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Dynamic Testing of an Elevated Water Tank

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Dynamic Testing of an Elevated Water Tank

Dynamic Testing of an Elevated Water Tank

A thesis submitted in partial fulfillment
of the requirements for the degree of
Master of Science in Civil Engineering

by

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University of Arkansas
Bachelor of Science in Civil Engineering, 2009

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This thesis is approved for recommendation to the Graduate Council.

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ABSTRACT

The objective of this thesis was to identify the modal properties of a pedesphere type, elevated water tank under different operating conditions relating to the water level in the tank. The research that was conducted included both numerical and experimental components. The numerical components consisted of a simple hand calculation to identify the fundamental frequency as well as numerical computer models to identify the natural frequencies and mode shapes for the first three bending modes. The experimental components consisted of characterizing the vibration of the elevated water tank in its current environment through ambient vibration testing.

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DEDICATION

This thesis is dedicated to my wife Samantha for her unwavering support and encouragement.

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CHAPTER 1. INTRODUCTION

1.1 BACKGROUND

The role of elevated water tanks as an integral part of the municipal water systems is crucial in the delivery of a constant supply of water to our nation's communities. These tanks may also prove a crucial asset in the event of any disaster which precipitates the immediate use of a large amount of water. Due to their consistent use it is paramount to maintain these structures in excellent operating condition. This requires an inspector to make scheduled inspections throughout the whole of the structure so as to locate any visible signs of distress or fatigue. Instances of structural fatigue that occur in elevated water towers may be caused from a variety of factors including wind gusts, standard operating procedures, or ground motion for seismically active areas. However, the effectiveness of maintenance and inspection programs is only as good as their timely ability to reveal problematic performance (Brownjohn 2007).

Experimental modal analysis methods have been used in numerous cases to identify and characterize the structural dynamic properties of bridges. For long span bridges, the dynamic properties of the structure, such as the natural frequencies, damping ratios, and mode shapes, are key parameters that reflect the operation and condition of the structure. Due to the flexible nature of long span bridges, dynamic loadings can sometimes result in severe or catastrophic damage, such as with the well known 1940 Tacoma Narrows bridge collapse. Although it is not as common for elevated water tanks to experience this catastrophic dynamic amplification, dynamic testing of these structures could be useful for characterizing the performance and condition of the structure.

Elevated water tanks are similar to long span bridges in that they are both relatively flexible structures. Determining the dynamic properties of these water tanks may be as beneficial as their bridge counterparts. The frequencies at which a structure resonates are a function of mass and stiffness, so as the mass and/or stiffness of the elevated water tank changes, the resonant frequencies will shift. If the dynamic properties of a structure are known, it is then possible to better understand the behavior of the structure under dynamic loadings, and could lead to the prevention of excitation amplification due to dynamic loadings.

In the area of structural health monitoring (SHM), structures are consistently monitored to note any changes which may indicate some level of degradation in structural integrity. With the use of a permanently installed sensor network, the structural integrity of an elevated water tank could be continuously monitored. SHM can potentially save time and money by allowing the inspector to place a higher priority on the location of the supposed damage, resulting in a more efficient inspection schedule. A quantitative characterization of the structure, such as what is obtained through global dynamic testing, is a necessary first step for developing a SHM regimen for elevated water tanks.

1.2 MOTIVATION

An elevated water tank located in Fayetteville, AR developed a fatigue crack during its operational service at the interface between the fill pipe and the water reservoir bowl located at the top of the structure. It was determined that a quantitative identification of the elevated water tank's structural characteristics may be beneficial for evaluating the future structural health and operation of the elevated water tank. A field vibration testing program was designed and implemented for the structure to identify a more quantitative description of the in-situ

performance characteristics of the tank. Such a characterization could be beneficial in the future to facilitate a more rational evaluation of the in-situ performance of this structure, which would enable a more reliable assessment of the structural integrity of the system. Due to the prevalence of this type of elevated water tank throughout the country, commonly referred to as a watersphere or pedisphere, any identified behavior through dynamic testing may provide valuable information to municipal owners of these type of elevated water tanks who may look to explore similar testing procedures.

1.3 OBJECTIVES AND SCOPE

The principal focus of this paper is the quantitative characterization of the in-service mechanical and performance characteristics of the elevated water tank as determined from a series of dynamic vibrations testing. The quantitative characterization will include three different operating conditions of the tank; full tank of water, half-full tank of water, and an empty tank. Although there are numerous examples in the literature related to vibration testing of full-scale civil infrastructures such as buildings, bridges, dams and chimneys, there is very limited information available related to vibration testing of elevated water tanks. The research objectives developed for this study are expected to contribute to the state of knowledge related to dynamic characterization of elevated water tanks by ambient vibration testing. The objectives are listed below:

- 1) Solve a simple hand-calculation numerical solution to identify the dynamic properties of the tank
- 2) Creation and analysis of numerical models

- 3) Dynamic characterization of an elevated water tank under different operating conditions through field vibration measurements

1.3.1 Numerical Solution

The simple hand-calculation numerical solution consists of a solved equation for calculating the fundamental natural frequency of a system with the known parameters of mass and stiffness defined for the system. The hand calculation provides a quick and simple evaluation for identifying the natural frequency of the first mode. The hand calculation can also serve the purpose of providing a comparison between the idealized solution and what is quantitatively obtained from the in-service full scale structure.

1.3.2 Numerical Models

In modern analysis of structures, much effort is devoted to the derivation of accurate models to be used in many applications of civil engineering structures such as damage detection, health monitoring, structural control, structural evaluation, and assessment (Bayraktar et al. 2007). In the development of finite element models, it is usual, and often necessary, to make simplifying assumptions. Inevitably, there will be differences in the results from the constructed numerical model and the results obtained from the field dynamic testing. These discrepancies originate from the uncertainties in simplifying assumptions of the geometry, materials, and boundary conditions of the structure. However, the limited number of structural connections and cantilever-like behavior of the elevated water tank help to eliminate the number of assumptions need to create a numerical model, which should result in accurate numerical models with reliable results.

Stick model

The elevated water tank was first simply modeled as a single concentrated mass attached at the top of a vertical cantilever with a given length, mass per unit length, and stiffness (Sadiku and Leipholz 1986; Chopra 2007). While this is a simplified model of the structure, it should be effective in determining the approximate modal properties, especially for the lowest order bending modes for the three different operating conditions.

3D model constructed from design drawings

Bayraktar et al. (2007) state “The finite element model of a structure is constructed on the basis of highly idealized engineering blueprints and designs that may or may not truly represent all the physical aspects of an actual structure.” Utilizing a very limited blueprint of the elevated water tank, a 3D numerical model will be constructed and subsequently analyzed by a 3D modeling program capable of determining the modal properties of a structure. The 3D numerical model will be able to identify the frequencies and mode shapes for higher level bending modes.

The initial results obtained from the 3D numerical model will assist in the design of the dynamic testing regimen. The number and location of the sensors used for the dynamic testing can be determined by evaluating the mode shapes identified by the 3D numerical model. This model will ultimately be validated using the results obtained from the field dynamic testing.

Comparison of the two numerical models

The dynamic characteristics, as determined from the different analytical models, will be compared. The simplified stick model should be more effective at identify the frequency and mode shape for the first bending mode, while the 3D numerical model will be able to identify the characteristics for higher level bending modes. This paper will look at the comparison of modal

parameters determined from both numerical models in addition to the results from the experimental dynamic testing.

1.3.3 Dynamic Characterization of an Elevated Water Tank

It is possible to determine the dynamic properties of the elevated water tank through a series of vibration tests. For this study, two different methods will be used to experimentally identify the dynamic system properties of the elevated water tank: experimental modal analysis and operational modal analysis. With experimental modal analysis, the structure is excited by an unmeasured input force, such as an impulse hammer or electrodynamic shaker. With operational modal analysis, also referred to as ambient vibration testing, the structure is excited by the natural environment under normal operating conditions, such as wind, ground motions, or the in-service operations of the structure. For this study, the modal characteristics identified from both types of testing will be compiled to present the overall identification of the modal parameters of the first three bending modes of the structure under the three different operating procedures; tank full, tank half-full, and tank empty.

CHAPTER 2. REVIEW OF LITERATURE

2.1 INTRODUCTION

There are many examples of previous work describing the experimental characterizations of large scale in-service civil infrastructures by vibration testing, however, the available literature involving the testing of elevated water tanks is somewhat limited. The research and application of vibration testing has been applied most often in the area of bridges. While it is well known that the United States bridge infrastructure is in serious need of successful and efficient structural health evaluation, which has likely pushed the emphasis of bridge applications in the field of dynamic testing, there is also a need to maintain a level of successful evaluation and maintenance of our country's water supply infrastructure.

The research that has been conducted on elevated water tanks is often conducted either under one operating condition, or while the structure is completely non-operational. The lack of research involving the testing of elevated water tanks under different operating conditions may be due to the complications of conducting such a test. The results from a series of dynamic vibrations testing of an in-service water tank under different operating conditions would provide valuable insight for designing, implementing and evaluating vibration testing programs for other elevated water tank structures.

The conducted research of elevated water tanks oftentimes focuses on the identification of the lowest order bending modes, or most often the fundamental bending mode, or first bending mode. The majority of the modal response of a cantilever structure is due to the response in its first bending mode. Thus, identifying the first bending mode yields the most pertinent information in the identification of the modal behavior of a structure.

Gentile and Saisi (2007) state that ambient vibration testing has recently become the main experimental method available for assessing the dynamic behavior of full-scale structures and has been demonstrated in recent studies to be especially suitable for flexible systems. While there are many techniques used for establishing natural frequencies and estimating mode shapes, the Peak Picking (PP) method is a simple, yet effective, technique that has been found to yield accurate results that correlate well with other, more intensive, techniques (Gentile and Saisi, 2007).

2.2 EXAMPLES OF PREVIOUSLY CHARACTERIZED IN-SERVICE STRUCTURES

2.2.1 Ambient Vibration Testing of a Historic Masonry Tower

Gentile and Saisi (2007) describe the procedure and results of ambient-vibration based investigations carried out on a historic masonry tower that dates back to the 18th century. The objective of the investigations were to dynamically characterize the tower as part of an extensive research program planned to evaluate the structural condition of the tower, which was characterized by the presence of major cracks on portions of the load-bearing walls. Ambient vibration testing was preferred for the characterization of the historic tower due to the ability to assess the structural parameters without having to introduce an excitation into the structure. As stated by the author, ambient vibration testing has recently become the main experimental method for evaluating the dynamic behavior of full-scale structures.

The tower was characterized using a roving instrumentation scheme that utilized two stationary sensors near the top of the structure for reference measurements. A total of nine piezoelectric sensors were used, in addition to the two stationary sensors, to record vibration

measurements in two different configurations for a total of 20 measurement points. Due to the low level of ambient excitation during the testing, the velocity measurements recorded were not in excess of about 0.15 mm/s. The tower vibrations were recorded for 38 minutes during each test setup at a sampling rate of 200 Hz.

The modal properties were extracted from the measured vibrations using two different output-only procedures: the Peak Picking method (PP) and the Frequency Domain Decomposition (FDD). Both methods are based on evaluation of the spectral matrix in the frequency domain. The auto-spectral densities (ASD) and cross-spectral densities (CSD) were estimated from the recorded time-histories using the modified periodogram method as originally outlined by Welch (1967). According to this approach, each recorded signal is divided into several frames that each consist of a specified number of samples, which are then averaged together before applying windowing and overlapping. For this study, the researcher divided each data record into 8192 points, then applied a Hanning window with 50% overlapping for spectral averaging, resulting in 100 periodograms. The averaging was completed on the periodograms, with a frequency resolution of 0.0244 Hz.

The modal parameters were first estimated using the Peak Picking method. The researcher stated that this method can lead to reliable results provided that the basic assumptions of low damping and well-separated modes are satisfied. For a lightly damped structure subjected to a white noise broad banded excitation, both the ASDs and CSDs reach a local maximum at the frequencies corresponding to the normal modes of the system, or at the resonant frequencies. The natural frequencies for the masonry tower were identified from resonant peaks in the ASDs and in the amplitude of CSDs, for which the cross-spectral phases are 0 or π . The mode shapes were obtained from the amplitude of square-root ASD curves and the phase information from the

CSDs. The spectral plots of the ASDs revealed well defined and consistent peaks; however, the drawbacks of this method are related to the difficulties in identifying closely spaced modes and damping ratios.

The modal parameters were then evaluated utilizing the Frequency Domain Decomposition technique. As described by the Gentile & Saisi (2007), the steps involved for this technique included: (1) the estimate of the spectral matrix; (2) the Singular Value Decomposition (SVD) of the spectral matrix at each frequency; (3) the inspection of the curves representing the singular values to identify the resonant frequencies and estimate the corresponding mode shapes using the information contained in the singular vectors of the SVD. This technique is explained in more detail by Golub and Van Loan (1996).

The researcher found the FDD technique to be a slightly more effective method to identify and evaluate mode shapes; however both techniques were found to be effective in identifying the modal parameters of the masonry tower. The two sets of mode shapes that were determined through the PP and FDD methods were compared by using the Modal Assurance Criterion (MAC) (Allemang & Brown, 1982). The MAC is one of the most widely recognized and used procedures to correlate two set of mode shape vectors.

The correlation is defined as follows:

$$MAC(\phi_{A,k}, \phi_{B,j}) = \frac{(\phi_{A,k}^T \phi_{B,j})^2}{(\phi_{A,k}^T \phi_{A,k})(\phi_{B,j}^T \phi_{B,j})}$$

where $\phi_{A,k}$ is the k th mode of data set A and $\phi_{B,j}$ the j th mode of the data set B . The MAC coefficient ranges from 0 to 1, where a value of 1 implies perfect correlation of the two

mode shape vectors and a value close to 0 indicates uncorrelated or orthogonal vectors. Gentile and Saisi (2007) state, “In general, a MAC value greater than 0.80 is considered a good match while a MAC value less than 0.40 is considered a poor match.”

A total of five vibration modes were identified from the ambient vibration data in the frequency range of 0-8 Hz. The resonant peaks revealed by the PP method were located at 0.59, 0.71, 2.46, 2.73, and 5.71 Hz. The peaks as revealed by the spectral plots of the ASDs were shown to be consistent throughout the sensors used for the testing. The plots, along with the calculated coherence values, suggested good quality of the data and the linearity of the dynamic response. The peaks, as determined through the FDD procedure, were found to be in agreement with those previously determined through the PP method. The corresponding mode shapes and scaled modal vectors obtained through the two different identification procedures were correlated using the MAC formula. For the five modes identified, the MAC values were found to be greater than 0.95, which suggests a high correlation between the information sets. The identified resonant frequencies were also found to be validated with results from a previous set of testing.

The experimental investigation was preceded by the development of a 3D finite element model (FEM) based on a geometric survey of the existing structure. The model contained a total of 4944 nodes, 3387 solid elements, and 80 shell elements with 14,286 active degrees of freedom. For formulation of the model, the tower foundation was considered fixed, a constant weight and Poisson’s ratio were assumed for the masonry, and rigid constraints were set to account for the connection of the tower to adjacent structures. The tower was divided into six regions for the purpose of assigning a separate Young’s modulus for the masonry contained within each region. There were five regions located at the base of the tower and one region to

represent all walls at the upper portion of the tower. In addition to the elastic moduli associated with the six regions, the spring constant representing the tower's connections to the adjacent structures was reviewed and updated. These seven structural parameters were estimated by minimizing the difference between the natural frequencies of the FEM and those identified through the experimental investigation. Through the process of updating the FEM, the modal behavior was found to correlate well with the experimental modal behavior. The first two modes were found to have a MAC correlation greater than 0.97. The correlation values for the higher modes were still in the range of 0.86-0.87.

The conclusions of the research summarize that the measurement of the structural response to the ambient levels of vibration proved to be an effective means for the identification of the dynamic properties of the masonry tower, although there were a few measurement points that resulted in a low signal-to-noise ratio. The results of the modes estimated by the PP method and FFD technique were found to be very similar. The dynamic-based assessment of masonry towers, and other flexible structures, appears to be a promising approach for evaluating damage in such structures, provided that an accurate geometric survey is available to establish the FE model.

2.2.2 Characterization of an Elevated Steel Water Tank

Sepe and Zingali (2001) describe the experimental characterization of an irregularly shaped elevated water tank as determined under ambient (wind) loading. The objective of the experiment was to identify both the static and dynamic structural properties due to wind, and to compare both the measured wind action and responses of the water tank structure to numerically calculated values. Due to the working status of the elevated water tank, the experiments were conducted while the tank was empty.

The shape and configuration of the elevated water tank were quite unique. The tank itself was described as a toroidal tank located 55 m (180.5 ft) above ground. The tank could best be described as a large circular ring oriented horizontally with the ground. The cross section dimensions were 4 m x 5.5 m (13 ft x 18 ft) and the overall diameter of the tank was 30 m (98.5 ft). Two pairs of cylindrical pillars supported the tank on each side of the ring. The pillars within each pair were 3 m (10 ft) in diameter and closely spaced at about 5 m (16.5 ft) center-to-center. The two pairs of pillars are diametrically opposed to each other on each side of the ring tank. One pair of pillars contains the stairs and lift which provide access to the water tank. There is also a large cylindrical element directly adjacent to these pillars that acts as a piezometer, which controls the water head in the tank. This piezometer element is 76 m (249 ft) tall and 7 m (23 ft) in diameter that tapers down to 3 m (10 ft) diameter at its base.

The experiment was conducted using four sensors located at the water tank level along with two cup anemometers to record the wind data. Two optical transducers, or spots, were used to record displacements. Two unidirectional accelerometers, each with a sensitivity of 0.02 g/V, were placed in a horizontal orientation at the top of the structure to record accelerations. All of the signals were relayed back to an analog-to-digital converter which was connected to a computer at the site. A remote connection established by modem enabled the researchers to control the data collection and processing remotely from off-site in real time. Each data record was composed of 4096 measurement points recorded at 25 samples/s, totaling about 164 seconds of data per record.

Because of the use of a limited number of sensors, the modal identification procedure conducted for this research was only able to identify the first three modes. More sensors would have been required along the height of the pillar supports to identify additional higher order

modes. The power spectral densities (PSD) were calculated and inspected for all of the recorded displacement and acceleration measurements. The actual identification of the modal properties was carried out on one of the displacement components, which exhibited the greatest response during testing. By making the assumption that the elevated water tank behaved as a single degree-of-freedom linear system, the natural frequency and critical damping ratio for the first mode were determined by utilizing the PSD of the recorded displacement data. The results for the first mode were as follows: $f_1 = 0.441$ Hz and $\xi_1 = 0.011$, where f_1 is the natural frequency and ξ_1 is the critical damping ratio. The following two modes were determined from the PSD plots to be $f_2 = 0.59$ Hz and $f_3 = 0.79$ Hz.

A numerical analysis of the structure was carried out prior to the experimental investigation using a three-dimensional finite element model with approximately 600 degrees of freedom. The elevated water tank was evaluated with both a full and empty water tank. As determined by the numerical model, the natural frequencies of the first three natural modes were significantly influenced by the presence of water in the tank, although the modal shapes appeared to be unaffected.

Even though the water tank was irregularly shaped, the investigation was able to determine the first three natural frequencies with close correlation between the experimental findings and the numerical calculations. The experimental investigation did not include the full water tank operating condition, however, the dynamic characteristics for the empty tank operating condition were well correlated with the numerical analysis. The researchers made a concerted effort to define the wind conditions during the numerical analysis and were able to validate their calculations through the wind data collected during the experimental investigation.

The natural frequency and damping ratio for the first mode were determined utilizing the displacement measurements and measured wind data.

2.3 SUMMARY

Ambient vibration testing has been shown to be effective in the dynamic characterization of large scale in-service structures. Results from modal identification conducted on ambient vibrations measurements have been found to correlate well with those extracted from forced vibration testing. Structures which exhibit linear lightly-damped behavior are ideal for this type of testing. Naturally occurring wind loading has been proven to serve as reliable sources of ambient excitation that exhibit the necessary characteristics for ambient vibrations testing, such as having broad banded white noise modal characteristics.

The amount of previous research that either included the dynamic testing of elevated water tanks under different operating conditions, or identified the higher level bending modes was very limited. It can be difficult to conduct vibrations testing on a full scale structure in operation, and it takes coordination with the ownership bodies to carry out testing on an elevated water tank under different operating conditions. The identification of higher level bending modes typically requires more sensor locations to properly identify the mode shapes, and thus the natural frequencies associated with the higher bending modes.

