstraping method proposed by Michaud (1998). The method develops the resampling statistic technique into the field of mean-variance optimisation. Assuming a stationary stochastic process behind the asset returns, the optimised portfolio weights are calculated as the average of the weights from n times of simulated samples.

In the competition between the Bayes and the resampling methods, Markowitz and Usmen (2003) designed a investment management game to test for their effectiveness and supported Michaud's strategy. However, Harvey, *et al.* (2008), in a follow up paper, revised the Bayes method used in Markowitz and Usmen (2003) and found out the Bayes method to be the winner. The mixed result seems to declare the superiority of one of the two over the other is difficult to establish with multiple influencing factors depends on situation.

The main reason for choosing the Bayes method in this chapter is due to calculation practicality. Barros Fernandes, *et al.* (2012) combines the resampling method with market equilibrium using the Black-Litterman method. In their study, the resampling method is feasible in terms of calculation because Gaussian distributions are assumed, even though the resampling technique is applied from the first part of the Black-Litterman method. In this study the C-vine copula method, proposed to emphasising on the non-Gaussian risk management, makes the repetitive resampling impractical,

if like Barros Fernandes, *et al.* (2012) the resampling start from the beginning of the model. In addition, the traditional resampling methodology, being an ad hoc methodology, has been criticised due to its lack of theoretical basis compared to the Bayes method..

The phenomenon of asymmetric dependence, where the returns are more correlated negatively than positively, and that of fat-tail, where extreme events happen more than the Gaussian distribution predicted, are well recognized nowadays in financial studies, and exert great influence on the portfolio choice problem (Ait-Sahalia and Brandt, 2001; Longin and Solnik, 2001; Ang and Chen, 2002; Bae *et al.*, 2003; Hong *et al.*, 2007; Ammann and Suss, 2009; Garcia and Tsafack, 2011). The copula dependence modelling method was introduced into the finance theories by Joe (1997), Embrechts *et al.* (1999) and Nelson (2006) to account for the complex dependence structure in the empirical evidence. The copula model separates the modelling of dependence and univariate time-dynamics in multivariate situations, and thus liberates us in utilising our strength of univariate series in multivariate cases. However, the bivariate copula models, especially the type that are good at capturing asymmetric effects, still lose some flexibility in a multivariate situation. Therefore, Joe (1997), Bedford and Cooke (2001, 2002), Kurowicka and Cooke (2006) and Aas *et al.* (2009) have developed the vine-structured pair-copula to model a multivariate dependence by pairs of bivariate copulas joined by conditional distributions.

$$F(x, y) = C(F_x(x), F_y(y)) \tag{4.11}$$

where $F(x, y)$ is a joint cumulative distribution of (x, y), and $F_x(x)$ and $F_y(y)$ are its respective partial cumulative distributions. Sklar's (1959) theorem states that if the

marginal density functions are continuous, the copula function $C(F_x(x), F_y(y))$ or $C(u, v)\ \ u, v \in [0,1]$ is uniquely determined. The theorem allows us to separate the margins with the dependence and thus enables us to model complex dependence.

In a multivariate situation, conditional density function decomposition of a multivariate density function can help us to use multiple bivariate copulas in modelling a multivariate case:

$$f(x_1, \dots, x_n) = f(x_n) \cdot f(x_{n-1}|x_n) \cdot f(x_{n-2}|x_{n-1}, x_n) \cdots f(x_1|x_2, \dots, x_n) \quad (4.12)$$

where $f(x_1, \dots, x_n)$ represents the density function of variables $(x_1, \dots, x_n)$, and $f(x_1|x_2, \dots, x_n)$ represents the density function of x_1 conditional on $(x_2, \dots, x_n)$,

with

$$f(x_1|x_2, x_3) = \frac{c_{13|2}[F(x_1|x_2), F(x_3|x_2)] \cdot f(x_1|x_2) \cdot f(x_3|x_2)}{f(x_3|x_2)}$$

$$= c_{13|2}[F(x_1|x_2), F(x_3|x_2)] \cdot f(x_1|x_2)$$

(4.13)

where $f(x_1|x_2)$ can be further decomposed using the same method, so:

$$f(x_1|x_2, x_3) = c_{13|2}[F(x_1|x_2), F(x_3|x_2)] \cdot c_{12}[F(x_1), F(x_2)] \cdot f(x_1) \quad (4.14)$$

A multivariate density function can be decomposed into the products of multiple bivariate copulas and the marginal density functions. Different orders of $(x_1, \dots, x_n)$ in the decomposition lead to different bivariate copula components. Among all sorts

of compositions the C-vine copula is well researched and its density function for parametric estimation is:

$$\prod_{k=1}^{n} f(x_k) \prod_{j=1}^{n-1} \prod_{i=1}^{n-j} c_{j,j+i|1,\dots,j-1}\{F(x_j|x_1,\dots,x_{j-1}), F(x_{j+i}|x_1,\dots,x_{i+j-1})\} \tag{4.15}$$

We use the C-vine copula in our analysis for the risk appraisal.

In the same field, several papers attempt to improve on asset allocation problems for the central banks in terms of the previous three aspects. Petrovic (2010) and Leon and Vela (2011) apply the Black-Litterman model for central banks. They recognize the potential of the Black-Litterman for allocation efficacy and combine the market equilibrium with investors' opinions. Barros Fernandes *et al.* (2012) use the Black-Litterman plus re-sampling techniques to deal with the estimation error. However, the re-sampling method is less intuitively appealing and less theoretically founded than the Bayesian method used in Jorion (1991) and others for estimation error. The method in their paper also lacks our copula risk appraisal ability. Another reason for choosing Jorion's shrinkage estimation over the re-sampling technique is that the estimation of vine-copula structures in high dimensional situations entails the high cost of computer power. In the re-sampling procedure, the repeated estimations of the copula parameters would take too much time to justify its advantage over the shrinkage estimation, even if such an advantage does exist.

4.3 Methodology

4.3.1 Bayesian linkage for three components

The three components we intend to incorporate in order to postulate a joint distribution are the above mentioned market equilibrium for robust portfolio, shrinkage estimation for estimation error and vine-copula for risk appraisal. It is important that these are connected in an intuitive manner. We are enlightened by the Black-Litterman approach for joining the market view and investors' views using the Bayesian theorem. The combination of the three components can be interpreted intuitively using the method and it is written as:

$$f\left(r\middle|r_{shrink},\pi,\Sigma,\theta_{copula}\right)=\frac{f_1\left(r_{shrink}|r,\theta_{copula};\pi,\Sigma\right)f_2(r|\pi,\Sigma;\theta_{copula})}{\int f_1\left(r_{shrink}|r,\theta_{copula}\pi,\Sigma\right)dr} \quad (4.16)$$

where $f\left(r\middle|r_{shrink},\theta_{copula},\pi;\Sigma\right)$ is the posterior probability density function for returns with combined views of the three components: shrinkage estimation r_{shrink}, copula risk appraisal θ_{copula} , and the market equilibrium return π . $f_2(r|\pi,\Sigma;\theta_{copula})$ is called the prior probability function and $f_1\left(r_{shrink}|r,\theta_{copula};\pi,\Sigma\right)$ is the investor's view expressing the copula risk dependence and shrinkage estimated returns.

In our theory of Bayesian connection for the three components, the prior distribution represents the market view of the returns. $f_2(r|\pi,\Sigma;\theta_{copula})$ is assumed to be Gaussian distribution with mean values as predicted by the market equilibrium. Based on the prior, the investor expresses her view conditional on the market view return from the prior. The return should follow a distribution with mean as the

market prior and a copula dependence structure as estimated from data. This means that the investor assumes that in the long run the returns should return to the equilibrium, but it is possible that the returns would deviate from the equilibrium in a manner predicted by the short run copula dependence pattern, and the shrinkage estimated returns represent the deviated short run returns. The Bayesian theorem approach of combination of different views is theoretically founded, compared to Meucci's (2009 and 2010) more subjective Black-Litterman copula opinion pooling method as well as the entropy minimization method.

