2013

Optimizing Wave Farm Layouts Under Uncertainty

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Optimizing Wave Farm Layouts Under Uncertainty

by

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Presented to the Graduate and Research Committee

of Lehigh University

in Candidacy for the Degree of

Master of Science

in

Industrial and Systems Engineering

Lehigh University

August 2013

This thesis is accepted and approved in partial fulfillment of the requirement for the master of science.

Date: August 2013

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Acknowledgments

Thanks for the instruction from Professor Snyder, members of the Lehigh University wave energy research group and help from friends in the Industrial and Systems Engineering department. Thanks for the encouragement from high school friends. I also want to thank my families for their constant support on all aspects.

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Abstract

Wave farms utilize wave energy converters (WEC) and related devices to generate electricity using ocean waves. Past research has shown that the layout of wave energy converters can have a dramatic impact on the total output of the wave farm, as evaluated by the q-factor. The q-factor expresses the efficiency of the mechanical power absorbed by the WECs, which can be used as an approximation for the electrical power produced by WECs, as a function of the locations of the WECs and their hydrodynamic properties. Past studies have proposed several procedures for optimizing wave farm layouts. However, the solutions obtained in previous research tend to degrade rapidly as the ocean state (wave heading direction and wave number) changes. This thesis presents a procedure to optimize the layout of a wave farm using a two-step genetic algorithm. The two-step genetic algorithm is introduced and tested. Furthermore, in order to improve the robustness of the solution, a preliminary study of wave farm layout under uncertainty is presented and computational results are discussed.

Chapter 1

Introduction

1.1 Wave Energy

Energy is one of the most important sources for social and economic development. The total U. S. energy consumption was 78.1 quadrillion BTU in 1980, 84.5 quadrillion BTU in 1990, 98.8 quadrillion BTU in 2000, 100.3 quadrillion BTU in 2005 and 97.7 quadrillion BTU in 2010 [7]. Although the energy consumption growth decreased in recent years due to the economic crisis [7], as the economy recovers and grows, more and more energy will be needed. The growing energy consumption results in the emission of by-products of using fossil fuels, which contributes greatly to global warming. As the effect of global warming get worse, more and more renewable and green energy is desired. Wave energy is one of the most promising renewable energy source for countries with rich ocean resources.

Wave power transfers the energy from sea surface waves to usable power, usually electricity, which is easy to transmit and utilize. Wave power technology is not mature and not commercially applied at present. The known first attempt to use wave power goes back to 1890 [18] and the first experimental wave farm was opened in

Portugal in 2008 [16].

A device for wave energy transformation is called wave energy converter (WEC). There are five categories of WECs in general. The first kind is known as an *attenuator*. This kind of converter is made of a series of floating sections. As waves pass, the sections will move up and down relative to each other. This makes the liquid (usually oil) within the sections flow and drives the electricity generator. One example of such a WEC is Pelamis [11]. The second kind of converter is a *point absorber*, for example the PowerBouy [12]. Usually, a point absorber consists of two parts. The two components move relative to each other as waves pass, which drives the electricity generator. The third kind of converter is called a *terminator*. One example of a terminator is the oscillating water column, such as WaveRoller [14]. The water enters a chamber and the wave motion drives the water column to move up and down. This forces the air go through the turbine that is used to generate electricity. The fourth kind is called an *overtopping device*, such as Water Dragon [13]. An overtopping device has containers to store water from waves and the level of the stored water is higher than the ocean surface. Then the water is released. The falling water will drive the electricity generator. The fifth kind is an *ocean thermal energy converter*. This kind of device uses the temperature difference between surface water and the water beneath to make water move and drive the electricity generator. However, this kind of WEC does not generate energy using the exciting forces of waves. The discussion in this thesis is mainly based on the study of transforming the exciting forces of the wave to electrical energy. We focus on point absorbers, but many of the contents discussed in this thesis can be adapted to other types of WECs.

Wave power can be viewed as a branch of hydropower. Hydropower is the practice of deriving power from falling or running water with corresponding devices, such as mills and dams. Another promising branch of hydropower is tidal power, which converts the energy of rising and falling tides into a useful form of power. Unlike wave power, tidal power is a more mature technology. Tidal power is more predictable than wind power, wave power and solar power. Although the cost of tidal power is high, new technology and research is addressing this problem.

1.2 Ocean Waves

Before discussing wave energy conversion in more detail, we first discuss how waves are modeled. Ocean waves are the movement of sea water driven by the wind over the water surface or earthquakes under the water. Ocean wave can be viewed as a sum of sine waves with their own frequencies, amplitudes and directions. Each component is described in terms of two parameters, wave heading direction β and wave number k. A wave with a single sine wave that has fixed and constant parameters is called a regular wave. Waves, in practice, are usually irregular waves that consist of multiple sine waves with their own parameters.

For the convenience of discussion, our attention will be restricted to regular waves in this thesis. For deterministic waves, this means that β and k are known. For stochastic waves, we assume that β and k follow a given probability distribution, such as normal or log-normal distribution. (Ocean waves are more commonly modeled using their spectral density, which gives a statistical description of the wave component frequencies. However, since we are considering only regular waves, there is only a single wave component, and we describe its parameters using common distributions for convenience.)

1.3 Literature Review

The study of wave energy conversion started in the 1970s. Several aspects of wave energy have been addressed in the literature.

Some researchers focus on the control of wave energy converters. Nolan et al. [20] have developed a semi-analytical solution methodology based on mathematical models to determine optimal damping profiles for a heaving WEC. Falcao [6] has applied a stochastic model to optimize the rotational speed control of an oscillating water column. Guang et al. [15] found that deterministic sea wave prediction combined with optimal constrained control can improve the efficiency of a WEC dramatically.

Some researchers focus on the characterization and forecast of the waves. For example, Gordon [21] has tested the ability of various time-series models to predict energy from sea waves.

Some researchers focus on energy storage and transmission. Li et al. [25] have simulated a novel hybrid power generation and energy storage system in both time-domain and frequency-domain. Tereke [1] has connected multiple WECs to a power distribution station to check if multiple WECs would stabilize the output and improve the integrity of the network.

The literature that is most relevant to this thesis is concerned with the layout of devices Evans [8] has introduced the concept of q-factor which is used to evaluate the layout. (See Section 1.4 for a more detailed explanation of the q-factor.) Evans [8] and Falnes [9] have formulated an expression for the absorbed power independently, which is used to approximate the q-factor. Those results became the foundation of later research. However, the calculation of absorbed power is of great difficulty, as we discuss in Section 1.4. An approximation is desired. A lot of research has been done on this aspect. For example, McIver [17] has presented the point absorber approximation. He also compared the point-absorber theory and the plane-wave theory. Fitzgerald and Thomas [10] have presented a numerical optimization procedure that can produce either symmetric or asymmetric layouts, with a particular focus on five-device problems under a small-body approximation. Rather than using approximate methods, some researchers have developed exact methods. For instance, Child and Venugopal [4] proposed an exact procedure to calculate the q-factor and two heuristics for optimizing the wave farm layouts to maximize the q-factor.

This thesis will focus on the wave farm layout problem.

1.4 q-factor

While several wave energy converters are located near one another in a wave farm, the devices do not generate electricity independently. They interact with each other in terms of generating new kinds of waves. When the *incident wave*, the wave that occurs due to a natural force, such as wind, hits the devices, two kinds of waves will occur. One of them is a *radiated wave.* A radiated wave happens when a device is hit by waves and begins to move up and down. The motion of the device produces the radiated wave. The other type of wave is a *scattered waves.* A scattered wave is the wave that the device reflects when the device is hit by waves. The effect of these three kind of wave, incident wave, radiated wave and scattered waves, could be either constructive or destructive to the overall electricity output.

