VAGUENESS AND ITS BOUNDARIES: A PEIRCEAN THEORY OF VAGUENESS

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ABSTRACT

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Many theories of vagueness employ question-begging assumptions about the semantic boundaries between truth and falsity. This thesis defends a theory of vagueness put forward by Charles S. Peirce and argues for a novel solution to the sorites paradox based upon his work. Contrary to widespread opinion, I argue that Peirce distinguished borderline vagueness from other related forms of indeterminacy, e.g. indefiniteness, generality, unspecificity, uninformativity, etc. By clarifying Peirce's conception of borderline vagueness, I argue for a solution to the sorites paradox based upon his logical semantics. In addition, I argue for this theory against the epistemic theory of vagueness, which makes controversial claims concerning the sharp semantic boundary between truth and falsity, and against the supervaluationist theory of vagueness, which is committed to the in principle impossibility of sharp semantics boundaries for propositions with vague terms.

Cornelis de Waal, Ph.D., Chair

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Chapter 1

An Introduction to Vagueness and the Sorites

> Tell me, do you think that a single grain of wheat is a heap? Thereupon you say: No.
> Then I say: What do you say about two grains?
> —Galen, *On Medical Experience*

0. Introduction

The goals of this thesis are multiple. First and foremost, my aim is to propose a solution to the sorites paradox. In order to do this, I argue that Charles S. Peirce did understand vagueness as it is articulated in contemporary literature (chapter 5) and argue that a novel solution can be formulated per his suggestion of existentially quantifying over legitimate senses of a vague predicate (chapter 6). In route to this solution, I offer a historical sketch of the sorites paradox (chapter 2) and evaluate two major contemporary theories of vagueness (epistemicism in chapter 3 and supervaluationism in chapter 4). The aim of this chapter is to introduce the topic of vagueness by distinguishing it from other types of indeterminacy and to summarize a number of the major responses to the sorites paradox.

1. Depicting Vagueness

Vagueness is pervasive. This thesis considers vagueness in one specific sense, namely vagueness as it is expressed about the semantic indeterminacy, or supposed indeterminacy, of *borderline conditions* specified by natural language predicates. The focus is on the effect of vagueness upon the semantics of language rather than upon ontology.[1] The classic case concerns predicating the property 'tall' of certain members of a group of human beings. Humans are monotonically arranged according to height in a row, and a reputable language-user is asked to determine which humans are tall and which are not tall. The two extremes of the row of humans afford easy classification. The shortest individuals are clearly not tall while the tallest individuals are clearly tall. Vagueness concerns the cases between the clearly tall and clearly not tall. These are known as 'borderline cases'. Determining how to classify these borderline cases is one of the principal concerns of a logic and semantics of vagueness. For borderline cases, the predicate 'tall' seems to both apply and not apply, yet it also could be said that 'tall' fails to apply with truth or falsity, or still yet it might be thought that it only applies to a

certain degree, and yet it may be the case that it applies (or fails to apply) but we humans will never know which. The semantic classification of borderline cases is thus a puzzle.

Vagueness is ubiquitous. A few monadic predicates infected by vagueness include: height and size predicates (e.g. 'tall', 'short', 'big', 'small', 'large'), color predicates (e.g. 'red', 'green', 'greenish', 'blue'), predicates for age, maturity, and development (e.g. 'young', 'middle-aged', 'old', 'boy', 'lady', 'girl', 'woman', 'elderly'), hair-covering predicates (e.g. 'bald', 'balding'), and predicates relating to wealth and generosity ('rich', 'poor', 'wealthy', 'miserly', 'stingy', 'generous'). The list goes on. Vagueness also infects nearly all the main terms and concepts of ethics, metaphysics, aesthetics, law, and social-science. While a monotonically increasing arrangement of beautiful paintings and people may not be possible, a painting or a person may be a borderline case of 'beautiful', 'pretty, 'handsome', 'ugly', etc. A deformed or depraved human may be a borderline case of a 'person'. Further, ethics and religion are troubled not merely with devils and angels but with humans whose actions are borderline cases of 'good', 'evil', 'supererogatory', and so forth. Vagueness is a problem not only for logic and language; it is a philosophical problem for anyone employing a terminology that admits borderline cases.

Although vagueness is widespread, it is not the only form of indeterminacy to infect language. It is a distinct type of indeterminacy. Consider the ambiguous utterance 'John went to the bank'. The ambiguity of the word 'bank' does not admit borderline cases in the same way that vagueness does. 'Bank' is ambiguous with respect to which *one* of two distinct senses is intended, which can be clarified by a disambiguation. However, after this is answered, 'bank' is still vague for there are borderline cases between financial institutions such that it is uncertain whether or not they are banks. A second form of indeterminacy distinct from vagueness are cases when language-users give a semantically precise but simply uninformative description. For example, say a language-user points to an isolated tree and says 'that tree is greater than 3 and less than 100 feet', the proposition is without a shred of vagueness (even though the exact height of the tree is not specified). There is no vagueness because 'greater than 3 and less than 100 feet' admits of no boundary cases and specifies its truth-conditions with precision. If

the tree is between 3 and 100 feet, the statement is true. If the tree falls outside of that specified range, the statement is false.

While having borderline cases is necessary, it is not a sufficient condition for vagueness. A semantically incomplete proposition can have borderline cases but these cases can be determined sharply. The classic example is from Sainsbury (1991) who argues that a peculiar usage of the word 'child*' can admit borderline cases yet be a sharply-defined predicate. Suppose that 'child*' truly applies to humans under 16 years old and fails to apply to humans over 18. A 17 year-old John is a borderline case of a child* yet 'child*' is a precise, yet semantically incomplete, term. Thus, vagueness fosters meta-level indeterminacy since what counts as a borderline case of a vague predicate can also be vague. For example, when a language-user evaluates the heights of human beings put into a monotonically increasing row, not only will there be uncertainty about whether a selected individual is tall but there will also be uncertainty about whether some humans are 'borderline cases of tall'. Further, when trying to distinguish *borderline cases of tall humans* from those that are *not borderline cases of tall humans*, there will be borderline cases between those that are borderline cases and those that are not. These are the borderline-borderline cases, or vagueness at the metalevel. Vagueness is thus a powerful beast, capable of infecting not only terms used to predicate things in the world (object-language) but also the language employed to predicate properties about our object-language. In other words, vagueness is also a higher-order phenomenon.

Finally, many wish to make vagueness more general than it is by associating it with other forms of indeterminacy, e.g. obtuse or hermetic utterances, meaninglessness, context-indeterminacy, broad epistemic uncertainty, generality, highly precise yet abstract discussion, etc. The general connection of vagueness to other forms of indeterminacy is drawn from everyday discourse where 'vagueness' and 'vague' are used in competing ways and in an inconsistent manner. What is focused upon in subsequent chapters is uncertainty about the application of certain predicates to certain objects insofar as the application of the predicates (1) admit borderline cases, (2) admit higher-order vagueness, and (3) are sorites-susceptible (see below).

Before addressing the sorites-susceptibility of vague terms, two relevant distinctions that apply to vague terms are *intensional and extensional vagueness (and*

precision) and the distinction between *linear and multi-dimensional vagueness.* A predicate *P* is *extensionally vague*—in a possible world *w* at a time *t*—if and only if the function that assigns the property semantically expressed by *P* does not completely determine the assignment of that predicate. For example, consider a possible world w_1 where w_1 consists of three objects *a*, *b*, *c*, and the precise heights of each object are: *a* = 50 feet, *b* = 25 feet, and *c* = 2 feet. Consider a function called 'monster-sized' that assigns the *P* in the following manner: an object is monster-sized if it is over 30 feet, and an object is not monster-sized if it is under 20 feet. In this world, *b* is extensionally vague since it is semantically undetermined whether *b* is monster-sized or not. A predicate is extensionally precise if all objects in a possible world are determined. In contrast, a predicate *P* is *intensionally vague* if and only if the function that assigns the property semantically expressed by *P* does not completely determine the assignment of that predicate in *some* possible world *w* at some time *t*. A predicate is *intensionally precise* if semantic extensions are determined in all possible worlds. From this it follows that a set of objects may be extensionally precise (non-vague) yet intensionally imprecise (vague). For example, consider a world w_2 consisting only of humans and horses, and assume there is some property *Q* applying to all humans and not applying to any horses. In w_2, *Q* is extensionally precise. But, if relative to w_2 is w_3—w_3 being a world consisting of human-like horses and horse-like humans—, then *Q* is intensionally vague, for the function that assigns the property semantically expressed by *P* does not completely determine a mutually-exclusive assignment of *Q* for *all* worlds *w* (i.e. w_3) at some time *t*. Part of the reason that vagueness is so pervasive is that for some predicates, there appears to be nothing in language-use or in the predicated-quality that completely determines extensions in all possible worlds. The focus of this thesis is on the more powerful, intensional interpretation of vagueness. The reason for delineating intensional/extensional distinction is to direct the discussion of vagueness toward the signification of *terms* in propositions rather than toward an analysis of certain contingent facts about the world. Since this thesis is concerned with the intensional interpretation of vagueness, while it may be the case that some predications are not vague because it just so happens that no borderline cases actually exist, it does not follow that the signification of predications is not vague since borderline cases *could* exist.

Secondly, a single vague predicate can be vague in more than one way. For example, Peirce and others have noted that color terms admit borderline cases with respect to hue, chroma, and/or luminosity (EP2:366, 394; CP4.159, 514, 6.526; CD4.1109, 4.2909, 2.986). A predicate that is taken to be vague in one way is a one-dimensionally vague predicate (or linearly vague). A vague predicate that can be vague in more than one distinct dimension is $(n+1)$-dimensionally vague (Burks 1946:481-2). Both distinctions are important for ensuring that many objections posed against a theory of vagueness are actually germane to it and it also helps to structure how a particular example of vagueness should be evaluated. For example, an object can be 'big' in a variety of different ways, and while vagueness may infect each one of these different dimensions, it is not the case that a theory of vagueness should address all of these dimensions at once. Specifying the manner in which a predicate is vague is thus important for evaluating a vague term.

2. Two Formulations of the Sorites

The connection of vagueness to the sorites paradox constitutes a third aspect of the phenomenon. A term is *sorites-susceptible* if it can be non-trivially employed in a sorites paradox. The connection of vagueness to the sorites situates it as a difficult philosophical puzzle that has been discussed from Eubulides of Miletus (fl. 4th century B.C.) to the present date. Countless scholars have tried to demolish the sorites paradox over the last two thousand years, yet the paradoxical nature of vagueness has proved unusually resilient. Crispin Wright and others project that the solution of the sorites is "light-years away" (2003:85). As a paradox, sorites arguments proceed from true premises to a false conclusion with a seemingly uncontroversial (valid) form of inference. Removing the paradoxical nature of the sorites is one of the central goals of a theory of vagueness for while vagueness by itself is tolerable, its paradoxical dimension is not. That is, the goal is vagueness without paradox. Before proceeding to a summary of such responses, below are two semi-formal versions of sorites.

2.1. The Many Modus Ponens Sorites

The modus ponens sorites begins with a paradigmatically true proposition, e.g. 'A man with 0 long, dark, and normally dispersed hairs on his head is bald'. It proceeds to a paradigmatically false conclusion—e.g. 'A man with 1,000,000 long, dark, and normally

dispersed hairs on his head is bald'—through an indefinite number of uses of modus ponens. Schematically, the sorites employing modus ponens is as follows:

A man with 0 hairs on his head is bald.

If a man with 0 hairs on his head is bald, then a man with 1 hair on his head is bald.
If a man with 1 hair on his head is bald, then a man with 2 hairs on his head is bald.
If a man with 2 hairs on his head is bald, then a man with 3 hairs on his head is bald.
.
.
.
∴   A man with 1,000,000 hair(s) on his head is bald.

Various vague predicates can be substituted in place of 'hairs' ('red', 'heaps', 'tall'), the conclusion can be suspended even further if 1,000,000 hairs is not sufficiently large enough, and more specification concerning the placement, color, visual conditions under which the hairs are examined, the method for removing the hairs or parts of hairs can be made explicit without affecting the paradoxical nature of the sorites. So as not to confuse the sorites paradox with some other unrelated form of indeterminacy, language-users are granted maximal understanding of the external world. Upon the addition of a hair, they are able to specify the exact length, number, and color of each hair. Further, it has been suggested that the location where hairs are placed will influence the determination of whether or not an individual is bald or not bald. For example, if the bulk of hairs are added to the top of a man's head, this would incline a language-user more quickly to say that a man is not bald. By contrast, if the same number of hairs were added to the back of a man's head, a language-user would be inclined to say that the man remains bald. Thus, considerations about the location of where the hairs are placed are thought to be just as relevant as the number of hairs, and this is thought to jeopardize the foundation of the sorites paradox in terms of its quantitative presentation. In order to preserve the quantitative approach, considerations concerning the removal of hair, the skin color and texture of the head from which the hairs are removed, and so forth are to be controlled as much as possible by removing the hair so as to preserve the impression that a man is not bald. That is, upon the removal of a single hair, the next hair to be removed ought to be taken from a location appropriately distant from the first. Secondly, it is not immediately

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clear whether these sorts of considerations apply to all forms of vagueness. The removal of hair from a man's head is subject to such considerations but it is not readily apparent whether such considerations also applies to color swatches, various heights of men, and other size predicates.

At root, every version of the paradox will require three features: (1) the initial premise must be paradigmatically true, (2) the conclusion must be paradigmatically false, and (3) the intermediate steps must be monotonically ordered and assert different yet sufficiently minute changes in the application of the predicate (see Wright 1975:333).[2]

2.2. Sorites with the Universally Quantified Premise

Another prevalent representation of the sorites paradox is one involving a universally quantified premise.

$$Pa_0$$
$$(n)(Pa_n \supset Pa_{n+1})$$
$$\therefore \quad (n)Pa_n$$

This argument involves a mathematical induction, which runs as follows: A man with zero hairs on his head is bald. However, since the addition of one hair can never make a difference between a man being bald and not bald, it follows that the following conditional is true: if a man has an arbitrary number of hairs and is bald, a man with an additional hair is also bald. Therefore, a man with a full head of hair is bald. This sorites shares the same key features as the modus ponens except condition (3) is satisfied by a universally quantified premise rather than an indefinite number of instances of modus ponens.

3. General Solutions

Removing the paradoxical feature from vagueness is no easy challenge. Five general attempts are briefly considered below.

The *first* is to deny that there is anything logically paradoxical about vagueness since it has no role in formal languages. The range of applicability of formal languages and logic ought to be restricted to artificial or scientific languages whose terms and concepts are precise. Vagueness is paradoxical only when vague terms are admitted into

formalized systems, and so what the sorites shows is that logic must not include vague terms. Frege (1960), Russell (1923), Lorenzo Valla (see chapter 2) and others have suggested something close to this option at one time or another. The solution is generally not met with much enthusiasm for the simple reason that there is no non-question-begging reason why natural language should have no underlying formal structure even if that formal structure and semantics wildly deviates from classical logic and semantics.[3]

A *second* option is to argue that the sorites is valid but unsound because one of the conditionals is false (in the case of the modus ponens sorites) or the universally quantified premise is false (in the case of the sorites with the universally quantified premise). This is the most conservative and popular option since the theorems of classical logic (and in some cases its semantics) can be retained. However, the major difficulty involves explaining how a minute difference in a vague predicate's extension can result in a change in truth value, or justifying the loss of other time-honored parts of classical logic (e.g. truth-functionality or the standard notion of validity). Two contemporary views are considered in greater detail in chapters 3 and 4, and my own solution pursues this line in chapter 6.

A *third* option is to argue that the argument is valid but unsound because the categorical premise is false (in the case of the modus ponens sorites and the sorites with the quantified premise). This option asserts that 'a man with no hairs on his head is bald' is false. After denying the categorical premise, the truth-values of the conditional or quantified premise are rendered irrelevant to solving the sorites. This is also known as the nihilistic alternative and was made popular by Peter Unger (Unger 1979; see also Wheeler 1975; 1979). Unger argues that to assume anything other than the nihilist solution to the paradox requires accepting one of two absurd views: (1) the *miracle of metaphysical illusion*, which is the dogmatic belief that nature inserts sharp semantic breaks into the division between 'bald' and 'not bald', or 'heap' and 'not heap', or (2) the *miracle of conceptual comprehension*, which is the absurd belief that language-users are sensitive to minute semantic adjustments in seemingly identical applications of a vague predicate (1979:125-126).[4] Since neither (1) nor (2) can be believed, Unger argues that the sorites entails the non-existence or non-reality of ordinary things. This, he argues, follows because a sorites for 'tall' leads to the truth of "a three-foot tall man is tall", and a

sorites for 'short' (non-tall) leads to the truth of "a ten-foot tall man is short". The sorites entails that it is true and false that a three foot man is tall. This is inconsistent, and therefore sorites-susceptible objects must not exist.

A *fourth* option is to argue that the argument is invalid. Subvaluationism—a paraconsistent approach—contends that classical consequence should not be unrestrictedly valid for vague terms.[5] This option is beset by usual problems facing any theory advocating inconsistency or paraconsistency: loss of truth-functionality, the invalidity of conjunction-introduction, and general disbelief concerning whether a proposition and its denial can both be determinately true (see Hyde 2008:93-104). Fuzzy and multi-valued logics also pursue the fourth option and claim to be buttressed by a more subtle semantic metatheory. It is claimed that the problem with the sorites and classical logic in general is that it butchers semantic theory by adherence to bivalence. The introduction of more semantic values aims at capturing the continuous movement from truth to falsity. The major problem with both many-valued and fuzzy logics is the contention that the semantics of a natural language can be rendered precise by semantic values (Goguen 1969:327; Machina 1976:54,61-76; Zadeh 1965; 1975). A precise semantic theory assumes both the absence of higher-order vagueness and what Unger calls the miracle of conceptual comprehension (Rolf 1984:222; Williamson 1994:111; Keefe 1998; 2000b). Both objections amount to asserting that the major difficulty that vagueness poses to classical semantics is not reducible to merely increasing the number of semantics values. Rather, vagueness poses a separate difficulty: how are semantic changes (between two values or many) possible without significant conceptual difference in the application of a predicate?

A *fifth* option is to argue that the argument is valid and sound. Dummett (1975), Quine (1981), and Rolf (1984) have argued for this option. Rolf argues that the sorites is valid within English because English itself is an analytically inconsistent language (1984:245). His suggestion is that the sorites cannot be solved within English for the principles of the language "make us steer into inconsistency and this cannot be avoided except by rejecting parts of the conceptual system of English" (1984:245). Quine (1981) regretfully advocates this option as well but largely because sacrificing the simplicity and elegance of classical logic and semantics is too great a price.

A number of other responses have been proposed, and all five responses put forward above could be, and have been, evaluated in greater detail.

4. Originality and Major Claims of the Thesis

This thesis is original for a variety of different reasons. Chapter 2 provides a historical sketch of the sorites paradox. Its originality is a result of its being more comprehensive than all previous historical examinations of the paradox. Two contemporary accounts are articulated in chapters 3 (epistemicism) and 4 (supervaluationism). There is a significant amount of research on each of these theories, so the originality is mostly confined to how these theories are rejected. The major objection to each stems from their respective views on the status of semantic boundaries. The epistemic theory contends that there exists a sharp, semantic boundary between truth and falsity. This has the effect of making 'John is tall' true and 'Mark is tall' false when Mark is only one micron shorter than John. In objecting to the epistemic theory, I contend that there is nothing in our use of words, in God's use of words, nor in nature that would entail sharp semantic boundaries. Supervaluationism, by contrasts, rejects sharp semantic boundaries yet contends that a number of classical relationships hold between propositions that are indeterminate. One effect of maintaining these relationships—or penumbral connections—is that certain propositions are supertrue (or true on all ways of making a term precise) yet none of its substitution instances are supertrue. This results in a rejection of truth-functionality. As a consequence, some existential propositions are supertrue even though it is *in principle impossible* to specify some substitution instance that would make it supertrue. This in principle impossibility is tantamount to arguing for the necessary non-existence of sharp semantic boundaries, a view which I contend is as equally question-begging as the epistemic theory.

Chapter 5 returns to the history of vagueness and argues against the claim—put forward by commentators of Peirce and standard historical treatments of vagueness—that Peirce failed to distinguish vagueness from other forms of indeterminacy. In that chapter, I argue that Peirce gave a clear account of vagueness as it is understood in contemporary analytic discussions. This claim is original for the standard view among Peirce scholars and vagueness theorists is that Peirce's theory of vagueness was equivalent to a theory of unspecificity or uninformativeness. The result of clarifying Peirce's account is that I am

able to sketch a solution to the paradox along Peircean lines in chapter 6. In short, my solution is that the sorites paradox can be solved conservatively by rejecting the universally quantified premise. I argue that this theory is possible without making any commitment to the existence or non-existence of sharp semantic boundaries. This distinguishes it from epistemicism (chapter 3) and supervaluationism (chapter 4), and is a reason for preferring my theory over these contemporary accounts.

Chapter 2

A Historical Sketch of the Sorites Paradox

> But you say that the *sorites* is erroneous. Demolish it then.
> —Cicero, *De Inventione* II. xxix.93

0. Introduction

The first logical and philosophical investigations into the topic of vagueness began with the emergence of the sorites paradox. This chapter embarks with a selective discussion of a few figures in the history of this paradox in order to contextualize the problem in the history of philosophy. Another principal concern here involves specifying the relation between the sorites paradox and sorites or "chain" syllogism. A comprehensive historical discussion of the problem will not be attempted, although one would be of great benefit since Timothy Williamson's history (1994) is short in breadth and depth, and Jonathan Barnes's focused history (1982) has limited scope. Additionally, Keefe's (2000b) account only sparingly considers the history of vagueness, opting for a thematic presentation of the *problem* to the neglect of how the sorites emerged and was revived. The aim here is to offer a broad exposition of the early history of the sorites paradox, touching on early solutions for explicative purposes, and indicating the motivation and use of the sorites paradox in the philosophical domain.

1. Zeno and Eubulides

While some scholars have tried to push the discovery of the sorites argument back to Zeno of Elea, Eubulides of Miletus—a contemporary of Aristotle and student of Zeno—is traditionally credited as the discover of the sorites argument (Williamson 1994:8; Mates 1961:5; Barnes 1982:36; Keefe 2000b:8). Taking Zeno as the discoverer of the paradox is argued for on the basis of his Millet Seed argument, which *prima facie* is adaptable on the basis of its soritical style, i.e. while one seed may not make a sound when dropped, neither two, nor three, but a handful of them will certainly make a sound (see Aristotle Phys. 250a19-22; Simplicius Phys. 1108, 18-28). But, as Barnes (1982:37) and Williamson (1994:9) point out, the focus of Zeno's argument is on the difference in the proportionality of the weight of the seed and not the boundary conditions of a term's extension.[6]

Very little is known for what purpose Eubulides's employed the sorites paradox, and little is known about Eubulides himself. Diogenes (II 109 and VII 187) contends that Eubulides authored, in addition to the sorites, a number of important dialectical arguments. But whether the use of the sorites paradox was to illustrate a logical feature of natural language or mercenarily employed for skeptical purposes—e.g. to undermine any of the following: the coherence of empirical conceptions, the principle of non-contradiction, pluralism, an item of Aristotelian thought, e.g. the doctrine of the Mean, or derail some thesis of Megarian thought—remains underdetermined given the writing extant (Empiricus M VII 13; Gomperz 1905:189; Barnes 1982:37n33; Gillespie 1911; Levi 1932; Reale 1976; Laertius 1959:II 109, VII 187; Moline 1969). In short, the available texts suggest a variety of different and possible hypotheses about the historical genesis of Eubulides's famed paradox but the available writing does not determinately favor one over any of the others.

2. Chrysippus and the Stoics

After Eubulides a number of Stoics are believed to have discussed the sorites and other Eubulides-created paradoxes (Barnes 1982:42-43; Kneale et al. 1962:114-115; Williamson 1994:10-12). The next principal character associated with it is Chrysippus.[7] The presentation of the sorites in the ancient context took the form of a series of questions and answers. For example, in the case of 'tall', a questioner asks whether a given paradigmatically tall individual is 'tall'. If the respondent answers 'yes', then the questioner asks the question about a similar, yet slightly shorter human being. The respondent again answers 'yes', and the series of questions and answers proceeds until the respondent answers 'no'. Say this occurs at 5'5. At this time, the questioner returns to the previously queried object (i.e. the one slightly taller than 5'5) and asks the respondent whether an individual slightly taller than 5'5 is tall. The respondent is then forced to decide whether the vagueness of his use of 'tall' actually corresponds to a precise usage, or whether to modify his/her original answers.

Chrysippus's solution to the paradox was to argue that there was an unknowable yet sharp boundary line between the true and false cases of tall such that the subtraction of an inch was sufficient to render a tall man no longer tall. The existence of such a boundary line between 'heap' and 'not heap' is suggested from various works on ethics

and causation. Diogenes (VII 127)—when discussing Stoic ethics—writes "[i]t is a tenet of theirs that between virtue and vice there is nothing intermediate" (see Bobzien 1998:61-74). Contra the Peripatetics, there were no degrees, shades, or intermediate stages in the development towards becoming more virtuous; one was either virtuous or vicious. Extrapolating this feature to the sorites-susceptible terms, the Stoics argued that there is no intermediate ground between a tall man and short man. One was either tall or not tall. However, despite being committed to the existence of such sharp boundaries, the Stoics argued that these boundaries were unknowable. Consider Cicero's explanation of this:

> No faculty of knowing absolute limits has been bestowed upon us by the nature of things to enable us to fix exactly how far to go in any matter; and this is not only in the case of a heap of wheat from which the name is derived, but in no matter whatsoever – if we are asked by gradual stages, is such and such a person a rich man or a poor man, famous or undistinguished, are yonder objects many or few, great or small, long or short, broad or narrow, we do not know at what point in the addition or subtraction to give a definite answer (Cicero Ac. II xxix 92).

While the Stoics argued for the unknowability of sharp boundary conditions, tactically, they argued that the proper response to sorites-style questioning consisted in a suspension of judgment *before* the soritical series of questions forced the respondent into the various problems that the sorites elicited.[8] It was typically suggested that this suspension of judgment should take the form of silence or reticence in the face of continued questioning about borderline cases between 'tall' and 'not tall'. This was thought to circumvent problems associated with openly admitting ignorance about borderline cases and was thought to be consistent with the Stoic epistemological commitment that knowledge involves "assent to what is clear" (Williamson 1994:15; Gould 1970:57-58, 62-63). Cicero again recounts Chrysippus's solution on the matter:

> But you say that the *sorites* is erroneous. Smash the *sorites* then, if you can, so that it may not get you into trouble, for it will if you don't take precautions. 'Precautions have been taken,' says he, 'for the policy of Chrysippus is, when questioned step by step whether (for example) 3 is few or many, a little before he gets to "many," to come to rest, or, as they term it, *hesychazein*.' (Cicero 1961 Ac. II, xxix 93).

So, when an individual is confronted with a question about the boundary line between 'heap or 'not heap', the Stoic position was that the question cannot be answered *with knowledge* and so to avoid the possibility of assenting to some proposition without knowledge, the respondent ought neither answer in the affirmative nor the negative, nor explicitly assert that one does not know, but refrain from answering the question altogether. Namely, Chrysippus contends that the Stoic should fall silent *before* the end of the clear cases, leaving a safe boundary where one might potentially make a few more affirmations without assenting to anything false (Williamson 1994:20; Burnyeat 1982:335; Barnes 1982:52).

Motivating this hesitation about affirming *all* clear cases is Chrysippus's notion of *anticipation*. The Chrysippan standard for determining whether a given presentation is or is not an illusory indication of a genuine object involves his notion of a 'common notion'. Generally put, a *common notion* is one's immediate acquaintance with an object or property. For example, my direct experience with certain properties associated with water indicates that the liquid in front of me is water. However, Diogenes (VII:54) argued that Chrysippus's view on what constituted a common notion was inconsistent. On the one hand, Diogenes contends that Chrysippus held that common notions are presentations that give a direct *apprehension* of the object, while, on the other hand, these notions were thought to be both these apprehensions and *preconceptions* or *anticipations* of certain objects or properties. Josiah Gould (1970:63) contends that these two views are not strictly contradictory. Instead, the full view of the Chrysippan notion of common or general notions involves both the employment of a family of past presentations together with the employment of anticipation of future phenomena. Thus, the Stoic suggestion that respondents should resort to silence upon approaching borderline cases is motivated by the thesis that certain common notions involve not only a direct encounter with tall things but an anticipation that tall things have unknowable borderline cases. That is, our common notion of 'tall' or 'heap' involves certain memories about objects previously regarded as tall but also anticipations concerning what will not be regarded as tall and what will be regarded as unknowable.

Despite this epistemological caution, Chrysippan silence was prone to a substantial objection. While regarded within the Stoic camp as having set down the final

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word on the sorites paradox, the Skeptical Academy was less than convinced.[9] Carneades ridiculed Chrysippus's answer to remain silent as a failed attempt at a solution for it avoids the real question of the sorites paradox. Do our terms and the objects they denote admit precise boundaries or do they not? A solution to the sorites paradox, contends Carneades, cannot be formulated by resorting to reticence when questioned about indeterminate cases. A positive solution to the sorites paradox is satisfactory only when it gives an *explanation* of these indeterminate cases. Thus, the Stoic resort to silence fails to quiet the skeptic's demand for an account of whether a borderline heap is a heap or a non-heap, whether three is few or many, or whether a given human being is tall or not tall. Carneades insists that silence is not a positive answer to the sorites solution:

> 'So far as I am concerned,' says Carneades, 'you may not only rest but even snore; but what's the good of that? For next comes somebody bent on rousing you from slumber and carrying on the cross-examination: "If I add 1 to the number at which you became silent, will that make many?" You will go forward again as far as you think fit.' Why say more? For you admit my point, that you cannot specify in your answers either where 'a few' stops or that where 'many' begins (Cicero Ac. II, XXIX).[10]

For Carneades, the Chrysippan resolve to turn reticent on the issue is not a positive solution to the paradox but an evasion of the puzzle.

