# Achievement, anxiety, self-concept and motivation among college algebra students within a cooperative learning structure 

Candace Oluwasanmi

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## B.S., MATHEMATICS

## THESIS

Submitted in Partial Fulfillment of the
Requirements for the Degree of
Master of Science Mathematics

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## DEDICATION

תודה לך אלוהים מלך היקום! האב ובנו רוה הקודש אתה הם כל הסיבות שלי!
To my husband Muyiwa, there are not enough words to express what a wonderful husband you are. Without your love, support and encouragement I know I would not be where I am today. And I know I would not have enjoyed the ride nearly as much without you. Mo ni ife re! To my parents, thank you both for doing your best with me and my brother, I know it wasn't easy. To my brother Lawerence, the smart one; you have continually found new ways to challenge me and therefore have made me a more critical thinker. By being yourself, through example, you have raised the bar for me, because of this I expect so much more of myself. Thank you for your love, support and encouragement throughout my life. To my Aunt LaVern you are not just my godmother but also my best friend. Thank you for all that you are to me.

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# ACHIEVEMENT, ANXIETY, SELF-CONCEPT AND MOTIVATION AMONG 

 COLLEGE ALGEBRA STUDENTS WITHIN A COOPERATIVE LEARNING STRUCTUREby

## Candace Oluwasanmi

B.S., Mathematics, University of New Mexico, 2007<br>M.S., Mathematics, University of New Mexico, 2011


#### Abstract

Cooperative learning methods have been shown to increase student achievement and social skills, therefore these methods have been promoted for the intervention of falling student achievement. This study was designed to examine the differences between a traditional lecture classroom structure and a cooperative learning structure in student achievement, anxiety, self-concept and motivation among College Algebra students. The sample is a convenience sample since participants were taken from pre-formed sections of College Algebra at a large urban university. This is a quantitative study, designed as a $2 \times 2$ factorial quasi-experiment. Using multivariate and univariate analysis of variance procedures, math achievement, anxiety, self-concept and motivation gains are examined. In addition, predictions about the target population's probability of passing the course for both treatment and comparison groups are estimated using a logistic regression analysis.


TABLE OF CONTENTS
Approval page ..... i
Title page ..... ii
Dedication. ..... iii
Acknowledgements ..... iv
Abstract ..... v
Table of Contents. ..... vi
LIST OF FIGURES ..... viii
LIST OF TABLES. ..... ix
CHAPTER 1: INTRODUCTION ..... 1
Contextual Background ..... 1
Overview of the Study ..... 3
Structure of the Study ..... 4
Overview of Results ..... 5
CHAPTER 2: REVIEW OF THE LITERATURE ..... 8
Five Key Elements for Success ..... 8
Conventional Implementations of Cooperative Learning ..... 9
Group Process Behavior ..... 10
Group Process Behavior and Psychological Factors ..... 11
CHAPTER 3: METHODS ..... 14
Research Design ..... 14
Participants ..... 16
Instrumentation and Measurement ..... 16
Procedure ..... 18
Application of the Cooperative Learning Treatment ..... 20
Timeline of the Study ..... 22
Scoring and Coding ..... 23
Data Analysis ..... 24
CHAPTER 4: RESULTS ..... 26
Response ..... 26
Descriptive Statistics ..... 26
Multivariate Analysis of Variance ..... 31
Algebra Preliminary and Final Exams ..... 32
Anxiety Pretest and Posttest ..... 40
Self-Concept Pretest and Posttest ..... 41
Motivation Pretest and Posttest ..... 43
Logistic Regression. ..... 45
Validity ..... 48
CHAPTER 5: DISCUSSION ..... 52
Restatement of the Study Purpose ..... 52
Summary of Procedures and Methods ..... 52
Result Implications ..... 54
Recommendations for Further Research ..... 59
APPENDICIES ..... 61
APPENDIX A: SUPPLEMENTARY FIGURES ..... 61
APPENDIX B: SUPPLEMENTARY TABLES ..... 64
APPENDIX C: ALGEBRA PRELIMINARY EXAM ..... 69
APPENDIX D: ALGEBRA FINAL EXAM ..... 73
APPENDIX E: ANXIETY INSTRUMENT ..... 78
APPENDIX F: SELF-CONCEPT INSTRUMENT ..... 79
APPENDIX G: MOTIVATION INSTRUMENT. ..... 80
APPENDIX H: GLOSSARY OF SPECIALIZED TERMS ..... 81
REFERENCES ..... 84

## LIST OF FIGURES

Figure 4.1 age distributions ..... 61
Figure 4.2 gender distributions ..... 61
Figure 4.3 ethnicity distributions ..... 62
Figure 4.4 distribution of students who tested into College Algebra ..... 62
Figure 4.5 distribution of students who completed intermediate algebra ..... 63
Figure 4.6 distribution of student's socio-economic status ..... 63
Figure 4.7 preliminary exam and algebra gain for comparison group ..... 37
Figure 4.8 preliminary exam and algebra gain for treatment group ..... 37
Figure 4.9 box plot of preliminary exam. ..... 39
Figure 4.10 box plot of final exam ..... 39
Figure 4.11 logistic regression equations ..... 47

## LIST OF TABLES

Table 4.1 response by population. ..... 64
Table 4.2 Chi-square test for gender ..... 27
Table 4.3 ethnicity by population. ..... 64
Table 4.4 condensed ethnicity cross tabulation ..... 28
Table 4.5 Chi-square test for ethnicity ..... 29
Table 4.6 condensed socio-economic status cross tabulation ..... 30
Table 4.7 Chi-square test for socio-economic status ..... 30
Table 4.8 multivariate tests ..... 64
Table 4.9 MANOVA tests between subjects ..... 65
Table 4.10 ANOVA tests between subjects for algebra gain ..... 33
Table 4.11 parameter estimates for algebra gain. ..... 34
Table 4.12 ANOVA tests between subjects for anxiety gain ..... 40
Table 4.13 parameter estimates for anxiety gain. ..... 41
Table 4.14 ANOVA tests between subjects for self-concept gain ..... 42
Table 4.15 parameter estimates for self-concept gain ..... 43
Table 4.16 ANOVA tests between subjects for motivation gain ..... 44
Table 4.17 parameter estimates for motivation gain ..... 44
Table 4.18 parameter estimates for logistic regression ..... 47
Table 4.19 test for normality ..... 67
Table 4.20 tests for homogeneity of covariance matrices ..... 67
Table 4.21 tests for correlations between instruments ..... 68

## CHAPTER 1

## Introduction

## Contextual Background

It is well documented that many students from elementary school through graduate school have an attitude of contempt, fear or aversion towards learning mathematics (Hersh \& John-Steiner, 2011). In conjunction with this negative disposition to learning mathematics, students are more inclined to avoid active engagement in math and often concede to poor academic achievement (2011). Common reasons students provide to account for their poor performance is that they have never been good at math, or don't see the use for it, or they just don't like it, and consequently they tend to avoid taking mathematics courses (2011). Along with these students' perceptions of mathematics, student achievement has decreased in the United States in comparison to other countries. One study on international mathematics literacy ranked the United States $27^{\text {th }}$ (of 39 participating countries) in combined mathematics literacy with performance averages of 20 other countries significantly higher than the United States (Lemke et al., 2004). Unless appropriate interventions are implemented, the educational outcomes of students in the United States will continue to fall short.

Academic achievement of College Algebra students at the university involved in this study is no exception. The average failure rate for College Algebra students from the fall semester of 1998 to the spring semester of 2009 was $47.6 \%$ (Institutional Data). For many students this is not their first attempt to pass the course. With an average enrollment of 968 students per semester, this means that if effective intervention is not implemented in the future, approximately 450 students will fail this course each semester.

With many universities and colleges facing similar problems, this university has been aware of the issue and has attempted to implement interventions over the years. In addition to instructor office hours, students have been offered many other opportunities for extra help. Below is a list of supplemental resources and interventions the university has provided to help increase the success rates for College Algebra students.

1. University sponsored peer-tutoring has been offered since 1980 in multiple sites across campus and online.
2. Department sponsored one-on-one tutoring is offered by course instructors.
3. Grant supported peer mentors inside the classrooms of many basic freshmen courses, including College Algebra. Mentors help their students maneuver the particulars of the university by providing social, cultural, and academic support inside and outside the classroom which includes, but is not limited to tutoring.
4. Peer-tutoring for minority students sponsored by the College of Engineering.
5. An online interactive homework application integrated with the textbook that shows examples to similar problems; breaks problems into smaller parts and references the exact page of the textbook that supports the concept or skill being learned. Administrators have continued to monitor the success/failure rates of College Algebra and in recent years decided to try a new approach. During the semester students were sampled for this study, the mathematics department, working in conjunction with a university office which supports faculty development, piloted a new cooperative learning initiative to improve the success rates for College Algebra students. Although this approach had not previously been implemented in College Algebra generally, it has been applied within discrete sections of the course as well as other university departments
where the study was conducted.
Within the past four decades there have been approximately 1,200 studies comparing individualistic and/or competitive methods (the standard pedagogical styles) with cooperative methods to help improve student learning and achievement (Johnson, Johnson, \& Smith 1998; Jacob, 1999). These studies have shown that despite some obstacles, cooperative learning can be an effective intervention to improve student achievement. In addition to this positive effect on student achievement, studies on cooperative learning have provided evidence of psychological and social benefits for students across various fields of study. This added benefit makes cooperative learning particularly advantageous. Not only are these benefits an essential quality in the work place, but interventions known to incorporate a positive influence on student's social, affective and motivational attributes (in addition to increased achievement) are desirable since they have also been shown to help improve mathematics performance (National Mathematics Advisory Panel, 2008)

## Overview of the Study

This study compared cooperative learning methods and lecture methods implemented in College Algebra classrooms at a large urban research university. The first hypothesis is that students in the treatment group (cooperative learning classrooms) will have lower anxiety, higher achievement, self-concept and motivation compared to the comparison group (traditional lecture classrooms). Previous research has suggested that gender differences may exist since gender is a predictor of group interaction styles for small groups (Valentino, 1988; Webb, 1986). Thus, the second hypothesis is that females
in the treatment group will have decreased anxiety and increased self-concept and motivation when compared to the comparison group. In addition gender differences are expected within the treatment group. Specifically males in the treatment group are expected to have a less significant change in anxiety than females in the treatment group, while a parallel increase between males and females in self-concept and motivation is expected. Lastly, students in the treatment group are expected to be more likely to pass the course than students in the comparison group.

## Structure of the Study

Students were not randomly assigned to the treatment group or the comparison group, rather, students registered for a section of College Algebra of their choosing without any prior knowledge of which sections were to be taught using the cooperative learning method. Prior to the beginning of the semester, instructors elected to implement a cooperative learning method or a standard lecture method. Subsequently instructors were placed into the section numbers that coincided with their schedule. Considering these conditions already in place, this study is structured as a $2 \times 2$ factorial quasiexperiment, where the two independent variables are teaching method and gender, supplemented with a logistic regression analysis comparing the probability of success in the course between students in cooperative learning sections and students in lecture sections. Analysis of variance procedures will compare the means of the treatment and comparison groups on four dependant variables. To obtain a more clear view of the participant population, demographic data was collected to help facilitate and monitor confounds.

Ten variables are measured for each participant to investigate whether students in the treatment group demonstrated lower anxiety, higher achievement, self-concept and motivation compared to the comparison group. The independent variables for the study are teaching method and gender for the $2 \times 2$ factorial analysis and logistic regression analysis. The dependent variables are the preliminary exam, final exam, anxiety (pretest and posttest), self-concept (pretest and posttest), motivation (pretest and posttest).

## Overview of Results

At the beginning of data collection 566 students consented to participate and completed pretests. By the end of data collection, the number of participants was 173 in total, with $N=86$ for the treatment group and $n=87$ for the comparison group. To determine if the two groups are comparable, descriptive statistics were collected and distribution comparisons are made where appropriate. Placement qualifications are reported and are comparable for the two groups. A two-sample $t$-test suggested the two groups are comparable in age distributions. The $\chi^{2}$ test is used to compare distributions of gender, ethnicity and socio-economic status. All three $\chi^{2}$ tests indicate the two groups are comparable with these distributions.

Univariate and multivariate analysis of variance procedures are conducted on all instrumentation. Backward elimination is applied to obtain a more parsimonious result for the analysis of variance procedures. Multivariate tests indicate the preliminary exam, anxiety pretest, self-concept pretest and treatment/comparison group (called group in subsequent writing) variables are significant, with $p$-values < 001 for all four variables. The univariate test for the algebra achievement gain indicates the preliminary
exam, self-concept pretest and group variables are significant, with $p$-values of < .001, .007 and .005 respectively. The univariate test for anxiety gain indicates the anxiety pretest, self-concept pretest and group variables are significant, with $p$-values of $<.001$, <. 001 and .063 respectively. The univariate test for self-concept gain indicates the selfconcept pretest and group variables are significant, with p-values of .001 , and .038 respectively. The univariate test for motivation gain indicates the motivation pretest and group variables are significant with $p$-values of .027 , and < .001 respectively. The logistic regression analysis indicates the overall probability of passing the course is greater for students in the treatment group and is also dependent upon preliminary (algebra pretest) exam performance.

The study validation is three-fold. First, internal consistency is examined for the preliminary exam, final exam, anxiety, self-concept and motivation instruments. By design, all internal consistency assessment values range between 0 and 1 , with a value of one indicating the instrument is consistent $100 \%$ of the time. Internal consistencies of the preliminary exam, final exam, anxiety, self-concept and motivation instruments are calculated at $.48, .71, .92, .98$ and .93 respectively. Second, analysis of variance test assumptions are examined. Analyses indicate the data do not satisfy all test assumptions of: a simple random sample, (multivariate) normal distribution, homogeneity of covariance matrices and independence of observations. Finally, the logistic regression test assumptions are inspected. With the exception of the non-randomization of the sample, test assumptions are generally satisfied with respect to the assumptions of: a dichotomous result, statistically independent outcomes, specificity, mutually exclusive
and collectively exhaustive categories and a minimum of fifty cases or more per predictor.

