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Calculating Vacancy Positions to Optiamally Assign Recruits of the ROK Army

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by

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Abstract

Since the number of troops in the ROK Army will gradually decrease, efficient personnel assignment is required to improve the level of combat power. Therefore, the process of assigning recruits will become more important. In the current system, the calculation of the vacancy positions for assigning recruits is performed manually by a person. Thus, the occurrence of mistakes in the process and the inefficiencies of the calculation results are inevitable problems. In particular, imbalances due to deviations among combat powers after the assignment of recruits can be a major problem. The purpose of the new model presented in this paper is to reduce these deviations among combat powers. Randomized data sets were used for the experiments. The difference between the result of applying the current system and the result of applying the new system were confirmed. Tables, graphs and statistical hypothesis testing were performed to compare the results. After these experiments, it was confirmed that there are significant differences between the results on the current system and those of the new system. Therefore, we can conclude that the proposed new system is more suitable for efficient assignments than the current system. These results imply that the application of the new system can help solve the imbalances among combat powers that occur during the assignment of recruits.

Chapter 1

Introduction

1.1 Background and motivation

Recently, the agenda regarding shortening the mandatory military service period has been proposed again in the Republic of Korea (ROK) [1]. It is one of the promises that the current President has made during his campaign. It is the official position of the government that they are currently discussing this topic and nothing is yet finalized. In the past, the ROK has shortened the service period multiple times. Some critics of this proposition claim that shortening the service period is dangerous. They expect that the shortening of the service period will reduce military capabilities and that will lead to the weakening of military combat power. Of course, these expectations may hold some truth, but as there is still no definite implementation of shortening the service period, nothing can be predicted with certainty as of now.

However, the reduction of military forces in the ROK has already been in place for a long period of time. Due to the new proposal of shortening the service period, the issue of military forces reduction is being highlighted once again. The proposed plan is called the National Defense Reform Plan 2020. Bennett [2] explains the outline of the plan in his paper. The most noticeable aspect of this plan is the change in the number of troops. As of 2020, the military capacity of the Air Force is expected to increase slightly compared to 2004 and the capacity of the Navy is expected to decline slightly, but there seems to be no significant difference. On the other hand, it can be seen that the Army's forces have been reduced from about 560,000 personnel to around 400,000. Therefore, it is necessary to focus specifically on finding ways to maintain and to improve the Army's combat power.

The reduction of military forces can weaken the Army's combat power. In their book, Tellis at el. [3] stated that the size of military forces has a significant relationship with the military combat power when describing military capabilities and national power. In this regard, some studies had been conducted to determine the appropriate size of military forces in various situations [4] [5] [6]. However, the reduction of military forces in the ROK is already underway and we need to focus on it. On the premise of a reduction of the force of the ROK Army, we need to find a way to strengthen its combat power.

As the number of troops decrease, the importance of each individual will increase. Assuming that a military unit's mission is unchanged, the decrease in the number of troops implies the greater importance of each individual in the unit's total combat power. For example, suppose you have replaced 100 soldiers who were watching the border area with 10 cameras. Suppose each camera will be operated by 10 people. The mission is unchanged, but due to the development of technology, the number of soldiers is reduced. When calculated mathematically, if one person makes a mistake while 100 people are watching, the probability of failing the border operation is 1%. However, if one camera fails to operate due to one soldier's mistake, the probability is 10%. Thus, we can say that as the number of troops decreases, the importance of each individual in the unit becomes greater.

The fact that each individual becomes more important means that the assignment of recruits is now more crucial than ever. In the future, it will be difficult for each unit to have enough soldiers due to the downsizing of military forces. The change in combat power in a unit occurs when a recruit is assigned or discharged. Of course, sometimes it is changed due to other reasons. However, they are not considered as variables because they are uncommon. Moreover, since the period of soldiers' service is precisely defined in the ROK Army, there is nothing we can do about it. As a result, the assignment of recruits has the greatest impact on the combat power of a unit. If one recruit is not assigned to a unit, then a certain function of the unit may not be performed. An improper assignment of one recruit may result in the imbalances among combat powers. Therefore, the assignment of recruits should be performed with great precision.

1.2 Current assignment system

1.2.1 Assigning recruits and calculating vacancy positions

The assignment of recruits is composed of two factors: giving recruits a military specialty, and placing them in a specific unit. First, each recruit is given a particular military specialty. Military specialties are decided in consideration of the vacancy positions and the individual characteristics such as the education level, physical condition and personal goals. Then, each recruit is placed in his respective subordinate unit within the selected specialty. They are randomly assigned units for fairness [7]. The ROK Army has an accurate and fair system for these procedures and the system is operated effectively.

But there is one aspect does not have its own system: the calculation of vacancy positions. Calculating the vacancy positions means determining how many recruits will be assigned to each unit and to each specialty. This can be calculated from the demands for military specialties of each unit. Of course, as the number of troops decreases, it cannot meet all the demands. Therefore, it is important to accurately calculate the vacancy positions. In the current system, these vacancy positions are calculated by the personnel officers of each unit. When calculating, they consider the conditions of each subordinate unit. These conditions are the required number of soldiers, the number of soldiers currently in service, and the estimated total number of soldiers that will be discharged in 3 months. Also, the personnel officers consider the combat powers of each subordinate unit. Table 1.1 below is a sample of the table currently used by the personnel officers of each unit when they assign recruits. This table represents one unit. As a personnel officer of this unit, let us suppose that we should assign recruits in each subordinate unit and each specialty. This unit has 10 subordinate units from A to J, and twenty specialties from 1 to 20. From specialties 1 to 7, recruits don't have their particular specialties. See Sector 1 in Table 1.1. When we assign 171 recruits of specialties 1 to 7 in each subordinate unit and each specialty, we can assign them in any subordinate unit and any specialty 1 to 7. For specialties 8 to 20, however, recruits cannot be dispersed randomly. See Sector 2 in Table 1.1. The number of recruits corresponding to the specialty 13 is 2. It means that the 2 recruits already have the specialty 13. Therefore, we have to assign them to specialty 13, but can assign them to any subordinate unit.

Table 1.1: A sample of the current assignment

	specialty	With no specialty							With specialty												
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	recruits	171							2	2	2	2	2	2	2	2	2	2	2	2	2
unit	specialty	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	Required	170	140	135	156	28	28	16	5	17	7	5	11	11	3	17	9	9	1	7	19
	Serving	175	137	140	150	29	27	16	5	17	7	5	12	11	3	17	9	9	1	7	19
A	Difference	5	-3	5	-6	1	-1	0	0	0	0	0	1	0	0	0	0	0	0	0	0
	Now Rate	102.9%	97.9%	103.7%	96.2%	103.6%	96.4%	100.0%	100.0%	100.0%	100.0%	100.0%	109.1%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	Discharge	24	20	19	22	4	4	2	1	2	1	1	2	2	0	2	1	1	0	1	3
100%	Assign				S									S							
	After Rate	88.8%	83.6%	89.6%	82.1%	89.3%	82.1%	87.5%	80.0%	88.2%	85.7%	80.0%	90.9%	81.8%	100.0%	88.2%	88.9%	88.9%	100.0%	85.7%	84.2%
	Required	170	140	135	156	10	16	25	7	15	21	19	13	15	17	13	11	11	7	19	7
	Serving	173	146	131	159	10	15	24	7	15	20	18	13	16	18	13	11	11	7	19	7
B	Difference	3	6	-4	3	0	-1	-1	0	0	-1	-1	0	1	1	0	0	0	0	0	0
	Now Rate	101.8%	104.3%	97.0%	101.9%	100.0%	93.8%	96.0%	100.0%	100.0%	95.2%	94.7%	100.0%	106.7%	105.9%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	Discharge	24	20	19	22	1	2	4	1	2	3	3	2	2	2	2	2	2	1	3	1
100%	Assign				E									E							
	After Rate	87.6%	90.0%	83.0%	87.8%	90.0%	81.3%	80.0%	85.7%	86.7%	81.0%	78.9%	84.6%	93.3%	94.1%	84.6%	81.8%	81.8%	85.7%	84.2%	85.7%
	Required	170	140	135	156	1	1	4	15	7	19	9	13	17	21	19	7	5	21	17	19
	Serving	173	139	131	154	1	1	4	16	7	20	9	13	16	22	19	7	5	21	17	20
C	Difference	3	-1	-4	-2	0	0	0	1	0	1	0	0	-1	1	0	0	0	0	0	1
	Now Rate	101.8%	99.3%	97.0%	98.7%	100.0%	100.0%	100.0%	106.7%	100.0%	105.3%	100.0%	100.0%	94.1%	104.8%	100.0%	100.0%	100.0%	100.0%	100.0%	105.3%
	Discharge	24	20	19	22	0	0	1	2	1	3	1	2	2	3	3	1	1	3	2	3
100%	Assign				C									C							
	After Rate	87.6%	85.0%	83.0%	84.6%	100.0%	100.0%	75.0%	93.3%	85.7%	89.5%	88.9%	84.6%	82.4%	90.5%	84.2%	85.7%	80.0%	85.7%	88.2%	89.5%
	Required	55	134	109	126	25	16	16	17	15	7	3	17	7	15	11	5	11	9	3	9
	Serving	59	154	123	139	27	17	18	19	17	7	3	19	8	17	12	5	12	10	3	10
D	Difference	4	20	14	13	2	1	2	2	2	0	0	2	1	2	1	0	1	1	0	1
	Now Rate	107.3%	114.9%	112.8%	110.3%	108.0%	106.3%	112.5%	111.8%	113.3%	100.0%	100.0%	111.8%	114.3%	113.3%	109.1%	100.0%	109.1%	111.1%	100.0%	111.1%
	Discharge	8	19	16	18	4	2	2	2	2	1	0	2	1	2	2	1	2	1	0	1
110%	Assign				T									T							
	After Rate	92.7%	100.7%	98.2%	96.0%	92.0%	93.8%	100.0%	100.0%	100.0%	85.7%	100.0%	100.0%	100.0%	100.0%	90.9%	80.0%	90.9%	100.0%	100.0%	100.0%
	Required	55	134	109	126	1	1	4	7	3	1	13	3	19	13	7	5	5	5	7	9
	Serving	62	151	122	140	1	1	5	7	3	1	14	3	22	14	8	6	5	5	8	10
E	Difference	7	17	13	14	0	0	1	0	0	0	1	0	3	1	1	1	0	0	1	1
	Now Rate	112.7%	112.7%	111.9%	111.1%	100.0%	100.0%	125.0%	100.0%	100.0%	100.0%	107.7%	100.0%	115.8%	107.7%	114.3%	120.0%	100.0%	100.0%	114.3%	111.1%
	Discharge	8	19	16	18	0	0	1	1	0	0	2	0	3	2	1	1	1	1	1	1
110%	Assign				O									O							
	After Rate	98.2%	98.5%	97.2%	96.8%	100.0%	100.0%	100.0%	85.7%	100.0%	100.0%	92.3%	100.0%	100.0%	92.3%	100.0%	100.0%	80.0%	80.0%	100.0%	100.0%
	Required	10	7	16	16	26	49	17	21	13	9	1	1	21	9	17	11	19	11	17	17
	Serving	10	7	16	16	26	51	16	20	13	9	1	1	22	9	18	11	20	11	16	17
F	Difference	0	0	0	0	0	2	-1	-1	0	0	0	0	1	0	1	0	1	0	-1	0
	Now Rate	100.0%	100.0%	100.0%	100.0%	100.0%	104.1%	94.1%	95.2%	100.0%	100.0%	100.0%	100.0%	104.8%	100.0%	105.9%	100.0%	105.3%	100.0%	94.1%	100.0%
	Discharge	1	1	2	2	4	7	2	3	2	1	0	0	3	1	2	2	3	2	2	2
100%	Assign				R									R							
	After Rate	90.0%	85.7%	87.5%	87.5%	84.6%	89.8%	82.4%	81.0%	84.6%	88.9%	100.0%	100.0%	90.5%	88.9%	94.1%	81.8%	89.5%	81.8%	82.4%	88.2%
	Required	31	10	25	16	26	49	17	17	15	3	19	13	13	5	3	13	5	13	3	11
	Serving	30	10	26	16	26	51	17	17	15	3	19	12	14	5	3	13	5	13	3	11
G	Difference	-1	0	1	0	0	2	0	0	0	0	0	-1	1	0	0	0	0	0	0	0
	Now Rate	96.8%	100.0%	104.0%	100.0%	100.0%	104.1%	100.0%	100.0%	100.0%	100.0%	100.0%	92.3%	107.7%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	Discharge	4	1	4	2	4	7	2	2	2	0	3	2	2	1	0	2	1	2	0	2
100%	Assign				1									2							
	After Rate	83.9%	90.0%	88.0%	87.5%	84.6%	89.8%	88.2%	88.2%	86.7%	100.0%	84.2%	76.9%	92.3%	80.0%	100.0%	84.6%	80.0%	84.6%	100.0%	81.8%
	Required	19	16	1	7	26	49	17	17	1	19	13	11	1	5	7	7	9	11	13	9
	Serving	19	16	1	7	25	49	18	17	1	20	13	11	1	5	7	7	9	10	14	9
H	Difference	0	0	0	0	-1	0	1	0	0	1	0	0	0	0	0	0	0	-1	1	0
	Now Rate	100.0%	100.0%	100.0%	100.0%	96.2%	100.0%	105.9%	100.0%	100.0%	105.3%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	90.9%	107.7%	100.0%
	Discharge	3	2	0	1	4	7	2	2	0	3	2	2	0	1	1	1	1	2	2	1
100%	Assign																				
	After Rate	84.2%	87.5%	100.0%	85.7%	80.8%	85.7%	94.1%	88.2%	100.0%	89.5%	84.6%	81.8%	100.0%	80.0%	85.7%	85.7%	88.9%	72.7%	92.3%	88.9%
	Required	16	1	4	31	3	68	39	15	5	7	11	9	21	21	1	1	5	13	3	15
	Serving	16	1	4	30	3	65	37	15	5	7	11	9	22	20	1	1	5	13	3	15
I	Difference	0	0	0	-1	0	-3	-2	0	0	0	0	0	1	-1	0	0	0	0	0	0
	Now Rate	100.0%	100.0%	100.0%	96.8%	100.0%	95.6%	94.9%	100.0%	100.0%	100.0%	100.0%	100.0%	104.8%	95.2%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	Discharge	2	0	1	4	0	10	6	2	1	1	2	1	3	3	0	0	1	2	0	2
100%	Assign																				
	After Rate	87.5%	100.0%	75.0%	83.9%	100.0%	80.9%	79.5%	86.7%	80.0%	85.7%	81.8%	88.9%	90.5%	81.0%	100.0%	100.0%	80.0%	84.6%	100.0%	86.7%
	Required	10	10	28	16	3	68	39	11	21	5	15	11	3	17	15	5	15	5	5	5
	Serving	10	10	28	16	3	67	39	12	22	5	15	11	3	16	15	5	15	5	5	5
J	Difference	0	0	0	0	0	-1	0	1	1	0	0	0	0	-1	0	0	0	0	0	0
	Now Rate	100.0%	100.0%	100.0%	100.0%	100.0%	98.5%	100.0%	109.1%	104.8%	100.0%	100.0%	100.0%	100.0%	94.1%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	Discharge	1	1	4	2	0	10	6	2	3	1	2	2	0	2	2	1	2	1	1	1
100%	Assign																				
	After Rate	90.0%	90.0%	85.7%	87.5%	100.0%	83.8%	84.6%	90.9%	90.5%	80.0%	86.7%	81.8%	100.0%	82.4%	86.7%	80.0%	86.7%	80.0%	80.0%	80.0%

1.2.2 Deficiencies of the current system

First, the procedure of calculating vacancy positions is done manually by a person. This means that we have to account for human errors regarding accuracy and consistency. If the calculations are performed for many units, specialties and recruits, then such problems could be more severe. In reality, those conditions often arise. Therefore, calculating vacancy positions should be conducted on a given system.

Second, the weight of each unit is not reflected in the calculation of combat power. Some units are encouraged to have more people than is required due to the importance of the unit's mission. This occurrence is to make sure that the unit is able to exert more than 100% of its combat power at all times. However, the current system does not reflect this when calculating combat power. This claim can be confirmed by the following example in Table 1.2. The demand for two units are the same and equal 50, and the numbers required for each specialty are the same. The first unit is encouraged to have 110% combat power, and the second unit is encouraged to have 100%. Current combat power is 86% for the first unit and 80% for the second unit. If there is one recruit, to which unit should we assign the recruit? We cannot know easily because the weight is not reflected. If weights are considered, the first unit's combat power will be 78.18% and the second unit's combat

Table 1.2: Reflection of unit weights

UNIT #1									
specialties	1	2	3	4	5	total	rate	new rate	weight
serving	9	5	16	8	5	43	0.86	0.78	110%
demand	10	5	20	10	5	50			
UNIT #2									
specialties	1	2	3	4	5	total	rate	new rate	weight
serving	7	4	15	9	5	40	0.8	0.8	100%
demand	10	5	20	10	5	50			

power will not be changed. If we assign the recruit to the first unit, its combat power will be 80%. Then, the two units will have the same combat power. In this way, it is necessary to reflect the weight of each unit to the combat power.

Third, the method for calculating combat power is too simple. In the current system, combat power is represented by the number of soldiers currently serving, compared to the required number of soldiers. Consider the following example in Table 1.3. These are the same units as Table 1.2, but the number of people currently serving has been changed. For both units, 35 people are currently serving and have a combat power of 70%. Their combat powers are the same in the current system. However, we can see that the combat power of the first unit's specialty 5 is extremely low at 40%. If a specialty of a unit has a significantly lower combat power and cannot perform its function, the combat power of the entire unit may be considered much lower than its surface value. In other words, the combat powers of each specialty in a unit should be at a similar level. Therefore, the calculation of combat power must be an average of combat powers of each specialty. The new combat power calculated using this method is 65% for the first unit and 73% for the second unit. The variance of the first unit is 0.02 and it of the second unit is 0.0036. This implies that the differences of the combat power of each specialty in the second unit are smaller. From these results, it can be seen that the second unit actually has more stable combat power. In this way, it is better to define the combat power as the average of the combat powers of each specialty. Of course, as with the third unit in Table 1.4, there could be a higher combat power with higher variance. The combat power of the third unit is 67%, which is higher than 65% of the first unit, but the variance of the third unit is also higher than it of the first unit. For this reason, the deviations among combat powers will be an important factor in this study.

Table 1.3: Calculating combat power

UNIT #1									
specialties	1	2	3	4	5	total	rate	new rate	variance
serving	8	3	15	7	2	35	0.7	0.65	0.02
demand	10	5	20	10	5	50			
rate	0.8	0.6	0.75	0.7	0.4	0.7			
UNIT #2									
specialties	1	2	3	4	5	total	rate	new rate	variance
serving	7	4	13	7	4	35	0.7	0.73	0.0036
demand	10	5	20	10	5	50			
rate	0.7	0.8	0.65	0.7	0.8	0.7			

Table 1.4: Another unit with a higher variance

UNIT #3									
specialties	1	2	3	4	5	total	rate	new rate	variance
serving	7	5	15	7	1	35	0.7	0.67	0.0676
demand	10	5	20	10	5	50			
rate	0.7	1	0.75	0.7	0.2	0.7			

1.3 Literature review

1.3.1 Military personnel assignment

A significant research effort has been devoted to studying the effective personnel assignment in the military. Because the word ‘efficiency’ is subjective, the research has been conducted in various directions depending on the criteria of each researcher. These directions can be narrowed to the following two topics. One focus of research could be to reduce the cost in the system of personnel assignment. Another could be the assignment of manpower while satisfying many conditions such as personal preferences, working areas, and personal experiences.

