1-28-2015

MODES OF ENGAGEMENT: MULTI-MODAL CURRICULUM AND INSTRUCTION; A SCHOOL-WIDE CURRICULAR DESIGN WITHOUT TEXTBOOKS

Gael Keyes

Gael Keyes
Candidate

Multicultural Teacher and Childhood Education
Department

This dissertation is approved, and it is acceptable in quality and form for publication:

Approved by the Dissertation Committee:

Dr. Diane Torres-Velásquez , Chairperson

Dr. Kersti Tyson

Dr. Vanessa Svihla

Dr. Stephen Preskill

MODES OF ENGAGEMENT:
MULTI-MODAL CURRICULUM AND INSTRUCTION;
A SCHOOL-WIDE CURRICULAR DESIGN WITHOUT TEXTBOOKS

BY

GAEL KEYES

B.S., Elementary Education, University of New Mexico, 1985
M.A., Elementary Education, University of New Mexico, 1987

DISSERTATION

Submitted in Partial Fulfillment of the
Requirements for the Degree of

Doctor of Philosophy

Multicultural Teacher and Childhood Education

The University of New Mexico
Albuquerque, New Mexico

July 23, 2014

DEDICATION

"All art is autobiographical; the pearl is the oyster's autobiography."

Federico Fellini

For Pearl Graeser, my grandmother, whose regret for never having gone to college taught me perseverance.

ACKNOWLEDGEMENTS

I would have never imagined this final product emerging from all my years of process. I owe my gratitude to my dissertation chair, Dr. Diane Torres-Velásquez and my committee members: Dr. Stephen Preskill, Dr. Kersti Tyson and Dr. Vanessa Svihla. Their single-mindedness to usher me through this enterprise inspired me to dig deeper and elevate the experience.

Without this once-in-a-lifetime opportunity to create and run a school for a quarter of a century, none of this study would have been able to occur. I am immensely grateful to those who were pivotal in guiding Family School through the years. They took a huge chance on a very inexperienced and naïve educator. For this, I must acknowledge the following: Jerry Anderson, Joan Heinsohn, Rhonda Seidenwurm, Brad Allison, Diego Gallegos, and Eddie Soto.

My Family School community of teachers, parents and students must be appreciated for their incredible, adventurous spirit, dogged commitment and visionary hopefulness to create a unique school where its complexity is a work of art. There are too many names to mention, though as I write these words, faces both old and new flash through as an album of warm and wonderful memories.

Patience does not even begin to describe the disposition needed in a husband during the process of a wife composing her dissertation. It was a dance, a poem, a monument erected to blurred lines between support and challenge. This study could not have been done without his unbounded belief in me.

MODES OF ENGAGEMENT:
MULTI-MODAL CURRICULUM AND INSTRUCTION;
A SCHOOL-WIDE CURRICULAR DESIGN WITHOUT TEXTBOOKS

BY

GAEL KEYES

B.S., Elementary Education, University of New Mexico, 1985
M.A., Elementary Education, University of New Mexico, 1987

To be awarded PhD., Multicultural Teacher and Childhood Education, University of New Mexico, 2014

ABSTRACT

One of the most challenging aspects of teaching and studying teaching is that the domain of teaching is so complex. This complexity has made it difficult for teachers to design curriculum and critique that evolving curriculum through ongoing reconsideration and revision in order to improve the effectiveness of instruction. An alternative curricular/instructional construct, called Modes of Engagement, was created and has been sustained for 24 years, by the principal and faculty at Desert Willow Family School, a K-8 Albuquerque Public Schools' alternative school. The Modes of Engagement, as they are employed in the teaching of mathematics at Family School, are explored in this study, from

their origins to their current iterations. By designing the mathematics curriculum in segments that reflect varying purposes, and simultaneously matching the modes' instructional practices to these purposes, students at the school have flourished in mathematics while experiencing themselves *as* mathematicians.

This self-study's autobiography and document analysis will reveal how these modes have stood up to the testing pressures and restrictive views of content driven by accountability based on the mandated testing under No Child Left Behind (NCLB). There were consequences for our students under this testing pressure. Within the two to three-year gap of having discontinued two of the school's math modes, students lost the ability to deal with ambiguity, challenges, tolerance for multiple methods, and enthusiasm for learning.

This study reveals the evolution of the Modes of Engagement, including the ways in which their development and implementation has been influenced by both internal and external forces of the school. The Modes of Engagement offer a unique framework for considering school-based curriculum design; the history of the evolution of the Modes offers insight into the forces that foster or impede teacher initiative in curriculum design.

Table of Contents

List of Figures

List of Tables

Chapter 1

Introduction

This study is about the Modes of Engagement, which are curricular/instructional constructs to capture students' engagement around the exciting process of seeing themselves work and think like experts in the fields of the particular subject matter the students are learning. These constructs were created at Desert Willow Family School, (often referred to as Family School). The school has been part of the Albuquerque Public Schools for 24 years and was created in 1990, when approval was granted to start this innovative, alternative program, which combined both public schooling and homeschooling. Since its inception, the school has grown to over 240 students and 12 teachers, from the original 16 students and myself as the only teacher. It now serves kindergarten through eighth grade in multi-age classrooms, using a specially-designed curriculum to meet the unique needs of our community and reflect our unusual classroom structure. All classrooms are multi-age, but the range of grades can vary from a primary classroom that includes kindergartners and first-graders to a classroom with third to eighth-graders.

The Modes of Engagement were a key construct in allowing us to design our own curricula. The teachers and I wanted our curricula and practices to deviate from the norm of the traditional schools since, in creating a new school, we hoped to demonstrate how much could be accomplished when teachers were empowered to design and enact innovative curricula. We designed our curricula to teach toward our chosen purposes rather than the objectives given in textbooks. We perceived the larger purposes of content to be more long-term. We asked the question, "Towards what ends is it important to have students learn this subject?" Textbooks outline academic objectives for students at each grade level, but we set out to identify more global and purposeful reasons for learning than those commonly identified academic objectives associated with traditional standards or benchmarks.

Years later, we gave the modes their name, Modes of Engagement, because we saw that our students were fully committed and engrossed in the material and the work. Having each mode designed to reach various purposes invited students to work with the subject matter differently in each mode, and this experiential variety kept students captivated and engaged. The Modes of Engagement had, in fact, provided the engagement factor in our teaching. Since then, we have come to understand the Modes of Engagement to be dynamic curricular/instructional constructs that have been a hallmark of our unique school, where we have been able to practice designing and implementing our own curriculum for 24 years.

Mode of Engagement Defined

A Mode of Engagement divides the content of one subject domain into smaller partitions by choosing the practices of experts or enthusiasts in the subject matter's field as the goal of the teacher's instruction; students engage in, and reach for, similar practices and similar outcomes of those practices an expert or enthusiast would experience. For instance, when partitioning the subject matter of mathematics, my teachers and I chose to partition teaching, learning and knowing mathematics by the purposes that mathematical experts and enthusiasts would practice. The instruction of each mode would then teach for a specific set of experiences common to experts in mathematical fields. Since mathematicians often work at the edge of their field, trying to solve mathematical quandaries that have not been solved, we chose brainteasers as an activity that would give students experience learning how to work with math at the far edges of their knowledge. We then identified four other purposes common to mathematicians in order to construct our other math modes: 1) solid foundations of basic algorithms, 2) application of math to the real world, 3) using data analysis to answer questions about the world, and 4) solving problems as a mathematics community. When all the math modes are presented to students throughout the year, students become well-rounded mathematicians who can see themselves functioning as mathematicians, having practiced and performed math in ways similar to the practices of real mathematical experts or enthusiasts.

Once the purposes are identified as the end goal of a mode, the modal instruction and decisions about what content are used in the mode are then designed as the mode's means, which will get the students to those ends. In this way, each Mode of Engagement becomes a curricular/instructional construct, because the content and instruction must be designed together in order to drive the mode's purposes. Having used the evolving modes for 24 years, we have come to see their elegance, effectiveness, and viability. This study will research the evolution of our math Modes of Engagement, over this 24-year period, to investigate how they responded to both internal and external pressures and how they have changed and survived over time as innovative, teacher-created, educational tools. Most educators are well-versed in dividing content into smaller content areas in which to use a variety of instructional practices, but the notion of dividing content according to specific and different teaching practices, in order to meet the various purposes of a field, is not a teaching concept commonly used by most educators.

The subject matter of mathematics is studied in this research project. The chosen modes in mathematics (which we call our math modes) are: the Algorithmic Math Group Mode, Math Brainteaser Mode, Mobius Math Mode, Real World Math Mode and Inquiry Mode. While all subject matter at Family School is designed and taught through the Modes of Engagement, only the math modes will be studied for this research project. The math modes were chosen because they all have roots in the beginning of the school, present a wide variety of distinct purposes around which they were designed, and have undergone a controversial evolution at our school.

Our definition and enactment of the modes inform the day-to-day work of our school. The Modes of Engagement are instructional/curricular constructs where a subject matter is parsed into smaller segments to enact and teach toward the distinctive purposes of the qualities an accomplished member in that field would demonstrate. Each Mode of Engagement functions in a very specific manner for having parsed the subject matter and its purposefulness into the various modes. In order to define, enact and teach the distinctive purposes of the qualities an accomplished member in that field would demonstrate, the modes must also be on-going and recursive, so that students can develop each distinct set of disciplinary habits, as enthusiasts or experts would develop. The figure below is a diagram of how the Modes of Engagement are designed to function as modes at our school.

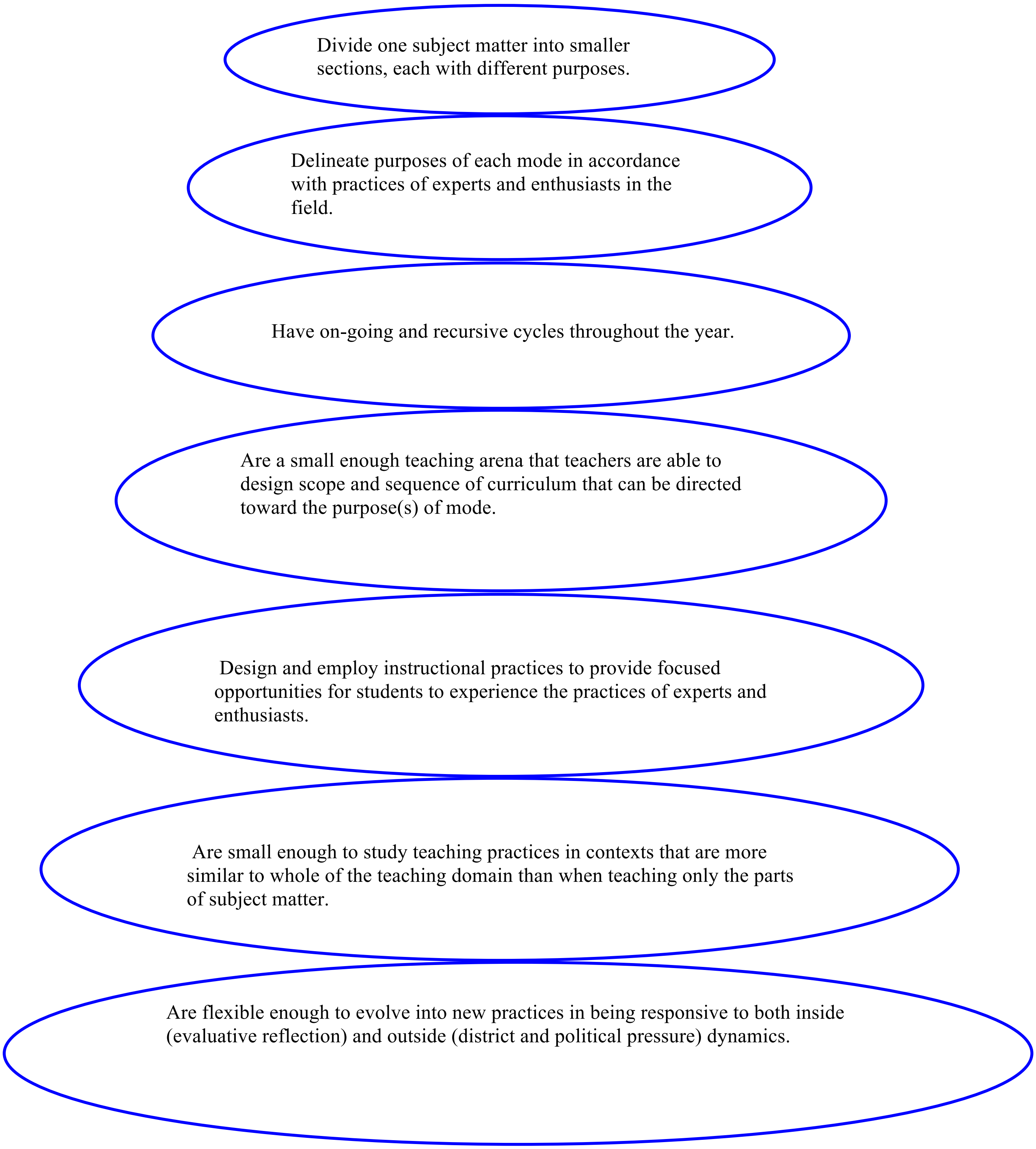


Figure 1. The properties of the Modes of Engagements at Family School.

Table 1 describes the various math modes. The purpose of the mode, in the first column, describes what mathematical practices are needed for students to function like experts or enthusiasts. The structure of the curriculum designed for these modes are described as linear, spiral, or neither, in which case the mode is periodically placed in the schedule throughout the year in small units, but the content is not connected from one unit to the next. The last column describes how the students are assessed in each math mode.

Table 1

How the Math Modes are used at Family School

Mode	Content	Purpose	Structure of Curr.	Performance Level
Algorithmic Math	Basic Mathematical Algorithms	Deep conceptual understanding of algorithms as the basic building blocks required of mathematicians.	Linear	Individual Mastery of each algorithm with 85% proficiency
Math Brainteasers	Math Puzzles or above grade-level- math concepts	Encouraging students to work at the edge of their mathematical knowledge by working their own way through to a solution. More emphasis on how to solve rather than on right answers.	Cycled yearly	Exposure and exit tickets for conceptual learning, Mastery of problem solving strategies
Mobius Math	Math grade level standards-based workbook	Mastering grade level requirements for testing. Mathematicians may have successful testing backgrounds in the field.	Spiral	Whole group of multi-age for exposure, but mastery of student's grade level only
Real World Math	Integrated math in conjunction with workshop unit themes	Investigating mathematics as it is integrated with other subjects, for example: science, social studies, and health	Linear	Individual Mastery of algorithmic level problems with 85% proficiency
Inquiry Data	Data used to answer students' inquiry designs	Learning the power of data and how it answers questions.	Cycled yearly	Yearly portfolio of inquiry samples or Inquiry Fair project

Background of the Study

> The word "complicated" is often used to describe what it means to fulfill the teacher's role. The various and overlapping tasks of responding individually and collectively to students, of "managing" material and human relationships, of organizing for instruction, and of assessing the merit of completed tasks, are all understood as activities that are enormously "complicated"… [W]e have come to understand that "complicated" does not offer an adequate description of the acts of teaching and learning. Rather, these are "complex" phenomena. (Davis and Sumara, 1997, p. 117)

Complex nature of teaching. Research in education tries to capture the intricacies of teaching and learning long understood by teachers and little appreciated by policy makers. While teachers must plan, implement, and evaluate lessons, simplified teaching practices don't begin to reveal the need to consider the context of differentiation, student interest, and motivation, learning styles, and the latest educational trends, not to mention school, district, and governmental demands placed on a teacher. These consequential matters influence a teacher's decisions continually. A complex dialogue about subject matter and how to extend that subject matter to students must consider many theories from the science of teaching and learning simultaneously. Mason (1998) asserts that teachers must build a capacity for attention to subject matter practices, student understanding, and also their own understanding of content as they teach. This level of attention will then lead to other complex issues. Schön (1987) suggests that effective teachers use "reflection-in-action," a process whereby teachers are rethinking and reshaping what they are teaching as they teach it. According to Jaworski (2004), the act of teaching requires "in-the moment decisions involving cognitive and sociosystemic factors relating to the diverse needs of pupils in class," (p. 22) which then requires a complex understanding and ability to act accordingly on the part of teachers.

Martinez (2009) goes so far as to say that "the contextual factors that are a part of today's classrooms far exceed the professional capacity of teachers, resources and the education system" (pp. 1-2). He contends that the overload of issues a teacher encounters results in teachers distancing themselves and "giv[ing] up ownership of many problems" (p.2) they encounter in the classroom.

Teachers have long tried to improve their teaching by solving any one of the many problems in their classrooms that are rooted in complex systems. Analyzing a problem embedded in a complex system can often lead to quick fixes that do not contextually address the complex system. Consequently, the quick fixes do not effectively address the issues in the classroom.

Recently, the pressure for teachers to improve their instruction because of the political pressure to increase teacher accountability has heightened this problem. The reauthorization of the Elementary and Secondary Education Act (ESEA), referred to as the No Child Left Behind Act [NCLB] of 2001, intensified the call to analyze teaching practices and hold teachers accountable for student learning more than ever before. Cochran-Smith (2003) asserted that this pressure makes the profession of teaching "unforgivingly complex" (p. 3). She contends that teaching should not be narrowed to "focus exclusively on test scores and ignore the incredible complexity of teaching and learning and the institutional realities inherent in the accountability context" (p. 4). Banks et al. (2005) also added the responsibility for preparing prospective teachers to be able to contend with the complexity of teaching. Davis and Sumara (1997) portray this complexity best in the following quote:

> When one considers that educationists are confronted with such contrasting theories as behaviorism and radical constructivism, for example, it is perhaps not surprising

> that they have been unable to settle on curricula and models of instruction that have met with the approval of more than a small portion of the population. The implications and emphases of the many understandings of cognition are simply too varied. We often find ourselves with programs of study and teaching strategies that attempt to address the interests and insights of as many theorists as possible, yet satisfy none. We thus find ourselves in a situation where, within virtually every speech or document intended to guide classroom practice, we can expect to hear calls for a renewed emphasis on the "basics" and for "student-centered instruction" and for the formal, standardized evaluation of individuals, and for the collective learning activities—all with no apparent awareness of the contradictory nature of many of these notions. (p. 107)

How do teachers or researchers study teaching and learning in such a way that the results can suggest changes which could be made to systematically improve classroom practice in such a complex environment?

Principles of complexity science as they pertain to complexity of teaching. The notion of complexity has also arisen in the context of *complexity science*, and the term complexity means something quite different than merely complicated. The metaphor that is used to describe something mechanical would be called complicated, because the machinery could be taken apart to be fixed (Stanley, 2009). In the new notion of complexity, this metaphor does not hold true. In fact, in complexity science, the whole is greater than the sum of its parts, so that it cannot be broken apart and fixed to get back to its original state. This notion of complexity is "true of learning and learning systems," (p.1) according to Stanley (2009). Davis and Sumara (2000) report that complexity science emphasizes the relational qualities of systems like neuronal assemblies, governance bodies, biological bodies, and ecological systems. Complexity science is not concerned with isolating the parts of these larger systems or organizations but rather more interested in understanding the relationships of the parts, and how they contribute to the whole. Because there is complexity in classrooms, studies should direct our attention as we research ways to improve our classrooms.

Principles of complexity science pertain also to teaching. Stanley (2009) highlights a few of these principles when he states, “We now have a few complexity principles with which to work: neighbour or local interactions, decentralized control, diversity and redundancy” (p. 2). When these principles are not in balance or are missing, then the learning organization is found to be unhealthy, meaning that it can’t share, change, or evolve. Davis and Sumara (1997) suggest that ecological boundaries don’t actually exist; they exist in gradients, and “[e]verything is inextricably intertwined with everything else” (p. 111), which is not a common way of thinking in a western society. These authors suggest that teachers should to be attentive to the ways in which teaching and learning deal with this complexity, by weaving together all of these strands required of teaching. Any attempts to change pedagogical practice would have to understand the simultaneity of events that are occurring constantly for both teachers and students.

Grossman and McDonald (2008), in considering the notion of studying the complexities of teaching, argue that researchers must “respect the difficulty of breaking apart such a complex system of activity and the dangers of doing irreparable harm to the integrity of the whole by making incisions at the wrong places.” They suggest that there is a need for a framework of teaching involving the “careful parsing of the domain” (p. 186) of teaching.

They assert that in order for teachers to improve amidst all this complexity, they must have a common framework with common language with which to talk about and understand their profession.

The following model, Figure 2, places the Modes of Engagement at the center of the parsing of the complexity of teaching. The modes parse the teaching into smaller components of curriculum and instruction, yet this parsing maintains a better integrity of the whole. By studying the teaching in the modes, teachers and researchers would be more able to address the complex system in which they are contextually based. The mode in the smaller-partitioned intersection is prescribed by more specific qualities of the larger domains of the model below. Thus, in the constraint of the intersection of these domains, the modes are characterized by more refined strategies and a common language to share among teachers.

How the Modes of Engagement focus the complexity of teaching.

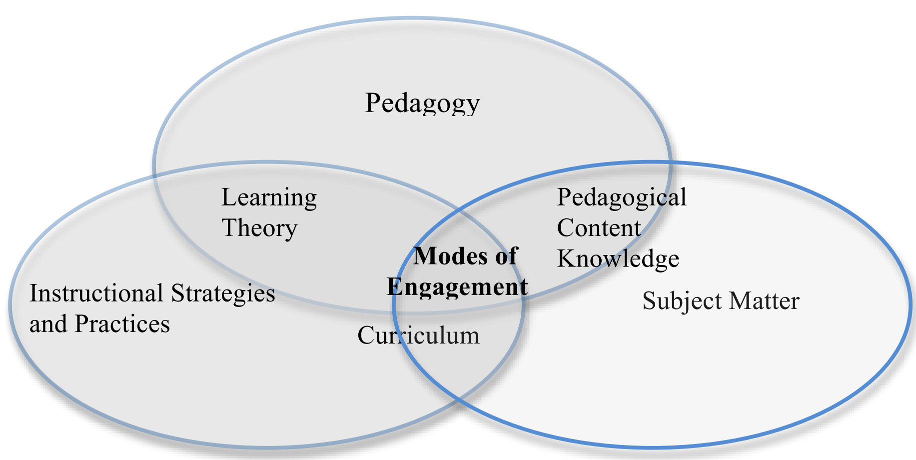


Figure 2. The Modes of Engagement address the intersection of the complexity in the classroom.

The Modes of Engagement can provide a window into viewing the complex nature of teaching. This window constrains, yet reveals, the interaction of curriculum, learning theory, pedagogical content knowledge, instructional practices, subject matter, and pedagogy. My definition of curriculum, for the purposes of this diagram, refers not only to the study of the written scope and sequence of content, but also to how curriculum is understood, and to the dynamic, enacted, teacher and student actions that bring it to life. At Family School, the context of our belief for teaching all of our students is that it must revolve around meeting the needs of all students, regardless of socioeconomic, cultural, and other individual differences. For our school, each of the areas in this diagram must address these student differences. As a result, any research done on teaching of the modes would then be representative of the whole of teaching, but done only in the smaller intersection of the whole. Research into the effectiveness of the modes could promote an excellent way for teachers to learn how to improve their teaching in the complex context of teaching. Our Modes of Engagement reflect teaching while wrestling with this complexity.

My teachers and I designed the modes with attention to content, instructional strategies reflective of learning theory, and pedagogical content knowledge of the curriculum to encourage students to be able to practice and think like domain experts. This study is the story of how one teacher, who is also the school principal used Modes of Engagement in her school to promote powerful learning among students, teachers, and parents alike. I am that teacher. I say I am a teacher, instead of the principal, because although I returned to the university to obtain my administrative license to continue to run the school, I had started the school as a teacher with only 16 students, and my heart has always been the heart of a teacher. Within three years of its creation, the school grew to 180 students and now averages 240 students each year. I have always continued to teach throughout the school’s 24 years. I am one of the few teaching principals in the district.

While this study incorporates my memories of this journey, the work of the document analysis has kept my memories humbly in check. The story is not just my story, but the story that the documents and my reflective, personal journal reveal about our school, the journey of the development of the Modes of Engagement, and what we have learned through teaching them.

Problem Statement

This study focuses on our mathematics Modes of Engagement and considers what the evolution and changes of the math modes over the years tell us about how they have operated, how they have been sustained, and how they have evolved. Since this study is about mathematics, it also includes how the modes influence the teaching of mathematics. While the context of classrooms is complex, the window of studying how the teaching of mathematics plays out by using these modes is less cumbersome than studying the Family School's entire educational program. Each mode narrows the window of mathematics instruction to smaller more manageable parts. By comparing the journey of each mode throughout the quarter of a century that the school has been in existence, this study investigates how the modes responded to the context of the internal and external pressures, those pressures the school community placed on the modes, or even the pressures of the political trends of the day. Having built our modes, the school needs to study their evolution throughout the years for continued improvement.

As Feiman-Nemser states (2012), "learning to teach outside of teaching is like trying to scratch an itch on the outside of your boot" (p. 239). The best place to learn about your teaching is in the context of the classroom and in conversation with staff members who have been brought together to study their teaching. When studying the teaching of one subject domain, teachers often have so many variations on the theme that they have difficulty seeing how other's practices could influence their own. However, by using the modes to break the subject matter of math into smaller segments, it narrows the focus, and the school is more easily able to study the domain of mathematics as each mode addresses various mathematical purposes to drive its instruction. The study of the evolution of the modes will be analyzed to continue to improve their use and design. The documents, autobiography, and reflective, personal journaling not only revealed the development of the modes but also revealed the themes of this journey, especially the impact of the modes on the improvement of teaching.

This research concerning the math modes will study the different teaching practices used for each mode as they uniquely fit in the intersection of curriculum, pedagogical content knowledge, and learning theory for mathematics. Another way of seeing this intersection for mathematics is summed up by Hersh (1997) when he lists the three essential criteria of mathematics education that are needed: "1.) Recognize the scope and variety of mathematics, 2.) Fit into general epistemology and philosophy of science, and 3.) Be compatible with mathematical practice—research, application, teaching, history, calculation, and mathematical intuition." Lakoff and Núñez (2000) portray a mathematician's view of mathematics eloquently in the following passage:

> Mathematics is every bit as conceptually rich as any other part of the human conceptual system. Moreover, mathematics allows for alternative visions and versions of concepts. There is not one notion of infinity but many, not one formal logic but tens of thousands, not one concept of number but a rich variety of alternatives, not one set theory or geometry or statistics but a wide range of them—all mathematics! Mathematics is a magnificent example of the beauty, richness, complexity, diversity,

> and importance of human ideas. It is a marvelous testament to what the ordinary embodied human mind is capable. (p. 379)

Such elegant ideals of mathematics should find their way into the teaching and learning of mathematics, yet many mathematical textbooks and mathematics teachers have not been able to portray the essential qualities of mathematics suggested by Hersh or Lakoff and Núñez. This research project examines Family School's math modes' evolution by identifying the critical features of each of the modes as they responded to this complexity of teaching. These features were revealed after investigating the purposeful design of instruction for the content of mathematics intended to offer students opportunities to experience the practice and cognitive strategies of domain experts.

Chapter organization. This research paper presents the literature review in Chapter 2 and the methodology in Chapter 3. The literature review is guided by studying the mathematical research that influenced or elucidates Family School's development and practices of the Modes of Engagement. Chapter 4 will present an overview of the school, its origins and development, to set a context for the reader about the *place* in which the modes were created, implemented, and nurtured. Chapters 5 through 9 reveal the narrative autobiographical research supported by both the document and personal journal analyses. These chapters reveal the evolution of each math mode from the roots of their origins to their present practice. Chapter 10 and Chapter 11 present the findings and conclusions that the Modes of Engagement's evolution reveal about their durability in supporting the school's continuous curricular renewal.

Chapter 2

Literature Review

This literature review will explore what practices that are important for a school community that participates in designing and evaluating their own curriculum. When the school involved with the designing is an alternative public school, it is essential that the design take into consideration the unique values of the school. This first section of the literature review discusses reflective practice and teacher as curricular designer, and the impact of this design on accelerated learning and student engagement.

The next section of the literature review presents the mathematical practices considered when specifically designing mathematics curriculum for a K-8 school, especially the making of mathematics curricula that support students in developing the practices that mathematicians use in their work, developing the metacognitive skill needed for mathematics, and the benefits of identifying threshold concepts in designed curriculum. Other considerations in mathematics design for the K-8 school are the use of mathematic manipulatives and building a curriculum toward the goal of proficiency in algebra. Lastly, while the curricular design of the Modes of Engagement is for mathematics, and in order to capture as much time for students in mathematics as possible, integration of math with other subject matter is considered, as well as the importance of real-world application.

Teacher Practices Important for Curricular Design

Reflective Practice

When teachers decide to design their own curriculum, it is important that reflective practice becomes a regular habit of the learning community, to measure the adequacy of the design and its effectiveness. Teachers don't come necessarily prepared to examine their practices. As Loughran (2002) eloquently writes:

> The danger for reflection is that if practice is limited to understanding it backwards, then forward practice may remain uninformed. If learning through practice matters, then reflection on practice is crucial and teacher preparation is the obvious place for it to be initiated and nurtured. (p. 41)

Loughran points out that reflection on what has been done is important for the planning of what one is to do, therefore, teachers' design of curriculum benefits from reflection on their design decisions for any alterations to improve its effectiveness. When the school is involved in the design, collaborative reflective practices should enhance the coordinated efforts to create improved curriculum.

Ferrero (2000) outlines three kinds of collaborative reflective practice: reflecting with peers, teacher mentors, or instructional coaches. She outlines the benefits to teachers, which include a deeper understanding of their own practices and effectiveness while wrestling with their own beliefs, understanding the status quo of educational practices and challenging their own practices to reach beyond those practices. Her review of the reflective practice literature indicates that inquiry is an effective method of reflective practice. Jay and Johnson (2000) suggest that reflection is a complex practice. They describe three types of processes for reflection: 1) Descriptive—which describes the topic or issue reflected, 2) Comparative—which reframes the topic of reflection, and 3) Critical—which considers the implications of the reflection and establishes a new perspective. By outlining the various ways to reflect, Jay and Johnson reveal essential actions that teachers undergo when developing the habit of reflective practice. These layers search more deeply within a teacher's intentions and beliefs as the habit for reflective practice becomes more deeply rooted in the teacher. Wide varieties of opportunities to reflect suggest that engaging all teachers in a reflective community invites many different levels of reflection to fit the variety of personalities of teachers. While the invitation to reflect is important, reflection can be prodded to go deeper than many teachers understand.

Larrivee's (2000) notion of critical reflective practice delves deeper into the notion of critical analysis by suggesting that teachers examine their *personal filtering system*, meaning that they should challenge their own value systems and assumptions. She delineates how philosophical beliefs impact the underlying principles of teachers' work, which influence their daily interpretive practice and inform their decision-making. She believes that if:

> …we are able to face the conflict, surrendering what is familiar, we allow ourselves to experience the uncertainty. This not knowing throws us into chaos. At this phase, if we *move into the eye of the storm*, we *weather* the turmoil and a deeper understanding emerges, moving us to the reconciling phase. In this final stage, we have had a shift in our thinking and sensing. (p. 305)

Unpacking teachers' philosophical beliefs is important work for teachers who are designing curriculum. Those deeper philosophies impact the educational decisions teachers make in their curricular design. The nature of design is creating curriculum that moves students toward some educational standard and ideal. Philosophical underpinnings direct design work toward ideals and values; these ideals and values cannot remain hidden for critical reflection to lead to substantive curricular change.

Lieberman (1995) suggests that in order for teachers to change the way they work, "teachers must have opportunities to talk, think, try, and hone new practices, which means they must be involved in learning about, developing, and using new ideas with their students" (p. 69). This can occur by working in small groups of teachers on new tasks, ultimately creating a culture of inquiry. When reflective practice becomes a habit, then inquiry in the classroom and continued design work is constantly questioned and evaluated. While this practice is important for any design, mathematical curricular design work also involves the added reflection into a teacher's own understanding and approach to mathematics.

Ball and Bass (2000) contend that the best way for teachers to improve their pedagogical content knowledge in mathematics is by grounding it in their practice, whereby the improvement "could help close the gaps that have plagued progress in teacher education" (p.101). While they were referring to the education of student teachers, it is also essential to consider that teachers continue to learn as they teach, and they should always continually learn to improve by studying their grounded practice. This suggests that the on-going reflective practice within a teacher's mathematics teaching will reveal much important pedagogical content knowledge specific to mathematics that will improve the mathematical curricular design.

Mason (1998) describes the complexity with which a teacher should study his/her own practice by differentiating various kinds of attention and awareness that will build a teacher's ability to improve math instruction. By building a better attention for what students are doing, together with attention to what the teacher herself is doing, the reflectivity needed to study one's teaching begins. However, he also contends that a teacher must build his/her awareness of the discipline of mathematics. "Mathematical thinking lies at the heart of teacher-pupil discourse, but can only be conducted effectively if it is informed by personal awareness-in-discipline and in-counsel [with an expert teacher]" (p. 262). Mason brings students into the conversation about teacher reflective practice. The context of learning from students is rich pedagogy that can deepen teachers' understandings of, and interest in, mathematics.

Mason's simultaneity of reflective practice demonstrates the importance of creating an arena for teacher discourse around similar teaching strategies in order to improve not only the students' abilities to learn but also the teachers' understandings of the subject matter and their ability to teach it. Professional development focused on both content and instruction should be well integrated, and this integration should inform our best practices of mathematics teaching and learning.

Stonewater (2005) interviewed students who had come from either a "watch-learn-practice" method or a "self-as-initiator" method of instruction in mathematics to analyze their perceptions after taking an inquiry course in mathematics. By the end of the course, two-thirds of the students in the study changed their views of the best method for teaching mathematics; they came to believe that the inquiry method was superior to the other methods. Just as students enjoyed using inquiry in their learning, perhaps teachers engaging in a community where reflective practice is a regular practice would come to prefer studying their own teaching through inquiry to the traditional *sit and get* professional development usually offered to teachers.

Another form of professional development that is very different from the traditional *sit and get* model is doing the actual design work. When teachers design the curriculum for the school, there is a great deal of research and practice that must precede the work of using it in the classroom with their students. Parke and Coble (1997) suggest that:

> Curriculum development is a vehicle for both the professional development of teachers and sustained school reform. The dialogue that occurs is necessary to create

> a common terminology understood by all so that decisions become a sense-making process for the teachers. (p. 785)

They also found that when teachers designed their own curriculum they were not able to point fingers at anyone else but themselves for any failings in the curriculum. More importantly, teachers bought into the curriculum more readily, having designed it.

Teacher as Designer

Teacher as curriculum designer is very much connected to reflective practice in that those deeper values held about the curricular design are often hidden from teachers' previous experience in schools.

Drake and Sherin (2006) researched the impact of teachers adapting mathematics curriculum in the context of the mathematical reform. In studying two teachers who were committed to making changes to their curriculum for mathematics reform, they found that the changes made in the curriculum may be connected to a teacher's previous mathematical experiences. For example, one teacher in the study believed that learning the new mathematical terms for the new reform would make the difference for student understanding because this was what she was lacking in her mathematical experience, and she didn't want this happening to her students. The other teacher's mathematical experience was connected to a high school geometry experience, where she was exposed to continual problem study in order to come to a mathematical understanding of the answer and its proof. Her adaptations to her reform activities were based on a continual study of her students and how they responded to her teaching. Drake and Sherin's (2006) study also reflected that the adaptations to the mathematic curriculum were not made "in a haphazard or ad hoc manner with the curriculum, but instead were engaging in deliberate ways with the curriculum materials" (p.

185). While Drake and Sherin found that it was important for teachers to *follow* curriculum, as new curricula were often used to implement reform changes, it was also important for teachers to make adaptations, though they observed that it was complex to root out reasons behind the adaptations that teachers chose to enact. Lastly, they supported the teachers' needs to adapt by suggesting that curriculum developers should be "explicit about the conceptual goals and intentions of a given lesson or set of lessons to connect those goals directly to the proposed activities" (pp.183- 184), in order for teachers to be able to adapt these activities for the right reasons.

Carlgren (1999) suggests that a reason teachers have not been seen as curriculum designers is the fact that their work *inside* the classroom is more valued than their largely-invisible planning stages. She argues that a teacher's reflection-in-practice does impact the planning phase and should be seen as a critical part of teacher work. She claims that much of the curriculum design for schools today does not have much to do with the real work of teaching; she believes that teachers should have a bigger part in their own curricular design. She writes:

> In order to develop professionalism in teachers' design work there is a need to develop a tradition of using language for conceptual meaning-making whereby the meanings of practice can be abstracted and dealt with in a way that is separate from the particular forms in which they are captured. (p. 53)

She also contends that if the planning phase of teaching is to be seen as competent design practice then students in teacher education programs should see their planning to teach treated as "a simulated practice with reflective backtalk as part of the planning, so that the students have experience naming and framing as well as reframing and become used to formulating and discussing the meaning of things" (p. 54). Again, the practice of teachers as designers is connected to reflective practice.

Both the design work and the reflection are not only connected to the teacher's values, but also are integrally connected to the values of the school. With values including the desire to have students attain success with an accelerated curriculum, student engagement becomes a priority to enact curriculum where students are actively involved in making meaning.

Accelerated Curriculum

Recent research into a middle school program that de-tracked its mathematics classes, so that all students could participate in accelerated mathematics classes, revealed much success for all students, even those students who would have been placed in lower classes before the de-tracking policy. Burris's (2006) research on heterogeneously-grouped students given an accelerated mathematics curriculum demonstrated that students who would have been traditionally tracked into lower-level classes were able to perform similarly to their higher-performing peers by the end of the course, thus passing the regents' examination in the eighth grade. They also found there was not any evidence that there were any increased numbers of students for whom this accelerated curriculum was too advanced, and, few students fell behind their grade level. Burris contends that her study "does not support concerns that the performance of high achievers will decrease in heterogeneously-grouped classes even if the high-track curriculum remains in place" (p. 131). Burris (2005) wrote of the same study where she followed three cohorts of de-tracked eighth-grade students into high school to track the level of mathematics courses that followed this eighth-grade experience. She found there were “significant increases in the probability of minority students studying advanced math courses” (p. 598).

Schmidt et al. (2005) wrestled with US mathematics curriculum and its lack of coherence. When they compared the curriculum for the US to the curriculum of the countries with an A+ (those countries that scored well on the Third International Mathematical and Science Study--TIMMS), they found that the US standards were taught in small sections over a great many years, whereas the A+ countries’ standards taught mathematics topics over fewer years, but for longer amounts of time. This less-fragmented curriculum provides much more coherence around the topics of mathematics and also allows students to make much deeper connections to each of the standards. Ultimately, this is a significant difference in teaching and is reflected in the scores on this international test (pp. 532-540). They found that fewer topics covered in greater depth led to an advanced understanding of mathematical concepts. The acceleration of material appears to be connected to the depth to which it is taught. Both of these considerations –fewer topics and greater depth—would greatly impact curricular design work of a mathematics curriculum.

Student Engagement

Designing curriculum to engage students is an important consideration. The kind of student practice required by the curriculum would ultimately make the instruction of the material more effective. Recent studies connect rigor and challenging curriculum to increasing levels of student engagement.

Shernoff, Csikszentmihalyi, Schneider and Shernoff (2003) conclude that high-school participants in their study experience engagement when they have challenging and relevant activities “in which [they] concentrate, experience enjoyment, and are provided with immediate, intrinsic satisfaction that builds a foundation of interest for the future" (p. 173). However, this study also found that about one-third of the students' time in school was recorded as being spent:

> …passively attending to information transmitted to the entire class. More than half of their instruction was spent on independent work that was somewhat active, structured, or intellectually challenging for at least some of the time. Approximately 14% of the students' time in class was spent in more interactive activities, such as class discussions and group activities. (p. 171)

It appears that school instruction alone does not ensure that student engagement will occur. The challenge and relevance of the material is important, as is the agency of the student. When students feel they are in charge of their learning, they become more engaged. However, Klem and Connell's (2004) research suggests that "by creating more personalized educational environments—an indicator of which would be increased experience of teacher support by students—student engagement, and higher attendance and test scores should result" (p. 271). This suggests that in order for students to be engaged, students need more support from teachers rather than relying on teachers to initiate the activity.

Both challenging and more personalized environments connecting student and teacher contribute greatly to student engagement, but other studies find that peer groups also create personalized environments. Traditional study groups developed to prepare for the rigor of challenging courses are mediated and personalized by the group interaction. Zhao and Kuh's (2004) research on university students participating in study groups and its effect on student engagement demonstrated that over the four years of their college experience, the students who participated in study groups were more likely to be involved in engagement activities

(activities that required extra participation on the part of the student). Participation in learning communities was "positively linked with more frequently interacting with faculty members, engaging in diversity-related activities, and having classes that emphasized higher-order thinking skills"(p.125). Ultimately, the study group participation was correlated with the students generally being more satisfied with their college experience. This study suggests that when students work with peers, student engagement occurs significantly more often.

Brewster and Fager (2000) present many techniques for schools and teachers to increase student engagement. Their focus is on motivational techniques that are represented in all of their ideas about engagement. They suggest that students need to build relationships with the teachers, be given challenging work about which they have some curiosity, and build relationships with other students as motivating factors. They also suggest the importance of communicating with the parents of the students. This literature also highlights four schools in the Northwest where student engagement is a predominant feature of the schools. They suggest that these schools' common features are "community, consistent discipline, and student-centered learning to motivate students day after day" (p. 13)

Yazzie-Mintz's (2007) analysis of the High School Survey of Student Engagement (HSSSE) suggest that there were:

> …a wide range of views – kudos and critiques, analysis and recommendation, frustration and excitement – the overwhelming number of comments from students that their efforts to express their views were 'pointless' since they were sure no action would come from this project speaks to the need for students to be taken seriously if they are to be engaged in school. (p.10)

Yazzie-Mintz advises that high schools should listen to the voices of these students, for students suggest that getting a degree to go on to college is not the only thing they want out of high school. Students are clamoring to be “intellectually, academically, socially, and emotionally engaged with the life and work of their high schools” (p. 11).

If challenging and personalized curriculum results in student engagement, designing mathematics curriculum should build mathematical prowess that develops students to direct their own personalized interaction in the class.

Mathematical Practices for Designing Mathematics Curriculum

In designing mathematical curriculum toward student engagement, it would be important to direct the curriculum toward students engaging in activities that are similar to the work of real mathematicians.

Developing Mathematical Identities

Many mathematics classrooms present curriculum by rote, procedural and repetitive methods, when the actual work of mathematicians requires much more problem-solving of challenging mathematical dilemmas and enigmas. Burton (2000) studied the qualities of research mathematicians to compare their practice to the experiences in a typical math education. She found that when mathematicians are working “at the cutting edge” of their field, they are immersed in a “creative endeavor, which demands a very different epistemological stance from the one which pervades the teaching and learning of mathematics” (p. 595). In Burton’s (2002) experience, she suggests that the more usual student experience in mathematics education is one of frustration, rather than the sense of wonder and delight in mathematics that mathematicians’ experience. Her interview of 70 research mathematicians revealed the manner by which these mathematicians belonged to a mathematics community, developed mathematical identities, practiced mathematics and made meaning using mathematics. Out of the 70 participants, only four mathematicians suggested that they worked alone. All others felt their roles and identities were connected to membership in a mathematical community by working collaboratively with others. Because of their work in these communities, they felt a sense of agency about their work. Burton (2002) believes that classrooms should create mathematical learning environments similar to the ones the research mathematicians described. She believes it important that students be able to collaborate with their peers around enticing mathematical problem-solving, so that they will develop a mathematical identity and become agentic members in their pursuit of mathematical meaning they are deriving.

While Burton (2002) believes this mathematical community should build common collaborative practices, she also states that the students should recognize the value of differences in the way various members of the community process mathematical problems. An important characteristic of working in groups is the opportunity to negotiate, which results in a more thorough understanding. Burton acknowledges how different these practices are from the more traditional classrooms, where teachers generally see their responsibility as knowledge transmission. Burton (2001) asserts that mathematics educators should strive to challenge the typical mathematics class where experiences often discourage students from looking any further in mathematics.

Metacognitive Development

The qualities needed for the work of mathematicians include metacognitive skills such as taking risks and not being afraid of making errors. Another behavior required is *search behavior,* which Burton (1980) suggests is an important practice employed by cutting-edge mathematicians. To strive for these ends, curricula would need to be designed differently than the standard mathematical curricula. Burton (2002) asserts that teachers must reconcile the mathematics curricula with an *active pedagogy* for richer mathematical experiences in the classroom. Thus, for example, math brainteasers would provide experiences where learners would be expected “to be agentic” and “build their own learning” (p. 172).

Paris and Newman’s (1990) discussion of the literature on metacognition reveals that students base their own theories of how they learn on many self-perceptions that may not be helpful to their learning. Paris and Newman also report that teachers can be very powerful in assisting students in increasing their ability to self-regulate their learning in order to change their impeding theories to “effective learning tactics. The shift from other-regulated to self-regulated learning is scaffolded and guided motivationally as well as cognitively. Teachers challenge, encourage, and support students as they internalize new strategies and persist in the face of difficulty” (p. 100).

Mathematical curricular design would benefit from teachers considering the use of metacognitive practices or the teaching of the metacognitive practices in the design. Schoenfeld’s (1992) research on metacognition and mathematics demonstrated that metacognitive instruction is most effective when a teacher, who has worked to develop the interaction between the mathematical concepts and metacognitive processes, teaches both in the domain of mathematics in a systematically-organized manner. For this reason, intertwining metacognitive coaching with the mathematical curriculum will be essential for students to become engaged in systematically addressing mathematical challenges.

Uncovering Recent Trends

Mathematical curricular design should also consider recent mathematical trends. One such important trend in designing curriculum is the notion of threshold concepts. Land et al. (2005) define threshold concepts as "concepts that bind a subject together, being fundamental with ways of thinking and practicing in that discipline" (p. 54). In their discussion paper on threshold concepts, in a text on course and program design for student diversity, they contend that upon acquiring a threshold concept, a student is able to change their use of the concept and integrate it into other aspects of the subject. Land et al. (2005) suggest that often students will need recursive experiences with these difficult threshold concepts. Defining the threshold concepts in the design work would be an important consideration. It would also be important to look to recent trends in curricular expectations. Recently in mathematics education, developing algebraic conceptual understanding has been an important trend in the elementary and middle school grades. By building arithmetic connections to algebra content, algebra concepts can be introduced in the primary and elementary grades. Traditional high school algebra curriculum is being pushed down to the middle school levels, making it all the more important to introduce students to algebra as early as possible.

Chazan (2008), a University of Maryland professor of education who wrote of the changing focus on algebra in the United State for the National Council of Teachers of Mathematics, points out that there was a push to make structural changes to the algebra placement, from being introduced in the ninth grade to being introduced in the eighth grade. He also reported that the content of algebra was changing to introduce equations as not only inputs of functions but also as expressions and comparative functions, as well as tables of values and graphs, as a larger part of the subject.

Kieran (2004) reports that the changing focus of algebra has also moved well into the elementary grade levels. She compares various mathematical curricula for her reaction paper, where she outlines recommendations for algebraic reasoning needed in the elementary grades. She suggests the importance of introducing students to letters as variables earlier in order for students to conceptualize algebraic thinking in the primary grades. She states that it is important for younger students to incorporate this algebraic thinking into the existing model so that it bridges mathematics to algebra. Other ways to incorporate mathematical practices to support algebraic thinking were to teach students to analyze relationships between quantities, study structure and change, and practice modeling, justifying and proving. When making curricular adjustments, it is important for teachers to revisit their own mathematical understanding of the newer material. The mathematical curricular design of the new material can become the focus of the professional development for teachers as they work to include this material.

Jacobs et al. (2007) studied the importance of using professional development to help teachers make the adjustment to encouraging elementary students to develop their algebraic reasoning. Their study of 19 urban elementary schools, 180 teachers, and over 3500 students in California, collected the results of providing on-going workshops with the teachers on key algebraic concepts important for students in the primary grades to learn. The first year's results of the study proved productive for teachers. Teachers demonstrated growth in their teaching of algebraic reasoning incorporated into the mathematical curriculum. Further evidence showed that mathematical thinking was more productive for students and teachers together as a result of providing the training for teachers on how to develop algebraic reasoning.

Provisioning with Manipulatives

Another important consideration in curricular design is providing the appropriate materials needed to assist in the teaching of the content. For mathematics in the elementary and middle school, those mathematical materials are manipulatives. While Uttal, Scudder, and DeLoache's (1997) review of the research literature on the use of manipulatives pointed out that some research showed students had great difficulty seeing manipulatives as representative of the mathematical concepts being taught, their recommendations suggested that the manipulatives be used to help the students discover mathematical meaning. The manipulatives make the abstract concrete. We chose our manipulatives carefully, as Uttal et al. suggested, so that they could represent the algorithms both mathematically and algebraically. What is important in choosing materials is making sure that those materials can be used to engage students, help move them toward meaning-making, and be used consistently throughout the curricular design.

Fusion and Briars (1990) found that students' use of blocks helped students develop the understanding of the algorithms, whereas the "typical textbook extension of multi-digit addition and subtraction problems over grades 2 through 4 or 5, adding one or two digits each year, underestimates what our children can learn" (p. 204). If the materials promote the content in such a meaningful engaging manner, then the likelihood of their frequent use will increase. Unlike Moyer's (2001) study, where she found most teachers' use of manipulatives to be for *fun,* we knew that our manipulatives were tools for deep understanding of key concepts in mathematics, such as place value, *names of one*, and multiplication and division represented as area problems with the blocks. While our students reported enjoying using the blocks, they do not use them only on “Fridays” or “fun days” or for “as playing, exploring or a change of pace,” (p. 188) as had the students in Moyer’s study. Curricular design is a time-consuming and challenging undertaking for a school staff. With these careful considerations taken by the teachers, the rewards are reflected in gains in student achievement, as well as other observable, but less testable, qualities and habits of mind. This is a study of a school where the continual development and evaluation of our math curriculum was our constant design work. Our commitment to this work is set in the context of the following theoretical framework.

Theoretical Framework

The theoretical framework within which this study was conducted was situated around investigating how one comes to know, what is known, and the perspectives of the teacher as a reflective practitioner. Recent trends in educational research suggest that it is essential that educators be able to translate what they have learned to improve educational practices from within the walls of education. It appears that there is frequently more political pressure from without our walls because of our hesitancy to legitimize our own understanding of research practices that are continually occurring in our classrooms. This statement reveals two underlying assumptions about schools and the work of our schools that impact this study: 1) schools are subject to politics; and 2) teachers’ voices are not valued in teacher research. Cochran-Smith and Lytle (2009) point out that “[r]ecent work in practitioner research at the school level continues the theme of teachers studying their own schools and making teacher research in school and district–based networks the primary mode of professional development” (p. 27). Yet Cochran-Smith and Lytle also point out that much of that school research is done in accordance with top-down pressure for research to inform teaching around required high-stakes test scores. It is important to understand the frameworks under which this self-study research is taking place. Shulman (1999) argues that "[r]esearch that renders one's own practice as the problem for investigation is at the heart of what we mean by professing the profession" (p. 11). The work of teachers themselves in classrooms should render the knowledge of what is to be known and understood about teaching.

In wrestling with a researcher's perspectives, it is important to consider the varied stances concerning who the research reflects. In self-study, the researcher's reflections will be made public, positioning the thinking, refining and reframing as a series of actions outside the self; the research reflects the stances of the researchers, revealing their story *in* and *on* their reflections. It is, therefore, important to value *teacher as researcher,* for who else can *voice* an analysis of the research in his/her own classroom more authentically than an insider, including the decisions as to what should be researched?

Clandinin (1986) defines knowledge as connections that we implicitly or explicitly express in our daily actions, influenced by our historical and cultural roots, and Howard (1989) contends that knowledge "arises between the inner impulses, interests, and qualities of the [person] and the physical and cultural world of which he or she is a part" (p. 229). Hamilton (1995) concludes that "all people produce knowledge; knowledge is no longer the domain of a special few" (p. 32). These scholars suggest that teachers' knowledge of their classrooms has great value as knowledge, yet teachers are often the last to be considered when considering how to gain knowledge about teaching and learning. Lytle and Cochran-Smith (1990) suggest that the neglect of teachers' knowledge is "exclusionary and disenfranchising" (p. 4). Is it any wonder why so much political pressure comes from outside the schools to improve them, when there is so little value on what goes on inside the schools? Duckworth (1991) contends that the "main thing wrong with the world of education is that there's this one group of people who do it—the teachers— and then there's another group who think they know about it—the researchers" (p. 34).

How one comes to know is important in understanding why the profession, and the knowledge generated within the profession, should be valued. Critical reflection and reflective practice provide teachers an opportunity to generate intimate knowledge of the practice of teaching. According to Brookfield (1995):

> [R]eflection becomes critical when it has two distinctive purposes: the first is to understand how considerations of power undergird, frame and distort educational processes and interactions. The second is to question assumptions and practice that seem to make our teaching lives easier but actually work against our own best long-term interests, and I would add those of our students. (p. 231)

Pedagogically speaking, the beliefs that are held about teaching and learning connect theory to practice. Fenstermacher (1994) argued that the "challenge for teacher knowledge research is not simply one of showing us that teachers think, believe, or have opinions but that they know. And even more important, that they know that they know" (pp. 50-51). Practitioner research into educational practice is the process of building knowledge about educational practice that is vital to the analysis of improvement and effectiveness. Reflective practice in the investigatory work of self-study challenges personal histories, values, and status quo practices.

The belief in the *social construction of knowledge*, including self-knowledge, informs our theories of the formation and reformation of identity. As teachers continue to study their teaching, they continually reformulate their interactions with their students and the content. This meaning-making continually sculpts not only their individual identities but also their roles as individuals in their educational communities. Bickman (2000) suggests that if teachers are “encouraged to join with their students in a pedagogical alliance founded on self-reflection and openness they will ‘re-form’ every educational situation” (p. 301).

Educational research methods are guided by our epistemological and pedagogical influences. As LaBoskey (2007) states, it is important to “recognize that the privileging of certain pedagogies and particular research methodologies is as much about *power* as it is about intellectual responsibility” (p. 833). So it is important to select research designs that “attend to the *insider* perspective, where all voices are listened to and heard, but also examined and questioned” (p. 833) because that story often reflects perspectives that are lost to the power structures in an academic institution. The issue of voice in research is a political one that reveals how discourse and power relationships are sculpted out of deeply-rooted struggles for voices to be recognized. It is important to recognize that voices have not been heard because these voices have often come from those who have traditionally not been heard. It is possible that teacher voices have long been ignored because the profession has been a predominantly female career, or the profession itself has not demanded the value it deserves. Teacher voices as researchers should question their own power and the influence of that privilege on their practices, especially when they are in relationship with their students, students’ parents, and fellow teachers.

Ham & Kane (2004) question whether the dominant form of research traditions in academia are the only appropriate research methods for teaching and teacher education. "Only relatively recently, since the 1980's, has research with, by and for, teachers, begun to give voice to teachers and teacher educators through the rise of qualitative approaches to research on teaching and teacher education" (p.133).

Reflective practice reveals teachers' stances and their motivation to maintain their focus on their teaching and their students' learning. This practice enables them to continually improve and refine their teaching practices as a result of this reflective framework. For this study, my theoretical framework of the reflective practitioner is broadened by the fact that as I reflect and study my own teaching, I am not doing this in isolation. I function with my staff as a *professional learning community* that reflects upon our practice as a school. Throughout the years, I have consistently reviewed the data and practices of the school, together with my fellow teachers, to focus on the continual improvement of our students. DuFour (2004) suggests that "when a school begins to function as a professional learning community, teachers become aware of the incongruity between their commitment to ensure learning for all students and their lack of coordinated strategy to respond when some students do not learn" (p. 8). Because of our commitment to studying ourselves, I collectively share our findings and continue to study our practices on an on-going basis. Stoll et al. (2006) suggest many characteristics that are critical for the function of a professional learning community, and many of these qualities are demonstrated at our school: shared values and vision, collaboration, group learning and *reflective professional inquiry.* Additionally, they contend that other conducive factors that promote and foster a better chance of success in professional learning communities include: an open orientation to change, supportive group dynamics, smaller school size and a supportive infrastructure. These conditions, too, are characteristic of Family School's professional learning communities. My staff and our school structure work with these factors, which demonstrate the theoretical framework of the reflective practitioner is not only important for myself but is also a framework well-known by my staff.

Practitioner reflection generates "knowledge-of-practice" where "practitioners across the professional life span can make problematic their own knowledge and practice as well as the knowledge and practice of others resulting in a different relationship to knowledge and action (Cochran & Lytle in Loughran, 2004, p.614). Cochran & Lytle distinguish knowledge-of-practice from knowledge-in-practice and knowledge-for-practice on the basis of exploring teacher knowledge in the context of a larger community of inquiry in order "to connect it to larger social, cultural and political issues" (p. 614). As an individual and a part of a larger professional learning community, conducting this research continued a long-standing practice at Family School to develop our knowledge of our practice and continue practicing from the stance that to be a teacher is to study one's teaching, and that studying one's teaching has an impact beyond just individual classrooms. It impacts our school and the greater community in which our students will one day serve as community members; our work as teachers impacts the greater educational community.

Considerations of teacher collaboration, teacher as designer, teacher as reflective practitioner and the school as learning community will be continually threaded throughout the five research chapters of each math mode. At or near the end of each mode chapter, designated by the icon in the following figure, these literature review connections will be continually developed as the evolution of the modes is revealed through their 24 years of being practiced.

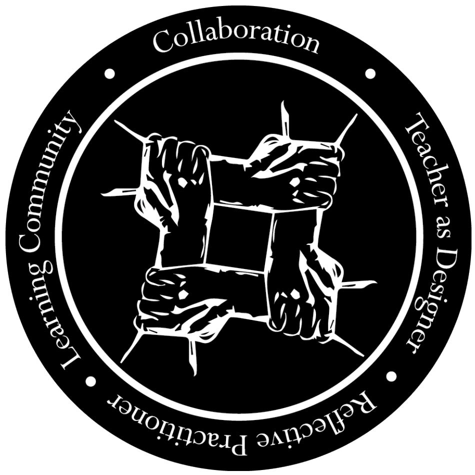


Figure 3: The literature review connection developed.

Our Modes of Engagement explored here in the context of the discipline of mathematics have long been a central focus of our practitioners' learning community at Family School. Our development and use of the modes occurred in response to both internal and external pressures put on the school during its evolution. At Family School, we came to value the *mode* as a curricular/instructional vehicle for teachers. This study will answer the following questions about the evolution of the Modes of Engagement at Family School:

1. How did the Modes of Engagement evolve over 25 years?

2. How did they evolve in response to dynamics within the school (i.e. teachers and student needs/interests)?

3. How did they evolve in response to dynamics outside of the school (i.e. political pressures)?

4. What are the elements (and/or structures) of the Modes of Engagement that were sustained over time?

Chapter 3

Overview of Methodology

> Every good poem begins in a language awake to its own connections—language that hears itself and what is around its gaze and knows more perhaps even than we do about who and what we are. It begins, that is, in the body and mind of concentration. (Hirshfield, 1998,p. 3)

While Hirshfield describes the mind and concentration of a poet in this epigraph, the same words could apply to the work of this study. It was in the concentration of the language used and analyzed that the real work of this study emerged. This particular study of the mathematics Modes of Engagement at Desert Willow Family School straddled the attention between objective documents and the subjective memory-work of self-study. Hirshfield's understanding of writing poetry works as a metaphor again when she states, "In the wholeheartedness of concentration, world and self begin to cohere. With that state comes an enlarging: of what may be known, what may be felt, what may be done" (p. 4). For this self-study, by analyzing the documents of the Modes of Engagement, while simultaneously writing the autobiography of the evolution of the modes throughout the life of the school, the research methods became so cohesive that they revealed analyses that neither method would have revealed separately. The autobiography would have revealed isolated, pivotal moments, while the document analysis reveals trends and patterns.

The following graphic suggests the intersection of the various methods of research that were used to orchestrate this inquiry.

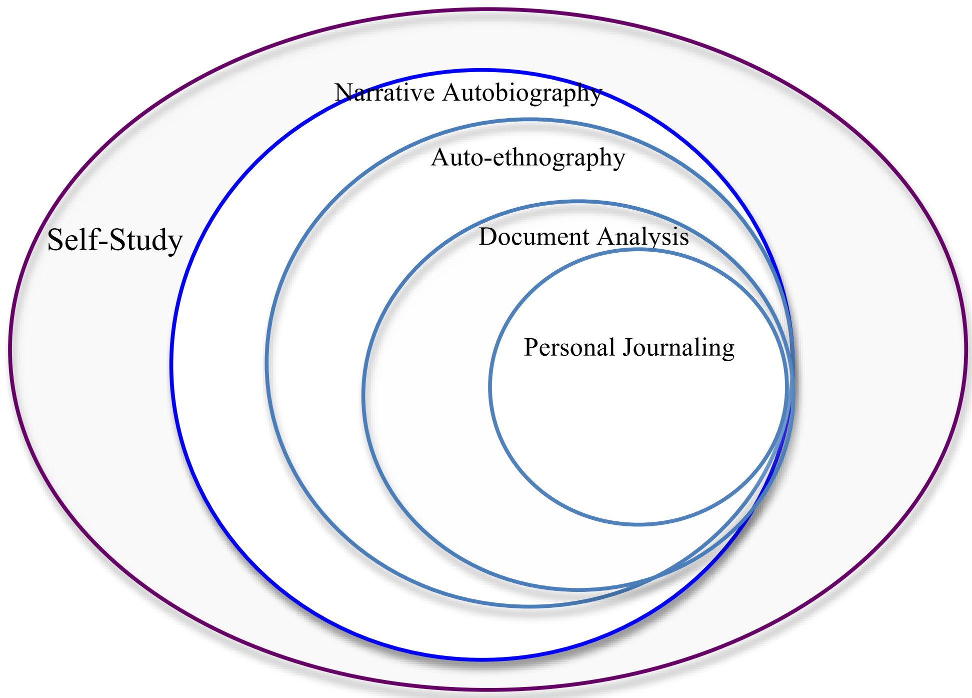


Figure 4. Model of the research methods needed for the project.

Context of Study

The purpose of this study was to understand the evolution of the Modes of Engagement and to determine what factors helped to sustain their use over the 24-year duration at Desert Willow Family School. This qualitative study focused on document analysis and autobiographical material used to reveal the development of the mathematic Modes of Engagement over the years the school has been in existence. As the principal and only teacher who has been at the school for the duration, I constructed the history of the modes from personal memory and through analysis of a variety of relevant documents. Personal journal entries about the math modes and how they are used presently in my classroom and across the school were also studied as a source of narration about the progression of the modes. The documents, autobiography, and journal entries were coded and analyzed for chronological patterns to help answer the research questions about the modes' evolution.

Rationale for the Research Design

Both teachers and researchers need to study classrooms across the nation with respect to instruction, curriculum, pedagogical content knowledge, educational theories, etc. To understand how to make improvements to a teacher's own instruction, or for a researcher to make recommendations for other teachers, this research needs to take place in an instructional segment that most closely represents the whole of teaching, to have the most impact on those research findings having significant implications for teaching and learning (Grossman and McDonald, 2008). The Modes of Engagement are a small enough, yet remain a whole enough segment of a teaching domain, where this research for teachers and researcher can take place. Yet, for this new curricular/instructional construct to become a viable construct for others, this study must reveal the staying power of the modes as both possible and enriching for the school environment. By studying the evolution of the Modes of Engagement at Family School over the 24-years of the school, this research can appraise the long-term value of the Modes of Engagement.

To reveal the key events in the evolution of the Modes of Engagement a self-study autobiography presents a sequential chronicle of their development. When this narrative is supported with the evidence from the document analysis, the key events in the evolution of the modes reveal patterns that inform us about their viability, strengths, weakness, and endurance.

Having the teachers study the Modes of Engagement, throughout the years, has been an on-going process at Family School that has led to many of our own school's discoveries, which have become key events in the journey of the modes. However, this study is focused on the events and processes that have led to their continuity, evolution, dissolution, and their revival.

Setting for the Study

This study was conducted at one *school of choice* in the Albuquerque Public School District called Desert Willow Family School. It is characterized as a *school of choice* because it is a school open to all of the district's families and their children. Presently, the school enrollment is just over 240 students with 12 teachers, one of whom is the administrator and the researcher of this study. The staff also includes: one secretary, one part-time special education teacher, one part-time counselor, a custodian, a nurse shared with a neighboring school and a part-time computer technician. Desert Willow Family School offers classrooms of kindergarten through eighth grade in a variety of multi-age classrooms (combining multiple grade levels).

While the school was predicated on the concept of sharing the instruction between public schooling and homeschooling, the changing nature of our community required the school to offer two kinds of program structures: the 50% and the 80%. The 50% program shares the responsibility, virtually equally, for the instruction— the school instructs the student for 16 hours per week, and the parent is required to instruct for 15 hours per week. The 80% program instructs students in the school setting for 24 hours per week, and requires 6-10 hours of homeschooling per week. This added program structure, requiring fewer hours of homeschooling per week, accommodated our parent community; some families needed to return to having two full-time working parents when the economic crisis developed.

The school student body consists of students participating in both general education and a combination of general and special education programs. Our students with learning disabilities are mostly instructed in their mainstream environment; occasionally, the special education teacher may work with them separately in her own classroom. Our school does not offer a gifted program, and our parents waive their students' gifted services while they attend our school. By having multi-age classrooms, our school has been able to offer our gifted students the required challenge their individualized educational programs outlined for the students prior to their attending our school. The following tables reveal the present student demographics at the school:

Table 2

2013/2014 Number of Students in Family School with Special Needs Breakdown

Regular Education Students	Students with IEP (not with gifted eligibility)	Gifted Students (services waived)
226	9	8

Table 3

2013/2014 Number of Students Enrolled and Percent of Total Population According to Students' Ethnicity in Family School

Anglo		African American		Hispanic		Native American		Asian		Total
Count	%	Count	%	Count	%	Count	%	Count	%	
155	64%	4	2%	76	31%	1	.5%	7	3%	243 students 141 families

Table 4

Student Breakdown into Grades at Family School 2013/2014

Kindergarten		First Grade		Second Grade		Third Grade		Fourth Grade		Fifth Grade		Sixth Grade		Seventh Grade		Eighth Grade		Total	
28		34		35		29		29		29		22		23		14		243	
10 Boys	18 Girls	16	18	11	24	15	14	16	13	15	14	13	9	12	11	6	8	99	115

I opened the school in 1990 and have been teaching for the entire span of the school, while also unofficially being the principal since 1993 and officially since 1996. The teachers' years of teaching experience at the school range from two to 25 years. Presently, two of our teachers are new to our school this year, and two have been at the school for fourteen years. All others have accumulated 2-13 years of experience teaching at our school.

The school is presently housed in a new facility built in the year 2010. Prior to having this building, we were housed in a group of portables for ten years, and prior to that we were scattered around the city as various isolated Family School programs in the empty rooms of traditional schools.

Role of Researcher and Limitations

I am greatly involved with the school and the Modes of Engagement, which is the reason for choosing a self-study research model as the design of this study. It is also the reason for implementing a document analysis with the autobiography, in order to be able to triangulate the memories with the objective nature of the documents on the Modes of Engagement. I recognize that I will not be able to remain completely impartial in the process of conducting this research. By using the documents with the narrative autobiography, I will be able to check my memories against the validity of the documents. However, the documents also have their limitations, in that some time periods of the research had many more documents from which to choose. I was able to find documents for all eras, though the computer not being extensively used in the first two eras made it more difficult to find original documents of those time frames. For this reason, the narrative autobiography was important to use cautiously when documents could not be found. When such times did occur in this research, I was careful to document my words as *assumptions* in the absence of documentation. Because the autobiography and documents can reveal much about the past, it was important to document the journey of the modes presently with the personal journal. Again the limitations of this method are that I am unable to remain completely impartial. However, this method of research was important to describing the present day use and value of the modes in order to determine their present day impact. In bringing all three kinds of research together, it was important to follow the criteria for self-study by Bullough & Pinnegar (2001) to enable this self-study to have the power of more objective research. Please refer to page 57 in the Methods section on Narrative Autobiography.

Methodology

This self-study dissertation employed three methods: autobiographical narrative, document analysis, and personal journal analysis. The auto-ethnographic work of this research is intricately embedded in the autobiographical narrative by giving voice of a teacher and principal, often overlooked in research. Not only is this the voice of a teacher, but the voice of an alternative teacher, which is even more often squelched as not belonging to the educational community. This voice is inextricably woven throughout the narrative analysis and interpretive work. The autobiography of the evolution of the Modes of Engagement over the 24 years of Family School was validated by the document analysis to verify my reflective work. The journal analyses revealed the current state of the modes and the direction towards which they are moving. Self-study is the preferred method of research for this study because I am the only person who has remained constant throughout the years of Family School, and also in the planning phase two years prior to starting the school. While the modes have evolved over the years, being a teacher and the administrator for the school throughout those years of evolution, the life of the modes are inextricably intertwined with my maturation as well. With this self-study, this research will leave a record of this alternative method of designing a curricular/instructional structure called *modes* for our professional learning community to use for its further growth as a school.

Self-Study

In Hamilton's (1995) self-study, she writes about her "Oz-dacious journey to Kansas" to analyze the importance of empowering teachers to have a voice. She cites many researchers who contend that teachers both know of their practice and study their practice, but she also contends that "teachers constantly create theory, as do all people, and they test it in their classrooms" (p. 33). She states that they also communicate their "frames of self: their interpretations, their histories, their personal understanding of the world." (p. 35). Through her support of self-study for teachers, she has found it to be important for herself as a teacher educator to promote self-study of her own teaching, in order to model for her students the process and need for teacher self-study.

Hamilton et al. (2008) discuss the elements of self-study. She found a *justification* for situating the research in the writing, *named the phenomenon* of her teaching practice, detailed her methods, and chronicled her *interpretive* processes. Loughran (2007) reports that this "moving beyond the self" assists researchers and teachers to uncover "deeper understandings of the relationship between teaching about teaching and learning about teaching" (p.12). He argues that the need for this in education would "enhance an articulation of a pedagogy of teacher education. I find this important for all teachers, not just prospective teachers, as teachers are always learning ways to improve their instruction, and the articulation of pedagogy is key to getting teachers to embrace change.

A more striking example of a teacher voice being developed through self-study is from Samaras (2002), where she began a self-study investigating her teaching by journaling how Vygotskian principles had resonated with her. Through her writing, she began to see herself changing and wanting to develop her own theories. She writes, "I was able to suggest practices in keeping with my intentions and values. This process will help me move my students toward formulating their own theories rather than simply parroting mine" (p.8). Most significantly, her self-study moved her to be able to claim, "I am a practicing theoretician, modeling and studying theory in practice" (p. 8).

Clark and Erikson (2004) reveal the importance of self-study in the research arena in the following excerpt:

> While the idea and concept of self-study of some type of practice, or more likely an institution, has been in existence for some time, its emergence as a research approach for studying particular educational practices has been attributed by Bullough and Pinnegar (2001) to the coalescence of a group of teacher educators at the American Educational Research Association (AERA) in 1992. Within the North American context, this forum illustrated a substantive interest by the academy in self-study scholarship. The importance of the turn cannot be underestimated as the potential impact on both teaching and teacher education was palpable for those present, as we were, and for the potential influence upon legitimizing new ways of thinking about, researching, and writing about professional practice. (p. 50)

Self-study is no longer in its early stages as a research practice in education. The present study aimed to meet Bullough & Pinnegar's, (2001) criteria for self-study by writing a study where 1) readable and engaging themes are evident and identifiable across the narrative presented, 2) a connection between autobiography and history are apparent, 3) the issues presented are central to teaching and teacher education, and 4) sufficient evidence is revealed so that readers don't have any difficulty recognizing the authority of the scholarly voice, not just its authenticity.

In deciding to analyze the practices of this school, my classroom, and both my teaching and administrative decisions over the years, I chose self-study as the basis of my methodology for this research. There is a wide range of possible ways that researchers place themselves in relation to the subject of what they are studying. Self-study allowed me the flexibility to use many methods to weave together an analysis that was both useful and justifiable. I found, similar to Ham and Kane (2004),that this wide range of relationships was: insider knowledge about my teaching, practices, classroom interactions, persona as a teacher, collaborative teaching, or institutional culture or practices (p. 122). Through this self-study I aimed toward the goals of Lighthall (2004): 1) making a new contribution to the field of teaching, 2) making conscious decisions to analytically study and design the self-study process, and 3) having both a practical and scholarly purpose for the field of education (pp. 194-198). Of the 125 self-studies on which he conducted a functional analysis, six features were coded 75% of the time. The story of Desert Willow Family School demonstrated these six elements: collaboration, practices, methods, autobiographies, school or program reforms, and conceptualizing theories. The methodology for studying my years at Family School, as a teacher and administrator, combined an autobiographical narrative, an analytical document study, and a reflective journal to reveal the self-study analysis of this work. An anatomization of themes, domains, metaphors, and patterns from these methods were categorized and organized to develop interpretations and translations (Cresswell, 2012).

Choosing self-study as a method was a natural outcome of my school community having researched our practices continually as a professional learning community. Our school echoes what Lighthall (2004) found suggestive of self-study:

> All of this bespeaks a spirit of community that supports and encourages and thus fosters release of enormous individual and collective energy. It is an energy that indicates deep professional commitment to the joint values of improving education and critically examining one’s self, one’s teaching, one’s ideas, and one’s self-examining process. (p. 250)

The Modes of Engagement research revealed, through self-study, that our school community developed a practice worth studying, but more importantly it revealed that our teacher community did in fact promote self-study of the modes throughout all our school years. Our staff discovered what Clarke and Erickson (2012) found self-study to be: the “cornerstone to professional practice,…the lifeblood of the teaching and learning dynamic” (p. 30). The authors believe that without this practice teaching becomes “repetitive not reflective”(p. 30). Though self-study is not a new phenomenon, this movement to legitimize a process for teacher voices to study their practice and thereby have an impact on their profession, should no longer be ignored. The educational landscape requires this professional research to counteract the lack of teacher research that still exists today.

Loughran (2007) states that self-study when done well is “disciplined and systematic inquiry that values professional learning as a research outcome” (p. 14). In wanting to better my knowledge of our Modes of Engagement so I can continue to improve them and study our teaching of them at the school, this project will actively engage in the methodology of self-study, and in doing so, find " learning about practice is more likely to be reality rather than rhetoric in teacher education (p.14).

In completing the self-study of our modes, I found the process to be more powerful for having woven all the parts of the study together to inform the story. More than once I was surprised to find that my narrative needed to be corrected by the validation of the document analysis work that accompanied it. My personal journaling revealed the inner dynamics of how our modes lived and evolved. The reflective personal journal provided a place to wonder about the next iteration of our math modes.

Auto-ethnography. The element of *voice* brings in another methodology: auto-ethnography. While auto-ethnography originally was used to describe cultural studies, it now also features stories about the self. Most of the auto-ethnographies focusing on self focus on cultural experiences, where it is important to "agitate and disrupt and contest views of the world" because of cultural differences (Hamilton et al. 2008, p. 22). Hamilton et al. (2008) contend that auto-ethnography has grown to include a way of knowing through self-reflection and the authors attempt to break away from the older notions of anthropology. Bennett (2009) suggests that an auto-ethnography is "[a]n analytical/objective personal account about the self/writer as part of a group or culture…often an analysis of being different or an outsider..[and] an explanation of how one is *othered"* (p. 1).

