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Multiple asset replacement under budget constraints

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Abstract

The replacement of equipment has been studied in various situations. Previous research has focused on models to determine replacement schedules for individual pieces of equipment while little work examines replacing numerous pieces of equipment under a budget constraint. We investigate both of these problems by looking at the costs associated with delaying or accelerating the replacement of the individual asset if the budget allocated for the suggested replacement period does not allow for replacement. We investigate the use of two previously developed dynamic programming models to determine the method which would more easily facilitate finding alternative replacement schedules through sensitivity analysis.

Keywords: Equipment replacement, sensitivity analysis, dynamic programming, medical equipment

Chapter 1

Introduction

This thesis considers the problem of replacing a large number of different assets under a common budget constraint. We focus our research on an application in medical equipment, although it is general for other replacement models. The problem to be analyzed has two parts. First, we seek to find the optimal replacement schedule for individual pieces of equipment that have been requested to be replaced. Our dynamic programming model recommends the optimal time for replacement of each piece. However, due to budget constraints, equipment may have to be replaced at an earlier or later time. Thus, we calculate the costs associated with delaying or accelerating the replacement of each individual piece of equipment through a designed sensitivity analysis. Second, we combine our individual asset solutions to determine the best solution for all assets under the budget constraint.

When considering the replacement of equipment, there are two major motivations: obsolescence and deterioration. Equipment can become obsolete due to new equipment being on the market which may be more technologically advanced.The rate at which technology advances and the improvements in the equipment is a reason for considering equipment replacement in the medical industry. Deterioration of equipment can also occur with time, as operating and maintenance costs increase and salvage values decrease. At some point the costs of maintaining equipment may exceed the costs of replacing the equipment. In this thesis we consider the case where assets deteriorate with time and savings can be made through periodic replacement.

1.1 Problem Description

In our problem, a set of heterogenous medical equipment is proposed to be replaced. Each piece of equipment is defined by its age i at time j with projected operating and maintenance costs (C_{ij}) and salvage values (S_{ij}) through its maximum age N. The asset must be replaced when it reaches age N. Furthermore, the purchase cost, p_j, estimated operating and maintenance costs and a salvage values are known for the potential replacement asset in each period $j \in T$, where T represents the problem horizon.

As stated previously there are two parts to the problem. First, the optimal replacement schedule for each equipment is identified. Second, due to budgetary constraints, we may not be able to replace all assets proposed to be replaced in a particular year. The use of sensitivity analysis will be investigated at this point. Hence, the costs associated with delaying or accelerating the replacement need to be calculated.

1.2 Literature Review

Christer and Scarf (1994) investigate a model which optimizes the replacement schedule of medical equipment. The model presented in this paper incorporates parameters and variables important to medical equipment. As mentioned by Christer and Scarf, the replacement decision for medical equipment has certain characteristic features:

1. The decision to replace equipment, may be one year, K years or only when forced by technical obsolescence.

2. Replacement may be driven by technical obsolescence, change in medical requirements or technological developments.

3. The replacement age of equipment should be related to usage.

4. All effects due to equipment failure and its unavailability are termed penalties.

5. Old equipment may not be scraped, but retained for use.

6. Equipment is always in demand.

The model presented by Christer and Scarf seeks find the values for K (remaining life of the asset) and L (economic life of the new asset) which minimizes the expected discounted cost per unit of usage over a period of $(K + L)$, such that, $\{C(N; K, L)\}$ is the total discounted cost of replacing the equipment of age N after K years and again after L years.

A budget constraint may prohibit the equipment from being replaced at the optimal time. Christer and Scarf propose a method of calculating the cost of delaying the replacement. Given the optimal replacement to be $(K*, L*)$, they propose the cost to delay by one year to be: $\{C(N; K*+1, L*)$ - $C(N; K*, L*)\}$.

We generalize the work of Christer and Scarf in that we allow for more than one replacement in the time $(K + L)$. Furthermore, we examine the cost of delaying or accelerating a decision to meet the budget constraint.

Karabakal et. al. (1994) introduces a model which includes the replacement of different assets under a budget constraint, called the Capital Rationing Replacement Problem (CRRP). They propose a finite horizon, deterministic version of CRRP as a zero-one integer program and use Lagrangian relaxation to solve this problem.

The formulation of their model can be represented by a network. In this representation, nodes represent the end of periods and arcs represent decisions to be made. Associated with each arc are two parameters, length (net present value benefit of replacement, π_{acij}) and resource consumption (purchase cost, P_{aci}). In their example they illustrate a piece of equipment with a three year planning horizon. The asset has two challengers: challenger 1 can be used until the end of the horizon time, and challenger 0 has a remaining life of two periods. The problem to be solved is to find the longest path from the initial asset's first node to the final assets's last node, so that no budget constraints are violated. In their model, they consider the replacement of equipment with multiple challengers. They suggest the use of Lagrangian relaxation to solve the integer program obtained from their model.

In a later paper, they discuss a dual heuristic for solving large, realistically sized problems. In this paper by Karabakal et.al (2000), they propose to solve the individual replacement problems ignoring the budget constraints. The next step would be to solve the Lagrangian dual problem in an attempt to eliminate or reduce budget violations. In their results they were able to solve problems with as many as 100 to 500 assets.

