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Analysis of Bank Failure and Size of Assets

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ANALYSIS OF BANK FAILURE AND SIZE OF ASSETS

by

Guancun Zhong

Master of Science
Dalian University of Technology
2009

A thesis submitted in partial fulfillment
of the requirements for the

Master of Science in Mathematical Sciences

**Department of Mathematical Sciences
College of Sciences
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**University of Nevada, Las Vegas
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THE GRADUATE COLLEGE

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Analysis of Bank Failure and Size of Assets

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August 2012

ABSTRACT

Analysis of Bank Failure and Size of Assets

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The financial health of the banking industry is an important prerequisite for economic stability and growth. Bank failures in the United States have run in cycles largely associated with the collapse of economic bubbles. The number of bank failures has increased dramatically over the last thirty years (Halling and Hayden, 2007). In this thesis, we try to address the following two questions: 1) What is the relationship, if any, between a bank's asset size and its likelihood of failures? 2) How can we use statistical tools to predict the numbers of bank failures in the future? Various modeling techniques are proposed and applied to bank failure data from Federal Deposit Insurance Corporation. For the first question, we find that there is a relationship between bank size and bank failure status based on the Pearson's chi-square test. To answer the second question, first, logistic regression is applied to the bank failure data, and the corresponding prediction rule and prediction results are obtained. Second, we develop the empirical recurrence rate (Ho, 2008) and empirical recurrence rates ratio time series for

the given data, and also perform corresponding theoretical and graphical analysis on both of them. We obtained much valuable information on the reason for, time period of, and trends of bank failures in the past thirty years. We perform pairwise bank failure rate comparisons using the conditional test (Przyborowski and Wilenski, 1940). Additionally, based on the smooth behavior of empirical recurrence rate and empirical recurrence rates ratio time series, we apply autoregressive fractional integrated moving average models to both of the series for forecasting purposes. Finally, some interesting results are discussed.

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CHAPTER 1

INTRODUCTION

A bank fails when it can no longer cover its obligations (liabilities) with its assets and must file for bankruptcy. Washington Mutual (WaMu) failed on September 25, 2008. WaMu reportedly had over \$30 billion in assets at the time of their failure; almost 300 banks have collapsed since then. During the two years since WaMu failed, the number of bank failures significantly increased compared to the previous six years, during which period only around 40 banks failed. In retrospect, the number of bank failures has increased dramatically over the last 25 years. Out of 3,879 total bank failures since 1934, when the Federal Deposit Insurance Corporation (FDIC) was established, nearly 3000 occurred between 1985 and 2010. The increase in bank failures is typically accompanied by high unemployment and reduced liquidity. Moreover, the survivors collect the market power by reducing competition and potentially harming consumers in the future (Levin and Coburn, 2011).

To reduce the risk of bank failures, the FDIC, which, since 1980, guaranteed to pay the first \$100,000 deposit in full to each account-holder if the bank failed, temporarily raised the amount to \$250,000 during the financial crisis in 2008. Additionally, Congress passed the Emergency Economic Stabilization Act to assist the banking industry during the financial crisis. The United States Treasury spent up to \$700 billion to support distressed assets from banks, which injected new capital into the banking system. Despite the aforementioned events, the number of bank failures increased. As more and more

analysts focus their attention on the banking industry, a widespread concern emerges: Will the situation worsen in the future?

Most previous studies of bank failures rely on bank-level accounting data, occasionally augmented with market-price data. These studies aim to develop models of an early warning system for individual bank failures (Cole and Wu, 2009). Since the 1980s, studies have been conducted using mathematical programming-based discriminant analysis. Theoretical studies on mathematical programming-based discriminant analysis were first conducted in the early 80s (Freed and Glover, 1981). These studies focused mostly on the applicability of mathematical programming techniques on discriminant analysis and their formulation (Glover, 1990). Also conducted were evaluation of the results of applications of these models (Wallin and Sundgren, 1995). Additionally, studies aimed at developing new models which were compatible with new mathematical programming-based discriminant analysis were also conducted, while goal-programming and mixed-integer programming were applied to combine discriminant analysis and data-envelopment analysis (Sueyoshi, 1999). Although there are several models in different fields in the literature, there is a consensus on “minimum sum of deviations model (MSD)” as the model which gives the most proper results in a significant portion of studies (Karacabey, 2003).

In this thesis, we extend the work of Cui (2011) in a few ways: First, the time period of data is different. Cui’s data starts from 1989, which is in the middle of Savings and Loan crisis (1980s to 1990s). We extend that back to 1980, to include the entire Savings and Loan crisis in our data. Second, different grouping strategies are applied; we separate bank data into four equal groups using first quartile, median and third quartile of the

adjusted assets, so we can hold the four population sizes the same. Third, we extend our modeling method to ARFIMA to get a better prediction in the application part in Chapter 6.

Additionally, we use a two way contingency table to find the dependency of bank status and bank size; We then perform simple logistic regression using adjusted assets as the solo independent variable; Next we transfer the raw data to empirical recurrence rate (ERR) and the empirical recurrence rates ratio (ERRR) time series, which is an extension of ERR. We perform some detailed analysis and exploration on both ERR and ERRR. Finally, we apply the autoregressive integrated moving average (ARIMA) and autoregressive fractional integrated moving average (ARFIMA) models on ERR and ERRR, including model selection, validation, and forecasting for the bank failures classified by the total assets.

Specifically, bank data and Pearson's chi-square tests are given in Chapter 2. Logistic regression is presented in Chapter 3. Empirical recurrence rate (ERR) and empirical recurrence rates ratio (ERRR) are introduced in Chapter 4. We have an introduction to autoregressive integrated moving average (ARIMA) and autoregressive fractional integrated moving average (ARFIMA) modeling in Chapter 5. In Chapter 6, we predict the ERR and ERRR time series with ARIMA and ARFIMA models. Chapter 7 concludes our work.

CHAPTER 2

EXPLORATORY DATA ANALYSIS

2.1 Data

The numbers of bank failures in the United States during 1980:Q1 to 2011:Q4 were obtained from the FDIC (Federal Deposit Insurance Corporation) failed banks list (<http://fdic.gov/bank/individual/failed/banklist.html>), which lists failed banks by name, location, charter type, total assets, and other characteristics. We count the number of bank failures on a quarterly basis. Based on this list, 3,212 banks were reported to fail over the 128 quarters (Fig. 1).

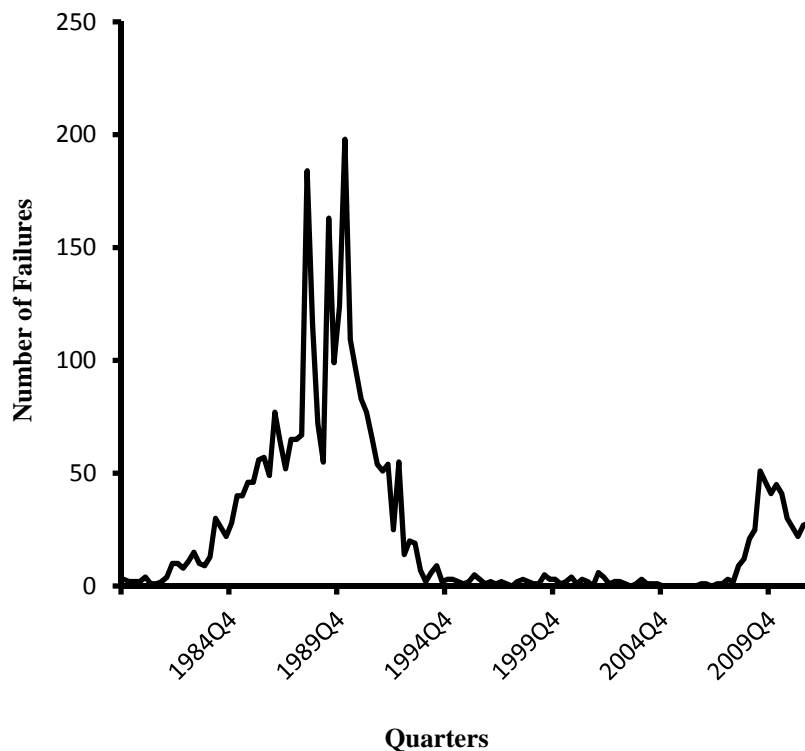


Fig. 1. Number of bank failures from 1980:Q1 to 2011:Q4.

2.1.1 CPI adjustment

In economics, the nominal level of prices of goods and services changes over a period of time. When the price level rises, each unit currency buys fewer amount of goods and services. The purchasing power of money – the real value in the internal medium of exchange and unit of account in the economy, changes over time. The Consumer Price Index (CPI) is used to bridge nominal values to real values. The total assets of banks are reported in terms of nominal price. To make the total assets in different time periods comparable, the total assets of banks are converted to the real values which are based on (Mankiw, 2002):

$$\text{Adjusted total assets} = \frac{\text{CPI}_b}{\text{CPI}_i} \times \text{Total assets}_i,$$

where Total assets_i is the nominal total asset at time i (the month a failure was reported); CPI_i is CPI at the i th month that bank failed; CPI_b is the CPI for the base month (taken as January 2011 in this thesis).

2.1.2 Bank classification

The data on bank failures will be divided into four groups, based on the adjusted total assets held by the banks at the time they failed. First, we have all bank assets adjusted by CPI index and ordered from smallest to largest. Then, we set the banks with adjusted assets (in millions) lower than the 1st quartile (67.43) of the adjusted assets list as our “Small” banks group (also known as G_1). Similarly, banks with adjusted assets between 1st quartile and the median (150) are referred to as “Medium” banks group (also known as G_2), banks with adjusted assets between median and 3rd quartile (360) are referred to as “Large” banks group (also known as G_3) and banks with adjusted assets higher than the

3rd quartile are referred as “Grand” banks group (also known as G₄). Counts of bank failures by status (“Yes” for failed and “No” for solvent) and Group (1 through 4) are summarized in Table 1. The time series plots by groups are illustrated in Fig. 2.

Table 1

Counts of banks by failure status and asset group during 1980:Q1-2011:Q4

		Group				Totals
		1 [3.0, 67.4]*	2 (67.4, 150]	3 (150, 360]	4 (360, 1810000]	
Status	Yes	1172	663	590	787	3212
	No	1456	1965	2038	1841	7300
Totals		2628	2628	2628	2628	10512

* In millions

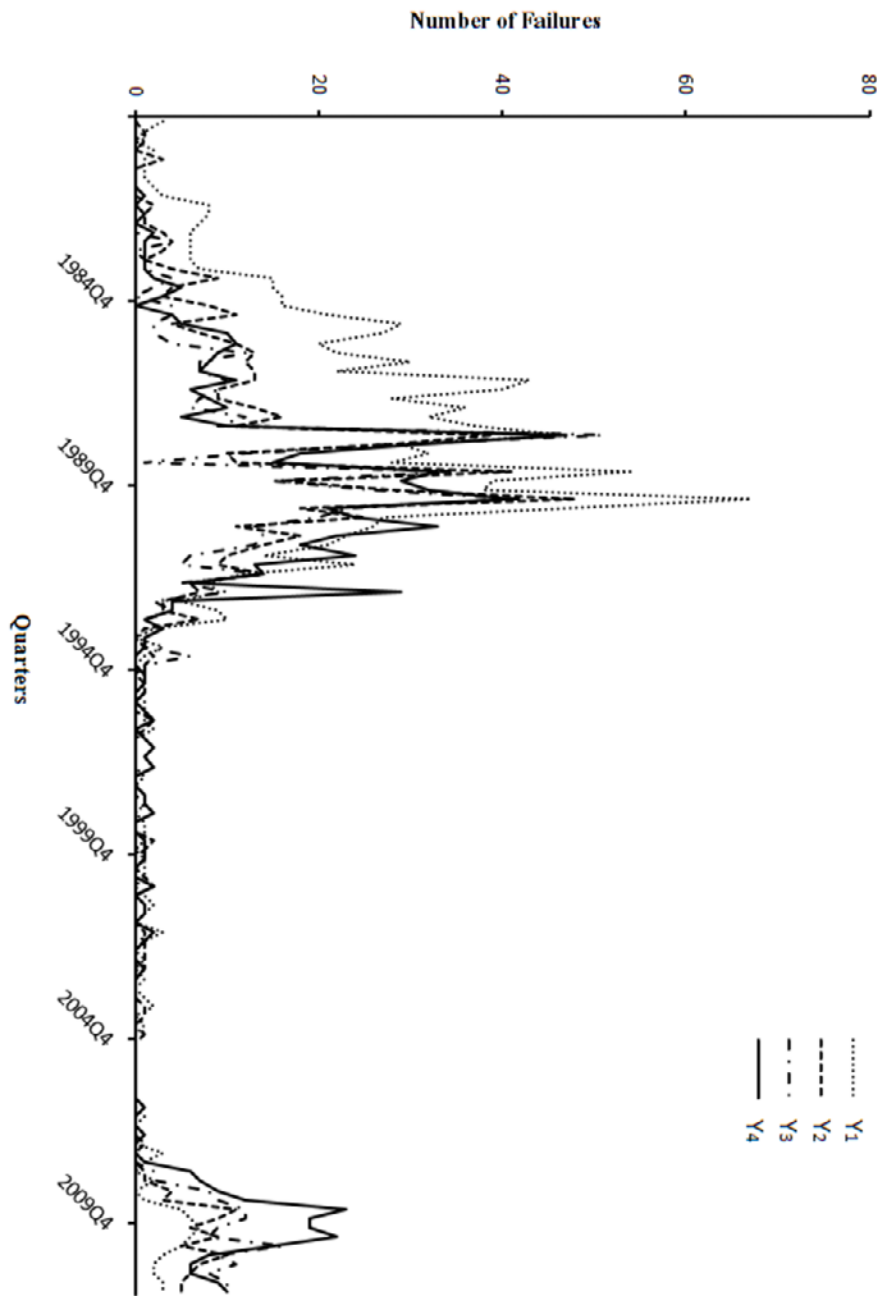


Fig. 2. Numbers of bank failures from 1980:Q1 to 2011:Q4 by groups.

2.2 Pearson's chi-square test for independence

We use Table 1 as a 2 x 4 contingency table and perform Pearson's chi-square test for independence between Status and Group. The null hypothesis and alternative hypothesis are stated as follows:

H_0 : Status and Group are independent.

H_a : Status and Group are dependent.

The chi-square test statistic is 361.141 at 3 df, resulting in a p-value ≈ 0 . Thus, we conclude that the data provides sufficient evidence to reject the null hypothesis at a 5% significance level. We conclude that there is an association between the failure status and asset size of a bank.

We then collapsed Table 1 to produce four 2 x 2 contingency tables for G_1 vs. the rest, G_2 vs. the rest, G_3 vs. the rest and G_4 vs. the rest with the same null hypotheses. The corresponding contingency tables are given in Table 2 and the results are shown in Table 3. Testing each of the groups, G_1 , G_2 and G_3 , against their corresponding complementary groups results in rejection of corresponding H_0 at $\alpha = 0.05$ leading us to conclude that there is some type of dependence between group membership and failure status. Testing G_4 against all the other banks results in that there is not enough evidence to reject the null hypothesis at $\alpha = 0.05$. It appears that the “Group” partitioned by G_4 and the others are independent of “Status”. This means that whether a bank is “Grand” or not has no bearing on its failure status.

Table 2

Collapsed 2x2 contingency tables based on Table 1

		Group							
		1	Rest	2	Rest	3	Rest	4	Rest
Status	Yes	1172	2040	663	2459	590	2622	787	2425
	No	1456	5844	1965	5335	2038	5262	1841	5459

Table 3

Results of Pearson's chi-square tests for independence

	Pairwise comparisons			
	1 vs. Rest	2 vs. Rest	3 vs. Rest	4 vs. Rest
Test statistic	325.6	46.9	108.5	0.61
<i>P</i> -value	0	0	0	0.434

2.3 Relative risk and odds ratio

The relative risk (RR) is the risk of an event (bank failure) relative to exposure (selected group). In other words, RR is a ratio of the probability of the event occurring in the exposed group (G_i) versus a non-exposed group (all the other groups combined). Let RR_i be the relative risk of bank failures occurring in G_i ($i = 1, 2, 3, 4$) vs. all the other groups combined (excluding G_i). In Table 4, F represents counts of failed banks and S is for solvent banks.

Table 4
Relative risk and odds
ratio

		Group	
		i	others
Status	Yes	F_i	F_{\cdot}
	No	S_i	S_{\cdot}

Then, we define $RR_i = \frac{F_i/(F_i + S_i)}{F_{\cdot}/(F_{\cdot} + S_{\cdot})}$,

The odds ratio (OR) is a measure of effect size, describing the strength of association or non-independence between two binary data values. Unlike the relative risk, the odds ratio treats the two variables being compared symmetrically, and can be estimated using some types of non-random samples (Sistrom and Garvan, 2004). For odds ratio, we first define odds for G_i by

$\omega_i = \pi_i/(1 - \pi_i)$, where $\pi_i = \text{Prob}(\text{bank failure for } G_i)$. Similarly, we define “ ω_{\cdot} ” for the complement of G_i (i.e., the others). The odds ratio is then defined as follows:

$$\Phi_i = \frac{\omega_i}{\omega_{\cdot}} = \frac{F_i \cdot S_{\cdot}}{F_{\cdot} \cdot S_i}, i = 1, 2, 3 \text{ and } 4.$$

For all four comparisons given in Table 2, we calculate the corresponding relative risk point estimators and their 95% confidence intervals, as well as odds ratio point estimators and 95% confidence intervals; the results are presented in Table 5 and Table 6.