The study of teaching and learning in the context of a school of choice has created a culture of *otherness,* as most everything we do in our classrooms is different from the norms of a traditional school. Having multi-age, self-contained classrooms for K-8, where students are in the classroom for only part-time, all the while developing our own curriculum in contrast to prescriptive dictates, set up opportunities for very different curricula, instructional strategies, assessments, and evaluations. While the school is still evaluated by the district and state using the same high-stake tests as every other school, Family School has yet to be required to follow the mainstream curricula or methods for implementing such curricula. Family School teachers and the administrator, required to attend trainings for various state and district expectations, often feel that sense of being *othered,* made invisible and irrelevant, despite the fact that our student achievement suggests that we have been meeting, and exceeding, the district's expectations for many years, in ways that the district shows no interest in exploring. To give voice to this story in a time when charter schools are on the rise is to value the meaning that the non-traditional schools are generating as *part of* a system that is presently not meeting the needs of our community or of our state; the mainstream educational community remains resistant to seeking answers from alternative educational opportunities that have demonstrated success.

This auto-ethnographhic method utilized *monologue* as a means of researching. Guilfoyle, Hamilton, Pinnegar and Placier (2004) suggested that when a reflection is recorded and is analyzed by the writer, the writer has engaged in a dialogue of sorts. When that monologue is publicly shared, it becomes a social-dialectic construct. Gurevitch (1990) contends that dialogue is a "dialogic connection between two (or more) individual selves" under the "obligation to speak, the obligation to listen and the obligation to respond" (p. 181). Gurevitch (2001) also suggests that the middle term of dialogue is where a separation for the speakers occurs. When this separation occurs for a monologic speaker, this is the finding of one's inner voice, and it also becomes dialogic. This goes one step further when the monologue's discussion results in a change of action: it was further considered that the selves had a separate conversation, and the action of one became the action of the other. Cochran-Smith and Lytle (2009) suggest that when research began into practice rather than into theory, which journaling about one's practice would encourage, the research findings resulted in a foundation for action or an action that could be expected to shift as a result of the internal dialogue.

While the publication of this research comes only with the completion of the project, throughout the process stories, results, patterns, and implications were revealed to our staff and students; this research became a dialogue before publication.

The authority of the *self* in self-research gets right at the heart of the validity of using such methods for research. Hammersly (1993) argues that teachers who research in their own environments have advantages, knowing more about the environment than outside researchers. He even suggests that "teachers have access to their own intentions and motives in a way that an observer does not, and so have a deeper understanding of their own behavior than an outsider could ever have" (1993, p. 218).

Houser (1990) and Lather (1986) argue that there has been a long history of teachers being seen as users of research knowledge, not generators of the research. Because of this, teachers have been seen as targets of the scientific research community. Santa and Santa (1995) contend that "in the past, *pure* research has been highly valued, whereas the practicing teacher has been viewed as a *technician* who applies esoterically generated knowledge" (p. 446). Because of being relegated to *second class,* teachers experience being out of control of the most important decisions of their profession. They are "prescribed to and directed by administrators, curriculum specialists, and textbook companies, who sometimes act as if the teacher has little right to an opinion and no status as a professional" (p. 446).

Hollingsworth (1990) even argues that teachers have been treated as a historically oppressed group, suggesting that by including teacher research, teachers might claim some of the control they have lost. Teachers who engage in teacher research come to trust themselves and their practice, while also making a contribution to their field.

Ultimately, self-study is a reflective professional practice whereby a teacher can inquire about his/her own practice, make meaning out of his/her own experience, and treat classroom experience as data for analysis, reflection, and translation.

Autobiographical narrative. Gamelin (2010) writes about her own self-study research into her career as a teacher educator. She reveals that "how [she] ha[s] become a writer, a teacher, and an academic is bound through autobiographical narratives" (p. 183). By studying her teaching with her students, she was able to see that "self-study does not determine who we become. Nevertheless, it can be an instrument that helps us to understand and embrace the transformational process by allowing us to get inside it" (p. 187). She found that her study of the values she attributed to investigating her artistic and academic identities changed her view of herself and her academic work. What she discovered was that:

> …putting what has been buried into a historical context… understanding the circumstances behind the sinking of women's treasures and examining what has been uncovered in some contextual light. At times, the feelings of fear and insecurity return, and the conflict between artist and academic resurfaces. (p.189)

Self-study using narrative autobiography requires strict guidelines to adhere to research standards. In writing the narrative of the evolution of the modes, I strove to meet all of Bullough and Pinnegar's (2001) ten guidelines for autobiography, where autobiographical studies should:

a.) Ring true and enable connection;

b.) Promote insight and interpretation;

c.) Engage history forthrightly and take an honest stand;

d.) Focuses on problems and issues that make someone an educator;

e.) Reveal an authentic voice;

f.) Have a researcher who has an obligation to seek to improve the learning situation, not only for the self but for the other;

g.) Portray character development and includes dramatic action;

h.) Attend carefully to persons in context or setting;

i.) Offer fresh perspectives on established truths; and

j.) Have interpretations of self-study data that should reveal and also interrogate the relationships, contradictions, and limits of the views.

(Bullough & Pinnegar, 2001, pp. 16-20)

I found the work of autobiographical narrative to echo O'Reilly-Scanlon's (2002) insight that the power of the narrative depends on the questions it generates:

> What do our stories tell us, not only about ourselves, but also about all of our lives? What structural and societal forces have helped to shape and perhaps continue to shape, whom we are – whom we all are – within the context of the larger community of which we are a part. Through the careful consideration of what was once there and what is there for us now, lies the potential to *re-invent* ourselves as we reflect upon and examine how our memories are manifested in our lives today. (p. 75)

Document analysis. The self-study focused the importance of the modes and the hard work that went into designing and implementing them. Patterns from the documents revealed the pivotal moments of the evolution of these modes, which is where document analysis begins to reveal the underlying perspective of its form and function. These moments will unfold as a:

> …good read [that] attends to the 'nodal moments' of teaching and being a teacher educator and thereby enables reader insight or understanding into self, reveals a lively conscience and balanced sense of self-importance, tells a recognizable teacher or teacher educatory story, portrays character development in the face of serious issues within a complex setting, gives place to the dynamic struggle of living life whole, and offers new perspectives. (Bullough & Pinnegar 2001, p. 19)

The patterns, as revealed through number of incidences found or through the weight of the implication of a document, have been woven into the autobiography of each math mode to reveal how the modes changed through our history and to determine how the modes were sustained. The document analysis was a crucial part of the self-study as a whole, for the voice of the documents, if they were to have one, speaks the truth and is as plain as what is seen on the paper.

Reflective journal analysis. In analyzing a reflective journal I kept about the math modes used today in my classroom, categorization by themes uncovered more patterns of these modes. The narrative autobiography covered many years of political pressure on the school to accommodate both federal and state mandates, which highlighted trends and themes of these mode practices. After analyzing the patterns of the journal, my thinking about the future of the modes came to reflect my commitment to restore some sense of control over my practice that the political pressures take from my classroom. The underlying tensions of political context and the modal development are dealt with in my journaling and show how I transformed my teaching of the modes in response to these pressures. The language of my journals translated the on-going enactment of the modes' evolution, and the interlacing of the journals with the document analysis and autobiography; my journaling voice developed its own language to describe the evolution in patterns and key events of this research.

> Translation occurs precisely in that moment of forgetfulness and dissolving, when everything already comprehended through great effort—grammar, vocabulary, meaning, background—falls away. In that surrendering instant, the translator turns from the known shore of the original to look into that emptiness where the outlines of the new poem begin to resolve, a changed landscape appearing through mist. (Hirshfield, 1998, p. 61)

As a translator of poetry written in a foreign language, Hirshfield (1998) wrestles with meaning-making while tight-rope walking between two languages and cultures. This utter transformation that occurs is very much akin to the translation of the research of a self-study, as one language, and document analysis, as another language. Each method challenges the meaning being made of the other. The humility with which memory is used is both astounding and disheartening. The translation comes from deep within, and emerges from places very familiar and autochthonous.

Ideology

Translation gets at the heart of ideology because it is in our meaning-making that our ideology is born. Before constructing my design for my study, it was important to consider the range of ideological beliefs that would constrict and define my understanding of what I was researching, how I would tell the story of the research, and how the analysis of my story would reveal my view of the world.

A basic educational ideology that is the foundation of all my other beliefs is a constructivist philosophy of education. Constructivism is a theory of knowledge focused on how we gain knowledge from new experiences by connecting to prior knowledge. I have come to view all persons as searching for meaning through constantly generating new, unique meanings based on their previously-made unique meanings. That being said, I believe that humans are never done making meaning and are constantly challenged with new knowledge-making, no matter how old or skilled. This socio-cultural lens of social constructivism suggests that since all meaning is made by humans, the roots of these meanings reflect their own biases.

My ideology is rooted in inquiry. If all humans are meaning-makers, then they have a means by which they make meaning. Lundberg and Young (2005) come the closest to what I have derived for myself in my understanding of inquiry. Explaining the cycle of belief modification, they suggest that individuals encounter anomalies and sustain a "botheration" and "curiosity" (p. 29) which ultimately motivates inquiry to reformulate their original belief. They also combine this cycle with a systematic model of how this cycle increases in sophistication of inquiry levels, from naïveté to scientific knowledge, as the individuals progress in their understanding of their need for more focused inquiry.

While inquiry is a way of approaching the world to figure out new meaning, the *what* that is inquired about is also important to consider. I whole-heartedly embrace both interdisciplinary and intradisciplinary practices for learning about subject matter. While this may not seem like an ideology that makes a difference, it is from this belief that my multiple approaches to curriculum and instruction are constructed. While I feel it is essential that students understand the importance of integrating subject matters together in order to view and solve a problem holistically, I also value the intradisciplinary passion for a subject for the subject's sake, not just for its application. Mathematicians are essential to scientific work, but they also do groundbreaking work around the puzzlements of pure mathematics. Often a discovery in one kind of disciplinary work will lead to work in another. I love for my students to find the passion in any subject by being exposed to as much of the whole range of experiences in that field as possible.

Any field of study is benefited by the realization that this integrated or specialized meaning is often made in communities, and I believe that a *community of learners* is the best environment in which to learn. This term is used very frequently in education and has become so mainstream that it is difficult to pin down one definition. Yet, for me, it is where the classroom becomes a place where teacher and all students learn together and share in creating a safe environment to withstand the struggles and risk-taking needed to learn. From this generic foundation, I derived a word that is essential to my being in a classroom: *simultaneity.* While I did discover some psychological roots of the word in teaching and learning, I did not know of them when I experienced this word as a core belief for myself. I have come to use it to represent my belief that the essential quality of a learning community is that all persons are learning, even if this learning is not openly available to each other. I have always felt that as a teacher I was always learning on multiple levels about the students, content, classroom as a whole, my teaching instruction, etc. I felt it essential that the students and I use the classroom as a place where we would not only learn the content but open ourselves up to, and share, these others levels of learning. Davis (2008) describes, very intricately, various simultaneities that have been uncovered in educational research, the most basic of which is the quandary of distinguishing between the knowing and the knowledge. Davis (2008) notes that even Dewey recognized the confusion between these, but he argued that understanding the distinction and the connection between them is important for research. In my ideology, by having knowledge and knowing it exists on many levels, I see them as separate, but in practice, as a class, we often cannot tell what is the objective of our lesson—knowledge or knowing. Since we are learning on so many levels, we cannot always tell whether it is the knowledge we are attaining or simply developing the knower to know himself better, which, in turn, would be knowledge.

To confuse matters more, I believe that as learners we are always reframing what we have come to know, which is another construct of Lundberg and Young's (2005) learning cycle, yet this reframing may not always be based on new evidence but rather a hope for new evidence. Again, this naïve notion of mine has its roots in a psychological realm. In cognitive therapy, cognitive reframing consists of people wanting to see another view, and purposefully engaging in that new view of the event or idea in order to see a change in mindset. This suggests for me that knowing and meaning-making are not always the result of prior knowledge, but prior knowledge can be something that gets in the way of our being and must be altered to reframe the experience. This is an active way of dealing with what may be hard for us to see or believe, or it may just be necessary because we can't see solutions until we reframe our experience. It is not a revision; it is a re-creation of meaning.

Meaning-making for self-study must also consider the constrictions within which meaning is made in a socio-cultural sense. Historically, there have been social and race constructs that have resulted in “structural inequities” in educational, legal, medical, and even publishing institutions (Brown, 2001, pp. 526-527). As researchers participate in discursive and curricular practices in these institutions, “the subjective experiences of the self and other” are viewed differently and are greatly impacted by the researchers’ stances to these institutional practices around race, gender, and social class (Brown, 2001, p. 527). Elliot (2007) suggests that the self is interpreted, challenged, and constructed in the context of these institutions. Elliot further explains that individuals interpret or mediate their social experiences as internalized meanings through their values of sexuality, race, gender, etc. It also follows that if our interpretations of our *selves* is impacted by our interaction in the world, our notion of our collected histories also continues any biases we have integrated into our understanding of our memory-work. It is essential to challenge our assumptions, attitudes, and motives when working with self-study.

Another important aspect of challenging a closely-held belief system is that often it is a transformational experience that will have the power to change our most rooted values. The evaluative and challenging stance required of a self-study has the potential to encourage transformation in the researcher. Griffiths, Bass, Johnston, and Perselli (2007) argue that self-study creates a unique opportunity where a teacher researcher is in a “space of possibility for both personal and social change that sometimes suddenly opens when members of social groups, between whom strict boundaries are normally drawn, intentionally or unintentionally come into contact” (p. 677).

Procedures

Assigning ages. In order to assist in telling the story of each math mode, the 24-year period of the school was divided into ages: the Dark Age, the Middle Age, the Renaissance, the Age of Exploration, and the Reconstruction. These eras help to delineate a quarter of a century into phases of the development of the program. They reflect the evolution of the math modes through the habit of our reflective practice, starting in the darkness and not really having much experience working together as teachers, moving to ages that reflect learning more about what we were doing, both creatively, and in response to school data. The last age suggests that, as with all growth and reflection, there is much to be built and rebuilt as evolution discovers new areas on which to focus. However, where there is a focused change, there are often overlooked omissions that will require further attention at some point. That is what did occur in our Reconstruction phase as some modes had evolved, dissolved and re-emerged due to our school research. The ages are as follows:

The Dark Age 1990-1994

The Middle Age 1995-1999

Renaissance 2000-2007

Age of Exploration 2008-2010

Reconstruction 2011-Present

At the end of every mode chapter, a timeline of major events or documents has been placed for that specific age. All five timelines have been included together in Appendix C. The timelines are color-coded for each age: Dark Age is blue-green, the Middle Age is orange, the Renaissance is green, the Age of Exploration is sky blue, and the Reconstruction is purple.

Setting. This study was conducted in a six-month period at the school, where documents were continually collected and culled for about three of those months. It would occur that as the autobiography was being written, it would call for a document, which had not been found in the original collection of documents. Because of being a researcher who was also still teaching and administering at the school, I revealed much of the information connected to the research to my fellow teachers as I was uncovering it. This historical perspective on our modes, brought to their attention, influenced and was an important part of the journaling about the modes in present day.

The teachers at Family School are expected to conduct their own inquiries about their instruction with their student. They were quite excited to embrace the emphasis of our study of the modes once again as an on-going expectation of our school because of this project, even though they were not participants of this self-study. The year of this study also merged with the first year of a new teacher evaluation system implemented by the state department of education, which required five observations of each teacher's classroom per year. We structured each observation around a different mode of the school. While this study focuses on the modes we use in mathematics, our school uses other modes for the subject domains at the school as well. The Modes of Engagement helped our teachers go through the new evaluation with a structure that was familiar to them in a time of radical changes in expectations of teaching.

This study took place when the school had been at our new site for four years; our community of families has become very stable. The school has had a waiting list of over 150 applicants to get into the school for the past three years, and we have not been able to get them all in because of our stability. This stability may reflect a commitment of the families to our curriculum and our use of the Modes of Engagement that the evolution of the modes may reveal. The application process is not a lottery process, but a waiting list process, meaning that once an application is filed and the applicant has attended our informational meeting about our school, the applicant is entered on our chronological waiting list. The students are admitted to the school, as openings are made available for the next applicant for that grade level. Attempts to balance gender are made, but not always possible, as allowing entire families to enter together can make that difficult. Since the families are being admitted who are committed to maintain some amount of homeschooling, the teachers and I feel it is important to admit all the students of a family, when possible. It sometimes occurs that this cannot happen, and siblings are then put on a sibling waiting list, which are given preference over the general waiting list.

Materials. For this study, three main kinds of materials were collected: documents, autobiographical material, and present-day journal entries. The documents that were collected were found in four main places: school files, my personal files, school-wide document collections maintained by the school community, and computer files on my present computer and some of my older computers. In those computers used before 2000, while saved documents were found, their files used older software and could not be converted to today's formats to be retrieved.

The autobiographical narrative was composed from memories, stories, restored and corrected memories from viewing the documents, and realizations from seeing the patterns of the modes' evolution. Though some of these materials are intangible, the words and patterns become the materials of the autobiographical work.

The reflective journal I kept on the modes being used in my classroom and across the school was another kind of evidence collected in this study. The journal was kept about twice a week for thirteen weeks. It was written on my computer, dated, logged and coded for use at the end of each of the mathematical modes' research.

Procedures for autobiographical narrative. The story of each math Mode of Engagement was written with an introduction to the particular math mode, then followed with the story of that mode as seen through the ages. Each math mode's story sifts through the ages with different emphases for each age, but in telling the story of the 24 years, there is always an origin, obstacles, short and long-term effects, development of school norms, and many creative leaps in designing and re-designing the modes for the school. The writing of the autobiography came with inspiration and many anecdotes of our school's Modes of Engagement. In rereading the narrative and rewriting it while interlacing evidence of the document analyses, new enlightenment about the evolution of the modes was developed and recorded. In rereading the narrative autobiography for a third time after completing this task, the conversation became more dialogic as the self-study moved toward publication, adding revisions while once again processing the story.

Procedures for collection process of the documents. Documents of 24 years were found in multiple locations: personal file cabinets, school file cabinets, present computer files, stored computer files, school notebooks, and communication emails. Hours of sifting through files and determining which files to collect occurred to find the documents of the study of the math modes. Initially, documents were collected for all math modes, but as the search found an abundance of documents, it was essential to begin to date each document and organize them according each particular math mode. Some of the documents were dated; others had to be dated using the computer's *properties* function to discover the date it was created.

Procedures for organization and coding of the documents. Once the documents were organized by modes, each mode document was then categorized according to kind of documents: classroom documents, school-wide documents, communication, or professional development agendas. Lastly, the categories were put in chronological order for the purposes of coding for themes for each era.

Generalized codes were assigned colors and the documents were highlighted with color-coding for these generalized codes. After completing the coding for all the documents once, the documents were then recoded to accommodate any new themes that were revealed after going through all of them the first time. Once all the documents were corrected with the new color-coding and a key of the color codes was set up, the excerpts of the documents were tallied on the key sheet according to their themes. The tallies were then re-tallied to reflect which tallies were in which time period, so that the themes reflected the patterns of their evolution through the ages. This was repeated for each math mode. Because the nature of the modes are quite different from each other, the categories and themes of the documents varied for each mode.

Procedures for analysis of the documents. The data of the documents was then analyzed to identify patterns and connections of themes during various time periods. Omissions were also found to be significant data for some of the modes. Looking for patterns of what is observable is common, but overlooking what is not observable is not as obvious. This data was then woven into the narrative story of each mode according the events presented for each mode. Key events or significant documents that were very important to the evolution of each mode were also added to the autobiographical information. Once the story and document analyses were integrated, support research literature topics were identified and were integrated into each mode chapter.

Procedures for personal reflective journal. Simultaneously, I also participated in reflective journaling twice weekly for thirteen weeks about my own present classroom experience in using and developing these math modes. About half the time, I found myself inspired by the day's events and their connections to the math modes; at other times, I assigned myself topics on which to write in my journal. The math modes as they were enacted in my classroom or responded to by my students were recorded in this journal. The day-to-day activities were given in detail, but also my thought process about the modes' structure and instruction was analyzed and wondered about. I also wrote about what the students were learning or qualities they were developing as a result of the math modes.

The themes of this journal were then organized by mode with a cut and paste method, organized by theme and analyzed for patterns in order to prepare to write the end of the story to each of the modes, with a view to their current manifestations in our curricula. The analyses of these personal journal patterns brought the evolution of each mode to a current natural conclusion.

Procedures for analysis of codes/themes to reveal story plot. The major coding of the autobiography, documents, and reflective journaling was divided into the major strands of what makes a mode: *specific content* chosen to meet a *higher purpose* of practices experienced by mathematicians or math enthusiasts, by utilizing a *strategic instructional* method. These codes were then organized into the themes that were prevalent in the research method. When analyzing the themes of each math mode, patterns of the themes could be said to *plot* a storyline of the evolution of the modes. This storyline in some way represents a *plotted line* derived from an analysis of the *nodal moments and documents* revealed by the research methods. Each mode creates a plotted line for its story, but there is also a story that is analyzed by analyzing all the modes together. That story is the NCLB story. The modes will reveal their major codes, their themes, and the plotted stories at the beginning of each chapter. The following figure reveals the icon that will be used to inform the reader of each mode's themes and story that will be revealed throughout the chapter. This icon represents the importance of how the three codes must be interwoven to create the mode's intentions and results in the mode's story.

Figure 5: The codes, themes, and plotted line icon.

The NCLB plotted storyline is then revealed when putting all the modes' stories together. This story is of particular interest to see how the pressures of NCLB high-stakes testing and mastery teaching influenced the evolution of the Modes of Engagement. Even though Family School is a high-performing school, this pressure had a great impact on the strategic instruction, which resulted in changed student engagement in the content. This NCLB story is constructed at or near the end of each mode and in the "Summative Analysis" of Chapter 10. Each time, the NCLB story is highlighted by the 🕮 icon.

Procedures for final analysis. Once the mode story had been rewritten with all the different methods of this study, a timeline was created to map the general chronological patterns for each of the modes. Generalized themes for all of the modes' growth through all of the different time periods were then analyzed for the findings to answer the research questions of this study. The analysis of patterns of themes that contributed to the modes being sustained throughout the 24 years were found first, which revealed what outside dynamics had impacts on the modes. In order to generate the genuine meaning and value of the modes, each of the concepts labeled in the coding of each part of the design was checked against the other concepts to see if they could be aggregated to reveal some *truth* about the use of the Modes of Engagement at Family School. In looking at what themes highlighted our internal dynamics that influenced the modes, other themes were identified that were different from initial analyses, though some themes did highlight both our outside and internal dynamics, impacting the modes similarly.

Ultimately, the themes were continually analyzed to improve the definition of a mode and identify the criteria that are essential for a mode to operate effectively.

This self-study was accomplished in many rewrites, many records of evidence tallied, and many analyses of the patterns through the ages. In the end, the story of the modes revealed successes and failures, but ultimately, in the words of Lundberg and Young (2005), it demonstrated:

> …a human spirit of inquiry. It is a place where learning and scholarship are revered, not primarily for what they contribute to personal or social well-being, but for the vision of humanity that they symbolize, sustain and pass on… Higher education is a vision, not a calculation. It is a commitment, not a choice. Students are not customers; they are acolytes. Teaching is not a job; it is a sacrament. Research is not an investment; it is a testament. (p. 460)

The research this study sets out to do tells one school’s story, but it is in the complexity of the research methods that the story can be challenged to reveal more about itself than any one person’s memory of the place.

Chapter 4

Context: Background of Family School

Creating Family School

Family School opened its door to a basement room in Monte Vista Elementary School. It took two years of planning, surveying, and knocking on many doors before I was given permission to begin the school.

The idea for the school came to me while teaching a third and fourth-grade combination class at Monte Vista. My daughter was in my class, and she struggled with her reading. I needed to find more time in the day to work with her. Because of working full-time, it was hard to work with her at night, both of us being so tired. Being one of twenty-eight students in my classroom, it was difficult to find the necessary time for her during the day. I began to think it would be nice to create a school where parents and teachers could work together to teach their students. Previously, I had done my master's thesis on prodigious children, some of whom were homeschooled, and through this research I became intrigued by homeschooling opportunities, which were common for many of the child prodigies. As a public school teacher, I valued learning in a school, but I also valued students having experiences outside the home. I knew I couldn't advocate for homeschooling totally because of the wondrous work that had happened in my classrooms for three years. To meet this need, to combine the benefits of home and school, I began to design half-day homeschooling/ half-day public schooling schedules, where it would be possible for a school to share educating students with parents.

I knocked on the doors of my district, State Department of Education, and the local university department of education offices. I believe it was this triad of support that ultimately led to the approval of this alternative school proposal. When presenting to one institution, there were always two other institutions whose prospective support I was able to present, which carried more weight than just reporting to two entities potentially vying with each other. I had to have the approval of the district to fund the program and its teacher and supplies. I had inquired into the university about finding a classroom on campus to house the program. Lastly, I needed the approval of the state department to begin a new kind of school in the state.

I went round and round with each of the entities, and then round again. I was just inexperienced enough to keep responding to the suggestions that were given to me from each person to whom I presented my proposal. I took each critique of my program and immediately redesigned my plans. The district finally gave me permission to open a program when all three entities found the proposition to have met all their suggestions, and it was given approval from the state department and a room at the university. Finally, once I had district approval, it was necessary to receive our local school board approval, and the school was born, but not without last minute complications. A few months before the scheduled start, the classroom at the university fell through and my principal at Monte Vista generously agreed to allow me to use a vacant basement room in the school and agreed to oversee the inception of the program.

The Premise of the School

Originally, the school plan was designed to be a half-time homeschooling, half-time public schooling educational opportunity for parents who wanted to have more time to educate their own children. The premise for the school was to have students attend four days of three and one-half hours of schooling each day. That comprised just over 50% of the educational time required in a public school setting. The other 50% were homeschooling hours that the parents fulfilled in the home. A full-time teacher could have half of her students (12) in the morning, and the other half (12) in the afternoon, which meant she was a full-time teacher. The *State of New Mexico's Statute Section 22-8-2M2 Definitions* states "a 'qualified student' means a public school student who: (2) is regularly enrolled in one-half or more of the minimum course requirements approved by the department for public school students." A qualified student is then also awarded full-time equivalent funding for each student from the state. Therefore the teacher was able to have a reduced pupil-teacher-ratio (PTR), while still receiving a full salary for serving a full class of 24 students. This reduced ratio was essential to the program because the classrooms were, and still are, multi-age.

Multi-age Classrooms

Initially, the multi-age structure of the classes was essential because it started out as one-room schoolhouses The first families who opted to do this program put siblings in the same room. I started the first classroom with 16 students, serving as a part-time teacher, to get the program going. Previously, I had surveyed all the homeschoolers who were registered with the district, which was well over 200 families, to see if they were interested in my proposed model and received 84 interested surveys back. Out of these 84 interested families, 16 students were the first to go on this venture with me. By the third year, Family School had grown to over 180 students and expanded to five various sites spread across the city. It was because of the fact that the district wanted this program to be dispersed and housed in other schools around the district that during the initial years that we continued to implement this multi-age model. A parent could not transport children to multiple sites. However, the third year also marked having two different kinds of programs: four were elementary programs, as grades 1-6, and one was a middle school program, grades 6-8. Some of our graduates from the first two years of the school wanted to continue into a similar middle school model.

Once the multi-age approach took hold in our classrooms, Family School found the challenge and reward of teaching multi-age classrooms to be of great benefit to our school goals and vision. Some factors about our multi-age classrooms that might be important to remember for this study are the lower pupil-teacher-ratio, the fact that the entire school is multi-age, some of our teachers have been doing this teaching for over 15 years and lend a great deal of experience to our newer teachers, and it is our school belief that learning happens in social learning communities of varied ages. Outside of school, students do not find themselves only around their same-age mates. Society more often creates opportunities for people to learn together in multi-age environments.

The First Years- Decentralized Sites

The first year with my sixteen students was great fun for me because everything I did was as an adventurer in education, but more importantly for the rest of the community who had expressed interest in the program, the district decided to support this new kind of school. The second year we grew to 26 students. I hired another teacher to teach with me in the morning to learn the program and have the other half of the students in the afternoon as her own classroom. I had written a 21st Century Grant that gave us funds to support me being a coordinator for the program for part of the day, which gave legitimacy to my role to continue as both part-time teacher and part-time coordinator, right from the onset. The third year demonstrated that Family School fulfilled a need in the district. This was what I had hoped would happen the first year, but it took the two years to demonstrate the value of the program when we finally grew to 180 students.

The first five years of the program were primarily about getting our footing and discovering who and what we were. I actually started the school with the name, *Community School,* which was changed after the state department suggested that there were already too many schools across the state with that same moniker. The name choice was taken to the parents, and they renamed it *The Family School*, which stayed with the school for 20 years, until a school building was built for the program and renamed: *Desert Willow Family School*. Upon the approval of this program by my district and the New Mexico State Department in 1990, the State Department went on to approve other *family school* models across the state. These other *family schools* are similar in their part-time public schooling/part-time homeschooling structures and concepts, but quite different in curriculum, instruction and philosophies.

Starting a school is full of excitement, wonder, and much hard work. Our start was no different. While only beginning with 16 students and increasing to only 26, I really had doubts about whether it would continue or not. By the end of the second year, the interest began increasing greatly, and so began the real work of making a difference for families. With the perseverance of parents, teachers, and students, we were able to direct our focus and dedication to creating this new elementary/middle school of choice in a public school setting. Our first few years had so few students that our real struggles didn't materialize until our third through fifth years. Yet, the whole community had a fighting spirit to continue to forge onward.

As Meier (2002) states about her desire to start Central Park East in the public school arena, "Schools embody the dreams we have for our children. All of them. These dreams must remain public property" (p. 11). When I was asked to make Family School a charter school setting by a state department employee after those five years, I knew my answer before the question was finished. Family School had made it through its first five years, with plenty of difficulty and little support, but we knew we wanted it to remain in the district to continue to prove our strength.

Those first five years set much of our vision right from the start. Working closely with parents, providing more individualized instruction to students, setting high expectation for learning in a learning community, delving into conceptual teaching, and focusing on critical thinking were just a few of our goals. While there are documents of this time frame, they are few in number, yellowed, and crisp in the back of file cabinet drawers, or buried in computers in a language no longer supported by today's software. When analyzing the documents that are accessible, it becomes clear how much we were in the dark, though we often proceeded as though we were very much illuminated in our vision. I have come to call these five years, 1990-1994, the Dark Age.

As the coordinator of the program from the start of adding new classrooms, I always taught in the morning session and was principal for the afternoon. I focused much of my work supervising other teachers, and orchestrating waiting lists, school policies and professional development. My role as principal was not solidified from the district's perspective because I did not have my administrative license until 1996. Still, in those first five years, I functioned as the principal, even completing the school program reviews for the district. I did describe my role as a teaching coordinator because of not having my license.

Not only did I act as principal, but I was also the secretary, trained to complete all the ordering of materials, entering student information in district software systems, and mediating parents and teachers who were having difficulties. I just had to dig in and *do it all* no matter how inexperienced I was or how much work it required. I finally was allocated a secretary toward the end of the first five years.

In starting the second five years, it was apparent to my district supervisor that I should get my administrative license. When I completed my program in 1996, my district credited me with four previous years of experience as an administrator.

The school evolved immensely in the next five years, yet our structure did not change much. As a group of teachers we were able to meet bi-monthly, on Fridays, as an entire staff for professional development, since that was the day we did not have students. Our agendas and staff meeting notes reveal that we functioned as a group, sharing many of our best practices and trying to have coherence by going over our rules and regulations often. Our curriculum evolved, and we seemed to have a larger network of common curricular practices that worked, to some extent, for us. I have come to call these years the Middle Age.

What the teachers and I knew was that as long as we were spread around the city in different sites, we could not function as a school. Three times during these first ten years, we requested to have portable classrooms moved together somewhere in the district so that we could be near each other. Finally, in 2000, it was approved.

Our focus of the Middle Age was to partner with our parents to educate their students. The teachers and I worked very closely with the parents in the twice-a-month parent classes. Our distinguishing feature was the camaraderie we established with families. It wasn't always easy, but it must have met a need that was not being met for all parents through the regular system because we engendered some trust to entice members of the homeschooling community into the public school system, but we also kept other families, who had become disillusioned with the district system, in the district by enrolling in our school. Creating a new school has the spirit of a quest for all constituents involved. We collaborated—parents, students and teachers— to forge a program that strove to measure up to the purpose of school. In defining this purpose, what was said was that the teachers at Family School had to answer the question of why students need to come to school at all, if homeschooling is a viable option for education. If Family School was to attract homeschoolers, we would have to have an answer to that question. From the start, the teachers and I identified the creation of a *social learning community* as our main goal. We felt it was important for students to learn how to learn *with, by, because, for, and even in spite of other children.* I remember my third year of teaching that a new second-grade student entered my room with a very hesitant and reluctant homeschooling parent. The mother expressed to me that she really wanted to continue homeschooling, but circumstances in her life were changing, and she needed to enroll her daughter in school. She also told me that her daughter didn't know how to read and didn't want to learn to read, nor did the parent want her child to be pushed into reading, for she believed that her daughter would read *when she was ready.* Within two weeks of entering the classroom and seeing all the students around her reading, the student wanted to read and began reading. It was belonging to a social learning community that was more powerful for her than her homeschooling environment.

Around the district our learning communities grew in our four elementary sites— Monte Vista, Mitchell, Hodgin, Chamisa— and in our middle school program at the Career Enrichment Center. My staff and I shared our curriculum at our meetings, but by being so separated, it was difficult to run our decentralized sites and still have similar practices. We were common in our alternative report cards, the communication forms to parents about student work, and some of the common beginnings of our Modes of Engagement began to emerge in all Family School sites, not just in our math modes but other modes for other subject matter, as well.

First Centralized Site

I have come to call the years of 2000-2007, the Renaissance. These years were pivotal in our growth as we came together to be a school at our centralized site centrally located in the city, and many of the practices we created then are still used today. Finally, being able to act as a school began the process of being able to develop more common practices together. The school was given six double portables for classrooms and one double portable for the administration building. We grew to 240- 260 students and also extended our grade range to kindergartners. Prior to this time, kindergarten in our district had already been a half-day program, so there wasn't a need for kindergarten in our school, but when the district went to a full-day kindergarten, parents clamored for kindergarten to be added to Family School. This really changed the face of our school, since kindergarten parents were often looking for a school for their children to spend their entire elementary experience. Prior to this, many parents chose our school because either full-time homeschooling or full-time public school was not successful for their children. With these new families searching for a *home* for their children from kindergarten, families started to stay longer with us.

Together, at our new site of portables, the teachers and I still kept our multi-age classrooms. Teachers had come to love teaching this way, and truly had become adept at it, though we did lower the range of grade-levels in each class. Some were K-3, 3-6, and 4-8. The range of grade-span changed each year as the placement of families' siblings created the need to create different ranges, but even today we are committed to multi-age classrooms. We are so committed that our middle school teachers must be elementary-certified, which means they are licensed to teach all middle school subjects in their classroom. Our middle school students do not change classrooms for different subjects, and are, therefore, taught by a teacher who knows the whole student as a learner.

At this time in the Renaissance, the teachers and I began the work responding to the call for clear of standards intending to close the achievement gap, but we did not feel extreme pressure for another six years, as our test scores were well above the rest of our district's scores. However, NCLB really changed our original intentions of working with parents. With teachers now holding so much accountability, my staff and I were no longer willing to claim some of the curriculum for school and leave other parts for home. It was time that all subjects be taught in the classroom and, consequently, have the parents use the homeschooling portion of the day to support *our curriculum* at home.

Modes of Engagement at Family School

While this study is about our math modes— the Algorithmic Math Group Mode, the Brainteaser Math Mode, the Mobius Math Mode, the Real World Math Mode, and the Inquiry Mode— Family School created modes for other subject matters, as seen below.

Table 5

Modes of Engagement at Family School

Reading Modes	Writing Modes	S.S / Sci. Modes	Math Modes
Long Vowel Method	Writing Brainteasers	Workshop Mode	Algorithmic Math Group Mode
Diagnostic Reading Tool Analysis Mode	Sophie/ Vocabulary Mode	Science Project	Brainteaser Math Mode
Reading Journal Mode	Reading Journal Mode	Writing Prompt Mode	Mobius Math Mode
Non-Fiction Reading Mode	Writing Project	Non-Fiction Reading Mode	Real World Math Mode
Book Clubs	Art/Poetry Night	Inquiry Mode	Inquiry Mode

As this study's explanation of the five math modes is complex and lengthy, so are the explanations of the modes of the other disciplines. It is not possible to explain briefly what these modes are or how they operate. If the content areas were listed for each mode, it would suggest they were divided for content. If the purposes were listed for each mode, it would not suggest enough information to get a feel for what the mode is. The focused instructional strategies for each of the modes require the explanations to be integrated with the content, purposes and structure. However, it is important to see that the math modes are placed in the context of a multi-modal classroom and school.

Our modes developed throughout the years as my teachers and I improved our curriculum and instructional design. In the years 2008 through 2011, this creativity gave way to the pressures of NCLB, as we madly dashed toward measuring student progress everywhere. This age is entitled the Age of Exploration, as this name represents a time when an overabundance of measuring our world resulted in many conquests.

School Structure

Presently, in the Reconstruction Age of our school, some of our students are still going four half-days, though my staff and I changed them from the original Monday through Thursday schedule to a Tuesday through Friday schedule. (That change in itself could be a dissertation!) The instructional day for this half-day program, which we call the 50% program, has grown from the original three and one-half hour sessions to four hours a day. To do both the morning and afternoon classes, we have to overlap both sessions an hour of the day from 11:00 to 12:00. This allows the teachers to still have two sessions of three hours with the smaller pupil-teacher ratio, and one hour with all 24 students in the middle of the day.

However, three years ago, the school needed to create an 80% program to accommodate the parents who loved our school, who, when the economic crisis hit, needed to go back to work and discovered the 50% program no longer was an option for their families. The 80% program requires students to attend three full days, 8:00 to 3:00, Tuesday through Thursday, and from 8:00-12:30 on Fridays. There is still a homeschooling requirement, but it is only 10 hours per week, as opposed to the 15 hours per week for the 50% program. The pupil-teacher-ratio in the 80% model is 18-to-1. The school has six classrooms that are in the 50% model, and six classrooms doing the 80% model.

When the school expanded to the 80% model, as their teaching principal, I felt it important to change my classroom from the 50% class I had been teaching and now teach an 80% classroom to learn the differences between the 50% and the 80% arrangements and better help the other teachers as they adjust to this new environment. I taught the 80% program for two years, but because of my increasing principal expectations for the district, I had to decrease my teaching hours this year. I presently have an amalgamation of the two programs, as many of my families were in the 80% because it met their family's needs. This year I taught a full day on Tuesdays, and five hours a day on the other three.

Teaching Principal

Some other aspects worth noting about the school are that while we are a school of choice, we are held to many of the same requirements as the more traditional public schools: the same calendar year, testing procedures, hiring and staffing requirements, teacher evaluation methods, teacher professional develop plans, budget constraints and procurement processes. Because we have been able to vary our instructional day and PTR, and create our own curriculum, we have been able to impact our teaching greatly. We are able to have one day a week without students, which allows me time as an instructional leader to offer professional development for my teachers on a weekly basis. I have a variety of meetings designed for various groups of teachers: 50% teachers, 80% teachers, all teachers, school data team, curricular teams, new teacher team, support teacher team, etc. Consequently, I am able to serve my teachers in ways that most principals cannot. It is important to restate that as a *teaching principal*, I am able to work with my staff around the improvement of teaching in ways unavailable to most principals who have not been in the classroom recently, or are not able to readily understand how the new district or state changes are really impacting teaching. While participating in both roles is quite difficult, it was, and is, a most important factor in being able to develop our Modes of Engagement and maintain the professional development it requires to bring new teachers up to par, as we have a shifting staff, like that of any school.

While I find many barriers in performing these dual roles and do agree that there is little support for doing multiple roles, I have found that the rewards of an improved staff relationship and their increased confidence in me is worth the effort. I often remember the first years when I had to be teacher, secretary, and principal for those 180 students and five other teachers, while raising five of my own children. Once the district finally awarded me a secretary, I felt like I could handle anything. Today, with over 240 students, we have 11 other classroom teachers, one special education teacher, a half-time technology assistant, a half-time counselor, one secretary, and we share a nurse with a neighboring school. I feel very blessed to have such support.

Parent Involvement

Another key component of our school is that the school requires both parents and teachers to meet monthly (twice a month for the 50% program and once a month for the 80%) in a parent class, where teachers teach parents about the school curriculum and how to teach it during the homeschooling portion of the days. In the beginning years of the program, the parent component to our school was very demanding and required parents to devote much time to their families. Presently, this demand has loosened. One reason the shift was made was because of NCLB. Once the school was not able to *share* teaching with the families because the school was going to be held accountable, the teachers and I shifted our *homeschooling* portion of the day by expecting parents to teach their lessons around our homework. This required much less training in teaching parents to teach as we had done before when they were teaching more of their own curriculum. After the economic crisis brought about the 80% program, 80% teachers and I found that parents were seemingly no longer as *involved* with their students as they were before in the 50% model. Despite the fact that we lowered the hours of homeschooling expectations for parents in the 80% program, we noticed that students were often working through their homeschooling on their own. We have definitely picked up the slack from this with teachers carrying more of the teaching responsibility in their classroom hours in this 80% program, where parents are too busy to maintain their commitment. The 80% teachers still meet regularly with parents in the parent classes, where we work to connect parents with their students' work and encourage them to participate when they can. We often prioritize for them where their attention might be needed with their students to get the support we may need.

With these expectations of our parents, one might be surprised to know the school maintains a waiting list to attend our school. To apply to our school, parents simply fill out an application and they are put on this waiting list. They must also attend a two-hour informational meeting, which moves the application up in the waiting list from those who have not attended this orientation. New students are selected from this list as openings occur. The list changes from year to year, as students who do not get into our school are placed by their parents in other schools. Some families wish to stay on our list for years until they decide they are ready to attend. Each year, between those staying and the new families applying, we have a waiting list of over 150 children.

The teachers and I have learned over the years that the informational meeting is important for prospective parents. Before charter schools became a trend, our elementary/middle school of choice was one of the few free alternatives to public schooling. We discovered that families had a great many different reasons for wanting an alternative. It was important for us to maintain our purposes and vision of the school in spite of this. We found that parents who used our school for their different purposes did not always have the same level of success as those parents who were in agreement with our philosophy of teaching and learning. The informational meeting offers us a platform for communicating to prospective parents our vision and how that vision plays out in the classroom. We have found that it is important for parents who choose the school to have the right information in order to make that choice appropriately. My staff presenters and I are very explicit about our vision, curriculum, and instruction at the informational meeting, so they can understand how we work. We also offer them an open-house day, where they can attend the school during regular hours to watch the classes in action. However, attending this day is not a requirement for getting into the school.

Student Composition

There are a wide variety of students at our school, comprised of regular education students and special education students. Our students are of many ethnicities, varying socio-economic status, and are also here for a variety of reasons.

Table 6

2013/2014 Number of Students in Family School with Special Needs Breakdown

Regular Education Students	Students with IEP (not with gifted eligibility)	Gifted Students (services waived)
226	9	8

Table 7

2013/2014 Number and Percent of Students' Ethnicity in Family School

Anglo		African American		Hispanic		Native American		Asian		Total
155	64%	4	2%	76	31%	1	.5%	7	3%	243 students 141 families

Table 8

Student Breakdown into Grades at Family School 2013/2014

Kindergarten		First Grade		Second Grade		Third Grade		Fourth Grade		Fifth Grade		Sixth Grade		Seventh Grade		Eighth Grade		Total	
28		34		35		29		29		29		22		23		14		243	
10 Boys	18 Girls	16	18	11	24	15	14	16	13	15	14	13	9	12	11	6	8	99	115

One aspect that we all have in common in our community is that parents, students, and teachers want to participate in an educational environment where our constituents can maintain some connection in having a say in what and how we teach and learn. The freedom to develop curriculum to meet the needs of our students has given us all an agentic purpose in raising our young people to see education as a place for lifting them up together as a community to meet their futures. The Modes of Engagement are one way, the teachers and I developed to enhance their learning to have higher purposes than high-stakes testing.

Main Features

In conclusion, here are seven features about Family School worth remembering when reading the research to come:

1) All students have reduced hours in our school, either 50% or 80% of the school week compared to those students in traditional schools;
2) All students are in multi-age classrooms, ranging form two grade-spans to five grade spans;
3) Students work in groups and are rarely in groups that are just put together for their grade level, and they are grouped according to ability or grouped heterogeneous;
4) Parents are regularly engaged in conversations around curriculum through the parent classes;
5) Parents are expected to maintain some level of engagement with their students around school curriculum in such tasks as homework or projects;
6) By being a school of choice, we have been given the freedom to develop our own curriculum; and
7) Throughout the twenty-four years of Family School, I have maintained being a teacher and a principal.

Chapter 5

Algorithmic Math Group Mode

Table 9

Major Codes and Their Themes for the Algorithmic Math Group Mode Creating Patterns of a Plotted Story Line

Mode: Algorithmic Math Group Mode		
Codes	Themes	Plotted Line
Higher Purpose	•Profound understanding of basic math •Meaning-making •Metacognitive strategies of connecting known to unknown, and learning at student's own rate	The Algorithmic Math Group Mode's themes create a plotted line that suggests that it is crucial to have some modes parsed for linear mastery teaching based on making meaning. In this way, students can build a solid understanding of basic mathematics without sacrificing the more open-ended teaching of the other math modes presented in the following chapters. This mode enables students to learn the value of performing mathematic concepts as mastered, while developing the habit of questioning their way through how the basic algorithms are designed to work. The recursive nature of the instruction leads to building habits of the importance of questioning and making meaning in mathematics.
Specific Content	•Meaning-making through manipulatives •Algorithms' procedures through meaning-making •Linear sequencing for mastery •Teachers teaching more than one grade-level content	
Strategic Instruction	•Mastery learning for all levels of students •Recursion of questioning •Habits of questioning	

The Algorithmic Math Group Mode is a mode where students in a classroom are grouped according to their ability to perform and understand the basic mathematical algorithms, such as addition and multiplication with whole numbers and with rational numbers. These small groups are taught daily by the teacher through a specific instructional method that includes the use of manipulatives, questions and student explanations. Each lesson lasts for 15-20 minutes and is designed to help the students to work their way through the unit until they are ready to take the unit test. Students are given a placement test to determine the algorithmic group to which they will belong. The groups move through the algorithm unit together, but must earn an 85% or better on the unit test in order to move on to the next algorithmic group.

The algorithmic groups are sequenced as follows: anti-counting (number sense), addition, subtraction, multiplication, division, fractions, decimals, percentages, geometry, and algebra. Once students have passed the unit tests, groups are reformed. Students can proceed at their own rates through the sequence of algorithms as they master the material. In this way the groups are flexible; students can repeat work in the same group if it is needed, or they can move on to the next group when they have demonstrated proficiency. Thus, it is mastery of the unit expectations, not grade-level expectations, that ensures the students' promotion to the next unit. Our sequence of algorithms does not match up with the normal sequence of algorithmic expectations of mathematical textbooks. Because these units emphasize learning the algorithm with the understanding of why the algorithm functions the way it does, it is possible for students to learn the entire content of any particular algorithm. For instance, once place value is understood in addition, to the tens' and hundreds' place, then a student can understand the part place value plays in adding in the millions or quadrillions. In this way, all of addition can be taught in one unit. The usual sequence for traditional mathematic programs is to teach increasing place values at increasing grade levels.

Revealed in this chapter, are the major lessons learned at Family School through the development and evolution of the Algorithmic Math Mode including:

1) All mathematics should be taught for meaning instead of memorization. This meaning should be understood and explained by all students without support or cues from the teacher.

2) Meaning-making in all mathematics should begin in kindergarten. Our kindergarten unit focuses on number sense, place value and learning mathematical concepts of odd and even through visual understanding rather than memorization.

3) We learned that teachers must know mathematics in a deep, meaningful manner themselves, so they can recognize the need for the students to demonstrate that deeper level of understanding.

4) The manipulative models and questions the staff has designed for each algorithmic group are the keys to helping students understand how the algorithm works. However, they must be used simultaneously with writing out the algorithm procedurally or the manipulative models will not translate into mastery of the algorithm.

5) When teachers ask questions, they should provide opportunities for the students to reveal their thinking about the mathematics through building models, writing or verbalizing and to provide direction about the procedure of the algorithm accordingly.

6) Collaboration as a staff, to design, implement and evaluate our algorithmic units of study and instructional design, had to be done in a uniform manner throughout the school for our students to benefit from this mathematical mode and its sequencing.

7) By developing the sequence of our mathematics curriculum, we came to understand its importance in building toward algebraic understanding, and we began to connect our understanding of our algorithms to the algorithms used in algebra. The importance of helping students translate their mathematical experience in the earlier grades to an algebraic one is important to transitioning students to algebra content in the later grades.

8) Having a solid foundation in the basic algorithms supports the fluency mathematicians need for other more puzzling or complex, enriching opportunities in mathematics.

A non-example of this mode would be to teach a math lesson on the algorithms from a prescribed textbook, where the material on the page is the focus of instruction, rather than the teacher knowing each student's level of understanding of the given algorithms. The content in this case would be driven by the grade level requirements without knowing what level of mastery each student has with regard to the given content.

What you will see is the chronological autobiographical story of this mode's development, starting with the origins of the algorithmic foundations from my years of teaching prior to the start of Family School. This story will have the analysis of the mode documents woven through it. As evolution of the mode reaches the present, my reflective personal journal will be used to tell the final iteration of the mode. The beginning seeds of the algorithmic mode were firmly planted in my ideas of teaching and learning mathematics right from the start of my teaching, and are critical to its creation in Family School.

Seeds of Algorithmic Math Group Mode Prior to Family School

Having only taught for three years prior to starting Family School, I didn't have too many tricks in my bag to bring to this innovative idea for a school, but there were a few. I had started my teaching in a first-grade classroom, followed by a third/fourth combination classroom. From the start, I realized the wide range of student capabilities in mathematics. I began my teaching career more interested in finding which limits student could reach rather than teaching to the prescribed limits of any given curriculum. I found my first graders could add and subtract in different base systems, manipulate positive and negative numbers on the number line, and devise their own methods of multiplication on their hands. Having seen the success of first-graders developing their own mathematical understanding, I was curious to bring this investigation to my next classroom, so in the third/fourth combination class, I was very interested in pushing their limits to figure out how to help students learn the various algorithms in meaningful ways rather than only procedurally. I worked with students to hear their explanations of the mathematical operations and how they made sense of them. The base ten blocks were a very popular set of manipulatives at the time, used to explain place value, which enabled reasonable explanations of addition and subtraction, and I was introduced to multiplication using the blocks, but none of the techniques used were able to translate from the concrete understanding of the blocks to an abstract explanation of the algorithm problem.

One of the parents of my students in the third/fourth combination class discovered a method, being taught at a Montessori school, called Mortensen Math and suggested I attend to see this method in action. This program showed me the missing link between the base ten blocks and the algorithms.

It was a very unique program that had many other components beside the base-ten blocks, but it was this link that was to influence me the most. The parents of the third/fourth combination class collected money to fund the use of the Mortensen Math program for that year's and the next year's classes. The students and I dug into the program earnestly to complete as much of it as we could in the two years. Unfortunately, as is often the case with much new mathematics curricula, there was little training that accompanied the texts, and the teacher edition delivered with the program was not easily comprehended. I set off to figure out the student texts with the students. The program was a highly graphic program with many pictures of how to use the manipulatives with very little explanations. I had some of my more *math–savvy* students work with me at recess to uncover the meaning behind the pictures. We soared through the algorithms of addition, subtraction, and multiplication and finally found out why the old method I had been taught with the blocks for multiplication didn't work: the suggested method for building the picture of the multiplication algorithm wouldn't match the problem because it was built upside down (as seen in Figure 6 below). With this program, I was able to see how to connect building the base ten blocks in such a way that it could also teach how to do the algorithm procedurally and conceptually.

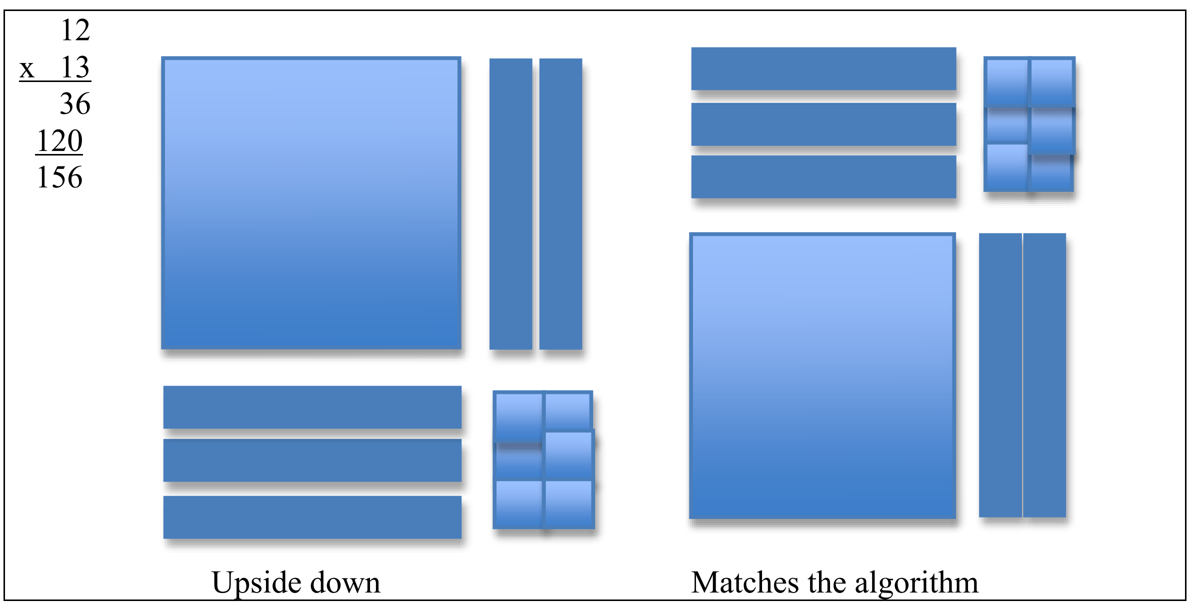


Figure 6. The multiplication algorithm built in base-ten blocks.

Through the attempts the students and I made to figure out what was on the pages of the texts, we developed a question-asking technique for building the algorithms. For instance: "What are they doing in the one's column? They are making tens. Which number do they want to make a ten out of? How much will they take from the other number? How much is left over? Did they make a ten, and where do we take that ten?"

This series of questions continued in the other operations as well. As learners together, the students and I were using these questions to move our understanding forward conceptually as to why, procedurally, the algorithm was done, which is why this questioning teaching method continues so pervasively to this day. We were building the base ten blocks to match the Mortensen Math program's pictures. As we built and asked the questions, we wrote out the algorithm, step by step. We were very excited to understand the why's and how's of these procedural mysteries. However, the division algorithm stumped us for years. We were able to make sense of the picture and see what was going on in division, but we were not able to ask the right questions that lead to understanding the division algorithm that we all know and use conceptually and procedurally. It was about eight years later, as I was teaching a division problem in another number base system as a brainteaser, that I finally understood what those Mortensen Math pictures were doing, and I was able to come up with the right questions to teach the understanding of the algorithm.

While I had not done the research on manipulatives in the late eighties when I began to teach, I knew there had to be a better way to teach mathematics than the way it was taught to me. Perhaps it was an assumption on both our parts, the parents and myself, that in order to excel in mathematics the main purpose of any mathematics program should focus a main part of its time and energy around making sure that the students could perform the algorithms accurately. However, after watching my first graders devise their own method of multiplying on their hands, I knew it was important for students to make sense of the algorithms. Moving to the third and fourth grade required that I teach both multiplication and division. The algorithms that were taught to me with tricks, like 0's as place holders, magically moving numbers over when doing the next row of multiplication, and using acronyms like *dead mice smell bad* to remember to divide, multiply, subtract and bring down in the division algorithm. Nothing made sense in these algorithms, nor did the text teach it in anyway that would help students make sense of it. At first I wasn't too concerned because, after all, that was how I was taught, and the students could learn the algorithms extremely quickly, but after moving on to other chapters, and a couple of months later, students could not remember how to do what they had memorized previously. That is when I felt that the algorithm should make sense, because if students forgot how to do the procedure, they could try to think their way through it, using meaning, not just memorized steps. With all the old tricks being used, there was no thinking, only memorization. It then became a purpose of this mode to have all mathematics be presented to students in such a way that they could make meaning out of what they were doing, even the basic operation algorithms. It seemed essential that the foundation of becoming a mathematician should come from meaning-making and not memorization.

Over the years, I learned that students might take a bit longer learning how to do the algorithm while also understanding why it worked, but the extra time was better than having to go back and memorize it again because you forgot how to do it the first time. But more importantly the message I intentionally wanted to send about mathematics was that we were all capable of making meaning in the world of numbers.

What I infer from this is that it was important for me to receive professional development in mathematics to improve my own understanding of mathematics. I was able to find this professional development both within and without the school. By attending a mathematics workshop on Mortensen Math, I experienced what it was like to have mathematics make meaning and was greatly influenced. The professional development in the school did not come from the principal or the district rather it came from working closely with the students as we made meaning together. This had a great influence on me because I learned how important it was to learn with my students right from the start. Not only did it highlight this teaching strategy, but the methods we developed became the seeds of the Algorithmic Math Mode. This work was further developed when I began Family School.

Seeds Take Root in Family School Through the Use of Manipulatives

Mortensen Math also showed pictures for fractions that explained the algorithms with fractions, but the fraction manipulatives that accompanied the explanations were extremely costly. While I was only working with my own students in my classroom, I had plenty of manipulatives from the one set I purchased, but when I moved to Family School, I knew I would need to purchase more for the other programs. The Family School parents of my students also wanted their own set for their homeschooling portion of the program, so we went into an assembly-mode and began to make our own homemade versions of the set. It was a year later that we found a set very similar to the ones we made being offered by Cuisinaire for a minimal price compared to the price of Mortensen's and worth not having to make them anymore. The Cuisenaire product is called Picture Grids for Fractions and Decimals for $16.95, a price well worth the conceptual understanding that can be achieved by using these tiles to demonstrate all operations with fractions and decimals.

It occurred to me that between a set of Cuisenaire Base Ten Blocks ($27.95) and the fraction grids, an entire algorithmic mathematic program from Kindergarten through fifth grade could be taught for less than $50.00. We ordered many and began our training of teachers and parents, doing away with the Mortensen texts, which were unwieldy, expensive and without explanations. This was the beginning of developing our own algorithmic mathematic curriculum.

While it was obvious to me that the use of manipulatives was the answer to bringing more understanding of the algorithms to the students, the research does not always reflect this. Kamii, Lewis, and Kirkland (2001) suggest quite the contrary. They suggest that in teaching place value base ten blocks "are usually not useful" (p.28) because most children "construct *tens* out of their own system of *ones* that are in their heads" (p. 30). Most importantly, they highlight that manipulatives are "useful or useless depending on the quality of thinking they stimulate" (p. 31). Perhaps Moyer (2001) found the reason such negative conceptions of manipulatives existed in the research. Her studies suggested that many of the teachers studied used the base ten blocks in their classes as a way of "playing, exploring, or a change of pace" (p. 188). Other teachers were quoted as talking about *real math* being taught at the beginning of the week, and then they used the manipulatives to be the *fun* mathematics that was done on Fridays. In conclusion, she states, " How and why manipulatives are used by teachers is a complex question. In previous studies where manipulatives were used, we viewed mere use itself as a positive sign. But simply using manipulatives is not enough if we do not consider *how* classroom teachers are using them" (p. 193). She claims the manipulatives' importance is unveiled when they "guide students to translate between representations in the form of mathematical objects, actions and abstract concepts, so students can see the relationship between their knowledge and new knowledge" (p. 194). This is exactly what we discovered at Family School about the best use for them. They become the tools that with the teacher's questions bridge the known to the unknown. As Moyer says, "manipulatives are not magic" (p. 176), their validity is enhanced when they are used well. She also states, "Important to consider is the significance of manipulatives as potential tools and their significance as a function of the task for which a teacher conceives them being used" (p. 176). This describes how, as a staff, we set out to use them explicitly for a purpose of walking a student through the meaning behind the algorithms.

When Fuson and Briars (1990) studied students learning to add and carry in two digit problems with and without the use of manipulatives, they discovered that both groups of students did learn to add correctly, but those students who had used manipulatives, when interviewed about their problems, demonstrated more understanding about what they had done mathematically. Our best work with students of Family School demonstrates what Uttal, Scudder, and Delache (1997) recommend for the best use of manipulatives: "Children need to perceive and comprehend relations between the manipulatives and other forms of mathematical expression" (p. 38). These authors further related that "manipulatives are not a substitute for instruction" (p. 51). They noted that many teachers have used manipulatives in a discovery approach, but they cautioned that the importance should be to teach students to understand how to use them in such a way that mathematical meaning is made more efficiently. Lastly, each kind of manipulative should be chosen carefully to assist in teaching the correct mathematical concepts.

I have discovered over the years that it is essential to instruct my teachers very carefully in how to use the blocks, so the manipulatives do the work that they are able to do if used properly. Some of these very specific techniques are to build the picture exactly as the problem is written, teach by pointing to both the blocks and written problem as the questions are being asked, and have students point to their blocks and the written problem as they give their responses.

What I infer from this is that the insistence to use the manipulative simultaneously with the algorithm was a function of ensuring that students were making meaning about the algorithm through learning how to use the blocks. This was an essential part of the mode's curriculum.

What you will see next is that as a new school, the Family School teachers worked together to develop these algorithms into a scope and sequence for all grades of our students.

The Dark Age and Middle Age- 1990-1999

It was probably not intentional in the Dark Age (1990-1994), but once my teachers and I began to teach the algorithms in order and truly understand why and how they worked, it became clear that there was no reason to continue to learn how to add or subtract throughout a student's first through eighth grade school experience as suggested in the mathematics texts of the time. It seemed a linear curriculum would work for our purposes of mastery, so that is what we settled on. We firmly decided that the algorithms were the foundation for mathematic prowess and should require mastery before moving on to the next operation. With this assumption, it was also clear to us that once addition was taught with a place value understanding, we knew that if students could add two digits with comprehension of place value, they could add any number of digits with the same understanding. We set out to teach our students to pass all levels of addition, including word problems, before moving onto subtraction. The manipulatives led the way as we used them to show how the algorithm operated. The blocks were used to build the problem as it was written. As a ten was carried in the problem, it was also carried in the blocks and placed where the carry is written in the problem.

Today, the blocks are initially used to question the students through the problem, without them focusing too much on the problems. As the concepts became more comfortable for the student, the connection of the blocks to the problem is highlighted. When the students are able to make the concrete connection more abstract on their own, they move away from the blocks to compute the problem with only pencil and paper. The blocks can also represent various items in the real world that need to be added, so that pictures can be used to help students translate mathematics to their own pictorial representation of word problems, as well.

What I infer here is that because our Family School classrooms were the multi-grades for first through sixth grade, as a staff, we needed to develop a scope and sequence for all the algorithms. By wanting our students to be taught from their individualized levels rather than just their grade level, we needed to organize the algorithms in such a way to group our students accordingly. This is probably how the sequencing of the algorithms in a linear manner came to be.

What you see next is the how development of place value in all the algorithms led the way to designing this scope and sequence.

Place value. When I began teaching mathematics in the late eighties and early nineties, there was much concern raised by many teachers that young children could not comprehend place value. It was evident to me that this was simply not the case, as long as the concepts were introduced with the concrete manipulatives. Also, there was concern that while the manipulatives could show the concepts, they did not translate to the algorithms to help students master the procedures any better. I also found this not to be the case, if the blocks were taught to mimic the algorithm, they actually taught the students to move from what they understood in using the blocks to what they were learning about the algorithm. Our questions were designed to help this to happen. It is not to say that our students did not wrestle with place value, because of course they did, but the blocks helped move them from not understanding to having visual representations of the new values of each place. I remember one very bright student wrestling with place value when I asked her how many units were represented in a one-unit, she stated, “One.” When I asked her how many units were represented in a ten-stick, she replied, “Forty-two.” I knew that she was not choosing the number randomly because her answers were always extremely thoughtful, and it wasn’t long before I saw that she had counted the four long sides as ten each and added the two ends of the stick to make forty-two. I just needed to go back to the one-unit, and remind her of its value and place two together, then three, and so forth, and she then understood the ten-stick’s value being ten of the units put together. The student and I talked about how they could have made the blocks flat but that it might have been more difficult for children to pick them up and easily to build with them. She had a little chuckle about her answer. I thought it brilliant. I love to see how children’s minds work, and perhaps that is why I wasn’t afraid to take on place value with young children.

In fact, in the Dark and Middle Ages, the idea of place value became so easily understood that my staff and I even included working in other number base systems in addition. Our first-graders found adding in other bases to actually be easier than adding in base ten, as there are fewer numbers in those bases (smaller than base ten). In those early years of the program, all of our students, despite their grade, completed our addition unit to make sure that they understood place value and its importance in the addition algorithm. I had created a placement test for my students to ascertain what concepts they understood about each of the algorithms they could perform, and if students were not solid in them, I would have them backtrack for a while to make sure the conceptual understanding was taught. This placement test explicitly looked for student conceptual understanding by having the students take the test while also recording their thought process on a tape-recorder. When evaluating their tests, I would listen to the tapes and write down student language as to whether their work also reflected conceptual understanding.

In the Middle Age, I remember one very tall fifth-grade student not performing well on his placement test and needing to start in addition. He did feel foolish and begged me to start him in fractions. I made a deal with him saying that if he could add in a different base system then I knew he understood the concepts of place value, and he would not have to be in the addition group. He suggested that it was not a fair proposition because he didn't know what another base system was. I replied that I would throw in that lesson for free and then give him a chance to add. Once I gave him the lesson on counting in other number systems, he was enthralled with how the systems worked, and how much more he understood about our base-ten system, so he was fine with being in the addition group, as long as I promised to teach him how to add in the other number systems. He passed out of addition easily, followed by subtraction, multiplication, division, and fractions in that one year. He clearly had an aptitude for mathematics that had been undermined by not having an understanding of what he was doing and why it worked. Making mistakes in problems that don't make sense in the first place is easy to do because it all seems like smoke and mirrors.

Some of our very early younger students helped us to see how quickly the place value method moved students to mental mathematics. One kindergarten boy, in particular, when asked one day how much nine plus seven makes, immediately blurted out sixteen. Amazed at his speed, for I figured he would need to count it out, I asked him how he got the answer so quickly. He replied that he knew the nine needed one more to make it a ten, that it would come from the seven, which would leave a ten and a six, making sixteen. My teachers and I immediately moved our place value comprehension to supporting mental mathematics.

I remember teaching another little first-grade boy how to simply think about five more than 68 by building the sixty-eight with the base-ten blocks. By building six tens and eight ones, the student could see that 68 was only two away from making 70. He then could see that if two were taken from the five, then there were three blocks left, making the answer 73. When he saw how easy this was to conceive, he jumped from his cross-legged position on the floor, screaming, “Give me another one. I think I can do it in my head!” I think he could have done it all day; he was so proud of himself.

These early days of the Family School were spent developing the major operation units by simultaneously teaching parents and teachers to learn this new system. Teachers were always expressing their disappointment at not having been taught to do mathematics this way because it made so much more sense than the way they had been taught. Soon it was understood that all of addition was to be associated with our first grade curriculum, subtraction in second grade, multiplication in third, division in fourth, and fractions in the fifth grade. Our middle school students retraced any of those units that were not mastered and then mathematics texts were used for decimals, percentages, ratios, geometry, and algebra. It wasn’t until the school’s centralization in portable buildings in the year 2000 that my teachers and I made a unit for decimals and then organized our own texts for the rest of the units. Though we assigned grade levels to the scope and sequence, we limited ourselves to mastery, so the grouping of students was more about their ability level than it was about their grade level. The placement tests were the key to grouping the students. However, each teacher used his/her own placement tests in the Dark and Middle Ages. We also did not want our groups to be stagnant; they needed to be flexible enough so that if students passed out of the groups, they could move to the next group, or if students needed more time in a group; they could stay until they were solid in their understanding.

What I can infer from this part of the autobiography is that our quest for meaning-making in mathematics encouraged a deep understanding of place value’s importance in the algorithm. Once we knew that our students could develop this conceptual understanding of place value, the entire algorithm could be taught in one unit.

What you will see next is how we developed our questions to encourage the students to make meaning of the algorithm as they proceed through the unit.

Questioning. Our staff-meeting agendas and summer-seminar notes show frequent entries about the training for our mathematics program. We were firmly committed as a school to the delineation of our algorithmic math groups from the start. This proceeded well except I couldn’t figure out the division algorithm well enough to design guiding questions. I could explain the building of the blocks, but it didn’t match the short-cut algorithm that the majority of people use to do division. I knew there had to be a way to use the blocks to teach the algorithm that we all use, but it escaped me for years. The reason was that the first question used in the short cut in a problem like 142 / 12 is: “Will 12 go into 1?” When it does not, one proceeds to ask whether it will go into 14, which it does, one time.

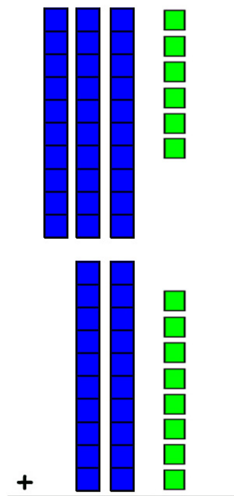
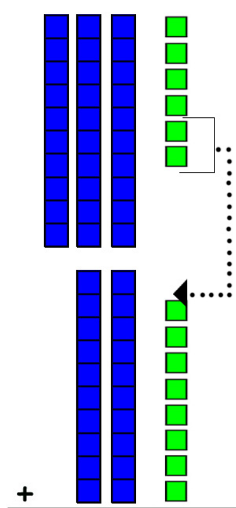
At one point, when working through a division problem in another base for a brainteaser, the base ten version flashed through my mind. I was then able to find the questions to support the block version of division. It became pointless to ask if 12 would go into 1. Since the answer to that question would have been put in the hundreds place, the real question was: “Does 1200 go into 100?” Next, the question would be: “Does 120 go into 140?” And so on. I needed to switch my thinking to building place value groups of 12 rather than just individual 12’s. Once that was discovered, I moved on to exploring fractions at great depth to develop that unit.

Right from the start of developing this mode, meeting the purpose of mastery of the algorithm through meaning-making provided focus for the instruction. It came through questioning of the student by connecting what is known to what is coming to be known. This became important in this linear sequence of mathematics. The students were tested for their abilities by finding what they did know in the algorithms. They were then grouped in ability groups flexible enough to allow them to move at their own pace, but with other students working on the same algorithm. With a wide range of small groups in this math mode, it meant that our teaching of each group would have to be done quickly, so the lessons were paired down to 15-minute lessons per day, with follow-up work assigned that could be continued in school and at home for homework. The small groups also facilitated the use of the manipulatives, allowing students to see how the blocks were built, while the teacher could observe everyone in the group build their own problems correctly.

The questioning method of teaching worked well with a small group of students with similar abilities. The questions could be delivered to the various children randomly to spot-check everyone’s understanding, but in close enough range to make sure that everyone was listening to the group lesson as if it was taught for an individual. The intent of our questioning was similar to Martino and Maher's (1999) justification for questioning when they suggest that "teacher questioning can invite students to reflect on their ideas; further, it can open doors for teachers and students to become more aware of each other's idea" (p. 75). Having the groups remain small allowed us to focus on how the students were developing their understanding of algorithms. Wimer, Ridenour, Thomas, and Place (2001) differentiate between higher-order and low-level questioning. They suggest that higher order questions encourage students to analyze, synthesize, and evaluate material, whereas low-level questioning encourages memorization, or uses rote memory. While I am aware that a series of questions can promote memorization, our questions were not designed to do this, but rather to help the student to learn how to connect the known to the unknown. Yet, despite the fact that many of our questions did not qualify as open-ended, there were a wide variety of questions, whose focus was on getting students to explain the reasoning behind the algorithms.

Franke et al. (2010) differentiate questioning by using the terms *funneling* and *focusing* questions. For them, funneling questions are in use when "teachers tak[e] on some of the mathematical work and mov[e] students in the direction teachers thought most critical" (p. 381), while focusing questions "encourag[e] students to do most of the mathematical work by focusing attention on particular aspects of students' explanations without guiding students in a specific, predetermined direction" (p. 381). According to these definitions of questioning, I would say that our list of questions fall on the side of funneling questions because our questions have a specific direction we are asking the students to move, though I do not go there unless the student finds meaning in the question and discovers the path independently. I would best describe our questions written for the mode as funneling, higher-order, minimally, open-ended questions, because while directing a student's thinking, they focus on conceptual understanding being constructed by the student. Occasionally, teachers must abandon the prescribed questions to ask more open-ended questions to hear the student's explanation of their conceptual understanding. While our questioning could be improved to encourage more higher level and more open-ended thinking, our questioning does result in our students' understanding of the algorithms. Redfield and Rousseau (1981) demonstrated that higher-order questioning resulted in higher achievement as compared to students who were not taught with higher-order questioning, but those results were not reflected in a testing performance, as both groups tested equally. The finding of higher achievement was determined by interviewing, which suggested that the higher-order questioned group had a better understanding of their work than those students who were taught through more rote methods. Our students are required to demonstrate their conceptual understanding of the algorithms, and also perform accurately on the test. Our funneling questions must work well enough for that to happen.

Addition 1: Level 3 (2 Digit + 2 Digit)

Level 3, Problem	**Build it**	**Teacher's Questions**	**Child's Answers**
36 +28		1. Where do you start? 2. Why? 3. Is this going to make a ten? 4. Which one do you want to make a ten?	1. Ones column. 2. Because 1s can make a 10. Another 10 will change the number of 10s. 3. Yes. 4. Eight.
36 +28		5. How many does the eight need to make a ten? 6. From where do you want to get the two? 7. How many 1s make a 10? 8. How many 1s are left?	5. Two. 6. Six. 7. Ten. 8. Four.

Addition 1: Level 3 (2 Digit + 2 Digit)

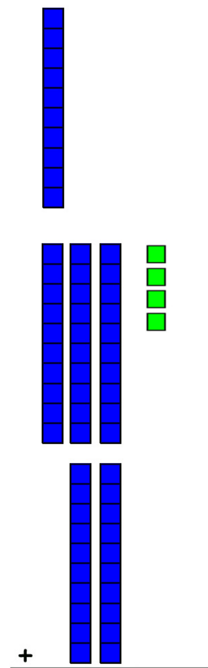
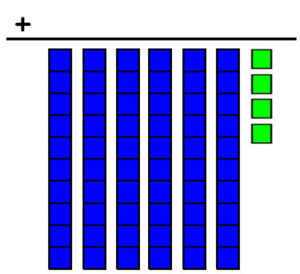
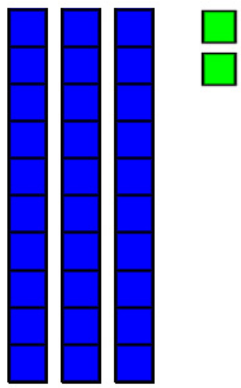
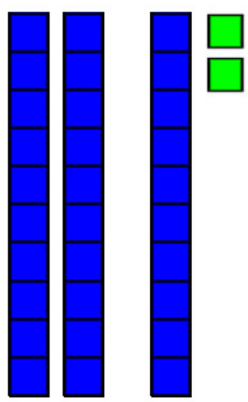
Level 3, Problem	**Build it**	**Teacher's Questions**	**Child's Answers**
1 36 +28 4		9. Where do you put your new ten? 10. How many ones are left? 11. How many tens do you have when we add our carry to the larger number of 10s? 12. And two more 10s make what?	9. Carry it to the tens column. 10. Four. 11. Four. 12. Six.
1 36 +28 64		13. Where do the ones go? 14. Where do your tens go? 15. What is your answer?	13. In the ones column. 14. In the tens column. 15. Sixty four.