4.3.2 Prior

The prior distribution expresses the market view. Its design is inspired by the Black-Litterman model for incorporating the market equilibrium. It assumes that the Capital Asset Pricing Model (CAPM) is established in the long run and the derivation of the equilibrium returns of the assets is a process of reverse optimisation of the market portfolio. If the CAPM is assumed to be valid, we have:

$$\pi = \beta(\mu_m - r_f) \tag{4.17}$$

where μ_m is the return of the market portfolio; r_f is the return of the risk free asset; β is a vector of asset betas representing the risk sensitivities of risky assets, where:

$$\beta = Cov(\mu, \mu' w_m)/\sigma_m^2 \tag{4.18}$$

where w_m is a vector for market weights of each asset; $r'w_m$ is the return of the market portfolio; and σ_m^2 is variance of the market portfolio. If we write $\Sigma = Cov(\mu, \mu')$ to be the covariance matrix of the risky assets, then:

$$\pi = \delta \Sigma w_m \tag{4.19}$$

where $\delta = (\mu_m - r_f)/\sigma_m^2$ is estimated from data. Also estimated from the data is the covariance matrix Σ, and the market weight w_m is known. Therefore the market equilibrium π is generated by the estimates of the risks Σ, the market preference δ and the market weights, which is more robust than the sample estimates of the returns.

4.3.3 Investor's View

The investor's view refers to the density distribution function, $f_1(r_{shrink}|r, \theta_{copula}; \pi, \Sigma)$. It contains the other two components of our model, the copula dependence and the shrinkage estimation of the returns. The incorporation of these two follows the Bayesian rule, and therefore the probability density function is a vine-copula function with parameters such as the copula coefficients, the return vector from the prior, r, and the shrinkage estimated returns r_{shrink} as function inputs. According to the C-vine copula density function, the investor's view density function states:

$$f_1(r_{shrink}|r, \theta_{copula}; \pi, \Sigma) =$$
$$\prod_{k=1}^{n} f(x_k) \prod_{j=1}^{n-1} \prod_{i=1}^{n-j} c_{j,j+i|1,\dots,j-1}\{F(x_j|x_1, \dots, x_{j-1}), F(x_{j+i}|x_1, \dots, x_{i+j-1})\} \tag{4.20}$$

where r_{shrink} is a vector composed by $\{x_1, \dots, x_n\}$; $f(x_k)$ is the marginal density function for kth elements in r_{shrink}, and $c_{j,i|\mathrm{k}}(\cdot)$ is a bivariate copula density function between jth and ith elements conditional on the kth.

For the estimation of r_{shrink}, the Bayesian-Stein method is described in Section 4.2.4. We follow Jorion (1986) and we have:

$$r_{shrink} = (1 - \hat{w})\bar{Y} + \hat{w}\bar{Y_0} \tag{4.21}$$

with

$$\bar{Y_0} = \frac{\mathbf{1}'\Lambda^{-1}\bar{Y}}{\mathbf{1}'\Lambda^{-1}\mathbf{1}}$$

$$\hat{w} = \frac{N+2}{(N+2) + (\bar{Y} - \bar{Y_0}\mathbf{1})'T\Lambda^{-1}(\bar{Y} - \bar{Y_0}\mathbf{1})}$$

$$\Lambda = \frac{\mathrm{T}-1}{\mathrm{T}-\mathrm{N}-2}\Sigma$$

(4.22)

where $\bar{Y}$ is the sample mean; Σ is the sample covariance matrix; T is the sample size and N is the number of returns.

In order to calculate the density function of Equation 4.20, we still need to determine the types and the parameters of the marginal densities of $f(x_k)$ and the bivariate copulas on each vine node for the C-vine structured dependence. ARMA – GARCH/APARCH – C-vine copula model combination is used for the task. The estimation contains two steps. In step 1 of the ARMA – GARCH/APARCH process, for each return series ARMA lag length parameters (u, v) are given choices from 0 up to 3. Two variance dynamics types are offered, GARCH and APARCH, with lag length parameters (p, q) also from 0 to 3. The residuals in the mean function are

given choices from three types of distributions, namely Gaussian, Student-t and the skewed Student-t. In the second step of the estimation process, each C-vine copula element is given the choice of 31 types of bivariate copulas. For both steps, the Akaike information criterion is applied for choosing the best fit models types, and maximized likelihood estimators are used for parameter values.

However, for the purpose of incorporating the shrinkage return and the copula dependence in this paper, not all the results from the above two steps are needed. In Equation 4.20, the copula parameters, θ_{copula}, derive from the estimation, but for the parameters in $f(x_k)$, the forecasted stationary mean values from the ARMA – GARCH/APARCH model are not needed. They should be based on the returns from the prior for compliance with the Bayesian assumption.

4.3.4 Posterior

In Bayesian probability theory, it is always difficult to calculate a posterior distribution. For ease of applying the Bayesian theory, analytic posterior distributions are given when the prior and the likelihood function, i.e. $f_1(r_{shrink}|r, \theta_{copula}; \pi, \Sigma)$ in Equation 4.16, take the forms of various usual continuous probability functions. These known analytic solutions of posterior and prior distributions are called conjugate distributions. However, in our case, in order to introduce the copula structure for better risk appraisal, the likelihood function is complex as well as flexible. The distribution function is a combination of marginal returns and copula dependence. In addition, there are 31 types of copula for each pair of returns in the vine structure and the number of types for each univariate return is 1536 (the product

of 2 types of variance model, 3 different residual distributions, 4^4 combinations of ARMA-GARCH lag length parameters u, v, p, q). It is extremely difficult to obtain an analytic posterior.

Cheung (2009) introduced a simulation method for general Bayesian posterior distributions. A simulated posterior for Equation 4.16 can thus be obtained in the following steps:

1. Prior distribution sampling. Sample $\{r^{(l)}\}_{l=1}^{L} \sim N(\pi, \Sigma)$, where L represents a large sample size, by applying the usual inverse probability integral transformation. The simulated distribution follows the prior distribution.
2. New probability vector calculation for the posterior distribution:

$$p^{(l)} = \frac{f_1\left(r_{shrink} | r^{(l)}, \theta_{copula}; \pi, \Sigma\right)}{\sum_{i=1}^{L} f_1\left(r_{shrink} | r^{(i)}, \theta_{copula}; \pi, \Sigma\right)} \tag{4.23}$$

3. The pair $\{p^{(l)}, r^{(l)}\}_{l=1}^{L}$ is the simulated posterior distribution with $r^{(l)}$ as a simulated value, $p^{(l)}$ is its probability.

It is worth noting that compared to a usual simulation applying the inverse probability integral transformation, the outcome pair $\left\{p^{(l)}, r^{(l)}\right\}_{l=1}^{L}$ here is different. For a usual simulation $\left\{r^{(l)}\right\}_{l=1}^{L} \sim N(\pi, \Sigma)$, it can be considered as a pair of $\left\{q^{(l)}, r^{(l)}\right\}_{l=1}^{L}$ where all $q^{(l)} = 1/L$, which means each $r^{(l)}$ is independent and equally important. This is not the case in the Bayesian posterior sampling. $\left\{p^{(l)}\right\}_{l=1}^{L}$ are the probability weights for each sample. The proof of the above procedure can be found in Cheung (2009).

4.3.5 Portfolio optimisation and performance assessment

The optimal asset allocation is solved based on the Bayesian distribution combining the above three components by maximizing an appropriate utility function. The chosen utility function must be able to reflect the investor's preference on higher moments other than mean and variance of the portfolio distribution and the asymmetric features of the assets' joint distribution. The Disappointment Aversion utility (DA utility hereafter) proposed by Gul (1991) is applied by Ang *et al.* (2005) and Hong *et al.* (2007) under asymmetric portfolio decisions similar to ours.

