5-2019

The Effect of Using Technology on Students' Understanding in Calculus and College Algebra

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The Effect of Using Technology on Students' Understanding in Calculus and College Algebra

A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy in Mathematics

by

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May 2019
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Abstract

This mixed qualitative and quantitative methods study addressed the effect of technology on college algebra and survey of calculus students' understanding. This research study was conducted in fall 2016 on eight college algebra classes with a total of 315 students, and in summer 2017, on two survey of calculus classes with a total of 40 students at the University of Arkansas.

Several sources were used to collect data. A pre- and post- student attitude survey was administered during the first and last week of the semester for both college algebra and survey of calculus courses. Students' scores and paper work on three written tests (review test 1, review test 2 and concept test) in college algebra and students' scores and paper work on two written tests (review test 1 and review test 2) in survey of calculus were collected. The concept test was the only paper test normally administered in college algebra. Quantitative and qualitative data analysis enabled discussion of the effect of technology on students' understanding and organization of their work. This research study was guided by the following research questions.

1. How does the use of technology affect college algebra and calculus students' understanding and performance?
2. What areas of college algebra and calculus are affected more by technology?
3. How does using technology affect the organization of college algebra and calculus students' written work?
4. Does the use of technology positively impact college algebra and calculus students' attitudes toward their mathematics skills?

The results from the study exposed evidence that use of technology (handheld graphing calculators, online graphing utility Desmos, and smartphone apps) in teaching and learning

increased college algebra students' understanding of several concepts such as domain, vertical and horizontal asymptotes, end behavior of a function, and logarithmic functions. In addition, college algebra students' skills such as logical reasoning, use of graph, organization including written order, and correct use of notation and symbols were significantly increased when they used technology. Survey of calculus students' understanding increased in several topics such as finding maximum/minimum for two variable functions, limits, and definite integrals when they used technology in their class activities and on written tests.

Acknowledgements

I would like to thank people who supported me by any means through this accomplishment. Foremost I would like to express my special thanks to my advisor, Dr. Bernard Madison, for all his help, guidance, enthusiasm, motivation and immense patronage. He trusted me and has given me the freedom to pursue my research dependently and helped me not deviate from the core of my research.

I would like to thank other committee members, Dr. Mark Arnold, Dr. Shannon Dingman, and Dr. Nama Namakshi for their precious advices and insights in my research studies. Especially Dr. Namakshi who provided her invaluable vision on the logarithm project.

I have great pleasure to acknowledge my gratitude to the coordinator and instructors of college algebra and survey of calculus courses, Ms. Kruse, Dr. Erdem, Mr. Mutanguha and Mr. Keyton who agreed that their classes be subjects of this study and for their time for giving surveys to the students and patiently helping me in collecting data.

In addition, I would like to thank Dr.Tipton, Dr. Zmhang, and Dr. Datta for discussion and introducing me to some of the statistical methods I used in this dissertation.

I would like to thank all the students who participated in this study and filled out the surveys and attended the interviews.

I would like to thank University of Arkansas for giving the permission to perform this research study.

Finally, my deep and sincere gratitude goes to my beloved husband Dr. Mehrshad Mehboudi, my son Ali, my family and friends for their continuous support, love, and help during these years. I am forever indebted to family who supported me emotionally and encouraged me to seek higher goals.

In loving memory of my sister Mehri Shahriari. Her memory is always with me and I live through her eyes.

ای مهربان تر از برگ در بوسه های باران
بیداری ستاره در چشم جویباران
آیینه نگاهت پیوند صبح و ساحل
لبخند گاهگاهت صبح ستاره باران
باز آ که در هوایت خاموشی جنونم
فریادها برانگیخت از سنگ کوهساران
ای جویبار جاری !زین سایه برگ مگریز
کاینگونه فرصت از کف دادند بیشماران
گفتی به روزگاری مهری نشسته گفتم
بیرون نمی توان کرد حتی به روزگاران
بیگانگی ز حد رفت ای آشنا مپرهیز
زین عاشق پشیمان سرخیل شرمساران
پیش از من و تو بسیار بودند و نقش بستند
دیوار زندگی را زینگونه یادگاران
این نغمه محبت، بعد از من و تو ماند
تا در زمانه باقیست آواز باد و باران
دکتر محمدرضا شفیعی کدکنی

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Overview

In this dissertation, the effects of using technology on students' understanding in calculus and college algebra were investigated. Chapter 1 is an introduction to the study and a review of the significance of the study. In addition, chapter 1 describes the purpose of the study, the problems and research questions. Two theoretical frameworks in the literature that are adapted to using graphing calculators in teaching mathematics are reviewed at the end of the chapter.

In chapter 2, the literature related to the effects of using technology in different aspects of learning and teaching mathematics is reviewed. In this chapter, previous studies of using technology in education and its relationship with theories of learning are discussed. Most of the research studies on the use of technology in teaching mathematics courses are limited to the use of a graphing calculator. Therefore, most of the chapter is dedicated to different aspects of the use of graphing calculators in teaching mathematics courses. These aspects include the effectiveness of using graphing calculators and the effect of graphing calculators on students' achievements and learning skills. In addition, in this chapter some research studies on the use of smartphone and tablet applications (apps) in mathematical education are reviewed.

Chapter 3 presents the research methodology. In this chapter, different sources of data are described, including results of review tests and attitude surveys. Samples of review test items and attitude survey questions are provided as well. In addition, this chapter describes the participants in the study and samples of methods of teaching with graphing calculators and smartphone and tablet apps.

Chapter 4 provides a detailed description of data analysis and interpretation of the results for both college algebra and survey of calculus courses. The first part of the chapter is devoted to the quantitative and qualitative analysis of data from college algebra and the second part to the survey of calculus. Data were collected and analyzed from different sources including students pre- and post- attitude surveys, the concept test, review tests 1 and 2, students' ACT scores, and student interviews. Summaries of the answers to the research questions are provided at the end of this chapter.

Chapter 5 is a report of a research study on the effectiveness of using apps in teaching logarithms to students in college algebra classes. Data from students' written tests and interviews were collected and analyzed qualitatively and quantitatively. Students' understanding of logarithms concepts categorized into five levels and students' errors were identified and categorized.

Chapter 6 provides a summery, discussions and findings of the research study. In addition, limitations and recommendations of this research study are discussed.

Chapter 1: Introduction

Today, technology has become a significant part of secondary and college mathematics classrooms. There is little doubt that appropriate use of technology can enhance learning and enliven the teaching environment. The general effects of technology on teachers, students, and the ways that they communicate are undeniable. However, how technology affects teachers, students and their communications is open to exploration.

Technology has influenced societies extensively in recent years, and its influence is increasing every year. Graphing calculators have had a broad impact on teaching and learning mathematics in the past few decades. More recently, teaching and learning of mathematics has been widely affected by computer software and tablet and smartphone applications (apps).

In 1975, the National Advisory Committee on Mathematical Education suggested that students in grade eight and above should have access to calculators for all assignments and tests. In 1980, the National Council of Teachers of Mathematics (NCTM) recommended that students in all grades take advantage of using calculators in mathematics classrooms.

Several research studies explored the effect of technology on teachers' instruction, students' learning skills, textbooks, and assessments. Many types of technology have been produced to improve various aspects of students' learning: problem-solving, reasoning, and conceptual understanding. Technology has allowed books and class activities to be designed such that students have more chances to explore and to visualize mathematical concepts. Assessments and class activities have changed in today's technological world. For example, problems that require long and complicated computations can be included because of using technology in secondary and college classrooms. The way that students produce and report answers can be affected by technology as well.

The possibilities of using technology to enhance the efficiency of learning mathematical concepts pointed research toward investigating the effect of using technology on s tudents' achievement in college algebra and calculus.

1.1 Purpose of the study

Based on the experience of teaching college algebra for a couple of years and discussion with more experienced instructors of the course, lack of understanding of some mathematical concepts by college algebra students was identified. For example, most college algebra students struggled with the concept of logarithms. In addition, college algebra students had difficulty making connections between mathematical concepts and representations of these concepts such as their graphs. Most of the research previously conducted in this area was based on overall students' performance, which did not identify the difficulty in college algebra students' understanding of mathematical concepts. Therefore, this study aimed to tackle this problem by investigating the effect of using technology (apps and graphing calculators). Graphing calculators and graphing utilities (for example, Desmos) also were used to help students make better connections between mathematical concepts and graphical representations. Unlike some previous research about this study, the written works of students were analyzed.

1.2 Statement of the problem

In college algebra class, students struggle to understand some concepts and mainly rely on memorizing procedures to solve a problem. Moreover, most of the time, they cannot justify their answers. College algebra and calculus students have difficulty making connections between mathematical concepts and their graphical representations. In addition, in both courses, students have difficulty defining concepts and using procedures to define concepts. Students in college algebra class have difficulty in understanding logarithm concepts. In this study, two educational apps were used to investigate whether students understand logarithms better by using the apps. Students used graphing calculators in both courses. However, it was not clear how students use graphing calculators for solving problems. Therefore, it was important to know how students approach a problem using technology. For this research, students' written works were analyzed to investigate how students used graphing calculators to solve problems and how graphing calculators affected their understanding. In addition, some students were interviewed about their approaches to solving problems using technology.

1.3 Research questions

This study was guided by the following research questions.

1. How does the use of technology affect college algebra and calculus students' understanding and performance?
2. What areas of college algebra and calculus are affected more by technology?
3. How does using technology affect the organization of college algebra and calculus students' written work?
4. Does the use of technology positively impact college algebra and calculus students' attitudes toward their mathematics skills?

1.4 Theoretical framework

The use of a graphing calculator as a tool to introduce and analyze mathematical concepts has been suggested by NCTM (National Council of Teachers of Mathematics, 2000; National Council of Teachers of Mathematics. Commission on Standards for School Mathematics, 1989). NCTM states that graphing calculators transform the classroom into an environment where students and instructors act as partners in developing mathematical understanding and enhancing students' problem-solving skills. This suggestion can also be applied to the use of other technologies such as online graphing calculators and apps in the classroom. In fact, online graphing calculators and apps not only are as effective as hand-held graphing calculators, but they also have extra features such as practice problems and are more user friendly than hand held graphing calculators. In addition, using different technologies in the classroom is aligned with the concept of multiple and external representation which enables students to implement one representation and link it to another representation (Goldin & Kaput, 1996).

R.E.Clark (1983, 1991, and 1994), author of several books on the effect of media on education, believes that the reason that teaching with one medium is more effective than another is not the medium itself but the methods that employ the media (Clark, 1994). Clark believes that media do not have effects on learning beyond being methods to convey information efficiently. Clark likens the effect of media on learning to the effect of a truck that transports groceries on people's nutrition. However, some researchers argue that although there are some students who learn with or without media, there are some students who will not learn some concepts without using media. They believe that educational technology is not a natural science, rather, a designed science (Glaser, 1976; Simon, 1996). Kozma (1994) argues that not observing a relationship between educational media and learning is because the designer does not make this relationship. This belief emphasizes the importance of how technology should be implemented to impact learning. Kozma objects to Clark's view of separating method from medium and states that medium and method are integrated parts of design. Medium makes method more powerful and adds concentration to it.

Clark (1994) has a view that suggests that teachers should not discard the current technology in favor of new technologies. He suggests that teachers should maximize the effect of current technology in their teaching. Therefore, this study on the effect of a graphing calculator is aligned with Clark's suggestion.

Cognitive load theory also has implication on the use of graphing calculators in the classroom. This theory, which is based on several studies (Cooper & Schleser, 2006; Sweller, 1988; Sweller, Chandler, Tierney & Cooper, 1990; Sweller, 1994; Sweller & Chandler, 1994) , states that the learning process has three steps: sensory memory, working memory, and long-term memory. Everyday, people are faced with a large amount of information that is stored in sensory memory. A small portion of sensory memory becomes working memory as people start processing some of this information. As learners practice the working memory and encode it into long-term memory, the learning process happens. Therefore, the only information that becomes part of people's knowledge is long-term memory. This theory holds that cognitive load has two types. Intrinsic cognitive load is the load that learners utilize to create links between their knowledge and new knowledge. Extrinsic cognitive load comes from materials - pedagogical tools and methods. The use of graphing calculators in teaching college algebra can reduce extrinsic cognitive load imposed on the learners (Chval & Khisty, 2001; Ellington, 2006).

1.5 Significance of the research study

A limited number of research studies have reported on students' written work in the presence of technology. Some research studies have compared the overall achievement of students with technology and students without technology (paper and pencil skills), but no comprehensive research study has reported on organization and content of students' written works. This research study is distinguished for several reasons. First, this research study is one of the first to explore the effect of hand-held graphing calculators, online graphing utilities (for example, Desmos), and educational apps on students understanding and organization of written work. The lack of research in this area provides the opportunity to explore the effect of technology on students' understanding and organization of written work. A second reason that makes this research study significant is analyzing students' papers in addition to analyzing students' grades. Analyzing students' papers helps explore the effect of technology on different aspects of teaching and learning. For example, how teachers design questions and what kind of answer they expect to observe would be affected by the presence of technology.

A third reason that distinguishes this research study is analyzing students' types of errors when they use technology. Analyzing students' papers produced in the presence of technology gives one an opportunity to find students' errors, thereby affecting teaching and learning. For example, when teachers become aware of students' errors, they consider alternate strategies for teaching the concepts. For example, teachers may use technology to improve visualization of the concepts, or teachers may present more examples related to the concepts. Teachers' understanding of students' types of errors can affect teacher and student interactions as well.

1.6 Summary

This dissertation reports on effects of technology on students' understanding in college algebra and calculus classes. In fall 2016, eight college algebra sections with 315 students and in summer 2017, two calculus sections with 40 students at the University of Arkansas were chosen. In addition to the effect of using technology on students' understanding, their organization of written work also was studied.

Chapter 2: Literature review

For several decades, technology has been used as a tool to enhance students' appreciation and understanding of mathematical topics. Teachers have used technologies such as computers, software packages, graphing calculators, educational tablets, and portable devices to facilitate teaching mathematical concepts.

This chapter provides a review of reports on the results of using technology in secondary and college mathematics classrooms. Because the focus of the research is on the graphing utilities, graphing calculator, Desmos, and a few apps will be emphasized. In addition, an overview of the existing literature as it relates to students' conceptual understanding of mathematics in the presence of technology is presented.

2.1 Technology in education

Technology is a combination of tools and processes that facilitate fulfilment of objectives. Technology has revolutionized every aspect of human life. With the advent of each new technology, people in different fields evaluate the effect of using the new technology to accomplish their objectives. Educators also have enthusiastically used the capacity of new technologies in teaching. They also have attempted to adapt new technologies to theories of learning. Technology in education has been viewed differently over time, although most view specified learning as the objective of using technology. In 1963 the view of technology in education had a focus on controlling the learning process (Ely, 1963). Januszewski (2001) believes that this view and specifically use of word control was a result of behaviorism ideas as the dominant theory of learning at that time. In addition, the notion of managing and controlling learning was widespread among educators. For example, Hoban (1965) placed the learning-teaching relationship as a part of learning management. The same idea prevailed in the work of Schewn (1977) and Heinich

(1984). They believed that technology should be used as a tool to control teaching and learning. There were some views that concentrated on different processes such as educational problem solving and design of educational process. For example, Silber (1970) had a definition that focuses on the problem-solving skills and not necessarily on increasing the possibility of learning. The Association for Educational Communications and Technology (1977) had a definition of educational technology that focused on activities without mentioning learning in the definition. The current definition of educational technology focuses on the facilitating role of educational technology in the learning process. The notion of the current definition is that educational technology does not cause learning and only has a facilitating role. This means that although teachers can help learners to learn better, learning is owned by the learners (Robinson, Molenda, & Rezabek, 2008).

2.2 Technology and theories of learning

Different theories of learning can be adapted to educational technology. Educational technology can be adapted to the three main theories of learning: behaviorism, cognitivism, and constructivism.

Behaviorism, developed in the early 20th century, is based on experiments performed on the learning behavior of animals (Skinner, 1938; Skinner, 1953). Constructivism focuses on the idea that students should be active in the learning process rather than being passive. This theory states that knowledge is constructed by students by adapting their existing knowledge to the new knowledge or creating representations for the new knowledge. Therefore, in constructivism, students construct the meaning (Piaget & Inhelder, 1969; Vygotski, 1987; Vygotsky, 1980).

The use of technology in education can be consistent with different theories of learning. Constructivism as the most recent and most dominant theory of learning emphasizes students' exploration. Educational technology improves learning because students can explore different concepts and construct cognition of different concepts. Using technology in education increases the exploratory potential and skills of students. For example, using graphing calculators allows students to explore how various functions behave. White and others (White-Clark, DiCarlo, & Gilchriest, 2008) suggest that graphing calculators should be used on a regular basis and this use should not be restricted to the graphing parts of courses. Graphing calculators or any apps that provide visual representations of functions are helpful and provide opportunities for students to explore different kinds of functions. For examples, graphing calculators can be used to find the points of intersection of two graphs and visualize the concept of solution or roots of an equation. These examples show how using a graphing calculator can support constructivist learning.

2.3 Overview of different types of graphing calculators

Handheld graphing calculators: A graphing calculator as used here is a handheld personal computer that can be used to perform calculations, plot graphs, and solve equations. In addition, there are many smartphone applications and websites that have the same and additional abilities. In 1985 Casio introduced the first commercial graphing calculator, the fx-700G. Many other companies have since produced graphing calculators with different features and abilities. Sharp produced its first graphing calculator in 1986, HP in 1988, and Texas Instrument in 1990. Most graphing calculators have functionality other than calculation and graphing. Examples of these functionalities are performing matrix algebra, computing statistics and describing distributions, finding roots, and evaluating symbolic derivatives and integrals. Some graphing calculators perform parametric algebra and find antiderivatives as well as derivatives of a given function. The most recent version of graphing calculators made by TI is TI-Nspire CX. This version has improved graphical representation and can show multiple views and exhibit graphical animations.

Online graphing calculators: There are numerous websites and smartphone applications that have the same and additional abilities as handheld graphing calculators. Some of these online utilities include other options such as mathematics practice at different levels. Desmos is one of the more popular online graphing utilities. Desmos is available as a website and as an app. Mathway is a similar online graphing utility that has many extra options with the ability to plot, solve for a parameter, and explore trigonometry and linear algebra. Mathway solves equations with intermediate explanations that play a tutorial role. Symbolab is also a user-friendly online graphing utility that is very similar to Desmos. There are many practice problems on Symbolab that allow students to practice in different areas such as algebra and calculus. There are many other online graphing utilities that share similar functionality.

2.4 Research on the effectiveness of using graphing calculators

Considerable research has assessed the effectiveness of using graphing calculators on different levels of education and on various mathematical topics. Four types of research on the effectiveness of using graphing calculators are reviewed here. These four types are students' overall achievement in mathematics classes, students' mathematical learning skills, students' performance in college algebra and calculus classes, and students' mathematical written work. Examples of mathematical learning skills are conceptual understanding, visualizing, problem-solving, and reasoning.

2.5 The effect of graphing calculators on students' achievement

Several research studies have been conducted to explore the effect of graphing calculators on students' overall performance in both college and secondary school mathematics. Harvey (1993) analyzed data from 55 schools by comparing the mean scores of a "calculus readiness (CR)" test using graphing calculators and computers in pre-calculus. He considered 22 schools as control sections in which students were taught by traditional methods and the rest of the schools as treatment sections where students were taught using graphing calculators. This study found statistically significant positive increases in the treatment schools on the CR test scores.

In another effort, Quesada and Maxwell (1994) compared the performance of pre-calculus students who used graphing calculators and a textbook that requires graphing tools to students who were taught by traditional methods, regular textbook and scientific calculators. Quesada and Maxwell repeated their study through three semesters. The results of Quesada and Maxwell's research showed that students who were taught using graphing calculators in pre-calculus classes had significantly higher grades on the comprehensive common final exams than students who were taught by traditional methods.

Dunham and Dicks' (1994) review of research studies indicated that students were more active and more involved in group activities in a classroom where students used graphing calculators. In another research study Heller and others (2005) explored a relationship between the use of graphing calculators and students' achievement in algebra 1. This research was performed on 458 high-school students in suburban Oregon and Kansas. Students in all classes used the same textbook and the same final exams. The results of the study indicated that students who used graphing calculators during class activities indicated higher achievement on the final exams. Furthermore, the results showed that the scores on the final exams were significantly higher in the classes where teachers explained how to use graphing calculators for solving a problem, compared to the other classes.

2.6 The effect of graphing calculators on students' learning skills

Several research studies have considered the effect of graphing calculators on students' learning skills such as problem-solving, conceptual understanding, visualizing problems, and reasoning. Dunham and Dick (1994) collected different research studies about the effect of graphing calculators on students' learning skills and categorized them into two types, the effect of graphing calculators on problem-solving and the effect on conceptual understanding. They mentioned that students who used graphing calculators during class activities and assessments had better understanding in reading and interpreting graphical information, obtaining more information, and finding better algebraic representations from the graphs. In addition, students had better understanding about making connections between graphical, numerical, and algebraic representations. Dunham and Dick's review of research indicated that students who used graphing calculators were more successful at problem-solving, used more flexible approaches, and were willing to engage and stay longer with a problem. Students also solved more nonroutine problems when they used graphing calculators.

Another research study on pr oblem-solving (Jones, 2008) that was performed on 46 students in pre-calculus algebra classes at Macon College showed that students did not understand graphs just because they have graphing calculators. One reason given was that students relied on graphing calculators for checking basic arithmetic.

Ellington's (2006) review of research provided some results including: students who used graphing calculators had a better conceptual understanding of functions, variables, and applications of algebra; using graphing calculators improved low ability students' performances; students who used graphing calculators spent more time in mathematical explorations and problem solving activities compared to students who did not use graphing calculators ; students were more likely to use graphing calculators in situations that they thought a graph would help problem solving processes, but students were less likely to use graphing calculators in situations where they thought a graph was not required (Ellington, 2003; Ellington, 2006).

Penglase and Arnold's (1996) research on the impact of graphing calculators in high school and college mathematics classrooms showed a positive correlation between the development of visualization skills and students' mathematical achievement, especially for female students.

2.7 Effect of graphing calculators on students' performance

Numerous research studies have been conducted in different areas of calculus, mainly concentrating on the effect of graphing calculators on students' conceptual and procedural knowledge. Research studies show positive impacts of using graphing calculators on students' learning of calculus concepts. A review by Hunter (2011) indicates that among varieties of technology, students benefited most using graphing calculators in a calculus class. Graphing calculators can be used in different ways for teaching and learning calculus concepts. For instance, increasing the efficiency of calculations and exploring abstract concepts that are difficult to visualize, such as end behavior, concavity, differentiability, and continuity.

Porzio (1997) performed a study on the effect of graphing calculators on students' understanding of numerical, graphical, and symbolic representations of calculus concepts. This study was performed on 100 students in three calculus sections. The results show that students performed better when they used graphing calculators.

In a study on the effect of graphing calculators on college algebra students, Smith and Shotsberger (1997) used four sections of a college algebra class where two were taught using classical methods and two with the aid of graphing calculators. This study did not show a significant difference in achievement between the different sections. However, a significant difference was observed on genders. In that study, female students' achievement significantly increased when using graphing calculators. This fact is also observed in other research studies (Dunham, 1995; Ruthven, 1990). This fact is possibly due to improvement of students' confidence by having graphing calculators. Some studies showed the positive effect of graphing calculators on problem-solving in college calculus courses (Bookman & Friedman, 1994). Some investigations have reported increased conceptual understanding of students in college calculus courses (Connors, 1995). In another study (Cunningham, 1991) researchers asked students to use computer software to perform symbolic manipulation in a first-year calculus course. They found that students who used the software performed significantly better than the other students on the same exam. This study also shows that students' performances did not change when they were deprived of the software package.

2.8 Students' written work in the presence of technology

A few research studies have been done on effect of graphing calculators on student written work. Some researchers compared students learning skills such as problem-solving in the presence of technology and without technology. Ellington (2003) studied the effect of graphing calculators on students' achievement in precollege classes. Students with low or average ability were investigated separately from students with high ability. Ellington's study shows that although graphing calculators had no effect on understanding of mathematical concepts of low or average ability students, their paper and pencil skills improved. Nevertheless, paper and pencil work of high ability students did not change when using graphing calculators. In addition, the problem-solving skills improved when using graphing calculators. A meta-analysis was performed by Hembree and Dessart (1986, 1992) in which the results of 79 studies were summarized into five conclusions. One of these conclusions is related to students' written work. It states that using graphing calculators simultaneously with traditional instruction not only does not harm students' paper-and-pencil skills but also improves their paper-and-pencil skills. This conclusion was based on the work on K-12 students. No other study was identified on the effect of graphing calculators on students' organization of written work.

2.9 Tablet and smartphone apps in education

Researchers have been investigating the potential of smartphone and tablet applications in mathematics education. Some researchers have performed case studies following quantitative or qualitative evaluation of the topic and using different grades and genders, while other researchers have gone further and tried to build a framework for teaching based on iPad or mobile-learning (m-learning). The concept of m-learning refers to using portable devices with ability to connect to the internet, such as tablets and smartphones (Park, 2011). Many studies have been done on different aspects of using iPad technology in education. Park has compared a learning model that uses mobile or wireless devices (m-learning) with a learning model that uses electronic technology (e-learning) for learning and teaching from remote sites. Park used transactional distance theory to analyze these comparisons (Moore, 1993). This theory defines the significant aspects of distance learning where the teachers and students are separated. Trying to adopt transactional distance theory to mobile learning, Park concluded that a new theoretical framework for reviewing mobile or wireless devices used is required.

Some researchers investigated the acceptance of m-learning among individuals by applying unified theory of acceptance and technology (UTAUT) (Wang, Wu, & Wang, 2009). They investigated many hypotheses on effect of various parameters such as performance expectation, perceived playfulness, effort expectation, social influence, and self-management of learning on behavioral purpose to use m-learning. The study was done on people from all range of ages and educational background and used UTAUT successfully by adding two new factors to the theory. It concluded that all the parameters are important in behavioral intention for using m-learning, for example, the performance expectation and perceived playfulness have great effect on behavioral intention, while the effect of gender was not visible. Effort expectancy was significant for older individuals and less significant for the young children

In another approach, Hargis et al. (2014) have presented strengths, weaknesses, opportunities, threats (SWOT) analysis for mobile learning based on s urveys. This research showed that students' engagement in learning is the main strength of m-learning and the weakness is the teachers' and parents' lack of technological skills.

Kearney et al. (2012) tried to build a framework that defines socio-cultural features of m-learning. They highlighted three features of m-learning: validity, personalization, and collaboration. Validity refers to the ability of m-learning to contextualize placed learning, personalization refers to ownership implication of m-learning and learning independently, and collaboration refers to connections and conversational aspect of m-learning.

In other research, Henderson and Yeow (2012) discussed two theories in education, behaviorism and constructivism. Behaviorists believe learning happens in a change in behavior. However, constructivists believe that knowledge is not transferred from teacher to student like transfer of a physical notion, but children learn by constructing the knowledge themselves rather than transfer from teacher to students. Henderson and Yeow claimed that only recently educational technology could accept a constructivist theory. They discussed three main qualities of iPad learning such as mobility, engagement, and collaboration. O'Malley et al. (2013) produced a case study to investigate the effect of iPad on basic mathematics learning of disabled students. The result of this study showed that iPad had a positive effect on learning, students' interest toward learning, and students' engagement.

Another interesting case study was reported by Vakil (2014) on increasing the fluency of urban youth in creating mobile and tablet apps. Although this work may not be relevant to mathematics education, the case study concludes that it is essential to teach people how to prepare their own apps instead of just letting them browse and use pre-made apps. This approach was designated as increasing digital fluency.

2.10 Mathematical apps in secondary and college courses

By the advent of first smart phone in 2007, the possibility of using apps in education was evaluated by educational researchers. The use of apps in education seems to be very exciting because, currently, almost everyone has a smartphone. In addition, apps can be designed for very specific purposes. Therefore, one can design apps to be used for just college algebra or for use by disabled students in mathematics. In addition, apps not only can be used in class but can engage students at other times. Much research has been directed at the effectiveness of apps on secondary and college mathematics courses.

One of the main problems that teachers face is the variety of apps, which makes the selection of an appropriate app difficult. In addition, most of the apps are not consistent with the theories of learning, which makes them not suitable to be used as a part of curriculum. In the light of this fact, Handal (2013) evaluated 100 mathematical educational applications and classified them into nine groups based on functionality. This effort helps teachers to choose suitable apps effectively from many available apps. For instance, apps are classified based on characteristics such as simulators, drawing or graphing, and informative apps. All mathematical educational apps are categorized into three main frameworks that are called explorative apps, instructive apps and productive apps. Handal stated that in explorative apps, teachers have a facilitator role and ensure that students do not digress while investigating. Activities that require designing products are in productive apps category and routine drill and problem-solving activities are in instructive apps.

Moyer-Packenham and her colleagues (2015) observed 100 students from grades 3 to 8 with different apps. This research used a clinical interview and tested performance by pretest apps, test apps, and posttest apps. Data were analyzed both qualitatively and quantitatively to evaluate children's performance and efficiency. The conclusion was that age is an important issue in learning with apps.

In another study, Falloon (2013) concludes that new technology such as computers in past years and iPad in recent years have failed to perform up to their potential. Two possible reasons for this were mentioned. First, there is a lack of mutual appreciation between teachers and pedagogical models with the potential performance or technology. The second reason is the potential performance of technology has been made unrealistic, and as various schools prepared technology indiscriminately, the possible success was not visible. Falloon examined the interaction of five-year old students with specific apps that were selected by an experienced teacher. These apps were related to literacy, numeric tasks and problem-solving skills. Rather than evaluating the performance of students by observation, recorded video of students working with apps was studied. This study concludes that careful attention is needed for designing the content of apps. Further, it s ummarizes the importance of apps as providing distracting free, communicative, prompt feedback. Falloon, finally, doubted the effectiveness of using apps to teach five-year old students.

Green et al. (2014) emphasized the difficulty of teachers in selecting an appropriate app for use in their classrooms. A rubric was suggested for selecting apps that rate the apps based on many features, and if the final grade of the app is high then it is proper for the class. This rubric that is designed for students in 5th to 12th grade is based on the content, feedback, scientific inquiry, and navigation.

While most researchers have tried to investigate the effectiveness of apps on learning, Blair (2013) has summarized basic principles for creating a conceptual mathematical app as follows:

- Apps should be designed based on simplicity.
- Apps should be designed based on specific needs.
- A meaningful educational model should be considered for designing an app. A good educational model allows students to develop fluency in the topics.
- Apps should support individual needs. The apps should be specifically designed to be self-paced, success oriented, user controllable, and produce low stress.
- Apps should support teachers and parents; these supports can be in terms of extra comments that suggest the prerequisites of an activity or application or any direction about implementation of apps in the classroom.

The potential use of apps in education is known to educators. However, teachers experience two challenges in using apps in class activities. The first challenge is the number of available apps that makes it difficult to choose among them. The second challenge is that most of the apps that are built as educational apps have financial purposes and more likely are not based on theories of learning. Therefore, it is essential that educators check the consistency of apps before recommending for use in the classrooms.

2.11 Summary

In summary, educational technology is defined as tools and procedures to facilitate learning. For a long time, these technologies were personal computers and graphing calculators. Large volumes of research exist on the effectiveness of graphing calculators. These studies were on all grades from 1st grade to college courses. Most of the studies showed a significant difference in students' achievement and learning when using graphing calculators. Therefore, based on the literature, it is known that graphing calculators can be effective, but it is important to know how they should be used. In addition, a few research studies have been performed on the effect of graphing calculators on students' written work.