Figure 7. A sample section of the parent booklet for addition as evidence of the questions used to teach the algorithm.

Subtraction Level 3 (2 Digits – 2 Digits)

Level 3, Problem	**Build it**	**Teacher's Questions**	**Child's Answers**
32 − 14		16. Can you build this?	16. Yes.
~~3~~2 12 − 1 4		17. Where are we going to start? 18. Why do we start there? 19. Do you want to take the 4 from the 2 or get a 10? 20. From where can you get a 10? 21. (Move one of the 10s to the ones column. Now, 12 are in the ones column.) 22. Do you want to take 4 from the 2 or the 10?	17. Ones column. 18. Because we may need to borrow a 10. 19. Get a 10. 20. From one of the 10s. 21. Record borrowing. 22. The 10.

Subtraction Level 3 (2 Digits – 2 Digits)

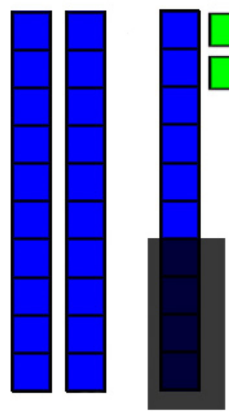
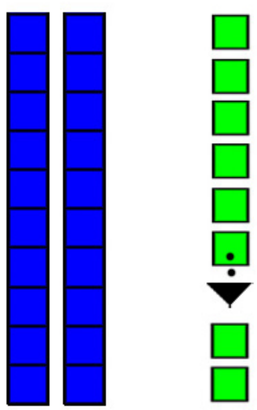
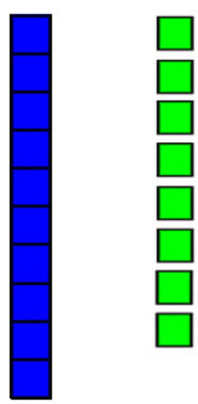
Level 3, Problem	Build it	Teacher's Questions	Child's Answers
~~3~~2 ¹2 − 1 4 8		23. Why do we start there?	23. 8 (Child should add 2 and 6 in her head.)
~~3~~2 ¹2 − 1 4 8		24. (Trade the 10 in for 6 ones.) Ask the child to line the 2 ones up with the 6 remaining ones.	24. (Child arranges the ones in a row.)
~~3~~2 ¹2 − 1 4 1 8		25. In the 10s column, do we have any to take away from the 10s? 26. How many tens are left? 27. How many are left altogether?	25. Yes. 26. 1 ten. (Move 10 down to the answer.) 27. 18.

Figure 8. A sample section of the parent booklet for subtraction as evidence of the questions used to teach the algorithm.

What I infer here is that developing the questions was an important curricular tool for teachers to help monitor the students' understanding of the algorithms as they are being learned. This close interaction during the lessons helped teachers see the need for constant formative assessment of their students' understanding.

What you will see next is how moving our school to a centralized site was pivotal in encouraging teachers to collaborate and develop out algorithmic curriculum even more. This next section will tell the story of our collaboration through a narrative autobiography of this time, but will also lace the memories with nodal documents that are important to the development of this mode in this era.

The Renaissance Uniformity- 2000-2007

Over the seven years of the Renaissance, the school came together in its first centralized location, and our curriculum became more uniform and consistent across classrooms. The teachers and I began to discuss how to unify our teaching and our Algorithmic Math Group Mode as a priority. Eventually, we designed tests to be used school-wide with a mastery cut-off score of 85%. We developed a school-wide placement test, and changed our previous fraction's unit to be expanded into two parts: Fractions 1 (for fractional numbers between 0 and 1) and Fractions 2 (for all mixed and improper fractions). We also formalized our own decimal unit and test. We agreed upon certain texts for percentages and ratios, found a geometry workbook we all liked, and we chose one algebra text. By 2007, the Parent Handbook reflects our explanation of this Mode of Engagement to the parents, as seen in Figure 9 below:

Mode: Math

At Family School, we teach math groups through a conceptual approach, emphasizing the use of manipulatives, such as base-10 blocks, and advanced questioning techniques. In this way, children construct their knowledge of numbers and operations. In the math progression, students transition into using traditional algorithms with pencil and paper calculation where ORKing (orderly recording of knowledge) is emphasized. We expect two things in math: The first is accuracy, where "silly mistakes" are considered "serious errors". The second is connective understanding as evaluated by their ability to express their thinking out loud. In our multiage setting, students are grouped by their math ability into small, fluid groups designed to meet the needs of individuals. We feel it is important to teach a concept in its entirety before progressing to the next one: for example, all of the addition, from single digit problems to adding in other bases, is mastered before subtraction is begun. We believe that this approach moves children most efficiently through number and operation skills. Students need to show proficiency on a written test before progressing to the next level. Below is the algorithmic math unit progression:

- Anticounting
- Addition 1
- Addition 2
- Subtraction
- Multiplication
- Division
- Fractions 1
- Fractions 2
- Decimals/Percentages
- Geometry
- Algebra

Figure 9. The parent handbook description of the algorithmic math group mode from the Family School Parent Handbook.

Anti-counting unit. In the beginning of the Renaissance, we first had to deal with a new glitch in our mathematic algorithmic system that we overlooked in the earlier ages. By the year 2000, the opportunity arose to start our own kindergarten program, which necessitated creating a new kindergarten mathematics curriculum. We decided to build it around number sense, and lead into place value addition, which we knew would have to emphasize place value as well. Of course, we knew that place value couldn't start until students knew their numbers up to ten. We felt it was important to develop number sense and not just rely on saying the number names in chronological order correctly. While counting is important, we wanted students to be able to manipulate those numbers under ten. It was important for them to be able to compose or decompose each number into other number combinations (though not calling this addition or subtraction). For example, a student would know the number five, when mastered, was made of combinations of one and four, two and three, and five and zero. We utilized a *hand game* where students were given a certain number of blocks on the table, such as five, and then the teacher would cover a certain part of the five blocks under his/her hand. The student could see the remaining blocks outside the hand, but the portion of the five under the hand is not seen. The child is asked to name the number hidden under the hand. For instance, in using five blocks, three may be placed outside the hand, and the child is asked to identify that there are two remaining under the hand. We created many versions of this game such as: using dot cards with formation of numbers in a variety of two different sets. Dot cards for the number five would be combinations of three and two, one and four, and five dots together for five and zero. Another game made was the pond game, where many blocks are placed on the table to represent a pond of fish. The rule to fish at this pond is established by the number the student is learning. If it is five, then only five fish could be removed from the pond, but in only two moves, to encourage the student to make combinations of five. This game is suggested for those students who are hesitant to guess the missing number under the hand because they cannot see it. One other game used to help students learn their number sense of numbers under ten uses two-colored poker chips. These chips with different colored sides are placed in a container and emptied onto the table to reveal the combinations of five in the two different colors. Students love to collect the chips each time, only to empty them onto the table many times, never knowing which combination would be revealed.

My teachers and I decided to call the year-long kindergarten unit *Anti-counting*, so that parents would understand the value of learning number sense by manipulating numbers in relationship to each other, rather than just counting. But once numbers one through ten were learned, we still had to think of a way to teach the numbers over ten in a meaningful way such that kindergartners could understand place value.

Years before this, I had had a student who had a processing difficulty and could not learn meaningless information. If information was packed into meaning, he could hold onto it. I knew this student would not be able to count to 100 if the numbers were a string of 100 meaningless words strung together. I created a “Base 10 Story,” so he could see the system that accompanied the numbers. I thought the storytelling approach would provide more meaning to the numbers to help him hold together the patterns that occur in numbers. The storyline is not that important, because since then I have told the story with many different characters and plots, but the following number concepts placed in the story are important. First, it is essential to count something with only one digit, meaning there is a number for each item being counted. Every new item would need its own number name. Students quickly see that it will be too hard to remember all those names. Second, the counting system evolves to pick a certain number of numbers that are easy to remember and then count by using them over and over. This is a problem because one can’t tell the difference between the different sets of numbers. Lastly, a hero in the story suggests using a small set of numbers over and over, but putting them in a different place so they have different values, and thus, our number system is created.

This story has one other important aspect that kept me from sharing it with other professionals for years. I created a way for some of the numbers beyond ten to be said in *baby-talk* fashion, and I was embarrassed to share this with peers. Even though I knew the story, and my specific language, worked to help children learn place value, it seemed unprofessional. Much later, I ran across a study that suggested that Chinese pre-schoolers were outperforming our American preschoolers in mathematics because they already understood place value. What the study failed to mention was that the Chinese language uses place value in the names of their numbers. For instance, twenty-one in Chinese translates as two-tens and one. It was then that I realized that my *baby talk* had reinvented the English names of numbers to contain the meaning, just like the Chinese. I began to share my story with the rest of the staff, including the parents of our students.

Since the story always starts with “A long time ago,” I imagined an era when people said four-ten for fourteen, and over the years it was slurred into fourteen. For 21, two tens and one was slurred into two-ty one, and 31 became three-ty one, and so forth. I always tell this story using a 0-100 chart and also by building the numbers with the base 10 blocks as we learn the numbers to one hundred. Immediately, students can count and build numbers to one hundred. They do use the words I made up, like five-ty seven, because they make more sense and are fun.

I remember one parent who had put four of her children through our school telling me how sad she was on the day her youngest corrected her by saying that the number was not really two-ty two, but twenty-two. She thought that would be the last time she would hear the silly number names from her children; they had all outgrown the story.

Another important story that brought the importance of the Base-10 Story out into the open was the day I was observing in another teacher's classroom. During teacher observations, I often choose students with whom to work to see how they are doing with the teacher's activity. This one student was working on his *5-More-Than Sheet*, a worksheet we are all familiar with at Family School, however, he was not able to complete the task or even know where to begin. It was clear to me that he needed to be told the Base 10 Story. I asked him, "Would you like me to tell you a story?" He replied with a very eager affirmative response. I began the story and he was appropriately engaged, but a female student from across the table scolded us by telling us that he was supposed to be doing his math. The little boy stated, "She is telling me a story." This appeased her, and I continued. Not long after, another little girl from across the room came stomping over and retorted, "He's supposed to be doing his math!" Again he simply said, "She is telling me a story." Again this appeased the other student. It was on that day that I discovered the power of story, and that our version of story problems in math is a poor excuse for storytelling in mathematics. Apparently, a student is quite appropriately occupied if being told a story. But the power of this example was that upon completion of the Base-Ten Story, this student could count to 100, build any number between 0 and 100 and also figure out five more than that number. It was evident that the Base-10 Story needed to be taught to all the teachers.

An agenda document from 2007 reveals the bones of the story being reviewed with the teachers at our May 31st Summer Seminar. Below Figure 10 gives the training notes of this by one of the teachers for everyone to share:

Base-Ten Story:
A long time ago…..
There were shepherds and they had sheep. They needed a way to record and count their sheep. They were trying to figure out how to count them. They figured for zero they would use 0, they would use 1 for one sheep, 2 for two sheep, ….all the way to 9 for nine sheep. They are making a symbol for every number. What symbol do we use for ten? What do we use for eleven? Continue until they get confused…Ask "How many symbols will we have to use?" So they see that we will need an infinite amount of symbols. What if we learn a certain amount of symbols and then reuse what we already know. 01234567890123345….how will you know when one is a three or a thirteen? So they go back to a drawing board, what if we use the same symbols but put them in a different place? So start using numbers and when you get to ten use "one" over again in a different place. So when it is moved over it means it is a "ten." So show "one ten", "two ten", "three ten"…. Now make a chart with 0 1 2 3 …across the top, and repeat the row at ten.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24					

So when get to 20….have two-ty, two-ty one, two-ty two….
Three-ty, three-ty one, three-ty two….
So need to know that 24 is made up of two tens and four ones.
Now should be able to make all numbers from 0 through 99.
They can make any number from hearing the word. They just need to learn 11 and 12 because you can't know 11 and 12 by just hearing them.
And then can move them to the More Game.
If they don't get the counting story, need to re-tell them the next day. Don't move on to the More Game until they understand how to
Important components:
A long time ago,…
First we count and make up numbers
Then pick some and use numbers over
Use numbers in different place with different values
Go to chart and start on 13
Use the blocks, verbal, and visuals
Have kids make numbers and talk it through…make 3-ty four. Have them count what they built so they understand that number is that value.

Figure 10. The agenda notes of the Base-Ten Story from computer records of summer seminar.

Not only was the kindergarten curriculum designed for the whole school in the Renaissance, but we also worked closely together to build each unit and their tests. The Renaissance documents reveal much discussion of the units and their placement according to grade levels. It soon became clear that while the addition unit was divided into two sections to make the tests more manageable for first graders, it did take a full year to master. With adding this kindergarten unit, we again reclaimed one of the main purposes of the mode: setting out a path of meaning-making for even our youngest students starting out in mathematics.

Scope and sequence for the rest of the algorithm units. As we had done with the kindergarten unit, we continued to formulate our units together for all the grades by creating unit checklists to be used in our student rubric notebooks. We created these notebooks in the Renaissance, but the exact year was not found. The earliest mention of the notebooks was in a State Department evaluative report document, dated 2006. The Rubric Notebooks are a collection of rubrics and checklists of all the school modes and projects for each child to collect examples of their work that demonstrated passing out of the units they had started at the beginning of each year. These notebooks travel with the student when they change teachers as a way to communicate to the new teacher the student's previous journey and provides documenting evidence of how well they did that year. The Rubric Notebooks are not cumulative, however, and are begun anew each year with the new year's level of rubrics and checklists. (A checklist is a list of content, while a rubric is a list of content written with performance evaluative grade level language.) Following, in Figures 11 and 12, are examples of the multiplication and division units' sequence on their checklists:

Multiplication Checklist

- Single digit multiplication
- Vocabulary (factors, array, product, zero property, identity property of one, commutative property, square numbers, cubed numbers, base, exponent, power)
- Build to solve and draw double digit multiplication problems
- Estimate answers to double digit multiplication problems (understand place value)
- Build to solve and draw double digit multiplications without regrouping using the "Great Shift"
- Build to solve and draw double digit multiplications with regrouping using the "Great Shift"
- Multiplication facts through five
- Multiplication facts through ten
- Build to solve and draw triple digit multiplications without regrouping using the "Great Shift"
- Build to solve and draw triple digit multiplications with regrouping using the "Great Shift"
- Decompose double digit problems into 4 parts
- Decompose double digit problems into 2 parts
- Solve multiplication problems of any length without blocks
- Mental math using multiples of ten – zero rule
- Missing numbers
- Order of operations
- Word problems (rate, combinations, exponential growth, area, perimeter, volume)

Figure 11. The Multiplication Checklist for Algorithmic Math Group Mode from the Family School *white notebook* of important school documents.

Division Checklist

- Show the relationship between multiplication and division (fact families) using numbers and symbols
- Vocabulary (dividend, divisor, quotient, prime number, composite number)
- Write equations to represent division principles (5/5 = 1; 8/2 ≠ 2/8)
- Build and solve division problems without remainders
- Build and solve division problems with remainders
- Solve division problems without blocks – side by side and algorithm
- Solve single digit division problems quickly – super shortcut
- Mental math with the division rule
- Word problems (single and multi-step, averages, rates, conversion)
- Missing numbers

Figure 12. The Division Checklist used in the Student Rubric Notebook from the Family School *white notebook* of important school documents.

At this time, the teachers and I still maintained the sequence of mathematic groups with one algorithmic group being expected to be accomplished each year, completing the percentage unit by grade six, to be followed up with the geometry and algebra in grades seven and eight.

This, in fact, worked with students we had from the beginning of their education, but many of our students came to us in the upper grades, and we were not able to get them on track right away. We feel it is better to have them retrace their steps according to our placement test, and move forward appropriately. In the beginning of the Renaissance, we focused our efforts towards mastery and moving the student forward according to the student's pace. However, by 2003/04, when the NCLB pressure was introduced, we knew that while making meaning was important, it was not as important as mastering content standards. Often content standards can be reached without much deep conceptual probing for which Family School teachers strive. With such a strong push to get all students to proficiency by 2014, we understood how misgivings about our mathematics curriculum would be lodged if our students could not perform as well as the rest of the district on standardized tests. At the time, the district had adopted two curricular mathematics texts to be used; we had not been made to make a choice because our test scores were already higher than the rest of the district. We did not want to lose ground in our scores, and began designing mastery unit tests for the entire school, and training the staff and parents in this effort.

Parent math classes. Every fall since the centralization of the school, being the administrator, I have taught parent math classes, each night a different algorithm. Parents sign up in advance and we work for two and half hours together going through each of the units. I mimic the same questioning technique with the parents that we use with the students. It is essential that the parents understand and use the questions we have chosen, so they can help their students. Over the years, I learned that the questions don't always come as easily to parents or teachers as they came to me with my fourth-grade students that first year using the Mortensen Math program. It soon became apparent that some folks needed to use the specific questions we designed. Parent booklets were written and copied for parent use, but this didn't occur until the end of the Renaissance. Perhaps, by then, the pressure from NCLB was so great that we knew we needed the parents to help, and the only way they could was to have the appropriate resources. These booklets transformed the parent and teacher training of this curriculum.

Another document reveals a different way the teachers and I worked with parents in the Algorithmic Math Mode to assist our teaching in the classroom. It was the *Math Home Study Checklist,* which prompted parents to communicate with us the level to which they had worked with their children on their mathematics homework. This short document was given to every student with every homework assignment. Figure 13 presents this 2008 document, for which the original version cannot be found because the properties indicate that it was created in the year 1970, which would have been impossible, obviously the computer had a date and time issue. The document properties did reveal that it was modified in 2008, which would suggests that this is similar to when the original or a completely new document would have been created.

Math Home Study Checklist

____ Checked and Corrected and can move on to next concept

____ Checked and Corrected but……

____ Accurately calculated, not conceptually understood

____ Needed another explanation and OK now

____ Needs another explanation and another assignment

____ Needs more practice problems

____ Shows no understanding

Parent Signature____

Figure 13. The Math Home Study Checklist for parents to fill out after working on math homework from my classroom files.

Math interventions. Not until the NCLB pressured us to have students proficient at grade level did we feel that it was important to work harder and faster than we had before to *catch up* the students who came to us from other schools and were well behind their grade level in mathematics. We had always felt that students needed mastery before moving onward in this linear approach to mathematics, though our Mobius Math Mode was teaching us to think about curriculum more spirally at this time. It wasn't until interventions (extra instructional time allotted to students who were not proficient) were required by the State Department that we felt the pressure that students needed to be on grade level. It was difficult to ascertain what was grade-level for us, since our scope and sequence didn't match the state standards. We decided that our school grade level expectations would be considered grade level, and that our students would be solid in algorithms; the rest of grade-level material we left for the Mobius Math Mode.

Our interventions for the Algorithmic Math Mode have been a wide variety of strategies throughout the years. In 2007, Family School created its own interventions by creating what we called AHS (pronounced OZ, so the students thought it interesting to go to OZ), which stood for Assisted Home Schooling. This was an afterschool session offered by teachers to help bring up students in their mathematic groups. Different teachers offered to teach a certain mathematic group and were assigned students from across the school to attend the intervention session. The parents really liked this design, but many of the teachers felt that it was more successful for some students than others, and it was often difficult to get all teachers to participate evenly. This was not a function of lack of desire. In any small school, it is difficult to prepare new teachers quickly enough for them to be the best intervention teachers. When our initial intervention design was discontinued, we moved to requiring intervention students to have a longer mathematics time in class, meaning that a different activity would be cut short for these students, for example, choosing between mathematics or writing brainteasers or workshops. This marks the transition to the Age of Exploration. It is interesting to note that the increased pressure of NCLB moved our community experience of intervening collectively with our struggling students, to making it the responsibility of each teacher to do their own interventions. It was clearly a call for teachers to step up and be accountable for each and every student in their classrooms.

What I infer from this is that becoming a centralized site was key to our teachers coming together to collaborate. Having had a solid foundation in making meaning with students continued as teachers worked together to develop a consistent curricular mode throughout the school. Creating documents that enabled the teachers to fill out the algorithm units was also important in building meaning of the function of the algorithm.
Creating stories, manipulative games, algorithmic scope and sequences, and modified interventions for students who were not proficient were developed to strengthen the Algorithmic Math Group Mode's value.

What you will see next is the analysis of the documents of this mode as we transitioned from the Renaissance to the Age of Exploration. There are also a few nodal documents that are highlighted to explore key changes that occurred during this time of transition.

The Themes of the Renaissance Transitioning to the Age of Exploration- 2007-2010

It is interesting to note the iterative coding of the documents for this mode as the teachers and I transitioned from the Renaissance to the Age of Exploration. The bulk of the documents (Table 10) were the written curricular materials for teachers and parents, like the unit tests and parent booklets. They clearly revealed all the themes of this mode, a focus on: 1) scope and sequence, 2) improving teaching, 3) evaluation, and 4) purposes of the unit. The other documents (Table 11), while fewer in number, do reflect a pattern of being more concerned with developing the scope and sequence and improved teaching rather than focusing on the evaluative aspect or the purposes. Perhaps the purposes were self-explanatory in the scope and sequence and instruction, but it wasn't until NCLB was initiated that we began to move into action designing and redesigning tests for the units. We also clearly needed to improve our teaching of the units with both teachers and parents, which resulted in the booklets.

Table 10

Number of Documents Analyzed that Reflect Recurring Themes for the Algorithmic Math Group Mode

	Focus on Scope and Sequence of Content	Focus on Improvement of Strategic Instruction	Focus on Evaluation	Focus on Purposes of the Mode	Total (excluding tests)
Agendas	6	5	0	0	11
School Docs	0	2	1	0	3
Recent Docs	0	1	0	2	3
EPSS's	2	5	2	0	9
School designed tests and parent booklets	(28)	(28)	(28)	(28)	(112)
Total	8	13	3	2	26

Table 11

Number of Documents Reflecting Themes of Scope and Sequence and Improvement of Instruction

	A Focus on Scope and Sequence of Content	A Focus on Improving Strategic Instruction	Total
Renaissance	3	4	6
Age of Exploration	3	1	5

Important documents of this era to set the tone for studying our teaching. Not only training our teachers to learn our curricular units more quickly became an important task to oversee and fine-tune the teaching of these fifteen-minute lessons. For this, I created the Math Lesson Observation Checklist to help teachers pace how to get through the mathematic lessons quickly using a strategic instructional method which would lead toward our purposes. In Figure 14 below, the document points out this pacing but also suggests how to teach from the known to the unknown by using the questioning technique.

Math Lesson Observation Checklist

Room Scan:

- ❑ Different Algorithm groups grouped around the room
- ❑ Students on task
- ❑ Students working on level appropriate work

Student Preparedness for Group Lesson

- ❑ Student has homework
- ❑ Student has necessary resources for lesson
- ❑ Student is ready for a lesson

Algorithm Lesson

1) Homework check

- ❑ Teacher conducts a student understanding check of the previous lesson
- ❑ Teachers uses a higher end problem
- ❑ Teacher is transparent about passing one lesson to move the next or not
- ❑ No time spent on parent responsibility of homework

2) Lesson on a new level problem

- ❑ Teacher understands the math content appropriately
- ❑ Teacher checks for student understanding of all students
- ❑ Teacher has students write as they learn
- ❑ Teacher uses the questioning approach

3) Assigning homework

- ❑ Homework is teacher-generated
- ❑ Homework strategy is explained and demonstrated
- ❑ Teacher checks for student understanding of homework assignment

Figure 14. Math Observation Checklist used for teachers to pace math lessons from the Family School *white notebook* of important school-wide documents.

This checklist was a powerful tool for teachers to develop the rhythm for teaching these mathematic groups quickly, yet powerfully. In a multi-age classroom, the range of mathematic groups can be quite wide, and it is essential to see each group everyday because of the quick pacing of the groups. It was most essential for teachers to learn that it was fine for the students to learn the algorithmic meaning of procedures through *little bites* of meaning-making, practice with that meaning (at home), and returning the next day ready to move forward. We were not going slower, but we were teaching in smaller chunks. Meaning-making made and students practiced good habits of thinking within these smaller steps.

Also in 2008, a series of other pacing documents were created to help teachers feel a natural rhythm to their strategically designed instructional lessons in such a simplistic manner as to be able to make each part of their lesson plan more powerful. Figure 15 shows the general document introducing the Before, During, and After basic structure of any lesson plan, and Figure 16 narrows the focus of this rhythm to small group instruction, which was important for this mode. Finally, Figure 17, specifically defines the characteristics of each part of the Before, During, and After lesson structure for the main modes at Family School. In Figure 17, the Algorithmic Math Group Mode is highlighted in yellow.

Before, During and After Lessons
Before
Example or Sample problems—can include Simple Simon
Brainstorming and Organizing
Asking Questions
Analyze Errors/ Pick a Focus
Read Directions (know what to research)
Go over Terminology (comprehension)
Have Student Explain What They Know
Quizzing
Budgeting Time
Model External Speech,
During
Conversation/Discussion
Direct Instructions (have student take notes)
Parallel Instruction
Peer Editing
Extreme White-boarding
Make up a test or quiz together
Ask questions to get at conceptual understanding
Use problem-solving strategies
Give expanding problems
Be Metacognitive Police
Help child to move external speech to internal
Partnering teaching
Objective P.O.V.
After
Reviewing/editing
Logging and Looking for Errors
Finding What's Right
"Tangent" lessons
Finding what worked or didn't work
Restructuring / redirected/ salvaging
Authentic discussion of discoveries
Ask for Evidence
Practice for tests
Strategizing—organizing procedures
Criteria check
Go over teacher comments
Doing Comparative Study
Celebrate
Generalized applications
Presentation

Figure 15. Before During, and After Lesson Examples from computer records of staff meeting notes.

Before	**During**	**After**
Establish group objective Predict group needs Set group norms Establish group scaffolding technique Encourage group empathy Encourage group memory Enlist generation of group prior knowledge Encourage peer assessment of prior lesson Encourage peer sharing of homework Establish material ritual for needed items Establish predictability habits	Establish value of listening to each other's learning Establish good habits for allowing others to learn Teach students to learn how to wonder about other's learning Establish sharing about how students learn themselves Establish group goals of everyone achieving objective and allowing them time and support to get that to happen Establishing multiple methods of accomplishing the same goal Learning to try on someone else's "way" Learning to articulate one's own thinking Encouraging students to take on the teaching role	Encourage acknowledgment of support Summing up all pieces of lessons to one whole concept Capturing assignments using the group whole concept Encouraging students to set goals and assignment expectations Encouraging students to set up support system methods for each other outside of school Encourage students to articulate lesson points Assign student(s) to write homework assignment

Figure 16. Before, During, and After Lesson structure for small group instruction from computer records of staff meeting notes.

Modes	Before	During	After
Sophie	Making clear the "whys" Convention why's Pre-error analysis Connection and context Predictions Individual and group	Spelling activity	Analysis at end of lesson Errors in sentences Improvements witnessed during Acknowledging individual stories and making them part of the class's story.
Math Group	Questions about the homework. Check to make sure that kids recall the lesson. Make sure that kids can question. Connect to the new lesson.	Look for parts of lesson creating difficulty for student, going back and prepping student for at home practice	Homework designed to address pitfalls witnessed during lesson
Reading Journal	Set up lesson (afters become todays' befores) Yesterday's lessons (on board) available to students as they begin the new day's lesson	Read each kids looking for how their writing is connecting to the lesson of the kid's reading journal rubric or processes	Take all individuals in their group and what they're working on Making clear what all are learning
Brainteaser	Really is not a before. (Purposeful omission of a before). Day 2's before is from day 1 – remembering what got figured out on day 1.	Huge amount of questioning (teacher and students), offering opportunities for in-depth investigation,	At-the-board examination, taking back and connecting to the conceptual understanding, facts,
Workshop	A pre- that reconnects kids to the conceptual question and the day's content	The activities that are mostly monitored, directed, guided by teacher	Coming back together to determine what questions are left for tomorrow, what knowledge was gained
Projects	Keep bringing the lesson (conceptual) to class	Help during individual working during class Keep bringing the lesson (conceptual) to class	
Inquiry	Keep bringing the lesson to the class	Help the individuals during the class Keep brining the lessons back to class	

Figure 17. Before, During, and After Lesson structure for the main modes of Family School from computer records of staff meeting notes.

What you see here is that as the units became designed with more understanding and connections. Students developed supporting math concepts to the algorithms. Also, the teaching of the mode became more uniformly structured. With multi-age classrooms, an average of four to six math groups occur in each classroom. A 15-20 minute lesson is about all the time each group receives, therefore teachers had to learn how to maximize the mode's teaching strategies. The documents were made to assist teachers think, plan and implement their lessons more effectively.

What you will see next is how the collaboration also involved evaluating the effectiveness of our teaching. It was clear that our first attempt to write our own tests for each unit was not reflecting the meaning-making emphasis of our teaching, so we rewrote the tests.

Rewriting our test documents. Another important change that began in the transition out of the Renaissance into the Age of Exploration was that we began to question our old unit tests, feeling that they were more traditional in nature and didn't reflect the conceptual understanding that we were teaching. In my observations of teachers in their mathematic groups, I could see teachers moving quickly through the concepts of the traditional algorithms. While it was always our intent to connect to the traditional algorithms, the point of the conceptual understanding would be lost if this was not connected carefully; it would be an easy omission for a teacher to make when the test only tested for the traditional algorithmic use. Together, all of the teachers worked to rewrite the tests to reflect each concept needed in each unit. If conceptual understanding was required in the test, then the teachers had to make sure to teach conceptual understanding for mastery. This school-wide task ushered in the Age of Exploration.

Figure 18 is an example of one of the fraction tests to demonstrate how we integrated traditional use of fraction algorithms with conceptual means to determine comprehension. Highlighted in yellow are items for assessing conceptual understanding; in blue are items assessing the traditional accuracy of algorithms.

Score:

Fractions I Unit Test A

Name ________________________________ Date _______________

ORAL COMPONENT

1) Student can verbally explain **and** show with the fraction tiles division of $\frac{2}{3}$ and $\frac{4}{12}$ in the following ways:

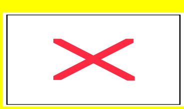
☐ bigger into smaller

☐ smaller into bigger

2) Complete each mathematical sentence with <, >, or = in each ○ .

a) $\frac{3}{5}$ ○ $\frac{5}{7}$ b) $\frac{9}{20}$ ○ $\frac{1}{3}$ c) $\frac{16}{18}$ ○ $\frac{24}{27}$

3) Write these fractions in order from greatest to least:



____ , ____ , ____ , ____

4) Write three fractions between $\frac{1}{3}$ and $\frac{4}{5}$: ____ , ____ , ____

5) Explain in words **and** with a picture why $\frac{5}{6} = \frac{15}{18}$.

6) Define in words, draw **and** locate the given fraction on a number line.

$$\frac{2}{9}$$

Define:

Draw:

Locate:

7) Use the given division problem to explain why you invert and multiply when you divide fractions.

$\frac{7}{10} \div \frac{2}{5}$

8) Draw the given multiplication problem. Explain the relationship of the product to the factors by labeling your diagram.

$\frac{1}{4}\ x\ \frac{1}{3}$

9) For the given problem, find the Least Common Denominator using the Greatest Common Factor method. Then, solve the problem. Your answer should be in lowest terms.

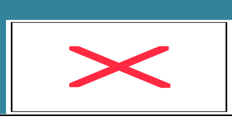
$\frac{2}{8} + \frac{3}{6}$ LCD=________ Solution =_______

10) For the given problem, find the Least Common Denominator using the Prime Factorization method. Then, solve the problem. Your answer should be in lowest terms.

$\frac{10}{84} - \frac{3}{132}$ LCD=________ Solution =_______

Solve each of the following in lowest terms.

11) $\frac{5}{12} + \frac{3}{12} =$ ____________ 12) $\frac{23}{24} - \frac{11}{24} =$ ____________

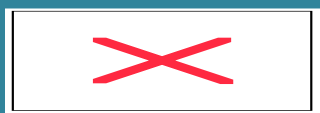
13)  ____________ 14) $\frac{31}{48} - \frac{5}{16} =$ ____________

15) $\frac{7}{24} \bullet \frac{32}{35} =$ ___________

16) $\frac{12}{56} \div \frac{18}{38} =$ ___________

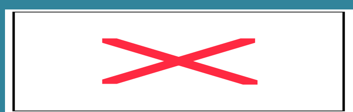
17) $\frac{5}{11}\ x\ \frac{22}{25} =$ ___________

18) ___________

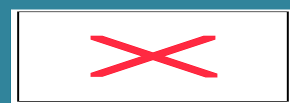
19)  ___________

____

20) $\frac{4}{9}\ x\ \frac{18}{19} \div \left(\frac{2}{5} - \frac{6}{15}\right) =$

21)  ___________

____

22) 

23) Write **and** solve a multiplication word problem using the following fractions.

$\frac{3}{8}$ and $\frac{2}{3}$

24) The animal shelter said that 200 of the animals adopted by families this year were younger than 5 years old. Of this group, $\frac{2}{7}$ were less than a year old; $\frac{1}{3}$ were between 1 and 2 years old. What fraction of this group was older than 2 years?

25) There are many beautiful flowers on the school campus. Two-thirds of the flowers on campus bloom in the Spring. Five-eighths of the flowers that bloom are yellow. What fraction of the flowers on campus are yellow flowers that bloom in the Spring?

26) Natalie has to finish $\frac{1}{3}$ of her math worksheet before gymnastics at 2:15 pm. By 1:00 pm, she had finished $\frac{2}{5}$ of what she was asked to complete before gymnastics. What part of the entire math worksheet does she have left to

finish before gymnastics?

27) The area rug in my living room is $\frac{3}{8}$ of a square yard. One side of the area rug is $\frac{2}{4}$ yd. long. What does the other side of the rug measure?

28) Emily has three pieces of red ribbon measuring: $\frac{1}{2}$ in., $\frac{1}{12}$ in., and $\frac{1}{6}$ in. She needs $\frac{2}{8}$ in. of ribbon to make a barrette. How many barrettes can Emily make?

Figure 18. Example of Fraction Test document for Family School.

As we transitioned out of the Renaissance, I can infer that our days of creative collaboration around meaning-making moved towards becoming efficient in our teaching for mastery. What you will see next is a focus on our middle school math level algorithms. And in order to accomplish more content in the older grades, the sequence had to progress more quickly in the lower grades. Once again, we reset our desired grade levels for each algorithm. We still encouraged students to advance through them at their own rate, but we tried to meet the new pacing of the lower grades to allow more time for the older grades in their algebra content.

The Age of Exploration- 2008-2010

In this era, the sequence was changed to: Kindergarten- Anti-counting unit, First Grade- Addition, Second Grade- Subtraction and ½ of Multiplication, Third Grade- Second ½ of Multiplication and Division, Fourth Grade-Fractions 1, Fifth Grade-Fractions 2 and

Decimals, Sixth Grade: Percentages and Geometry (subsuming ratios), Seventh and Eighth Grade- Algebra.

Our new versions of the algorithmic unit tests were finalized in 2008. The NCLB pressure of teaching toward mastery forced our hand to make sure that the teachers and I were testing what we were teaching. We also felt the need to push back, making sure that our mastery represented conceptual understanding and not just a surface level understanding of the many standards required of the NCLB benchmarks. We also renegotiated our sequence to ensure that our students would not be penalized for our curriculum not being in alignment with the testing company.

Emphasis on algebra. From the Dark Age into the Renaissance we knew there was a connection of the elementary algorithms to algebra. The Mortensen Math program had shown us how kindergartners and first graders could manipulate x's and x^2's with addition and subtraction. It taught the basic operations of addition, subtraction, multiplication, and division of whole numbers right alongside performing these operations with algebra terms as well. When my teachers and I saw the connection algebra had to basic mathematics, we knew how important it was to introduce algebra many years before high school. This was also borne out in the new standards that were introduced by the NCLB raised expectations. We had already been introduced to this idea by the time these standards came out.

Chazan (2008) responds to the conflicts that arose from teachers and community when this shift towards learning algebra in the early grades began to happen. He suggested that mathematics teachers were "at a locus of cultural conflict between those who insist on raising standards and those, not usually active participants in the policy debates, who do not understand why what was good enough for them is not deemed sufficient for their children"(p. 30). He recommends that "we need tools and arguments that will help us explain to our students and to their families why our society is requiring this mathematics of them"(p. 30). The Family School teachers did not face these issues with parents, but it was an adjustment for our primary teachers to be required to think algebraically. They simply asked for suggestions from our middle school teachers as to how to add algebraic concepts to their curriculum and which ones would be the most beneficial with which to begin.

Cai, Lew, Morris, Ng, and Schmittau's (2005) research on various international curricula and how they utilized algebra concepts in the elementary grades reveal many differences, but most surprisingly was the lack of symbolic representation in the American curriculum. China and Korea used this abstraction very early on, while the Russian curriculum only used letter symbols to be represented in applied problems.

Our teachers began small by playing around with the relational value of the equal sign and writing some simple mathematical sentences with variables. Algebraic understanding for Family School had its origins in our very first tests for the addition and subtraction units. From the beginning, we had a notion to create the connection of arithmetic to algebra. Figure 19 is an example of a problem taken from one of our newer tests, because the older tests were destroyed, but this concept has remained constant over the years.

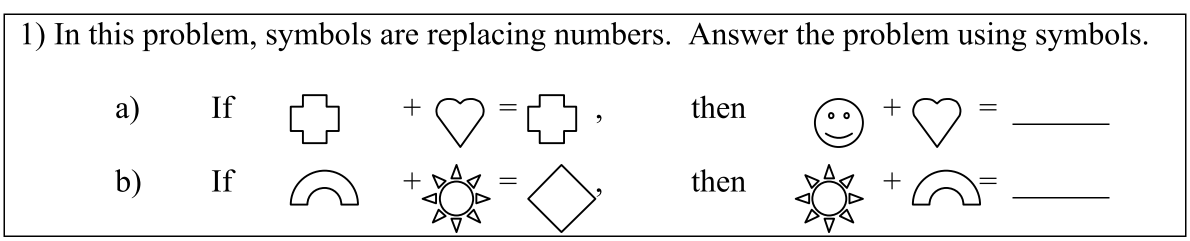


Figure 19. An example of a symbol problem on the addition test from the Family School addition unit test.

It is interesting to note how the differences among math curricula around the world reflect the view that there is no single, right sequence. As a school designing its own curricular scope and sequence, it is reassuring that there are many orders and priorities that can be justified, but in the end, we have learned that it is not so much about the *what's* of what is taught and their order, as it is in the *how's* of what is taught that makes the difference. The *what's* are the curricular decisions in choosing what mathematical content is taught, scope and sequence. The *how's* of mathematics are the mathematical curricular decisions in teaching students metacognitive mathematical practices that cannot necessarily be tested as content, yet, more valuable that students should learn them in order to be able to manipulate mathematical content more adeptly.

However, the algebra strand caught our attention. Because of the origins of this mode stemming from Mortensen Math and its commitment to algebra, my staff and I had always felt this to be an important connection to our algorithmic mathematic groups. While this connection existed, it was tenuous at best and really depended on the teacher's level of skill with mathematics. With NCLB bringing algebra to the forefront, we made an effort to solidify our curriculum to this strand. NCLB even required algebra to be introduced earlier and more effectively at the middle school level. The State Department of Education required the Educational Plans for Student Success (EPSS) documents to state how middle schools would address this need. Prior to this expectation, we had used a variety of algebra books to teach algebra, but we collaboratively chose a particular text as this algebra strand became more highlighted. Because of this EPSS requirement, we went into full preparation for the chapters of this newly-chosen text.

The text chosen was actually a college textbook used to teach the 120 level of algebra at our local university at the time. We chose it because it covered all the same content of the algebra texts written for high school, and it covered it in a much faster pace and at the bigger-picture level conceptually, so that students could see how the scope of algebra was held together. The body of knowledge seemed attainable and not something that would take years to learn. Our students who completed most of these chapters had a solid understanding of the algebraic concepts needed for each of these steps of algebra content. Many of our students took algebra placement tests upon entering high school and qualified for honors algebra one classes, or even skipped that course and moved to algebra two courses. Some of our students, if they came to our school in the upper grades, had only begun to get through chapters one and two of the book, which still set them up for success in a regular algebra high school program. This text has remained with us from the Age of Exploration until today. Our students enjoy its pace and rigor.

One aspect of our mathematic groups that changed upon mastering the decimal group is that our use of manipulatives and the questioning techniques lessen. Once the students have graduated from all the basic algorithms, my middle school teachers and I felt they needed to develop a sense of how to use textbooks more appropriately for habits of study. The text chosen for percentages introduced the students to reading about explanations for formula usage. The geometry text was a self-paced text with the answers in the back of the book. The algebra book, being a standard college text, had a fair amount of explanation of new concepts before given practice exercises with the answers to the odd problems in the back of the book. At each level, we raised the expectation for students to read the material before a mathematic lesson and also check the answers of their homework practice problems.

The students were required to ask us questions rather than us asking them. They were given quizzes along the way to gauge their mastery of concepts before being given chapter tests. Manipulatives were used as a resource for explaining conceptual barriers to the material but not as a requirement. By the time most of our students reach algebra, they have developed a solid, abstract, conceptual base to mathematics. However, we have also accumulated algebra tiles, Hands-on Equations scales, and geometric figures to aid in the teaching of these units.

While my middle school staff and I focused our attention to teaching algebra through an algebra textbook, we also took note of the research about Common Core suggesting that high performance in fractions led to increased performance in high school algebra. Booth and Newton (2012) identified the document that mades this claim to be made by the National Mathematics Advisory Panel (NMAP, 2008), but have found very little research to support such a claim. Their study found that increased knowledge of fraction magnitude did correlate with having more skill in algebra, but they cautioned that much more research is needed to understand this relationship. Others, such as Wu (2008) and Brown and Quinn (2007) suggest that whether there is a connection or not, depends on how fractions are taught. They state that fractions can be taught deliberately to have a connection to algebra, which would enhance students' understanding of algebra but also enrich their understanding of fractions. Whether or not the mastery of fractions is the *doorman* to algebra, which is the *gatekeeper* to success in college, we were determined to have deep understanding of both topics. We heightened our awareness of teaching algebra and focused our attention on the assessment of fractions to be more able to reflect what students understood.

What I infer from this is that it was essential to engage my teachers in choosing the algebra text. It was also important to train them to understand how to relate algebra back to all the early work they had done with younger students. The text choice continued our commitment to high expectations for meaning, since we chose a higher-level book that presented units more quickly and in bigger concepts than the usual high school texts. By retracing our algebra knowledge to our elementary algorithmic knowledge, the teachers were more excited to see the continuity and share this algebra connection with the lower-grade teachers.

What you will see next is how the mode evolved as we moved to our new building and the 100% proficiency goal of NCLB quickly approached, only to be interrupted by the initiation of the Common Core.

Reconstruction- 2011-Present

Once our staff moved to our new school building, Desert Willow Family School, designed for our particular instruction, we pulled together even more than when we were grouped together in portable barracks. We had been doing intervention programs for our students in the old building, and these interventions had changed through the years, but once at our new site we changed them again. Whether it was the result of the continued pressure to bring students to grade level proficiency, or whether it was the result of our feeling more cohesive in our new building, or some combination of both, one new plan was to bring together a group of students from across the school who were not able to pass the multiplication unit. I taught this group. However, the teachers wanted to attend with their students in order to learn how to teach the algorithm more conceptually. Once a week, we met for an hour and a half with the students and then spent another half-an-hour to an hour debriefing the instruction with the teachers. This was an amazing learning opportunity for us all!

Intervention insights. This topic could be a thesis in itself, but, to make a long story short, we discovered three very important points about teaching our lower students. 1) The teachers had accepted much more superficial understanding of each of the concepts laid out by the curriculum than they would have accepted from higher-achieving students. Because it does take longer for struggling students to achieve a deep level of understanding, the teachers were fearful that it would take too long to complete the unit if they went as slowly as these students required. 2) The teachers *enabled* the lower students in their mathematic progress by giving away clues to right and wrong moves with the slightest movements in their facial features or body language. These students were masters at reading their teachers' body language to alter their answers or work, even using their peripheral vision of the teacher as she watched the student work through problems. We had to move the teachers out of the students' line of vision in order for me to accurately gauge what the students knew and what they didn't know. I was able to develop a dead-pan look as I watched them work, and I didn't give clues about what is right or wrong in my questioning. Sometimes I questioned for further explanation when students answered correctly; sometimes I did it when they were wrong, so they could find their error and correct it. 3) Students took substantially longer to learn concepts only at the beginning of the unit; these students, once solid in the underlying basic concepts of the algorithm, actually picked up speed in the latter parts of the unit. This knowledge reassured teachers that it was not going to take *forever* for students to complete the units. In fact, taking extra time at the start of a unit actually shortened the overall time required for mastery. In the past, when hurried through the units, these students often did not master 85% of the material and would be required to repeat the unit, which ultimately took more time than it would have taken to complete the unit at a slower pace with complete conceptual understanding in the first place. Ultimately, this intervention helped us to come up with a quote we have repeated many times that guides us greatly today, though it has not been written out as a document: “Each student deserves to understand each step of the lesson as solidly as your highest student understands it, no matter how long it takes them to get there. They should have the same confidence in their explanations as the higher students.” In fact, we have proven this to be true over and over with other intervention students. When students finally demonstrated solid conceptual understanding through their explanations of their work or use of the manipulatives, it is almost always accompanied by a huge smile and a confident tone in their speech patterns. They can’t help but feel the accomplishment of their comprehension and then we know its time to move on to the next step.

This does not make the notion of higher and lower students irrelevant, however. It is essential that students be taught in a variety of groups. Our algorithm groups are ability-based, so that students have a solid foundation in each group before moving into the next. Yet, even in the same ability group, one student does not always learn at the same rate as another. No matter the individual rate, each student should develop the same deep level of understanding of the concepts. Also, since the Algorithm Math Group Mode is only one of our math modes, our other modes provide heterogeneous groups for students to learn math with all levels of students. We have designed our instructional modes to meet the varying needs of students. Some modes will teach right to the student’s level while some modes are designed to have all student levels work together. It is always essential for teachers to know each of their student’s abilities in order to provide the appropriate support for the various types of groups while reaching for mastery of a subject. Many teachers’ mistakes were around expecting less of their lower students' quality of responses, without even realizing they were.

The political pressure pushed the teachers and I to study ourselves in very important ways. For instance, until we studied it, we didn't realize the very meaning we were emphasizing as the main purpose of the Algorithmic Math Group Mode was lost on those students who needed it the most. Since the pacing for the Algorithmic Math Group Mode is fast, teachers sacrificed depth for the lower students. They didn't realize lowered expectations prevented them from seeing how this mode is structured for these students to develop habits of conceptual thinking in small steps. Teachers learned that the questions required in the algorithm lessons might need to be broken down into smaller questions and even then, the teacher might also need to request for more open-ended explanation from students to make sure the comprehension is mastered. The algorithm groups could easily start out with five to six students in a group, but because of the varying rates of learning the material, it could easily be split into two groups. As we learned how flexible we needed to be about grouping, more groups began to emerge in our classrooms, which become unwieldy. We are still committed to having multi-age classrooms, but having wide grade spans makes it almost impossible for our 50% program to get through all the mathematic groups each day. To solve this problem, to move teachers strategically next door to each other, with the shared teacher office in between both classrooms. Paired teachers will have a four-grade span between them. This way, the teachers can use each other to group students more flexibly and maintain the preferred limit of six groups. If some students in the lower-grade class needed a higher mathematic group, they could go to the neighboring teacher's room with the next two higher-grade spans. And if the teacher of the higher grades had a student who needed more time in a lower group, that student could join the other teacher's. We are hoping this enables us to provide the pacing students need.

What I can infer from this section is that the pressure of NCLB did bear down on us heavily. Though we did not abandon our Algorithmic Math Group Mode, we had to continue to evaluate our teaching strategies and designed curriculum of the mode to make sure we were reaching all students. I was able to demonstrate that students who were not passing our algorithm groups were able to pass with more effective teaching within the mode. More careful attention paid to the quality of students' answers explanations of the manipulatives was the key to this improvement.

📖 The pressure for our students to perform for NCLB testing accountability issues had both positive and negative impacts on the Algorithmic Math Group Mode.

☺ It certainly influenced our desire to improve our instruction of the basic algorithms for all levels of students. We more closely scrutinized our strategic instruction of our struggling students in mathematics. We redesigned our math tests to reflect our students' meaning-making and upheld our desire for 85% mastery on our math unit tests. Our linear scope and sequence of the content in this mode continued to support students to learn the basic algorithmic procedures in small-steps while supported by manipulatives to reveal the meaning behind the procedural moves.

☹ However, the pressure of NCLB also influenced teachers to see this mode as more important than other modes with purposes more akin to how mathematicians practice and with content less prevalent in high-stakes testing. Consequently, this mode's importance enabled teachers to spend more time on this mode, which eventually led to eliminating some of our other, more non-traditional, math modes.

Figure 20: NCLB story revealed from the Algorithmic Math Group Mode

What you will see next is how more analysis will be needed to be done with our algorithmic curriculum and instruction when Common Core is implemented in our school district.

Common core alignment. Because the Common Core curriculum required more conceptual understanding than NCLB, my teachers and I took another look at our scope and sequence. The Common Core curriculum suggested that fraction expertise in the early grades was a good predictor of algebra success in high school. I had already noticed that our fraction units were the most difficult units for students to master and had begun to relate to teachers and parents how important many fraction concepts (like multiplying by the name of 1 to make equivalents) are to mastering it in algebra. Between the Mortensen connection and our own, we, again, had set mathematic standards well in advance of the required standards being set by the Common Core curriculum.

All along our algorithmic mathematic program was shaped by these four specific elements: use of manipulatives for conceptual understanding to accompany the procedural knowledge; questioning techniques to move the learning from what the students knew to what they didn't know; early algebra connections at the elementary level; and high expectations for mastery of fractions because of their connection to algebra. The staff and I checked our curriculum to see how well it aligned with the Core's expectations. By splitting up our teachers to work in multi-grade- level groups, like primary, intermediate and secondary grade levels, they evaluated the Core documents. Because the Core aligned all the strands beyond just algorithms, the final analysis for our teachers impacted both the Algorithmic Math Group Mode and the third mode to be discussed, the Mobius Math Mode. It is important to note for this mode, however, that the Common Core encouraged us to change our scope and sequence of the algorithms one more time. We had been covering our fraction unit in two years, which we collapsed to one— the fourth-grade year. While we realized this was a huge expectation, we decided that the scope of fractions presented in the Common Core, which taught both fractions smaller than one and larger than one at the same time, created a problem for our fourth-graders. Since fractions would be introduced to them in the Mobius Math workbooks, we decided we should probably do our best to cover both in the same year. We figured if the unit took longer and went into the fifth-grade year, it was better than not having started to teach it at all. This change will shift all the other units to earlier grades, making it possible for the algebra unit to be started in sixth grade.

I have already seen a few fifth-grade students begin the algebra text, which has been unheard of in the past at our school. There is no rush when this happens and the middle school teachers and I move through the chapters slowly and conceptually. We have never been able to get any of our students through this whole text (which was not even accomplished in the 120 college course either), but I'm assuming these few students who are getting started on it so early will make it through more chapters than any other students before them.

What I can infer from this section is that because my staff and I worked so hard to develop a mode that focused on meaning –making and commensurate instruction, that the transition into Common Core was not very difficult. A few adjustments in sequence were made, but for the most part, this mode has stayed in tact throughout the years of the school.

What you will see next is an iterative analysis of the themes in evaluative documents for Family School, which will reveal an interesting pattern that emerged when teachers were given the opportunity to design their own curriculum. The nodal documents used for this are the documents written for our state department. Though they changed from Program Reviews to Educational Plans for Student Success, they outline our strategies and analysis of our data throughout the years to reveal an interesting pattern about our disclosure of this mode.

Theme analysis. The Algorithmic Math Group Mode, closely aligned with traditional mathematic programs, remained a main ingredient in these state documents. Seven documents were found and analyzed. The 1996/7 Program Review only mentioned that our mathematics was taught in small groups but did not mention our innovative sequence. Those teachers working with me to draft these documents and I were not brave enough to mention our own varied scope and sequence until 2008/9, where we analyzed grade level dips that resulted from being out of sync with the state curriculum. In Figure 21 below there is an excerpt from the 2008 EPSS, which reveals our first discussion of how our fourth-graders might be below proficient because our scope and sequence did not coincide with the district's curricular map. We also mentioned how we were planning to remedy this dip so that the students could become proficient according the state accountability. This document also shows this admission for our seventh-graders, and can be found in the Appendix A.

Reporting Period 2 2008/2009

Because our 4th graders' scores dipped significantly on the 2008 SBA, we chose the 4th and 5th math workbook curriculum as a major school-wide data point on which to build a data dialogue school goal. We identified this goal project in November at the data dialogue training, to which my whole staff attended. We designed a team of teachers to gather the materials in December, then distributed the materials and trained teachers in early Jan. The focused school-wide instruction took place for 3.5 weeks in January and the Winter A2L math scores demonstrated the goals of this project. It was important to look at both 4th and 5th grade students for the following reasons: 1) the 4th grade students who scored low in spring of 2008 are now 5th graders, 2)the 4th grade dip seemed to be a dip district-wide, therefore we thought it could be a jump in the curriculum between 3rd and 4th, and lastly, 3) because our school is divided between K-4th and 4th –8th grade multi-age classrooms; it seemed that the entire school carried this problem together and it would make a great "first" school-wide effort to bring all the teachers together in their efforts. Because our school curriculum is designed to move students along at an accelerated pace through a deep conceptual understanding of the individual algorithms, it is essential that they participate in the traditional "survey" math curriculum. We have our students participate in grade level test prep workbook materials for this reason. We have had to develop instructional methods of delivery using the grade level workbook in our classes that we call "mobius" lessons—meaning simply that the spiraling of the concepts happens right in the lesson for several grades obtaining the lesson simultaneously, as opposed to the traditional spiral lessons that spiral back on each other in the future grade levels. Between the focus materials and instruction, we were able to make significant gains as demonstrated in the data.

Figure 21. Excerpt from 2008 EPSS Document filed with the State Department of Education

The 2003 EPSS document did indicate that my teachers and I had designed our own tests, but it neglected to mention how the units were designed in scope and sequence. In 2007, we chose to focus our mathematics data on how well we moved our newer students to proficiency, using the district short cycle testing data (Assess 2 Learn, A2L). When we collected our data, we saw that our Hispanic non-FAY students (not having attended Family School for a full academic year yet) had improved from 64% proficient in the Fall to 92% in the Spring. Our 2010 EPSS describes how our Professional Learning Community of teachers works together to successfully develop our stellar mathematic curriculum. In writing this year's EPSS, we returned to the need to improve our teaching of our mathematic groups by utilizing new techniques to help students develop their learning habits.

While our Algorithmic Math Group Mode contributed throughout the years to our students' performance on state-mandated tests, my staff and I were not forthcoming in explaining our non-traditional approach to teaching mathematics. Our 2007 data, which was kept in-house on short-cycle assessments, proved to us that what we were doing worked. We have yet to declare on state documents that our algorithm mode is self-designed. Although it does not necessarily align perfectly with the state or district requirements, our students perform well on their tests.

What I can infer from this is that while we have developed the confidence in this mode, we only feel confident enough to mention select aspects of it on district-level documents, such as generating tests, creating Professional Learning Communities to study our teaching, and refining the effectiveness of our teaching through interventions.

Concluding Remarks on the Algorithmic Math Group Mode

Table 12:

Examples of Literature Review Connections Developed

Collaboration	• Teachers immediately began to build cohesive and consistent algorithm math units for the entire school, once the school was centralized.
Teacher as Designer	• Teachers designed scope and sequences of algorithm math units. • Teachers designed unit tests for school-wide use of the math units. •Teachers wanted to have a common mathematical experience and self-made school curricula for the entire school.
Reflective Practitioner	• Teachers studied algorithmic procedures through their design of the math units. • Teachers studied the algebraic connections to mathematics. • Teachers studied how to connect manipulatives to the procedural mathematics of the algorithms. • Teachers' study of math units builds depth of mathematical knowledge.
Learning Community	• Teachers taught parents the scope and sequence of algorithmic units in parent classes. • School-wide algorithmic classes taught to both teachers and parents at the beginning of each year.

Presently this mode is on-going in our classrooms. While I journaled extensively on this mode in my reflective personal journal, all the references to this mode were about how well the mode supports and is supported by the other modes. (For that reason, I have not included these references here.) Instead I will conclude with the following remarks.

The Algorithmic Math Group Mode has taught our school much about the need for deep conceptual understanding of the very basic building blocks of mathematics. Even at the level of simple addition problems, the deeply embedded concepts of place value and number sense can make mathematics come alive for students. Their interest and excitement for math is greatly enhanced by understanding patterns in numbers and how these patterns help them manipulate mathematical operations in their minds. Many a teacher or parent that I have taught our Algorithm curriculum has uttered the words, “I think if I were taught this way when I was a student, I would have done well in math and loved it!” As it is, far too many adults were exposed to brutal memorization techniques for solving algorithms, and from the beginning of their exposure to mathematics, many believed math wasn’t supposed to make sense. Reading would never be taught in such a way, to suggest that reading is not for meaning, but for word-call memorization is ridiculous. Yet, mathematics, for many adults, was an endless memorization of addition, subtraction, multiplication, and divisions facts. By the time fractions were introduced, it was no wonder that no foundation for the conceptual understanding existed. Consequently, many just multiplied the numerators and denominators, never understanding why they multiplied nor wondering why their answer had gotten smaller. I find so many of these misconceptions in teachers and parents, but they can be easily changed with the Algorithm Math Group Mode strategies.

From the beginning of our mathematical exposure to our students, the teachers and I communicated to them that every little step in mathematics is full of meaning, and that they must make their meaning, not the teacher. It was also important for us to understand how critical it was to hear students communicate their understandings. Through story, manipulatives, answering questions, math games and practice, our students developed the basic number operations while understanding that thinking like a mathematician does not come from mere memorization.

These basic algorithmic operations are the foundation on which mathematicians must build their work. If these building blocks are taught through sheer memorization with no understanding, then the language of mathematics becomes untranslatable and the fascinating and mysterious mathematical puzzles and enigmas are impossible to understand and enjoy.

The Algorithm Math Group Mode was also significant because it was the first mode that our school staff designed. Because it focused only on the algorithms and not all math topics, it was a small enough part of mathematics for a staff to wrestle with in designing the scope and sequence. When we saw our mode make a difference in our families' mathematical experiences, we expanded conceptual understanding for other mathematical purposes.

The themes of this mode suggest that on-going redesigning of the units was important in developing success with this material. By continuing to question our previous work, we uniformly came together as a staff to retool the work and consequently improve our previous understanding. Professional development opportunities were the actual redesign work, which was done in small groups of teachers at similar grade levels and also done by individual teachers who were willing to teach others what they had learned. A sense of community was built around creating our mode's curriculum, shared by teachers, parents and students.

The themes also suggest that the school-wide documents we created are valued. The teaching booklets for each of the algorithms have been found invaluable to the entire community. The more visual cues for both parents and teachers to share, the more the curriculum is embedded in the community.

Lastly, while the political pressures that have created certain issues with curricular decisions across the nation, at Family School, we use the pressure as an opportunity to re-evaluate our curriculum and redesign where necessary. There have been good outcomes with the NCLB pressure, such as relooking at how we reach all levels of students. In our desire to be compassionate, we found we often accepted less meaning-making of our lower students, and that was not acceptable. With interventions, we were able to adapt our Algorithm Math Group Mode to reach all students.

One story I remember a parent sharing with me was about how her son struggled with the subtraction algorithm for days. He frowned each day he came home from school. He was determined to understand what the base-ten blocks meant, and, finally, one day he came home elated because he understood the regrouping of place values when subtracting. He proceeded to dance around the house while singing a song he made up, entitled, “I Am the Subtraction Man.” It may have been that after the Algorithm Mode became a success for us, we, too, could have sung, “We Are the Mode Designers.” Next, we began our design of our other math modes.

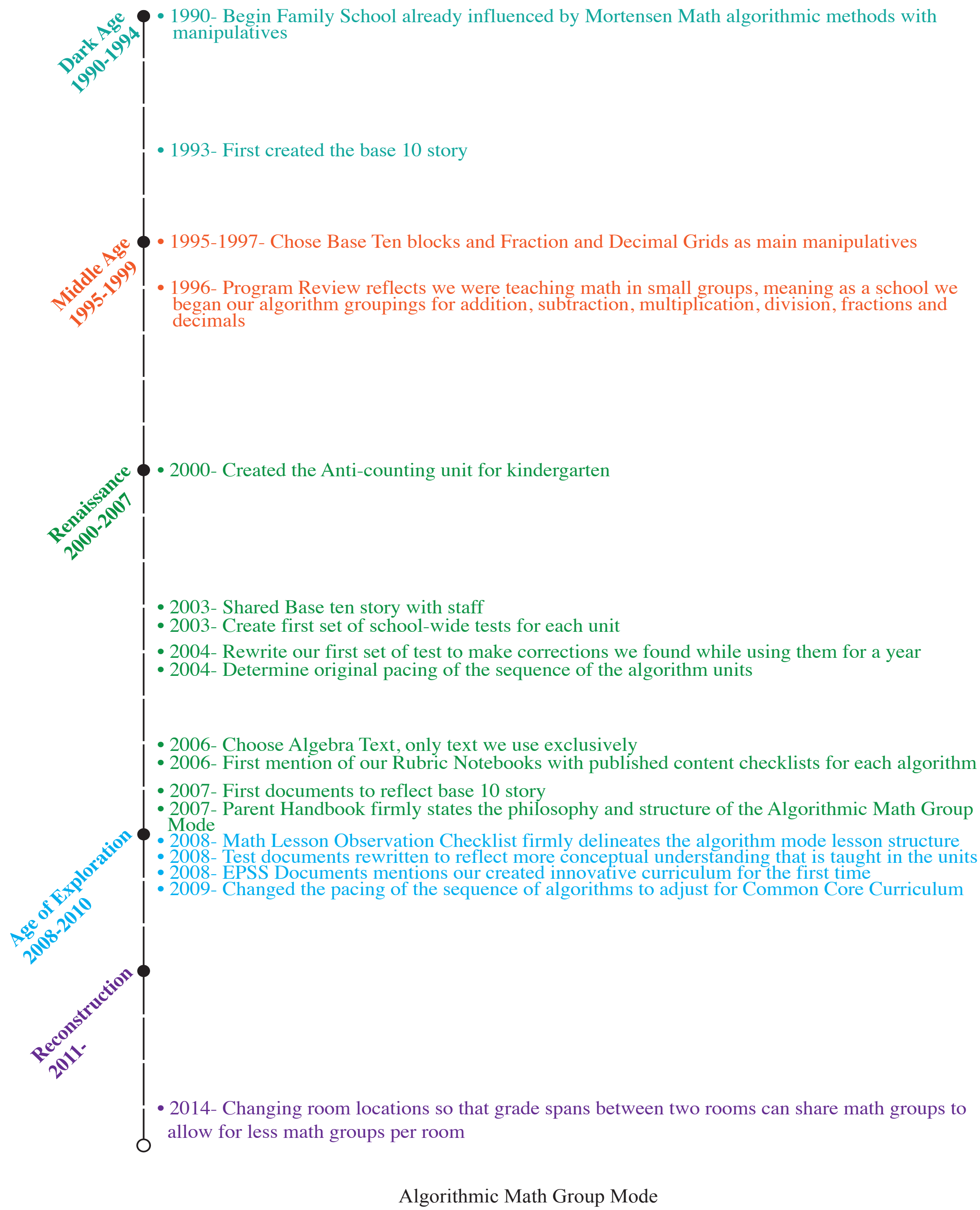


Figure 22. Algorithmic Math Group Mode Timeline

Chapter 6

The Math Brainteaser Mode

Table 13

Major Codes and Their Themes for the Math Brainteaser Mode Creating Patterns of a Plotted Story Line

Mode: Math Brainteaser Mode		
Codes	Themes	Plotted Line
Purpose	•Development of math community • Metacognitive strategies of developing: the *how's* over the *what's;* a student's sense of agency; a tolerance for ambiguity and working in the unknown	The Math Brainteaser Mode's themes created a plotted line that suggests that while we know it is important to teach students at their ability-level, this mode also allows us to teach math well beyond students grade-level. This promotes teaching students to become comfortable with new practices that are aligned with the practices of *real mathematicians.* We are able to promote both linear and non-linear approaches to students to give them the best of both worlds. Teaching brainteasers showed us how important it was for the teachers to be learning with the students as a genuine learning community where all participates learn together. Brainteasers also revealed how important it is for students to direct their own way through solving complex mathematical problems.
Content	•Non-linear scope and sequence • Content that allows students to work at the edges of their math proficiency	
Strategic Instruction	•Solving problems in more than one way • Importance of teachers to learn with students •Student- directed instruction •Recursive instruction to promote habits of *leaping* and *search practices* in mathematics	

The Math Brainteaser Mode provides on-going opportunities for students to periodically work with units of mathematical brainteasers of similar structure and content. Students are asked to solve them on their own, as a community of learners, as if they were mathematicians solving a cutting-edge mathematical conundrum. The teacher's role is to encourage students by participating more as learners than as teachers. Teachers ask students questions to move them forward to solutions to the puzzles. Teachers are not to solve the problem for students. Brainteaser lessons usually last for an hour, but have been known to engage students for two hours. Even when students discover initial solutions, they work to find multiple ways to solve, explain and extend the problem and lesson.

A non-example of this mode is giving the students a brainteaser as a single challenge activity without embedding it in a unit of similar brainteasers, nor having an on-going, regularly-scheduled time to teach brainteaser units. Another non-example of this mode is when teachers give students who have finished with their *regular* class work a brainteaser to give those students something to do while waiting for the rest of the class to finish. In both of these cases, brainteasers are seen as supplemental, enriching, but non-essential for all learners. In the second example, independent problem-solving contributes nothing to shared mathematical understandings.

After a brief introduction of brainteasers and the purposes of this mode, the chapter will follow the chronological order of the development of this Math Brainteaser Mode, beginning with its origin in my traditional classroom, before Family School was created. I will tell the autobiographical story of this mode through the Family School ages, while also weaving the document analysis through this story. Important documents illuminating the evidence of the evolution of the mode will be highlighted throughout the chapter. The present use of the Math Brainteaser Mode will be described at the end of the chapter by using excerpts and analysis from my reflective, personal journal.

This chapter will reveal that brainteasers have provided an important way for students to think, together, like mathematicians. Not only is it important for students to experience this, it is also important for them to experience the teacher entering the problem as a co-learner. Because I am a teaching principal, I almost always took the initial risk in teaching brainteasers in order to encourage other teachers to take these teaching risks, as well. Teaching brainteasers, especially when I introduced brainteasers for which I did not know the answer, helped me to identify and develop teaching tools that caused me to reflect on my own teaching for improvement. When I worked with the rest of the staff, our experiences teaching brainteasers helped us to develop some of these teaching tools further, such as our "Taxonomy of the Pedagogy of Teaching." [See Figure 21 on page 189] This tool, the direct result of our conversations about the thinking we observed and experienced while using the Brainteaser Mode, enabled teachers to better design small teaching maneuvers needed by students during the unfolding of a complex brainteaser. These discussions also allowed teachers to reflect on the roots of the philosophical underpinnings of their teaching. For example, did they believe that it was possible to "teach" without knowing an answer beforehand? What might such teaching look like? Could they learn to suspend their desire to show and tell? Could they learn to pose a question that made the next, incremental piece of a problem accessible to a student without giving everything away, hence deepening thinking rather than foreclosing it with a solution not earned by the students?

While the many benefits of the Math Brainteaser Mode were evident, the story of brainteasers will reveal that due to NCLB testing pressures, this mode was discontinued because we felt pressured to spend more time focused on the curriculum that we knew would be tested. However, it was not only external pressure that impacted the mode. As a principal, I pushed my teachers to become more accountable by asking them to share the results of their brainteaser unit tests in a data-dialoguing process. Teachers had to present, to their peers, their students' results of the brainteaser unit tests. This was threatening to teachers. An honest account of what had happened revealed failures to teach brainteasers through the teacher moves to which we aspired, especially our desire let the students discover and co-create as much of the mathematical meaning as possible. Brainteasers posed another problem for us to ponder. What did it mean for a student to reach “mastery” of brainteasers? Each brainteaser unit was very different, in math content and structure. Within a unit, from problem to problem, there was some progression of complexity and difficulty, but units were not created in a linear sequence. One unit did not lead to the next. The surrounding context emphasized standardized testing, so it made sense that we wondered about how we would test for mastery, but in the end the mode was more about collaborative thinking moves, and learning from and with peers, and we eventually decided that we did not have to push for “proof” that brainteaser math had been mastered. Ultimately, what we clarified for ourselves was that not all modes, or teaching opportunities, should be tested for mastery. Math brainteasers were an opportunity for students to learn a process for thinking and acting like mathematicians, qualities more important than mastering brainteaser content.

It took my staff and I years to feel confident enough in our Math Brainteaser Mode to reveal it in our state and district-required documents. Where, after all, was the evidence of the mode’s impact? Why were we departing from linear, textbook mathematics? Yet, we knew that the mode was teaching mathematical habits of mind. That confidence was helpful in bringing back the mode to classrooms after its two-year absence. The math brainteaser journey provides an excellent example of teachers adapting and reconsidering a mode in response to the pressures without and within the school’s four walls, yet the mode’s value was reaffirmed, in the end.

Introduction

The Math Brainteaser Mode was developed very early in my teaching career, before the creation of Family School, however it took years before it became a math mode for our entire school. Originally, in my traditional school classroom, I wanted to do brainteasers with my students because the traditional mathematics program seemed so linear and boring. As I immersed myself in brainteasers, I found that both the students and I couldn't help but be enthralled in the mystery of a puzzle. There were days when I chose brainteasers to write on the board that I had no idea of how to solve, yet with my students, we were able to conquer them together. Being learners together made us feel like we were at the new frontier of mathematics itself, and no one else had discovered our processes and solutions before.

In fact, the way I had chosen to do brainteasers also captivated other students outside my classroom. I would write the puzzle on a piece of whiteboard and place it in the hall outside my 3rd-4th grade classroom door. This way, my students would see it and read it before entering the classroom. They were very eager to put away their coats and lunches and hurry to the front of the room to begin their solving together. However, I would have other students from other rooms come and knock on my door at lunch to share with me their solutions, or beg me for an explanation. This is how I came to understand the importance of the math brainteasers and their appeal to children. Because of the interest, and mathematical thinking, that they generated in my traditional classroom and my wing of the school, I knew that my Family School classroom would have to have brainteasers as an essential piece of the curriculum.

Some examples of these brainteasers are:

Table 14

Brainteaser Examples

XYZ XYZ -AB +AB CDEF BGA	At a special meeting, everyone shakes hands exactly once with each other person present. Altogether there are 45 handshakes. How many people attended the meeting.	I think of 3 numbers; I work out that:- 2P +Q + R = 22 P + 2Q +R = 20 P + Q + 2R + 18 What numbers did I think of?	What comes next: 30,31,35,44, 60__ __ Make an equation using: 2 2 3 4 = + x x ( )
Berloquin, P. (1976). *100 Numerical Games.* Barnes & Noble, Inc.	Wells, D. (1982). *Can You Solve These?.* Ancient House Press, Ipswich, England.	Wells, D. (1982). *Can You Solve These?.* Ancient House Press, Ipswich, England.	Orleans, S. (1987). *The Great Big Book of Pencil Puzzles.* Penguin.

What you see here are the kinds of brainteasers used to set the context for how different the content is in this mode as compared to the Algorithmic Math Group Mode. Because the content is so challenging, this mode's purposes were developed to be quite different from the algorithmic mode. Developing the qualities of expert mathematicians when they work to solve mathematical, cutting-edge dilemmas were chosen as the purposes this mode would aim to develop in students.

Qualities of Mathematicians

Since my Family School classroom was going to be a multi-age classroom for grades one through six, I knew that the students in this one-room school were going to have to get used to being around subject matter that was well above their own cognitive abilities. Brainteasers were one method available that would provide for this opportunity. What I learned from doing math brainteasers in this multi-age classroom was nearly the same discovery I had made in my traditional classroom. I again discovered, in my multi- age classroom, how important it was to be a student with my students, which included the possibility of the older students learning from the younger students, as it certainly included my learning from my students. While the older students often thought they would solve the brainteaser before the younger students, this was not always the case. Often the younger minds were not tainted by mindsets, not searching for some vaguely-remembered procedural formula that would cut to a solution without having to think. The young ones could often ask questions that the older students could not find. It was not age or knowledge that propelled our ability to solve these brainteasers but a willingness to take risks. Younger students seemed more open to taking these leaps than older students. Perhaps school had not yet taught them that the primary expectation was to get only right answers, or perhaps they were still not afraid of not knowing something.

Tall (1996) suggests that this *leaping* is an important feature for mathematics curriculum. More importantly it is a "damaging assumption" (p. 10) to think that students must learn mathematics curriculum in small, specified stages. He suggested:

> Democracy in education does not therefore mean giving every child the same sequences of learning, but at different paces. It means giving each child the education that best suits the child's individual needs appropriate for his or her growing cognitive structure. (p.11)

Brainteasers allowed me a place in the curriculum that wasn't linear, where I could see how far students' minds could stretch and leap in their mathematical thinking when they attacked the problems. It wasn't long until our classroom learning community was evaluating different methods, both older and younger students alike, in their quest for solutions to puzzles. In describing the qualities of the work of mathematicians, Burton (1999) found that the culture of mathematicians' research has moved from working as individuals to working collaboratively. She interviewed 70 mathematicians, and they claimed to have witnessed a changing environment in their professional working styles over the years.

This suggests that to adequately prepare students to become mathematicians, it is important for students to learn to work collaboratively around problems, where they can all come to the table, both knowing something about the problem and, more importantly, not knowing something.

As Family School grew, all teachers used brainteasers in this pursuit—learning the joy of the unknown, not being afraid to be stumped, and learning to work together with the *community brain,* as we called it. Later, because some teachers seemed to revel in brainteasers and others did not, we started to collaborate on them. While some teachers gravitated to the challenge of brainteasers, others were very fearful of them. For teachers who liked doing brainteasers, there was also the issue of teaching something that was difficult to accomplish. Often in brainteasers, teachers will take risks with the students and work the brainteaser out without knowing the answer. This is an uncommon practice for teachers, as they like to know how to teach for correct answers. However, it is exciting for the students when everyone in the room is working to solve the mystery. In order to overcome some of the teachers' hesitancy toward brainteasers, we began to work together to develop brainteaser units and teach each other what we had learned when doing them with our students. It was then that they became a Mode of Engagement because we had to define the purposes for using brainteasers and claim them as a valuable component of our school curricula.

The following is an excerpt written by me in 2004 as a part of a Family School writing group. A group of teachers, including myself, of our own volition, had decided to write about our experiences of teaching at our school. We met weekly to share our writing and set new "assignments" for ourselves for the next week. We discovered that our writing about our teaching helped us to further learn from our practice. But initially, we just wanted to experience the writing. One week, my assignment was to write about brainteasers and what they represented to Family School. It is important to remember that my focus in this study is on mathematical brainteasers, but we were experimenting with the effectiveness of the Brainteaser Mode in other content areas, especially language arts.

