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INVERSE PROBLEM FOR NON-VISCOUS MEAN FIELD CONTROL: EXAMPLE FROM TRAFFIC

by

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A thesis submitted in partial fulfillment
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ABSTRACT

Inverse Problem for Non-viscous Mean Field Control: Example from Traffic

by

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This thesis presents an inverse problem for mean field games where we find the mean field problem statement for which the given dynamics is the solution. We use distributed traffic as an example and derive the classic Lighthill Whitham Richards (LWR) model as a solution of the non-viscous mean field game. We also derive the same model by choosing a different problem where we use travel time, which is a distributed parameter, as the cost for the optimal control. We then study the stationary versions of these two problems and provide numerical solutions for the same.

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CHAPTER 1

INTRODUCTION

Mean field games (MFG) involve the study of Nash equilibrium among infinitely many players where the interplay between individual dynamics and the continuum limit of the players is studied ([1], [2], [3]). Mean field framework has been used in many applications such as consensus building ([4]), complex networks ([5]), electric vehicles using smart grid ([6], [7]). There has been some work in utilizing the mean field principles in transportation problems in one (vehicular traffic) and two dimensions (pedestrian traffic) ([8], [9], [10]).

The solution to a calculus of variations problem involves solving ordinary differential equations ([11]) arising from the corresponding Euler-Lagrange equations. There is a great deal of interest in finding the Lagrangian in the calculus of variations problem that results in a given ordinary differential equation. This is the inverse problem that has a classic result called Helmholtz condition ([12]) and also has had some recent results ([13]).

Vehicular traffic on the highways can be viewed microscopically ([14]), i.e. in terms of each vehicle, or macroscopically, i.e. in terms of aggregate variables such as traffic density and flow. The microscopic dynamics of vehicles such as the car-following models, result in the evolution of macroscopic dynamics ([15]), such as the LWR (Lighthill

Whitham Richards) model ([16], [17]). Macroscopic traffic modeling [18] is very useful in developing effective controls using ramp metering [19], observability analysis [20], financial modeling [21] and other useful analysis such as [22] and [23]. Additionally, researchers have worked on mesoscopic models to evaluate transportation systems for infrastructure improvements [24]. The inherent interdependence of transportation systems with other systems such as Economic, Environmental and Social systems was also studied [25] [26]. Furthermore, the dynamic nature of transportation systems was analyzed using system dynamics and other modeling approaches [27]. The outcomes of such studies have helped decision makers to design appropriate control mechanisms for policy making [24].

In this thesis, we solve the inverse problem of deriving the LWR model from a non-viscous mean field game, and also provides basic analysis and numerical solutions of the stationary viscous and non-viscous cases. We then extend the model by adding a distributed parameter to the model in terms of travel time which was developed in ([28], [29]).

The contributions of this thesis work are as follows. We derive the traffic dynamics solving an inverse problem providing a link between the microscopic behavior of drivers to the macroscopic behavior of traffic. This is similar to how Newton's laws and other physical fundamental laws for instance are derived from variational principles of mechanics [12]. We further utilize the recently developed framework of the mean field games to connect the optimizing behavior of the drivers to the evolving macroscopic traffic behavior. There is some recent literature where mean field games are used to study pedestrian and crowd dynamics ([8], [9] and [30]) as well as steady state traffic behavior ([10]). To our knowledge, this framework has not been used before to derive the fundamental LWR traffic dynamics. Moreover, the travel time dynamics developed by the first author, is being used to link driver behavior to the evolved traffic behavior for the first time. This connection between the microscopic and macroscopic behavior can lead researchers to design traffic controllers from both sides.

The remainder of this thesis is organized as follows. Chapter 2 presents the fundamentals of the mean field games, and an introduction to the LWR model with the corresponding Greenshield's model for traffic along with an account of the Liouville equation and the hyperbolic Hamilton Jacobi Bellman (HJB) equation which are the non-viscous versions of the model obtained from the mean field equations. The first main results of this thesis are developed in chapter 3 where the LWR model is derived as the solution of the inverse mean field problem. These results are in the form of lemma 1 and theorem 2. The derivation of the travel time Hamilton Jacobi partial differential equation is provided in chapter 4 and the stationary versions of our models are studied in chapter 5, where we also present their numerical simulations. Chapter 6 provides the concluding remarks.

CHAPTER 2

MATHEMATICAL BACKGROUND

2.1 Mean Field Games

We consider agent stochastic differential model on a probability space $(\Omega, \mathcal{A}, P)$ with a filtration $\mathcal{F}^t$ generated by $w(t)$, the n-dimensional standard Wiener process ([31]). Here Ω is the sample space which consists of all outcomes for the state x at any given time t, $\mathcal{A}$ is the sigma algebra of all events that the state at t can be in, and P is the probability measure on the sigma algebra. The system is causal, and hence is adapted to the filtration of progressively increasing sigma algebras generated by the Wiener process.

$$\begin{aligned} dx = v(x(t), u(x(t)), \rho(x,t))dt + \sigma(x(t))dw(t), \\ x(0) = x_0, \end{aligned} \tag{2.1}$$

where $v : \mathcal{R}^n \times \mathcal{R}^d \times \mathcal{R}^n \to \mathcal{R}^n$, and $\sigma : \mathcal{R}^n \to \mathcal{L}(\mathcal{R}^n; \mathcal{R}^n)$ are measurable functions, ρ is the probability density of the state $x(t)$, and $u(x(t))$ is the state feedback control. For existence and uniqueness, at least locally, we have that ([32]) $\exists C \in \mathcal{R}$ and $C \in \mathcal{R} || f(x,u,\rho)| + |\sigma(x)| \leq C(1+|x|)$, and $|f(x,u,\rho) - f(y,u,\rho)| + |\sigma(x) - \sigma(y)| \leq D|x-y|$. The initial state x_0, in general, is the random variable independent of the σ-algebra

$\mathcal{F}^t$, $t \geq 0$ with $E[|x_0^2|] < \infty$.

The objective for the control law design for the agent is the expected cost for a given initial condition x and time t.

$$J_{x,t}[u(\cdot)] = E\left\{\int_t^T r(x(s), u(x(s)), \rho(s))ds + k(x(T))\right\}, \tag{2.2}$$

where, r and k are known functions representing the running cost and the terminal cost respectively.

The value function for this problem is

$$\mathcal{V}(x,t) = \inf_{u(\cdot)\epsilon\,\mathcal{U}} J_{x,t}[u(\cdot)]. \tag{2.3}$$

The PDE satisfied by the value function is the stochastic Hamilton-Jacobi-Bellman (HJB) equation, given as

$$\begin{aligned} \mathcal{V}_t(x,t) + \frac{\sigma^2}{2}\Delta\mathcal{V}(x,t) + \min_{u(\cdot)\epsilon\,\mathcal{U}} \{v(x,u,\rho,t)\nabla_x\mathcal{V}(x,t) + r(x,u)\} = 0, \\ \mathcal{V}(x,T) = k(x). \end{aligned} \tag{2.4}$$

The corresponding Kolmogorov or Fokker Planck (FP) equation for the evolution of the state probability density is given by

$$\begin{aligned} \rho_t(x,t) - \frac{\sigma^2}{2}\Delta\rho(x,t) + \nabla\cdot(\rho v(x,u,\rho,t)) = 0, \\ \rho(x,0) = \rho_0(x). \end{aligned} \tag{2.5}$$

The combined two point boundary value problem given by equations (2.4) and (2.5) is referred as the HJB-FP system or the coupled PDE system of the Mean Field Games.