The use of apps as educational technology has been investigated by many researchers. Some of these studies are fundamental studies that build a framework for using apps as educational technology, and some others are case studies. Most of these studies concluded that apps are effective in teaching mathematics courses.

Chapter 3 Methodology

This chapter presents the details of analyzing qualitative and quantitative data to examine the effect of technology on the written work of college algebra and survey of calculus students in fall 2016 and summer 2017 at the University of Arkansas. The instruments that were used to collect and evaluate data are described.

College algebra: College algebra was a three-semester credit hours course. During the fall and spring semester students participate in three, fifty-minute class meetings every week. College algebra students were mostly freshman. College algebra topics were: functions and transformation of functions, linear and quadratic equations, exponential and logarithmic functions, and system of linear and nonlinear equations.

Survey of calculus: Survey of calculus was a three-semester credit hours course in polynomial calculus. During the fall and spring semester students participate in three, fifty-minute class meetings every week. Survey of calculus was taken by business students or students who do not plan to take further calculus courses. Survey of calculus concepts were: limits, derivatives, and definite and indefinite integrals.

3.1 Data collection instruments

The primary goals of the study were to explore the effects of using technology, such as graphing calculators and apps, on students' organization of written work, and students' understanding of specific concepts. Examples of these concepts are logarithms in college algebra and limits, derivatives, and integrals in survey of calculus. Therefore, to discover the effects, it was essential to assess students' written work as fully as possible. To probe students' abilities to answer questions properly and to evaluate students' written work in the presence of technology, students' explanatory answers were needed. However, students used the MyLabsPlus (hereafter, MLP) website for all their assignments and tests for both college algebra and calculus courses; therefore, students' written works were not accessible.

MLP questions were mostly multiple-choice items with four choices or short answers, but evidence was needed to show how students used technology, specifically the graphing calculators, to produce an answer, and how this technology affected their work. Therefore, the current MLP exams were not sufficient for evaluating; other tools were required. After reviewing related literature, especially, chapter 17 of Fey, et al. (Fey, Cuoco, Kieran, McMullin, & Zbiek, 2003) two written tests were designed. Review test 1 and review test 2 were constructed and included open-ended questions to determine the effect of the graphing calculators on different aspects of students' written work, such as organization, reasoning, and ability to solve a problem. Details of participants, the review tests, and course content follow below. Another test, called the concept test, was part of the regular class. A sample of the concept test, review test 1 and review test 2 questions are included in appendix A, and complete details of the results and analysis are provided in chapter 4.

3.2 Participants

In fall 2016, eight college algebra sections with a total of 315 students and in the summer of 2017, two sections of survey of calculus class with a total of 40 students were chosen. College algebra students were freshmen; calculus students were a mix of freshmen, sophomores, and juniors. Students in both college algebra and calculus comprised various majors, and races, and both genders. College algebra was taught by two female and two male instructors, three of them international and all senior PhD students with similar experience in teaching college algebra courses. Each instructor taught one control and one treatment section. All instructors had experience in teaching other undergraduate courses. The principal researcher and one of the instructors taught calculus in summer 2017.

3.3 Student attitude survey

A previously validated student attitude survey (SAS) meeting certain requirements was needed to examine students' attitudes toward technology in mathematics courses. The student attitude survey should:

- Have been previously implemented in other studies.
- Have been administrated to algebra students.
- Have been used in the USA.
- Have been used at the college level.
- Evaluate students' attitude toward using technology on mathematics problems.

In search of a survey meeting all criteria, different reports in the literature were reviewed. One example was a r esearch study that was conducted by Smith and Shotsberger (1997) on assessing the use of the TI-82 graphing calculators in college algebra. Even though they applied a student attitude survey to college algebra students, the questions were mostly about students' attitude toward mathematics, not about using technology for solving problems. Therefore, other attitude surveys such as one by Korey, Brookstein et al. (2011), and Tharp (1999) were reviewed. Some of the surveys reviewed were used to evaluate high school students' attitudes about using technology in mathematics courses. A few surveys focused on measuring college students' attitude toward technology in mathematics (Brookstein et al., 2011). However, these were mostly administered to college students outside of the USA.

Finally, a survey that was developed by Tharp (1992) and was used by Merriweather and Tharp (1999) for their research study was selected. This survey was designed to investigate the effect of use of the TI-82 graphing calculators on eighth grade general mathematics students, but it has not been used at the college level. The chosen survey had 23 items with five levels of response: strongly agree (SA), agree (A), neutral or undecided (N/U), disagree (D), and strongly disagree (SD). One item was added to the student attitude survey to explore students' proficiency in using a graphing calculator, rated from 0 to 10. Sample items of Tharps' student attitude survey are shown in Table 1.

Table 1. Sample items of Tharp SAS

Sample Questions	SA	A	N/U	D	SD
A graphing calculator can be used as a tool to solve problems I could not solve before.					
I would try harder in math if I had a graphing calculator.					
I would do better in math if I could use a graphing calculator.					
Since I have a graphing calculator, I do not need to learn to make graphs by hand.					

3.4 Supplementary information on college algebra course content

College students participated in three, fifty-minute class meetings every week. Instructors taught following the book by Blitzer, Mayne and Pietro (2004). The electronic version of the textbook was included in MLP. MLP is a computer provided system that students use for quizzes, exams, final exams, and projects. MLP is used in several undergraduate mathematics courses including college algebra and survey of calculus. Table 2 shows grade weighting for each section of class activities and tests. In addition to materials of Table 2, grades of review test 1 and review test 2 were considered as a replacement for the two lowest grade quizzes.

Table 2.Grade weights for each portion of the syllabus.

	Percentage of the total
Homework	8%
Quizzes	15%
Wiki projects	8%
Test 1	12%
Test 2	12%
Test 3	12%
Test 4	8%
Participation	5%
Final test	20%

Students could use graphing calculators approved for use on ACT tests. For examples, the TI-83 and TI-84 were approved for all quizzes, homework, class activities, and MLP tests. Instructors used both traditional lecturing and PowerPoint slides to teach lessons. In addition, they used online graphing calculators or smartphone apps in some lessons. Worksheets were also used as a class activity for each lesson. Students were required to complete worksheets. The sections taught earlier in the day were considered control sections in which students were not allowed to use a graphing calculator on the concept tests, and those sections taught later in the day were considered treatment sections in which students were allowed to use a graphing calculator on the concept tests. The detailed information about the concept tests and review test 1 and review test 2 are presented below.

3.5 Concept test, review tests 1 and 2

The concept test: The concept test was the only paper test normally administered in college algebra. There were seven open-ended questions, designed by the coordinator of the course. The topics on the concept test were functions and combinations of functions, transformation of a graph, linear, quadratic and polynomial equations, exponential and logarithms functions, linear and nonlinear system of equations, and inequalities. Students completed the concept test two weeks before the final exam.

After negotiating with the coordinator of college algebra, two questions of the concept test were changed such that students had more opportunities to use graphing calculators for answers. However, questions were graphing calculator neutral, that is, could be answered with or without a graphing calculator; therefore, it was unclear whether students would use graphing calculators. Since the primary goal of the study was to explore students' understanding of the concepts, the way that they produce and record answers in the presence of technology, some tools to measure their knowledge were needed. Using the concept test as the only source of students' written work for college algebra was not enough to aid the principal researcher in answering all research questions. Therefore, precise evidence was needed to address these questions. For this reason, two written tests were designed.

Review test 1: Two written tests, review test 1 and review test 2, were designed to give the principal researcher more data to answer the research questions. The review test 1 was a graphing calculator-based test in which questions were designed in a way that students had to use a graphing calculator to produce an answer. Review test 1 included five open-ended problems that were designed to examine students' understanding of zeros and domains of rational functions, x- and y-intercepts, holes, turning points, end behavior of functions, and asymptotes. Review test 1 was completed by students in both the control and treatment sections as a review for the midterm exam. Some questions of the review test 1 are shown in the Table 3.

Review test 2: Review test 2 was a non-graphing calculator-based test that had five open-ended problems. The problems were designed to measure students' abilities to derive information from the graph of a function and use that information to solve a problem. These problems were used to measure students' understanding of the mathematical concepts. Therefore, for most of the questions in the review in the review test 2, graphs were given.

Table 3. Sample questions of the review test 1

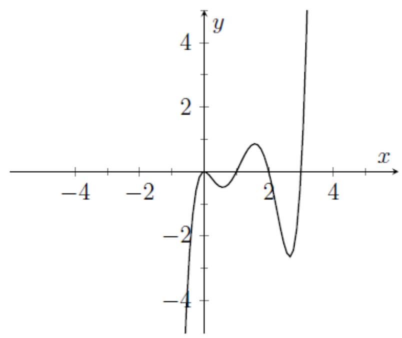
Q3	Graph the polynomial $\boldsymbol{p(x) = 3x^4 + 9x^3 - 3x^2 - 9x + 6}$ and answer the following: a) Label the zeros of the polynomial on the graph (with exact values). b) Label the turning points. c) Describe the end behavior.
Q4	Let $\boldsymbol{f(x) = \frac{(x^2-4)}{(x^2+5x+6)}}$ be a rational function. Find the following: a) x- and y-intercepts. b) vertical and horizontal asymptotes. c) holes. d) domain.
Q5	Find the value of x that solves $\boldsymbol{\log_3 2x - \log_3(x-3) = 1}$

The review test 2 examined students' understanding of similar concepts as the review test 1 and was given as a review for the final exam. Some common concepts were placed on both the review test 1 and the review test 2 to allow the principal researcher to compare students' understanding of a concept with and without graphing calculators. Comparing student performances on review test 1 and review test 2 ways aimed at finding answers to the research questions. Students in the control and treatment sections were not allowed to use graphing calculators during the review test 2. Examples of the review test 2 questions are shown in Table 4.

3.6 Teaching college algebra with the aid of technology

Different technology such as a h and-held graphing calculator, online graphing utility (Desmos), and apps were used to help students understanding of mathematical concepts in college algebra.

Table 4. Sample questions of the review test 2

Q1	Let $f(x) = \frac{2x^2+12x-14}{x^2+2x-15}$ be a rational function. Answer the following questions and explain how you use graphing calculators to answer each part. a) y-int: b) x-int: c) vertical asymptote(s): d) horizontal asymptote: e) domain: f) hole:
Q3	Use the graph of the polynomial at the right to answer the following:  What is the minimum degree of polynomial? Why? What is the sign of leading coefficient? Why? What are the zeros and local max/min for this polynomial? Use the information you can gather from the graph to write the equation of the polynomial.
Q5a	Solve the following equation: log6 (x + 4) + log6 (x + 3) = 1

TI-84 in college algebra: Instructors taught students to use a graphing calculator for different concepts such as domain and range of functions, function transformations, and solving system of equations. Figure 1a shows the graph of system of two equations and their intersections. Figure 1b shows using graphing calculators to find the transformation of function.

Desmos in college algebra: Instructors used the online graphing utility Desmos to help students' graphical understanding of the college algebra concepts such as transformation of functions, zeros of functions, piecewise defined functions, and linear and quadratic equations.

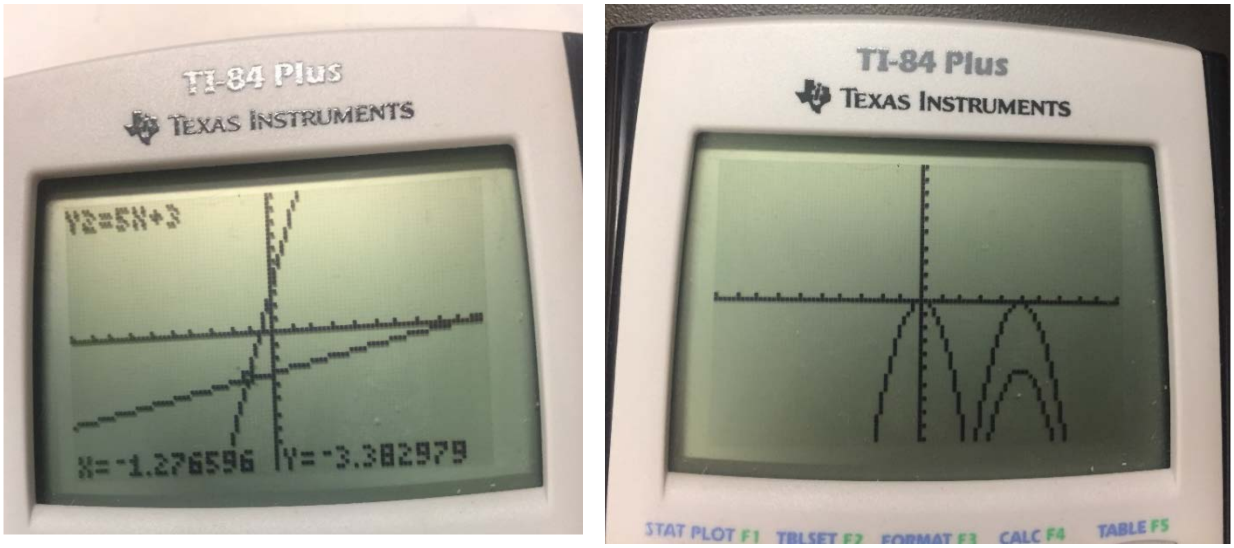


Figure 1.a) System of two equations using GC b) Transformation of function using GC

Students can use Desmos to visualize the effect of horizontal and vertical transformations. They also can easily observe how function's graph can be stretched or compressed horizontally or vertically. Samples of transformation of function $f(x) = x^2$ are illustrated in Figure 2a. In addition, students can observe the behavior of a piecewise-defined function and find the value of the function at any point. For example, Figure 2b shows the graph of piecewise-defined function defined below. $f(x) = \begin{cases} x^2 - 1 & x \leq 0 \\ \sqrt{x} + 5 & x > 0 \end{cases}$

With the use of graphing calculators students can make connection between graph of a

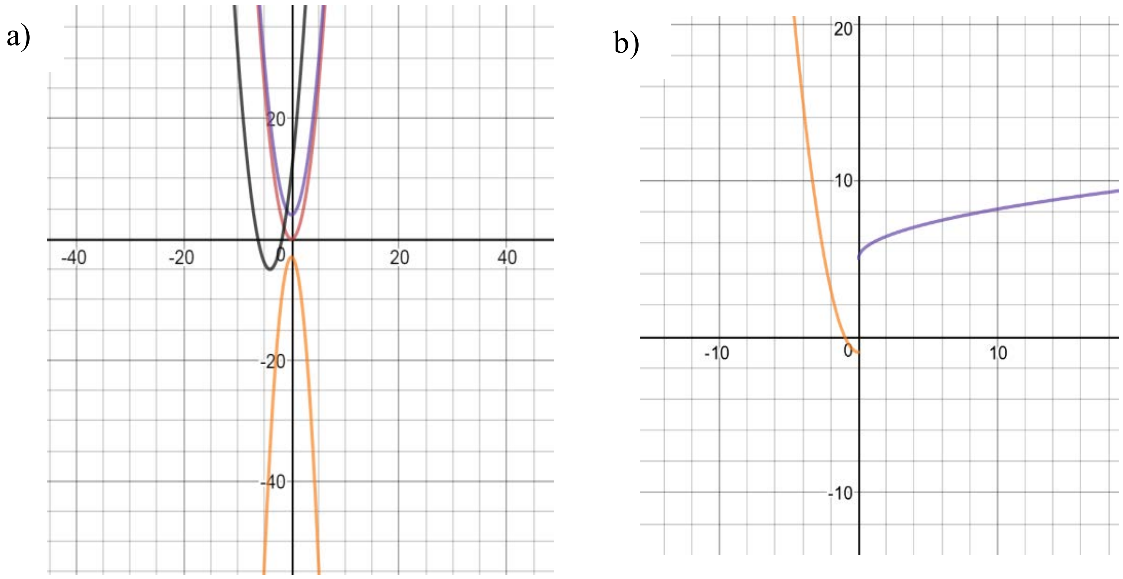


Figure 2. a) Transforming function by Desmos b) Graph of a piecewise-defined function

function and algebraic definition of asymptotes, holes, and end behavior of the function. Figure 3a shows the graph of a rational function. Using Desmos, students can visualize zeros, multiplicity of zeros and turning points of graphs of functions. Figure 3b shows zeros and turning points of polynomial $p(x) = 6x^3 + 9x^2 - 6x$.

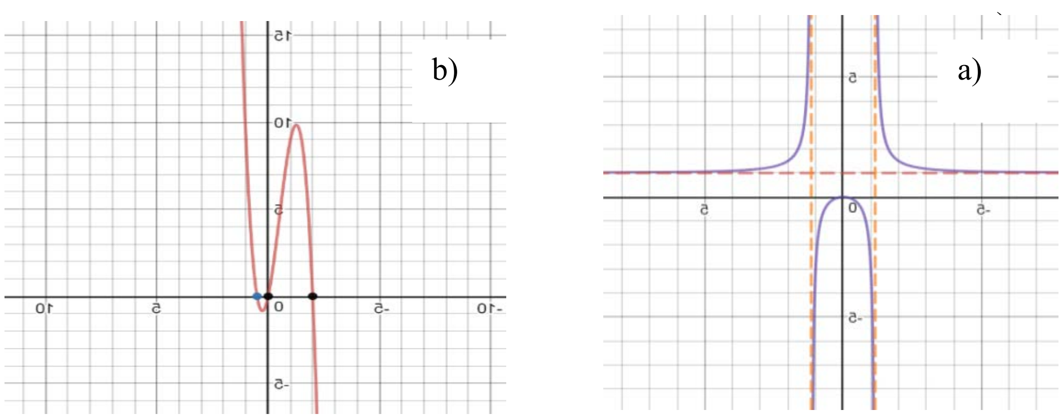


Figure 3. a) Graph of a rational function, b) Zeros of the polynomials

Apps in college algebra: Logarithm is an important concept in mathematics, and anecdotal evidence from instructors has confirmed that students in college algebra have difficulties understanding and using logarithms. Instructors used smartphone applications for teaching logarithms. Instead of worksheet activities, students were asked to complete activities in the pre-specified apps of "Logtrainer" and "Logarithms." Students were also asked to send screenshots of their results to their instructors. The Logtrainer app was used at the first session. The Logtrainer app is a t utorial and practice-based app, containing multiple-choice questions. In this app, questions were mostly about converting logarithms to exponentials. After clicking on one of the answer choices for a question, learners could see the correct answer and a complete explanation of a similar problem. In this app, similar problems are repeated several times, supposedly to help learners understand logarithms using their knowledge of exponentials. A sample problem from this app is shown in Figure 4.

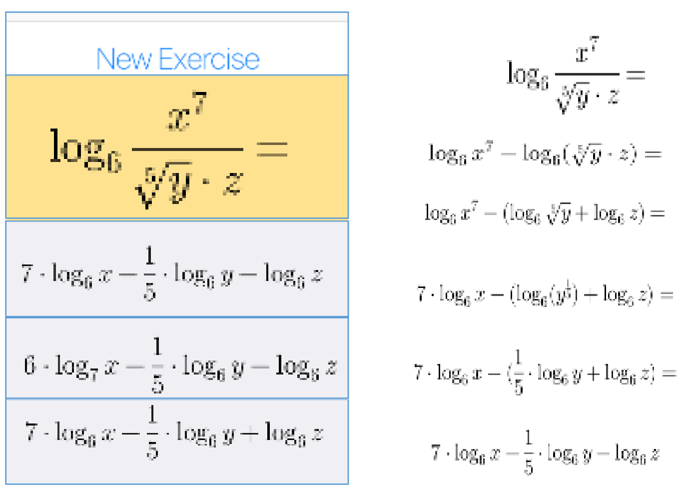


Figure 4. Sample problem from Logarithms app

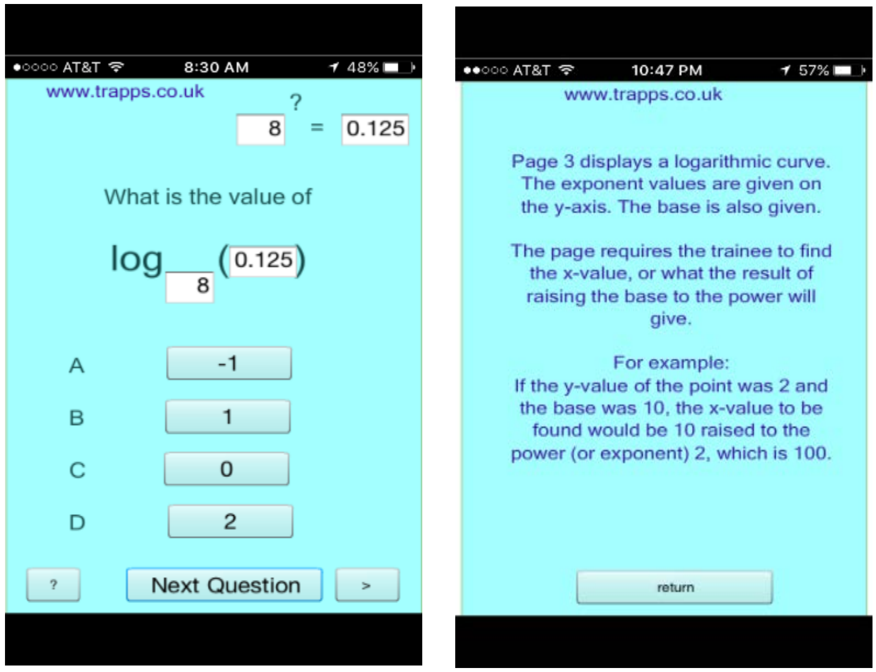


Figure 5. A Sample problem on apps

The Logarithms app consists of four parts: logarithm rules, simplification, expansion, and solving logarithmic equations. Each part includes some multiple-choice problems relating to the subject (examples are shown in Figure 5.). Students were able to see correct answers immediately after picking one of the answer options. The complete solution to each question was provided in this app; therefore, students were able to review their work and correct themselves.

Survey of calculus

After working on college algebra, the principal researcher decided to examine the same questions on the more advanced course, survey of calculus. Using technology might be more efficient for understanding concepts such as limit, derivative, and integral. Therefore, a similar study was conducted to examine how using technology, specifically a graphing calculator, would affect students' understanding of mathematical concepts and organization of written work. In the summer of 2017, two survey of calculus classes with a total of 40 students were chosen. The section that was taught by the principal researcher was considered the control section and the section that was taught by another instructor was considered the treatment section. Students in both sections were allowed to use ACT-approved graphing calculators, such as the TI-83 or TI-84, for doing homework, class activities, and all assessments except review test 1 and the review test 2 for this course. Clicker quizzes as developmental evaluations were included in the PowerPoint slides presented in class. Students worked on their quizzes, exams, and final exams on the MLP website.

3.7 Supplementary information of survey on calculus

In the summer section, survey of calculus was a five-week course meeting five ninety-minute periods a week. Instructors taught calculus following the book *Calculus with Application* (Lial, Greenwell, & Ritchey, 2013). The e-version of the textbook was included on the MLP website. The final grades were evaluated based on students' performance in homework, quizzes, and tests. The grade weightings are shown in Table 5.

Table 5. Final grade weightings

	Percentage of the total
Homework	12%
Quizzes	10%
Test 1	16%
Test 2	16%
Test 3	16%
Clicker quiz and review test	10%
Final test	20%

All the resource materials including the textbook, PowerPoint slides, worksheets, quizzes, and tests were the same for both sections. Instructors used PowerPoint slides for teaching mathematical concepts. They also used Desmos to help students with graphical understanding of the concepts.

3.8 Teaching calculus with the aid of a graphing calculator and Desmos

Instructors used hand-held graphing calculators and Desmos to promote students' understanding of mathematical concepts such as limit, derivative, and definite integral. Examples of teaching with hand-held graphing calculators and Desmos are shown in Figure 6.

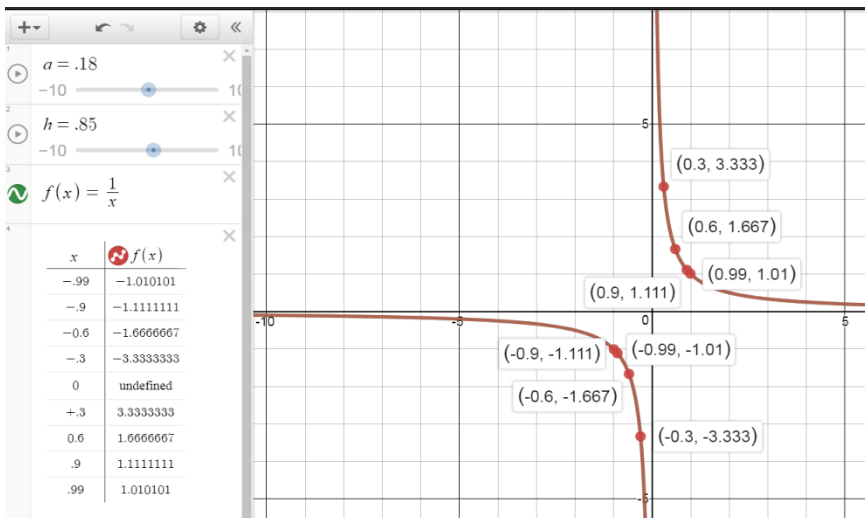


Figure 6. Exploring limits of function by table using Desmos

Survey of calculus with Desmos: Instructors used Desmos for sketching graphs and visualizing discontinuities of a function. Figure 6 shows how Desmos can visualize the right side and left side limit of a piecewise-defined function at x= -1. Students used tables and graphs to investigate limits. An example of investigating a limit problem using a table is shown in Figure 7. Students plug in different values for x and explore its effect on y and observe the behavior of the function around a point of discontinuity (in this example, around x= 0). Other topics taught using technology were local and absolute extrema, graph of the first and second derivative of a function, and relations of derivatives to increasing/decreasing, concave upward/downward. Figure 8a shows the graphs of function $f(x) = 3x^3 - x^2 - 4x$. The graphs of first and second derivatives are also shown. Figure 8b indicates instantaneous rate of change of function $\mathrm{g(x) = ln(x^2 + 1)}$.

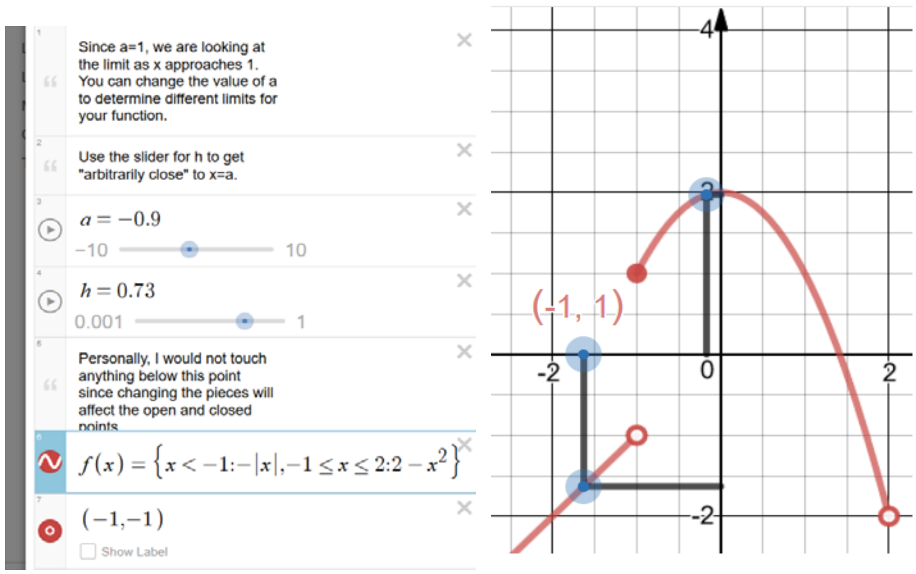


Figure 7. Limit with graph using Desmos

a) b)

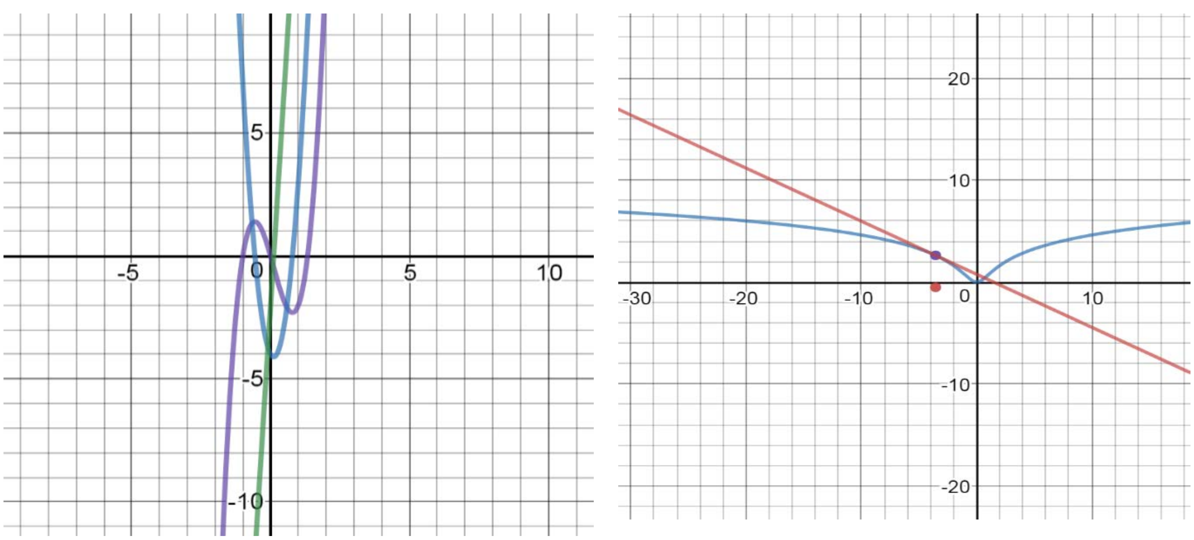


Figure 8. a) Derivatives using Desmos b) Instantaneous rate of change of a function

Instructors used Desmos to visualize boundaries of areas between two curves to help students have a better understanding of the definite integral. Figure 9a illustrates the value of the integral of functions $f(x) = x^2 - 12, g(x) = 11x,\ x = -2, and\ x = 2$, and Figure 9b shows examples of integral for functions $f(x) = x^2, and\ g(x) = x + 20$ by Desmos.

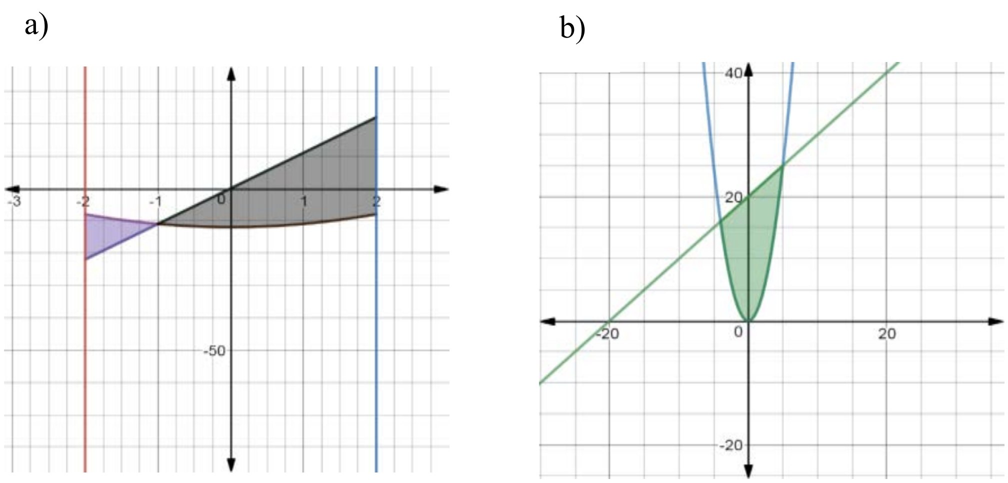


Figure 9. Example of visualizing definite integral using Desmos

TI-84 in a survey of calculus: Instructors taught students how to use a graphing calculator (TI-84) for solving various problems. For example, students could calculate the value of a definite integral with the TI-84. Figure 10a shows an example of solving definite integral

$f(x) = \int_1^5 \left(\frac{2}{(5x+1)^3}\right) dx$ with the TI-84.