Liang and Thompson [8] studied a large-scale personnel assignment model for the Navy and presented it as a multi-objective model. They solved the problem using a network model. Based on the transportation model, they proposed a solution by weighing each policy. This study suggested an assignment model that satisfies 11 policies, such as minimizing travel costs or maximizing personal preferences. This study was meaningful in that it presented a suitable multi-objective model that could satisfy-many conditions at the same time.

Maskos [9] studied the optimal assignment of marine recruits to occupational training. He proposed a binary multi-objective optimization model that could meet the four goals required. It is solved through two steps: first, it used integrality relaxation and second, it used upper and lower bounds to find integer solutions in a network model. The purpose of this study is to place each individual at an optimal location according to given conditions.

Enoka [10] conducted a study on optimizing marine security guard assignments. The model in this study was presented as an integer linear program, with the goal of efficiently assigning marine security guards (MSG) to billets. Specifically, it minimized the assignment cost and balanced the experience level of MSGs across detachments. Also, it pursued to reduce working hours by presenting an Excel-based decision support tool. In this study, weights that can be adjusted by the user were introduced to ensure flexibility.

Hooper [11] studied optimizing marine corps personnel assignments using an integer programming model. This study pointed out that the existing assignment system operated well without any problems, but it could not minimize the cost of the assignment. Therefore, the study aimed at minimizing the assignment cost while satisfying the conditions such as military specialty, billet vacancy, and personal preference.

As mentioned earlier, these studies are aimed at minimizing costs or satisfying various conditions. However, my particular focus of study is to find out how many people should be assigned to each unit and how many people should be assigned to each military specialty for even combat powers. My conclusions indicate that each specialty and each unit maintains a similar level of combat power. Much of the existing research in this field does not overlap with my focus of study.

1.3.2 Equitable assignment

Sabado [12] conducted a study on the equitable assignment of recruits. His model was presented as a nonlinear integer programming model. In this model, he used variables that could be changed by the decision maker. This allowed users to gain flexibility as in previous studies. The purpose of his study is to calculate the quality of MSG when they are placed and to assign them so that the quality level of all regions is as similar as possible based on their quality. The purpose of this study is similar to the purpose of my research, in which the combat power of each unit must remain at a similar level. Sabado's study proposed a method to minimize the sum of squared differences to make the quality level of the regions equal. The solution could be an optimal way to achieve the goal of creating an equivalent standard. However, the process of solving this problem is somewhat inconvenient. He calculated all squared differences manually to determine values of the objective function. If this method is used in my research, the process may not be appropriate. This calculation can take a lot of time if the number of units increases. Additionally, in my study the calculation process is divided into two steps to achieve two objectives, while the model he presented was for one objective. If it was used in my research, then I could have accomplished only one of two objectives. Therefore, applying the solution as proposed by him may be inefficient for my study. So I opted for a slightly simpler alternative.

Li et al. [13] studied the subcarrier assignment and power allocation problem. This is not a study of personnel assignment, but it has been helpful in my research as a study of the equitable assignment. What is important in this study is the concept of max-min fairness. This is to reduce the deviation between the highest value and the lowest value by maximizing the lowest value. As a result, all the values can be maintained at a similar level as they exist within the deviation. This principle can be applied to my research as follows. In the second step, we will assign personnel not assigned in the first step, by assigning them first in the unit with the lowest combat power. The levels of combat powers of each specialty are similar to each other throughout the first step, and the combat power deviation between the units can be reduced by additional assignment to the unit with the lowest combat power following the second step. In my study, I solved the problem using max-min fairness as proposed by Li et al.

1.4 Purpose of thesis

The purpose of this study is to create an efficient system of calculating vacancy positions. According to the National Defense Reform Plan 2020, the ROK Army faces the problem of massive force reduction. Therefore, it is necessary to improve the level of combat power through the efficient assignment of recruits. This study will contribute to improving the

ROK Army's combat power by presenting a new system for calculating vacancy positions when assigning recruits of the ROK Army. It has the following assumptions.

First, this model can be applied in a broad sense. In other words, in this study, we will not deal with only specific units like regiments or divisions. So I will only use two words ‘unit’ and ‘subordinate unit’. In Table 1.1, the unit represents all of the data, and subordinate units represent data corresponding to the alphabet. And the actual assignments of recruits are carried out in a variety of units; from Army training camps to division and brigade [7]. Also, all units’ cycle of assignment, number of recruits, number of units and number of specialties they have may differ. Therefore, this study will account for the most general type of system.

Second, the word ‘combat power' as used in my thesis has a slight difference from the actual meaning. Combat power is commonly defined as “The total means of destructive and/or disruptive force that a military unit/formation can apply against the opponent at a given time.” [14]. In the general sense, there are a number of factors besides the number of soldiers that can be used to calculate actual combat power. However, my study only discusses the relationship between the number of soldiers and combat power. Therefore, the use of combat power in my thesis can be accounted as simply the number of soldiers.

Third, since the background of this paper focuses on the reduction of military forces, the total number of recruits is less than the total number required by the unit. In addition, the number of losses to be considered in assigning recruits is the estimated amount loss for three months in total. In other words, the number of personnel required by the unit means that the personnel must be supplemented for a total of three months. Normally, the assignment of recruits is conducted several times over three months. Therefore, it is appropriate to assume that the total number of recruits is less than the total number required by the unit.

1.5 Outline of thesis

In Chapter 2, I will explain the conditions that should be applied when assigning recruits of the ROK Army, and describe the new system that satisfies the conditions above. In Chapter 3, I will show the data required and use it to experiment both the current system and the new system. I will then compare the results of the experiments and explain what information we can derive from it. In Chapter 4, I will discuss the results of this study, its limitations, and future research.

Chapter 2

A model for calculating vacancy positions

2.1 Purpose and conditions

The purpose of the assignment of the recruits is to make each combat (unit) power similar. More specifically, after the assignment, the combat powers of each specialty must remain similar. In order for a unit to perform its normal combat power, all functions must be exercised. However, different specialties function differently. Therefore, it is necessary to reduce the deviations among the combat powers of each specialty. Likewise, the combat powers of each subordinate unit must remain similar. Each subordinate unit is assigned a different mission. Failure of a subordinate unit's mission will also affect the whole unit. Therefore, it is also necessary to reduce the deviations among the combat powers of each subordinate unit.

There are several conditions that must be considered when assigning recruits. The current system is operated under the currently specified conditions, and the new system will also be realized considering these conditions. Some units, however, also necessitate special conditions for their situations: such conditions are excluded because they are unusual and cannot be applied to all units. The following conditions are core contents that are common for all units.

First, every time recruits are assigned, the cycle is not regular and the number of recruits is not constant. In addition, the number of annual execution of assignments varies from unit to unit. It is therefore difficult to consider the impact of the previous or next assignment of recruits. In other words, all assignments are based on only the information given at the time, without considering the results of a previous assignment or an expected assignment.

Second, the estimated discharged number is the sum of the anticipated discharged within three months. Sometimes, many soldiers are discharged at one time. In this case, the unit should anticipate and prepare for massive manpower losses. Therefore, through several assignments, it is possible to prevent the manpower loss in the future by considering the 3-month loss beforehand.

Third, the required combat power of each unit may be different. In the ROK Army, each unit has a table showing the number of soldiers it must have. It is called the Table of Organization and Equipment (TOE). The TOE is defined as “A document that authorizes a unit’s formation, personnel, and equipment and prescribes its mission.”[15]. It indicates the total number and a number for each specialty of soldiers according to the duty and character of the unit. Most units are required to maintain a 100% combat power based on the TOE. However, some units are required to maintain a level higher than 100%. The reasoning behind this is that, in all units, there are personnel that are not included in the actual combat power due to circumstances such as vacation, illness, etc. Considering this fact, to ensure 100% or higher combat power for those unit, some units have to have more soldiers than the others. This has been illustrated above on section 1.2.2.

Fourth, recruits can be divided into two types depending on whether they have specialties. Some of the recruits do not have particular specialties, therefore we assign them to certain specialties. The other type of recruits are those who were already given specialties and thus we cannot assign them to any other specialty. Of the total recruits, there are more recruits who do not already possess specialties.

In addition, the calculation of combat power has been changed in the new model. I mentioned earlier that there are two problems with how to calculate the combat power in the current system on section 1.2.2. It was a matter of considering the weight and sub-steps. So the new model will use a new calculation method to solve these two problems as described in the following section.

2.2 Formulation

In this model, two steps are required to calculate vacancy positions. It can be expressed as shown in Figure 2.1. The first step is to minimize the deviations among the combat powers of each specialty, and the second step is to minimize the deviations among the combat powers of each subordinate unit.

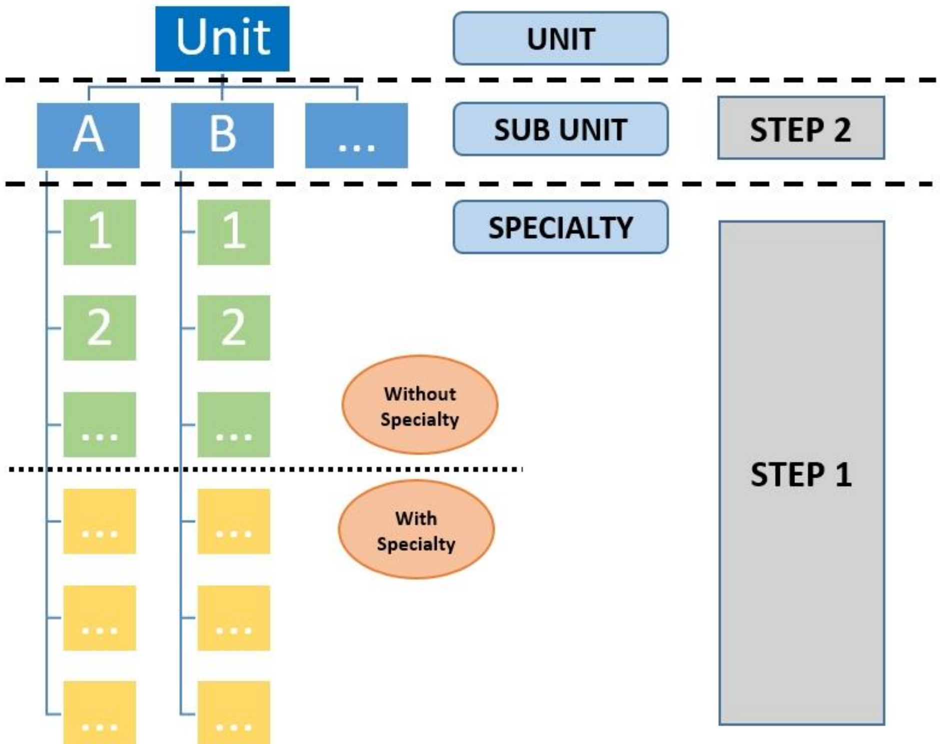

Figure 2.1: Procedures of the new system

2.2.1 Step 1

Parameters

- $\boldsymbol{I}$ Set of subordinate units, indexed by $i \in \{1, \dots, m\}$
- $\boldsymbol{J}$ Set of specialties, indexed by $j \in \{1, \dots, k\}$ for α_1 / $j \in \{k+1, \dots, n\}$ for α_{2j}
- $\boldsymbol{m}$ Total number of subordinate units
- $\boldsymbol{k}$ Number of specialties that we can assign
- $\boldsymbol{n}$ Total number of specialties
- $\boldsymbol{W_i}$ Weight for subordinate unit i, for each $i \in I$
- $\boldsymbol{R_{ij}}$ Number in the TOE for subordinate unit i, specialty j, for each $i \in I, j \in J$

$\boldsymbol{S_{ij}}$ Number of soldiers currently in service for subordinate unit i, specialty j, for each $i \in I, j \in J$

$\boldsymbol{E_{ij}}$ Number of estimated loss of soldiers in 3 months for subordinate unit i, specialty j, for each $i \in I, j \in J$

$\boldsymbol{C}$ Total number of recruits that are newly coming without specialty for $j \in \{1, \ldots, k\}$

$\boldsymbol{C_j}$ Number of recruits that are newly coming with specialty j, for each $j \in \{k+1, \ldots, n\}$

Variable

$\boldsymbol{\alpha_1}$ Highest combat power can be made with C for all subordinate unit and specialty for $j \in \{1, \ldots, k\}$

$\boldsymbol{\alpha_{2j}}$ Highest combat power can be made with C_j for all subordinate unit in specialty j, for each $j \in \{k+1, \ldots, n\}$

Objective Function

For α_1, $j \in \{1, \ldots, k\}$

$$\boldsymbol{Maximize} \quad \alpha_1$$

For recruits without specialties, we should find a maximum value of α_1. α_1 is equal to combat power. When solving this problem with the following constraints and obtaining α_1, the combat power of all specialties is at least α_1. It means they will become as similar as possible for all subordinate unit with all specialty.

For $\alpha_{2j,}$ $j \in \{k+1, \ldots, n\}$

$$\boldsymbol{Maximize} \quad \sum_{j=k+1}^{n} \boldsymbol{\alpha_{2j}}$$

For recruits with specialties, we should find a maximum value of summation for all α_{2j}. We actually need to maximize each α_{2j}. However, each α_{2j} is calculated independently for each j. Therefore, with the objective function above we can get each α_{2j} value. When solving this problem with the following constraints and obtaining α_{2j}, the combat power of all specialties with j specialty is at least α_{2j}. It means they will become as similar as possible for all subordinate unit with each specialty j.

Constraints

For $\alpha_1, j \in \{1, \dots, k\}$

$$\sum_{i=1}^{m}\sum_{j=1}^{k}\boldsymbol{max}\ \left(\left\lceil\left[\boldsymbol{W_i} \cdot \boldsymbol{R_{ij}}\right] \cdot \boldsymbol{\alpha}_1\right\rceil - \boldsymbol{S_{ij}} + \boldsymbol{E_{ij}}, \boldsymbol{0}\right) \leq \boldsymbol{C}$$

$\left\lceil[W_i \cdot R_{ij}] \cdot \alpha_1\right\rceil - S_{ij} + E_{ij}$ equals the number of soldiers required to achieve combat power α_1 for all subordinate unit i and all specialty j. This number can be negative if there are enough soldiers already serving. By using the max function, we can obtain a certain value if it is positive or 0 if it is negative. The purpose of this constraint is to find α_1 within a range that the summation of all the demands does not exceed C, the total number of recruits we can assign. This can be changed to an Integer Programming (IP) problem through the following process. For this, we need new variables and parameters.

$\boldsymbol{M_{ij}}$	Rounding $(W_i \cdot R_{ij})$ for subordinate unit i, specialty j, for each $i \in I, j \in \{1, \dots, k\}$
$\boldsymbol{t}$	Sufficiently small positive parameter

- $[W_i \cdot R_{ij}] \Longrightarrow M_{ij}$

$$M_{ij} \in \mathbb{Z}, \qquad i \in I, j \in \{1, \dots, k\}$$

$$M_{ij} \le W_i \cdot R_{ij} + 0.5, \qquad i \in I, j \in \{1, \dots, k\}$$

$$M_{ij} \ge W_i \cdot R_{ij} - 0.5 + t, \qquad i \in I, j \in \{1, \dots, k\}$$

$\boldsymbol{Y_{ij}}$ Ceiling $(M_{ij} \cdot \alpha_1)$ for subordinate unit i, specialty j, for each $i \in I, j \in \{1, \dots, k\}$

- $\lceil M_{ij} \cdot \alpha_1 \rceil \Longrightarrow Y_{ij}$

$$Y_{ij} \in \mathbb{Z}, \qquad i \in I, j \in \{1, \dots, k\}$$

$$Y_{ij} \le M_{ij} \cdot \alpha_1 + 1 - t, \qquad i \in I, j \in \{1, \dots, k\}$$

$$Y_{ij} \ge M_{ij} \cdot \alpha_1, \qquad i \in I, j \in \{1, \dots, k\}$$

$\boldsymbol{V_{ij}}$ max $(Y_{ij} - S_{ij} + E_{ij}, 0)$ for sub unit i, specialty j, for each $i \in I, j \in \{1, \dots, k\}$

$\boldsymbol{Z_{ij}}$ $\begin{cases} 0, & \text{if } Y_{ij} - S_{ij} + E_{ij} \le 0 \\ 1, & \text{otherwise} \end{cases}$ for sub unit i, specialty j, for each $i \in I, j \in \{1, \dots, k\}$

$\boldsymbol{L}$ Sufficiently large positive parameter

- max $(Y_{ij} - S_{ij} + E_{ij}, 0) \Longrightarrow V_{ij}$

$$\sum_{i=1}^{m} \sum_{j=1}^{k} V_{ij} \le C$$

$$V_{ij} = (Y_{ij} - S_{ij} + E_{ij}) \cdot Z_{ij}, \qquad i \in I, j \in \{1, \dots, k\}$$

$$Z_{ij} \in \{0, 1\}, \qquad i \in I, j \in \{1, \dots, k\}$$

$$Y_{ij} - S_{ij} + E_{ij} \leq L \cdot Z_{ij}, \qquad i \in I, j \in \{1, \dots, k\}$$

$$Y_{ij} - S_{ij} + E_{ij} \geq L \cdot (Z_{ij} - 1), \qquad i \in I, j \in \{1, \dots, k\}$$

$$\mathbf{0 \leq \boldsymbol{\alpha}_1 \leq 1}$$

Since α_1 is the combat power, it is a ratio: α_1 should be greater than or equal to 0 and less than or equal to 1.

For $\alpha_{2j}, j \in \{k+1, \dots, n\}$

$$\boldsymbol{\sum_{i=1}^{m} \max\left(\lceil [W_i \cdot R_{ij}] \cdot \alpha_{2j} \rceil - S_{ij} + E_{ij}, 0\right) \leq C_j, \qquad j \in \{k+1, \dots, n\}}$$

$\lceil [W_i \cdot R_{ij}] \cdot \alpha_{2j} \rceil - S_{ij} + E_{ij}$ equals the number of soldiers required to achieve combat power α_{2j} for all subordinate units with specialty j. This number can be negative if there are enough soldiers already serving. Therefore, using the max function we can obtain a certain value if it is positive or a 0 if it is negative. The purpose of this constraint is to find α_{2j} within a range that the summation of all the demands for each specialty does not exceed C_j, the total number of recruits we can assign for each j. This can be changed to an IP problem by the same process as applied in α_1.

$$\boldsymbol{0 \leq \alpha_{2j} \leq 1, \qquad j \in \{k+1, \dots, n\}}$$

Again, since α_{2j} is the combat power: it is a ratio. So it should be greater than or equal to 0 and less than or equal to 1.

