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Mitigating Portfolio Downside Risk Using VIX-Based Products

by

Michael I. Arak

A Thesis

Presented to the Graduate and Research Committee

of Lehigh University

in Candidacy for the Degree of

Master of Science

in

Management Science and Engineering

Lehigh University

May 2013

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Michael I. Arak

Certificate of Approval

This thesis is accepted and approved in partial fulfillment of the requirements for the Master of Science.

Date

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Abstract

The purpose of this thesis is to investigate the benefits of allocating part of a portfolio to exchange-traded products (ETPs) based on the Chicago Board Options Exchange's Volatility Index (VIX). Due to the highly negative correlation of the VIX to the S&P 500, many professionals and academics have researched the VIX and VIX related products over the last ten years. Dash and Moran (2005) set up an original framework of incorporating the VIX spot into hedge fund portfolios. They looked at three different portfolios, one having no allocation to the VIX spot, one having a constant 5% allocation to the VIX spot, and one having a 0-10% tactical allocation to the VIX spot depending on its movement in the previous month. This methodology has been replicated for VIX futures and VIX options, but not yet for VIX ETPs.

This thesis extends the original Dash and Moran framework by allocating two different VIX exchange traded products, the iPath S&P 500 VIX Short-Term Futures ETN (ticker: VXX) and the iPath S&P 500 VIX Mid-Term Futures ETN (ticker: VXZ) to an equity portfolio represented by the SPDR S&P 500 ETF (ticker: SPY) and a bond portfolio represented by the iShares Core Total U.S. Bond Market ETF (ticker: AGG) respectively. Moreover, I expand upon the original model by implementing optimization and two additional metrics, the reciprocal of the coefficient of variation (ICV), and the reciprocal of the downside coefficient of variation (DICV). Furthermore, I compare the results of the two VIX ETPs to that of another negatively correlated asset over the period, U.S. Treasury securities. For these securities, I chose the iShares 10-20 Year Treasury ETF (ticker: TLH) and the iShares 20+ Year Treasury Bond ETF (ticker: TLT).

Over the entire period from February 27, 2009 to March 1, 2013, investing in VIX ETPs would have been beneficial with respect to risk-adjusted returns for only certain sub periods and of the two VIX ETPs, VXZ was the more likely one to result in a benefit to risk-adjusted portfolio returns. However, a static allocation to either ETP over this period would have reduced risk-adjusted portfolio returns far more often than it would have increased them. In comparison to the two Treasury ETFs over this period, static allocations to VXZ outperformed TLT, but grossly underperformed TLT.

In conclusion, this thesis finds that, in general, VIX ETPs are beneficial only during specific time periods and that a static allocation to a VIX exchange-traded product is more detrimental than beneficial. Therefore, extensions of this thesis should attempt to develop a tactical allocation scheme in order to take advantage of the negative correlations of the VIX exchange-traded products, without subjecting the portfolio to its relatively high probability of negative returns and large volatility.

Introduction

Rational investors always desire a higher expected return and a lower risk of not meeting that expected return. One of the most commonly used techniques to accomplish this feat is diversification. Assuming that past performance has some indication of future performance, holding assets that have low or negative[1] correlations with each other may enable a portfolio to either increase its expected return while keeping risk constant or decrease risk while keeping the expected return constant. Although this is a simple and fundamental aspect of modern portfolio theory, finding negatively correlated assets is typically challenging. For this reason, the Chicago Board Options Exchange's (CBOE) Volatility Index (VIX) has received a lot of attention from both professionals and academics. According to Dash and Moran (2005), the VIX tends to have a strong negative correlation with the S&P 500 Index. This begs the question: will investing in VIX products improve risk-adjusted portfolio returns? There have been several studies on incorporating VIX spot, VIX futures, and VIX options into a portfolio; however, this thesis answers the question for a relatively new VIX product, VIX Exchange-Traded Products (ETPs).

Relevance

The VIX, with its negative correlation to equities, has been an increasingly relevant topic in asset management; however, the literature on the effect on the risk-adjusted return of incorporating VIX ETPs into a portfolio is relatively sparse. Moreover,

[1] The range of the correlation between two assets is from negative one to positive one. The closer the correlation is to negative one, the more potential diversification exists.

ETPs in general are one of the most tax-efficient investment vehicles, especially compared to mutual funds. According to ETF Database (2012):

> "*One of the biggest advantages of ETFs is their tax-efficient structure; ETPs generally maintain lower capital gains distributions than mutual funds thanks to the nuances of the underlying creation/redemption mechanism. Unlike mutual funds, when redemptions occur in ETFs they are done so 'in-kind' and aren't considered sales. As such, these transactions <u>don't trigger a taxable event</u>. It's important to note that the tax-efficient features of ETFs don't allow investors to skip out on their obligations; gains on positions in exchange-traded products will ultimately be taxed at the applicable rate. However, the ETF wrapper does give investors more control over their tax situations, since most ETFs avoid incurring capital gains during the normal course of their operations*."

This feature allows investors to choose when they would like to realize their capital gains and thus gives these investors an opportunity to minimize their capital gains taxes. On the other hand, both VIX futures and VIX options must be continually rebalanced, which forces investors to continually realize capital gains (or losses) at their income tax rate instead of 20%[2]. Moreover, the continual rebalancing of futures or option incurs a higher net transaction cost than a buy and hold ETP investment. Therefore, if investing in VIX ETPs does improve the risk-adjusted return of a portfolio, it would be beneficial from a transaction cost and tax perspective as well.

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[2] The long-term capital gains tax rate depends on the level of income of that particular investor. For single taxpayers with income over $400,000 or married ones with joint income over $450,000, the long-term capital gains tax rate is 20%. Refer to the U.S. tax code for more information.

Research Contributions

Building on the framework of Dash and Moran (2005), I investigate the optimal allocations between a VIX ETP and both an equity and a bond portfolio respectively. The optimization tool that I use to do this analysis is Excel Solver. Furthermore, I look at static allocations to the VIX ETPs, as in the original Dash and Moran analysis, but I include four test statistics: the Sharpe ratio, the Sortino ratio, the reciprocal of the coefficient of variation (ICV), and the expected portfolio return divided by the downside standard deviation — which I will refer to as the reciprocal of the downside coefficient of variation (DICV) — instead of only the Sharpe and Sortino ratios that Dash and Moran studied. I find that, in general, VIX ETPs are beneficial only during specific time periods and that a static allocation to a VIX exchange-traded product is more detrimental than beneficial.

Literature Review

VIX History

The first known derivative dates back to about 1700 B.C.E. during Biblical times. For the following 3,700 years derivatives continuously evolved until it was dramatically changed by perhaps the most famous formula in finance, the Black-Scholes formula. This equation developed by Fisher Black and Myron Scholes set up the mathematical framework for pricing derivative securities, which lead to the creation of many new derivatives and ultimately to a major expansion within the derivatives market.

In order to sell and monitor this new and expanding market of derivative securities, many new entities were created. One of these entities was the Chicago Board Options Exchange (CBOE), which was established in 1973. Ten years later, this organization created options on the CBOE 100 Index (now the S&P 100 Index).

Portfolio managers began using these call and put options in order to protect against negative stock market movements. In 1993, Robert E. Whaley and the Chicago Board Options Exchange used the implied volatilities of these S&P 100 options in order to create the VIX index. According to Whaley (2009), they created this new index with two purposes in mind: "to provide a benchmark of expected short-term market volatility and an index on which futures and options could be written."

The original VIX calculation was based on eight at-the-money calls and puts of the S&P 100 Index. At that time, the options on the S&P 100 were the most actively-traded index options in the U.S., accounting for 75% of the total index option volume. Moreover, at-the-money options were the most actively traded among the S&P 100 options. Since the at-the-money calls and puts on the S&P 100 provided the most liquid and thus most accurate prices, these options were chosen for the original VIX calculation. As time passed, however, options on the S&P 500 surpassed those on the S&P 100 as the most actively traded index options and trading on out-of-the money options increased as well. Whaley ascertains some possible contributing factors for the increased activity in S&P 500 options include "the fact that the S&P 500 index is better known, futures contracts on the S&P 500 are actively traded, and S&P 500 option contracts are European-style (i.e., exercisable only at expiration), making them easier to value." According to Whaley (2009), not only had activity on S&P 500 options increased, but the trading volume on S&P 100 options had significantly decreased. With respect to increased activity in out-of-the money options, Whaley believes this increase was due to an increase in portfolio insurers, who routinely buy out-of-the-money and at-the-money index puts for insurance purposes. For these two reasons, the VIX calculation was changed on September 22, 2003. The new and current formulation of the VIX uses both out-of-the-money and at-the-money calls and puts on the S&P 500 and the full calculation appears in Appendix A.

This change in calculation helped to make the VIX less sensitive to any single option price, hence less susceptible to manipulation, while not devaluing historical VIX prices as a volatility benchmark. Moreover, for all intents and purposes, the S&P 100 and S&P 500 index portfolios are perfect substitutes. Whaley cites that over the period January 1986 through October 2008, the mean daily returns of the S&P 100 and S&P 500 were nearly identical, 0.0263% and 0.0266%, respectively, and the standard deviations of

S&P 100 daily returns was only slightly higher than the S&P 500 returns, 1.182% and 1.138%, respectively. From the perspective of trading derivatives contracts on the VIX, the change in methodology also provided a means of trading VIX by passively using SPX option contracts. This provides market makers in VIX futures and options with a less expensive means of hedging their inventory and promotes narrower bid/ask spreads in the VIX futures and options markets. Moreover, Whaley cites that the two formulations have a near perfect correlation between their daily returns, 0.9898, implying that, holding other factors constant, OEX and SPX options are equally effective from a risk management standpoint. However, from the standpoint of maintaining the VIX as a timely and accurate reflection of expected stock market volatility, SPX is more effective because its option market has more depth and liquidity. For these reasons, Whaley and CBOE switched from OEX to SPX option prices for the VIX calculation.

Since the change in 2003, several VIX related securities have been constructed. The first product released was VIX futures contracts in May 2004. Next, in 2006, CBOE launched VIX option contracts. Finally, VIX exchange traded products launched in 2009. Later we will see how these products relate to different asset classes.

VIX and Portfolio Allocation

One of the most noteworthy characteristic of the VIX and its related products is that there is a negative correlation between VIX and VIX products with equity indices. From this characteristic, many papers have focused on the benefit of incorporating VIX and VIX-based products into portfolios. Dash and Moran (2005) set up an original framework of incorporating the VIX spot into hedge fund portfolios. They looked at three different portfolios, one having no allocation to the VIX spot, one having a constant 5% allocation to the VIX spot, and one having a 0-10% tactical allocation to the VIX spot depending on its movement in the previous month. Dash and Moran found that the tactical allocation had the greatest Sharpe and Sortino ratios, then the constant 5% allocation to the VIX spot, and then no allocation. Although allocating part of a portfolio to the VIX seems like it would yield beneficial results to a portfolio, the major drawback of this paper is the inability to invest in the VIX spot.

Szado (2009) improved upon the Dash and Moran framework by testing VIX futures and options. He also incorporated different combinations of stocks, bonds, and alternatives into his base portfolios. Szado found that holding VIX products during a major market downturn seems to be beneficial; however, he only takes into consideration the Sharpe ratio.

Jones (2011) furthers the analysis of Szado and Dash and Moran by testing the benefits of VIX futures to a portfolio with different combinations of stocks and bonds over a wider time period. He also incorporates his own tactical allocation strategy. Like Szado, the main drawback of Jones' analysis is that he only looks at the Sharpe ratio, not the Sortino ratio or ICV.

Looking at the existing literature, there has not yet been an analogous analysis of VIX-based ETPs. This thesis will construct a similar Dash and Moran framework to test the benefit of incorporating VIX futures ETPs into a portfolio of stocks and bonds with respect to the Sharpe ratio, Sortino ratio, ICV and DICV.

Experiment Design

In this section, I discuss the assumptions, methodology, and equations used to analyze the effect of allocating part of a portfolio to a VIX ETP has on the risk adjusted return of a that portfolio. I build upon previous work by Dash and Moran (2005), Szado (2009), and Jones (2011) and extend their framework by analyzing the portfolios with respect to the Sharpe ratio, Sortino ratio, ICV, and DICV. Also, I use Excel Solver to optimize the portfolio allocation by maximizing the respective metrics. For the equity and bond indices, I use the SPDR S&P 500 ETF (ticker: SPY) and the iShares Core Total U.S. Bond Market ETF (ticker AGG) respectively. As for the VIX, I use the iPATH S&P 500 VIX Short-Term Futures ETN (ticker: VXX) and iPATH S&P 500 VIX Mid-Term Futures ETN (ticker: VXZ). Furthermore, I compare the results of the VIX ETPs to two different Treasury ETFs, iShares 10-20 Year Treasury Bond ETF (ticker: TLH) and iShares 20+ Year Treasury Bond ETF (ticker: TLT). A detailed description of each of these assets is provided in Appendix B. According to a Bloomberg article by David and Veronesi (2012), the correlation of Treasury securities and stocks has been mainly negative since the onset of the financial crisis in 2008. "Volatility of stock returns increased dramatically and the equity markets plunged while, at the same time, U.S. Treasury bond prices shot up." The article continues to say that this is not always the case and that this negative correlation depends on the macroeconomic environment. However, during the time period of my historical data set, Treasury securities and stocks were typically negatively correlated. Therefore, if the VIX were to add risk-adjusted benefit to a portfolio, I wanted to know if it performed better than the “safe-haven” of Treasury securities.