Data collection instruments for calculus

Calculus was an MLP course; therefore, students' paper tests were not available. Since the goal of this research was to evaluate students' mathematical understanding and their organization of written work using technology, paper tests were needed. Therefore, like college algebra, in addition to MLP tests, two review tests were designed.

Review test 1 for survey of calculus students: Review test 1 was the first paper test in calculus that covered the concept of limits and derivatives. This test was administered before the midterm exam and had four open-ended questions. Table 6 shows an example of the review test 1 questions.

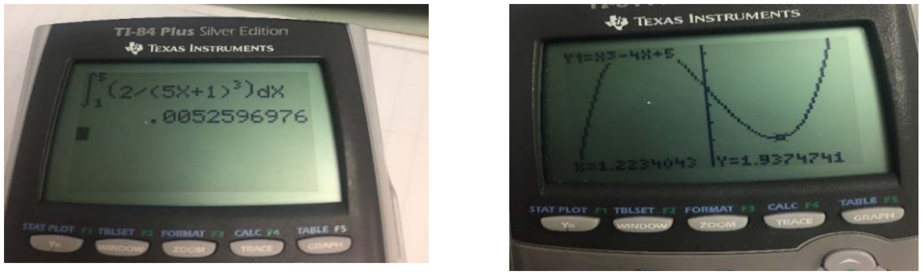


Figure 10. a) Value of definite integral b) Exploring maximum of a function

Table 6. Sample question of review test 1

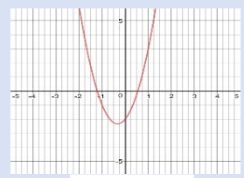
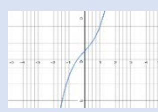
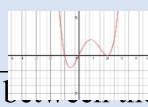
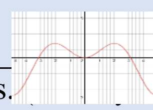
Q1	Find the open interval where the function $f(x) = -2x^3 + 12x^2 + 170x - 6$ is concave upward or concave downward. Find any inflection point.
Q2	a) Let f(x,y) be a function that has (6,7) as a critical point. We determine that $f_{xx}(6,7) = -2\ ,\ f_{yy}(6,7) = 2\ and\ f_{xy}(6,7) = -10$ What D test tells us about the function f? b) Find the partial derivative $\frac{\partial z}{\partial y}$ of $z = 8x + 7xy^3 - 6y^2$
Q3	Find all the local maxima, local minima, and saddle points of the given function: $f(x,y) = 4x^2 + 6xy + 8y^2 + 4x - 20y$

Review test 2 for calculus students: Review test 2 was a comprehensive paper test over the concepts of limits, derivatives, and integrals. This test was given just before the final exam. Some questions from review test 2 are shown in Table 7.

3.9 Primary data collection

Data from multiple sources were collected to address the research questions. Some of these sources were standardized tests that were taken before students began attending college and some of them were tests designed by coordinators that were part of the material for the courses.

Table 7. Sample questions of review test 2.

Q3	a) Sketch a graph for the below function. (10 points) f(x)= $\begin{cases} 3 & x < 0 \\ x^2 + 1 & 0 \leq x \leq 3 \\ 10 & x > 3 \end{cases}$ b) Find all values of x where the function f is discontinuous. (Show all steps of your work). c) For which x value in the interval [0,3] limit of f(x) exists? Why?
Q4	a) Find the average rate of change of the function y over the given points. (10points) y= $\sqrt{(7x + 7)}$ between x=0 and x=6 b) Find the slope and the equation of the tangent line to the graph of the function f(x) at the given value of x. (show all steps of your work). f(x)= $x^2(3 - x)$; x=-2
Q5	The graph of $f'(x)$ is given below. Determine which of the following graphs is an approximate sketch of the f(x). Explain your reason. (5  points) f(x) **a**  b  c  d 
Q7	Find the area b[illegible] following curves. [illegible] ır work). (5 points) X=-4, x=3, y=0, and $y = 2x^2 + 4$

The following are all the data sources of this study.

Data from student attitude surveys: Meetings were arranged with the coordinators and instructors of college algebra and calculus courses a week before college algebra class began in fall 2016, and a week before calculus class began in summer 2017. These meetings were arranged to discuss the research purposes and procedures. Instructors who agreed to participate in the research were asked to send their class rosters to the principal researcher. A four-digit code was assigned to each student in the participating classes. Students used the assigned code to remain anonymous when they completed pre-surveys and post-surveys.

- **Data from ACT scores**: ACT scores of college algebra and calculus students were collected for both control and treatment sections.
- **Data from concept test:** college algebra students' concept test scores were collected for analyzing in college algebra.
- **Data from review test 1**: college algebra students' review test 1 scores were collected for analyzing.
- **Data from review test 2**: college algebra students' review test 2 scores were collected for analyzing
- **Data from review test 1:** calculus students' review test 1 scores were collected for analyzing.
- **Data from review test 2:** calculus students' review test 2 scores were collected for analyzing.

Data from MLP tests 1, 2, 3, and final exam in survey of calculus: In addition to the review test 1 and review test 2 scores, calculus students' MLP tests 1, 2, 3, and final exams were collected and organized in different classifications of the treatment and control section to be prepared for final analyzing.

Data from student interviews: A week before the final exam, after all the written tests were taken, students in both control and treatment sections of college algebra and calculus were interviewed, one-to-one and face-to-face. Students were asked general questions about their attitude toward using technology in mathematics courses and specific questions about the way they used the graphing calculators for solving the problems on written tests. In addition, questions were asked to clarify how students understood concepts such as end behavior of a function, limits, derivatives, and integrals. Some of the interview questions are shown in Table 8.

Table 8. A sample of interview questions

Do you think using graphing calculators can help you to understand the mathematics concepts better? In which area of this course?
Does using a graphing calculator affect the way that you write a solution? How?
What does the $\lim_{x \to 2} f(x) = 7$ mean? Explain in your own words.
How do you use graphing calculators to solve definite integral?

3.10 Primary evaluations

All test questions were grouped into three categories based on Bloom's six levels of learning (Bloom, 1984). These six levels are shown in Figure 11. Questions were categorized based on the levels of learning needed. These categories are a procedure, application, and conceptual questions. Questions that need recalling the concepts were categorized in the procedural category. These questions require limited understanding and mostly need memorizing solution steps. Questions that require the application of learning are categorized accordingly. To solve application questions students, need to know and use a combination of rules and definitions. Conceptual questions are those questions that need a higher level of learning: analyzing, evaluating and creating.

Evaluation of data began by comparing the students' performances in each category between the control and treatment sections. This comparison includes both qualitative and quantitative differences between students' performances in the presence of technology. In addition, students' organizations of written work were evaluated. Since this study was performed on two different courses, college algebra and survey of calculus, there were some specific subjects that were considered for each course separately.

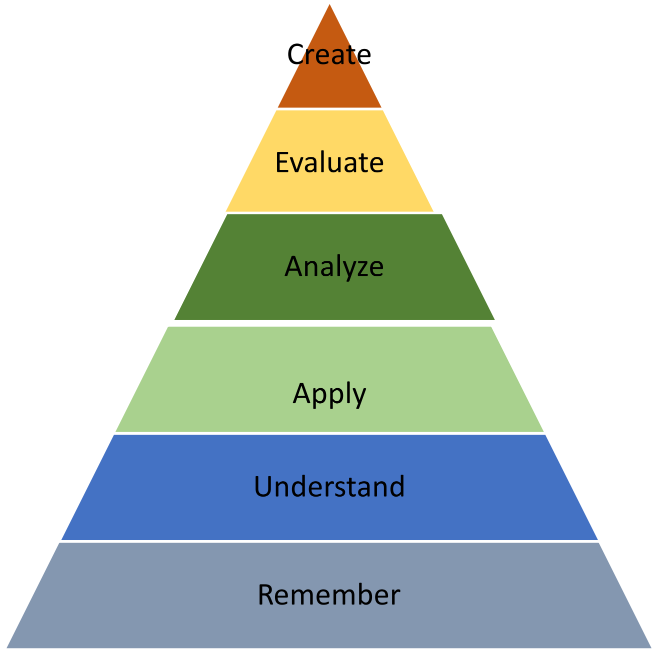


Figure 11. Bloom's six levels of learning

College algebra: In college algebra, the numerical scores of students on the concept test, review test 1 and review test 2 were evaluated and compared between control and treatment sections. Moreover, each question for both treatment and control sections were compared separately to find out how or whether students' performances were affected by using a graphing calculator. Students' written work was another aspect of this study. For this purpose, the difference between students' written work on the concept test, and review test 1 and review test 2 in the control and treatment sections in areas such as organization, accuracy, and length of answers were analyzed. Finally, the students' graphical understanding of the concepts such as zeros, holes, turning points and end behavior of functions were compared.

Survey of calculus: In survey of calculus, the numerical scores of students on MLP exams, review test 1 and review test 2 were evaluated and compared between control and treatment sections. The students' paper tests were analyzed as in college algebra. In addition, students' understanding of several concepts such as derivative and integral were analyzed between control and treatment sections.

This study also investigated how using a graphing calculator affects producing a graph, evaluating definite integral, writing an equation for a tangent line, and understanding of derivatives qualitatively. Finally, students' confidences in solving a problem in the presence of technology were investigated.

3.11 Summary

This chapter explained the primary sources of data collection for investigating the effect of technology on s tudents' performances in college algebra and survey of calculus. For college algebra, three tests were used: concept test, review test 1 and review test 2. For survey of calculus, data were collected from review test 1 and review test 2. Data from student attitude surveys were collected for both college algebra and survey of calculus.

Data from the concept test were used to evaluate students' performances in the control and treatment sections quantitatively. In addition, this test was used to evaluate the students' understanding of mathematics and organization of written work. Review test 1and review test 2 were used to compare each student's performance in the presence of technology and without using technology. Further, review test 1and review test 2 were used to perform a qualitative comparison of each student's understanding of mathematics and organization of written work with and without technology. The same analysis was applied to review test 1and review test 2 of survey of calculus course. In addition, students' attitude surveys were analyzed.

Chapter 4 Data analysis and results

This chapter provides a detailed description of data analysis and interpretation of the results for both college algebra and survey of calculus courses. This study was conducted in fall 2016 on eight college algebra sections with a total of 320 students and in summer 2017 on two survey of calculus classes with a total of 40 students at the University of Arkansas Mathematical Sciences Department. The primary focus of this study was the effect of technology on college algebra and calculus students' understanding. Four college algebra sections were chosen as control groups in which students were not allowed to use graphing calculators on the concept test, and four classes were considered as treatment groups in which students could use a graphing calculator on the concept test. Data were collected from different sources including students pre- and post- attitude surveys, the concept test, review tests 1 and 2, students' ACT scores, and student interviews.

This study was guided by the following research questions:

1. How does the use of technology affect college algebra and calculus students' understandings and performances?
2. What areas of college algebra and calculus are affected more by technology?
3. How does using technology affect the organization of college algebra and calculus students' written work?
4. Does the use of technology positively impact college algebra and calculus students' attitudes toward their mathematics skills?

The stage of data analysis was guided by Corbin and Strauss (2008), Miles, Huberman, and Saldana (2013), and Thompson (2012). Data were analyzed qualitatively and quantitatively to answer research questions. The data sources and result are described in the following sections. How the data sources related to the research questions is shown in the Table 9.

Table 9. Sources that were used to answer each research question

Research question	Data sources
1. How does using technology affect college algebra and calculus students' performances and mathematical understandings?	• Student's grades on review tests 1 and 2 and concept test. • For understanding, data from student interviews and analytic rubric.
2. What areas of college algebra and calculus are affected more by technology?	• Students grades in review test 1 and 2 and concept test • Data from the analytic rubric • Data from interviews
3. How does using technology affect the organization of college algebra and calculus students' written work?	• Data from analytic rubric on review test 1 and review test 2 as well as the concept test. • Data from student interviews.
4 Does the use of technology positively impact college algebra and calculus students' attitudes toward their mathematics skills?	Data from SAS and interviews

This research study was conducted on two different courses. The first part of the chapter is devoted to the analysis of college algebra quantitatively and qualitatively, and, in the second part, a survey of calculus will be discussed.

4.1 Quantitative data analysis and result for college algebra

Quantitative result of review tests 1 & 2: As mentioned before, two written tests, review test 1 (hereafter, RT1) and review test 2 (RT2), were designed by the principal researcher. RT1 was a graphing calculator (hereafter, GC) based test, and students used TI-84 to complete the test. RT1 was completed by students in both the control and treatment sections as a review for the midterm exam. RT2 was a non-graphing calculator-based test that had five open-ended problems. RT2 examined students' understanding of concepts similar to those in RT1 and was given as a review for the final exam in both the control and treatment sections. Students were given 50 minutes to complete each review test and were informed that the result would be a replacement for their lowest quiz grades or lowest wiki project grades. Students papers for RT1 and RT2 were graded by the same rubric and were reviewed by the principal researcher and another instructor of the college algebra. To pair students result a 4-digit SAC was used. Students' numerical grades for RT1 (GC-based) and RT2 (non-GC-based) for all sections were collected. The histograms for RT1 and RT2 grades are shown in Figure 12.

Boxplot and the summary of statistical results for RT1 and RT2 are shown in Figure 13 and the

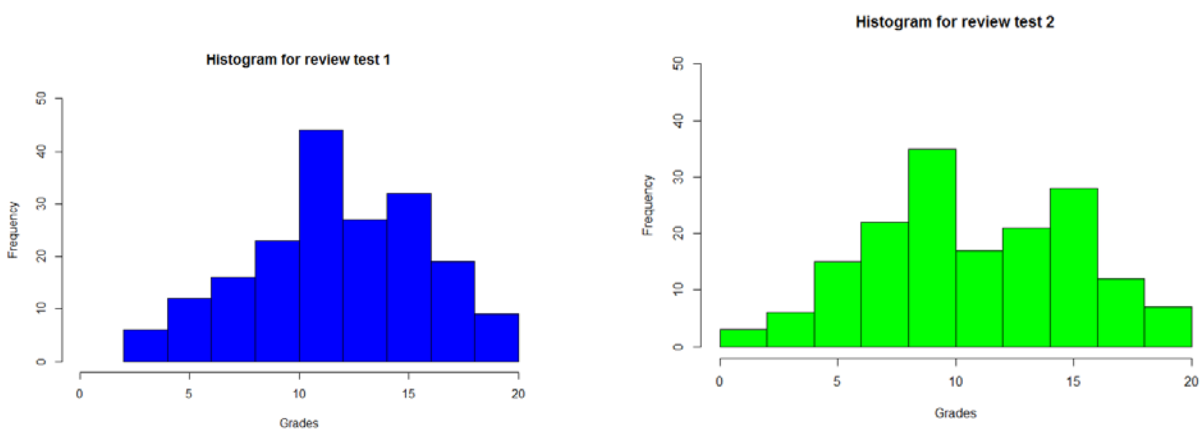


Figure 12. Histogram of the grades of review tests 1 and 2

Tables 10 and 11.

Table 10. Summary of RT1 (GC- based).

Number of the question in RT1	Q1	Q2	Q3	Q4	Q5	Total
Median	1.5	3	4	3	0	11.5
Mean	1.4	2.73	3.78	2.92	1.12	11.95

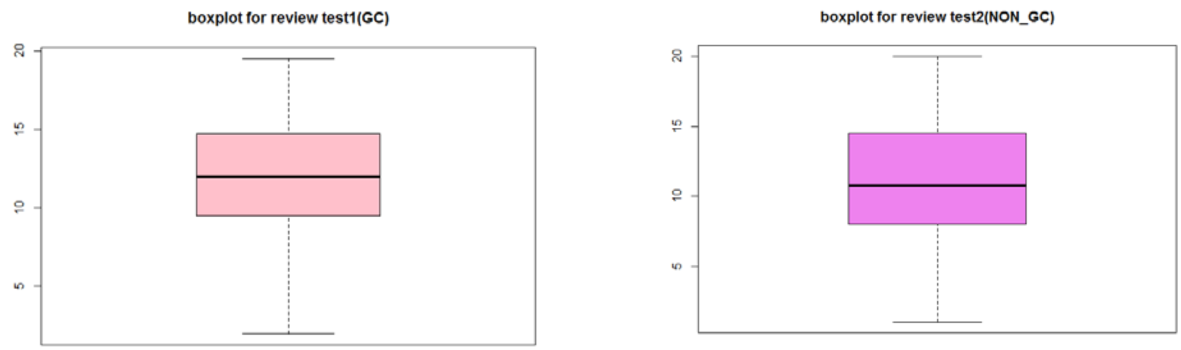


Figure 13. Grade range of review test 1 and 2

Table 11. Summary for RT2 (Non- GC based) for all the sections.

Number of the question in RT2	Q1	Q2	Q3	Q4	Q5a	Total
Median	3	2	2	3.5	1	11.5
Mean	3.06	1.76	2.58	2.33	1.24	10.97

Both boxplots of students' grades in the RT1 and RT2 show that there is no outlier in the data.

Students mean score for RT1 (GC based) was 11.95 while the mean score of RT2 (non-GC based) for these students was 10.97. In other words, students had a better performance when they used a graphing calculator on the similar mathematical concepts compared to when they did not use a graphing calculator. The median for both review test scores is 11.5.

In next step independent two-sample t-test with 95 percent confidence intervals on the result of RT1(GC) and RT2 (non-GC) were applied and shown in Table 12.

Table 12. Two sample t-test over a mean of RT1 and RT2.

Summary of t-test over RT1&RT2	Mean of RT1 N=170	Mean of RT2 N=190	df	P-value
Overall grades	11.95	10.98	340.15	.00245

This result gives additional evidence to propose that the GC (TI-84) used by a student in RT1 had a significant impact on student performance compared to the performance of the non-GC based test. In other words, p-value = .00245 < α= .05 indicates that there is a significant difference between overall student grades on RT1 and RT2 with a confidence interval of 95 percent. The mathematical concepts were similar between question number 3 of RT1 and question number 3 of RT2. Question number 4 of RT1 and question number 1 of RT2 had the same concepts. Therefore, two-sample t-test over the mean of these questions was applied to find if there is any significant difference between student performance on the common concepts with and without technology. The result of the t-test is shown in the Table 13.

Table 13. Two sample t-test over a mean of Q3, 4, of RT1 with Q3, 1 of RT2 respectively.

Number of the questions in RT1	Mean of the questions in RT1 (N=170)	Number of the questions in RT2 (N=190)	Mean of the questions in RT2	df	p-value
Q3	3.78	Q3	2.58	329.74	0.00000209
Q4	3.50	Q1	3.06	350.41	0.0187

There is a significant difference between students' performance in Q3 and and Q4 of RT1 and Q3 and Q1 of RT2 with a significance level of α= .05 and confidence interval of 95 percent. The p-value is 0.0187, which is less than significance level α= .05. This result gives enough evidence to support alternative hypotheses. Students showed a better performance on s imilar concepts when they used graphing calculator compare to their performance without a graphing calculator.

4.2 Quantitative results of the concept tests

As a reminder for the reader, the concept test was the only paper test normally administered in college algebra. There were seven open-ended questions, designed by the coordinator of the course. The topics on the concept test were functions and combinations of functions, transformation of a graph, linear, quadratic and polynomial equations, exponential and logarithms functions, linear and nonlinear system of equations, and inequalities. Students completed the concept test two weeks before the final exam. Four college algebra classes were considered as treatment groups in which students used the TI84 GC to complete the concept test and four control sections in which students did not use the TI84 GC to complete the concept test. Student papers for the concept test were graded by all instructors of the college algebra courses (each instructor one question) with the same rubric designed by the coordinator of college algebra. There were 131 students in the control sections and 135 students in the treatment sections who completed the concept test. The first phase of statistical analysis of the concept test was started by finding a statistical summary for control and treatment groups and boxplot for showing the distribution of data and identifying any outliers which is illustrated in Figure 14.

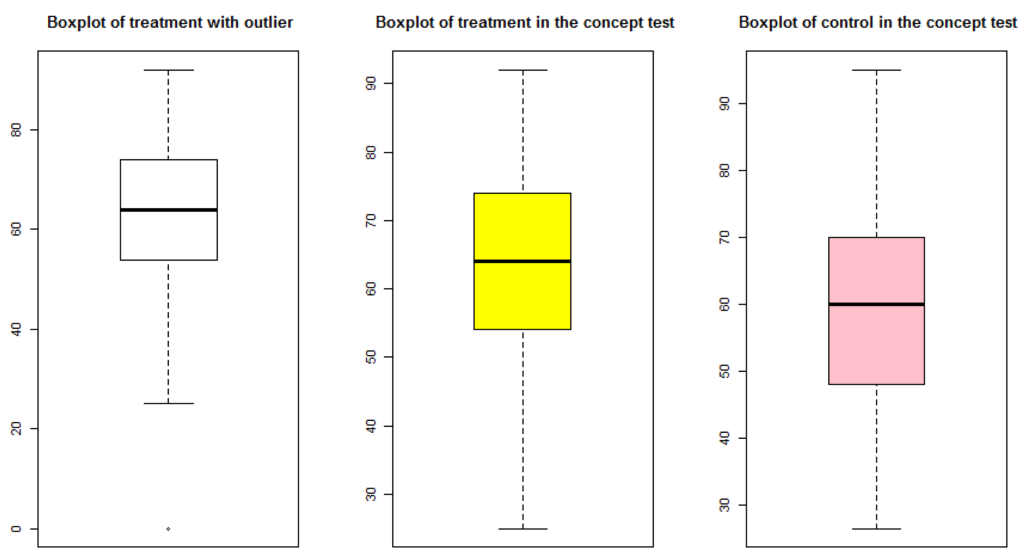


Figure 14. Grade range of the concept tests

The first boxplot corresponds to students' grades on the concept test from treatment sections and as it shows in the plot there is an outlier (a zero). The second plot represents students' grades on the treatment sections without outlier (outlier was removed) and the third plot belongs to students' grades of the concept test in the control sections. Here is the statistical summary of the students' grades on the concept test for control sections and treatment sections.

Statistical summary for control sections. N= 131

Min.	1st Qu	Median	Mean	3rd Qu	Max
26.50	48.00	60.00	59.97	70.00	95.00

Statistical summary for treatment sections. N= 135

Min.	1st Qu.	Median	Mean	3rd Qu	Max
25.00	54.00	64.00	63.21	74.00	92.00

The result of statistical summary between control and treatment sections on the concept test shows that students in treatment sections have a higher mean (63.21) and median (64) scores compared to mean (59.97) and median (60) of the control sections. Histograms of the grades of control and treatment groups are shown in Figure 15. In the next step, the researcher conducted a two-sample t-test to compare the concept test scores between the control and the treatment groups. The result of the t-test is shown in the Table 14.

Table 14. Two sample t-test over the mean of the concept test.

t-test over the concept test between control and treatment sections	Mean of the concept test for control sections N=131	Mean of the concepts test for treatment sections N=135	df	P-value
	59.97	63.21	259.13	0.06783

With a significance level of α= 0.1 and 90 percent confidence interval, we reject null hypotheses which says there is no difference between the mean of the control group and mean of treatment group and accept alternative hypotheses which says "there are true differences between the mean of students' grades in the control and treatment groups."

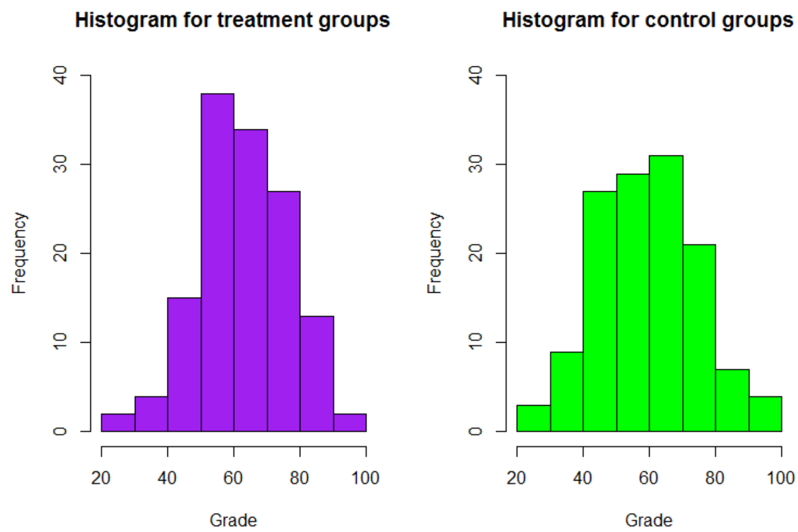


Figure 15. Histograms of concept test grades for treatment and control groups.

In other words, students who used a GC on the concept test performed better compared to the students who did not use a GC on the same test with a 90 percent confidence interval and a significance level of $\alpha = 0.1$.

4.3 Qualitative data analysis and result for college algebra

The result of RT1 and RT2 by the analytic rubric: To find if the use of technology affects students' understanding and organization of written work we used a qualitative rubric. After researching similar studies and reviewing literature the following rubric was designed. This rubric was used to investigate whether the use of GC influences students' performance on the following skills: logical reasoning; organization including intellectual order, written order and use of symbol and notation; and use of a graph. The rubric was designed and applied to the papers of several college algebra students after it was revised multiple times. This rubric is shown in Table 15.

Table 15. Qualitative Rubric (QR) for qualitative analyses.

Level	Logical reasoning	Organization including Intellectual order, written order, correct use of symbols and notation.	Use of graph
0	No relevant argument to support conclusion **or** no relevant conclusion	No relevant organization	No relevant evidence of use of graph
1	Some reasoning to support an incomplete solution **or** poor reasoning to support correct solution or strong reason for the poor conclusion	Some organizations for incomplete or incorrect solutions or poor organization for a correct solution or well organization for the poor conclusion	use of a graph for an incomplete or incorrect solution
2	Incomplete reasons to support the complete correct solution **or** complete reason to support correct (or incorrect) incomplete solution	well Organized description, argument, and notation for correct (or incorrect) incomplete conclusion or incomplete argument, description, and notation for complete correct (or incorrect) conclusion	Relevant use of graph in parts
3	Complete logic argument to support complete correct conclusion	Well organized description, argument, and complete correct notation for correct conclusion	complete use of graph for complete correct conclusion

The population size (students who have both RT1 and RT2) was 125, which was too large to evaluate by the rubric. To save time, and have a precise analysis, a sample of students was selected. The following conditions were considered for data sampling. All students who have only one of the review tests grades were removed. Our goal was to compare students with themselves to see if the use of technology influences their answer. Students belong to different categories including good, medium and poor, and researcher wanted to analyze students' papers in all these categories, but the numbers of students in each of these categories were not equal; therefore, the researcher used proportional stratified random sampling.

4.4 Proportional stratified random sampling

Students were stratified into three groups (good grade, average grade, and low grade) but the number of students was not equal, which means that if an equal number of students from each stratum would be selected it would bias the result. We have 3 strata with 21, 70, and 30 population sizes, and we chose a sampling fraction of 1/5 of population size in each stratum which means 4, 14, and 7 population size respectively. Table 16 shows the information about sampling.

Table 16. Proportional stratified random sampling.

Students grade /100	Students grade /20	Qualitative grade /4	Total number of students in each stratum for N_GC test	Final sample size
80 -100	16-20	A, B	N_1=21	n_1=4
60 -79.99	12-15.9	C, D	N_2=70	n_2=14
0-59.99	0-11.9	F	N_3=33	n_3=7

After stratified random sampling, the researcher collected the students' papers in RT1 and RT2 for each class into separate folders. Some questions were common between RT1 and RT2. By comparing the equivalent problem solved with the same student with and without technology, we might have a more precise view about the influence of technology on students learning mathematics. Therefore, the rubric was applied to questions with common concepts in RT1 and RT2. The questions were listed in the Table 17.

Each question has several parts and they were picked from different levels of learning. The solution rubric for the selected question from the RT1 and RT2 are shown in the following rubric.

Table 17. Questions of RT1 and RT2.

Question numbers in RT 1	Question numbers in RT2
3a	3c(z) (first part)
3b	3c(M)
3c	3b
4d	1e

Questions

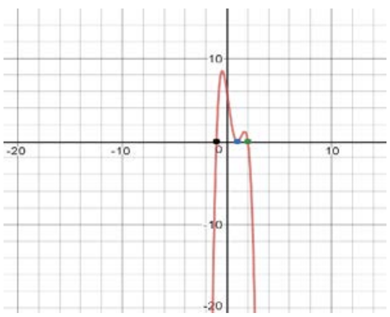
Q3. Graph the polynomial $\boldsymbol{p(x) = -3x^4 + 9x^3 - 3x^2 - 9x + 6}$ and answer the following:

a. Label the zeros of the polynomial on the graph (with exact values).
b. Label the zeros of the polynomial on the graph (with exact values).
c. Describe the end behavior

Q4. Let $\boldsymbol{f(x) = \frac{(x^2-4)}{(x^2+5x+6)}}$ be a rational function. Find the following:

e. Domain

Solutions	**Point allocated**
Q3_a: A= (-1, 0), B= (1, 0), C= (2, 0)	$3: \begin{cases} 2: \textit{corrct and exact x values} \\ 1: \textit{labaling on the graph} \end{cases}$



Q3_b: A= (-.443, 8.5) B= (1.693, 1.191) C= (1, 0)	$3: \begin{cases} \text{1: correct and exact x valus} \\ \text{1: correct and exact y value} \\ \text{1: labaling on the graph} \end{cases}$
Q3_solution1: As x → $\pm\infty$, P(x) → $-\infty$ or as x goes toward infinity from the right and left, p(x) fell. Solution 2: since polynomial has degree 4 and leading confection is positive therefore p(x) goes toward negative infinity as x goes toward either positive or negative infinity.	$3: \begin{cases} \text{2: correct explanation and reasoning} \\ \text{1: correct notation} \end{cases}$
Q4_e: Solution 1: Domain= $(-\infty, -3)\cup(-3, -2)\cup(-2, \infty)$ $f(x) = \frac{(x-2)(x+2)}{(x+3)(x+2)}$	$3: \begin{cases} \text{1.5: finding VA and hole} \\ \text{1.5: corect answer and notation} \end{cases}$

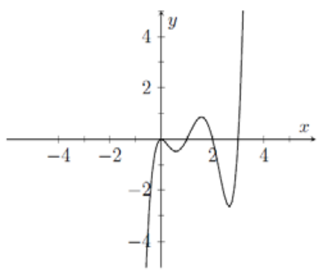
First find hole if it exists: x= -2,
Then find VA, in this case, x=-3, then remove hole and VA from all real number.
Or by using graph

Table: Solution Rubric _RT2 (Non-GC)

Questions

Q1. Let $f(x) = \frac{2x^2+12x-14}{x^2+2x-15}$
be a rational function. Answer the following questions and explain how you find the answer for each part.
e) domain

Q3. Use the graph of the polynomial at the right to answer the following:



b) What is the sign of the leading coefficient? Why?

c) First part) What are the zeros for this polynomial? (second part of c) What is local max/min for this polynomial?

