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Galois Groups of CM Fields in Degrees 24, 28, and 30

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Abstract

Given a CM field K of degree $2n$, there is a triple (G, H, ρ) called an imprimitivity structure in which G is the Galois group of the Galois closure of K, H is the subgroup of G that fixes K, and $\rho \in G$ is a distinguished central order 2 element that is induced by complex conjugation. Dodson showed in [2] that imprimitivity structures may be identified under the action of G into equivalence classes called ρ-structures. He determined all possible ρ-structures for $n = 3, 4, 5$ and 7 and a partial list for $n = 6$. In [15], Zoller completed the list of ρ-structures for $n = 6$, produced complete lists for $n = 8, 9$, and 10, and laid the foundation for the completion of $n = 12$. In the present investigation we continue the study for $n = 12$ as well as begin the study for $n = 14$ and 15.

Nondegenerate CM types have rank $n+1$ and make up the majority of CM types for a given CM field. In contrast, degenerate CM types have rank less than $n+1$ and occur less frequently. Dodson was concerned with the identification of degenerate CM types and their relationship with CM fields that contain an imaginary quadratic subfield. Zoller extended this investigation from $n = 6$ to $n = 8, 9, 10$, and 12. He also found and characterized degenerate CM types arising from a CM field that do not contain an imaginary quadratic subfield for $n = 8, 9$, and 10. In the case $n = 12$ he determined the types of CM fields that contain an imaginary quadratic subfield.

We continued the study in degree $2n = 24$ by looking at ρ-minimal groups and using a subgroup analysis. The difficulty in getting a complete picture is that there are over 19,000 Galois groups of CM fields in degree 24. So we use these special methods along with Zoller's study of CM fields containing an imaginary quadratic subfield to find bounds on the number of degenerate types. For $n = 14$, we find and characterize degenerate CM types arising from a CM field that may or may not contain an imaginary quadratic subfield. For the larger order Galois groups in degree $2n = 28$, we adapt the subgroup analysis from degree 24 to lessen computation time. For $n = 15$, we begin to find and characterize degenerate CM types arising from a CM field that may or may not contain an imaginary quadratic subfield. We find similar results to the $n = 9$ case.

Chapter 1

Introduction

1.1 Notation

Recall that a permutation group of degree d is a subgroup of S_d. The programming library GAP was used to create a database of transitive permutation groups of degree at most 32. We used this database extensively in the course of this investigation. See [5] for more information. The database was accessed using the computational algebra system Magma [1]. We adopt the following notation to make our discussion compatible with the entries in the database: Let $T_{d,k}$ denote the group numbered k in the GAP database of degree d transitive permutation groups.

It may be useful to identify transitive permutation groups using more familiar notation. When the degree $d \leq 19$, we may use the database of Klüners and Malle, available at [7]. Unless otherwise noted, all such identifications were made using this resource.

1.2 Theory

In order to study Galois groups of CM fields, we first want to establish some of the theory. A CM field is a totally imaginary quadratic extension of a totally real field. A more precise definition is as follows:

Definition 1.2.1. An extension F of $\mathbb{Q}$ is called *totally real* if the image of F is contained in $\mathbb{R}$ under any embedding $F \to \mathbb{C}$. The field K is a *CM field of degree* $2n$ if K contains a totally real field K_0 with $[K_0 : \mathbb{Q}] = n$ and $[K : K_0] = 2$ such that the image of K is not contained in $\mathbb{R}$ under any embedding $K \to \mathbb{C}$.

1.2.1 Examples of CM Fields

For a first example of a CM field, let $d \in \mathbb{Z}^+$ be squarefree and $K_0 = \mathbb{Q}\left(\sqrt{d}\right)$. Then K_0 is a totally real field with $[K_0 : \mathbb{Q}] = 2$. Let $K = K_0\left(\sqrt{-1}\right) = \mathbb{Q}\left(\sqrt{d}, \sqrt{-1}\right)$. Then K is a CM field of degree 4.

Next, let ζ_m be a primitive m^{th} root of unity, and let $K_0 = \mathbb{Q}(\zeta_m + \zeta_m^{-1})$. Then K_0 is totally real with $[K_0 : \mathbb{Q}] = \frac{\phi(m)}{2}$, where ϕ is the Euler-ϕ function. Let $K = K_0(\sqrt{(\zeta_m + \zeta_m^{-1})^2 - 4}) = \mathbb{Q}(\zeta_m)$. Then K is a CM field of degree $\phi(m)$.

1.2.2 The Imprimitivity Sequence of a CM Field

For the remainder of this chapter let K be a CM field of degree $2n$ with a totally real subfield K_0. Let K^C and K_0^C be the Galois closures of K and K_0 respectively. Let G =Gal$(K^C/\mathbb{Q})$ and G_0 =Gal$(K_0^C/\mathbb{Q})$. Fix an embedding of K^C into $\mathbb{C}$ and identify K^C with its image under this embedding. We note that because K is an imaginary extension of a totally real field, the center of G must contain an element ρ of order 2 corresponding to complex conjugation. Moreover, there is an exact sequence associated to any CM field, which we present based on a slightly different form Dodson gave in [2].

Lemma 1.2.2. There is an exact sequence relating G and G_0:

$$0 \to \mathrm{Gal}(K^C/K_0^C) \to G \to G_0 \to 1.$$

Moreover, there is a positive integer v with $1 \leq v \leq n$ such that $\mathrm{Gal}(K^C/K_0^C) \cong (\mathbb{Z}_2)^v$.

Galois theory accounts for the existence of this exact sequence. We call such a sequence an *imprimitivity sequence*.To show that the isomorphism $\mathrm{Gal}(K^C/K_0^C) \cong$

$(\mathbb{Z}_2)^v$ holds, we use the argument and notation in [2]. Let $\delta \in K_0$, let $K = K_0(\sqrt{-\delta})$, and let $\delta_1, \ldots, \delta_n$ be the conjugates of δ over $\mathbb{Q}$. Then we have $K^C = K_0^C(\sqrt{-\delta_1}, \ldots, \sqrt{-\delta_n})$. Each automorphism ι of K^C that fixes K_0^C is determined by the images of the maps $\sqrt{-\delta_j} \mapsto \epsilon_j\sqrt{-\delta_j}$ with $\epsilon_j = \pm 1$ and $1 \leq j \leq n$. Thus, $\mathrm{Gal}(K^C/K_0^C)$ may be identified with the image in $(\mathbb{Z}_2)^n$ of the map $\iota \mapsto (e_1, e_2, \ldots, e_n) \in (\mathbb{Z}_2)^n$, where e_j is defined by $\epsilon_j = (-1)^{e_j}$. Recall that the center of G always contains an element ρ of order 2 corresponding to complex conjugation. The image of ρ in $(\mathbb{Z}_2)^n$ is $(1, 1, \ldots, 1)$, so we must have $v \geq 1$.

We now turn to a theorem that describes G, G_0, and $(\mathbb{Z}_2)^v$ in greater detail. Following Dodson in [2], we refer to it as the Imprimitivity Theorem. Recall that a permutation group G acting on a set X is called *imprimitive* if the action of G produces nontrivial partitions of X. The nontrivial partitions of X are called *sets of imprimitivity.*

Theorem 1.2.3. Suppose K is a CM field of degree $2n$. Then:

1. $G = \mathrm{Gal}(K^C/\mathbb{Q})$ may be represented as an imprimitive permutation group of degree $2n$ with n sets of imprimitivity of order 2.

2. $G_0 = \mathrm{Gal}(K_0^C/\mathbb{Q})$ may be represented as a transitive permutation group of degree n and may be identified as the group of permutations of the sets of imprimitivity in the representation of G as a permutation group.

3. $(\mathbb{Z}_2)^v \cong \mathrm{Gal}(K^C/K_0^C)$ is identified with the group of permutations preserving the sets of imprimitivity and is acted upon by G_0 by permutation of coordinates under an inclusion $i : (\mathbb{Z}_2)^v \hookrightarrow (\mathbb{Z}_2)^n$.

1.2.3 Imprimitivity Structures

The Imprimitivity Theorem specifies the structure of the non-identity groups of an imprimitivity sequence. We may abbreviate an imprimitivity sequence by listing the triple (G, G_0, ρ), where $\rho \in (\mathbb{Z}_2)^v$. In the course of the following discussion we may refer to an imprimitivity sequence by referring to this triple instead. We now define another triple that will be useful in the course of our investigation.

Definition 1.2.4. Suppose K is a CM field of degree $2n$ and that $G =$Gal$(K^C/\mathbb{Q})$ is a transitive permutation group of degree $2n$. Let H be the subgroup of G that fixes K, and let $\rho \in G$ be the central order 2 element induced by complex conjugation. Then we call the triple (G, H, ρ) an *imprimitivity structure* for K.

Note that G may have more than one central order 2 element, for which the corresponding CM fields may have different structures.

1.2.4 CM Types

The study of CM fields is interesting in its own right, but it also has important connections to the study of certain objects in algebraic geometry called abelian varieties (which are projective varieties that generalize elliptic curves). Suppose A is a simple abelian variety with endomorphism ring $End(A)$. Then the center of the endomorphism algebra $End(A) \otimes \mathbb{Q}$ is either a totally real field or a CM field. We may also construct certain abelian varieties from CM fields if we consider the notion of a CM type.

Definition 1.2.5. A *CM type* (K, Φ) consists of a CM field K and a set $\Phi = \{\phi_1, \ldots, \phi_n\}$ of n non-conjugate embeddings $K \to \mathbb{C}$.

Following the example of previous treatments of CM fields, we may refer to Φ as a type when the CM field K is understood from the context.

Given a type (K, Φ), it is possible to construct an n-dimensional abelian variety A so that K is isomorphic to a subalgebra of $End(A) \otimes \mathbb{Q}$. See [8] or [14] for a careful treatment of the connections between abelian varieties and CM fields.

Given a CM field K, we may construct 2^n types on K as follows. Let $\Phi = \{\phi_1, \ldots, \phi_n\}$ be a set of n non-conjugate embeddings $K \to \mathbb{C}$. Note that $\Phi \subset \{\phi_1, \overline{\phi}_1, \ldots, \phi_n, \overline{\phi}_n\}$, where $\overline{\phi}$ denotes the complex conjugate of ϕ. A type (K, Φ) is specified by making n choices from among the sets $\{\phi_i, \overline{\phi}_i\}$, for $1 \leq i \leq n$. Thus, K has one of 2^n types (K, Φ).

The Galois group $G =$Gal$(K^C/\mathbb{Q})$ acts on Φ in the following way. Recall that we have identified K^C with its image under an embedding $K^C \to \mathbb{C}$. Then each

$\phi \in \Phi$ may be written as an embedding $K \to K^C$. The group G acts on each Φ in the collection of types by sending Φ to $\Phi^g = \{\phi_1^g, \dots, \phi_n^g\}$: given $\phi \in \Phi$ and $x \in K$, $\phi^g(x)$ denotes the result of g acting on $\phi(x)$ under the fixed embedding.

We consider two examples of CM types. First, let $K = \mathbb{Q}(\sqrt{d}, \sqrt{-1})$ for some squarefree positive integer d. For $x = a_1 + a_2\sqrt{d} + a_3\sqrt{-1} + a_4\sqrt{-d} \in K$, set $\phi_1(x) = x$ and $\phi_2(x) = a_1 - a_2\sqrt{d} + a_3\sqrt{-1} - a_4\sqrt{-d}$. Then $\Phi = \{\phi_1, \phi_2\}$ is a CM type on K.

Our second example comes from the work of Shimura and Taniyama [14]. Let $p \in \mathbb{N}$ be an odd prime, let $n = \frac{p-1}{2}$, let ζ_p be a primitive p^{th} root of unity, and let $K = \mathbb{Q}(\zeta_p)$. For each $i = 1, \cdots, n$, set $\phi_i(\zeta_p) = \zeta_p^i$. Then the ϕ_i are pairwise non-conjugate; hence, $\Phi = \{\phi_1, \cdots, \phi_n\}$ is a type on K.

A CM field K may contain a subfield K_1 that is also a CM field. When this occurs, we would like to know whether or not a type on K is determined by a type on K_1. This leads us to the following definition:

Definition 1.2.6. Let (K, Φ) be a CM type, and suppose that (K_1, Φ_1) is a CM type with K_1 a subfield of K. We say Φ is a *lift* of Φ_1 if the restriction of any embedding $\phi \in \Phi$ to K_1 is contained in Φ_1. A type (K, Φ) is *primitive* if K is not a lift of any of its CM subfields. Otherwise, (K, Φ) is called *reducible.*

Primitive types are minimal with respect to field containment, so they merit special attention in the study of CM types.

Definition 1.2.7. Let (K, Φ) be a CM type with $\Phi = \{\phi_1, \dots, \phi_n\}$. Let M_1 be the $\mathbb{Z}$-module spanned by $\{\phi_1, \bar{\phi_1}, \dots, \phi_n, \bar{\phi_n}\}$. Let M_2 be the sub-module of M_1 spanned by $\{\Phi^g | g \in G\}$. Then the *rank* of Φ, $t(\Phi)$, is the rank of M_2.

Definition 1.2.8. If (K, Φ) is primitive and $t(\Phi) < n + 1$, then (K, Φ) is called *degenerate.* If $t(\Phi) = n + 1$, then (K, Φ) is primitive and is called *nondegenerate.*

We sometimes refer to the orbit of a type as degenerate or nondegenerate, so we may find the following definition useful:

Definition 1.2.9. Let (K, Φ) be a CM type, let G be the Galois Group of K^C, and let $\mathcal{O}$ be an orbit of Φ under the action of G. Then $\mathcal{O}$ is called *degenerate* (or *reducible* or *nondegenerate*) if (K, Φ) is degenerate (or reducible or nondegenerate).

The problem of finding degenerate types is motivated by questions in geometry and number theory. Degenerate types correspond to abelian varieties with special properties. Among them are the algebraic relations between certain transcendental numbers p_K studied by Shimura in [12] and [13]. When (K, Φ) is degenerate, these relations are non-trivial. In particular, Shimura's Theorem 2.5 of [13] gives $(n+1-t(\Phi))$ relations among the transcendental numbers arising as the periods of abelian integrals.

In [9], Pohlmann explains Tate's reasoning that degeneracy of an abelian variety is the condition for there to be exceptional Hodge cycles, which are those not in the subring generated by divisors. These cycles are the ones not known to be algebraic. For such homology classes, we do not know that there is a representative given by an algebraic variety, the conjecture of Hodge. Pohlmann then gives an explicit construction of the example of Mumford of a dimension 4, degenerate abelian variety, the existence of which was proposed by Tate.

Additionally, in [10], Ribet proposes another application of the rank of a type. Let $A[p^n]$ be the points of order p^n on the abelian variety A with complex multiplication, and $P(x)$ be the function for which $P(n)$ is the degree of the field extension given by adjoining the points of $A[p^n]$, as in the theory of complex multiplication. Then Ribet says that $P(x)$ is a polynomial of degree d, where d is the rank of the type. In most cases $d = \dim(A) + 1$, or nondegenerate. Thus, the growth rate is being cut down by the extra Hodge cycles in the degenerate case. Also, Ribet discusses three examples of degenerate types different than Mumford's example. In [15], Zoller verified these results in his study of CM types in degrees 16 and 18.

Reflex Fields

In addition to the ranks of types, the sizes of orbits of types are also of interest. The sizes of orbits are directly related to reflex fields, which we define as follows:

Definition 1.2.10. The *reflex field* of K is the subfield K' of K^C fixed by the stabilizer of Φ under the action of G.

In [8], Lang shows that K' is a CM field and that (K, Φ) determines a type $(K, \Phi)'$ which we also denote by (K', Φ'). In general, for $((K, \Phi)')' = (K', \Phi')' = (K'', \Phi'')$, we have $K'' \subseteq K$ with equality if and only if (K, Φ) is primitive. It is worth noting that $[K' : \mathbb{Q}]$ is the index of the stabilizer of the orbit of Φ under the action of G, and hence, the size of the orbit of Φ under this G-action. For more information about reflex fields and their relationship to abelian varieties, see [8].

1.2.5 Analysis of Non-Cyclic Centers

Suppose that G is a transitive permutation group of degree $2n$ with a center of even order. An isomorphism of G with the Galois group of the Galois closure of a CM field determines a unique central element ρ of order 2 corresponding to complex conjugation. If the center of G is cyclic, then G has a unique central element of order 2. Thus, G has a unique imprimitivity structure (G, H, ρ).

However, if the 2-Sylow subgroup of the center of G is non-cyclic, then G has more than one central element of order 2. In this situation, the central order 2 elements may be distinguished according to whether or not they are induced by complex conjugation under an isomorphism with the Galois group of the Galois closure of a given CM field.

ρ-Structures

To motivate the notion of a ρ-structure, we present an example from Zoller in [15] motivated by [2]. Let $G = T_{8,2} \cong \mathbb{Z}_2 \times \mathbb{Z}_4$ in degree 8. G has three central elements of order 2, which we label ρ_1, ρ_2, and ρ_3, where $\rho_1 \in \mathbb{Z}_2 \times \langle 0 \rangle, \rho_2 \in \langle 0 \rangle \times \mathbb{Z}_4$, and $\rho_3 = \rho_1 \cdot \rho_2$. There are three imprimitivity structures corresponding to each ρ, (G, H, ρ_i) for $i = 1, 2, 3$. Since G is abelian, $H = \langle 0 \rangle \times \langle 0 \rangle$ is the identity subgroup of G.

We can also identify the imprimitivity sequences $(G, G_0^i, \rho_i), i = 1, 2, 3$ associated to G. These sequences are not distinct with respect to the image group G_0 and the value of v. We have $G_0^1 = G_0^3 = T_{4,1} \cong \mathbb{Z}_4$ with $v = 1$ for both and $G_0^2 = T_{4,2} \cong \mathbb{Z}_2 \times \mathbb{Z}_2$ with $v = 1$. Additionally, both (G, G_0^1, ρ_1) and (G, G_0^3, ρ_3) give identical partitions of 2^4, namely $2+2+4+8$, whereas (G, G_0^2, ρ_2) gives the partition $4+4+8$.

From this, it appears that (G, G_0^1, ρ_1) and (G, G_0^3, ρ_3) are equivalent in some sense, while (G, G_0^2, ρ_2) clearly differs. Since this phenomenon occurs frequently in our study, we define it in the following way relevant to our work, though a more general definition can be given:

Definition 1.2.11. Let G be a transitive permutation of degree $2n$ with an even order center. Let H be a subgroup of G such that $(G : H) = 2n$ and G acts effectively on the cosets of $H \subset G$, and let $\rho_1, \rho_2 \in G$ be central elements of order 2. Then (G, H, ρ_1) is said to be *ρ-equivalent* to (G, H, ρ_2) if there exists an automorphism ψ of G such that $\psi(H) = H$ and $\psi(\rho_1) = \rho_2$. A *ρ-structure* of degree $2n$ is an equivalence class of triples (G, H, ρ) under ρ-equivalence.

It is important to note that different ρ-structures correspond to unique CM fields and hence unique CM types.

Type Structures

To motivate the notion of a type structure, we again turn to an example presented in [15]. Let $G = T_{12,26} \cong A_4 \times (\mathbb{Z}_2)^2$, and let $\rho_i \in G, i = 1, 2, 3$ be central elements of order 2. We have $H \cong (\mathbb{Z}_2)^2$ for each imprimitivity structure (G, H, ρ_i) where H is a non-normal subgroup of A_4. The imprimitivity sequences (G, G_0^i, ρ_i) share the same image group $G_0^i = T_{6,6} \cong A_4 \times \mathbb{Z}_2$, the same value for v ($v = 1$), and the same partition of 2^6, $8 + 8 + 24 + 24$. Thus, we would expect them all to be ρ-equivalent. However, Zoller found that (G, H, ρ_2) is ρ-equivalent to (G, H, ρ_3) but not (G, H, ρ_1). To account for this subtle difference, we have the following definition:

Definition 1.2.12. Let G be a transitive permutation of degree $2n$ with an even order center. Let H be a subgroup of G such that $(G : H) = 2n$ and G acts effectively on the cosets of $H \subset G$, and let $\rho_1, \rho_2 \in G$ be central elements of order 2. For $i = 1, 2$, let Ω_i be the collection of orbits of types for the ρ-structure (G, H, ρ_i).

Then (G, H, ρ_1) is said to be *type-equivalent* to (G, H, ρ_2) if there exists an automorphism ψ of G such that $\psi(H) = H$ and $\psi(G_0^1) = G_0^2$ and for each $\mathcal{O} \in \Omega_1$, the corresponding $\mathcal{O}' \in \Omega_2$ has the same size and rank. A *type-structure* of degree $2n$ is an equivalence class of triples (G, H, ρ) under type-equivalence.

We have seen an example that shows that type equivalence is a weaker equivalence than ρ-equivalence. We do have the following lemma:

Lemma 1.2.13. If two imprimitivity structures are ρ-equivalent, then they are type equivalent.

Proof. Suppose (G, H, ρ_1) and (G, H, ρ_2) are ρ-equivalent. Then there is an automorphism ψ of G fixing H so that $\psi(\rho_1) = \rho_2$. For $i = 1, 2$, let Ω_i be the collection of orbits of the subgroup of G generated by ρ_i. Because $\psi(\rho_1) = \rho_2$, when we compare Ω_1 to Ω_2, we find a $1-1$ correspondence between orbits, orbit sizes, and orbit ranks. $\square$

In [2], Dodson began looking into the problem of finding sizes and ranks of orbits of types for CM fields of degree up to $2n = 12$. The case of finding degenerate types in degree $2n = 8$, dimension $n = 4$, verified the result of Mumford described by Pohlmann in [9]. In [15], Zoller continued this study for composite dimensions up to $n = 12$. We summarize their results in Chapter 2. I have continued this study in higher dimensions by analyzing a more complete picture in degree $2n = 24$, in Chapter 3, and finding results on degenerate types in degrees 28 and 30 in Chapter 4 and Chapter 5, respectively.

Chapter 2

Ranks of Types

2.1 Ranks of Types in Degree $2n$

There has been some work done on the problem of finding ranks of types in degree $2n$. Theorem 1.0 of [3] and the discussion following it give Lenstra's lower bounds r_0 on the ranks of primitive degenerate types. They are $r_0^1 = \log_2 n + 2$ when n is a power of 2 and $r_0^2 = \lfloor \log_2 n \rfloor + 3$ when n is composite but not a power of 2.

We now present a theorem by Ribet, in [11], in the case when $n = p$ is a prime:

Theorem 2.1.1 (Ribet)**.** Let (K, Φ) be a CM type with K a CM field of degree $2n$. If $n = p$ is prime and (K, Φ) is primitive, then (K, Φ) is nondegenerate.

In [2], Dodson found a converse of Ribet's Theorem that he presented in the language of abelian varieties. We restate it here using our notion of CM fields:

Theorem 2.1.2 (Dodson)**.** Let $n > 4$ be composite and factor n as $n = kl$, with $k > 2, l \geq 2$. Then there exist primitive degenerate CM types (K, Φ) with K a CM field of degree $2n$. In particular, $t(\Phi) = n - l + 2$.

2.2 Ranks of Types in Degrees 8, 12, 16, 18, and 20

As we mentioned in Chapter 1, we are also interested in specific ranks and sizes of orbits of types. Dodson in [2] and Zoller in [15] began working on this problem in the first 5 degrees with n a composite number. We will present summaries of their findings, but to find the specific information about orbits of types in these degrees, see [2] and [15].

2.2.1 Ranks of Types in Degree 8

The first example of finding a degenerate simple Abelian variety in the case of $n = 4$ was due to Mumford following a footnote in a paper of Tate. Dodson [2] continued the study by finding the possible values of $t(\Phi)$ for $n = 4$. Mumford's example, $2n = 8$ is characterized as occurring only when the Galois group, G_0, of the Galois closure, K_0^C, of the totally real field, K_0, is the alternating group, A_4, or the symmetric group, S_4, with just one degenerate orbit of size 6. Also, the CM-field contains an imaginary quadratic subfield. We summarize this information with the following theorem:

Theorem 2.2.1 (Dodson)**.** *Let K be a CM field of degree 8. Then K has a degenerate type if and only if K contains an imaginary quadratic subfield, and G_0 is either A_4 or S_4 with just one degenerate orbit of size 6.*

2.2.2 Ranks of Types in Degree 12

For $2n = 12$, Dodson began finding possible ranks and sizes of orbits in [2] and gave a complete list of orbits of types in [4]. In [15], Zoller confirmed that degenerate types only occur in the following cases: there is an imaginary quadratic subfield, and there is a degenerate orbit for 15 of the 16 possible G_0 groups, with the only exception being $G_0 \cong S_3$. Moreover, the only degenerate rank that occurs is 6. We summarize this information with the following theorem:

Theorem 2.2.2 (Dodson, Zoller)**.** Let K be a CM field of degree 12. Then K has a degenerate type if and only if K contains an imaginary quadratic subfield and $G_0 \ncong S_3$. There are 15 ρ-structures with 21 degenerate orbits of types, all of which have rank 6.

2.2.3 Ranks of Types in Degree 16

In [15], Zoller found that there are 1,746 transitive permutation groups in degree 16 with even order centers. They produce 1,940 type structures. One issue that Zoller had to consider regarding primitive degenerate types was whether orbits of size 8 and rank 5 were degenerate or reducible. Lenstra's lower bound for degree 16 gives that the smallest rank a degenerate orbit could have is 5. To check, he had to consider the existence of a reflex field, which would be in degree 8 and have an orbit of size 16 and rank 5. In general, we know the following:

Proposition 2.2.3. Let K be a CM field of degree $2n$ with Galois group G and with an orbit $\mathcal{O}$ of size $2s$, $s < n$, and rank $r, r_0 \leq r \leq n+1$ from Lenstra's lower bound. If there exists a reflex field K' of degree $2s$ with an orbit of size $2n$ and rank r and with Galois group $G' \cong G$, then $\mathcal{O}$ is degenerate.

This proposition stems from our discussions on reflex fields and primitive types in the previous chapter. Notice that if no such reflex field exists, then the orbit is reducible. From Zoller's work, we find that there is a Galois group of a CM field of degree 16 containing an imaginary quadratic subfield without a primitive degenerate type. The group $T_{16,3}$ has reducible orbits of size 8 and rank 5. We now make a substantial clarification to the theorem that Zoller established that characterizes degenerate types in degree 16:

Theorem 2.2.4 (Zoller)**.** There are 50 Galois groups of CM-fields of degree 16 that contain an imaginary quadratic subfield, and all but one of them have a primitive degenerate type. There are 207 degree 16 transitive permutation groups that are Galois groups of CM-fields not containing an imaginary quadratic subfield but having a primitive degenerate type. The degenerate orbits have rank 5, 7, or 8.

2.2.4 Ranks of Types in Degree 18

Zoller found that there are 138 transitive permutation groups in degree 18 with even order centers, each having a unique ρ-structure. He also found that not all possible values of v occur; the Magma computations showed that only odd values of v occur, $v = 1, 3, 5, 7, 9$. Additionally, not all Galois groups of CM fields containing an imaginary quadratic subfield correspond to a CM field with a primitive degenerate type. The following theorem characterizes degenerate types in degree 18:

Theorem 2.2.5 (Zoller)**.** There are 34 degree 18 transitive permutation groups that are Galois groups of CM-fields containing an imaginary quadratic subfield, 25 of which correspond to a CM-field having a primitive degenerate type, and 9 of which do not. Altogether, there 31 ρ-structures corresponding to primitive degenerate types. The degenerate orbits have rank 6 or 8.

2.2.5 Ranks of Types in Degree 20

There are 452 transitive permutation groups in degree 20 with even order centers. Zoller found that they produce 458 distinct type structures. He found that the only possible values of v are $1, 2, 5, 6, 9$ and 10. Following a careful review of Zoller's work, we have the following theorem characterizing degenerate types in degree 20:

Theorem 2.2.6 (Zoller)**.** There are 45 Galois groups of CM-fields of degree 20 that contain an imaginary quadratic subfield, and all of them have a primitive degenerate type. There are 71 degree 20 transitive permutation groups that are Galois groups of CM-fields not containing an imaginary quadratic subfield but having a primitive degenerate type. The degenerate orbits have rank 6, 7, or 10.

Chapter 3

Analysis of Degree 24

3.1 Imaginary Quadratic Subfields

In [15], Zoller began to look at Galois groups of CM fields in degree 24. There are 19,126 transitive permutation groups in degree 24 with even order centers. Due to the large number of types, 2^{12}, and the significant increase in number of groups, Zoller wanted to begin looking at this case by working on a specific subset of these groups. He began by finding all 301 groups that are Galois groups of CM fields containing an imaginary quadratic subfield, which we list in Appendix B.1.3. These groups are of the form $G \cong G_0 \times \langle \rho \rangle$. The presence of an imaginary quadratic subfield, K_1, is indicated by an orbit of size 2. In [15], he found the sizes and ranks of orbits for the first 291 of the 301 groups. For the remaining cases, we found a degenerate orbit for the first 7 of these 10, as indicated in B.1.5. See below for the final 3.

The study of imaginary quadratic subfields has led us to the following principle:

Conjecture 3.1.1. Let G be a transitive permutation group with an even order center in degree $2n$, where $n = 2m, m > 3$. If G is the Galois group of a CM field containing an imaginary quadratic subfield, then the CM field has a primitive degenerate type, except for a few small groups G.

To see this, we consider the orbits of weight (m, m). By this, we mean the convention taken in the constant weight criterion in §3 of [2]. The conjecture is that there will be at least one degenerate orbit from this weight. In particular, we know that if there is a primitive type of weight (m, m), then it is degenerate. This follows from the discussions of the constant weight criterion in [2], which uses concepts from §1 regarding an alternate form of the Imprimitivity Theorem and from §2 regarding a more complete discussion of ρ-structures. The only missing step in this heuristic is that if $m > 3$, then there exists a primitive type in all but a few exceptional cases. In practice, degenerate orbits for small G are known. In the remaining cases of groups G, we are left with large, multiply-transitive G_0. For such G, we can construct specific types of weight (m, m) and rank $2m$.

In Appendix B.1.3, we list the groups Zoller found, in [15], that are Galois groups of CM fields of degree 24 containing an imaginary quadratic subfield. As we explained, we were able to find degenerate orbits for the first 4 groups of the 10 remaining after Zoller's work. For the others, it suffices to explain the existence of a primitive type. Reducible types only occur for orbits of size less than $2n$, or, specifically in this case, 24. The largest groups, $T_{24,24815}$ and $T_{24,24747}$, have S_{12} and A_{12} as respective image groups G_0, which are 12- and 10-transitive. More specifically, these groups are 6-transitive on the 12 coordicates of $\mathbf{f} = (1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0)$ corresponding to the type ϕ^0 from the trivial ρ-structure defined in [2]. Thus, the orbit $G_0(\mathbf{f})$ will have rank $\dim([\ ]_{6,6}) = 12$ and size $\binom{12}{6} = 924 > 24$. The next largest group is $T_{24,22335} \cong \mathbb{Z}_2 \times (S_6 \wr \mathbb{Z}_2)$. The image group G_0 is isomorphic to $S_6 \wr \mathbb{Z}_2$, and so we can view one S_6 as permuting the first 6 coordinates and the other S_6 permuting the other coordinates. Thus, the size of the orbit will be larger than 24 and so the type will be primitive degenerate.

It is worth noting the exception we have found to the above principle. In Zoller's study of degree 16 in [15], he found that $G = T_{16,3}$ had orbits of size 2 and rank 2, size 8 and rank 5, and size 16 and rank 9. However, we see from [7] that $G = T_{16,3} \cong (\mathbb{Z}_2)^4$. Thus, G is not isomorphic to any degree 8 transitive permutation group and the orbits of rank 5 are reducible. This is then an example of a Galois group of a CM field containing an imaginary quadratic subfield without a primitive degenerate type. From this, we might also suspect a similar result for $G \cong (\mathbb{Z}_2)^5$ in degree 32.

3.2 ρ-Minimal Groups

In [2], Dodson found that one way to study the partitions of 2^n that can arise from orbits of CM-types of a CM-field with Galois closure G, a transitive permutation group, is to study the partitions of 2^n that arise from certain transitive subgroups of G. He explored this by looking at the minimally transitive permutation groups in degree 6 to study the case $n = 6$, or degree 12. In [15], Zoller discussed how to translate this idea to the transitive permutation groups that are minimal with respect to properties of the Galois group of a CM-field. We begin with some definitions.

Definition 3.2.1. A *minimally transitive permutation group*, M, is a transitive permutation group with no proper transitive subgroups.

Definition 3.2.2. A *ρ-minimal transitive permutation group*, G, is a transitive permutation group of degree $2n$ that has a distinguished element $\rho \in Z(G)$ of order 2 such that for any proper transitive subgroup, M, of G, $\rho \notin M$ and $G = M \cup \rho M$.

Note that M must be a minimally transitive group. Otherwise, for $M' \subset M \subset G$, $G \neq M' \cup \rho M'$.

These groups are of particular interest because of the following proposition:

Proposition 3.2.3. Let (G, H, ρ) be an imprimitivity structure of degree $2n$, and let (G', H', ρ') be an imprimitivity structure of degree $2n$ such that G is a transitive subgroup of G'. If a partition of 2^n for (G, H, ρ) is $2^n = a_1 + \cdots + a_k$ then the partition of 2^n for (G', H', ρ') must have summands that are either in the set $\{a_1, \ldots, a_k\}$ or are sums of these numbers.

Proof. The summands in the partition of 2^n describe the orbits of types for a CM field associated to each imprimitivity structure. The number of summands in the partition is the number of orbits of types, and the size of each orbit is the value of each summand. Since G' contains G as a transitive subgroup, the orbits of types for a CM field with imprimitivity structure given by (G', H', ρ') must be unions of orbits of types for a CM field with imprimitivity structure given by (G, H, ρ). $\square$

We also take advantage of this proposition when doing the subgroup analysis in the next section.

3.2.1 Constructing ρ-Minimal Groups in Degree 24

Since G must have only an index 2 minimally transitive subgroup, we begin constructing ρ-minimal groups by finding minimally transitive groups M in degree 24 without a central order 2 element. In [6], Hulpke found the minimally transitive groups in all degrees through 30. Due to the Imprimitivity Theorem, we need G_0 to be a minimally transitive group in degree 12 that acts on the sets of imprimitivity of G, and consequently M. Moreover, the kernel of the action of M on the sets of imprimitivity corresponds to the value of v. Thus, we construct the order 2 permutation ρ from a partition of 24 comprised of 12 sets of size 2. Letting $G = \langle \rho \rangle \times M$, we verify that G has only M as a proper transitive subgroup.

We note that if G_0 is not minimally transitive in degree 12, then G will have a subgroup corresponding to the subgroup of G_0. Also, if $v > 1$ but G_0 is minimally transitive, we may still have a proper transitive subgroup of G of index greater than 2. Hence, verifying that M is the only subgroup is of great importance. The Magma program we used to construct ρ-minimal groups is in Appendix A.2.

Example of ρ-Minimal Groups in Degree 24

Zoller originally looked at the minimally transitive permutation group $T_{24,1489}$ in order to build a ρ-minimal group. We expand on his findings here.

Proposition 3.2.4. There is one ρ-minimal group whose only transitive subgroup is $T_{24,1489}$.

Proof. First, we note that $T_{24,1489}$ has a trivial center, and hence does not contain ρ. Now, we examine the action of $T_{24,1489}$ on its minimal partitions. The group $T_{24,1489}$ has 3 such partitions, each containing 12 blocks of size 2, which we label P_i for $i = 1, 2, 3$. Magma can find the image and kernel of the action of $T_{24,1489}$ on each of these block systems, which we label I_i and K_i, respectively, for $i = 1, 2, 3$. Let $\rho_i \in S_{24}$ be the order 2 element such that if $j, k \in P_i$ then $\rho_i(j) = k$ and $\rho_i(k) = j$.

Magma calculates that $|K_1| = |K_3| = 1$ and $|I_1| = |I_3| = 576$. However, these abstractly isomorphic images are the distinct transitive permutation groups of degree 12, $T_{12,160}$ and $T_{12,162}$, respectively. Note that, by abstractly isomorphic, we mean the groups are isomorphic but are not conjugate subgroups of S_{12}. We find in [6] that only $T_{12,162}$ is minimal. Magma also calculated that $|K_2| = 16$ and $|I_2| = 36$, and we found that I_2 is the degree 12 minimal transitive permutation group $T_{12,17}$ of order 36.

Let $G_i = M \cup \rho_i M$ for $M = T_{24,1489}$, as in the second part of the definition of ρ-minimal groups. We used Magma to find the proper transitive subgroups of G_1, G_2, and G_3 and found that both G_1 and G_2 had $T_{24,76}$ of order 72 as a subgroup, failing condition 2 of the definition. Conversely, G_3 contained only $T_{24,1489}$ as a proper transitive subgroup, and so G_3 is ρ-minimal. Using the Magma command TransitiveGroupIdentification, which finds the number of the group in the Transitive Group Database, we see that G_3 is the group $T_{24,2799}$. $\square$

Remark 1. During this exploration, we found that $T_{24,2799}$ is abstractly isomorphic to the transitive groups $T_{24,2781}$, $T_{24,2798}$, $T_{24,2800}$, $T_{24,2801}$, and $T_{24,2802}$, meaning they are isomorphic groups but not conjugate subgroups of S_{24}. First, using the identification in Magma, we found that G_1 is the group $T_{24,2802}$ and G_2 is the group $T_{24,2798}$, both not ρ-minimal. Moreover, $T_{24,2801}$ is found in [6] as a minimally transitive permutation group of degree 24, and hence it is ρ-minimal by condition 1.

In order to try to classify the last 2 groups, we found the groups abstractly isomorphic to $T_{24,1489}$, which are $T_{24,1491}$, $T_{24,1505}$, $T_{24,1506}$, and $T_{24,1508}$. We started by running through the same calculations for $T_{24,1491}$ as we found for $T_{24,1489}$.

Proposition 3.2.5. There is one ρ-minimal group whose only transitive subgroup is $T_{24,1491}$.

Proof. The group $T_{24,1491}$ has minimal partitions P_1, P_2, P_3; however, only P_1 has 12 sets of imprimitivity of size 2, and thus is the only candidate for ρ. Defining ρ in the same manner as before, and letting $G = M \cup \rho M$ for $M = T_{24,1491}$, we obtain the group $T_{24,2781}$. This group has only $T_{24,1491}$ as a proper transitive subgroup, and so it is ρ-minimal. $\square$

None of the groups $T_{24,1505}$, $T_{24,1506}$, $T_{24,1508}$ had a minimal G-invariant partition that contained a candidate for ρ. So, we did a proper transitive subgroup analysis on $T_{24,2800}$ and found that its only proper transitive subgroup is $T_{24,76}$ of order 72.

3.2.2 Ranks of Types

Combining this information, we have the following theorem:

Theorem 3.2.6. Together, there are 131 minimal and ρ-minimal groups that are Galois groups of CM fields in degree 24. Of these, 17 are Galois groups of CM fields containing an imaginary quadratic subfield, all of which have a primitive degenerate type. There are 47 that are Galois groups of CM-fields not containing an imaginary quadratic subfield but having a primitive degenerate type. The degenerate orbits have ranks 6, 7, 8, 9, 10, 11, and 12, which are all possible degenerate ranks.

3.3 Subgroup Analysis

With the previous 2 subsets of the 19,126 groups in degree 24, namely those corresponding to imaginary quadratic subfields and ρ-minimal groups, we have found the sizes and ranks of orbits for 348 groups. This leaves much of the degree 24 case unknown. To continue the study a bit more, we used Magma to compute the sizes and ranks of orbits of types for groups of order up to $9{,}216 = 2^{10} \cdot 3^2$, which we summarize in Appendix B.1.5. This accounts for groups with an index between 1 and 9,960, leaving groups with an index between 9,961 and 25,000 without any information other than the 2 above cases.

For the remaining groups, we used Proposition 3.2.3 about unions of orbits. We found 58 groups of order 96, 120, and 192 with all nondegenerate orbits. There are

30 of these groups having no proper transitive subgroups with all nondegenerate orbits, and every group of order less than 96 has a reducible or degenerate orbit, as seen in Appendix B.1.1. Then we checked whether the groups from index 9,961 to 25,000 had one of these 30 groups as a subgroup. If so, then they will have only nondegenerate orbits. The remaining groups may or may not have a primitive degenerate type.

This process left us with well under one half of the groups of large order. For example, for indexes between 18,000 and 20,000, 1,187 groups had at least one of the 30 groups as a subgroup while 502 did not. This reduction allowed us to compute sizes and ranks of orbits for groups with indexes up to 19,997 and to find those with a degenerate orbit, as seen in Appendix B.1.5. In Appendix B.1.4, we list all groups without one of the 30 groups with all nondegenerate orbits as a subgroup for which we were unable to find sizes and ranks of orbits.

3.3.1 Examples of Unions of Orbits

We will consider 2 unions of orbits exemplifying Proposition 3.2.3 in the case where G' is a group with only nondegenerate orbits. First let $G' = T_{24,85}$ of order 96. Then G' has only one proper transitive subgroup, $G = T_{24,1} = \mathbb{Z}_{24}$. The group G has 2 reducible orbits of size 8 and rank 5 and 170 nondegenerate orbits of size 24 and rank 13. The group G' has all nondegenerate orbits, 2 of which are of size 32 and 42 of size 96. Thus, we see that the unions occurred in the following ways: 2 instances of an orbit of size 8 union an orbit of size 24 and 42 instances of a union of 4 orbits of size 24.

