

2017

# Attitude Estimation of Quadcopter through Extended Kalman Filter

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# **Attitude Estimation of Quadcopter through Extended Kalman Filter**

by

Wenjing Liang

Presented to the Graduate and Research Committee

of Lehigh University

in Candidacy for the Degree of

Master of Science

in

Mechanical Engineering

**Lehigh University**

**(May 2017)**

This thesis is accepted and approved in partial fulfillment of the requirements for the Master of Science.

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Date

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Professor Terry J. Hart, Thesis Advisor

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Dr. Gary Harlow, Chairperson of Department

## **Acknowledgments**

I would like to thank my advisor Professor Terry Hart, for his constant help and his patience. When I first meeting with Professor Hart, I have no idea what direction can I work on. After know what I'm interested in, professor provided me several subject that they are researching on. And I picked this topic, the attitude estimation of quadcopter. Professor Hart gave me a lot of very useful advice and lent me several relevant book on this topic. I really appreciate for his support and help during these semesters.

I would like to thank my parents for their love and support. Although they are in China, but they always care about me and encourage me when I felt disappointed.

And also, I want to thank my friend Jenny, she helped me to modify the expression and grammar problem of this thesis. I felt so lucky to have you. Thank you for the help.

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## **Abstract**

The aim of this paper is to estimate the attitude of the quadcopter using the sensors: 3-axes accelerometer, 3-axes gyroscope, 2-axes compass.

At first I introduce some basic conception of quadcopter, such as the three main factor: roll, pitch, yaw, and the coordinate system that are used to implement the next calculations. Then according to the mathematical model, I simulated the quadcopter in Simulink. The sensors are also modeled using the real sensor measurements to correctly estimate the measurement noise.

After finished the model, I gave it a step input and get the output from the scope. Then I add the Gaussian noise on to it and use this as the input of Extended Kalman Filter. And compare some different type of Kalman Filter to conclude that the EKF is the best strategy.

Finally we can conclude that the standard extended Kalman filter is the best estimator. If all of the parameters can be set correctly, The EKF can have a better result. But since it is not implement on the embedded system, it can be used only as a reference and provide satisfying result in most situations.

**Keywords:** Quadcopter, Extended Kalman Filter, Euler angle



# 1. Introduction

## 1.1 Quadcopter history

Quadcopter is not a new concept in nowadays. Etienne Oehmichen was the first scientist who experimented with rotorcraft design in 1920. Among the six design he underwent, his second prototype with four rotors and eight propellers are mounted on a single engine. Oehmichen used a cross shaped frame with two rotors on top of four arms. Due to the change in angle of the blades, so that the propellers spinning in the horizontal plane, can achieve lift and stability. For two of the propellers used for forward propulsion and others for steer. The quadcopter was able to make a few minutes' flight and a close circuit control.

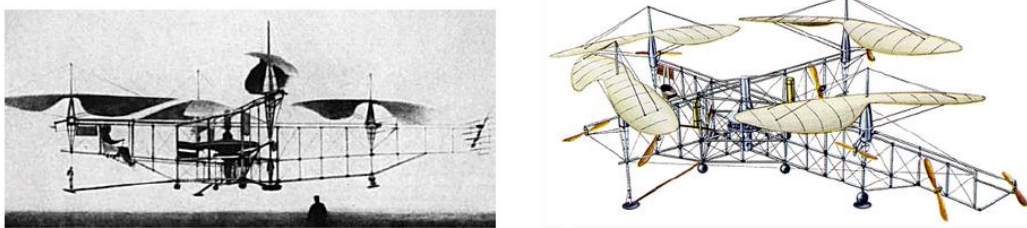


Figure 1.1 Oehmichen No.2 Quadcopter

But the development of quadcopter was once abandoned in favor of research for what have become out traditional helicopter. Only in 1956 would the concept be refreshed with the “ Convertawings Model A”, as shown in Figure 1.2 inspired by both Oehmichen and Bothezat, a four rotor aircraft was built. With proper power and simplified mechanism, this vehicle flew successfully.

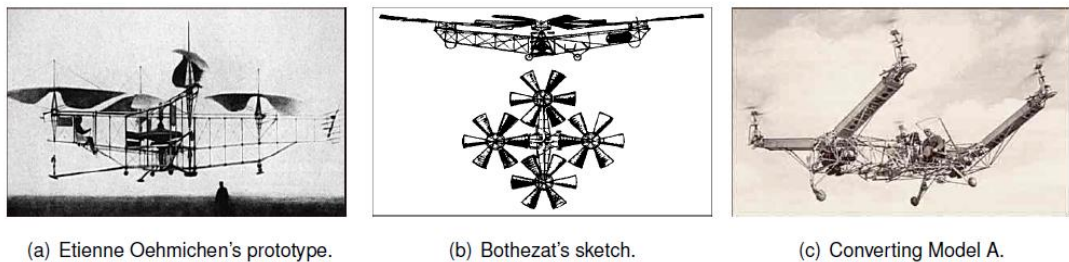


Figure 1.2 Development of the Quadcopter Concept

## **1.2 Structure of the thesis**

The work is divided into two parts. The first part deals with the modeling of the quadcopter, presenting the conventions used throughout the work, the dynamics and kinematics describing the quadrotor motion and the model of the sensors, including the evaluation of the parameters of the model. The choice of sensor is explained.

In the second part, a control model is presented and implemented in Matlab to generate data useful for the estimation purposes. Then different estimation approaches are discussed and compared using the Matlab.

Finally, the conclusions are drawn and some possible improvements for the stabilization of the quadcopter attitude are presented.

## 2. Modeling the Dynamic of a Quadrotor

### 2.1 Basic Conception

The concept of the quadcopter is not a new word to most of us. It is a helicopter with four rotors. The rotors are directed upwards and placed in a square formation with equal distance from the center of mass of quadcopter. The quadcopter is controlled by adjusting the angular velocities of the rotors. In this section, I will introduce some important factors, that are used for simulate a quadcopter.

#### 2.1.1 Roll, Pitch and yaw

To discuss the motion of quadcopter, it is necessary to mention three main factors: roll, pitch and yaw. These three motion can lead to the quadcopter's uplift and downfall. The space motion of the rigid body quadcopter can be divided into two parts: the barycenter movement and the movement around the barycenter. Six degree freedom are required in describing all of the space motion, which can be expressed as three rotation movement along three axes and three translation. We can control the six degree movements through adjusting the rotational speed of the motors.

- Roll

Rotation around the front-to-back axis is called roll. Roll is making the quadcopter fly sideward, either to left or right. Roll is controlled with the aileron stick.

- Pitch

Rotation around the side-to-side axis is called pitch. Pitch is the movement of quadcopter either forward or backward. Pitch is controlled by elevator.

- Yaw

Rotation around the vertical axis is called yaw. Yaw is the deviation/rotating the head of quadcopter either to right or left. Yaw can be controlled by the throttle stick, also called rudder.

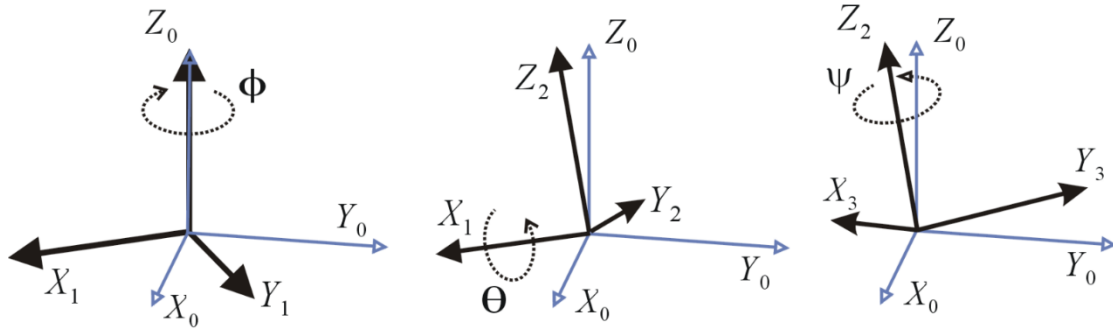


Figure 2.1 yaw, roll and pitch

### 2.1.2 Euler angle

The Euler angle are three angles introduced by Leonhard Euler, which used to describe the orientation of a rigid body. To describe such an orientation in the 3-dimensional Euclidean space, three parameters are required. And they can be described in several ways. We will use ZYX Euler angle. They are also used to describe the orientation of a frame of reference relative to another and they transform the coordinate of a point in a reference frame.  $\phi \in [-\pi, \pi], \theta \in [-\frac{\pi}{2}, \frac{\pi}{2}], \psi \in [-\pi, \pi]$ . Euler angle represents a sequence of three elemental rotations, since any movement can be achieved by combining three elemental rotations. The combination used is described by the following rotation matrix.