We use the following formulas for confidence intervals (Agresti, 2002):

$$CI(RR_i) = \exp [CI(\log RR_i)],$$

where $CI(\log RR_i) = \log RR_i \pm z_{\alpha/2} SE(\log RR_i)$, and

$$SE(\log RR_i) = \sqrt{\frac{1}{F_i} - \frac{1}{F_i + S_i} + \frac{1}{F_{\cdot}} - \frac{1}{F_{\cdot} + S_{\cdot}}}$$

$$CI(\Phi_i) = \text{Exp} [CI(\log \Phi_i)],$$

where $CI(\log \Phi_i) = \log \Phi_i \pm z_{\alpha/2} SE(\log \Phi_i)$, and

$$SE(\log \Phi_i) = \sqrt{\frac{1}{F_i} + \frac{1}{S_i} + \frac{1}{F_i} + \frac{1}{S_i}}$$

As an example, we look at the comparison of G_1 vs. the rest (first comparison in Table 2). $RR_1 = 1.724$ (Table 5) indicates that the relative risk of bank failure for G_1 is 1.724 times higher than the risk of the others. We are also 95% confident that the relative risk of failure for banks in G_1 is between 1.629 and 1.824 compared to the others. Similarly, $\Phi_1 = 2.306$ means that the odds of bank failure for banks in G_1 are 2.306 times the odds of those not in G_1 . Thus, there is strong evidence that membership in G_1 results in more failures compared to the others.

The results of the relative risk and odds ratios are consistent with the results of the Pearson's tests. If the groups are independent of the bank Status we would have expected the relative risk and the odds ratio to be close to one. It can be seen that the G_1 banks have much higher odds of failure than the other categories. The confidence intervals are consistent with these findings in that the value 1 is only included in the confidence intervals for relative risk and odds ratio for the contingency table of G_4 vs. the others. So it seems like being in G_1 increases the chances of failure, while being in G_2 or G_3 decreases. Being in G_4 has no significant effect on chances of failure.

Table 5
Point estimator and 95% confidence interval for relative risk

	Relative Risk	Confidence Interval	
	Point Estimator	Lower Bound	Upper Bound
G_1 vs. Rest	1.724	1.629	1.824
G_2 vs. Rest	0.781	0.726	0.840
G_3 vs. Rest	0.675	0.625	0.730
G_4 vs. Rest	0.973	0.915	1.041

Table 6

Point estimator and 95% confidence interval for odds ratio

	Odds Ratio	Confidence Interval	
	Point Estimator	Lower Bound	Upper Bound
G ₁ vs. Rest	2.306	2.103	2.528
G ₂ vs. Rest	0.707	0.639	0.781
G ₃ vs. Rest	0.581	0.525	0.644
G ₄ vs. Rest	0.961	0.873	1.059

CHAPTER 3
LOGISTIC REGRESSION

3.1 Methodology

3.1.1 Simple logistic regression

Let us begin by considering the distribution of the dependent variable Y , bank failure status. We assume that the dependent variable is binary, taking the values of 1 (failed) and 0 (solvent) with probabilities of $\pi = P(Y = 1)$ and $1 - \pi = P(Y = 0)$. Thus $Y \sim \text{Bernoulli}(\pi)$. Here we use X_i to express the adjusted asset of a bank. The simple logistic regression is:

$Y_i \sim \text{Bernoulli}(\pi)$, where

$$\pi = \frac{e^{\beta_0 + \beta_1 X_i}}{1 + e^{\beta_0 + \beta_1 X_i}}, \quad i = 1, \dots, n.$$

3.1.2 Maximum likelihood estimation

Here the dependent variable follows a Bernoulli distribution. The probability distribution is given as follows:

$$f_i(y_i) = (\pi_i)^{y_i} (1 - \pi_i)^{1 - y_i} \text{ where } y_i = 0, 1; i = 1, \dots, n$$

Assuming all the y_i 's are independent, their joint probability function (likelihood) is:

$$L(\beta_0, \beta_1) = \prod_{i=1}^n (\pi_i)^{y_i} (1 - \pi_i)^{1 - y_i} = \prod_{i=1}^n \left[\left(\frac{\pi_i}{1 - \pi_i} \right)^{y_i} (1 - \pi_i) \right]$$

Now, we take the natural log on both sides of the above likelihood function.

$$\begin{aligned}\ln L(\beta_0, \beta_1) &= \ln \prod_{i=1}^n \left[\left(\frac{\pi_i}{1-\pi_i} \right)^{y_i} (1-\pi_i) \right] \\ &= \sum_{i=1}^n \left[y_i \ln \left(\frac{\pi_i}{1-\pi_i} \right) \right] + \sum_{i=1}^n \ln(1-\pi_i)\end{aligned}$$

Since $\pi_i = \frac{e^{\beta_0 + \beta_1 x_i}}{1 + e^{\beta_0 + \beta_1 x_i}}$, it follows that $1 - \pi_i = [1 + e^{\beta_0 + \beta_1 x_i}]^{-1}$. Also, we have

$\ln \left(\frac{\pi_i}{1-\pi_i} \right) = \beta_0 + \beta_1 x_i$. We can now plug this into the log-likelihood function to get:

$$\ln L(\beta_0, \beta_1) = \sum_{i=1}^n [y_i(\beta_0 + \beta_1 x_i)] - \sum_{i=1}^n \ln(1 + e^{\beta_0 + \beta_1 x_i})$$

To find the maximum likelihood estimates, we take derivative with respect to β_0 and β_1 .

The derivatives are given by:

$$\frac{\partial \ln L}{\partial \beta_0} = \sum_{i=1}^n \left[y_i - \frac{e^{\beta_0 + \beta_1 x_i}}{1 + e^{\beta_0 + \beta_1 x_i}} \right] = \sum_{i=1}^n [y_i - \pi_i]$$

$$\frac{\partial \ln L}{\partial \beta_1} = \sum_{i=1}^n \left[x_i y_i - \frac{x_i e^{\beta_0 + \beta_1 x_i}}{1 + e^{\beta_0 + \beta_1 x_i}} \right] = \sum_{i=1}^n x_i [y_i - \pi_i]$$

where $\pi_i = E[y_i]$. Now we set these derivatives equal to 0 to get the likelihood equations:

$$\begin{aligned}\sum_{i=1}^n y_i &= \sum_{i=1}^n \frac{e^{\beta_0 + \beta_1 x_i}}{1 + e^{\beta_0 + \beta_1 x_i}} \\ \sum_{i=1}^n x_i y_i &= \sum_{i=1}^n \frac{x_i}{1 + e^{\beta_0 + \beta_1 x_i}}\end{aligned}\tag{3.1}$$

We need a computer-intensive numerical search procedure to find the actual maximum likelihood estimates b_0 and b_1 of β_0 and β_1 , respectively, which are solutions

to (3.1). When we get the estimates, we plug them into the response function $\frac{e^{\beta_0 + \beta_1 x_i}}{1 + e^{\beta_0 + \beta_1 x_i}}$

to get the fitted response function; denoted as:

$$\hat{\pi}_i = \frac{e^{b_0 + b_1 x_i}}{1 + e^{b_0 + b_1 x_i}}$$

Once the fitted logistic response function is obtained, we can examine the appropriateness of the fitted response function and calculate predictions (Mathis, 2011).

Also, note that $\ln\left(\frac{\pi_{x_i}}{1-\pi_{x_i}}\right) = \beta_0 + \beta_1 x_i$ is the log-odds for given independent variable $x = x_i$, and $\ln\left(\frac{\pi_{x_i+1}}{1-\pi_{x_i+1}}\right) = \beta_0 + \beta_1(x_i + 1)$. Therefore, the difference of the two is the log of the ratio of the two odds as shown below: $\ln\left(\frac{\pi_{x_i+1}}{1-\pi_{x_i+1}}\right) - \ln\left(\frac{\pi_{x_i}}{1-\pi_{x_i}}\right) = \ln\left(\frac{odds_{x_i+1}}{odds_{x_i}}\right) = \beta_1$. If we take antilogs, we obtain the odds ratio: $OR = e^{\beta_1}$. The log odds ratio can thus be interpreted as the log-change in odds associated with a unit increase in the value of the predictor variable. In general, the odds ratio associated with a change of d units in the predictor variable is $e^{d\beta_1}$ (Douglas et al, 2012).

3.2 Modeling

3.2.1 Application and interpretation

Next, we try to build a simple logistic regression model for the binary response (bank failure status) during 01/01/1980 to 12/31/1995 (covering the period of Savings and Loan crisis, 1980s to 1990s) using log of adjusted assets (a continuous variable) as the solo predictor. We take logarithms of the adjusted assets since the range of the original values is too wide (from 3 million to 1,810,000 million). Note that, the data are ungrouped. Computer output is given below:

```

> failbank<-glm(status~asset,data=bank,family=binomial)
> summary(failbank)

Call:
glm(formula = status ~ asset, family = binomial, data = bank)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-1.1987  -0.8993  -0.8052   1.3668   2.1168

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept)  3.68912    0.33079   10.61  <2e-16 ***
asset       -0.24776    0.01763  -12.91  <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 10829  on 10005  degrees of freedom
Residual deviance: 10650  on 10004  degrees of freedom
AIC: 10654
Number of Fisher Scoring iterations: 4

```

So the estimated model is:

$$\ln\left(\frac{\hat{\pi}}{1-\hat{\pi}}\right) = 3.689 - 0.248 \ln(\text{Adjusted total assets}) \tag{3.2}$$

Thus, $\hat{\pi} = \frac{\exp(3.689-0.248x_i)}{1+\exp(3.689-0.248x_i)}$, so we have estimated OR = $e^{b_1} = e^{-0.248} = 0.78$

The 95% confidence interval for β_1 is given by:

$$b_1 \pm z(0.975) \cdot s(b_1) = (-0.283, -0.213),$$

and the 95% confidence interval for the odds ratio is given by:

$$\exp(b_1 \pm z(0.975) \cdot s(b_1)) = (0.754, 0.808).$$

This means that with additional unit of log adjusted assets, the odds of bank failure are $e^{-0.248} = 0.78$ of the prior value. That is, the odds of bank failing are 22% smaller for each additional unit increase of log (adjusted bank assets).

3.2.2 ROC curve and prediction on new observations

A classification model is a mapping of instances into certain classes/groups. The classifier or diagnosis result can be a real value, in which case the classifier boundary between classes must be determined by a threshold value, or it can be a discrete class label, indicating one of the classes.

In a two-class prediction problem, the outcomes can be labeled either as positive (p) or negative (n). There are four possible scenarios from such a binary classifier. If the outcome from a prediction is p and the actual value is also p, then it is called a true positive (TP); however if the actual value is n then it is said to be a false positive (FP). Conversely, a true negative (TN) is said to have occurred when both the prediction outcome and the actual value are n, and false negative (FN) is said to have occurred when the prediction outcome is n while the actual value is p.

A contingency table can be evaluated using various methods. One of them is a Receiver Operating Characteristic (ROC) curve, which plots true positive rate (TPR) on the y-axis against false positive rate (FPR) on the x-axis. The TPR defines how many correct positive results occur among all positive samples available during the test. FPR, on the other hand, defines how many incorrect positive results occur among all negative samples available during the test.

An ROC depicts relative trade-offs between true positive and false positive. Since TPR is equal to sensitivity and FPR is equal to $1 - \text{specificity}$, the ROC graph is sometimes also known as the sensitivity vs. $(1 - \text{specificity})$ plot.

The best possible prediction method would yield a point in the upper left corner i.e., (0, 1) coordinate of the ROC space, representing 100% sensitivity (no false negatives) and 100% specificity (no false positives). The (0, 1) point is also called a perfect classification. A completely random guess would give a point along the 45° diagonal line from the left bottom to the top right corner. An example of random guessing would be a decision reached by flipping coins.

Points above the 45° diagonal represent good classification results, while points below the line represents poor results (worse than random). Note that the output of a consistently poor predictor could simply be inverted to obtain a good predictor. Also, we can use AUC (area under curve) to measure the goodness of a classification model. AUC is calculated as the area covered by the ROC curve and x-axis in the ROC space. AUC is a number between 0.5 and 1. Since we would like the ROC curve to be far from the diagonal for good predictions, a model with AUC between 0.7 and 0.9 is considered a good model, while one with AUC greater than 0.9 is considered perfect (Swets, 1996).

The ROC curve of model (3.2) shown in Fig. 3. In the graph, we can see the sensitivity is 40.8%, specificity is 79.5%, so 1 - specificity is 20.5%. Sensitivity is the probability of correctly identifying a failed bank. Specificity is the probability of correctly identifying a solvent bank and hence 1 - specificity is the probability of incorrectly concluding a solvent bank as a failed bank. We find the AUC to be 0.602, which indicates that the fitted logistic regression model is not a satisfactory predictor.

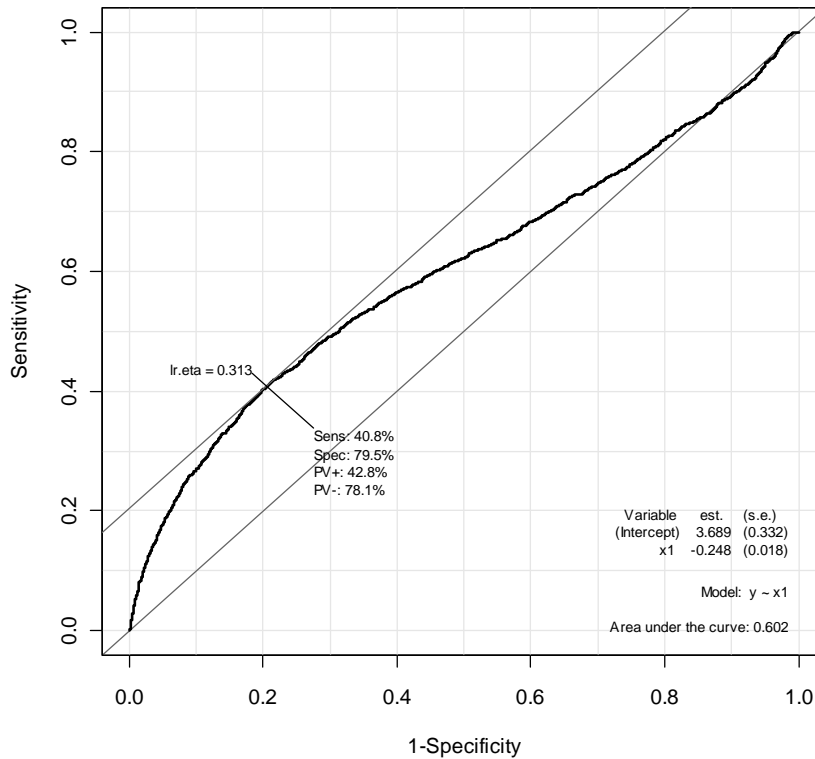


Fig. 3. ROC curve for the logistic regression model fitted to bank failure from 1980:Q1 to 1995:Q4.

The optimal prediction rule given by the ROC curve turns out to be:

Predict 1(Bank Failure) if $\hat{\pi} \geq 0.313$ and predict 0 if $\hat{\pi} < 0.313$

Although Fig.2 is difficult to read, it is clear that there were more failures for large banks during the period of the Great Recession (2007 ~). Therefore, for comparison, we also performed a simple logistic regression modeling based on the data during 01/01/1996 to 12/31/2011 (covering the period of Great Recession crisis) using log of adjusted assets (a continuous variable) as the solo predictor. The corresponding computer output is given below:

```

> failbank<-glm(status~asset,data=bank,family=binomial)
> summary(failbank)

Call:
glm(formula = status ~ asset, family = binomial, data = bank)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-0.7910  -0.3518  -0.3247  -0.3004   2.6492

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept) -6.75729    0.60115  -11.241  <2e-16 ***
asset         0.20392    0.03077   6.628  3.41e-11 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 3331.8  on 7731  degrees of freedom
Residual deviance: 3292.0  on 7730  degrees of freedom
AIC: 3296
Number of Fisher Scoring iterations: 5

```

The resulting fitted model is:

$$\ln\left(\frac{\hat{\pi}}{1-\hat{\pi}}\right) = -6.76 + 0.204 \ln(\text{adjusted total assets})$$

Thus, $\hat{\pi} = \frac{\exp(-6.76+0.204(\ln(\text{adjusted total assets})))}{1+\exp(-6.76+0.204(\ln(\text{adjusted total assets})))}$, so we have estimated OR = $e^{b_1} =$

$$e^{0.204} = 1.23$$

The 95% confidence interval for β_1 is given by:

$$b_1 \pm z(0.975) \cdot s(b_1) = (0.144, 0.264),$$

and the 95% confidence interval for the odds ratio is given by:

$$\exp(b_1 \pm z(0.975) \cdot s(b_1)) = (1.154, 1.302).$$

This means that each additional unit of log adjust assets changes the odds of bank failure by a factor of $e^{0.204} = 1.23$. This indicates that the odds of a bank failing are 23% higher

for each additional unit of log (adjusted bank assets). When compared with the previous odds ratio we obtained for the first period 01/01/1980 - 12/31/1995, ($e^{-0.248} = 0.78$), this one implies that in the Great Recession period, the situation reversed; the bigger banks were now more likely to fail. We will next explore some graphical methods to explain the details of how the tables turned; the empirical recurrence rate (ERR) and empirical recurrence rate ratio (ERRR) and the corresponding graphical analyses will be presented in the following Chapter.

CHAPTER 4
GRAPHICAL ANALYSIS

4.1 Empirical recurrence rate

A key parameter desired by economists is the recurrence rate of failures of the targeted bank group. Let t_1, \dots, t_n be the times of the n -ordered bank failures during an observation period $(t_0, 0)$, where t_0 is the time-origin and 0 is the present time. If h is the time-step, a discrete time series $\{Z_l\}$ is generated sequentially at equidistant time intervals $t_0 + h, t_0 + 2h, \dots, t_0 + lh, \dots, t_0 + Nh = 0$, using the empirical recurrence rate (Ho, 2008) as follows:

$$z_l = \frac{n_l}{lh} = \frac{\text{total number of bank failures in } (t_0, t_0 + lh)}{lh},$$

where $l = 1, 2, \dots, N$. z_l can be regarded as the observation at time $t (= t_0 + lh)$, for the bank failures to be modeled. Note that z_l evolves over time and is simply the maximum likelihood estimator (MLE) of the mean, if the underlying process observed over $(t_0, t_0 + lh)$ is a homogeneous Poisson process. The time-plot of the empirical recurrence rate (ERR plots) offers the possibility of further insights into the data. If we have data up to time T , the value Z_{T+k} , $k \geq 1$ needs to be predicted based on the sample observation (Z_1, \dots, Z_T) of an ERR time series.

