12-2010

Poisson Process Monitoring, Test and Comparison

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POISSON PROCESS MONITORING, TEST AND COMPARISON

by

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Bachelor of Science
University of Science and Technology of China
2004

A thesis submitted in partial fulfillment
of the requirements for the

Master of Science in Mathematical Sciences
Department of Mathematical Sciences
College of Sciences

Graduate College
University of Nevada, Las Vegas
December 2010

THE GRADUATE COLLEGE

We recommend the thesis prepared under our supervision by

Qing Chen

entitled

Poisson Process Monitoring, Test and Comparison

be accepted in partial fulfillment of the requirements for the degree of

Master of Science in Mathematical Sciences

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December 2010

ABSTRACT

Poisson Process Monitoring, Test and Comparison

By

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The task of determining whether a sudden change occurred in the generative parameters of a time series generates application in many areas. In this thesis, we aim at monitoring the change-point of a Poisson process by method, which is characterized by a forward-backward testing algorithm and several overall error control mechanisms. With the application of this proposed method, we declare that Mount Etna is not a simple Poissonian volcano, because two different regimes divided by the change point, January 30th 1974, are identified. The validation procedures, used in a complementary fashion, by the formal hypothesis tests and graphical method will be discussed. In conclusion, the proposed method is easy to implement, and its assessment studies could be conducted based on large scale simulation.

ACKNOWLEDGEMENTS

I would like to say thank you to everyone who helped me throughout this journey. Especially, I would like to show appreciation to my advisor, Dr. Ho, for everything. His encouragement, guidance, and support kept me on the right track. His excellence in both research and teaching will always be a great example for me.

In addition, I am very thankful to these committee members, Dr. Westveld, Dr. Amei and Dr. Bai, for their positive inputs and mentoring during my graduate studies.

Last but not least, I would also like to thank my family for their love and support.

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CHAPTER 1

INTRODUCTION

Poisson processes are one of the most important classes of stochastic processes, with many properties, finding applications in diverse areas. Methods of determining whether a sudden change occurred in the generative parameters of a Poisson process is an active research field.

The Z test of a Poisson process is defined by Bain et al. (1985), and is modified for a failure truncated process by Bain et al. (1991). Ho (1993) develops the backward Z test and shows the asymmetric performance of the forward and backward Z tests in detecting an alternative which is increasing or decreasing step-function intensity. In addition, he suggests that both tests could be performed on the same data to monitor the process and identify instability and unusual circumstances.

Mulargia et al. (1987) address the importance of quantitative identification of different regimes of a volcano via the change points. They apply a sequential testing procedure based on the two sample Kolmogorov-Smirnov (K-S) statistic. Ho (1992) demonstrates a procedure for regime identification of a Poisson process based on a simple control chart. He also recommends adjusting the significance levels because multiple tests are performed.

In this thesis, we monitor a single Poisson process by implementing the forward and backward Z tests based on an algorithm to detect the change points. In the detecting procedure, different error control methods are available when multiple tests are compared. The change points identified will be verified via several tests such as the F test, the Conditional test by Przyborowski and Wilenski (1940), and the empirical recurrence rate plot (Ho, 2008).

The fundamental tools and related theory are introduced in Chapter 2. Chapter 3 describes the methods to detect the change points and the methods of validation. The proposed method is implemented to identify different regimes of Mount Etna in Chapter 4. We then conclude our studies in Chapter 5.

CHAPTER 2

FUNDAMENTAL TOOLS

A point process is a stochastic model that describes the occurrences of events. These occurrences are thought of as points on the time axis. Let N(t) be the random variable that denotes the number of events in the interval (0, t]. The intensity function of the process is defined as $\lambda(t) = \lim_{\Delta t \to 0} \frac{P(N(t, t+\Delta t] = 1)}{\Delta t}$. A counting process N(t) is called a Poisson process, if and only if it satisfies the three conditions: (1) N(0) = 0; (2) The random variables N(a, b] and N(c, d] are independent, for any a < b ≤ c < d; And (3) for any a < b, N(a, b] has the Poisson distribution with mean $\int_a^b \lambda(x)dx$.

A family of Poisson Processes with the intensity $\lambda(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1}$, for $\beta > 0, \theta > 0$ are called the power law processes. They provide models for many repairable systems which study the occurrence rate of failures. A repairable system is said to be deteriorating (or improving) if the intensity of the process is increasing (or decreasing). If $\beta = 1$, it is a homogeneous Poisson process (HPP). Otherwise it is a non-homogeneous Poisson process (NHPP). If $\beta > 1$, the derivative of the intensity function $\frac{\partial \lambda}{\partial t}$ is always positive, so the intensity function $\lambda(t)$ keeps increasing. If $\beta < 1$, the intensity function $\lambda(t)$ is decreasing.

2.1 Forward and Backward Tests

It may be reasonable to assume that the intensity of a Poisson process, $\lambda(\cdot)$, is constant, so tests of $H_0 : \lambda(\cdot)$ is constant versus $H_a : \lambda(\cdot)$ is not constant are of interest.

The results of such tests indicate whether the simple HPP may be adequate or whether a more general NHPP is required in modeling the occurrences of stochastic phenomena.

2.1.1 The Z Test

Bain et al. (1985) define the Z test of a Poisson process. It is modified for the failure truncated case by Bain and Engelhardt (1991). The Z test is discussed for the smooth alternatives and step functions with one or three (regular or irregular) jumps by Bain et al. (1985) and Engelhardt et al. (1990). It is the UMPU test for the power law process with the intensity $\lambda(t) = (\beta/\theta)(t/\theta)^{\beta-1}$, for β and θ > 0.

A power law process is referred to as "failure truncated" if it is truncated at the nth event. Likewise, a time truncated power law process is truncated at a prescribed time point t, where n events are observed before time t. The maximum likelihood estimator (MLE) of the parameter β is $\hat{\beta} = n \Big/ \sum_{1}^{n_w} \log \frac{t_w}{t_i}$, where $n_w = n-1$ and $t_w = t_n$ for the failure truncated process; and $n_w = n$ and $t_w = t$ for the time truncated process. Note that $\frac{t_{(1)}}{t_w} < \frac{t_{(2)}}{t_w} < \Lambda < \frac{t_{(n_w)}}{t_w}$ are ordered statistics from a uniform distribution on (0, 1). It is easy to show that $\sum_{1}^{n_w} \log \frac{t_w}{t_i}$ is distributed as a Gamma with parameters n_w and 1, and $\frac{2n}{\hat{\beta}} = 2\sum_{1}^{n_w} \log \frac{t_w}{t_i}$ follows a chi-square distribution with $2n_w$ degrees of freedom. Therefore, for

$$H_0: \text{ The process is an HPP}$$

$$H_a: \text{ The process is not an HPP,}$$

the forward Z test statistic is $Z = \frac{2n}{\hat{\beta}} = 2\sum_{1}^{n_w} \log \frac{t_w}{t_i}$, we reject the null hypothesis, if

$Z \le \chi^2_{1-\alpha/2}(2n_w)$ or $Z \ge \chi^2_{\alpha/2}(2n_w)$.

