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Novice versus Expert Algebraic Problem-Solving Strategies: An Eye Tracking Approach

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NOVICE VERSUS EXPERT ALGEBRAIC PROBLEM-SOLVING STRATEGIES: AN EYE TRACKING APPROACH

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Abstract

Algebra continues to be an important point along the educational spectrum. It is often the point at which we see a deviation of educational trajectories for those who are interested in pursuing additional math and science courses and those who are not. Understanding how college algebra students perform is a crucial step in further understanding the difficulties that students often encounter, as well as understanding other potential factors that may contribute to their performance. The novice/expert paradigm is one way to examine performance differences. While existing literature indicates that novices and experts perceive, sort, and solve problems differently across a variety of domains, this paradigm has yet to be applied to algebra. It has also been shown that cognitive processing can guide an individual's eye movements. Evidence is lacking in this domain with regard to what performance differences look like in terms of these eye movements (e.g., number of fixations, length of fixations) during more complex math tasks. Using an algebraic problem-solving task, this thesis examined performance differences between novices (undergraduate College Algebra students) and experts (graduate students with a substantial background in mathematics). Given the role that working memory plays in mental arithmetic, another purpose of this thesis was to assess how working memory might impact performance when solving algebraic equations and if working memory is a good predictor of performance.

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Chapter 1: Introduction

Number sense is a topic that has generated research expanding over several disciplines including developmental psychology, neuroscience, educational psychology, and cognitive psychology (Ashcraft & Stazyck, 1981; Jordan, Kaplan, Ramineni & Locuniak, 2009; Petersen & Hyde, 2015; Suarez-Pellicioni, Nunez-Pena, & Colome, 2013). Despite its rich history, much of the existing research has focused on the fundamental concepts of arithmetic (e.g., basic facts; Faust, Ashcraft, & Fleck, 1996). Such studies have examined a variety of ages from early elementary school students to college-aged adults (Hecht, 2002; Moore & Ashcraft, 2015). These studies have built a solid foundation of research defining how basic facts are learned, how performance differs based on different types of problems, and what performance looks like in terms of the common cognitive measures of reaction time (RT) and accuracy. Recently, the study of numerical cognition has grown to include the more in-depth method of measuring eye movements with an eye tracking device (Hartmann & Fischer, 2016; Mock, Huber, Klein, & Moeller, 2016). The use of eye tracking in psychological research has equipped researchers with more accurate measures of the underlying cognitive processes taking place during a given task. Where participants are fixating and the length of a fixation can be an indication of the active processing of that information (Hartmann & Fischer, 2016; Just & Carpenter, 1980), and because of that, such measures provide a more precise depiction of participants' performance beyond the typical measures of RT and accuracy.

Research is still lacking in more advanced mathematical concepts such as algebra. Expanding this research to include this more complex type of math can begin to shed some light on the cognitive processes involved in these more difficult types of problems. It can also begin to unearth some of the problem-solving strategies involved in these more challenging, multi-step math problems.

Furthermore, typical math cognition studies examine one age group or grade level, or even multiple grade levels to examine the developmental trajectory of acquiring such conceptual knowledge (Moore & Ashcraft, 2015; Trezise & Reeve, 2014). One approach that remains underutilized, however, is the study of comparison groups such as novices and experts. This approach has proved to be useful in other domains such as physics, chemistry, and athletic performance (Heyworth, 1999; Milton, Solodkin, Hlustik & Small, 2007; Priest & Lindsay, 1992), but has yet to be thoroughly applied to the area of numerical cognition. Comparing these different groups, which exist on different levels of the educational spectrum, can help us better understand what performance looks like at these two extreme points and what differences exist between these two groups.

In the following sections I will review the importance of algebra in the educational spectrum as well as the limited research in this area. Additionally, relevant research with working memory will be outlined; specifically, the role that working memory plays in mathematics. A thorough background of the use of eye tracking in psychological research will also be discussed in addition to its applicability to more difficult areas of numerical cognition. Lastly, the novice/expert approach will be examined in detail and applied to an understudied area of math cognition, namely algebra. All of which will culminate with a study that takes a novice/expert approach to studying eye movements during an algebraic problem-solving task.

Number Sense

Number sense is defined as one's ability to understand and manipulate numbers (Dehaene, 2011). It develops over time as additional general number knowledge and specific knowledge of mathematical concepts are acquired via experience. This knowledge includes everything from basic number recognition, counting, the ability to discriminate numbers, use of the four basic mathematic operations (addition, subtraction, multiplication, and division), and also more difficult concepts such as algebra, geometry, calculus, etc. The foundation of number sense is something that is built early on in childhood and we continuously build upon this foundation as we acquire additional knowledge. Studies investigating what people know about numbers typically have two variables of interest, RT and accuracy. These measures can begin to tell a researcher how familiar a person is with a particular math task or problem and whether or not they have mastered that given task.

Perhaps one of the most common effects that is often found in the study of numerical cognition is the problem size effect. This is defined by an increase in RT and errors as the size of the problem increases (Ashcraft & Stazyk, 1981). The problem size effect is characterized by the type of strategy employed to solve a given problem. Small problems are more often than not solved via a retrieval method, therefore resulting in faster RTs and fewer errors. Large problems, on the other hand, are characterized by more procedural methods, resulting in longer latencies and more errors (Ashcraft & Guillaume, 2009). This effect can be seen in performance involving basic arithmetic as well as more complex arithmetic problems (Faust, et al., 1996). We would expect the problem size effect to also be present in more difficult math, such as algebra.

Algebra

Algebra, a mathematical concept that uses abstract representations, is built upon an arithmetic foundation (Campbell, 1992). It too is often considered a foundation not only for more difficult mathematical concepts (e.g. trigonometry, calculus), but also for additional educational opportunities in higher education (Geary, Hoard, Nugent & Rouder, 2015; Stein, Kaufman, Sherman & Hillen, 2011). Algebra involves defining relationships among numbers as well as unknown variables (Stacey & MacGregor, 2000). This abstract concept is typically introduced to students during late middle school or early high school years, although there has been a recent push for earlier implementation into elementary school curriculum and universal access to algebra courses in middle school and high school (Carraher, Schliemann, Brizuela & Earnest, 2006; Stein, et al., 2011).

Algebra continues to be an important point along the continuum of mathematics education. Recent research highlights surprising disparities between those who do and do not take algebra courses in middle school and even throughout college. The educational trajectory for those who take algebra early on begins to deviate from the rest of the student population soon after enrollment in an algebra course. Middle school students who take algebra in the eighth grade are more likely to take future math courses as well as additional science courses such as chemistry and physics (Paul, 2005). Many universities consider college algebra to be the lowest math course for which institutional credit can be given (Herriott & Dunbar, 2009) and it is often the first math course that is taken after any required remedial courses are completed. However, class retention and enrollment in future math courses is difficult to attain beyond the level of college algebra. Data collected from the University of Nebraska at Lincoln showed that 20% of the students enrolled in college algebra had to retake the course in the future and nearly all of the students enrolled in a college algebra course did so because the course was required for their major (Herriott & Dunbar, 2009). Similar research shows that less than half of the students who enroll in a college algebra course will enroll in future math courses (Herscovics & Linchevski, 1994; Swafford & Brown, 1989). By just these data alone, we can see that algebra remains a critical point along the educational spectrum, influencing future mathematics course enrollment as well as future science course enrollment, which in turn can have lasting impacts on future career choices.

Existing research has provided a thorough basis for the understanding of algebra performance and algebraic problem-solving. When teaching algebra, it has been suggested that worked out examples of varying algebraic equations can be more beneficial than having students work through conventional algebra problems (Sweller & Cooper, 1985). Providing students with problems that are already worked out allows them to focus less on the end goal and more on the intermediate steps involved in the problem-solving process, thereby adding to their schema for those types of problems. Researchers propose that algebraic knowledge and problem-solving skills are acquired via schema acquisition (Sweller & Cooper, 1985). Schemas are defined as mental constructs of knowledge about things that aid in our ability to categorize those things that we encounter (Sweller & Cooper, 1985). These schemas allow students to categorize problems in order to more efficiently apply appropriate problem-solving strategies; the more schemas one has, the more efficient one would be at solving that particular type of problem.

There is also research that suggests that students' arithmetic knowledge can interfere with their ability to use appropriate algebraic problem-solving methods (Stacey & MacGregor, 1999). Students between the ages of 13 and 19 were given four different algebra problems to solve and given specific instructions to use algebra to solve the given problems. Students were later interviewed regarding the problem-solving methods that they used. Results indicated that despite being given specific instructions on the problem-solving method that they were to use, most of the students used arithmetic reasoning and non-algebraic strategies to guide their methods. This included everything from trial and error, to immediately beginning to solve the problem without consideration of any algebraic component. For example, when asked to solve $3x + 14 = 44$

algebraically, several students used the trial and error method of substituting different numbers for *x*, until they were able to get to a value of 44, instead of algebraically solving for *x* in the given equation. Researchers concluded that perhaps students lack a fundamental understanding of what algebra is and what it means to solve a problem algebraically. They suggest that when teaching algebraic problem-solving strategies, easy problems must be mastered so that students gain a solid understanding of why algebraic methods are important. As problem difficulty increases, teachers should encourage the use of those same algebraic problem-solving methods and not allow students to revert back to easier arithmetic methods, such as trial and error (Stacey & MacGregor, 1999).