Miguel and Rodriguez (2006) introduce a model based on an artificial neural network that guarantees a warning when a piece of medical equipment requires replacement. The model was developed using event tree theory. The model included factors, such as, usage time over useful life, service cost over acquisition cost, and unavailability. The model, however, could not predict certain cases:

- Items which have already reached the end of their useful life, but maintenance costs and maintenance parameters are of adequate intervals.
- Medical equipment which have not reached its useful life, but maintenance costs or maintenance parameters are not in adequate intervals.

The main purpose of this article was to propose a more robust model for the retirement of a piece of equipment from hospital inventory, using artificial neural network. The model was used to test medical equipment to determine with greater precision whether the equipment should be repaired or replaced.

1.3 Research Motivation

The motivation for this research project comes from a local hospital trying to ensure the optimal replacement schedule for their equipment replacement. It was obvious from historical data that some equipment in inventory should have already been replaced. For example, operation and maintenance costs for some equipment was exceeding the cost of replacing them. In this thesis we seek to propose a model which will help in deciding the optimal replacement schedule for each individual piece of equipment.

Along with trying to decide on the optimal replacement for each equipment, the hospital also has a limited budget each year for equipment replacement. So our motivation is to find the optimal replacement schedule for a number of pieces of equipment when given a budget constraint in each year. Due to the budget constraints equipment suggested to be replaced in a particular may not be replaced. Hence, sensitivity analysis is used to determine the costs associated with delaying the replacement or replacing the equipment early as the budget allows.

This thesis proceeds as follows. In Chapter 2, we discuss our sub-problem of solving the optimal replacement solution for each piece of medical equipment. In Chapter 3, the optimal replacement results found in Chapter 2 are used to formulate the model to analyze the replacement of all equipment under a budget constraint. Sensitivity of the model is also considered to determine the cost of delaying or accelerating the replacement of the equipment. In Chapter 4, case study results are presented using data collected from medical equipment. Finally, conclusions and future studies are stated in Chapter 5.

Chapter 2

Single Asset Replacement Problem

In this chapter we focus on two previously developed methods using dynamic programming to find the optimal replacement schedule for equipment. First, we examine the Bellman model, proposed in 1955. We then look at an alternative approach given by Wagner (1975) to calculate the optimal replacement for an asset. Finally we compare both models to determine on the best model to be used when considering calculating the costs of delaying or accelerating replacement.

2.1 Bellman's Model

In a representative network for Bellman's model, a node represents the age of the asset, which is the state of the system. When moving from period to period, the decision is whether to keep or replace the asset. If a decision to keep the asset is made, then the state transitions from an asset of age n to age $n+1$. If a decision to replace the asset is made then the state transitions from n to 1. This means that the old asset is sold and a new asset replaces it and is used for one period. Figure 2.1 shows a representation of the Bellman model, the nodes represent the states of the asset, i.e. the age of the asset and the arcs represent the decisions. The decision to keep an asset is represented by an upward arc from any node and the decision to replace is represented by a downward arc from any node labeled with age 1.

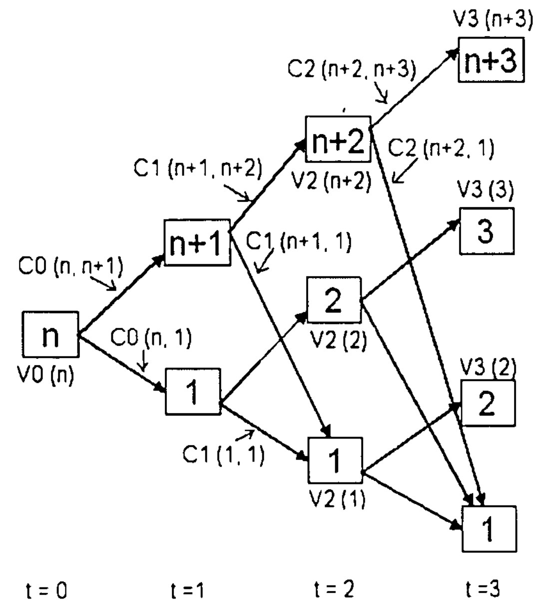

Figure 2.1: Bellman's dynamic programming model for single asset replacement problem

Defining $V_t(i)$ as the minimum expected net present value cost of an asset of age i at time t, when employing optimal replacement decisions through the horizon T. The recursive formulation can be written as:

$$V_t(i) = \min \begin{cases} Keep: & \alpha[C_t(i) + v_{t+1}(i+1)] \\ Replace: & P_t - S_t(i) + \alpha[C_t(0) + f_{t+1}(1)] \end{cases} \tag{2.1}$$

The boundary condition for the final period (where the asset is sold) is:

$V_t(i) = -S_T(j)$

In the model, the decision at each state i in period j is whether to keep the asset, at $C_t(i)$, the cost of operating and maintaining equipment at time t for a machine of age i, or replacing it for salvage value $S_t(i)$ and purchase cost P_t. Keeping the asset results in a state of $i+1$ while replacement results in state 1. Operating and maintenance costs are assumed to occur at the end of the period and all costs are discounted with the periodic discount rate α.

2.2 Wagner's Model

Wagner (1975) presented an alternative dynamic programming formulation in which the state of the system is the time period. In this representation there is only one state per period, but the number of decisions per state increases. The decisions in the Wagner model can be seen in Figure 2.2. The nodes represent the time period and the arcs represent decisions of how long to retain the asset. K_{01} is the decision to keep the initial (old) asset for one period and R_{01} represents the decision to replace the old asset with the new one in period one. The optimal decision is found by calculating the minimum cost path from node 0 to node 4.

