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# Calculus & Technology: Conceptual Explorations and Applications

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Calculus & Technology: Conceptual Explorations and Applications

By

Kellie Keiser

A Thesis

Presented to the Graduate and Research Committee

of Lehigh University

in Candidacy for the Degree of

Master of Science

in

Applied Mathematics

Lehigh University

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This thesis is accepted and approved in partial fulfillment of the requirements for the Master of Science.

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Date

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## ABSTRACT

Graphing calculators made their first appearance in classroom settings in the early 1980's and have grown to have a valuable place in the instruction of mathematics. The technology has advanced to a point where the calculators are more similar to software programs that have been available on the computer, but are far less expensive compared to the price of a computer and the accompanying software. Many experts in the field of mathematics education advocate the use of graphing calculators. In fact, the National Council of Teachers of Mathematics (NCTM) advocates the integration of graphing calculators into the classroom beginning at the elementary school level.

One of the areas of concern with the implementation of lessons in the classroom using graphing calculators is that not all instructors share the same pedagogical view on how the graphing calculators should be used and, therefore, do not use the graphing calculators in the same manner. Since there are no standards for implementation, graphing calculator usage is varied from classroom to classroom. It has been shown in studies discussed in this document that student performance in mathematics is linked to student ability to model mathematical situations graphically, numerically, and algebraically. With this in mind, the graphing calculator activities Exploring the Relationship Between Average and Instantaneous Rates of Change with the TI-*n*spire CX Handheld and Exploring the Relationship Between Derivatives and Integrals with the TI-*n*spire CX Handheld were created to help students discover two major concepts in Calculus, the derivative and the integral.

The results presented in this document represent a small sample of students taking an introductory college- level Calculus course. Although results appear to indicate that

student competency in graphical, numerical, and algebraic modeling, as well as technological competency, may be linked to student success in the graphing calculator lab activities, more research on a larger sample group of students will need to be completed to substantiate the results. In informal discussions with students while they were completing the lab activities, many of the lower-performing students attributed their inability to complete the labs without assistance based on the fact that they were initially unable to link the graphical, numerical, and algebraic representation together to draw the correct conclusions. When students had completed the labs and results were discussed as a group, this lower-performing group was able to see the connections between the different representations and was able to understand the concepts that were being explored.

Without the use of a graphing calculator, explorations like those presented in this document would be very difficult to implement. The graphing calculator allows students to perform computations that would require a great deal of time and would detract from the overall purpose of the labs. In addition, the data collection features on the graphing calculators allow students to explore data in the world around them, allowing them to make mathematical connections by focusing on topics that are important and meaningful to them. The graphing calculators not only carry out mathematically difficult computations, which are often time-consuming, they also allow students to easily represent data graphically, numerically, and algebraically. As the graphing calculator continues to make its way into the mathematics classroom and students are able to explore mathematical models in multiple representations, researchers are optimistic that student performance in mathematics will continue to improve.

## THE ROLE OF THE GRAPHING CALCULATOR IN EDUCATION

As a student of calculus in the late 1980's and early 1990's, my learning experience was one that seemed to mirror what is thought of as the "traditional approach" to learning calculus. Success was determined, primarily, by the ability to perform symbolic manipulation to achieve the correct result. Assignments consisted mainly of numerous problems designed to provide reinforcement of the given topic. The ability to be successful at these tasks, and to receive high marks, did not necessarily require an in-depth understanding of the concepts being covered, but rather the procedures required to obtain the correct solution. Samuels (2011, p.10) notes that although many students are able to maintain high marks employing this tactic, the conceptual understanding is lacking, which can cause additional difficulties in the continuing study of calculus.