4.2 Empirical recurrence rate plots

In Fig. 4, we have plotted the bank failures during 1980:Q1 to 2011:Q4 (Y) and its corresponding ERR time series (Z) in the same graph. It's clear that in the raw data there

are dramatic ups and downs. The raw data set reaches its maximum of 198 at the 43rd time-step, which is 1990:Q2, the middle of Savings and Loan crisis. Also, there are many zeros in the data set, which indicate no bank failures during that particular quarter. It is quite hard to approximate and forecast such data. In comparison, the ERR tends to smooth the data because of its nature as a cumulative function. It is very slow to grow with the savings and loan crisis. It peaks after the crisis and although it declines, it never captures the extremely low levels between the Savings and Loan crisis and the Great Recession. It barely registers the Great Recession at all. Also, ERR is also a very smooth and stationary time series, so many techniques from time series models can be used.

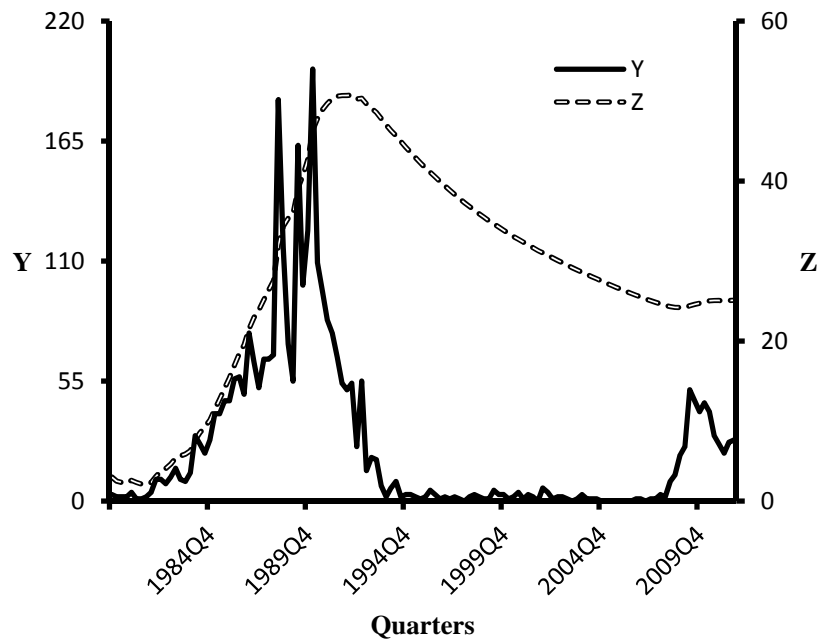


Fig. 4. Plots of bank failures and ERRs.

We just mentioned that we don't like the huge fluctuations in the raw data graph because they are hard to approximate or forecast. However, we still need them, as the

fluctuations reflect the economic cycles, exactly what we want to forecast. Here, the ERR plot shows its power once again; it can retain the trends of the raw data. The raw data plot from the beginning to the early 1990s show a rapid increase because of the Savings and Loan crisis. The ERR plot shows the same trend but is much smoother. Then we have stable period because of the economic recovery, the ERR plots show us a smooth slow down; In the ending part, ERR plots show a little rebound which can precisely explain the Great Recession from 2007; Since the number of bank failures is a lot less when compared with the Savings and Loan crisis, the rebound on ERR is very limited. Also, the ERR is smoothed out – the further away from the first data point the smaller the hump.

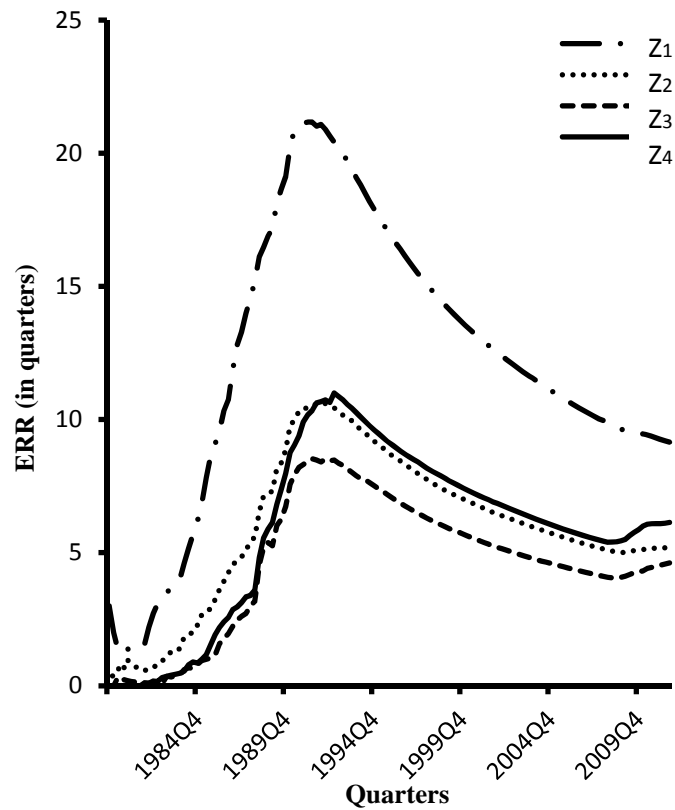


Fig. 5. ERR plots by groups.

ERR plots eliminate the disadvantages of the raw data but retain the good points. The ERR plots for bank failures by assets groups is given in Fig. 5. Z_1, Z_2, Z_3 and Z_4 are used to denote ERRs of the four groups G_1, G_2, G_3 and G_4 respectively. We see that G_1 has the highest while G_3 has the fewest bank failures among all four groups. If we take a closer look at G_2 and G_4 , we see that G_2 has more bank failures than G_4 before 1991:Q3 and this situation has reversed after 1991:Q3; more bank failures from G_4 . We then separate the time period into two parts: 1980:Q1 to 1995:Q4 and 1996:Q1 to 2011:Q4; so we can include the entire Savings and Loan crisis in the first part. The ERR plots for the second period is shown in Fig. 6, and corresponding counts is given in Table 8. From Fig. 6, we can see that at the beginning, the numbers of bank failures for all the four groups are quite small, a reflection of the stable period after Savings and Loan crisis. A closer look at the end part of the graph shows that there are rapid increases for all groups from 2008, especially for G_4 . This is a reflection of the Great Recession from 2007, and it indicates that banks in G_4 are more likely to fail than other groups. We will verify all information we obtained here in the ERRR plots again in Section 4.5. Pairwise comparisons are given in the next section.

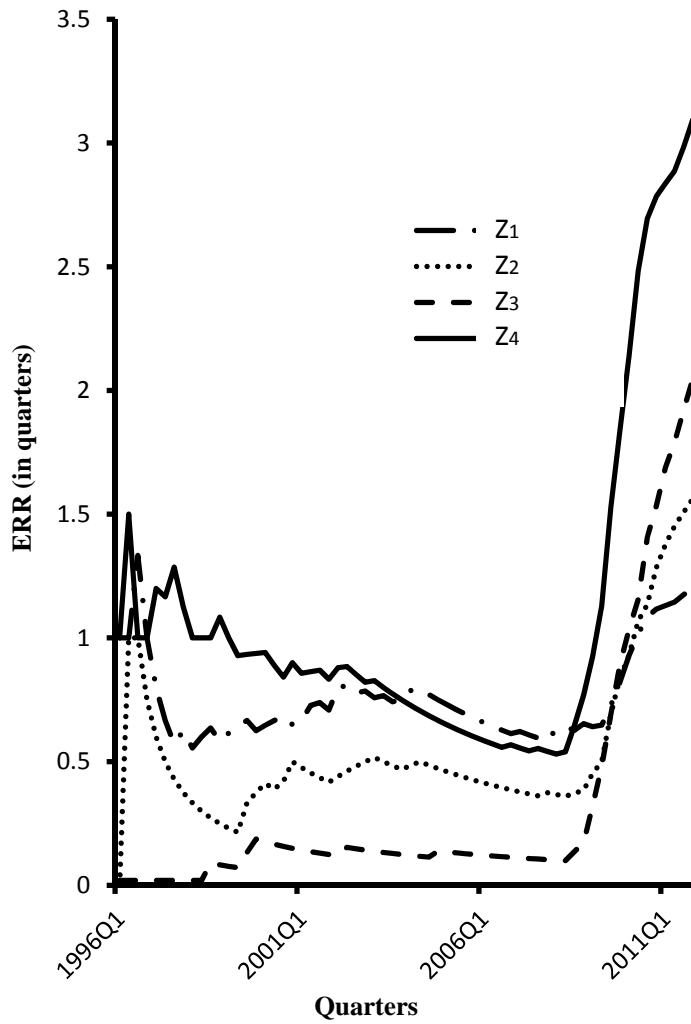


Fig. 6. ERR plots by groups for the second period (1996:Q1 to 2011:Q4).

4.3 Pairwise comparisons

4.3.1 Conditional test

Let X_1 and X_2 be independent observations from Poisson (λ_1) and Poisson (λ_2) distributions respectively. Then, the joint distribution of X_1 and X_2 is:

$$f(x_1, x_2) = \left[\frac{\lambda_1^{x_1} e^{-\lambda_1}}{x_1!} \right] \left[\frac{\lambda_2^{x_2} e^{-\lambda_2}}{x_2!} \right] = \frac{\lambda_1^{x_1} \lambda_2^{x_2}}{x_1! x_2!} e^{-(\lambda_1 + \lambda_2)}, \quad x_1 = 0, 1, 2, \dots; \quad x_2 = 0, 1, 2, \dots$$

Note that $S = X_1 + X_2 \sim \text{Poisson}(\lambda_1 + \lambda_2)$.

A well-known method of testing the difference of two Poisson means is the conditional

test, which is based on the fact $X_1 | (S = s) \sim \text{Bin}(s, p_{12})$, where $p_{12} = \frac{\lambda_1}{\lambda_1 + \lambda_2} = \frac{\rho_{12}}{1 + \rho_{12}}$

with $\rho_{12} = \frac{\lambda_1}{\lambda_2}$.

The proof goes as follows. Consider the conditional distribution of X_1 given $S = s > 0$.

The probability mass function of the conditional distribution of X_1 given $S = s$ is given

by:

$$\begin{aligned} f(x_1 | S = s) &= \frac{P(X_1 = x_1, X_1 + X_2 = s)}{P(X_1 + X_2 = s)} \\ &= \frac{e^{-\lambda_1} \frac{\lambda_1^{x_1}}{x_1!} \cdot e^{-\lambda_2} \frac{\lambda_2^{s-x_1}}{(s-x_1)!}}{e^{-(\lambda_1 + \lambda_2)} \frac{(\lambda_1 + \lambda_2)^s}{s!}} \\ &= \binom{s}{x_1} \left(\frac{\lambda_1}{\lambda_1 + \lambda_2} \right)^{x_1} \left(\frac{\lambda_2}{\lambda_1 + \lambda_2} \right)^{s-x_1} \\ &= \binom{s}{x_1} \left(\frac{\rho_{12}}{1 + \rho_{12}} \right)^{x_1} \left(\frac{1}{1 + \rho_{12}} \right)^{s-x_1} \sim \text{Binomial} \left(s, \frac{\rho_{12}}{1 + \rho_{12}} \right) \end{aligned}$$

Testing $H_0: \lambda_1 = \lambda_2$ vs. $H_1: \lambda_1 \neq \lambda_2$ is equivalent to testing $H_0: \rho_{12} = 1$ vs. $H_1: \rho_{12} \neq 1$,

which is also equivalent to testing $H_0: p_{12} = \frac{1}{2}$ vs. $H_1: p_{12} \neq \frac{1}{2}$.

When $X_1 = k$ is observed, the conditional test (C-test) rejects H_0 , if the

p -value = $2 \cdot \min\{P(X_1 \leq k|S = s), P(X_1 \geq k|S = s)\} < \alpha$, where α is the level of significance. Of course, normal approximation can be implemented for the above binomial test for large s (Przyborowski and Wilenski, 1940).

4.3.2 Application

As before, we divide the banks into four groups based on the levels of total assets of the banks. For each bank group, we assume that the number of bank failures follows a homogeneous Poisson process. Based on the classification criterion described in Chapter 2, let λ_i be the average failure rate of banks in the i th group from 1980:Q1 to 2011:Q4, $i = 1, 2, 3, 4$. Also, let $\rho_{ij} = \frac{\lambda_i}{\lambda_j}$ and $p_{ij} = \frac{\rho_{ij}}{1 + \rho_{ij}}$, $1 \leq i < j \leq 4$. Then the hypotheses for bank failure rates comparison between any two groups i and j can be presented as follows:

$$H_0: \rho_{ij} = \rho_{ij}^0 \text{ vs. } H_1: \rho_{ij} \neq \rho_{ij}^0,$$

where $\rho_{ij}^0 = 1$, since we have the same marginal total for all bank groups. The corresponding C-test is then

$$H_0: p_{ij} = p_{ij}^0 \text{ vs. } H_1: p_{ij} \neq p_{ij}^0,$$

$$\text{where } p_{ij}^0 = \frac{\rho_{ij}^0}{1 + \rho_{ij}^0} = 0.5.$$

For example, in comparing G_1 and G_3 , the total numbers of bank failures during the entire time period are 1,172 and 590 for G_1 and G_3 , respectively. Based on the C-test, if we set our $H_0: \rho_{13} = 1$, we have:

$$p\text{-value} = 2 \cdot \min\{P(X_1 \leq 1172 \mid S = 1762), P(X_1 \geq 1172 \mid S = 1762)\}$$

$$= 2 \cdot \left\{ \sum_{k=1172}^{1762} \binom{1762}{k} (0.5)^k (0.5)^{(1762-k)} \right\} \approx 0,$$

The null hypothesis is thus rejected, indicating that compared with G_3 , banks in G_1 are not equally likely to fail during the observation period. Therefore, the result of the above C-test implies that banks in G_1 and G_3 have significantly different survival rates during the observation period. We have performed similar C-tests for all such pairwise comparisons, the results of which are presented in Table 7. Note that \hat{p}_{ij} denotes the estimated value of p_{ij} . We conclude that there is significant evidence to say G_1 banks are not as likely to fail as banks in G_2 , G_3 or G_4 . Any two groups of comparison, we conclude that the banks have significant difference likelihood of failure.

Table 7

Conditional tests for pairwise comparisons for
 $H_0: p_{ij} = 0.5, 1 \leq i < j \leq 4.$

\hat{p}_{ij}	P -value
$\hat{p}_{12} = 0.64$	≈ 0
$\hat{p}_{13} = 0.67$	≈ 0
$\hat{p}_{14} = 0.6$	≈ 0
$\hat{p}_{23} = 0.53$	0.042
$\hat{p}_{24} = 0.46$	0.001
$\hat{p}_{34} = 0.43$	≈ 0

We then built the same C-tests for the second period (1996:Q1 to 2011:Q4), time periods separation is the same way as mentioned in Section 4.2. The numbers of bank failures by groups for second period are given in Table 8, the corresponding C-test results are given in Table 9. Since the population size is changed and different for groups in the second

period, so we have different H_0 for different comparisons, which are also shown in Table 9. From the p-values, we can see that for comparisons between G_1 , G_2 and G_3 , the rate of bank failures contributed by smaller banks is same as that contributed by larger banks. The rate of bank failures contributed by G_1 , G_2 or G_3 is different from that contributed by G_4 .

Table 8
Counts of bank failures by Status and Group for the second period (1996:Q1 to 2011:Q4)

		Group				Totals
		1	2	3	4	
Status	Yes	77	100	130	199	506
	No	1456	1965	2038	1841	7300
Totals		1533	2065	2168	2040	7806

Table 9
Conditional tests for pairwise comparisons – 1996 - 2011

H_0	\hat{p}_{ij}	P -value
$p_{12} = 0.43$	0.44	0.849
$p_{13} = 0.42$	0.37	0.234
$p_{14} = 0.43$	0.28	≈ 0
$p_{23} = 0.49$	0.43	0.129
$p_{24} = 0.50$	0.33	≈ 0
$p_{34} = 0.51$	0.40	≈ 0

4.4 Empirical recurrence rates ratio

The C-test examines the relationship between means of two homogeneous Poisson processes, which have constant expected values. Motivated by the ideas of the C-test and the empirical recurrence rate developed by Ho (2008), we now use empirical recurrence

rates ratio (ERRR) time series to measure the bank failure rates ratio between G_i and G_j . Let t_1, t_2, \dots, t_n be the n -ordered bank failure times during an observation period $(t_0, t_{0+Nh}]$ from the past to the present, where $n = Nh$. The ERRR at time $t_0 + lh$ is defined as:

$$R_{ij,l} = \frac{\sum_{k=1}^l X_{ik}}{\sum_{k=1}^l (X_{ik} + X_{jk})}, \quad 1 \leq i < j \leq 4, \quad l = 1, 2, \dots, N,$$

where X_{ik} = number of failures in G_i during $(t_0, t_{0+kh}]$, for $i = 1, 2, 3, 4$ and $k = 1, 2, \dots, l$. Then a discrete time series is generated sequentially at the points $t_0 + h, t_0 + 2h, \dots, t_0 + lh, \dots, t_0 + Nh$. Here, "h" presents the time-step. We call this the ERRR time series.

Both the ERR and ERRR offer the possibility of developing a model for monitoring and predicting bank failure rate ratios. Moreover, if both of the targeted processes are homogeneous Poisson processes, then the ERRR is the maximum likelihood estimator (MLE) of ρ_{ij} , which can be used to find the MLE of ρ_{ij} using the invariance property of the MLE.

4.5 Empirical recurrence rates ratio plots

Empirical recurrence rates ratio plots (ERRR plots) for all six pairwise comparisons, referred to as, $R_{12}, R_{13}, R_{14}, R_{23}, R_{24}$ and R_{34} , respectively are presented in Fig. 7. Because R_{ij} is a ratio, it will be undefined if the denominator is 0 (treated as a burn-in period). In Fig. 7, all the six ERRR plots start at lag-3 (by deleting the first 2 time-steps as the burn-in period).