The DA utility is defined by the following equation:

$$DA(W) = \frac{1}{K}\left(\int_{-\infty}^{\mu_w} u(W)dF(W) + A\int_{\mu_w}^{\infty} u(W)dF(W)\right) \tag{4.24}$$

where $u(\cdot)$ is the felicity function in the form of CRRA utility here, i.e.

$$u(W) = \begin{cases} (1-\gamma)^{-1}\cdot(W)^{1-\gamma} & if\ \gamma \neq 1 \\ \ln(W) & if\ \gamma = 1 \end{cases}, \tag{4.25}$$

μ_w is the certainty equivalent according to the Constant Relative Risk Aversion (CRRA) power utility; $F(\cdot)$ is the cumulative distribution function of the wealth; and K is a constant scalar given by:

$$K = P(W < \mu_w) + AP(W > \mu_w). \tag{4.26}$$

The disappointment aversion parameter A in the above equations gives asymmetric preference on gains over losses. The risk preference parameter, γ, represents the investor's risk appetite. We consider the risk preference $\gamma = 5$, and disappointment

aversion $A = 0.45$ as appropriate levels representing China's SWF preference. The asset allocation is optimised by:

$$\max_w DA(W) \tag{4.27}$$

$$W = 1 + w'R \tag{4.28}$$

where the distribution of the asset returns R is modelled by the Bayesian method described previously.

For the purpose of assessing the optimal portfolio performance and the effectiveness of the Bayesian distributional method proposed in this paper, three dimensions of evaluation measures are devised, namely financial performance, risk predictability, and allocation efficacy. Financial performance is assessed by in-sample and out-of-sample DA utilities of the optimal allocation. Risk predictability is assessed by the difference between in-sample and out-of-sample skewness and the difference between in-sample and out-of-sample excess kurtosis. The allocation efficacy comprises the allocation diversification and stability, and these are evaluated respectively by the mean Herfindahl index, given by the sum of the squared asset weights as suggested in Barros Fernandes *et al.* (2012), and the average turnover given by the sum of changes of each asset between two consecutive years divided by the value of the portfolio.

4.4 Empirical Analysis

4.4.1 Data and comparison procedure

According to its annual report, financial assets account for the majority of CIC's investment portfolio, with public equities taking 32%, fixed-income securities 19.1%, and cash and others 3.8% as of 31 December 2012. Among the fixed-income securities investment, sovereign bonds of advanced and emerging economies account for 54.7% and 17.5% respectively, and another big chunk is investment grade corporate bonds, which takes 25.1%. Equity investment comprises three basic categories: US equities take 49.2%, other advanced economies equities 27.8% and emerging market equities 23%.

We follow these disclosed asset classes, using a total of 15 representative indices. For the fixed-income investments, six Bank of America Merrill Lynch Bond indices are selected. Four are sovereign bonds for advanced and emerging economies, while the other two are US corporate bonds and EMU AAA graded bonds. Six FTSE equities indices are used for the public equities investment, with three representing developed regions and three for the emerging economics. In addition to these 12 financial assets, there are three exchange-traded fund (ETFs) indices of real estate, oil and gold to represent the non-financial investments partially disclosed in the CIC annual reports. Details of the indices are in Table 4.1.

Table 4.1 Data Source Description

Name	Type	Source	Mnemeric Code	Frequency
FTSE AW NORTH AMERICA	Stock Indices	Thomson Reuters Datastream	AWNAMR$(RI)	Daily
FTSE AW EUROPE			AWEROP$(RI)	
FTSE AW DEV ASIA PAC.			AWDVAP$(RI)	
FTSE EMERGING ASIA PAC.			AWAEPA$(RI)	
FTSE EMERGING LATIN AMER			AWAELA$(RI)	
FTSE AW MIDDLE EAST & AFRICA			AWMEAF$(RI)	
BOFA ML GLB GVT G7	Bond Indices		MLGGVG7(RI)	
BOFA ML USD EMRG SOV ASIA			MLIGDA$(RI)	
BOFA ML USD EM SOV LTN AM			MLIGDL$(RI)	
BOFA ML USD EMRG SOV EUR/ME/AFR			MLIGDE$(RI)	
BOFA ML US CORP AAA			MLC3ART(RI)	
BOFA ML EMU CORP LGE CAP AAA			MLELA0$(RI)	
ISHARES US REAL ESTATE	Commodity ETFs		U:IYR(RI)	
UNITED STATES OIL FUND			U:USO(RI)	
SPDR GOLD SHARES			U:GLD(RI)	

Notes:
'FTSE AW' refers to the FTSE all world indices. 'DEV' is short for developed countries. 'ASIA PAC.' is the abbreviation for Asian Pacific. 'BOFA ML' refers to Bank of America, Merrill Lynch. ‘Emerging countries’ is abbreviated to 'EM' or 'EMRG'. 'GLB', 'GVT', 'SOV', 'CORP', and 'LGE CAP' refer to global, government, sovereign bonds, corporate bonds, and large capitalization respectively. 'EUR/ME/AFR' refers to Europe, Middle East and Africa.
Source: Compiled by the author.

The data frequency is daily and the coverage period is from the beginning of 2006 until the end of 2012. A three-year rolling window approach of allocation optimisation and evaluation is applied. This means that a three-year data window is used for the estimation of the next year's distribution and at the end of the next year the three-year window rolls a year forward to exclude the earliest year data and include the latest year data for the next estimation. The eight years' data coverage allows us to make such optimisations five times.

In addition to the Bayesian method comprising the market equilibrium, estimation errors and the copula risk appraisal techniques, there are four other estimation methods for comparison to manifest the advantage of our proposed method. These are listed in Table 4.2. Three of these methodologies exclude one of the three components, to reflect the effects of the missing component. The fourth method is the simple sample mean-variance estimation as a benchmark. The five methodologies are compared across three dimensions: financial performance, risk predictability and allocation efficacy as described in section 3.5. It is worth noting that the third method, EsEq, is just the Black-Litterman model with the investor's views as the shrinkage estimated returns from the data.

Table 4.2 Denotation of Five Models

EsCoEq	Three-component model of Estimation Error, Copula and Market Equilibrium
CoEq	Two-component model of Copula and Market Equilibiurm
EsEq	Two-component model of Estimation Error and Market Equilibrium
EsCo	Two-component model of Estimation Error and Copula
Sample	Simple Mean-Variance model by historical returns

Source: Compiled by the author.

A robustness test of the proposed method is carried out after the initial comparison. This confirms the combination of the three components, and we then provide analysis of the optimal allocation outcome.

4.4.2 Comparison of methods

Table 4.3 displays the criteria statistics results according to the method described above. The investment universe contains all 15 asset classes across 5 years. The table shows the comparison of 10 criteria across the 5 methods. The first two criteria are the DA utilities of the optimal asset allocation according to a particular method. The values of the utility function represent the financial objective of the investor, and therefore they are used as the criterion for assessing the financial performance of the allocation. The in-sample DA utility is calculated based on the estimation using the data window. The out-of-sample DA utility is obtained by holding the optimal allocation from the estimation through the next year and using the daily data of that year as an empirical returns distribution. The same logic of these in-sample and out-of-sample statistics applies in the skewness and excess kurtosis case. In terms of risk management, it is important to provide accurate estimate of the future risks, especially the non-Gaussian risks such as fat-tails and asymmetries. In order to test for the effectiveness of the copula model built within some of the 5 methods, the differences between the in-sample and out-of-sample skewness and excess kurtosis are provided as criteria for the asymmetric and fat-tail risk prediction.

The remaining two criteria are the turnover and the Herfindahl index, to reflect allocation stability and diversification respectively. We follow Barros Fernandes, *et al.* (2012) in using these two criteria, with turnover defined as the sum of changes in

allocation compared with previous period's allocation divided by the value of the portfolio, and with Herfindahl index defined as the sum of the squared asset allocation weights for each period. The turnover statistic needs the allocation information of the previous period, and therefore the values are zero in the first year.

As to the out-of-sample statistics, data from next year are needed as the realised empirical distribution. Hence, in the last year there is no out-of-sample statistic. In the following analyses, the financial performance of a method is represented by the in-sample and out-of-sample DA utilities. The skewness and excess kurtosis differences are used as the criteria for the risk predictability. With regard to allocation efficacy, the turnover and the Herfindahl index reveal stability and diversification.