Next, let $G' = T_{24,328}$ of order 192. The group G' has 5 proper transitive subgroups, one of which is $G = T_{24,85}$. We know the orbit sizes and ranks of G. The group G' has all nondegenerate orbits, as well: 2 orbits of size 32, 10 orbits of size 96, and 16 of size 192. Thus, there were 16 instances of a union of 2 orbits of size 96.

3.3.2 Bounds on Number of Degenerate Types

We have found 265 groups of order at most 192 with degenerate orbits. Next, we found that there are 3,425 groups of order between 192 and 9,216 without a subgroup of order 96, 120, or 192 having only nondegenerate orbits. The other 4,604 groups had at least one such subgroup. Of the 3,425 groups, 1,581 had a degenerate type. Then, among the groups of order greater than 9,216, there are 3,602 without a subgroup of order 96, 120, or 192 having only nondegenerate orbits. The other 7,049 groups had at least one such subgroup. There are 381 groups of order between 9,216 and 393,216 with a degenerate orbit. Finally, we found 12 groups of order greater than 393,216 with a degenerate orbit, 588 without a degenerate orbit, and 21 for which we were unable to compute any orbits. We know 3 of the 21 groups correspond to CM fields with imaginary quadratic subfields and 8 groups have $v = 11$ or 12. The order 393,216 corresponds to groups of index up to 19997 in Appendix B.1.4.

Summarizing the previous information, we have the following theorem:

Theorem 3.3.1. There are 19,126 transitive permutation groups in degree 24 with even order centers. There are at least 2,248 groups that are Galois groups of CM fields with a primitive degenerate type. Additionally, there are at least 16,868 groups in degree 24 that are Galois groups of CM fields without a primitive degenerate type.

Remark 2. We note that it would have been sufficient to only consider order 96 subgroups. Of the 11,653 groups with a subgroup of order 96, 120, or 192 having only nondegenerate orbits, roughly 10,000 of them had one of the order 96 groups as a subgroup. This would have still reduced the original problem to a feasible computation. Moreover, all of the groups with such a subgroup must have one of the following lists of orbits or unions of them: 2 orbits of size 32 and 42 of size 96; 8 orbits of size 32 and 40 of size 96; or 4 orbits of size 24, 2 of size 32, 14 of size 48, and 34 of size 96.

Chapter 4

Analysis of Degree 28

4.1 Galois Groups of Order up to $2^{17} \cdot 7$

In degree 28, each CM field has 2^{14} types, so as the sizes of the Galois groups increase, the computation time becomes unwieldy. Although there are only 957 groups with even order centers in degree 28 as opposed to 19,126 in degree 24, it is still unreasonable to compute the orbits for every one of these groups. So, we wanted to compute as many as possible before adapting the subgroup analysis in degree 24 to the final cases in degree 28.

There are 641 groups of order up to 917,504 $= 2^{17} \cdot 7$. We were able to compute the sizes and ranks of orbits of types in these cases, which we summarize in Appendix C.1. Once the order of the groups exceeds this, the computation time significantly increases. We were able to identify 284 of the remaining 316 groups as having all nondegenerate orbits. Of the remainders, we hope to find those corresponding to CM fields with an imaginary quadratic subfield, as well as those with $v = 13$ or 14.

There are 56 groups of the 641 that are Galois groups of CM fields containing an imaginary quadratic subfield. All of them had a primitive degenerate type. The remaining 7 Galois groups of CM fields containing an imaginary quadratic subfield have order greater than 917,504. Additionally, of the 641 groups, 68 are Galois groups of CM fields not containing an imaginary quadratic subfield but having a primitive degenerate type.

4.2 Subgroup Analysis

We adapted the subgroup analysis described in degree 24 to the final 316 groups in degree 28. There are 32 of these that do not have a subgroup with all nondegenerate orbits. Again, 7 of these correspond to CM fields containing an imaginary quadratic subfield. We know these will have a primitive degenerate type. For the remaining 25 groups, we can find the image group G_0 and the value of v. If $v = 13$ or 14, then the partition of 2^{14} will be either $2^{13} + 2^{13}$ or 2^{14}.

4.2.1 Analysis of Large Groups

The first large order group without one of the designated groups as a subgroup is $G = T_{28,1238}$ of order 1,376,256 $= 2^{16} \cdot 3^1 \cdot 7^1$. The group G has image group $G_0 = T_{14,50}$ and $v = 7$. Although our hope is to see large values of v, this only guarantees us that the smallest orbit has size 128. This is one of the few groups of order greater than 917,504 for which we were able to compute complete information on the sizes and ranks of orbits of types. For this group, there are 2 degenerate orbits of size 128 and rank 8.

Next, we have $G_1 = T_{28,1272}$ and $G_2 = T_{28,1273}$ of order 1,605,632 $= 2^{15} \cdot 7^2$. Both groups have image group $G_0 = T_{14,12}$ and $v = 13$. Thus, one is the split group with partition $2^{13} + 2^{13}$ and the other is nonsplit with partition 2^{14}. The Magma computations completed and show that G_1 is split and G_2 is nonsplit.

The next group is $G = T_{28,1360}$ of order 2,752,512 $= 2^{17} \cdot 3^1 \cdot 7^1$. The group G has 3 possible choices for ρ, and each ρ-structure has image group $G_0 = T_{14,51}$ and $v = 7$. Similar to $G = T_{28,1238}$, the Magma computations show that for each ρ-structure, there are 2 orbits of size 128 and rank 8.

Next, for $G = T_{28,1376}$ of order 3,211,264 $= 2^{16} \cdot 7^2$, we find $G_0 = T_{14,12}$ and $v = 14$. Thus, G must have partition 2^{14}. Furthermore, we find that G has both $G_1 = T_{28,1272}$ and $G_2 = T_{28,1273}$ as subgroups.

The next 4 groups, $G_1 = T_{28,1408}$, $G_2 = T_{28,1409}$, $G_3 = T_{28,1410}$, and $G_4 = T_{28,1411}$, are all of order 4,816,896 $= 2^{15} \cdot 3^1 \cdot 7^2$. The groups G_1 and G_2 both have image group $G_0 = T_{14,22}$ and $v = 13$. Thus, one is the split group with partition $2^{13} + 2^{13}$ and the other is nonsplit with partition 2^{14}. The groups G_3 and G_4 both have image group $G_0 = T_{14,23}$ and $v = 13$. Thus, we have that one is split and the other is nonsplit.

Next, we consider $G = T_{28,1545}$ of order 20,643,840 $= 2^{16} \cdot 3^2 \cdot 5 \cdot 7$. The group G has one ρ and has image group $G_0 = T_{14,53}$ with $v = 7$. This is the first case where we might have a degenerate orbit, in particular of size 128, but were unable to compute the orbits of types.

We next consider the 3 groups $G_1 = T_{28,1606}$, $G_2 = T_{28,1613}$, and $G_3 = T_{28,1614}$ all of order 41,287,680 $= 2^{17} \cdot 3^2 \cdot 5 \cdot 7$. The group G_1 has 3 possible choices for ρ, all with the same image group $G_0^1 = T_{14,56}$ and $v = 7$. Both G_2 and G_3 have a unique ρ and have $G_0^2 = T_{14,54}, G_0^3 = T_{14,55}$ with $v = 7$ for both. Since G_1 has 3 possible choices for ρ, we believe that each will correspond to a CM field having a primitive degenerate type.

Now, let $G = T_{28,1659}$ of order $82,575,360 = 2^{18} \cdot 3^2 \cdot 5 \cdot 7$. The group G has 3 possible choices for ρ, all with the same image group $G_0 = T_{14,57}$ and $v = 7$. Since G has 3 possible choices for ρ, we believe that each will correspond to a CM field having a primitive degenerate type.

Finally, we have $G = T_{28,1682}$ of order 203,212,800 $= 2^{11} \cdot 3^4 \cdot 5^2 \cdot 7^2$. G has a unique ρ and has image group $G_0 = T_{14,61}$ and $v = 2$. The remaining 17 groups may or may not correspond to CM fields containing an imaginary quadratic subfield, so we analyze them further now.

Imaginary Quadratic Subfields

First, we see that $T_{28,1217}$ is the only group with $G_0 = T_{14,57}$ and $v = 1$. Thus, $T_{28,1217}$ is the Galois group of a CM field containing an imaginary quadratic subfield. Moreover, $G = T_{28,1217}$ was one of the few for which we were able to compute the orbits. We found that G has 2 choices for ρ with the same type structure having primitive degenerate orbits of rank 8.

Both $T_{28,1573}$ and $T_{28,1574}$ have image group $G_0 = T_{14,58}$ and $v = 1$. However, the computations took to long to find an orbit of size 2 and rank 2, so until we find a more efficient program, we are unable to distinguish which one corresponds to the CM field with an imaginary quadratic subfield.

Both $T_{14,59}$ and $T_{14,60}$ have the same order, 25,401,600 $= 2^8 \cdot 3^4 \cdot 5^2 \cdot 7^2$. There are 6 groups with order 50,803,200 $= 2^9 \cdot 3^4 \cdot 5^2 \cdot 7^2$. The indexes are $1628, 1629, 1630, 1631, 1632,$ and 1633. The group $T_{28,1628}$ has image group $G_0 = T_{14,58}$ and $v = 2$. Both $T_{28,1629}$ and $T_{28,1630}$ have image group $G_0 = T_{14,59}$ and $v = 1$. The other 3 groups have image group $G_0 = T_{14,60}$ and $v = 1$.

The group $T_{14,61}$ has order 50,803,200 $= 2^9 \cdot 3^4 \cdot 5^2 \cdot 7^2$. There are 6 groups with order 101,606,400 $= 2^10 \cdot 3^4 \cdot 5^2 \cdot 7^2$. The indexes are $1665, 1666, 1667, 1668, 1669,$ and 1670. The group $T_{28,1665}$ has image group $G_0 = T_{14,59}$ and $v = 2$ and $T_{28,1666}$ has image group $G_0 = T_{14,60}$ and $v = 2$. The rest have image group $G_0 = T_{14,61}$ and $v = 1$.

We know that $T_{14,62} = A_{14}$ and $T_{14,63} = S_{14}$. Moreover, there is only one group that has each of these as an image group G_0, $T_{28,1754}$ and $T_{28,1769}$, respectively. Thus, these must correspond to CM fields with an imaginary quadratic subfield. This completes the study of Galois groups of CM fields with an imaginary quadratic subfield with our current capabilities.

4.2.2 Bounds on Number of Degenerate Types

There are 124 groups of order at most 917,504 with degenerate orbits. We know there are an additional 7 corresponding to CM fields containing an imaginary quadratic subfield but were only able to find 3 of them. There are 517 groups of order at most 917,504 with no degenerate orbits. Additionally, there are 284 groups of order greater than 917,504 with a subgroup having only nondegenerate orbits. Other than these, we found 7 groups with $v \geq 13$, so they also do not have degenerate orbits.

We summarize this information in the following theorem:

Theorem 4.2.1. There are 957 transitive permutation groups in degree 28 with even order centers. There are 131 groups that are Galois groups of CM fields with a primitive degenerate type that are of order at most 917,504 or that correspond to a CM field containing an imaginary quadratic subfield. There are at least 808 groups in degree 28 that are Galois groups of CM fields without a primitive degenerate type.

Chapter 5

Analysis of Degree 30

5.1 Galois Groups of Order up to $2^9 \cdot 3^2 \cdot 5$

In degree 30, each CM field has 2^{15} types, so as the sizes of the Galois groups increase, the computation time becomes unwieldy. There are 643 groups in degree 30 with even order centers. Although there are even fewer groups than in degree 28, the Magma programs began timing out much sooner in the list of groups than in the list of groups for degree 28.

We were able to get complete information on the ranks and sizes of orbits of types for groups of order up to $23{,}040 = 2^9 \cdot 3^2 \cdot 5$, which we summarize in Appendix D.1.1. There are 176 such groups that are Galois groups of CM fields in degree 30. This leaves 467 groups for which we still have work to do.

Degree 30 is the first case since degree 18 with n an odd, composite number. Due to this, we expected to see some similar results. One of the most interesting results Zoller found in [15] was that there were 34 degree 18 groups corresponding to CM fields with an imaginary quadratic subfield, but 9 of them did not have a primitive degenerate type. Among the first 176 groups in degree 30, there are 64 groups that are Galois groups of CM fields with an imaginary quadratic subfield. This leaves 40 such groups among the remaining 467. There was one group of the 64 that had no degenerate orbits, $T_{30,566}$.

5.2 Subgroup Analysis

We adapted the subgroup analysis described in degree 24 to the final 467 groups in degree 30. There are 176 of these that do not have a subgroup with all nondegenerate orbits. Of these, 40 correspond to CM fields containing an imaginary quadratic subfield. We do not know, a priori, whether these will have a primitive degenerate type or not. For the remaining 136 groups, we can find the image group G_0 and the value of v. If 15, then the orbit must be of size of 2^{15}.

We initially considered the first 33 groups with only nondegenerate orbits of types. These groups were $T_{30,108}$ of order 480; $T_{30,179}$ of order 720; $T_{30,212}$ of order 960; $T_{30,261}$ of order 1440; $T_{30,325}$, $T_{30,326}$, $T_{30,327}$, $T_{30,335}$, $T_{30,344}$, and $T_{30,355}$ of order 1920; $T_{30,493}$, $T_{30,497}$, $T_{30,498}$, $T_{30,500}$, $T_{30,502}$, $T_{30,506}$, $T_{30,513}$, $T_{30,518}$, $T_{30,524}$, and $T_{30,525}$ of order 3840; $T_{30,569}$ of order 5760; and $T_{30,677}$, $T_{30,679}$, $T_{30,680}$, $T_{30,682}$, $T_{30,687}$, $T_{30,688}$, $T_{30,689}$, $T_{30,690}$, $T_{30,696}$, $T_{30,701}$, $T_{30,704}$, and $T_{30,705}$ of order 7680.

5.2.1 Analysis of Large Groups

From our previous list, we have eliminated 301 groups because they have subgroups with all nondegenerate orbits. These groups are the following:

{ 1076, 1077, 1078, 1082, 1089, 1097, 1098, 1099, 1188, 1189, 1190, 1191, 1196, 1197, 1273, 1277, 1282, 1285, 1291, 1292, 1305, 1307, 1308, 1370, 1422, 1426, 1428, 1472, 1473, 1474, 1475, 1476, 1477, 1481, 1484, 1488, 1491, 1493, 1503, 1598, 1600, 1608, 1609, 1655, 1656, 1657, 1659, 1663, 1665, 1668, 1730, 1733, 1735, 1748, 1750, 1751, 1836, 1842, 1845, 1846, 1893, 1894, 1908, 1911, 1913, 1914, 1918, 1922, 1923, 1924, 1925, 1926, 1928, 2014, 2015, 2016, 2023, 2122, 2126, 2127, 2129, 2235, 2237, 2240, 2244, 2325, 2374, 2375, 2380, 2489, 2525, 2526, 2600, 2602, 2605, 2609, 2663, 2664, 2667, 2864, 2871, 2876, 2967, 2968, 2971, 2980, 3018, 3019, 3023, 3024, 3026, 3238, 3241, 3244, 3253, 3254, 3255, 3261, 3366, 3367, 3369, 3370, 3397, 3398, 3654, 3657, 3659, 3665, 3666, 3673, 3677, 3678, 3683, 3684, 3687, 3717, 3781, 3785, 3788, 3792, 3794, 3804, 3813, 3818, 3819, 3993, 3994, 3998, 3999, 4000, 4007, 4013, 4016, 4017, 4019, 4022, 4024, 4026, 4027, 4051, 4053, 4057, 4106, 4107, 4111, 4112, 4118, 4121, 4122, 4124, 4128, 4135, 4138, 4140, 4142, 4145, 4147, 4150, 4283, 4286, 4288, 4289, 4292, 4294, 4295, 4303, 4305, 4306, 4307, 4309, 4310, 4333, 4389, 4390, 4391, 4397, 4402, 4403, 4405, 4409, 4410, 4412, 4414, 4489, 4490, 4491, 4495, 4498, 4402, 4403, 4405, 4409, 4410, 4412, 4414, 4489, 4490, 4491, 4495, 4498, 4503, 4504, 4505, 4508, 4512, 4517, 4634, 4635, 4637, 4638, 4639, 4641, 4688, 4689, 4690, 4695, 4696, 4697, 4711, 4717, 4718, 4824, 4826, 4831, 4859, 4860, 4861, 4874, 4875, 4883, 4884, 4885, 4886, 4887, 4890, 4893, 4962, 4964, 4966, 4986, 5014, 5015, 5016, 5053, 5054, 5055, 5140, 5142, 5199, 5226, 5227, 5234, 5263, 5264, 5265, 5280, 5286, 5287, 5289, 5297, 5322, 5330, 5331, 5333, 5338, 5339, 5341, 5342, 5344, 5345, 5348, 5371, 5372, 5373, 5377, 5398, 5399, 5402, 5420, 5694, 5696 }.

We still have 178 groups to analyze. These groups are the following:

{ 962, 965, 966, 967, 969, 972, 973, 978, 980, 984, 985, 994, 995, 1000, 1002, 1008, 1017, 1116, 1118, 1119, 1124, 1126, 1132, 1142, 1143, 1154, 1200, 1201, 1203, 1209, 1210, 1213, 1214, 1217, 1221, 1225, 1226, 1229, 1231, 1233, 1236, 1238, 1253, 1303, 1329, 1330, 1331, 1332, 1337, 1338, 1349, 1351, 1353, 1354, 1358, 1359, 1360, 1361, 1368, 1435, 1436, 1437, 1440, 1442, 1447, 1449, 1458, 1533, 1534, 1535, 1542, 1543, 1547, 1548, 1552, 1553, 1555, 1556, 1619, 1620, 1622, 1706, 1707, 1709, 1711, 1713, 1714, 1792, 1812, 1818, 1835, 1883, 1885, 1887, 1987, 1991, 1992, 1994, 1997, 1998, 2003, 2005, 2007, 2008, 2064, 2065, 2067, 2075, 2076, 2078, 2092, 2215, 2219, 2220, 2224, 2225, 2226, 2302, 2303, 2308, 2311, 2314, 2320, 2323, 2328, 2329, 2330, 2507, 2511, 2512, 2585, 2587, 2593, 2595, 2601, 2616, 2618, 2623, 2629, 2631, 2890, 2902, 2960, 2961, 2962, 2983, 2987, 2988, 2991, 2992, 2995, 2997, 2999, 3000, 3002, 3278, 3297, 3299, 3304, 3363, 3381, 3382, 3384, 3388, 3693, 3696, 3699, 3714, 3822, 3824, 3827, 4039, 4042, 4045, 4151, 4316, 5468, 5498 }.

We now try to get as much information about these groups as we can, although we do not have the computation time or power to find all the sizes and ranks of orbits of types. However, we can find G_0 and the value v for each of these. We know that they all have a unique ρ-structure. The information, organized according to G_0, is presented in Appendix D.1.2. Let us look at a few cases:

First, consider the three groups $G_1 = T_{30,2960}$, $G_2 = T_{30,2961}$, and $G_3 = T_{30,2962}$. From Table D.1, we see that they have image groups $G_0^1 = T_{15,19}$, $G_0^2 = T_{15,17}$, and $G_0^3 = T_{15,18}$, respectively, with $v = 15$ for each. Thus, they all have one nondegenerate orbit of size 2^{15}.

The groups $G_0^1 = T_{15,17}$, $G_0^2 = T_{15,18}$, and $G_0^3 = T_{15,19}$ are also the image groups for $G_1 = T_{30,2311}$, $G_2 = T_{30,2308}$, and $G_3 = T_{30,2302}$, each with $v = 13$.

5.2.2 Imaginary Quadratic Subfields

We noted that we are missing the information for 40 groups that are Galois groups of CM fields containing imaginary quadratic subfields. From the information in Table D.1, we can only specify 14 such groups. In order to determine the remaining 26, we would need to find an orbit of size 2 and rank 2. We present some information on what we have found.

The 14 groups are $T_{30,1017}$, $T_{30,1132}$, $T_{30,1154}$, $T_{30,1236}$, $T_{30,1238}$, $T_{30,1253}$, $T_{30,1458}$, $T_{30,2092}$, $T_{30,2328}$, $T_{30,2329}$, $T_{30,2330}$, $T_{30,2631}$, $T_{30,5468}$, and $T_{30,5498}$. Unlike when n is even, we have already seen an example here of a Galois group of a CM field containing an imaginary quadratic subfield without a primitive degenerate type.

As for $G = T_{30,566}$, the Galois group of a CM field containing an imaginary quadratic subfield but not having a primitive degenerate orbit, we would like to know why there is no degenerate type and possibly classify which groups might also fall into this category.

First, we note that G has image group $G_0 = T_{15,47} \cong A_7$, which is a not solvable, primitive, irreducible permutation group. In degree 18, of the 9 groups corresponding to CM fields containing an imaginary quadratic subfield but having no degenerate type, 4 of them have not solvable, primitive, irreducible image groups. The other 5 were solvable and primitive. There are 6 groups in degree 15 that are not solvable, primitive, and irreducible. They are $G_0^1 = T_{15,20} \cong A_6$, $G_0^2 = T_{15,28} \cong S_6$, $G_0^3 = T_{15,47} \cong A_7$, $G_0^4 = T_{15,72} \cong A_8$, $G_0^5 = T_{15,103} \cong A_{15}$, and $G_0^6 = T_{15,104} \cong S_{15}$.

Both A_6 and S_6 are image groups for degree 12 CM fields with imaginary quadratic subfields. Thus, K_1 and K_2 in degree 30 with Galois groups $G_1 = T_{30,172} \cong$

$T_{12,180}$ and $G_2 = T_{30,260} \cong T_{12,219}$ have reflex fields with an orbit of size 30. We can find these orbits and see they have rank 7, meaning K_1 and K_2 both have degenerate orbits of rank 7, which were also computed using the Magma programs.

We already commented on $G = T_{30,566}$ with $G_0 = T_{15,47} \cong A_7$. When computing image groups G_0 and values of v for all degree 30 groups with even order centers, we found that $G = T_{30,1154}$ is the Galois group of a CM field with image group $G_0 = T_{15,72} \cong A_8$. Magma was able to find the orbit sizes and ranks for this solitary group, and there were no degenerate orbits.

5.2.3 Analysis of Degenerate Orbits

We recall Lenstra's lower bound for ranks of degenerate types. For $n = 15$, we have that the lower bound is 6. Every degenerate orbit $\mathcal{O}$ from the first 176 groups has exactly one of the following two properties:

1. $\mathcal{O}$ is an orbit of a type under the action of a Galois group G with $v = 1$.
2. $\mathcal{O}$ is an orbit of size 32 and rank 6 for a Galois group G with $v = 5$ for which every other orbit is nondegenerate.

5.2.4 Bounds on Number of Degenerate Types

There are 83 groups of order at most 23,040 corresponding to CM fields with a degenerate type. We found 93 groups of order at most 23,040 without a degenerate orbit. All 104 groups with $v = 15$ are of order greater than 23,040. We also found 301 groups that have subgroups of order at most 7,680 with only nondegenerate orbits.

We summarize this information in the following theorem:

Theorem 5.2.1. There are 643 transitive permutation groups in degree 30 with even order centers. There are 83 groups that are Galois groups of CM fields with a primitive degenerate type that are of order at most 23,040. There are at least 498 groups in degree 30 that are Galois groups of CM fields without a primitive degenerate type.

Chapter 6

Conclusions

We have had much to say about Galois groups of CM fields of degrees 24, 28, and 30. Our analysis of degree 24 has left only 10 groups out of the total 19,126 for which we have to determine degeneracy. In degree 28, we have only 18 groups left. If we can find an efficient manner of classifying Galois groups of CM fields containing an imaginary quadratic subfield other than computing every orbit of types, we would be able identify the last 4 imaginary quadratic cases. We still have 97 groups in degree 30 that we have not analyzed yet. We have more to say about our thoughts regarding these groups.

6.1 Future Directions

We started each case by having Magma compute sizes and ranks of orbits of types for as many groups as possible with the time constraints through Lehigh's High Performance Computing. For degree 24, due the vast number of groups, we computed all sizes and ranks of orbits for every group of order up to 192. We then used the groups having only nondegenerate orbits to limit the computations to possible groups corresponding to degenerate orbits. We were then able to compute sizes and ranks of types for the limited number of groups of order up to 393,216.

For degrees 28 and 30, we computed sizes and ranks of orbits of types until the Magma programs could not finish running. We then found which remaining groups had no subgroups with only nondegenerate types. We were able to analyze this small number of groups in each degree, leaving few unknown cases.

Our work has thus focused primarily on sizes and ranks of orbits of types for CM fields having a primitive degenerate types. It is worth noting that the groups we eliminated from consideration in degrees 24, 28, and 30 might be useful when studying larger degrees. These groups might be useful in determining whether an orbit in degree $2n > 30$ is degenerate (by comparing the orbit and group to those of the reflex field) or reducible.

6.1.1 Degenerate Orbits

It is difficult to characterize Galois groups of CM fields in degree $2n$ with a primitive degenerate type. However, there seems to be similarities in the cases when n is odd.

Degeneracy in Odd Dimensions

In Zoller's study of degree 18 in [15], we can see that every degenerate orbit occurs for Galois groups with $v = 1$. In our study of degree 30, we found that among the first 176 groups in degree 30 with even order centers, all but 18 of the groups corresponding to CM fields with a primitive degenerate type have $v = 1$. So, we consider the following question:

Question 6.1.1. For degrees 18 and 30, almost every degenerate orbit occurred for a Galois groups G with $v = 1$. There were a limited number of degenerate orbits in degree 30 for which the Galois groups had $v = 5$. Do most degenerate orbits occur in an odd dimension for a Galois group with $v = 1$? Is there a way to characterize a group G or image group G_0 with $v > 1$ corresponding to CM field having a degenerate orbit?

We would like to look further into this phenomenon. This would involve further analysis of the remaining groups in degree 30. Also, we could consider the next cases with odd, composite dimension n, which are degrees $2n = 42$ and $2n = 50$.

6.1.2 Imaginary Quadratics in Even Dimensions

Another part of our study of each degree was looking at Galois groups of CM fields with an imaginary quadratic subfield. We mentioned in Chapter 3 that if n is an even composite number, a primitive type of weight $\frac{n}{2}, \frac{n}{2}$ will be degenerate, except in a few cases. In degrees 20, 24, and 28 every group found corresponding to a CM field with an imaginary quadratic subfield had a degenerate orbit. This motivates the following conjecture:

Conjecture 6.1.2. Let n be an even composite number with $n > 6$. Then every CM field in degree $2n$, except a few with small Galois groups G, containing an imaginary quadratic subfield has a primitive CM type of weight $\left(\frac{n}{2}, \frac{n}{2}\right)$ and hence a degenerate type.

Further study in larger even dimensions could provide us with more evidence. However, if we study the relationships between image groups for each n, we might find a better justification.

6.1.3 Imaginary Quadratics in Odd Dimensions

Regarding Galois groups of CM fields containing imaginary quadratic subfields in odd dimensions, we found similar results in our study of degree 30 as Zoller found in degree 18. We were able to identify 2 groups with no degenerate orbits. Both groups had primitive image groups, just as did those in degree 18. This leads us to ask the following question:

Question 6.1.3. For degrees 18 and 30, we found examples of Galois groups of CM fields containing imaginary quadratic subfields but having no degenerate types. The examples we found all had primitive image groups G_0. Is there a way to further classify the image groups of Galois groups of CM fields containing imaginary quadratic subfields but having no degenerate types?

We could get a better idea of the image groups of Galois groups of CM fields containing imaginary quadratic subfields but having no degenerate types if we look into degrees 42 or 50. As we have noted, not every primitive group in degree n corresponds to such a Galois group. We could also study these primitive groups to see why they provide exceptions in the cases we have studied.

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Appendix A

Magma Programs

A.1 Ranks and Sizes of Orbits of Types

In order to compute the ranks and sizes of orbits in degree $2n$, we used the following program adapted from Zoller in [15]:

```
n:=;
function OrbitsOfTypes(gpNum)
G:=TransitiveGroup(2*n,gpNum);
S:=SubgroupClasses(G:OrderEqual:=2);
RhoGroups:=[];
L:={@ @};
for i in [1..#S] do
if S[i]`length eq 1 then
Append( RhoGroups,S[i]`subgroup);
end if;
end for;
if #RhoGroups eq 1 then
print "There is a unique rho for this group.";
print " ";
```

```
else
print ”There are”,#RhoGroups,”rhos for this group.”;
print ” ”;
end if;
TypeSizes:=[];
OrbitSizes:=[];
SizeRankList1:=[];
SizeRankList2:=[];
count:=[];
for r in [1..#RhoGroups] do
print ”Information for rho #”,r,”:”;
print ” ”;
SetsOfTypes:={@ s : s in Orbits(RhoGroups[r]) @};
//makes an enumerated set of the partitions
Types:={@ {@ a1,...,an @} : a1 in SetsOfTypes[1],..., an in SetsOfTypes[n] @};
f:=Action(GSet(G));
O:=[];
// array containing at most 2^n orbits of types
Ob:={@ @};
// set containing distinct orbits of types
orbitSum:=0;
for i in [1..2^n] do
O[i]:={@ Types — @};
end for;
for i in [1..2^n] do
if orbitSum lt 2^n then
for g in G do
U:={@ @};
for j in [1..n] do
Include( U,f(Types[i][j],g));
end for;
```

```
Include( O[i],U);
end for;
// Calculate sum of sizes of orbits
Include( Ob,O[i]);
OrbitSizes[r]:=[ #Ob[i] : i in [1..#Ob] ];
Sort( OrbitSizes[r]);
orbitSum:=+OrbitSizes[r];
end if;
end for;
SizeRankList2[r]:=[];
count[r]:=[];
for i in [1..#Ob] do
vect:=[];
orbitSize:=#Ob[i];
// print "Orbit #",i,"of size",orbitSize,"is:",Ob[i];
// print " ";
vect[i]:=[];
for j in [1..orbitSize] do
vect[i,j]:=[0,...,0]; // must have 2*n zeroes
end for;
for a in [1..orbitSize] do
for b in [1..2*n] do
if b in Ob[i,a] then vect[i,a,b]:=1;
end if;
end for;
end for;
A:=Matrix(vect[i]);
// print "The corresponding matrix is A =",A;
// print "Orbit #",i,"of size",orbitSize,"has rank",Rank(A);
// print " ";
Append(  SizeRankList2[r], [ orbitSize,Rank(A) ] );
```

```
end for;
SizeRankList1[r]:={@ d : d in SizeRankList2[r] @};
Sort( SizeRankList1[r]);
Sort( SizeRankList2[r]);
// SizeRankList1[r];
TypeSizes[r]:=[ #Ob[i] : i in [1..#Ob] ];
Sort( TypeSizes[r]);
// print "Rho #",r,"has the following sizes of orbits of types:",TypeSizes[r];
// print " ";
print "Here is a list of orbit sizes and ranks.";
print " ";
for i in [1..#SizeRankList1[r]] do
count[r,i]:=0;
end for;
for i in [1..#SizeRankList1[r]] do
for j in [1..#SizeRankList2[r]] do
if SizeRankList1[r,i] eq SizeRankList2[r,j] then
count[r,i]+:=1;
end if;
end for;
end for;
// print "The size of SizeRankList1[",r,"] is",#SizeRankList1[r];
// print "The count array looks like",count[r];
for i in [1..#SizeRankList1[r]] do
Include( L,[SizeRankList1[r,i,1],SizeRankList1[r,i,2],gpNum]);
if count[r,i] gt 1 then
print "There are",count[r,i],"orbits of size",SizeRankList1[r,i,1],"and rank",SizeRank
List1[r,i,2];
else
print "There is",count[r,i],"orbit of size",SizeRankList1[r,i,1],"and rank",SizeRank
List1[r,i,2];
```

```
end if;
end for;
print " ";
end for;
Obts:={@ TypeSizes[i] : i in [1..#RhoGroups] @};
// print "The following array shows the sizes of orbits of types:", Obts;
if #RhoGroups gt 1 then
if #Obts gt 1 then
print "There are",#Obts,"rhos that produce distinct lists of sizes of orbits of types.";
print " ";
rhoCount:=[];
for j in [1..#Obts] do
rhoCount[j]:=[];
end for;
for i in [1..#RhoGroups] do
for j in [1..#Obts] do
if TypeSizes[i] eq Obts[j] then
Append( rhoCount[j],i);
end if;
end for;
end for;
for j in [1..#Obts] do
if #rhoCount[j] gt 1 then
print "The",#rhoCount[j],"rhos",rhoCount[j],"share this list of sizes of orbits of types: ",Obts[j];
print " ";
else
print "Rho #",rhoCount[j],"has this list of sizes of orbits of types:",Obts[j];
print " ";
end if;
end for;
```

```
else
print "Each rho produces the same list of sizes of orbits of types.";
end if;
end if;
print "————————————————————————————————";
return L;
end function;
ListOfGroups:=[];
OrbRankList:=[];
for i in [ ] do
G:=TransitiveGroup(2*n,i);
Z:=Center(G);
if IsDivisibleBy(2^n*Factorial(14),Order(G)) then
if IsEven(Order(Z)) then
Append( ListOfGroups,i);
end if;
end if;
end for;
for k in ListOfGroups do
print "Here is the information for T",2*n,"N",k,"of order",Order(TransitiveGroup
(2*n,k)),"=",FactoredOrder(TransitiveGroup(2*n,k)),":";
OrbRankList[Index(ListOfGroups,k)]:=OrbitsOfTypes(k);
end for;
S:={@ @};
for k in [1..#OrbRankList] do
for i in [1..#OrbRankList[k] ] do
Include( S,OrbRankList[k,i]);
end for;
end for;
T:=SetToSequence(S);
Sort( T);
```

```
U:=@ T[i,1] : i in [1..#T] @;
V:=@ T[i,2] : i in [1..#T] @;
L:=[];
for i in U do
L[Index(U,i)]:=[];
for j in V do
L[Index(U,i),Index(V,j)]:=[];
for k in [1..#T] do
if T[k,1] eq i and T[k,2] eq j then
Append( L[Index(U,i),Index(V,j)],T[k,3]);
end if;
end for;
end for;
end for;
for i in U do
for j in V do
if IsDefined(L[Index(U,i),Index(V,j)],1) then
print "The following groups have an orbit of size",i,"and rank",j,":", L[Index(U,i),
Index(V,j)];
end if;
end for;
end for;
quit;
```

A.2 ρ-Minimal Groups

I made the following program to compute the rho-minimal groups in degree $2n$:

```
n:=;
```

```
MinGroups:=[ ];
//List of minimal groups in degree 2n
for i in [1..100] do
if IsRegular(TransitiveGroup(2*n,i)) then
Append( MinGroups,i);
Sort( MinGroups);
end if;
end for;
//Add regular groups (order 2n) to the minimal group list
TrivCenter:=[];
EvenCenter:=[];
OtherCenter:=[];
for i in MinGroups do
if IsEven(Order(Center(TransitiveGroup(2*n,i)))) then
Append( EvenCenter,i);
else
if Order(Center(TransitiveGroup(2*n,i))) eq 1 then
Append( TrivCenter,i);
else
Append( BadCenter,i);
end if;
end if;
end for;
T:=#(TrivCenter);
print "There are" ,T, "groups with trivial center, which are:";
TrivCenter;
E:=#(EvenCenter);
print "There are" ,E, "groups with even order center, which are:";
EvenCenter;
B:=#(OtherCenter);
print "There are" ,O, "groups with any other center, which are:";
```

```
OtherCenter;
Gen2:=[];
//Gen3:=[];
//Gen4:=[];
//Add in extra lists if the minimal groups have more than 2 generators
for GpNum in TrivCenter do
if #(Generators(TransitiveGroup(2*n,GpNum))) eq 2 then
Append( Gen2,GpNum);
else
if #(Generators(TransitiveGroup(2*n,GpNum))) eq 3 then
Append( Gen3,GpNum);
else
if #(Generators(TransitiveGroup(2*n,GpNum))) eq 4 then
Append( Gen4,GpNum);
end if;
end if;
end if;
end for;
RhoMin2Gen:={};
//RhoMin3Gen:={};
//RhoMin4Gen:={};
S:=Sym(2*n);
for GpNum in Gen2 do
print "——————————————————";
print "For group number" ,GpNum, "we have the following information:";
print "——————————————————";
G:=TransitiveGroup(2*n,GpNum);
P:=MinimalPartitions(G);
MinParts:=[];
for i in [1..#P] do
if #(P[i]) eq n then
```

```
Append( MinParts,i);
end if;
end for;
for k in MinParts do
rho:=S!(1);
print ”——————————————-”;
print ”Partition ” ,k;
print ”——————————————-”;
for i in [1..2*n] do
for j in [i..2*n] do
if {i,j} in P[k] then
rho:=rho*S!(i,j);
end if;
end for;
end for;
I:=BlocksImage(G,P[k]);
K:=BlocksKernel(G,P[k]);
print ”The order of the image of the action of G on this partition is” ,Order(I);
print ”The order of the kernel of the action of G on this partition is” ,Order(K);
print ”The GAP index of the image is” ,TransitiveGroupIdentification(I);
print ”————————————-”;
print ”We now add rho to the generators of G”;
gen:=SetToSequence(Generators(G));
Gnew:=PermutationGroup¡2*n — gen[1],gen[2],rho¿;
RhoMin2Gen:=RhoMin2Gen join {TransitiveGroupIdentification(Gnew)};
for i in [1..#(Subgroups(Gnew:IsTransitive))] do
print ”for transitive subgroup number” ,i, ”of the new group, we have the following
information:”;
print ”The order of the subgroup is” ,Subgroups(Gnew:IsTransitive)[i]‘order;
print ”The GAP index is” ,TransitiveGroupIdentification(Subgroups(Gnew: IsTran-
sitive)[i]‘subgroup);
```

```
print "The center is" ,Center(Subgroups(Gnew:IsTransitive)[i]`subgroup);
if not IsIsomorphic(Subgroups(Gnew:IsTransitive)[i]`subgroup,Gnew) then
if Order(Center(Subgroups(Gnew:IsTransitive)[i]`subgroup)) ne 1 then
Exclude( RhoMin2Gen,TransitiveGroupIdentification(Gnew));
end if;
end if;
print "——————————";
end for;
end for;
end for;
//Repeat this process for groups with more than 2 generators
print "The following groups are rho-minimal:";
RhoMin2Gen join RhoMin3Gen join RhoMin4Gen;
quit;
```

A.3 Subgroup Analysis

I made the following program to determine if a selection of groups, specifically in degree 24, had at least one small order group (with all nondegenerate orbits) as a subgroup:

```
WithSubgroup:={};
List96:={85, 86, 87, 88, 89, 129, 131};
List120:={201};
List192:={290, 293, 298, 299, 300, 301, 305, 306, 307, 308, 309, 310, 311, 312, 313,
315, 316, 317, 426, 427, 428, 429};
//Groups of a given order with all nondegenerate orbits
Pool:=[];
//For groups from the following interval with even order center
```

```
for i in [ ] do
if IsEven(Order(Center(TransitiveGroup(24,i)))) then
Append( Pool,i);
end if;
end for;
TheDiff:={};
for i in Pool do
EvenCenterSubgroups96:={};
EvenCenterSubgroups120:={};
EvenCenterSubgroups192:={};
for j in [1..#Subgroups(TransitiveGroup(24,i):IsTransitive,OrderEqual:=96)] do
if IsEven(Order(Center(Subgroups(TransitiveGroup(24,i):IsTransitive,OrderEqual:=
96)[j]‘subgroup))) then
EvenCenterSubgroups96:=EvenCenterSubgroups96 join {TransitiveGroupIdentification
(Subgroups(TransitiveGroup(24,i):IsTransitive,OrderEqual:=96)[j]‘subgroup)};
if not IsEmpty(EvenCenterSubgroups96 meet List96) then
WithSubgroup:=WithSubgroup join {i};
end if;
end if;
end for;
for j in [1..#Subgroups(TransitiveGroup(24,i):IsTransitive,OrderEqual:=120)] do
if IsEven(Order(Center(Subgroups(TransitiveGroup(24,i):IsTransitive,OrderEqual:=
120)[j]‘subgroup))) then
EvenCenterSubgroups120:=EvenCenterSubgroups120 join {TransitiveGroupIdentification
(Subgroups(TransitiveGroup(24,i):IsTransitive,OrderEqual:=120)[j]‘subgroup)};
if not IsEmpty(EvenCenterSubgroups120 meet List120) then
WithSubgroup:=WithSubgroup join {i};
end if;
end if;
end for;
for j in [1..#Subgroups(TransitiveGroup(24,i):IsTransitive,OrderEqual:=192)] do
```

```
if IsEven(Order(Center(Subgroups(TransitiveGroup(24,i):IsTransitive,OrderEqual:=
192)[j]`subgroup))) then
EvenCenterSubgroups192:=EvenCenterSubgroups192 join {TransitiveGroupIdentification
(Subgroups(TransitiveGroup(24,i):IsTransitive,OrderEqual:=192)[j]`subgroup)};
if not IsEmpty(EvenCenterSubgroups192 meet List192) then
WithSubgroup:=WithSubgroup join {i};
end if;
end if;
end for;
end for;
ThePool:=SequenceToSet(Pool);
WithoutSubgroup:=ThePool diff WithSubgroup;
print "The following groups have subgroups in the List:";
WithSubgroup;
print "The following groups do not:";
WithoutSubgroup;
quit;
```

Appendix B

Ranks and Sizes of Orbits of Types in Degree 24

B.1 Degree 24

We computed ranks and sizes of orbits of types for the minimal and ρ-minimal groups as well as for groups of order up to 2,304. Due to the size of the output, we will give the complete information for groups of order up to 72 and the minimal and ρ-minimal groups of order greater than 72. Unless otherwise noted, there is a unique ρ. Also, it is worth noting that an orbit of size 2 corresponds to an imaginary quadratic subfield.