Solutions	**Point allocated**
Q1: f(x)= $\frac{2(x+7)(x-1)}{(x+5)(x-3)}$ VA: x= -5 x=3 Domain: (-∞, -5)∪ (-5,3)∪(3, ∞)	$3:\begin{cases} 1.5: finding\ VA\ and\ hole \\ 1.5: corect\ answer\ and\ notation \end{cases}$
Q3_b: This graph is a polynomial with odd degree. 3: The end behaver of polynomial as $x \to +\infty$ p(x)→+∞ and as $x \to -\infty$ p(x)→-∞ which means the leading coefficient is positive.	$\begin{cases} 2: correct\ explanation\ and\ reasoning \\ 1: correct\ notation \end{cases}$

Q3_c_ first part: (0,0), (1,0), (2,0), (3,0)
Or x= 0, x=1, x=2, x=3

$$3:\begin{cases} 2: corrct\ and\ exact\ x\ values \\ 1: corrcet\ notation \end{cases}$$

Q3_c_secod part: Approximately,

local max: (0,0), (1.5, 1)
Local min: (.5, -.5), (2.5, -2.5)

$$3:\begin{cases} 1: correct\ x\ valus \\ 1: correct\ \ y\ value \\ 1: labaling\ on\ the\ graph \end{cases}$$

4.5 Validating the analytic rubric

For re-labeling and validating the qualitative rubric, the analytic rubric was applied to 20 papers and the rubric was revised based on different types of solutions that appeared. In addition, the researcher asked another colleague to use the rubric for some papers in RT1 and RT2 and asked him to explain how he scored. He mentioned that he did not have difficulty with scoring organization and the use of a graph, and it covers all possible type of students' answers. For scoring level zero and three of organization, there was not any ambiguity for him as well. He mentioned that when he wanted to score logical reasoning if students' logical reasoning was not either irrelevant or complete he had difficulty to score between level one and two. To remove the ambiguity between logical reasoning levels one and two researchers had to add more conditions and revise the rubric. If the answer is completely correct with incomplete reasoning it belongs to level 2, or if complete reasoning but incomplete solution (more than half of the answer) again it belongs to level 2.

4.6 Guideline and memo for using the analytic rubric over RT1

To ensure that the analytic rubric gives the same output independent of the person who is applying it, a sample of ten students paper were chosen. The rubric was applied by two different persons and was adjusted accordingly. In addition, the following scoring guideline was considered to ensure that the rubric is reliable.

Example one:

a. Since this student has a comprehensive logical argument to support complete correct solution the logical reasoning score is 3. An example is shown in Figure 16.

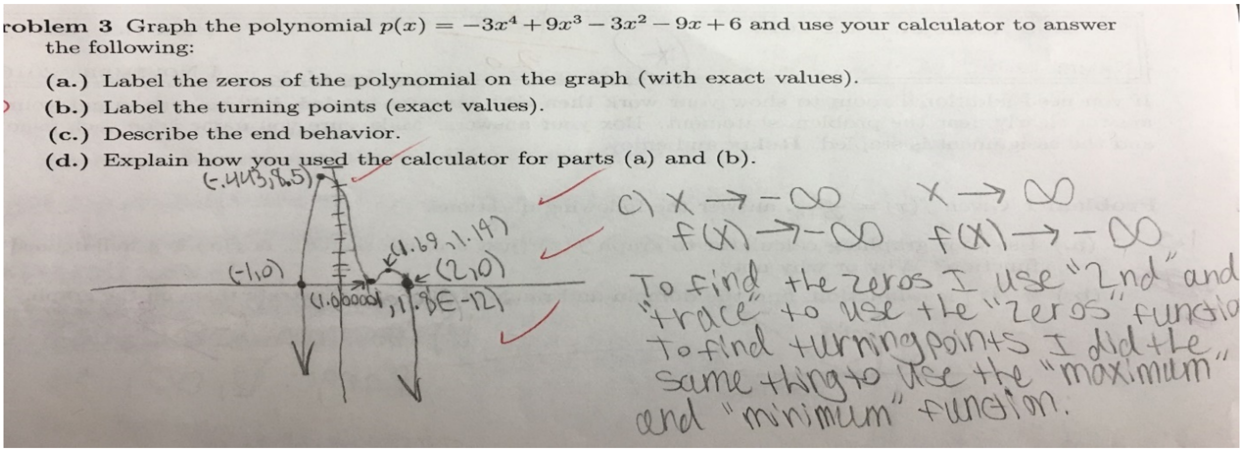


Figure 16. Example students work that all categories all complete.

b. This student has a well-organized argument and complete correct notation for a complete solution, the organization level is 3.

c. This student has a complete use of a graph for producing a complete correct conclusion, so the use of graph level is 3 as well.

Example two:

a. An incomplete logical argument to support complete correct solution yields logical reasoning score of 2. An example is shown in Figure 17.

b. An incomplete argument and notation for a completely correct solution yields organization level of 2.

c. This student did not draw a graph, nor did he/she mention use of a graphing calculator for making the conclusion, so there is no relevant evidence of the use of a graph for solving part c. Thus, the level of use of graph is 0.

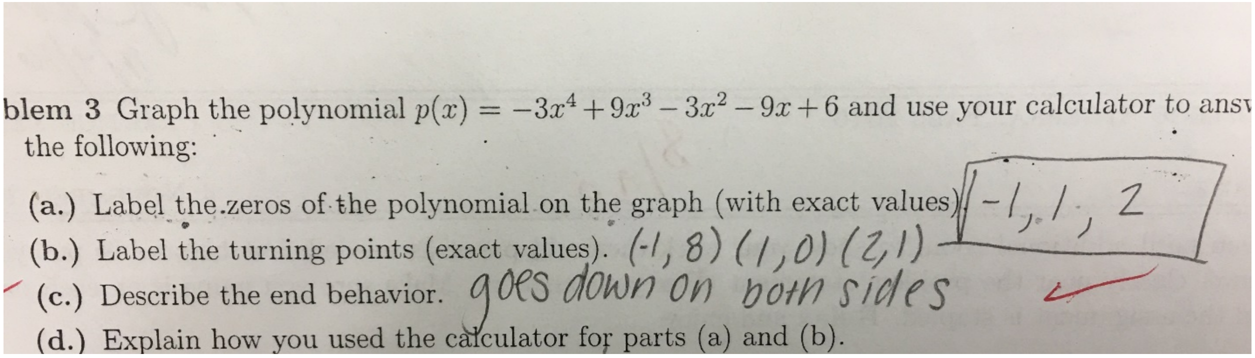


Figure 17. An example of level two solution.

Example three:

a. A poor reasoning to support an incomplete solution; therefore, the logical reasoning level is 1. An example is shown in the Figure 18.

b. Some organization for the incomplete conclusion, gives the organization level 1.

c. Use of graph to produce an incomplete solution, so the level of use of graph is 1.

4.7 Memos

Memo 1: Logical reasoning deals with a logical argument to support a conclusion, but organization deals with an understanding of the concepts (based on written work) as well as correct use of notations and symbols. For example, for question 3(a) one may find the correct values of zeros of polynomial but not write the zeros as ordered pairs or x equal forms; therefore, logical reasoning level would be 3 but organization would be 2.

Memo 2: There are some cases where the scores for all the levels are equal. But it does not mean that a student who has a poor reasoning level also has a poor use of graph or organization level.

Memo 3: Because of repetition of the following case between student answers for question number 4(d) this memo was made. The question asked to find the domain. Students usually find asymptotes and holes, but to show domain, students mostly removed vertical asymptote from real

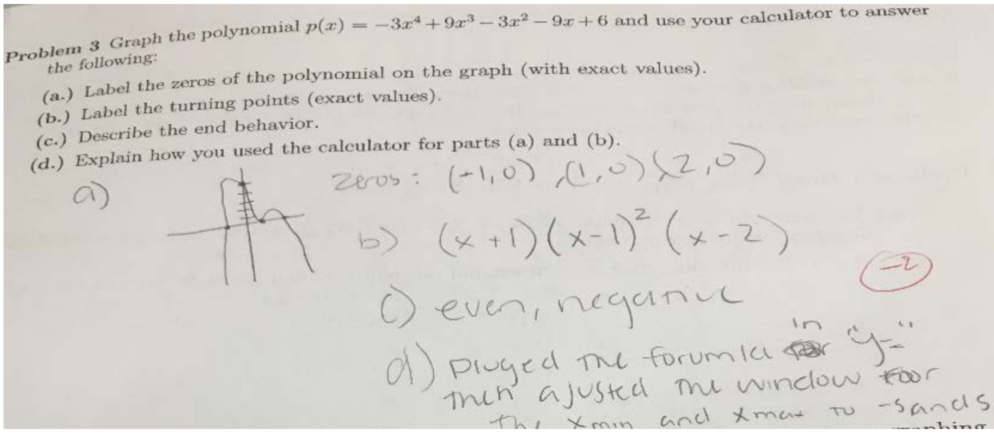


Figure 18. An example of level 1 solution.

numbers but did not remove hole. In this situation, the logical reasoning level is 2 and it satisfied complete reason to support an incomplete solution. Organization level is 2 because it satisfied well-organized argument and notation for a correct incomplete solution. An example is shown in the Figure 19.

4.8 The result of the analytic rubric applied to RT1 and RT2

In the next phase, the analytic rubric was applied to the selected samples. The results of scoring

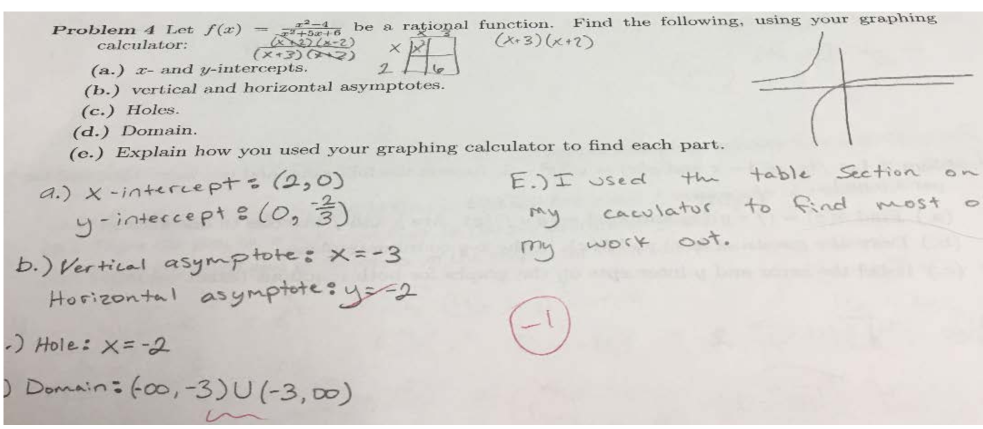


Figure 19. An example of student work for finding domain

RT1 and RT2 by the analytic rubric were tabulated, which can be seen in Table 18 and 19. SAC stands for students' assigned code, LR stands for logical reasoning, or stands for organization and UG stands for use of a graph.

Table 18. RT1 by the analytic rubric.

SAC	LR_3a	OR_3a	Ug_3a	LR_3b	Or_3b	UG_3b	LR_3c	OR_3c	UG_3c	LR_4d	OR_4d	UG_4d
3209	3	3	3	3	3	3	2	2	3	1	1	0
3234	3	2	3	3	3	3	2	2	3	2	2	2
3231	3	2	3	2	2	3	2	2	3	2	2	3
3208	2	2	3	3	3	3	1	1	2	1	1	1
3223	3	3	3	3	3	3	2	2	3	2	2	0
3201	1	1	3	3	3	3	2	2	1	2	2	0
2433	3	3	3	3	3	3	2	2	3	2	2	3
2404	3	3	3	3	3	3	2	2	3	2	2	3
2417	3	2	2	2	2	1	2	2	3	3	2	3
2413	0	1	1	0	0	0	2	2	3	1	1	1
1420	3	3	3	3	3	3	2	2	3	3	2	3
1435	3	3	3	2	2	3	2	2	3	1	2	0
1428	2	2	3	1	1	1	2	2	3	1	1	3
1406	3	3	3	3	3	3	2	2	3	2	2	3
1407	3	3	3	3	3	3	2	2	3	2	2	2
1403	2	2	2	2	2	1	2	2	3	2	2	1
1528	2	2	3	3	3	3	2	2	3	1	2	1
1513	3	3	3	0	1	0	1	1	2	1	2	1
1503	3	3	3	1	1	3	1	1	1	2	2	3
1624	3	3	3	1	2	2	1	1	0	2	2	0
1629	3	2	3	3	3	3	3	2	3	2	2	1
1610	1	1	2	1	1	1	3	2	0	3	3	3
9017	3	2	3	0	0	0	1	1	3	2	2	2
9033	3	3	3	1	1	1	2	1	1	1	1	1
9009	3	2	3	1	1	1	2	2	3	1	1	0

4.9 The result of t-test over mean score obtained using the analytic rubric

As mentioned previously, some common concepts between RT1 and RT2 were selected for a sample of student papers. The qualitative rubric was applied for these papers for both RT1 and RT2. In the next phase, the two-sample t-test over the mean scores of the qualitative rubric grades was applied to find if the use of graphing calculator has a significant effect on students' logical reasoning, organization including intellectual order, written order and correct use of symbol and notation, and use of a graph. A t-test with $\alpha = 0.05$ and 95 percent confidence interval was used to compare the following questions from RT1 versus RT2 as well as total logical reasoning, total organization and total use of graph, which includes the mean scores archived by qualitative rubric of these skills for all questions in RT1 versus RT2.

Table 19. RT2 by the analytic rubric.

SAC	LR_1e	OR_1e	UG_1e	LR_3b	OR_3b	UG_3b	LR_3z	OR_3z	UG_3z	LR_3m	OR_3m	UG_3m
3209	3	2	0	1	1	2	3	2	3	1	1	2
3234	3	3	0	1	1	1	3	2	3	1	1	1
3231	2	2	0	1	1	1	2	2	3	0	0	0
3208	1	1	0	3	2	3	3	3	3	3	3	3
3223	3	3	0	1	1	1	3	2	3	0	0	0
3201	2	2	0	1	1	2	3	2	3	1	1	1
2433	3	3	0	3	3	3	3	2	3	2	2	3
2404	1	1	0	1	1	1	3	2	3	2	2	1
2417	0	1	0	1	0	0	3	2	3	1	1	2
2413	3	3	0	1	1	0	3	2	3	0	0	0
1420	3	3	0	2	2	2	3	2	3	1	1	1
1435	1	1	0	2	2	3	3	3	3	1	1	1
1428	3	3	0	0	0	1	3	3	3	1	1	2
1406	3	3	3	1	2	1	2	1	2	0	0	0
1407	3	3	0	1	0	0	3	3	3	1	1	1
1403	3	3	0	1	1	1	3	1	3	0	0	0
1528	3	3	0	1	1	1	3	2	3	0	0	0
1513	3	3	0	2	2	3	3	2	3	1	1	1
1503	1	1	0	0	0	0	2	1	3	0	0	0
1624	1	1	0	1	0	0	3	3	3	0	0	0
1610	1	0	0	1	1	3	3	2	3	0	0	0
9017	0	0	0	0	0	0	1	1	1	0	0	0
9033	2	2	0	2	1	2	3	3	3	0	0	0
9009	0	0	0	1	1	1	3	2	3	2	1	2

Table 20. t-test over mean scores of the qualitative rubric on review test.

t-test over mean scores of different categories of the qualitative rubric	df	P-value
Logical Reasoning	47.828	0.01914
Organization	46.445	0.0001978
Use of graph	45.734	9.113e-08

Overall, there is a significant difference in logical reasoning, organization, and use of a graph of students' when they used a graphing calculator (RT1) compared to when the same students did not use a graphing calculator (RT2). Significance level is $\alpha = 0.05$ and p-values are less than .05 for all categories of the qualitative rubric. Therefore, with 95 percent confidence interval the null hypotheses (the mean scores of the qualitative rubric in review RT1 and RT2 are equal) is rejected and alternative hypotheses (the mean scores of qualitative rubrics in RT1 and RT2 are not equal) is accepted. In addition, the researcher conducted a two-sample t-test over the mean score by qualitative rubric for a similar concept in RT1 and RT2. The results of t-test show that there are significant differences in students' logical reasoning, organization, and use of graph skills in question 3b versus 3m, question 3z versus 3b and use of a graph, and organization level of question 4d versus 1e. But there is not a significant difference between logical reasoning, organization, and use of a graph for question 3a of RT1 versus 3z of RT2.

4.10 Estimating the population mean and confidence interval of the analytic rubric

The total number of students in the population is N=125. We divided the population into three strata which contain N_1=21, N_2=70 and N_3=33 and one outlier observations. Samples of n_1=4, n_2=14 and n_3=7 students are chosen from stratum one, two and three respectively. Let $\overline{y}_h$ denote the mean of each stratum (where h runs from 1 to 3). Therefore, one can write them as:

$$\overline{y}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} y_{hi} \qquad \text{Equation 1}$$

where y_{hi} is each observation in the stratum h.

The finite-population variance from each stratum is:

$$\sigma_h^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}(y_{hi} - \mu_h)^2 \qquad \text{Equation 2}$$

The sample variance from stratum h can be written as:

$$s_h^2 = \frac{1}{n_h - 1}\sum_{i=1}^{n_h}(y_{hi} - \overline{y}_h)^2 \qquad \text{Equation 3}$$

With the stratified sampling method, one can find the unbiased estimator of the population mean as

$$\overline{y}_{st} = \frac{1}{N}\sum_{h=1}^{L} N_h \overline{y}_h \qquad \text{Equation 4}$$

where L =3 is the number of strata.

The variance of the stratified population mean is

$$var(\overline{y}_{st}) = \sum_{h=1}^{L} \left(\frac{N_h}{N}\right)^2 \left(\frac{N_h - n_h}{N_h}\right) \frac{\sigma_h^2}{n_h} \qquad \text{Equation 5}$$

Therefore, the unbiased estimate of the variance of the stratified population mean is

$$v\widehat{ar}(\overline{y}_{st}) = \sum_{h=1}^{L} \left(\frac{N_h}{N}\right)^2 \left(\frac{N_h - n_h}{N_h}\right) \frac{s_h^2}{n_h} \qquad \text{Equation 6}$$

To find the confidence interval one can use the t distribution with a modified degree of freedom. The 100 (1-α) % confidence interval of the estimator of the mean population will be

$$\overline{y}_{st} = \hat{\overline{y}}_{st} \pm t_{\alpha/2}\sqrt{v\widehat{ar}(\hat{\overline{y}}_{st})} \qquad \text{Equation 7}$$

The degrees of freedom of t distribution can be estimated as

$$d = \frac{\left(\sum_{h=1}^{L} a_h s_h^2\right)^2}{\left(\sum_{h=1}^{L} \frac{(a_h s_h^2)^2}{n_h - 1}\right)} \qquad \text{Equation 8}$$

where a_h is defined as:

$$a_h = \frac{N_h(N_h - n_h)}{n_h} \qquad \text{Equation 9}$$

With the mentioned formulation of stratified sampling the estimator of the mean and variance of the population, and confidence interval of the mean of three features of the students RT1 and RT2 were tabulated and can be seen in Tables 21 and 22 respectively. These three features are logical reasoning, organization of written work and the use of a graph.

Table 21. RT1.

	Estimated mean	Estimated standard error	Stratified degrees of freedom	The confidence interval of the mean of the population
Logical Reasoning	8.20	0.34	20	(7.62, 8.79)
Organization of written work	8.00	0.28	20	(7.52, 8.48)
Use of graph	8.94	0.31	20	(8.40, 9.47)

Table 22. RT2.

	Estimated mean	Estimated variance	Stratified degrees of freedom	The confidence interval of the mean of the population
Logical Reasoning	6.7972350	0.35	20	(6.19, 7.4)
Organization of written work	5.8306452	0.32	20	(5.27, 6.4)
Use of graph	5.1932604	0.30	20	(4.68, 5.7)

Interpretation of confidence interval: Based on the results shown in tables 21 and 22 in all cases of logical reasoning, organization of written work and use of graph the estimated mean of students' skill is larger in the GC test. This also can be seen in the 95% confidence interval of the mean which is shown in the last columns of the two tables. For example, the confidence interval of the mean of the logical reasoning skills is (7.62, 8.79) for GC test and (6.19, 7.4) for the none-GC test. These two 95% confidence interval do not intersect, which shows that the mean of the logical reasoning skills of the students increases when they use GC on the test. The same analogy is applicable to the organization of written work and the use of graph skills where confidence intervals of the mean of the skills scores increase when students use GC.

4.11 A detailed qualitative description and discussion of students' review tests

Application of the analytic rubric to the review tests shows that students have better logical reasoning, organization, and use of graph skills when they used a GC compared to not using a GC for similar problems. Below is a discussion of why and how GC might influence students' skills. Some sample papers in RT1 and RT2 will be used in the discussion. Question 3 with multiple subsets and 4 in RT1 as well as question 1-part e and 3 with multiple subsets in RT2 will be analyzed. The following themes will be considered for analyzing students' written work. These themes were adapted from the analytic rubric. The researcher used a mixed qualitative-quantitative method (Creswell, 2002) in which both quantitative data such as test scores and qualitative data such as interviews and students 'written work collected, analyzed and reported.

Question 3a, b, and c: Zeroes of polynomial, turning point, and end behavior: Question 3 asked students to find and explain zeroes, turning point and end behavior of a polynomial. Students were also asked to explain how they produced their answers. The RT1and RT2 were designed with common concepts on the two tests to give the possibility to compare students' performance in

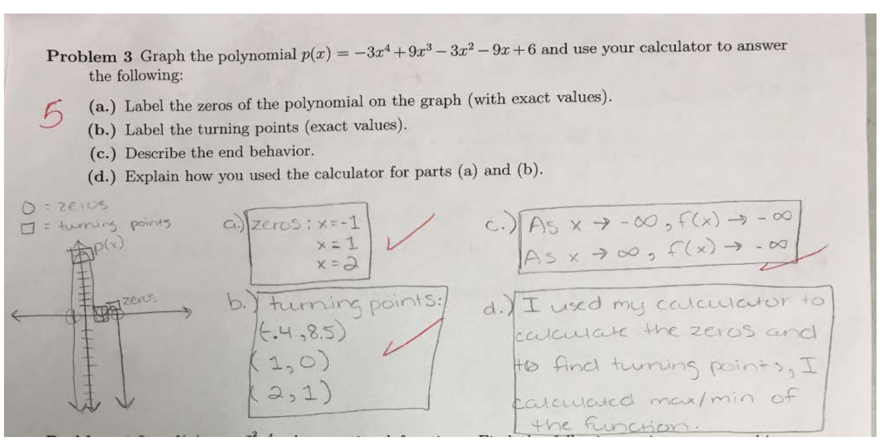


Figure 20.An example of a student solution with using GC

presence of technology and without technology. A sample of students' written work on question 3 in RT1 and RT2 are shown in figures 20 and 21, and the details of the differences between their solutions with and without technology will be discussed.

Case 1:

1a. Sara's written work on RT1 (GC) question 3

1b. Sara's written work on RT2 (Non-GC) question 3

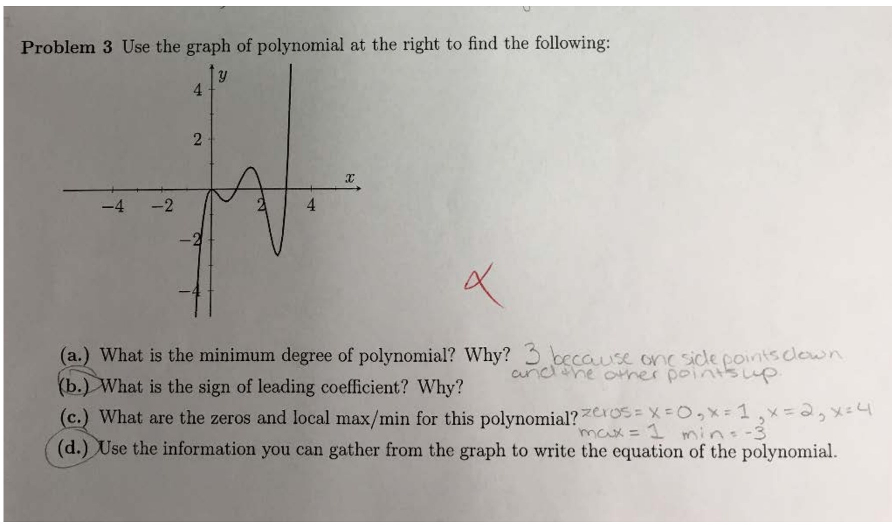


Figure 21. An example of the student solution without using GC

case 1a: The first picture shows Sara's answer to question 3 of the RT1. Sara used the graphing calculator to produce a graph for the polynomial. Then she labeled the zeros of the polynomial, which were the points that the graph meets the x-axis. She labeled the turning point with exact values on the graph that were the points at which the graph changed direction. Sara described the end behavior of the polynomial by using the graph, and she showed her understanding by correct notation. For subparts of question 3 in RT1, she had correct and appropriate use of the graph, and she was able to make a good connection between these concepts and their graphical representation.

She used the correct notation for showing zeroes, she used order pairs to show turning points. For this question, the logical reasoning level is 3 because she used the complete logical argument to support a completely correct solution. The organization of her written work is 3 because her written work is well organized and she has completed correct notation for the correct conclusion, and the use of graph level is 3 because she has a complete use of a graph for solving all parts of question 3.

Case 1b. This shows Sara's written work on question 3 in RT2 (no GC). Therefore, the graph of the polynomial was given in question 3 to minimize the effect of not having a graphing calculator. Students were asked to find the degree of the polynomial and the sign of leading coefficient which was an indicator of end behavior of polynomial. In addition, students were asked to find zeroes and maximum and minimum (like turning points in RT1). Sara mentioned that the polynomial has degree 3 because the polynomial goes down on one side and goes up on the other side. She was able to make a connection between the end behavior of the polynomial and the degree. But she did not have enough reason to support why she picked degree 3. T he logical reasoning level for question 3(a) is 1 b ecause she has some reasoning to support an incomplete solution; her organization for this part is 1 because she has incomplete reason to support incomplete solution; her use of graph level for part a is 2 because she has relevant use of the graph in parts. Although she had a graph of the polynomial, she was not able to make a connection between the graphical representation of polynomial and the sign of the leading coefficient. She did not answer part b, so logical reasoning and organization and use of graph level are 0. In question 3(c) (first part c, she wrote zeros in the following form: x=0, x=1, x=2, x=4. She probably understood that zeros of the polynomial are the points that the graph meets the x-axis. The logical reasoning level for this part is 1 because she has some reasoning to support the correct solution. The organization level is

2 because she used the correct notation for a correct incomplete solution, and the level of use of graph is 2 because she has relevant use of the graph in parts. For the second part, she showed the max=1 and min= -3. The polynomial has two local maxima at points (0,0) and approximately (1.5,1) and two local minima at points (0.5, -5) and (2.5, -2.5) approximately. The organization level for this part is 1 because she has some reasoning for an incomplete solution. The organization level for the answer is 1 because she does not have a well-organized argument or complete use of symbols and notation, and the use of graph level is 1 because she used the graph for an incomplete solution.

Comparing Sara's work for Q3 in RT1 and Q3 in RT2: Based on observation from Sara's written work in RT1 and RT2, her levels of producing an answer for similar concepts with and without technology are different. In one view she had a better performance when she used a GC. For example, in question 3 of RT1, Sara's written work is well organized, neat and coherent. She has appropriate use of a graph for producing complete correct answers. When she used a graphing calculator, she answered all parts with apparent confidence (see Figures 22 and 23) and she used completely correct notations and symbols for all parts. She made a good connection between the concepts of zeros, turning points and end behavior of a polynomial and their graphical representation. In contrast, in question 3 of RT2, although she had a graph of the polynomial, she was not able to derive enough information from the graph to answer different parts of question 3. She was not able to make a connection between the graphical representation of polynomial and the degree and the sign of leading coefficient. She did not use correct notation to show local maxima and minima. It seems that she does not have enough confidence to answer the questions when she does not have a graphing calculator or when she did not produce the graph of the polynomial herself.

Question 4 and 1: Find the domain of a rational function

Case1

1c. Sara's paper on RT1 (GC)

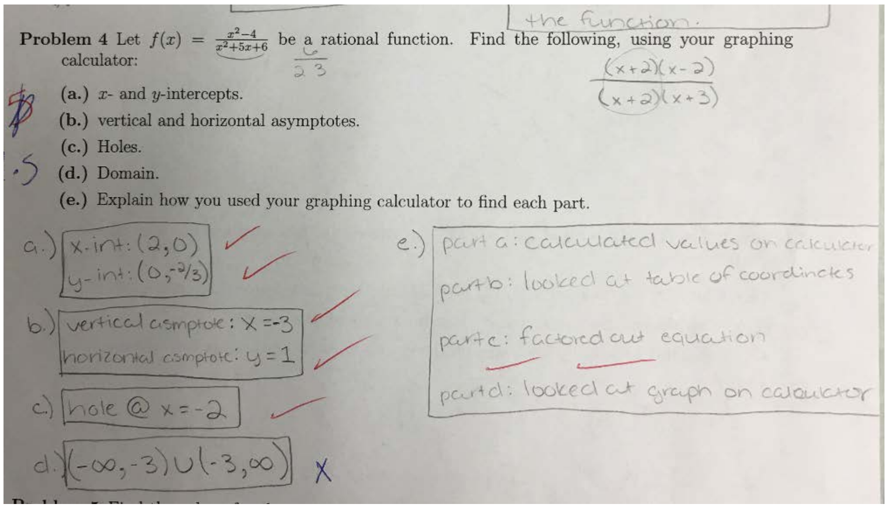


Figure 22. Sara's paper on RT1 with GC

1d. Sara's paper in RT2 (no GC)

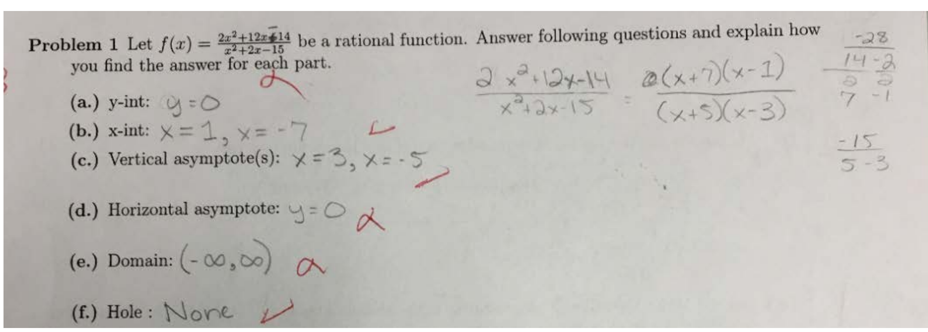


Figure 23. Sara's paper in RT2 without GC

1c: The first picture shows Sara's answer to question 4 of RT1. In this question, Sara did not use a GC to produce answers. Written work for part d as well as parts b, c and e should produce information enough to find the domain. In part e of question 4, Sara mentioned that she used a graphing calculator to find a vertical asymptote and domain, and she used factoring methods to find a hole. She found the vertical asymptote correctly, and she used correct notation to show it. She also found a hole, and she removed vertical asymptote from the domain but not the hole. Visualizing the rational function could help Sara to remove vertical asymptote from domain, but the graphing calculator does not show a hole in the graph. If students check the table in GC, then they can find a hole. The hole is the points in the table that in front of x value for which y value is undefined. This might be one of the reasons that Sara did not remove the hole because only by looking at the graph of the rational function, it seems continuous everywhere except at the vertical asymptote. In this case, the logical reasoning level is 2 because it was complete reasoning to support incomplete correct solution. The organization level is 2 because it has a well-organized argument and correct notation for the correct incomplete conclusion, and the use of graph level is 2 because it has relevant use of the graph in parts.

