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Robust dynamic pricing of perishable products

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Abstract

This research is going to solve robust dynamic pricing of perishable products using different models and techniques. Then, the insights and efficiency we gain from these different models will be compared.

Chapter 1 Introduction & Literature

1.1 Introduction

We are going to study dynamic pricing for perishable products when demand is uncertain and the underlying probabilities are not known precisely. We are going to use two different models and techniques. Both models considers a linear price-response function with additive uncertainty, $D =$ Market Size - Price Elasticity * Price $+ \varepsilon_S$, with $E[\varepsilon_S] = 0$. However, the two models exploit different assumptions of the fluctuations of the demand. The first model deals with demand volatility by assigning nominal values (i.e., mean value of the interval forecast) to both market size and price elasticity and applying probabilities to different realizations of ε_S. The second model assigns distributions to both market size and price elasticity with some means and standard deviations and simulates all possible demands with the above parameters. Moreover, the first model assumes that price elasticity of a product be only related with its own price, while the second supposes that price elasticity of a product might be a matrix related with both its own price and other products.

1.2 Literature Review

Studies on yield management, overbooking and pricing have been appearing since 1971. However, the three topics generally have been done separately. Therefore, Weatherford and Bodily [11] developed a framework categorizing the types of problems within the three topics and with regard to interrelatedness of the three topics. Furthermore, this paper proposed a term, perishable-asset revenue management (PARM), to replace the term yield management. Weather and Bodily listed fourteen elements to distinguish the comprehensive taxonomy. Also, they surveyed the problems which have been found solutions up to 1992 and gave recommendations for the after-then research. The following table is the comprehensive taxonomy by Weatherford and Bodily ([11])

Elements	Descriptors
A. Resource	Discrete/Continuous
B. Capacity	Fixed/Nonfixed
C. Prices	Predetermined/Set optimally/Set jointly
D. Willingness to Pay	Buildup/Drawdown
E. Discount Price Classes	1/2/3/.../I
F. Reservation Demand	Deterministic/Mixed/Random-independent/Random-correlated
G. Show-Up of Discount Reservations	Certain/Uncertain without cancellation/Uncertain with cancellation

H. Show-Up of Full-Price Reservations	Certain/Uncertain without cancellation/Uncertain with cancellation
I. Group Reservations	No/Yes
J. Diversion	No/Yes
K. Displacement	No/Yes
L. Bumping Procedure	None/Full-price/Discount/FCFS/Auction
M. Asset Control Mechanism	Distance/Nested
N. Decision Rule	Simple static/Advanced static/Dynamic

Comprehensive Taxonomy (Weatherford and Bodily [8])

Later in 1995, Feng and Gallego [6] proposed a method to decide the optimal starting and stopping times for a single price change from a given initial price when facing the problem of selling a fixed stock of items over a finite horizon. The method suggested a time threshold dynamically applied depending on the number of yet unsold items sequentially.

Bitran and Mondschein [4] suggested pricing policies as functions of time and inventory based on a continuous time model where a seller deals with a stochastic arrival of customers. Feng and Gallego defined the time-to-go, a time threshold, depending on the quantities of yet unsold inventory. Bitran and Mondschein complied their study with the real practices by retail stores.

In 1999, McGill and Van Ryzin [8] reviewed the research on transportation revenue management (a.k.a. yield management) beyond the efforts of Weatherford and Bodily. They covered developments not only in overbooking and pricing but also forecasting and seat inventory control.

Bitran and Caldentey [2] examined the research of dynamic pricing policies and the impact on revenue management in summer 2003, while Elmaphraby and Keskinocak [5] gave an overview in the research in dynamic pricing with inventory considerations.

In 2006, Bertsimas and Thiele [1] gave a tutorial describing the robust and data-driven optimization when making decisions under uncertainty without perfect information. Actually, robust optimization is booming around late 1990s as Ben-Tal and Nemirovski [2] started to study on the computationally tractable and explicit robust counterparts of uncertain problems.

1.3 Contributions

In this thesis, we develop the approaches giving insights on the impact of uncertainty level on the optimal solutions, which is the percentage of price reduction, for both one-product systems and multiple-product systems. Furthermore, in the approach for the multi-product systems, we suggest the introduction of correlation between products.

Chapter 2 Pricing with Uncertain Probabilities

2.1 Problem Setup

We apply linear regression to the forecast of the demand. According to the regression assumption, the residual value is random variable which is normally distributed. Then, we use discretization of the random variable to solve the problem with different residual values with different probabilities. We will use the following notation for this model.

N: the total number of items available at the beginning of the season (assumed given for now and no reorder allowed)

T: the total length of the season (e.g., T=12 weeks)

At each time period (beginning of each week), the manager looks at the inventory in stock and decides whether he is going to put the item on sale at a discounted price (20, 30, or 40% discount, for instance) or keep selling it at the initial price p (assume p is given, imposed by the manufacturer).

If he decides to sell at a discount, he must decide on the size of the discount (finite number of strategy available, for example, 20%, 30%, etc.)

The problem is that the manager does not quite know how customers will react.

2.2 Simplified Formulation Model

At first, we build a model with only two time periods and one product. Here are the related assumptions and settings.

1. Time 0: No sale

   Original selling price per unit = p

   End-of-period inventory = x_0

2. Time 1: Sale or no sale?

   Discount rate?

   Start-of-period inventory = x_0

3. Demand without sale = $D_1\ (p)$

Distribution of D_1 is known, say, π_D

$\pi_D^i \in [0, 1]$ and $\sum_{i=1}^{N} \pi_D^i = 1$

4. Demand with sale = $S_1\ ((1-\alpha)p)$

Distribution of S_1 is unknown, say, π_S

$\pi_S^{\omega} \in [0, 1]$ and $\sum_{\omega=1}^{2T+1} \pi_S^{\omega} = 1$

π_S has odd number of scenarios, say, $2T+1$, i.e. $\sum_{\omega=1}^{2T+1} \pi_S^{\omega} = 1$.

$S_1 = a^s - b^s * (1-\alpha) * p + \varepsilon_S^{\omega}$, which is mean demand with a^s and b^s known and with ε_S^{ω} having $2T+1$ values at interval of M (i.e. $[-M, M]$)

Let u be the indicator whether putting on sale or not

α be the discount rate

The classical optimization model is as follows:

$$\begin{aligned}
\text{Maximize} \quad & (1-u) * p * E[\min(x_0, D_1)] + u * (1-\alpha) * p * E[\min(x_0, S_1)] \\
\text{Subject to} \quad & \pi_S^{\omega} \in [0, 1], \forall \omega \\
& \sum_{\omega=1}^{2T+1} \pi_S^{\omega} = 1 \\
& 0 < \alpha < 1 \\
& u \in \{0, 1\}
\end{aligned}$$

Then we can rewrite the expected values in the objective into the as follows:

$$\begin{aligned}
\text{Maximize} \quad & (1-u) * p * \sum_{i=1}^{N} [\pi_D^i * \min(x_0, D_1^i)] + u * (1-\alpha) * p * \sum_{\omega=1}^{2T+1} [\pi_S^{\omega} * \min(x_0, S_1^{\omega})] \\
\text{Subject to} \quad & \pi_S^{\omega} \in [0, 1], \forall \omega \\
& \sum_{\omega=1}^{2T+1} \pi_S^{\omega} = 1 \\
& 0 < \alpha < 1 \\
& u \in \{0, 1\}
\end{aligned}$$

To solve it, we decompose the above model into two parts regarding $u=0$ and $u=1$:

1. $u=0 \rightarrow$ baseline case (no optimization)

2. $u=1 \rightarrow$ the classical optimization model is like the following:

$$\text{Maximize} \quad (1-\alpha)*\sum_{\omega=1}^{2T+1}\pi_S^{\omega}*\min(x_0, a^s - b^s*(1-\alpha)*p+\varepsilon_S^{\omega})$$

$$\text{Subject to} \quad \sum_{\omega=1}^{2T+1}\pi_S^{\omega}*\varepsilon_S^{\omega}=0$$

$$\sum_{\omega=1}^{2T+1}\pi_S^{\omega}=1$$

$$\pi_S^{\omega}\geq 0, \forall\omega$$

$$\pi_S^{\omega}\leq 1, \forall\omega$$

$$\underline{\underline{\pi_S^{\omega}}}\leq \pi_S^{\omega}\leq \overline{\overline{\pi_S^{\omega}}}, \forall\omega$$

$$\underline{\underline{\pi_S^{\omega}}}=(1-\gamma)\pi_s^{\omega 0}, \forall\omega, \quad \pi_s^{\omega 0} \text{ is nominal } \pi_s \text{ under scenario } \omega$$

$$\overline{\overline{\pi_S^{\omega}}}=(1+\gamma)\pi_s^{\omega 0}, \forall\omega, \quad \pi_s^{\omega 0} \text{ is nominal } \pi_s \text{ under scenario } \omega$$

$$\varepsilon_S^{\omega}\in[-M, M], \forall\omega$$

$$\varepsilon_S^{\omega}=-M+\frac{2M}{2T}*(\omega-1), \forall\omega$$

$$0\leq\alpha\leq 1$$

$$0.05\leq\alpha\leq 0.95$$

Since p is given, we can eliminate it from the objective function. Also, because we care about the robustness under some given uncertainty set, we are trying to do worst case analysis. Therefore, the classical optimization model is again rewritten to the robust optimization counterpart model:

$$\text{Maximize}_{0\le\alpha\le1} \quad \min_{\pi_s \in \pi_s} (1-\alpha)*p*\sum_{\omega=1}^{2T+1} \pi_S^{\omega} * \min(x_0, a^s - b^s*(1-\alpha)*p+\varepsilon_S^{\omega})$$

$$\text{Subject to} \quad \sum_{\omega=1}^{2T+1} \pi_S^{\omega} * \varepsilon_S^{\omega} = 0$$

$$\sum_{\omega=1}^{2T+1} \pi_S^{\omega} = 1$$

$$\pi_S^{\omega} \ge 0, \forall\omega$$

$$\pi_S^{\omega} \le 1, \forall\omega$$

$$\underline{\underline{\pi_S^{\omega}}} \le \pi_S^{\omega} \le \overline{\overline{\pi_S^{\omega}}}, \forall\omega$$

$$\underline{\underline{\pi_S^{\omega}}} = (1-\gamma)\pi_S^{\omega 0}, \forall\omega, \quad \pi_s^{\omega 0} \text{ is nominal } \pi_s \text{ under scenario } \omega$$

$$\overline{\overline{\pi_S^{\omega}}} = (1+\gamma)\pi_S^{\omega 0}, \forall\omega, \quad \pi_s^{\omega 0} \text{ is nominal } \pi_s \text{ under scenario } \omega$$

$$\varepsilon_S^{\omega} \in [-M, M], \forall\omega$$

$$\varepsilon_S^{\omega} = -M + \frac{2M}{2T}*(\omega-1), \forall\omega$$

Here we have to add one constraint for the natural rule that demand be always nonnegative, which is $a^s - b^s*(1-\alpha)*p+\varepsilon_S^{\omega} \ge 0$. Then, we replace ε_S^{ω} with the worst value $-M$, and we get $\alpha \ge 1-\frac{a^s-M}{b^s p}$. We rewritten the model as follows:

$$\underset{0\le\alpha\le 1}{\text{Maximize}} \quad \min_{\pi_s \in \pi_s} (1-\alpha) * p * \sum_{\omega=1}^{2T+1} \pi_S^{\omega} * \min(x_0, a_S - b_S * (1-\alpha) * p + \varepsilon_S^{\omega})$$

$$\text{Subject to} \quad \sum_{\omega=1}^{2T+1} \pi_S^{\omega} * \varepsilon_S^{\omega} = 0$$

$$\sum_{\omega=1}^{2T+1} \pi_S^{\omega} = 1$$

$$\pi_S^{\omega} \ge 0, \forall \omega$$

$$\pi_S^{\omega} \le 1, \forall \omega$$

$$\underline{\underline{\pi_S^{\omega}}} \le \pi_S^{\omega} \le \overline{\overline{\pi_S^{\omega}}}, \forall \omega$$

$$\underline{\underline{\pi_S^{\omega}}} = (1-\gamma)\pi_S^{\omega 0}, \forall \omega, \quad \pi_s^{\omega 0} \text{ is nominal } \pi_s \text{ under scenario } \omega$$

$$\overline{\overline{\pi_S^{\omega}}} = (1+\gamma)\pi_S^{\omega 0}, \forall \omega, \quad \pi_s^{\omega 0} \text{ is nominal } \pi_s \text{ under scenario } \omega$$

$$\varepsilon_S^{\omega} = -M + \frac{2M}{2T} * (\omega - 1), \forall \omega$$

$$\alpha \ge 1 - \frac{a^s - M}{b^s p}$$

Then, we have to write the dual of the min part to make it a max problem and obtain a big, pure max problem thereafter. It's a tractable way to solve this problem and it also can help us get theoretical insights about the influence of parameters on the optimal solution of α.

To make it less complicated, we replace $\min(x_0, a_S - b_S * (1-\alpha) * p + \varepsilon_S^{\omega})$ with y_s in the objective function. Then, the primal is as follows:

$$\underset{\pi_S}{\text{Minimize}} \quad \sum_{S=1}^{2T+1} (1-\alpha) * \pi_S^{\omega} * y_S$$

$$\text{Subject to} \quad \sum_{\omega=1}^{2T+1} \pi_S^{\omega} = 1 \qquad : q$$

$$\sum_{\omega=1}^{2T+1} \pi_S^{\omega} * \varepsilon_S^{\omega} = 0 \quad : r$$

$$\pi_S^{\omega} \le \overline{\overline{\pi_S^{\omega}}}, \forall \omega \qquad : \overline{\overline{u_S^{\omega}}}$$

$$\pi_S^{\omega} \ge \underline{\underline{\pi_S^{\omega}}}, \forall \omega \qquad : \underline{\underline{u_S^{\omega}}}$$

$$\pi_S^{\omega} \ge 0, \forall \omega$$

$$\pi_S^{\omega} \le 1, \forall \omega$$

q, r, $\overline{\overline{u_S^{\omega}}}$, and $\underline{\underline{u_S^{\omega}}}$ are the dual variables introduced for all the constraints in the primal.

Then, the dual is as follows:

$$\underset{q,\, r;\, \underline{\underline{u_S^{\omega}}},\, \overline{\overline{u_S^{\omega}}}}{\text{Maximize}} \quad q + \sum_{\omega=1}^{2T+1} \left(\underline{\underline{\pi_S^{\omega}}} * \underline{\underline{u_S^{\omega}}} + \overline{\overline{\pi_S^{\omega}}} * \overline{\overline{u_S^{\omega}}} \right)$$

Subject to q, r is free

$$\underline{\underline{u_S^{\omega}}} \ge 0, \forall \omega$$

$$\overline{\overline{u_S^{\omega}}} \le 0, \forall \omega$$

$$\begin{bmatrix} q & r & \underline{\underline{u_S^{\omega}}} & \overline{\overline{u_S^{\omega}}} \end{bmatrix} \begin{bmatrix} 1 \\ \varepsilon_S^{\omega} \\ 1 \\ 1 \end{bmatrix} \le (1-\alpha) y_S, \forall s$$

$$y_S = \min(x_0, a^s - b^s * (1-\alpha) * p + \varepsilon_S^{\omega})$$

Then we can plug this dual into the original big problem as the following:

$$\underset{0\le\alpha\le 1}{\text{Maximize}}\ \underset{q,\,r,\,\underline{\underline{u_S^\omega}},\,\overline{\overline{u_S^\omega}}}{\text{Maximize}}\quad q+\sum_{\omega=1}^{2T+1}\left(\underline{\underline{\pi_S^\omega}}*\underline{\underline{u_S^\omega}}+\overline{\overline{\pi_S^\omega}}*\overline{\overline{u_S^\omega}}\right)$$

Subject to q, r is free

$$\underline{\underline{u_S^\omega}}\ge 0,\ \forall\omega$$

$$\overline{\overline{u_S^\omega}}\le 0,\ \forall\omega$$

$$q+\varepsilon_S^\omega*r+\underline{\underline{u_S^\omega}}+\overline{\overline{u_S^\omega}}\le(1-\alpha)*\min(x_0,\ a^s-b^s*(1-\alpha)*p+\varepsilon_S^\omega),\ \forall\omega$$

$$\alpha\ge 1-\frac{a^s-M}{b^s p}$$

Here, now, we should replace the min part in the constraint with some linear expression.

$$\underset{0\le\alpha\le 1}{\text{Maximize}}\ \underset{q,\,r,\,\underline{\underline{u_S^\omega}},\,\overline{\overline{u_S^\omega}}}{\text{Maximize}}\quad q+\sum_{\omega=1}^{2T+1}\left(\underline{\underline{\pi_S^\omega}}*\underline{\underline{u_S^\omega}}+\overline{\overline{\pi_S^\omega}}*\overline{\overline{u_S^\omega}}\right)$$

Subject to q, r is free

$$\underline{\underline{u_S^\omega}}\ge 0,\ \forall\omega$$

$$\overline{\overline{u_S^\omega}}\le 0,\ \forall\omega$$

$$q+\varepsilon_S^\omega*r+\underline{\underline{u_S^\omega}}+\overline{\overline{u_S^\omega}}\le(1-\alpha)*x_0,\ \forall\omega$$

$$q+\varepsilon_S^\omega*r+\underline{\underline{u_S^\omega}}+\overline{\overline{u_S^\omega}}\le(1-\alpha)*\left[a^s-b^s*(1-\alpha)*p+\varepsilon_S^\omega\right],\ \forall\omega$$

$$\alpha\ge 1-\frac{a^s-M}{b^s p}$$

$$\underline{\underline{\pi_S^\omega}}=(1-\gamma)\pi_S^{\omega 0},\ \forall\omega$$

$$\overline{\overline{\pi_S^\omega}}=(1+\gamma)\pi_S^{\omega 0},\ \forall\omega$$

$$\varepsilon_S^\omega=-M+\frac{2M}{2T}*(\omega-1),\ \forall\omega$$

Now, it's a quadratic problem since in the fifth constraint we've got the square of α. We can solve it by some quadratic solver. However, is there a more efficient way to solve it since the problem with nonlinear objective and linear constraints is better? Here, we're going to introduce the lower and upper limits to replace the quadratic function of α in the fifth constraint since $\alpha \in [0, 1]$, which is as the following formulation:

$$1-2\alpha \le (1-\alpha)^2 \le 1-\alpha .$$

That is, we are going to rewrite the model to two formulations. One is to replace the $(1-\alpha)^2$ part of the fifth constraint by $1-2\alpha$, which will give an upper bound of α, α_u. The other is to replace the $(1-\alpha)^2$ part of the fifth constraint by $1-\alpha$, which will give a lower bound of α, α_l. α will be the consequence of $\frac{\alpha_u+\alpha_l}{2}$ then.

Also, we need to get some insights on optimal α related with different assumptions of uncertainty level. Therefore, we will solve the model by choosing some numerical values and look on an example what the influence of the parameters is.

2.3 Numerical Experiment

We want to get some insights on optimal α. Therefore, we try to solve the model in the previous section by choosing some numerical values for M, γ, T and several distributions for ε_S^{ω} such as triangular distribution, uniform distribution, and Gaussian distribution so that we can look on an example what the influence of the parameters is.

The following table contains the portfolios of the parameters we choose for our experiments.

Case#	M	T	γ	Distribution	q	r	α	Objective
1	90	3	10%	Uniform	666.429	0.2143	57.14%	675.107
2	90	3	10%	Triangle	666.429	0.2143	57.14%	678.161
3	90	3	20%	Uniform	666.429	0.2143	57.14%	674.124
4	90	3	20%	Triangle	666.429	0.2143	57.14%	676.856
5	90	5	10%	Uniform	666.858	0.2588	56.87%	675.244
6	90	5	10%	Triangle	666.429	0.2143	57.14%	678.058
7	90	5	20%	Uniform	666.858	0.2588	56.87%	674.312
8	90	5	20%	Triangle	666.429	0.2143	57.14%	676.766
9	180	3	10%	Uniform	647.143	0.2143	57.14%	664.500
10	180	3	10%	Triangle	647.143	0.2143	57.14%	670.607
11	180	3	20%	Uniform	647.143	0.2143	57.14%	662.533
12	180	3	20%	Triangle	647.143	0.2143	57.14%	667.997
13	180	5	10%	Uniform	647.757	0.2605	56.58%	664.637
14	180	5	10%	Triangle	647.143	0.2143	57.14%	670.401
15	180	5	20%	Uniform	647.757	0.2605	56.58%	662.761
16	180	5	20%	Triangle	647.143	0.2143	57.14%	667.817

Using Xpress-MP, we obtain the optimal percentage of price reduction and the optimal objective values under each case. We can see that with larger M and larger γ , the optimal objective value will be smaller though the optimal percentage of price reduction remains the same. It implies that the higher the uncertainty level, the lower the optimal objective values. Another observation is that the optimal objective values will be larger under the assumption that the demand is triangular distributed than under the assumption that the demand is uniformly distributed though the percentage of price reduction remain the same. However, the impact of T will be reverse for Triangle distribution and Uniform distribution assumptions. If we assume that the demand is triangular distributed, the optimal objective value will be larger with smaller T . If we assume that the demand is uniformly distributed, the optimal objective value will be larger with larger T .

Chapter 3 Pricing with Uncertain Parameters

3.1 Problem Setup

At each time period (beginning of each week), the manager looks at the inventory in stock and decides whether he is going to put the item on sale at a discounted price (20, 30, or 40% discount, for instance) or keep selling it at the initial price p (assume p is given, imposed by the manufacturer).

It he decides to sell at a discount, he must decide on the size of the discount (finite number of strategy available, for example, 20%, 30%, etc.)

The problem is that the manager does not quite know how customers will react. In this chapter, we'll discuss a model dealing with customer reaction with some parameters, such as mean and standard deviation, instead of probabilities, of the demand volatility corresponding to price change.

3.2 Constructing the Formulation Model – One-Time, Inventory Sufficiently Large

At first, we're trying to solve a three-product and one-time model without considerations of the inventory. That is, we're assuming the inventory is sufficiently large so that the profit is totally based upon the customer demand.

This model deals with some correlations, which is that the demand volatility of one product is not only related to its price change but also other products by the same vendor or the competitors. However, we don't use statistical correlation matrix. Instead, for example, we model the relationship for a model with three products like the following:

$$d = a - Bp$$

$$d = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}, \quad a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}, \quad B = \begin{bmatrix} b_{11} & b_{12} & 0 \\ b_{21} & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix}, \quad p = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}$$

$$\text{i.e.} \quad \begin{aligned} d_1 &= a_1 - (b_{11} p_1 + b_{12} p_2) \\ d_2 &= a_2 - (b_{21} p_1 + b_{22} p_2) \\ d_3 &= a_3 - b_{33} p_3 \end{aligned}$$

This way, we can see that the demand of product 1 and product 2 are both related to the price of each other, while the demand of product 3 is only related to its own price. In other words, product 1 and product 2 are correlated, while product 3 is independent. Moreover, all the elements of matrix a and B are random except the 0s. Besides, b_{12} and b_{21} are both negative.

Then, we're going to build the complete model for finding the optimal solution the discount price and the corresponding objective value, profit.

$$\max_{0\le\alpha_i\le 1} \sum_{i=1}^{n}(1-\alpha_i)[a_i-[B\cdot diagp\cdot(1-\alpha)_i]]$$

Subject to $[a_i-[B\cdot diagp\cdot(1-\alpha)_i]]\ge 0, \quad \forall i$

$$0\le\alpha_i\le 1, \quad \forall i$$

$$diagp \text{ is a diagonal matrix} = \begin{bmatrix} p_1 & 0 & \cdots & \cdots & 0 \\ 0 & p_2 & & & \vdots \\ \vdots & & \ddots & & \vdots \\ \vdots & & & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & p_n \end{bmatrix}$$

$$a_i, b_i \ in\ B \sim some\ continuous\ distribution(\mu, \sigma^2)$$

In the above model, each a_i and b_i is the nominal values, i.e. μ of the distribution they are assumed, and $\sigma=(1-\gamma)\mu,\ 0\le\gamma\le 1$. When we expand the above matrix calculation for the example with three products, we can find demand volatility like the following,

$$d' = \begin{bmatrix} a_1-\alpha_1' * b_{11} * p_1 - \alpha_2' * b_{12} * p_2 \\ a_2-\alpha_1' * b_{21} * p_1 - \alpha_2' * b_{22} * p_2 \\ a_3-\alpha_3' * b_{33} * p_3 \end{bmatrix}$$

where d' is the demand matrix corresponding to price discount
α_i' is (1 − percent off) of product i

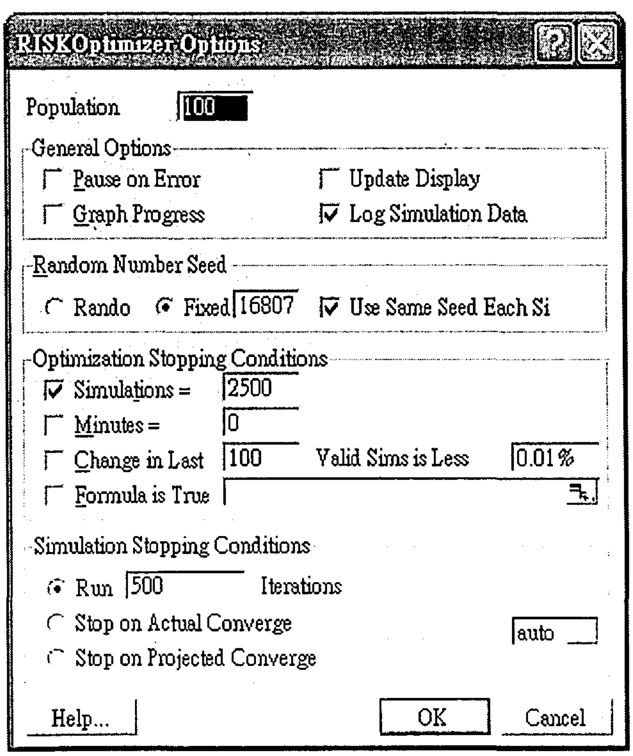
3.3 Numerical Experiments - Solving by Simulation Using Decision Tool Software

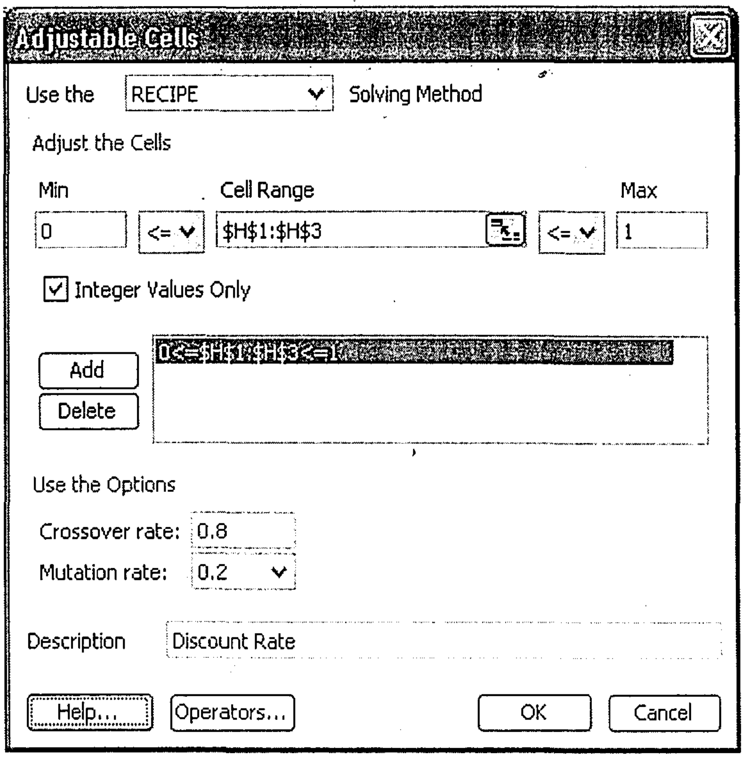
We use some decision tool software, RiskOptimizer, to simulate different standard deviations for the elements in the matrix a and B and to find different percentiles of the maximum objectives. The numerical experiment assumes that we have three products and matrix a and B are assumed Gaussian distribution.

3.3.1 Environmental Settings

The software deploys genetic algorithm. Therefore, we have to define some parameters such as population, cross-over rate, mutation rate, and stopping condition to be confident of our solutions. Moreover, we have to define random number seed to make sure the simulations are thoroughly comparable.

The following is the authentic setting window of the software related with the genetic algorithm parameters and the random number seed.





There are several points in the settings. First, the population should have been over 500 to work more properly, avoiding convergence too quickly, according to genetic algorithm. However, with population of 500, the stopping conditions of numbers of simulations and iterations should also be large enough to ensure the population work appropriately, which will increase the running time as much as ten times or more. With experiments, to save time, we will decrease the population to 100. Second, the mutation rate should have been less than 0.01. However, to get round the process of convergence, the rate of mutation must be very high for small population. Since the population here, 100, is pretty smaller than required one, 500, we're going to lift up the mutation rate to 0.2. Third, to control the repeatability of the simulation system, we have to define the random number seed of each simulation to be the same. Besides, to insure a full period of random numbers generated, we choose 16807 as the random number seed. It has been extensively tested by Learmonth, G.P. and P.A.W. Lewis [6]. Fourth, to avoid extreme values, despite the constraints $0 \le \alpha \le 1$, we start our algorithm with initial values of α as 10%. Also, to ensure each optimization to be comparable, we start each optimization with the same initial values of α, though starting with intuitively higher (better) initial values different from optimizations to optimizations might produce higher optimal objective values. Fifth, the genetic algorithm doesn't guarantee global optimum. We conclude the above critical options based upon recommendations from some researchers ([9], [10]).

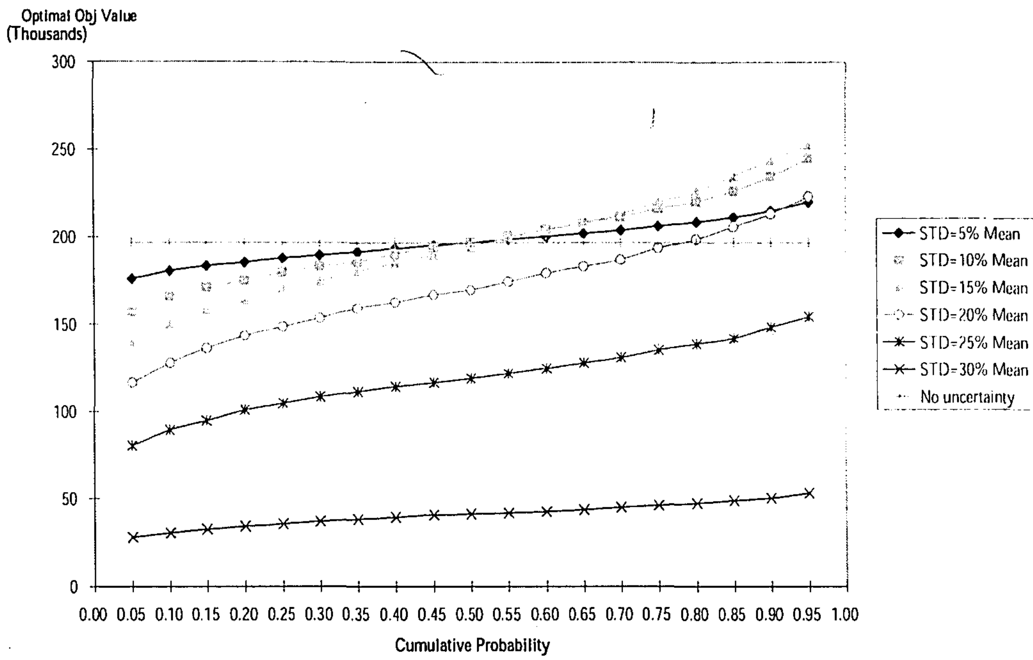
3.3.2 Solution Analysis

$$a = \begin{bmatrix} 3000 \\ 2500 \\ 2000 \end{bmatrix}, \quad B = \begin{bmatrix} 40 & -1 & 0 \\ -2 & 30 & 0 \\ 0 & 0 & 12 \end{bmatrix}, \quad p = \begin{bmatrix} 50 & 0 & 0 \\ 0 & 50 & 0 \\ 0 & 0 & 100 \end{bmatrix}$$

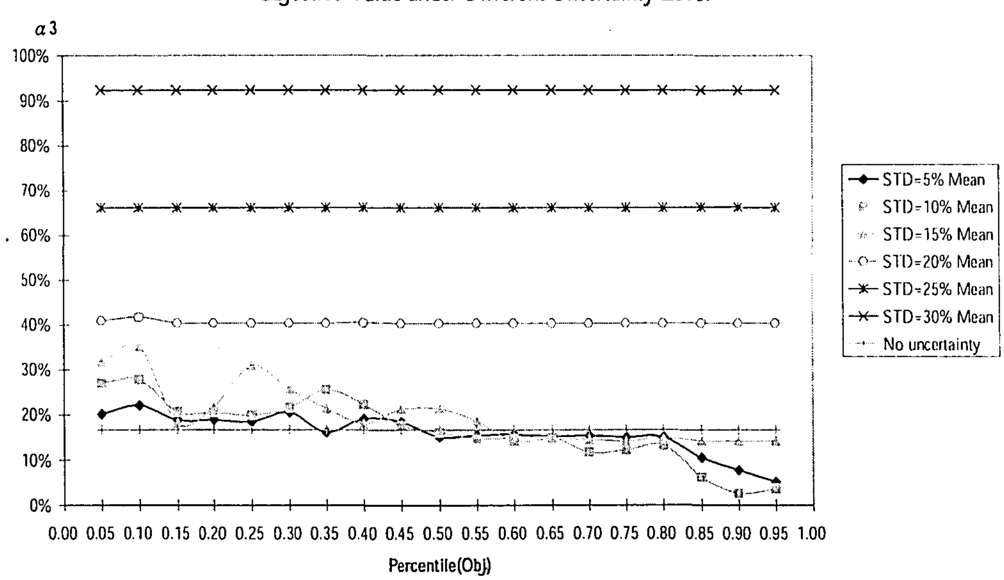
All the values shown above are the means of those elements and we are going to assign different $1-\gamma$ of the means to their standard deviations. Then, we are going to solve different percentiles, from 0.05 to 0.95, of the maximum objective.

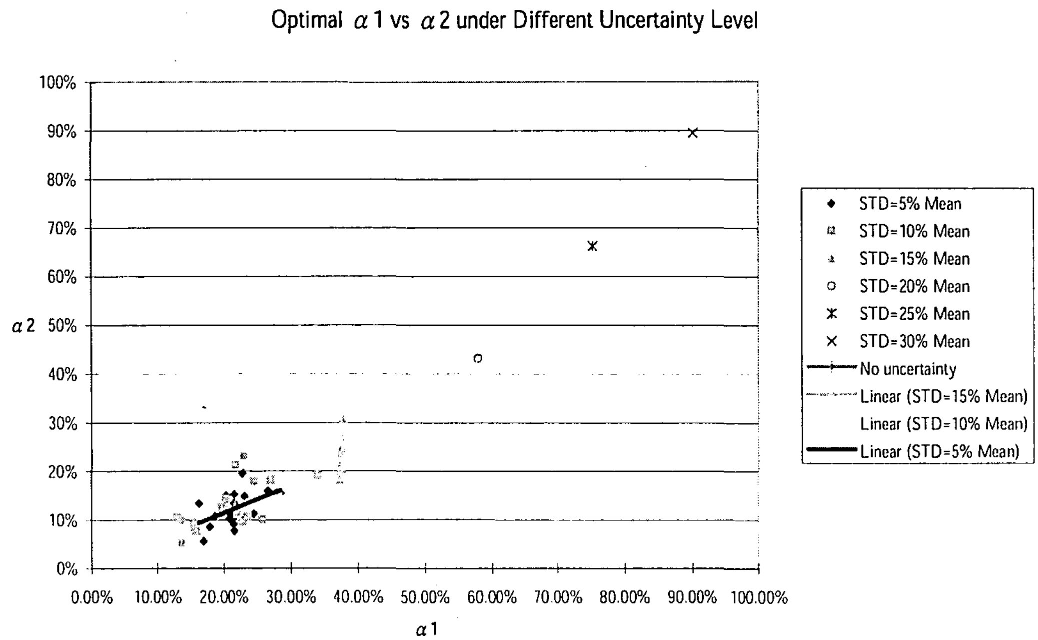
From RiskOptimizer, we get the results as the following graphs. It implies that the higher the uncertainty level, the lower the optimal objective values. We also get a conclusion that when the standard deviations are larger than 30% mean, there exists no optimal solution.