The dynamic programming formulation is:

$$C(t) = \min_{j=1 \ldots min\{t,M\}} \{C(t-j)) + c_{t-j,t}\} t > 0 \tag{2.2}$$

The boundary condition for the final period: C(0) = 0

Where:

C (t) = the minimum present value cost of reaching node t from any other node.

M = maximum service life of the asset.

To solve the recursion forward, the solution from the previous nodes are required. The state C($i-j$) is used in the solution of state C(t),where j represents the number of periods of service foi the asset. The cost of this decision is c.

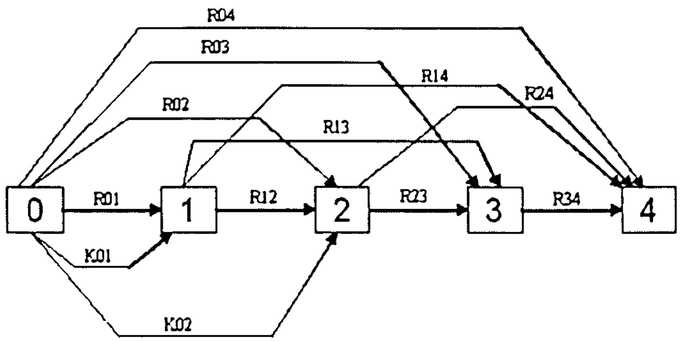

Figure 2.2: Wagner's dynamic programming model for single asset replacement problem

The nodes represent the time period and the arcs represent decisions of how long to retain the asset. K_{01} is the decision to keep the asset for one period and R_{01} represents the decision to replace the old asset with the new one in period one. The optimal decision is found by calculating the minimum cost path from node 0 to node 4.

2.3 Determining Alternative Replacement Solutions

Both dynamic programming models can solve the replacement problem efficiently. The Bellman model has at most N states in any of T periods with two decisions per state, leading to a worst run time of $0(2NT)$. For Wagner's model, there is at most 1 state in T periods with N maximum decisions, defining $0(NT)$. While Wagner's is (slightly) more efficient, it may not be best for our application, as shown below.

2.3.1 Sensitivity Analysis in Bellman's Model

An example of the Bellman model is shown in Figure 2.3. $V_0(n)$ defines the optimal solution value and the red lines indicate the optimal decisions for that piece of equipment through time t =3. In Figure 2.3 the optimal replacement schedule is to keep the equipment for two periods and then replace in it (KKR). Note the from our recursion equation (2.1), we can write this as:

$$V_0(n) = \frac{C_0(n, n+1)}{\alpha} + \frac{C_1(n+1, n+2)}{\alpha^2} + \frac{V_2(n+2)}{\alpha^2} \quad (2.3)$$

where $C_0(n, n+1)$ is the cost of keeping the equipment from age n to $n+1$.

The blue lines depict an alternative the path to be taken if the asset replacement was delayed by one year. The cost of delaying a replacement by one year (i.e. replacing the equipment after the third period (KKK)) is calculated as follows:

$$\textit{Delay Cost}\ (1\ \textit{year}) = \frac{C_0(n, n+1)}{\alpha} + \frac{C_1(n+1, n+2)}{\alpha^2} + \frac{C_2(n+2, n+3)}{\alpha^3} + \frac{V_3(n+3)}{\alpha^3} \quad (2.4)$$

The value of $V_3(n+3)$ is known from the solution of Bellman's model.

The green lines show the path if the asset was replaced at in an earlier year. The cost of replacing equipment one year earlier (i.e. replacing the equipment after the first period (KRK)) is calculated as follows:

$$\textit{Replace Early}\ (1\ \textit{year}) = \frac{C_0(n, n+1)}{\alpha} + \frac{C_1(n+1, 1)}{\alpha^2} + \frac{C_2(1, 2)}{\alpha^3} + \frac{V_3(2)}{\alpha^3} \quad (2.5)$$

Again, the value of $V_3(2)$ is known from the solution of Bellman's model. Thus, alternative replacement schedule can be formed quite easily. If budget constraints exist of the first $T\prime$ periods, we can easily determine alternative replacement schedules over $T\prime$.

