12-1-2014

Modeling and Analysis of Pedestrian Flows

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MODELING AND ANALYSIS OF PEDESTRIAN FLOWS

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Bachelor of Technology

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2008

A thesis submitted in partial fulfillment
of the requirements for the

Master of Science - Mathematical Sciences

Department of Mathematical Sciences
College of Sciences
The Graduate College

University of Nevada, Las Vegas
December 2014

THE GRADUATE COLLEGE

We recommend the thesis prepared under our supervision by

Romesh Khaddar

entitled

Modeling and Analysis of Pedestrian Flows

is approved in partial fulfillment of the requirements for the degree of

Master of Science - Mathematical Sciences

Department of Mathematical Sciences

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December 2014

ABSTRACT

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by

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According to the Traveler Opinion and Perception Survey of 2005, about 107.4 million Americans regularly use walking as a mode of transport during their commute, which amounts for 51% of the total American population. In 2009, 4092 pedestrian fatalities were reported nationwide, out of 59,000 pedestrian crashes. This amounts for 12% of the fatalities in the total traffic accidents recorded, and shows an over-representation of pedestrians incidents. Thus, it is imperative to understand the causes behind such statistics, and conduct a comprehensive research on pedestrian walking behavior and their interaction with surroundings.

A lot of researches on pedestrian flows have been conducted with respect to crowd dynamics in various situations like evacuation simulations. In this thesis, we investigate the Hughes model for pedestrian flows, which is governed by a coupled system of a scalar conservation law and an eikonal equation. The Hughes model considers the pedestrians as a continuum fluid and describes the motion of pedestrians in a densely crowded region. For the one-dimensional Hughes model with a single turning point (the origin), the governing equation can be be decomposed into two classical conservation laws on two sub-domains around the origin. We study various commonly observed interactions of pedestrian flows for tracking and understanding their movement on a mesoscopic level.

In this thesis, the conservation law for pedestrian flows and the Hughes model are introduced in Chapter 2. We then summarize some existing theoretical work on the well-posedness and existence of the entropy solutions of the Hughes model in Chapter 3. In Chapter 4, we study the one-dimensional Hughes model with a single turning point (the origin) and the given pedestrian potential which governs pedestrian flow tendency around the origin, and investigate 18 different cases. An interesting phenomena of dual shocks is observed, and remains to be investigated further in the future work.

ACKNOWLEDGEMENTS

Every project depends on the right guidance and enough motivation to ensure quality and completion. I would like to express my sincere thanks and gratitude towards Dr. Zhonghai Ding for inspiring, motivating and guiding me throughout my masters degree and the thesis. I cannot say enough words to appreciate his help. I am also grateful to Dr. David Costa, Dr. Hossein Tehrani, Dr. Anjala Krishen and Dr. Pushkin Kachroo for their insightful remarks.

I would like to take this opportunity to show my appreciation towards the love and support of my family.

I am also grateful to my friends for support, motivation and help during the period of my work, without whom this would have never been possible.

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CHAPTER 1

INTRODUCTION AND LITERATURE SURVEY

Pedestrian safety is a primary concern in mixed traffic situations as it is related to human life. According to a report by NHTSA, 4,092 pedestrians were killed and an estimated 59,000 were injured in traffic crashes in United States in 2009. The numbers are significant as they account for 12% of fatalities in crash data [25]. Each pedestrian injury or fatality has serious implications in terms of cost to those affected directly and indirectly. A great deal of research has been done in pedestrian and vehicle interactions [28, 30]. A slight improvement in pedestrian safety can lead to increased mobility and in turn encourage pedestrian friendly environments. As a result, it becomes extremely important to create ways that improves the interactions between driver and pedestrians.

Traffic congestion is a very important aspect of transportation planning within city limits and boundaries. The costs attributed to congestion are at multiple levels and can be broadly classified in direct and indirect costs. Direct costs include fuel, vehicle operations and maintenance; while the indirect costs include inability to calculate precise travel time i.e. delays, air pollution and societal concerns. But the trend doesnt stop here and traffic congestion creates tertiary effects like road rage, anger, and slow emergency vehicle response [24].

Traditionally, larger part of travel planning is done with the aim to minimize travel time. In addition, preference is given to vehicle traffic over pedestrian traffic [14]. Some studies have attempted to explain the effect of reducing traffic congestion by optimizing travel time costs to both pedestrians and vehicles [26]. These studies are more relevant to traffic in heavily traveled areas, but point towards a more subtle area of pedestrian safety. After the completion of planning, study on pedestrian safety is required to understand the effect of the new improvements; such as effects of countermeasures using before and after studies. Such studies are incomplete in a broader sense of safety unless actual human subjects experience such systems. Some methods used for study of pedestrian safety are surveys and observations on an actual implemented system, over the course of time.

Pedestrian safety is also attributed to drivers of vehicles traveling through the traffic system. Furthermore, pedestrian safety is a mutual relationship between both drivers and pedestrians. If any of the two does not understand it or fail to respect others right, both of them have to face the repercussions. The study of such issues is under the broad topic of human factors research in transportation engineering.

Studies have shown that the socio-economic, demographics, and level of pedestrian activity of an area affect the transportation behavior [30]. Transportation behavior in general discusses the interaction between transportation system and people; ranging from mode choice to trip frequency and distance as well as the ways citizens affects the transportation policy. Demographics is a broad term and includes many variables such as religion, income, race, sex, marital status, etc. that can help put members of a population into smaller groupings. With respect to the transportation behavior, certain variables have been found in high correlation such as age distribution, race and ethnicity, education level etc. [10]. Statistically, a set of individuals can be identified as the representative of demographics for a region to conduct human factors research. Such research is an important topic in the field of transportation since it assesses effects on transportation systems subject to variations in user behavior due to their personal traits.

Due to the complexity involved between transportation behavior and demographics, individual behavior cannot be truly studied or analyzed for the entire population. Statistical methods provide the capability to represent demographical information with a smaller set of individuals, thereby creating a significant representation of the population inhabiting in that region. Such methods are based on surveys in a safe and controlled environment of a lab and have widely been accepted for study of human factors research [32].

As mentioned earlier, pedestrian safety has been mostly studied on established transportation systems by surveys and observations at the locations/site. Such methods though address in understanding many real life problems, however they have certain implied assumptions and therefore lack on a few grounds. For example, since such surveys and observations are taken on an existing system and the results are used for suggesting modifications to it. This incidentally implies the system might be running in a potentially unsafe condition.

The standards in conducting pedestrian Level of Service (LOS) analysis is explained Highway Capacity Manual (HCM) in USA. Although, a standardized set of practices is defined for data collection and quantifying congestion in pedestrian facilities, many studies identify amendments and new methods for HCM to analyze LOS [4].

According to HCM, LOS for pedestrian part of a transportation system can be improved upon three primary areas namely pedestrian characteristics, sidewalk environment and flow characteristics. The relationships between these categories have emerged in the literature for pedestrian studies and can be illustrated as shown in figure 1.1 [4].

Pedestrian characteristics can be broadly classified as personal characteristics, trip purposes, and expectations and behavior. Personal characteristics relate variables like pedestrian speed and sidewalk widths with age, gender, group size and other demographic factors [5, 18, 34]. Trip purpose and pedestrian perceptions like

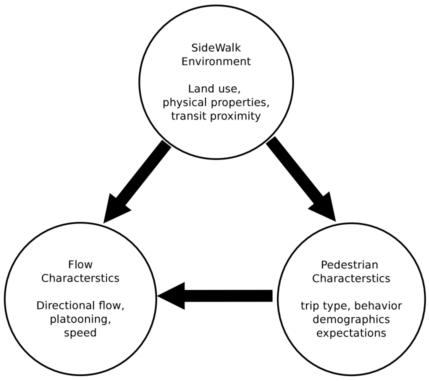

Figure 1.1: Relationship between pedestrian and traffic environment

safety, comfort and convenience have also been found to affect their behavior, though they have not been addressed in HCM. Researchers have confirmed that pedestrians perception of environment affect their behavior significantly [33, 16, 23]. In general, they have a tendency to put a cost to each sidewalk facility for a destination on their personal expectations [12]. Similarly, individual behaviors like use of music players and mobile handsets during walking, has been criticized by various authors [3] but researchers merely have anecdotal evidences for the same and wish to understand it more.