Further, the commitment to the existence of an unknowably sharp borderline between tall and not tall was problematic because there is no account of how this was known. Cicero reports that Chrysippus, contra Epicurus, "exerts every effort to prove the view that every *axioma* is either true or false."[11] But, as Bobzien (1998:62) notes, "we are nowhere told how Chrysippus backed the principle up." If the Stoic epistemological principle is that knowledge involves "assent to what is clear," the Stoic needs to justify or make clear the implausible notion of precisely-bounded sorites-susceptible terms.

One remaining aspect of the sorites paradox was how best to characterize the Stoic commitment to the logical structure of its presentation. The traditional presentation came in the form of repeated questioning, but Williamson (1994:22) and Barnes (1982:27-28) both contend that the Stoics were the first to recognize a logical structure to the sorites in the form of a series of conditionals and applications of modus ponens. This is important in the history of the paradox for it expands the presentation of the paradox to

an explicitly logical structure and a logical problem, rather than one cased in a dialectical façade. Diodorus Cronus and Philo of Megara are regarded as the first to have seriously debated conditional statements.[12] For Philo a conditional statement is true if and only if it does not have a true antecedent and a false consequent, namely P→Q is equivalent to '~(P&~Q)'. Diodorus adopted a stronger view with regard to the truth of conditional statements. A conditional is true if and only if it neither is nor never was possible that the antecedent is true and the consequent false, namely P→Q is equivalent to 'Not ever: P&~Q.' The Diodorian conditional might be understood as an "omni-temporally true Philonian conditional" (LaBarge 2002:242). According to Cicero, Chrysippus opted for a third type of conditional, i.e. what is now known as the strict implication.[13] Diogenes (VII 73-75) contends that this "hypothetical proposition is therefore true, if the contradictory of its conclusion is incompatible with its premiss" and "false, if the contradictory of its conclusion does not conflict with the premiss." Williamson (1994:24) interprets the Chrysippan conditional to be a stronger version than the Philonian or Diodorian conditionals, taking it to mean "true if and only if its antecedent is incompatible with the negation of its consequent. Thus 'If P then Q' becomes equivalent to 'Not possible: P and not Q.'" But there is a problem in interpreting the Stoic conditional in terms of strict implication. Bobzien (1998:94-95) contends that the truth-criterion of the connection of the Chrysippan conditional is founded on a notion of incompatibility or conflict between the antecedent and the consequent that is not clearly specified in the texts. This difficulty is spelled out by LaBarge (2002:242) as follows:

> On the one hand, the Stoics seem to have defined truth-conditions of implications much along the lines of modern relevance conditionals; for a conditional to be true there must be some kind of logically necessary connection between the contents of the antecedent and the consequent. On the other hand, we know from the works of Cicero […] that the Stoics also accepted conditionals that would seem to be unable to meet such strong conditions; in particular, they endorse some medical inferences that seem clearly to be at best connected by some kind of empirical necessity.

While the importance of noting the Chrysippan conditional figures in understanding the logical structure of the sorites paradox, how Chrysippus understood the sorites paradox will not be continued here for it is unclear exactly how to interpret the Chrysippan conditional.[14] Both Barnes (1982:28) and Williamson (1994:24-25) contend that

Chrysippus would often rewrite conditionals as negated conjunctions to indicate a weaker interpretation (presumably the Philonian conditional) than the Chysippan conditional. One example is found in Diogenes, who contends that the Stoics represented the sorites as follows:

> It cannot be that if two is few, three is not so likewise, nor that if two or three are few, four is not so; and so on up to ten. But two is few, therefore so also is ten (Laertius 1959:VII 82).[15]

The advantage to rewriting conditionals in this fashion is that it provides a more challenging paradox than one formulated by the Chrysippan conditional, and it allows for connecting the Stoic presentation of the sorites paradox to the contemporary formulation. By rewriting conditionals in this fashion, both Williamson (1994:22) and Barnes (1982:29) contend that we can formulate the following logical presentation of the sorites paradox where the argument involves repeated applications of modus ponens.[16]

P_1

$P_1 \supset P_2$

$P_2 \supset P_3$

.

.

.

$P_{n-1} \supset P_n$

$\therefore P_n$

If, in fact, the Stoic solution involved a commitment to bivalence, then the Stoic affirms the validity of the above argument, but rejects its soundness. On the Stoic account, the initial premise is clearly true, the conclusion clearly false, and one of the intermediate conditional premises is false, i.e. some P_i is true while P_{i+1} is false. And, not knowing exactly which one of these conditionals is false, the Stoic only affirms as true those that are paradigmatically true.

3. Galen

The next major figures to address the sorites are found in Galen's *On Medical Experience*. The sorites argument is employed by both the Empirical Doctors (Empiricists) and the Dogmatic Doctors (Rationalists) for the purpose of undermining

each other's claim. Galen, who serves as the recorder of the debate, contends that the Rationalist camp is led by Asclepiades, who is frequently labeled as a maniac, and the Empiricist camp is backed by a variety of authorities including Menodotos, Serapion, and/or Theodosius (Galen 1985:51). The empirical doctors argue that medical knowledge is completely learned through experiential acquaintance with symptoms and experiential acquaintance with treatments, while the rationalist doctors argue that medical knowledge requires, in addition to experience, inference from observable symptoms to their invisible causes. The sorites is mercenarily employed by both sides. The empiricist doctor claims that medical knowledge is not possible upon a single observation of symptoms, nor upon a few, but a claim concerning the cause of a set of symptoms can be regarded as true if and only if these symptoms have been observed *many times*. The rationalist reply is that 'many times' is objectionable on account of its vagueness:

> [C]an you tell us, Empiricists, how many times very many times is? For we desire to gain knowledge through observation the way you do. Hence, to make sure that we do not, for a lack of measure, miss the appropriate amount, either because we think that we have come to the end before we have observed the matter sufficiently, or because, out of our ignorance of the proper measure, we extend our observation far beyond what is appropriate, we ask you to show us, too, what the measure is, so that we, too, can learn something from observation (Galen 1985:58).

The rationalist argument is that the inability to specify the requisite number of times a certain set of symptoms need be observed for knowing that a set of symptoms is disease X rather than Y—or should be treated by remedy A rather than B—shows that the point at which the empirical doctor acquires knowledge is not sharply defined. Further, while no common measure can be given for every kind of disease, the rationalist doctor argues that no common measure can be given for even one kind of disease. The vagueness of 'many times' is claimed to be objectionable through an employment of the sorites:

> I would ask them [empiricists] therefore, if that which has been observed ten times is included in that which has been seen very many times, and their answer to this is 'No'. Then I would say to them: 'And what has been seen eleven times?'—and they say 'No'. [...] And so I never cease asking and adding another number to each until I reach a high number. Nothing remains for him thus questioned except either to deny at a given time that the number has reached the limit when one can say it constitutes very many times, or, should he admit that it has, to make himself a laughing-stock for men, since he would thus require them to allow him a number

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reached solely by a usage fixed by himself, and a decision made by him alone (Galen 1985:59).

The empirical doctor is caught in a dilemma. Either claim that a near infinite number of observations is insufficient for medical knowledge (and thereby contradict the claim that knowledge is obtained upon 'many' observations) or stipulate some specific number of observations that corresponds to 'many' and make knowledge objectionably arbitrary and subjective, as well as the semantics for knowledge claims to turn on the addition of one experience. The rationalist argues that stipulation is objectionable because the empiricist cannot give an answer for why the stipulation was made at one point rather than some other. The rationalist argues, "Why, for example, should anything that was seen fifty times be regarded as having been seen very many times, and that which was seen forty-nine times is not regarded as having been seen very many times." (Galen 1985:59).

The empirical doctor argues that sorites arguments are equally applicable to the rationalist doctor. The empiricist argues that there are two consequences to the sorites. The first is "that the being and not-being of a heap is determined by a grain" (Galen 1985:76). The second is that objects like heaps, mountains, armies, and flocks do not exist (Galen 1985:77). The second consequence is said to partially follow from the first and is articulated by the empiricist as follows:

> The second result of this admirable argument [the sorites], which follows from the first, is that we find there is no such thing as a mountain, and then a third and fourth and fifth and sixth result show that there is no row, no city and no flock, no army, no crowd and no nation, for not one of these is formed by the union of one or two things which causes this to become a people, and this a row, and this a flock, and this an army, and this something else, but rather because union of many individuals must take place, if one wishes this to be a row of people and that, a nation, also that not an inconsiderable number of sheep must come together if one wishes to call this collection a flock, and not a few houses are necessary if one wishes their conglomeration to constitute a city (Galen 1985:77).

Part of the reply from the empirical doctors is that rationalists must also deal with vagueness insofar as rationalists affirm the reality of ordinary things. In affirming that there are cities, flocks, and armies, the empiricist contends that this conglomeration must be determined by some precise number or else the sorites argument can be employed to show, for example, that no number of sheep is sufficient to form a flock.

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The empiricist's rebuttal is that experience shows that conglomerates like flocks and heaps do exist, and they claim their existence is absolutely self-evident (see Barnes 1982:58). Rather than denying or resorting to skepticism about the reality of common sense objects, the empiricist argues that sorites-susceptible terms contain limits that are *intrinsically indeterminate* (see Barnes 1982:59). Galen writes:

> Do you impute this against us because we cannot state with exactitude the precise number contained in each of these, but are only able to give a general notion of what they are and of what is formed of each of them in the mind or in the imagination? Since each has always been capable of expansion and augmentation and without limit or end at which its being stops, it is therefore impossible for us to say how large is the number of each one of them (1985:77).[17]

The empiricist doctor aims to dissolve the demand for the location of sharp semantic boundaries by claiming that such a demand is unreasonable. The argument by which this claim is worked-out is not very explicit but it amounts to either (1) a rhetorical appeal to common sense and/or (2) the claim that the demand for a precise semantic boundary is only applicable to a rationalist conception of knowledge. Both are summarized in the following remark: "if you say of something which people see very many times under the same conditions throughout their lives, that it is non-existent, you will not be helped at all. For you reject it and declare it to be invalid only by argument [logos] and not in reality" (Galen 1985:78). Part of the claim then turns on common sense. The empirical doctor argues that the extension of 'flock' or 'army' or 'heap' need not be known yet experience seems to obviously tell us that these things exist. The other part of the claim turns on whether both theories must solve the paradox. The empirical doctors claim that theirs need not because the demand for a precise semantic boundary stems from some intellectual demand to explain *how* or *why* we have knowledge independent from experience. Experience, contend the empirical doctors, does not give us knowledge of how many sheep are necessary for a flock nor how many grains are necessary for a heap. It only indicates that more than one is required but less than some extreme amount is needed.[18]

4. After Antiquity: Medievals & Lorenzo Valla

There is surprisingly little discussion of the sorites paradox in the medieval period. Williamson writes that after antiquity a "thousand years may be passed over in a few

words. Sorites puzzles formed no part of the medieval logical curriculum" (1994:31-2; see also Jardine 1977:166). What occurred during this period were a flurry of commentaries devoted to Aristotle's *Organon*, some dealing with individual books, others dealing with one its two main divisions (the *Logica vetus* and the *Logica nova*), but the majority focusing on the *Organon* as a whole (Ashworth 1988:143-4). Since the *Organon* does not make the sorites paradox a central concern, little to no discussion of it is extent. Even changes that occurred during the sixteenth century with respect to work done on the *Organon*—ranging from increased emphasis on Averroes, Aquinas, an influx of new translations, and publication of the Greek commentators on Aristotle's logic—did little to reorient logical discussion to take the sorites paradox as a serious problem.[19] Finally, while semantic paradoxes, such as the liar, aroused considerable discussion between 1400–1700, the sorites paradox remained absent (Ashworth 1972).

The sorites was ultimately revived in humanist logic in its focus on oratory, debate, and clear thinking in asyllogistic discourse. Lorenzo Valla suggested that much of the debate concerning the intersection of natural language and formal language might be obviated by detaching the two, sectioning the range of formal validity to syllogistic reasoning and leaving natural language to handle paradoxes that emerge from its use in practical reasoning. Jardine writes that "Valla suggests, one might prefer to concede formal logic's limited sphere, and tackle ordinary language problems from the point of view of the linguistic specialist" (1988:179; 1983). The motivation for this was rooted in the view that syllogistic reasoning was not the only form of rationality. In order to justify this claim, Valla needed to dislodge the applicability of validity from natural language so that he could substantiate other forms of *rational* argumentation. One way to do this was to chip away at a few central elements of the syllogism itself. Since the throne of rationality was occupied by formal analysis of propositions and terms in the context of the syllogism, and medieval interest in these features occupied a significant part of the overall curriculum, Valla's tactic was to undermine the role of the *term* and thereby jeopardize the *universal applicability* of the syllogism. What medieval logic deemphasized was the role *dialectical arguments* played in rational inquiry or rational communicative practices. Specifically with respect to oratory, the mode of reasoning emphasized by constructing the syllogism was ineffective when tied to formal validity.

The sorites paradox was a perfect candidate for attacking syllogistic reasoning since no syllogism could guarantee the success of the conclusion, because according to Valla, many syllogisms require that terms be interpreted in univocal fashion (see Jardine 1977:166). However, sorites-susceptible terms like 'tall', 'heap', and for many figures in antiquity, even natural kind terms like 'water', resist univocal treatment. So, if terms could not be treated univocally in common discourse, then the applicability of the syllogistic analysis to everyday speech is suspect. And, in sum, Valla's interest in and employment of the sorites was mercenary for its aim was to limit the scope of syllogistic reasoning and make room for a separate kind of rationality found in natural language.

Many of the old paradoxes investigated by Stoic logicians—including the sorites—were thus reborn in his *Dialecticae disputationes*. The work was apparently effective since, according to Jardine, it "is a mark of the wide influence of Valla's *repastinatio* of dialectic that virtually every post-fifteenth-century humanist dialectic manual includes some minimal treatment of classic[al] *sorites*, in spite of its not finding a place in any medieval manuals" (1988:181). And also that "by the end of the sixteenth century humanist dialectic had ousted medieval dialectic from the Arts' curricula of almost all the major European universities" (Jardine 1977:145; see also Ashworth 1972; Ashworth 1982:790-1). So, while the dethroning of formal validity as the sole form of rationality led to a shift in focus as to how arguments should be evaluated, no definitive solution was offered by Valla as how to solve the sorites.

5. Three Moderns: Locke, Leibniz, & Hume

The role of the sorites in the early modern period requires some initial disambiguation. Syllogisms strung or chained together were called "sorites syllogisms" despite there being no explicit connection to paradox. For example, in the 17th century *Port-Royal Logic*, its authors wrote that arguments composed of several propositions, "in which the second depends on the first, and so forth, are called *sorites*" (Arnauld et al. 1996:137, 177). After defining the sorites as a syllogism composed of more than three propositions, the *Port-Royal Logic* went on to classify three types of sorites, namely "gradations", "dilemmas", and "epicheiremata" (1996:177; see Gassendi 1981:135-6). But, in these logic texts, the sorites appears to be treated in a completely syllogistic context that is detached from its roots as an ancient paradox.

However, some moderns were concerned with vagueness and sorites-style reasoning in a context that was detached from formal logic. For example, Locke, Leibniz, and Hume all considered the sorites paradox in the context of their respective analyses of the human understanding.[20] For Locke, the sorites was employed against the reality of universals. The argument went something as follows. General ideas and terms (like those for biological species) are vague insofar as language-users are unable to specify a sharp boundary between it and some other species. The inability to distinguish between one species and another entailed, for Locke, the absence of any real boundary between species and the absence of any knowledge of the real essence of a species. The argument generally proceeds by reductio. That is, consider the realist position that general terms indicate real classes that are distinct. If this is the case, then humans ought to be able to describe a property that functions as a boundary-line between one general term and another. That is, Locke identifies the recognition of the real essence of a species as equivalent to a recognition of the real boundary that separates it from another species. He writes that the "measure and boundary of each sort, or *species*, whereby it is constituted that particular sort, and distinguished from others, is that we call its *essence*, which *is* nothing but that *abstract* idea *to which the name is annexed*" (III.vi.2). In connection with Locke's thesis that words confound a proper understanding of things, Locke contends that names for physical species lead us to believe that animals within a species both have a common property and those in different species are distinct from each other. But Locke insists that the arbitrariness of classificatory schemes is made evident through their sorites susceptibility of our terms. If classifications of natural species were regulated by an understanding of a real essence, then there would be no inability to detect the boundary between two adjacent species. That is, if humans knew the real essence of a species, then there would be no vagueness in natural language since knowledge of the class of things would correspond to language use. Instead, our ideas and terms for different species are confused because they do not properly distinguish themselves by the inclusion of a difference in the meaning of a term grounded in simple ideas.

In subject-predicate language, Locke argues that our understanding of a *species* is a generalized subject (or complex idea) that is obtained not only through the unity of a number of simple predicates, but also through an abstraction to a more

generalized subject via the understanding (e.g. the aggregate of a number of ideas of horses to produce the general idea *horse*). The understanding plays a role in the formation of the species that cannot, says Locke, be ignored. When we take a substantial notion such as *horse* and affix a name to it 'horse', we do not grasp the real essence of *horse* by having an experience of some common property. Instead, we simply give linguistic denomination to an idea formulated in the workshop of our personal understanding whose materials are individuals. While this idea has its foundation in similarities that exist between simple ideas, the creation of a general idea requires abstraction upon these, and this abstraction requires that we *draw* a boundary between one species and another.

Thus, Locke's argument against the real essence of general ideas and words is that in the realm of words and ideas, we are incapable of pointing to a simple property that serves as the boundary line between one general idea/word and another. Locke chides his reader to produce a real boundary between one species and another when he writes "[i]f any one will regulate himself herein by supposed *real* essences, he will, I suppose, be at a loss: and he will never be able to know when anything precisely ceases to be of the species of a *horse* or *lead*" (III.iii.13). A similar argument against real essences was proposed years later by Wittgenstein who argued that there is no common property to 'games' although the similarity between different particular games and the very fact that we call this constellation of similarities 'games' may cause the illusory idea that games have a real, common essence (Wittgenstein 1953:§§65-71). Much like Locke before him, Wittgenstein took his reader to task with a sorites-type challenge: "For how is a game bounded? What still counts as a game and what no longer does? Can you give the boundary? No. You can *draw* one; for none has so far been drawn" (§68).[21] Our inability to indicate a boundary leads Locke and Wittgenstein to contend that we cannot be assured that there is a real boundary between them or a real common property shared by all horses or games. Locke's corrective to the problem is one of the central anti-scholastic projects of the *Essay Concerning Human Understanding*: the distinctness of names should not lull us into forgetting that names refer to abstract ideas created in the workshop of the understanding, and if these abstract ideas stand for anything real,

they stand for our simple ideas of sensation and reflection. We cannot let the virtual and arbitrary elements found in the signification of words for substances play a role in regulating our understanding since these elements do not reduce to simples.

One serious problem Leibniz had with the Lockean account of language is that it would eliminate a large amount of representation commonly taken to be significant. Under the Lockean view, the proper signification of any word is the corresponding idea in the speaker's mind. According to Leibniz, however, this precludes the possibility that words can stand for *another individual's ideas* as well as *real things*. With respect to the latter, Leibniz writes that "words indicate the things as well as the ideas" (RB 287). But if, on Locke's account, words can only refer to private ideas then Locke's skeptical use of the sorites undermines not only the realist position but the possibility of communication altogether.

How exactly Leibniz substantiates his anti-skeptical position is unclear. Contrary to the view that Leibniz's consideration of the sorites is confined to his reply to Locke in the *New Essays*, Leibniz is unique for his consideration of vagueness (Williamson 1994:33-4). Samuel Levey (2002) contends that in the early modern period "it is Leibniz above all who takes interest in the topic of vagueness." This is evidenced by Leibniz's consideration of the sorites paradox outside of things that would normally be classified as "natural kinds", e.g. chemical elements, to the consideration of heaps and colors.

Leibniz's thought on the sorites paradox is explicitly put forward in three texts: the *Pacidius Philalethi* (1676), "Chrysippus' Heap" (1678), and the *New Essays on Human Understanding* (1704). In *Pacidius Philalethi*, Leibniz holds that the boundary between vague terms is sharp and the solution to the sorites lies in denying the inductive premise of the sorites argument. Namely, the change from 'tall' to 'not tall' or 'poor' to 'rich' occurs, not gradually, but with the subtraction of one mikron in height or the further acquisition of a penny. In this text, Leibniz's solution is similar, if not identical, to the early Stoic solution in their claim of unknowably sharp boundaries for terms. In "Chrysippus' Heap", Levey argues that Leibniz changes his mind from a "sharp-boundary thesis" to a nihilist view on vagueness (2002:34). In this text, Leibniz remarks that notions such as 'wealth' or 'baldness', when taken

absolutely, are vague imaginary notions of which we have no corresponding idea. Levey clarifies this remark with a passage in the *Discourse on Metaphysics* (1989:art.25) where Leibniz writes "we have no idea of a notion when it is impossible." The lack of vague ideas is thought to indicate vagueness admits of inconsistency, or hidden impossibilities. Therefore, ideas and things that are sorites-susceptible, rather than being sharply bounded, do not exist. Finally, Leibniz's *New Essays* are thought by Levey to merely extend Leibniz's nihilistic views on vagueness but introduce some minimal tension with his earlier thought (2002:37, 39). On his account, Leibniz contends that vague objects do not exist but a *species* could have real essences independent of *our* knowledge. This is problematic because if species can have real essences independent of our knowledge, it unclear why vague terms such as color and height cannot have them as well.[22] And, if some objects that admit vagueness to our finite understanding can have sharp boundaries, then Leibniz abandons his supposed nihilism for an earlier sharp-boundaries theory.

The purposed tension that Levey suggests in Leibniz's thought might be alleviated in a number of different ways. One way is simply to reject the interpretation that Leibniz argues for a nihilistic conclusion in "Chrysippus' Heap" and thereafter. Levey is correct in contending that Leibniz does not argue for a sharp-boundaries solution since Leibniz contends that "[i]f *poverty*, taken absolutely, were a true notion, it ought to be defined by a certain number of pennies, because it is necessary for someone who is not poor to become poor on the removal of one penny" (Aiv23, 69). Leibniz explicitly rejects this option and thereby rejects an equation of vagueness with the sharp-boundary view. However, the rejection of this view does not entail a nihilist view on vagueness. Levey defines nihilism about vague terms as involving either (a) logical or semantic incoherence or (b) that vague terms are deficient in some way that precludes them from belonging to a well-formed proposition (2002:31). Levey contends that for the nihilist "[v]ague notions are thus never *true* of anything" (2002:31).

I argue that Leibniz did not adopt a nihilist view. Instead, propositions involving vague terms are indexed to speaker-relative specifications and are to be treated modally. That is, Leibniz can be seen as offering a revision of vagueness that aims at retaining the indeterminacy of vagueness without affirming its absolute non-reality. In *New Essays*,

Leibniz contends that when truth or falsity are assigned to ideas, as opposed to statements, it is the possible ideas that are true and the impossible ones that are false (RB 269). So, when Leibniz writes that vague notions are false in "Chrysippus' Heap", it is perhaps natural to read Leibniz as saying that vague notions are impossible, which would be an affirmation of nihilism. But Leibniz writes that vague notions are false when they are "taken absolutely". He does not contend that vague notions should ever be understood in this fashion. Immediately after he notes that when vague notions are taken absolutely they are false, he writes "Precisely those notions to which the Stoics' objection cannot be made are understood purely and transparently by us. That is to say, the above notions indicate something with respect to our opinion, which varies" (Aiv23, 69). Rather than using the sorites paradox as a touchstone for an observer-independent or speaker-independent notion of truth, he uses it as a negative criterion for what notions can or cannot be made transparent independent of human opinion. While anything susceptible to the sorites cannot be understood purely, this is not to say that vagueness cannot be understood at all nor is it to say that vague notions are, in every respect, false (impossible). Instead, Leibniz's response is that the transparency of vague notions is always qualified with respect to the vacillating opinion from which it originates. What this means is that any determination of the possibility of a proposition involving a vague notion cannot be determined independently of the perspective from which it emanates. What it does not entail is that vague terms are automatically precluded from belonging to a well-formed proposition, nor does it entail that they are never true of anything..

> To use the earlier example of the pennies, Leibniz writes:
>
> If *poverty*, taken absolutely, were a true notion, it ought to be defined by a certain number of pennies, because it is necessary for someone who is not poor to become poor on the removal of one penny; or he will never become poor at all (Aiv23, 69, emphasis added).

Leibniz claims that if 'poverty' were taken absolutely, the consequence would be a sharp, one penny boundary between 'poor' and 'not poor'. As noted earlier, Leibniz regards vague-notions-taken-as-absolute as defective, but this is not to preclude their possibility if they are taken from the standpoint of the speaker. Nevertheless, it does not seem clear that Leibniz ever formulated a well-worked out solution to the sorites for nowhere does

he explain how his articulation of the semantics of vague terms should solve the paradox.[23]

A final instance where the sorites emerged is in a well-known passage of section 2 of Hume's *Enquiry*. Hume admits that there is one phenomenon that contradicts his position that all ideas are derived from impressions. This is the capacity of individuals to fill-in un-experienced shades of color in a color continuum. Hume's example runs roughly as follows: There are a collection of shades of blue, starting from dark blue to light blue with a number of intermittent shades. Since every idea is a mere copy of a sense impression, the gradation of color must be produced by having experienced each of these shades *distinctly*. Supposing that there is at least one shade of blue in the continuum that has not been experienced, the question is whether or not the mind can independently supply it? Hume writes:

> I believe it will be readily allowed, that the several distinct ideas of colour, which enter by the eye, or those of sound, which are conveyed by the ear, are really different from each other; though, at the same time, resembling. Now if this be true of different colours, it must be no less so of the different shades of the same colour; and each shade produces a distinct idea, independent of the rest. For if this should be denied, it is possible, by the continual gradation of shades, to run a colour insensibly into what is most remote from it; and if you will not allow any of the means to be different, you cannot, without absurdity, deny the extremes to be the same (1999:sec.2).

Hume has two main points here. The first is that 'blue' is meaningless as a general term. It is meaningless because it does not refer to any distinct impression but to a continuum of distinct impressions collected together under a general idea. If the meaning of 'blue' is not reduced to such distinct impressions, the sorites paradox can be applied and individuals are forced to admit contradiction in their use of the term. The second point is that if the idea of 'blue' has any content at all it must reduce down to some *distinct* sense impression. This deflates the possibility of applying the sorites paradox since, by hypothesis, all terms reduce down to discrete impressions that admit of no boundaries.

6. A Prelude to the Rise of Modern Logic: Early 19th Century Discussion

Whereas the concern with the sorites for Locke, Leibniz, and Hume was primarily epistemological and metaphysical, the sorites reemerged in more logical contexts in the 19th century. But before mathematical logic rose to full-swing, the use of the word

"sorites" still was tied to its application to chain syllogisms. For example, it was traditional for logic textbooks like Gassendi's 1658 *Institutio Logica* and the *Port-Royal Logic* to contrast enthymemes with chain syllogisms. An enthymeme is an abridged version of a syllogism insofar as it has a missing a premise (see Whately 1854:138; Gassendi 1981:134-5; Arnauld et al. 1996:175-8). Because enthymemes required the addition of a missing premise, Whately claimed that it is "*not strictly syllogistic*; *i.e.* its conclusiveness is not apparent from the mere form of expression, till the suppressed premiss shall have been, either actually or mentally, supplied" (1854:138). In contrast to enthymemes where the expressed premise may be true but the conclusion false, the sorites was "strictly syllogistic" and therefore capable of being valid (1854:138). Whately and early logic texts state that syllogisms could be strung or put into a chain whereby the predicate of the first premise was made the subject of the second, and then the predicate of this second premise could be made the subject of a third premise. This procedure could proceed until ultimately "the predicate of the last of the premises is predicated (in the conclusion of the subject of the first" (1854: 138). The example Whately gives is the following: every A is B, every B is C, every C is D, every D is E; therefore A is E.

Thus defined, the sorites is a valid syllogism with an unlimited number of intermediate propositions linking the first premise and the conclusion. One interesting aspect of Whately's discussion of the sorites is the recognition that the first premise of a sorites is actually the minor premise. To illustrate this point, take an abbreviated form of the chain (sorites) syllogism given a few lines above, i.e.

> Every A is B
> Every B is C
> Therefore, every A is C

In the traditional construction of the syllogism, there are three terms (major, minor, middle) and three propositions (major, minor, conclusion). Traditionally, the subject of the conclusion is the *minor* term, the predicate of the conclusion is the *major* term, and the term occurring in both premises is the *middle* term (e.g. Bain 1880:134). The three propositions are denoted as follows: the major premise is the proposition containing both major and middle terms; the minor premise is the proposition containing the minor and middle terms; the conclusion is the proposition containing the major and middle terms

(e.g. Bain 1880:135). Up to this point in our illustration, the sorites does not deviate from the traditional syllogism.

Minor Premise	Every A is B
Major Premise	Every B is C
Conclusion	Therefore, every A is C

Minor Premise	Every minor is middle
Major Premise	Every middle is major
Conclusion	Therefore, every minor is major

The purported difference emerges in the traditional *ordering* of the premises. Typically, the major premise is presented first, the minor premise presented second, and the conclusion at the end (Bain 1880:138; Whately 1854:139). Namely, rather than the above figure, the syllogism was traditionally represented as follows:

Major Premise	Every B is C
Minor Premise	Every A is B
Conclusion	Therefore, every A is C.