With the onset of rapid changes in technology in the late 1980's and early 1990's, came changes in the way that calculus was being presented in the classroom. The graphing calculator was first introduced in 1985 and has since become commonplace in the calculus classroom. As the graphing calculator made its way into the calculus curriculum, there were several factors that were of concern when considering the manner in which the device would be integrated into the traditional curriculum. The three main concerns were accessibility to the graphing calculator, the manner in which the instructor integrated the graphing calculator into the curriculum, and the pedagogical concerns over the manner in which it had been observed that students tended to model the instructor's method of solution, rather than seeking alternate methods of solution and approaches to problem solving on their own (Kastberg, 2005).

As far as accessibility is concerned, graphing calculators are readily available in large chain stores and via the internet. Texas Instruments and Casio are currently the largest competitors in the graphing calculator market. The TI 84 graphing calculator model is widely available to students and is relatively affordable. The calculator can be purchased new, with accompanying software in PC and Mac formats, for approximately \$100. In addition, free operating system updates are available for download via the internet. As mentioned, several of the graphing calculator models can be purchased with accompanying software. The software allows the user to represent the mathematical model under consideration graphically, numerically, and algebraically on the same screen. This allows the user flexibility in problem solving. It has also been shown that being well-versed in solving problems using various representations of a mathematical model is indicative of better performance in mathematics (Samuels, 2011. p.4).

The manner in which the graphing calculator is implemented into the calculus curriculum plays a significant role in the students' ability to use the device effectively. Usage in calculus instruction ranges from the graphing calculators being used as a scientific calculator with a larger screen, to a highly-portable and affordable hand-held computer with mathematical programs built in. Other than the NCTM standards, there are no other widely-accepted rules or recommended methods of integrating the technology, which could possibly account for this disparity. Although the graphing calculator has been in existence for over twenty five years, there is no collection of standardized texts that have shown successful implementation over a long period of time.



The graphing calculator is also integrated into calculus instruction in many different ways as well. The manner in which the graphing calculator has been integrated into calculus instruction also manifests itself as a result of the instructor's pedagogical approach to teaching calculus. Its use varies from being a tool used to simplify numeric computations to a tool that is used to present the model under consideration using different representations. Successful use of the graphing calculator in calculus instruction has seen it being used allow students to discover important concepts in calculus that originally had been too tedious to perform, and also to provide both graphical and symbolic representations of the model under consideration. Trends in one representation of the model are able to be used to highlight trends in the other representations of the model. This allows the connections between the different representations of the model to be used to enhance the student's conceptual knowledge of the topic being discussed. Calculus instruction today generally includes students being given graphing calculator lab activities that encourage students to discover concepts using graphical, numerical, and algebraic representations of a mathematical model (Santos-Trigo & Reyes Rodríguez, 2011).

Despite concerns about the place of the graphing calculator in calculus instruction, graphing calculators continue to make their presence known in mathematics instruction. Many new calculus texts have integrated graphical, numerical, and algebraic approaches to problem solving and have placed a greater emphasis on conceptual understanding as opposed to the rigors of symbolic manipulation (Waits & Demana, 1998). In addition, students learning calculus in a setting that includes the integration of the graphing

calculators and technology have been found to possess more positive attitudes about learning calculus than students learning calculus in a setting that does not include the integration of technology into instruction (Samuels, 2011, p.4).

While it appears that the use of the graphing calculators in calculus instruction has continued to receive increasingly wide-spread support and acceptance, the manner in which the integration of the technology is implemented varies widely. With graphing calculators being able to perform the majority of numeric computations, more time has been available to focus on conceptual development through exploration and discovery (Kissane, 1995). By focusing on problem solving techniques through multiple representations of mathematical models, students are able to develop more of a conceptual understanding of the model, than merely a procedural understanding. This type of problem solving is more representative of the type of work that is done by mathematicians in the field (Samuels, 2011, p.3). The use of problem solving in calculus instruction also allows the student to see the usefulness of the skill being taught at the time that it is being presented, rather than as a skill-building exercise that will be used for future problem solving (Pomerantz & Waits, 1997). The use of graphing calculators allows students to be a part of interactive learning. Rather than sitting in a traditional lecture-style classroom, students are able to be actively involved in their own learning in a cooperative learning environment. This setting allows students the opportunity to engage in discussion of problem solving strategies and encourages the exchange of ideas in a more relaxed setting (Wolfram). Additionally, the instructor's willingness to incorporate technology into the curriculum with a positive attitude has been shown to have a direct, positive correlation with a student's success in