Rotation about the x-axis by angle  $\phi$  is

$$R_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & \sin(\phi) \\ 0 & -\sin(\phi) & \cos(\phi) \end{bmatrix} \quad (2.1)$$

Where  $\theta > 0$  indicates a counterclockwise rotation in the plane  $x=0$ . The observer is assumed to be positioned on the side of the plane with  $x > 0$  and looking at the origin. Rotation about the y axis by angle  $\theta$  is

$$R_y(\theta) = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix} \quad (2.2)$$

Where  $\theta > 0$  indicates a counterclockwise rotation in the plane  $y=0$ . The observer is assumed to be positioned on the side of the plane with  $y > 0$  and looking at the origin.

Rotation about the z axis by angle  $\psi$  is

$$R_z(\psi) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2.3)$$

The inertial position coordinate and the body reference coordinate are relate to the matrix

$R_{zyx}(\phi, \theta, \psi)$

$$\begin{aligned} P = R_{zyx}(\phi, \theta, \psi) &= R_z(\psi) \cdot R_y(\theta) \cdot R_x(\phi) \\ &= \begin{bmatrix} c(\theta)c(\psi) & s(\phi)s(\theta)c(\psi) - c(\phi)s(\psi) & c(\phi)s(\theta)c(\psi) + s(\phi)s(\psi) \\ c(\theta)s(\psi) & s(\phi)s(\theta)s(\psi) + c(\phi)c(\psi) & c(\phi)s(\theta)s(\psi) - s(\phi)c(\psi) \\ -s(\theta) & s(\phi)c(\theta) & c(\phi)c(\theta) \end{bmatrix} \end{aligned} \quad (2.4)$$

Where  $c(\phi) = \cos(\phi)$ ,  $s(\phi) = \sin(\phi)$ ,  $c(\theta) = \cos(\theta)$ ,  $s(\theta) = \sin(\theta)$ ,  $c(\psi) = \cos(\psi)$ ,  $s(\psi) = \sin(\psi)$ . The matrix describes the rotation from the body reference system to the inertial reference system.

The  $R_{zyx}(\phi, \theta, \psi)$  matrix describes the transformation Earth-fixed coordinate to the body fixed coordinate. The first set of state equation is describing the change of position according to quadcopter's attitude in its velocity measured in the body frame:

$$\begin{bmatrix} \vec{b}_1 \\ \vec{b}_2 \\ \vec{b}_3 \end{bmatrix} = P \begin{bmatrix} \vec{i} \\ \vec{j} \\ \vec{k} \end{bmatrix} \quad (2.5)$$

## 2.2 Coordinate system

Before describing the mathematical model for a quadcopter, it is necessary to introduce the reference coordinate system in which we describe the movement and the attitude. For the quadcopter, we need

to use two possible reference system. The first is the fixed and the second is mobile. The fixed system is also known as the inertial coordinate system. As fixed coordinate system, which is also called NED frame, for the frame centered on O, is set on the ground and point towards North, East and Down. The mobile reference system refers to the frame fixed on the quadcopter, and is united with the barycenter, the center of mass of the quadcopter. Thus is called the Body-fixed coordinate system or the Aircraft-Body-Centered frame. As shown in the Figure 2.2, are the two coordinate system.

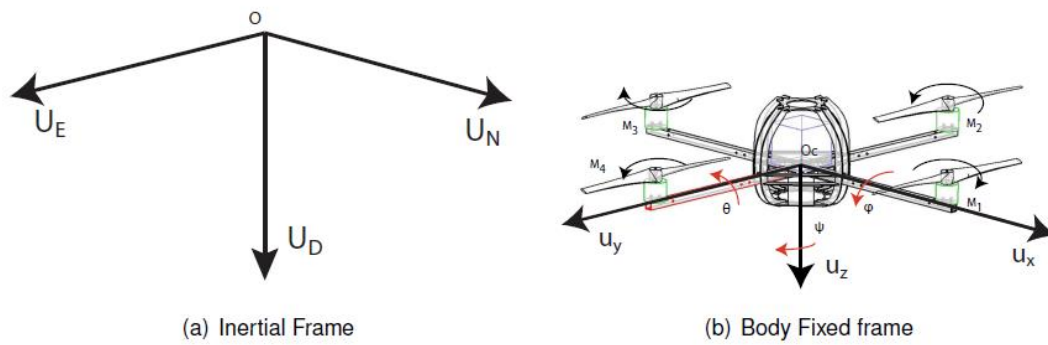


Figure 2.2 Inertial and Body-Fixed Frame

### 2.3 Mathematical model

We provide a mathematical model of quadcopter in this chapter, exploiting Newton and Euler equation for the 3D motion of a rigid body. The aim of this section is to further explain the movement of quadcopter and providing an reliable model that can be used for simulating and controlling the its behavior. We first set a sequence vector  $[x \ y \ z \ \phi \ \theta \ \psi]^T$  contains the linear and angular position of the quadcopter in the earth frame and  $[u \ v \ w \ p \ q \ r]^T$  the vector contains the linear and angular velocities in the body frame. From the 3D body dynamics, it says that these two frame has the following linked relation:

$$v = R \cdot v_B \tag{2.6}$$

$$\omega = T \cdot \omega_B \quad (2.7)$$

Where  $v = \begin{bmatrix} \dot{x} & \dot{y} & \dot{z} \end{bmatrix}^T \in R^3$ ,  $\omega = \begin{bmatrix} \dot{\phi} & \dot{\theta} & \dot{\psi} \end{bmatrix}^T \in R^3$ ,  $v_B = [u \ v \ w]^T \in R^3$ ,  $\omega_B = [p \ q \ r]^T \in R^3$ .

Figure 2.3 shows the free body diagram of the quadcopter.

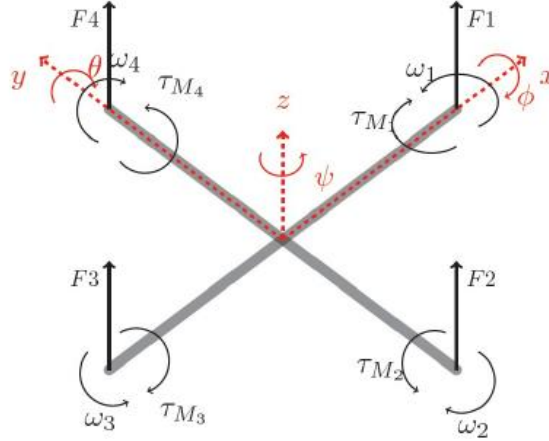


Figure 2.3 Free body diagram of quadcopter

### 2.3.1 Linear acceleration

The linear acceleration in Earth-fixed frame is described by the Newton's Second Law:

$$F = ma \quad (2.8)$$

Then,

$$\vec{F} = \left( \sum_{i=1}^{i=4} F_i \right) \vec{e}_3 - mg \vec{k} = m \frac{d^2}{dt^2} \vec{r} = m \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} \quad (2.9)$$