2.1.2 The Backward Z Test

It is known that the Z test has asymmetric performance in detecting an alternative which is increasing or decreasing step-function intensity in a Poisson process. Ho (1993) proposed the backward Z test and shows that it is more powerful than the forward test in detecting an increasing step-intensity alternative for the failure truncated process or a decreasing step-intensity alternative for the time truncated process. Both of the tests are robust against an abrupt change in the process.

Suppose $t_{(1)} < t_{(2)} < \Lambda < t_{(n_w)}$ are the cumulative events times on $(0, t_w)$ of the Poisson process, the new cumulative event times $t_w - t_{(n_w)} < t_w - t_{(n_w - 1)} < \Lambda < t_w - t_{(1)}$ based on the reversed order of the original inter-event time are obtained. Applying the Z test on the backward cumulative event times, we have the backward Z test statistics $Z_B = -2\sum_{1}^{n_w} \log(1 - \frac{t_i}{t_w})$. For the following hypothesis test,

$H_0:$ The process is an HPP

$H_a:$ The process is not an HPP

the backward Z test statistics is $Z_B = -2\sum_{1}^{n_w} \log(1 - \frac{t_i}{t_w})$, we reject the null hypothesis, if

$Z \le \chi^2_{1-\alpha/2}(2n_w)$ or $Z \ge \chi^2_{\alpha/2}(2n_w)$.

2.2 Other Hypothesis Tests

Besides the forward and backward Z tests, there are many well-known hypothesis tests for Poisson processes. Both of the Z tests introduced above are for testing if a single process is an HPP. The F test and the Conditional test to be described below provide methods to determine whether the parameters of two HPPs are identical.

2.2.1 The F Test

It may be the case that the HPP is a reasonable model for more than one system, but the systems are not identical in the sense that their means may be different. The F test (Rigdon and Basu, 2000) used to be test whether the occurrence rates of two failure-truncated HPPs are identical.

If two HPPs are truncated at n_1^{th} and n_2^{th} events, respectively, the MLE of the intensities are $\hat{\lambda}_i = n_i / T_{i,n_i}$, for i =1, 2. The statistic $2T_{i,n_i}\hat{\lambda}_i$ is distributed as a chi-square distribution with $2n_i$ degrees of freedom, for i = 1, 2, and they are independent. So the ratio $\lambda_1\hat{\lambda}_2 / \lambda_2\hat{\lambda}_1$ has an F distribution with $2n_1$ numerator and $2n_2$ denominator degrees of freedom. When the null hypothesis is true, then F = $\hat{\lambda}_2 / \hat{\lambda}_1$ is distributed as F($2n_1$, $2n_2$). Thus, for

$$H_0 : \lambda_1 = \lambda_2$$

$$H_a : \lambda_1 \neq \lambda_2$$

the test statistic is F = $\hat{\lambda}_2 / \hat{\lambda}_1$; we reject the null hypothesis, if $F \le F_{1-\alpha/2}(2n_1, 2n_2)$ or $F \ge F_{\alpha/2}(2n_1, 2n_2)$.

2.2.2 The Conditional Test

The Conditional test of comparing two Poisson means is provided by Przyborowski and Wilenski (1940). Suppose independent random variables $X_1, \cdots, X_{n_1}$ are distributed as Poisson with mean α_1, and another group of independent random variables $Y_1, \cdots, Y_{n_2}$ are distributed as Poisson with mean α_2, and the two groups of random variables are also independent. Then $X = \sum_1^{n_1} X_i$ is distributed as Poisson with mean $n_1\alpha_1$, $Y = \sum_1^{n_2} Y_i$ is distributed as Poisson with mean $n_2\alpha_2$, and X and Y are independent. So, condition on X + Y = k, X has a Binomial distribution with success probability $\frac{n_1\alpha_1/n_2\alpha_2}{1+n_1\alpha_1/n_2\alpha_2}$. Hence, for

$$H_0 : \alpha_1 = \alpha_2$$

$$H_a : \alpha_1 \neq \alpha_2$$

if k_1 is the observed value of X, then the p-value is $2 * \min\{P(X \le k_1), 1 - P(X \le k_1)\}$. We reject the null hypothesis, if the p-value is less than the set threshold. Or, we reject the null hypothesis if $k_1 \le c_1$ or $k_1 \ge c_2$, where

$$P(k_1 \le c_1 \text{ or } k_1 \ge c_2 \mid H_0) = \sum_0^{c_1} \binom{k}{k_1} 0.5^k + \sum_{c_2}^{k} \binom{k}{k_1} 0.5^k \le \alpha.$$

The number of events on any unit time interval of a Poisson process has a Poisson distribution, so the Conditional hypothesis test described above can be used to test if two HPPs have the same rate. Note that, testing $H_0 : \lambda_1 = \lambda_2$ is equivalent to testing $H_0 : \alpha_1 = \alpha_2$.

2.3 Controlling the Error Rate

The p-value of a hypothesis test is the probability of obtaining a test statistic at least as extreme as one that was actually observed, assuming that the null hypothesis is true. Usually we reject the null hypothesis if the p-value is less than 0.05 or 0.01, corresponding to a 5% or 1% threshold of type I error. And in hypothesis testing, it is possible to have two kinds of errors. Rejecting a true null hypothesis is called the type I error or the false positive. The type II error is failing to reject a null hypothesis that is not true. Moreover, the power of a test is the probability of not making a type II error.

When we consider simultaneously testing a family of hypotheses, a problem occurs resulting from the increase of the type I error. When we set a p-value threshold of, for example, 0.05, we are saying that there is a 5% chance that the result is a false positive. While 5% is acceptable for one individual test, if we do lots of tests on the data, then this 5% can result in a large number of false positives. Techniques have been developed to solve this problem.

2.3.1 Controlling the Family-Wise Error Rate

The control of the family-wise error rate (FWER) is to maintain the chance of making even a single type I error for the family of hypothesis tests at the desired α level, by performing individual tests at error rates that are a fraction of the overall α. For example the classical Bonferroni procedure (CB, Holm 1979) is to conduct each individual test at a significance level of $\frac{\alpha}{m}$, given that the significance level for the whole family of m tests is α. Let $P(A_i)$ be the probability of no type I errors occurring and α_0 be the significance level for each of the m tests. So $P(\bigcap_{i=1}^{m} A_i)$ is the probability of no type I errors for the family of tests. The Bonferroni inequality $P(\bigcap_{i=1}^{m} A_i) \geq \sum_{i=1}^{m} P(A_i) - (m-1)$ implies 1 – P(FWER) $\geq m(1-\alpha_0) - (m-1)$. In order to satisfy the family wise error rate of the multiple testing to be no longer than α, it is easy to show that $\alpha_0 \leq \frac{\alpha}{m}$.