calculus (Samuels, 2011, pp.31-32). The use of the graphing calculator in calculus instruction gives the student more responsibility in their own learning process. Instead of receiving instruction in the classroom and processing the information on their own time, students are now subject to demonstrating proficiency during instructional time in the classroom (Samuels, 2011, p.35).

Perhaps the greatest advantage of the use of the graphing calculator in calculus instruction is the graphing calculator's ability to represent data graphically, numerically, and algebraically. Students that are able to link these representations of mathematical models using technology have also formed strong links between the representations in their minds (Samuels, 2011, p.35). It has also been shown that the links between the different representations are made even stronger through applications to real-world situations. With this in mind, two graphing calculator lab activities have been developed with the purpose of forming a strong conceptual knowledge of the topic being covered. Each of the labs is focused on exploratory learning using graphical, numerical, and algebraic techniques of problem solving for each of the mathematical models being considered.

## STUDENT ASSESSMENT IN CALCULUS USING EXPLORATORY GRAPHING CALCULATOR LAB ACTIVITIES

The remainder of this document will focus on these exploratory graphing calculator lab activities that have been designed to highlight the major concepts covered in an introductory college-level Calculus course. The activities will involve graphical, numerical, and algebraic representations of mathematical models. In addition, open-ended questions have been included as part of the activities to encourage students to summarize the results of the activities and to promote discussion and the exchange of ideas. These reflective components of the activities will allow students the opportunity to review their method of solution, stream-line the process where possible, and learn to justify solutions both coherently and concisely.

The graphing calculator lab activities presented in this document are designed for use with the TI-*n*spire CX handheld. This calculator is the latest graphing calculator released by Texas Instruments, and has been available for purchase by the public since late May of 2011. The calculator is an updated version of the TI-*n*spire handheld. Although the upgraded operating system is available for free download via the internet, the new TI-*n*spire has a thinner design, as well as a color view screen with touchpad capabilities. Perhaps the most exciting feature of the new TI-*n*spire CX handheld is its ability to import full color images to be analyzed. The ease in which the images can be imported into the operating system allows for flexible analysis of real-world situations. The current average retail value of the TI-*n*spire CX handheld is \$159.00 and includes the accompanying student software for PC or Mac platforms. The price is comparable to popular cell phone

models, making it affordable to the general population. In addition to being available in large retail stores, this model is also available for purchase via the internet.

Prior to beginning the graphing calculator lab activities, instructions need to be given on the usage of the graphing calculator and the basic features of the calculator that will be used during the lab activities. The graphing calculator is intended to enhance learning, not to become the focus of the activities. If students are not familiar with the basic functions of the calculator, the process of operating the graphing calculator can overshadow the concepts being explored. Although integration of technology is recommended throughout mathematics instruction, many students are not fluent in the basic mechanics of the technology. Prior to beginning the graphing calculator lab activities, students are to complete additional activities that review the technical aspects of the labs. The following section contains “Tech Sheets” which have been designed to demonstrate each of the calculator processes that will be used in the graphing calculator lab activities. In practice, students are to complete the corresponding Tech Sheets required for each of the graphing calculator lab activities prior to completing the activities themselves. The Tech Sheets include instruction on how to graph functions, import images, create lists of data and spreadsheets with the data capture feature available on the graphing calculator, perform regression analysis, refine regression analysis with the quick regression feature, and collect data using the Vernier EasyTemp temperature probe.