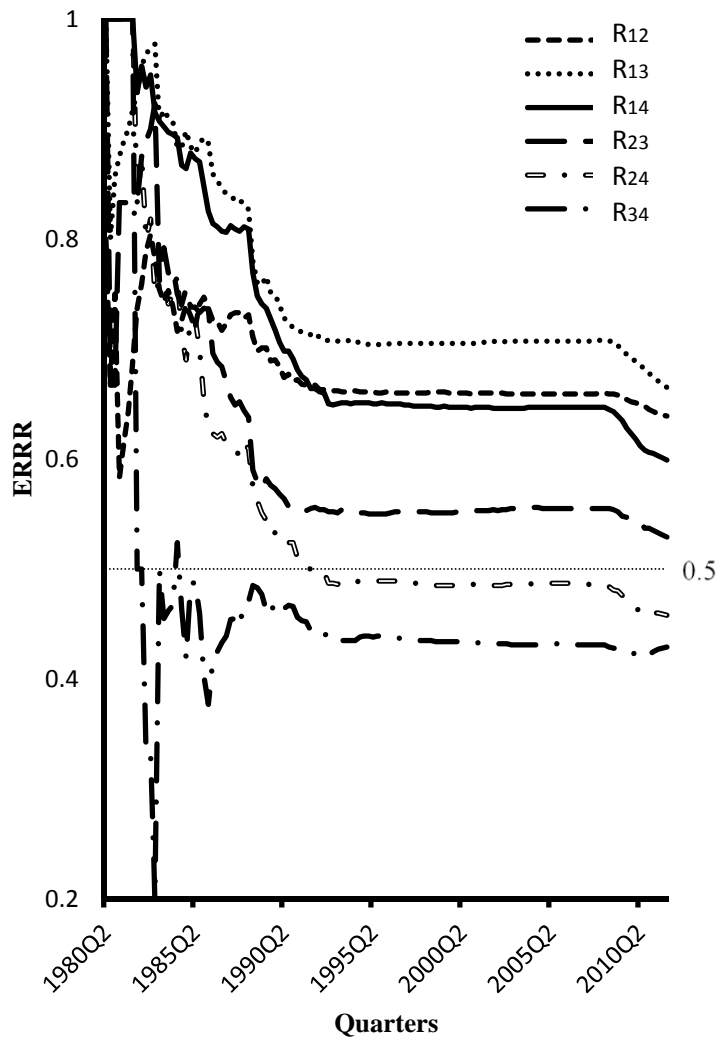


Fig. 7. ERRR plots for pairwise comparisons.

As defined in the previous section, $R_{ij} = 0.5$ means that there are same numbers of bank failures in i th group and j th group. If $R_{ij} < 0.5$, there are more bank failures in G_i than in G_j , while $R_{ij} > 0.5$ indicates that there are more bank failures in G_j than that in the G_i . We take R_{24} as an example. In the ERRR plots, R_{24} is greater than 0.5 before 1991:Q3, which means that there are more bank failures in G_2 than that in G_4 , the situation reversed after 1991:Q4, we got more G_4 bank failures than G_2 . This is the same information as we get

from the ERR plots analysis. The middle period, 1995 to 2007, was a pretty stable one. G_1 banks still failed at a somewhat greater rate than the other groups but the rates of failure of the larger classes were pretty similar (R_{ij} close to 0.5). The final period, after 2007, reversed the earlier conditions. In each pairwise comparison, the G_3 banks were found to fail at a much higher rate than the smaller class of banks they were being compared to. Given the historically stable relationship of failure rates, this rapid change in the G_3 banks would suggest that the G_3 banks were in fact engaged in activities outside of their normal risk tolerance. Perhaps the G_3 banks engaged in very risky behavior and expected to be bailed out in case of a problem. This is the period that we call the Great Recession.

We separate the ERRR plots into two sections based on the economic cycles; ERRR plots during 1980:Q1 to 1995:Q4 and ERRR plots during 1996:Q1 to 2011:Q4 (Fig. 7 and Fig. 8). A closer look at the beginning of the observation period shows that for all the comparisons R_{ij} , the curves are higher than 0.5. This indicates that G_1 banks seemed to contribute more to the failure rates ratio. The larger banks in G_3 and G_4 had the lowest failure rates ratio, implying that during this period, most banks failing were from G_1 . In fact, from 1980 to 1995, was the Savings and Loan crisis. It was dominated mostly by banks in G_1 and a few of the larger ones. The 1980 to 2011 chart helps clarify that except for the G_3 compared to G_4 , the smaller class of banks failed at a greater rate than the larger class it was compared to.

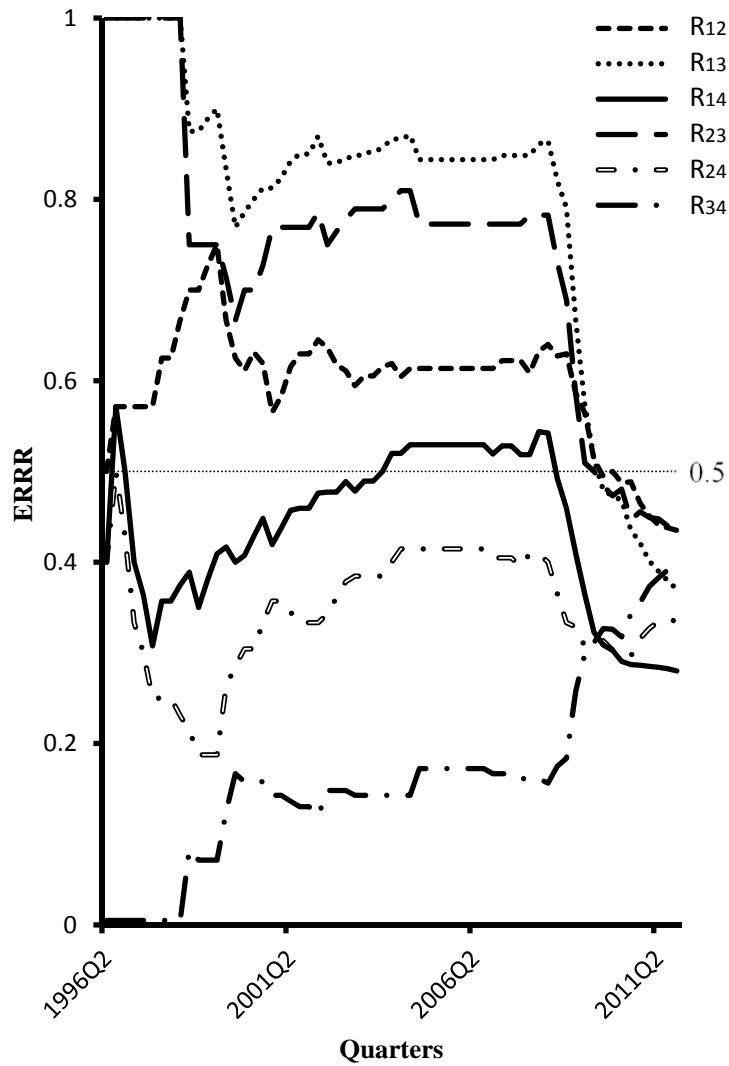


Fig. 8. ERRR plots for pairwise comparisons for second period (1996:Q1 to 2011:Q4).

Next, we focus on the second part, ERRR plots during 1996:Q1 to 2001:Q4 (Fig. 8), with burn-in period deleted (1 time-step). First, we can see that R_{12} , R_{13} and R_{23} are greater than 0.5 before 2009:Q3, indicating that in the first three groups (1, 2 and 3), the smaller assets banks always have more bank failures than bigger ones. After 2009:Q3, all three R_{ij} s are smaller than 0.5, meaning the situation reversed; more bigger banks failed. We

also find from this chart that both R_{24} and R_{34} are smaller than 0.5 in the period during 1996:Q1 to 2011:Q4. This means that the G_2 and G_3 banks always have fewer failures when compared with G_4 .

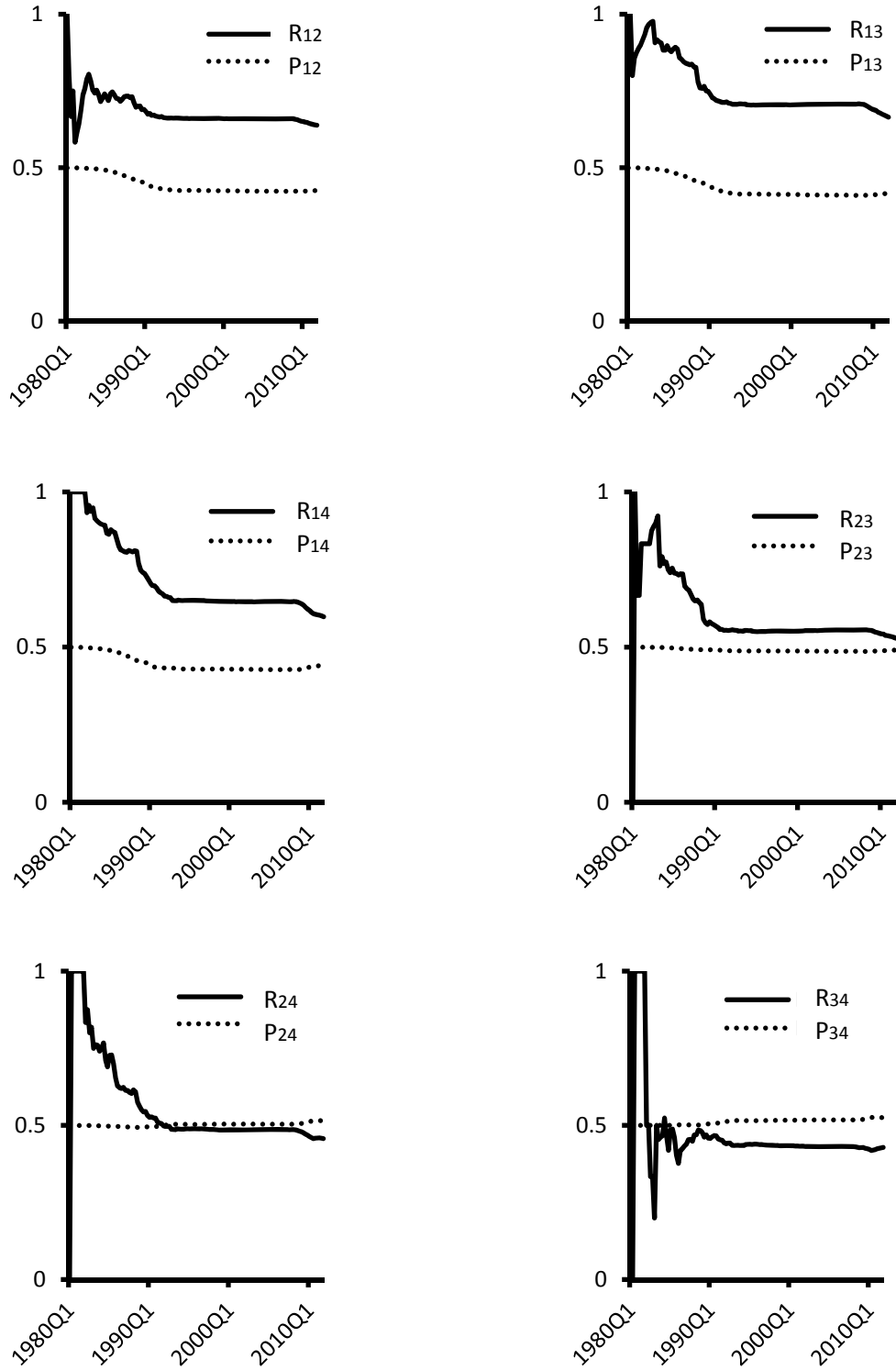


Fig. 9. Plots of R_{ij} vs. P_{ij}^0 (reference line) for $1 = i < j = 4$ during 1980:Q1 and 2011:Q4.

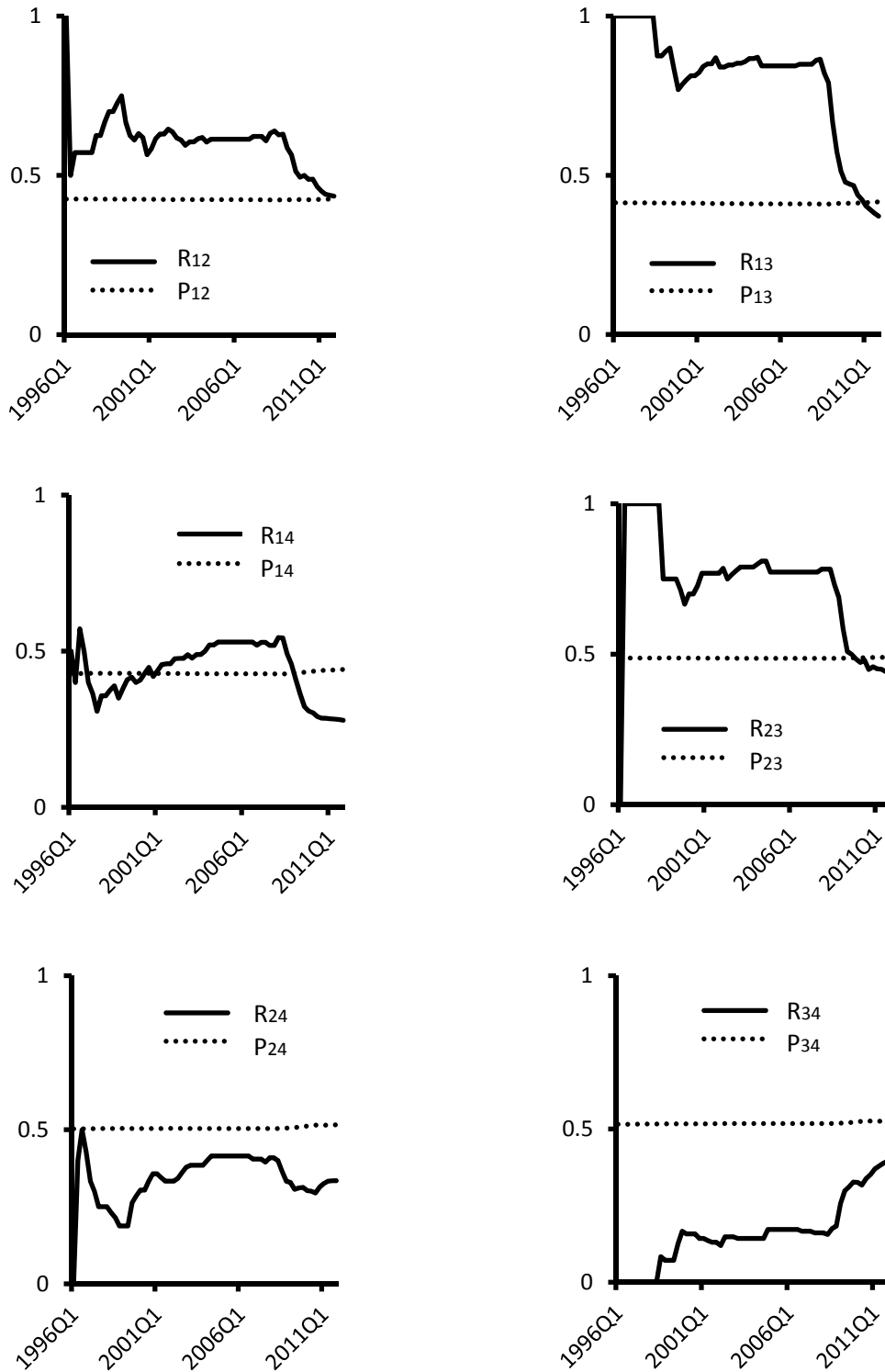


Fig. 10. Plots of R_{ij} vs. P_{ij}^0 (reference line) for $1 = i < j = 4$ during 1996:Q1 and 2011:Q4.

Based on the previous detailed analysis using ERR and ERRR plots, we now have a general understanding of bank failures at different periods and for different groups. Also, for additional information of the ERRR plots and C-tests, including pairwise C-tests H_0 and \widehat{P}_{ij} , we built all single ERRR plots with corresponding reference line comparison for every time-steps (see Fig. 9 and Fig. 10). In these graphs, the reference lines are not 0.5 anymore, because the population sizes are always change for different time-steps and groups, we have the new reference lines based on the quotient of population sizes in two groups. For further study and forecast on bank failures, we will proceed with the autoregressive integrated moving average (ARIMA) and autoregressive fractional integrated moving average (ARFIMA) modeling of ERR and ERRR time series in the following two chapters.

CHAPTER 5

ARIMA AND ARFIMA MODELING

5.1 Autoregressive integrated moving average models

In this chapter, we try to predict the numbers of bank failures in the future, so autoregressive integrated moving average (ARIMA), autoregressive fractional integrated moving average (ARFIMA) models and corresponding data transformation skills are introduced.

The acronym ARIMA, stands for autoregressive integrated moving average. The original key reference is from Box and Jenkins (1976). It is used to model the dynamics of a time series data set. The basic processes of the Box–Jenkins ARIMA model consist of the following: the autoregressive process, the integrated process, and the moving average process. The autoregressive model is analogous to the regression model, based on the idea that the current value of the series X_t is a linear combination of the p most recent past values of itself plus an “innovation” term Z_t that incorporates everything new in the series at time t that is not explained by the past values. An autoregressive model of order p , is of the form:

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \dots + \phi_p X_{t-p} + Z_t$$

where $t = 1, 2, \dots, N$, and X_t is a mean-zero stationary process. The quantities ϕ_1, \dots, ϕ_p are called the autoregressive coefficients for a p th order process, Z_t is a Gaussian white noise series with mean zero and variance σ^2 , independent of $\{X_t\}$ for every t .

A moving average (MA) process $\{X_t\}$ of order q is a linear combination of the current white noise term and the q most recent past white noise terms Z_t and is defined by

$$X_t = Z_t + \theta_1 Z_{t-1} + \dots + \theta_q Z_{t-q}$$

where $t = 1, 2, \dots, N$, X_t is mean-zero stationary time series, Z_t is Gaussian white noise process with mean zero and variance σ^2 . The quantities $\theta_1, \dots, \theta_q$ are called the MA parameters of the model.