Researchers have studied modeling techniques that involves interactions of pedestrians and drivers. Pedestrians are affected largely by transportation, environmental, and social systems surrounding them whereas drivers are inherent part of transportation systems [29, 22]. The benefit of such a system is the need to create a pedestrian friendly environment for the planning of sustainable transportation systems and livable communities. Additionally, optimizations and site selection techniques have been used to identify high crash locations as well as to design experiments to implement the countermeasures [31]. Despite recent efforts by notable researchers, there is still a need to understand the pedestrian and driver behavior through simulation techniques such as the use of driving simulator, video games, and animations. This can help both the driver and the pedestrian understand the effects of safe driving habits as soon as they get their drivers license.

This research primarily addresses some of the concerns and focus mainly on simulating the pedestrian behavior. Vehicles usually travel in a single line of motion, i.e. they travel in only one direction. As opposed to vehicular traffic, pedestrians are treated as crowd as they do not follow a particular line of motion (or lane in general). There are multiple approaches suggested for such kind of problem and are primarily solved based on two concepts.

- Pedestrians are treated as discrete elements passing through a domain, generally in a computer simulation. Approaches covered are:
  1. Using a granular material analogue (rare)
  2. Modeling the path taken assuming pedestrians optimize their immediate local behavior
  3. Assuming they attempt to move along predefined globally determined paths
- The crowd is treated as a whole, generally applicable for a large crowd. Ap-

proaches covered are:

1. A fluid (now rare)

2. A continuum responding to local influences

3. In a continuum, individuals optimize behavior to reach non-local objectives

With this idea, a better interactive pedestrian simulator was envisioned. This simulator is expected to allow a human subject to traverse a virtual space consisting of computed pedestrians. Such implementation requires a better representation of background pedestrian traffic. This thesis, a mathematical model is proposed to compute background pedestrian interactions and the pedestrian traffic flow. The objective was to better capture the effect of the pedestrian flow as well as validate the method corresponding to it.

An expected result by this work allows towards development of a module using which a platform (human centered pedestrian simulator) can be constructed for conducting studies on pedestrian related transportation systems.

With the above mentioned concepts, this research envisages a better interactive pedestrian simulator. This simulator is expected to allow a human subject to traverse a virtual space consisting of computed pedestrians. Such implementation requires a better representation of background pedestrian traffic. In this research, a mathematical model to compute background pedestrian interactions and the pedestrian traffic flow was proposed. The objective was to better capture the effect of the pedestrian flow through a computationally faster approach. The results show the development of a module to construct a platform (human centered pedestrian simulator) for conducting studies on pedestrian related transportation systems.

The thesis is structured as follows. Chapter 2 introduces the conservation law for pedestrian flows and the Hughes model, which forms an important component of this research work. Chapter 3 summarizes some existing theoretical work on the well-posedness and existence of the entropy solutions of the Hughes model. Chapter 4 focuses on the one-dimensional Hughes model with a single turning point (the origin) and the given pedestrian potential which governs pedestrian flow tendency around the origin, and investigate 18 different cases. Chapter 5 summarizes the conclusions and future work.

CHAPTER 2

CONCEPT REVIEW - CONSERVATION LAW IN 1D

In this thesis, the modeling of pedestrian flow is done in a similar manner as the modeling of traffic flow. Traffic flow is modeled based on the conservation law, i.e. given a roadway segment, difference between numbers of vehicles leaving and entering equals the number of vehicles on the roadway segment. This chapter details the derivation of scalar conservation law for traffic models and nature of corresponding solutions.

2.1 Conservation of Mass

The conservation of mass is discussed in context of flow across a region or boundary. In case of single dimension, let us consider a section between $x = x_1$ and $x = x_2$ referenced from the origin on the x-axis as shown in figure 2.1. Assuming this section contains fluid with scalar field $\rho(t, x)$, and that the fluid enters the region from left i.e. at $x = x_1$, and leaves at $x = x_2$. Therefore the flux of fluid entering at any time is $q(t, x_1)$, and leave at $q(t, x_2)$. Flux flowing through a given point at any time is the product of density and velocity at that point equation (2.1).

$$q(t, x) = \rho(t, x)v(t, x) \tag{2.1}$$

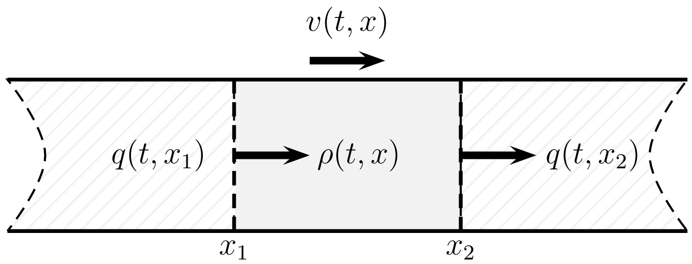

Figure 2.1: Conservation of Mass

Since the conservation of mass implies the net change in flux, it is equal to the mass contained inside an arbitrary boundary. hence in one dimensional case is defined by x_1 and x_2. The mass in the section from $x = x_1$ to $x = x_2$ at time t is given by

$$\text{mass in } [x_1, x_2] \text{ at time t } = \int_{x_1}^{x_2} \rho(t,x)dx \tag{2.2}$$

The total mass that enters the section from the edge at $x = x_1$ is given by

$$\text{Inflow at } x_1 \text{ from time } t_1 \text{ to } t_2 = \int_{t_1}^{t_2} \rho(t,x_1)v(t,x_1)dt \tag{2.3}$$

Similarly, the total mass that leaves the section from the edge at $x = x_2$ is given by

$$\text{Outflow at } x_2 \text{ from time } t_1 \text{ to } t_2 = \int_{t_1}^{t_2} \rho(t,x_2)v(t,x_2)dt \tag{2.4}$$

The conservation law states that the change in mass in the section $[x_1, x_2]$ from time $[t_1, t_2]$ is equal to the mass that enters through the flux at x_1 from which the mass that exits through the flux at x_2 has been subtracted. This is stated below as the conservation law in the *first integral form.*

$$\int_{x_1}^{x_2} \rho(t_2, x)dx - \int_{x_1}^{x_2} \rho(t_1, x)dx = \int_{t_1}^{t_2} \rho(t, x_1)v(t, x_1)dt - \int_{t_1}^{t_2} \rho(t, x_2)v(t, x_2)dt \quad (2.5)$$

Alternatively, this can also be written in the *second integral form* as:

$$\frac{d}{dt}\int_{x_1}^{x_2} \rho(t, x)dx = \rho(t, x_1)v(t, x_1) - \rho(t, x_2)v(t, x_2) \quad (2.6)$$

Equation (2.5) can be written as

$$\int_{x_1}^{x_2} [\rho(t_2, x) - \rho(t_1, x)]\, dx = \int_{t_1}^{t_2} [\rho(t, x_1)v(t, x_1) - \rho(t, x_2)v(t, x_2)]\, dt \quad (2.7)$$

If $\rho(t, x)$ and $v(t, x)$ are differentiable functions then we get

$$\rho(t_2, x) - \rho(t_1, x) = \int_{t_1}^{t_2} \frac{\partial}{\partial t}\rho(t, x)dt \quad (2.8)$$

and

$$\rho(t, x_2)v(t, x_2) - \rho(t, x_1)v(t, x_1) = \int_{x_1}^{x_2} \frac{\partial}{\partial x}(\rho(t, x)v(t, x))dx \quad (2.9)$$

Using equations (2.8) and (2.9) in (2.7) gives the following equation.

$$\int_{x_1}^{x_2}\int_{t_1}^{t_2}\left\{\frac{\partial}{\partial t}\rho(t,x)+\frac{\partial}{\partial x}[\rho(t,x)v(t,x)]\right\}dtdx=0 \tag{2.10}$$