Whately noted that what constitutes the sorites is a change in the order of premises of a syllogism, coupled with the insertion of any number of additional premises. He further noted that when the chain of reasoning was broken into distinct syllogisms, the conclusion of each syllogism always served as the minor premise for the next. Thus:

Minor Premise	Every A is B
Major Premise	Every B is C
Conclusion	Therefore, every A is C.
	↓
Minor Premise	Every A is C
Major Premise	Every C is D
Conclusion	Therefore, every A is D

However, calling these strings of syllogisms the "sorites" is somewhat of an extension of the term beyond the context of the sorites *paradox*. Traditionally, as we have seen, the sorites is employed as a puzzle or in a mercenary application to undermine realism about general properties, natural kinds, or secondary qualities. Here we find the term "sorites" being applied neutrally and without reference to the difficulty of isolating the limit or

bounds of a term's extension. The rightful employment of the term "sorites" in the context of the *problem* of vagueness is not merely one concerning syllogisms with more than three propositions. However, this application of the term is not irrelevant since the sorites paradox was often employed in the context of valid syllogisms with more than three propositions (or questions). Whately and some other logicians during his time (e.g. Hamilton, Mill) seemed to try and connect the sorites qua syllogism with the sorites paradox in the form of what Whately called the "destructive syllogism". Whately wrote that a destructive syllogism is a syllogism involving more than three propositions where "you, of course, go back from the denial of the last consequent to the denial of the first antecedent: "G is not H; therefore A is not B"" (1854:140). In this case, we have the typical chain syllogism except that one of subsequent premises has a different *quality* than a preceding premise. That is

Premise	A is B
Premise	B is C
Premise	C is D
Premise	D is not E

Interpreted as a series of conditionals, we could write this as follows:

P1 S_h is tall
P2   if S_h is tall, then a man one millimeter shorter is tall (S_{h-1})
P3   if (S_{h-1}) is tall, then a man one millimeter shorter is tall (S_{h-2})
P4   if (S_{h-2}) is tall, then a man one millimeter shorter is tall (S_{h-3})
             .
             .
             .
P_n if $(S_{n-1,000,000})$ is tall, then $(S_{n-1,000,001})$ is tall
P_{n-1} $(S_{n-1,000,001})$ is not tall

Taking P1 as intuitively true, the destructive syllogism is designed is to lead us to the conclusion that P_{n-1} is false. However, P_{n-1} is intuitively true. And, through a series of modus tollens, we are led to take P1 as false. The destructive sorites syllogism was thus designed to charge rational language-users back and forth between the two extremes.

It must be said that logicians in the 18th and early part of the 19th century generally did not focus on "destructive syllogisms" of this sort. They seemed to be more interested in how different combinations of syllogisms could be combined or how the

premises were ordered. The term "sorites" in this context became wholly restricted to syllogisms involving more than three propositions, i.e. trains of, heaps, or chains of syllogisms (see De Morgan 1926:143; Hamilton 1874a:366; Veitch 1885:443-7; Fowler 1892:110-2; Devey 1854:138-141). One clear instance of the raucous discussion of the sorites occurred between William Hamilton, in his *Lectures on Metaphysics and Logic*, and John Stuart Mill, in his *An Examination of Sir William Hamilton's Philosophy*, both adopting the general tendency of logicians during this period to treat the sorites as a form of chain syllogism. Hamilton argued that while logicians have readily identified the sorites in the first figure of the syllogism—e.g. All A is B, All B is C, therefore All A is C—"[a]ll logicians have overlooked the Sorites of Second and Third Figures" (1874b:403).[24] But this extension of the sorites syllogism was roughly rebuked by Mill, accusing Hamilton not only of logical ignorance but also of being infected by a "passion which appears to have seized him, in the later years of his life, for finding more and more new discoveries to be made in Syllogistic Logic" (1865:227-8). Mill states that Hamilton was thoroughly confused as to the meaning of "sorites". Mill argued that Hamilton only thought he had proposed two instances of a sorites that have been ignored by logicians. The sorites syllogism in the second figure is No B is A, No C is A, No D is A, No E is A, All F is A, therefore no B, or C, or D, or E, is F (Mill 1865:227). The sorites syllogism in the third figure is A is B, A is C, A is D, A is E, A is F, therefore some B, and C, and D, and E, are F (Mill 1865:227). Mill scoffed that either of these inference chains were a sorites since neither is a "chain argument" since it "does not ascend to a conclusion by a series of steps, each introducing a new premise. It does not deduce one conclusion from a succession of premises, all necessary to its establishment" (1865:227). In any event, the discussion between Mill and Hamilton never seems to make its way toward the sorites paradox.

One notable exception from this time period is Alexander Bain. Bain argued, much like Locke and Leibniz before him, that the sorites (or what he termed the "sophisma polyzeteseos") posed a particular difficulty for definitions of terms insofar as they often did not sharply define their extensions (Bain 1874:717). On the one hand, there was no great difficulty in listing a number of sufficient representatives that fell into the extension of "solid". He writes that quite a number of objects are "compatible with the

name—metals, rocks, woods, bones, and all the products of vegetable and animal life denominated solid" (1874:390). Yet, Bain noted that when we define "solid" as the "resistance to force" we are met with the difficulty that the extension "shade[s] insensibly into the state called 'liquid,' where solidity terminates" (Bain 1874:390). And just like the Greeks before him, Bain queried about the limits of the extensions of terms: "Now, at what point does solidity end, and the opposite state begin? Is a paste, a glue, a jelly, solid or not? Is Hamlet right in talking of 'this too, too solid *flesh*?'" (1874:1890). And the scope of the sorites problem was, for Bain, relatively ubiquitous. He writes that not only did it infect the transition from night to day, sleeping and waking, and small heap to large heap, but also to definitions for natural entities in the sciences. He writes that "there has always been a doubt as to the exact individual that ends the animal series, and is neighbour to the beginning of the plant series" (1874:390). Even further, he writes that the "great chemical sub-division into metals and non-metals has an ambiguous border in the substances arsenic and tellurium" (1874:390).

As for a solution to the paradox, Bain argued that there could be only one. Rather than make a determination about the precise borderline between extension and anti-extension of a vague term, Bain argued that we must simply agree to disagree about its status. He writes that a "certain *margin* must be allowed as *indetermined*, and as open to difference of opinion" (1874:390-1). And that cases that fell into this "margin of transition" ought not to be classified into one extension or the other. In short, borderline cases fell into a no-man's land of neither A nor ~A. But, for Bain, this margin must not be allowed to corrupt the intelligibility of the term (or its opposite). The sorites ought neither to lead us into semantic or ontological nihilism. The status of the borderline cases between metal and non-metal should not undermine the fact that certain metals fit into the extension of "solid" for they clearly do have the quality of resisting force and that certain non-metals do not fit into the extension of "solid" for the lack of such quality. In other words, the nihilist fails to seize the day because the existence of the borderline case—while making the exact point of the transition between any two contrasting terms difficult or impossible to identify—did not undermine the "radical contrast" between the poles. That is, despite the existence of the borderline cases, the radical difference between 'tall' and 'not tall' was sufficient for substantiating the reality of both.

7. Conclusion

From the ancient period up until the middle of the 19ᵗʰ century, the sorites was employed in diverse contexts, principally as a troubling paradox and as a type of chain syllogism. Certainly no confusion ought to exist between the *sorites paradox* and a *sorites syllogism*. The two items were kept distinct throughout the history of philosophy even when a connection between the two was tentatively made.[25] However, further steps were required to solidify the bond between vagueness and the sorites paradox. Increased attention to the formal structure of logic and language allowed for logicians and philosophers of language (such as Frege, Peirce, and Russell) to rigorously distinguish vagueness from other forms of indeterminacy, specifically ambiguity, inexactness, and generality. The historical analysis of these figures is not presented here, although chapter 5 treats Peirce's analysis of vagueness.

Secondly, from the above sketch of the sorites paradox, it should be clear that the sorites has had a long-standing presence in philosophical, logical, and linguistic contexts. In the skeptical context it has served to circumscribe knowledge of what vague terms mean, the reality of universals, and the applicability of syllogistic or formal procedures to natural language. Stoics and others argued that knowledge of the semantic boundary of vague terms was unknowable for certain terms. For Locke this further implied that there is no such thing as a real essence for general terms, and so all terms denoting species were rooted in the arbitrary line-drawing of the human understanding. In logical and linguistic contexts, the sorites was employed to resist formal analysis altogether. The vagueness and sorites-susceptibility of terms was thought to undermine syllogistic reasoning for rational arguments cannot, by themselves, guarantee that terms are treated in an univocal fashion. Since vague terms resisted univocal or absolute treatment, the paradoxical nature of vagueness served as a potential threat to the scope of formal logic.

The sorites is a strange monster, howling near the limits of any seemingly determinate concept. In the following two chapters, I present two contemporary theories of vagueness that claim to have chained the beast so as to keep vagueness free from paradox.

Chapter 3

The Epistemic Theory of Vagueness

> Vagueness issues from our limited powers of conceptual discrimination.
> —Timothy Williamson, *Vagueness* 1994, p.237

0. Introduction

A theory of vagueness aims to explain both the source of vagueness and free it from paradox. One contemporary theory of vagueness is the epistemic theory. It contends that propositions involving vague terms require greater conceptual discrimination than language users have available. This lack of conceptual discrimination results in an inability to know the sharp semantic boundary-line between truth and falsity, i.e. an inability to know whether 'John is tall' is true when John is a borderline case of tall. The epistemic theory solves the sorites paradox by arguing for the existence of a sharp semantic boundary. Intensionally, there is some man who is tall, but the loss of one micron of height would make him not tall. This renders both versions of the sorites paradox mentioned in chapter 1 valid but unsound since—in the case of the many modus ponens sorites—there is a false conditional.

The epistemic theory and its solution to the sorites can be summarized in two thesis: (1) epistemic deficiency and (2) semantic optimism.[26] The latter contends that vague predicates determine "sharp" semantic extensions. In the context of the epistemic theory, this is explained as a commitment to the thesis that bivalence holds for propositions with vague predicates. The former thesis contends that our inability to know this sharp boundary-line accounts for the source of vagueness. If we did have exact knowledge of the boundary-line, vagueness would be eradicated.

Section 1 offers a brief history of the epistemic theory. Sections 2 and 3 lay out the basic tenets of the epistemic view, which are semantic optimism and vagueness as ignorance. Section 4 and its subsections address the epistemic theory's most problematic and controversial thesis, namely its commitment to the existence of sharp semantic boundaries (semantic optimism). The epistemic theory argues that our understanding of omniscient language usage, properties in nature, or our own language usage demands commitments to such boundaries, but I argue that reasons for this are question-begging.

1. The History of the Epistemic View of Vagueness

The early history of the epistemic approach is found in Stoic responses to the sorites paradox (Bobzien 2002:447; Williamson 1994:22-7). As a contemporary theory of vagueness, it was revitalized by Cargile (1969), Campbell (1974), Sorenson (1988; 2001), and most notably by Williamson (1994; 1997a; 1997b). With the rise and development of the theory, there have been a number of disputes, modifications, and divergences. In presenting and criticizing the epistemic view, I confine the majority of my discussion—as others have done—to the *locus classicus*, namely Timothy Williamson's *Vagueness* (1994) and related articles that articulate and object to his view.[27]

2. Semantic Optimism

One benefit of espousing semantic optimism is that it renders the logic of vagueness classical in both semantics and logic. Williamson contends that "properly understood as an epistemic phenomenon, vagueness provides no motive for revising classical semantics or logic, and in particular no motive for denying bivalence" (1994:186). One of the more troubling consequences of this account for vagueness is how to control various counter-intuitive results associated with non-classical logics, e.g. the denial of the universality of the principle of non-contradiction in paraconsistent logics or the denial of the principle of excluded middle in multi-valued logics.[28] However, adherence to classical semantics for propositions with vague predicates comes with its own price. The cost is the implausibility of postulating sharp-boundaries on the truth conditions of vague terms.

According to Williamson and others, commitment to bivalence entails the espousal of the least number principle. This principle contends that every non-empty set of non-negative integers has a least member. Applied to a semantics of vague terms, it can be restated as follows: every non-empty extension of a vague term has a least member. For example, take the semantic extension of the proposition 'Some man is bald'. According to the least number principle, there is some least member n where the addition of a hair would render n no longer part of the semantic extension. In a semantics that is bivalent, this would involve a change in an extension from truth to falsity. This is implausible because the least-number principle for a bivalent semantics entails that a vague predicate that previously applied to an object can fail to apply in the case of a

minute change to the object, e.g. gaining one hair, removing one grain of sand, etc. Further, this also affects the application of a vague predicate modified by adjectives like "clearly" and "determinately".[29] For example, for some "clearly tall" object, upon the loss of one micron, the new object is not "clearly tall" but a borderline case of tall. A "sharp" semantic change occurs with respect to the truthful predication of the vague term despite there being no noticeable change in the language-user's use, no sharp determinant in physical theory, and no substantial phenomenal difference between the clearly tall object and one that is not clearly tall. Thus, adopting the least-number principle for the semantics of vague terms entails the implausible consequence of a sharp semantics without a sharp determinant.

Commitment to the semantic optimism thesis comes with an advantage. Since semantic optimism eradicates vagueness by supposing that there is a complete and corresponding set of truth-valuations, sharply divided, for all vague statements, it follows that the sorites paradox can be solved in one stroke. Namely, classical semantics entails the consequence of accepting the existentially quantified (minor) premise of the sorites paradox (see 3 below). The semantic optimism of the epistemic view (and the solution to the sorites paradox) can be expressed by the following theses:

(1) A man without a single hair is paradigmatically bald.
(2) A man with a full head of hair is paradigmatically not bald.
(3) There is some man that the addition of an additional hair would turn him from a bald man into a non-bald man, i.e. $(\exists k)[P(k)\&{\sim}P(k+1)]$.
(4) For any man n with less hairs than k, n is bald $(\forall n<k)(Pn)$, and for any man n with more hairs than k, n is not bald $(\forall n>k)({\sim}P(n))$.

Thesis (3) ensures that there is some number of hairs k that the addition of a hair will turn a bald man into a man who is not bald. In short, the epistemic theory's commitment to it makes some conditional in the many modus ponens sorites false, and falsifies the universally quantified premise.

3. Vagueness as Ignorance and Inexact Knowledge

With a straightforward solution to the sorites paradox, the epistemic theory ought to further clarify the source of vagueness and *why* we are committed to sharp extensions, or, to put this differently, *what* makes extensions sharp.

The source of the indeterminacy is simply this: vagueness is a form of ignorance. In short, vagueness is a form of *ignorance*. If there is a sharp boundary between the extension and anti-extension of a proposition involving a vague predicate, then vagueness stems from our inability to know this sharp boundary. For example, vagueness is our inability to know where to draw the line when faced with various instances of 'X is tall' when various borderline cases are substituted for X. (see Williamson 1996d:39). The specific form of ignorance is described as a conceptually-based epistemological inability that is manifested in human agents as the expressed indecision about whether a given member is truly or falsely included in the extension or anti-extension of a term. So, contra semanticist theories that contend that vagueness emerges from different truth conditions that are found in language itself, the epistemic theory contends that vagueness emerges from our inability *to know*, as Williamson puts it, "whether vague terms apply in borderline cases" (Williamson 1994:216-47; 1996d:39). In short, any given proposition with vague terms is true or false (not both) but due to vagueness knowing whether it is true or false is humanly impossible.

This specific form of ignorance is articulated by Williamson via his "margin for error principle". For pedagogical reasons, this principle is typically illustrated through an example that has nothing to do with vagueness and then applied to vagueness. Imagine you are scanning a large crowd at a major athletic event with the intent of calculating the total number of people in the stadium. The epistemicist contends that the following two statements are undeniable:

> "for no number m do I know that there are exactly m people" nor do "I know that there [are] not, say, exactly $m – 1$" or $m + 1$ people (Williamson 1994:217,218).

That is, when trying to determine the exact number of people in a crowd, the number of grains of sand in a sandbox, or the number of hairs on a woman's head, we cannot—with through causal observation—know with precision because there exist cases similar to our guess that are not equally justifiable. That is, we are not justified, in having confidence in—nor claiming knowledge of—the *exact* number of people in the stadium—even though we are justified in postulating a range—for competing guesses are equally good candidates.

Williamson renders these two postulates in a general principle, called the "margin for error" principle:

M_w: "a principle of the form: 'A' is true in all cases similar to cases in which 'It is known that A' is true" (1994:227).

The motivation for adopting a margin-for-error principle emerges out of the more general epistemological principle that knowledge shouldn't reduce to luck; or, alternatively, that if knowledge is justified true belief, then it should be reliable. Sainsbury terms this epistemological principle the "Reliability Conditional", namely "[i]f you know, you couldn't easily have been wrong" (1997:907).[30] Essentially, if I hold the true belief p, but am unable to discriminate it from similar false cases $p + 1$ or $p - 1$ in some relevant respect, then my belief was produced by a method that could have easily produced a different and false result, and therefore should not count as knowledge. For example, I hold the true belief that there are more than 1,000, but less than 100,000 spectators. When applying the margin-for-error principle, this statement counts as knowledge, because "there are more than 1,000, but less than 100,000 spectators" is true in all similar cases, e.g. "there are more than 1,001, but less than 99,999 spectators." If, however, I am pressed to be more exact in the number of people in attendance, and I correctly guess that there are 83,343 spectators, my method for determining the exact number of people could easily have rendered 83,344 or 83,342. That is, I could have been easily wrong. Both of 83,344 and 83,342 are false beliefs and both are so closely related to the correct answer—given my method—such that I could have easily chosen either of those two rather than the correct answer. Thus, according to the margin-for-error principle, my true belief concerning the precise number of spectators cannot count as knowledge.

The margin-for-error principle is not a broadly negative semantic principle in that it does not reduce statements about the number of people in a crowd to gibberish, nor does it preclude any hopeful optimism about (or statistical asymmetry in) guessing right more than not. Instead, it is a principle about what fails to constitute knowledge. Stemming from it, neither inexact knowledge nor luck are forbidden. There are, indeed, certain limitations to our perceptual or information-gathering abilities that restrict our capacity to accurately count the number of people in a large crowd. But, on the other hand, the epistemicist will argue that while we do not know the exact number of people

present, we do acquire information of the *inexact* stripe about the number of people in the crowd. While I do not know that there are exactly m or $m + 1$ or $m - 1$ people, I do know that there is more than one person and less than a million people in the crowd.

The margin-for-error principle and the above example concerning knowledge of the number of people in the crowd is meant to apply analogously to our knowledge of propositions involving vague predicates. While we do not have exact knowledge over the sharp boundaries between truth and falsity for propositions that have vague predicates, we do have *inexact* knowledge of the semantics of propositions involving vague predicates.[31] That is, we know that 'John is tall' is true when John is eight feet tall, and we know that 'John is tall' is false when John is two feet tall. But, as for the borderline cases, no exact knowledge can be had.

The example of the crowd characterizes vagueness as a perceptual and computational limitation that is capable of being overcome through adopting a more rigorous method than casually viewing a crowd. There are, no doubt, quite a few obstacles toward justifiably pinning down the exact number of attendees. We may not be able to identify the number of people in a large crowd because our visual apparatus has limited sensitivity, e.g. some people may not be counted because they are too far away, and we are unable to discriminate one hollering fan from another, or also, certain people may be hidden from view because they are obscured by other spectators. In addition to perceptual limitations, there may also be a failure to count individuals because we cannot keep track of who has been counted and who hasn't. Or, there may simply be too many people to count in the time-frame we are given. But this type of indeterminacy is not vagueness. Although, the "roughness" of vagueness is analogous to a variety of limitations and problems with information gathering, the real epistemic block to recognizing the transition from heaphood to non-heaphood is not perceptual nor computational, but *conceptual* (Williamson 1994:237; 1997a:220; 2000:76). Williamson (1997a) illustrates this by imagining a heap of twenty large stones (S_{20}). With great patience, a series of stages begins, each marked by the removal of a stone (S_{19} then S_{18} then S_{17} to S_0). While each stage can be perceptually discriminated from the next, it still remains indeterminate as to the precise cut-off for 'heap'. While I may not be able to identify the number of people in a large crowd or the number of hairs on a passing

individual's head, I am easily able to distinguish S_{20} from S_{19}, S_{19} from S_{18}, and distinguishing S_{20} from S_{18} proves even less of a difficulty. But despite each stage being distinguishable from the next, determining the exact number of stones for a heap remains a conceptual task.

A further illustration that the epistemic theory is addressing a purely conceptual phenomenon is that the epistemic theory will not countenance our inability to judge what a vague term means based upon a *lack of information about the thing* in question. The term 'tall' is not vague because we do not have enough information about the exact height of a collection of basketball players we are calling 'tall'. As far as vagueness is concerned, we can suppose an abundance of data about the object and still fail to accurately judge borderline cases correctly. In other words, vagueness is not an issue of substantial indeterminacy. Take for instance the exact measurements of ten billion men, all different heights, ranging from five feet to ten feet tall. The struggle to parse out who is tall and who isn't—or who might be neither or both—remains despite having, at our conceptual hands, an abundance of precise data concerning numerous individuals. As Sainsbury writes "[y]ou may know how tall someone is to the millimetre, yet be unable to say whether or not he is tall. You may see a shade under perfect conditions for assessing its colour, yet be unable to say whether or not it is red" (1995:590). Further, Keefe (2000b:67) notes that this is part of the reason why the margin-for-error principle is not formulated in terms of *a minute change in the object*. That is, the type of margin-for-error principle employed is *not* of the form:

> M_k: If A is a borderline case for 'tall', and you believe that 'A is tall'; this cannot count as knowledge since if A were marginally shorter, you would have formed the same, although this time false, conclusion.

Instead, Williamson's margin-for-error principle is articulated on the basis of the manner in which individuals make *judgments* about the *truth values* of propositions with vague predicates. What matters for the epistemicist's commitment to margin-for-error principles is that the judgment that a certain man is 'tall' does not count as knowledge when a slight (and perhaps undetectable) deviation in the *truth* of the term's application would have resulted in a false judgment. This differs from undetected alterations in the object.

I take the epistemic theory to have offered a thoughtful and convincing articulation for *why* human beings, with limited conceptual powers, are incapable of knowing the sharp divide between the truth and falsity of propositions involving vague terms. Sainsbury (1995:590) writes that the main idea of (Williamson 1994) is "to use margin for error principles to explain the ignorance postulated by the epistemic theory". Exactly what our ignorance consists of is the inability to know certain facts about borderline cases. Williamson writes "one should not assume without argument that our inability to decide the matter does not depend on ignorance. Of what fact could we be ignorant? There is an obvious answer: we are ignorant either of the fact that TW is thin or the fact that TW is not thin (our ignorance prevents us from knowing which)" (1992b:151-2). However, a second claim must be established in order to make the epistemic theory of vagueness convincing. Namely, that there are sharp semantic boundary lines. So far, the epistemic view has failed to provide a compelling justification for whether *there is* a fact to the matter of whether 'TW is thin' or whether 'TW is not thin'. In short, margin-for-error principles only establish the source of vagueness if and only if there are sharp semantic boundaries. If there are no sharp boundaries, or there is no compelling justification for believing in them, then the epistemic view has failed to establish the source of vagueness. The essential question for the epistemic theory is *what* determines sharp semantic boundaries or *what* reason is there for believing that vague terms have such boundaries? We may be ignorant of the fact of whether TW is thin or TW is not thin, but is there even a fact that we, because of a margin-for-error principle, do not know? If there is, what determines this fact?

4. What Determines Sharp Boundaries?

In the following sections, I investigate claims concerning the determination of sharp truth conditions for vague terms. If the following investigation were to be characterized by a question, it would be: *what* determines that truth values are sharply bounded for propositions involving vague terms? Three answers for the determination of sharp boundaries are as follows: (1) omniscient speakers are capable of knowing them, so they are determined by the beliefs of such omniscient speakers, (2) something in nature determines them, and (3) commitment to rational language usage determines them. I

argue that none of these are convincing reasons for justifying a commitment to sharp boundaries between truth-values.

4.1. Omniscient Speakers

If vagueness is an epistemic phenomenon, one involving a limitation on our conceptual powers, then it seems that an omniscient speaker would be able to identify the sharp cut-off and put borderline cases in their proper place. If it is the case that vagueness creates an unavoidable stumbling block for human agents, then a superhuman being might be able to place these indeterminate patches of red, conglomerates of sand, and borderline cases of tall men in their proper place. However, if vagueness is not an epistemic phenomenon, if it is not a matter of knowing, then even an omniscient speaker would not be able to identify the cut-off point since there is no sharp boundary to be known. One way of deciding which is the case is through a thought-experiment involving a series of questions to a group of omniscient speakers. If this group of omniscient speakers employ vague terms in a consistent manner across the continuum—or are all able to point to that one single exact member that the additional of a hair, grain, or micron of growth would result in the change of extension—then vagueness is, in fact, an epistemic phenomenon. However, if the omniscient speakers produced different results as to the extension of "heap" or "bald man" then vagueness should not be characterized by sharp boundaries and therefore should not be characterized epistemically.

Williamson contends that "[i]n some cases it is mandatory to apply the term 'heap'; in others it is permissible but not mandatory. In the latter cases, some but not all omniscient speakers answer 'Yes'" (1994:200). The idea is that when omniscient speakers are allowed to use their own discretion, there will be a range of answers as to how the extension of a vague predicate should be determined. This alone would undermine the epistemic theorist's contention that there are hidden boundaries since disagreement among the omniscient speakers would seem to imply either there is simply no sharp cut-off to be known or that the sharp cut-off is unknowable even to an omniscient speaker. But Williamson thinks that evidence for the hidden boundary of a vague term emerges when omniscient speakers are directed in how they ought to use their discretion:

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For example, you can instruct them to use it conservatively, so that they answer 'Yes' to as few questions as is permissible. They will still answer 'Yes' to the first few questions, for the same reason as before: at that stage, it is mandatory to apply the term 'heap'. Now if two omniscient speakers stop answering 'Yes' at different points, both having been instructed to be conservative, the one who stops later has disobeyed your instructions, for the actions of the other show that the former could have used her discretion to answer 'Yes' to fewer questions than she actually did (Williamson 1994:200).

It is Williamson's contention that when the omniscient speakers are directed to be conservative, they will all stop at the same point, and this point will be the hidden boundary line that is unavailable to epistemically-limited language users.

Gómez-Torrente (1997:238-240) contends that Williamson's reasoning is fallacious: "it relies on the question-begging assumption that there is a sharp boundary between being conservative and not being conservative." Gómez-Torrente argues that if there were no sharp boundary line between "conservative" and "non-conservative" then omniscient speakers might disagree over where to draw the line. Further directions as to the discretionary application of the term do not matter. An instruction of "be conservative in your conservativeness" might prompt the omniscient speakers to be less likely to include permissible cases, but without assuming that all omniscient speakers interpret "conservative" in a precise (non-vague) way, we cannot assume that the omniscient speakers would reach the same conclusion about the extension of the term. Our standard understanding of the term "conservative" is vague, but Williamson stipulates a precise definition that precludes its vagueness.

Williamson's reply to Gómez-Torrente runs as follows:

But to know the linguistic meaning of an expression is not to set its boundary on a particular occasion; a sentence with a given vague meaning can express different propositions on different occasions. The omniscient speakers certainly know the linguistic meaning of the sentence 'Use any discretion you have to minimize the number of times you answer "Yes"'. They still have to set boundaries to 'discretion', but if 'they may set the boundaries at many different points, within some limits', then to do so in a way which would not minimize the number of times they answer 'Yes' would clearly violate the instruction, given the linguistic meaning of the sentence by which it was expressed (1997c:259).

I contend that this reply is not adequate. Omniscient speakers are directed to use their discretion conservatively, and to understand "conservative" as minimizing the number of times they answer "Yes". But Williamson discredits any omniscient speaker that doesn't interpret "conservative" in the most conservative way. "Conservative" is understood as "most conservative", and the understanding of "most conservative" precludes vagueness when it occurs in a group. This is clear from two of Williamson's statements:

> (1) "if 'they [omniscient speakers] may set the boundaries at many different points, within some limits', then to do so in a way which would not minimize the number of times they answer 'Yes' would clearly violate the instruction" (1997c:259).

> (2) "Now if two omniscient speakers stop answering 'Yes' at different points, both having been instructed to be conservative, the one who stops later has disobeyed your instructions" (1994:200).

No agreement concerning the borderline cases has been achieved, for Williamson establishes the agreement about what qualifies as conservative discretion by defining "conservative" sharply. That is, by precluding the possibility of disagreement through charging any omniscient speaker with linguistic incompetency for failing to be as conservative as his most conservative peer.

In addition, instructing omniscient speakers in this manner is to reduce agreement about borderline cases to agreement about paradigmatic cases. For example, I hire four omniscient painters to paint four rooms. Their payment for service will be rendered if they are conservative in their choice of red. Most likely, each painter will be paid since each will choose a qualitatively identical instance of red. But this is not the type of agreement that establishes the existence of a sharp boundary-line among borderline cases. In order to establish the existence of a sharp boundary, there needs to be agreement among omniscient painters about what counts as the boundary line among borderline cases. Returning to the example, to the four omniscient painters I say 'I'd like you to repaint the rooms red but not a prototypical or paradigmatic instance of red. I would like it red but not too red. Also, be conservative in your color selection. I don't want pink or fuchsia.' *Per hypothesis*, no omniscient painter can be wrong in their color selection. But it does not follow that any of the painters are under command to treat "conservative" as

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"most conservative". In order to establish the existence of a sharp boundary-line, what is required is the question-begging assumption that a determination by an omniscient speaker is conservative if and only if it is the most conservative. However, this is the same assumption that there exists a sharp boundary-line between truth and falsity for the proposition 'John is tall' where John is a borderline case of tall. And so, Williamson begs the question.