Assumptions

The premise of this paper assumes that both VXX and VXZ will be negatively correlated to SPY. Therefore, I must first check and verify this to be true. Additionally, I must also check that TLH and TLT are negatively correlated to SPY so that I can compare my results for VIX ETPs to the results for the Treasury ETPs. I also would like to check the correlations of VXX and VXZ with AGG. If the correlations are positive, then there should be no benefit of adding either VXX or VXZ and removing AGG. In order to use the general equations for the Sharpe ratio, Sortino ratio, ICV, and DICV, I assume that returns are independently and identically distributed normal random variables. To monitor this, I create normal plots for the returns of each respective ETF.

Methodology

I downloaded daily price close data from February 27, 2009 to March 1, 2013 from Bloomberg for all of the ETPs. From there I perform several different tests on the data which are explained below.

Optimization

For the optimization, I calculate the four respective metrics over 252 day periods and use Excel Solver to select the portfolio allocation that maximized the respective metric. This tactic id performed on four different portfolios: AGG & VXX, AGG & VXZ, SPY & VXX, and SPY & VXZ. Furthermore, using VBA, I loop the Solver algorithm through all of the data. For each iteration, I add one new data point and remove the oldest data point. If some of the trials allocate to a VIX ETP, this means that there exists time periods where it is beneficial to invest some of an equity or a bond portfolio into a VIX ETP.

Static Portfolios

After identifying that there are some instances in which allocating to a VIX ETP is beneficial, the next question is: how much should be allocated? The optimal allocation tends to heavily weight assets that performed well over the respective period and thus these optimal allocations will most likely underperform different data sets. Therefore, I construct several static portfolios, starting with 0% in VIX, ending with 100%, and changing by 1% in an attempt to identify a long-term optimal allocation. For each of the time periods, I test how a static percentage invested in a VIX ETP performs over the period. Comparing these results to the portfolio without VIX, I hope to find a static VIX investment that would consistently outperform the no VIX portfolio. After finding this portfolio, I test its performance relative to other negatively correlated assets, namely the two Treasury ETFs. Consequently, I finished my analysis by constructing the same static portfolio experiment with TLH and TLT and comparing the results with that of VXX and VXZ.

Equations

For my analysis, I use four main metrics, the Sharpe Ratio, the Sortino ratio, ICV, and DICV; this section covers the equations and formulations I use to obtain these metrics. Firstly, both the Sharpe and Sortino ratios require a risk-free rate. Since I employ a holding period of one year for all of my analysis, I chose the U.S. 1 year Treasury Bill as my risk-free security. The next section describes how I converted the given yield to annual and daily yields. After, I describe the equations for my four metrics.

Risk-Free Calculations

As stated above, I used the U.S. 1-year Treasury Bill as my risk-free rate. According to the U.S. Department of the Treasury, the yield curve, which relates the yield on a security to its time to maturity, is based on the closing market bid yields on actively traded Treasury securities in the over-the-counter market. In the U.S. market, Treasury Bills are quoted as a bank discount yield, which is a method of quoting the yield on a debt security in which the amount of the discount from face value is divided by the security's face value with the result annualized. Therefore, in order to convert the historical value into an effective annual rate (EAR), I had to make the following transformation:

$$d = i * \frac{t}{360} = i * \frac{360}{360} = i, where\ i = the\ bid\ yield$$

$$P = F - d = 1 - i$$

$$EAR = \left(1 + \frac{d}{P}\right)^{\frac{365}{t}} - 1 = \left(1 + \frac{i}{1-i}\right)^{\frac{365}{365}} - 1 = \frac{i}{1-i}$$

Then to make the EAR an effective daily rate (EDR), I divided by 252, the average number of trading days in a year. From this EAR and EDR, the Sharpe ratio and Sortino ratios can be computed, which I will do in the next two sections.

Sharpe Ratio

Firstly, historical data is being used; therefore, I will calculate historical or ex-post Sharpe ratios. Moreover, I am assuming that the portfolio returns are normally distributed, which will be verified later. Looking at Sharpe (1994), the ex-post Sharpe ratio assuming normal returns is calculated as follows. Let R_{pt} be the return on the portfolio in period t, R_{ft} the risk-free rate of return at period t (calculated in the previous section), and E_t the excess return in period t:

$$E_t = R_{pt} - R_{ft}$$

Let $\bar{E}$ be the average value of E_t over the historic period from t=1 through T:

$$\bar{E} = \frac{1}{T}\sum_{t=1}^{T} E_t$$

Finally, let σ_E be the standard deviation over the period:

$$\sigma_E = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(E_t - \bar{E})^2}$$

Then, the ex-post, or historic Sharpe Ratio, $S_{h,}$ is:

$$S_h = \frac{\bar{E}}{\sigma_E}$$

According to Sharpe (1994), this version of the ratio is a risk-adjusted return measure that identifies the historic average excess return per unit of historic variability of the excess return. If the risk-free rate is assumed to be a constant equal to the average of the risk-free rates over the period, the Sharpe ratio becomes:

$$S_h = \frac{\overline{R_p} - \overline{R_f}}{\sigma_p},$$

where $\overline{R_p}$ is the average return of the portfolio and $\overline{R_f}$is the average risk-free rate over the time period. I use both formulations in my analyses. The second formulation is slightly less accurate, but allows the programs written in MATLAB and VBA to run slightly faster. Considering the relatively low and consistent interest rate environment, the increase in processing speed of the second formulation should outweigh the potential error due to the assumption.

Sortino Ratio

The Sortino ratio is a risk-adjusted return measure that identifies the historic average excess return per unit of downside historic variability of the excess return. The only difference between the Sharpe ratio and the Sortino ratio is that the standard deviation of returns is replaced by the downside standard deviations of returns. The rationale behind this is that active portfolio managers desire returns that are higher than expected, but want to minimize returns that are lower than expected. Downside standard deviation, a measure of the variability of not meeting the expected return, is calculated as:

$$\sigma_d = \sqrt{\frac{1}{T}\sum_{t=1}^{T} \max\{0, \bar{R} - R_t\}^2},$$

Where $\bar{R}$ is the average return over the time period and R_t is the return at time t. Therefore, the Sortino ratio is:

$$Sort_h = \frac{\bar{E}}{\sigma_{Ed}},$$

or in the simplified case:

$$Sort_h = \frac{\overline{R_p} - \overline{R_f}}{\sigma_{pd}}.$$

As for Sharpe, these equations are valid as long as the returns are normal. Also, the simplification increases the speed of the optimization without a large increase in error.

ICV

The inverse of the coefficient of variation is essentially the Sharpe ratio without the risk-free rate. This metric is similar to the Sharpe ratio, but does not require estimation of the risk-free rate. This should reduce the estimation error of the risk-adjusted return metric. However, the Sharpe ratio is the more prevalent ratio used in industry; thus, I include both in my analyses. The equation is:

$$ICV = \frac{\overline{R_p}}{\sigma_p}$$

DICV

Just like ICV, the reciprocal of the downside coefficient of variation is essentially the Sortino ratio without the risk-free rate. Since I include both the Sharpe and Sortino ratios in my analyses, I also include both ICV and DICV. The equation for this is as follows:

$$DICV = \frac{\overline{R_p}}{\sigma_{pd}}$$

Since both the Sharpe ratio and Sortino are extremely similar to ICV and DICV respectively, I include only the results of the Sharpe and Sortino in the body of the report. The figures for ICV appear in Appendix C while the figures for DICV are located in Appendix D.

Empirical Results and Analysis

Assumption Verification

As stated in the previous section, before I begin any analysis, I first must test the correlations between, SPY, VXX, VXZ, and AGG. Because I plan on comparing the VIX ETP results to the two Treasury ETFs, I also include TLH and TLT in the correlation analysis. In order to do this, I separated the returns into yearly intervals starting from March 1, 2009 and ending March 1, 2013. From *Table 1* below, it is evident that all five ETPs are negatively correlated to SPY. These negative correlations increase across all five assets from 2009 to 2011 and decrease slightly in 2012. VXX seems to be the most negatively correlated to SPY, then VXZ, followed by the two Treasury ETFs and finally AGG. The negative correlations of the VXX and VXZ are due to their price formulation while the negative correlations for the bond ETFs are most likely due to the macroeconomic environment since the 2007-2008 financial crisis. Since all five of the assets are negatively correlated to SPY, it is not surprising that they have positive correlations among each other. Furthermore, because the VIX tends to have a negative average return and relatively high volatility, I expect that there will be no benefit of adding an allocation of VXX or VXZ to a bond portfolio.

Table 1: Asset Correlations 2009 – 2012

Asset Correlations 2009

	SPY	AGG	VXX	VXZ	TLH	TLT
SPY	1.000	-0.165	-0.824	-0.775	-0.292	-0.315
AGG	-0.165	1.000	0.197	0.247	0.801	0.772
VXX	-0.824	0.197	1.000	0.892	0.296	0.288
VXZ	-0.775	0.247	0.892	1.000	0.324	0.319
TLH	-0.292	0.801	0.296	0.324	1.000	0.948
TLT	-0.3151	0.77245	0.28791	0.31883	0.94833	1.000

Asset Correlations 2010

	SPY	AGG	VXX	VXZ	TLH	TLT
SPY	1.000	-0.343	-0.874	-0.839	-0.539	-0.559
AGG	-0.343	1.000	0.247	0.278	0.868	0.816
VXX	-0.874	0.247	1.000	0.930	0.443	0.471
VXZ	-0.839	0.278	0.930	1.000	0.468	0.492
TLH	-0.539	0.868	0.443	0.468	1.000	0.965
TLT	-0.5587	0.8156	0.47101	0.49176	0.96477	1.000

Asset Correlations 2011

	SPY	AGG	VXX	VXZ	TLH	TLT
SPY	1.000	-0.470	-0.859	-0.848	-0.724	-0.716
AGG	-0.470	1.000	0.391	0.359	0.831	0.797
VXX	-0.859	0.391	1.000	0.951	0.636	0.621
VXZ	-0.848	0.359	0.951	1.000	0.609	0.592
TLH	-0.724	0.831	0.636	0.609	1.000	0.969
TLT	-0.7157	0.7972	0.62117	0.59229	0.96916	1.000

Asset Correlations 2012

	SPY	AGG	VXX	VXZ	TLH	TLT
SPY	1.000	-0.442	-0.815	-0.799	-0.651	-0.655
AGG	-0.442	1.000	0.320	0.321	0.848	0.827
VXX	-0.815	0.320	1.000	0.916	0.480	0.484
VXZ	-0.799	0.321	0.916	1.000	0.483	0.485
TLH	-0.651	0.848	0.480	0.483	1.000	0.979
TLT	-0.6555	0.82707	0.48426	0.48471	0.97857	1.000

The second major assumption is that the return of each ETP is an independently and identically distributed normal random variable. As stated in the previous section, this assumption is needed in order to use the equations for the Sharpe ratio, Sortino ratio, ICV, and DICV. To check this assumption, I create normal plots for the returns of each respective ETF over the entire time frame (March 1, 2009 to March 1, 2013) using MATLAB. From the normal plots below in *Figures I – VI*, the distributions for the ETF returns tend to have fatter tails than the normal distribution. However, most of the distribution for each ETP, .15 - .90, .05 - .99, .02 - .99, .02 - .99, .02 - .98, and .02 - .98 for SPY, AGG, VXX, VXZ, TLH, and TLT respectively is approximately normal. This especially makes sense for the two VIX ETPs and the bond rates because the VIX, like interest rates, typically has a mean reverting characteristic similar to that of the normal distribution. On the other hand, stocks are typically characterized by a log-normal distribution; therefore, it also makes sense that SPY has the least normal returns. However, assuming the returns from my data set are within the .15 to .90 of its respective cumulative distribution function (CDF), I can proceed with my assumption that the returns of SPY are normal. Since I am assuming the returns of all six ETFs are approximately normal, the returns of a combination of these six assets should also have a normal distribution.