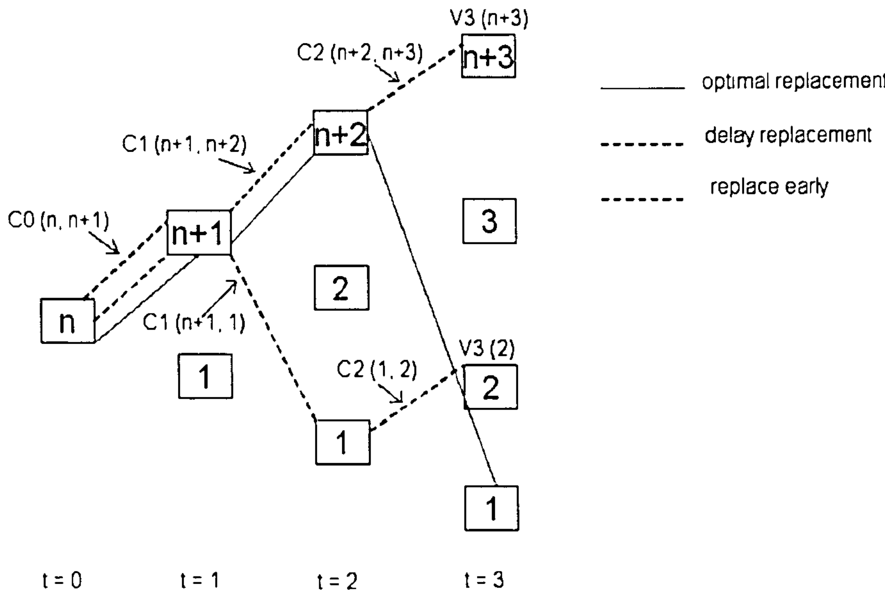

Figure 2.3: Alternative replacement schedules from Bellman's Model

2.3.2 Sensitivity Analysis in Wagner's Model

An example of the Wagner model is shown in Figure 4.1. The red lines show the optimal solution for that piece of equipment through time t =3. In Figure 4.1 the optimal solution shown in the Wagner model is to replace the equipment at the end of period two (KKR), as in our previous example. As indicated in Figure 4.1 by the red path, the optimal solution is the summation of the path c_{02} and the minimum of all paths from node 2 through the end of the horizon, or:

$$C(2) = c_{02} + \min \begin{cases} \frac{c_{24}}{\alpha^4} + \frac{C(4)}{\alpha^4} \\ \frac{c_{25}}{\alpha^5} + \frac{C(5)}{\alpha^5} \\ \vdots \\ \frac{c_{2T}}{\alpha^t} + \frac{C(T)}{\alpha^t} \end{cases} \tag{2.6}$$

The blue lines depict the path to be taken if the asset replacement was delayed by one year. The cost of delaying a replacement by one year is calculated as follows:

$$Delay\ Cost\ (1\ year) = c_{03} + \min \begin{cases} \frac{c_{34}}{\alpha^4} + \frac{C(4)}{\alpha^4} \\ \frac{c_{35}}{\alpha^5} + \frac{C(5)}{\alpha^5} \\ \vdots \\ \frac{c_{3T}}{\alpha^T} + \frac{C(T)}{\alpha^T} \end{cases} \tag{2.7}$$

The green lines show the path if the asset was replaced at in an earlier year. The cost of replacing equipment one year earlier as shown by the green path in Figure 4.1 is calculated as follows:

$$Replace\ Early\ (1\ year) = c_{01} + \min \begin{cases} \frac{c_{13}}{\alpha^3} + \frac{C(3)}{\alpha^3} \\ \frac{c_{14}}{\alpha^4} + \frac{C(4)}{\alpha^4} \\ \frac{c_{15}}{\alpha^5} + \frac{C(5)}{\alpha^5} \\ \vdots \\ \frac{c_{1T}}{\alpha^T} + \frac{C(T)}{\alpha^T} \end{cases} \tag{2.8}$$

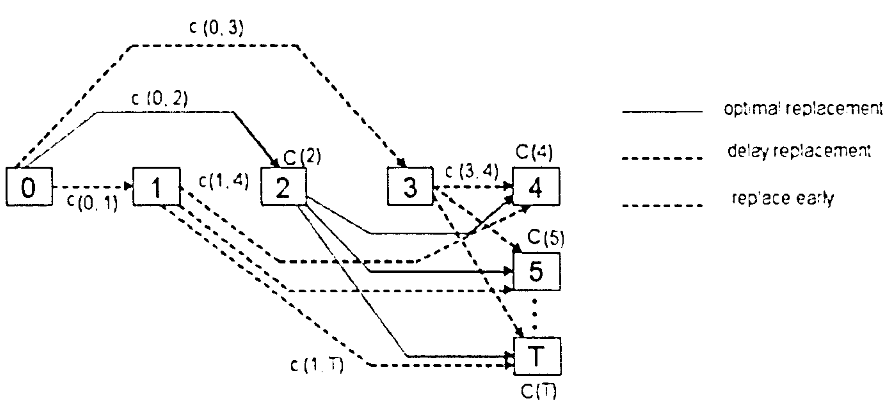

Figure 2.4: Alternative Replacement Schedules from Wagner's Model

2.3.3 Bellman Model used in our analysis

When comparing equations (2.4) and (2.7) it should be clear that more calculations are required if the sensitivity analysis is performed using Wagner's model than with Bellman's. This is due to the fact that Bellman's model has a larger state space and thus more information is saved.

If Wagner's model were to be used to calculate the cost of delay, a minimum from the set must be determined.

$$Replace\ (R\ year) = C_{0R} + \min_{j}\{\frac{C_{Rj}}{\alpha^j} + \frac{V(j)}{\alpha^j}\} \tag{2.9}$$

Where R is the year chosen for the equipment to be replaced, and the minimum path is chosen given that α is the discount factor. When considering the general case for this replacement model, it can easily be seen that calculating the minimum cost after choosing to replace at a particular time would be very tedious as the cost over all paths from the replacement node must be known through the end of the horizon.

When considering Bellman's model it can easily be seen in Figure 2.3 that the paths can easily be identified and the cost for replacing at a particular time can be calculated using one path. No minimum cost path from a set must be identified and the calculation is very simple and can easily be calculated. Thus, we move forward using Bellman's model.