While Williamson contends that his "omniscient speakers argument" is not "a knockdown refutation of the non-epistemic view", he does think that it carries weight against those that contend vague terms do not have sharp hidden boundaries (1997c:259).[32] It is my position that the initial intuition found in Gómez-Torrente article is effective against the omniscient-speakers argument for the epistemic view articulated by Williamson, and retains its bite despite Williamson's reply.

4.2. Nature

Another option for the epistemic view is to contend that some property in nature fixes sharp boundaries. For example, examining a beaker of H_2O, a language-user says 'the liquid in this beaker is H_2O.' The statement is true or false depending upon whether or not the molecular constitution of the liquid is, in fact, H_2O. Likewise, whenever someone utters 'John is tall' or 'Frank is bald', the predications 'is tall' and 'is bald' map onto physical properties that precisely determine whether John is tall and whether Frank is bald.

The analogy between natural kind terms and vague terms is a false one for no properties can be assigned to vague terms like 'red', 'tall', or 'heap' such that their use in predication would fix truth extensions in a sharp fashion. Goldstein (1988:447) writes, "while we know from chemistry that a water molecule minus one atom is not water, *chemistry* cannot tell us whether Jones minus one atom is Jones, or is a person."[33]

One reason nature does not fix sharp semantic boundaries for vague terms is because there seems to be no single property or complex of properties that would fix a sharp boundary and that vague terms could latch onto. Take a monadic, linear predicate, such as 'tall'. There are a variety of different heights that 'tall' could latch onto but it does not appear to latch onto any one of these heights more than another. Assume for a moment that 'tall' latches onto the natural property of 'being over six feet'. When Mark

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says 'John is tall' and John is less than six feet, Mark has said something false. But why does 'tall' correspond to 'over six feet' rather than some other height determinant, such as 'over 5'11' or 'over 5'10'? No natural explanation can be given. There thus appears to be no non-question-begging reason why 'tall' latches onto one given property (over six feet tall) over another (over 5'11), or why the predicate should correspond to *one* physical property over an indeterminate set. Thus, the claim that there are natural properties that map onto vague terms and these natural properties determine a sharp semantics for vagueness cannot be justified.

4.3. The Argument from Meaning and Use: Williamson's Master Argument

Williamson has a third reason for retaining bivalence. This relies on the relationship between the extension of a predicate and its *use*. That is, whenever we utter a declarative sentence asserting something to be the case, we are committed to sharply-defined truth-conditions even when the predicates in the proposition are vague.[34] Assuming use determines meaning, Williamson claims that rational use of language commits us to certain uncontroversial forms of inference and minimal commitments to the notion of truth and falsity.[35] These, in turn, commit us to a belief in the principle of bivalence and require the postulation of sharp semantic boundaries. The argument runs as follows:

1	*u* says that P	Speech Act: an utterance says P is the case
2	Not: either *u* is true or *u* is false	Rejection of Bivalence
3	*u* is true if and only if *P*	T-schema
4	*u* is false if and only if not *P*	F-schema
5	Not: either *P* or not *P*	(3) and (4) Substitution of Equivalents into (2)
6	Not *P* and not not *P*	5, De Morgan's Laws[36]

According to Williamson, any attempt to deny bivalence leads to absurdity. That is, bivalence cannot be denied if truth and falsity are defined as they are in (3) and (4), since "the supposition of a counterexample to bivalence leads to a contradiction" (1994:189). Williamson states that the formulation of the truth and falsity conditions in (3) and (4) are used to link the denial of bivalence in (2) to a denial of the law of excluded middle in (5). Furthermore, Williamson contends that the inference from (2), (3), and (4) to (5) should not be controversial when the biconditionals in (3) and (4) are read as equating their two sides in semantic value (1994:189; see also 1992:146). In other words, when the truth

conditions of statements are articulated in terms of (3) and (4), the principle of bivalence and the law of excluded middle are equivalent.

The argument is subject to criticism on three distinct points. The first is whether (3) and (4) are persuasive in equating (2) and (5), and therefore most of the weight of the argument falls upon whether a plausible semantics can be proposed that denies or modifies (3) and (4). I argue that there is nothing inherently wrong with (3) and (4). A second is that the metalanguage rejection of bivalence is not equivalent to the object language rejection of excluded middle. Since (2) and (5) are not equivalent because u is some third value or neither true nor false or undefined, Williamson begs the question against a semantic theory that denies bivalence.

4.3.1 Vagueness and the T-schema

There is nothing inherently problematic about Williamson's use of the T-schema in (3) and (4). Thus, I have no issue with Williamson's argument from (1)–(4). The use of the T-schema and F-schema would be an adequate formulation of truth and falsity if the semantics of vagueness turned out to be bivalent in the classical sense. Tarski's (1944) T-schema is well-motivated in its employment for the construction of a semantic definition of truth and its ability to avoid the liar paradox by rejecting that exactly specified languages must be semantically closed. All of this is acceptable, provided (a) Tarski's semantic *definition* of truth and falsity are not confused with the function of the T-schema or (b) the T-schema and corresponding F-schema are not treated as exhausting the semantics for a non-bivalent formalized language. Assuming (a) or (b) would presuppose bivalence. Tarski famously proposed a semantic definition of truth and falsity when he wrote

> It turns out that for a sentence only two cases are possible: a sentence is either satisfied by all objects, or by no objects. Hence we arrive at a definition of truth and falsehood simply by saying that *a sentence is true if it is satisfied by all objects, and false otherwise* (1944:353).

The notion of satisfaction as a semantic concept is problematic insofar as it is unclear how objects satisfy sentences. One suggestion Tarski had was that "we might try to define it by saying that given objects satisfy a given function if the latter becomes a true sentence when we replace in it free variables by names of given objects" (1944:353). For

example, *snow* satisfies the sentential function "*x* is white". But Tarski remarked that "this method was not available to us" and instead proposed the following:

> We indicate which objects satisfy the simplest sentential functions; and then we state the conditions under which given objects satisfy a compound function—assuming that we know which objects satisfy the simpler functions from which the compound one has been constructed. Thus, for instance, we say that given numbers satisfy the logical disjunction "*x is greater than y or x is equal to y*" if they satisfy at least one of the function "*x is greater than y*" or "*x is equal to y*" (1944:353).

What is left undefined in the above is how an object is capable of satisfying atomic propositional function. Instead, what is defined by Tarski is how compound propositional functions are satisfied in terms of these propositional functions. Thus, the notion of satisfaction of a compound sentence is achieved by (a) ignoring the question about how atomic propositions are satisfied and (b) assuming that compound propositions are satisfied based upon a classical interpretation of logical connectives. Both (a) and (b) are problematic because in the former case, there is no way of knowing whether the T-schema or F-schema is adequate for the satisfaction of an atomic proposition. The latter case is problematic because the tacit assumption is that compounds involving connectives and the logic of these connectives are to be interpreted classically. Tarksi sometimes indicates that this involves adopting bivalence as true, i.e. "a sentence is either satisfied by all objects, or by no objects" (1944:353).

But even if the only reasonable semantics for classical logic is bivalent, we are not compelled to accept classical logic as a logic of vagueness. Arguably there are cases where bivalence does not apply and this may be one of them. This is not a reason for arguing against the T-schema and F-schema for they quite minimally provide conditions for a sentence being true and false. They do, however, only provide the semantic conditions for a bivalent semantics. They do not specify how a sentence might fail to be neither true nor false. Therefore, we might propose a corresponding schema, called the N-schema, which provides the conditions for a sentence being neither true nor false. This would read something like the following: "Snow is white" is neither true nor false if and only if snow is neither white nor not white. Returning to Williamson's argument, in (1) where *u* says that P, the truth conditions for P in the object language are not exhausted by

T and F schemas since some value ought to be employed to specified if T and F are not exhaustive of semantic value.[37]

4.3.2. Excluded Middle and Bivalence

Perhaps the most conservative objection to Williamson's argument is to note that (5) and (2) are not equivalent. Williamson's argument makes the rejection of the principle of bivalence, which occurs in the metalanguage, equivalent to the rejection of excluded middle, which occurs in the object language. These two are not equivalent since the latter can be true in a case where the former is not. Bivalence is always false for three-valued logics but when P takes some third value, $\sim(P \vee \sim P)$ can be neither true nor false (although never true). That is, the rejection of bivalence in (2) can be true while the rejection of excluded middle in (5) can be neither true nor false, especially whenever u is neither true nor false. Examples of conservative three-valued logics where $\sim(P \vee \sim P)$ expresses a quasi-contradiction are Kleene's Strong 3-valued logic (K^S_3), Łukasiewicz's 3-valued logic (L_3), and Dmitri Bochvar's internal 3-valued logic (B^I_3) (Bochvar 1937; Bergmann 2008:71-85). All three of these logics take the rejection of excluded middle to be a quasi-contradiction (i.e. it can never be false) but none of them involve the claim that every proposition is either true or false. Thus, since the rejection of bivalence in (2) and the rejection of excluded middle in (5) are not equivalent, treating them as if they are equivalent begs the question against a non-bivalent semantics.[38]

5. Conclusion

The principal aim of this chapter was to evaluate Williamson's argument for the epistemic theory, and the principal result was that Williamson has not put forward convincing reasons for belief in sharp semantic boundaries. While the use of the T-schema is uncontroversial, the argument begs the question by (1) assuming that omniscient speakers use vague terms sharply, (2) arguing that nature might determine sharp semantic boundaries for propositions with vague terms, and (3) making the rejection of excluding middle equivalent to the rejection of bivalence. From the preceding, it seems that the appeal to sharp semantic boundaries to rid vagueness of paradox is unwarranted and question-begging. In the next chapter, I investigate the theory of *supervaluationism*, a theory of vagueness that rejects bivalence and the existence of

sharp semantic boundaries by employing vague metalanguage concepts and a semantics that admits of truth-value gaps.

Chapter 4

Supervaluationism

> 'Truth is super-truth' can be the supervaluationist's slogan.
> —Rosanna Keefe, *Theories of Vagueness* 2000, p.202

0. Supervaluationism: An Introduction

In the previous chapter, the epistemic theory of vagueness was shown to argue for classical semantics and its logic. One such consequence in accepting the principle of bivalence was a commitment to sharp semantic boundaries. That is, there is some number of hairs h such that it is true that a man with h hairs is bald and it is false that a man with $h + 1$ hairs is not bald. On this view, the semantics are *complete*—every proposition is true or false—but our knowledge of whether a proposition is true or false is lacking. However, commitment to the existence of a sharp boundary between truth and falsity was shown to be question-begging since there seems to be nothing in nature, nothing in the mind of God, nor in our use of terms that would determine their existence.

Another option for explaining vagueness is to modify classical semantics so as to eliminate sharp semantic boundaries.[39] One powerful and popular theory is supervaluationism. This is a theory of the logic and semantics of vagueness that has a clear solution to the sorites paradox, lacks sharp semantic boundaries by employing vague concepts in its metalanguage, and captures intuitions about there being no determinate fact to whether a borderline-case of 'red' is red. In what follows, I offer a brief history of the view, explain the basic features of the theory (including how it solves the sorites paradox), and propose a number of objections to the theory. In short, the theory promotes a version of *semantic pessimism*. That is, a proposition involving a vague term is *supertrue* and *superfalse* if and only if *all* of admissible different ways in which the vague term could be made precise come out true or false, respectively. All other cases fall into a truth-value gap, where propositions are neither true nor false. Ultimately, I argue that this theory is unsatisfactory because (1) it assumes that sharp boundaries *do not* exist and (2) the metalanguage notion of *admissibility* in 'admissible precisification' is objectionably indeterminate.

1. The History of the Supervaluational View of Vagueness

There are at least two reasons why the first informal presentation of supervaluationism is found in Mehlberg's 1958 *The Reach of Science*. The first is his general attempt to try and save the syntax of classical logic from deviance. Mehlberg contends that propositions with vague terms constitute a violation of the principle of excluded middle (PEM) in its *metalogical sense* as opposed to its *logical sense* (1958:258). He characterizes the metalogical version of PEM as a form of the principle of bivalence, formulating it as "every statement is either true or false" (1958:258). His proposal was to retain classical syntax but modify its semantics by denying bivalence. The second relevant feature are his claims concerning how a modified semantics is supposed to look and what consequence such a modification has upon other logical notions. For instance, Mehlberg contends that the result of abandoning bivalence involves associating truth with true on all admissible interpretations, that truth-functionality of composite statements (particularly disjunctions) must be abandoned, and that the semantics for vague terms involve statements that are neither true nor false:

> Since a statement with vague terms is true provided it remain true under every admissible interpretation of its terms, a disjunction with vague terms may well happen to be true, even if its members are indeterminate. This will obviously be the case whenever neither member of the disjunction is true under every admissible interpretation, whereas the disjunction itself always remains true, owing to a suitable interrelatedness of its members (1958:259; see 257-8).

While Mehlberg's presentation is the first informal presentation of the theory, the first formal presentation is found in a number of related papers by Bas van Fraassen (1966; 1968; 1969). Van Fraassen saw his own work emerging out free logic analysis, i.e. a logic that addresses singular constant terms that may or may not have existential import but his aim was not to apply the supervaluational approach to vagueness (e.g. Hailperin 1953; Quine 1954; Lambert 1963). Instead, his strategy for treating terms without existential import have been co-opted and developed for a language that admits vague terms (see Dummett 1975; Fine 1975; Kamp 1975; Lewis 1970; Przelecki 1969; 1976). While Fine (1975) is regarded as the *locus classicus* on the topic, this chapter will focus on the more recent and systematic account by Rosanna Keefe in her *Theories of Vagueness* (2000). However, it must be mentioned that a number of alternative strands of

supervaluation exist, most notably that of McGee and McLaughlin (1995; see also Burgess et al. 1987; Hyde 1997; 2008).[40]

2. *Supervaluationism: The Basics*

Supervaluationism aims to take hold of two central intuitions about propositions containing vague predicates: (1) there are a *number of equally admissible ways of making a vague term precise* and (2) *we should consider them all in the determination of the semantics of vagueness.*[41] Consider the following proposition: 'John is tall' (J) where John is a borderline case of tall and stands at exactly 5'11. According to supervaluationism, the predicate 'tall' can be made precise in a number of possible, different, yet admissible ways. Some of these will make J true, while others will make J false. For example, precisifying[42] the predicate 'tall' at exactly six feet will result in $v(J)=F$, while precisifying the predicate 'tall' at five-foot-ten will result in $v(J)=T$. In short, individual precisifications determine classical extensions. While J is true on some ways of making 'tall' precise and false on other ways of making 'tall' precise, supervaluationism contends that the semantics for propositions involving vague terms are *supertruth* (T_{SV}) and *superfalsity* (F_{SV}). These two semantic values result from universally quantifying over the entire set of classical values rendered by the set of admissible precisifications.[43] Thus, in the case where *all* precisifications of a proposition involving a vague predicate come out true, the proposition is *supertrue*, in the case where *all* precisifications of a proposition involving a vague predicate come out false, the proposition is *superfalse*.[44] However, a truth-value gap emerges when a proposition is neither *supertrue* nor *superfalse,* i.e. when some admissible precisifications render a proposition classically true and some admissible precisifications render it false. Propositions falling in the truth-value gap are *neither supertrue nor superfalse.*[45] In sum, while supervaluationism is classical when understood strictly from the standpoint of the valuations of individual precisifications (all of these are bivalent), universally quantifying over the set of these precisifications in order to extract the notion of supertruth for vague statements is non-classical insofar as its admits truth-value gaps (a rejection of bivalence).

A second point that distinguishes supervaluationism from other theories concerns the source of vagueness. In the previous chapter, the epistemic theory argued that

vagueness is *epistemic* since every proposition with a vague term is true or false, and our uncertainty about predications of vague predicates to borderline cases is the result of our inability to know whether the proposition is, in fact, true or false. Other theories account for vagueness in different ways. Some pragmatic theories of vagueness argue that vagueness is *pragmatic*, i.e. it is the result of our indecision about "*which* (precise) language a community uses" or the indeterminate relation between language-users and their choice between a variety of possible precise languages (Keefe 2000b:140; Burns 1986; 1991; Van Kerkhove 2003). A semantic multi-valued theory of vagueness contends that uncertainty about predicating a vague term to a borderline case is the result of a monolithic notion of truth (Edgington 1997). Vagueness results from trying to valuate certain propositions as true or false when, in fact, they are indeterminate or partially true. Supervaluationism's account of vagueness is semantic in the sense of the multi-valued theorist but vagueness results from there being *no definite fact* to the matter about whether a borderline case of tall is tall. That is, when language-users cannot decide whether 'John is tall', it is not because they don't know the fact of the matter, nor because they cannot decide how to make 'tall' precise, nor because the proposition has an intermediate semantic value between truth and falsity. Instead, language-users are indecisive because the proposition 'John is tall' is neither definitely true (supertrue) nor definitely false (superfalse). In other words, there is no fact to the matter about the truth or falsity of 'John is tall' .

A third point about supervaluationism that needs to be grasped for a basic understanding is the absence of sharp boundaries between supertrue, superfalse, and neither supertrue nor superfalse. This demand can be alternatively articulated as a need to accommodate higher-order semantic vagueness. That is, cases where a proposition—e.g. 'John is tall' (J)—is not merely a borderline case of 'tall', but a borderline-borderline case of 'tall', or a borderline-borderline-borderline case of 'tall', and so on. The epistemic theory of vagueness has little problem addressing higher-ordered forms of vagueness since while every proposition is true or false, our knowledge about whether they are true or false is not. So, given a proposition J, it can not only be the case that I do not know the truth-value of J, i.e. $\sim Kv(J)=T$, but also that I do not know that I do not know that $\sim K\sim Kv(J)=T$, and so forth.

Higher-order vagueness is addressed by supervaluationism by an appeal to a vague semantic metalanguage (Keefe 2000b:ch.8). For the supervaluationist, a statement involving a vague term is supertrue if and only if it is true not merely upon any precisification—since a number of ways of making terms precise are absurd (e.g. that a tall basketball player is three feet tall)—but true upon all *admissible* precisifications. The *admissible* nature of a precisification is itself *vague*, and occurs as a part of the supervaluationist's metalanguage. By articulating notion of admissibility as also being vague, supervaluationism is able to ward off objections concerning sharp boundaries for it does not involve quantifying over a determinate set of precisifications. That is, as Keefe puts it, the supervaluationist theory does not quantify over a "precise and unique set of complete and admissible specifications" (2000b:202). The boundary line between admissible and non-admissible specifications is thus not sharp, and—if we are to maintain that supervaluationism is a complete system—ought to be treated according to the supervaluationist approach. In order to determine whether a given set of precisifications are admissible or non-admissible, supervaluationist semantics are reapplied. Consider the sentence 'John is tall'. 'John is tall' is supertrue if and only if it is true on all admissible precisifications, is superfalse if and only if it is false on all admissible precisifications, and is neither supertrue nor superfalse otherwise. But since what counts as an 'admissible precisification' is itself vague, it follows that there may be a borderline case of an admissible precisification that makes 'John is tall' false, while all other admissible precisifications make 'John is tall' true. According to the supervaluationist, we cannot contend that 'John is tall' is *supertrue* because all *non-borderline cases* of admissible precisifications came out true. This would be *ad hoc* insofar as it merely discounts borderline cases of admissible precisifications. If 'John is tall' cannot be treated simply as *supertrue*, another option is to valuate 'John is tall' as *neither true nor false* because 'John is tall' is neither true on all admissible precisifications (even those that are borderline) nor false on all admissible precisifications (especially those that are borderline). In both cases, there is the temptation to make supervaluationism a theory that quantifies over a precise set of admissible precisifications.

Keefe, however, rejects both of these options, contending that supervaluationism "denies the assumption that there is a precise and unique set of complete and admissible specifications" (2000b:202). If the supervaluationist quantifies over a set of admissible precisifications that is not precise, then statements involving borderline cases of admissible precisifications with respect to 'John is tall' are left *unvaluated*. As Keefe puts it:

> For the indeterminacy over whether the truth-condition is fulfilled implies that we should not conclude that *p* is true, but nor should we call *p* neither true nor false as we would do if the condition (determinately) failed to be fulfilled. The truth-status of *p* (whether true, false or lacks a value) remains unsettled. And sharp boundaries between the true predications and the borderline cases of 'tall' are avoided (2000b:203).

The supervaluationist postulates, in its aim to circumvent sharp-boundaries for the extensions of vague statements and to address higher order vagueness, a set of statements that fail to fulfill any of the specified semantic (true/false) or non-semantic (neither true nor false) values.

The fourth, and final, point on the supervaluationist theory pertains to its solution to the sorites paradox. Keefe contends that "[i]n short, supervaluationism solves the sorites paradox in all its forms" (2000b:165). The manner in which this feat is performed involves a different understanding of the truth and falsity of existential and universal statements.

Consider the following existential statement:

($H_\exists$): there is a height *x* such that people of height *x* are tall while people 0.01 inches shorter are not tall.[46]

On the supervaluationist account, ($H_\exists$) is supertrue. The reason is as follows. Consider a single admissible precisification, e.g. substitute 5'11 for *x* in ($H_\exists$). For this substitution instance, there is a height *x* such that people of height *x* are tall while people 0.01 inches shorter are not tall. There is such a height, and, in this case, it is 5'11. Now consider another substitution instance, e.g. 5'8. For this substitution instance, there is a height *x* such that people of height *x* are tall while people 0.01 inches shorter are not tall. There is such a height, and, in this case, it is 5'8. Since admissible precisifications have classical

values, every individual precisification has a sharp semantic boundary. And, so since ($H_\exists$) is *true* on *all* admissible precisifications of ($H_\exists$), ($H_\exists$) is supertrue.

This is extraordinary since ($H_\exists$) is supertrue even though no single substitution instance for x is supertrue. This is a puzzling position, so let us consider a substitution instance for x:

($H_{5'11}$): there is a height 5'11 such that people of height 5'11 are tall while people 0.01 inches shorter are not tall.

On the supervaluationist account, while ($H_{5'11}$) is *true* for at least one precisification of 'tall', it is *false* on a variety of others. One admissible precisification of 'tall' is 'anyone taller than or equal to 6'1'. On this admissible precisification, ($H_{5'11}$) is *false*. Therefore, ($H_{5'11}$) is *neither true nor false*. Thus, supervaluationism claims that with respect to borderline cases, no substitution instance of ($H_\exists$) will be *supertrue* or *superfalse* but ($H_\exists$) is *supertrue*. The reason ($H_\exists$) is supertrue is because ($H_\exists$) comes out as *true* for every admissible precisification that is a substitution instance of it. So, while it is not the case that there is some supertrue substitution instance making ($H_\exists$) supertrue, it is the case that ($H_\exists$) is true for every substitution instance that is an admissible precisification.[47] To rephrase a line from Tappenden (1993:564), in each complete admissible extension there is a different x such that $[P(x) \wedge \sim P(x + .01)]$ is true. To put this same concept in terms of logical connectives rather than quantifiers—and since existential quantifiers are often treated as compound disjunctions—supervaluationism is committed to cases where certain disjunctions are supertrue despite the fact that neither of the disjuncts are supertrue.

This opens a unique avenue for solving the sorites paradox without having to strongly deny the universal generalization that occurs in one of the premises of the sorites paradox. Typically, there is a universally quantified statement found in sorites arguments—that is $(\forall x)F(x)$—and a standard negation of the sorites premise would be $\sim(\forall x)F(x)$. The negation of a universal statement allows for the classically equivalent statement involving an existential quantifier, i.e. $(\exists x)\sim F(x)$. Rather than rejecting the logical duality between the two quantifiers, supervaluationism contends that in both cases—the rejected sorites premise and its logical dual—are true but they are not true for

any substitution instance of them. In order to solve the sorites paradox, let us consider the following articulation, which is the version of the paradox with the quantified premise:

(1) Fx_1
(2) For all i, if Fx_i then Fx_{i-1}
(3) Fx_n

In the classical framework, (3) is thought to follow directly from the intuitively true premises (1) and (2) and also standard ideas about validity. The epistemic theory blocks this argument by rejecting (2) and arguing that some substitution of ($H_\exists$) is true and the next one is false (although we don't know which one). Alternatively, the supervaluationist contends that (1) is supertrue, (3) is superfalse, and (2) is superfalse although none of its substitution instances are false. Additionally, since (2) is superfalse, supervaluationism is committed to the supertruth of the negation of (2). That is, that "there is a height i such that people of height i are tall while people 0.01 inches shorter are not tall", i.e. ($H_\exists$). Rather than involving a commitment to sharp boundaries, (2) is superfalse but it is superfalse despite the fact that none of the substitution instances of it are superfalse. Thus the sorites paradox (in this form) is valid but not sound.

In summary, the first basic point to consider is that in order to formulate an appropriate definition of truth for statements involving vague terms, it is necessary that we appeal to the different ways in which vague terms are made precise, and then quantify over all admissible precisifications. The second is that vagueness is purely semantic and its source is located in their being no determinate fact to the matter about whether borderline cases are tall, heaps, or bald. The third is that supervaluationism is not committed to sharp boundary lines for its semantics and is capable of addressing higher-order vagueness if it contends that the definition of truth employs a vague concept of 'admissible precisification'. Finally, supervaluationism, in its unique definition of truth, is capable of solving the sorites paradox by rejecting the universally-quantified premise as superfalse. This, however, comes with the consequence that certain propositions will be supertrue/superfalse even though no substitution instance makes it supertrue/superfalse.[48]

3. Objections

The basics have been given. Supervaluationism is a viable theory of vagueness insofar as it solves the sorites paradox without postulating a sharp semantics for sentences involving vague terms and it offers an explanation concerning the source of vagueness.[49] A number of objections have been levied at supervaluationism. In the following sections, I present four objections to supervaluationism.

3.1 Supervaluationism and Validity

One objection to supervaluationism concerns whether valid arguments are *truth-preserving* (Montminy 2008). Keefe notes the worry concerning validity as follows:

> if sentences without classical values are admitted (e.g., to accommodate borderline cases), then it looks possible to have an argument whose premises cannot be true while its conclusion is false, but which does not guarantee preservation of truth, since its premises might be true while its conclusion is neither true nor false (Keefe 2000a:93).

The idea is relatively simple. Supervaluationism universally quantifies over classical values rendered by admissible precisifications. An argument involving an uncontroversial rule of inference can be invalid when validity is defined in terms of truth-preservation. This is possible in the case where an argument has premises that are neither supertrue nor superfalse but its conclusion is superfalse. In such a case, the classical truth-values that are quantified over in the premises is not preserved, therefore supervaluational validity is not truth-preserving.

This objection is easily defused since valid supervaluational arguments are supertruth-preserving, not truth-preserving. Keefe (2000a:95) writes that an argument involving vague terms is:

> valid iff necessarily whenever the premises are true (i.e., is true on all those specifications) the conclusion is also true (i.e., is true on all those specifications).[50]

Thus, supervaluationism contends that given that all premises are true on all precisifications, then the conclusion will be true on all precisifications. This objection to supervaluationism is not sufficient since it fails to explain why classical semantic values ought to be preserved in arguments involving vague terms.

3.2. Vagueifiers

A 'vagueifier' is a word that introduces more vagueness into a statement. In English and in other languages, vagueifiers typically take the form of a set of adjectives or suffixes appended to a word, but they can also be qualifying phrases appended to statements (e.g. 'John is tall, *sort of*'). With little difficulty, vagueifiers can be attached to predicates that are already determinate (e.g. 'a perfect geometric circle' to 'roughly a perfect geometric circle') or to predicates that are already vague (e.g. 'red' to 'reddish'). There is seemingly no end as to how vague a predicate can be made since vagueifiers can be reiterated and different vagueifiers can be compounded to the point of absurdity, e.g. 'John is kind of tallish, sort of'. Vagueifiers ('-ish', 'roughly', 'sort of') in propositions often change the semantic value of sentences. For example, 'John is tall' is supertrue when John is a clear-cut case of tall. However, if I say 'John is tallish' when John is a clear-cut case of tall, there is somewhat of a compelling reason to valuate the sentence as neither true nor false since John is a determinate case of tall. The reason for the semantic shift, in the above example, is because the addition of the vagueifier '-ish' is thought to indicate that John is, in fact, a *borderline case* of tall. In that case, 'John is tallish' would only be true if John were a borderline case of tall.

Matti Eklund has objected to supervaluationism's thesis that the multitude of admissible assignments of semantic values "correspond to possible *completions* of the meanings of vague expressions" (2001:363-4). He writes that according "to the supervaluationist analysis, vagueness is a matter of what we have *failed* to lay down. But it appears that the conventions associated with 'roughly' and '-ish' say exactly that constructions with these expressions are supposed to be vague' (Eklund 2001:366). In other words, what the vagueifying function shows is that not all vagueness corresponds to different ways in which that predicate *fails* to be supertrue or superfalse. Instead, vagueifiers, in pointing out their own vagueness, correspond to the fact that propositions with these functions are supposed to be vague. The central problem then, according to Eklund, is that if vagueified statements are necessarily borderline and therefore statements that are necessarily incompletable (neither capable of being made true nor false), then supervaluationism's claim that it is consistent with classical logic is overturned (2001:368). The reason for this is because to retain the inherent

incompletability of the proposition means giving up on the possibility of an acceptable assignment of semantic values that are bivalent (2001:368).[51]

The objection is either the result of misunderstanding the modality of semantic completeability or can be explained away by giving an alternative account of the function of vagueifiers. First, the semantics of a proposition with a vague predicate are formulated by universally quantifying over, not merely the ways in which a vague predicate *fails* to be precise, but the resulting classical truth-values from the admissible ways in which the predicate *could be made precise*. On this account, any vagueified proposition *could* be made precise even though the utterer's aim is that the proposition remain imprecise. Victor says 'John is tallish'. The semantics are completeable provided it is possible for there to be an imprecise set of admissible precisifications of 'tallish'. An example is considered below. Second, I see no reason why vagueifiers cannot be treated much like other vague predicates, except that they tend to require disambiguation. Take an example of a proposition with a vagueified predicate, e.g. 'John is tallish'. The *inclusive* sense of 'tallish' is where all tall objects count as 'tallish'. The more natural sense is the *exclusive* sense where some tall objects are not tall. It is important to note the different senses of 'tallish' because the vagueness of each sense is different. The borderline cases in the *inclusive* sense of 'tallish' is between tallish objects and objects that are not tall. The borderline cases in the *exclusive* sense of 'tallish' are the same as those in the inclusive sense but there are also borderline cases between 'tallish' objects and 'tall' objects.