Final model form

For α_1, $j \in \{1, \dots, k\}$

$$\boldsymbol{Maximize} \quad \boldsymbol{\alpha}_1$$

$$\boldsymbol{Subject\ to} \quad \sum_{i=1}^{m}\sum_{j=1}^{k} \boldsymbol{V_{ij}} \le \boldsymbol{C}$$

$$\boldsymbol{V_{ij}} = \left(\boldsymbol{Y_{ij}} - \boldsymbol{S_{ij}} + \boldsymbol{E_{ij}}\right) \cdot \boldsymbol{Z_{ij}}, \qquad \boldsymbol{i} \in \boldsymbol{I}, \boldsymbol{j} \in \{\boldsymbol{1}, \dots, \boldsymbol{k}\}$$

$$\boldsymbol{Z_{ij}} \in \{\boldsymbol{0}, \boldsymbol{1}\}, \qquad \boldsymbol{i} \in \boldsymbol{I}, \boldsymbol{j} \in \{\boldsymbol{1}, \dots, \boldsymbol{k}\}$$

$$\boldsymbol{Y_{ij}} - \boldsymbol{S_{ij}} + \boldsymbol{E_{ij}} \le \boldsymbol{L} \cdot \boldsymbol{Z_{ij}}, \qquad \boldsymbol{i} \in \boldsymbol{I}, \boldsymbol{j} \in \{\boldsymbol{1}, \dots, \boldsymbol{k}\}$$

$$\boldsymbol{Y_{ij}} - \boldsymbol{S_{ij}} + \boldsymbol{E_{ij}} \ge \boldsymbol{L} \cdot \left(\boldsymbol{Z_{ij}} - \boldsymbol{1}\right), \qquad \boldsymbol{i} \in \boldsymbol{I}, \boldsymbol{j} \in \{\boldsymbol{1}, \dots, \boldsymbol{k}\}$$

$$\boldsymbol{M_{ij}} \in \mathbb{Z}, \qquad \boldsymbol{i} \in \boldsymbol{I}, \boldsymbol{j} \in \{\boldsymbol{1}, \dots, \boldsymbol{k}\} \tag{2.1}$$

$$\boldsymbol{M_{ij}} \le \boldsymbol{W_i} \cdot \boldsymbol{R_{ij}} + \boldsymbol{0.5}, \qquad \boldsymbol{i} \in \boldsymbol{I}, \boldsymbol{j} \in \{\boldsymbol{1}, \dots, \boldsymbol{k}\}$$

$$\boldsymbol{M_{ij}} \ge \boldsymbol{W_i} \cdot \boldsymbol{R_{ij}} - \boldsymbol{0.5} + \boldsymbol{t}, \qquad \boldsymbol{i} \in \boldsymbol{I}, \boldsymbol{j} \in \{\boldsymbol{1}, \dots, \boldsymbol{k}\}$$

$$\boldsymbol{Y_{ij}} \in \mathbb{Z}, \qquad \boldsymbol{i} \in \boldsymbol{I}, \boldsymbol{j} \in \{\boldsymbol{1}, \dots, \boldsymbol{k}\}$$

$$\boldsymbol{Y_{ij}} \le \boldsymbol{M_{ij}} \cdot \boldsymbol{\alpha_1} + \boldsymbol{1} - \boldsymbol{t}, \qquad \boldsymbol{i} \in \boldsymbol{I}, \boldsymbol{j} \in \{\boldsymbol{1}, \dots, \boldsymbol{k}\}$$

$$\boldsymbol{Y_{ij}} \ge \boldsymbol{M_{ij}} \cdot \boldsymbol{\alpha_1}, \qquad \boldsymbol{i} \in \boldsymbol{I}, \boldsymbol{j} \in \{\boldsymbol{1}, \dots, \boldsymbol{k}\}$$

$$\boldsymbol{0} \le \boldsymbol{\alpha}_1 \le \boldsymbol{1}$$

$For\ \alpha_{2j}, j \in \{k+1, \ldots, n\}$

$$\boldsymbol{Maximize} \quad \sum_{j=k+1}^{n} \boldsymbol{\alpha_{2j}}$$

$$\boldsymbol{Subject\ to} \quad \sum_{i=1}^{m} \boldsymbol{V_{ij}} \leq \boldsymbol{C_j}, \qquad \boldsymbol{j} \in \{\boldsymbol{k+1}, \ldots, \boldsymbol{n}\}$$

$$\boldsymbol{V_{ij}} = \left(\boldsymbol{Y_{ij}} - \boldsymbol{S_{ij}} + \boldsymbol{E_{ij}}\right) \cdot \boldsymbol{Z_{ij}}, \qquad \boldsymbol{i} \in \boldsymbol{I}, \boldsymbol{j} \in \{\boldsymbol{k+1}, \ldots, \boldsymbol{n}\}$$

$$\boldsymbol{Z_{ij}} \in \{\boldsymbol{0}, \boldsymbol{1}\}, \qquad \boldsymbol{i} \in \boldsymbol{I}, \boldsymbol{j} \in \{\boldsymbol{k+1}, \ldots, \boldsymbol{n}\}$$

$$\boldsymbol{Y_{ij}} - \boldsymbol{S_{ij}} + \boldsymbol{E_{ij}} \leq \boldsymbol{L} \cdot \boldsymbol{Z_{ij}}, \qquad \boldsymbol{i} \in \boldsymbol{I}, \boldsymbol{j} \in \{\boldsymbol{k+1}, \ldots, \boldsymbol{n}\}$$

$$\boldsymbol{Y_{ij}} - \boldsymbol{S_{ij}} + \boldsymbol{E_{ij}} \geq \boldsymbol{L} \cdot \left(\boldsymbol{Z_{ij}} - \boldsymbol{1}\right), \qquad \boldsymbol{i} \in \boldsymbol{I}, \boldsymbol{j} \in \{\boldsymbol{k+1}, \ldots, \boldsymbol{n}\}$$

$$\boldsymbol{M_{ij}} \in \mathbb{Z}, \qquad \boldsymbol{i} \in \boldsymbol{I}, \boldsymbol{j} \in \{\boldsymbol{k+1}, \ldots, \boldsymbol{n}\} \tag{2.2}$$

$$\boldsymbol{M_{ij}} \leq \boldsymbol{W_i} \cdot \boldsymbol{R_{ij}} + \boldsymbol{0.5}, \qquad \boldsymbol{i} \in \boldsymbol{I}, \boldsymbol{j} \in \{\boldsymbol{k+1}, \ldots, \boldsymbol{n}\}$$

$$\boldsymbol{M_{ij}} \geq \boldsymbol{W_i} \cdot \boldsymbol{R_{ij}} - \boldsymbol{0.5} + \boldsymbol{t}, \qquad \boldsymbol{i} \in \boldsymbol{I}, \boldsymbol{j} \in \{\boldsymbol{k+1}, \ldots, \boldsymbol{n}\}$$

$$\boldsymbol{Y_{ij}} \in \mathbb{Z}, \qquad \boldsymbol{i} \in \boldsymbol{I}, \boldsymbol{j} \in \{\boldsymbol{k+1}, \ldots, \boldsymbol{n}\}$$

$$\boldsymbol{Y_{ij}} \leq \boldsymbol{M_{ij}} \cdot \boldsymbol{\alpha_{2j}} + \boldsymbol{1} - \boldsymbol{t}, \qquad \boldsymbol{i} \in \boldsymbol{I}, \boldsymbol{j} \in \{\boldsymbol{k+1}, \ldots, \boldsymbol{n}\}$$

$$\boldsymbol{Y_{ij}} \geq \boldsymbol{M_{ij}} \cdot \boldsymbol{\alpha_{2j}}, \qquad \boldsymbol{i} \in \boldsymbol{I}, \boldsymbol{j} \in \{\boldsymbol{k+1}, \ldots, \boldsymbol{n}\}$$

$$\boldsymbol{0} \leq \boldsymbol{\alpha_{2j}} \leq \boldsymbol{1}, \qquad \boldsymbol{j} \in \{\boldsymbol{k+1}, \ldots, \boldsymbol{n}\}$$

Figure 2.2 shows the results before Step 1. We can see the differences among the combat powers. Figure 2.3 shows the results after Step 1 using the data in Table 1.1. It can be seen that the deviations of the combat powers of each specialty have been reduced.

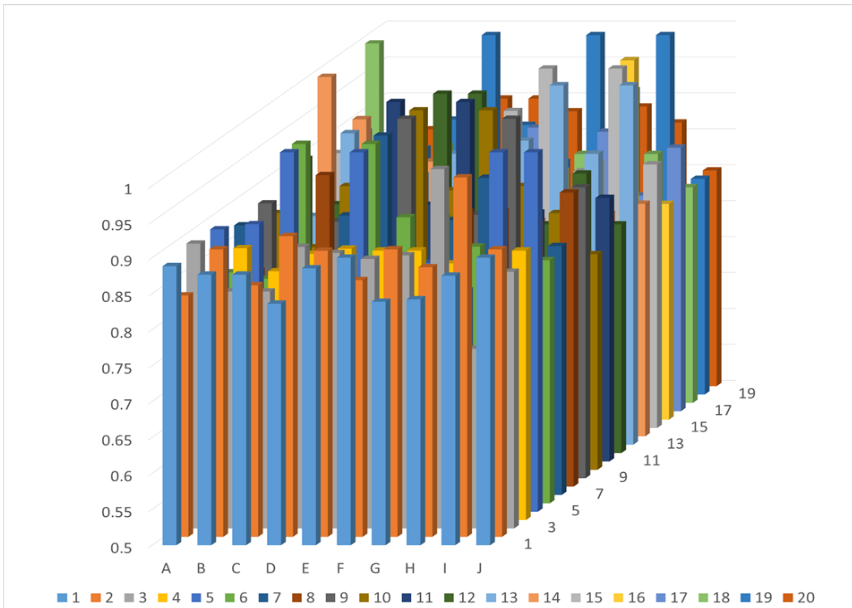

Figure 2.2: Results before step 1

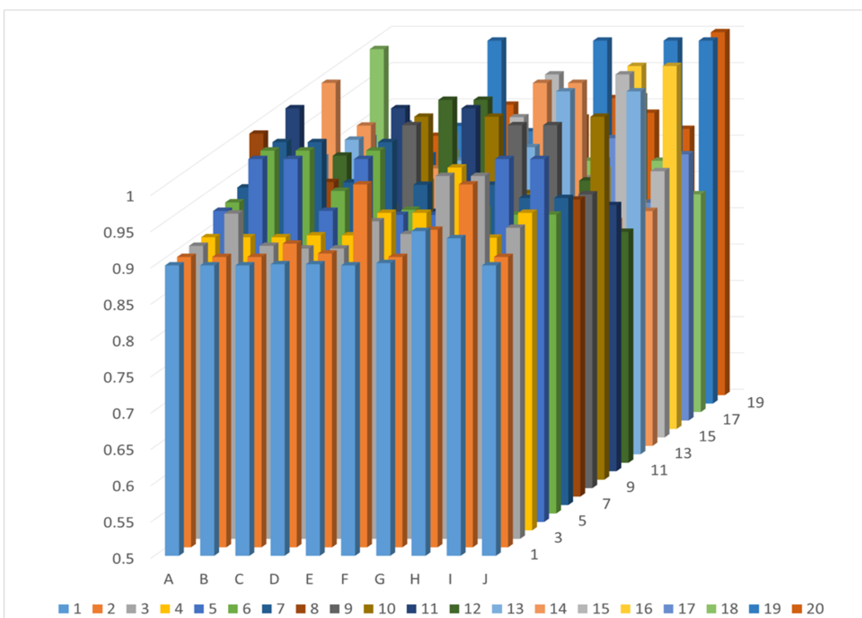

Figure 2.3: Results after step 1

2.2.2 Step 2

Parameters

$\boldsymbol{I}$ Set of subordinate units, indexed by $i \in \{1, \dots, m\}$

$\boldsymbol{J}$ Set of specialties, indexed by $j \in \{1, \dots, k, \dots, n\}$

$\boldsymbol{m}$ Total number of subordinate units

$\boldsymbol{k}$ Number of specialties that we can assign

$\boldsymbol{n}$ Total number of specialties

$\boldsymbol{W_i}$ Weight for subordinate unit i, for each $i \in I$

$\boldsymbol{R_{ij}}$ Number in the TOE for subordinate unit i, specialty j, for each $i \in I, j \in J$

$\boldsymbol{S_{ij}}$ Number of soldiers currently in service for subordinate unit i, specialty j, for each $i \in I, j \in J$

$\boldsymbol{E_{ij}}$ Number of estimated loss of soldiers in 3 months for subordinate unit i, specialty j, for each $i \in I, j \in J$

$\boldsymbol{C}$ Total number of recruits that are newly coming without specialty for $j = \{1, \dots, k\}$

$\boldsymbol{C_j}$ Number of recruits that are newly coming with specialty j, for each $j = \{k+1, \dots, n\}$

$\boldsymbol{\alpha_1}$ Highest combat power can be made with C for all subordinate unit and specialty for $j = \{1, \dots, k\}$

$\boldsymbol{\alpha_{2j}}$ Highest combat power can be made with C_j for all subordinate unit in specialty j, for each $j = \{k+1, \dots, n\}$

Variable

$\boldsymbol{X_{ij}}$ Number of recruits that assigned to subordinate unit i, specialty j, for each $i \in I, j \in J$

Objective Function

$$\boldsymbol{Maximize}\ \ \min_{i \in I} \frac{\mathbf{1}}{|J|}\sum_{j=1}^{n}\left(\frac{\boldsymbol{S_{ij} - E_{ij} + X_{ij}}}{[\boldsymbol{W_i \cdot R_{ij}}]}\right)$$

Here $\frac{S_{ij}-E_{ij}+X_{ij}}{[W_i \cdot R_{ij}]}$ is equal to the combat power of sub unit i, specialty j; and $\frac{1}{|J|}\sum_{j=1}^{n}\left(\frac{S_{ij}-E_{ij}+X_{ij}}{[W_i \cdot R_{ij}]}\right)$ equals the average of the combat power of all the specialties of the sub unit i, that is, the combat power of unit i. In Step 1, the combat power of each specialty has been maintained at a similar level. Since the combat powers of each specialty do not vary much, the combat powers of each unit will remain somewhat similar. '$Maximize\ min$' will make them more similar by maximizing the lowest combat power among the subordinate units. This can be changed to an IP problem through the following process, by introducing a new variable.

P Minimum combat power among all subordinate units' combat power

- $\min_{i \in I} \frac{1}{|J|}\sum_{j=1}^{n}\left(\frac{S_{ij} - E_{ij} + X_{ij}}{[W_i \cdot R_{ij}]}\right) \Rightarrow P$

$$Maximize \quad P$$

$$Subject\ to \quad P \leq \frac{1}{|J|}\sum_{j=1}^{n}\left(\frac{S_{ij} - E_{ij} + X_{ij}}{[W_i \cdot R_{ij}]}\right), \qquad i \in I$$

Constraints

$$\sum_{i=1}^{m}\sum_{j=1}^{k}\boldsymbol{X_{ij}} = \boldsymbol{C}$$

$$\sum_{i=1}^{m} \boldsymbol{X_{ij}} = \boldsymbol{C_j}, \qquad \boldsymbol{j} \in \{\boldsymbol{k+1}, \dots, \boldsymbol{n}\}$$

All recruits must be assigned.

$$\frac{\boldsymbol{S_{ij} - E_{ij} + X_{ij}}}{[\boldsymbol{W_i \cdot R_{ij}}]} \geq \boldsymbol{\alpha_1}, \qquad \boldsymbol{i} \in \boldsymbol{I}, \boldsymbol{j} \in \{\boldsymbol{1}, \dots, \boldsymbol{k}\}$$

$$\frac{\boldsymbol{S_{ij} - E_{ij} + X_{ij}}}{[\boldsymbol{W_i \cdot R_{ij}}]} \geq \boldsymbol{\alpha_{2j}}, \qquad \boldsymbol{i} \in \boldsymbol{I}, \boldsymbol{j} \in \{\boldsymbol{k+1}, \dots, \boldsymbol{n}\}$$

The combat power of each specialty is greater than or equal to the α values obtained in step 1, so that the deviation of combat power of all specialty is minimized. This can be changed to an IP problem through the following process, by introducing a new variable and a new parameter.

$\boldsymbol{M_{ij}}$	Rounding $(W_i \cdot R_{ij})$ for subordinate unit i, specialty j, for each $i \in I, j \in J$
$\boldsymbol{t}$	Sufficiently small positive parameter

- $[W_i \cdot R_{ij}] \Longrightarrow M_{ij}$

$$\frac{S_{ij} - E_{ij} + X_{ij}}{M_{ij}} \geq \alpha_1, \qquad i \in I, j \in \{1, \dots, k\}$$

$$\frac{S_{ij} - E_{ij} + X_{ij}}{M_{ij}} \geq \alpha_{2j}, \qquad i \in I, j \in \{k+1, \dots, n\}$$

$$M_{ij} \in \mathbb{Z}, \qquad i \in I, j \in J$$

$$M_{ij} \leq W_i \cdot R_{ij} + 0.5, \qquad i \in I, j \in J$$

$$M_{ij} \geq W_i \cdot R_{ij} - 0.5 + t, \qquad i \in I, j \in J$$

$$\boldsymbol{X_{ij} \leq \max\left(\lceil W_i \cdot R_{ij} \rceil - S_{ij} + E_{ij}, 0\right), \qquad i \in I, j \in J}$$

$\lceil W_i \cdot R_{ij} \rceil - S_{ij} + E_{ij}$ is a demand for 100% combat power. As in the previous case, this number can be negative if there is a large number of soldiers serving, so we can adjust it with the max function. The number to be placed must be less than or equal to the demand. This can be changed to an IP problem through the following process, by introducing a new variable.

$\boldsymbol{Z_{ij}}$	$\begin{cases} \mathbf{0}, & \textbf{if } \boldsymbol{M_{ij} - S_{ij} + E_{ij} \leq 0} \\ \mathbf{1}, & \textbf{otherwise} \end{cases}$	for sub unit $\boldsymbol{i}$, specialty $\boldsymbol{j}$, for each $\boldsymbol{i \in I, j \in J}$
$\boldsymbol{L}$	Sufficiently large positive parameter	

- Binary variable Z_{ij}

$$X_{ij} \leq M_{ij} - S_{ij} + E_{ij} + L \cdot \left(1 - Z_{ij}\right), \qquad i \in I, j \in J$$

$$X_{ij} \leq L \cdot Z_{ij}, \qquad i \in I, j \in J$$

$$Z_{ij} \in \{0, 1\}, \qquad i \in I, j \in J$$

$$\boldsymbol{X_{ij} \in \mathbb{Z}, \qquad i \in I, j \in J}$$

$$\boldsymbol{X_{ij} \geq 0, \qquad i \in I, j \in J}$$

Final model form

$$\boldsymbol{Maximize \quad P}$$

$$\textit{Subject to}\quad P \le \frac{1}{|J|}\sum_{j=1}^{n}\left(\frac{S_{ij}-E_{ij}+X_{ij}}{M_{ij}}\right), \qquad i \in I$$

$$\sum_{i=1}^{m}\sum_{j=1}^{k} X_{ij} = C$$

$$\sum_{i=1}^{m} X_{ij} = C_j, \qquad j \in \{k+1, \dots, n\}$$

$$\frac{S_{ij}-E_{ij}+X_{ij}}{M_{ij}} \ge \alpha_1, \qquad i \in I, j \in \{1, \dots, k\}$$

$$\frac{S_{ij}-E_{ij}+X_{ij}}{M_{ij}} \ge \alpha_{2j}, \qquad i \in I, j \in \{k+1, \dots, n\}$$

$$M_{ij} \in \mathbb{Z},\ i \in I, j \in J \tag{2.3}$$

$$M_{ij} \le W_i \cdot R_{ij} + 0.5, \qquad i \in I, j \in J$$

$$M_{ij} \ge W_i \cdot R_{ij} - 0.5 + t, \qquad i \in I, j \in J$$

$$X_{ij} \le M_{ij} - S_{ij} + E_{ij} + L \cdot (1 - Z_{ij}), \qquad i \in I, j \in J$$

$$X_{ij} \le L \cdot Z_{ij}, \qquad i \in I, j \in J$$

$$Z_{ij} \in \{0, 1\}, \qquad i \in I, j \in J$$

$$X_{ij} \in \mathbb{Z}, \qquad i \in I, j \in J$$

$$X_{ij} \ge 0, \qquad i \in I, j \in J$$

Figure 2.4 shows the results before and after Step 2 using the data in Table 1.1. The combat power of subordinate unit B was significantly lower before applying Step 2, but became much higher after Step 2 was applied. Step 2 thus reduces the variance among combat powers of each subordinate unit. In Step 2, the combat powers of each subordinate unit are set at a similar level.