Where F refers to the external forces that worked on the quadcopter, m refers to the mass of the quadcopter, and v is the velocity of the quadcopter,  $F_i$  is the lift force on each rotor. From the transformation matrix (2.4) and (2.5), we can know that:

$$\vec{b}_3 = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \end{bmatrix} \begin{bmatrix} \cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi \\ \sin \psi \sin \theta \cos \phi - \sin \phi \cos \psi \\ \cos \theta \cos \phi \end{bmatrix} \quad (2.10)$$

Substitute (2.10) into (2.9), we can get:

$$\left(\sum_{i=1}^{i=4} F_i\right) \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \end{bmatrix} \begin{bmatrix} \cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi \\ \sin \psi \sin \theta \cos \phi - \sin \phi \cos \psi \\ \cos \theta \cos \phi \end{bmatrix} - mg \vec{k} = m \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} \quad (2.11)$$

From the equivalence of two matrix:

$$\begin{aligned} \ddot{x} &= \sum_{i=1}^4 K_i w_i^2 (\cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi) / m \\ \ddot{y} &= \sum_{i=1}^4 K_i w_i^2 (\sin \psi \sin \theta \cos \phi - \sin \phi \cos \psi) / m \\ \ddot{z} &= \sum_{i=1}^4 K_i w_i^2 (\cos \theta \cos \phi) / m - g \end{aligned} \quad (2.12)$$

### 2.3.2 Angular acceleration

Application of an external torque will change the angular momentum of the quadcopter:

$$\vec{M} = \frac{d\vec{H}}{dt} \quad (2.13)$$

But the angular momentum vector changes its direction , thus the total derivative of vector H is applied:

$$\vec{M} = \left. \frac{d\vec{H}}{dt} \right|_b + \vec{\omega} \times \vec{H} \quad (2.14)$$

For

$$\vec{M} = \vec{M}_1 + \vec{M}_2 \quad (2.15)$$

Where H is the angular momentum, M is the resultant external momentum on the quadcopter,

M1 is the momentum caused by the lift force, M2 is the momentum caused by the air drag, and

$M_2 = K_d \omega_i^2$  ,  $K_d$  is drag coefficient and  $\omega_i^2$  is the rotate speed of motor.

$$\vec{M}_1 = \sum_{i=1}^4 \vec{r}_i \times \vec{F}_i = l(F_3 - F_1) \vec{b}_2 + l(F_4 - F_2) \vec{b}_1 \quad (2.16)$$

$$\vec{M}_2 = K_d (\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \vec{b}_3 \quad (2.17)$$



$$\vec{M} = \vec{M}_1 + \vec{M}_2 = (\vec{b}_1, \vec{b}_2, \vec{b}_3) \begin{bmatrix} l(\mathbf{F}_4 - \mathbf{F}_2) \\ l(\mathbf{F}_3 - \mathbf{F}_1) \\ K_d(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \end{bmatrix} \quad (2.18)$$

Because the quadcopter can be viewed as a rigid body, so its inertia matrix is a diagonal matrix:

$$J = \begin{bmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{bmatrix} \quad (2.19)$$

The angular momentum matrix of quadcopter is :

$$\vec{H} = (\vec{b}_1, \vec{b}_2, \vec{b}_3) \begin{bmatrix} J_x \omega_x \\ J_y \omega_y \\ J_z \omega_z \end{bmatrix}$$

$$\frac{d\vec{H}}{dt} \Big|_b + \vec{\omega} \times \vec{H} = (\vec{b}_1, \vec{b}_2, \vec{b}_3) \begin{bmatrix} J_x \dot{\omega}_x + (J_z - J_y) \omega_y \omega_z \\ J_y \dot{\omega}_y + (J_x - J_z) \omega_x \omega_z \\ J_z \dot{\omega}_z + (J_y - J_x) \omega_x \omega_y \end{bmatrix} \quad (2.20)$$

We substitute (2.18) and (2.20) into (2.14) can get:

$$(\vec{b}_1, \vec{b}_2, \vec{b}_3) \begin{bmatrix} J_x \dot{\omega}_x + (J_z - J_y) \omega_y \omega_z \\ J_y \dot{\omega}_y + (J_x - J_z) \omega_x \omega_z \\ J_z \dot{\omega}_z + (J_y - J_x) \omega_x \omega_y \end{bmatrix} = (\vec{b}_1, \vec{b}_2, \vec{b}_3) \begin{bmatrix} l(\mathbf{F}_4 - \mathbf{F}_2) \\ l(\mathbf{F}_3 - \mathbf{F}_1) \\ K_d(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \end{bmatrix} \quad (2.21)$$

From the equivalence of two matrix:

$$\dot{\omega}_x = \frac{[l(\mathbf{F}_4 - \mathbf{F}_2) + (J_z - J_y) \omega_y \omega_z]}{J_x}$$

$$\dot{\omega}_y = \frac{[l(\mathbf{F}_3 - \mathbf{F}_1) + (J_x - J_z) \omega_x \omega_z]}{J_y} \quad (2.22)$$

From Euler dynamical equation:

$$\omega = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \dot{\psi} \sin \phi \sin \theta + \dot{\theta} \cos \phi \\ \dot{\psi} \cos \phi \sin \theta - \dot{\theta} \sin \phi \\ \dot{\phi} + \dot{\psi} \cos \theta \end{bmatrix} \quad (2.23)$$

$$\omega = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \dot{\phi} \\ \dot{\psi} \\ \dot{\phi} \end{bmatrix} \quad (2.24)$$

We define  $U_1, U_2, U_3, U_4$  as the input of the four control channel of the quadcopter, it can be simplified as

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} F_1 + F_2 + F_3 + F_4 \\ F_4 - F_2 \\ F_3 - F_1 \\ F_2 + F_4 - F_1 - F_3 \end{bmatrix} = \begin{bmatrix} K_t \sum_{i=1}^4 w_i^2 \\ K_t (w_4^2 - w_2^2) \\ K_t (w_3^2 - w_1^2) \\ K_d (w_1^2 + w_3^2 - w_2^2 - w_4^2) \end{bmatrix} \quad (2.25)$$

Where  $U_1$  is the control of the vertical input,  $U_2$  is the control of the roll input,  $U_3$  is the control of the pitch input, and  $U_4$  is the control yaw.  $\omega_i$  is the rev of the propeller.

From (2.12),(2.22) and (2.25) , the dynamic model of the quadcopter in the inertial frame is:

$$\begin{aligned} \ddot{x} &= (\cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi) U_1 / m \\ \ddot{y} &= (\sin \psi \sin \theta \cos \phi - \sin \phi \cos \psi) U_1 / m \\ \ddot{z} &= (\cos \theta \cos \phi) U_1 / m - g \\ \ddot{\phi} &= \frac{[l U_2 + \dot{\theta} \psi (J_z - J_x)]}{J_y} \\ \ddot{\omega}_x &= \frac{[l U_3 + \dot{\phi} \psi (J_y - J_z)]}{J_x} \\ \ddot{\omega}_z &= \frac{[U_4 + \dot{\phi} \dot{\theta} (J_x - J_y)]}{J_z} \end{aligned}$$

### 2.3.3 Simulation of quadcopter

According to the mathematical model of the quadcopter, I built a model in simulink.