An improvement called the sequential Bonferroni procedure (SB, Holm 1979) performs tests based on the ranked p-values while maintaining the FWER at the desired level. Rank the these m p-values in the increasing order. For a given overall significance level, find the largest r such that $p_{(r)} \leq \frac{\alpha}{m+1-r}$ and reject all $H_{(i)}$ for i = 1, 2, Λ , r. The idea of the sequential Bonferroni is to order the p-values and compare the smallest p-value to $\frac{\alpha}{m}$. If it is smaller than $\frac{\alpha}{m}$, we reject it and start to test the remaining $(m-1)$ hypotheses by using the same rule. Continue doing this until the hypothesis cannot be rejected. At that point, stop and accept the rest hypotheses.

Sometimes when the number of tests is very big, or the test statistics are highly dependent, the power for an individual test may become unacceptably low following the FWER controlling method. That is a consequence of minimizing the chance of making even a single type I error. Let us think about the extreme case, that the tests are perfectly dependent. Assume that the testing contains 1000 identical individual tests, and the prescribed significance level is 0.05, then the classical Bonferroni method would require p-values to be smaller the 0.05/1000.

2.3.2 Controlling the False Discovery Rate

The classical and sequential Bonferroni procedures focus on controlling the FWER, resulting in more type II errors and a reduction in the power. An alternative way of controlling FWER is to control the false discovery rate (FDR) which is proposed by Benjamini and Hochberg (1995). The FDR is defined as the expectation of the proportion of the type I errors among all the significant hypotheses. The error rate controlling procedures are shown in Table 1 and Figure 1.

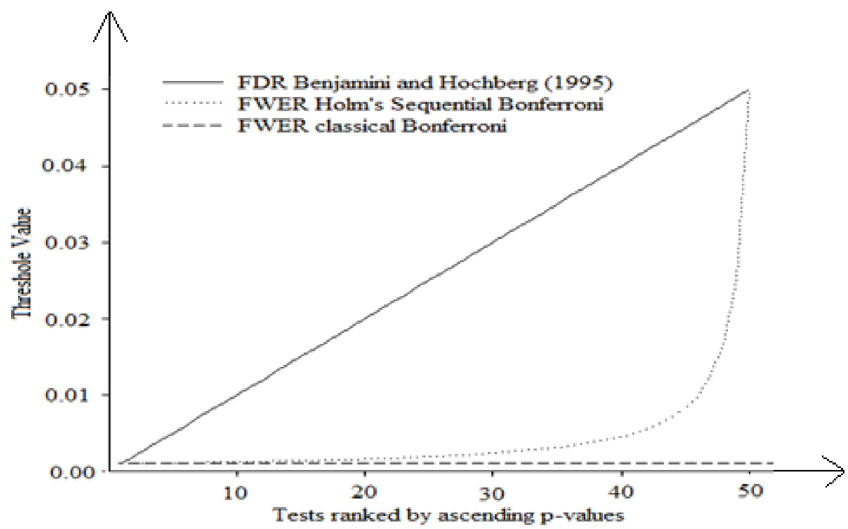

Fig 1. Individual test threshold (Verhoeven et al., 2005) performing 50 tests and FWER = FDR = 0.05

In practice, the Benjamini and Hochberg procedure (BH) is easy to apply. Let $H_1, H_2, \ldots, H_m$ be the null hypotheses and $p_1, p_2, \ldots, p_m$ their corresponding p-values. Order these values in increasing order and denote them by $p_{(1)}, p_{(2)}, \ldots, p_{(m)}$. For a given α, find the largest *k* such that $P_{(k)} \le \frac{k}{m \cdot c(m)}\alpha$ .Then reject (i.e. declare positive) all $H_{(i)}$ for i = 1, 2, $\ldots$, k. When the m tests are independent or positively correlated then $c(m) = 1$

; if they are negatively correlated, then $c(m)=\sum_{1}^{m}\frac{1}{i}$. The FDR control is to strike a more balanced compromise between type I error and type II error.

Table 1. Error correction procedures when m tests are performed

Tests ranked by p-value	Bonferroni	Sequential Bonferroni	BH
1	α/m	α/m	α/m
2	α/m	$a/(m-1)$	$2\alpha/m$
3	α/m	$a/(m-2)$	$3\alpha/m$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
i	α/m	$a/(m-i+1)$	$i\alpha/m$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
m	α/m	a	a

CHAPTER 3

METHOD

Change-point detection is a problem for discovering time points where the property of a time series, based on data, has changed. The analysis of change points attracts active research due to its wide application in lots of fields. For example, people may want to detect when there was a shift in the key parameter that measures the quality of products in industrial control. Given time series data modeled by an NHPP, we are interested in where the change points are, if they exist. If there are no such change points, then an HPP is adequate for modeling the occurrence of the events. Hypothesis testing could be a good statistical tool for detecting change points. In this thesis, we apply the forward and backward Z tests to monitor a single Poisson process and identify its change points. Type I errors controlling procedures are needed for the proposed technique because multiple tests are performed. We choose to demonstrate our method using the FDR procedure. Results using the classical and sequential Bonferroni procedures will also be compared.

3.1 Forward-Backward Testing Algorithm

Before applying the tests, we need to choose the method of adjusting the type I error. Two versions, simple and detailed, of the forward-backward testing algorithm are present below.

3.1.1 A Simple Version

1. Determine the maximum number of tests for the chosen error rate control mechanism;

2. Rank all the p-values generated by the sequential Z tests and record the earliest

test, which is significant and is a candidate for a true change point;

3. Repeat Step 2 using the Z_B test;

4. Claim the first change point identified by the Z test if the trend is decreasing, because the forward test is more powerful than the backward test in detecting an NHPP with decreasing step-function intensities in the failure-truncated framework (Ho, 1993). Otherwise, declare the change point produced by the Z_B test as the real change point;

5. Start the search of next change point by repeating Step 1- 4 with a new time origin, which is the end of the previous regime;

6. Continue doing Step 5 until no more change points can be detected.

3.1.2 A Detailed Version

(1) We need at least two inter-event time intervals to perform either test. Thus, we need to conduct $(n-1)$ tests, if there are n events in the Poisson process. The first one is to test whether the process from the initial time to the second event is an HPP.

(2) Suppose $p_{(1)}, p_{(2)}, \Lambda, p_{(n-1)}$ are the ascending ranked p-values by the forward Z test and the prescribed significance level is α. Find the largest j such that $p_{(j)} \le \frac{j}{n-1}\alpha$. Then all $H_{(i)}$ are rejected by the forward Z tests, for i = 1, 2, Λ , j.