B.1.1 Groups of Order Up to 72

Here is the information for T 24 N 1 of order 24 = $[< 2, 3 >, < 3, 1 >]$:

This group is minimally transitive.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 8 and rank 5 and 170 orbits of size 24 and rank 13

Here is the information for T 24 N 2 of order 24 = $[< 2, 3 >, < 3, 1 >]$:

This group is minimally transitive.

There are 3 ρs for this group.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 4 and rank 3, 1 orbit of size 8 and rank 5, 10 orbits of size 12 and rank 7, 2 orbits of size 24 and rank 9, 28 orbits of size 24 and rank 11, and 135 orbits of size 24 and rank 13

Information for ρ # 2 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 2 and rank 2, 1 orbit of size 4 and rank 3, 2 orbits of size 6 and rank 4, 1 orbit of size 8 and rank 5, 2 orbits of size 12 and rank 6, 7 orbits of size 12 and rank 7, 2 orbits of size 24 and rank 8, 4 orbits of size 24 and rank 9, 18 orbits of size 24 and rank 10, 22 orbits of size 24 and rank 11, 22 orbits of size 24 and rank 12, and 97 orbits of size 24 and rank 13

Information for ρ # 3 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 2 and rank 2, 1 orbit of size 4 and rank 3, 2 orbits of size 6 and rank 4, 1 orbit of size 8 and rank 5, 2 orbits of size 12 and rank 6, 7 orbits of size 12 and rank 7, 2 orbits of size 24 and rank 8, 4 orbits of size 24 and rank 9, 18 orbits of size 24 and rank 10, 22 orbits of size 24 and rank 11, 22 orbits of size 24 and rank 12, and 97 orbits of size 24 and rank 13

There are 2 ρs that produce distinct lists of sizes of orbits of types.

ρ # [1] has this list of sizes of orbits of types:

[4, 4, 8, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24,

24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24,
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24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24,
24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24]

The 2 ρs $[2, 3]$ share this list of sizes of orbits of types:

[2, 2, 4, 6, 6, 8, 12, 12, 12, 12, 12, 12, 12, 12, 12, 24, 24, 24, 24, 24, 24, 24, 24,
24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24,
24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24,
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24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24,
24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24]

Here is the information for T 24 N 3 of order 24 = $[< 2, 3 >, < 3, 1 >]$:

This group is minimally transitive.

There are 7 ρs for this group.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 2 and rank 2, 4 orbits of size 6 and rank 4, 1 orbit of size 8 and rank 5, 12 orbits of size 12 and rank 6, 12 orbits of size 12 and rank 7, 6 orbits of size 24 and rank 9, 28 orbits of size 24 and rank 10, 20 orbits of size 24 and rank 11, 28 orbits of size 24 and rank 12, and 75 orbits of size 24 and rank 13

Information for ρ # 2 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 2 and rank 2, 4 orbits of size 6 and rank 4, 1 orbit of size 8 and rank 5, 12 orbits of size 12 and rank 6, 12 orbits of size 12 and rank 7, 6 orbits of size 24 and rank 9, 28 orbits of size 24 and rank 10, 20 orbits of size 24 and rank 11, 28 orbits of size 24 and rank 12, and 75 orbits of size 24 and rank 13

Information for ρ # 3 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 2 and rank 2, 4 orbits of size 6 and rank 4, 1 orbit of size 8 and rank 5, 12 orbits of size 12 and rank 6, 12 orbits of size 12 and rank 7, 6 orbits of size 24 and rank 9, 28 orbits of size 24 and rank 10, 20 orbits of size 24 and rank 11, 28 orbits of size 24 and rank 12, and 75 orbits of size 24 and rank 13

Information for ρ # 4 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 2 and rank 2, 4 orbits of size 6 and rank 4, 1 orbit of size 8 and rank 5, 12 orbits of size 12 and rank 6, 12 orbits of size 12 and rank 7, 6 orbits of size 24 and rank 9, 28 orbits of size 24 and rank 10, 20 orbits of size 24 and rank 11, 28 orbits of size 24 and rank 12, and 75 orbits of size 24 and rank 13

Information for ρ # 5 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 2 and rank 2, 4 orbits of size 6 and rank 4, 1 orbit of size 8 and rank 5, 12 orbits of size 12 and rank 6, 12 orbits of size 12 and rank 7, 6 orbits of size 24 and rank 9, 28 orbits of size 24 and rank 10, 20 orbits of size 24 and rank 11, 28 orbits of size 24 and rank 12, and 75 orbits of size 24 and rank 13

Information for ρ # 6 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 2 and rank 2, 4 orbits of size 6 and rank 4, 1 orbit of size 8 and rank 5, 12 orbits of size 12 and rank 6, 12 orbits of size 12 and rank 7, 6 orbits of size 24 and rank 9, 28 orbits of size 24 and rank 10, 20 orbits of size 24 and rank 11, 28 orbits of size 24 and rank 12, and 75 orbits of size 24 and rank 13

Information for ρ # 7 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 2 and rank 2, 4 orbits of size 6 and rank 4, 1 orbit of size 8 and rank 5, 12 orbits of size 12 and rank 6, 12 orbits of size 12 and rank 7, 6 orbits of size 24 and rank 9, 28 orbits of size 24 and rank 10, 20 orbits of size 24 and rank 11, 28 orbits of size 24 and rank 12, and 75 orbits of size 24 and rank 13

Each ρ produces the same list of sizes of orbits of types.

Here is the information for T 24 N 4 of order 24 = $[<2,3>,<3,1>]$:

This group is minimally transitive.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 8 and rank 5, 16 orbits of size 24 and rank 9, and 154 orbits of size 24 and rank 13

Here is the information for T 24 N 5 of order 24 = $[<2,3>,<3,1>]$:

This group is minimally transitive.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 8 and rank 5 and 170 orbits of size 24 and rank 13

Here is the information for T 24 N 6 of order 24 = $[<2,3>,<3,1>]$:

This group is minimally transitive.

There are 3 ρs for this group.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 4 and rank 3, 1 orbit of size 8 and rank 5, 10 orbits of size 12 and rank 7, 2 orbits of size 24 and rank 9, 28 orbits of size 24 and rank 11, and 135 orbits of size 24 and rank 13

Information for ρ # 2 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 2 and rank 2, 1 orbit of size 4 and rank 3, 6 orbits of size 6 and rank 4, 1 orbit of size 8 and rank 5, 7 orbits of size 12 and rank 7, 6 orbits of size 24 and rank 8, 8 orbits of size 24 and rank 9, 30 orbits of size 24 and rank 10, 34 orbits of size 24 and rank 11, 6 orbits of size 24 and rank 12, and 81 orbits of size 24 and rank 13

Information for ρ # 3 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 2 and rank 2, 1 orbit of size 4 and rank 3, 6 orbits of size 6 and rank 4, 1 orbit of size 8 and rank 5, 7 orbits of size 12 and rank 7, 6 orbits of size 24 and rank 8, 8 orbits of size 24 and rank 9, 30 orbits of size 24 and rank 10, 34 orbits of size 24 and rank 11, 6 orbits of size 24 and rank 12, and 81 orbits of size 24 and rank 13

There are 2 ρs that produce distinct lists of sizes of orbits of types.

ρ # [1] has this list of sizes of orbits of types:

[4, 4, 8, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 24]

The 2 ρs $[2, 3]$ share this list of sizes of orbits of types:

[2, 2, 4, 6, 6, 6, 6, 6, 6, 8, 12, 12, 12, 12, 12, 12, 12, 24]

Here is the information for T 24 N 7 of order 24 = $[< 2,3 >, < 3,1 >]$:

This group is minimally transitive.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 8 orbits of size 8 and rank 5, 8 orbits of size 24 and rank 9, and 160 orbits of size 24 and rank 13

Here is the information for T 24 N 8 of order 24 = $[< 2,3 >, < 3,1 >]$:

This group is minimally transitive.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 8 and rank 5, 24 orbits of size 24 and rank 9, and 146 orbits of size 24 and rank 13

Here is the information for T 24 N 9 of order 24 = $[< 2,3 >, < 3,1 >]$:

This group is minimally transitive.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 2 and rank 2, 5 orbits of size 6 and rank 4, 3 orbits of size 8 and rank 4, 4 orbits of size 8 and rank 5, 3 orbits of size 12 and rank 6, 21 orbits of size 12 and rank 7, 3 orbits of size 24 and rank 7, 36 orbits of size 24 and rank 10, 4 orbits of size 24 and rank 11, 28 orbits of size 24 and rank 12, and 84 orbits of size 24 and rank 13

Here is the information for T 24 N 11 of order 24 $= [< 2, 3 >, < 3, 1 >]$:

This group is minimally transitive.

There are 3 ρs for this group.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 2 and rank 2, 12 orbits of size 6 and rank 4, 1 orbit of size 8 and rank 5, 24 orbits of size 12 and rank 6, 28 orbits of size 12 and rank 7, 14 orbits of size 24 and rank 9, 36 orbits of size 24 and rank 10, 28 orbits of size 24 and rank 11, 12 orbits of size 24 and rank 12, and 51 orbits of size 24 and rank 13

Information for ρ # 2 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 2 and rank 2, 12 orbits of size 6 and rank 4, 1 orbit of size 8 and rank 5, 24 orbits of size 12 and rank 6, 28 orbits of size 12 and rank 7, 14 orbits of size 24 and rank 9, 36 orbits of size 24 and rank 10, 28 orbits of size 24 and rank 11, 12 orbits of size 24 and rank 12, and 51 orbits of size 24 and rank 13

Information for ρ # 3 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 2 and rank 2, 12 orbits of size 6 and rank 4, 1 orbit of size 8 and rank 5, 24 orbits of size 12 and rank 6, 28 orbits of size 12 and rank 7, 14 orbits of size 24 and rank 9, 36 orbits of size 24 and rank 10, 28 orbits of size 24 and rank 11, 12 orbits of size 24 and rank 12, and 51 orbits of size 24 and rank 13

Each ρ produces the same list of sizes of orbits of types.

Here is the information for T 24 N 12 of order $24 = [<2,3>,<3,1>]$:

This group is minimally transitive.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 4 and rank 3, 1 orbit of size 8 and rank 5, 30 orbits of size 12 and rank 7, 6 orbits of size 24 and rank 7, 6 orbits of size 24 and rank 9, 12 orbits of size 24 and rank 11, and 131 orbits of size 24 and rank 13

Here is the information for T 24 N 13 of order $24 = [<2,3>,<3,1>]$:

This group is minimally transitive.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 4 and rank 3, 60 orbits of size 12 and rank 7, 12 orbits of size 24 and rank 7, 36 orbits of size 24 and rank 11, and 92 orbits of size 24 and rank 13

Here is the information for T 24 N 14 of order $24 = [<2,3>,<3,1>]$:

This group is minimally transitive.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 4 and rank 3, 40 orbits of size 12 and rank 7, 14 orbits of size 24 and rank 7, 44 orbits of size 24 and rank 11, and 92 orbits of size 24 and rank 13

Here is the information for T 24 N 15 of order $24 = [<2,3>,<3,1>]$:

This group is minimally transitive.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 4 and rank 3, 20 orbits of size 12 and rank 7, 8 orbits of size 24 and rank 7, 60 orbits of size 24 and rank 11, and 92 orbits of size 24 and rank 13

Here is the information for T 24 N 16 of order 48 = $[< 2, 4 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 16 and rank 5 and 85 orbits of size 48 and rank 13

Here is the information for T 24 N 17 of order 48 = $[< 2, 4 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 8 and rank 5, 10 orbits of size 24 and rank 13, and 80 orbits of size 48 and rank 13

Here is the information for T 24 N 18 of order 48 = $[< 2, 4 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 8 and rank 5, 20 orbits of size 24 and rank 13, and 75 orbits of size 48 and rank 13

Here is the information for T 24 N 19 of order 48 = $[< 2, 4 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 8 and rank 5, 30 orbits of size 24 and rank 13, and 70 orbits of size 48 and rank 13

Here is the information for T 24 N 20 of order 48 = $[< 2, 4 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 16 and rank 5 and 85 orbits of size 48 and rank 13

Here is the information for T 24 N 21 of order 48 = $[< 2, 4 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 16 and rank 9 and 84 orbits of size 48 and rank 13

Here is the information for T 24 N 22 of order 48 = $[< 2, 4 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 16 and rank 5, 2 orbits of size 16 and rank 9, 2 orbits of size 48 and rank 9, and 82 orbits of size 48 and rank 13

Here is the information for T 24 N 23 of order 48 = $[< 2, 4 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 8 and rank 5, 10 orbits of size 24 and rank 13, and 80 orbits of size 48 and rank 13

Here is the information for T 24 N 24 of order 48 = $[< 2, 4 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 8 and rank 5, 20 orbits of size 24 and rank 13, and 75 orbits of size 48 and rank 13

Here is the information for T 24 N 25 of order 48 = $[< 2, 4 >, < 3, 1 >]$:

There are 3 ρs for this group.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 4 and rank 3, 1 orbit of size 8 and rank 5, 2 orbits of size 12 and rank 7, 4 orbits of size 24 and rank 7, 2 orbits of size 24 and rank 9, 4 orbits of size 24 and rank 11, 17 orbits of size 24 and rank 13, 12 orbits of size 48 and rank 11, and 59 orbits of size 48 and rank 13

Information for ρ # 2 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 2 and rank 2, 1 orbit of size 4 and rank 3, 2 orbits of size 6 and rank 4, 1 orbit of size 8 and rank 5, 2 orbits of size 12 and rank 6, 3 orbits of size 12 and rank 7, 2 orbits of size 24 and rank 7, 2 orbits of size 24 and rank 8, 4 orbits of size 24 and rank 9, 10 orbits of size 24 and rank 10, 6 orbits of size 24 and rank 11, 2 orbits of size 24 and rank 12, 11 orbits of size 24 and rank 13, 4 orbits of size 48 and rank 10, 8 orbits of size 48 and rank 11, 10 orbits of size 48 and rank 12, and 43 orbits of size 48 and rank 13

Information for ρ # 3 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 2 and rank 2, 1 orbit of size 4 and rank 3, 2 orbits of size 6 and rank 4, 1 orbit of size 8 and rank 5, 2 orbits of size 12 and rank 6, 3 orbits of size 12 and rank 7, 2 orbits of size 24 and rank 7, 2 orbits of size 24 and rank 8, 4 orbits of size 24 and rank 9, 10 orbits of size 24 and rank 10, 6 orbits of size 24 and rank 11, 2 orbits of size 24 and rank 12, 11 orbits of size 24 and rank 13, 4 orbits of size 48 and rank 10, 8 orbits of size 48 and rank 11, 10 orbits of size 48 and rank 12, and 43 orbits of size 48 and rank 13

There are 2 ρs that produce distinct lists of sizes of orbits of types.

ρ # [1] has this list of sizes of orbits of types:

[4, 4, 8, 12, 12, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48]

The 2 ρs $[2, 3]$ share this list of sizes of orbits of types:

[2, 2, 4, 6, 6, 8, 12, 12, 12, 12, 12, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48]

Here is the information for T 24 N 26 of order 48 = $[< 2, 4 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 8 and rank 5, 30 orbits of size 24 and rank 13, and 70 orbits of size 48 and rank 13

Here is the information for T 24 N 27 of order 48 = $[< 2, 4 >, < 3, 1 >]$:

There are 3 ρs for this group.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 4 and rank 3, 1 orbit of size 8 and rank 5, 6 orbits of size 12 and rank 7, 2 orbits of size 24 and rank 7, 2 orbits of size 24 and rank 9, 12 orbits of size 24 and rank 11, 25 orbits of size 24 and rank 13, 8 orbits of size 48 and rank 11, and 55 orbits of size 48 and rank 13

Information for ρ # 2 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 2 and rank 2, 1 orbit of size 4 and rank 3, 2 orbits of size 6 and rank 4, 1 orbit of size 8 and rank 5, 2 orbits of size 12 and rank 6, 5 orbits of size 12 and rank 7, 1 orbit of size 24 and rank 7, 2 orbits of size 24 and rank 8, 4 orbits of size 24 and rank 9, 6 orbits of size 24 and rank 10, 10 orbits of size 24 and rank 11, 2 orbits of size 24 and rank 12, 15 orbits of size 24 and rank 13, 6 orbits of size 48 and rank 10, 6 orbits of size 48 and rank 11, 10 orbits of size 48 and rank 12, and 41 orbits of size 48 and rank 13

Information for ρ # 3 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 2 and rank 2, 1 orbit of size 4 and rank 3, 2 orbits of size 6 and rank 4, 1 orbit of size 8 and rank 5, 2 orbits of size 12 and rank 6, 5 orbits of size 12 and rank 7, 1 orbit of size 24 and rank 7, 2 orbits of size 24 and rank 8, 4 orbits of size 24 and rank 9, 6 orbits of size 24 and rank 10, 10 orbits of size 24 and rank 11, 2 orbits of size 24 and rank 12, 15 orbits of size 24 and rank 13, 6 orbits of size 48 and rank 10, 6 orbits of size 48 and rank 11, 10 orbits of size 48 and rank 12, and 41 orbits of size 48 and rank 13

There are 2 ρs that produce distinct lists of sizes of orbits of types.

ρ # [1] has this list of sizes of orbits of types:

[4, 4, 8, 12, 12, 12, 12, 12, 12, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48]

The 2 ρs $[2, 3]$ share this list of sizes of orbits of types:

[2, 2, 4, 6, 6, 8, 12, 12, 12, 12, 12, 12, 12, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48]

Here is the information for T 24 N 28 of order 48 = $[< 2, 4 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 8 and rank 5, 30 orbits of size 24 and rank 13, and 70 orbits of size 48 and rank 13

Here is the information for T 24 N 29 of order 48 = $[< 2, 4 >, < 3, 1 >]$:

There are 3 ρs for this group.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 4 and rank 3, 1 orbit of size 8 and rank 5, 6 orbits of size 12 and rank 7, 2 orbits of size 24 and rank 7, 2 orbits of size 24 and rank 9, 12 orbits of size 24 and rank 11, 25 orbits of size 24 and rank 13, 8 orbits of size 48 and rank 11, and 55 orbits of size 48 and rank 13

Information for ρ # 2 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 2 and rank 2, 1 orbit of size 4 and rank 3, 2 orbits of size 6 and rank 4, 1 orbit of size 8 and rank 5, 2 orbits of size 12 and rank 6, 5 orbits of size 12 and rank 7, 1 orbit of size 24 and rank 7, 2 orbits of size 24 and rank 8, 4 orbits of size 24 and rank 9, 14 orbits of size 24 and rank 10, 10 orbits of size 24 and rank 11, 10 orbits of size 24 and rank 12, 11 orbits of size 24 and rank 13, 2 orbits of size 48 and rank 10, 6 orbits of size 48 and rank 11, 6 orbits of size 48 and rank 12, and 43 orbits of size 48 and rank 13

Information for ρ # 3 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 2 and rank 2, 1 orbit of size 4 and rank 3, 2 orbits of size 6 and rank 4, 1 orbit of size 8 and rank 5, 2 orbits of size 12 and rank 6, 5 orbits of size 12 and rank 7, 1 orbit of size 24 and rank 7, 2 orbits of size 24 and rank 8, 4 orbits of size 24 and rank 9, 14 orbits of size 24 and rank 10, 10 orbits of size 24 and rank 11, 10 orbits of size 24 and rank 12, 11 orbits of size 24 and rank 13, 2 orbits of size 48 and rank 10, 6 orbits of size 48 and rank 11, 6 orbits of size 48 and rank 12, and 43 orbits of size 48 and rank 13

There are 2 ρs that produce distinct lists of sizes of orbits of types.

ρ # [1] has this list of sizes of orbits of types:

[4, 4, 8, 12, 12, 12, 12, 12, 12, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48]

The 2 ρs $[2, 3]$ share this list of sizes of orbits of types:

[2, 2, 4, 6, 6, 8, 12, 12, 12, 12, 12, 12, 12, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48]

Here is the information for T 24 N 30 of order 48 $= [< 2, 4 >, < 3, 1 >]$:

There are 7 ρs for this group.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 2 and rank 2, 4 orbits of size 6 and rank 4, 1 orbit of size 8 and rank 5, 12 orbits of size 12 and rank 6, 12 orbits of size 12 and rank 7, 6 orbits of size 24 and rank 9, 12 orbits of size 24 and rank 10, 12 orbits of size 24 and rank 11, 4 orbits of size 24 and rank 12, 13 orbits of size 24 and rank 13, 8 orbits of size 48 and rank 10, 4 orbits of size 48 and rank 11, 12 orbits of size 48 and rank 12, and 31 orbits of size 48 and rank 13

Information for ρ # 2 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 2 and rank 2, 4 orbits of size 6 and rank 4, 1 orbit of size 8 and rank 5, 12 orbits of size 12 and rank 6, 12 orbits of size 12 and rank 7, 6 orbits of size 24 and rank 9, 12 orbits of size 24 and rank 10, 12 orbits of size 24 and rank 11, 4 orbits of size 24 and rank 12, 13 orbits of size 24 and rank 13, 8 orbits of size 48 and rank 10, 4 orbits of size 48 and rank 11, 12 orbits of size 48 and rank 12, and 31 orbits of size 48 and rank 13

Information for ρ # 3 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 2 and rank 2, 4 orbits of size 6 and rank 4, 1 orbit of size 8 and rank 5, 12 orbits of size 12 and rank 6, 12 orbits of size 12 and rank 7, 6 orbits of size 24 and rank 9, 12 orbits of size 24 and rank 10, 12 orbits of size 24 and rank 11, 4 orbits of size 24 and rank 12, 13 orbits of size 24 and rank 13, 8 orbits of size 48 and rank 10, 4 orbits of size 48 and rank 11, 12 orbits of size 48 and rank 12, and 31 orbits of size 48 and rank 13

Information for ρ # 4 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 2 and rank 2, 4 orbits of size 6 and rank 4, 1 orbit of size 8 and rank 5, 12 orbits of size 12 and rank 6, 12 orbits of size 12 and rank 7, 6 orbits of size 24 and rank 9, 12 orbits of size 24 and rank 10, 12 orbits of size 24 and rank 11, 4 orbits of size 24 and rank 12, 13 orbits of size 24 and rank 13, 8 orbits of size 48 and rank 10, 4 orbits of size 48 and rank 11, 12 orbits of size 48 and rank 12, and 31 orbits of size 48 and rank 13

Information for ρ # 5 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 2 and rank 2, 4 orbits of size 6 and rank 4, 1 orbit of size 8 and rank 5, 12 orbits of size 12 and rank 6, 12 orbits of size 12 and rank 7, 6 orbits of size 24 and rank 9, 12 orbits of size 24 and rank 10, 12 orbits of size 24 and rank 11, 4 orbits of size 24 and rank 12, 13 orbits of size 24 and rank 13, 8 orbits of size 48 and rank 10, 4 orbits of size 48 and rank 11, 12 orbits of size 48 and rank 12, and 31 orbits of size 48 and rank 13

Information for ρ # 6 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 2 and rank 2, 4 orbits of size 6 and rank 4, 1 orbit of size 8 and rank 5, 12 orbits of size 12 and rank 6, 12 orbits of size 12 and rank 7, 6 orbits of size 24 and rank 9, 12 orbits of size 24 and rank 10, 12 orbits of size 24 and rank 11, 4 orbits of size 24 and rank 12, 13 orbits of size 24 and rank 13, 8 orbits of size 48 and rank 10, 4 orbits of size 48 and rank 11, 12 orbits of size 48 and rank 12, and 31 orbits of size 48 and rank 13

Information for ρ # 7 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 2 and rank 2, 4 orbits of size 6 and rank 4, 1 orbit of size 8 and rank 5, 12 orbits of size 12 and rank 6, 12 orbits of size 12 and rank 7, 6 orbits of size 24 and rank 9, 12 orbits of size 24 and rank 10, 12 orbits of size 24 and rank 11, 4 orbits of size 24 and rank 12, 13 orbits of size 24 and rank 13, 8 orbits of size 48 and rank 10, 4 orbits of size 48 and rank 11, 12 orbits of size 48 and rank 12, and 31 orbits of size 48 and rank 13

Each ρ produces the same list of sizes of orbits of types.

Here is the information for T 24 N 31 of order 48 = $[< 2, 4 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 16 and rank 5 and 85 orbits of size 48 and rank 13

Here is the information for T 24 N 32 of order 48 = $[< 2, 4 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 8 and rank 5, 30 orbits of size 24 and rank 13, and 70 orbits of size 48 and rank 13

Here is the information for T 24 N 33 of order 48 = $[< 2, 4 >, < 3, 1 >]$:

There are 3 ρs for this group.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 4 and rank 3, 1 orbit of size 8 and rank 5, 4 orbits of size 12 and rank 7, 3 orbits of size 24 and rank 7, 2 orbits of size 24 and rank 9, 13 orbits of size 24 and rank 13, 14 orbits of size 48 and rank 11, and 61 orbits of size 48 and rank 13

Information for ρ # 2 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 4 and rank 3, 1 orbit of size 8 and rank 5, 6 orbits of size 12 and rank 7, 2 orbits of size 24 and rank 7, 2 orbits of size 24 and rank 9, 4 orbits of size 24 and rank 11, 21 orbits of size 24 and rank 13, 12 orbits of size 48 and rank 11, and 57 orbits of size 48 and rank 13

Information for ρ # 3 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 4 and rank 3, 1 orbit of size 8 and rank 5, 4 orbits of size 12 and rank 7, 3 orbits of size 24 and rank 7, 2 orbits of size 24 and rank 9, 4 orbits of size 24 and rank 11, 21 orbits of size 24 and rank 13, 12 orbits of size 48 and rank 11, and 57 orbits of size 48 and rank 13

There are 3 ρs that produce distinct lists of sizes of orbits of types.

ρ # [1] has this list of sizes of orbits of types:

[4, 4, 8, 12, 12, 12, 12, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48]

ρ # [2] has this list of sizes of orbits of types:

[4, 4, 8, 12, 12, 12, 12, 12, 12, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48]

ρ # [3] has this list of sizes of orbits of types:

[4, 4, 8, 12, 12, 12, 12, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48]

Here is the information for T 24 N 34 of order $48 = [< 2, 4 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 8 and rank 5, 30 orbits of size 24 and rank 13, and 70 orbits of size 48 and rank 13

Here is the information for T 24 N 35 of order $48 = [< 2, 4 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 16 and rank 5 and 85 orbits of size 48 and rank 13

Here is the information for T 24 N 36 of order $48 = [< 2, 4 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 16 and rank 5, 4 orbits of size 48 and rank 9, and 81 orbits of size 48 and rank 13

Here is the information for T 24 N 37 of order 48 = $[< 2, 4 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 8 and rank 5, 10 orbits of size 24 and rank 13, and 80 orbits of size 48 and rank 13

Here is the information for T 24 N 38 of order 48 = $[< 2, 4 >, < 3, 1 >]$:

There are 3 ρs for this group.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 4 and rank 3, 1 orbit of size 8 and rank 5, 2 orbits of size 12 and rank 7, 4 orbits of size 24 and rank 7, 2 orbits of size 24 and rank 9, 4 orbits of size 24 and rank 11, 7 orbits of size 24 and rank 13, 12 orbits of size 48 and rank 11, and 64 orbits of size 48 and rank 13

Information for ρ # 2 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 2 and rank 2, 1 orbit of size 4 and rank 3, 2 orbits of size 6 and rank 4, 1 orbit of size 8 and rank 5, 2 orbits of size 12 and rank 6, 3 orbits of size 12 and rank 7, 2 orbits of size 24 and rank 7, 2 orbits of size 24 and rank 8, 4 orbits of size 24 and rank 9, 10 orbits of size 24 and rank 10, 2 orbits of size 24 and rank 11, 6 orbits of size 24 and rank 12, 21 orbits of size 24 and rank 13, 4 orbits of size 48 and rank 10, 10 orbits of size 48 and rank 11, 8 orbits of size 48 and rank 12, and 38 orbits of size 48 and rank 13

Information for ρ # 3 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 2 and rank 2, 1 orbit of size 4 and rank 3, 2 orbits of size 6 and rank 4, 1 orbit of size 8 and rank 5, 2 orbits of size 12 and rank 6, 3 orbits of size 12 and rank 7, 2 orbits of size 24 and rank 7, 2 orbits of size 24 and rank 8, 4 orbits of size 24 and rank 9, 10 orbits of size 24 and rank 10, 2 orbits of size 24 and rank 11, 6 orbits of size 24 and rank 12, 21 orbits of size 24 and rank 13, 4 orbits of size 48 and rank 10, 10 orbits of size 48 and rank 11, 8 orbits of size 48 and rank 12, and 38 orbits of size 48 and rank 13

There are 2 ρs that produce distinct lists of sizes of orbits of types.

ρ # [1] has this list of sizes of orbits of types:

[4, 4, 8, 12, 12, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48]

The 2 ρs $[2, 3]$ share this list of sizes of orbits of types:

[2, 2, 4, 6, 6, 8, 12, 12, 12, 12, 12, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48]

Here is the information for T 24 N 39 of order 48 $= [< 2, 4 >, < 3, 1 >]$:

There are 3 ρs for this group.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 4 and rank 3, 1 orbit of size 8 and rank 5, 2 orbits of size 12 and rank 7, 4 orbits of size 24 and rank 7, 2 orbits of size 24 and rank 9, 3 orbits of size 24 and rank 13, 14 orbits of size 48 and rank 11, and 66 orbits of size 48 and rank 13

Information for ρ # 2 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 4 and rank 3, 1 orbit of size 8 and rank 5, 6 orbits of size 12 and rank 7, 2 orbits of size 24 and rank 7, 2 orbits of size 24 and rank 9, 8 orbits of size 24 and rank 11, 27 orbits of size 24 and rank 13, 10 orbits of size 48 and rank 11, and 54 orbits of size 48 and rank 13

Information for ρ # 3 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 4 and rank 3, 1 orbit of size 8 and rank 5, 6 orbits of size 12 and rank 7, 2 orbits of size 24 and rank 7, 2 orbits of size 24 and rank 9, 8 orbits of size 24 and rank 11, 27 orbits of size 24 and rank 13, 10 orbits of size 48 and rank 11, and 54 orbits of size 48 and rank 13

There are 2 ρs that produce distinct lists of sizes of orbits of types.

ρ # [1] has this list of sizes of orbits of types:

[4, 4, 8, 12, 12, 24, 24, 24, 24, 24, 24, 24, 24, 24, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48]

The 2 ρs $[2, 3]$ share this list of sizes of orbits of types:

[4, 4, 8, 12, 12, 12, 12, 12, 12, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48]

Here is the information for T 24 N 40 of order 48 = $[< 2, 4 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 8 and rank 5, 10 orbits of size 24 and rank 13, and 80 orbits of size 48 and rank 13

Here is the information for T 24 N 41 of order 48 = $[< 2, 4 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 16 and rank 5 and 85 orbits of size 48 and rank 13

Here is the information for T 24 N 42 of order 48 = $[< 2, 4 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 16 and rank 5 and 85 orbits of size 48 and rank 13

Here is the information for T 24 N 43 of order 48 = $[< 2, 4 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 8 and rank 5, 30 orbits of size 24 and rank 13, and 70 orbits of size 48 and rank 13

Here is the information for T 24 N 44 of order 48 = $[< 2, 4 >, < 3, 1 >]$:

There are 3 ρs for this group.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 4 and rank 3, 1 orbit of size 8 and rank 5, 6 orbits of size 12 and rank 7, 2 orbits of size 24 and rank 7, 2 orbits of size 24 and rank 9, 3 orbits of size 24 and rank 13, 14 orbits of size 48 and rank 11, and 66 orbits of size 48 and rank 13

Information for ρ # 2 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 4 and rank 3, 1 orbit of size 8 and rank 5, 8 orbits of size 12 and rank 7, 1 orbit of size 24 and rank 7, 2 orbits of size 24 and rank 9, 8 orbits of size 24 and rank 11, 27 orbits of size 24 and rank 13, 10 orbits of size 48 and rank 11, and 54 orbits of size 48 and rank 13

Information for ρ # 3 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 4 and rank 3, 1 orbit of size 8 and rank 5, 8 orbits of size 12 and rank 7, 1 orbit of size 24 and rank 7, 2 orbits of size 24 and rank 9, 8 orbits of size 24 and rank 11, 27 orbits of size 24 and rank 13, 10 orbits of size 48 and rank 11, and 54 orbits of size 48 and rank 13

There are 2 ρs that produce distinct lists of sizes of orbits of types.

ρ # [1] has this list of sizes of orbits of types:

[4, 4, 8, 12, 12, 12, 12, 12, 12, 24, 24, 24, 24, 24, 24, 24, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48]

The 2 ρs $[2, 3]$ share this list of sizes of orbits of types:

[4, 4, 8, 12, 12, 12, 12, 12, 12, 12, 12, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48]

Here is the information for T 24 N 45 of order 48 $= [< 2, 4 >, < 3, 1 >]$:

There are 3 ρs for this group.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 4 and rank 3, 1 orbit of size 8 and rank 5, 6 orbits of size 12 and rank 7, 2 orbits of size 24 and rank 7, 2 orbits of size 24 and rank 9, 12 orbits of size 24 and rank 11, 15 orbits of size 24 and rank 13, 8 orbits of size 48 and rank 11, and 60 orbits of size 48 and rank 13

Information for ρ # 2 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 2 and rank 2, 1 orbit of size 4 and rank 3, 6 orbits of size 6 and rank 4, 1 orbit of size 8 and rank 5, 5 orbits of size 12 and rank 7, 1 orbit of size 24 and rank 7, 6 orbits of size 24 and rank 8, 8 orbits of size 24 and rank 9, 18 orbits of size 24 and rank 10, 2 orbits of size 24 and rank 11, 6 orbits of size 24 and rank 12, 21 orbits of size 24 and rank 13, 6 orbits of size 48 and rank 10, 16 orbits of size 48 and rank 11, and 30 orbits of size 48 and rank 13

Information for ρ # 3 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 2 and rank 2, 1 orbit of size 4 and rank 3, 6 orbits of size 6 and rank 4, 1 orbit of size 8 and rank 5, 5 orbits of size 12 and rank 7, 1 orbit of size 24 and rank 7, 6 orbits of size 24 and rank 8, 8 orbits of size 24 and rank 9, 18 orbits of size 24 and rank 10, 2 orbits of size 24 and rank 11, 6 orbits of size 24 and rank 12, 21 orbits of size 24 and rank 13, 6 orbits of size 48 and rank 10, 16 orbits of size 48 and rank 11, and 30 orbits of size 48 and rank 13

There are 2 ρs that produce distinct lists of sizes of orbits of types.

ρ # [1] has this list of sizes of orbits of types:

[4, 4, 8, 12, 12, 12, 12, 12, 12, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48]

The 2 ρs $[2, 3]$ share this list of sizes of orbits of types:

[2, 2, 4, 6, 6, 6, 6, 6, 6, 8, 12, 12, 12, 12, 12, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48]

Here is the information for T 24 N 46 of order $48 = [< 2, 4 >, < 3, 1 >]$:

This group is ρ-minimal.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 8 and rank 4, 8 orbits of size 12 and rank 7, 2 orbits of size 16 and rank 7, 8 orbits of size 24 and rank 7, 24 orbits of size 24 and rank 10, 16 orbits of size 24 and rank 13, 4 orbits of size 48 and rank 10, and 54 orbits of size 48 and rank 13

Here is the information for T 24 N 47 of order 48 = $[< 2, 4 >, < 3, 1 >]$:

This group is minimally transitive.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 2 and rank 2, 6 orbits of size 6 and rank 4, 2 orbits of size 8 and rank 4, 4 orbits of size 12 and rank 6, 6 orbits of size 12 and rank 7, 2 orbits of size 16 and rank 9, 6 orbits of size 24 and rank 7, 4 orbits of size 24 and rank 9, 16 orbits of size 24 and rank 10, 16 orbits of size 24 and rank 13, 6 orbits of size 48 and rank 10, 16 orbits of size 48 and rank 11, 6 orbits of size 48 and rank 12, and 32 orbits of size 48 and rank 13

Here is the information for T 24 N 48 of order 48 = $[< 2, 4 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 2 and rank 2, 3 orbits of size 6 and rank 4, 1 orbit of size 8 and rank 4, 2 orbits of size 8 and rank 5, 1 orbit of size 12 and rank 6, 4 orbits of size 12 and rank 7, 1 orbit of size 16 and rank 7, 1 orbit of size 16 and rank 8, 3 orbits of size 24 and rank 7, 1 orbit of size 24 and rank 9, 13 orbits of size 24 and rank 10, 2 orbits of size 24 and rank 11, 2 orbits of size 24 and rank 12, 14 orbits of size 24 and rank 13, 6 orbits of size 48 and rank 10, 1 orbit of size 48 and rank 11, 13 orbits of size 48 and rank 12, and 45 orbits of size 48 and rank 13

Here is the information for T 24 N 49 of order 48 = $[< 2, 4 >, < 3, 1 >]$:

There are 3 ρs for this group.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 2 and rank 2, 1 orbit of size 6 and rank 4, 3 orbits of size 8 and rank 4, 3 orbits of size 12 and rank 6, 11 orbits of size 12 and rank 7, 2 orbits of size 16 and rank 8, 7 orbits of size 24 and rank 7, 6 orbits of size 24 and rank 10, 16 orbits of size 24 and rank 13, 2 orbits of size 48 and rank 10, 2 orbits of size 48 and rank 11, 14 orbits of size 48 and rank 12, and 48 orbits of size 48 and rank 13

Information for ρ # 2 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 8 and rank 4, 16 orbits of size 12 and rank 7, 2 orbits of size 16 and rank 7, 8 orbits of size 24 and rank 7, 4 orbits of size 24 and rank 10, 16 orbits of size 24 and rank 13, 4 orbits of size 48 and rank 10, and 62 orbits of size 48 and rank 13

Information for ρ # 3 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 2 and rank 2, 1 orbit of size 6 and rank 4, 3 orbits of size 8 and rank 4, 3 orbits of size 12 and rank 6, 11 orbits of size 12 and rank 7, 2 orbits of size 16 and rank 8, 7 orbits of size 24 and rank 7, 6 orbits of size 24 and rank 10, 16 orbits of size 24 and rank 13, 2 orbits of size 48 and rank 10, 2 orbits of size 48 and rank 11, 14 orbits of size 48 and rank 12, and 48 orbits of size 48 and rank 13

There are 2 ρs that produce distinct lists of sizes of orbits of types.

The 2 ρs $[1, 3]$ share this list of sizes of orbits of types:

[2, 6, 8, 8, 8, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 16, 16, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48]

ρ # [2] has this list of sizes of orbits of types:

[8, 8, 8, 8, 12, 12, 12, 12, 12, 12,12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 16, 16, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48]

Here is the information for T 24 N 50 of order 48 $= [< 2, 4 >, < 3, 1 >]$:

This group is minimally transitive.

There are 3 ρs for this group.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 2 and rank 2, 2 orbits of size 6 and rank 4, 2 orbits of size 8 and rank 4, 6 orbits of size 12 and rank 6, 6 orbits of size 12 and rank 7, 2 orbits of size 16 and rank 9, 6 orbits of size 24 and rank 7, 8 orbits of size 24 and rank 10, 16 orbits of size 24 and rank 13, 10 orbits of size 48 and rank 11, 16 orbits of size 48 and rank 12, and 40 orbits of size 48 and rank 13

Information for ρ # 2 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 2 and rank 2, 2 orbits of size 6 and rank 4, 2 orbits of size 8 and rank 4, 6 orbits of size 12 and rank 6, 6 orbits of size 12 and rank 7, 2 orbits of size 16 and rank 9, 6 orbits of size 24 and rank 7, 8 orbits of size 24 and rank 10, 16 orbits of size 24 and rank 13, 10 orbits of size 48 and rank 11, 16 orbits of size 48 and rank 12, and 40 orbits of size 48 and rank 13

Information for ρ # 3 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 2 and rank 2, 2 orbits of size 6 and rank 4, 2 orbits of size 8 and rank 4, 6 orbits of size 12 and rank 6, 6 orbits of size 12 and rank 7, 2 orbits of size 16 and rank 9, 6 orbits of size 24 and rank 7, 8 orbits of size 24 and rank 10, 16 orbits of size 24 and rank 13, 10 orbits of size 48 and rank 11, 16 orbits of size 48 and rank 12, and 40 orbits of size 48 and rank 13

Each ρ produces the same list of sizes of orbits of types.