This research aims to provide an example of both a characterization of an elevated water tank under different operating conditions and the identification of the frequencies and mode shapes for the first three bending modes.

CHAPTER 3. AMBIENT VIBRATION TESTING OVERVIEW

3.1 INTRODUCTION

There are several forms of testing that can be used to experimentally characterize the dynamic properties of a full scale, in-service structural system. These include forced-vibration testing, free vibration testing and ambient vibration testing. Each form of testing has its advantages and disadvantages, depending upon the type of structure to be characterized, in regards to practicality of application and reliability of results.

In forced-vibration testing, a controlled force, or excitation, is applied to the structure during testing. The excitation is typically applied to the structure in a controlled fashion through the use of linear mass shakers, eccentric mass shakers, or instrumented hammers. Both the input excitation and the vibration responses of the structure being evaluated are simultaneously measured. These measurements are used to determine the modal properties of the structure. This form of testing can become expensive or complicated due to the need for a controlled, measureable input which can adequately excite the resonant frequencies of the structure.

In free vibration testing, the structure is subjected to some initial conditions, such as an applied displacement, which induces a free vibration response in the structure. The measured free vibration response of the structure is used to identify the modal properties of the structure. This form of testing is somewhat impractical for evaluating large-scale and in-service systems. Because the input excitation with forced-vibration and free vibration testing is controllable in terms of force magnitude, force direction, and frequency, the signal-to-noise ratio of the measured vibration responses of the structure is increased, or of better quality.

In ambient vibration testing, the vibration responses of the structure due to the natural (ambient) sources of excitation acting on the structure are used to identify the modal properties. The ambient sources of excitation for an elevated water tank would typically include wind, operation of the tank, and ground motions. In contrast to the other forms of testing, these ambient sources are unmeasured components. Due to the associated costs and difficulty of properly carrying out either forced-vibration or free vibration testing, ambient vibration testing is generally more commonly used for characterizing the dynamic properties of large in-service structural systems. Ambient vibration testing also oftentimes allows the test specimen to maintain in-service operations during the duration of testing.

This chapter provides a basic overview of the ambient vibration testing method utilized for this study. The necessary assumptions made with respect to the unmeasured input excitation, and the structure under consideration, are described. A description of some of the more important test considerations are then presented and described. Finally, the methods used in this study for estimating the modal properties are briefly described.

3.2 BASIC ASSUMPTIONS

With ambient vibration testing, the dynamic characteristics of the ambient sources of excitation cannot be identified, and assumptions must be made. In addition, ambient vibration testing, like the other dynamic test methods, requires that certain assumptions are made with respect to the structure being characterized. These assumptions are briefly described as follows:

- The ambient dynamic excitation is assumed to be broad banded Gaussian white noise for the frequency band of interest. This assumption implies that the power spectral

density of the unmeasured excitation is constant for the frequency band where the modes are expected to exist.

- The unmeasured ambient excitation and the structural responses are assumed to be stationary processes. This assumption implies that the average properties of both the ambient excitation and the structure may be computed over any length and number of time histories. This assumption applies to the isolated testing carried out for this specific study.
- The ambient excitation and the structural responses are assumed to be in a state that is closely similar to previous and future states. This assumption implies that the responses, when observed over an interval of sufficient duration, are assumed to represent the typical state of the ambient excitation and of the structure.
- The structure being tested is assumed to be linear.
- The structure being tested is assumed to be observable, meaning that the dynamic behavior is able to be identified through the testing program.
- Normal mode behavior (proportional damping) is generally assumed for the response of the structure.

In ambient vibration testing, it is generally understood that these assumptions are not always strictly true. It is also understood that any violations of these assumptions, if significant, will affect the reliability of the identified dynamic properties.

3.3 EXPERIMENTAL CONSIDERATIONS

There are several factors to consider when designing an ambient vibration experiment. Of the most important is that the responses measured during ambient vibration testing are

generally very low level vibrations that may be easily corrupted by noise. This can lead to errors associated with digitization of the analog measurement signals. To counter this, the accelerometers used must be very sensitive, and care must be taken to eliminate sources of experimental noise in the measurements. In addition to using sensitive accelerometers (large volt/g output), the resolution of the measured digitized bits of vibration responses may be increased by amplifying the analog signals to occupy a larger percentage of the range of the analog-to-digital (ADC) converter and/or by using an A/D converter with a higher number of bits.

The minimum sampling rate is usually at least twice the highest frequency of interest, as defined by the Nyquist criterion. For example, the frequency band of interest for this study consisted mostly of frequencies under 50 Hz. Thus, the raw data was assembled at 100 Hz before processing. The frequency band of interest may be established by a numerical model or preliminary measurements on the structure. A sampling rate of 10 times the Nyquist frequency was common for the tests conducted with this research.

3.4 IDENTIFICATION OF MODAL PROPERTIES

The modal properties (natural frequencies and mode shapes) can be evaluated from the measured vibration responses using a variety of modal identification methods. However, it should be noted that the mode shapes obtained from ambient vibration testing are considered to be operating deflection shapes. Operating deflection shapes approximate mode shapes, but mass normalized mode shapes cannot be extracted since modal scaling cannot be directly obtained through ambient vibration testing. The measured input excitation must be known to be able to determine the modal scale factor.

There are several modal identification methods that are conducted in the frequency domain. One of the more common methods is the Peak Picking (PP) method, which was used in this study for identifying the modal parameters of the elevated water tank. The peak picking method is considered one of the more basic approaches that can be used to identify the modal properties; however, the results have been proven to be accurate when applied to lightly damped structures. The details of this approach were originally outlined by Bendat and Piersol (1980). The basic premise of this approach is that when a lightly damped structure is subjected to random excitation, the output autospectrum, at any response point, will reach a maximum at frequencies where peaks occur in the frequency response function for the structure.

There are several characteristics that may be considered to help distinguish between the output spectral peaks that are due to structural modes and those that are due to peaks in the excitation spectrum or other noise. One characteristic is that all measurement points on a structure responding in a lightly damped normal mode of vibration will be either in phase or 180 degrees out of phase with one another. The direction of the phase depends merely on the shape of the normal mode. At frequencies where a peak in the output spectra is the result of a peak in the excitation spectra, the phase between any pair of outputs will usually be some value other than zero or 180 degrees. The phase between any pair of output measurements can be determined from the cross spectrum estimated between them. The magnitude of the cross spectrum estimated between the two output measurements will also peak at the locations of the normal mode frequencies. The ordinary coherence functions will also tend to peak at the normal modes since the signal-to-noise ratio in the measurements is maximized at these frequencies.

The normal mode shape at the identified resonant frequencies can be estimated from the measurements in a given direction using the following expression from Bendat and Piersol (1980):

$$\phi_i(y_i) = \left[G_{y_j y_j}(f_i) \right]^{1/2} \quad \begin{array}{l} i = 1, 2, 3, \dots \\ j = 1, 2, \dots, r \end{array} \quad (3.1)$$

where $G_{y_j y_j}(f_i)$ is the output autospectral density value at the i th normal mode frequency and the j th location. A minimum number of output sensor locations equivalent to the desired number of identified normal modes is required. However, more sensor locations may be desired to define the mode shape in more detail. The amplitude of the mode shape is defined at each degree of freedom by the relative difference in the magnitude between each output location and a specified reference location. The phase information for each measurement degree of freedom is determined from the cross spectra function between each output and the reference. The equations used for estimating these spectra are given in Chapter 4 of this thesis.

Another approach used for constructing the mode shapes is made by utilizing the transfer function computed between each output sensor location and the reference sensor. The transfer function is based on the frequency response function (FRF) is used when a calibrated input measurement is available with the output measurements. As shown by Bendat and Piersol (2000), an estimate of the FRF would be in the form shown:

$$\dot{H}(f) = \frac{\dot{G}_{xy}(f)}{\dot{G}_{xx}(f)} \quad (3.2)$$

where the numerator is the cross spectrum between the input and the output measurements, and the denominator is the autospectrum of the calibrated input measurement. Since there is no

measurement of the input available in an ambient vibration test, the reference sensor, $x(t)$ is selected as the input. The transfer function between the input $x(t)$ and each output $y(t)$ can then be described by the following expression:

$$\dot{T}_{xy}(f) = \frac{\dot{G}_{xy}(f)}{\dot{G}_{xx}(f)} = \frac{X^*(f)Y(f)}{X(f)X^*(f)} = \frac{Y(f)}{X(f)} \quad (3.3)$$

The magnitude of the transfer function is the ratio of the amplitudes of the output sensor locations to the reference sensor. The phase factor in turn provides the phase information for each output sensor location in relation to the reference sensor.

Although the peak picking method has been successfully used to identify the modal properties of many large scale constructed systems by ambient vibration testing, there are some limitations associated with this approach. One of the main limitations with the peak picking method can be the subjectivity in identifying the peaks if the peak amplitudes are not very large. In an attempt to reduce the subjectivity associated with this method, Felber (1993) developed an automated implementation of the peak picking method. Another limitation with this approach is related to the modal identification being completed in the frequency domain. The natural frequencies are identified from spectral peaks within the frequency spectra. As a result, the accuracy of the identified spectral peaks is determined by the frequency resolution of the spectra. The peak picking method also does not identify the modal participation for multiple degree of freedom systems. This can adversely affect the accuracy of the identification if the modes are closely spaced since several modes will contribute to the response at each peak. The peak picking method is also typically limited to structures that are lightly damped. The mode shapes identified by this approach are actually operating deflection shapes. The operating deflection

shapes reflect not the shape of a single mode but rather the contributions of multiple modes.

However, the operating deflection shapes will be close approximations of the normal modes if

the system is lightly damped and the modes are well-separated.

CHAPTER 4. AMBIENT VIBRATION TEST OF THE ELEVATED WATER TANK

4.1 INTRODUCTION

This chapter describes the experimental characterization of the elevated water tank by ambient vibration testing. The experimental characterization was performed to investigate the related causes to fatigue cracking at the interface between the water tank and the fill pipe. The characterization consisted of identifying the dynamic properties (frequencies and mode shapes) for the elevated water tank. The measured natural frequencies and the corresponding mode shapes of the elevated water tank are directly correlated to its mass and stiffness characteristics, the internal connections of the various components that make up the water tank structure, and the boundary conditions of the elevated water tank. The main objective for undertaking the experimental characterization of the in-service structure was to better understand the operating behavior of the elevated water tank in its natural environment and to thus create a calibrated analytical model with which to characterize the structure.

The plan to carry out a series of ambient vibration tests for determining the dynamic properties of the elevated water tank was determined based upon the assumptions required for conducting ambient vibration testing. The configuration and behavior of the elevated water tank is inherently flexible. From initially considering the elevated water tank analytically as a SDOF inverted pendulum, it is reasonably assumed that the mode shapes of the structure would be well separated and not significantly complex. The elevated water tank can be considered a linear structure, in that all of its components are linked in a fixed, ordered, and direct succession. Thus, the elevated water tank structure can be treated as a lightly and classically damped structure,

which lends itself to being able to be experimentally characterized through ambient vibration testing.

The execution and analysis of the ambient vibration testing program that was implemented for the elevated water tank are described in the following sections of this chapter. The ambient vibration environment at the elevated water tank is also discussed. The vibration data was analyzed using the peak-picking technique that has been used in previous experimental characterizations of elevated water tanks. The results of the data analysis using this approach are presented and discussed.

4.2 DESCRIPTION OF THE ELEVATED WATER TANK

The elevated water tank, shown in Figure 4.1, is located in Fayetteville, AR, and is currently owned and operated by the city of Fayetteville. The elevated tank is a 75,000 gallon steel watersphere built by Chicago Bridge and Iron Co. in 1976. The water tank has an inside diameter of 27 feet and sets on a 6.5 feet diameter steel shaft at 88.83 feet above finished grade. The water line elevation varies from 92.33 feet to 117.5 feet above finished grade depending if the water tank is operating anywhere with between an empty to full tank. The shaft is supported by a bell that is roughly 17.6 feet tall. The bell diameter increases from 6.5 feet diameter at the top of the bell to approximately 15 feet diameter at the base. The structure is supported on a circular concrete foundation that bears on the naturally occurring shale 9 feet below the finished grade.

Worthy of note, several sets of antennas have been added to the elevated water tank since the original construction of the structure. At the time of this writing there was a copy, made available to the researcher, of the load check conducted in 1994 for the increased wind forces on

the elevated water tank due to antennas being installed at the top of the structure above the water tank. As noted by the engineer who conducted the load checks, the result of the calculations indicated that two minor overstresses were apparent at the foundation level. These were concluded, however, to not be serious enough to warrant any structural strengthening of the elevated water tank. It should also be noted that an array of antennas are currently installed on the outside of the shaft just below the water tank. These can be seen in the image shown in Figure 4.1. These antennas were neither mentioned nor accounted for in the previously mentioned calculations.

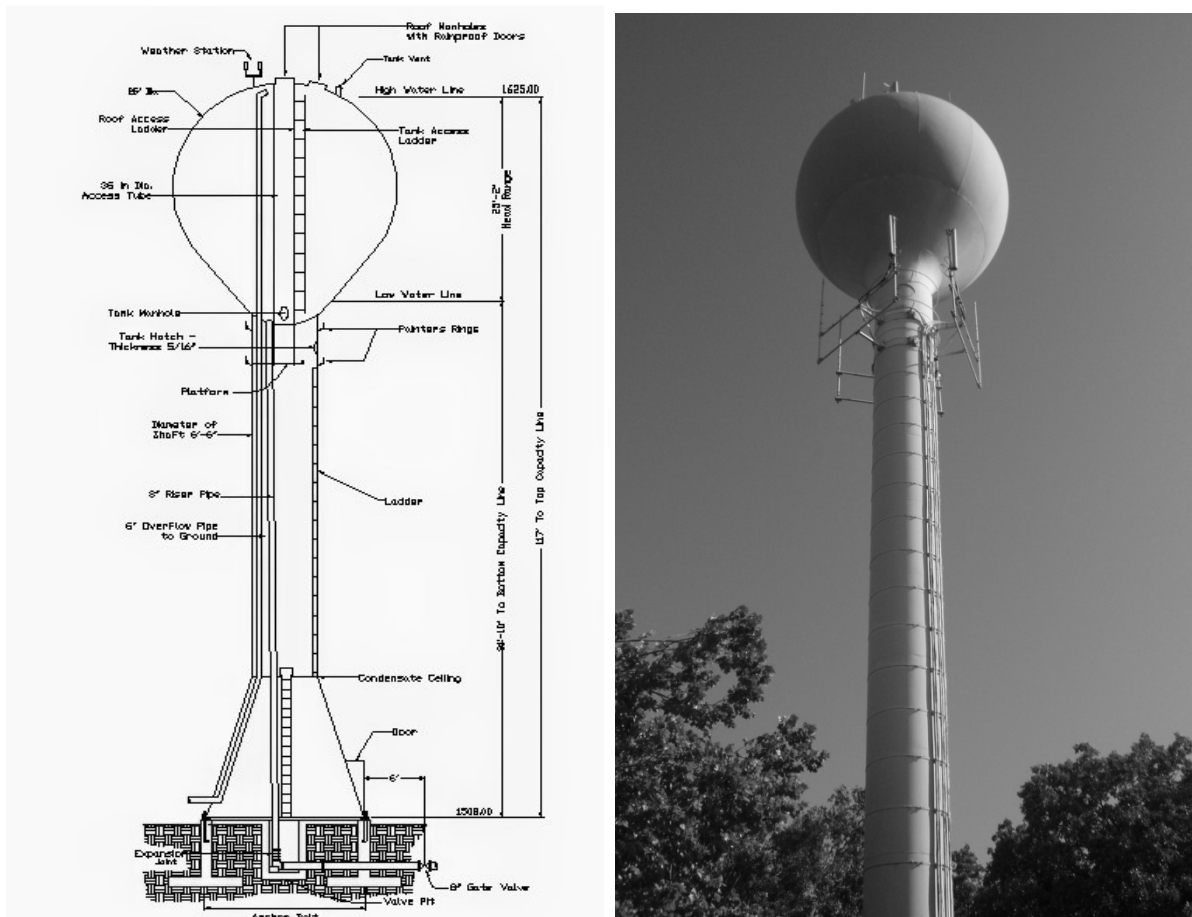


Figure 4.1. CAD Drawing (left) and picture (right) of Elevated Water Tank (Photo by Author)

4.3 TEST DESCRIPTION

4.3.1 Proposed Test Procedure

Due to the configuration of the structure and the difficulty in properly conducting a forced vibration testing program, an experimental program was devised that would consist of a series of ambient vibration tests on the water tank under different operating conditions. The ambient vibration testing would be used to determine the natural frequencies of the tower. The results of the experimental testing could then be used to validate the associated finite element model (FEM) of the elevated tank. The major concern with utilizing the ambient vibration testing to determine the modal parameters of the elevated water tank is the inherent low signal-to-noise ratio that is the result of not introducing an excitation input into the structure in addition to the natural environment (typical wind conditions, normal tank operation, etc.). This was expected to possibly lead to some uncertainty related to the identified modal characteristics.

A linear mass shaker was used during a portion of the vibration testing. The output from the linear mass shaker was found to increase the amplitude of the measured vibrations of the structure. The output from the linear mass shaker was typically set to operate at a frequency outside of the frequency range of interest, as determined from the analytical study. The increased amplitude of the vibration response of the elevated water tank due to the shaker input would likely affect the identification of the damping ratio for the structure. In addition, it is difficult to determine the damping ratio of a structure through ambient vibration testing alone, so the damping ratio of the elevated water tank was not evaluated for this research.

The ambient vibration testing would be conducted using a system of accelerometers installed at several locations along the height of the structure. The locations of the

accelerometers were determined by identifying nodes of interest from the FEM and ease of accessibility.

4.3.2 Instrumentation Scheme

A total of 17 accelerometers, 3 triaxial sensors and 14 uniaxial sensors (Figure 4.2), were installed at different measurement levels throughout the elevated water tank, as shown in Figure 4.3 and Figure 4.4. The individual accelerometers were installed at various locations along the height of the elevated water tank to measure horizontal vibration responses along two different axes that were orthogonal to each other. The accelerometers were installed so that all of their reference axes were aligned with each other.

A triaxial accelerometer was installed at the top of the water tank to measure vibration responses in the longitudinal, transverse, and vertical directions. Two more triaxial accelerometers were installed, one at the mid-height and one at the base of the water tank, to measure vibration responses in the longitudinal and transverse directions. A total of eight uniaxial accelerometers were installed along the fill pipe at different measurement levels to record the longitudinal and transverse vibrations. An additional six uniaxial accelerometers were installed to the interior of the shaft and bell structure at different measurement levels to record the longitudinal and transverse vibrations.

The accelerometers were installed to the different components of the elevated water tank using magnets that adhered to the steel structure. The accelerometers were installed by utilizing the access ladders and platforms within the interior of structure as shown in Figure 4.5. The accelerometer cables were routed down through the inside of the shaft to the data acquisition system located at the base of the bell shaft of the structure.