Entering the Mind of Teaching

> Every good day begins with a brainteaser. The mind that enters the room in foggy memories of last night's dreams or eager anticipation of a new social event quickly raises the curtain to puzzlement, wonder and the challenge of an adventure. The mathematical patterns or the grammatical cryptograms awaken me, and my students, but not in the same way. I study the faces and body language of my students while they unpack their gear and read the crisply etched directions of today's torture. If it's math brainteasers, they are more apt to breed energy in their engagement with each other; and if it's writing brainteasers, their quiet support offers caverns of space for imagination and formulation. I, on the other hand, stubborn in my non-directive manner, am dying to enter their minds and hear their thoughts, probe their questions, and maybe even secretly ease their frustrations, just a little. But I would only do such a thing in a way that they understood I hadn't really done it at all; they had really solved the dilemma on their own.

So what do I do if I'm not teaching; and I dare not teach. For that moment of teaching would suggest they needed teaching, and what would be the point of brainteasers if not to be able to fumble around while being teased or be taught only by the light of one's own illumination. I watch, I listen, I think. I'm actually building the most salient lesson to be born of their efforts and suffering and deliverance.

Always the moment comes all too quickly, when I must capture the tension from the solved and unsolved, and begin to turn back unto them what they have just given to me. I saw all their strategies being unveiled, from one student to another. I heard their questions shape the words on their pages. I thought about their thinking strategies. And then I approach the board. It's white like this page, and it's also as daunting. I'm going to reveal the answers that I have not even solved. I must be crazy, and yet, day after day I find the solution or the magic that is needed to work the brainteaser to its natural end with the students. "What do we know?" We start from the obvious, which always weaves to the not so obvious. Upon hearing all the knowns, an unknown is catapulted into the known from a student who unknowingly didn't have access to this idea moments ago. It's beginning to happen: the magic.

I need to listen even more closely to this whole group discussion, which means that I enter the mind of teaching. It feels like the room is of one mind that exists for all of us; and our ideas and questions are all a whirl inside. We all have access to this mind. I'd like to think that I translate the mind's meditations on this conundrum, but in truth it happens in the realm of simultaneity. It speaks because they speak, I speak because it speaks, and the mind moves the class through to the end framed in many lessons of content, strategies and building a community of learners.

This writing example demonstrates how entrenched I was in the importance of studying teaching and learning through brainteasers. I see myself wrestling with the fact that teaching often meant “not teaching” at all. In fact, it meant that my students were steering the work more than me. It is in this design that we found the purpose of brainteasers and related it back to one of the domain-specific practices and learning strategies of mathematicians: the collaborative search for a solution to a problem for which no one person holds the entire answer.

Burton (1999) suggested that the collaborative model of instruction places more importance on students being involved in “exploring and negotiating meaning, assuming that such meaning is negotiable and non-homogeneous” (p. 138). Brainteasers lend themselves well to this negotiable discussion about how to solve a problem, or even whether there is only one right answer. Many of the mathematicians in Burton’s study were also mathematics instructors at the college level. While many of them revealed the value of their own personal growth when working collaboratively, many of these college-level teachers did not allow time for their own students to collaborate but rather taught using only a lecture format. While they valued collaboration as professionals, they hadn’t figured out how to make that happen in their own classrooms for the students of mathematics.

What I can infer from the introduction and purposes of this mode is that brainteasers, while often and easily seen as nothing more than diversionary puzzles, came to provide us with the content of a mode that worked well to present opportunities to students that mimicked what a community of expert mathematicians would do when solving new mathematical problems. In deciding to create a mode towards this purpose, we created an opportunity to design different instructional strategies than the strategies employed by most teachers when teaching mathematics. By creating a mode around such content, Family School offered its teachers a context where they could study their innovative mathematical instruction.

Documents of the Eras

My writing excerpt about math brainteasers was one of 97 documents analyzed for this brainteaser mode, spanning the years of Family School, 1990-present. These documents were categorized and sorted by different kinds of documents: actual brainteasers; unit tests and brainteaser rubrics for unit tests; professional development documents, such as staff meeting agendas or minutes; presentation notes; administrative documents used for instructional improvement; and general school documents, such as student narrative report cards and documents for the state department of education. The documents were then analyzed for the various themes associated with the Brainteaser Mode as seen in Table 15 below. This data was disaggregated into the significant time periods in the evolution of our school. Some of the recurrent themes that were prevalent clustered around the following foci: 1) Determining the content of brainteasers; 2) Determining the metacognitive strategies of brainteasers; 3) Determining the grade level of brainteasers; 4) Understanding the process of brainteasers; and 5) Determining what should be evaluated in brainteasers, as seen on Table 16 below. These themes were then analyzed for patterns and frequency to reveal the story of the brainteaser mode throughout the years of our school. While doing this research has revealed a great many surprises in revisiting these past documents, the most surprising to me is that sometimes the absence of data, during a particular time period, raises the most questions and supports the most compelling interpretations.

The statistics will be further disaggregated later in the chapter to reveal more about the evolution of the brainteaser mode. From the wide variety of documents analyzed and coded, it can be inferred that, during the Renaissance and Age of Exploration, the contextual pressure to focus on student performance led us to consider ways to assess student learning connected to the Brainteaser Mode. For this period, I located nine documents focused on rubrics for assessing student performance and learning associated with this mode. No document mentioning rubrics for the Brainteaser Mode exists from any other period. Clearly, from 2002-2010, we were wrestling with whether we ought to be using brainteasers to teach for demonstrated mastery, a stance we had not taken originally, and a stance we abandoned, after thoughtful deliberation, some time after 2010, concluding that not all modes needed to be taught for mastery. What you will see next is the chronological account of this mode, beginning with the Dark and Middle Ages of the school.

Table 15

The Number of Documents Studied for Each Time Period of Family School.

Brainteasers	1990-1995 Dark Age	1996-2001 Middle Age	2002-2007 Ren-aissance	2008-2010 Explora-tion	2011-present Reconstruc-tion	Total
Prof. Dev.	1	19	10	6	1	37
Rubrics	0	0	5	4	0	9
Examples of Brainteasers	11	2(3)	11(3)	6(3)	6(3)	36 (3 repeated docs. also)
General Doc.	2	2	5	1	5	15
Total	14	23	31	17	12	97

Table 16

Themes of Brainteasers Found on Various Family School Documents

Codes	**Themes**	General Documents	Brainteaser Rubrics	Agendas	Total
Specific Content	Focuses on determining content	**0**	**56**	**10**	**66**
	Focuses on improvement of instruction	**4**	**0**	**34**	**38**
	Focuses on determining grade level expectations or groupings	**0**	**35**	**5**	**40**
Higher Purposes	Focuses on the determining metacognitive strategies	**11**	**12**	**27**	**50**
Strategic Instruction	Focuses on the process of doing brainteasers	**6**	**21**	**5**	**32**
	Focuses on the performance of doing brainteasers	**7**	**42**	**17**	**66**
		28	**166**	**98**	**292**

The Dark and Middle Ages of the Math Brainteasers Mode- 1990-1999

The story of the Math Brainteaser Mode begins before 1990. As I have stated, I have done brainteasers since before the creation of Family School, certainly since my second year of teaching (1987-88), yet documents of the first ten years of Family School, 1990-1999, show no mention of brainteasers. I celebrated doing brainteasers every single day with my students, which is seen in eleven actual examples of brainteasers I used during this time. I recall outside observers, and even my supervisors, visiting the classroom to find out a bit of what was happening in my classroom and seeing the students and I engage in animated brainteaser time, yet, in more formal documents, such as the district-mandated, annual Program Review of the school, and the Evaluation of the Program for the 21st Century Grant I was awarded, no mention is made of the Brainteaser Mode. Nor is it mentioned in program fliers, or professional development agendas. There is not one mention of doing brainteasers. It is clear from the documents of this time that there are themes referring to *the math curriculum* topics, and they were well discussed and delineated, but there is no mention of the brainteaser part of that curriculum. After the first year of teaching Family School, during which time I was the only teacher, I had many discussions with the newly-added Family School teachers about how to teach the mathematical concepts in our curriculum, especially during the third year. However, there is no record of my discussing brainteasers, much less teaching the new teachers how to teach using brainteasers. It suggests that I was still figuring out a language, and collecting examples, that would allow me to begin to share my evolving strategies. It is interesting to note this absence during these Dark and Middle Ages, in those first ten years, because brainteasers are much more visible in the documents associated with the next ages, roughly the second ten years, with much discussion about the Brainteaser Mode at our staff meetings. For the first ten years, I must have been unsure, as a leader, as to whether I ought to suggest, or require, their use in other teachers' classrooms. For those teachers who did use them, it was next to impossible to share what we were learning. We were too far apart physically, since the Family School existed not as a single, centralized location, but disbursed to several, small, district sites, some separated by many miles. There was little to no collaboration between teachers at discrete sites. Brainteasers, where they were used, served a function of challenging students and bringing the various ages to work together, but they had not been identified as a mode, nor had we identified their true purposes, yet.

For those first ten years of our school, each teacher was housed at a different school site throughout the district, and while we did have meetings together on Fridays, it is clear from the minutes of these meetings that our main concerns were about individual students. When we discussed curriculum, we were developing our language to talk about what we meant by *best practices*. For example, the Sept. 5, 1997 agenda notes reveal our small group of teachers sharing the following mathematical "best practices" at the four Family School sites: the Mitchell Elementary School site was playing dice games and wrestling with place value; Hodgin Elementary School site was doing fractions with color tiles; the Monte Vista Elementary School site (my site) was doing a series of differentiated problems having the students articulate their thinking and problem solving into a tape recorder; and the Chamiza Elementary School site was working on measurement with various measurement tools. It was clear that the focus was on discussing a wide variety of topics in our mathematics curriculum. The format of these teacher meetings, for which I set the agendas, generally revealed a more *show and tell* approach to considering our professional development rather than a more reflective, self- critical discussion about improving our instruction and curriculum. Again, this demonstrates that I did not want to impose what I had been doing in my classroom as a requirement for the other teachers. I was interested in having everyone share their best practices. I was not able to bring any real sense of cohesion to our various practices in the early days of the school.

That did change in the summer, when we held our first Summer Seminar (a professional development opportunity for professional development of all of our teachers), where the teachers and I focused on curriculum and instruction. Still, the instruction that held our attention the most in mathematics was how best to teach, for conceptual understanding, the basic algorithms— addition, subtraction, multiplication, division, fractions, decimals, and percentages. Often, we had to explore our own conceptual understandings as a foundation for our work with children.

The various sites were doing math brainteasers, but teachers described the purpose in terms of *waking up* the brain to start the day to stimulate problem- solving strategies. I did brainteasers, too, but the fact that they were not mentioned at all in the documents could mean that I felt they would not be received well by my supervisors in the district or in the State Department of Education. The brainteasers that we did were well above any standards written in either district or state requirements. I'm sure that as a beginning teacher, I felt that it was inappropriate to do brainteasers rather than the required curriculum. However, since brainteasers were able to introduce students to so much math content, while energizing and engaging the students, I continued to find them very useful as an instructional tool, no matter how difficult and challenging the daily problems were. The beauty and complexity of solving brainteasers appealed to me, but I was not sure others would agree with me at the district and state level. Even Davis and Hersh (1995), mathematicians themselves, contend that "blindness to the aesthetic element in mathematics is widespread and can account for a feeling that mathematics is dry as dust, as exciting as a telephone book" (p. 185). The fact that brainteasers went unmentioned in official reports suggests that the teachers and I thought our reviewers were more likely to approve a traditional mathematics curriculum, holding fast to the dusty ways of what was required in the mathematical textbooks of the early nineties, or that we lacked the confidence to include brainteasers in our discussion of mathematics until we were better able to articulate what we were getting out of doing them with our students. In the analysis of these documents, teaching metacognition is mentioned, as are the beginnings of discussions about improving our teaching during the summer seminars. These seeds bore fruit in our professional development program and influenced the creation of our Math Brainteaser Mode.

In analyzing the actual brainteasers that were used during the Dark Age, it is clear that they were chosen from brainteaser books (*popular* trade books, usually featuring multiple kinds of brainteasers). During the Middle Age, the sources of brainteasers included some mathematics texts. Some of the original brainteasers came from Wells's (1982) Can You Solve This?, Berloquin's (1995) 100 Numerical Games, and Stuben's (1987) Intelligence Games. My collection of brainteaser books grew almost immediately, these being some of my favorites, mostly because they were compelling enough to start my journey into the world of mathematical puzzles.

Originally, my teachers and I would do just one puzzle per day, but somewhere they grew into lessons that might involve multiple problems of the same sort, then larger units, meaning that we spent days, and even weeks, on the same sort of brainteasers, unpacking the structure of these problems, honing our strategies. This evolution made sense. We had to do more than one brainteaser a day to reveal the procedural knowledge at the heart of a set of similar brainteasers. A unit of brainteasers in this era would string together similar brainteasers, which often meant finding them in multiple sources or creating our own versions of the same type. Very few brainteaser books put similar brainteasers in a chapter. In preparation for doing a unit, a teacher would need to determine strategies that might help students solve the brainteasers, and also some procedural math tricks that are common for that kind of brainteaser.

A good example of developing procedural knowledge for a set of brainteasers would be to look at patterns of numbers. If given a series of numbers such as: 1,3,6,10,15, and asked for the next two numbers that would logically follow in the series, one discovers a pattern; expressing this pattern in words is cumbersome, which is one of the points of learning to write equations. But there is great value in having students try, before equations, to explain the patterns they discover, to demonstrate their thinking, to try prove that their hypothesis fits the evidence. In this example, each number after the first has been arrived at by adding a number, beginning with two, that increases by one in each interval between numbers. Two is added to one to equal three, then three is added to three to equal six, then four is added to six to make ten, and so forth. The mathematical thrill is in the discovery, next in the explication, and then the reward of having others follow your thinking. There is also struggle and failure and frustration, but these are framed as stepping-stones toward breakthroughs. Now, suppose that pattern was our first brainteaser. All number patterns do not use this pattern, nor are all patterns based on addition only, nor are they always based on something occurring regularly between each number. After working through many sets of number-pattern brainteasers, the procedural knowledge of the various ways mathematical patterns can be made become more apparent, such as: patterns for every other number; patterns using kinds of numbers (square numbers, triangular numbers); and patterns of every third number. There are endless possibilities. Yet a procedural knowledge for this category of brainteasers becomes more sophisticated as these brainteasers are explored as a unit rather than just doing one of them. In the early days, a brainteaser on one day might have nothing to do with the next brainteaser introduced. Think of how much mathematical value was being lost by not expanding and extending the thinking about process.

Burton's (1980) article on math brainteasers suggests two opposing arguments in using brainteasers in classrooms. At first she appears to spell out an argument against their use. She stated, “Problems are usually serious and demanding on a cognitive level... Puzzles, on the other hand, are diversionary and, as they do not create too much tension and frustration, they are ‘fun’” (p.21). This seems to suggest that using brainteasers in the classroom would not provide students with the experience provided by *real* mathematical problems such as those faced by mathematicians. However, later in the same article, she suggested that puzzles may be used in the classroom if five phenomena are contrived or discovered in doing the brainteaser. My earliest ventures into using math brainteasers with my multi-age students did, in fact, uncover the five qualities Burton outlined: 1) finding a problem; 2) using multiple methods to solve it; 3) generating new questions; 4) communicating one’s thinking; and 5) focusing on search behaviors (searching out strategies for solving puzzles). These habits of engagement with brainteasers emerged because I was genuinely discovering what math brainteasers were all about. I’m sure I was very wary of what I was doing with them, because I was so green, yet my mathematical knowledge and thinking processes expanded greatly the more I practiced them. More importantly, the more I accomplished, the more I was able to mine them for the benefit of students. This took some years, as evidenced by our documents. It was not only my students who were learning to think like mathematicians.

I can infer from this that initially brainteasers were used more as a response to teaching a multi-age class than as a math mode. In the early years of the school, they were a compelling way to capture children’s attention, from the 1st-graders to the 6th- graders. However, once we shifted from stringing together dissimilar brainteasers to following one brainteaser with another related to it, we began to string together units consisting of similar brainteasers in order to discover generalizable solution strategies. In a sense, the Brainteaser

Mode was teaching students to think like mathematicians by deriving procedures from experience with problem sets. This can bee seen as the beginning of brainteasers becoming the Brainteaser Mode, representing a profound shift in the deliberate, metacognitive ways in which teachers approached their teaching when working with brainteasers. Initially, our purposes were more about setting up contexts for multi-age students to work as a community, but this community work eventually grew into a deliberate effort to model, and experience, expert practice as a purpose for the mode.

What you are about to see is how the mode developed once our school was centralized and teachers could collaborate on creating their brainteaser units.

Brainteasers in the Renaissance- 2000-2007

The brainteasers, at this long-awaited uniting of our school and the entrance into the Renaissance, reveal a great bubbling of excitement. The kinds of brainteasers my teachers and I chose and developed expanded to include more teacher-generated brainteasers. Below is an example of a teacher-generated brainteaser focused on box and whisker plots.

I'm a little box plot, short and tall,
Here is my midpoint,
Here is my whisker……
MY Q_3 is thrice my minimum
My maximum is 3 more than twice the Q_2
My interquartile range is 1.5 times the size of the third quartile
Q_2 is 65 more than the minimum
If my fourth quartile whisker's range is 93
How long is my other whisker?
And…
My Q_2 is 300% more than the minimum
When I add the digits of my minimum, I get the whisker length of my fourth quartile whisker,
The difference between the range of the second quartile and the minimum is 2.
The third quartile equals a whisker.
The range of the first quartile is a palindrome between 55 and 77.
The last quartile whisker is $1/7^{th}$ of the last two quartiles.
Find all my data points on my box plot.

Figure 23. Example of a teacher-generated math brainteaser from my computer files, dating September, 2006.

This brainteaser aims to teach the content of box plots, a concept frequently occurring on standardized tests. Unlike most textbook problems that are typically front-loaded with a data set but, this brainteaser provided information about the final box plot that would require students to work backwards to be able to solve the puzzle.

In order to generate our own brainteasers, we had many discussions about each brainteaser unit's content. One of the reasons we began this development was because of the NCLB push for new mathematical benchmarks. The heightened requirements of the data, measurement, geometry, and algebra benchmarks needed coverage in our classrooms. Some of the genuine brainteasers from brainteaser books were moved aside and replaced by brainteaser units that we developed around the new standards.

Purpose and structure of brainteaser lessons. While designing the brainteasers, the teachers and I also solidified the purpose and structure of brainteaser lessons. Brainteasers lessons always began by being posted on the white board for all students to view, or on individual worksheets to be passed out to students. However, students did not work alone. Students chose their own group members or were grouped by the teacher. Students were not given any instruction or prompting from the teacher as to how to proceed. At first, this lack of teacher direction might have seemed daunting to a student who had never been taught in this way; direct teaching was never used in this mode.

For many students, staring at a brainteaser forced them to focus on what they didn't know. One of our first problem-solving strategies was to ask the question, "What *do* you know?" This question was a strategy, which required students to switch their focus from the unknown to the known. Usually, they realized they knew more than they thought about the puzzle, and as a result, they became immersed in trying to solve it. It was often very hard not to intervene while the students struggled in this part of the brainteaser, but it improved teachers' questioning abilities, because the teachers and I agreed that we would only question students during this time. This design of instruction was constrained by one of the purposes of the brainteaser: students must work at the edge of their knowledge. If a teacher gave a direct lesson on how to begin and accomplish the brainteaser, then the students would already have been given support and would now be working in the zone of proximal development with teacher supporting their zone. With no direct teacher support, we hoped students would feel they were mathematicians at the edge of their knowledge. We allowed ourselves to ask questions that encouraged students to justify their methods, make connections to other methods, think outside the box and wonder aloud what questions they had about the problem.

Usually, we let students work together without any teacher-coordination of their efforts for about 20-30 minutes. Many dynamic elements occurred during this *incubation* period. Students discovered ways into the brainteaser, and the groups shared their ideas with enthusiasm, expressing their findings more loudly as their insights expanded. Sometimes, non-productive groups asked to work with more productive groups. I did have one rule that I tried to teach to the students and enforce when possible, and that was never tell someone a solution that you have found. Instead, question the other students into the method that led you to the solution. In other words, I asked students to practice a strategy I was practicing. At other times, I joined a non- productive group to model how to ask ourselves questions and find our way into the problem. Also, I chose to work with a productive group and speak loudly, so other students could hear, but also got the message out that the effortful groups got my attention. I didn’t want anyone to learn that they had to rely on me.

I recall a fourth-grader joining my class for her first time in our school. She was a gifted student, but her initial experience with a brainteaser was the first time in her school career that she had ever been thrown a curve, and she didn’t know what to do. She stated, “I was not taught how to do this at my old school!” I confirmed that this was probably true, but she could certainly give it a good try and figure out what to do. She indignantly replied that she couldn’t do it because she was never taught how. With patience and questions she made it through that day, but she was quite angry with me. She was my student for the next five years, and we still laugh about this story today. She graduated from our school in 8th grade, proceeded to a public high school and was awarded a full scholarship to a prestigious university. I had hoped that what she had learned from brainteasers was that in her educational experience she was bound to come across content she was going to have to figure out more on her own than from a teacher.

Many observers who have come to our school have shared with us that they didn’t think it was right for students to be expected to do a problem without a lesson beforehand — that students should be shown how to do the problem before attempting it on their own. But such an approach would defeat the whole purpose of the brainteasers.

After 20-30 minutes of searching for solutions, the students and I come together as a whole group to share our thinking. This is where students generate multiple methods, proofs for their methods, and reveal their thinking processes, whether intuitive, using prior knowledge or connecting to new knowledge. Students explain all their thinking until all other students understand. The questioning and answering is predominately done by the students.

The teacher simply guides the discussion and checks for understanding. This section is usually scheduled for 30-50 minutes, but it happens often that it goes on for an additional 30 minutes. This extension stems from all of us becoming so immersed in solving and understanding the problem that we don't want to put it down until we feel satisfied that we have learned enough.

Teachers and students learned that one day's brainteaser lesson, including each day's debriefing of the lesson ("How did we do what we did?") suggested the needed lesson for the next day's brainteaser, as these were held together in units of similar brainteasers. When this procedure began all over again the next day, the students had more understanding of how to *mess* with the brainteaser. This experience with the complex process of problem-solving as a group was cumulative, and with each new day in the unit, students began from a stronger foundation. There were days when I assigned homework connected to the brainteaser, but generally this was not a mode that insisted on daily homework. I often asked students to explain what they learned to their parents, or simply review their notes as a way to solidify their understanding.

If there was a brainteaser that stumped all of the students (which didn't happen often), the teacher used more guiding questions to break open the thought process. For the most part, we solved the brainteasers, but there were days when we couldn't, and we went home and asked our families to jump into the fun and reported back the next day. I loved this modeling of having fun with the unknown, the challenge, the higher concepts of mathematics, and seeing the joy constantly reflected by my students, and often their parents, even though they struggled with the problems.

What I infer from these sections is that it was important for teachers to collaborate on deciding the structural format of a brainteaser lesson, once having determined its purpose. We now had a mode than functioned very differently that the algorithm mode. While there were adjustments for some students, and challenges for teachers, to become acquainted with the Math Brainteaser Mode, students' and teachers' enthusiasm for the mode, as well as everyone's continued improvement in solving brainteasers, helped the mode become entrenched in our classrooms, for the time being. Some teachers struggled to become comfortable teaching brainteasers, which resulted in more teacher collaboration.

Teacher collaboration. Because of brainteasers' inherent challenge, some teachers had more success than others. Asking teachers to teach, on occasion, around problems they do not fully understand themselves, can be frightening. Yet, we were committed. Many teachers thought that we needed to collaborate on developing units in order to assist those teachers who were struggling. The structure of our school was such that teachers taught four days each week and prepared for the next week, or met in staff development, on the fifth day. We had plenty of time for collaboration as a whole staff or in grade- level groups. Our focus on collaboration time was needed to develop the curriculum we were designing, and we practiced this community effort well before the district and state required us to provide for teacher collaboration. Our collaboration time often happened in our staff meetings, which we were able to have every week for two or more hours each time on that fifth day. The staff meetings were designed as teachers suggested what they would like to work on. We often looked at student work examples to see how students were developing concepts. Sometimes, we collected data from the performances of units we designed for data dialoguing and sharing ideas with each other about how to address the issues presented in the data. Other times, we broke up work into task groups to have small groups of teachers designing units for the rest of the similar grade levels. Book study, classroom inquiries, studying videotapes of our teaching, and teacher presentations were also frequent methods for collaboration. For us, the Renaissance brought the culture of a *real school* being together, and we took advantage of it. We were finally able to build our curriculum and instruction in ways we could not while we were housed in different locations.

What I infer is that teacher collaboration in designing the brainteaser units supported teachers who were struggling with the brainteaser mode. Prior to the centralizing of the school at a single location, this weekly support for implementing a complex and challenging curriculum was missing. In the next section, I explain how I developed a teaching tool to help teachers develop their ability to make deliberate, mode- specific, teaching maneuvers to enhance the effectiveness of their instruction,

Taxonomy of the pedagogy of teaching. Collaboration was not the only answer to helping each other learn to teach brainteasers. Because I have been a teaching principal for all these years, my teachers looked to me for direction in their instruction. We have always done a fair amount of observing one another teaching. We have even set money aside in the school budget for parents to be substitutes in our classrooms when teachers want to do observations in each other's classrooms. After observing me teach, or listening to my stories of my teaching in our once-a-week staff meetings, teachers often asked me how I knew how to make decisions so quickly to move a teaching moment in one direction or another. We called these small teaching decisions *maneuvers* -- maneuvers to increase student understanding, meaning-making or mastery. I don't like teachers to merely take my maneuvers and repeat them; the moves, for me, are always contextually- based. I would rather teachers learn to read and understand the context of their own lessons with students to better generate their own maneuvers, or at least understand how the ones I generate arise from, and fit into, a specific context for a specific reason. I could simply tell them my maneuvers, but the old fish adage comes to mind, only adapted to learning to teach: Give teachers a maneuver and you solve their problems for that day; teach teachers how to study their teaching, students, and context, and you teach them how to continuously construct their own effective maneuvers, improving their teaching throughout their lifetimes." Because of wanting my fellow teachers to understand the deep context from which my teaching maneuvers and decisions were made, in 2006 I created a document to help map out the roots of maneuvers made in teaching. I knew that reasons for my maneuvers could be traced back to my basic educational philosophical underpinnings. Having considered other taxonomies, I devised my own Taxonomy of the Pedagogy of Teaching (Figure 24), so teachers would be able to see the strength and origins of one's own teaching practices. It traces the foundation of one's teaching from one's philosophy, to one's method, then to the specific approaches to, and support for, students, in all the decisions of planning a lesson. It branches out quickly when the planned lesson hits the classroom floor to include teaching techniques, devices and student strategies, ultimately narrowing into teaching maneuvers.

When I presented this document to teachers, we discussed examples of maneuvers they had made and traced their roots to their philosophies. We also started with their philosophies of learning and traced them upwards towards maneuvers, so that teachers could become comfortable with the coherence of their teaching decisions. What is significant about this document as it pertains to the Modes of Engagement is that the modes existed on the second level of the taxonomy where the specific methods were chosen to shape entire lessons. This document crystallized our shared understanding that at our school we perceived the modes to carry the basic shape of the lesson at a very early stage in the taxonomy. The modes were intended to shape all the decisions for planning and instruction that followed. Clearly, by the time this document was created (2006) we understood that our modes were built for different purposes and these purposes were essential in differentiating the kind of instruction they would require, while also acknowledging that through all the levels of crafting curriculum and instruction, teaching always comes down to small *maneuvers.* Today, as in the past, our teachers, instead of choosing the subject of mathematics, choose a particular Mode of Engagement for math, based on our philosophy for how to teach mathematics, rather than just choosing a page in a mathematics text to teach.

For example, if a teacher believes philosophically in a constructivist approach to teaching mathematics, then designing a mode for brainteasers must be based on students constructing their own knowledge while solving the brainteaser. The approach we chose was using perplexing puzzles to captivate their attention and require meaning-making to occur in order to solve the puzzle. The teacher support required for this specific mode is primarily non-directive; the teacher intervenes with the students' construction of meaning almost solely by asking questions. As teachers observe students' work, they may use their questions to make connections to the students' previous knowledge, or teachers may employ manipulatives to assist student understanding. Having listened closely to the meaning expressed by the students, teachers might provide students with particular questions or strategies they could use themselves without her prompting. Lastly, the teachers would have to specifically design the maneuvers that each child might need to grasp connections, using manipulatives or other strategies the teacher has chosen. These maneuvers rely on a teacher's understanding of each student's learning and how it works in concert with the taxonomic design of teaching. Figure 24 reveals our Taxonomy of the Pedagogy of Teaching.

Taxonomy of the Pedagogy of Teaching

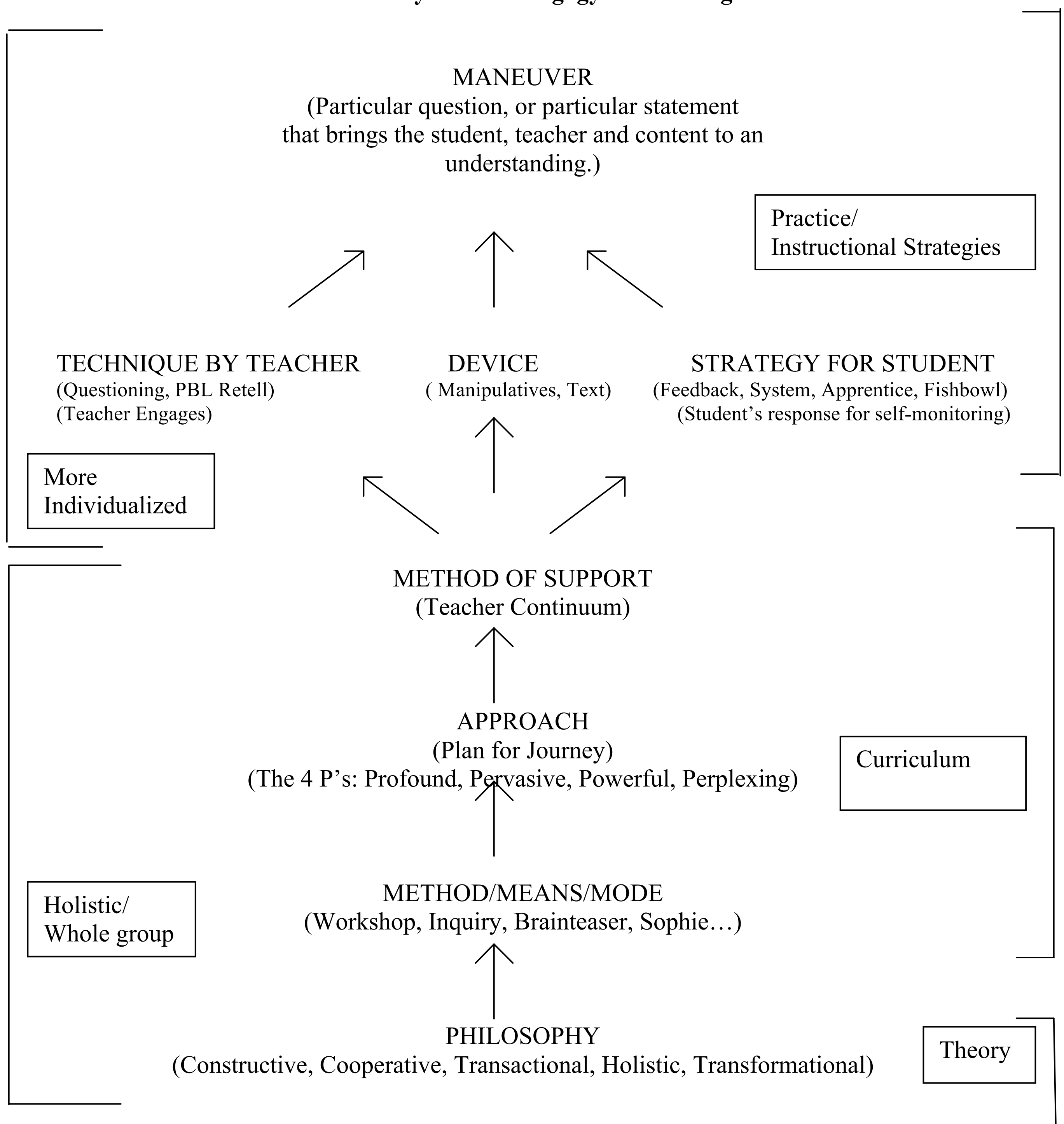


Figure 24. Taxonomy of the Pedagogy of Teaching from the Family School *white notebook* of school-wide important documents.

Though our Renaissance brainteasers are similar to the brainteasers of today in structure, there are differences. In the Renaissance, the teachers and I alternated math brainteasers with Mobius Math, alternating two weeks of each. Today, we have not been able to get to the brainteasers as often as before. To understand this change, we will return to an analysis of what the documents show.

Themes in the transition from Dark and Middle Ages to the Renaissance

Metacognitive strategies emphasized first. In contrasting the themes of the documents between the first two periods with the Renaissance period, it is revealed that our beginning brainteasers were mined for problem-solving methods, metacognitive strategies, and brainteaser-specific procedural knowledge as the teachers and I *accidentally* discovered math content and mathematical skills that could be developed. In the 1990s, encouraged by the SCANS Report for 2000 (1991), we were focusing on preparing students to contribute to the workforce by learning to be able to think in teams to solve problems. The SCANS Report connected school-to-work skills that were needed for students to have upon graduation from high school. It asserted, "All American high school students must develop a new set of competencies and foundation skills if they are to enjoy a productive, full, and satisfying life"(p. i). It proposed five competencies in the areas of: resources, interpersonal, information, systems and technology, all of which we appropriately incorporated throughout our curriculum.

We also realized a need for students to experience curriculum that let them see themselves practicing the actual work of professionals. We have called it our *Self-As* strategy. The origins probably came out of our writing, reading and workshop (integrated social studies and science units) curricula. Our writing curriculum had a strong component for getting children to experience themselves *as authors.* In workshop, which is the mode for teaching integrated science and social studies hands-on activities and projects, we were again very influenced by the SCANS Report for 2000 (1991). The SCANS report encouraged us to create a rubric for our school around students learning how to organize resources, work with others, organize and interpret information, work in groups, and use technology. We attached this rubric to our Workshop Mode, which demonstrated our acceptance of our responsibility for helping our students to see themselves as having the skills needed for professional effectiveness. While learning content standards, students could also experience *self-as biologist* or *self-as historian.* Self-as mathematician was not far behind. Brainteasers allowed us an opportunity to create a mode where students could operate as mathematicians.

What I infer from this section is that the brainteasers' origins in our classrooms, in the early eras of the school, had a primary emphasis on teaching students metacognitive strategies for problem-solving. While students and teachers discussed mathematical content in the solving of the brainteasers, mathematical content was not the primary focus of the lesson.

In the Renaissance era, this was no longer the case. As the teachers collaborated on their brainteaser units, the mathematical content took on more prominence. Metacognitive strategies were still taught, in addition to the mathematical content emphasis.

Supplementing with more content development next. During the Renaissance, our brainteasers began to be mined more for content, as we collaborated as teachers to create units together. For example, when teachers built a brainteaser unit on patterns of numbers, the rubrics called for students to create t-charts, graph patterns from the data sets on their t-charts, demonstrate different scales of numbers on their graphs, describe the rule for the pattern, and name the expression of that rule, just to name a few. Prior to this, patterns of numbers were taught in such a way that students had many ways to display their method of solution, and we focused more on how students came up with their thinking, or justified their thinking, than we did mastering the content on the rubric. The Renaissance Age rubrics reveal that brainteasers began to be evaluated for varying levels of mastery: novices, apprentices, practitioners, and experts. In addition, as a remnant of the Middle Age, some rubrics still required metacognitive strategies to be demonstrated.

Table 17 below shows that rubrics began in the Renaissance and only existed for two eras.

Table 17

Number of Brainteaser Rubric Documents Analyzed

Brainteasers	1990-1995 Dark Age	1996-2001 Middle Age	2002-2007 Renaissance	2008-2010 Exploration	2011- present Reconstruction	Total
Number of Rubrics Analyzed	0	0	5	4	0	9

The new rubrics indicated the listed criteria of content needed to attain a desired performance at each level, yet still focusing on the processes needed to demonstrate how to solve them, and the metacognitve strategies a student would need to demonstrate during the unit. One document example of this time period reveals a focus on the process of solving brainteasers by requiring students to: demonstrate multiples methods, write out their thinking, and/or label and play a mathematical game with oneself for verification of some idea. Another 2005 rubric required the student to be able to state the purpose of brainteaser problems and what they were teaching conceptually about the major operations. In order to meet these expectations, students would have to rely on more than just their content knowledge, using deep understanding of the mathematics, as well as playing with the process to further their understanding. Sometimes these rubrics were encouraging students to rely on practices in mathematics that could move from guesses to proof, or from intuition to definable steps of problem solving.

Burton (2001) contended that mathematicians value the use of intuitive or insightful process in solving mathematical problems. Her study also revealed that there was a wide variety of thinking styles across the 70 mathematicians she interviewed, suggesting that problem-solving includes many different methods of seeing and solving problems. Lastly, she found that mathematicians valued connectivity from one area of mathematics to other areas of mathematics or to the real world. Burton (1999) lays out her argument that classroom mathematics should value these metacognitive strategies, as well. She believes that making mathematical arguments requires knowledge, but that learning to make mathematical arguments "is a more important part of learning mathematics and, in any case, cannot be done in the absence of content (p.31).

She observed from her interviews that "the practicing research mathematicians speak with such enthusiasm and joy of their practices," yet many of those practices were "absent from mathematical literature"(p.32). It is complex and daunting to begin to teach students to frame mathematical arguments, but the brainteaser mode provides the unstructured *messiness* of real mathematics to provide opportunities for students to develop capacities not fostered by textbook mathematics.

By looking at our rubric requirements in the Renaissance, it appears that the first ten years of Family School while practicing brainteasers with no real curriculum built around them did develop our ability to build units that were able to capture Burton's spirit of mathematical problem solving. This was followed by the evolution of additional layers of brainteaser techniques and learning strategies, including a shift in the direction of mastery of content and the assessment of that content knowledge. These layers may have been influenced by Cognitive Apprenticeship as a teaching model, a model that reflected what we believed about the complexity of educational environments.

Collins (2006. p. 50) provides an excellent overview of the four dimensions of Cognitive Apprenticeship: content, method, sequencing and sociology. It is the way in which Cognitive Apprenticeship stretches the notion of content to include metacognitive strategies that corresponds to our model of teaching in the brainteaser mode:

Table 18

Cognitive Apprenticeship Strategies

Domain knowledge	Subject matter specific concepts, facts, and procedures
Heuristic strategies	Generally applicable techniques for accomplishing tasks
Control strategies	General approaches for directing one's solution process
Learning strategies	Knowledge about how to learn new concepts, facts and procedures

Table 18 shows the various levels of teaching proposed by the Cognitive Apprenticeship model. Teachers must employ multiple strategies depending on their purposes. One type of strategy is employed when teaching concepts, and a distinct approach when teaching students how to hone their metacognitive learning skills. Our brainteaser units had taught us just this about instruction. The Cognitive Apprenticeship model confirmed for us that the students could also be involved in understanding the various levels on which they can learn to operate as they learn.

One other aspect suggested by Collins (2006) is important in sequencing curriculum so that it is delivered with the global concepts before the local skills. Our brainteasers were built around the bigger pictures of mathematics, such as place value in other number bases, modular mathematics, “names of one”, transformations in graphing equations, complex measurement, and conversion units, designed to pull students into these topics well before they had mastered the needed skill levels to deal with them. Yet, these challenging concepts captivated their thinking and the skill needed were scaffolded with technology or through the help of older students. Subsequently, the students encountered and manipulated mathematical concepts well before any curriculum would have asked it of them.

From this I infer that effective strategic mathematical teaching integrates the teaching of content and metacognitive learning strategies for students. This time of the Renaissance was a very generative period for the Math Brainteaser Mode as evidenced by the multitude of documents. The documents begin to reveal our growing confidence in the Brainteaser Mode.

School and state documents. While the district-mandated Program Reviews from the previous ages did not mention brainteasers, the EPSS (Education Plan for Student

Success), the Renaissance goal-oriented school document for the State Department of New Mexico, during the raging race to reach the 2014 goals of NCLB, did describe brainteasers as a Family School strategy for raising our percentage of proficient students in mathematics in the school years 2004-2005 and 2006-2007. With 82% of our students proficient in mathematics, in 2006 we stated that with teachers doing lesson studies in brainteasers, and having students participate in brainteasers, we would be able to meet 85% proficiency. By 2008, our EPSS stated that 87.9 % of our students were proficient in mathematics. While it cannot be assumed that a single strategy determined the improvement, it is interesting to see brainteasers finally made it into professional documents about the school, especially one so public, in a time of great pressure to meet the standards of the infamous NCLB march to 100% proficiency. Our shared belief in, and commitment to, the Brainteaser Math Mode was finally public.

Our own general school documents in the Renaissance also touted brainteasers regularly in: report cards, self-assessment checklists, mathematics boot-camp topics, instructional improvement documents, principal observation sheets, and even the school's Parent Handbook. Family School narrative report cards are a good source of documented evidence that reveal our purposes for the Brainteaser Math Mode. Below are a few examples from our narrative report cards revealing how important it is for students to work interactively with other students in the learning community:

•She enthusiastically jumps into the brainteasers for each day and settles into an alert posture at the front of the room. She works bent over extremely close to her page, only coming up for an occasional breath of air to check her thinking with those close by. •She has developed her ability to think through brainteasers with deftness and tries every problem solving strategy taught. She enjoys thinking through problems with students her own age, but will not seek out teaching opportunities of the younger students, however, she is eager to work with them in a parallel way when they ask her. This may be due to the fact

ask the specific questions other students' need for their thinking.
•She worked hard in the measurement brainteaser unit to be able to learn how to do conversions more easily. She learned how to utilize the resources available in the room. She also made resources for herself to be able to use in the future—on tests that allow you to bring in some notes. She worked on how to organize her thinking this year by using mind maps.

Figure 25. Narrative report card excerpts from my computer files.

Other narrative reports reveal how students develop important learning habits to improve their ability to solve brainteasers:

•He also seems to be getting more out of brainteasers this year, by being more actively engaged in solving them, studying them, and asking questions about them. He is also learning more how to utilize the classroom environment to learn. Other students enjoy working with him, as he becomes more engaged in learning from them.
•During brainteaser math, he was able to explore the unit on major operations unit on bases, algebra and abacus operations by exploring his strengths and weaknesses.
•He loves to work through math brainteasers and will often take them home to work through them again or correct the errors he had made during the day.
•In math brainteasers, she struggles to stretch to the high level of content, but with careful prodding she is most capable of seeing the connections. It's as if the strangeness of the new math is itself a hindrance in understanding, as if they don't really exist, though she is building bridges to these new concepts, no matter how tentative they are.
•He enjoys the math brainteasers and will work diligently to get through a problem with whatever strategy is taught. He is becoming most adept at these problems and loves to share his understanding of them with his peers. He always has good questions to ask about conceptual understanding of the math.

Figure 26. Narrative report card excerpts from my computer files.

The performance of the content of mathematics comes through as also important in the teaching of brainteasers in the following excerpt:

•She has had much previous experience with algebra in brainteasers and feels comfortable with the basic concepts of the discipline. She thoroughly enjoys the brainteaser math and will eagerly work through the problems, but is demonstrating some difficulty performing on the tests. It will require her to carefully understand the rubric of the test and work in devising an effective study method for the test's objectives.
•For math brainteasers, he is still learning to work meticulously in order to head off his knack for making careless errors in his enthusiasm to solve the puzzle. He is really developing in his ability to be much more thorough in his thinking in problem solving.
•In math brainteasers she may start off slowly, but usually finishes with a bang. She talks about being uncomfortable with the unknown, which she has obviously learned to deal with in story problems, but not in math puzzles.

Figure 27. Narrative report card excerpts from my computer files.

These examples demonstrate the purposes we set out to reach and highlight in our teaching and in our assessments of how students responded to brainteasers. Predominately, they reveal our focus on concepts, problem solving, critically thinking and collaborating with peers. For some students, this takes time and is quite difficult; for other students, the first few times are incredibly difficult, and then they grow to love the ambiguity; still others love doing brainteasers right from the start. Those narrative report cards reveal the Renaissance period to be filled with great passion and commitment for the teaching of brainteasers.

The interesting pattern of brainteasers becoming a more collaboratively-planned, school-wide activity resulting in the teacher-generated brainteaser units showed that the teachers and I traded the *authentic* brainteasers from brainteaser trade books for units we designed. These teacher-generated brainteaser units allowed staff to use high-level mathematical content from geometry, numbers and operations, data and probability, measurement, and algebra to further the student's understanding of mathematics. While these topics were covered in other math modes, we wanted brainteasers to teach them in a deeper and more problem-based manner. Because of wanting brainteasers to meet this new purpose of the new curriculum standards, yet still meet our purposes of providing challenges for the students, this teacher-designed content emerged as a prominent type of brainteaser document.

Table 19

Kinds of Brainteasers Used Throughout the Ages

Types of Brainteasers Sampled	Dark Age	Middle Age	Renais-sance	Age of Ex-ploration	Recon-struction	To-tal
Authentic Brainteasers	11		5	3	5	24
Harder Material form General Math Books	0	5	5	0	4	14

Teacher-generated Brainteasers	0	0	4	6	1	11
Total	11	5	14	9	10	49

I infer the documents reveal a growing sense of school pride in our brainteaser units, feeling that our brainteaser strategies and content were translating into improved math abilities for our students as evidenced by their performance on the state- mandated tests. Teachers became more comfortable with the instruction of brainteasers as they collaborated on the units they designed. Student report cards reveal the many benefits the students gained from their participation in this mode.

Even with this success, the Math Brainteaser Mode was about to take a turn. In the next three sections, you will see how the pressure to perform and reach proficiency eventually turned the tide on our brainteaser mode. Three vital documents, *The Performance Cycle,* the *8 Points,* and the *Data Dialogue* documents, which I created, with input from the other teachers, contributed to the end of brainteasers.

Teaching brainteasers for mastery using the performance cycle. It should also be noted that the brainteaser rubric emerged for the first time in 2005, which means that it was considered to be important to put the Math Brainteaser Mode through the school- designed *Performance Cycle* (Figure 28), requiring a performance opportunity for the students to demonstrate their mastery of a brainteaser unit. Prior to this, brainteaser was not taught for mastery. The emphasis had been on the problem-solving and metacognitive work as being more important than the mathematics content, and now, with the organization of the mode involving collaborative, school-wide agreement, accompanied by the added pressure to raise proficiency rates, everything seemed to focus us toward demonstrating mastery. While we thought the Performance Cycle was a great tool to help our students become better prepared to perform, unbeknownst to us at the time, the cycle must be used cautiously, or it can have negative impacts, as the brainteaser story will reveal.

The Performance Cycle is another document I created in 2008 for my teachers to help us move from the *process only* orientation of our Dark and Middle Ages. While focusing on teacher best practices in those first ten years of the program, we developed an extreme affinity for providing a great deal of process time for students to learn. We provided so much hands-on, project-oriented, and inquiry-based instruction, that we were not very attentive to helping the students perform their newly-found knowledge on tests, quizzes or authentic task performances, as in demonstrations or portfolios. While there were high-stakes tests in those days, they were based on percentiles, and we all knew that getting 100% of our students to the front of the line was not feasible, so we couldn't take the test seriously. Although we knew 100% was probably not a reality, improving all students' proficiencies was possible. However, having focused so much on process originally put our students at a disadvantage when testing. The Performance Cycle was created to get teachers and students to understand that one cannot be prepared for, and improve in, performance without going through all the stages of the Performance Cycle. We likened it to getting ready for the big game in sports. There is process time to improve skills, develop understanding of the game, learn how your coach coaches, and learn how to interact with your teammates, but one cannot go into the big game prepared without going through more structured rehearsal, such as scrimmages. Then players are ready for the game, which is the performance. After the game, a team debriefs, closely considering its performance in order to know how better to repeat the cycle, and improve the performance, for the next game.

Once we had this Performance Cycle document, we began making sure that each mode was run through the entire Cycle so that we could show that our students were not just immersed in process but could rehearse and ultimately perform whatever they had come to know. The Renaissance presented this notion rather innocuously because the rubrics for performance still focused on process, but it was to have grave effects in the Exploration Age, which will be explained in the next section.

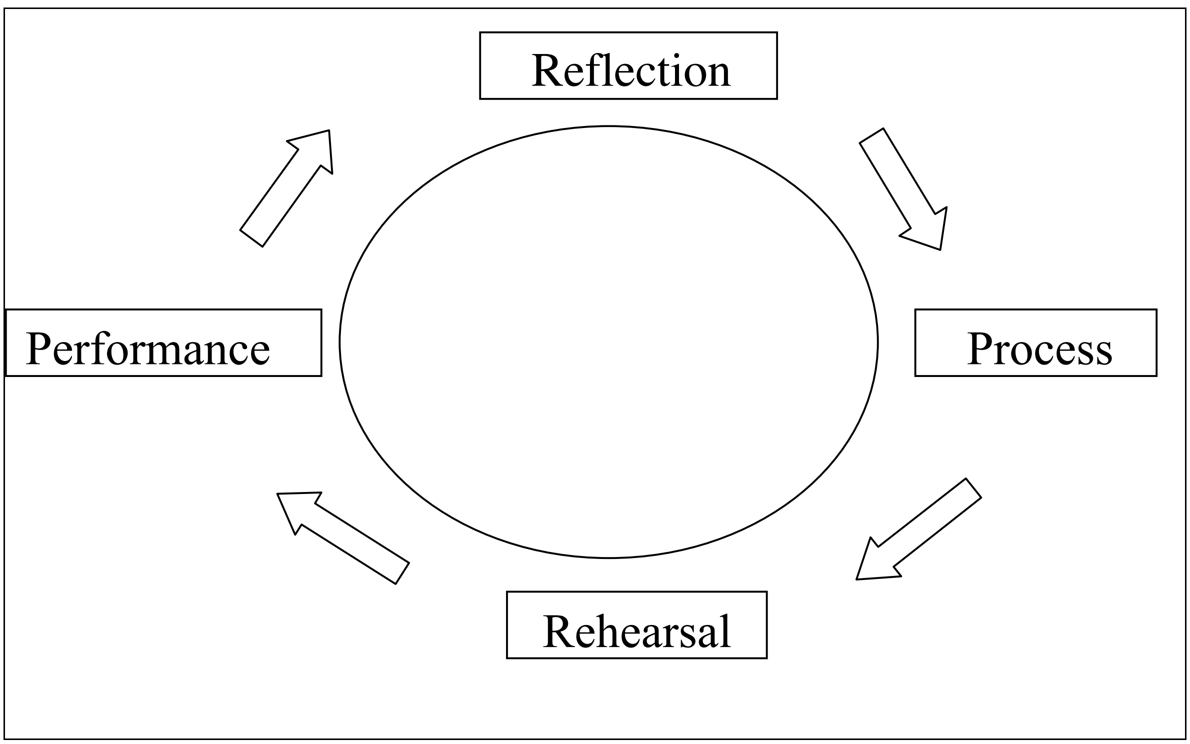


Figure 28. The Performance Cycle demonstrates the importance of curriculum going through the entire cycle of learning while adequately preparing for performances.

While the creation of this document was well-intended and meant to help prepare students for taking tests, applying it across the board to everything that we taught was a bad decision on my part. In my enthusiasm to hold teachers accountable to what they taught, I mistakenly believed that all teaching must result in mastery learning.

I no longer believe this, and now believe that some content should be taught by only proceeding from process to rehearsal, then back to reflection and back to process. This cycle allows some students an opportunity to develop process skills that can become stifled if always tested.

Themes in the Renaissance Transition to the Age of Exploration- 2006- 2010

There are some commonalities in the Renaissance and Exploration eras, as well as some significant differences. It is important to focus on the commonalities first to demonstrate the gradual impact that the political pressure was having on the school. As implied in the name of *Renaissance,* it is evident that the school experienced great freedom to create and explore the brainteaser units. With more emphasis on metacognitive and problem-solving strategies, and less on content, it is easy to see that we used brainteasers to fulfill a part of the mathematical curriculum where students can experience the joy of playing with mathematical concepts while dealing with the unknown. A process that mimics the *search* behavior that Burton (1980) talks about in her article about how to use brainteasers in the classroom is the main focus of the Mode of Engagement. There is not any directed teaching in brainteasers, and students must work together to generate the curriculum to be discussed that day. With the beginning of our collaboration efforts as teachers, we did begin to focus more on the mathematical content of the brainteasers, but mostly as a way to discuss what the brainteaser units could be about, and not so much about mastering this content through the brainteaser.

Selected rubric themes from Table 20 show this progression from an emphasis on content-free, process-oriented brainteasers to much more heavily-embedded, content-rich brainteasers:

Table 20

Rubric Analysis for the Renaissance and Age of Exploration

Rubrics	Renaissance	Age of Exploration	Total
Content	23	20	43
Process	20	1	21
Performance	24	4	28
Metacognitive	9	3	12
Scope and Sequence	21	22	43
Total	97	50	147

Evidence of content was just as frequent in both eras, and the evidence of a documented focus on metacognitive strategies also decreased from the Renaissance to the Exploration Age. Some of these changes in the shift from process to performance began toward the end of the Renaissance, around 2005. This shift may indicate the pressure NCLB was placing on our school and that the teachers and I perceived it was necessary to use rubrics to document mastery to make sure that no child *was* being left behind. By shifting our focus to content on our rubrics, it shifted our teaching. We lessened the time spent on process and spent more time teaching the mathematical content, which was often a higher-level content required for district and state testing. Some teachers rushed through the brainteasers to make room for other mathematical concepts that were needed for standardized tests. Still, we forged ahead, striving for mastery on brainteasers, as a response to the NCLB pressure that the school was feeling as the 2014 deadline date was fast approaching.

This is also mirrored in the school's professional development agenda data, which reveal themes of assessment agenda items increasing over 100% from the Middle Age to both the Renaissance and Exploration Ages. These agenda also reveal a new resurgence in our discussions about how to improve our teaching. Our *8 Points* document, which I composed in 2006, really put the pressure on teachers to realize what effects their teaching had on students' performance. I formulated the points as a series of interrelated propositions. Figure 29 presents our *8 Points* document:

1. Students can perform only as much as the teacher knows.
2. Students can perform only as well as the teacher's instruction can teach.
3. Students can perform only as well as the teacher is able to know the students' thinking and learning.
4. Students can perform only as well as teachers are able to teach them about their responsibility to their own learning and metacognition patterns.
5. Students can perform only as well as any particular mode of engagement's intent and purposes are clearly taught to students and parents.
6. Students can perform only as well as the teacher evaluates the students' work and communicates effectively to the parents and students what they have already performed and where the need for improvement is.
7. Students can perform only as well as the teacher is able to create an environment and culture that promotes excellence in performance.
8. Students can perform only as well as the teacher prepares students for performance.

Figure 29. 8 Points document created to increase teacher's awareness of NCLB accountability from the Family School *white notebook* of important school-wide documents.

While now I cringe a bit when reading this document because of its pedantic tone, it reveals an obvious truth. In the midst of the pressure I was feeling from the district and state, I needed to convey to my teachers how much they needed to work on their accountability. Teachers often have a tendency to have the processors (the students) carry all the responsibility. Principals have heard, all too often, teachers exclaim that they had taught the subject matter, and it was the students' fault they hadn't learned it. With this document, I hoped to raise the teachers' awareness of their own agency in their students' learning.

However, with all honesty, it does really put the pressure on teachers, and because of the emphasis on mastery learning, and the push for 100% proficiency, it was hard to swallow all this responsibility, too.

Here again, in my enthusiasm to help teachers become more accountable in their teaching, I created a formal, *official,* school document, which, while it served a purpose for teachers to see the need to improve, ironically conflicted with the fact that an essential condition of the Brainteaser Mode was that much of what was learned (especially metacognitive, collaborative strategies -- critical thinking strategies -- are not easily testable or measurable.

The Great Divide between the Renaissance and Exploration Ages

The dividing moment between the Renaissance and the Exploration ages occurred in 2008. The pressure to perform and see increased data points for NCLB encouraged the creation of *data dialoguing* the brainteaser performances. The data dialogue document was a form used to collect all the performance trends of brainteasers at a whole-group staff meeting where teachers would discuss how to respond to the success and failure of students' performances at the end of each brainteaser unit. My memory of those discussions is that there were many tears and hurt feelings when the teachers' class scores were analyzed and discussions for reteaching methods exceeded the teachers' limits. The document directed our discussion around the following categories: 1) Content, 2) Instruments, 3) Strategies, 4) Maneuvers, and 5) Barriers. Requiring students to master the brainteaser content was too much for some teachers to bear. They felt that since the content of brainteasers was not reflected on the state test, that their students should not have to master the content of the brainteasers. Their frustration was that the brainteasers required too much of the class time, which was a precious commodity in a half-day program. They also felt that the complex problem-solving approach to teaching took too much time as opposed to a more strategic, directive approach like that of the Algorithm Math Group Mode.

By 2010, one agenda finally revealed the end of brainteasers. Changes were being made to our curriculum to allow teachers more time to teach content that was *on the test.* Math brainteasers were cut from our suggested schedule. I continued to squeeze them into my classroom when I could, to prove a point to my teachers and students about what I believed to be essential mathematical habits, but, for the most part, brainteasers were absent for two to three years.

One of the reasons my teachers and I acknowledged that brainteasers should re-emerge as a Mode of Engagement was because of an incident a couple years back. I had noticed that my students were no longer tolerant of solving problems that were difficult, and I mentioned this at a staff meeting. One of my teachers chuckled and remarked that a student in his class that day had stated the very same thing. This student volunteered to work with another younger student in the class on the mathematics content that the teacher had been trying to teach. It was apparent that the younger student was not able to comprehend the lesson. After the older student spent some significant time with the younger Family School student, he told the teacher that he felt it was not easy for some of these younger Family School students to learn how to do these more difficult mathematics problems because they did not have the experience of solving math brainteasers, as he had *in the old days* of the school.

I didn't find it surprising that both the student and I had uncovered this issue, and I found it a great opportunity to open the discussion with my teachers about bringing brainteasers back. There was a shared excitement about this possibility, but at just this point, the Common Core state standards were adopted, and we needed to spend many hours learning how that would impact our modes. We are just now (2013-14) getting back on track to bring brainteasers back on a more regular basis. They occupy part of my schedule in my classroom, and my journaling indicates many new themes concerning their re-emergence in my room. Professional development agendas reveal that we are providing support for teachers to begin doing brainteasers in other classrooms, as well.

What I infer from this is that because of the external pressure of the NCLB race to 100% proficiency and the internal pressure of my over-enthusiastic administration, Math Brainteaser Mode was not able to withstand the political agendas of the time. Again, being well-intentioned, we abandoned brainteasers to afford more time to improve test scores, but even then, brainteasers were resurrected due to the power of their purpose.

How do brainteasers function in my classroom presently? My reflective, personal journal excerpts and analysis will reveal the current status of the Brainteaser Mode at Family School.

Reconstruction -- 2011-2014

In my analyses of all of the modes of today, my journaling reveals most clearly the importance of bringing the Math Brainteaser Mode back into our repertoire. Originally, my staff and I had known the value of its purposes, though those purposes were eroded with the legislated purposes of NCLB and even Common Core assessment. Though the Common Core curriculum provides much that suggests that critical thinking, such as that promoted by brainteaser math, is required, the assessment design encourages teachers to madly teach to a particular kind of problem, most likely, in form and content, to appear on a standardized test.

How is it that those political purposes take hold with such a death grip that schools can lose sight of what they know works? On January 13, 2014, I wrote of returning to some of the brainteasers I had used in the very beginning of my career. I rediscovered what I knew then—that students loved to be tickled with these problems, and that children must experience the excitement of doing mathematics.

During this journaling, I came across Hersh and John-Steiner's (2010) book challenging myths about mathematical lives, and I was reminded of the importance of standing firm against political whims. In my journal, I wrote about Hersh and John- Steiner's first chapter about the childhoods of mathematicians:

> The first chapter about their childhoods reveals a great love for playing with numbers and solving puzzles. Their persistence, dedication, and curiosity are completely transparent, yet our children do not display these qualities. I do believe I can reawaken that with both brainteaser and inquiry, if I have not *mastered* them to death from all that mastery teaching!

Allowing children the possibility to find challenging excitement through mathematical puzzles at the edge of their ability seemed so risky to my teachers and I at the beginning of our journey that we didn't even think it important enough to talk about. Yet, we began to see how doing away with brainteasers impacted students' performance in other math modes, I wrote:

> Our roots were solid and while it is important to grow, sometimes we grow astray of our own important standards. It sort of reminds me of my overlooked roses in the front yard. Because of neglect, they grew so tall that they outgrew their *roseness* and had to be cut back to nothing. I don't even know if any of them will grow back. It is

> not that we neglected inquiry or brainteasers (at least for very long), but we have pushed their value so far back, it reveals that there was something very profound that was neglected.

My teachers and I wouldn't have noticed that brainteasers shouldn't have been neglected if we hadn't seen how the other modes were impacted by their absence. It was seeing our students' inability to stretch beyond their grade-level mathematics, their lack of stamina for wrestling with the unknown, and their lackadaisical affect related to mathematics that had us longing for the brainteaser wonders of the days of yore. Much of my journaling about the Brainteaser Mode of today reflects the discovery of how all of the modes support one another, and each serves an essential set of purposes. We may well discover and invent new modes, and we will certainly continue to deepen our understandings of our existing, ever-evolving, modes, but it is clear that mathematical thinking is fostered by the entire array of modes, and that the loss of one of the modes weakens the overall coherence of the math program at Family School.

My March 5, 2014, journal expresses two major examples of how brainteasers had supported the other math modes. In following the students' understanding of all the transformations of algebraic functions in one brainteaser unit, it was revealed to me that students could not adequately explain a deep understanding of the function unless it was done graphically, formulaically and algorithmically. This triangulated explanation of functions was turned into a protocol for students to use for mathematical explanations of their problem-solving in other math modes. I wrote:

> In the algebraic functions brainteaser today, I noticed that my high students navigated through all three representations (graphical, formulaic, and algorithmic) while

explaining their thinking without being requested to by me. They needed to connect all three to generate the meaning they wanted. My middle students, however, might connect two, but mostly would gravitate to using one representation only to make meaning. My lower students could only use one, but could not make meaning with only one. It was with them that I realized how important it is to use all three, then they could see the meaning that could be made out of each, but not until they saw how they came together.

The graphic representation, whether a diagram, table or chart, is really what made the comprehension begin, yet they (the lower students) will insist on NEVER doing a graphic representation in their work. It is almost like those poor readers who do not see a movie in their head as they read, so they are just reading words and a few phrases here and there.

If it were not for the brainteaser emphasis on student-initiated problem-solving strategies and explanations, I would not have been able to see the importance of students needing to represent their knowledge in three ways.

The next discovery I had was that while brainteasers built up the students' abilities to be able to dig deeper into their mathematics, and have more tolerance for ambiguity in mathematics, it also impacts the teachers' teaching when they see their students growing in mathematics differently than before. In other words, the teacher sees another means of working with a student that seemed closed to that student before being observed in the brainteaser. Also, student-learning habits begin to change, and the student pursues mathematics differently than before being exposed to brainteasers.

My journal also provided insight into how my reflection about my brainteaser teaching had changed over the years, and not always for the better. In reviewing the purposes of brainteasers, I discovered a flaw in my teaching that had developed over the years. It is important for me to see that different purposes stretch me as much as the students. I wrote about a day when I saw I had short-changed the students in brainteasers because I had grown so adept at teaching and solving brainteasers over the years that I had forgotten the purposes by rushing them through some of *my* steps in *my* process too quickly. I wrote:

> I remember not knowing how to solve these a long time ago and now that I do know, I think I give way too many clues. It's not because I don't want to see the students be challenged, because I see enough of that already in other ways. But I think it is because I am rushed. I feel like there is so much material to get through in a day, I don't have that relaxed attitude you need to do brainteasers. I am definitely going to play around with this tomorrow.

Not only has my reflection in my personal journal brought my own teaching to center stage, but my self-reflection has also been focused on all of the teachers. The teachers and I chose to do school-wide observations and evaluations focusing on different modes for each evaluation required. To prepare the teachers for the new, state- mandated, teacher evaluation process, we reviewed the rubric used. We then videotaped our lessons to have discussions about how to improve their teaching in order to meet the state rubric for proficient teaching. My journaling about the brainteaser videotapes reveals that though brainteaser was a brand new mode for some of them, and only minimally-experienced by others, they were very excited to improve their teaching. For one, the students responded so enthusiastically to the brainteasers that the teachers wanted to find out how to improve their teaching. Secondly, they saw themselves as *real teachers,* watching their students enthusiastically want to investigate, use multiple methods, discuss their various processes, and take pride in developing their mathematics' skills. The teachers were as excited as their students to share their videotapes with each other. I summed up the professional development activity by writing: "They [the teachers] are hooked now and want to bring back brainteasers more than ever."

The discussion of teaching brainteasers at staff meetings shows that, when we discuss teaching toward the brainteasers' purposes, we realize that teaching within a mode does constrain our teaching, but this, in turn, allows us to be able to have a common language about our teaching. It is easier to help teachers to improve when we all understand each other as we discuss our teaching successes and dilemmas within a bounded mode. Although the purposes constrain the instructional strategies, they require a certain flexible structure, which ultimately frames those strategies.

The most important change to the structure of the new Brainteaser Mode was taking it off the Performance Cycle that required mastery learning. It was clear that while highlighting learning targets within the brainteaser process did not take away from the purpose of discovery and challenge, unit tests of brainteasers were not acceptable for this mode's purposes. The other change to the structure has been in how often we can find the time to fit in the brainteaser units, given the continued emphasis on testing. Thus far, I have been able to do many more brainteaser units than the other teachers, even teachers who have used them in the past, which might mean that they have not seen as much value as I have seen in having brainteasers return to the classroom. However, many teachers have revealed that their student self-assessments showed them how much the students loved brainteasers and that they did want to use them more often. We will have to discuss highlighting their placement more for next year (2014-15).

The professional development that was focused on what we could learn from the videotapes of ourselves teaching the various math modes demonstrated that we still use the original structure for the brainteaser lessons, and their success is revealed in the student engagement. Based on these videotapes, it appeared that the purposes of the mode are being borne out in the structure of the lesson and the engagement of the students. We continue to play around with ways to improve instruction of content, metacognitive strategies, and formative assessment of learning targets. Understanding the difference in learning target mastery and brainteaser mastery is a very important lesson for us. On this I wrote:

> The lesson we had to learn is that edgy purposes are not a good mix with the Performance Cycle. We can have targets be about pieces of it around their grade level or close to grade level mathematics used, but it is pointless to test them on problems that are at the end of their zone of proximal development unless they are given teacher support to reach the end of that zone during the performance.

These journal excerpts reveal the multifaceted context for learning about teaching that the Math Brainteaser Mode provides. Most likely, this occurs because it is a mode where the students can take the lead, which leads to more unpredictability. With students leading, teachers must learn to follow the thinking of their students. In teaching-as- telling, it is easy for a teacher to lose all awareness of actual student thinking. This mode provides a rich context to learn about teaching.

🕮 NCLB Story Revealed from the Math Brainteaser Mode: ☹The pressure for our students to perform for NCLB testing accountability issues had mostly negative impacts on the Math Brainteaser Mode. Because mastery learning became so prevalent with NCLB and we wanted to make sure that students could perform what they had learned in brainteasers as far as the math content they could teach, we shifted our brainteaser units to be focus on content more than process. Before NCLB, we were more interested in using the brainteaser unit to teach students about the fascinating process of solving problems at the edge of their understanding of mathematics. Since the testing pressures required mastery, we thought it important to use every mathematical opportunity to move students forward in their mastery of math content and our rubrics demonstrated that our teaching became more focused on students learning the content of brainteasers and less focused on learning the various strategies for solving them. Our instruction moved toward mastery learning and, eventually, the content seemed so out of alignment with the expectations of the high-stakes test that teachers eliminated the Math Brainteaser Mode altogether. The Math Brainteaser Mode returned to the classroom when both students and teachers realized that while the metacognitive strategies used in brainteasers could be applied to our more traditional math modes, they were not as effectively helping students to develop those mathematical community practices without being embedded in the above-level material of math brainteasers. The Math Brainteaser Mode then returned to the school now that the teachers understood that the math modes operated as complex systems that worked together.

Figure 30: NCLB Story Revealed from the Math Brainteaser Mode

Conclusion

Table 21

Examples of Literature Review Connections Developed

Collaboration	•Once our school was centralized, our teachers became more collaborative about creating brainteaser units to support teachers who were intimidated by the brainteaser content and also created cohesive and consistent practices in this mode across the school.
Teacher as Designer	•Teachers designed scope and sequences of brainteaser units. •Teachers wanted to have a common mathematical experience and self-made brainteaser units for the entire school. •Teachers designed more content-oriented brainteasers.
Reflective Practitioner	• Teachers study brainteaser procedures for solving through their design of the brainteaser units. •Teachers' recursive instruction helps teachers to improve in their teaching of brainteaser procedures for solving problems. •Teachers learn how to design their own brainteasers.
Learning Community	• Teachers learn the value creating and sustaining a true learning community with their students when they learn with their students.

In summing up the story of the Brainteaser Math Mode, there is one brainteaser document worth noting. At Family School, it is a common practice, particularly at the end of the year, for teachers to create whimsical but poignant awards for each student. It has become a way for a teacher to let each individual student know that she has noticed something beautiful, unique, and powerful about each student. This practice of conferring imaginary awards has been taken up by the students, as well. In this case, a student was nominated, by her classmates, for an in-class award for being a "Family School Brainteaser Kid" (2006), which is already suggestive of how, by 2006, the mode had captivated the enthusiasm of students, no matter how hard the puzzles. This award process was also connected to the school's annual Self-Assessment Project, where students are taught to evaluate their own learning for strengths and weaknesses and collect their work to show growth in their learning. This student, having been *nominated,* did not simply get to claim the award. That would be too easy! Instead, she was required to write a description of herself as a learner to persuade others that she was deserving of the title "Brainteaser Kid." Here, she explains the criteria she believes her classmates used in order to determine the award: "Though some math brainteasers might be difficult, the Math Brainteaser Kid won her title by always striving to conquer the dangerous, yet sophisticated Realm of Math Brainteasers. No matter how difficult (and strange) a Math Brainteaser may be, The Math Brainteaser kid never loses her cool."

This student demonstrated that she understood the value of *intellectual courage* and *revising her thinking* from doing brainteasers; two important traits that Lampert ((1990) suggests are essential for students to experience in mathematics in order to build meaning which is "congruent with the disciplinary discourse" (p.58). Her essay ends with a very modest understanding that knowing how to work brainteasers may not be earth- shattering, but she hoped she might serve as inspiration to other students, thus revealing another one of Lampert's qualities: intellectual modesty. She understood how humble one must be regarding one's ability to do brainteasers, for there is always another mindset in which to be "trapped." She continues:

> Right now, The Math Brainteaser Kid is just a normal kid. But in the future, who knows what she can end up being. Perhaps she will grow up to be a great teacher of Math Brainteasers, passing down her knowledge to unknowing little children. Maybe she will tell people how great and mighty [brainteasers are], so that people will be less scared of the Realm of Math Brainteasers, and instead rule there themselves. Whatever she does, many a child could be inspired by this awesome power [of] the Math Brainteaser Kid.

This student reminded me that while the teachers may have felt the NCLB pressure to end this mode, for students, this mode fostered important growth in themselves as learners, and the students themselves identified the mode as developing a unique set of qualities needed to be conversant in that mode. Brainteasers make students feel like mathematicians. For that reason, I was inspired to include these awards in this year's self-assessment unit. My students who were selected for this year's Math Brainteaser Kids beamed with pride as they read their acceptance speeches.

My experience of the joy of brainteasers returning to my classroom was also the experience of other teachers, as they requested discussion about brainteasers at our most recent Summer Seminar (summer 2014). They discussed how to choose brainteasers, what procedural lessons could be found for various brainteasers, and agreed that we should videotape brainteasers lessons again in the coming school year. Having returned our discussion to the purposes of brainteasers for the newer teachers, the staff collaborated on strategies that could be designed into the lesson structure to highlight those purposes. They even discussed the need for teaching the purposes to the students. Our veteran teachers told stories of student enthusiasm for problem-solving, and this reinforced, for everyone, the importance of the Math Brainteaser Mode. We have all agreed that the brainteasers will not utilize the full Performance Cycle, and with that agreement, brainteasers have been scheduled into the school's yearly calendar. Teachers grouped according to their grade-levels and years of experience and began discussion of what brainteasers they would like to use.

We can once again see the value of the process of making meaning. We understand Burton (2002) when she argues that the "purpose of learning is to make mathematical meaning. This does not mean that the purpose of learning is to acquire the mathematical meaning of others but to position oneself as the agent of one's own learning" (p. 158). The Math Brainteaser Mode can respond to the absence of challenge in traditional mathematics education and teach students the value of belonging to a mathematical community. Teachers can also learn the value of teaching mathematics for the purpose of creating a mathematical community in their classroom. This mode offers the opportunity for both students and their teachers to experience mathematics as an exploration of their boundaries in their mathematical thinking.