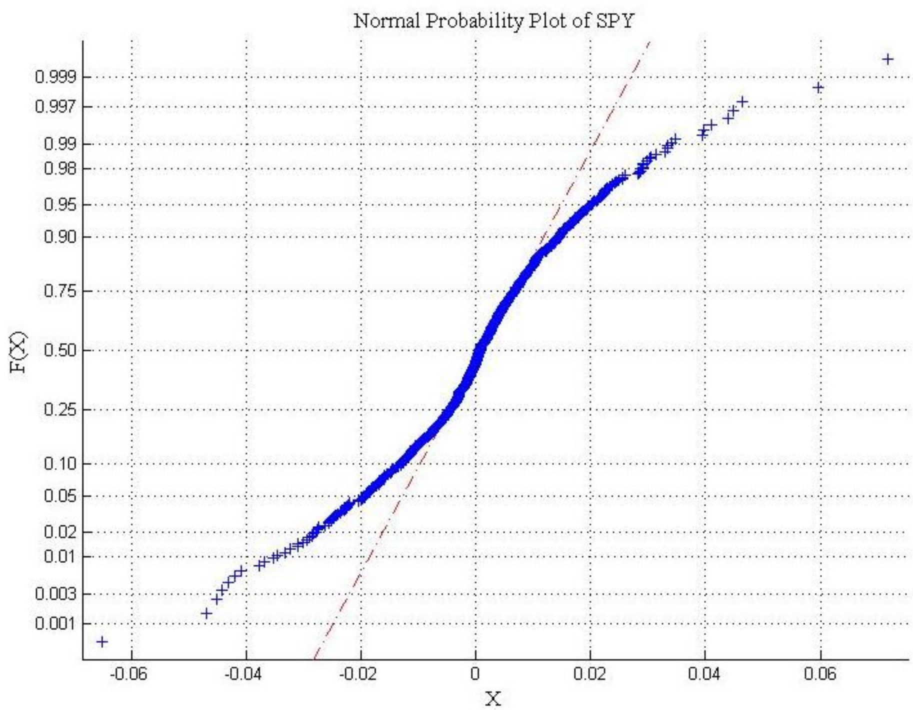


Figure I: SPY Normal Probability Plot

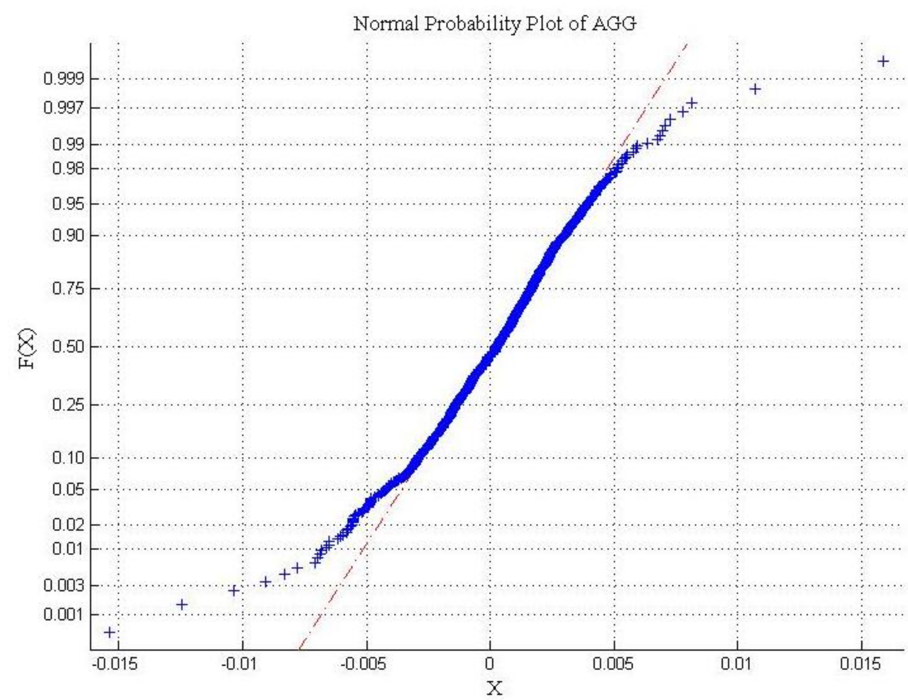


Figure II: AGG Normal Probability Plot

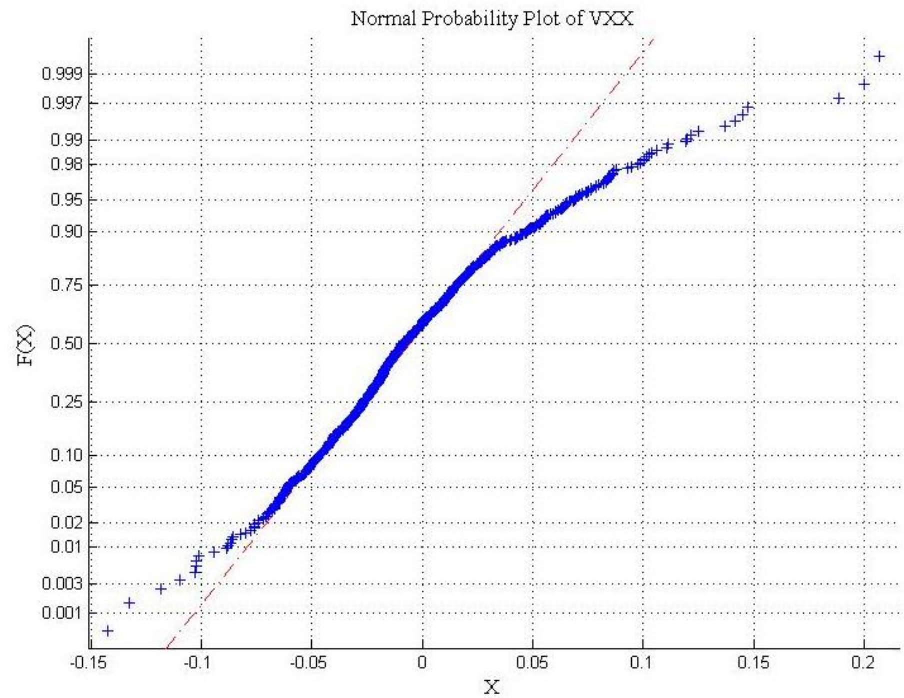


Figure III: VXX Normal Probability Plot

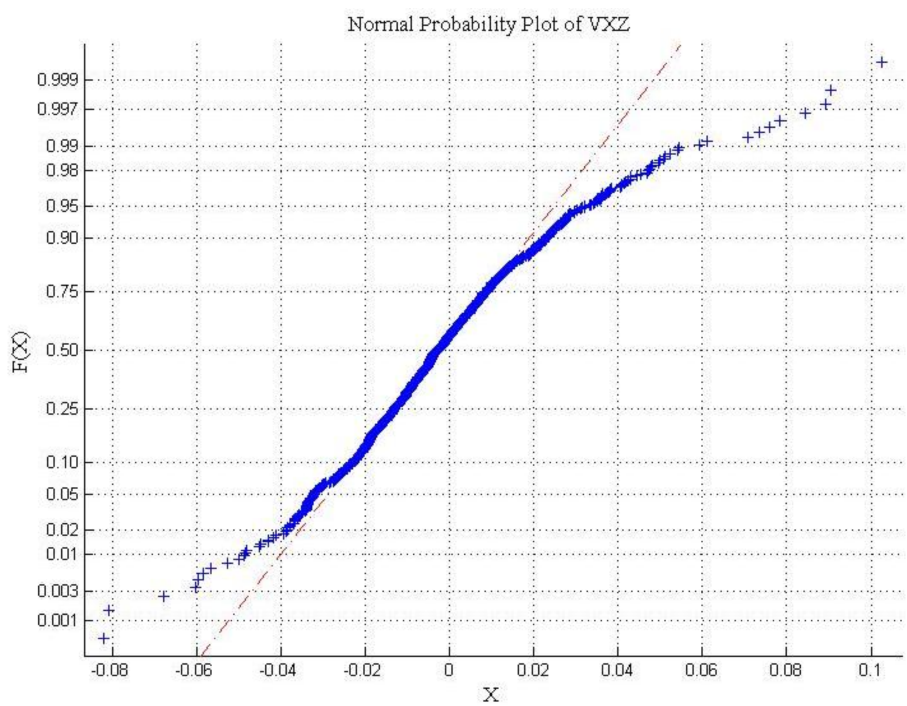


Figure IV: VXZ Normal Probability Plot

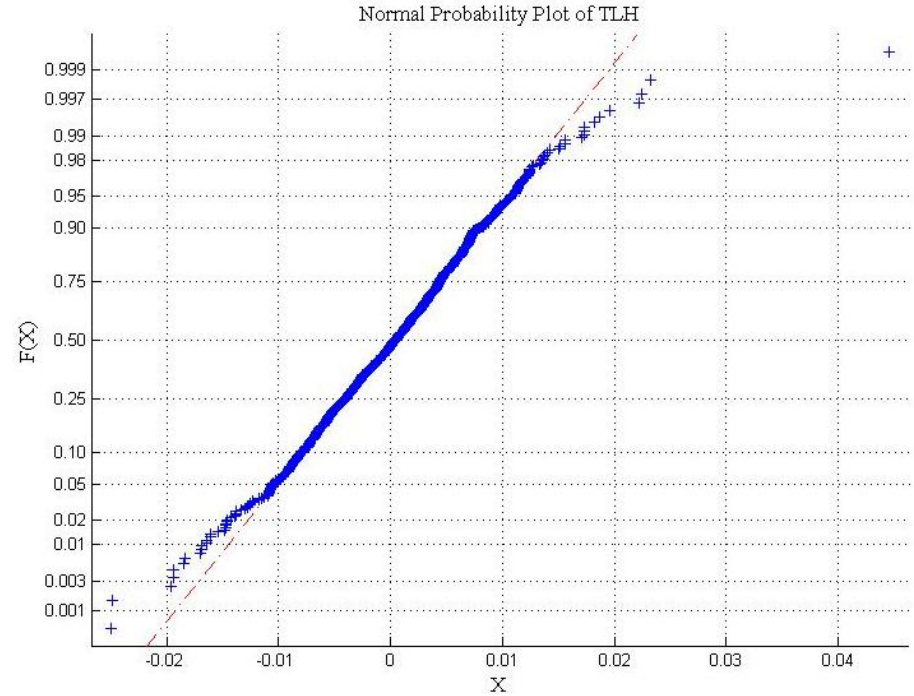


Figure V: TLH Normal Probability Plot

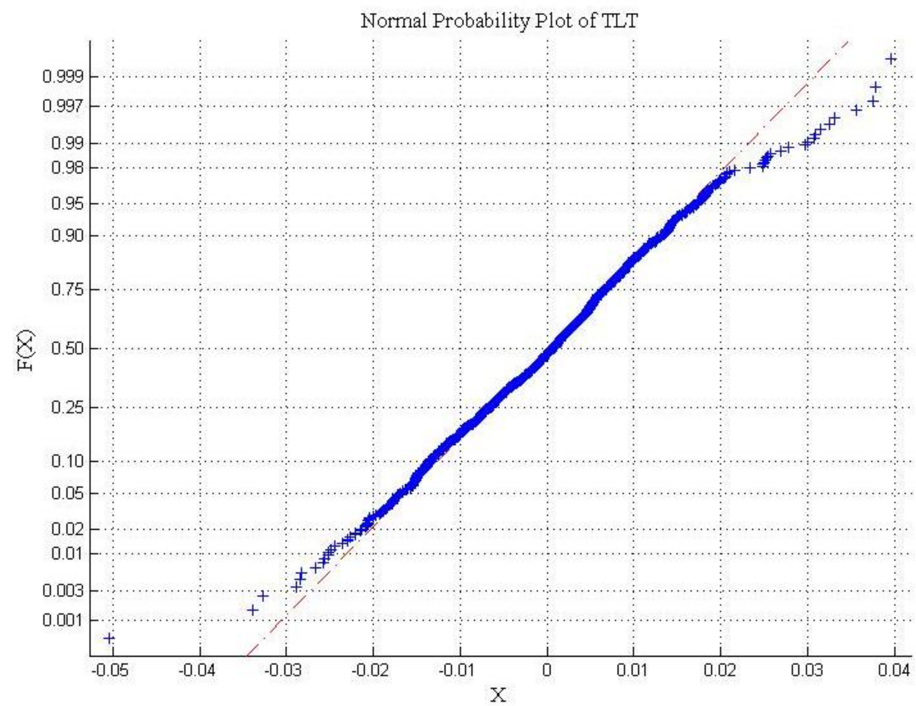


Figure VI: TLT Normal Probability Plot

Now that the normality assumptions have been verified, I want to check mean the return and standard deviation of all the assets over each respective year. This will help to hypothesize the likelihood that the VIX ETFs will be beneficial in a portfolio. The mean returns for each respective year reside in *Table 2* and the standard deviations in *Table 3* below. Since AGG tends to have a higher average return and a lower standard deviation than both VXX and VXZ and both are positively correlated to AGG, allocating to VIX products from a bond portfolio under this data will most likely add no benefit with respect to the four metrics. On the other hand, because VXX and VXZ are negatively correlated to SPY, the return and volatility metrics give limited insight into whether allocating to the VIX products from an equity portfolio will be beneficial. Before I move into the analysis of VIX and SPY, I first want to test my hypothesis with respect to AGG; I do this in the following section.