Chapter 3

Multiple Asset Replacement with Budget Constraints

In this chapter, we take the results obtained from our dynamic programming model to construct an integer program to find the optimal replacement schedules for a number of assets under a budget constraint. Using the Bellman dynamic programming model, the costs for each individual equipment replacement schedule was found. The costs associated with the optimal solution and, as shown in section 2.3.1 the costs of delaying the replacement or replacing the equipment at an earlier time is also calculated. We make an assumption that a piece of equipment, i, can only be replaced once over the periods in which a budget constraint exists.

C_{ij} is the cost associated with replacing equipment i at time j. The optimal cost is found in our dynamic programming model and it includes all costs over the horizon, discounted to time zero. The costs associated with a delay in the replacement or replacing earlier is also calculated using our Bellman network as seen in the previous chapter. Thus, a piece of equipment i may be defined by a number of variables, such as x_{i0}, x_{i1}, etc., signifying replacements at time zero, one, etc. with associated costs C_{i0}, C_{i1}, etc. P_{ij} is the purchase cost of equipment i at time. It is the cost of purchasing new equipment if the old asset needs to be replaced. M is number of pieces of equipment being investigated. N_i is the maximum age of equipment i while, n_i is the current age of equipment i and T is the maximum horizon considered in each problem. The integer program is as follows:

$$min \sum_{i=1}^{M} \sum_{j=0}^{min(N-n_i,T)} C_{ij}x_{ij} \tag{3.1}$$

subject to:

$$\sum_{i=1}^{M} P_{ij}x_{ij} \leq B_j \qquad \forall j \tag{3.2}$$

$$\sum_{j=0}^{min(N-n_i,T)} x_{ij} = 1 \qquad \forall i \tag{3.3}$$

$$x_{ij} \in \{0,1\} \tag{3.4}$$

x_{ij} is a zero - one decision variable:

$$x_{ij} = \begin{cases} 1 & \text{if replace equipment i at time j} \\ 0 & \text{otherwise} \end{cases} \tag{3.5}$$

The objective of this integer program (3.1) is to find the minimum cost replacement schedules for all equipment i within the given j time periods. Every time period has a respective budget constraint B_j. The first constraint (3.2) ensures that the purchase costs of new equipment acquired at that particular time j should be less than the budget allocated for that time j. The second constraint (3.3) ensures that equipment i is replaced only once over the horizon. (This is similar to an assignment constraint.) The fourth constraint (3.4) ensures that the results for replacement is given in an integer form.

For a T period problem with M assets having a maximum age N, there is a maximum of MxT integer variables. Furthermore, there is a maximum of T budget constraints and M assignment constraints.

Chapter 4

Case Study

We illustrate our two-stage approach with a case study from our hospital partner. We use data given to us from the hospital along with assets requested to be replaced at time zero. We then analyze each individual asset to find it's optimal replacement schedule. Then we calculate the cost to delay and accelerate replacement of the equipment. We then use this results to form our integer program to analyze the optimal replacement schedule of all pieces of equipment given a budget constraint.

4.1 Single Asset Solution

To illustrate the single asset solution, we chose a 13-year old Infant warmer, that could be retained a total of 20 years. Using Bellman's model with the costs given in Table 4.1, the optimal replacement schedule was determined to be after one or more period of use (age 14). Now, we demonstrate how the cost to replace a year earlier and the cost to delay replacement by a year are calculated. In a similar manner, costs for the equipment to be replaced in two, three years etc. can also be calculated. The purchase cost of new equipment (challenger) is \$12,920.00.

The optimal solution is to replace at time 1, with the total net present value cost ($V_0(1)$) given as \$12,324.00. The cost to replace one period earlier, over a 20 period horizon (i.e. at time zero) is calculated as follows:

$$Replace\ Early\ (1\ year) = \frac{C_0(13,1)}{\alpha} + \frac{C_1(1,2)}{\alpha^2} + \frac{V_2(2)}{\alpha^2} \tag{4.1}$$

$$Replace\ Early\ (1\ year) = \frac{((\$12,920 - \$1.35) + \$641)}{(1.05)} + \frac{\$64}{(1.05)^2} + \frac{\$1,564}{(1.05)^2} = \$14,390. \tag{4.2}$$

The cost to replace at one period later, (i.e. at time two) is calculated as follows:

$$Delay\ Cost\ (1\ year) = \frac{C_0(13,14)}{\alpha} + \frac{C_1(14,15)}{\alpha^2} + \frac{C_2(15,1)}{\alpha^3} + \frac{C_3(1,2)}{\alpha^4} + \frac{V_4(15)}{\alpha^4} \tag{4.3}$$

$$\begin{aligned} Delay\ Cost\ (1\ year) = & \frac{\$641}{(1.05)} + \frac{\$668}{(1.05)^2} + \frac{((\$12,920 - \$0.34) + \$695)}{(1.05)^3} + \\ & \frac{\$64}{(1.05)^4} + \frac{\$1,151}{(1.05)^4} = \$13,977 \end{aligned} \tag{4.4}$$