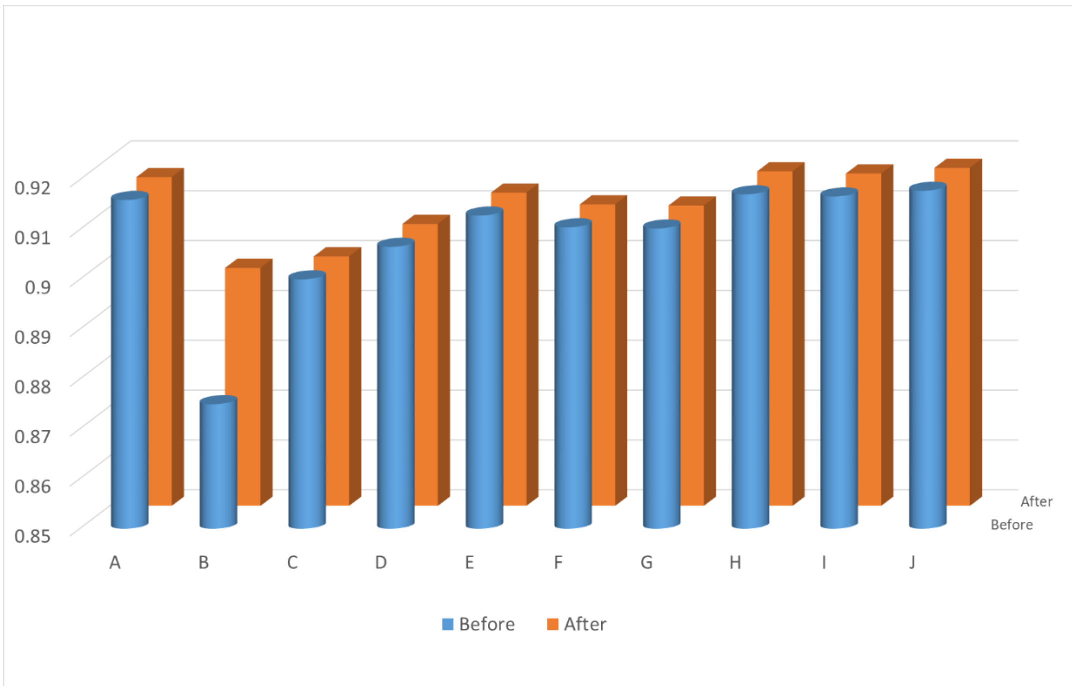

Figure 2.4: Results before and after step 2

Chapter 3

Experiments and results

3.1 Data sets

In this study, I could not use real data sets: since each unit's TOE is considered to be a military secret, the actual assignment results of the recruits could not be obtained. However, as I need to compare the results from the current system with from the new system, I have made use of applying other methods. The information data format is the same as the one currently used in the ROK Army, but the fictitious data values are randomly generated for it. The form is the same as shown in Table 1.1. The data were created on a spreadsheet, using the following assumptions.

1) The number of subordinate units is 10 from A to J.

2) The number of specialties that we can assign is 7 from 1 to 7.

3) The number of specialties already assigned is 13 from 8 to 20.

4) The number of recruits without specialties is around 180: this might differ.

5) The number of recruits with their specialties is around 25: this might differ.

6) Subordinate unit D and E must have 110% of combat power compared to the TOE.

7) A total of 20 assignments are conducted, which means we have 20 sheets.

3.2 Current system

I received help from two people to get results of the current system. One is currently serving as a personnel officer, and the other is a former personnel officer. They assigned recruits of 20 sheets by applying the criteria considered in the current system.

3.2.1 Subordinate unit

In Table 3.1, Subunit variance is the variance of combat powers of each specialty. The large variances imply that the combat powers are not at a similar level. Subunit max-min is the difference between the highest and the lowest combat power for each specialty. A large max-min value means that some combat powers are very high or very low. If both of these indicator values are small, then we can determine that the combat powers are at a similar level. Table 3.1 shows where the variance is 0.005 or more, and where the max-min value is 0.3 or more. In most cases, the two conditions are met as they are related. However, when we look at the subordinate units D and E of the 20th unit, the subordinate unit D has a larger variance even though the max-min value is not large. In this case, we can say that the combat powers are uneven. Figures 3.1 and 3.2 show the combat powers using box plots. We can identify some units with wide distribution or with very high or very low value.

Table 3.1: Current system - Statistics on the combat powers of specialties

Unit	Sub unit	Subunit Variance	Subunit MAX-MIN	Unit	Sub unit	Subunit Variance	Subunit MAX-MIN	Unit	Sub unit	Subunit Variance	Subunit MAX-MIN	Unit	Sub unit	Subunit Variance	Subunit MAX-MIN
1	A	0.0031	0.1818	6	A	0.0020	0.1818	11	A	0.0025	0.1905	16	A	0.0032	0.1818
	B	0.0023	0.2105		B	0.0041	0.2000		B	0.0038	0.1818		B	0.0031	0.2000
	C	0.0027	0.1579		C	0.0035	0.1818		C	0.0048	0.2105		C	0.0022	0.1765
	D	0.0023	0.1667		D	0.0044	0.2500		D	0.0018	0.1765		D	0.0056	0.3333
	E	0.0067	0.3333		E	0.0024	0.2000		E	0.0028	0.1667		E	0.0013	0.0967
	F	0.0039	0.1905		F	0.0027	0.1905		F	0.0021	0.1818		F	0.0028	0.2000
	G	0.0037	0.2000		G	0.0029	0.1818		G	0.0034	0.1818		G	0.0025	0.1765
	H	0.0041	0.2000		H	0.0043	0.2000		H	0.0023	0.1765		H	0.0033	0.2308
	I	0.0048	0.2000		I	0.0025	0.1538		I	0.0022	0.1579		I	0.0029	0.1579
	J	0.0044	0.2000		J	0.0044	0.1765		J	0.0037	0.1818		J	0.0023	0.1818
2	A	0.0026	0.1429	7	A	0.0036	0.2000	12	A	0.0021	0.1818	17	A	0.0019	0.1538
	B	0.0036	0.2000		B	0.0042	0.2000		B	0.0031	0.2000		B	0.0010	0.1538
	C	0.0027	0.1579		C	0.0028	0.1765		C	0.0032	0.2105		C	0.0029	0.2000
	D	0.0018	0.1667		D	0.0029	0.2000		D	0.0032	0.1739		D	0.0028	0.1765
	E	0.0025	0.1905		E	0.0015	0.1238		E	0.0044	0.2692		E	0.0035	0.2500
	F	0.0034	0.1818		F	0.0032	0.2000		F	0.0027	0.1579		F	0.0037	0.2000
	G	0.0044	0.2000		G	0.0034	0.1579		G	0.0028	0.2000		G	0.0034	0.2000
	H	0.0024	0.2000		H	0.0046	0.1905		H	0.0034	0.2000		H	0.0028	0.1818
	I	0.0023	0.2105		I	0.0032	0.2000		I	0.0035	0.1765		I	0.0025	0.2000
	J	0.0037	0.1765		J	0.0027	0.1579		J	0.0030	0.2000		J	0.0033	0.1818
3	A	0.0017	0.1818	8	A	0.0021	0.1818	13	A	0.0029	0.1538	18	A	0.0027	0.1579
	B	0.0026	0.1538		B	0.0029	0.2000		B	0.0023	0.1579		B	0.0029	0.2000
	C	0.0022	0.1579		C	0.0023	0.1579		C	0.0035	0.1538		C	0.0036	0.1818
	D	0.0042	0.2500		D	0.0052	0.2909		D	0.0027	0.1667		D	0.0029	0.1765
	E	0.0021	0.1765		E	0.0066	0.3409		E	0.0038	0.2500		E	0.0040	0.2500
	F	0.0032	0.2000		F	0.0034	0.1818		F	0.0035	0.1818		F	0.0034	0.1765
	G	0.0031	0.2000		G	0.0032	0.1765		G	0.0033	0.1818		G	0.0022	0.1579
	H	0.0040	0.1905		H	0.0018	0.1538		H	0.0028	0.2000		H	0.0026	0.2000
	I	0.0022	0.1905		I	0.0033	0.1765		I	0.0038	0.2000		I	0.0012	0.1579
	J	0.0022	0.2000		J	0.0022	0.1579		J	0.0031	0.1538		J	0.0034	0.1818
4	A	0.0017	0.2000	9	A	0.0028	0.1818	14	A	0.0014	0.1579	19	A	0.0022	0.1905
	B	0.0018	0.1818		B	0.0014	0.1818		B	0.0019	0.1579		B	0.0029	0.1818
	C	0.0035	0.2000		C	0.0039	0.2000		C	0.0021	0.1818		C	0.0021	0.1818
	D	0.0043	0.1765		D	0.0036	0.2500		D	0.0031	0.1905		D	0.0022	0.1739
	E	0.0035	0.1905		E	0.0016	0.1667		E	0.0024	0.1765		E	0.0024	0.1765
	F	0.0043	0.2500		F	0.0026	0.1818		F	0.0014	0.1429		F	0.0019	0.2000
	G	0.0043	0.2000		G	0.0030	0.1538		G	0.0040	0.1765		G	0.0022	0.2000
	H	0.0039	0.2105		H	0.0036	0.1818		H	0.0022	0.1538		H	0.0039	0.2000
	I	0.0065	0.3247		I	0.0035	0.1818		I	0.0028	0.2000		I	0.0027	0.1579
	J	0.0018	0.1781		J	0.0037	0.1818		J	0.0024	0.2000		J	0.0035	0.2000
5	A	0.0025	0.1538	10	A	0.0033	0.1579	15	A	0.0028	0.1818	20	A	0.0035	0.2000
	B	0.0031	0.2000		B	0.0036	0.1818		B	0.0031	0.2105		B	0.0021	0.2000
	C	0.0024	0.2000		C	0.0026	0.1538		C	0.0028	0.1818		C	0.0026	0.1818
	D	0.0036	0.2500		D	0.0065	0.3333		D	0.0028	0.1905		D	0.0051	0.2500
	E	0.0057	0.3333		E	0.0022	0.1905		E	0.0031	0.1905		E	0.0036	0.2500
	F	0.0043	0.2000		F	0.0037	0.2000		F	0.0026	0.1538		F	0.0031	0.2000
	G	0.0037	0.1818		G	0.0033	0.1579		G	0.0024	0.1818		G	0.0035	0.1818
	H	0.0040	0.2000		H	0.0022	0.2000		H	0.0037	0.2000		H	0.0020	0.1579
	I	0.0043	0.2000		I	0.0040	0.2000		I	0.0037	0.2000		I	0.0037	0.2000
	J	0.0034	0.2105		J	0.0038	0.1579		J	0.0030	0.1429		J	0.0029	0.1905

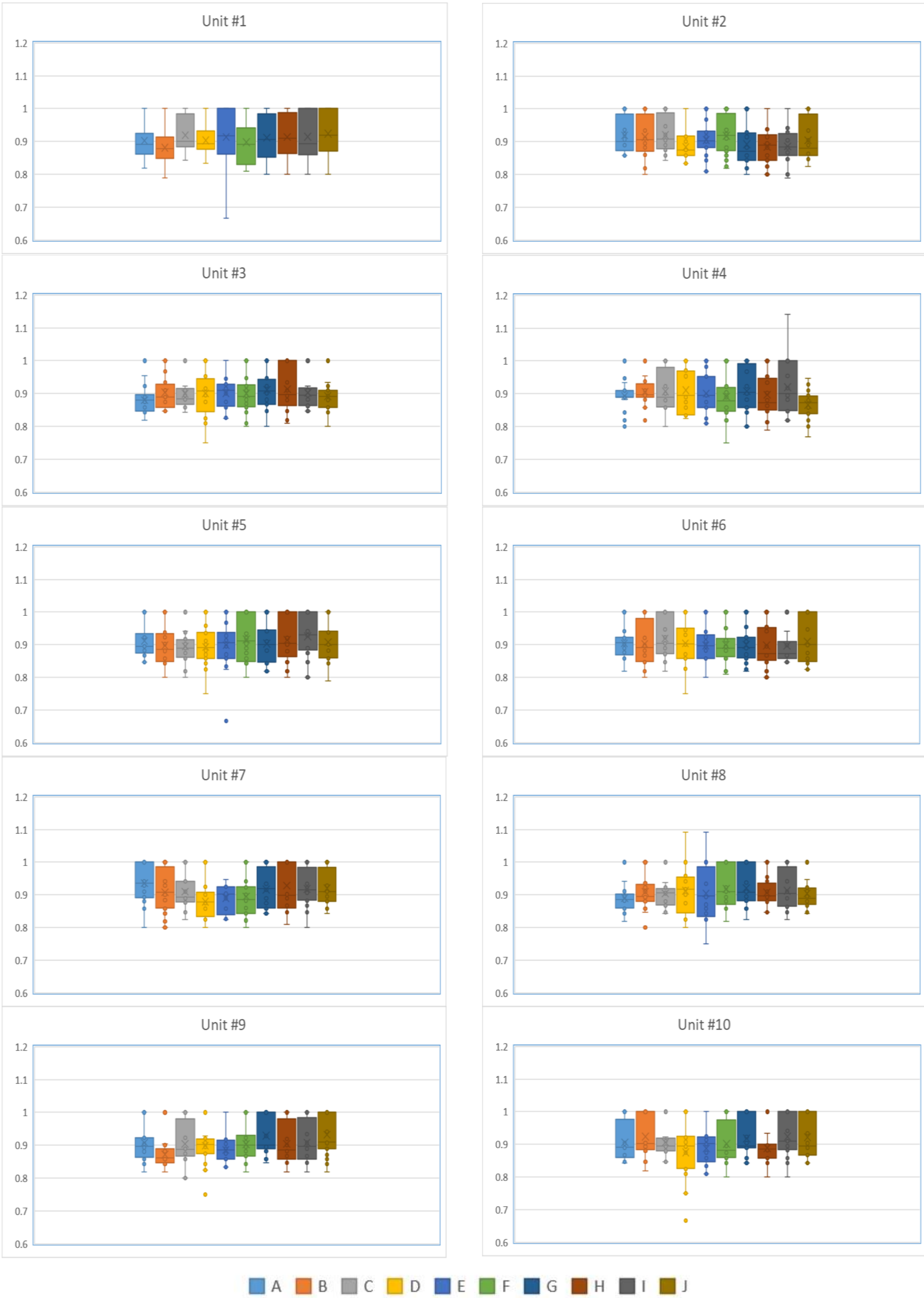

Figure 3.1: Current system - Box plots 1 of combat powers of specialties

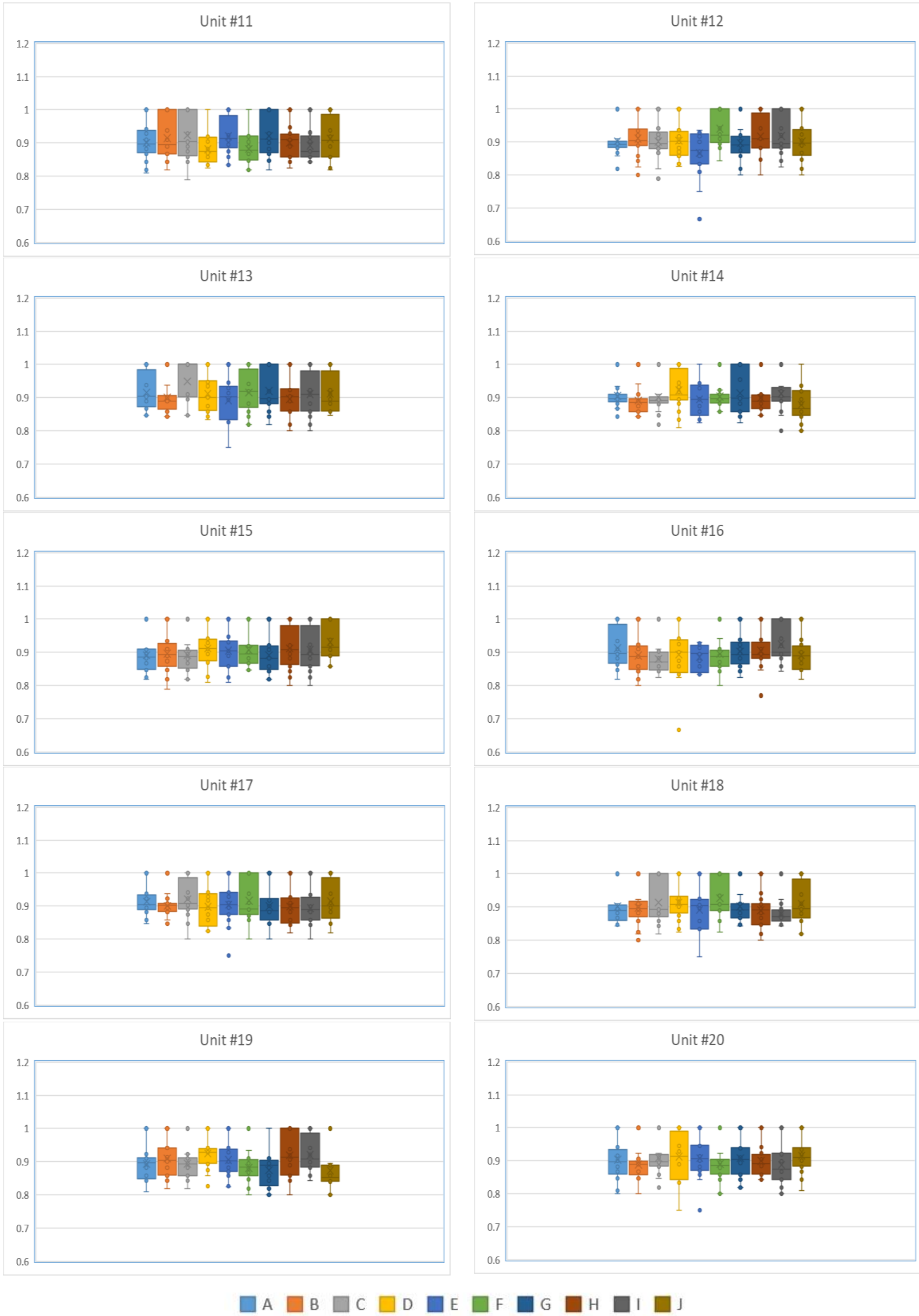

Figure 3.2: Current system - Box plots 2 of combat powers of specialties

3.2.2 Unit

Table 3.2 shows statistical data of the combat powers for each subordinate unit. The table shows that some variances are 0.00025 or more and some max-min values are 0.05 or more. The 7th, 9th, 10th, 12th, and 19th units have relatively large variances, so we can say that their combat powers are relatively uneven. The 4th and 13th units do not have larger variances but their max-min values are over 0.05. This means that there are subordinate units that have a difference of 5% or more among the combat powers of each subordinate unit. The combat powers possessed by these subordinate units can also be said to be relatively uneven. On the other hand, the 6th and 17th units have small variances and max-min values. Compared to other units, we can conclude that the combat powers of these units are at a similar level. Figure 3.3 shows the combat powers of each subordinate unit in the box plot. In the 4th and 13th units, most of the data are at a similar level, but one of their data is at a very different level. We can see why the variances of these units are smaller and the max-min values are larger. Similar analyses can also be visually confirmed in this manner.