Two similar layouts of $N = 5$ are shown in Figure 1.1. The red layout is obtained by moving every WEC in blue layout slightly. They are very similar but with very different q-factors (which will be defined later in this section). The blue layout has $q = 2.7770$ while the red layout has $q = 0.9796$. Due to the interaction, the layout of devices has a considerable effect on the total amount of absorbed power. And the relative location of the devices determines whether the interaction effect between each pair of devices is constructive or destructive.

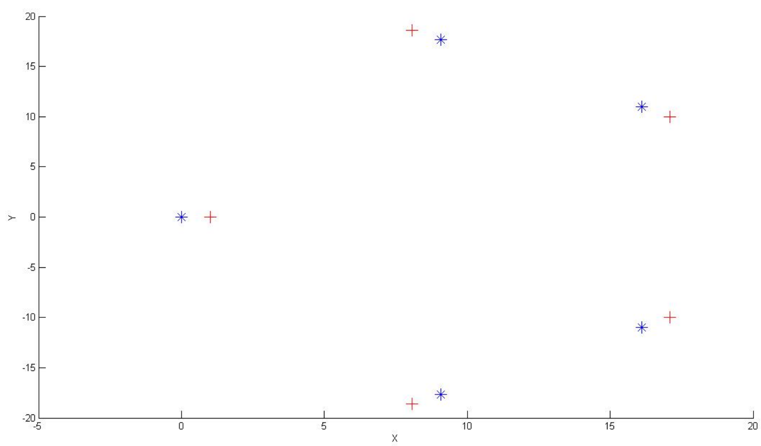

Figure 1.1: Two Similar Layouts with $N = 5$

The q-factor, also called interaction factor, is used to measure the quality of the layout. The q-factor is calculated with (1.1).

$$q = \frac{\sum_{j=1}^{N} P_j}{N \times P_0} \tag{1.1}$$

In this function, P_j is the mean mechanical power absorbed by jth wave energy converters and P_0 represents the mean mechanical power absorbed by a single isolated wave energy converters. N is the number of WECs. The actual quantity needed here is the output electrical power for each device, rather than the absorbed mechanical power. However, the calculation of output electrical power is of great difficulty, since it requires configuration information, such as mechanical and electrical properties, of the devices. On the other hand, the calculation of absorbed mechanical power needs no such properties. In using the q-factor, an assumption has been made implicitly that a constant portion of the absorbed mechanical power is transferred to the output electrical power. Therefore, the absorbed mechanical power of each device can be used to approximate the output electrical power.

The mean power absorbed during a wave period by a layout with N devices has been shown by Evans [8] and Falnes [9] independently. If the hydrodynamic coefficients, the coefficients used to represent the hydrodynamic forces acting on the device, of all the devices in the layout are known, the mean absorbed power P is given by

$$P = \frac{1}{4}(\boldsymbol{X}^*\boldsymbol{U} + \boldsymbol{U}^*\boldsymbol{X}) - \frac{1}{2}\boldsymbol{U}^*\boldsymbol{B}\boldsymbol{U} \tag{1.2}$$

In this formula, $\boldsymbol{U}$ is the complex velocity vector $(N \times 1)$. Complex velocity is the derivative of complex potential, which is used to describe fluid in fluid mechanics, in terms of an ideal fluid. $\boldsymbol{X}$ is the complex wave exciting force vector $(N \times 1)$. Wave exciting force is the force that causes the motion of devices. The $*$ denotes the complex conjugate transpose. $\boldsymbol{B}$ is the radiation damping matrix $(N \times N)$. Radiation damping means that vibrating energy of motion is converted and emitted in the form of radiated waves or other types of waves.

There are actually two optimization problems while optimizing the absorbed power P. One of them is the layout problem for the devices. The other is the control problem, represented in (1.2) by the matrix $\boldsymbol{U}$, the control variables. However, (1.2) is very non-convex and nonlinear. It's very difficult to optimize both the layout problem and the control problem at the same time. It also requires the hydrodynamic coefficients, which are hard to calculate, to solve this problem. As Fitzgerald and Thomas [10] mentioned, it's very challenging to determine the exciting force and analytic solutions are only available for simple geometries. For problems with two optimization problems, one option is to optimize one of the sub-problems first and then optimize the other sub-problem.

It's easy to solve if a fixed layout is given. When the devices are unconstrained and the control of each device is optimal, the maximum absorbed mechanical power, also shown by Evans [8], is given by

$$P = \frac{1}{8}\boldsymbol{X}^{*}\boldsymbol{B}^{-1}\boldsymbol{X} \tag{1.3}$$

This optimal value is achieved by

$$\boldsymbol{U} = \frac{1}{2}\boldsymbol{B}^{-1}\boldsymbol{X} \tag{1.4}$$

The layout optimization problem is the goal of this thesis. The objective function for this problem is (1.3). However, this problem is difficult to optimize, because (1.3) is non-convex, and also because it requires the calculation of the hydrodynamic coefficients $\boldsymbol{X}$ and $\boldsymbol{B}$, which are difficult to compute. Therefore, a simpler approximation is desired.

1.5 The Point-absorber Approximation

In the point-absorber approximation, the devices are assumed to be small enough and widely spread. In this case, the scattered waves are very weak due to the small size of the devices and the scattered waves will fade while traveling from one device to others. Therefore, the scattered waves can be neglected when they hit another device. The calculation of the q-factor using point-absorber theory is presented as following. This function is given by Evans [8]:

$$q = \frac{1}{N} \boldsymbol{L}^* \boldsymbol{J}^{-1} \boldsymbol{L}, \tag{1.5}$$

where the column vector $\boldsymbol{L} = \{L_m, m = 1, 2, ...N\}$ has elements

$$L_m = e^{ikd_m \cos(\beta - \alpha_m)} \tag{1.6}$$

and the matrix $\boldsymbol{J} = \{J_{mn}, m = 1, 2, ...N \; n = 1, 2, ...N\}$ has elements

$$J_{mn} = J_0(kd_{mn}) \tag{1.7}$$

Here $J_0(x)$ is the Bessel function of the first kind of order zero. d_{mn} indicates the relative distance between the mth and nth devices. d_m and α_m are the polar coordinates of the mth device related to a fixed origin.

This function indicates that the q-factor only depends on the location information (polar coordinates) of each buoy d_m and α_m, the wave number k and the wave heading direction β. It's worth noting that this function is independent of the hydrodynamic coefficients $\boldsymbol{X}$ and $\boldsymbol{B}$, which are difficult and time-consuming to calculate and require specialized software and tools, such as WAMIT [24]. Typically, it takes ten minutes or more to calculate $\boldsymbol{X}$ and $\boldsymbol{U}$ for a single layout. Usually, in order to get a better solution, there are many possible layouts to evaluate, which will take a considerable amount of time. Rather than (1.3), (1.5) is more attractive and convenient to calculate.

When $q > 1$, the constructive effect is greater than the destructive effect, which is desired. When $q = 1$, the constructive effect is equal to the destructive effect.

When $q < 1$, the destructive effect is greater than the constructive effect. Typically, the q-factor is less than 2 or 3, indicating that a wave farm with a good layout can absorb 2 or 3 times as much power as the same number of WECs operating in isolation.

1.6 Alternative q-factor Expression

One natural approach that comes to mind for any optimization problem is to solve it with optimization software, such as AMPL [3]. In order to do that, a simpler expression, in polynomial form, is needed due to the existence of complex numbers.