Table 2: Annualized Mean Returns 2009 – 2012

Annualized Mean Returns

	SPY	**AGG**	**VXX**	**VXZ**	**TLH**	**TLT**
2009	44.170%	3.720%	-132.320%	-41.080%	-1.980%	-9.530%
2010	19.840%	1.150%	-106.480%	-17.750%	3.310%	2.130%
2011	6.470%	5.060%	3.210%	4.890%	16.860%	26.660%
2012	11.160%	-0.220%	-119.550%	-86.160%	1.480%	1.240%

Table 3: Annualized Standard Deviation of Returns 2009 – 2012

Annualized Standard Deviation of Returns

	SPY	**AGG**	**VXX**	**VXZ**	**TLH**	**TLT**
2009	23.310%	5.370%	48.650%	25.870%	12.260%	16.710%
2010	17.210%	4.140%	59.780%	30.810%	10.520%	16.370%
2011	22.840%	3.700%	74.040%	38.180%	11.520%	20.230%
2012	13.100%	2.700%	66.510%	31.300%	7.690%	13.720%

VIX and Bond Portfolios

In the previous section, I found that the VIX ETFs have a positive correlation to AGG as well as a lower expected return and a higher standard deviation. Due to this, I believe that adding VIX to a bond portfolio would decrease all four metrics. In order to test this theory, I use Excel Solver to loop through sets of 252 trading days and change the asset weights to maximize each respective metric.[3] Throughout all of the 757 trials and across all four metrics, the optimal solution is almost always to invest 100% in AGG and 0% in either VIX ETF. In less than 1.6% and 0.5% of the scenarios, Solver invests in VXZ and VXX respectively. Only 3 times out of the 757 trials and across all four metric an optimal solution invests more than 5% in VXZ and 0 times an optimal solution invests more than 5% in VXX. Furthermore, the largest resulting investments in VXZ and VXX are 6.07% and 1.03%, respectively. These results are both under the Sortino ratio. The results appear to verify my hypothesis that under this data set, VIX ETPs should not replace bond allocation. In the next section, I perform the same analysis on an equity portfolio.

VIX and Equity Portfolios

In the previous section, I found that under this data set, VIX and VIX products should not replace bond allocation. In this section, I will construct the same analysis for an equity portfolio. From section 4.1, the two VIX ETFs have negative correlations to SPY; therefore, adding VIX to an equity portfolio may increase the Sharpe ratio, Sortino ratio, ICV, and DICV of a portfolio. In order to test this theory, I again used Excel Solver

[3] This Excel workbook will be available upon request from Lehigh University.

to loop through sets of 252 trading days and change the asset weights to maximize each respective metric.[4] Unlike the bond portfolio, there are many cases when Excel Solver invests in the VIX ETFs. However, across all four metrics, when Solver does invest in a VIX ETF, it tends to invest in VXZ. With respect to the Sortino ratio, Solver invests in VXX in 15.32% of the 757 trials and VXZ in 38.57%. Looking at the Sharpe ratio, ICV, and DICV, the optimal solution chooses VXX 15.72%, 0.92%, and 15.32% of the time respectively. On the other hand, Solver invests in VXZ 36.86%, 37.38%, and 39.23% of the trials for the Sharpe ratio, ICV, and DICV respectively. *Figure VII* and *Figure VIII* below show histograms of allocations weights for VXX and VXZ. For the most part, when Solver invests in VXX, it allocates 100% to it; otherwise, Solver invests 0% in VXX. In only 25 cases out of 3,028 (757 trials times four metrics) did Solver allocate a weight other than 0% or 100%. On the other hand, Solver invests a weight other than 0% or 100% in VXZ in 29.62% of the 3,028 cases. It appears that if a 0% or 100% weight in VXZ is not optimal, the optimal weight is between 20% and 50%. From this analysis and this data set, it seems that there can be some benefit in allocating part of an equity portfolio to a VIX ETF. Furthermore, allocating to S&P 500 VIX Mid-Term Futures ETF, VXZ, appears better than allocating to S&P 500 VIX Short-Term futures ETF, VXX. This section looked at optimal solutions for each 252 day period. The next part of the paper, looks to see how static allocations of equity and a VIX ETF perform over the four year period.

[4] This Excel workbook is also available upon request from Lehigh University.

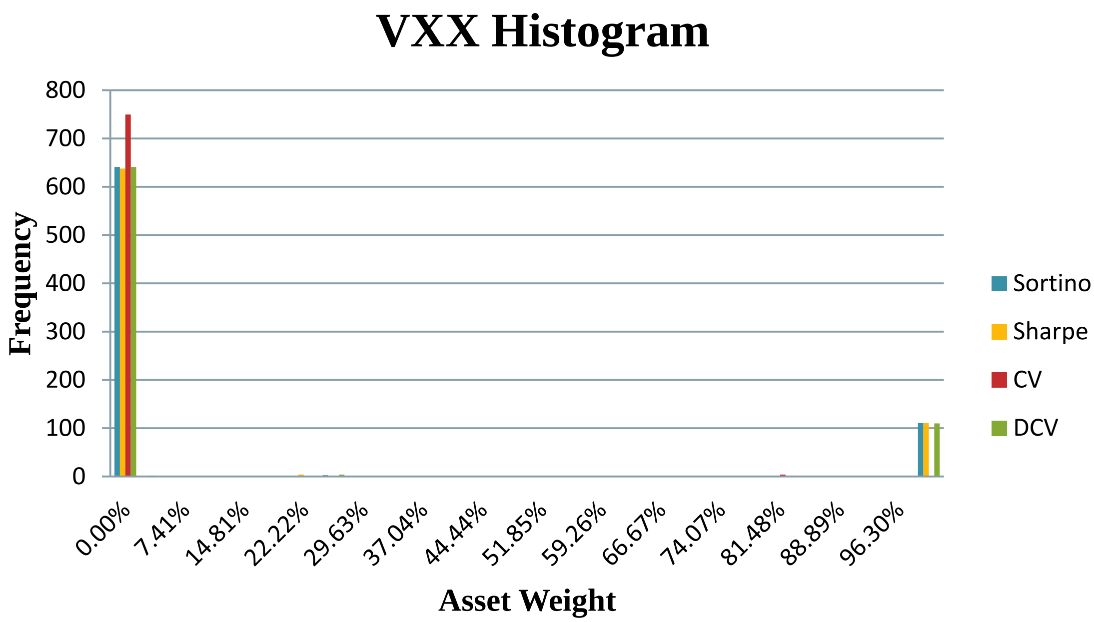


Figure VII: Histogram of Optimal Allocations for VXX in an Equity Portfolio

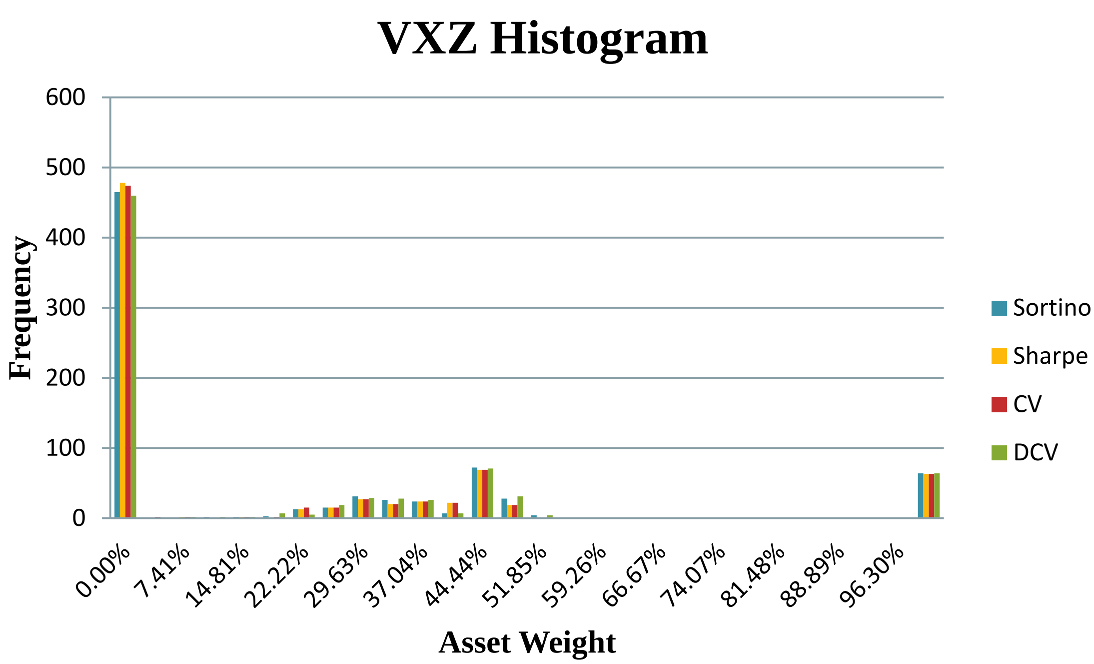


Figure VIII: Histogram of Optimal Allocations for VXZ in an Equity Portfolio

Static Allocations

The last section illustrates that there are some cases where it is optimal to allocate part of an equity portfolio to a VIX ETP. This section attempts to find an optimal long-run static allocation to a VIX ETP. To accomplish this, I enumerate all of the possible combinations of SPY and the VIX ETP between a 0% and 100% allocation to the VIX ETP with 1% intervals using MATLAB. Then I calculate the resulting metrics for that allocation over several 252 day periods. For the time intervals, I start at February 27, 2009 and then for each subsequent time period begin 15 trading days later for a total of 40 different trials. The 3D results for VXX and VXZ under the Sharpe and Sortino ratios appear in *Figure IX* and *Figure X*. From these figure, it appears that an optimal static allocation for VXX is practically 0%, while for VXZ, it tends to be in the range of 30% - 50%. To better display the results, I construct histograms for the number of times each percentage allocation outperforms the 0% allocation. I then create a histogram for the average outperformance of these instances, which appear in *Figures XI* and *XII*.[5] Looking at these graphs, VXZ clearly outperforms VXX. Furthermore, it appears that across all four metrics, holding roughly 45% in VXZ over from March 2009 to March 2013 would have netted a portfolio the largest average risk-adjusted return outperformance. However, holding a static position in VXZ that high is fairly risky. The lower echelon of VXZ allocations tends to outperform more times, but at a lower amount. Still, the maximum amount of times that any percentage across all four metrics outperformed the 0% allocation to VIX was 242 times out of 757, which is roughly 32% of the time. Therefore, it seems like allocating to VIX ETPs can be beneficial during periods of turmoil, but a static allocation is not beneficial. Moreover, if a manager decides to invest in a VIX ETP, the iShares S&P 500 VIX Mid-Term Futures ETN, VXZ, is likely to perform better than the iShares S&P 500 VIX Short-Term Futures ETN, VXX. Now that we know that there are cases when allocating to VIX ETPs can improve risk-adjusted returns, is it better than the typical method of investing in Treasury securities? The next section answers this question.

[5] Graphs for ICV and DICV appear in Appendix C and D respectively.