2.2 LWR and Greenshield's Models for Traffic

The macroscopic traffic flow model formulates the relationship among the key traffic flow parameters such as density, flow etc. The classic LWR (Lighthill-Whitham-Richards) model was proposed in 1956. It is a one-dimensional macroscopic traffic model named after the authors in [16] and [17]. The dynamics of traffic flow using this model is given by equation (2.6),

$$\rho_t(t,x) + f_x(t,x) = 0, \tag{2.6}$$

where, ρ is the traffic density and f is the flux. Traffic flux is defined as the product of traffic density and the traffic speed v , i.e. $f = \rho v$. There are many models which link traffic density to traffic speed. One of them is Greenshield's model which proposes a linear relationship between traffic density and traffic speed, [33]. This model is given by equation (2.7),

$$v(\rho) = v_f \left(1 - \frac{\rho}{\rho_m}\right), \tag{2.7}$$

where v_f is the free flow speed and ρ_m is the maximum possible density or jam density. Free flow speed is the traffic speed when the traffic density is zero. This means that an unimpeded single vehicle on the highway will have the free flow speed. Traffic jam density is the density at which the traffic speed is zero. In other words, jam density refers to that density of traffic when vehicles are most tightly packed resulting in zero speed.

Traffic flow using Greenshield's model is given by equation (2.8),

$$f(t) = v_f \rho(t) \left(1 - \frac{\rho(t)}{\rho_m}\right), \tag{2.8}$$

and the fundamental diagram of traffic flow is shown in figure 2.1.

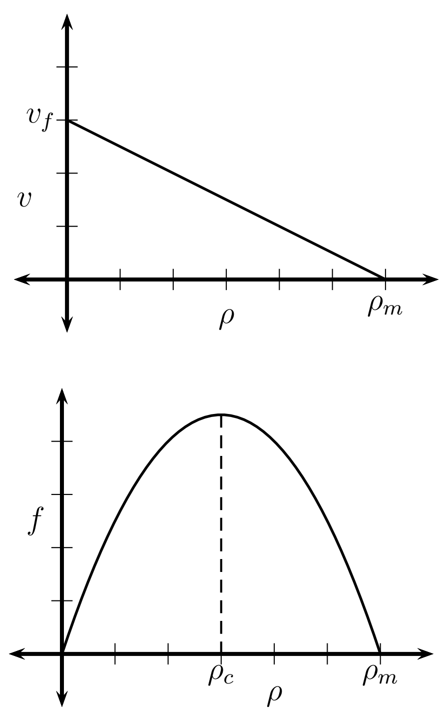

Figure 2.1: Traffic Fundamental Diagram using Greenshield's Model

Next, we present generalized and weak solutions for the scalar conservation laws and then state the initial boundary value problem.

2.2.1 Generalized Solutions

For a conservation law

$$\rho_t + f_x(\rho) = 0 \tag{2.9}$$

with initial condition

$$\rho(x,0) = \rho_0(x), \tag{2.10}$$

where $\rho_0(x) \in L^1_{loc}(\mathcal{R};\mathcal{R}^n)$, solution in the distributional sense is defined below for a given smooth vector field $f : \mathcal{R}^n \to \mathcal{R}^n$, (see [34]).

Definition 1. A measurable locally integrable function $\rho(t,x)$ is a solution in the distributional sense of the Cauchy problem (2.9) if for every $\phi \in C_0^\infty(\mathcal{R}^+ \times \mathcal{R}) \to \mathcal{R}^n$

$$\iint_{\mathcal{R}^+\times\mathcal{R}} [\rho(t,x)\,\phi_t(t,x) + f(\rho(t,x))\,\phi_x(t,x)]\,dx\,dt + \int_{\mathcal{R}} \rho_0(x)\,\phi(x,0)\,dx = 0 \tag{2.11}$$

2.2.2 Weak Solutions

A measurable locally integrable function $\rho(t,x)$ is a weak distributional solution of the Cauchy problem (2.9) if it is a distributional solution in $(0,T)\times\mathcal{R}$ satisfying (2.10) and if ρ is continuous as a function from $[0,T]$ into L^1_{loc}. We assume $\rho(t,x) = \rho(t,x^+)$ and the continuity condition implies

$$\lim_{t\to 0}\int_R |\rho(t,x) - \rho_0(x)|\, dx = 0 \tag{2.12}$$

2.2.3 Scalar Initial-Boundary Problem

Consider the scalar conservation law given by

$$\rho_t + f_x = 0, \tag{2.13}$$

with initial condition

$$\rho(0,x) = \rho_0(x), \tag{2.14}$$

and boundary conditions

$$\rho(t,a) = \rho_a(t) \text{ and } \rho(t,b) = \rho_b(t). \tag{2.15}$$

The boundary conditions cannot be prescribed point-wise, since characteristics from inside the domain might be traveling to outside at the boundary. In that case, the data at the boundary influences the local dynamics at the boundary but does not become equal to the value at the boundary. This is shown in Figure 2.2 where for some time boundary data on the left can be prescribed when characteristics from the boundary can be *pushed in* (see [35]). However when the characteristics are coming from inside, the boundary data can not be prescribed.

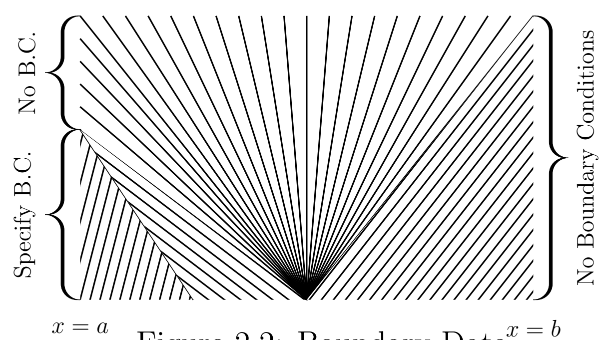

Figure 2.2: Boundary Data

For the traffic density equation, it should satisfy the entropy Kruzkov solution, [36].

Definition 2 (Kruzkov Solution)**.** The Kruzkov entropy solution is a function $\rho : [0, \infty) \to L^{\mathcal{R}}_{\text{loc}}$, such that $\forall k > 0, \phi > 0 \in C^1_c(\mathcal{R}^2)$ with the compact support of ϕ is in $t > 0$, we have

$$\iint [|\rho - k|\phi_t + (f(\rho) - f(k)) \operatorname{sgn}(\rho - k)\phi_x] dxdt \geq 0 \tag{2.16}$$

and there exists a set E of zero measure on $[0, T]$, such that for $t \in [0, T] - E$, the function $\rho(t, x)$ is defined almost everywhere in $\mathcal{R}$, and for any ball $K_r = \{|x| \leq r\}$

$$\lim_{t \to 0} \int_{K_r} |\rho(t, x) - \rho_0(x)| dx = 0. \tag{2.17}$$

It has been shown that entropy solutions such as Kruzkov are equivalent to vanishing viscosity solutions for hyperbolic conservation laws ([34], [37]).