Since this must be satisfied for all intervals of time and x then it must be true that the following *differential form of the conservation law* is satisfied.

$$\frac{\partial}{\partial t}\rho(t,x)+\frac{\partial}{\partial x}[\rho(t,x)v(t,x)]=0 \tag{2.11}$$

In terms of the mass flux, this equation can be written as

$$\frac{\partial}{\partial t}\rho(t,x)+\frac{\partial}{\partial x}q(t,x)=0 \tag{2.12}$$

And this equation can also be written as

$$\frac{\partial}{\partial t}\rho(t,x)+\triangledown(\rho(t,x)v(t,x))=0 \tag{2.13}$$

2.2 Pedestrian Conservation Law

In terms of pedestrians, they do not move one behind the other as the vehicles do. Pedestrians do not follow lanes and neither do they follow direction of movement. It can be argued that pedestrians tend to go out of way as deemed fit. Therefore, the same lane can be used for bidirectional traffic. This can be modeled as a net direction of movement on a given segment. Since a pedestrian flow is being discussed, in which only one type of pedestrian is involved, following qualities are taken into account:

- Density, q, of the flow, which is the expected number of individuals located within unit area of road segment at a given time, t, and location x, and
- Velocity, v, of the flow, which is the expected velocity of individuals at a given time, t, and location, x.

It is also assumed that small variations from the expected value can be there, and therefore are negligible, so q and v may be taken as their local mean values. Thus, conservation of pedestrians implies from equation (2.12). For further analysis, it is necessary to make following assumptions about the nature of pedestrian motion. Three major assumptions are made here:

1. The behavior characteristics and density of surrounding pedestrians determine the speed of pedestrians. Thus the velocity components for a single type of pedestrian are given by

$$v = f(\rho)\hat{\phi}_x \tag{2.14}$$

   where $f(\rho)$ is the speed and $\hat{\phi}_x$ is direction cosine of the motion. This is a standard assumption. For the crowds of interest here i.e. the density is not extreme, pedestrian speed is established by surrounding pedestrians in a similar way as to Greenshield's (1934) model of vehicular flow. As it will become clear later, this assumption is fundamental to use of the Lighthill and Whitham (1955) model, which has been verified in various studies. However, an uniformly accepted form of the function relating the density and speed cannot be established, because of multiple extrinsic factors and their varying effect which are individual dependent. Some examples can be the psychological state of pedestrians or the conditions of ground under foot.

2. The pedestrians in a group aim to reach a common destination without any preference to location. i.e. they have a common sense of task ϕ (called potential). There is no perceived advantage of moving along a line of constant potential. Therefore, the only direction for motion of pedestrians is perpendicular to that of the potential, i.e.

$$\hat{\phi_x} = \frac{-\frac{\partial \phi}{\partial x}}{|\frac{\partial \phi}{\partial x}|} \tag{2.15}$$

   This assumption is not applicable for vehicular traffic but when pedestrian flows are considered, they appears to be quite applicable. This is due to the reason that pedestrians ave the ability for visual assessment of the situation. There is also another implicit assumption in this that shorter pedestrians take a cue for their direction from the tallest pedestrians in the vicinity as they have a better overall view of the situation. It should also be noted that most crowds follow this assumption but not all types can do the same. However, even in situations when it is not applicable an acceptable approximation is provided.

3. Pedestrians avoid high density area at the same time optimizing travel time to be minimum. This can be assumed separable such that product of travel time and density is minimized. This leads to the situation that two pedestrians having

same potential should be end up at the same new potential at some later time. Thus, it can be noted that time is a measure of potential. Hence, the pedestrian speed has to be proportional to distance between potentials irrespective of the starting position of a pedestrian on same potential.

$$\frac{1}{|\frac{\partial \phi}{\partial x}|} = g(\rho)|f(\rho)| \tag{2.16}$$

where $g(\rho)$ is a factor to allow for the discomfort at very high densities.Generally, the factor $g(\rho)$ is equal to unity for most densities but rises for high densities.

The equations from above assumptions (2.14) - (2.16) together form the governing equations for pedestrian flow. This converges to:

$$-\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho g(\rho) f^2(\rho) \frac{\partial \phi}{\partial x}) = 0 \tag{2.17a}$$

$$g(\rho) f(\rho) = \frac{1}{|\frac{\partial \phi}{\partial x}|} \tag{2.17b}$$

The system (2.17) requires explicit boundary conditions for every particular situation. Sometimes, ρ is explicitly defined on the open boundaries that correspond to entrances. However, by specifying ρ and the speed, $f(\rho)$, the flow $\rho f(\rho)$, is specified automatically. Usually, the potential ϕ, is considered zero at exits and the normal derivative of ϕ is specified as zero on closed boundaries. For any slowly moving boundary, such as next to a slowly moving vehicle, the normal components of velocity of both the pedestrians and the boundary must be equal. For a rapidly moving boundary, safety issues are important and the boundary condition depends on psychological influences.

Together the set of equations in (2.17) is a generalization in single dimension of Hughes Model [13]. This model can also be written as:

$$-\frac{\partial \rho}{\partial t} + div(\rho f^2(\rho) \bigtriangledown \phi) = 0 \tag{2.18a}$$

$$|\bigtriangledown \phi| = \frac{1}{f(\rho)} \tag{2.18b}$$

2.3 Solution Properties

The flow of pedestrian traffic is dependent on density and flow velocity. This is given by:

$$q = \rho f(\rho) \tag{2.19}$$

Since $f(\rho)$ and $g(\rho)$ can be any function in model (2.17), there is a problem to identify the correct one for obtaining solutions and is a difficult choice. However, they are expected to have following properties:

- $f(0)$ is finite
- $f(\rho_{max}) = 0$

- $\frac{df(\rho)}{d\rho} \leq 0$
- $g(\rho) \geq 1$
- $\frac{dg(\rho)}{d\rho} \geq 0$

where ρ_{max} is the density at which pedestrians can't move anymore. For example, based on Greenshield's model, $f(\rho)$ and $g(\rho)$ can be defined as:

$$g(\rho) = 1 \tag{2.20}$$

$$f(\rho) = A - B\rho \tag{2.21}$$

where A and B are positive constants. The generates the flow as:

$$q = \rho(A - B\rho) \tag{2.22}$$

and the resulting characteristics are quadratic in nature and can be visualized as in the figure 2.2

Therefore the maximum flow q is:

$$q_{max} = \frac{A^2}{4B}$$

at,

$$\rho_{q_{max}} = \frac{\rho_m}{2} = \frac{A}{2B}$$

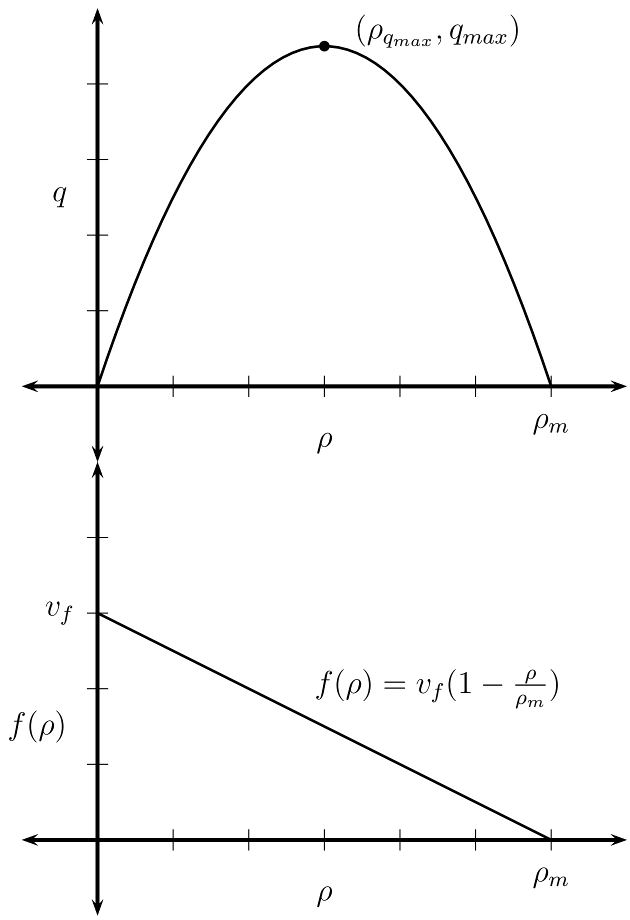

Figure 2.2: Greenshield's Model characteristics

There are two possible values for pedestrian density for any value of flow less than the maximum possible value, viz, supercritical ($\rho < \rho_{q_{max}}$) and sub-critical ($\rho > \rho_{q_{max}}$). Thus, for simplicity, we set