Table 3.2: Current system - Statistics on the combat powers of subordinate units

Unit	Unit Variance	Unit MAX-MIN	Unit	Unit Variance	Unit MAX-MIN
1	0.00014	0.04275	11	0.00023	0.04232
2	0.00021	0.03593	12	0.00031	0.07293
3	0.00008	0.03351	13	0.00022	0.05674
4	0.00019	0.05386	14	0.00014	0.04368
5	0.00010	0.03248	15	0.00014	0.04050
6	0.00004	0.02251	16	0.00015	0.04326
7	0.00028	0.05639	17	0.00008	0.02691
8	0.00007	0.03041	18	0.00017	0.04627
9	0.00028	0.06030	19	0.00033	0.05151
10	0.00026	0.04866	20	0.00010	0.02815

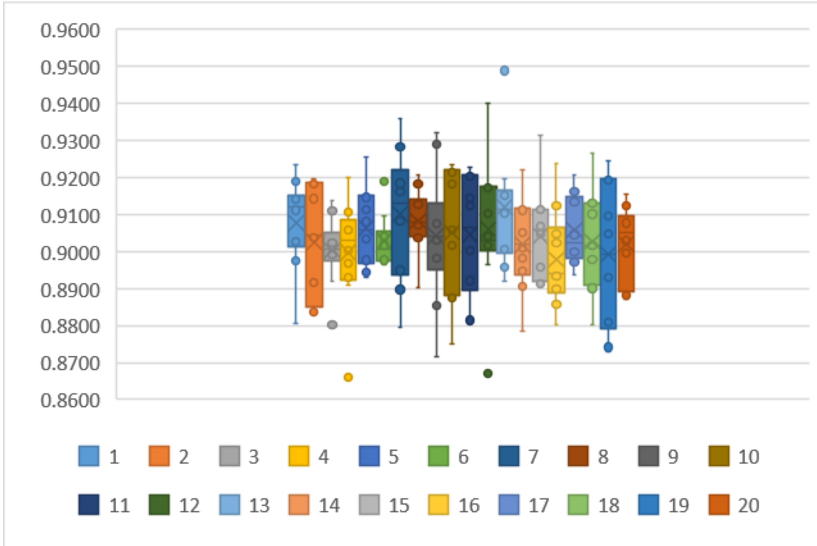

Figure 3.3: Current system - Box plots of combat powers of subordinate units

3.3 New system

AMPL (with solver Gurobi) and Excel (with its add-in Solver) were used to conduct our numerical experiments. Excel was used to review and organize results. Its solver can be used for simple calculations but it was not enough for this experiment. Therefore, AMPL was used for the calculations.

3.3.1 Subordinate unit

In Table 3.3, the largest subunit variance is 0.0049, which is less than 0.005. The largest subunit max-min value is 0.2308, which is less than 0.3. Compared with the other units in the new system, these can be said not to be at a similar level. But compared to the units in the current system, we cannot make the same claim. Figures 3.4 and 3.5 show the combat powers in the box plot. We can see most of the data is located between 0.8 and 1.0.

Table 3.3: New system - Statistics on the combat powers of specialties

Unit	Sub unit	Subunit Variance	Subunit MAX-MIN	Unit	Sub unit	Subunit Variance	Subunit MAX-MIN	Unit	Sub unit	Subunit Variance	Subunit MAX-MIN	Unit	Sub unit	Subunit Variance	Subunit MAX-MIN
1	A	0.0023	0.1579	6	A	0.0023	0.1818	11	A	0.0030	0.2105	16	A	0.0033	0.2000
	B	0.0033	0.1905		B	0.0041	0.2000		B	0.0043	0.2105		B	0.0030	0.2000
	C	0.0025	0.2000		C	0.0040	0.1818		C	0.0043	0.2105		C	0.0028	0.1765
	D	0.0018	0.1667		D	0.0029	0.1739		D	0.0020	0.1765		D	0.0033	0.1765
	E	0.0038	0.1667		E	0.0017	0.1429		E	0.0030	0.1667		E	0.0018	0.1667
	F	0.0029	0.1905		F	0.0024	0.1818		F	0.0028	0.1818		F	0.0033	0.2000
	G	0.0033	0.2000		G	0.0032	0.1818		G	0.0042	0.1818		G	0.0020	0.1765
	H	0.0036	0.1818		H	0.0036	0.2000		H	0.0025	0.1818		H	0.0025	0.1538
	I	0.0049	0.2000		I	0.0035	0.1818		I	0.0022	0.1579		I	0.0041	0.2000
	J	0.0041	0.2000		J	0.0034	0.1765		J	0.0037	0.1818		J	0.0023	0.1818
2	A	0.0033	0.2000	7	A	0.0038	0.2000	12	A	0.0021	0.1818	17	A	0.0022	0.1538
	B	0.0034	0.2000		B	0.0039	0.2000		B	0.0034	0.2000		B	0.0020	0.1538
	C	0.0032	0.2000		C	0.0027	0.1765		C	0.0028	0.2000		C	0.0029	0.2000
	D	0.0031	0.1667		D	0.0030	0.1765		D	0.0036	0.1739		D	0.0024	0.1765
	E	0.0023	0.1905		E	0.0022	0.1765		E	0.0025	0.1667		E	0.0023	0.1667
	F	0.0025	0.1765		F	0.0029	0.2000		F	0.0046	0.2000		F	0.0041	0.2000
	G	0.0039	0.1818		G	0.0030	0.2000		G	0.0026	0.2000		G	0.0047	0.2000
	H	0.0026	0.1579		H	0.0048	0.1905		H	0.0038	0.2000		H	0.0031	0.1818
	I	0.0017	0.1538		I	0.0030	0.2000		I	0.0034	0.1765		I	0.0026	0.1579
	J	0.0036	0.1765		J	0.0027	0.2000		J	0.0034	0.2000		J	0.0036	0.1818
3	A	0.0039	0.2105	8	A	0.0025	0.1818	13	A	0.0035	0.2000	18	A	0.0029	0.2000
	B	0.0024	0.1538		B	0.0027	0.2000		B	0.0024	0.1579		B	0.0035	0.2000
	C	0.0010	0.1429		C	0.0027	0.1765		C	0.0036	0.1538		C	0.0039	0.1905
	D	0.0020	0.1905		D	0.0027	0.1765		D	0.0028	0.1667		D	0.0025	0.1667
	E	0.0017	0.1739		E	0.0027	0.1765		E	0.0029	0.1739		E	0.0029	0.1667
	F	0.0031	0.2000		F	0.0035	0.1818		F	0.0030	0.1818		F	0.0033	0.1765
	G	0.0031	0.2000		G	0.0032	0.1765		G	0.0039	0.2000		G	0.0022	0.1579
	H	0.0035	0.1818		H	0.0024	0.1818		H	0.0028	0.1818		H	0.0027	0.1818
	I	0.0021	0.1538		I	0.0031	0.1765		I	0.0038	0.2000		I	0.0025	0.1579
	J	0.0025	0.1818		J	0.0024	0.1579		J	0.0031	0.1538		J	0.0036	0.2000
4	A	0.0017	0.2000	9	A	0.0029	0.1818	14	A	0.0019	0.1818	19	A	0.0028	0.1905
	B	0.0017	0.1818		B	0.0028	0.1818		B	0.0018	0.1579		B	0.0034	0.2000
	C	0.0030	0.2000		C	0.0036	0.2000		C	0.0024	0.1818		C	0.0021	0.1818
	D	0.0036	0.1765		D	0.0018	0.1579		D	0.0023	0.1429		D	0.0022	0.1739
	E	0.0026	0.1905		E	0.0023	0.1667		E	0.0023	0.1765		E	0.0018	0.1429
	F	0.0043	0.2105		F	0.0026	0.1818		F	0.0022	0.1818		F	0.0024	0.1818
	G	0.0037	0.2000		G	0.0033	0.1579		G	0.0041	0.1765		G	0.0036	0.2000
	H	0.0034	0.2105		H	0.0037	0.1818		H	0.0022	0.1538		H	0.0041	0.2000
	I	0.0035	0.1818		I	0.0038	0.1905		I	0.0028	0.1579		I	0.0027	0.1579
	J	0.0018	0.2000		J	0.0039	0.2000		J	0.0021	0.1579		J	0.0034	0.2000
5	A	0.0030	0.1579	10	A	0.0033	0.1579	15	A	0.0043	0.2000	20	A	0.0039	0.2000
	B	0.0034	0.2000		B	0.0039	0.2000		B	0.0028	0.1818		B	0.0033	0.2000
	C	0.0023	0.1818		C	0.0028	0.1765		C	0.0030	0.1818		C	0.0041	0.1818
	D	0.0023	0.1667		D	0.0020	0.1905		D	0.0026	0.1667		D	0.0027	0.1667
	E	0.0030	0.1765		E	0.0023	0.1905		E	0.0025	0.1765		E	0.0024	0.1579
	F	0.0040	0.2000		F	0.0037	0.2000		F	0.0029	0.1765		F	0.0033	0.2000
	G	0.0036	0.2308		G	0.0033	0.1579		G	0.0022	0.1818		G	0.0038	0.2000
	H	0.0042	0.2000		H	0.0027	0.2000		H	0.0044	0.2000		H	0.0020	0.1579
	I	0.0042	0.2000		I	0.0045	0.2000		I	0.0037	0.2000		I	0.0047	0.2000
	J	0.0039	0.2000		J	0.0039	0.1579		J	0.0036	0.2000		J	0.0037	0.1905

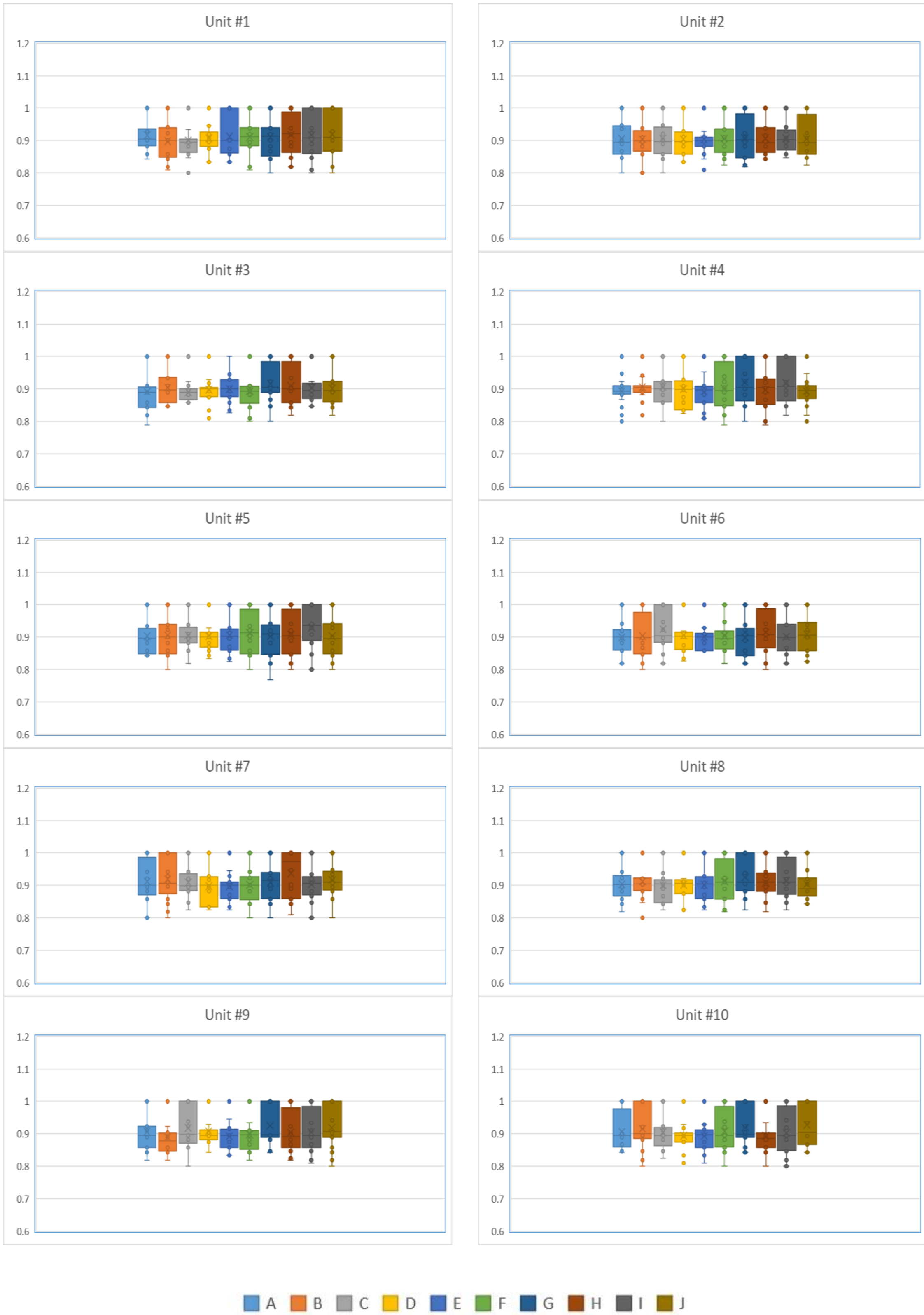

Figure 3.4: New system - Box plots 1 of combat powers of specialties

Figure 3.5: New system - Box plots 2 of combat powers of specialties

3.3.2 Unit

In Table 3.4, the largest variance is 0.00019, which is less than 0.00025. The largest max-min value is 0.04071, which is less than 0.05. Compared with the other units in the new system, these can be said not to be at a similar level. But compared to the units in the current system, we cannot make the same claim. Figure 3.6 shows the combat powers in the box plot. We can see that most of the data are located between 0.89 and 0.93.

Table 3.4: New system - Statistics on the combat powers of subordinate units

Unit	Unit Variance	Unit MAX-MIN	Unit	Unit Variance	Unit MAX-MIN
1	0.00005	0.02008	11	0.00019	0.03606
2	0.00000	0.00384	12	0.00003	0.01981
3	0.00006	0.02608	13	0.00013	0.04071
4	0.00009	0.02755	14	0.00011	0.02720
5	0.00006	0.02727	15	0.00009	0.02789
6	0.00006	0.02599	16	0.00004	0.01788
7	0.00013	0.03918	17	0.00003	0.01472
8	0.00005	0.02402	18	0.00013	0.04063
9	0.00012	0.03496	19	0.00013	0.02968
10	0.00012	0.03470	20	0.00002	0.01244

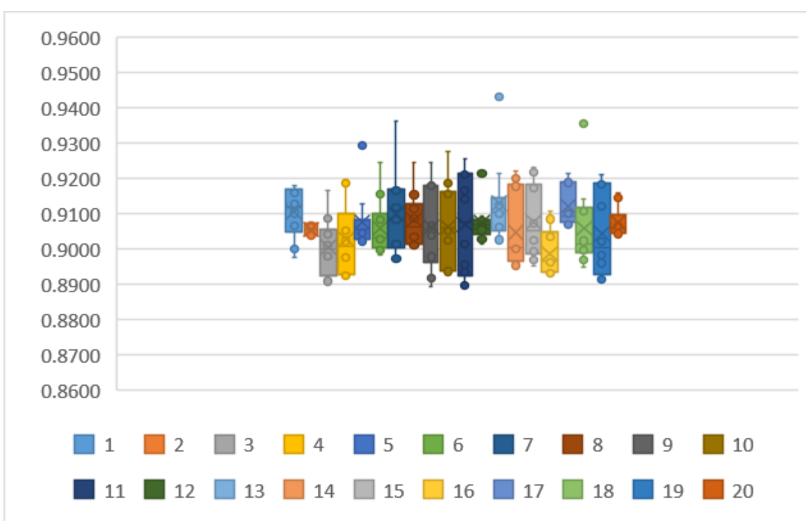

Figure 3.6: New system - Box plots of combat powers of subordinate units

3.4 Comparisons

Comparisons were made in the order of using graphs and using statistical tests. Minitab was used for performing statistical tests. Excel also allows basic statistical tests, but I used Minitab to use additional functions such as the normality test. We will be able to see the distribution of the data through graphs. However, since it is difficult to conclude the exact result with it, we will apply the t-test statistical method. Since this experiment is based on the same sample and we are comparing the results on the current system with the results after applying the new system, we will use the paired t-test. The significance level chosen for all experiments is 0.05. The main data are the variances and max-min values in each system. If we can statistically verify that the variances and max-min values in the new system are smaller than in the current system, then this implies that the new system gives a noticeable improvement.

3.4.1 Subordinate unit

Figure 3.7 is a graph of subunit variances in the current and the new system using Tables 3.1 and 3.3. We need to find out in which system the variances are smaller on average. The average of the variances in the current system is 0.0031, while in the new system it is 0.0030. The difference is so small that it can be considered insignificant. However, this graph shows that the results from the new system are more consistent.

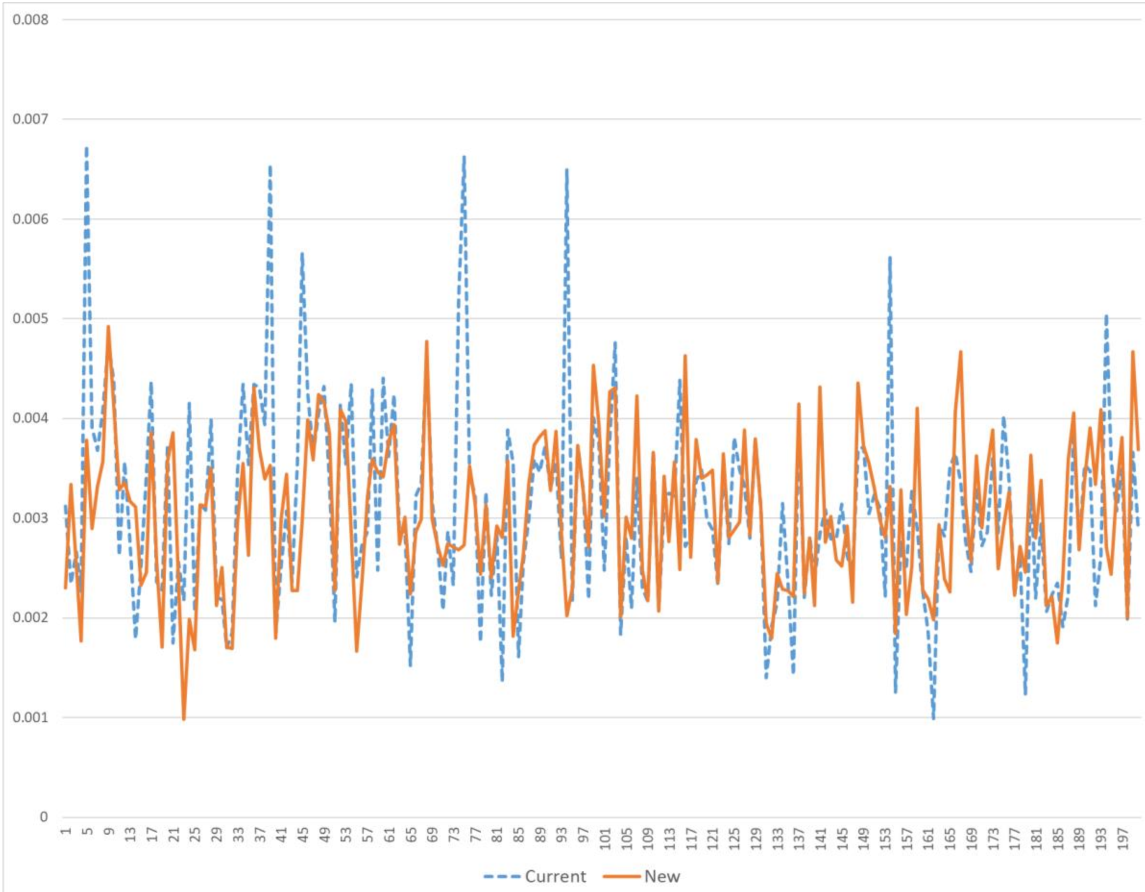

Figure 3.7: Subunit variances in the current and new system

Before conducting the t-test, we should make sure that the data are normally distributed. Although in general we could use the normality test, but here the number of samples is 200. If the size of the sample data set is large enough, the data can be assumed to follow the normal distribution by the Central Limit Theorem (CLT), and the size of the sample should usually be at least 30 [16]. Therefore, both data sets can be regarded as following the normal distribution.