Here is the information for T 24 N 51 of order 48 = $[<2,4>,<3,1>]$:

This group is minimally transitive.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 8 orbits of size 12 and rank 7, 4 orbits of size 16 and rank 7, 4 orbits of size 24 and rank 7, 24 orbits of size 24 and rank 13, 2 orbits of size 48 and rank 7, and 66 orbits of size 48 and rank 13

Here is the information for T 24 N 52 of order 48 = $[<2,4>,<3,1>]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 4 and rank 3, 1 orbit of size 8 and rank 5, 6 orbits of size 12 and rank 7, 12 orbits of size 24 and rank 7, 6 orbits of size 24 and rank 9, 6 orbits of size 24 and rank 11, 17 orbits of size 24 and rank 13, 6 orbits of size 48 and rank 11, and 57 orbits of size 48 and rank 13

Here is the information for T 24 N 53 of order 48 = $[<2,4>,<3,1>]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 4 and rank 3, 12 orbits of size 12 and rank 7, 8 orbits of size 24 and rank 7, 24 orbits of size 24 and rank 11, 20 orbits of size 24 and rank 13, 20 orbits of size 48 and rank 11, and 36 orbits of size 48 and rank 13

Here is the information for T 24 N 54 of order 48 = $[< 2, 4 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 4 and rank 3, 1 orbit of size 8 and rank 5, 6 orbits of size 12 and rank 7, 4 orbits of size 24 and rank 7, 2 orbits of size 24 and rank 9, 14 orbits of size 24 and rank 11, 11 orbits of size 24 and rank 13, 6 orbits of size 48 and rank 11, and 62 orbits of size 48 and rank 13

Here is the information for T 24 N 55 of order 48 = $[< 2, 4 >, < 3, 1 >]$:

This group is minimally transitive.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 16 and rank 7, 32 orbits of size 24 and rank 13, 2 orbits of size 48 and rank 7, and 66 orbits of size 48 and rank 13

Here is the information for T 24 N 56 of order 48 = $[< 2, 4 >, < 3, 1 >]$:

This group is minimally transitive.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 4 and rank 3, 5 orbits of size 12 and rank 7, 1 orbit of size 16 and rank 7, 2 orbits of size 16 and rank 9, 2 orbits of size 24 and rank 11, 22 orbits of size 24 and rank 13, 1 orbit of size 48 and rank 7, 6 orbits of size 48 and rank 11, and 64 orbits of size 48 and rank 13

Here is the information for T 24 N 57 of order $48 = [< 2, 4 >, < 3, 1 >]$:

This group is minimally transitive.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 4 and rank 3, 5 orbits of size 12 and rank 7, 1 orbit of size 16 and rank 7, 2 orbits of size 16 and rank 9, 2 orbits of size 24 and rank 11, 22 orbits of size 24 and rank 13, 1 orbit of size 48 and rank 7, 6 orbits of size 48 and rank 11, and 64 orbits of size 48 and rank 13

Here is the information for T 24 N 60 of order $72 = [< 2, 3 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 4 and rank 3, 1 orbit of size 8 and rank 5, 20 orbits of size 24 and rank 13, 10 orbits of size 36 and rank 11, 6 orbits of size 72 and rank 11, and 39 orbits of size 72 and rank 13

Here is the information for T 24 N 61 of order $72 = [< 2, 3 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 4 and rank 3, 20 orbits of size 24 and rank 13, 20 orbits of size 36 and rank 11, 16 orbits of size 72 and rank 11, and 24 orbits of size 72 and rank 13

Here is the information for T 24 N 62 of order $72 = [< 2, 3 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 8 and rank 5, 20 orbits of size 24 and rank 13, and 50 orbits of size 72 and rank 13

Here is the information for T 24 N 63 of order 72 = $[< 2, 3 >, < 3, 2 >]$:

This group is minimally transitive.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 8 and rank 5, 20 orbits of size 24 and rank 13, and 50 orbits of size 72 and rank 13

Here is the information for T 24 N 64 of order 72 = $[< 2, 3 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 8 and rank 5, 20 orbits of size 24 and rank 13, and 50 orbits of size 72 and rank 13

Here is the information for T 24 N 65 of order 72 = $[< 2, 3 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 4 and rank 3, 1 orbit of size 8 and rank 5, 20 orbits of size 24 and rank 13, 10 orbits of size 36 and rank 11, 6 orbits of size 72 and rank 11, and 39 orbits of size 72 and rank 13

Here is the information for T 24 N 66 of order 72 = $[< 2, 3 >, < 3, 2 >]$:

There are 3 ρs for this group.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 4 and rank 3, 1 orbit of size 8 and rank 5, 4 orbits of size 12 and rank 7, 2 orbits of size 24 and rank 9, 4 orbits of size 24 and rank 11, 12 orbits of size 24 and rank 13, 2 orbits of size 36 and rank 7, 8 orbits of size 72 and rank 11, and 41 orbits of size 72 and rank 13

Information for ρ # 2 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 2 and rank 2, 1 orbit of size 4 and rank 3, 1 orbit of size 8 and rank 5, 4 orbits of size 12 and rank 7, 2 orbits of size 18 and rank 6, 2 orbits of size 24 and rank 9, 4 orbits of size 24 and rank 11, 12 orbits of size 24 and rank 13, 1 orbit of size 36 and rank 7, 4 orbits of size 72 and rank 10, 4 orbits of size 72 and rank 11, 10 orbits of size 72 and rank 12, and 31 orbits of size 72 and rank 13

Information for ρ # 3 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 2 and rank 2, 1 orbit of size 4 and rank 3, 1 orbit of size 8 and rank 5, 4 orbits of size 12 and rank 7, 2 orbits of size 18 and rank 6, 2 orbits of size 24 and rank 9, 4 orbits of size 24 and rank 11, 12 orbits of size 24 and rank 13, 1 orbit of size 36 and rank 7, 4 orbits of size 72 and rank 10, 4 orbits of size 72 and rank 11, 10 orbits of size 72 and rank 12, and 31 orbits of size 72 and rank 13

There are 2 ρs that produce distinct lists of sizes of orbits of types.

ρ # [1] has this list of sizes of orbits of types:

[4, 4, 8, 12, 12, 12, 12, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 36, 36, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72]

The 2 ρs $[2, 3]$ share this list of sizes of orbits of types:

[2, 2, 4, 8, 12, 12, 12, 12, 18, 18, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 36, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72]

Here is the information for T 24 N 67 of order 72 = $[< 2, 3 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 4 and rank 3, 20 orbits of size 24 and rank 13, 20 orbits of size 36 and rank 11, 16 orbits of size 72 and rank 11, and 24 orbits of size 72 and rank 13

Here is the information for T 24 N 68 of order 72 = $[< 2, 3 >, < 3, 2 >]$:

There are 3 ρs for this group.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 2 and rank 2, 1 orbit of size 8 and rank 5, 4 orbits of size 12 and rank 7, 4 orbits of size 18 and rank 6, 2 orbits of size 24 and rank 9, 4 orbits of size 24 and rank 11, 12 orbits of size 24 and rank 13, 8 orbits of size 36 and rank 10, 8 orbits of size 36 and rank 11, 4 orbits of size 72 and rank 10, 12 orbits of size 72 and rank 12, and 25 orbits of size 72 and rank 13

Information for ρ # 2 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 2 and rank 2, 1 orbit of size 8 and rank 5, 4 orbits of size 12 and rank 7, 4 orbits of size 18 and rank 6, 2 orbits of size 24 and rank 9, 4 orbits of size 24 and rank 11, 12 orbits of size 24 and rank 13, 8 orbits of size 36 and rank 10, 8 orbits of size 36 and rank 11, 4 orbits of size 72 and rank 10, 12 orbits of size 72 and rank 12, and 25 orbits of size 72 and rank 13

Information for ρ # 3 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 2 and rank 2, 1 orbit of size 8 and rank 5, 4 orbits of size 12 and rank 7, 4 orbits of size 18 and rank 6, 2 orbits of size 24 and rank 9, 4 orbits of size 24 and rank 11, 12 orbits of size 24 and rank 13, 8 orbits of size 36 and rank 10, 8 orbits of size 36 and rank 11, 4 orbits of size 72 and rank 10, 12 orbits of size 72 and rank 12, and 25 orbits of size 72 and rank 13

Each ρ produces the same list of sizes of orbits of types.

Here is the information for T 24 N 69 of order $72 = [< 2, 3 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 8 and rank 5, 20 orbits of size 24 and rank 13, and 50 orbits of size 72 and rank 13

Here is the information for T 24 N 70 of order $72 = [< 2, 3 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 8 and rank 5, 4 orbits of size 24 and rank 9, 10 orbits of size 24 and rank 13, and 52 orbits of size 72 and rank 13

Here is the information for T 24 N 71 of order $72 = [< 2, 3 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 2 and rank 2, 2 orbits of size 6 and rank 4, 1 orbit of size 8 and rank 5, 1 orbit of size 18 and rank 10, 4 orbits of size 24 and rank 10, 2 orbits of size 24 and rank 11, 1 orbit of size 24 and rank 12, 6 orbits of size 24 and rank 13, 5 orbits of size 36 and rank 10, 1 orbit of size 36 and rank 12, 2 orbits of size 36 and rank 13, 1 orbit of size 72 and rank 10, 1 orbit of size 72 and rank 11, 9 orbits of size 72 and rank 12, and 37 orbits of size 72 and rank 13

Here is the information for T 24 N 73 of order 72 = $[< 2, 3 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 2 and rank 2, 1 orbit of size 8 and rank 5, 12 orbits of size 12 and rank 7, 12 orbits of size 18 and rank 6, 6 orbits of size 24 and rank 9, 8 orbits of size 24 and rank 13, 12 orbits of size 36 and rank 10, 8 orbits of size 36 and rank 11, 12 orbits of size 72 and rank 12, and 25 orbits of size 72 and rank 13

Here is the information for T 24 N 74 of order 72 = $[< 2, 3 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 4 and rank 3, 12 orbits of size 12 and rank 7, 6 orbits of size 24 and rank 7, 8 orbits of size 24 and rank 13, 6 orbits of size 36 and rank 7, 22 orbits of size 36 and rank 11, 8 orbits of size 72 and rank 11, and 28 orbits of size 72 and rank 13

Here is the information for T 24 N 75 of order 72 = $[< 2, 3 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 4 and rank 3, 1 orbit of size 8 and rank 5, 12 orbits of size 12 and rank 7, 6 orbits of size 24 and rank 9, 8 orbits of size 24 and rank 13, 6 orbits of size 36 and rank 7, 12 orbits of size 36 and rank 11, and 41 orbits of size 72 and rank 13

Here is the information for T 24 N 76 of order 72 = $[< 2, 3 >, < 3, 2 >]$:

This group is minimally transitive.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 2 and rank 2, 1 orbit of size 4 and rank 3, 1 orbit of size 8 and rank 5, 12 orbits of size 12 and rank 7, 6 orbits of size 18 and rank 6, 6 orbits of size 24 and rank 9, 8 orbits of size 24 and rank 13, 3 orbits of size 36 and rank 7, 6 orbits of size 36 and rank 10, 6 orbits of size 36 and rank 11, 10 orbits of size 72 and rank 12, and 31 orbits of size 72 and rank 13

Here is the information for T 24 N 77 of order 72 = $[< 2, 3 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 4 and rank 3, 4 orbits of size 12 and rank 7, 2 orbits of size 24 and rank 7, 8 orbits of size 24 and rank 11, 8 orbits of size 24 and rank 13, 2 orbits of size 36 and rank 7, 10 orbits of size 36 and rank 11, 16 orbits of size 72 and rank 11, and 28 orbits of size 72 and rank 13

B.1.2 Minimal and ρ-Minimal Groups

There are 99 minimally transitive permutation groups that are Galois groups of CM fields, which are:

{ 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 47, 50, 51, 55, 56, 57, 63, 76, 93, 94, 96, 174, 179, 180, 213, 214, 215, 216, 238, 239, 240, 241, 255, 257, 258, 259, 263, 267, 268, 273, 307, 308, 309, 310, 311, 312, 315, 316, 317, 424, 460, 468, 470, 481, 483, 496, 506, 596, 597, 598, 620, 622, 945, 1371, 1392, 1410, 2128, 2129, 2130, 2788, 2801, 3075, 3098, 5509, 5535, 5693, 5872, 5873, 7443, 7444, 7445, 7446, 7447, 7448, 7688, 7690, 7692, 7694, 7695, 7696, 7697, 7882, 7905, 10036, 10283 }.

The following 32 groups are ρ-minimal:

{ 46, 173, 175, 256, 260, 264, 306, 442, 447, 453, 471, 476, 823, 831, 946, 1620, 1781, 1830, 2127, 2781, 2799, 3072, 3078, 7449, 7687, 7689, 7691, 7693, 7698, 12368, 12369, 12370 }.

Here is the information for T 24 N 93 of order 96 $= [< 2, 5 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 16 and rank 5, 5 orbits of size 48 and rank 13, and 40 orbits of size 96 and rank 13

Here is the information for T 24 N 94 of order 96 $= [< 2, 5 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 16 and rank 5, 15 orbits of size 48 and rank 13, and 35 orbits of size 96 and rank 13

Here is the information for T 24 N 96 of order 96 $= [< 2, 5 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 16 and rank 5, 5 orbits of size 48 and rank 13, and 40 orbits of size 96 and rank 13

Here is the information for T 24 N 174 of order $96 = [< 2, 5 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 2 and rank 2, 1 orbit of size 6 and rank 4, 1 orbit of size 8 and rank 4, 1 orbit of size 12 and rank 6, 1 orbit of size 12 and rank 7, 1 orbit of size 24 and rank 7, 1 orbit of size 24 and rank 9, 3 orbits of size 24 and rank 10, 4 orbits of size 24 and rank 13, 2 orbits of size 32 and rank 7, 4 orbits of size 32 and rank 11, 8 orbits of size 48 and rank 13, 2 orbits of size 96 and rank 10, 7 orbits of size 96 and rank 12, 25 orbits of size 96 and rank 13

Here is the information for T 24 N 179 of order $96 = [< 2, 5 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 8 orbits of size 32 and rank 7 and 40 orbits of size 96 and rank 13

Here is the information for T 24 N 180 of order $96 = [< 2, 5 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 24 and rank 13, 4 orbits of size 32 and rank 7, 4 orbits of size 32 and rank 13, 14 orbits of size 48 and rank 13, and 32 orbits of size 96 and rank 13

Here is the information for T 24 N 213 of order $144 = [< 2, 4 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 4 and rank 3, 1 orbit of size 8 and rank 5, 12 orbits of size 24 and rank 13, 6 orbits of size 36 and rank 11, 4 orbits of size 48 and rank 13, 4 orbits of size 72 and rank 11, 15 orbits of size 72 and rank 13, 2 orbits of size 144 and rank 11, and 12 orbits of size 144 and rank 13

Here is the information for T 24 N 214 of order 144 = $[< 2, 4 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 4 and rank 3, 1 orbit of size 8 and rank 5, 12 orbits of size 24 and rank 13, 6 orbits of size 36 and rank 11, 4 orbits of size 48 and rank 13, 15 orbits of size 72 and rank 13, 4 orbits of size 144 and rank 11, and 12 orbits of size 144 and rank 13

Here is the information for T 24 N 215 of order 144 = $[< 2, 4 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 4 and rank 3, 12 orbits of size 24 and rank 13, 12 orbits of size 36 and rank 11, 4 orbits of size 48 and rank 13, 8 orbits of size 72 and rank 11, 8 orbits of size 72 and rank 13, 6 orbits of size 144 and rank 11, and 8 orbits of size 144 and rank 13

Here is the information for T 24 N 216 of order 144 = $[< 2, 4 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 8 and rank 5, 12 orbits of size 24 and rank 13, 4 orbits of size 48 and rank 13, 18 orbits of size 72 and rank 13, and 16 orbits of size 144 and rank 13

Here is the information for T 24 N 238 of order 144 = $[< 2, 4 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 4 and rank 3, 12 orbits of size 24 and rank 13, 12 orbits of size 36 and rank 11, 4 orbits of size 48 and rank 13, 4 orbits of size 72 and rank 11, 8 orbits of size 72 and rank 13, 8 orbits of size 144 and rank 11, and 8 orbits of size 144 and rank 13

Here is the information for T 24 N 239 of order 144 = $[< 2, 4 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 8 and rank 5, 12 orbits of size 24 and rank 13, 4 orbits of size 48 and rank 13, 18 orbits of size 72 and rank 13, and 16 orbits of size 144 and rank 13

Here is the information for T 24 N 240 of order 144 = $[< 2, 4 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 4 and rank 3, 1 orbit of size 8 and rank 5, 12 orbits of size 24 and rank 13, 6 orbits of size 36 and rank 11, 4 orbits of size 48 and rank 13, 15 orbits of size 72 and rank 13, 4 orbits of size 144 and rank 11, and 12 orbits of size 144 and rank 13

Here is the information for T 24 N 241 of order 144 = $[< 2, 4 >, < 3, 2 >]$:

There are 3 ρs for this group.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 2 and rank 2, 1 orbit of size 8 and rank 5, 4 orbits of size 12 and rank 7, 4 orbits of size 18 and rank 6, 2 orbits of size 24 and rank 9, 4 orbits of size 24 and rank 11, 8 orbits of size 24 and rank 13, 8 orbits of size 36 and rank 11, 2 orbits of size 48 and rank 13, 8 orbits of size 72 and rank 10, 4 orbits of size 72 and rank 12, 9 orbits of size 72 and rank 13, 4 orbits of size 144 and rank 12, and 8 orbits of size 144 and rank 13

Information for ρ # 2 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 2 and rank 2, 1 orbit of size 8 and rank 5, 4 orbits of size 12 and rank 7, 4 orbits of size 18 and rank 6, 2 orbits of size 24 and rank 9, 4 orbits of size 24 and rank 11, 8 orbits of size 24 and rank 13, 8 orbits of size 36 and rank 11, 2 orbits of size 48 and rank 13, 8 orbits of size 72 and rank 10, 4 orbits of size 72 and rank 12, 9 orbits of size 72 and rank 13, 4 orbits of size 144 and rank 12, and 8 orbits of size 144 and rank 13

Information for ρ # 3 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 2 and rank 2, 1 orbit of size 8 and rank 5, 4 orbits of size 12 and rank 7, 4 orbits of size 18 and rank 6, 2 orbits of size 24 and rank 9, 4 orbits of size 24 and rank 11, 8 orbits of size 24 and rank 13, 8 orbits of size 36 and rank 11, 2 orbits of size 48 and rank 13, 8 orbits of size 72 and rank 10, 4 orbits of size 72 and rank 12, 9 orbits of size 72 and rank 13, 4 orbits of size 144 and rank 12, and 8 orbits of size 144 and rank 13

Each ρ produces the same list of sizes of orbits of types.

Here is the information for T 24 N 255 of order 144 $= [< 2, 4 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 2 and rank 2, 1 orbit of size 4 and rank 3, 1 orbit of size 8 and rank 5, 2 orbits of size 18 and rank 10, 8 orbits of size 24 and rank 13, 5 orbits of size 36 and rank 11, 6 orbits of size 72 and rank 12, 13 orbits of size 72 and rank 13, 2 orbits of size 144 and rank 10, 2 orbits of size 144 and rank 11, 2 orbits of size 144 and rank 12, and 10 orbits of size 144 and rank 13

Here is the information for T 24 N 257 of order 144 = $[< 2, 4 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 2 and rank 2, 1 orbit of size 8 and rank 5, 4 orbits of size 18 and rank 10, 8 orbits of size 24 and rank 13, 12 orbits of size 36 and rank 11, 4 orbits of size 72 and rank 12, 11 orbits of size 72 and rank 13, 4 orbits of size 144 and rank 10, 4 orbits of size 144 and rank 12, and 8 orbits of size 144 and rank 13

Here is the information for T 24 N 258 of order 144 = $[< 2, 4 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 4 and rank 3, 1 orbit of size 8 and rank 5, 8 orbits of size 24 and rank 13, 6 orbits of size 36 and rank 11, 19 orbits of size 72 and rank 13, 4 orbits of size 144 and rank 11, and 12 orbits of size 144 and rank 13

Here is the information for T 24 N 259 of order 144 = $[< 2, 4 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 4 and rank 3, 8 orbits of size 24 and rank 13, 12 orbits of size 36 and rank 11, 4 orbits of size 72 and rank 11, 12 orbits of size 72 and rank 13, 8 orbits of size 144 and rank 11, and 8 orbits of size 144 and rank 13

Here is the information for T 24 N 263 of order 144 $= [< 2, 4 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 2 and rank 2, 1 orbit of size 8 and rank 5, 4 orbits of size 12 and rank 7, 4 orbits of size 18 and rank 6, 2 orbits of size 24 and rank 9, 4 orbits of size 24 and rank 11, 8 orbits of size 24 and rank 13, 8 orbits of size 36 and rank 11, 2 orbits of size 48 and rank 13, 8 orbits of size 72 and rank 10, 4 orbits of size 72 and rank 12, 9 orbits of size 72 and rank 13, 4 orbits of size 144 and rank 12, and 8 orbits of size 144 and rank 13

Here is the information for T 24 N 267 of order 144 $= [< 2, 4 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 4 and rank 3, 1 orbit of size 8 and rank 5, 4 orbits of size 12 and rank 7, 2 orbits of size 24 and rank 9, 4 orbits of size 24 and rank 11, 8 orbits of size 24 and rank 13, 2 orbits of size 36 and rank 7, 2 orbits of size 48 and rank 13, 4 orbits of size 72 and rank 11, 17 orbits of size 72 and rank 13, 2 orbits of size 144 and rank 11, and 12 orbits of size 144 and rank 13

Here is the information for T 24 N 268 of order 144 $= [< 2, 4 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 4 and rank 3, 4 orbits of size 12 and rank 7, 2 orbits of size 24 and rank 7, 4 orbits of size 24 and rank 11, 8 orbits of size 24 and rank 13, 2 orbits of size 36 and rank 7, 6 orbits of size 36 and rank 11, 2 orbits of size 48 and rank 11, 6 orbits of size 72 and rank 11, 12 orbits of size 72 and rank 13, 6 orbits of size 144 and rank 11, and 8 orbits of size 144 and rank 13

Here is the information for T 24 N 273 of order 144 = $[< 2, 4 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 4 and rank 3, 4 orbits of size 12 and rank 7, 2 orbits of size 24 and rank 7, 4 orbits of size 24 and rank 11, 8 orbits of size 24 and rank 13, 2 orbits of size 36 and rank 7, 6 orbits of size 36 and rank 11, 2 orbits of size 48 and rank 11, 10 orbits of size 72 and rank 11, 12 orbits of size 72 and rank 13, 4 orbits of size 144 and rank 11, and 8 orbits of size 144 and rank 13

Here is the information for T 24 N 307 of order 192 = $[< 2, 6 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 32 and rank 13, 2 orbits of size 64 and rank 13, 4 orbits of size 96 and rank 13, and 18 orbits of size 192 and rank 13

Here is the information for T 24 N 308 of order 192 = $[< 2, 6 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 64 and rank 13 and 20 orbits of size 192 and rank 13

Here is the information for T 24 N 309 of order 192 = $[< 2, 6 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 64 and rank 13 and 20 orbits of size 192 and rank 13

Here is the information for T 24 N 310 of order 192 = $[< 2, 6 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 64 and rank 13 and 20 orbits of size 192 and rank 13

Here is the information for T 24 N 311 of order 192 = $[< 2, 6 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 64 and rank 13 and 20 orbits of size 192 and rank 13

Here is the information for T 24 N 312 of order 192 = $[< 2, 6 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 64 and rank 13 and 20 orbits of size 192 and rank 13

Here is the information for T 24 N 315 of order 192 = $[< 2, 6 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 32 and rank 13, 2 orbits of size 64 and rank 13, 12 orbits of size 96 and rank 13, and 14 orbits of size 192 and rank 13

Here is the information for T 24 N 316 of order 192 = $[< 2, 6 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 64 and rank 13 and 20 orbits of size 192 and rank 13

Here is the information for T 24 N 317 of order 192 = $[< 2, 6 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 64 and rank 13 and 20 orbits of size 192 and rank 13

Here is the information for T 24 N 424 of order 192 = $[< 2, 6 >, < 3, 1 >]$:

There are 3 ρs for this group.

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 32 and rank 7, 2 orbits of size 64 and rank 13, 4 orbits of size 96 and rank 13, and 18 orbits of size 192 and rank 13

Information for ρ # 2 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 2 and rank 2, 1 orbit of size 6 and rank 4, 1 orbit of size 8 and rank 4, 1 orbit of size 12 and rank 6, 1 orbit of size 12 and rank 7, 1 orbit of size 24 and rank 7, 1 orbit of size 24 and rank 9, 1 orbit of size 24 and rank 10, 6 orbits of size 24 and rank 13, 2 orbits of size 32 and rank 7, 1 orbit of size 48 and rank 10, 11 orbits of size 48 and rank 13, 2 orbits of size 64 and rank 11, 2 orbits of size 96 and rank 10, 3 orbits of size 96 and rank 12, 11 orbits of size 96 and rank 13, 2 orbits of size 192 and rank 12, and 6 orbits of size 192 and rank 13

Information for ρ # 3 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 2 and rank 2, 1 orbit of size 6 and rank 4, 1 orbit of size 8 and rank 4, 1 orbit of size 12 and rank 6, 1 orbit of size 12 and rank 7, 1 orbit of size 24 and rank 7, 1 orbit of size 24 and rank 9, 1 orbit of size 24 and rank 10, 6 orbits of size 24 and rank 13, 2 orbits of size 32 and rank 7, 1 orbit of size 48 and rank 10, 11 orbits of size 48 and rank 13, 2 orbits of size 64 and rank 11, 2 orbits of size 96 and rank 10, 3 orbits of size 96 and rank 12, 11 orbits of size 96 and rank 13, 2 orbits of size 192 and rank 12, and 6 orbits of size 192 and rank 13

There are 2 ρs that produce distinct lists of sizes of orbits of types.

ρ # [1] has this list of sizes of orbits of types:

[32, 32, 32, 32, 64, 64, 96, 96, 96, 96, 192, 192, 192, 192, 192, 192, 192, 192, 192, 192, 192, 192, 192, 192, 192, 192, 192, 192, 192, 192]

The 2 ρs $[2, 3]$ share this list of sizes of orbits of types:

[2, 6, 8, 12, 12, 24, 24, 24, 24, 24, 24, 24, 24, 24, 32, 32, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48, 64, 64, 96, 96, 96, 96, 96, 96, 96, 96, 96, 96, 96, 96, 96, 96, 96, 192, 192, 192, 192, 192, 192, 192, 192]

Here is the information for T 24 N 460 of order 192 = $[< 2, 6 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 32 and rank 7, 2 orbits of size 64 and rank 13, 4 orbits of size 96 and rank 13, and 18 orbits of size 192 and rank 13

Here is the information for T 24 N 468 of order 192 = $[< 2, 6 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 32 and rank 13, 2 orbits of size 64 and rank 7, 4 orbits of size 96 and rank 13, and 18 orbits of size 192 and rank 13

Here is the information for T 24 N 470 of order 192 = $[< 2, 6 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 64 and rank 7, 2 orbits of size 64 and rank 13, and 20 orbits of size 192 and rank 13

Here is the information for T 24 N 481 of order 192 = $[<2,6>,<3,1>]$:

Information for $\rho \ \# \ 1$:

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 64 and rank 7, 2 orbits of size 64 and rank 13, and 20 orbits of size 192 and rank 13

Here is the information for T 24 N 483 of order 192 = $[<2,6>,<3,1>]$:

Information for $\rho \ \# \ 1$:

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 16 and rank 7, 2 orbits of size 48 and rank 7, 8 orbits of size 48 and rank 13, 2 orbits of size 64 and rank 7, 16 orbits of size 96 and rank 13, and 10 orbits of size 192 and rank 13

Here is the information for T 24 N 496 of order 192 = $[<2,6>,<3,1>]$:

Information for $\rho \ \# \ 1$:

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 64 and rank 7, 2 orbits of size 64 and rank 13, 16 orbits of size 96 and rank 13, and 12 orbits of size 192 and rank 13

Here is the information for T 24 N 506 of order 192 = $[<2,6>,<3,1>]$:

Information for $\rho \ \# \ 1$:

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 64 and rank 7, 2 orbits of size 64 and rank 13, and 20 orbits of size 192 and rank 13

Here is the information for T 24 N 596 of order 288 = $[< 2, 5 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 16 and rank 5, 4 orbits of size 48 and rank 13, 3 orbits of size 96 and rank 13, 3 orbits of size 144 and rank 13, and 11 orbits of size 288 and rank 13

Here is the information for T 24 N 597 of order 288 = $[< 2, 5 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 16 and rank 5, 5 orbits of size 96 and rank 13, 5 orbits of size 144 and rank 13, and 10 orbits of size 288 and rank 13

Here is the information for T 24 N 598 of order 288 = $[< 2, 5 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 16 and rank 5, 5 orbits of size 96 and rank 13, 5 orbits of size 144 and rank 13, and 10 orbits of size 288 and rank 13

Here is the information for T 24 N 620 of order 288 = $[< 2, 5 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 16 and rank 5, 2 orbits of size 48 and rank 13, 4 orbits of size 96 and rank 13, 1 orbit of size 144 and rank 13, and 12 orbits of size 288 and rank 13

Here is the information for T 24 N 622 of order 288 = $[< 2, 5 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 16 and rank 5, 6 orbits of size 48 and rank 13, 2 orbits of size 96 and rank 13, 9 orbits of size 144 and rank 13, and 8 orbits of size 288 and rank 13

Here is the information for T 24 N 945 of order 384 = $[< 2, 7 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 8 orbits of size 128 and rank 13 and 8 orbits of size 384 and rank 13

Here is the information for T 24 N 1371 of order 576 = $[< 2, 6 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 16 and rank 5, 3 orbits of size 96 and rank 13, 3 orbits of size 144 and rank 13, 1 orbit of size 192 and rank 13, 3 orbits of size 288 and rank 13, and 4 orbits of size 576 and rank 13

Here is the information for T 24 N 1392 of order 576 = $[< 2, 6 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 16 and rank 5, 3 orbits of size 96 and rank 13, 3 orbits of size 144 and rank 13, 1 orbit of size 192 and rank 13, 3 orbits of size 288 and rank 13, and 4 orbits of size 576 and rank 13

Here is the information for T 24 N 1410 of order $576 = [< 2, 6 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 16 and rank 5, 2 orbits of size 96 and rank 13, 3 orbits of size 144 and rank 13, 4 orbits of size 288 and rank 13, and 4 orbits of size 576 and rank 13

Here is the information for T 24 N 2128 of order $768 = [< 2, 8 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 256 and rank 13 and 4 orbits of size 768 and rank 13

Here is the information for T 24 N 2129 of order $768 = [< 2, 8 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 256 and rank 13 and 4 orbits of size 768 and rank 13

Here is the information for T 24 N 2130 of order $768 = [< 2, 8 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 256 and rank 13 and 4 orbits of size 768 and rank 13

Here is the information for T 24 N 2788 of order $1152 = [< 2, 7 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 2 and rank 2, 1 orbit of size 6 and rank 4, 4 orbits of size 24 and rank 13, 1 orbit of size 36 and rank 12, 1 orbit of size 36 and rank 13, 3 orbits of size 72 and rank 10, 3 orbits of size 72 and rank 13, 1 orbit of size 96 and rank 12, 3 orbits of size 96 and rank 13, 1 orbit of size 128 and rank 11, 4 orbits of size 144 and rank 13, 3 orbits of size 288 and rank 13, 1 orbit of size 384 and rank 13, 1 orbit of size 576 and rank 12, and 1 orbit of size 576 and rank 13

Here is the information for T 24 N 2801 of order 1152 = $[< 2, 7 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 2 and rank 2, 2 orbits of size 12 and rank 7, 2 orbits of size 16 and rank 9, 2 orbits of size 18 and rank 6, 2 orbits of size 24 and rank 13, 2 orbits of size 32 and rank 7, 5 orbits of size 48 and rank 13, 2 orbits of size 72 and rank 12, 2 orbits of size 72 and rank 13, 2 orbits of size 96 and rank 13, 1 orbit of size 144 and rank 11, 2 orbits of size 144 and rank 12, 1 orbit of size 144 and rank 13, 1 orbit of size 192 and rank 11, 2 orbits of size 192 and rank 13, 2 orbits of size 288 and rank 12, 3 orbits of size 288 and rank 13, and 1 orbit of size 576 and rank 13

Here is the information for T 24 N 3075 of order 1536 = $[< 2, 9 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 512 and rank 13 and 2 orbits of size 1536 and rank 13

Here is the information for T 24 N 3098 of order 1536 = $[< 2, 9 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 512 and rank 13 and 2 orbits of size 1536 and rank 13

Here is the information for T 24 N 5509 of order $3072 = [< 2, 10 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 1024 and rank 13

Here is the information for T 24 N 5535 of order $3072 = [< 2, 10 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 512 and rank 13 and 2 orbits of size 1024 and rank 13

Here is the information for T 24 N 5693 of order $3072 = [< 2, 10 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 1024 and rank 13

Here is the information for T 24 N 5872 of order $3072 = [< 2, 10 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 512 and rank 13 and 2 orbits of size 1024 and rank 13

Here is the information for T 24 N 5873 of order $3072 = [< 2, 10 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 1024 and rank 13

Here is the information for T 24 N 7443 of order $4608 = [<2,9>,<3,2>]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 512 and rank 13 and 2 orbits of size 1536 and rank 13

Here is the information for T 24 N 7444 of order $4608 = [<2,9>,<3,2>]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 512 and rank 13 and 2 orbits of size 1536 and rank 13

Here is the information for T 24 N 7445 of order $4608 = [<2,9>,<3,2>]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 512 and rank 13 and 2 orbits of size 1536 and rank 13

Here is the information for T 24 N 7446 of order $4608 = [<2,9>,<3,2>]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 384 and rank 13, 2 orbits of size 512 and rank 13, and 2 orbits of size 1152 and rank 13

Here is the information for T 24 N 7447 of order $4608 = [<2,9>,<3,2>]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 512 and rank 13 and 2 orbits of size 1536 and rank 13

Here is the information for T 24 N 7448 of order 4608 = $[< 2, 9 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 384 and rank 13, 2 orbits of size 512 and rank 13, 1 orbit of size 1152 and rank 13, and 1 orbit of size 1536 and rank 13

Here is the information for T 24 N 7688 of order 5184 = $[< 2, 6 >, < 3, 4 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 16 and rank 5, 2 orbits of size 96 and rank 13, 2 orbits of size 144 and rank 13, 2 orbits of size 288 and rank 13, 2 orbits of size 864 and rank 13, and 1 orbit of size 1296 and rank 13

Here is the information for T 24 N 7690 of order 5184 = $[< 2, 6 >, < 3, 4 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 16 and rank 5, 2 orbits of size 96 and rank 13, 2 orbits of size 144 and rank 13, 2 orbits of size 288 and rank 13, 2 orbits of size 864 and rank 13, and 1 orbit of size 1296 and rank 13

Here is the information for T 24 N 7692 of order 5184 = $[< 2, 6 >, < 3, 4 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 16 and rank 5, 2 orbits of size 96 and rank 13, 2 orbits of size 144 and rank 13, 2 orbits of size 288 and rank 13, 2 orbits of size 864 and rank 13, and 1 orbit of size 1296 and rank 13

Here is the information for T 24 N 7694 of order $5184 = [< 2,6 >,< 3,4 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 4 and rank 3, 8 orbits of size 24 and rank 13, 8 orbits of size 36 and rank 11, 4 orbits of size 72 and rank 11, 4 orbits of size 72 and rank 13, 8 orbits of size 216 and rank 13, and 4 orbits of size 324 and rank 11

Here is the information for T 24 N 7695 of order $5184 = [< 2,6 >,< 3,4 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 2 and rank 2, 1 orbit of size 8 and rank 5, 8 orbits of size 24 and rank 13, 12 orbits of size 36 and rank 11, 6 orbits of size 72 and rank 13, 4 orbits of size 162 and rank 10, 4 orbits of size 216 and rank 12, 4 orbits of size 216 and rank 13, and 1 orbit of size 648 and rank 13

Here is the information for T 24 N 7696 of order $5184 = [< 2,6 >,< 3,4 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 4 and rank 3, 1 orbit of size 8 and rank 5, 8 orbits of size 24 and rank 13, 4 orbits of size 36 and rank 11, 10 orbits of size 72 and rank 13, 8 orbits of size 216 and rank 13, 2 orbits of size 324 and rank 11, and 1 orbit of size 648 and rank 13

Here is the information for T 24 N 7697 of order $5184 = [< 2,6 >,< 3,4 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 8 and rank 5, 8 orbits of size 24 and rank 13, 12 orbits of size 72 and rank 13, 8 orbits of size 216 and rank 13, and 2 orbits of size 648 and rank 13

Here is the information for T 24 N 7882 of order 6144 = $[< 2, 11 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 2048 and rank 13

Here is the information for T 24 N 7905 of order 6144 = $[< 2, 11 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 1024 and rank 13 and 1 orbit of size 2048 and rank 13

Here is the information for T 24 N 10036 of order 10368 = $[< 2, 7 >, < 3, 4 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 16 and rank 5, 2 orbits of size 96 and rank 13, 2 orbits of size 144 and rank 13, 2 orbits of size 288 and rank 13, 2 orbits of size 864 and rank 13, and 1 orbit of size 1296 and rank 13

Here is the information for T 24 N 10283 of order 12288 = $[< 2, 12 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 4096 and rank 13

Here is the information for T 24 N 173 of order 96 = $[< 2, 5 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 32 and rank 7, 4 orbits of size 32 and rank 13, and 40 orbits of size 96 and rank 13

Here is the information for T 24 N 175 of order 96 = $[< 2, 5 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 8 and rank 4, 4 orbits of size 12 and rank 7, 2 orbits of size 24 and rank 10, 6 orbits of size 24 and rank 13, 2 orbits of size 32 and rank 7, 4 orbits of size 32 and rank 10, 2 orbits of size 48 and rank 10, 6 orbits of size 48 and rank 13, 2 orbits of size 96 and rank 10, and 32 orbits of size 96 and rank 13

Here is the information for T 24 N 256 of order 144 = $[< 2, 4 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 8 and rank 5, 8 orbits of size 24 and rank 13, 22 orbits of size 72 and rank 13, and 16 orbits of size 144 and rank 13

Here is the information for T 24 N 260 of order 144 = $[< 2, 4 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 8 and rank 5, 8 orbits of size 24 and rank 13, 22 orbits of size 72 and rank 13, and 16 orbits of size 144 and rank 13

Here is the information for T 24 N 264 of order 144 = $[< 2, 4 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 4 and rank 3, 4 orbits of size 12 and rank 7, 2 orbits of size 24 and rank 7, 4 orbits of size 24 and rank 11, 8 orbits of size 24 and rank 13, 2 orbits of size 36 and rank 7, 6 orbits of size 36 and rank 11, 2 orbits of size 48 and rank 11, 10 orbits of size 72 and rank 11, 12 orbits of size 72 and rank 13, 4 orbits of size 144 and rank 11, and 8 orbits of size 144 and rank 13

Here is the information for T 24 N 306 of order 192 = $[< 2, 6 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 32 and rank 13, 2 orbits of size 64 and rank 13, 4 orbits of size 96 and rank 13, and 18 orbits of size 192 and rank 13

Here is the information for T 24 N 442 of order 192 = $[< 2, 6 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 16 and rank 7, 4 orbits of size 32 and rank 7, 2 orbits of size 48 and rank 7, 8 orbits of size 48 and rank 13, 20 orbits of size 96 and rank 13, and 8 orbits of size 192 and rank 13

Here is the information for T 24 N 447 of order 192 = $[< 2, 6 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 32 and rank 7, 2 orbits of size 64 and rank 13, 4 orbits of size 96 and rank 13, and 18 orbits of size 192 and rank 13

Here is the information for T 24 N 453 of order 192 = $[< 2, 6 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 32 and rank 7, 2 orbits of size 64 and rank 7, 20 orbits of size 96 and rank 13, and 10 orbits of size 192 and rank 13

Here is the information for T 24 N 471 of order 192 = $[<2,6>,<3,1>]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 32 and rank 7, 2 orbits of size 64 and rank 13, 12 orbits of size 96 and rank 13, and 14 orbits of size 192 and rank 13

Here is the information for T 24 N 476 of order 192 = $[<2,6>,<3,1>]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 32 and rank 7, 2 orbits of size 64 and rank 7, 12 orbits of size 96 and rank 13, and 14 orbits of size 192 and rank 13

Here is the information for T 24 N 823 of order 384 = $[<2,7>,<3,1>]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 128 and rank 13 and 10 orbits of size 384 and rank 13

Here is the information for T 24 N 831 of order 384 = $[<2,7>,<3,1>]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 128 and rank 13 and 10 orbits of size 384 and rank 13

Here is the information for T 24 N 946 of order 384 = $[< 2, 7 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 8 orbits of size 128 and rank 13 and 8 orbits of size 384 and rank 13

Here is the information for T 24 N 1620 of order 768 = $[< 2, 8 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 256 and rank 13 and 4 orbits of size 768 and rank 13

Here is the information for T 24 N 1781 of order 768 = $[< 2, 8 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 256 and rank 13 and 4 orbits of size 768 and rank 13

Here is the information for T 24 N 1830 of order 768 = $[< 2, 8 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 256 and rank 13 and 8 orbits of size 384 and rank 13

Here is the information for T 24 N 2127 of order 768 = $[< 2, 8 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 128 and rank 13, 2 orbits of size 256 and rank 13, and 4 orbits of size 768 and rank 13

Here is the information for T 24 N 2781 of order $1152 = [< 2, 7 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 128 and rank 13, 4 orbits of size 384 and rank 13, and 4 orbits of size 576 and rank 13

Here is the information for T 24 N 2799 of order $1152 = [< 2, 7 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 4 and rank 3, 2 orbits of size 12 and rank 7, 2 orbits of size 16 and rank 9, 2 orbits of size 24 and rank 13, 1 orbit of size 36 and rank 7, 5 orbits of size 48 and rank 13, 1 orbit of size 64 and rank 7, 2 orbits of size 72 and rank 11, 2 orbits of size 72 and rank 13, 2 orbits of size 96 and rank 13, 4 orbits of size 144 and rank 13, 1 orbit of size 192 and rank 11, 2 orbits of size 192 and rank 13, 3 orbits of size 288 and rank 13, and 2 orbits of size 576 and rank 13

Here is the information for T 24 N 3072 of order $1536 = [< 2, 9 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 128 and rank 13, 2 orbits of size 384 and rank 13, and 4 orbits of size 768 and rank 13

Here is the information for T 24 N 3078 of order $1536 = [< 2, 9 >, < 3, 1 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 512 and rank 13 and 2 orbits of size 1536 and rank 13

Here is the information for T 24 N 7449 of order 4608 = $[< 2, 9 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 384 and rank 13, 2 orbits of size 512 and rank 13, 1 orbit of size 1152 and rank 13, and 1 orbit of size 1536 and rank 13

Here is the information for T 24 N 7687 of order 5184 = $[< 2, 6 >, < 3, 4 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 16 and rank 5, 2 orbits of size 96 and rank 13, 2 orbits of size 144 and rank 13, 2 orbits of size 288 and rank 13, 2 orbits of size 864 and rank 13, and 1 orbit of size 1296 and rank 13

Here is the information for T 24 N 7689 of order 5184 = $[< 2, 6 >, < 3, 4 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 16 and rank 5, 2 orbits of size 96 and rank 13, 2 orbits of size 144 and rank 13, 2 orbits of size 288 and rank 13, 2 orbits of size 864 and rank 13, and 1 orbit of size 1296 and rank 13

Here is the information for T 24 N 7691 of order 5184 = $[< 2, 6 >, < 3, 4 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There is 1 orbit of size 16 and rank 5, 2 orbits of size 96 and rank 13, 2 orbits of size 144 and rank 13, 2 orbits of size 288 and rank 13, 2 orbits of size 864 and rank 13, and 1 orbit of size 1296 and rank 13

Here is the information for T 24 N 7693 of order 5184 = $[< 2, 6 >, < 3, 4 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 4 and rank 3, 8 orbits of size 24 and rank 13, 8 orbits of size 36 and rank 11, 4 orbits of size 72 and rank 11, 4 orbits of size 72 and rank 13, 8 orbits of size 216 and rank 13, and 4 orbits of size 324 and rank 11

Here is the information for T 24 N 7698 of order 5184 = $[< 2, 6 >, < 3, 4 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 4 and rank 3, 1 orbit of size 8 and rank 5, 8 orbits of size 24 and rank 13, 4 orbits of size 36 and rank 11, 10 orbits of size 72 and rank 13, 8 orbits of size 216 and rank 13, 2 orbits of size 324 and rank 11, and 1 orbit of size 648 and rank 13

Here is the information for T 24 N 12368 of order 18432 = $[< 2, 11 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 512 and rank 13 and 2 orbits of size 1024 and rank 13

Here is the information for T 24 N 12369 of order 18432 = $[< 2, 11 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 4 orbits of size 1024 and rank 13

Here is the information for T 24 N 12370 of order 18432 = $[< 2, 11 >, < 3, 2 >]$:

Information for ρ # 1 :

Here is a list of orbit sizes and ranks.

