

2016

# A DEA-Based Approach to Evaluate the Efficiency of Non-Homogeneous Service Locations

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**A DEA-Based Approach to Evaluate the Efficiency of Non-Homogeneous Service Locations**

by

Amer H. Asiri

A Thesis

Presented to the Graduate and Research Committee of Lehigh University

in Candidacy for the Degree of

Master of Science

in

Industrial and Systems Engineering

Lehigh University

May, 2016

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Amer H. Asiri

Thesis is accepted and approved in partial fulfillment of the requirement for the Master of Science in Industrial and Systems Engineering

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## **Acknowledgements**

The author expresses his gratitude to Prof. Tamás Terlaky for taking the time to review this study. Also, I gratefully acknowledge the helpful comments and valuable suggestions of my thesis advisor, Prof. George Wilson. In addition, many thanks to my family and friends for their patience and support.

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## **Abstract**

This study aims at evaluating the performance of a company, ‘XYZ Company’, that has 115 service locations. Because of its ability of handling large numbers of inputs and outputs, and removing the need of predefining the factors’ weights, Data Envelopment Analysis (DEA) is used. DEA is benchmark tool that measures the efficiency of entities with respect to each other by assessing their performance of utilizing inputs to produce outputs. Researchers have developed several DEA models, all of which have different characteristics.

A main assumption of DEA is that the entities are homogeneous – i.e. operating under similar conditions, which is not applicable sometimes. Thus, various approaches have been introduced to relax the homogeneity assumption. In this study, we propose an approach that estimates the efficiency over some stages, obtains efficiency scores from each stage, and then calculates the final weighted score by assigning a higher weight to the stage that represents the actual conditions of the entity more clearly.

We apply three DEA models, utilizing the proposed approach to overcome the entities’ heterogeneity, to the data set of XYZ Company. Then, we compare the results of the three models, analyze the efficiency scores of the 115 service locations, and provide some major findings.

# 1. Introduction

An international company which we refer to in this paper as “XYZ Company” has customers all over the world. In the United States, there are tens of thousands customers who use its products on a daily basis. It doesn't only sell the products but also provides post-sale services.

Consequently, it has high demand of maintenance orders which could be for repairs or spare parts installation. XYZ Company's products are mainly different kinds of machines. Each machine belongs to a platform and contains several parts. The number of machines exceeds 200 which results in huge number of spare parts to be available upon customers' requests. In order to store all these parts, different service locations are needed. XYZ Company managed to locate these service locations such that logistics and services are optimized. Currently, there are 115 service locations servicing over 7447 cities in 50 states.

The process starts with signing a contract with the new customer, and then assigning that customer to a service location, most likely the nearest one. However, various factors are taken into account in order to do the assignment. For example, the contentious availability of spare parts based on how many customers are using the same kind of machine in that area. Another factor is the availability of experienced technicians since some machines require a higher level of skill, unlike some other machines, which could be repaired by any technician. A key consideration is the Part Delivery Time (PDT), which classifies the contracts into three types. The service timeframes for the three types, “High Priority”, “Normal Priority”, and “Low Priority”, are 2, 4 and 12-hour service delivery windows, respectively. Therefore, some service locations are assigned to all types of contracts, while others are only assigned to 2, or even 1 type.

XYZ Company is seeking an efficiency tool to measure the performance of each service location, evaluate the effect of the three service performance PDT types and conduct an internal benchmark analysis. One objective of the study is to find out how revenue could be increased. In other words, what are the cost reduction techniques that are implemented in some service locations and can possibly be also done in the others? Another objective is to investigate the relationship between the service locations' efficiency and the assigned PDT types at a particular location. In some locations, technicians need to travel for hours to fulfill a service order, while it is a matter of few steps in other locations. Such a difference could definitely impact the service level and associated costs.

In this study, we implement Data Envelopment Analysis (DEA) using real data provided by XYZ Company to achieve the mentioned objectives. DEA is selected from among a number of other efficiency tools because of its capability of dealing with both homogeneous and non-homogeneous entities, which are entities operate under either similar or different conditions. Moreover, DEA can deal with a large number of variables, and come up with weights for performance factors.