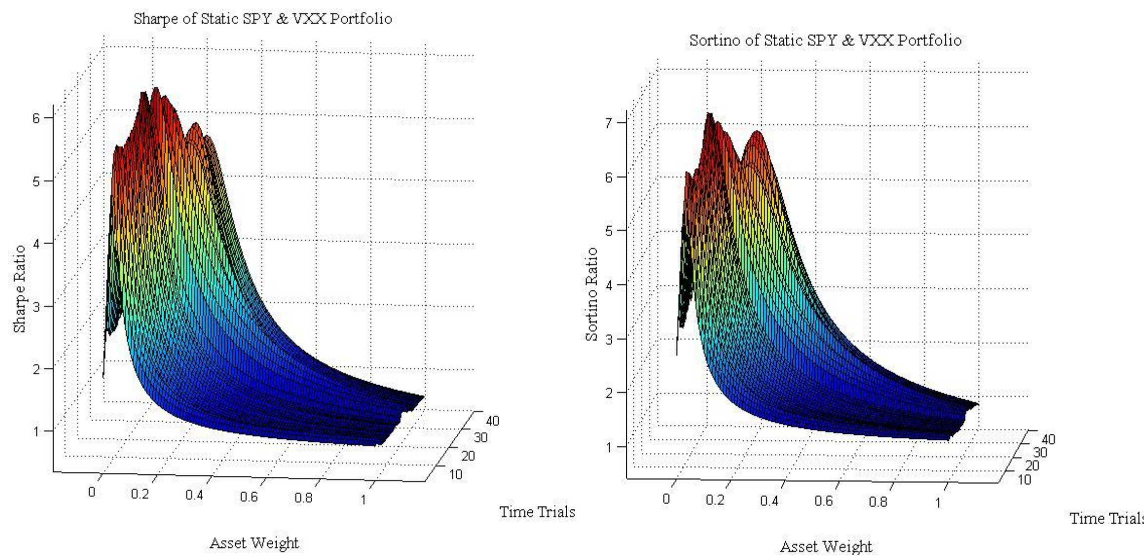


Figure IX: 3D Sortino & Sharpe Ratios for the Static Allocation of VXX

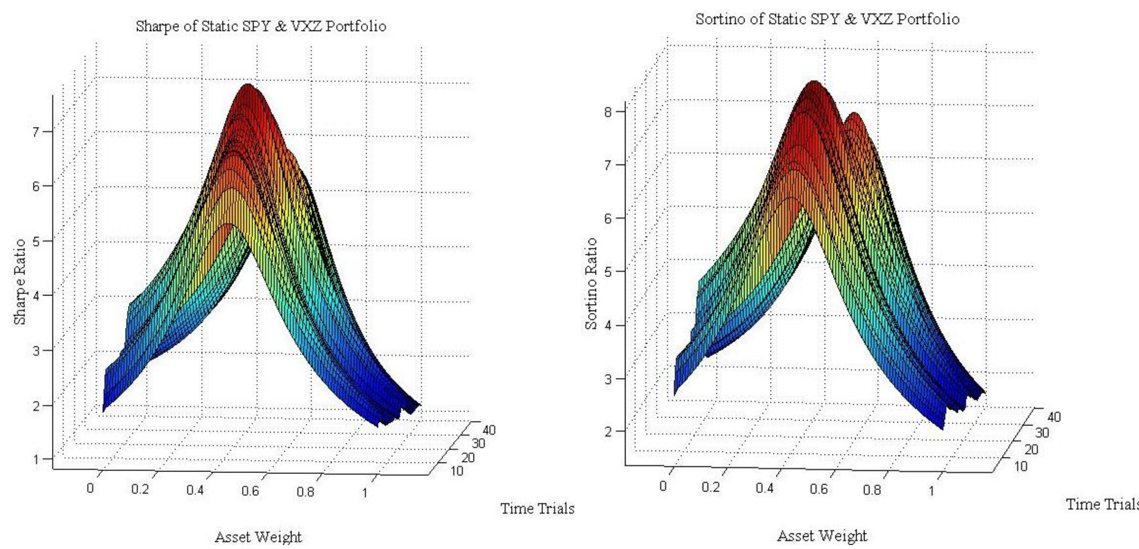
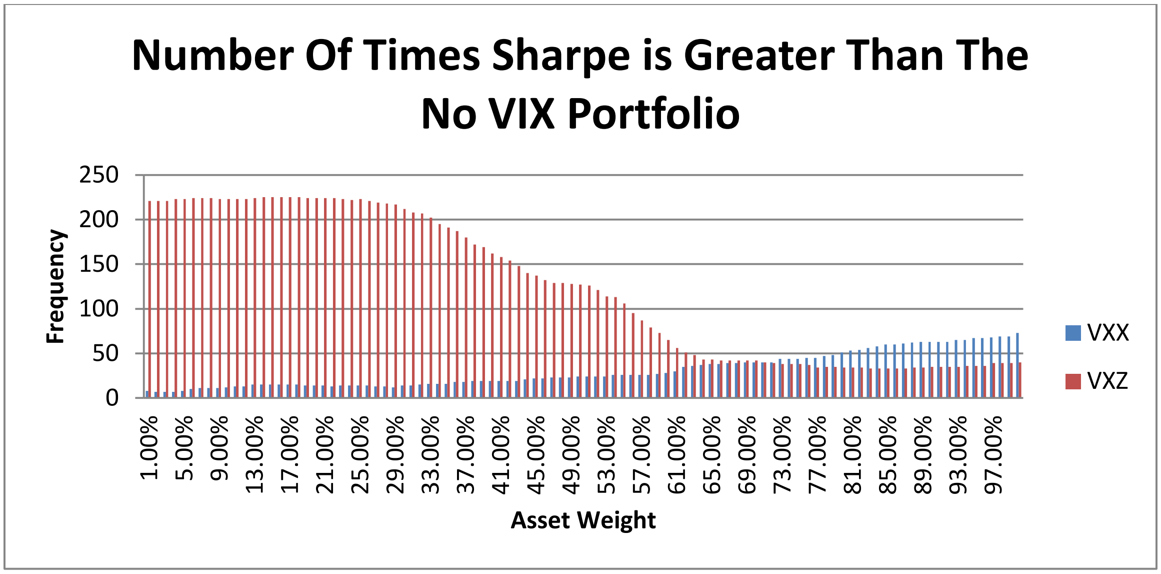


Figure X: 3D Sortino & Sharpe Ratios for the Static Allocation of VXZ



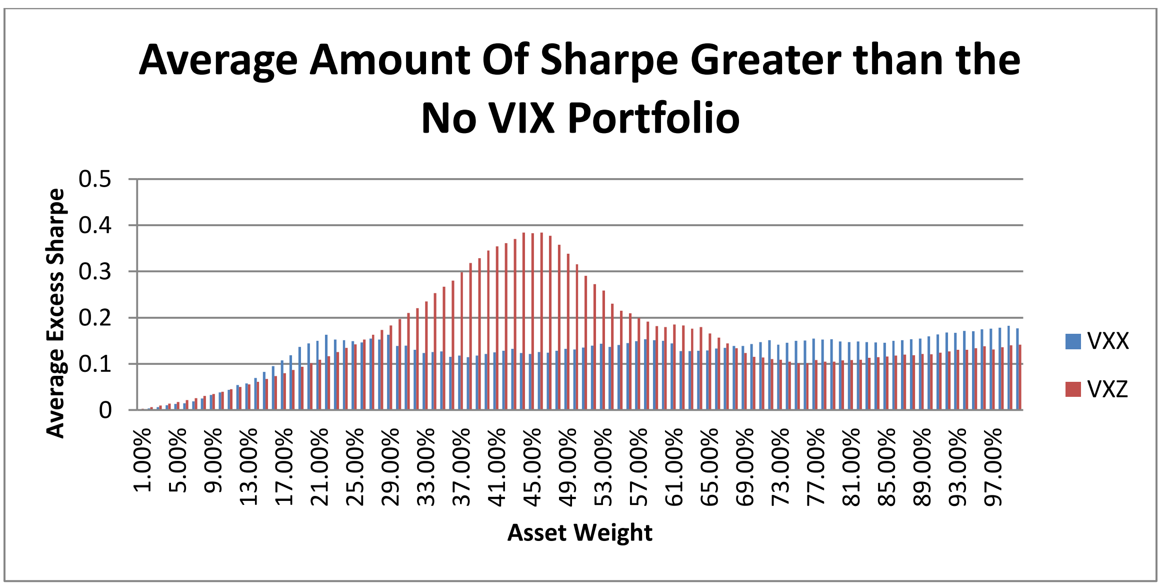
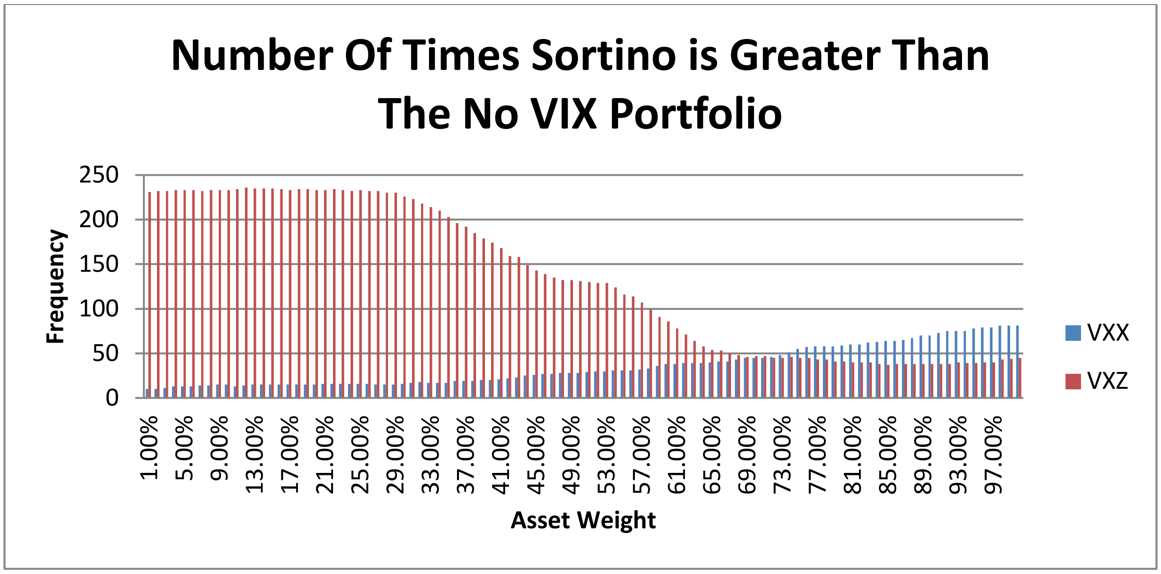


Figure XI: Static Sharpe Histograms of VXX & VXZ



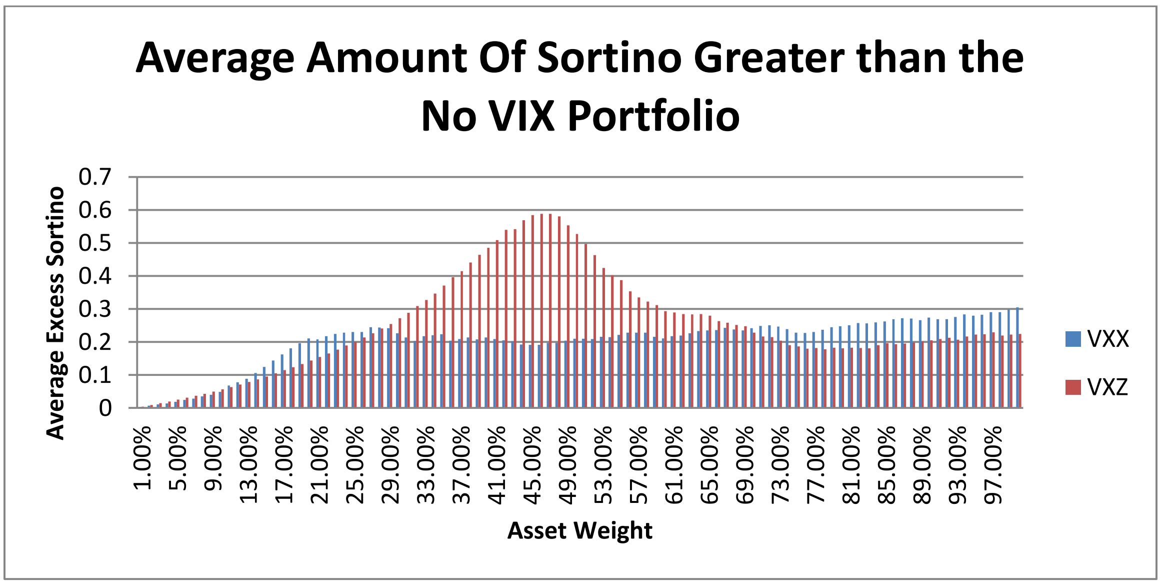


Figure XII: Static Sortino Histograms of VXX & VXZ

VIX and Treasury ETP Comparison

In the assumptions section above, I showed that both TLH and TLT are negatively correlated to SPY and that both have approximately normal returns. From here, I construct and identical static allocation test for the two Treasury ETFs. The resulting 3D graphs of the Sharpe and Sortino ratios for TLH and TLT appear in *Figure XIII* and *Figure XIV*. Then, *Figures XV* and *XVI* compare the resulting histograms for VXZ and the two Treasury ETFs. I compare TLH and TLT only to VXZ because VXZ clearly outperformed VXX over this time period from the previous section. Looking at these graphs, the iShares 20+ Year Treasury Bond ETF, TLH noticeably outperforms both VXZ and TLT with respect to all metrics. In fact, over this time period investing around 70% of a portfolio in TLH would have yielded outperformance in around 80% of the scenarios by about .9 for the Sharpe ratio and ICV and by about 1.3 for the Sortino ratio and DICV. Therefore, although VXZ would have provided some downside protection for an equity portfolio, TLH accomplishes this task better with respect to the four metrics and is undoubtedly the best asset to have invested in over this time period among those examined in this thesis.