After writing down the polynomial form of the q-factors in the cases of $N = 2, 3, 4$, some patterns can be found. The expressions for the q-factors if $N = 2, 3, 4$ are shown in (1.8), (1.9) and (1.10), respectively. Those equations are obtained by expanding the matrix algebra in (1.5).

$$q(2) = L_1 L_1^* H_{11} + L_1 L_2^* H_{21} + L_2 L_1^* H_{12} + L_2 L_2^* H_{22} \tag{1.8}$$

$$\begin{aligned} q(3) = {} & L_1 L_1^* H_{11} + L_1 L_2^* H_{21} + L_1 L_3^* H_{31} \\ & + L_2 L_1^* H_{12} + L_2 L_2^* H_{22} + L_2 L_3^* H_{32} \\ & + L_3 L_1^* H_{13} + L_3 L_2^* H_{23} + L_3 L_3^* H_{33} \end{aligned} \tag{1.9}$$

$$\begin{aligned} q(4) = & L_1L_1^*H_{11} + L_1L_2^*H_{21} + L_1L_3^*H_{31} + L_1L_4^*H_{14} \\ & +L_2L_1^*H_{12} + L_2L_2^*H_{22} + L_2L_3^*H_{32} + L_2L_4^*H_{24} \\ & +L_3L_1^*H_{13} + L_3L_2^*H_{23} + L_3L_3^*H_{33} + L_3L_4^*H_{34} \\ & +L_4L_1^*H_{41} + L_4L_2^*H_{42} + L_4L_3^*H_{43} + L_4L_4^*H_{44} \end{aligned} \tag{1.10}$$

Extending this pattern, a general expression for N devices can be obtained, as in (1.11).

$$q = \sum_{m=1,2,\ldots N, n=1,2,\ldots N} L_m L_n^* H_{mn}, \tag{1.11}$$

We conjecture that (1.11) holds for all N, though we have been unable to prove it rigorously.

In these expressions, L_m^* stands for the mth element in matrix $\boldsymbol{L}^*$, where $*$ denotes complex conjugate transpose. H_{mn} stands for the element in the mth row and nth column in the matrix $\boldsymbol{H}$, where $\boldsymbol{H}$ is the inverse matrix of $\boldsymbol{J}$. As we know, L_m and L_m^* can be expressed as (1.12) and (1.13), where $A_m = kd_m \cos(\beta - \alpha_m)$.

$$L_m = e^{iA_m} = \cos A_m + i \sin A_m \tag{1.12}$$

$$L_m^* = e^{-iA_m} = \cos A_m - i \sin A_m \tag{1.13}$$

Therefore, we have

$$L_m L_m^* H_{mm} = e^{iA_m} e^{-iA_m} H_{mm} = e^0 I_{mm} = H_{mm} \tag{1.14}$$

$$L_m L_n^* H_{nm} = e^{iA_m} e^{-iA_n} H_{nm} = e^{i(A_m - A_n)} H_{nm} \tag{1.15}$$

As a consequence, in general, the terms can be combined as

$$L_n L_m^* H_{mn} + L_m L_n^* H_{nm} = H_{mn} \cos(A_m - A_n) + H_{nm} \cos(A_n - A_m) \tag{1.16}$$

(1.11) can be simplified using (1.16). To obtain

$$q = \sum_{m=1,2,\ldots N, n=1,2,\ldots N} H_{mn} \cos(A_m - A_n) \tag{1.17}$$

However, optimization software can't handle integration, inverting matrices and Bessel functions. And there is no efficient approximation for $\boldsymbol{H}$ for $N \geq 5$. Therefore, (1.17) is useful for optimization software that can handle the calculation mentioned above or that is able to interact with other mathematical software, such as Matlab. So the original expression for the q-factor (1.5) is used in the following discussion.

In addition to the point-absorber approximation, there are also other methods of approximation. McIver [17] has compared the point-absorber theory to the plane wave theory [22]. And Child and Venugopal [4] have addressed an exact procedure to determine the q-factor.

1.7 Symmetric Layout vs. Asymmetric Layout

Although there is no proof yet, the results of our two-step genetic algorithm, which will be introduced in Section 2.2, indicate that symmetric layouts tend to perform better than asymmetric ones.

The q-factors for the top five solutions obtained by our two-step genetic algorithm for both symmetric and asymmetric layouts with $N = 5$ devices, with fixed wave heading direction and wave number are shown in Table 1.1. The symmetric layouts tend to have better q-factors than the asymmetric layouts. The results presented by Fitzgerald and Thomas [10] also show this trend. So only symmetric layouts will be included in the following discussion.

Table 1.1: Symmetric Layouts vs. Asymmetric Layouts

Ordinals	Symmetric	Asymmetric
1th	2.7777	2.6666
2nd	2.6730	2.5828
3rd	2.5892	2.5425
4th	2.5502	2.5381
5th	2.5425	2.4946

Chapter 2

Wave Farm Layout Problem

The wave farm layout problem is an optimization problem that tries to maximize the power output of the wave farm by optimizing the layout of WECs. The output is approximated using q-factor and the point-absorber approximation discussed in Chapter 1. In order to satisfy the precondition of the point-absorber approximation, a minimum distance between any pair of WECs is required.

The objective of this problem varies when attention is paid to different aspects. When there is no uncertainty, the wave direction and wave number are constants and the maximum q-factor is desired. However, in reality, waves are usually stochastic. Since WECs represent long-term investments, wave farm operators will typically want to maximize the average output over a long time horizon, during which the ocean environment will change stochastically. In this case, one wishes to maximize the expected q-factor. In some cases, wave farm operators may be risk averse and may want to optimize the worst-case performance. In this case, the minimum q-factor is the objective to be maximized.

As mentioned in Section 1.5, there is no effective approximation foe the wave farm layout problem that can be used in commercial optimization software. A customized algorithm is needed to solve the problem. Figure 2.1 plots the objective function (q-factor) as the location of device 1 changes, keeping the locations of the other four devices fixed, for the best layout of $N = 5$ reported by our two-step genetic algorithm. The surface is very non-convex and nonlinear, which means that we are likely to find local optimal solutions when a convex optimization algorithm is used. Although there are some global optimization solvers, such as LGO [2], that are designed for non-convex problems, those solvers can't solve the wave farm layout problem, not only because they can't handle the calculation mentioned in Section 1.6 (actually, this could be solved by connecting solvers to other mathematical software) but also because there are too many local maxima to find the global optima. Therefore, a heuristic that is good at finding the global optimal solution is needed to solve this problem. In this case, a genetic algorithm is chosen.

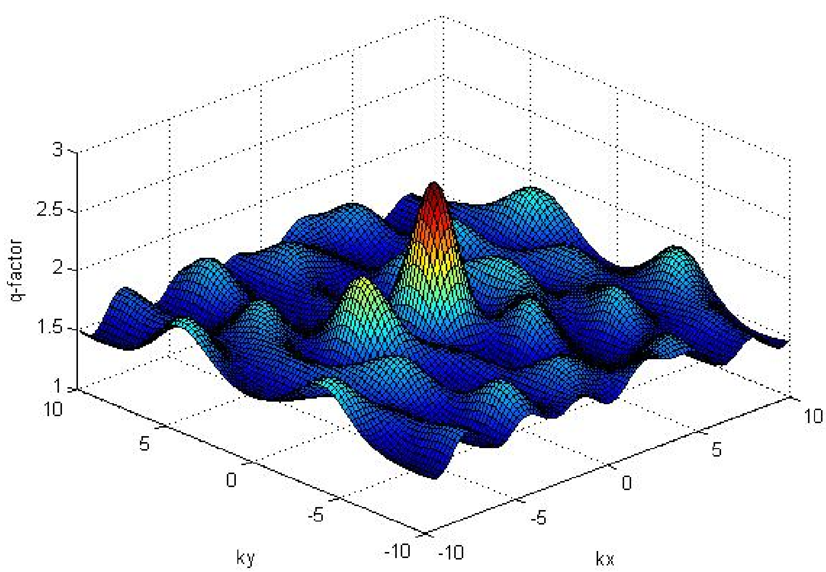

Figure 2.1: q-factor vs. location of device 1

2.1 Genetic Algorithm

A genetic algorithm is a heuristic that simulates the process of evolution. This heuristic (also sometimes called a metaheuristic) is routinely used to generate useful solutions to optimization and search problems [19]. A genetic algorithm usually works in four steps. First of all, a *population*, namely a set of random feasible solutions, is generated. Usually, a selected objective function is used to evaluate the quality of each solution. Secondly, part of the population, usually the solutions with the better objective function values, will be selected and become *parents*. Thirdly, those parents will exchange their *genes*, i. e. part of the solution, randomly with each other to generate a set of new solutions, called *children*. This step is called *crossover*. Then *mutation* may happen to the new solutions so that the solutions may change by chance. In addition, a procedure is needed to check and guarantee the feasibility of the solution. Fourthly, the parents and the new children constitute the new population. Then selection, crossover and mutation will happen to the new population again and again until a termination criterion is met. As the problem changes, the objective function and crossover procedure vary accordingly.