A recent study took a comprehensive approach to the difficulties students encounter in algebra and examined six different types of errors (variables, negative sign, equality/inequality, operations, fraction, and mathematical properties) in order to identify which were the most prevalent in a typical algebra course (Booth, Barbieri, Eyer & Pare-Blagoev, 2014). Researchers looked at data from 565 students enrolled in 7^{th} to 10^{th} grade. Data were collected from assignments throughout the school-year and students were given an end-of-the-year assessment with 10 algebra-related problems from standardized tests. Results show that of the six different categories of errors, by the end of the school-year, students were making the most errors with negatives, variables, and equalities/inequalities. Although the frequency of these errors fluctuated throughout the year, resolving these mistakes early on could lead to a better overall understanding of the concept and aid in the general understanding of algebraic problem-solving. It is concerning, however, that at the end of the year, students were still having issues with variables and equalities, both of which sit at the heart of the algebraic foundation. Given the broad range of topics covered in a typical algebra course, additional research that focuses on specific types of algebraic problem-solving could help better understand these and perhaps additional difficulties that are commonly encountered.

Working Memory

Working memory (WM) is a mental workbench where cognitive effort is applied to keep track of mental processes in our immediate environment (Radvansky & Ashcraft, 2014). More specifically, it is responsible for "maintaining, manipulating, and retrieving" the necessary information for a given task (Unsworth, Redick, Heitz, Broadway & Engle, 2009). Working memory is limited in capacity and plays an essential role in executive function (Carpenter & Just, 1989; Unsworth et al., 2009).

Working memory capacity can be measured by using a variety of complex span tasks. These WM tasks require participants to hold information, which will later be recalled, while manipulating other information. Commonly administered tasks include: the reading span task (RSPAN) and the operation span task (OSPAN) (Daneman & Carpenter, 1980; Turner & Engle, 1989). The RSPAN is a task in which participants are asked to read sentences and remember the terminal word of each sentence. They are later asked, after a series of sentences, to recall the terminal words in the order that they appeared. It has been concluded that participants' reading span scores are related to reading comprehension (Daneman & Carpenter, 1980). The OSPAN has participants solve a series of simple math problems. After solving each problem, a letter appears, and participants are asked to remember those letters and later recall them sequentially in the order that they were presented. It has been concluded from this task that operation span scores are related to verbal abilities as well as reading comprehension (Turner & Engle, 1989; Unsworth, et al., 2009).

Perhaps two of the more underused WM tasks are the symmetry span and the rotation span tasks. These two tasks have been shown to have greater predictive ability than the operation span alone (Draheim, Harrison, Embretson & Engle, 2017). The symmetry span has participants perform a sequence recall task during a symmetry judgment task. After being asked if an image is symmetrical, participants are presented with a 4x4 matrix with one filled square and later asked to sequentially recall the positions of the filled squares (Unsworth, et al., 2009). This task, unlike some of the other WM tasks, taps into spatial WM capacity. The rotation span task asks participants to determine if a rotated letter is in the correct position or if it is shown as a mirror image. This is followed by either a short or long arrow in one of eight possible positions. Participants are asked to recall those arrows in the order that they were presented (Foster, Shipstead, Harrison, Hicks, Redick, & Engle, 2015). Using multiple complex span tasks together has been shown to be a better, more reliable measure of WM capacity (Foster, et al., 2015)

Research on WM in numerical cognition suggests that it plays a crucial role in those mathematical concepts that go beyond the automatic retrieval of basic facts (Ashcraft & Guillaume, 2009). Evidence indicates that factors such as the size of the operands in a problem or the number of steps required to solve a problem are important when considering the role of WM (Ashcraft & Guillaume, 2009). A 2001 study examined working memory and its role in an arithmetic task using a dual task procedure (Ashcraft & Kirk, 2001). Dual task involves the use of two tasks, both of which require WM resources. As these tasks become more difficult, there should be observable decrements in performance as the two tasks compete for available WM resources. In this study, participants were presented with addition problems that varied in size (small, medium, and large) and varied in whether or not carrying was involved in order to solve the problem. A secondary letter recall task was also used, consisting of either two letter or six letter sets. Results indicated that problems that were larger in size and required carrying resulted in poorer performance, evidence of problem size and carrying effects. Additionally, in the more difficult memory load sets (the six letter sets), problem size and carrying effects were even more pronounced, resulting in significantly longer latencies and higher error rates for larger problems and problems that required carrying compared to performance in the two letter sets (Ashcraft & Kirk, 2001). This is a clear indication that the role of WM is contingent upon problem-related factors such as problem size and the number of steps involved in the problem-solving process. The more components and/or steps in a problem, the more WM is taxed, thereby leaving less WM capacity available for other necessary manipulations (e.g. additional tasks).

The concept of WM has also been applied to algebraic expressions. Another 2001 study assessed 8^{th} and 9^{th} graders' ability to expand brackets in an algebraic expression. For example, participants were asked to expand the brackets in the expression on the left to obtain the correct answer on the right:

$$-2(-3 - 4x) - 3(-4x - 5) = 6 + 8x + 12x + 15$$

Results indicated that errors were not normally distributed. Participants made more errors within the second bracket compared to the first, as a result of cognitive load being more heavily strained during manipulations within the second set of brackets. Similar results were found when comparing errors within each bracket (more errors with the second operation compared to the first). There was less WM capacity available to accurately perform computations as participants moved through the problem (Ayres, 2001). There is clear evidence that WM plays a crucial role throughout various mathematical concepts, yet additional research is still needed to further investigate its role in online algebraic problem-solving.

Eye tracking

One methodological approach to numerical cognition, with its origins in reading research, is the use of eye tracking. This approach is based on the eye-mind assumption, the idea that an individual's cognitive processing guides their visual fixations, as well as the immediacy assumption, the notion that each word that is encountered is processed/interpreted immediately as opposed to processing groups of words together (Just & Carpenter, 1980; Rayner, 1998). Based on these two assumptions, eye movements are suitable measures to help better understand online/in the moment processing of information. The eyes move across the visual field in short, fast movements called saccades. Saccades take place between short pauses known as fixations (Mock, et al., 2016). It is during these short pauses (fixations) when attention is given to a particular part of the visual field and information is encoded (Mock, et al., 2016). Also important is the length of these fixations. How long a person is fixated on a stimulus is indicative of the active processing of that information. With today's technological advances, we now have remote eye trackers that are able to gather accurate data on a participant's eye movements using a reflection of the cornea and video-based techniques (Mock, et al., 2016).

Research nearly four decades old suggests that the more difficult a particular text, the longer participants fixated. Participants were also more likely to have regressive looks and longer saccades (Jacobson & Dodwell, 1979; Rayner & Pollatsek, 1989). This suggests that performance differences were not just due to individual differences in ability, but to text-related components as well. This is an important finding that may prove to carry over into other domains such as numerical cognition.

Implementing eye tracking methodology into numerical cognition research has proven to be especially effective at uncovering information about the underlying cognitive processes that are taking place during a specific task. This methodology can also reveal information that typical behavioral measurements of RT and accuracy can only begin to explain. Eye tracking has been used in several basic math tasks, including solving basic arithmetic facts and number line estimation (Sullivan, Juhasz, Slattery, & Barth, 2011; Yu, Liu, Li, Liu, Cui & Zhou, 2016). Another recent study examined fixation patterns across the four basic arithmetic operations (addition, subtraction, multiplication, and division) (Curtis, Huebner & LeFevre, 2016). Researchers presented adult participants with small and large problems from the four basic operations and monitored their eye movements during the problem-solving process. Addition and multiplication problems resulted in similar fixation patterns: one short fixation to each operand and about half of the total fixation time spent on the operator (the addition, subtraction, multiplication, or division sign) suggesting active calculating. Subtraction and division problems were more complex with varying eye movement patterns. Typical problem size effects were also found in this study, with participants performing significantly slower and making more errors with the large problems compared to the small problems. The problem size effect was also observable in the eye tracking measurements, with longer gaze durations and more fixations associated with the larger problems (Curtis, et al., 2016). I expected similar results in the current study, as larger problems typically require more cognitive effort and I expected this to be reflected in RT and accuracy as well as the eye tracking measurements.

Novice versus Expert

Understanding that performance on any given task falls on a continuum is not only beneficial when studying mathematics performance, but also has its place in several different domains. A common approach that is used to compare performance between two different groups, such as novices compared to experts, is described as the *relative approach* (Chi, 2006). This approach is predicated on one principal assumption: that novices can ultimately attain an expert's level of proficiency (Chi, 2006). The objective of this approach goes beyond just identifying how experts outperform their novice counterparts to also include how this select group gained their expertise and why they are able to outperform others (Chi, 2006). Such an approach has been applied to everything from sports, to the arts, and continues to benefit areas investigating the process of learning and acquiring knowledge as well as teaching and education (Alexander, 2003).