But once this disambiguation is made, I see no reason why a vagueified term like 'tallish' cannot be made precise. For example, consider three ways of making 'John is tallish' precise (in the exclusive sense).

> (1 tallish) 'John is tallish' if he is not over 6'3 and not under 5'5.
> (2 tallish) 'John is tallish' if he is not over 6'2 and not under 5'6.
> (3 tallish) 'John is tallish' if he is not over 6'1 and not under 5'7.

This expresses a set of precisifications that capture the exclusive sense of 'John is tallish'. The inclusive sense can be represented by dropping the left conjunct in all three cases. Provided the relevant disambiguation is made, the semantic completeness of a vagueified predicate is *possible* because precisifications of vagueifiers are possible. Vagueifiers thus present no real problem for supervaluationism.

3.3. No Truth-Functionality, the Argument from Upper-case Letters

As explained earlier, supervaluationism solves the sorites paradox without appealing to sharp semantic boundaries by arguing that an existentially quantified proposition can be supertrue even though none of its substitution instances are supertrue. The same is the case with disjunction where some disjunctions, e.g. P ∨ ~P, can be supertrue despite neither of the disjuncts being supertrue (see McGee et al. 1995:212). One consequence for supervaluationism is the failure of truth-functionality for compound statements, e.g. if P is neither supertrue nor superfalse, and ~P is neither supertrue nor superfalse, the conjunction of the two gives us the superfalse P · ~P. Likewise, the addition of ~P to the neither supertrue nor superfalse P yields the supertrue P ∨ ~P.

There is then a famous objection, called the 'argument from upper-case letters', to the supervaluationist claim that complex statements involving vague terms can fail to be truth-functional. The 'argument from upper-case letters' is often ascribed to Tappenden (1993:564), but Tappenden references Sanford (1976), and it can be found as early as Mehlberg (1958:258-9; see also Kamp 1981). Tappenden writes:

> Despite the ingenuity of the supervaluational story, truth-value shift does not fit very well with the way we normally understand disjunction and existential quantification. One might describe the objection as the "objection from upper-case letters," since it is hard to resist the temptation to type 'When I say that there is a number *n* such that ... I mean that THERE IS a number *n* such that'. [...] [Likewise,] [w]e are told that '*p* or *q*' is true and we naturally ask, 'Well, which one is it (if not both)?" So, too, if we are told that there was a number with an interesting property *P* we might think to ourselves, "Let's investigate further to find out which particular number it is." (1993:564).

The argument against a non-truth functional semantics for composite statements is not merely that it violates linguistic-intuitions with respect to the compositional structure of complex statements. Instead, the problem with this account, according to Tappenden, is that it conflicts with natural responses to inquiring about the truth of a compound statement. In being told that there is a number *n* with a particular property, inquiry is generally directed or regulated by the belief that there is a determinate—or ultimately specifiable—number *n* that makes this proposition true.

Independent of these intuitions, Tappenden suggests that the intuitions underpinning excluded middle and non-contradiction demand truth-functionality. He

contends that while the principle of non-contradiction involves a "no overlap" condition—whereby the principle of non-contradiction "says that the set of objects of which *P* is true and the set of objects of which *P* fails to be true are to be disjoint"—the law of excluded middle functions as a "sharp boundaries" condition (1993:565). Consequently, when we utter statements like 'John is tall or not tall', we are committed to the view that John is one way or another.

Tappenden's argument, however, about the principle of excluded middle is not convincing since it is not excluded middle that captures the sharp boundaries condition, but excluded middle in coordination with bivalence and strict negation. These commitments capture our intuitions that something must be one way or another, i.e. that a proposition must be T or F. Viewed independently from bivalence, excluded middle captures intuitions about covering all possible semantic possibilities by a complex expression in the object-language. For example, Ladd-Franklin writes that while the principle of non-contradiction involves a *no-overlap* condition or a condition that contends the sentences are exclusive, the law of excluded middle is articulated as a disjunction that might better be defined in terms of *exhaustion* ($\infty < x + \sim x$). Ladd-Franklin contends that terms can be exclusive without being exhaustive, as in the case of 'prime number' and 'even number' (except in the case of 2), while terms can be exhaustive without being exclusive (Ladd-Franklin's example is 'not even' and 'not prime').

The question then is whether the supervaluationist can contend that a complex statement can be both exhaustive and exclusive of semantic possibility yet lack a commitment to a semantics involving sharp boundaries. The supervaluationist contends that it can, and all that is required is that we abandon the truth-functional intuition that would be forced upon us if we were committed to *bivalence* (or sharp semantic conditions) and not to *excluded-middle*.[52] Consider Tappenden's example:

> Say you have the job of sorting color samples on an assembly line. The samples come along the line in varying shades of red or orange. No other colors are sent rolling out. You are to drop the orange samples into the bin and the red ones into another. Every so often an indeterminate case comes along and you cannot make up your mind about it, so you set it aside. The foreman notices the growing pile of samples by your side and says, "Every one of these samples is either red or orange." Among the effects of this

utterance is to set high standards for resolving indeterminacy: indeterminate cases are to be put into one bin or another, so that 'red' and 'orange' function as predicates with sharp boundaries because the contextual standards are such as to get you to make them behave as if they have sharp boundaries (1993:565-6).[53]

The context of the foreman's utterance suggests that (s)he means bivalence and not the excluded middle. There are only two bins and your foreman's utterance is something equivalent to "you better take each of those samples you set aside and drop them into one and only one bin, and no sample should be put aside." If we view the foreman's command as semantically underdetermined, the supervaluationist would contend that there will be a set of samples that do not definitely belong into one bin or another. All excluded-middle does is say something to the effect of "any item that belongs in a bin ought to be put in one and only one bin", but this is not the same as bivalence, which adds the dictum that all samples *do* belong in one and only bin.

The argument for the truth-functionality of composite statements cannot be made on the grounds that our language intuitions dictate statements be truth-functional nor on an appeal to excluded-middle that is viewed as a sharp-boundary condition. The former is unphilosophical insofar as it is grounded in intuitions transferred from non-vague contexts and the latter is bivalence disguised as excluded-middle. McGee and McLaughlin put the same point quite forcefully with respect to the fact that supervaluationism is concerned with the definite truth of complex statements. They write

> The definite truth of a compound sentence need not be compositionally grounded [...] in the definite truth or falsity of its atomic components. A disjunction can be definitely true without any disjunct being definitely true, an existential sentence can be definitely true without any instance being definitely true, and a biconditional can be definitely true without either component being either definitely true or definitely false. This is not to say that a sentence can be definitely true for no reason at all—for a sentence to be definitely true, there must be something we do, think, or say which, together with nonlinguistic facts, makes it true—but the grounding relation is more complicated than simply compound sentences being grounded in atomic facts (1995:216).

The supervaluationist rejection of the truth-functional intuition is, while perhaps implausible when dealing with precise terms, a plausible conjecture when handling vague terms. The logic may seem to be grounded on a necessary equivocation (or truth-value

shift), but it is instead rooted in the notion that the supertruth of vague statements are not obtained atomically, and some composite statements, while capable of covering all semantic possibility, are not capable of being further decomposed.

However, along similar lines as Tappenden, Hyde has objected that what is counter-intuitive in the supervaluationist interpretation of existential quantification is that there is not only an epistemic-block for specifying substitution instances but the "existential claims we are asked to accept […] are ones for which we are not only unable to produce a witness but for which no witness could *in principle* ever be produced" (1994:259, my emphasis). Thus, on the supervaluational theory, it is *impossible* for there to be any supertrue substitution instance for a sorites-solving supertrue existentially quantified proposition. If this is the case, then supervaluationism involves the question-begging assumption that sharp semantic boundaries are *neither actual nor possible*. That is, since it is in principle impossible for a supertrue specification of a sharp-boundary line, it follows that supertruth-value gaps are not merely a contingent part of the semantics of supervaluationism but a necessary feature. The impossibility of specifying a sharp semantic boundary entails that no such boundary could exist in principle. In chapter 3, I argued that the epistemic theory's claim concerning sharp semantic boundaries was question-begging. Here I make the same objection to supervaluationism and claim that similar objections posed in the previous chapter attach to it. For example, although God is not under the logical necessity to use vague words in a precise way, there is no logical necessity preventing a perfect language-user or group of language-users from using vague terms in a precise way. That is, would determine the in-principle impossibility of a sharp boundary specification, especially when language-users quite frequently render their vague terms precise?

3.4. Supertruth as Quantification over Admissible Precisifications

Two key positions of supervaluationism are its treatment of vagueness as a modal phenomenon and its commitment to the view that language usage determines a vague range where precisifications would be located if language determined them. For supervaluationists, "[v]agueness arises because no choice is made between a number of alternative precisifications" yet though "*our* practices do not determine a precise extension to 'tall', they do determine a (vague) range within which the precise extension

would have to be if there were one" (Keefe 2000:25; 153, my emphasis). So while nothing in our use of vague determines that propositions having vague predicates must have sharp semantic boundaries, if there were sharp boundaries they would be located within the vague extension which our language use determines. I take no issue with this feature of the theory, nor with the semantics of individual precisifications (which are classical), but argue that the quantification over these precisifications is objectionable. Without further specification about what makes a precisification 'admissible' rather than 'unadmissible', the theory is incomplete for it gives no account of how the semantics or logic could ever be correctly applied. The theory must take a stance on what would determine an admissible precisification.

The semantics of supervaluationism are obtained by universally quantifying over *admissible* precisifications. This set over which supervaluationism quantifies is an imprecise set due to the vagueness of what counts as 'admissible'. The notion of admissibility in Fine's (1975) work is a primitive notion, one which he unofficially states as being defined in terms of its *appropriateness* for some precisification (1975:272). Williamson (1994:158) defines "admissibility" as "consistency with the semantic rules of the language." He contends that the concept might be associated with reasonableness: "[a]n interpretation is reasonable if it does not license misuses of the language (from the standing of an ordinary understanding of it)" (1994:158). Keefe usually repeats Fine's phrase on the matter or gives examples that appeal to common sense (Keefe 2000b:203; McGee et al. 1995:205-8). While one important feature of supervaluationism is that admissibility is vague, one unhelpful feature is that admissibility is infected with another form of indeterminacy making it too indeterminate for criticism. In what follows, I argue that the notion gives no criterion for distinguishing between what counts as paradigmatically admissible as opposed to paradigmatically non-admissible. I argue that the supervaluationist theory must at least take a stance on the determinant of admissibility.

Consider a set of precisifications of 'tall' in 'John is tall' where the context consists of North American men, and John is exactly 4'11. Assume that Victor, who utters 'John is tall', believes it to be true, is well-aware of his exact height and the average heights of other North American men, and is willing to consistently valuate all

men taller than John as tall. Is Victor's precisification of 'tall' as '4'11 and over' an admissible precisification? Supervaluationism has two options. The *first* is to argue that precisifying 'tall' as '4'11 and over' should not count as an *admissible precisification* since it is contrary to typical usage, practices, conventions, reasonableness, etc. Keefe writes, as an example, that "it is acceptable to make 'tall' precise by drawing a boundary at 6 feet 0 inches but not by drawing one at 5 feet 0 inches" (2000b:203). Thus, on admissible precisifications of 'tall', 'John is tall' is *superfalse*. The second option is to regard the utterer's unconventional precisification of tall as an admissible precisification. If it is so regarded, then it figures in one of the precisifications quantified over, and thus 'John is tall' is *neither true nor false* despite the fact that John is shorter than five feet tall.

In the *first* option, speaker-meaning is severed from the determination of the semantics, and the supervaluationist commitment to language practices determining a vague range where precisifications would be located is partially compromised. If the possible precisifications of the utterer of the proposition involving the predicate are not allowed to count as admissible, then they do not figure among the set that is quantified over in the determination of truth. And if this is the case, the semantics of supervaluationism are determined independently of how vague predicates are used by speakers.

In the *second* option, supervaluationism takes a stance on what counts as an admissible interpretation/precisification by arguing that how a speaker makes a vague predicate precise should count among the admissible precisification. If this is the case, then alternative admissible precisifications from other language-users either do or do not form part of the quantified set. If they do, then we are still left with admitting the utterer's unconventional precisification of 'John is tall', and the truth value of the proposition as *neither true nor false* despite the fact that all other precisifications within the set may valuate the proposition as *false*. Just as it only takes one bad apple to spoil the bunch, it only takes one renegade language-user (even the speaker) to undermine the super in supertruth. If the only relevant precisifications worth considering are the speaker's own, then the semantics for propositions with vague terms will be relativized to individual speakers, much like Leibniz suggested (see chapter 2). However, confining the

admissibility of precisifications to those made by utterers allows for situations where token sentences have divergent truth values. That is, cases of a similar context where it is supertrue that 'John is tall' when utterered by an unconventional language user, superfalse for an individual who uses the term in a more conventional fashion, and neither true nor false for a third member.

To summarize, one problematic feature of supervaluationism concerns the admissibility property of the non-precise set of precisifications that is quantified over. If the notion of admissibility excludes those precisifications made by the utterer of the proposition, then the supervaluationist partially abandons a commitment to language use as the determinant of the vague range in which a precise semantics would be located if there were one. If the notion of admissibility includes precisifications made by the utterer as well as precisifications from other language users, then the theory must reconcile itself with a majority of paradigmatically supertrue or superfalse propositions falling within a truth-value gap. Finally, if the notion of admissibility is restricted only to those possible precisifications that would be made by the utterer, then supervaluationism must explain the implausibility of how different token instances of the same proposition can be supertrue, superfalse, and neither supertrue nor superfalse that 'John is tall' at the same time in the same context. The supervaluationist must take a stance on this issue otherwise the theory is objectionably vague for no account can be given as to how the logic could ever be applied.

4. Conclusion

The rejection of epistemic theory resulted in the need to find a theory of vagueness without a sharply-defined semantic theory. Supervaluationism provides such an account through a semantics that admits of truth-value gaps and where these gaps are formed by quantifying over a non-precise set of classical values. While this theory is resistant to a number of objections ranging from its rejection of truth-functionality to its redefinition of global validity as the preservation of supertruth, there are two problems with it. The first objection is that its account of supertruth and superfalsity yields the implausible scenario that there are disjunctions and existential statements that are supertrue even though it is impossible, in principle, for any disjunct or substitution instance to be supertrue. In short, even though sorites-solving existential propositions are supertrue, it is impossible for any

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substitution to be supertrue. This is tantamount to claiming sharp semantic boundaries are impossible, and such a view can only be substantiated by some question-begging position as to their non-existence. The reasons given above suggest that a semantic theory of vagueness ought to be *weakened* so as to avoid controversial claims about the necessary non-existence or actual existence of sharp semantic boundaries.

Secondly, although supervaluationism argues that the set of admissible precisifications is vague, the vagueness of the set of admissible precisifications does not explain why paradigmatic cases of admissible precisifications are as indeterminate as they are. The notion of admissibility is left so indeterminate (not merely vague) that it is unclear how a supervaluationist logic would ever be applied. The reasons given above suggest that a semantic theory of vagueness ought to be *strengthened* so as to at least provide some hint as to which precisifications are admissible and which are not admissible.

One option for consideration is the theory of vagueness proposed by Charles S. Peirce. In chapter 6, I will argue that a Peirce-inspired theory can be both agnostic about sharp semantic boundaries and take a stand on the determinant of admissibility. However, before giving a defense of his view, it is necessary to correct a widespread misapprehension that Peirce's semantics for vague predicates is only applicable to a form of indeterminacy called uninformativeness or inexactness. Upon showing that Peirce's theory was aimed at accounting for a variety of different forms of determinacy, including vagueness, I conclude in the final chapter with a theory prompted by some of his suggestions on vagueness.

Chapter 5

Peirce and the Specification of Vagueness

> Yet the only thing thoroughly real is the present state of things and that is vague.
> —Charles Sanders Peirce, NEM3:913

0. Introduction

The principal aims of this chapter are to correct a misconception that Peirce, in connecting vagueness to the existential quantifier, did not address the modern phenomenon of vagueness but rather conflated it with a number of other forms of indeterminacy. This chapter shows that Peirce understood borderline vagueness while the next chapter sketches an answer to the sorites paradox along Peircean lines. Randall Dipert has written that "the nature of Peirce's greatest contribution to thought [is] his logic and his ideas about the nature of logic and its relation to the world and to the mental" (2004:318). In addition to this, he adds that Peirce scholars have yet to grasp the leading ideas of Peirce's logic sufficiently well (2004:318). This is perhaps the case with Peirce's logic of vagueness.[54] In what follows I argue: (a) that Peirce claimed that there could be a logic and semantics for vagueness, (b) that vagueness qua borderline cases was understood by Peirce as objective indefiniteness in essential depth, and (c) that Peirce's notion of vagueness is not reducible—as has been claimed—to unspecificity or to uninformativeness.

1. Peirce had a logic of vagueness

That Peirce thought there could be a logic of vagueness is clear from his well-known and often-cited claim that "[l]ogicians have been at fault in giving Vagueness the go-by, so far as not even to analyze it. The present writer has done his best to work out the Stechiology (or Stoicheiology), Critic, and Methodeutic of the subject" (EP2:350). But knowing *that* Peirce's had a logic of vagueness and not knowing *what* his logic of vagueness is, is like knowing that Peirce was a logician but not knowing anything about his logic. That Peirce thought there could be a logic of vagueness distinguishes his view from those that quarantine vagueness from logic. Max Black summarizes the logical commitment of such a view nicely when he writes that its "resulting policy is tantamount to *forbidding* such reasoning" (1970:7). Far from forbidding reasoning with vague terms,

Peirce sometimes remarked that vagueness excelled in eliminating doubt where precision stumbled. "I will only add", Peirce writes, "that though precise reasoning about precise experiential doubt could not entirely destroy doubt, any more than the action of finite conservative forces could leave a body in a continuous state of rest, yet vagueness, which is no more to be done away with in the world of logic than friction in mechanics, can have that effect" (CP5.512).[55] If reasoning with vague terms offers this sometime additional benefit, what logic and semantics reaps such rewards?

Peirce claimed to have "worked out the logic of vagueness with something like completeness" but scholars have been puzzled whether such a manuscript has been lost or whether it requires some clever reading of extent material (CP5.506 [c.1905]). On the mysterious whereabouts of Peirce's logic of vagueness, Cohen writes that "Peirce seems to have concluded not that formal logic is intrinsically inapplicable to ordinary discourse but rather that a new logic, 'a logic of vagueness,' was required, […] and though Peirce's logic of vagueness has never been found, various attempts have since been made to replace its loss" (1962:266). Jared Brock has suggested more ambitiously that vagueness is ubiquitous in Peirce's logical theory and philosophy. He writes "[o]nce we become clear about that project, there is no difficulty in "finding" the logic of vagueness. It is "everywhere"!" (1969:3). But Brock's claim that the logic of vagueness is everywhere doesn't clarify much since Peirce made many logical advances that have nothing to do with vagueness and because generalizing the logic of vagueness in this manner renders it invisible. Brock signals this sentiment when he writes that "[i]ts parts are scattered, fragmentary, obscure, and sometimes even contradictory. But they unquestionably exist and just as unquestionably fit into a grand plan" (1969:3).

While many have made valuable attempts in tidying up Peirce's logic of vagueness, a complete and detailed Peircean theory is not yet to be had.[56] However, despite its incompleteness, the presence of Peirce's logic of vagueness is apparent through its influence on a variety of authors. Some of these include those responsible for elements of the modern treatment of vagueness itself, e.g. Bertrand Russell, Max Black, Carl Hempel, Frank Ramsey, Wittgenstein, Arthur Burks, and Charles Morris.[57] And, even if Peirce's theory is thought to be cryptic or improperly directed toward the

phenomenon, his theory still receives attention by contemporaries in their historical account of vagueness (see Williamson 1994:46-52; Keefe et al. 1996).

2. *Borderline Vagueness is Essential Depth*

One problem with Peirce's theory of vagueness involves how to rectify his numerous remarks about the form of indeterminacy particular to vagueness. Modern accounts of vagueness put strict prohibition upon conflating vagueness with other forms of indeterminacy, e.g. ambiguity, indefiniteness of reference, unspecificity (see chapter 1). Peirce seems to lump many of these different types of indeterminacy under the heading vagueness, or imprecision, or indefiniteness, and it is rare to find passages where 'vagueness' is used in a precise way.

Some agreement can be found in the scholarly literature but much of this is owing to a few explicit remarks made by Peirce himself. One universally agreed upon point is that Peirce thought that vagueness and generality were distinct forms of indeterminacy that should not be confused. There are countless texts where Peirce insists on differentiating the two through the non-applicability of the law of excluded middle (LEM) in the case of generality and the law of non-contradiction (LNC) in the case of vagueness (e.g. EP2:351). Further, Peirce was extremely critical of those who did confuse these two forms. In a letter to E.H. Moore, Peirce writes

> It is indispensible in these matters to avoid all confusion between what is *general* and what is *vague*. It might seem almost impossible to confuse the two concepts, which are truly as wide apart as the poles. Yet we all do so continually (L229:NEM3:913).[58]

And, elsewhere, Peirce writes

> It is plain that a proposition cannot be both vague and general in the same respect; for if the right to determine the sense belongs to the utterer of it, that liberty is thereby forbidden to the interpreter. This remains true when one holds converse with oneself; for the self which signifies is always other than the self to whom the thought is signified. Hence, although Locke was right enough when he said of the triangle, in general, that it is neither equilateral, isosceles, nor scalene, he was quite wrong in adding that it is all of these at once. For he thereby denies the applicability of contradiction and thus makes his general triangle to be at the same time vague. But nothing can be vague and general at once and in the same respect (R530:16, 2nd pagination).

The confusion between the two types of indeterminacy corresponds to a confusion between the logical relations between universal and particular propositions. That is, while the truth conditions for propositions involving vague terms correspond to those of existentially quantified expressions, propositions involving general terms have truth conditions that correspond to universally quantified expressions. On Peirce's account, while each of the following are false—(1) 'all triangles are isosceles', (2) 'all triangles are scalene', (3) 'all triangles are equilateral'—the falsity of (1)–(3) does not entail 'all triangles are scalene, isosceles, and equilateral'. Peirce's point is that Locke confuses contrary and subcontrary relations insofar as the falsity of two contrary propositions does not entail that a conjunction of these propositions is true. In other words, the falsity of both of the following $(\forall x)(Tx \supset Ex)$ and $(\forall x)(Tx \supset {\sim}Ex)$ does not entail the truth of $(\forall x)[Tx \supset (Ex \,\&\, {\sim}Ex)]$ even though both of the following can be true: $(\exists x)(Tx \,\&\, {\sim}Ex)$ and $(\exists x)(Tx \,\&\, Ex)$.

Further, Peirce thinks this logical error is equivalent to violating the rights an interpreter and utterer have in confirming or disconfirming propositions. Peirce claims that the *utterer* has the right to specify or select a substitution instance for vague or existentially quantified sentences, and the *interpreter* has the right to specify or select a substitution instance for general or universally quantified sentences. Locke's claim that 'a general triangle is scalene, isosceles, and equilateral' is tantamount to playing both sides. That is, Locke reserves his right to specify *some* general triangle that is at once isosceles, scalene, *and* equilateral, yet restricts the right of the interpreter to choose *any* triangle that is not isosceles, scalene, and equilateral.

A second relatively uncontroversial aspect of Peirce's theory of vagueness is that vagueness is indeterminacy relating to a term's *depth*. Peirce, however, had a number of different senses of depth, and which one best connects with vagueness qua borderline cases is not clear in his early work. For example, in his 1867 "Upon Logical Comprehension and Extension", Peirce refers to *substantial depth* ("the real concrete form which belongs to everything of which a term is predicable with absolute truth"), *essential depth* ("the really conceivable qualities predicated of it [a term] in its definition"), and *informative depth* ("all the real characters (in contradistinction to mere names) which can be predicated of it (with logical truth on the whole) in a supposed state

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of information") (W2:81,80,79).[59] Under these types of depth, Peirce offered a horde of modifiers to cover additional senses in which these types might be expressed. For example, Peirce wrote that "depth, like the breadth, may be certain or doubtful, actual or potential, and there is a comprehensive distinctness corresponding to extensive distinctness" (W2:79). Without continued use of these modifiers in his later work and without explicitly and consistently linking vagueness to one form of depth, subsequent commentary on Peirce's logic of vagueness has been subject to a textual impasse. Commentators have taken Peirce's understanding of vagueness as a form of indeterminacy subsuming all other forms of indeterminacy (e.g. Brock 1979:41), have concluded that Peirce's work on vagueness was only restricted to borderline-cases or fuzziness (Khatchadourian 1965:120; see also Khatchadourian 1962), have argued that it is not related to borderline-cases or fuzziness (Lane 1997), and have tried to extract a specific theory of vagueness from one of the above, modified forms of depth (e.g. Nadin 1983; Brock 1979:45).[60]

My aim here is not to tabulate the different senses of indeterminacy that Peirce proposed, nor is it directed at sorting out the different ways in which others have interpreted Peirce's theory of vagueness. Here the aim is to link Peirce's theory of vagueness to contemporary discussion by tying one particular sense of indeterminate depth to the type of indeterminacy now called "vagueness". There are a variety of different benefits to doing this. First, it is important historically for Peirce's work on vagueness antedates Russell's, and the latter is typically credited as the progenitor of the modern form of vagueness. Second, it is important to a contemporary analysis of vagueness if Peirce offers a unique solution to the sorites paradox. Finally, it is important for a complete understanding of Peirce's logic and philosophy since vagueness and indeterminacy are recurrent themes in Peirce's semiotic, pragmaticism, and logic.

Scholarly literature that aims at connecting vagueness to the contemporary form of borderline-vagueness is sparse. Nadin contends that vagueness qua borderline cases is *informed depth*.[61] The logic by which this claim is worked out is not made explicit, but it seems to have been gleaned from Peirce's remarks in R283:141,138-9 [1905] where Peirce writes "indefiniteness in depth may be termed vagueness" and from various reflections in his 1867 "Upon Logical Comprehension and Extension" (Nadin 1983:156).

However, Nadin's view stands in contrast to a number of Peirce's other remarks. Consider the same manuscript page cited by Nadin used to substantiate the claim that vagueness is informed depth (R283:138, rejected). Peirce writes that we "may use the term *indefiniteness in depth*, or *vagueness*, to denote any indefiniteness which primarily affects the essential depth of a sign". Thus, in opposition to Nadin's view, this manuscript sheet articulates vagueness as a type of indefiniteness affecting *essential*, not *informed* depth.[62]

The case for vagueness qua borderline-cases should be connected to essential depth for a variety of other reasons. First, as far as this author knows, Peirce never explicitly writes that indeterminacy in either substantive depth or informed depth is vagueness. Second, Peirce explicitly does connect vagueness with essential depth. Third, vagueness understood in terms of *informed* depth better corresponds to unspecifity (or uniformativity), while indefiniteness in essential depth better corresponds to the contemporary understanding of vagueness. In order to substantiate this final claim, consider the three senses in which depth was articulated by Peirce in 1867:

> (1) "By the informed depth of a term, I mean all the real characters (in contradistinction to mere names) which can be predicated of it (with logical truth, on the whole) in a supposed state of information; no character being counted twice over knowingly in the supposed state of information" (W2:79).
>
> (2) "By the *essential depth* of a term, then, I mean the really conceivable qualities predicated of it in its definition" (W2:80).
>
> (3) "*Substantial depth* is the real concrete form which belongs to everything of which a term is predicable with absolute truth" (W2:81).

Whereas the different senses of *breadth* involve a relation between a sign (or term) and objects to which it applies, all three types of depth involve a relation between a sign (or term) and the potential interpretations the sign elicits. Types of indeterminacy emerge as a result of a restriction on the type of interpretation viably produced by the sign. Determining which form of depth best corresponds to vagueness is a matter of determining the unique restriction involved with a particular form of depth. Naturally, informed and essential depth are the best candidates for vagueness because both at

minimum involve indeterminacy as a result of the meaning of terms, of which the sorites paradox and modern discussions of vagueness are principally concerned. One reason for choosing essential depth over informed depth as the candidate for vagueness is that the former pertains only to *the meaning* of a term and not to indeterminacy relating to gathering information or present information states. In the case of essential depth, there is no demand that the predication take into account factual, substantive, or informational states and variables that are not directly related to the *conceivable meaning of the term*. This is different from the informed depth where predication requires not only that a given quality can be consistently predicated of an object but also that the current state of information permits such a predication. With respect to the informed depth of a term, Peirce requires that all the information at hand must be taken into account, and that those characters of which there is not on the whole reason to believe can be predicated of it are not to be reckoned as part of its depth (see W2:79). Informed depth therefore involves all the known *real* characters that can be truly predicated of a term and not merely *conceivable* characters in the term's meaning.