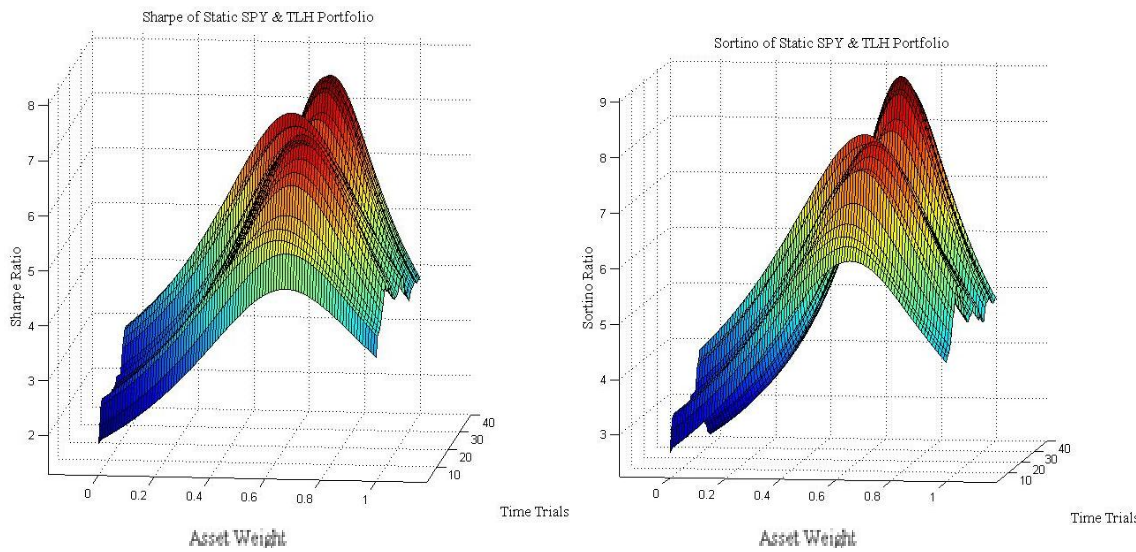


Figure XIII: 3D Sortino & Sharpe Ratios for the Static Allocation of TLH

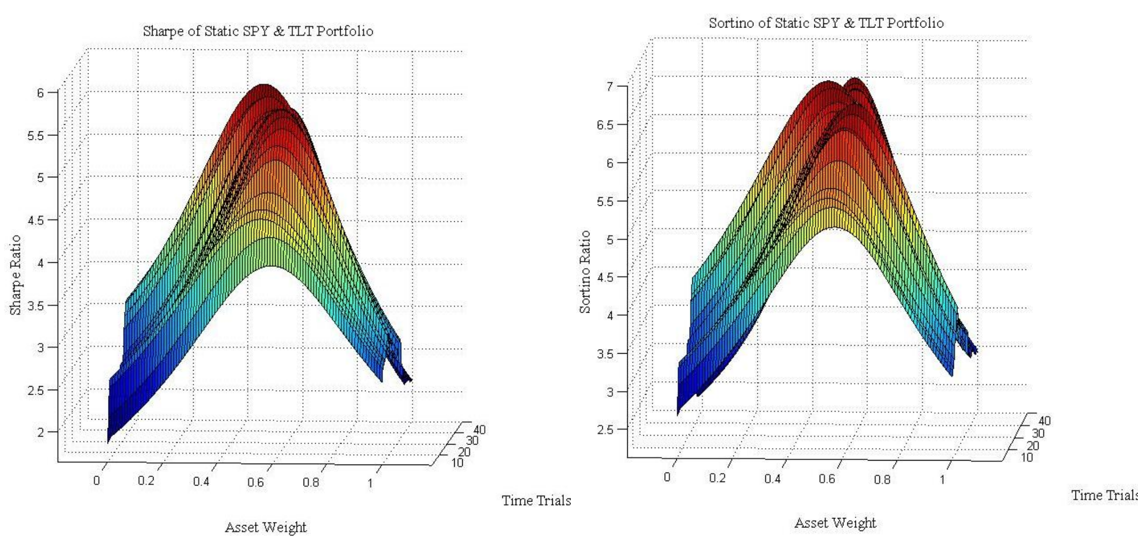
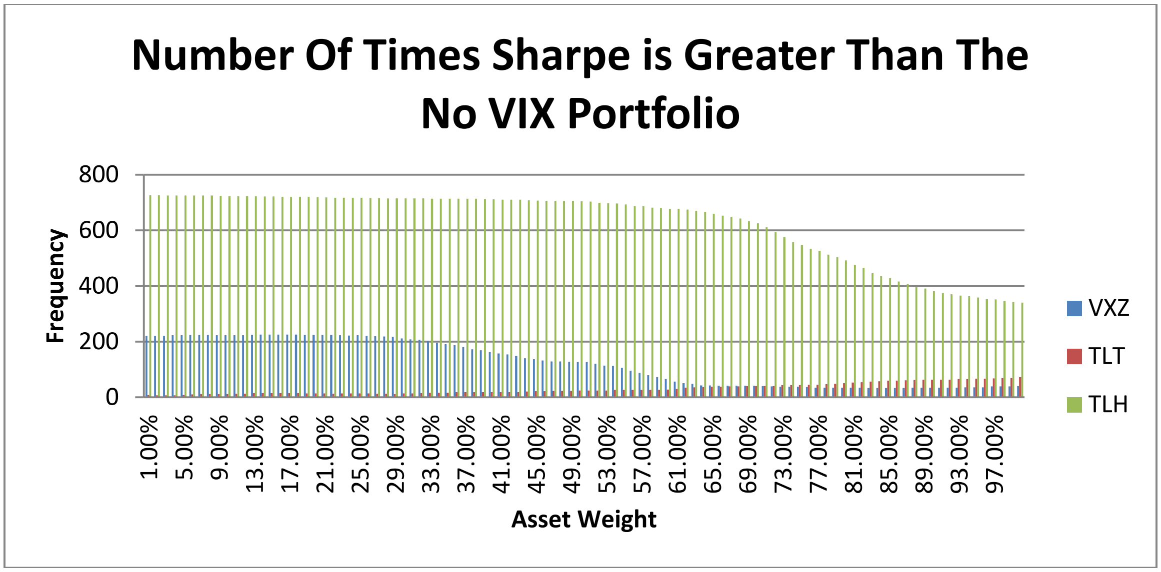


Figure XIV: 3D Sortino & Sharpe Ratios for the Static Allocation of TLT



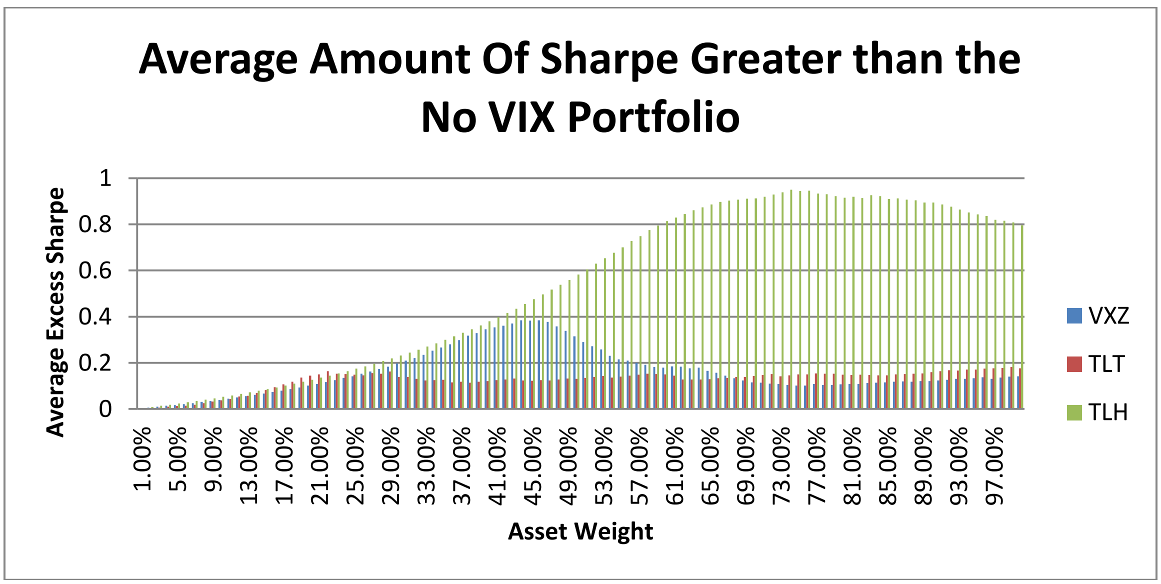
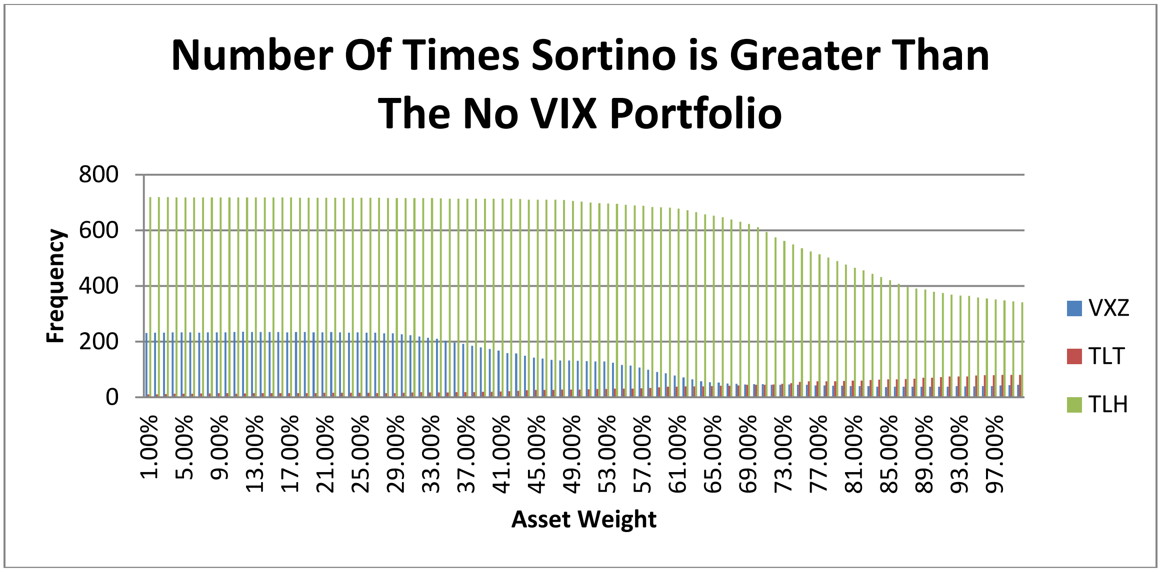
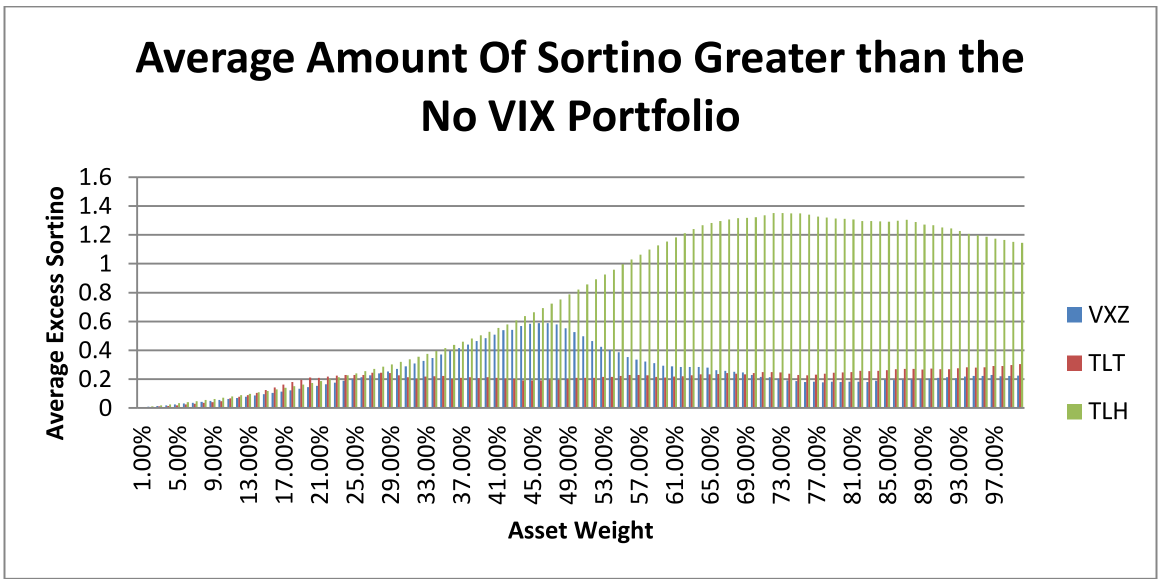


Figure XV: Sharpe Histograms of VXZ, TLT, & TLH


Number Of Times Sortino is Greater Than The No VIX Portfolio
800
600
400
200
0
Frequency
1.00%
5.00%
9.00%
13.00%
17.00%
21.00%
25.00%
29.00%
33.00%
37.00%
41.00%
45.00%
49.00%
53.00%
57.00%
61.00%
65.00%
69.00%
73.00%
77.00%
81.00%
85.00%
89.00%
93.00%
97.00%
Asset Weight
VXZ
TLT
TLH


Average Amount Of Sortino Greater than the No VIX Portfolio
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Average Excess Sortino
1.00%
5.00%
9.00%
13.00%
17.00%
21.00%
25.00%
29.00%
33.00%
37.00%
41.00%
45.00%
49.00%
53.00%
57.00%
61.00%
65.00%
69.00%
73.00%
77.00%
81.00%
85.00%
89.00%
93.00%
97.00%
Asset Weight
VXZ
TLT
TLH

Conclusions

Expanding the Dash and Moran framework to VIX ETPs and including optimization, I have shown that VIX ETPs can improve risk-adjusted portfolio returns; however, only in certain scenarios. Moreover, the iPath S&P 500 VIX Mid-Term Futures ETN, VXZ, appears more likely to improve these risk-adjusted returns in comparison to the iPath S&P 500 VIX Short-Term Futures ETN, VXX. Looking at the period from February 27, 2009 to March 1, 2013, there were certain times when investing in VIX ETPs would have been beneficial with respect to a risk-adjusted return metric; however, a static allocation to either ETP over this period would have reduced risk-adjusted portfolio returns more often than increased them. In comparison to Treasury ETFs over this period, static allocations to VXZ outperformed the iShares 20+ Year Treasury Bond ETF, TLT, but grossly underperformed the iShares 10-20 Year Treasury Bond ETF TLT.