2.3 Vanishing Viscosity Mean Field

In this section we study the connection between the mean field formulation with the stochastic (semilinear parabolic) HJB-FP system and the system governed by the hyperbolic HJB and the Liouville equation.

2.3.1 Liouville Equation

In this section, we review the Liouville equation for control systems and provide its solution.

Let the control system be described by

$$\dot{x}(t) = v(x(t), u(t)) \tag{2.18}$$

where $x = (x_1, \dots, x_n)$ and u is the control input. The time evolution of the probability density function of the initial state of the system is a function of the control input. Given a point x and time t, the initial condition Ξ can be found if the control input

$u(\cdot)$ is known for $[0, t]$. In such a case, the system can be considered an autonomous system and the backward dynamics of (2.18) can be solved to find Ξ. Thus,

$$\Xi = \Xi(x, t, u(\cdot)), \tag{2.19}$$

and the Liouville equation becomes

$$\rho(x, t, u(\cdot))_t + [\rho(x, t, u(\cdot))v(x(t), u(t))]_x = 0. \tag{2.20}$$

The solution to the Liouville equation can be obtained by the method of characteristics as in [38] and [39] by letting

$$\frac{dx}{ds} = v(x(t), u(t)) \tag{2.21}$$

$$\frac{dt}{ds} = 1 \tag{2.22}$$

$$\frac{du}{ds} = 0. \tag{2.23}$$

Then the partial differential equation (2.20) becomes the ordinary differential equation

$$\frac{d\rho(x, t, u(\cdot))}{ds} = -\psi(x, u(t))\rho(x, t, u(\cdot)) \tag{2.24}$$

where

$$\psi(x(t), u(t)) = \sum_{i=1}^{n} \frac{\partial v_i(x, u(t))}{\partial x_i} \tag{2.25}$$

and $v_i(x, u(t))$ is the ith component of $v(x(t), u(t))$. The solution to (2.20) is

$$\rho(x,t,u(\cdot)) = \rho_0(\Xi(x,t,u(\cdot))) \exp\left[-\int_0^t \psi(\hat{x}(\tau),u(\tau))\,d\tau\right] \tag{2.26}$$

where $\hat{x}(\tau)$ denotes the trajectory starting at $\Xi_x(x,t,u(\cdot))$ at time zero and ending at x at time t.

2.3.2 Hyperbolic HJB and the Liouville equation

Letting $\sigma \to 0$ in equation (2.4) and equation (2.5), we get vanishing viscosity solutions for the following set of equations.

$$\begin{aligned} \mathcal{V}_t(x,t) + \min_{u(\cdot)\epsilon\mathcal{U}} \{v(x,u,\rho,t)\nabla_x\mathcal{V}(x,t) + r(x,u)\} = 0, \\ \mathcal{V}(x,T) = k(x), \end{aligned} \tag{2.27}$$

$$\begin{aligned} \rho_t(x,t) - \nabla\cdot(\rho v(x,u,\rho,t)) = 0, \\ \rho(x,0) = \rho_0(x). \end{aligned} \tag{2.28}$$

The second equation is the Liouville equation instead of the Fokker Planck one from before with the difference that in this case pde is not semilinear in general anymore. We will show that the LWR model matches this set of vanishing viscosity mean field model for the system.

CHAPTER 3

LWR Model from HJB equation

We take the agent model to be

$$dx = u(t)dt + \sigma(x(t))dw(t), \;\; x(0) = x_0. \tag{3.1}$$

Define a function $h(x(t), t)$ as

$$h := -\frac{v_f}{2}(1 - \frac{\rho(x(t), t)}{\rho_m}), \tag{3.2}$$

where v_f is the traffic free flow speed, ρ_m is the traffic jam density and $\rho(x(t), t)$ is the density at time t and position x.

Now we define the running cost $r(x(t), t, u(t))$ as

$$r(x(t), t, u(t)) := \frac{1}{2}u^2(t), \tag{3.3}$$

where $u(t)$ is the control variable and the cost function as

$$J_{x,t}[u(\cdot)] = E\left\{\int_{t_0}^{t_f} r(x(s), s, u(s))ds + \int_{x}^{x_f} h(x(s), s)ds\right\}, \tag{3.4}$$

where t_0 is the initial time, t_f is the final time and x_f is the final position. Next we attempt to determine the control $u(t)$ that minimizes $J_{x,t}[u(\cdot)]$, for all admissible $x(t)$ and for all $t < t_f$. We will now present the main result that shows the LWR model as the solution of the mean field vanishing viscosity problem. In order to prepare the main result, we first provide a supporting lemma.

Lemma 1 (Inverse Derivation of the Greenshield's Model)**.** *For the stochastic differential equation model given by* (3.1) *the optimal control for the cost function given by* (3.4) *is given by*

$$u(t) = v_f(1 - \frac{\rho}{\rho_m}). \tag{3.5}$$

Proof. The value function for the system given by equation (3.1) whose cost function is given by equation (3.4) is

$$\mathcal{V}(x(t), t) = \min_{u(\cdot)} E\left\{\int_{t_0}^{t_f} r(x(s), s, u(s))ds + \int_{x}^{x_f} h(x(s), s)ds\right\}. \tag{3.6}$$

By subdividing the interval we obtain

$$\mathcal{V}(x(t), t) = \min_{u(\cdot)} E\left\{\int_{t}^{t+\Delta t} rd\tau + \int_{t+\Delta t}^{t_f} rd\tau + \int_{x}^{x+\Delta x} hds + \int_{x+\Delta x}^{x_f} hds\right\}. \tag{3.7}$$

The principle of optimality requires that

$$\mathcal{V}(x(t),t) = \min_{u(t)} E\left\{\int_t^{t+\Delta t} r d\tau + \int_x^{x+\Delta x} h ds + \mathcal{V}(x+\Delta x, t+\Delta t)\right\}, \tag{3.8}$$

where $\mathcal{V}(x+\Delta x, t+\Delta t)$ is the minimum value of the process for the time interval $t+\Delta t < \tau < t_f$, and with initial state $x(t+\Delta t)$.