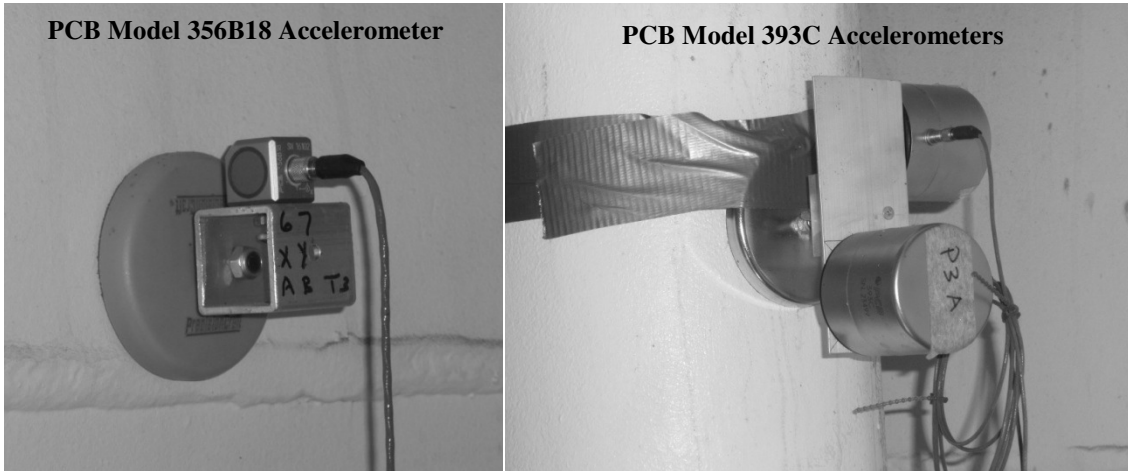


Figure 4.2. Triaxial (left) and uniaxial (right) accelerometers installed on Elevated Water Tank
(Photo by Author)

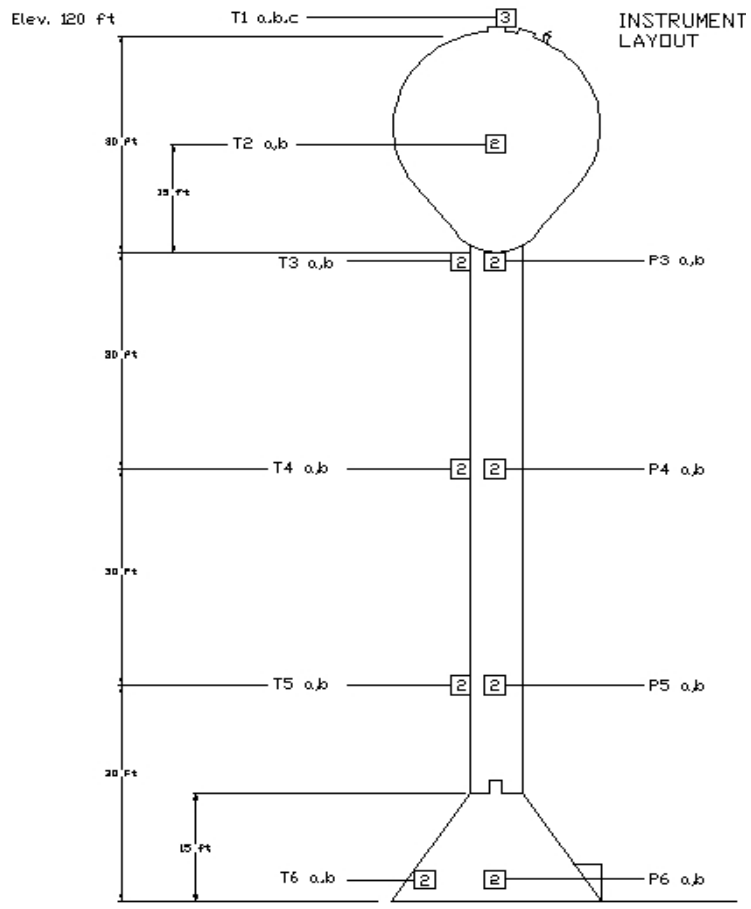
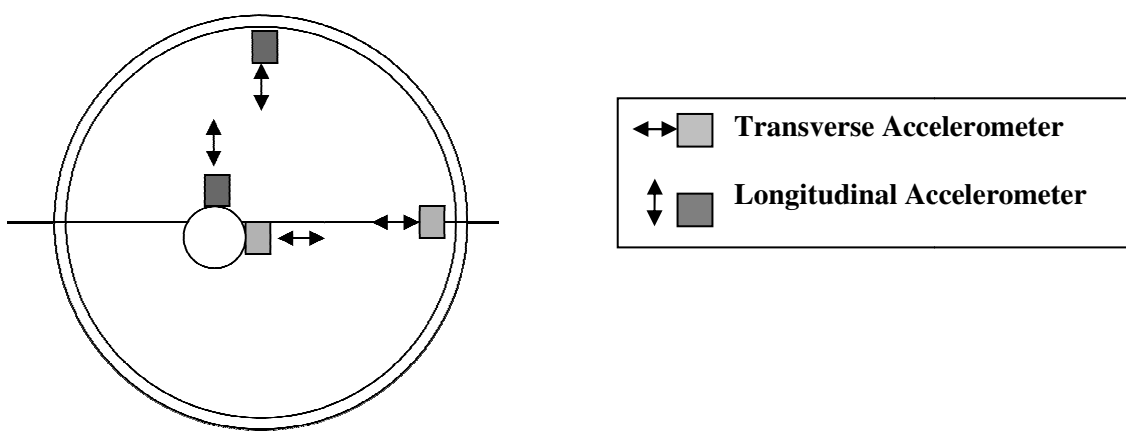


Figure 4.3. Measurement levels on the Elevated Water Tank



Shaft Levels 3, 4, & 5

Figure 4.4. Accelerometer layout at shaft of Elevated Water Tank



Figure 4.5. Installation of accelerometers on the Elevated Water Tank (Photo by Author)

4.3.3 Sensors and Data Acquisition

The instrumentation scheme developed for the ambient vibration testing of the elevated water tank included sensors for measuring the vibration responses and an anemometer for measuring wind speed and direction. The sensors were installed to the fill pipe and elevated water tank structure at various different levels. The cables for the accelerometers were all routed to a single data acquisition device located at the base of the structure. The sensors were left in

their installed locations for the entirety of the of the ambient vibration testing program. More specific information concerning the sensors and data acquisition system that were used for the experimental program are described in the following sections.

4.3.3.1 Accelerometers

Two different types of accelerometers were used to measure the vibration responses of the elevated water tank. The first type was the Model 393C seismic accelerometer from PCB Piezotronics, Inc. The second type was the Model 393B05 seismic accelerometer; also from PCB Piezotronics, Inc. Both types of accelerometers are integrated circuit piezoelectric (ICP) sensors, and feature built-in signal conditioning elements. Piezoelectric accelerometers use the piezoelectric effects of certain materials to directly convert acceleration to a low impedance voltage signal. The voltage signal produced is directly proportional to the force experienced by the accelerometer and can be transmitted over long cable distances with minimal loss in signal quality. The Model 393C accelerometer utilizes a quartz compression element to sense vibration, and the Model 393B05 accelerometer utilizes a ceramic shear element. The Model 393C accelerometer is a uniaxial sensor, which means that it can only measure accelerations in one direction. The Model 356B18 accelerometer is a triaxial accelerometers and is capable of measuring vibrations in three orthogonal directions at any given time. The specific performance characteristics for both of these sensors are summarized in Table 4.1.

Table 4.1. Performance characteristics of the accelerometers

Parameter	Model 393C (Uniaxial)	Model 356B18 (Triaxial)
Sensitivity	1000 mV/g ($\pm 15\%$)	1000 mV/g ($\pm 10\%$)
Measurement Range	2.5 g peak	5.0 g peak
Frequency Range	0.025 to 800 Hz ($\pm 5\%$)	0.5 to 3000 Hz ($\pm 5\%$)
Broadband Resolution	0.0001 g rms (1 to 10,000Hz)	0.00005 g rms (1 to 20,000 Hz)
Nonlinearity	$\leq 1\%$	$\leq 1\%$
Transverse Sensitivity	$\leq 5\%$	$\leq 5\%$

4.3.3.2 Data Acquisition System

The data acquisition system used for the project consisted of three hardware components: (1) multi-channel accelerometer signal conditioners from PCB Piezoelectronics, Inc., (2) a Model PXI-1042Q chassis which contains a Model PXI-8106 controller and Model PXI-4472B input modules from PCB Piezoelectronics, Inc., and (3) a laptop computer. Figure 4.6 shows a graphical diagram of the data acquisition system.

The signal conditioners used were specific for use with piezoelectric accelerometers. Each signal conditioner could accommodate 16 accelerometers and supplied the excitation voltage required to operate the sensors.

The analog to digital conversion of the acceleration signals is accomplished using Model PXI-4472B input modules which are installed into open slots on the PXI chassis. Each input module is an 8 channel, 102.4 kSamples/second digitizer with 24 bit resolution for amplitude and dynamic range. The digital signal processing (DSP) modules were configured to collect the

acceleration measurements using DC coupling, so that frequencies below 1 Hz would be observable from the vibration measurements.

A short patch cable (SMB 120) connected each output channel from the signal conditioners to the input modules on the PXI chassis. A laptop computer connected to the PXI-8106 controller on the PXI chassis was used to control the data acquisition system and to store the measurement data.

The data acquisition system was organized at the base of the elevated water tank structure during the testing, and removed when testing was not being completed. A separate RG58U coaxial cable was used to connect each accelerometer to the signal conditioner.

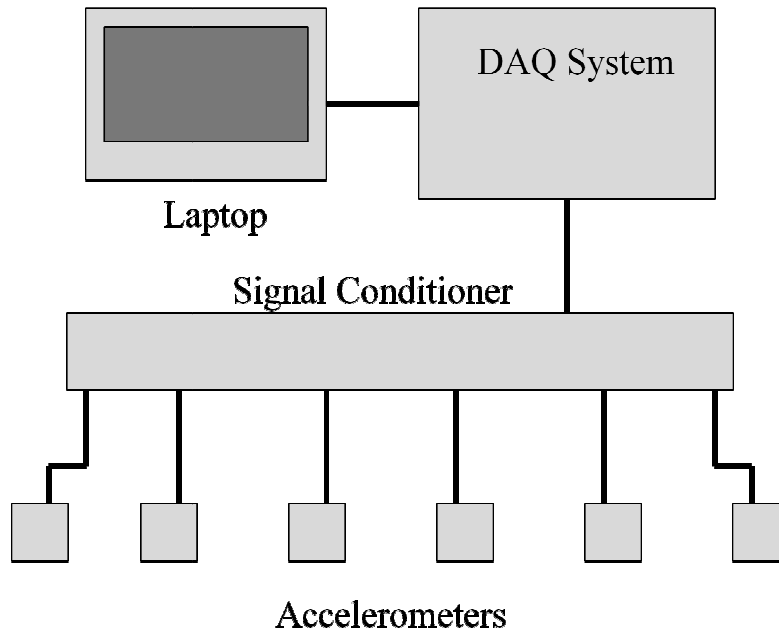


Figure 4.6. Data acquisition system components

4.3.4 Test Execution

The ambient vibration testing was conducted over the course of three days; September 29, October 8 & 15 of 2009. The data acquisition system was manually operated from the base of the structure. The data acquisition system was also set to operate for an extended period while unattended on September 29, 2009. The acceleration signals were measured at 1000 samples/second (1000 Hz), and were recorded in individual files containing 60,000 samples, or 60 seconds of data. These tests were often run consecutively to record approximately a total of ten minutes worth of data. On one occasion, the data acquisition system was left to collect a continuous stream of data over an approximately six hour time span.

4.4 DATA PROCESSING AND ANALYSIS

4.4.1 Data Pre-Processing

The data pre-processing consisted of a few steps to ensure the quality of each measured acceleration signal and to prepare these signals for the subsequent data processing steps needed to identify the dynamic properties for the elevated water tank. The data pre-processing procedure applied to the acceleration response data included the following steps:

- Data quality analysis
- Data filtering
- Signal cleaning

The first step of the procedure consisted of visually inspecting the raw time domain signals from each output signal. was used to initially evaluate the quality of the signals recorded by each output channel. The remaining steps were applied once the data had been collected and brought back to the lab. The removal of noise and bias errors from the raw time domain signals

was a significant step during the data filtering, since these errors can affect the estimating of the modal parameters. Removing the malfunctioning sensor channels during the signal cleaning step also served to avoid errors during the modal parameter estimating of the subsequent data analysis.

4.4.1.1 Data Quality Analysis

The quality of the measured data was initially evaluated by visually inspecting the untreated time domain signals for each channel. Large spikes in the signal, drift of the signal over time, or a lack of any identifiable response are all possible characteristics of the recorded time domain signals that might signify noisy or malfunctioning sensors. The frequency domain signals were also inspected to identify any malfunctioning sensors, as well as to establish the frequency range of interest for testing by identifying the range that contained the majority of the significant response. In the event of a noisy or malfunctioning sensor, the cabling and associated hardware could be checked for related issues. In some cases, malfunctioning sensors were left to operate during testing and the related signal was removed from the processing at a later step.

4.4.1.2 Data Filtering

After the testing was completed to record the measured vibrations, the signals were filtered to remove any DC bias or drift from the signals. This digital filter was achieved by using the detrend command in Matlab. As defined by Matlab, the detrend command identifies a linear trend within a specified group of data and sets the mean value, or straight line vector, of the trend equal to zero. This command can be completed on any specified number of data points. For the measured data from the ambient vibration testing, the detrend command was completed on each individual block of recorded data, composed of 60,000 data points for each sensor channel. A plot of the raw time data before and after completing the detrend command is shown in Figure

4.7. As can be seen in Figure 4.7, the responses for each channel are centered at 0 g after completing the detrend command on the data. The detrend command also helps to remove the DC, or 0 Hz, bias. The DC bias will result in a large peak in the PSD function located at 0 Hz. This could result in effectively drowning out any natural modes of the structure located close to 0 Hz. A plot of the PSD function estimated from the acceleration response both before and after completing the detrend command on the time domain signal is shown in Figure 4.8. A large response at 0 Hz can be seen for the PSD plot of the non-detrended data. The spectral peaks of the structure, which can be seen on the PSD plot of the detrended data, are not visible next to the large peak at 0 Hz.

4.4.1.3 *Signal Cleaning*

The initial data analysis identified some malfunctioning accelerometers during the ambient vibration testing. One of the malfunctioning sensors was located at the top of the fill pipe, labeled P3X as shown on Figure 4.3. The other malfunctioning sensor was located on the shaft of the tank structure, labeled T5X as shown on Figure 4.3. Visual inspection of the raw time domain signals clearly showed a malfunction with the sensors, as can be seen with the sensor channel, T5X, in Figure 4.7. A PSD plot of the malfunctioning sensors on the tank and pipe are shown in Figure 4.10 and Figure 4.12, respectively. The responses from these accelerometers were removed from the subsequent data processing and analysis.