Experts can be defined as those who have "acquired extensive knowledge that affects what they notice and how they organize, represent, and interpret information in their environment…(affecting) their abilities to remember, reason, and solve problems" (Bransford, Brown & Cocking, 1999, p. 31). Studies across various different domains have helped identify the most common ways in which experts differ from non-experts. Experts often generate the most effective strategy and the best solution while doing this more accurately than novices (Chi, 2006; Klein, 1993; Lemaire & Siegler, 1995). A study examining expert and novice performance differences in chemistry tested students on basic chemistry problems involving volumetric analysis (e.g., calculating moles of a solute and solution concentration) (Heyworth, 1999). During the problem-solving window, experts solved the problems more quickly and more accurately, while also identifying the correct strategy that was to be applied to the given problem. Novices, on the other hand, took longer to solve the same problems, while attempting to apply multiple different strategies (Heyworth, 1999). This is a clear example of how additional knowledge and experience can aid in the recognition of a particular problem type and ultimately lead to more effective and efficient problem-solving strategies. This research suggests that differences between these two groups begin to emerge immediately (at the onset of problem presentation) and persist throughout the problem-solving window.

Experts are also able to more quickly retrieve domain-specific knowledge with greater automaticity and less cognitive effort (Alexander, 2003; Bransford, et al., 1999). This is a combination of not only having greater knowledge, but also the organization of that knowledge that aids in the ability to better recognize and identify problem types (Bransford et al., 1999). While, by definition, an expert is one who has acquired additional knowledge in a given domain, expertise goes beyond that to encompass what that individual can do with the vast knowledge that they have acquired and how the organization of that knowledge influences the way which they plan and approach various situations. Similarly, expertise plays a significant role in problem recognition and perception. Problem perception varies based on level of expertise and this perception becomes more expert-like as proficiency increases (Schoenfeld, 2014). When asked to sort different math problems, experts focused more on problem relatedness and how each problem would be solved compared to novices who focused more on surface level subjects and the items in the problems (Schoenfeld, 2014). For example, when presented with the problem: "If you add any five consecutive whole numbers, must the result have a factor of 5?", novices sorted this problem based on the surface structure of "factors of 5", while experts sorted it based on an understanding of "number representations." (Schoenfeld, 2014). Novices sort problems based on the words and items in the actual problem, while experts go beyond the problem itself and focus on how the problem would be solved or the theory/law that the problem is based upon. We are not only seeing differences in performance, but also in participants' initial perception and processing of the problems in which they are presented.

Evidence also suggests that brain activation differs in novices and experts. Novice and expert golfers participated in a study in which they were asked to perform simulated golf swings (Milton, et al., 2007). Results showed that all participants (both novices and experts) reported the use of motor planning and imagery during the simulation. Experts, however, showed less volume of brain activation than non-experts, suggesting a more precise motor planning neural network. Different regions were also activated during the simulation: novices showed activation in the limbic areas and basal ganglia, while experts did not show activation in these areas. Similar reciprocal activations were shown between these two groups: novices showed significant activation of the posterior cingulate gyrus and minimal activation of the superior parietal lobule and the dorsolateral premotor cortex, while experts showed the exact opposite (significant activation on the superior parietal lobule and the dorsolateral premotor cortex and no activation in the posterior cingulate gyrus). Researchers concluded that such differences were related to the dissimilarities in attentional focus; novices lacked the selective attention that experts exhibited in the activation of their motor system (Milton et al., 2007).

Observable differences can also be found between experts and non-experts in the form of eye movements. Chess players of varying levels of expertise were tested on computer-based chess board simulations. Results showed that experts had fewer fixations per trial, suggesting better perceptual encoding of their visual field than their non-expert counterparts (Reingold, Charness, Pomplun & Stampe, 2001). Similar results were found in reading research. When encountering text that was difficult, good readers were better at moving their eyes back to a specific location in the difficult text. Poor readers typically had longer and more frequent fixations as well as more regressive eye movements. The strategies employed during this visual search of previously read text suggests differences in the ability to spatially encode information (Murray & Kennedy, 1988). Similar differences have been found in fixation length, number of fixations, and number of regressive eye movements in other reading research (Everatt, Bradshaw & Hibbard, 1998; Rayner, 1978; Underwood, Hubbard & Wilkinson, 1990). Collectively, these findings imply that experts have acquired the ability to better spatially encode the information that they are presented with, which results in differences across these various eye tracking measures. It is possible that we may see similar differences between novices and experts in the realm of mathematics. Novices may require more regressive eye movements while trying to decide on an appropriate problem-solving strategy, while experts may require fewer fixations and fewer regressive eye movements overall.

Understanding that differences not only exist between these two groups, but how and why these differences exist is imperative in pursuit of a better understanding of the continuum of knowledge and the trajectory of learning. It is not enough to know that experts outperform novices in a given math-related task. We need to have a better grasp on where these differences occur and what these differences look like in terms of measures of performance and potential influential factors.

Current Study

Using an algebraic problem-solving task, this study examined performance differences between novices and experts. This study addresses the following: (1) if and where performance differences occur between the two groups, (2) if different problem-solving strategies are employed, and (3) what these potentially different problem-solving strategies look like in terms of eye movements. Additionally, given the role that working memory plays in mental arithmetic, another purpose of this study was to assess how working memory might impact performance when solving algebraic equations and if working memory is a good predictor of performance in this type of problem-solving.

An increase in RT and decrease in accuracy were expected for novices and experts for the easy problems compared to hard problems (e.g., $Ax + B = C$ versus $Ax + B = Cx + D$) (Sweller

& Cooper, 1985). However, as problem complexity and problem size increase, experts were expected to have faster RTs and fewer errors compared to their novice counterparts. Within each expertise group, faster performance and fewer errors were expected on the easy compared to hard problems as well as the small compared to large problems. There should be clear evidence of a problem effect when comparing novice performance across problem complexity and problem size. Experts, however, should not significantly differ in behavioral measures across the different problem types; they should view the easy/hard and small/large problems as being equally as easy. Performance differences were also expected between the novice and expert groups based on the value of the unknown variable x. Problems where x is a whole number should be easier than those problems where x is not a whole number (e.g., $2x + 4 = 8$, where $x = 2$ versus $2x + 3 = 6$, where $x = 1.5$ or $3/2$) and novices should display more difficulty in the form of slower RTs and more errors when solving problems where the correct answer is not a whole number.

Eye tracking measures, including the number of fixations and length of fixations to each area of interest (AOI), were used to identify patterns in eye movements indicative of the problem-solving strategies used by participants in each of the expertise groups. Overall, experts should choose more effective problem-solving strategies (Heyworth, 1999). This should be expressed by having fewer fixations overall, fewer fixations to each AOI, and shorter fixation durations (Curtis, et al., 2016). Novices were expected to exhibit opposite patterns in eye movements compared to their expert counterparts, defined by more fixations overall, more fixations to each AOI, and longer fixation durations. These eye movements represent any difficulty encountered while solving the algebraic equations and any potential inability to efficiently apply effective problem-solving strategies. Additionally, novices should have more regressive looks overall, indicating any difficulty encountered during the problem-solving process (Murray & Kennedy, 1988). It was expected that novices would make more regressive eye movements to the operands during the problem-solving window, representing any recalculations that needed to be performed or any encoding that needed to be redone due to loss of information in WM. These eye movements were anticipated to be less prevalent when solving easy and small problems compared to hard and large problems, as the easy and small problems should generally be easier and require less cognitive effort. These results should mirror previous findings from original reading research showing that more difficult text resulted in more fixations, longer fixation durations, and more regressive eye movements (Murray & Kennedy, 1988). Results from the current study should be parallel in that as problem difficulty increased, participants should begin to show more fixations, longer fixation durations, and more regressive eye movements among the different AOIs.

In terms of WM capacity, it was expected that WM would be a good predictor for performance in terms of accuracy. The hard problems require multiple steps in order to solve them, so because this is a mental task, I expected those with greater WM capacity to outperform those with lower WM capacity, thereby making WM a good predictor.

Chapter 2: Method

Participants

A total of 53 students participated in this study. Novice participants (21 female, 9 male; M_{Age} = 19.61 years) were undergraduate students currently enrolled in a College Algebra course in the Department of Mathematical Sciences at the University of Nevada, Las Vegas. College Algebra, at this university, is the first college level math course that undergraduates take after any required remedial math courses. Expert participants (10 female, 13 male; M_{Age} = 30.95 years) were current graduate students or PTIs at the University of Nevada, Las Vegas, from one of the following departments: Department of Mathematical Sciences, Department of Physics & Astronomy, or the College of Engineering. All participants were recruited via direct contact through their department or via announcements in an online newsletter available to all faculty and graduate students. Previous research has defined novices as college students with no formal college level math experience and experts as advanced graduate students; these guidelines were used to determine the samples for this study (Priest & Lindsay, 1992). Participants from this sample self-identified as the following: 39.22% Asian/Pacific Islander, 25.49% Caucasian, 15.69% Hispanic/Latino, 7.84% African American, and 11.76% Multi-racial/Other. Compensation included either $10 or $15 cash, depending on the point in which the individual participated in the study. Compensation increased from $10 to $15 about half way through the study in order to better incentivize participation and overcome recruitment difficulties.