Each of the subsequent graphing calculator lab activities has been presented in a calculus class on multiple occasions. The lab activities that are being presented have been refined over a several year period of time based on analysis of student performance,

analysis to determine that the target concept was effectively understood, as well as consideration of student and teacher reflective components, to ensure that the desired outcome has been achieved. Each of the lab activities has been designed with the goal of presenting each of the concepts graphically, numerically, and algebraically, in an effort to form cohesive bonds between the representations. It has been the goal of the lab activities to foster these links between the representations in an effort to provide a stronger conceptual knowledge base for the topics being addressed.

In addition to the tech sheets and the graphing calculator lab activities, a comprehensive unit overview is provided as a resource for instruction located in Table 1. Within the Unit Lesson Plan the instructor is provided with a Unit Overview, Essential Questions and Enduring Understandings, Evidence of Learning, and a Lesson Plan outline.

Table 1  
Introduction to Calculus  
Exploring Relationships to Develop Critical Concepts

<b>Unit Overview</b>
<b>Content Area:</b> Calculus
<b>Unit Title:</b> Introduction to Calculus: Exploring Relationships to Develop Critical Concepts
<b>Target Course:</b> Calculus I
<p><b>Unit Summary:</b> The students will be able to reason mathematically using algebraic, numeric, and graphical representations to discover major concepts in calculus. These topics include the relationship between average and instantaneous rates of change (the derivative) and The Fundamental Theorem of Calculus (the integral).</p> <p><b>21<sup>st</sup> Century Themes:</b></p> <p>Critical Thinking and Problem Solving</p> <p>Creativity and Innovation</p> <p>Communication and Media Fluency</p> <p>Collaboration, Teamwork and Leadership</p> <p>Accountability, Productivity and Ethic</p>

Unit Essential Questions	Unit Enduring Understandings
<ul style="list-style-type: none"> <li>• How do we analyze and understand the relationships between graphical, numerical, and algebraic representations of mathematical models?</li> </ul>	<ul style="list-style-type: none"> <li>• Modeling involves analyzing real-world situations, representing those situations graphically, numerically, and algebraically, analyzing and testing the mathematical model for reasonableness.</li> </ul>
<ul style="list-style-type: none"> <li>• How can data analysis tools be utilized most effectively and efficiently?</li> </ul>	<ul style="list-style-type: none"> <li>• Use of calculator and computer technologies for analyzing data graphically, numerically, and algebraically requires critical thinking to determine which tool will provide the most appropriate results in the most efficient manner.</li> </ul>
<ul style="list-style-type: none"> <li>• How do begin to solve a problem using mathematical models?</li> <li>• How do you identify appropriate problem solving strategies for the given situation?</li> <li>• How do you assess that you have accurately solved the problem in question?</li> </ul>	<ul style="list-style-type: none"> <li>• Problem solving skills require: <ul style="list-style-type: none"> <li>○ Knowledge of problem solving strategies</li> <li>○ Mathematical content knowledge appropriate to the given topic</li> <li>○ Appropriate self-assessment to determine that the most effective solution has been identified</li> </ul> </li> </ul>
<p><b>Unit Learning Targets</b></p> <ul style="list-style-type: none"> <li>• Focus on appropriate use of technology to solve problems.</li> <li>• Understand patterns, relations and functions.</li> <li>• Using generalized patterns, create explicitly defined models, including appropriate notation.</li> <li>• Construct suitable mathematical models to represent a situation focusing on data.</li> <li>• Draw conclusions about a situation using a mathematical model.</li> <li>• Use appropriate units and scales when graphing on the Cartesian plane.</li> <li>• Understand that there are multiple ways to solve a problem and apply those methods to real-world situations.</li> <li>• Monitor problems solving, assess achievement of goals, and alter strategies, where necessary.</li> <li>• Formulate extensions to a problem or model a new situation to predict an outcome.</li> </ul>	



## Evidence of Learning

### **Summative Assessment:**

Students will complete two graphing calculator lab activities with accompanying Tech Sheets to discover two major concepts in calculus: the relationship between average and instantaneous rates of change (the derivative) and The Fundamental Theorem of Calculus (the integral). Students will model data and look for patterns amongst the results. They will formulate hypotheses to describe the patterns they have discovered (see scoring rubrics).