## CHAPTER 2

## Review of the Literature

Cooperative learning is a type of Problem Based Learning, which is a pedagogy that centers on the student. In a Problem Based Learning course students are divided into groups of approximately two to five and given a structured task or problem. The students are then expected to interpret the problem, gather the needed information, identify possible solutions, evaluate options and present conclusions with the teacher acting only as a guide (Roh, 2003). At the heart of Problem Based Learning is the cooperative learning method; which has its tertiary origins from social interdependence, cognitivedevelopmental and behavioral learning theories (Johnson, Johnson, \& Smith, 1998).

## Five Key Elements for Success

Research on cooperative learning methods has increased over the last 40 years, and they have been shown to be an effective intervention for the success of students in mathematics (Johnson, Maruyama, Johnson, Nelson, \& Skon, 1981; Slavin, 1990; Townsend \& Wilton, 2003). Johnson, Johnson, and Smith derived the following five key elements critical for successful implementation of the cooperative learning method (1998).

1. First, the instructor must ensure each student perceives that his/her success is reliant on the success of the group; this is called positive interdependence and can be achieved by group rewards, dividing resources among students within the group, or assigning complementary roles to each group member (Slavin, 1983; Johnson et al.,1998).
2. The second element is individual accountability. This is where each student should
be individually assessed, by quizzes, exams, observations, etc. (Slavin, 1983; Johnson et al., 1998).
3. The third key element is to create an environment of cooperation, where students promote each other's success within each group. This is done by students helping, supporting, encouraging and praising classmates' efforts face to face (Johnson et al., 1998).
4. The fourth element needed is social skills, both interpersonal and small-group (Johnson et al. 1998). The social skills that should be modeled are leadership, decision making, trust building, communication and conflict management. In short, students need to be guided to cooperate with each other.
5. Finally the students need to be provided time to work on group processing (Johnson et al. 1998). Group processing is time students take to evaluate themselves and their cooperation with each other. They will identify positive and negative actions and make decisions on what actions to change so that group processes become more efficient (1998).

## Conventional Implementations of Cooperative Learning

Johnson, Johnson and Smith's five key elements for success are the foundation of many common types of cooperative classroom structures. The most commonly recognized implementations first divide students into assigned heterogeneous groups of approximately four to six members which are chosen from groups of high, medium and low achievers, ethnicity, gender and handicap. After groups are formed in class, typically a Student Team Learning (STL), Jigsaw, Learning Together (LT), Group Investigation or

Team Assisted Individualization (TAI) structure is implemented (Slavin, Sharon, Kagan, Lazarowitz, Webb, \& Schmuck, 1985; Slavin, 1990; Jacob, 1999). Within STL, there are, different methods of implementation consisting of Student Teams-Achievement Divisions (STAD), Teams-Games-Tournament (TGT), and Jigsaw II, a modification of the original Jigsaw structure (Slavin, Sharon, Kagan, Lazarowitz, Webb, \& Schmuck, 1985; Slavin, 1990; Jacob, 1999). The TAI implementation is of particular interest because it was developed specifically for implementation in mathematics classes due to the difficulty of using previously developed standard CL structures (Slavin, Sharon, Kagan, Lazarowitz, Webb, \& Schmuck, 1985).

## Group Process Behavior

Apart from of the type of cooperative learning structure an instructor chooses to implement, student learning and achievement is dependent upon group dynamics. Past research has shown that student actions within a group, including seating position, talkativeness, personality and leadership, are important variables for the successful cooperation of the group (Jaques, 1991). Another study on group process behaviors found that the outcome task performance (such as the learning and/or achievement of students on a given task or assignment) of a group is a product of the potential productivity minus the process loss (Hurley, \& Allen, 2007). Potential productivity is a combination of the resources available to students along with the task demands put upon the students. For instance, resources may include books, information, preparedness for the course, and supplies available such as rulers, compasses or calculators; demands put upon the students may be tasks assigned, teacher expectations, class schedule, etc. Process loss can
be thought of as the time taken from student learning and instead is used for the administration of the teaching method. For instance, students need to take time to form groups, realize task demands, communicate and coordinate strategies/solutions to produce a desirable group outcome. In general, the process loss is composed of the coordination loss, difficulties of coordinating multiple participants and student motivation loss (2007).

Hurley and Allen, through a qualitative analysis identified seven categories of student process loss behaviors (2007). The first two categories include preoccupation with the authority figure, i.e. the professor, or preoccupation with others in the group (2007). Students were also observed to be off-task in two ways, either passive, i.e. spacing out, or aggressive, i.e. attempting to distract others from the task (2007). In addition, three types of controlling behaviors were also identified. Directing-controlling was demonstrated by students' attempts to control or structure roles or behaviors of other students (2007). Resistant aggressive behaviors were displayed as non-constructive criticism of other students (2007). Resistant passive aggressive students would exhibit behaviors that disrupt cooperation within their group (2007).

## Group Process Behavior and Psychological Factors

Student behaviors and leadership are derived from personal views and beliefs as well as environmental reinforcements (Dowd \& Kelly, 2005; Jaques, 1991). The process loss behaviors of preoccupation with the authority or other students can be an indication of student anxiety with the course content. Here anxiety refers to "feelings of tension that interfere with the manipulation of numbers and the solving of mathematical problems in a
wide variety of ordinary and academic situations" from Tobias (as cited in Townsend, \& Wilton, 2003). Similarly the off-task passive and aggressive behaviors can be resulting from a lack of motivation. Motivation is "the internal state of the student that arouses, directs and sustains goal-oriented behavior" (Glynn, Taasoobshirazi, Brickman, 2009). The three remaining process loss behaviors of directing-controlling, resistant aggressive and resistant passive aggressive can all be derivatives of a students' self-concept relative to the content area studied, where self-concept refers to "a student's beliefs, feelings, attitudes and/or perceptions regarding one's ability to understand and perform tasks in mathematics" (Townsend, \& Wilton, 2003; Gourgey, 1982). However, self-concept, anxiety and motivation may be contributing to any or all of the seven process loss behaviors.

Despite the presence of these seven process loss behaviors in cooperative learning classrooms, students enrolled in these classrooms have demonstrated higher academic and social gains when compared to students in a competitive and/or an individualistic structure (Akinoğlu \& Tandoğan, 2006; (Johnson \& Johnson, 1981; Schmuck \& Schmuck, 1983; Sharan \& Sharan, 1976; Slavin, 1983, 1989) as cited in Mulryan, 1995; Johnson, Maruyama, Johnson, Nelson \& Skon, 1981; Valentino, 1988). In addition to these separate studies, a comparative meta-analysis of 168 studies on cooperative learning in college revealed that cooperative learning students repeatedly demonstrated academic gains greater than students in competitive or individualistic classrooms (Johnson, Johnson \& Smith, 1998). Two other studies on tertiary students in the West Virginia (studying College Algebra) and in New Zealand (studying statistics) demonstrated lower anxiety in cooperative classrooms (Townsend \& Wilton, 2003;

Valentino, 1988). Students in New Zealand also experienced higher motivation when instructors use a cooperative learning method (Townsend \& Wilton, 2003). Bouris, Creel \& Stortz found motivation gains are greater in cooperative classrooms in comparison to traditional classrooms, since there is a direct correlation between motivation and the active involvement of the learner (1998). Furthermore, students report a higher selfconcept in cooperative classrooms (Townsend, Tuck, Moore, \& Wilton, 1998). Other studies have suggested that math self-concept is related to math anxiety in college students, and students who exhibit higher anxiety will also demonstrate lower motivation (Marsh \& Tapia, 2002; Gourgey, 1982).

## CHAPTER 3

## Methods

## Research Design

The study design is three-fold. From the outset of this project the intention has been to use a 2 (Group) $\times 2$ (Gender) factorial design alongside a logistic regression analysis. The factorial portion of the design utilizes analysis of variance procedures to determine if there is a difference among (dependent) variable means on Factor 1 (Group) and Factor 2 (Gender). As the name ( $2 \times 2$ factorial design) implies, there are two categorical choices for each factor. That is, for Factor 1 (Group) students can be either in the treatment group (cooperative learning) or the comparison group (traditional lecture); for Factor 2 (Gender) students are classified as either male or female. The factorial design utilizes a 2 (Group) x 2 (Gender) MANOVA procedure to investigate possible multivariate effects and interactions. Since sample data and instrumentation do not maximally satisfy MANOVA test assumptions, a series of ANOVA procedures is conducted, one on each dependent variable. Backward elimination is then applied to both multivariate and univariate analysis to obtain a more parsimonious result. Logistic regression analysis is used to determine if treatment students are more likely to pass the course, based on relevant predictor variables. However, because the sample was not randomized, a static-group pretest-posttest structure along with descriptive statistics are used to clarify and qualify the former analysis (Fraenkel \& Wallen, 2009). The static group analysis is applied to only the algebra instrumentation to further detail treatment effects on this dependent variable.

As in all static-group pretest-posttest experiments, the treatment and comparison groups are sampled at the same time (Fraenkel \& Wallen, 2009). Standard analysis of static-group experiments involves mapping an individuals' pretest score against their respective posttest gain. In this case, the pretest is the preliminary exam and the posttest is the final exam; the gain calculated is called the algebra achievement gain. Achievement gain is calculated by subtracting the students' preliminary exam score from their final exam score (2009). This type of description is preferable because it provides a better description of the change in individual student performance. It is also worth noting that for this type of analysis, a student's potential for achievement gain is inversely related to their preliminary exam performance. Because of this inverse relationship, students who performed very well on the preliminary exam have little opportunity, if any to increase their performance on the final exam.

Comparing pretest performance to gain does provide a useful description of the change in individual achievement; although this description is limited since only one variable is evaluated at a time. Another more comprehensive analysis will be useful in illuminating main effects and interactions between variables. For this purpose a MANOVA test was chosen. The MANOVA test examines possible differences among (dependent variable) mean gains on two factors. A series of ANOVA tests are performed post-hoc on each dependent variable, to check for differences among mean gains on Factor 1 and Factor 2. This series of ANOVA tests are included to support MANOVA results, since the algebra, anxiety, self-concept and motivation instruments are somewhat intercorrelated. To predict if students in the treatment group are more likely to pass the course, predictor coefficients are provided using logistic regression.

## Participants

The target population was all students enrolled in College Algebra at a large urban university. The accessible population was students enrolled in College Algebra during fall of 2009, approximately 1,200 students. The treatment and comparison groups were both sampled from this accessible population. The treatment sample is taken from the group of students enrolled in any section using cooperative learning methods. The comparison sample is taken from the group of students enrolled in any traditional lecture section. A majority ( 21 sections of 23 total sections) of instructors teaching sections of College Algebra at the university's main campus allowed student recruitment. After student recruitment was complete, the total number of students consenting to participate was 566.

## Instrumentation and Measurement

This study includes two independent variables, eight dependant variables as well as demographic measures to help monitor confounds. The independent variables are group and gender. Group was determined by the teaching method in the class the student was enrolled in and was established by section number. Section number was reported by each student and confirmed by the university Registrar. Gender was self-reported through a demographic survey. Anxiety, self-concept and motivation pretests and posttest were also self-reported and measured by Likert scale. The math anxiety inventory (see Appendix E) has a range of total summed scores of 20-80 on the original inventory (Betz, 1978). The math self-concept inventory (see Appendix F) has a range of total summed scores of 27-135 on the original inventory (Gourgey, 1982). The science motivation
questionnaire for non-science majors inventory (see Appendix G) has a range of total summed scores of 30-150 on the original inventory (Glynn, Taasoobshirazi, Brickman, 2009).

Algebra gain is the (raw percent) gain earned from the algebra preliminary exam (see Appendix C) to the algebra final exam (see Appendix D), ranging from -100 to 100 . To prevent any scoring inconsistencies, the preliminary and final exams were scored by scantron at the university's Information and Technology Test Scoring department, not by individual instructors. Results for both algebra exams were rescored in Microsoft Excel after the end of the semester to ensure accuracy.

The online pretests and posttests were verbatim; however the algebra pretest and posttest were not. The intent of the preliminary algebra exam is to measure the basic prerequisite knowledge, providing a baseline measurement of Intermediate Algebra competency for each student. Familiarity with topics such as linear equations, inequalities, polynomials, factoring, exponents, radicals, fractional expressions and equations, quadratic equations, perimeters, areas of simple geometric shapes, logarithms and problem solving skills is assumed upon entering College Algebra and is tersely assessed by the preliminary exam (Institutional Data). The intent of the algebra final exam is to measure College Algebra competency. Upon successful completion, competency in evaluating equations, functions, graphs, polynomial, rational, exponential, logarithmic functions and particularly linear and quadratic functions is expected along with application and problem solving skills involving simple geometric objects (Institutional Data). While calculating achievement gain is helpful for seeing a change in individual performance, the exact meaning of achievement is somewhat elusive here
since these two exams tested different content knowledge. The preliminary exam tested for Intermediate Algebra proficiency while the final exam tested for College Algebra proficiency. Using the same pretests and posttests would have been ideal. However the preliminary and final exams were the measures in place during data collection. For this study these two exams are assumed to have similar scale measurements.

## Procedure

All twenty-three section instructors of College Algebra at the university's main campus were recruited to participate. Of these, twenty-one section instructors chose to participate in the study. Instructors were then given a synopsis of the study including expectations, procedures and rights of students and instructors. During the second week students were recruited in class, as a group, within each section. Students were given an oral synopsis of the study's purpose, expectations and participant rights during this recruitment.

After recruitment, students were sent an email invitation directing them to the online survey. After students logged into the survey site, they were provided with a written informed consent on the first page. Each student was asked if they agreed and still wished to participate. Student participation was encouraged through the incentive of extra credit. The amount of extra credit offered was selected by individual section instructors. To prevent coercion, an alternative math assignment was offered for the same amount of extra credit (within each student's respective section) to students who wished not to participate. The synopsis provided on the informed consent page was as nebulous as
possible to help minimize students purposely skewing data. The synopsis included the following:

- Research Purpose: "The purpose of this study is to investigate teaching methods to help improve the quality of mathematics instruction at the university."
- Research Focus: "Teaching methods are the focus of this study."
- Participant Requirements: "Students will need to complete an online demographic survey, in class preliminary and final exam, and an online entry and exit inventory."
- Informed Consent: "When you login online to take the survey, the first page will be your informed consent to participate. If you agree, click "Yes" and you will be taken to the survey. If you do not agree, click "No", you will be logged out immediately and your data will not be recorded."
- Notice of ability to drop out of the study: "Participants will reserve the right to skip any questions they do not wish to answer and/or withdraw from the study without any adverse consequences."