Materials

All participants received the same measures. The computer–based problem-solving task and the working memory tasks were presented on a desktop computer monitor, using E-Prime 2.0 software (Schneider, Eschman, & Zuccolotto, 2012). The math computation subtest of the

Wide Range Achievement Test – 3 (WRAT) was administered to measure level of math achievement. This subtest is a 40-item pencil and paper math assessment in which participants have fifteen minutes to complete as many problems as possible. Problems range from single-digit addition and subtraction to more difficult algebraic equations. Scores were based on the number of correct answers out of a total of 40 possible points.

During the problem-solving task, participants' monocular eye movements for each trial were recorded with a sampling rate of 1250 Hz, using an SMI Eyelink iView X Hi-speed eye tracker. The SMI eye tracker included a chin and forehead rest to help minimize participant movement during the study.

Tasks

Problem solving task. The algebraic problem-solving task consisted of algebraic equations ranging in complexity (easy and hard). Presentation of the easy problems included following format: $Ax + B = C$, where A was the coefficient of the unknown variable x, B was a constant, and C was a constant different than the value of B. Hard problems were presented in the following format: $Ax + B = Cx + D$, where A and C were coefficients of the unknown variable x (where A does not equal C), and B and D were constants (where B does not equal D). Easy and hard problems consisted of a subset of small and large problems. Small and large problems were defined by the size of the coefficient of x in the reduced versions of the algebraic equations. For example, easy problems can be reduced from $Ax + B = C$ to $Ax = C – B$; small problems were those where "A" ranged from 2 to 5 and large problems were those where "A" ranged from 6 to 9. Hard problems can be reduced from $Ax + B = Cx + D$ to $Ax - Cx = D - B$; small problems were those where "A-C" ranged from 2 to 5 and large problems were those where "A-C" ranged from 6 to 9 (See Table 1). Within the subsets of small and large problems, there was an additional subset of problems based on the value of *x* (whole or decimal). Whole problems consisted of those equations where *x* was a rational whole number (less than 10) and decimal problems consisted of those equations where *x* was not a whole number (a decimal or fraction less than 10). There was a total of 40 problems, with an equal number of easy/hard problems, small/large problems, and whole/decimal problems (problem complexity x problem size x value of *x*). Problems were visually presented on a computer monitor, in black, size 44 font on a white background.

Table 1

Examples of Problems from Each Category

	Easy Problems	Hard Problems
Small (whole *x*)	$2x + 5 = 17$	$10x + 4 = 7x + 13$
Small (decimal *x*)	$4x + 5 = 15$	$12x + 5 = 8x + 15$
Large (whole *x*)	$9x + 6 = 69$	$10x + 7 = 2x + 39$
Large (decimal *x*)	$8x + 6 = 18$	$11x +7 = 5x + 28$

Working Memory Task. Following completion of the problem-solving task, participants were presented with the working memory tasks: the symmetry span and the rotation span. In the symmetry span task, participants were asked to perform a sequence recall task during a symmetry judgment task. An 8x8 matrix of black and white squares appeared on the screen and participants were asked to determine whether or not the image was vertically symmetrical.

Following each 8x8 matrix, a 4x4 matrix was shown with one filled square; participants were asked to remember the location of the filled square. Set-sizes ranged from two to five, with three trials for each set-size, for a total of 12 trials. Upon completion of each set, participants were asked to recall the filled squares from the 4x4 matrix in the order that they were shown. Scores were determined by the number of squares recalled in the correct order and in the correct location (Unsworth, et al., 2009). The rotation span was similarly presented and asked participants to determine if a letter was presented in its correct position or if it was a mirror image. After each letter, an arrow (short or long) appeared in one of eight possible positions; participants were asked to recall the length and position of the arrows in the order that they were presented. Similar to the symmetry span task, set-sizes ranged from two to five, with three trials for each set-size, for a total of 12 trials. Scores for this task were determined by the number of arrows recalled in the correct order and in the correct position (Foster, et al., 2015).

Procedures

All participants received all the measures in the same order. Upon obtaining informed consent, participants filled out a demographics questionnaire that included questions about their formal math education and experience. Participants were set up in the eye tracker and presented with a 13-point calibration. Upon successful calibration, participants were presented with the problem-solving task. Participants were asked to mentally solve for *x* in each problem as quickly and as accurately as possible. Prior to the onset of each problem, a random rectangular visual cue appeared in one of the four corners of the screen (Curtis, et al., 2016). Participants were instructed to locate and fixate on the cue. The visual cue ensured that participants were not already fixating on any part of the screen that would contain the problem, as to not inflate the number of fixations or the length of fixation duration to that particular area of interest. The fixation cue remained on the screen for 2s, until the problem appeared. Participants were asked to indicate, via a keypress, once they solved the problem and knew the answer. Following each keypress, participants said their answer out loud, and the researcher entered their answer into the response box on the screen. Once their answer was entered into the response box, the subsequent problem's random visual fixation cue appeared. Participants completed five practice problems before beginning the study to ensure that they understood the study procedures.

Upon completion of the problem-solving task, participants were asked to rate, on a scale from 1 to 7, the amount of effort that they put forth on the previous task. This scale was given to measure participants' engagement in the given task. After this, they were given a two-minute break and then asked to complete the working memory tasks. Following completion of the working memory tasks, the math subset of the WRAT-3 was administered. After the allowed 15 minutes for the WRAT-3, participants were debriefed and compensated for their time and voluntary participation.

Areas of Interest

For analysis purposes, each algebraic equation was divided to obtain separate AOIs for each operand and operator. Easy problems were divided into five AOIs, while hard problems were divided into seven AOIs (Curtis, et al., 2016) (See Figure 1).

Ax	+	B	=	C

Ax	+	B	=	Cx	+	D

Figure 1. Sample AOIs for Analyses (Easy and Hard Problems)

Chapter 3: Results

Data Analyses

Behavioral measures and eye tracking measures were analyzed separately using expertise group (novice/expert) as the between-subjects factor. For the behavioral measures, easy and hard problems were analyzed independent of one another by using separate 2 (novice/expert) x 2 (small/large) x 2 (whole/decimal) mixed model ANOVAs for RT and percent accuracy. Percent accuracy was defined as the number of correct problems out of a total of 20 problems (20 easy and 20 hard). For the eye tracking measures, easy problems were analyzed using separate 2 (novice/expert) x 2 (small/large) x 2 (whole/decimal) x 5 (AOIs) mixed model ANOVAs. Hard problems were analyzed using separate 2 (novice/expert) x 2 (small/large) x 2 (whole/decimal) x 7 (AOIs) mixed model ANOVAs. Respectively, these analyses were repeated for each of the eye tracking measures -- number of fixations, fixation duration, and number of regressive fixations. Number of fixations (or fixation count) was defined as the number of fixations inside an AOI. Fixation duration was the sum of all fixation durations in a particular AOI. Regressive fixations (or refixations) were defined as the number of looks in an AOI originating from outside that AOI, minus one. A multiple regression was also conducted to determine if WM span, effort, and expertise were good predictors of performance in terms of RT and accuracy.

One novice was excluded from analyses due to poor calibration. Three experts were excluded (two participants did not meet the minimum eligibility requirements and one did not follow task instructions). Outlier analyses were performed on RT data, any values beyond +/- 2.5 SDs were excluded from analyses. This resulted in the exclusion of an additional 60 trials from the remaining 49 participants.

Tracking ratio, a percentage measure of tracking of non-zero gaze positions across the duration of the entire study, was also examined to ensure accurate eye tracking for each participant. No additional participants were excluded due to low tracking ratios.

Math Achievement and Effort

Level of math achievement was obtained from scores on the math subtest of the WRAT-3 in order to substantiate differences in expertise. An independent samples *t*-test was conducted to compare level of math achievement between the two expertise groups. Results show that experts ($M = 35.90$, $SD = 2.02$) scored significantly higher than novices ($M = 29.62$, $SD = 4.81$) on the math assessment, thereby confirming their group membership, $t(47) = -5.49$, $p < .001$, $d = 1.60$. Scores from the effort scale were also compared to assess group differences. Results indicate that experts ($M = 4.05$, $SD = 1.82$) demonstrated less effort than novices ($M = 5.90$, $SD = 1.21$), $t(47) = 4.28$, $p < .001$, $d = 1.20$. It is clear that group differences are inherently present beyond the algebra task that was being administered.

Baseline Problems

Baseline problems in the format Ax = B were included in the problem-solving task to obtain a baseline measure of performance. There were no differences between expertise groups for accuracy or length of fixation duration on baseline problems. There were significant differences, however, for RT, ($M_{\text{Expert}} = 1643.99$, $SD_{\text{Expert}} = 578.15$; $M_{\text{Novice}} = 2395.16$, $SD_{\text{Novice}} = 1319.35$), $t(47) = 2.39$, $p < .05$, $d = .74$, and number of fixations, ($M_{\text{Expert}} = 2.47$, $SD_{\text{Expert}} = .55$; $M_{\text{Novice}} = 3.41$, $SD_{\text{Novice}} = 1.16$), $t(46) = 3.30$, $p < .05$, $d = 1.04$. Experts were significantly faster than novices and they also had fewer fixations per problem. This finding was interesting given that even at a rather basic level of algebra, expertise differences have started to emerge.