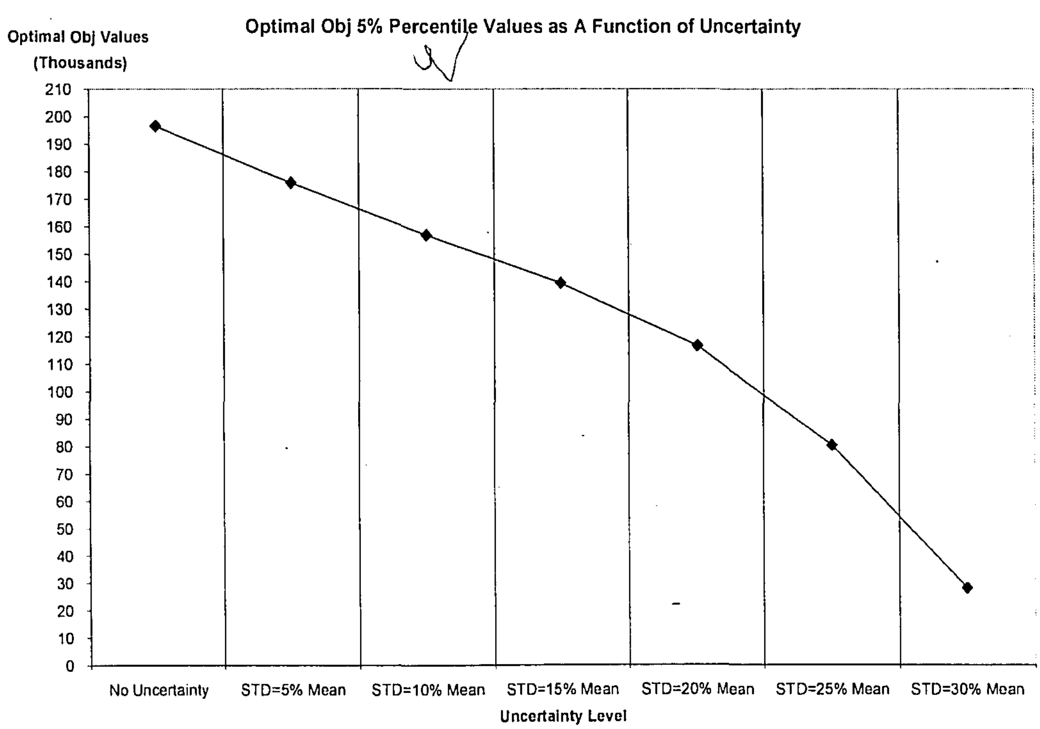
Optimal Objective Percentile Values under Different Uncertainty Level



Optimal Percentiage of Price Reduction of Product 3 (α 3) for Different Percentiles of the Objective Value under Different Uncertainty Level

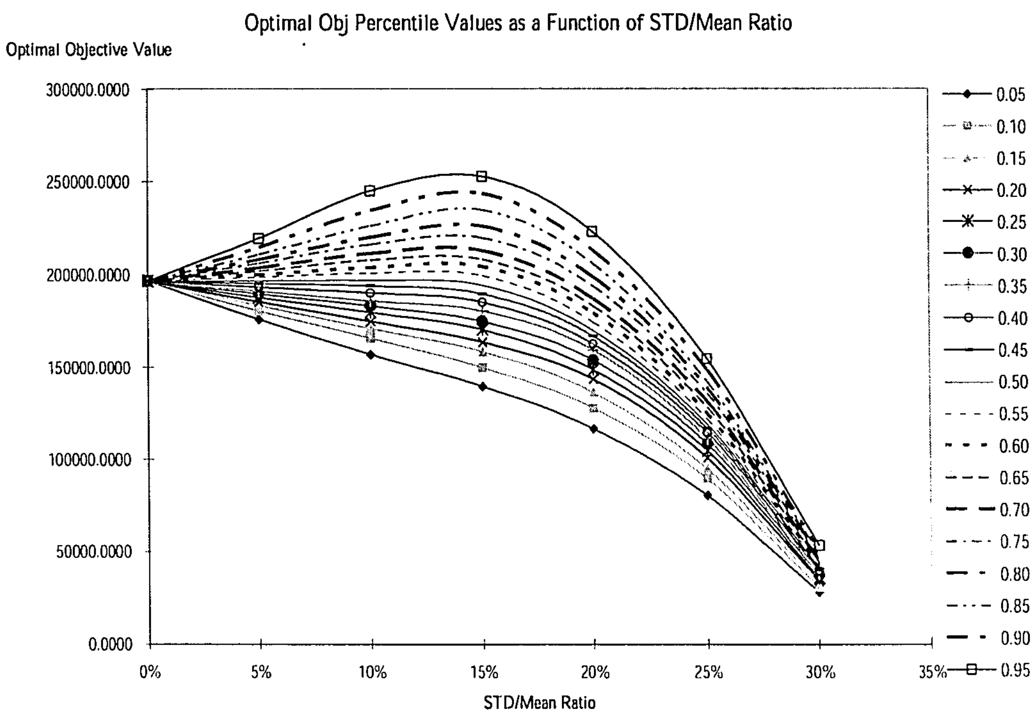






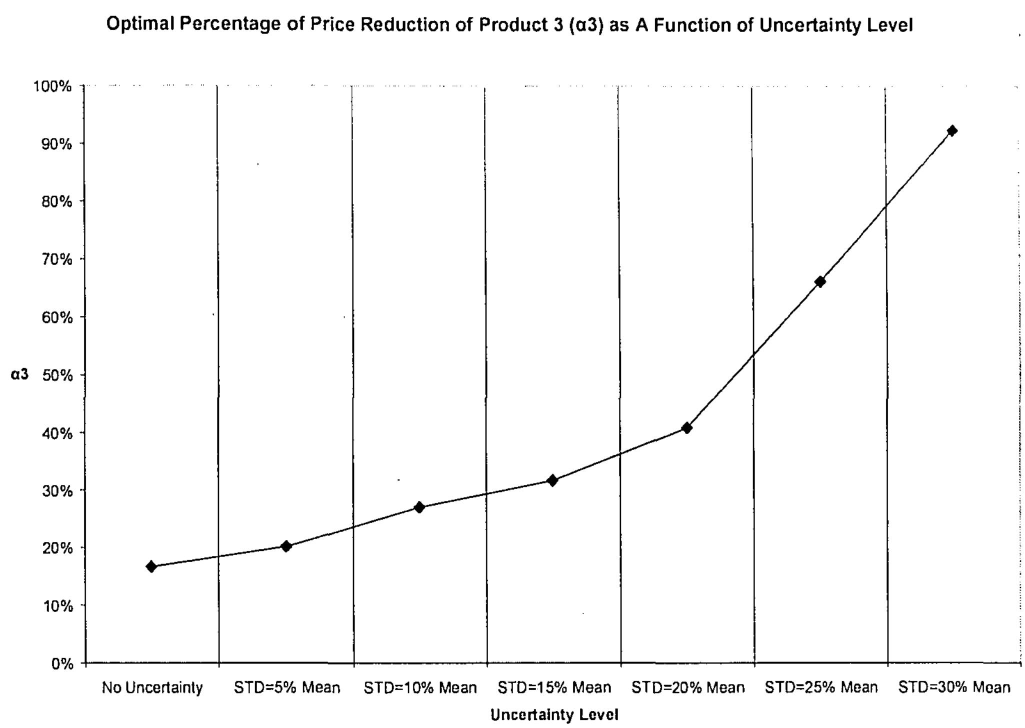
We can see from the graphs that when the standard deviations are higher, i.e., the demand fluctuations are more volatile, the optimal solutions for the discount are more stable and the maximum profits less than or equal to 50% percentiles are smaller. Also, we can see that from the uncertainty level, Standard Deviation = 20% Mean, the slope of the decrease of optimal objective values gets sharper, which we can say that this uncertainty level might be a threshold of the impact of decision.

Since there seems to be different trends for the maximum profits at different percentiles, we draw the graph in the view of different percentiles instead like the following:

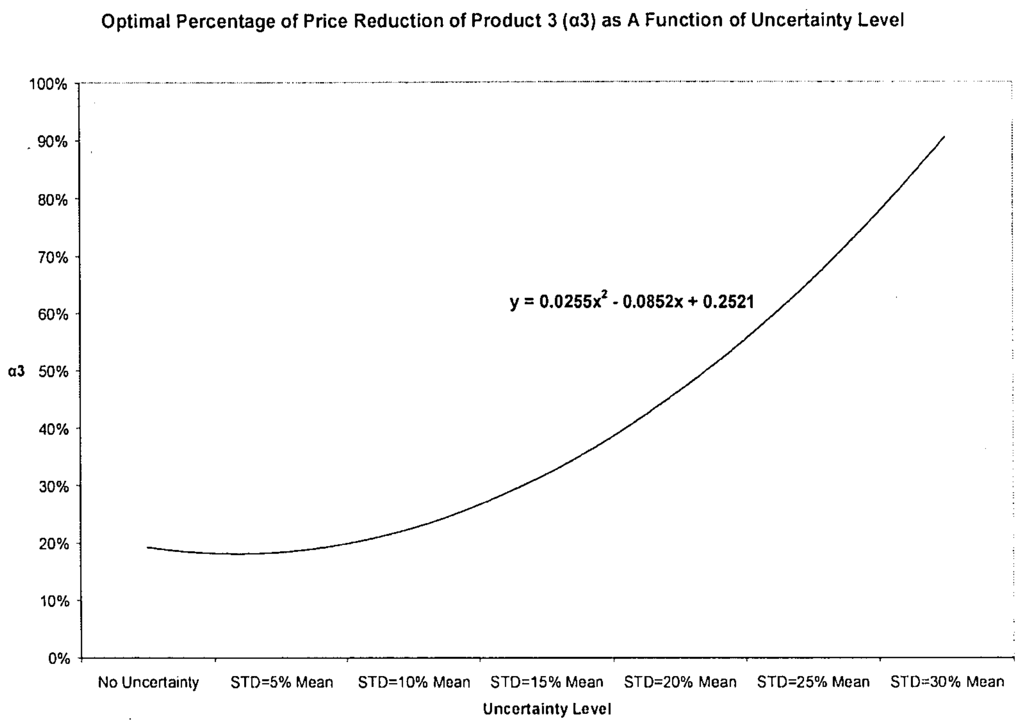


We can see that all the graphs for different desired percentiles are convex, which means that as the standard deviations get from no uncertainty to the extreme large, we get smaller profits.

We can see that as the desired percentiles get higher, the maximum profits are more sensitive to the standard deviation. However, roughly after 75% percentile, the maximum is no longer under the situation of no uncertainty. We will say that to make sure the robustness of our optimal solution, we will choose some percentiles below 50% to be our desired objective.



From the above graph, we can see sharp upward slope for the percentage of price reduction of product 3 after the standard deviations greater than or equal to 20% of mean. Since genetic algorithms are heuristics which converge towards a local not necessarily global maximum, we draw trend lines for the relationships between discount and standard deviation based on power-2 polynomial functions shown on the following graph:



Now, we are going to take STD/Mean=20% for an example. From the above graph, we can say no matter what percentiles of the objective value we're going to maximize, the optimal solutions for α_1, α_2, and α_3 remain almost the same., which is 57.97%, 43.12%, and 40.37%. However, in practice, the manager selects discounts from a discrete set, e.g., {10%, 15%, 20%, ...}. Therefore, we are going to round the price discount of the optimal solution to 55%, 45%, and 40% for each product respectively.

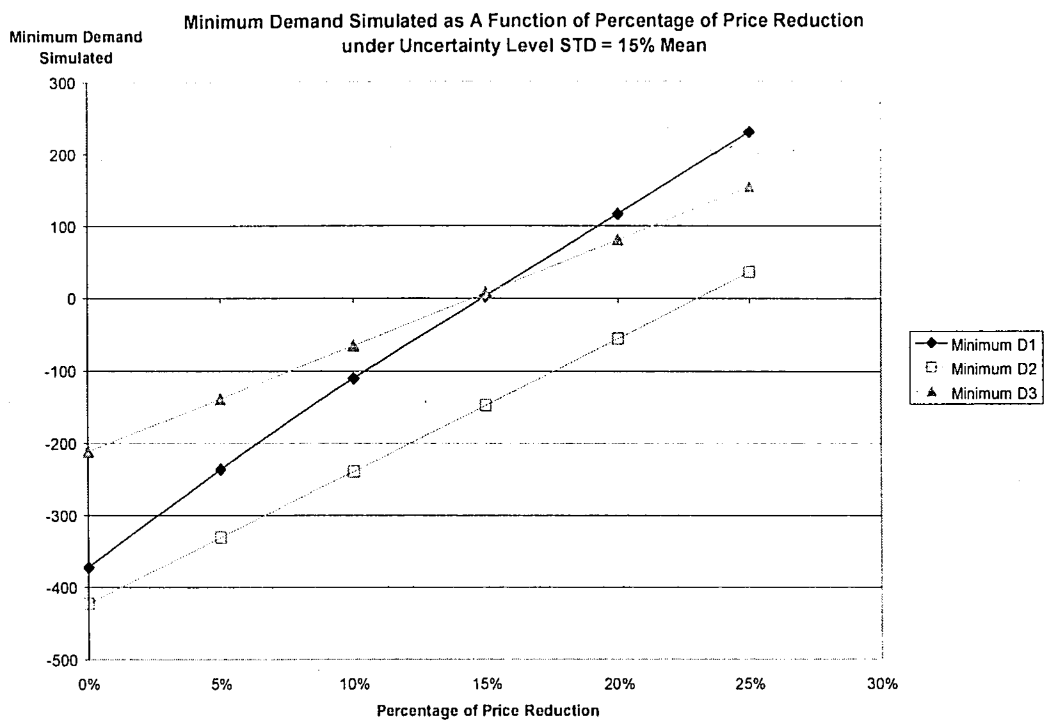
Now, we have to verify whether the decision by the genetic algorithm is truly the best strategy or not. We use @Risk to simulate. What's very important here is that we should choose the same random number seed as the one used in RiskOptimizer.

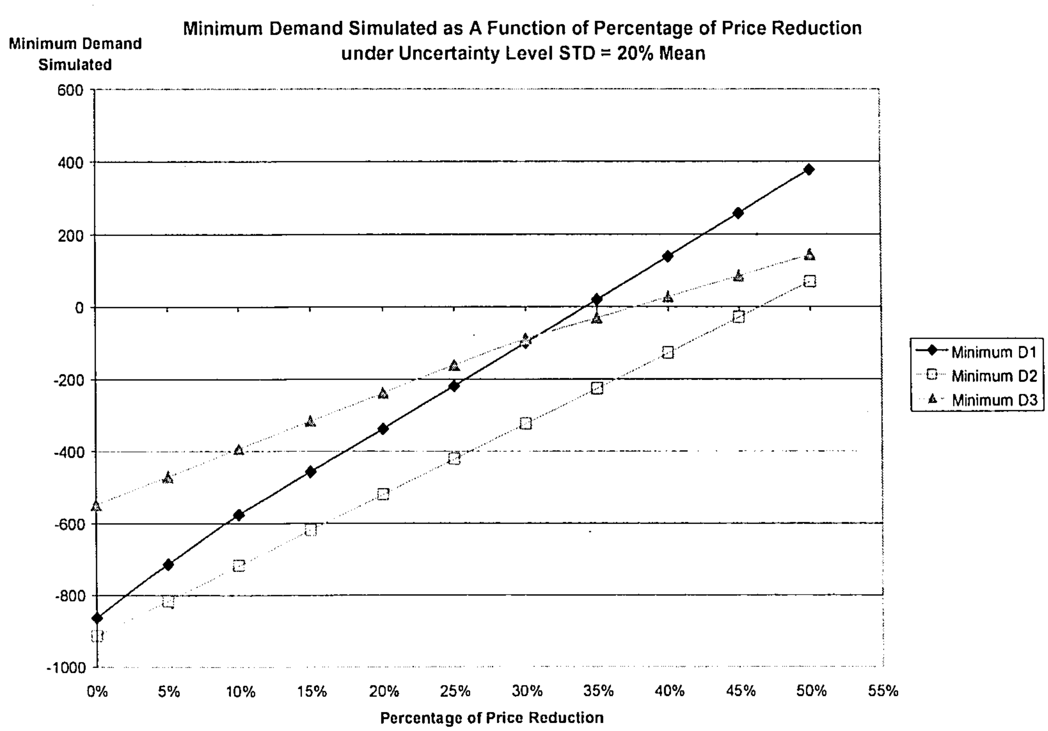
We find that the optimal solution we have got is not the global optimum according to the simulation results because the settings of RiskOptimizer might not meet the requirement of the genetic algorithm to find a global optimum deployed in this software. Also, we have to pay close attention to the violation of constraints. That is, under certain uncertainty level, the percentage of price reduction should exceed some threshold so that the constraints depicting demand is usually larger than or equal to zero are always satisfied. However, from the ranking results in the following tables, we still can say that the portfolio of the rounded discounts does not perform badly amongst all the alternative portfolios.

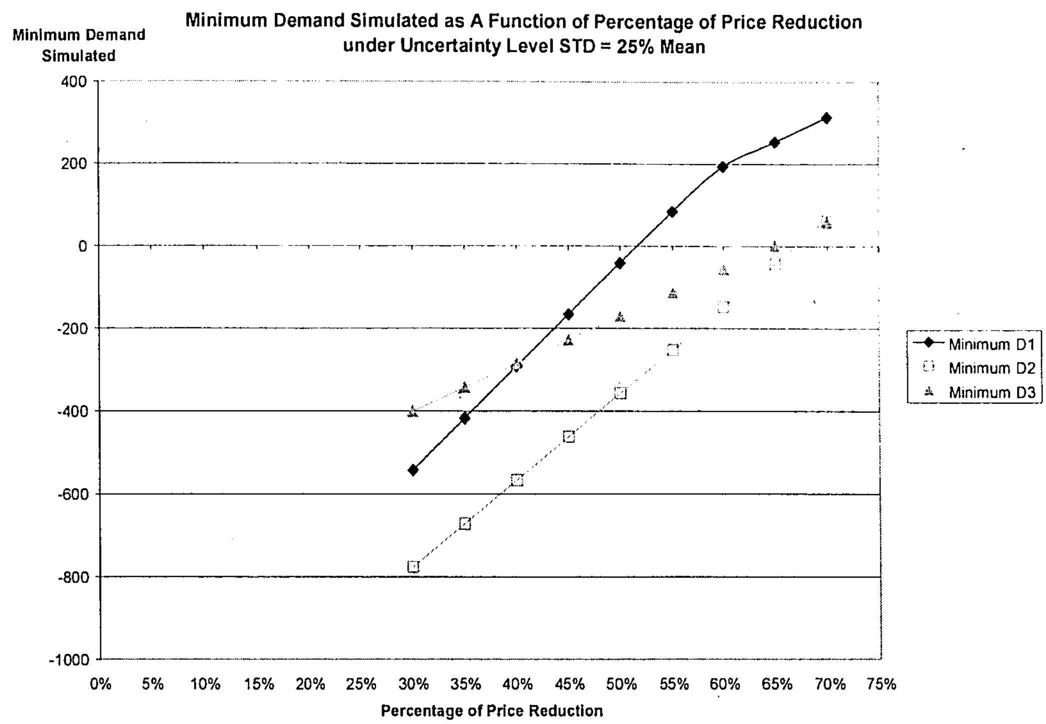
Simulation Portfolio #	Percentage of Price Reduction
1	40, 50, 40
2	40, 50, 45
3	50, 50, 40
4	50, 50, 45
5	55, 50, 40
6	55, 50, 45
7	60, 50, 40
8	60, 50, 45

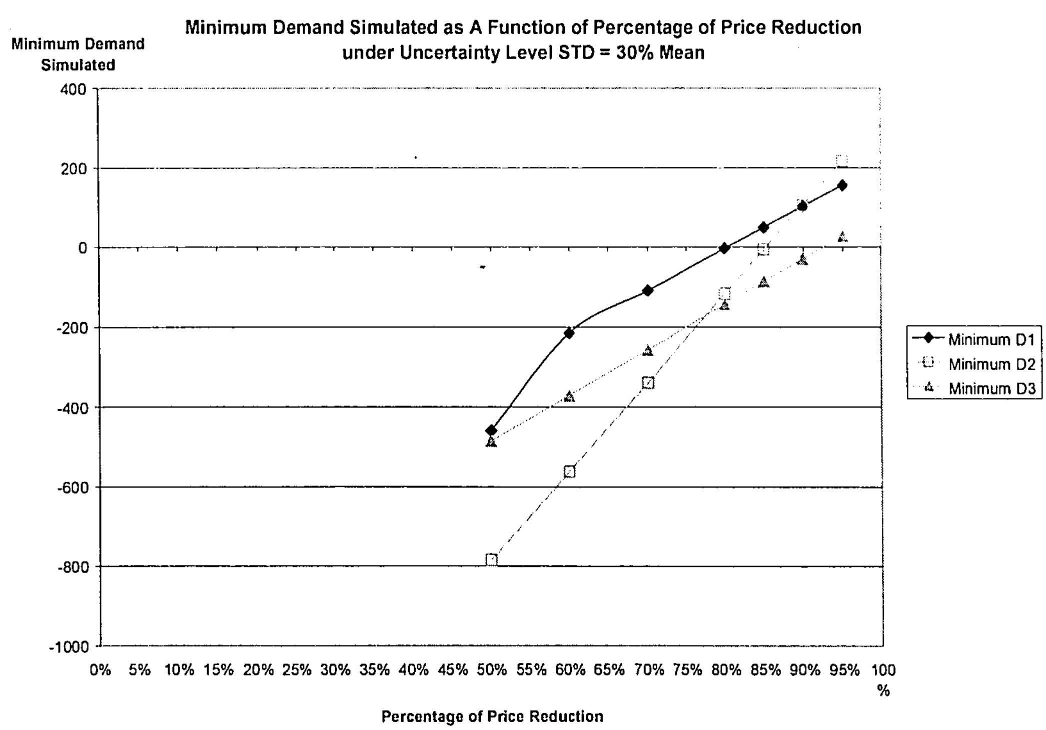
	Simulation Portrfolio #							
Percentile	1	2	3	4	5	6	7	8
0.05	3	1	4	2	6	5	8	7
0.10	1	2	3	4	5	6	7	8
0.15	1	2	3	4	5	6	7	8
0.20	1	2	3	4	5	6	7	8
0.25	1	2	3	4	5	6	7	8
0.30	1	2	3	4	5	6	7	8
0.35	1	2	3	4	5	6	7	8
0.40	1	2	3	4	5	6	7	8
0.45	1	2	3	4	5	6	7	8
0.50	1	2	3	5	4	6	7	8
0.55	1	2	3	4	5	6	7	8
0.60	1	2	3	5	4	7	6	8
0.65	1	2	3	5	4	6	7	8
0.70	1	2	3	5	4	7	6	8
0.75	1	2	3	5	4	7	6	8
0.80	1	2	3	5	4	7	6	8
0.85	1	2	3	5	4	7	6	8
0.90	1	2	3	5	4	7	6	8
0.95	1	2	3	5	4	7	6	8
Total Ranking	1	2	3	4	5	6	7	8

We also check the threshold of percentage of price reduction exceeding which the constraints depicting demands are larger than or equal to zero are always satisfied.


Minimum Demand Simulated as A Function of Percentage of Price Reduction under Uncertainty Level STD = 15% Mean
Minimum Demand Simulated
300
200
100
0
-100
-200
-300
-400
-500
0%
5%
10%
15%
20%
25%
30%
Percentage of Price Reduction
Minimum D1
Minimum D2
Minimum D3


Minimum Demand Simulated as A Function of Percentage of Price Reduction under Uncertainty Level STD = 20% Mean
Minimum Demand Simulated
600
400
200
0
-200
-400
-600
-800
-1000
0%
5%
10%
15%
20%
25%
30%
35%
40%
45%
50%
55%
Percentage of Price Reduction
Minimum D1
Minimum D2
Minimum D3


Minimum Demand Simulated as A Function of Percentage of Price Reduction under Uncertainty Level STD = 25% Mean
Minimum Demand Simulated
400
200
0
-200
-400
-600
-800
-1000
0% 5% 10% 15% 20% 25% 30% 35% 40% 45% 50% 55% 60% 65% 70% 75%
Percentage of Price Reduction
Minimum D1
Minimum D2
Minimum D3


Minimum Demand Simulated as A Function of Percentage of Price Reduction under Uncertainty Level STD = 30% Mean
Minimum Demand Simulated
400
200
0
-200
-400
-600
-800
-1000
0% 5% 10% 15% 20% 25% 30% 35% 40% 45% 50% 55% 60% 65% 70% 75% 80% 85% 90% 95% 100 %
Percentage of Price Reduction
Minimum D1
Minimum D2
Minimum D3

We can see that as the uncertainty level is higher, the price reduction percentage threshold for the demand constraints saying that demands are always larger than or equal to zero is higher.

3.4 Constructing the Formulation Model – Two-Time, Inventory Considered

In this section, we're going to discuss the optimal solutions with considerations of inventory.

$$
\begin{aligned}
\max \quad & (1-\alpha)\cdot p \cdot \min(d_2, I_1) \\
\text{Subject to} \quad & d_2 = [a - B\cdot diagp \cdot (1-\alpha)] - d_1 \\
& d_1 = \frac{1}{2}(a - Bp) \\
& I_1 = I_0 - d_1 \\
& d_1 \geq 0 \\
& d_2 \geq 0 \\
& 0 \leq \alpha \leq 1
\end{aligned}
$$

$$
diagp \text{ is a diagonal matrix} = \begin{bmatrix} p_1 & 0 & \cdots & \cdots & 0 \\ 0 & p_2 & & & \vdots \\ \vdots & & \ddots & & \vdots \\ \vdots & & & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & p_n \end{bmatrix}
$$

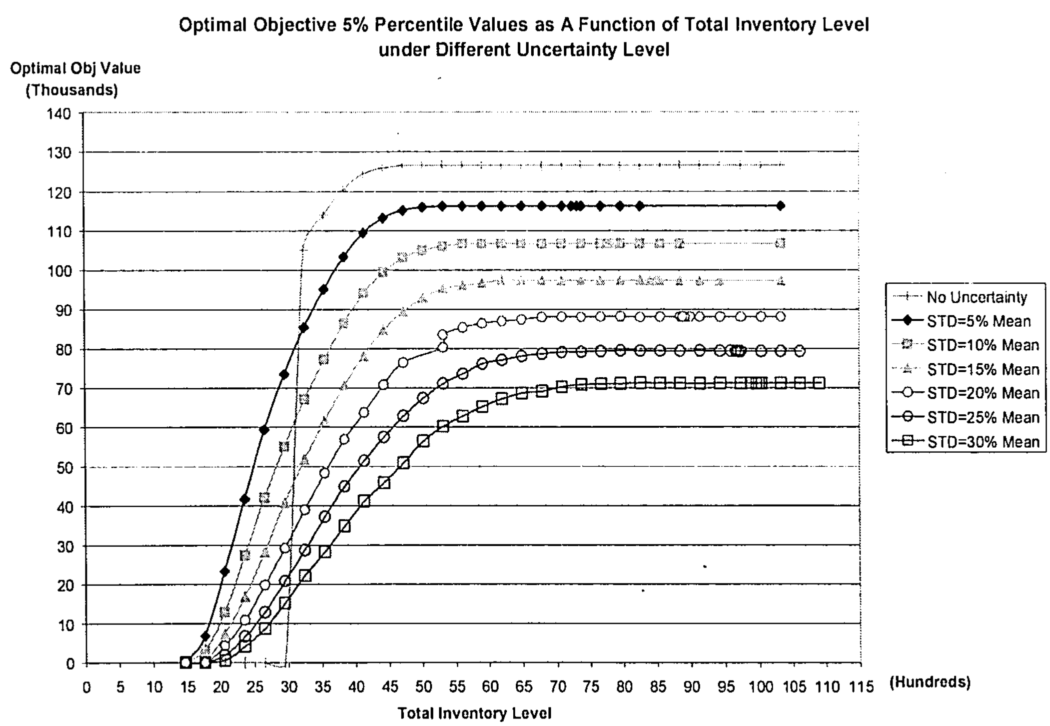
$$
a_i,\, b_i \textit{ in } B \sim \textit{some continuous distribution}(\mu, \sigma^2)
$$

3.5 Numerical Experiments - Solving by Simulation Using Decision Tool Softwar

$$
a = \begin{bmatrix} 3000 \\ 2500 \\ 2000 \end{bmatrix}, \quad B = \begin{bmatrix} 40 & -1 & 0 \\ -2 & 30 & 0 \\ 0 & 0 & 12 \end{bmatrix}, \quad p = \begin{bmatrix} 50 & 0 & 0 \\ 0 & 50 & 0 \\ 0 & 0 & 100 \end{bmatrix}
$$

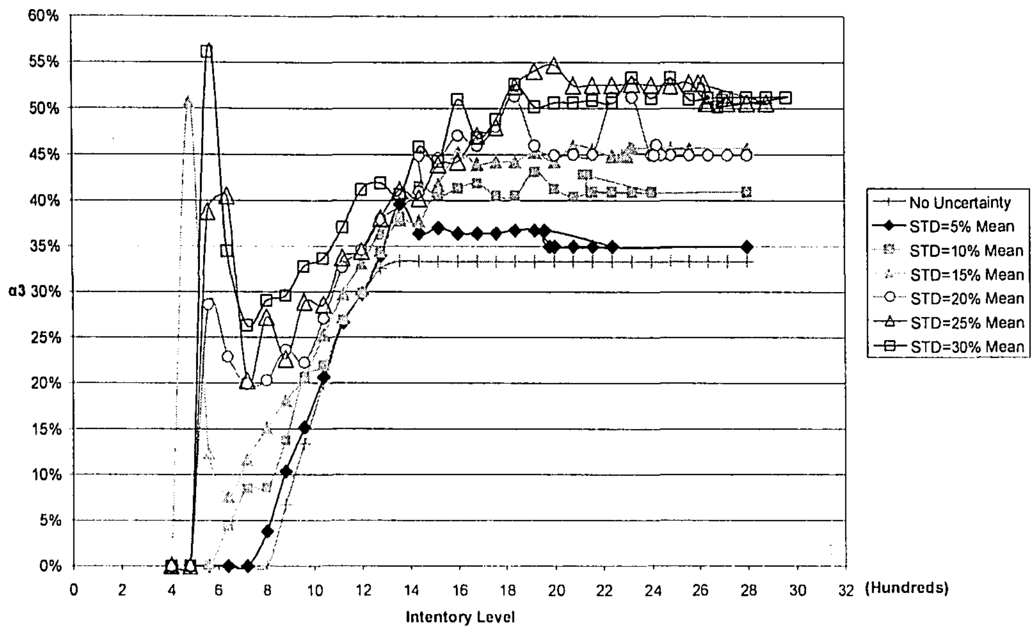
All the values shown above are the means of those elements and we are going to assign different $1-\gamma$ of the means to their standard deviations. Then, we'are going to solve different percentiles, from 0.05 to 0.95, of the maximum objective.

From RiskOptimizer, we get the results as the following graphs. The first graph is the optimal objective value under different standard deviation. We can see that, with higher standard deviation, we have lower optimal objective value. Also, with higher standard deviation, the threshold passing which we do not have to consider optimal solutions for different inventory levels is higher.

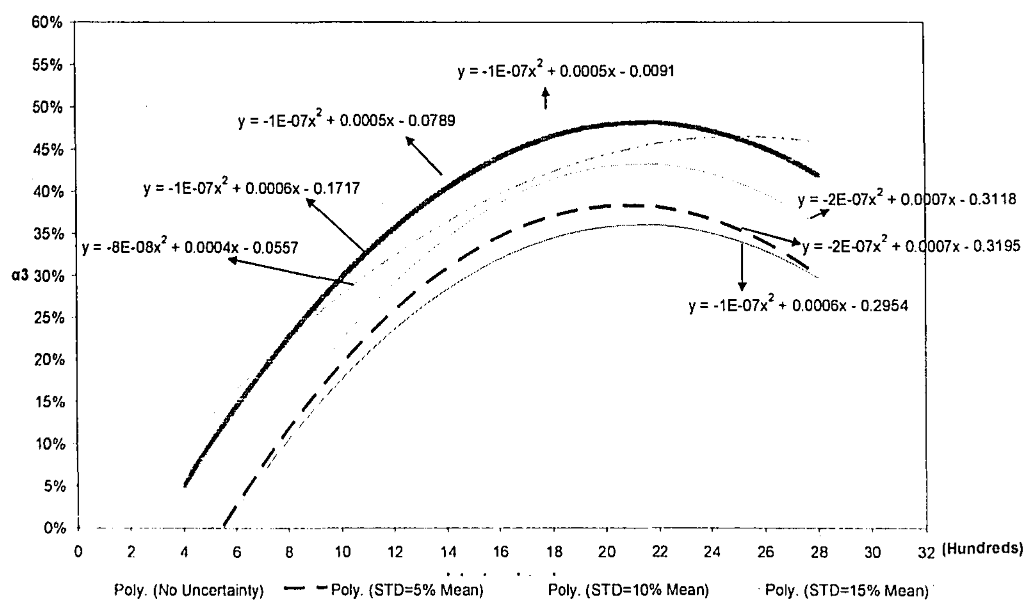


The second graph is the discount for product 3 under different standard deviation because this product is independent of the others. We can see that, with higher standard deviation, we will have to give higher discount.

Optimal Percentage of Price Reduction of Product 3 (α3) as A Function of Inventory Level under Different Uncertainty Level

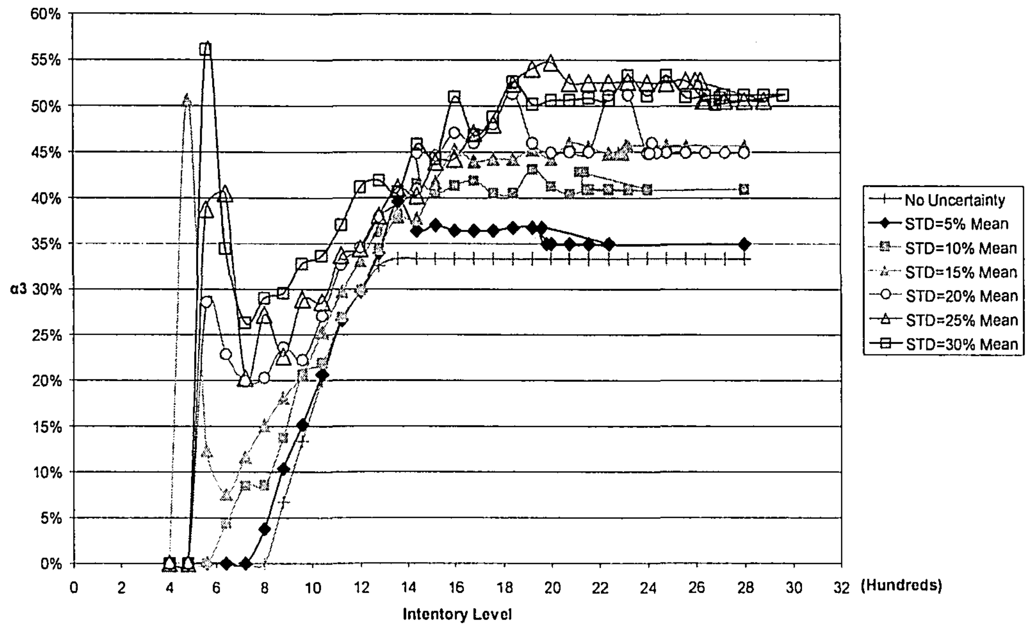


Optimal Percentage of Price Reduction (α3) of Product 3 as A Function of Inventory Level under Different Uncertainty Level

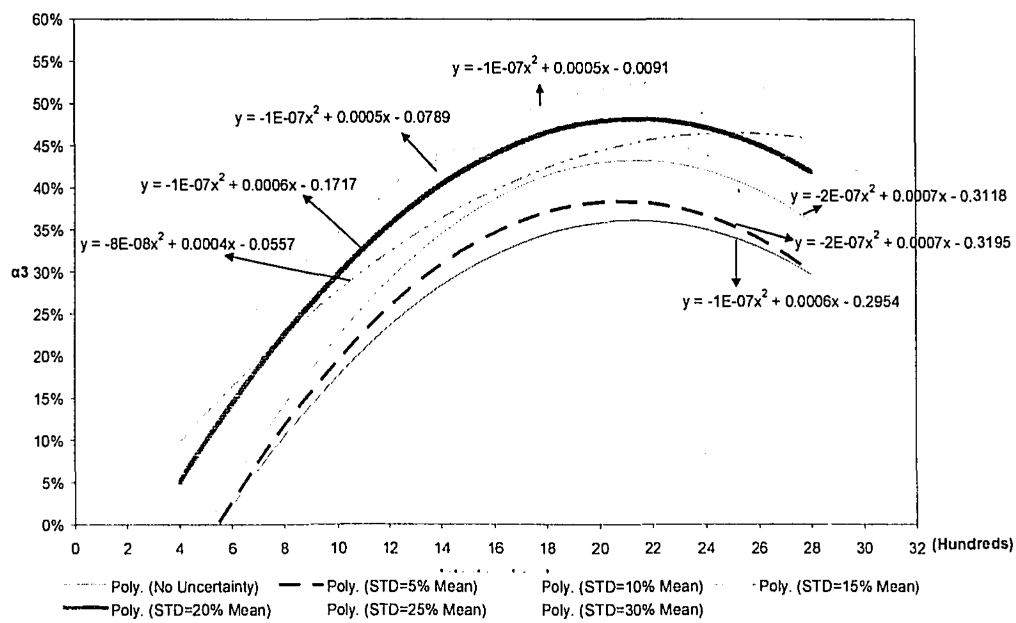


have to give higher discount.