Students received individual email invitations to the online surveys on the second and fifteenth weeks of the semester. Students were given approximately one week to complete the online surveys. Student responses to the online surveys and in class exams were maintained for less than one year by the university's Information and Technology department. In addition to the data maintained at the Information and Technology department, the principal investigator collected and stored students' final course grade, GPA and section number, which were provided by the university's Registrar in order to ensure the accuracy of self-reported data.

## Application of the Cooperative Learning Treatment

The cooperative learning treatment was applied in two separate ways. The first treatment application was designed by an experienced cooperative learning instructor who implemented the application in her section alone. Her design is straight forward. Students are expected to prepare for class by completing handwritten notes from the textbook on the section that will be studied in class. A typical class begins with a review of common mistakes from students' previous work and an introduction of the day's concepts and goals. Next students are assigned a list of problems to complete within their groups, with priority given to a different problem for each group. After students have collaborated with their peers and the instructor, they present their priority problems to the entire class. After class, students synthesize their learning by completing an online homework assignment which reinforces the concepts learned in class. The second cooperative learning application is more fluid and explicit, since the other twenty section instructors are considered novice instructors because their experience using cooperative learning methods is minimal.

To provide support, resources and save preparation time for the novice cooperative instructors, curriculum was prepared before the beginning of the semester by mathematics faculty and divided into ten teachable units. Each teachable unit set a mental stage for the instructor to prepare and guide student learning and provided a listing of pre-class, in-class and after class activities. Instructor's then tailored these teachable units to design their classes based on the skills and needs of their students. Each teachable unit also included a list of what students should be able to do after completing the unit and how the unit contributes to the overall importance and relevancy of the course objectives.

In addition, the teachable units included the activities and expectations students should satisfy in order to demonstrate their mastery of the unit and course objectives.

Students were expected to be prepared for in-class activities by completing specific reading assignments, homework and quizzes prior to attending class. During class, students were encouraged to work problems themselves, work and discuss strategies with peers in small groups and make direct contact with the instructor. After class, students synthesized their learning by completing an online quiz that assessed their knowledge of unit objectives. Direction was also provided in the teachable unit to the instructors, so they could judiciously spend their time in class facilitating the cooperative learning method and explaining how students should work within their groups. Finally a schedule suggestion was included in each teachable unit. The schedule recommended instructors:

1. At the beginning of class, address misconceptions and common mistakes from previous work.
2. Introduce goals for the next class session and a plan to achieve these goals.
3. At the end of class provide a brief conclusion and summary that integrates the day's concepts and introduces concepts for the next class session.

Although these two applications were not based on any of the common cooperative classroom structures, they did incorporate Johnson and Johnson's five key elements for success. Positive interdependence was achieved in both applications through small group work and in some cases was also encouraged by the grading scheme used. Individual accountability was attained by assigning individual homework, quizzes and exams. An environment of cooperation was promoted by instructors encouraging and
coaching students to further cooperation within groups and between groups whenever possible. In addition, the experienced instructor (in her section only) also met with each student individually at the beginning of the semester to get to know each student and to answer their questions on the class structure. Social skills were promoted in the first application by the instructor coaching students to work together while in their groups and were reinforced during student presentations. Social skills were promoted in the second application by encouraging instructors to coach students to work together while in their groups. Group processing was encouraged in the first application through an extra credit incentive by completing a study plan outlining changes in study habits, class participation and seeking outside help. Group processing time was minimal in the second application and took place while students discussed strategies with peers and the instructor of how to solve problems. One important caveat to the second application is that the teachable units, which encouraged the use of the five key elements for success, were used as resources which instructor's tailored to the skills and needs of their students and did not necessarily include all activities and guidelines.

## Timeline of the Study

Sampling of students took place during the fall semester of 2009. Pretests, including the in class preliminary exam, online demographic survey, anxiety, selfconcept and motivation instruments were administered during the second week of class. The latter four instruments were measured concurrently as an online survey. The online posttest included the anxiety, self-concept and motivation instruments which were administered during the fifteenth week. Students were given one week to complete both
online surveys. The in class final exam was administered on the sixteenth week of the semester, which completed the data collection for this study.

## Scoring and Coding

Upon completion of the data collection, coders confirmed the accuracy of online survey data, as well as scanned preliminary and final exam data. Subsequently, a list of participants was sent to the university Registrar's office for verification of student section number, final grade in the course and cumulative GPA. The two data files were then combined and the names of students replaced with randomly generated numbers to ensure student privacy. The preliminary and final exams were then rescored in Microsoft Excel to authenticate accurate scores from the university's Test Scoring department. Sampled student data were then divided into treatment and comparison groups according to section number. To ensure students were classified into the proper group, the self-reported section number from the online survey was re-checked against the student's section number reported by the university Registrar.

Next, the number of student responses for the preliminary and final exams was determined for the two groups. Students not returning both preliminary and final exams were excluded from the study because a change in achievement could not be determined without record of these two exams. Student responses for the anxiety, self-concept and motivation surveys were then imported into SPSS and scored. Scoring of negatively stated questions on the anxiety survey was reversed so that a lower score on the survey suggests higher anxiety (Betz, 1978). Scoring on the self-concept survey was also reversed for negatively worded items, so that a higher score indicates a more favorable
mathematical self-concept (Gourgey, 1982). Similarly, negatively stated items on the motivation survey were reversed scored so that a lower score indicates a less motivated student (Glynn, Taasoobshirazi, Brickman, 2009).

During recruitment students were directed to skip any questions within the online survey they did not feel comfortable providing answers to. For this reason there are occasional missing values within the data set. To help provide a more accurate analysis of student anxiety, self-concept and motivation, the mean scores for these pretests and posttests were used instead of a summed scores as in the original instrumentation. The raw mean scores were also used for the algebra pretest and posttest to calculate the algebra gain.

## Data Analysis

Since randomization was not a part of the research design, descriptive statistics were collected so that the treatment group can be compared with the comparison group. The number of students responding to instrumentation is reported in table form. The percent of students who tested in with an ACT/SAT and/or passed the prerequisite course are also reported. A two-sample $t$-test is used to determine if the treatment and comparison groups have a significant difference in age distributions. The Chi-Square test is used to determine if there is a significant difference among gender, ethnicity and socioeconomic status distributions between the treatment and comparison groups.

The MANOVA is chosen over multiple univariate analysis procedures for two primary reasons. The MANOVA procedure reduces the risk of a Type I error over performing multiple univariate procedures. In addition the MANOVA accounts for
correlations between variables and can look for interactions between these variables.
These relationships are an important aspect of analysis and is unseen by multiple univariate tests. Because sample data do not ideally satisfy MANOVA test assumptions, this test is followed by a series of ANOVAs. The ANOVA tests are intended to supplement and moderate MANOVA results since correlations between dependent measures are not ideal. In addition descriptive statistics are provided to facilitate a monitor of possible confounding factors. Finally, predictor coefficients to estimate the probability of students passing the course are provided using logistic regression.

## CHAPTER 4

## Results

## Response

Approximately $51 \%$ of the treatment and $22 \%$ of the comparison groups completed both preliminary and final exams. See Table 4.1 in Appendix B for the response by population. This begs the question: are these two groups inherently different? Perhaps students in the treatment group have distribution differences in age, gender, ethnicity or socio-economic status. Perhaps the treatment group has more students who prepared for the course in high school. Maybe more of the treatment group completed intermediate algebra, the prerequisite to College Algebra. The following descriptive statistics examine whether the treatment and comparison groups are comparable in these aspects and, if possible, how they compare to the university's general population.

## Descriptive Statistics

Age
Response was high for the age demographic, with only two students in the comparison group not reporting their age, thus $n=85$. However, all students in the treatment group reported their age, thus $N=86$. The mean age for the treatment group is 19.55 years with a standard deviation of 3.445 . The mean age for the comparison group is 19.22 years with a standard deviation of 3.093 . A two sample $t$-test is calculated at $t=$ .645 with a sig. (2-tailed) of $p=.520$. So there is no evidence to support that the mean ages are significantly different among the treatment and comparison groups. See Figure
4.1 in Appendix A, for a graph of age distributions. No additional bias should be contributed since the two groups are comparable in age.

## Gender

The treatment and comparison groups have slightly different distributions of male and female students (see Figure 4.2 in Appendix A). The treatment group is composed of $63 \%$ female and $37 \%$ male students while the comparison group is composed of $54 \%$ female and $46 \%$ male students. Among the general population of 19,610 (undergraduate) students enrolled in fall 2009, 55\% are female and 45\% are male (Institutional Data). So the comparison group appears to be a closer reflection of the institution's gender proportion than the treatment group. Despite this, the question still remains: is there a significant difference between these two distributions?

Response was high for this demographic with only one student in the comparison group not reporting his/her gender, thus $n=86$. However, all students in the treatment group reported their gender accordingly, $N=86$. The Chi-Square test is used to determine if there is a significant distribution difference among males and females between the treatment and comparison groups. Results show the Pearson Chi-Square $=1.542$, with a (2-sided) $p$-value of .214. So there is not evidence to support a significant difference of gender distributions between the treatment and comparison groups. Accordingly, gender should not introduce additional bias to the study.

## Chi-Square Tests

|  | Value | df | Sig. (2-sided) |
| :--- | ---: | ---: | ---: |
| Pearson Chi-Square | $1.542^{2}$ |  | 1 |

a. 0 cells $(.0 \%)$ have expected count less than 5 . The minimum expected count is 35.00 .

Table 4.2 Chi-square test for gender distributions

## Ethnicity

There are some differences among the two largest ethnic groups between the treatment and comparison groups; see Figure 4.3 in Appendix A. For the ethnic composition of the institution (for undergraduate students at main campus during fall of 2009) alongside the treatment and comparison groups see Table 4.3 in Appendix B (Institutional Data). Since it is unclear if these differences are significant, the Chi-Square test is used to check for significant distribution differences among ethnic groups between the treatment and comparison groups. Due to restrictions on cell counts for the ChiSquare test, the African American and Native American categories are compressed into one category while the Asian/Pacific Islander and Other ethnicity categories are compressed into another for this test. Results show the Pearson Chi-Square $=1.562$, with a (2-sided) $p$-value of .668 . So there is not evidence to support a significant difference of ethnicity distributions between the treatment and comparison groups. Therefore, ethnicity should not introduce additional bias to the study.
(Condensed) Ethnicity Cross Tabulation

|  | Ethnicity |  |  |  | Total |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | African and Native American | Caucasian | Hispanic | Asian/Pacific Islander \& Other |  |
| Comparison | 6 | 34 | 39 | 8 | 87 |
| Treatment | 6 | 41 | 31 | 8 | 86 |
| Total | 12 | 75 | 70 | 16 | 173 |

Table 4.4 Condensed ethnicity cross tabulation.

Chi-Square Tests

|  | Value | df | Sig. (2-sided) |
| :--- | ---: | ---: | ---: |
| Pearson Chi-Square | $1.562^{\mathrm{a}}$ |  | 3 |
| N of Valid Cases | 173 |  | .668 |

a. 0 cells $(.0 \%)$ have expected count less than 5 . The minimum expected count is 5.97 .

Table 4.5 Chi-square test for ethnicity.

## Placement qualifications

There are two main avenues for placement into College Algebra. The first avenue is to test directly into College Algebra by either an ACT or SAT exam. Fifty-nine percent of students in the treatment group and $61 \%$ of students in the comparison group tested into College Algebra (see Figure 4.4 in Appendix A). So the treatment and comparison groups are similar to each other. The second avenue for placement is for students to complete Intermediate Algebra, the prerequisite course. Approximately 37\% of the treatment group and 39\% of the comparison group completed Intermediate Algebra as a prerequisite (see Figure 4.5 in Appendix A). So the treatment and comparison samples are again quite comparable.

## Socio-Economic Status

The comparison and treatment groups have a similar distribution of students among each socio-economic class, except for the lowest. There is a difference between the treatment and comparison groups among students whose family income is less than $\$ 30,000$ per year. In the comparison group, $18 \%$ of students reported having a family income of less than $\$ 30,000$ per year, while $28 \%$ of students in the treatment group reported this to be their family income. This is of some concern since there may be more students within the treatment group maintaining scholarships or other merit based
funding as a necessity to staying in school and therefore may be more motivated to perform well in the course. See Figure 4.6 in Appendix A for a more detailed description of the socio-economic status within both the treatment and comparison groups.

To determine if there is a significant distribution difference of socio-economic class between the treatment and comparison groups the Chi-Square test is used. Due to restrictions on cell counts for the Chi-Square test, the over $\$ 250,000$ and no response categories were compressed into one category labeled "Other" for this test. Results show the Pearson Chi-Square $=3.229$, with a ( 2 -sided) $p$-value of .520 . So there is not evidence to support a significant difference of socio-economic status distributions between the treatment and comparison groups. As a result, socio-economic status should not introduce additional bias to the study.

Condensed SES Cross Tabulation

|  | Condensed SES |  |  |  |  | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Less than $30,000$ | 30,000-60,000 | $\begin{aligned} & 60,000- \\ & 100,000 \end{aligned}$ | $\begin{aligned} & 100,000- \\ & 250,000 \end{aligned}$ | Other |  |
| Comparison | 16 | 24 | 19 | 19 | 9 | 87 |
| Treatment | 24 | 23 | 19 | 15 | 5 | 86 |
| Total | 40 | 47 | 38 | 34 | 14 | 173 |

Table 4.6 Condensed socio-economic status cross tabulation.

Chi-Square Tests

|  | Value | df | Sig. (2-sided) |
| :--- | ---: | :--- | ---: |
| Pearson Chi-Square | $3.229^{\mathrm{a}}$ |  | 4 |
| $N$ of Valid Cases | 173 |  | .520 |

a. 0 cells $(.0 \%)$ have expected count less than 5 . The minimum expected count is 6.96 .

Table 4.7 Chi-square test for socio-economic status.