Assuming that the partial derivatives of the $\mathcal{V}$ exist and are bounded, we can expand $\mathcal{V}(x+\Delta x, t+\Delta t)$ using Ito's chain rule about the point $(x(t),t)$ to obtain

$$\begin{aligned}\mathcal{V}(x(t),t) = \min_{u(t)} E\Bigg\{\int_t^{t+\Delta t} r d\tau + \int_x^{x+\Delta x} h ds + \mathcal{V}(x(t),t) + \\ \left[\mathcal{V}_t(x(t),t) + \frac{\sigma^2}{2}\Delta\mathcal{V}(x(t),t)\right]\Delta t + \nabla_x\mathcal{V}(x(t),t)\Delta x + \text{h.o.t.}\Bigg\},\end{aligned} \tag{3.9}$$

where h.o.t. stands for the higher order terms. For small Δt we can write

$$\begin{aligned}\mathcal{V}(x(t),t) = \min_{u(t)} E\Bigg\{r\Delta t + h\Delta x + \mathcal{V}(x(t),t) + \Bigg[\mathcal{V}_t(x(t),t) + \frac{\sigma^2}{2}\Delta\mathcal{V}(x(t),t) + \\ \nabla_x\mathcal{V}(x(t),t)u(t)\Bigg]\Delta t + \text{h.o.t.}\Bigg\},\end{aligned} \tag{3.10}$$

here we have utilized the fact that $u(t) = \Delta x/\Delta t$.

Now getting the terms involving $\mathcal{V}(x(t),t)$ and $\mathcal{V}_t(x(t),t)$ out of the min term as they do not depend on $u(t)$ and then canceling $\mathcal{V}(x(t),t)$ from both sides, we get

$$\begin{aligned}0 = \mathcal{V}_t(x(t),t)\delta t + \min_{u(t)} E\Bigg\{r\Delta t + h\Delta x + \\ \left[\frac{\sigma^2}{2}\Delta\mathcal{V}(x(t),t) + \nabla_x\mathcal{V}(x(t),t)u(t)\right]\Delta t + \text{h.o.t.}\Bigg\}.\end{aligned} \tag{3.11}$$

Substituting $\Delta x = u(t)\Delta t$, dividing by Δt and letting $\Delta t \to 0$ yields

$$0 = \mathcal{V}_t(x(t), t) + \min_{u(t)} E \left\{ r + hu(t) + \frac{\sigma^2}{2} \Delta \mathcal{V}(x(t), t) + \nabla_x \mathcal{V}(x(t), t) u(t) \right\}. \quad (3.12)$$

Now we define the Hamiltonian $\mathcal{H}$, where we have used the same notation for the optimal control and corresponding $\mathcal{V}$ variables.

$$\mathcal{H} = r + hu(t) + \nabla_x \mathcal{V}(x(t), t) u(t) + \frac{\sigma^2}{2} \Delta \mathcal{V}(x(t), t). \quad (3.13)$$

Using the above equations (3.12) and (3.13) we obtain the stochastic Hamilton-Jacobi equation as

$$0 = \mathcal{V}_t(x(t), t) + \mathcal{H}. \quad (3.14)$$

Please note that the only point where the stochastic nature of the problem enters into the HJB is in the last term of equation (3.13), which involves the second derivative of the value function. In order to calculate the optimal value of the control $u(t)$, we differentiate $\mathcal{H}$ with respect to $u(t)$ and equate to zero

$$\frac{\partial \mathcal{H}}{\partial u} = \frac{\partial}{\partial u} \left[\frac{1}{2} u^2(t) + hu(t) + \frac{\sigma^2}{2} \Delta \mathcal{V}(x(t), t) + \nabla_x \mathcal{V}(x(t), t) u(t) \right] = 0. \quad (3.15)$$

This gives the control law $u(t)$ as

$$u(t) = -(\nabla_x \mathcal{V}(x(t), t) + h). \tag{3.16}$$

Now replacing the values of $\nabla_x \mathcal{V}(x(t), t)$ and h in the above equation yields Greenshield's traffic flow formula,

$$u(t) = v_f(1 - \frac{\rho}{\rho_m}). \tag{3.17}$$

□

Next, we replace the value of the optimal control $u(t)$ in equation (3.12) to obtain the HJB equation as follows

$$0 = \mathcal{V}_t(x(t), t) + \frac{\sigma^2}{2}\Delta \mathcal{V}(x(t), t) + v_f(1 - \frac{\rho}{\rho_m})\nabla_x \mathcal{V}(x(t), t). \tag{3.18}$$

The Fokker Planck equation for the evolution of the probability density function (pdf) of the state is given by

$$\begin{aligned} \rho_t(x, t) - \frac{\sigma^2}{2}\Delta\rho(x, t) + \nabla \cdot (\rho v(x, u, \rho, t)) = 0, \\ \rho(x, 0) = \rho_0(x). \end{aligned} \tag{3.19}$$

Equations (3.18) and (3.19) represent the coupled PDE system of the Mean Field Games.

Theorem 2 (Inverse Derivation of the LWR Model)**.** *For the stochastic differential equation model given by equation* (3.1) *the vanishing viscosity solution for the mean* *field game for the cost function given by equation* (3.4) *is the LWR model*

$$\rho_t + f_x(\rho) = 0 \tag{3.20}$$

where, $f = \rho v$ *and*

$$v(\rho) = v_f(1 - \frac{\rho}{\rho_m}) \tag{3.21}$$

Proof. The lemma 1 showed that the optimal control, which is the vehicle speed according to the model given by equation (3.1) is

$$v(\rho) = v_f(1 - \frac{\rho}{\rho_m}). \tag{3.22}$$

Moreover, the Fokker Planck equation for the evolution of the pdf of the state is given by

$$\begin{aligned} \rho_t(x,t) - \frac{\sigma^2}{2}\Delta\rho(x,t) + \nabla \cdot (\rho v(x,u,\rho,t)) = 0, \\ \rho(x,0) = \rho_0(x). \end{aligned} \tag{3.23}$$

Letting $\sigma \to 0$ gives us the conservation law of the LWR model. □

CHAPTER 4

Travel Time PDE using HJB equation and Pontryagin's Minimization Principle

We can also derive the LWR model by minimizing the travel time for an agent. This strategy not only develops the LWR model, but also produces the travel time dynamics in terms a PDE that models the evolution of the travel time field. The travel time PDE developed is the same one developed for the first time in [28] and [29], but in this case using the vanishing viscosity mean field framework.

We take the agent model once again to be

$$dx = u(t)dt + \sigma(x(t))dw(t), \;\; x(0) = x_0. \tag{4.1}$$

We define the cost function to be the expected value of the travel time $T(x(t), t)$ of the agent to a fixed location,

$$J_{x,t}[u(\cdot)] = E\{T(x(t), t)\}. \tag{4.2}$$

The value function for the problem becomes

$$\mathcal{V}(x(t), u(t), t) = \min_{u(t)} E\left\{T(x(t), t)\right\}. \tag{4.3}$$

By subdividing the interval we obtain

$$\mathcal{V}(x(t), t) = \min_{u(t)} E\left\{\Delta t + \mathcal{V}(x(t+\Delta t), t+\Delta t)\right\}, \tag{4.4}$$

where $\mathcal{V}(x+\Delta x, t+\Delta t)$ is the minimum expected travel time of the process for the time interval $t+\Delta t < \tau < t_f$, and with initial state $x(t+\Delta t)$.