### **Equipment needed:**

Students will need access to a TI-*n*spire CX Handheld graphing calculator or corresponding software, Vernier EasyTemp temperature probe, a computer, and Microsoft Office.

**Instructor Resources:** Same as student

**Internet:** Websites containing free downloadable trial software, if necessary

**Books:** Mathematical texts as resources found at the local library and research centers

### **Formative Assessments:**

- Graph equations using the TI-*n*spire graphing calculator or the accompanying software.
- Specify the appropriate domain and range for equations being graphed.
- Import images using the TI-*n*spire.
- Capture data from imported images using the TI-*n*spire.
- Perform various types of regression analysis on sample data collected using the TI-*n*spire.
- Collect data using the Vernier Easy-Temp temperature probe and the TI-*n*spire.
- Calculate the average rate of change between sets of points.
- Estimate the instantaneous rate of change between sets of points.
- Calculate the area of known geometric figures.
- Estimate area under a curve.

Lesson Plans	
Lesson	Timeframe
Lesson 1: <u>Exploring the Relationship Between Average and Instantaneous Rates of Change with the TI-<i>n</i>spire CX Handheld</u> Tech Sheet: “Data Collection Using the Vernier EasyTemp Temperature Probe”	35 minutes
Lesson 2: <u>Exploring the Relationship Between Average and Instantaneous Rates of Change with the TI-<i>n</i>spire CX Handheld</u> Tech Sheet: “Regression Analysis with the TI- <i>n</i> spire CX Handheld”	45 minutes
Lesson 3: <u>Exploring the Relationship Between Average and Instantaneous Rates of Change with the TI-<i>n</i>spire CX Handheld</u> Complete Part I of Graphing Calculator Lab Activity	45 minutes
Lesson 4: <u>Exploring the Relationship Between Average and Instantaneous Rates of Change with the TI-<i>n</i>spire CX Handheld</u> Complete Part II of Graphing Calculator Lab Activity	45 minutes
Lesson 5: <u>Exploring the Relationship Between Derivatives and Integrals Using the TI-<i>n</i>spire CX Handheld</u> Tech Sheet: “Graphing Functions with the TI- <i>n</i> spire CX Handheld”	30 minutes
Lesson 6: <u>Exploring the Relationship Between Derivatives and Integrals Using the TI-<i>n</i>spire CX Handheld</u> Tech Sheet: “Importing Images with the TI- <i>n</i> spire CX Handheld”	35 minutes

Lesson 7: <u>Exploring the Relationship Between Derivatives and Integrals Using the TI-<i>n</i>spire CX Handheld</u> Tech Sheet: “Creating Lists Using Data Capture on an Image with the TI- <i>n</i> spire CX Handheld”	45 minutes
Lesson 8: <u>Exploring the Relationship Between Derivatives and Integrals Using the TI-<i>n</i>spire CX Handheld</u> Complete Part I of Graphing Calculator Lab Activity	45 minutes
Lesson 9: <u>Exploring the Relationship Between Derivatives and Integrals Using the TI-<i>n</i>spire CX Handheld</u> Complete Part II of Graphing Calculator Lab Activity	45 minutes
Lesson 10: Student Reflective Component Complete Student Reflective Component for Graphing Calculator Lab Activities	45 minutes

## TECH SHEET – GRAPHING FUNCTIONS WITH THE TI-*n*spire CX HANDHELD

This tech sheet is designed to provide the user with instruction on how to graph functions using the TI-*n*spire CX handheld graphing calculator. Work through the tutorial and complete the graphing exercises, noting any additional information on this sheet that you found helpful. This sheet should be completed prior to beginning the accompanying graphing calculator lab activity.

- To begin, turn the calculator on and the home screen will appear. Using the arrow keys or the touchpad, scroll down to highlight the graphing icon as indicated below in Figure 1.