$$f(\rho) = v_f(1 - \frac{\rho}{\rho_{max}}),$$

$$\rho_{q_{max}} = \frac{\rho_m}{2},$$

$$q_{max} = \frac{v_f \rho_m}{4}.$$

CHAPTER 3

MATHEMATICAL ANALYSIS OF HUGHES MODEL

In chapter 2 the basics of traffic conservation law was touched upon by introduction of Hughes model. In this chapter the Hughes model is discussed more deeply and the solutions are approached.

3.1 Hughes Model

In the Hughes Model, the pedestrians are treated as continuum as the derivation stems from the continuity equation (2.13). However, the introduction of a potential function was to identify that all the pedestrians together had a common destination but the immediate destination may or may not be the same. This allowed for notion that the crowd is taking decisions to optimize its flow velocity. The Hughes Model is described as:

$$-\frac{\partial \rho}{\partial t} + div(\rho f^2(\rho) \bigtriangledown \phi) = 0 \tag{3.1a}$$

$$| \bigtriangledown \phi| = \frac{1}{f(\rho)} \tag{3.1b}$$

Here x denotes the position variable with $x \in \Omega$, a bounded domain in $\mathbb{R}^d$ with smooth boundary $\partial\Omega$, $t \geq 0$ is time and $\rho = \rho(x,t)$ is the crowd density. The function $f(\rho)$ is given by $f(\rho) = 1 - \rho$, modeling the existence of a maximal density of individuals which can be normalized to 1 by a simple scaling. The boundary and initial conditions are:

$$\phi(x,t) = 0, x \in \partial\Omega, t \geq 0 \tag{3.2}$$

and

$$\rho(x,0) = \rho_1(x) \geq 0 \tag{3.3}$$

In (3.1) if the system is decoupled by equating $f(\rho)$ to 1, it is converted into a non-linear conservation law with discontinuous flux. Such equations have been studied in [15] and [17]. The Hughes model share multiple features with such class of equations, however, it is much more challenging methodically. The implicit time dependence of the potential $\bigtriangledown\phi$ and the non-linearity of the equation are the cause behind it. Regularity corresponding to Lipschitz continuity only can be expected for the unique viscosity solution ϕ.

Since the density of pedestrians satisfy the continuity equation (2.13), the velocity vector can be written as

$$V(t,x) = |V(t,x)|Z(t,x) \tag{3.4a}$$

$$|Z(t,x)| = 1 \tag{3.4b}$$

The relationship between $|V|$ and ρ can be assumed to be linear (3.5). This is also a form of Greenshield's Model.

$$|V(t,x)| = 1 - \rho \tag{3.5}$$

The directional unit vector $Z(x,t)$, is assumed to be parallel to the gradient of the potential $\phi(x,t)$. Such potential is determined by solving the equation in (3.1). Here the potential ϕ signifies the common sense of the task (the task is represented by the boundary $\partial\Omega$). In other words, pedestrians try to minimize their travel time estimate, which can be modeled by following equation:

$$|\bigtriangledown \phi| = 1, \quad \phi|_{\partial\Omega} = 0$$

The above equation has the unique semi-concave solution $\phi(x) = dist(x, \partial\Omega)$ at least in a convex domain Ω. Hence, a reasonable assumption is that individuals attempt to manipulate their travel time by avoiding high densities, therefore we can safely assume:

$$|\bigtriangledown \phi| = \frac{1}{1-\rho}, \quad \phi|_{\partial\Omega} = 0$$

This leads to

$$Z(t,x) = \frac{\bigtriangledown \phi(t,x)}{|\bigtriangledown \phi(t,x)|} = (1-\rho) \bigtriangledown \phi(t,x)$$

which implies that equation (3.1) is continuous in nature.

Developing a mathematical theory for the model (3.1) has not been successful till now. The inherent non-linearity in the continuity equation requires to go for entropy solution in scalar conservation laws. In such cases, the weak L^{∞} solutions of equations are not unique in general. Moreover, the vector field $\bigtriangledown\phi$ can have time varying discontinuities in subsets of Ω.

It can be argued that the subsets of discontinuity depend on ρ both non-linearly and non-locally and can be validated in examples for single dimension problems. It can also be seen that $f(\rho) = 1-\rho$ generates problems when the crowd density approaches $\rho = 1$ as the magnitude of the potential function $| \bigtriangledown \phi|$ blows up. Therefore, the model is highly non-trivial, even in single dimension as, even if the model can be decoupled by integration.

In [8] an approximations to the Hughes model (3.1) has been proposed which regularizes the potential thereby avoiding the discontinuity in gradient of potential $| \bigtriangledown \phi|$. This has been achieved by approximating the potential equation by addition of a small viscosity:

$$\delta \triangle \phi + | \bigtriangledown \phi|^2 = \frac{1}{f^2(\rho)}, \quad \delta > 0 \tag{3.6}$$

This would still create a problem of blow up at over-crowdedness i.e. $\rho = 1$. However, it can be considered as shown below thereby elimination the unintentional blowup at the boundary.

$$\delta \triangle \phi + f^2(\rho)| \bigtriangledown \phi|^2 = 1, \quad \delta > 0 \tag{3.7}$$

Still this complicates the development of a satisfactory existence and uniqueness theory by using the coupling (3.7) due to density dependent coefficient for Hamilton-Jacobi term $| \bigtriangledown \phi|^2$. Therefore, a better method was proposed as:

$$\delta_1 \bigtriangleup \phi + | \bigtriangledown \phi|^2 = \frac{1}{(f(\rho) + \delta_2)^2}, \quad \delta_1, \delta_2 > 0 \tag{3.8}$$

the potential ϕ in (3.8) would satisfy:

$$| \bigtriangledown \phi| = \frac{1}{1 - \rho + \delta_2} \tag{3.9}$$

The velocity field polar decomposition introduced in (3.4) converts to:

$$V = |V|Z, \quad |Z| = 1$$

$$|V| = f(\rho)^2 | \bigtriangledown \phi| = \frac{f(\rho)^2}{\delta_2 + f(\rho)} = \frac{(1-\rho)^2}{\delta_2 + (1-\rho)}, \quad Z = \frac{\bigtriangledown \phi}{| \bigtriangledown \phi|} \tag{3.10}$$

$|V|$ has a logistic profile in (3.10), similar to that of original Hughes Model. However, it has a residual velocity at the half of maximum density. Also the gradient of unit vector Z is very high in value but not infinite. Therefore, the behavior of $|V|$ in the additional viscosity solution is close to that of original Hughes Model.