Optimal Percentage of Price Reduction of Product 3 (α3) as A Function of Inventory Level under Different Uncertainty Level



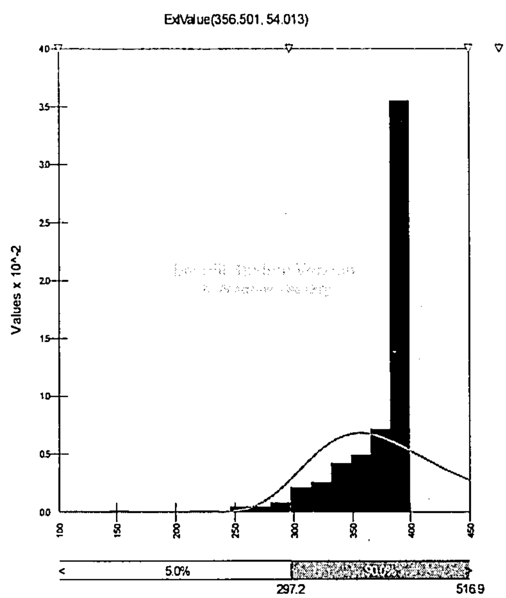
Optimal Percentage of Price Reduction (α3) of Product 3 as A Function of Inventory Level under Different Uncertainty Level

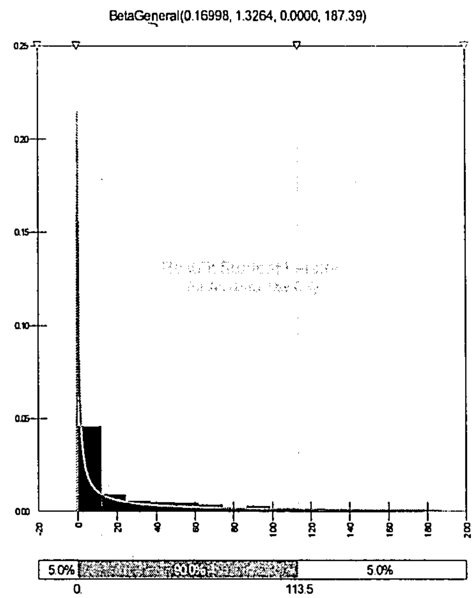


In the graph of the original solutions without converting to the trend lines, we can see that when the uncertainty levels get higher, the price reduction percentage of product 3 is pretty volatile between the inventory level 400 and 800. It is because that the simulated demands for product 3 in stage 2 are pretty volatile in terms of the distribution types. Here are the verification graphs of the distribution types of the demands simulated under different uncertainty levels and inventory levels:

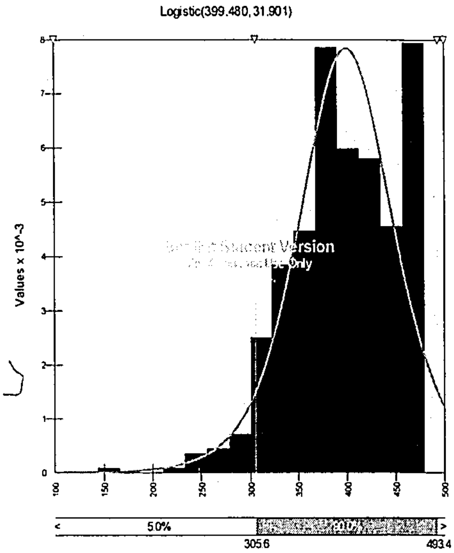
Uncertainty level Standard Deviation = 5% Mean

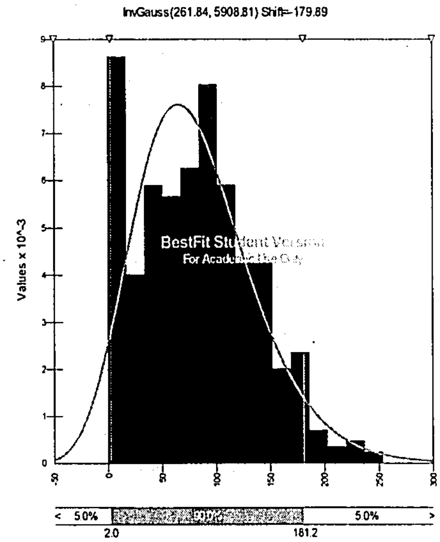
I1, D13, D23



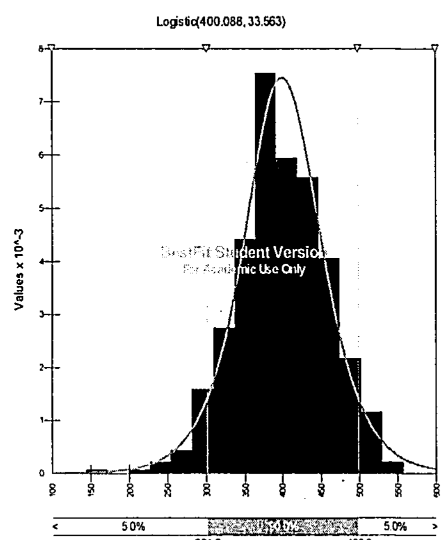


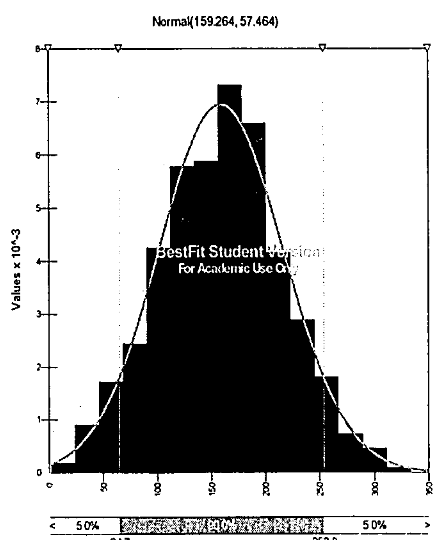
I2, D13, D23



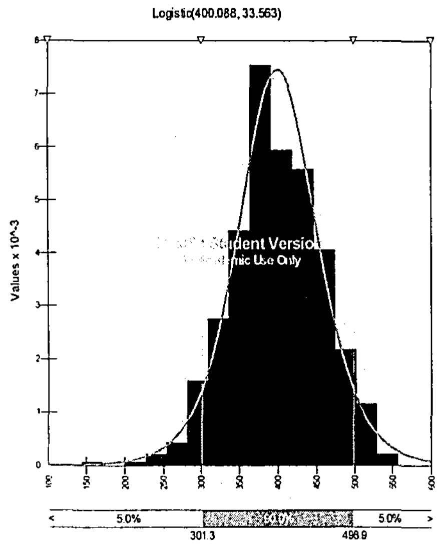


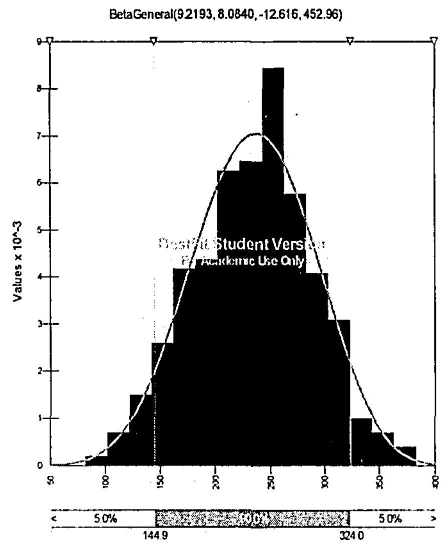
I3, D13, D23



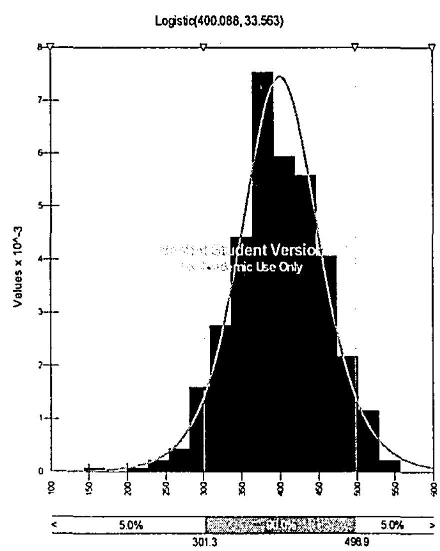


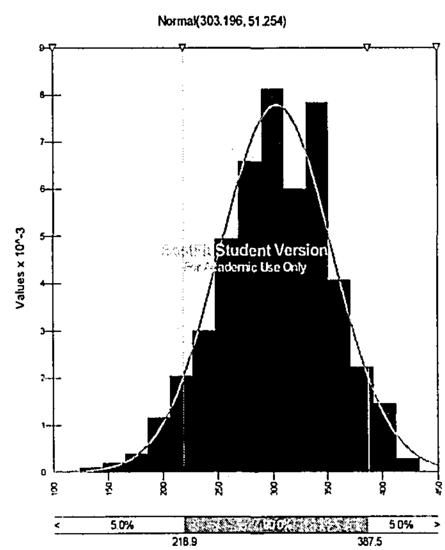
I4, D13, D23



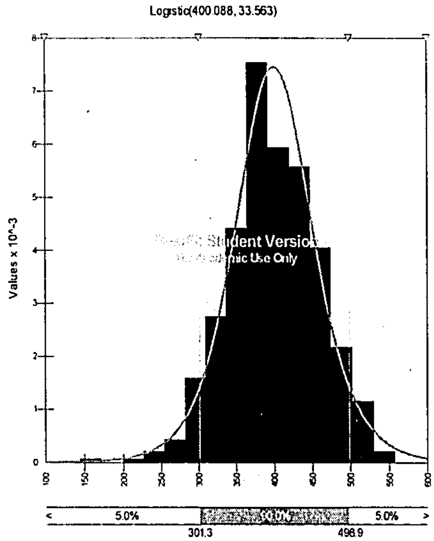


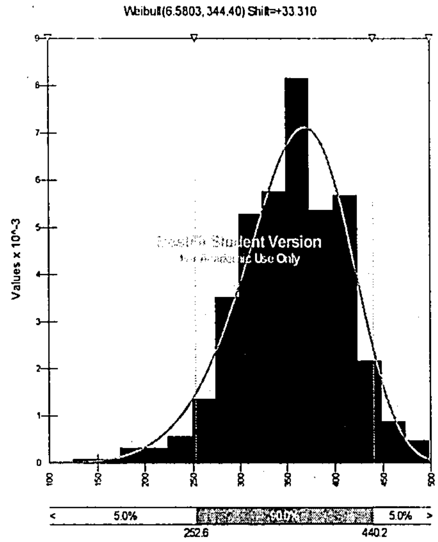
I5, D13, D23



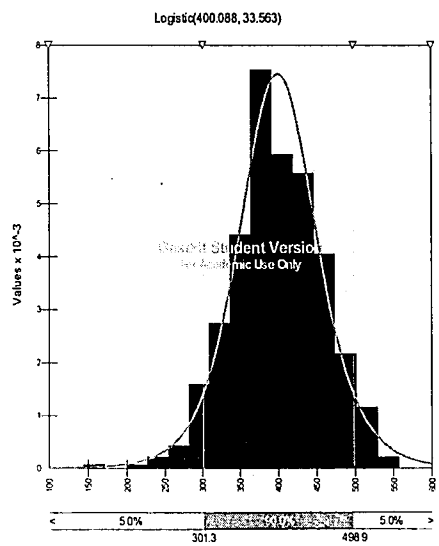


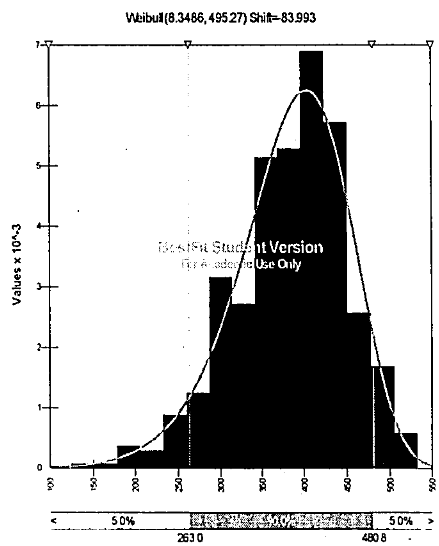
I6, D13, D23



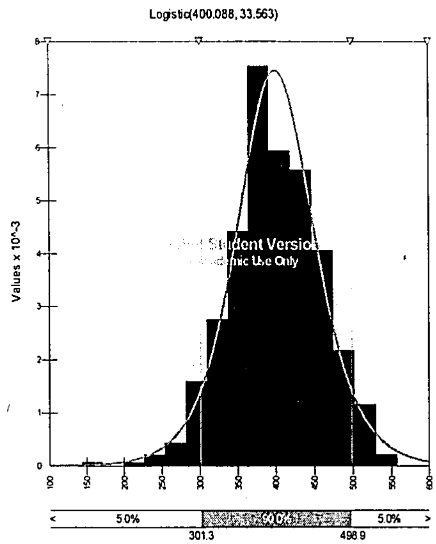


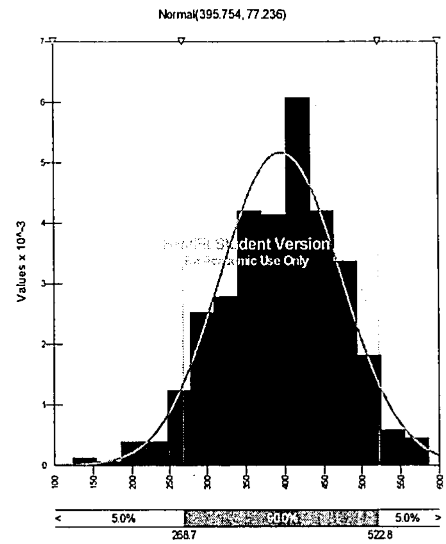
I7, D13, D23



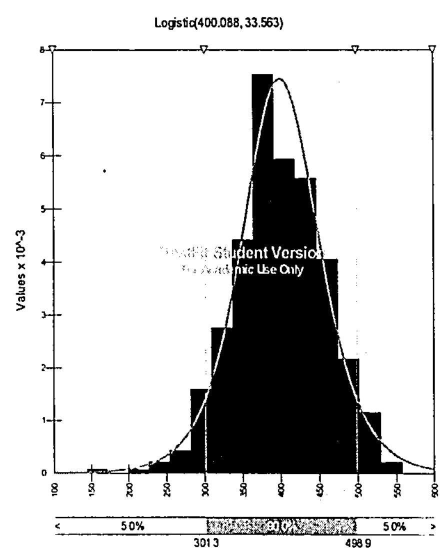


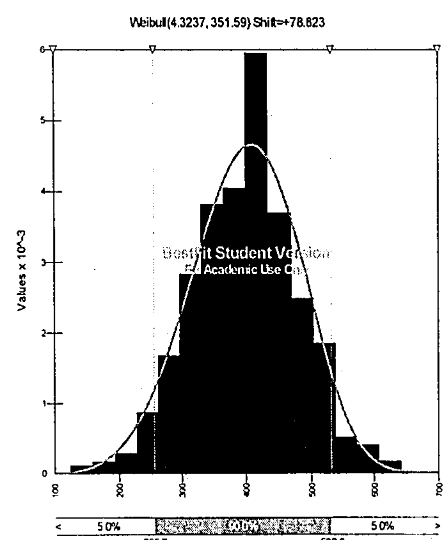
18, D13, D23



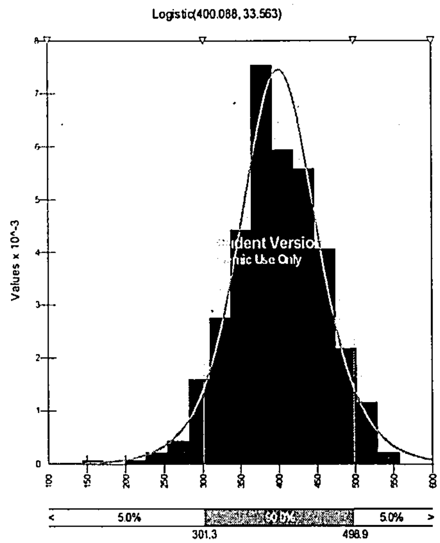


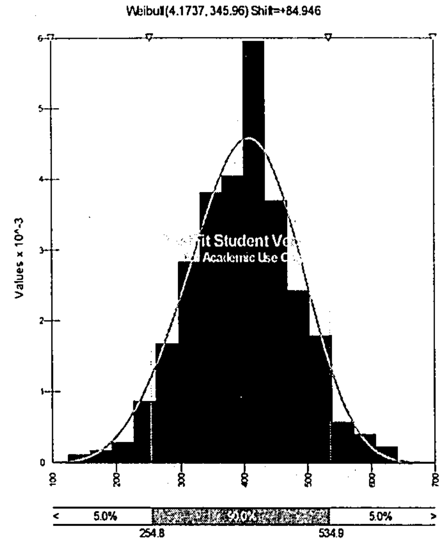
19, D13, D23



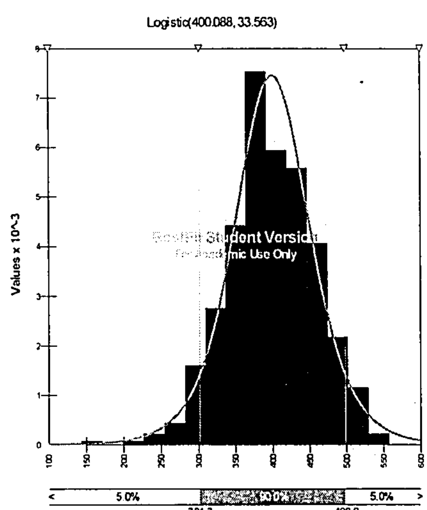


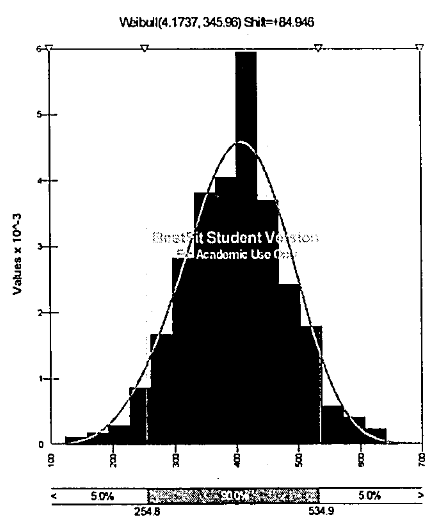
I10, D13, D23



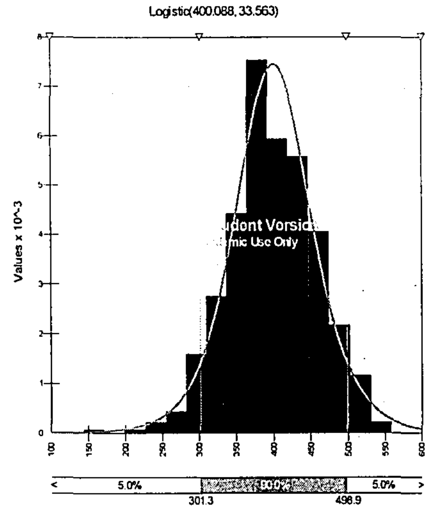


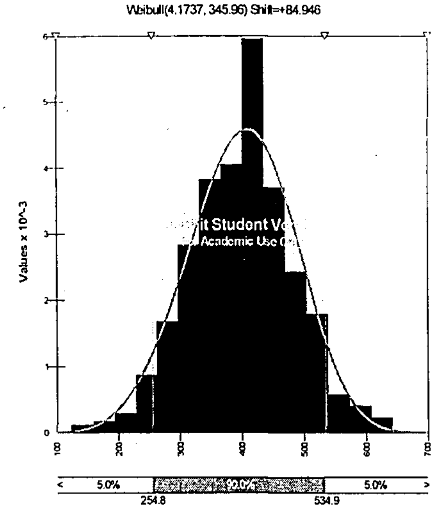
I11, D13, D23





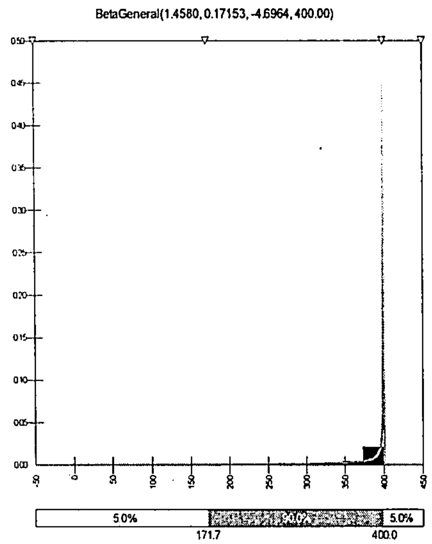
I12, D13, D23

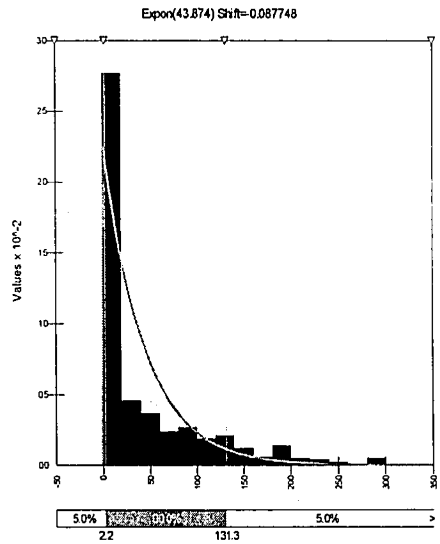

Logistic(400.088, 33.563)
Values x 10^-3
5.0%
90.0%
5.0%
301.3
498.9


Weibull(4.1737, 345.96) Shift=+84.946
Values x 10^-3
5.0%
90.0%
5.0%
254.8
534.9

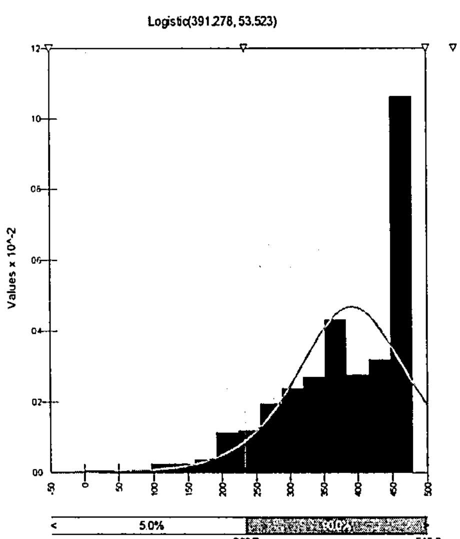
Uncertainty level Standard Deviation = 10% Mean