The speed of a genetic algorithm depends on the complexity of the objective function, the size of the problem (number of variables) and the parameters (number of iterations and termination condition).

2.2 Two-step Genetic Algorithm

Our preliminary implementation of a basic genetic algorithm showed that the quality of the results depend heavily on the quality of the solutions in the initial population. Motivated by this, we devised a two-step genetic algorithm in which the best results from several runs of the basic genetic algorithm become the initial population for another basic genetic algorithm. Table 2.1 compares the q-factors of the best solutions found by ten runs of the basic genetic algorithm and the two-step genetic algorithm. The basic genetic algorithm solutions from the first column form the initial population of the two-step genetic algorithm. Note that the results in the second column are much better than those in the first column.

Table 2.1: Basic Genetic Algorithm vs. Two-step Genetic Algorithm

Number	Basic GA	Two-step GA
1	2.1418	2.7770
2	1.8457	2.6730
3	1.8023	2.5892
4	1.7177	2.5425
5	1.7042	2.5391
6	1.6470	2.3716
7	1.6121	2.3069
8	1.5826	2.3068
9	1.5525	2.2154
10	1.4987	2.1865

2.2.1 Basic Genetic Algorithm for Wave Farm Layout Problem

We first discuss the basic genetic algorithm in this section, then in Section 2.2.2 discuss the two-step genetic algorithm. Usually, the implementation of a genetic algorithm highly depends on the problem it deals with. The implementation details for the wave farm layout problem are introduced in the following steps.

Population Encoding

For the wave farm layout problem, a solution contains the locations of N devices. The location of a device is described in a coordinate system. Therefore, two $(N \times 1)$ vectors, $\boldsymbol{PX1}$ and $\boldsymbol{PY1}$, are used to store the population. $\boldsymbol{PX1}$ is used to store x-coordinates and $\boldsymbol{PY1}$ is used to store y-coordinates. As mentioned in Section 1.7, only symmetric layouts are discussed in this thesis. So the devices that are symmetric to each other will be called a pair. And the locations of a pair are defined as a piece of gene. Once the location of a device changes due to feasibility checking, crossover or mutation, the location of the other device in the same pair will also change accordingly.

Initial Population

The initial population is commonly generated randomly. For our genetic algorithm, the location of devices is generated randomly within a 40×40 square. After the initial population generation, a check will be performed to ensure the feasibility of solutions. In the wave farm layout problem, feasibility means a device is not too close to other devices. We check from the first device to the last device one by one and regenerate the location of a device if it is too close to the previous ones until the minimum distance is met. We choose the size of initial population as one hundred.

Selection

The objective function values of each solution in the sets $\boldsymbol{PX1}$ and $\boldsymbol{PY1}$ are calculated and stored in the set $\boldsymbol{PQ1}$. The elements in $\boldsymbol{PQ1}$ will be sorted in decreasing order. The best fifty solutions will be selected as parents and stored in $\boldsymbol{BX}$ and $\boldsymbol{BY}$.

Crossover

We pair parents randomly. The paired parents swap odd-number genes to produce new solutions. Those new solutions are called children. The parents and the children will constitute the one hundred individuals in the new population $\boldsymbol{PX2}$ and $\boldsymbol{PY2}$.

Mutation

Mutation will happen to every gene in every individual in $\boldsymbol{PX2}$ and $\boldsymbol{PY2}$ with probability 20%. After mutation, feasibility will be checked and infeasible solutions will be fixed by re-mutating the genes that cause the infeasibility. Finally, the solutions in $\boldsymbol{PX2}$ and $\boldsymbol{PX2}$ will be saved as $\boldsymbol{PX1}$ and $\boldsymbol{PY1}$.

Termination Condition

The selection, crossover and mutation procedures continue until the termination condition is met. The termination condition is that the number of iterations in which no improved solution is found exceeds $L1$.

2.2.2 Two-step Genetic Algorithm

The two-step genetic algorithm will use $\boldsymbol{PX3}$ and $\boldsymbol{PY3}$ to store the top ten solutions from ten runs of the basic genetic algorithm. This is the first step. In the second step, another run of the basic genetic algorithm will be executed. This second run of the genetic algorithm will use $\boldsymbol{PX3}$ and $\boldsymbol{PY3}$ as the initial population. The limit on the is with no improvement in the second step is $L2$. The termination condition for the basic genetic algorithm in the second step is more rigorous, which means $L2 \geq L1$. The solution found by the two-step genetic algorithm will be stored in $\boldsymbol{PX4}$ and

$\boldsymbol{PY4}$.

2.2.3 Improvement Step

The way the genetic algorithm works is to search for the best random points from the solution surface. Those points, usually, are not precisely the local optimal solutions. The solutions in $\boldsymbol{PX4}$ and $\boldsymbol{PY4}$ are just the best solutions from the random points. Those points are close to local optima. So a search for locally optimal solutions is implemented after the two-step genetic algorithm. The procedure for the search is to try find a better solution around the genetic algorithm solution. For each solution, a search for better solutions is performed by moving the solution within a (2×2) square. If better solutions are found, the best solution will replace the present solution and we keep searching until no better solution is found within the square. Otherwise, the search will stop and store the present solution as the local maximum.

2.3 Greedy Algorithm

In order to verify the effectiveness of the genetic algorithm, a greedy algorithm has also been implemented to find a solution. A greedy algorithm makes the locally optimal choice at each stage [5]. In our case, the greedy algorithm searches for a solution to the N-WEC problem by keeping the solution to the $(N-1)$-WEC problem fixed and finding the best single WEC to add to it. With the objective function of maximizing the q-factor for $N = 2, 3, ..., 7$, the solutions found by both algorithms are listed in Table 2.2.

It shows that the solutions found by the genetic algorithm are much better than

Table 2.2: Genetic Algorithm vs. Greedy Algorithm

N	Greedy Alg.	Two-step Genetic Alg.
2	1.6744	1.6744
3	1.8230	1.9880
4	1.9076	2.1776
5	1.9732	2.7770
6	1.9907	2.7955
7	2.0359	3.0703

the ones found by the greedy algorithm, especially for $N \geq 3$. A plot of the device locations of solutions for $N = 2, 3, 5$ found by the two-step genetic algorithm is given in Figure 2.2.

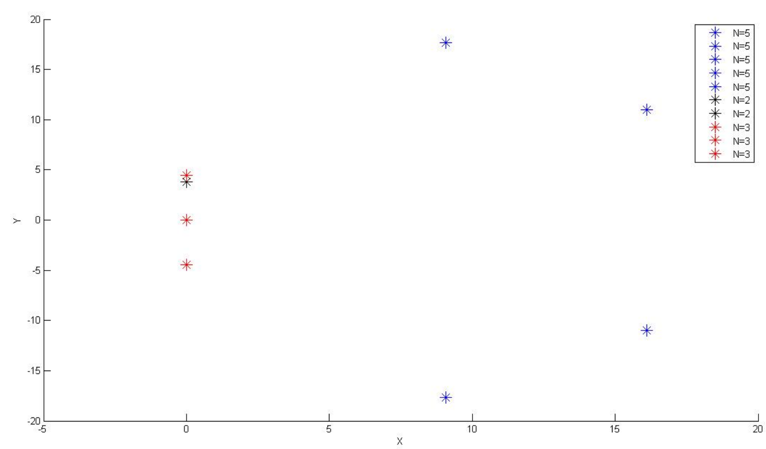

Figure 2.2: Best Layouts of $N = 2, 3, 5$

The figure shows that the layouts for different N are not similar to each other. This means that the solutions to the problem are not nested. The greedy algorithm is therefore not suitable for this problem and genetic algorithm works better.