## 2. Literature Review

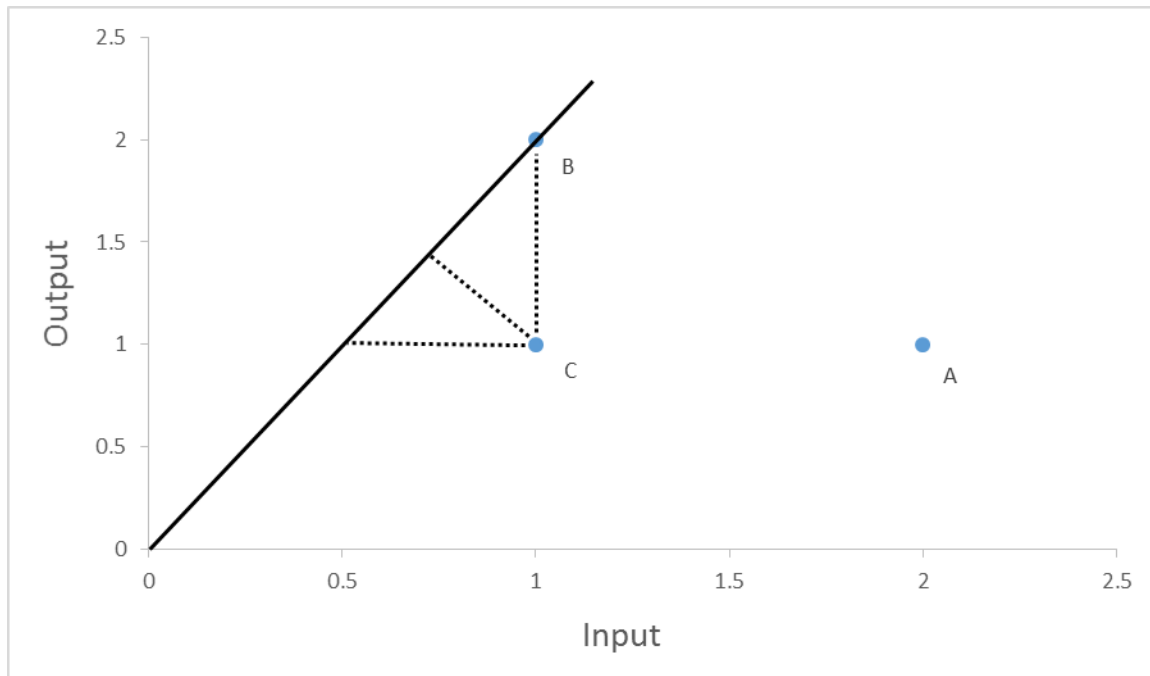
### 2.1. DEA

In 1978, Charnes et al (Charnes et al., 1978) introduced a measurement tool known as Data Envelopment Analysis (DEA). This tool is capable of measuring 'decision making efficiency'. They defined the entities of DEA as Decision Making Units (DMUs) which are the center of interest in the analysis. DMU is any unit or processor that consumes certain amount of resources

to produce some outcomes. Those resources and outcomes are the inputs and outputs of the model, respectively.

DMUs are evaluated with respect to each other. Hence, DEA gives a relative efficiency score, but not an absolute one. More specifically, DMUs are compared with the ‘best’ DMU, and that is what distinguishes DEA from other statistical measures which compare entities with the ‘average’ one.

Simply put, if a given DMU, “DMU1”, can produce a certain amount of output, “output1”, utilizing a certain amount of input, “input1”, then any DMU that has the same amount of “input1” should be capable of producing the same amount of “output1”. If a DMU is operating “better” than all other DMUs, the model defines it as an efficient DMU with a relative efficiency score of 100%. Therefore, those efficient DMUs form the Efficiency Frontier which envelops all DMUs. In other words, all data are either touched or contained by the frontier. The score of an inefficient DMU is represented by the distance from the DMU to the frontier. Figure 1 shows an example of an efficiency frontier that touches entity “B” which makes it efficient, and contains the other inefficient entities “A” and “C”. It also shows ways of improving the efficiency of entity “C” by reducing the distance between the entity and the frontier.



*Figure 1: Efficiency Frontier*

Each DMU is given an efficiency score between zero and unity based on the ratio of its outputs to its inputs. So, a DMU is given a score of less than unity if there is a linear combination of other DMUs that can produce more outputs with a similar quantity of inputs, or produce a similar quantity of outputs with less inputs.

DEA is able to process large numbers of inputs and outputs since it is based on a mathematical programming formulation. Also, it is a non-parametric method which removes the need of specifying the weights before estimating the frontier. Moreover, it is capable of dealing with multiple inputs and outputs that have different units of measure.

The first DEA model, CCR (Charnes et al., 1978), has some assumptions that limit the use of the method in some real cases. Since its introduction, many researchers have studied DEA to relax these assumptions and introduce extended models. Some of the assumptions are mentioned and discussed in this paper, such as model orientation, non-discretionary factors, return to scale,

types of efficiencies, and homogeneity (Banker et al., 1985; Cooper et al., 2007; Sean et al., 2005; Gregoriou et al., 2007; Haas et al., 2003; Hua, 2006; Ruggiero, 1996).

## 2.2. Model Orientation

In general, the efficiency can be declared as the ratio of produced outcomes to utilized resources. In DEA, it is referred to as the output\input ratio. Traditionally, there are two ways to increase the efficiency; input reduction or output augmentation. Simply put, either the cost of resources should be reduced while maintaining, at least, the same level of production or the outcomes of production should be increased while utilizing, at most, the same level of resources. So, the model that is formulated to reduce costs is called input-oriented model, while the one formulated to increase outcomes is called output-oriented model (Cook et al., 2013; Cooper et al., 2007; Cooper et al., 2004; Dyson et al., 2001; Gomes et al., 2012). However, it may happen that reducing inputs or increasing outputs is insufficient to make a DMU efficient, which motivated introducing a new direction of DEA modeling in which inputs get minimized and outputs get maximized, simultaneously. Cooper et al. (Cooper et al., 2007) mention Charnes-Cooper-Rhodes (CCR) and Banker-Charnes-Cooper (BCC) as examples of input-oriented and output-oriented models and The Additive Model and The Slacks-Based Measure (SBM) as examples of combined input\output-oriented models. These models will be defined and explained in DEA Models section.

It is worth mentioning that empirical results show that the efficiency scores differ according to the choice of the model orientation, except under the assumption of Constant Return to Scale(CRS), as Meza et al. mention (Meza et al., 2002). Therefore, the choice should be made carefully and reasonably. Here are some tips for choosing:

- Control of change over inputs and outputs (Banker et al., 1999): some inputs could be exogenous and uncontrollable or some outputs could be limited to a certain level by policies.
- Ease of change: in some cases, changing either inputs or outputs is possible but dealing with one of them is much easier than dealing with the other one because of, for example, the length of the change process or the reaction of the affected people.

As a bottom line, it is the management's call to decide which direction to go, considering all conditions and consequences.

### 2.3. Non-Discretionary

Identifying the variables of the model should be done carefully. The model should include all possible inputs and outputs. Doing so would increase the possibility of having more accurate and representative results. However, this may lead to include some factors that cannot be varied, i.e. increased or decreased, since they are out of the DMUs' control. In the literature, these factors are referred to as 'Non-Discretionary' factors, or as Banker et al (Banker et al., 1985) first called them, "exogenously fixed factors". Ignoring such factors or not considering them as non-discretionary factors result in infeasible values and misleading efficiency scores. Examples may include the budget allocated to a department by the corporation, the population around a market, or targeted level of production. Therefore, non-discretionary factors are to be involved in assessing the efficiency but not in the improvement actions taken after the analysis because it is impossible for a department to go beyond its budget, a market to change the population or a manufacturer to exceed the production limit (Cooper et al., 2007; Hua et al., 2007).