This distinction is important in contemporary accounts of vagueness, for successful predication of a vague predicate is always articulated as insurmountable by any increase of information. This is sometimes explained by contending that vagueness is a purely conceptual phenomenon not reducible to a lack of *information about the thing* in question. For example, the semantic or epistemic uncertainty about whether the vague predicate 'tall' applies to a borderline case of a tall man is not vague simply because there is a question about the exact height of the man. Even if the height of a borderline case of a tall man were precisely known, the application of the vague predicate to him would still be indeterminate because there is uncertainty about the qualities a *tall* person is supposed to comprise. Sainsbury puts this point as follows: "[y]ou may know how tall someone is to the millimetre, yet be unable to say whether or not he is tall. You may see a shade under perfect conditions for assessing its colour, yet be unable to say whether or not it is red" (1995:590).

Assuming then that essential depth best corresponds to vagueness, one major problem in assessing Peirce's account of vagueness is that many of his examples of vagueness treat the phenomenon as dispensable by an increase in information. This is

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indeterminacy qua informed breadth/depth. Take Peirce's example of a two-person conversation. Both participants are reputable language-users and both have a man in view. One of the language-users says "A man whom I could mention seems to be a little conceited" (EP2:351). For the other language-user, who takes the standpoint of the interpreter, the informed *breadth* of "a man" is in question. Exactly which man has the property of being conceited? The interpreter has to take into account only the information at hand and leave outside the informed breadth that cannot be predicable of "a man" with *logical truth on the whole*. "The *suggestion* here", Peirce writes, "is that the man in view is the person addressed; but the utterer does not authorize such an interpretation or *any* other application of what she says" (EP2:351). According to the informed breadth, the utterer does not authorize the interpreter to select the man in view because the interpreter does not have information that authorizes the selection. The utterer may well have someone in mind that the interpreter neither sees nor knows. Peirce continues "She [the utterer] can still say, if she likes, that she does *not* mean the person addressed" (EP2:351). Here the implication is that the informed breadth of the original statement differs with respect to utterer and the interpreter because the utterer is *unspecific* or *uninformative* about the conceited man to which she is determinately referring. That is, the utterer has some specific man in mind but does not specify who he is. In such a case, while the expression is indeterminate, the indeterminacy is not a function of the vagueness of the utterer's expression since the indeterminacy is dispensible if the interpreter were to know everything that the utterer knows.

A variety of other forms of indeterminacy might be subsumed by informed depth/breadth. These include cases where utterers are coy, cryptic, or hiding information that would otherwise make the expression clear. However, this sort of indeterminacy is not vagueness as it is articulated in contemporary literature (as having borderline cases or being sorites-susceptible) since the modern phenomenon cannot be dispensed by an increase in information about either the world or a language-user's intentions.[63] It is my position that (a) given Peirce's explicit connection between essential depth and vagueness, plus (b) the recognition that informed depth better corresponds to unspecifity, and since (c) vagueness is not an informational phenomenon, then (d) vagueness qua borderline cases is best understood as essential depth. In the next section, I show that

treating vagueness as essential depth is also profitable for not only does it make Peirce's approach to indeterminacy more versatile but it also renders his connection of vagueness to existential quantification coherent.

3. Vagueness is neither Unspecify nor Existential Quantification

While the definition of essential depth given in the previous section does not preclude other forms of indeterminacy, it does shirk the incorrect association of vagueness with cases where language users are simply being uninformative or unspecific. This distinction is important because vagueness qua informed depth is subject to an objection to which vague qua essential depth is immune. Associating Peirce's theory of vagueness with a theory of uninformativeness or unspecificity has become a commonplace interpretation. This puts Peirce at a strange place in the history of vagueness. Timothy Williamson writes that the "kinds of determination at issue [in Peirce's theory] are too disparate. […] Inquiry could not progress until vagueness was distinguished from unspecificity" (1994:52). Williamson points to Russell as the key figure who "made the distinction between vagueness and unspecificity" (1994:52). The making of this distinction amounted to connecting vagueness to essential doubtfulness about whether a given term applied or failed to apply to a borderline cases.[64] Russell puts this point as following in his 1923 essay:

> [i]t is perfectly obvious, since colours form a continuum, that there are shades of colour concerning which we shall be in doubt whether to call them red or not, not because we are ignorant of the meaning of the word "red," but because it is a word the extent of whose application is essentially doubtful (1923:85).

Russell's understanding of vagueness is thus articulated in terms of the inherent doubtfulness concerning predication about borderline cases.[65] Williamson's view is that Peirce didn't see the essential doubtfulness in the application of terms as distinct from merely being unspecific or uttering statements that could be clarified with an increase of information. Such a view is also a commonplace among Peirce scholars. Haack invokes Alston to say that Peirce commonly uses "vague" and "unspecific" interchangeably (Haack 1996:110; Alston 1964:85).[66] Additionally, Tom Short interprets Peirce's understanding of vagueness as unspecificity. He writes that "[t]o lack specificity is

another meaning of 'vague', and vagueness in that sense is distinct from fuzziness" (2007:274).[67]

Showing how Peirce's view is compatible with vagueness as it is contemporarily distinguished from other forms of indeterminacy involves two problems. The first, attributed to Peirce by Williamson and others, is that Peirce did not differentiate vagueness from *unspecificity* or *underspecificity*. The second is that Peirce did not distinguish between vagueness and *uninformativeness*.

Let us take each in turn. Consider Rosanna Keefe's articulation of unspecificity. She writes that the "remark 'Someone said something' is naturally described as vague (who said what?). Similarly, '*X* is an integer greater than thirty' is an unhelpfully vague hint about the value of *X*. Vagueness in this sense is underspecificity" (2000b:10). Vagueness qua underspecificity lacks the marquee characteristics of sorites-susceptibility, having borderline cases, and semantic tolerance (see Wright 1975). The predicate "is an integer greater than thirty" is sharply-bounded, even if the value of *X* is not clearly indicated because the sense of the statement is clearly understood. In contrast, the predicate 'tall' in the statement 'John is tall' lacks an explicitly identifiable sharp boundary, is susceptible to the sorites paradox, and is tolerant to small changes in meaning. In short, 'John is tall' does not mean the same thing as 'John is shorter than 10 feet'; the former is by contemporary standards *vague*, while the latter is *underspecific*. The important contrast between the two is not the issue of information but that the predicate 'is an integer greater than thirty' identifies a sharply-bounded set of possible referents, while the boundary for the application of 'tall' although also indeterminate, does not clearly involve a sharp-yet-indeterminate set of references.

A number of texts reflect the fact that Peirce understood this distinction. I quote one at length:

> I will begin by explaining the difference between *precise* and *vague*, and between *definite* and *indefinite*. If I tell you anything; for instance, if I say "A certain friend of mine has only a hundred and twenty-three hairs on his pate at most," there will always be much that I neither affirm nor deny. I do not mean to tell you, for example, what the color of his hair is. That is left 'indeterminate.' Nor do I say positively that he has over a hundred hairs. In that respect my statement is not positively 'determinate.' But the statement may fairly be called quite 'precise,' since it leaves no doubt what I mean to assert about the person in question. Had I said that his hair

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was red, that would not be quite 'precise,' but a little 'vague,' since there are shades of hair between sandy and red which I might one day call red while on another day I might say, "No, that is reddish, but not red." Had I said that his hair was *not* red, that would be vague in the extreme, since it might be black, white, golden, or chestnut. A negative proposition is usually vaguer than an affirmative one; and a mathematically exact statement is the most precise possible (R48:8-10).

In this passage, Peirce contends that the predicate "has only a hundred and twenty-three hairs on his pate at most" is not a vague expression, despite the fact that it does not indicate—much like Keefe's example—the exact number of hairs on the person's head. The predicate is indeterminate but specific with respect to its *meaning*. It is specific insofar as the *statement*, as Peirce says, *leaves no doubt about what is meant* to be asserted about the person. The marquee characteristics of vagueness mentioned earlier are not applicable to the non-vague statement "A certain friend of mine has only a hundred and twenty-three hairs on his pate at most", but are applicable to the vague statement "A certain friend of mine has red hair on his pate". In the case of "red", Peirce says that even one individual will equivocate as to whether a certain patch of color counts as "red".[68] And, this equivocation concerns borderline cases of red, i.e. language users will have doubt about whether a given patch of 'red' is red, or 'scarlet', or some other shade of red.

Now for the second objection. While Peirce knew the difference between vagueness and unspecificity, his examples and explanations of vagueness often frequently confuse it with other types of indeterminacy. He may have been *aware* of the relevant distinction between vagueness and unspecificity/uninformativeness, but his explanations about the *logic of vagueness* undo this distinction, especially those involving existential quantification. What his examples show instead is that vagueness can be equated with being *uninformative*, which makes vagueness dispensable when language-users acquire more information. This objection can be substantiated since Peirce frequently connected vagueness to existentially quantified propositions whose substitution instances could be determined by an increase in information. For example, the vagueness of 'a man' in the existentially quantified expression "A man *whom I could* mention seems to be a little conceited" is thought to illustrate vagueness but the particular man could be determined if the information available to the interpreter was equal to the

information available to the speaker of the proposition (see EP2:351). On this account, vagueness reduces to informational constraints placed upon interpreters.

This sort of objection has been put forward by Williamson who writes "[a]n utterance is vaguely true if some determination of it results in a truth, and vaguely false if some determination of it results in a falsehood. Thus 'The number of bald men is even' is both vaguely true and vaguely false. In this sense the principle of contradiction does not apply" (1994:52). In relation to Peirce's treatment of vagueness and generality in terms of ordinary quantified sentences, Williamson continues: "a sentence such as 'A woman wrote *Middlemarch*' is vague, and the principle of contradiction does not apply to it. Yet the sentence is straightforwardly true." (1994:52). In other words, Peirce treats existentially quantified statements as vague and claims that the law of non-contradiction (LNC) does not apply to them. However, LNC does apply to existential propositions since many of them are known to be true (such as 'A woman wrote *Middlemarch*'). Therefore either existentially quantified expressions are not vague or LNC does apply to existentially quantified expressions.[69]

Williamson's rationale for this interpretation turns on Peirce's remarks in "Issues of Pragmaticism" (CP5.448; EP2:351 [1905]). The text reads:

> Perhaps a more scientific pair of definitions would be that anything is *general* in so far as the principle of excluded middle does not apply to it and is *vague* in so far as the principle of contradiction does not apply to it. Thus, although it is true that "Any proposition you please, *once you have determined its identity,* is either true or false"; yet *so long as it remains indeterminate and so without identity,* it need neither be true that any proposition you please is true, nor that any proposition you please is false. So likewise, while it is false that "A proposition *whose identity I have determined* is both true and false," yet until it is determinate, it may be true that a proposition is true and that a proposition is false (CP5.448, see also R641:24 2/3).

Williamson's charge is that LNC does hold for the vague statement "Some woman wrote *Middlemarch*" since Mary Anne Evans wrote *Middlemarch*. On the interpretation of vagueness as informed depth, Williamson's objection holds since "A woman wrote *Middlemarch*" is clearly true, and LNC as a principle of logic ought to hold independent of the interpreter's lack of information. In short, Peirce confuses epistemology with semantics since the non-applicability of LNC is the result of a lack of information about

whether or not Mary Anne Evans wrote *Middlemarch*. That is, on the vagueness qua informed depth interpretation, Williamson is correct to contend that Peirce confuses unspecificity or uninformativeness with vagueness.

The problem with Williamson's interpretation is that Peirce connects vagueness to different types of possibility when calling an existentially quantified statement "vague". For example, Peirce writes:

> Fully to understand this [real vagueness], it will be needful to analyze modality, and ascertain in what it consists. In the simplest case, the most subjective meaning, if a person does not know that a proposition is false, he calls it *possible.* If, however, he knows that it is *true,* it is much more than possible. Restricting the word to its characteristic applicability, a state of things has the Modality of the possible—that is, of the merely possible—only in case the contradictory state of things is likewise possible, which proves possibility to be the vague modality. (CP5.454, EP2:354-5 [1903]).

The difficulty that this passage poses for Williamson and others who understand propositions involving vague terms as existentially quantified statements is that statements concerning vagueness qua informed depth pertain to the *subjective meaning of possibility*, while vagueness qua essential depth pertains to the *objective meaning of possibility*. In the above passage, Peirce contends that something is *subjectively possible* if it is *not known* to be false, while something is *objectively possible* only if the contradictory state of things is likewise possible.[70] In distinguishing the two different forms of modality associated with existential quantification, it is easy to show why Williamson's objection only applies to existentially quantified statements that are subjectively possible. Despite examples that seem to indicate confusion on Peirce's part, the distinction between the two forms of modality was crystal clear in Peirce's mind around 1897, if not earlier. The explanation he gives for not always clarifying the specific type of depth and possibility he is analyzing is usually related to brevity (see EP2:394). And, in many of the post 1897 texts where Peirce gives a concentrated explanation of vagueness, vagueness is explicitly linked to objective possibility. Consider Peirce's well-known and often-cited 1902 definition of "vagueness" in the *Baldwin Dictionary*:

> Indeterminate in intention. A proposition is vague when there are possible states of things concerning which it is intrinsically uncertain whether, had they been contemplated by the speaker, he would have regarded them as

excluded or allowed by the proposition. By intrinsically uncertain we mean not uncertain in consequence of any ignorance of the interpreter, but because the speaker's habits of language were indeterminate; so that one day he would regard the proposition as excluding, another as admitting, those states of things. Yet this must be understood to have reference to what might be *deduced* from a perfect knowledge of his state of mind; for it is precisely because these questions never did, or did not frequently present themselves that his habit remained indeterminate (1902:748).

Here Peirce reaffirms what is clear in the initial example of the 1905 "Issues of Pragmatism" article concerning vagueness and stresses the "essential doubtfulness" insisted upon by Russell. Namely, real vagueness is not a product of the *ignorance* of the interpreter (not an issue of subjective possibility or information states). If "A woman wrote *Middlemarch*" is *objectively vague*, the vagueness of the statement cannot be attributed to the interpreter *not knowing* the fact that Mary Anne Evans wrote *Middlemarch*. Or, in the case of *underspecificity*, such as 'a man who I could mention seems conceited', the indeterminacy is not attributable to the failure to know the identity of the intended man. Vagueness, as Peirce defines it above is neither of the two, for Peirce writes that even if the interpreter had *perfect knowledge* of what might be deduced from the speaker's state of mind, indeterminacy would remain.

This point can further clarified by distinguishing two interpretations of existential quantification. On the substitutional approach, it is in principle possible to substitute a name—determinate singular—for every variable within a true existentially quantified proposition. Peirce mentions this substitutional treatment of existential quantified propositions in his "The Categories Defended" [1903]. There, he calls the existential quantifier the "vague" quantifier and claims that quantified variables can be replaced by substitution instances provided the interpreter is sufficiently knowledgeable.

> "Something" means that *sufficient knowledge would enable us to replace* the "something" by a monstrative index and still keep the proposition true" […] Logicians confine themselves, apart *[from]* monstrative indices themselves, to "Anything" and "Something," two descriptions of what monstrative index may replace the subject, the one description *vague*, the other *general*" (EP2:173, my emphasis).[71]

On a more realist interpretation of existential quantification, there are true existential propositions that cannot be determined by even an omniscient being. No increase in

information about the utterer's use of a term or about the present state of the world would yield a true substitution instance since neither the utterer's intension nor the state of the world determines that the existentially-quantified proposition is true of a specified object.

Peirce's connection of vagueness to existential quantification has no relevance to the modern discussion on the vagueness if vagueness is understood as informed depth because it treats the indeterminacy of vague terms as dispensable upon sufficient increase of information and because it renders his claims about the non-application of LNC absurd. However, treating vagueness as essential depth and calling existentially quantified propositions as vague provided they express *objective, not subjective, possibility*, renders the objection that Peirce did not have a precise understanding of vagueness null and void. It does so insofar as Peirce thinks it can be objectively indeterminate whether a certain borderline patch of the color is red is red.

4. A Historical Interlude on Peirce's Modal Shift

In this section, I comment on, although I do not defend, why Peirce decided that a non-informational (or non-epistemic) definition of possibility was unacceptable. It has come to the fore that Peirce shifted or broadened his view on modality between 1896 and 1897. Initially, Peirce characterized modality relative to a particular information states (the IR-account), then latter appealed to experimentation that takes place in an ideal world for making determinations prior to experience (the IW-account).[72] This shift in modality was first documented by Peirce himself in 1897 and in a number of other works (see CP8.308). Many commentators have thematized this modal shift. In the early 1960s Murphey gestured toward the shift as a reconceptualization of the categories as modes of being rather than mere classes of phenomena (1961:391-3). In writing on the gamma part of the existential graphs, Don Roberts (1973:84-5) remarks that Peirce attached cross marks to broken cuts to indicate that modalities were indexed to information states. This was necessary to prevent modalities from collapsing into each other upon the increase of information states. In Peirce's later work on the graphs, different kinds of modalities were tinctured, and one such tincture was for a non-epistemic, objective possibility (Peirce 1906:526; Roberts 1973:94; Zeman 1997).[73] Furthermore, Morgan (1979; 1981) explicates the shift in some detail, offering a logic that employs an analogue function (similar to the accessibility function for possible worlds), while Noble (1989:161-162)

has articulated the shift in the context of Peirce's theory of continuity. Lastly, and most recently, Robert Lane (2007) has used the shift to draw out certain consequences for making pragmatism more intelligible. In this section, I characterize the shift briefly, before proposing its importance for a theory of vagueness.

On Peirce's initial IR-account, the varieties or senses of modality are expressed in the different types of information states. This view extends back to Peirce's 1867 articulation in "Upon Logical Comprehension and Extension" where the sense of any term (including those of modality)—be it the essential, substantial, or informed—is understood as a state of information (see e.g. R736; NEM4:104). The last place Peirce explicitly pledges his commitment to the IR-account is in his 1896 *Monist* article "The Regenerated Logic".

> it is best to mention that *possibility* may be understood in many senses; but they may all be embraced under the definition that that is possible which, in a certain state of information, is not known to be false. By varying the supposed state of information all the varieties of possibility are obtained. Thus, *essential* possibility is that which supposes nothing to be known except logical rules. *Substantive* possibility, on the other hand, supposes a state of omniscience (CP3.442; Peirce 1896:32-33).

In the above quotation, Peirce construes possibility as purely a matter of not *knowing* something to be false given one's state of information. Something is essentially or substantially possible given the type of information at hand. In CP4.67 [1893], Peirce writes something "is *essentially* or *logically possible* which a person who knows no facts, though perfectly *au fait* at reasoning and well-acquainted with the words involved, is unable to pronounce untrue" (see also NEM4:104 [c.1893]; W5:330 [1886]). Something is thought to be *essentially possible* provided that it cannot be pronounced false, not on the basis of any known fact, but solely on the basis of the *meaning* of the word. For instance, it is essentially possible for the *Basiliscus basiliscus* to be found in North America because there is nothing in the meaning or within the logical consequences of that meaning that would make its residing in North America untrue. Alternatively, consider the mythological basilisk. A statement is *essentially necessary* provided that the person knows the statement to be true independent of the fact that the basilisk ever existed. Peirce writes that a person would "not know whether there was or was not such an animal as a *basilisk*, or whether there are any such things as serpents, cocks, and eggs;

but he would know that every basilisk there may be has been hatched by a serpent from a cock's egg. That is essentially necessary; because that is what the word *basilisk* means" (CP4.67 [1893]).

Peirce, however, became dissatisfied with this characterization of modality. His modal shift occurred between a two-part article that appeared *The Monist* as "The Regenerated Logic" (October 1896) and in "The Logic of Relatives" (January 1897).[74] While it is between these two articles in R787 that Peirce articulates the IR-account of modality as only covering the negative (or "ignorantial") senses of possibility, it is in the "Logic of Relatives" where Peirce argues that the supposed states of information cannot cover the substantive sense of possibility (Peirce 1897:206-7; CP2.346). Peirce's shift away from the IR-account to an IW-account is prompted primarily through a need to develop a logic to solve a question relating to the cardinal comparability of sets.[75] The motivating question that led Peirce to make the revision was: "[i]s it, or is it not, logically possible for two collections to be so multitudinous that neither can be put into a one-to-one correspondence with a part or the whole of the other? To resolve this problem demands, not a mere *application* of logic, but a further *development* of the conception of logical possibility" (1897:206). In the end, Peirce argued that it is logically impossible for two collections to be so multitudinous that neither can be put into a one-to-one correspondence with a part or the whole of the other. But given Peirce's previous IR-account of possibility, how can it be substantively determined—given differing states of information—that it is logically impossible for two collections to be so large that neither can be put into a one-to-one correspondence? Peirce's answer was that this impossibility is *not determined by a given state of information*. The result was an ideal-world account that rearranged both the substantive meaning of terms and Peirce's theory of modality (see Hintikka 1997:18-9).[76] Noble represents the changing sense of possibility in the IR-account to the IW-account as a transition from saying that "the statement 'X is possible' is warranted by 'It is not known that X is not true' to one where the "statement 'It is not known that X is not true' follows from 'X is possible'" (1989:163). Peirce put the same point perspicuously when he wrote: "It is not that certain things are possible because they are not known not to be true, but that they are not known not to be true because they are, more or less clearly, seen to be possible" (CP6.367). The claim amounts to a rearranging

of the conditions for something being possible. Previously, Peirce took a nominalistic (subjectivist or epistemic) approach to possibility by contending that X is possible because—in a hypothetical state of information—we do not know that X is false. The conditions for whether a proposition is or is not possible lies completely in the given hypothetical state of information. But the recognition that there are some possibilities independent of this information changes why something is possible. Namely, the old view—where X is possible because I do not know X to be false—is supplemented by cases where I do not know X to be false because X is possible.

Thus Peirce's IW-account charges his previous IR-account with, as Lane puts it, getting things backwards (2007:562). A proposition is IR-substantively possible if and only if a subject who knows both the present facts, laws, and their consequences would not know the proposition to be false. This makes substantive possibility a condition of an actual or hypothetical state of information. For Peirce, this ignores the fact that "[w]e know in advance of experience that certain things are not true, because we see they are impossible" (1897:206).[77] What is of crucial significance is how the modal shift changed Peirce's understanding of essential possibility. While the modal shift occurred as a result of Peirce's attempt to further develop his theory of substantial modality (specifically substantial possibility), the shift had important implications for essential possibility. Before the shift, essential possibility was simply knowledge of logical laws coupled with an understanding of the definition of the term, but after the shift the meaning of essential possibility was principally conditioned by whether or not a given proposition contained an explicit contradiction as a part of its meaning. In the 1897 "Logic of Relatives", Peirce spelled out his view on essential possibility when writing that "[v]ery many writers assert that everything is logically possible which involves no contradiction. Let us call that sort of logical possibility, *essential*, or *formal*, logical possibility. […] [F]or in this sense, two propositions contradictory of one another may both be severally possible, although their combination is not" (1897:207). Unlike in substantial possibility where the contradictory of a possible proposition is impossible, essential possibility allows for severally possible propositions that are contradictory, but not a possible proposition that is contradictory. So while $\Diamond_e P$ (it is *essentially* possible that P) could be true, and $\Diamond_e \sim P$ could also be true, and even $\Diamond_e \sim P \wedge \Diamond_e P$ could be true, $\Diamond_e (P \wedge \sim P)$ would never be true. Further, it should be clear

that essential possibility is not associated with, or relativized to, any information states since its objective possibility is not derived from one's a lack of knowledge about a given predicate. Thus, the essential possibility was derived from the fact that no contradiction is derivable solely from the definition of the word.

5. *After the Interlude: Vagueness is Objective Essential Possibility*

In connecting Peirce's work on vagueness to the contemporary discussion, the relevant connection is to the objective indeterminacy affecting the essential logical depth of a term. On Peirce's account, vagueness was both distinct from underspecificity and uninformativeness. Concerning the former, Peirce claimed that vagueness was an indeterminacy of a term's essential depth, and this he clarified as an indeterminacy about whether a term *could be* predicated of certain borderline cases. Furthermore, vagueness was not uninformativeness since no amount of increase of information about the utterer's intention in using the word or knowledge about the object under discussion would eliminate the indeterminacy. And so, regarding vagueness as a form of essential depth concerning whether it was objectively possible whether a predicate could be affirmed of a certain subject, Peirce is in line with the relevant aspects of the modern treatment of vagueness. And, still further, Peirce was well-aware of the connection of vagueness to a number of other features of vagueness, including the sorites, multi-dimensional vagueness, and higher-order vagueness. In the next chapter, I argue that Peirce's proposed semantics for vagueness suggests a logical semantics that is agnostic about the existence of sharp boundaries.

Chapter 6

Solving the Paradox

> Every concept that is vague is liable to be self-contradictory in those
> respects in which it is vague.
> —Charles Sanders Peirce, CP6.496 [1906]

0. Introduction

One profitable way for understanding how Peirce would have solved the sorites paradox involves his claim that the law of non-contradiction (LNC) does not apply when propositions involve vague predicates (or predications). Lane argues that this amounts to asserting the uncontroversial claim that two subcontrary propositions can both be true, not the controversial claim that there are true contradictions. In terms of the logical semantics, Lane and Brock have both argued that Peirce's logical semantics can be construed as existential quantifications over legitimate senses of vague predicates (Lane 1997; Brock 1969; 1979; 1980; 1997:567). For example, Lane writes—citing R530:15, 2nd pagination— that "Peirce construes imprecise predications as covert existential quantifications, with their quantifiers ranging over "legitimate senses" of the predicate term" (1997:695). From the text of R530:14-5 and others, this seems to be Peirce's preferred explanation of the logical semantics, but what answer does it give the sorites? The previous chapter argued that many Peirce scholars were skeptical that any account of vagueness could be given. Here I sketch a solution in five sections. The first addresses the logical semantics and distinguishes it from two other semantic views (supervaluationism and subvaluationism). The second section specifies the source of vagueness. The third section argues that Peirce's semantic theory does not make any commitments to sharp semantic boundaries. The fourth section solves the sorites paradox and the final section suggests some advantages of my theory over other competing theories of vagueness.

1. Logical Semantics — No Gaps or Gluts

On Peirce's account, the truth or falsity of a proposition with a vague predicate is determined by whether or not there is *some* legitimate sense that the utterer *could* choose such that the proposition would be rendered true. The appeal to "legitimate senses" is drawn from R530:14-5, where Peirce also writes that the utterer has the right to

understand a vague term in "any sense the word would bear and that he might choose."[78] While little is given in this text to clarify what constitutes the *legitimacy* of a *sense*, I offer three criteria for a sense being legitimate. First, a legitimate sense is treated as equivalent to an "admissible precisification" (see Chapter 4). That is, a *legitimate sense* is a possible sharpening of a vague term, e.g. 'tall' is 'over five feet and eleven inches'. Second, in the previous chapter I claimed that it was Peirce's view that the application of a vague term is *essentially (objectively) possible* provided the application of the predicate *P* to the borderline case *S* is without any explicit contradiction locatable within the definition or use of the term. Finally, for a sense to be legitimate, it must be capable of being chosen by the utterer of the term. This makes it clear that the use of the term must be consistent with the language practices of the utterer of the term, and not consistent with a set of admissible precisifications chosen indeterminately.

While the semantics for legitimate senses—like those of admissible precisifications—are classical, the semantics for vague propositions are defined by *existential* and *universal* quantifications over "legitimate senses" of a vague predicate. Thus, a proposition with a vague predicate *P* is *legitimately true* if and only if the utterer *u* of *P* could choose some legitimate sense *s* for the vague predicate such that *P* is true. This is represented more tersely by the following schema:

> (Tv) '*P*' is *legitimately true* if and only if *P* for *some* legitimate sense *s*
> capable of being chosen by *u*.

Using the schema, a proposition with a vague predicate is substituted for *P* within the quotation marks, a proposition with the legitimate sense as its predication is substituted for the *P* to the right of the biconditional, a 'legitimate sense' chosen by the utterer is substituted for *s*, and the name of the utterer of the proposition is substituted for *u*. For example, consider a case where John points at a borderline tall man and says, 'that man is tall'.

> 'that man is tall' is *legitimately true* if and only if that man is tall for some
> legitimate sense of tall (5'11 and over) that is capable of being chosen by
> John.

A corresponding schema for falsity involves *universally* quantifying over legitimate senses, similar to the definition for falsity in supervaluationism, i.e.

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(Fv) '*P*' is *legitimately false* if and only if *not-P* for *all* legitimate senses *s* capable of being chosen by *u.*

The semantics of this theory are distinct from supervaluationism and subvaluationism, its logical dual. On the supervaluational account, truth and falsity simpliciter are defined by *universally* quantifying over admissible precisifications. This is clearly not Peirce's theory since truth is defined as *existentially* quantifying over legitimate senses. In existentially quantifying over admissible precisifications, a Peircean theory might be thought to yield paraconsistent results and a *subvaluational semantics*. This is not the case. Subvaluationism contends that, in addition to *supertruth* (true on all admissible precisifications) and *superfalsity* (false on all admissible precisifications), there is a class of semantic values that better corresponds to vague sentences (Hyde 1997; 2008; 1999; Varzi 1994; Akiba 1999). These are sentences that are *subtrue* (true on some admissible precisification) and *subfalse* (false on some admissible precisification). Rather than truth-value gaps, subvaluationism yields truth-value gluts. This is because a proposition *P* and its negation ~*P,* both can be true, i.e. *P*, ~*P* $\nVdash \varnothing$.

Peirce's semantic account differs from subvaluationism insofar as there is not a corresponding semantic category for *subfalsity*. That is, while both theories understand *P* as having the semantic force of $\Diamond P$—i.e. $\Diamond P$ is true if *P* is true at some possible world—, for Peirce legitimate falsity is defined as the strict negation of $\Diamond P$. That is, ~*P* has the same force as ~$\Diamond P$ or $\Box$~*P*. So, while Peirce's semantics and subvaluationism share a similar feature insofar as they both define truth by existentially quantifying over legitimate senses of a vague term, Peirce's theory does not countenance contradictions.