(3) Repeat the steps above using the backward Z test. Suppose that we find $H_{(i)}$ are rejected by the backward Z tests, for i = 1, 2, Λ , u.

(4) Assume the time of the $n_1 th$ event is the earliest truncated time among the significant hypotheses by the forward tests, and $n_2 th$ event is the earliest truncated time in the significant hypotheses by the backward test. We could get the idea of the trend of the

intensity via the MLEs of the parameter β (see Section 2.1). If MLEs of β are greater than 1, we conclude that the time of the $(n_2 - 1)th$ event is the change point. Otherwise we take the time point of the $(n_1 - 1)th$ event to be the change point, because the backward Z test is more powerful than the traditional forward test if the process is increasing (Ho, 1993).

(5) Other possible change points are obtained by implementing the same procedure to the rest of the process which starts from the time point of the first change point.

(6) Continue doing Step 5 until no more change points can be detected. All the different regimes of the Poisson process are divided by the change points.

3.2 Validation - Used in a Complementary Fashion

The change points identified using the forward-backward testing procedure and the type I errors controlling methods can be further validated via additional tests and graphing techniques such as the empirical recurrence rate plot (ERR-plot), to be described below.

3.2.1 Validation via Empirical Recurrence Rate Plot

For a Poisson process truncated at t, we assume that t_1, t_2, Λ, t_n are the times of the n ordered events during the period (t_0, t). Suppose t_0 is the initial time, and $t_0 + Qh$ is equal to t, the empirical recurrence rates are defined as (Ho, 2008)

$$z_l = \frac{\text{Total number of events in } (t_0, t_0 + lh)}{lh}, \text{ where } l = 1, 2, \Lambda, Q.$$

The ERR-plot is a good technique to study a Poisson process. An example of the ERR-plot is shown in Figure 2. The ERR-plot of an HPP should lie roughly along a horizontal line, indicating a constant intensity. If the ERR-plot is increasing, the intensity of the process is increasing and a power law process with the parameter β greater than one may be a good model for this process. And a power law process with a parameter β smaller than one provides a reasonable model for the process whose ERR-plot is decreasing. Additionally, based on the ERR-plot of the regime identified, we could learn whether a change point is due to an abrupt change or a gradual change. For instance, if the ERR-plots of the two regimes before and after the change point are roughly horizontal, then there is an abrupt change in the process and the process may have a step-intensity.

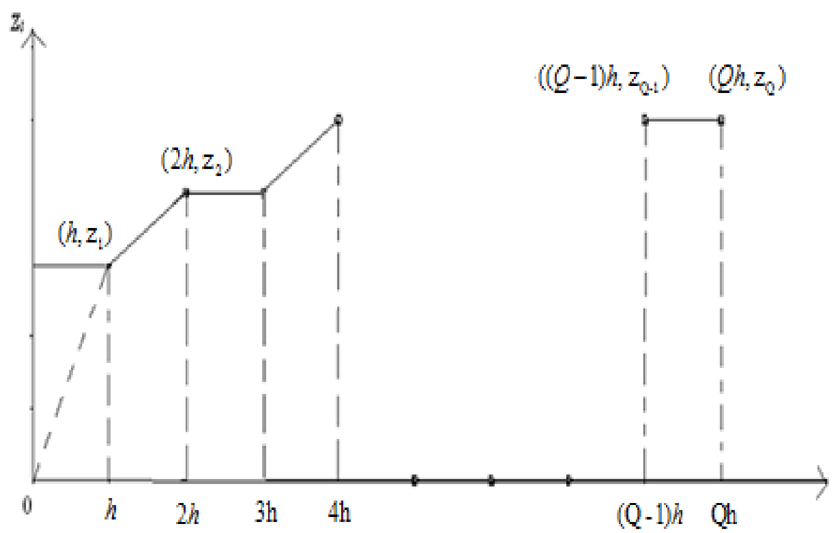

Fig 2. Example of an ERR-plot

CHAPTER 4

APPLICATION: REGIME IDENTIFICATION FOR A VOLCANIC TIME SERIES

Mulargia et al. (1987) apply a sequential testing procedure to detect the change points of Mount Etna based on the two-sample Kolmogorov-Smirnov (K-S) test, which is a general-purpose test that discriminates between two data sets as belonging to two different regimes. Ho (1992) constructs a table of control limits, and demonstrates a procedure of regime identification based on a simple control chart, that shows a point outside the control limits almost as soon as the process enters a new regime. He uses the idea of statistical process control to distinguish between the variation inherent in the observed repose times and the extraordinary variation that signals a real change in the regimes.

Following the forward-backward testing algorithm described in Chapter 3, an updated data set of Mount Etna (Smethurst et al., 2009) is used to demonstrate the change point(s) search process, including validation and comparison using additional graphical and quantitative methods.

4.1 Empirical Example: The Case of Mount Etna

The importance of quantitative and objective identification of different regimes of a volcano is addressed by Mulargia et al. (1987). Mount Etna is one of the famous volcanoes in the world, attracting many scientists to study its activities. There are many available and reliable records of its history of eruptions from mid-seventeenth century. The latest data set, provided by Smethurst et al. (2009), records the eruptive activities of

Mount Etna, which contains dates, volume, and other information of the eruptions from 1669 to 2008.

Table 2. Eruptive activity on Mount Etna

Date	Time between events	Date	Time between events	Date	Time between events
1669-03-11	4922	1874-08-29	1731	1975-02-24	278
1682-09-01	2386	1879-05-26	1396	1975-11-29	882
1689-03-14	4741	1883-03-22	1154	1978-04-29	117
1702-03-08	19359	1886-05-19	2243	1978-08-24	86
1755-03-09	2891	1892-07-09	5772	1978-11-18	258
1763-02-06	132	1908-04-29	693	1979-08-03	592
1763-06-18	197	1910-03-23	536	1981-03-17	741
1764-01-01	847	1911-09-10	2638	1983-03-28	715
1766-04-27	5135	1918-11-30	1660	1985-03-12	288
1780-05-18	2600	1923-06-17	1965	1985-12-25	3
1787-07-01	1791	1928-11-02	4988	1985-12-28	306
1792-05-26	3824	1942-06-30	1700	1986-10-30	1060
1802-11-15	2324	1947-02-24	1012	1989-09-24	3
1809-03-27	944	1949-12-02	358	1989-09-27	808
1811-10-27	2769	1950-11-25	1923	1991-12-14	3503
1819-05-27	4906	1956-03-01	2893	2001-07-17	467
1832-10-31	0	1964-02-01	1436	2002-10-27	0
1832-10-31	4034	1968-01-07	154	2002-10-27	681
1843-11-17	3199	1968-06-09	1030	2004-09-07	675
1852-08-20	4546	1971-04-05	1031	2006-07-14	668
1865-01-30	1700	1974-01-30	40	2008-05-12	
1869-09-26	1798	1974-03-11	350		

We assume that the time of the first eruption is the initial time (see Table 2), so that the second event in the original data set becomes the first event of the process. In the original data set, there are two eruptions recorded on each of the following dates: October 31st of 1832 and October 27th of 2002. For the following analysis, we treat both pairs of eruptions occurred on the same date as a single event. Therefore, using March 11, 1669 as the time origin, we generate 62 inter-event time intervals (in days, Table 2).