Combining the above two ideas, one obtains a general autoregressive moving average (ARMA) model, denoted ARMA (p, q), and given by:

$$X_t - \phi_1 X_{t-1} - \dots - \phi_p X_{t-p} = Z_t + \theta_1 Z_{t-1} + \dots + \theta_q Z_{t-q}.$$

A time series $\{X_t\}$ is said to follow an autoregressive integrated moving average model (ARIMA) if the d th difference $Y_t = \nabla^d X_t$ (to be defined in Section 5.2.2) is an ARMA process. If Y_t follows an ARMA (p, q) model, we say that X_t is an ARIMA (p, d, q) process. In fitting an ARIMA model, we go through 3 stages: identification, estimation, and diagnostic checking. In the identification stage, preliminary estimates for q, p and d are obtained using the plots of the sample autocorrelation function (ACF) and sample partial autocorrelation function (PACF). Sometimes identification is done by an auto fit procedure – fitting many different possible model structures and orders and using a goodness-of-fit statistic to select the best model. The second stage is to estimate the coefficients of the identified model. In this step, we adopt the maximum likelihood

estimation method. The last stage is model diagnostic checking. In ARIMA modeling, this is done using residuals of the fitted model. This usually consists of a group of tests including tests for normality using the residuals. Moreover, it is necessary to test that all the model parameters are statistically significant. The fitting process is often guided by the principle of parsimony, by which the best model is one that has the fewest parameters among all models that fit the data. (For details, see Cryer and Chan, 2008; Box and Jenkins, 1976; Shumway and Stoffer, 2005).

5.2 Data transformation

ARMA model requires that the realized data follow a stationary process, which means that the statistical properties such as mean, variance, autocorrelations, etc. remain constant over time. Some mathematical transformations will be employed, if the process is not stationary. Two common transformations that will be discussed are the following:

5.2.1 Box-Cox transformation

The Box-Cox procedure automatically identifies a transformation from the family of power transformations on Y . If the variability of the data set increases or decreases over time, the Box-Cox transformation will be employed to make the variance constant. This transformation converts original observations Y_1, Y_2, \dots, Y_n to $f_\lambda(Y_1), f_\lambda(Y_2), \dots, f_\lambda(Y_n)$, where:

$$f_\lambda(y) = \begin{cases} \frac{y^\lambda - 1}{\lambda}, & \lambda \neq 0, \\ \log(y), & \lambda = 0. \end{cases}$$

Suitable value of λ will be chosen to make the variability of $f_\lambda(y)$ a constant.

5.2.2 Differencing

Differencing is a data-processing technique that is used to remove trends or seasonal components. In this, one simply considers the difference between pairs of observations with appropriate time separations, such as, the first difference, which is denoted as:

$$\nabla X_t = X_t - X_{t-1} = (1 - B)X_t ,$$

where B is the backward shift operator. Differencing of order d is given by

$$\nabla^d X_t = (1 - B)^d X_t .$$

Single differencing is used to remove linear trend, while double differencing is used to eliminate quadratic trend. As mentioned earlier, ARIMA processes can be reduced to ARMA processes by differencing a time series.

The differencing technique adopted to deal with the seasonality of period d is the lag- d difference operator ∇_d , which is defined as:

$$\nabla_d X_t = X_t - X_{t-d} = (1 - B^d)X_t .$$

For example, differencing at lag-4 will remove the annual effect in a quarterly time series.

We use the software ITSM2000 for our model fitting. The software ITSM2000 (Brockwell and Davis, 2002) uses a zero-mean ARMA process as the default setting.

After removing apparent deviations from stationarity by differencing, we work with the corresponding mean-subtracted data for all our analysis.

5.3 Autoregressive fractional integrated moving average models

Autoregressive fractionally integrated moving average (ARFIMA) models are time series models that generalize ARIMA (autoregressive integrated moving average) models by allowing non-integer values of the differencing parameter and are useful in modeling time series with long memory. The acronyms "ARFIMA" or "FARIMA" are often used, although it is also conventional to simply extend the "ARIMA (p, d, q)" notation by simply allowing the order of differencing, d , to take fractional values.

The conventional ARIMA (p, d, q) process is often referred to as a short memory. When the sample ACF of a time series decays slowly, this indicates a long term memory. Long-term memory is considered as an intermediate compromise between short memory ARMA type models and the fully integrated nonstationary processes. Thus, there may be a problem of over-differencing of the original process when we use an integer difference parameter. In the previous section, a time series X_t is said to follow an integrated autoregressive integrated moving average model (ARIMA) if the d^{th} difference $Y_t = \nabla^d X_t = (1 - B)^d X_t$ is a stationary ARMA process. In particular, if Y_t follows an ARMA (p, q) process, we say that X_t has an ARIMA (p, d, q) process. When we allow the differencing parameter d to be a fraction, we can get an ARFIMA (p, d, q) process, with $0 < |d| < 0.5$ (Shumway and Stoffer, 2006). ARIMA modeling will be addressed in the next chapter.

5.4 Model building

Box and Jenkins (1976) proposed an iterative model-building strategy that has been widely adopted by practitioners. In search of the best ARIMA model for our ERR time series, the following strategy, consisting of three main phases, is followed:

1. Tentative specification or identification of a model;
2. Efficient estimation of model parameters;
3. Diagnostic checking of fitted model for further improvement.

5.4.1 Sample ACF/PACF of the residuals

If the model fit is correct, when the sample size n is large enough, the residuals sequence Y_1, \dots, Y_n with finite variance are approximately independent and identically distributed (iid) with distribution $N(0, \frac{1}{n})$. Whether the observed residuals are consistent with the iid noise can be tested by examining the sample autocorrelations of the residuals. The null hypothesis of iid noise will be rejected if more than two or three out of 40 fall outside the bounds $\pm 1.96/\sqrt{n}$ or if one falls far outside the bounds (Brockwell and Davis, 2002).

5.4.2 Tests for randomness of the residuals

A popular test, formulated by Ljung and Box (1978), called Ljung-Box test, is commonly used to check whether the residuals of a fitted ARIMA model are observed values of independent and identically distributed random variables. It is referred to as a portmanteau test, since it is based on the entire autocorrelation plot of the residuals and tests the overall independence based on a few lags. The Ljung-Box test proceeds as follows:

H_0 : The residual sequence data are iid

H_a : The residual sequence data are not iid

After a model has been fitted to a series z_1, \dots, z_n , we get the residuals $\hat{a}_1, \dots, \hat{a}_n$. If no model is being fitted, then $\hat{a}_1, \dots, \hat{a}_n$ are the “mean corrected” values of z_1, \dots, z_n .

Here the test statistic is:

$$\hat{Q}(\hat{r}) = n(n+2) \sum_{k=1}^m (n-k)^{-1} \hat{r}_k^2,$$

where $\hat{r}_k = \frac{\sum_{l=k+1}^n \hat{a}_l \hat{a}_{l-k}}{\sum_{l=1}^n \hat{a}_l^2}$, the estimated autocorrelation at lag k , n = sample size and m = number of lags being tested.

As a rule of thumb, the sample ACF and PACF are good estimates of the ACF and PACF of a stationary process for lags up to about a third of the sample size (Brockwell and Davis, 2002).

If the sample size n is large, the distribution of $\hat{Q}(\hat{r})$ is roughly χ_{m-p-q}^2 under the null hypothesis, where $p+q$ is the number of parameters of the fitted model. The null hypothesis will be rejected at level α if $\hat{Q} > \chi_{1-\alpha; m-p-q}^2$. Consequently, the sequence data are not independent, implying a poor fit of the model.

5.4.3 AIC, BIC and AICC statistics

Another approach to model selection is the use of information criteria such as Akaike Information Criterion (AIC), or the Bayesian Information Criterion (BIC), which is a Bayesian modification of the AIC statistic. The bias-corrected version of the AIC statistic, the AICC statistic, introduced by Akaike in 1974, is employed in this thesis as

information criterion to select appropriate models using the ITSM2000 package. The three information criteria are defined as follows:

$$AIC_{p,q} = n \log \hat{\sigma}_{\varepsilon}^2 + 2r,$$

$$AICC_{p,q} = n \log \hat{\sigma}_{\varepsilon}^2 + 2rn/(n - r - 1),$$

$$BIC_{p,q} = n \log \hat{\sigma}_{\varepsilon}^2 + r \log n,$$

where $\hat{\sigma}_{\varepsilon}^2$ is the maximum likelihood estimator of σ_{ε}^2 , and $r = p + q + 1$ is the number of parameters estimated in the model, including a constant term. The second term in all the three equations is a penalty for increasing r . Thus, minimizing the number of parameters is one of the ways to minimize the values of these criteria. The best model should be the model that has the fewest parameters yet still sufficiently describes the data, giving rise to small residual variance by graphing the raw data (Fig. 1), we notice that there are a lot of zeroes in the time series as we mentioned in Chapter 4. The ordinary ARIMA modeling techniques are not be able to handle such series with many zeros, since the stationarity may be difficult to achieve. Hence we will perform ARIMA and ARFIMA modeling on the ERR and ERRR time series in the following chapter.

CHAPTER 6
APPLICATION

6.1 Modeling for ERR time series

ERR plots for all the four ERR plots for groups are given in Fig. 5. We will use Z_3 as an example to present the ARIMA modeling. First, we split the ERR time series into training sample and prediction set. Our training sample is the full data set excluding the last four ERRs, which will form the prediction set, as shown in Fig. 11. These four ERR values in the prediction set, representing the most recent four quarters of ERR plots, will be compared with those of the four predictions produced by a candidate model. The size of a prediction set is quite flexible as long as it fits a common goal of model selection. Fig. 12 shows the time plot, sample ACF and sample PACF plots of the training sample. They indicate non-stationary behavior of the ERR series. A difference at lag-1 was taken and Fig. 13 shows the results of the seasonal difference. It appears as though a trend is still present after differencing. Thus, a further differencing at lag-2 was taken, Fig. 14 shows the corresponding plots of the twice-differenced ERR series, its sample ACF and sample PACF.

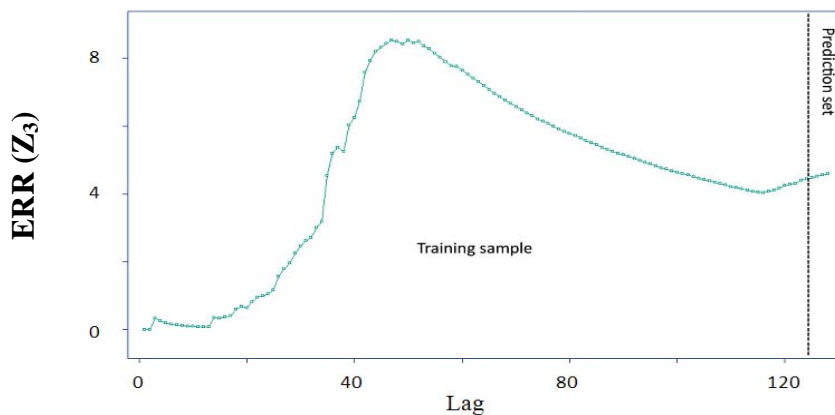


Fig. 11. ERR plots (Z_3) with training sample and prediction set.

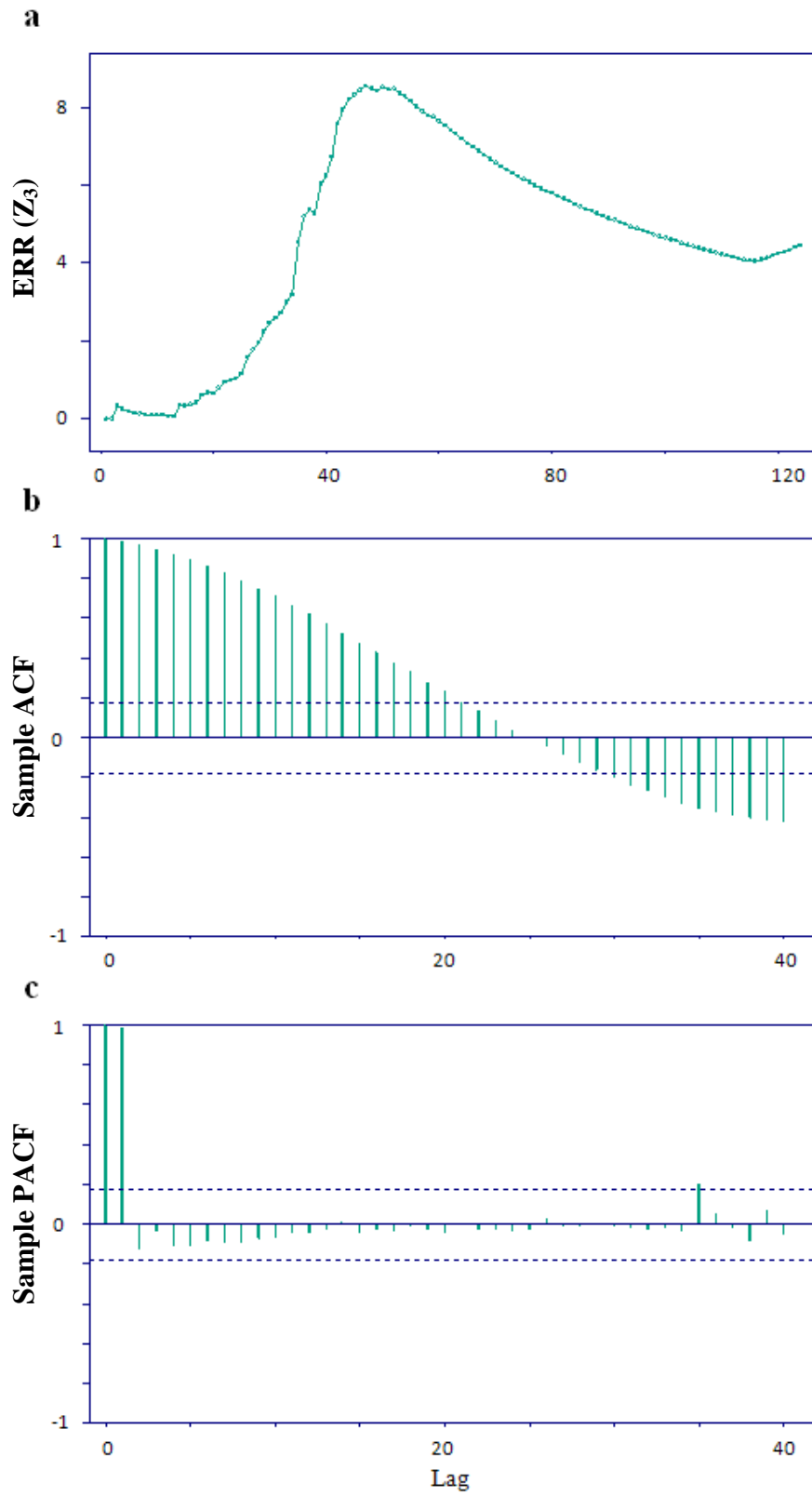


Fig. 12. **a.** Time plot; **b.** ACF; **c.** PACF of the training sample of Z_3 .

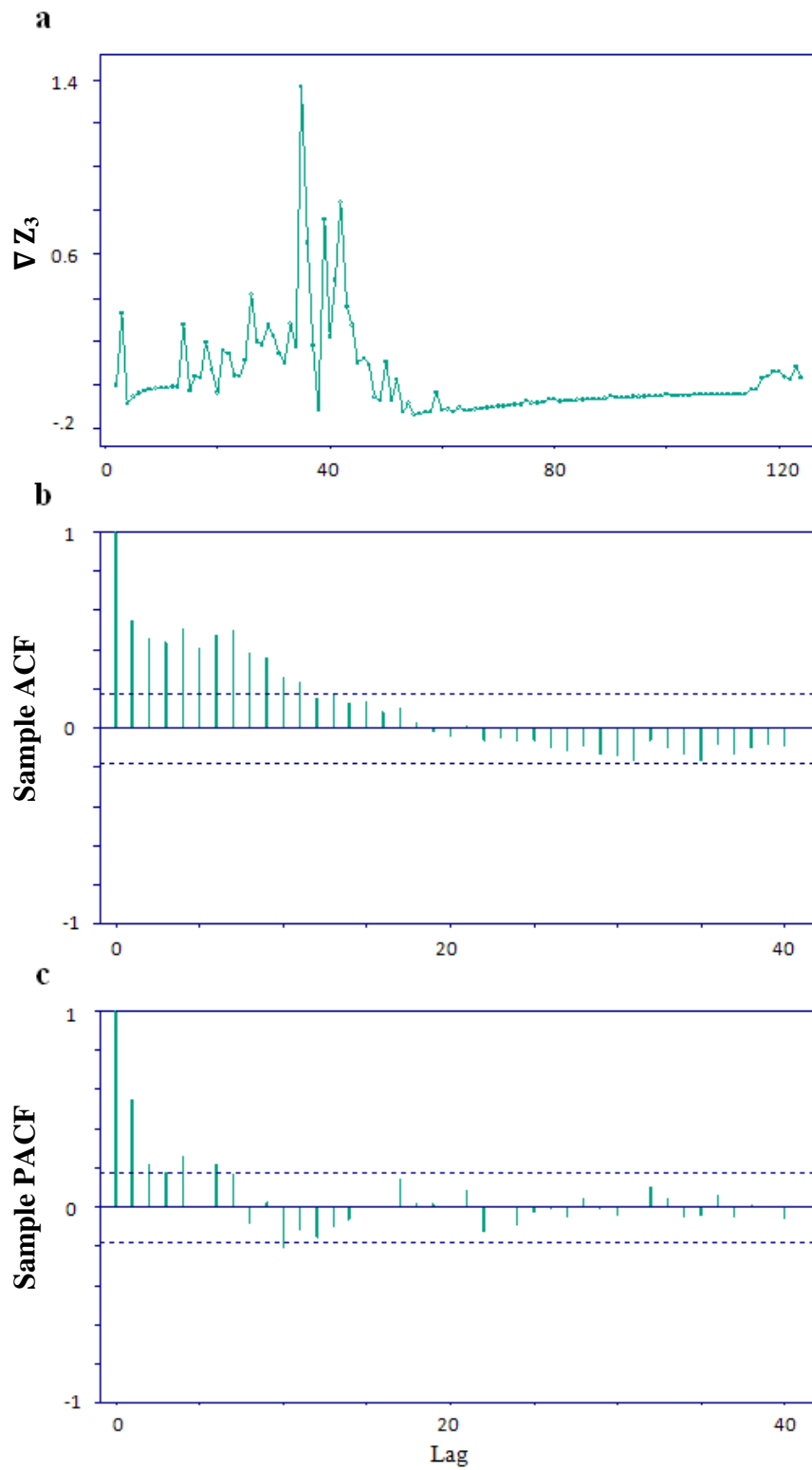


Fig. 13. **a.** Time plot; **b.** ACF; **c.** PACF of the lag-1 differenced training sample.