Behavioral Results

Easy Problems. Reaction time analyses for easy problems, as predicted, indicated a main effect of problem size, $F(1,47) = 19.04$, $p < .001$, $\eta_p^2 = .288$, and x-value, $F(1,47) = 29.01$, $p < .001$, $\eta_p^2 = .382$. There was a significant interaction between x-value and expertise, $F(1,47) = 4.10$, $p < .05$, $\eta_p^2 = .080$, in that novices and experts both took longer to respond to easy problems if the answer was a decimal/fraction, but novices took significantly longer compared to experts for both problem types, those with a whole number answer as well as those with a decimal/fraction answer (See Figure 2). Percent accuracy analyses showed no expertise effects, but a main effect of x-value, $F(1,47) = 7.29$ $p < .05$, $\eta_p^2 = .134$. Overall, problems where *x* was a decimal/fraction resulted in lower accuracy rates compared to problems where *x* was a whole number.

Hard Problems. Reaction time analyses for hard problems reveal similar effects as seen with the easy problems. There was a main effect of problem size, $F(1,46) = 62.81$, $p < .001$, $\eta_p^2 = .577$, and x-value, $F(1,46) = 37.18$, $p < .001$, $\eta_p^2 = .447$, that was superseded by an interaction between problem size and x-value, $F(1,46) = 27.34$, $p < .001$, $\eta_p^2 = .373$. This suggests that small problems resulted in comparable RTs for problems where *x* was a whole number as well as problems where *x* was a decimal/fraction, but x-value was particularly effective with the large problems, resulting in a significant increase in RTs between whole number and decimal/fraction problems. There was also a significant interaction between x-value and expertise, $F(1,46) = 5.64$, $p < .001$, $\eta_p^2 = .109$. Pairwise comparisons show that although problems with a decimal/fraction answer resulted in significantly longer RTs for both expertise groups, experts outperformed novices across both problem types (See Figure 2). Percent accuracy analyses revealed a significant main effect of problem size, $F(1,47) = 14.99$, $p < .001$, $\eta_p^2 = .242$, and a significant interaction between problem size and x-value, $F(1,47) = 5.16$, $p < .001$, $\eta_p^2 = .109$. Accuracy for small problems did not differ based on the value of x, but for large problems we saw a significant decrease in accuracy between problems where x was a whole number versus problems where x was a decimal/fraction (See Figure 3).

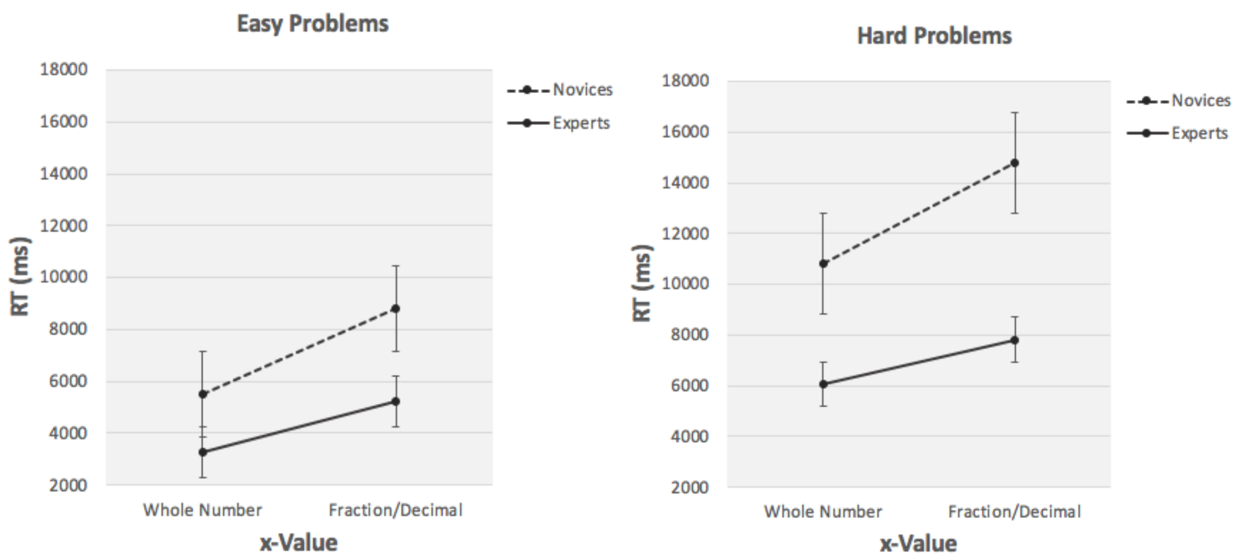


Figure 2. RT Results for Easy and Hard Problems. Significant interaction between expertise and x-value for easy problems, $F(1,47) = 4.10$, $p < .05$, $\eta_p^2 = .080$, and hard problems, $F(1,46) = 5.64$, $p < .001$, $\eta_p^2 = .109$.

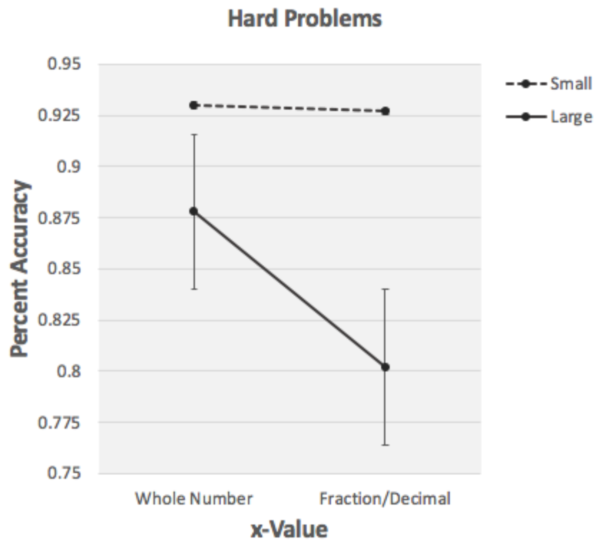


Figure 3. Percent Accuracy Results for Hard Problems. Significant interaction between problem size and x-value, $F(1,47) = 5.16$, $p < .001$, $\eta_p^2 = .109$.

Eye Tracking Results

Easy Problems. Analyses of the length of fixation duration resulted in several predicted outcomes. There was a main effect of x-value, $F(1,46) = 7.65$, $p < .05$, $\eta_p^2 = .143$, and AOI, $F(4, 184) = 14.63$, $p < .001$, $\eta_p^2 = .241$, both of which were superseded by a significant interaction between x-value and AOI, $F(4, 184) = 2.58$, $p < .05$, $\eta_p^2 = .053$. These results indicated that, as expected, problems where *x* was a decimal/fraction resulted in longer fixation durations to the AOIs containing the operands, compared to problems where *x* was a whole number. A significant interaction between problem size and AOI was also present, $F(4, 184) = 3.64$, $p < .05$, $\eta_p^2 = .073$, showing that large problems resulted in an increase in fixation durations to the operands as well. There was also a significant interaction between AOI and expertise, $F(4, 184) = 4.17$, $p < .05$, $\eta_p^2 = .083$. Novices had significantly longer fixations toward the operands than the operators compared to their expert counterparts. This suggests that experts' greater experience allowed them to not only solve problems faster, but also more efficiently in terms of eye movements (See Figure 4).

Number of fixations were also analyzed and although these analyses did not result in any expertise effects, several predicted main effects and interactions were evident. There were main effects of problem size, $F(1, 46) = 4.95$, $p < .05$, $\eta_p^2 = .097$, x-value, $F(1, 46) = 20.20$, $p < .001$, $\eta_p^2 = .305$, and AOI $F(1, 46) = 27.62$, $p < .001$ $\eta_p^2 = .375$. There were also significant interactions between problem size and AOI, $F(4, 184) = 7.93$, $p < .001$, $\eta_p^2 = .147$ (See Figure 5), and between x-value and AOI, $F(4, 184) = 4.30$, $p < .05$, $\eta_p^2 = .086$. These significant interactions show that as problem size increases, so do the number of fixations to the operands. Equally as important, the number of fixations to the AOIs is significantly lower for problems where *x* is a whole number compared to problems where *x* is a decimal/fraction. This suggests that the value of the *x* variable appears to impact the difficulty of the problems, as does problem size.

We predicted that number of regressive looks (i.e., refixations) would significantly differ between expertise groups. Although results show that this may not be true for easy problems, the analysis did result in several main effects (See Table 2). There were also significant interactions between problem size and AOI, $F(4, 128) = 3.08$, $p < .05$, $\eta_p^2 = .088$ as well as x-value and AOI, $F(4, 128) = 3.15$, $p < .05$, $\eta_p^2 = .090$. These results show that the number of refixations that participants make to each AOI increased between small and large problems as well as between problems where *x* was a whole number versus where *x* was a decimal/fraction. These refixations are an indication that problem difficulty is increasing and refixations are necessary in order to finish solving the problems.