2.4 Disadvantage of Genetic Algorithm

The genetic algorithm can not guarantee the optimality of the solution with a single run. In this case, a number of runs are needed before we can be confident that a near-optimal solution has been obtained. However, this becomes a serious issue when the size of the problem increases. The run time of the genetic algorithm highly depends on the size of the problem (the number of variables, or the number of devices). And the solution surface becomes more non-convex and nonlinear as the number of variables increases, which means it is harder to find optimal and near-optimal solutions.

2.5 q-factor Conjecture

If we take a layout for N WECs and duplicate it, the q-factor for the $2N$-WEC layout will equal the q-factor for the N-WEC layout if we move the two sets of N devices very far from each other (so they don't have hydrodynamic interactions). This also means that the $2N$-WEC layout has a feasible solution that is at least as good as the N-WEC layout. Plots of q-factor as the duplicate set moves away for $N = 2, 5$ are in Figure 2.3 and Figure 2.4. The straight line stands for the best q-factor values. The optimality of the solution for $N = 2$ had been proved by Snyder [23] mathematically. The solution of $N = 5$ is the best solution reported by the two-step genetic algorithm and also found by Fitzgerald and Thomas [10]. The points above the line indicate that there are many better feasible solutions for the $2N$-WEC layout than simply moving the duplicated set far away.

If the q-factor is expressed as a function of the number of devices $q(N)$, we can conclude that a problem with $2N$ devices always has a better solution than the

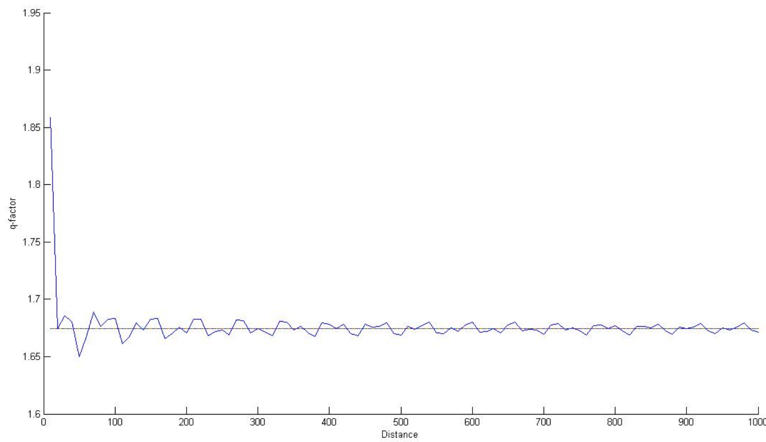

Figure 2.3: Move added set as $N = 2$

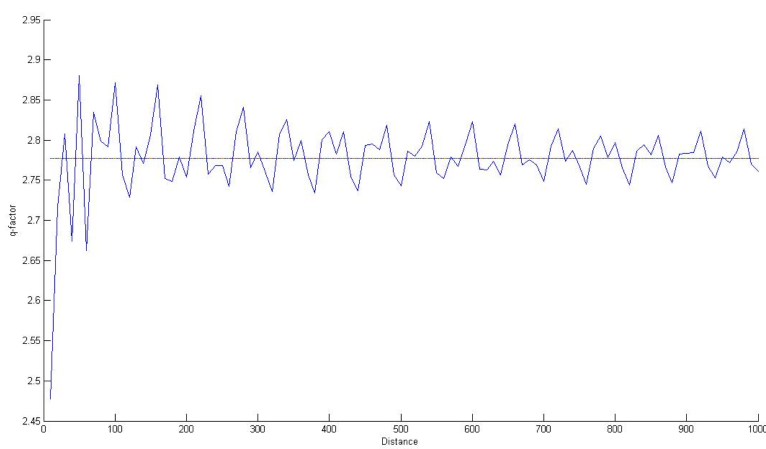

Figure 2.4: Move added set as $N = 5$

problem with N devices, namely

$$q_{opt}(2N) \geq q_{opt}(N) \tag{2.1}$$

where the subscript opt stands for optimal solution.

If the conclusion above is extended in a more general way, another conjecture can be made: the optimal q-factor in the case of $N+1$ devices is always at least as good as the optimal q-factor in the case of N devices, i. e.

$$q_{opt}(N+1) \geq q_{opt}(N) \tag{2.2}$$

Chapter 3

Computational Analysis

For a given number of devices, the optimal layout changes when attention is paid to different aspects (different objective function). The case of five devices has been a major object of study in the past. The discussion in Section 3.2, Section 3.3 and Section 3.4 will therefore focus on the case of five devices ($N = 5$).

All the solutions in this chapter are reported by our two-step genetic algorithm and encoded with Matlab. The version of Matlab is Version 7.11.0.584 (R2010b) run on an Intel Core i5 CPU, 4.00GB RAM and 32-bit system.

3.1 Maximizing q-factor

In past research, the majority of attention has been paid to the maximum output that a layout could produce under a deterministic regular wave. We discussed our own result in this section.

A table and a plot of the best solutions found by the two-step genetic algorithm for layouts with 2 to 15 devices is shown in Table 3.1 and Figure 3.1. The solution

Table 3.1: Best Solutions Found by Two-step GA for $N = 2, 3, ...15$

N	q-factor	Average Run-time (seconds)
2	1.6744	5.28
3	1.9880	6.37
4	2.1776	12.72
5	2.7777	45.82
6	2.7954	44.93
7	3.0703	86.39
8	2.9979	102.53
9	3.3938	130.87
10	3.2913	148.56
11	3.3670	167.84
12	3.1742	254.29
13	3.1905	938.20
14	3.0290	1025.60
15	2.9364	1563.52

for $N = 2$ has been proved to be optimal by Snyder [23]. The solution of $N = 5$ has also been found by Fitzgerald and Thomas [10] and believed to be optimal.

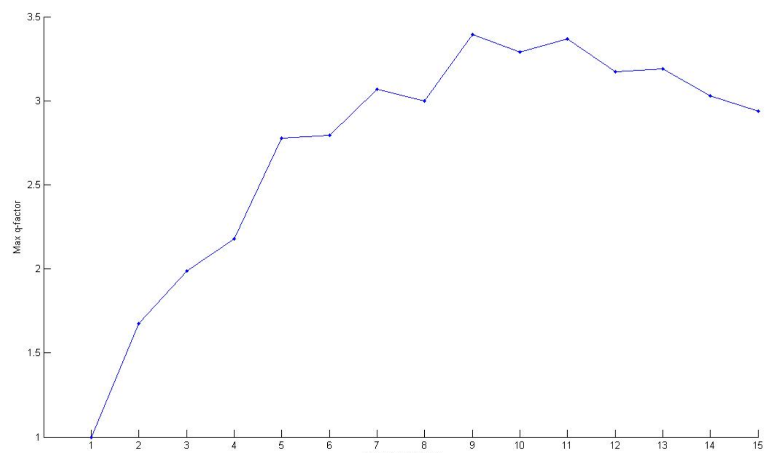

Figure 3.1: q-factors of $N = 2, 3, ...15$

The plot shows that q increases with N for $N = 2, 3, , 7$. This trend stops for $N > 7$, in contrast to our conjecture in Section 2.5, but we believe this is because our GA failed to find optimal solutions for the larger problems. In addition, we make the additional conjecture that increase in the optimal q-factor will decrease as N increases. That is, if we define $c = q_{opt}(N+1) - q_{opt}(N)$, then we conjecture that c decreases as N increases (and is always non-negative if our conjecture in Section 2.5 is correct).