Dyson et al (Dyson et al., 2001) summarize the techniques to enable accommodating non-discretionary factors in DEA models:

Performing an output-oriented model, while all exogenous factors are in the inputs side.

Performing an input-oriented model, while all exogenous factors are in the outputs side.

Using DEA models that are extended and formulated to include exogenous factors. See (Cooper et al., 2007) for examples such as Non-controllable Variable Model (NCN) and Non-discretionary Variable Model (NDSC).

## 2.4. Return to Scale

Returns, at DMU's, are of different kinds based on size, all of which have their own characteristics and can be found by different models. Data could be analyzed for returns to scale, or, a priori information could be used. One of the returns types is Constant Returns to Scale (CRS), which, as an alternative, could instead be either Increasing Returns to Scale (IRS) or Decreasing Returns to Scale (DRS). CRS implies a proportional relationship between inputs and outputs. Simply put, if an activity belongs to the Production Possibility Set (PPS) which is the set of all inputs and outputs, then any activity multiplied by a positive scalar should belong as well. On the other hand, Variable Returns to Scale (VRS) doesn't assume proportionality and exhibits convexity, instead. VRS was developed by Banker, et al (Banker et al., 1985), to take in scale effects in analysis. Basically, VRS allows both IRS and DRS to be combined. In general, efficiency scores resulting from CRS are lower than those resulting from VRS, since the frontier that exhibit VRS envelops the data more tightly (Cooper et al., 2007; Dyson et al., 2001; Gomes et al., 2012; Gregoriou et al., 2007; Martić et al., 2009).

## 2.5. Types of Efficiencies

In DEA, efficiency measures can be classified into two types; radial measures and non-radial measures. The radial measures are based on an assumption that all inputs or outputs are changed (increased or decreased) in which their proportion remains the same. On the other hand, non-radial measures not only increase or decrease inputs or outputs but also may alter the mix of them in order to reach efficiency. Two examples of radial measures are overall technical efficiency and pure technical efficiency. The difference between them is that the overall technical efficiency combines both technical efficiency and scale efficiency, whereas the scale size is not involved in the pure technical efficiency. So, technical efficiency could bring the associated DMU to the frontier but doesn't necessarily make it efficient because of the inefficient proportion of its inputs or outputs. Hence, the importance of the non-radial measure (Cooper et al., 2007; Hua, 2006; Koltai et al., 2015; Martić et al., 2009).

## 2.6. DEA Model Types

Over the years, many DEA models have been developed. Many of them are extensions of the basic ones to relax some assumptions and address associated issues. Each model has specific characteristics and might share some similarities with other models. Hence, selecting a model to use depends on characteristics of the data and the objective of the analysis. Here, we shed light on the most popular basic models and provide a brief comparison.

### 2.6.1. CCR

The first DEA model, CCR, was introduced by Charnes, et al (Charnes et al., 1978) in 1978. It was proposed as an efficiency measurement tool that evaluates the overall technical efficiency of a DMU. A DMU is CCR-efficient if it has an efficiency score of 1 and has no slacks, which

means no excess inputs or outputs shortfall. However, CCR includes only the radial efficiency and ignores the impact of non-radial efficiency. Also, CCR assumes a Constant Return to Scale and requires semi-positive variables which means at least one element of every input and output is positive while the others are non-negative (Cooper et al., 2007).

### **2.6.2. BCC**

To relax the assumption of Constant Return to Scale in CCR, Baker (Banker, 1984) introduced BCC model which assumes Variable Return to Scale, instead. The relaxation was achieved by imposing a new constraint into the formulation to impose a convexity condition. So, CCR efficiency is always less than or equal to BCC efficiency. BCC is similar to CCR in most characteristics yet it measures only pure technical efficiency. In other words, it neglects the impact of both scale size and non-radial efficiency (Cooper et al., 2007).

### **2.6.3. The Additive Model**

Charnes, et al (Charnes et al., 1985) introduced another model, The Additive Model, that involves the slacks in the objective function and measures the mix efficiency, i.e. considers the non-radial efficiency. Another feature of this model is that it has the ability to combine both input-oriented and output-oriented models and analyze the data considering both goals. However, it lacks the scalar measure that provides the efficiency score (Cooper et al., 2007).

### **2.6.4. SBM**

In addition to dealing with the slacks directly in the objective function, SBM model give an efficiency score, unlike The Additive Model. SBM was proposed by Tone (Tone, 2001) and is also known as Enhanced Russell Measure. It measures the mix efficiency and its efficiency score can be interpreted as the product of input and output inefficiencies. Furthermore, it has the ability

to deal with variables measured in different units which is called Unit Invariance (Cooper et al., 2007).

In Table 1, the main characteristics of the above models are summarized:

Model		CCR	BCC-I	BCC-O	Additive	SBM
Data	X	Semi-positive*	Semi-positive*	Free**	Free**	Semi-positive*
	Y	Free**	Free**	Semi-positive*	Free**	Free**
Efficiency Score Range		[0-1]	[0-1]	[0-1]	NA	[0-1]
Type of Efficiency		Overall Technical	Pure Technical	Pure Technical	Mix	Mix
Return To Scale		CRS	VRS	VRS	Both	Both

*Table 1: DEA Models Comparison*

*\*Semi-positive: at least on element of every input and output is positive*

*\*\*Free: free in sign; positive, negative or zero*

## 2.7. Non-Homogeneity

DMU's evaluated by DEA are fundamentally assumed to be homogeneous (Charnes et al., 1978). Homogeneous units should, typically, operate under similar conditions. Researchers state different ways to explain the homogeneity conditions. For example, Soteriou, et al (Soteriou et al., 1999) mention having similar size and operating in similar business. However, most

publications refer to what Dyson, et al (Dyson et al., 2001) suggest as homogeneity conditions.