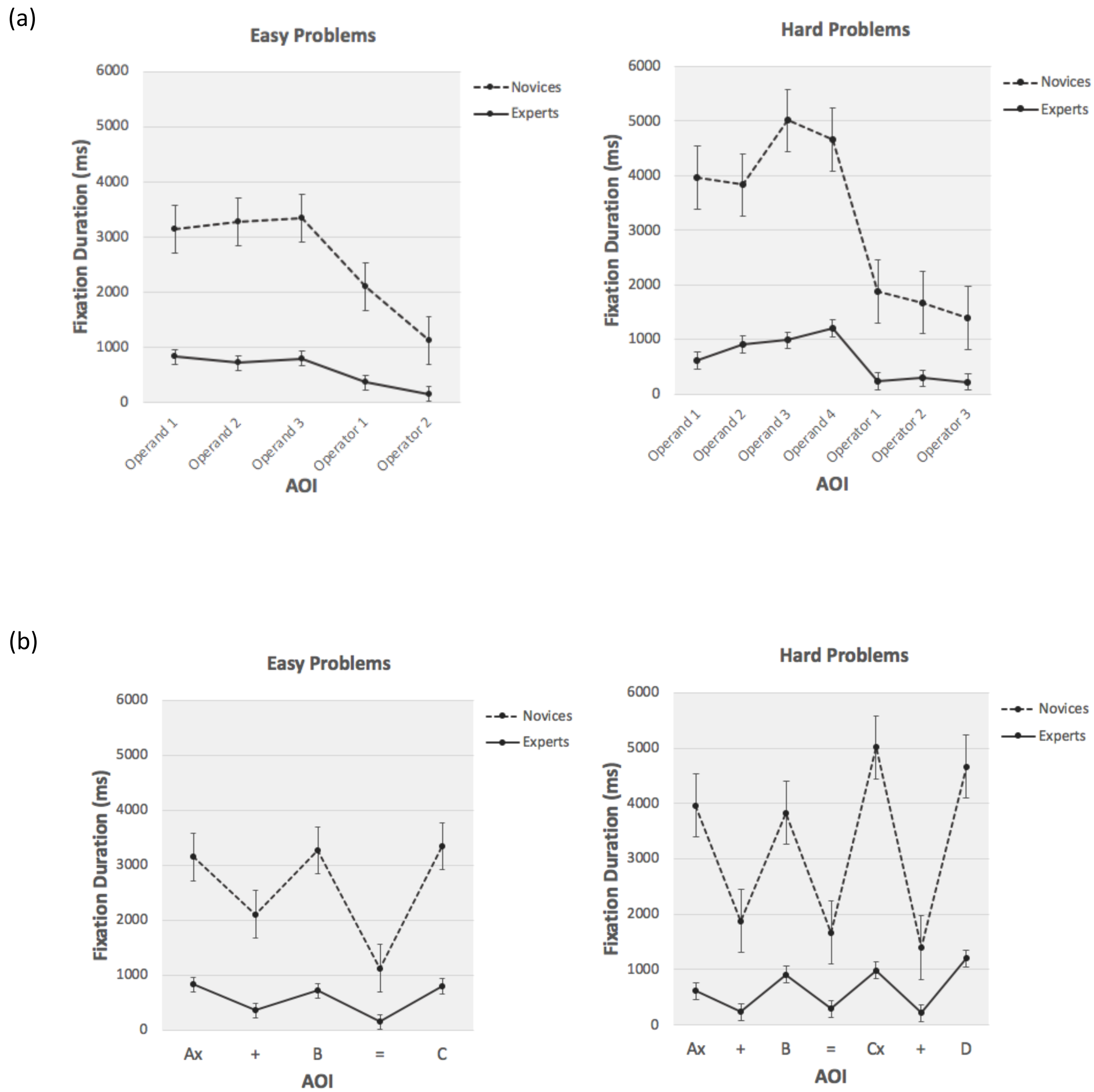


Figure 4. Mean Fixation Duration for Easy and Hard Problems. (a) Significant interactions between expertise and AOI for easy problems, $F(4, 184) = 4.17$, $p < .05$, $\eta_p^2 = .083$, and hard problems, $F(6, 270) = 7.86$, $p < .001$, $\eta_p^2 = .149$, plotted with operands and operators grouped together. (b) Significant interactions between expertise and AOI for easy and hard problems, with AOIs plotted in the order in which the problems were presented, from left to right, Ax + B = C and Ax + B = Cx + D.

Hard Problems. Fixation duration for hard problems revealed significant main effects of problem size, $F(1, 45) = 6.75$, $p < .05$, $\eta_p^2 = .130$, and AOI, $F(6, 270) = 21.68$, $p < .001$, $\eta_p^2 = .325$, along with a significant interaction between problem size and AOI, $F(6, 270) = 5.55$, $p < .001$, $\eta_p^2 = .110$. These findings, as predicted, show that as problem size increases, so do the length of fixations to the operands. Similar to the fixation duration results of the easy problems, we again have a significant interaction between AOI and expertise, $F(6, 270) = 7.86$, $p < .001$, $\eta_p^2 = .149$. Novices, again, are fixating significantly longer toward the operands compared to the operators than the experts and this effect is more pronounced compared to the same effects found with the easy problems (See Figure 4). Analyses concerning number of fixations for hard problems, did not result in any expertise effects. We do, however, have the predicted main effects of problem size, $F(1, 45) = 22.12$, $p < .001$, $\eta_p^2 = .330$ x-value, $F(1, 45) = 13.90$, $p < .05$, $\eta_p^2 = .236$, and AOI, $F(6, 270) = 35.33$, $p < .001$, $\eta_p^2 = .440$. There were also significant interactions between problem size and x-value, $F(1, 45) = 8.03$, $p < .05$, $\eta_p^2 = .151$, problem size and AOI, $F(6, 270) = 9.61$, $p < .001$, $\eta_p^2 = .176$, (See Figure 5), as well as between x-value and AOI, $F(6, 270) = 3.67$, $p < .05$, $\eta_p^2 = .075$. These interactions support previous results indicating that our prediction of problem size and x-value impacting overall problem difficulty continues to hold true in terms of fixation count for hard problems, in addition to the previously discussed behavioral results.