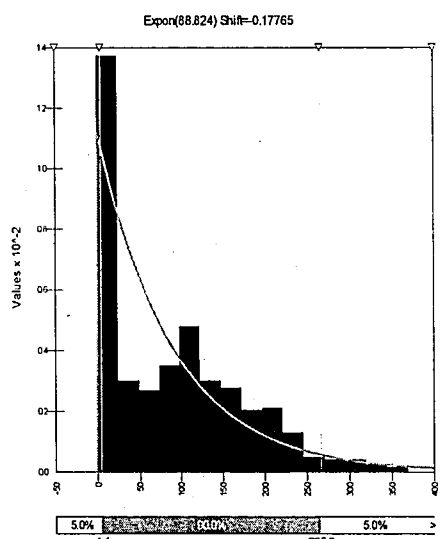
I1, D13, D23



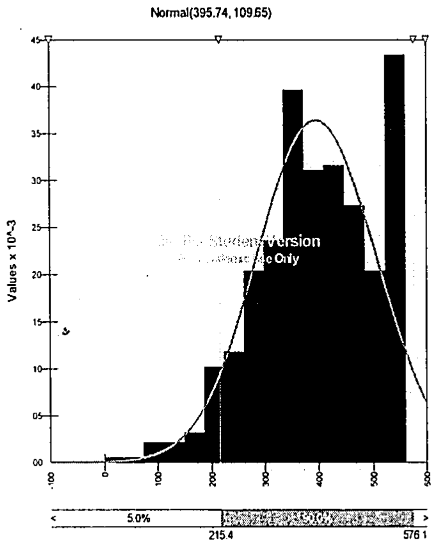


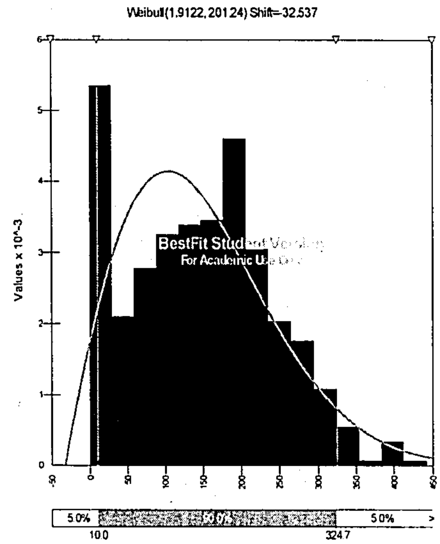
I2, D13, D23



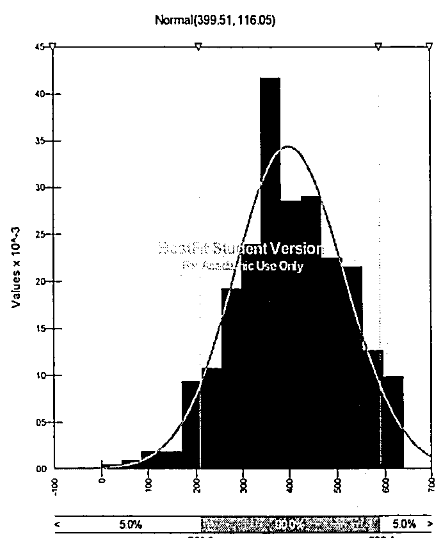


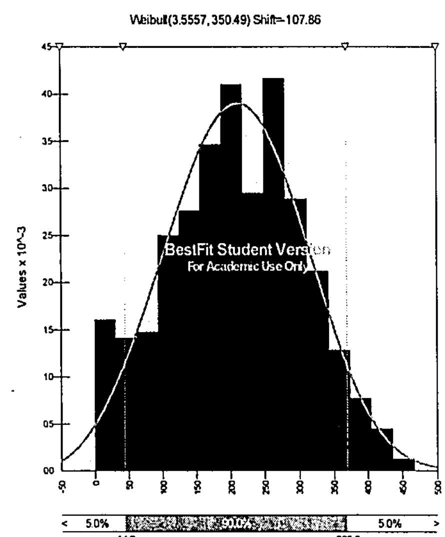
I3, D13, D23



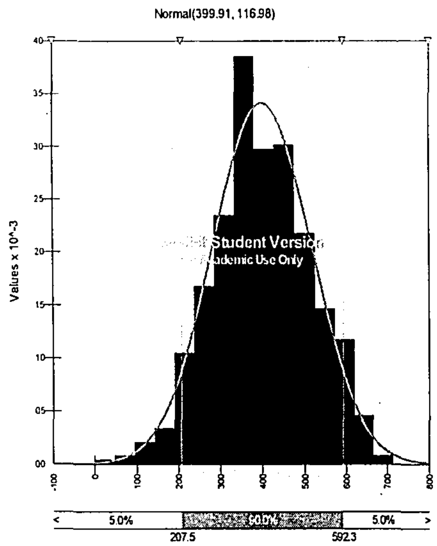


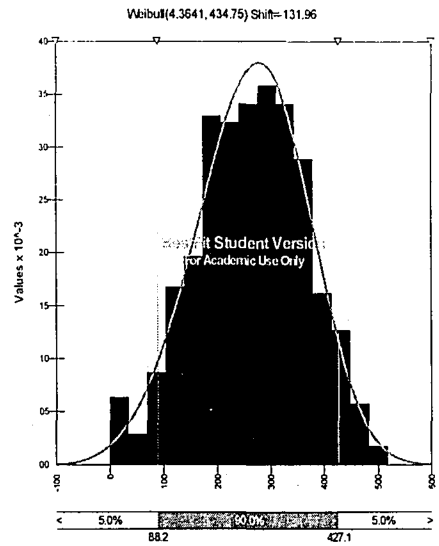
I4, D13, D23



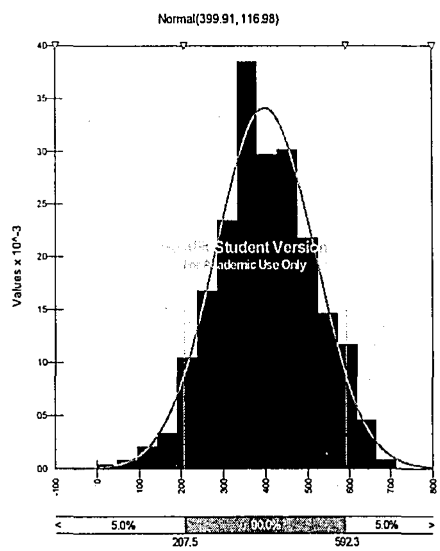


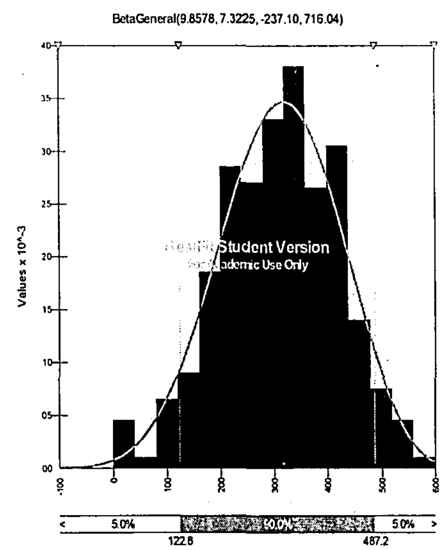
I5, D13, D23



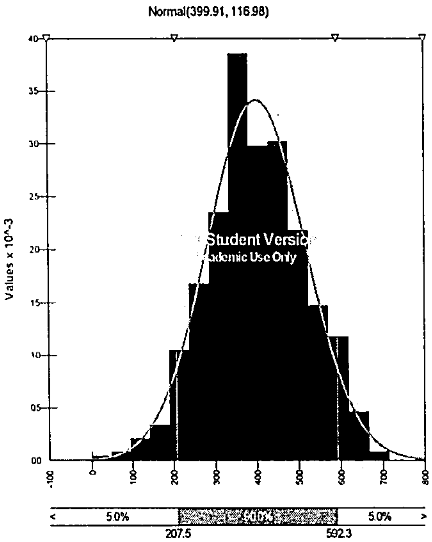


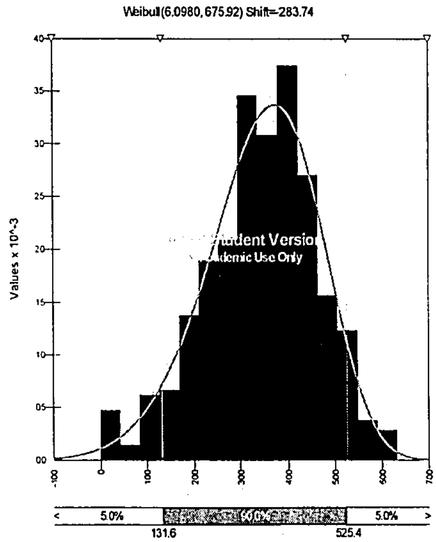
I6, D13, D23



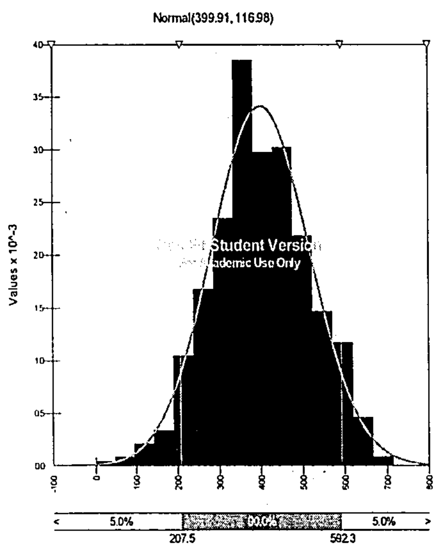


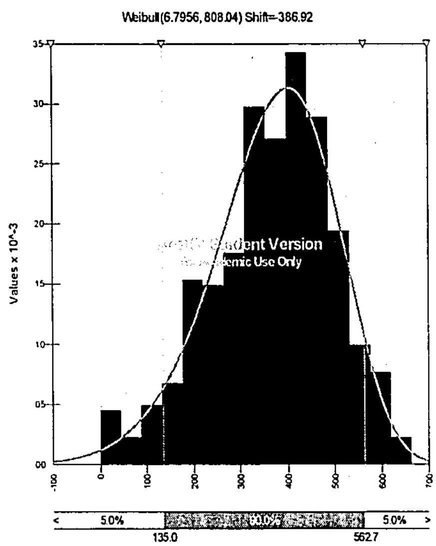
I7, D13, D23



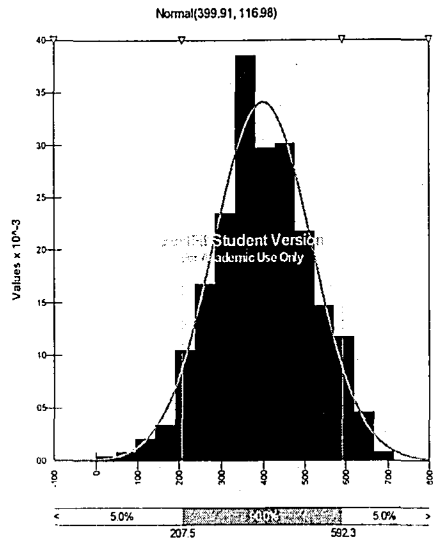


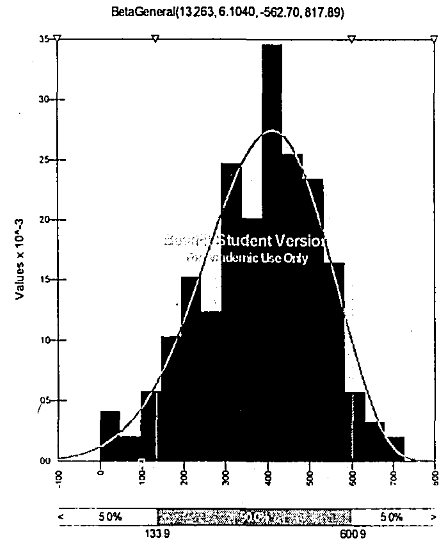
I8, D13, D23



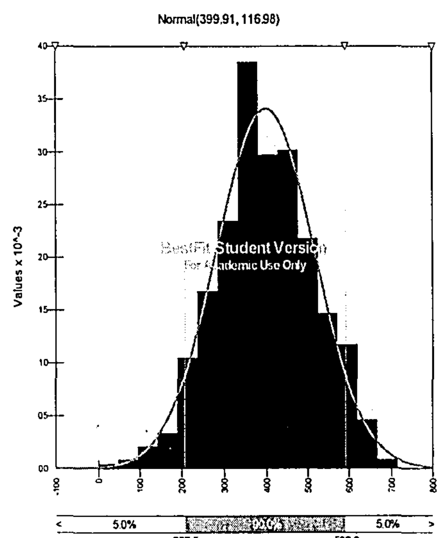


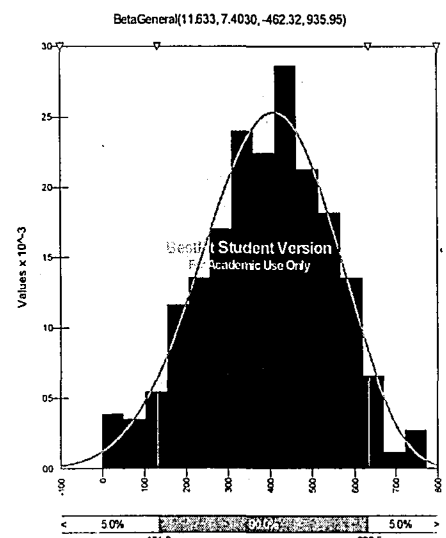
I9, D13, D23



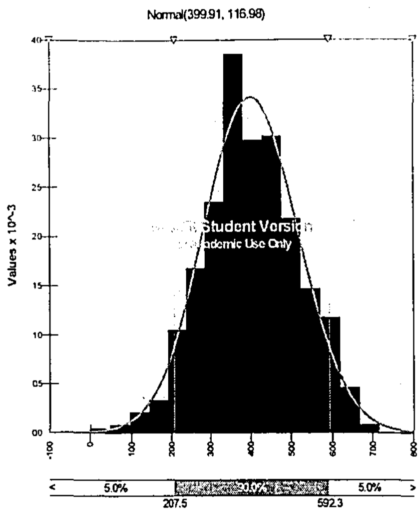


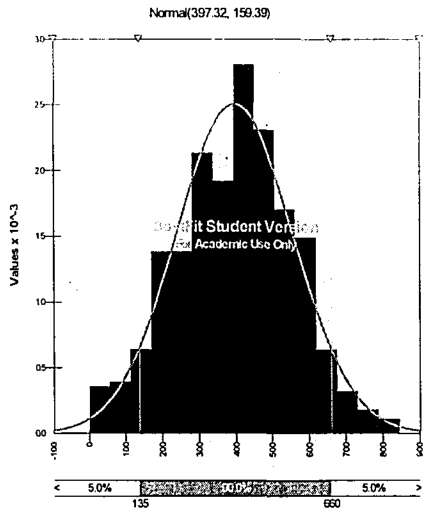
I10, D13, D23



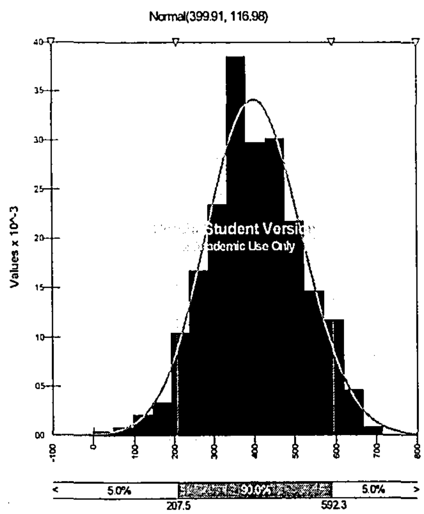


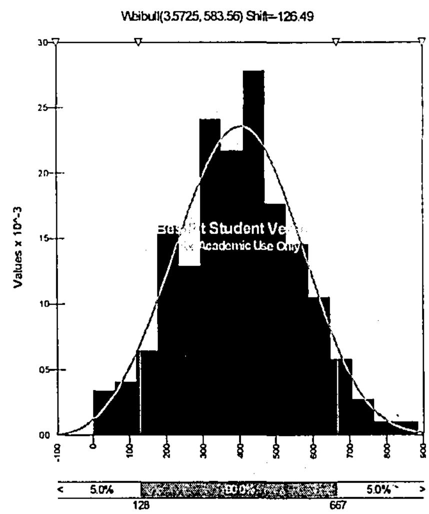
I11, D13, D23



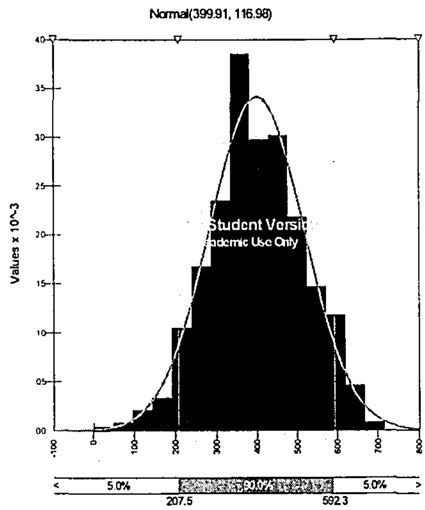


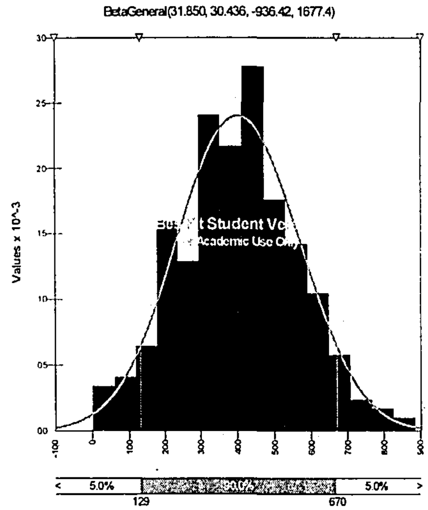
I12, D13, D23



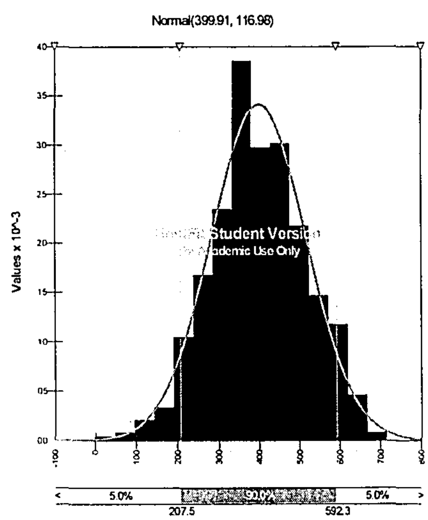


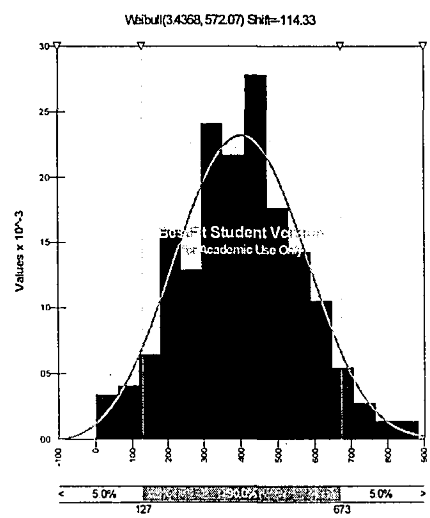
I13, D13, D23



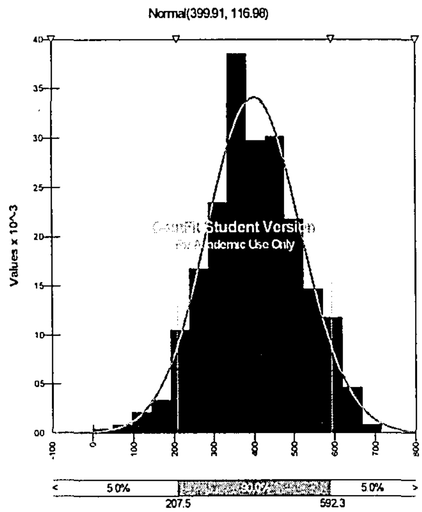


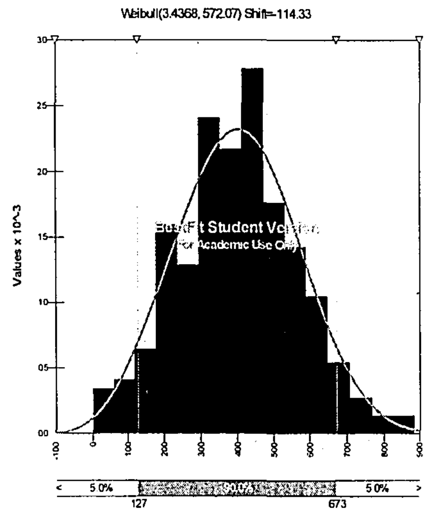
I14, D13, D23



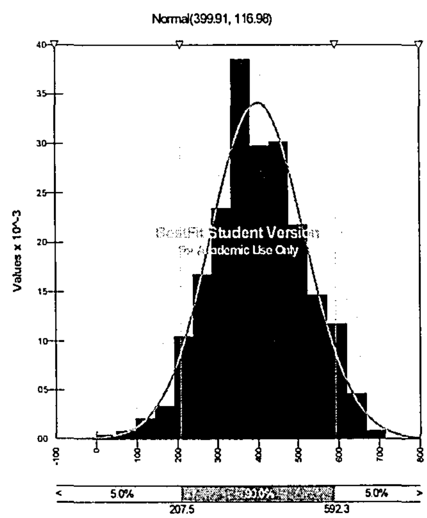


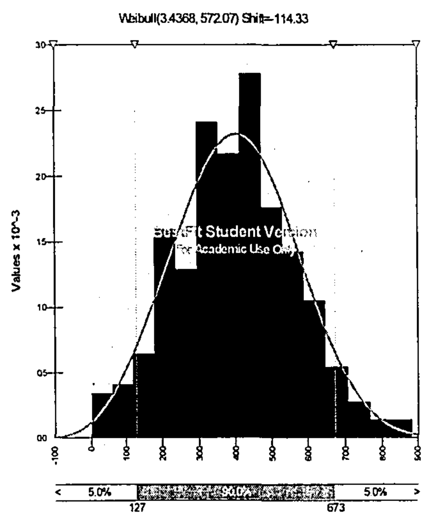
115, D13, D23





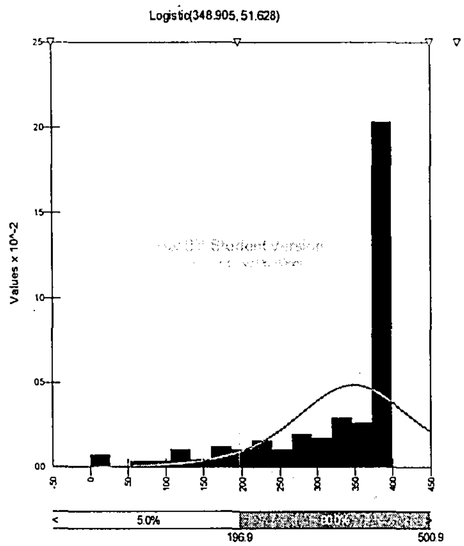
116, D13, D23

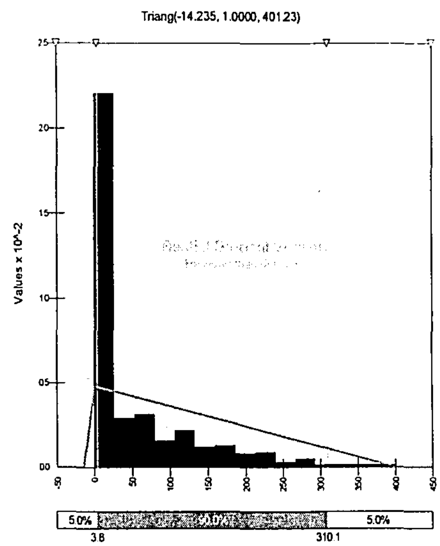




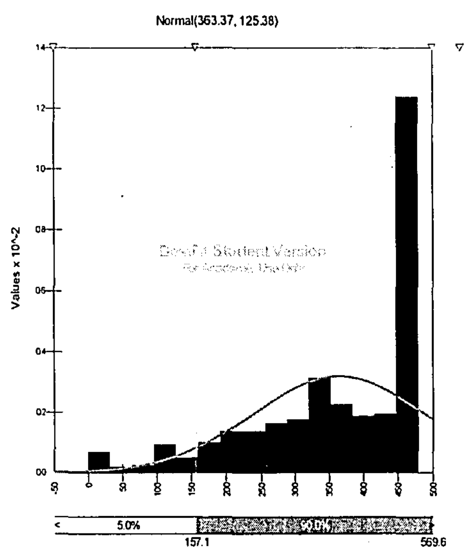
Uncertainty level Standard Deviation = 15% Mean