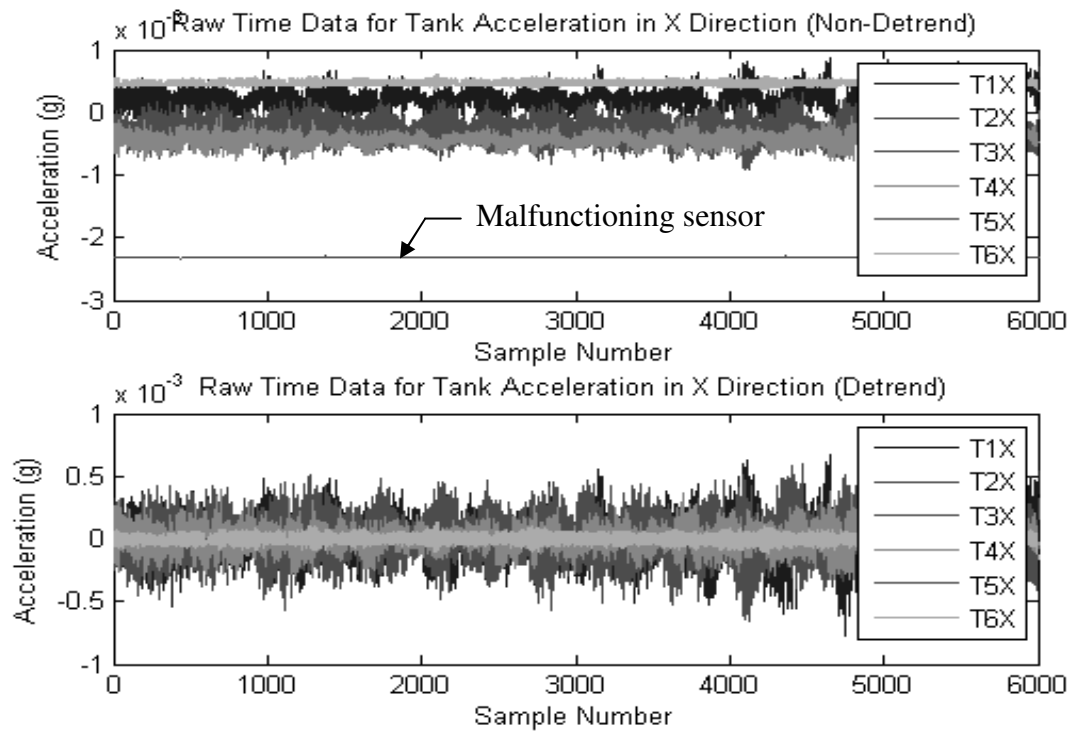


Figure 4.7. Raw time data before and after applying the Matlab detrend command

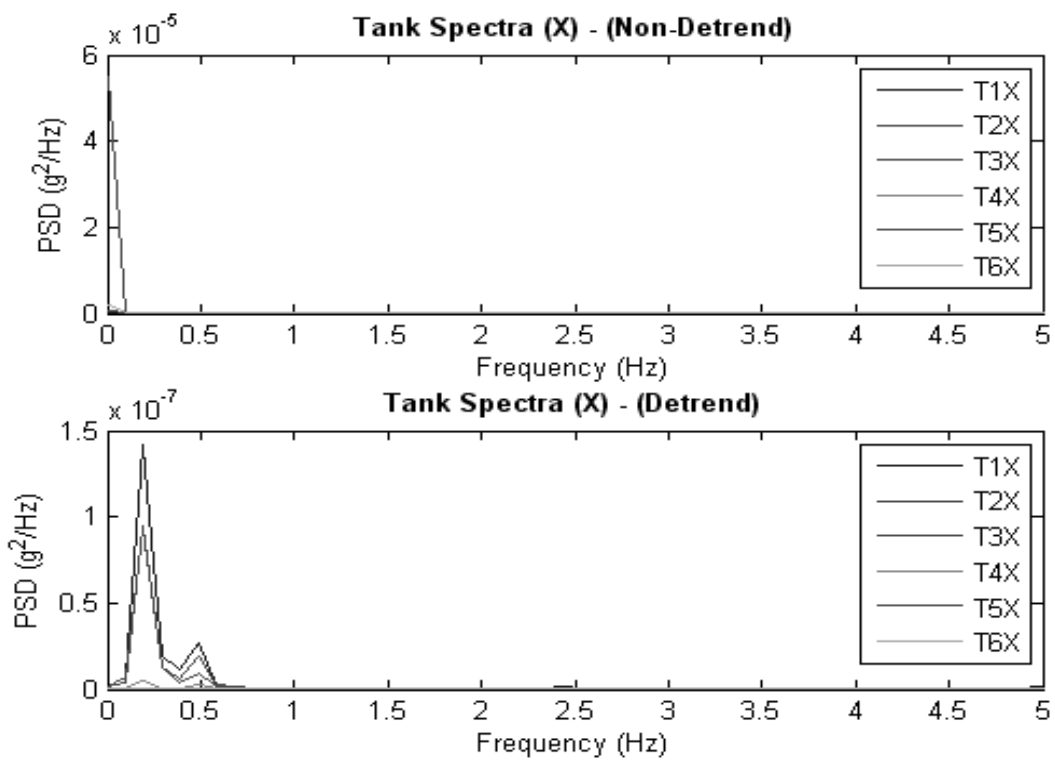


Figure 4.8. PSD plot before and after applying the Matlab detrend command

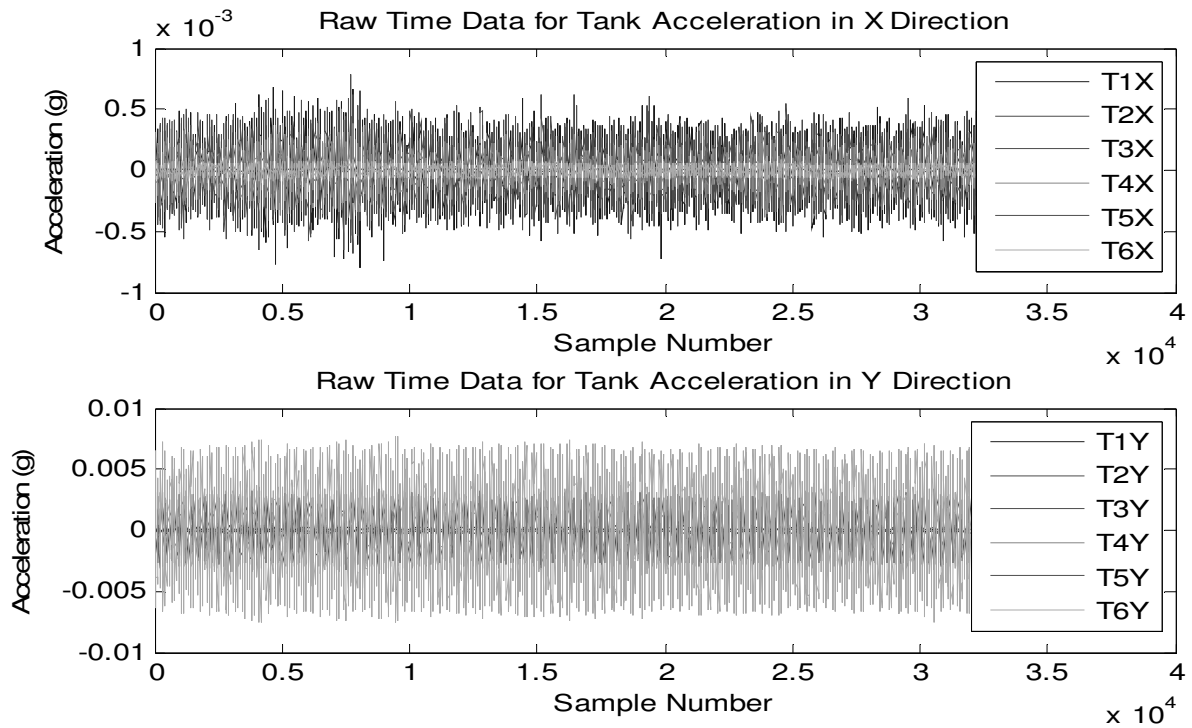


Figure 4.9. Raw time data for sensor channels located on the tank

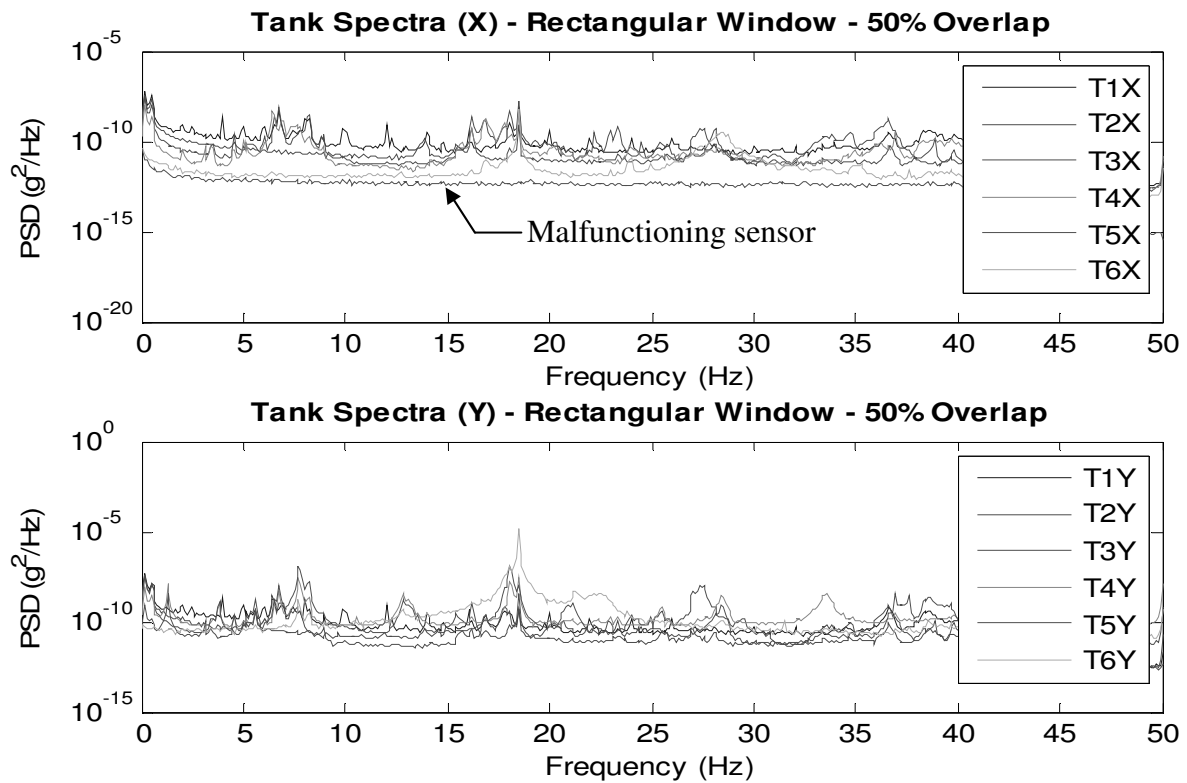


Figure 4.10. Initial PSD plot of the sensor channels located on the tank

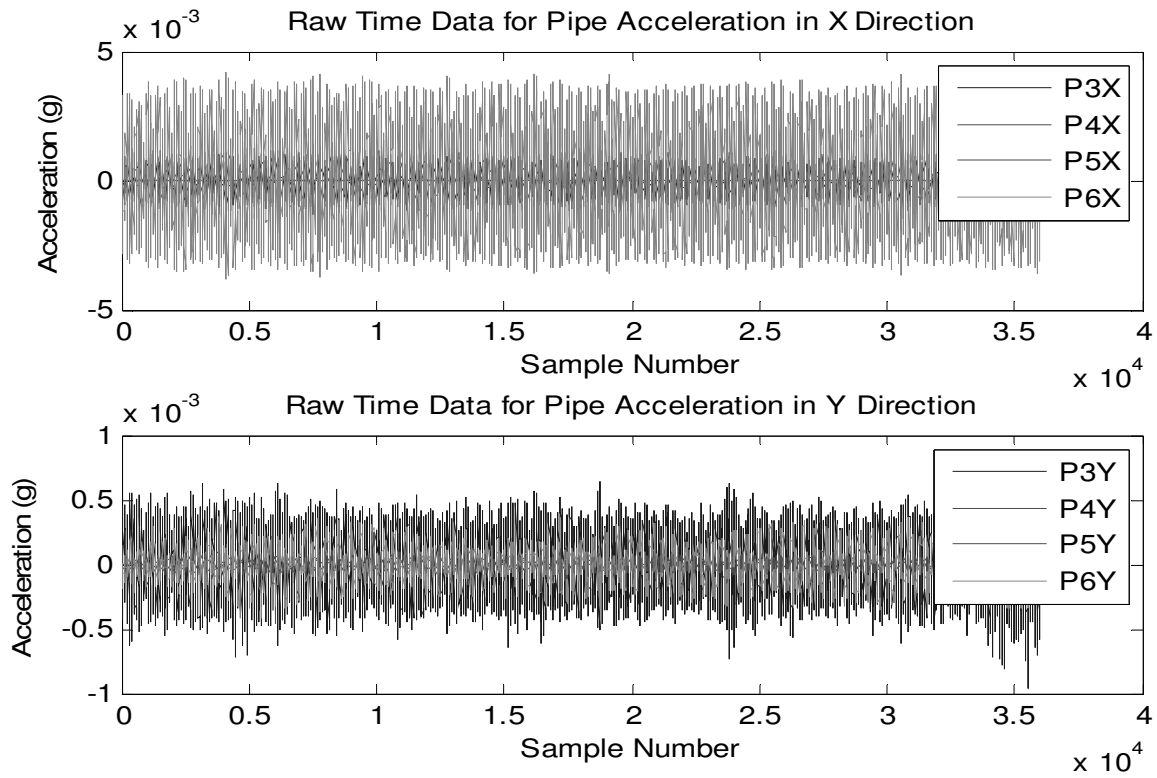


Figure 4.11. Raw time data for sensor channels located on the pipe

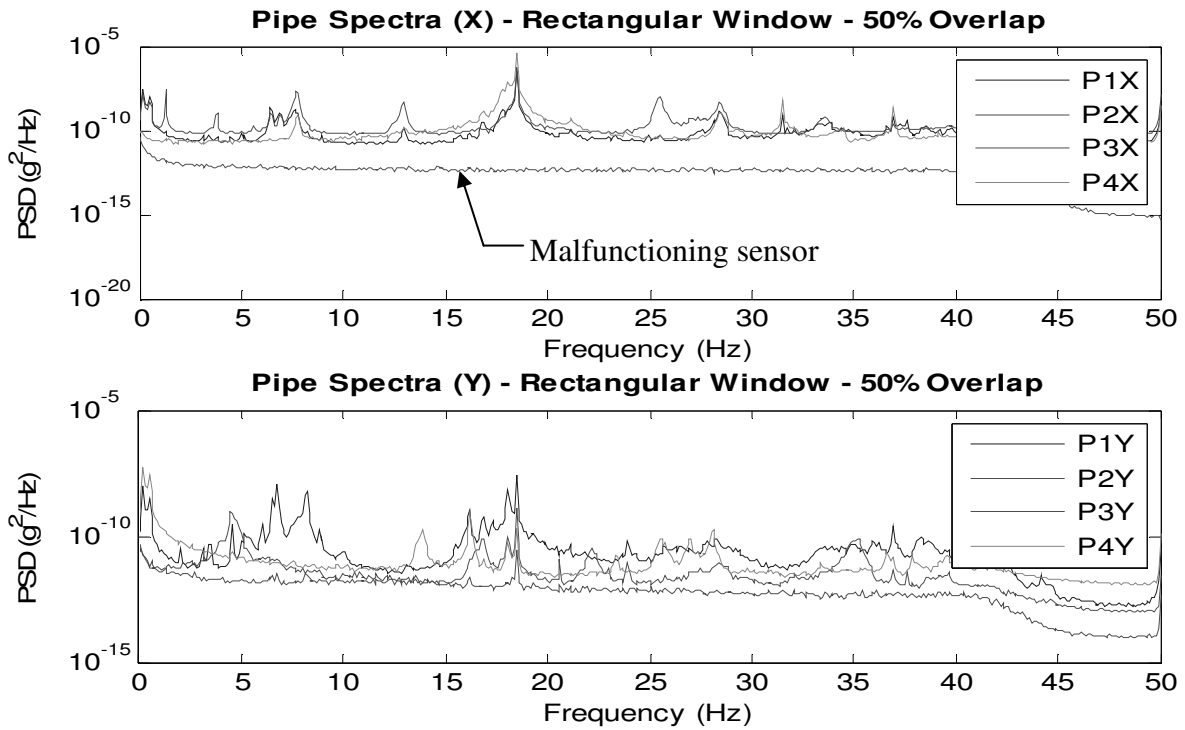


Figure 4.12. Initial PSD plot of the sensor channels located on the pipe

4.4.2 Modal Parameter Identification by Peak-Picking Method

The frequencies and mode shapes were identified from the measured responses using the peak-picking (PP) method. For the PP method, the modal properties of the elevated water tank can be identified using the power spectral density (PSD) functions and cross spectral density (CSD) functions as described by Bendat and Piersol (1980). The natural frequencies of the structure were identified from the peaks in the PSD functions calculated for each measured acceleration response. The mode shapes at each of the identified spectral peaks were estimated from the magnitude and phase information from the estimated CSD functions. The CSD functions were estimated for each signal response by comparing the response in a particular direction at each measurement location with the measured response in the same direction at a selected reference measurement location. A CSD function was estimated for each measurement location in reference to each of the other possible reference measurement locations. For this study, the mode shapes were determined by actually estimating the operating deflection shapes. Different from an actual mode shape, an operating deflection shape at any particular spectral peak may include the contributions of more than one mode in the vicinity of the spectral peak. These operating deflection shapes may be considered reasonable approximations of the undamped mode shapes if the modes are not too closely spaced and the damping ratios are small.

4.4.2.1 Estimation of Power Spectral Density and Cross Spectral Density Functions

The PSD and CSD functions were estimated using the average modified periodogram method as originally outlined by Welch (1967), which is also referred to as the direct FFT method. The PSD and CSD estimates were computed from the pre-processed data using 50% overlap processing and a Rectangular Window to minimize spectral leakage effects. The blocksize used in computing these estimates was 4096. Since the measured data was sampled at

1000 Hz and later decimated by a factor of 10 to 100 Hz, the frequency resolution came out to be 0.024 Hz, which was assumed to be adequate for accurately identifying the natural modes. The following equations (4.1) and (4.2) were used to estimate the PSD functions equations (4.3) and (4.4) were used to estimate the CSD functions.

$$GXX_{pp} = \sum_1^{N_{avg}} X_p X_p^* \quad (4.1)$$

$$GYY_{qq} = \sum_1^{N_{avg}} Y_q Y_q^* \quad (4.2)$$

$$GXY_{pq} = \sum_1^{N_{avg}} X_p Y_q^* \quad (4.3)$$

$$GYX_{qp} = \sum_1^{N_{avg}} Y_q X_p^* \quad (4.4)$$

4.4.2.2 *Identification of Natural Frequencies and Mode Shapes*

The natural frequencies of the elevated water tank were determined by first identifying the spectral peaks from the PSD functions. The identified peaks from the PSD functions were compiled together along with a modal indicator function (MIF). The MIF used for this study was the Average Normalized Power Spectral Density (ANPSD) function that was developed by Felber (1993). The ANPSD function is determined by first estimating the power spectral densities for each output channel. The resulting PSD functions are then normalized for each

output channel (i) by dividing the PSD value for each frequency by the sum of the PSD values at all frequencies, according to the following expression:

$$NPSD_i(f_k) = \frac{PSD_i(f_k)}{\sum_{k=0}^{N/2} PSD_i(f_k)} \quad k = 0, 1, 2 \dots (N/2) \quad (4.5)$$

The normalized PSD values are finally summed at each frequency for all of the output channels (l) and divided by the total number of output channels.

$$ANPSD(f_k) = \frac{1}{l} \sum_{i=1}^l NPSD_i(f_k) \quad k = 0, 1, 2 \dots (N/2) \quad (4.6)$$

An ANPSD function was calculated for the structural response in each recorded direction (x and y). The peaks were automatically identified utilizing the Matlab software by defining a peak as any ANPSD value at a frequency line in which the values at each of the adjacent frequency lines, immediately preceding and following, are less than the value in question. Due to the normal fluctuating variance in the spectral response of the structure across the several frequency lines, it was more than likely that this peak selection algorithm would identify multiple peaks in the ANPSD function that were not relevant to the modes at the natural frequencies of the structure. However, the spectral peaks at the natural frequencies of the structure would, by definition, have a larger amplitude than those peaks which were identified by the selection algorithm but irrelevant to the main modal response of the structure. A minimum ANPSD amplitude level was selected and applied to the identified spectral peaks. This minimum amplitude criterion was determined through an iterative process that consisted of first

choosing a possible minimum amplitude level and then examining how many spectral peaks applied to the criterion. An adequate minimum amplitude criterion resulted in a number of spectral peaks somewhere between 30 and 100.

The mode shapes were calculated for each measured direction at every identified spectral peak from the magnitude and phase information contained in the CSD functions. A CSD function was computed between each output channel in a given direction and a selected reference output channel in that same direction. For example, six channels on the elevated water tank recorded the vibration response in the x-direction. A CSD function was calculated between each of these six channels with one of the other five channels as a selected reference output channel, making for 30 CSD functions in the x-direction.

The calculated phase values were utilized to categorize the measured sensors into one of two absolute categories; in-phase or out-of-phase with the reference sensor. Calculated phase factors between zero and ± 90 degrees were forced to a value of zero degrees, meaning in-phase with the reference sensor. Calculated phase factors between 90 and 180 (or -90 and -180 degrees) were forced to a value of 180 degrees, meaning out-of-phase with the reference sensor.

4.5 RESULTS

The following results were computed from multiple data sets recorded during the dynamic testing of the elevated water tank that took place on September 9, October 8, and October 15, 2009. These measurements were collected at a 1000 Hz sampling rate. The average duration of the analyzed data sets was approximately ten minutes. The different data sets included testing that took place during the three different operating conditions of the elevated water tank: empty, half-full, and full.

4.5.1 Amplitudes of Acceleration Signals

The acceleration amplitudes were fairly consistent as recorded by the different accelerometers throughout the elevated water tank. The sensors located at the top of the elevated water tank generally recorded higher acceleration amplitudes. This is to be expected for this cantilever-type of structure, with a fixed base and free end at the top of the water tank. Figure 4.13 shows some of the recorded acceleration response from the sensors located at the top of the elevated water tank.

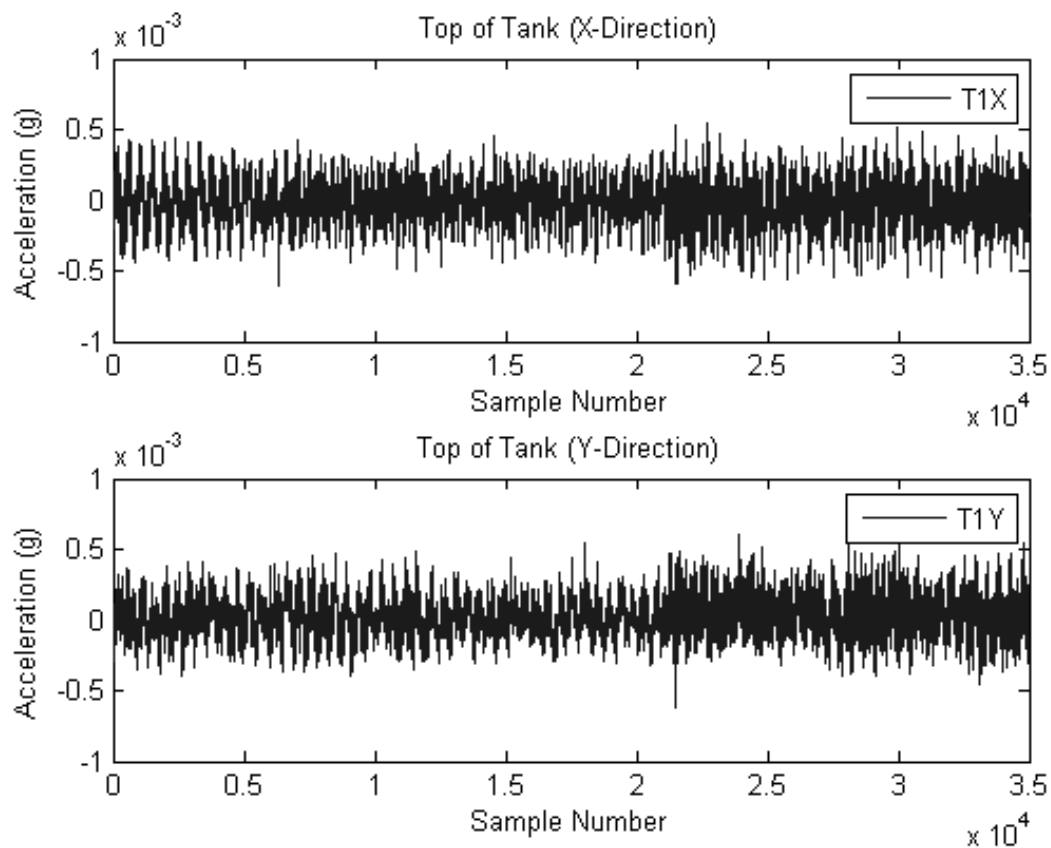


Figure 4.13. Acceleration amplitudes for x- and y-direction tank vibrations

4.5.2 Spectral Content of Acceleration Signals

The power spectral densities computed for the different sensor channels for each tank operating condition in both the x- and y-direction are shown in the following figures. The power spectral densities computed for the vibrations of the full tank condition are shown in Figure 4.14, Figure 4.15, Figure 4.16, and Figure 4.17. Figure 4.15 shows some significant peaks between 0 Hz and 0.5 Hz. The preliminary analytical models of the elevated water tank for the full-tank condition indicated the fundamental mode would be in this range. Figure 4.16 shows several spectral peaks in the frequency range of 6 Hz to 9 Hz. The 3D numerical model indicated that the second bending mode would be located somewhere in this range. Figure 4.17 shows one significant spectral peak between 18 Hz and 19 Hz. The 3D analytical model indicated that the third bending mode would be located somewhere in the range of 20 Hz to 25 Hz. It may be possible that this spectral peak identified in the PSD plot from the experimental investigation could be the third bending mode.

The power spectral densities computed for the vibrations of the half-full tank condition are shown in Figure 4.18, Figure 4.19, Figure 4.20, and Figure 4.21. Figure 4.19 shows several spectral peaks in the frequency range of 0 Hz to 1 Hz. The numerical models indicated that the first bending mode would be located somewhere in this range; however this range should not contain several modes related to the structure. Some of these peaks likely correspond to something other than the natural frequencies of the elevated water tank. Figure 4.20 shows several spectral peaks for the vibrations in the x-direction in the frequency range of 6 Hz to 9 Hz. However, the PSD plot of the vibrations measured in the y-direction indicate one significant spectral peak near 8 Hz. The analytical models indicated that the natural frequencies of the structure should be consistent in both the x- and y-direction. Figure 4.21 shows a significant

spectral peak between 18 Hz and 19 Hz. This spectral peak likely corresponded to the third bending mode.

The power spectral densities computed for the vibrations of the empty tank condition are shown in Figure 4.22, Figure 4.23, Figure 4.24, and Figure 4.25. Figure 4.23 shows one very significant spectral peak located near the frequency of 1 Hz. The analytical models all indicated that the first bending mode for the empty tank condition would be located near 1 Hz. Figure 4.24 shows several spectral peaks in the frequency range of 6 Hz to 9 Hz. Figure 4.25 shows one significant peak between 18 Hz and 19 Hz, as well as several peaks in the frequency range of 17 Hz to 25 Hz. The analytical models indicated that the third bending mode would likely be located in this range.