These conditions could be summarized as follows:

- Performing the same activities and engaging in the same process. Using common technology can be included here, as well.
- Utilizing similar resources (considered as inputs) and producing similar outcomes (considered as outputs).
- Operating in common environments, which enables the model to minimize the impact of external factors (Soteriou et al., 1999).

However, homogeneity conditions might not exist in many real life cases. DMUs could have different sizes, operate under different environmental conditions, or lack some inputs or outputs. Comparing heterogeneous entities without considering the differences may result in inaccurate conclusions. Therefore, extending DEA to relax the assumption of homogeneity has been a rich research area recently. Researchers have discussed this topic from different perspectives depending on the reason for heterogeneity. For instance, Sarrico (Sarrico, 1998) applied an approach which seeks external comparators in the first stage and compare the standing of the DMUs in the second stage. Athanassopoulos, et al (Athanassopoulos et al., 1995) clustered the DMUs into homogeneous entities and then performed a typical DEA. Another approach was proposed by Sexton, et al (Sexton et al., 1994) to integrate DEA with a regression model which adjusts the outputs according to the various conditions of the DMUs. However, these approaches do not take into account the case of heterogeneity that is caused by having different sets of inputs or outputs. To address this issue, Cook, et al (Cook et al., 2012) proposed a 3-step process to measure the efficiency. In the first step, they classify the outputs into subgroups in which one of the subgroups contains the common outputs, and then the inputs are split among the outputs

subgroups. Then, a standard DEA analysis is performed only for the DMUs that have common outputs. In the last step, DEA analysis is performed again only for the DMUs which have extra outputs. For DMUs that have been analyzed twice, the final efficiency score is calculated using weighted average methods. To illustrate the approach, let's say we have three DMUs; DMU#A, DMU#B and DMU#C. All three DMUs produce four common outputs; set1 {Y1, Y2, Y3, Y4}. Only DMU#A and DMU#B produce two more outputs; set2 {Y5, Y6}. The approach is performed as follows:

For DMU#A and DMU#B, a portion of the inputs is assigned to set1 and the other portion to set2. On the other hand, all inputs of DMU#C are assigned to set1 since it doesn't produce set2.

DEA analysis is carried out to evaluate all DMUs over set1 only, the common outputs.

Another DEA analysis is carried out to evaluate only DMU#A and DMU#B over set2, the extra outputs.

At the end, the efficiency score of DMU#C is the score measured in step 2, while a weighted average method is used to calculate the scores of DMU#A and DMU#B using the scores measured in step 2 and step 3.

Based on the Cook, et al (Cook et al., 2012), we claim, in this paper, that the more outputs we involve in each step the more accurate result we get. Also, more weight and credit should be given to the step that involves more outputs. Our proposal will be provided and discussed in the Methodology section.

## 3. Methodology

In this section, we introduce and discuss our proposal and explain the configuration of the model and the characteristics of the analysis we are applying.

### 3.1. Proposed Approach

There are several factors that increase the accuracy of the efficiency score, two of which are including all possible variables and evaluating all DMUs together. Considering both factors at the same time is not completely achieved by the methods proposed in the publications that have been discussed. What usually happens is one of two approaches:

- Grouping DMUs into homogeneous subgroups, and then applying a conventional DEA analysis for each group separately
- Comparing DMUs considering only the common outputs they all have.

Cook, et al (Cook et al., 2012) applied the second approach through multiple steps. The first step compares all DMUs considering the common outputs while the second step compares the DMUs that have additional outputs considering only the outputs which aren't considered in the first step. We claim that in each step all possible common outputs should be included because this represents the performance of the DMUs more realistically. Another way to look at it is that the method proposed by Cook, et al (Cook et al., 2012) is based on splitting the inputs and considering the portions separately. However, there are some cases where splitting an input impacts the effect of having the whole amount altogether. For example, let's assume a factory has 6 workers who produce 3 products. We may split them into 2 groups; 4 workers produce 2 products and 2 workers produce 1 product. However, the two groups may not be able to accomplish production as efficiently as if all workers work together. Reasons could be human

factors, management issues, or experience lacking. Hence, what we are pointing to is that we should reduce the effect of splitting as much as we can by involving all possible outputs with their associated inputs in each step. So, we modify the approach as follows:

Let's say we have three DMUs; DMU#A, DMU#B and DMU#C. All three DMUs produce four common outputs; set1 {Y1, Y2, Y3, Y4}. Only DMU#A and DMU#B produce two more outputs; set2 {Y5, Y6}. The modified steps are:

- For DMU#A and DMU#B, a portion of the inputs is assigned to set1 and the other portion to set2. On the other hand, all inputs of DMU#C are assigned to set1 since it doesn't produce set2.
- DEA analysis is carried out to evaluate all DMUs over set1 only, the common outputs.
- Another DEA analysis is carried out to evaluate only DMU#A and DMU#B over set1 and set2, all common outputs, without excluding set1.