Combining (3.10) with Hughes Model results in following model:

$$\begin{cases} \rho_t - (\rho f^2(\rho)\phi_x)_x = 0, \\ -\delta_1 \phi_{xx} + |\phi|^2 = \frac{1}{(f(\rho)+\delta_2)^2} \end{cases} \tag{3.11}$$

3.2 Existence of Solutions

Let:

$$g(\rho) := \rho f^2(\rho)$$

with initial condition:

$$\rho(x,0) = \rho_1(x) \geq 0 \tag{3.12}$$

and Dirchlet Boundary condition:

$$\min_{k\in[0,tr(\rho)]} g(tr(\rho)) - g(k) = 0 \tag{3.13}$$

$$\phi(\pm 1, t) = 0 \tag{3.14}$$

Here $tr(\rho)$ denotes the trace of ρ on the boundary. Therefore,

$$\begin{cases} tr(\rho(-1,t)) = \lim\limits_{x\to -1^+} \rho(x,t) \\ tr(\rho(1,t)) = \lim\limits_{x\to 1^-} \rho(x,t) \end{cases} \tag{3.15}$$

A more detailed proof is covered in [2] which establishes that (3.13) and (3.14) are the correct ways for scalar conservation law. Thus boundary condition reduces to

$$g(tr(\rho)) \geq g(k) \text{ on } x = \pm 1, \quad \forall k \in [0, tr(\rho)] \tag{3.16}$$

and implies the fact that on the boundary the function g is non-decreasing for allowed densities. Introduce the space of bounded variation functions by

$$BV([-1,1]) = \{f \in L^1([-1,1]) | V_{-1}^1(f) < \infty\}$$

where,

$$V_{-1}^1(f) = \sup_{p\in\mathcal{P}} (\sum_{n=1}^{n_p} |f(x_{i+1}) - f(x_i)|)$$

and P is the set of all the partitions of $[-1,1]$. If $f'(x)$ is integrable, then

$$V_{-1}^1(f) = \int_{-1}^{1} |f'(x)| dx.$$

Definition 3.2.1. *Entropy Solution: let $\rho_1 \in BV([-1,1])$. A couple (ρ,ϕ) is a weak entropy solution to the system* (3.11) *if:*

- $\rho \in BV([-1,1]\times[0,T)) \cap L^\infty([-1,1]\times[0,T))$
- $\phi \in W^{2,\infty}[-1,1]$
- *ρ and ϕ must satisfy the following inequality:*

$$\iint_{\Omega_T} |\rho - k|\psi_t dxdt + \int_{-\infty}^{\infty} \rho_1\psi_0 dx - \iint_{\Omega_T} sgn(\rho-k)[g(\rho)-g(k)]\psi_x\phi_x dxdt$$
$$+ \iint_{\Omega_T} sgn(\rho-k) g(k)\psi\phi_{xx} dxdt - sgn(k)\int_0^T [g(tr(\rho)) - g(k)]\phi\psi|_{\pm 1} dt \geq 0 \tag{3.17}$$

for every Lipschitz continuous test function ψ in $[-1,1]\times[0,T)$ having compact *support.*

- *ρ and ϕ satisfy the second equation in* (3.11) *almost everywhere in x and t.*

In context of conservation law, equation (3.11) can be approximated via a vanishing viscosity approach into a system:

$$\rho_t - (\rho f^2(\rho)\phi_x)_x = \epsilon \rho_{xx} \tag{3.18a}$$

$$-\delta_1 \phi_{xx} + |\phi_x|^2 = \frac{1}{(f(\rho) + \delta_2)^2} \tag{3.18b}$$

for small $\epsilon > 0$. The system (3.18) is coupled with homogeneous boundary conditions:

$$\rho(x,t)|_{x=\pm 1} = 0,$$

$$\phi(x,t)|_{x=\pm 1} = 0$$

with initial conditions

$$\rho(x,0) = \rho_1(x)$$

Existence of unique (smooth) solutions to the above regularized problem follows from standard results [20] and [35].

Theorem 3.2.1. *(Existence of entropy solutions). There exists an entropy solution* (ρ, ϕ) *to system* (3.11) *with initial condition* (3.12) *and boundary conditions* (3.13)-(3.14) *in the sense of Definition 3.2.1. Such solution is the limit as $\epsilon \to 0$ of the solution ρ^ϵ to system* (3.18).

This theorem leads to the situation where we establish the uniqueness of entropy solution in sense of Definition 3.2.1.

Theorem 3.2.2. *(Uniqueness of entropy solutions). There exists at most one entropy solution* (ρ, ϕ) *to the system* (3.11) *with initial condition* (3.12) *and boundary conditions* (3.13)-(3.14) *in the sense of Definition 3.2.1.*

Using the above definition and theorems, which are proved in [8], we can say that the system described in (3.11) has a unique solution (ρ, ϕ) in a weak sense.

However, during this exercise we see two major things as follows:

- The actual Hughes Model has not been proven to have unique solution, although modifications like system (3.11) based on equation (3.1) have been proven as seen above.
- Due to the nature of actual Hughes Model, the existence and uniqueness is not easy to prove, and many other modifications to the model are possible as per the requirements within the purview of Hughes Model.

In the next chapter, another such proposed modification on (2.17) following the solution properties is discussed.

CHAPTER 4

MODIFIED PEDESTRIAN FLOW MODEL

In Chapter 2 we discussed the basics of traffic conservation law and touched upon the Hughes model. However in Chapter 3 the challenge to prove the existence and uniqueness of solutions to the Hughes model were discussed. Thereby, an established modification to the Hughes model was discussed and a theoretical exercise was completed as well as established its continuity. In this chapter we cover the modification to the Hughes model and how the solutions are approached.

4.1 Modified Hughes Model

In the Hughes Model, the pedestrians are treated as continuum as the derivation stems from the continuity equation. Hence, we can assume that the pedestrians are treated as a non-porous fluid. The introduction of a potential function was to identify the common direction for pedestrians. This common destination allowed them to move in a path that culminated at the set destination. Moreover, the model inherently assumed that the crowd moves in a particular direction and there is no cross traffic.

However, in a city environment there is no mandate and pedestrians can move around freely. In other words, each pedestrian has a different final destination, but choose path based on the Hughes Model hypotheses. Therefore it is very important that a traffic network model is defined for such a city to understand how pedestrians interact with each other.

4.1.1 Traffic Network Model

A traffic network model is defined as a grid comprising of set or defined paths. Each of the path can be straight line or a curve. However we assume that there are negligible curve paths and can be approximated as a collection of straight lines. This allows for multiple intersections to be present in the traffic network whenever two of such paths cross.

Therefore, a traffic network can be modeled as a system of linear equations:

$$Y = M \times X + C \quad \text{where}(x, y) \in \Omega \tag{4.1}$$

The solutions to the set of equation 4.1 represent the intersections of the roadway systems. This in turn divides each line into a set of line segments marked by intersections, and each segment contains pedestrians. Each point of intersection can be thought of as a traffic light and since the effects of curve are assumed negligible, it is valid to assume that both the line segments are collinear. The pedestrians exist on both the side of the considered traffic light. Note that at this point the focus has been shifted from a two-dimensional physical space to a single dimensional space which considers pedestrians on a straight line.

4.1.2 Modeling the movement

Since a single dimensional movement is being considered, it can be assumed that the potential is merely a sign of direction of movement for a particular pedestrian. However, since pedestrians can intermingle and not all of them will have the same direction of movement individually, we treat them as continuum and assign attributes to a group. Hence it can be assumed that a group of pedestrians have a common destination that involves crossing the next line segment and the decision is taken based on Hughes Model assumptions.

However, the model considers a generalized set of functions $f(\rho)$ and $g(\rho)$ which follow the solution properties 2.3, and establish the necessary bounds on (f, g). The functions chosen within these bounds will adhere to Hughes Model, hence will follow the same semantics, however, will depend on certain assumptions. Therefore we define following assumptions:

- There is a net direction of movement for a group of pedestrians, however, individual pedestrian may go in any direction and are negligible in number.
- A group of pedestrians can move in any direction, i.e. positive or negative.
- The dynamics are tracked from the moment when red light switches to green.

Based on the above assumptions, the following functions can be taken:

$$g(\rho) = \frac{1}{v(t,x)|\frac{d\phi}{dx}|} \tag{4.2}$$

$$f(\rho) = v(t,x) \tag{4.3}$$

where $v(t,x)$ is the velocity of pedestrians through the road segment and ϕ is a C_c^1 curve for determination of direction of movements. It is assumed C_c^1 under the assumptions stated above. The gradient of this curve determines the tendency to move in a particular direction. However, quantification of such a tendency is very hard. Therefore, by taking the sign of this tendency curve, we can judge the direction of movement for the pedestrian at a point at any given time.