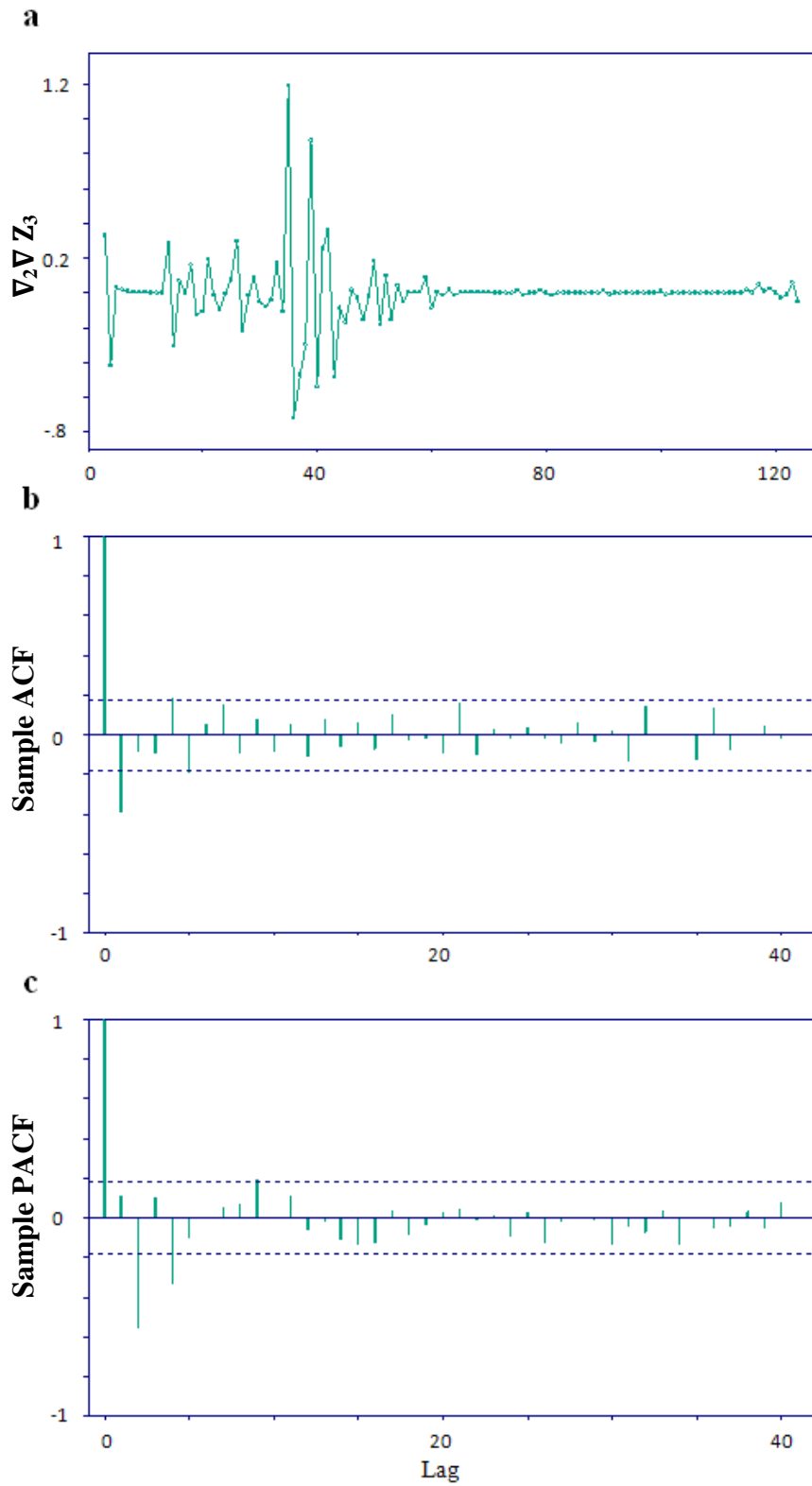


Fig. 14. a. Time plot; b. ACF; c. PACF of the twice differenced training sample.

Next ARIMA modeling and computational techniques are used to fit the twice differenced ERRs. We set AR parameter between 0 and 5 and MA parameter between 0 and 5, after traversing all possible model combinations, we find the best ARMA model is MA (5). Fig.15 is a set of diagnostic plots produced by ITSM2000 package, which show the ACF and PACF of residuals of training sample obtained after fitting the MA (5) model. The AICC statistic is -94.13. And the Ljung-Box test is not significant (p -value = 0.31). The estimated model is given in the following box:

MA(5) Model:

$$X_t = Z_t + .1969 Z_{t-1} - .7467 Z_{t-2} - .1936 Z_{t-3} + .2623 Z_{t-4} + .07214 Z_{t-5}$$

$$\text{WN Variance} = 0.027562$$

After a closer look at the coefficients of MA(5), we found that the last one is really small. Hence, we drop the last coefficient to get the subset model of MA(5). The diagnostics for the subset model is given in Fig. 16, and the AICC statistic is -81.31. The Ljung-Box test is not significant (p -value = 0.15). Model is given in the following box:

MA(5) subset Model:

$$X_t = Z_t + .3130 Z_{t-1} - .7858 Z_{t-2} - .02617 Z_{t-3} + .07264 Z_{t-4}$$

$$\text{WN Variance} = 0.026362$$

Since we drop the last coefficient in MA(5), we only have first four coefficients in the model as states in the above box, which has a same form of MA(4), so we also built a MA(4) for comparison. Model is given in the following box:

MA(4) Model:

$$X_t = Z_t + .3064 Z_{t-1} - .7950 Z_{t-2} - .03636 Z_{t-3} + .06897 Z_{t-4}$$

$$\text{WN Variance} = 0.026447$$

The diagnostics for the MA (4) model is given in Fig. 17. The corresponding AICC statistic is -81.23 and the Ljung-Box test is not significant (p -value = 0.14272). We produce Fig. 18 to compare the observed ERRs in the prediction set with the forecasted counterparts obtained using the three models we discussed earlier.

The root-mean-square deviation (RMSD) or root-mean-square error (RMSE) is a frequently used measure of the accuracy of a prediction model. The root mean squared errors (RMSEs) for a particular forecasting method are summarized across series by (e.g., Armstrong and Collopy, 1992):

$$\text{RMSE} = \left(\frac{1}{4} \sum_{i=1}^4 (z_i - \hat{z}_i)^2 \right)^{1/2}, \text{ where } z_i \text{ is the actual value and } \hat{z}_i \text{ is its forecast.}$$

Because $\text{RMSE}_{\text{MA}(5)} = 0.036$, $\text{RMSE}_{\text{MA}(5) \text{ subset}} = 0.009$ and $\text{RMSE}_{\text{MA}(4)} = 0.006$, we conclude that the MA (5) subset and MA (4) have a better predictive ability. Since MA(4) gets the prediction results a little better than subset model of MA (5), we choose MA(4) as our final model in this thesis. We present the prediction results for ERR using MA(4) and real counts in Table 10.

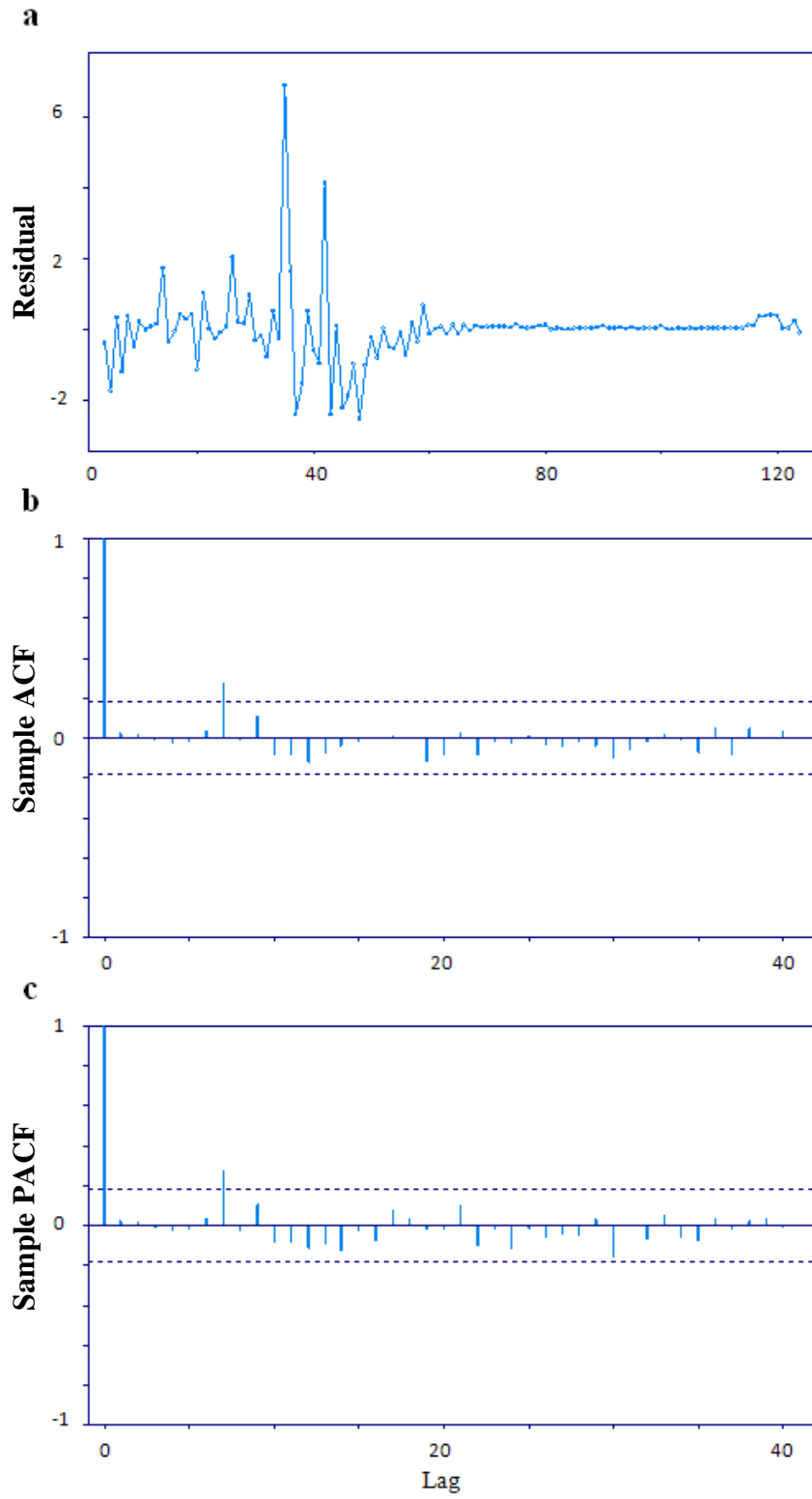


Fig. 15. Diagnostics for the MA (5) fitted to the mean-corrected and twice differenced training sample. **a.** time plot; **b.** ACF, and **c.** PACF of the residuals.

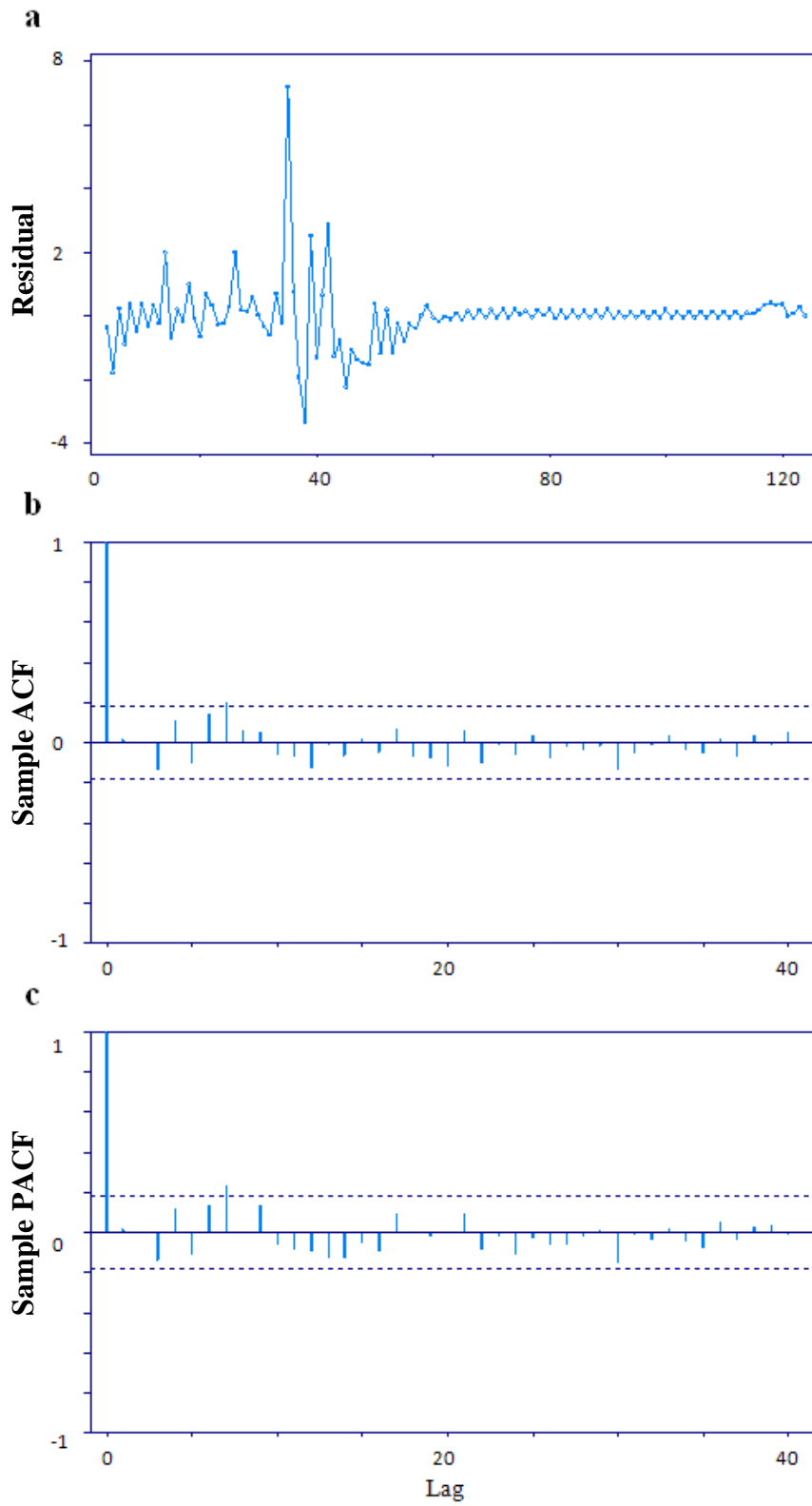


Fig. 16. Diagnostics for the subset model of MA (5) fitted to the mean-corrected and twice differenced training sample. **a.** time plot, **b.** ACF, and **c.** PACF of the residuals.

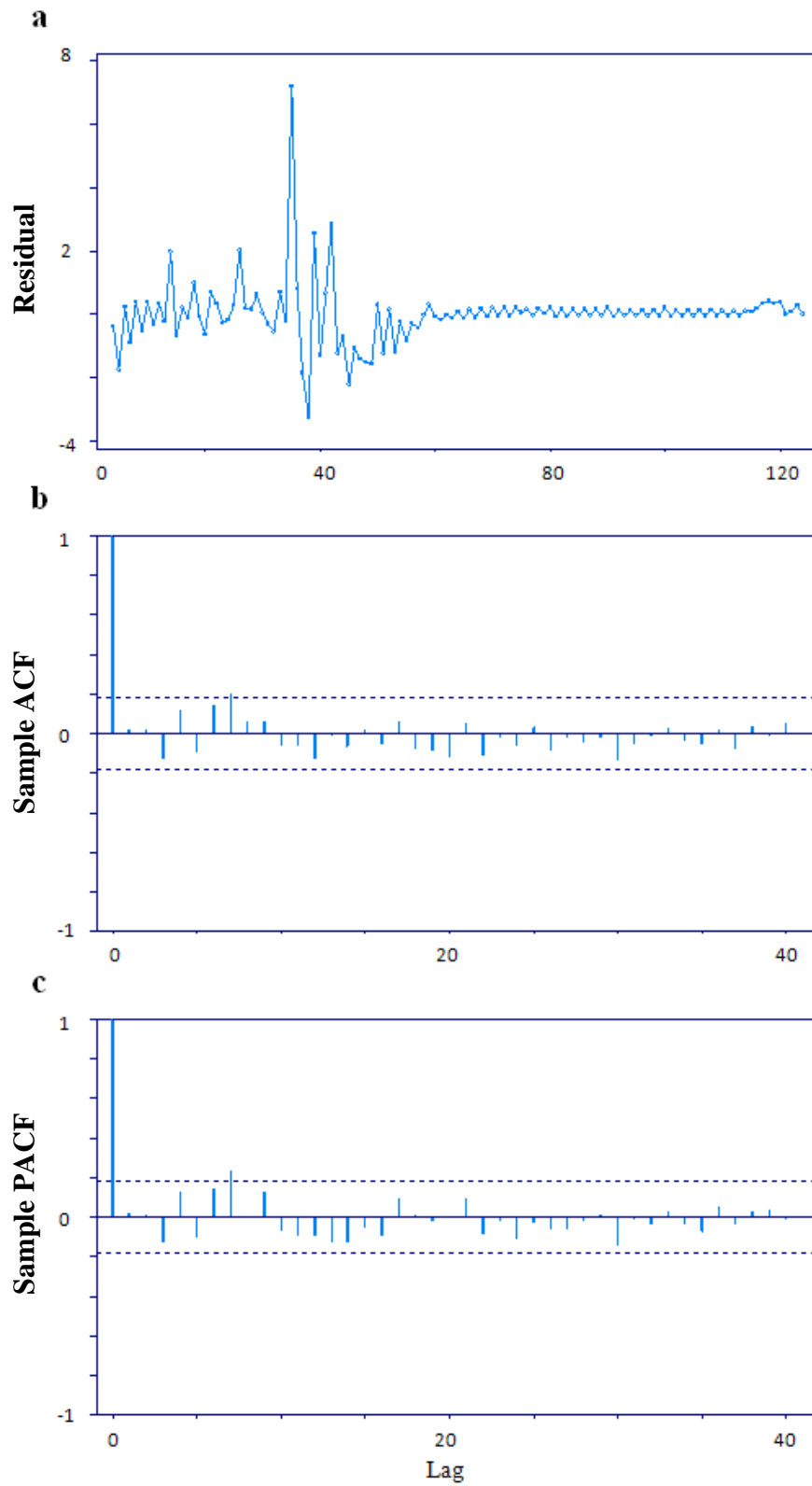


Fig. 77. Diagnostics for MA (4) fitted to the mean-corrected and twice differenced training sample. **a.** time plot, **b.** ACF, and **c.** PACF of the residuals.