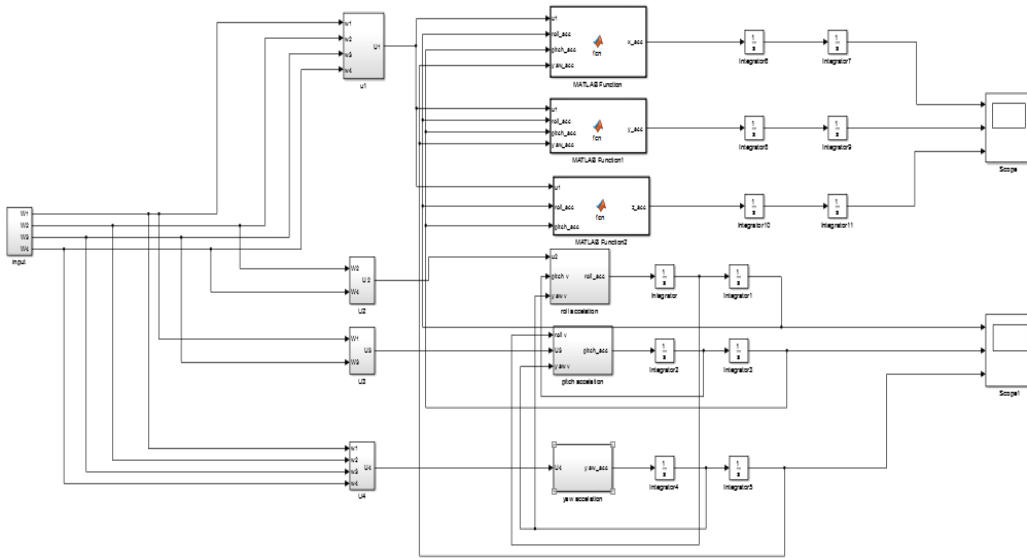


Figure 2.4 Simulation of the quadcopter

The model basically is divide into for part: input, acceleration on x,y,z direction, acceleration on roll, pitch and yaw, and the output. The input of the quadcopter are the angular velocity of each rotor.