Assuming that the partial derivatives of the $\mathcal{V}$ exist and are bounded, we can expand $\mathcal{V}(x+\Delta x, t+\Delta t)$ using Ito's chain rule about the point $(x(t), t)$ to obtain

$$\mathcal{V}(x(t), t) = \min_{u(t)} E\left\{\Delta t + \mathcal{V}(x(t), t) + \ \mathcal{V}_t \Delta t + \mathcal{V}_x \Delta x + \frac{\sigma^2}{2}\Delta \mathcal{V} \Delta t + h.o.t.\right\}. \tag{4.5}$$

Simplifying and canceling $\mathcal{V}(x(t), t)$ from both sides we get

$$0 = \min_{u(t)} E\left\{\Delta t + \mathcal{V}_t \Delta t + \mathcal{V}_x u(t) \Delta t + \ \frac{\sigma^2}{2}\Delta \mathcal{V} \Delta t + h.o.t.\right\}. \tag{4.6}$$

Now dividing by Δt and taking the limit as $\Delta t \to 0$ we get

$$0 = \min_{u(t)} \left\{1 + \mathcal{V}_t + \mathcal{V}_x u(t) + \frac{\sigma^2}{2}\Delta \mathcal{V}\right\}. \tag{4.7}$$

Here we define the Hamiltonian $\mathcal{H}$ as

$$\mathcal{H} := \min_{u(t)} \left\{ 1 + \mathcal{V}_x u(t) \right\}. \tag{4.8}$$

The stochastic Hamilton-Jacobi-Bellman equation is

$$0 = \mathcal{V}_t + \frac{\sigma^2}{2} \Delta \mathcal{V} + \mathcal{H}. \tag{4.9}$$

Pontryagin's minimum principle asks to minimize $\mathcal{H}$ as a function of $u \in [0, v_f(1 - \rho/\rho_m)]$ at each fixed time t for this problem, as the speed is constrained by the given traffic density in this formulation we are taking. Since $\mathcal{H}$ is linear in u, it follows that the minimum occurs at one of the endpoints $u = 0$ or $u = v_f(1 - \rho/\rho_m)$, hence the control is bang-bang. Since $\mathcal{V}_x$ is negative (travel time value decreases as a function of x), hence to minimize the travel time we choose the maximum possible speed, which is

$$u(t) = v(\rho) = v_f(1 - \frac{\rho}{\rho_m}). \tag{4.10}$$

The dynamics of the expected travel time are given by the PDE

$$\mathcal{V}_t + \mathcal{V}_x u(t) + \frac{\sigma^2}{2} \Delta \mathcal{V} + 1 = 0. \tag{4.11}$$

Replacing the value of optimal control $u(t)$ in the above equation we obtain the HJB equation for expected travel time as follows

$$T_t + v_f(1 - \frac{\rho}{\rho_m})T_x + \frac{\sigma^2}{2}\Delta T + 1 = 0, \tag{4.12}$$

and the Fokker Planck equation for the evolution of the pdf of the state is given by

$$\begin{aligned} \rho_t(x,t) - \frac{\sigma^2}{2}\Delta\rho(x,t) + \nabla \cdot (\rho v(x,u,\rho,t)) = 0, \\ \rho(x,0) = \rho_0(x). \end{aligned} \tag{4.13}$$

Equations (4.12) and (4.13) represent the coupled PDE system for the Mean Field Games.

Now in the case of vanishing viscosity equation (4.12) becomes the PDE shown here, for the travel time field.

$$T_t(x,t) + T_x(x,t)v(\rho) + 1 = 0. \tag{4.14}$$

Hence, in this chapter we have derived the travel time dynamics as well as the Greenshield's traffic velocity, which provides the LWR model by minimizing the travel time for each agent in the mean field.

CHAPTER 5

Stationary Mean Field Games and Traffic

In order to get some understanding of the system behavior we study the stationary dynamic problem (where the time variation has become zero) instead of the time varying one, as the time varying system is much more complex. This gives us a solution in steady state and we can understand how the system would be after transients have settled down. In this chapter we will discuss the stationary mean field games for the two inverse problems of traffic formulated in previous sections.

5.1 Stationary MFG for LWR Model

Using equations (3.18) and (3.19) we can write the stationary MFG equations. Stationary HJB equation for inverse problem of deriving LWR model is given as

$$\frac{\sigma^2}{2}\frac{d^2}{dx^2}\mathcal{V}(x) + v_f(1 - \frac{\rho}{\rho_m})\frac{d}{dx}\mathcal{V}(x) = 0, \tag{5.1}$$

and the corresponding stationary FPK equation is given as

$$-\frac{\sigma^2}{2}\frac{d^2}{dx^2}\rho(x) + \frac{d}{dx}(\rho v(\rho)) = 0. \tag{5.2}$$

Equations (5.1) and (5.2) show that the system in the steady state has position invariants. Equation (5.1) produces the following position invariant (a constant) K_1,

$$\frac{\sigma^2}{2}\frac{d}{dx}\mathcal{V}(x) + v_f(1 - \frac{\rho}{\rho_m})\mathcal{V}(x) = K_1. \tag{5.3}$$

Similarly, equation (5.2) produces the following position invariant (a constant) K_2,

$$-\frac{\sigma^2}{2}\frac{d}{dx}\rho(x) + \rho v(\rho) = K_2. \tag{5.4}$$

Equation (5.4) can be rearranged in the form

$$\int \frac{d\rho}{a\rho^2 + b\rho + c} = \int dx$$

and hence its closed form solution can be obtained from

$$x = \frac{2\tan^{-1}\left((2a\rho + b)/(4ac - b^2)^{1/2}\right)}{(4ac - b^2)^{1/2}} + \text{constant term},$$

where

$$a = \frac{-2v_f}{\rho_m\sigma^2},\ b = \frac{2v_f}{\sigma^2} \text{ and } c = \frac{-2K_2}{\sigma^2}.$$

5.1.1 Existence of Unique Solutions

Equations (5.3) and (5.4) can be rearranged as follows:

$$\mathcal{V}\prime(x) = -\frac{2}{\sigma^2}v_f(1 - \rho/\rho_m)\mathcal{V}(x) + \frac{2}{\sigma^2}K_1 =: F_1(\rho, \mathcal{V}) \tag{5.5}$$

$$\rho\prime(x) = \frac{2}{\sigma^2}v_f(1-\rho/\rho_m)\rho - \frac{2}{\sigma^2}K_2 =: F_2(\rho,\mathcal{V}) \tag{5.6}$$

and accompanied by initial conditions $(\rho_0, \mathcal{V}_0)$.

We have the existence of a unique solution to the above initial value problem (5.5)-(5.6), since $F_1, F_2, \frac{\partial F_1}{\partial \rho}, \frac{\partial F_1}{\partial \mathcal{V}}, \frac{\partial F_2}{\partial \rho}, \frac{\partial F_2}{\partial \mathcal{V}}$ are continuous in a region that encloses the initial condition.