The major limitation to this study was lack of data. Since VIX exchange-traded products first began trading in 2009, there was only four years of available data. Additionally, over those four years, the macroeconomic environment was relatively stable. Furthermore, more advanced products such as iPath S&P 500 Dynamic VIX ETN (ticker XVZ), an adjusting combination of short and mid-term futures, have even less data available.

From these limitations, several extensions of this work could prove useful and interesting. Firstly, since VIX ETPs appear to have some benefit with respect to risk-adjusted portfolio return, but not as a static allocation, one avenue for future research would be to construct models and simulations in order to devise a tactical allocation strategy. Hopefully a method could be devised to take advantage of the negative correlation that these ETPs have with equities without exposing the portfolio to the potential negative returns and high volatility. Secondly, conducting this study with more data and in a different macroeconomics environment could yield new and interesting results, especially if the study was performed in an environment in which bonds are positively correlated with equities. Finally, comparing new VIX ETPs such as XVZ and different Treasury ETFs such as short-term ETFS could yield different results as well. Despite the negative returns, high volatility, and crazy macroeconomic environment, VIX exchange-traded products do, in the most general sense, help to improve risk-adjusted portfolio return. The next question is: what is the best strategy in which to implement this investment vehicle?

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Appendix A – VIX Calculation

Stock indexes, such as the S&P 500, are calculated using the prices of their component stocks. Each index employs rules that govern the selection of component securities and a formula to calculate index values. VIX is a volatility index comprised of options rather than stocks, with the price of each option reflecting the market's expectation of future volatility. Like conventional indexes, VIX employs rules for selecting component options and a formula to calculate index values. The generalized formula used in the VIX calculation from the CBOE website is:

$$\sigma^2 = \frac{2}{T}\sum_i \left[\frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i)\right] - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2, where$$

- σ = VIX/100
- T = Time to expiration
- F = Forward index level derived from index option prices
- K_0 = First strike below the forward index level, F
- K_i = Strike price of the i^{th} out-of-the-money option; a call if $K_i > K_0$, a put if $K_i < K_0$, and both a call and a put if $K_i = K_0$
- $\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2} = The\ interval\ between\ strike\ prices\ on\ either\ side\ of\ K_i$
- $\mathrm{R = Risk - free\ interest\ rate\ to\ expiration}$
- $Q(K_i)$ = The midpoint of the bid-ask spread for each option with strike K_i

VIX measures 30-day expected volatility of the S&P 500 Index. The components of VIX are near- and next-term put and call options, usually in the first and second SPX contract months. "Near-term" options must have at least one week to expiration; a requirement intended to minimize pricing anomalies that might occur close to expiration. When the near-term options have less than a week to expiration, VIX "rolls" to the second and third SPX contract months. For example, on the second Friday in June, VIX would be calculated using SPX options expiring in June and July. On the following Monday, July would replace June as the "near-term" and August would replace July as the "next-term."

The VIX calculation measures time to expiration, T, in calendar days and divides each day into minutes in order to replicate the precision that is commonly used by professional option and volatility traders. For the purpose of calculating time to expiration, SPX options are deemed to "expire" at the open of trading on SPX settlement day (8:30 AM on the third Friday of the month). The time to expiration is given by the following expression:

$$T = \frac{M_{Current\ day} + M_{Settlement\ Day} + M_{Other\ days}}{Minutes\ in\ a\ year}, where$$

- $M_{\text{Current day}}$ = minutes remaining until midnight of the current day
- $M_{\text{Settlement day}}$ = minutes from midnight until 8:30 am on SPX settlement day = 510 minutes
- $M_{\text{Other days}}$ = Total minutes in the days between the current day and settlement day = 1,440 minutes times the number of days in between the current day and settlement day.
- Minutes in a year = 525,600 minutes

The risk-free interest rate, R, is the bond-equivalent yield of the U.S. T-bill maturing closest to the expiration dates of relevant SPX options. As such, the VIX calculation may use different risk-free interest rates for near- and next-term options.

STEP 1 – Select the options to be used in the VIX calculation

The selected options are out-of-the-money SPX calls and out-of-the-money SPX puts centered around an at-the-money strike price, K0. Only SPX options quoted with non-zero bid prices are used in the VIX calculation. One important note: as volatility rises and falls, the strike price range of options with nonzero bids tends to expand and contract. As a result, the number of options used in the VIX calculation may vary from month-to-month, day-to-day and possibly, even minute-to-minute.

For each contract month:

- Determine the forward SPX level, F, by identifying the strike price at which the absolute difference between the call and put prices is smallest.

$$F = Strike\ Price + e^{RT} * (Call\ Price - Put\ Price)$$

- Determine $K_{0,t}$ – the strike price immediately below the forward index level F_t (F_1 for the near-term and F_2 for the next-term).
- Select out-of-the-money put options with strike prices < K0. Start with the put strike immediately lower than K0 and move to successively lower strike prices. Exclude any put option that has a bid price equal to zero (i.e., no bid).
- Next, select out-of-the-money call options with strike prices > K0. Start with the call strike immediately higher than K0 and move to successively higher strike prices, excluding call options that have a bid price of zero. As with the puts, once

two consecutive call options are found to have zero bid prices, no calls with higher strikes are considered.

- Finally, select both the put and call with strike price K0. Notice that two options are selected at K0, while a single option, either a put or a call, is used for every other strike price.

STEP 2 – Calculate volatility for both near-term and next-term options

Applying the VIX formula (1) to the near-term and next-term options with time to expiration of T1 and T2, respectively, yields:

$$\sigma_1^2 = \frac{2}{T}\sum_i \left[\frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i)\right] - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2$$

$$\sigma_2^2 = \frac{2}{T}\sum_i \left[\frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i)\right] - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2$$

STEP 3 – Calculate the 30-day weighted average of σ_1^2 and σ_2^2. Then take the square root of that value and multiply by 100 to get VIX.

$$VIX = 100 * \sqrt{\left(T_1\sigma_1^2\left[\frac{N_{T_2} - N_{30}}{N_{T_2} - N_{T_1}}\right] + T_2\sigma_2^2\left[\frac{N_{30} - N_{T_1}}{N_{T_2} - N_{T_1}}\right] + \frac{N_{365}}{N_{30}}\right)} \ \ , where$$

- N_{T1} = number of minutes to settlement of the near-term options
- N_{T2} = number of minutes to settlement of the next-term options
- N_{30} = number of minutes in 30 days = 30 * 1,440 = 43,200
- N_{365} = number of minutes in a year = 525,600

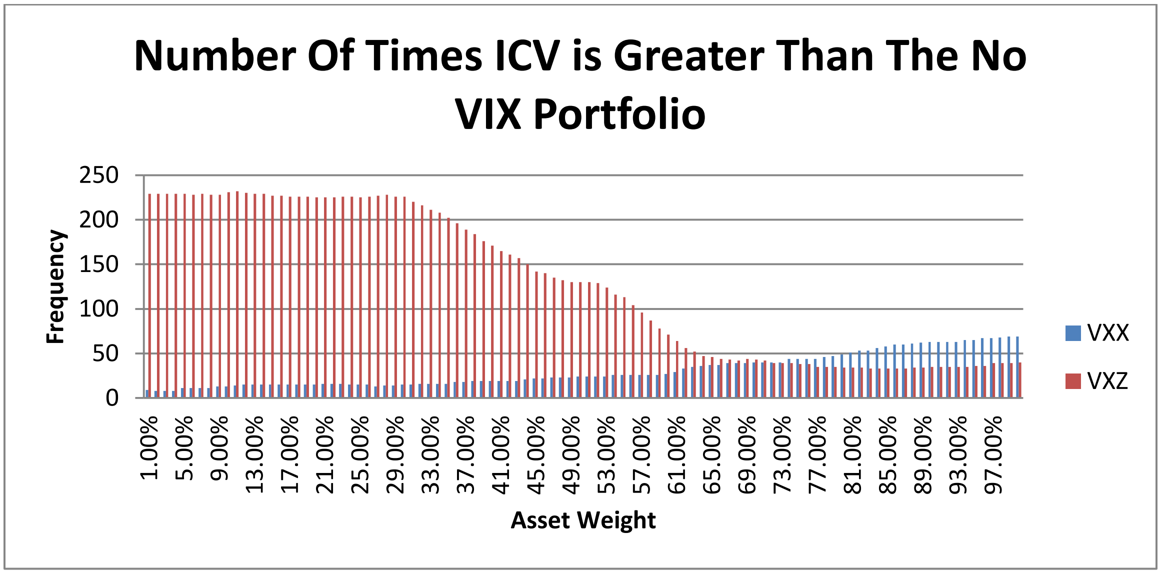
Appendix B – ETP Descriptions

- **SPDR S&P 500 ETF (SPY):** an ETF managed by State Street Global Advisors with the objective that the fund, before expenses, generally corresponds to the price and yield performance of the S&P 500® Index.
- **iShares Core Total U.S. Bond Market ETF (AGG):** an ETF managed by Blackrock with the objective that the fund, before fees and expenses, corresponds generally to the price and yield performance of the Barclays U.S. Aggregate Bond Index.
- **iPath S&P 500 VIX Short-Term Futures ETN (VXX):** an ETN managed by BlackRock that is designed to provide access to equity market volatility through CBOE Volatility Index futures. The ETN offers exposure to a daily rolling long position in the first and second month VIX futures contracts and reflects the implied volatility of the S&P 500 at various points along the volatility forward curve.
- **iPath S&P 500 VIX Mid-Term Futures ETN (VXZ):** an ETN managed by BlackRock that is designed to provide access to equity market volatility through CBOE Volatility Index futures. The ETN offers exposure to a daily rolling long position in the fourth, fifth, sixth and seventh month VIX futures contracts and reflects the implied volatility of the S&P 500 at various points along the volatility forward curve.
- **iShares 10-20 Year Treasury Bond ETF (TLH):** an ETF managed by Blackrock with the objective that the fund, before fees and expenses, corresponds

generally to the price and yield performance of the long-term sector of the United States Treasury market as defined by the Barclays U.S. 10-20 Year Treasury Bond Index.

- **iShares 20+ Year Treasury Bond ETF (TLT):** an ETF managed by Blackrock with the objective that the fund, before fees and expenses, corresponds generally to the price and yield performance of the long-term sector of the United States Treasury market as defined by the Barclays U.S. 20+ Year Treasury Bond Index.

Appendix C – ICV Figures



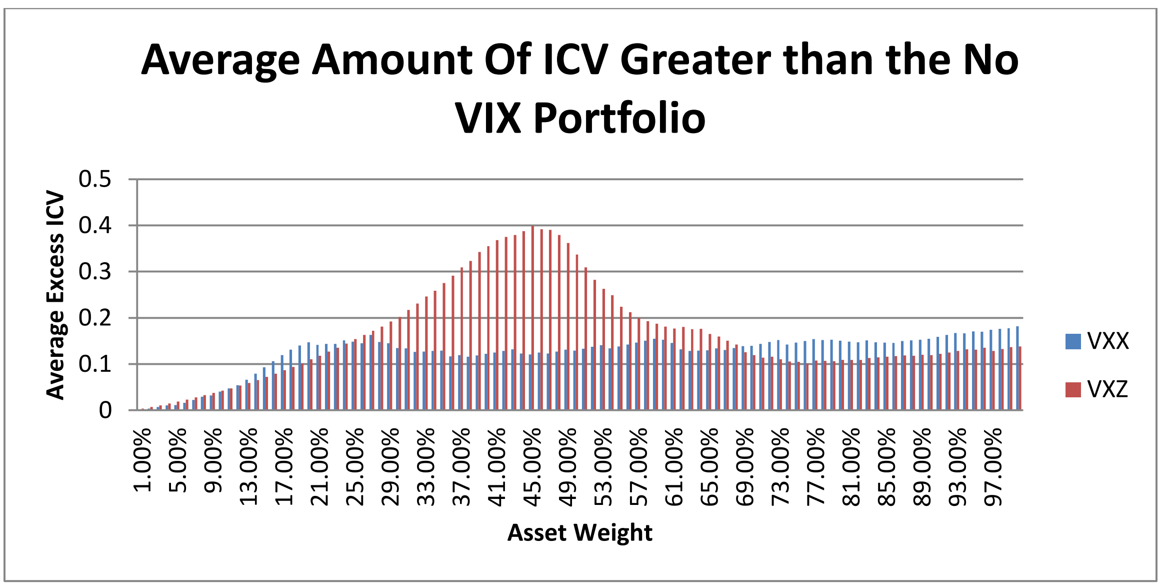
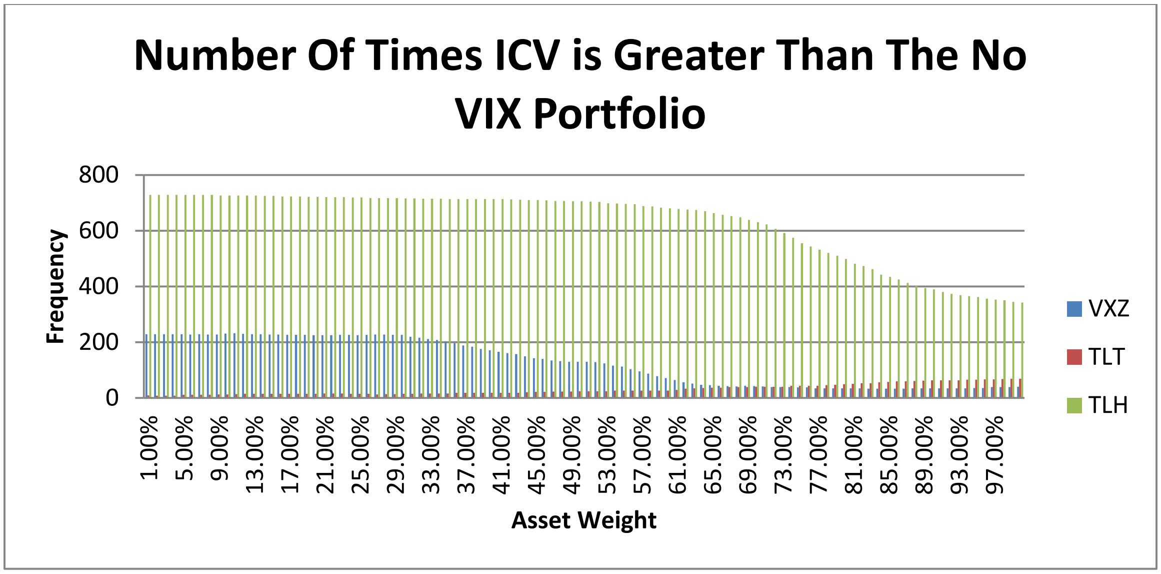