2. The Source of Vagueness

On my account, vagueness and the semantic indecision associated with it emerges for one reason. It emerges when there is a question about whether, for a given proposition *P*, *P* is legitimately true or legitimately false. Since the semantic value of *P* is determined by whether or not there is *some* conceivable legitimate sense *s'* for *P*, borderline cases for *P* are cases where *s'* is a borderline case of a legitimate sense. This can be settled by a reapplication of the semantic definition for legitimate true and falsity. That is, is there some conceivable legitimate sense *t'* such that the '*s'* is a legitimate sense for *P*' is true?

Or, alternatively, is 's' is a legitimate sense for P' legitimately true? If the answer is *yes*, then P is legitimately true. If the answer is *no*, then P is legitimately false. If it is *borderline* whether there is some conceivable legitimate sense t', then another application of the semantic definition for legitimate truth and falsity is required. Thus, vagueness stems from an indeterminacy over the number of applications required to settle whether a given proposition P is legitimately true or legitimately false. In practice, questions about the legitimate truth or falsity of P are settled at a meta-level that is much higher than semantically required. This occurs because language-users are unaware that their language practices allow for a more inclusive set of legitimate senses than they are consciously prepared to admit. Thus, rather than stating 's is a legitimate sense of 'tall' that would render P true', they ascend to a higher meta-level and say 'there is a legitimate sense of s, which is t, such that s is a legitimate sense of 'tall' that would render P true'. So while vagueness emerges from an indeterminacy over semantic ascent, this account in no ways reduces vagueness to semantic laziness since it is possible for the number of applications to be objectively indeterminate on account of there being no sharp semantic boundary between propositions that are legitimately true and those that are legitimately false.

3. Logical Semantics — No Sharp Boundaries

One may wonder about the limit to the recursive applications of the semantic criteria for legitimate truth and falsity. Will all propositions turn out either legitimately true or legitimately false upon a sufficient number of applications of the semantic theory? That is, upon analysis of all possible legitimate senses for the vague metalanguage concept of 'legitimate', do all propositions come out legitimately true or legitimately false? No answer to this question is necessary for solving the sorites paradox. On *practical* grounds, language-users can prevent the absurdity of the sorites conclusion by choosing one legitimate sense from an assortment of other legitimate senses without commitment to sharp semantic boundaries. That is, the capacity of language-users to sharpen vague terms and render a precise semantics is possible without commitment to either of the following theses: (a) the thesis that language-users cannot stipulate a more inclusive extension or (b) the thesis that there is a determinate boundary between the legitimate senses of a vague predicate and the illegitimate senses. Solving the sorites without either of these

commitments is tantamount to removing the paradoxical nature of vagueness without any strong commitment to the status of sharp semantic boundaries.

In chapter 3, I argued against the epistemic theory of vagueness in its appeal to the existence of sharp semantic boundaries. In that chapter, I claimed that there was no reason to suppose they exist. In chapter 4, I argued against supervaluationism in its claim that sharp semantic boundaries are impossible. In that chapter, I claimed that there was no reason to suppose they did not exist. Toward the end of chapter 4, I suggested that a semantic theory of vagueness should remain agnostic about such boundaries. In my theory, this is possible because the boundary between propositions that are legitimately true and those that are legitimately false is left *undefined* but there is no in-principle impossibility to its indetermination. Nevertheless, the various versions of the sorites can be solved. How is this possible? Consider a monotonically-increasing row of men, from those that are clearly bald at one end to those that are clearly not bald at the other. For this set,

> (1) it is *legitimately true* that for some h, a man with h hairs is bald and a man with $h+1$ hairs is not bald.

While this is a sharp boundary, it is not a sharp boundary in the counterintuitive sense of a sharp *semantic* boundary. What is needed for bivalence, and its accompanying sharp partitions, is the following:

> (2) there is some h such that it is *legitimately true* that a man with h hairs is bald and it is *legitimately false* that a man with $h+1$ hairs is bald.

Existentially quantifying over legitimate senses of 'bald' in (1) amounts to language-users quantifying over different ways that they *could* specify the term. For example, a language-user could say, "a man with more than thirty, two-inch hairs is bald and a man with one more hair is not bald" but (s)he could have also said "a man with more than forty, two-inch hairs is bald and a man with one more hair is not bald". Either will do from the standpoint of appealing to a legitimate sense of 'bald'. What (2) says, however, is that there is some number of hairs h that a language-user could conceivably specify for 'bald' such that calling a particular man 'bald' would be true on some legitimate sense of 'bald' but calling a man with one extra hair (i.e. $h + 1$) 'bald' would be false on every conceivable and legitimate sense of 'bald'. Asking whether (2) is true or false is

tantamount to asking for a boundary-line between the legitimately true and legitimately false cases. Asking whether (1) is true or false is only to ask for some conceivable specification between some possible positive and negative extension of a vague term.

The reason (1) does not entail (2) is because the semantic intension fixing the semantic extension of "legitimate sense" is vague. That is, since the metalanguage concept of a "legitimate sense" is vague, the semantics of a proposition are determined by an existential quantification over an *imprecise set* of legitimate senses of a vague term. This means that the semantic value of some propositions in the object language will depend on what a language-user means by the metalanguage concept of a "legitimate sense". Whether or not a borderline case of a legitimate sense is a legitimate sense, however, will be determined by whether it is *legitimately true* whether a given borderline case of a legitimate sense is a legitimate sense. This, however, is determined by existentially quantifying over another legitimate senses t, t', t'' of a legitimate sense s. From here it should be obvious that this procedure employs the meta-metalanguage concept of a legitimate sense t, which is also vague, and borderline cases of it require further consideration in another meta-meta-metalanguage concept of a legitimate sense.

Even when the logical semantics are recursively applied to higher-order metalanguages, there is no determinate answer concerning the question of sharp boundaries. For example, assuming the notion of 'legitimate sense' is vague in (1), we have

> (3) it is *legitimately true* that for some legitimate sense t of 'bald', a man with h hairs is bald and a man with $h+1$ hairs is not bald.

which is equivalent to the following,

> (3*) there is some legitimate sense s of 'legitimate sense' such that there is some legitimate sense t of 'bald', such that a man with h hairs is bald and a man with $h+1$ is not bald.

But neither of these entails a univocal commitment to counterintuitive sharp semantic conditions because neither entails that there is a sharp-boundary between *all* legitimately true cases and legitimately false ones.

4. Solving the Paradox — Yes

Given an absence of truth-value gaps and gluts, and without a commitment to sharp

semantic boundaries, does the above semantics yield a solution to the sorites paradox?

Consider both versions of the sorites paradox presented in chapter 1. These were the

'Many Modus Ponens Sorites':

> A man with 0 hairs on his head is bald.
>
> If a man with 0 hairs on his head is bald, then a man with 1 hair on his head is bald.
> If a man with 1 hair on his head is bald, then a man with 2 hairs on his head is bald.
> If a man with 2 hairs on his head is bald, then a man with 3 hairs on his head is bald.
> .
> .
> .
>
> ∴   A man with 1,000,000 hair(s) on his head is bald.

And, the 'Sorites with the Universally Quantified Premise':

> Pa_0
>
> $(n)(Pa_n \supset Pa_{n+1})$
>
> ∴ $(n)Pa_n$

As my theory is articulated above, the paradox is valid but unsound. First, the categorical

premise is *legitimately true* and the conclusion is *legitimately false* because (1) on *some*

legitimate sense of the word 'bald', a man with 0 hairs on his head is bald, and (2) on *no*

legitimate sense of the word 'bald' is a man with a full-head of hair bald. Second, *at least*

one (if not more) of the many conditionals is legitimately false and the universally

quantified premise is legitimately false. Consider a falsifying instance of the universally

quantified premise:

> (H∃) There is a height *n* such that people of height *n* are *short* while people
> .001 inches taller are *not short*.

Showing (H∃) to be legitimately true only requires that at least one substitution instance

of it is legitimately true. On the supervaluational account, (H∃) was shown to be supertrue

without any supertrue substitution instance. In chapter 4, I argued that this was an

implausible scenario. On my account, a number of different, legitimately true substitution

instances can be evoked to confirm that ($H_\exists$) is legitimately true despite the fact that each

of these possible substitution instances are inconsistent. That is, while one and only

substitution instance could ever be definitely substituted for ($H_\exists$), each substitution

instance serves equally well as a confirming instance of ($H_\exists$). That such substitution

instances exist can be easily shown whenever a language-user does make his or her

language more precise by specifying how a vague term should be taken. Consider the

following two substitution instances:

> ($H_{5'6}$) There is a height *5'6* such that people of height *5'6* are short while
> people .001 inches taller are not short.

> ($H_{5'7}$) There is a height *5'7* such that people of height *5'7* are short while
> people .001 inches taller are not short.

Assuming 5'6 and 5'7 are legitimate substitutions, both of the above substitution

instances are inconsistent yet they separately serve to confirm the legitimately truth of

($H_\exists$). This is the case because either of them (but not both) could be conceivably

substituted for *n* in ($H_\exists$). Perhaps one counterintuitive aspect of my theory is that since

both substitution instances above render ($H_\exists$) legitimately true, the set of possible

substitution instances should not be inconsistent. For example, in "Some *x* is an elephant"

($\exists x$)Ex, provided a substitution instance makes the proposition true, additional

substitution instances for the proposition should not introduce inconsistency. That is, the

conjunction of all confirming substitution instances should be true. So where *a* = pink

elephant and *b* = gray elephant, *a* & *b* should never be truth-functionally false. This is not

the case for propositions involving vague terms since existential quantification ranges

over an inconsistent set of *possible* substitution instances. These possible substitution

instances serve to *separately* confirm ($H_\exists$) since all that goes into a language-user uttering

a legitimately true proposition with a vague predicate is that he *could* specify some

legitimate sense for the vague predicate, not that he *does* specify some nor that he is

committed to all such substitution instances.

5. Vaguely Better than Alternatives? — No

In this section, I briefly state why this theory is a better alternative than two of its rivals.

First, it should be noted that my Peirce-developed theory of vagueness solves the

sorites paradox in both its forms by conservatively rejecting the soundness of the sorites

argument. This feature of my theory puts it on par with its rivals for each shows how to bar the paradoxical feature of vagueness by proposing a solution to the sorites paradox.

Second, the strength of this theory over epistemicism is its *agnosticism* about sharp semantic borderline cases. The epistemic theory argues that there is a fact to the matter whether 'John is tall' or not, we simply don't know what it is. Vagueness is our inability to know this fact, and higher-order vagueness is our inability to know whether we know this fact. My theory posits no such sharp-semantic boundaries and contends that vagueness is not a function of our inability to know but an objective indeterminacy stemming from the number of meta-levels needed to determine whether or not a proposition is legitimately true or legitimately false. It might be thought that such a commitment to sharp-boundaries is beneficial for it solves a version of the paradox that mine cannot. That is, on my account there is no determinate answer concerning whether or not every proposition comes out as legitimately true or legitimately false. Since it is possible for some proposition *P* to be semantically undefined, my theory cannot account for what should happen in a scenario where a language-user must continually appeal to a higher and higher metalevel specification of 'legitimate' to determine the truth-value of *P*. The epistemic theory does have an answer through its commitment to a precise metalanguage. However, in the second half of chapter 3 I argued that the epistemic theory's reasons for such a precise metalanguage is question-begging.

Third, there are certain affinities between my theory and that of supervaluationism insofar as both involve quantifying over admissible precisifications or legitimate senses. However, there is an important and relevant difference between this theory and my Peirce-developed theory. One advantage of my theory concerns the interpretation of existentially quantified expressions. Supervaluationism claims that (H∃) is supertrue even though it is not possible to specify any supertrue substitution instance (Keefe 2000b:162-5; 2008:316; McGee et al. 1995:212). That is, supervaluationism is committed to a realist interpretation of the existential quantifier, since (∃x) means "there exists an *x*" not that "some *x* is specifiable". Hyde has objected to this on two points. The *first* is that there is not only an epistemic-block for specifying substitution instances but the "existential claims we are asked to accept […] are ones for which we are not only unable to produce a witness but for which no witness could *in principle* ever be produced" (1994:259, my

emphasis). The second objection is that, even granting a nonconstructive interpretation of quantifiers, "in the absence of any explanation of the supposed barrier to knowledge, it is reasonable to remain skeptical of the existence of such unknowable instances. They appear, after all, quite extraordinary and the extraordinary begs for explanation" (1994:259). My theory is immune to both of these objections. First, on my theory, while existential quantification should be understood in a realist sense, there is no in principle impossibility of providing a witness. Language-users regularly do so by selecting a substitution instance from the many possible different substitution instances that separately confirm $(H_\exists)$. Secondly, since any complete determination on the object level will require an appeal to a vague metalanguage concept, my theory remains agnostic about the boundary-line between legitimately true and legitimately false. That is, it leaves the question as to whether all propositions are either legitimately true or legitimately false open.

Chapter 7

Semi-Formal Clarification of Semantics

0. Introduction

In this section, I clarify my position that the semantics of a vague language can be defined in terms of legitimate truth and legitimate falsity without a commitment to sharp semantic boundaries or bivalence. This is achieved by appealing to the vagueness of the accessibility relation in possible world semantics. Below I argue that while every proposition can be valuated as true or false at a world, this does not require any commitment to the thesis that every proposition is legitimately true or legitimately false (see Kripke 1963).

1. Vagueness and the Accessibility Relation

Different conceivable legitimate senses, or possible admissible precisifications, can be understood to correspond to different possible worlds. Let a non-empty yet arbitrary set of these worlds be represented as **W**. In possible world semantics, possible worlds in an interpretation are ordered under an *accessibility* relation. The two-place accessibility relation **R** forms the basis for determining the truth-value of any modal sentence. A *frame* is an ordered pair consisting of an accessibility relation **R** and an arbitrary set of worlds **W**, i.e. **Fr = ⟨W, R⟩**. A *valuation-function* **v** assigns truth values to propositions in a frame, and its addition to a frame forms an *interpretation* **I**, i.e. **I = ⟨W, R, v⟩**. Bivalence can hold with respect to sentences so that a sentence in any world is either true or false, although not both. That is, if *P* is a sentence, then either $v_I(P, w) = T$ or $v_I(P, w)=F$, but not both. This reflects the fact that valuations are classical for specific senses or precisifications of sentences with vague predicates. The semantics for legitimate truth and legitimate falsity are defined by quantifying over accessible worlds. A sentence *P* is *legitimately true* $v_I(\Diamond P, w)=T$ if and only if $v_I(P, w_i)=T$ at some world w_i in I such that Rww_i. Likewise, a sentence is *legitimately false* if and only if $v_I(\Diamond P, w)=F$, i.e. $v_I(P, w_i)=F$ at all worlds w_i in I such that Rww_i.

Since a proposition with a vague term is *legitimately true* only if there is some legitimate sense in which the vague term can be taken by the utterer so as to make the proposition true, determining whether there is such a legitimate sense is partially

determined by the scope of the accessibility relation. For example, consider the following three worlds w_1, w_2, and w_3, such that proposition P in $v(w_1)=T$, $v(w_2)=F$, $v(w_3)=T$. The legitimate truth or falsity of P at any world is determined by the two-place accessibility relation. Representing the ordered pairs of these worlds so that the first member of the pair is a world and the second member of the pair is a world accessible to the first world, a set of ordered pairs can be given. For example, $\{\langle w_1, w_2 \rangle, \langle w_1, w_3 \rangle, \langle w_2, w_3 \rangle\}$. So, the accessibility relation, along with the valuations at specific worlds, determines whether P is legitimately true or legitimate false. Thus, $v_I(\Diamond P, w_1)=T$ since $v_I(P, w_2)=T$ and Rw_1w_2. However, $v_I(\Diamond P, w_3)=F$ since there is no world accessible to w_3 such that $\Diamond P$ is true (this is because w_3 is not accessible to any worlds).

On my account, while bivalence holds for propositions, sharp-boundaries are avoided because the set of accessible worlds existentially quantified over is imprecise. Another way of putting this is that the accessibility relation of sentences with vague predicates does not determine a precise set of ordered pairs. Consider the following example. A sentence P is *legitimately true* $v_I(\Diamond P, w)=T$ if and only if $v_I(P, w_i)=T$ at some world w_i in I such that Rww_i. However, whether or not $v_I(\Diamond P, w)=T$ depends upon whether or not Rww_i. In the preceding chapter, my argument was that the range of accessibility hinges on the vagueness of the metalanguage concepts of 'admissibility' or 'legitimacy'. In that chapter, I argued that a recursive application of the semantics to the metalanguage is sufficient to solve the sorites without any commitment to bivalence. That is, consider the case where it is vague whether $Rw_{49}w_{50}$. Let Q be the proposition 'world 49 is accessible to world 50'. Q is legitimately true provided there is some legitimate sense such that Q is true. That is,

$$v_{I*}(\Diamond Q, w)=T \text{ if and only if } v_{I*}(Q, w_i)=T \text{ at some world } w_i \text{ such that } Rww_i$$

This is a higher-order expression formulated by reapplying the semantics of legitimate truth and falsity to Q.[79] A sufficient number of reapplications of the semantic definition are, I argued, sufficient for solving the sorites paradox. Nevertheless, the resolution does not make use of sharp semantic boundaries because the accessibility relation permits further borderline cases that can only be resolved by an appeal to higher-level metalanguage.

2. Conclusion

Truth-value gaps, gluts, and bivalence are unnecessary for a semantics of vagueness. By my account, valuating statements as bivalent at specific worlds works fine but this does not imply a commitment to sharp semantic boundaries because there is no commitment to a precise metalanguage. Instead, there is a commitment to the capacity of language-users to make vague terms and the conditions under which they are true increasingly more precise. This is all that is required to keep vagueness free from paradox.

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Notes

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[1] For a variety of sources that do consider whether there are vague objects, see (Evans 1978; Noonan 1982; 2004; Inwagen 1988; Lewis 1988; Burgess 1989; Pelletier 1989; Over 1989; Tye 1990; Burgess 1990; Garrett 1991; Zemach 1991; Cowles et al. 1991; Heck Jr 1998; Simons 1999; Akiba 2004).

[2] Changes between steps need not be uniform; they only need to be monotonic and sufficiently small. Consider a monotonically-increasing row of men, and the predicate 'tall'. Starting with a man with a full head of hair named 'John', a sorites could be constructed beginning with John, and then proceed through the removal or reduction of a small, but irregular number of hairs. For example, first 1 hair, then .5 hairs, then 1.5 hairs, then .3 hairs, etc.

[3] Wright writes the following "Of course, we have long since abandoned the Frege-Russell view of the matter. We no longer see the vagueness of ordinary language as a defect. But we retain a second-order wraith of the Frege-Russell view in the notion that even if the senses of many expressions in natural language are not exact, there is a precise semantical description for a given natural language, i.e., a theoretical model of the information assimilated in learning a first language or, equivalently, of the conceptual equipment in whose possession mastery of the language may be held to consist. Even if 'bald', say, is imprecise, this does not require any inexactitude in an *account* of its sense" (Wright 1975:325-6).

[4] Specifically, the *miracle of conceptual comprehension* is directed at the multi-valued theorist and those believing that semantic values have precise limits (see Unger 1979:125-126).

[5] Subvaluationists argue that propositions involving borderline cases should be valuated as both true and false since for a given proposition, (1) it can be regarded as true on *some* legitimate interpretation (or way of making it precise) and (2) it can be regarded as false on *some* legitimate interpretation. Hyde writes "Consider the sentence "A pile of n grains of sand is a heap" where a pile of n grains counts as a borderline case for "heap". The sentence is true and false, so it is true. Since it is also false, then the material conditional "If a pile of n grains of sand is a heap, then a pile of $n - 1$ grains is a heap" is true by virtue of the falsity of its antecedent. Nonetheless, a pile of $n - 1$ grains of sand might be determinately not a heap, thus making the sentence "A pile of $n - 1$ grains of sand is a heap" false." (Hyde 2008:98; see also Hyde 1997; Beall et al. 2001; Akiba 1999).

[6] The biblical Abraham is also sometimes credited as the originator (see e.g. Barnes 1982). In the book of Genesis, after God announced to the elderly Abraham and Sarah that they would have a son, Abraham set out for Sodom. Upon arriving and learning of God's plan to destroy Sodom and Gomorrah, Abraham pleaded for saving the cities provided there were fifty righteous individuals within the city. God agreed. The cities would not be destroyed if fifty righteous could be found. Abraham then asked God as to whether he would destroy the cities if only forty-five righteous could be found. God agrees, the cities would not be destroyed if only forty-five could be found. Abraham then asks about thirty, then twenty, then ten. The point is that Abraham understands the stated conditions for the destruction of Sodom and Gomorrah to unspecific, so Abraham questions God on this matter. But it is unclear what this has to do with vagueness since the conditions for destroying the city could be highly precise in God's mind but God

simply does not indicate them to Abraham. Ultimately, the underlying conditions were never made precise. Abraham's last plea is that the cities be saved if there are ten righteous individuals but there appears to be four righteous individuals at most: Lot, his two daughters, and his wife (see Genesis 18-19). But it is unclear exactly how righteous Lot's wife and daughters were since his wife was rendered into salt for having turned back to look upon the carnage and his daughters later intoxicate Abraham and engage in incestuous relations (see Genesis 19:26, 31-38).

[7] Although in discussing Chrysippus or Stoic logic in general, some modern critics regard the sorites paradox hardly worthy of mention (e.g. Mates 1961:85), Chrysippus's solution has met a resurgence in thinkers adopting the epistemic solution (Williamson 1994:14). Williamson (1994:12-14, 22-27) contends that the Chrysippan solution contains many features found in modern analysis, i.e. the Stoic adoption of bivalence, the standard monotonically increasing approach to answering the sorites paradox, the acceptance of sharp cut-off points, the validity of modus ponens, the adoption of the Philo conditional, and the resolution that the inability to decisively answer questions that circle around the cut-off point is a product of our ignorance.

[8] On the Chrysippan solution to suspend judgment, see (Empiricus 2000:PH II 252-3): "If a road is leading us to a precipice, we do not drive ourselves over the precipice because there is a road leading to it; rather, we leave the road because of the precipice: similarly, if there is an argument leading us to something agreed absurd, we do not assent to the absurdity because of the argument – rather, we abandon the argument because of the absurdity. Thus, when an argument is propounded to us in this way, we shall suspend judgment over each proposition; and then, when the whole argument has been propounded, we shall introduce what seems to be the case. And if Chrysippus and his fellow Dogmatists say that, when the sorites is being propounded, they ought to halt and suspend judgment while the argument is advancing in order not to fall into absurdity, so much the more appropriate is it for us, who are skeptics, when we suspect an absurdity, not to be rash while the assumptions are being propounded but to suspend judgment about each of them until the whole argument is propounded." Also see (Empiricus 2005 M VII 416): "For in the case of the "heap" problem, since the last apprehensive appearance lies next to the first non-apprehensive one, and is just about impossible to distinguish from it, Chrysippus says that, in the case of appearances where the difference between them is so small, the wise person will hold fast and keep quiet, whereas in the case where a greater difference strikes him, he will assent to one of them as true. So if we establish that there are many false and non-apprehensive things lying next to the apprehensive appearance, it is clear that we will be in a position of having proved the necessity of not assenting to the apprehensive appearance, so that we do not, by agreeing to it, also fall into assenting to the false and non-apprehensive ones because they are in the same neighborhood – however much difference in the appearances seem to strike us."

[9] See (Cicero Ac. II xxviii 91-xxix 92). See also (Cicero 1961 Ac. II, xxix 93) and (Sextus M VII 416). Cicero writes "when the last apprehensive presentation lies next to the first non-apprehensive presentation and is hard to distinguish from it, Chrysippus and his school say that in the case of presentations where the difference is small in this way, the wise man will stop and fall quiet (hēsuchasei), but in cases where it strikes him as greater, he will assent to the one as being true". Sextus reports that "when the sorites is

being propounded, they [Chrysippus and his fellow Dogmatists] ought to halt and suspend judgment while the argument is advancing in order not to fall into absurdity" (Empiricus 2000:PH II 253).

[10] In Cicero's *Ac.*, II, xxix, "'So far as I am concerned,' says Carneades, 'you may not only rest but even snore; but what's the good of that? [F]or next comes somebody bent on rousing you from slumber and carrying on the cross-examination: "If I add 1 to the number at which you became silent, will that make many?" — [Y]ou will go forward again as far as you think fit.' Why say more? [F]or you admit my point, that you cannot specify in your answers either where 'a few' stops or that where 'many' begins; and this class of error spreads so widely that I don't see where it may not get to. 'It doesn't touch me at all,' says he, 'for like a clever charioteer, before I get to the end, I shall pull up my horses, and all the more so if the place they are coming to is precipitous: I pull up in time as he does,' says he, 'and when captious questions are put I don't reply any more.' If you have a solution of the problem and won't reply, that is an arrogant way of acting, but if you haven't, you too don't *perceive* the matter; if because of its obscurity, I give in, but you say that you don't go forward till you get to a point that is obscure. If so, you come to a stop at things that are clear. If you do so merely in order to be silent, you don't score anything, for what does it matter to the adversary who wants to trap you whether you are silent or speaking when he catches you in his net? [B]ut if on the contrary you keep on answering 'few' as far as 9, let us say, without hesitating, but stop at 10, you are withholding assent even from propositions that are certain, nay, clear as daylight; but you don't allow me to do exactly the same in the case of things that are obscure. Consequently that science of yours gives you no assistance against a *sorites*, as it does not teach you either the first point or the last in the process of increasing or diminishing."

[11] See (Cicero De Fato x 21). See also (Cicero De Fato xvi 38): "For if anything propounded is neither true nor false, it certainly is not true; but how can something that is not true not be false, or how can something that is not false not be true? We shall therefore hold to the position maintained by Chrysippus, that every proposition is either true or false;"

[12] See (Kneale et al. 1962:128).

[13] See (Cicero De Fato vi 12) and (Laertius 1959:VII 73-75). For more on the relation of the controversy of truth-conditionals to the Stoic theory of vagueness, see (Williamson 1994:23-27).

[14] See (Bobzien 2002; Williamson 1994:24) for more on the relation of the Chrysippan conditional to the epistemic theory of vagueness.

[15] There are lacunae before and after this passage, but it surely refers to the sorites paradox based upon context. Barnes (1982:27-28) translates this passage as follows "It is not the case that two are few and three are not also; it is not the case that these are and four are not also (and so on up to ten thousand). But two are few: therefore ten thousand are also." Williamson (1994:25) contends that the above passage is an indication that the standard Stoic sorites accounts were formulated in terms of negated conjunctions. Williamson follows the translation of (Long et al. 1987:222), which reads "Not: 2 are few, but not 3 as well. Not: the latter but not 4 as well. And so on up to 10. But 2 are few. Therefore 10 are few as well."

[16] The Stoics are thought to have acknowledged the validity of modus ponens (Laertius 1959:VII 73-75).

[17] The remaining portion of the text reads: "If you wish, speak, it will not cause me to be angry with you; if, however, you should say of something which people continually see under the same conditions throughout their lives, that it is non-existent, it will not help you at all. For you reject it and declare it to be invalid only by the logos, but not in reality, since you only contradict yourself and prove yourself in the wrong. Since, however, every logos is bad if contradicted by one single point in a thing that is plain to the senses, consequently yours is a bad and a wrong logos. And how should this not be the worst and most erroneous of all logoi, since so many facts contradict it?"

[18] Although not mentioned here, the sorites was also addressed in a very passing manner by Macrobius, Horace, and Aulus Gellius (Jardine 1977:162).

[19] These Greek commentators include the following: Alexander of Aphrodisias, Themistius, Ammonius, Philoponus, and Simplicius.

[20] Abbreviations for Leibniz's work adhere to the following conventions: G = (Leibniz 1875-90); GM = (Leibniz 1849-63); A = (Leibniz 1923-); DSR = (Leibniz 1992); AG = (Leibniz 1989); PLP = (Leibniz 1966); C = (Leibniz 1903); RB = (Leibniz 1996); L = (Leibniz 1969); References to Locke all refer to the *Essay Concerning Human Understanding*, which is abbreviated (Bk.#, Ch.#, Sec.#).

[21] Unlike Locke, however, the failure to designate a real boundary does not entail that the use of words fails to be reference to things, i.e. the lack of sharp boundaries does not entail that words cannot stand for things or for our private idea (Wittgenstein 1953:§§70-1; Hallett 1977:140-157; McGinn 1984:3). With respect to the latter, scientific theories with non-referential explanatory concepts are frequently tallied in the philosophy of science as prime examples of false theories that were once successful in scientific discourse (Lauden 1981; Lyons 2002; 2003:898-9).

[22] Levey's point is not exactly this. He contends that "the most natural and compelling ground for denying the existence of sharp boundaries to vague notions—namely, the thought that if our vague notions *did* have sharp boundaries, we should be able to know what they are—now seems to be lost, and Leibniz appears to have no reason at all to motivate his nihilism about vague notions" (2002:40).

[23] Levey's analysis and my own might benefit from a broader consideration of texts. For example, (1) Leibniz remarks that the knowledge of color is distinct, yet inadequate (see G IV 422-26; VE V 1075-81). Also (2) Leibniz writes that the perception of color is so numerous and so very small that our mind cannot distinctly consider each individual component, and in the consideration of a mixture of yellow blue, we do not see the fine mixture, but 'fashion some new thing for ourselves' (AG:23-7).

[24] The second figure being of the form P-M, S-M, therefore S-P; the third figure being of the form M-P, M-S, therefore S-P.

[25] Generally, the two are always kept distinct. For example, the "sorites" entry in *Baldwin's Dictionary of Philosophy and Psychology*, two entries are given. One is for the sorites syllogism, the other for the sophism, and nothing concerning their connection (1902:507).