## Multivariate Analysis of Variance

A $2 \times 2$ MANOVA was conducted on the dependent variables to determine if the treatment and comparison groups achieved differently, and if gender effects and/or interactions are present. In an effort to obtain a more parsimonious result, backward elimination is applied (recursively) to the MANOVA analyses. This model selection algorithm only eliminated variables which contributed either a negligible or no effect for results. The variables not eliminated were the preliminary exam, anxiety pretest, selfconcept pretest and group which all have a $p$-value of $<.001$ for the multivariate tests. The effect size was moderate for the preliminary exam with the partial $\eta^{2}=.523$. The effect size was somewhat moderate for the anxiety and self-concept pretests with the partial $\eta^{2}=.440$ and the partial $\eta^{2}=.435$ respectively. For the group, the effect size was small, with the partial $\eta^{2}=.150$. As shown in Table 4.8 in Appendix B, these four variables do have a statistically significant effect between subjects (to varying degrees) on achievement, anxiety, self-concept and motivation gains (Table 4.9 in Appendix B).

Since gender and interactions were eliminated by the backward elimination process, they did not have a significant effect between subjects. Results show that preliminary exam performance does have a significant effect on achievement gain with $p$ $<.001$ and a moderate effect size with the partial $\eta^{2}=.495$. Results show that anxiety pretest performance does have a significant effect on anxiety gain with $p<.001$ and a smaller effect size with the partial $\eta^{2}=.242$. Results show that self-concept pretest performance does have a significant effect on achievement, anxiety and self-concept gains with $p=.054, p<.001$ and $p=.002$ respectively. However effect sizes were small
for achievement, anxiety and self-concept with the partial $\eta^{2}=.022$, the partial $\eta^{2}=.146$ and the partial $\eta^{2}=.054$ respectively.

Results show that the group variable had the most consistent effects on all four dependent variables; albeit with small effect sizes for the between subjects tests. The $p$ values are calculated at $p=.005, p=.064, p=.030$ and $p=.001$ for achievement, anxiety, self-concept and motivation gains respectively. Given a larger sample size, a random sample, or a change in other dynamic factors, such as instructor experience with cooperative learning, etc. the $p$-value for anxiety may approach statistical significance more closely. Group effect sizes were small for the achievement, anxiety, self-concept and motivation variables with the partial $\eta^{2}=.045$, the partial $\eta^{2}=.020$, the partial $\eta^{2}$ $=.028$ and the partial $\eta^{2}=.068$ respectively.

## Algebra Preliminary and Final Exams

As a follow-up to the MANOVA test, an ANOVA was conducted on the mean algebra gains against the preliminary exam and the anxiety pretest, self-concept pretest and motivation pretest, group and gender to determine if there is a difference in achievement. Backward elimination was also used to obtain a more parsimonious result. This model selection algorithm only eliminated variables which contributed a negligible or no effect for results. Anxiety pretest, motivation pretest, gender and interactions were not significant contributors to achievement gain. The variables not eliminated were the preliminary exam, self-concept pretest and group which have $p$-values of $<.001, .007$ and .005 respectively. The effect size was moderate for the preliminary exam with the partial $\eta^{2}=.498$. Effect size was small for the self-concept and group variables with the
partial $\eta^{2}=.043$ and the partial $\eta^{2}=.047$ respectively. Results concur with multivariate tests for the preliminary exam, self-concept pretest and group variables.

Tests of Between-Subjects Effects
Dependent Variable: Gain from Preliminary to Final Exam

| Source | Type III <br> Sum of <br> Squares | df | Mean Square | F | Sig. | Partial Eta <br> Squared |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Corrected Model | $34300.494^{\text {a }}$ | 3 | 11433.498 | 58.497 | . 000 | . 509 |
| Intercept | 33815.463 | 1 | 33815.463 | 173.010 | . 000 | . 506 |
| Preliminary Exam | 32714.593 | 1 | 32714.593 | 167.378 | . 000 | . 498 |
| Self-Concept | 1471.321 | 1 | 1471.321 | 7.528 | . 007 | . 043 |
| pretest |  |  |  |  |  |  |
| Group | 1612.816 | 1 | 1612.816 | 8.252 | . 005 | . 047 |
| Error | 33031.664 | 169 | 195.454 |  |  |  |
| Total | 68493.360 | 173 |  |  |  |  |
| Corrected Total | 67332.158 | 172 |  |  |  |  |

a. R Squared $=.509($ Adjusted R Squared $=.501)$

Table 4.10 ANOVA tests between subjects for algebra gain.

Parameter estimates indicate a negative relationship between the preliminary exam and algebra achievement gain with $B=-.799$ and $p<.001$. This is tenable since the greater achievement on the preliminary exam, the less opportunity there is to gain points. Conversely, if a student earns a low score on the preliminary exam, they then have a greater opportunity to increase achievement gain. Parameter estimates indicate the selfconcept pretest has a positive relationship with achievement gain where $B=.151$ and $p=$ .007. In addition, parameter estimates indicate a positive relationship between achievement gain and the treatment group with $B=6.158$ and $p=.005$.

| Parameter Estimates |  |  |  |
| :---: | :---: | :---: | :---: |
| Dependent Variable: Gain from Preliminary to Final Exam |  |  |  |
| Parameter |  |  |  |
|  | B | Std. Error | Sig. |
| Intercept | 59.651 | 4.440 | . 000 |
| Preliminary Exam | -. 799 | . 062 | . 000 |
| Self-Concept Pretest | . 151 | . 055 | . 007 |
| Comparison Group | $0^{2}$ |  |  |
| Treatment Group | 6.158 | 2.144 | . 005 |

Table 4.11 Parameter estimates for algebra gain.

Figures 4.7 and 4.8 map each participant's preliminary exam score ( $x$-axis) against their respective achievement exam gain ( $y$-axis). Achievement gain is calculated by taking the raw percent grade on the final exam and subtracting the raw percent grade on the preliminary exam. Note any marker falling below the $x$-axis indicates the final exam score is lower than the respective preliminary exam score. For example, a student earning $100 \%$ on the preliminary exam was unable to increase their grade on the final exam and therefore can only have a corresponding $y$-value on or below the $x$-axis. In the case where this student earns less than $100 \%$ on the final, the student marker on the scatter plots will have negative gain, meaning their final exam score was lower than their preliminary exam score. If a student earned the same grade on the final and preliminary exams their marker will be on the $x$-axis which indicates a gain of zero.

Reference lines are provided for Figures 4.7 and 4.8, so one can easily see the percentage of achievement gain earned from the preliminary to final exam. For example, if a student earned a score of $50 \%$ on the preliminary exam, this student then had the opportunity to increase their gain on the final exam by 50 points. In this case, the student
marker would be located on reference line 100 (which is the line joining points $(0,100)$ and $(100,0))$ and above $x=50$, meaning the student earned all fifty points possible on the final exam. Seldom does it happen that a student earning a failing grade on the preliminary exam earns a perfect score on the final exam. So suppose instead that this same student earned a score of $80 \%$ on the final exam, in this instance the student marker would lie on reference line 60 , above $x=50$. This means the student gained $60 \%$ of possible points with respect to their achievement on the preliminary exam (30 points gained / 50 possible points to gain $=60 \%$ actually earned).

When the graphs in Figures 4.7 and 4.8 are compared generally, it is apparent that the center of the treatment group scatter plot lies higher on the $y$-axis than does the center of the comparison group scatter plot. Although the ranges for these two scatter plots are similar, one can see the treatment group scatter plot is denser than the comparison group scatter plot. Closer inspection reveals that more student markers lay on top of each other within the treatment group than the comparison group. For this reason it appears there are fewer student markers in the treatment group scatter plot. In fact, in some instances two or three student markers occupy the same coordinates on the treatment group scatter plot.

Closer examination also reveals a difference in gain between the two groups. This can be seen particularly for students who earned $70 \%$ or less on the preliminary exam. Among students earning approximately $70 \%$ on the preliminary exam, there are fewer students with a negative gain in the treatment group than the comparison group. This continues to be true for the remainder of students earning less than $70 \%$ in the treatment group when compared against the comparison group. Among students earning 80\% or more on the preliminary exam, in general, students within the treatment group have a
smaller negative gain in comparison to the comparison group. The overall impression is students within these two groups earning comparable grades on the preliminary exam have a greater gain (to varying degrees) on the final exam in the treatment group than do students in the comparison group, with the greatest gain earned by students having the greatest opportunity for improvement. In other words, there is a negative association between grade earned on the preliminary exam and gain increase, which is larger in the treatment group than the comparison group.


Figure 4.7 Preliminary exam and algebra gain for comparison group


Figure 4.8 Preliminary exam and algebra gain for treatment group

The box plots in Figure 4.9 for the preliminary exam show both the treatment and comparison groups have the same median values. However the spread between the first and third quartiles is somewhat different with the range of the treatment group shifted slightly lower than the comparison group. The mean values on the preliminary exam for the treatment group is $66.99 \%$ while the mean for the comparison group is $67.55 \%$. The overall implication is the treatment and comparison groups began statistically identical. Inspection of the box plots in Figure 4.10 for the final exam shows the median is higher for the treatment group than for the comparison group. In addition, the spread of the first and third quartiles is shifted higher, indicating greater achievement compared to the comparison group. The most notable difference among these two box plots is the difference in median. The median of the treatment group surpassed the comparison group by ten points. The mean of the final exam for the treatment group is $72.51 \%$ while the mean for the comparison group is $67.21 \%$. Comparing Figure 4.9 to Figure 4.10 clearly indicates the treatment did increase overall achievement outcomes.


Figure 4.9 Box plot of preliminary exam


Figure 4.10 Box plot of final exam

## Anxiety Pretest and Posttest

As a follow-up to the MANOVA test, an ANOVA was conducted on the mean anxiety gains against the preliminary exam, anxiety pretest, self-concept pretest and motivation pretest, group and gender to determine if there is a difference in anxiety. Backward elimination was also used to obtain a more parsimonious result. This model selection algorithm, again, only eliminated variables which contributed a negligible or no effect for results. Preliminary exam, motivation pretest, gender and interactions were not significant contributors to anxiety gain. The variables not eliminated were the anxiety pretest, self-concept pretest and group which have p-values of $<.001,<.001$ and .063 respectively. The effect size was somewhat moderate for the anxiety pretest with the partial $\eta^{2}=.239$. Effect size was small for the self-concept and group variables with the partial $\eta^{2}=.147$ and the partial $\eta^{2}=.020$ respectively. Results concur with multivariate tests for the anxiety pretest, self-concept pretest and group variables.

Tests of Between-Subjects Effects
Dependent Variable: Anxiety Gain

| Source | Type III Sum <br> of Squares | df | Mean Square | F | Sig. | Partial Eta <br> Squared |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Corrected Model | $1315.056^{\mathrm{a}}$ | 3 | 438.352 | 20.384 | .000 | .266 |
| Intercept | 97.446 | 1 | 97.446 | 4.531 | .035 | .026 |
| Anxiety pretest | 1144.429 | 1 | 1144.429 | 53.217 | .000 | .239 |
| Self-Concept | 624.554 | 1 | 624.554 | 29.042 | .000 | .147 |
| pretest |  |  |  |  |  |  |
| Group | 75.590 | 1 | 75.590 | 3.515 | .063 | .020 |
| Error | 3634.366 | 169 | 21.505 |  |  |  |
| Total | 5103.000 | 173 |  |  |  |  |
| Corrected Total | 4949.422 | 172 |  |  |  |  |

a. R Squared $=.266$ (Adjusted R Squared $=.253$ )

Table 4.12 ANOVA tests between subjects for anxiety gain.

Parameter estimates indicate a negative relationship between the anxiety pretest and anxiety gain with $B=-.663$ and $p<.001$. This is tenable since the greater baseline anxiety, the less opportunity there is to increase anxiety. Conversely, if a student begins with less anxiety, they then have a greater opportunity to increase anxiety gain. Parameter estimates indicate the self-concept pretest has a positive relationship with anxiety gain with $B=.158$ and $p<.001$. In addition, parameter estimates indicate a positive relationship between anxiety gain and the treatment group with $B=1.341$ and $p=.063$.

Parameter Estimates
Dependent Variable: Anxiety Gain

| Parameter | B | Std. Error | Sig. |
| :---: | :---: | :---: | :---: |
| Intercept | 1.602 | . 551 | . 004 |
| Comparison Group | $0^{2}$ |  |  |
| Treatment Group | 1.341 | . 715 | . 063 |
| Anxiety Pretest | -. 663 | . 091 | . 000 |
| Self-Concept Pretest | . 158 | . 029 | . 000 |

Table 4.13 Parameter estimates for anxiety gain.

## Self-Concept Pretest and Posttest

As a follow-up to the MANOVA test, an ANOVA was conducted on the mean self-concept gains against the preliminary exam, anxiety pretest, self-concept pretest and motivation pretest, group and gender to determine if there is a difference in self-concept. Backward elimination was also used to obtain a more parsimonious result. This model selection algorithm, again, only eliminated variables which contributed a negligible or no effect for results. Preliminary exam, anxiety pretest, motivation pretest, gender and interactions were not significant contributors to self-concept gain. The variables not eliminated were the self-concept pretest and group which have $p$-values of .001 and .038
respectively. Effect size was small for the self-concept pretest and group variables with the partial $\eta^{2}=.066$ and the partial $\eta^{2}=.025$ respectively. Results concur with multivariate tests for the anxiety pretest, self-concept pretest and group variables.

## Tests of Between-Subjects Effects

Dependent Variable: Self-Concept Gain

| Source | Type III Sum <br> of Squares | df | Mean Square | F | Sig. | Partial Eta <br> Squared |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Corrected Model | $3613.461^{2}$ | 2 | 1806.730 | 9.227 | .000 | .098 |
| Intercept | 747.100 | 1 | 747.100 | 3.815 | .052 | .022 |
| Self-Concept | 2335.723 | 1 | 2335.723 | 11.928 | .001 | .066 |
| pretest |  |  |  |  |  |  |
| Group | 852.990 | 1 | 852.990 | 4.356 | .038 | .025 |
| Error | 33288.632 | 170 | 195.815 |  |  |  |
| Total | 37494.000 | 173 |  |  |  |  |
| Corrected Total | 36902.092 | 172 |  |  |  |  |

a. R Squared $=.098($ Adjusted R Squared $=.087)$

Table 4.14 ANOVA tests between subjects for self-concept gain.

Parameter estimates indicate a negative relationship between the self-concept pretest and self-concept gain with $B=-.188$ and $p=.001$. This is tenable since the greater baseline self-concept, the less opportunity there is to increase self-concept. Conversely, if a student begins with a lower self-concept, they then have a greater opportunity to increase their self-concept gain. In addition, parameter estimates indicate a positive relationship between self-concept gain and treatment group, with $B=4.479$ and $p=.038$.