Figure XVI: ICV Histograms of VXX & VXZ



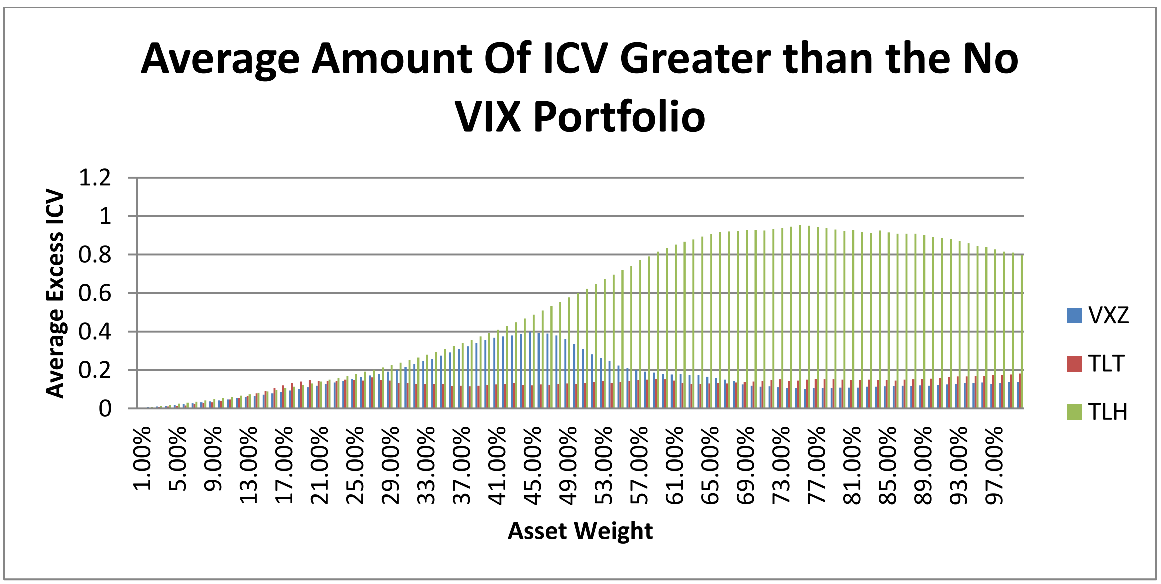
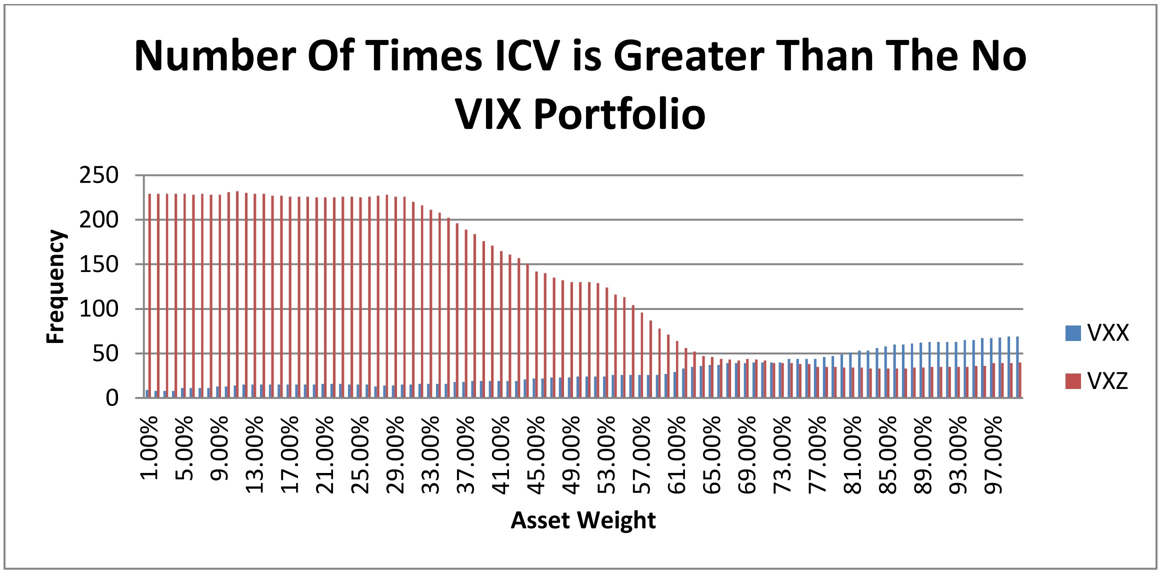


Figure XVII: ICV Histograms of VXZ, TLT, & TLH

Appendix D – DICV Figures



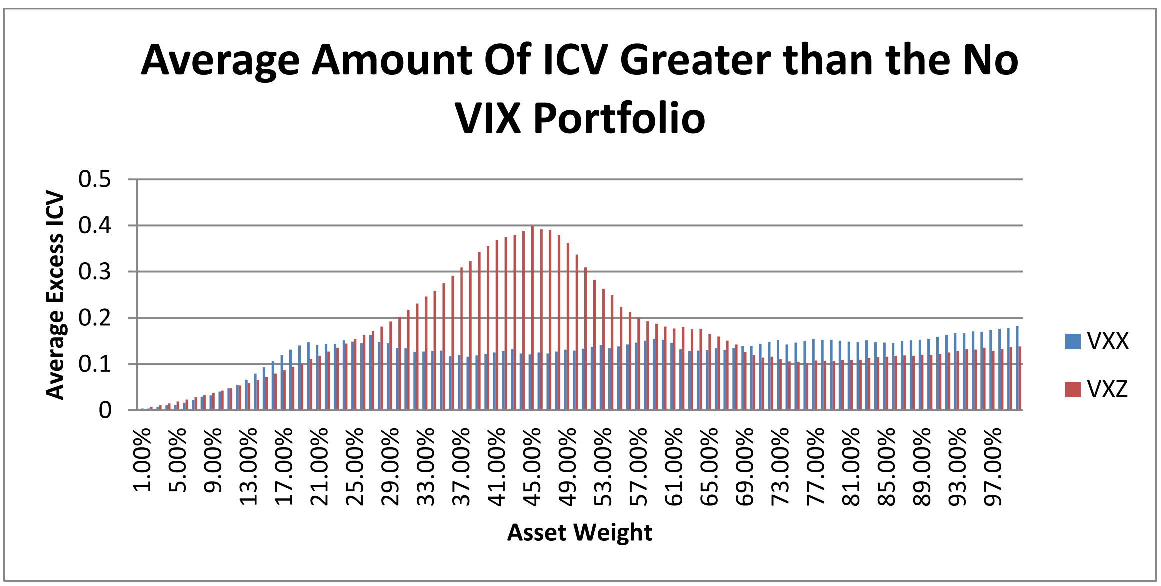
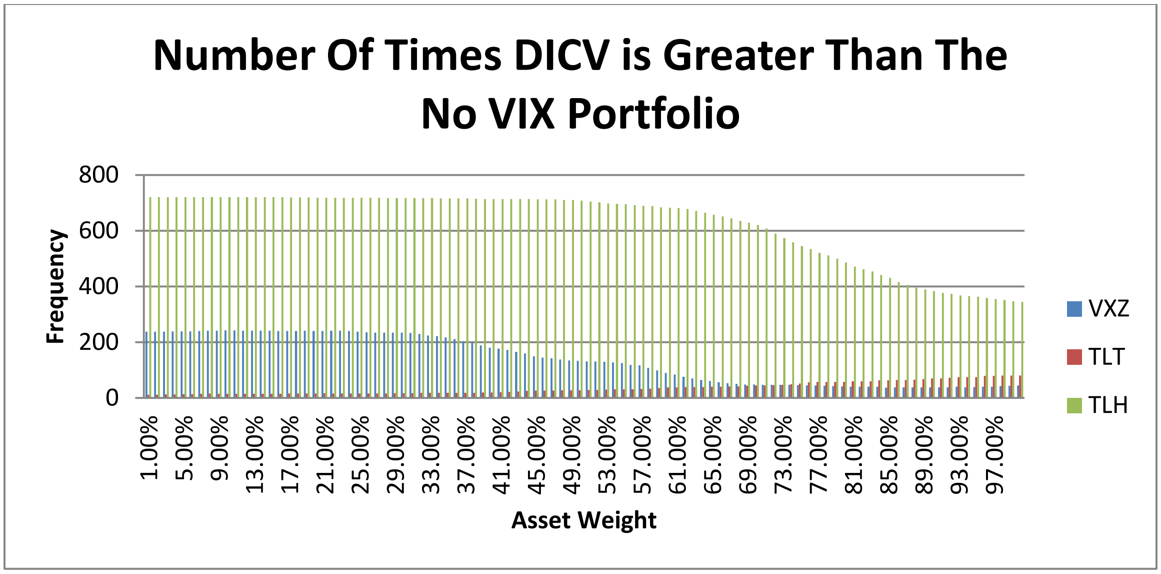


Figure XVIII: Static DICV Histogram of VXX & VXZ



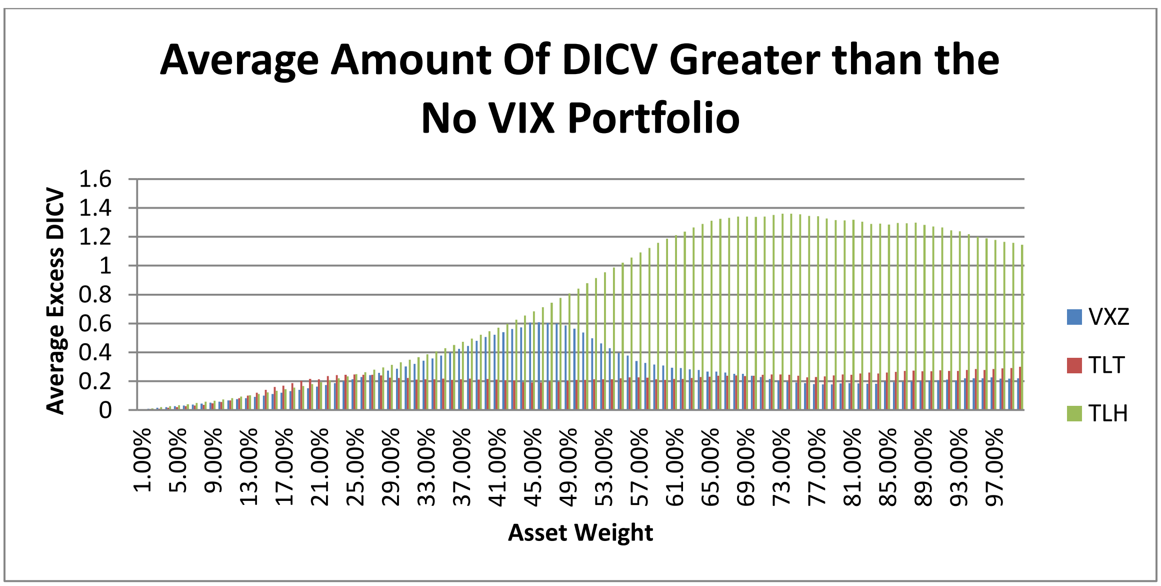


Figure XIX: DICV Histograms of VXZ, TLT, & TLH

Vita

Michael I. Arak was born in 1990 in Princeton, New Jersey. He graduated in 2008 from Robbinsville High School in Robbinsville, New Jersey. He received a Bachelor of Science in Integrated Business & Engineering with a concentration in Financial Engineering in May 2012. He was awarded a Presidential Scholarship from Lehigh University for the academic year 2012-2013, and plans to graduate in May 2013 with a Master of Science in Management Science and Engineering.