5.1.2 Stability Analysis

Using the parameters listed in table 5.1 (column two), we solve for the critical points (i.e. equilibrium solutions) of (5.3)-(5.4) and the resulting direction field is shown in figure 5.1.

We can analyze the stability of this non linear system near a critical point. To obtain the corresponding locally linear system we will try to find the Jacobian as follows:

$$J = \begin{bmatrix} F_{1_\mathcal{V}} & F_{1_\rho} \\ F_{2_\mathcal{V}} & F_{2_\rho} \end{bmatrix} = \begin{bmatrix} -\frac{2}{\sigma^2}v_f(1-\frac{\rho}{\rho_m}) & \frac{2}{\sigma^2}v_f\frac{\mathcal{V}}{\rho_m} \\ 0 & \frac{2}{\sigma^2}v_f - \frac{4}{\sigma^2}v_f\frac{\rho}{\rho_m} \end{bmatrix}$$

and evaluating at the critical point (67.9,67.9) we get

$$J = \begin{bmatrix} -0.294 & 1.105 \\ 0 & -0.811 \end{bmatrix}$$

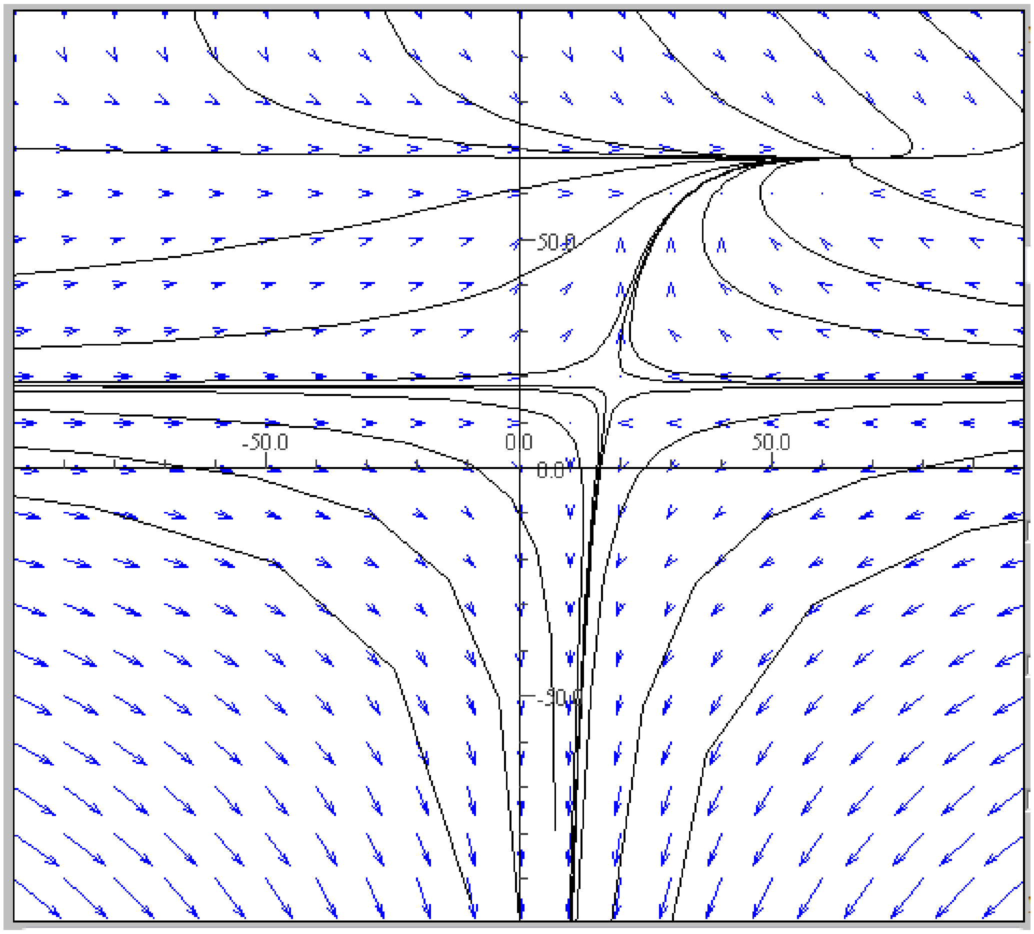

Figure 5.1: Direction Fields for Inverse Problem-1

Let r_1, r_2 be the eigenvalues of the linear system described above. We find

$$r_1 = -0.294, r_2 = -0.811$$

As $r_1 < 0$ and $r_2 < 0$ we can conclude that the critical point $(67.9, 67.9)$ is an asymptotically stable nodal sink.

Now evaluating the above equation at the critical point $(18.09, 18.09)$ and calculating the eigenvalues we get

$$r_1 = 0.811, r_2 = -1.105$$

As $r_2 < 0 < r_1$ we can conclude that the critical point $(18.09, 18.09)$ is an unstable saddle point.

For solving (5.3) and (5.4), we need to provide the initial conditions $\mathcal{V}(0)$ and $\rho(0)$ as well as the values of the two constants K_1 and K_2. The solution obtained from the second order ODEs (5.1) and (5.2) is the same as the one obtained from the first order ODEs (5.3) and (5.4) provided the values of the boundary derivatives and the constants are chosen to match each other at the boundary. The matching boundary conditions become:

$$\frac{\sigma^2}{2}\frac{d}{dx}\mathcal{V}(x)\Big|_{x=0} + v_f(1 - \frac{\rho}{\rho_m})\mathcal{V}(0) = K_1 \tag{5.7}$$

and

$$-\frac{\sigma^2}{2}\frac{d}{dx}\rho(x)\Big|_{x=0} + \rho v(\rho(0)) = K_2. \tag{5.8}$$

Using the parameters listed in table 5.1, numerical results are obtained for (5.3) and (5.4) and the results are shown in figure 5.2. We used MATLAB to numerically solve the ODEs. ODE45, the inbuilt ODE solver in MATLAB was used which is based on an explicit Runge-Kutta (4,5) formula with a variable time step for efficient computation. The $\sigma = 0$ column of table 5.1 corresponds to the equilibrium solution.

Table 5.1: Parameters for Simulation 1

Parameter	Values			
σ	0	10	20	30
$\rho(0)$	67.9	40	30	20
$\mathcal{V}(0)$	67.9	80	90	120
K_1	1000	1000	1000	1000
K_2	1000	1000	1000	1000
ρ_m	86	86	86	86
v_f	70	70	70	70

In figure 5.2 we plot the value function $\mathcal{V}(x)$, traffic density $\rho(x)$, the control action $u(x)$, and the traffic flow $f(x)$ with respect to the spatial coordinate x, as the dependence on time t is gone in the steady state. Notice that equation (5.4) is a first order nonlinear ordinary differential equation with a quadratic drift term. On the other hand equation (5.3), although has first order derivative in terms of the value $\mathcal{V}(x)$, but because of the presence of the density ρ, it shows a second order behavior with respect to x. We also notice the convergence of the plots towards the non-viscous case ($\sigma = 0$), as $x \to \infty$.