There are 2 orbits of size 2048 and rank 13

B.1.3 Galois Groups of CM Fields with Imaginary Quadratic Subfields

The groups are Galois groups of CM fields with imaginary quadratic subfields in degree 24:

{2, 3, 6, 9, 11, 25, 27, 29, 30, 38, 45, 47, 48, 49, 50, 66, 68, 71, 73, 76, 126, 132, 135, 136, 143, 147, 148, 150, 151, 152, 174, 178, 203, 225, 230, 232, 241, 242, 248, 250, 251, 254, 255, 257, 261, 262, 263, 399, 400, 406, 413, 416, 418, 419, 421, 424, 444, 451, 455, 458, 466, 473, 474, 479, 480, 487, 491, 492, 511, 547, 548, 553, 555, 572, 573, 577, 645, 651, 654, 657, 671, 674, 679, 681, 684, 1076, 1078, 1089, 1092, 1095, 1097, 1098, 1101, 1102, 1105, 1107, 1110, 1111, 1113, 1115, 1116, 1119, 1120, 1130, 1134, 1189, 1190, 1195, 1196, 1202, 1203, 1205, 1206, 1211, 1212, 1292, 1296, 1304, 1310, 1312, 1316, 1320, 1345, 1346, 1476, 1480, 1481, 1484, 1486, 1510, 1514, 1516, 1518, 2454, 2475, 2481, 2494, 2495, 2499, 2501, 2502, 2505, 2506, 2514, 2517, 2522, 2524, 2525, 2526, 2528, 2530, 2531, 2533, 2537, 2539, 2646, 2651, 2772, 2776, 2785, 2786, 2788, 2790, 2791, 2800, 2801, 2839, 2843, 2850, 2854, 2857, 2864, 2867, 2872, 2874, 2876, 2878, 2880, 2948, 2952, 2954, 2956, 2959, 4782, 4785, 4787, 4790, 4799, 4800, 4806, 4810, 4813, 4826, 4948, 5080, 5084, 5087, 5089, 5090, 5095, 5096, 5097, 5098, 5101, 5108, 5109, 5113, 5116, 5190, 5203, 5216, 5219, 5223, 5243, 5245, 5249, 5255, 5323, 5331, 5333, 7158, 7164, 7168, 7171, 7174, 7176, 7181, 7214, 7215, 7250, 7257, 7258, 7261, 7262, 7499, 7502, 7503, 7505, 7506, 7512, 7516, 7659, 7665, 7676, 7684, 7695, 7705, 7711, 7716, 9614, 9619, 9620, 9621, 9622, 9672, 9674, 9676, 9683, 9684, 9937, 10073, 10087, 10096, 10108, 10109, 10113, 10128, 12104, 12114, 12139, 12151, 12204, 12576, 12590, 12593, 12594, 12617, 13989, 13990, 14016, 14020, 14022, 14589, 14590, 14629, 14630, 14631, 16043, 16056, 16586, 16588, 16590, 16619, 18407, 18439, 20409, 21431, 21432, 22335, 24747, 24815}

B.1.4 Groups without Subgroups Having All Nondegenerate Orbits

There are 30 groups of order 96, 120, or 192 that have only nondegenerate orbits and have no subgroups with only nodegenerate orbits. These groups are the following:

{85, 86, 87, 88, 89, 129, 131, 201, 290, 293, 298, 299, 300, 301, 305, 306, 307, 308, 309, 310, 311, 312, 313, 315, 316, 317, 426, 427, 428, 429}.

The final 21 groups without one of the above groups as a subgroup are:

{ 22332, 22333, 22335, 22782, 23169, 24747, 24760, 24763, 24765, 24815, 24825, 24826, 24828, 24829, 24830, 24831, 24869, 24958, 24970, 24971, 24979 }

The following groups correspond to imaginary quadratics: {22335, 24747, 24815}.

The following groups have $v = 11$ or 12: { 24760, 24825, 24826, 24869, 24958, 24970, 24971, 24979 }.

B.1.5 Groups with Degenerate Orbits

We now list the groups that have an orbit with rank $6 \leq r \leq 12$. We present them in the order in which we performed the computations.

The following groups have order up to 192:

{2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15, 21, 22, 25, 27, 29, 30, 33, 36, 38, 39, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 60, 61, 65, 66, 67, 68, 70, 71, 73, 74, 75, 76, 77, 112, 116, 123, 124, 125, 126, 127, 128, 130, 132, 133, 134, 135, 136, 137, 138, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 203, 204, 208, 213, 214, 215, 224, 225, 226, 229, 230, 231, 232, 235, 238, 240, 241, 242, 247, 248, 250, 251, 252, 254, 255, 257, 258, 259, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 360, 385, 386, 393, 394, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515}

The following groups have order greater than 192 and up to 2,304:

{543, 544, 545, 546, 547, 548, 549, 550, 553, 555, 570, 571, 572, 573, 575, 577, 578, 579, 592, 594, 606, 618, 626, 630, 632, 643, 645, 647, 649, 651, 652, 653, 654, 655, 657, 661, 671, 672, 673, 674, 675, 679, 680, 681, 684, 685, 686, 687, 688, 689, 690, 691, 839, 902, 971, 972, 973, 974, 1003, 1004, 1005, 1006, 1007, 1008, 1010, 1012, 1013, 1014, 1015, 1016, 1017, 1022, 1028, 1029, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1105, 1106, 1107, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1139, 1142, 1144, 1150, 1151, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1282, 1284, 1286, 1288, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1304, 1305, 1310, 1311, 1312, 1313, 1316, 1317, 1320, 1341, 1343, 1345, 1346, 1347, 1350, 1352, 1362, 1364, 1366, 1412, 1444, 1452, 1460, 1475, 1476, 1477, 1478, 1479, 1480, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1509, 1510, 1511, 1514, 1516, 1518, 1932, 1942, 1951, 1973, 1974, 1981, 1982, 1990, 2004, 2005, 2122, 2202, 2204, 2205, 2287, 2288, 2289, 2290, 2291, 2292, 2293, 2298, 2299, 2300, 2301, 2302, 2303, 2306, 2307, 2308, 2313, 2314, 2317, 2318, 2319, 2320, 2321, 2322, 2323, 2324, 2325, 2326, 2327, 2328, 2329, 2330, 2333, 2334, 2336, 2337, 2341, 2342, 2344, 2345, 2347, 2348, 2349, 2350, 2351, 2354, 2359, 2360, 2367, 2377, 2383, 2386, 2394, 2395, 2396, 2397, 2398, 2399, 2400, 2401, 2402, 2403, 2404, 2447, 2448, 2449, 2450, 2451, 2452, 2453, 2454, 2455, 2456, 2457, 2458, 2459, 2460, 2461, 2462, 2463, 2464, 2465, 2466, 2467, 2468, 2469, 2470, 2471, 2472, 2473, 2474, 2475, 2476, 2477, 2478, 2479, 2480, 2481, 2482, 2483, 2484, 2488, 2493, 2494, 2495, 2496, 2498, 2499, 2500, 2501, 2502, 2503, 2504, 2505, 2506, 2507, 2508, 2509, 2510, 2511, 2512, 2513, 2514, 2515, 2516, 2517, 2518, 2519, 2520, 2521, 2522, 2523, 2524, 2525, 2526, 2527, 2528, 2529, 2530, 2531, 2532, 2533, 2534, 2535, 2536, 2537, 2538, 2539, 2540, 2541, 2542, 2543, 2544, 2545, 2546, 2547, 2548, 2549, 2550, 2551, 2552, 2553, 2554, 2555, 2556, 2557, 2558, 2559, 2560, 2561, 2562, 2563, 2564, 2565, 2566, 2567, 2568, 2569, 2570, 2571, 2572, 2573, 2574, 2575, 2576, 2577, 2578, 2579, 2580, 2581, 2582, 2583, 2584, 2585, 2586, 2587, 2588, 2589, 2590, 2591, 2592, 2593, 2594, 2595, 2596, 2597, 2598, 2599, 2600, 2601, 2602, 2603, 2604, 2605, 2606, 2607, 2608, 2618, 2622, 2626, 2630, 2634, 2638, 2646, 2647, 2648, 2649, 2651, 2665, 2697, 2701, 2742, 2772, 2773, 2774, 2775, 2776, 2785, 2786, 2787, 2788, 2789, 2790, 2791, 2792, 2793, 2794, 2795, 2796, 2797, 2798, 2799, 2800, 2801, 2802, 2815, 2816, 2817, 2818, 2819, 2833, 2835, 2839, 2840, 2841, 2842, 2843, 2844, 2849, 2850, 2853, 2854, 2855, 2856, 2857, 2858, 2860, 2861, 2862, 2864, 2865, 2866, 2867, 2869, 2870, 2872, 2874, 2876, 2878, 2880, 2948, 2952, 2953, 2954, 2955, 2956, 2957, 2958, 2959, 3552, 3792, 3941, 3942, 3943, 3944, 3946, 3950, 3952, 3954, 3959, 3960, 3961, 3962, 3963, 3964, 3975, 3978, 3981, 3983, 3996, 4006, 4009, 4010, 4011, 4188, 4191, 4192, 4193, 4194, 4202, 4204, 4215, 4218, 4219, 4223, 4230, 4238, 4256, 4323, 4391, 4395, 4396, 4401, 4402, 4405, 4494, 4495, 4496, 4497, 4498, 4499, 4500, 4501, 4502, 4541, 4543, 4544, 4564, 4565, 4566, 4622, 4623, 4624, 4625, 4626, 4627, 4628, 4629, 4630, 4631, 4632, 4633, 4634, 4635, 4636, 4637, 4642, 4643, 4644, 4645, 4650, 4651, 4652, 4653, 4658, 4659, 4660, 4661, 4663, 4664, 4665, 4666, 4669, 4670, 4673, 4674, 4676, 4677, 4678, 4679, 4680, 4683, 4686, 4687, 4688, 4691, 4692, 4693, 4695, 4696, 4697, 4698, 4699, 4701, 4702, 4704, 4707, 4708, 4709, 4710, 4711, 4714, 4717, 4718, 4719, 4720, 4721, 4722, 4723, 4724, 4725, 4726, 4729, 4730, 4731, 4732, 4735, 4736, 4737, 4738, 4739, 4744, 4749, 4750, 4751, 4752, 4753, 4754, 4755, 4756, 4757, 4758, 4761, 4764, 4769, 4770, 4771, 4772, 4773, 4774, 4775, 4776, 4777, 4778, 4779, 4780, 4781, 4782, 4784, 4785, 4786, 4787, 4788, 4789, 4790, 4791, 4792, 4793, 4794, 4795, 4796, 4797, 4798, 4799, 4800, 4801, 4802, 4803, 4804, 4805, 4806, 4807, 4808, 4809, 4810, 4811, 4813, 4814, 4815, 4816, 4817, 4818, 4819, 4820, 4821, 4822, 4823, 4824, 4825, 4826, 4827, 4828, 4829, 4830, 4831, 4832, 4833, 4834, 4835, 4836, 4837, 4838, 4839, 4840, 4841, 4842, 4843, 4844, 4845, 4846, 4847, 4848, 4849, 4850, 4851, 4852, 4853, 4854, 4855, 4856, 4857, 4858, 4859, 4860, 4861, 4862, 4863, 4864, 4865, 4866, 4867, 4868, 4869, 4870, 4871, 4872, 4873, 4874, 4875, 4876, 4877, 4878, 4879, 4880, 4881, 4882, 4883, 4884, 4885, 4886, 4887, 4888, 4889, 4890, 4891, 4892, 4893, 4923, 4946, 4948, 5040, 5044, 5051, 5079, 5080, 5081, 5084, 5085, 5086, 5087, 5088, 5089, 5090, 5095, 5096, 5097, 5098, 5099, 5100, 5101, 5102, 5103, 5104, 5105, 5106, 5107, 5108, 5109, 5110, 5111, 5112, 5113, 5114, 5115, 5116, 5117, 5118, 5119, 5120, 5121, 5122, 5123, 5124, 5125, 5126, 5127, 5128, 5132, 5133, 5134, 5135, 5136, 5137, 5138, 5139, 5140, 5141, 5142, 5143, 5144, 5145}

The following groups have order greater than 2,304 and up to 9,216:

{5159, 5163, 5166, 5170, 5174, 5177, 5179, 5182, 5184, 5186, 5190, 5191, 5192, 5195, 5196, 5197, 5200, 5201, 5202, 5203, 5204, 5205, 5209, 5210, 5211, 5213, 5216, 5217, 5218, 5219, 5223, 5224, 5227, 5230, 5231, 5232, 5233, 5235, 5237, 5238, 5239, 5241, 5242, 5243, 5244, 5245, 5247, 5248, 5249, 5255, 5256, 5257, 5323, 5325, 5326, 5327, 5328, 5329, 5330, 5331, 5332, 5333, 5334, 5335, 5336, 6002, 6023, 6027, 6031, 6032, 6145, 6152, 6155, 6161, 6211, 6215, 6245, 6255, 6291, 6333, 6342, 6405, 6413, 6457, 6511, 6518, 6520, 6521, 6533, 6549, 6562, 6564, 6584, 6586, 6590, 6623, 6635, 6638, 6646, 6647, 6654, 6656, 6678, 6694, 6697, 6698, 6701, 6702, 6705, 6706, 6709, 6710, 6713, 6714, 6717, 6718, 6722, 6724, 6729, 6732, 6736, 6738, 6739, 6740, 6741, 6742, 6743, 6744, 6745, 6746, 6765, 6767, 6770, 6772, 6783, 6789, 6790, 6812, 6814, 6818, 6821, 6825, 6826, 6828, 6836, 6843, 6847, 6848, 6853, 6856, 6858, 6864, 6865, 6874, 6875, 6878, 6879, 6882, 6883, 6904, 6905, 6906, 6914, 6919, 6921, 6922, 6923, 6924, 6925, 6934, 6935, 6936, 6940, 6941, 6947, 6950, 6951, 6952, 6953, 6955, 6956, 6957, 6958, 6968, 6969, 6994, 7002, 7007, 7009, 7010, 7011, 7036, 7038, 7041, 7043, 7052, 7053, 7058, 7067, 7068, 7095, 7096, 7097, 7098, 7100, 7101, 7102, 7103, 7104, 7105, 7106, 7107, 7108, 7109, 7110, 7111, 7112, 7113, 7114, 7115, 7116, 7117, 7118, 7121, 7124, 7127, 7130, 7131, 7132, 7133, 7134, 7135, 7136, 7137, 7138, 7139, 7140, 7141, 7142, 7143, 7144, 7145, 7146, 7147, 7148, 7149, 7150, 7151, 7152, 7153, 7154, 7155, 7156, 7157, 7158, 7159, 7160, 7161, 7162, 7163, 7164, 7165, 7166, 7167, 7168, 7169, 7170, 7171, 7172, 7173, 7174, 7175, 7176, 7177, 7178, 7179, 7180, 7181, 7182, 7183, 7184, 7185, 7186, 7187, 7188, 7189, 7190, 7191, 7192, 7193, 7194, 7195, 7196, 7214, 7215, 7238, 7248, 7249, 7250, 7251, 7252, 7253, 7254, 7255, 7257, 7258, 7259, 7261, 7262, 7263, 7336, 7338, 7340, 7342, 7353, 7359, 7363, 7364, 7369, 7371, 7377, 7378, 7380, 7381, 7384, 7385, 7386, 7388, 7389, 7390, 7391, 7392, 7393, 7394, 7395, 7396, 7399, 7401, 7407, 7413, 7429, 7432, 7441, 7450, 7458, 7459, 7466, 7467, 7496, 7497, 7498, 7499, 7500, 7501, 7502, 7503, 7504, 7505, 7506, 7507, 7508, 7509, 7510, 7511, 7512, 7513, 7514, 7515, 7516, 7517, 7518, 7519, 7520, 7521, 7522, 7523, 7524, 7525, 7526, 7527, 7528, 7529, 7530, 7587, 7595, 7597, 7599, 7603, 7605, 7611, 7615, 7617, 7619, 7621, 7623, 7627, 7631, 7639, 7640, 7641, 7642, 7643, 7644, 7645, 7646, 7652, 7654, 7657, 7659, 7661, 7665, 7666, 7667, 7668, 7670, 7675, 7676, 7680, 7683, 7684, 7693, 7694, 7695, 7696, 7697, 7698, 7701, 7705, 7706, 7707, 7711, 7712, 7713, 7714, 7715, 7716, 7722, 7790, 7791, 8491, 8494, 8757, 8760, 8971, 8973, 8974, 8977, 8978, 8979, 8984, 8985, 8989, 8992, 8993, 8994, 9164, 9169, 9171, 9179, 9195, 9228, 9282, 9284, 9315, 9316, 9336, 9379, 9410, 9411, 9414, 9415, 9426, 9427, 9428, 9429, 9434, 9435, 9436, 9437, 9454, 9459, 9460, 9465, 9472, 9473, 9474, 9475, 9476, 9477, 9484, 9485, 9486, 9487, 9488, 9489, 9490, 9491, 9496, 9497, 9498, 9502, 9503, 9504, 9511, 9516, 9520, 9521, 9522, 9528, 9534, 9536, 9538, 9543, 9544, 9545, 9546, 9547, 9548, 9553, 9554, 9555, 9556, 9562, 9563, 9565, 9566, 9567, 9568, 9569, 9570, 9571, 9572, 9573, 9574, 9575, 9576, 9577, 9578, 9579, 9580, 9581, 9582, 9583, 9584, 9585, 9586, 9587, 9588, 9589, 9590, 9591, 9592, 9593, 9594, 9595, 9596, 9597, 9598, 9599, 9600, 9601, 9602, 9603, 9604, 9605, 9606, 9607, 9608, 9609, 9610, 9611, 9612, 9613, 9614, 9619, 9620, 9621, 9622, 9672, 9673, 9674, 9675, 9676, 9678, 9679, 9680, 9683, 9684, 9855, 9856, 9885, 9890, 9894, 9902, 9911, 9913, 9931, 9932, 9933, 9934, 9935, 9936, 9937, 9938 }

The following groups have order greater than 9,216:

{10013, 10018, 10024, 10029, 10034, 10039, 10043, 10047, 10057, 10071, 10072, 10073, 10074, 10075, 10076, 10087, 10088, 10095, 10096, 10107, 10108, 10109, 10113, 10114, 10115, 10128, 10129, 10130, 10418, 10422, 10451, 10470, 11363, 11364, 11383, 11384, 11447, 11453, 11460, 11462, 11627, 11643, 11770, 11772, 11774, 11775, 11865, 11872, 11873, 11895, 11899, 11920, 11921, 11935, 11950, 11951, 11952, 11953, 11958, 11959, 11960, 11961, 11968, 11969, 11970, 11971, 11972, 11973, 11982, 11983, 11984, 11985, 11986, 11987, 11994, 11995, 11996, 11997, 11998, 11999, 12008, 12009, 12010, 12011, 12012, 12013, 12018, 12019, 12020, 12021, 12026, 12027, 12028, 12029, 12034, 12035, 12040, 12041, 12046, 12047, 12060, 12061, 12062, 12063, 12068, 12069, 12070, 12071, 12076, 12077, 12078, 12079, 12080, 12081, 12082, 12083, 12084, 12085, 12086, 12087, 12088, 12089, 12090, 12091, 12093, 12095, 12104, 12114, 12138, 12139, 12151, 12204, 12349, 12351, 12406, 12408, 12409, 12420, 12422, 12424, 12434, 12444, 12464, 12466, 12517, 12518, 12520, 12521, 12522, 12524, 12525, 12526, 12527, 12528, 12529, 12532, 12533, 12535, 12537, 12538, 12539, 12540, 12541, 12542, 12543, 12544, 12545, 12546, 12547, 12548, 12550, 12551, 12552, 12553, 12555, 12556, 12557, 12558, 12559, 12561, 12562, 12563, 12564, 12567, 12569, 12570, 12571, 12572, 12576, 12577, 12578, 12579, 12580, 12581, 12583, 12584, 12585, 12586, 12587, 12588, 12589, 12590, 12591, 12592, 12593, 12594, 12617, 12618, 12619, 12620, 13329, 13353, 13750, 13752, 13789, 13791, 13794, 13811, 13813, 13837, 13838, 13839, 13840, 13876, 13878, 13883, 13888, 13889, 13901, 13902, 13910, 13911, 13932, 13933, 13934, 13935, 13936, 13937, 13938, 13939, 13948, 13949, 13950, 13951, 13952, 13953, 13954, 13955, 13967, 13985, 13988, 13989, 13990, 13991, 13992, 14016, 14020, 14022, 14324, 14332, 14333, 14342, 14355, 14365, 14416, 14417, 14460, 14463, 14464, 14465, 14468, 14470, 14471, 14472, 14476, 14477, 14478, 14483, 14485, 14486, 14489, 14490, 14493, 14494, 14495, 14496, 14538, 14567, 14589, 14590, 14591, 14592, 14629, 14630, 14631, 14632, 14633, 14634, 15727, 15795, 15897, 15898, 15905, 15906, 15913, 15918, 15919, 15928, 15949, 15972, 15973, 15976, 15977, 16024, 16040, 16041, 16043, 16044, 16045, 16046, 16049, 16056, 16349, 16434, 16435, 16463, 16467, 16474, 16501, 16502, 16503, 16510, 16511, 16524, 16525, 16528, 16529, 16530, 16531, 16534, 16568, 16570, 16572, 16586, 16587, 16588, 16589, 16590, 16591, 16592, 16619, 17670, 17705, 17751, 17811, 17812, 17813, 17814, 17918, 17933, 17935, 17940, 18269, 18285, 18332, 18333, 18334, 18335, 18398, 18400, 18407, 18408, 18439, 19046, 19254, 19270, 19271, 19609, 19611, 19613, 19629, 20393, 20409, 20410, 20673, 20706, 20708, 20710, 21428, 21429, 21430, 21431, 21432, 21433, 21838, 21849, 21850, 22334, 22337 }

We have the following summary for groups of order up to 9,216 with orbits of rank less than 13:

The following groups have an orbit of size 2 and rank 2 : [2, 3, 6, 9, 11, 25, 27, 29, 30, 38, 45, 47, 48, 49, 50, 66, 68, 71, 73, 76, 126, 132, 135, 136, 143, 147, 148, 150, 151, 152, 174, 178, 203, 225, 230, 232, 241, 242, 248, 250, 251, 254, 255, 257, 261, 262, 263, 399, 400, 406, 413, 416, 418, 419, 421, 424, 444, 451, 455, 458, 466, 473, 474, 479, 480, 487, 491, 492, 511, 547, 548, 553, 555, 572, 573, 577, 645, 651, 654, 657, 671, 674, 679, 681, 684, 1076, 1078, 1089, 1092, 1095, 1097, 1098, 1101, 1102, 1105, 1107, 1110, 1111, 1113, 1115, 1116, 1119, 1120, 1130, 1134, 1189, 1190, 1195, 1196, 1202, 1203, 1205, 1206, 1211, 1212, 1292, 1296, 1304, 1310, 1312, 1316, 1320, 1345, 1346, 1476, 1480, 1481, 1484, 1486, 1510, 1514, 1516, 1518, 2454, 2475, 2481, 2494, 2495, 2499, 2501, 2502, 2505, 2506, 2514, 2517, 2522, 2524, 2525, 2526, 2528, 2530, 2531, 2533, 2537, 2539, 2646, 2651, 2772, 2776, 2785, 2786, 2788, 2790, 2791, 2800, 2801, 2839, 2843, 2850, 2854, 2857, 2864, 2867, 2872, 2874, 2876, 2878, 2880, 2948, 2952, 2954, 2956, 2959, 4782, 4785, 4787, 4790, 4799, 4800, 4806, 4810, 4813, 4826, 4948, 5080, 5084, 5087, 5089, 5090, 5095, 5096, 5097, 5098, 5101, 5108, 5109, 5113, 5116, 5190, 5203, 5216, 5219, 5223, 5243, 5245, 5249, 5255, 5323, 5331, 5333, 7158, 7164, 7168, 7171, 7174, 7176, 7181, 7214, 7215, 7250, 7257, 7258, 7261, 7262, 7499, 7502, 7503, 7505, 7506, 7512, 7516, 7659, 7665, 7676, 7684, 7695, 7705, 7711, 7716, 9614, 9619, 9620, 9621, 9622, 9672, 9674, 9676, 9683, 9684, 9937]

The following groups have an orbit of size 4 and rank 3 : [2, 6, 12, 13, 14, 15, 25, 27, 29, 33, 38, 39, 44, 45, 52, 53, 54, 56, 57, 60, 61, 65, 66, 67, 74, 75, 76, 77, 112, 116, 124, 133, 143, 144, 145, 146, 158, 159, 163, 164, 165, 166, 168, 169, 171, 172, 204, 208, 213, 214, 215, 224, 225, 226, 229, 230, 231, 235, 238, 240, 242, 247, 248, 255, 258, 259, 261, 262, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 360, 394, 397, 398, 403, 404, 407, 409, 410, 412, 414, 415, 417, 437, 438, 439, 440, 503, 515, 543, 544, 545, 546, 549, 550, 553, 555, 571, 575, 592, 594, 606, 618, 626, 630, 632, 643, 645, 647, 649, 651, 652, 653, 655, 657, 661, 671, 672, 673, 674, 675, 680, 681, 686, 687, 688, 689, 690, 691, 971, 973, 1070, 1071, 1072, 1073, 1074, 1075, 1077, 1079, 1131, 1164, 1169, 1178, 1181, 1241, 1242, 1270, 1271, 1278, 1279, 1280, 1282, 1284, 1286, 1288, 1292, 1293, 1294, 1295, 1297, 1298, 1299, 1304, 1305, 1310, 1311, 1312, 1313, 1316, 1317, 1341, 1343, 1347, 1350, 1352, 1412, 1444, 1452, 1460, 1475, 1476, 1477, 1478, 1488, 1509, 1511, 1514, 2202, 2453, 2455, 2456, 2473, 2474, 2476, 2477, 2479, 2483, 2518, 2521, 2535, 2597, 2598, 2603, 2604, 2618, 2622, 2626, 2630, 2634, 2638, 2646, 2647, 2648, 2649, 2665, 2742, 2774, 2793, 2794, 2796, 2797, 2799, 2802, 2816, 2818, 2833, 2835, 2839, 2840, 2841, 2842, 2844, 2849, 2850, 2853, 2854, 2855, 2856, 2858, 2860, 2861, 2862, 2865, 2866, 2867, 2869, 2870, 2953, 2955, 2957, 2958, 4494, 4499, 4816, 4818, 4820, 4822, 4824, 4828, 4830, 4923, 5079, 5081, 5085, 5086, 5088, 5099, 5100, 5102, 5110, 5111, 5114, 5115, 5120, 5121, 5122, 5123, 5125, 5126, 5127, 5128, 5133, 5134, 5135, 5136, 5140, 5141, 5142, 5143, 5159, 5163, 5166, 5170, 5174, 5177, 5179, 5182, 5184, 5186, 5190, 5191, 5192, 5195, 5196, 5197, 5200, 5201, 5202, 5204, 5205, 5209, 5211, 5216, 5217, 5218, 5219, 5224, 5227, 5230, 5231, 5232, 5233, 5237, 5238, 5239, 5241, 5242, 5244, 5245, 5248, 5249, 5255, 5256, 5257, 5325, 5326, 5327, 5328, 5329, 5330, 5332, 5334, 5335, 5336, 7067, 7466, 7496, 7497, 7498, 7500, 7501, 7504, 7507, 7508, 7509, 7510, 7511, 7513, 7514, 7515, 7520, 7521, 7522, 7523, 7524, 7525, 7526, 7527, 7587, 7595, 7599, 7605, 7611, 7615, 7619, 7623, 7627, 7631, 7639, 7644, 7646, 7652, 7654, 7657, 7665, 7666, 7667, 7668, 7675, 7676, 7683, 7684, 7693, 7694, 7696, 7698, 7705, 7706, 7711, 7712, 7713, 7714, 7715, 7716, 7790, 7791, 9911, 9913, 9931, 9932, 9933, 9934, 9935, 9936, 9938]

The following groups have an orbit of size 6 and rank 4 : [2, 3, 6, 9, 11, 25, 27, 29, 30, 38, 45, 47, 48, 49, 50, 71, 126, 132, 135, 136, 143, 147, 148, 150, 151, 152, 174, 178, 250, 251, 254, 399, 400, 406, 413, 416, 418, 419, 421, 424, 444, 451, 455, 458, 466, 473, 474, 479, 480, 487, 491, 492, 511, 679, 684, 1076, 1078, 1089, 1092, 1095, 1097, 1098, 1101, 1102, 1105, 1107, 1110, 1111, 1113, 1115, 1116, 1119, 1120, 1130, 1134, 1189, 1190, 1195, 1196, 1202, 1203, 1205, 1206, 1211, 1212, 1320, 1480, 1481, 1484, 1516, 1518, 2454, 2475, 2481, 2494, 2495, 2499, 2501, 2502, 2505, 2506, 2514, 2517, 2522, 2524, 2525, 2526, 2528, 2530, 2531, 2533, 2537, 2539, 2651, 2785, 2786, 2788, 2874, 2876, 2878, 2880, 4782, 4785, 4787, 4790, 4799, 4800, 4806, 4810, 4813, 4826, 4948, 5095, 5096, 5097, 5098, 5223, 7158, 7164, 7168, 7171, 7174, 7176, 7181, 7214, 7215, 7257, 7258, 7261, 7262, 7506, 9614, 9619, 9620, 9621, 9622, 9683, 9684]

The following groups have an orbit of size 8 and rank 4 : [9, 46, 47, 48, 49, 50, 123, 124, 125, 126, 132, 133, 134, 135, 136, 150, 151, 152, 174, 175, 176, 177, 178, 394, 397, 398, 399, 400, 405, 407, 411, 412, 420, 421, 423, 424, 432, 445, 448, 451, 452, 455, 456, 457, 458, 459, 465, 466, 473, 474, 479, 480, 486, 487, 490, 491, 492, 493, 508, 509, 579, 839, 902, 1016, 1022, 1077, 1080, 1082, 1089, 1090, 1091, 1092, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1105, 1107, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1121, 1122, 1125, 1126, 1132, 1133, 1187, 1188, 1195, 1196, 1202, 1203, 1204, 1205, 1206, 1211, 1212, 1362, 1364, 1366, 1932, 1942, 1951, 1974, 1982, 2005, 2122, 2291, 2300, 2306, 2308, 2317, 2333, 2336, 2341, 2349, 2360, 2377, 2383, 2401, 2458, 2461, 2470, 2472, 2482, 2494, 2495, 2498, 2499, 2501, 2502, 2503, 2504, 2505, 2506, 2507, 2510, 2511, 2512, 2513, 2514, 2515, 2516, 2522, 2524, 2525, 2526, 2528, 2530, 2531, 2533, 2536, 2538, 2697, 2701, 3942, 3950, 3963, 3975, 3996, 4006, 4009, 4010, 4011, 4192, 4204, 4218, 4219, 4230, 4238, 4256, 4323, 4391, 4395, 4405, 4635, 4637, 4645, 4653, 4677, 4678, 4680, 4683, 4687, 4692, 4781, 4782, 4785, 4787, 4788, 4789, 4790, 4791, 4792, 4793, 4794, 4795, 4796, 4797, 4798, 4799, 4800, 4801, 4802, 4803, 4804, 4805, 4806, 4807, 4810, 4811, 4813, 4814, 4825, 5040, 5044, 5051, 6701, 6714, 6732, 6740, 6767, 6783, 6789, 6825, 6828, 6843, 6847, 6848, 6864, 6875, 6904, 6922, 6936, 6950, 6953, 7011, 7052, 7058, 7158, 7159, 7160, 7161, 7162, 7163, 7164, 7165, 7168, 7169, 7170, 7171, 7174, 7175, 7176, 7177, 7180, 7181, 7432, 7441, 7450, 9411, 9427, 9454, 9473, 9498, 9511, 9521, 9534, 9544, 9548, 9613, 9614, 9885, 9890, 9894, 9902]

The following groups have an orbit of size 8 and rank 5 : [1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 17, 18, 19, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 37, 38, 39, 40, 43, 44, 45, 48, 52, 54, 60, 62, 63, 64, 65, 66, 68, 69, 70, 71, 73, 75, 76, 91, 92, 95, 97, 98, 100, 101, 102, 103, 108, 111, 112, 115, 116, 119, 139, 143, 144, 145, 146, 204, 205, 206, 208, 209, 210, 213, 214, 216, 218, 219, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 232, 234, 235, 237, 239, 240, 241, 242, 244, 246, 247, 248, 249, 250, 251, 254, 255, 256, 257, 258, 260, 261, 262, 263, 265, 266, 267, 270, 271, 272, 274, 333, 335, 341, 345, 349, 352, 357, 360, 363, 364, 367, 542, 544, 545, 547, 548, 549, 551, 552, 553, 554, 555, 582, 584, 588, 589, 591, 592, 593, 594, 595, 599, 602, 605, 606, 607, 608, 609, 610, 611, 614, 615, 618, 619, 621, 623, 626, 629, 630, 632, 633, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 653, 654, 655, 656, 657, 659, 661, 664, 665, 667, 668, 671, 672, 673, 674, 675, 676, 679, 680, 681, 683, 687, 688, 689, 690, 691, 967, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1288, 1290, 1292, 1293, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1308, 1310, 1311, 1312, 1313, 1314, 1316, 1317, 1318, 1320, 1321, 1368, 1373, 1377, 1384, 1386, 1390, 1394, 1395, 1399, 1403, 1405, 1407, 1409, 1411, 1412, 1413, 1414, 1415, 1418, 1419, 1422, 1423, 1428, 1429, 1432, 1442, 1444, 1445, 1450, 1451, 1452, 1455, 1460, 1463, 1472, 1475, 1476, 1477, 1478, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 2610, 2612, 2615, 2616, 2618, 2621, 2622, 2623, 2624, 2625, 2626, 2629, 2630, 2631, 2634, 2637, 2638, 2641, 2642, 2646, 2647, 2648, 2649, 2651, 2706, 2711, 2716, 2721, 2727, 2731, 2737, 2741, 2742, 2745, 2754, 2757, 2833, 2834, 2835, 2837, 2839, 2840, 2842, 2843, 2844, 2845, 2846, 2848, 2849, 2850, 2852, 2853, 2854, 2857, 2858, 2859, 2860, 2861, 2863, 2864, 2865, 2867, 2868, 2869, 2871, 2872, 2874, 2875, 2876, 2878, 2880, 4900, 4902, 4907, 4912, 4917, 4921, 4922, 4923, 4924, 4929, 4930, 4947, 4948, 5054, 5154, 5155, 5157, 5158, 5159, 5162, 5163, 5164, 5165, 5166, 5169, 5170, 5171, 5174, 5175, 5177, 5178, 5179, 5181, 5182, 5183, 5184, 5186, 5187, 5188, 5190, 5191, 5192, 5193, 5194, 5195, 5196, 5197, 5198, 5199, 5202, 5203, 5204, 5205, 5206, 5207, 5208, 5209, 5210, 5211, 5213, 5215, 5216, 5217, 5218, 5219, 5221, 5223, 5224, 5225, 5226, 5227, 5228, 5229, 5230, 5231, 5232, 5233, 5234, 5235, 5236, 5238, 5239, 5240, 5241, 5243, 5244, 5245, 5247, 5248, 5249, 5251, 5252, 5255, 5256, 5257, 7208, 7212, 7257, 7258, 7259, 7261, 7262, 7263, 7555, 7561, 7563, 7567, 7569, 7574, 7576, 7577, 7580, 7582, 7583, 7585, 7587, 7593, 7595, 7596, 7597, 7598, 7599, 7602, 7603, 7604, 7605, 7608, 7611, 7614, 7615, 7616, 7617, 7618, 7619, 7620, 7621, 7622, 7623, 7624, 7627, 7630, 7631, 7632, 7637, 7638, 7640, 7641, 7642, 7643, 7644, 7645, 7646, 7647, 7648, 7649, 7650, 7651, 7652, 7653, 7654, 7655, 7656, 7657, 7658, 7659, 7660, 7661, 7665, 7666, 7667, 7668, 7670, 7673, 7674, 7675, 7676, 7680, 7683, 7684, 7685, 7686, 7695, 7696, 7697, 7698, 7701, 7703, 7704, 7705, 7706, 7707, 7711, 7712, 7713, 7714, 7715, 7716, 7717, 7718, 7722, 9681, 9683, 9684]

The following groups have an orbit of size 12 and rank 6 : [2, 3, 9, 11, 25, 27, 29, 30, 38, 47, 48, 49, 50, 126, 132, 135, 136, 143, 147, 150, 151, 152, 174, 178, 399, 400, 413, 416, 418, 419, 421, 424, 444, 451, 455, 458, 466, 473, 474, 479, 480, 487, 491, 492, 1076, 1078, 1089, 1092, 1095, 1097, 1098, 1101, 1102, 1105, 1107, 1110, 1111, 1113, 1115, 1116, 1119, 1120, 1189, 1190, 1195, 1196, 1202, 1203, 1205, 1206, 1211, 1212, 2454, 2481, 2494, 2495, 2499, 2501, 2502, 2505, 2506, 2514, 2517, 2522, 2524, 2525, 2526, 2528, 2530, 2531, 2533, 2537, 2539, 4782, 4785, 4787, 4790, 4799, 4800, 4806, 4810, 4813, 4826, 7158, 7164, 7168, 7171, 7174, 7176, 7181, 9614]

The following groups have an orbit of size 12 and rank 7 : [2, 3, 6, 9, 11, 12, 13, 14, 15, 25, 27, 29, 30, 33, 38, 39, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 57, 66, 68, 73, 74, 75, 76, 77, 112, 116, 124, 126, 132, 133, 135, 136, 143, 144, 145, 146, 147, 148, 150, 151, 152, 158, 159, 163, 164, 165, 166, 168, 169, 171, 172, 174, 175, 177, 178, 203, 225, 230, 232, 235, 241, 242, 247, 248, 261, 262, 263, 264, 265, 266, 267, 268, 269, 273, 360, 394, 397, 398, 399, 400, 403, 404, 406, 407, 409, 410, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 423, 424, 437, 438, 439, 440, 444, 451, 452, 455, 457, 458, 465, 466, 473, 474, 479, 480, 487, 491, 492, 493, 503, 511, 515, 572, 573, 577, 578, 645, 651, 654, 657, 661, 671, 673, 674, 675, 686, 971, 973, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1089, 1090, 1092, 1095, 1097, 1098, 1099, 1100, 1101, 1102, 1105, 1107, 1110, 1111, 1113, 1114, 1115, 1116, 1119, 1120, 1130, 1131, 1134, 1164, 1169, 1178, 1181, 1189, 1190, 1195, 1196, 1202, 1203, 1205, 1206, 1211, 1212, 1241, 1242, 1270, 1271, 1278, 1279, 1345, 1346, 1476, 1478, 1486, 1488, 2202, 2453, 2454, 2455, 2456, 2473, 2474, 2475, 2476, 2477, 2479, 2481, 2483, 2494, 2495, 2499, 2501, 2502, 2505, 2506, 2511, 2514, 2517, 2518, 2521, 2522, 2524, 2525, 2526, 2528, 2530, 2531, 2533, 2535, 2537, 2539, 2597, 2598, 2603, 2604, 2772, 2774, 2776, 2790, 2791, 2796, 2797, 2799, 2800, 2801, 2802, 2952, 2959, 4494, 4499, 4782, 4785, 4787, 4790, 4796, 4799, 4800, 4806, 4810, 4813, 4816, 4818, 4820, 4822, 4824, 4826, 4828, 4830, 5080, 5081, 5084, 5086, 5087, 5088, 5089, 5090, 5099, 5101, 5108, 5109, 5113, 5116, 5120, 5123, 5127, 5128, 5331, 7067, 7158, 7164, 7168, 7171, 7174, 7176, 7181, 7248, 7250, 7497, 7499, 7500, 7502, 7503, 7505, 7508, 7512, 7514, 7516, 9614, 9672, 9673, 9674, 9676, 9933, 9937]