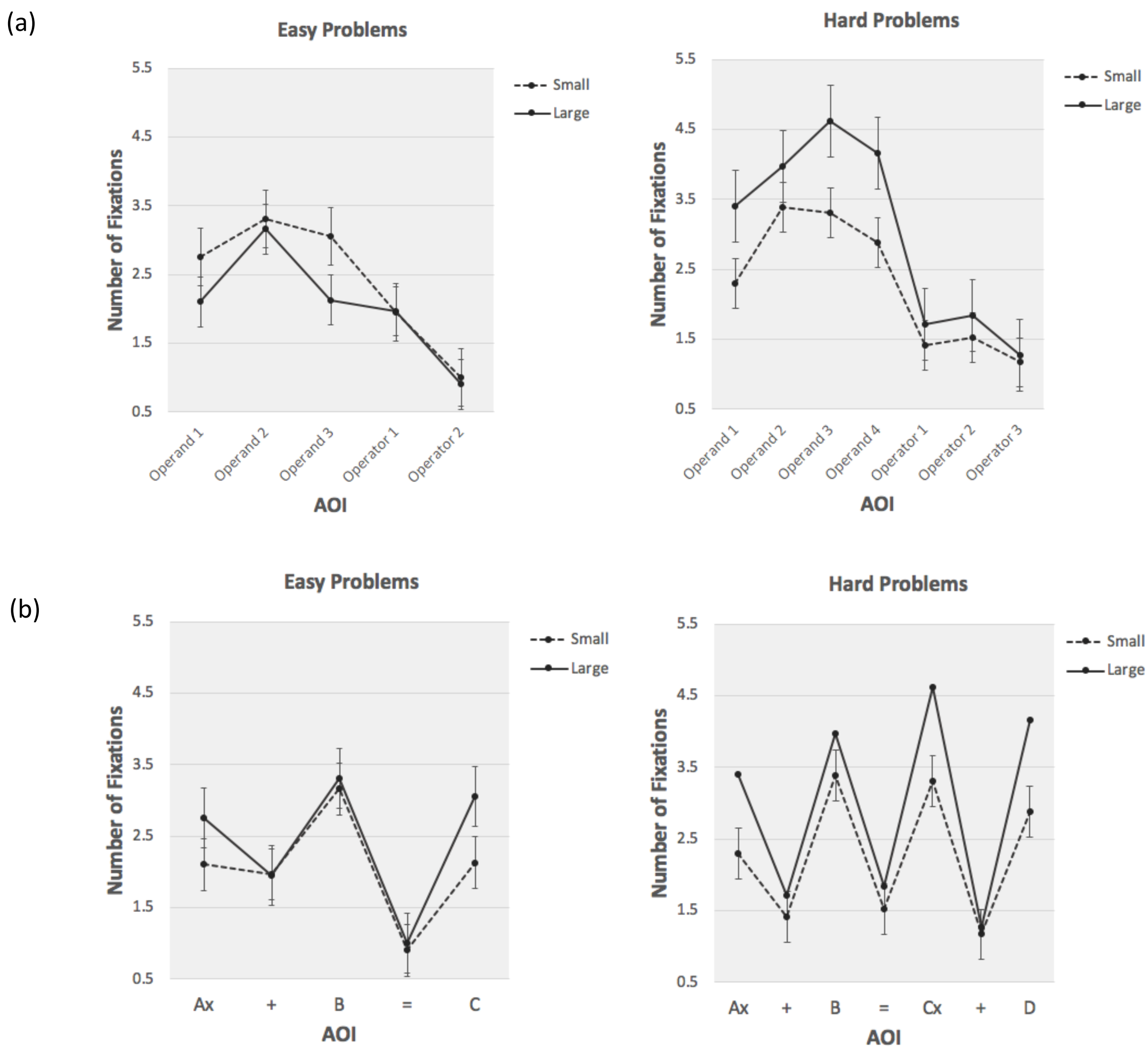


Figure 5. Number of Fixations for Easy and Hard Problems. (a) Significant interaction between problem size and AOI for easy problems, $F(4, 184) = 7.93$, $p < .001$, $\eta_p^2 = .147$, and hard problems, $F(6, 270) = 9.61$, $p < .001$, $\eta_p^2 = .176$, plotted with operands and operators grouped together, (b) Significant interactions between expertise and AOI for easy and hard problems, with AOIs plotted in the order in which the problems were presented, from left to right, Ax + B = C and Ax + B = Cx + D.

Regressive looks were also analyzed for hard problems. Although significant results echoed those found for the easy problems (See Table 3), there was also a significant interaction between AOI and expertise, $F(6, 162) = 4.48$, $p < .05$, $\eta_p^2 = .090$. As expected, the expert group exhibited fewer refixations to the operands than the operators compared to the novice group. With a noticeable increase in refixations for novices specifically for operand three. It is possible that experts' greater fluency with algebra resulted in more efficient eye movements and they were able to correctly identify the answer to the problems with fewer refixations overall. Novices, on the other hand, appear to be exhibiting more regressive looks to the operands while actively completing the mathematical operations (See Figure 6).

Table 2

Refixations Results for Easy Problems: Significant Main Effects and Interactions

	F-value	*p*-value	η_p^2
Problem Size	4.219	.048	.116
x-Value	17.218	.000	.350
AOI	27.002	.000	.458
Problem Size x AOI	3.077	.019	.088
x-Value x AOI	3.149	.017	.090

Table 3

Refixations Results for Hard Problems: Significant Main Effects and Interactions

	F-value	*p*-value	η_p^2
Problem Size	11.450	.002	.298
x-Value	15.844	.000	.370
AOI	24.937	.000	.480
Problem Size x AOI	4.478	.000	.142
x-Value x AOI	2.306	.037	.079
Expertise x AOI	2.682	.017	090

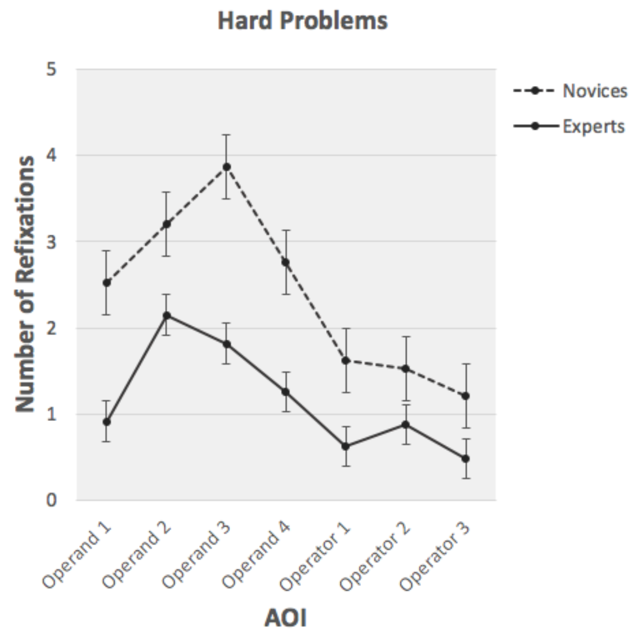
Regression

A step-wise multiple regression was conducted to determine if expertise (novice/expert), working memory (composite WM span score), and effort were good predictors of performance in terms of RT. Composite WM span scores were computed by averaging participants' partial span scores for the rotation and symmetry span tasks. The mean composite score for the novice group was 30.80, and the mean composite score for experts was 28.19. Regression results yielded an overall significant model, $F(1, 47) = 19.37$, $p < .001$., $R^2 = .292$. Expertise was the only significant predictor of RT performance, $t(47) = 15.00$, $p < .001$, $\beta = -.540$. WM span ($t(47) = -1.15$, $p = .255$) and effort ($t(47) = -.25$, $p = .806$) were not significant predictors in this model. A second step-wise multiple regression was conducted to determine if the same variables (expertise, WM, and effort) were good predictors of performance in terms of accuracy. Results yielded an overall significant model, $F(1, 47) = 6.65$ $p < .001$., $R^2 = .124$. Expertise was the only significant predictor of accuracy, $t(47) = 54.23$, $p < .001$, $\beta = -.352$. Again, WM span ($t(47) = 1.01$, $p = .320$) and effort ($t(47) = .45$, $p = .652$ were not significant predictors in this model.

Given the influence that expertise appears to have on performance, additional regression analyses were conducted on each expertise group separately. Step-wise multiple regressions were conducted to determine if WM and effort were good predictors of performance in terms of RT and accuracy. Results from these analyses yielded an overall significant model, $F(1, 28) = 5.30$, $p < .05$, $R^2 = .164$, for RT only for the novice group alone. WM span, $t(28) = 5.43$, $p < .001$, $\beta = -.405$, was a significant predictor in this model for RT, while effort was not a significant predictor $t(28) = .77$, $p = .447$. These results suggest that the novice group, given their lack of expertise in algebra, are more reliant on WM than their expert counterparts. Experts, on the other hand, have the experience and knowledge to work through these problems rather efficiently.

These problems are more challenging for the novice group, thereby requiring more cognitive effort and more reliance on WM capacity.

(a)



(b)

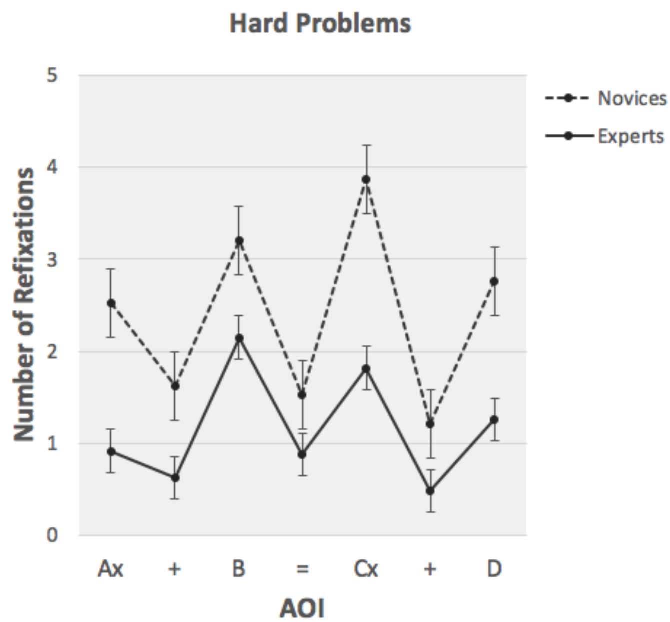


Figure 6. Refixation Results for Hard Problems. (a) Significant interaction between expertise and AOI, $F(6, 162) = 4.48$, $p < .05$, $\eta_p^2 = .090$, plotted with operands and operators grouped together. (b) Significant interaction between expertise and AOI for hard problems, with AOIs plotted in the order in which the problems were presented, from left to right, A*x* + B = C and A*x* + B = C*x* + D.

Chapter 4: Discussion

The primary aim of this study was to determine what performance looks like between novices and experts on an algebraic problem-solving task; specifically identifying, (1) if and where performance differences occur between the two groups, (2) if different problem-solving strategies are employed, and (3) what these potential performance differences look like in terms of eye movements. Given the critical role that working memory plays in mathematics, a secondary objective was to assess if working memory span was a good predictor of performance for the novice and expert groups.

Our findings are consistent with our predictions regarding the presence of the problem size effect and effects of problem complexity, however, our predictions regarding expertise effects were not as clear. Results produced significant problem size effects for both behavioral measures and eye tracking measures. Performance in terms of RT and accuracy were impacted by the size of the problem as well as the value of the x variable. Participants displayed longer latencies as well as lower accuracy rates for large compared to small problems as well as for problems where x was a decimal/fraction compared to x being a whole number. These factors contributed to the overall difficulty of the problems, thereby requiring more cognitive effort, and ultimately resulting in longer RTs for both groups. Regarding the eye tracking measures, we see a pronounced problem size effect especially for the large, hard problems with a significant increase in the number of fixations to each operand compared to the small problems. Fixation duration also demonstrates the problem size effect as we have increasing lengths of fixations between small and large problems for both the easy and hard manipulations. It is promising that we are able to see these effects in both typical behavioral measures as well as eye tracking measures, as this helps strengthen the argument for the presence of these effects in more difficult mathematical concepts, such as algebra.