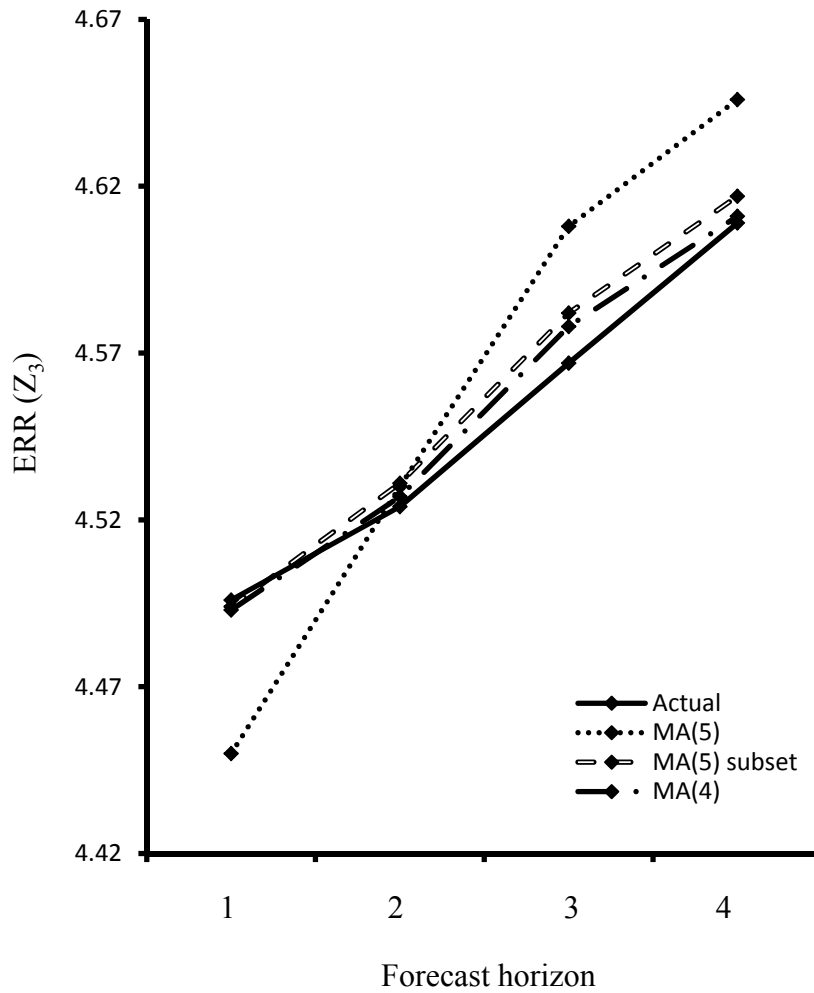


Fig. 18. Comparison of the forecasted ERRs using MA (5), subset MA (5) and MA (4) model with the observed data.

Table 10

Numerical values of observed ERRs, corresponding observed counts, predicted ERRs (using MA(4)) and corresponding predicted counts for the prediction set

Forecast Horizon	ERRs		Counts	
	Actual	Prediction	Actual	Prediction
125	4.496	4.493	11	11
126	4.524	4.527	8	9
127	4.567	4.578	10	11
128	4.609	4.611	10	9

6.2 Modeling for ERRR time series

We have six ERRR time series as illustrated in Chapter 4, Fig. 7. In this section, we will use the R_{23} series as an example to present the ARIMA and ARFIMA modeling. The modeling process is similar to that detailed earlier. Fig. 19 shows the ERRR plots with training sample and prediction set. As in the previous section, we use the last 4 ERRR values as the prediction set and the remaining as the training sample. Fig. 20 shows the corresponding time plot, sample ACF and sample PACF, which indicates nonstationary behavior. A difference at lag-1 was taken and Fig. 21 shows the time plot, sample ACF and sample PACF of the lag-1 differenced series.

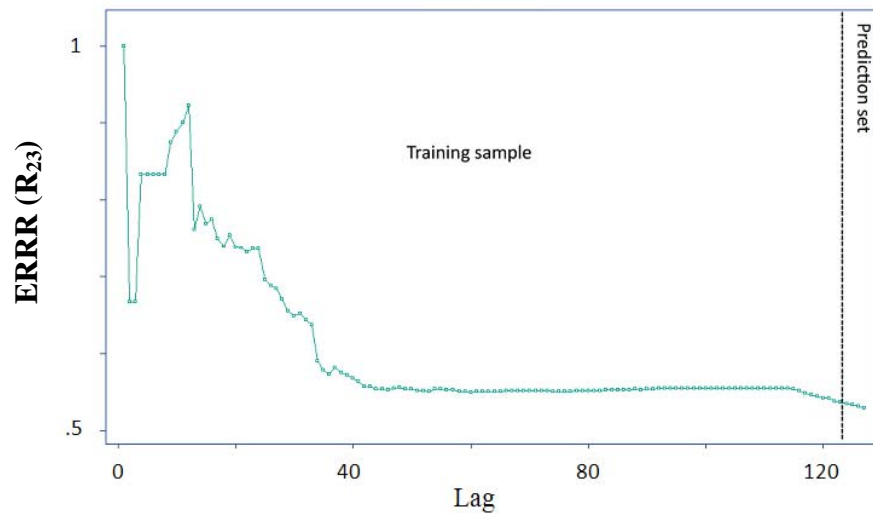


Fig. 19. ERRR plots (R_{23}) with training sample and prediction set.

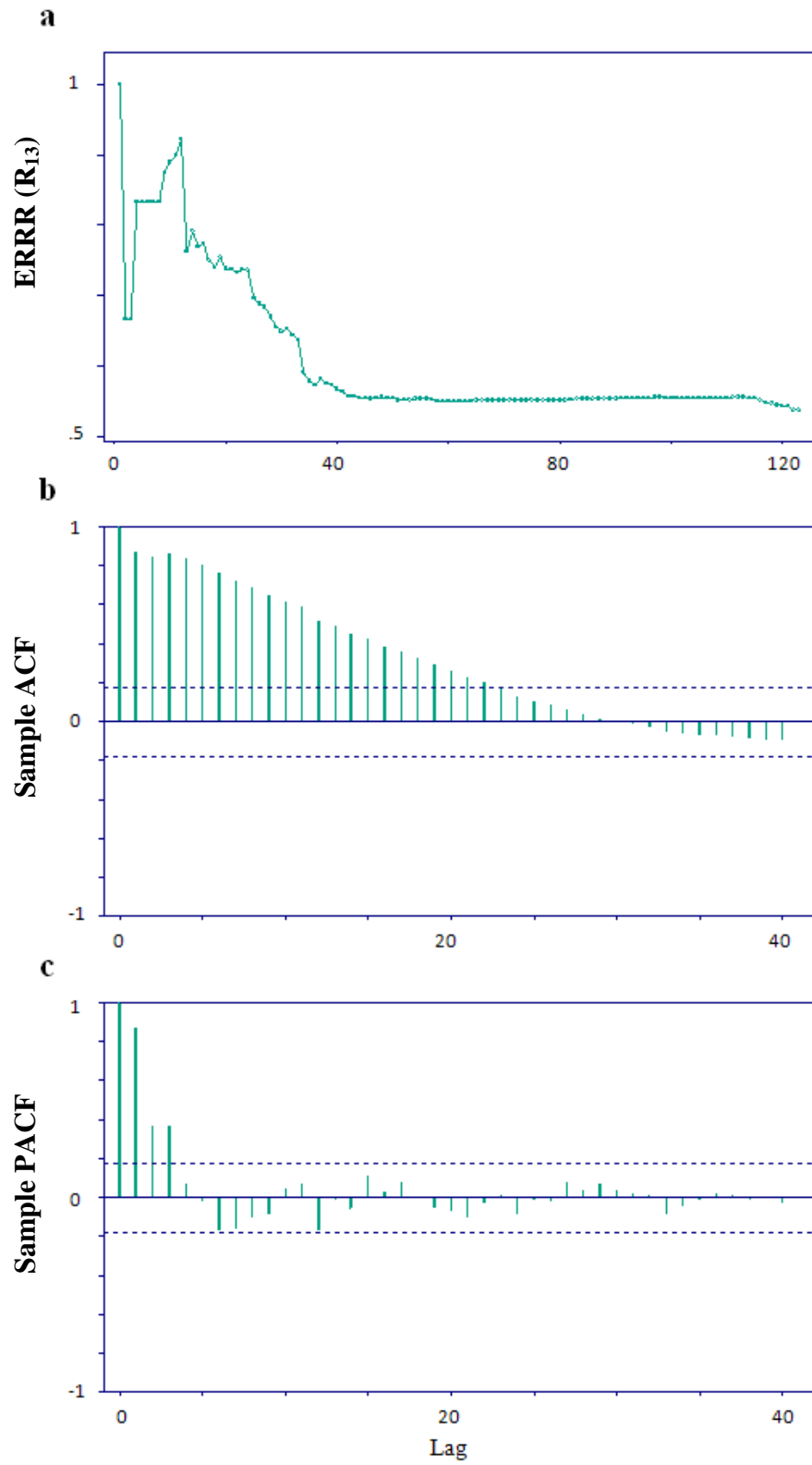


Fig. 20. a. Time plot; b. ACF; c. PACF of R_{23} in the training sample.

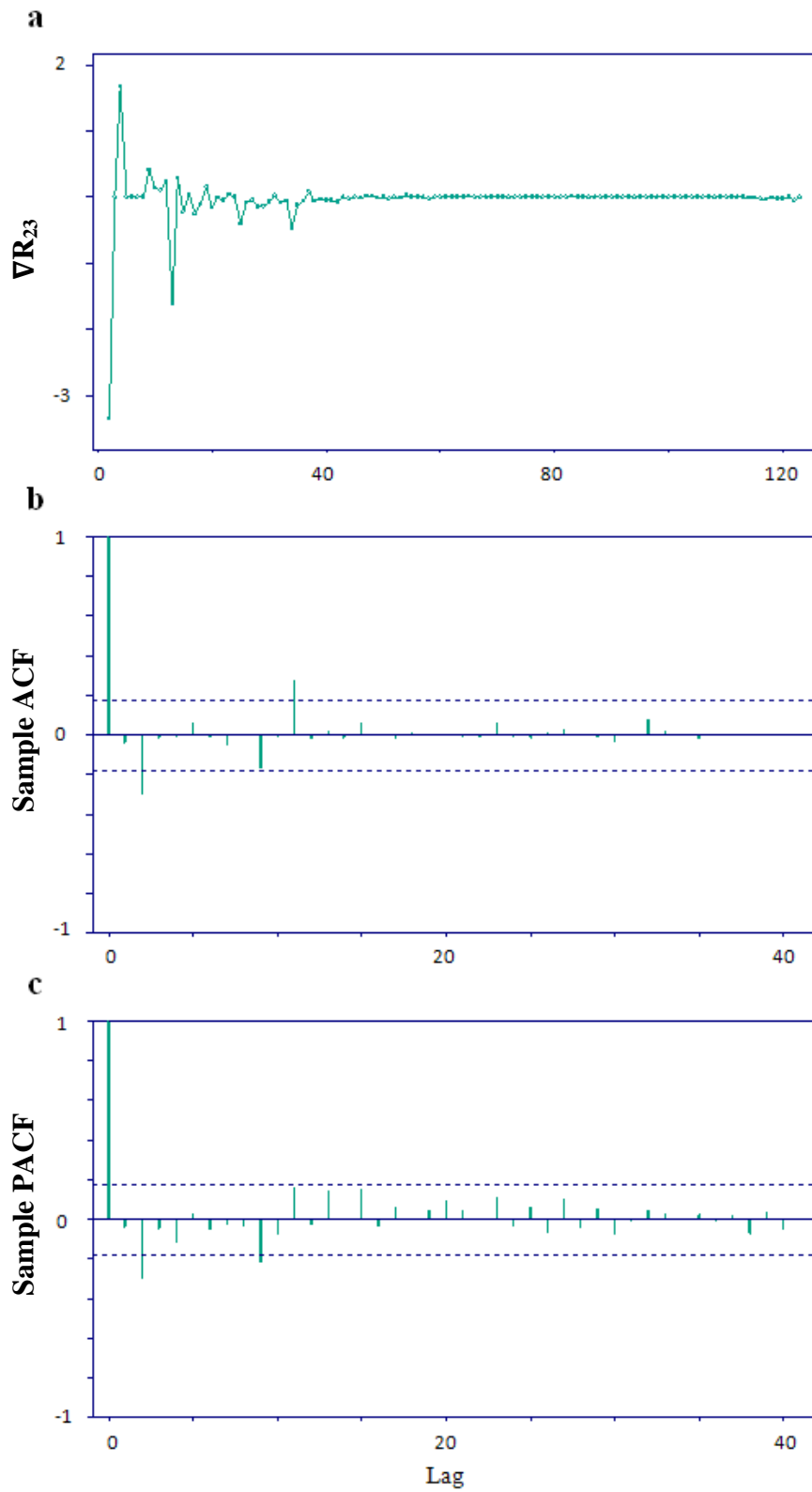


Fig. 21. a. Time plot; b. ACF; c. PACF of the lag-1 differenced R_{23} series in the training sample.

Next, ARIMA modeling and computational techniques are used to fit the differenced ERRRs. We set the AR and MA parameters between to be 0 and 5, and after traversing all possible models, we find the best model is ARMA(4, 2). Fig. 22 is a set of diagnostic plots produced by ITSM2000 package, which show the ACF and PACF of residuals of training sample obtained after fitting ARMA (4, 2) model. The AICC statistic is -463.15 and the Ljung-Box test is not significant (p -value = 0.45). The estimated model is given in the following box:

ARMA(4, 2) Model:

$$X_t = - .3176 X_{t-1} + .1690 X_{t-2} - .1078 X_{t-3} + .04347 X_{t-4} + Z_t + .2380 Z_{t-1} - .5709 Z_{t-2}$$

WN Variance = 0.001159

Looking at the raw data sample ACF in Fig. 20 b, we see that the autocorrelation decreases very slowly, which indicates a long-term memory. We thus consider using a fractional differencing parameter, which gives rise to the ARFIMA modeling. We find that the best such model is ARFIMA (1, 0.498, 1). Fig. 23 is a set of diagnostic plots produced by ITSM2000 package, which show the ACF and PACF of residuals obtained from the fitted ARFIMA (1, 0.498, 1) model applied to the training sample. The AICC statistic is -469.82 and the Ljung-Box test is not significant (p -value = 0.47). The estimated model is given in the following box.

ARFIMA(1, 0.498, 1) Model:

$$(1-B)^{0.4985} [X_t + .7287 X_{t-1}] = Z_t + 1.0000 Z_{t-1}$$

WN Variance = 0.001056

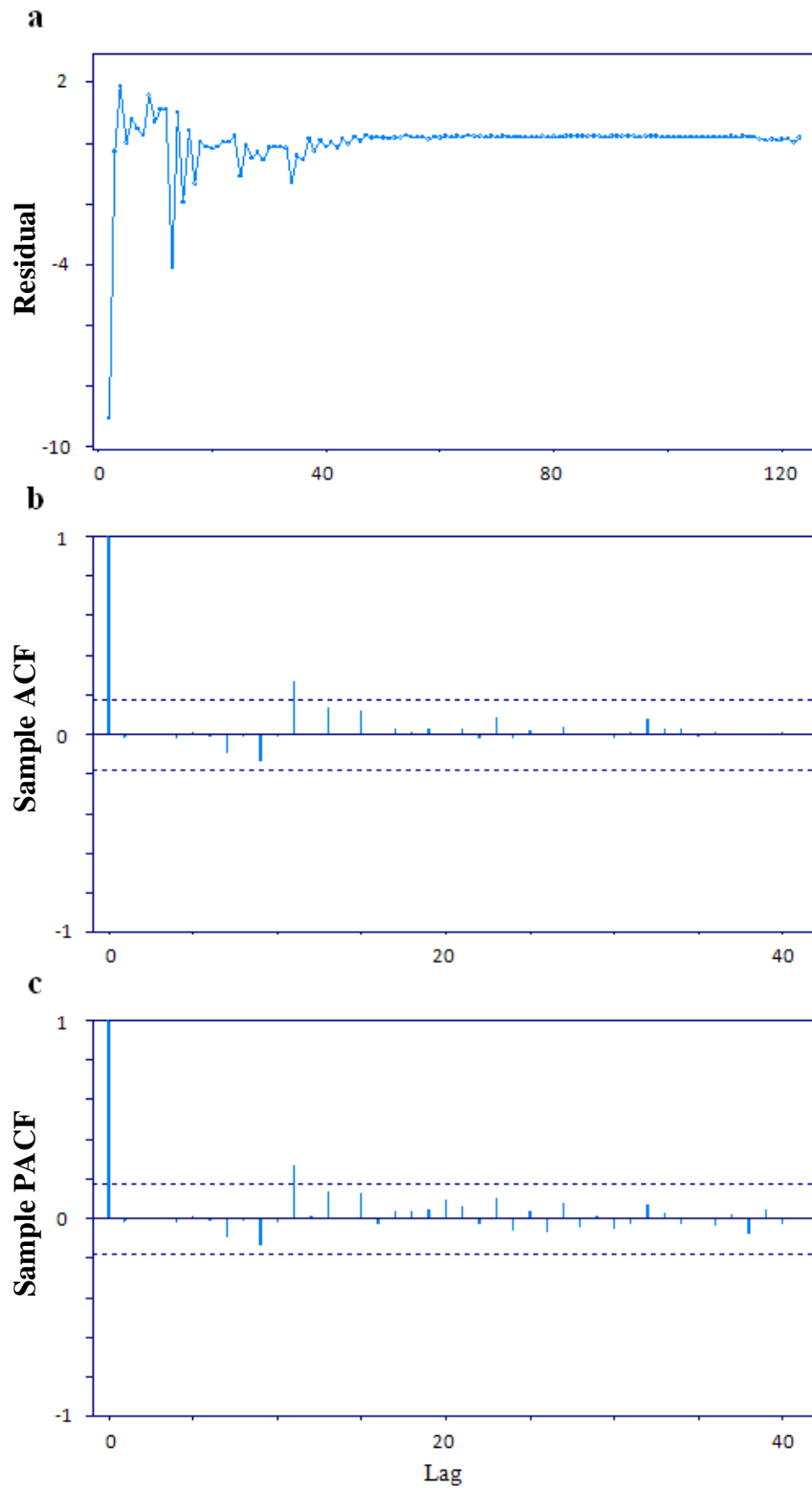


Fig. 22. Diagnostics for ARIMA(4,2) fitted to the mean-corrected and lag-1 differenced training sample. **a.** time plot, **b.** ACF, and **c.** PACF of the residuals.