## Parameter Estimates

Dependent Variable: Self-Concept Gain

| Parameter |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Intercept |  | Std. Error | Sig. |  |  |
| Comparison Group |  | .156 | 1.524 |  | .918 |
| Treatment Group | $0^{2}$ |  | . |  | .038 |
| Self-Concept pretest | 4.479 | 2.146 |  | .001 |  |

Table 4.15 Parameter estimates for self-concept gain.

## Motivation Pretest and Posttest

As a follow-up to the MANOVA test, an ANOVA was conducted on the mean motivation gains against the preliminary exam, anxiety pretest, self-concept pretest and motivation pretest, group and gender to determine if there is a difference in motivation. Backward elimination was also used to obtain a more parsimonious result. This model selection algorithm, again, only eliminated variables which contributed a negligible or no effect for results. Preliminary exam, anxiety pretest, self-concept pretest, gender and interactions were not significant contributors to motivation gain. The motivation pretest and group variables were not eliminated which have p-values of .027 and $<.001$ respectively. Effect size was small for the motivation pretest and group variables with the partial $\eta^{2}=.028$ and the partial $\eta^{2}=.073$ respectively. These results are supplementary to the MANOVA since the motivation pretest was eliminated from the multivariate analysis during the backward elimination process.

## Tests of Between-Subjects Effects

Dependent Variable: Motivation Gain

| Source | Type III Sum <br> of Squares | df | Mean Square | F | Sig. | Partial Eta <br> Squared |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Corrected | $10789.249^{2}$ |  | 2 | 5394.624 | 8.778 | .000 |
| Model |  |  |  |  |  | .094 |
| Intercept | 502.293 | 1 | 502.293 | .817 | .367 | .005 |
| Motivation | 3060.642 | 1 | 3060.642 | 4.980 | .027 | .028 |
| pretest |  |  |  |  |  |  |
| Group | 8239.416 | 1 | 8239.416 | 13.407 | .000 | .073 |
| Error | 104474.300 | 170 | 614.555 |  |  |  |
| Total | 130191.000 | 173 |  |  |  |  |
| Corrected Total | 115263.549 | 172 |  |  |  |  |

a. R Squared $=.094($ Adjusted R Squared $=.083)$

Table 4.16 ANOVA tests between subjects for motivation gain.

Parameter estimates indicate a negative relationship between the motivation pretest and motivation gain with $B=-.224$ and $p=.027$. This is tenable since the greater baseline motivation, the less opportunity there is to increase motivation. Conversely, if a student begins with less motivation, they then have a greater opportunity to increase motivation gain. Parameter estimates indicate the treatment group has a negative relationship with motivation gain with $B=-13.823$ and $p<.001$.

## Parameter Estimates

Dependent Variable: Motivation Gain

| Parameter | B | Std. Error | Sig. |
| :---: | :---: | :---: | :---: |
| Intercept | -. 190 | 7.577 | . 980 |
| Comparison Group | $0^{2}$ |  |  |
| Treatment Group | -13.823 | 3.775 | 000 |
| Motivation pretest | -. 224 | . 100 | . 027 |

Table 4.17 Parameter estimates for motivation gain.

## Logistic Regression

Logistic regression is used to determine if students in the treatment group have a greater probability of passing the course than students in the comparison group. Passing the course is defined by the student earning credit for the course and receiving an overall grade of C or better. Since preliminary exam performance does have a significant effect on students passing, it was included as a predictor in the logistic regression model. Results show a significant difference for the treatment group with a regression coefficient of $B_{1}=1.140$ and the corresponding $p$-value of .005 . Results also show a significant difference for the preliminary exam with a regression coefficient of $B_{2}=0.024$ with a corresponding $p$-value of .029 . Since both $B_{1}$ and $B_{2}$ are positive coefficients, both the group and preliminary exam predictors contribute positively to students' probability of passing the course. Because the overall probability of passing the course is dependent upon preliminary exam performance, which is a continuous variable, the probability of passing cannot be interpreted by group membership alone. The overall probability for each student can be calculated by the following formula (see Figure 4.11 for a graph of these equations):

$$
\text { Probability of passing the course }=f(z)=\frac{e^{z}}{e^{z}+1}
$$

$$
\text { Where } z=-1.089+1.140(\text { Group })+.024(\text { Prelim. } . \text { Grade })
$$

*Note that for computational purposes, the treatment group $=1$ and the comparison group $=0$.

For instance, the probability of two students, one from each group, earning $67 \%$ on the preliminary exam (the mean preliminary exam score for both the treatment and comparison groups) have the following probabilities of passing the course:

Comparison Group:

$$
\begin{aligned}
& \text { Probability of passing the course }=f(-1.089+1.140(0)+.024(67)) \\
& \qquad=\frac{e^{-1.089+1.140(0)+.024(67)}}{e^{-1.089+1.140(0)+.024(67)}+1}=.63
\end{aligned}
$$

Treatment Group:
Probability of passing the course $=f(-1.089+1.140(1)+.024(67))$

$$
=\frac{e^{-1.089+1.140(1)+.024(67)}}{e^{-1.089+1.140(1)+.024(67)}+1}=.84
$$

So this means, a student who earned the average preliminary exam score in the comparison group has an estimated $63 \%$ probability of passing the course. Similarly, a student who earned the average preliminary exam score in the treatment group has an estimated $84 \%$ probability of passing the course. So there is evidence to suggest that students within the treatment group do have a greater probability of passing the course (dependent upon their preliminary exam score) than students within the comparison group.

Parameter Estimates

| Parameter | B | Std. <br> Error |  |  |  | 95\% Confidence <br> Interval for $\operatorname{Exp}(\mathrm{B})$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | df | Sig. | $\operatorname{Exp}(\mathrm{B})$ | Lower | Upper |
| (Intercept) | -1.089 | . 7707 | 1 | . 158 | . 337 | . 074 | 1.525 |
| Treatment Group | 1.140 | . 4046 | 1 | . 005 | 3.126 | 1.415 | 6.909 |
| Comparison Group | $0^{a}$ |  |  |  |  |  |  |
| Preliminary Exam | . 024 | . 0112 | 1 | . 029 | 1.025 | 1.003 | 1.047 |
| Grade |  |  |  |  |  |  |  |
| (Scale) | $1{ }^{\text {b }}$ |  |  |  |  |  |  |

Dependent Variable: Passed Course (with NR grade coded as did not pass)
a. Set to zero because this parameter is redundant.
b. Fixed at the displayed value.

Table 4.18 Parameter estimates for logistic regression.


Figure 4.11 logistic regression equations. The $x$-axis is preliminary exam score while the $y$-axis is the students' probability of passing the course.

## Validity

Two assessments were used to determine the internal consistency of the instrumentation. The Kuder-Richardson approach, particularly the KR21 test, is used to calculate reliability of the preliminary and final algebra exams (Fraenkel and Wallen, 2009). Reliability estimates for the preliminary and final exams are calculated at .48 and .71, respectively. The alpha coefficient is used to determine reliability of the anxiety, self-concept and motivation instruments. The alpha coefficient is calculated from the general form of the Kuder-Richardson KR20 test (2009).The reliability coefficient (Chronbach's alpha) for the math anxiety, self-concept and motivation instruments are reported at $\alpha=.92, \alpha=.98$ and $\alpha=.93$, respectively (Betz, 1978; Gourgey, 1982; Glynn, Taasoobshirazi, Brickman, 2009). The reliability estimates for the algebra exams are as expected. The reliability of the preliminary exam is low for an in-class exam; however this is expected since the exam has only thirteen questions. Reliability estimates for the anxiety, self-concept and motivation exams are high (the range for both the KR20 and KR21 tests are 0-1).

The MANOVA test assumes sample data satisfies three conditions prior to analysis. The first assumption is that sample data is multivariate normally distributed (Bryant et al., 2000). Both the Kolmogorov-Smirnov and Shapiro-Wilk tests for univariate normality indicate the algebra instrument is normally distributed, however the anxiety, self-concept and motivation instruments are not for the treatment group (see Table 4.19 in Appendix B). These tests also indicate that the algebra, anxiety and motivation instruments are normally distributed; however, the self-concept instrument is not for the comparison group. Therefore the sample data is not multivariate normally
distributed for either the treatment or comparison groups (Looney, 1995). The multivariate normally distributed condition is nonessential, since MANOVA is considered a robust test and violation of this assumption minimally affects the probability of a Type I error (Bryant et al., 2000).

Homogeneity of covariance matrices is the second MANOVA test assumption (Bryant et al., 2000). Levene's test indicates the algebra ( $p=.048$ ) and motivation ( $p=$ .015) instruments do not have equal variances (because they are < .05) thus the homogeneity of covariance matrices assumption is not met (see Table 4.20 in Appendix B). Since the sample sizes for treatment and comparison groups are approximately equal, with 86 and 87 participants respectively, the MANOVA test is largely unaffected by this violation, however statistical power is slightly reduced (2000).

Lastly, the independence of observations is assumed (Bryant et al., 2000). This means each participant's responses are not influenced by another participant within their group. MANOVA results can be skewed due to violations of this assumption; however to what degree is unclear (2000). This assumption is not directly violated since students completed each instrument independently. Nonetheless, consideration must be taken since the treatment group worked together within small groups during class and may indirectly influence the full realization of this assumption. Despite these limitations, the MANOVA test is the best tool for analysis. Overall validity of the MANOVA test is slightly reduced because data do not absolutely satisfy multivariate normality and independence of observations assumptions.

The MANOVA test is chosen in place of performing multiple ANOVA tests to increase validity. The use of MANOVA increases validity three ways. First by
concurrently analyzing both independent and dependent variables, the test investigates any possible multivariate effects and/or interactions occurring within the sample. In addition, the MANOVA accounts for possible correlation effects among dependent variables. The final advantage is that the MANOVA test reduces the risk of a Type I error, over conducting multiple ANOVA tests. Since a multivariate effect was found, multiple ANOVA tests were conducted post-hoc. These ANOVA procedures do not pose an additional threat to increasing the probability of falsely rejecting the null hypothesis when qualified by the MANOVA test (Bryant et al., 2000).

In addition to former assumptions of the MANOVA test, this test also requires dependent measures be statistically correlated (Bryant et al., 2000). The Pearson's correlation coefficient $(\boldsymbol{r})$ is calculated post-hoc and is of interest (see Table 4.21 in Appendix B). The algebra and self-concept instruments are minimally correlated with Pearson's $r=.225$, and significance (2-tailed) of .008 . The motivation and self-concept instruments are slightly more correlated with Pearson's $r=.598$, and significance (2tailed) of $p<.001$. The anxiety instrument is not significantly correlated with other dependent measures. Among the correlated dependent measures results are at best considered low, reducing test validity. Because of this, multiple ANOVA procedures were conducted on each dependent variable to clarify and substantiate MANOVA test results.

Since analysis of variance test assumptions are not maximally satisfied, it will be beneficial to consider descriptive statistics for the two groups. Because of this, and since sampled groups were taken from pre-formed sections, a change in individual student achievement was considered. Since no random selection or random assignment was
applied, the sample introduces a subject characteristic bias which can skew observed relationships (Fraenkel, \& Wallen, 2009). Therefore, individual student performance is examined by comparing individual student pretest (algebra) scores to gains earned. This analysis is used because it reduces the possibility that participant's individual characteristics will skew observations, increasing internal validity (2009). In addition to this descriptive data, logistic regression is used to determine which of the two groups are more likely to pass the course.

Similar to the analysis of variance procedures, the absence of randomization affects the logistic regression validity however; variables under analysis are dichotomous as assumed (Bryant et al., 2000). The second assumption that outcomes are statistically independent is satisfied since students cannot pass and fail the course simultaneously (2000). In an endeavor to satisfy the specificity assumption, the backward elimination method (used for the analysis of variance procedures) is helpful for selecting relevant predictors. As usual, it is uncertain if this assumption is absolutely satisfied (2000). The fourth test assumption is that categories under analysis are mutually exclusive and collectively exhaustive (2000). This condition is satisfied since participants were enrolled in either a treatment or comparison section but not both simultaneously. Lastly, a sample of approximately fifty cases or more per predictor is assumed (2000). The sample for this study is sufficient to satisfy this test assumption. It is important to note that the external validity is by no means far-reaching since the sample was not randomized. This study is only applicable to the accessible population of this particular university.

## CHAPTER 5

## Discussion

## Restatement of the Study Purpose

This study was conducted to examine the effects of a cooperative learning treatment on College Algebra students. Thee hypotheses were examined. First, the treatment group was expected to have lower anxiety, higher achievement, self-concept and motivation compared to the comparison group. Second, gender differences were expected within the treatment group. Specifically, the females in the treatment group were expected to have a more significant change in anxiety than males in the treatment group. In addition, the males and females in the treatment group were expected to have a parallel increase in self-concept and motivation. Finally, students in the treatment group were expected to be more likely to pass the course than students in the comparison group.

## Summary of Procedures and Methods

Students were recruited during the second week of the semester, following which they were sent an email invitation to the combined informed consent and online survey at the beginning and end of the semester. The treatment and comparison groups were both sampled from students enrolled in College Algebra during the fall semester of 2009. The total number of students consenting to participate was 566 from an accessible population of approximately 1,200 students. This study included gender and group (treatment or comparison) as the only two independent variables. There were eight dependant variables, namely the preliminary algebra exam, final algebra exam, anxiety, self-concept
and motivation pretests and posttests. In addition demographic data was included to investigate possible differences among the two groups.

After the data collection process was completed, coders confirmed the accuracy of the eight dependent instruments. Student section number was then verified with the university Registrar. Students were subsequently divided into two groups, either treatment or comparison according to their course section number. To authenticate accurate scoring of the preliminary and final exams, these two instruments were then rescored by the PI. Students not returning both preliminary and final exams were then excluded from the study; accordingly the final number of participants was 173 students in total. There were occasional missing values within the final data set since students were directed to skip any questions they did not feel comfortable answering. In an effort to provide the most accurate analysis possible, the mean score of all five instruments was used in analysis instead of the summed scores used for the original anxiety, self-concept and motivation instruments.