1d. The second picture shows Sara's written work on question 1 in RT2. In this question, Sara did not use a GC to produce an answer. Part d of this question will be discussed. In addition, parts c and f will be considered to see if she was able to use information from these parts to find the domain. Sara used a factoring method to find the domain. She found vertical asymptote by putting denominator equal zero. She mentioned that there is no hole probably because she did not see any common term in numerator and denominator, but she did not remove the vertical asymptote from the domain. It might be because she did not have the graph that she was not able to see that the rational function does not continue at vertical asymptote points. In other words, she was not able to visualize the graph of a rational function to see the function does not have any y value at vertical asymptotes. The logical reasoning level is 0 because of no relevant argument to support a conclusion. The organization level is 1 because it has some organization and some correct notation for an incomplete incorrect solution. The use of graph level is 0 because there is no relevant evidence of the use of a graph.

4.12 Comparing Sara's work for Q4 in RT1 and Q1 in RT2

Sara's written work on finding a domain with and without technology is discussed. In the test with a graphing calculator, she was able to find vertical asymptote and hole, but she only removed the vertical asymptote from a domain, not a hole. As mentioned before this mistake could be happening because of the way that GC produces a graph. The TI-84 does not show a hole in the graph. The only way that one can find the hole by GC, is by looking at the window (table and by checking the value that is undefined). She was able to make the good connection between the concept of domain and the graphical representation of it. In contrast, in the question 1 of RT2 in which Sara did not use a graphing calculator to find the domain, she found vertical asymptote by putting denominator equal zero and she also found a hole, but she did not remove them from the domain. She was not able to visualize the domain of the function, and she was not able to make the connection between the concept of vertical asymptote and domain as well as a hole. Since the non-GC test was taken after the GC based test, it is surprising that she was able to find the domain in first test partially correct and in second test completely wrong. Perhaps since she did not have a clear image of the rational function in the second test it might influence finding the domain for the function. She was not able to see the image of a rational function on the vertical points which could cause not removing them from the domain. In the first test, she was not able to see the hole on the graph because the way that TI-84 produces the graph, and that could be the reason that she did not remove the hole from the domain. Sara's performance on the concept of domain was better when she used GC compared to when she did not use GC. As a tool GC could help students to visualize different mathematical concepts including vertical asymptote, horizontal asymptote, and domain, which can help students to have a better conceptual understanding of these concepts.

4.13 The result of the analytic rubric applied to the concept test

Data from the concept test were collected as another source for answering the research questions. Therefore, students' papers from the concept test were analyzed with the same methods as were the review tests. As a reminder for the reader concept test was the only written test in college algebra. It was administered two weeks before the final exam and was designed by the college algebra coordinator. It constituted 10% of the total student grade in college algebra. Four college algebra sections were considered as treatment sections where students were allowed to use a GC on the concept test, and four sections were considered as control because they were not allowed to use a GC on the concept test. In this phase, the researcher used a stratified sampling method to select sample size for both the control and treatment groups. Three strata were considered: students with grades 80 and above, students with grades 60 to 80, and students with grades under 60. Some questions were common between the concept test and RT1 and RT2. Therefore, the rubric was applied to questions with concepts common with the review tests to give the researcher an opportunity to explore students' understanding based on multiple sources. These questions are 2b, 3c, and 3g.

The result of the analytic rubric applied to control group: The total population of control group was 131 students with N_1=12, N_2=55 and N_3=64. One-eighth of the population was stratified into 3 strata (as defined above) with 2, 7, and 8 students. Table 23 shows the information about sampling.

After stratified random sampling, the researcher collected the students' concept test papers for control and treatment groups. In the next phase, the analytic rubric was applied to the selected samples.

Table 23. Number of students in each stratum.

Students grade /100	Qualitative grade /4	Population size in non-GC test. N=131	Sample size n=17
80 -100	A, B	N_1=12	n_1=2
60 -79.99	C, D	N_2=55	n_2=7
0-59.99	F	N_3=64	n_3=8

The following memos were used based on the repetition of them between students answers to help the researcher apply the rubric equally on the concept test for both control and treatment sample size.

Memo 1: In question 2c, if the upper and lower bound of the interval, and interval notation and

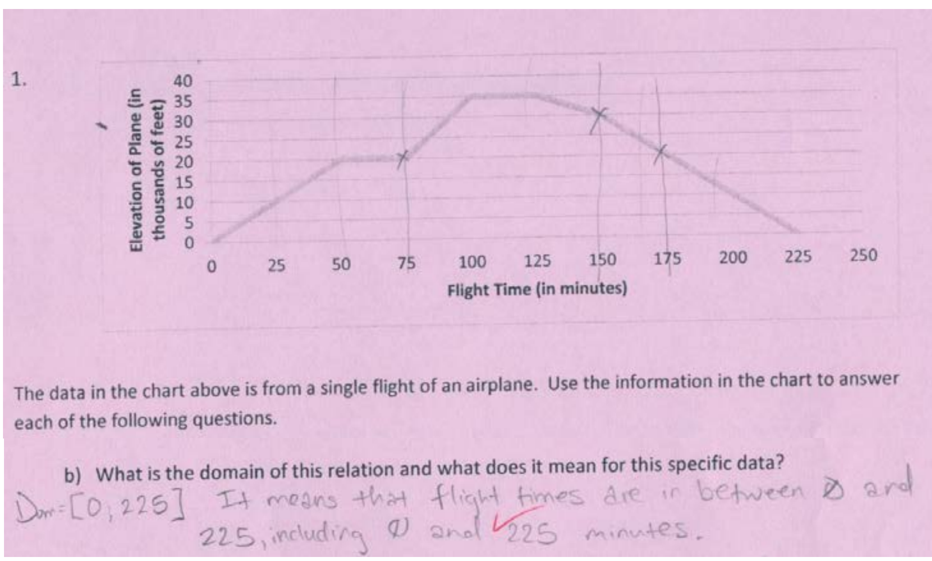


Figure 24. An example of solution with level 3 score by analytic rubric

reasoning to describe domain were correct then LR=3, OR=3, and UG=3.

Memo 2: In question 2, if a student incorrectly showed the interval of [0,225] as [0, 25, 50, 5, 225] then the given LR=1, and OR=1.

Memo 3: If students used parentheses or curly brackets to show domain instead of the bracket, but the reasoning part was correct then: LR=3 OR=2.

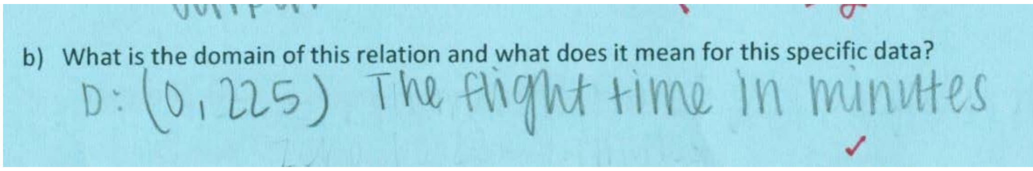


Figure 25. An example of solution scored by analytic rubric

Memo 4: In question 2, which asked about the domain, if the upper and lower bound of the interval and interval notation were correct but the explanation was not correct then: LR=1, OR= 2.

Memo 5: For question 3c, if a student defines zero as x-intercept without any more explanation then LR=2, OR=2.

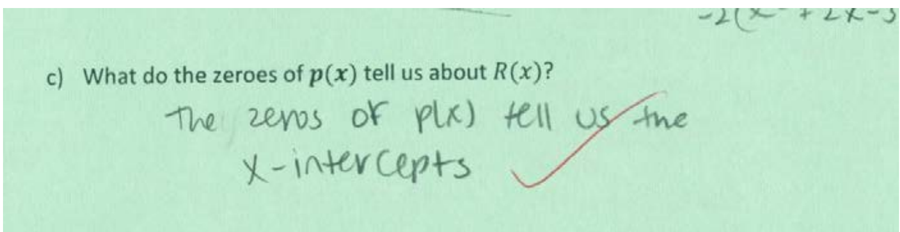


Figure 26. A solution with logical reasoning and organization of written work scores of 2

Memo 6. For question 3c shown in Figure 27, if the reasoning does make sense for describing the zeros and the notation is appropriate, then LR=3 and OR=3. For UG we will check student explanation in part f to see if the graph was used for answering part c. In this case UG=3.

Memo 7: In question 3g shown in Figure 28, if students have complete correct reasoning with complete correct use of notation then LR=3 and OR=3. For UG, part f of question 3 will be check. If a student mentioned use of GC to describe end behavior by making the graph, then UG=3.

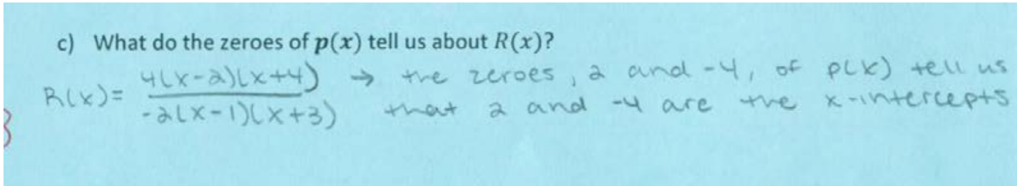


Figure 27. Question 3c

Memo 8: For question 3g shown in Figure 29, if a student described the end behavior of the graph

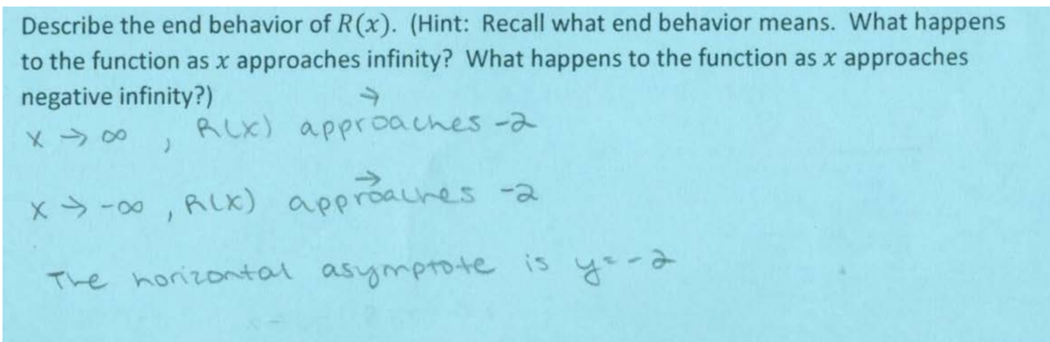


Figure 28. End behavior of a function

by indicating that it falls to the left and the right then LR=1, OR=0.

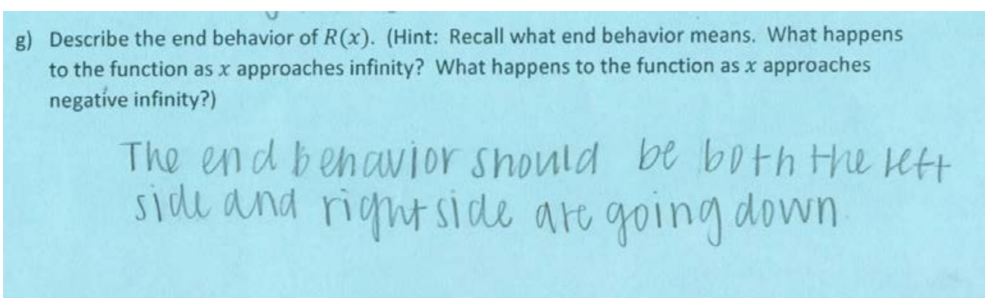


Figure 29. Question 3g

Memo 9: For question 3g shown in Figure 30, if a student explained the end behavior as x approaches negative infinity, y approaches to -2, and as x approaches positive infinity, y (instead r(x)) approaches to -2 then LR=3 but OR= 2 because for OR correct use of notation is required.

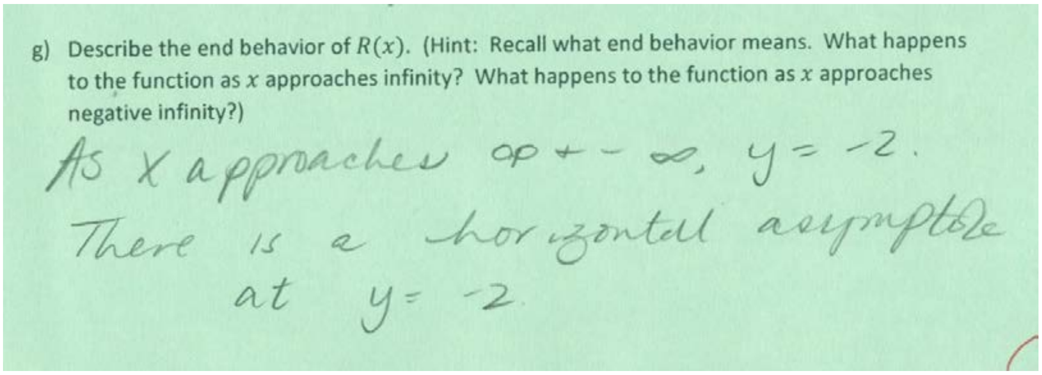


Figure 30. Question 3g with LR=3 but OR= 2

The results of scoring the concept test for the control group by the analytic rubric were tabulated, which can be seen in the Tables 24 and 25. SAC stands for student assigned code, LR stands for logical reasoning, OR stands for organization and UG stands for use of a graph.

Table 24. The results of applying the rubric.

SAC	LR_2b	OR_2b	UG_2b	LR_3c	OR_3c	UG_3c	LR_3g	OR_3g	UG_3g
1426	3	3	3	3	3	0	3	3	3
2433	2	3	3	3	3	0	1	1	0
2432	2	2	3	1	1	2	1	1	2
2207	3	3	3	1	1	0	1	0	0
1403	3	2	3	0	0	1	1	0	0
1410	3	2	3	3	2	3	2	1	2
2438	3	2	3	1	0	0	1	0	0
2203	2	3	3	1	1	2	1	0	0
9005	3	2	0	0	0	1	1	1	0
2403	1	1	1	2	2	0	1	1	0
2424	2	3	3	0	1	0	1	1	0
2209	1	1	3	1	1	0	2	1	2
9002	3	2	2	3	2	0	0	0	0
9024	1	2	2	0	0	2	0	0	1
1435	3	2	3	0	0	0	0	1	0
1420	3	3	3	3	3	3	3	3	3
1411	1	1	1	2	2	2	1	0	1
1413	0	0	1	2	2	3	1	0	0

4.14 The result of the analytic rubric applied to treatment group

Treatment groups had three strata with population sizes 16, 65, and 88. A fraction of (approximately) 1/8 of population in each stratum was selected for samples. The sample sizes for treatment groups are 2, 8, and 7 respectively. Table 25 shows the information about sampling.

Table 25. Sampling information.

Students grade /100	Qualitative grade /4	Population size in GC test N=136	sample size n=17
80 -100	A, B	N_1=16	n_1=2
60 -79.99	C, D	N_2=65	n_2=8
0-59.99	F	N_3=55	n_3=7

In the next phase, the analytic rubric was applied to the selected samples. The results of scoring the concept test for treatment group by the analytic rubric were tabulated as seen in the Table 26 and 27.

Table 26. The results of applying the rubric.

SAC	LR_2b	OR_2b	UG_2b	LR_3c	OR_3c	UG_3c	LR_3g	OR_3g	UG_3g
3131	3	3	3	1	1	3	3	2	3
1529	3	2	3	3	3	3	1	0	0
3204	3	3	3	2	3	3	2	2	2
1616	3	3	3	1	1	3	3	3	3
1633	3	3	3	1	1	3	1	2	0
1538	3	3	3	1	1	2	1	1	0
1527	3	3	3	3	3	3	1	1	1
1638	2	2	2	3	3	3	2	1	2
1605	3	2	3	1	1	3	1	1	0
1631	3	3	3	0	0	0	1	0	0
1615	3	3	3	3	3	3	0	1	0
1602	2	3	3	2	2	3	3	3	3
1528	1	1	2	3	3	2	1	0	0
3235	3	3	3	3	3	3	3	3	3
3123	3	3	3	2	2	3	2	2	2
3107	3	3	3	3	2	3	3	2	3
3206	3	3	3	3	3	3	3	3	3

Result of t-test over mean score of the analytic rubric in the concept test

The rubric was applied to questions with concepts in common with the review tests to give the researcher an opportunity to explore student understanding. In the next phase, the two-sample t-test over the mean scores of the qualitative rubric grades was applied with significance level $\alpha = 0.05$ and 95 percent confidence interval was applied to the concept test for control and treatment sections. The results are shown in Table 27.

Table 27. t-test over mean scores of the qualitative rubric on concept test.

t-test over mean scores of different categories of the qualitative rubric for the concept test	df	P-value
Logical Reasoning	28.25	0.0068
Organization	30.47	0.0046
Use of graph	31	0.00055

The result of the two-sample t-test shows that there are significant differences in logical reasoning, organization, and use of graph skill of treatment students verses control sections. Students who used a graphing calculator in the concept test have better logical reasoning, organization including well organized argument, and correct use of notation and symbol compared to the students who did not used a GC in the same test.

4.15 Estimating the population mean and confidence interval

The total number of students in the control group is N=131. We divided the population into three strata that contain N_1=12, N_2=55 and N_3=64 observations. The sample size is n=17 including n_1=2, n_2=7 and n_3=8 students chosen from stratum one, two and three respectively. With the mentioned formulation of stratified sampling in the previous part the estimator of the mean and variance of the population, and confidence interval of the mean of three features of the students in the control and treatment section in the concept test were calculated and tabulated as seen in Table 28 and 29 respectively.

Table 28. Concept test control group.

	Estimated mean	Estimated standard error	Stratified degrees of freedom	The confidence interval of the mean of the population
Logical reasoning	4.61	0.39	10.10	(3.09,5.33)
Organization of written work	4.05	0.38	8..06	(3.45,4.77)
Use of graph	4.34	0.49	12.00	(3.46, 5.22)

The total number of students in the treatment sections is N=136. We divided the population into three strata that contain N_1=16, N_2=65 and N_3=55 observations. The sample size is n=17 including n_1=2, n_2=8 and n_3=7 students chosen from stratum one, two and three respectively.

Table 29. Concept test treatment group.

	Estimated mean	Estimated variance	Stratified degrees of freedom	The confidence interval of the mean of the population
Logical reasoning	6.65	0.32	8.76	(6.05, 7.25)
Organization of written work	6.36	0.32	8.96	(5.77 ,6.95)
Use of graph	7.07	0.45	3.31	(6.04, 8.09)

Interpretation of confidence intervals for the concept test

Based on t he results shown in Tables 28 and 29 in all cases of logical reasoning, organization of written work and use of graph, the estimated mean of students' skill in the treatment sections are higher than students in the control sections. This also can be seen in the 95% confidence interval of the mean which are shown at the last columns of the two tables. For example, the confidence interval of the mean of the organization skills in the control section is (3.45, 4.75) and (5.77, 6.95) for treatment sections. These two 95% confidence interval do not intersect, which shows that the mean of the organization skills of the students very likely increases (probability more than 95%) when they use GC on the concept test. The same analogy is applicable to logical reasoning and use of graph skills.

4.16 A detailed qualitative description of students' concept tests

In this section different students' answers are qualitatively compared and summarized.

Question 2: Domain

For this question about function and meaning of the domain in terms of the problem, 8 students in control section answer correctly with complete correct reasoning and complete correct notation to show the domain. Five students had correct reasoning, but they used parentheses instead of brackets to show the domain. One student used curly braces to show the domain. One student showed domain as a set of discrete points and two students had an irrelevant solution. A sample of work of a student in control section is shown in Figure 31.

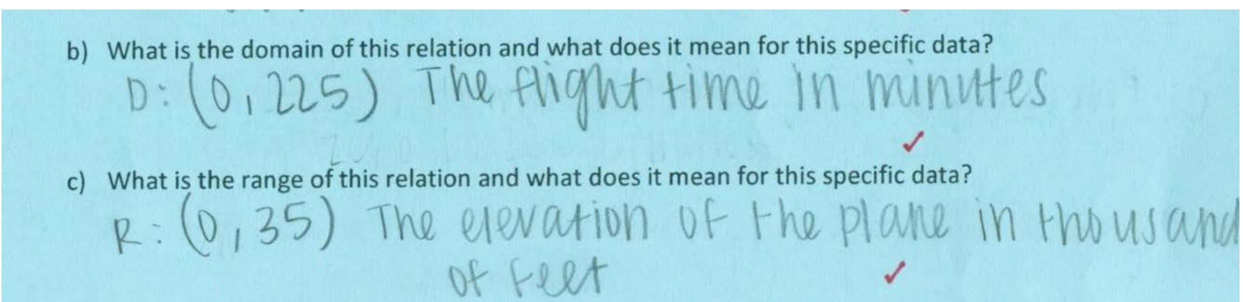
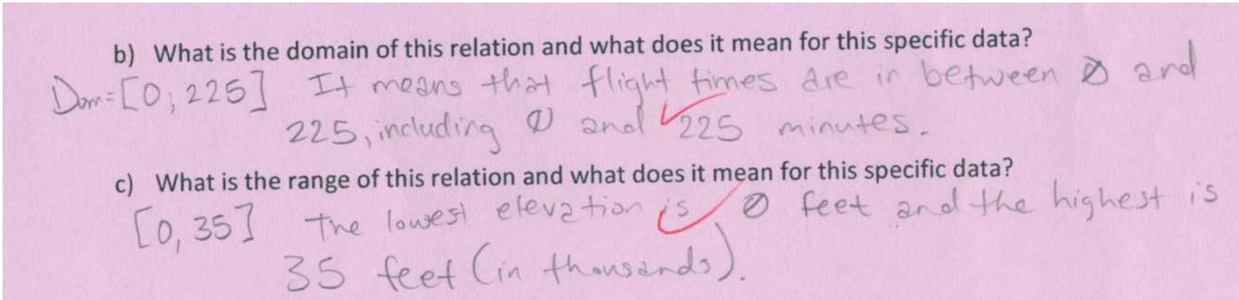


Figure 31. A sample of work of student in control section on domain problems.

In treatment sections 13 students used correct logical reasoning and correct use of notation to explain function and the domain of function. Two students had correct logical reasoning, but they used parentheses to show the domain, and two students used a set of discrete numbers. A sample of work of a student in the treatment section is shown in Figure 32.



b) What is the domain of this relation and what does it mean for this specific data?

Dom=[0,225] It means that flight times are in between Ø and 225, including Ø and 225 minutes.

c) What is the range of this relation and what does it mean for this specific data?

[0, 35] The lowest elevation is Ø feet and the highest is 35 feet (in thousands).

Figure 32. A sample of work of a student in the treatment section on domain problems.

More students in the treatment sections were able to present well-organized reasoning and correct use of notation to define the function and to find the domain of the function compared to students in the control sections (13 vs 7). More students in control sections misused notation to show the domain of the function compared to the students in the treatment section; five students in control sections used parentheses while two students in the treatment section did so.

Question 3: Zeros and end behavior of a functions

Zeros: First part of this question asked what the zeros of p(x) tell us about r(x) when

r(x) = $\frac{p(X)}{q(x)}$. In treatment sections 12 students described the zeros as x-intercepts or stated that the graph goes through those points on the x-axis. One student mentioned that "zeros tell us that the numerator for r(x) are (x-2) (x+4). These are the x-intercept for r(x). Only 4 students have irrelevant solutions for this part. In control sections 10 students described zeros as x-intercept, 6 students mixed up zeros with either vertical asymptote or multiplicity or they had irrelevant solutions. Sample of student answer in treatment sections are shown in Figures 33 and 34.

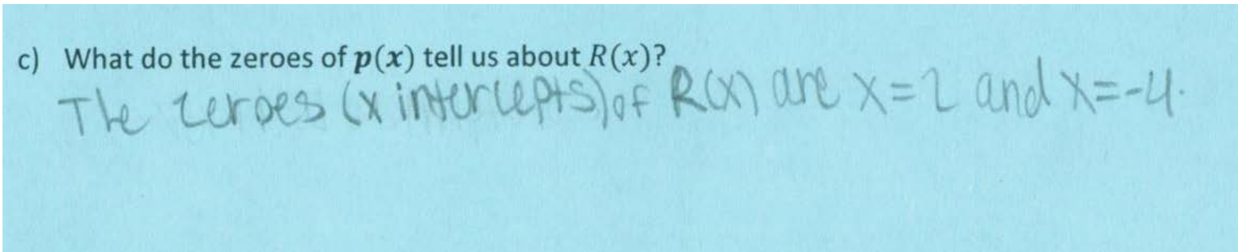


Figure 33. Sample of students answer on finding zeros in the treatment section.

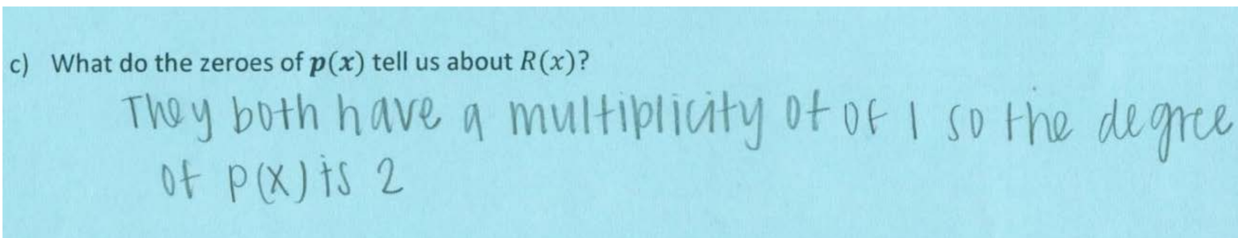
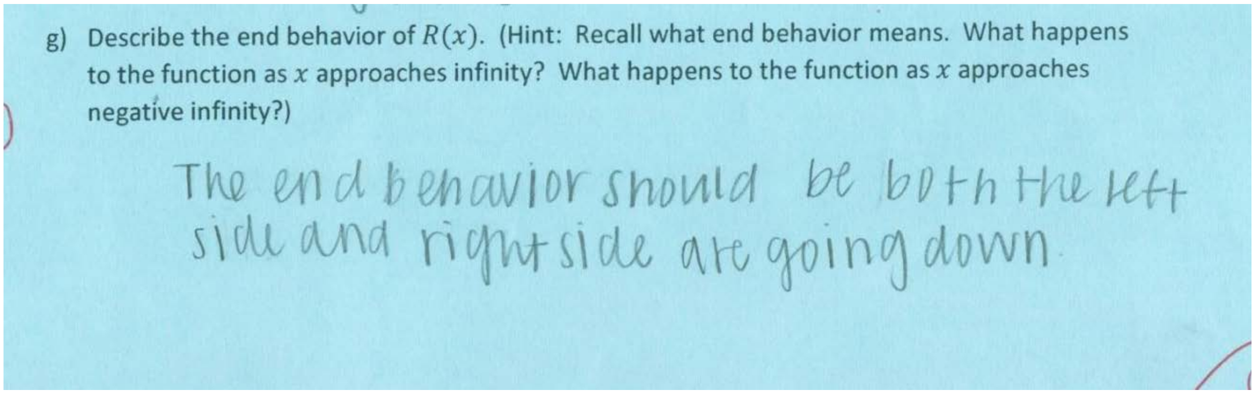


Figure 34. Sample of students answer in control section

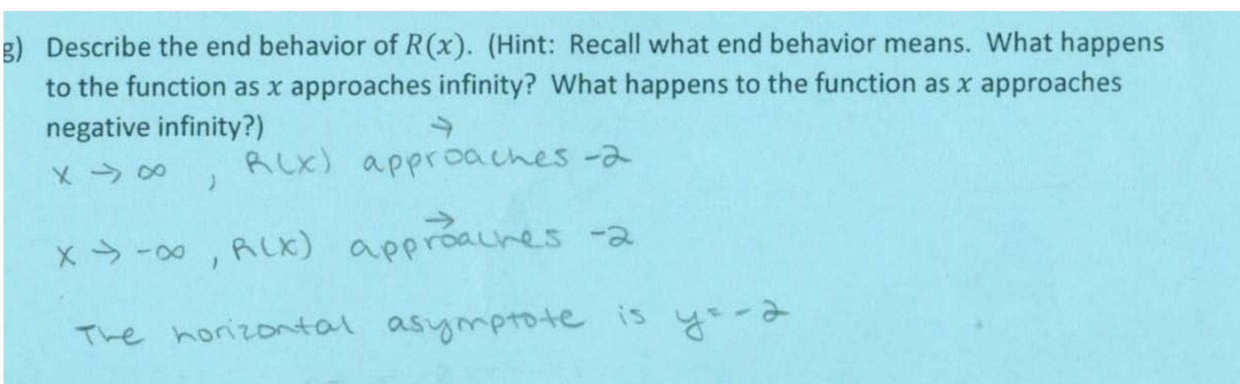
End behavior of the function: Last part of this question shown in Figure 35 asked to describe the end behavior of function r(x). Only two students in control section described end behavior of the function correctly with the correct notations and symbols. Four students drew a correct graph manually based on the given information about zeros and vertical and horizontal asymptotes but they could not describe the end behavior of the function. They were not able to find logical connection between the graph of the function and the end behavior of the function. Eight students in the control sections mentioned as x approaches to positive infinity the function grows and as x approaches to negative infinity the function decay. Three students conclude the following answer: "the lading confection is negative, and the highest degree is even therefore the function falls to the right and to the left." A sample of students answer to this question in shown in Figure 35.


g) Describe the end behavior of $R(x)$. (Hint: Recall what end behavior means. What happens to the function as x approaches infinity? What happens to the function as x approaches negative infinity?)

The end behavior should be both the left side and right side are going down.

Figure 35. A sample of student answer in control section

In treatment section 6 students were able to produce correct answer with correct notation. Two students had correct reasoning buy they used y instead r(x). They mentioned that they used GC to produce a graph and they used the graph to answer to this question. Four students mentioned the function falls to the right and to the left and three students mentioned as x goes toward positive and negative infinity R(x) goes toward positive infinity. A sample of student answer is shown Figure 36.


g) Describe the end behavior of $R(x)$. (Hint: Recall what end behavior means. What happens to the function as x approaches infinity? What happens to the function as x approaches negative infinity?)

$x \to \infty$, R(x) approaches -2

$x \to -\infty$, R(x) approaches -2

The horizontal asymptote is y = -2

Figure 36. A sample of student answer in treatment section.

In summary, students who used a GC in the concept test showed more skills in reasoning, organization including well-organized argument and correct use of notation and symbol, and use of graph in defending domain, zeros and endeavor of a function compare to the students who did not use a GC in the same test.