4.1.3 Proposed Modified Model

Using the above described assumptions for the involved dynamics on the Hughes Model (2.17), along with equations (4.3) and (4.2) the following model is established

$$-\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho \ v \ \text{sgn}(\phi_x)) = 0 \tag{4.4}$$

Where sgn is the signum function to obtain sign of the slope of potential curve. This sign determines the direction of movement towards negative or positive infinity.

It is important to note that a signum function has been introduced, which is discontinuous at 0, hence there are necessary changes required around discontinuity. This changes the above equation into the following model:

$$\begin{cases} -\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho f(\rho)\text{sgn}(\phi_x)) = 0, \text{when} \frac{\partial \phi}{\partial x} \neq 0 \\ \frac{\partial \rho}{\partial t} = 0, \text{ when } \phi_x = 0 \end{cases} \tag{4.5}$$

which is required from the equation (4.2) and therefore requires the explicit definition in (4.5).

4.2 Solution of Proposed Modified Hughes Model

By observing Hamilton-Jacobi type equations (e.g. [6]), it is evident that (4.5) may feature more than one weak solution. In other words, ϕ_x can change its sign at infinite points within its domain. Since the model of pedestrian interactions is chosen as described in the section 4.1.1, the possible solution is chosen such that there is an extremal point at origin.

It is clear in the proposed model (4.5) that there may be a discontinuity introduced into the system. Therefore, the system requires to consider that in approaching for a solution. To solve this set of equations, a case wise approach is used. In this approach, various possible combinations of potential are considered such that $\frac{\partial \phi}{\partial x}$ is compared with 0. These combinations are applied with respect to the Modeling of movement as covered in the previous section.

Therefore, the problem statement can be re-stated as: on a stretch of a road segment, with traffic light at the center in stop state (red light), has pedestrians on both sides. However, pedestrians on each side of the red light are following a potential function of their own, and is common to their side.

This problem is solved from the time instant the red light switches to green, thereby signaling the traffic to move. At this point following combinations of potential for the two way traffic emerge (later referred as cases).

1. $\phi_x^\ell < 0$ and $\phi_x^r > 0$
2. $\phi_x^\ell < 0$ and $\phi_x^r = 0$
3. $\phi_x^\ell < 0$ and $\phi_x^r < 0$
4. $\phi_x^\ell = 0$ and $\phi_x^r > 0$
5. $\phi_x^\ell = 0$ and $\phi_x^r = 0$
6. $\phi_x^\ell = 0$ and $\phi_x^r < 0$
7. $\phi_x^\ell > 0$ and $\phi_x^r > 0$
8. $\phi_x^\ell > 0$ and $\phi_x^r = 0$
9. $\phi_x^\ell > 0$ and $\phi_x^r < 0$

where ϕ_x^ℓ, ϕ_x^r are the derivative with respect to x for $x < 0$ and $x > 0$ respectively, and $x = 0$ marks the position of the traffic light. This simplifies the movement of pedestrians by determining their tendency for a particular direction.

However, this also leaves another problem in the system. From classical analysis of density equations, $\rho_0^\ell \geq \rho_0^r$ because the primary assumptions were that the flow happens from left to right(unidirectional flow) and the boundaries are always allowing free flow. But in the above described scenarios, the flow can be bidirectional, however, the boundaries still behave the same.

For each of nine cases listed above, following two subcases occur and need to be investigated:

- Case A: $\rho_0^\ell \leq \rho_0^r$
- Case B: $\rho_0^\ell > \rho_0^r$

At this point, each of the initial conditions under Case A or B are eventually leads to

$$-\frac{\partial \rho}{\partial t} + k\frac{\partial(\rho v)}{\partial x} = 0 \tag{4.6}$$

where $k \in \{-1, 0, 1\}$. This converts the Hughes model (4.5) into a set of two typical Witham-type equations with opposite directions on both sides of origin in single dimension as shown in equation (2.11). The solutions to this set of equations can be computed by the method of characteristics, which is a standard procedure to solve Riemann-type problems, and is covered widely in literature like [11].

4.2.1 Entropy Solutions

From above it can be seen that the conservation law (4.5) splits into two separate conservation laws represented in (4.6). It is well known that the Riemann problem may feature, in general, more than one weak solution. Thus, one needs the concept of entropy solution, [19, 27]. It is known that Lax admissibility criterion for shocks [21] implies that admissible shocks are decreasing on $x < 0$ and increasing on $x > 0$. Thus, for most cases, a solution (weak in nature) is either a rarefaction or a shock wave as shown in figures 4.1, and 4.2. Note that the velocity $v(t, x)$ follows the Greenshield's model, i.e. the velocity and density of the flow follow a quadratic relationship expressed as $v = v_f(1 - \frac{\rho}{\rho_{max}})$ where v_f is free flow velocity of the traffic and ρ_{max} is the maximum possible density which occurs at traffic jam.

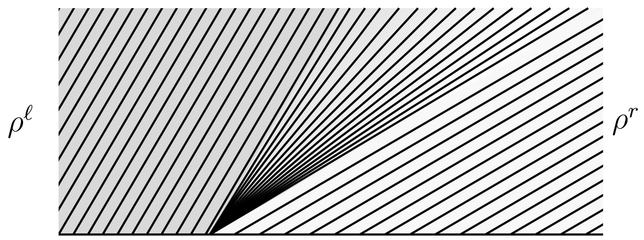

Figure 4.1: Rarefaction Characteristics Solution

The entropy solution for (4.5) is introduced as follows

Definition 4.2.1. *(Weak entropy solutions) Let* $0 < \delta < 1$ *and* $\rho_0 \in BV \cap L^\infty([-1, 1])$ *with* $0 < \rho_0 \leq \rho_{max}(1 - \delta)$. *A function*

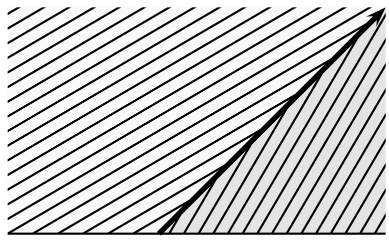

Figure 4.2: Shock wave Characteristics Solution

$\rho \in L^\infty([0,+\infty)\times[-1,1]) \cap BV_{loc}([0,+\infty)\times[-1,1])$ is a BV weak entropy solution to the problem (4.5) with initial datum $\rho_0(0,x) = \rho_0(x)$ if and only if $\rho(t,x) \in [0, 1-\delta]$ for all $x \in [-1,1]$ and $t \geq 0$, satisfies the following conditions:

1. *For all test functions $\varphi \in C_c^\infty([0,T)\times(-1,1))$ we have*

$$\int_0^T \int_{-1}^1 \rho(t,x)\varphi_t(t,x)dxdt + \int_{-1}^1 \rho_0(x)\varphi(0,x)dx - \int_0^T \int_{-1}^1 \rho(t,x)v(t,x)sgn(-x)\varphi_x(t,x)dxdt = 0 \quad (4.7)$$

2. *$Tr(\rho(t, x = \pm 1)) \in [0, \bar{\rho}] \quad \forall t > 0$.*

3. *For each convex function $e : [0, \rho_{max}(1-\delta)] \to \mathbb{R}$, there exists a Lipschitz function $p : [0, \rho_{max}(1-\delta)] \to \mathbb{R}$, such that:*

$$e(\rho)_t + kp(\rho)_x \leqslant 0 \text{ on } x \in [-1,1] \setminus \{0\} \quad (4.8)$$

where the above inequality converts into two inequalities, depending on $k \in$

$\{-1, 0, 1\}$, *and the two inequalities are satisfied in the sense of distributions.*

The BV condition is required here because the initial finiteness of the total variation is propagated along the solution. This issue is strictly related with the effectiveness of the wave front tracking strategy for this problem [7]. The condition $\rho \leq \rho_{max}(1 - \delta)$ $\forall t \geq 0$ is used to avoid the singularity in (4.2), due to the condition $f(\rho_{max}) = 0$ (i.e., null velocity at maximum density). Assume this condition on the initial data and expect that, by the maximum principle, it is satisfied at later time:

$$\|\rho(t)\|_{L^\infty} \leq \|\rho_0\|_{L^\infty} \quad \forall t \geq 0.$$

At the boundary points $x = \pm 1$, the behavior of the solution can be determined similar to that in [9], by solving two Riemann problems: at $x = -1$ with $\rho^\ell = 0$ and $\rho^r = Tr(\rho(x = -1))$, at $x = 1$ with $\rho^\ell = Tr(\rho(x = 1))$ and $\rho^r = 0$. Since g is concave, the two boundary layers at $x = \pm 1$ have to be solved by means of a rarefaction wave. The rarefaction fan generated at the boundary enters the domain $[-1, 1]$ when the trace of ρ (on both sides $x = \pm 1$) satisfied $Tr(\rho) \in [\bar{\rho}, 1]$ and leaves the domain otherwise.