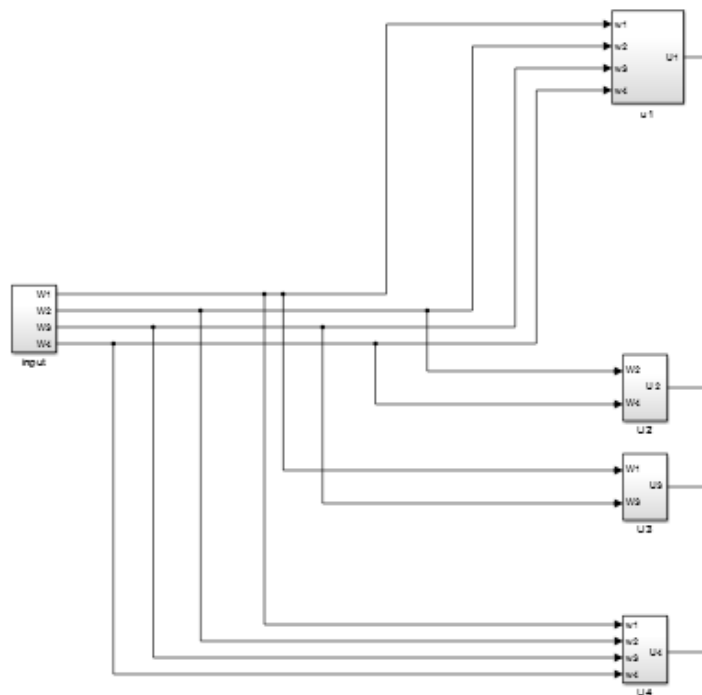


Figure 2.5 Input of the quadcopter

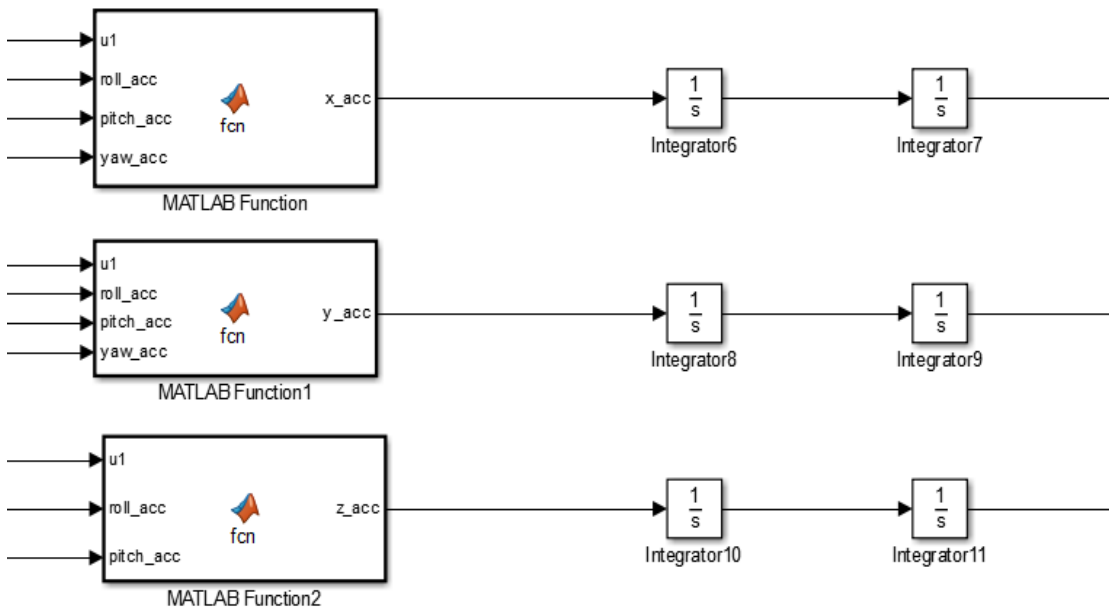


Figure 2.6 X, Y, Z acceleration

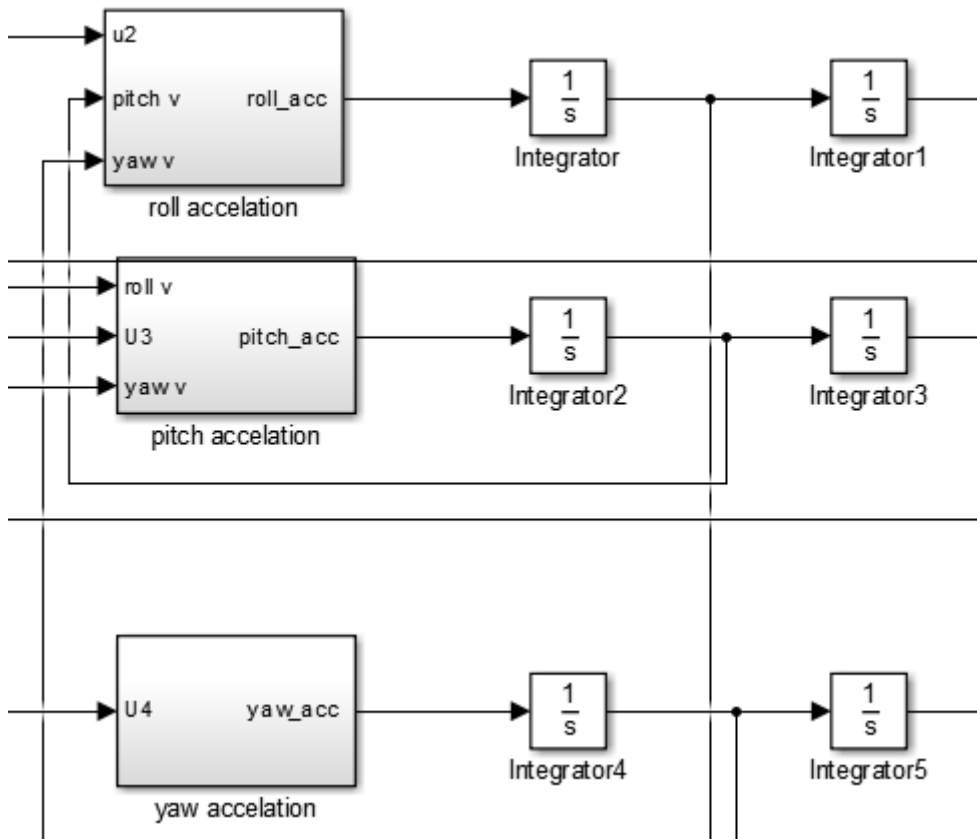


Figure 2.7 Roll, pitch and yaw acceleration

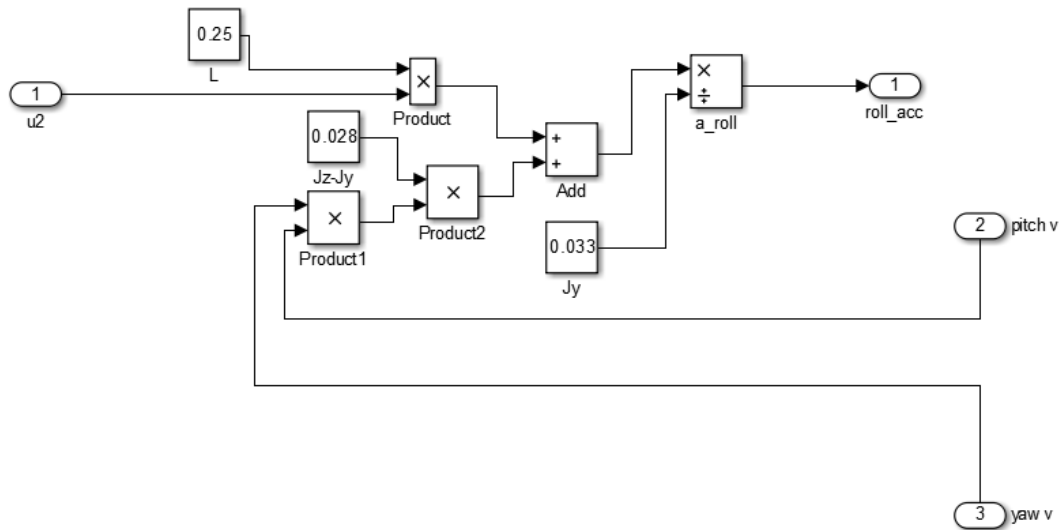


Figure 2.8 Detailed composition inside the roll acceleration

From this simulation, we can simply control the angular velocity of each rotor as the input, and get the displacement on x, y, and z axis and the angular change on roll, pitch, and yaw as the output.

### **3. Sensor**

In this section, I will give a brief description of the option of sensor, and explain their principle of operation. The noise associated to the sensors is measured on the prototype for greater accuracy of the simulation.

#### **3.1 Choosing the sensor**

In order to justify the choice of the sensors and explain what the sensors must be sensitive to, I have to quote a reference to the basic concepts. As said in Wahba's paper in 1966, the solution for estimating the attitude of the satellite lies on finding a matrix which brings a first set of two non-collinear vectors into the best coincidence with a second set of non-collinear vectors. The important idea is that, for a correct estimation, the measurement of two non-collinear vector are needed. For example, the main factors in selecting a position sensor are excitation, filtering, environment, and whether line of sight or a direct, physical connection is required to measure distance. Both the preference and application play an important role in making a decision.

In this case, we use the gyroscopes provide the main information which is updated by a gravity sensor and a magnetic sensor. The accelerometer is used for the gravity vector and the magnetic compass is used for the magnetic vector. In consequence, the chosen sensor are:

- A 1-axis Gyroscope LY530ALH for  $U_z$  and a 2-axis Gyroscope LPR530AL for  $U_x$  and  $U_y$
- A 3-axis accelerometer ADXL330
- A2-axis compass Honeywell HMC6352

#### **3.2 Model of the sensor**

The sensors played an important role in measure the state variable. Naturally, they carry limitations directly linked to their inherent functioning principle. By understanding it, it becomes

possible to reduce the noise affecting the measurement and guarantee the correct behavior of the quadcopter.

### 3.2.1 Gyroscope

The gyroscope was created according to miniaturization and micro machining techniques. It can be divided into several categories due to their sensing elements. According to [1] the simplest MEMS gyroscopes are vibratory rate will is similar as the chosen one. The sensor LPR530AL data sheet presents a value for the noise density and [2] suggest a value for bias. These theoretical characteristics of the sensor are presented in Table 3.1:

measured range	$\pm 300^\circ/s$
sensitivity	$3.33mV/(\circ/s)$
noise	$0.035(\circ/s)/\sqrt{Hz}$
bias	$2.8(\circ/s)/s$
bias varying rate	$0.1Hz$
resolution	$0.9678^\circ/s$

Table 3.1 Gyroscopes Characteristics

### 3.2.2 Accelerometer

To define the pitch and yaw angle, we use the accelerometer is to measure the direction of the gravity vector,  $g$ . At first we set it as a constant, and pointing down along  $U_d$  with an intensity  $g_0 = 9.8m/s^2$  and set  $\bar{a}^B$  denote the accelerometer vector.

There are different types of MEMS accelerometers based on different functioning principles, some use the piezoelectric effect to create an electric signal due to accelerative forces. According to [3], for the chosen frame system, we can write:  $\bar{a}^B = g^B - a^B$ . As the Gaussian measurement mentioned in [2], the general model can be written as:

$$\bar{a}^B = Sg^J - a^B + \mu_a + b_a$$

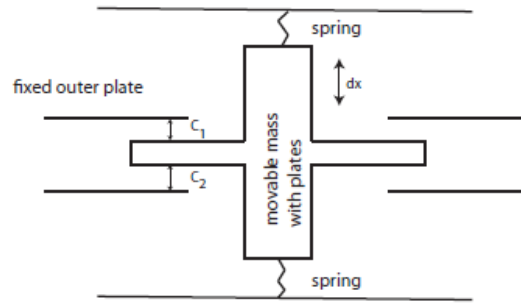


Figure 3.2 Functioning Principle of Accelerometer

Where  $\bar{a}^B$  is the sensor output in  $m/s^2$ ,  $S$  is the rotation matrix,  $g^J$  is the gravity vector define in the inertial frame,  $a^B$  is the acceleration of the craft due to its movement defined in the body fixed frame,  $\mu_a$  is a Gaussian noise component,  $b_a$  is the constant bias of the accelerometer.

The theoretical values for the chosen sensor is shown below:

measured range	$\pm 3g$
sensitivity	$330mV/g$
noise $X_{out}, Y_{out}$	$150\mu g/\sqrt{Hz}rms$
noise $Z_{out}$	$300\mu g/\sqrt{Hz}rms$
resolution	$0.0958m/s^2$

Table 3.3 Accelerometer Characteristics

### 3.2.3 Compass

The compass is a sensor designed to detect the magnetic North direction, written as  $N^t$ . By the definition of the reference frame and neglecting the magnetic inclination or magnetic dip,

$$N^t = [1 \ 0 \ 0]^T.$$

For the chosen compass Honeywell HMC6352, is a two axis magneto-resistive sensor; It is made of thin strips of permalloy, whose electrical resistance varies with a change in the applied magnetic field [4]. By measuring the electric resistance, the orientation of the magnetic field can



be estimated. Let  $N^B$  be the sensor measurement, considering the frame system, a Gaussian measurement noise,  $\mu_m$  and a bias term,  $b_m$ . As in [5], the compass measurement corresponds to:

$$N^B = SN^t + \mu_m + b_m$$

Where  $N^B$  is the measured magnetic vector,  $N^t$  is the magnetic vector,  $\mu_m = [\mu_{m-x} \quad \mu_{m-y} \quad \mu_{m-z}]^T$  is the Gaussian measurement noise,  $b_m = [b_{m-x} \quad b_{m-y} \quad b_{m-z}]^T$  is the measurement bias.

The Gaussian noise term is given on the data sheet of the sensor and presented in Table 3.4

measured range	360°
heading resolution	1°
noise	2.5° <i>rms</i>

Table 3.4 Compass Characteristics

### 3.2.4 Discussion of the choice of sensor

We need to combine the characteristics of the model and the advantage of the sensor when we making a choice. The acceleration corresponding to the movement of the craft, to gravity and to vibrations of motors, this will make the accelerometer estimation difficult. Moreover, these vibrations are usually considered in the noise term of the model. These vibrations affect the high frequency measurements of the sensor. On the other hand, the gyroscopes measure angular rates which are subjected to a low frequency bias.