In symbols,

$$E_A = \frac{set1 \{Y1+Y2+Y3+Y4\} + set2 \{Y5+Y6\}}{X}$$

$$E_B = \frac{set1 \{Y1+Y2+Y3+Y4\} + set2 \{Y5+Y6\}}{X}$$

$$E_C = \frac{set1 \{Y1+Y2+Y3+Y4\}}{X}$$

- A portion of DMU#A and DMU#B inputs is assigned to produce set1 of their outputs, denoted as  $X_{set1}$ .
- DEA analysis for:

$$E_{A1} = \frac{set1 \{Y1+Y2+Y3+Y4\}}{X_{set1}}, E_{B1} = \frac{set1 \{Y1+Y2+Y3+Y4\}}{X_{set1}}, E_C = \frac{set1 \{Y1+Y2+Y3+Y4\}}{X}$$

- DEA analysis for:



$$E_{A2} = \frac{\text{set1} \{Y1+Y2+Y3+Y4\} + \text{set2} \{Y5+Y6\}}{X_{\text{set1}} + X_{\text{set2}}}, E_{B2} = \frac{\text{set1} \{Y1+Y2+Y3+Y4\} + \text{set2} \{Y5+Y6\}}{X_{\text{set1}} + X_{\text{set2}}}.$$

Note that the final efficiency score of DMU#C is the score calculated in step 2 ( $E_C$ ). However, we have two scores for DMU#A, ( $E_{A1}$ ) and ( $E_{A2}$ ), and two score for DMU#B, ( $E_{B1}$ ) and ( $E_{B2}$ ).

To calculate the final score for them, we use the Weighted Average method to assign weights to the scores they obtain in each step. We suggest the following approach for determining the weights:

The weight of a score obtained in step i:

$$W_i = \frac{\text{number of outputs included in step } i}{\text{number of outputs in all steps}}$$

This approach gives more weight to the step that includes more outputs.

So, the weights for the two steps of the above example:

$$W_1 = \frac{\text{number of outputs included in step 1}}{\text{number of outputs in all steps}} = \frac{4}{10} = .4$$

$$W_2 = \frac{\text{number of outputs included in step 2}}{\text{number of outputs in all steps}} = \frac{6}{10} = .6$$

And the final score will be:

$$E_f = \frac{\sum_i (W_i * E_i)}{\sum_i W_i}$$

Therefore, the final efficiency scores for DMU#A and DMU#B are:

$$E_{Af} = \frac{W_1 * E_{A1} + W_2 * E_{A2}}{W_1 + W_2}, E_{Bf} = \frac{W_1 * E_{B1} + W_2 * E_{B2}}{W_1 + W_2}$$

In short, the method we propose aims at involving as many variables as possible in each step, and then gives a higher weight to the step that considers more outputs.

To compare both methods and investigate how the results could differ, we applied the proposed method in this study to the data provided in Cook's study (Cook et al., 2012). The problem was to examine the efficiency of 32 fabrication plants. All plants have 4 common inputs. Regarding outputs, 20 plants, forming the set  $\{N_1\}$ , produce 4 outputs while 12 plants, forming the set  $\{N_2\}$ , produce only 3 outputs. The 3 common outputs comprise the set  $\{R_1\}$ , and the additional output forms the singleton set  $\{R_2\}$ . Cook, et al (Cook et al., 2012) did the following steps:

- 1) The inputs of  $N_1$  were split between the production of  $R_1$  and  $R_2$ . No split was required for  $N_2$  because they don't produce  $R_2$ .
- 2) They carried out a DEA analysis for all 32 plants with respect to  $R_1$ . The scores of  $N_2$  derived from this step were final.
- 3) They carried another DEA analysis for  $N_1$  with respect to  $R_2$ .
- 4)  $N_1$  plants have two efficiency scores derived from step 2 and 3. So, they used some multipliers to come up with weighted scores.

Now, the modified method is applied to the same data with the following steps:

- 1) The inputs of  $N_1$  are split between the production of  $R_1$  and  $R_2$ . No split is required for  $N_2$  because they don't produce  $R_2$ .
- 2) We carry out a DEA analysis for all 32 plants with respect to  $R_1$ . The scores of  $N_2$  derived from this step are final.

- 3) We carry another DEA analysis for  $N_1$  with respect to both  $R_1$  and  $R_2$  with all associated inputs.
- 4)  $N_1$  plants have two efficiency scores derived from step 2 and 3. So, we use a weighted average method that gives more weight to the score derived from the step that involves more outputs. In this case, the weight of scores obtained in step 2 is  $(\frac{3}{7}) = 0.429$ , and the weight of scores obtained in step 3 is  $(\frac{4}{7}) = 0.571$ .

Table 2 shows the final results of both the original and the modified methods:

DMU	Efficiency Score - Original Method	Efficiency Score - Modified Method
1	0.589426	0.702066314
2	0.651497	0.887596375
3	0.60023	0.626344611
4	0.634821	0.887049201
5	0.593688	0.692476623
6	0.64081	0.680558189
7	0.952717	1
8	0.782894	0.86499406
9	0.842819	0.907116573
10	0.839719	0.981876333
11	0.756237	0.906615835
12	0.684909	0.99495232
13	0.689408	0.775522061
14	0.662163	0.841740498
15	0.948293	0.968196509
16	0.88325	1
17	0.886883	1
18	0.86166	0.904345679
19	1	1
20	1	1
21	1	1
22	1	1
23	0.719863401	0.719863401
24	0.942626355	0.942626355
25	0.977759344	0.977759344
26	0.719732313	0.719732313
27	0.671795203	0.671795203
28	0.590300401	0.590300401
29	0.747786	0.747786
30	0.509065645	0.509065645
31	0.423697156	0.423697156
32	1	1

*Table 2: Efficiency Scores of The Original and The Modified Methods*

Clearly, the modification, we propose, makes some differences. In order to conclude which method is more appropriate, systematic guidance is needed. Previous literature has not provided this, as mentioned by Schaar, et al (Schaar et al., 2008). However, we point out some observations based on our comparison study:

- The modified method tends to give slightly higher scores. As a result, it increases the confidence of identifying the inefficiencies. Sherman, et al (Sherman et al., 2006) considered this tendency of understating the inefficiencies as the nature of DEA which makes it a powerful tool that managers can use with confidence. In other words, it is more capable of determining the losers. However, it is not an absolute advantage because it may result in considering all units as efficient units. But, this is not the case with our method since the average percentage of increase in the scores is only 15%.
- Using the original method, the average score of both  $N_1$  and  $N_2$  is 77.5%. However, using the modified method, the average scores for  $N_1$  and  $N_2$  are 88.1% and 77.5%, respectively. This difference shows that the modified method is more able to discriminate between the performance of the plants that produce 4 outputs and those that produce only 3 outputs.
- The scores of 3 plants, [2, 4, and 12], have significantly increased. By reviewing the method steps, we may assume that the analysis that evaluates these plants with respect to  $R_1$ , step 2, represents 75% of their total production since  $R_1$  includes 3 outputs out of 4. Therefore, their overall score should be very close to the one obtained in this step. The results of the modified method comply with this assumption very well.