However it should be noted that here each case and subcase differs in nature and therefore is required to be handled differently. Nature of solution to each of them is dependent on the left and right density of pedestrians as well as their tendency to move towards a particular direction. Based on this, the following subsections aim to capture the various possible scenarios that are derived, and visualizes the solutions.

4.2.2 Case A: $\rho^\ell \le \rho^r$

This is the case when $\rho^\ell \le \rho^r$ which implies that the absolute value of slope of characteristics on the left section is less than that of right hand side. The nine cases listed above (from 1 to 9) about various combinations of left and right potentials will generate solutions that displayed in the following figures. For the case where a shock wave occurs, and an appropriate equation is provided for the progress of shock wave ($\dot{s}$ is the shock wave speed).

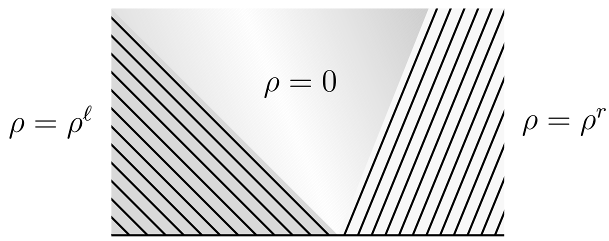

Figure 4.3: Case 1: Vacuum Solution

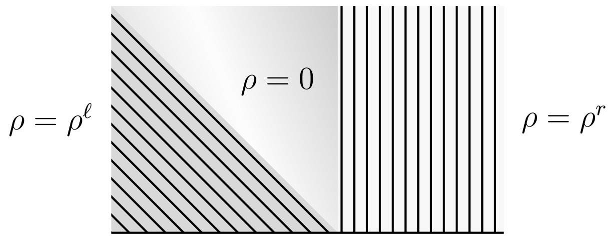

Figure 4.4: Case 2: Vacuum Solution

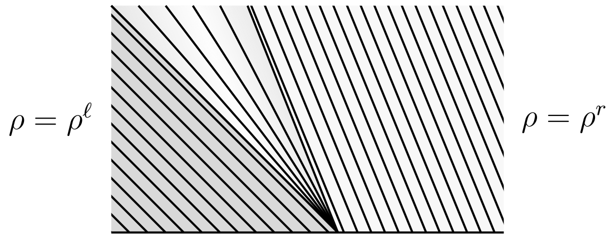

Figure 4.5: Case 3: Rarefaction Solution

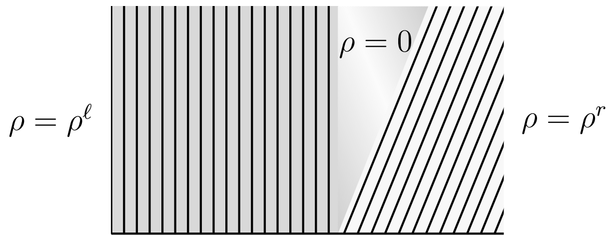

Figure 4.6: Case 4: Vacuum Solution

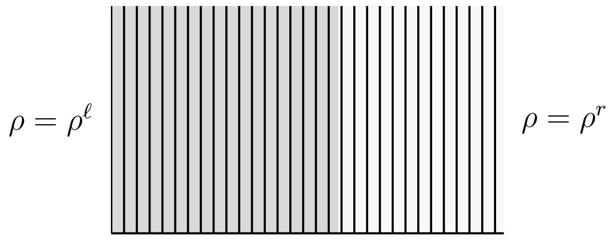

Figure 4.7: Case 5: Still Solution

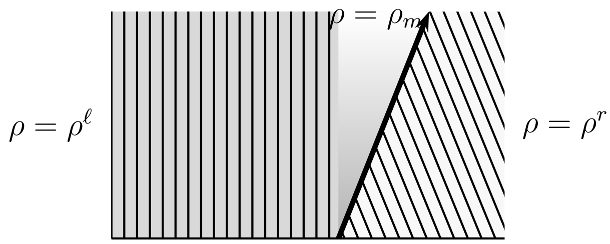

Figure 4.8: Case 6: Shock wave Solution

$$\dot{s} = \frac{q^r}{\rho_m - \rho^r}$$

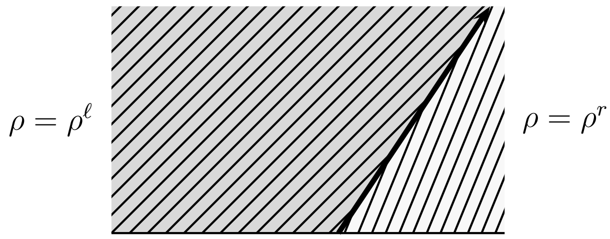

Figure 4.9: Case 7: Shock wave Solution

$$\dot{s} = -\frac{q^\ell - q^r}{\rho^\ell - \rho^r}$$

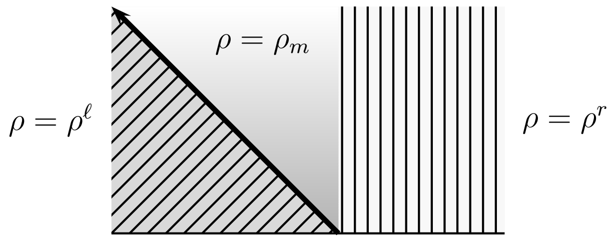

Figure 4.10: Case 8: Shock wave Solution

$$\dot{s} = \frac{q^\ell}{\rho^\ell - \rho_m}$$

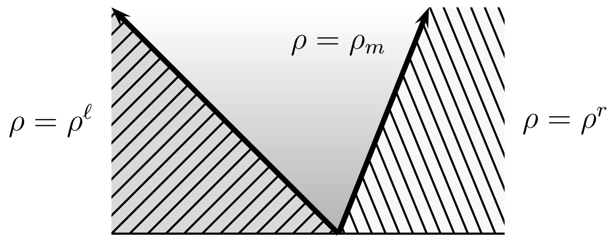

Figure 4.11: Case 9: Dual Shock wave Solution

$$\dot{s}^\ell = \frac{q^\ell}{\rho^\ell - \rho_m},$$

$$\dot{s}^r = -\frac{-q^r}{\rho_m - \rho^r}$$

4.2.3 Case B: $\rho^\ell > \rho^r$

This is the case when $\rho^\ell > \rho^r$ which implies that the absolute value of slope of characteristics on the left section is greater than that of right hand side. In this case, the above discussed cases (from 1 to 9) about various combinations of left and right potential will generate solutions that can be represented in the following set of figures. In certain cases, shock wave occurs, and an appropriate equation has been provided.