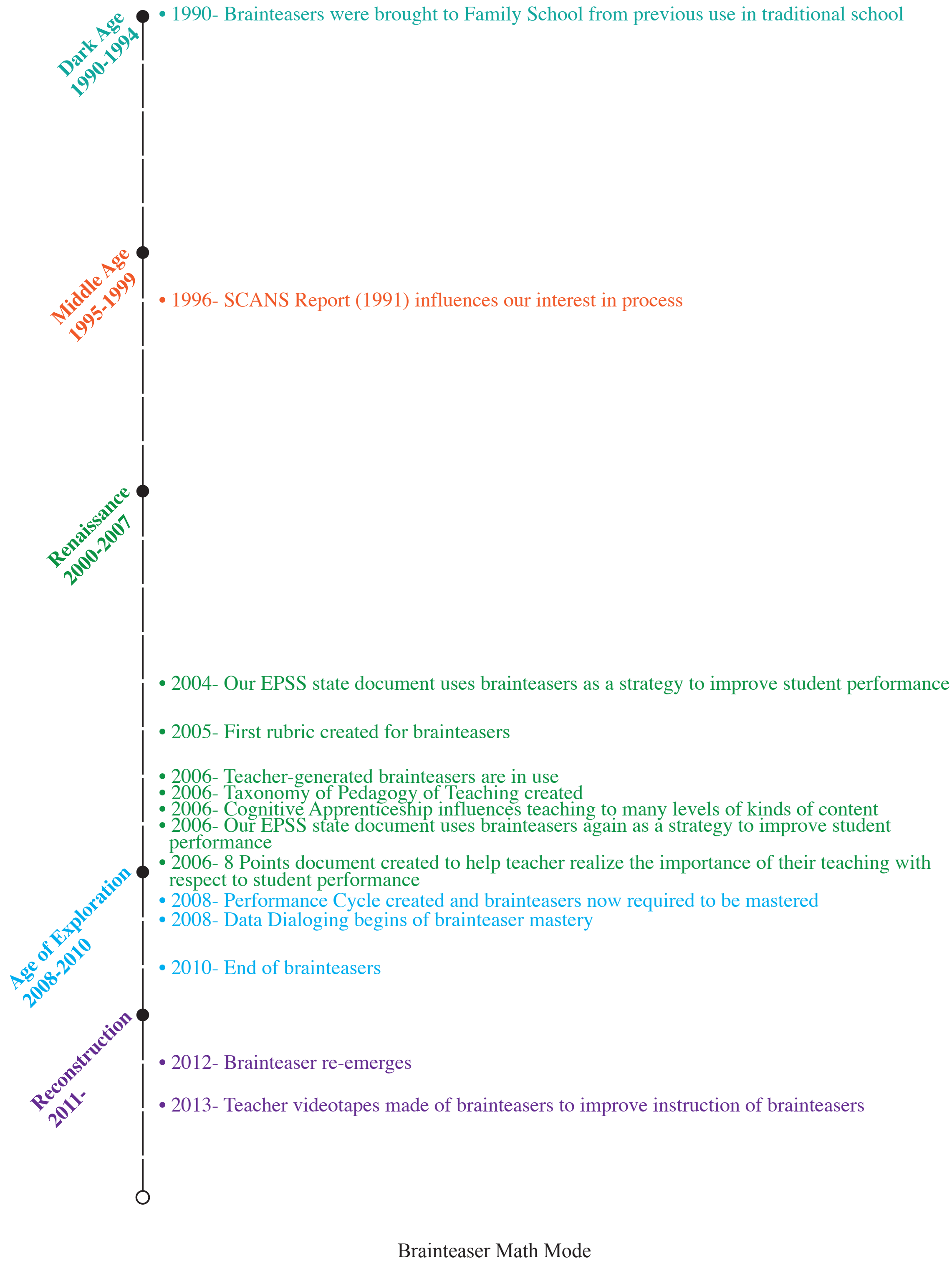


Figure 31. Math Brainteaser Mode Timeline

Chapter 7

The Mobius Math Mode

Table 22

Major Codes and Their Themes for the Mobius Math Mode Creating Patterns of a Plotted Story Line

Mode: Mobius Math Mode		
Codes	Themes	Plotted Line
Purpose	•Simulate a mathematical learning community learning with all ability levels engaged around a problem • Recursive metacognitive strategies used to learn how to engage in above grade-level material	The themes of the Mobius Math Mode create a pattern of a plotted line that reveals the importance of allowing students opportunities to learn mathematics that is well above their ability level. While this mode required the students to master their grade-level material, it also required them to participate in learning above grade-level content. This mode taught both teachers and students the value of working with harder material to help solidify the easier concepts, as well as developing profound understanding of the connections between a variety of math concepts. As teachers improved their strategic instruction to teach a wide range of grade-spans at one time, they became better instructors for finding multiple ways for solving problems and starting with the bigger mathematical concepts first to invite in all levels of learners. The recursive instruction enabled teachers to improve instruction, and the recursive content of the Mobius Key Concepts helped students to develop deep conceptual understanding of mathematics.
Content	•Recursive Mobius Key Concepts developed •Content utilizes both linear and non-linear scope and sequences •Students expected to master linearly-designed, grade-level material, but not above-grade-level material	
Strategic Instruction	•Teachers instruct using larger math concepts first •Students direct multiple methods of solving problems •Teachers teach metacognitive strategies to engage all levels of students	

The Mobius Math Mode provides on-going teaching opportunities for students of a multi-grade classroom to periodically work in their varied grade-level mathematics workbooks as part of a whole group. The challenge of this mode lies in the teacher's challenge to simultaneously teach many different grade-level expectations while practicing whole-group teaching. The workbooks for this mode were selected because they chunked similar kinds of problems into units that matched the curricular benchmarks required of the state standards; for example, problems focused on measurement or data and statistics were clustered together in each grade-level of the workbook. Teaching each unit to the variety of grade levels holds together the learning experience for the multi-age level students. There are three main characteristics of teaching that were designed to meet this structural challenge of whole- group teaching to multi-grade expectations. 1) Teachers present students with metacognitive strategies to learn how to maintain interaction with material that may be either above or below their grade levels. 2) Teachers identify key mathematical concepts that hold together the various grade-level expectations through deep conceptual understanding of how to connect multiple mathematical concepts. 3) Students participate in a self-assessment process that enables them to link together the metacognitive strategies learned with the content of the key concepts to reveal and analyze their learning patterns.

Professional development, in order for teachers to improve their delivery of this varied content, is focused on teaching teachers to present mathematical lessons from the larger concepts as opposed to the procedural skill level. This method should be used instead of teaching what teachers perceive to be as the one-and-only first step into solving the problem, when there are usually many ways into a problem. Using videotapes of their teaching helped teachers to design an Observation Checklist of the structure of the instruction teachers needed to understand and master in order to maximize this whole-group teaching mode towards its purposes.

A non-example of this mode would be to have students working on the same page of a mathematics textbook at only one grade level that is taught to the whole class. Another non-example would be teaching a two-grade-level, multi-age class using two different grade-level workbooks, taking turns teaching two separate groups.

The chapter will follow the chronological order of the development of the Mobius Math Mode, beginning with its origin in the early eras of Family School. I will tell the autobiographical story of this mode through the Family School ages, while also weaving the document analysis through the story. Important documents illuminating the evidence of the evolution of the mode will be highlighted throughout the chapter. The present use of the Mobius Math Mode will be described at the end of the chapter by using excerpts and analysis from my reflective, personal journal.

While this mode was not initially created to reach toward the higher purposes of mathematicians' work, the evolution of the mode presents the dilemma for teachers to consider how to teach a multi-age group of students in various grade-level material at the same time. This dilemma led to interesting questions, such as: What should students get out of attending to lessons that are well above their grade level? What responsibility do students above the grade level being taught have for contributing to the lesson? How can we teach students to become personally responsible for their learning when the material is not aimed at their level? As the story unfolds, the Mobius Math Mode reveals how teachers' responses to these questions revealed more about student learning than teaching to required grade-level expectations. The bulk of this mode's journey occurs in the Renaissance, but the chapter will begin with the origins of the mode in the Dark and Middle Ages.

Dark and Middle Ages- 1990-1999

Mixed-purposes. While one of the main purposes of the Brainteaser Math Mode was to have students work at the edge of their knowledge, the Mobius Math Mode was created to fill in the gaps in their grade-level knowledge that were left behind by the Algorithmic Math Mode and brainteasers. Neither of these modes limited students to work at the grade level to which they were assigned, as the algorithmic mode met the students at their ability level, whether above or below grade level, and the brainteaser mode usually focused above grade level, or even dealt with material that was never to be covered by standards. The purpose of the Mobius Math Mode wasn't really revealed until the Renaissance because in the 1990s, we were not as focused on students improving in the percentile-based high-stake tests. Though the Mobius Math Mode does not legitimately begin until then, documents do reveal some of the Mobius Math Mode roots extending back into the Dark and Middle Ages. One of those roots, as I have suggested in brainteaser, connected to seeing themselves as mathematicians (*Self-As*), is having a solid foundation in all grade-level requirements of mathematics.

Mobius Math Mode documents for this study could only be found in 1998 to confirm this conversation about using *Self-As* in mathematics. These documents were called *Autobiography Logs* and were filled out by students as they thought about the evidence that supported their emerging conception of themselves as mathematicians. To have documents from 1998 using the phrase of *Autobiography Log*s with the students makes it quite conceivable that, as a group of teachers, we were beginning to develop our notion of the Mobius Math Mode, setting the origins of this practice before 1998.

So while brainteasers met this practice of *Self-As* when it came to puzzling out complex mathematical problems, it was clear that brainteasers alone were not sufficient for students to see themselves as mathematicians. Our students also had to score well on high-stakes mathematics tests, just as all students in the public system of schools who are accomplished mathematicians are expected to do. The teachers and I didn't want our students to be ill-prepared for such experiences and regard themselves as anything less than solid mathematicians. Without exposure to the "prescribed" curriculum, our students' knowledge might compare poorly to knowledge demonstrated by other students on standardized tests. We wanted our students to know at least as much as other students when traditional mathematical assessments were required.

To handle this dilemma, we knew that we would need to have our students participate partially in some sort of traditional mathematics curriculum. Yet with the Algorithmic Math Groups mode, and the Brainteaser mode already generating much mathematical knowledge, we wanted this new mode to fill in any gaps appropriately and purposefully. Our initial response was to resort to daily practice, rather than adopt another mode.

Our answer for this practice at that time was to order workbooks that were aligned with the high-stakes tests, initially starting with Terra Nova, which eventually changed to SBA and then again to the Partnership for Assessment of Readiness for College and Careers (PARCC). In the late 1990s, we scoured available workbooks and chose those that would be the least invasive in terms of time in our classrooms. We did not order mathematics workbooks to be the basis of our mathematics program because we had already adapted the algorithm groups from Mortensen Math and developed brainteasers as a mode, and there was no need to use a textbook in addition to all that coverage. Yet, there were strands of mathematics conspicuously missing in our curriculum, including measurement, data and probability, geometry, and ratios and proportions, which a workbook would cover more quickly than using a whole mathematics textbook.

The scope and sequence of the workbooks influenced our decisions as to which one to choose. Since their purpose was to prepare students for testing, it was decided to give students their grade-level workbooks, even though in the algorithmic groups, we believed that students ought to work at their ability level, which is not always their grade level. This workbook's purpose would be to expose them to grade-level material the best we could. One example of why this was apparently necessary was when one year we noticed that our students did poorly on a state test that kept asking students about *nets.* This was a new term for the curriculum for that year, and by not using the newly-adopted, prescribed curriculum, our students were not introduced to this concept of nets, hence missing test items about this concept. While our students were working through our mathematics curriculum at their own rate, it generally meant that they were above grade level in some mathematical concepts and at, or below, grade-level concepts in others. We wanted the mode to assist our students to be prepared for the grade-level expectations of the state-mandated testing.

Since this mode was being designed for testing purposes, we chose a workbook that mimicked the test. Each page had a wide variety of different mathematics strands, just as seen in the testing materials. In the Dark and Middle Ages of our school, students completed the workbook independently first and then the teachers would periodically check for accuracy and decide the content in which the students would need lessons. You could say this was a *flipped* version of teaching before this strategy became popularized. For us, it simply was a matter of convenience and efficiency, to see where the gaps of our classes were in regards to the grade-level curriculum and to make sure the students would learn that content.

What I infer from this section is that while we knew we were creating a mode for the purposes of preparing students for testing, we also felt it was important that students see themselves as well-prepared mathematically if they were to see themselves as mathematicians. These skimpy beginnings as a mode, while seemingly thin in substance, were the beginning of some very important lessons about the benefits of teaching multi-age classrooms that we would never have known if this mode hadn't come about.

What you will see next are the very productive periods of the Renaissance and Age of Exploration, as we wrestled with the importance of this mode for NCLB testing and how to teach the mode effectively.

Renaissance Age- 2000-2007

It wasn't until the Renaissance Age and the pressure once again of NCLB that my teachers and I sculpted Mobius Math into a formal Mode of Engagement. We realized that we needed to check for students' mastery of the workbook's concepts instead of just making sure we were teaching it. Once we began to use the Performance Cycle (Figure 28, page 197) with the workbooks, we began setting up various checkpoints throughout the workbook to check for mastery. With the initial ITBS, Terra Nova, and SBA workbooks, which contained the wide variety of types of problems on each page, it was difficult to create a cohesive unit around which to assess mastery. These types of curricula are spiral in nature where the "intent of the spiral curriculum is to add successive depth, but the practical result is the rapid, superficial coverage of a large number of topics each year" (Engelmann, Carnine, and Steely, 1991, p. 292). In creating most of our own mathematics curriculum, we knew that we needed to adequately fill the gaps with the right workbook curriculum to make the mode have a more meaningful impact on students and be more easily assessable. While we may have tried to *deemphasize* the role of the textbook, Crawford and Snider's (2000) research shows a teacher is "able to achieve substantially better student outcomes by changing the curriculum" (p. 141) in response to required textbooks meeting the student needs. So in 2008 (again timely to suggest the push to meet mastery standards of NCLB), we acquired a new workbook that featured various sections of the mathematics requirements for the standard-based curriculum of NCLB: algebra, geometry, numbers and operations, measurement, and data and probability to better help us cohesively organize and improve our teaching to be stronger than before. This was the beginning of our most recent version of the Mobius Math Mode of Engagement.

Once this classroom practice had a better workbook with a structure of units, the teachers and I began to consider the expectations, structure of lessons, and purposefulness in our classrooms. The previous *checking and lecturing* instruction had now grown into a full scheduled period (ranging anywhere between 45 to 90 minutes) of interactive instruction in the classroom. Our multi-age classrooms, a significant dimension of our school, were now impacted in such a way that we needed to go back to the drawing board to design how to use this time effectively and to what ends? With workbooks distributed by grade level to each student, and the limited time available to teach, it was neither sufficient nor practical to break students up into groups as we had done in the algorithm groups; we had to negotiate new purposes and instruction for keeping them together as a whole group and teaching all grade-level questions at once. It was with this new collaboration of designing lessons to accompany the new unit workbooks that Mobius Math became more of a math mode than ever because not only did we design the purposes of doing the mode, but the design of the instruction had to be designed to directly meet those purposes as well.

Specifically, the challenge was to create an opportunity for students as a whole group, in a multi-age class to be taught the math content for all levels of mathematical standards, not just the basic, grade level operations that were required by new NCLB curriculum. Immediately, it made sense to us that the unit separation was the cohesive factor to hold the teaching together for all grades. So when the geometry section was taught, for example, all the grades of my third through eighth-grade students would be working through their geometry sections of their workbooks individually, but as a whole, heterogeneously-grouped classroom we would discuss all of the geometry unit material. This required some strategic thinking about instruction.

Initially, my staff and I all taught the lessons without really thinking this through, and it was not long before students at the different grade levels for which the lesson was not required would simply turn off their attention, since they would not be required to demonstrate mastery of this material on the performance. We had simply hoped that all students would be curious enough to attend, which some were, but it was clear that students did not understand what was required of them, and this, together with our non-graded approach to teaching, made us realize that we had underestimated the power that the minimum requirements would have on students. We had stumbled on a problem with the multi-age classroom. While our small groups worked for the Algorithm Math Group Modes, and whole group was perfect for the *community brain* needed for brainteasers, teaching grade-leveled curriculum for a multi-grade classroom had proven to be a challenge.

The research on multi-age classrooms is quite contradictory and spotty. Song, Spradlin, and Plucker's (2009) review of the research states that most of the first wave of research was done in the early 1990's, when there were a growing number of schools putting in place such programs. "The multi-age education philosophies have been supported by much of the historical research and adopted by many schools all over the world"(p. 1). However, Veenman (1995) finds that multi-age classes did not show improved cognitive gains as compared to their single grade counterparts. Mason and Burns (1996) argue that finding no gains in a multi-age classroom actually could be translated into negative affects, because the evidence suggests that higher achieving or more cooperative students were usually placed in multi-age classrooms. They also find that requiring teachers to cover multiple grade curricula resulted in less attention for student needs and lower quality of instruction. However, Ong, Allison, and Haladyna's (2000) study reveals increases in reading, writing and mathematics where "for each ability, multiage students scored higher on the state's integrated performance assessment. The effects were substantial for mathematics but less substantial for reading and writing" (p. 6). Logue's (2006) research states that student behavior and language development is better in multi-age classrooms. Milburn's (1981) study reveals higher vocabulary scores than the single grade classroom. It is interesting to note that the research does suggest most often that multi-age classrooms are created to meet the grade-level student population of the school, which does not translate into having classrooms where the teacher is trained to teach to various grades, nor even request such a position. At Family School, multi-age has been a strong preference of the teachers from the beginning, and any conversation we have had to go to single grades is always shut down immediately. Although teaching a multi-age classroom is harder, it has so many more benefits to learning and more organically imitates how children learn in the world naturally.

This drove us to design a strategic instructional method for the multi-age, whole-group lesson in Mobius to be utilized in such a way as to have benefits for all learners and deal with the *zoning out* of lessons not directed at the student's specific grade-level material. While diving in to teach these units, the teachers and I realized that some concepts in each of the units that needed to be taught over and over in order for the units to be understood at a conceptual level by all students and, consequently, functioning more like a community of learners. These particular key concepts kept arising in our teaching of the units across multiple grade levels at multiple times within a unit, but also throughout the year when connecting mathematical concepts to other units. The discovery of these key concepts were to be the unifying feature of this mode, and we were able to better design our instruction to meet the purpose of this mode (Table 25 on page 241).

To address both issues of holding students' attention and the re-teaching of key, poorly grasped concepts, we realized that it was essential that all students learn how to attend to all the lessons, even if the lessons were above their grade-level requirements, because these overlapping concepts would be strengthened by this recursive nature of the mode. Instead of calling this type of curriculum spiral, linear or strand, we invented the name: Mobius Math, after the mobius strip discovered by August Ferdinand Möbius. It is a unique structural phenomenon where a two-dimensional strip of paper is twisted in the middle, circled around and then attached at its ends to complete a circle. It's twisted circular shape now creates the possibility for a pencil to begin drawing a line on the outside of the loop, hit the twist in the paper, and move to the inside of the paper. Continuing on the inside, the pencil will again loop to the outside at the twist without ever having been lifted from the paper. The teachers and I felt it represented the recursive nature the students would be experiencing in having those repeated lessons so many times in a year, and from grade level to grade level each year. One time their understanding of the lesson of this key concept would be at one point on the mobius strip, and the next time they would experience it, they would have traveled along the strip and returned to the same spot on the strip, but on the opposite side. Since this context was usually associated with a different grade-level understanding, this second visit to that content was more deeply learned, resulting in an experience of the key concept as a new dimension of their understanding— hence the name Mobius Math for this Mode of Engagement. With this new name, my teachers and I had purposes to hold this mode together and moved into the Exploration Age.

From this I infer that it was the structure of the multi-age class and the use of various grade-level books that presented the dilemma of teaching various expectations to a single whole group of students. In finding the benefits of this structure, we were about to stumble on important teaching concepts that allow for teaching well above grade-level concepts to some students, while simultaneously discovering and providing challenges for students being presented with below-grade-level concepts. The Mobius structure created a means of finding great value in the repetitive nature of teaching mathematical key concepts as they occurred periodically for students in different mathematical contexts. We were also about to find out that by developing these reoccurring, key concept lessons, our students were able to participate in learning material that was above their grade-level expectations. This elevated the mode to become an accelerated curriculum, not just for the students being introduced above their grade level but, surprisingly for students findng new and deeper, conceptual meanings in previously *mastered* math concepts.

Renaissance to Exploration Age

Throughout the Renaissance Age, our multi-age classroom was a huge contributing factor in the making of this mode and its name. This has remained consistent with the structure of Mobius even today. Because of the age spans in the classrooms, students can learn material well above their grade level. This accelerated approach's benefits to learning have been researched in a number of ways, though at Family School our idea of teaching an accelerated curriculum is done for different benefits for a multi-age classroom. It is interesting to note our students are able to receive accelerated learning in the design of our Mobius Math Mode even though it is taught around grade-level expectations. Perhaps brainteasers have warmed them up to conquering *the difficult,* and this multi-grade-level material is not usually as hard as a brainteaser. This also could explain why our students do well on the high-stakes tests.

Burris, Heubert, and Levine (2006) did research to study whether using heterogenous grouping with accelerating mathematics had any relationship to improving scores for both low and high achievers. The school district they studied eliminated the tracked mathematics courses in the middle schools, implemented newly purchased accelerated curriculum, created common planning periods to learn the material together as a group of teachers, and scheduled the instruction to include four accelerated classes and two workshop periods for each teacher. The achievement data of these students were analyzed for grades five through twelve. They found that low-achieving students improved greatly in response to this change, and overwhelmingly more students of color and low-achieving students took more advanced mathematic courses in high school. This research also indicated that there was not a drop, but an increase in high-achieving students taking higher mathematics courses in high school. Burris and Welner's (2005) research states that, "By the time the cohort of 1999 graduated in 2003, the gap had closed dramatically—82% of all African American or Hispanic and 97% of all white or Asian American graduates earned Regents diplomas" (p. 597). They concluded that "by dismantling tracking and providing the high-tract curriculum to all, we can succeed in closing the achievement gap on important measures of learning" (p. 595).

While Family School did not create Mobius to deliberately detrack the school, the nature of the multi-age classrooms throughout our school does that organically. If it weren't for wanting to teach Mobius Math to the whole group of multi-age students, we simply would have divided the students into small groups and worked with the different grade levels separately. We never would have discovered this mode's instructional tools and benefits of teaching to a whole group with the purpose of including all ability levels around these key concepts. Our Mobius Math lessons were taught to both the highest and lowest achievers in the class, requiring both to comprehend the material. It is also important to note that teachers must also stretch their understanding of mathematics to the highest level in the class, which better prepares them to teach the lower grades. Because each grade will most likely have students performing above grade level, so does the highest-grade level have students who can reach well beyond that, so the teacher is usually stretched even beyond their highest-grade level of the class. As these students bring their knowledge to the discussion and questions of the problems, the entire classroom learning community is enriched. Sometimes my highest students ask me questions that I have to take the weekend researching. Their ability to look for efficiency of problem solving, or even wonder about the bigger picture of a problem can get my mind exploring much higher high school mathematical concepts.

Themes of the transition from Renaissance to Exploration eras

Documents between the Renaissance and Exploration eras reveal that in the Exploration Age there was an explosion of discovery about Mobius. In these eras, many documents were found pertaining to defining the mode more specifically, school-wide training about this mode and references to using this mode to improve student test scores, as well. Once again the impending NCLB goals encouraged us to buckle down and use this mode to effectively meet the requirements of testing. Table 23 below reveals that the number of documents increased mentioning the themes from the Renaissance to the Age of Exploration. What the teachers and I learned in this time about Mobius was that the learning needed to be both metacognitive, because of our heterogeneous teaching method, and cognitive, to be able to improve student mastery for the improved school percentages required of proficient students for NCLB.

Table 23

Number of Documents Analyzed

Themes	2002-2007 Renaissance	2008-2010 Exploration	Total
Communication	3	15	18
School-wide Documents	5	9	14
My Classroom Documents	2	6	8
School Forms	0	9	9
Total	10	39	49

What I infer from this data is that the increase in documents from the Renaissance to the Age of Exploration is indicative of our needing to be effective in this mode to students to improve in testing. While these intentions don't seem to serve the higher ideals of mathematicians, how we created the effectiveness becomes much more purposeful in seeing the bigger picture of mathematical problems and developing self-monitoring, metacognitive skills to be able to be responsible to an enhanced community of learners.

What you will see next is how our teachers of multi-age classrooms teach *learner lessons* to help students learn to do this self-monitoring, metacognitive work.

Metacognitive skills developing learning habits. While my class grade-span ranges six grades levels, most of the other classrooms at Family School only range two to three grade levels. Because of these grade-spans, the structure for Mobius Math had to deal with teaching whole group multi-age curriculum. While brainteasers are above their grade level, they are able to work collaboratively there. In the Mobius lessons, the new lesson starts after students having graded their workbooks against the answer keys and come to a whole group lesson to ask questions about those problems that were found to be incorrect. The students are responsible for directing the lessons for the day by asking their questions and engaging in discussions about how to understand and perform the problems. If students need reteaching from the day before, teachers will review those concepts before going on to the students' new questions for the day. Keeping the attention of the students from the other grade levels while going over a problem that is not in their workbook has to become essential for this mode to work. Family School teachers use our community of learners' language to discuss the importance of learning all material presented in this mode, and often, older students have to prove to the younger students that they can comprehend the higher, more difficult material.

We teach that it is the responsibility of all the students to ask questions or report when they lose meaning. This requirement pushes our Mobius Math into teaching mathematical metacognitive skills, and also puts the student back in the driver's seat of his/her learning, which ultimately includes strategies for engaging with material that is either above or below the grade level of the student. This promotes their agency for learning. The students' questioning, meaning-making and discussion drive the lesson. Yet, the teacher is responsible for teaching students to learn how to act when they notice they are not making meaning, whether it is grade-level material or not. Many students have previously developed learning habits that preclude them from wanting to reveal that they don't understand, or they convince themselves they do understand. It is the metacognitive work that teachers must accomplish with students that helps them develop agency to be responsible for their own learning. Teachers design various tools to allow the students to practice these metacognitive tools. One such tool was a set of small flags teachers created. Each student had a set of half a dozen flags, and each different flag represented a reason students wanted to enter the math conversation. Instead of raising his or her hand, a student could display one of the flags. The teacher could then see, at a glance, the various reasons and purposes that students had for wanting to participate. One flag indicated that a child had a question, but another flag informed everyone that a child had a connection to make. Other flags let the teacher know that a student was lost or needed to time to talk with his elbow partner. Most importantly, the flags gave students a concrete way to realize that following a mathematical conversation was a complex and demanding enterprise requiring that they monitor their experience through metacognitve awareness. Once students practiced ways to interject themselves into the lessons, the individual students became more accountable to their own learning, and the class as a whole began to contribute to each other more readily.

Family School teachers have also discovered from teaching these various grade-level objectives to a single whole group that the grade level requirements often seem arbitrary and teaching both above and below the students' grades allows for more connection of the content and a deeper understanding of the problems presented. Teachers have acknowledged that by teaching multi-grades at the same time, they realize they teach for deeper understanding than if they had been teaching to one grade level. By teaching this way, we are often confronted with the profound awareness that to really *know and understand* a concept required at a particular grade level, that concept can be explored more deeply than one grade-level's expectations require.

Once the students suggest they have no more questions or confusion over the material at their own grade level, and the teachers are also confident they have addressed all those students who were learning to have agency over their learning, the students are given the unit tests. While they are required to learn the material beyond their grade level, they will only be tested on their grade-level material. This allows meaning-making to occur consistently throughout subsequent years in Mobius without always adding the pressure of performance for everything that is learned, and only of the content required for each grade level. This repetitive, spiraling nature of the content helps the students gain mastery more easily, as in the following years they will have already been exposed to the higher expectations from their previous years and will have a distinct advantage over students experiencing the content for the first time.

What I infer about metacognitive teaching is that it requires a great deal of time, effort and creativity, so that students can function as a better learning community. However, once classrooms develop these habits, the instruction of the content is able to present deeper conceptual understanding than before by connecting all the grade-level concepts together with the metacognitive strategies.The structure of the Mobius lesson was also put into place to assist teachers in improving their instruction toward meeting a well-functioning learning community.

Observation checklist. The structure of the Mobius lesson was formed and utilized in the Renaissance Age, but by the Exploration Age in 2008 (Figure 32 below), it was written down and agreed upon by teachers as a checklist for my teacher observations. It was designed to help teachers understand what I was looking for during my observations of Mobius lessons as their principal. I could easily check off the observed items on the list and add notes quickly as a tool to help teachers organize their instruction for this mode. The checklist is divided into four sections— Review Check, Teacher Preparedness, New Lesson, and Check for Understanding. This allowed me to check whether teachers had prepared for the three-pronged lesson—*before, during, and after* structure of the mode's lesson. The *new lesson* portion of the checklist looked for continual student participation, the expectation being that all students would remain engaged throughout the discussion. It also required teachers to differentiate the lesson of key concepts to all grade levels through metacognitive strategies, called *Learner Lessons*, which demonstrate how the math pertains to all grade levels. This document is copied below:

Math Workbook Lesson Observation Checklist

Review Check

- ❑ Individually accountable
- ❑ Teacher acknowledgment of how teacher is using the review data—re-teach or move on with individual reasons for not re-teaching

Teacher Preparedness

- ❑ Teacher has chosen the lesson in advance and uses prepared lesson
- ❑ Or Teacher is fluent in Family School Mobius Math Lessons

New Lesson

- ❑ Student participation is at a rate of _____per minute
- ❑ Students actively engaged in note-taking
- ❑ Teacher is reaching a wide range of learners
- ❑ Teacher can differentiate the lesson for all levels of students
- ❑ Teacher checks for conceptual understanding in students
- ❑ Teachers teaches at least one Mobius Key Concept Lesson
- ❑ Teachers makes pervasive math transparent
- ❑ Teacher gives or monitors at least one Learner Lesson
- ❑ Teacher understands the math content appropriately

CYU (Check Your Understanding)

- ❑ Teacher gets students to identify concepts
- ❑ Teachers uses an instrument to check for all students' levels of understanding
- ❑ Teachers states the re-teach method to be used

Figure 32. Observation Checklist for Mobius Math Lesson from Family School *white notebook* of important school-wide documents.

Metacognition is established. Documents were collected for analysis in Mobius Math in the following categories: my classroom documents, my communication documents, Mobius forms used across the school, and school-wide documents, such as agendas and evaluative documents. The Renaissance Age documents demonstrate that the metacognitive work was already well established as an important feature of Mobius. In the above document, Figure 32, it is called a *Learner Lesson*. Some documents refer to a variety of specific metacognitive skills, like using connective thinking; other documents refer to metacogniton more generally, like *self-inquiry* or *self-practice*. In either case, metacognitive development has been very much connected to the teaching of Mobius Math throughout both these periods as exemplified in the document theme table below. In the Renaissance, 33% of the themes coded were about developing metacognitive skills, and in the Age of Exploration 32 percent of the themes were coded for metacognitive skill work with a total of 40% of the themes coded for the documents of both ages.

Table 24

Number of Times the Themes Were Found in Documents

Themes	2002-2007 Renaissance	2008-2010 Exploration	Total
Specific Content	1 (3% of total documents)	31	32
Strategic Instruction	3 (5% of total documents)	56	59
Assessment	4 (8% of total documents)	49	53
Mathematical and Metacognitive Purposes	31 (33% of total documents)	64	95

Research on the use of metacognition needing to be developed in conjunction with teaching mathematics echoes what we have learned about how needed this is. In Lucangeli and Cornoldi's (1997) research of third and fourth grade classrooms, they report that there is evidence that "metacognitive components are highly associated with mathematical achievement" (p. 128). Their study also indicates that for both grades, both good and poor problem solvers scored high in their ability to problem solve after learning metacognitive strategies. Though there was a discrepancy for fourth grade, with poor problem-solvers only improving in geometry problems, they still did improve. Cornoldi's (1997) research indicates that the “specific knowledge of the procedure is not a particularly critical factor of the relationship that we found between success in the metacognitive and in the mathematical tasks” (p. 130). Yet, they also note that the association is not close for more automatised mathematical tasks.

Zimmerman (2002) asserts that effective problem solving is influenced by a student’s ability and willingness to use strategies to monitor, change and self-reflect upon the process. These self-regulatory strategies are associated with self-efficacy— that is, the belief in one’s ability to organize one’s actions to meet desired outcomes. Mathematics has shown an important connection between problem solving and domain-specific self-efficacy, In other words, the more students becomes comfortable with monitoring and adjusting their thinking in problem solving, the more effective they become in learning the patterns of problem solving for both particular kinds of problems and those related. Hoffman and Spatariu (2007) researched a way to help students improve this effectiveness by using Metacognitive Prompting. Some of the prompting questions used in that study were:

1. Have you solved similar problems before?
2. What strategies can you use to solve these problems?
3. What steps are you using to solve the problem?
4. Can your answer be checked for accuracy?
5. Are you sure your answers are correct?
6. Can the problem be solved in steps?
7. What strategy are you using to solve the problems?
8. Is there a faster method to solve the problem?
9. Are these problems similar to addition in any way?

10. What is the best method to solve the problem? (p.890)

They found “that under conditions of increasing complexity, metacognitive prompting may induce greater cognitive awareness and the utilization of typically unmindful problem-solving strategies”(p.888), although they also found that these strategies were not helpful for less complex problems. Schoenfeld (1992) asserts that problem solving in mathematics is “learning to think mathematically” (p. 3). This requires developing a means for valuing the processes of mathematics, the abstraction to apply them, and developing *tools of the trade* to use them towards the goal of understanding the structure, which he called *mathematical sense-making*. Lester, Garafalo, and Kroll, (1989) suggest that students of mathematics must solve a variety of problems on a regular basis and over a prolonged time span, but also that metacognitive instruction is the most effective strategy when it is built around domain specific context. Lastly, they believe both mathematics and metacognitive instruction is most effective when it is systematically structured under the direction of a teacher.

Our teachers found that the reason metacognitive instruction is best when structured by the teachers is that they are best at evaluating the skills and habits their students display and which metacognitive skills are lacking. They can also create a means of teaching metacognitive skills in a manner that is accessible to students at a particular grade level, perhaps even matching them to metaphors being used in the room for other units.

How we were able to get students to attend to lessons beyond their expectation required a multitude of metacognitive strategies and teaching strategies. Teaching strategies included, and still include: exit tickets, entrance tickets, responding 1-5 on fingers to indicate understanding, pop quizzes (not for summative evaluative purposes but to show what was learned formatively), mathematical written explanations, and especially discussions that include all voices. Playful means have been created for students to express their metacognitive awareness and strategies: flags for students to raise when they have “gone over the edge” of understanding, or for asking to go over the problem with their elbow partner. They can count off other connections to the problem, receive pink feathers for being able to laugh at their mistaken understandings and are now tickled pink by them, find the knowns and unknowns, use “simple Simon” (create a simpler version of the problem), and write aha’s on our Breaking News at 5 bulletin board. The strategies are endless for teaching them that they are capable of learning anything they put their minds to with the help of our own unique metacognitive prompting. The purpose of the emphasis on these strategies is encouraging students to be more agentic in their approach to learning mathematics. Teachers continue to utilize their teaching maneuvers to encourage student mastery of the material, but ultimately our aim is to develop students who learn how to engage themselves in the lessons more actively.

There have been only a few occasions when I have actually said that I was going to explore an explanation with an older student, without making sure the younger students could find an entry point into that material. Usually that is only because there is not enough time, not because I thought they couldn’t understand the material or that I couldn’t teach them a way to understand the material.

What I infer from these sections is that we intended to expand the students’ repertoire of metacognitive strategies, and the documents for the transition from the Renaissance to the Age of Exploration bear that out. With that firmly represented in our teaching, we then moved into focusing on our instruction and assessment in this mode in the Age of Exploration. We still continued our strong emphasis on metacognitive work. While there were not many documents that were generated around content, one of the most influential documents of the Mobius Mode the lesson checklist was created in this time period.

Mobius key concepts lessons. In 2009, because of the recursive nature of the Mobius mode, we decided to delineate what we call *Mobius Key Concept Lessons*. Teachers wanted to make sure they were dealing with those major key concepts that everyone was experiencing as being both difficult to master and also pivotal to understanding in that mathematics strand. Recently, Meyer and Land (2006) defined threshold concepts as concepts that hold together ideas and practices in fundamental ways that transform the student's understanding of the subject matter. Land et al. (2005) also delineate qualities needed in courses to encourage the mastery of threshold concepts to transform a student's experience. These concepts are: identifying the jewels of the curriculum, student engagement, listening for understanding, reconstitution of the self, tolerating uncertainty, and recursiveness and excursiveness. In Mobius Math, the Mobius Key Concepts are able to meet many of these criteria. We have identified the jewels of standard-based mathematics curriculum, require student engagement, listen for understanding in our discussions and tolerate uncertainty as we require students to wrestle with above grade-level material. What is interesting to add to this is that we also discovered the recursive and excursive nature of these threshold concepts. Land et al. (2005) state:

> Given the often troublesome nature of threshold concepts it is likely that many learners will need to adopt a recursive approach to what has to be learned, attempting different 'takes' on the conceptual material until the necessary integration and connection discussed early begins to take place. (pp. 59-60)

We have also found that many of our discussions of the problems students are having difficulty understanding required additional mathematics lessons in order to explore behind these Mobius problems. These additional lessons may seem to be on tangents, but they result in deeper connections being made. We find these jaunts important to get at the comprehension of these threshold concepts. Land et al. (2005) suggest that excursive learning is “a journey or excursion; [where] there will be digression and revisiting (recursion) and possible further points of departure and revised direction. The eventual destination may be reached, or it may be revised” (p. 60). While our Mobius Key Concept Lessons may not always address threshold concepts because our mathematics concepts don’t meet all the criteria, they do meet many of them and seem to function very similarly. At Family School, teachers have come to define our notion of threshold concepts as concepts that are key to unlocking deep mathematical understanding to a wide variety of connections in the mathematics content for K-8 standards. These concepts are recursive in the curriculum, and may also require excursive lessons to appreciate how they are situated so importantly in the content. Below is a table of the Mobius Key Concept Lessons we would consider our threshold concepts from my computer files, dated June 2009.

Table 25

2009 Mobius Math Key Concept Lessons and their Connections Delineated by Our Teachers.

Key Concept Lessons	Connections that can be made with Concept Lessons
Multiply by names of 1	-Equivalent fractions -Division of fractions -Reducing or simplifying fractions -Cross simplifying of multiplying fractions -Ratio -Infinite names of 1 and each fraction -Multiplicative Identity -Simplifying Algebraic Rational Expressions -Comparing fractions -Wholes in terms of parts
Preservation of Equality	-Equality -Equations -Variables -Inverse operations -Missing number -Inequality, < , > -Creating zero -Operation distinction for term or expression -Word problems -Formulas-example Pythagorean Theorem
Commutative/ Non-Commutative	-Addition/Subtraction -Geometric model: result in greater whole, parts (equal/non-equal) -Commute means "move", not "remove" -Linear direction is basically negative (∅ comm.) or positive (comm.) -Cross reduction of fractions -Rate, ration, proportion
4 ways to name a part	-Fraction, decimals, percentages, and negative exponents - "The ant crossing the table" story
Place value chart	-Ordering decimals (place value of) -Words -Exponents -Place value of fractions -Bases -Names of parts -Expanded notation
Deconstructing/Modeling Word	-Reverse thinking/backward working

Problems	-Vocabulary -Different ways of writing; changing context -Different ways to model mathematically
Properties and Order of Operations	-Associative, commutative, distributive, identity -Accurate modeling of word problems -Performing any calculation with multiple operators -Calculating/solving any problem that has higher level organization: parenthesis, brackets, fraction bars, etc.
Rounding/Estimating	-Application to addition, subtraction, multiplication, division, fractions, decimals, and percentages -Basic understanding of number theory, operations -Checking for reasonableness of answer -Real world intention and purpose -Difference between rounding and estimating (whether answer should be estimated or real solution)
Ratio/Rate/Proportion	-Definition of ratio, proportion, rate -Linear measurement -Word problems -Division/Fractions -Presences of equality -Conversions - Setting up proportions/relations
Linear relationships/ Functions/Points	-Number line: negative, positive numbers, directionality -Relation of addition and subtraction -Map coordinates -Quadrants -Linear number patterns -Slope, negative slope -Line of best fit -Functions -Data representation -Independent/dependent variables -Real world data, word problem representation
Zero	-Different property in different operations: add/sub, multip., division (by zero = undefined; into zero = zero), fractions -Exponent -Place value (where needed before or after number) -Bases -Vertex in coordinate system -Zero degrees -Inverse operations

The recursive nature of these lessons can occur in two ways: by having the lesson taught several times in one year as it arises for the different grade levels, or from year to year as students experience a lesson they had above grade level the year before. For example, in this year's classroom, all my students have learned the Pythagorean Theorem with conceptual lessons and deep discussion at several different points throughout the year. Only those students in 7th and 8th grade will be required to master it. Yet my third grader and 4th grader have a very good understanding of it. When they return next year, they will experience it with another level of their mathematical maturity and begin to understand it even more deeply. Recently, they understood it well enough to know that when we were trying to apply it to two cities on the globe to figure out the linear distance between them, that there was no longer a right angle when the triangle was placed on a sphere. Still, their knowledge of algebra, geometry, and measurement will deepen their understanding of the theorem as they progress through each of the units each year.

This year my third, fourth and fifth, graders have a thorough understanding of how to solve algebraic equations, and they work easily through right triangles and even similar triangles. They also have a solid foundation of one, two, and three dimensional measurement units. Their understanding of squaring numbers is solid, but for my one third-grader, taking the square root of a square number is difficult to really comprehend this year, though inverse operations, like subtraction being related to addition, and division being the inverse of multiplication, the idea of *unsquaring* did not settle well for her. I'm sure she will experience a transformational moment next year when she matures into this understanding.

Another concept for the Mobius Key Concept Lessons could be what Ma (1999) calls "profound understanding of fundamental mathematics" (p. 107). Ma asserts that teachers must have this profound understanding of all mathematical algorithms and how they interact in order to appropriately teach it to the students. We have found that by just creating this Mobius Key Concept Lessson list, it began a journey to improve our own understanding of how all these items connect to threshold concepts. Ma contends that thoroughness is the key to pulling the ideas into the coherent whole.

It is interesting to note that the themes found in the documents for this section could appear to be contradictory. In fact, they demonstrate that as a teacher of this material, I distinguish between content themes and the themes of mathematical purposes in a very unique manner, which suggests my understanding of this deep level of teaching and learning requires the need to pull together metacognitive and conceptual understanding. For example, when I evaluated the themes of the *Mobius Key Concept Lessons* chart with connective lessons as seen in Table 25 (p. 241), I did not evaluate it as a theme labeled *content* but as *mathematical purposes*. When I evaluated another document that was a only a list of the Mobius Key Concept Lessons without the connective lessons listed on the document at all, I evaluated it as the theme of *content*. I argued with myself on several occasions about this and finally left the analysis as is. My reasoning for this is that without the connective lessons, those identified Mobius Key Concept Lessons alone appear to be nothing more than content topics. While I knew the difference myself, many teachers had difficulty teaching them beyond their surface level understanding until the key concept lessons chart with the connecting concepts was created at a Summer Seminar in 2009. This chart made clear all the other concepts that could be connected to the key concept, which added more depth of understanding. To me, the expanded chart is demanding of the teacher to teach the connectivity of concepts, which lends itself to more mathematical purposes and metacognitive strategies.

Table 26

Theme Distribution for Mobius Math Mode

Themes	Forms used in Mobius (9)	School-wide documents on Mobius (14)	Communication documents about Mobius (18)	My classroom documents on Mobius (8)	Total (49)
Math content items or collaboration of content	20	5	3	4	32
Strategic Instruction for teachers	26	17	9	7	59
Assessment Discussion or criteria	31	12	3	7	53
Mathematical and Metacognitive Purposes	22	44	9	20	95
Total	99	78	24	38	239

Table 26 demonstrates that mathematical and metacognitive purposes are the most important feature of our Mobius mode by finding an overwhelmingly increased number of incidences of them in the documents. The second most common coding analyzed was *strategic instruction for teachers*. Revealing another unique importance of the structure of this mode: that it is easy to study for the improvement of teaching because it is a reasonable size parsing of teaching that can be dissected, designed, and evaluated with teachers for the sake of improvement. The mode allows us to create a common language, structure and expectations for the lessons involved in Mobius. Our discussions, observations and evaluation of Mobius lessons on videotape have revealed how, as a school, we strive to share our commonalities as teachers in order to learn from each other and improve our instruction. It is clear that teaching Modes of Engagement is quite different than the experience typical of a traditional classroom, and teachers need time to develop strategies to support its complexity.

While this section again reveals data that suggests content was not a common theme in our Mobius Mode documents, the Mobius Key Concept lessons changed the face of our instruction. This is revealed by the increase in the *strategic instruction* coding found in the documents of these eras.

What you will see next is how our creation of student self-assessment brought together the metacognitive emphasis with the content of the Mobius Key Concept lessons to benefit the students' learning.

Self-assessment brings together metacognition and mathematical proficiency. Students develop the awareness that this mode is key to pulling a great deal of their mathematical proficiency together through doing self-assessment on their work in Mobius. When they conquer the challenge of bringing together their metacognitive skills with their mathematical prowess, they are amazed at the growth of their mathematical concepts, mental facility, and competence. Last year's Self Assessment report cards of my students reveal that students have utilized metacognitive strategies, and an accelerated learning and understanding of threshold concepts to reveal their progress in mathematics. The Self Assessment offers the students of Family School an opportunity to collect their pivotal work, throughout the year, that they believe demonstrates their growth in learning. They present their portfolio in March at an hour-long presentation to their parents and teacher. The report cards paraphrase the students' words as they present their portfolios; the teacher quickly types the document while the student is presenting. Though the words are spoken by the student in the first person, the report cards are written in third person by the teacher. Here are some examples:

> She worked her way through her 5th grade math book this year and has learned how to evaluate her errors and even laugh at her mistakes.
>
> She connected the problems to find a more efficient means to do the work since the grade-level math is really easy, she finds other ways to deepen her knowledge.
>
> He has learned the value of asking questions while learning to do his percents. He feels that working slowly back over his lessons and communicating with the teacher helps him know what he does and does not understand. He has been doing much more mental math with his long multi-stepped problems. He has been working to build his trust in his own ability to make sense of his math work.
>
> She is learning to trust her mathematical instincts. She is working to stand on her own and develop her own ideas more. She is finding that she has quite a bit of math knowledge that she can build the new knowledge on. She has made the conclusion that all math connects if you think about it enough.
>
> It has been important to her this year to listen to others' ideas while she is processing the concepts behind the math.
>
> She is working to develop her attention to difficult material, and discovered that it really was not as difficult as she thought. While her workbook only had her plotting points, she is able to find the equation of a line for the older workbooks.
>
> In graphing lines, he learned how to use very specific language to be able to craft his questions in order to correct his misconceptions. He has learned to value solving problems in multiple methods because he develops a deeper understanding of his math concepts.

Figure 33. Narrative self-assessment report card excerpts from my computer files.

This collection of excerpts of students' self-assessment report cards reveal that students have used the metacognitive strategies, including: listening to others' ideas, laughing at their mistakes, attending to difficult material, clarifying with specific mathematical language, and trusting one's instincts. The students integrally connect these metacognitive strategies to math content they have chosen for their self-assessments, such as: multi-stepped percentage problems, equations of line, and plotting points on a graph.

Table 27

Examples of Literature Review Connections Developed

Collaboration	•Teachers collaborated to design the Key Concept Lessons that they found to be recursive when teaching the Mobius Math Mode. •Teachers collaborated on sequencing the Mobius Math units throughout the school year.
Teacher as Designer	•Teachers designed sequences of Mobius math units. • Teachers designed the flow and pacing of the strategic instruction.
Reflective Practitioner	• Teachers studied Key Concepts in the math units. • Teachers studied the conceptual connections to Key Concepts. • Teacher studied how to build the metacognitive strategies into the strategic instructions of Mobius Math lessons.
Learning Community	•Teachers, students and parents learned together how to teach this strategic instructional method by starting with the larger mathematical concepts. •Teachers and students work together to find multiple methods for solving mathematical problems.

Present-day Mobius

Today the metacognitve work of Mobius Math has become highlighted more than ever. My journaling reveals how each new class has their own culture of learning that requires me to always invent new ways of reaching them. This year's group is particularly quiet compared to other groups of students, and so I worked to break the whole group up by levels of participation to make sure they are participating and not being quiet because of others' rambunctious participation. This is demonstrated in my March 8, 2014 entry:

> I am finally getting this quiet, passive group to participate as a group, whether in whole group or small group. My taking out the talkers to go work together has now moved to taking out the non-talkers to go work together because the talking group is so big they have to stay at the front table. Then, as I watch the non-talkers work together, I reward the new talkers with returning to the big table.

I have even moved to not needing students to raise their hands to talk as we are able to have discussions using polite gestures and comments to move the conversation along. What was most interesting to me about breaking the group into the more active learners is that it wasn't always the lower students that were in the more passive groups. When reporting this strategy to the other teachers, it was clear they didn't understand it, when trying it out in their rooms, they simply ability-grouped their students. My lower students get to stay with the active group as long as they participate by asking questions for explanations, or simply stating they don't understand. As long as the students put their learning out on the table for everyone to wrestle with, they were considered active. In analyzing the student participation on the first day of trying this, it was clear that when a student had suggested a wrong answer to the group, it helped another student question it and figure out how to solve the problem doing the correct method. Questions and wrong answers still move the group forward.

Another metagcognitive strategy I needed to put into place for this group this year was to not only be accountable to themselves as learners but to take a responsibility for the group as a whole. On January 13, 2014, I wrote:

> However, with my new community focus for self-assessment, I am holding them responsible for other's learning as well as their own. And when I tell them they must go back into their groups until everyone understands the lesson we just taught in mobius, it really takes time. Time enough to even pull a small mathematics group there, too. They love this opportunity though. Just when I got them all to be responsible for their own learning, I go and say, now you have to be responsible for somebody else's.
>
> I've never done this before, and it is having a slow start, but I see them beginning to check each other's notes to make sure they have written it down correctly. I changed my metacognitive learning flags to learning buttons, but they have more of a community component. It is the perfect mode for this because even though each grade level has different lessons, all grade levels are responsible for learning the other grades' material, therefore they could be responsible for each other.
>
> I also just think the students this year need to step outside themselves and help others more!

This is an example of how the modes evolve. While the structure and purpose of the mode stays consistent, the need to develop and change the metacognitve strategies always changes with each new group of students. This particular group of students' needs were very different than other students I had taught previously and required strategies I had not needed to create for prior classes.

Another theme that persisted throughout my journaling about Mobius Math was how the modes have come to support each other. Mobius Math journals revealed this much more about the student's level. One student was noticed to have been able to fly through her algebra math group chapters because of the algebra concepts taught in Mobius Math, most of them for the 8th grade workbook, which is above her grade level.

Another student is able to feel he is able to retrace content that he is shaky in by going over the workbook lessons of the grade levels below him, but he is also able to learn his grade level as well. But having experienced above-grade level material in the preceding years also helped, in this journal on March 26th, I wrote:

> Usually mastery may take two or three tests [for him], but it does come. The other good news about this mode is that it does expose him to above grade level concepts, which allowed him to get a *sneak peak* at them before he was going to be held to mastery. Concepts like Pythagorean and exponents were in his repertoire well before he was supposed to be introduced to them.

For this particular student, that advanced exposure really helped him learn new content each year.

The support for other modes was also demonstrated by my teaching decisions to connect content between the modes to further solidify the students' mastery of the content. This happened in January this year when, upon completing the Geometry section of the workbook, I chose to follow it up with geometry brainteasers to see how well the students could apply their general knowledge of the benchmark to brainteaser work.

While there was some evidence in my journals that a theme for Mobius was to consider its importance to testing, there were only a few instances. Mostly, I was aware that with the change to Common Core curriculum, the new workbook had fewer problems and allowed more time for the other modes to occur. It did seem easier for students to master fewer concepts and get prepared for the high-stakes test, though they were given more time with brainteasers and Real World Math, so it is hard to tell which had the most impact on their testing. It was also noted in my journal that we added Math Performance Tasks as designed by the district to some of the problems in the Mobius workbook to help prepare students for that kind of testing situation that will be more prevalent in the new PARCC testing next year.

Lastly, the theme of how the *structure* of the Mobius Math Mode *focuses* the teaching of the mode was discussed at length in my journaling. I think that this is because the structure of the mode has not changed much, except for the length of the workbook requiring less time in the year, and using some of the problems as occasional mathematics task performances to prepare for the PARCC test. This year it was important to study Mobius Math instruction through our videotaping. It was evident that it was difficult for teachers to understand how to teach to a multi-age whole group as a whole group. One particular issue that the videotapes often revealed was when teachers insisted on entering a mathematical problem given in the workbook with a step-by-step approach as to how to solve it. It appears that this is a very prevalent idea about good teaching, and most teachers want to start with the most obvious and particular entry points. However, there are usually many entry points to a problem, and when focusing on one small one, many students are turned off, bored, or simply don't relate to that approach. It is suggested that Mobius teaching start with collecting the bigger concepts from all of the students, so as see the whole gamut of what is needed to solve the problem. Usually within minutes of doing so, the problem's many ways of being solved are revealed and students are eager to jump in and choose the one they see more easily than others. I explored this in my journal entry on March 26, 2014:

> Most teachers start with the proverbial step 1 of the problem. Yes, it is true that a student who asked the question of how to do it probably didn't know how to get into the problem correctly, so he is looking for step one, and if we gave him step 1 he would be delighted, but he clearly doesn't have some other skill to work with, which is why he didn't have step one. They are big picture thinking concepts, and teachers always think it is a small step like looking for a specific word or clue in the problem that will give away step 1. However, what we fail to understand is that even if they are looking for that word or clue, there is usually a big idea concept behind that word or clue that they fail to understand and therefore don't see it to be the blinking sign to step 1.
>
> Also because the other grades are there, a big picture sweep of the room for concepts invites everyone into the discussion of the problem. Sometimes the older students will connect this easier problem to one of their harder concepts. After the students and I generate a collection of information we know about being useful to the problem, we choose one direction to solve it, however, we will also show other methods based on all the knowledge that was collected, so even the older students will see how to apply their harder material. In this way the mathematics becomes mobius instead of linear.

This is an excellent example of how the structure of the instruction and purposes of the mode constrain our teachers in order for the appropriate outcomes to occur. The videotaping of teachers in Mobius showed improvement in student engagement as they moved to teaching for the bigger concepts first.

Another strategy that has helped our teachers develop their teaching of this mode is working together as a staff to identify the Mobius Key Concept lessons. Teachers were asked to evaluate which mathematics lessons had the most connections to other mathematical concepts. In working through these decisions, teachers improved their own mathematical development of deeper conceptual understanding of various mathematical content. This list of lessons also encouraged us to develop a checklist for teachers to evaluate the breadth of the teaching they were doing. The Observation Checklist document was requested by teachers to help them gauge the flow of the lessons. The effectiveness of the checklist was supported by the discussions of the videotaping of their teaching. Teachers appreciated identifying patterns of teaching because they were able to discern improvement by comparing the patterns of instructional design from one video tape to another. Lastly, teachers have improved their teaching by listening to their students' self-assessments. When the students give their presentations about the value of what they have learned in the Mobius Math Mode, teachers hear the value of all the metacognitive teaching that they have done. Metacognitive awareness does not often get revealed in the usual math performance testing required at the end of teachers' Mobius units.

While the Mobius Math Mode began more as a stop-gap practice, its growth into a full-blown math mode helps us see the value of having modes, as they help us improve our teaching, respond to pressures both inside and outside our classrooms, and prepare well-rounded students of mathematics. Once this mode was determined, it has remained relatively stable because of its connection to testing. I am not sure how stable it would be if the testing pressures were made less prominent. I am fairly certain, however, that the mathematics, like the key concept lessons, and metacognitive strategies we have built into Mobius Math, would shift into our teaching of brainteasers, real world, or inquiry modes.

What this mode has achieved is a development of teachers' conceptual understanding of mathematical key concepts by teaching teachers that there are plenty of connections in mathematics to be made. In addition to building depth of knowledge about mathematics, the most important achievement for this mode is that teachers understand now that while teaching to a child's ability level is one method of differentiating, another viable method is using metacognitve skills to enable students to connect to ability levels both above and below their own. In a sense, it is arming the students with a way to take responsibility for differentiating the instruction for themselves. Students' self-assessments demonstrated that they had developed the language to describe the metacognitive skills they had used to progress through the mathematical content. Similarly, teachers found that developing a common language and guided patterns for their lessons in the Mobius Math Mode was helpful in improving their instruction.

Testing is not going to be made less prominent, and may become even more prominent as New Mexico's PARCC will now even be given at two times in the year next year, instead of once a year, as all previous high-stakes tests were given. The teachers and I were confronted with another important decision for this mode this year. The workbook we had been using before the Common Core was required was chosen for its unit structure, which was a unique workbook structure for the mathematics books of its day. At the time, we ordered them from a relatively obscure publisher. We found an abundance of mathematical errors throughout the books and answer keys. At the time, on staff, we had a teacher who completed all of the mathematics books and made a correction sheet for the school. I informed the publisher of these errors, for which he gave us new mathematics books to replace the old ones at no charge. He, of course, asked for the corrections, which I gave him with the teacher's approval, suggesting he compensate our teacher for her work, which he did. When this publisher updated the workbooks to align with the Common Core curriculum, the books, however, upon return still retained more errors than were acceptable to our staff. The errors were not just accuracy errors but conceptual errors and we were no longer interested in maintaining this workbook. We searched and found a different publisher that has a workbook somewhat structured in the same unit method. We are sure that this will alter our mode slightly as we venture into this new text. We will continue our work of comparing our chosen key concept lessons to the content of the new workbooks. We will also collaboratively discuss how to group the various units in the workbooks as units for the class, as they are not aligned between elementary and middle school levels, and we have multi-age classrooms that cover the breadth of both. We will continue to discuss our whole group instruction around using this new book and make sure our chosen instructional strategies work with the new content. When new curricular changes are imposed on schools, as Common Core and many iterations of standards have been, the modes will be tweaked by spending the year in discussion about meeting each mode's purpose and instructional intent. I am sure we will continue the Mobius Mode with eagerness to bring about its multi-grade, whole-group teaching toward student agency in their own learning within our classroom communities of mathematics.

📖 NCLB Story Revealed from the Mobius Math Mode:

The pressure for our students to perform for NCLB testing accountability had both positive and negative impacts on the Mobius Math Mode.

☺ Positively, the NCLB pressure influenced our breadth of mathematics content we utilized in the mathematics workbooks that we used for this mode. Had we not needed to figure out a way to teach multiple grade levels through whole-group instruction, we would not have figured out the value of teaching one math concept to multiple levels. This enabled the teachers to see multiple ways into a problem and multiple ways of solving problems.

☹ However, conversely, the same NCLB pressure greatly eroded our way of teaching, and the study of this mode demonstrated that erosion. As mastery teaching pressured the teachers to be sure that students were mastering the concepts in their math workbooks, the teachers became more accountable to specific math proficiencies. Teachers' teaching came to be more *directive* and *linear* rather than complex. Teachers fell back on teaching the formulaic problem-solving strategies characteristic of how they had been taught mathematics. This kind of teaching eroded the previous strategic instruction in which students needed to work their own way through the problems rather than rely on the teacher demonstrating it. When testing pressure impacts the classroom, teachers feel crunched for time, and walking students through a single method requires much less time for *learning* than studying multiple methods. However, having students direct their own learning through devising their own multiple methods is much more successful in supporting the development of students who are competent in mathematics.

Figure 34: NCLB Story Revealed from the Mobius Math Mode

Figure 35. Mobius Math Mode Timeline

Chapter 8

Real World Math Mode

Table 28

Major Codes and Their Themes for the Real World Math Mode Creating Patterns of a Plotted Story Line

Mode: Real World Math Mode		
Codes	Themes	Plotted Line
Higher Purpose	• Applied mathematics •Integrated mathematics •Improved numeracy	The themes of the Real World Math Mode create patterns that reveal a plotted line that suggests that this mode enables our students to see the importance that math holds in real world encounters. While this math mode struggled to stay active in our classrooms, its strength was always in the importance of application, precision and integration. Due to the fact that its strategic instruction was originally in a pre- and post-testing format, the Real World Math Mode was eliminated because NCLB was bringing too much testing into the classroom as it was. The new mode design connects us back to the original intent of students interacting with content of mathematics in the real world, but it now creates a learning community where the students generate their interest in topics and questions. They work as a community to solve the problems they pick, working toward accuracy as a group, rather than as individuals on a test.
Specific Content	•Utilizing data from charts, graphs, diagrams, and embedded in text •Importance of precision in mathematics •Recursive use of integration enables students to see that math is an everyday language of world affairs	
Strategic Instruction	Old mode used: • Pre- and post-testing format to teach toward precision and it was eliminated from our classrooms . New mode used: •Student-directed learning; •Student-generated curriculum; •Teaching towards precision as outgrowth of real world connection; and •Metacognitive strategies to develop learning community that applies mathematics.	

In a *Scientific American* column on innumeracy, the computer scientist Douglas Hofstadter cites the case of the Ideal Toy Company, which stated on the package of

> the original Rubrik cube that there were more than three billion possible states the cube could attain. Calculations show that there are more than 4×10^{19} possible states, 4 with 19 zeroes after it. What the package says isn't wrong; there are more than three billion possible states. The understatement, however is symptomatic of a pervasive innumeracy which ill suits a technologically based society. It's analogous to a sign at the entrance to the Lincoln Tunnel stating: New York, population more than 6; or McDonald's proudly announcing that they've sold more than 120 hamburgers. (Paulos, 1988, p. 8)

Paulos (1988) demonstrates the importance of mathematics literacy as he reveals how *innumerate* our society can be. The world is full of various literacies, and yet schools focus most of their instruction on written and oral language. The language of mathematics is often taught in such a way that if the subject of reading was taught this way, children would never make meaning out of what they had read. Much of our mathematics curriculum feels meaningless because it revolves around algorithms and stagnant word problems that have little to do with the world in which we live.

The Real World Math Mode was developed specifically to provide students with on-going opportunity to wrestle real-world math applications. As teachers developed science and social studies curriculum for their classrooms, they found the mathematical questions that authentically relate to those topics. They then developed these real-world math applications into questions for students. The questions were directly related to understanding science or social studies topics of study. They also integrated mathematic subjects, such as geometry with measurement and statistics. Because Real World Math questions were deeply embedded in context, they were complex, multi-step problems that were deeply engaging to students.

Non-examples of this mode are haphazardly placed, unrelated real-world problems presented to students occasionally for fun, or real-world worksheets given to students to do while other students are finishing up their work. The Real World Math Mode is a regularly scheduled math block of time devoted to exploring and solving the types of math problems that scientists and other professionals might encounter in their fields.

The most important attribute of Real World Math Mode is the way it teaches students to connect a wide variety of real-world concepts, a commendable quality of mathematicians. For our school, that meant integrating mathematics with our yearly science or social studies units. This integration allowed the mathematics to bring more meaning to the content, but it also meant that the content brought more meaning to the mathematics. Integration was fairly easy for Family School teachers, because they teach all subjects (unlike traditional middle school teachers) and because they have low PTRs. Family School teachers can connect their knowledge of their students' abilities in mathematics with the other content areas for enrichment.

After a brief introduction to the purposes of Real World Math, this chapter describes the development of Real World Math Mode in chronological order. As this chapter will show, the students were enthusiastic about Real World Math Mode, but the testing format ultimately led to this mode's demise. Because it was embedded in other content areas, this mode probably lasted longer than it would have if it had been a "stand-alone" mode. As it was, the number of real-world testing cycles decreased greatly throughout the Renaissance, but the mode continued, mainly due to the fact it was integrated into other subjects. Eventually, however, the testing component was too great a problem for teachers to overcome, and feeling that students already had too many tests in the year, teachers discontinued it. Fortunately, knowing that the purpose of mathematical application is important, teachers have recently revived interest in the mode. It has been re-introduced to Family School curriculum with its original purpose of connecting to math in the real world, but without integration and without a testing format. In its new form, this mode's cycle of units will not be repeated as often throughout a year, but will be back in the yearly schedule.

Brief Overview

The Real World Math Mode was created to meet the mathematical purpose of building numeracy around how mathematics is experienced in everyday life. However, this particular mode has lived a rocky existence at Family School. Though it started early in the Middle Age in the late 1990s, it was not pervasive in the school until the teachers and I were moved together to our portable site. Once there, evidence in the documents suggests that by 2005, it was firmly planted as a mode and periodically placed on staff professional development agendas. I was only able to find my own classroom documents that date back to the year 2000, though I know that I had begun our use of Real World Math somewhere in the Middle Age, when I was housed at Monte Vista. All those computer records for Real World tests are not retrievable from that time, and I'm sure that many of the real world documents were handwritten and not saved. The real world physical file in my filing cabinet was full of documents dating from the year 2000. When the school was moved together as a centralized site, the teachers and I were eager to discover our school-wide curriculum. That was initiated with the Algorithmic Math Group Mode and the Brainteaser Mode, but I had done Real World Math in my room and shared it with a few others. When more of the teachers heard about it and tried it out, it soon became a school-wide mode. I might have trained those who were first interested, until it became more of interest to everyone by the year 2005. It must have felt important for me to save the early documents as examples for this training.

The other evidence, revealing that the Real World Math mode was not as solidly established in our school, is that it is also difficult to ascertain its purposes from the documents. There is not much mention of training on the topic of Real World Math, and no important school-wide documents have been generated around the Real World Math. Similarly, school professional development agendas reveal it was difficult to find a place to squeeze it into our daily and weekly schedules. One agenda suggests one way to integrate it, and another year's agenda suggests a different practice altogether.

Still, it lasted from 2000- 2010, where it dwindled into the abyss of forgotten curricula, until its resurgence with the recent Common Core curricular changes. So what is it, and why was there all this confusion?

Finding a Place for the Real World Math Mode in the Dark and Middle Ages

Originally, I wanted a mode that addressed the fact that students need to be able to apply their mathematical prowess in the real world, because much of what is taught in class seems quite distant from that reality. There are many kinds of application of mathematics to the world, and many levels of expectation to accomplish this through curricular means. Gainsburg's (2008) review of the literature around using real world mathematics points out that "[t]he K-12 mathematics-education community is virtually united on the importance of connecting classroom mathematics with the real world" (p.199). Her study evaluates the use of real world mathematical problems used by secondary mathematics teachers.

While teachers agree on the importance of real world math, they seem to have trouble implementing it. For example, Gainsburg (2008) also found that while 80% of the 60 teachers surveyed suggested that using real world problems motivates students, and 75% of the teachers believed that it made the mathematical concepts easier to understand, the five teachers she observed used real world problems infrequently or superficially, and not to the depth they needed to be taught in order to make an impact on students. Forty-seven percent of the surveyed teachers suggested that real world problems were too time intensive, and teachers required more training to use them more effectively and accurately.

Another interesting aspect of her study was that her teacher observations revealed that teachers were more inclined to worry about “over-challenging students than under-challenging them” (p. 216). She found that “[a] reluctance to challenge students would influence many teacher practices, not just connections, and would significantly impact learning opportunities” (p. 217).

Effective implementation of real world math curriculum solves the problem of under-challenging students because it provides students with opportunities to study complex problems. Boaler (1993) proposes that real world context has:

> …the power to form a barrier or bridge to understanding and it is this realization which prompts consideration of the range and complexity of influences upon a student’s transfer of mathematics. It is also this realization that should ensure that contexts are used in learning, in order to provide bridges and to help break down barriers. …

Additionally, math problems that students encounter in their lives are likely to be complicated and integrated into a wide scope of contexts. Real world context readies students for such problems, as Boaler (1993) further points out:

> For mathematics is widely thought of as certain and probably the most important school subject with respect to real world demand. Yet when students are faced with problems in the real world, they find that the certainty they have learned is not applicable. This leads them to perceive themselves as mathematical failures and perceive mathematics as ultimately difficult. This discontinuity is not caused by misunderstanding but by students learning techniques without knowing what they mean, what they are for, when they can be used, how they may be derived and most importantly, where they fit in the overall mathematical picture. …Problems in the real world are generally complicated, not in terms of their mathematical demand, but in the relative influence of subjectivity, experience, communication, process and content. (pp. 370-371)

Lastly, Boaler also recommends that real world mathematics can bring about a more "culturally based" (p. 371) curriculum that can connect students' meaning in a social environment. She states:

> If the students' social and cultural values are encouraged and supporting the mathematics classroom, through the use of contexts or through an acknowledgement of personal routes and directions, then their learning will have more meaning for them. Their social, cultural, personal, 'folk' or 'ethno' mathematics will be given enhanced mathematical recognition in a social setting, and this in turn will enable connections to be made with the mathematics of the classroom, which may make this

> more meaningful when students are faced with the demands of the "real world." (Boaler. 1993, p. 17)

Saxe (1988) further illustrated this when he compared the mathematics results of child street sellers to urban students who held candy seller jobs outside of school, to rural students, who did not have this real world experience. He found that the urban students and the children who were street sellers scored much higher on the tasks that involved addition and subtraction of bills and complex ratio problems. The more removed the mathematics lessons were from the real world experience of the students or children, the more errors were made in their understanding of mathematical concepts. His study shows that there is a:

> …relation between children's engagement with everyday practices and their developing mathematical understandings. Children generate mathematical problems as they participate in cultural practices such as candy selling. These problems are linked to both larger social processes, such as the inflated monetary system, and to local conventions that arise in the practice, such as the ratio conventions for retail pricing. (p. 1424)

Another importance for using real world mathematics problems is the connection found with studying the metacognitive resources of students. Stillman and Galbraith (1998) find that there is a "symbiotic relationship" (p. 187) between the cognitive, metacognitive and the affective domain of problem solving. They find that when students are unable to problem solve real world problems, they lack the metacognitive skills to maintain their perseverance to discover and apply other methods that might work to find a solution.

It cannot be overlooked that it might be difficult for teachers to teach real world mathematical application because this type of mathematics was not presented to them in their schooling. Davis and Petish (2005) suggest that what stands in the way of preservice teachers making those connections is that they need additional training in applying textbook concepts to the real world. It appears as though the tradition of teaching from textbooks passes from a generation of teachers to generation of students, who then become prospective teachers.

What I can infer from the research and history presented in this section is I had good reason to create a Real World Math Mode. The time spent developing this mode was well-spent, as I experimented widely with choosing everyday math topics and integrating techniques. The next section will describe in detail how I integrated Real World Math into Workshop Mode. It will also reveal the pre- and post-testing structure we used for this mode.

Renaissance- 2000-2007

At Family School, my teachers and I wanted to break the pattern of adherence to textbook math, and do something to make a connection to the real world with mathematics. Though our efforts were small, it was one way to help our teachers begin to see possibilities and grow with their students. We chose to integrate this mode with our Workshop Mode—our hands-on, thematic, social studies and science units. We created a pre- and post-test for a few multi-stepped mathematical problems based on using the graphs, maps, readings or charts of information that were attached to the workshop activities already designed for the students. This allowed students to stretch their understanding of these science or social studies units by building a numerical understanding of these topics as well as a literate understanding.

Many examples of real world mathematics are utilized when teachers connect mathematics with the students' utility of performing mathematics that is connected to their everyday experience in the real world, such as shopping, sports, travel, etc. Our connection to real world mathematics was connecting the world of what we were studying in our workshop topic for the year. These unit topics cover social studies and science unit that are designed to do in-depth learning about one topic and how it relates deeply to benchmark standards as opposed to covering all science and social studies benchmarks superficially. By integrating the mathematics with this unit study, it deepened the understanding of the unit even further. Our unit topics have ranged from ecosystems, to world poverty, to ocean life. Each topic can be integrated with mathematics and make the world of the unit carry more meaning because of the mathematical connections.

Following are two examples of Real World Math tests (there are four other example tests collected in Appendix B) to demonstrate the various topics that were studied and how the problems were written to use a wide variety of resources from which the students were required to get the data to be able to compute the problems. They had to learn how to read tables, charts, graphs, diagrams, and even pull some numerical data from text in order to be able to solve the following problems. The charts of data have not been included because of their length and, in some cases they were not kept in the files.

1.Brazil's original extent of forest cover was 2,860,000 square kilometers and now only presently covers 1800,000 square kilometers. Its present rate of deforestation is 2.3 % per year. Make a graph that shows the decline of the rainforest from what is presently there today and determine what year there would be no rainforest if we continued to cut down the forest at this rate.
(Obviously we cannot figure in new growth each year.) If you are not in percentages, use the fact that 42000 square kilometers are being lost each year and make a graph show 10 years of deforesting.

2.Choose two points from the graph and determine the rate of ozone depletion from your point A to point B. At that rate, determine how many Dobson units would be measured in 2010?

3.The global rate of destruction of rainforest is 2.5 acres per second. At this rate how many acres are destroyed a year and how many square miles is that? (640 acres are in 1 square mile)

4.In California the government issued statistics indicating that it would take 2000 years to build up 1 inch of topsoil, and that its large-scale agricultural business was depleting as much as 1 inch every 25 years. If this is true, how many times faster is California using up its topsoil than it takes to make it?

5. Write the equation of the line that you created in problem # 2.

Figure 36. Real World Math Test from my computer files dated April, 2004.

1)If the data for population growth in China is as follows: (in millions)

2000	2001	2002	2003
1267	1276	1284	1292

What might be a pattern and a prediction for 2004 and 2005?

2)While at least 200 million Indians go hungry, In 1995 India exported $625 million worth of wheat and flour and $1.3 billion worth of rice. The two staples of the Indian diet." If their own food was distributed evenly among their poor, how much would it have cost India to feed each individual both rice and wheat and flour.

3) "Of the price U.S. consumers pay for bananas, only 14% actually return to Central America in the form of wages and taxes levied by the governments. A startling 86% ends up in the hands of corporations that control production, shipping, ripening, distributing, and retailing. If Costa Rica exports 2.2 tons of bananas to us, at an average price of $1.19 a pound. How much money goes back to Costa Rica

4) If 7/10 of 5 billion hectares (approximately 2 1/2 acres)of dry lands used for agriculture around the world is at risk of being turned into deserts, how many acres used for agricultural purposes are at risk?

5)"In the early 1908's media coverage of starvation in Ethiopia reinforced the notion that drought is to blame for famine. We visited Ethiopia's highland villages in January and February of 185, seeking out the causes of the widespread human suffering we had seen on television.

First, we learned how wrong we were to assume that the Ethiopian drought prevailed over the entire country or even most of it. As our jeep wound over mountain roads, we saw one valley in which the rains had brought a good harvest, followed by a valley where obviously little rain had fallen. The next valley had only enough rainfall to grow drought-tolerant sorghum. Ethiopia's sheer size (twice the size of Texas-268,601 square miles) and its varied geography make it highly unlikely that a drought or any other climatic condition would be nation wide.

Several agriculturists confirmed our impression, estimating that the 1982-85 drought affected at most 30 percent of Ethiopia's farmland. If "**Land Use:** Of the total land area, about 20 percent is under cultivation, " how many square feet were affected by the drought?

Figure 37. Real World Math Test from my computer files dated September, 2006.

Students were given a set of these real world problems as a pre-test and then participated in lessons going over how to accomplish them by accurately deciphering the readings or charts and discussing how that numerical analysis impacted their understanding of the material with which they had previously interacted. The problems from the test (pp.269-270) or any related problems were the only problems that were connected to the three to six-week time period needed to cover the unit. Upon the completion of the unit workshop activities, students were given the same or a similar test as a post-test to demonstrate their mastery of using the mathematical resources, such as maps, tables, and charts, and their algorithmic prowess in mastering several-stepped, complex problems. The grading for this test was used as a way to emphasize the importance of accuracy, labeling, demonstrating all the steps, using the charts correctly, and writing out a sentence explanation of the answer. These problems were taught in a whole group process to be able to answer all their questions, but they were also required to go over the problems in their small, heterogeneous workshop groups as well. If they felt they needed more practice, the students assigned themselves homework and followed up on these problems the next day.

It was the practice that each of the different mathematical problems used various mathematical algorithms. For instance, problem one might use multiplication, problem two use division, and so forth. Though this was a test for the whole class, the students were only required to master the problems that were at the level of Algorithmic Math Group they were in. Thus this mode was taught at their level, and mastery was expected at their mathematical ability level. However, they were invited to do the other problems on the test for extra credit. While this was one small effort to integrate mathematics to the world they were studying, it enlivened the unit in a way that was previously not highlighted, and it also allowed a means of pushing the sense of accuracy needed in mathematics, especially in the *real world.*

After the new mathematics reform standards had already been adopted and approved by the American Mathematical Society, Wu (1996) reminded mathematicians that perhaps they should speak for themselves as to whether to support the new reforms, since the new standards promoted *process over product.* He felt the reforms did not emphasize the need for precision, technique, and the importance of "getting a single correct answer" (p. 1534) in mathematics. Not that this is the ultimate end of mathematics, but he argued that the reform efforts of that time had completely eliminated these expectations from the standards' document.