The study design utilized three discrete methods to examine data. To determine if there was a significant difference on the dependent variable gains, for the treatment and comparison groups, a 2 (Group) $\times 2$ (Gender) factorial design was used for both multivariate and univariate analyses. Since data did not optimally satisfy test assumptions, descriptive statistics were provided to augment the analysis of variance procedures. Finally, logistic regression analysis was used to determine if students in the treatment group were more likely to pass the course than students in the comparison group. To help interpret results, internal reliability tests were conducted on each instrument. In addition, test assumptions were statistically examined where possible, to
determine the validity and power of the analysis of variance and logistic regression procedures.

## Result Implications

Eighty-six students ( $51 \%$ of the initial participants) in the treatment group completed both preliminary and final exams, while 87 students ( $22 \%$ of the initial participants) in the comparison group completed both exams. In addition to the five instruments collected, participants also reported their age, gender, ethnicity, type of placement into the course and family socio-economic status on a demographic survey. A two sample $t$-test was conducted on the age distributions of the two groups. The test calculated $t=.645$ with a sig. (2-tailed) of $p=.520$, so there was no evidence to support a significant age distribution difference between the two groups. The Chi-Square test was used to determine if there was a significant difference in the gender, ethnicity and socioeconomic distributions. The test calculated $\chi^{2}=1.542$, with a ( 2 -sided) $p$-value of .214 for gender, so there was no evidence to support a significant difference of gender distributions. The Chi-Square test for ethnicity calculated $\chi^{2}=1.562$, with a ( 2 -sided) $p$ value of .668 , so there was no evidence to support a significant difference of ethnicity distributions. The Chi-Square test for socio-economic status calculated $\chi^{2}=3.229$, with a (2-sided) $p$-value of .520 , so there was no evidence to support a significant difference of socio-economic status distributions. Approximately $37 \%$ of the treatment group and $39 \%$ of the comparison groups completed Intermediate Algebra prior to entering College Algebra. Despite the non-random sample, the treatment and comparison groups did not
significantly differ on the distribution differences for any of the collected demographic data.

A 2 (Group) $\times 2$ (Gender) MANOVA was conducted on the dependent variables to determine if the treatment and comparison groups achieved differently. This test was then followed by four post-hoc ANOVA tests to support MANOVA results. To obtain a more parsimonious result, backward elimination was applied to both the MANOVA and ANOVA analyses. The MANOVA analysis showed the preliminary exam, anxiety pretest, self-concept pretest and group were all significant with $p$-values of less than .001 for multivariate tests. These four variables did have a statistically significant effect between subjects (to varying degrees) on algebra, anxiety, self-concept and motivation gains. Results showed that the group variable had the most consistent effects on exam, anxiety, self-concept and motivation gains. Gender and interactions did not have a significant effect between subjects since they were eliminated by the backward elimination process.

The results for the univariate test on algebra gain supported the multivariate test results for the preliminary exam, self-concept pretest and group variables. The results for the univariate test on anxiety gain also supported the multivariate test results for the anxiety pretest, self-concept pretest and group variables. The results for the univariate test on self-concept gain also supported the multivariate test results for the anxiety pretest, self-concept pretest and group variables. The results for the univariate test on motivation gain were supplementary to the multivariate test results since the motivation pretest was eliminated during the backward elimination process. The variables not eliminated in the
univariate analysis were the motivation pretest and group, which were both significant for motivation gain.

Since the multivariate and univariate tests did indicate significant differences for the former variables, parameter estimates were also given along with the univariate analyses to indicate the type of relationships present. Parameter estimates indicated the preliminary exam had a negative association with the algebra gain, which was expected. Parameter estimates also indicated the self-concept pretest had a positive association with algebra achievement gain, which was unexpected. In addition the treatment group did, as expected, have a positive association with algebra achievement gain. The treatment group did demonstrate higher achievement than the comparison group as hypothesized.

The second univariate test examined anxiety gain. Parameter estimates indicated the treatment group had a positive association with the anxiety gain, which was expected since a higher score on the anxiety test indicates lower anxiety. These estimates also indicated the anxiety pretest had a negative association with anxiety gain, which was also an expected result. In addition the self-concept pretest unexpectedly had a positive association with anxiety gain. The treatment group did demonstrate lower anxiety than the comparison group as hypothesized.

The third univariate test examined self-concept gain. Parameter estimates indicated the treatment group had a positive association with the self-concept gain, which was expected since a higher score on the self-concept test indicates higher self-concept. These estimates also indicated the self-concept pretest had a negative association with self-concept gain, which was also an expected result. The treatment group did demonstrate higher self-concept than the comparison group as hypothesized.

The final univariate test examined motivation gain. Parameter estimates indicated the treatment group had a negative association with the motivation gain, which was unexpected since a higher score on the motivation test indicates higher motivation. These estimates also indicated the motivation pretest had a negative association with motivation gain, which was an expected result. The treatment group did not demonstrate higher motivation than the comparison group as hypothesized.

A logistic regression analysis was conducted with the preliminary exam and treatment group as predictor variables to determine if students in the treatment group had a greater probability of passing the course than students in the comparison group. Results indicated a significant difference for the treatment group and preliminary exam predictors with $p$-values of .005 and .029 respectively. Since both predictor coefficients were positive, both the treatment group and preliminary exam predictors contributed positively to students' probability of passing the course. As hypothesized students in the treatment group were more likely to pass the course than students in the comparison group.

The internal consistency assessments indicated the preliminary exam, final algebra exam, anxiety instrument, self-concept instrument and motivation instrument reliabilities were $.48, .71, .92, .98$ and .93 respectively. Since the range of consistency assessments is zero to one, reliabilities for the anxiety, self-concept and motivation instruments were quite good, however the final algebra exam, and particularly the preliminary exam were relatively low. Risk of a Type I error was slightly increased for the analysis of variance procedures since the data were not entirely normally distributed. In addition power was slightly reduced since the data did not have homogeneous covariance matrices. Dependent measures were moderately correlated except for the
anxiety instrument, which reduced test validity for the multivariate analysis. However post-hoc univariate analyses did support multivariate results. Satisfied test assumptions for the logistic regression analysis were: assumptions of statistically independent outcomes, mutually exclusive and collectively exhaustive categories and sample size. It is unclear if the specificity assumption was completely satisfied. Randomization was not part of the study design however it was assumed by analysis of variance and logistic regression procedures.

Examination of the demographic data indicated the treatment and comparison groups did have similar distributions of age, gender, ethnicity, placement qualifications and socio-economic status. This demographic information was valuable since data did not optimally satisfy test assumptions for the MANOVA, ANOVA and logistic regression procedures. However the multivariate procedures were reasonably unaffected by assumption violations since this procedure is considered robust. In addition use of the backward elimination method increased the power of the analysis of variance procedures.

Analysis of variance procedures indicated the treatment group did have lower anxiety, higher achievement and self-concept as hypothesized and demonstrated by Townsend, Tuck, Moore, \& Wilton's and Valentino's studies. However, the treatment group demonstrated lower motivation which was contrary to the hypothesis and previous studies such as Bouris, Creel \& Stortz found. Perhaps this motivation loss was due to Hurley and Allen's process loss which includes student motivation loss due to difficulties administering the treatment coupled with minimally experienced cooperative learning instructors (2007). The backward elimination method indicated that no gender differences were significant which was also contrary to previous studies. This result was also
contrary to the hypothesis that females in the treatment group would have a more significant change in anxiety than males in the treatment group. Logistic regression analysis did indicate that the treatment group had a greater probability of passing the course than students in the comparison group, as hypothesized.

## Recommendations for Further Research

More work still needs to be done on cooperative learning and its effects on student outcomes. This study resulted in two unexpected outcomes which should be examined more closely. First, the self-concept pretest was positively correlated with algebra gain. Previous research has shown that students in cooperative classrooms did demonstrate higher self-concept. However studies on correlations of mathematics achievement and self-concept among individualistic, competitive and cooperative structures are still needed to examine why these variables are related and what conditions may change this relationship.

The second unexpected outcome of this study was that the treatment group did not demonstrate higher motivation than the comparison group as hypothesized and previously found in other studies. Although this effect may be explained by Hurley and Allen's process loss along with novice cooperative learning instructors (2007). Perhaps a study examining the relationship between the five key elements for success and motivation, which may change according to the method or amount each key element is applied in the treatment. Since twenty of the twenty-one sections were taught by novice instructors then perhaps a study comparing student outcomes and instructor experience with cooperative learning methods may shed some light on more effective applications. In addition to these
recommendations, perhaps a similar study to this project with a stronger design may also be in order.

As with most social science research, a random sample is difficult to obtain however remains the ideal to be strived for. The external validity and population generalizability of the current study would be increased if a random sample was drawn from the accessible population. Ecological generalizability would also be improved if the sample was taken from a random selection of schools during a regular semester. Perhaps a similar study that utilizes a cross-sectional or cohort design would have a more parsimonious result and reveal variable interactions better.

## APPENDIX A



Figure 4.1 Histogram of age by year. Years that have zero values are not displayed.


Figure 4.2 Bar graph of gender distributions for treatment and comparison groups.


Figure 4.3 Bar graph of ethnicity distributions for treatment and comparison groups.


Figure 4.4 Bar graph of students who tested into College Algebra.


Figure 4.5 Bar graph of students who took intermediate algebra.


Figure 4.6 Bar graph of socio-economic distributions.

## APPENDIX B

Tables

Response by population

| Instrument | Total Sample | Treatment | Comparison |
| :--- | :---: | :---: | :---: |
| Preliminary $\underline{O R}$ Final Exam | 227 | 50 | 177 |
| Preliminary $\underline{A N D}$ Final Exam | 173 | 86 | 87 |
| Online Pretest | 566 | 169 | 397 |
| Online Posttest | 357 | 118 | 239 |

Table 4.1 Participant response by population.

Ethnicity by population

| Ethnicity | Institution in general | Treatment | Comparison |
| :--- | :---: | :---: | :---: |
| African American | $3.4 \%$ | $4 \%$ | $0 \%$ |
| Asian/Pacific Islander | $3.9 \%$ | NR | NR |
| Native American | $6.7 \%$ | $4 \%$ | $7 \%$ |
| Hispanic | $37 \%$ | $36 \%$ | $45 \%$ |
| Caucasian | $43.2 \%$ | $48 \%$ | $39 \%$ |
| No Response | $4.8 \%$ | $0 \%$ | $<1 \%$ |
| Other | $1 \%$ | $9 \%$ | $8 \%$ |

*NR denotes not a reported value
Table 4.3 Ethnicity by population.

## Multivariate Analysis of Variance

Multivariate Tests ${ }^{\text {b }}$

| Effect |  |  |  |  |  |  | Partial Eta <br> Squared |
| :--- | :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Intercept | Pillai's Trace | .548 | $49.932^{\mathrm{a}}$ | 4.000 | 165.000 | .000 | .548 |
|  | Wilks' Lambda | .452 | $49.932^{\mathrm{a}}$ | 4.000 | 165.000 | .000 | .548 |
|  | Hotelling's Trace | 1.210 | $49.932^{\mathrm{a}}$ | 4.000 | 165.000 | .000 | .548 |
|  | Roy's Largest Root | 1.210 | $49.932^{\mathrm{a}}$ | 4.000 | 165.000 | .000 | .548 |
| PRELIM | Pillai's Trace | .523 | $45.202^{\mathrm{a}}$ | 4.000 | 165.000 | .000 | .523 |
|  | Wilks' Lambda | .477 | $45.202^{\mathrm{a}}$ | 4.000 | 165.000 | .000 | .523 |


|  | Hotelling's Trace | 1.096 | $45.202^{\mathrm{a}}$ | 4.000 | 165.000 | .000 | .523 |
| :--- | :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Roy's Largest Root | 1.096 | $45.202^{\mathrm{a}}$ | 4.000 | 165.000 | .000 | .523 |  |
| ANXT_PRE Pillai's Trace | .440 | $32.398^{\mathrm{a}}$ | 4.000 | 165.000 | .000 | .440 |  |
|  | Wilks' Lambda | .560 | $32.398^{\mathrm{a}}$ | 4.000 | 165.000 | .000 | .440 |
|  | Hotelling's Trace | .785 | $32.398^{\mathrm{a}}$ | 4.000 | 165.000 | .000 | .440 |
|  | Roy's Largest Root | .785 | $32.398^{\mathrm{a}}$ | 4.000 | 165.000 | .000 | .440 |
| SC_PRE | Pillai's Trace | .435 | $31.726^{\mathrm{a}}$ | 4.000 | 165.000 | .000 | .435 |
|  | Wilks' Lambda | .565 | $31.726^{\mathrm{a}}$ | 4.000 | 165.000 | .000 | .435 |
|  | Hotelling's Trace | .769 | $31.726^{\mathrm{a}}$ | 4.000 | 165.000 | .000 | .435 |
|  | Roy's Largest Root | .769 | $31.726^{\mathrm{a}}$ | 4.000 | 165.000 | .000 | .435 |
| GROUP | Pillai's Trace | .150 | $7.252^{\mathrm{a}}$ | 4.000 | 165.000 | .000 | .150 |
|  | Wilks' Lambda | .850 | $7.252^{\mathrm{a}}$ | 4.000 | 165.000 | .000 | .150 |
|  | Hotelling's Trace | .176 | $7.252^{\mathrm{a}}$ | 4.000 | 165.000 | .000 | .150 |
|  | Roy's Largest Root | .176 | $7.252^{\mathrm{a}}$ | 4.000 | 165.000 | .000 | .150 |

Exact statistic
b. Design: Intercept + PRELIM + ANXT_PRE + SC_PRE + GROUP

Table 4.8 Multivariate tests.