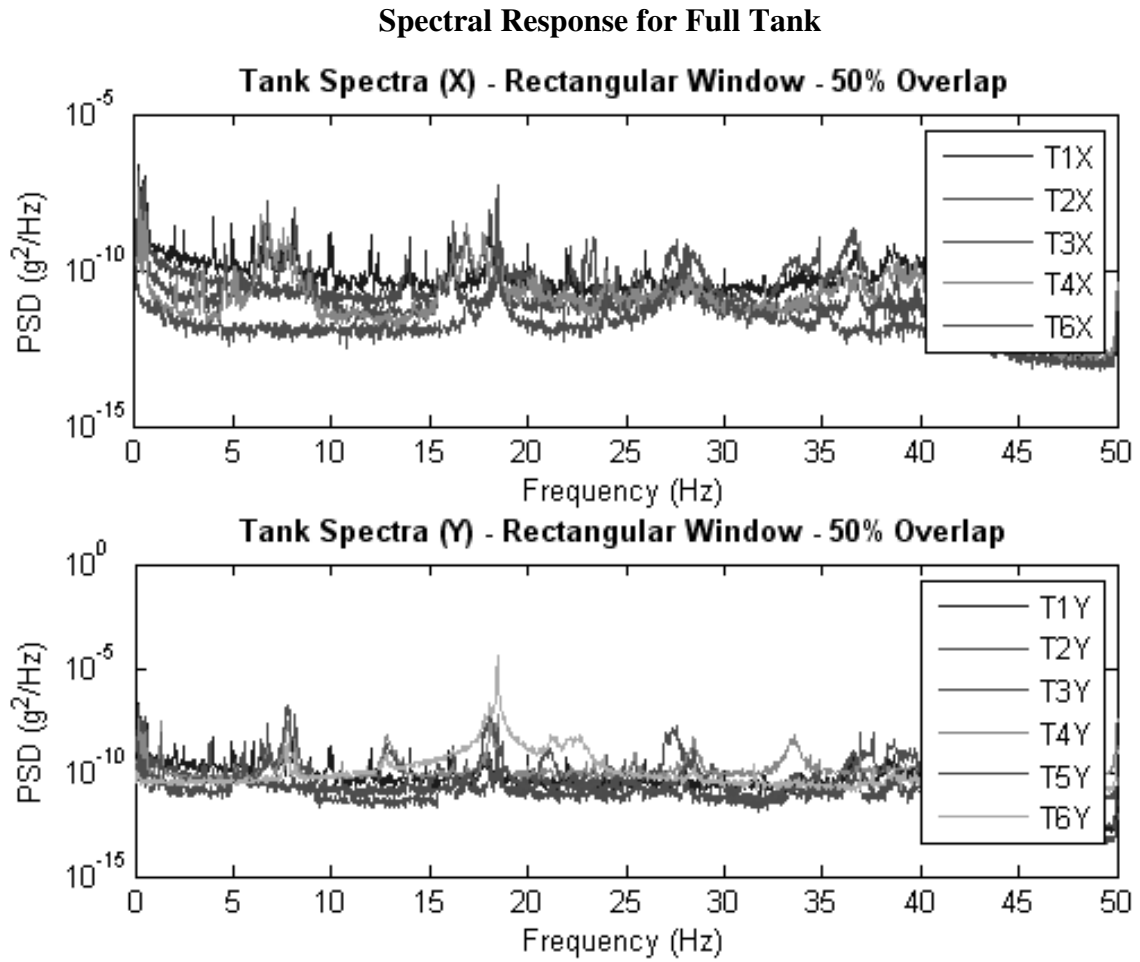


Figure 4.14. Power spectral densities for the vibration response of the elevated water tank

Spectral Response for Full Tank

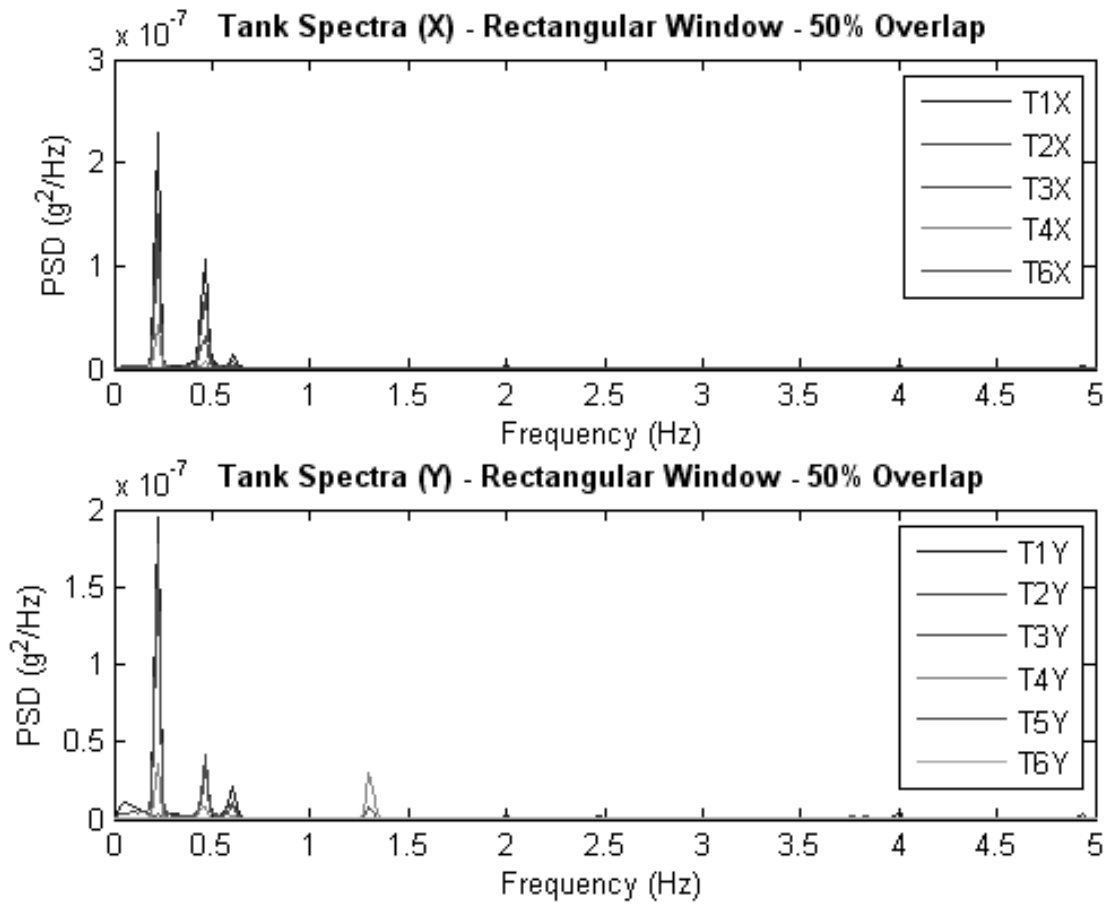


Figure 4.15. Power spectral densities for the vibration responses (0 to 5 Hz)

Spectral Response for Full Tank

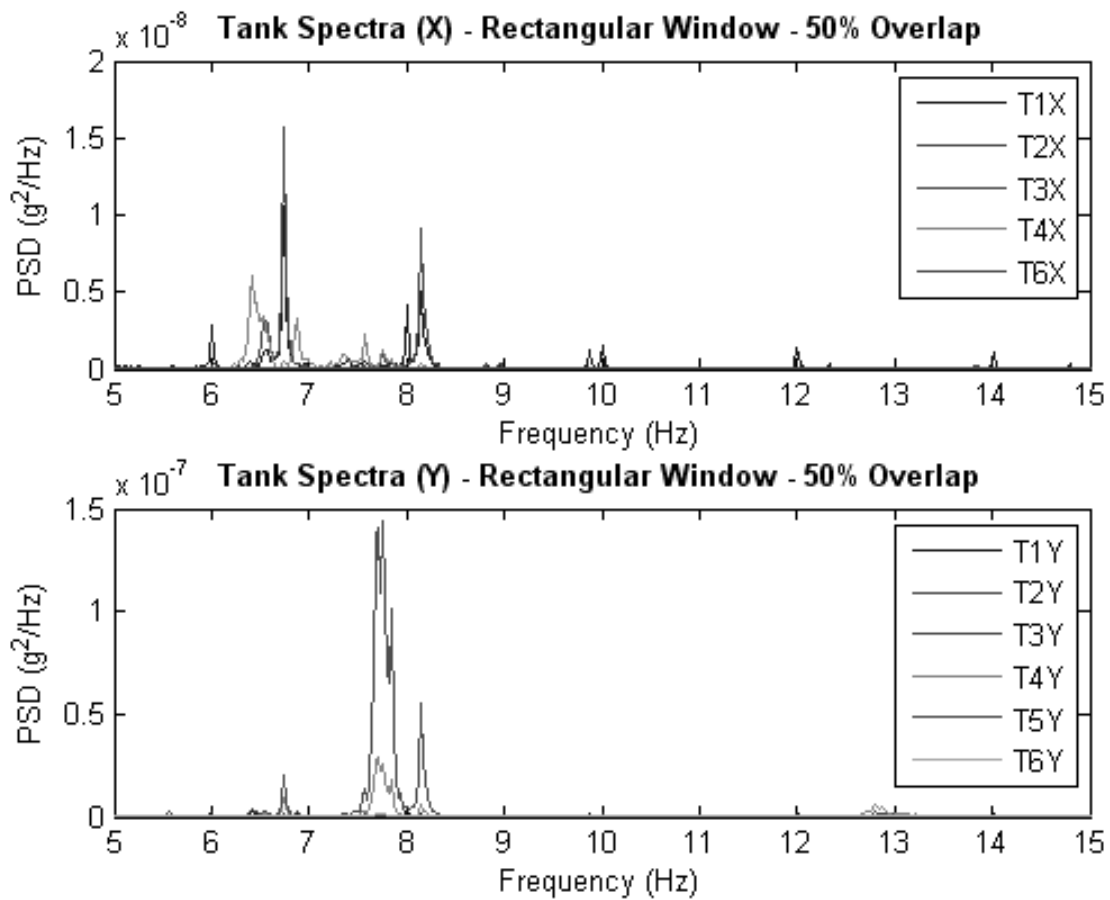


Figure 4.16. Power spectral densities for the vibration response (5 to 15 Hz)

Spectral Response for Full Tank

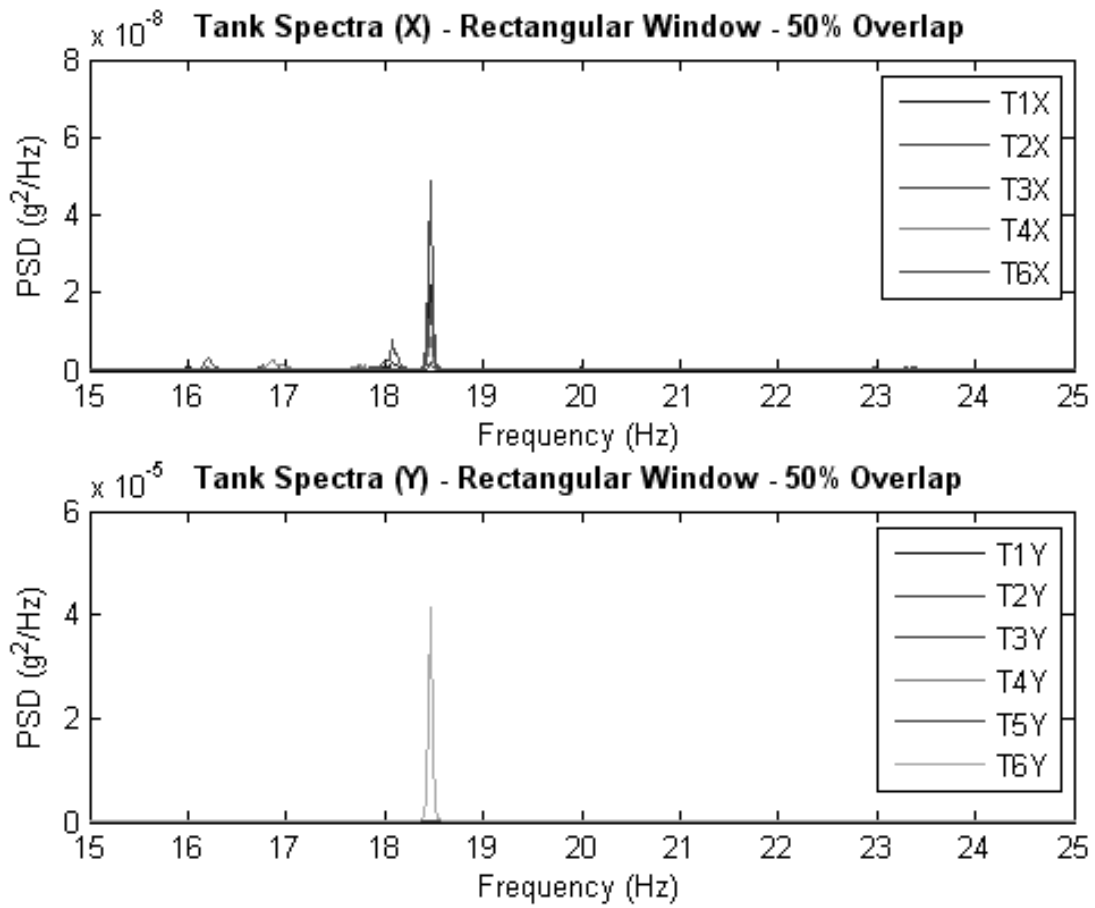


Figure 4.17. Power spectral densities for the vibration response (15 to 25 Hz)

Spectral Response for Half-Full Tank

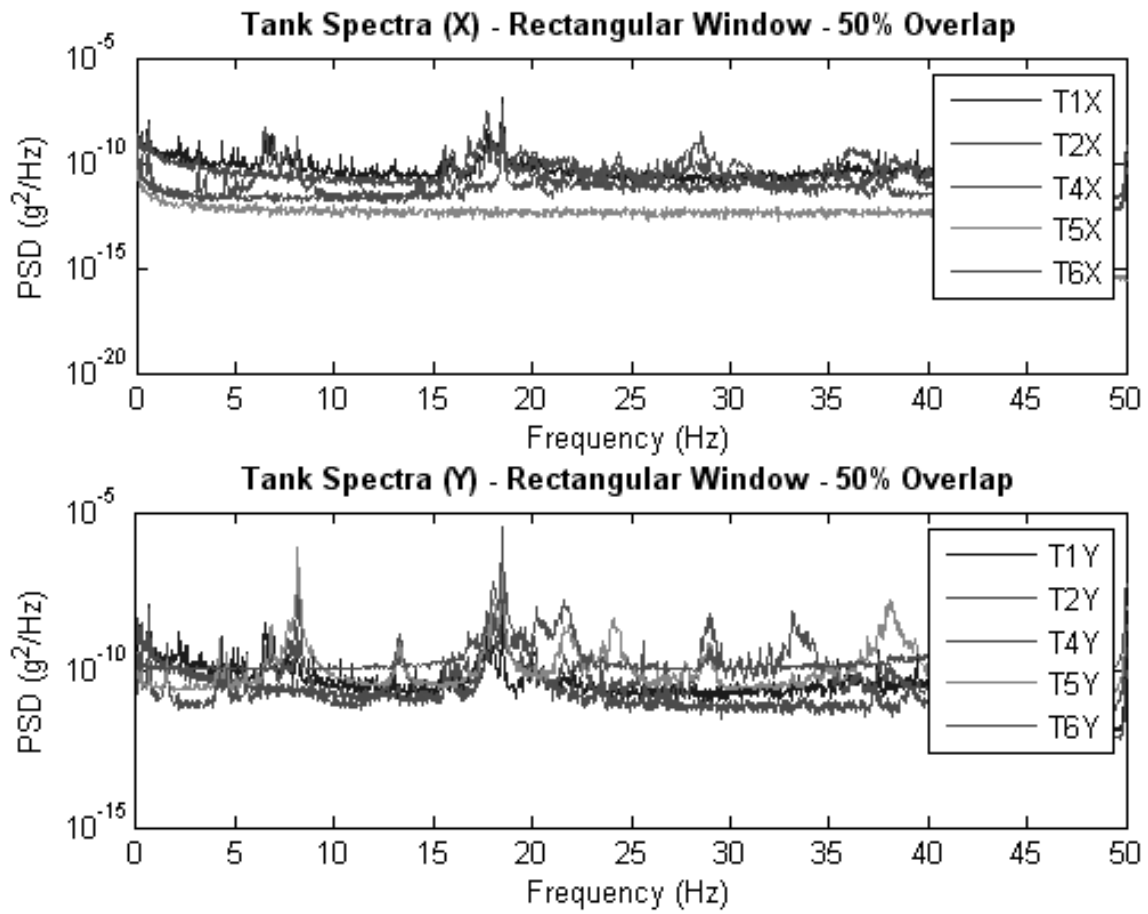


Figure 4.18. Power spectral densities for the vibration response of the elevated water tank

Spectral Response for Half-Full Tank

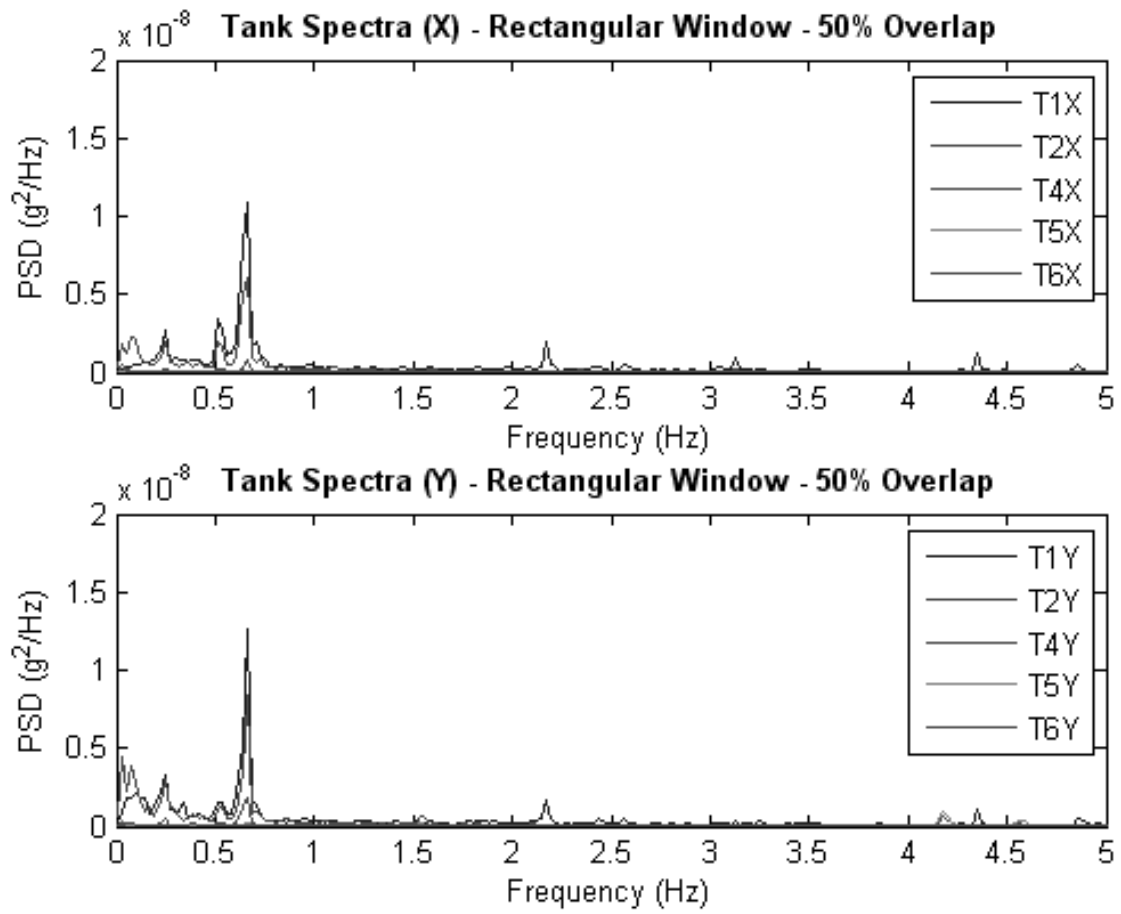


Figure 4.19. Power spectral densities for the vibration response (0 to 5 Hz)