The following groups have an orbit of size 16 and rank 5 : [16, 20, 22, 31, 35, 36, 41, 42, 90, 93, 94, 96, 99, 104, 105, 106, 107, 109, 110, 113, 114, 117, 118, 140, 141, 142, 207, 211, 217, 220, 233, 236, 243, 245, 252, 253, 285, 286, 287, 288, 289, 332, 334, 336, 337, 338, 339, 340, 342, 343, 344, 346, 347, 348, 350, 351, 353, 354, 355, 356, 361, 362, 365, 366, 581, 583, 585, 586, 587, 590, 596, 597, 598, 600, 601, 603, 604, 612, 613, 616, 617, 620, 622, 624, 625, 627, 628, 631, 658, 660, 662, 663, 666, 669, 670, 677, 678, 682, 708, 709, 710, 711, 712, 713, 714, 962, 963, 964, 965, 966, 968, 969, 1287, 1289, 1291, 1307, 1309, 1315, 1319, 1357, 1358, 1359, 1360, 1361, 1369, 1370, 1371, 1372, 1374, 1375, 1376, 1378, 1379, 1380, 1381, 1382, 1383, 1385, 1387, 1388, 1389, 1391, 1392, 1393, 1396, 1397, 1398, 1400, 1401, 1402, 1404, 1406, 1408, 1410, 1416, 1417, 1420, 1421, 1424, 1425, 1426, 1427, 1430, 1431, 1433, 1434, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1443, 1446, 1447, 1448, 1449, 1453, 1454, 1456, 1457, 1458, 1459, 1461, 1462, 1471, 1473, 1474, 1553, 2609, 2611, 2613, 2614, 2617, 2619, 2620, 2627, 2628, 2632, 2633, 2635, 2636, 2639, 2640, 2643, 2644, 2645, 2650, 2672, 2673, 2674, 2675, 2676, 2677, 2678, 2679, 2680, 2681, 2682, 2683, 2684, 2685, 2704, 2705, 2707, 2708, 2709, 2710, 2712, 2713, 2714, 2715, 2717, 2718, 2719, 2720, 2722, 2723, 2724, 2725, 2726, 2728, 2729, 2730, 2732, 2733, 2734, 2735, 2736, 2738, 2739, 2740, 2743, 2744, 2746, 2747, 2748, 2749, 2750, 2751, 2752, 2753, 2755, 2756, 2836, 2838, 2847, 2851, 2873, 2877, 2879, 4894, 4895, 4896, 4897, 4898, 4899, 4901, 4903, 4904, 4905, 4906, 4908, 4909, 4910, 4911, 4913, 4914, 4915, 4916, 4918, 4919, 4920, 4925, 4926, 4927, 4928, 4991, 4992, 4993, 4994, 4995, 4996, 4997, 4998, 5053, 5055, 5056, 5057, 5058, 5059, 5060, 5153, 5156, 5160, 5161, 5167, 5168, 5172, 5173, 5176, 5180, 5185, 5189, 5212, 5214, 5220, 5222, 5246, 5250, 5253, 5254, 7197, 7198, 7199, 7200, 7201, 7202, 7203, 7204, 7205, 7206, 7207, 7209, 7210, 7211, 7213, 7256, 7260, 7373, 7551, 7552, 7553, 7554, 7556, 7557, 7558, 7559, 7560, 7562, 7564, 7565, 7566, 7568, 7570, 7571, 7572, 7573, 7575, 7578, 7579, 7581, 7584, 7586, 7588, 7589, 7590, 7591, 7592, 7594, 7600, 7601, 7606, 7607, 7609, 7610, 7612, 7613, 7625, 7626, 7628, 7629, 7633, 7634, 7635, 7636, 7662, 7663, 7664, 7669, 7671, 7672, 7677, 7678, 7679, 7681, 7682, 7687, 7688, 7689, 7690, 7691, 7692, 7699, 7700, 7702, 7708, 7709, 7710, 7719, 7720, 7721, 9615, 9616, 9682, 9685]

The following groups have an orbit of size 16 and rank 7 : [46, 48, 49, 51, 55, 56, 57, 123, 124, 126, 128, 130, 132, 133, 134, 147, 148, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 385, 386, 393, 394, 397, 398, 399, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 433, 434, 435, 436, 437, 438, 439, 440, 442, 446, 449, 450, 451, 452, 475, 477, 478, 482, 483, 484, 485, 486, 487, 488, 489, 493, 494, 495, 504, 505, 507, 512, 513, 839, 902, 971, 972, 973, 974, 1003, 1004, 1005, 1006, 1007, 1008, 1010, 1012, 1013, 1014, 1015, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1081, 1083, 1089, 1090, 1106, 1109, 1112, 1113, 1114, 1115, 1116, 1117, 1123, 1128, 1129, 1132, 1139, 1142, 1144, 1150, 1156, 1158, 1159, 1162, 1166, 1168, 1173, 1174, 1176, 1177, 1179, 1185, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1213, 1214, 1220, 1221, 1222, 1223, 1237, 1239, 1240, 1243, 1244, 1248, 1249, 1250, 1251, 1252, 1253, 1262, 1263, 1264, 1265, 1266, 1267, 1272, 1273, 1276, 1277, 1932, 1942, 1951, 1973, 1974, 1981, 1982, 1990, 2202, 2204, 2205, 2287, 2288, 2289, 2290, 2298, 2299, 2300, 2301, 2302, 2303, 2328, 2329, 2344, 2345, 2350, 2354, 2359, 2360, 2367, 2386, 2397, 2403, 2457, 2459, 2460, 2462, 2463, 2464, 2465, 2466, 2468, 2469, 2470, 2478, 2480, 2484, 2488, 2493, 2494, 2496, 2498, 2500, 2504, 2505, 2519, 2520, 2522, 2523, 2525, 2527, 2529, 2531, 2532, 2533, 2534, 2552, 2553, 2554, 2555, 2556, 2557, 2558, 2599, 2600, 2601, 2602, 3552, 3792, 3941, 3942, 3943, 3944, 3954, 3961, 3962, 3963, 3964, 3983, 4009, 4191, 4215, 4223, 4256, 4323, 4401, 4497, 4498, 4501, 4502, 4541, 4543, 4544, 4624, 4627, 4629, 4632, 4642, 4643, 4644, 4645, 4650, 4651, 4652, 4653, 4658, 4659, 4660, 4661, 4663, 4664, 4665, 4666, 4669, 4670, 4673, 4674, 4696, 4698, 4707, 4708, 4711, 4714, 4723, 4724, 4729, 4730, 4731, 4732, 4735, 4738, 4739, 4744, 4753, 4754, 4755, 4756, 4761, 4764, 4769, 4770, 4772, 4773, 4775, 4778, 4779, 4780, 4784, 4786, 4787, 4808, 4809, 4815, 4817, 4819, 4821, 4823, 4827, 4829, 6333, 6562, 6647, 6656, 6702, 6705, 6706, 6713, 6717, 6718, 6729, 6736, 6738, 6739, 6740, 6741, 6743, 6744, 6745, 6765, 6770, 6772, 6789, 6825, 6847, 6865, 6874, 6882, 6883, 6904, 6905, 6906, 6921, 6925, 6947, 6951, 6952, 6953, 6956, 6957, 6968, 6969, 6994, 7009, 7010, 7053, 7068, 7096, 7097, 7098, 7101, 7102, 7103, 7110, 7111, 7112, 7113, 7118, 7121, 7124, 7127, 7150, 7151, 7154, 7155, 7156, 7157, 7166, 7167, 7172, 7173, 7178, 7179, 9164, 9169, 9179, 9228, 9282, 9284, 9315, 9316, 9336, 9415, 9426, 9428, 9429, 9434, 9435, 9436, 9437, 9454, 9459, 9460, 9465, 9476, 9504, 9536, 9545, 9546, 9547, 9553, 9554, 9555, 9556, 9611, 9612]

The following groups have an orbit of size 16 and rank 8 : [48, 49, 126]

The following groups have an orbit of size 16 and rank 9 : [21, 22, 47, 50, 56, 57, 151, 158, 163, 166, 168, 172, 437, 444, 503, 511, 515, 1169, 1181, 1189, 1242, 1271, 1278, 1486, 1488, 2603, 2790, 2793, 2796, 2799, 2801, 2816, 5110, 5113, 5120, 5125, 5133, 5140, 7524]

The following groups have an orbit of size 18 and rank 6 : [66, 68, 73, 76, 225, 230, 232, 241, 242, 248, 261, 262, 263, 645, 651, 654, 657, 671, 674, 1476, 1486, 2772, 2776, 2790, 2791, 2800, 2801, 5080, 5084, 5087, 5089, 5090, 5101, 5108, 5109, 5113, 5116, 7499, 7502, 7503, 7505, 7512, 7516, 9937]

The following groups have an orbit of size 18 and rank 10 : [71, 250, 251, 254, 255, 257, 679, 681, 1320, 2651]

The following groups have an orbit of size 20 and rank 6 : [203, 572, 573, 577, 1345, 1346, 2952, 2959, 5331, 7250, 9672, 9674, 9676]

The following groups have an orbit of size 20 and rank 7 : [203, 570, 572, 7248, 9673]

The following groups have an orbit of size 20 and rank 11 : [2954, 2957, 5201, 5203, 5204, 5210, 5211, 5213, 5216, 5218, 5219, 5223, 5232, 5233, 5235, 5237, 5242, 5243, 5245, 5247, 5248, 5249, 5255, 5257, 5323, 5325, 5328, 5329, 5330, 5331, 5332, 5333, 5335, 5336]

The following groups have an orbit of size 24 and rank 7 : [9, 12, 13, 14, 15, 25, 27, 29, 33, 38, 39, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 74, 77, 112, 116, 123, 124, 125, 126, 132, 133, 134, 135, 136, 143, 144, 145, 146, 150, 151, 152, 158, 159, 163, 164, 165, 166, 168, 169, 171, 172, 174, 176, 177, 178, 230, 235, 247, 248, 261, 262, 264, 265, 266, 268, 269, 270, 271, 272, 273, 274, 360, 394, 397, 398, 399, 400, 404, 405, 407, 409, 411, 412, 414, 415, 417, 421, 424, 432, 437, 438, 439, 440, 445, 448, 451, 455, 456, 457, 458, 459, 466, 473, 474, 475, 477, 478, 479, 480, 482, 486, 487, 488, 489, 490, 491, 492, 493, 495, 508, 509, 578, 626, 651, 657, 661, 671, 672, 673, 674, 675, 686, 687, 688, 689, 690, 691, 839, 902, 971, 973, 1016, 1022, 1070, 1071, 1072, 1073, 1074, 1075, 1077, 1080, 1082, 1089, 1091, 1092, 1094, 1095, 1096, 1097, 1098, 1101, 1102, 1103, 1105, 1106, 1107, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1121, 1122, 1125, 1126, 1132, 1133, 1164, 1169, 1178, 1181, 1187, 1188, 1195, 1196, 1197, 1198, 1202, 1203, 1204, 1205, 1206, 1211, 1212, 1213, 1214, 1241, 1242, 1270, 1271, 1278, 1279, 1346, 1444, 1452, 1475, 1476, 1477, 1478, 1932, 1942, 1951, 1974, 1982, 2005, 2122, 2202, 2291, 2300, 2306, 2308, 2317, 2333, 2336, 2341, 2349, 2360, 2377, 2383, 2401, 2453, 2455, 2458, 2461, 2470, 2472, 2474, 2476, 2477, 2479, 2482, 2483, 2493, 2494, 2495, 2498, 2499, 2500, 2501, 2502, 2503, 2504, 2505, 2506, 2507, 2510, 2512, 2513, 2514, 2515, 2516, 2518, 2521, 2522, 2523, 2524, 2525, 2526, 2527, 2528, 2530, 2531, 2533, 2535, 2536, 2538, 2597, 2598, 2603, 2604, 2742, 2793, 2794, 2816, 2818, 3942, 3950, 3963, 3975, 3996, 4006, 4009, 4010, 4011, 4192, 4204, 4218, 4219, 4230, 4238, 4256, 4323, 4391, 4395, 4405, 4494, 4499, 4635, 4637, 4645, 4653, 4677, 4678, 4680, 4683, 4687, 4692, 4780, 4781, 4782, 4785, 4787, 4788, 4789, 4790, 4791, 4792, 4793, 4794, 4795, 4797, 4798, 4799, 4800, 4801, 4802, 4803, 4804, 4805, 4806, 4807, 4808, 4810, 4811, 4813, 4814, 4816, 4818, 4820, 4822, 4824, 4825, 4828, 4830, 5079, 5085, 5100, 5102, 5110, 5111, 5114, 5115, 5121, 5122, 5125, 5126, 5133, 5134, 5135, 5136, 5140, 5141, 5142, 5143, 6701, 6714, 6732, 6740, 6767, 6783, 6789, 6825, 6828, 6843, 6847, 6848, 6864, 6875, 6904, 6922, 6936, 6950, 6953, 7011, 7052, 7058, 7067, 7158, 7159, 7160, 7161, 7162, 7163, 7164, 7165, 7168, 7169, 7170, 7171, 7173, 7174, 7175, 7176, 7177, 7179, 7180, 7181, 7466, 7496, 7498, 7501, 7504, 7507, 7509, 7510, 7511, 7513, 7515, 7520, 7521, 7522, 7523, 7524, 7525, 7526, 7527, 9411, 9427, 9454, 9473, 9498, 9511, 9521, 9534, 9544, 9548, 9613, 9614, 9675, 9911, 9913, 9931, 9932, 9934, 9935, 9936, 9938]

The following groups have an orbit of size 24 and rank 8 : [2, 6, 25, 27, 29, 38, 45, 124, 133, 143, 394, 397, 398, 407, 412, 1077]

The following groups have an orbit of size 24 and rank 9 : [2, 3, 4, 6, 7, 8, 11, 12, 25, 27, 29, 30, 33, 38, 39, 44, 45, 47, 48, 52, 54, 66, 68, 70, 73, 75, 76, 112, 116, 126, 132, 143, 144, 145, 146, 147, 148, 150, 174, 225, 230, 232, 235, 241, 242, 247, 248, 261, 262, 263, 265, 266, 267, 270, 271, 272, 274, 360, 399, 406, 413, 416, 418, 419, 421, 424, 444, 451, 455, 466, 473, 474, 479, 480, 487, 511, 626, 645, 651, 654, 657, 661, 671, 672, 673, 674, 675, 687, 688, 689, 690, 691, 1076, 1078, 1089, 1092, 1097, 1098, 1101, 1102, 1105, 1107, 1110, 1111, 1113, 1115, 1116, 1119, 1120, 1130, 1134, 1189, 1190, 1195, 1196, 1202, 1203, 1211, 1212, 1444, 1452, 1475, 1476, 1477, 1478, 2454, 2475, 2481, 2494, 2495, 2501, 2502, 2505, 2506, 2514, 2517, 2522, 2524, 2525, 2526, 2528, 2530, 2531, 2533, 2537, 2539, 2742, 4782, 4785, 4787, 4790, 4799, 4800, 4806, 4810, 4813, 4826, 7158, 7164, 7168, 7171, 7174, 7176, 7181, 9614]

The following groups have an orbit of size 24 and rank 10 : [2, 3, 6, 9, 11, 25, 27, 29, 30, 38, 45, 46, 47, 48, 49, 50, 71, 124, 125, 126, 132, 133, 135, 136, 143, 147, 148, 150, 174, 175, 176, 177, 178, 251, 254, 394, 397, 398, 399, 406, 407, 412, 413, 416, 418, 419, 420, 421, 423, 424, 444, 445, 451, 452, 455, 457, 458, 459, 465, 466, 473, 474, 475, 477, 478, 479, 480, 486, 487, 488, 489, 493, 508, 511, 579, 684, 839, 1022, 1076, 1077, 1078, 1080, 1089, 1090, 1092, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1105, 1106, 1107, 1110, 1111, 1113, 1114, 1115, 1116, 1117, 1119, 1120, 1125, 1130, 1134, 1187, 1188, 1189, 1190, 1195, 1196, 1197, 1198, 1202, 1203, 1211, 1212, 1213, 1214, 1362, 1364, 1366, 1480, 1481, 1484, 1932, 2308, 2360, 2454, 2475, 2481, 2482, 2493, 2494, 2495, 2498, 2500, 2501, 2502, 2505, 2506, 2511, 2514, 2515, 2517, 2522, 2524, 2525, 2526, 2528, 2530, 2531, 2533, 2537, 2539, 2697, 2701, 2785, 2786, 4010, 4256, 4323, 4780, 4781, 4782, 4785, 4787, 4789, 4790, 4796, 4799, 4800, 4806, 4810, 4811, 4813, 4826, 5040, 5044, 5051, 5095, 5096, 5097, 6789, 7158, 7164, 7168, 7171, 7174, 7176, 7177, 7179, 7181, 7432, 7441, 7506, 9614, 9885, 9890, 9894]

The following groups have an orbit of size 24 and rank 11 : [2, 3, 6, 9, 11, 12, 13, 14, 15, 25, 27, 29, 30, 33, 38, 39, 44, 45, 48, 52, 53, 54, 56, 57, 66, 68, 71, 77, 124, 133, 143, 146, 158, 159, 163, 164, 165, 166, 168, 169, 171, 172, 225, 230, 232, 235, 241, 242, 247, 248, 251, 262, 263, 264, 266, 267, 268, 269, 271, 273, 274, 394, 397, 398, 407, 412, 437, 438, 439, 440, 645, 651, 654, 657, 661, 671, 673, 674, 675, 686, 688, 1077, 1476, 1478]

The following groups have an orbit of size 24 and rank 12 : [2, 3, 6, 9, 11, 25, 27, 29, 30, 38, 45, 48, 71, 143, 250, 251, 254, 679]

The following groups have an orbit of size 30 and rank 7 : [203, 572, 573, 577, 1345, 1346, 2952, 2959, 5331, 7250, 9672, 9674, 9676]

The following groups have an orbit of size 30 and rank 12 : [203, 577]

The following groups have an orbit of size 32 and rank 7 : [173, 174, 175, 179, 180, 420, 421, 422, 423, 424, 441, 442, 443, 444, 445, 446, 447, 448, 453, 454, 455, 456, 460, 461, 462, 471, 472, 473, 474, 475, 476, 480, 485, 508, 509, 510, 511, 1016, 1017, 1028, 1029, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1091, 1092, 1093, 1096, 1097, 1105, 1106, 1109, 1110, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1199, 1200, 1201, 1202, 1203, 1231, 1232, 1233, 1234, 1235, 1485, 1486, 2004, 2005, 2122, 2204, 2291, 2292, 2293, 2301, 2306, 2307, 2313, 2314, 2320, 2325, 2326, 2329, 2330, 2333, 2334, 2336, 2337, 2348, 2349, 2399, 2400, 2401, 2447, 2448, 2451, 2453, 2457, 2462, 2469, 2471, 2476, 2477, 2478, 2479, 2480, 2481, 2482, 2483, 2484, 2495, 2496, 2508, 2509, 2510, 2511, 2512, 2513, 2514, 2515, 2540, 2541, 2542, 2543, 2544, 2559, 2560, 2561, 2562, 2563, 2564, 2565, 2566, 2567, 2588, 2590, 2591, 2772, 2773, 2774, 2775, 2789, 2790, 2791, 2798, 2801, 2802, 3946, 3950, 3952, 3954, 3962, 3964, 3975, 3978, 3981, 3983, 4006, 4188, 4191, 4193, 4202, 4215, 4218, 4223, 4230, 4395, 4396, 4401, 4402, 4634, 4635, 4636, 4637, 4644, 4652, 4676, 4677, 4678, 4679, 4688, 4691, 4721, 4722, 4779, 4780, 4781, 4782, 4788, 4789, 4790, 4791, 4792, 4793, 4808, 4809, 4810, 4811, 4823, 4824, 4877, 4878, 4879, 4880, 4881, 4882, 4883, 4946, 5079, 5080, 5081, 5088, 5090, 5102, 5104, 5106, 5107, 5112, 5113, 5114, 5115, 5116, 6027, 6031, 6697, 6705, 6710, 6718, 6729, 6732, 6736, 6738, 6739, 6765, 6767, 6770, 6772, 6878, 6882, 6905, 6924, 6934, 6935, 6936, 6940, 6941, 6952, 7104, 7105, 7106, 7107, 7108, 7109, 7130, 7131, 7132, 7133, 7144, 7145, 7166, 7167, 7176, 7177, 7248, 7249, 7250, 7255, 7498, 7500, 7501, 7502, 7511, 7515, 8491, 8494, 8757, 8760, 8971, 8985, 9410, 9411, 9414, 9415, 9434, 9472, 9473, 9474, 9475, 9476, 9477, 9484, 9485, 9486, 9487, 9488, 9489, 9490, 9491, 9496, 9497, 9502, 9503, 9504, 9516, 9520, 9522, 9528, 9534, 9536, 9538, 9543, 9556, 9587, 9588, 9589, 9590, 9591, 9592, 9593, 9594, 9675, 9676, 9855, 9938]

The following groups have an orbit of size 32 and rank 9 : [124, 127, 133, 135, 137, 138, 147, 148, 149, 150, 152, 159, 164, 165, 169, 171, 394, 397, 398, 400, 403, 404, 406, 407, 409, 410, 412, 413, 414, 415, 416, 417, 418, 419, 425, 438, 439, 440, 685, 971, 973, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1120, 1130, 1131, 1134, 1164, 1178, 1190, 1241, 1270, 1279, 1483, 2202, 2453, 2454, 2455, 2456, 2473, 2474, 2475, 2476, 2477, 2479, 2481, 2483, 2517, 2518, 2521, 2535, 2537, 2539, 2597, 2598, 2604, 2772, 2774, 2776, 2791, 2794, 2797, 2800, 2802, 2818, 4494, 4499, 4816, 4818, 4820, 4822, 4824, 4826, 4828, 4830, 5079, 5080, 5081, 5084, 5085, 5086, 5087, 5088, 5089, 5090, 5099, 5100, 5101, 5102, 5108, 5109, 5111, 5114, 5115, 5116, 5121, 5122, 5123, 5126, 5127, 5128, 5134, 5135, 5136, 5141, 5142, 5143, 6145, 6152, 6155, 6161, 6211, 6245, 6255, 6291, 6342, 6405, 6413, 6457, 6511, 6518, 6520, 6549, 6564, 6584, 6586, 6590, 6623, 6635, 6638, 6646, 6654, 6678, 6694, 6814, 6818, 6821, 6836, 6856, 6858, 6914, 6919, 6955, 6958, 7002, 7007, 7036, 7038, 7041, 7043, 7067, 7068, 7466, 7496, 7497, 7498, 7499, 7500, 7501, 7502, 7503, 7504, 7505, 7507, 7508, 7509, 7510, 7511, 7512, 7513, 7514, 7515, 7516, 7520, 7521, 7522, 7523, 7525, 7526, 7527, 9911, 9913, 9931, 9932, 9933, 9934, 9935, 9936, 9937, 9938]

The following groups have an orbit of size 32 and rank 10 : [175, 177, 452, 465, 475, 478, 489, 493, 1197, 1213]

The following groups have an orbit of size 32 and rank 11 : [132, 136, 174, 178, 399, 451, 466, 473, 479, 487, 491, 684, 1195, 1211, 1480, 1481, 1484, 2786]

The following groups have an orbit of size 36 and rank 7 : [66, 74, 75, 76, 77, 225, 230, 235, 242, 247, 248, 261, 262, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 626, 645, 651, 657, 661, 671, 672, 673, 674, 675, 686, 687, 688, 689, 690, 691, 1444, 1452, 1475, 1476, 1477, 1478, 1488, 2742, 2774, 2793, 2794, 2796, 2797, 2799, 2802, 2816, 2818, 5079, 5081, 5085, 5086, 5088, 5099, 5100, 5102, 5110, 5111, 5114, 5115, 5120, 5121, 5122, 5123, 5125, 5126, 5127, 5128, 5133, 5134, 5135, 5136, 5140, 5141, 5142, 5143, 7466, 7496, 7497, 7498, 7500, 7501, 7504, 7507, 7508, 7509, 7510, 7511, 7513, 7514, 7515, 7520, 7521, 7522, 7523, 7524, 7525, 7526, 7527, 9911, 9913, 9931, 9932, 9933, 9934, 9935, 9936, 9938]

The following groups have an orbit of size 36 and rank 10 : [68, 71, 73, 76, 232, 242, 250, 251, 254, 261, 262, 657, 679]

The following groups have an orbit of size 36 and rank 11 : [60, 61, 65, 67, 68, 73, 74, 75, 76, 77, 204, 208, 213, 214, 215, 224, 226, 229, 231, 232, 238, 240, 241, 255, 257, 258, 259, 262, 263, 264, 265, 266, 268, 269, 271, 273, 274, 543, 544, 545, 546, 547, 548, 549, 550, 553, 555, 592, 594, 606, 618, 630, 632, 643, 647, 649, 652, 653, 654, 655, 680, 681, 686, 688, 1292, 1293, 1294, 1296, 1298, 1304, 1305, 1310, 1312, 1313, 1316, 1317, 1412, 1460, 1509, 1510, 1511, 1514, 2646, 2648, 2839, 2840, 2841, 2843, 2844, 2849, 2850, 2854, 2855, 2856, 2857, 2858, 2860, 2861, 2862, 2864, 2865, 2866, 2867, 2869, 2870, 2872, 5190, 5191, 5200, 5201, 5203, 5204, 5211, 5216, 5218, 5219, 5232, 5233, 5237, 5242, 5243, 5245, 5248, 5249, 5255, 5257, 7639, 7644, 7659, 7665, 7667, 7676, 7684, 7693, 7694, 7695, 7696, 7698, 7705, 7706, 7711, 7713, 7715, 7716]

The following groups have an orbit of size 36 and rank 12 : [71, 250, 251, 254, 679, 684, 1480, 1481, 1484, 2785, 2786, 2788, 5095, 5096, 5097, 5098, 7214, 7215, 7506, 9619, 9620, 9621, 9622]

The following groups have an orbit of size 40 and rank 7 : [578, 1346, 9675]

The following groups have an orbit of size 40 and rank 11 : [571, 575, 1341, 1343, 1347, 1350, 1352, 2665, 2953, 2955, 2956, 2958, 5325, 5326, 5327, 5328, 5329, 5330, 5332, 5333, 5334, 5335, 7790, 7791]

The following groups have an orbit of size 48 and rank 7 : [51, 55, 56, 57, 123, 124, 133, 134, 147, 148, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 385, 386, 394, 397, 398, 399, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 433, 434, 435, 436, 437, 438, 439, 440, 442, 449, 450, 451, 452, 483, 484, 485, 486, 487, 488, 489, 494, 495, 504, 507, 512, 513, 839, 902, 971, 972, 973, 974, 1003, 1004, 1005, 1006, 1007, 1008, 1010, 1012, 1013, 1014, 1015, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1081, 1083, 1089, 1090, 1112, 1113, 1114, 1115, 1116, 1117, 1123, 1128, 1129, 1132, 1158, 1159, 1162, 1166, 1168, 1173, 1174, 1176, 1177, 1179, 1185, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1220, 1221, 1222, 1223, 1237, 1239, 1240, 1243, 1244, 1248, 1249, 1250, 1251, 1252, 1253, 1262, 1263, 1264, 1265, 1266, 1267, 1272, 1273, 1276, 1277, 1932, 1942, 1951, 1973, 1974, 1981, 1982, 1990, 2202, 2204, 2205, 2287, 2288, 2289, 2290, 2298, 2299, 2300, 2301, 2302, 2303, 2328, 2329, 2344, 2345, 2350, 2354, 2359, 2360, 2367, 2386, 2397, 2403, 2457, 2459, 2460, 2462, 2463, 2464, 2465, 2466, 2468, 2469, 2470, 2478, 2480, 2484, 2493, 2494, 2496, 2498, 2504, 2505, 2519, 2520, 2522, 2523, 2525, 2529, 2531, 2532, 2533, 2534, 2552, 2553, 2554, 2555, 2556, 2557, 2558, 2599, 2600, 2601, 2602, 3552, 3792, 3941, 3942, 3943, 3944, 3954, 3961, 3962, 3963, 3964, 3983, 4009, 4191, 4215, 4223, 4256, 4323, 4401, 4497, 4498, 4501, 4502, 4541, 4543, 4544, 4624, 4627, 4629, 4632, 4642, 4643, 4644, 4645, 4650, 4651, 4652, 4653, 4658, 4659, 4660, 4661, 4663, 4664, 4665, 4666, 4669, 4670, 4673, 4674, 4696, 4698, 4707, 4708, 4711, 4714, 4723, 4724, 4729, 4730, 4731, 4732, 4735, 4738, 4739, 4744, 4753, 4754, 4755, 4756, 4761, 4764, 4769, 4770, 4772, 4773, 4775, 4778, 4779, 4784, 4786, 4787, 4809, 4815, 4817, 4819, 4821, 4823, 4827, 4829, 6333, 6562, 6647, 6656, 6702, 6705, 6706, 6713, 6717, 6718, 6729, 6736, 6738, 6739, 6740, 6741, 6743, 6744, 6745, 6765, 6770, 6772, 6789, 6825, 6847, 6865, 6874, 6882, 6883, 6904, 6905, 6906, 6921, 6925, 6947, 6951, 6952, 6953, 6956, 6957, 6968, 6969, 6994, 7009, 7010, 7053, 7068, 7096, 7097, 7098, 7101, 7102, 7103, 7110, 7111, 7112, 7113, 7118, 7121, 7124, 7127, 7150, 7151, 7154, 7155, 7156, 7157, 7166, 7167, 7172, 7178, 9164, 9169, 9179, 9228, 9282, 9284, 9315, 9316, 9336, 9415, 9426, 9428, 9429, 9434, 9435, 9436, 9437, 9454, 9459, 9460, 9465, 9476, 9504, 9536, 9545, 9546, 9547, 9553, 9554, 9555, 9556, 9611, 9612]

The following groups have an orbit of size 48 and rank 9 : [22, 36, 252]

The following groups have an orbit of size 48 and rank 10 : [25, 27, 29, 30, 38, 45, 46, 47, 48, 49, 123, 124, 125, 126, 132, 133, 134, 143, 147, 148, 150, 151, 152, 175, 250, 251, 254, 394, 397, 398, 399, 400, 405, 406, 407, 411, 412, 413, 416, 418, 419, 420, 423, 424, 432, 448, 451, 452, 456, 465, 466, 479, 486, 487, 490, 491, 492, 493, 509, 679, 839, 902, 1016, 1076, 1077, 1082, 1089, 1090, 1091, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1107, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1121, 1122, 1126, 1132, 1133, 1195, 1196, 1204, 1205, 1206, 1211, 1212, 1362, 1366, 1481, 1484, 1932, 1942, 1951, 1974, 1982, 2005, 2122, 2291, 2300, 2306, 2317, 2333, 2336, 2341, 2349, 2360, 2377, 2383, 2401, 2458, 2461, 2470, 2472, 2494, 2498, 2499, 2501, 2502, 2503, 2504, 2505, 2506, 2507, 2510, 2511, 2512, 2513, 2514, 2515, 2516, 2522, 2524, 2525, 2526, 2528, 2530, 2531, 2533, 2536, 2538, 2697, 2786, 3942, 3950, 3963, 3975, 3996, 4006, 4009, 4011, 4192, 4204, 4218, 4219, 4230, 4238, 4256, 4323, 4391, 4395, 4405, 4635, 4637, 4645, 4653, 4677, 4678, 4680, 4683, 4687, 4692, 4781, 4782, 4785, 4787, 4788, 4789, 4790, 4791, 4792, 4793, 4794, 4795, 4796, 4797, 4798, 4799, 4800, 4801, 4802, 4803, 4804, 4805, 4806, 4807, 4810, 4811, 4813, 4814, 4825, 5044, 5051, 5096, 5097, 6701, 6714, 6732, 6740, 6767, 6783, 6789, 6825, 6828, 6843, 6847, 6848, 6864, 6875, 6904, 6922, 6936, 6950, 6953, 7011, 7052, 7058, 7158, 7159, 7160, 7161, 7162, 7163, 7164, 7165, 7168, 7169, 7170, 7171, 7174, 7175, 7176, 7177, 7180, 7181, 7441, 7506, 9411, 9427, 9454, 9473, 9498, 9511, 9521, 9534, 9544, 9548, 9613, 9614, 9890, 9894]

The following groups have an orbit of size 48 and rank 11 : [25, 27, 29, 30, 33, 38, 39, 44, 45, 47, 48, 49, 50, 52, 53, 54, 56, 57, 112, 116, 124, 126, 133, 135, 143, 144, 145, 146, 147, 148, 150, 151, 152, 158, 159, 169, 171, 172, 230, 235, 247, 248, 250, 254, 264, 268, 269, 273, 360, 394, 400, 403, 404, 407, 409, 410, 413, 414, 415, 416, 417, 418, 419, 444, 503, 515, 626, 651, 657, 661, 671, 672, 673, 674, 675, 679, 686, 688, 689, 690, 691, 971, 973, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1078, 1079, 1120, 1131, 1164, 1169, 1178, 1181, 1189, 1190, 1241, 1242, 1270, 1271, 1278, 1279, 1444, 1452, 1475, 1476, 1477, 1478, 2202, 2453, 2454, 2455, 2456, 2473, 2474, 2476, 2477, 2479, 2481, 2483, 2517, 2518, 2521, 2535, 2537, 2539, 2597, 2598, 2603, 2604, 2742, 4494, 4499, 4816, 4818, 4820, 4822, 4824, 4826, 4828, 4830]

The following groups have an orbit of size 48 and rank 12 : [25, 27, 29, 30, 38, 47, 48, 49, 50, 126, 135, 143, 147, 148, 150, 151, 152, 400, 406, 413, 416, 418, 419, 444, 511, 1076, 1078, 1120, 1130, 1134, 1189, 1190, 2454, 2475, 2481, 2517, 2537, 2539, 4826, 7067]

The following groups have an orbit of size 54 and rank 10 : [547, 548, 553, 555, 1292, 1296, 1304, 1310, 1312, 1316, 1518, 2646, 2874, 2876, 2880, 5223]

The following groups have an orbit of size 54 and rank 12 : [1320, 2651]

The following groups have an orbit of size 60 and rank 11 : [573, 577, 1345]

The following groups have an orbit of size 60 and rank 12 : [203, 572, 573, 577, 1345, 2952, 2959, 5331]

The following groups have an orbit of size 64 and rank 7 : [453, 461, 463, 464, 465, 466, 467, 468, 469, 470, 472, 476, 477, 478, 479, 481, 482, 483, 494, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 512, 513, 514, 515, 1079, 1081, 1083, 1085, 1087, 1088, 1093, 1098, 1099, 1100, 1101, 1102, 1103, 1106, 1107, 1109, 1111, 1118, 1119, 1120, 1121, 1122, 1127, 1128, 1131, 1132, 1133, 1134, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1200, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1245, 1246, 1247, 1248, 1249, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1264, 1265, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1487, 1488, 2205, 2299, 2303, 2317, 2318, 2319, 2320, 2321, 2322, 2323, 2324, 2325, 2326, 2327, 2328, 2330, 2341, 2342, 2344, 2345, 2347, 2350, 2351, 2394, 2395, 2396, 2397, 2398, 2402, 2403, 2404, 2447, 2448, 2449, 2450, 2451, 2452, 2453, 2454, 2455, 2456, 2457, 2458, 2459, 2460, 2461, 2462, 2463, 2464, 2465, 2466, 2467, 2468, 2469, 2470, 2471, 2472, 2473, 2474, 2475, 2476, 2477, 2478, 2479, 2480, 2483, 2484, 2496, 2500, 2501, 2502, 2503, 2506, 2507, 2516, 2517, 2518, 2519, 2520, 2521, 2524, 2526, 2527, 2528, 2529, 2530, 2534, 2535, 2536, 2537, 2538, 2539, 2541, 2544, 2545, 2546, 2547, 2548, 2549, 2550, 2551, 2553, 2561, 2568, 2569, 2570, 2571, 2572, 2573, 2574, 2575, 2576, 2577, 2578, 2579, 2580, 2581, 2582, 2583, 2584, 2585, 2586, 2587, 2588, 2589, 2592, 2593, 2594, 2595, 2596, 2597, 2598, 2599, 2600, 2601, 2602, 2603, 2604, 2605, 2606, 2607, 2608, 2774, 2775, 2776, 2792, 2793, 2794, 2795, 2796, 2797, 2798, 2799, 2800, 2815, 2816, 2817, 2818, 2819, 3952, 3954, 3959, 3960, 3962, 3964, 3981, 3983, 3996, 4011, 4188, 4191, 4192, 4193, 4194, 4202, 4204, 4215, 4219, 4223, 4238, 4391, 4401, 4402, 4405, 4494, 4495, 4496, 4497, 4498, 4499, 4500, 4501, 4502, 4541, 4544, 4564, 4565, 4566, 4622, 4623, 4624, 4625, 4626, 4627, 4628, 4629, 4630, 4631, 4632, 4633, 4642, 4650, 4658, 4660, 4663, 4665, 4669, 4673, 4686, 4687, 4688, 4691, 4692, 4693, 4695, 4696, 4697, 4698, 4699, 4701, 4702, 4704, 4707, 4708, 4709, 4710, 4717, 4718, 4719, 4720, 4723, 4724, 4725, 4726, 4730, 4732, 4735, 4736, 4737, 4738, 4749, 4750, 4751, 4752, 4753, 4754, 4755, 4756, 4757, 4758, 4771, 4772, 4773, 4774, 4775, 4776, 4777, 4778, 4779, 4780, 4784, 4785, 4794, 4795, 4796, 4797, 4798, 4799, 4800, 4801, 4802, 4803, 4804, 4805, 4806, 4807, 4808, 4809, 4813, 4814, 4815, 4816, 4817, 4818, 4819, 4820, 4821, 4822, 4823, 4824, 4825, 4826, 4827, 4828, 4829, 4830, 4831, 4832, 4833, 4834, 4835, 4836, 4837, 4838, 4839, 4840, 4841, 4842, 4843, 4844, 4845, 4846, 4847, 4848, 4849, 4850, 4851, 4852, 4853, 4854, 4855, 4856, 4857, 4858, 4859, 4860, 4861, 4862, 4863, 4864, 4865, 4866, 4867, 4868, 4869, 4870, 4871, 4872, 4873, 4874, 4875, 4876, 4880, 4884, 4885, 4886, 4887, 4888, 4889, 4890, 4891, 4892, 4893, 5079, 5081, 5084, 5085, 5086, 5087, 5088, 5089, 5099, 5100, 5101, 5102, 5103, 5104, 5105, 5106, 5107, 5108, 5109, 5110, 5111, 5117, 5118, 5119, 5120, 5121, 5122, 5123, 5124, 5125, 5126, 5127, 5128, 5132, 5133, 5134, 5135, 5136, 5137, 5138, 5139, 5140, 5141, 5142, 5143, 5144, 5145, 6023, 6032, 6215, 6333, 6647, 6697, 6698, 6701, 6702, 6705, 6706, 6709, 6710, 6713, 6714, 6717, 6718, 6729, 6736, 6738, 6739, 6742, 6743, 6746, 6765, 6770, 6772, 6783, 6828, 6843, 6864, 6865, 6874, 6875, 6878, 6879, 6882, 6883, 6905, 6906, 6922, 6923, 6924, 6925, 6947, 6950, 6951, 6952, 6957, 6968, 6969, 6994, 7010, 7011, 7052, 7053, 7058, 7067, 7068, 7095, 7096, 7097, 7100, 7102, 7103, 7108, 7109, 7110, 7111, 7112, 7113, 7114, 7115, 7116, 7117, 7132, 7133, 7134, 7135, 7136, 7137, 7138, 7139, 7140, 7141, 7142, 7143, 7146, 7147, 7148, 7149, 7150, 7151, 7152, 7153, 7154, 7155, 7156, 7157, 7158, 7159, 7160, 7161, 7162, 7163, 7164, 7165, 7166, 7167, 7168, 7169, 7170, 7171, 7172, 7173, 7174, 7175, 7178, 7179, 7180, 7181, 7182, 7183, 7184, 7185, 7186, 7187, 7188, 7189, 7190, 7191, 7192, 7193, 7194, 7195, 7196, 7251, 7252, 7253, 7254, 7466, 7467, 7496, 7497, 7498, 7499, 7500, 7501, 7503, 7504, 7505, 7507, 7508, 7509, 7510, 7511, 7512, 7513, 7514, 7515, 7516, 7517, 7518, 7519, 7520, 7521, 7522, 7523, 7524, 7525, 7526, 7527, 7528, 7529, 7530, 8494, 8760, 8973, 8974, 8977, 8978, 8979, 8984, 8989, 8992, 8993, 8994, 9164, 9171, 9179, 9195, 9282, 9284, 9315, 9336, 9379, 9414, 9415, 9426, 9427, 9428, 9429, 9434, 9435, 9436, 9437, 9472, 9475, 9476, 9485, 9486, 9488, 9489, 9490, 9491, 9496, 9497, 9498, 9502, 9503, 9504, 9511, 9516, 9520, 9521, 9522, 9536, 9538, 9543, 9544, 9545, 9546, 9547, 9548, 9553, 9554, 9555, 9556, 9562, 9563, 9565, 9566, 9567, 9568, 9569, 9570, 9571, 9572, 9573, 9574, 9575, 9576, 9577, 9578, 9579, 9580, 9581, 9582, 9583, 9584, 9585, 9586, 9591, 9592, 9593, 9594, 9595, 9596, 9597, 9598, 9599, 9600, 9601, 9602, 9603, 9604, 9605, 9606, 9607, 9608, 9609, 9610, 9611, 9612, 9613, 9614, 9672, 9673, 9674, 9675, 9678, 9679, 9680, 9856, 9911, 9913, 9931, 9932, 9933, 9934, 9935, 9936, 9937, 9938]