### 3.2. Analysis Characteristics

The approach proposed earlier is applied to measure the performance of 115 service locations of XYZ Company. The primary management objective of the company is to increase the revenue and improve the service level, which are the outcomes analyzed by the approach. Hence, the model will be output-oriented. However, we will examine the model with the combined

orientation, input\output, to figure out if we can improve the situation by working on the inputs and the outputs at the same time.

One of the inputs of the model is the “Demand” which represents the frequency of ordering a specific part to be replaced in a specific machine. The company doesn’t have a policy to limit the number of service or maintenance orders, which takes the level of demand out of its control.

Therefore, in the model, we consider the “Demand” to be a non-discretionary factor.

Analyzing the data set shows that there is no proportional relationship between inputs and outputs. Simply put, increasing the service level requires more cost in general. On the other hand, spending more money decreases the revenue which is the other output that the company is trying to augment. So, a model that allows Variable Returns to Scale would fit the data very well.

As mentioned earlier, the customers have three options of contracts depending on the “Part Deliver Time”. The data we are using is categorized, accordingly. For example, we have three groups of revenue sources resulting from having three kinds of customers: “High Priority”, “Normal Priority” and “Low Priority”. This categorization is applied to all inputs and outputs. Knowing that some service locations lack some inputs and outputs since they are assigned to only one or two types of customers results in treating them as “non-homogenous” entities in this study.

Taking the characteristics, mentioned above, into account, the model type we implement should be output-oriented, or, we might choose a combined orientation model to analyze the results of manipulating both inputs and outputs. Moreover, the model should allow for Variable Returns to Scale. Hence, we will tackle the data utilizing three models; BCC-O, Non-oriented SBM, and SBM-O.

## 4. The Application

### 4.1. The Data

To conduct the benchmark analysis for the service locations of XYZ Company, we group them into three homogenous sets: locations that have all types of contracts, locations that have 2 types, and locations that have only one. Among the 115 service locations, 9 locations have all types of contracts, while 103 locations have 2, and 3 locations have only one type.

The model has 4 inputs and 2 outputs, all of which are classified based on the contract type. The first input is the “Demand” which is the annual level of maintenance orders requested by customers who are assigned to this location. Another input is “Holding Cost” of storing the spare parts. Another associated cost is “Replenishment Cost”, which is the cost of ordering spare parts to refill the inventory. Also, there is a “Transportation Cost” resulting from moving the parts from the service location to the customer’s site. On the output side, the “Revenue” represents the annual income of each service location. The other output is the annual “Service” level reached by the service location (i.e., the satisfied demand).

In Table 3, all inputs and outputs are shown with the classification based on the Part Delivery Time (PDT) for 2, 4 and 12-hour service delivery timeframes.

DMU	Inputs												Outputs					
	D			H			RP			T			RV			S		
	PDT			PDT			PDT			PDT			PDT					
	2	4	12	2	4	12	2	4	12	2	4	12	2	4	12	2	4	12
9 SLs - All TCs	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
103 SLs - Two TCs	X	X		X	X		X	X		X	X		X	X		X	X	
3 SLs - One TC	X			X			X			X			X			X		

*Table 3: Model's Inputs and Outputs*  
*D=Demand, H=Holding Cost, RP=Replenishment Cost, T=Transportation Cost*  
*RV=Revenue, S=Service*  
*SL=Service Location, TC=Type of Contract*

## 4.2. Results and Analysis

Three models are applied to the data mentioned above, BCC (output oriented), SBM (not oriented), and SBM (output-oriented). The results yielded some variations that is worth investigating. XYZ Company is more concerned about the service locations which are not performing well, since they provide the focus for improvement. Using DEA enables management to discover the inefficient locations and improve them by applying the best practices that are done by the efficient locations. To be more focused, we follow the 80-20 rule and point out the best 20% and the worst 20% of the service locations. The following tables, Table 4, Table 5, and



Table 6, shows those locations and their efficiency scores, utilizing the 3 DEA models mentioned, earlier.

BCC-The Worst 20%		BCC-The Best (all efficient)							
DMU	Efficiency Score	DMU	Efficiency Score	DMU	Efficiency Score	DMU	Efficiency Score	DMU	Efficiency Score
LOC89	0.897559736	LOC26	1	LOC80	1	LOC4	1	LOC91	1
LOC14	0.951697765	LOC27	1	LOC88	1	LOC41	1	LOC92	1
LOC15	0.972545064	LOC34	1	LOC97	1	LOC44	1	LOC93	1
LOC40	0.984709357	LOC48	1	LOC98	1	LOC45	1	LOC94	1
LOC114	0.987276775	LOC56	1	LOC103	1	LOC50	1	LOC104	1
LOC109	0.990059778	LOC57	1	LOC110	1	LOC53	1	LOC11	1
LOC99	0.990410414	LOC10	1	LOC1	1	LOC58	1	LOC64	1
LOC31	0.992253259	LOC101	1	LOC108	1	LOC62	1	LOC96	1
LOC47	0.995169082	LOC102	1	LOC111	1	LOC65	1	LOC36	1
LOC30	0.995169082	LOC105	1	LOC16	1	LOC67	1	LOC71	1
LOC90	0.995972211	LOC106	1	LOC17	1	LOC69	1	LOC75	1
LOC112	0.996394502	LOC107	1	LOC2	1	LOC74	1		
LOC12	0.997071889	LOC19	1	LOC20	1	LOC76	1		
LOC38	0.997493734	LOC42	1	LOC21	1	LOC78	1		
LOC13	0.998313304	LOC46	1	LOC22	1	LOC79	1		
LOC6	0.998607635	LOC49	1	LOC23	1	LOC8	1		
LOC43	0.998722861	LOC5	1	LOC24	1	LOC81	1		
LOC18	0.998883305	LOC51	1	LOC25	1	LOC82	1		
LOC77	0.999036645	LOC54	1	LOC29	1	LOC83	1		
LOC63	0.999061033	LOC59	1	LOC33	1	LOC85	1		
LOC66	0.999084507	LOC60	1	LOC35	1	LOC86	1		
LOC68	0.999101526	LOC7	1	LOC37	1	LOC87	1		
LOC113	0.999102495	LOC72	1	LOC39	1	LOC9	1		