Figure 3.8 shows the result of the paired t-test using Minitab. An alternative hypothesis is that the average value in the current system is larger than the average value in the new system. As a result, the p-value is 0.141, which is greater than the significance level 0.05, so the null hypothesis cannot be rejected. There is no significant difference between the two data averages. Therefore, even if the new system is applied, the mean of variances is not changed, at least as indicated by the statistical test. As per this finding, the new system does not sufficiently reduce the deviations among the combat powers of each specialty.

Paired T-Test and CI: Current, New

Descriptive Statistics

Sample	N	Mean	StDev	SE Mean
Current	200	0.003104	0.000983	0.000070
New	200	0.003037	0.000757	0.000053

Estimation for Paired Difference

Mean	StDev	SE Mean	95% Lower Bound for μ_difference
0.000067	0.000878	0.000062	-0.000036

μ_difference: mean of (Current - New)

Test

Null hypothesis	H_0: μ_difference = 0
Alternative hypothesis	H_1: μ_difference > 0

T-Value	P-Value
1.08	0.141

Figure 3.8: T-test on subunit variances

Figure 3.9 is a graph of subunit max-min values in the current system and in the new system using Tables 3.1 and 3.3. As with the previous graph of variances, we need to find out on which system the max-min values are smaller on average. The average of the max-min values in the current system is 0.1912 and it in the new system is 0.1830. The difference is so small that it is difficult to see if it is meaningful. Again, the graph shows that the results from the new system are more consistent. The results in Figure 3.9 show this more clearly than the results in Figure 3.7. The max-min values in the new system are mostly distributed between 0.15 and 0.2, but the values in the current system show that some of them are much higher than others.

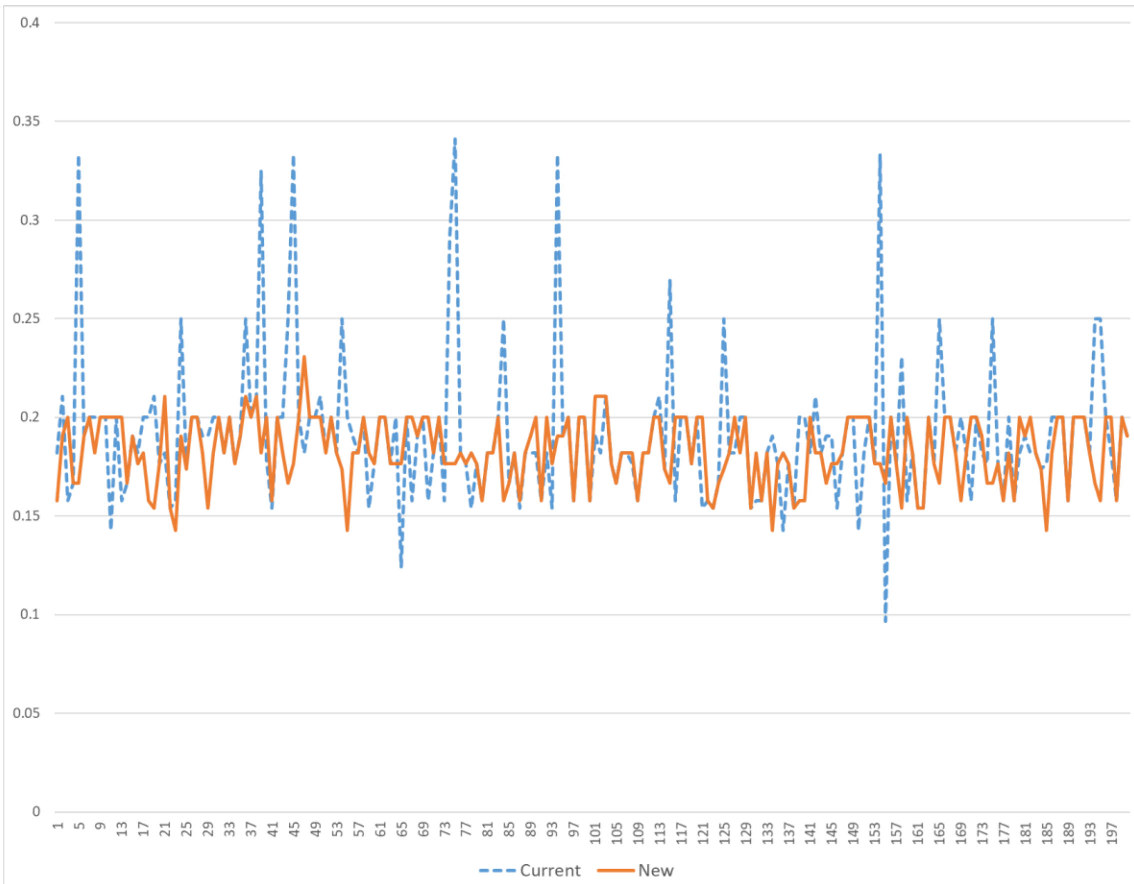

Figure 3.9: Subunit max-min values in the current and new system

Again, since the number of sample data is 200, both data can be regarded as following the normal distribution by CLT.

Figure 3.10 shows the result of the paired t-test using Minitab. An alternative hypothesis is that the average value in the current system is larger than the average value in the new system. As a result, the p-value is 0.001, which is smaller than the significance level of 0.05. So we reject the null hypothesis. There is a significant difference between the two data. Therefore, applying the new system indicates that the average of max-min values is smaller. This means that the new system is effective.

Paired T-Test and CI: Current, New

Descriptive Statistics

Sample	N	Mean	StDev	SE Mean
Current	200	0.19120	0.03567	0.00252
New	200	0.18300	0.01711	0.00121

Estimation for Paired Difference

Mean	StDev	SE Mean	95% Lower Bound for μ_difference
0.00820	0.03807	0.00269	0.00376

μ_difference: mean of (Current - New)

Test

Null hypothesis	H_0: μ_difference = 0
Alternative hypothesis	H_1: μ_difference > 0

T-Value	P-Value
3.05	0.001

Figure 3.10: T-test on subunit max-min values

3.4.2 Unit

Figure 3.11 shows a graph of unit variances in the current system and in the new system using Tables 3.2 and 3.4. We need to find out in which system the variances are smaller on average. The average of the variances in the current system is 0.00017, and the value in the new system is 0.00008: thus, the difference is not small. In addition, looking at the graph in Figure 3.11, the data in most current systems is larger than the data in the new system. However, we can see that some data in the current system are smaller than the data in the new system. Therefore, we cannot conclude based on this graph alone.

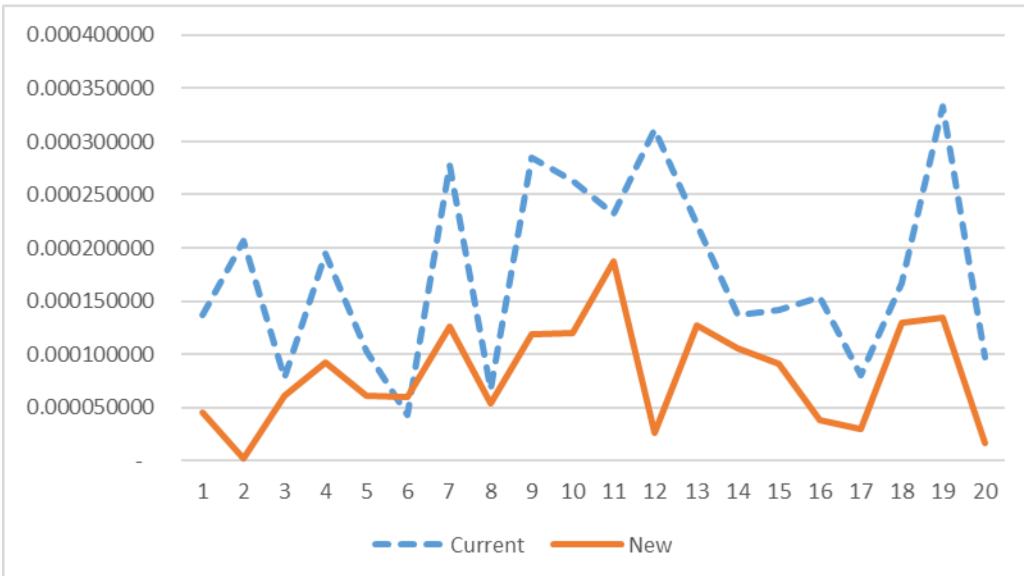

Figure 3.11: Unit variances in the current and new system

In this case, the number of samples is 20. Therefore, I performed the normality test using Minitab. Figures 3.12 and 3.13 show the results. Since the p-values are 0.557 for the current system and 0.351 for the new system (both being greater than the significance level 0.05), both data sets can be regarded as following the normal distribution.

Figure 3.14 shows the result of the paired t-test using Minitab. The alternate hypothesis states that the mean in the current system is greater than the mean in the new system. As a result, the p-value of 0.000 is less than the significance level of 0.05, so we reject the null hypothesis. That is, the average value in the new system is smaller than it in the current

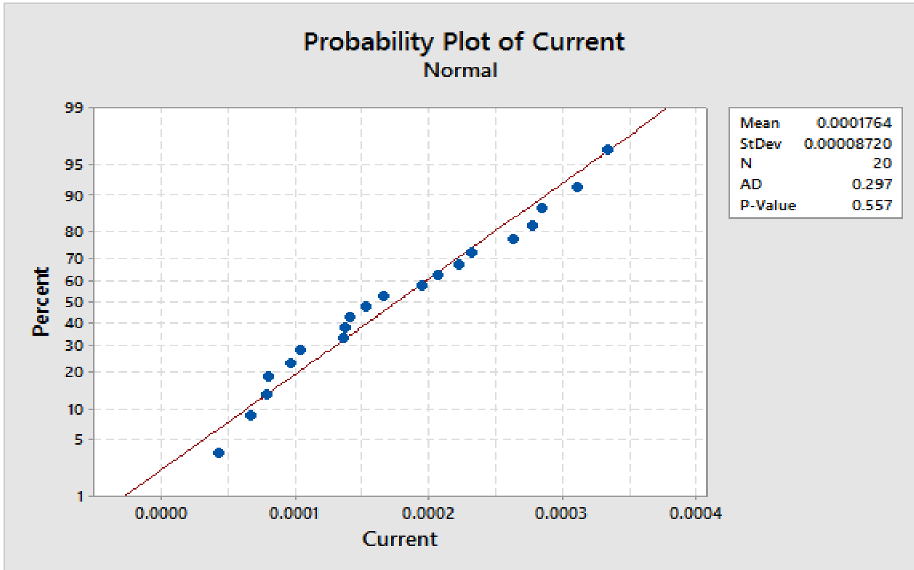

Figure 3.12: Normality test on unit variance in the current system

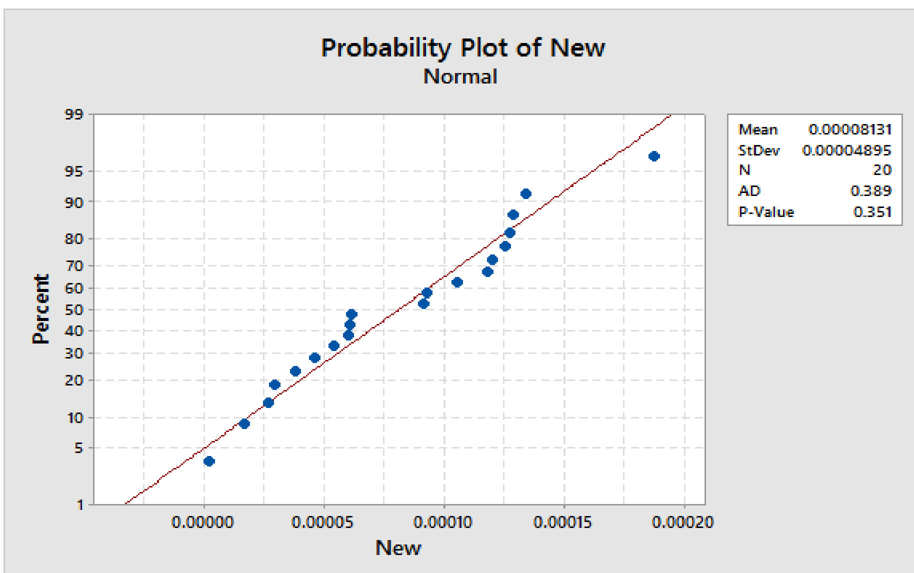

Figure 3.13: Normality test on unit variance in the new system

system. These findings indicate that the variances of the combat powers of each subordinate unit in the new system are smaller. Ultimately, this means that the introduction of the new system reduces the differences in combat powers of subordinate units.

Paired T-Test and CI: Current, New

Descriptive Statistics

Sample	N	Mean	StDev	SE Mean
Current	20	0.000176	0.000087	0.000019
New	20	0.000081	0.000049	0.000011

Estimation for Paired Difference

Mean	StDev	SE Mean	95% Lower Bound for μ_difference
0.000095	0.000077	0.000017	0.000065

μ_difference: mean of (Current - New)

Test

Null hypothesis	H_0: μ_difference = 0
Alternative hypothesis	H_1: μ_difference > 0

T-Value	P-Value
5.52	0.000

Figure 3.14: T-test on unit variances

Figure 3.15 is a graph of subunit max-min values in the current system and in the new system using Tables 3.2 and 3.4. The average of the max-min values in the current system is 0.0434 and 0.0265 in the new system. Since it almost exactly follows the same shape as Figure 3.11, the same conclusion can be drawn.

Figures 3.16 and 3.17 show the results of the normality test. Since each p-value is 0.913 for the current system and 0.601 for the new system (both being greater than the significance level 0.05), both data can be regarded as following the normal distribution.

Figure 3.18 shows the result of the paired t-test using Minitab. The alternate hypothesis states that the mean in the current system is greater than the mean in the new system. As a

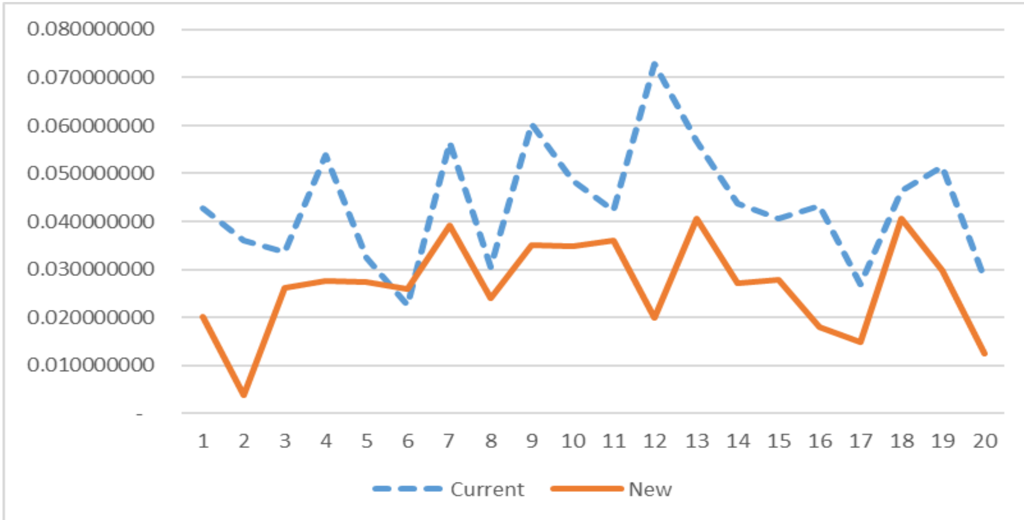

Figure 3.15: Unit max-min values in the current and new system

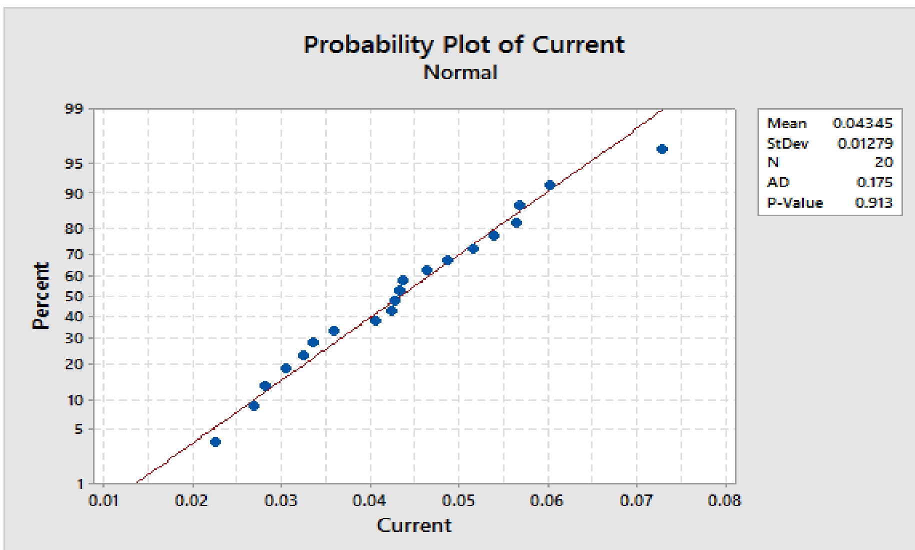

Figure 3.16: Normality test on unit max-min value in the current system

result, the p-value of 0.000 is less than the significance level of 0.05, so we reject the null hypothesis. That is, the average in the new system is smaller than the average in the current system. These findings show that the max-min values of the combat powers of each subordinate unit in the new system are smaller. Ultimately, this means that the introduction of the new system reduces the differences in combat powers of subordinate units.

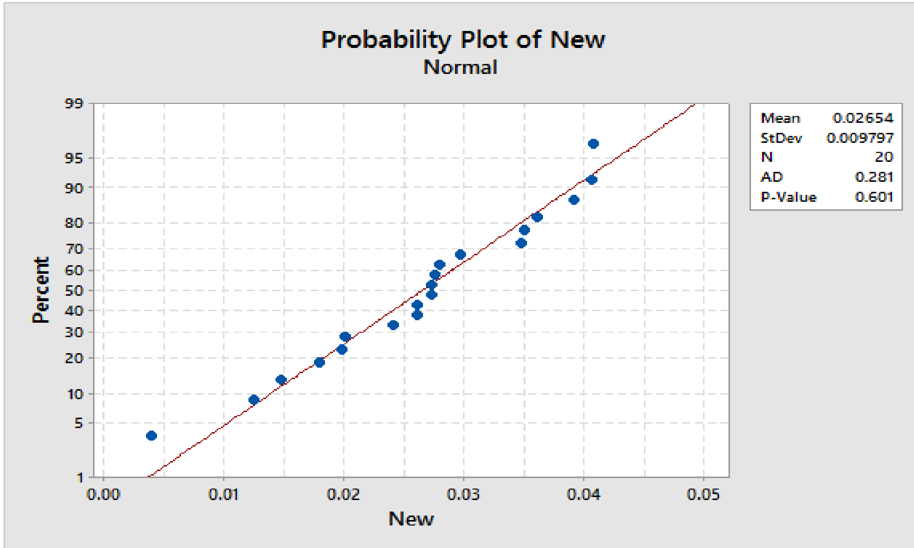

Figure 3.17: Normality test on unit max-min value in the new system

Paired T-Test and CI: Current, New

Descriptive Statistics

Sample	N	Mean	StDev	SE Mean
Current	20	0.04345	0.01279	0.00286
New	20	0.02654	0.00980	0.00219

Estimation for Paired Difference

Mean	StDev	SE Mean	95% Lower Bound for μ_difference
0.01692	0.01230	0.00275	0.01216

μ_difference: mean of (Current - New)

Test

Null hypothesis H_0: μ_difference = 0
Alternative hypothesis H_1: μ_difference > 0

T-Value	P-Value
6.15	0.000

Figure 3.18: T-test on unit max-min values

3.5 Analysis and summary

The purpose of the new assignment system is essentially the same as the purpose of the current system. First, the combat powers of each specialty need to be at a similar level. Second, the combat powers of each subordinate unit should be at a similar level. We can derive the following conclusions from the results of the experiments presented here (in the form of tables, graphs and statistical tests).