4.17 The result of student attitude survey (SAS)

Results of SAS in college algebra: As a reminder for readers, a survey that was developed by Tharp (1992) and was used by Merriweather and Tharp (1999) was selected. The chosen survey had 23 items with the format of typical five-level Likert items with responses of strongly disagree (SD), disagree (D), neutral or undecided (N/U), agree (A), and strongly agree (SA). One item was added to the student attitude survey to explore students' proficiency in using a graphing calculator, rated from 0 to 10. This survey was completed by college algebra students in the first week and last week of the semester. After collecting the completed survey, the primary researcher changed the five-level responses to a range of numerical values from 1 to 5. For example, SD= 1, and SA= 5.

In the first phase of analyzing SAS responses, data from pre-and post-survey in the control and treatment sections were visualized to have a clear image of students' responses. The images are shown in Figures 37 to 38 below.

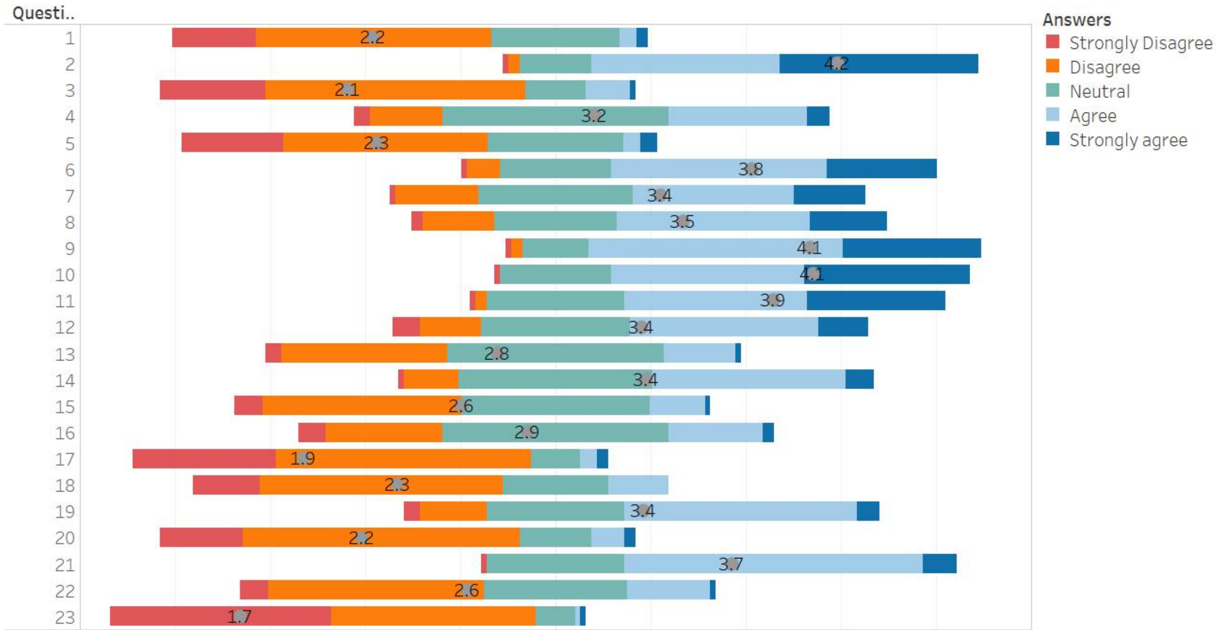


Figure 37. Control Pre-SAS

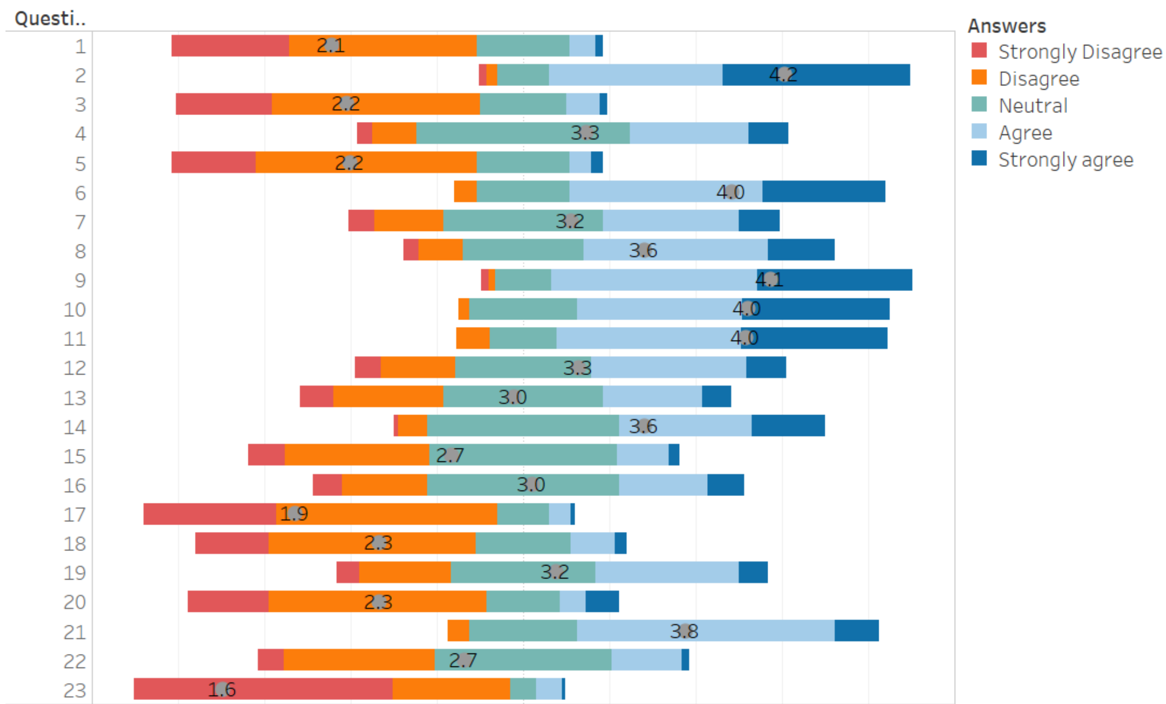


Figure 38. Treatment pre-SAS

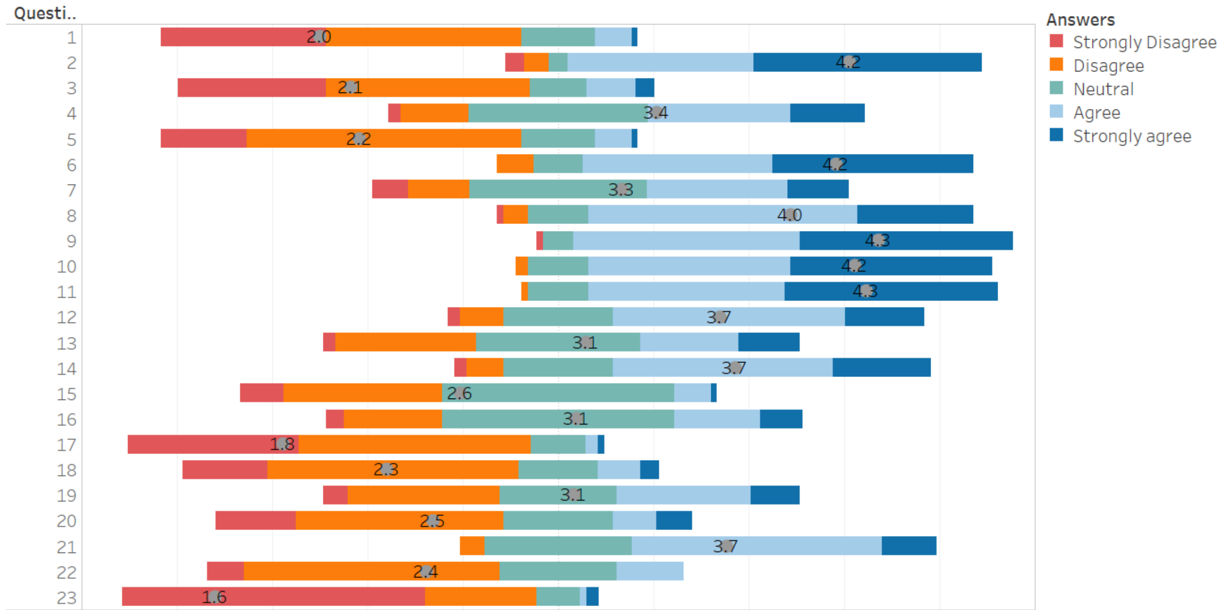


Figure 39. Control post-SAS

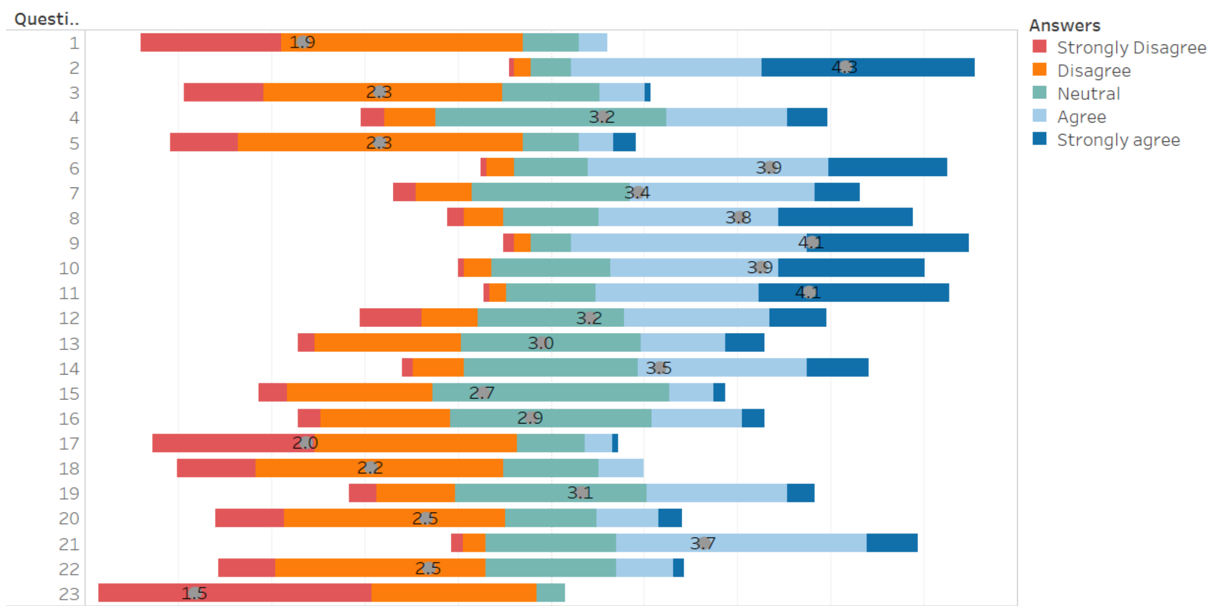


Figure 40. Treatment post-SAS

In the next stage, out of 24 survey items, 13 items were selected based on relevancy to the research questions. The selected items divided into two groups based on their nature: general items and personal items. General items derived to three subcategories including the positive view in use of a GC, positive view about mathematics, and negative view about mathematics. Items from each category are as the following:

General items

Subcategory 1: Positive view about the use of a GC

Q 4. Graphing calculator makes math fun.
Q 6. Learning algebra is easier if a graphing calculator is used to solve problems.
Q 9. It is important that everyone learn how to use a graphing calculator.

Subcategory 2: Positive view about mathematics

Q 21. Learning mathematics means exploring problems to discover patterns and generalize.

Subcategory 3: Negative view about mathematics

Q 13. Mathematics is boring.
Q 19. Learning mathematics is mostly memorizing a set of facts and rules.

Personal items, which refers to items that have the subject "I", divided into three subcategories. The first subcategories included items that have a negative view of the use of a GC. For example, the GC can be a hindrance and a tool which reduces visualization skill. The second subgroups are the items that show the level of an individual skill of use of a GC. The third subcategory shows students impression of their mathematics abilities.

Personal questions

Subcategory 1: Negative view about the use of a GC

Q 3. The graphing calculator will hinder my ability to understand basic computation.
Q 17. I feel I am cheating myself out of a chance to learn when I use a graphing calculator.
Q 18. If I use a graphing calculator my ability to visualize problems will decrease.
Q 22. I rely on my graphing calculator too much when solving problems.

Category 2: An individual skill of use of a GC

Q 8. I know how to use a graphing calculator very well.
Q24. How much experience of using a calculator in math courses have you had?

Category 3: Personal impression of mathematics ability

Q 12. I am good at mathematics.

At this stage of survey data analyzing, means of responses in the above-mentioned categories in both pre- and post- survey for both control and treatment groups were computed and a two-sample t-test with a significance level of $\propto = 0.05$ and confidence interval of 95 present was applied. In the second stage, the students' responses to the general category in the pre-survey and post-survey for control and treatment sections were compared. The results are given in the tables 30 and 31.

Table 30. T-test of means of responses for pre-survey between control and treatment.

Categories	# of questions	Mean(C-Pre)	Mean(T-Pre)	df	p-value
Positive view about use of GC	4,6,9	3.68	3.93	162	0.127
Positive view about math	21	3.74	3.75	192	0.93
Negative view about math	13,19	3.06	3.06	195	0.93
Female		52	65		
male		34	52		

The result of the two-sample t-test over the mean scores of general category questions of pre-survey does not show any significant differences between students' positive view in use of a GC, positive view about mathematics and negative view about mathematics between control and treatment sections. Both groups have a positive view of the use of GC (the mean is 3.68 in control sections vs 3.93 out of 5 in the treatment section). Thus, students in both control and treatment sections believed that use of a GC makes mathematics fun, learning algebra is easier with a GC and learning how to use a GC is important. In addition, the number of students who believed that mathematics means exploring problems to discover patterns and generalize is similar in both groups; the number of students who believed mathematics is boring and is a set of memorizing rules were equal in control and treatment sections.

A two-sample t-test over mean responses to the same items between control and treatment students' post-survey was conducted. The mean responses to items 4, 6, and 9 for the control group is 3.97 vs 3.76 for treatment groups. The results of t-test show significant differences between students' positive views in control and treatment groups. The results are shown in the Table 31.

Table 31. Students' positive views in control and treatment groups

Categories	# of questions	Mean(C-Post)	Mean(T-Post)	df	p-value
How does using technology affect the organization of college algebra and calculus students' written work?	4,6,9	3.97	3.76	159	0.022
Positive view about mathematics	21	3.7	3.58	159	0.7
Negative view about mathematics	13,19	3.09	3.05	148	0.74
Female		14	46		
Male		22	30		
Unknown gender		12	7		

Although students in both control and treatment groups had a positive view of the use of GC, students who were not able to use a GC on the concept test had a more positive view about the use of GC. They believed that having GC would make algebra easier, more fun and the use of GC is important. Results are in Table 32. There is not any significant difference between control and treatment sections positive view in post-survey as well as a negative view of mathematics between the control and treatment sections (subcategory 2, and subcategory 3).

Table 32. T-test over means of the scores, for post-survey between control and treatment.

Categories	# of questions	Mean(C-Pre)	Mean(T-Pre)	df	p-value
Negative view of use of a GC	3,18,22,17	2.25	2.29	190	0.6639
Self-impression of the use of GC skill	8,24	3.54	3.62	177	0.5582
Self-impression of mathematics skills	12	3.36	3.26	184	0.48
Female		52	65		
male		34	52		

Personal questions: In the next stage, a two-sample t-test was applied to items in the personal category between results of students' pre- and post-survey in the control and treatment sections. The results do not show any significant difference between students' negative view of the use of a GC, students in both groups had an average 2.25 out of 5 for the questions that mentioned GC is a hindrance or GC reduced the visualization skill. The summary of results is shown in Table 33. Students in both groups have similar skill (at least 3.5 out of 5) of the use of GC. Students in both control and treatment section had a positive impression of their mathematics skills.

The same analysis was done on the results of the post-survey between control and treatment sections. The results show that students in the control section had a more positive self-impression of their mathematics skill. The results are below in Table 33. There is a significant difference (p-value = $.003<.05$) between students' impression about their mathematics skills in the control and treatment sections. The average of students' responses in the control sections who believed that "I am good at math" is 3.67 in post-survey while this average is 3.16 for the treatment sections. The students' impression of their skill in using GC in the control group is slightly higher than in treatment sections. But there are not any significant differences between their negative views on the use of GC.

Control pre- and post-SAS, general and personal categories, between the control sections

In the next phase, the results of pre-SAS and post-SAS in control sections were compared. Students' positive view on the use of a GC increased for students in the control section. Students in control sections believed that if they would use a GC, then mathematics is more fun, algebra is easier, and learning how to use a GC is important. Their positive view about the use of GC increased at the end of the semester. Students' positive view about mathematics does not change from the beginning and the end of the semester, but more students in control sections believed “math is boring”, and “learning mathematics is just memorizing the rules” at the end of the semester. Results are in the Table 34.

Table 33. Students' positive self-impression of their mathematics skill.

Categories	Items	Mean(C-Post)	Mean(T-Post)	df	p-value
Negative view of use of GC	3,18,22,17	2.17	2.23	158	0.59
Self-impression of the use of GC skill	8,24	3.99	3.74	157	0.76
Self-impression of mathematics skills	12	3.67	3.16	154	0.003
Female		44	46		
Male		22	30		
Unknown		12	7		

The same analysis methods applied to the personal question of the pre- and post-SAS, between control groups show that students’ impression of their skill in the use of a GC increased at the end of the semester in control sections. But there is not any significant difference about student’s impression on their mathematics skill as well as their negative view of the use of GC.

Table 34. Students positive view about mathematics

Categories	# of questions	Mean(C-Pre)	Mean(C-post)	df	p-value
View about use of GC	4,6,9	3.68	3.97	162	0.003
Positive view about math	21	3.74	3.7	153	0.72
Negative view about math	13,19	3.06	3.09	154	0.003
Negative view of use of GC	3,18,22,17	2.25	2.17	159	0.38
Self-impression of the use of GC skill	8,24	3.54	3.99	161	0.002
Self-impression of mathematics skills	12	3.36	3.67	161	0.59

Treatment pre-SAS and post-SAS in general and personal categories

In the next phase, the results of pre- and post-SAS on the general and personal question in the treatment sections were compared by the same analysis as previous parts. Results are shown in Table 35.

Table 35. Results of pre- and post-SAS on the general and personal question.

Categories	#of questions	Mean(T-Pre)	Mean(T-post)	df	p-value
Positive view about use of GC	4,6,9	3.93	3.76	161	0.28
Positive view about math	21	3.75	3.58	158	0.4
Negative view about math	13,19	3.06	3.05	177	0.94
Negative view of use of GC	3,18,22,17	2.29	2.23	170	0.49
An individual skill of the use of a GC	8,24	3.62	3.74	170	0.43
Self-impression of mathematics skills	12	3.26	3.16	161	0.59

The results of t-test over all subcategories of general questions as well as personal questions do not show any significant difference between treatment students' view from pre-and post-survey for the stated categories.

No analysis was attempted based on gender and ethnicity. This could be a topic for future study.

Correlation

In the last stage of SAS data analyzing, students' level and skill of use of GC and students' mathematics skill were compared with their positive/negative view about mathematics as well as their positive/negative views about the use of GC to explore if there is any direct or indirect correlation between these characteristics. Correlations were calculated by using equation 10.

$$correlation = \frac{Cov(x,y)}{\sigma_x \sigma_y} = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2}\sqrt{\sum(y-\bar{y})^2}} \qquad \text{Equation 10}$$

where $Cov(x, y)$ is the covariance of x and y data, σ_x and σ_y are the standard deviations of x and y data respectively. Further, $\bar{x}$ and $\bar{y}$ stand for the mean of data x and mean of y respectively.

The number of surveys that were analyzed to check the correlation between the stated categories was 365 in total including 207 females, 139 males, and 19 students whose gender was not given. The results are shown in Table 36.

Table 36. Survey Results.

Categories	A positive view of mathematics	The negative view of mathematics	A positive view of the use of a GC	The negative view of the use of a GC
student's skill of using GC	-2%	-6.2%	24%	
Student self-impression of mathematics skills	13.7%	-26%	9.4%	8.6%

The result show that students' skills of use of a GC have a positive correlation (24 %) with students' positive view of the use of a GC. That means students who have a higher skill in the use of a GC have a more positive view of the use of GC. Students who were skillful in the use of a GC believed that GC can make mathematics more fun, make learning algebra easier, and knowing how to use a GC is important.

By applying equation 10, t he correlation between students' impression of their mathematics skills and positive view of mathematics such as "Learning mathematics means exploring problems to discover patterns" was 13.7 percent. This shows that students who had a higher positive self-impression of mathematics ability had a more positive view of mathematics. Students who have a positive impression of their mathematics skills think more positively about mathematics.

There was a direct correlation between students' mathematics skills and their positive view of the use of a GC (positive correlation 9.4 %). This value was calculated by equation 10 and shows that students who believed "I am good at math" are more likely to believe that GC can make math more fun and makes learning algebra easier. There is a negative correlation (-6.2%) between an individual skill of use of GC and negative view of mathematics. As students' skill of use of a GC increased their negative view of mathematics, such as math is boring, and math is only memorizing the rules, decreased. Students' positive view of their mathematics ability is indirectly dependent on students' negative view about mathematics with -26% correlation. As students' self-estimate of mathematics ability increased the negative view of mathematics such as math is boring, or mathematics is only memorizing the rules decreased.

4.18 Summary of results of pre- and post-survey in college algebra

Out of 24 s urvey items, 13 items were selected based on relevancy to the research questions. The selected items divided into two groups based on their nature: general items and personal items. General items were placed in three subcategories: the positive view of use of a GC, positive view about mathematics, and negative view about mathematics. Personal items, which refers to the items that have the subject "I", divided into three subcategories: a negative view of the use of a GC, an individual skill of use of a GC, and students' impressions of their mathematics abilities. A two-sample t-test with a significance level of $\propto = 0.05$ and confidence interval of 95% was applied to the mean scores of students' responses to the selected items in the pre-survey between control and treatment section as well as post-survey between control and treatment sections. In addition, pre- and post-survey in control, and pre- and post-survey in treatment sections were compared. The results do not show any significant differences between students' responses in the control and treatment section in pre-survey for all categories as well as the treatment students' responses in pre- and post-survey. The results of t-test over mean scores of general and personal categories in pre- and post-survey in control sections show that students' positive view on the use of a GC increased for students in the control section. Students in control sections believed that if they would use a GC, then mathematics is more fun, algebra is easier and learning how to use a GC is important. Their positive view about the use of GC increased at the end of the semester. More students in control sections believed “math is boring”, and ‘learning mathematics is just memorizing the rules” at the end of the semester. The results show that students’ impression on their skill at use of a GC increased at the end of the semester in control sections. But there is not any significant difference on students’ impression of their math skills as well as their negative view of the use of GC. The same analysis was done on the results of the post-survey between control and treatment sections. The results show that students in the control section had a more positive self-impression of their mathematics skill. The average of students’ answers in the control sections who believed that “I am good at math” is 3.67 in post-survey while this average is 3.16 for the treatment sections. The students’ impression of their skill at use of GC in the control group is slightly better than treatment sections. But there are not any significant differences between their negative views on the use of GC. At the last stage of SAS data analyzing in college algebra, students’ level and skills of use of GC and students’ mathematics skill were compared with their positive/negative view about mathematics as well as their positive/negative views about the use of GC. The results show that students’ skills at use of a GC have a positive correlation (24 %) with students’ positive view of the use of a GC. The correlation between students’ impression of their mathematics skills and positive view of mathematics is 13.7%. Students who have a positive impression of their mathematics skills think more positively toward mathematics.

Students’ mathematics skills are directly dependent on students’ positive view of the use of a GC (positive correlation 9.4 %). Students who believed “I am good at math” believed that GC can make mathematics more fun and makes learning algebra easier. Students’ mathematics skills are directly dependent on students’ positive view of the use of a GC (positive correlation 9.4 %).

Students who believed "I am good at math" believed that GC can make mathematics more fun and makes learning algebra easier. As students' skill of use of a GC increased their negative view of mathematics such as math is boring, and math is only memorizing the rules, decreased. Students' positive views of their mathematics ability is indirectly dependent on students' negative view about mathematics with a correlation of -26%. As students' level of self-estimate of mathematics ability increased the negative view of mathematics such as math is boring, or mathematics is only memorizing the rules decreased.

4.19 Results of pre- and post-survey in survey of calculus

The results of pre- and post-survey on the general and personal question between students in survey of calculus were compared by the same analysis as SAS in college algebra. Twenty-one students completed the pre-survey -- 13 females and 9 males. Seventeen students completed the post-survey -- 6 males, 9 females, and 2 unknowns. A t-test over means of students' responses in pre- and post-survey was applied. Results are shown in the Table 37.

Students' positive view about mathematics increased insignificantly at the end of the semester. The results do not show any significant differences between students view in all subcategories of general items as well as personal items from beginning and the end of the semester. Since the sample size is small, other analysis such as finding a correlation between subcategories was ignored and could be a topic for future research.

4.20 Quantitative data analysis and result in survey of calculus

As a reminder to the readers, in the summer of 2017, two survey of calculus classes with a total of 40 students were chosen and two review tests were administered. The first review test, RT1, which was designed by the principal researcher, was over the derivative and was taken before the midterm exam. It has three open-ended questions.

Table 37. Students' responses in pre- and post-survey

Categories	# of questions	Mean_pre	Mean_post	df	p-value
Positive view about use of GC	4,6,9	3.60	3.63	30	0.92
Positive view about math	21	3.45	3.70	35	0.44
Negative view about math	13,19	3.18	3.00	37	0.48
Negative view of use of a GC	3,18,22,17	2.01	2.01	31	0.98
An individual skill of use of a GC	8,24	3.80	3.79	28	0.98
Student self-impression of mathematics skills	12	3.17	2.77	34	0.25

The second review test 2, RT2, which was over continuity, limit, derivative and integrals, was taken before the final exam and it had 7 open-ended questions. One class was considered as the control in which students were not allowed to use a GC in RT1 and RT2, and one class was the treatment section in which students used a GC for both review tests. Sample questions of RT 1 and RT2 are shown in below.

RT1

Q1: Find the open interval where the function $f(x) = -2x^3 + 12x^2 + 170x - 6$ is concave upward or concave downward. Find any inflection point. (5 points)

Q2: a) Let f (x, y) be a function that has (6, 7) as a critical point. We determine that

$f_{xx}(6,7) = -2$, $f_{yy}(6,7) = 2$ and $f_{xy}(6,7) = -10$

What D test tells us about the function f? (5 points)

b) Find the partial derivative $\frac{\partial z}{\partial y}$ of $z = 8x + 7xy^3 - 6y^2$

Q3: Find all the local maximum, local minimum, and saddle point of the given function:
$f(x, y) = 4x^2 + 6xy + 8y^2 + 4x - 20y$

RT2

Q2: Find the below limit. Which method did you use (table, graph, or limit properties. Show your work). (5 points)

$$\lim_{x \to -6} \frac{x^2-36}{x+6}$$

Q3: a) Sketch a graph for the below function. (10 points)

$$(x)=\begin{cases} 3 & x < 0 \\ x^2+1 & 0 \leq x \leq 3 \\ 10 & x > 3 \end{cases}$$

b) Find all values of x where the function f is discontinuous. (Show all steps of your work)

c) For which x value in the interval $[0,3]$ limit of f(x) exists? Why?

Q6: Evaluate the following integrals. (Show all steps of your work). (10 points)

a) $\int (x+1)e^{3x^2+6x}$dx

b) $\int_1^5 \frac{2}{(5x+1)^3}\,dx$

Q7: Find the area between the following curves. (Show your work). (5 points)

X=-4, x=3, y=0, and $y = 2x^2 + 4$

After grading RT1 and RT2 following the same rubric, in the next step, the principal researcher conducted a two-sample t-test with a significant level of α= 0.05 and 95 percent confidence interval over the mean scores of RT1 of the control and the treatment section. The result of the t-test is shown in the Table 38.

Table 38. Two sample t-test over the mean of the RT1.

Number of questions	Mean scores in the control section	Mean scores in the treatment section	df	p-value
Q1	2.87	3.625	14	.3942
Q2	2.75	4.37	11	.13
Q3	2.5	4.25	10	.0422

There is a significant difference between students' performance on Q3 in the control and treatment section (p-value = .0422<.05). which means students who used a GC in RT1 had a better result in finding maximum/minimum or saddle point compared to the students who did not use a GC on the same question with a 95 percent confidence interval and a significance level of α = 0.05. Although the mean scores on Q1 and Q2 are higher in treatment sections compared to the control sections, there are not any significance differences between students' performance on Q1 and Q2 for both groups. With the same method, data from RT2 were compared between the control and treatment section. The result of the t-test over RT2 is shown in the Table 39.

Table 39. The result of the t-test over RT2.

questions	Mean scores of control sections	Mean in the treatment section	df	p-value
Q2	3.6	2.18	19	.17
Q3	1.2	4	17.1	0.02535
Q6	4	4.45	17.3	.7745
Q7	1	2.7	17	.03745

There is a significant difference between students' performance on Q3 and Q7 of RT2 in the control and treatment sections. The p-value for questions 3 and 7 is less than 0.05 which indicates that students who used a GC on RT2 had better results on the concept of limit and integral compared to the students who did not use a GC on the same question with a 95 percent confidence interval and a significance level of α = 0.05. There are not any significant differences between students' mean scores on Q2 and Q6 of both groups.

4.21 The result of the analytic rubric applied to RT1 and RT2

RT1: The results of the t-test over RT1 show that students who used GC on the test had better achievement on Q3. To explore the effect of the use of a GC on calculus students' understanding and organization of their written work in Q3 of RT1 between control and treatment sections the qualitative rubric was applied to Q3. Since the sample size is small, the qualitative rubric was applied to all student papers for Q3 in both sections. The results are in Table 40.

Table 40. RT1 control and treatment sections by the analytic rubric.

C-LR-3	C-Or-3	T-Ug-3	T-LR-3	T-Or-3	T-Ug-3
1	1	0	3	3	3
2	2	0	3	3	3
1	1	0	2	2	0
0	0	0	3	2	0
1	1	0	3	2	0
1	1	0	3	3	0
3	3	0	1	1	0
3	3	0	3	3	0

RT2

The results of t-test over RT2 show that students in treatment sections had a higher mean score on Q3 and Q7. To dig deeper on how the use of GC affected students' understanding and organization of written work on Q3 and Q7 of RT2, the qualitative rubric was applied to Q3 and Q7 of students' papers in both control and treatment sections. Since the sample size is small, the qualitative rubric was applied to all papers for both sections. The results are shown in the Table 41 and Table 42.

Table 41. RT2 control group by the analytic rubric.

LR-3a	OR-3a	UG-3a	LR-3c	OR-3c	UG-3c	LR-7	OR-7	UG-7
1	0	1	0	0	0	0	0	0
1	0	0	0	0	0	1	1	0
1	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
2	2	2	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0
2	2	2	2	1	2	1	1	0
2	2	2	3	3	3	3	3	3
0	0	0	0	0	0	0	0	0

The results of the two-sample t-test over the results of the analytic rubric applied to RT1 and RT2 do not show any significant differences between students' logical reasoning, organization, and use of graph skills in control and treatment sections.

Table 42. RT2 treatment group by the analytic rubric.