Spectral Response for Half-Full Tank

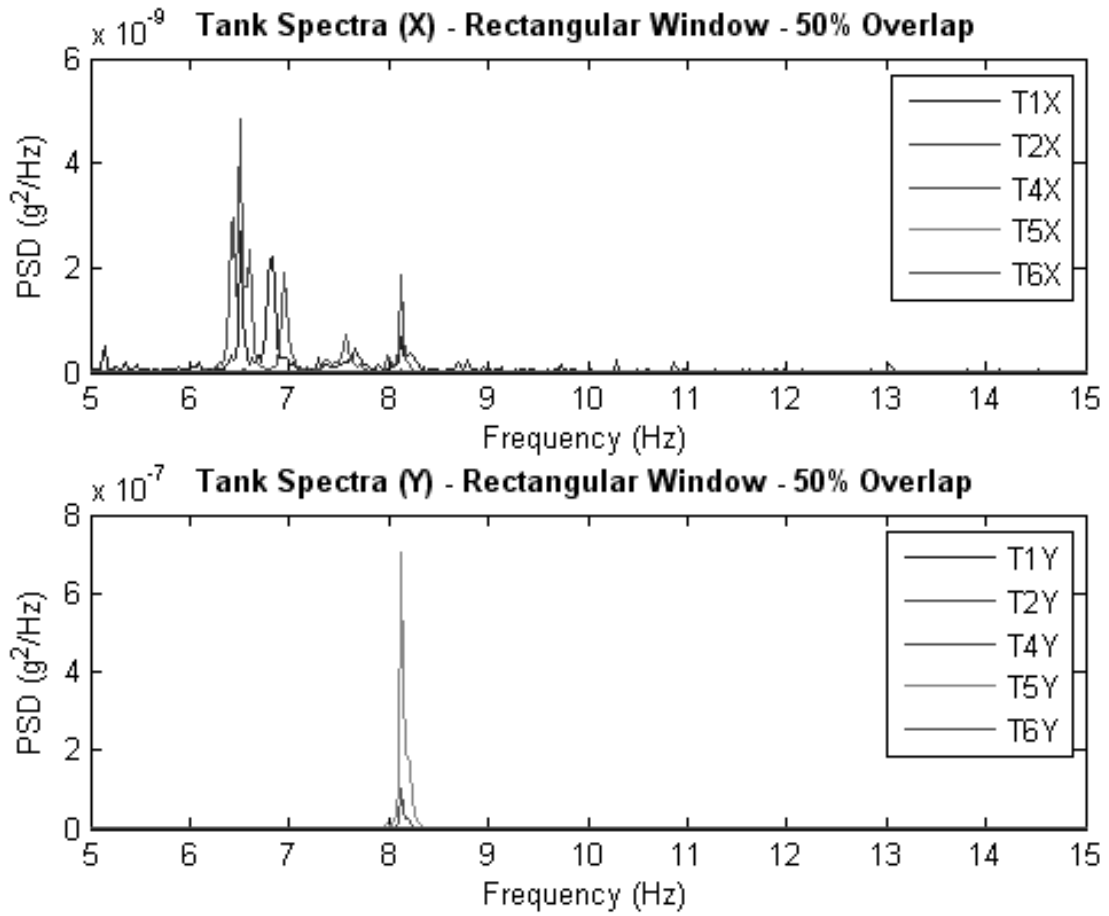


Figure 4.20. Power spectral densities for the vibration response (5 to 15 Hz)

Spectral Response for Half-Full Tank

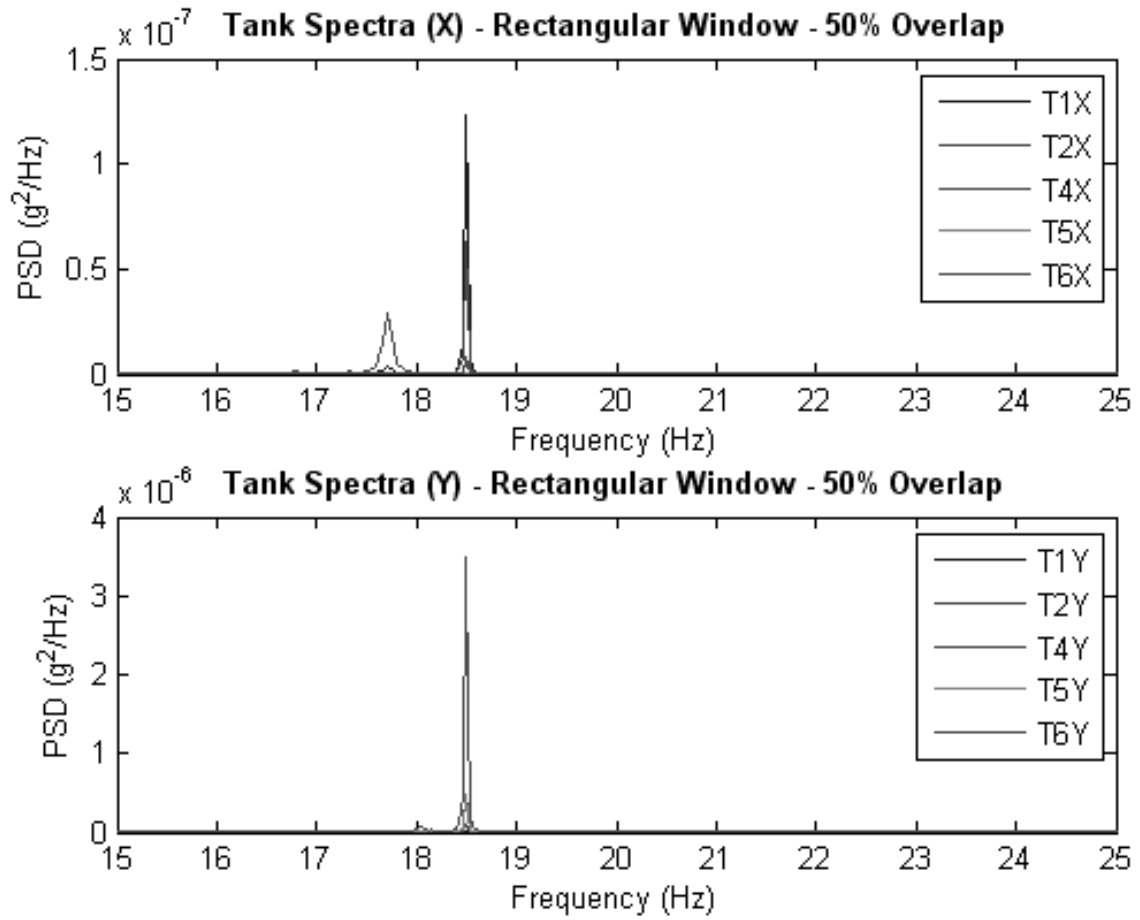


Figure 4.21. Power spectral densities for the vibration response (15 to 25 Hz)

Spectral Response for Empty Tank

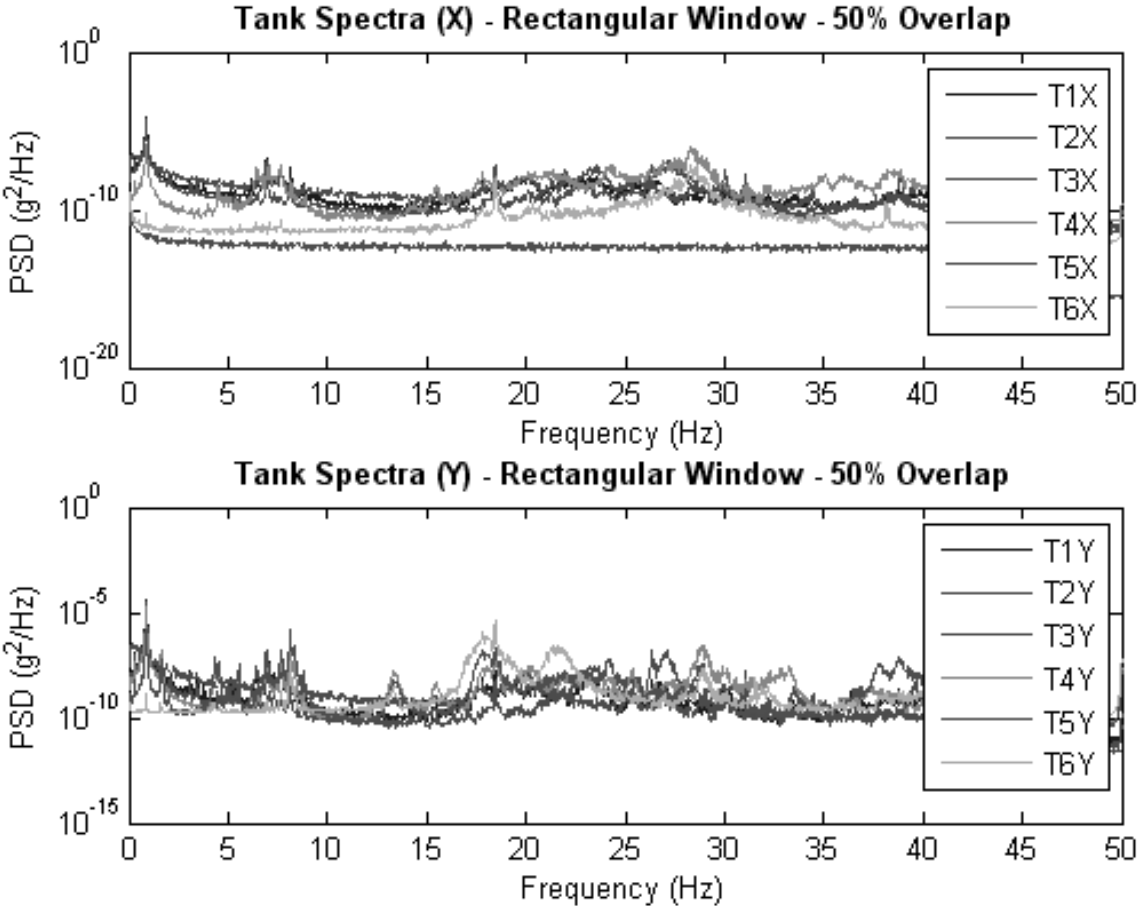


Figure 4.22. Power spectral densities for the vibration response of the elevated water tank

Spectral Response for Empty Tank

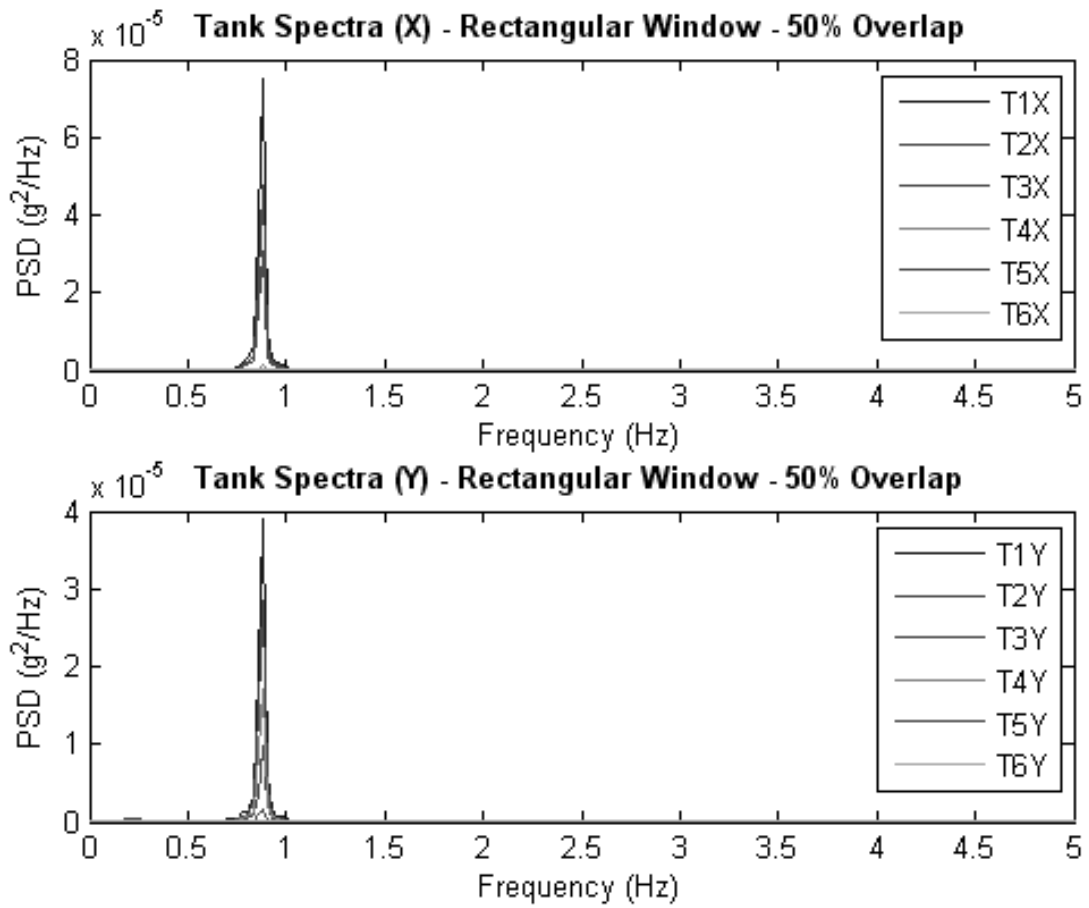


Figure 4.23. Power spectral densities for the vibration response (0 to 5 Hz)

Spectral Response for Empty Tank

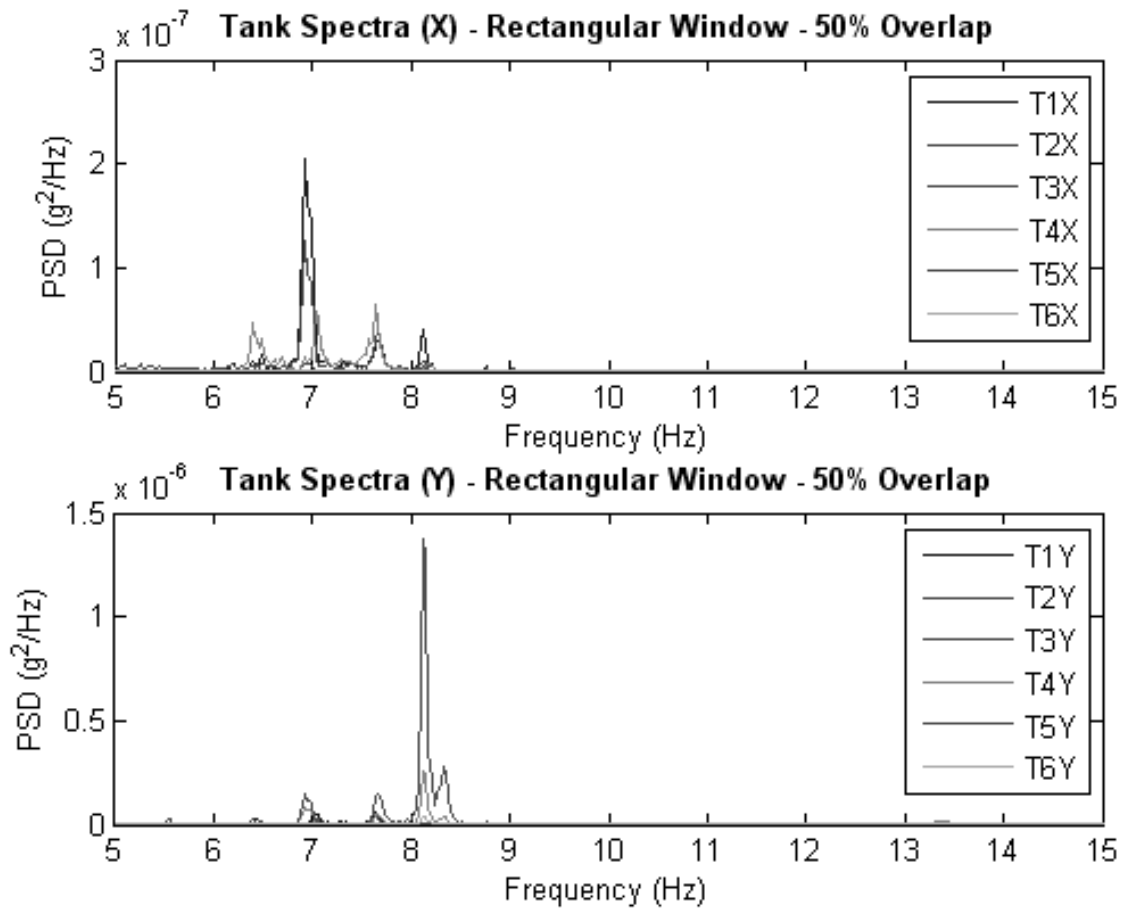


Figure 4.24. Power spectral densities for the vibration response (5 to 15 Hz)

Spectral Response for Empty Tank

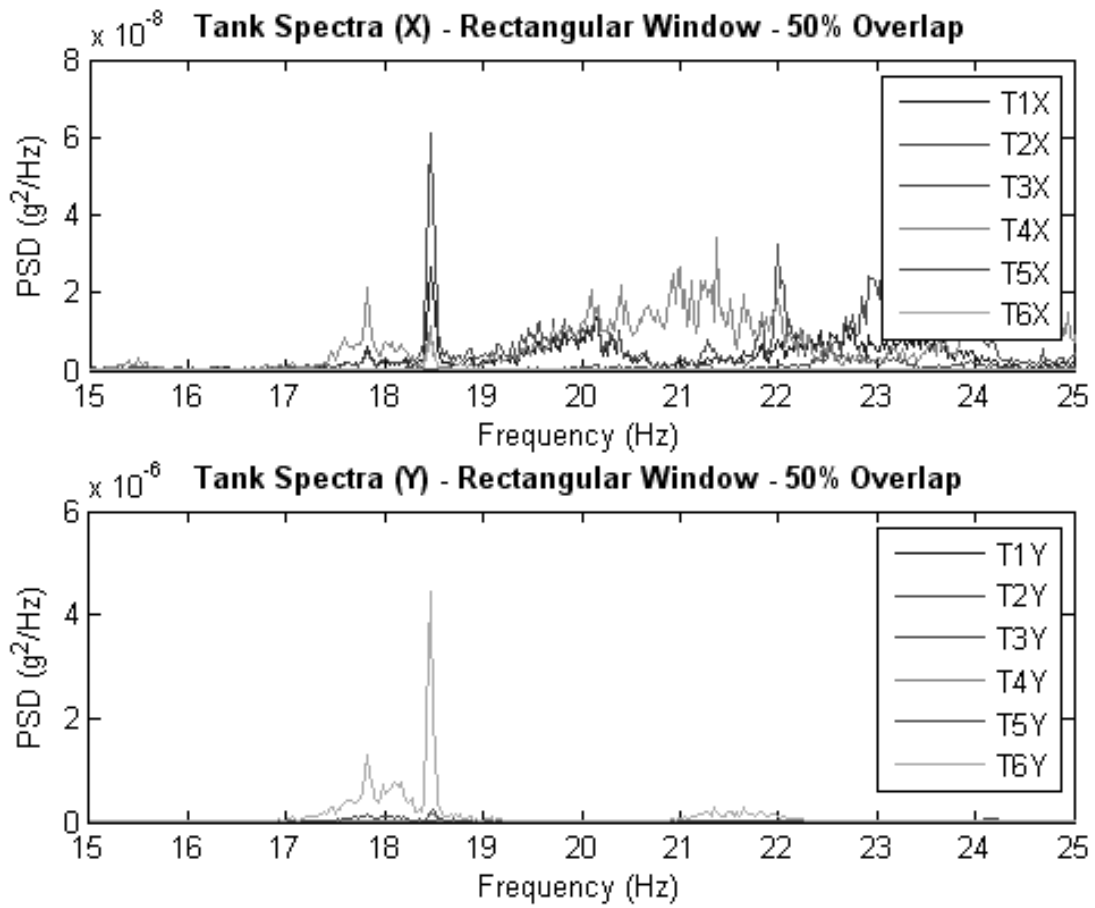


Figure 4.25. Power spectral densities for the vibration response (15 to 25 Hz)

4.5.3 Natural Frequencies and Mode Shapes

The frequencies of the spectral peaks were identified by utilizing an automated peak picking procedure which identified peaks in the ANPSD functions that corresponded to the estimated PSD and CSD functions. The peaks were identified by having a higher magnitude than the values immediately preceding and succeeding itself. The amplitudes in the ANPSD functions of the identified peaks were inspected along with the plotted mode shapes to determine the likely natural frequencies of the elevated water tank. The results of determining the natural frequencies for the first three bending modes for the different operating conditions are shown in Table 4.2. The plotted mode shapes for the three bending modes are shown in Figure 4.26, Figure 4.27, and Figure 4.28.