Table 4: The Best and the Worst 20% Service Locations Using BCC

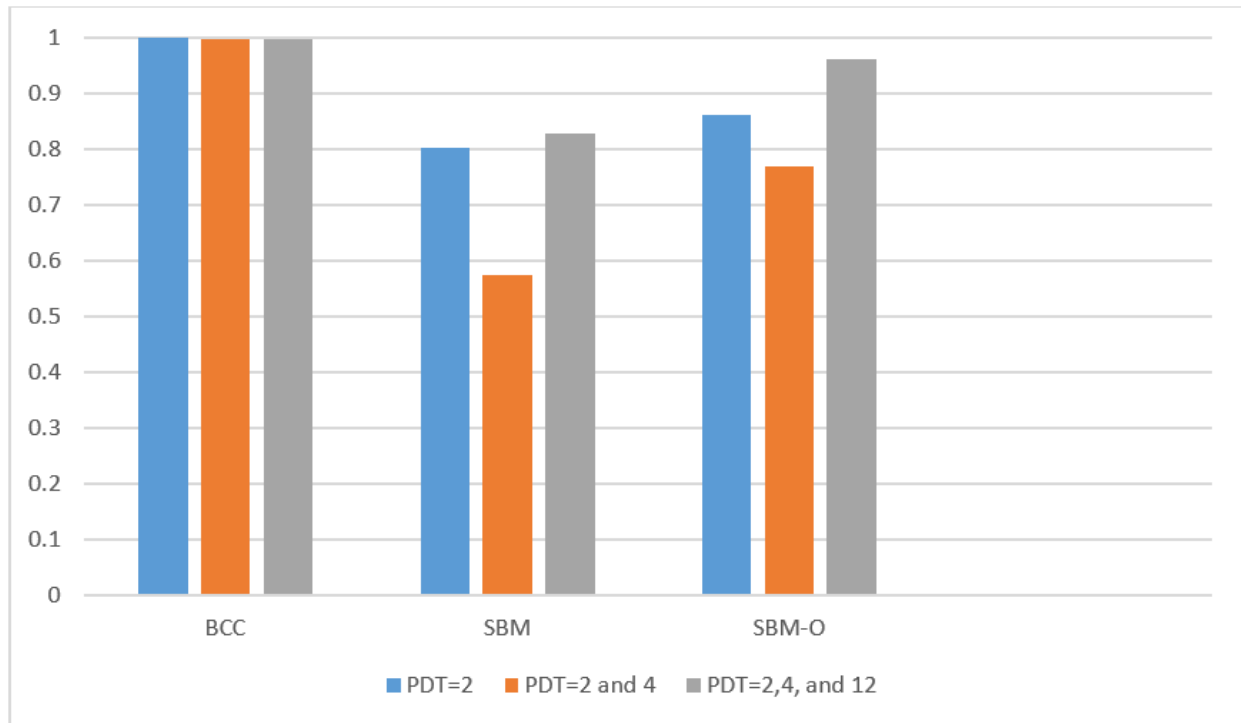
SBM-The Worst 20%		SBM-The Best 20%	
DMU	Efficiency Score	DMU	Efficiency Score
LOC13	0.166458987	LOC93	1
LOC70	0.236364605	LOC110	1
LOC63	0.26078329	LOC56	1
LOC43	0.26794472	LOC101	1
LOC52	0.281400653	LOC115	1
LOC46	0.305890194	LOC96	1
LOC19	0.313899366	LOC11	1
LOC82	0.317364151	LOC1	1
LOC35	0.31952687	LOC106	1
LOC32	0.334792416	LOC111	1
LOC45	0.336864915	LOC87	1
LOC77	0.345987649	LOC104	1
LOC6	0.354058392	LOC67	1
LOC9	0.35409101	LOC64	1
LOC58	0.355907985	LOC14	1
LOC68	0.368519422	LOC114	0.907419661
LOC51	0.370142041	LOC8	0.901312686
LOC34	0.377503304	LOC2	0.893036223
LOC62	0.379197217	LOC102	0.866970615
LOC79	0.381881063	LOC105	0.865693805
LOC31	0.385421684	LOC41	0.85058217
LOC47	0.38589744	LOC109	0.846384961
LOC4	0.386964591	LOC92	0.825368987

Table 5: The Best and the Worst 20% Service Locations Using SBM

The Worst 20%		The Best 20%	
DMU	Efficiency Score	DMU	Efficiency Score
LOC13	0.434630231	LOC110	1
LOC70	0.451052142	LOC96	1
LOC45	0.458512743	LOC1	1
LOC82	0.467506896	LOC101	1
LOC52	0.470175171	LOC106	1
LOC3	0.479842677	LOC111	1
LOC46	0.484459756	LOC115	1
LOC51	0.48547726	LOC14	1
LOC62	0.48708233	LOC56	1
LOC58	0.531806422	LOC67	1
LOC9	0.541392891	LOC87	1
LOC89	0.54598624	LOC93	1
LOC35	0.556927036	LOC104	1
LOC42	0.564769634	LOC11	1
LOC79	0.564804734	LOC64	1
LOC103	0.580631388	LOC109	0.989906911
LOC27	0.580861962	LOC114	0.983923553
LOC60	0.586786878	LOC29	0.981556993
LOC43	0.603192913	LOC2	0.980479172
LOC47	0.605945269	LOC113	0.977646
LOC53	0.616186082	LOC32	0.968825
LOC4	0.627989285	LOC92	0.967204669
LOC17	0.634609405	LOC55	0.962932007

Table 6: The Best and the Worst 20% Service Locations Using SBM-O

To show the different performances of the DEA models, we calculate the average score of the locations that have all types of contracts, 2 types, and only one type, and compare them in Figure 2.