However, a wave farm is a long-term investment. A plot of the q-factor as β changes for the best layout for $N = 5$ is shown in Figure 3.2. The q-factor for this layout is high only as the wave heading direction $\beta = 0$. It degrades rapidly as ocean state changes. Rather than high output in one certain sea state, overall output or average output may be desired. Robustness is an issue that has rarely been covered in past studies. The objective functions with robustness are discussed in Section 3.2 and Section 3.3.

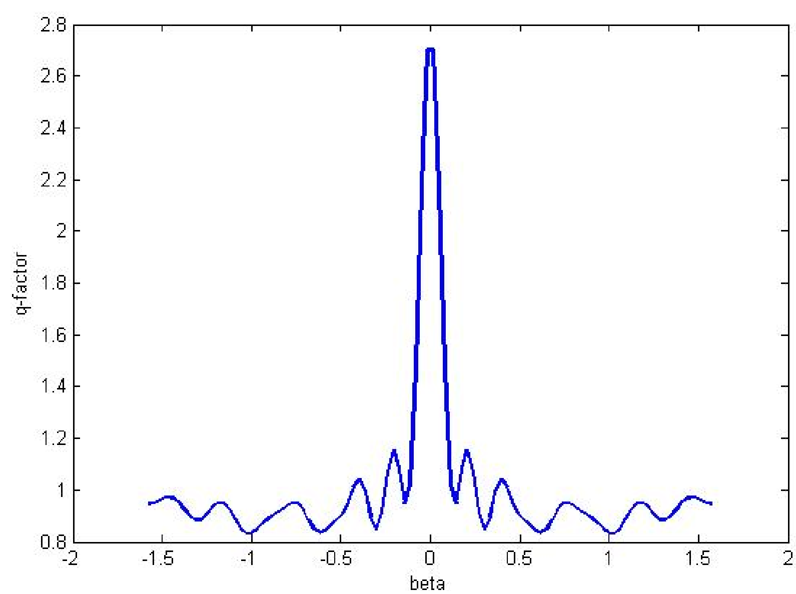

Figure 3.2: q-factors of $N = 5$ Best Layout as β Changes

3.2 Maximizing Expected q-factor

As we know, waves are not deterministic in reality. A good overall output is desired under uncertainty. In this case, an expected overall output is needed to be maximized when the wave heading direction and wave number changes. The wave heading direction and wave number under uncertainty are modeled using both normal and log-normal distribution in order to test both symmetric and asymmetric probability distribution. In this section, three situations will be discussed, wave heading direction under uncertainty, wave number under uncertainty and both wave heading direction and wave number under uncertainty.

3.2.1 Normal Distribution

In the following discussion, the factors under uncertainty follow a normal distribution, and the mean value of the normal distribution is 0 when β is stochastic and 2.5 when k is stochastic, while the standard deviation varies. In the following figures, SD stands for standard deviation.

The plot of q-factors vs β for the solutions found when the wave heading direction is under uncertainty and follows different normal distributions is shown in Figure 3.3.

Although the optimality of solutions can't be guaranteed, the tendency that the curves are shorter and flatter as the variance increases is obvious. The larger the variance is, the larger the chance that the wave heading direction will be very different from the mean value. This means the more regular (the variance is smaller) the waves are, the more energy is obtained.

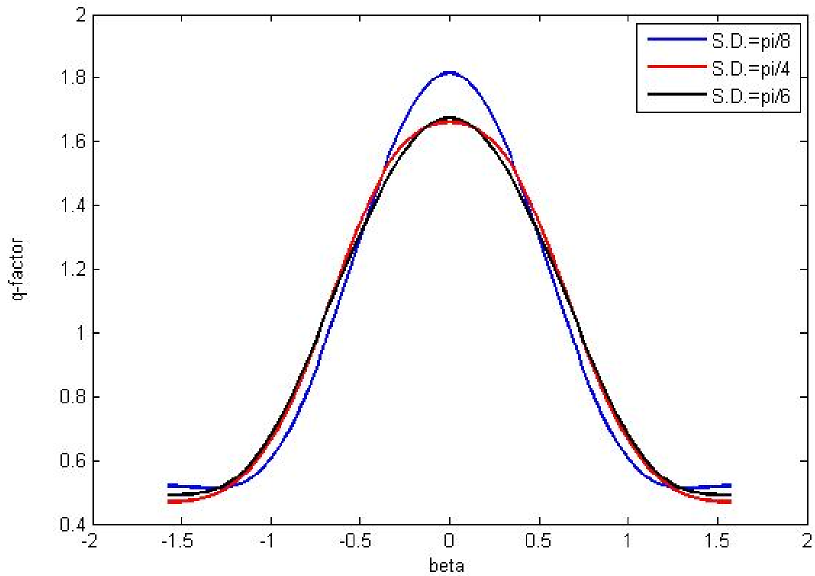

Figure 3.3: q-factors vs. β Under Uncertainty

The plot of q-factors vs. k for the solutions found when wave number is under uncertainty and follows different normal distributions is shown in Figure 3.4. The curves remain above the $q = 1$ line for a broader range of k values when the SD of the distribution increases.

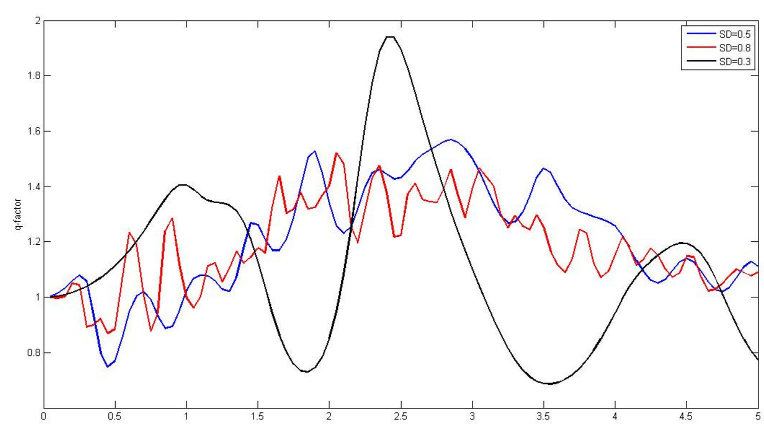

Figure 3.4: q-factors vs. k Under Uncertainty With Normal Distribution

The surface of q-factors is presented in Figure 3.5 when both the wave heading direction and the wave number is under uncertainty. And the surface of q-factor obtained with the objective function of maximizing the q-factor under the fixed sea state $k = 2.5$ and $\beta = 0$ is also shown in Figure 3.6. When uncertainty is taken into account, the q-value for $\beta = 0$, $k = 2.5$ is smaller than it is for the deterministic case, but on the other hand the solution is more robust, which we can see from the fact that $q > 1$ for a larger set of β and k values in Figure 3.4 than in Figure 3.6.

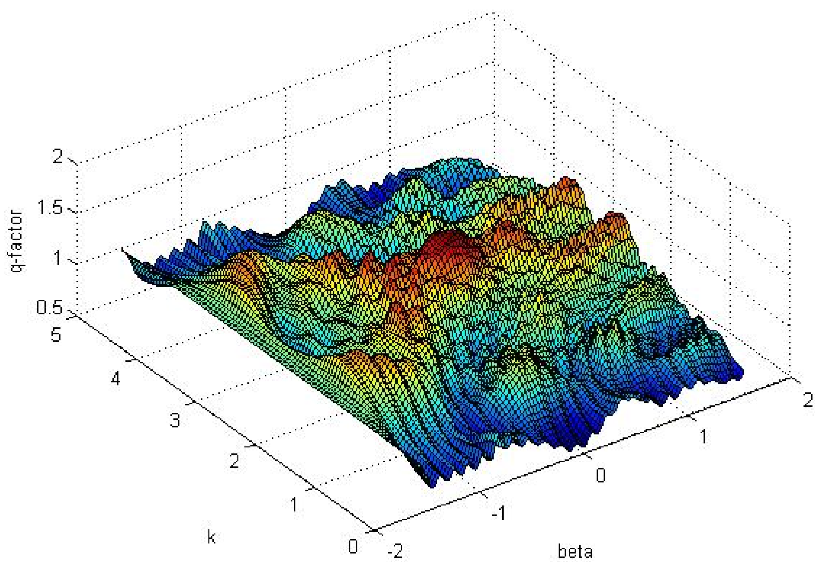

Figure 3.5: q-factors vs. β and k Under Uncertainty With Normal Distribution

3.2.2 Log-normal Distribution

In this section, the factors under uncertainty follow a log-normal distribution, and the scale value of log-normal distribution is zero while the shape value σ varies.