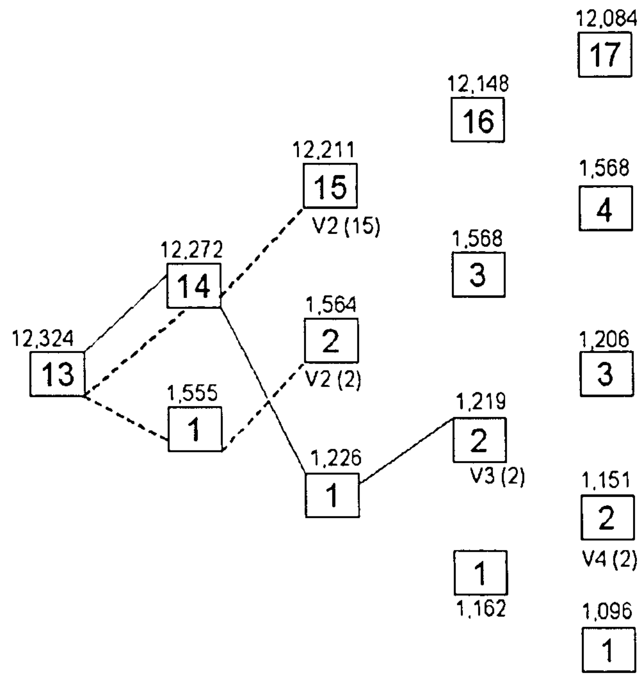

Figure 4.1: Case example using Bellman's Model

Table 4.1: Costs and salvage values for old and replacement assets

Age (i)	$C(i)$	$S(i)$	$C(i)$	$S(i)$
1	317	5522	64	6460
2	344	2761	69	4199
3	371	1380	74	2729
4	398	690	79	1774
5	425	345	84	1153
6	452	173	89	750
7	479	86	94	487
8	506	43	99	317
9	533	22	104	206
10	560	11	109	134
11	587	6	114	87
12	614	3	119	57
13	641	2	124	37
14	668	1	129	24
15	695	0	134	16
16	722	0	139	10
17	749	0	144	7
18	776	0	149	4
19	803	0	154	3
20	830	0	159	2

From this solution we define C_{10}, C_{11} and C_{12} as $14,390, $12,324 and $13,977 for the integer program.

4.2 Solutions to 10 Multiple Asset Problem

Using the results from section 4.1 we now construct the integer program. The replacement costs for each asset i are placed in the objective function to be minimized. The values for the purchase costs and budget are entered into the constraints.

The following is an example with 10 pieces of equipment from our case study. The optimal replacement cost suggested by our dynamic program for each piece of equipment is given in bold, all other costs were calculated by computing the cost to delay or accelerate replacement to that particular time period. Recall that x_{ij} is a zero - one decision variable, where it is one if equipment i is replaced at time j and zero otherwise. For our initial solution, we assumed the following budgets:

Table 4.2: Budgets used for years 0-17 for initial solution

Year	*Budget*
0	11000
1	11000
2	12000
3	15000
4	15000
5	16000
6	18000
7	19000
8	18000
9	18000
10	18000
11	13000
12	13000
13	10000
14	10000
15	10000
16	10000
17	10000

min

$\{ 3920x_{10} + \mathbf{3740}x_{11} +$

$4670x_{20} + \mathbf{3190}x_{21} +$

$4630x_{30} + \mathbf{3290}x_{31} +$

$3670x_{40} + \mathbf{3530}x_{41} + 3730x_{42} + 3980x_{43} + 4270x_{44} + 4600x_{45} + 4950x_{46} + 5330x_{47} + 5730x_{48}$

$+ 6150x_{49} + 6590x_{410} + 7040x_{411} +$

$15420x_{50} + \mathbf{14520}x_{51} + 15390x_{52} + 16450x_{53} + 17730x_{54} + 19250x_{55} + 21000x_{56} + 22950x_{57}$

$+ 25090x_{58} + 27400x_{59} + 29860x_{510} + 32440x_{511} + 35120x_{512} + 37880x_{513} + 40700x_{514} +$

$43540x_{515} + 46370\, x_{516} + 49130x_{517} +$

$14070x_{60} + \mathbf{13900}x_{61} + 14010x_{62} + 14170x_{63} + 14430x_{64} + 14820x_{65} + 15340x_{66} + 15990x_{67}$

$+ 16760x_{68} + 17630x_{69} + 18590x_{610} + 19640x_{611} + 20750x_{612} + 21910x_{613} + 23100x_{614} +$

$24290x_{615} + 25460x_{616} + 26570x_{617} +$

$10930x_{70} + \mathbf{10800}x_{71} + 10920x_{72} + 11130x_{73} + 11400x_{74} + 11730x_{75} + 12120x_{76} + 12560x_{77}$

$+ 13040x_{78}\ 13560x_{79} + 14110x_{710} +$

$3030x_{80} + \mathbf{2950}x_{81} + 3040x_{82} + 3140x_{83} + 3270x_{84} + 3420x_{85} + 3590x_{86} + 3770x_{87} + 3960x_{88}$

$+ 4170x_{89} + 4380x_{810} + 4610x_{811} + 4840x_{812} +$

$3320x_{90} + 3250x_{91} + \mathbf{3230}x_{92} +$

$14080x_{100} + 13950x_{101} + 13790x_{102} + 13270x_{103} + \mathbf{12960}x_{104}\ \}$

subject to:

$2160x_{10} + 2340x_{20} + 2340x_{30} + 2360x_{40} + 8670x_{50} + 8670x_{60} + 8100x_{70} + 2060x_{80} + 2060x_{90}$

$+ 12620x_{100} \leq 11000$

$2160x_{11} + 2340x_{21} + 2340x_{31} + 2360x_{41} + 8670x_{51} + 8670x_{61} + 8100x_{71} + 2060x_{81} + 2060x_{91}$

$+ 12620x_{101} \leq 11000$

$2360x_{42} + 8670x_{52} + 8670x_{62} + 8100x_{72} + 2060x_{82} + 2060x_{92} + 12620x_{102} \leq 12000$