Our real world tests required the precision needed to enhance the research they accompanied. The real world tests inspired students to see history or science through another lens, and they seemed to enjoy this lens more than just reading about it. When our mathematics workbooks were ordered in 2008 to create the more politically acceptable mode of Mobius Math was when the Real World Tests started to vanish from our classrooms. 2007 agendas and 2008 yearly unit plans suggest that the teachers and I tried to fit Real World Math in occasionally with our workshop units without doing the pre- and post-test, but just exploring the mathematics revealed in the activity research. Some agendas suggested we were able to find half-hours here and there where we could squeeze in some real world problems. Even in our narrative student report cards, their mention was in support of other math modes. For example, "He has also worked to build real world knowledge of mathematical application which really helped in his understanding of the percentage unit" of the Algorithmic Math Group mode. Another example shows Real World Math is merely mentioned within another mode: “In her multiplication she was able to demonstrate procedural, conceptual knowledge and application in real world problems.” By 2010, the only evidence was one problem in a placement test we designed for 8th graders that year.

What I can infer from this section is that the pre- and post-testing format of the mode had mixed results. On one hand, testing added a sense of import to the mode. After all, math in the real world must be done with accuracy or have real-world consequences, so students should experience some feeling of consequence as well. However, with increased emphasis on testing under the NCLB regime, the pre-and post-testing format became both overused and onerous for teachers. One of the only reasons that Real World Math Mode survived was its integration into other modes. The next section will present how that integration changed over time.

Integration with other subject matter. This particular math mode did not stand alone as its own subject matter mode; it was integrated with our science and social studies units. It was this integration that made it more real, yet it was the integration that made it more liable to disappear. The teachers and I had created the Workshop Mode long before the Real World Math Mode, and we knew science and social studies had to have their own integrated but *real world* focus. Since we were already integrating science, social studies and language arts, it was a natural question to ask, “why not integrate the mathematics also?” The Real World Math test was a quick method to add in the mathematics to workshop’s already complex structure, that made its activities offering another *Self-As* opportunity: self-as biologist, self-as historian, self-as writer, and now: self-as mathematician in this Real World Math Mode as well.

These workshop social studies and science units are planned for the entire year and are prepared by the teachers over the summer. In the beginning, we all planned our units separately, but as we became stronger as a school unit, we moved to joint planning, and today all of the upper-grade teachers plan one unit together, as do the lower-grade teachers. Our unit plans are orchestrated to show the integration and various layers of topics to be studied. Following are some saved unit plans for my multi-age classroom of third grade through eighth grade over the three years when Real World Math was struggling to be maintained in the Renaissance. In 2006, I was already substituting the test format with practice problems (but one test for that unit was found); in 2007, Real World Math was only a possibility, and in 2008, I scheduled it, but it did not occur. The highlighted lines indicate where Real World Math was built into the unit plan. The Real World Math Mode and the Inquiry Mode were the only math modes that were integrated into the Workshop Mode.

	Aug	Sept.- Oct.	Jan. –Feb.	March
African Biomes	Introduction	Grasslands	River and Lakes	Forests
Workshop-				
PBL	Mysterious Radioactive Tail tale	Eco-tourism on the move	Rising tempers and temperatures	The Burning Issues
Rubric	Pre	Interdependence	Habits of Mind	Post/ Organizational skills
Real World Math		Week out of math workbook to take pre and post and to study		
Eco/ Etho	Org. of biosphere, Biomes, Food web, food chain, Mammal and bird, Patterns and processes	Energy flow in ecosystems,Competition, Mutualism, Exploitation, Adaptation to envir. Value of wild species, biodiversity,	water cycle, photosyn. Carbon cycle, nitrogen cycle,	Succession and stability, population equilibrium, response to disturbance
Sophie—Animal Issues				
Topic	Exotic pets, Changing understanding	Poaching, Cultural needs and beliefs,	Global warming, Urban sprawl, Pollution	Intro of non-native species, Experimentation
Website	Explore and choose	Connect to Sophie issues	Letter to contact from site	Ideas for expanding website, or creating a new website
Projects—Animal Study				
Goal-Nonfiction reading of Individual Animal Study	Non-fiction testing and developing reading strategies	Implementing and evaluating effectiveness of strategies	Finish the book	Finish the book
Writing–Fiction Animal Story			Story about ind. Animal using info to inform story	
Science--	Sign up for science presentation, each presentation to include a connection (exception: space first)			
Inquiries		Life zones of Sandias	Weather of Alb. Thermal inversion Wind. Mesa	Bio profile of Middle Rio Grande Valley and River
Regular	Inq/pbl	Two inq.	1 poet inq. / One inquiry	Inquiry Fair Packet
Advanced	Inq/pbl	Packet (freeform, parts)		

Figure 38. Enough is Enough-Life in the Balance Unit- an African animals and ecosystems unit from my computer files dated August, 2008. Real World was scheduled in its pre- and post-testing format, but Mobius Math had to be canceled for a week for each unit to do them. This suggests that by 2008, we had only scheduled four Real World Math units per year.

	Aug.	Sept –Oct	Nov-Dec	Jan- Beg. Of March	April
	Intro IDEA	IDEAS AS THE ROAD NOT TAKEN	WRONG TURNS	PERSEVERANCE-STICKING WITH THE JOURNEY	Conclusion-RESULTS-BRANCHES OFF THE IDEAS
	How to generate ideas Collection of ideas Idea notebook	Goal Project Student inquiries Idea notebook	Poetry and Art	Writing Project—Screenplay/ Short Fiction Home inquiries from idea notebook	Inquiry Fair Patent Application/Magazine Article
Hist. of the IDEA of our Nation	George Washington Ben Franklin Myth of Greatness	Thomas Paine Pamphleteering Washington's Farewell Address	American Indians Slavery	The Great Depression	The Green Revolution
American Invention	Kid patents	Machine and Tools	Electricity	Speed	Environment
Invention	What purpose does invention serve a society and individual	Who benefits	What is its infrastructure	What are consequences	How is it impacted by the political processes
Innovation	Myth of Innovation	History of innovation	People love new ideas Good ideas are hard to find	The Best ideas win Innovation is always good	Discovery Methods and Me
Inspiration	Creative Thinking	Discovery Methods	Women who have inspired	Leadership and Inspiration	What inspires me?
Inquiry	Ben Franklin	Eli Whitney/ McCormack	Edison		
Tinkering	Telegraph Make magazine				
Non-fiction Reading	Reframers Retell	Reframers, Vocabulary	Context, Emotional	Writing, Physical	Writing, Context
Workshop Rubric	Interpendence	Organizational Skills, Content	Habits of Mind	Integrated, Interdependence Content	Integrated, Habits of Mind
Real World Math	Comparing data	Calculating benefit	none	Using data to make decisions	
Workshop Writing	Short Biographical Journal Writing	Pamphlets Letters	Charters	Patent Application Process	News Story
Accountability	Personal Plan	Parent Day Short Essay	Short Answer	Parent Day Study booklet	Personal Analysis Final Exam & Essay

Figure 39. Inventions Unit from my computer files dated August, 2007. In 2007, it appears that only three Real World Math units were scheduled in three parts of this invention unit.

	Introduction: the call to be involved, and the definitions	Which comes first? Which is the leading cause of extreme poverty?	Who's on second? The network of institutions and individuals making a difference	The third eye: a new vision for a global economy
Poverty	P1 –In, 1, 18 def.p.20 P2-In,16 P5-8 B1-p264	P1-3, P2- 1,2 P5-p9,69,B1-7,B2-1	P4-15, B1-8,,B3-8,E1-1,P2-3-7,P1-11,12,P5-part3,	B1-12,13,B2-8,9,B3-10,P2-8-14,P1-14-17
Hunger	H2-In, Ep, H3-In,Con H4-19	P5-6,P4-15,H1-part 1H3- part 1, H2-1-6,H4-4,9, B1-2,4,B2-5,E1-2,3,E2-1,5,6	B1-9,E1-2H3-6,7,H2-7-10,H1-part 3	H4-11,13,14,16,H2-11,12,H1-part 2
Drought	D1-In, D2, for. Part of intro, P2-p. 49-50,E1-p.67	P5-p284,H4-7,B1-3,4,5,B2-6,D1-2,4	B1-10'D1-8,D2-p80,48,120,142	B3-4, E1-4,E2-7,D2-any
Geography	Identifying place, researching place	Identifying culture, researching culture	Identifying government, researching government	Integrating all three
Science		Seeds Malaria Desertification Nutrition, Hydration	Energy, technology, agricultural practices, Water infrastructure	Inquiry on lifestyle changes, sustainable development
Math	Isolated problems	Comparing problems	Building and comparing budgets	Inquiry involving self data to make decisions
Writing	Describes, paraphrases, monitors comprehension, makes connections	Application, press release, memo, proposal Makes connections to prior knowledge, cross references	Minutes of a meeting, resume, grant, Writes self into existence, makes connections beyond the text	case study, annotated bibliography, article, Does extended research, unpacks the meaning of own writing
Reading	Describes, paraphrases monitors comprehension, makes connections,	Makes connections to prior knowledge, cross references	Reads self into existence, makes connections beyond the text	Does extended research, conducts reading discussions unpacking the meaning
Web Connection	Research and choose website About us, Communications	Features, News, Topics	Education, Careers, Data and Research	Projects, Events, How to get involved

P1-The End of Poverty P5- Rethinking Globalization H4-Harvest for Hope B2- Outgrowing the Earth P20-Ending Global Poverty	H1-The Paradox of Plenty E1- Earth Democracy B3- State of the World 2006 P3-The Wealth of the World and the Poverty of Nations	H2-World Hunger 12 Myths E2- Stolen Harvest D1- Water Wars	P4-Teaching Economics As If People Mattered H3-Ending Hunger in our Lifetime B1-Plan B 2.0 D2- Dry, Life Without Water

Figure 40. Understanding World Poverty Unit from my computer files dated August, 2006. This 2006 unit schedule four units of Real World Math units to be built into the various sub-units.

What you see in these unit plans is overview of the types of real-world problems that I planned to integrate into each section of the overall unit.   At the earliest planning stages, we were already considering what kinds of Real World math problems to include. For instance, in the world poverty unit, I considered teaching isolated problems, then comparing problems and budgets.

Integration of subject matter has been in a cycling pattern throughout the twentieth century. Mathison and Freeman's (1998) review of the literature on integrated curriculum clearly indicates a flourish of interest in integration in the 30s, 50s early 70s, late 80s and again in the reform literature of the mid 90s. Clearly, integration of disciplines continues to push against the more traditional, fragmented curriculum of the outmoded factory model of education. Much research into integrated curriculum finds various way to categorize how curricula can be integrated (Mathison and Freeman, 1998, Kiray, 2012, Davison, Miller and Metheny,1995, Fogarty, 1991). Mathison and Freeman (1998) use the three categories of interdisciplinary, integrated and integrative models, and these categories are a more generalized than Kiray's (2012) five categories, Davison, Miller and Metheny's (1995) use five types, or Fogarty's (1991) ten ways to integrate subject matter. Mathison and Freeman also reveal in their various justifications for interdisciplinary curriculum that there is an "intellectual argument, which suggests that any field is enriched by ideas or methods from other fields," (p. 16) as well as a practical reason, "which suggests that the real-world of knowledge is connected and new ties are formed every day" (p.17). Lastly, they contend that the "pedagogical argument … suggests that learning is seriously hindered by the current fragmented system" (p.17). Their review of the integration literature reveals over forty references throughout the twentieth century that also suggest benefits in educational outcomes and benefits for teachers, as well.

However, Steen (1994) argues against integration of mathematics and science. Her fear is that by giving up the traditional curriculum, some key fundamental concepts of both disciplines would be lost in the integration of both. She feels that a coherence or “suitable order” (p. 10) is needed to teach each subject adequately, and this would be interrupted or ignored in an interdisciplinary approach to curriculum. She encourages integrating methodologies rather than curriculum.

While the abundance of literature is in support of integration of curriculum, Czerniak, Ahern, Sandmann and Weber (1999) report many problems with the research. They state that while there is a “plethora of literature about curriculum integration, there is little research evidence that curriculum integration is a better way to provide instruction than traditional discipline-specific methods” (p. 8). They report a wide variety of models, but curriculum designed for any of these models tends to focus on *process skills* rather than subject content. Finally, they ironically attest that while an interest in integration has once again arisen, it has done so at a time where increasing emphasis on standards remain separate in their disciplines and require even more strictly-disciplinary accountability than before.

Our multi-modal instruction answers these concerns. At Family School, Real World Math promotes integration, the Algorithmic Math Mode focuses on coherent, ordered mathematics curriculum without integration, and yet the Mobius Math Mode works to address standards for testing purposes.

Documenting the Renaissance to the Present- 2000-Present

So what went wrong? If Real World had purpose, and is well supported by research, why did it disappear, and with only a whimper, not a declaration by the teachers who had vetoed brainteasers out of testing frustration.

Table 29

Number of Real World Math Documents Found and Evaluated for this Study

Year 200_	0	1	2	3	4	5	6	7	8	10	12	13	Total
Agendas						1		4					5
Examples	1	3	1	4	4		1						14
Unit Plans					1		2	2	1				6
Rubrics											1	1	2
Student Reports				1					1	1			3
Parent Communication					1				1				2
Total	1	3	1	5	6	1	3	6	3	1	1	1	32

Table 29 indicates that the Real World Math Tests were used regularly from 2000-2007, and then the agendas suggest that we were trying to find other ways to do Real World Math rather than in the pre- and post-testing format. A variety of documents were found that referred to Real World Math, many were not found. School workshop rubrics suggested that Real World Math should be brought back in an integrated manner and taught with workshop again, but without pre and post-tests.

Personally, I can attest to the fact that the mathematics workbooks for Mobius Math took so much more mathematics time each day that Real World Math was literally squeezed out of my classroom, as the agendas suggest. But what the documents don't reveal is how time-consuming it was to collect the data charts or readings and write word problems that were at the edge of understanding—including the teacher's edges—of the real world material we were studying.

And in our new world that emerged out of constant testing, it just didn't feel like the mode structure of pre- and post-testing was a good idea anymore. The teachers and I knew we wanted to emphasize precision because the mathematics that lived in the real world should reflect precision, but there was already too much emphasis on right answers in all the short-cycle and high-stakes testing. However, when the documents of the real world tests that I have collected over the years are examined, the collection of real world problems makes a much more interesting workbook than the one the school has purchased. The problems from the school workbook are out of context, often even silly in their inability to represent real world application. For example, one problem that makes my students laugh is when Bob, who is standing at a corner of a football field, is trying to figure out which way is the fastest way for him to get across the field– to go around two sides, or cut across the field diagonally.

The real world tests require students to wrestle with data given in context of the unit they are studying and apply their mathematical knowledge to the content in which they are immersed in their workshop activities. For example, while studying ecological issues of the world, they wrestled with the following problem: In California the government issued statistics indicating that it would take 2000 years to build up 1 inch of topsoil, and that its large-scale agricultural business was depleting as much as 1 inch every 25 years. If this is true, how many times faster is California using up its topsoil than it takes to make it? Distilling these kinds of questions into a workbook could make them seem as isolated and fragmented as the workbook problems are now. The integration of working through mathematical concepts while studying a topic of interest is an important quality of the Real World Math Mode that captures the students' interests and deepens their understanding of the material of both subjects: math and the integrated topic of study. Of all the tests found, (which is not all of tests created), the number that were integrated into various disciplines are on the chart that follows:

Table 30

Number of Real World Tests in Various Content Areas

Science Integrated RW tests	Social Studies RW tests	Language Arts RW test
6	13	2

The kinds of resources used with each test, often multiple kinds for a single test, while other test resources were not saved, are as follows:

Table 31

Number of Kind of Resources Used with Real World Tests

Charts	Maps	Readings	Tables	Misc.
8	5	6	17	4

What I infer about this section is that, from 2000 to 2007, Real World Math Mode worked well *because* it was integrated. As a result of being embedded within our yearly science and social studies Workshop Mode, it was simple to add a self-as-mathematician component. Consequently, Real World Math consisted of rich, genuinely important questions about the science or social studies fields we studied. However, the difficulty of finding, understanding, and refining these questions within our units was time-consuming for teachers. When added to the pressures of testing preparation that NCLB brought, the time commitment required for Real World Math was too great. Real World Math was literally squeezed out of the school day.

Reconstruction- 2011-Present

No Real World Tests were found past 2007. As the increased pressure of NCLB ensued, we found our time being more consumed with modes that were more aligned to the tests. While our Math Brainteaser Mode held onto 2008, the Real World Math Mode ended sooner, due to the fact that it was always embedded in the Workshop Mode. It was difficult to find the time to teach the content of both the Workshop Mode and the Real World Math Mode with so little time left in our days from the attention given to math and language arts standards for testing. Quite surprisingly, the Real World Math Mode would still come up in conversation when teachers and I collaboratively planned our workshop units. We would reminisce about their importance but plan without them. This year, with one of the units being the history of money, talk ensued about Real World Math again, which resulted in redesigning a new version of the Real World Math Mode.

Since the unit was planned without any Real World Tests like those we had used before, we decided to use a chart on the internet about the changing value of gold over the years. I found that the chart was stagnant and didn't have as much information as I was used to accessing to construct my real world problems. Together, the students and I started searching the internet to answer our questions about gold and its changing value. We journeyed from Fort Knox to a site that gave us up-to-the-minute gold and silver prices. Using those sites, the students and I generated mathematical problems that created a flurry of excitement. They calculated the amount of money that the bullion in Fort Knox (last counted in their 1953 audit) would be worth. By the time they had figured out its worth, the gold prices had risen, and so they figured it out again and again. Then they wondered to what number the percent increase on the gold and silver site was referring. They calculated it and found it wasn't the previous number they had just seen a minute ago. They concluded the percentage was from the day before, and another chart revealed they were correct. They generated several conversion problems between US dollars and other currency. They even calculated if they could make more money by buying gold or silver over the same time period of the week. They were so excited to do this; they could have kept doing it for weeks, but it was time to fill that scheduled slot with another math mode. It was clear that we had hit on another structure for a new Real World Math. We could still integrate it with our workshop unit, and use charts, etc. from the internet, (even better than before because the internet is much more interactive), but not make it a pre- and post-test. The idea of having students help generate problems of their own, in addition to the ones I generated, covered more mathematics than the old tests. They worked collaboratively all over their small group white-boards, and couldn't wait to generate the next problem. The students and I could produce several similar problems to improve their conceptual understanding and accuracy. This first time I did not test them on what they had learned, but formative assessment could have easily been added to make sure that skills and concepts were mastered.

My journals reveal my excitement at bringing this mode back to my teaching. I write:

> Right now, Real World Math is making a comeback in my room with a vengeance. It is reshaping itself in structure but not in purpose…. The [mathematics] problems we came up with were endless. I could keep going, but I ran out of time. We had a blast. They love it as much as brainteaser. I was humbled once again about the days of yore.

Find the purposes, and go with it, march forward carrying the banners of the enthusiasts, forget the tests."

Because the students and I were working together to generate curriculum, it felt like *real teaching and learning*, like the days in the Middle Age and early Renaissance, before the testing mania began. I even felt as if the Common Core mathematical practices were being highlighted as we scanned mathematics content from percents to ratios, to conversions, to order of operations. I wondered as I watched us race through mathematics content like there was no end, and I saw connections being made by the students. What was all the fuss about regarding the barriers to integration, the need for coherence or order, the fear of not covering enough curriculum or the problems of enacting integrated curriculum into scheduling? What is most important to remember as a Family School teacher is that those problems don't exist when we have the other modes to answer them.

It is interesting to note that available time has opened up the possibility to bring back both Brainteaser Math and Real World Math because the workbook for the Common Core standards is much shorter than our previous workbooks, thus we were able to easily complete it by mid-January. Even though the testing mania has not lessened because we are still being bombarded with district short-cycle assessments and test-readiness documents, I still find myself with time to be able to explore these old modes once again. The new PARCC test will not begin until next year, and perhaps after seeing how my students do on the test, I may feel as though they do need more time preparing for it. Without the knowledge of that test, it feels as if we are preparing our students adequately, and there is still ample time to explore the other modes. It is also possible that I should be considering that both Brainteaser Math and

Real World Math are *preparation for the mandated tests,* though that is not their modal purpose.

Because of the excitement that Real World Math has brought to my classroom, it has planted the seed of a new math mode. The new mode came directly out of the new structure of the Real World Math mode. Without having the pre- and post-testing structure, and by using the internet to find the charts and data needed to create problems, I have discovered how students want to take charge of their questions about these charts. They loved making up their own problems so much that I asked them what they wanted to learn about in mathematics and their answer was trigonometry.

We had explored the very basics of trigonometry to find lengths of sides or angles of a triangle, but that must have been just enough to tickle their imaginations about the rest of trigonometry. It has been decades since I fully studied trigonometry, but because of our learning community becoming so strong in our desire to ask and answer our own questions, I agreed to the challenge.

The students and I started by going over what we knew about trigonometry, which was how to use SOH, CAH, TOA (three functions for solving for sides or angles: sine A= opposite/hypotenuse, cosine A= adjacent/hypotenuse, and tangent A= opposite/adjacent). We really didn't know why or how they came to be or worked. We started with these as the source of our basic questions, which led us into studying the unit circle. As we explored a breadth of resources to answer what we didn't know, we kept uncovering new concepts to wonder about. I am genuinely intrigued to see my fourth and fifth graders as engaged around this as my sixth through eighth graders, but then I feel like I am right alongside my students. We have generated questions for which we couldn't find answers; fortunately our computer technician has a major in mathematics, so we were able to access him. He didn't know the answers either but asked his previous professor and brought information to us the following day.

This trigonometry experience was not Real World Math Mode. It differed in several respects. First, it was pure math, for the sake of math. Real World Math is by definition embedded in real world contexts. Second, while it definitely generated student connection-making within trigonometry, it did not increase their connection-making between math and the world at large – the primary goal of Real World Math Mode. However, the students and I shared a genuine enthusiasm for exploring and solving trigonometry questions, which I believe wouldn't have been possible without our Real World Math experiences.

I'm not expecting to maintain this idea for a new mode, but it is fascinating to watch students take this mathematics and run with it. In my last journal entry, I have some fun thinking about this new mode:

> So what would we call the mode: Self-initiated mathematics? Give them what they want? Teach what you are not allowed to teach? Over their heads? Hang on for your life? Learning together: students and teacher. …
>
> All fun aside, it gives me a chance to play around with why or how it could be a mode.
>
> Let's analyze its purposes: 1) It was a student choice, so I think that has to be key. 2) I think the next part is that I am learning with them. There is something very important about the teacher being a learner with the students, especially in this program because we tout all this metacognitive stuff to the students; it is high time they see us act on *our* metacognitive strategies. 3) The metacognitive piece then

follows nicely as a purpose. I absolutely must model the strategies I have been preaching to the students. And, 4) it must be above grade level, but that is the charm that is appealing to them to be able to say they can understand something that is normally not taught to their ability level.

In my journal on April 6, 2014, I also wonder about the instruction, and whether those purposes would direct the kind of instruction needed for the mode:

> I believe that the teaching of a lesson is now constrained by those purposes. 1) Because the content is above grade level, and I don't know it, I have to teach it in such a way that I function like a student. I ask them for direction, what questions they have. How do they want to go about figuring it out? They can take more of a lead, but I'm not sitting back watching them learn something I already know, so I get to be a participant and model good student learning behaviors that accompany not knowing and sticking with it. 2) Working as a whole group seems to work well with this, but I have broken them up to go into smaller groups to restate something we just learned. 3) I don't imagine this should be on the Performance Cycle, but I don't see why it couldn't have learning targets for formative assessment, and quite honestly, I wouldn't mind us making up our own test and testing ourselves about our journey. 4) I haven't assigned homework because I didn't want to deal with parent complaints about why I was sending home trig homework with their third-grader, but I wouldn't mind taking class time to work through something I might have sent home for homework. 5) One thing that I haven't done, but I think would be essential to the purposes, would be to keep a record of the all the questions we ask while in pursuit of our knowledge, and the various resources and metacognitive strategies that we use to

> answer them. It would seem that being transparent about our learning would home in on that purpose more. 6) It requires time- so I can't see it being taught in less than an hour a day, and scheduling it as a unit for a period of about 3-4 weeks would be good. (I suppose I could see two-weeks as sufficient just to be assured of getting it in). 7) It feels like one of those experiences that I would do towards the end of the year after I have established some sort of learning community, however, in just writing that, I wonder sometimes why we make those decisions, and don't use something like this to teach how to be a learning community at the beginning of the year. This part of the teaching constraint would have to be tested. 8) Oddly enough, one key piece the trigonometry keeps coming back to is how it is used in the real world. Students seem more willing to push through difficult concepts when they see what those concepts apply to.

I can infer, by studying the above journal entry, my enthusiasm for teaching just bursting off my journal pages, and, yet, I know that I will probably not act on this new mode. Why? I suppose I think that we have enough modes as it is, and to take on another one with all of the teachers is asking a great deal of the school. It may also be a repeat of the history of brainteasers, and I don't think that it would be valued to be studying something that is considered to be so far above the students' capabilities. I have discovered that the end of the students' capabilities is usually the end of my capabilities, which are where our zones of proximal development intersect, yet I have also discovered that if I can understand it, so can they, and we don't move on until I see the smiles of comprehension spread across the students' faces.

I am sure the argument would be lodged that trigonometry is not on the test, and I would have to worry that I had not spent enough time covering what is needed for the test. I shouldn't have to worry about coverage, because all of the students, grades 3-8, are in mathematic algorithmic groups that meet their developmental needs for building those algorithms conceptually, and these students are also in Mobius Math, where they have become completely comfortable with working on higher-level mathematic concepts with scaffolding. The brainteaser units have also contributed to these students by teaching them to not be flustered when presented with a problem above their grade level comprehension. Whether we explore this new mode, or we simply accept the new Real World Math Mode as a place where they can explore their own direction in mathematics, the *purposefulness* of student-directed instruction has been revealed to be a key element for a math mode.

What can I infer from this section is that the demise and comeback of the Real World Math Mode enhanced my classroom; the enthusiasm this mode brought into my class for solving real-life problems was amazing. It even inspired us to conceive a possible new math mode. Although I'm thrilled with that prospect, finding the time to regularly schedule the new Real World Math Mode will be challenging enough. Ultimately, the preparation for testing might be enthusiastically received when the modes of real mathematical purpose take hold and are not *scared away* by political pressures for student performance.

For these students, real world matters, and happens, and inspires, even if it is in small doses or parsed curricular units. Because of how our modes work separately and together, their whole is greater than their parts, and we are able to battle Paulos' innumeracy.

Table 32

Examples of Literature Review Connections Developed

Collaboration	•As teachers collaborated on their workshop unit plans, the Real World Math Mode opportunities integrated into those units were collaborated.
Teacher as Designer	•Teachers designed the pre- and post-tests of the Real World Math Mode. •Teachers designed the new mode format and content to bring Real World Math Mode back to the classroom.
Reflective Practitioner	• Teachers studied how Real World enhance workshop curriculum. • Teacher studied how new mode used student-directed instruction.
Learning Community	•Teachers will be designing and learning a new format for the Real World Math Mode. •Teachers and students will work together to find multiple methods for applying and solving mathematical problems.

🕮 NCLB Story Revealed from the Real World Math Mode:

The pressure for our students to perform for NCLB testing accountability issues had only negative impacts on the Real World Math Mode.

☹ Because we had designed our Real World Math Mode to be a pre- and post-testing format, this mode was eliminated due to NCLB emphasizing too much testing. The testing pressures did not at all influence us to teach more real world application or teach it in another manner. It simply required too much time for teachers away from what they perceived as essential to help move students towards demonstrable (tested) proficiency.

However, because of our desire to study ourselves, we have seen that this mode is essential to improving all the math modes to work together in the best interest of the students. Though Common Core emphasizes more use of non-fiction material for students to analyze, as our real world charts and graphs were doing, it was not due to Common Core that this mode was revived. Students were not benefiting from true application of mathematics to the world. The Real World Math Mode proved its value as a part in the whole of all the math modes needing to work together for the benefit of all of the students.

Figure 41: NCLB Story Revealed from the Real World Math Mode

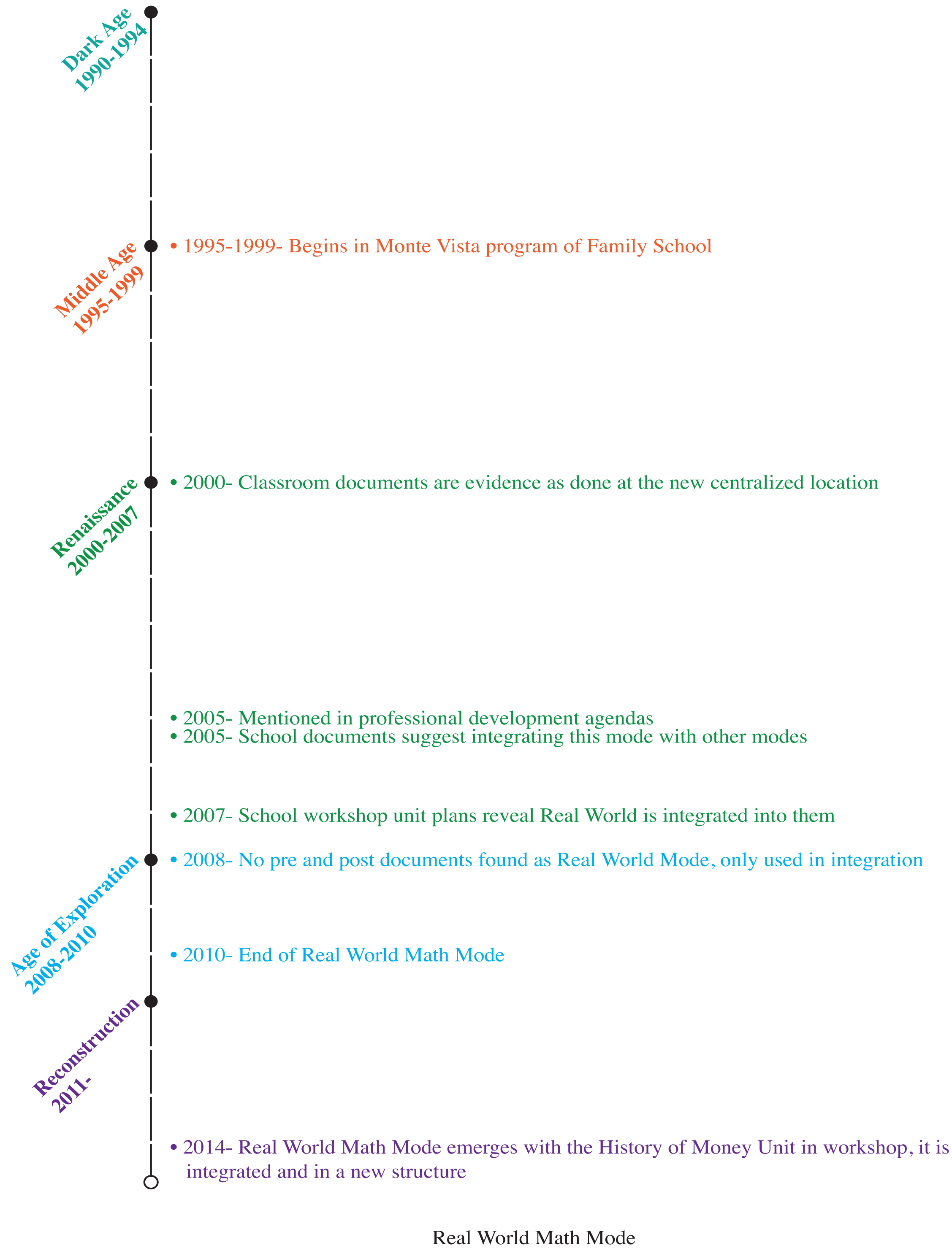


Figure 42. Real World Math Mode Timeline

Chapter 9

Inquiry Mode

> The fatal pedagogical error is to throw answers like stones at the heads of those who have not yet asked the questions. (Tillich, in Brown, 1971, p. 8)

The Inquiry Mode is not specifically a math mode, since it can be used across disciplines. However, some of the most significant mathematical experiences occur during inquiry. Students access mathematics by including data collection and analysis for questions in any subject matter. Consequently, we identify Inquiry as a math mode because we know that as the students' mathematical understanding of statistical analysis increases from tally marks to algebraic functions, their sophistication of the inquiry process enhances their understanding of the world. It is also true that some of the questions students ask are mathematical in nature from the onset, for example, "Could the population of the world fit in New Mexico? Yet, every inquiry requires a systematic plan for the collection of evidence, which most often results in mathematical analysis of data.

This chapter will evaluate three important characteristics of the Family School Inquiry Mode. First, it was important for us to make inquiry a mode as opposed to an activity. My teachers and I teach the process of inquiry in connection to any topic a student wants, instead of using inquiry merely as an activity to teach specific content knowledge. Practice with the process of inquiring is what improves students' ability to develop a design and to collect data for analysis. At Family School, we have designed a form to structure the inquiry process, but throughout the year inquiry steps can be taught while working up to completing the entire form. Secondly, when creating a community of inquirers, it is important to include parents and teacher with the students. In order for students to learn that interconnectedness is a natural part of learning, everyone must be involved in the process. Because inquiry is done both in class and at home, it is essential to train parents about this mode. Lastly, the process of doing inquiry presents an ideal opportunity to teach critical thinking. Family School's Critical Reframer document became an invaluable tool for this. The Critical Reframers enabled students to structure their questions in ways more manageable to answer. Once students learned how to reframe, they had critical thinking skills in their toolbox to help in their answering of the their questions.

A non-example of this mode is using inquiry to teach a particular activity, not providing an on-going opportunity for students to develop their inquiry skills. Another non-example of this mode is using inquiry in one subject or giving the students the questions about which to inquire. It is important for students to learn to find their own questions in the world and set up their own designs to generate the answers.

Some of the basic fundamental concepts of doing inquiry can get lost if not done on a regular basis. For this reason, it is important to study the evolution of the Inquiry Mode. This chapter chronicles the evolution of Inquiry Mode and analyzes the factors that make the Inquiry Mode effective. Early in its inception, inquiry focused on asking and answering a wealth of questions. As the Inquiry Mode evolved, the standards for thinking and writing increased. Eventually, the expectations of inquiry became so sophisticated that the number of inquiries decreased in a year. Consequently, the habit of inquiry was not developed as it had been in the earlier iterations of the mode. It became necessary to vary the lengths of inquiries done, so that smaller, more frequent inquiries could be accomplished while working towards a longer, more sophisticated inquiry by the end of the year.

Table 33

Major Codes and Their Themes for the Inquiry Mode Creating Patterns of a Plotted Story Line

Mode: Inquiry Mode		
Codes	Themes	Plotted Line
Higher Purpose	•Mathematicians generate their own theories based on their analysis of their data collection. •Mathematics has power when used to inform the generation of theories. •Mathematics is meaning-making.	The themes of the Inquiry Mode created a pattern that suggested a plotted line which demonstrates that we learned that the formalized inquiry form of all it parts and expectations is less important than the function of the inquiry process. As this recursive instruction improved to such a particular formalized paper report, the inquiries were done less frequently. Consequently, the recursion would not occur enough to develop good inquiry habits in students. We returned students to doing the older versions of inquiry, now called, *mini-inquiries,* so that students could accomplish many more inquiries, thus, being more recursive—and habits abounded.
Specific Content	•Inquiry process is the content. •Teachers taught students to use a variety of data types and analyze the data collected.	
Strategic Instruction	• This mode should emphasize mastery of the process of inquiry. •Teaching inquiry is emphasized instead of using inquiry to teach content.	

This chapter will reveal the autobiographical narrative of this evolution, while analyzing the inquiry documents throughout the story. The present-day use of the Inquiry Mode is presented at the end of the chronological narrative by using excerpts and analysis from my reflective personal journal this year. A conclusion to this chapter reveals the words of graduates of our school when given the opportunity to discuss the importance of Inquiry.

Middle Age- 1995-1999

It was in the earlier years of Family School when the Inquiry Mode started in my classroom at Family School, but I cannot find any evidence to confirm the starting day or year. I do remember being concerned that my students were not very inquisitive and did not have very many questions about the world in which they lived. As a mother who lived through my own three- and four-year-old children's stages, I remembered them asking non-stop questions. I was concerned that this questioning seemed to disappear completely by second grade.

It occurred to me that perhaps, as parents and teachers, we were overzealous in our desire to answer their questions. It could be that in answering them, we took the power and enjoyment of asking questions and pursuing answers away from the children. How satisfying is it to ask a question, if everyone else always knows the answers, and no one wants to go on the investigatory journey of figuring it out? It also could have been that in a few short years in school that students learned that asking questions reveals to everyone else in their class that they do not know something, suggesting that they are not as smart as they want others to think.

I had a deep desire to have all my students love wondering about the world, and it was important to me that they not only felt safe about asking questions but also couldn't wait to think up more. While in Math Brainteasers and the Algorithmic Math Group Mode, I had wanted students to explore the world of mathematics; I also wanted students to be inquisitive in all subject areas. Even more importantly, I wanted them to want to investigate finding the answers for themselves. I decided to create a space in my curriculum to teach inquiry.

Edwards (1997) reports that the *Institute of Education Sciences* encouraged inquiry-based learning for science "to guide students to fashion their own investigations" (p. 18). The standards state that students should "formulate their own questions and devise ways to answer them. They also collect data, decide how to represent them, and test the reliability of the knowledge they have generated" (p. 18). Edwards points out that in the science laboratory experience in traditional schools that "true inquiry is usually forfeited in the process" (p.18). He acknowledges that to begin inquiry teaching, teachers must take on the challenge of engaging passive students who may balk at the new expectation. He also argues that getting students to ask questions is difficult. He offers three strategies to help students find questions through reading articles, engaging in activities, and providing various topics.

By 2003, the *Institute of Education Sciences* (2003) reported that many kinds of inquiries had been used in science classes and categorized them into three categories: 1) Structured inquiry— where students follow the precise instructions of a teacher-directed activity; 2) Guided inquiry— where the question is teacher-generated, but the students direct their own inquiry investigation, and 3) Student-initiated inquiry— where students create their own questions and design their own investigation to find their answers.

While the intent of our inquiry is the third, there have been times throughout the years where I have incorporated the other two methods during the process of teaching them how to do inquiry.

One of my inquiry documents for a parent presentation I gave years later in 2008 about inquiry reveals that I distinguished our Inquiry Mode from other notions of inquiry by explaining that others were"[t]eaching inquiry" while we were employing "inquiry as teaching." In its inception, Inquiry was not assigned to any one discipline of study. I wanted students to be able to ask questions in whatever field they wanted. I did collect all the students' questions they had used for their inquiries over the first couple of years the Inquiry Mode existed in my own classroom. I literally collected over 250 student questions and categorized them by topics or themes to document the figures in each category. The results are posted in Table 34 below:

Table 34

Student Inquiry Question Topics

People	46
Measurement	45
Earth Science	30
Chemistry	29
Food	26
Plants	20
Animals	14
Forces	14
Design	12
Art	10
Misc.	10
Technology	9
PE	4
Total	269

This table shows that when given the opportunity to ask questions, students find questions everywhere. Although their favorite questions are about people, these topics cover a wide range of academic subjects. Inquiry was chosen as a mode for developing the habits of inquirers in all subjects.

Below is a sample of those questions, generated by first through fifth grade students:

What has more life: river water, swamp water, or old rainwater? What kinds of endings does Silverstein use in his poems? What part of the windmill does the wind need to hit to make it go fastest? Does a candle burn differently at different altitudes? Do children follow rules better than adults or is it the other way around? What percent of people like their jobs? What freezes fastest hot or cold water? If Earth is 24,901 miles around the equator, then what is the percentage of circumference decrease for every 10 degrees of latitude? What is an ant's favorite color? How many miles would it take to get to LA from Albuquerque to go the pretty way, the shortest way, and what is the time difference? How tall is the average tree? What percent of time does each person in my family have on the phone?

Figure 43. Sample inquiry questions asked and answered by students from my classroom files.

Clearly, students had many questions whirling in their minds. I just had to figure out how to get them out. Once that happened, inquiry became one of their favorite modes.

Getting students to generate these questions, however, was not an easy task in the beginning. I created a wide range of activities to get them to begin to wonder about these activities, such as: exploring how a bicycle works, playing with a wide variety of balls, watching *pill bugs*, isopods, react to different stimuli, and even taking the students on a bike ride around the neighborhood to get them to notice questions they could find outside. The day I brought in a sack full of 25 different balls, their eyes widened. When I brought a couple of bikes into the class for them to study, they began to think I was strange, but when I arranged for them all to bring their bikes to tour the school's community, they really knew that they could ask questions and feel safe.

The next problem was that their questions were not all easily answerable. Because I had thought the reason they had stopped asking questions was because we answered them, I wanted them to be able to answer their own questions without looking up the answers at all. I was, and still am, more concerned with empowering them to set up their own ways to find their own answers, even if the answers were wrong. It felt like the process was more important than the answer. If another student disagreed with the discovered answer, they were invited to do their own inquiry to refute the original one. I justified this idea to myself by saying, "There have been hundreds of generations of people who have lived under wrong answers for hundreds of generations, (like thinking the Earth was flat) and I wonder what wrong answers we are living under right now that we don't know about yet." So, we continued to allow students to ask and answer their own questions by designing their own investigations, collecting their own data and generating their own analyses of the data. This was a good objective, but some of the questions were so open-ended, students couldn't come up with a design for the investigation.

That is when I discovered some critical thinking skills in one of Marzano's (1988) publications. I used these eight critical thinking skills to teach the children to reframe their questions. They were: comparing, classifying, deducing, inducing, abstracting, error analysis, perspective analysis, sequencing, and constructing support. We were then able to take a question like: Why is a tomato a fruit, and change it to a comparing question such as: How is a tomato similar and different from other fruits? This reframing allowed students to see the way to go about collecting evidence to answer this more specific question. This particular reframed question infers that the student will compare a tomato to the characteristics and qualities of other fruits. Sometimes the reframing moved a child away from their original question, but because it was still very close to it, they were eager to get started on their new, reframed question.

On a side note, these Critical Reframers, as we have come to call them, changed my teaching and my thinking approach. I was able to utilize them to improve my questioning of my students in all subjects, as facility at reframing their questions became a natural part of my process. After about a year of using those original eight, the students told me there had to be more reframers, because when they started reframing their questions, they came up with questions that did not fit the original eight on the list. So together, we came up with seven more. Ten years later, we added the sixteenth. It was ironic to us that we had omitted *omission analysis*. Below is the chart of all the Critical Reframers still used for this mode at present and also for many of the other modes in other subjects at the school:

Table 35

Critical Reframers Developed by Family School

Comparing	Identifying and stating similarities and differences between things.	How are these things alike? What particular characteristics are similar? How are these things different? What particular characteristics are different?
Classifying	Grouping things into definable categories on the basis of their attributes.	What groups would you put these things in? What are the rules for membership in each group? What are the defining characteristics of each group?
Inducing	Creating generalization (or principle) from observation or analysis. *Detail to general*	Based on these details, what can you conclude?
Deducing	Predicting details or conditions that would lead to general principles. *General to detail.*	Given this general principle, what would you predict? If___, what would you conclude?
Constructing Support	Building a system of proof for a claim.	What arguments support that assumption? How would you organize evidence / reasons to support your case?
Abstracting	Identifying and stating the underlying theme or general pattern of information.	What is the underlying pattern? What other situation does this pattern apply to?
Perspective Analysis	Identifying and stating how perspectives affect understanding of information.	What would this be like from a different point of view? Why would that person or group think that? What is a different way to think about this?
Error Analysis	Identifying and stating how errors affect understanding of information or results.	What isn't right? How is this inaccurate and misleading? How could this be corrected or improved?
Means/End	Identifying and stating how method/procedure affects the result.	If I alter the method, how will it affect the result? If I want this result, how will I need to alter the method/procedure?
Form/Function	Identifying and stating how structure influences purpose and how purpose/operation informs the structure.	How does organization or form affect function? How does function require a necessary form?
Utility	Identifying and stating usefulness.	What is the purposefulness or usefulness?
Sequencing	Identifying and stating order.	What is the order? What happened first?
Prioritizing	Identifying, arranging and stating order based on importance.	Why is it essential to do it in that order? What is most important?
Omission Analysis	Identifying what was left out and how the lack of inclusion impacts the understanding of the information.	What can I infer from the exclusions? Why was it excluded?
Parameters	Considering effects of changing the limits, boundaries, or perspectives.	How does _____occur within this range? What is the range where ____ will occur?
Defining	Identifying and distinctly describing the basic nature or qualities.	What's the definition? What is it?

Critical thinking has always been a strong focus for our school because of these Critical Reframers— even more so as Common Core curriculum arrived during Reconstruction. The Common Core's push for building critical thinking popularized the reframers in our community, and even in other schools. There have been individuals who had contact with our reframers from our school, who have started spreading them around other schools.

It took some time to teach them how to ask questions and critically think, but it also changed our roles as teachers as we learned how to answer them. According to the Center for Science Education, (2003) the teacher should promote peer interaction and class collaboration to investigate and process findings. The teacher's role changes to encourage students to reflect rather than to evaluate. The students' roles change to become much more active in the process of learning as they "develop habits of [the] inquiry process and thinking" (p. 5).

Once our students were able to generate questions, they were writing many inquiries. At first they just journaled about what they had done in a spiral notebook, but it wasn't long until they wanted to know what a good inquiry had in it. The students and I compared the students' inquiry pages together and realized that there were parts to an inquiry that made it feel more complete. They identified the parts and a form was created for them to follow for each inquiry they did. Originally, I had not created a form for them because I wanted them to have freedom to follow the flow of their own thinking. But it did appear that both students and parents were more comfortable having a formatted booklet in which to do their inquiries. That is when I created the first bound notebook for the students. The format outline is as follows:

Pre- Investigation
- Found Question
- Reframed Questions
- Chosen Question
- Brainstorming
- Present Knowledge
- Postulate
- Design

Investigation Stage
- Journaling of Investigation
- Data Collection
- Data Presentation
- Conclusion

Post Investigation Stage
- Post Analysis
- New Questions

Figure 44. Inquiry Parts as Designed by Family School Students.

At the time of organizing this, I had no knowledge of any structured inquiry parts suggested by research and was simply analyzing the students' writing to see how that writing logically made sense or helped me, as a reader, make sense of their student inquiry. Lundberg (2005) outlines the various activities in inquiry as: project conception, design, data surfacing, data processing, data examination, dissemination of research and termination of the project (p. 36). While I was no expert in inquiry, our own analysis of the best parts of the student inquiry matched up fairly well with the experts. One reason I was hesitant to label the parts was that I wanted more of an organic process to the students' research. I wanted students to feel they could progress through the inquiry as their method and thinking encouraged them to proceed, rather than following a specified format. Lundberg (2005) also acknowledges this as part of the process when he states:

> While the sequence of activities outlining the research process is arguably what experienced researchers do, not all steps are always performed or performed consciously…Some activities are ignored or glossed over because of naiveté or personal proclivities of the researcher or because of resource constraints. (p. 37)

Student inquiry, especially grades 1-6 are a far cry from the inquirers of expert researchers, yet even the students felt there was an urgency to do their research, to *do it right or best*. Though the structured inquiry outline pages were created, it was not a requirement to fill them out completely until the Renaissance. Once we perfected the inquiry process, we encouraged all students to execute the entire process. Students' inquiry improved with the added structure, but the continual use of the entire structure did lead to decreased inquiry journals, which will be discussed in Reconstruction age.

What I can infer from this section is that the Inquiry Mode began with great inspiration and was driven by the students' eventual inquisitive nature. Their desire to evaluate their journals led to the structure of the Inquiry format, and their insistence that other questions existed led to our important Critical Reframers. As with brainteasers, the *process* of learning was far more important to Inquiry than the content.

Teaching inquiry instead of inquiry to teach. To return to the notion that what I wanted to do was teach inquiry instead of using inquiry to teach, I had now begun to find that teaching the inquiry process might be helping students to become better learners, if only to be open to asking questions and more comfortable with searching for answers. White and Frederiksen (1998) completed a study to analyze the metacognitive skills students developed by being taught the inquiry process. They found that through the "process of scientific inquiry and modeling … you also teach them how to monitor and reflect on their inquiry processes" (p. 6). Students who were interviewed after learning the inquiry process stated that they felt this process would be applicable other subjects other than science. This is the main reason that I wanted to teach inquiry as a mode applicable any subject the students were interested in pursuing. Some students have a penchant for other topics outside science that are very worthy of investigating. Some quoted excerpts from some of the students' inquiries from the Middle Age demonstrate their wide interests.

In 1997, in the example below, a student is wrestling with how to have consistent measurement of water pressure in order to be able to measure her suds in relation to hot and cold water:

> Today I retested a lot. I ran water hot at full water pressure to figure out what I think includes a lot of measuring. I did hot at full water pressure about four times in a row. After I finished that I did hot water at about medium water pressure. I did it over about a billion times because I kept getting different measures of suds. Then I moved to cold water. What I did is I did cold water with high water pressure. …

In this inquiry sample, she learns the value of retesting and how to change her design several times in figuring out how to measure the water pressure. The lesson of restricting her variables is becoming apparent to her.

After tallying how many ants went to collect hard-boiled egg pieces off different-colored construction paper placed by the anthill, this first-grade student wrote:

> The ants rileay likte the yeloe. Gold the ong. And the wite. Rilee not the blak. Or the red. Or the broun or the grean. Or bloo.
>
> (The ants really liked the yellow. Gold the orange. And the white. Really not the black. Or the red. Or the brown or the green. Or blue.)

This young student learned the value of collecting data to find her answer. In seeing her tally marks, it must have been a difficult and rushed process to get them all down. However by taping the different color strips onto the journal page, she was easily able to identify quickly where to collect her tally marks.

Here a fourth-grader wrestles with population and land area by trying to see if the entire world population could fit into New Mexico:

> I have learned that you could easily fit all the people into New Mexico if everyone had 2 square feet. Now I know that you could fit multiple worlds into New Mexico. The problem is that if each of us had 2 square feet, we couldn't move. 1,694,968,821,200 people could fit into New Mexico and the whole world would take up .0003 of New Mexico.

This student learned the value of mathematical calculation with her inquiry. She had done the work twice to verify her answer, and even went on to do the extra problem to figure out how much of New Mexico the population would use.

These student excerpts indicate that these students' interest in the world is full of questions in subjects other than science. Inquiry is an essential process for all learning and is not limited to science or math. It is important to impart the process of inquiry to students, instead of using inquiry as an occasional strategy to teach a concept in science or any other academic subject. Perhaps this is why this mode started with an emphasis on the inquiry process, rather than as activity for one of our other modes.

Renaissance- 2000-2007

While it is important to note that although I began these inquiries in the Middle Age, and reported this new idea I had been teaching my students to other teachers in 1997, it wasn't until our school moved to the centralized, portable site that we began to incorporate them as a school mode. It took me some years of experimenting with their purpose and structure before I was ready to suggest that inquiry could be considered as a mode for the whole school.

It is also important to discuss what all this has to do with being considered a math mode. While some of the inquiry questions could be actual mathematical questions, as is seen in some of the previous sample questions and sample excerpts, that is not why it became considered a math mode. It is because the inquiries rely on the students' collecting evidence from which to analyze their answers. It was the data collection and data presentation that the teachers and I realized was a missing part of mathematics instruction that had not been highlighted in any of the other math modes. We then began thinking of it as a modal opportunity for mathematics.

In thinking about the purpose inquiry had for mathematics that was different from the other modes, my teachers and I began to connect inquiry to the math modes. It still stands as its own mode for helping students improve their thinking and wondering about the world, but it supports our mathematical purposes by bringing *power* to mathematics. What we wanted students to see about mathematics is that numbers can help them make decisions about what they know and how data informs the questions they pose.

In 2008, when I was preparing to give a presentation to a group of middle school mathematics teachers in Arizona, I was wrestling with describing, simply, what was essential for teaching a good mathematics program. I knew I couldn't present our math mode structure, because the Arizona school had a prescribed curriculum that teachers were required to use. I decided to put together a presentation on what I had come to understand as the best qualities of our modes. Those qualities became outlined as The 4 P's. They are: puzzling (for brainteaser), profound (for brainteaser, and real world), pervasive (for doing all the modes and making sure that mathematics is taught more often and in an integrated manner), and powerful (for inquiry). The 4 P's have remained engrained in our minds at our school as we make curricular decisions today for any subject. Having just completed our school-wide Inquiry Fair this year, and seeing the parents pour into the classrooms as students gave their presentations for the other classes, it is evident that the power of inquiry stands as strong today, and we have explored how to encourage both inquiries i*nto* mathematics or inquiries *using* mathematics.

For students who struggle with mathematics, the power of collecting and analyzing data in their inquiry can be the impetus for them to understand the purpose for improving in mathematics. Anderson's (2002) review of the research literature finds that many studies have shown inquiry to support students with learning disabilities, promote higher achievement, and create more motivation and discovery for students. I wonder if the notion of empowerment encourages students to move from being passive learners to becoming more active.

Inquiry grew over the Renaissance as all the teachers began implementing inquiries school-wide. C*ritical thinking* and *thinking for themselves* were the qualities of inquiry that drew our students to love it. This mode was a long way from any activities in the traditional classroom at the time, but it felt right to move our students in this direction. Mason (1994) addresses researchers who critique inquiry learning by suggesting there is no one way to teach mathematics as there is a "growing realization that single perspectives are neither possible nor desirable" (p. 192). According to Mason, mathematical inquiry enables people to shift from "*talking-about* (using vocabulary in discourse but not in-dwelling what they are saying) to *seeing through* (using vocabulary as a means of describing what they experience" (p. 194). Anderson's research (2002) suggests that textbooks, assessments, fear of lack of coverage and inadequate professional development steer teachers away from using inquiry. He also suggests that a great shift for teachers is to teach students to be able to work in groups successfully, and be able to handle new student roles. The teacher role shifts also, which can be a barrier for some teachers. Also he suggests that sometimes *parental resistance* has been noted to keep inquiry from taking hold in classrooms when parents do not support this more innovative technique to learn.

The defining characteristic, which makes Inquiry a math mode is data analysis. Through analyzing their data, students improve their mathematical thinking. Inquiry brings power to math. As students progress, they improve their data analysis skills. This ever-developing sophistication can better inform students about their world. In the next two sections, you will see this became a powerful tool for parents and teachers to realize as well.

Parent inquiry. In order for parents to support us through this Renaissance, right from the beginning in 2000, it was clear my teachers and I needed to teach our parents the reasons for inquiry, as well as how to support their children doing inquiries when they were assigned to implement them at home. Many parents have not had experience with doing their own inquiries, so there have been times when we have asked parents to do their own inquiries. By doing this, parents were able to experience the importance of asking and answering their own questions. It is important for parents to remember they are supporting their own children, and ought not take over the child's inquiry, nor evaluate it too harshly. They are the support for the students to follow their own methods and construct their own answers. By doing their own parent inquiry, they learn the ownership and the enthusiasm that is generated by that ownership. One such parent inquiry from 2001 was long hidden in my files. Below are some excerpts from her inquiry where she asked: “How fat would I get if I ate all the Ben & Jerry’s ice cream in all the Wild Oats stores, and what would I have to do to lose the weight again?”:

> To find the weight gain formula, I called my nutritionist friend who told me to use 3500 calories eaten to one-pound weight gained as an average figure. She informed me that all calories are equal when it comes to weight gain. I will therefore use the *total calorie* figure from the Ben & Jerry’s.
>
> New question/side note- How long would it take me to eat all that? Estimate: assuming no satiety, I could probably eat 2 pints per hour. I figured out this would be 40 days of nonstop eating.
>
> Second side note- how much would it cost me to replace my wardrobe?

Her inquiry demonstrates the fun and enthusiasm she developed for her process. What we discovered is that inquiry is a natural process by which we learn, and teaching inquiry to both our students and parents helped them to recapture this love of learning and being an inquirer.

Teacher inquiry. Not only was it, and is it, important for parents to do inquiry, but we also have our teachers do inquiries on their teaching. Recently, we have them present two of them each year, one in the fall and the other in the spring. It is key that they use inquiry to model for the students the power of answering their own questions with the data that is collected. Often teachers announce the inquiry question to the students and the students help the teacher to collect the evidence. Sometimes a teacher will not be transparent and will reveal her findings to the students when she has completed it. Some inquiry questions that have been presented by teachers are presented below in Figure 45:

Is the size of the zone of proximal development an indicator of student achievement level? What qualities do the students I interact most with possess and how do they compare to the students who receive less contact? What are the impacts of the increased or decreased interaction? What does this information tell me about how I prioritize interactions in the classroom? What is the result when I increase or decrease my interactions with a given student? Will teaching this student the critical reframers related to reading group and structuring the required response around each reframer result in this student internalizing these reframers and initiating a thoughtful response about what is being read in terms of the reframers taught ,without prompting? How will different types of struggling readers benefit from the same strategies? What strategies will work best to improve student independence and attention to task? If I change frequency of feedback, how will it affect student performance? How will changing the type of question asked affect student engagement in lesson and student learning?

Figure 45. Sample teacher inquiry questions done for professional development.

These questions awaken the teacher to her classroom in the same way they awaken the student to the world. Teachers feel empowered to see how the collection and analysis of data can solve their own issues, and then share and borrow ideas from each other after presenting their inquiry results to the staff. One of the origins of teachers studying their classrooms through inquiry probably originated with teacher interviews that I did with our Middle Age teachers for their Professional Development Plans for the district. Though we were given specific forms to use for this plan, it seemed lacking, and we chose to have teachers use the interview to get insights into their classroom environment and practices. They enjoyed this interview's gentle, non-threatening manner for probing their thinking about their teaching. It was this document that helped us to firmly plant the notion that as a group of teachers, we would seek a means for studying our teaching beyond the district expectations, which ultimately evolved into teacher inquiries.

Lotter, Harwood, and Bonner's research (2007) revealed that when teachers were given the opportunity to have summer professional development around doing their own inquiry in science, they were more likely to adventure into trying it in their classrooms during the school year. Though our teacher inquiries are not necessarily done on the curriculum they teach, they might have even more impact because they may focus on how to improve their teaching. They get two benefits from one experience, seeing the value of inquiry, and solving a classroom dilemma.

Truthfully, having done inquiry for so long at Family School, I have come to believe it is a natural state of learning, and I have discovered others who believe as I do. Siegel and Borasi (1994) also associate inquiry with a very natural state of learning as a human activity. They find connections to this state going back over 100 years when they quote Pierce's (1877) theory of inquiry:

> [W]e never have firm rock beneath out feet; we are walking on a bog and we can be certain only that the bog is sufficiently firm to carry us for the time being. Not only is this all the certainty that we can achieve, it is also all the certainty that we can rationally wish for, since it is precisely the tenuousness of the ground that constantly prods us forward, ever closer to our goal. Only doubt and uncertainty can provide a motive for seeking new knowledge. (Siegel and Borasi, 1994, pp. 202-203)

What can be inferred from these sections is that inquiry is as important for adults as it is for students. Reawakening the power of questions in our parents and teachers helped us to realize that we, along with the students, had become a community of inquirers.

Community of inquirers. Sometimes students will develop inquiries about their own learning and collect data to make decisions as to how to improve in their student skills. All ways that mathematics is used in the inquiries empower students to see the influences of data collection and data presentation. But what is most powerful in inquiry is sharing it with the classroom community. They love to express their findings with each other. The audience is full of questions and comments when they present, and often they will generate further questions that another student would like to follow up. Some students love to prove their postulate was right, though those are few. Most of the students love the surprise of finding meaning in the most unusual ways. Because of this enthusiasm for having a learning community with whom to share, I will often employ partners or triads to work together on their questions or similar questions around the same topic. They love this opportunity and could spend days entranced by their work. Jaworski (2006) states: "In constructivist terms, inquiry can be seen to stimulate accommodation of meanings central to individual growth. In sociocultural terms it is a way of acting together that is inclusive of the distributed ways of knowing in a community" (p. 204). Inquiry hits at the heart of Piaget's (1952) cognitive theory of intellectual development. When students' inquiry experiences transform their thinking of an event, they have accommodated the information from their previous understanding. When they are not convinced in their findings, they may have only assimilated the information.

Often the students and parents will engage in inquiries together, genuinely engaging in co-discovery. Many of the teachers also conduct their teacher inquiries with their classes, and have students help them collect data. As students see everyone around them involved in inquiry, they see the value of *wanting to know* over the value *of thinking that they know.*

The Renaissance Feels the Pressure of the Coming Age of Exploration

Inquiry has remained stable in our curriculum throughout the Middle Age, Renaissance, Age of Exploration and Reconstruction, with the exception of one year. Once we moved together to the portable site, inquiry became a staple of our curriculum as a school. There was so much support for it from teachers and parents that we began to hold an Inquiry Fair each year, in lieu of a Science Fair. The teachers and I felt that students should have an opportunity to present their inquiries in any field, not just science. It was important that students see how data analysis can be used in all domains. The Family/Teacher Organization (FTO) an organization akin to a PTA, even collected funds for participation awards. However, in 2006, we cancelled Inquiry Fair due to parents asking to decrease the homework load. Parents' concern over too much homework may have been a response to the teachers responding to accountability stress under NCLB by sending home more homeschooling to help students make gains. It may also have been the result of the inquiries improving so much that they became more like college-level theses than young students' inquiries. The documents suggest that Inquiry Fair was brought back by the popular demand of the parents, but the staff found ways to keep the inquiries from overloading the homework hours. Below are some of the ways the documents showed how the staff made efforts to help with the homeschooling issue to help keep inquiry as one of our modes. The February 2007 staff meeting notes indicate parents were wrestling with doing inquiry at home as part of their homeschooling, especially in families with multiple children, and our response was:

> As Family School curriculum has developed across time, we notice that families with multiple children are struggling with how to keep homework meaningful and manageable.
>
> Can we allow some at-home projects (Sophie, inquiry, projects) to be done as a group?
>
> We may want to encourage kids at the older grades to start taking more on themselves.
>
> We do not want to lighten the load or change our expectations, but offer a means to work together as a family to engage in these rich experiences in a meaningful way.
>
> We will be willing to work together to collaborate with other teachers to coordinate components of projects for these families as the family sees a need and it fits.

Further addressing the homeschooling difficulties with inquiry in our Summer Seminar, which is a yearly professional development opportunity for our teachers to get three days of training for our upcoming year, our Summer Seminar notes from May 2007, indicate that we wanted to integrate more inquiry into the classroom instead of parents working with their children during their homeschooling.

INQUIRY

- How can we teach inquiry in class?
- Measurement brainteasers through the process of inquiry
- Workshop inquiries – whole group inquiries
- Taught in reading
- Integrated with Non-fiction writing
- Connected to Sophie Letters
- Through Real World Math

Figure 46. Summer seminar notes on Inquiry from my computer files dated May, 2007.

It is also important to note that the inquiries of the Renaissance did not really resemble those of the Middle Age. Having continued the inquiry process over eight years, the sample inquiries below show how much the writing improved and how much higher the expectations were for students. Their ability to articulate the designs of their journal have developed, as well as their acute sense of communicating a detailed process of their investigation. Here are some excerpts from students in 2004.

This is a paragraph out of an eight-page, single -spaced document of a seventh-grader.

> As I have now fingerprinted 10 people and having to redo a few, my skills are improving greatly. However, I am noticing as I finger print people from ages 8-53, the way the print shows up is of great difference, even though I am the only fingerprinter. At the lower end of the spectrum, I am getting a larger amount of smearing. My guess would be this is happening because their fingers are so much smaller than mine that when I go to roll it in the page it is difficult for me to maneuver their hand. A few have also been giving resistance when I go to print them, not giving their hand the full range of motion that it is capable of. As I move towards the older spectrum the prints get bigger and thus so are the fingers. Based on my previous comment, as I can now easily control the finger and maneuver it, the prints are getting better. However, occasionally I have some problems with people trying to rest their elbows and not being able to turn their hand and with one person in particular, I had great difficulty fingerprinting the right hand as the person said they weren't very flexible in that arm and was having great difficulty performing the motion necessary. The final problem I have had so far is when I forget which way the thumb and fingers are supposed to roll, often rolling the thumb out and the fingers in, when in fact I am supposed to be doing it the other way around.

Figure 47. Student Inquiry excerpt from my computer files dated August, 2004.

In analyzing the marble game of Zertz, a seventh-grade student looked for the maximum number of Zertz marbles that can be played on the board before a jump became the required move. This particular student demonstrated how he was able to integrate his knowledge of algebra into helping him solve a strategic question about the game of Zertz. He also demonstrated an ability to follow through on his question more deeply than he anticipated, and then place parameters on his question from having figured that issue out with his thought process. His investigation was thorough, and he was able to articulate the intricacies of his inquiry. He wrote:

> By the inconsistencies in the points, 3-6-8-9, they will not follow a line and thus will not work with the y=mx+b equation. However, if I do a smaller board, such as a three by three by three hexagon to get an equation that hopefully will have a pattern that reaches to a four by four by four. A 3x3x3 has points 3,2 and 19,7, which then allows me to plug that into y_2+y_1/x_2+x_1 . 7-2/19-3 translates to 5/16 that I can now plug into y=mx+b. My equation now is y=5/16x+b. To solve for b I plug in my known Xs and Ys. Thus, I have 2=(5/16)(3)+b which translates to y=5/16x+2 2/15. Now I can plug 32 in for my y, which then gives me my answer for my maximum number of marbles… 12 2/15. However, I can't play 2/15 of a marble and it so insignificant that I will round down to 12 marbles for my maximum number of marbles. Now I will test my math.
>
> My dad and I went at it not to win but to strategically place the marbles so as to get the highest number on the board possible. We played it through and the first two got us 12 and 13 where the 3rd game got 16. This was a substantial difference that made us want to play another game to see if we could get higher. We did, we managed 17 twice but then decided that it would most likely hold 18 so we went again and with a very strategic plan (mirroring, whatever we do to one side we do it to the other) and we pulled off eighteen. However, we can guarantee that eighteen is the most because once the eighteenth ball was placed it pulled a piece that split the board into two halves, which guaranteed a win because of the number of balls on the half.
>
> To correct my math I now have the points 4,3 and 37, 18 which translates to 18-3/37-4 or 15/33 and makes my equation y=15/33x+b. This makes my final equation y=.454x+1.55. This probably only works for the 37 game piece board, but that's all I wanted.

Figure 48. Student Inquiry excerpt from my computer files dated August, 2004.

The sophistication of both the students' and teachers' understanding of inquiry have grown since the Middle Age, and Renaissance expectations have also increased. All of the inquiries took on higher expectations because of having Inquiry Fair. Teachers felt it necessary to move from the process orientation of their roots to being more product-oriented. It meant teaching inquiry throughout the year with a more product-oriented goal. The product, or performance, was presented at the Inquiry Fair. It is no wonder that parents became stressed with inquiry carrying so much performance value. Since much of the inquiry work is done individually, most inquiry work was done in the home for their homeschooling portion. Teachers taught the structure of inquiry and the intention of each of the parts of inquiry in class, but when it came to following up on the students' own questions as an inquiry, this was done in the home. If a family had several children, this placed quite a burden on the parent. It was clear that the school was going to need to pick up more of the implementation of inquiry in the class, or come up with a means of lightening the load for parents at home. The changes after 2006 of doing more inquiry in class, allowing siblings to do joint inquiries and even revisiting the *old days* and doing *mini inquiries* for part of the year has helped maintain the Inquiry Mode throughout all the ages of Family School.

The integration of teaching inquiry more in the classroom was evidenced in my own teaching documents of my unit plans and school-wide documents created for inquiry. Below is a table of the coded themes of those documents:

Table 36

Number of Examples of Themes Found in Inquiry Documents

	Scheduling Inquiry	Strategic Teaching of Inquiry	For Evaluating Inquiry	Demonstrated Integration of Inquiry into Other Modes
Renaissance	0	6	4	3
Age of Exploration	1	0	1	3
Reconstruction Era	1	1	0	0
Total	2	7	5	6

Despite the 2006 glitch of canceling Inquiry Fair, the document theme analysis through the ages suggest that we maintained a healthy discussion of the mode by creating documents to improve our teaching of inquiry, evaluate our students' inquiries, and integrating it into our other modes. It was this integration that was the key for teachers to be able to maintain inquiry in the classroom.

What I can infer from this section is that with improved instruction in inquiry came more sophisticated inquiry journals, which in turn had increased expectations. With increased expectations came more pressures and time constraints. The frequency of inquiries done in a year diminished and because of this, the habits of inquiring decreased as well. Inquiries became harder for the parents to fit into their busy homeschooling schedules, and they became harder for teachers to fit in their busy class schedules. We wrestled with these problems for years, but despite inquiries' demands, the Inquiry Mode has remained constant.

Reconstruction- 2011-Present

Except for 2006, the Inquiry Mode has prominently remained as a highlighted mode for Family School. The structure today of the Inquiry is very similar to its previous renditions, and, in fact, today the teachers and I borrow from all versions in the interest of time and motivation. We learned that we needed to value the *mini-inquiry* version for the part of the year that we teach the parts of the inquiry to the students. As they become more competent with all the parts, we move into small group or partner inquiries. We save the long performance full-form of the inquiry for the final projects for the Inquiry Fair.

When we teach how to do the mini-inquiries in class, we often integrate it into our workshop unit activities. However, we have also created time to teach inquiry as its own separate period as well. The mini-inquiries are usually taught throughout the first semester of school. In January, we begin to develop our students to take on more of the whole process with partners and build to doing an independent inquiry by the spring, when Inquiry Fair is held.

The inquiry work is often a joint effort between home and school. The 50% program, which has more homeschooling hours, allows for more inquiry to be done at home than in school. The 80% program often needs to carry more of the inquiry in school, and we more often have students work in small groups or partners. Still, the 80% relies on some of the inquiry either being completed at home or written up at home.

There is a wide variety of parent support for inquiry. Some parents work extremely closely with their students, and some students work independently throughout the entire process. When done in class, the teacher is more of a supporter than director of the activity. Teachers question students into thinking through their designs, help gather materials, and question their analyses of their findings.

Another important part of the structure of inquiry that is essential for classrooms is providing time to allow students to orally present their findings to their classmates. One of my favorite times for discussions is around inquiry presentations. Usually the students become very animated about extending one student's idea into their own reframed idea of what they have heard.

This year, my class was composed a number of new students to the school, and they didn't have as much of a background in inquiry as other classes I have had in the past. Some of the parents for my students expressed a strong desire for me to emphasize my teaching of inquiry more. I interpreted that to mean they wanted more opportunities for students to ask questions. I decided to increase the rate of the mini inquiries greatly from previous years. They were required to do two mini-inquiries every week. The students' eyes grew very wide when I told them this requirement, but they relaxed when I showed them a very mini-form for the mini-inquiries. Once they got into the rhythm of doing them regularly, they had stacks of questions they were waiting to get to. In my January 21, 2014 journal, I wrote:

> Inquiry is coming alive in my class again. I am requiring two a week but with a very small form that is doable. I feel that this class needs to ask tons of questions and play around with it more. I am taking time to have them share them with the whole class like the old days. It is very exciting. We all generate new questions based on their presentation, and it breeds more inquiry. We have had math, literature, psychology, science, art, writing, and, of course, food questions. I really just want to keep this up instead of going to the long form for Inquiry Fair. We are generating lots of interest in numbers and data and how they inform your thinking on a topic.

My class did move to the long form for the Inquiry Fair, and, surprisingly to me, they were ready for this rise in the expectation. I believe this was in part due to my extra work with parents at the parent meeting about supporting their children in the inquiry project for Inquiry Fair. In teaching the 80% program, teachers must always be cognizant of the amount of work they require their parents to handle, as they are limited in their homeschooling hours due to both parents' employment. I presented a power point my old students had written about the power of inquiry, and they were very eager to begin working with their students. On March 6, 2014, I wrote about this experience:

> Next, I pumped up the parents with the old student power point from my graduated students on how important inquiry was in their lives. I suggested to the parents that I

> wanted them to step in and work with the kids on inquiry. The first set of inquiries I got back from the students was so much better and used a great deal of math. I guess I am saying that if we are going to be quantitative beings then we should quantify in more sophisticated ways than just counting. Counting is okay, and useful, and sometimes very meaningful. But what was the response to doing more math? More enthusiasm, of course.

One student in particular demonstrated great growth in her inquiries with this increase in mathematical expectation. I wrote of one student:

> Lastly, in inquiry, she is amazing. This is the first year where inquiry finally came alive for her, and she puts in effort that is beyond expectation. Every single one of her mini-inquiries is an example for other students, and they love to hear about what she has done. I thought that perhaps she was a math kid, but now I am seeing her ability and interest expand into the sciences and social sciences.

However, for another student, I noticed in looking at her pie chart on the modes and how they illustrated her image of herself as a mathematician:

> Her inquiry slice is only 6%--- again because of her hesitance to work with the unknown. She is surely troubled when dealing with the unknown. This is important for me to realize about her, and I don't think I put it together until this moment. It makes complete sense about her.

I am sure that I will work more closely with her on her inquiries to see what can improve her image of them. It is also possible that she doesn't see them contributing that much to her idea of herself as a mathematician because they do not only teach mathematics. I will be interested to see how much enthusiasm she presents about her three volume inquiries she is presenting for the Inquiry Fair because this set is full of much more mathematics than she has ever done all year in inquiry.

Lastly, my journal also demonstrated a similar theme to the other modes in demonstrating that the modes support each other. I wrote about an experience while teaching inquiry that sparked Real World Math possibilities. While I was working with some students who were generating questions for an inquiry journal about the standard paper sizes used in the paper industry and wondering about how many sheets of the various sizes would be used to cover a billboard, I began to wonder about the mathematics of billboards. We started looking up the cost to rent and to buy. Immediately, we had a Real World experience and wanted to jump into creating problems with all the information we found. I began to see more possibilities for next year's Real World activities being connected to careers. My imagination took me away from the inquiry question, and the students realized they had to bring me back to the moment of their inquiry question. We logged our idea with the rest of the class, got the nod of approval and went back to inquiry. Even my students can see our modes supporting each other.

It is always energizing to see inquiry come alive with a group of students who previously had not been exposed or had not found the interest in doing it before. The frequency of inquiries necessary to generate a habit of inquiring was far greater than I had ever imagined. We did two mini-inquiries a week for about two months before I could observe their habit of finding questions everywhere. Sharing student inquiries with their classmates was also important. This community aspect of inquiry allowed students to both model good inquiry habits for others, and probe students' inquiries with more questions, generating more enthusiasm in further inquiries.

Table 37

Examples of Literature Review Connections Developed

Collaboration	•Teachers collaborated in learning how to teach the parts of the inquiry process to both students and parents. •Teachers collaborated in learning how to teach the critical reframers to both students and parents.
Teacher as Designer	•Teachers designed the coherent and consistent content and expectations of each section of the inquiry form. •Teachers designed a variety of mini-inquiry forms.
Reflective Practitioner	• Teachers did inquiry on their classrooms about their teaching or their students. • Teachers taught their students to become reflective learners by doing inquiry on their learning.
Learning Community	•Inquiry is where we become a true learning community, as it can involve all constituents doing inquiry, and demonstrating their willingness to learn together, around their own self-generated questions and scientific design for answering those questions.

🕮 NCLB Story Revealed from the Inquiry Math Mode:

The pressure for our students to perform for NCLB testing accountability issues had only negative impacts on the Inquiry Math Mode.