The plot of q-factors vs. β for the solutions found when the wave heading direction is under uncertainty and follows different log-normal distributions is shown in Figure 3.7. Figure 3.8 and Figure 3.9 show similar plots as the wave number (Figure 3.8)

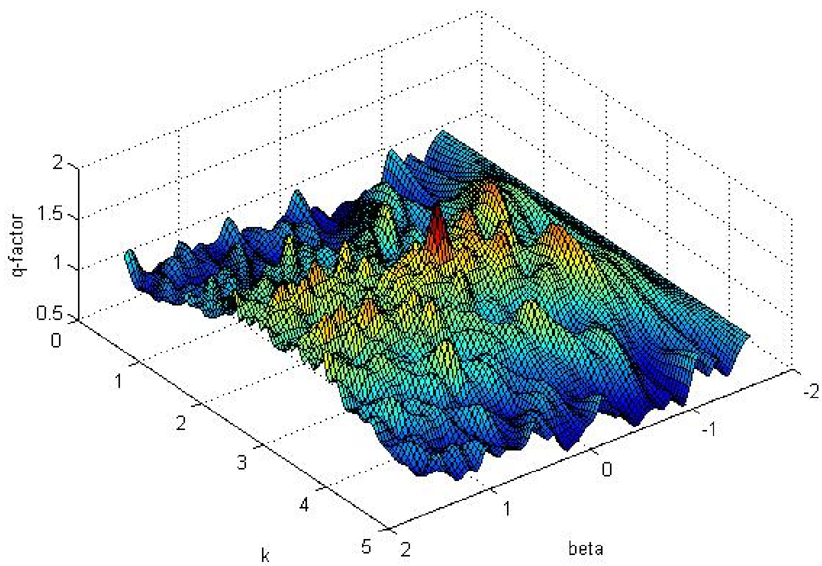

Figure 3.6: q-factors vs. β and k Under No Uncertainty

and both the wave heading and the wave number (Figure 3.9) are under uncertainty. As in the case of the normal distribution, increasing the variance of the distribution tends to lower the peaks but increase the range of values for which $q > 1$.

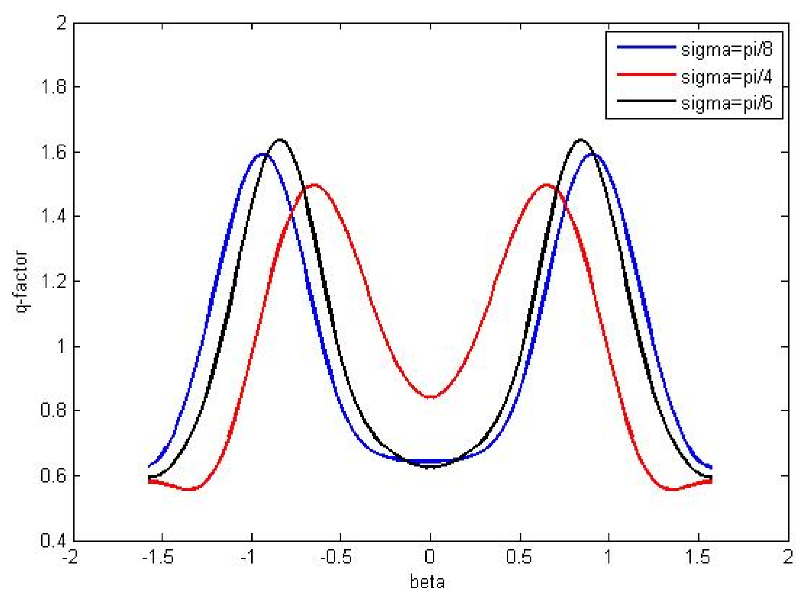

Figure 3.7: q-factors vs. β Under Uncertainty With Log-normal Distribution

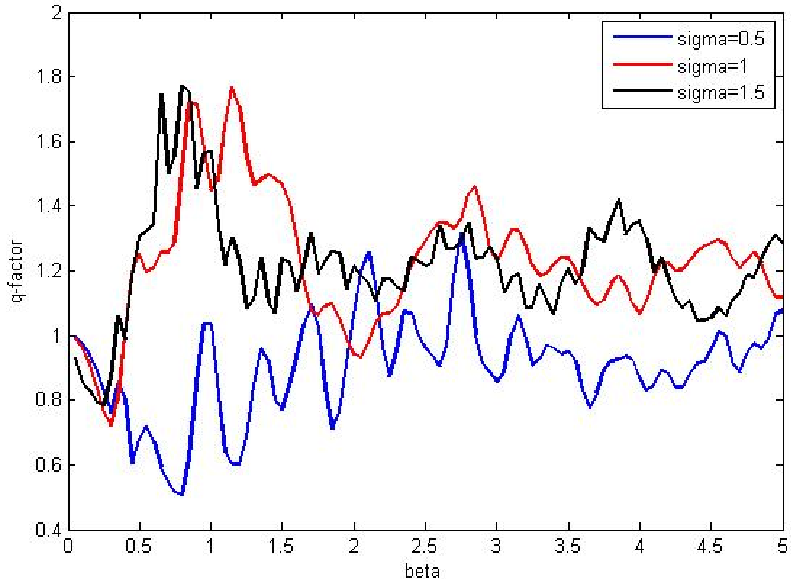

Figure 3.8: q-factors vs. k Under Uncertainty With Log-normal Distribution

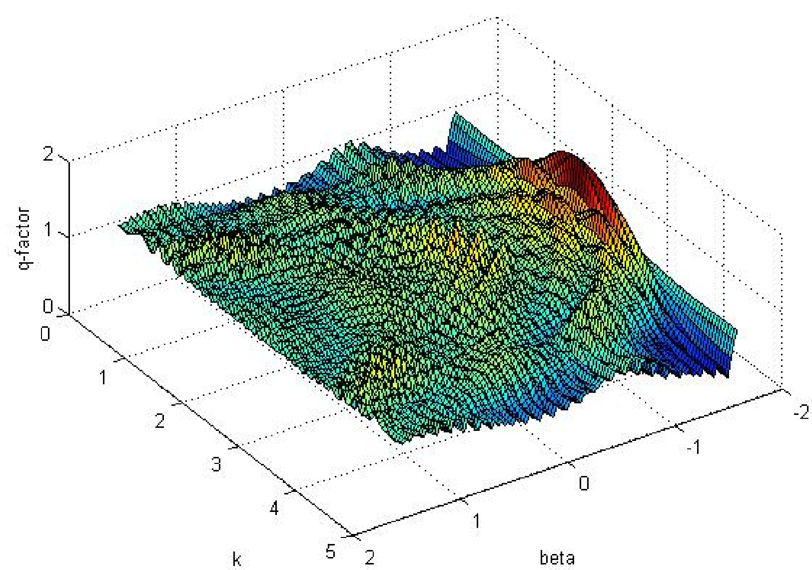

Figure 3.9: q-factors vs. β and k Under Uncertainty With Log-normal Distribution

3.2.3 Discussion of Stochastic Cases

The plots in Figure 3.3 and Figure 3.7 are of great interest. The shapes of the curves are very smooth and extraordinarily similar to the probability density curves of the corresponding distributions that the uncertainty follows. Although the plots in Figure 3.4 and Figure 3.8 are less smooth, the trends of the curves are still similar to the probability density curves of the corresponding distributions that the uncertainty follows. This may suggest that there are relatively simple relationships between k and β and the q-factor. Unlike the solution shown in Figure 3.2, those solutions have better q-factors though the most of the ocean states.