Tests of Between-Subjects Effects

\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline Source \& Dependent Variable \& \begin{tabular}{l}
Type III \\
Sum of \\
Squares
\end{tabular} \& df \& \[
\begin{gathered}
\text { Mean } \\
\text { Square } \\
\hline
\end{gathered}
\] \& F \& Sig. \& \begin{tabular}{l}
Partial Eta \\
Squared
\end{tabular} \\
\hline \multirow[t]{4}{*}{\begin{tabular}{l}
Corrected \\
Model
\end{tabular}} \& \multirow[t]{4}{*}{\begin{tabular}{l}
Gain from \\
Preliminary to Final \\
Exam \\
Anxiety Gain \\
Self-Concept Gain \\
Motivation Gain
\end{tabular}} \& \(34318.241^{\text {a }}\) \& 4 \& 8579.560 \& 43.659 \& . 000 \& . 510 \\
\hline \& \& \(1328.358^{\text {b }}\) \& 4 \& 332.090 \& 15.407 \& . 000 \& . 268 \\
\hline \& \& \(4053.805^{\circ}\) \& 4 \& 1013.451 \& 5.183 \& . 001 \& . 110 \\
\hline \& \& \(9770.518^{\text {d }}\) \& 4 \& 2442.630 \& 3.890 \& . 005 \& . 085 \\
\hline \multirow[t]{5}{*}{Intercept} \& \multirow[t]{5}{*}{\begin{tabular}{l}
Gain from \\
Preliminary to Final \\
Exam \\
Anxiety Gain \\
Self-Concept Gain
\end{tabular}} \& \multirow[t]{5}{*}{33655.671} \& \multirow[t]{5}{*}{} \& \multirow[t]{5}{*}{33655.671

.211

465.786} \& \multirow[t]{3}{*}{171.266} \& \multirow[t]{3}{*}{. 000} \& \multirow[t]{3}{*}{$$
.505
$$} <br>

\hline \& \& \& \& \& \& \& <br>
\hline \& \& \& \& \& \& \& <br>
\hline \& \& \& \& \& . 010 \& . 921 \& . 000 <br>
\hline \& \& \& \& \& 2.382 \& . 125 \& . 014 <br>
\hline
\end{tabular}



| Corrected | Gain from | 67332.158 | 172 |  |  |  |
| :--- | :--- | ---: | ---: | :--- | :--- | :--- |
| Total | Preliminary to Final |  |  |  |  |  |
|  | Exam | 4949.422 | 172 |  |  |  |
|  | Anxiety Gain | 36902.092 | 172 |  |  |  |
|  | Self-Concept Gain | 115263.549 | 172 |  |  |  |
|  | Motivation Gain |  |  |  |  |  |

a. R Squared $=.510($ Adjusted R Squared $=.498)$
b. R Squared $=.268($ Adjusted R Squared $=.251)$
c. R Squared $=.110($ Adjusted R Squared $=.089)$
d. R Squared $=.085($ Adjusted R Squared $=.063)$

Table 4.9 Tests between subjects.

Tests of Normality

|  | Group | Kolmogorov-Smirnov ${ }^{\text {a }}$ |  |  | Shapiro-Wilk |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Statistic | df | Sig. | Statistic | df | Sig. |
| Algebra Gain | Comp <br> Treat | .071 .048 | $\begin{aligned} & 87 \\ & 86 \end{aligned}$ | $\begin{aligned} & .200^{*} \\ & .200^{*} \\ & \hline \end{aligned}$ | $\begin{array}{r} .983 \\ .992 \\ \hline \end{array}$ | 87 86 | .302 <br> .905 |
| Anxiety Gain | Comp <br> Treat | .070 .098 | $\begin{aligned} & 87 \\ & 86 \\ & \hline \end{aligned}$ | $\begin{gathered} .200^{*} \\ .040 \\ \hline \end{gathered}$ | $\begin{array}{r} .990 \\ .979 \\ \hline \end{array}$ | 87 86 | $\begin{aligned} & .756 \\ & .165 \\ & \hline \end{aligned}$ |
| Self-Concept Gain | Comp <br> Treat | $\begin{array}{r} .109 \\ .120 \\ \hline \end{array}$ | 87 86 | $\begin{aligned} & .012 \\ & .004 \\ & \hline \end{aligned}$ | $\begin{array}{r} .972 \\ .967 \\ \hline \end{array}$ | 87 86 | $\begin{aligned} & .057 \\ & .026 \\ & \hline \end{aligned}$ |
| Motivation Gain | Comp <br> Treat | .065 .152 | 87 86 | $\begin{gathered} .200^{*} \\ .000 \\ \hline \end{gathered}$ | $\begin{aligned} & .983 \\ & .916 \end{aligned}$ | 87 86 | .302 .000 |

a. Lilliefors Significance Correction

* This is a lower bound of the true significance.

Table 4.19 Tests for normality.

Levene's Test of Equality of Error Variances ${ }^{\text {a }}$

|  | F | df1 | df2 | Sig. |
| :--- | ---: | ---: | ---: | ---: |
| Algebra Gain | 2.710 |  | 3 | 115 |
| Anxiety Gain | .210 | 3 | .048 |  |
| Self-Concept Gain | .755 | 315 | .889 |  |
| Motivation Gain | 3.625 | 3 | 115 | .521 |

Tests the null hypothesis that the error variance of the dependent variable is equal across groups.
a. Design: Intercept + Group + Gender + Group * Gender

Table 4.20 Test for the homogeneity of covariance matrices.

## Correlations

|  |  | Algebra ExamGain | Self-Concept Gain | Motivation Gain | Anxiety <br> Gain |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Algebra | Pearson Correlation | 1 | . $225{ }^{* *}$ | . 077 | . 023 |
| Exam Gain | Sig. (2-tailed) |  | . 008 | . 366 | . 786 |
|  | N | 173 | 140 | 141 | 143 |
| Self- | Pearson Correlation | . 225 ** | 1 | .598** | -. 017 |
| Concept | Sig. (2-tailed) | . 008 |  | . 000 | . 851 |
| Gain | N | 140 | 140 | 139 | 120 |
| Motivation Gain | Pearson Correlation | . 077 | . $598 *$ | 1 | . 001 |
|  | Sig. (2-tailed) | . 366 | . 000 |  | . 992 |
|  | N | 141 | 139 | 141 | 120 |
| Anxiety <br> Gain | Pearson Correlation | . 023 | -. 017 | . 001 | 1 |
|  | Sig. (2-tailed) | . 786 | . 851 | . 992 |  |
|  | N | 143 | 120 | 120 | 143 |

**. Correlation is significant at the 0.01 level (2-tailed).

Table 4.21 Test for correlations between instruments.

## APPENDIX C

In class algebra preliminary exam:
Version A

1. $\frac{8}{15}+\frac{3}{10}$ is equal to:
a) $\frac{11}{25}$
b) $\frac{11}{30}$
c) $\frac{5}{6}$
d) $\frac{9}{16}$
2. If $x=-2$ and $y=-1$, then $\sqrt[3]{(2 x)^{2} 4 y}$ is equal to:
a) 16
b) -4
c) 4
d) No real solution
3. If $x=-3$ and $y=5$, then $2|x+2 y|$ is equal to:
a) 26
b) 4
c) -14
d) 14
4. The radical $\sqrt{54 x^{3}}$ is written in simplified form is:
a) $9 x \sqrt{2 x}$
b) $3 \sqrt{6 x^{3}}$
c) $3 x \sqrt{2 x}$
d) $3 x \sqrt{6 x}$
5. $(3 x-2)^{2}$ is equal to:
a) $9 x^{2}+4$
b) $9 x^{2}-12 x+4$
c) $3 x^{2}-4$
d) $4 x^{2}$
6. The simplified form of $\left(\frac{6 x^{3}}{2 x^{-4}}\right)^{2}$ is:
a) $9 x^{2}$
b) $\frac{32}{x^{12}}$
c) $9 x^{14}$
d) $\frac{9}{x^{2}}$
7. The simplified form of $(-27)^{\frac{2}{3}}$ is:
a) -18
b) 9
c) $3^{\frac{4}{3}}$
d) Cannot be simplified
8. Approximated to the nearest tenth, $2.1^{2}$ is equal to:
a) 4.4
b) 4.1
c) 4.2
d) 2.2
9. If $A, B$ and $C$ are points on a number line with coordinates $-3,-2$ and 5 , respectively, then the distance between $A$ and $C$ is:
a) 2
b) 0
c) -5
d) 8
10. The product $(2 x-5)\left(x^{2}-3 x+2\right)$ is equal to:
a) $x^{2}-x-3$
b) $2 x^{3}-6 x+4 x$
c) $2 x^{3}-11 x^{2}+19 x-10$
d) $2 x^{3}-5 x+15 x-10$
11. The domain of the variable $x$ in the expression $\frac{x-1}{x+3}$ is:
a) $-1<x<3$
b) $x \neq 1$ and $x \neq-3$
c) $x \neq-3$
d) $x \geq 0$
12. The complete factorization of $2 x^{2}+3 x-2 x-3$ is:
a) $(x-1)(2 x+3)$
b) $(2 x-3)^{2}$
c) $(2 x-3)\left(x^{2}-1\right)$
d) Cannot be factored
13. When $2 x^{3}+7 x^{2}+9 x-2$ is divided by $x+2$ the answer is:
a) $2 x^{2}+3 x-5, \mathrm{R} 1$
b) $2 x^{2}-3 x$
c) $x^{2}+2$
d) $2 x^{2} 3 x+3, \mathrm{R}-8$

Version B

1. $\frac{7}{10}-\frac{3}{8}$ is equal to:
a) 2
b) $\frac{13}{40}$
c) $\frac{5}{16}$
d) $\frac{2}{3}$
2. If $x=-1$ and $y=-2$, then $\sqrt[3]{(x+2)^{2} 4 y}$ is equal to:
a) -2
b) -4
c) $\sqrt[3]{-7}$
d) No real solution
3. If $x=-2$ and $y=-4$, then $2 x|x-y|$ is equal to:
a) 2
b) 8
c) -24
d) -8
4. The radical $\sqrt{44 x y^{5}}$, written in simplified form is:
a) $2 x^{2} y \sqrt{11 x}$
b) $11 x y \sqrt{2 y}$
c) $2 y^{2} \sqrt{11 x y}$
d) $11 y^{2} \sqrt{2 x y}$
5. The factors of $9 x^{2}-4$ are:
a) $x=\frac{2}{3}$
b) $(3 x+2)^{2}$
c) $\frac{3}{2}(x-1)^{2}$
d) $(3 x-2)(3 x+2)$
6. The simplified form of $\left(\frac{15 x^{-2}}{3 x^{3}}\right)^{2}$ is:
a) $25 x^{2}$
b) $9 x^{10}$
c) $\frac{25}{x^{10}}$
d) $\frac{1}{9 x^{2}}$
7. The simplified form of $(16)^{\frac{3}{4}}$ is:
a) 8
b) 12
c) $4^{\frac{4}{3}}$
d) Cannot be simplified
8. Approximated to the nearest tenth, $2.4^{2}$ is equal to:
a) 4.8
b) 3.6
c) 6.2
d) 5.8
9. If $A, B$ and $C$ are points on a number line with coordinates $-5,-2$ and -1 respectively, then the distance between $A$ and $C$ is:
a) 6
b) 4
c) -6
d) 3
10. The product $(2 x+3)\left(2 x^{2}-x-5\right)$ is equal to:
a) $8 x^{2}-7 x-15$
b) $4 x^{3}+4 x^{2}-13 x-15$
c) $4 x^{3}+6 x^{2}-3 x-15$
d) $4 x^{3}-9 x^{2}-15 x$
11. The domain of the variable $x$ in the expression $\frac{x+4}{x-1}$ is:
a) $x>1$
b) $-1 \leq x \leq 4$
c) $x \neq 1$ and $x \neq-4$
d) $x \neq 1$
12. The complete factorization of $x^{3}-2 x^{2}-x+2$ is:
a) $(x-2)(x+1)^{2}$
b) $(x-1)(x+1)(x-2)$
c) $(x-2)^{3}$
d) Cannot be factored
13. When $2 x^{3}-9 x^{2}+10 x-2$ is divided by $x-3$, the solution is:
a) $2 x^{2}-15 x-25, \mathrm{R} 5$
b) $2 x^{2}-3 x+1, \mathrm{R} 1$
c) $2 x^{3}-9 x^{2}+9 x+1$
d) $2 x^{2}+3 x+1$

## APPENDIX D

In class algebra final exam:

1. The solution(s) to the equation $9^{2 x} \cdot 3^{x^{2}}=\frac{1}{27}$ is/are:
a) $x=3, x=1$
b) $x=-3$
c) $x=-3, x=-1$
d) $x=1$
2. The logarithm $\log _{3}\left(\frac{1}{9}\right)$ is equivalent to:
a) $1 / 3$
b) -2
c) 3
d) $1 / 2$
3. The solution(s) to the equation $\log _{4} x+\log _{4}(x+3)=1$ is/are:
a) $x=4, x=1$
b) $x=1$
c) $x=0$
d) $x=1 / 4$
4. The graph of the basic exponential equation $y=e^{x}$
a) has a vertical asymptote of $x=0$
b) has a horizontal asymptote of $y=0$
c) has an $x$-intercept at $(1,0)$
d) has a $y$-intercept at $(0,-1)$
5. The function $f(x)=|x+2|$ is increasing over the interval:
a) $(-\infty,-2)$
b) $(-\infty, 2)$
c) $(-2, \infty)$
d) $(2, \infty)$
6. Sam has 400 yards of fencing and wishes to enclose a rectangular area. Express the area as a function of the width. After you find the width that gives the maximum area, find the maximum area. The maximum area is:
a) $400 \mathrm{yds}{ }^{2}$
b) 160,000 yds. $^{2}$
c) $10,000 \mathrm{yds}{ }^{2}$
d) $40,000 \mathrm{yds} .^{2}$
7. The inverse of $f(x)=\ln (x)$ is:
a) $f(x)=-\ln (x)$
b) $f(x)=\log (x)$
c) $f(x)=e^{x}$
d) $f(x)=\frac{1}{\ln (x)}$
8. The vertex of the quadratic function $f(x)=2 x^{2}-12 x-4$ is:
a) $(-3,46)$
b) $(3,-22)$
c) $(-3,50)$
d) $(3,-4)$
9. The one true statement about the graph of the function $f(x)=4 x+x^{3}$ is:
a) It is an increasing function with three $x$-intercepts.
b) Its only intercept is the point $(0,0)$.
c) It is a parabola, opening up, with a vertex at $(0,0)$.
d) It passes through the point $(-2,-4)$.