LR-3a	OR-3a	UG-3a	LR-3c	OR-3c	UG-3c	LR-7	OR-7	UG-7
1	1	1	0	0	0	3	3	3
0	0	0	0	0	0	3	3	3
1	0	0	0	0	0	0	0	0
1	0	0	2	2	2	1	1	2
0	0	0	0	0	0	2	2	2
1	0	0	1	0	1	1	0	0
1	0	0	1	0	0	0	0	0
3	3	3	1	1	1	3	3	3
3	3	2	2	2	2	3	3	3
2	2	2	0	0	0	3	2	3
1	0	1	0	0	0	0	0	0

The number of calculus students who took RT1 in the treatment sections was 8 and 10 in the control sections. Eleven students in treatment sections took RT2. Since the sample size is small the t- test would not help to draw any conclusion.

4.22 Discussion of RT1 and RT2 for survey of calculus

Discussion of Q3 of RT1: Find all the local maxima, local minima, and saddle points of the given function: $f(x, y) = 4x^2 + 6xy + 8y^2 + 4x - 20y$.

In the control section four students produced a completely correct solution while five students in the treatment sections did so. Students in both sections used the second partial derivative test to solve the problems. One student in control and one in the treatment section used the second partial derivative test but incorrectly said (-2,2) is a local maximum. The sample of students' work is shown in Figure 41 and 42.

Although most of the students in both groups were able to answer this question correctly students' written work in the treatment section was more organized, neater, and shorter than the students in the control section.

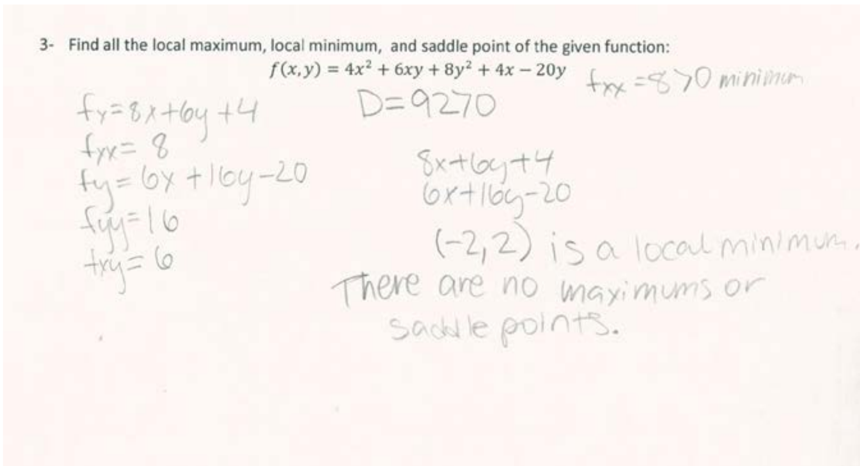


Figure 41. A sample of student papers in the treatment section

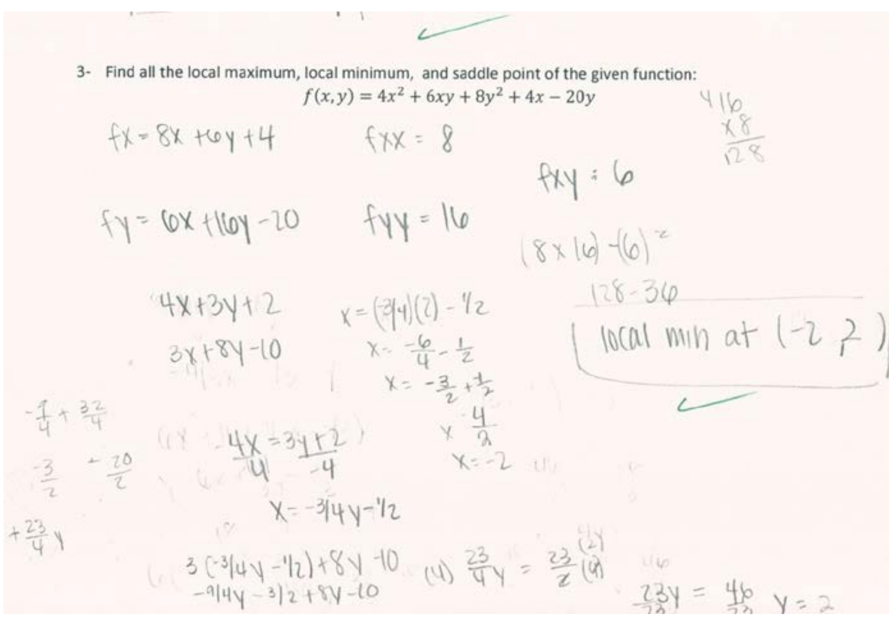


Figure 42. A sample of student papers in the control section

4.23 Discussion for Q3 and Q7 of RT2

Q3: a) Sketch a graph for the function below.

$$f(x)=\begin{cases} 3 & x < 0 \\ x^2+1 & 0 \leq x \leq 3 \\ 10 & x > 3 \end{cases}$$

d) For which x-values in the interval $[0, 3]$ do the limit of f(x) exists? Why?

In part (a) of Q3, most of the students in both control and treatment section had difficulty in drawing the graph correctly. Most of the students drew the graph of y=3, y=x^2+1 , and y=10 for all real numbers not for the stated domains.

Students in both groups had difficulty graphing the piecewise function. They were able to graph the lines and parabola separately but fail to combine these using the given intervals. A sample answer is given in Figure 43.

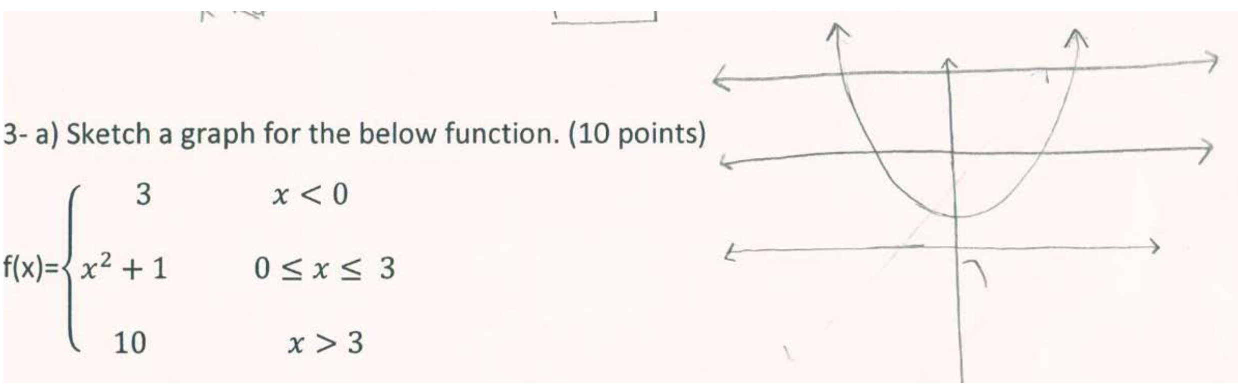


Figure 43. Sample of students answer for piecewise function

In part c, although some of the students were able to find the limit they did not use correct natation as seen in Figure 44. Students used MLP, which is computer-based testing. They learn by focusing on the answer, and they do not need correct notation, and they do not need to show their work for credit.

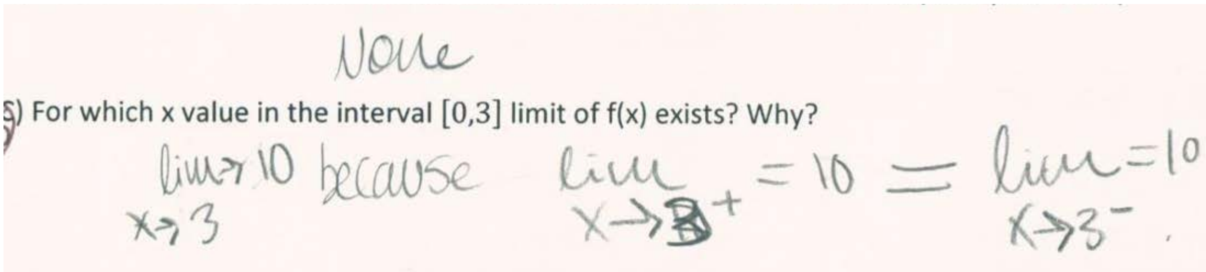


Figure 44. Sample of students' answer of limits

Q7: Find the area between the following curves. (Show your work). (5 points)

X=-4, x=3, y=0, and $y = 2x^2 + 4$

Most of the students in the control section even did not try to answer the question. Only 2 students out of 10 students in control sections responded to Q7, while almost all students in treatment sections tried to answer this question. Seven students in treatment sections produced a correct solution with correct notation. Some of the students in treatment sections drew the graph of quadratic using their calculator to shade the area under the curve. By using the graph, they were

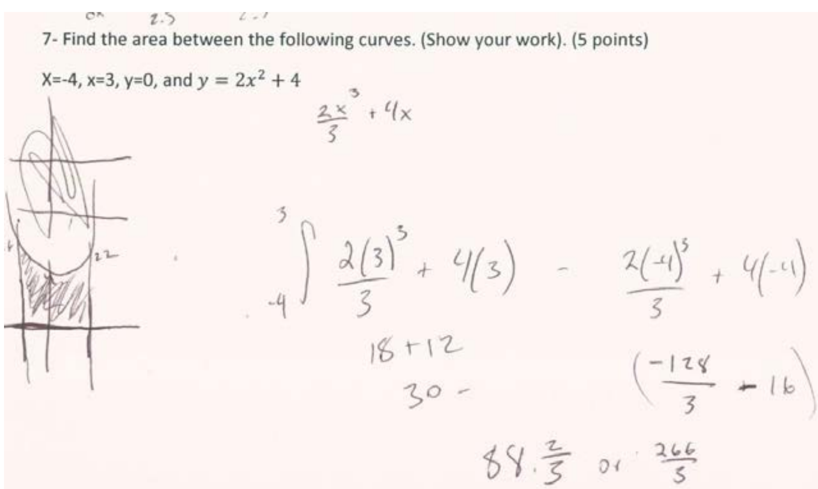


Figure 45. Sample of students answer in treatment section on definite integral.

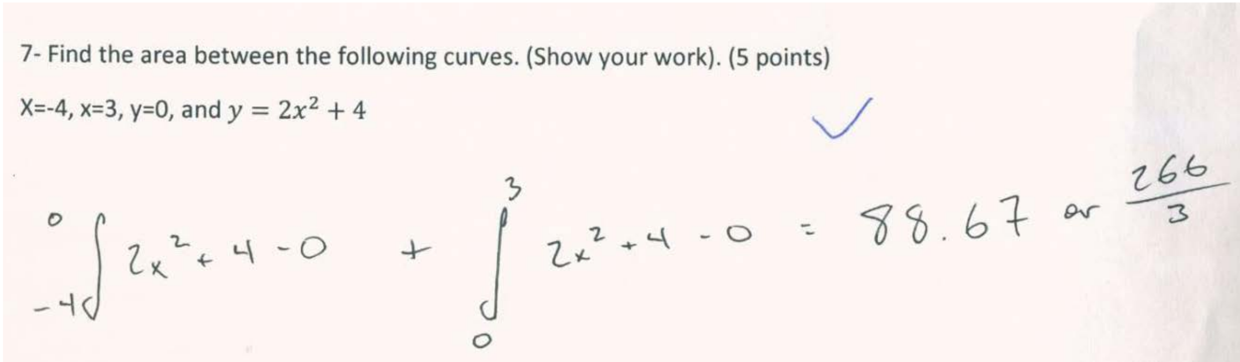


Figure 46. Sample of students answer in treatment section on definite integral.

able to find the lower and upper bounds for the integral. A sample of work is shown in Figure 45. Students in the treatment section used TI-84 to find the answer for the definite integral. Samples of student work are shown in Figure 46. Giving the final answer to two decimal places that they used GC. One can infer that the restriction on calculator usage in the control sections significantly affected

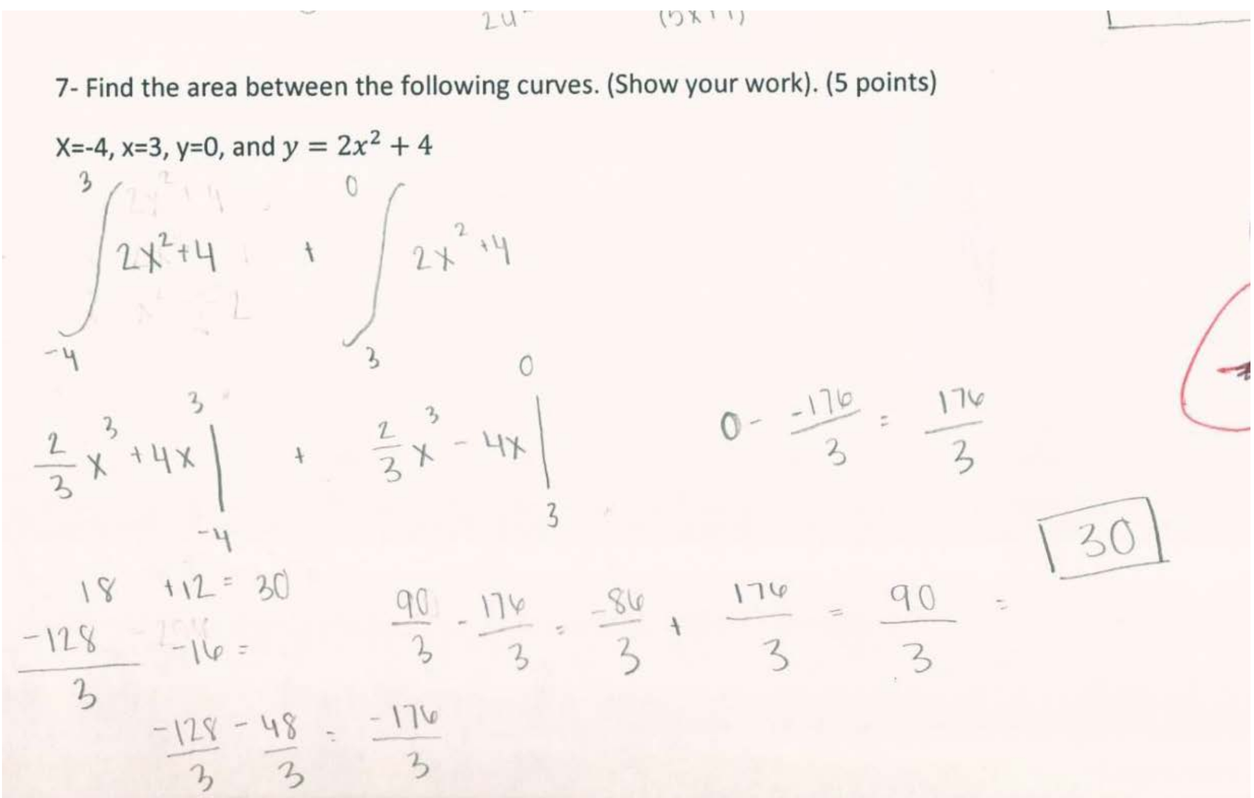


Figure 47. Sample of students answer in control section on definite integral.

students' self-confidence or success in evaluating the definite integral. A sample of student work in the control sections is shown in Figure 47.

In this sample, although the student was able to find the correct limits and the antiderivatives the calculation is incorrect. One can safely say that using GC will help students in evaluating definite integrals. Also, GC can help students find the limits of integrals efficiently, and GC can help students visualize the curves and have a good understanding of the area above or below the curve.

4.24 Summary of results in the survey of calculus

In summary, there is a significant difference between students' performance on Q3 in the control and treatment which means students who used a GC in the RT1 had a better result in finding maxima/ minima or saddle points compared to the students who did not use a GC on the same question. The results show a significant difference between students' performance on the Q3 and Q7 of RT2 between the control and treatment sections. Students who used a GC on RT2 had a better result in the concept of limit and integral compared to the students who did not use a GC. The results of the qualitative rubric applied to RT1 and RT2 do not show any significant differences between students' logical reasoning, organization, and use of graph skills in control and treatment sections. The qualitative analyzing of students' papers on the stated problems show that although most of the students in both groups were able to answer Q3 on review test correctly, students from the treatment section had more organized, neater, and shorter work than the students in the control section. Question 3 (Q3) of RT2 asked students to graph a piecewise function and find the limit. Students were able to graph the line, parabola and constant line separately but failed to combine these using the defined intervals. Although some of the students were able to find the limit, they did not use correct notation. For example, they used $\lim_{x\to 3} = 10$ which lacks the function expression after limit sign. One reason for not writing the correct notation could be the use of

MLP, which is a computer-based testing. This may lead students to focus on the answer rather than correct notations. In addition, on MLP students do not need to show their work for credit.

For the definite integral question, only 2 students out of 10 in the control section were able to produce a correct answer, and the rest of students did not try to answer it. While almost all students in the treatment sections tried to answer this question and 7 of them produced a completely correct solution. Some of the students in treatment sections drew the graph of quadratic using their calculator and shaded the area under the curve. By using the graph, they were able to find the lower and upper bound for the integral. One can say using GC would help students in evaluating definite integrals. Thus GC can help students to find the bounds of integrals and calculate the values of definite integrals. In addition, GC could help students visualize the curves and have a good understanding of the area above or below the curve.

4.25 Interview

As a reminder to the readers, after all the written tests were taken, some students in both control and treatment sections of college algebra and calculus were interviewed. Students were asked general questions about their attitude toward using technology in mathematics courses and specific questions about the way they used the graphing calculators for solving the problems on written tests. Five college algebra and three survey of calculus students were interviewed. The audios were transcribed and coded to summarize students' attitudes toward GC and their approach to solving problems. The following codes, important to the research questions, were noted in the transcripts of college algebra interviews.

- Zeros and y-intercept of a function.
- Domain, vertical and horizontal asymptotes, and holes of a function
- Students' views of the effect of GC on the organization of written work

- Logarithm
- Were students encouraged to use GC?
- Use of GC in MLP and concept test

Zeros and y-intercept: As the first question was about zeros and y-intercepts, the interviewer asked about the definition of zeros and y-intercept. Most of the students had a clear understanding of the definitions of zeros and y-intercept of a function. For example, they mentioned that zeros are the x-intercepts, i.e. when the graph touches the x axis. The y- intercept is when the graph touches the y axis. This means that most of the students have a graphical view of zeros and y-intercept. To find the zeros most of them preferred to use GC and especially using the zero finder of GC except one of them who calculated manually mentioned that for complicated functions he uses GC. To find the y-intercept, all of them evaluated the value of the function at x = 0.

Domain: Students defined the domain of a function as the values of x that give appropriate output. All of them knew that holes and vertical asymptotes should be excluded from the domain. Students unanimously defined asymptotes as a straight line that the graph cannot touch. Only one added that the function becomes infinitely close to the line but never touches it. To find the domain of a function three of them found a hole and asymptotes manually and excluded them from the domain. Two others used GC to graph the function and used the graph to find the domain. When the students were asked how to find the domain using GC, one mentioned he would graph it and look at the graph for holes and asymptotes. Some knew that GC has a limitation in showing the holes in the function and mentioned that they knew they should be cautious and therefore found it manually. One mentioned that:

> "*I just plug it that equation into the y =, and there where shows vertical and horizontal asymptote. But I think it does not show the hole unless you look at the table function.* "

The notation was another feature that students were asked about. One mentioned that:

> *"I actually don't like the interval notation, so I say x is the elements of real numbers except it cannot be equal -2 and 03. "*

In addition, because of range limitation on t he GC screen some of them mentioned finding asymptotes using GC is not easy. Nevertheless, most of them checked their work with GC and even for holes they plugged in the hole value in the function to check their work.

The effect of GC on the organization of written work: All the interviewees had the feeling that using GC would help them to have more organized written works. This includes those who did not use GC heavily. However, one of them added:

> *"I think it will be more organized because you will not work them out all. I feel it will be more organized but if somebody wants to follow your work maybe hand-written is better because some steps will be skipped when you use GC"*

Students' encouragement to use GC: All the students mentioned that their instructor encouraged them to produce answers both manually and using GC.

Using GC in MLP and concept test: Interviewed students mentioned a different issue about using GC in MLP. One of them mentioned he did not use GC heavily on MLP because MLP software does not like the format of the numbers from GC, which leads to problems. Another student mentioned that he tried to use GC on every problem just to check his result. One mentioned that he used GC more than he does normally and mainly because he wanted to check the results. The other mentioned that for some concepts such as systems of equations he used GC on the MLP test.

The concept of logarithm: Students were asked about the definition of logarithms. Only two out of five stated that logarithm is the inverse of an exponential function. One of them knew there is a connection between logarithms and exponential functions but did not exactly know the relation.

She talked about the similarity of converting log of a multiplication to the sum of the logarithms with converting the multiplication of two exponentials with the same basis to one exponential. The other two students could not define logarithm and did not know about its relationship with exponential function. They just mentioned they know the properties and mentioned that the level of the difficulty of logarithms is like other concepts of this course.

The following codes were noted in the transcripts of survey of calculus interviews.

- Limit
- Derivatives
- Integrals
- General view about GC and Desmos
- Organization of written work

The limit was the first subject of the interview for survey of calculus students. All three of them mentioned that they did not use GC to answer the limit question. Instead they used the limit rules. For the question where the graph was given, they thought that GC was not needed; however, they used the graph to answer the question. Another question about limit was answered with three different approaches. One answered it using the rules and without a graphing calculator. Another solved it by GC using a table. He mentioned:

> *"I put the whole equation in the GC and then used the table and looked around 1 and made sure there is not any hole near the values. "*

The last one graphed the function and calculated the limit manually by rules and checked them together.

Derivatives are the next concept that was discussed. The first question asked students to find the equation of the tangent line to the graph of a function at a certain point. All three students solved the problem manually. They found the derivative and evaluated the derivative function to find the slope. Students were asked to make a connection between extrema of a function and its derivative. To find the extrema of the functions one graphed the function and traced the graph and visually located the extremum (unaware of the fact that there may be another extremum out of the screen boundaries). The other two set the derivative equal zero and found the extrema. Students were asked to find the slope of the tangent line at a local minimum or maximum and they correctly mentioned it will be zero. However, when they are asked to talk about the graphical correlation of the derivative and the original function, one of them did not efficiently say that the value of the derivative at each point is equal to the slope of the tangent line to the graph. Concavity was also another feature that was explored. Two of the students used the second derivative to determine whether the graph of the function is concave down or up. One used a graphical approach where she graphed the function and looked at the graph to see if it is concave down or up (again unaware of the fact that there maybe something relevant exists out of the boundaries of the screen). She stated that:

> *"I would use my GC and graph the original function and then try to solve it the and then I do not do the second derivative and just look at the graph to find concave down or up"*

Students were also asked about their view of integrals. Two of them could make a connection between the integral and the area under the graph of a function. When they were asked about the unknown integral all three of them mentioned it means finding the anti-derivative.

One student believed that anti-derivative was the hardest part of the entire course because she must remember lots of rules, and when she was asked why she did not have a problem with derivative which has the same number of rules she mentioned that:

"Because my mind was working forward, and it was hard to turn it around!"

Two of the students were asked about their opinion on the effect of GC on the organization of their written work. Both believed that GC and Desmos had a positive effect on the organization of their written work. However, one mentioned that sometimes she forgets to write things down, but she generally likes to write things down, which sometimes introduces some error in the writing formulas.

In general, all three believe that GC could be helpful in their understanding and they did not heavily depend on i t. One student in control section also mentioned that integral was harder for him compared to other topics because they did not use technology that much.

4.26 Summary of response to the four research questions:

In summary, multiple data sources were collected to address the research questions. Different methods of data analysis and statistical tools were used for analyzing the qualitative and quantitative data. The findings are evidenced by students written work, student interviews, and students' grades on several tests, and students' responses to attitude survey for college algebra and survey of calculus courses.

Research question 1. Research question one is about the effect of the use of technology on students' understanding and achievements in college algebra and survey of calculus courses. The results of the concept test, RT1 and RT2 indicate that college algebra students have a greater mean score when they used a graphing calculator compared to the time that they did not use a graphing calculator on the similar mathematical concept. Students who used a GC in the concept test had a higher mean score compared to students who did not use a GC in the same test. In addition, survey of calculus students who used a graphing calculator had a higher mean score in RT1 and RT2 compared to those who did not use a graphing calculator on the same tests.

Research question 2. Research question 2 asks "what areas of college algebra and calculus are affected more by technology?" There are several areas that were affected significantly by using technology. College algebra students who used graphing calculators have a better understanding of x-intercepts and y-intercept, domain of a function, end behavior, vertical and horizontal asymptote. However, the performance of students on function composition was similar.

Survey of calculus students who used a graphing calculator have a better understanding of finding maximum and minimum for two variable functions. They also have a better understanding of the concept of limit and definite integrals. However, students had similar performances on derivative problems, indefinite integrals and limits that need the use of rules.

Research question 3. Question 3 asks how using technology affects the organization of college algebra and survey of calculus students' written work. The designed qualitative rubric has three aspects of students written work which is i) reasoning ii) written order iii) use of symbol and notation. The results of qualitative rubric applied to RT1 and RT2 and the concept test show that there is a significant difference between logical reasoning, written order, and correct use of symbol and notation of students when they used graphing calculator compared to the time that they did not use. Students' written work is more organized, neater, and they use more correct notation when they used a GC in their test. They also able to derive more information from a graph that they produced by a GC themselves compared to the time that the graph was given in the test. However, no significance difference was observed for the case of students in the survey of calculus class. In addition, all the interviewed students believe that use of technology enhances the organization of their written work.

Research question 4. Does the use of technology positively impact college algebra and calculus students' attitudes toward their mathematics skills? The results of pre- and post- surveys of the control sections show that students' negative view of the use of GC, self-impression of the use of GC skills, and their positive view of mathematics did not change significantly. Nevertheless, their positive view of the use of GC increased. In the treatment section no significance difference was observed between pre- and post-survey in all mentioned features. No significance difference between students view to all sub-categories of pre- and post-survey was observed.

The results reveal that there is a p ositive correlation between students' self-impression of mathematics skills and their positive view of the use of a GC.

Chapter 5 Logarithm

5.1 Introduction

Mobile learning (m-learning) is the use of mobile or wireless devices such as smartphones, tablets, and laptops for learning and teaching (Traxler & Kukulska-Hulme, 2005). With the mobile revolution in the recent years, researchers in different academic areas have considered the potential use of mobile devices in education. However, the use of these devices in educational research has not been as rapid as technological development. In fact, the development of apps in everyday life has been so fast that apps are widely used by students in their education, and often teachers are behind the students in using apps. Nevertheless, research on the effectiveness of apps in education has gained momentum in recent years (Park, 2011; Handal et al., 2013; Hargis et al., 2014).

5.2 Purpose of the study

From our own teaching experience and through discussions with other instructors, we found that most college algebra students struggle with the topic of logarithms. Therefore, this study was designed to figure out a more effective way to teach the concept of logarithms. The purpose of this research is to investigate the effect of using tablet and smartphone apps on student learning when teaching the concept of logarithms to college algebra students.

5.3 Research questions and hypotheses

The research questions for our study are:

1. Does the use of mobile apps in teaching influence college algebra students' learning achievement?
2. In what areas of students' understanding of logarithms is the use of mobile apps most and least effective?

The null hypothesis (H_0) states that the mean score of college algebra students on a written test is unchanged by using mobile apps. ($\mu c = \mu e$). Alternatively (H_a) states that the mean score of college algebra students in a written test is greater when they did use mobile apps (experiment) compare to the time that they did not use this technology (control) in a written test $\mu e > \mu c$.

5.4 Literature Review

The use of technology in education traces back to many studies on the effect of graphing calculator on students' learning. Mobile devices have many features that potentially can be used to enhance learning. Visualization of the concepts, providing instant feedback, student engagement, and self-directed learning are some of the features of mobile learning (Sung, Change, & Liu, 2016). An emerging number of research studies have explored the effectiveness of mobile devices and specifically smartphone apps in mathematics education (Blair, 2013; Handal, El-Khoury, Campbell, & Cavanagh, 2013b; Hargis et al., 2014). Smartphone or tablet apps have more potential compared to fixed computers in lab because they are more aligned with the preferred method of students as they have physical touch, trial and error and ease of use. Many studies have been conducted on the quality of apps in mathematics education. However most of these studies are general review of numerous apps (Highfield & Goodwin, 2013; Larkin, 2013) or suggestion on the methods and criteria to choose apps for classroom (Cherner, Dix, & Lee, 2014; Handal, El-Khoury, Campbell, & Cavanagh, 2013c; Park, 2011). For example, Handal et al. (2013) categorized mobile apps in mathematics education based on functionality. These efforts help teachers in selecting effective apps from the many that are available. Most of the studies on the effect of apps on teaching math are in K-12 grades. The research on apps in college mathematics education is much limited compared to the use of apps in other disciplines such as engineering and language. Therefore, in this study we focus on the use of apps in teaching college level mathematics.

Based on our teaching experience and as also reported in the literature (Kenney, 2005; Larkin, 2013), we found that college algebra students have difficulty in learning logarithms. To address this issue some research studies focused on students' view of logarithm. Weber (Weber, 2002) introduced constructions that help students understand the concept of logarithm. This study concluded that most of students cannot think of logarithm as a process. Berezovski and Zazkis (2006) argued that most students do not treat a logarithm as a number, and they try to simplify terms like $\log_2 3$. They may try to reduce it to a fraction. These authors suggest that logarithms such as $\log_2 3$ should be treated as an object. They also suggested using the exponent definition for logarithms and developing the logarithmic rules based on exponents. Kenney and Kastberg (2013) interviewed students about their knowledge of logarithms and found that those who memorize logarithm rules are more likely to make mistakes in reconstructing or applying the rules. After reviewing the literature, one can see that logarithms have many features that are confusing for students. Logarithm notation is different from other concepts and because it is the inverse of exponential function, the concept of inverse is also a source of confusion for many students (Weber, 2002). Therefore, the dual nature of logarithm function, as a process and an object, makes the understanding of logarithm difficult for new learners (Kinzel, 1999; Sajka, 2003). In this study, we have investigated the effect of teaching logarithms with mobile apps on students' understanding of this concept.

5.5 Method

Participants and Setting: One hundred forty-three undergraduate students enrolled in four different sections of the college algebra course participated in this study. These course sections were randomly selected and consent from the students to participate in this study was then obtained. Participants were freshmen with various majors from a diverse student body. Two sections of the course were identified as treatment groups and were asked to use educational apps during class activities when teaching the topic of logarithms. The two remaining sections served as control groups. The study took place in the fall 2016 at the University of Arkansas.

Design of the study: Students received five weeks (total of 15 hrs.) instruction in logarithmic functions as part of their normal college algebra topics sequence during this study. Concepts that were covered during this five-week period included: exponential and logarithmic functions, logarithms and their properties, graphs of exponential and log functions, solving logarithmic and exponential equations. These concepts were introduced to students by lecture and PowerPoint slides for both treatment and control groups, which started with basic concepts such as definition, graphs, and properties. Two instructors taught these four sections. Each instructor taught one control section and one treatment section. Both treatment and control sections were given the same test for assessment. Most of the examples presented during the lecture were the same for all sections except when the instructors used websites (e.g. Khan Academy).

Control sections: In the two control sections traditional lecture method of teaching was used. Power-point slides were used to introduce definitions and show examples. Worksheets were given to students for independent or group practice. Instructors introduced logarithmic concepts including definitions, relation to exponential functions, domain, and basic properties. Product rules, quotient rules, and power rules were taught. Instructors worked out examples on the board to show students how to solve logarithmic and exponential equations. The growth and decay applications of logarithms to real life problems were also presented. In addition, expanding and simplifying logarithmic expressions were covered. At the end, students were assessed and interviewed.