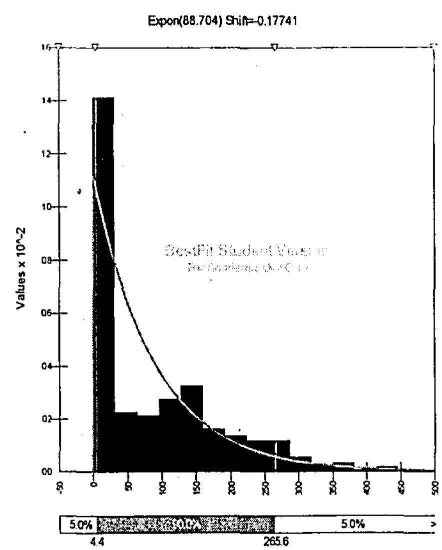
I1, D13, D23



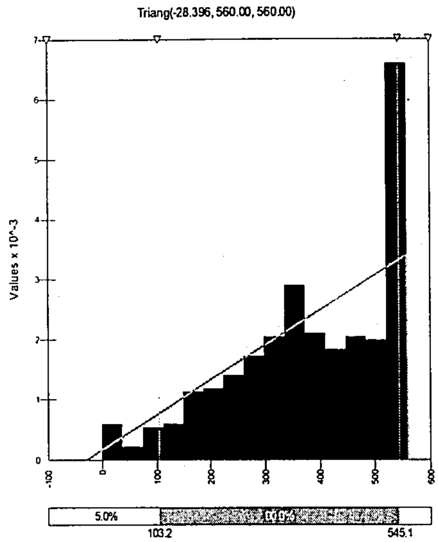


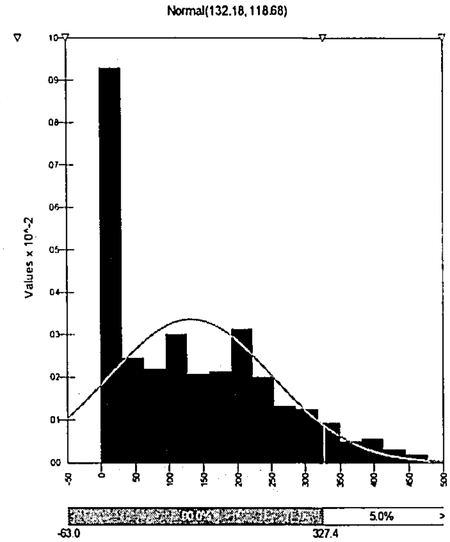
I2, D13, D23



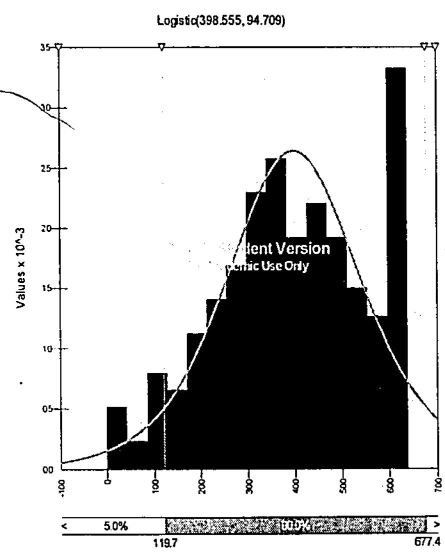


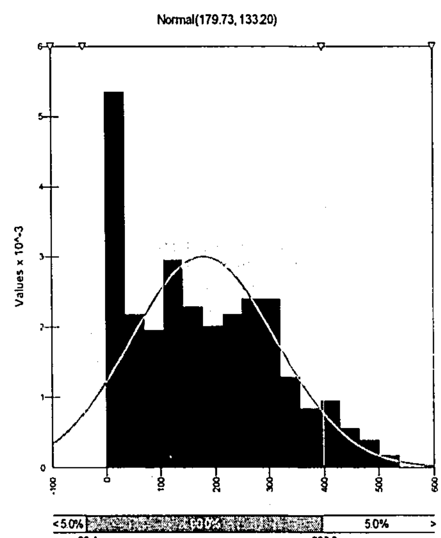
I3, D13, D23



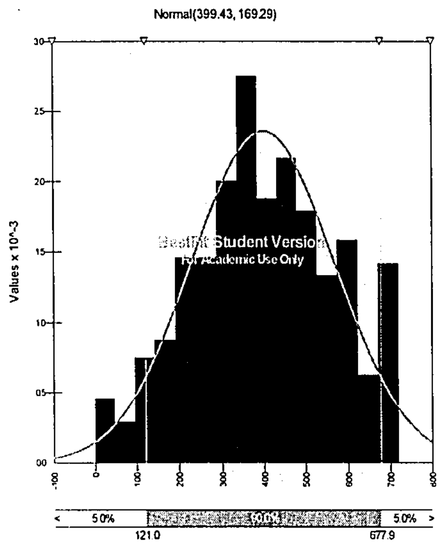


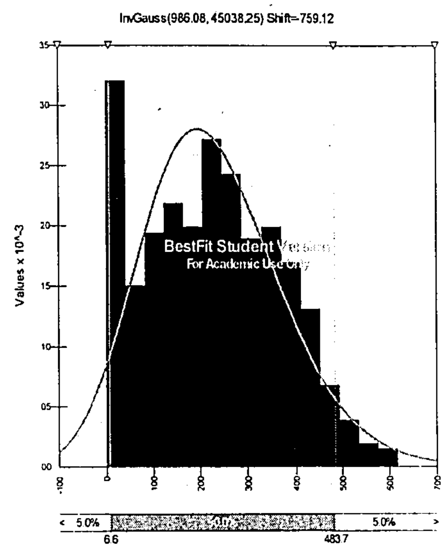
I4, D13, D23



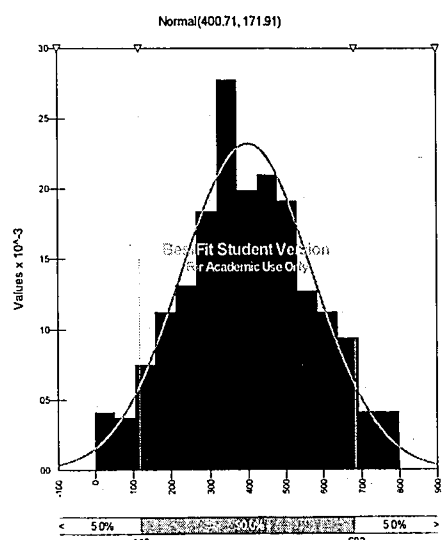


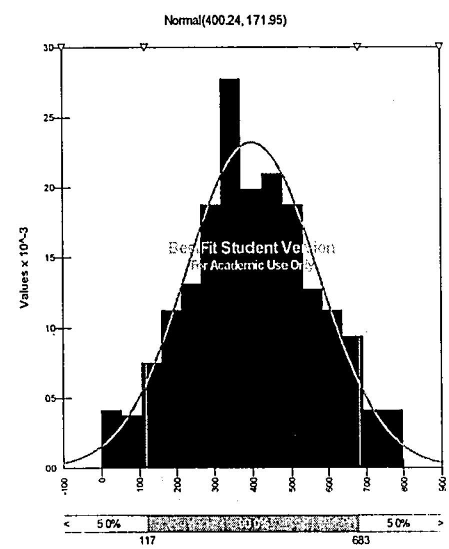
I5, D13, D23



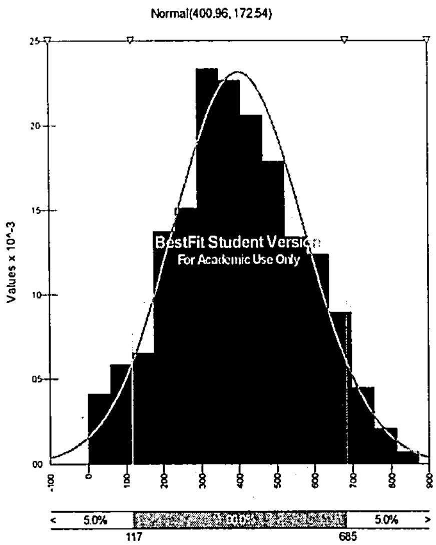


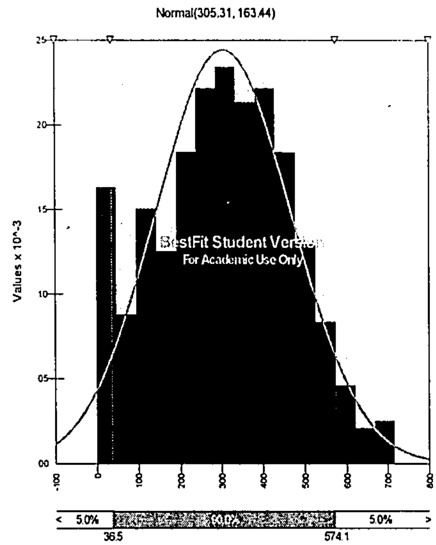
I6, D13, D23



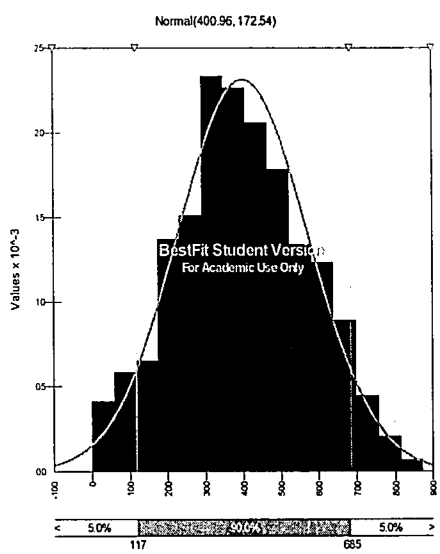


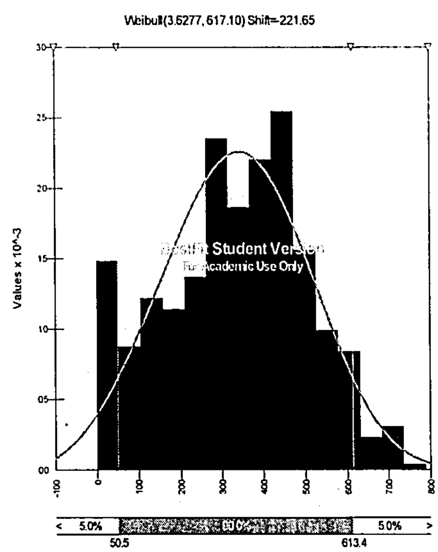
I7, D13, D23



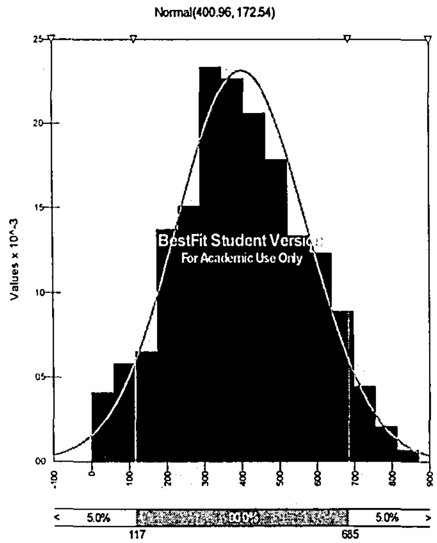


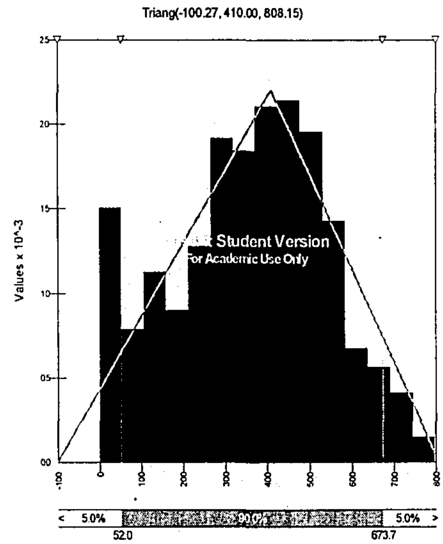
I8, D13, D23



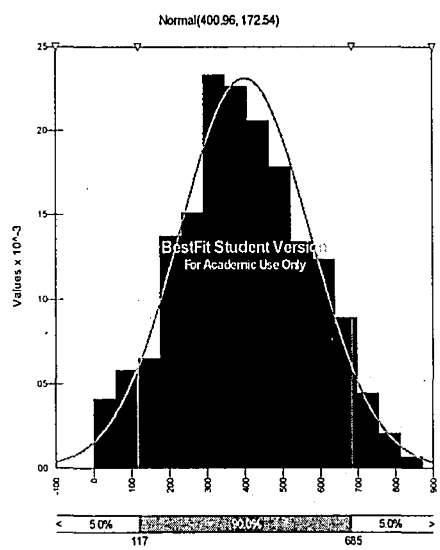


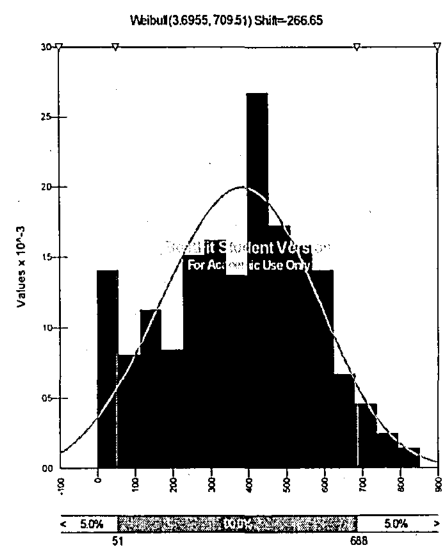
I9, D13, D23



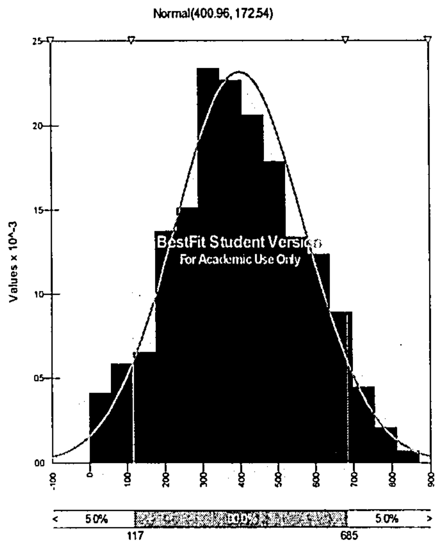


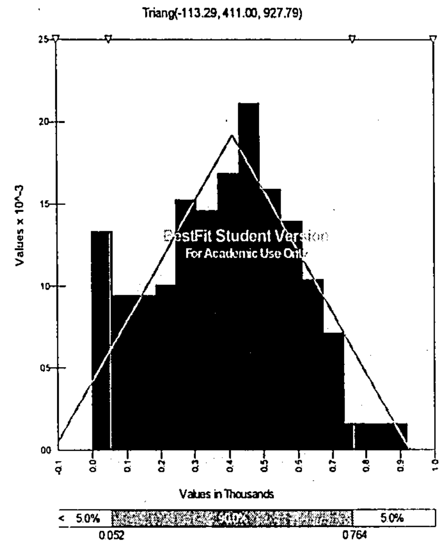
I10, D13, D23



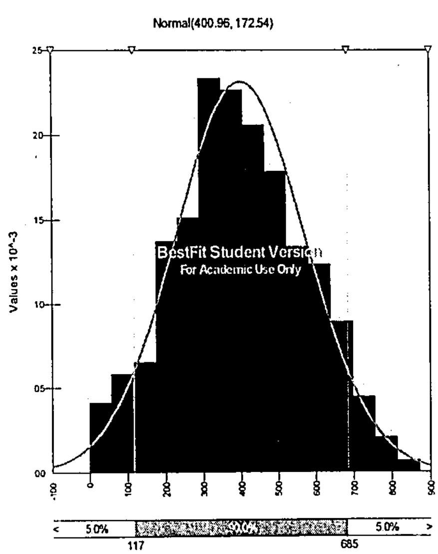


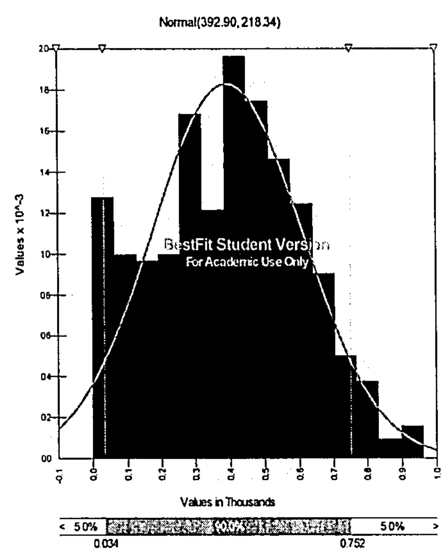
I11, D13, D23



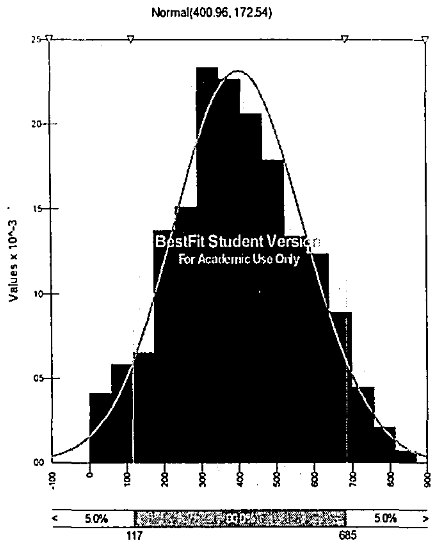


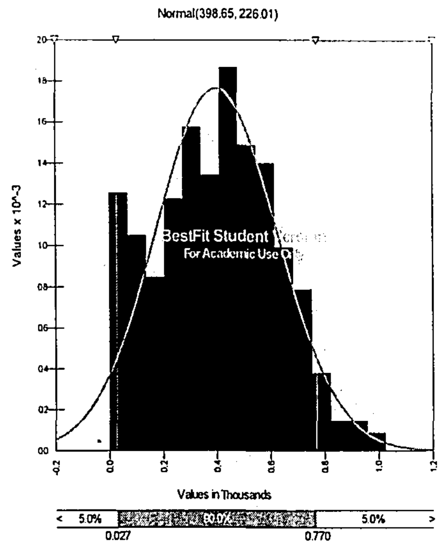
I12, D13, D23



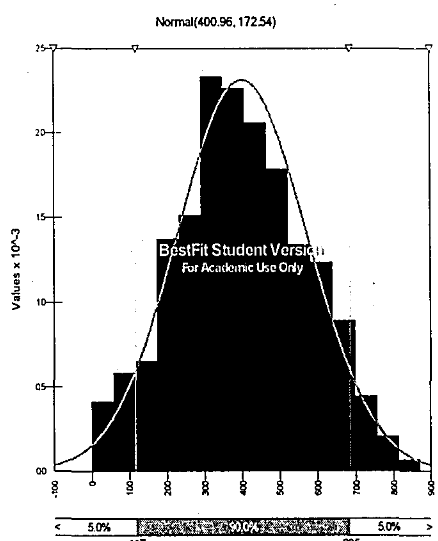


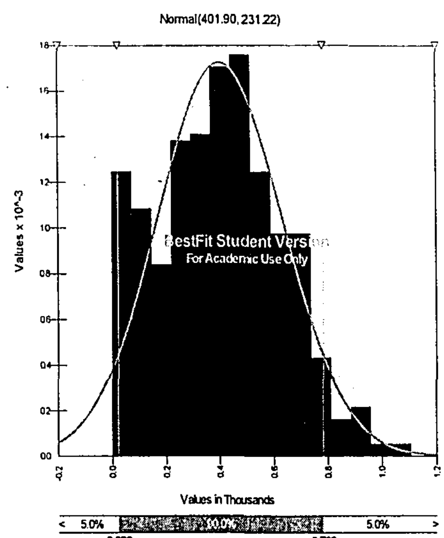
I13, D13, D23



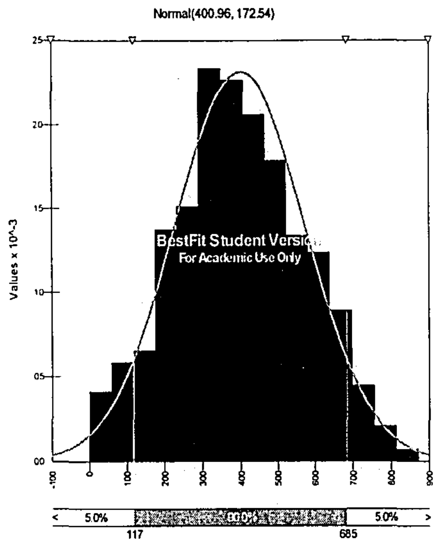


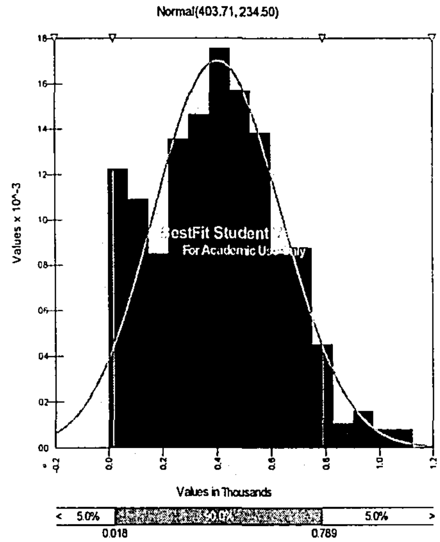
I14, D13, D23



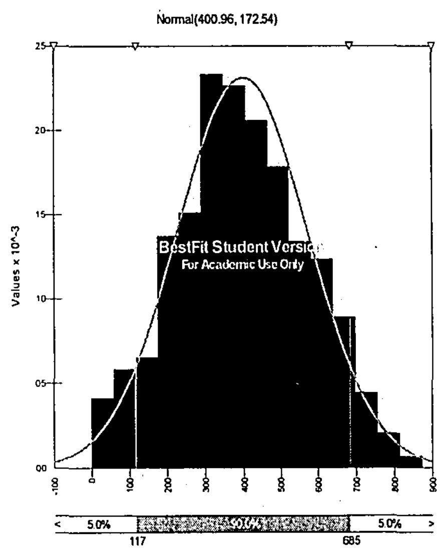


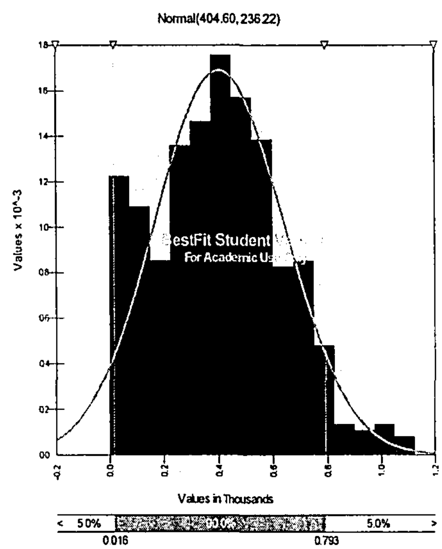
I15, D13, D23



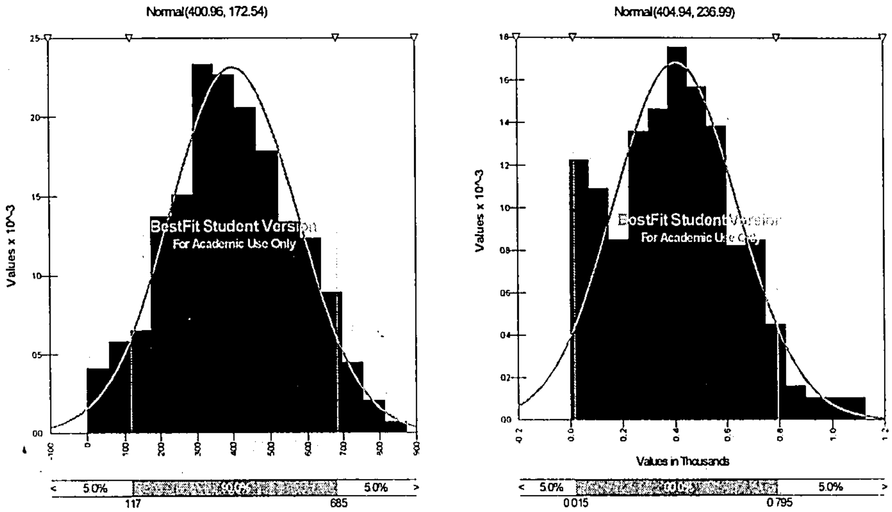


I16, D13, D23

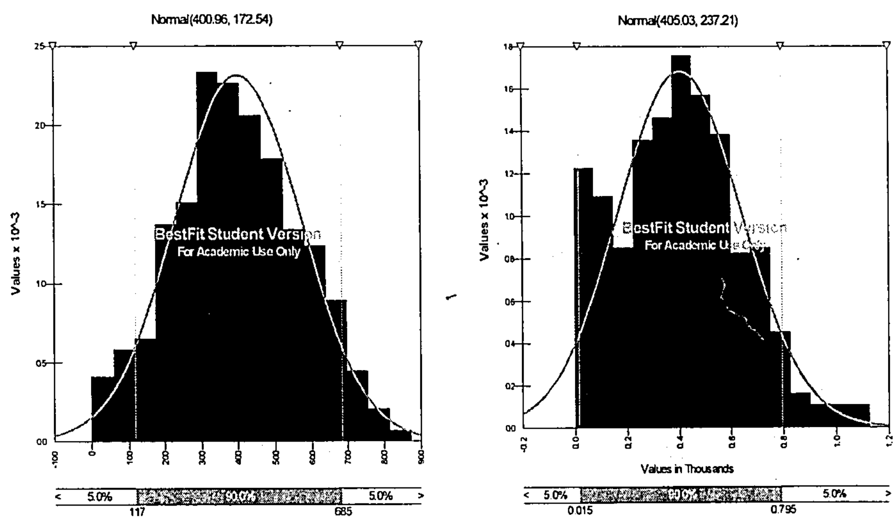




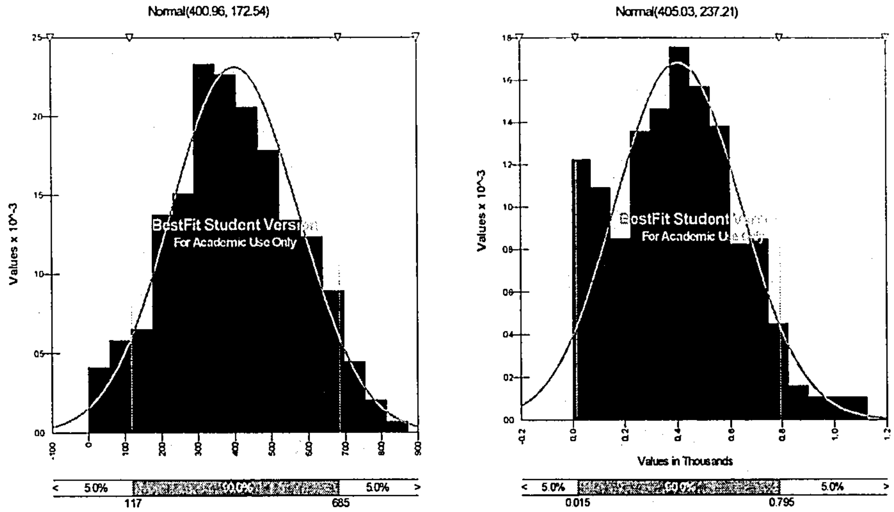
I17, D13, D23



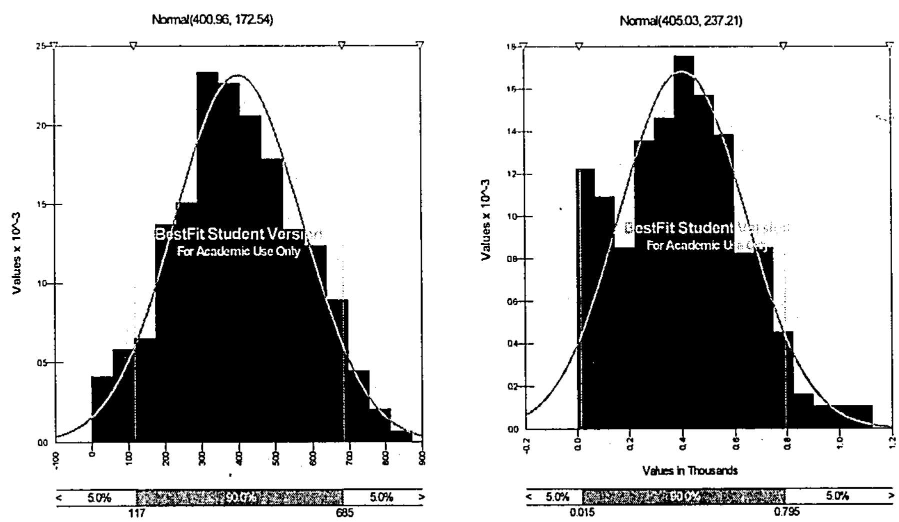
I18, D13, D23



I19, D13, D23

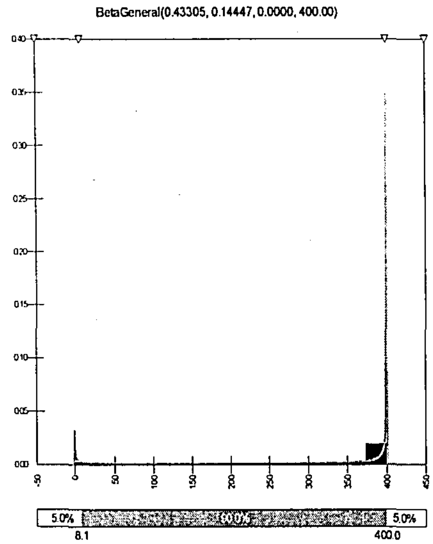


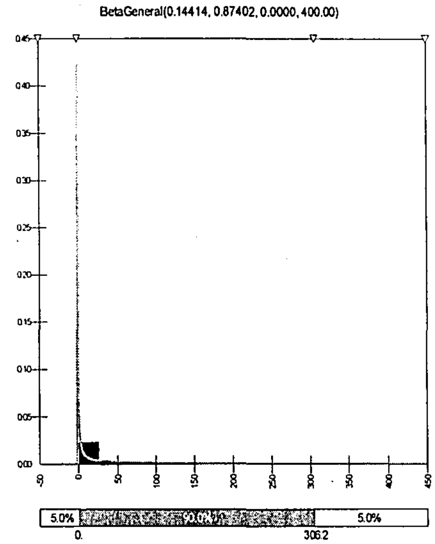
I20, D13, D23



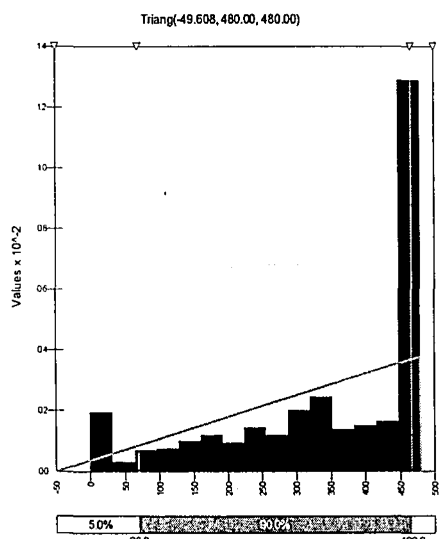
Uncertainty level Standard Deviation = 20% Mean