The predicted expertise effects were only present for the behavioral measure of RT and the eye tracking measure of fixation duration. Experts were generally faster than novices, and although the problems where the answer was a decimal/fraction took experts longer to solve than the problems where the answer was a whole number, experts consistently outperformed the novice group. No differences in accuracy indicate that perhaps these problems were easy enough for both groups to accurately answer given their respective response times, but the additional knowledge and experience of the expert group allowed them to complete these problems significantly faster. The behavioral and eye tracking results align and provide support for one another in that if experts are solving problems faster than novices, then they should also exhibit shorter fixation durations – both of which are reinforced by our results. These results collectively support previous research indicating that experts in a given area, in this case math, not only have more experience in the subject, but this additional experience leads to greater efficiency in retrieving domain-specific knowledge, and ultimately leads to requiring less cognitive effort for the task at hand (Alexander, 2003; Bransford, et al., 1999, Heyworth, 1999).

Existing reading literature has shown that the number of regressive looks and length of fixation duration increase as text difficulty increases (Everatt, Bradshaw & Hibbard, 1998; Murray & Kennedy, 1988; Rayner, 1978; Underwood, Hubbard & Wilkinson, 1990). Results from the present study support and relate these original findings to the area of numerical cognition. While we did not find any expertise effects in regard to regressive looks for easy problems, we did see group differences for the hard problems. Experts made fewer refixations to the AOIs of the hard problems compared to the novices. These differences are indicative of greater fluency and mirror the reading literature in that as text difficulty increases (in this case, the difficulty of the algebra problems), novices require additional refixations in order to efficiently process the information for accurate solving.

Problem-solving strategies employed by these two groups become evident via the eye tracking measures of fixation duration and refixations. Both groups spend significantly less time looking at the operators of a problem compared to the operands. This is consistent with existing literature showing comparable differences (Curtis, et al., 2016). In Figures 3 and 4, we can see consistent patterns of fixation durations. Overall, novices and experts engage in the same pattern of looking across the entirety of a given problem, however, experts appear to be more fluent and more efficient in processing – resulting in overall shorter fixations to each AOI. For the hard problems, we see an increase in the length of fixations and the number of refixations for operand 3. This suggests that for a problem such as Ax + B = Cx + D, novices are spending significantly more time looking at the Cx term. This is the term that needs to be subtracted first during the problem-solving process. Perhaps novices are exposing their lack of expertise by having to revisit operand 3 and by looking longer at this operand during the calculation window. These longer fixations and revisits may be a direct indicator of the need for additional cognitive resources in order for novices to solve these types of problems.

Research supports the idea that WM plays a crucial role in math concepts beyond basic facts, especially those that involve active calculations and/or multiple steps (Ashcraft & Guillaume, 2009). Results from the regression analyses indicate that for this particular task, WM span was a good predictor of performance in terms of RT for the novice group alone. This suggests that novices are more reliant on WM than experts, given their lack of experience and knowledge of the subject. Experts, given their experience, already know how to solve these types of problems and are not as reliant on WM during the problem-solving process as these problems may be fairly easy. Novices, on the other hand, may find these types of problems rather challenging and therefore WM becomes a crucial factor in their overall performance. It is possible that with greater expertise, the role that WM plays in math begins to diminish. With expertise comes experience, additional knowledge, and additional practice, all of which contribute to overall performance.

It is worth briefly addressing the group differences in the amount of effort applied to the algebra task. According to the results, experts exhibited less effort than novices. One would naturally expect experts to display less effort, as this task likely requires less cognitive effort on their part. In the future, it might be useful to include a confidence scale in lieu of or in conjunction with an effort scale to better gauge group experiences.

Additional research is needed to help further understand the performance disparities that exist between novices and experts. This study only used two general types of algebra problems (equations with one or two unknowns), so additional research is warranted in order to examine different aspects of algebraic problem-solving. Different types of equations and different mathematical operations should also be considered in order to get a more holistic interpretation of algebra performance. It may worth examining differences in more basic mathematical concepts in order to develop an understanding of fundamental differences between these two groups. From there, additional research can expand into other, more difficult mathematical concepts, such as fractions.

This study emphasizes the importance and utility of using eye tracking to help better understand problem-solving and performance differences. Eye tracking measures help uncover otherwise unexplored results in terms of where participants are looking and for how long, during the calculation window. This information is not accessible via typical behavioral measures of RT and accuracy and although participant self-reports can inform researchers of the general strategies that are used, number of fixations and length of fixations to each AOI are not typical of self-report data. The use of behavioral measures in conjunction with eye tracking measures help obtain a more complete story of the cognitive processes employed during the problem-solving process and can help uncover other potentially unexplored group differences.

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Curriculum Vitae

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EDUCATION

Expected 2020	University of Nevada, Las Vegas Ph.D. Experimental Psychology
2018	University of Nevada, Las Vegas M.A. Experimental Psychology
2012	University of Nevada, Las Vegas B.A. Psychology

RESEARCH EXPERIENCE

University of Nevada, Las Vegas
Graduate Research Assistant
August 2015 – Present
Faculty Advisor: Mark H. Ashcraft, Ph.D., Math Cognition Lab

University of Nevada, Las Vegas
Graduate Research Assistant
August 2015 – August 2016
Research Assistant
April 2014 – August 2015
Faculty Advisor: Jennifer L. Rennels, Ph.D., Baby and Child Rebel Lab

TEACHING EXPERIENCE

Fall 2018 – Present	Instructor, *Foundations of Cognitive Psychology* University of Nevada, Las Vegas
Fall 2017 – Spring 2018	Instructor, *General Psychology (Introduction to Psychology)* University of Nevada, Las Vegas
Fall 2016 – Spring 2017	Teaching Assistant, *Statistics for Psychologists I* University of Nevada, Las Vegas

CONFERENCE PRESENTATIONS

Kamekona, K. & Ashcraft, M. H. (2017, November). *The Effects of Math Anxiety and Math Achievement on Basic Arithmetic Facts: A Pupillometry Approach.* Poster session presented at the Psychonomic Society 58th Annual Meeting. Vancouver, BC, Canada.

Rennels, J.L., Kulhanek K., Kayl A.J., Cummings, A.J., & **Kamekona, K**. (2017, May). *Infants' Preference for Female Faces Varies Based on Motor Skills and Facial Expressions.* Poster session presented at the 2017 Society for Research in Child Development Biennial Meeting. Austin, TX.

Rennels, J.L., Cummings, A.J., Kayl, A.J., **Kamekona, K**., & Kulhanek, K. (2016, May). *Infants' Advantage in Locating Female Faces in a Visual Search Task.* Poster session presented at the 20th Biennial International Conference on Infant Studies. New Orleans, LA.

Rennels, J.L., Kayl, A.J., Cummings, A.J., **Kamekona, K**., & Kulhanek, K. (2016, May). *Age Differences in Infants' Visual Preference for Female Faces*. Poster session presented at the 20th Biennial International Conference on Infant Studies. New Orleans, LA.

Glover, V.A., Rennels, J.L., Valdez, V.W., & **Kamekona, K.** (2013, January). *Using a Learning Task to Alter Implicit Associations for African American Males.* Poster session presented at the 14th Annual Meeting of the Society for Personality and Social Psychology. New Orleans, LA.

TALKS

Glover, V.A., & Rennels, J.L., Valdez, V. W., & **Kamekona, K.** (2014, April). *Racial Implicit Bias for African American and Caucasian Males*. Presented at the University of Nevada, Las Vegas.

Glover, V.A., & Rennels, J.L., Valdez, V. W., & **Kamekona, K.** (2013, March). *Using a Learning Task to Alter Implicit Associations for African American Males*. Presented at the annual meeting of the Graduate and Professional Student Association research forum, Las Vegas, NV.

AWARDS AND GRANTS

2018 - 2019	Patricia Sastaunik Scholarship – University of Nevada, Las Vegas ($2,500)
2017 - 2018	Graduate College Summer Research Fellowship – University of Nevada, Las Vegas ($7,000)
2017 - 2018	OUMP Outstanding Mentor Award – University of Nevada, Las Vegas ($75)
2016	Graduate & Professional Student Association Travel Grant – University of Nevada, Las Vegas ($500)

ACADEMIC SERVICE

2018 – 2019	Reviewer – Association for Psychological Science Student Caucus, Student Grant Competition
2018 – Present	Diversity & Inclusion Liaison – Experimental Student Committee, University of Nevada, Las Vegas
2016 – 2017	Secretary – Experimental Student Committee, University of Nevada, Las Vegas
2016 – 2017	Cohort Representative – Experimental Student Committee, University of Nevada, Las Vegas
2015 – Present	Mentor – Outreach Undergraduate Mentoring Program (OUMP), University of Nevada, Las Vegas

PROFESSIONAL AFFILIATIONS

Association for Psychological Science
Psychonomic Society
Western Psychological Association