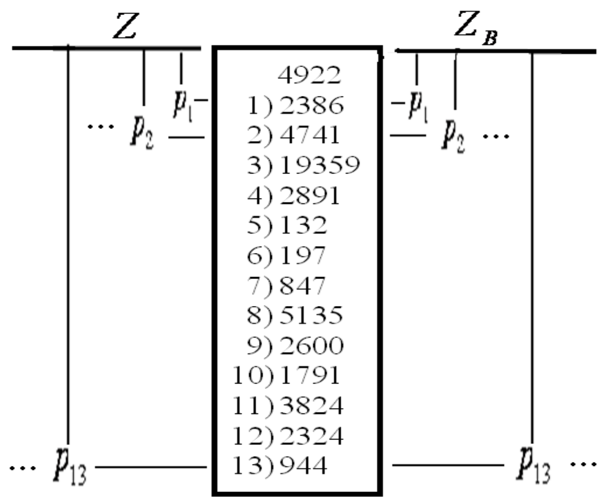

Fig 3. Tests in searching the first change point

4.2 Implementation and Data Analysis

A step by step forward-backward testing procedure, closely resembles the algorithm in Chapter 3, is applied to Mount Etna.

4.2.1 Step One

First, we decide to adopt the Benjamini and Hochberg procedure (BH) (Benjamini and Hochberg, 1995) to control the false discovery rate (FDR). Because both the forward and backward Z tests require at least two inter-event times, our initial search requires a total of 61 sequential tests, respectively, for each test. Therefore, consistent with the notations developed in Chapter 2, for a given α, we need to locate the largest k such that

$P_{(k)} \le \frac{k}{61}\alpha$. Then reject all $H_{(i)}$ for i = 1, 2, Λ , k. Also, α is set at 0.05. All of the tests in search of the first change point are shown in Figure 3.

4.2.2 Step Two

The performance statistics generated by the sequential forward Z test are summarized in Table 3, and are shown in Figure 4. According to the ranked p-values and the FDR testing guidelines, there are 23 significant sequential tests with the 39th test as the earliest (Figure 5). In other words, the forward Z test detects 1968-06-09 as the first change point, which marks the end/beginning of the first/second regime.

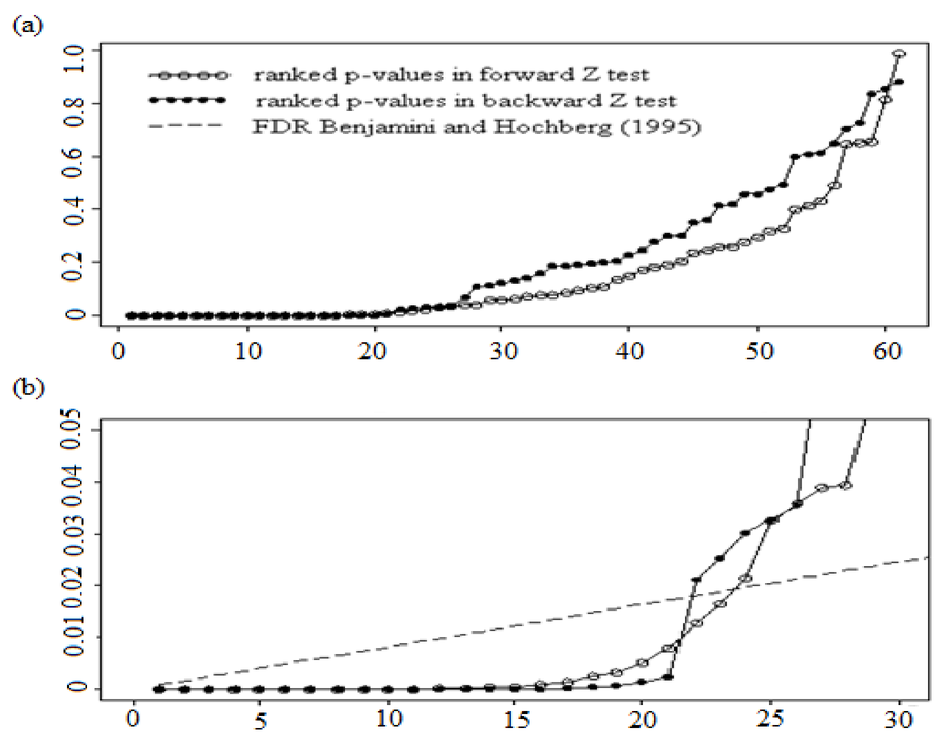

Fig 4. Ranked p-values (the BH procedure) based on (a) all the p-values, and (b) the p-values below 0.05

Table 3. Ranked p-values by the Z test on the whole process (the BH procedure)

P-value rank	Test ID	Ranked p	BH	P-value rank	Test ID	Ranked p	BH
1	61	4.94E-06	8.20E-04	32	32	7.36E-02	2.62E-02
2	60	7.10E-06	1.64E-03	33	28	7.61E-02	2.70E-02
3	56	9.36E-06	2.46E-03	34	27	7.65E-02	2.79E-02
4	59	1.02E-05	3.28E-03	35	31	8.52E-02	2.87E-02
5	55	1.28E-05	4.10E-03	36	24	9.55E-02	2.95E-02
6	58	1.46E-05	4.92E-03	37	23	1.04E-01	3.03E-02
7	57	2.30E-05	5.74E-03	38	26	1.08E-01	3.11E-02
8	54	2.52E-05	6.56E-03	39	22	1.36E-01	3.20E-02
9	53	3.11E-05	7.38E-03	40	25	1.48E-01	3.28E-02
10	52	5.29E-05	8.20E-03	41	21	1.71E-01	3.36E-02
11	51	1.01E-04	9.02E-03	42	14	1.81E-01	3.44E-02
12	50	1.70E-04	9.84E-03	43	13	1.92E-01	3.52E-02
13	49	2.43E-04	1.07E-02	44	20	2.06E-01	3.61E-02
14	48	3.44E-04	1.15E-02	45	15	2.36E-01	3.69E-02
15	47	5.13E-04	1.23E-02	46	19	2.46E-01	3.77E-02
16	46	8.50E-04	1.31E-02	47	17	2.58E-01	3.85E-02
17	45	1.46E-03	1.39E-02	48	16	2.61E-01	3.93E-02
18	44	2.46E-03	1.48E-02	49	12	2.77E-01	4.02E-02
19	43	3.31E-03	1.56E-02	50	18	2.98E-01	4.10E-02
20	42	5.22E-03	1.64E-02	51	10	3.17E-01	4.18E-02
21	41	7.96E-03	1.72E-02	52	11	3.27E-01	4.26E-02
22	40	1.29E-02	1.80E-02	53	3	4.02E-01	4.34E-02
23	39	1.66E-02	1.89E-02	54	9	4.13E-01	4.43E-02
24	38	2.14E-02	1.97E-02	55	7	4.31E-01	4.51E-02
25	37	3.26E-02	2.05E-02	56	8	4.92E-01	4.59E-02
26	35	3.58E-02	2.13E-02	57	4	6.44E-01	4.67E-02
27	36	3.89E-02	2.21E-02	58	1	6.53E-01	4.75E-02
28	34	3.93E-02	2.30E-02	59	6	6.54E-01	4.84E-02
29	33	5.71E-02	2.38E-02	60	2	8.13E-01	4.92E-02
30	30	5.81E-02	2.46E-02	61	5	9.89E-01	5.00E-02
31	29	6.47E-02	2.54E-02				