☹ Because we had designed our Inquiry Mode to allow students to ask and answer their own questions in any field, and generate results based on their own design, we were not concerned with whether they got correct or incorrect answers. This mode is not aligned with mastering content available on any high-stakes testing. Therefore, NCLB had no positive influence on this mode. The negative impacts were, and still are, greatly experienced in the classroom. The inquiry process is a long and involved one and consumes much class time. Because inquiry does not benefit testing, teachers found it hard to devote much classroom time to it. Consequently, much of inquiry was delegated to homework, where parents could spend more time with their children doing it. However, NCLB also led to other homework increasing, to make sure that students were adequately practicing material required for the testing. One year, the increase in homework load might have contributed to the cancellation of the school's Inquiry Fair. Teachers and parents worked together to find ways to continue inquiry more equitably in the next years, and inquiry has remained a steady mode. The most important lesson learned in revisiting inquiry in shorter more accessible ways, after the fair's cancellation, was that the formalized paper form used for inquiry was too long and arduous to do for every inquiry a student was accomplishing. More frequent use of a *mini-inquiry* form encouraged more recursive content and instruction that could enable students to develop good inquiry habits. So again, our desire to have students perform more on our long inquiry forms, which had been influenced by the focus on the tested performance of our students, almost put too much emphasis on inquiry performance and would have resulted in our eliminating inquiry, as well. Returning to our more process-oriented roots allowed the stabilization of this mode.

Figure 49: NCLB story revealed from the Inquiry Math Mode.

In the next section, I present the reflections of former Family School students on Inquiry Mode. These students typically had been at Family School for many years and had "grown up on" inquiry. Now, as high school students, they shared their thoughts.

The Power of Inquiry

In 2008, years after some of my previous students went to high school, I asked some of them to meet with me to discuss inquiry and what it had meant to them. I was going to give a presentation on inquiries to a group of teachers with Golden Apple, and I was interested in having a student voice be a part of the presentation. They decided to put that discussion into a power point quickly for me that day. The following words of the students show the power of inquiry, of which mathematics plays a great part.

1. *Attitude*
    - *Inquiry encourages the student to respect the teacher who works with them, rather than above them.*
    - *Inquiry allows students to formulate their own questions and testing process; much more fulfilling then a uniform lab set forth by the teacher.*

2. *Perseverance*
    - *When many inquiries are focused around one subject, the student learns perseverance and the ability to work towards an ultimate answer through many different methods and questions.*
    - *Inquiries teach the student that if they stick with it a little longer, they will always learn a little more.*

3. *Passion to Ask Questions*
    - *Inquiry is based on the ability to ask questions, and to reform and critically rethink these questions, so as to make uninteresting subjects interesting.*
    - *Inquiry eventually evolves into a passion where everything is seen as a question, and thus a possible quest for knowledge.*
    - *Inquiry teaches the student to respect other questions and utilize them to expand their knowledge.*

4. *Belief You Can Find the Answer*
    - *IF students' answers are incorrect, the teacher guides the students to analyze for their errors so as to reform their process.*
    - *Inquiry gives students the confidence to search for their own answers.*

5. *Connective Thinking*
    - *Inquiry gives the student the ability to connect what they learn in class to what they already know, so as to solidify it into their mind.*
    - *Ex. Mathematics, one of the most integrated subjects in academics, is a matter of connecting what you learn daily to what you already know and understand.*

6. *Pre-Introduction to the Scientific Method*
    - *Inquiry is loosely based to the scientific method in the formulation of hypothesis and the investigative process.*
    - *Inquiry is an earlier introduction to such a method and makes it much easier to adapt to in high school.*

7. *Ability to Thrive in Cooperative Activities*
    - *Cooperative inquiries prepare students for joint future operations in which multiple individuals or companies work together.*
    - *Such inquiries also teach the student to appreciate and employ other student's thinking processes and personal areas of expertise*

Figure 50. Student powerpoint text on inquiry from my computer files dated February, 2008.

When students tell you how powerful something can be in their school careers, teachers should listen. I was so thrilled to hear them articulate how inquiries had served them even after they left our school. Their own traditional high schools didn't use inquiry, but the benefits of having years under their belt stayed with them. There have been studies to see if inquiry has led to higher achievement, and there has been no conclusive evidence that it does. Kirschner, Sweller, and Clark's (2006) research explains that student may fail to adequately learn when using constructivist methods due to the fact that the cognitive load during the construction of schemata may be too heavy to adequately move knowledge into long-term memory. "The advantage of *guidance* [teaching] begins to recede only when learners have sufficiently high prior knowledge to provide *internal* guidance (p. 75). However, these students were not trying to say they knew more, were smarter or achieved more than other students, they were revealing the power they felt about their ability to think, gather data and evaluate new evidence to learn. Perhaps these studies only gauged whether the minimal guided methods versus the guided teaching methods resulted in *learned* material, rather than improved methods of the ability *to learn*.

Perhaps improved test scores are not the only measured achievement needed from our schools, our students learn so much more about themselves as learners, and accomplish great tasks as inquirers. Such important processes are not assessed on these achievement tests. Of the documents studied for this mode, there were writing pieces I had written about my teaching in the Renaissance period when our teachers decided to have a writing group to write about our experience of teaching at Family School. In 2003, I was inspired by one of my fourth-grade students as I was writing about teaching students think. I wrote:

> " I believe that good life is a life full of thoughts." This was stated by fourth-grader, [xxxxxx], who certainly does his share of thinking, but this year I have seen him become even more connected to his work."

I continued in my writing to connect all the modes used in the classroom that encouraged this student *to think*, which included inquiry. I concluded my writing piece on thinking with:

> "Having the ability to think doesn't have anything to do with being smart. In fact as I have grown to be a better thinker, I have found that it is more interesting to know less and want to know more.
>
> [xxxxxx] has seen that. How to think about what you do is much more fun and motivating and leaves you with the feeling of a full life of forward momentum rather than the retrospective feel of memorizing."

Perhaps those returning students from high school promoted the Inquiry Mode the best when they concluded their power point with:

We Believe:

- *That inquiry is useful to every teacher in every subject*
- *That teachers should study themselves as a teacher*
- *That teachers should do inquiry for themselves in their own area and involve students in those inquiries*
- *That as students, inquiry has changed our lives and the way that we think about the world.*

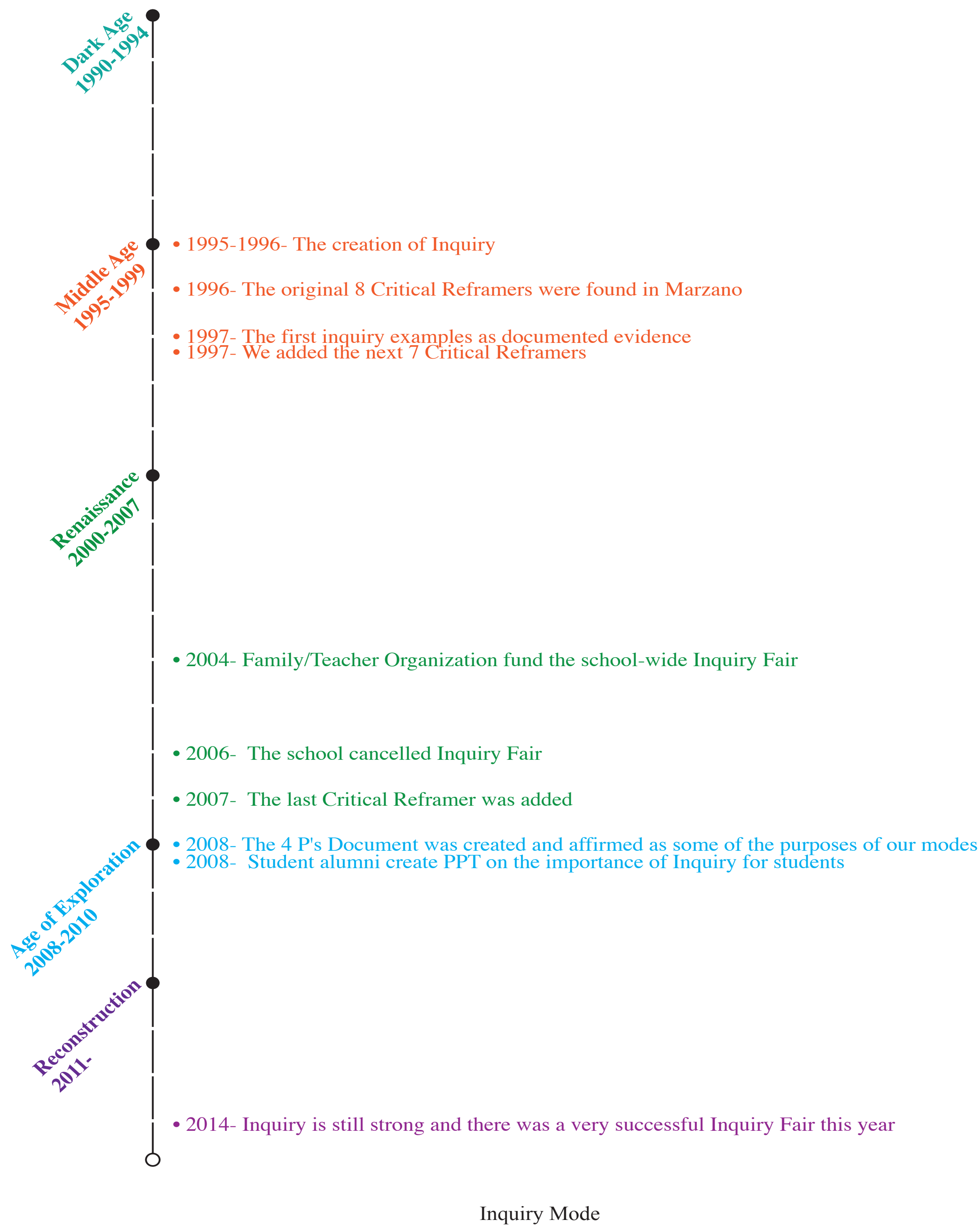


Figure 51. Inquiry Mode Timeline

Chapter 10

Summative Analysis

> With the roads to the exalted places we all want to visit more crowded than ever, we look more and more, but see less and less. But we don't need more gimmicks and gadgets; all we need do is *re-imagine* the way we travel. If we truly want to know the secret of soulful travel, we need to believe that there is something sacred waiting to be discovered in virtually every journey. (Cousineau, 1998, p. xxii)

A self-study blended with a document analysis of twenty-four years is a journey, a journey back in time, rife with nostalgia pitted against the chronicled veracity of documents. The evolution of a school, as seen through those methodological lenses, casts an account that, at every turn in the road, must live up to the word, *account,* and be *accountable.* This study is seen more clearly to be the soulful journey of the institution. This study set out to discover the patterns of change, the interchange of internal and external dynamics, and the tenacious qualities of the Modes of Engagement at Family School. In truth, this study not only evaluated the modes as if they were ancient ruins, or a lasting natural wonder, but also encouraged this newly enlightened traveler to re-imagine more powerful versions of the modes than ever before.

I learned of our failings, and I learned of our strengths, for both the autobiographical work and the document study echoed these failings and strengths in the multiple modes. I uncovered ideas that I would not have known if it were not for the undertaking of my personal journey the self-study. Moreover, by sharing much of the process of this work with my staff, students and their parents, I have rediscovered our continual desire to be soulful travelers as a community of learners. To be soulful comes from living a life enriched with finding deeper meaning, usually in response to self-reflection. As a community, our continual reflection on our work elevates our consciousness of our responsibilities, our promise to each other, and our profound desire to always want to know more.

What are the elements (and/or structures) of the Modes of Engagement that sustained their use over time?

Parsing

Answering this question was a twenty-four year journey, where we set out to create a curriculum with a unique set of purposes. Much of what I have discovered from doing this study were the seeds of the origins of the Modes of Engagement. I may have identified the direction I wanted to teach right from the beginning of the school, and I may have reflected on my practice throughout my career, but in doing this rigorous and concentrated research project, I am re-discovering, re-learning, and re-imagining nuances about these modes that I had not seen before. The Modes of Engagement function in a very unique way in our school to excite our students into excellence, inspire our teachers to improve, and invite our parents into our learning community. Why have the modes sustained their practice at the same time that other educational practices have come into vogue and then vanished? Even though our school suspended the Math Brainteaser Mode and the Real World Math Mode for a couple of years, both modes returned to our classrooms as requested by teachers and students. What was it about the modes that aided in their constancy? Most districts re-evaluate but adopt new textbooks every seven years, so how is it that the Family School modes have lasted so long?

One of the most important features that the story and documents of the modes reveals is the importance of parsing the whole of teaching in three integrated ways: purpose, curriculum, and instructional strategy. Very early in our school's development, we developed the notion of *self-as* and knew this was a purpose toward which our teaching should aim. It grew into creating understanding as a whole set of purposes for each area of subject matter that we wanted to build our curriculum to achieve. For instance, if self-as mathematician was the goal, then we needed to decide the purposes for which a mathematician would aim. We decided that experts would purposely see to: unravel cutting-edge mathematical conundrums, find mathematical explanations for how things in the world work, develop profound understanding of mathematical concepts, find the beauty in the world of numbers, etc.

Once we had parsed our various purposes, we then parsed out the content that would align with such purposes. Math Brainteasers are a perfect match for mathematical conundrums, inquiry's data collection and analysis are a match for discovery of mathematical explanation for how the world operates, and the algorithms are a match for profound understanding of mathematical concepts.

The third parsing comes in choosing the instructional strategies that define and structure the teaching to harness the purpose and the content to move student engagement toward the proficiency of mathematics, and more importantly toward a metacognitive proficiency whereby students see themselves as mathematicians precisely because they have the parsed purposes. It has taken me this entire research project to be able to say that so clearly. It is not that I haven't understood such connections, because having operated in the world of modes at our school, this is just how we live and how we intuit what we build and how we parse. But the parsing is difficult to explain to other educators who have not had the experience of designing modes, or have designed activities for their content but have not considered the integration of these three parsed educational variables. To us, the modes work, they get improved, they work better, they provide us with insights, they challenge us, they surprise us, and, again, they work. The study of their evolution champions the questions why and how, and would they work elsewhere.

Other schools that order new texts as new standards are adopted re-create the wheel every time this change occurs. Our math modes have adapted to newly-chosen standards and newly-chosen grade-level expectations. For example, when we found new workbooks for our Mobius Math Mode, we adapted the Mobius Mode content. The Algorithmic Math Group Mode has remained constant because the algorithms remain fairly stable themselves. Other texts have taken students through various trends in algorithmic procedures, such as the lattice method of multiplication, but ultimately, no matter how many methods are shared for an algorithm, a procedure that is most efficient is the best method to ultimately leave in the student's possession. Because our teachers have designed our unit tests, and the accompanying support booklets, we can, and do, update them when we feel the winds of the times carving out the new waves of mathematical maneuvers with their weathering pressures.

Grossman and McDonald (2008) describe the need for this parsing in the domain of teacher education as well. They suggest the "careful parsing of the domain" (p.186) of teacher education would then find underlying structures of the practice of teaching, as well as to develop a common language for teachers to identify "key components of teaching" (p. 186). Grossman and McDonald would like to see core practices and fundamental pedagogies of enactment parse the domain of teaching in order to better help teachers grasp a fundamental core understanding in the complex field of teaching. By doing so, they argue that no matter the route to teach, whether alternative or traditional, teachers would develop a practice with this core enactment of pedagogy and instruction. Similar to this notion for teacher education, the Modes of Engagement create the purposes, the instructional strategies and the chosen curricular content so that the enactment of this teaching and learning can be mined for the interaction of its core practices, content and pedagogical knowledge. These modes enable teachers to design curricular scope and sequence in less overwhelming ways than for an entire subject area, but bigger than a single activity, so as to hold together the integrity of the whole of the specific content discipline. The mode focuses the instructional strategies needed to reach the specific goals that are the purposes experts or enthusiasts would propose for their chosen discipline.

I learned that the danger of parsing teaching into practices only is that it ignores the conversation about how these practices interact with content in specific ways. The danger of only addressing the parsing of the content is not including the instructional practices that promote the learning of some content over others. The Modes of Engagement go even farther, and parse the curriculum and instructional practices into smaller parcels, and also restrict those choices by aiming their instruction toward identified learning purposes. This intersection narrows the enactment of this curricular/instructional construct to an even smaller window within which to study teaching and learning. This parsing is not only valuable for teacher education but for all teachers and schools. My reflective journal revealed that the Modes of Engagement are an effective construct to allow teachers to study their teaching in a smaller context, while integrally still connecting the research findings to the systematic complexity of the day-to-day operations of our classroom. In my journal, I found that the study of these modes showed that what we learned in one mode could impact or be applied to another mode, thus, demonstrating the parsing was integrally connected to the whole of all the math modes.

The Modes of Engagement have also demonstrated the nature of their flexibility, allowing them to be reiterative, yet changing and altering to meet the demands of our 24 years, the influences within and without our school. This research identified the themes of this unique construct to address the dynamic of the ease with which teachers can design and re-design the modes.

Structure

Size. One of the most important features that the story and documents of the Modes of Engagement reveal is the importance of their compact size. The size of the modes' chosen foci creates more opportunity for teacher design and re-design. By having a smaller size of content around which to build a scope and sequence, teachers can more easily see linear and spiraling connections of the material. They feel more confident about being able to master the material of the mode. Many of the documents revealed themes about scope and sequence. Teachers' conversations about content also occurred around developing better understanding of the content or pedagogical knowledge of the content. Likewise for students, they also can see the content of the mode in these digestible chunks and feel like the mode is more manageable than learning the broad subject of math.

What I have seen from doing this study is that the documents reveal that there has been a great deal of time and effort spent on professional development, training and re-training in and around the modes. Because of teacher turnover, our school hires a fair number of new teachers from year to year. I do train the new teachers in the summer before they begin the school year, but much of the professional development re-training also provides the continued work needed to keep our modes effective for both new and returning teachers.

Here, again, the smaller size of the mode allows for training to occur in more manageable time spans as well.

The study of the specific, significant documents highlighted in the modes' ages reveal that teaching concepts for study and improvement of teaching can be captured in single documents to have profound effects in the entire modular structure. The autobiography and documents reveal that I was able to observe teachers' instruction of modes through observation or video tapes, then structure the professional development for the whole school by creating for, or designing with, the teachers, documents that frame the manner of improvement needed.

Recursive. Schmidt et al. (2007) did a comparative study on content standards between the United States and the countries around the world that received an A+ on the TIMSS test (Third International Mathematics and Science Study). They found that the other countries had a scope and sequence of single content standards that were taught in fewer years than the United States, whereas our standards were covered in a very surface- level manner and spread over many more years between kindergarten and eighth grade. It appears that A+ countries teach fewer standards per year, but teach them more deeply than we do. This highlights the difference between a spiral curriculum, which builds from year to year for many years, and the more comprehensive programs, which are more like our Mobius or Algorithm modes.

Related to the importance of size is the fact that the modes repeat a lesson structure that was designed to engage students around *self-as* purposes. Several documents reveal this patterning to assist teachers to change their teaching accordingly. The narrative autobiography speaks of various teaching maneuvers that were pivotal to a mode's effectiveness. For example, in Mobius Math, key concept lessons had to be taught first, *gathering* the bigger mathematical concepts from students before starting to address how to solve the problem in the workbook that depended on an understanding of the key concepts.

The Family School teachers requested that I provide, with their input, documents that delineate the recursive nature of the lesson structures, and this indicated that they benefited from understanding each modal structure's form and function. The recursive nature of Mobius Math was found in the Mobius story and documents in the discussion about creating the key concept lessons. These lessons were identified as the lessons that frequently occurred, not because students couldn't learn them, but because they were fundamentally connected to key meaning-making in many mathematical concepts.

The Inquiry story reveals that the decrease in the recursive nature of this mode actually decreased the likelihood that students would benefit from the purpose of the mode. When students retraced the frequent use of *mini-inquiries* as the original intent of the mode was designed to do, students were more likely to acquire the qualities of inquirers. When inquiry was done infrequently, the product may have appeared to have quality, but it became more of an assignment than a practice.

Multi-age. For Family School, the modes are specifically designed to address the multi-age classroom, and this research illuminated that the structure of the classroom environment cannot be separated from the mode. The brainteaser mode story revealed that the mode was originally created to provide an opportunity for a classroom of multi-age students to learn how to come together as a *community brain* to solve the brainteasers. The Mobius Math Mode was also specifically designed to maximize whole-group instruction for the wide range of learners in a multi-age class. Inquiry documents reveal that all ages, students, parent and teachers, are full of questions, and once over the initial fear of sharing questions with the community, our school developed a culture of inquiry. More academically speaking, the Algorithmic Math Group Mode story reveals that it was designed to teach students at their specific math level by grouping students according to the algorithm the student needs to learn next.

I think what was most surprising to me to learn about our multi-age classrooms is how well the modes responded to creating different learning structures for learning in a multi-age setting. I don't think I would have ever considered a mastery teaching lesson structure for a multi-age whole group instruction at the beginning of these 24 years. Originally, our process-oriented approach to learning encouraged us to teach multi-age whole group for exposure and heightened awareness but not mastery. Mobius Math and Real World Math have taught us a great deal about how to change this.

Lesson structure. Key documents in both the Mobius and the Algorithm Math Group modes have presented lesson structures for teachers. Brainteasers did not reveal a document delineating such a lesson structure, but the brainteaser is most definitely with a clear lesson structure. It might be helpful to have the teachers generate a documented structure for the Brainteaser Mode in the future. The fact that each mode prescribes a flow to the structure of the teaching is an interesting feature of the mode. Inquiry and Real World have not done this. Teachers have expressed a comfort in having the lesson structure. Our understanding of the lesson structure deepens as we improve our teaching, and it could change, but these examples have not demonstrated much change in their structure over the years.

However, as chapter 8 reveals, next year (2014-15) the Real World Math Mode is going to change significantly in its structure. While it will still present mathematics content that is integrally connected to a real world context, my reflective, personal journaling reveals that it will no longer be structured in a pre- and post-testing format as decided by the teachers when we discontinued real world math. Based on the description of our work to bring it back this year, we discovered that it should shift back to its original content of math's application in today's world rather than integrated content with our science or social studies workshops. In this instance, the purposes will remain the same, the content will shift slightly, but the teaching strategies will change considerably.

Common language. The mode documents and story reveal the importance of a group of teachers having a common language with which to consider their teaching with their peers. In some cases, the documents themselves created the language, like Mobius Key Concept Lessons; or in other cases, the language came out of our practice in the classroom, such as *self-as* or *community brain.*

Staff meeting agendas and student narrative report cards reveal much discussion has taken place about metacognitive strategies for specific modes and how to name the qualities needed in a student's repertoire to enable him/her to improve his/her learning. *Tickled pink*, for example, is a phrase we use with children (and with one another) to mean being able to laugh at one's mistakes; *going over the edge* means a student has lost meaning in a lesson, and *surveying the landscape* reveals a problem-solving strategy of *playing around* with the different parts of a brainteaser before trying to solve it.

My observation of teachers becomes more strategic when I can use documents to create a language that the teachers understand to represent goals in their instruction. Similarly, teachers' observations and formative assessments of their students are more strategic when the teacher and the students have developed a common language with which to identify behavioral, learner-lesson, and academic objectives.

Elements

Student learning. Some of the themes that came up around student learning are very much connected to the evolution of the modes as revealed by this study. The continual tension around process and product, or the difference between learning and mastery learning, has been enlightening. As the testing focus of NCLB pressured schools to have students demonstrate proficiency in mandated standards, our emphasis on process learning shifted greatly. Trends in the documents reveal a great flurry of designing tests, rubrics and checklists to make sure that students were learning the appropriate concepts. Other documents, like the Performance Cycle and the 8 Points document, were created to urge teachers to take this shift seriously and find transparent methods to enable students to master the material. Before these times, we were more concerned with *how* they learned, and that shifted to "did they learn?" The benefit of this for students is that the modes provided an environment where we could focus on student mastery while still embedding the goal of mastery in a mode with a higher purpose for learning than testable mastery evidenced by test performance.

Students' learning actually improved with this focus, if the test scores are the mark by which this is indicated. But in actuality, the story and documents of this study indicate a different trend. Students seemed to be learning differently. Dropping brainteasers for our menu of modes revealed that students were not learning risk-taking learning habits, nor were they building up stamina or tolerance for challenging material. Themes in the documents reveal more accountability in our rubrics to content as opposed to developing metacognitive learning habits. It is surprising to me that students absolutely gained more proficiency, and I mean not to be sarcastic about this, because our teachers were improving and students were learning more deeply than before the content that was chosen for mastery learning. The surprising factor was how greatly lacking the other qualities became and how visible it was to both students and teachers who had been at the school for a long while. Clearly, the value of this lesson is that it is not a process *versus* product issue because both are valuable and need either integration or equal time in their pursuits.

Documents revealed student enthusiasm for the modes. Narrative report cards, award speeches, and student writing suggest that students were intimately aware of the benefits of both the what's and how's of their learning in the modes. When a student understands the scope and sequence, they are enthusiastic to continue to work their way through the linear progression. In fact, at the end of this past school year (May 2014), a student in my room passed her algorithmic math unit test, and two other students in the class were so proud of her they started teaching her the next unit right away, that day, without me. I was busy teaching another group, so I handed them the beginning exercises, and they went to work. If the necessity ever arose, I do believe that the students would just carry on the Algorithmic Math Group Mode without me. They are familiar with the scope and sequence so well from having seen it taught multiple times in a multi-age class. The completion of these units is similar to obtaining merit badges as Boy or Girl Scouts, and they are eager to collect their rewards.

In the modes that don't require conquering unit tests, such as the Brainteaser and Real World Math modes, the students are just enthusiastic to *play* with the mathematics of the real world or brainteasers. The themes of these two modes clearly reveal the importance of *searching, playing, wrestling, and triumphing over* the challenge of puzzles and applied mathematics. These modes connect to the students' interests but also focus on developing learning around metacognitive habits. Learning how to solve problems and develop their critical thinking enables the students to become better meaning-makers. Once they uncover their ability to do so, they are hooked on continuing those modes.

Meaning-making. Making meaning is key to student learning, yet it merits is own findings because it intricately weaves together student and teacher learning. What the story and documents reveal is that it has been a cycle of teachers making meaning out the content and how they understand students to make meaning out the content that improves the design of the modes. For example when teaching brainteasers, teachers had to learn how to learn with their students when the brainteasers were too difficult for them to solve alone. The *community brain* usually conquers all.

The Algorithmic Math Group Mode's most basic kindergarten concepts were designed for meaning-making. The kindergarten algorithmic unit is called Anti-counting because of our realization that learning how to count can be mere memorization of a song-like wording rather than an authentic development of number sense. The story and documents reveal our constant discussion through agenda notes and key school-wide documents about the importance of making meaning in this algorithm mode. Parents were trained, teachers reconstructed tests to reveal more student-meaning-making, and students, even our lowest students in our interventions, kept revealing how important it was for teachers to ascertain that students were making meaning out of every step of the algorithm procedure.

While I believe that what we would have said about ourselves very early was that we wanted to engender habits in our students, the evidence of a very strong metacognitive theme in our documents portrays the wealth of strategies that encouraged this. The notion of habits has become so deeply engrained, not only the metacognitive strategies but also with the purposes of the modes. These habits are developed because of the parsed interplay of the characteristics of the three-pronged mode: purpose, content and instructional strategy. The modes promote habits of mind, habits of practice, habits of being, and habits of change.

Most importantly, demonstrated by the Mobius Math Mode, we have found that once student meaning-making becomes the goal, there is no end to making meaning. The Mobius key concept lessons demonstrated that once one connection was found, we could continue to make meaning by making conceptual connections more profoundly coherent. This was not only true for our students; as teachers, teaching for this coherence, we began to see our ever-sophisticated notions of our mathematical conceptual understanding flourishing.

Teacher learning. Because the Inquiry Mode has been an important part of our curricular modes from the beginning, we have always been a school that believes it is essential to conduct inquiry into our classroom practices. Even in the early eras of the school, before Inquiry was fully implemented throughout all of the classrooms, we recognized the need to study ourselves, and begin to get teachers to question and analyze their teaching practices through an interview process. These interviews paired with teaching students to conduct their own inquiries, led to our teachers conducting their own inquiries within their classrooms. These inquiries often developed into the formal professional development goals required by the district of all teachers.

Perhaps one important feature of the school that encouraged our continual study of our teaching is often taken for granted at Family School but does not occur frequently in the traditional schools, and that feature is the multi-age classroom. Teachers are rarely trained to be able to teach in multi-age classrooms, especially those that range across more than two-grade spans. While we all started with first through sixth-grade combinations, during the first ten years of Family School before being centralized, since that time, these age ranges constantly have changed for teachers. I have held onto my third through eighth-grade class for at least fourteen years. Most recently, I have decreased the grade-span for teachers to only two or three years. Increasingly, with more accountability to meet the state standards and the national movement for more nationally-approved curricular standards, it becomes difficult for any school that doesn't follow a curricular map, aligned with their own state and district testing. While it might seem more difficult to cover more than one grade level's worth of material in a year, with careful study of instruction, like what we have done in these modes, multi-age teaching practices improve a teacher's understanding of how the curriculum is structured throughout the grade levels and develops a teacher's understanding of a student's developing maturity to learn various concepts. The multi-age classroom has kept us on our toes because the range of the curricular stretch is always changing with each new class of students.

It is important for teachers to have a broad view of the scope and sequence of the content because it better prepares them to teach any one concept. By understanding how one concept fits into the sequence of concepts of multiple grades, teachers can teach each grade-level concept in a manner that will better prepare them to connect it to the next level.

Community of learners. Our school's commitment to being a community of learners is a most important element of the modes' staying-power. The story reveals that even before I created Family School, I found it essential to work closely with parents and students. The parents of my students in Monte Vista recommended that I attend the Mortensen Math training, and they supported me buying the program when I demonstrated to them its value. I couldn't have made sense of the Mortensen program's workbooks without working closely with my students using the materials. This set the tone for my work with Family School.

However, once we added other teachers to Family School, I was not able to really promote a learning community with the other teachers because of our decentralized structure and my administrator inexperience. In those ages, I structured the staff meetings to share best practices from the different sites. However, once we were centralized, documents reveal we began work immediately to develop the school-wide curriculum in our modes. Because students would now be able to pass from one teacher's room to another, the teachers wanted a consistency of modal experiences. This promoted the teachers to begin to learn from each other, just as we had already been learning from our students and parents.

Documents reveal that opening those doors to each other involved paying substitutes to cover teachers' classrooms so that teachers could observe each other's teaching, work in small task groups to develop school-wide documents, and share their video tapes of their teaching in staff meetings.

Video-taping is a very powerful substitute for observing each other's classrooms. We save money by not having to have someone take over a classroom to allow one teacher to visit another, and the students lose not time with their teacher. Documents show that a Mode of Engagement would be scheduled into the calendar over a month-long period and teachers would be expected to video this mode. Staff meetings up to the end of the month would be structured around the teaching of that mode. At the end of the month, teachers would bring their videos to the staff meeting, and we would watch and evaluate them for the highlights of what we had discussed during the month. For those teachers who were shy about sharing their videos, they could schedule a private viewing with me at another time. Interestingly, it was the new teachers who were eager to share with each other, while the veteran teachers needed much encouragement from me to discontinue our private viewings and share what they had learned with the newer teachers.

Our agenda documents reveal much discussion of our parent meetings, where we have had to figure out how to improve our community of learners. Inquiries required much discussion about how to share the teaching and learning of this mode. Having all constituents of the school involved in inquiry developed us the most as a community of learners. The parent math classes held over six weeks at the beginning of each year also bring teachers and parents together to deepen their understanding of our Algorithmic Math Group Mode. Perhaps most important to the community are our critical reframers. We have presented many parent classes using this document for inquiry, reading journals, writing, and across the curriculum in general. The reframers have developed such a reputation at our school that the FTO (Family-Teacher Organization, akin to a PTA) sold bumper stickers about our students using them, and they also designed t-shirts to sell that said, “Reframe This” on the front with the whole document of reframers on the back.

Testing controversy. Our ITBS, Terra Nova, and SBA testing has continually shown that our students score well above the rest of our district and state averages. This fact is not taken for granted at our school, and we continually use these scores to evaluate how to improve and alter our curriculum and teaching practices of our modes. The early Program Reviews of our school in the Dark and Middle Ages (1990-1999) document our study of the ITBS scores. They suggest we built our instruction generally around working towards improvement, but this became more of a collaborative practice as we were moved together and were able to function as a school.

From the Renaissance on, we were much more focused as a school around our test scores and designed our own school-wide documents to mark the progress of our work with our students. The controversy that ensued goes back to the student learning issues presented about process and product. What is important to document here is that the story and documents reveal that, due to testing, we lost two modes. More importantly, we almost lost a way of understanding learning and the importance of the purposes we had set out to achieve. Yet the power of the remaining modes, and the scrutiny of our learning community eventually articulated the need to bring back those two modes.

We live in a time when this testing monopolization of our time is not going to disappear readily, but our confidence in our different methods of teaching through the different modes has strengthened our resolve to offer the full menu of mathematical opportunities. Since the modes are applied to all content areas, modal thinking has strengthened our resolve not to narrow or simplify the complexity of our curriculum.

One document that was not uncovered in the math modes helps to explain what we learned from this journey. I was trying to explain to the teachers why it was important to bring brainteasers back, so I created a document on convergent and divergent teaching. I had decided that the testing mania had pushed us into convergent teaching, which specifically targets objectives and teaches directly toward mastery of that objection. Divergent teaching is more obtuse and teaches for the sake of teaching, with no special objective in mind. It is possible to identify an objective in divergent teaching once a teacher has already begun the lesson and sees a gap for the students, which needs to be addressed. It is also possible that an objective is presented and abandoned because a tangent lesson reveals a concept much more powerful to teach at that moment.

Some of our modes are designed towards convergent teaching, such as the Algorithmic Math Group Mode, and other modes are more inclined to divergent teaching, such as the Math Brainteaser Mode. This document helped teachers deal with the testing controversy because, while it is important to prepare our students for the tests, it is also important they learn the value of learning that comes from other educational experiences. The figure below presents the document presenting the pros and cons of divergent and convergent teaching.

	Pros	Cons
Convergent—coming together to move you closer to the standards and aims of the grade level expectations.	Pros Aimed, greater chance of hitting the target Selective, choice in line with district or national expectations Measurable- target more defined, so can measure how far off a student is Realistic- majority of students can reach Specific- usually smaller target Good for individual grade classes	Cons Encourages surface level knowledge Requires in-step development Discourages tangential learning Often boring Often repetitive Often boring Often repetitive
Divergent—teaching for an individual's possibilities of many levels of learning, with much larger aims in mind.	Pros Greater chance of improving something More individualized and meaningful Transferable lessons- because more globalized Encourages variety and more in-depth study Usually more interesting topics in which to embed skills and concepts Great for teaching long term learning lessons Good for multi-age classes	Cons Requires more time Targets not as reliably reached Harder to assess and maintain on-going assessments Isn't calendar mapped with district or nationally Target is not easily defined for either students or teachers
Modes of Engagement	Convergent : Mobius Math Diagnostic Reading Assessment/Lesson Reading Workbook Long Vowel Reading Method Goal Projects Algorithmic Math Group Real World Math (Old version of the mode)	Divergent: Mobius Math Reading Journals Math Brainteasers Writing Brainteasers Workshops Sophie Inquiry Project Portfolio Self-Assessement Real World Math (Added new version of the mode)

Figure 52. Convergent vs. Divergent teaching found in the Family School *white notebook* of important school-wide documents.

How did the Modes of Engagement evolve in response to dynamics from outside of the school (i.e. political pressures)?

Influences

Othered. The story reveals this in ways that are difficult to see. Our hesitancy to share our curriculum and ideas in our publicly-shared documents suggests a host of reasons for this particular question. Specifically identified with feeling *othered* is the fact that the school started in a basement of a school after two years of knocking on doors and overcoming challenge after challenge from educators who said that the school would not work. Being told never to request a *brick and mortar* school kept me from asking to be centralized during the first five years. During the second five years, we went to the local school board two times before the centralization was approved on the third try. We spent ten years at this site, a set of portable building, where we were often touted as one of the highest scoring schools in the district and the state, before the most lovely, real school building (frame stucco as opposed to brick and mortar) was built for us in 2010.

Just recently we have been redesignated from a district *alternative school* to a *school of choice*. In the past, alternative Albuquerque Public Schools have been associated with students who can't make it in traditional schools. Alternative schools existed to help struggling students complete their education. Our school was not targeting these students, but was targeting the families who had left the district, or were planning to leave, abandoning the public school system to homeschool their children. Some of our students had very special needs, while others just preferred to be homeschooled, or had special interests, like swimming, gymnastics or music, that required large amounts of time in their day that would interfere with traditional public school hours. Still being considered *alternative* yet not fitting into the district's notion of alternative showed me over many years, that much of my work has not been valued by other administrators in the district. Over the years, as the school has become more established, and with the re-designation that suggests that we are more akin to a *magnet* school than an alternative school, we seem to have gained a bit more acceptance.

It is hard for documents to reveal this pattern of being overlooked by the district because central office often overlooks our school when sending documents. We have had our textbook money taken and spent on the rest of the district's purchase of adopted mathematics texts, without notifying us to make other arrangements for our workbooks. Just recently, we were not allocated district rubric protocols for teachers to grade their students' performances on the district primary short-cycle assessments. Because the school is not allocated an instructional coach, as every other elementary school in the district is, we could not obtain the documents. They were only given to trained instructional coaches, which of course we did not have. We ultimately copied them from teacher connections at other schools. This kind of omission often sends the message that we are *othered.*

This does impact our practice, which get addressed in the next two sections. Our insecurities about our work often impede our confidence, and yet, the freedoms we have been afforded have granted us a very important creative license for our modes.

Insecurities. Because our modes created innovative experiences for our students, we often felt the increased pressure to make sure they were working, since we were not doing what the rest of the district was doing. In the Renaissance, we developed our own short cycle assessments of our various modes before the district was required to do this by the state. Even when the district was required to submit short-cycle assessments, we did both the district's short cycle assessments and our own assessments to make sure our analysis of our curricula would continue to help us improve our teaching.

One surprising, yet important, discovery of this study is that one of the features that helped sustain the modes was that in having those insecurities about our work, we may have stumbled on one of the features that inadvertently helped to sustain them. Perhaps because of our lack of confidence in our *alternative methods* and not reporting clearly what methods we were using, we were able to stay *under the radar.* We believed if we kept them hidden, then perhaps we wouldn't be asked to change them. But keeping them hidden was not going to keep us from being scrutinized if our scores on the mandated testing were not high enough to keep everybody from looking too closely at our practices. Between our insecurities and the threat of losing our precious work, we worked harder to re-design our modes.

Innovative freedom and compliance. The most important element of being an alternative school or school of choice, whatever designation we were assigned, we were given freedoms to create our own curriculum and instructional strategies as long as our students performed well. Presently, our school is ranked number one in the state for both elementary and middle school testing for language arts and math. I never knew this because we were never told this by the state department. But several websites, which have organized the test scores in different ways, report this. I was made aware of this through information from an acquaintance. This is not the main goal of our instruction, though the pressure to perform well on our tests is considered important to keep our modes alive and well at our school. Our ultimate evaluation of whether our modes are heading in the right direction is whether students are learning, engaged, increasingly demonstrating self-initiated learning, and exploring *self-as* moments. We don't, for a minute, believe our scores on the test reflect the whole picture of our aims, yet we also embrace helping our students deal with this reality as part of our holistic view of what our school accomplishes.

Economic changes. The economic pressures of the nation also pushed against our modes, and they were able to respond. When our families needed to return to two full-time working parents, they did not want to leave our school. We responded by creating the 80% program. In this way, parents' home schooling responsibilities were greatly decreased, which allowed parents to return to work without feeling that they were short-changing their children's homeschooling. With the decreased parent support in the home, the modes shifted to encompass more time for class and homework processing in the classroom. This shifted our learning community to lessen its reliance on the parents and increased the expectations of students to explore and broaden the scope of their learning in the classroom. The modes were now provided more time in a day, but the teacher carried more accountability for the students' learning in the classroom. I became an 80% teacher the first year of the new program so that I could help other teachers adjust to the change. I found that the 80% day allowed more time to evaluate the modes and learn even more about teaching and learning. My reflective, personal journal reflects my enthusiasm for that investigation.

Response

Economic. An economic response of the modes is that there was very little cost value needed to support the changes required by the district. Because we did not need to adopt the district's new mathematics texts, replacing one mathematics workbook with another was a very inexpensive shift. We also provided all of our own professional development for any shifts in content for which the teachers felt they needed support. In our continual design of the major instructional documents that define and redefine our school, we have been able to respond to outside pressures by making meaningful alterations to our needs as appropriate. Perhaps meeting our specific needs makes our professional development more meaningful.

Our school functions without all the extras services, personnel or facilities that other schools have. We have no PE instructor, no libraries, no cafeteria, and no busses. We have the bare bones of a school; the documents and story reveal that the focus of our school is on teaching and learning. Perhaps the minimal budget and minimal distractions from other services have finely tuned our attention to our students. Documents did reveal our FTO parent organization raising money to support our Inquiry Fair, but that have also supported the school copying funds and provide teachers with $400 dollars a year to spend on classroom supplies. They run three fund-raising events each year. This supplements our budget nicely and our teachers feel supported in our innovative efforts to create and supply our own materials, like the parent math booklets for the Algorithm Math Group Mode or brainteaser books needed for the classrooms.

Documents. This study clearly points to the fact that when the external pressures were put on the school, documents were created to translate those directives into our own philosophically-driven translations, the best we can. Sometimes I create the documents for the teachers, and sometimes we create them collaboratively. Not all documents are created equally, or equally well. But internally, we respond to these external forces in our own way. The Performance Cycle document was initially very powerful in improving our strategic teaching. In our enthusiasm to utilize this concept, we made the mistake of getting too wrapped up in that frenzy of testing our students, which resulted in the elimination of two of the modes. But other documents created later, like the convergent and divergent teaching document, helped to explain the need of different educational opportunities for students.

What we came to understand was that when tests are given such incredible weight and value, the curricular choices become more and more restricted, more than we ever thought we would restrict ourselves. Teachers became too focused on content performance, and the students' ability to think and develop their learning habits diminished.

🕮 **The NCLB story.** In summarizing all the modes' themes, the NCLB story emerges. It is clear that there are two main divisions of math modes at our school: the grade or ability-level content modes and the above-grade or above-ability-level content modes. The pressure to perform for NCLB impacted both divisions dramatically, but, in fact, caused the elimination of the above- grade or ability-level math modes. Most importantly, what the research reveals is that the metacognitive strategies taught around the grade-level content were essential for these modes (Brainteaser and Real World) but they also proved essential to the other modes. The metacognitive strategies could be applied to the grade-level modes, but they didn't function in the same manner. It was learned that students were struggling to learn the grade-level math because they were not experiencing the metacognitive strategies designed for the above- grade-level material. Consequently, they were not building a tolerance for the unknown, the ability to take risks and comfort with ambiguity. Without these dispositions, they were inhibited in their ability to learn even the grade-level material.

The instructional strategies that were designed around a linear approach to mathematics, while they were developed for profound depth of conceptual understanding, were not building the same experiences of profundity that brainteasers and real world math could. Because the strategic instruction was not directed at those purposes that demanded risk-taking, discovery, and invention, students could not translate those experiences to the fundamental math modes of the Algorithmic Math Group Mode and the Mobius Math Mode.

Also, the content of the grade-level modes was designed to be taught in a step-by-step manner which presented students with a mindset that all math could be delivered to them in palatable piece-meal chunks of meaning- making, a mindset not conducive to deeper understandings and appreciation of mathematics. The specific content themes of the modes also revealed another erosion of our teaching because of the pressure of NCLB. As mastery teaching became more prevalent as a means of assuring that students had mastered a concept, mastery teaching started to diminish the kind of teaching where students could direct their own learning, or discover their own approaches, or even find a different method than the teacher. We discovered that when teachers know how to master a math concept, they tend to teach the way they have mastered it, rather than discovering the multiple ways that students can find for themselves. Teachers tend to teach a math problem by presenting what should be done as the first step of solving the problem, followed by all the other steps. Instead, we found it was most important to start out by teaching all the larger math concepts that could be involved in the problem first, rather than the small, procedural first-step.

Lastly, NCLB's pressure also influenced the amount of homework sent home and almost eliminated the Inquiry Mode because it was too much for parents to do on top of all the other practice homework for testing material. These revelations from this research were not readily available to see but were found by comparing the shift in themes, such as the shift from process to mastery of content in our brainteaser rubrics. Before NCLB, we used authentic brainteaser books, and our rubrics, reflecting our goals, emphasized metacognitive strategies. With the advent of NCLB, we began designing or own brainteasers, in part to use the brainteasers to teach toward mastery (problem-solving efficiency) rather than metacognition. Our rubrics reflected this shift in focus from processes guided by metacognitive strategies to mastery of math procedures and content. Prior to NCLB, if teachers couldn't solve the brainteasers themselves, they joined their learning community with their students and worked together to solve them, and the rubrics most emphatically, looked for metacognitive strategies that students (and teachers) used to work through the brainteasers.

Even though we are a high-performing school when it comes to our testing scores, and we design our own curricula, our own instruction and classroom learning was eroded by the NCLB regime. Our best intentions to improve our modes spiraled out of control as we tried to demonstrate that our students had mastered everything they had been taught. It was clear, after three years of the loss of brainteasers and real world that our multiple math modes needed to act as a system together. The whole was definitely greater than the sum of its parts; the modes improved the students' performances in each as they were all utilized together, whether they met NCLB standards or not.

Curricular change. What are the consequences of a decade of instructional decisions around such a restrictive curriculum? What are the consequences for a nation of students who have only been taught to have their lessons spell out every step of their educational journey? We found the consequences to be grave. Within the two to three-year gap of having discontinued two of our math modes, we found our students lost the ability to deal with ambiguity, challenges, and tolerance for multiple methods, and they lost their enthusiasm for learning. It seemed the performance grade or rubric was their only motivation.

It was not only the teachers, but students also, who lamented the loss of more divergent teaching methods. Math Brainteasers and Real World Math were brought back into the classrooms with adjustments and gusto. The most important lesson we learned about the modes was that not all modes should be made to run the gamut of the entire performance cycle. If all the modes are made to go through the entire performance cycle, students end up forgoing self-initiated learning and the thrill of exploring the boundaries of their knowledge. Also, it is important for students to understand that mastery-learning is only one of many forms of learning, though the only form presently for which teachers are being held accountable.

My reflective, personal journal accounts of my students in brainteaser and real world math modes revealed the joy of self-initiated learning. Students were completely wrapped up in furthering their knowledge, whether it was at grade level or not. The journal also revealed the students choosing to learn more about trigonometry, for example, for a self-chosen unit, suggesting a deepening desire to advance their math knowledge. I played around with creating a new mode where students could choose any topic to study together. While we actually did do this little unit, the journal allowed me a place to intellectually play with turning it into a new mode. This exercise was important to see the how modal construct might again be used to pull together the purposes, strategies, and content to see what it would create.

This particular change will probably not occur, and while it seems to be in response to internal influences (the students), I offered this opportunity during the testing window because I wanted the students to be able to feel that they had some semblance of control during the three weeks of mandated testing.

Other evidence of this study also suggests that teachers used the modes to feel in control of dealing with external pressures. Teachers made the decision to both eliminate and reinstate two of the math modes. When they chose to focus on the math modes that supported the testing, they felt they were able to adjust them to meet the urgency of the times. It was clear we did not need to embrace a completely different mathematics curriculum or even new trendy mathematical instructional strategies, which would required retraining of all our teachers. We simply continued to study our remaining modes and modify them more effectively to meet the new expectations. In this fashion, our school felt it had a means to push back at the system, which had created such a restrictive view of educational practices and standards.

Presently, in bringing back the Brainteaser and Real World Math Mode, teachers will have to challenge themselves to appreciate the more divergent approach and learn to see how the modes support the whole picture of the student as mathematician.

How did the Modes of Engagement evolve in response to dynamics within the school? (i.e. teacher and student needs/interests)

Some of these internal forces have already been revealed in answering the first two questions because the internal forces are inextricably connected to outside forces. For example, parent requests to increase the school time with students created the 80% program, but it was originally a response on the parents' part to the external economic pressure. However, it is important to classify some of the differences between these two forces, internal and external, rather than just see them as connected.

It is our own evolution as teachers, which is most often reflected in the study. We couldn't have made any of the changes needed to meet the external forces if it weren't for our desire to improve. To be able to move from a process-oriented school to a performance-oriented school couldn't have happened without our investigating the problems with teaching only process. And by the same notion, our discovering we needed to move back to reclaim some of that process came from within our own four walls. Our need to continue to study our teaching reveals the need to change more.

Keeping Our Eyes on the Students

Perhaps it is in the roots of our program that the focus of responding to our students began. When teachers are allowed to create their own curriculum and have no texts to outline the limits of the curriculum that they teach, teachers must look to the students to identify student capabilities rather than assume prescribed texts know the student limits to these capabilities. When we discover some students are able to do so much more than the prescribed texts suggest and can also explain how they are doing it, teachers learn valuable information about how far students can reach. When we discover students who need more time to learn a concept and can explain their difficulties, we learn other ways we need to intervene. Most importantly, we learned that the students needed to use tools to help them explain their learning process more clearly to teachers. This meant that teachers needed to embed our modes with metacognitive strategies that would enable students to self-monitor their learning and articulate where they needed assistance. Not only did we want to be more responsive to students, we wanted the students to be responsive to themselves.

For each math mode, there were documents that revealed how important it was that we let students' learning, and their accomplished work, guide our design of our Modes of Engagement. In Mobius Math, it was our watching closely how the students were learning when material was being taught beyond their grade level that encouraged us to figure out how to teach that mode differently in order that they actively attend. In Brainteaser, it was clear that our most important job was to watch the students problem-solve without our direct teaching. The Algorithmic Math Group Mode revealed the importance of deliberate design of the questions needed to prod for a student's explanation of each and every step of making meaning out of the algorithms. Watching students answer their own questions in inquiry was clearly a method of our discovering what our students wanted to know about the world, and more importantly, how they could design ways to go about finding the answers. My reflective, personal journal reveals that the new version of the Real World Math Mode is now designed to follow the direction that the students want to take the mathematical information, data or statistics from which we build our mathematical investigation for the day. By looking to the students for evidence of their abilities, their requests for intervention, and their direction of curricular choices, the students' learning becomes our training ground for curricular design work in the modes.

Responding to Parents

Our parents create the *community* in our community of learners. The story reveals that it was the parents in my traditional classroom that encouraged both my curricular professional development and also the true creation of the school. Parents are an integral and vital part of our program at Family School. Our parents have made a commitment to dedicate some part of the home life to participating in their child's education. We have always responded to the various parent motivations for wanting the program as it grew in response to our waiting list. Once again, the school awaits the construction of a new wing because our waiting list continues to increase yearly.

The mode documents that reveal the close relationship we share with parents are the Algorithmic Math Group Mode and Inquiry. Both of these modes' documents reveal the parent training that is essential for our students to benefit from these modes. The fact that the teachers have worked diligently to create documents for assisting the parents in their efforts for both of these modes attests to the fact that the teachers and parents work well together to create and support this learning community.

Professional Development

The iterative analysis for the theme of professional development in our school agendas was prominent in three out of the five modes, the Algorithmic Math Mode, the Brainteaser Math Mode, and the Mobius Math Mode. For Real World Math and Inquiry, while there was evidence of our professional work on the agendas, there was not as much attention to or discussion about, improving our teaching or better defining our purposes for these modes.

However, this is one of the most important themes in the documents. The Modes of Engagement create very strategically designed instruction toward the higher purposes of education. Because the modes instructional strategies are so strategically designed, their training becomes directed and specific. The school's various major documents, like observational lesson checklists, the Mobius Lessons, and the inquiry forms, help to direct teachers' choices of strategies. Professional development is made easier because of the modes' focused instructional strategies and prioritized focal points. Another continual theme throughout the modes in support of their being able to be sustained, is the fact that our community of teachers is continually studying and improving our teaching of the modes and rewriting or refocusing our curriculum for the modes to best meet the needs of the community. It is interesting to note that it is not always the same teachers pursuing this improvement. Family School has had to deal with a natural attrition of teachers throughout the years. Family School, like all schools, has maintained a cadre of teachers for many years but also hires new teachers for those who have left. This variable rate is fairly steady, and continual training of new teachers occurs regularly. While this continual professional development may seem difficult or taxing for a small school with self-designed curriculum, it is more of a blessing. For as we continue to teach these modes to our new teacher learners, we also discover that we continually improve our own understanding of the modes as we teach them to others. The new teacher force always brings with it new questions and perspectives that are good for us to synthesize along with our own understanding of what we are trying to achieve.

Collaboration. How is it that we have developed a culture of collaborative learning at our school? It is the school's commitment to heavily support its newer teachers, since our home-grown Modes of Engagement cannot be found anywhere else for new teachers to learn. We have hired some of our student teachers, since they had learned the material during their student teaching assignment at our school. However, most teachers come to us via our district's Human Resource standard advertising.

We have an inquiry interview process whereby a panel of teachers and I interview a small group of candidates. In the group interview, we encourage candidates to build on each other's answers to interview questions. We often give the group of applicants a math brainteaser so we can observe for community learning behaviors among the teacher candidates. Right from the start, we are looking for those teachers who are willing to learn with others. The culture of our school doesn't provide for teachers to hide behind their closed classroom doors and teach whatever curriculum they choose. Being a new teacher in our school has a really steep learning curve. I train them as much as possible the summer before the beginning of the school year, but throughout the year we rely on every teacher mentoring the newer teachers together.

Over the years, I have participated in hiring many teachers that have gone through the interview process; the panel of teachers and I have chosen them for a wide variety of reasons. We have looked for exceptional knowledge of content areas, which has a good predictor of success for some teachers but not for others. Sometimes knowing a subject too well leads to a refusal to learn in new ways. We have looked for extreme caring for children, which has resulted in similar contradictions. Sometimes extreme nurturing behaviors in a teacher do not work well with the level of challenge we provide for our students. It is hard to generate a list of qualities that can be seen in an interview that actually translates into success in teaching at Family School, with the exception of the willingness to be collaborative. My most inexperienced, brand new teachers have done a beautiful job transitioning to the profession when they have worked extensively with their peers. Some veteran teachers, who had no desire to learn with us, have failed miserably at Family School.

To lessen the steep learning curve, newer teachers learn one or two modes at a time. Once those first modes are learned, the other modes can be added throughout the rest of the year. As an administrator, I have always given newer teachers a year to become acquainted with the curriculum. Experienced teachers offer support to different modes, so that a single teacher is not burdened with teaching everything to a newer teacher. In years where I have had more that one new teacher, I offered a “Newbie” staff meeting all year long to help answer their questions. Ultimately, sustaining all the modes of a curricular subject requires much on-going training in order to sustain them. Ultimately, it is the culture of the school that enlists them to participate fully in collaboration.

Documents analyses and the story reveal collaboration to be an essential theme in the evolution of the modes. The teachers’ design work for the algorithm, Mobius, brainteaser and inquiry modes have resulted in the improvement of their teaching. The collaboration often concerned developing the scope and sequence of the mode. Other times collaboration standardized the unit materials for the whole school. Much collaboration is done around evaluating students' work to design grade-level expectations or unit expectations for their rubrics. Scheduling the modes and the modal events, such as Inquiry Fair, in the yearly calendar also showed evidence of collaboration also suggested in the documents.

Teacher as designer. This research revealed many benefits to teacher as designer, not only as designer of modes' curricula and strategic instructional strategies but also as designer of the continual reflective process for improvement and adaptation to the educational political pressures. When the teachers designed the modes, they had buy-in to the choices of content and instruction that were occurring in their rooms. They were more inclined to continue to improve their practice to make the modes the most effective they could be. One of the inherent benefits to teacher as designer is that by being involved in the design, teachers are professionally developing their understanding of the conceptual nature of the content and the practical nature of the instructional strategies. The benefit of self-developed professional development is that teachers personalizes their training to be aimed at their weaknesses.

While teachers have their own uniqueness, so does each class of students. Teachers as designers ultimately look to their students to improve their design, and so teachers can personalize their design to more specifically meet the needs of their own students. Because of our collaborative practices at the school, the group of teachers as designers enjoys the benefit of designing a consistent and cohesive mode across the school that still can be flexible in response to student differences.

Particular to mode design is the ultimate aim of the instruction toward those higher purposes of mathematical expert practices. Because of this design work, metacognitive strategies are developed, and our classrooms generate learning communities, whereby teachers share some of the responsibility for design with the students.

Reflective practitioner. It is a natural practice of design work to reflect on the work that has been created because of the natural buy-in that teachers have in their own design. By including an emphasis on inquiry at our school, our teachers have been exposed to continual reflective practice. In thinking about school-wide design work, such as the modes, it is essential that teachers learn to function as a group of reflective practitioners. Our staff meetings function as a forum for our designing, thinking and reflecting together. In order for this to be a natural practice of a school community, this practice must be on-going, pervasive, powerful, puzzling and profound, just as we have identified the 4 P's needed for curriculum, since for teachers, our teaching is our own curriculum.

Many of our documents have been designed to empower teachers to reflect better on all stages of their design work, planning, in-class decisions, evaluative practices, and in re-design work. Documents such as the "Before, During, After," or the "Taxonomy of the Pedagogy of Teaching" documents provide examples of how we have generated tools to enable teachers to reflect about all parts of their practice.

Teacher inquiry. This research study fed into the culture of my teachers because we have always done inquiry, and this study was just another inquiry for us. As I have explored these modes, my teachers began to explore them, historically and currently, with me. In retracing their evolution, we retraced our own purposes and teaching of the modes once more.

Our culture of questioning can seem disputatious; we incessantly push our own understanding of what we are doing to reach all of our students. We notice our students changing, and the roles their parents play in the homeschooling also changing, and we know we must wonder how our teaching the modes must shift to meet these new challenges. It is this skeptical nature that promotes the evolution of the modes responding to the internal dynamics. The documents' theme for how to improve our teaching demonstrated that on many occasions documents were written and rewritten for improvement. The scope and sequences of modes was mapped and re-mapped on several occasions. The story of the multiplication intervention class the teachers asked me to teach with their struggling students suggested that we were willing to ask challenging questions of our own teaching, often revealing profound answers and reasons for change.

Modes Interaction

My journaling also reveals another very important element that contributes to the modes' endurance. The theme of the modes working together to support students' growth as well-rounded mathematicians came through often. The power of having some modes that promote performance-mastery and others that promote process-mastery, is essential to the entirety of the mathematics program. It is evident that the students experience momentum in the wholeness of *self-as* mathematician, if these modes are designed in such a way as to reach the various goals of the practices of mathematical experts or enthusiasts of mathematics. When this momentum is created, the teachers are able to see that even though some modes won't directly work towards improved testing, they do have an indirect power of helping students attack those mathematical performances with more confidence, innovation, stamina and deeper understanding of mathematics content.

Enigma of Improvement

One interesting theme that the modes point out, all too ironically, is that while our continual inquiry into improvement has contributed to the power of the modes and contributed to their *staying-power*, it has also contributed to their demise. It is an interesting phenomenon of improvement that when something is improved, it usually changes in some aspect more than others, and often those other aspects are diminished or even lost due to the new focus. When continuous improvement does this, time after time, the original intent is often lost in part or completely. In our case, the original intent of our modes to provide a well-rounded experience of the practices of math experts was lost completely when choosing to eliminate brainteasers and real world math experiences.

My Leadership

The narrative autobiography, woven together with the document analysis, presented many dynamic elements of self-reflection throughout this process. I have experienced many moments of humility. Other times, I was greatly surprised by my professional insight evidenced in documents I hadn't remembered. I was most embarrassed by the revelations of the early documents. While I remember that time through the joy of having created the program, that I was greatly disappointed in the contents of our staff meeting notes. It was clear, as a new principal, I was not able to bring the staff together in any real cohesive manner. Many of our notes at this time reflect going over procedural information, time and time again. This didn't maintain itself as a theme at all in the rest of the ages. I am thankful for those beginning ten years for having planted the seeds of much great work to come, once we were centralized.

During the rest of the ages, I began to show some vision as a principal. It seemed tentative in the first couple of years after the centralization of the school. It grew steadily, from 2004 to 2010. These are the years of which I am most proud. The number of documents from this time is large, and they reveal thoughtful work and meaningful curricular decisions. Even though some of the decisions during this time were to be regretted later, the work was powerful and fully committed. My teachers and I seemed to be riding a wave of fearlessness and competence. Perhaps some decisions were rashly made, but I don't remember it that way. I do remember feeling a shift in our work with the continued political pressures. State documents began requiring more accountability and standardization of services and materials. The state department began grading the schools and threatening state takeovers of failing schools. Morale plummeted at the end of these years.

Over the last four years we have been in our brand new facility. My leadership has been bittersweet. It is amazing to have our wonderfully designed building. It attests to the fact that our feeling *othered* for so long may finally be beginning to transition into acceptance. Recently, the district has re-categorized our school as a School of Choice as opposed to being *alternative,* meaning that we are a school that families from throughout the district can choose to attend rather than attend their neighborhood school. Our district has recently realized this trend is imperative to support because charter schools have begun to deplete the district's student enrollment. My teachers' and my work over this last four years is no less committed or thoughtful, but the generativity of new documents has decreased. Perhaps it is a function of having solid ideas, which we keep referring, or perhaps it is due to the fact that as we collaborate more, our discussions serve more as documents and are not recorded. What I wonder is whether the spirit of the school in these times of political pressure continues to truly impact our desire to continue to reach toward the profound.

This year the State Department of Education rolled out a new teacher evaluation system. It has been the most demoralizing political effort to take control of the public school system yet. We have had some of the most incredible discussions at our staff meetings this year in response to this system, but no new documents were created, except those the State is forcing down our throats with their rubrics. These rubrics promote their criteria for teaching excellence, which is compelling great teachers to leave the profession. My staff and I have all decided to hang in there together. None of my teachers are leaving the profession. We incorporated this new evaluation system into our yearly calendar of professional development trying to translate these mandates in meaningful ways for our school. But mode documents were not created this year.

My journal, on the other hand, reveals great excitement in my classroom. My tone often seems to want to revel in the excitement of creating new modes so completely in the face of recommended teaching strategies. I express that my students are yet again the directional compass for which I will turn to generate the enthusiasm needed to pull myself out of this morale slump. When I read my journal entries, I am certain I have already begun.

Perhaps the most important document of this year is not in this study because it *is* this study. This dissertation has served as the beacon of the lighthouse lamp, signaling my return to the fog-lifted shores. I had planned on retiring at the end of this last year, but I can feel this research concentration rekindle the fight to continue to hold out for educational differences that make a difference in preparing our students for futures we can no longer even imagine.

I'm not retiring, and I am looking forward to next year with my amazing students and teachers. This dissertation helped me to realize I want to hold out for the modes' ideals. We need to prepare students with the *how's* of learning, because we have no idea what *what's* are going to be needed when these kindergartners are ready to enter the work world twenty-some years from now.

How did the modes evolve over the 24 years?

Dark Age

In determining the range of years for the eras, the patterns of the documents reveal the meaning behind the era names. Evidence from the first five years of the program was hard to come by. There are yellowed newspaper articles, a program review document, an evaluation for a 21st Century Grant I was awarded to start the program, and old Mortensen Math pamphlets. Many of the documents used during that time had long been culled out of the file cabinets to make room for the newer documents. Similarly, the memories of those five years seem slim, dark and sketchy. The program review document is many pages long and reflects those years most accurately. It was humbling to retrace those years because I was not impressed with the evidence of my leadership. In fact, it felt like I acted more as a fellow teacher than an administrator, which is, in fact, the truth, since I didn't become the designated administrator until 1996.

Even in the beginning, I did act more as the coordinator of the teachers. Some of our staff meeting practices were to swap journals with each other to get individualized attention to our teaching concerns and share best practices at our Friday meetings. These practices do suggest that I acted like a fellow teacher.

We do have the documents we used to discuss difficult students who we wanted to get some advice about teaching, called the Descriptive Review Sheet. This was the beginning of our intense study of our students. I remember these discussions as lively, thought-provoking, and very helpful. While this document didn't focus on mathematics, it was designed to have teachers describe the student through descriptors of five dimensions: academic, family, physical, disposition, and social realms. It also focused on the student's relationship with the teacher and peers. Other than this practice, which did emerge into one of our early narrative report cards, most of the practices of these five years were only seeds of a few of the math modes that were to come: 1) Mortensen Math evolved into the Algorithmic Math Group Mode, and 2) Brainteasers evolved into brainteaser units.

Middle Age

During this period, we were successful in maintaining a program where parents had more input into their children's education as we shared this experience in the pre-NCLB days with the luxury to partition the curriculum between parents and teachers. The Middle Age for us was more akin to the actual historical High Middle Ages where population increased along with innovations and trade. Our decentralized programs began to plant more seeds of innovation that connected more directly to some of the modes that still exist today: 1) The Inquiry Mode has changed over the years, but is very similar to the intent of the very first days of the mode, 2) The Mobius Math Mode began with a mathematics workbook and still uses a workbook to this day, though we altered our instructional strategies of this mode greatly, and 3) Real World Math began in this age and continued well into the Renaissance until it dwindled out of existence by 2008 to be redesigned completely for its comeback today. Surviving to tell of the Middle Age are two program reviews that are hundreds of pages long with much evidence to reveal how our cadre of teachers very intently developed the program into a relationship with our parents and the district.

The Middle Age documents suggest our innovative spirit in creating these new math modes. This kind of creative spirit marked the alternative nature with which we came together as a staff in our new location marking the beginning of the Renaissance.

Renaissance

The documents of the Renaissance are abundant and reflect a turn in my leadership. My creative spirit became more of the vision of the school as I shared our seedlings with the staff at our new school. We brought some of the teachers from our decentralized locations, but we hired new teachers also, since our new location could enroll a larger population of 240. The Renaissance brought with it the development of our philosophies to hold us together. Our innovative modes seemed to spring forward as if it was our own language. The community came together culturally to embrace our innovation and our modes became the language of our curriculum. The parent organization even humorously put together a dictionary of Family School (Appendix D) terms in the parent handbook for new incoming parents, so they could translate our lingo at the parent meetings.

Our math modes grew by leaps and bounds in the Renaissance, the documents revealing many discussions about scope and sequence, how to improve our teaching of them, how to teach students to develop metacognitve skills in them, and how to develop conceptual understanding of the mathematics. It was when the term *self-as* became formalized into our curriculum during the Renaissance that our modes began to build toward achieving the practices used by experts or enthusiasts in the field. Our modes' popularity increased with much of our parent training that was able to occur more easily now that we were a centralized location. The parent mathematics classes began, as well as parent inquiry classes. The parents were trained in the other modes in their regular bi-monthly parent meetings.

Perhaps it was because of the fact that our school was made up of a series of portable barrack buildings, making it evident every day of our ten years there that portable buildings can be moved away as easily as they were brought together, that our community developed the spirit of our unique community as trailblazers. We all worked hard and pushed our creative curriculum forward. The documents begin to reveal that we pushed harder and harder as the Renaissance continued. The work was beginning to reflect the pressures of NCLB. Not only documents of checklist, rubrics, and school-wide tests, but also documents that were written expressly to assist teachers to study and improve their teaching can be found in this period. The biggest shift in the documents in that latter part of the Renaissance was demonstrated by the themes moving from a process-orientation of the modes to a performance-orientation of the modes. Being consistent with the Renaissance spirit of exploration, we embraced the change by constructing our own meaningful way of approaching the accountability in our own innovative manner with the Performance Cycle and the 8 Points document.

The real historical Renaissance may have been supplanted by the *Enlightenment*, but our Renaissance was supplanted by an overzealous response to measuring student achievement. Student achievement is an important goal in education, but what we have learned most emphatically is that it is also important to define achievement in other ways than just the results of student testing. Under the pressure of NCLB, this fact was obscured from our view and document analysis reveals that we continued to measure our students with our own assessments as well as the district and state mandated testing.

Age of Exploration

This unenlightened perspective of education closed our Renaissance days and led us into the Age of Exploration, the age where the measurement of the world became important to expand empires. Our measurement didn't expand empires intentionally, but our improved test scores did bring more permanence to our school as our curriculum continued to outscore the rest of the district. The documents of the Age of Exploration reveal our honesty in conveying our innovative curriculum to the district and state department for the first time, data dialoguing all the results of our added school tests, and our increased study of our mandated test scores. Such constant measurement does bear results, as we demonstrated higher student achievement, yet often an overbearing emphasis in a one area causes repercussions in another. That repercussion for us was the fact that our students were losing the benefits of our out-stretching reach for the best part of mathematics that pushed for those practices of the experts and enthusiasts.

Reconstruction

With this realization began the period of Reconstruction where our documents and my personal journal reveal patterns of new discussions to bring back Math Brainteasers and the Real World Math Mode. We enhanced our Mobius instruction with the videotape analyses, and the math modes were back together again. Reconstruction, while this period's name, is really the theme of all the modes throughout all our years, and it began with our rebuilding from the damage of the NCLB days.

From this work, the six most important findings about our modes are that they provided opportunities to promote:

1) A community of learners, where we have discovered that continual collaboration is essential in sustaining the work that is created by the school;

2) A common language for the community to share that enables teachers, students and parents to more effectively use the modes;

3) A continuous cycle of meaning-making by members of our community of learners, who have all come to profoundly experience that deeper meaning can always be made;

4) The notion of *self-as,* so as a staff we could determine the purposes which we would design our modes to attain;

5) The notion that the *how's* of the metacognitive learning strategies are more important than the *what's* of the chosen content; and,

6) The power of the modes is engendering and sustaining student habits of the mind and habits of learning practices that inspire and motivate learning.

I have often been asked why I am doing a Ph.D. so close to my retirement, and yet, having done the work of this dissertation, I can't imagine a better time to strive for such an achievement. I have already shared so much of my work for this research, the rough drafts of the dissertation, the research of the literature review, and the transformation of my scholastic journey, with my community, and I know the value of leaving this school a document to help prepare them for their next journey. Cousinineau (1998) writes about what he called *The Pilgrim's Law*: "A soulful traveler replenishes the camp before moving on for those who will follow, and you must share whatever wisdom you've been blessed with on your journey with those who are about to set out on their own journey" (p.215-216). I have worked hard to support our teachers to be able to begin this transition to carry on the school without my leadership for some time, but this research has done more than I could have put together in any professional development opportunities we still have left together. To read a piece of this school's history, to see the roots of our practices of today, to study our response to the political pressures that continually push on our teaching parameters, and to analyze our curricular changes to improve our teaching are the boon of this pilgrimage and the best way to show the teachers, families and students that they have all they need to carry forward the school's vision and excellence. As Campbell (2011) states, "The ultimate aim of the quest, if one is to return, must be neither release nor ecstasy for oneself, but the wisdom and the power to serve others." To serve my school community with this life-long endeavor and also leave them with this analysis of our history is my homage to a learning community within which I was honored to be a participant.

Discussion

The request to *tell this story* accompanied with document analyses, supportive research literature, and personal journal analyses provided much needed checks and balances to the self-study autobiography. However, there are still points to be made that a reader should consider when reading this research.

Many times in the writing of the autobiography, I was proven wrong in my memory of the events, especially when it came to the dates of these events. I had to correct the autobiography when this occurred. I was surprised in both directions of the timelines in that some memories I found to be older than I remembered, and others were corrected when I discovered their origins were to be more recent than I remembered. It was important to be able to correct these errors in the autobiography in order to analyze the document patterns more carefully.

Loughran (2005) suggests that self-study encourages "educators to look into their practice with new eyes so that their understandings of teaching and learning about teaching become more meaningful" (p. 13). Had I just told my memories without the rigor of triangulating the other methods, this venture would have had different findings. I would not have determined the patterns that the documents revealed. Self-study should find issues in the classroom to solve, as Loughran suggests, making the teaching *problematic*. My self-study represented my teaching as *problematic* so that it could be seen and "appropriately interrogated [which] is fundamental to moving beyond a tips-and-tricks approach" (p. 9).

These 24 years were long and create a complex story to tell, complicated by the three location moves of the school and three major political eras of pressure on the schools: the Scans Report, NCLB, and Common Core. It was difficult for me to hold on to the quarter of a century's worth of curricular evolution. There has been much push and pull through those years, and a great shift on our nation's educational landscape. It is a challenge to tell that story thoroughly yet in a transparent way for a reader to understand the intricacies of the changes undergone by this innovative school of choice.

Traditional public schools have long been thought to inhibit educational innovations, trends, research, and policies, yet the movement toward schools of choice is broadening in our nation. With the ability for *schools of choice* to envision the structures, protocols, and environments of school in such radically different ways, the stories of these schools will sound radically different from the experiences of more traditional schools. It is often difficult to communicate these stories when they are more often than not translated through traditional school memories. This proved most difficult for me in this autobiography. Being a member of this alternative community from the beginning I often assumed that my experience was common to many other educators, which is most significantly not the case.

It was essential to work with the complexities of our school and try to translate our innovative experience because the story revealed important lessons not only for our school but for educators at large. Educators need to share their learned experiences, especially those such as the Modes of Engagement. This story of complexity is inspired by Cochran-Smith (2003), who urges educators to embrace the complexities in our classrooms because:

> [W]e must avoid what is too simple— isomorphic equations between teaching quality and test scores and between students learning and test scores—because they are grossly inadequate to the task of understanding (and ultimately improving) teaching and learning in a diverse but democratic society in the 21^{st} century.(p. 5)

It is also problematic to write this autobiography having two roles in the organization. Being a teaching principal was reflected in my writing. At times I wrote from a teaching perspective and at other times from an administrative prospective. It was often impossible for me to tell from which voice I was speaking. I have been doing both roles for the entire 24 years and do not think of myself as being one or the other. Also, it is important to understand that while many of my memories and documents are of my work, they are also of *our (*my teachers' and my) work. When working collaboratively, the voice of the community is heard in our documents.

Much of what I present to the staff comes directly from the teaching in my classroom, and still other topics and guidance come from my administrative duties, which require me to translate the district and state directives. I must often convince myself, as a teacher, to undertake some of these mandates. In the end, whichever hat I wear when working with the staff or my students, I have come to worry less about which hat it is because I trust that we will all work together as a learning community. We are all learners trying to improve our ability to teach and learn: students, parents, teachers and administrator.

Throughout this process, I found that I also wrestled with my identity as a woman, and as an administrator more than my identity as a teacher. I found myself hesitant to acknowledge my power or realize that I was an inspiration for my teachers, and I also found it difficult to value my work. I resonated with Gamelin's (2010) suggestion as to the importance of women conducting autobiographical self-study when she cited that the outcome of such work is to:

> …recognize and acknowledge the alternative ways of women's knowing, being, [and] doing. This cannot be accomplished by replicating what is already in the mainstream, academic picture, but by imagining the importance of what has been left out. In retrieving vestiges of women's knowing, their novels, letters, diaries, quilts, and pottery, we understand how women have transformed their lives into artful creations, when no other instruments to express their knowing were available" (p. 190-191)

My work on this narrative has long been a silenced piece of work; as the creator of an alternative educational environment, my work is often overlooked, ignored, misunderstood, and even represented in falsehoods. Despite being recognized by the district and state for our record of student achievement, neither the district nor the state has expressed any interest in understanding our curriculum and instruction. To tell this narrative was a labor of love, after which I appreciated Gamelin's words upon the completion of her narrative:

> I stand back and look at my experiences as a farmer would look at her fields. There are not straight lines in mine. There are curves, loops, and zigzags. What others see as chaos, I see as the particular design of my life. What others see as mundane, I see as the narrative of daily living. I know that there is harvest and abundance. So that I may maintain a sense of integrity in my work and a sense of authority in my voice, I must take this way of being and seeing into account when I write" (p. 191)

I found that in writing my autobiography I could not assume that my memories were correct; instead, I reminded myself that "[w]hat and how we remember things provides clues to how we think now, windows of sorts into our emotional and cognitive re-structuring of experience and testimony to how the past shapes us" (Weber, 2002, p. 122).