Table 4.2. Elevated water tank modes identified by ambient vibration testing

	Description	Natural Frequency (Hz)		
		Empty Tank	Half-Full Tank	Full Tank
Mode 1	1 st Bending	0.8789	0.6592	0.2197
Mode 2	2 nd Bending	8.130	8.130	7.764
Mode 3	3 rd Bending	18.457	18.481	18.457

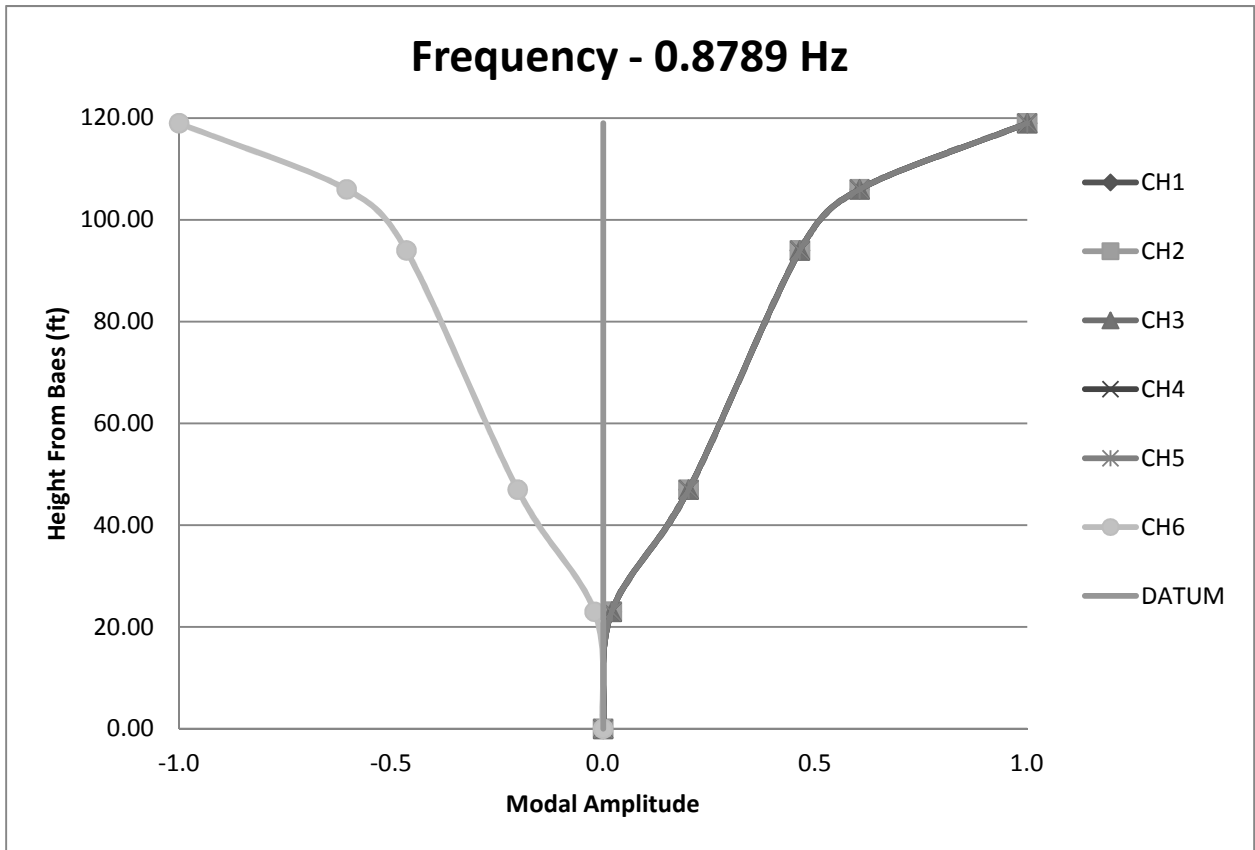


Figure 4.26. Operating deflection shape at 1st bending mode for empty tank condition

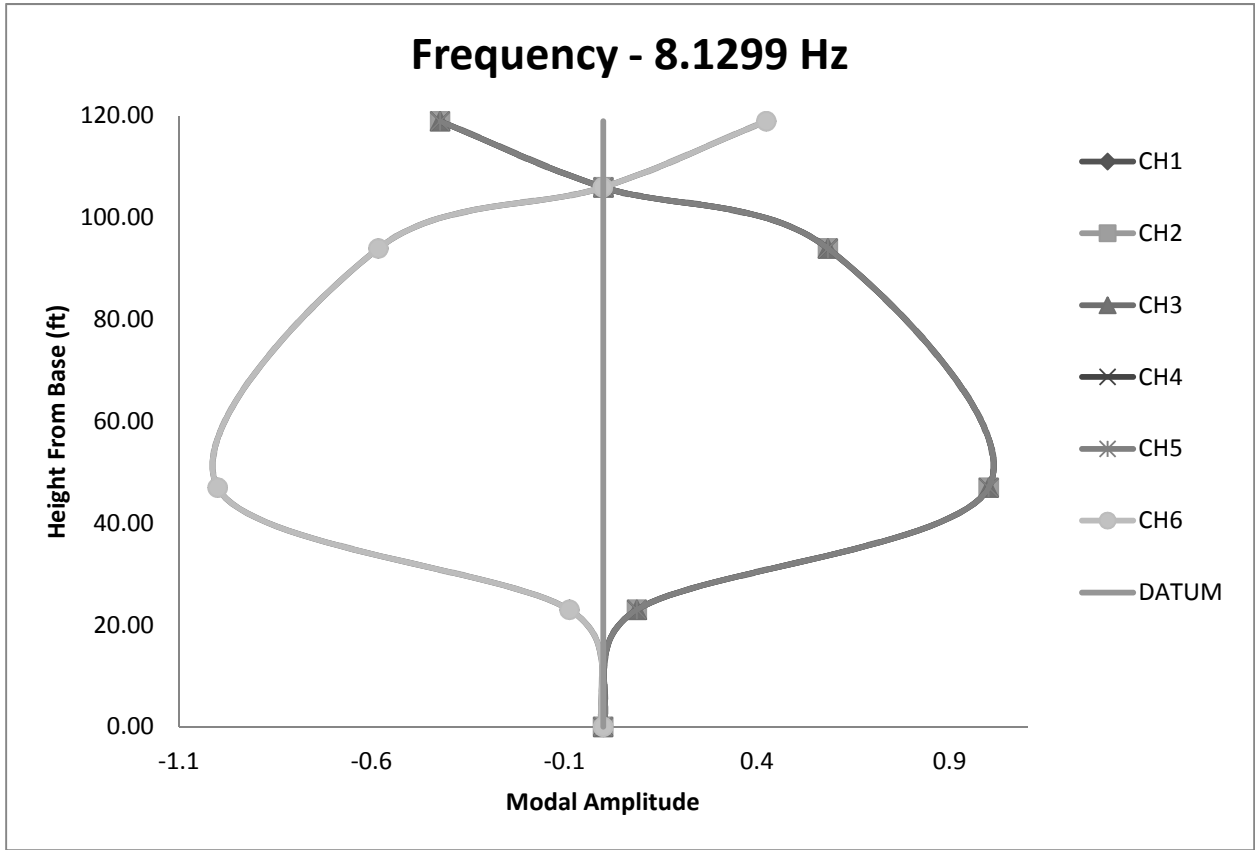


Figure 4.27. Operating deflection shape at 2nd bending mode for empty tank condition

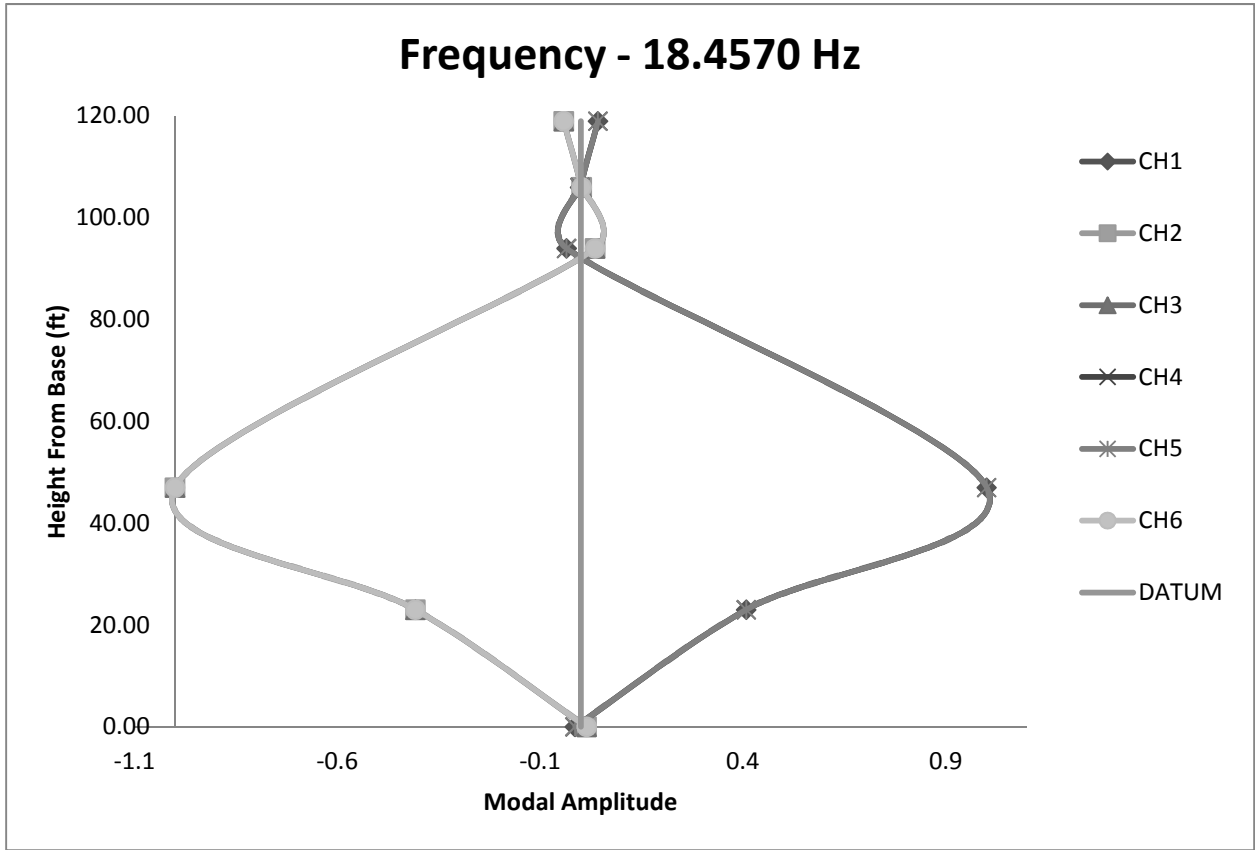


Figure 4.28. Operating deflection shape at 3rd bending mode for empty tank condition

4.6 DISCUSSION

The natural frequencies and mode shapes of the elevated water tank identified by the ambient vibration testing correlated well with the results from the numerical models. The natural frequency of vibration as identified by the numerical models for each of the different operating conditions correlated well with the results obtained from the ambient vibration testing. This may be due in part to the simple approach required to determine the natural frequency of vibration, or the first mode. The main components of mass and stiffness, along with the length of the structure, define the frequency of the first mode. These components are easily determined. A more in-depth description of the elevated water tank, in terms of the distributed mass and stiffness over the full height of the structure, is more difficult to estimate. The complexity of the higher modes may require the analytical models to be more finely calibrated to accurately estimate the natural frequencies of the higher modes of the structure.

The geometry of the tank is symmetrical about 360 degrees around the center of the tank. As such, the vibration response of the structure should be similar in all possible measurement directions. There are some access ladders and other small components contained with the structure of the elevated water tank that could throw off the symmetry, but the mass of these components and their effect on the overall characteristics of the structure is very small in relation to the mass and stiffness of the water tank and water operation. With that being said, the location of the largest spectral peaks near the frequencies identified for the 2nd bending mode were consistently different between the measured vibrations in the x- and y-direction. This relationship was not repeated for the first and third bending modes, in which the identified natural frequencies were identical in both directions. For many of the ambient vibration tests, it was discovered that the sensor located at the mid-height of the tank that was measuring

vibrations in the x-direction was malfunctioning or recording very noisy measurements. As a result, it is possible that the resonant frequencies of the 2nd bending mode in the x-direction for the different operating conditions were not accurately identified. The reported results for the resonant frequencies at the 2nd bending modes were taken from the measurements recorded in the y-direction.

As shown in Table 4.2, the normal trend for the natural frequencies of each operating condition is for the natural frequencies to decrease as mass is added to the water tank. This relationship between the natural frequencies identified for the third bending mode at each operating condition appears to not follow this trend. In reality, these identified values are essentially the same. The frequency resolution from the data analysis process was 0.024 Hz. During the data processing, each computed data point for the response of the structure in the frequency domain is rounded to the nearest frequency line, with each frequency line occurring at 0.024 Hz intervals. The frequency of 18.481 Hz identified for the half-full tank operating case is 0.024 Hz greater than the 18.457 Hz identified for the empty and full tank operating conditions. The difference could be attributed to a simple rounding error at some point during the data processing. Thus, it is acceptable to assume that the peak representing the third bending mode occurs at the same frequency for each operating condition.

CHAPTER 5. ANALYTICAL STUDY OF THE ELEVATED WATER TANK

5.1 INTRODUCTION

The experimental investigation of large scale in-service structures is typically accompanied by an analytical study of the structure. Oftentimes, the analytical study is completed beforehand to help guide the analysis and interpretation of the results from the ambient vibration testing of the structure. The analytical study of the elevated water tank was particularly helpful in identifying the quantity and order of the natural modes of the structure that should exist within the frequency range of interest. A large number of spectral peaks were identified from the experimental investigation of the elevated water tank, and the dynamic behavior information gathered from the analytical study assisted in determination of which of these identified spectral peaks actually represented the natural frequencies of the structure.

Identifying the initial frequencies of interest from the analytical model also served to define the frequency band of interest for the experimental investigation. The minimum sampling frequency could also be determined from defining the frequency band of interest. The minimum sampling rate is usually at least twice the highest frequency of interest, according to the Nyquist criterion.

The instrumentation scheme and sensor layout were designed by examining the estimated mode shapes determined from the analytical study. By inspecting the estimated mode shapes, the inflection points of the mode shapes (where the shape crosses through the vertical datum) could be determined, and the sensor locations could be chosen so as to limit their occurrence at these inflection points. While it is sometimes unavoidable, it is generally advisable to avoid locating sensors at the modal inflection points, because the vibration response at the modal

inflection points will be extremely limited for those corresponding modes. The sensors, more or less, cannot see a particular mode of the structure if they are located at the mode's shape inflection point.

For this project, two forms of idealized analytical models were created and a simple numerical analysis was also conducted. The two forms of models consisted of a simple stick model idealization of the structure in addition to a more detailed three-dimensional (3D) finite element model. The numerical analysis was conducted by utilizing well known equations developed for characterizing idealized single degree-of-freedom systems. The modal behavior of the fill pipe was not examined as part of the analytical study.

5.2 DESCRIPTION OF ANALYTICAL STUDIES

5.2.1 Numerical Analysis

For the numerical analysis, the elevated water tank was considered as an undamped single degree-of-freedom system (SDF) subjected to free vibration. As discussed by Chopra (2007), the analytical results describing the free vibration of a structure provide a basis to determine the natural frequency of vibration. The motion of linear SDF systems, visualized as a mass-spring-damper system, subjected to external force $p(t)$ is written as follows:

$$m\ddot{u} + c\dot{u} + ku = p(t) \quad (5.1)$$

This same equation of motion subjected to no external force ($p(t) = 0$) and without damping ($c = 0$) becomes:

$$m\ddot{u} + ku = 0 \quad (5.2)$$

Free vibration is initiated by disturbing the system from its static equilibrium position by introducing a displacement $u(0)$ and velocity $\dot{u}(0)$ to the mass. This is defined at the moment the motion is initiated by the following:

$$u = u(0) \qquad \dot{u} = \dot{u}(0) \tag{5.3}$$

The solution to the homogenous differential equation of free vibration motion is:

$$u(t) = u(0)\cos\omega_n t + \frac{\dot{u}(0)}{\omega_n}\sin\omega_n t \tag{5.4}$$

where,

$$\omega_n = \sqrt{\frac{k}{m}} \tag{5.5}$$

Equation (5.4) is shown plotted below in Figure 5.2. When plotted, the homogenous differential equation shows that the system undergoes oscillatory motion about its static equilibrium position. This motion repeats itself every $2\pi/\omega_n$ seconds. Furthermore, the state of the mass is identical at two time instants, t_1 and $t_1 + T_n$, where $T_n = 2\pi/\omega_n$. This motion is known as simple harmonic motion.

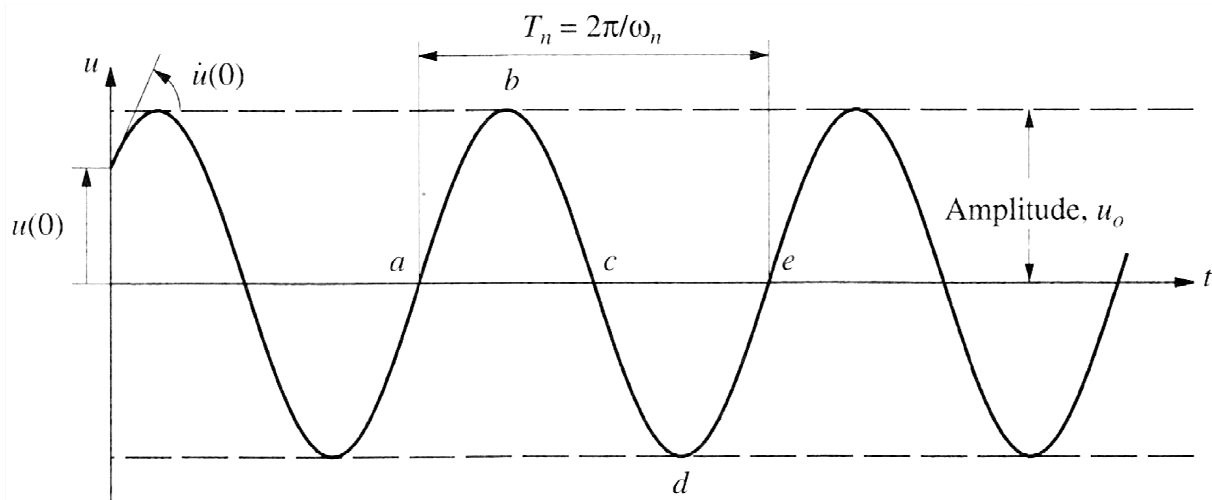


Figure 5.2. Free vibration of a system without damping (Chopra 2007)

The time required for the undamped system to complete one cycle of free vibration is known as the *natural period of vibration* of the system, denoted as T_n . A system executes $1/T_n$ cycles for each second. This is known as the *natural cyclic frequency of vibration* and is denoted by:

$$f_n = \frac{1}{T_n} \quad (5.6)$$

The units of f_n are in Hertz (Hz), and f_n is related to ω_n , the *natural circular frequency of vibration*, through the following relationship:

$$f_n = \frac{\omega_n}{2\pi} \quad (5.7)$$

The term *natural frequency of vibration* applies to both ω_n and f_n . The natural vibration properties ω_n , T_n , and f_n depend only on the mass and stiffness of the structure, as shown in Equation (5.5). Considering two SDF systems with equal mass, the SDF system with higher stiffness will have the higher natural frequency and the shorter natural period. Similarly, considering two SDF systems with equal stiffness, the SDF system with more mass will have the lower natural frequency and the longer natural period.

The natural frequency of vibration of a system is identified as the first occurring natural mode of the system, or fundamental mode. The total modal response of a structure is characterized by the cumulative responses of the structure at all of the naturally occurring modes. The fundamental mode contains a significant portion of this total modal response. As such, identifying the fundamental mode, and the response of the system at this identified mode, can provide a valuable portion of the information pertaining to the modal response of the structure. Thus, while the proposed numerical analysis of the elevated water tank will identify just the first

mode, the fundamental mode is of great significance in characterizing the vibration response of the structure.