Treatment sections: In the two treatment sections the instructors introduced smartphone applications as part of teaching and used them during classroom activities. The students, after having worked on exercises that existed in the apps, were asked to send the screenshots of their results to their instructors. Details of the applications used in the treatment sections are included below.

The *Logtrainer* application is a tutorial and practice-based app that has multiple-choice questions. In this app, questions were mostly about converting logarithms to exponentials. After clicking on one of the answer choices for a question, the students could see the correct answer and a complete explanation of a similar problem. In this app, similar problems were repeated several times to help students understand logarithms by using their knowledge of exponentials. Sample problem from this app are shown in Figure 49.

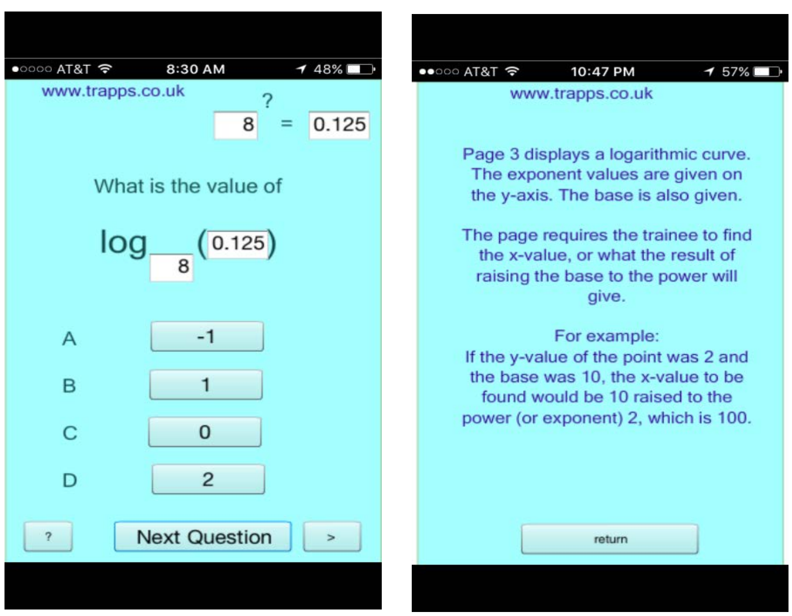


Figure 48. A Sample problem on apps.

In other sessions, students used the *Logarithms* app's activities. This app consists of four parts: logarithm rules, simplification, expansion, and solving logarithmic equations. Each part includes some multiple-choice problems relating to the topic. Students were able to see correct answers immediately after picking one of the answer options. The complete solution to each question was provided in this app; therefore, students were able to review their work and correct themselves. Sample problems from this app are shown in Figure 50.

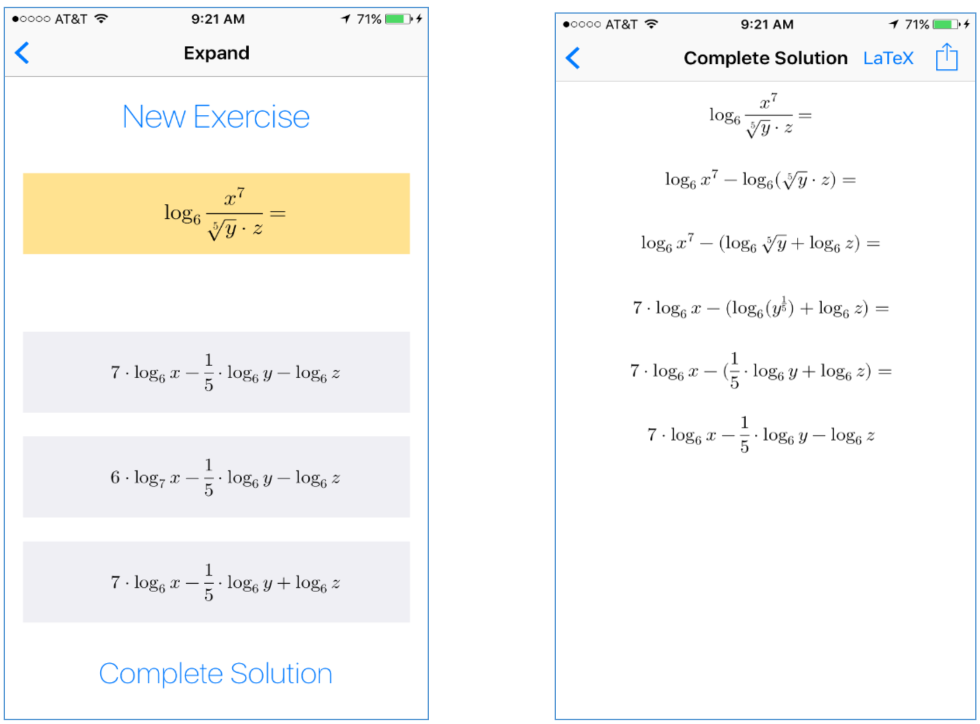


Figure 49. Sample problem from Logarithm app.

5.6 Data collection

Data from multiple sources were collected to address the research questions. Some of these sources were standardized/placement tests that were taken before students began attending college and some of them were designed by the course coordinators as part of the material for the college algebra course. The following are all the data sources of this study:

Data from ACT scores: ACT scores of college algebra students were collected for both control and treatment sections.

Data from written test: Students' written exam that assessed the topic of logarithmic functions was collected for both control and treatment sections. Only 59 students in treatment and 49 in the control group section took the test. The written test was designed by two instructors and the portion of the test which assessed the topic of logarithms consisted of three questions shown in Figure 3. The questions were designed in view of Bloom's definition of six levels of intellectual behaviors (Krathwohl, 1956). The questions were valued at 10, 5, a nd 5 poi nts, respectively. All three questions were open response and students were not allowed to use a graphing calculator, but they could use scientific calculators. This test was graded based on the same rubric for all sections.

Q1	What is a logarithm? (10 points)
Q2	Produce an argument that could convince a friend of the following. (5 points) a) $\log_b(M+N) \neq \log_b M + \log_b N$ b) $\frac{\log_2 27}{\log_2 9} \neq \log_2(\frac{27}{9})$
Q3	Write an expression as a sum and/or difference of logarithms. (5 points) $a)$ $\log_4(\frac{x^3}{z.\sqrt{y}})$ b) $\log_5(2x8y^3)$

Figure 3. Test Questions

Data from student interviews: Ten students in control sections and ten from treatment sections of college algebra were interviewed separately. These semi-structured interview sessions were 20-minutes long, conducted face-to-face, and were audio-taped with students' permission.

The purpose of interviews was to explore the level of students' understanding of the concept logarithms and exponential functions. In addition, the interviews examined students' attitudes toward using technology, specifically, apps in learning logarithms. General questions about using technology in mathematics courses were asked. In addition, students were asked specific questions about the way that they used technology for solving a logarithm problem and problems in the written test. Some questions were asked to clarify how students understood concepts such as functions and logarithms. Student interviews were transcribed and analyzed qualitatively using open coding (Corbin & Strauss, 2008).

5.7 Data analysis

To answer the first research question (Does the use of mobile apps in teaching influence college algebra students' learning achievement?), a statistical two-sample t-test with significance level of $\alpha=0.05$ was performed on mean of students written test scores of the treatment sections versus the control sections. In addition, the same t-test was performed on the ACT scores of the treatment and control sections to check if the control sections and the treatment sections have the same level of pre-knowledge.

To answer the second research question (In what areas of students' understanding of logarithms is the use of mobile apps most and least effective?), students' written exams were qualitatively analyzed using open coding (Corbin & Strauss, 2008). For this purpose, we collected different types of students' answers and coded them. Codes were then combined and categorized into five groups based on frequency of answers. Moreover, the mistakes were categorized in the same way into two common types. Interview data were also used to answer research question 2. Students' interview transcripts were also coded from which three main categories emerged i) students' understanding of logarithmic functions, ii) effect of technology on learning logarithms, and iii) students' understanding of logarithmic properties.

5.8 Results and discussion

Result from written test and ACT scores: The mean of the scores on the written test for the control section was 12.42 while for the treatment section was 14.16 which shows improvement in the scores of the treatment section. These results show that there is a significant difference between the mean score of treatment and control sections, which implies that students who used apps in class activities performed better on the written test. Most improvement was observed on the first question. However, there is not enough statistical evidence to show significant difference in the performance of students on the other two test questions. The summary of t-tests is shown in Table 43.

Table 43. Mean and p-value for the control and treatment sections

Number of Questions:	Mean of the control sections	Frequency of the control sections	Mean of the treatment sections	Frequency of the treatment sections	Degrees of freedom	P-value
Q1	4.82	49	6.31	59	103.92	0.0109
Q2	3.42	49	3.64	59	101	0.5117
Q3	4.17	49	4.20	59	101.96	0.8693

The means of ACT math scores were also compared, and we found no significant difference between the pre-knowledge of students for both groups.

5.9 Qualitative findings

Types of students' answers: On the first exam question (Q1) students were asked to describe a logarithm. All students' written tests were coded for both control and treatment sections. Students' answers for Q1 were coded as one of the five types: i) logarithm as a function, ii) logarithm as an inverse of an exponential function, iii) logarithm as an exponent, iv) logarithm as an equation and v) logarithm as properties and rules. The frequency of students' answers into each category are shown in Table 44.

The number of students who described logarithms as an inverse of the exponential function were significantly different between the control and treatment sections. More students in the control sections described logarithm as an equation.

Table 44. Classification of students' correct responses for Q1.

Q 1: What is a logarithm?	Number of students from the treatment sections (N=59)	Number of students from the control sections (N=49)
Defined logarithm as a function	20	19
Described logarithm as an inverse of an exponential function	33	21
Described logarithm as an exponent	13	10
Described logarithm as an equation	4	7
Used logarithm rules to describe logarithms	7	7

Students who used apps in class activity were able to make a better connection between logarithms and exponential functions while students who were taught using worksheets and traditional lecture method described logarithms as an equation.

Students' answers on the second test question were coded and classified as one of two categories – counter-examples and logarithmic rules. Answers that used a counter-example to verify each statement were classified under category 1, and answers that depended on logarithm rules (product and quotient rules) were classified under category 2. Part a of the 2nd test question asked students to present an argument to verify $\log_b(M+N) \neq \log_b M + \log_b N$. A few students tried to compare $\log_b M + \log_b N$ with the product rule and they came up with $\log_b(MN)$; therefore, they explained that the left hand side of the statement does not represent the product rule and therefore the statement, $\log_b(M+N) \neq \log_b M + \log_b N$, is correct. Four students in each of the control and treatment sections used a counter-example to verify the statements. The number of students who used logarithm rules were similar for the control and treatment sections, 36 versus 33. Frequency of the solution types is shown in the Table 45.

Table 45. Categories of students' responses for Q2.

Q 2: $a)$ $\log_b(M+N) \neq \log_b M + \log_b N$ b) $\frac{\log_2 27}{\log_2 9} \neq \log_2(\frac{27}{9})$	Number of students from the treatment sections (N=59)	Number of students from the control sections (N=49)
Answer that used a counter-example to verify the statement.	4	4
Answer that used logarithm rules to verify statement.	36	33

In question 3, students were asked to write an expression as a sum and/or difference of logarithms. Students commonly misapplied the power rule. Table 46 gives a frequency of students' responses.

Table 46. Students' most common mistake

Q3: $\log_5(2x8y^3)$	Most common mistake: $\log_5(2x8y^3)$= $\log_5(2x)$+ 3$\log_5(8y)$
Number of students in the treatment sections who made a mistake applying the power rule.	20 students
Number of students in the control sections who made a mistake applying the power rule.	30 students

Students either misapplied the power rule or they did not simplify the logarithm. In the control sections 30 out of 49 (61%) students misused the power rule, while only 20 out of 59 (34%) of students in the treatment sections misapplied the power rule.

5.10 Interview findings

A sample of ten volunteer students (5 from control and 5 from treatment) were interviewed about the three questions on the test. Students were asked to explain their understanding about the concept of logarithms. Five students in the control sections mentioned that they just do not really know what logarithm means, they only know logarithm properties, and they can apply logarithm rules to solve problems. For example, one student mentioned

"I *don't know what it is, but I know the logarithm properties."*

Four students from the treatment sections defined logarithm based on exponential functions, and they were aware of the relationship between logarithms and exponentials. For example, one mentioned that:

> *"A logarithm is the inverse operation to an exponential function. It represents a power to which the base is raising".*

One student used the graph of logarithm to define it. Students in the treatment sections were asked if using apps helped them to have a better understanding of logarithms and logarithm rules. All interviewed students mentioned apps helped them to learn better but one of them mentioned that:

> *"I think I used them and they were useful for me but when you are on your phone and doing math you will get distracted*"

When students in the control and treatment sections were asked to explain why $\log_b(M+N)$ *and* $\log_b M + \log_b N$ are not equal, all student responses referred to the product rule. They mentioned that they only memorized the rules and do not know reasons behind them. Students were asked about the definition of logarithms. Only two out of five stated that logarithm is the inverse of an exponential function. One of them knew there is a connection between logarithms and exponential functions but did not exactly know the relation. She talked about the similarity of converting log of a multiplication to the sum of the logarithms with converting the multiplication of two exponentials with the same basis to one exponential.

5.11 Discussion and Conclusion

To investigate the effect of using tablet and smartphone apps on student learning when teaching the concept of logarithms, we analyzed written tests, data from ACT scores, and interviews that were collected from students enrolled in four different college algebra sections. Our aim was to figure out a more effective way to teach the concept of logarithms in introductory college math courses. The results of this study reveal that smartphone applications can facilitate learning of mathematical concepts as they helped students' learning of logarithms and exponential functions.

During, this study, researchers were aware of students' difficulties in understanding the concept of logarithms. The results of the study indicate that teaching logarithms using apps will have positive effects on students' understanding of logarithms. One of the positive effects is making a better connection between logarithms and exponentials. In this study 56% of the students in experimental sections (using apps) described logarithms as an inverse of an exponential function, while the percentage for the control sections were 43%. Furthermore, there was a significant difference between students' performance in the control and experimental sections in applying the power rule. The power rule was misapplied by 34% of students in the treatment sections, and 61.2% by students in the control sections. In addition, the findings of the interview data indicate that students felt that the using apps helped them better understand the concept of logarithms.

Smartphone applications are a relatively new technology that can potentially help students in understanding challenging mathematical concepts. Despite the use of smartphone apps in K-12 mathematics and college level in non-mathematics courses, the use of this technology in college level mathematics is very limited. Our study shows that smartphone applications can enhance students' understanding of logarithms, which has been reported and observed as a challenging concept. The dual nature of logarithms as an object as well as a process makes the understanding of this concept even more problematic for students in introductory math courses (Kinzel, 1999; Sajka, 2003). Our study provides a starting point for considering the use of apps in college-level math courses such as college algebra to help students develop a better understanding of mathematical concepts such as functions and logarithms.

Chapter 6 Summary, conclusions, and recommendations

6.1 Summary

This mixed qualitative and quantitative methods study addressed the effect of technology on college algebra and survey of calculus students' understanding. This research study was conducted in fall 2016 on eight college algebra classes with a total of 315 students, and in summer 2017, in two surveys of calculus classes with a total of 40 students at the University of Arkansas. College algebra students who were mostly freshman, participated in three, fifty-minute class meetings every week. Survey of calculus was taken by business students or students who do not plan to take further calculus courses. Both college algebra and survey of calculus were three-semester credit hour courses. Four college algebra classes were considered as control sections in which students did not use a graphing calculator (GC) on a concept test and four sections as treatment in which students used a GC on the same concept test. All college algebra sections used a GC on one review test, RT1, and did not use a GC on a second review test, RT2.

Several sources were used to collect data. A pre- and post- student attitude was administrated during the first and last week of the semester for both college algebra and survey of calculus courses. Students' scores and paper work on three written tests (RT1 and RT2 and concept test) in college algebra and students' scores and paper work on two written tests (RT1 and RT2) in survey of calculus were collected. The concept test was the only paper test normally administered in college algebra. There were seven open-ended questions designed by the coordinator of the course. Both RT1 and RT2 were designed to give the principal researcher more data on students' written work. The RT1 was a graphing calculator-based test and RT2 was a non-graphing calculator-based test including some open-ended problems. Some common concepts were included on RT1 and RT2.

The RT1 was taken just before the midterm exam and the RT2 was taken just before the final exam. A week before the final exam, after all the written tests were taken, a few students were interviewed in both college algebra and survey of calculus.

A pre- and post- student attitude survey had 24 items was completed during the first and last week of the semester for both college algebra and survey of calculus courses. Thirteen of the 24 survey items were selected for analysis based on relevancy to the research questions. These 13 were divided into general items and personal items. General items were devided into three subcategories: positive view in use of a GC, positive view about mathematics, and negative view about mathematics. Personal items, items that have the subject "I", divided into three subcategories including a negative view of the use of a GC, an individual skill of use of a GC, and students' importation of their mathematics ability. The results of the mentioned categories were compared in several ways such as control vs treatment sections.

Quantitative and qualitative data analysis enables the researcher to discuss on the effect of technology on students' understanding and organization of their work. This research study was guided by the following research questions.

1. How does the use of technology affect college algebra and calculus students' understanding and performance?
2. What areas of college algebra and calculus are affected more by technology?
3. How does using technology affect the organization of college algebra and calculus students' written work?
4. Does the use of technology positively impact college algebra and calculus students' attitudes toward their mathematics skills?

To answer research question one, two sources of data were used. First, qualitative data from students' interviews that were transcribed and coded, based on open coding methods. The following codes were noted in the transcripts of college algebra interviews: zeros and y-intercept of a function, domain, vertical and horizontal asymptotes, and holes of a function, students' views about the effect of GC on the organization of written work, logarithm, how many students were encouraged to use GC, use of GC in MLP and concept test. For survey of calculus the following codes were noted: students understanding of limits, derivatives, and integrals; students' general views about the use of GC and Desmos, and students' views on the effect of GC on the organization of their written work. In addition, data from the analytic rubric that was applied to students' written work was used. The designed qualitative rubric has three aspects of students' written work, which are i) reasoning ii) written order iii) use of symbol and notation. This analytic rubric was used to investigate whether the use of GC influences students' performances on the mentioned skills. To apply the analytic rubric on students' papers proportional stratified random sampling was used to select the sample size. A memo on how to use an analytic rubric to score students' papers was provided and a statistical two-sample t-test was conducted over mean score obtained using the analytic rubric between control and treatment sections. To answer the second part of research question one, the effect of technology on students' performances from students' grades on RT1, RT2 and the concept test between control and treatment sections were compered by statistical two-sample t-test.

In answering research questions two and three, multiple sources of data were used. For example, the result of data from students' grades in RT1 and RT2, data from analytic rubric applied to students' written work and data from students' interviews.

The fourth research question, which asked about the effect of technology on students' impression of their mathematics skills, was answered by the result of data from students pre- and post-survey as well as students' interview.

Results from research question one showed that the college algebra students had a greater mean score when they used a GC compared to the time that they did not use a GC on the similar mathematical concepts. Students who used a GC on the concept test had a higher mean score compared to students who did not use a GC in the same test. In addition, survey of calculus students who used a GC had a higher mean score on RT1 and RT2 than those who did not use a GC on the same tests.

The results applied to research question two reveals that college algebra students who used GCs have better understanding of x- and y-intercept, domain of a function, end behavior, vertical and horizontal asymptote. However, the performances of students on function composition and word problems were similar. In addition, college algebra students who used smartphone applications in their class activities were able to make better connections between logarithms and exponential functions. They also were able to use the power rule more accurately compared to students who did not use apps.

Survey of calculus students who used a GC have a better understanding of finding maximum and minimum for functions of two variables. They also have a better understanding of the concepts of limit and definite integrals. However, students had similar performances on derivative problems, indefinite integrals and limits that need the use of rules.

The results applied to research question three, based on the results obtaining by applying analytic rubric to RT1 and RT2, and the concepts test, show that there is a significant difference between logical reasoning, written order, and correct use of symbol and notation of students when they used

GC compared to the time that they did not use a GC on the similar tests. Students' written work is more organized, neater, with more correct notation when they used a GC on their test. They also able to derive more information from a graph that they produced by a GC themselves compared to when the graph was given in the test. However, no significance difference was observed for the case of students in the survey of calculus class. In addition, all the interviewed students in college algebra and survey of calculus courses believed that use of technology enhances the organization of their written work. Even students who mentioned that they preferred to work on problems manually still believed that use of a GC would positively affect the organization of their written work.

The answer to research question four that asked about the effect of technology on students' impression of their mathematics skills shows that students' self-impression of the use of GC skills, negative view of the use of GC, and positive view about their mathematics skills did not change significantly from pre-survey to post-survey in the control sections. Nevertheless, their positive view of the use of GC increased. In the treatment sections no significance difference was observed between pre- and post-survey in all mentioned features. No significant difference between students' views to all sub-categories of pre- and post-survey were observed as well.

The results of analyzing students' responses to surveys reveal that there was a positive correlation between students' self-impression of mathematics skills and their positive view of the use of a GC. Students who believed "I am good in math" believed GC can make math more fun and makes learning algebra easer. Students' mathematics skills appear directly correlated to students' positive view of the use of GC. In addition, students' skills on use of a GC have a positive correlation with students' positive view of the use of GC. As students' skill of use of a GC increased their negative view of mathematics, such as math is boring, and math is only memorizing the rules, decreased.

Students who had higher skill in the use of a GC believed that GC can make mathematics more fun and makes algebra easier.

In summary, results from the study exposed evidence that used of technology (GC, Desmos, and apps) in teaching and learning increased college algebra students' understanding of several concepts such as domain, vertical and horizontal asymptotes, end behavior of a function, and logarithms functions. In addition, college algebra students' skills such as logical reasoning, use of graph, and organization including written order, and correct use of notation and symbols were significantly increased when they used technology. Survey of calculus students' understanding increased in several topics such as finding maximum/minimum for two variable functions, limits, and definite integrals when they used technology in their class activities and on written tests.

6.2 Limitations

There are several limitations on the results of this research study. Four different teachers taught eighth college algebra sections with different methods and knowledge and different ways to use technology. In this research study the effect of teacher knowledge was neglected as well as the effect of teacher methods. This research study was conducted on both female and male students from various ethnicities. The effects of gender and ethnicity were ignored in this study.

The concept test, which was the only written test in college algebra, was designed by the coordinator of college algebra. Therefore, the primary researcher did not have control on the design of the tests and some of the questions were GC neutral.

The sample size in college algebra courses was large for qualitative analyses. Therefore, only students' understanding on some of the concepts were qualitatively analyzed.

This research study was conducted on survey of calculus during summer classes of five weeks, which was the short timeframe. Students participated in class every weekday for 90 minutes. Therefore, students did not have enough time to go over some concepts in depth. This could influence students understanding of a concept. The sample size in survey of calculus was too small to allow a t-test to show any conclusion on RT1, RT2, SAS, and the qualitative rubric.

Both survey of calculus and college algebra courses were MLP courses, which means these courses were more computer based, and the students' written works were not available. The other two review tests that were designed by the primary instructor were replacements for the students' lowest quiz scores; therefore, students did not have strong motivation to take these tests.

Teaching based on apps requires smartphones or tablets, but some students do not have access to this technology, which makes it difficult. Finding specific free educational apps whose designs are based on theories of learning is not easy and requires time as well.

6.3 Implications and recommendations for future research

This research study adds to the limited experiential literature that reports on the effect of technology on students' understanding and organization of their written work in college level mathematics. This study also extends information on the effect of specific technology such as apps on students' learning in college mathematics courses. The findings from this research study have several implications for teaching and learning college and secondary mathematics based on new technology. Students can be at an advantage in understanding abstract mathematical concepts by using technologies such as online GC, handheld GC, and apps. University teachers can benefit from new technology to enhance teaching mathematics courses. Further, teachers need access to learning about technology-based curricula in order to teach higher mathematics courses for conceptual understanding. Findings from this research study suggest that teachers can benefit from new technology-based mathematics teaching approaches for other undergraduate mathematics courses beyond introductory calculus.

The sample size in the survey of calculus was small and the timeframe of the research on this course was short. In addition, a qualitative analysis was applied to only a few topics in survey of calculus course. Future studies could be conducted qualitatively and quantitatively on the effect of technology on students understanding of other topics of calculus with larger sample sizes and longer timeframes.

The findings from this research study showed that college algebra students had more organized written work and more correct use of notation and symbols when they used a GC on the test compared to the time that they did not use a GC for similar concepts. But there is not any clear reason why this happened, and there is not any information on the states of students' minds. Future research could be conducted on cognitive issues associated with the use technology and how it affects their organization of written work.

Although in this research study data on gender and ethnicity of the participants were collected, the effect of technology on gender and ethnicity was not analyzed. This can be a topic for future research and could be worthwhile information about the differences on the effect of technology on female and male students while learning mathematics concepts.

It would take time and effort to find appropriate educational technology, tools, and apps that would consider learning theories and specific topics from college algebra and survey of calculus. Therefore, mathematics education research could explore new technology and tools and provide a list of educational technologies and teaching methods that could facilitate college students learning mathematics as well as college teachers teaching mathematics.

Finding from this research study showed that survey of calculus students benefited in learning of the limit and integrals using GCs. In addition, survey of calculus students mentioned in their interviews that they understand integrals and limits better when they used Desmos. Desmos has great potential for visualizing calculus concepts such as integrals and limits and derivatives. Future research could conduct a mixed quantitative and qualitative methods study on the effect of Desmos on calculus students understanding that could yield valuable finding to help students to understand calculus concepts better.

6.4 Conclusion

In this study, mixed qualitative and quantitative methods were used to investigate the effectiveness of teaching with hand-held GCs, online GCs and smartphone applications on the understanding of students in college algebra and survey of calculus classes. This study aimed to investigate i) what areas of college algebra and survey of calculus are affected more by technology ii) how technology affects the organization of students written work and, iii) the effect of technology on the attitude of the students toward mathematics. Data were collected from different sources such as pre- and post- student attitude surveys, scores on three written tests, and interviews with students. The findings from this study revealed that college algebra students who used GCs had a better understanding of x and y-intercept, domain of a function, end behavior, and vertical and horizontal asymptote. However, the performance of students on function composition and word problems was similar with or without GCs. In addition, college algebra students who used smartphone application in their class activities were able to make a better connection between logarithms and exponential functions. The results of the qualitative analysis showed that students' written work is more organized when they use technology on their tests. Students who used a GC for the test in survey of calculus showed a better understanding of maximum and minimum for functions of two variables as well as the concept of limit and definite integrals. However, students had similar performance in derivative problems, indefinite integrals and limits that need the use of rules. As a case study on the effect of technology on college level mathematics courses, this study supports the need for future research on other undergraduate mathematics courses, especially calculus sequences on a larger scale.

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Appendix A: IRB Approval Letters

UNIVERSITY OF ARKANSAS

Office of Research Compliance
Institutional Review Board

August 23, 2016

MEMORANDUM

TO:	Razieh Shahriari Bernard Madison
FROM:	Ro Windwalker IRB Coordinator
RE:	PROJECT MODIFICATION
IRB Protocol #:	16-08-035
Protocol Title:	*The Effect of Using a Graphing Calculator in Students' Work in a College Algebra Class*
Review Type:	☒ EXEMPT ☐ EXPEDITED ☐ FULL IRB
Approved Project Period:	Start Date: 08/23/2016 Expiration Date: 08/17/2017

Your request to modify the referenced protocol has been approved by the IRB. **This protocol is currently approved for 280 total participants.** If you wish to make any further modifications in the approved protocol, including enrolling more than this number, you must seek approval *prior to* implementing those changes. All modifications should be requested in writing (email is acceptable) and must provide sufficient detail to assess the impact of the change.

Please note that this approval does not extend the Approved Project Period. Should you wish to extend your project beyond the current expiration date, you must submit a request for continuation using the UAF IRB form "Continuing Review for IRB Approved Projects." The request should be sent to the IRB Coordinator, 109 MLKG Building.

For protocols requiring FULL IRB review, please submit your request at least one month prior to the current expiration date. (High-risk protocols may require even more time for approval.) For protocols requiring an EXPEDITED or EXEMPT review, submit your request at least two weeks prior to the current expiration date. Failure to obtain approval for a continuation *on or prior to* the currently approved expiration date will result in termination of the protocol and you will be required to submit a new protocol to the IRB before continuing the project. Data collected past the protocol expiration date may need to be eliminated from the dataset should you wish to publish. Only data collected under a currently approved protocol can be certified by the IRB for any purpose.

If you have questions or need any assistance from the IRB, please contact me at 109 MLKG Building, 5-2208, or irb@uark.edu.

UNIVERSITY OF ARKANSAS

Office of Research Compliance
Institutional Review Board

April 28, 2017

MEMORANDUM

TO: Razieh Shahriari
Bernard Madison

FROM: Ro Windwalker
IRB Coordinator

RE: PROJECT MODIFICATION

IRB Protocol #: 16-08-035

Protocol Title: *The Effect of Using a Graphing Calculator in Students' Work in a Survey of Mathematics Classes*

Review Type: ☒ EXEMPT ☐ EXPEDITED ☐ FULL IRB

Approved Project Period: Start Date: 04/28/2017 Expiration Date: 08/17/2017

Your request to modify the referenced protocol has been approved by the IRB. **This protocol is currently approved for 380 total participants.** If you wish to make any further modifications in the approved protocol, including enrolling more than this number, you must seek approval *prior to* implementing those changes. All modifications should be requested in writing (email is acceptable) and must provide sufficient detail to assess the impact of the change.

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If you have questions or need any assistance from the IRB, please contact me at 109 MLKG Building, 5-2208, or irb@uark.edu.

109 MLKG • 1 University of Arkansas • Fayetteville, AR 72701-1201 • (479) 575-2208 • Fax (479) 575-6527 • Email irb@uark.edu

Appendix B: Pre and post-Students attitude survey Items

#	Questions:	SA	A	N/U	D	SD
1	Calculator should "only" be used to check work.					
2	A graphing calculator can be used as a tool to solve problems I could not solve before.					
3	The graphing calculator will hinder my ability to understand basic computation.					
4	Graphing calculator make math fun.					
5	Since I have a graphing calculator, I do not need to learn to make graphs by hand.					
6	Learning algebra is easier if a graphing calculator is used to solve problems.					
7	I understand mathematics better if I solve problems with pencil and paper first before I use a graphing calculator.					
8	I know how to use a graphing calculator very well.					
9	It is important that everyone learn how to use a graphing calculator.					
10	I would do better in math if I could use a graphing calculator.					
11	I prefer working problems with a graphing calculator.					
12	I am good in mathematics.					
13	Mathematics is boring.					
14	I would appreciate math better if I had a graphing calculator.					
15	Using a graphing calculator to solve statistics problems is confusing.					
16	I would try harder in math if I had a graphing calculator.					
17	I feel I am cheating myself out of a chance to learn when I use a graphing calculator.					
18	If I use a graphing calculator my ability to visualize problems will decrease.					
19	Learning mathematics is mostly memorizing a set of facts and rules.					
20	When doing mathematics, it is only important to know how to do a process and not why it works.					
21	Learning mathematics means exploring problems to discover patterns and generalize.					
22	I rely on my graphing calculator too much when solving problems.					
23	I feel graphing calculators should not be used while taking mathematics tests.					

24- How much experience of using calculator in math courses have you had? Give your response based on a scale of 0 to 10(0 for none, 5 for 3 courses, and 10 for all of your previous math courses).