The following groups have an orbit of size 64 and rank 9 : [401, 430, 431, 1151, 1479, 1482, 2787]

The following groups have an orbit of size 64 and rank 10 : [420, 423, 452, 457, 465, 475, 477, 488, 493, 1090, 1099, 1100, 1106, 1114, 1197, 1198, 1213, 1214, 2493, 2500, 2511, 4780, 4796, 6002, 6023, 6027, 6031, 6032, 6161, 6521, 6533, 6722, 6724, 6790, 6812, 6826, 6853, 7153, 7155, 7178, 7179, 7180, 7193, 7238]

The following groups have an orbit of size 64 and rank 11 : [421, 424, 451, 455, 458, 466, 474, 479, 480, 487, 492, 1089, 1098, 1101, 1105, 1107, 1115, 1195, 1196, 1202, 1205, 1211, 1212, 2494, 2501, 2514, 4782, 4799, 4806, 7181]

The following groups have an orbit of size 72 and rank 10 : [66, 68, 71, 225, 230, 232, 241, 242, 248, 250, 251, 254, 262, 263, 579, 645, 651, 654, 657, 671, 674, 679, 684, 1362, 1476, 1481, 2701, 2785, 2788, 5044, 5097, 5098, 7215, 7259, 7261, 7262, 7263, 7432, 7450, 7497, 7499, 7500, 7502, 7503, 7505, 7506, 7508, 7510, 7512, 7514, 7516, 7587, 7595, 7597, 7599, 7603, 7605, 7611, 7615, 7617, 7619, 7621, 7623, 7627, 7631, 7639, 7640, 7641, 7642, 7643, 7644, 7645, 7646, 7652, 7654, 7657, 7659, 7661, 7665, 7666, 7667, 7668, 7670, 7675, 7676, 7680, 7683, 7684, 7693, 7694, 7695, 7696, 7697, 7698, 7701, 7705, 7706, 7707, 7711, 7712, 7713, 7714, 7715, 7716, 7722, 9619, 9620, 9621, 9894, 9902]

The following groups have an orbit of size 72 and rank 11 : [60, 61, 65, 66, 67, 71, 74, 77, 204, 208, 213, 215, 224, 225, 226, 229, 230, 231, 235, 238, 242, 247, 248, 250, 251, 254, 259, 261, 262, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 543, 546, 550, 592, 594, 606, 618, 630, 632, 645, 647, 649, 652, 653, 655, 657, 661, 671, 673, 675, 679, 680, 681, 686, 687, 688, 689, 1280, 1282, 1284, 1286, 1288, 1292, 1293, 1294, 1295, 1297, 1299, 1304, 1305, 1311, 1312, 1313, 1316, 1317, 1412, 1460, 1475, 1478, 1486, 1509, 2618, 2622, 2626, 2630, 2634, 2638, 2646, 2647, 2648, 2649, 2772, 2776, 2790, 2791, 2799, 2802, 2833, 2835, 2839, 2840, 2841, 2842, 2849, 2850, 2853, 2855, 2856, 2862, 2866, 2870, 4923, 5080, 5084, 5087, 5088, 5101, 5110, 5111, 5114, 5115, 5159, 5163, 5166, 5170, 5174, 5177, 5179, 5182, 5184, 5186, 5190, 5191, 5192, 5195, 5196, 5197, 5200, 5201, 5202, 5205, 5209, 5217, 5218, 5219, 5224, 5227, 5230, 5231, 5237, 5238, 5239, 5241, 5242, 5244, 5248, 5249, 5255, 5256, 5257, 7498, 7504, 7512, 7514, 7587, 7595, 7599, 7605, 7611, 7615, 7619, 7623, 7627, 7631, 7639, 7646, 7652, 7654, 7657, 7665, 7666, 7667, 7668, 7675, 7683, 7693, 7694, 7705, 7706, 7711, 7712, 7713, 7714, 7715, 7716, 9935]

The following groups have an orbit of size 72 and rank 12 : [66, 68, 71, 73, 76, 225, 230, 232, 241, 242, 248, 250, 251, 254, 255, 257, 261, 262, 263, 553, 555, 645, 651, 654, 657, 671, 674, 679, 681, 1292, 1304, 1310, 1312, 1316, 1320, 1476, 1486, 1514, 2646, 2651, 2772, 2776, 2790, 2791, 2800, 2801, 2839, 2850, 2854, 2867, 2954, 2956, 5080, 5084, 5087, 5089, 5090, 5101, 5108, 5109, 5113, 5116, 5190, 5216, 5219, 5245, 5249, 5255, 5333, 7499, 7502, 7503, 7505, 7512, 7516, 7665, 7676, 7684, 7705, 7711, 7716, 9937]

The following groups have an orbit of size 90 and rank 12 : [2954, 2956, 5333]

The following groups have an orbit of size 96 and rank 10 : [123, 148, 174, 175, 405, 406, 420, 421, 423, 424, 444, 445, 448, 455, 456, 465, 466, 473, 474, 479, 480, 508, 509, 511, 1016, 1078, 1080, 1082, 1091, 1092, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1105, 1107, 1110, 1111, 1118, 1119, 1120, 1121, 1122, 1125, 1126, 1130, 1132, 1133, 1134, 1187, 1188, 1189, 1190, 1202, 1203, 1211, 1212, 2005, 2122, 2291, 2306, 2317, 2333, 2336, 2341, 2349, 2401, 2454, 2458, 2461, 2470, 2472, 2475, 2481, 2482, 2495, 2501, 2502, 2503, 2506, 2507, 2510, 2511, 2512, 2513, 2514, 2515, 2516, 2517, 2524, 2526, 2528, 2530, 2536, 2537, 2538, 2539, 3950, 3975, 3996, 4006, 4011, 4192, 4204, 4218, 4219, 4230, 4238, 4391, 4395, 4405, 4635, 4637, 4677, 4678, 4687, 4692, 4781, 4782, 4785, 4788, 4789, 4790, 4791, 4792, 4793, 4794, 4795, 4796, 4797, 4798, 4799, 4800, 4801, 4802, 4803, 4804, 4805, 4806, 4807, 4810, 4811, 4813, 4814, 4825, 4826, 6701, 6714, 6732, 6767, 6783, 6828, 6843, 6864, 6875, 6922, 6936, 6950, 7011, 7052, 7058, 7158, 7159, 7160, 7161, 7162, 7163, 7164, 7165, 7168, 7169, 7170, 7171, 7174, 7175, 7176, 7177, 7180, 7181, 9411, 9427, 9473, 9498, 9511, 9521, 9534, 9544, 9548, 9613, 9614]

The following groups have an orbit of size 96 and rank 11 : [112, 124, 132, 133, 135, 136, 143, 144, 145, 146, 147, 148, 150, 151, 152, 158, 159, 163, 164, 165, 166, 168, 169, 171, 172, 360, 394, 397, 398, 399, 400, 404, 406, 407, 409, 412, 413, 414, 415, 416, 417, 418, 419, 437, 438, 439, 440, 444, 503, 511, 515, 684, 973, 1070, 1071, 1072, 1073, 1075, 1076, 1077, 1078, 1120, 1130, 1134, 1164, 1169, 1181, 1189, 1190, 1241, 1242, 1270, 1278, 1279, 1480, 1481, 1484, 1486, 1488, 2202, 2453, 2454, 2455, 2475, 2477, 2481, 2483, 2517, 2521, 2535, 2537, 2539, 2597, 2598, 2603, 2604, 2786, 2791, 2794, 2797, 2800, 2802, 2816, 4494, 4816, 4818, 4820, 4824, 4826, 4828, 4830, 5108, 5115, 5122, 5127, 5134, 5141, 7067, 7525]

The following groups have an orbit of size 96 and rank 12 : [126, 132, 135, 136, 143, 147, 151, 152, 174, 178, 399, 400, 416, 418, 419, 421, 424, 451, 455, 458, 466, 473, 474, 479, 480, 487, 491, 492, 684, 1076, 1089, 1092, 1095, 1097, 1098, 1101, 1102, 1105, 1107, 1110, 1111, 1113, 1115, 1116, 1119, 1189, 1190, 1195, 1196, 1202, 1203, 1205, 1206, 1211, 1212, 1480, 1481, 1484, 2481, 2494, 2495, 2499, 2501, 2502, 2505, 2506, 2514, 2517, 2522, 2524, 2525, 2526, 2528, 2530, 2531, 2533, 2537, 2539, 2785, 2786, 2788, 4782, 4785, 4787, 4790, 4799, 4800, 4806, 4810, 4813, 4826, 5095, 5096, 5097, 5098, 7158, 7164, 7168, 7171, 7174, 7176, 7181, 7214, 7215, 7353, 7359, 7363, 7364, 7369, 7371, 7377, 7378, 7380, 7381, 7384, 7385, 7386, 7388, 7389, 7390, 7391, 7392, 7393, 7394, 7395, 7396, 7399, 7401, 7407, 7413, 7429, 7441, 7458, 7459, 7466, 7467, 7496, 7497, 7498, 7499, 7500, 7501, 7502, 7503, 7504, 7505, 7506, 7507, 7508, 7509, 7510, 7511, 7512, 7513, 7514, 7515, 7516, 7517, 7518, 7519, 7529, 7530, 9614, 9619, 9620, 9621, 9622]

The following groups have an orbit of size 108 and rank 10 : [547, 555, 1296, 1304, 1310, 1312, 1316, 2646, 2876, 2880, 5223]

The following groups have an orbit of size 108 and rank 11 : [543, 544, 545, 546, 549, 550, 553, 555, 1280, 1282, 1284, 1286, 1288, 1292, 1293, 1294, 1295, 1297, 1298, 1299, 1304, 1305, 1310, 1311, 1312, 1313, 1316, 1317, 2618, 2622, 2626, 2630, 2634, 2638, 2646, 2647, 2648, 2649, 4923]

The following groups have an orbit of size 108 and rank 12 : [1320, 1516, 1518, 2876, 4948, 7258, 7262]

The following groups have an orbit of size 110 and rank 12 : [2948, 5323]

The following groups have an orbit of size 120 and rank 11 : [571, 573, 575, 577, 1341, 1343, 1345, 1347, 1350, 1352, 2665]

The following groups have an orbit of size 120 and rank 12 : [203, 572, 573, 577, 1345, 1346, 7250, 9672, 9674, 9676]

The following groups have an orbit of size 128 and rank 11 : [1092, 1095, 1097, 1102, 1110, 1111, 1113, 1116, 1119, 1203, 1206, 2495, 2499, 2502, 2505, 2506, 2522, 2524, 2525, 2526, 2528, 2530, 2531, 2533, 2785, 2788, 4785, 4787, 4790, 4800, 4810, 4813, 5095, 5096, 5097, 5098, 7158, 7164, 7168, 7171, 7174, 7176, 7214, 7215, 7506, 9614, 9619, 9620, 9621, 9622]

The following groups have an orbit of size 132 and rank 12 : [2948]

The following groups have an orbit of size 144 and rank 10 : [255, 257, 681, 1320, 1364, 1366, 1480, 1484, 2651, 2697, 2786, 5040, 5051, 5095, 5096, 7441, 7506, 9619, 9621, 9885, 9890]

The following groups have an orbit of size 144 and rank 11 : [208, 213, 214, 215, 224, 226, 230, 231, 235, 238, 240, 247, 248, 255, 258, 259, 264, 267, 268, 269, 273, 592, 594, 606, 618, 626, 643, 645, 647, 649, 651, 652, 653, 655, 657, 661, 671, 672, 673, 674, 675, 680, 686, 688, 689, 690, 691, 1412, 1444, 1452, 1475, 1476, 1477, 1478, 1488, 2742, 2774, 2793, 2794, 2796, 2797, 2800, 2801, 2816, 2818, 5079, 5081, 5085, 5086, 5089, 5090, 5099, 5100, 5102, 5108, 5109, 5113, 5116, 5120, 5121, 5122, 5123, 5125, 5126, 5127, 5128, 5133, 5134, 5135, 5136, 5140, 5141, 5142, 5143, 7466, 7496, 7497, 7499, 7500, 7501, 7502, 7503, 7505, 7507, 7508, 7509, 7510, 7511, 7513, 7515, 7516, 7520, 7521, 7522, 7523, 7524, 7525, 7526, 7527, 9911, 9913, 9931, 9932, 9933, 9934, 9936, 9937, 9938]

The following groups have an orbit of size 144 and rank 12 : [225, 232, 241, 242, 248, 250, 251, 254, 255, 257, 262, 263, 645, 651, 654, 657, 671, 674, 679, 681, 1320, 1476, 1486, 2651, 2772, 2776, 2790, 2791, 2800, 2801, 5080, 5084, 5087, 5089, 5090, 5101, 5108, 5109, 5113, 5116, 7499, 7502, 7503, 7505, 7512, 7516, 9937]

The following groups have an orbit of size 162 and rank 10 : [1510, 1514, 1516, 2839, 2843, 2850, 2854, 2857, 2864, 2867, 2872, 2878, 4948, 5190, 5203, 5216, 5219, 5243, 5245, 5249, 5255, 7257, 7258, 7261, 7262, 7659, 7665, 7676, 7684, 7695, 7705, 7711, 7716, 9683, 9684]

The following groups have an orbit of size 162 and rank 12 : [1516, 1518, 2874, 2876, 2878, 2880, 5223]

The following groups have an orbit of size 180 and rank 11 : [2952, 2955, 2956, 2959, 5331, 5334]

The following groups have an orbit of size 180 and rank 12 : [2952, 2959, 5331]

The following groups have an orbit of size 192 and rank 11 : [394, 398, 403, 404, 406, 407, 409, 410, 412, 413, 414, 415, 416, 417, 419, 437, 439, 971, 973, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1120, 1130, 1131, 1134, 1164, 1178, 1189, 1242, 1271, 1278, 2202, 2453, 2454, 2455, 2456, 2473, 2474, 2475, 2476, 2477, 2479, 2481, 2483, 2517, 2518, 2521, 2535, 2537, 2539, 2597, 2598, 2603, 2772, 2774, 2776, 2790, 2793, 2796, 2799, 2801, 2818, 4494, 4499, 4816, 4818, 4820, 4822, 4824, 4826, 4828, 4830, 5079, 5080, 5081, 5084, 5085, 5086, 5087, 5088, 5089, 5090, 5099, 5100, 5101, 5102, 5109, 5110, 5111, 5113, 5114, 5116, 5120, 5121, 5123, 5125, 5126, 5128, 5133, 5135, 5136, 5140, 5142, 5143, 7067, 7466, 7496, 7497, 7498, 7499, 7500, 7501, 7502, 7503, 7504, 7505, 7507, 7508, 7509, 7510, 7511, 7512, 7513, 7514, 7515, 7516, 7520, 7521, 7522, 7523, 7524, 7526, 7527, 7639, 7642, 7644, 7659, 7665, 7667, 7670, 7676, 7680, 7684, 7693, 7694, 7695, 7696, 7697, 7698, 7705, 7706, 7711, 7713, 7715, 7716, 9911, 9913, 9931, 9932, 9933, 9934, 9935, 9936, 9937, 9938]

The following groups have an orbit of size 192 and rank 12 : [399, 413, 416, 418, 421, 424, 444, 451, 455, 458, 466, 473, 474, 479, 480, 487, 491, 492, 1076, 1078, 1092, 1095, 1097, 1102, 1105, 1107, 1110, 1113, 1115, 1119, 1120, 1189, 1190, 1195, 1196, 1202, 1203, 1205, 1206, 1211, 1212, 2454, 2481, 2494, 2495, 2499, 2501, 2502, 2505, 2506, 2517, 2524, 2525, 2528, 2530, 2531, 2533, 2537, 2539, 4782, 4785, 4787, 4790, 4800, 4806, 4826, 7158, 7164, 7168, 7174, 7176, 7181, 9614]

The following groups have an orbit of size 216 and rank 10 : [7214, 7257, 7258, 7259, 7261, 7262, 7263, 9622]

The following groups have an orbit of size 216 and rank 11 : [543, 545, 546, 1280, 1282, 1288, 1294, 1295, 1298, 1299, 1305, 1310, 1311, 1312, 1313, 1316, 1317, 2622, 2626, 2630, 2634, 2638, 2646, 2647, 2648, 2649, 4923]

The following groups have an orbit of size 216 and rank 12 : [547, 548, 553, 555, 1292, 1296, 1304, 1310, 1312, 1316, 1510, 1514, 1516, 1518, 2646, 2651, 2839, 2843, 2850, 2854, 2857, 2864, 2867, 2872, 2874, 2876, 2878, 2880, 4948, 5190, 5203, 5216, 5219, 5223, 5243, 5245, 5249, 5255, 7257, 7258, 7261, 7262, 7659, 7665, 7676, 7684, 7695, 7705, 7711, 7716, 9683, 9684]

The following groups have an orbit of size 220 and rank 12 : [5323]

The following groups have an orbit of size 240 and rank 11 : [571, 575, 1341, 1343, 1347, 1350, 1352, 2665]

The following groups have an orbit of size 240 and rank 12 : [577, 1345, 1346, 2952, 2959, 5331, 7250, 7336, 7338, 7340, 7342, 9672, 9674, 9676]

The following groups have an orbit of size 264 and rank 12 : [5323]

The following groups have an orbit of size 288 and rank 11 : [630, 632, 680, 681, 1460]

The following groups have an orbit of size 288 and rank 12 : [684, 1484, 1486, 2772, 2776, 2790, 2791, 2800, 2801, 5080, 5084, 5087, 5089, 5090, 5101, 5108, 5109, 5113, 5116, 7499, 7502, 7503, 7505, 7512, 7516, 9676, 9937]

The following groups have an orbit of size 324 and rank 11 : [1509, 1511, 1514, 2833, 2835, 2839, 2840, 2841, 2842, 2844, 2849, 2850, 2853, 2854, 2855, 2856, 2858, 2860, 2861, 2862, 2865, 2866, 2867, 2869, 2870, 5159, 5163, 5166, 5170, 5174, 5177, 5179, 5182, 5184, 5186, 5190, 5191, 5192, 5195, 5196, 5197, 5200, 5201, 5202, 5204, 5205, 5209, 5211, 5216, 5217, 5218, 5219, 5224, 5227, 5230, 5231, 5232, 5233, 5237, 5238, 5239, 5241, 5242, 5244, 5245, 5248, 5249, 5255, 5256, 5257, 7587, 7595, 7599, 7605, 7611, 7615, 7619, 7623, 7627, 7631, 7639, 7644, 7646, 7652, 7654, 7657, 7665, 7666, 7667, 7668, 7675, 7676, 7683, 7684, 7693, 7694, 7696, 7698, 7705, 7706, 7711, 7712, 7713, 7714, 7715, 7716]

The following groups have an orbit of size 324 and rank 12 : [2876, 2878, 2880, 5223]

The following groups have an orbit of size 330 and rank 12 : [2948, 5323]

The following groups have an orbit of size 360 and rank 11 : [2953, 2954, 2957, 2958, 5325, 5326, 5327, 5328, 5329, 5330, 5332, 5333, 5335, 5336, 7790, 7791]

The following groups have an orbit of size 360 and rank 12 : [2954, 2956, 5333]

The following groups have an orbit of size 384 and rank 12 : [1089, 1097, 1098, 1101, 1102, 1107, 1111, 1113, 1115, 1116, 1195, 1211, 2494, 2501, 2502, 2505, 2506, 2514, 2522, 2524, 2525, 2526, 2528, 2530, 2531, 2533, 4782, 4785, 4787, 4790, 4799, 4800, 4806, 4810, 4813, 7158, 7164, 7168, 7171, 7174, 7176, 7181, 9614]

The following groups have an orbit of size 432 and rank 12 : [1320, 2651]

The following groups have an orbit of size 480 and rank 12 : [7250, 9672, 9674, 9676]

The following groups have an orbit of size 486 and rank 12 : [4948, 7257, 7258, 7261, 7262, 9683, 9684]

The following groups have an orbit of size 576 and rank 12 : [1480, 1481, 2785, 2786, 2788, 5095, 5096, 5097, 5098, 7214, 7215, 7506, 9619, 9620, 9621, 9622]

Appendix C

Ranks and Sizes of Orbits of Types in Degree 28

C.1 Degree 28

Due to the size of the output files, we will only give a summary of the ranks and sizes of orbits and which groups corresponded to such orbits. The following summary is for groups of order up to 917,504 = $2^{17} \cdot 7$:

The following groups have an orbit of size 2 and rank 2 : [2, 4, 9, 14, 15, 20, 24, 34, 39, 43, 44, 48, 52, 72, 74, 81, 84, 85, 88, 94, 118, 123, 125, 129, 132, 167, 189, 196, 197, 200, 208, 212, 229, 232, 236, 246, 250, 270, 282, 310, 312, 314, 316, 317, 330, 361, 363, 386, 431, 462, 539, 668, 946, 1096, 1097, 1099]

The following groups have an orbit of size 4 and rank 3 : [1, 3, 5, 6, 7, 8, 10, 12, 13, 18, 21, 22, 23, 25, 26, 33, 41, 47, 49, 50, 51, 71, 73, 83, 87, 90, 91, 92, 93, 95, 121, 122, 124, 126, 127, 128, 130, 131, 147, 149, 166, 168, 202, 203, 204, 205, 206, 207, 209, 210, 211, 213, 245, 247, 248, 249, 283, 284, 325, 326, 327, 328, 329, 360, 362, 397, 427, 428, 429, 430, 432, 501, 666, 868]

The following groups have an orbit of size 14 and rank 8 : [2, 4, 9, 14, 15, 20, 24, 39, 43, 44, 84, 85, 88, 118, 189, 196, 197, 229, 232, 236, 270, 310, 312, 314, 316, 317, 361, 363, 386, 431, 462, 539, 946, 1096, 1097, 1099]

The following groups have an orbit of size 16 and rank 8 : [19, 20, 38, 39, 43, 44, 80, 84, 85, 102, 104, 112, 114,116, 179, 187, 188, 222, 226, 231, 232, 233, 234, 235, 299, 309, 311, 312, 313, 343, 412, 456, 459, 460, 464, 530, 536, 538, 540, 837, 1017]

The following groups have an orbit of size 28 and rank 8 : [4, 9, 15, 24, 196, 197, 270, 316, 317, 386]

The following groups have an orbit of size 28 and rank 14 : [2, 4, 9, 14, 20, 24, 39, 44, 85, 118, 189, 196, 197, 236, 270, 314, 316, 317, 386]

The following groups have an orbit of size 28 and rank 15 : [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 18, 20, 21, 22, 23, 24, 25, 26, 33, 34, 39, 41, 43, 44, 48, 49, 51, 52, 71, 72, 73, 74, 81, 83, 84, 85, 87, 88, 92, 94, 118, 123, 124, 125, 126, 128, 129, 131, 132, 149, 167, 168, 189, 196, 197, 199, 200, 206, 208, 210, 212, 229, 232, 236, 246, 247, 249, 250, 270, 282, 310, 312, 314, 316, 317, 328, 330, 360, 361, 362, 363, 386, 427, 428, 429, 430, 431, 432, 462, 501, 539, 666, 668, 946, 1096, 1097, 1099]

The following groups have an orbit of size 32 and rank 15 : [37, 59, 79, 82, 86, 145, 146, 148, 178, 180, 184, 186, 266, 272, 292, 294, 296, 300, 303, 307, 308, 373, 376, 378, 387, 529, 535, 653]

The following groups have an orbit of size 42 and rank 8 : [14, 15, 24, 43, 44, 84, 85, 88, 229, 232, 236, 310, 312, 314, 316, 317, 361, 363, 386, 431, 462, 539, 946, 1096, 1097, 1099]

The following groups have an orbit of size 42 and rank 14 : [15, 81]

The following groups have an orbit of size 56 and rank 8 : [43, 84, 88, 229, 232, 310, 312, 462, 539]

The following groups have an orbit of size 56 and rank 14 : [9, 20, 81, 196, 197, 270]

The following groups have an orbit of size 56 and rank 15 : [5, 6, 7, 8, 9, 10, 18, 20, 21, 22, 23, 24, 25, 26, 39, 41, 43, 44, 47, 48, 49, 50, 51, 52, 90, 91, 92, 93, 94, 95, 118, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 147, 149, 166, 189, 196, 197, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 231, 232, 245, 246, 247, 248, 249, 250, 270, 281, 283, 284, 325, 326, 327, 328, 329, 330, 397, 427, 428, 430, 432, 501, 868]

The following groups have an orbit of size 70 and rank 8 : [361, 363, 431, 946, 1096, 1097, 1099]

The following groups have an orbit of size 84 and rank 14 : [14, 15, 24, 43, 44, 85, 88, 236, 314, 316, 317, 361, 363, 386, 431]

The following groups have an orbit of size 84 and rank 15 : [12, 13, 14, 15, 21, 22, 23, 24, 25, 26, 41, 43, 44, 71, 72, 73, 74, 81, 83, 84, 85, 87, 88, 123, 124, 125, 126, 128, 129, 131, 132, 147, 149, 167, 168, 206, 208, 210, 212, 229, 232, 236, 246, 247, 249, 250, 310, 312, 314, 316, 317, 328, 330, 360, 361, 362, 363, 386, 427, 428, 429, 430, 431, 432, 462, 501, 539, 666, 668]

The following groups have an orbit of size 98 and rank 14 : [34, 48, 52, 72, 74, 94, 123, 125, 129, 132, 167, 208, 212, 246, 250, 330, 668]

The following groups have an orbit of size 112 and rank 8 : [19, 20, 38, 39, 43, 44, 84, 85, 102, 104, 112, 114, 116, 179, 187, 188, 222, 226, 231, 232, 233, 234, 235, 299, 309, 311, 312, 313, 343, 412, 456, 459, 460, 464, 530, 536, 538, 540, 837, 1017]

The following groups have an orbit of size 112 and rank 14 : [20, 39, 43, 81, 84, 88, 118, 189, 196, 197, 229, 232, 270, 310, 312, 462, 539]

The following groups have an orbit of size 112 and rank 15 : [18, 19, 20, 39, 41, 43, 44, 80, 81, 83, 84, 85, 87, 88, 90, 91, 93, 95, 110, 118, 147, 149, 166, 189, 196, 197, 202, 203, 204, 205, 207, 209, 211, 213, 229, 232, 233, 234, 236, 270, 283, 284, 309, 310, 311, 312, 314, 325, 326, 327, 329, 397, 462, 464, 539, 540, 666, 668]

The following groups have an orbit of size 128 and rank 8 : [110, 111, 112, 113, 114, 115, 116, 117, 118, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 229, 230, 231, 235, 236, 269, 270, 309, 310, 313, 314, 315, 316, 317, 343, 344, 345, 386, 411, 412, 455, 456, 460, 461, 462, 463, 464, 530, 538, 539, 540, 591, 746, 760, 761, 833, 834, 835, 836, 837, 914, 946, 1005, 1015, 1016, 1017, 1018, 1096, 1097, 1098, 1099, 1133]

The following groups have an orbit of size 128 and rank 15 : [99, 173, 175, 182, 217, 287, 289, 298, 302, 306, 395, 449, 489, 491, 519, 523, 532, 663, 667]

The following groups have an orbit of size 140 and rank 15 : [360, 361, 362, 363, 427, 428, 429, 430, 431, 432, 501]

The following groups have an orbit of size 156 and rank 14 : [200, 282]

The following groups have an orbit of size 156 and rank 15 : [199]

The following groups have an orbit of size 168 and rank 14 : [24, 43, 44, 81, 84, 88, 229, 232, 310, 312, 316, 317, 386, 462, 539]

The following groups have an orbit of size 168 and rank 15 : [21, 22, 23, 24, 25, 26, 41, 43, 44, 81, 83, 84, 85, 87, 88, 121, 122, 127, 130, 147, 149, 202, 203, 204, 205, 207, 209, 211, 213, 229, 232, 236, 245, 248, 283, 284, 310, 312, 314, 316, 317, 325, 326, 327, 329, 360, 361, 362, 363, 386, 397, 427, 428, 429, 430, 431, 432, 462, 501, 539, 868, 946, 1096, 1097, 1099]

The following groups have an orbit of size 182 and rank 15 : [200, 282]

The following groups have an orbit of size 196 and rank 14 : [34, 48, 52, 72, 74, 94, 129, 132, 167, 250]

The following groups have an orbit of size 196 and rank 15 : [33, 34, 47, 48, 49, 50, 51, 52, 71, 72, 73, 74, 90, 91, 92, 93, 94, 95, 121, 122, 124, 126, 127, 128, 130, 131, 166, 167, 168, 202, 203, 204, 205, 206, 207, 209, 210, 211, 213, 245, 247, 248, 249, 283, 284, 325, 326, 327, 328, 329, 397, 666, 668, 868]

The following groups have an orbit of size 224 and rank 14 : [39, 196, 197, 270]

The following groups have an orbit of size 224 and rank 15 : [37, 38, 39, 59, 79, 80, 82, 84, 85, 86, 102, 104, 110, 112, 113, 114, 115, 116, 118, 145, 146, 147, 148, 149, 178, 179, 180, 183, 184, 185, 186, 187, 188, 189, 191, 192, 194, 195, 196, 197, 222, 226, 229, 231, 232, 233, 234, 235, 236, 266, 269, 270, 272, 292, 294, 296, 299, 300, 303, 307, 308, 309, 310, 311, 312, 313, 314, 316, 317, 343, 373, 376, 378, 386, 387, 412, 456, 459, 460, 462, 464, 529, 530, 535, 536, 538, 539, 540, 653, 837, 868, 1017]

The following groups have an orbit of size 256 and rank 15 : [171, 172, 174, 176, 177, 181, 183, 185, 255, 256, 257, 258, 259, 260, 261, 265, 271, 285, 286, 288, 290, 291, 293, 295, 297, 301, 304, 305, 331, 364, 365, 366, 367, 368, 369, 370, 374, 375, 377, 379, 380, 396, 403, 404, 434, 468, 473, 490, 492, 505, 508, 509, 512, 517, 518, 522, 526, 528, 531, 537, 621, 622, 633, 634, 637, 638, 641, 651, 652, 664, 665, 704, 710, 716, 722, 866, 867, 882, 888, 897, 906, 907, 915, 972, 977, 981, 985, 987, 993, 998, 1006, 1055, 1094, 1095, 1114, 1116, 1118, 1124, 1126, 1129, 1130]

The following groups have an orbit of size 280 and rank 14 : [361, 363, 431, 946, 1096, 1097, 1099]

The following groups have an orbit of size 280 and rank 15 : [427, 428, 430, 432, 501]

The following groups have an orbit of size 294 and rank 14 : [72, 74, 123, 125, 129, 132, 208, 212]

The following groups have an orbit of size 312 and rank 15 : [281]

The following groups have an orbit of size 336 and rank 14 : [43, 44, 81, 84, 85, 88, 229, 232, 236, 310, 312, 314, 316, 317, 386, 462, 539]

The following groups have an orbit of size 336 and rank 15 : [41, 43, 44, 80, 81, 83, 84, 85, 87, 88, 147, 149, 229, 231, 232, 233, 234, 236, 309, 310, 311, 312, 314, 316, 317, 386, 427, 428, 430, 432, 462, 464, 501, 539, 540]

The following groups have an orbit of size 364 and rank 14 : [200, 282]

The following groups have an orbit of size 364 and rank 15 : [199, 200, 282]

The following groups have an orbit of size 392 and rank 14 : [48, 52, 94, 123, 125, 208, 212, 246, 330]

The following groups have an orbit of size 392 and rank 15 : [47, 48, 49, 50, 51, 52, 90, 91, 92, 93, 94, 95, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 166, 204, 205, 206, 208, 209, 210, 212, 213, 245, 246, 247, 248, 249, 250, 326, 328, 329, 330, 868]

The following groups have an orbit of size 420 and rank 14 : [361, 363, 431]

The following groups have an orbit of size 420 and rank 15 : [360, 361, 362, 363, 429, 431, 946, 1096, 1097, 1099]

The following groups have an orbit of size 448 and rank 14 : [118, 189, 196, 197, 270]

The following groups have an orbit of size 448 and rank 15 : [59, 102, 104, 110, 111, 112, 113, 114, 115, 116, 117, 118, 145, 146, 148, 178, 179, 180, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 222, 226, 229, 230, 231, 232, 233, 234, 235, 266, 269, 270, 271, 272, 292, 294, 296, 299, 300, 307, 308, 309, 310, 311, 312, 313, 316, 317, 343, 373, 376, 378, 386, 387, 412, 456, 459, 460, 462, 464, 529, 530, 535, 536, 538, 539, 540, 653, 837, 1017]

The following groups have an orbit of size 546 and rank 15 : [200, 282]

The following groups have an orbit of size 560 and rank 15 : [360, 361, 362, 363, 427, 428, 429, 430, 431, 432, 501, 946, 1096, 1097, 1099]

The following groups have an orbit of size 588 and rank 14 : [72, 74, 125, 129, 167, 212]

The following groups have an orbit of size 588 and rank 15 : [71, 72, 73, 74, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 167, 168, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 246, 247, 249, 250, 283, 284, 328, 330, 666, 668]

The following groups have an orbit of size 672 and rank 14 : [84, 85, 88, 232, 312, 316, 317, 386]

The following groups have an orbit of size 672 and rank 15 : [79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 145, 146, 147, 148, 149, 222, 226, 229, 231, 232, 233, 234, 235, 236, 292, 296, 299, 303, 309, 310, 311, 312, 313, 314, 316, 317, 373, 378, 386, 456, 459, 460, 462, 464, 530, 536, 538, 539, 540, 837, 1017]

The following groups have an orbit of size 728 and rank 14 : [200, 282]

The following groups have an orbit of size 728 and rank 15 : [199, 281, 282]

The following groups have an orbit of size 784 and rank 14 : [668]

The following groups have an orbit of size 784 and rank 15 : [90, 91, 92, 93, 95, 166, 202, 203, 204, 205, 206, 207, 209, 210, 211, 213, 283, 284, 325, 326, 327, 328, 329, 397, 666, 668]

The following groups have an orbit of size 840 and rank 14 : [361, 363, 431]

The following groups have an orbit of size 840 and rank 15 : [360, 361, 362, 363, 427, 428, 429, 430, 431, 432, 501, 946, 1096, 1097, 1099]

The following groups have an orbit of size 882 and rank 14 : [167, 246, 250, 330, 668]

The following groups have an orbit of size 896 and rank 15 : [99, 102, 104, 111, 112, 113, 114, 115, 116, 117, 118, 173, 175, 178, 179, 180, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 217, 222, 229, 230, 231, 233, 234, 235, 236, 266, 269, 270, 271, 272, 287, 289, 291, 292, 294, 295, 296, 298, 299, 300, 302, 303, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 343, 344, 345, 373, 376, 378, 386, 387, 411, 412, 449, 455, 456, 460, 461, 462, 463, 464, 523, 529, 530, 532, 535, 538, 539, 540, 591, 653, 746, 760, 761, 833, 834, 835, 836, 837, 914, 946, 1005, 1015, 1016, 1017, 1018, 1096, 1097, 1098, 1099, 1133]

The following groups have an orbit of size 1024 and rank 15 : [332, 333, 334, 398, 406, 407, 410, 435, 436, 450, 503, 511, 514, 515, 521, 525, 534, 549, 553, 558, 559, 566, 686, 690, 726, 731, 740, 794, 796, 798, 799, 802, 803, 804, 806, 960, 962, 966, 968, 995, 1000, 1001, 1002, 1007, 1049, 1180]

The following groups have an orbit of size 1092 and rank 14 : [200, 282]

The following groups have an orbit of size 1092 and rank 15 : [199, 200, 282]

The following groups have an orbit of size 1120 and rank 15 : [427, 428, 430, 432, 501, 946, 1096, 1097, 1099]

The following groups have an orbit of size 1176 and rank 14 : [123, 125, 129, 132, 208, 212, 246, 250, 330]

The following groups have an orbit of size 1176 and rank 15 : [121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 245, 246, 247, 248, 249, 250, 283, 284, 325, 326, 327, 328, 329, 330, 397, 868]

The following groups have an orbit of size 1260 and rank 14 : [361, 363, 431]

The following groups have an orbit of size 1344 and rank 14 : [229, 232, 236, 310, 312, 314, 316, 317, 386, 462, 539, 946, 1096, 1097, 1099]

The following groups have an orbit of size 1344 and rank 15 : [145, 146, 147, 148, 149, 222, 226, 229, 230, 231, 232, 233, 234, 235, 236, 292, 294, 296, 299, 300, 303, 307, 308, 309, 310, 311, 312, 313, 314, 316, 317, 373, 376, 378, 386, 387, 456, 459, 460, 462, 464, 529, 530, 535, 536, 538, 539, 540, 653, 837, 946, 1017, 1096, 1097, 1099]

The following groups have an orbit of size 1456 and rank 15 : [281]

The following groups have an orbit of size 1568 and rank 14 : [668]

The following groups have an orbit of size 1568 and rank 15 : [868]

The following groups have an orbit of size 1680 and rank 14 : [946, 1096, 1097, 1099]

The following groups have an orbit of size 1680 and rank 15 : [360, 361, 362, 363, 427, 428, 429, 430, 431, 432, 501, 946, 1096, 1097, 1099]

The following groups have an orbit of size 1764 and rank 15 : [167, 168, 245, 246, 247, 248, 249, 250, 325, 326, 327, 328, 329, 330, 397, 666, 868]

The following groups have an orbit of size 1792 and rank 15 : [171, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 184, 186, 193, 194, 195, 255, 256, 257, 258, 259, 260, 261, 265, 266, 269, 270, 271, 272, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 300, 301, 302, 304, 305, 306, 315, 331, 364, 365, 366, 367, 368, 369, 370, 373, 374, 375, 376, 377, 378, 379, 380, 386, 387, 395, 396 , 403, 404, 434, 449, 473, 489, 490, 491, 492, 505, 508, 509, 512, 517, 518, 519, 522, 523, 526, 528, 531, 532, 537, 633, 634, 637, 641, 651, 652, 663, 664, 665, 667, 704, 710, 716, 722, 760, 761, 882, 888, 897, 906, 907, 914, 915, 972, 977, 981, 985, 987, 993, 998, 1006, 1015, 1016, 1055, 1094, 1095, 1114, 1116, 1118, 1124, 1126, 1129, 1130, 1133]

The following groups have an orbit of size 2048 and rank 15 : [399, 400, 401, 402, 405, 408, 409, 469, 470, 471, 472, 504, 506, 507, 510, 513, 516, 520, 524, 527, 533, 631, 632, 635, 636, 639, 640, 650, 677, 681, 687, 691, 694, 698, 703, 732, 737, 738, 878, 880, 894, 895, 902, 953, 955, 957, 961, 963, 967, 969, 973, 976, 978, 979, 982, 994, 996, 997, 999, 1004, 1106, 1108, 1110, 1111, 1117, 1121, 1122, 1123, 1125, 1175, 1179]

The following groups have an orbit of size 2184 and rank 15 : [200, 281, 282]

The following groups have an orbit of size 2240 and rank 15 : [946, 1096, 1097, 1099]

The following groups have an orbit of size 2352 and rank 15 : [202, 203, 204, 205, 207, 209, 211, 213, 283, 284, 325, 326, 327, 329, 397, 666, 668]

The following groups have an orbit of size 2520 and rank 15 : [360, 362, 427, 428, 429, 430, 432, 501]

The following groups have an orbit of size 2688 and rank 15 : [217, 222, 226, 229, 231, 233, 234, 235, 287, 291, 292, 294, 295, 296, 299, 300, 303, 307, 308, 309, 310, 311, 312, 313, 314, 315, 373, 376, 378, 386, 387, 455, 456, 460, 461, 462, 463, 464, 519, 529, 530, 535, 538, 539, 540, 653, 833, 834, 835, 836, 837, 1005, 1015, 1016, 1017, 1018, 1096, 1098, 1133]

The following groups have an orbit of size 3136 and rank 15 : [666, 868]

The following groups have an orbit of size 3360 and rank 15 : [427, 428, 430, 432, 501, 946, 1096, 1097, 1099]

The following groups have an orbit of size 3528 and rank 15 : [245, 248, 325, 326, 327, 329, 397]

The following groups have an orbit of size 3584 and rank 15 : [256, 258, 259, 260, 261, 265, 331, 364, 366, 367, 369, 370, 375, 379, 380, 434, 450, 455, 456, 459, 463, 468, 519, 526, 530, 531, 536, 539, 540, 621, 622, 638, 641, 651, 834, 835, 836, 837, 866, 867, 882, 888, 907, 915, 1017, 1018, 1055, 1114, 1116, 1130]

The following groups have an orbit of size 4480 and rank 15 : [1096, 1098]

The following groups have an orbit of size 4704 and rank 15 : [868]

The following groups have an orbit of size 5376 and rank 15 : [285, 286, 288, 289, 290, 292, 293, 294, 296, 297, 298, 300, 301, 302, 304, 305, 306, 307, 308, 364, 365, 366, 367, 368, 369, 373, 374, 376, 377, 378, 387, 434, 449, 505, 508, 509, 512, 517, 518, 523, 528, 532, 537, 633, 634, 637, 638, 652, 972, 977, 981, 985, 987, 993, 998, 1006, 1094, 1095, 1114, 1116, 1118, 1124, 1126, 1129, 1130]

The following groups have an orbit of size 6272 and rank 15 : [395, 489, 491, 663, 667]

The following groups have an orbit of size 7168 and rank 15 : [332, 333, 334, 398, 406, 407, 410, 435, 436, 503, 505, 509, 511, 514, 515, 517, 518, 521, 525, 527, 528, 529, 533, 534, 535, 537, 549, 553, 558, 559, 566, 633, 637, 638, 652, 653, 686, 690, 726, 731, 740, 794, 796, 798, 799, 802, 803, 804, 806, 960, 962, 966, 968, 972, 977, 993, 995, 998, 1000 , 1001, 1002, 1007, 1049, 1118, 1124, 1180]

The following groups have an orbit of size 8192 and rank 15 : [550, 551, 552, 554, 555, 556, 557, 560, 561, 562, 563, 564, 565, 672, 674, 683, 684, 689, 707, 708, 712, 717, 718, 719, 720, 723, 724, 728, 733, 735, 736, 739, 742, 745, 795, 797, 800, 801, 805, 873, 911, 912, 913, 947, 950, 958, 964, 970, 984, 988, 989, 991, 1003, 1048, 1050, 1051, 1052, 1104 , 1132, 1167, 1173, 1182, 1183, 1185]

The following groups have an orbit of size 8960 and rank 15 : [1094, 1095]

The following groups have an orbit of size 10752 and rank 15 : [370, 375, 379, 380, 522, 526, 531, 641 , 651]

The following groups have an orbit of size 12544 and rank 15 : [396, 468, 490, 492, 621, 622, 664, 665, 866, 867]

The following groups have an orbit of size 14336 and rank 15 : [399, 400, 401, 402, 405, 408, 409, 469, 470, 471, 472, 504, 506, 507, 510, 513, 516, 520, 524, 631, 632, 635, 636, 639, 640, 650, 677, 681, 687, 691, 694, 698, 703, 732, 737, 738, 878, 880, 894, 895, 902, 953, 955, 957, 961, 963, 967, 969, 973, 976, 978, 979, 982, 994, 996, 997, 999, 1004, 1106, 1108, 1110, 1111, 1117, 1121, 1122, 1123, 1125, 1175, 1179]

The following groups have an orbit of size 16384 and rank 15 : [671, 673, 678, 679, 680, 682, 685, 688, 692, 693, 695, 696, 697, 699, 700, 701, 702, 705, 706, 709, 711, 713, 714, 715, 721, 725, 727, 729, 730, 734, 741, 743, 744, 747, 870, 871, 872, 874, 875, 876, 877, 879, 881, 883, 884, 885, 886, 887, 889, 890, 891, 892, 893, 896, 898, 899, 900, 901, 903, 904, 905, 908, 909, 910, 916, 917, 918, 948, 949, 954, 956, 959, 965, 971, 974, 975, 980, 983, 986, 990, 992, 1008, 1047, 1053, 1054, 1056, 1101, 1102, 1103, 1105, 1107, 1109, 1112, 1113, 1115, 1119, 1120, 1127, 1128, 1131, 1166, 1172, 1174, 1176, 1177, 1178, 1181, 1184, 1186]

Appendix D

Ranks and Sizes of Orbits of Types in Degree 30

D.1 Degree 30

Due to the size of the output files, we will only give a summary of the ranks and sizes of orbits and which groups corresponded to such orbits. We now summarize the orbit information for groups of order up to 23,040 = $2^9 \cdot 3^2 \cdot 5$.