However, it is difficult to determine the merits of each method, since there are many criteria and many years. For convenience in comparison, we have devised a ranking method for the statistics. The ranking contains two steps. In the first step, we rank the 5 methods based on the 6 criteria. For example, with respect to DA in-sample utility in 2008 the best utility method, EsEq, is ranked 1, and the worst method, Sample, has the lowest ranking, 5. The rank index for each distribution method is recorded for the six criteria we are interested in and across five years. Therefore, taking the DA in-sample utility criterion as an example, for each year from 2008 to 2012 there is a set of DA in-sample utility rank indices. In the second step, these rank indices across five years for the same criteria are summed and then ranked again from the smallest number summed to the largest. The idea is to summarise the rankings across all five years. The smaller the sum of the rankings, the better the performance of this

particular method in terms of a particular criterion. Again looking at the DA in-sample utility as an example, from the step 1 we get five rankings for each year from 2008 to 2012, and they represent the order of the five methods in terms of the criterion DA in-sample utility. After the second step, the five years' rankings are added up to generate a new single ranking for the overall performance across five years. The final ranking of method EsCoEq of the second position means that over the five years, this method has been ranked the second best in terms of the DA in-sample utility. This example of DA in-sample utility ranking procedure is demonstrated in Fig. 4.1.

Table 4.3 Allocation Criteria across 5 Methods

2008										
	DAinsample	DAoutsample	skewinsample	skewoutsample	skewdiff	exkurinsample	exkuroutsample	exkurdiff	Turnover	Herfindahl
EsCoEq	-0.11165	-0.11256	0.132312	0.085298	0.047014	0.169559	3.711341	3.541782	0	0.56849
CoEq	-0.11135	-0.11258	-0.01991	0.265078	0.284988	0.359389	3.89628	3.536891	0	0.62192
EsEq	-0.1115	-0.11269	0.018831	-1.44797	1.466803	0.021254	12.52276	12.50151	0	0.563322
EsCo	-0.1119	-0.11134	-6.28925	0.182246	6.471497	419.573	0.716303	418.8567	0	0.304174
Sample	-0.11175	-0.11197	0.001672	-0.45244	0.454111	0.002001	6.248988	6.246987	0	0.325619
2009										
	DAinsample	DAoutsample	skewinsample	skewoutsample	skewdiff	exkurinsample	exkuroutsample	exkurdiff	turnover	Herfindahl
EsCoEq	-0.11163	-0.11155	0.011059	-0.27598	0.287036	0.397809	0.234974	0.162835	0.640811	0.337697
CoEq	-0.11167	-0.11143	0.12662	-0.3342	0.460822	0.335493	0.370403	0.03491	0.908177	0.349243
EsEq	-0.11181	-0.11182	-0.00868	-0.23894	0.23026	-0.03189	-0.05582	0.023927	0.367264	0.465653
EsCo	-0.11159	-0.11136	-0.19485	-0.3683	0.173452	12.09569	0.920028	11.17566	1.082556	0.360958
Sample	-0.11183	-0.11142	0.000753	-0.28396	0.284711	-0.00942	0.645569	0.654988	0.458316	0.299011
2010										
	DAinsample	DAoutsample	skewinsample	skewoutsample	skewdiff	exkurinsample	exkuroutsample	exkurdiff	turnover	Herfindahl
EsCoEq	-0.11157	-0.11146	-0.17458	-0.55172	0.377147	0.221506	1.791427	1.569921	0.48821	0.243843
CoEq	-0.11171	-0.11152	0.09301	-0.51828	0.61129	0.285407	1.739939	1.454531	0.21619	0.299138
EsEq	-0.11451	-0.11377	-0.01043	-0.56461	0.554176	0.046608	2.393497	2.346889	1.659972	0.066667
EsCo	-0.11167	-0.11144	-0.21587	-0.62116	0.405295	4.437041	1.874159	2.562881	0.769485	0.230415
Sample	-0.11188	-0.11153	0.014467	-0.48701	0.50148	0.028068	1.495352	1.467285	0.06884	0.28655
2011										
	DAinsample	DAoutsample	skewinsample	skewoutsample	skewdiff	exkurinsample	exkuroutsample	exkurdiff	turnover	Herfindahl
EsCoEq	-0.11132	-0.11117	-0.04417	-0.04432	0.00015	0.307915	0.442138	0.134222	0.722214	0.344264
CoEq	-0.11122	-0.11109	-0.0556	-0.05689	0.00129	0.618172	0.710675	0.092503	1.52243	0.573431
EsEq	-0.11143	-0.1111	0.000862	-0.01005	0.010908	0.009243	0.757508	0.748264	1.317486	0.213773
EsCo	-0.1114	-0.1112	-8.73061	-0.00847	8.722145	845.5981	0.610053	844.9881	0.496482	0.208899

Sample	-0.11141	-0.11111	-5.14E-05	-0.0318	0.031751	-0.00984	0.470388	0.48023	0.795511	0.210576
2012										
	DAinsample	DAoutsample	skewinsample	skewoutsample	skewdiff	exkurinsample	exkuroutsample	exkurdiff	turnover	Herfindahl
EsCoEq	-0.11122	0	0.014964	0	0	0.080257	0	0	0.825921	0.249259
CoEq	-0.11121	0	-0.04542	0	0	0.084172	0	0	1.095843	0.228651
EsEq	-0.11133	0	-0.0129	0	0	0.029853	0	0	0.3518	0.246443
EsCo	-0.11126	0	-0.16954	0	0	2.040057	0	0	0.48981	0.242937
Sample	-0.11128	0	-0.00746	0	0	0.036474	0	0	0.481838	0.268293

Notes:
(i). The turnover is defined as the sum of purchases and sales of the assest in portfolio compared with the previous year's level divided by the value of the portfolio in this period.(ii). The Herfindahl index is defined as the sum of square asset allocation weights.
Source: Compiled by the author.

In Table 4.4, the six criteria are further summarized into three categories. Financial performance contains the DA in-sample and out-of-sample utilities. Its ranking is obtained by considering the two criteria as one. Similarly, risk predictability treats the skewness difference and excess kurtosis difference as one criterion, and allocation efficacy includes stability and diversification. The first column records the overall ranking covering the six criteria of each method.

It can be seen from the table that the proposed three-component method does perform best overall. It ranks second for financial performance and first for risk predictability. It confirms our prediction that the combination of copula for risk appraisal, market equilibrium for allocation stability and Bayesian-Stein for estimation error reduction outperforms other methods, i.e. those with only two components or the naked naïve MV analysis. The sample MV method only ranks second to last.