Table 4. Ranked p-values of the backward Z test (the BH procedure)

P rank	Test ID	Ranked p	BH	P rank	Test ID	Ranked p	BH
1	55	1.94E-08	8.20E-04	32	37	1.42E-01	2.62E-02
2	52	5.04E-08	1.64E-03	33	23	1.57E-01	2.70E-02
3	53	1.60E-07	2.46E-03	34	30	1.86E-01	2.79E-02
4	56	4.63E-07	3.28E-03	35	29	1.86E-01	2.87E-02
5	54	1.69E-06	4.10E-03	36	13	1.92E-01	2.95E-02
6	51	4.31E-06	4.92E-03	37	36	1.94E-01	3.03E-02
7	47	5.39E-06	5.74E-03	38	33	2.00E-01	3.11E-02
8	48	9.72E-06	6.56E-03	39	24	2.05E-01	3.20E-02
9	46	1.05E-05	7.38E-03	40	28	2.27E-01	3.28E-02
10	61	1.46E-05	8.20E-03	41	22	2.47E-01	3.36E-02
11	49	1.57E-05	9.02E-03	42	26	2.79E-01	3.44E-02
12	50	1.71E-05	9.84E-03	43	10	2.99E-01	3.52E-02
13	60	2.09E-05	1.07E-02	44	14	3.02E-01	3.61E-02
14	59	3.09E-05	1.15E-02	45	32	3.49E-01	3.69E-02
15	58	4.65E-05	1.23E-02	46	21	3.57E-01	3.77E-02
16	45	9.66E-05	1.31E-02	47	12	4.15E-01	3.85E-02
17	57	1.44E-04	1.39E-02	48	9	4.21E-01	3.93E-02
18	43	4.35E-04	1.48E-02	49	20	4.58E-01	4.02E-02
19	44	7.94E-04	1.56E-02	50	8	4.60E-01	4.10E-02
20	42	1.43E-03	1.64E-02	51	31	4.78E-01	4.18E-02
21	41	2.33E-03	1.72E-02	52	11	4.96E-01	4.26E-02
22	6	2.12E-02	1.80E-02	53	19	6.00E-01	4.34E-02
23	40	2.52E-02	1.89E-02	54	25	6.10E-01	4.43E-02
24	38	3.02E-02	1.97E-02	55	15	6.16E-01	4.51E-02
25	39	3.27E-02	2.05E-02	56	1	6.53E-01	4.59E-02
26	7	3.57E-02	2.13E-02	57	17	7.07E-01	4.67E-02
27	34	6.57E-02	2.21E-02	58	16	7.28E-01	4.75E-02
28	35	1.11E-01	2.30E-02	59	4	8.40E-01	4.84E-02
29	27	1.14E-01	2.38E-02	60	2	8.56E-01	4.92E-02
30	5	1.23E-01	2.46E-02	61	18	8.82E-01	5.00E-02
31	3	1.32E-01	2.54E-02				

Table 5. MLEs of β

Regime 1				Regime 2	
# of events	MLE	# of events	MLE	# of events	MLE
2	5.060	23	1.490	2	0.878
3	2.150	24	1.520	3	0.895
4	0.937	25	1.520	4	0.680
5	1.080	26	1.430	5	0.810
6	1.290	27	1.470	6	0.934
7	1.500	28	1.510	7	0.966
8	1.650	29	1.500	8	0.884
9	1.520	30	1.510	9	0.814
10	1.540	31	1.510	10	0.781
11	1.600	32	1.450	11	0.816
12	1.540	33	1.460	12	0.889
13	1.560	34	1.480	13	0.909
14	1.630	35	1.520	14	0.825
15	1.610	36	1.520	15	0.883
16	1.510	37	1.500	16	0.844
17	1.460	38	1.510	17	0.658
18	1.440	39	1.540	18	0.676
19	1.390	40	1.560	19	0.685
20	1.420	41	1.580	20	0.693
21	1.440	42	1.620	21	0.701
22	1.460				