*Figure 2: Average Scores of Three types of Service Locations*

In order to achieve the main objective of XYZ Company, which is to find out the service locations that are not performing well, we analyze Table 4, Table 5, Table 6, and Figure 2 and provide the following observations:

- ✓ BCC consider 80 locations out of 115 as efficient (See Table 4). Also, the scores of the inefficient locations are close 1. As Figure 2 shows, these results imply that BCC is not the most capable model to distinguish between the DMUs' efficiencies. A main reason for this pattern is that the scores, given by BCC, only represent the Pure Technical Efficiency. Thus, we may conclude that the inefficiency of these locations is caused by

their size of operations. In other word, the combination of inputs and outputs is not efficient.

- ✓ On average, the service locations that have 2 types of contract have the lowest scores. In addition, they make up the majority of the worst 20% locations and only 10% of them are efficient. On the other hand, they form more than 60% of the best 20% lists. This distinction indicates a high performance variation within the set of locations that have similar conditions.
- ✓ Based on the results of SBM and SBM-O, none of the service locations that have three types of contracts appears in the worst 20% lists. However, more than 50% of them are in the best 20% lists.
- ✓ Most of the service locations that have only one type of contracts appear in the best 20% lists, and considered as efficient.
- ✓ All service locations tend to have higher scores when evaluated from the output side. Thus, the management should give some attention to not only augmenting the outputs but also reducing the inputs.
- ✓ The same 15 service locations are considered as efficient by both SBM and SBM-O.
- ✓ For SBM and SBM-O, 65% of the worst locations on the 20% lists are similar, while 6 locations, (17, 27, 42, 53, 60, and 103), are not doing well from the outputs side.
- ✓ All models agree that service locations 13, 43, and 47 have very low efficiency scores. Therefore, these locations should be one the first locations to investigate.
- ✓ Although there are some similarities between the results of SBM and SBM-O, they vary, mainly, because of the different orientations of them. Simply put, SBM-O aims at

increasing the outputs while keeping, at most, the same level of inputs, whereas SBM manipulates both inputs and outputs, simultaneously, in order to, jointly, optimize both.

- ✓ By analyzing the weights of inputs and outputs, generated by the Linear Programming in the DEA, we notice that most of the best performers give more importance to “Transportation Cost”. In contrary, the worst performers give more importance to “Replenishment Cost”, which suggests that their “Transportation Cost” is relatively high. Therefore, optimizing the “Transportation Cost” is a critical factor of increasing the efficiency of the service locations.

Seeking a response regarding the reasonableness of the results, we have discussed them with the management of XYZ Company and we list their reactions:

- All main service locations are in the list of the best performers, which indicates that the company is performing well at the most important locations. In addition, it shows that the DEA analysis is, reasonably, representative.
- Many bad performers are located in the mid region of US, which makes sense to them since the management has noted a low service level at that area.
- Some results are “surprising”. The management think that some service locations are doing well, while their scores are low. Uncovering such hidden weaknesses is an advantage of DEA, because it involves several factors, unlike the management’s judgment, which, mainly, is based on the density of demand and service. However, the correctness of the efficiency studies is not, always, assured. There is a chance of deviation due to the inaccuracy of the data or the factors are correlated.

## 5. Conclusion

This study evaluates the efficiency of 115 service locations of XYZ Company utilizing DEA. The locations are non-homogenous as they operate under different conditions. Basically, they share a lot of factors, but some of them lack a few of the factors. To overcome that, a method of dealing with non-homogeneous DMUs is needed. We apply a method, proposed by Cook, et al (Cook et al., 2012), which groups the DMUs into homogeneous sets, and assesses them through multiple stages. However, we modify the method in which we keep the DMUs close to their actual set of inputs and outputs as much as we can. Then, in order to calculate the final efficiency score, we give more weight to the factors that represents the DMU the best. Even though the literature has not suggested clear guidance to compare DEA methods, we examine our approach using the data of Cook's study (Cook et al., 2012), and point out some observations.

To apply the proposed approach to the data of XYZ Company, we select three DEA models, BCC (output oriented), SBM, and SBM (output oriented). The selection relies on the characteristics of the data and the objective of management. The models yield quite different results because they have different orientations and focus on different kinds of efficiencies. However, the variation allows for insightful interpretations from different perspectives. For example, using both output-oriented and non-oriented models show that some locations are performing well in terms of producing outputs but their expenses and use of revenues are considerably higher than what would be desired.

The results show that the service locations that have all types of contracts perform better, on average. Furthermore, beside "Demand" and "Service", a primary factor of being efficient is to optimize the "Transportation Cost". On the other hand, some scores do not conform to the

perception of the management regarding some locations. We recommend that management conduct further investigations at those locations and consider all possible factors that might affect performance.

For future work, having guidance or a framework, which helps compare the DEA models, would be useful, indeed. Although data characteristics may direct the selection among of the models, the decision is somewhat subjective and, consequently, can be misleading. Moreover, for calculating the final efficiency score using the scores obtained through the stages, it would be helpful to develop a systematic weighting method that considers not only the number of outputs involved, but also the weights generated by the LP for them.

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# Vita

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