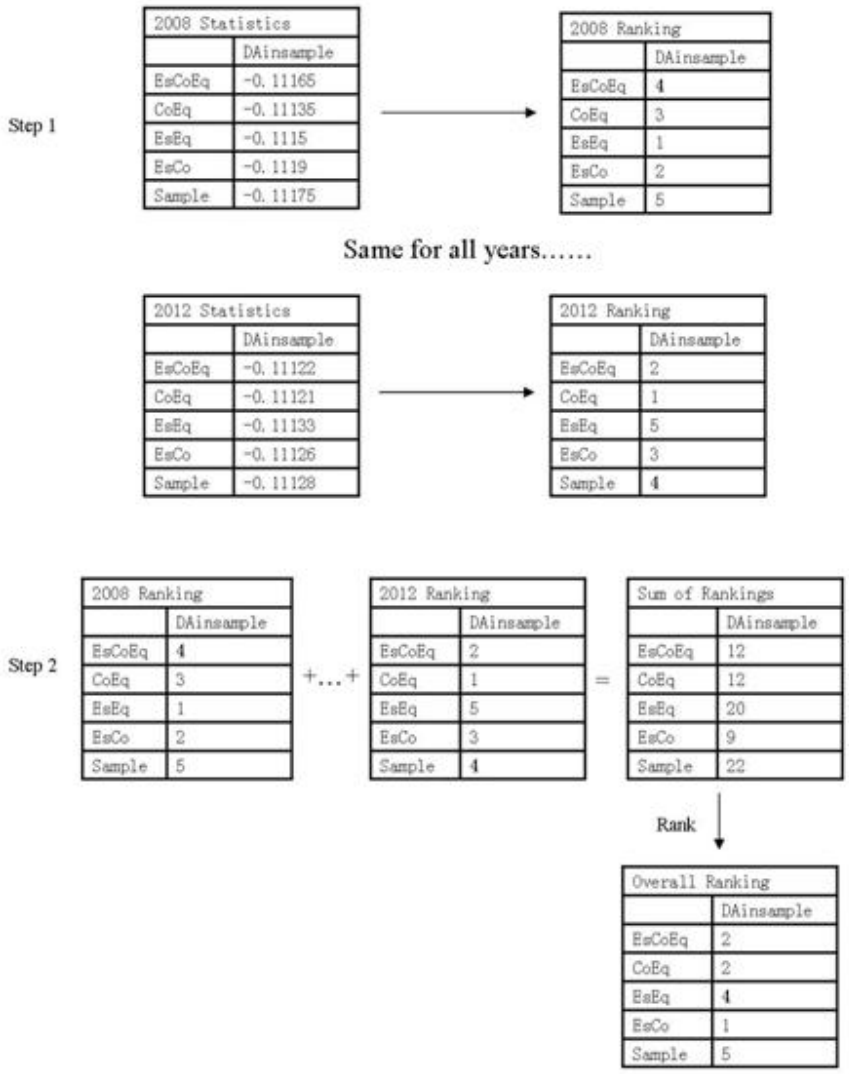


Fig. 4.1 Procedure for Deriving Performance Rankings

However, the result in Table 4.4 only contains five years. The merit of the three-component method may be just by chance. Also, the incorporation of the market equilibrium does not seem to improve the allocation efficacy. In contrast, the two methods without the market equilibrium are ranked first and second in this regard.

To find out the reason for this, and to test the robustness of the proposed method, we continue with more analyses of the methods. In addition, the robustness test result in the following section can also tell us the effects of each of the three components proposed.

Table 4.4 Performance Ranking in Three Categories

	All	Financial Performance	Risk Predictability	Allocation Efficacy
EsCoEq	1	2	1	4
CoEq	2	1	2	5
EsEq	5	5	3	3
EsCo	3	3	5	1
Sample	4	4	4	2

Notes:
The numbers indicate the rankings of each method compared with other methods according to a specific criterion indicated by the column caption.
Source: Compiled by the author.

4.4.3 Method robustness

In order to test for the robustness of the proposed method, we divide the data into four separations, and apply the same procedure as for method comparison. In addition to the 15 asset classes in section 4.2, there are three further asset allocation portfolios. We group the 12 financial assets together as the first separation. The second and third separations are six bonds as the fixed-income securities group and six stocks plus three commodity ETFs as the high risk securities group. In the following analyses we label these as bonds and stocks separations respectively.

Table 4.5 shows the overall rankings across the four separations. For each method there are 20 sub-rankings (4 separations times 5 years) summarized for the criteria of

stability and diversification, while for financial performance and risk predictability there are 40 sub-rankings, because each of these contains two specific criteria. The table synthesizes all four situations and ranks the three-component method as best overall. The relative lack of performance in the allocation efficacy criterion leads us to reinstate its original two criteria format. In terms of stability, the proposed three-component method is ranked third. From the comparisons between the methods, the effects on stability of the three components, i.e. estimation error, copula and equilibrium, can be revealed. By comparing EsCoEq and CoEq, it is clear that the omission of estimation error has deteriorated the stability. Similarly, by observing the rankings in stability between EsCoEq and EsEq, and between EsCoEq and EsCo, it can be seen that the incorporation of copula has weakened the stability, whereas the equilibrium has strengthened it. In terms of the criterion of diversification, the first three methods with equilibrium incorporated have lower rankings, compared to the last two methods without. This is due to the fact that the market value weights of each asset class are not very averagely allocated.

Table 4.5 Performance Rankings Summarized from Four Sample Separations

	All	Financial Performance	Risk Predictability	Stability	Diversification
EsCoEq	1	2	1	3	4
CoEq	2	1	2	4	5
EsEq	5	5	3	2	3
EsCo	3	3	4	5	1
Sample	4	4	5	1	2

Notes:
The numbers indicate the rankings of each method compared with other methods according to a specific criterion indicated by the column caption. The result is reached by summarizing rankings across the four sample separations.
Source: Compiled by the author.

From Table 4.6 to Table 4.9, the specific rankings of the four separations are listed. The overall dominance of the three-component model is shown inTable 4.10. In the specifics here, we can see that the proposed model does not perform poorly in any of the situations. The result shows the robustness of the proposed model.

We have expectations when including each of the components, i.e. the estimation error, the copula or the market equilibrium, into the model. The copula should help with the risk prediction. The market equilibrium should be able to improve the allocation efficacy, and the estimation error should have a positive overall impact across the criteria of financial performance, risk predictability and allocation efficacy. The effects of each component can be revealed by comparing the three-component model with each of the two-component models. The two-component models each lack the effect of a particular missing component. Therefore the changes of rankings in each criterion are considered to be mainly due to the missing component. We use upward or downward pointing arrows beside the rankings of the three two-component models to indicate their changes compared with the proposed three-component model.

Table 4.10 is a summary of Tables 4.6 to 4.9. It groups the changes of rankings by the three two-component methods. If the ranking of a criterion is lowered, this means that the lack of a particular model component deteriorates the criterion performance, and thus proves the importance of that component. We focus on Table 4.10 to explain the advantages of the three-component model as follows.

For the CoEq method, a combination of the copula and the market equilibrium, we expect that compared to the three-component model EsCoEq, it should manifest the

characteristics of the estimation error factor. The incorporation of estimation error is supposed to improve the criteria in all three aspects systemically, and this is what we see in the result. In all four situations, i.e. all assets, financial assets, bonds and stocks, the number of times a criterion ranking falls is higher than or at least equal to the number of times the ranking rises. For example, in the case of bonds, all rankings decrease, which means improvement in all aspects. For stocks, two rankings fall and two rise, which simply indicates that the benefits and disadvantages are balanced. Across all cases, if the estimation error factor is missing, more damage is done than benefit received.

The effects of the other components, the copula for risk predictability and the market equilibrium for allocation efficacy, are more evident. The EsEq method demonstrates the copula impact whereas the EsCo shows the market equilibrium. In all four situations, all assets, financial assets, stocks and bonds, the inclusion of the copula component is proved to increase the risk predictability, and incorporating market equilibrium can improve allocation stability, as highlighted by the downward pointing arrows in bold text. These effects are unlikely to be by chance, due to their consistent presence in all four robustness testing situations. Other causalities, between copula and stability for example and indicated by other arrows not in bold text, might be false, and depend on the situation.

Above all, the confirmation of our expectations for the three model components, i.e. the estimation error, copula risk incorporation, and market equilibrium, renders us confidence in the model robustness and in its application for China's SWF strategic asset allocation decisions.

Table 4.6 Ranking Indices for All 15 Assets

	All	Financial Performance	Risk Predictability	Stability	Diversification
EsCoEq	1	2	1	1	4
CoEq	2	1(↑)	2(↓)	5(↓)	5(↓)
EsEq	5	5(↓)	3(↓)	3(↓)	3(↑)
EsCo	3	3(↓)	5(↓)	4(↓)	1(↑)
Sample	4	4	4	2	2

Notes:
The numbers indicate the rankings of each method compared with other methods according to a specific criterion indicated by the column caption. The upward and downward pointing arrows represent the rising or falling of the method's ranking compared to the proposed three-component method in the first row.
Source: Compiled by the author.

Table 4.7 Ranking Indices for 12 Financial Assets

	All	Financial Performance	Risk Predictability	Stability	Diversification
EsCoEq	1	2	2	1	1
CoEq	2	1(↑)	1(↑)	4(↓)	4(↓)
EsEq	4	5(↓)	3(↓)	2(↓)	5(↓)
EsCo	3	3(↓)	4(↓)	5(↓)	2(↓)
Sample	5	4	5	3	3

Notes:
The numbers indicate the rankings of each method compared with other methods according to a specific criterion indicated by the column caption. The upward and downward pointing arrows represent the rising or falling of the method's ranking compared to the proposed three-component method in the first row.
Source: Compiled by the author.

Table 4.8 Ranking Indices for Stocks

	All	Financial Performance	Risk Predictability	Stability	Diversification
EsCoEq	3	3	2	4	5
CoEq	5	4(↓)	3(↓)	3(↑)	4(↑)
EsEq	4	5(↓)	4(↓)	2(↑)	3(↑)
EsCo	1	1(↑)	1(↑)	5(↓)	1(↑)
Sample	2	2	5	1	2

Notes:
The numbers indicate the rankings of each method compared with other methods according to a specific criterion indicated by the column caption. The upward and downward pointing arrows represent the rising or falling of the method's ranking compared to the proposed three-component method in the first row.
Source: Compiled by the author.