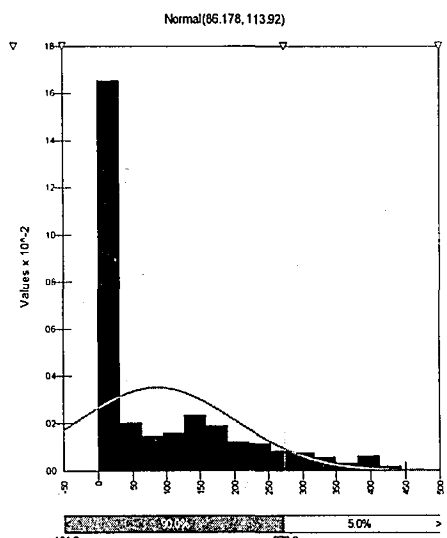
I1, D13, D23



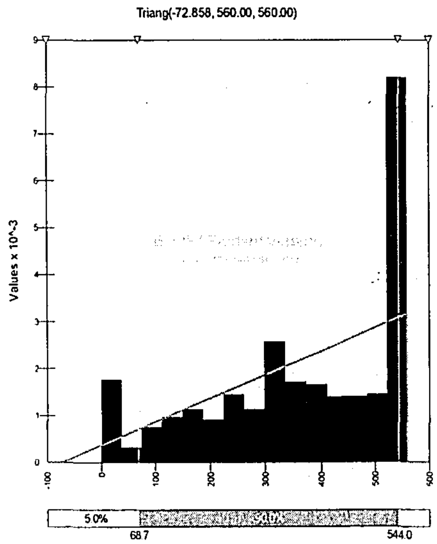


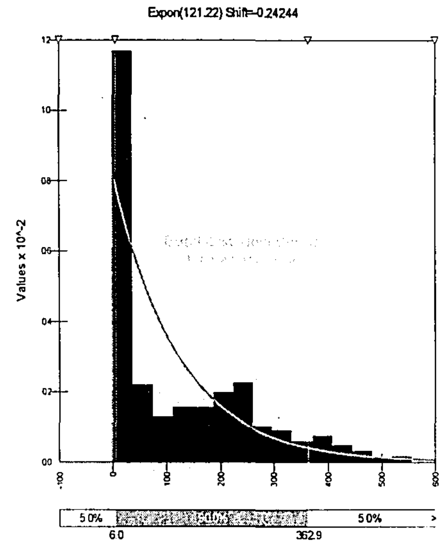
I2, D13, D23



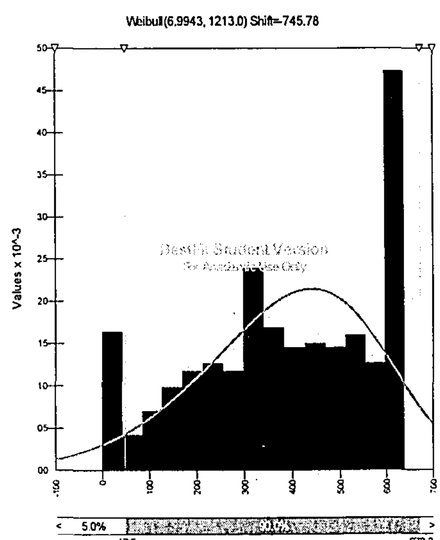


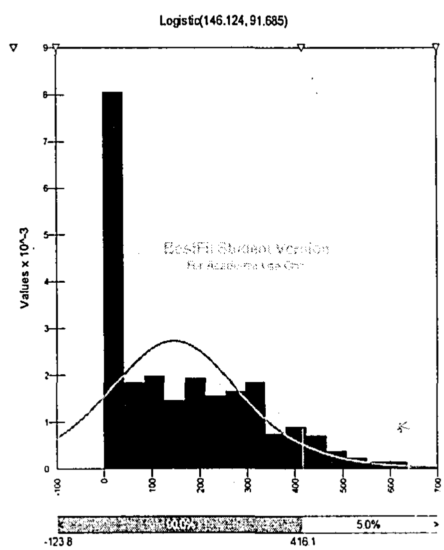
I3, D13, D23



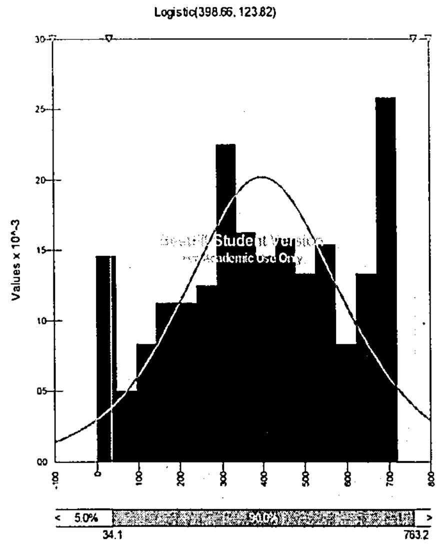


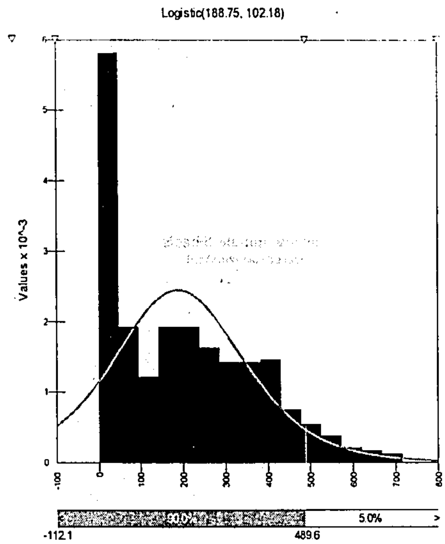
I4, D13, D23



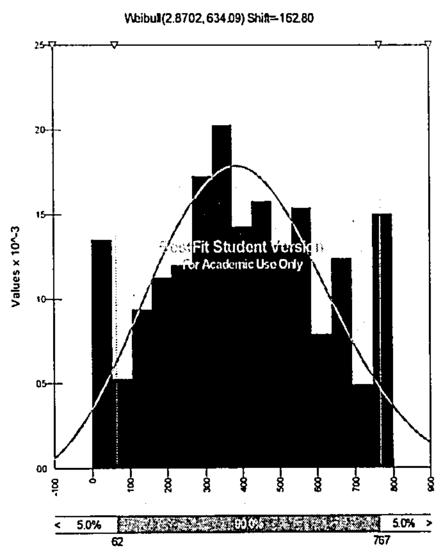


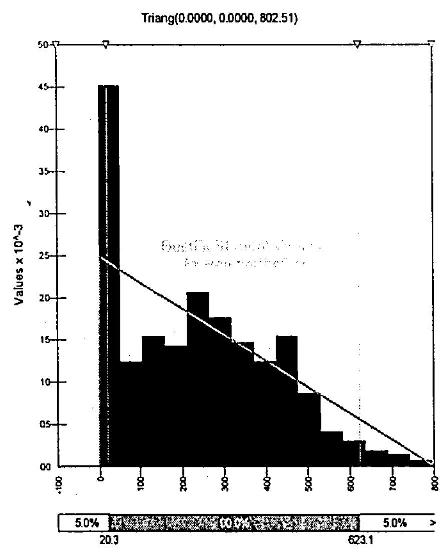
I5, D13, D23



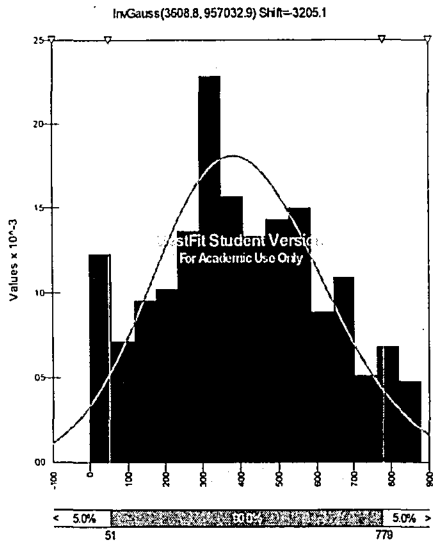


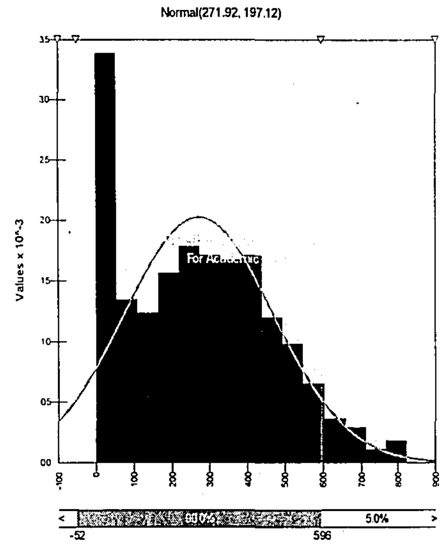
I6, D13, D23



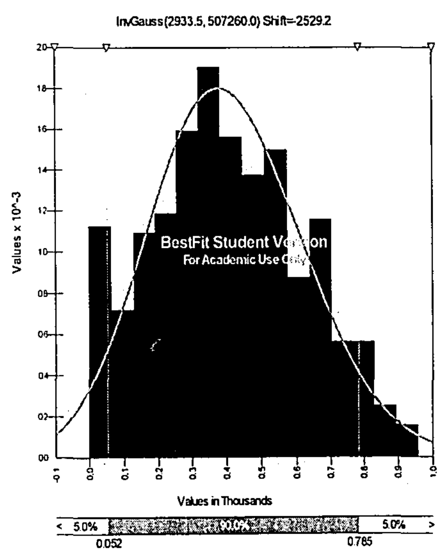


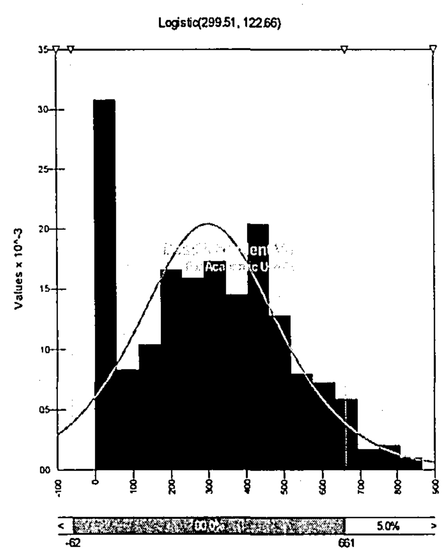
I7, D13, D23



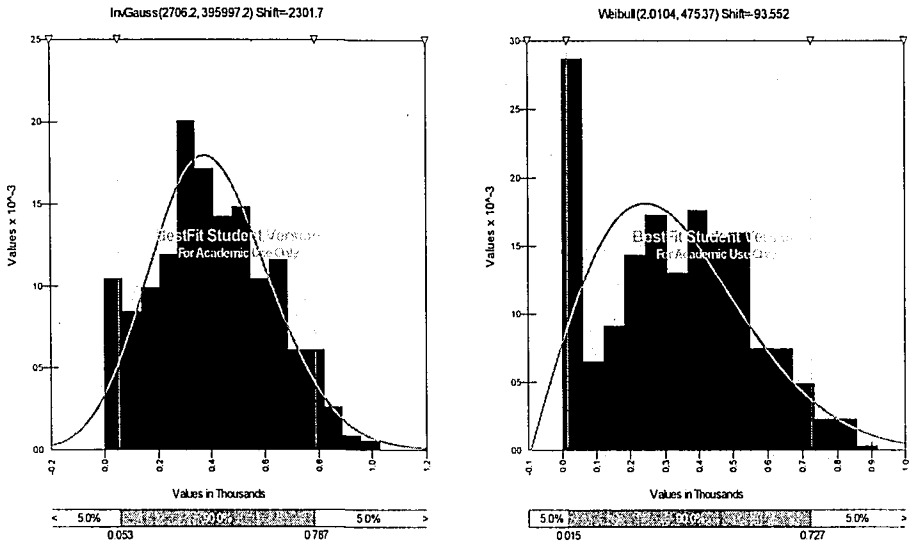


I8, D13, D23

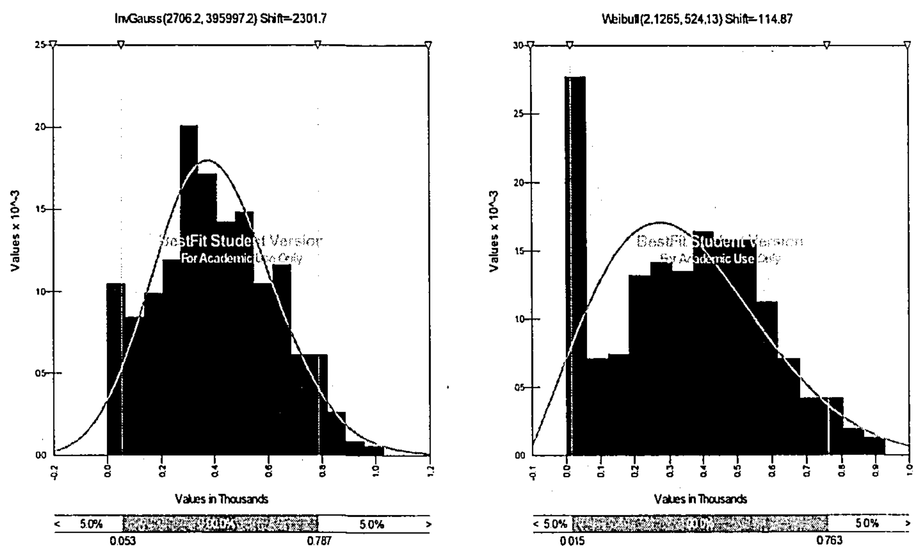




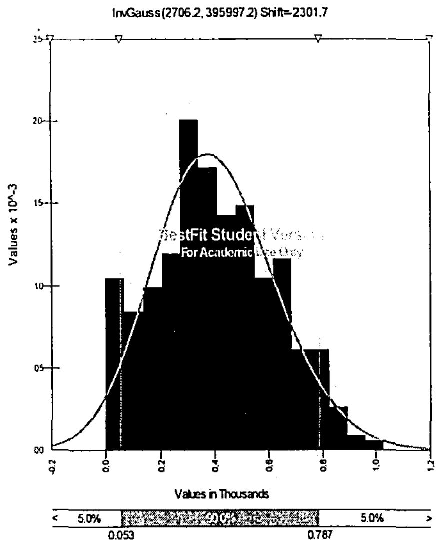
I9, D13, D23

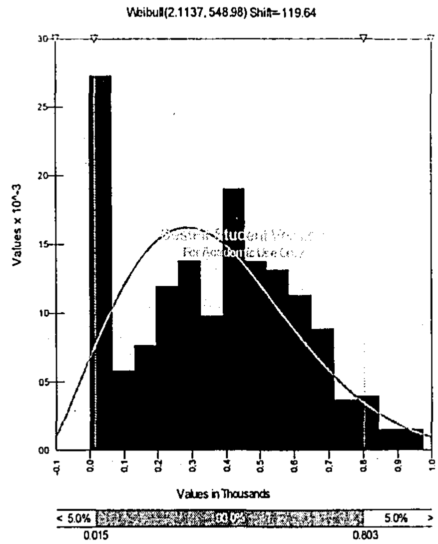


I10, D13, D23

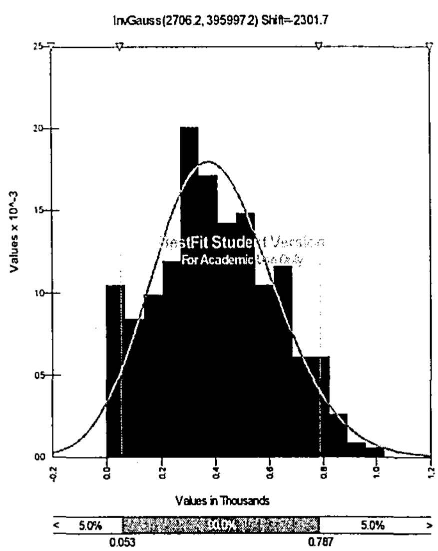


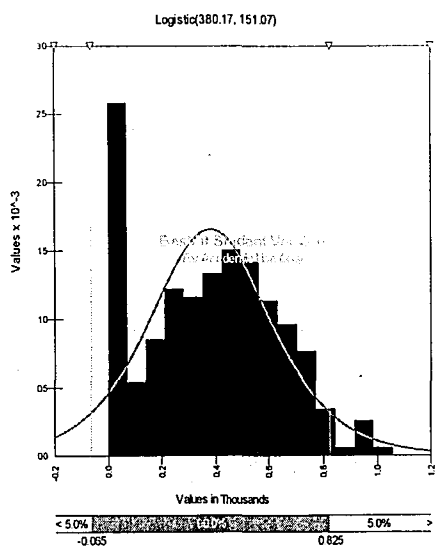
I11, D13, D23



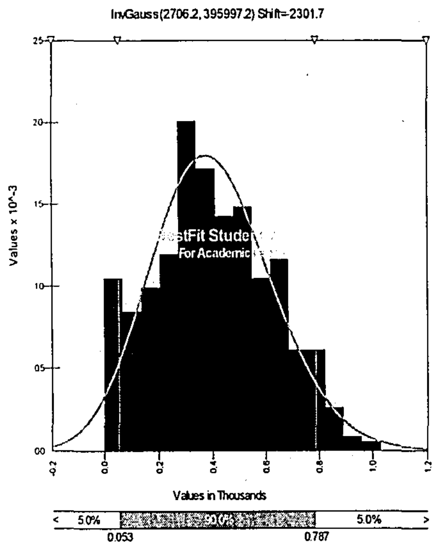


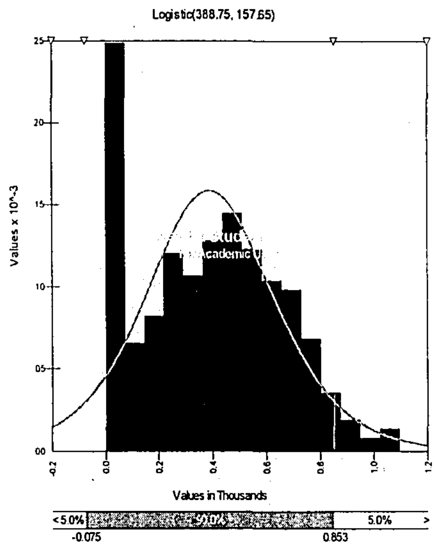
I12, D13, D23



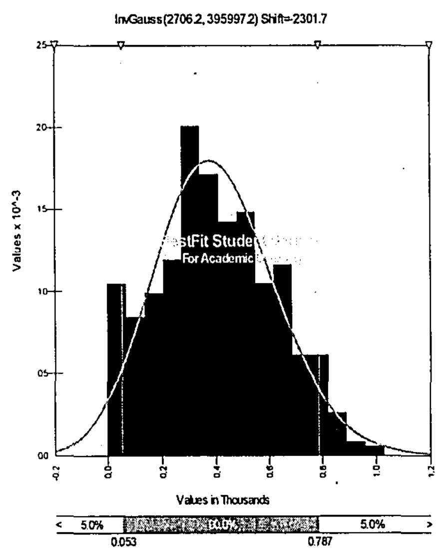


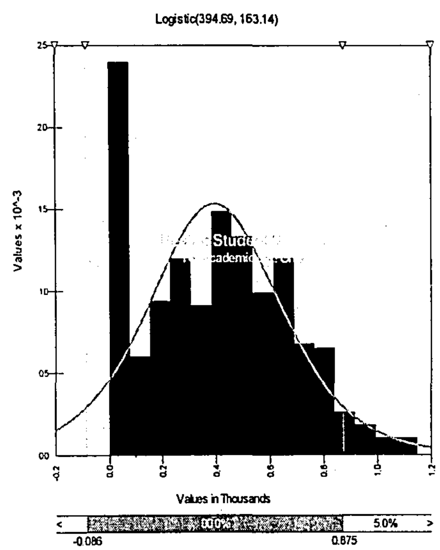
I13, D13, D23



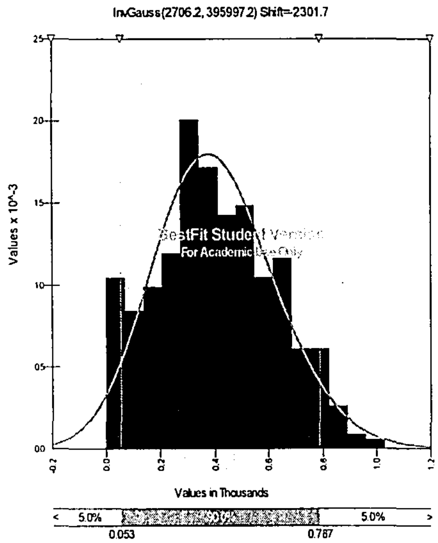


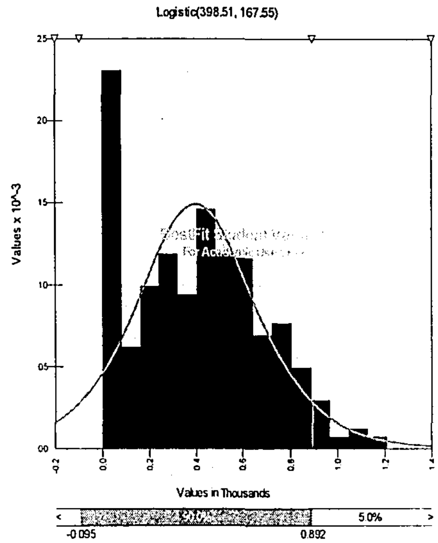
I14, D13, D23



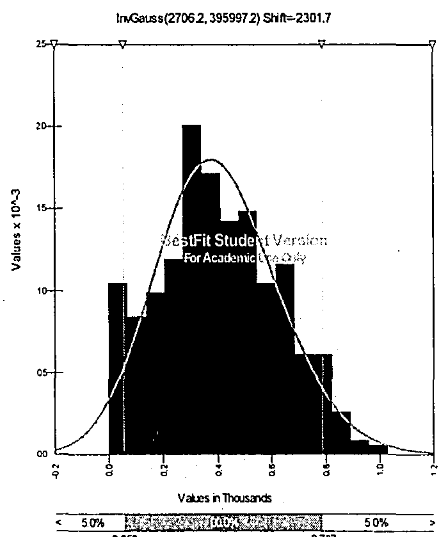


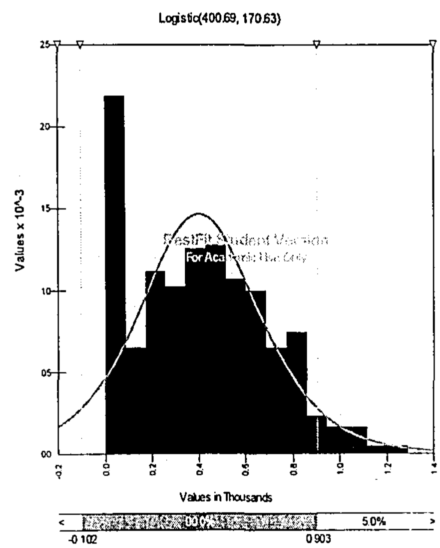
I15, D13, D23



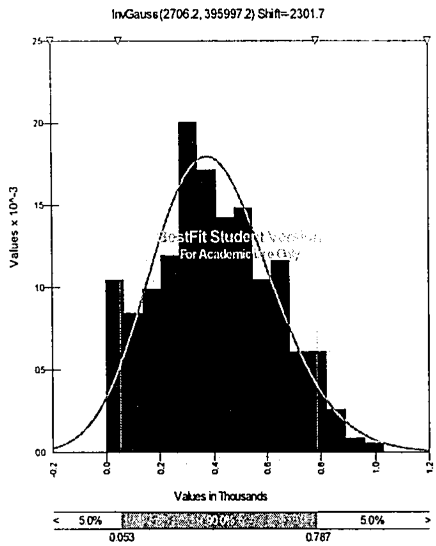


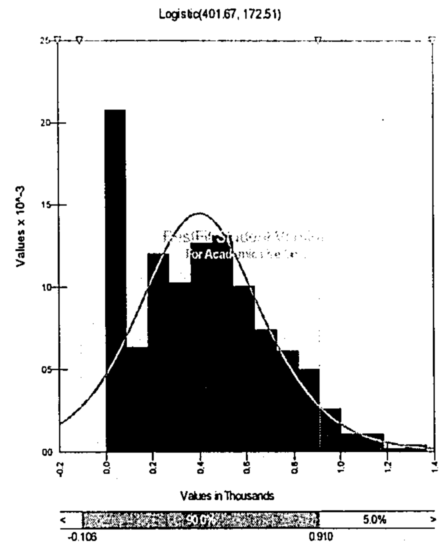
I16, D13, D23



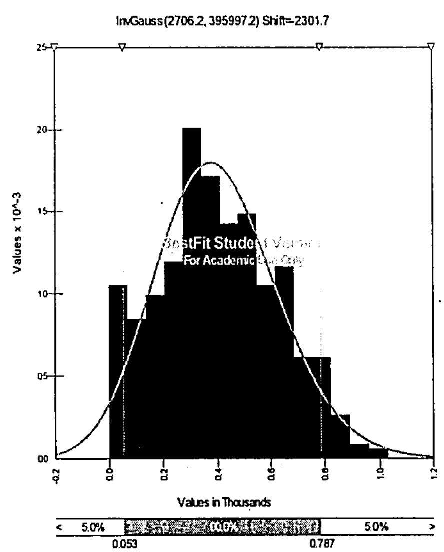


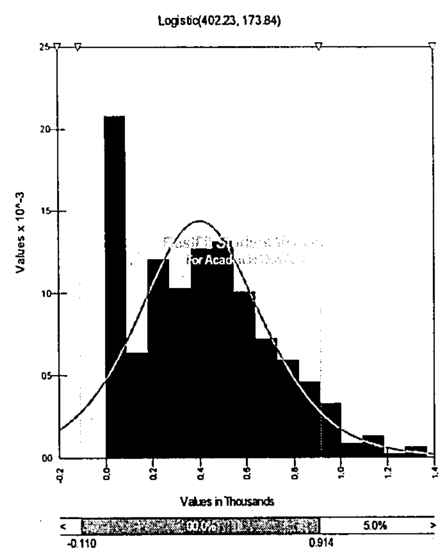
I17, D13, D23



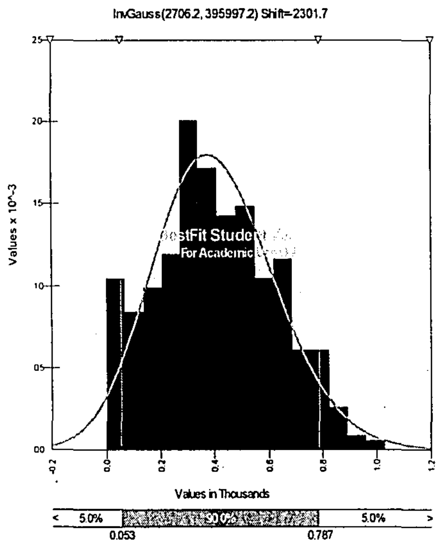


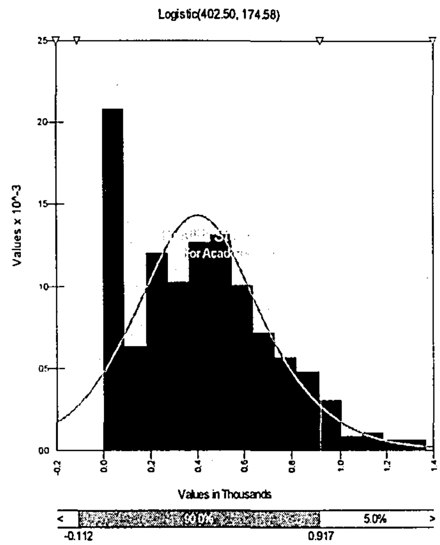
I18, D13, D23



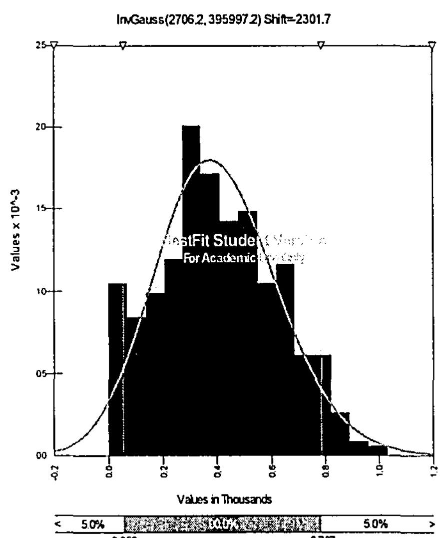


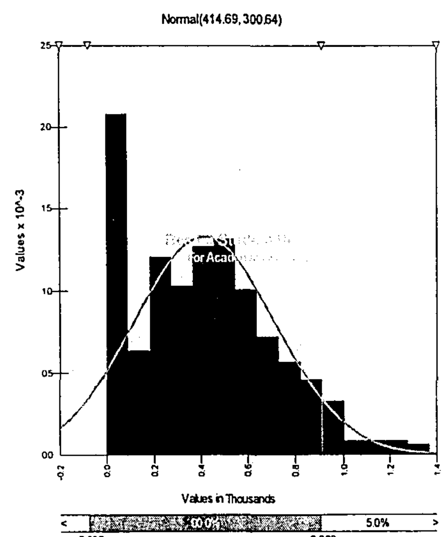
I19, D13, D23



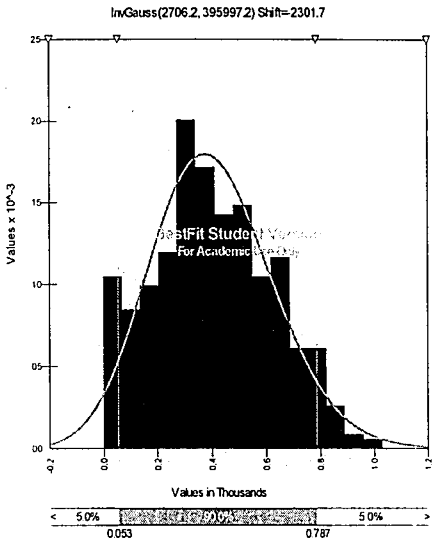


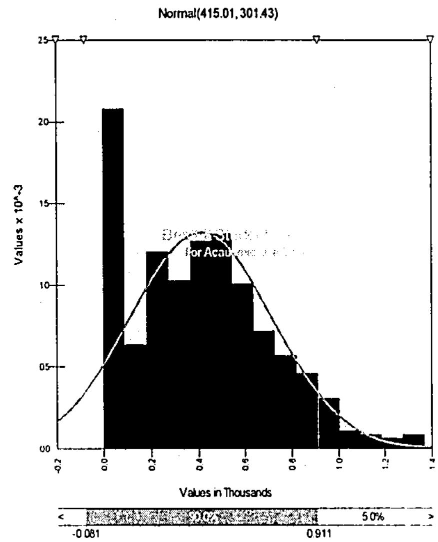
I20, D13, D23



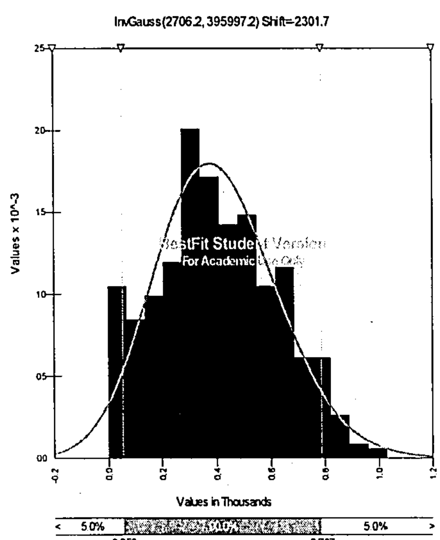


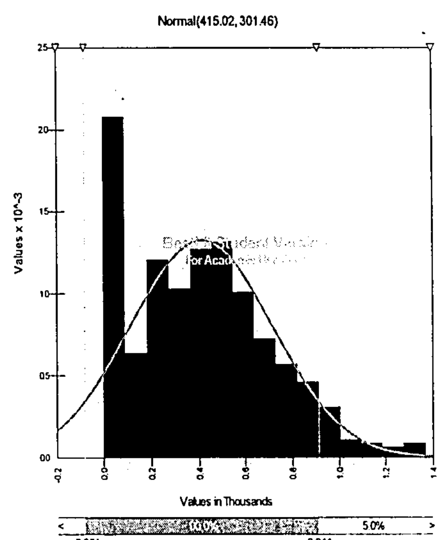
I21, D13, D23



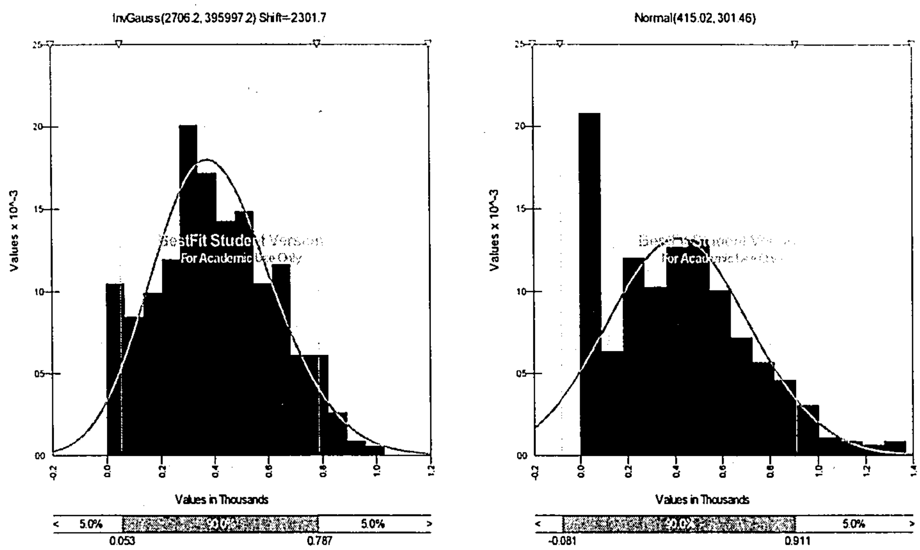


I22, D13, D23

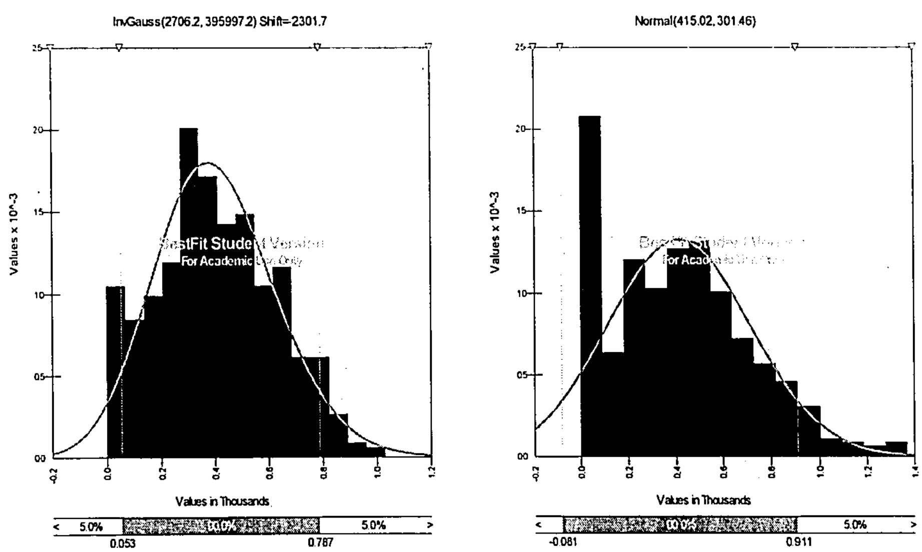




I23, D13, D23

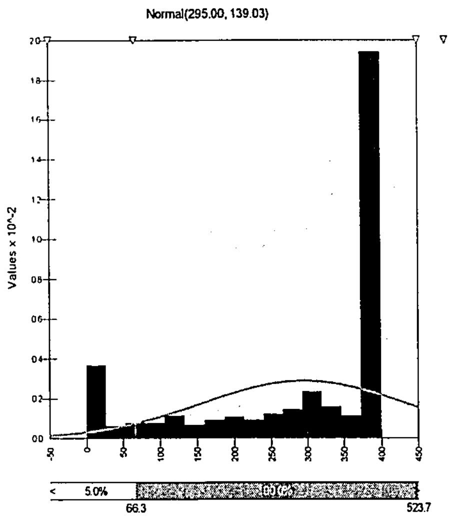


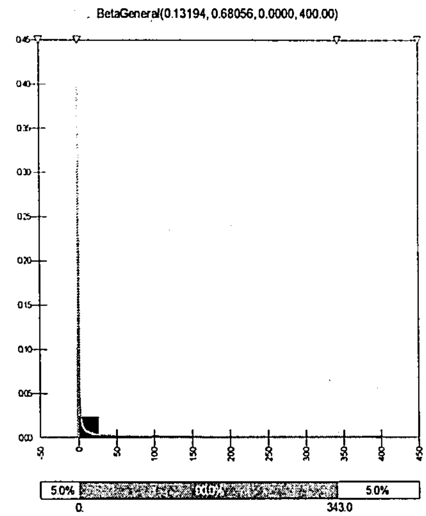
I24, D13, D23



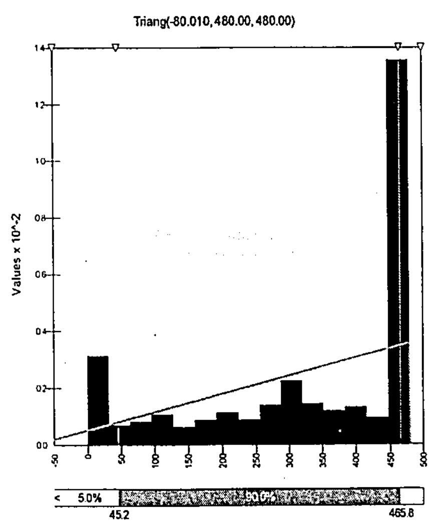
Uncertainty level Standard Deviation = 25% Mean