For questions 10-13: Let $g(x)=\frac{x-2}{x+1}$
10. The vertical asymptote of $g(x)$ is:
a) $x=2$
b) $x=-1$
c) $y=x-2$
d) $x=0$
11. The horizontal asymptote of $g(x)$ is:
a) $x=3, x=1$
b) $x=-3$
c) $x=-3, x=-1$
d) $x=1$
12. The $x$-intercept(s) of $g(x)$ is/are:
a) $y=x-2$
b) $y=0$
c) $y=1$
d) $y=2$
13. $g(-2)=$
a) 0
b) Undefined
c) 4
d) $-\frac{4}{3}$
14. Given the one-to-one function $f(x)=\frac{2 x}{2 x+3}$, the range of its inverse is $y=g^{-1}(x)$ is:
a) $y \neq 1$
b) $y \neq 0, y \neq-\frac{3}{2}$
c) $y \neq 0$
d) $y \neq \frac{3}{2}$
e) all $y$
15. If the data $(0,2),(1,3),(3,10),(4,15),(1,2),(3,8),(2,5) \&(4,18)$ is modeled as an exponential relationship, the best equation is:
a) $y=2^{-x}$
b) $y=2^{-x}+1$
c) $y=2^{x+3}$
d) $y=(x+2)^{2}$
16. The graph of the equation $y=-2 x^{2}-2 x+1$ is:
a) a parabola, opening down, with a vertex at $\left(-\frac{1}{2}, \frac{3}{2}\right)$.
b) a parabola, opening up, with a vertex at $\left(\frac{1}{2},-\frac{1}{2}\right)$.
c) a straight line with slope of -2 and a $y$-intercept of 1 .
d) none of the above
17. In order to compute her tax, Janet must subtract $\$ 5,000$ from her income, then divide the result by 3.781356 . According to that formula, an income of $\$ 41,500$ would result in a tax of:
a) $\$ 138,019.49$
b) $\$ 9,652.62$
c) $\$ 3,711.49$
d) $\$ 5,974.90$
18. Sid's Sub Sales are modeled by the function $\mathrm{S}(x)=x^{3}-35 x^{2}+300 x$, were S represents sales per day (in dollars) and $x$ equals the number of different choices he provides his customers. If Sid is set up to provide at most 25 choices of sandwich, in order to maximize his daily sales, he should provide about:
a) 6 choices
b) 12 choices
c) 18 choices
d) 25 choices
19. Quimi starts from home at 8 am riding 11 mph on his bike. Thirty minutes later, his sister Adela starts down the same road riding 13 mph . Adela would catch up to Quimi at:
a) 9:25 a.m.
b) 10:00 a.m.
c) $11: 15 \mathrm{a} . \mathrm{m}$.
d) 12:05 p.m.
20. The amount of money Jeremy should invest now in order for his account to be worth $\$ 15,000$ after 5 years at $3.5 \%$ yearly interest, compounded monthly, is about:
a) $\$ 13,250$
b) $\$ 12,595$
c) $\$ 17,864$
d) $\$ 12,630$
21. A biologist began an experiment with a population of 500 insects. After 4 days had gone by, there were 650 insects. Assuming natural exponential growth, the doubling time for this insect population is about:
a) 1100 days
b) 10 days, 14 hours
c) 6 days, 8 hours
d) 12 days

For questions 22-24, let $f(x)=x^{2}$ and $g(x)=\frac{2 x}{x-3}$
22. $g(f(x))=$
a) $\frac{4 x^{2}}{x^{2}-6 x+9}$
b) $\frac{2 x^{3}}{x-3}$
c) $\frac{2}{x^{2}-3 x}$
d) $\frac{2 x^{2}}{x^{2}-3}$
23. $f(g(-2))=$
a) $\frac{16}{25}$
b) 8
c) $-\frac{8}{7}$
d) -16
24. $g^{-1}(2)$
a) -4
b) 0
c) $-\frac{1}{2}$
d) Undefined
25. When a gas station sold 600 gallons of gas per day, its operating costs were $\$ 750$. When it sold 1150 gallons of gas a day, its operation costs were $\$ 975$. Assuming a linear relationship between gas sold $(x)$ and operating costs $(y)$, the gas station manager calculates that when 1000 gallons are sold per day, the operating costs would be:
a) $\$ 750.00$
b) $\$ 913.64$
c) $\$ 950.25$
d) $\$ 2150.00$
26. The solution(s) to the equation $3-\frac{2}{x}=\frac{x}{x-1}$ is/are:
a) $x=\frac{1}{2}, x=2$
b) $x=-5, x=1$
c) $x=-1 \pm \sqrt{6}$
d) $x=\frac{5 \pm \sqrt{33}}{2}$
27. For the function $(x)=1+\frac{2}{x}$, the average rate of change from $x=-2$ to $x=3$ is:
a) $\frac{2}{15}$
b) $-\frac{2}{5}$
c) $\frac{1}{3}$
d) -1
28. The graph of the piecewise-defined function $f(x)=\left\{\begin{array}{c}{ }^{x+1}-1 \text { if } x \leq-1 \\ x^{2}+1 \text { if } x>1\end{array}\right.$
a) has $x$-intercepts at $(-1,0)$ and $(1,0)$.
b) is symmetrical about the $x$-axis.
c) has a vertical asymptote of $x=2$.
d) includes the point $(2,-3)$.
29. The graph of the function $f(x)=-3^{x+2}$
a) has been shifted 2 to the right and flipped upside-down.
b) includes the point $(0,-9)$.
c) has a vertical asymptote of $x=1$.
d) cannot be graphed.
30. The graph of the function $f(x)=\frac{1}{2} \log _{2}(x+1)$
a) is increasing over the interval $(-1, \infty)$.
b) is symmetrical about the $y$-axis.
c) has a horizontal asymptote of $y=3$.
d) includes the point $(-1,-1)$.

## APPENDIX E

Math anxiety inventory (Betz, 1978). Note that this inventory is a modified version of the Fennema Sherman Mathematics Anxiety Scale (Betz, 1978; Fennema \& Sherman, 1976, 1986 as cited in Bessant, K., 1995). The 4-item Likert scale used to measure this instrument is almost never - sometimes - often - almost always. Items that were reverse scored are marked with an "R".

1. It wouldn't bother me at all to take more math courses.
2. I have usually been at ease during math tests.
3. I have usually been at ease in math courses.
4. I usually don't worry about my ability to solve math problems.
5. I almost never get uptight while taking math tests.
6. I get really uptight during math tests. $\mathbf{R}$
7. I get a sinking feeling when I think of trying hard math problems. $\mathbf{R}$
8. My mind goes blank and I am unable to think clearly when working mathematics. $\mathbf{R}$
9. Mathematics makes me feel uncomfortable and nervous. R
10. Mathematics makes me feel uneasy and confused. $\mathbf{R}$

## APPENDIX F

Math self-concept inventory (Gourgey, 1982).The 5-item Likert scale used to measure
this instrument is disagree strongly - disagree somewhat - undecided - agree somewhat -
agree strongly. Items that were reverse scored are marked with an "R".

1. It takes me much longer to understand mathematical concepts than the average person. $\mathbf{R}$
2. I have never felt myself incapable of learning math.
3. I have a mental block when it comes to math. $\mathbf{R}$
4. I have a good mind for math.
5. I can understand a math problem then it must be an easy one. $\mathbf{R}$
6. It has always seemed as if math required brain cells I didn't have. $\mathbf{R}$
7. I can understand math better than most people.
8. Whenever I am exposed to math, I feel that it is beyond me. $\mathbf{R}$
9. I don't ask questions in math classes because mine sound so stupid. $\mathbf{R}$
10. I have no more trouble understanding math than any other subject.
11. I just don't have a mathematical mind. R
12. When I have difficulties with math, I know I can handle them if I try.
13. My mathematical ability is above average.
14. I have never been able to think mathematically. $\mathbf{R}$
15. I always feel like a dummy in my math classes. $\mathbf{R}$
16. I don't have a good enough memory to learn math. $\mathbf{R}$
17. I get very tense when I see a math problem because I know I will not be able to do it. R
18. I never feel like a mathematical incompetent.
19. Whenever I do math problems, I am sure that I have made a mistake. $\mathbf{R}$
20. I feel secure in my ability to do math.
21. If my eating depended on my ability to do math, I would undoubtedly starve to death. R
22. I have no facility with numbers. $\mathbf{R}$
23. When I have to take math, I worry about whether I can pass. R
24. When I have to do math problems, I do not worry about whether I will be able to do them.
25. Whenever I do math problems, I end by giving up in despair. R
26. I never worry about failing math.
27. When I do math, I feel confident that I have done it correctly.

## APPENDIX G

Science motivation inventory by Glynn and Koballa 2006 (as cited in Glynn,
Taasoobshirazi, Brickman, 2009). Note that the original inventory said "science" instead
of "math". The 5-item Likert scale used to measure this instrument is never - rarely -
sometimes-usually - always. Items that were reverse scored are marked with an "R".

1. I enjoy learning the math.
2. The math I learn relates to my personal goals.
3. I like to do better than other students on the math tests.
4. I am nervous about how I will do on the math tests. $\mathbf{R}$
5. If I am having trouble learning the math, I try to figure out why.
6. I become anxious when it is time to take a math test. $\mathbf{R}$
7. Earning a good math grade is important to me.
8. I put enough effort into learning the math.
9. I use strategies that ensure I learn the math well.
10. I think about how learning the math can help me get a good job.
11. I think about how the math I learn will be helpful to me.
12. I expect to do as well as or better than other students in the math course.
13. I worry about failing the math tests. $\mathbf{R}$
14. I am concerned that the other students are better in math. $\mathbf{R}$
15. I think about how my math grade will affect my overall grade point average.
16. The math I learn is more important to me than the grade I receive.
17. I think about how learning the math can help my career.
18. I hate taking the math tests. $\mathbf{R}$
19. I think about how I will use the math I learn.
20. It is my fault, if I do not understand the math.
21. I am confident I will do well on the math labs and projects.
22. I find learning the math interesting.
23. The math I learn is relevant to my life.
24. I believe I can master the knowledge and skills in the math course.
25. The math I learn has practical value for me.
26. I prepare well for the math tests and labs.
27. I like math that challenges me.
28. I am confident I will do well on the math tests.
29. I believe I can earn a grade of " $A$ " in the math course.
30. Understanding the math gives me a sense of accomplishment.

## APPENDIX H

Glossary of specialized terms.

## A

Alpha coefficient: see "Cronbach Alpha"
Anxiety refers to "feelings of tension that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary and academic situations" from Tobias (as cited in Townsend, \& Wilton, 2003)

## B

C
Cronbach Alpha: "An internal consistency or reliability coefficient for an instrument requiring only one test administration." (Fraenkel \& Wallen, 2009)

## D

## E

Effect Size: "An index used to indicate the magnitude of an obtained result or relationship." (Fraenkel \& Wallen, 2009) Usually ranges between zero and one. Environment of Cooperation Is when students promote each other's success within each group (Johnson et al., 1998).
External Validity: "Is the degree to which results are generalizable or applicable, to groups and environments outside the research setting" (Fraenkel \& Wallen, 2009)

## F

Factorial Design: "An experimental design that involves two or more independent variables (at least one of which is manipulated) in order to study the effects of the variables individually, and in interaction with each other, upon a dependent variable." (Fraenkel \& Wallen, 2009)
Five Key Elements for Success Five elements critical for successful implementation of the cooperative learning method which include positive interdependence, individual accountability, environment of cooperation, social skills and group processing (Johnson et al., 1998).

## G

Group Processing Is the time students take to evaluate themselves and their cooperation with each other (Johnson et al., 1998).

## H

I
Individual Accountability When each student is individually assessed, by quizzes, exams, observations, etc. (Johnson et al., 1998).

Kuder-Richardson Approaches: "Procedures for determining an estimate of the internal consistency reliability of a test or other instrument from a single administration of the test without splitting the test into halves." (Fraenkel \& Wallen, 2009) Kuder-Richardson KR21 Test: This test assumes that instrument items are of equal difficulty. The formula used to calculate this reliability coefficient is:
$K R 21$ reliability coef ficient $=\frac{K}{K-1}\left[1-\frac{M(K-M)}{K\left(S D^{2}\right)}\right]$
where $\mathrm{K}:=$ the number of items on the test, $\mathrm{M}:=$ the mean of the set of test scores and SD:= the standard deviation of the set of test scores. (Fraenkel \& Wallen, 2009)

## L

## M

Motivation "is the internal state of the student that arouses, directs and sustains goaloriented behavior" (Glynn, Taasoobshirazi, Brickman, 2009)

## N

## 0

Outcome Task Performance: Is a product of the potential productivity minus the process loss (Hurley, \& Allen, 2007).

## P

Partial Eta Squared: Denotes effect size. See effect size.
Pearson Correlation Coefficient ( $r$ ): "An index of correlation appropriate when the data represent either interval or ratio scales; it takes into account each pair of scores and produces a coefficient between 0.00 and either $\pm 1.00$." (Fraenkel \& Wallen, 2009) Positive Interdependence When each student perceives that his/her success is reliant on the success of the group (Johnson et al., 1998).
Potential productivity: Is a combination of the resources available to students along with the task demands put upon the students (Hurley, \& Allen, 2007).
Problem Based Learning Is a teaching method that centers on the student. Students are divided into groups of approximately two to five and given a structured task or problem. The students are then expected to interpret the problem, gather the needed information, identify possible solutions, evaluate options and present conclusions with the teacher acting only as a guide (Roh, 2003).
Process Loss: Is the time taken from student learning and used for the administration of the teaching method (Hurley, \& Allen, 2007).

## Q

Quasi-Experimental Design: "A type of experimental design in which the researcher does not use random assignment of subjects to groups." (Fraenkel \& Wallen, 2009)

Reliability Coefficient: "An index of the consistency of scores on the same instrument. There are several methods of computing a reliability coefficient, depending on the type of consistency and characteristics of the instrument." (Fraenkel \& Wallen, 2009)

## S

Self-Concept "refers to a student's beliefs, feelings, attitudes and/or perceptions regarding one's ability to understand and perform tasks in mathematics" (Townsend, \& Wilton, 2003; Gourgey, 1982)
Social Skills Includes both modeling interpersonal and small-group social skills including leadership, decision making, trust building, communication and conflict management, in short, students need to be guided to cooperate with each other. (Johnson et al., 1998).

## T

Type I Error: "Rejecting the null hypothesis when it is true; also called an alpha error." (Bryant et al., 2000) and (Fraenkel \& Wallen, 2009)
Type II Error: "The failure to reject a null hypothesis that is false; also called a beta error." (Fraenkel \& Wallen, 2009)
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