Table 4.9 Ranking Indices for Bonds

	All	Financial Performance	Risk Predictability	Stability	Diversification
EsCoEq	2	1	1	3	4
CoEq	4	2(↓)	2(↓)	5(↓)	5(↓)
EsEq	3	5(↓)	4(↓)	2(↑)	3(↑)
EsCo	5	3(↓)	5(↓)	4(↓)	2(↑)
Sample	1	4	3	1	1

Notes:
The numbers indicate the rankings of each method compared with other methods according to a specific criterion indicated by the column caption. The upward and downward pointing arrows represent the rising or falling of the method's ranking compared to the proposed three-component method in the first row.
Source: Compiled by the author.

Table 4.10 Components' Effects

	Financial Performance	Risk Predictability	Stability	Diversification
CoEq (Missing Estimation Error)				
All Asset	(↑)	**(↓)**	**(↓)**	**(↓)**
Financial	(↑)	(↑)	**(↓)**	**(↓)**
Stocks	**(↓)**	**(↓)**	(↑)	(↑)
Bonds	**(↓)**	**(↓)**	**(↓)**	**(↓)**
EsEq (Missing Copula)				
All Asset	(↓)	**(↓)**	(↓)	(↑)
Financial	(↓)	**(↓)**	(↓)	(↓)
Stocks	(↓)	**(↓)**	(↑)	(↑)
Bonds	(↓)	**(↓)**	(↑)	(↑)
EsCo (Missing Market Equilibrium)				
All Asset	(↓)	(↓)	**(↓)**	(↑)
Financial	(↓)	(↓)	**(↓)**	(↓)
Stocks	(↑)	(↑)	**(↓)**	(↑)
Bonds	(↓)	(↓)	**(↓)**	(↑)

Notes:
This table is a summary of the arrow indicators from the previous 4 tables.
Source: Compiled by the author.

4.4.4 Result analysis

The robust merit of the proposed three-component method can help us discover the optimal asset allocation for CIC's international investment.Table 4.11 andTable 4.12 report the optimal combinations of the 15 asset classes considered using the three-component method and the sample mean-variance method respectively. A prominent feature in both tables is that the majority of the wealth is concentrated in fixed-income securities. This result is not due to a particular chosen method. In fact, the same phenomenon exists across all five methods. It is due to the fact that in the years under investigation, from 2008 to 2012, the performance of stocks and commodity ETFs is much poorer compared to that of bonds. The first six indices of stocks around the world and the last three commodity ETFs possess higher volatilities but also have lower returns. Therefore, in the optimisation processes, no wealth is allocated on these risky assets without justification of risk premium returns.

Although only small weights are assigned to stocks and ETFs, Tables 11 and 12 reveal the favourite fixed-income securities. The preferred choice in both methods is the US corporate bonds, which is also the most steady asset class, accounting for around 30% from 2008 to 2012. The other asset class with constant allocation during the period is the G7 government bonds. However, the sample mean-variance method significantly underestimates the importance of the G7 government bonds. With the three components taken into consideration, i.e. estimation error, copula and market equilibrium, the G7 government bonds take up 69.19% and 37.99% in 2008 and 2009 respectively. In these two years, when the financial crisis was at its height, it was sensible to choose the G7 government bonds as the safest option. Without the

copula risk appraisal mechanism, the simple mean-variance method fails to notice this. The three-component method is also the first to pick up the importance of emerging Pacific countries in the aftermath of the crisis. Heavier weights are assigned on the emerging Pacific countries' government bonds from 2010 onwards.

The optimal allocation result in Table 4.11 can also be compared with CIC's actual composition in fixed-income securities disclosed in their annual report. Fig. 4.2 shows this information as of 31 December 2012. With respect to the percentage of the investable corporate bonds, the 25.1% in the actual allocation differs little from our analysis across the five years, with an average of 28.68%. The difference lies in the allocation of sovereign bonds of advanced economies and emerging economies. In the actual CIC allocation, the proportion of advanced economies' government bonds is 54.7%, whereas the proportion for emerging economies is just 17.5%. However our analysis shows that the sum of emerging economies, mainly the Asia Pacific emerging markets and Europe, Middle East, and Africa emerging markets, takes an average of 43.55% from 2010 onwards, whereas the G7 bonds only account for an average of 24.95% in the same period of 2010, 2011, and 2012. Our analysis suggests that in the fixed-income securities investment there should be further diversification from the advanced economies' government bonds to the emerging economies in the period after the financial crisis.

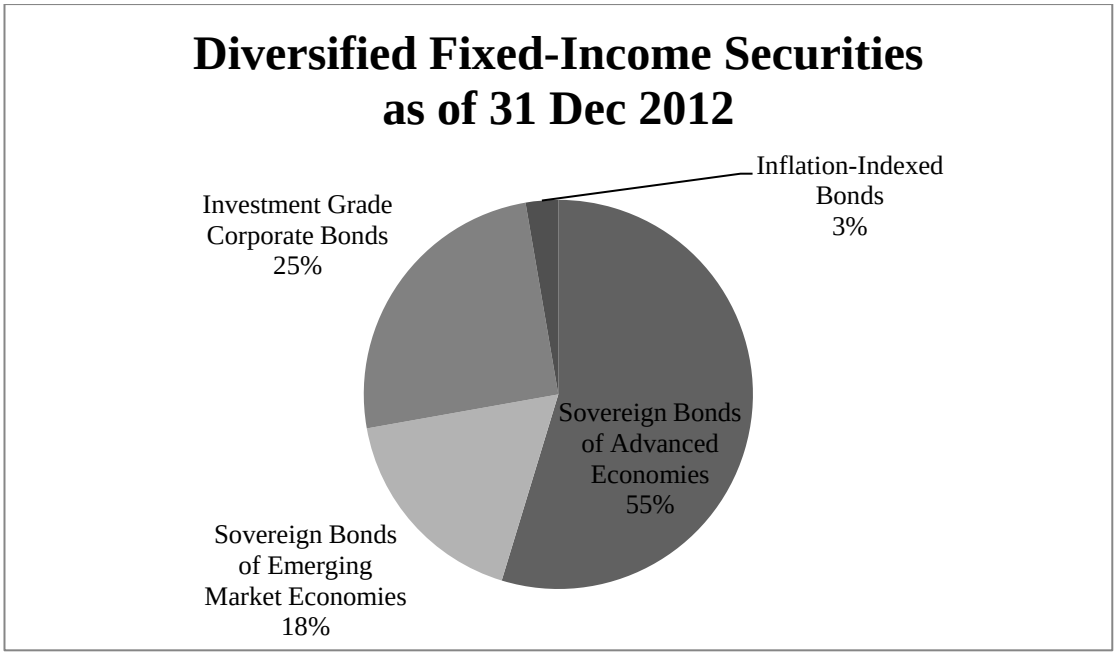


Fig. 4.2 Investments in Fixed-Income Securities by China Investment Corporation

Source: China Investment Corporation Annual Report 2012

Table 4.11 Optimal Strategic Asset Allocation under the Three-Component Method

	North America	Europe	Asian Pacific	EM Pacific	EM Latin	EM EMEA	G7 GOV	EM PACIFIC GOV	LATIN GOV	EMEA GOV	US COP	EMU COP	US ESTATE	US OIL	GLD
2008	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	69.19%	0.86%	0.00%	0.00%	29.94%	0.00%	0.00%	0.00%	0.00%
2009	0.00%	0.00%	0.01%	0.01%	0.00%	0.00%	37.99%	0.03%	1.03%	24.16%	36.73%	0.01%	0.01%	0.01%	0.01%
2010	0.02%	0.01%	0.01%	0.01%	0.01%	0.02%	28.69%	23.76%	1.54%	23.43%	22.34%	0.05%	0.02%	0.03%	0.05%
2011	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	19.83%	50.89%	10.52%	0.04%	18.68%	0.01%	0.00%	0.00%	0.01%
2012	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	26.33%	16.62%	3.52%	15.91%	35.39%	0.00%	2.22%	0.00%	0.00%

Source: Compiled by the author.