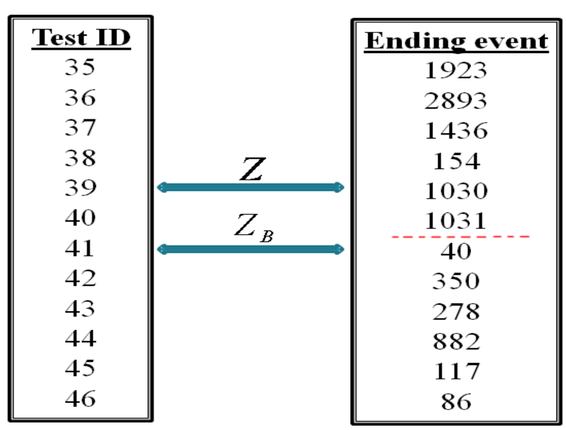

Fig 5. Results of searching the first change point

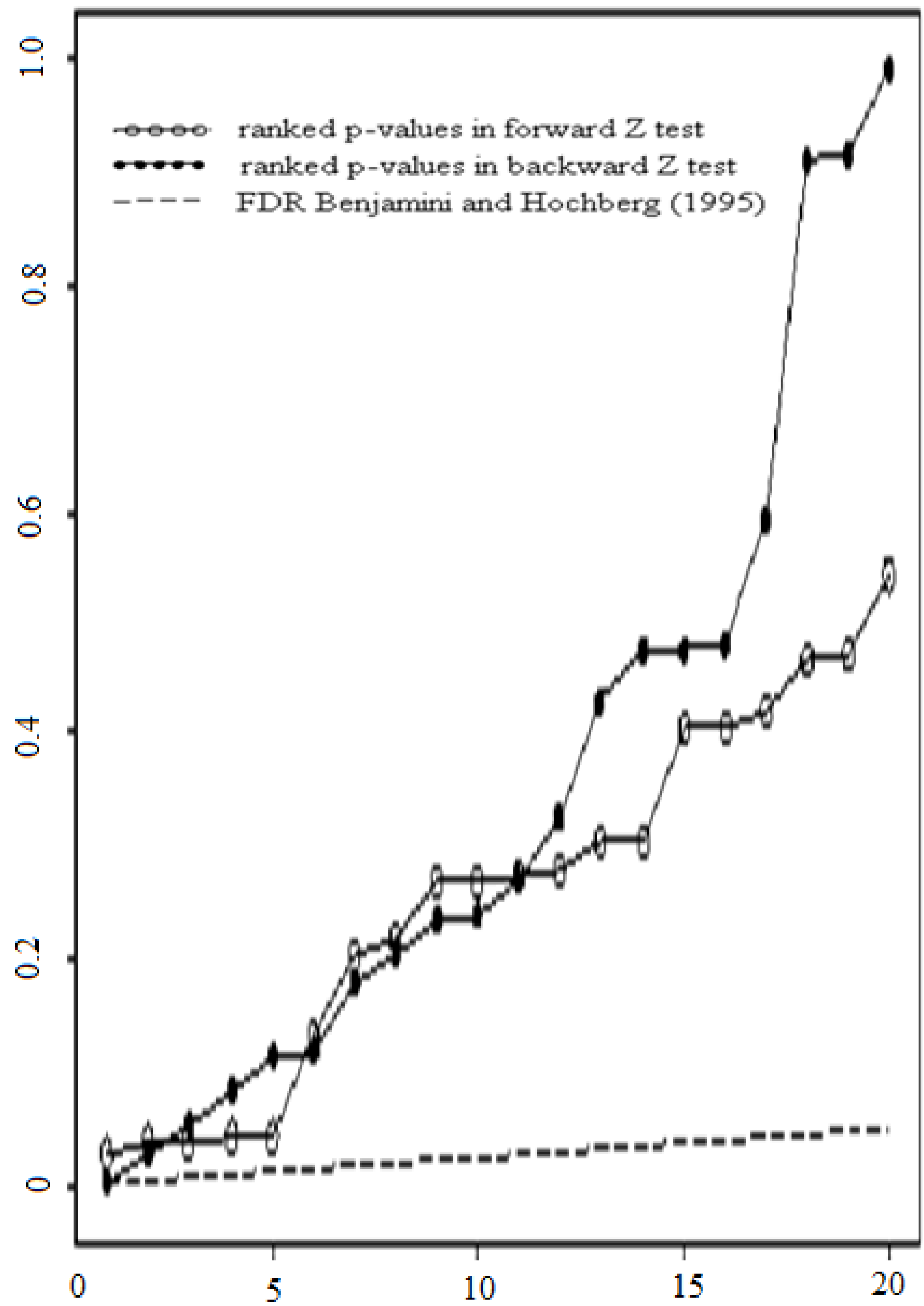

Fig 6. Ranked p-values on the second regime (the BH procedure)

Table 6. Ranked p-values of the second regime (the BH procedure)

Forward Z test		BH	Backward Z test	
Test order	ranked p-values		Test order	Ranked p-values
16	0.031	0.003	16	0.003
17	0.038	0.005	17	0.030
18	0.040	0.008	18	0.054
19	0.042	0.010	19	0.084
20	0.045	0.013	20	0.116
3	0.135	0.015	3	0.119
9	0.205	0.018	1	0.181
1	0.219	0.020	9	0.205
8	0.272	0.023	10	0.235
13	0.273	0.025	13	0.239
11	0.273	0.028	4	0.269
15	0.279	0.030	8	0.328
10	0.303	0.033	15	0.430
14	0.304	0.035	2	0.475
7	0.404	0.038	14	0.476
2	0.404	0.040	7	0.483
5	0.424	0.043	11	0.590
4	0.465	0.045	5	0.909
6	0.470	0.048	12	0.913
12	0.541	0.050	6	0.990

4.2.3 Step Three

The performance statistics generated by the sequential backward Z test are summarized in Table 4, and are shown in Figure 4. According to the ranked p-values and the FDR testing guidelines, there are 21 significant sequential tests with the 41st test as the earliest this time (Figure 5). In other words, the backward Z test detects January 30, 1974 as the first change point, which is two events later than the one detected by the forward Z test.

4.2.4 Step Four

The MLEs of the parameter β of the point process truncated at the 40th, 41st, 42nd event are 1.56, 1.58, and 1.61 (Table 5), respectively, which indicate that the trend is increasing. Consequently, January 30, 1974 is declared as the first change point, which is detected by the backward test.

Table 7. Two regimes of Mount Etna

	Date		Date		Date
1	1682-09-01	23	1883-03-22	5	1978-04-29
2	1689-03-14	24	1886-05-19	6	1978-08-24
3	1702-03-08	25	1892-07-09	7	1978-11-18
4	1755-03-09	26	1908-04-29	8	1979-08-03
5	1763-02-06	27	1910-03-23	9	1981-03-17
6	1763-06-18	28	1911-09-10	10	1983-03-28
7	1764-01-01	29	1918-11-30	11	1985-03-12
8	1766-04-27	30	1923-06-17	12	1985-12-25
9	1780-05-18	31	1928-11-02	13	1985-12-28
10	1787-07-01	32	1942-06-30	14	1986-10-30
11	1792-05-26	33	1947-02-24	15	1989-09-24
12	1802-11-15	34	1949-12-02	16	1989-09-27
13	1809-03-27	35	1950-11-25	17	1991-12-14
14	1811-10-27	36	1956-03-01	18	2001-07-17
15	1819-05-27	37	1964-02-01	19	2002-10-27
16	1832-10-31	38	1968-01-07	20	2004-09-07
17	1843-11-17	39	1968-06-09	21	2006-07-14
18	1852-08-20	40	1971-04-05	22	2008-05-12
19	1865-01-30	41	1974-01-30		
20	1869-09-26		1974-03-11		
21	1874-08-29		1975-02-24		
22	1879-05-26		1975-11-29		

4.2.5 Step Five and Beyond

Analogously, our search at the second stage requires a total of 20 sequential tests. Note that the new time origin is January 30, 1974. Again, we need to locate the largest k such that $P_{(k)} \leq \frac{k}{20}\alpha$, and reject all $H_{(i)}$ for i = 1, 2,Λ, k. Also, the performance statistics generated by the sequential forward and backward Z tests are summarized in Table 6 and Figure 6, which conclude that there are no more change points. The two regimes identified are shown in Table 7.