$2360x_{43} + 8670x_{53} + 8670x_{63} + 8100x_{73} + 2060x_{83} + 12620x_{103} \leq 15000$

$2360x_{44} + 8670x_{54} + 8670x_{64} + 8100x_{74} + 2060x_{84} + 12620x_{104} \leq 15000$

$2360x_{45} + 8670x_{55} + 8670x_{65} + 8100x_{75} \leq 16000$

$2360x_{46} + 8670x_{56} + 8670x_{66} + + 8100x_{76} \leq 18000$

$2360x_{47} + 8670x_{57} + + 8670x_{67} + 8670x_{67} + 8100x_{77} + 2060x_{87} \leq 18000$

$2360x_{48} + 8670x_{58} + 8670x_{68} + 8100x_{78} + 2060x_{88} \leq 18000$

$2360x_{49} + 8670x_{59} + 8670x_{69} + 8100x_{79} \leq 18000$

$2360x_{410} + 8670x_{510} + 8670x_{610} + 8100x_{710} \leq 18000$

$2360x_{411} + 8670x_{511} + 8670x_{611} + 2060x_{811} \leq 13000$

$8670x_{512} + 8670x_{612} + 2060x_{812} \leq 13000$

$8670x_{513} + 8670x_{613} \leq 10000$

$8670x_{514} + 8670x_{614} \leq 10000$

$8670x_{515} + 8670x_{615} \leq 10000$

$8670\ x_{516} + 8670x_{616} \leq 10000$

$8670x_{517} + 8670x_{617} \leq 10000$

$x_{10} + x_{11} = 1$

$x_{20} + x_{21} = 1$

$x_{30} + x_{31} = 1$

$x_{40} + x_{41} + x_{42} + x_{43}\ x_{44} + x_{45} + x_{46} + x_{47} + x_{48} + x_{49} + x_{410} + x_{411} = 1$

$x_{50} + x_{51} + x_{52} + x_{53} + x_{54} + x_{55} + x_{56} + x_{57} + x_{58} + x_{59} + x_{510} + x_{511} + x_{512} + x_{513} + x_{514}$

$+ x_{515} + x_{516} + x_{517} = 1$

$x_{60} + x_{61} + x_{62} + x_{63} + x_{64} + x_{65} + x_{66} + x_{67} + x_{68} + x_{69} + x_{610} + x_{611} + x_{612} + x_{613} + x_{614}$

$+ x_{615} + x_{616} + x_{617} = 1$

$x_{70} + x_{71} + x_{72} + x_{73} + x_{74} + x_{75} + x_{76} + x_{77} + x_{78} + x_{79} + x_{710} = 1$

$x_{80} + x_{81} + x_{82} + x_{83}\ x_{84} + x_{85} + x_{86} + x_{87} + x_{88} + x_{89} + x_{810} + x_{811} + x_{812} = 1$

$x_{90} + x_{91} + x_{92} = 1$

$x_{100} + x_{101} + x_{102} + x_{103} + x_{104} = 1$

$x_{ij} \in \{0, 1\}$

The optimal solution found as \$ 73,460.00 using AMPL and CPLEX. The optimal replacement schedule is given in Table 4.3:

Table 4.3: Optimal replacement periods for each asset.

Equipment	*Replacement year*
1	1
2	1
3	1
4	1
5	2
6	3
7	0
8	0
9	2
10	4

This optimal decision suggests that assets 7 and 8 should be replaced in year 0, assets 1-4 should be replaced after the 1st year, assets 5 and 9 should be replaced in year 2 and asset 10 replaced in year 4. In the next section we further investigate the optimal replacement schedule when the budget for each year is changed.

4.3 Sensitivity of Budget Constraints

In this section we investigate changes in the optimal replacement schedule if the budget constraints are adjusted. The sensitivity analysis of the budget constraint is important as the budget may vary from year to year. The budget in a later year may be uncertain and hence, there must be an easy way of evaluating the replacement schedule if budget constraints were to change.

The following tables show various budget constraints used and the replacement schedule obtained because of these changes in the budget.

Table 4.4: Four different sets of budgets used for years 0-17

Year	*Budget* 1	*Budget* 2	*Budget* 3	*Budget* 4
0	9000	13000	9000	5000
1	9000	13000	9000	5000
2	10000	14000	7000	6000
3	13000	17000	8000	7000
4	13000	17000	17000	13000
5	14000	18000	16000	13000
6	16000	20000	15000	12000
7	16000	20000	15000	12000
8	16000	20000	15000	12000
9	16000	20000	15000	12000
10	16000	20000	15000	12000
11	11000	15000	12000	10000
12	11000	15000	12000	10000
13	8000	12000	5000	3000
14	8000	12000	5000	3000
15	8000	12000	5000	3000
16	8000	12000	5000	3000
17	8000	12000	5000	3000

Table 4.5: Optimal Replacement Schedule for Budgets 1 - 4

Equipment	*Budget* 1	*Budget* 2	*Budget* 3	*Budget* 4
1	1	1	1	0
2	1	1	1	1
3	1	1	1	1
4	0	1	2	0
5	2	2	0	5
6	3	3	5	6
7	5	0	6	7
8	1	1	1	2
9	0	2	2	2
10	4	4	4	2
solution cost	$74,410	$73,380	$75,450	$80,450

The budget for each year was changed in four different instances and each time the optimal replacement schedule changed. It is obvious from this analysis that our model can easily calculate the new optimal replacement schedule if the budget constraints were to be changed for any reason.