Table 4.12 Optimal Strategic Asset Allocation under the Mean-Variance Method

	North America	Europe	Asian Pacific	EM Pacific	EM Latin	EM EMEA	G7 GOV	EM PACIFIC GOV	LATIN GOV	EMEA GOV	US COP	EMU COP	US ESTATE	US OIL	GLD
2008	2.41%	0.00%	0.00%	0.07%	0.00%	0.00%	17.22%	11.18%	45.31%	0.00%	23.69%	0.00%	0.00%	0.03%	0.08%
2009	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	14.15%	0.01%	14.03%	53.88%	17.59%	0.00%	0.05%	0.00%	0.27%
2010	5.45%	0.00%	0.00%	0.00%	0.00%	0.00%	18.76%	16.40%	1.85%	27.92%	29.50%	0.00%	0.00%	0.11%	0.00%
2011	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	18.54%	20.50%	22.20%	12.27%	26.10%	0.00%	0.37%	0.00%	0.01%
2012	5.82%	0.00%	0.00%	0.00%	0.00%	0.00%	28.48%	15.01%	4.77%	11.08%	34.84%	0.00%	0.00%	0.00%	0.00%

Source: Compiled by the author.

There is one major difference in terms of the investment proportion of the Equities between our analysis and the reported actual CIC diversification. In Tables 11 and 12, there is virtually no wealth allocation in any of the stocks or the commodities compared to the fixed-income securities. This is due to the fact that in the sample period these Equities and commodity ETFs have higher volatilities, but without higher expected returns as compensation. The indices we choose for strategic asset allocation are well diversified to reflect the representative markets. The high percentage of investment in public Equities, 32% as disclosed in the CIC's annual report 2012, indicates the company's confidence in identifying and selecting the high performance equities out of the markets. It is likely that in their strategic asset allocation process they have used their own selective equities indices to decide the proportion between equities and the fixed-income securities.

Despite this difference, if we exclude the fixed-income securities from the portfolio for the moment, out market representative equity indices can still reflect the relative importance of the asset classes if such CIC expertise in selecting equities is not relied on, and market weighted equities are simply held.Table 4.13 shows the optimal allocation using the three-component method, and there are two evident features. First, we find that different from the situation in fixed-income securities, the equities in advanced economies, i.e. North America and advanced Asian Pacific countries, take the dominant role against the equities in emerging economies. This indicates the stronger positions of the corporate environment in advanced economies, although the government bonds of the G7 are weaker than expected. Second, the gold ETF performs well consistently across the five years, taking an average share of 52.56%. This manifests the importance of gold as a formidable competitor for investments.

Table 4.13 Optimal Strategic Asset Allocation in the Non-Fixed-Security Section

	North America	Europe	Asian Pacific	EM Pacific	EM Latin	EM EMEA	US ESTATE	US OIL	GLD
2008	38.10%	0.00%	19.86%	0.00%	0.00%	0.04%	0.01%	7.68%	34.30%
2009	1.15%	0.00%	20.63%	0.01%	0.00%	0.00%	0.01%	0.01%	78.20%
2010	30.67%	0.01%	22.21%	0.04%	0.01%	0.07%	0.02%	0.01%	46.96%
2011	8.94%	0.00%	18.78%	4.54%	0.00%	0.00%	5.31%	0.00%	62.42%
2012	12.90%	0.03%	12.03%	11.10%	0.05%	0.05%	22.71%	0.19%	40.93%

Source: Compiled by the author.

4.4 Conclusions

This chapter contributes both as a case study for China's SWF investment allocation decisions and in terms of methodology to innovate forecasting of asset returns. The method and the case are consistent, as the proposed three-component forecasting method suits the investment needs of CIC, China's SWF.

To provide insights on the strategic asset allocation decisions of the CIC, we first analyse the investment objectives through its history, comparison with other types of SWF and its investment environment. We summarize that the CIC needs much higher returns than does SAFE, which is responsible for the liquidity tranche of China's foreign reserves, but also in-depth risk appreciation due to its poor pre-crisis performance experience. Then, keeping these objectives in mind, the final optimised allocation results yield three suggestions for CIC. First, when diversifying fixed-income securities, more emphasis should be put on the sovereign government bonds in emerging market economies instead of the sovereign bonds in advanced economies. Second, on the equities side, the focus is reversed. Corporate performance in the advanced economies is superior to that in the emerging markets. Third, using the commodity ETFs of gold to represent the significance of gold in the

portfolio, it is discovered that gold is a formidable competitor to investment in equities.

This chapter also proposes an innovative method custom-made for the double emphasis on return and risk. The method for forecasting the asset class returns combines three components, i.e. estimation error, copula and market equilibrium, using the Bayesian theorem, in order to deal with the well documented problems in mean-variance optimisation, such as difficulty in estimating the proper parameters, lack of capability to handle non-Gaussian distributions, and the often extreme allocations. With regard to estimation error, Jorion (1985, 1986 and 1991) represents the direction of using Bayesian rule to incorporate the estimation risk. For the non-Gaussian returns, Hong *et al.* (2007) and other papers point out the importance of noticing asymmetries in individual assets and their dependence on the asset allocation decisions. In response to the unintuitive allocations of the mean-variance method, Black and Litterman (1991 and 1992) and subsequent papers propose models to incorporate the market equilibrium asset weights as a benchmark for analysis. We discover that a combination of the three is well suited to the CIC's investment requirements on both returns and special attention to extreme risks.

In order to test for the effectiveness and the robustness of the proposed method, we rank it with other comparable methods in the three aspects most important to CIC: financial performance, risk management, and allocation efficacy. In various situations, the proposed three-component method gives the overall best performance.

In the future research, improvements can be made in respect both of data and of methodology. With regard to the dataset utilised here, currently indices from FTSE

and Merrill Lynch represent the financial asset classes around the world. However, if it were possible to use a customized set of indices reflecting the views of CIC's analysts, the allocation result would be more informative. The diversification decision and the relative importance of each asset class can provide more guidance as to the strategic asset allocation decision. In the methodology aspect, firstly the Bayes-Stein method for reducing estimation error can be improved with other diffusing prior. Alternatively, the resampling or bootstrapping method can be incorporated with the other two elements in the model for comparison with the Bayes method. Secondly, the proposed model offering good financial performance, risk appraisal and allocation efficacy should be widely applicable in other asset allocation situations. For some insurance and pension management funds, as well as some university endowments, their strategic asset allocation objectives resemble the investment-centred SWFs such as CIC. Therefore, the method should be tested in a wider range of applications, and with consideration of the performance in assets with different risk regimes and different durations. In addition, the robustness test can be enhanced further. A bigger dataset, longer horizon, and more data divisions should be attempted to confirm the proposition of wider applicability.

CHAPTER FIVE

This chapter presents summaries and implications of each of the three aspects of the structure management for China's foreign reserves, and of the thesis overall. In addition, limitations of the research and future improvements on the topic are suggested.

CHAPTER 5

CONCLUSIONS

5.1 Research Conclusions and Implications

5.1.1 Optimal currency composition

An appropriate currency structure is an essential aspect of sound management of foreign reserves. It is the first step in managing investment in the liquidity tranche of the foreign exchange reserves of China, where the emphasis is on the liquidity demand for foreign trade and financing activities and the priority is risk concerns. In Chapter 2, we set up a flexible framework based on pair-copula construction. This approach allows us to model critical features of currency returns, including the asymmetry, fat-tails and complex dependence structure. In the context of China, we apply the copula model to analyse how these features affect the currency returns and to derive an optimal currency structure for China's reserves management.

Each currency return is first modelled using a variety of ARMA-GARCH filters with different residual distributions to best suit dynamics in univariate returns series. The dependency structure to connect each currency return is then modelled by pair-copula construction with two different vine structures. Based on the established distribution we use the preference under the disappointment aversion effect as the optimising objective to obtain the optimal currency composition. Our comparison shows that the mean-variance method cannot reflect the skewness, whereas the pair-copula method can capture the features of higher moments such as skewness and

kurtosis. Our further comparison shows the economic value of switching to the pair-copula models from the mean-variance framework. Considering the enormous amount of international reserves held by emerging economies such as China, the central bank in our model can achieve sizeable gains.