5.2 Stationary MFG for Travel Time PDE

Using equations (4.12) and (4.13) we can write the stationary MFG equations . Stationary HJB equation for inverse problem of deriving Travel Time dynamics is

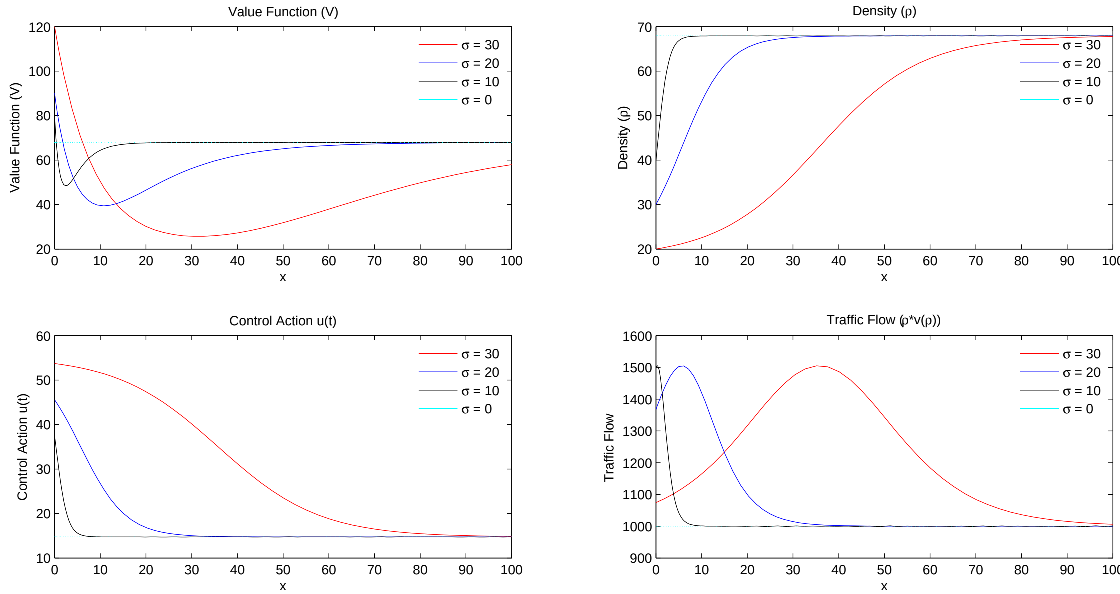

Figure 5.2: Numerical Results for Inverse Problem-1

given as

$$\frac{\sigma^2}{2}\frac{d^2}{dx^2}T(x) + v_f(1 - \frac{\rho}{\rho_m})\frac{d}{dx}T(x) + 1 = 0, \tag{5.9}$$

and the corresponding stationary FPK equation is given as

$$-\frac{\sigma^2}{2}\frac{d^2}{dx^2}\rho(x) + \frac{d}{dx}(\rho v(\rho)) = 0. \tag{5.10}$$

Equations (5.9) and (5.10) also show that the system in the steady state has position invariants. Equation (5.9) produces the following position invariant (a constant) K_3,

$$\frac{\sigma^2}{2}\frac{d}{dx}T(x) + v_f(1 - \frac{\rho}{\rho_m})T(x) + x = K_3. \tag{5.11}$$

Similarly, equation (5.10) produces the following position invariant (a constant) K_4,

$$-\frac{\sigma^2}{2}\frac{d}{dx}\rho(x) + \rho v(\rho) = K_4. \tag{5.12}$$

5.2.1 Existence of Unique Solutions

Equations (5.11) and (5.12) can be rearranged as follows:

$$T\prime(x) = -\frac{2}{\sigma^2}v_f(1-\rho/\rho_m)T(x) + \frac{2}{\sigma^2}(K_3 - x) =: F_1(\rho, T) \tag{5.13}$$

$$\rho\prime(x) = \frac{2}{\sigma^2}v_f(1-\rho/\rho_m)\rho - \frac{2}{\sigma^2}K_4 =: F_2(\rho, T) \tag{5.14}$$

and accompanied by initial conditions (T_0, ρ_0).

We have the existence of a unique solution to the above initial value problem (5.13)-(5.14), since $F_1, F_2, \frac{\partial F_1}{\partial \rho}, \frac{\partial F_1}{\partial T}, \frac{\partial F_2}{\partial \rho}, \frac{\partial F_2}{\partial T}$ are continuous in a region that encloses the initial condition.

5.2.2 Stability Analysis

Using the parameters listed in table 5.2 (column two) and for $x = 100$, we solve for the critical points (i.e. equilibrium solutions) of (5.11)-(5.12) and the resulting direction field is shown in figure 5.3.

For (5.11) and (5.12), we need to provide the conditions $T(\ell)$, which is 0, and $\rho(\ell)$