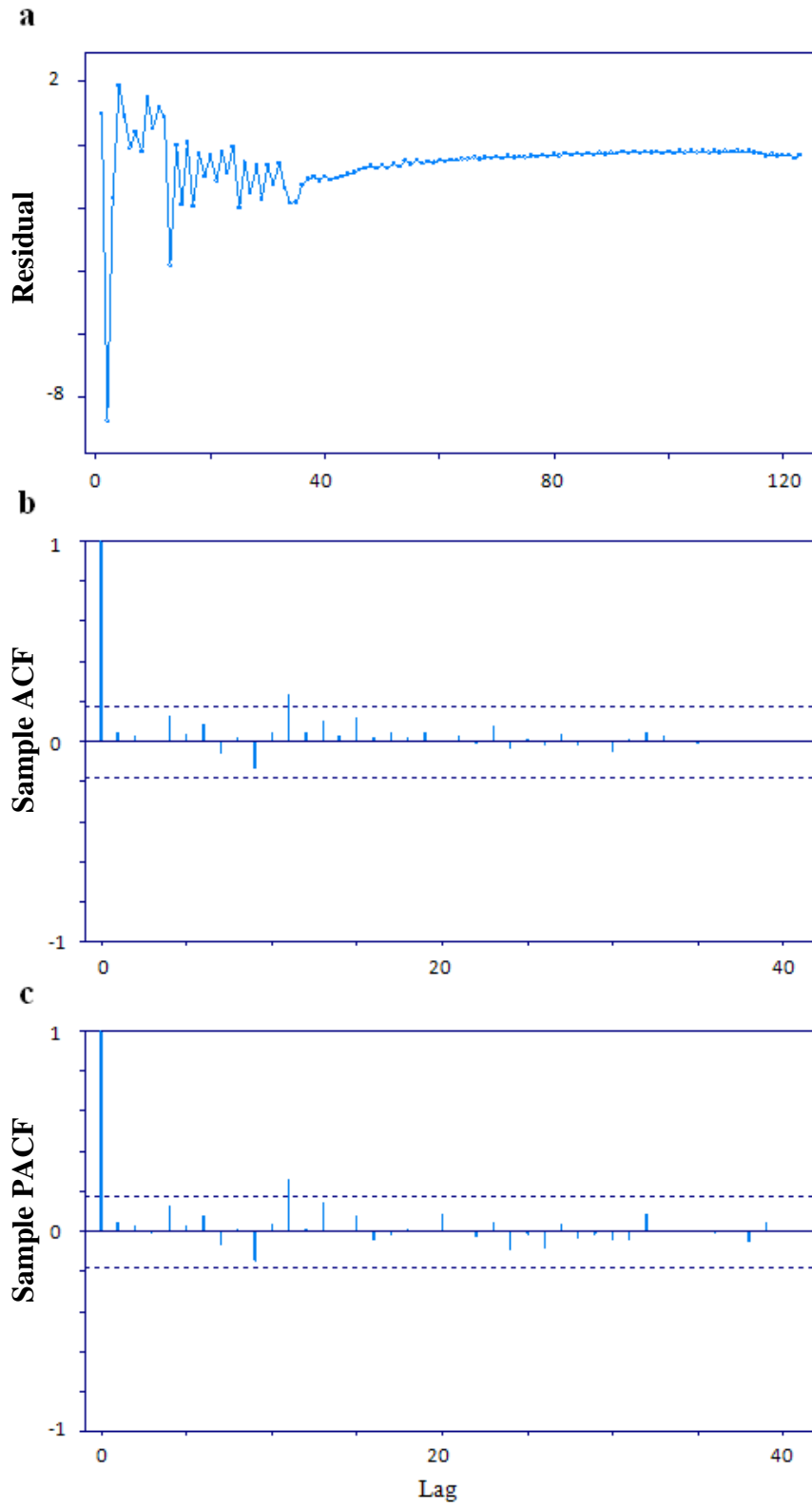


Fig. 23. Diagnostics for ARFIMA(1,0.498,1) fitted to the mean-corrected training sample. **a.** time plot, **b.** ACF, and **c.** PACF of the residuals.

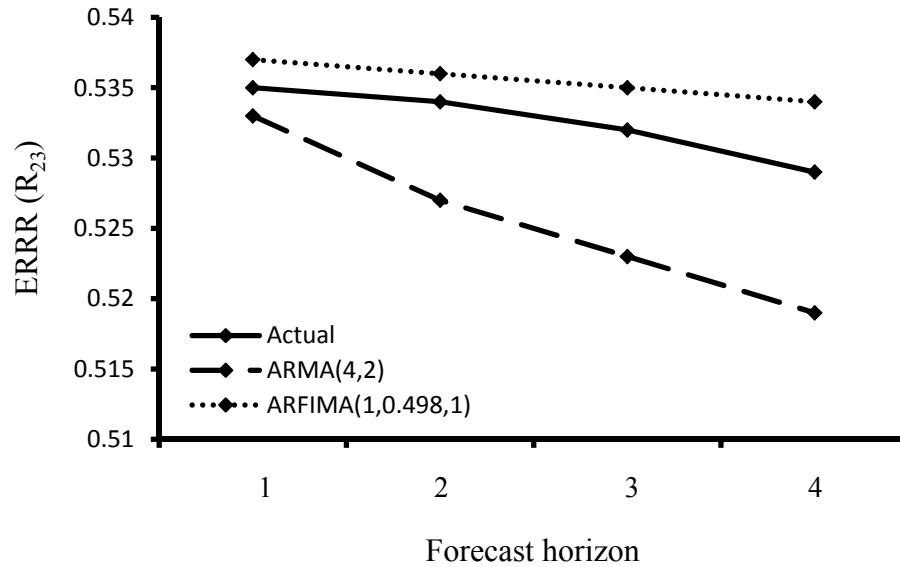


Fig. 24. Comparison of the forecasted ERRRs using ARMA (4, 2) and ARFIMA (1, 0.498, 1) models with the observed values in the prediction set.

We use Fig. 24 to compare the forecasted ERRRs using ARMA (4, 2) and ARFIMA (1, 0.498, 1) model with the observed values in the prediction set. As defined in the previous section, we calculated the root mean square error (RMSE) to compare the accuracy of prediction for these models, and obtained $RMSE_{ARMA(4, 2)} = 0.008$, $RMSE_{ARFIMA(1, 0.498, 1)} = 0.003$. Hence we choose ARFIMA (1, 0.498, 1) as our final model and corresponding predicted ERRRs are presented in Table 11.

Table 11

Actual ERRRs of small-large bank failure comparison and their predictions

Forecast horizon	ERRRs	
	Actual	Prediction
125	0.535	0.537
126	0.534	0.536
127	0.532	0.535
128	0.529	0.534

6.3 Full data set prediction

Since we obtained acceptable models for both ERR and ERRR time series based on the training data set, we want to apply this procedure to the full data set (combination of the training data set and prediction data set) and predict numbers of bank failures and rates ratios for 2012. The results are summarized in Table 12. Note that some of the ERR lower confidence bounds were adjusted to reflect the nature of the ERRs.

Table 12
Full data (1980:Q1 – 2011:Q4) time series ($Z_0, \dots, Z_4, R_{12}, \dots, R_{34}$) modeling and forecasting for 2012:Q1 – Q4

Time Series	Transformed series	Model	2012 Forecast (95% Prediction Intervals.)			
			Q1	Q2	Q3	Q4
Z	VVlogZ	ARMA(2,3)	(24.121, 26.110)	(23.418, 26.857)	(22.662, 27.656)	(21.841, 28.521)
Z ₁	VVlogZ ₁	ARMA(4,4)	(8.741, 9.473)	(8.342, 9.772)	(7.887, 10.129)	(7.379, 10.537)
Z ₂	VlogZ ₂	MA(2)	(4.933, 5.443)	(4.779, 5.601)	(4.673, 5.740)	(4.510, 5.929)
Z ₃	V ₂ VZ ₃	MA(4)	(4.280, 5.020)	(4.117, 5.271)	(3.997, 5.569)	(3.802, 5.811)
Z ₄	VlogZ ₄	AR(1)	(5.907, 6.420)	(5.682, 6.616)	(5.481, 6.790)	(5.321, 6.963)
R ₁₂	VVR ₁₂	MA(2)	(0.582, 0.692)	(0.550, 0.723)	(0.502, 0.768)	(0.445, 0.823)
R ₁₃	VVR ₁₃	MA(2)	(0.623, 0.705)	(0.610, 0.716)	(0.600, 0.724)	(0.588, 0.732)
R ₁₄	VVR ₁₄	MA(4)	(0.578, 0.613)	(0.569, 0.617)	(0.560, 0.621)	(0.552, 0.624)
R ₂₃	R ₂₃	ARFIMA(1,0.498,1)	(0.495, 0.561)	(0.492, 0.560)	(0.490, 0.558)	(0.476, 0.571)
R ₃₄	VVR ₃₄	ARMA(4,4)	(0.423, 0.488)	(0.414, 0.492)	(0.407, 0.499)	(0.402, 0.499)
R ₃₄	VVR ₃₄	ARMA(1,5)	(0.316, 0.540)	(0.277, 0.580)	(0.246, 0.611)	(0.219, 0.637)

CHAPTER 7

CONCLUSIONS

The original objective of this study was to examine the relationship between bank failure status and bank asset size; we wanted to know if they are related or not. We hoped to use this information to predict the future bank failures. The Pearson's chi-square test for contingency tables gave us a positive response. For further study on bank failure and its prediction based on size, we applied the logistic regression modeling. We obtained the corresponding prediction rules and results. However, the results were not very satisfactory.

We then transferred the raw data to ERR and ERRR time series, which provided us with additional ideas of data analysis. We obtained much valuable information on the reason for, time period of, and trends of bank failures during the past thirty years. First, we performed a detailed analysis of the ERR and ERRR time series plots. We separated 1980 to 2011 into three periods: Savings and Loan crisis, the stable period and the Great Recession crisis. We studied the features of failed bank types during these different periods. Smaller banks failed more often during the Savings and Loan crisis. However, banks with larger assets failed more often between 2007:Q1 to 2011:Q4 (Great Recession).

Additionally, we performed pairwise bank failure rate comparisons using the conditional test (Przyborowski and Wilenski, 1940). We found that the ERR and ERRR not only smooth and reduce the volatility of a financial system modeled by a stochastic process, but also operate as a linking bridge between a classical time series and a point

process, which became the most important factor helping us to get our predictions. Based on the good behaviors of ERR and ERRR time series, a variety of time series modeling techniques could be applied. We fitted ARIMA and ARFIMA models to the ERR and ERRR time series and found the resulting predictions of failure counts to be quite accurate.

APPENDIX I

Detailed bank data with bank Status, Periods and Assets (Data file is available upon request, <http://fdic.gov/bank/individual/failed/banklist.html>)

Bank	Status	Period	Group	Assets	Log-Assets
1	0	0	1	3015000	14.9191104
2	1	1	1	3140075.9	14.9597575
3	0	0	1	3491000	15.0656988
				.	
				.	
				.	
10510	0	0	4	1451969302000.00	28.0039419
10511	0	0	4	1542984268826.00	28.0647395
10512	0	0	4	1811678000000.00	28.2252746

Definitions of variables

Status: "1" for failed bank and "0" for solvent.

Period: "0" for solvent bank, "1" for banks failed between 1980:Q1 and 1995:Q4, "2" for banks failed between 1996:Q1 and 2006:Q4, and "3" for banks failed between 2007:Q1 and 2011:Q4.

Group: "1" for G₁, "2" for G₂, "3" for G₃, and "4" for G₄.

APPENDIX II

Number of Bank failures during 1980:Q1 to 2011:Q4 by groups.

Quarter	Total	Small	Medium	Large	Grand
1980Q1	3	3	0	0	0
1980Q2	2	1	1	0	0
1980Q3	2	0	1	1	0
1980Q4	2	2	0	0	0
1981Q1	4	1	3	0	0
1981Q2	1	1	0	0	0
1981Q3	1	1	0	0	0
1981Q4	2	2	0	0	0
1982Q1	4	3	0	0	1
1982Q2	10	8	2	0	0
1982Q3	10	8	1	0	1
1982Q4	8	7	1	0	0
1983Q1	11	6	3	0	2
1983Q2	15	6	4	4	1
1983Q3	10	6	3	0	1
1983Q4	9	6	1	1	1
1984Q1	13	7	4	1	1
1984Q2	30	15	9	4	2
1984Q3	26	15	4	2	5
1984Q4	22	16	3	0	3
1985Q1	28	16	8	4	0
1985Q2	40	21	11	4	4
1985Q3	40	29	4	2	5
1985Q4	46	27	7	2	10
1986Q1	46	20	11	4	11
1986Q2	56	22	13	12	9
1986Q3	57	30	12	7	8
1986Q4	49	22	13	7	7
1987Q1	77	43	13	10	11
1987Q2	64	40	9	9	6
1987Q3	52	28	9	7	8
1987Q4	65	36	13	6	10
1988Q1	65	32	16	12	5
1988Q2	67	37	11	9	10

1988Q3	184	47	39	51	47
1988Q4	116	29	28	28	31
1989Q1	72	32	10	12	18
1989Q2	55	28	11	1	15
1989Q3	163	54	41	35	33
1989Q4	99	39	16	15	29
1990Q1	124	38	28	26	32
1990Q2	198	67	48	42	41
1990Q3	109	47	18	23	21
1990Q4	96	27	25	20	24
1991Q1	83	26	11	13	33
1991Q2	77	23	18	14	22
1991Q3	66	21	14	13	18
1991Q4	54	14	10	6	24
1992Q1	51	24	9	5	13
1992Q2	54	12	14	14	14
1992Q3	25	8	6	5	6
1992Q4	55	9	7	10	29
1993Q1	14	5	3	2	4
1993Q2	20	9	3	4	4
1993Q3	19	10	7	1	1
1993Q4	7	2	1	1	3
1994Q1	2	0	0	1	1
1994Q2	6	3	1	1	1
1994Q3	9	1	2	6	0
1994Q4	2	0	1	1	0
1995Q1	3	1	0	1	1
1995Q2	3	1	1	0	1
1995Q3	2	0	1	1	0
1995Q4	1	1	0	0	0
1996Q1	2	1	0	0	1
1996Q2	5	1	2	0	2
1996Q3	3	2	1	0	0
1996Q4	1	0	0	0	1
1997Q1	2	0	0	0	2
1997Q2	1	0	0	0	1
1997Q3	2	0	0	0	2
1997Q4	1	1	0	0	0
1998Q1	0	0	0	0	0
1998Q2	2	1	0	0	1
1998Q3	3	1	0	1	1

1998Q4	2	0	0	0	2
1999Q1	1	1	0	0	0
1999Q2	1	1	0	0	0
1999Q3	5	1	2	1	1
1999Q4	3	0	1	1	1
2000Q1	3	1	1	0	1
2000Q2	1	1	0	0	0
2000Q3	2	1	1	0	0
2000Q4	4	0	2	0	2
2001Q1	1	1	0	0	0
2001Q2	3	2	0	0	1
2001Q3	2	1	0	0	1
2001Q4	0	0	0	0	0
2002Q1	6	3	1	0	2
2002Q2	4	1	1	1	1
2002Q3	1	0	1	0	0
2002Q4	2	1	1	0	0
2003Q1	2	0	1	0	1
2003Q2	1	1	0	0	0
2003Q3	0	0	0	0	0
2003Q4	1	1	0	0	0
2004Q1	3	2	1	0	0
2004Q2	1	0	1	0	0
2004Q3	1	1	0	0	0
2004Q4	1	0	0	1	0
2005Q1	0	0	0	0	0
2005Q2	0	0	0	0	0
2005Q3	0	0	0	0	0
2005Q4	0	0	0	0	0
2006Q1	0	0	0	0	0
2006Q2	0	0	0	0	0
2006Q3	0	0	0	0	0
2006Q4	1	0	0	0	1
2007Q1	1	1	0	0	0
2007Q2	0	0	0	0	0
2007Q3	1	0	0	0	1
2007Q4	1	0	1	0	0
2008Q1	3	3	0	0	0
2008Q2	2	1	0	0	1
2008Q3	9	0	1	2	6
2008Q4	12	2	1	2	7

2009Q1	21	0	4	8	9
2009Q2	25	1	3	9	12
2009Q3	51	5	11	12	23
2009Q4	46	6	9	12	19
2010Q1	41	7	6	9	19
2010Q2	45	6	9	8	22
2010Q3	41	5	5	15	16
2010Q4	30	3	10	9	8
2011Q1	26	2	7	11	6
2011Q2	22	2	6	8	6
2011Q3	27	3	5	10	9
2011Q4	28	3	5	10	10

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