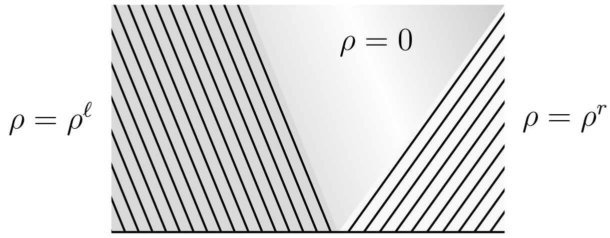

Figure 4.12: Case 1: Vacuum Solution

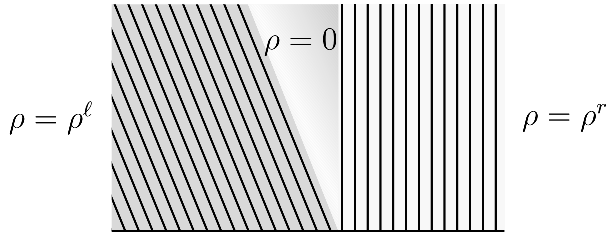

Figure 4.13: Case 2: Vacuum Solution

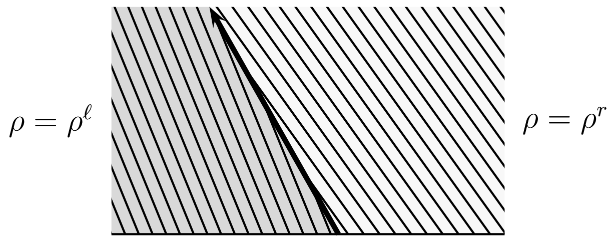

Figure 4.14: Case 3: Shock wave Solution

$$\dot{s} = \frac{q^\ell - q^r}{\rho^\ell - \rho^r}$$

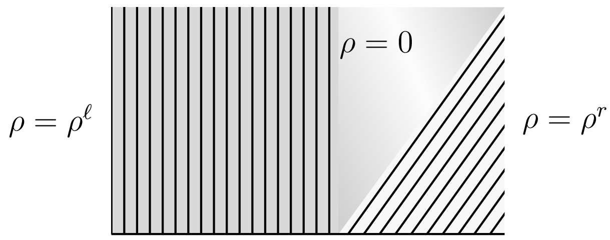

Figure 4.15: Case 4: Vacuum Solution

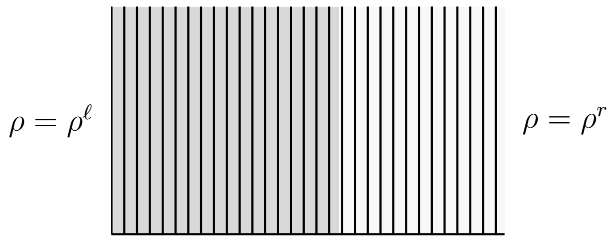

Figure 4.16: Case 5: Still Solution

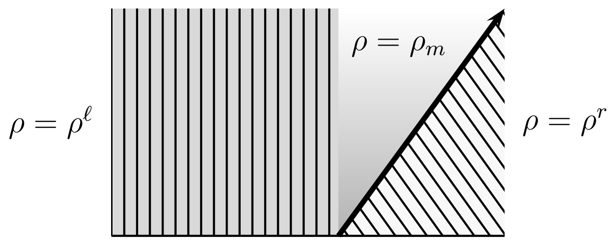

Figure 4.17: Case 6: Shock wave Solution

$$\dot{s} = \frac{q^r}{\rho_m - \rho^r}$$

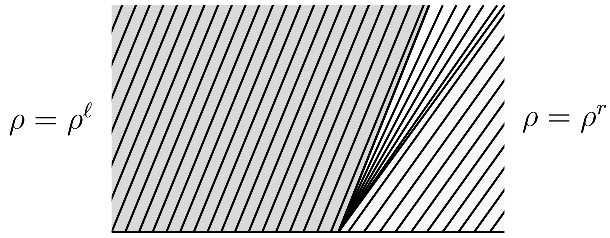

Figure 4.18: Case 7: Rarefaction Solution

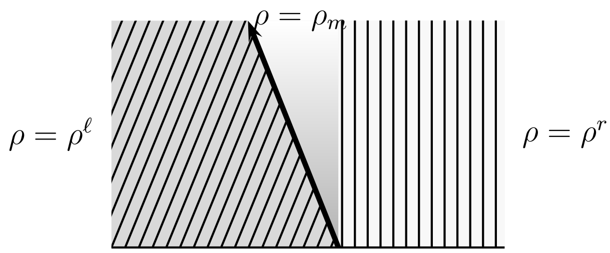

Figure 4.19: Case 8: Shock wave Solution

$$\dot{s} = \frac{q^\ell}{\rho^\ell - \rho_m}$$

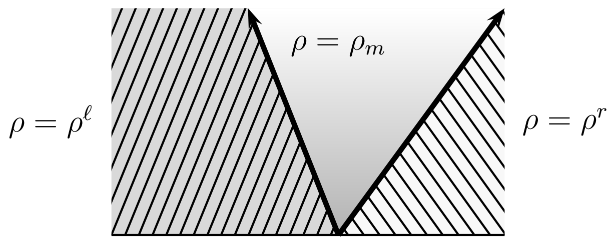

Figure 4.20: Case 9: Dual Shock wave Solution

$$\dot{s}^\ell = \frac{q^\ell}{\rho^\ell - \rho_m}, \ \ \dot{s}^r = -\frac{-q^r}{\rho_m - \rho^r}$$

4.3 Further discussion of solutions

From last section, the nature of one dimensional pedestrian flow across a red light is analyzed. Based on our investigation, we have following two observations:

- The nature of solutions is similar in most cases when compared from *Case A* to *Case B* barring a few cases of potential distribution (viz case 3 and 7). In these cases, the rarefaction and shock wave solutions were interchanged. In case of dual shock wave solution, the shock speed changed in both sides due to the change in relative densities.

- The case of dual shock wave solution is fairly interesting as it generates the situation of congestion at the junction that needs mathematical justification. This is relevant because of the consideration of a traffic network model. In case of a city, there might be common point interest for pedestrians, which implies all the pedestrians aim to reach a particular location and therefore, can create congestion situations.

CHAPTER 5

CONCLUSION AND FUTURE WORK

5.1 Conclusion and further discussions

In this thesis, the primary investigation was about pedestrian flows in single dimension as well as the pedestrian behavior in such situations. Various cases were covered when a single fixed turning point was considered, where a turning point is the point where the crowd differs in the tendency to move in a particular direction at time $t = 0$. In this case, the location of turning point (point at which the pedestrians change their direction) is known beforehand along with the potential distribution (tendency to move in any particular direction) along the domain.

However, a similar problem was addressed in literature [1], but aimed to identify the turning point as it was not fixed. The literature aimed to address mathematical difficulty in the discontinuous gradient of the solution to the eikonal equation (2.17b) appearing in the flux of the conservation law for the Hughes model (2.17).This was particularly for one dimensional interval with zero Dirichlet conditions (the two edges of the interval are interpreted as targets), the model can be decoupled in a way to consider two classical conservation laws on two sub-domains separated by a turning point.

In this thesis, the system of equation (2.17) was investigated for a fixed turning point and known tendency which yielded a few interesting observations. It was realized that a case of dual shocks is now possible as a weak solution for the system. It implies the situation where in a traffic network model, whenever a multitude of pedestrians aim to reach a specific point, a stagnant crowd will be generated, which confirms with general observation. In the finer details, even though a lot of cases look similar, they are having a subtle difference between them as they are different in nature due to the definition of problem itself. The problem changes considerably as soon as the left and right hand side densities have various relations. Another important observation in this investigation was that the pedestrians can emulate the situation of maximum density because of their tendency (potential distribution), even though the jam density has not been reached on a roadway segment.

5.2 Further work

It will be interesting to pursue mathematically the following four cases as a part of further work.

- In the situation when the tendency of pedestrians is to stand and not move, it creates a situation where jam density is emulated at the interface.
- A situation of dual shocks was observed in chapter 4. This situation presents an interesting case where the turning point emulates jam density.
- There can be more than one turning point on the same road segment under

consideration, and their location can be determined by the eikonal equation in [1]. Under the discussed framework, it will generate interesting situations for analysis.

- A brief introduction of the network traffic model was given in the chapter 4, however, a complete analysis in that direction is also of mathematical importance as it will generate a more opportunities in analyzing the evolution of density profile on a network under various events. Moreover such analysis can be helpful in simulation field for systems as discussed in literature [28].

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