*Figure 1.* Screen shot of home screen from TI-*n*spire CX Handheld.

- Once the graphing icon has been selected, press the “enter” key and the main graphing screen will appear. Using the keypad, enter a function for “ $f1(x)=$ ” and then press the “enter” key. The function will appear on the screen as shown in the diagram below. For this example, use  $f1(x)=x^2$ . The screen image appears in Figure 2.

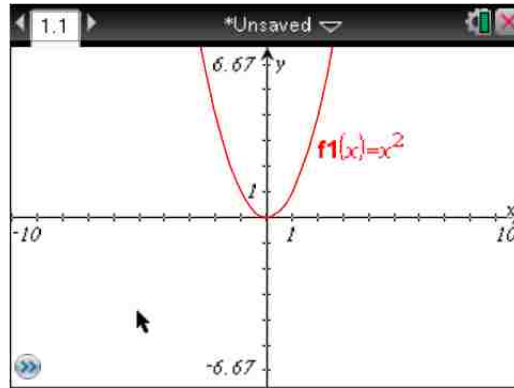


Figure 2. Screen shot of graph of  $f_1(x)$  from TI-nspire CX Handheld.

- In many mathematical models, more than one function needs to be viewed on the same set of axes. In this case, the user may enter multiple equations by selecting the arrows on the bottom left-hand side of the screen and then selecting the “enter” key. Once this has been completed, a blank entry line will appear for the next function number that can be entered. The user must input the function using the keypad. The variables “x”, “y”, and “z” may be selected directly from the keypad also. To view all of the functions that appear on the screen, the user should highlight the arrows that appear on the bottom right-hand side of the screen and then press the “enter” key. Figure 3 shows the functions  $f_1(x) = x^2$  and  $f_2(x) = x^2 - 2$ .

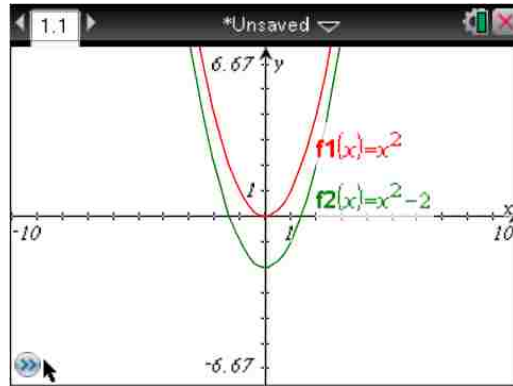


Figure 3. Screen shot of  $f_1(x)$  and  $f_2(x)$  from TI-nspire CX Handheld.

- In addition to entering functions on the graphing screen, the axes will need to be adjusted based on the functions that are being analyzed. To adjust the axes, select the “menu” key, followed by selecting “4: Window/Zoom” and followed by “1: Window Settings”. Enter the desired domain and range, and select “OK” to return to the graphing window with the adjusted axes. The screen shot in Figure 4 shows the key sequence for adjusting the axes.

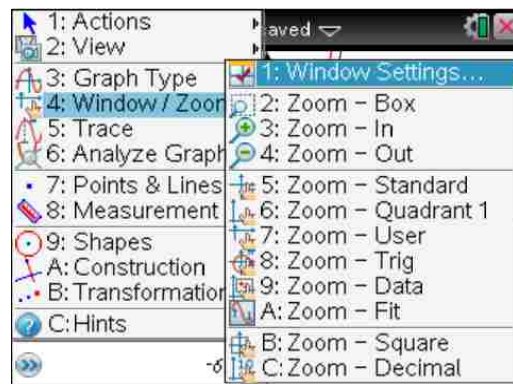


Figure 4. Screen shot of Window/Zoom menu from TI-nspire CX Handheld.

- The final graph of  $f1(x) = x^2$  and  $f2(x) = x^2 - 2$  with a domain of  $[-10,10]$  and a range of  $[-10,10]$  appears in Figure 5 below.