[26] See (Gómez-Torrente 1997:237-8; 2002:108) for an alternative articulation. Schiffer (1999:482) defines the epistemic theory in three theses: (1) the existence of vague

propositions, (2) bivalence holds for all propositions, and (3) we are ignorant about whether a given borderline falls into the extension or anti-extension. We take (1) for granted since this thesis is only with theories that hold this view.

[27] There are, of course, some important differences between Williamson's work and Sorenson's version of epistemicism. Particularly, see Sorenson's articulation of those differences in (2001). There, he writes, "Williamson takes vagueness to arise directly from the indiscriminability relation. In contrast, I prefer an account in which vagueness arises from a semantic feature (such as truthmaker gaps) which precipitates ignorance as a necessary byproduct" (2001:13). For other valuable resources on the epistemic theory, see (Alter 2004; Armour-Garb et al. 2005b; Cargile 2005; Deas 1989; Gómez-Torrente 1997; 2002; Graff 2002; Raffman 2005; Sainsbury 1995; Simons 1992; Sorensen 1988; 1991; 1995; 1997; 2001; 2005; Varzi 2003; Weatherson 2003; Williamson 1990; 1992a; 1992b; 1994; 1995; 1996a; 1996c; 1996d; 1996b; 1997c; 1997a; 1997b; 1999b; 1999a; 2000; 2002a; 2002b; 2002c; 2004)

[28] See (Cargile 1969:194-5) for difficulties pertaining to the denial of the Law of Excluded Middle and the least number principle. See also (Sorensen 1988; Williamson 1994).

[29] Or, a tadpole in a fishbowl, whose actions are continually watched by a camera, becomes, in less than a blink of an eye, a fish (Cargile 1969). A bald man, with the addition of a hair, becomes non-bald, etc. (Russell 1923).

[30] In addition, we might describe it in terms of counterfactual situations. Gómez-Torrente (2002:111) writes "According to Williamson, if I know that p then I am reliably right when I utter 'p', and this implies that I would be right if I uttered 'p' in a counterfactual situation in which 'p' had only a slightly different meaning." Williamson (2002a:143) likewise puts the same point as follows: "Suppose that in fact Harry is bald; the argument is similar if he is not. Then we speak truly if we say "Harry is bald". But since the case is borderline, we might easily have uttered the same words even if our overall pattern of use had been slightly different so as slightly to shift the boundary of 'bald' in that context, making 'Harry is bald express a different and false proposition."

[31] In calling vagueness a species of inexact knowledge, I am thinking of (Williamson 1994:216), which reads "[i]gnorance in borderline cases will be assimilated to a much wider phenomenon, a kind of ignorance that occurs wherever knowledge is *inexact*." Keefe contends that this point of the epistemic theory is poorly articulated, and a better representation would be as follows: "I suggest that the phenomenon of inexact knowledge is better captured by reference to a cluster of instances of ignorance of certain facts and knowledge of others, along with the role of a margin for error in explaining the former (2000b:66)

[32] Although the omniscient speakers argument is not claimed to be one that it is a knock-down argument, it is an argument nonetheless, and it used in frequent support of the epistemic view, even if only in passing. See (Williamson 1994:198-201; 1997b:926; 1997c:5-10). See (Williamson 1999b:508) for a close analogue in the discussion of the Goldbach conjecture.

[33] The whole quotation reads as follows: "Such paradoxes [sorites] depend crucially on the employment of what have come to be called 'observational' terms—those the application of which does not wholly rest on testable matters of fact. Thus, while we

know from chemistry that a water molecule minus one atom is not water, *chemistry* cannot tell us whether Jones minus one atom is Jones, or is a person. Neither the name 'Jones' nor the sortal 'person' is a term of chemistry. The predicate 'is a person' is applied on the basis of discretionary judgements—and the making of such a judgement may be difficult in the case of a Jones who borders on the state of a vegetable" (Goldstein: 1988:447).

[34] Williamson purportedly links our inability to deny bivalence with the idea that reasonable *use* determines commitment to sharp boundaries for vague terms. Williamson (1994:202) writes "[t]hat the facts are specified by use of the word 'thin' is just what one would expect in the light of Section 7.2" (this is the section involving the discussion of bivalence, which we have been addressing). Williamson makes this explicit when he writes "It [the principle of bivalence] does not say that everything is either true or false, for no one supposes that a drop of water is true or false. The principle does not even apply to every meaningful sentence, or use of one to perform a speech act, for there is no need to suppose that a question or command is true or false. Nor does it apply to every well-formed declarative sentence. If a teacher pronounces 'He was there then' as a sample sentence of English, leaving 'he', 'there' and 'then undetermined in reference, nothing has been said to be the case, truly or falsely. *The principle of bivalence claims truth or falsity when, and only when, something has been said to be the case*" (1994:187, my emphasis). Williamson writes that the principle of bivalence "is explicitly restricted to occasions when someone uses an utterance to say that something is the case, in brief (if again with a little artificiality), when the utterance says that something is the case" (1994:187; see also Schiffer 1999:483).

[35] Williamson contends that this violation of language-users' intuitions ignores the more philosophical point. "Native speakers", Williamson writes, "may well feel that it would be wrong of them to assert or deny 'TW is thin'. Such a feeling might simply be caused by their knowledge that they do not know whether TW is thin, so they are in no position to make either claim. [...] The dispute is a philosophical one, on which the views of native speakers are not authoritative" (1994:207-208). Williamson's point is that the objector to epistemicism—who points to native speakers' articulated intuitions about their language—may have established that our intuitions find a sharp divide between the truth conditions of declarative sentences involving vague terms to be a somewhat implausible one, but the objector has not established the alternative position "that there is no fact of the matter" to be more intuitive or that these intuitions dispel intellectual misgivings about the nature of the progress from truth to falsity. Whatever the statistical breakdown of what language-users think to be the truth-implications of their language use, Williamson contends our standard intuitions about our language-use are not sufficient to establish an anti-epistemic or epistemic view. In short, what we say and how we feel about our the truth-conditions of our language usage does not solidly refute the epistemic theory, nor does it crown non-epistemic theories.

[36] This argument is put forward in (Williamson 1992b:145-149; 1994:187-198; 1997b:925-926). Earlier versions of this argument can be found in (Dummett 1978; Heidelberger 1968:213; Horwich 1990:80; 1998:76-77). Dummett writes, "A popular account of the meaning of the word 'true', also deriving from Frege, is that 'It is true that *P*' has the same sense as the sentence *P*. [...] If, as Frege thought, there exist sentences

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which express propositions but are neither true nor false, then this explanation appears incorrect. Suppose that *P* contains a singular term which has a sense but no reference: then, according to Frege, *P* expresses a proposition which has no truth-value. This proposition is therefore not true, and hence the statement 'It is true that *P*' will be *false*. *P* will therefore not have the same sense as 'It is true that *P*', since the latter is false while the former is not. It is not possible to plead that 'It is true that *P*' is itself neither true nor false when the singular term occurring in *P* lacks a reference, since the *oratio obliqua* clause 'that *P*' stands for the proposition expressed by *P*, and it is admitted that *P* does have a sense and express a proposition; the singular term occurring in *P* has in 'It is true that *P*' its indirect reference, namely its sense, and we assumed that it did have a sense" (1978:4-5). The critique by Dummett bears more on Frege's view of the equivalence between saying 'p' and 'p is true', or the so-called redundancy thesis generally attributed to Frank Ramsey (Ramsey 1927; Williams 1988:424; 2005:303; David 2005:409,411). This Fregean view dates back to William of Sherwood, see *Introductiones in logicam*, p.33. The origin of the disquotational or assorted deflationary views can be found in varying places, including (Quine 1970:12; Tarski 1944; 1983; Ramsey 1927). See also (Leeds 1978:121). On the idea that the function of a truth predicate is "to serve certain expressive purposes" (specifically those involving the expression of infinite conjunctions and disjunctions) through its disquotational character (in virtue of undoing quotation marks), see (Gupta 2005:203; Armour-Garb et al. 2005a:6-11).

[37] For minimalist views on truth, see (Holton 1993; Smith 1994; Jackson et al. 1994). For more on the T-schema, see (Gupta 2005:205; Horwich 1998:36-38,52,116,125), and even further see (Horwich 1998:40-41; David 2005:385-6). For more on Williamson's take on truth, see (Andjelkovic et al. 2000). The fact that epistemicism accommodates the T-schema is thought to be a reason for it over other views. For example, Wright writes that the "wide reception of supervaluational semantics for vague discourse is no doubt owing to its promise to conserve classical logic in territory that looks inhospitable to it. The downside, of course, rightly emphasized by Williamson and others, is the implicit surrender of the T-Scheme. In my own view, that is already too high a cost" (Wright 2003:88; see Williamson 1994:162).

[38] Without much justification, I ignore further objections involving the need to reject excluded middle, which would require lengthy analysis (see Verma 1970; Keefe 1998; 2000b:118-119; Tye 1989; Sayward 1989).

[39] This excludes many multi-valued theories, see (Williamson 1994:135; Keefe 2000b:96). There is the additional criticism for logics whose semantics express truth values in the form of continuous gradual change by employing non-numerical degree approaches to vagueness. Some of these are guilty of positing a sharp boundary between the first completely true (i.e. T or truth-value of 1) and the next degree that takes on an indeterminate truth value less than T or value of 1. See (Horgan 1994; Sanford 1976; Wright 1976).

[40] For other sources of discussion on supervaluationism, see (Lewis 1970; Sanford 1976; Kremer et al. 2003; Varzi 1995; Morreau 1999; Rasmussen 1990; Varzi 2007; 2000; Tye 1989; Keefe 2000a; Eklund 2001; Hardegree 2005; McKinnon 2002; McLaughlin 1997; Ballweg 1983; Keefe 2008; Fodor et al. 1996).

[41] This practice involves fixing a sharp boundary between the positive and negative extensions, and makes use of the supervaluational notion of a *precisification*. A precisification is a way that language-users determine or draw a sharp boundary between cases in which the vague expression applies and cases where it fails to apply. Keefe alternatively puts this as follows: "[r]eplacing vagueness by precision would involve fixing a sharp boundary between the predicate's positive and negative extensions and thereby deciding which way to classify each of the borderline cases: this is to give a 'precisification' or 'sharpening' of the predicate" (2000b:154). Thus, in the case of the vague predicate "tall", to precisify the term would be to propose a sharp cut-off that would make its use in a given context 'true' , where anyone precisely equal to or taller than six feet is tall, while anyone shorter than six feet is not tall. Thus, for any particular precisification bivalence holds and sharp-cut offs exist.

As mentioned in the previous chapter, there doesn't appear anything in the conception, our use, or nature that would justify one particular way of precisifying a vague term over another. Thus the relevant aspect of vagueness is found in that there are a number of ways of making a vague predicate precise, and no justifiable method for determining which, if any of them, ought to be privileged.

[42] What it means to precisify a statement involving a vague predicate is just what it means to ascribe it a truth-value, and this involves positing a fixed extension for the vague term employed in the statement.

[43] Keefe writes that "according to supervaluationism, by taking account of *all* precisifications we can provide the logic and semantics of vague language. It is proposed that a sentence is true iff it is true on all precisifications, false iff false on all precisifications, and neither true nor false otherwise" (Keefe 2000a:154).

[44] One example is the following. Consider the proposition: 'Robert is tall' (R), which is contextualized to the class of late 20th century men and Robert is a striking eight feet tall. While the predicate 'tall' can be made precise in a number of possible, different yet admissible ways, all of these different ways of making 'tall' precise will result in $v(R)=T$.

[45] The notion of completeability and admissibility (or appropriateness) are probably best understood in terms of the specification-space approach, which can be found in (Fine 1975). A specification space is a non-empty collection of specification points at which some or all sentences are given a truth value. A specification is "complete" if it assigns a definite truth value (True or False) to a point or space of points, and a specification is "partial" if it deviates from assigning definite truth values, leaving the assignment of some or all points indeterminate (Fine 1975:268; Keefe 2000:166). A specification space consists of a base point and an extension relation. Within the specification space, a base-point is selected from which connecting points extend it. The base point is an appropriate specification, seemingly the initial point first assigned a valuation. *Specifications* are defined as "appropriate" or "admissible" if it is in accordance with the well-understood meaning of the vague predicate. So in the case of "This wall is red", assuming that the wall is a prototypical instance of red, an appropriate specification would be valuate it True. *Specification spaces* are defined as "appropriate" if there is a corresponding precisification for each point within the space. Thus, a specification space is appropriate only if its specifications are admissible. The notion of supertruth and superfalsity can be articulate n terms of the specification-space theory, where supertruth is the following: a

"sentence is true simpliciter if and only if it is true at the appropriate specification point, i.e. at all complete and admissible specifications" (Fine 1975:273).

[46] This example and view are directly from (Keefe 2000b:164-165) although examples and statements of this feature of supervaluationism can be found elsewhere. For example, McGee and McLaughlin (1994:212) use the following example: "'$(\exists x)((x=0 \wedge$ Harry is bald) $\vee$ (x=1 $\wedge$ Harry is not bald))' is definitely true, but none of its instances is definitely true." In addition, they write that an "existential sentence can be definitely true without any instance being definitely true" and more exactly "if every member of the universe of discourse has a name, then an existential sentence is true if and only if at least one of its substitution instances is true" (1994:212, 246n20).

[47] McGee and McLaughlin characterize this feature of quantifiers as their failure to commute the D-operator (1994:212). McGee and McLaughlin write, "It is the failure of the 'definitely' operator to commute with the existential quantifier that blocks the fallacious inference from the definite truth of the existence claim to the absurd conclusion that there is a sharp partition" (1995:212).

[48] Another similar, although slightly different, way of articulating the principal theses of supervaluationism is to say that supervaluationism contends: (a) vagueness in language is a semantic phenomenon that allows for multiple admissible assignments of semantic values to statements, typically characterized by semantic indeterminacy in the form of incompleteness or indecision (Eklund 2001:364; Lewis 1986:212; Varzi 2000:279; Keefe 2008:315); (b) these admissible assignments correspond to possible extensions (or completions) of the meaning of the vague statement (Keefe 2008:315); (c) truth is supertruth, or truth under all admissible assignments/precisifications (Eklund 2001:364; Keefe 2008:315).

[49] Furthermore, the logic marshalling those statements in arguments is classical and therefore avoids a number of counterintuitive aspects of deviating from classical modes of inference (Keefe 2000b:175-6; Eklund 2001:364-5).

[50] The type of validity employed by supervaluationism is a type of *global validity*, articulated by Williamson (1994:148) as follows: "an argument is valid just in case necessarily if every admissible valuation makes its premises true then every admissible valuation makes its conclusion true, in other words, necessarily if its premises are supertrue then it sconclusion is also supertrue." More formally, Varzi (2007:647) articulates this particular version of validity as follows: "$\Sigma \vDash_A \Gamma =_{df}$ Necessarily, if every $\varphi \in \Sigma$ is: T on all precisifications, then some $\psi \in \Gamma$ is: T on all precisifications."

[51] This is partially indicative of the more general criticism that supervaluationism, in its employment of precisifications, does not address the phenomenon in question. Crispin Wright contends that "supervaluationism insists that truth and valid inference among vague statements operate *as if* there were no such indeterminacy, as if we had to deal only with full precise concepts and definite situations" (Wright 2003:88). Furthermore, Tappenden writes that it is contrary to our understanding of the meaning of the modifier 'roughly' "to imagine drawing a sharp boundary for a predicate like '*x* is roughly a handful of sand'. In cases like this it is not merely lethargy that prevents the drawing of new, completely sharp boundaries; such boundaries appear to be ruled out by the very principles governing the use of the predicates" (1993:567).

[52] Another logician siding with Ladd-Franklin in characterizing excluded middle as exhausting semantic possibility is Couturat (1905:24), although he is drawing his view from Ladd-Franklin. He writes "Comme l'a fait justement remarquer Mrs. Ladd-Franklin (Baldwin, *Dictionary of Philosophy and Psychology*, art. *Laws of Thought*), le principe de *contradiction* ne suffit pas à définir les contradictoires; il faut lui adjoindre le principe du milieu exclu, qui mériterait tout aussi bien ce nom. C'est pourquoi Mrs. Ladd-Franklin propose de les appeler respectivement *principe d'exclusion* et *principe d'exhaustion:* en vertu du premier, deux termes contradictoires sont *exclusifs* (l'un de l'autre); en vertu du second, ils sont *exhaustifs* (de l'univers du discours)."

[53] Furthermore, Tappenden writes that while supervaluationism also respects the principle of non-contradiction, "an instance of excluded middle is always at risk: having uttered it, one is committed, at least in principle, to stipulating sharper boundaries to resolve indeterminacies whenever they are produced or arise on their own. Such stipulations may seem pointless, given the standards of precision in a given context" (1993:566).

[54] Peirce himself continuously touted the importance of logic for guiding a variety of other disciplines throughout his life. In his 1896 review of Schröder's *Exact Logic*, he wrote that "questions of logic ought *not* to be decided upon philosophical principles, but on the contrary, that questions of philosophy ought to be decided upon logical principles" (1896; EP2:385-7).

[55] See CP4.237. At one time, Peirce thought that vague statements had very little value (see W6:175-6).

[56] For example, see Hookway (2002), Lane (1997), Nadin (1975).

[57] See Black (1937), Burks (1946), Russell (1923), Hempel (1939), and Morris (1971). Hookway (2002) notes Frank Ramsey's influence over Wittgenstein.

[58] The remainder of the quotation reads: "The typical general is the substance of a wish or purpose. I wish for a young horse. I do not wish for a stallion, a mare, or a gelding,—I just wish for a young horse. The characteristic of the general is that the principle of excluded middle does not apply to it. The type of the vague is that state of things at the present instant. It is as settled a fact as the past, yet all that we can change is the present instant. The only thing we are quite certain of is what is this instant experienced. Yet nothing is so hopelessly occult. The characteristic of the vague is that the principle of contradiction does not apply to it. It is this or that; one or other and neither. Or, better, it is either not this or not that, and it is both, like a point where a continuous variable makes a *saltus*. Of course, the vague is mere fiction. Yet the only thing thoroughly real is the present state of things and that is vague. The merely possible is vague, since the only mode of being it has consists in its being possibly so, and possibly not so." (L229:NEM3:913).

[59] Nor is it always clear how these distinctions are meant to relate to some of the distinctions Peirce made later in life. For example, Peirce (1906:511) writes "Indefiniteness in Breadth may be either Implicit or Explicit."

[60] This attribution to Brock is somewhat misleading, partly because (a) Brock understands Peirce's "logic of vagueness" as covering his whole logic of indeterminacy, and (b) not explicitly connected to borderline cases. But Peirce makes it abundantly clear that he is at least partially considering vagueness as it is contemporarily understood for he writes "What do we mean by 'vague'? The stock instances are those of 'much' and

'little'. Some things are definitely much: others are definitely little. Between them is a border to which a man might at one hour apply the word 'much', at another other the word 'little', in the endeavor to express the same truth. Does a man who applies either word, say 'much', to a case falling within that border tell the truth or not?" (R530:13, 2nd pagination).

[61] Explicitly, Nadin writes that "[v]agueness concerns the informed depth" (1983:156).

[62] Brock has also argued that vagueness qua borderline cases is indeterminacy of essential depth (1979:46).

[63] In addition, unlike natural-kind terms, no amount of further investigation into the denoted objects will produce sharply-defined meanings for vague terms. See (Goldstein 1988:447; Williamson 1992b:155-7; Keefe 2000b:76-8).

[64] Williamson takes Russell's definition as follows: "a word is vague just in case it can have a borderline case, in which its application is 'essentially doubtful'" (1994:55).

[65] This is further reinforced by Russell when he writes that the "fact is that all words are attributable without doubt over a certain area, but become questionable within a penumbra, outside which they are again certainly not attributable" (1923:87, 89).

[66] Haack, of course, qualifies this statement, see (1996:110).

[67] While Short does not actually say much about Peirce's understanding of vagueness, it is clear from his discussion referenced above that this alternate sense is how we should view Peirce's use of the word. Robert Lane offers more of a warning, writing that "there is some danger of confusing what Peirce means by "vagueness" or "imprecision" of the predicate with fuzziness or borderline vagueness. Peirce's predicate vagueness is not the same as borderline vagueness" (1998:89). While understanding of Peirce's theory of vagueness as unspecity is not entirely without textual evidence nor has it failed to produce interesting results, it is tenable only when vagueness is treated as informed depth. For example, Short uses Peirce's conception of vagueness qua unspecificity as a way of translating between specific theories (2007:333-7). This is also Quine's thesis concerning the vagueness of color in *Word and Object* (1960:41).

[68] Peirce also tells us that the statement he uses in this example is indeterminate in another way. In this same document, Peirce writes "The statement, "A certain friend of mine has only a hundred and twenty-three hairs on his head, at most" is certainly remarkably precise, as assertions go. But while it is not "vague," which simply means "not precise," it is *indefinite*, since it abstains from letting you know just who it is that I am referring to" (R48:42).

[69] Peirce commonly connected the definition of vagueness with the non-application of the law of non-contradiction. For example, Peirce writes

> Perhaps a more scientific pair of definitions would be that anything is
> *general* in so far as the principle of excluded middle does not apply to it
> and is *vague* in so far as the principle of contradiction does not apply to it.
> Thus, although it is true that "Any proposition you please, *once you have*
> *determined its identity*, is either true or false"; yet *so long as it remains*
> *indeterminate and without identity*, it need neither be true that any
> proposition you please is true, nor that any proposition you please is false.
> So likewise, while it is false that "A proposition *whose identity I have*

determined is both true and false," yet until it is determinate, it may be true that a proposition is true and that a proposition is false" (CP5.448).

Or again, that,

> The *vague* might be defined as that to which the principle of contradiction does not apply. For it is false neither that an animal (in a vague sense) is male, nor that an animal is female (CP5.505).

And again, that,

> in a continuum, "there is a possible, or potential, point-place wherever a point might be placed; but that which only *may be* is necessarily thereby indefinite, and as such, and in so far, and in those respects, as it is such, it is not subject to the principle of contradiction, just as the negation of a may-be, which is of course a *must-be*, (I meant that if "S *may be* P" is untrue, then "S *must-be* non-P" is true), in those respects in which it is such, is not subject to the principle of excluded middle" (CP6.182).

And to Paul Carus, he writes,

> a *can be* may be defined as that which is not subject to the principle of contradiction. On the contrary, if of anything it is only true that it *can be X* [then] it *can be not X* as well (CP8.216).

See also (L224:NEM3:815). Peirce also rejected the idea of denying LNC when propositions involve precise terms. He writes,

> no philosopher has yet been found to maintain that any proposition is in precisely the same sense absolutely true and false at once. Hegel, it is true, professes to do this; but that was because he mistook the relation actually existing between his own thought and that of ordinary men (R748; NEM3:753).

Also, Peirce states that "The relation of truth and falsity as formal logic conceives it is defined in two clauses, as follows: 1st, No proposition is both true and false. Every proposition is either true or false. The first clause is called the principle of contradiction, the second the principle of excluded middle" (R748; NEM3:751,752). Peirce took LNC to universally apply to propositions whose sense is entirely determined (R748; NEM3:753).

[70] It is important to recognize that Peirce was aware, not only of the two different types of possibility, but also that the indeterminacy of a term can be taken in a number of different senses (see EP2:356). The two different types of interpretation Peirce is referring to are the subjective and objective interpretations of possibility (see EP2:354-356). In addition, Peirce notes in EP2:394 that with a view to brevity he did not make adequate time to

articulate how vagueness and generality differentially affect logical depth and breadth. See (Morgan 1979:65-66; 1981; Lane 2007:555). Peirce gives the following example of a case of *objective possibility*: "A man says, "I *can* go to the seashore if I like." Here it is implied, to be sure, his ignorance of how he will decide to act. But this is not the point of the assertion. It is that, the complete determination of conduct in the *act* not yet having taken place, the further determination of it belongs to the subject of the action regardless of external circumstances" (EP2:355). In this example, Peirce contends that the man's statement is emblematic of *subjective* possibility insofar as the man is *ignorant* of how he will act, but, in addition, it also signifies an *objective* possibility. It is objectively possible that the man *can* decide to go to the seashore and it is objectively possible that he *can* decide *not* to go to the seashore. Peirce's point is that both are *really* possible because the determination of the action lies within the power of the agent (or utterer, or object) who, in his act, unknowingly decides which one is to become an actuality.

[71] But see also R9:4 [c.1903] where Peirce says that terms can be used vaguely in existential quantifications. Dipert wrote in 1981 that a "thorough and accurate account of Peirce's quantifiers cannot be said to exist" (1981:585). See (Beatty 1969; Martin 1976; Turquette 1981; Zeman 1986:6-7; Hintikka et al. 1994:119).

[72] See (Morgan 1979:65-66; 1981; Lane 2007:555).

[73] Illustrating the collapse of modalities is informative because (1) it suggests why Williamson's objection is convincing since the non-application of LNC for an existentially-quantified proposition is absurd when the truth-values of propositions are evaluated in differing information states, (2) it further reinforces why Peirce took precautions to index gamma graphs to information states and why this procedure was later abandoned for tinctured graphs. Consider the following:
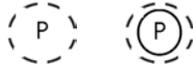
In the above example, the two graphs represent the possibility of a proposition *P* that may be true or false in a given information state. Suppose then, the information state is increased so that *P* is now known to be false. This gives the following three graphs:

Next, the broken cut-P can be iterated into the cut-P, giving the following

This graph reads: it is not the case that it is not necessary that P. That is, P is necessary. This consequence is undesirable because from not P, P is necessary can be deduced, yet unpreventable given (1) the lack of notation governing graphs in different information states and (2) the gamma graph rules (the only restriction on deiteration and iteration in

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the gamma graphs is that graphs cannot be iterated or deiterated *across* broken cuts) (see Roberts 1973:82, R478:158).

[74] Morgan identifies late 1896 and early 1897 as the dates where Peirce makes his modal shift (1981:208). R787 is identified by Lane as marking the first step away from the IR-account of modality, Lane writes that the location of Peirce's modal shift occurs in "The Logic of Relatives" because it "was there that he first insisted on a sort of modality that cannot be defined in epistemic terms" and it is there that "Peirce himself eventually identified "The Logic of Relatives" as the work in which his thinking about modality took an important leap forward" (2007:561).

[75] See CP5.527, R291. Peirce himself attests that questions about multitude, specifically that of the cardinal comparability of sets, led him to reevaluate modality. For example, in a letter to James, Peirce writes that he "reached this truth by studying the question of possible grades of multitude" (CP8.308).

[76] Peirce calls the IR-account "nominalistic" (see CP6.367).

[77] The illustration that Peirce offers of this point concerns the hypothetical example of two chemists reporting to Peirce upon the contents of a hundred bottles. The first chemist tests the hundred bottles for fluorine, finds the chemical element present in the majority, and sends his report to Peirce. The second chemist tests the same hundred bottles for oxygen, finds the chemical element present in the majority, and also sends his report to Peirce. Now consider the claim that "it is *impossible* that fluorine and oxygen are found together in at least one bottle of the hundred bottles tested". On the IR-account, this claim is *false* (i.e., it is possible) because Peirce, who is in a hypothetical state of absolute intuition of all the facts and laws in the world of sensible experience, would judge—on the basis of what has been reported by both chemists—the claim to be false. IR-modality is conditioned by how we *would* judge provided we had all the facts *at hand*. On the IW-account, the statement is known to be *false* in advance of any experience because the statement is seen to be preliminarily impossible, not only in our actual world (the world of sensible experience) but in any world. Namely, it is false that "it is *impossible* that fluorine and oxygen are found in at least one bottle of the hundred bottles tests" because the claim is *impossible* if "fluorine is present in the majority" and "oxygen is present in the majority" are true.

[78] The relevant passage reads: "Suppose that after a person had said that something was much, reserving, of course, his natural right to understand 'much' in any sense the word would bear and that he might chose, a second person were to declare that he was in the wrong. This would be tantamount to declaring the thing not to be much, while renouncing the right to take 'much' in a sense to suit himself, but, on the contrary, allowing the interpreter of his speech to take the word in any legitimate sense, that *he* might choose" (R530).

[79] An asterisk is used to indicate that the valuation does not occur in the same interpretation.

EDUCATION
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Ph.D. Candidate: Philosophy	The Pennsylvania State University
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PUBLICATIONS

The UFAIL Approach: Unconventional Weapons and their 'Unintended' Medical Effects. Forthcoming in 2010. *Bulletin of Science, Technology & Society*.

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PRESENTATIONS

April 8, 2006   Commentator on "Am I My Brother's Keeper" by Albert Spencer – The Sixth Annual Donald G. Wester Conference: Josiah Royce on Ethics and Community. Oklahoma City, OK.

Sept. 29, 2006   The Role of Replication in the Growth of Symbols. 2006. Annual Meeting of the Semiotic Society of America. West Lafayette, IN.

Nov. 7, 2005   The Symbolic Self. 2005. 8th International Meeting on Pragmatism. Pontifical Catholic University of Sao Paulo, Brazil.

Oct. 28, 2005   Peirce and the Symbolic Self. 2005. Institute for American Thought. Indianapolis, IN.

AWARDS, GRANTS, FELLOWSHIPS

University Fellowship – The Pennsylvania State University, 2007–2012.

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TEACHING EXPERIENCE

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Ethics & Values in Science & Technology, 2008 – Steven Walton
Technology and War, 2008 – Steven Walton
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Philosophy of Science, 2005 – Timothy D. Lyons

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Descartes, Locke, Leibniz, & Hume, 2008 – Emily Grosholz
Kant's Moral & Political Philosophy, 2007 – John Christman
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Symbolic Logic, 2009 – Emily Grosholz
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European Philosophy Seminar, 2007 – Dennis Schmidt
Merleau-Ponty, 2007 – Vernique Foti
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