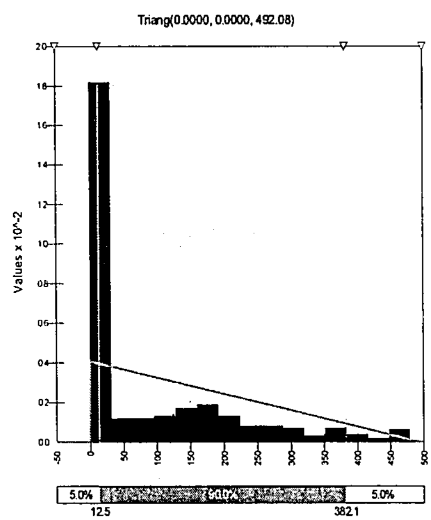
I1, D13, D23



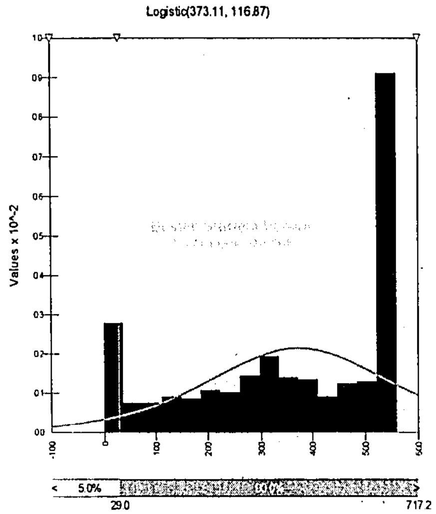


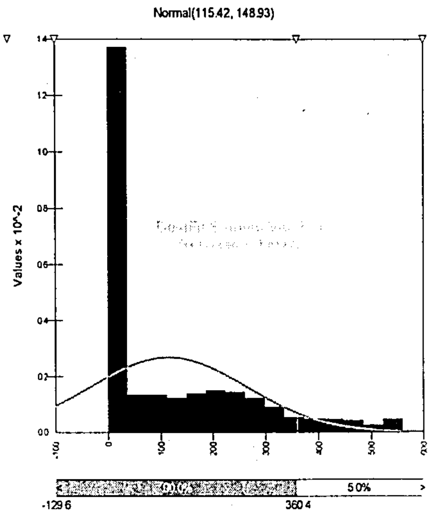
I2, D13, D23



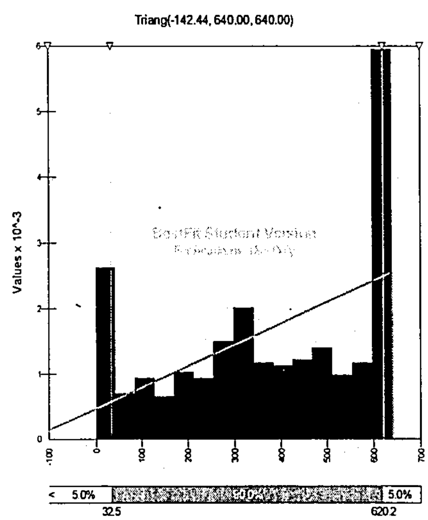


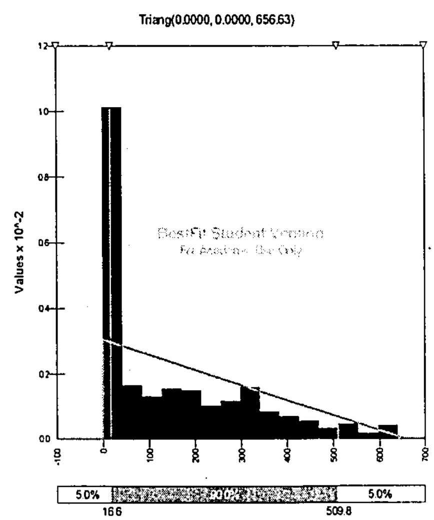
I3, D13, D23



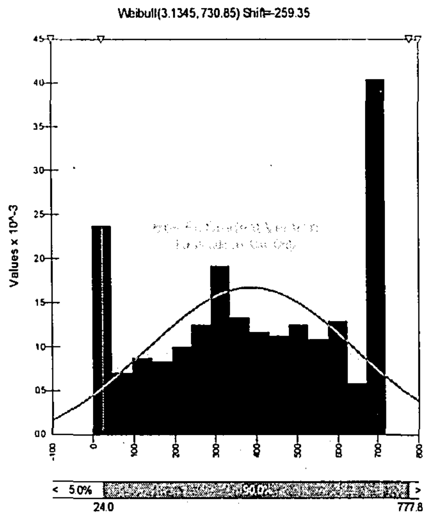


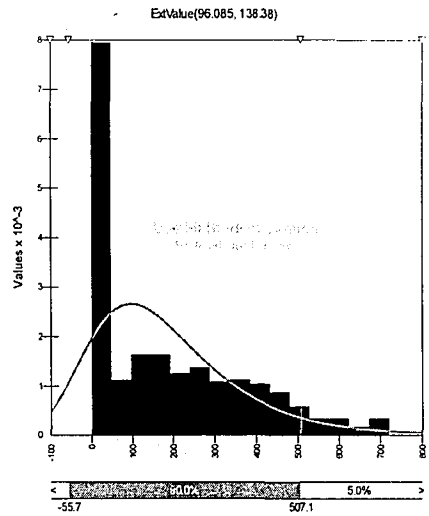
I4, D13, D23



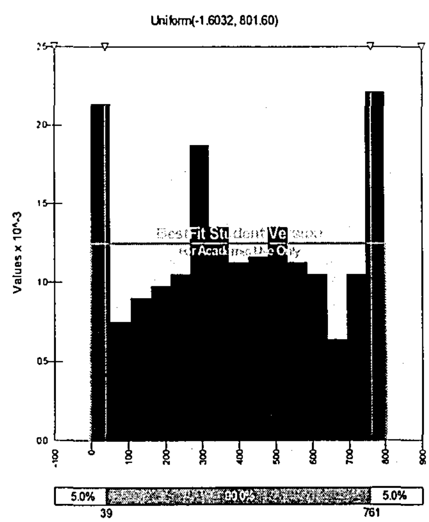


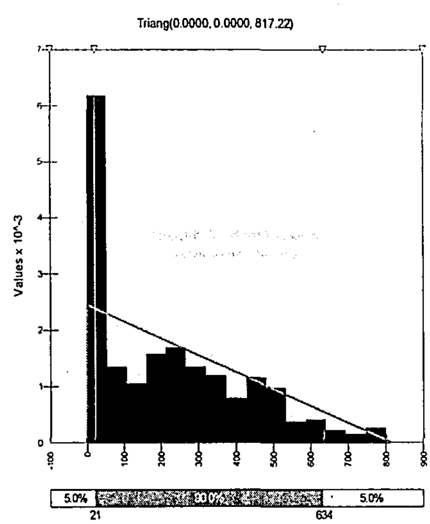
I5, D13, D23



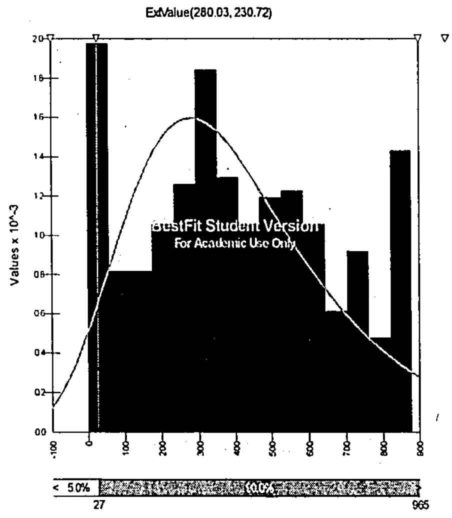


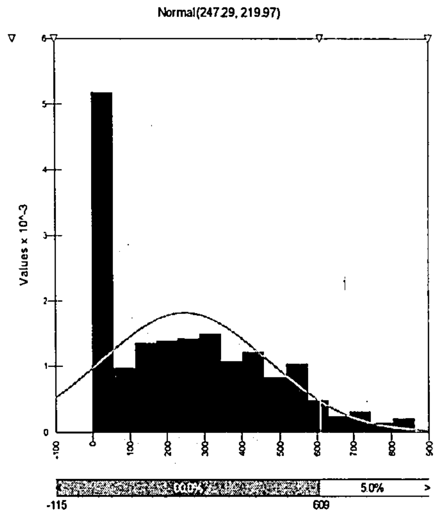
I6, D13, D23



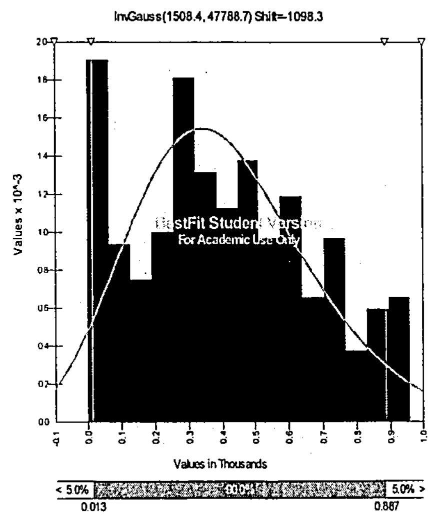


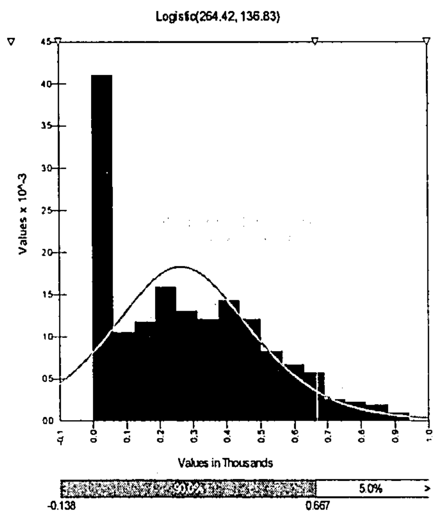
I7, D13, D23



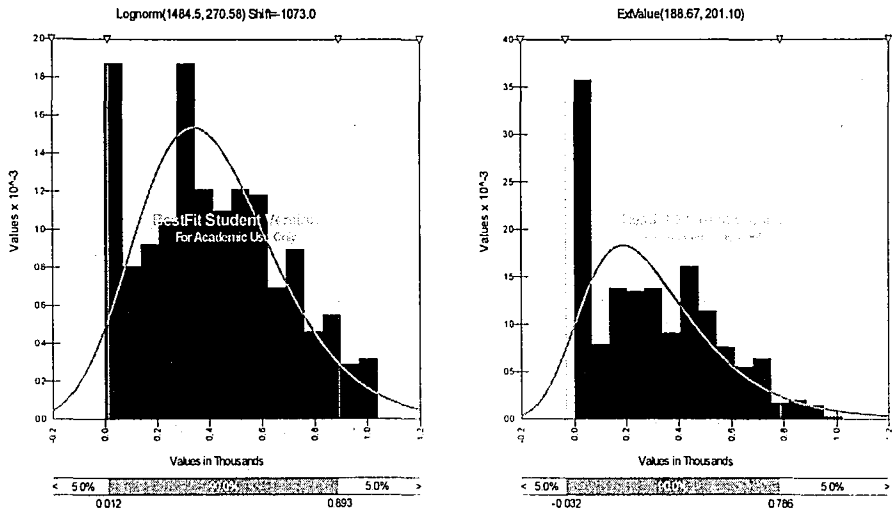


I8, D13, D23

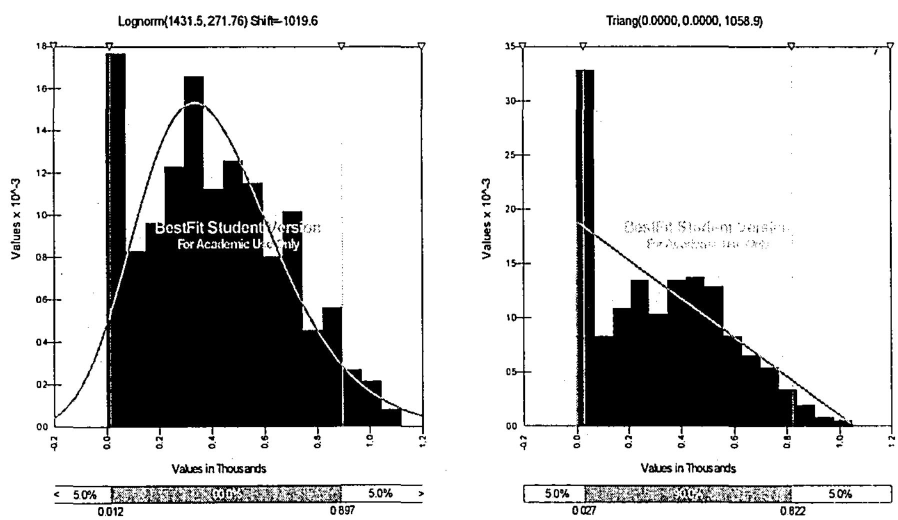




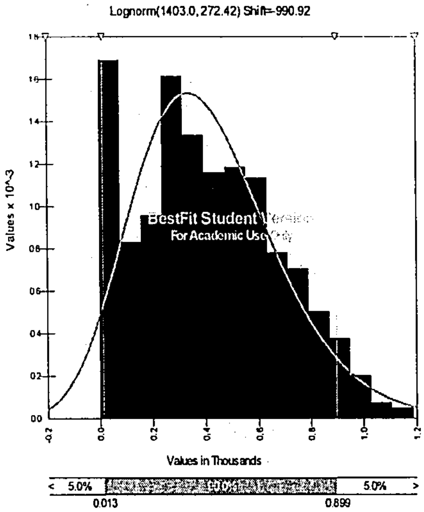
I9, D13, D23

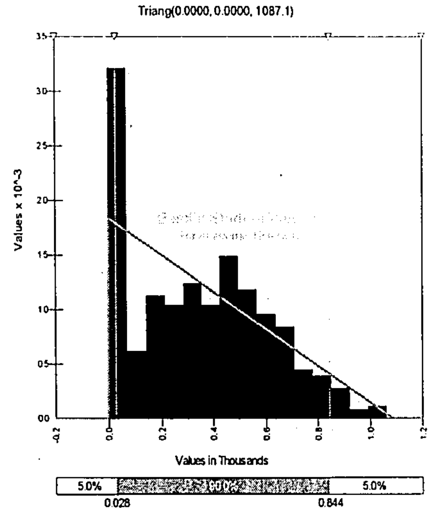


I10, D13, D23

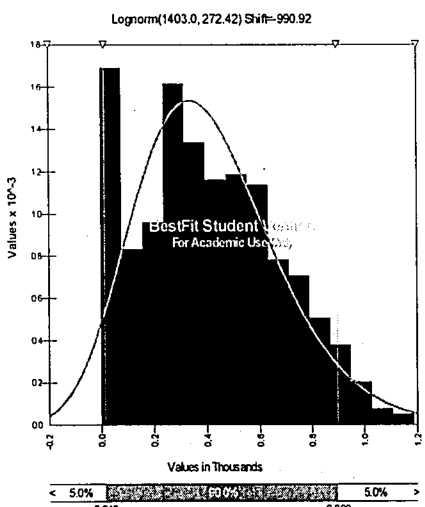


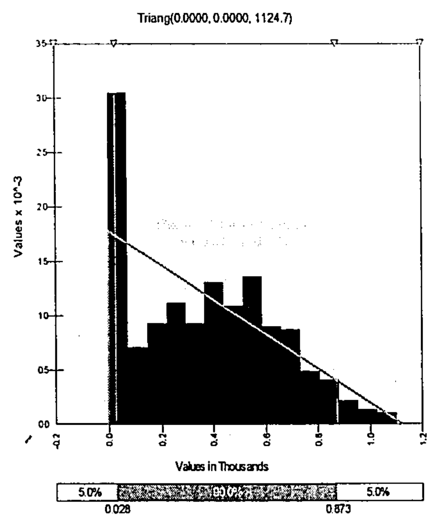
I11, D13, D23



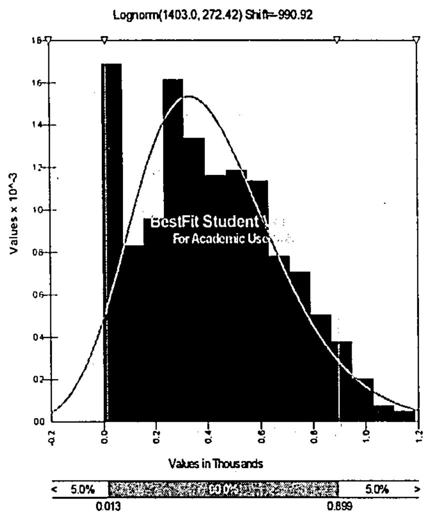


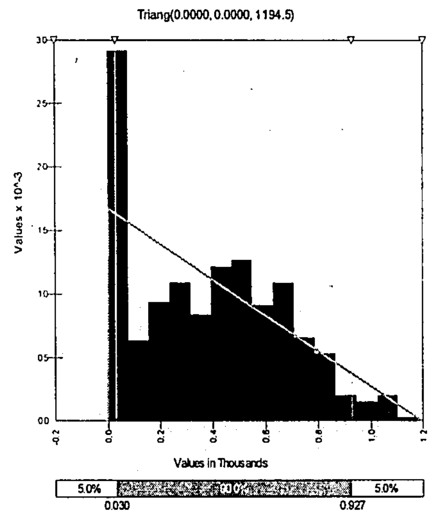
I12, D13, D23



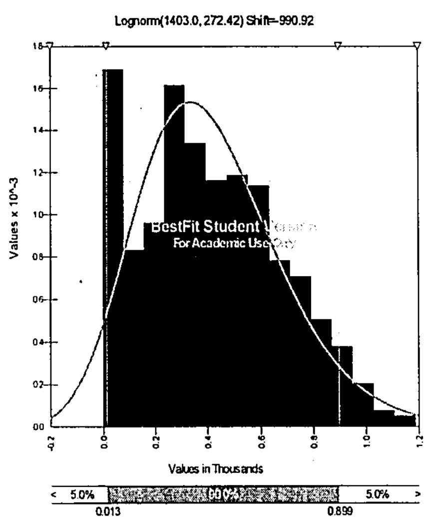


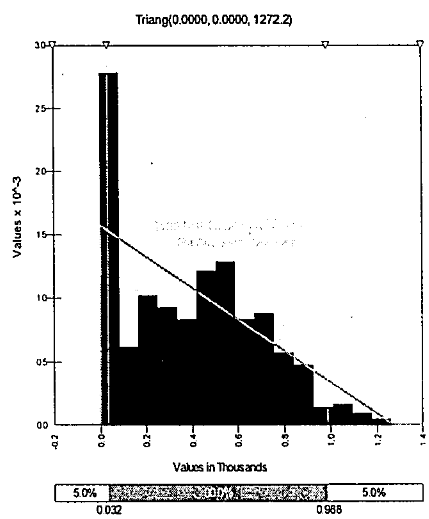
I13, D13, D23



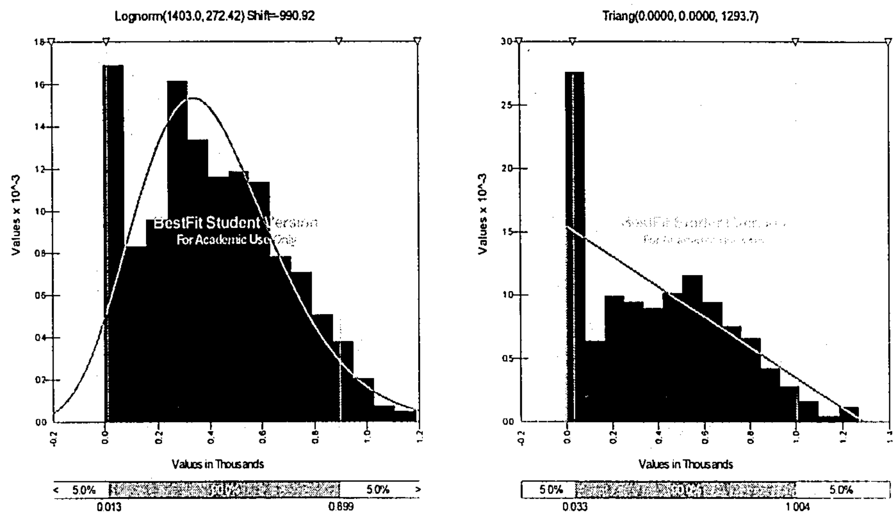


I14, D13, D23

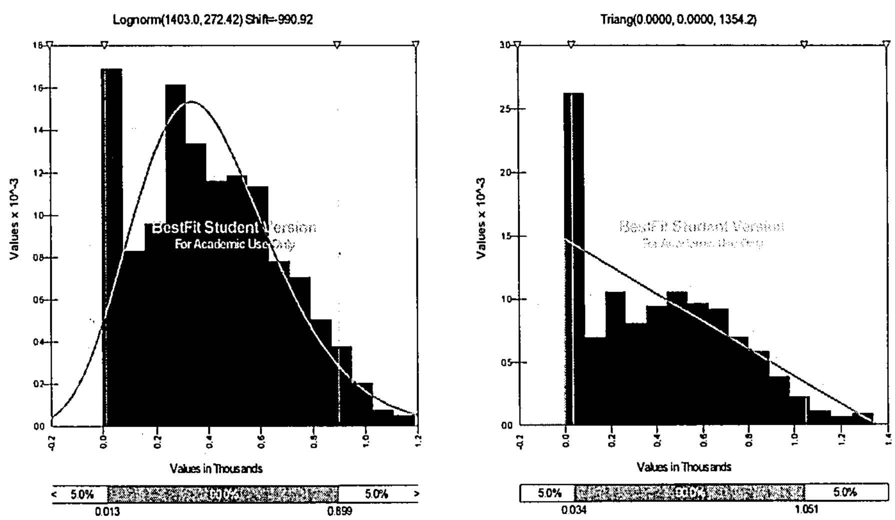




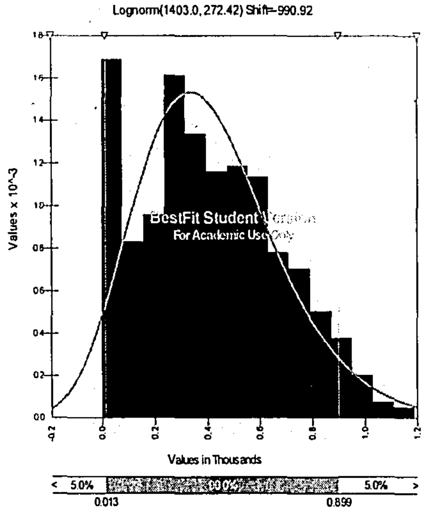
I15, D13, D23

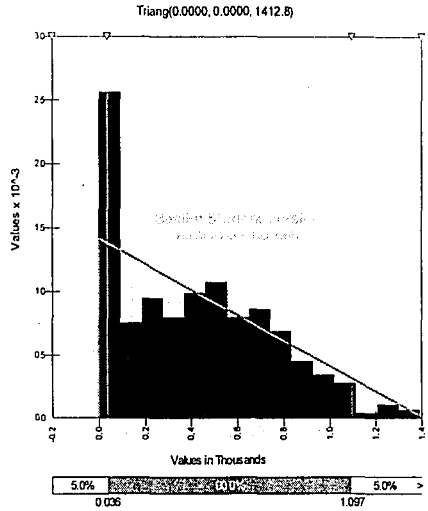


I16, D13, D23

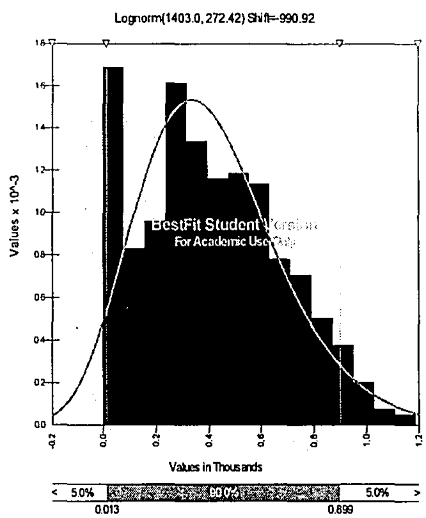


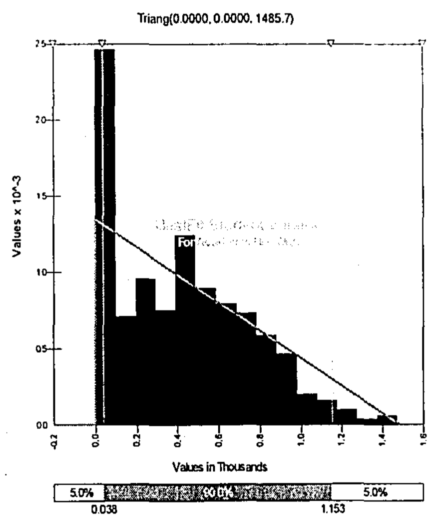
I17, D13, D23



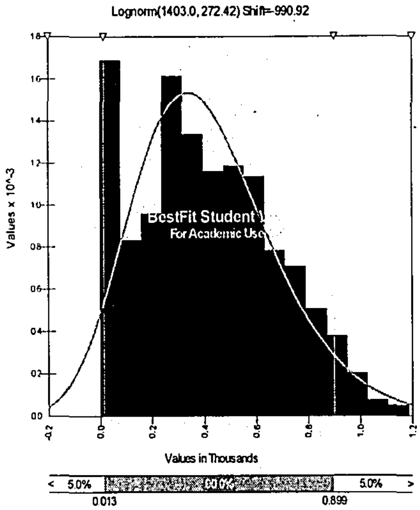


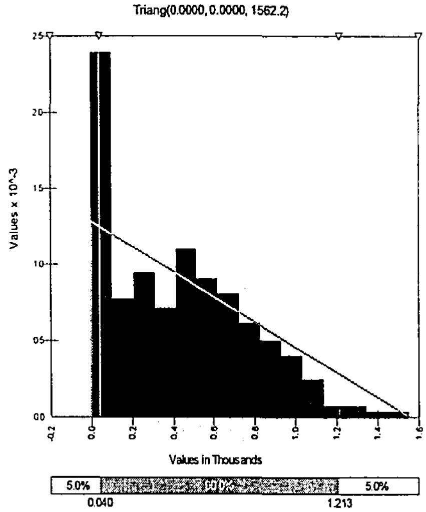
I18, D13, D23



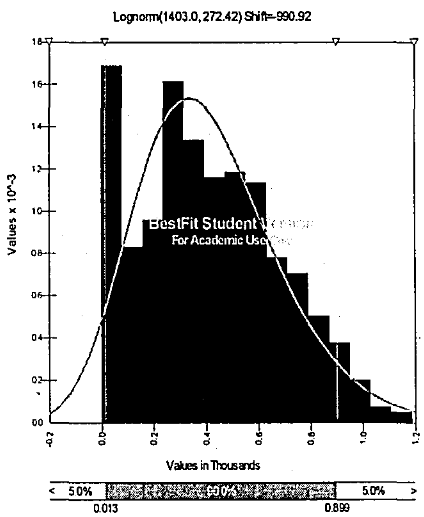


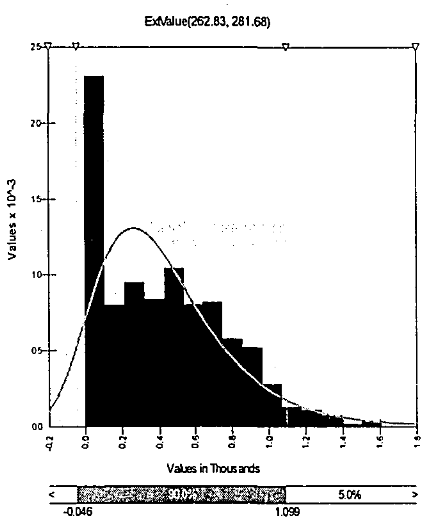
I19, D13, D23



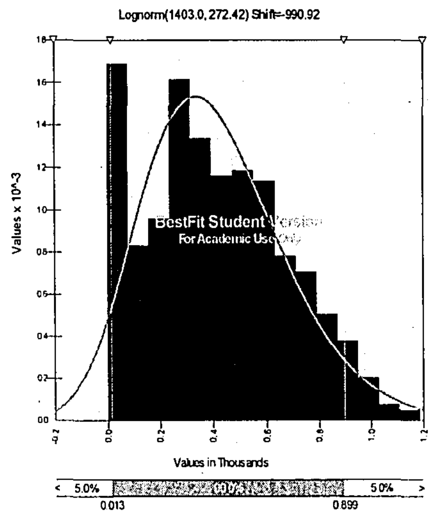


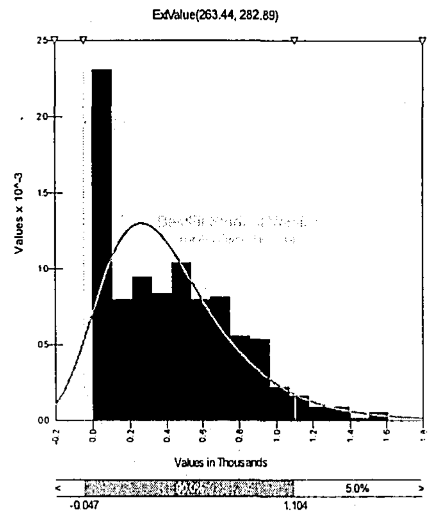
I20, D13, D23



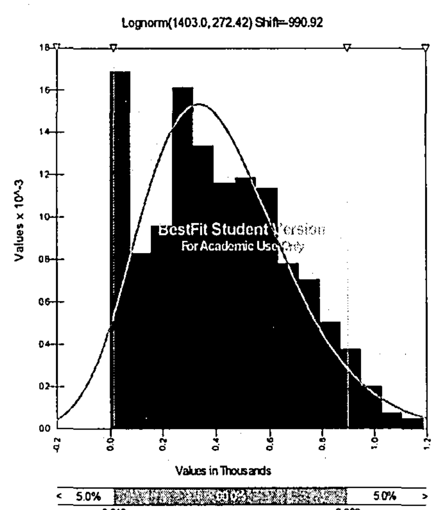


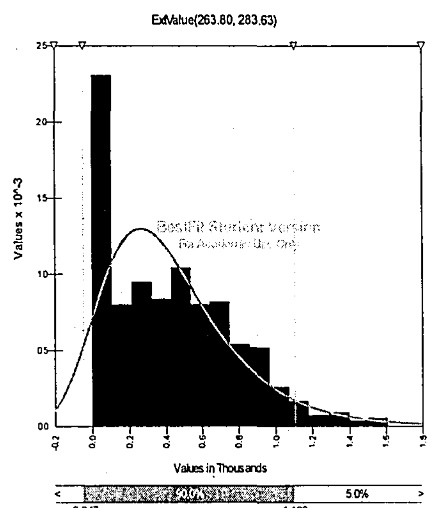
I21, D13, D23



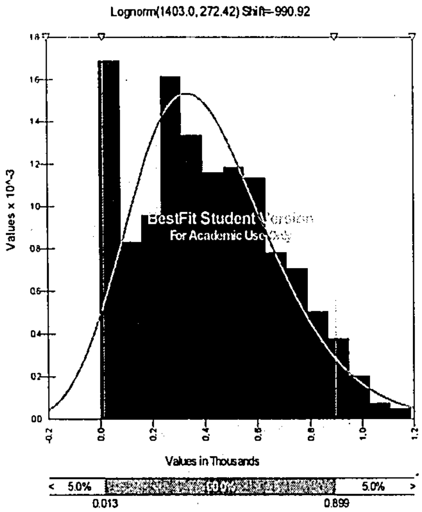


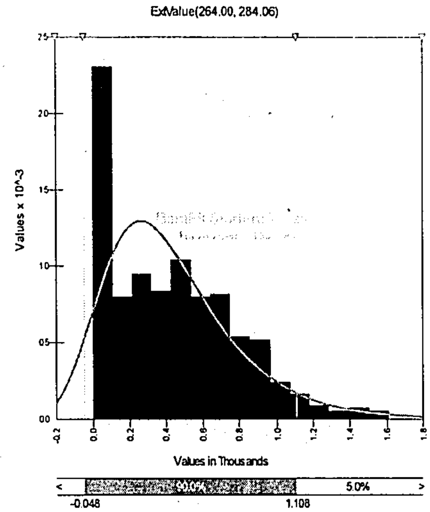
I22, D13, D23



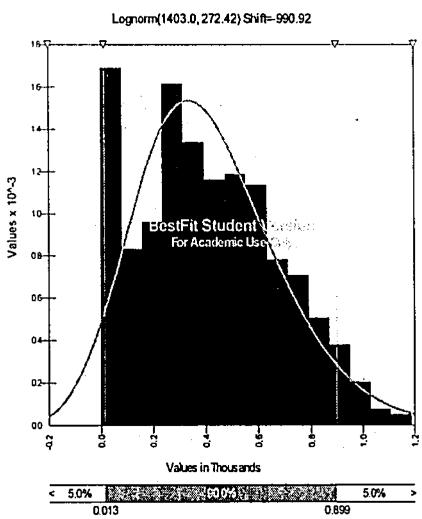


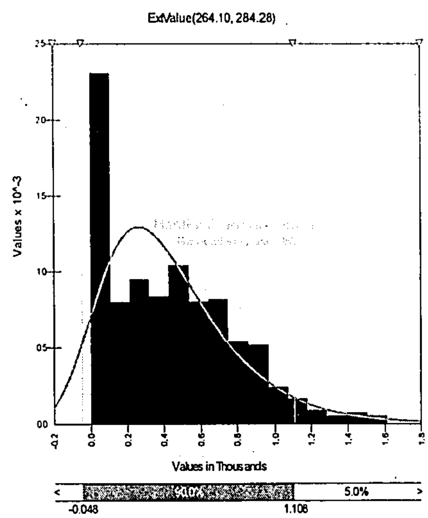
I23, D13, D23



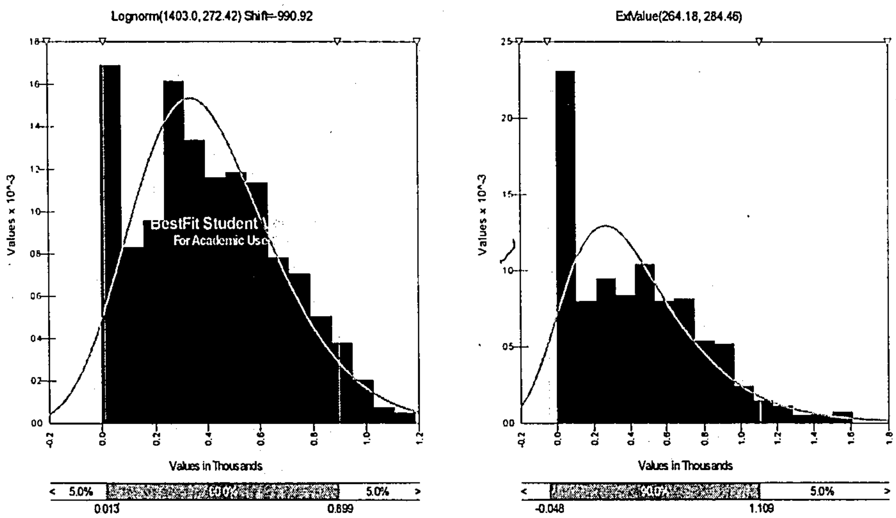


I24, D13, D23

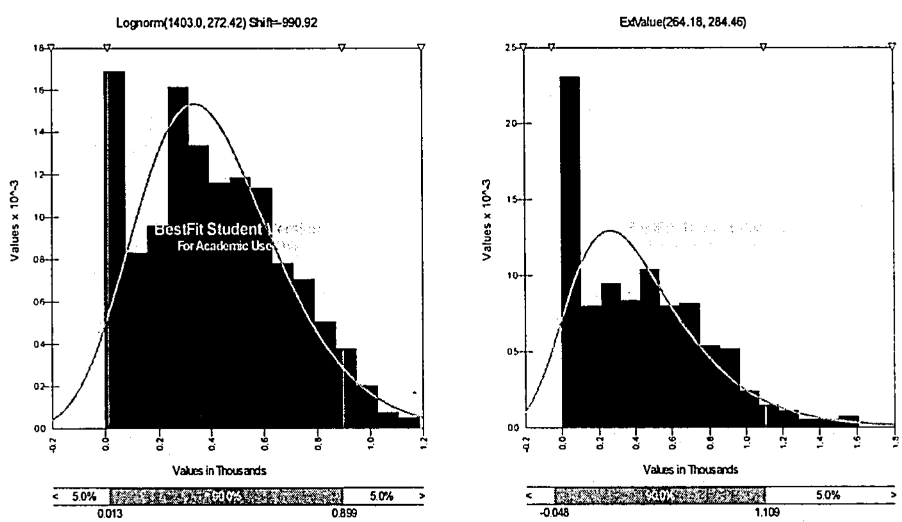




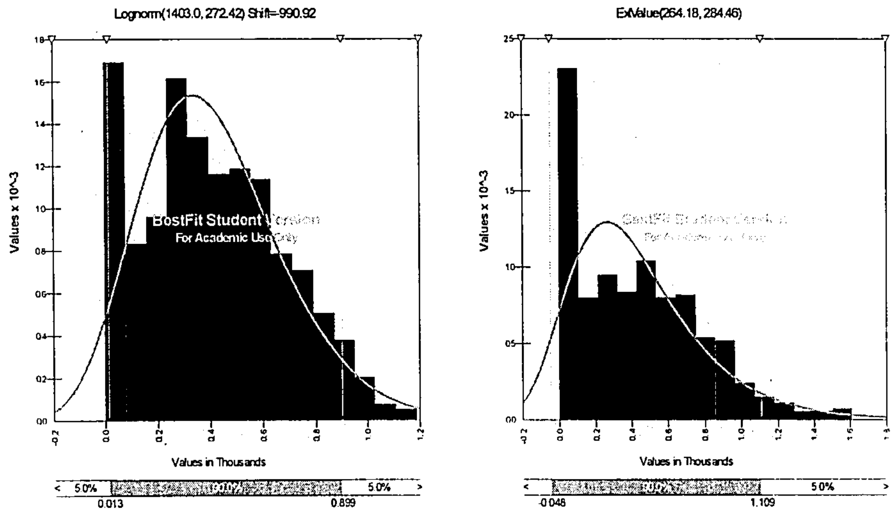
I25, D13, D23



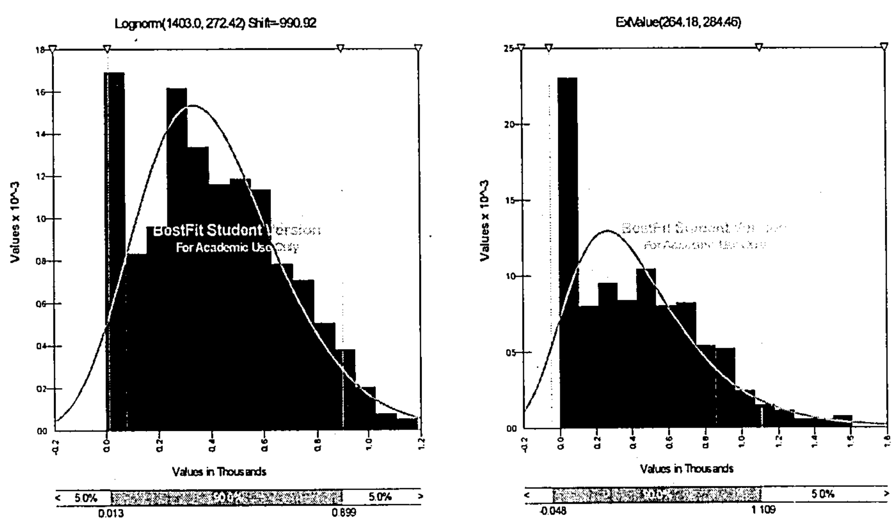
I26, D13, D23



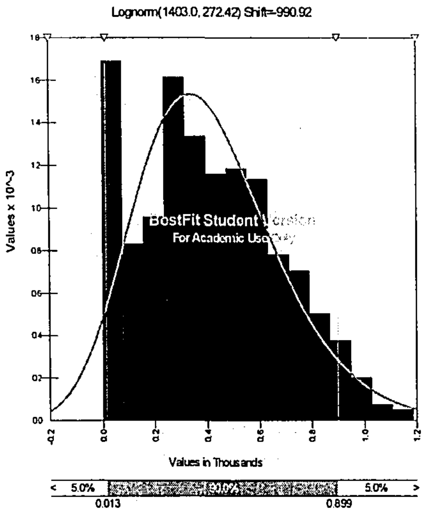
I27, D13, D23

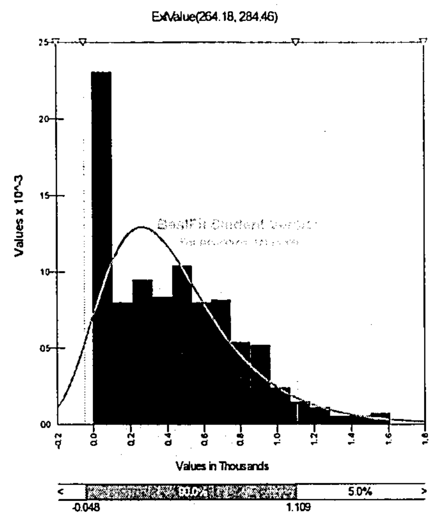


I28, D13, D23

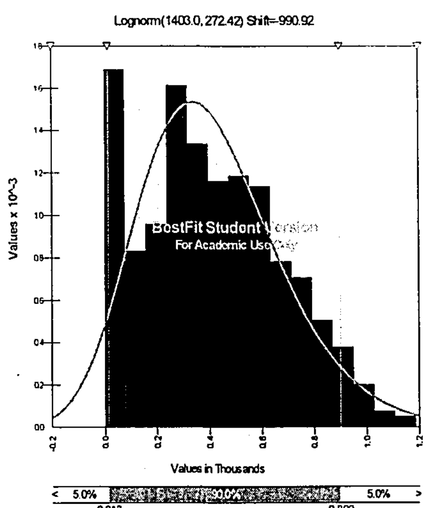


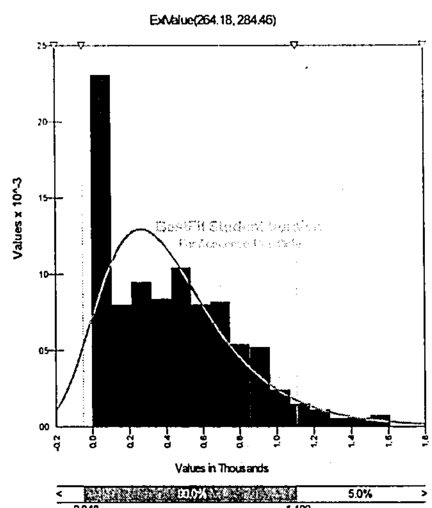
I29, D13, D23





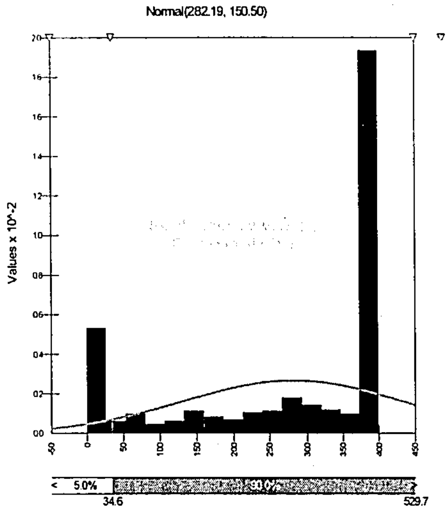
I30, D13, D23

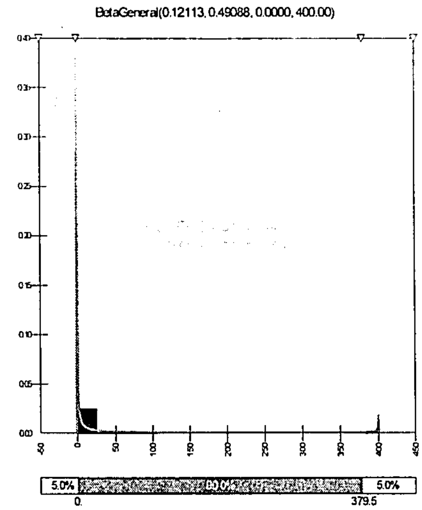




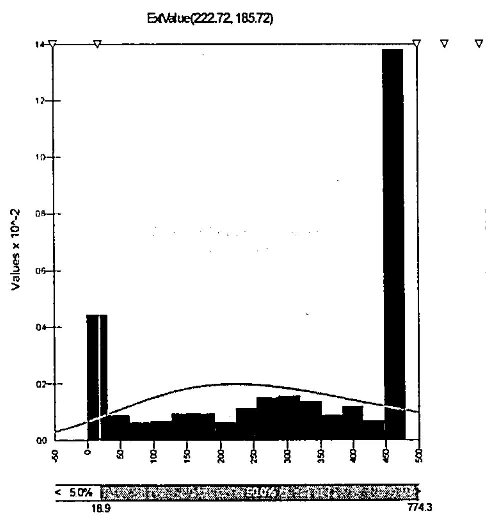
Uncertainty level Standard Deviation = 30% Mean