It becomes clear that only the accelerometer is not efficient in providing information about pitch and roll because of its inaccuracy on high frequencies. The gyroscope fails at low

frequencies. The fusion comes as a good idea. This explains the difficulties experience by Jorge Domingues [6] and justifies the acquisition of the gyroscopes. For security reasons it is desired to have the quadcopter above a certain threshold but also not flying too high, which might probably become a threat. Therefore, the range finder was acquired.

## 4. Attitude Estimation

During the quadcopter's navigation, neither the accelerometer nor the gyro could be used by itself to get accurate reading. The accelerometer can be used to get estimate the direction of the gravity acceleration vector while stationary but it will become difficult when the quadcopter is moving. The gyro reading on the other hand can be integrated over time to get angle relative to starting point but each measurement error will also integrate into angle estimation. So we can use the Kalman filter to merge sensor reading of a gyro and an accelerometer into pitch and roll of the quadcopter.[9]

### 4.1 Introduction of Kalman Filter

The Kalman filter uses a dynamical model for the time development of the system and a model of the sensor measurements to obtain the most accurate estimate possible of the system state using a linear estimator based on present and past measurements. [14] It is ideally suited to both ground-based and on board attitude determination. However, the applicability of the Kalman filtering technique rests on the availability of an accurate dynamical model.

The dynamic equation for the spacecraft attitude pose many difficulties in the filter modeling. In particular, the external torques and the distribution of momentum internally due to the use of rotating or rastering instruments lead to significant uncertainties in the modeling. For autonomous spacecraft permits the circumvention of these problems.[7] In this representation the angular velocity of the spacecraft is obtained from the gyro data. The kinematic equations are used to obtain from the attitude state and this is augmented by means of additional state-vector components for the gyro biases. Thus, gyro data are not treated as observations and the gyro noise appears as state noise rather than as observation noise.

The Kalman filter consists of six steps, executed over and over again with a sample time between the executions [12] . As shown in Figure 4.1:

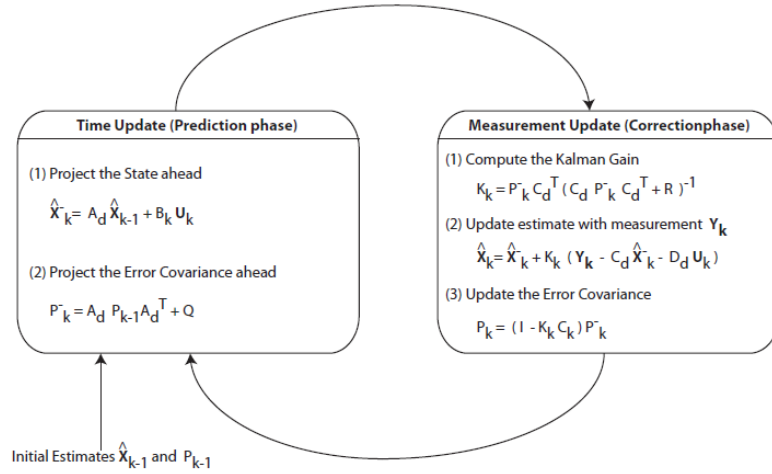


Figure 4.1 Mathematical Formulation of Kalman Filter

- **State prediction based on dynamic model**

This is one of the most important parts of Kalman filter. The model is used to predict the next state based on the current state and control signals. Since we separate the estimation of pitch and roll, state  $x$  according to one of the filter can be described by three different variables (two identical filters are created).

$$x = \begin{bmatrix} \theta \\ \dot{\theta} \\ \dot{\theta}_b \end{bmatrix}$$

Three rows refer to the angle, the angular velocity and the bias in the angular velocity. Using this state representation the equation:

$$x_k = F x_{k-1} + B u_k + w_k \quad (4.1)$$

Can be used to make a good guess of the states in this sample based on the state in the previous sample and the current control signal.

The matrix represents the dynamics of the system. Multiplied with the old states the result is a guess of the new states. For the quadcopter the following matrix is used:

$$F = \begin{bmatrix} 1 & \Delta t & -\Delta t \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Expand the matrix equation for  $\mathbf{x}_k$  the result will be:

$$\begin{aligned} \theta_k &= \theta_{k-1} + \Delta t(\dot{\theta}_{k-1} - \dot{\theta}_{b(k-1)}) \\ \dot{\theta}_k &= \dot{\theta}_{k-1} \\ \dot{\theta}_{bk} &= \dot{\theta}_{b(k-1)} \end{aligned} \quad (4.2)$$

The angle in the next sample will be the angle in the sample before plus the unbiased angular velocity multiplied with the sampling time. The model also shows that the other two states will remain the same as in the previous sample which means the angular velocity and angular velocity measuring bias will change slow enough to be barely noticeable between two sample times.[9]

The next part of equation (4.1) involves the control signal input to the system. The multiplication with the  $\mathbf{B}$  matrix can be expanded in exactly the same way as the  $\mathbf{F}$  matrix. Feed the filter with the input to the motors affecting the filter angle  $u_1$  and  $u_2$  and you also know that the force generated by the motor is linearly dependent of the input by the constant  $k$ . You also know the inertia  $J$  around the axis of the angle. So  $u$  and  $B$  can be represented like this:

$$B = \begin{bmatrix} 0 & 0 \\ \frac{k}{J} & \frac{k}{J} \\ 0 & 0 \end{bmatrix}$$

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Resulting in a model for the estimation of the angular velocity looking like this

$$\dot{\theta}_k = \dot{\theta}_{k-1} + \frac{k}{J}(u_1 - u_2) \quad (4.3)$$

Since the model is pretty coarse there is a need for something more to take model errors into consideration. It is easy to see that the model for the 2<sup>nd</sup> and 3<sup>rd</sup> state will not be 100% accurate all the time. Then we introduce  $w$ , which is known as model noise. A zero mean Gaussian noise is added to each state to represent changes that doesn't agree with the model. In mathematical terms

$$w \sim N(0, Q)$$

$Q$  is the covariance matrix of the noise, For the quadcopter the noise on each state is considered to be independent which makes  $Q$  diagonal

$$Q = \begin{bmatrix} q_1 & 0 & 0 \\ 0 & q_2 & 0 \\ 0 & 0 & q_3 \end{bmatrix}$$

The elements on the diagonal represent how large model errors we expect in the different states and will be an important tuning parameter in the filter.

- **State covariance matrix update based on model**

Another important part of the Kalman filter is the matrix  $P$  which represents how much we trust the current values of the state variables. A small  $P$  matrix means that the filter has converged close to the real value. When the model predictions in step 1 have been done the  $P$  matrix has to be updated according to the uncertainties induced by the model noise  $w$  with the covariance  $Q$ . This is done with the equation

$$P_k = F P_{k-1} F^T + Q \tag{4.4}$$

- **Model and measurement differences**

The next part of the filter is the new measurements come in. In this step the difference between the states calculated from the model is compared to what is measured using the equation

$$\mathbf{y}_k = \mathbf{z}_k - \mathbf{H}\mathbf{x}_k + \mathbf{v}_k \quad (4.5)$$

H matrix maps the current state on the measurements. We measure the angle based on the gravity vector by using the arctan(b, a) on the acceleration vectors perpendicular to the filters rotational axis and the angular rate measured by the gyro. This results in a measurement vector

$$\mathbf{z} = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}$$

To map or state vector  $\mathbf{x}$  onto the measurement vector  $\mathbf{z}$  we multiply by the matrix

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

The vector  $\mathbf{v}$  is similar to  $\mathbf{w}$  it represents errors in the measurements which are assumed to be zero mean and Gaussian distributed as well

$$\mathbf{v} \sim N(0, \mathbf{R})$$

$\mathbf{R}$  is similar to  $\mathbf{Q}$  the covariance matrix of the measurements. This can be directly related to the sensor quality. As with the  $\mathbf{Q}$  matrix the sensor errors are assumed to be independent which make the  $\mathbf{R}$  matrix diagonal as well

$$\mathbf{R} = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix}$$

- **Measurement covariance update**

After including new data in form of measurements we need to calculate the covariance matrix of this data which is known as  $S$  and is similar in many ways to  $P$ . This is calculated with the equation

$$S_k = HP_kH^T + R \quad (4.6)$$

As we can see from the equation, the value of  $S$  depends on the covariance of the previous model predictions transformed to the measurement vector through  $H$  plus the covariance of the sensor readings.