The stiffness of the elevated water tank was estimated using the information provided by the construction drawings. If the elevated water tank is considered as a cantilevered element, the stiffness can be determined by the following equation:

$$k = \frac{3EI}{L^3} \quad (5.8)$$

where E is Young's modulus (29,000 ksi for steel), I , is the moment of inertia, and L is the total height of the structure. The shaft of the structure was identified as ½" steel with an inside diameter of 6.5 feet. Using the equation for the moment of inertia of a circle, $I = (\pi/4)r^4$, the I was calculated as 94985 in⁴ for the shaft of the elevated water tank. The height of the elevated water tank was considered as the height from the base of the structure to the center of the mass of the tank. This height was estimated as 104'-5". This height was used for both the empty and full tank operating conditions. However, the height was adjusted to approximately the elevation of the center of mass for the half-full tank condition. The mass of the water makes up a significant portion of the combined masses of the water and the steel tank, so the center of the combined masses would likely be much closer to the center of mass of just the water component. To simplify this calculation, the mid-height between the base of the tank and the height of the center of the tank was used. The height used for the half-full tank operating condition was 98'-2".

The shaft of the elevated water tank was considered as a rigid massless element. The lumped mass located at the top of the shaft element was idealized to represent the mass of the steel tank in addition to the mass of the water for the different operating conditions; empty, half-full, and full. The weight of the actual steel portion of the tank was estimated to be

approximately 35,000 lb. The 75,000 gallon capacity tank was estimated to hold 625,500 lb of water at full capacity. The weight of the water at half-full capacity was estimated to be 312,750 lb. The corresponding masses were calculated by dividing the aforementioned weights by the gravitational acceleration constant, g , which is equal to 32.2 ft/s^2 (386.2 in/s^2). The natural frequency of vibration, ω_n , was determined for the different operating conditions of the elevated water tank by using equation (5.5).

5.2.2 Stick Model

An idealized numerical model constructed as a cantilever-like system that consisted of a single lumped mass at the top of a stick element with specified stiffness. The lumped mass located at the top of the shaft element was idealized to represent the mass of the steel tank in addition to the mass of the water for the different operating conditions. This lumped mass was located 104'-5" above the base of the elevated water tank, as determined from the available structural drawings. The shaft element was considered as a rigid, massless element. The stiffness of the shaft element was defined as being consistent from the base to the mass element located on top. The stiffness variation due to the bell element located below the straight shaft near the base of the structure was not accounted for within this model. An increasing number of nodes were used to define the model and refine the identification of the modes. The initial stick model was created with one node to represent the lumped mass with defined stiffness in the shaft element. Utilizing the capabilities of the analyzing software, the nodes were increased exponentially up to from one node up to 64 nodes while still maintaining the same mass and stiffness characteristics. The creation and analysis of this model was completed with the SAP2000 software. A representation of the stick model is shown in Figure 5.1.

5.2.3 3D Finite Element Model

A multi degree-of-freedom finite element model was created from construction drawings and on-site observations, consisting of approximately 800 defined nodes. The objective of creating this analytical model was to construct the closest approximation of the actual geometry of the elevated water tank. As with the stick model, the mass representing the tank and its different operating conditions was located at approximately the center of the tank, at 104'-5" above the base. The stiffness was determined by the analyzing software from the geometry of the constructed model.

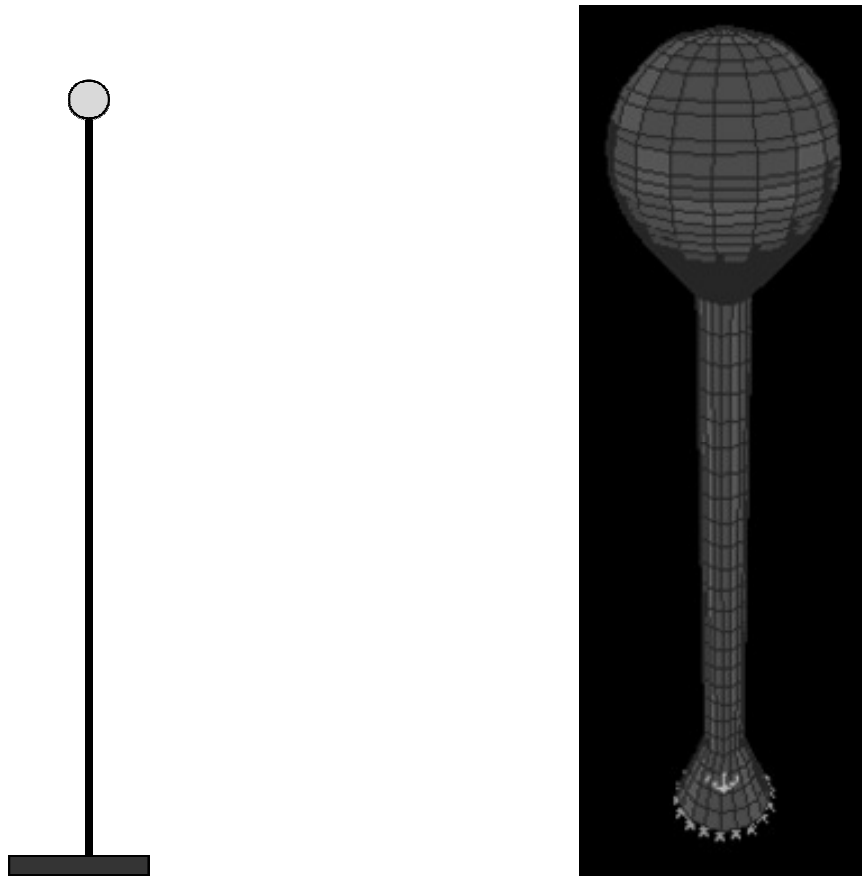


Figure 5.1. Stick model (left) and 3D model (right) of elevated water tank

5.3 RESULTS

The results of the numerical analysis are given in Table 5.1. The numerical analysis was used to identify the first mode, or the fundamental mode, of the elevated water tank. Mode shapes were not determined from the numerical analysis. The results from the most defined stick model are given in Table 5.2. The stick model was analyzed with an increasing number of nodes. The results for the fundamental mode from each of the corresponding stick models, with increasing node count, for each operating condition are given in Table 5.3. The relationship between these determined values for the fundamental mode are shown to converge as the number of nodes is increased. This convergence can be seen in Figure 5.2. The effect of the increasing number of nodes used is more noticeable with the empty tank condition.

Table 5.1. Elevated water tank fundamental mode for different operating conditions (Numerical)

	Description	Natural Frequency (Hz)		
		Empty Tank	Half-Full Tank	Full Tank
Mode 1	1 st Bending	1.082	0.3771	0.2491

Table 5.2. Elevated water tank modes for different operating conditions (Stick Model)

	Description	Natural Frequency (Hz)		
		Empty Tank	Half-Full Tank	Full Tank
Mode 1	1 st Bending	1.083	0.3708	0.2702
Mode 2	2 nd Bending	11.87	11.44	11.41
Mode 3	3 rd Bending	35.06	34.60	34.57

Table 5.3. Fundamental mode for increasing number of nodes (Stick Model)

Description	Natural Frequency (Hz)		
	Empty Tank	Half-Full Tank	Full Tank
1 Node	0.9995	0.3670	0.2688
2 Node	1.061	0.3699	0.2699
4 Node	1.077	0.3705	0.2701
16 Node	1.083	0.3708	0.2702
64 Node	1.083	0.3708	0.2702

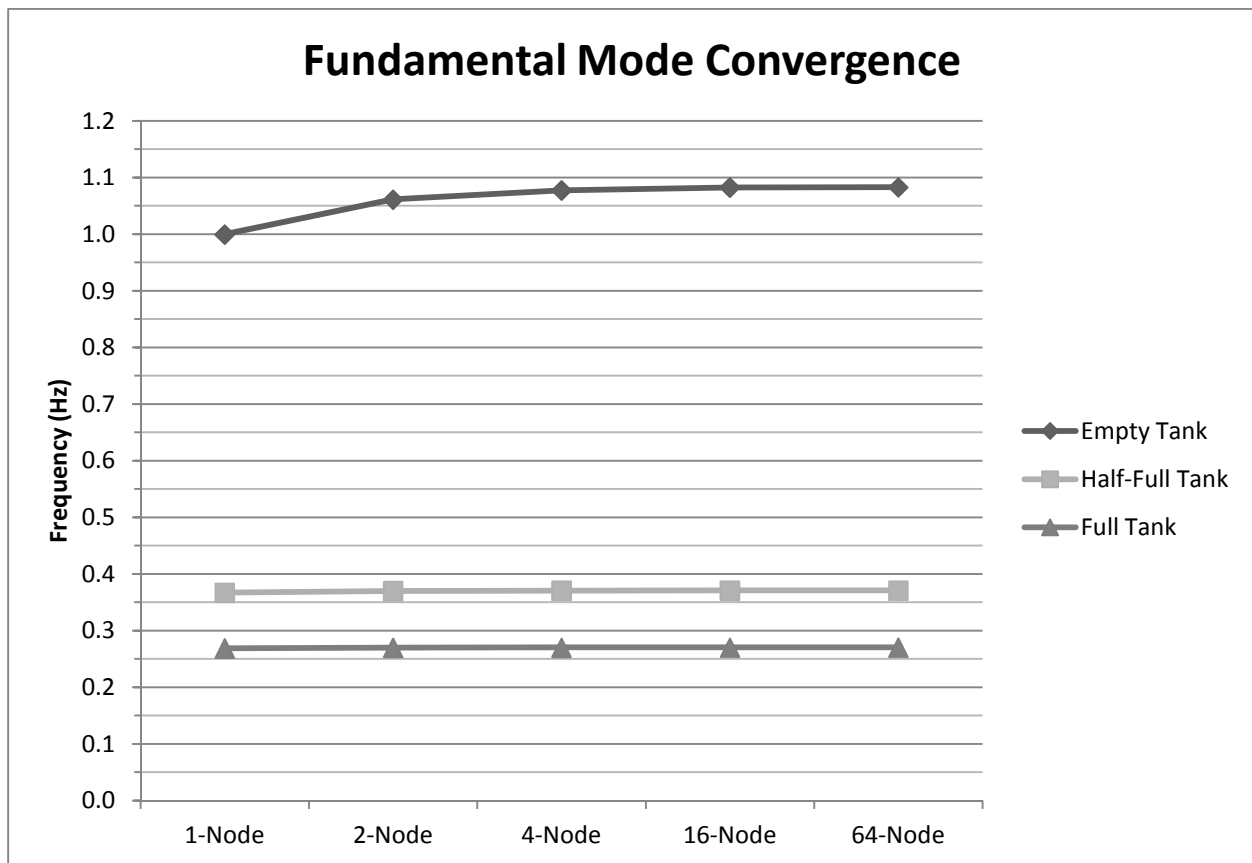


Figure 5.2. Convergence of stick model results

The results from the 3D finite element model are given in Table 5.4. The mode shapes for the first three bending modes, as identified by the 3D model, are shown in Figure 5.3. The mode shapes were very similar for all of the different operating conditions. The mode shapes determined from the 3D model that indicated axial bending (bending in one direction) matched those determined from the stick model. Due to the simplicity of the stick model, additional complex modes, such as torsional or rotational modes, were not able to be identified as with the 3D model.

Table 5.4. Elevated water tank modes for different operating conditions (3D Model)

	Description	Natural Frequency (Hz)		
		Empty Tank	Half-Full Tank	Full Tank
Mode 1	1 st Bending	1.007	--	0.2626
Mode 2	2 nd Bending	8.020	--	7.904
Mode 3	3 rd Bending	21.72	--	21.62

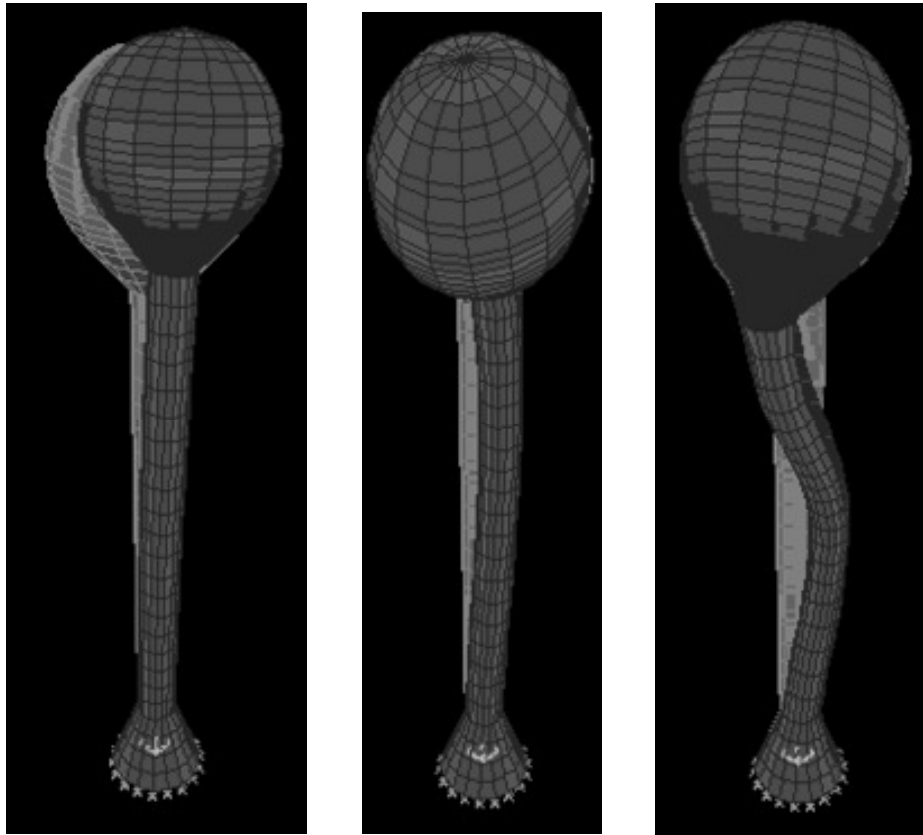


Figure 5.3. 1st bending (left), 2nd bending (middle), 3rd bending (right) from 3D model

5.4 DISCUSSION

Overall, the results from the different analytical studies were relatively consistent in determining the fundamental natural frequency of the elevated water tank. The numerical analysis and the stick model study both idealized the elevated water tank as a single degree-of-freedom system. The correlation between the results from the numerical analysis and those of

the stick model should be expected, since the analysis for both was conducted with the same assumptions and approach.

The results from the analytical models both indicated that the first three bending modes should occur within the 0 to 50 Hz frequency range that was chosen as the frequency range of interest for the experimental investigation. The fundamental mode for the different operating conditions as determined by the 3D model also correlated very well with the results from the other analytical studies. It appears that the assumptions made about the elevated water tank for using the more simplistic approach conducted for the numerical analysis were validated by the close correlation of the fundamental mode identified by all three studies.

The second and third bending modes did not correlate as well between the stick model and the 3D model. There could be a number of reasons for this disparity. The stick model was created as a simply idealized system with a lumped mass at the top of a rigid massless stick element. While this is a reasonable idealized model to estimate the behavior of the elevated water tank, there are some characteristics of the structure that may become too generalized and limit the accuracy of the results. For the stick model, the location of the lumped mass was set at the center of the tank. While this is likely a good approximation for the location of the center of mass, the actual location of the interface between the water tank the shaft is lower; at 91'-10" above the base. The 3D model more accurately represented the relationship between the mass element of the water tank and the resisting stiffness element of the shaft which could have had an effect on the overall stiffness of the structure. The conical base of the shaft, as can be seen in Figure 5.3 was included in the creation of the 3D model. The lack of the stiffness characteristics of this element in the stick model may have also had an effect on the overall stiffness of the

structure as represented in the stick model. As mentioned previously, the difference in stiffness would have a direct effect on the calculated modal frequency values.

Overall, the analytical studies served to provide a base understanding of the modal behavior of the elevated water tank. The design and implementation of the experimental investigation were directly impacted by the findings from the analytical studies. The correlation of the simple hand calculation and stick model with the more defined 3D model indicate that the elevated water tank can accurately be approximated to behave as a single degree-of-freedom system. For future work with elevated water tanks of similar geometry and operation, the SDF approximation can serve as a simple, yet accurate, initial investigation of the modal characteristics of the structure.

CHAPTER 6. CONCLUSIONS

6.1 INTRODUCTION

The objective of the research detailed in this thesis was the dynamic characterization of an elevated water tank under different operation conditions, as determined through a series of vibration tests. The vibration testing regimen and instrumentation scheme were designed to be able to identify the higher level bending modes, according to the results from a series of numerical models. The numerical models were created from available construction drawings for the structure. The modal parameters determined from the numerical models were compared to those determined from the experimental investigation of the elevated water tank.

6.2 NUMERICAL STUDY

The numerical study consisted of a hand calculation solution and analysis of two idealized numerical models that were used as a basis to estimate the modal parameters of the elevated water tank. The results from the numerical study helped to initially define the dynamic behavior of the elevated water tank as well as to design the experimental investigation.

The hand calculation solution was effective in determining the fundamental mode of vibration for the elevated water tank. The structural parameters (mass and stiffness) used with the numerical solution were easily determined from the available construction drawings. The results were determined for the three different examined operating conditions (empty, half-full, and full) by simply adjusting the mass parameter. The natural frequencies determined for the different operating conditions were found to be quite accurate when compared to the analysis of the numerical models and the results from the experimental investigation.

The two numerical models constructed, the stick model and the 3D finite element model, were both very effective in determining the fundamental natural frequency. The natural frequencies determined for the 2nd and 3rd bending modes varied considerably between the results from the stick model and those from the 3D finite element model. After reviewing the results from the ambient vibration testing, it was determined that the natural frequencies estimated by the 3D finite element model were closer to those estimated from the experimental investigation of the actual elevated water tank. This discrepancy in the results from the stick model and the 3D finite element model is likely due to the difference in the number of degrees-of-freedom that the 3D finite element model employs versus those that makes up the stick model. However, the stick model does provide an effective means to determine the fundamental modal information for the structure with a simple modeling approach.

The 3D finite element model was found to provide a good estimate of the modal parameters for the elevated water tank. The natural frequencies determined from the experimental investigation helped to validate those estimated by the 3D finite element model. The 3D finite element model could be further calibrated to align better with the dynamic behavior of the actual in-service elevated water tank by adjusting the mass and stiffness characteristics of the model. A concerted effort at calibrating the 3D finite element model from the results of the experimental investigation is beyond the scope of this thesis.

6.3 EXPERIMENTAL INVESTIGATION

The experimental investigation of the elevated water tank consisted of a series of ambient vibration and output-only vibration testing. The elevated water tank was evaluated at different common operating conditions, including an empty tank, half-full, and full tank. For the most

part, the measurements recorded by the accelerometers were found to be of good quality.

Unfortunately, a few sensors were found to be malfunctioning or of poor quality during the data pre-processing phase. The measurements from the malfunctioning sensors were removed prior to processing the data.

The natural frequencies and mode shapes determined from the experimental investigation appeared to provide a good estimate for the dynamic behavior of the elevated water tank. The determined natural frequencies from the recorded measurements of the ambient vibration testing correlate well with the estimated natural frequencies from the 3D finite element model, particularly the frequencies for the 1st and 2nd bending modes. The first three bending modes were the primary modes identified by the experimental investigation, although additional modes make up the complete modal behavior of the elevated water tank. The 3D finite element model alluded to the existence of these additional modes, but the scope of the experimental investigation was limited to identifying the first three bending modes.

6.4 CONCLUSION

The frequencies and mode shapes for the first three bending modes of elevated water tank under three different operating conditions were successfully identified through dynamic vibrations testing. The results from the experimental investigation were validated by the numerical models created from the construction drawings, and vice versa. The hand calculation solution provided a good estimate for the fundamental mode of vibration of the elevated water tank.

The natural mode of vibration, as determined by the vibration testing, was estimated to be 0.2197 Hz with a full tank and 0.8789 Hz with an empty tank. A worthwhile investigation would

be to estimate the displacements at the top of the structure under the different operating conditions. If the assumption is made that higher displacements induce greater stress on the structure, and more specifically the interface between the fill pipe and the water tank, estimating the displacement of the water tank under the different operating conditions might be able to lead to recommendations for future operations of the elevated water tank. Unfortunately, a recommended operating program was beyond the scope of this thesis.

Additional efforts could be made to further calibrate the 3D finite element model so that it better estimates the dynamic behavior of the actual in-service elevated water tank. A calibrated finite element model could be beneficial in the continued progress monitoring of the structural integrity of the elevated water tank.

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