Next we investigate an even more realistic problem. In this problem we consider that the budget constraints for the first 5 years are known, the later years have not been assigned a budget and so we allow for zero budget in those time periods. The following table shows 4 different budgets used, for all cases the budget is zero after time = 4.

Table 4.6: Four different sets of budgets used for years 0-4 and with unknown budgets for years 5 - 17

Year	*Budget* 5	*Budget* 6	*Budget* 7	*Budget* 8
0	10000	13000	9000	11000
1	10000	13000	9000	12000
2	10000	14000	7000	13000
3	13000	17000	8000	10000
4	13000	17000	17000	10000
5	0	0	0	0
6	0	0	0	0
7	0	0	0	0
8	0	0	0	0
9	0	0	0	0
10	0	0	0	0
11	0	0	0	0
12	0	0	0	0
13	0	0	0	0
14	0	0	0	0
15	0	0	0	0
16	0	0	0	0
17	0	0	0	0

Table 4.7: Optimal Replacement Schedule for Budgets 5 - 8

Equipment	*Budget* 5	*Budget* 6	*Budget* 7	*Budget* 8
1	1	1	infeasible solution	1
2	1	1		1
3	1	1		1
4	3	1		1
5	0	2		0
6	2	3		4
7	3	0		3
8	3	1		1
9	1	2		0
10	4	4		2
solution cost	\$74,110	\$73,380	no solution	\$74,790

In these results again we see that the optimal replacement schedule is for each different set of budget constraint is varied. As can be seen from the results, due to zero budget in years 5 - 17 all equipment are forced to be replaced at times 0 - 4. For the budget constraint set 7 the results were given to be an infeasible solution. No basis was found when trying to solve and using these budget constraints.

Next we investigate an even more another case. In this problem we consider that the budget constraints for the first 5 years are known, the later years have a budget of $1,000,000. The following table shows 4 different budgets used:

Table 4.8: Four different sets of budgets used for years 0-4 and with large budgets for years 5 - 17

Year	*Budget* 9	*Budget* 10	*Budget* 11	*Budget* 12
0	10000	13000	9000	11000
1	10000	13000	9000	12000
2	10000	14000	7000	13000
3	13000	17000	8000	10000
4	13000	17000	17000	10000
5	1000000	1000000	1000000	1000000
6	1000000	1000000	1000000	1000000
7	1000000	1000000	1000000	1000000
8	1000000	1000000	1000000	1000000
9	1000000	1000000	1000000	1000000
10	1000000	1000000	1000000	1000000
11	1000000	1000000	1000000	1000000
12	1000000	1000000	1000000	1000000
13	1000000	1000000	1000000	1000000
14	1000000	1000000	1000000	1000000
15	1000000	1000000	1000000	1000000
16	1000000	1000000	1000000	1000000
17	1000000	1000000	1000000	1000000

Table 4.9: Optimal Replacement Schedule for Budgets 8 - 12

Equipment	*Budget* 9	*Budget* 10	*Budget* 11	*Budget* 12
1	1	1	1	1
2	1	1	1	1
3	1	1	1	1
4	0	1	2	1
5	2	2	0	0
6	3	3	5	4
7	5	0	5	3
8	1	2	2	1
9	0	2	3	0
10	4	4	4	2
solution cost	$74,410	$73,380	$75,060	$74,790

In these results again we see that the optimal replacement schedule is for each different set of budget constraint is varied. For these cases we only changed the amount of the budget in years 5 – 17. When compared to the previous results we observe budget 9 to have a different replacement schedule from budget 5. Six out of the ten pieces of equipment were suggested to be replaced at a different time. Comparing the results of budget 10 and 6, it is observed only equipment eight is suggested to have a different replacement time. From the results of budget 7 we saw that the solution was infeasible but when the budgets are changed to budget 11 a solution is found. The solution for budget 12 is the same as the solution for budget 8.

Chapter 5

Conclusions and Directions for Future Research

The importance of sensitivity analysis is obvious when budget constraints are involved. As in our case, some of the equipment being used cost more for the hospital to operate and maintain than it would cost to replace the equipment. Determining the optimal replacement schedule for each individual piece of equipment is essential as money can be saved if the equipment is replaced at the right time.

We were able to successfully determine the most efficient method to use when trying to determine the costs associated with sensitivity analysis. We were also able to successfully determine the optimal replacement schedule for a set of equipment suggested to be replaced at a given time under budget constraints. In trying to determine the most efficient method to investigate sensitivity analysis we illustrated that Bellman's classical model is more efficient than Wagner's method.

In our case study we investigated the replacement schedule of 10 pieces of equipment. Analyze the replacement schedules under different budget constraints. The integer program developed in this thesis can be used to analyze multiple pieces of equipment under a budget constraint.

Further analysis into the sensitivity analysis can be done to find an easier method of calculating the delay or acceleration costs. Also, the sensitivity analysis can further be analyzed to determine the minimum amount of funds which needs to be allocated each year. Future work in this area can also analyze larger amounts of equipment for replacement under budget constraints, using the integer program suggested in chapter 3 to create a model and analyze a larger data set with varying costs and budget constraints. We did not encounter computational problems but these are possible with larger problems as integer programming has been proposed.

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