D.1.1 Orbits of Size s and Rank r

The following groups have an orbit of size 2 and rank 2 : [1, 5, 12, 14, 17, 21, 26, 30, 40, 51, 60, 68, 77, 81, 87, 92, 127, 139, 152, 168, 172, 175, 177, 180, 189, 193, 228, 260, 263, 269, 278, 283, 292, 294, 301, 387, 411, 415, 434, 441, 445, 446, 450, 544, 550, 556, 566, 584, 586, 599, 609, 630, 715, 722, 723, 734, 764, 774, 787, 789, 914, 916, 921, 922]

The following groups have an orbit of size 6 and rank 4 : [1, 5, 12, 14, 17, 21, 26, 40, 51, 68, 77, 81, 87, 127, 139, 152, 168, 177, 180, 189, 228, 263, 269, 278, 283, 411, 415, 434, 441, 584, 586, 599, 609, 764, 774, 787, 789]

The following groups have an orbit of size 8 and rank 4 : [18, 55, 61, 65, 107, 109, 121, 148, 213, 235, 243, 256, 268, 374, 375, 382, 396, 399, 404, 407, 412, 437, 542, 573, 583, 597, 598, 608, 611, 617, 620, 763, 767, 768, 786, 790, 791, 793, 796, 803, 807]

The following groups have an orbit of size 10 and rank 6 : [1, 5, 12, 14, 17, 21, 26, 30, 51, 60, 87, 92, 168, 175, 177, 180, 193, 263, 292, 294, 301, 387, 445, 446, 450, 544, 550, 556, 630, 715, 722, 723, 734, 914, 916, 921, 922]

The following groups have an orbit of size 12 and rank 7 : [30, 60, 172, 260]

The following groups have an orbit of size 20 and rank 6 : [17, 26, 30, 51, 60, 87, 92, 168, 175, 177, 180, 263, 445, 446, 630, 722, 723, 734, 914, 916, 921, 922]

The following groups have an orbit of size 20 and rank 11 : [30, 60, 172, 260]

The following groups have an orbit of size 24 and rank 12 : [29, 60]

The following groups have an orbit of size 30 and rank 11 : [172, 260]

The following groups have an orbit of size 30 and rank 12 : [1, 5, 14, 17, 26, 30, 60, 87, 180]

The following groups have an orbit of size 30 and rank 14 : [1, 5, 12, 14, 21]

The following groups have an orbit of size 30 and rank 16 : [1, 5, 12, 14, 17, 21, 26, 30, 40, 51, 60, 68, 77, 81, 87, 92, 127, 139, 152, 168, 172, 175, 177, 180, 189, 193, 228, 260, 263, 269, 278, 283, 292, 294, 301, 387, 411, 415, 434, 441, 445, 446, 450, 544, 550, 556, 566, 584, 586, 599, 609, 630, 715, 722, 723, 734, 764, 774, 787, 789, 914, 916, 921, 922]

The following groups have an orbit of size 32 and rank 6 : [105, 200, 201, 218, 322, 323, 354, 356, 495, 517, 568, 581, 744, 748, 751, 758, 812, 940]

The following groups have an orbit of size 32 and rank 16 : [108, 212, 325, 355, 506, 569, 742, 757, 953]

The following groups have an orbit of size 36 and rank 12 : [92, 175]

The following groups have an orbit of size 40 and rank 11 : [30, 60]

The following groups have an orbit of size 40 and rank 12 : [29, 58]

The following groups have an orbit of size 40 and rank 16 : [18, 29, 55, 58, 60, 61, 65, 107, 109, 121, 172, 213, 260, 268, 396, 399, 404, 573]

The following groups have an orbit of size 48 and rank 12 : [58]

The following groups have an orbit of size 50 and rank 14 : [40, 68, 77, 81, 127, 139, 152, 228]

The following groups have an orbit of size 60 and rank 11 : [30, 60]

The following groups have an orbit of size 60 and rank 12 : [5, 12, 14, 17, 21, 26, 30, 51, 60, 87, 168, 177, 180, 263]

The following groups have an orbit of size 60 and rank 14 : [5, 12, 14, 17, 21, 26, 51]

The following groups have an orbit of size 60 and rank 16 : [5, 12, 14, 17, 21, 26, 30, 51, 60, 77, 81, 87, 92, 127, 139, 152, 168, 172, 175, 177, 180, 228, 260, 263, 269, 278, 294, 301, 411, 415, 441, 445, 446, 450, 544, 550, 586, 599, 609, 630, 715, 723, 734, 764, 774, 787, 789, 922]

The following groups have an orbit of size 64 and rank 16 : [497, 506, 688, 950, 952]

The following groups have an orbit of size 70 and rank 16 : [566]

The following groups have an orbit of size 72 and rank 16 : [172]

The following groups have an orbit of size 80 and rank 16 : [58, 60, 107, 121, 179, 213, 261, 268, 396, 399, 404, 573]

The following groups have an orbit of size 84 and rank 16 : [566]

The following groups have an orbit of size 90 and rank 12 : [92, 175]

The following groups have an orbit of size 90 and rank 16 : [92, 172, 175, 193, 260, 292, 294, 301, 387, 450, 544, 550, 556, 715]

The following groups have an orbit of size 96 and rank 16 : [105, 108, 200, 201, 212, 218, 322, 323, 325, 356, 495, 568, 569, 744, 748, 751, 757, 940]

The following groups have an orbit of size 100 and rank 14 : [68, 77, 127, 139, 152, 228]

The following groups have an orbit of size 120 and rank 11 : [172, 260]

The following groups have an orbit of size 120 and rank 12 : [17, 26, 51, 60, 87, 168, 177, 180, 263]

The following groups have an orbit of size 120 and rank 14 : [17, 21, 26, 51, 87, 168, 177, 180, 263]

The following groups have an orbit of size 120 and rank 16 : [17, 18, 21, 26, 29, 30, 51, 55, 58, 60, 61, 65, 87, 92, 107, 109, 121, 148, 168, 172, 175, 177, 180, 213, 228, 235, 243, 256, 260, 263, 268, 374, 375, 382, 396, 399, 404, 407, 412, 437, 445, 446, 542, 573, 583, 597, 598, 608, 609, 611, 617, 620, 630, 722, 723, 734, 763, 767, 768, 786, 790, 791, 793, 796, 803, 807, 914, 916, 921, 922]

The following groups have an orbit of size 128 and rank 16 : [326, 327, 493, 500, 502, 513, 679, 680, 696, 701, 704, 705, 888, 893, 942, 955]

The following groups have an orbit of size 140 and rank 16 : [566]

The following groups have an orbit of size 144 and rank 16 : [179, 260]

The following groups have an orbit of size 150 and rank 14 : [40, 68, 77, 81, 127]

The following groups have an orbit of size 150 and rank 16 : [40, 68, 77, 81, 127, 139, 152, 189, 228, 269, 278, 283, 411, 415, 434, 441, 584, 586, 599, 609, 764, 774, 787, 789]

The following groups have an orbit of size 160 and rank 16 : [108, 212, 325, 355, 506, 569, 757]

The following groups have an orbit of size 162 and rank 12 : [193, 292, 294, 301, 445, 446, 450, 630, 722, 914, 916, 921]

The following groups have an orbit of size 180 and rank 12 : [87, 92, 168, 177, 180, 263]

The following groups have an orbit of size 180 and rank 14 : [87, 168, 177, 180, 263]

The following groups have an orbit of size 180 and rank 16 : [87, 92, 168, 172, 175, 177, 180, 260, 263, 294, 301, 445, 446, 450, 544, 550, 630, 715, 722, 723, 734, 914, 916, 921, 922]

The following groups have an orbit of size 192 and rank 16 : [354, 517, 742, 952]

The following groups have an orbit of size 200 and rank 14 : [139, 152, 228]

The following groups have an orbit of size 200 and rank 16 : [148, 235, 243, 256, 374, 375, 382, 542]

The following groups have an orbit of size 210 and rank 16 : [566]

The following groups have an orbit of size 240 and rank 14 : [51, 177, 180, 263]

The following groups have an orbit of size 240 and rank 16 : [51, 55, 58, 60, 61, 65, 107, 109, 121, 168, 172, 175, 177, 179, 213, 260, 261, 263, 268, 375, 382, 396, 399, 404, 542, 566, 573, 786, 793, 796, 803, 807]

The following groups have an orbit of size 250 and rank 14 : [189, 269, 278, 283, 411, 415, 434, 441, 584, 586, 599, 609, 764, 774, 787, 789]

The following groups have an orbit of size 252 and rank 16 : [566]

The following groups have an orbit of size 270 and rank 16 : [193, 292, 294, 301, 387, 450, 544, 550, 556, 715]

The following groups have an orbit of size 288 and rank 16 : [261]

The following groups have an orbit of size 300 and rank 14 : [68, 77, 127, 139, 152, 228]

The following groups have an orbit of size 300 and rank 16 : [68, 77, 81, 127, 139, 152, 228, 269, 278, 411, 415, 441, 586, 599, 609, 764, 774, 787, 789]

The following groups have an orbit of size 320 and rank 16 : [325, 354, 355, 497, 506, 517, 569, 688, 757]

The following groups have an orbit of size 324 and rank 12 : [301, 445, 450, 630, 916, 921]

The following groups have an orbit of size 360 and rank 12 : [175]

The following groups have an orbit of size 360 and rank 14 : [87, 168, 177, 180, 263]

The following groups have an orbit of size 360 and rank 16 : [87, 92, 168, 172, 175, 177, 180, 260, 263, 445, 446, 630, 722, 723, 734, 914, 916, 921, 922]

The following groups have an orbit of size 384 and rank 16 : [335, 344, 517, 525, 690, 950, 953]

The following groups have an orbit of size 400 and rank 16 : [235, 256, 374, 375, 382, 542]

The following groups have an orbit of size 420 and rank 16 : [566]

The following groups have an orbit of size 480 and rank 16 : [105, 107, 108, 109, 121, 200, 201, 212, 213, 218, 261, 268, 322, 323, 325, 354, 355, 356, 396, 399, 404, 495, 506, 517, 568, 569, 573, 581, 742, 744, 748, 751, 757, 758, 812, 940, 953]

The following groups have an orbit of size 486 and rank 12 : [387, 544, 550, 556, 715, 723, 734, 922]

The following groups have an orbit of size 500 and rank 14 : [415, 441, 609, 764, 774]

The following groups have an orbit of size 512 and rank 16 : [677, 682, 889, 891, 902, 903]

The following groups have an orbit of size 540 and rank 16 : [294, 301, 445, 446, 450, 544, 550, 630, 715, 722, 723, 734, 914, 916, 921, 922]

The following groups have an orbit of size 576 and rank 16 : [581, 758]

The following groups have an orbit of size 600 and rank 14 : [139, 152, 228]

The following groups have an orbit of size 600 and rank 16 : [127, 139, 148, 152, 228, 235, 243, 256, 374, 375, 382, 407, 412, 437, 542, 583, 597, 598, 608, 609, 611, 617, 620, 763, 767, 768, 786, 789, 790, 791, 793, 796, 803, 807]

The following groups have an orbit of size 630 and rank 16 : [566]

The following groups have an orbit of size 640 and rank 16 : [327, 335, 344, 497, 498, 506, 513, 517, 524, 525, 688, 689, 696, 701, 705, 742, 893, 942, 950, 952, 953]

The following groups have an orbit of size 720 and rank 14 : [168, 180]

The following groups have an orbit of size 720 and rank 16 : [168, 172, 175, 177, 179, 180, 260, 261, 263, 268, 396, 399, 404, 573]

The following groups have an orbit of size 750 and rank 14 : [189, 269, 278, 283, 411, 434, 584, 586, 599, 787]

The following groups have an orbit of size 750 and rank 16 : [189, 269, 278, 283, 411, 415, 434, 441, 584, 586, 599, 609, 764, 774, 787, 789]

The following groups have an orbit of size 768 and rank 16 : [498, 518, 524, 687, 689, 898]

The following groups have an orbit of size 800 and rank 16 : [375, 382, 542]

The following groups have an orbit of size 810 and rank 16 : [193, 292, 294, 301, 387, 445, 446, 450, 544, 550, 556, 630, 715, 722, 723, 734, 914, 916, 921, 922]

The following groups have an orbit of size 840 and rank 16 : [566]

The following groups have an orbit of size 960 and rank 16 : [200, 201, 212, 213, 218, 322, 323, 325, 354, 355, 356, 396, 399, 404, 495, 497, 506, 517, 568, 569, 573, 581, 688, 744, 748, 751, 757, 758, 940, 950, 952]

The following groups have an orbit of size 1000 and rank 14 : [789]

The following groups have an orbit of size 1000 and rank 16 : [407, 412, 437, 608, 611, 617, 620, 763, 767, 768, 786, 790, 791, 793, 796, 803, 807]

The following groups have an orbit of size 1080 and rank 16 : [445, 446, 583, 597, 598, 630, 722, 723, 734, 914, 916, 921, 922]

The following groups have an orbit of size 1200 and rank 16 : [228, 235, 243, 256, 374, 375, 382, 542, 583, 598, 608, 617, 767, 786, 790, 793, 796, 803, 807]

The following groups have an orbit of size 1260 and rank 16 : [566]

The following groups have an orbit of size 1280 and rank 16 : [498, 518, 525, 687, 689, 690, 696, 701, 705, 753, 893, 898, 941, 942, 943, 945]

The following groups have an orbit of size 1440 and rank 16 : [260, 261, 263, 268, 396, 399, 404, 569, 573, 581, 742, 757, 758, 812, 953]

The following groups have an orbit of size 1500 and rank 14 : [415, 441, 609, 764, 774]

The following groups have an orbit of size 1500 and rank 16 : [269, 278, 411, 415, 441, 586, 599, 609, 764, 774, 787, 789]

The following groups have an orbit of size 1536 and rank 16 : [677, 682, 889, 891, 902, 903]

The following groups have an orbit of size 1620 and rank 16 : [292, 294, 301, 445, 446, 450, 544, 550, 630, 715, 722, 723, 734, 914, 916, 921, 922]

The following groups have an orbit of size 1680 and rank 16 : [566]

The following groups have an orbit of size 1920 and rank 16 : [322, 323, 325, 326, 327, 335, 344, 354, 355, 356, 493, 495, 497, 498, 500, 502, 506, 513, 517, 524, 525, 568, 569, 581, 679, 680, 688, 689, 690, 696, 701, 704, 705, 742, 744, 748, 751, 757, 758, 888, 893, 940, 942, 953, 955]

The following groups have an orbit of size 2000 and rank 16 : [786, 793, 796, 803, 807]

The following groups have an orbit of size 2304 and rank 16 : [753, 943]

The following groups have an orbit of size 2400 and rank 16 : [374, 375, 382, 542]

The following groups have an orbit of size 2430 and rank 16 : [387, 544, 550, 556, 715, 723, 734, 922]

The following groups have an orbit of size 2520 and rank 16 : [566]

The following groups have an orbit of size 2560 and rank 16 : [682, 687, 690, 889, 891, 898, 902]

The following groups have an orbit of size 2592 and rank 16 : [812]

The following groups have an orbit of size 2880 and rank 16 : [396, 399, 404, 568, 569, 573, 581, 742, 744, 748, 751, 757, 758, 940, 950, 952]

The following groups have an orbit of size 3000 and rank 16 : [407, 412, 437, 583, 597, 598, 608, 609, 611, 617, 620, 763, 767, 768, 786, 789, 790, 791, 807]

The following groups have an orbit of size 3240 and rank 16 : [445, 446, 630, 722, 723, 734, 914, 916, 921, 922]

The following groups have an orbit of size 3840 and rank 16 : [493, 495, 497, 498, 500, 502, 506, 513, 517, 518, 524, 525, 679, 680, 687, 688, 689, 690, 696, 701, 704, 705, 744, 748, 753, 758, 888, 893, 898, 940, 941, 943, 945, 950, 955]

The following groups have an orbit of size 4320 and rank 16 : [812]

The following groups have an orbit of size 4608 and rank 16 : [941, 945]

The following groups have an orbit of size 4800 and rank 16 : [542]

The following groups have an orbit of size 4860 and rank 16 : [722, 723, 734, 914, 916, 921, 922]

The following groups have an orbit of size 5040 and rank 16 : [566]

The following groups have an orbit of size 5760 and rank 16 : [568, 569, 581, 742, 744, 748, 751, 757, 758, 940, 950, 952]

The following groups have an orbit of size 6000 and rank 16 : [583, 598, 608, 617, 767, 786, 790, 793, 796, 803, 807]

The following groups have an orbit of size 7680 and rank 16 : [677, 679, 680, 682, 687, 690, 696, 701, 705, 888, 889, 891, 893, 898, 902, 903, 941, 942, 945, 955]

The following groups have an orbit of size 11520 and rank 16 : [753, 941, 942, 943, 945, 950, 952, 955]

The following groups have an orbit of size 12960 and rank 16 : [812]

D.1.2 Galois Groups According to G_0 and v

We now present all Galois groups of CM fields in degree 30 with their order and value of v, organized according to their image groups G_0:

Table D.1: Galois Groups of CM Fields in Degree 30

$T_{15,1}$ occurs as an image group for 12 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
1	30	1
18	120	3
105	480	5
108	480	5
326	1920	7
327	1920	7
677	7680	9
682	7680	9
1077	30720	11
1078	30720	11
1472	122880	13
1836	491520	15

$T_{15,2}$ occurs as an image group for 8 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
14	60	1
61	240	3
201	960	5
502	3840	7
889	15360	9
1308	61440	11
1656	245760	13
2016	983040	15

$T_{15,3}$ occurs as an image group for 12 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
5	60	1
55	240	3
212	960	5
218	960	5
493	3840	7
513	3840	7
891	15360	9
903	15360	9
1273	61440	11
1307	61440	11
1665	245760	13
2015	983040	15

$T_{15,4}$ occurs as an image group for 8 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
12	60	1
65	240	3
200	960	5
500	3840	7
902	15360	9
1285	61440	11
1657	245760	13
2023	983040	15

$T_{15,5}$ occurs as an image group for 20 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
29	120	1
30	120	1
335	1920	5
344	1920	5

$T_{15,5}$ occurs as an image group for 20 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
354	1920	5
355	1920	5
497	3840	6
518	3840	6
525	3840	6
687	7680	7
705	7680	7
1082	30720	9
1099	30720	9
1291	61440	10
1292	61440	10
1305	61440	10
1475	122880	11
1481	122880	11
1488	122880	11
2240	1966080	15

$T_{15,6}$ occurs as an image group for 12 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
17	120	1
121	480	3
323	1920	5
325	1920	5
680	7680	7
701	7680	7
1089	30720	9
1098	30720	9
1474	122880	11
1503	122880	11

$T_{15,6}$ occurs as an image group for 12 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
1845	491520	13
2244	1966080	15

$T_{15,7}$ occurs as an image group for 8 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
21	120	1
109	480	3
356	1920	5
704	7680	7
1097	30720	9
1473	122880	11
1846	491520	13
2237	1966080	15

$T_{15,8}$ occurs as an image group for 8 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
26	120	1
107	480	3
322	1920	5
679	7680	7
1076	30720	9
1477	122880	11
1842	491520	13
2235	1966080	15

$T_{15,9}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
40	150	1
148	600	3
1883	614400	13
2303	2457600	15

$T_{15,10}$ occurs as an image group for 22 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
58	240	1
60	240	1
498	3840	5
506	3840	5
517	3840	5
524	3840	5
688	7680	6
689	7680	6
690	7680	6
696	7680	6
893	15360	7
898	15360	7
1277	61440	9
1303	61440	9
1476	122880	10
1484	122880	10
1491	122880	10
1493	122880	10
1659	245760	11
1663	245760	11
1668	245760	11
2525	3932160	15

$T_{15,11}$ occurs as an image group for 8 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
51	240	1
213	960	3
495	3840	5
888	15360	7

$T_{15,11}$ occurs as an image group for 8 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
1282	61440	9
1655	245760	11
2014	983040	13
2526	3932160	15

$T_{15,12}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
68	300	1
235	1200	3
2065	1228800	13
2587	4915200	15

$T_{15,13}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
81	300	1
243	1200	3
2064	1228800	13
2593	4915200	15

$T_{15,14}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
77	300	1
256	1200	3
2067	1228800	13
2595	4915200	15

$T_{15,15}$ occurs as an image group for 8 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
92	360	1
569	5760	5
581	5760	5

$T_{15,15}$ occurs as an image group for 8 degree 30 groups G

GAP index of G	$\lvert G \rvert$	Value of v
942	23040	7
1428	92160	9
1748	368640	11
1750	368640	11
2663	5898240	15

$T_{15,16}$ occurs as an image group for 8 degree 30 groups G

GAP index of G	$\lvert G \rvert$	Value of v
87	360	1
268	1440	3
568	5760	5
955	23040	7
1426	92160	9
1733	368640	11
2129	1474560	13
2667	5898240	15

$T_{15,17}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\lvert G \rvert$	Value of v
139	600	1
375	2400	3
2311	2457600	13
2961	9830400	15

$T_{15,18}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\lvert G \rvert$	Value of v
127	600	1
374	2400	3
2308	2457600	13
2962	9830400	15

$T_{15,19}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
152	600	1
382	2400	3
2302	2457600	13
2960	9830400	15

$T_{15,20}$ occurs as an image group for 11 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
172	720	1
179	720	1
742	11520	5
753	11520	5
943	23040	6
950	23040	6
1730	368640	10
1735	368640	10
1908	737280	11
1918	737280	11
3019	11796480	15

$T_{15,21}$ occurs as an image group for 8 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
175	720	1
757	11520	5
758	11520	5
1190	46080	7
1609	184320	9
1923	737280	11
1925	737280	11
3024	11796480	15

$T_{15,22}$ occurs as an image group for 8 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
168	720	1
396	2880	3
748	11520	5
1191	46080	7
1598	184320	9
1914	737280	11
2374	2949120	13
3024	11796480	15

$T_{15,23}$ occurs as an image group for 8 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
177	720	1
404	2880	3
744	11520	5
1197	46080	7
1608	184320	9
1911	737280	11
2380	2949120	13
3018	11796480	15

$T_{15,24}$ occurs as an image group for 8 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
180	720	1
399	2880	3
751	11520	5
1196	46080	7
1600	184320	9
1926	737280	11
2375	2949120	13
3026	11796480	15

$T_{15,25}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
189	750	1
437	3000	3
2489	3072000	13
3261	12288000	15

$T_{15,26}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
193	810	1
812	12960	5
1987	829440	11
3278	13271040	15

$T_{15,27}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
228	1200	1
542	4800	3
2585	4915200	13
3363	19660800	15

$T_{15,28}$ occurs as an image group for 15 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
260	1440	1
261	1440	1
941	23040	5
945	23040	5
952	23040	5
953	23040	5
1188	46080	6
1189	46080	6
1913	737280	10

$T_{15,28}$ occurs as an image group for 15 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
1922	737280	10
1924	737280	10
1928	737280	10
2126	1474560	11
2127	1474560	11
3397	23592960	15

$T_{15,29}$ occurs as an image group for 8 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
263	1440	1
573	5760	3
940	23040	5
1422	92160	7
1751	368640	9
2122	1474560	11
2664	5898240	13
3398	23592960	15

$T_{15,30}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
283	1500	1
620	6000	3
2876	6144000	13
3659	24576000	15

$T_{15,31}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
278	1500	1
617	6000	3
2871	6144000	13
3665	24576000	15

$T_{15,32}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
269	1500	1
583	6000	3
2864	6144000	13
3687	24576000	15

$T_{15,33}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
292	1620	1
1002	25920	5
2215	1658880	11
3693	26542080	15

$T_{15,34}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
294	1620	1
1000	25920	5
2219	1658880	11
3699	26542080	15

$T_{15,35}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
301	1620	1
1008	25920	5
2220	1658880	11
3696	26542080	15

$T_{15,36}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
387	2430	1
1143	38880	5
2325	2488320	11
3804	39813120	15

$T_{15,37}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
441	3000	1
786	12000	3
3244	12288000	13
4022	49152000	15

$T_{15,38}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
415	3000	1
793	12000	3
3254	12288000	13
4024	49152000	15

$T_{15,39}$ occurs as an image group for 8 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
407	3000	1
412	3000	1
434	3000	1
763	12000	3
3238	12288000	13
3241	12288000	13
3253	12288000	13
3994	49152000	15

$T_{15,40}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
411	3000	1
790	12000	3
3255	12288000	13
4026	49152000	15

$T_{15,41}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
446	3240	1
1229	51840	5
2511	3317760	11
4042	53084160	15

$T_{15,42}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
445	3240	1
1231	51840	5
2512	3317760	11
4045	53084160	15

$T_{15,43}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
450	3240	1
1233	51840	5
2507	3317760	11
4039	53084160	15

$T_{15,44}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
556	4860	1
1332	77760	5
2600	4976640	11
4135	79626240	15

$T_{15,45}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
550	4860	1
1359	77760	5
2605	4976640	11
4142	79626240	15

$T_{15,46}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
544	4860	1
1353	77760	5
2602	4976640	11
4145	79626240	15

$T_{15,47}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
566	5040	1
1370	80640	5
2609	5160960	11
4150	82575360	15

$T_{15,48}$ occurs as an image group for 6 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
586	6000	1
598	6000	1
978	24000	3
3673	24576000	13
3684	24576000	13
4307	98304000	15

$T_{15,49}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
609	6000	1
966	24000	3
3654	24576000	13
4288	98304000	15

$T_{15,50}$ occurs as an image group for 8 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
584	6000	1
597	6000	1
611	6000	1
962	24000	3
3657	24576000	13
3677	24576000	13
3683	24576000	13
4306	98304000	15

$T_{15,51}$ occurs as an image group for 6 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
599	6000	1
608	6000	1
967	24000	3
3666	24576000	13
3678	24576000	13
4310	98304000	15

$T_{15,52}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
630	6480	1
1449	103680	5
2890	6635520	11
4316	106168320	15

$T_{15,53}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
722	9720	1
1543	155520	5
2968	9953280	11
4405	159252480	15

$T_{15,54}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
734	9720	1
1552	155520	5
2971	9953280	11
4414	159252480	15

$T_{15,55}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
715	9720	1
1535	155520	5
2967	9953280	11
4397	159252480	15

$T_{15,56}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
723	9720	1
1555	155520	5
2980	9953280	11
4389	159252480	15

$T_{15,57}$ occurs as an image group for 8 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
768	12000	1
789	12000	1
791	12000	1
1201	48000	3
4016	49152000	13
4017	49152000	13
4019	49152000	13
4505	196608000	15

$T_{15,58}$ occurs as an image group for 6 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
774	12000	1
807	12000	1
1210	48000	3
4013	49152000	13
4027	49152000	13
4498	196608000	15

$T_{15,59}$ occurs as an image group for 8 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
764	12000	1
796	12000	1
803	12000	1
1214	48000	3
3993	49152000	13
4000	49152000	13
4007	49152000	13
4491	196608000	15

$T_{15,60}$ occurs as an image group for 6 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
767	12000	1
787	12000	1
1225	48000	3
3998	49152000	13
3999	49152000	13
4503	196608000	15

$T_{15,61}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
921	19440	1
1714	311040	5

$T_{15,61}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
3370	19906560	11
4637	318504960	15

$T_{15,62}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
916	19440	1
1709	311040	5
3369	19906560	11
4641	318504960	15

$T_{15,63}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
914	19440	1
1713	311040	5
3366	19906560	11
4638	318504960	15

$T_{15,64}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
922	19440	1
1707	311040	5
3367	19906560	11
4635	318504960	15

$T_{15,65}$ occurs as an image group for 6 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
984	24000	1
985	24000	1
1442	96000	3
4292	98304000	13
4294	98304000	13
4697	393216000	15

$T_{15,66}$ occurs as an image group for 6 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
965	24000	1
994	24000	1
1447	96000	3
4283	98304000	13
4309	98304000	13
4690	393216000	15

$T_{15,67}$ occurs as an image group for 8 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
972	24000	1
973	24000	1
980	24000	1
1436	96000	3
4286	98304000	13
4289	98304000	13
4305	98304000	13
4688	393216000	15

$T_{15,68}$ occurs as an image group for 6 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
969	24000	1
995	24000	1
1435	96000	3
4295	98304000	13
4303	98304000	13
4695	393216000	15

$T_{15,69}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
1017	29160	1
1818	466560	5
3717	29859840	11
4718	477757440	15

$T_{15,70}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
1132	38880	1
1887	622080	5
3813	39813120	11
4824	637009920	15

$T_{15,71}$ occurs as an image group for 14 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
1116	38880	1
1118	38880	1
1119	38880	1
1124	38880	1
1126	38880	1
1142	38880	1
1885	622080	5
3781	39813120	11
3785	39813120	11
3788	39813120	11
3792	39813120	11
3794	39813120	11
3818	39813120	11
4826	637009920	15

$T_{15,72}$ occurs as an image group for 5 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
1154	40320	1
1893	645120	5
1894	645120	5
3819	41287680	11
4831	660602880	15

$T_{15,73}$ occurs as an image group for 6 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
1209	48000	1
1213	48000	1
1620	192000	3
4495	196608000	13
4512	196608000	13
4861	786432000	15

$T_{15,74}$ occurs as an image group for 6 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
1200	48000	1
1226	48000	1
1622	192000	3
4508	196608000	13
4517	196608000	13
4859	786432000	15

$T_{15,75}$ occurs as an image group for 8 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
1203	48000	1
1217	48000	1
1221	48000	1
1619	192000	3
4489	196608000	13

$T_{15,75}$ occurs as an image group for 8 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
4490	196608000	13
4504	196608000	13
4860	786432000	15

$T_{15,76}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
1238	58320	1
1994	933120	5
4051	59719680	11
4890	955514880	15

$T_{15,77}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
1253	58320	1
1997	933120	5
4053	59719680	11
4874	955514880	15

$T_{15,78}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
1236	58320	1
1991	933120	5
4057	59719680	11
4886	955514880	15

$T_{15,79}$ occurs as an image group for 8 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
1349	77760	1
1360	77760	1
1368	77760	1

$T_{15,79}$ occurs as an image group for 8 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
2078	1244160	5
4111	79626240	11
4118	79626240	11
4122	79626240	11
4966	1274019840	15

$T_{15,80}$ occurs as an image group for 8 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
1338	77760	1
1354	77760	1
1358	77760	1
2076	1244160	5
4128	79626240	11
4138	79626240	11
4147	79626240	11
4962	1274019840	15

$T_{15,81}$ occurs as an image group for 14 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
1329	77760	1
1330	77760	1
1331	77760	1
1337	77760	1
1351	77760	1
1361	77760	1
2075	1244160	5
4106	79626240	11
4107	79626240	11
4112	79626240	11

$T_{15,81}$ occurs as an image group for 14 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
4121	79626240	11
4124	79626240	11
4140	79626240	11
4964	1274019840	15

$T_{15,82}$ occurs as an image group for 6 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
1437	96000	1
1440	96000	1
1792	384000	3
4689	393216000	13
4696	393216000	13
4986	1572864000	15

$T_{15,83}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
1458	116640	1
2224	1866240	5
4333	119439360	11
5014	1911029760	15

$T_{15,84}$ occurs as an image group for 6 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
1533	155520	1
1556	155520	1
2323	2488320	5
4403	159252480	11
4410	159252480	11
5053	2548039680	15

$T_{15,85}$ occurs as an image group for 6 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
1542	155520	1
1553	155520	1
2320	2488320	5
4402	159252480	11
4412	159252480	11
5055	2548039680	15

$T_{15,86}$ occurs as an image group for 8 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
1534	155520	1
1547	155520	1
1548	155520	1
2314	2488320	5
4390	159252480	11
4391	159252480	11
4409	159252480	11
5054	2548039680	15

$T_{15,87}$ occurs as an image group for 6 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
1706	311040	1
1711	311040	1
2601	4976640	5
4634	318504960	11
4639	318504960	11
5140	5096079360	15

$T_{15,88}$ occurs as an image group for 6 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
1812	466560	1
1835	466560	1

$T_{15,88}$ occurs as an image group for 6 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
2902	7464960	5
4711	477757440	11
4717	477757440	11
5199	7644119040	15

$T_{15,89}$ occurs as an image group for 6 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
1992	933120	1
2005	933120	1
3304	14929920	5
4883	955514880	11
4887	955514880	11
5264	15288238080	15

$T_{15,90}$ occurs as an image group for 6 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
2003	933120	1
2007	933120	1
3299	14929920	5
4875	955514880	11
4885	955514880	11
5263	15288238080	15

$T_{15,91}$ occurs as an image group for 6 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
1998	933120	1
2008	933120	1
3297	14929920	5
4884	955514880	11
4893	955514880	11
5265	15288238080	15

$T_{15,92}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
2092	1296000	1
2623	5184000	3
5142	5308416000	13
5297	21233664000	15

$T_{15,93}$ occurs as an image group for 6 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
2225	1866240	1
2226	1866240	1
3714	29859840	5
5015	1911029760	11
5016	1911029760	11
5322	30576476160	15

$T_{15,94}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
2328	2592000	1
2997	10368000	3
5234	10616832000	13
5348	42467328000	15

$T_{15,95}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
2329	2592000	1
3002	10368000	3
5226	10616832000	13
5331	42467328000	15

$T_{15,96}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
2330	2592000	1
2992	10368000	3
5227	10616832000	13
5338	42467328000	15

$T_{15,97}$ occurs as an image group for 4 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
2631	5184000	1
3381	20736000	3
5280	21233664000	13
5377	84934656000	15

$T_{15,98}$ occurs as an image group for 8 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
2616	5184000	1
2618	5184000	1
2629	5184000	1
3384	20736000	3
5286	21233664000	13
5287	21233664000	13
5289	21233664000	13
5373	84934656000	15

$T_{15,99}$ occurs as an image group for 6 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
2987	10368000	1
3000	10368000	1
3827	41472000	3
5342	42467328000	13
5344	42467328000	13
5398	169869312000	15

$T_{15,100}$ occurs as an image group for 6 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
2988	10368000	1
2995	10368000	1
3824	41472000	3
5333	42467328000	13
5345	42467328000	13
5402	169869312000	15

$T_{15,101}$ occurs as an image group for 8 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
2983	10368000	1
2992	10368000	1
2999	10368000	1
3822	41472000	3
5330	42467328000	13
5339	42467328000	13
5341	42467328000	13
5399	169869312000	15

$T_{15,102}$ occurs as an image group for 6 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
3382	20736000	1
3388	20736000	1
4151	82944000	3
5371	84934656000	13
5372	84934656000	13
5420	339738624000	15

$T_{15,103}$ occurs as an image group for 2 degree 30 groups G

GAP index of G	$\|G\|$	Value of v
5468	1307674368000	1
5694	21424936845312000	15

$T_{15,104}$ occurs as an image group for 2 degree 30 groups G

GAP index of G	$\lvert G \rvert$	Value of v
5498	2615348736000	1
5696	42849873690624000	15

Curriculum Vitae

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Citizenship: United States.

Research interests:

Computational Algebra, Number Theory, Cryptography

Education:

Lehigh University — Bethlehem, PA
August 2013-May 2017
Ph.D., Mathematics, May 2017.
Dissertation *Galois Groups of CM Fields in Degrees 24, 28, and 30* supervised by Bruce Dodson.

Lehigh University — Bethlehem, PA
August 20011-May 2013
M.S., Mathematics, May 2013.

University of Scranton — Scranton, PA
August 2007-May 2011
B.S., Mathematics, May 2011

Professional history:

LEHIGH UNIVERSITY Bethlehem, PA

January 2015-December 2015

Precalculus & Introduction to Mathematical Thought. Responsible for first semester precalculus and stand alone math course for graduation requirement. Prepared and presented lectures, wrote and graded exams and quizzes.

Math 05	Introduction to Mathematical Thought	Spring 2015
Math 00	Preparation for Calculus	Fall 2015

LEHIGH UNIVERSITY Bethlehem, PA

August 2011-May 2017

Teaching Assistant. Teaching assistant for various levels of calculus, including multivariate, ran recitation sessions once a week for four sections per semester, graded papers/homework and exams, held office hours.

UNIVERSITY OF SCRANTON Scranton, PA

August 2008-May 2011

Tutor and Supplemental Instructor. Peer tutor for various math courses, from calculus to discrete structures. As a supplemental instructor, held review sessions for Pre-Calculus and Calculus I designed for a specific class and developed practice quizzes and worksheets.

JOHNS HOPKINS UNIVERSITY CENTER FOR TALENTED YOUTH

Baltimore, MA

Summer 2015

Instructor for 2 sessions of Cryptology. Prepared and presented lectures and designed worksheets, group activities, and projects. Met with 15 students for about 100 hours a week for 3 weeks.

Grants and awards:

LEHIGH UNIVERSITY Bethlehem, PA

Summer 2016

CAS Summer Research Fellowship.

Presentations:

Invited Talks

Mathematics Seminar,
University of Scranton September 2016

The $\Delta \|x\|$ Mathematics and Computer Science Lecture Series,
DeSales University September 2016

Graduate Student Intercollegiate Mathematics Seminar (GSIMS),
Lehigh University April 2016

GSIMS, Lehigh University November 2014

GSIMS, Lehigh University January 2014

Contributed Talks

Joint Mathematics Meetings, Atlanta January 2017

Other activities:

- Fall 2014 - Spring 2017: Member, Graduate Student Mentorship Program
- 2012 - 2015, 2017: Volunteer, Lehigh University High School Math Contest
- Fall 2014 - Fall 2016: Volunteer, Lehigh Calculus Committee scheduling meetings
- Summer 2013 - Summer 2016: Speaker, orientation for new math department graduate students
- Summer 2015: Leader, orientation for new math department graduate students