The surfaces in Figure 3.5 and Figure 3.9 show that the q-factors with high values tend to appear near the mean values of k and β. Compared to Figure 3.6, in which the only obvious peak appears at the ocean state with $k = 2.5$ and $\beta = 0$, the solutions obtained under uncertainty exhibit more robustness.

3.3 Maximizing Minimum q-factor

As mentioned above, sometimes we need a more stable output and want to optimize the worst-case performance. In this case, the minimum q-factor in certain range of wave heading directions is maximized. A plot of q vs. β for the solution obtained by maximizing minimum q-factor for different range of wave heading directions is shown in Figure 3.10. The plot shows that the wider the range of the wave heading is, the lower the level of the q-factor is in the range of wave heading but the wider the range is in which $q > 1$.

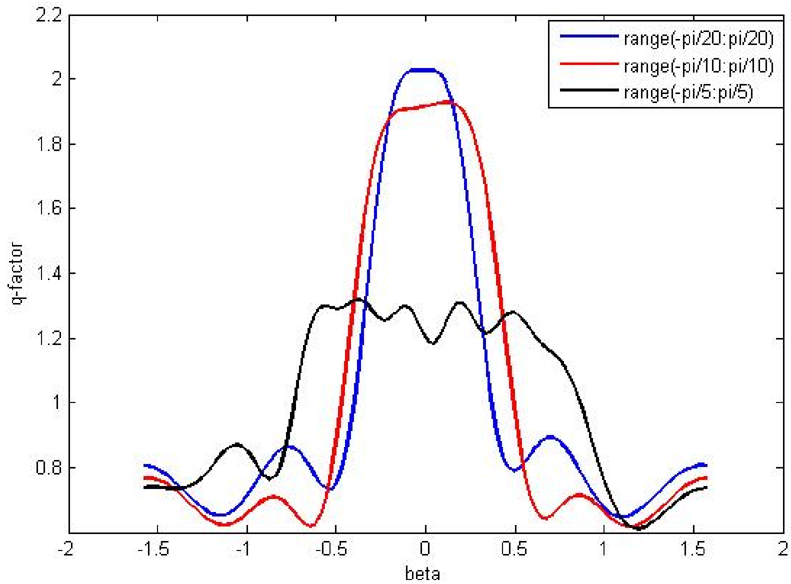

Figure 3.10: q-factors for Different Ranges

3.4 Layout Compare

The best solutions with various objective functions are compared in this section. A plot of the q-factors vs. β for the best solutions for different objective functions is shown in Figure 3.11.

The curves of q-factor tend to be flatter when the objective function emphasizes more robustness.

A plot of the locations of WECs for different objective functions with $N = 5$ is shown in Figure 3.12. The blue stars represent the solution for maximizing q-factor. The red crosses represent the solution for maximizing the expected q-factor. The black circles represent the solution for maximizing the minimum q-factor. The layouts differ greatly. The objective function has a heavy impact on the layout of the WECs.

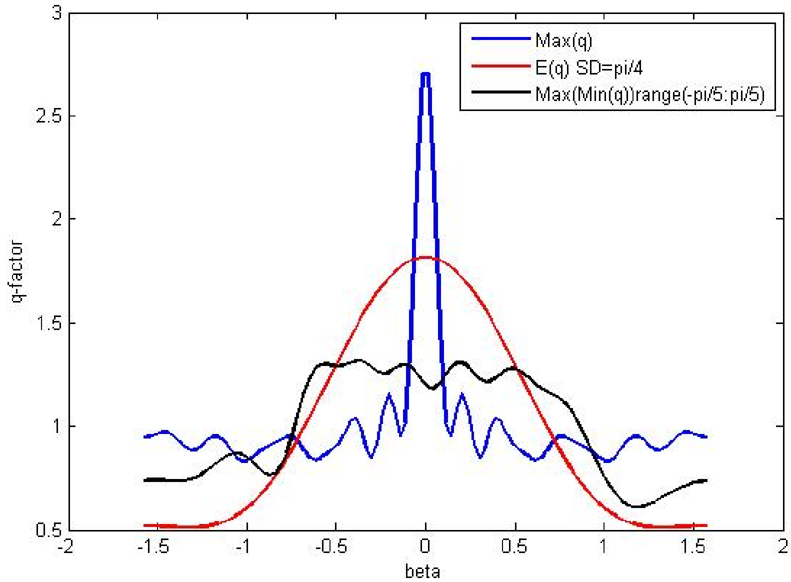

Figure 3.11: q-factors for Different Objective Functions

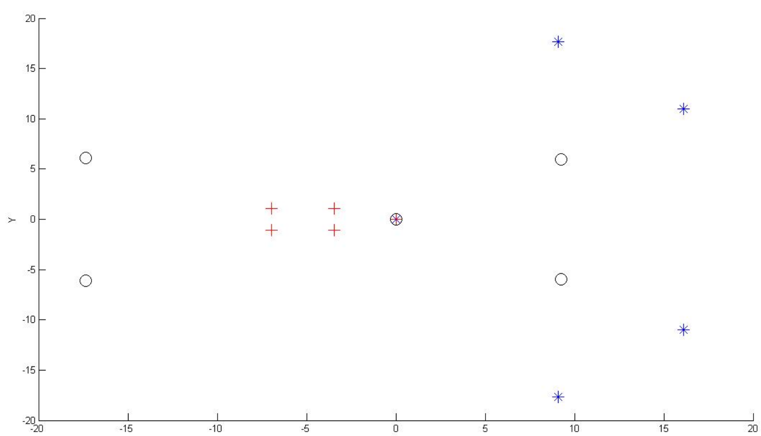

Figure 3.12: Layouts for Different Objective Functions

Chapter 4

Conclusion

This thesis first introduces wave energy briefly. Concepts related to wave energy converters and wave models have been presented. Secondly, an overview of key theories, including the q-factor and the point-absorber approximation, are shown. We focus on the point-approximation because it makes the calculation of absorbed power much simpler. An alternative expression of the q-factor has been shown. This expression may be useful if an efficient approximation is found or powerful optimization software is designed. Thirdly, the wave farm layout problem and our two-step genetic algorithm used to solve the problem have been introduced. The two-step genetic algorithm shows strong compatibility to multiple objective functions and efficiency for small scale wave farm layout problem. Several conjectures have been addressed regarding the q-factor based on the data reported by the two-step genetic algorithm. Although there are no proofs for the conjectures yet, the study of these conjectures can illustrate the relationship between k, β and N and the q-factor. Fourthly, a computational analysis is performed. Results for different objective functions are shown. In addition to maximizing the q-factor as has been study in the past, a preliminary study of layouts with more robustness have been shown. The solutions for maximizing the minimum q within some range of wave heading direction and a stochastic study on the expected q-factor when the wave number and wave heading direction are under uncertainty have also been presented. The robustness of the solutions has rarely been mentioned in the past.

In the future, many studies need to be deepened and extended. More efficient code is desired to shorten the run time of the two-step genetic algorithm for large-scale problem. If efficient approximations can be found for the alternative expression of the q-factor, the calculation and optimization of the q-factor will be much simpler. Proofs and more discussion are needed for the conjectures. Those conjectures could provide more knowledge on the layout problem. The study of robustness needed to be deepened that more wave models and uncertainties needed to be discussed. Rather than simple distributions, it is more valuable to study more realistic models of ocean waves.

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