- **Calculate Kalman gain**

In this part we merge the knowledge from the model with the measurements. This is done through a matrix called the Kalman gain,  $K$ . This matrix will help us weigh the measurements and the model together.  $K$  is calculated by

$$K_k = P_kH^T S_k^{-1} \quad (4.7)$$

- **Improve model prediction**

Now it is time to improve the model prediction by adding the difference between measurements and predictions to the states predicted in step 1 after scaling it with the gain matrix  $K$ :

$$\mathbf{x}_k = \mathbf{x}_k + K_k \mathbf{y} \quad (4.8)$$



The result will be the filter output at this sample time.

- **Update state covariance**

Since we added data in form of measurements to the state vector we need to update the state covariance matrix  $P$  which is done by the equation

$$P_k = (I - K_k H) P_k \quad (4.9)$$

Where  $I$  is the identity matrix

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

After this step is done we can wait for the next sample and then start over at first step .

## 4.2 Extended Kalman Filter used in attitude estimation

When we want to use matlab to illustrate the whole process, we need the following steps to achieve it.[8]

First we need to initiate state variable:

$$\begin{aligned} & \left[ w_x \quad w_y \quad w_z \quad wa_x \quad wa_y \quad wa_z \quad ze_x \quad ze_y \quad ze_z \quad mu_x \quad mu_y \quad my_z \right]^T \\ = & \left[ gyro_x \quad gyro_y \quad gyro_z \quad gyro_{accx} \quad gyro_{accy} \quad gyro_{accz} \quad acc_x \quad acc_y \quad acc_z \quad mag_x \quad mag_y \quad mag_z \right]^T \end{aligned}$$

where  $w$  is the angular velocity,  $wa$  is the angular acceleration,  $zw$  is the gravity vector and  $mu$  is the magnetic vector.

Then we need to compute the current state estimate from the previous estimation body angular

accelerations.

$$wak = \begin{bmatrix} wa_x \\ wa_y \\ wa_z \end{bmatrix} + J^{-1} * \begin{bmatrix} -wa_x \\ wa_y \\ wa_z \end{bmatrix} \otimes (J * \begin{bmatrix} wa_x \\ wa_y \\ wa_z \end{bmatrix}) * dt$$

The body angular rate is :

$$wk = \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} + dt * wak$$

Then calculate derivative of the prediction rotation matrix:

$$O = \begin{bmatrix} 0 & -w_z & w_y \\ w_z & 0 & -w_x \\ -w_y & w_x & 0 \end{bmatrix}$$

Calculate the prediction of the earth z vector:

$$zek = \left( \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + O * dt \right) \begin{bmatrix} ze_x \\ ze_y \\ ze_z \end{bmatrix}$$

$$muk = \left( \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + O * dt \right) \begin{bmatrix} mu_x \\ mu_y \\ mu_z \end{bmatrix}$$

Compute the error covariance estimate from the previous estimation:

$$EZ = \begin{bmatrix} 0 & ze_z & -ze_y \\ -ze_z & 0 & ze_x \\ ze_y & -ze_x & 0 \end{bmatrix}^T$$

$$MA = \begin{bmatrix} 0 & mu_z & -mu_y \\ -mu_z & 0 & mu_x \\ mu_y & -mu_x & 0 \end{bmatrix}^T$$

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

A state convert matrix is :

$$A_{lin} = \begin{bmatrix} Z & E & Z & Z \\ Z & Z & Z & Z \\ EZ & Z & O & Z \\ MA & Z & Z & O \end{bmatrix}$$

The process noise matrix is :

$$Q = \begin{bmatrix} q_{rs} & q_{rs} & q_{rs} \\ q_{ra} & q_{ra} & q_{ra} \\ q_{acc} & q_{acc} & q_{acc} \\ q_{mag} & q_{mag} & q_{mag} \end{bmatrix}$$

Where  $q_{rs}$  is process of gyro,  $q_{ra}$  is the process noise of gyro acceleration,  $q_{acc}$  is the process noise of acceleration and  $q_{mag}$  is the process noise of magnetometer.

The process covariance matrix is:

$$P_{apr} = A_{lin} * P_{apa} * A_{lin}^T + Q$$

The measurement noise matrix is:

$$R_v = \begin{bmatrix} r_{gyro} & r_{gyro} & r_{gyro} & r_{acc} & r_{acc} & r_{acc} & r_{mag} & r_{mag} & r_{mag} \end{bmatrix}$$

The observation matrix is:

$$H_k = \begin{bmatrix} E & Z & Z & Z \\ Z & Z & E & Z \\ Z & Z & Z & E \end{bmatrix}$$

Then using the Kalman Filter to get the estimation of the attitude. The model is used to predict the next state based on the current state and control signals:

$$Y_k = z(1:9) - H_k * X_{apr}$$

State covariance matrix update based on model:

$$S_k = H_k * P_{apr} * H_k^T$$

Model and measurement differences:

$$S_k(1:9+1:end) = S_k(1:9+1:end) + R_v$$

Measurement covariance update:

$$K_k = (P_{apr} * H_k^T) / S_k$$

Calculate Kalman gain:

$$X_{apo} = X_{apr} + K_k * Y_k$$

Improve model prediction:

$$P_{apo} = (I_{12 \times 12} - K_k * H_k) * P_{apr}$$

Euler angle extraction:

$$Z_n^b = \frac{-X_{apo}(7:9)}{|X_{apo}(7:9)|}$$

$$M_n^b = \frac{X_{apo}(10:12)}{|X_{apo}(10:12)|}$$

$$X_n^b = Y_n^b \otimes Z_n^b$$

$$X_n^b = \frac{X_{apo}}{|X_{apo}|}$$

$$T = [X_n^b \quad Y_n^b \quad Z_n^b]$$

$$\begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix} \arctan\left(\frac{T_{23}}{T_{33}}\right) \\ -\arcsin(T_{13}) \\ \arctan\left(\frac{T_{12}}{T_{11}}\right) \end{bmatrix}$$

### 4.3 Simulation and discussion about Kalman Filter

The estimations obtained with the different kinds of Kalman Filter models are presented in

Figure 5.1.