Firstly, regarding the combat powers of each specialty, we have confirmed that the application of the new system does not change the variances on the average. The reason for this can be as follows. In Step 1 of the new system, we assign the recruits and equalize the combat powers of each specialty. We then assign the remaining recruits in Step 2 to make the combat powers of each subordinate unit even. As a result of Step 2, the results after Step 1 are partially changed to make the combat powers of subordinate units similar. Therefore, the similarity among the combat powers of specialties is not guaranteed. However, we can still find some additional meaningful information from the tables and the graphs. We saw that some data in the current system have very high variances. This implies that the corresponding subordinate units have unstable combat powers. The number of such data is reduced in the new system. This implies that the number of subordinate units with unstable combat powers has decreased in the new system. Although we cannot conclude that the variances are decreased on the average in the new system, the new system reduces the number of data with higher variances. Applying the t-test, we have confirmed that the application of the new system reduces the max-min values on the average. From the results presented by graphs, we have found that the combat powers of the current system are either significantly higher or significantly lower than the average; the results of the new system are more stable. This result can be explained by the process of obtaining α_1 and α_{2j} in Step 1. Each α value guarantees at least the minimum combat power of all specialties. The process of maximizing these numbers is Step 1, so extreme results may not occur. As a result, the number of data with higher variances is reduced and the max-min values decrease on average in the new system. So the introduction of the new system can be concluded to make the combat powers of each specialty more even.

Secondly, concerning the combat powers of each subordinate unit, significant differences were found in both the variances and the max-min values. By using the new system, it can be said that the variances and the max-min values of the combat powers of each subordinate unit are reduced. This means that the combat powers of each subordinate unit are more even in the new system.

Based on the above findings, it can be concluded that the assignment of recruits in the new system is more effective than it is in the current system.

Chapter 4

Conclusion

In this paper, we have discussed the new model for calculating vacancy positions that can create uniformity among combat powers. The new model introduces two α values which can guarantee minimum combat powers, and max-min fairness [13] which is applied to reduce the deviations between the results. From the experimental results, we can see the differences between the results in the current system and those in the new system. Regarding the combat powers of specialties, it has been confirmed that the number of data with higher variances is reduced, and that the max-min values are decreased on average in the new system. Additionally, the variances and the max-min values of combat powers of the subordinate units are reduced on average. Based on the results of the experiments, we confirmed that the application of the new system can help solve the Army's equitable assignment problem.

Due to confidentiality issues, we used randomly generated data sets in our experiments. However, all 20 data sets were created with nearly identical conditions. All data sets possessed the same number of subordinate units and the same number of specialties. They also had the same subordinate unit weights. If the experiments were conducted using a variety of data sets, the validity of the results would be further ensured. Moreover, if real data sets were used, the results would support the findings of this study even more. Experiments using real data sets may reveal problems that we did not anticipate. Therefore, in order to apply the new model proposed in this thesis, experiments using real data sets must be made beforehand.

Future research projects related to this study may be considered to reflect the weights of the specialties. In the model presented here, only the weights of each subordinate unit were reflected. However, when assigning recruits, the conditions required for each situation may vary. Therefore, we can gain flexibility by considering the weights of the specialties. In order to reflect such weights, it is necessary to reconsider the conditions required to calculate the combat powers and to compare them. If this study is continued, then it will be possible to present a new model that satisfies various conditions in the assignment of recruits.

Appendix A

AMPL implementation of the new system

A.1 Step 1 for α_1

The following AMPL model (***mod***) file is used for all data sets from 1 to 20. For the AMPL data (***dat***) file, data set #1 was used which is the same as Table 1.1. The ***mod*** file for α_1 at (2.1) is shown below.

Mod file: exa1.mod

```
reset;
set I;
param k;
set J = 1..k;
param t;
param L;

var a_1 >= 0 <= 1;
var Y{i in I, j in J} integer;
var M{i in I, j in J} integer;
var z{i in I, j in J} binary;

param R{i in I, j in J};
param S{i in I, j in J};
param E{i in I, j in J};
param c;
param W{i in I};

var V{i in I, j in J} = (Y[i,j]-S[i,j]+E[i,j])*z[i,j];

maximize alpha : a_1;
subject to c1 : sum{i in I, j in J} V[i,j] <= c;
subject to c2{i in I, j in J} : Y[i,j]-S[i,j]+E[i,j] <= L*z[i,j];
subject to c3{i in I, j in J} : 0 <= Y[i,j]-S[i,j]+E[i,j] + L*(1-z[i,j]);
```

```
subject to c4{i in I, j in J} : M[i,j]*a_1 <= Y[i,j];
subject to c5{i in I, j in J} : Y[i,j] <= M[i,j]*a_1+1-t;
subject to c6{i in I, j in J} : W[i]*R[i,j]-0.5+t <= M[i,j];
subject to c7{i in I, j in J} : W[i]*R[i,j]+0.5 >= M[i,j];
option solver gurobi;
```

The ***dat*** file using data #1 for α_1 is the following:

Dat file: ex1a1.dat

```
data;
set I := A B C D E F G H I J;
param k := 7;
param t := 1e-4;
param L := 1e+5;
param  R :
  1 2 3 4 5 6 7 :=
A      170    140    135    156    28     28     16
B      170    140    135    156    10     16     25
C      170    140    135    156    1      1      4
D      55     134    109    126    25     16     16
E      55     134    109    126    1      1      4
F      10     7      16     16     26     49     17
G      31     10     25     16     26     49     17
H      19     16     1      7      26     49     17
I      16     1      4      31     3      68     39
J      10     10     28     16     3      68     39 ;
param  S :
  1 2 3 4 5 6 7 :=
A      175    137    140    150    29     27     16
B      173    146    131    159    10     15     24
C      173    139    131    154    1      1      4
D      59     154    123    139    27     17     18
E      62     151    122    140    1      1      5
F      10     7      16     16     26     51     16
G      30     10     26     16     26     51     17
H      19     16     1      7      25     49     18
I      16     1      4      30     3      65     37
J      10     10     28     16     3      67     39 ;
param  E :
  1 2 3 4 5 6 7 :=
A      24     20     19     22     4      4      2
B      24     20     19     22     1      2      4
C      24     20     19     22     0      0      1
D      8      19     16     18     4      2      2
E      8      19     16     18     0      0      1
F      1      1      2      2      4      7      2
G      4      1      4      2      4      7      2
H      3      2      0      1      4      7      2
I      2      0      1      4      0      10     6
J      1      1      4      2      0      10     6 ;
param c := 171;
```

```
param W :=
A 1  B 1  C 1  D 1.1  E 1.1  F 1  G 1  H 1  I 1  J 1 ;
solve;
display a_1;
display V;
display sum{i in I, j in J} V[i,j];
```

A.2 Step 1 for α_{2j}

As shown in section A.1, the following ***mod*** file is used for all data sets from 1 to 20 and data #1 was used for the following ***dat*** file. Data # 1 is the same as Table 1.1. The AMPL demo version was used in the experiments. Since the demo licenses for AMPL is limited to 300 constrains for nonlinear problems, I split the ***dat*** file into two. The ***mod*** file for α_{2j} at (2.2) is the following:

Mod file: exa2.mod

```
reset;
set I;
param k;
param n;
set J = k+1..n;
param t;
param L;

var a_2{j in J} >= 0 <= 1;
var Y{i in I, j in J} integer;
var M{i in I, j in J} integer;
var z{i in I, j in J} binary;

param R{i in I, j in J};
param S{i in I, j in J};
param E{i in I, j in J};
param C{j in J};
param W{i in I};
var V{i in I, j in J} = (Y[i,j]-S[i,j]+E[i,j])*z[i,j];

maximize alpha : sum{j in J} a_2[j];
subject to c1{j in J} : sum{i in I} V[i,j] <= C[j];
subject to c2{i in I, j in J} : Y[i,j]-S[i,j]+E[i,j] <= L*z[i,j];
subject to c3{i in I, j in J} : 0 <= Y[i,j]-S[i,j]+E[i,j] + L*(1-z[i,j]);
subject to c4{i in I, j in J} : M[i,j]*a_2[j] <= Y[i,j];
```

```
subject to c5{i in I, j in J} : Y[i,j] <= M[i,j]*a_2[j]+1-t;
subject to c6{i in I, j in J} : W[i]*R[i,j]-0.5+t <= M[i,j];
subject to c7{i in I, j in J} : W[i]*R[i,j]+0.5 >= M[i,j];
option solver gurobi;
```

The ***dat*** files using data #1 for α_{2j} are the following:

<u>Dat file: ex1a2.dat</u>

```
data;
set I := A B C D E F G H I J;
param k := 7;
param n := 14;
param t := 1e-4;
param L := 1e+5;
param  R :
  8 9 10 11 12 13 14  :=
A      5      17     7      5      11     11     3
B      7      15     21     19     13     15     17
C      15     7      19     9      13     17     21
D      17     15     7      3      17     7      15
E      7      3      1      13     3      19     13
F      21     13     9      1      1      21     9
G      17     15     3      19     13     13     5
H      17     1      19     13     11     1      5
I      15     5      7      11     9      21     21
J      11     21     5      15     11     3      17 ;
param  S :
  8 9 10 11 12 13 14  :=
A      5      17     7      5      12     11     3
B      7      15     20     18     13     16     18
C      16     7      20     9      13     16     22
D      19     17     7      3      19     8      17
E      7      3      1      14     3      22     14
F      20     13     9      1      1      22     9
G      17     15     3      19     12     14     5
H      17     1      20     13     11     1      5
I      15     5      7      11     9      22     20
J      12     22     5      15     11     3      16 ;
param  E :
  8 9 10 11 12 13 14  :=
A      1      2      1      1      2      2      0
B      1      2      3      3      2      2      2
C      2      1      3      1      2      2      3
D      2      2      1      0      2      1      2
E      1      0      0      2      0      3      2
F      3      2      1      0      0      3      1
G      2      2      0      3      2      2      1
H      2      0      3      2      2      0      1
I      2      1      1      2      1      3      3
J      2      3      1      2      2      0      2 ;
param C :=
8 2      9 2     10 2     11 2     12 2      13 2     14 2 ;
```

```
param W :=
A 1  B 1  C 1  D 1.1  E 1.1  F 1  G 1  H 1  I 1  J 1 ;
solve;
display{j in J} a_2[j];
display V;
```

<u>Dat file: ex1a2_2.dat</u>

```
data;
set I := A B C D E F G H I J;
param k := 14;
param n := 20;
param t := 1e-4;
param L := 1e+5;
param  R :
   15 16 17 18 19 20 :=
A      17     9      9      1      7      19
B      13     11     11     7      19     7
C      19     7      5      21     17     19
D      11     5      11     9      3      9
E      7      5      5      5      7      9
F      17     11     19     11     17     17
G      3      13     5      13     3      11
H      7      7      9      11     13     9
I      1      1      5      13     3      15
J      15     5      15     5      5      5 ;
param  S :
  15 16 17 18 19 20 :=
A      17     9      9      1      7      19
B      13     11     11     7      19     7
C      19     7      5      21     17     20
D      12     5      12     10     3      10
E      8      6      5      5      8      10
F      18     11     20     11     16     17
G      3      13     5      13     3      11
H      7      7      9      10     14     9
I      1      1      5      13     3      15
J      15     5      15     5      5      5 ;
param  E :
  15 16 17 18 19 20 :=
A      2      1      1      0      1      3
B      2      2      2      1      3      1
C      3      1      1      3      2      3
D      2      1      2      1      0      1
E      1      1      1      1      1      1
F      2      2      3      2      2      2
G      0      2      1      2      0      2
H      1      1      1      2      2      1
I      0      0      1      2      0      2
J      2      1      2      1      1      1 ;
param C :=
15 2     16 2     17 2      18 2      19 2       20 2;
```

```
param W :=
A 1  B 1  C 1  D 1.1  E 1.1  F 1  G 1  H 1  I 1  J 1;
solve;
display{j in J} a_2[j];
display V;
```

A.3 Step 2

As described above, the following ***mod*** file is used for all data sets from 1 to 20 and data #1 was used for the following ***dat*** file. Data # 1 is the same as Table 1.1. The ***mod*** file for step 2 at (2.3) is the following:

Mod file: ex.mod

```
reset;
set I;
param k integer;
param n integer;
param t;
param L;

var X{i in I, j in 1..n} >= 0 integer;
var P;
var M{i in I, j in 1..n} integer;
var z{i in I, j in 1..n} binary;

param R{i in I, j in 1..n};
param S{i in I, j in 1..n};
param E{i in I, j in 1..n};
param c;
param C{j in k+1..n};
param W{i in I};
param a_1;
param a_2{j in k+1..n};

maximize minpower : P;
subject to c1{i in I} : P <= (1/card(1..n))*sum{j in 1..n} ((S[i,j]-
E[i,j]+X[i,j])/M[i,j]);
subject to c2 : sum{i in I, j in 1..k} X[i,j] = c;
subject to c3{j in k+1..n} : sum{i in I} X[i,j] = C[j];
subject to c4{i in I, j in 1..k} : ((S[i,j]-E[i,j]+X[i,j])/M[i,j]) >= a_1;
subject to c5{i in I, j in k+1..n} : ((S[i,j]-E[i,j]+X[i,j])/M[i,j]) >=
a_2[j];
subject to c6{i in I, j in 1..n} : M[i,j] >= W[i]*R[i,j]-0.5+t;
subject to c7{i in I, j in 1..n} : M[i,j] <= W[i]*R[i,j]+0.5;
```

```
subject to c8{i in I, j in 1..n} : X[i,j]<=M[i,j]-S[i,j]+E[i,j]+L*(1-z[i,j]);
subject to c9{i in I, j in 1..n} : X[i,j]<=L*z[i,j];
option solver gurobi;
```

The ***dat*** file using data #1 for step 2 is the following:

<u>Dat file: ex1.dat</u>

```
data;
set I := A B C D E F G H I J;
param k := 7;
param n := 20;
param t := 1e-4;
param L := 1e+5;
param a_1 := 0.9;
param a_2 :=
8 0.809      11 0.818    14 0.809    17 0.8      20 0.842;
9 0.857      12 0.818    15 0.846    18 0.8
10 0.809     13 0.875    16 0.818    19 0.842
param  R :
  1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 :=
A      170     140     135     156     28      28      16      5       17      7       5
       11      11      3       17      9       9       1       7       19
B      170     140     135     156     10      16      25      7       15      21      19
       13      15      17      13      11      11      7       19      7
C      170     140     135     156     1       1       4       15      7       19      9
       13      17      21      19      7       5       21      17      19
D      55      134     109     126     25      16      16      17      15      7       3
       17      7       15      11      5       11      9       3       9
E      55      134     109     126     1       1       4       7       3       1       13
       3       19      13      7       5       5       5       7       9
F      10      7       16      16      26      49      17      21      13      9       1
       1       21      9       17      11      19      11      17      17
G      31      10      25      16      26      49      17      17      15      3       19
       13      13      5       3       13      5       13      3       11
H      19      16      1       7       26      49      17      17      1       19      13
       11      1       5       7       7       9       11      13      9
I      16      1       4       31      3       68      39      15      5       7       11
       9       21      21      1       1       5       13      3       15
J      10      10      28      16      3       68      39      11      21      5       15
       11      3       17      15      5       15      5       5       5 ;
param  S :
  1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 :=
A      175     137     140     150     29      27      16      5       17      7       5
       12      11      3       17      9       9       1       7       19
B      173     146     131     159     10      15      24      7       15      20      18
       13      16      18      13      11      11      7       19      7
C      173     139     131     154     1       1       4       16      7       20      9
       13      16      22      19      7       5       21      17      20
D      59      154     123     139     27      17      18      19      17      7       3
       19      8       17      12      5       12      10      3       10
```

```
E      62     151    122    140    1      1      5      7      3      1      14
       3      22     14     8      6      5      5      8      10
F      10     7      16     16     26     51     16     20     13     9      1
       1      22     9      18     11     20     11     16     17
G      30     10     26     16     26     51     17     17     15     3      19
       12     14     5      3      13     5      13     3      11
H      19     16     1      7      25     49     18     17     1      20     13
       11     1      5      7      7      9      10     14     9
I      16     1      4      30     3      65     37     15     5      7      11
       9      22     20     1      1      5      13     3      15
J      10     10     28     16     3      67     39     12     22     5      15
       11     3      16     15     5      15     5      5      5 ;
param  E :
  1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 :=
A      24     20     19     22     4      4      2      1      2      1      1
       2      2      0      2      1      1      0      1      3
B      24     20     19     22     1      2      4      1      2      3      3
       2      2      2      2      2      2      1      3      1
C      24     20     19     22     0      0      1      2      1      3      1
       2      2      3      3      1      1      3      2      3
D      8      19     16     18     4      2      2      2      2      1      0
       2      1      2      2      1      2      1      0      1
E      8      19     16     18     0      0      1      1      0      0      2
       0      3      2      1      1      1      1      1      1
F      1      1      2      2      4      7      2      3      2      1      0
       0      3      1      2      2      3      2      2      2
G      4      1      4      2      4      7      2      2      2      0      3
       2      2      1      0      2      1      2      0      2
H      3      2      0      1      4      7      2      2      0      3      2
       2      0      1      1      1      1      2      2      1
I      2      0      1      4      0      10     6      2      1      1      2
       1      3      3      0      0      1      2      0      2
J      1      1      4      2      0      10     6      2      3      1      2
       2      0      2      2      1      2      1      1      1 ;
param c := 171;
param C :=
8 2     11 2    14 2    17 2    20 2;
9 2     12 2    15 2    18 2
10 2    13 2    16 2    19 2
param W :=
A 1  B 1  C 1  D 1.1  E 1.1  F 1  G 1  H 1  I 1  J 1;

solve;
display P;
display {i in I} (1/card(1..n))*sum{j in 1..n} ((S[i,j]-
E[i,j]+X[i,j])/M[i,j]);
display (1/card(1..n))*(1/card(I))*sum{i in I, j in 1..n} ((S[i,j]-
E[i,j]+X[i,j])/M[i,j]);
option display_transpose -11;
display M;
display{i in I, j in 1..n} max(M[i,j]-S[i,j]+E[i,j],0);
display X;
```

Appendix B

Reviewing the results of the new system with Excel

B.1 Step 1 for α_1

After creating Table B.1, we can verify the feasibility of this problem using Solver in Excel.