4.3 Validation

4.3.1 By Formal Hypothesis Tests

As we have mentioned in Chapter 2, both the F test and the Conditional test are designed to investigate whether two HPPs have the same rate/mean. As expected, both of the p-values (= 5.38e-07 for the F test and 9.97e-07 for the Conditional test) obtained by performing the tests are highly significant.

4.3.2 By the ERR-Plot

The ERR-plot offers the possibility of further insight into the data and provides valuable technical basis for model developments. ERR-plots produced for the whole process, regime one, and regime two are presented as Figure 7. According to Figure 7(a), there is an apparent slope change of the ERR time series at the detected change point, January 30, 1974. Moreover, the opposite trends depicted by the ERR-plots of both regimes provide additional justification for the outcome. Interestingly enough, both the sub-ERR plots are not near horizontal. Clearly, it indicates that the change in trend is gradual. In other words, a near HPP ERR-plot signals an abrupt change between two regimes, which is not the case for Mount Etna.

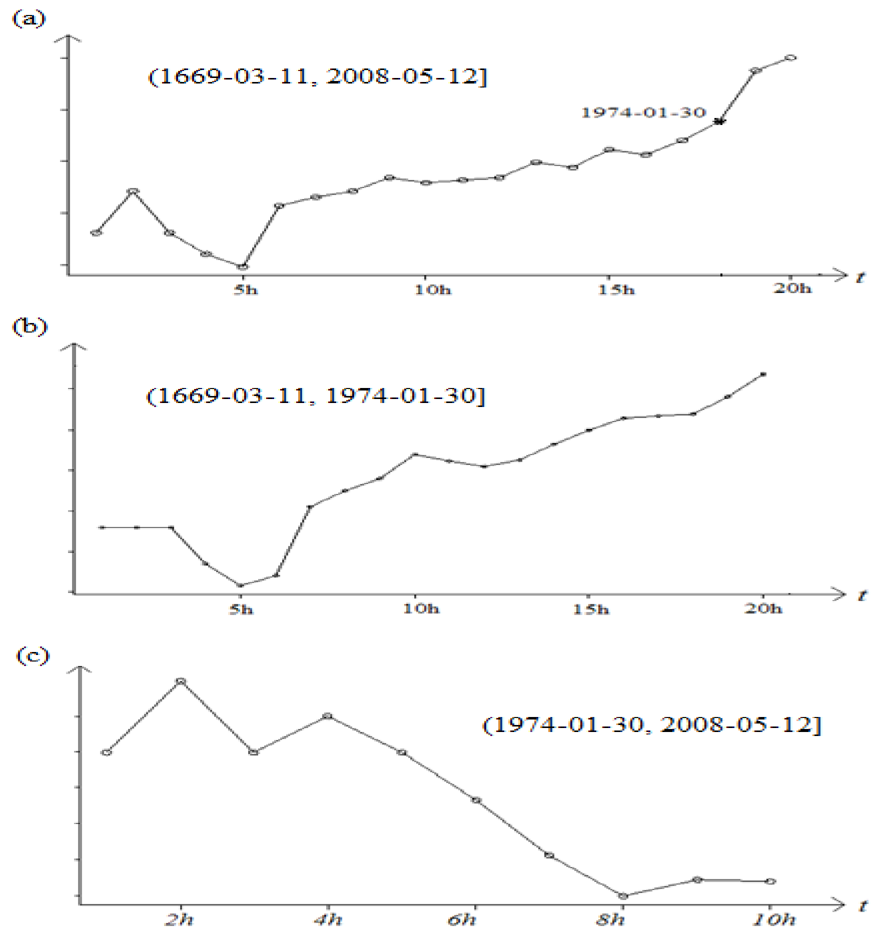

Fig 7. ERR plots. (a) The whole process, (b) regime 1, and (c) regime 2

4.4 Comparisons with Other Error Controlling Methods

The implementation of the proposed technique using the FDR error rate controlling mechanism has been shown in a great detail. For the sake of comparisons, both the classical Bonferroni (CB) and sequential Bonferroni (SB) FWER controlling procedures are adopted to detect the change points for Mount Etna as well. The results are rather interesting: (a) Both of the FWER controlling procedures produce the same change point. (b) In chronological order, the change point selected by the CB and SB is two events later than that previous detected using their counterpart (FDR). And (c) the backward test picks all regardless of the controlling mechanism. Figure 8 compares the results using the three error rate controlling procedures.

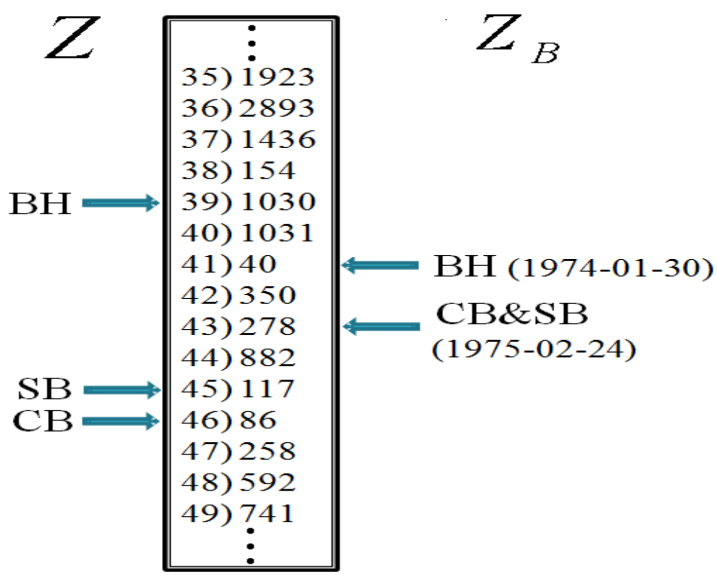

Fig 8. Results using different error rate controlling methods

CHAPTER 5

CONCLUSIONS

The forward and backward Z tests are applied sequentially on a single Poisson process to detect its change points. Different regimes of the process are then classified by the detected change points. Results of the testing procedure are investigated using additional methods. We choose a reliable volcanic data set to apply the methods, because objective identification of regimes of a volcano is of great importance to the volcanological community.

The testing algorithm which combines both of the forward and backward tests is more efficient to detect the change points, due to the different performance of the Z test of detecting the change points of the process with the increasing or decreasing intensity. The adjustment of the significance levels should be considered in the testing procedure because multiple tests are performed. The forward-backward testing procedure, coupled with the error control methods, is simple to apply and can be extended to detect change points for different purposes. For instance, the proposed method can solve problems of identifying the time point at which the occurrence rate of car accidents in an area has decreased, detecting the time when the dust storms start to occur more frequently than before, discovering when an economic entity begins to recover, and so on.

In conclusion, our effort for future studies will be devoted to proposing a simpler testing algorithm, which can be evaluated with a large-scale simulation. The selection criterion is set to be: declare the first point (with respect to the forward-pass), identified by either test (Z and Z_B), as the true change point.

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