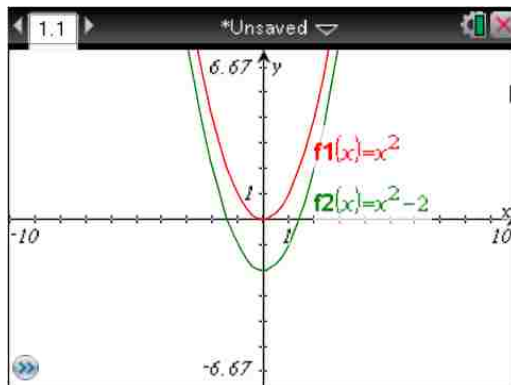


Figure 5. Screen shot of  $f1(x)$  and  $f2(x)$  with adjusted window from TI-nspire CX Handheld.

- Within the “4: Window/Zoom” menu are other options for setting the domain and range including, but not limited to, trigonometric, parametric, and polar graphing options.

## **EXERCISES**

As a check, graph the following functions with the given domain and range. The final results are shown below the exercises. If you experience any difficulty producing the graphs, be sure to ask for assistance prior to beginning the accompanying graphing calculator lab activity.

1. Graph  $f1(x) = e^x$ ,  $f2(x) = 2e^x$ , and  $f3(x) = .5e^x$  in a standard viewing window.
2. Graph  $f1(x) = \sin(x)$ ,  $f2(x) = \sin(2x)$ , and  $f3(x) = \sin(.5x)$  in a trigonometric viewing window.

SOLUTIONS  
TECH SHEET – GRAPHING FUNCTIONS  
WITH THE TI-*n*spire CX HANDHELD

**Exercise #1**

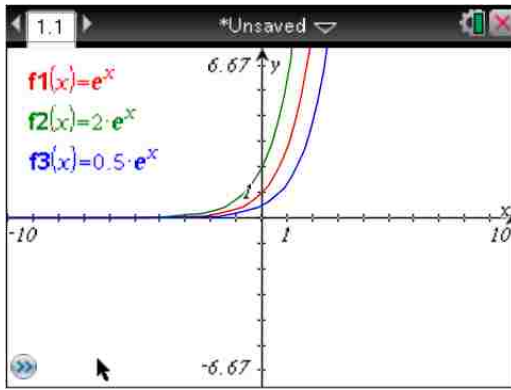


Figure 6. Screen shot of  $f1(x)$ ,  $f2(x)$ , and  $f3(x)$  from Exercise 1.

**Exercise #2**

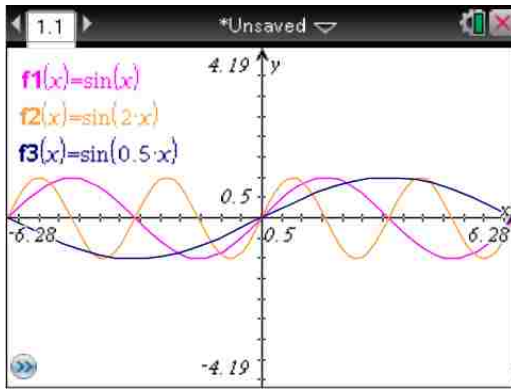


Figure 7. Screen shot of  $f1(x)$ ,  $f2(x)$ , and  $f3(x)$  from Exercise 2.



## TECH SHEET – IMPORTING IMAGES WITH THE TI-*n*spire CX HANDHELD

This tech sheet is designed to provide the user with instruction on how to import images using the TI-*n*spire CX handheld graphing calculator. Work through the tutorial and complete the accompanying exercises on importing images, noting any additional information on this sheet that you found helpful. This sheet should be completed prior to beginning the accompanying graphing calculator lab activity.

Images may be imported into the TI-*n*spire CX handheld graphing calculator, but several steps must be taken in order to complete the process. To begin, the image that is to be downloaded onto the handheld must be copied onto the computer that contains the accompanying TI-*n*spire CX software. Images must be converted to the file format that is compatible with the handheld (\*.tns). Images are unable to be imported to the handheld without completing the conversion process.