To analyse the Chinese case, we mimic China's currency shares of external payments by imposing *ad hoc* weight restrictions according to China's foreign trade and debt relations. Evidence shows that the pair-copula model with the D-vine structure has advantages over other methods. In this approach, the US dollar consistently takes the largest share in China's reserve currency composition. However, incorporation of the features of asymmetry, fat tails and complex dependence structure would allow more room for other currencies to be chosen for currency diversification of China's reserves. It is therefore desirable and feasible for China to adopt the copula approach to the currency composition of its reserves. Diversification is important for countering dependence complexities to manage currency composition of its huge and growing reserves.

5.1.2 Optimal asset allocation for foreign reserves

Strategic asset allocation is an essential part of foreign reserve management. It is also a natural sequence decision after the currency composition optimisation. Together they can serve for the management of the liquidity tranche of the foreign reserves. In a time of financial turmoil, it is of paramount importance to base the strategic asset allocation on robust risk management. In Chapter 2, we look at four aspects of this management: investment universe; the dependence structure; risk measure and asset allocation optimisation; and the decision on flight to safety. We apply the copula

approach to the risk-based management of foreign reserves in terms of strategic asset allocation. Special emphasis is placed on the impacts of asymmetries and fat tails on the asset allocation decisions.

In examining the dependence structure of the returns on the selected asset classes, we first analyse the univariate returns using an ARMA-GJR-GARCH model. A two-state regime-switching copula model for multiple asset classes is then developed to further analyse the dependence. A C-Vine copula is used to connect the seven representative asset classes that form China's investment universe. Twenty-one bivariate Clayton copulas are used as elements to form the joint dependence. The difference between the two regimes is that they have different pivotal variables in the first tier of the C-Vine structure. Each regime uses one safe asset as the protagonist, so that its asymmetric dependence with other assets can be better manifested.

Taking CVaR as the risk measure, two optimal asset allocation strategies are performed: the CVaR minimization and the DA utility maximization. They represent respectively the situations where the central bank is concerned only with the risk for the level of returns that can counter inflation, and the situation where the stance of the central bank is still conservative, but trade-off is allowed between higher returns and higher risk.

We deploy a regime-switching pair-copula multivariate model to highlight the features of safe assets. The two dependence regimes in our model allow us to focus on two safe assets, short- and long-term treasury bonds, respectively. The interchange between the two regimes is governed by a Markov chain. We find that if the central bank is focused solely on risk, the asymmetries would encourage the

flight to safety. However, if higher risks are allowed in trading for higher returns, even if the exchange is very conservative, the asymmetries would discourage the flight to safety. This indicates possible changes in the pattern of China's reserve investment. With the gradual passing of the recent global financial crisis, Chinese reserve managers may start to moderately increase their pursuit of returns by way of bolder investment in the classes of assets that are not among those traditionally believed to be safe. Given the massive size of China's reserve assets, this may bring about a new era of international investment.

5.1.3 Optimal asset allocation for sovereign wealth funds

The discussions of currency composition in Chapter 2 and asset allocation in Chapter 3 explore the structure management of foreign reserves in the vertical direction, where they represent different layers for managing the liquidity tranche of the reserves. In the horizontal direction, the parallel question to the management of the liquidity tranche is the asset allocation problem for the return tranche of the foreign reserves. Sovereign Wealth Funds are built largely to fulfil this function. Chapter 4 contributes both as a case study for China's SWF investment allocation decisions and in terms of methodology to innovate the forecast of the asset returns. The method and the case are consistent, as the proposed three-component forecasting method suits the investment needs of China's SWF, the CIC.

To provide insights on the strategic asset allocation decisions of the CIC, we first analyse the investment objectives through its history, comparison with other types of SWFs, and its investment environment. The findings show that the CIC needs much higher returns than the liquidity tranche of China's foreign reserves such as the

SAFE, but also in-depth risk appreciation due to its poor pre-crisis performance experience. Then, keeping these objectives in mind, the final optimised allocation results give three suggestions for the CIC. First, when diversifying fixed-income securities, more emphasis should be put on the sovereign government bonds in emerging market economies, instead of the sovereign bonds in advanced economies. Second, on the equities side, the focus is reversed. The corporate performance in the advanced economies is superior to that in the emerging markets. Third, using the commodity ETFs of gold to represent the significance of gold in the portfolio, it is discovered that gold is a formidable competitor to the investment in equities.

Chapter 4 also proposes an innovative method custom-made for the double emphasis on return and risk. The method for forecasting the asset class returns combines three components, i.e. estimation error, copula and market equilibrium, using the Bayesian theorem, in order to deal with the well-documented problems in mean-variance optimisation, such as the difficulty in estimating the proper parameters, lack of capability to handle non-Gaussian distributions, and the frequent occurrence of extreme allocations. In the aspect of estimation errors, Jorion (1985, 1986 and 1991) represents the direction of using Bayesian rule to incorporate the estimation risk. With regard to the non-Gaussian returns, Hong *et al.* (2007) and other papers point out the importance of noticing asymmetries in individual assets and their dependence on the asset allocation decisions. In response to the unintuitive allocations of the mean-variance method, Black and Litterman (1991 and 1992) and subsequent papers propose models to incorporate the market equilibrium asset weights as a benchmark for analysis. We find that a combination of the three provides a good fit for the investment requirements of the CIC on both returns and attention to extreme risks.

In order to test for the effectiveness and robustness of the proposed method, we rank it with other comparable methods in the three aspects most important to the CIC: financial performance, risk management, and allocation efficacy. In various situations, the proposed three-component method gives the overall best performance.

5.2 Limitations and Future Improvements

In the future research, there are several directions where improvements can be made. With respect to the management of the safety tranche of foreign reserves, which is covered in Chapter 2 and Chapter 3, the first suggested modification is to apply the asset-liability management method.

The very reason for the safety tranche of foreign reserves to be conservative is the consideration of likely withdrawals from it. The possible sources of withdrawals are summarized in the literature (Roger, 1993; Jeanne and Ranciere, 2011) as being from three directions: international trading needs, financing demands and sudden changes in the capital account. If more information on these can be obtained and future fluctuations can be reasonably forecasted, the optimisation of the allocation structure should incorporate the composition of the elements belonging to the liability side of the reserve, and the net position should be optimally allocated rather than just the overall asset side as in this current research.

Second, further research should look for better ways to quantify and incorporate various sources of transaction costs arising from transferring from one currency/asset to another currency/asset. This improvement still points at the core distinction of the

safety tranche of foreign reserves. It covers the precautionary need for funding from various perspectives.

The currency/asset structure of such precautionary demands can be very different from the optimal composition in terms of investment considerations such as risk and return prospects. The current manner of reconciling the difference between the structure of precaution demand and investment is to put *ad hoc* restrictions on currency/asset weights summarized from international trading of financing activities. However, this is an oversimplification. If the cost of conversion from an ample currency/asset relative to the emergency needs to a deficient one can be more accurately captured, the model will be more custom-made. The benefits or damages for a mismatch in composition between the precautionary demands and investment needs can be calculated and better managed.

With respect to the management of the return tranche of foreign reserves, i.e. the strategic asset allocation of the SWF, improvements can be made in both data and methodology. The current dataset comprises indices from the FTSE and Merrill Lynch to represent the financial asset classes around the world. However, if a customized set of indices reflecting the views of CIC analysts could be used, the allocation result would be more informative. The diversification decision and the relative importance of each asset class would give more guidance as to the strategic asset allocation decision. In terms of methodology, the proposed model offering good financial performance, risk appraisal and allocation efficacy should be widely applicable in other asset allocation situations. For some insurance and pension management funds, as well as some university endowments, the strategic asset

allocation objectives resemble those of investment-centred SWFs like the CIC. Therefore, the method should be tested in wider applications and with consideration of the performance of assets with different risk regimes and different durations. In addition, the robustness test can be enhanced further. A larger dataset, longer horizon, and more data divisions should be attempted to confirm the proposition of wider applicability.

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