Table B.1: Table for α_1

specialty		1	2	3	4	5	6	7	
alpha		0.9							①
minimum demand for alpha	A	2	9	1	13	1	3	1	②
	B	4	0	10	4	0	2	3	
	C	4	7	10	9	0	0	1	
	D	4	0	1	5	3	2	1	
	E	1	1	2	4	0	0	0	
	F	0	1	1	1	2	1	2	
	G	2	0	1	1	2	1	1	
	H	2	1	0	1	3	3	0	
	I	1	0	1	2	0	7	5	
	J	0	0	2	1	0	5	3	
sum		161							③
C		171							④

① Alpha: α_1, the number calculated by AMPL

② Minimum demand: $max\ (\left\lceil\left[W_i \cdot R_{ij}\right] \cdot \alpha_1\right\rceil - S_{ij} + E_{ij}, 0),\ i \in I,\ j \in \{1, \dots, 7\}$

③ Sum: $\sum_{i=A}^{J}\sum_{j=1}^{7} max\ ([[W_i \cdot R_{ij}] \cdot \alpha_1] - S_{ij}\ + E_{ij}\ , 0)$

④ C : The number of recruits

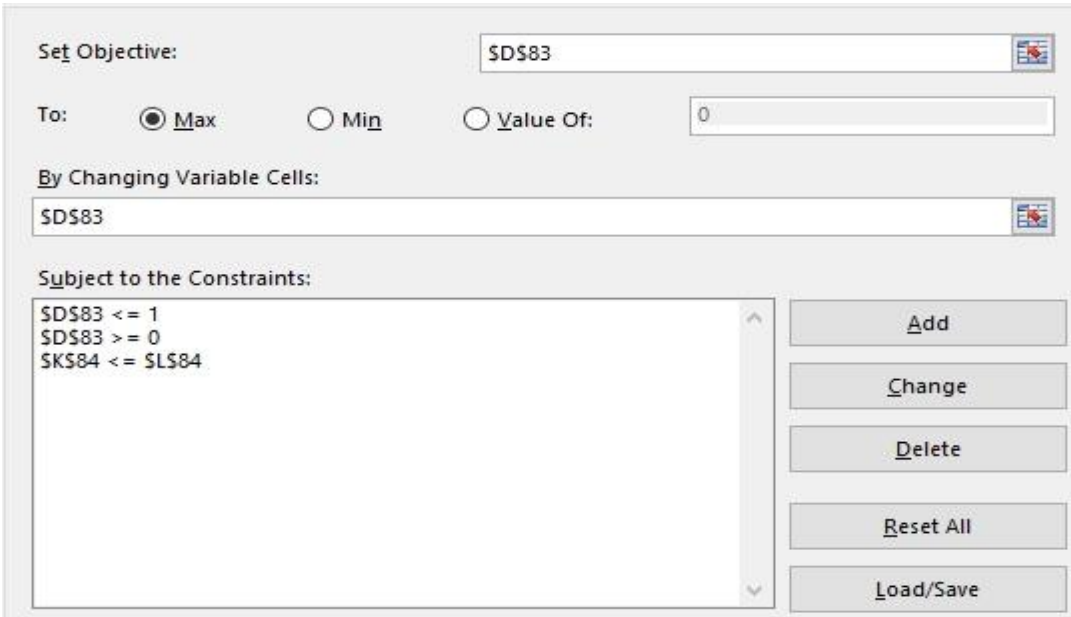

Figure B.1: Solver settings to find α_1

In order to run the Solver as shown in Figure B.1, we can use Table B.2.

Table B.2: Information for Figure B.1

	Cell	**Equation**
Objective	$D$83	α_1
		①
Variable	$D$83	α_1
		①
Constraint	$D$83 <= 1	$\alpha_1 \leq 1$
		① ≤ 1
	$D$83 >= 0	$0 \leq \alpha_1$
		$0 \leq$ ①
	$K$84 <= $L$84	$\sum_{i=A}^{J}\sum_{j=1}^{7} \max\left([[W_i \cdot R_{ij}] \cdot \alpha_1] - S_{ij} + E_{ij}, 0\right) \leq C$
		③ $\leq$ ④

B.2 Step 1 for α_{2j}

We need the maximum of each α_{2j}; however, here we maximize the sum of α_{2j}. As explained earlier, we can obtain the same result, since the values α_{2j} do not affect each other. After we create Table B.3, we can verify the feasibility of this problem using the Excel Solver.

① Alpha: $\alpha_{2j}, j \in \{8, \dots, 20\}$, the numbers calculated by AMPL

② Minimum demand: $max\ (\lceil \lceil W_i \cdot R_{ij} \rceil \cdot \alpha_{2j} \rceil - S_{ij} + E_{ij}, 0), i \in I,\ j \in \{8, \dots, 20\}$

③ Sum: $\sum_{i=A}^{J} max\ (\lceil \lceil W_i \cdot R_{ij} \rceil \cdot \alpha_{2j} \rceil - S_{ij}\ + E_{ij}\ , 0), j \in \{8, \dots, 20\}$

④ C_j: The numbers of recruits with specialty j, $j \in \{8, \dots, 20\}$

⑤ Sum alpha: $\sum_{j=8}^{20} \alpha_{2j}$

Table B.3: Table for α_{2j}

specialty		8	9	10	11	12	13	14	15	16	17	18	19	20	
alpha		0.809	0.857	0.809	0.818	0.818	0.875	0.809	0.846	0.818	0.8	0.8	0.842	0.842	①
minimum demand for alpha	A	1	0	0	1	0	1	0	0	0	0	0	0	0	②
	B	0	0	0	1	0	0	0	0	0	0	0	0	0	
	C	0	0	0	0	0	1	0	1	0	0	0	0	0	
	D	0	0	1	0	0	0	0	1	1	0	0	0	0	
	E	1	0	0	0	0	0	0	0	0	1	1	0	0	
	F	0	1	0	0	0	0	0	0	0	0	0	1	0	
	G	0	0	0	0	1	0	1	0	0	0	0	0	1	
	H	0	0	0	0	0	0	1	0	0	0	1	0	0	
	I	0	1	0	0	0	0	0	0	0	0	0	0	0	
	J	0	0	1	0	0	0	0	0	1	0	0	1	1	
sum		2	2	2	2	1	2	2	2	2	1	2	2	2	③
Cj		2	2	2	2	2	2	2	2	2	2	2	2	2	④
sum alpha		10.743													⑤

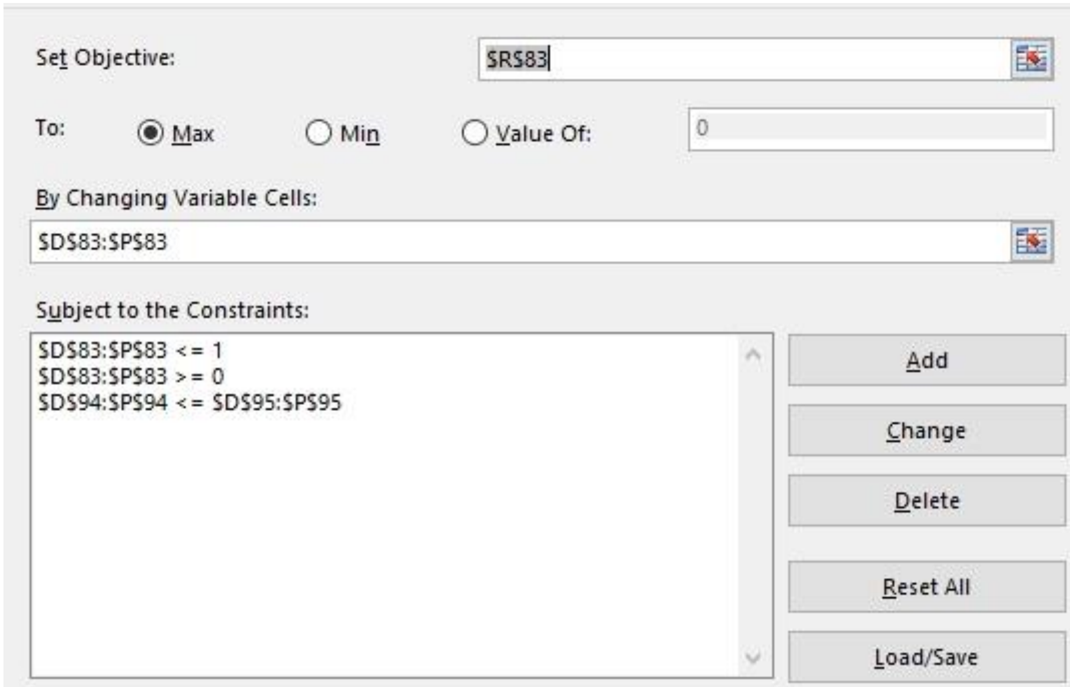

Figure B.2: Solver settings to find α_{2j}

In order to run the Solver as shown in Figure B.2, we can use Table B.4.

Table B.4: Information for Figure B.2

	Cell	**Equation**
Objective	$R$83	$\sum_{j=8}^{20} \alpha_{2j}$
		⑤
Variable	$D$83:$P$83	$\alpha_{2j}, j \in \{8, \ldots, 20\}$
		①
Constraint	$D$83:$P$83 <= 1	$\alpha_{2j} \leq 1, j \in \{8, \ldots, 20\}$
		① ≤ 1
	$D$83:$P$83 >= 0	$0 \leq \alpha_{2j}, j \in \{8, \ldots, 20\}$
		$0 \leq$ ①
	$D$94:$P$94 <= $D$95:$P$95	$\sum_{i=A}^{J} \max\left(\left[\left[W_i \cdot R_{ij}\right] \cdot \alpha_{2j}\right] - S_{ij} + E_{ij}, 0\right) \leq C_j,$ $j \in \{8, \ldots, 20\}$
		③ $\leq$ ④

B.3 Step 2

After we create Table B.5, we can verify feasibility using the Excel Solver.

Table B.5: Table for X_{ij}

units(i)	specialty(j)	1	2	....	19	20		total non fix	total fixed	total	rate of unit	
A	Required(Rij)	170	140	....	7	19		673	121	794	91.596%	⑦
	M. Required(Mij)	170	140	....	7	19	①	673	121	794		
	Serving(Sij)	175	137	....	7	19		674	122	796		
	Exp. Dischar(Eij)	24	20	....	1	3		95	17	112		
	Demand(Dij)	19	23	....	1	3	②	94	16	110		
100%	assign(xij)	2	9	....	0	0	③	30	3	33		
	rate(after assign)	0.9	0.9	....	0.85714	0.84211	④					
					⋮							
J	Required(Rij)	10	10	....	5	5		20	10	30	95.000%	⑦
	M. Required(Mij)	10	10	....	5	5	①	20	10	30		
	Serving(Sij)	10	10	....	5	5		20	10	30		
	Exp. Dischar(Eij)	1	1	....	1	1		2	2	4		
	Demand(Dij)	1	1	....	1	1	②	2	2	4		
100%	assign(xij)	0	0	....	1	1	③	0	2	2		
	rate(after assign)	0.9	0.9	....	1	1	④					
total	M. Required(Mij)	718	758	....	95	122		1476	217	1693	91.388%	
	Serving(Sij)	727	771	....	95	123		1498	218	1716		
	Exp. Dischar(Eij)	99	103	....	12	17		202	29	231		
	assign(xij)	20	19	....	2	2	⑨	39	4	43		
	rate(after assign)	0.90251	0.90633	....	0.89474	0.88525		⑩				
MIN		0.9		....	0.84211	0.84211	⑤			MIN	89.774%	⑧
Alpha		0.9		....	0.842	0.842	⑥					

① M. Required (M_{ij}): $[W_i \cdot R_{ij}]$, round$(W_i \cdot R_{ij})$, $i \in I, j \in J$

② Demand (D_{ij}): $\max([W_i \cdot R_{ij}] - S_{ij} + E_{ij}, 0)$, $i \in I, j \in J$

③ assign (X_{ij}): X_{ij}, $i \in I, j \in J$, the numbers calculated by AMPL

④ rate (after assign): $\frac{S_{ij}-E_{ij}+X_{ij}}{[W_i \cdot R_{ij}]}$, $i \in I, j \in J$

⑤ MIN: $\min_{i \in I}\left(\frac{S_{ij}-E_{ij}+X_{ij}}{[W_i \cdot R_{ij}]}\right), j \in \{1, \ldots, 7\}$ / $\min_{i \in I}\left(\frac{S_{ij}-E_{ij}+X_{ij}}{[W_i \cdot R_{ij}]}\right)$, for each $j \in \{8, \ldots, 20\}$

⑥ Alpha: α_1 and α_{2j}, $j \in \{8, \ldots, 20\}$, the numbers calculated by AMPL

⑦ rate of unit: $\frac{1}{20}\sum_{j=1}^{20}\left(\frac{S_{ij}-E_{ij}+X_{ij}}{[W_i \cdot R_{ij}]}\right), i \in I$

⑧ MIN: $\min_{i \in I} \frac{1}{20}\sum_{j=1}^{20}\left(\frac{S_{ij}-E_{ij}+X_{ij}}{[W_i \cdot R_{ij}]}\right)$

⑨ total-assign (X_{ij}): $\sum_{i=A}^{J} X_{ij}\,, j \in \{8, \ldots, 20\}$

⑩ total-assign (X_{ij}): $\sum_{i=A}^{J}\sum_{j=1}^{7} X_{ij}$

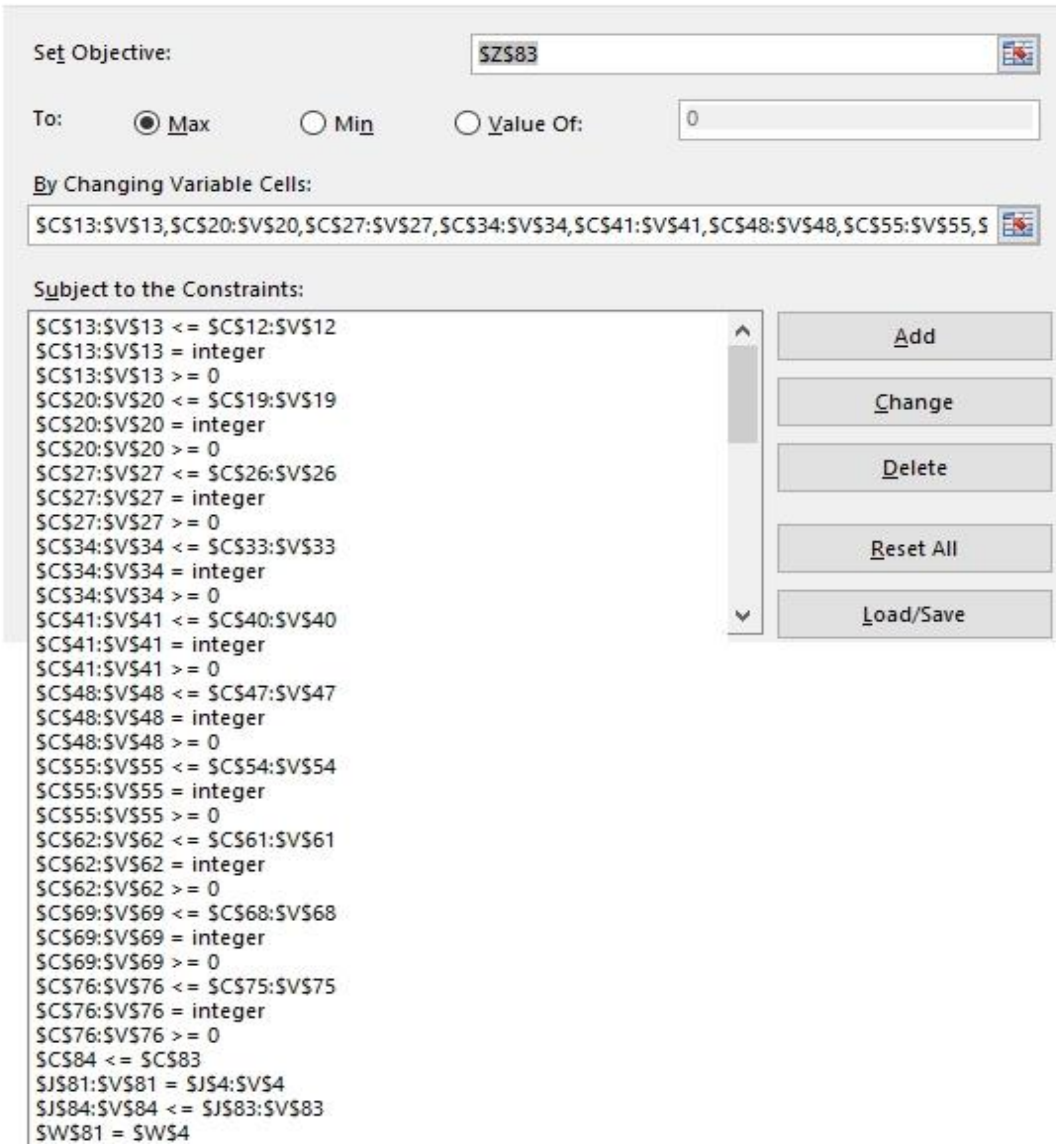

Figure B.3: Solver settings to find X_{ij}

In order to run the Solver as shown in Figure B.3, we can use Table B.6.

Table B.6: Information for Figure B.3

	Cell	Equation
Objective	$Z$83	$\min\limits_{i \in I} \frac{1}{20} \sum_{j=1}^{20} \left(\frac{S_{ij} - E_{ij} + X_{ij}}{[W_i \cdot R_{ij}]} \right)$
		8
Variable	$C$13:$V$13, $C$20:$V$20 . . $C$76:$V$76	$X_{ij}, i \in I, j \in J$
		3
Constraint	$C$13:$V$13 <= $C$12:$V$12 $C$20:$V$20 <= $C$19:$V$19 . . $C$76:$V$76 <= $C$75:$V$75	$X_{ij} \leq \max\left(\left[W_i \cdot R_{ij}\right] - S_{ij} + E_{ij}, 0\right)$ $, i \in I, j \in J$
		3 $\leq$ 2
	$C$13:$V$13 = integer $C$20:$V$20 = integer . . $C$76:$V$76 = integer	$X_{ij} \in \mathbb{Z}, i \in I, j \in J$
		3 $\in \mathbb{Z}$
	$C$13:$V$13 >= 0 $C$20:$V$20 >= 0 . . $C$76:$V$76 >= 0	$X_{ij} \geq 0, i \in I, j \in J$
		3 ≥ 0
	$C$84 <= $C$83	$\frac{S_{ij} - E_{ij} + X_{ij}}{[W_i \cdot R_{ij}]} \geq \alpha_1, \ i \in I, \ j \in \{1, \dots, 7\}$
		5 $\geq$ 6
	$J$81:$V$81 = $J$4:$V$4	$\sum_{i=A}^{J} X_{ij} = C_j \ , j \in \{8, \dots, 20\}$
		9 $= C_j \ , j \in \{8, \dots, 20\}$
	$J$84:$V$84 <= $J$83:$V$83	$\frac{S_{ij} - E_{ij} + X_{ij}}{[W_i \cdot R_{ij}]} \geq \alpha_{2j}, i \in I, \ j \in \{8, \dots, 20\}$
		5 $\geq$ 6
	$W$81 = $W$4	$\sum_{i=A}^{J} \sum_{j=1}^{7} X_{ij} = C$
		10 $= C$

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Biography

Doheon Han was born in 1987 in Icheon, South Korea. In March 2010, he graduated from the Korea Military Academy with a Bachelor of Science degree in Applied Physics and a Bachelor of Military Science degree. He had served in the Korea Army for 7 years. In 2016, he began his graduate studies at the Department of Industrial and Systems Engineering, Lehigh University. He will graduate with a Master of Science degree in Industrial and Systems Engineering in May 2018.