Additionally, interlacing my teacher self-knowledge with Shulman's (1986) teacher knowledge brings another dimension of analysis. This added dimension of context and social history is a natural addition in narrative, as storytelling most adeptly considers setting in its expository formula. Analyzing context adds the importance to teacher knowledge that should be embedded in the narrative told about our work as teachers. Brookfield (1995) suggests the importance of context in proposing that reflection needs to have two important agendas: to wrestle with the social constraints such as power undertones and to question and challenge the assumptions and existing practices that seem to pull us away from our best intentions and the needs of our students.

Finally, the narrative work of this study proved to be extremely dynamic, in that while writing about the past, I was, in fact, rewriting the present. I was continuously sharing, with the other teachers, what I was writing. Some of them became very interested in the history of the modes. The modes were seen in a new light by highlighting their history. The narrative helped to re-envision some of the extinguished math modes: specifically, the Brainteaser Math Mode and the Real World Math Mode, which were already returning to the school but had returned without much vision about their future. This narrative reminded us of their origins and obstacles, which spurred on innovations to design them to be more effective than before.

The document analysis process for this research was a very complex and laborious process. I had initially hypothesized that there would not be many documents, which was impossibly understated. Once I had searched my file cabinets, the secretaries' file cabinets and my computer's files, which contained two previous computer's files that had been transferred onto this hard-drive, there were more documents than I knew. Still, occasionally I had memories of a document that I was not able to find, especially those in the first ten years. It was clearly easier to find documents for the last three eras of the school, and so those years are over-represented, while the Dark and Middle Ages are under-represented. Choosing documents from the first two eras was not difficult because the number was so low, but to select key documents in the last three became problematic. I tried to collect as many as possible to include in the iterative analysis of themes, but there were some documents that spoke more loudly to the evolution of the math modes and the philosophy of the school. Those documents were specifically chosen and highlighted throughout the study. The criteria for choosing them was that they had a connection to: 1) the math modes, 2) school philosophy, 3) the evolution of the math modes, and lastly 4) that they were used school-wide.

Our documents can become problematic in that they are self-made and often use our own lingo to promote our concepts in education. This is important for a school because in creating them we articulate what we understand our philosophy to be and think about how to enact it in our practices, but it is onerous for new teachers and outsiders to comprehend the documents until they see them in action. When substitutes come to our school, they are amazed, because our students are generally left with much of the direction of the class for the day, since we know how hard it is to communicate our modes to the general population.

Cochran-Smith (1999) reveals the importance of teacher research to a full understanding of lives in schools:

> The concept of teacher as researcher can interrupt traditional views about the relationships of knowledge and practice and the roles of teachers in educational change, blurring the boundaries between teachers and researchers, knowers and doers, and experts and novices. It can also provide ways to link teaching and curriculum to wider political and social issues. When this happens, teacher research creates dissonance, often calling attention to the constraints of the hierarchical arrangements of schools and universities as well as to the contradictions of imperatives for both excellence and equity." (p. 22)

It is also important to note that the choice to study the math modes was also a difficult one, as it was not my intention to suggest that our school is any more math-oriented than it is oriented toward any other discipline. We have Modes of Engagement in Language Arts, and Science, and Social Studies. We do not have Modes of Engagement in the arts or physical education. Our instructional time in Family School is limited, given our part-time schedule, and when we do the arts or PE, we do not have time to parse it into modes.

Lastly, my personal journal research for this project raises a few questions. It was an on-going practice, which included the time in which this research was being conducted. It was evident that my research was impacting my journaling, just as it was clear that my journaling was also carving out some of my research. As I wrote in my personal journal, while conducting this study, I was most impacted in thinking about my students, and I would often discuss my thoughts about the modes with my students and teachers. It is in this crossing of boundaries that the research became more transparent to others in the school. This is what surprised me the most. The transparency heightened my students' interest in the modes. Students knew we had these things we called *modes* and practiced the modes' expectations and rituals without really thinking about them being *modes.* With our discussions about the topic, the students began to heighten their awareness of the modes' differences and their appreciation of the modes' differences. This transparency has re-committed my staff to reclaiming the history of the modes, to heighten their understanding of teaching the modes. This was not the intent of the study, nor was there ever a thought that it needed to happen, for I would have said that the staff was very cognizant of the modes to begin with. What I discovered was that this was not the case, and by retracing some of the autobiography and documents with my colleagues, they became extremely eager to learn more and re-energized their efforts to understand and employ the modes.

Drevdahl et al. (2003) report that journaling can be an important method of self-study to reveal the personal, practical and formal knowledge of the teaching profession. Kniskon (2000) suggests that journaling in self-study is a beneficial "means of stimulating dialogue to explore thoughts and ideas and address the uncertainty that accompanies new learning" (p.259). She also suggests that journals make it possible "to use language to learn and reflect upon, clarify extend and broaden your thinking in regard to the ideas inherent in curriculum development and implementation" (p. 259).

I have never been one to keep a journal, and I knew that it would be difficult for me to do this. I had hoped to journal more frequently, but with doing the research for this study, teaching and leading the school, I was only able to journal twice a week for thirteen weeks. I was acutely aware that I needed to be disciplined to accomplish it, and found myself choosing topics around which to journal. I was unable to journal in such a way that I imagine those who journal would. I was only inspired to write entries about fifty percent of the time. The rest of the entries were those written around topics I had assigned myself.

The outcome of the journaling was more positive than I had thought, in that I did become intensely interested in my journal writing, especially because the journal was used to process ideas and observations about the modes. The journal entries were important to the evolutionary work of the autobiography. They enabled the modes to be brought up to date with my musings about their standing today. They revealed the classroom experimentation that goes into redesigning and re-creating modes. One journal entry did not get used because I got so excited about a new science mode that I couldn't wait to think about it on paper. This new mode was connected to the math modes abstractly, but its science focus made it only tangentially relevant to the study of the math modes. Still, I couldn't stop myself from constructing the parameters, the instructional strategies, the purposes, and the goals this new mode might achieve like that of science experts. Writing about the existing modes, I could not help but to imagine new modes.

This research is very personal work, and consequently full of biases, the inexactitude of memory, and challenges to the impartial interpretive analyses of themes. I found the power of the primary sources of the documents to be the strongest impartial voice and, therefore, infused them throughout this research document to expand the discourse of the autobiography. What I had hoped to do within this research was to have *the modes speak for themselves*. The documents would reveal the language of the document world: agendas, report cards, scope and sequences and guiding protocols—a language of induced details, deduced abstractions, anatomized classifications, and discovered omissions. The *modes' autobiography* would reveal their secrets, their essence, and their recognition, demonstrating the ways in which the modes could *articulate* the content of mathematics into viable, significant, teaching landscapes. If successful, other teachers would see that such modes could be created, sustained and improved. Lastly, my personal journaling would give voice to the modes' responsiveness to the pressures at hand, and, as such become more documents to speak for themselves.

Chapter 11

Concluding Remarks

The conclusion of this study of the Modes of Engagement must include the discussion of the generation of the new modes that are now being formed and articulated at Family School. The two newest modes are redesigns of older modes, but they are barely recognizable in their structure, purpose and focused instructional strategies. These two new modes are the Science Lens Mode and the Real World Math Mode. Teachers implemented both modes this year in an *experimental* phase. The redesign of Real World Math Mode has been previously addressed in this study, but the science mode has not. The teachers designed the science mode to have a specific structure and purpose, but the instructional strategies were left wide open for investigation.

My mode journal of Dec. 30th explores my great enthusiasm for this experimental phase in designing this new mode. One part of the structure that we designed was using different scientific lenses through which to teach the various science benchmarks, such as using mathematics, careers, and networking lenses to teach the science content. We began with physics being taught through a mathematical lens, followed by astronomy using an internet networking lens, and chemistry was taught through careers in chemistry. Each lens presented the science material in different ways and allowed the students to experience the science in a more dynamic manner than just learning typical science content. I began the year somewhat apprehensive about this new mode, but with each unit it was clear that the students held on to the material more enthusiastically using the lenses. I wrote in my journal, "The most powerful thing about designing this mode is the power I feel in learning how curriculum can be shaped in much more interesting ways than linearly setting out concepts for mastery in test-taking."

This was most apparent in the networking lens. While using astronomers' blogs as the entry point for the astronomy lessons, the material on these blogs was often about content at the *edges* of the bloggers' interests. Usually this level of material would not be introduced in a beginning astronomy course that was linearly developed for students. Yet, with our investigatory curiosity, the students, a volunteer science enthusiast assisting in this mode, and I were able to research their questions and make meaning out of these new findings in the field of astronomy.

The design of this new mode demonstrates the importance of a school having the opportunity to design their curricular decisions in this reasonably-sized teaching arena –the modes. The enactment of the design revealed nuances about content and instruction that would not have been studied if not for creating the mode. By very distinctly setting out the purposes of science experts, who infuse their understanding of science concepts through many lenses, we set some abstract objectives for teaching the science material to students in a new way.

The power of the modes is easily observed during this experimentation phase, yet our work as a staff will continue as we collaborate about our findings and adjust the mode based on what did and didn't work this year. The modes bring teachers power, learning and research opportunities, and multiple lenses through which to observe their students interact with subject matter. The modes keep our teacher inquiry alive and dynamic at our school. One inquiry leads into many questions for others. Similarly, this research of the Modes of Engagement would be benefitted by further research into other opportunities for teachers to parse curricular strands for the purposes of the vision for their schools. It would be interesting to compare the modes that would be designed for various kinds of schools: traditional, private, charter and parochial. It would seem that the purposes for various school visions leave room for schools to design modes to meet those particular needs in powerful ways. Teachers could feel their own classroom connection to the school vision by having this opportunity to design toward these goals.

Many educational goals or mission statements are written in generalized lingo, which in a few succinct sentences should guide the actions of the organization. It would be interesting to research whether the Modes of Engagement could reflect the actions of a school's mission. Most schools use texts from which to teach and don't consider their school's ideals for the purposes of their curriculum other than to become proficient in it. Parents and students could more easily observe such activities, and it would enable the entire community to actively work toward more visionary school goals. Research into the school's modes can help prioritize a school's special niche. With the wider acceptance of schools of choice, charter schools, and magnet schools across the nation, it would be important that individual schools identify their philosophies and greater purposes. The Modes of Engagement are designed to teach toward these goals and should be a priority of the school. While I don't believe most schools have practices like the Modes of Engagement, I do believe students and teachers would benefit from trying to create one or two just to start. Schools would see teachers rally round their modes to study and improve their teaching, and students would be engaged in learning toward larger purposes.

Practical Assistance to Begin Designing a School's Own Modes

If a school wanted to implement Modes of Engagement, the teachers should choose the purposes of your educational program that match the uniqueness of the school's community and environment. A school can have a variety of missions such as developing ecologically-minded citizens, embodying democratic practices, or carrying out community projects. Whatever the higher aim, higher purposes should be found to accompany those aims. Once the purposes have been identified, start small. Don't try to create all the modes at once. Choose one subject matter, like math, reading, or science. Identify the purposes for that subject and choose the purpose that is most different from the apparent purposes of the traditional program occurring at the school. A group of teachers should then decide on the kind of content that would best reach the particular new purpose chosen. It is often helpful to break teachers into primary or intermediate-level task forces to build scope and sequence for the content that was chosen. It is essential that in building this sequence there is time to analyze student work around this material. Teachers often underestimate what students are capable of when building scope and sequence. Having student work examples from various lessons can assist teachers in honoring the design of the scope and sequence.

The strategic instruction is the most difficult part of the work and requires teachers to be able to share lessons, videos, or get into each other's classrooms to observe. It is key that teachers be able to discuss the instruction clearly and articulate how the instruction moves students to experience the purposes while learning the content. It is helpful to identify the pacing and flow of one strategic instructional lesson pattern that engages students in the manner the school is seeking. When each teacher tries to emulate this lesson in a recursive manner, they will be better able to articulate what works and what needs improvement. Study the kind of mistakes that students make. Each new approach to teaching will pull out errors in students. Many teachers are frightened about seeing new kinds of errors, thinking that there is something wrong with the instruction. Errors are students' ways of learning how to get on the right path of instruction, not something that students or teachers should be hesitant to embrace. A mode's value is truly in its fostering student engagement. When students are engaged, developing their own meaning, and building a sense of agency about their learning, then the mode's design has lived up to it name: Mode of Engagement. Make sure to share the mode with all constituents of the school community.

Once a school has successfully designed one Mode of Engagement, teachers can begin to build their second and even third. It is essential that, as the modes are developed, the old, traditional subject matter program become minimized. If it stays as the school's priority, there will never be enough time to implement the newer modes. It must also be suggested that if a single teacher is interested in building modes for his/her own classroom, the same process would be recommended, such as starting with one, and working up to two or three only after success with one has been accomplished. It is important to remember that most of our school's modes did start in one teacher's classroom.

Continued Research

This study found that our modes were uniquely designed to follow the students' far-reaching abilities. The modes' capacity to engage with students by connecting them to their potential is another feature that would benefit from further research as our nation continues to wrestle with the test-score gap in our country. Could the modes be directed to have specific instructional strategies for particular learner needs to improve targeted learning goals? Would these instructional strategies be better designed in the parsed size of a mode rather than just utilizing the strategy? Such research questions would be interesting to study considering the modes' resilience and staying-power.

Having gone through a 24-year cycle of the modes responding and changing in response to those outside pressures, and learning that students do not benefit from forcing all the modes to be made to teach toward mastery, and testing, of specific standards, research would be important to understand the reason behind this and the consequences of having students be required to be so continually tested. Research into the importance of the modes' emphasis on process could be used to demonstrate the importance of students *mastering process,* as well as content. The metacognitive theme of the mode was essential at Family School to meet the purposes of the modes, and it would be interesting to discover whether the metacognitive focus led to improved learning as well as improved learning processes.

It is crucial to understand what makes a mode and what is not a mode. In further defining the features of this educational curricular/instructional tool, the integrity of the whole of teaching can be better defined to fit into this kind of sectioning of the teaching arena. Just as Schulman's (1986) term – pedagogical content knowledge—was further defined as continued research studied how these practices were enacted in classrooms, the term of Mode of Engagement could also be refined, stretched and abstracted to have more powerful meaning and parameters for teachers and students. Ultimately, as more research on the modes is done, not only will its usefulness be highlighted, but also attention toward what specific modes are not able to accomplish would also be important in finding each mode's niche in the educational landscape.

Our modes have demonstrated that the best elements of them have survived through the political pressures that continue to enforce the public schools to perform in mandated ways. They allow a small enough arena where teachers are comfortable to design and study their work. But most importantly to this issue is that by having a variety of modes, some can more directly meet the political trends, while others give permission to teachers to hold out for higher purposes in education than just test results. The complexity of our classrooms is better studied and understood when observed through the purview of the Modes of Engagement. As we no longer are restricted in our use of texts because of the new Common Core Curriculum standards encouraging the use of a variety of many texts, we need to have reasonable teaching frameworks within which to revitalize teachers to design exciting and critically engaging curriculum for our citizens of the future.

References

Anderson, R. D. (2002). Reforming science teaching: What research says about inquiry. *Journal of Science Teacher Education*, *13*(1), 1-12.

Ball, D. L., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. *Multiple perspectives on the teaching and learning of mathematics*, 83-104.

Ball, D. L., Sleep, L., Boerst, T. A., & Bass, H. (2009). Combining the development of practice and the practice of development in teacher education. *The Elementary School Journal*, *109*(5), 458-474.

Banks, J., Cochran-Smith, M., Moll, L., Richert, A., Zeichner, K., LePage, P., Darling-Hammond, L.,Duffy, H. & McDonald, M. (2005). Teaching diverse learners. *Preparing teachers for a changing world: What teachers should learn and be able to do*, 232-274.

Bennett, S. (2009). Susan Bennett on autoethnography, Retrieved on Oct. 19, 2013, from http://www.scribd.com/doc/51303236/Susan-Bennett-on-Autoethnography.

Bickman, M. (2000). Reforming all the time: Recuperating the tradition of the active mind for teacher education. *Phi Delta Kappan*, *82*(4), 300-303.

Bloom, B. S., Engelhart, M. D., Furst, E. J., Hill, W. H., & Krathwohl, D. R. (1956). Taxonomy of educational objectives: Handbook I: Cognitive domain. *New York: David McKay*, *19*, 56.

Boaler, J. (1993). Encouraging the transfer of ‘school’mathematics to the ‘real world’ through the integration of process and content, context and culture. *Educational Studies in Mathematics*, *25*(4), 341-373.

Boaler, J. (1993). The role of contexts in the mathematics classroom: Do they make mathematics more “real”. *For the learning of mathematics*, *13*(2), 12-17.

Booth, J. L., & Newton, K. J. (2012). Fractions: Could they really be the gatekeeper’s doorman?. *Contemporary Educational Psychology*, *37*(4), 247-253.

Brewster, C., & Fager, J. (2000). *Increasing student engagement and motivation: From time-on-task to homework*. Portland, OR: Northwest Regional Educational Laboratory.

Brookfield, S. D. (1995). *Becoming a Critically Reflective Teacher. Jossey-Bass Higher and Adult Education Series*. Jossey-Bass, Inc., San Francisco, CA 94104.

Brown, E. (2004). The significance of race and social class for self-study and the professional knowledge base of teacher education, In Loughran, Hamilton, LaBoskey, & Russell (Eds.), *International handbook of self-study of teaching and teacher education practices* (pp. 483-517). Dordrecht, Netherlands: Springer.

Brown, G. I. (1971). Affectivity, Classroom Climate, and Teaching.

Brown, M., & Edelson, D. C. (2001, April). Teaching by design: Curriculum design as a lens on instructional practice. In *Annual meeting of the American Educational Research Association,* Seattle, WA.

Brown, M., & Edelson, D. (2003). Teaching as design: Can we better understand the ways in which teachers use materials so we can better design materials to support their changes in practice. *Design Brief). Evanston, IL: Center for Learning Technologies in Urban Schools*.

Brown, G., & Quinn, R. J. (2007). Fraction proficiency and success in algebra: What does research say?. *Australian Mathematics Teacher, The*, *63*(3), 23.

Bullough, R. V., & Pinnegar, S. (2001). Guidelines for quality in autobiographical forms of self-study research. *Educational researcher*, *30*(3), 13-21.

Burris, C. C., Heubert, J. P., & Levin, H. M. (2006). Accelerating mathematics achievement using heterogeneous grouping. *American Educational Research Journal*, *43*(1), 137-154.

Burris, C. C., & Welner, K. G. (2005). Closing the achievement gap by detracking. *The Phi Delta Kappan*, *86*(8), 594-598.

Burton, L., & Burton, M. (1980). Problems and puzzles. *For the Learning of mathematics*, *1*(2), 20-23.

Burton, L. (1999). The practices of mathematicians: What do they tell us about coming to know mathematics?. *Educational Studies in Mathematics*, *37*(2), 121-143.

Burton, L. (1999). Why is intuition so important to mathematicians but missing from mathematics education? *For the Learning of mathematics*, *19*(3), 27-32.

Burton, L. (2001). Research mathematicians as learners—and what mathematics education can learn from them. *British Educational Research Journal*, *27*(5), 589-599.

Burton, L. (2002). Recognizing commonalities and reconciling differences in mathematics education. *Educational Studies in Mathematics*, *50*(2), 157-175.

Cai, J., Lew, H. C., Morris, A., Moyer, J. C., Ng, S. F., & Schmittau, J. (2005). The development of studients' algebraic thinking in earlier grades. *Zentralblatt für Didaktik der Mathematik*, *37*(1), 5-15.

Campbell, J. (2011). *Myths to live by*. San Anselmo,CA: Joseph Campbell Foundation.

Carlgren, I. (1999). Professionalism and teachers as designers. *Journal of curriculum studies*, *31*(1), 43-56.

Chazan, D. (2008). The shifting landscape of school algebra in the United States. *Algebra and algebraic thinking in school mathematics*, *70*, 19-33.

Clandinin, D. J. (1986). Classroom practice: Teacher images in action. London: Falmer Press.

Clarke, A., & Erickson, G. (2004). The Nature of Teaching and Learning in Self-Study. In Loughran, Hamilton, LaBoskey, & Russell (Eds.), *International handbook of self-study of teaching and teacher education practices* (pp. 41-68). Dordrecht, Netherlands: Springer.

Clarke, A., & Erickson, G. (2012). Self-study: The fifth commonplace. In Kitchen & Russell (eds.), *Canadian Perspectives on the Self-study of teacher education practices* (pp. 23-34). Ottawa, Canada: Canadian association for teacher Education.

Cochran-Smith, M., & Lytle, S. L. (1990). Research on teaching and teacher research: The issues that divide. *Educational researcher*, *19*(2), 2-11.

Cochran-Smith, M., & Lytle, S. L. (1999). The teacher research movement: A decade later. *Educational researcher*, *28*(7), 15-25.

Cochran-Smith, M. (2003). The unforgiving complexity of teaching: Avoiding simplicity in the age of accountability. *Journal of Teacher Education*, *54*(1), 3-5.

Cochran-Smith, M., & Lytle, S. L. (2004). Practitioner inquiry, knowledge, and university culture. In *International handbook of self-study of teaching and teacher education practices* (pp. 576-600). Dordrecht, Netherlands: Springer.

Cochran-Smith, M., & Lytle, S. L. (2009). *Inquiry as stance: Practitioner research for the next generation.* New York: Teachers College Press.

Collins, A. (2006), Cognitive apprenticeship, Sawyer, R. K. (Ed.), *The Cambridge handbook of the learning sciences* (Vol. 2, No. 5). New York: Cambridge University Press.

Congress, U. S. (2009). American recovery and reinvestment act of 2009. *Public Law*, (111-5), 111.

Cousineau, P. (2012). *The art of pilgrimage: the seeker's guide to making travel sacred*. Conari Press: Berkeley,CA.

Cornoldi, D. L. C. (1997). Mathematics and metacognition: What is the nature of the relationship?. *Mathematical cognition*, *3*(2), 121-139.

Crawford, D. B., & Snider, V. E. (2000). Effective mathematics instruction: The importance of curriculum. *Education and Treatment of Children*, *23*(2), 122-142.

Creswell, J. W. (2012). *Qualitative inquiry and research design: Choosing among five approaches*. London, UK: Sage.

Czerniak, C. M., Weber, W. B., Sandmann, A., & Ahern, J. (1999). A literature review of science and mathematics integration. *School Science and Mathematics*, *99*(8), 421-430.

Davis, B., & Sumara, D. J. (1997). Cognition, complexity, and teacher education. *Harvard educational review*, *67*(1), 105-126.

Davis, B., Sumara, D. J., & Luce-Kapler, R. (2000). *Engaging minds*. Mahwah, NJ: Erlbaum

Davis, B. (2008). Complexity and education: Vital simultaneities. *Educational Philosophy and Theory*, *40*(1), 50-65.

Davis, E. A., & Petish, D. (2005). Real-world applications and instructional representations among prospective elementary science teachers. *Journal of Science Teacher Education*, *16*(4), 263-286.

Davis, P. J., Hersh, R., & Marchisotto, E. A. (1995). *The mathematical experience*. Woodbine, NJ: Birkhauser.

Davison, D. M., Miller, K. W., & Metheny, D. L. (1995). What does integration of science and mathematics really mean?. *School science and mathematics*, *95*(5), 226-230.

Drake, C., & Sherin, M. G. (2006). Practicing change: Curriculum adaptation and teacher narrative in the context of mathematics education reform. *Curriculum Inquiry*, *36*(2), 153-187.

Drevdahl, D. J., Louie, B. Y., Purdy, J. M., & Stackman, R. W. (2003). Advancing the scholarship of teaching through collaborative self-study. *The Journal of Higher Education*, *74*(2), 150-171.

Driscoll, M. P., & Driscoll, M. P. (2005). Psychology of learning for instruction. Boston, MA: Pearson.

Duckworth, E. (1996). *The having of wonderful ideas and other essays on teaching and learning*. New York, NY:Teachers College Press.

DuFour, R. (2004). What is a" professional learning community"?. *Educational leadership*, *61*(8), 6-11.

Edwards, C. H. (1997). Promoting student inquiry. *SCIENCE TEACHER-WASHINGTON-*, *64*, 18-21.

Elliott, A. (2007). *Concepts of the self*. Cambridge, UK: Polity.

Engelmann, S., Carnine, D., & Steely, D. G. (1991). Making connections in mathematics. *Journal of Learning Disabilities*, *24*(5), 292-303.

Feiman-Nemser, S. (2012). *Teachers as Learners*. Cambridge, MA: Harvard Education Press.

Fenstermacher, G. D. (1994). The knower and the known: The nature of knowledge in research on teaching. In L. Darling-Hammond (Ed.) *Review of research in education*, *20*, 3-56.

Ferraro, J. M. (2000). Reflective practice and professional development. ERIC Digest. *Washington, DC: ERIC Clearinghouse on Teaching and Teacher Education, American Association of Colleges for Teacher Education*.

Fogarty, R. (1991). Ten ways to integrate curriculum. *Educational Leadership*, *49*(2), 61-65.

Franke, M. L., Webb, N. M., Chan, A. G., Ing, M., Freund, D., & Battey, D. (2009). Teacher questioning to elicit students' mathematical thinking in elementary school classrooms. *Journal of Teacher Education*, *60*(4), 380-392.

Fuson, K. C., & Briars, D. J. (1990). Using a base-ten blocks learning/teaching approach for first- and second-grade place-value and multidigit addition and subtraction. *Journal for research in mathematics education*, 180-206.

Gallard Martínez, A. J. (2009). Complexity and the universe of education. In *Forum on Public Policy Online*.

Gainsburg, J. (2008). Real-world connections in secondary mathematics teaching. *Journal of Mathematics Teacher Education*, *11*(3), 199-219.

Gamelin, A. K. (2010). The sand diaries: Visions, vulnerability & self-study. In Mitchell, O'Reilly-Scanlon & Weber (Eds.), *Just who do we think we are: Methodologies for autobiography and self-study in teaching* (pp. 183-192), New York: Rutledge Falmer.

Gess-Newsome, J. (2002). Pedagogical content knowledge: An introduction and orientation. In *Examining pedagogical content knowledge* (pp. 3-17). Dordrecht, Netherlands: Springer.

Griffiths, M., Bass, L., Johnston, M., & Perselli, V. (2004). Knowledge, social justice and self-study. In Loughran, Hamilton, LaBoskey, & Russell (Eds.), *International handbook of self-study of teaching and teacher education practices* (pp. 651-707). Dordrecht, Netherlands: Springer.

Grossman, P., & McDonald, M. (2008). Back to the future: Directions for research in teaching and teacher education. *American Educational Research Journal*, *45*(1), 184-205.

Grossman, P. L. (1990). *The making of a teacher: Teacher knowledge and teacher education*. New York: Teachers College Press.

Guilfoyle, K., Hamilton, M. L., Pinnegar, S., & Placier, P. (2004). The epistemological dimensions and dynamics of professional dialogue in self-study. In *International handbook of self-study of teaching and teacher education practices* (pp. 1073-1109). Dordrecht, Netherlands: Springer.

Gurevitch, Z. (2001). Dialectical dialogue: the struggle for speech, repressive silence, and the shift to multiplicity. *The British journal of sociology*, *52*(1), 87-104.

Gurevitch, Z. (1990). The dialogic connection and the ethics of dialogue. *British Journal of Sociology*, 41, 2, 181-196

Ham, V., & Kane, R. (2004). Finding A Way Through The Swamp: A Case For Self-Study As Research*. In Loughran, Hamilton, LaBoskey, & Russell (Eds.), *International handbook of self-study of teaching and teacher education practices* (pp. 103-150). Dordrecht, Netherlands: Springer

Hamilton, M. L. (1995). Confronting self: Passion and promise in the act of teaching or my Oz-dacious journey to Kansas. *Teacher Education Quarterly*, *22*(3), 29-43.

Hamilton, M. L., Smith, L., & Worthington, K. (2008). Fitting the methodology with the research: An exploration of narrative, self-study and auto-ethnography. *Studying Teacher Education*, *4*(1), 17-28.

Hammer, D. (1995). Student inquiry in a physics class discussion. *Cognition and Instruction*, *13*(3), 401-430.

Hammersley, M. (1993). On the teacher as researcher. *Educational Action Research*, *1*(3), 425-445.

Hersh, R. (1997). *What is mathematics, really?* Oxford, UK: Oxford University Press.

Hersh, R., & John-Steiner, V. (2010). *Loving and hating mathematics: Challenging the myths of mathematical life*. Princeton University Press.

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, 372-400.

Hirshfield, J. (1998). *Nine gates: Entering the mind of poetry*. New York, NY: HarperCollins.

Hoffman, B., & Spatariu, A. (2008). The influence of self-efficacy and metacognitive prompting on math problem-solving efficiency. *Contemporary educational psychology*, *33*(4), 875-893.

Hollingsworth, S. (1990). Teachers as researchers: Writing to learn about ourselves and others. *Quarterly of the National Writing Project and Center for the Study of Writing, 12,* 10-18.

Houser, N. (1990). Teacher-researcher: The synthesis of roles for teacher empowerment. *Action in Teacher Education, 12,* 55-60.

Institute of Education Sciences, National Center for Educational Statistics: Retrieved from https://nces.ed.gov/fastfacts/display.asp?id=30

Jacobs, V. R., Franke, M. L., Carpenter, T. P., Levi, L., & Battey, D. (2007). Professional development focused on children's algebraic reasoning in elementary school. *Journal for research in mathematics education*, 258-288.

Jaworski, B. (2004). Grappling with complexity: Co-learning in inquiry communities in mathematics teaching development. In *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 17-36).

Jaworski, B. (2006). Theory and practice in mathematics teaching development: Critical inquiry as a mode of learning in teaching. *Journal of mathematics teacher education*, *9*(2), 187-211.

Jay, J. K., & Johnson, K. L. (2002). Capturing complexity: A typology of reflective practice for teacher education. *Teaching and teacher education*, *18*(1), 73-85.

Jones, M., & Yonezawa, S. (2002). Student voice, cultural change: Using inquiry in school reform. *Equity &Excellence in Education*, *35*(3), 245-254.

Kamii, C., Lewis, B. A., & Kirkland, L. (2001). Manipulatives: When are they useful?. *The Journal of Mathematical Behavior*, *20*(1), 21-31.

Kieran, C. (2004). Algebraic thinking in the early grades: What is it. *The Mathematics Educator*, *8*(1), 139-151.

Kiray, S. A. (2012). A new model for the integration of science and mathematics: The balance model. *Energy Education Science and Technology*, *Social and Educational Studies*. *4*(3), 1181-1196.

Kirschner, P. A., Sweller, J., & Clark, R. E. (2006). Why minimal guidance during instruction does not work: An analysis of the failure of constructivist, discovery, problem-based, experiential, and inquiry-based teaching. *Educational psychologist*, *41*(2), 75-86.

Klem, A. M., & Connell, J. P. (2004). Relationships matter: Linking teacher support to student engagement and achievement. *Journal of school health*, *74*(7), 262-273.

Kniskon, J.A. (2000) The myth of the well-planned project or how research is like life: It doesn't always turn out the way you expected. In Loughran & Russell (eds.), *Exploring Myths and Legends in Teacher Education (pp. 257-262),* East Sussex, UK.

Knowles, J. G. & Cole, A. I. (2012). Researching the "good life": Reflections on professional practice. In Kitchen & Russell (eds.), *Canadian Perspectives on the Self-study of teacher education practices* (pp. 35-46*)*. Ottawa, Canada: Canadian association for teacher Education.

LaBoskey, V. K. (2004). The methodology of self-study and its theoretical underpinnings. In *International handbook of self-study of teaching and teacher education practices* (pp. 817-869). Dordrecht, Netherlands: Springer.

Lakoff, G., & Núñez, R. E. (2000). *Where mathematics comes from: How the embodied mind brings mathematics into being*. New York, NY: Basic books.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American educational research journal*, *27*(1), 29-63.

Land, R., Cousin, G., Meyer, J. H., & Davies, P. (2005). Threshold concepts and troublesome knowledge (3): implications for course design and evaluation. *Improving Student Learning–equality and diversity, Oxford: OCSLD*.

Larrivee, B. (2000). Transforming teaching practice: Becoming the critically reflective teacher. *Reflective Practice*, *1*(3), 293-307.

Lather, P. (1986). Research as praxis. *Harvard Educational Review, 56,* 257-278.

Lester Jr., F. K. (1989). The Role of Metacognition in Mathematical Problem Solving: A Study of Two Grade Seven Classes. Final Report.

Lieberman, A. (1995). Practices that support teacher development: Transforming conceptions of professional learning. *Innovating and Evaluating Science Education: NSF Evaluation Forums, 1992-94*, 67.

Lighthall, F. F. (2004). Fundamental features and approaches of the s-step enterprise, In Loughran, Hamilton, LaBoskey, & Russell (Eds.), *International handbook of self-study of teaching and teacher education practices* (pp. 151-193). Dordrecht, Netherlands: Springer.

Logue, M. E. (2006). Teachers Observe to Learn: Differences in Social Behavior of Toddlers and Preschoolers in Same-Age and Multiage Groupings. *Young Children*, *61*(3), 70-76.

Lotter, C., Harwood, W. S., & Bonner, J. J. (2007). The influence of core teaching conceptions on teachers' use of inquiry teaching practices. *Journal of Research in Science Teaching*, *44*(9), 1318-1347.

Loughran, J. J. (2002). Effective reflective practice in search of meaning in learning about teaching. *Journal of teacher education*, *53*(1), 33-43.

Loughran, J. (2005). Researching teaching about teaching: Self-study of teacher education practices. *Studying Teacher Education*, *1*(1), 5-16.

Loughran, J. (2007). Researching teacher education practices responding to the challenges, demands, and expectations of self-study. *Journal of Teacher Education*, *58*(1), 12-20.

Lundberg, C. C., & Young, C. A. (2005). *Foundations for inquiry: Choices and trade-offs in the organizational sciences*. Stanford, CA: Stanford University Press.

Mariano, L. T., & Kirby, S. N. (2009). Achievement of Students in Multigrade Classrooms.

Martino, A. M., & Maher, C. A. (1999). Teacher questioning to promote justification and generalization in mathematics: What research practice has taught us. *The Journal of Mathematical Behavior*, *18*(1), 53-78.

Gallard Martínez, A. J. (2009). Complexity and the universe of education. In *Forum on Public Policy Online*.

Marzano, R. J. (1988). *Dimensions of Thinking: A Framework for Curriculum and Instruction*. The Association for Supervision and Curriculum Development, Alexandria, VA : ASCD.

Mason, D. A., & Burns, R. B. (1996). "Simply no worse and simply no better" may simply be wrong: A critique of Veenman's conclusion about multigrade classes. *Review of Educational research*, *66*(3), 307-322.

Mason, J. (1998). Enabling teachers to be real teachers: Necessary levels of awareness and structure of attention. *Journal of Mathematics Teacher Education*, *1*(3), 243-267.

Mason, J. (1994). Enquiry in mathematics and in mathematics education. In Ernest, P. (Ed.). *Constructing mathematical knowledge: Epistemology and mathematics education*. London, UK: Falmer Press.

Mathison, S., & Freeman, M. (1998). *The logic of interdisciplinary studies*. National Research Center on English Learning & Achievement, University at Albany, State University of NY.

Meek, A. (1991). On thinking about teaching: A conversation with Eleanor Duckworth. Educational Leadership, *48*(6), 30.

Meier, D. (2002). *The power of their ideas: Lessons for America from a small school in Harlem*. Boston, MA: Beacon Press.

Meyer, J. H., & Land, R. (2006). Threshold concepts and troublesome knowledge. *Overcoming barriers to student understanding: threshold concepts and troublesome knowledge*, 3-18.

Milburn, D. (1981). A Study of Multi-Age or Family-Grouped Classrooms. *Phi Delta Kappan*, *62*(7), 513-14.

Morine-Dershimer, G., & Kent, T. (2002). The complex nature and sources of teachers' pedagogical knowledge. In *Examining pedagogical content knowledge* (pp. 21-50). Dordrecht, Netherlands: Springer.

Moyer, P. S. (2001). Are we having fun yet? How teachers use manipulatives to teach mathematics. *Educational Studies in Mathematics*, *47*(2), 175-197.

Nemser, S. (2012). *Teachers as learners*. Cambridge, Mass.: Harvard Education Press.

No Child Left Behind (2002). Act of 2001, Pub. L. No. 107-110, § 115. *Stat*, *1425*, 107-110.

Ong, W., Allison, J., & Haladyna, T. M. (2000). Student achievement of 3rd-graders in comparable single-age and multiage classrooms. *Journal of Research in Childhood Education*, *14*(2), 205-215.

O'Reilly-Scanlon, K. (2002). Muted echoes and lavender shadows: Memory work and self-study. *Making a difference in teacher education through self-study*, *2*, 74-78.

Parke, H. M., & Coble, C. R. (1997). Teachers designing curriculum as professional development: A model for transformational science teaching. *Journal of research in science teaching*, *34*(8), 773-789.

Paris, S. G., & Newman, R. S. (1990). Development Aspects of Self-Regulated Learning. *Educational Psychologist*, *25*(1), 87.

Paulos, J. A. (1988). *Innumeracy: Mathematical illiteracy and its consequences*. New York, NY: Hill and Wang.

Perrone, V. (1991). *A Letter to Teachers: Reflections on Schooling and the Art of Teaching*. Jossey-Bass, Inc., Publishers, 350 Sansome Street, San Francisco, CA 94104.

Piaget, J., & Cook, M. T. (1952). The origins of intelligence in children. New York, New York: International Universities Press, Inc.

Pierce, C.S. (1877/1994) 'The fixation of belief', in Siegel, M., & Borasi, R. (1994). Demystifying mathematics education through inquiry. In Ernest, P. (Ed). *Constructing mathematical knowledge: Epistemology and mathematics education*, London, UK: Falmer Press, pp. 201-214.

Pratt, D. (1986). On the merits of multiage classrooms. *Research in Rural Education*, *3*(3), 111-115.

Price, E. Document R= ED 131 062 95 SP 010 622.

Redfield, D. L., & Rousseau, E. W. (1981). A meta-analysis of experimental research on teacher questioning behavior. *Review of educational research*, *51*(2), 237-245.

Remillard, J. T. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of Educational Research*, *75*(2), 211-246.

Samaras, A. (2002). Self-study for teacher educators Crafting a pedagogy for educational change. *New York: Peter Lang*.

Santa, C. M., & Santa, J. L. (1995). Teacher as researcher. *Journal of Reading Behavior*, *27*(3), 439-51.

Sawyer, R. K. (Ed.). (2006). *The Cambridge handbook of the learning sciences,* Cognitive Apprenticeship (Vol. 2, No. 5). New York: Cambridge University Press.

Saxe, G. B. (1988). The mathematics of child street vendors. *Child Development*, 1415-1425.

Schmidt, W. H., Wang, H. C., & McKnight, C. C. (2005). Curriculum coherence: An examination of US mathematics and science content standards from an international perspective. *Journal of Curriculum Studies*, *37*(5), 525-559.

Schön, D. A. (1987). Teaching artistry through reflection-in-action. *Educating the reflective practitioner*, 22-40.

Schön, D. A. (Ed.). (1991). *The reflective turn: Case studies in and on educational practice*. New York: Teachers College Press.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. *Handbook of research on mathematics teaching and learning*, 334-370.

School of Education, University of Michigan*, High leverage practice.* Retrieved from *http://www.soe.umich.edu/academics/bachelors_degree_programs/uete/uete_hlp/*

Secretary's Commission on Achieving Necessary Skills (1991). *What work requires of schools: A SCANS report for America 2000.* Washington, DC: U.S. Department of Labor.

Shernoff, D. J., Csikszentmihalyi, M., Shneider, B., & Shernoff, E. S. (2003). Student engagement in high school classrooms from the perspective of flow theory. *School Psychology Quarterly*, *18*(2), 158.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching.

Educational researcher, *15*(2), 4-14.

Shulman, L. S. (1999). Taking learning seriously. *Change: The Magazine of Higher Learning*, *31*(4), 10-17.

Siegel, M., & Borasi, R. (1994). Demystifying mathematics education through inquiry. In Ernest, P. (Ed). *Constructing mathematical knowledge: Epistemology and mathematics education*, London, UK: Falmer Press, pp. 201-214.

Smith, B. O., Stanley, W. O., & Shores, J. H. (1957). *Fundamentals of curriculum development*. New York: Harcourt, Brace & World.

Song, R., Spradlin, T. E., & Plucker, J. A. (2009). The Advantages and Disadvantages of Multiage Classrooms in the Era of NCLB Accountability. Education Policy Brief. Volume 7, Number 1, Winter 2009. *Center for Evaluation and Education Policy, Indiana University*.

Stanley, D. (2009). WHAT WORKS? Research into Practice.

Steen, L. A. (1994). Integrating school science and mathematics: Fad or folly. In *DF Berlin, NSF/SSMA Wingspread Conference: A Network for Integrated Science and Mathematics Teaching and Learning. Conference plenary papers* (pp. 7-12).

Stillman, G. A., & Galbraith, P. L. (1998). Applying mathematics with real world connections: Metacognitive characteristics of secondary students. *Educational Studies in Mathematics*, *36*(2), 157-194.

Stoll, L., Bolam, R., McMahon, A., Wallace, M., & Thomas, S. (2006). Professional learning communities: A review of the literature. *Journal of educational change*, *7*(4), 221-258.

Stonewater, J. K. (2005). Inquiry teaching and learning: The best math class study. *School Science and Mathematics*, *105*(1), 36-47.

Taba, H., & Spalding, W. B. (1962). *Curriculum development: Theory and practice* (p. 529). New York: Harcourt, Brace & World.

Tall, D. (1996). Can all children climb the same curriculum ladder. *The Mathematical Ability of School Leavers, Gresham Special Lecture, Gresham College, London*.

United Nations Educational, Scientific and Cultural Organization, (2003). *Visiting a high school inquiry classroom: How to prepare and observe.* Retrieved from: http://otp.unesco-ci.org/training-resource/scientific-research/visiting-high-school-inquiry-classroom-how-prepare-and-observe.

Uttal, D. H., Scudder, K. V., & DeLoache, J. S. (1997). Manipulatives as symbols: A new perspective on the use of concrete objects to teach mathematics. *Journal of applied developmental psychology*, *18*(1), 37-54.

Veenman, S. (1995). Cognitive and noncognitive effects of multigrade and multi-age classes: A best-evidence synthesis. *Review of Educational Research*, *65*(4), 319-381.

Vygotsky, L. (1997). Interaction between learning and development. In Gauvain, M. & Cole, M., (Eds.), *Readings on the development of children*, New York: Freeman and Company, pp. 29-36.

Weber, S., & Mitchell, C. (2002, August). Academic literary performance, embodiment, and self-study: When the shoe doesn't fit: Death of a salesman. In *Making a difference in teacher education through self-study. Proceedings of the Fourth International Conference on Self-Study of Teacher Education Practices, Herstmonceux Castle, Sussex, UK* (Vol. 2, pp. 121-124).

Westbury, I., & Steimer, W. (1971). Curriculum: A discipline in search of its problems. *The School Review*, *79*(2), 243-267.

White, B. Y., & Frederiksen, J. R. (1998). Inquiry, modeling, and metacognition: Making science accessible to all students. *Cognition and instruction*, *16*(1), 3-118.

Wiggins, G. P., & McTigue, J. A. (2005). *Understanding by design*. Alexandria, VA: ASCD.

Wimer, J. W., Ridenour, C. S., Thomas, K., & Place, A. W. (2001). Higher order teacher questioning of boys and girls in elementary mathematics classrooms. *The Journal of Educational Research*, *95*(2), 84-92.

Wu, H. (1996). The mathematician and the mathematics education reform. *Notices of the AMS*, *43*(12), 1531-1537.

Wu, H. (2008). Fractions, decimals, and rational numbers. *Retrieved August*, *20*, 2008.

Yazzie-Mintz, E. (2007). Voices of students on engagement: A report on the 2006 high school survey of student engagement. *Center for Evaluation and Education Policy, Indiana University*, *12*.

Zhao, C. M., & Kuh, G. D. (2004). Adding value: Learning communities and student engagement. *Research in Higher Education*, *45*(2), 115-138.

Zimmerman, B. J. (2002). Becoming a self-regulated learner: An overview. *Theory into practice*, *41*(2), 64-70.

Zimmerman, B., Lindberg, C., & Plsek, P. (1998). A Complexity Science Primer: What is Complexity Science and why should I learn about it. *Adapted From: Edgeware: Lessons From Complexity Science for Health Care Leaders, Dallas, TX: VHA Inc*.

Appendices

Appendix A

EPSS Document Excerpt

Figure A1: 2008 EPSS Document Excerpt

There is now a dip in the 7th and 8th grade A2L scores, which clearly represents the newly curriculum-mapped testing and changed testing proficiency score at that level. Again, our Family School curriculum was caught off-guard with this change, but still managed to score with at least a 80% proficiency rate. Consequently, we have shifted our expectations to move up a half-year in our own Family School math unit expectations, and we feel this will make a significant change next year. In the meantime, we have been working with these 7th and 8th grade students in Assisted Home Schooling sessions to bring up their focus in math. The 100% scores in the Fall A2L are significant, because they represent that these students were 100% on their math skills from the previous year, as that is what the mid-school Fall test was supposed to test—the last grade's material.

Our Non-Fay students (new to Family School) scored at a 50% proficiency test in the Fall and increased to an 83% proficiency rate by the winter. Already their journey into Family School has improved their performance. A closer look at this reveals some interesting ideas. Our curriculum is divided up by algorithms: K- anti-counting (number sense to 1000's), 1st grade-addition (all place values), 2nd grade- Subtraction (all place values), 3rd grade all Multiplication, 4th grade- all division, 5th grade- all fractions, 6th grade- all decimals, ratios and percentages, 7th grade- all geometry (mid school level only) , 8th grade- algebra. When new students come to Family School in K and 1st grade it is reasonable to take them back through our curriculum from the beginning, because 1st graders from other schools do NOT come with number sense taught in our Anti-Counting curriculum. But when second graders come and cannot pass into our Subtraction unit which our second graders are ready for, they can't just go back and learn addition, but also always inevitably need to go though our Anti-Counting Unit as well. This takes time and it is a tough request to do in one year. This usually works its way out in their second year with us. This exact pattern happens in the 5th grade. Students who have not mastered multiplication and division with their old schools, must go back to our third grade multiplication groups to conceptually understand both multiplication and division (since they are inverse operations, they have similar conceptual understanding demonstrated through the base 10 manipulatives), and they will take extra time to get into the fraction groups. All of these students are exposed to these grade level concepts in the math workbook math, but they will not have a strong conceptual understanding needed of these math concepts until they have passed our algorithm math tests with an 85% or better. So it is reasonable to see the dip in 2nd and 5th grade for our new students

Appendix B

Real World Math Tests

Figure B1: Real World Math Test

Real World

1) Come up with a combination of states that would add up to 270 electoral votes with the least amount of states.
2) The Electoral College gives disproportionate voting power to the states, favoring the smaller states with more Electoral votes per person. The small states were given additional power to prevent politicians from only focusing on issues which affect the larger states. The fear was that without this power, politicians would only focus on the big states and major cities.
If Wyoming has 3 Electoral votes for a population of 493,782, and Texas has 32 Electoral votes for a population of over 20 million people, what does each state have as the number of people per each elector. How many mores times does an individual vote count in Wyoming than in Texas?

3) "Faithless Electors" are members of the Electoral College who, for whatever reason, do not vote for their party's designated candidate.

Since the founding of the Electoral College, there have been 156 faithless Electors. 71 of these votes were changed because the original candidate died before the day on which the Electoral College cast their votes. Three of the votes were not cast at all as three Electors chose to abstain from casting their Electoral vote for any candidate. The other 8 Electoral votes were changed on the personal initiative of the Elector.
If this rate of "Faithless Electors" per elections continued in the next 50 year's worth of elections, how many more "Faithless Electors" would we have? (The first election was in the year 1789, but every four years since 1792.)

4) Since the Electoral College excludes candidates who do not win pluralities in any individual states from its total, a candidate can win the Electoral College without winning a majority of the popular vote of the nation.

This has happened 16 times since the founding of the Electoral College, most recently in 2000. In every one of these elections, more than half of the voters voted *against* the candidate who was elected.
In the year2000, 50,461,092 people voted for Bush, 50,994,086 voted for Gore. Bush got 271 electoral votes, and Gore received 266 electoral votes. What percentage of the popular vote voted for Gore and what percentage of the electoral vote voted for Bush?

5) Swing states are the one in which recent presidential elections have been decided by a narrow margin. In 2000, New Mexico, Florida, Iowa, Oregon and Wisconsin had the closest margins, with the vote in each state decided by less than one percent. Florida's famous disputed vote came down to 537 votes or 0.01 percent. With Florida worth 25 electoral votes at the time, those were 537 incredibly important votes. How many people voted in Florida in 2000?

Figure B2: Real World Math Test

Real World

1) The Empire State building was built in 1931 and held the title of the tallest building from 1931-1972. They began excavation for the building on January 22 1930, and it was opened on May 1, 1931. How many days did it take from start to finish?
2) There are approximately 18 steps for each floor of the Empire State Building with 102 stories. How many steps are there for the entire building?
3) It has been figured that it took approximately 7 million man-hours to build the Empire State Building. Assuming that a work day was an 8-hour work day, what was the average amount of workers used to build the skyscraper each day? How does that compare to the stated peak periods of building having 3400 workers a day? How do you explain this difference?
4) The cost for the land that the building was built on was $24,718,000 and the cost for the building was $40,948,900. The skyscraper is said to be 37 million cubic feet, what was the cost of each cubic yard?
5) According to conversion charts on inflation, a dollar spent in 1930 was 9.1% of what it would be now. So, how much would the total cost of the building and the land be today and how much is each cubic yard worth today?

Figure B3: Real World Math Test

Real World

1.)The altitude of the Machu Picchu is 6750 feet above sea level, while right off the coast of Peru's west coast is a ocean trench going to the depth of 6866 meters below sea level. What is the distance from the lowest point of the trench to the highest point of Machu Picchu?

2.)The site of Machu Picchu is 5 square kilometers, How many square meters is this and how many square feet is this?

3.)It is believed that Machu Picchu held 750 people in it. What was the population density in meters if this fact is true?

4) Researchers believe Machu Picchu to have existed from 1440 – 1532. What fraction of a century is this?

5) Some research suggests that 135 bodies were uncovered, of which 109 were female. Make a pie chart using the 750 population number as the whole. Why do you think not all the bodies were not found? What is your theory for the decline of the civilization.

Figure B4: Real World Math Test

Real World

The idea of democracy was based on representation- that is that the government would be made up by elected representatives from each state to meet as Congress to enact laws. The way the representation was figured as to how many representatives there should be per person is called apportionment. This Real World will look at the differences in the numbers represented from the first Congress apportionment of 1789 to today's numbers.

1) The average ratio for the apportionment in 1790 was 1 representative for every 37,068 people. Based on this information and using the apportionment chart for the year 1790, approximately what the population for the top 3 most represented states?

2) The population for 1790 was 3,929,214, and population used for apportionment today is 281,424,177. How many times bigger is the population today and how many times bigger is the # of representatives of today compared to 1790 (find the answer to the nearest whole number)? If the ratio were to stay the same today as it was in 1790 (see question 1), how many representatives should we have in Congress today (round the problem to the nearest thousand)?

Let's examine the difference in the apportionment methods for the Webster and Hamilton methods of apportionment. (You can use a calculator from here on.)

3) Webster- Consider the following population numbers as representing some amount of population 59, 26, 16, and 7,and they would only be only allotted 11 seats in the Congress. Using the Webster Method and knowing that the fixed ratio is figured by dividing the total population by the 11 seats, how many seats would each population group receive in Congress.

4) Hamilton – Consider the same population numbers as above and the same fixed ratio to utilize the Hamilton way to figure the apportionment of the 11 seats.

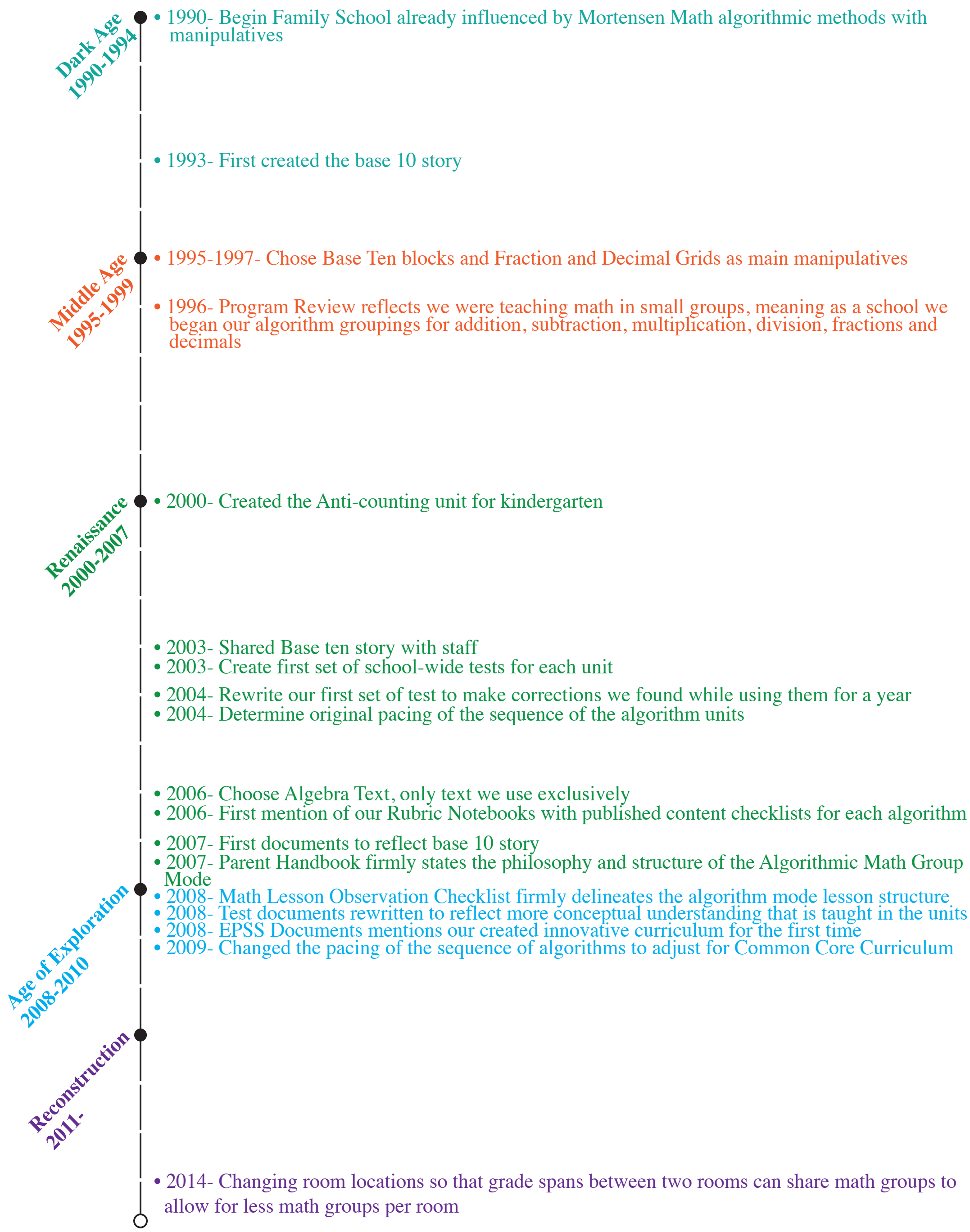
5) Make a table of the original states of the US that shows the apportioned representatives for 1789, and 2000, and the percent of increase or decrease for each state.

6) For homework find a New Mexico city that has the population of the average ratio of people to representative for 1790, and a total of more than one city for 2000.

Appendix C

Mode of Engagment Timelines

Figure C1: Algorithmic Math Group Mode Timeline

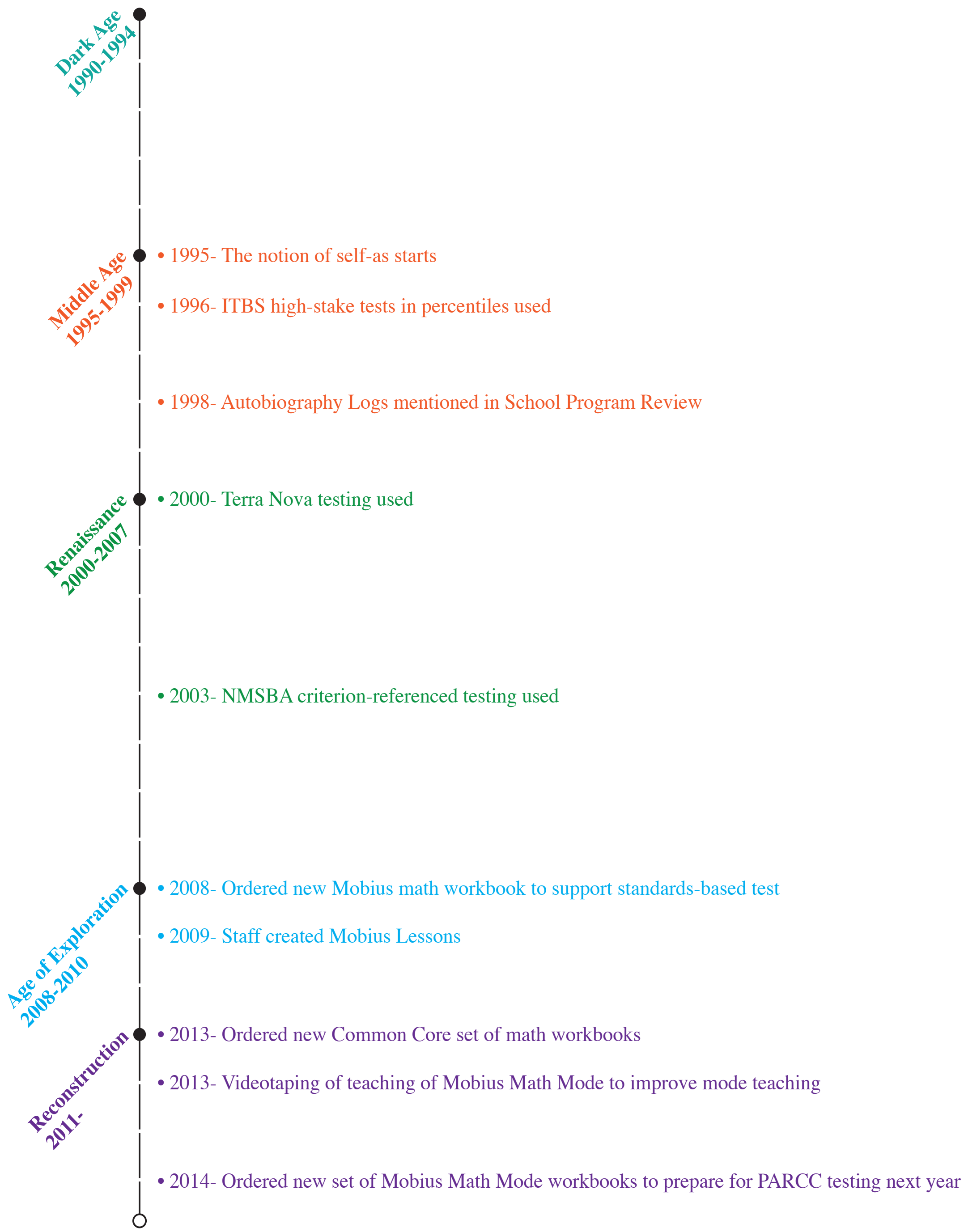


Algorithmic Math Group Mode

Figure C2: Math Brainteaser Mode Timeline

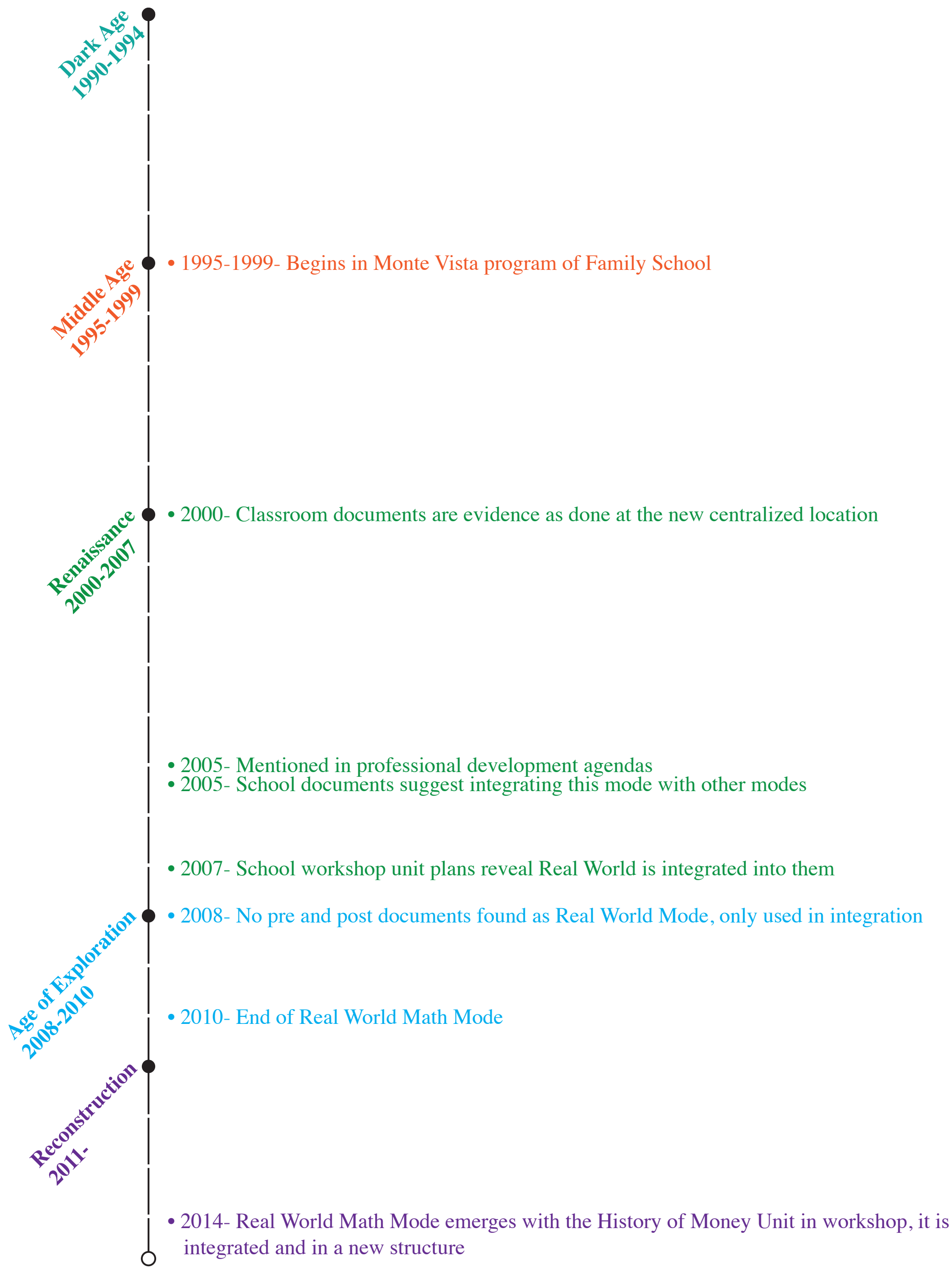
Brainteaser Math Mode

Figure C3: Mobius Math Mode Timeline



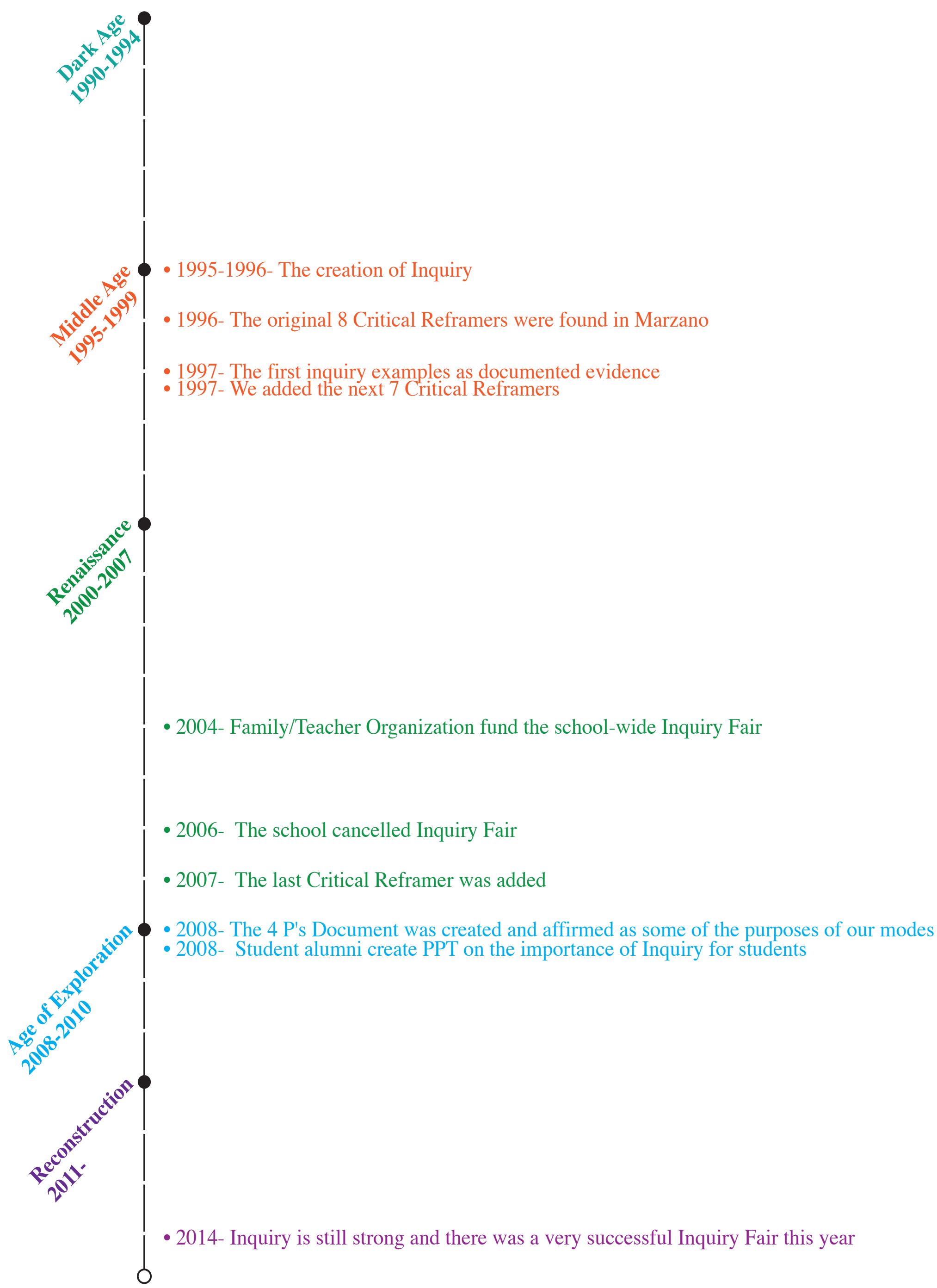
Mobius Math Mode

Figure C4: Real World Math Mode Timeline



Real World Math Mode

Figure C5: Inquiry Mode Timeline



Inquiry Mode

Appendix D

Parent Handbook Glossary

(Written by the parents at Family School for the new parents of Family School as referred to on page 370)

A2L: Assess2Learn**.** Web based survey tests for grades 1 – 8. A2L looks at levels of student understanding and compares student skills in reading, language arts and math to state and national standards. Students test on A2L three times a year. Grades 1 and 2 are tested on math only.
AIP: Academic Improvement Plan. Based on below-grade- level standardized testing results. Teacher, parent and student devise a plan to bring performance to grade level standards within a specified period of time.
Amnesia: A loss of memory, often temporary but known to affect large numbers of family school parents during recommitment night.
Anti-counting: An introduction to basic math concepts stressed during kindergarten and into first grade. Tools used include the number bone, the hand game, and base ten blocks. Skills include counting by 2s, 5s and 10s, mental number sense, odds and evens.
Art and Poetry Night: A celebration of student art and poetry held in mid-December. A school "open house" on a grand scale, Art and Poetry night includes self-guided tours of each class display, a silent auction of student art, a bake sale and extensive socializing.
Associate Team Teacher: An associate that works with a specific teacher in a specific classroom for two hours a week. These two hours allow the primary teacher time for professional and faculty development.
Attitude: In metacognitive theory, it is understood that attitude effects behavior and that people have some control over their attitude.
Attention: In met cognitive theory, an aspect of self-regulation is being aware of and controlling ones attention level.

Base Ten Story: A tale used to examine the origin and meaning of base ten counting – of the concepts of zero, ten and place value.
Base Ten Blocks: Tactile and visual manipulatives used to reinforce and concretize important mathematic concepts. Base Ten blocks can be used in the teaching of number and place value, addition, subtraction, multiplication, division, fractions, decimals and measurement.
Brain Teaser: A holistic class exercise meant to expose younger students to new concepts and to challenge older students into fresh ways of thinking and problem solving. Brain Teasers provide conceptual context for math and writing skills.

Commitment: An aspect of metacognitive self-regulation, commitment is a major determinant of student success. Commitment is neither feeling nor inclination – but rather a conscious decision to work hard (or not).
Community Brain: It is said that two (or more) heads are often better than one. Used to describe the synergy that can occur in workshop or brainteaser groups, the community brain is also called upon when a student is "stuck" and requests input and fresh perspective from classmates.
Critical Reframers: 1) Approaching an issue from different aspects. 2) Another way of asking the same question.

Descriptive review: 1) 1st trimester "report card." 2) A written analysis of the student's academic attributes. Also known as the Student Profile.
Dog bone (see number bone)
Dictation: A holistic, integrated educational tool that teaches spelling, capitalization, grammar and punctuation in context. The essay/paragraph usually complements a current workshop theme. The students take both a pre -and post-test of the passage within a week's time. How it looks: Teacher reads slowly. Student writes furiously.

Fit: Compatibility of approach, philosophy and temperament between teacher and parents and student.
FTO: 1) Family Teacher Organization (akin to PTA). Functions to raise school funds and provide teacher, student and school support. 2) Many hands make light work.

Goal Project: Typically, a four-week project geared toward improved performance in a particular subject area. Ideally student lead, often parent/teacher inflicted.

Habits of Mind: Core behaviors exhibited by successful learners. Behaviors include persistence, questioning, and accuracy.
Hand game: 1) A major component in anti-counting and in developing "number sense." 2) Not to be confused with" hand jive."
Homework Log: A record of home schooling activities and home schooling hours. In some classrooms, used as an avenue of parent-student/teacher communication and reflection.

IEP: **I**ndividualized **E**ducation **P**lan devised to service students with special needs.
Image Grammar: 1) Book upon which Family School writing curriculum is based. 2) Cure for insomnia.
3) Available by special order at any locally owned bookstore.
Inquiry: A hands on, integrated student lead process in which students are able to ask, and answer their own question(s) through their own discovery and analysis of the process. Appropriate inquiry topics include anything a young mind might wonder about.
Inquiry Fair: Students share their Inquiry Projects with their own and other classes.
Integrated Project: A project that integrates multiple academic disciplines. The teacher determines the nature of the project.
Interviews: Meeting with your child's potential teacher. A teacher presentation followed by parent questions, the interview is invaluable in making a good match for your family. The interview is also required if you want to request / rank that teacher for your child. Held toward the end of the school year, interviews are attended by new families and by current Family School families seeking to place their child in a new class.

Landscape Day: Armed with gardening implements, squads of students, teachers and parents descend upon unsuspecting profligate shrubs and rioting weeds to restore order and vegetative harmony to the Family School Campus. Landscape Days are scheduled throughout the year.
Learning Cycle: Process – Rehearsal – Performance - Reflection

Metacognition: 1) Being aware of your thinking as you perform specific tasks and then

using this awareness to control/chose what you are doing. 2) Components of metacognition include commitment, attitude, effort and attention.
Metacognitive Review: see **Self Assessment**
Number Bone: A manipulative tool used in anti-counting. A small scroll with a blank grid that children progressively fill in as they stack blocks into groups of ten and then 100s on up to 1,000, the Number Bone is used to teach place value and tedium.
Number Sense: You know it when you see it.

Open House: Shopping for a teacher and a class. Held twice a year for families prospecting for a new school or for a new teacher. Parents sign in at the office then go to different classes to observe. Based upon these observations, parents determine which teachers to interview in May. See Interviews.

Parent Classes (Gael's Classes): Interesting, enlightening and tremendously encouraging classes offered at the beginning of each school year. The Math Series and the Reading Journal Series cover different grade level concepts. There is also a class on the Inquiry Process. Though classes are optional, attendance will save you from weeks (and, in some cases, months) of perplexing bafflement in your home schooling.
Performance Project Portfolio: 3rd Trimester "report card". In the PPP, a student prepares a portfolio containing examples of her work in different subject areas.

Ranking: What parents do after teacher interviews? Parents are allowed to list and rank their 3 teacher preferences. While every attempt is made to honor a parent's first choice, student placement is ultimately determined by class size and student seniority.
Reading Journal: A reading, writing and thinking tool, the reading journal is a yearly progression of skills developed to ensure comprehension and critical thinking. Beginning with Retelling in the third grade, Reading Journal skills progress to Personal Comment, Writing Technique, Critical Thinking, and Structure and, in the 8th grade, Research and Thesis.
Recommitment: A Family School Rite of spring. Teachers and families decide whether or not to return to Family School the following academic year. A time of reflection and evaluation, the process also facilitates enrollment planning for the next year.
Rubric: 1) Criteria for assessing achievement
2) A scoring guide for a test or other assessment task.
3) Stanley: Famous director of landmark films 2001 Space Odyssey and Dr. Strangelove

Standards -Based Assessment (SBA): A standards-based test, the New Mexico SBA measures specific skills defined for each grade by the state. Students in grades 3 – 9 are tested in reading, math and science and are then rated at one of four performance levels: beginning proficient, nearing proficient, proficient or advanced. The obvious goal is for all students to score at or above proficient on the test.
Self-assessment: 2nd Trimester "Report Card". The student describes himself as a learner.
SLANT: An acronym describing, "what attention looks like." **Smile** + **L**ean forward + **Ask questions** + **N**od to indicate interest or agreement + **Take notes**
Sophie Experience: The Sophie Experience is an integrated, multi- purpose learning tool. Depending on the classroom, the Sophie Experience varies in its depth of overlapping

components. At the very least, the Sophie Experience includes the **dictation** of a passage that is connected to a classroom unit of study and the pen pal letters to a secret classroom parent. Letters may focus on the dictation topic or on another student project. The dictation studies and the yearlong exchange of letters encourage a real and personal writing experience and put writing mechanics in a meaningful context.
Specialized project: Different in every classroom – past specialized projects have included an illustrated fictional story and an in-depth and yearlong study into a student's particular interest. The specialized project has to address a grade level standard.

SSP: Student Success Plan: A plan of support for a student not meeting Family School requirements. Teacher, parent and student devise a plan to bring student up to a certain performance level within a specified period of time.

Used Book Sale: FTO fundraiser held in the Spring. Families donate and families purchase used books.

Workshop: Students work together in heterogeneous teams to explore issues related to the year's theme.