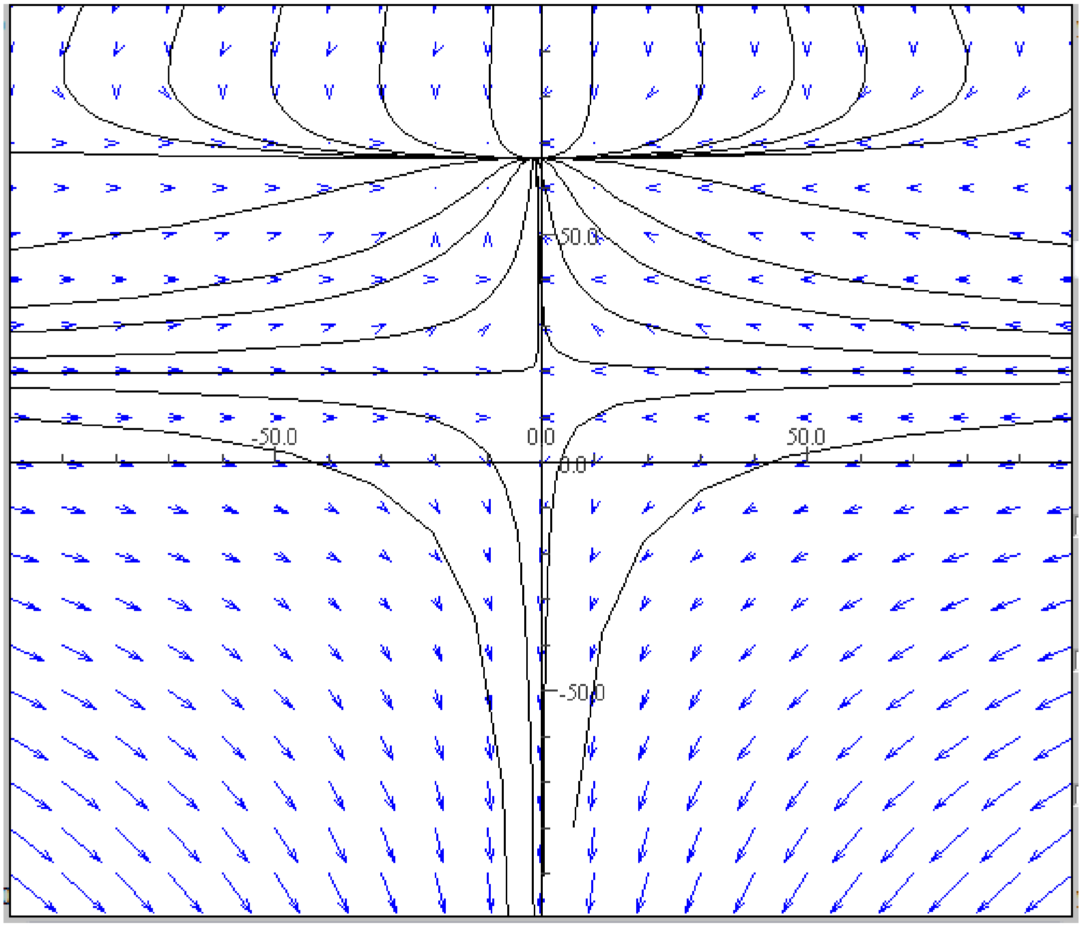

Figure 5.3: Direction Fields for Inverse Problem-2

as well as the values of the two constants K_3 and K_4. We can provide the matching conditions for this case as well so that the second order and the first order ODEs become equivalent.

We can also provide the corresponding initial conditions $T(0)$ and $\rho(0)$ as well as the values of the two constants K_3 and K_4. The solution obtained from the second order ODEs (5.9) and (5.10) is the same as the one obtained from the first order ODEs (5.11) and (5.12) provided the values of the boundary derivatives and the constants are chosen to match each other at the boundary. The matching boundary conditions become:

$$\frac{\sigma^2}{2}\frac{d}{dx}T(x)\Big|_{x=0} + v_f(1-\frac{\rho}{\rho_m})T(0) = K_3 \tag{5.15}$$

and

$$-\frac{\sigma^2}{2}\frac{d}{dx}\rho(x)\Big|_{x=0} + \rho v(\rho(0)) = K_4. \tag{5.16}$$

Similarly as in stationary MFG of LWR, for the travel time stationary case, we have equations (5.11) and (5.12) that are parallel to equations (5.3) and (5.4). We observe that equation (5.12) is the same as equation (5.4) and hence will have the same solution as we provided for that. However, equation (5.11) has an additional term x in the left hand side. This equation is still a linear ODE, although space varying. The closed form solution of this ODE will be more complex than that of equation (5.3).

Using the parameters listed in table 5.2, numerical results are obtained for (5.11) and (5.12) and the results are shown in figure 5.4. We used MATLAB to numerically solve the ODEs. ODE45, the inbuilt ODE solver in MATLAB was used which is based on an explicit Runge-Kutta (4,5) formula with a variable time step for efficient computation. The $\sigma = 0$ column of table 5.1 corresponds to the equilibrium solution. For the case of zero viscosity, we get a constant value of density ρ (66.6). Using this value in the non-viscous version of equation (5.11), we get the stationary travel time

$T(x)$ as a linear function of x as compared to a constant function that we obtained for the equation (5.3), and as expected at the right boundary, $T(x)$ is equal to zero. The convergence results are also similar as for the stationary LWR case. Which means that the behavior of the model for a given viscosity, i.e. for a given σ, as $x \to \infty$, is the same as in the non-viscous case.

Table 5.2: Parameters for Simulation 2

Parameter	Values			
σ	0	10	20	30
$\rho(0)$	66.6	40	30	20
$T(0)$	5	20	40	60
K_3	80	80	80	80
K_4	1050	1050	1050	1050
ρ_m	86	86	86	86
v_f	70	70	70	70

It is very interesting to note that for the non-viscous case the travel time function is a monotonic function of x as vehicles can not cross each other according to the deterministic model for drivers, which leads to the speed being a function of density. However, in the stochastic differential equations case, since there is stochasticity, two vehicles at the same location can have different speeds as they would be two different samples. Hence, in that case, the travel time function does not have to be monotonic.

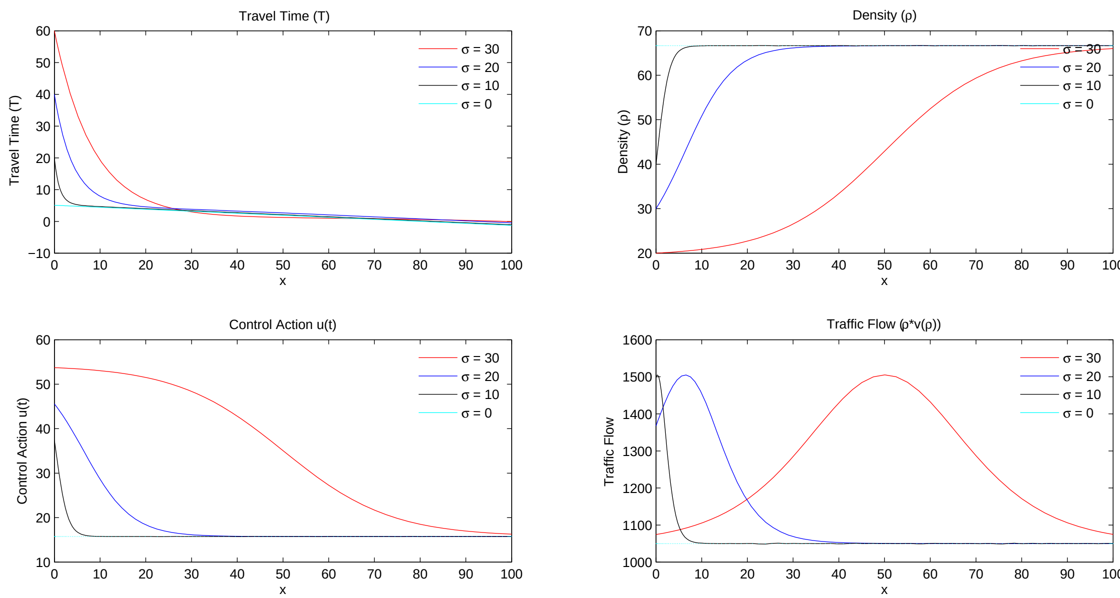

Figure 5.4: Numerical Results for Inverse Problem-2

CHAPTER 6

Conclusion

This thesis solved two inverse problems using Mean Field Games. In order to derive the classic traffic LWR model based on specific driver behavior cost leading to that model, we identified the costs functions whose solutions through mean field games lead to the derivation of LWR model for traffic. We also derived the travel time spatio-temporal model obtained as a solution to an inverse problem. The paper then discussed the stationary mean field games and solved the two inverse problems numerically for the stationary case.

In our models, we have shown how the microscopic driver behavior leads to the classic Greenshield's fundamental relationship between traffic density and traffic speed, as well as the well known LWR conservation law dynamics for traffic. We then enhanced the formulation to also show how microscopic driver behavior based on travel time considerations also produce the very significant distributed parameter model for travel time dynamics. The analysis of the stationary versions of the models showed behavior that is consistent with long term expectation of the evolution.

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