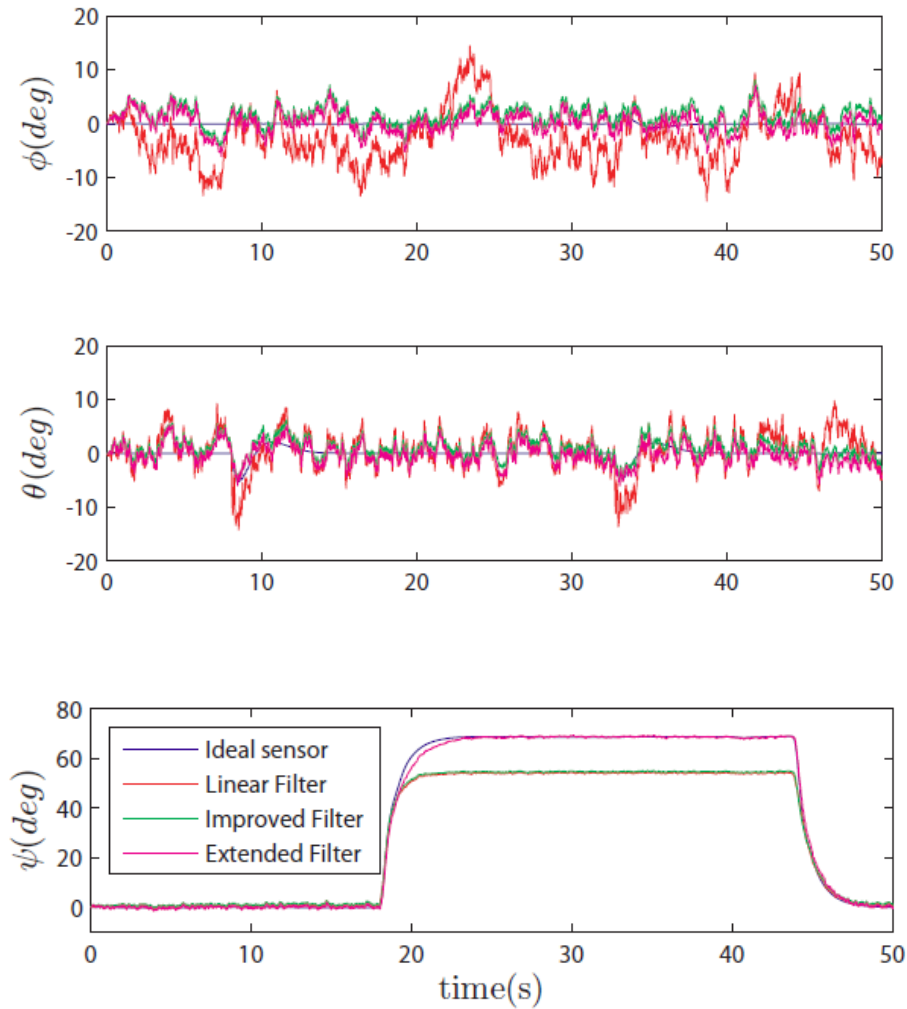


Figure 5.1 Attitude Estimation with Kalman Filters

Estimator	Estimated Angles								
	$\phi$			$\theta$			$\psi$		
	mse (deg) <sup>2</sup>	std (deg)	mean (deg)	mse (deg) <sup>2</sup>	std (deg)	mean (deg)	mse (deg) <sup>2</sup>	std (deg)	mean (deg)
Linear Filter	29.91	4.80	2.62	9.70	3.09	0.40	104.47	7.38	7.06
Improved Filter	5.08	1.85	1.28	4.22	1.59	1.32	99.22	7.31	6.75
Extended Filter	4.20	2.02	0.33	3.19	1.78	0.12	1.07	1.02	0.13

Table 5.2 Comparison of Extended Kalman Filter with accurate data

The linear filter has the advantage of estimating all necessary states whereas the improved filter has no processing angle rates. On the other side, the improved filter is computationally more efficient and presents a better estimate. The advantage of Extended Kalman Filter is to overcome the  $\psi = 1rad$  limitation. Therefore, considering that for the hovering situation, the angles  $\phi$  and  $\theta$  do not change considerably, the system is only linearized around  $\psi_k$ . It suggests that the Extended Kalman Filter filters can reaches a very good stationary estimation.

The figure below shows the Kalman Filter applied on practical model. I introduce the Gaussian White noise to the input signal. And let it pass through the Extended Kalman Filter, then we can get the plot of the original signal, estimated signal and the mean square error between these two. The result are shown below:

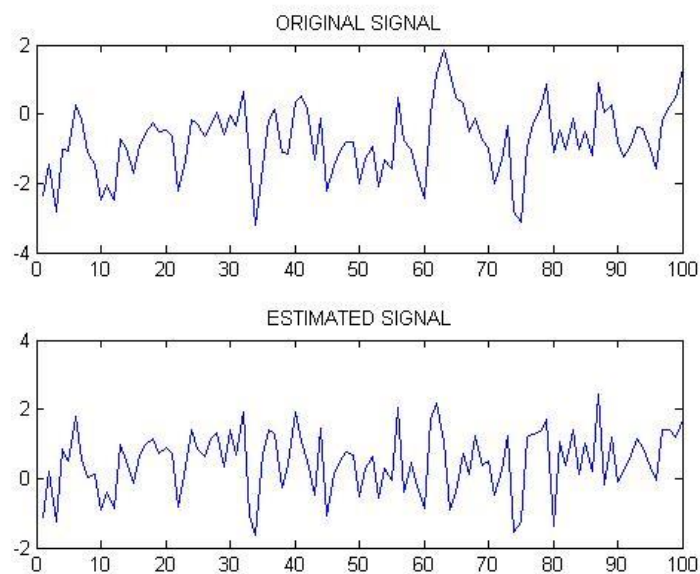


Figure 5.3 Original signal and Estimated signal

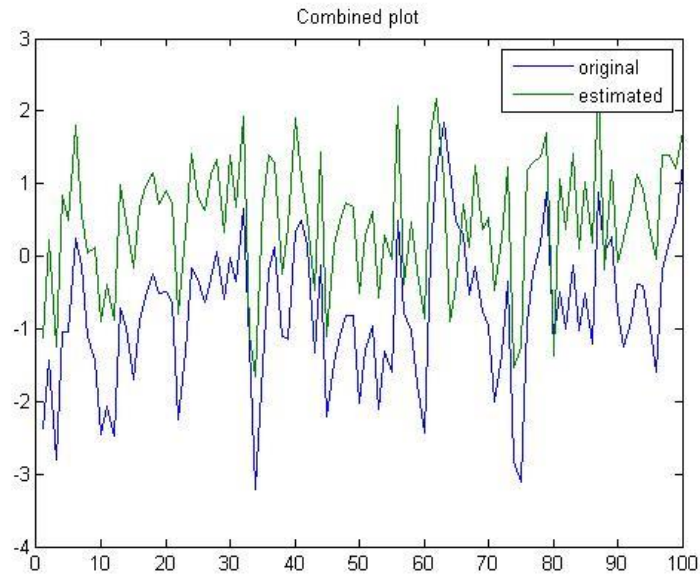


Figure 5.4 Combination of two signal

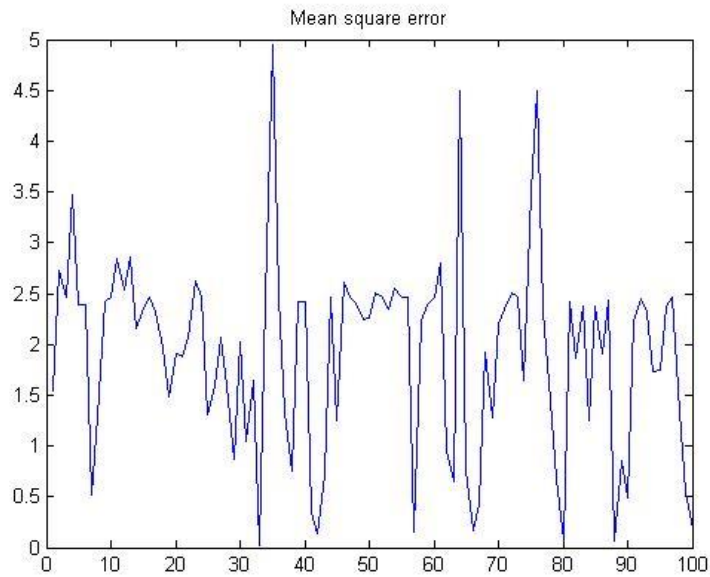


Figure 5.5 Mean square error between two signals

From the figure, we can conclude that the standard extended Kalman filter is the best estimator. If all of the parameters can be set correctly, The EKF can have a better result. But since it is not implement on the embedded system, it can be used only as a reference and provide satisfying result in most situations.

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## **Vita**

Wenjing Liang was born on August 19, 1993 in Shandong, China, daughter of Dangwei Liang and Huiying Zhang. After finishing high school in 2011, she entered Huazhong University of Science and Technology and studied in the field of Mechanical Engineering in China. She completed the Bachelor of Science in Mechanical Engineering in 2015. After completing the B.S., he entered Lehigh University in 2015. She is currently a graduate student at Lehigh University.