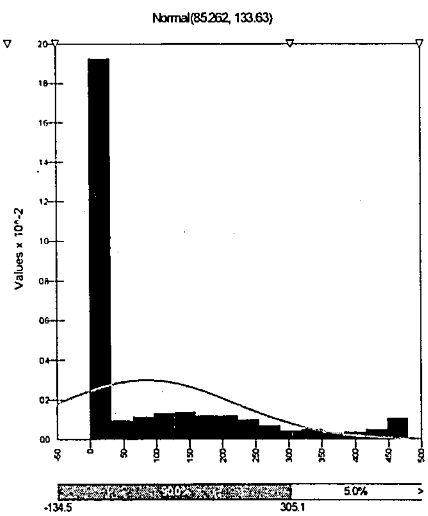
I1, D13, D23



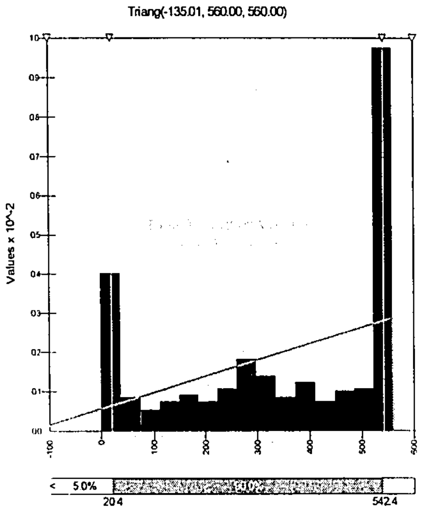


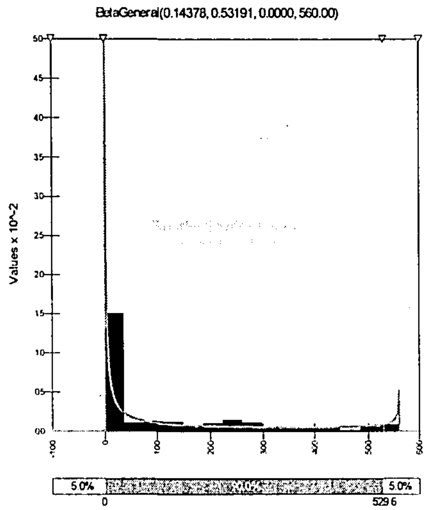
I2, D13, D23



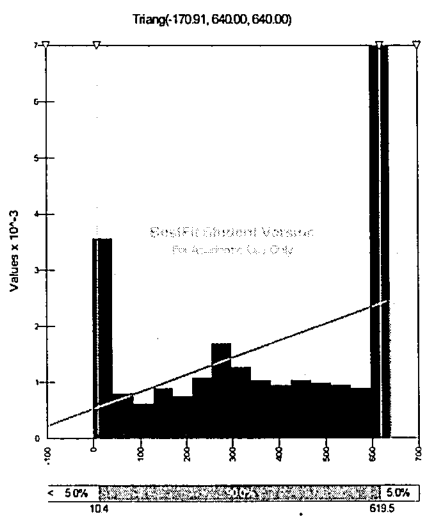


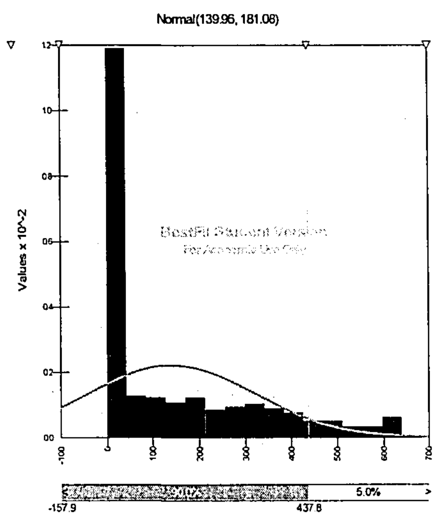
I3, D13, D23



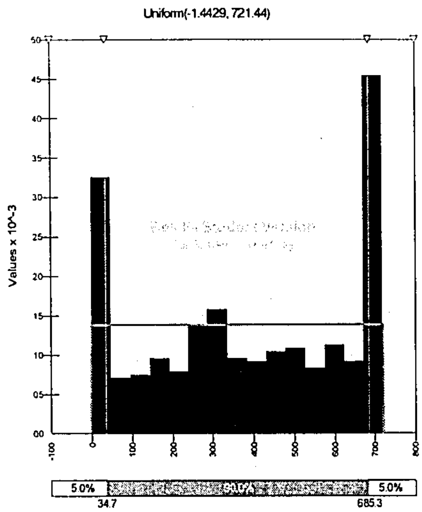


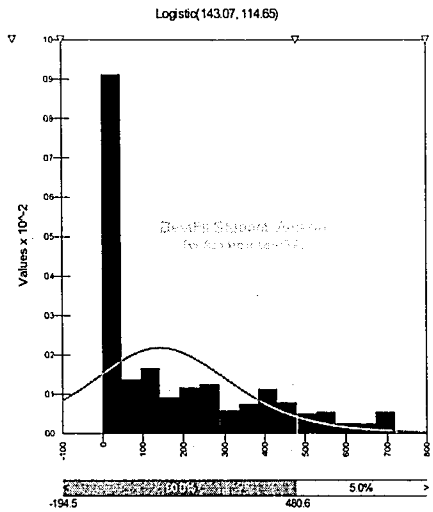
I4, D13, D23



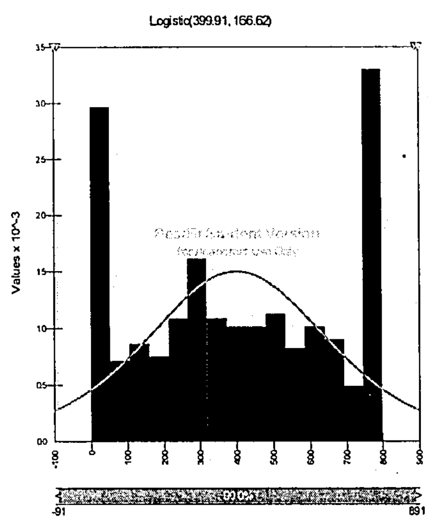


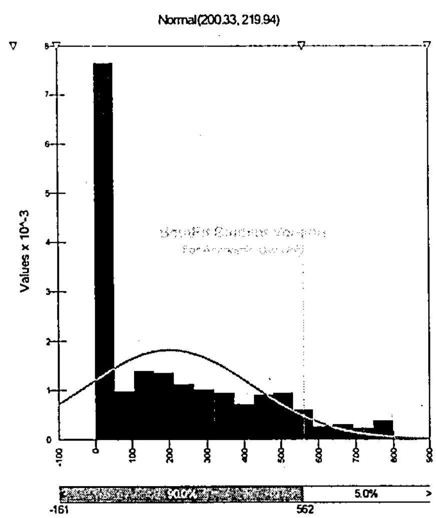
I5, D13, D23



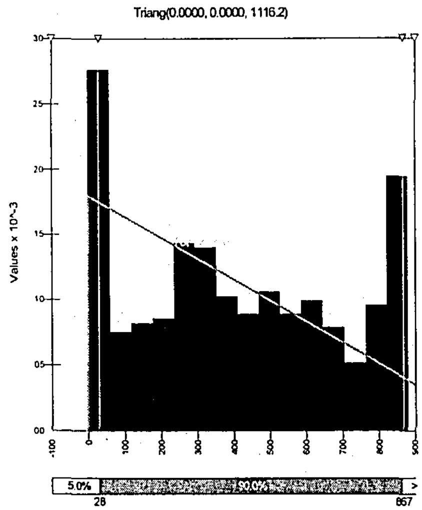


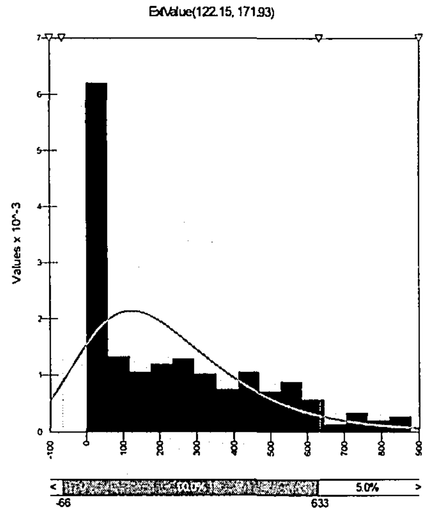
I6, D13, D23



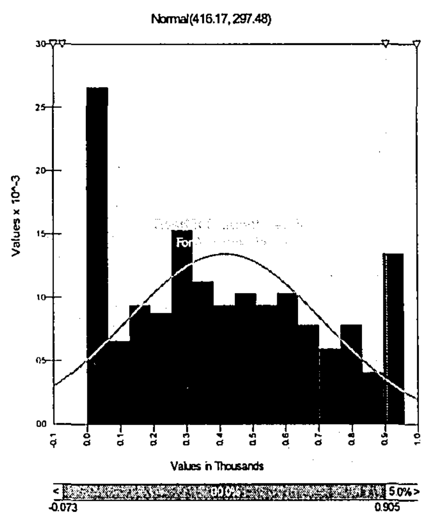


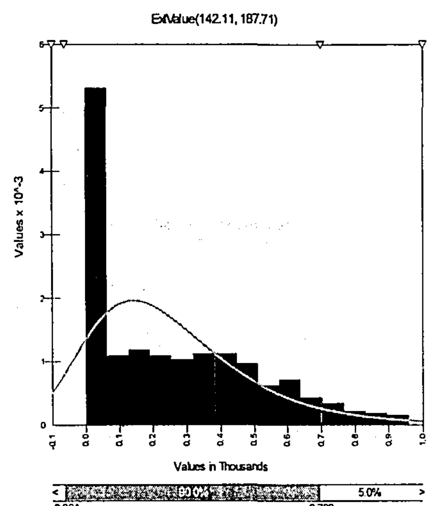
I7, D13, D23



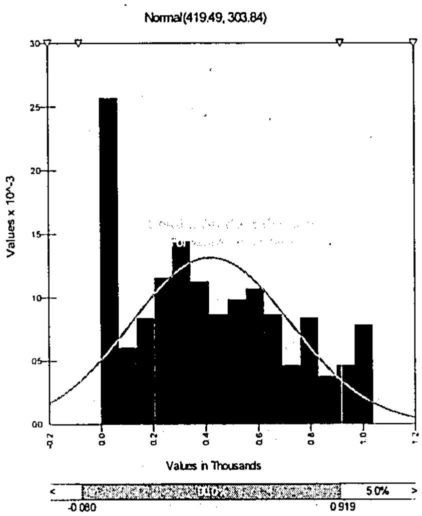


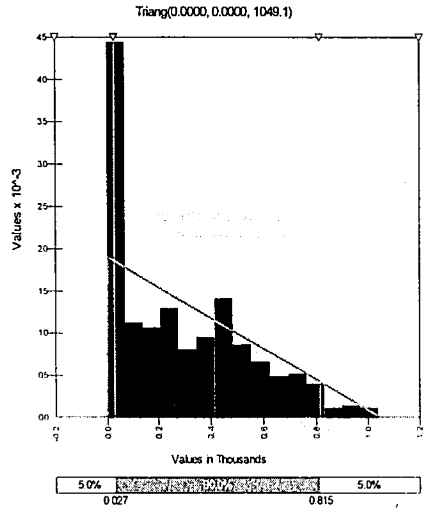
I8, D13, D23



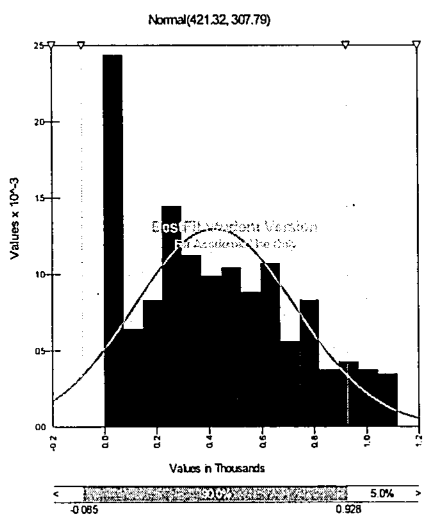


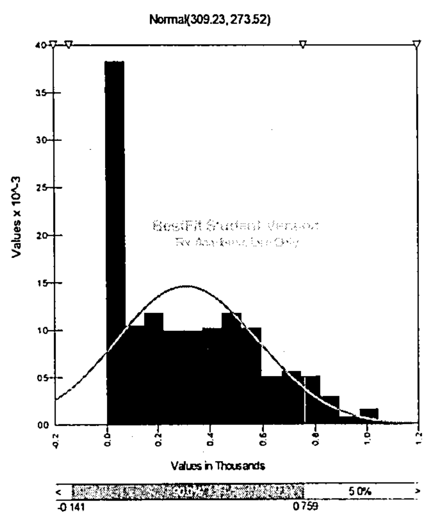
I9, D13, D23



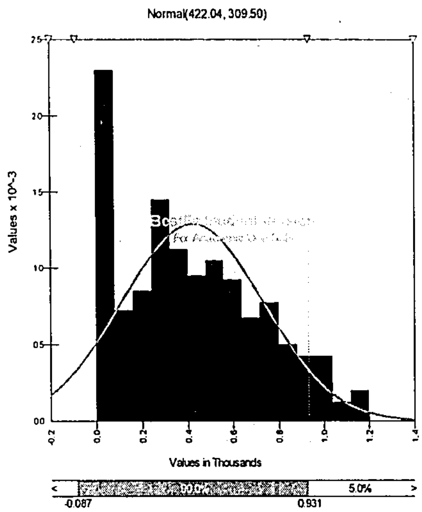


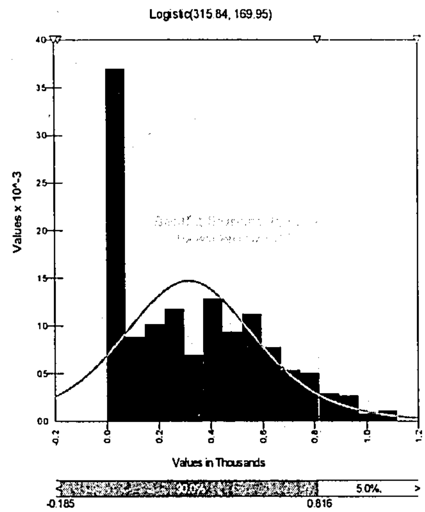
I10, D13, D23



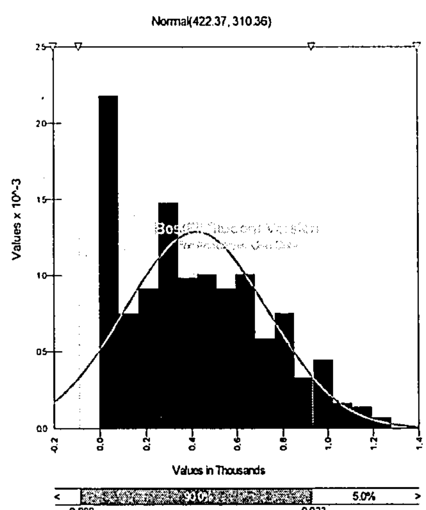


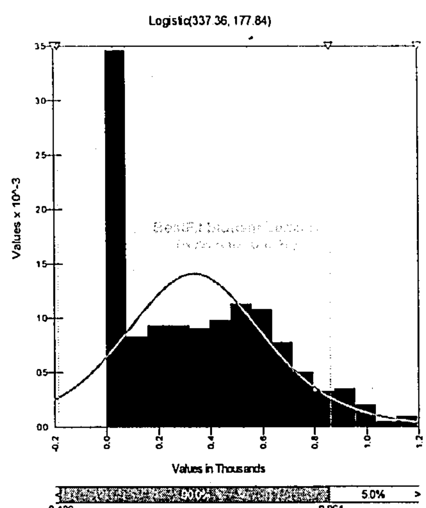
I11, D13, D23



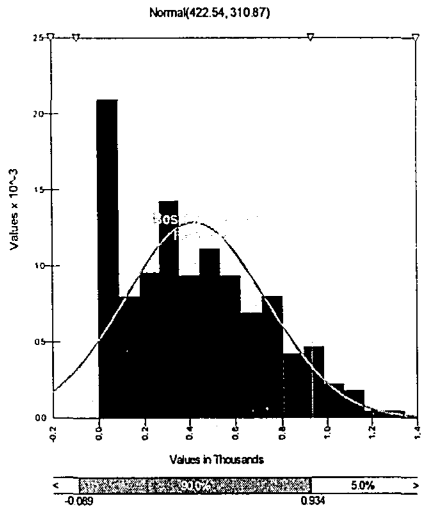


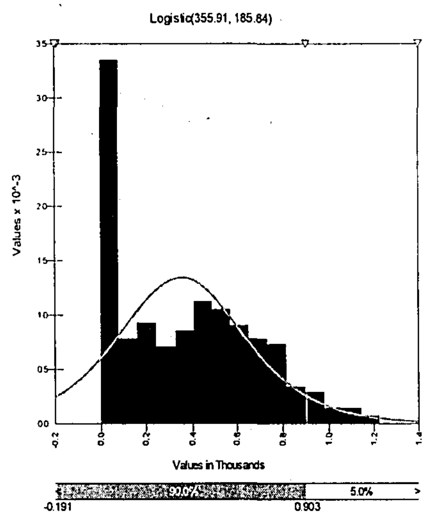
I12, D13, D23



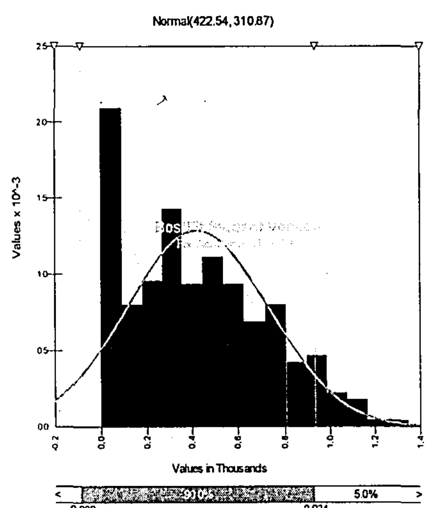


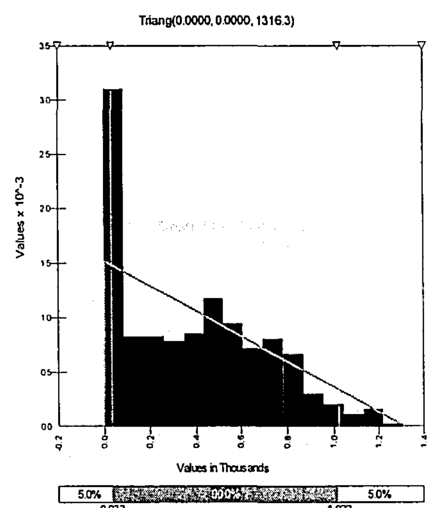
I13, D13, D23



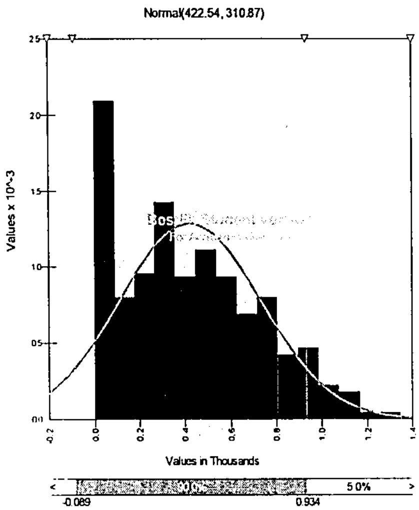


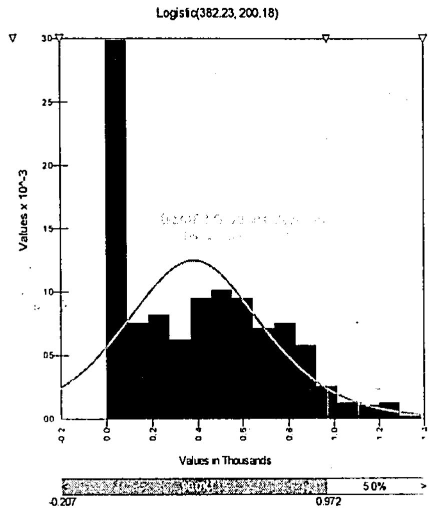
I14, D13, D23



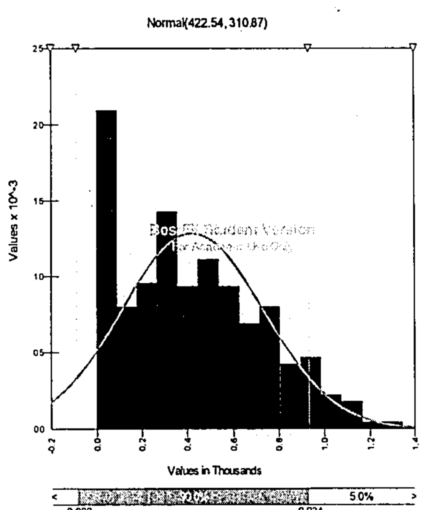


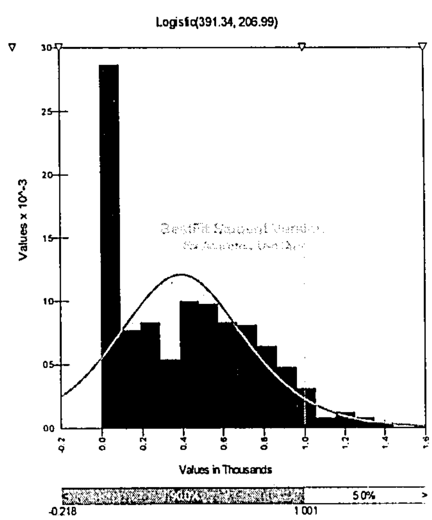
I15, D13, D23



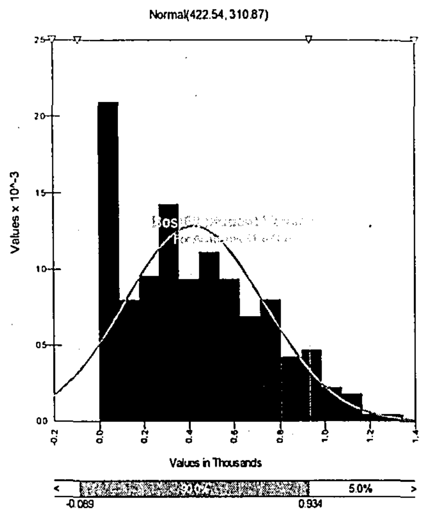


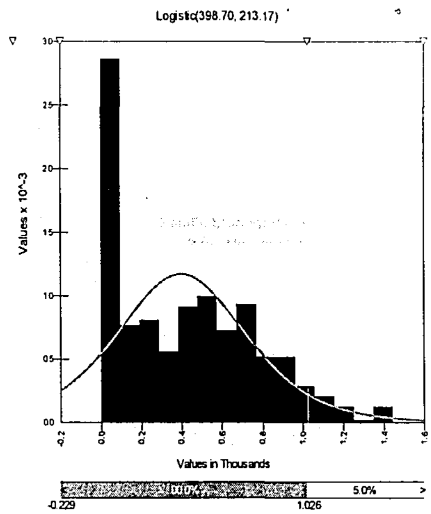
I16, D13, D23



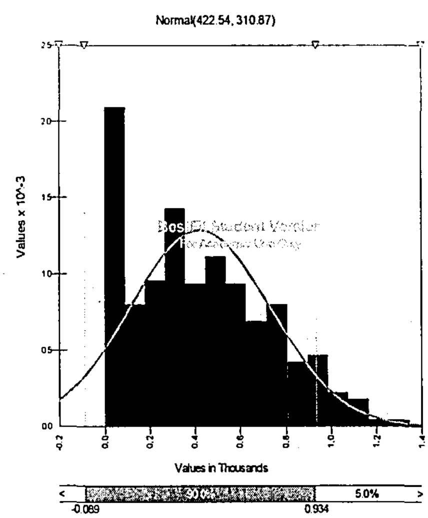


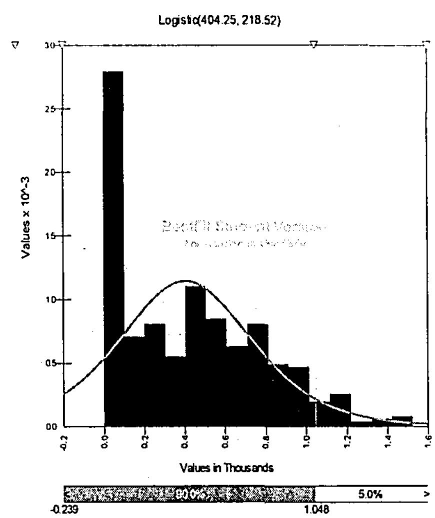
I17, D13, D23



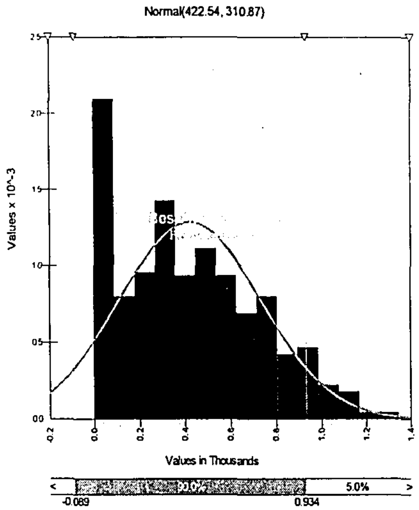


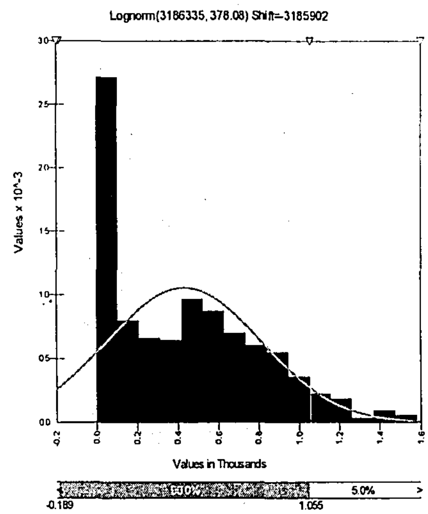
I18, D13, D23



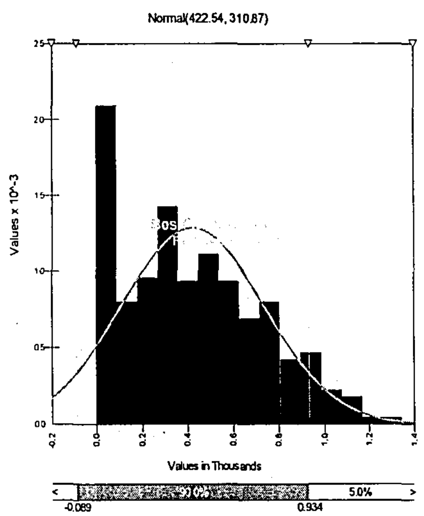


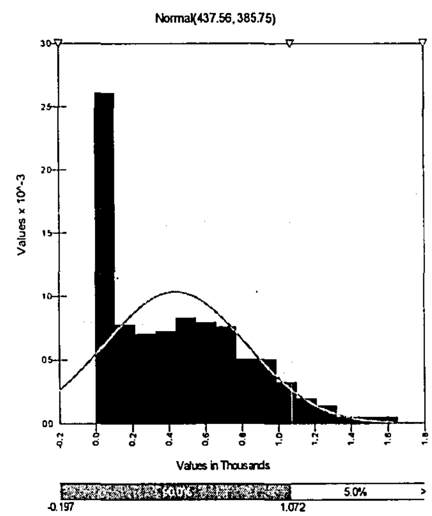
I19, D13, D23



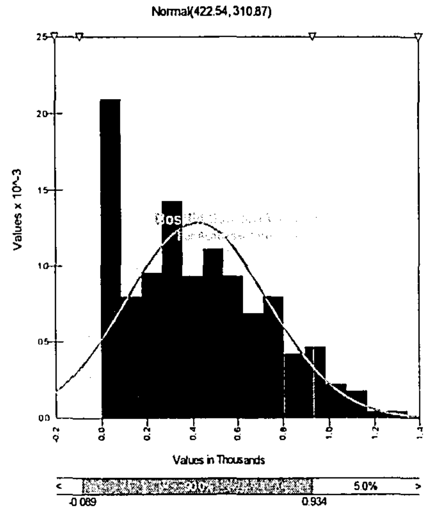


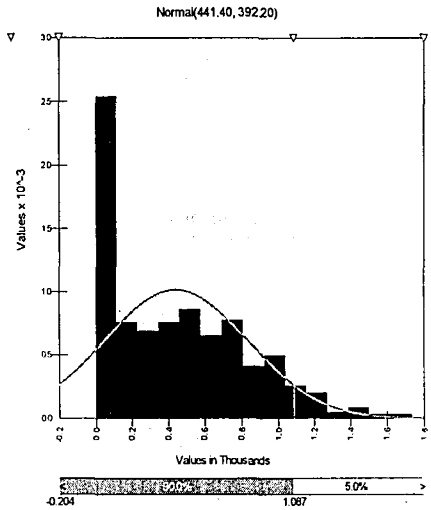
I20, D13, D23



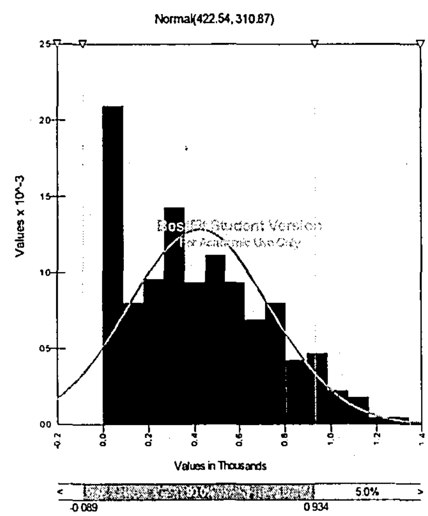


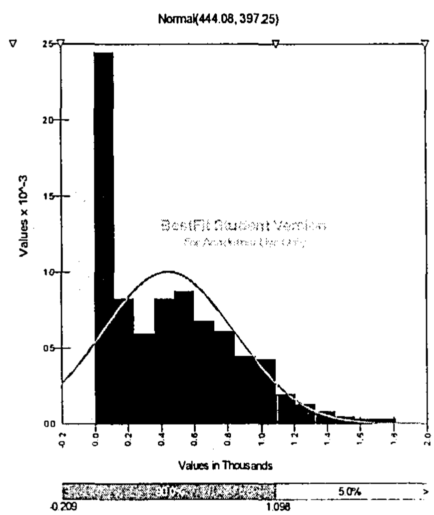
I21, D13, D23



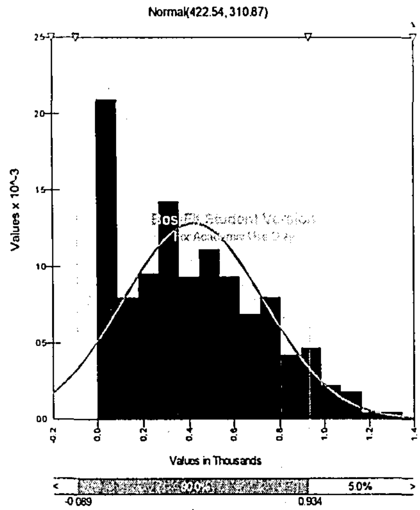


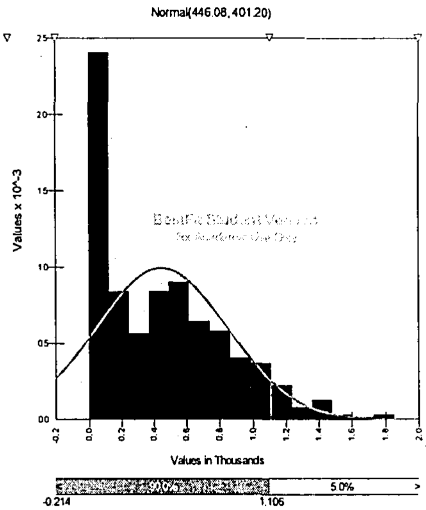
I22, D13, D23



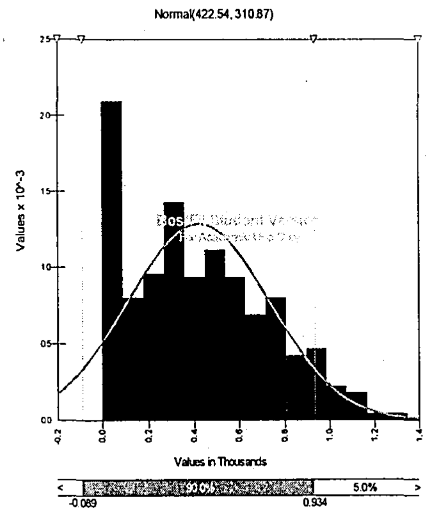


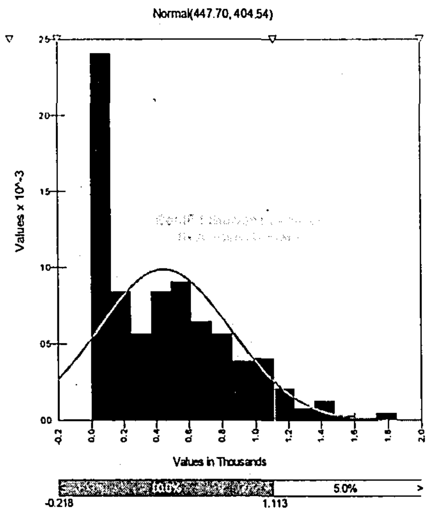
I23, D13, D23



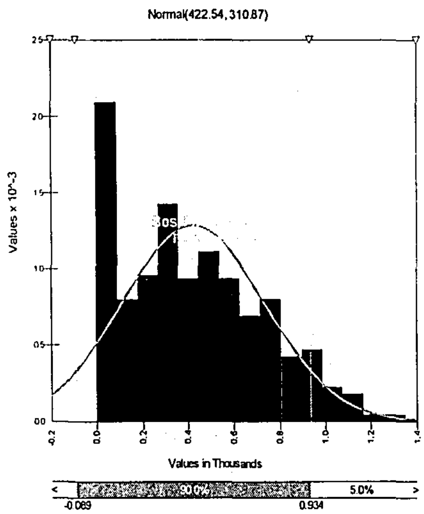


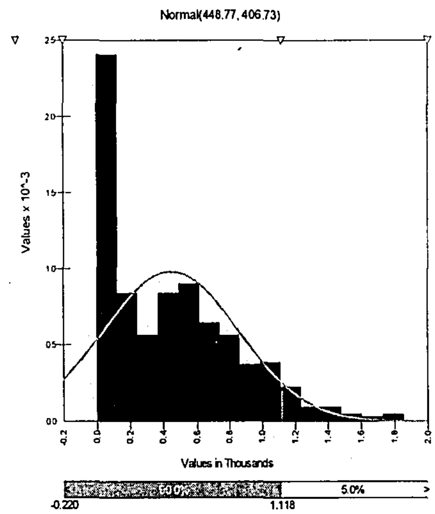
I24, D13, D23



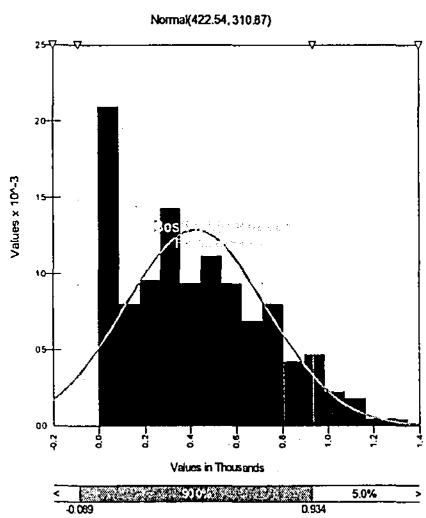


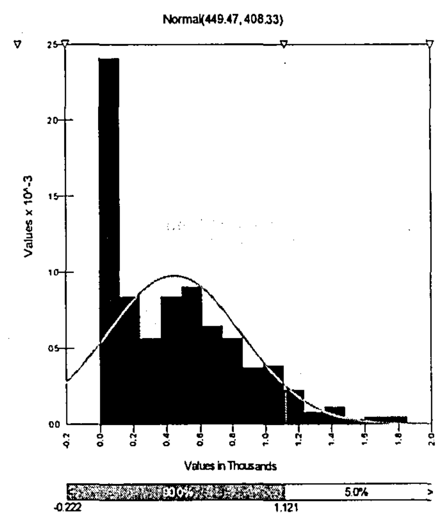
I25, D13, D23



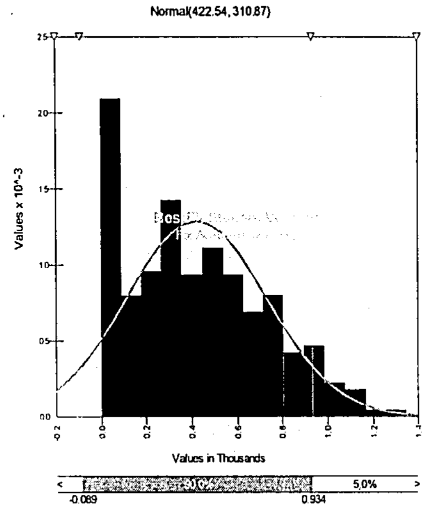


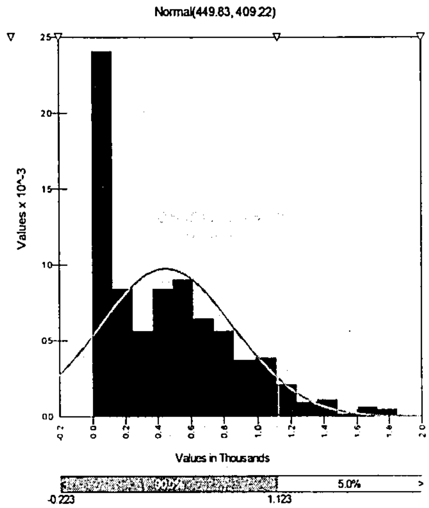
I26, D13, D23



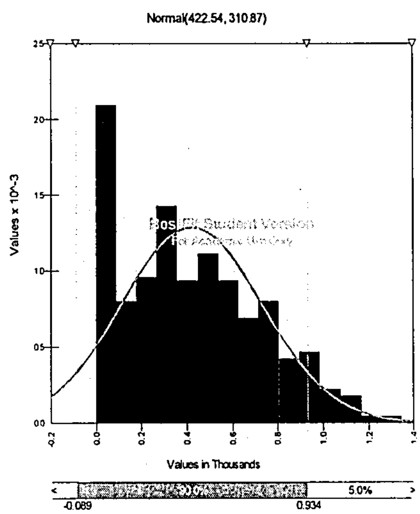


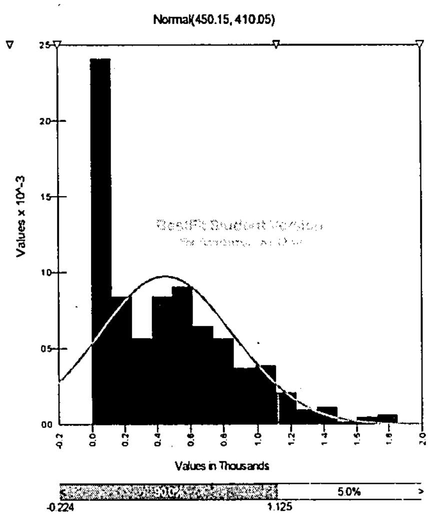
I27, D13, D23



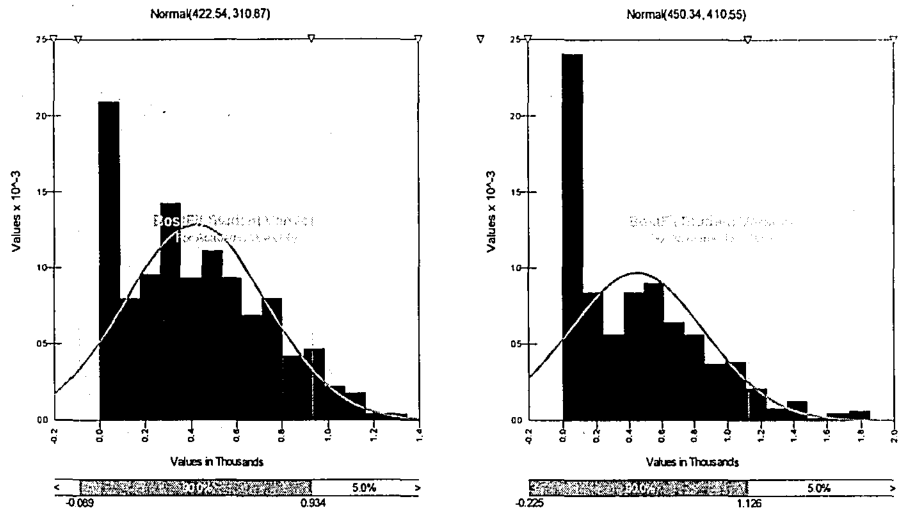


I28, D13, D23

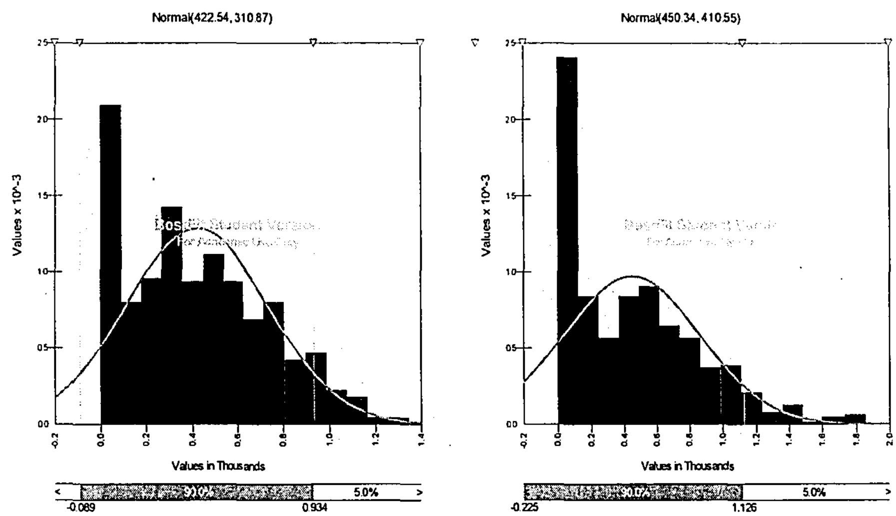




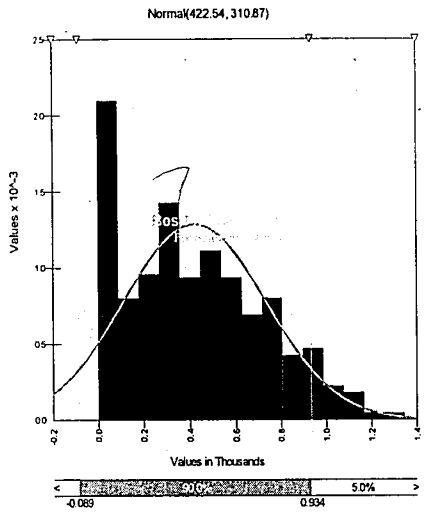
I29, D13, D23

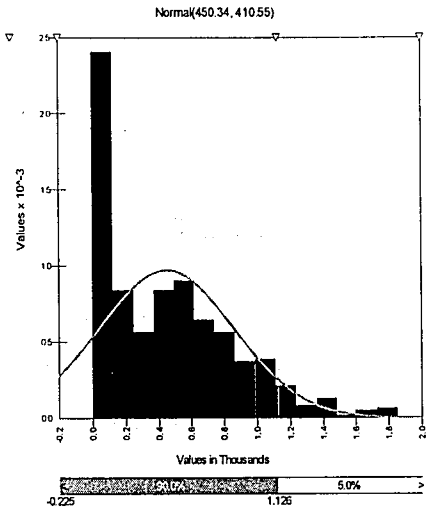


I30, D13, D23

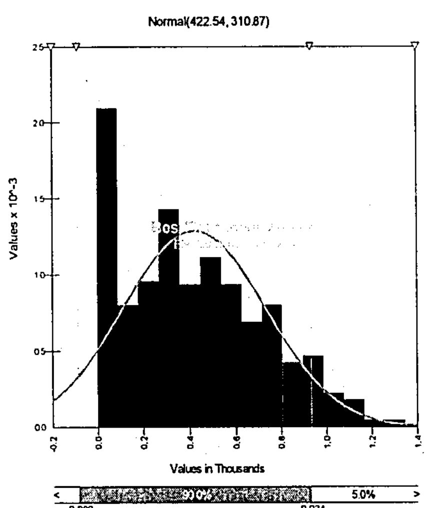


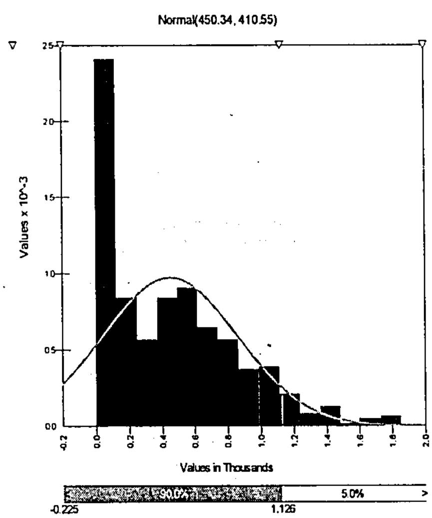
I31, D13, D23



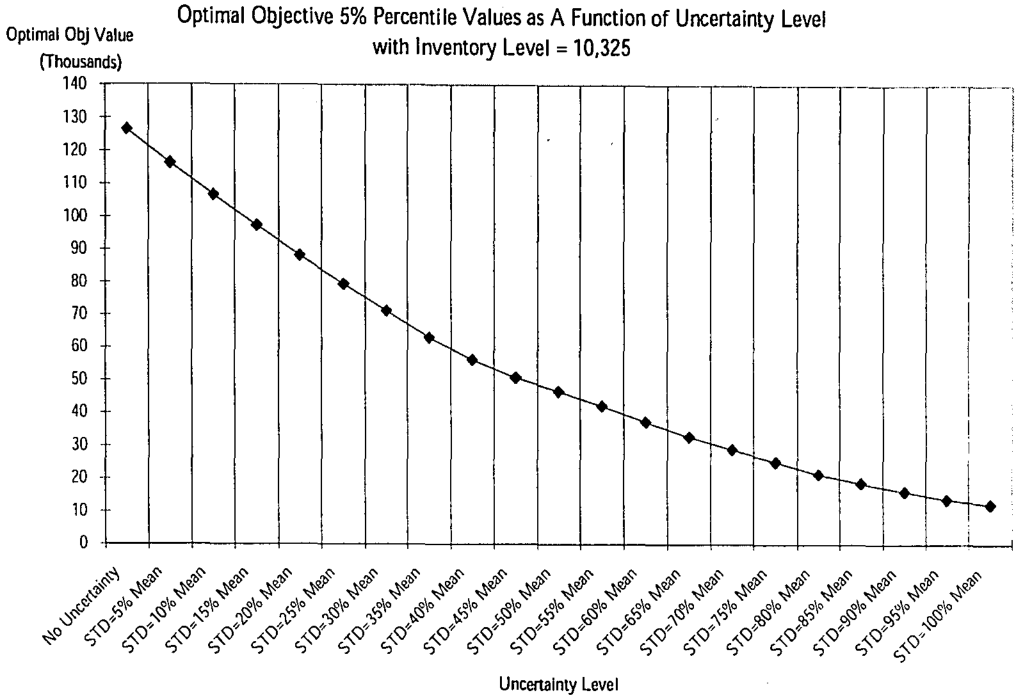


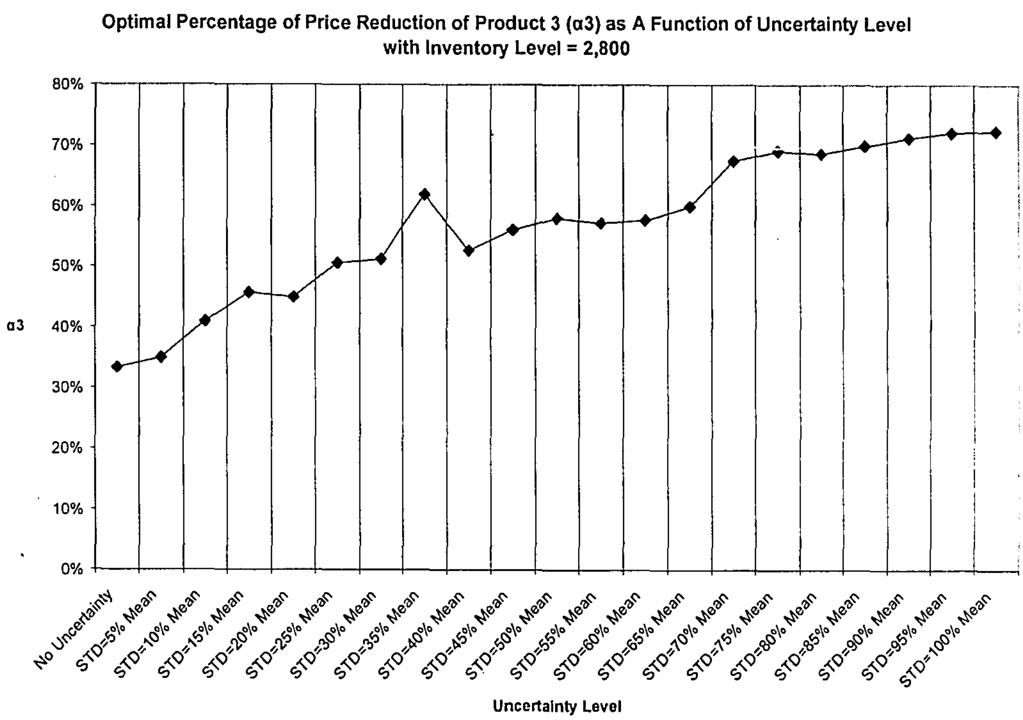
I32, D13, D23



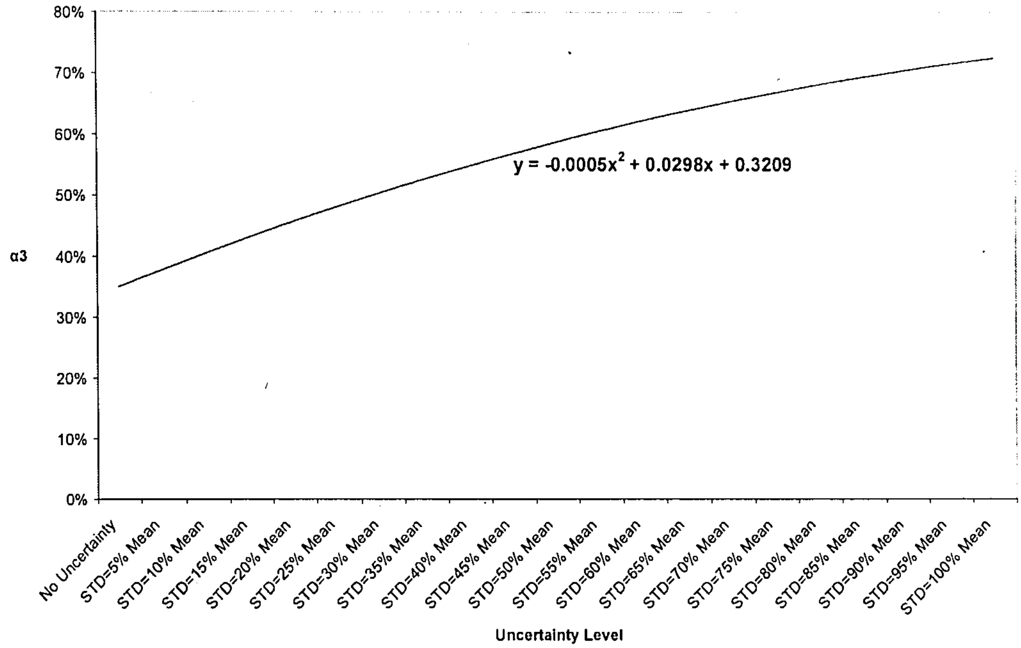


We also observe that under certain inventory level, under which total inventory = 10,325 and inventory of product 3 = 2,800, the optimal objective 5% percentiles get smaller while the price reduction percentage of product 3 gets larger when the uncertainty level becomes higher. The results observed are as the following graphs.


Optimal Objective 5% Percentile Values as A Function of Uncertainty Level
with Inventory Level = 10,325
Optimal Obj Value
(Thousands)
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
No Uncertainty
STD=5% Mean
STD=10% Mean
STD=15% Mean
STD=20% Mean
STD=25% Mean
STD=30% Mean
STD=35% Mean
STD=40% Mean
STD=45% Mean
STD=50% Mean
STD=55% Mean
STD=60% Mean
STD=65% Mean
STD=70% Mean
STD=75% Mean
STD=80% Mean
STD=85% Mean
STD=90% Mean
STD=95% Mean
STD=100% Mean
Uncertainty Level


Optimal Percentage of Price Reduction of Product 3 (α3) as A Function of Uncertainty Level
with Inventory Level = 2,800
α3
80%
70%
60%
50%
40%
30%
20%
10%
0%
No Uncertainty
STD=5% Mean
STD=10% Mean
STD=15% Mean
STD=20% Mean
STD=25% Mean
STD=30% Mean
STD=35% Mean
STD=40% Mean
STD=45% Mean
STD=50% Mean
STD=55% Mean
STD=60% Mean
STD=65% Mean
STD=70% Mean
STD=75% Mean
STD=80% Mean
STD=85% Mean
STD=90% Mean
STD=95% Mean
STD=100% Mean
Uncertainty Level

Optimal Percentage of Price Reduction of Product 3 (α3) as A Function of Uncertainty Level
with Inventory Level = 2,800



Chapter 4 Conclusions

We have proposed two approaches to address uncertainty level and to find the optimal solutions for the percentage of price reduction of perishable products and the optimal VaR of the profits we can make with the decision made based on the optimal price reduction percentage. Both approaches have shown that the higher the uncertainty level, the higher the optimal price reduction percentage and the lower the profit inferred from the optimal solutions. However, the first model finds the optimal solutions for the worst-case values of the uncertainty set, while the second finds the optimal solutions based on heuristic simulation of all the possible values within the uncertainty set. Also, since the second one is pursuit by genetic algorithms, which does not guarantee the global optimum, we recommend that we verify the solutions by exploiting the true distribution of the uncertainty set to make sure all the constraints are satisfied, which might increase the inefficiency of the model. Moreover, we suggest that we do further research on the parametric study to enhance genetic algorithm's roulette wheel selection performance so that we can get closer to the global optimum. These parameters include the population size, the mutation and the crossover rates. Further research directions also include capturing quadratic programming and exploiting convex uncertainty set to give more insight of the robustness of the solutions.

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Biography

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