- To import an image onto the TI-*n*spire CX handheld, the file must be in one of the following formats: “\*.jpg”, “\*.jpeg”, “\*.bmp”, or “\*.png”.
- From the main screen of the accompanying software, choose the graphing icon. The graphing window must be open to import an image to be analyzed.
- To select an image from the computer, select “Insert” from the main tool bar, select “Image”, and then locate the file to be imported. Once the file has been selected, choose “File” from the main tool bar, and then “Save as”. The document will be saved in the correct file format (\*.tns) as the default file format.

- In order for the file to be shared with other CX handhelds, connect the handheld with the associated USB cable to the computer that is being used with the accompanying TI-*n*spire CX software.
- Choose the “Documents” option under the main tool bar and select the “Handheld + SideScreen” viewing option. Open the content explorer, which is the icon with the file folder and magnifying glass. The image below in Figure 8 shows the Content Explorer open, the image file in “\*.tns” form, and an attached CX handheld.

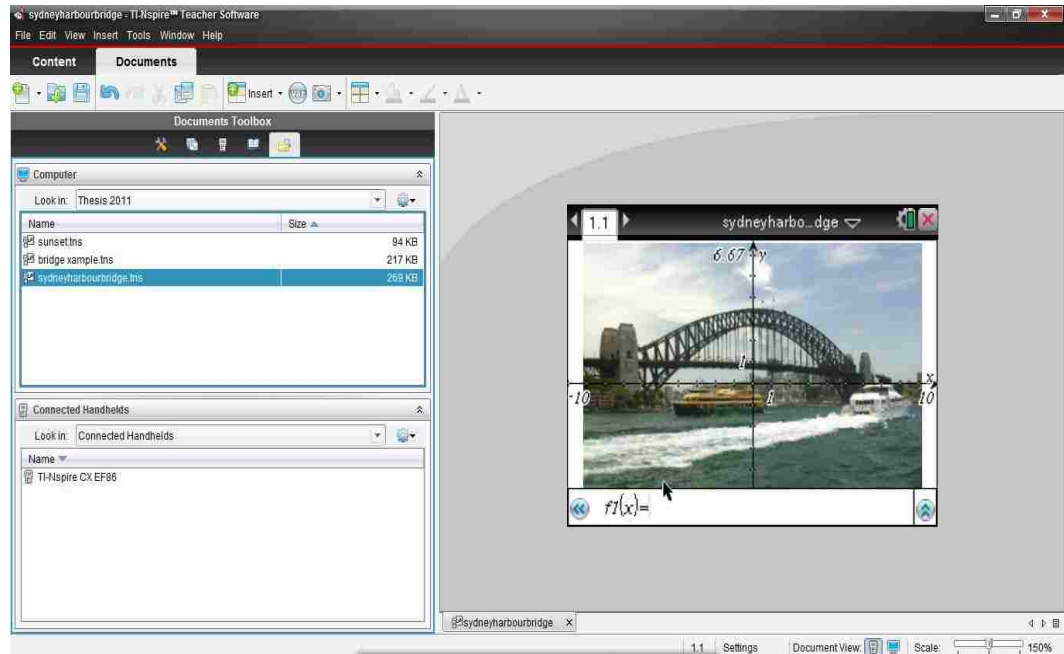


Figure 8. Screen shot of the Sydney Harbour Bridge in the TI-*n*spire CX software.

To transfer the image file to the CX handheld, simply drag and drop the selected file to the handheld directory and the image will be transferred.

## **EXERCISE**

As an exercise, locate an image to be imported onto the TI-*nspire* CX handheld using the procedure described above. Your performance on this task will be assessed by means of an opening activity one day prior to beginning the accompanying graphing calculator lab activity. You will receive an e-mail at the beginning of class that will contain the file to be imported onto your CX handheld. Without using this tech sheet for reference, you will have to download the file to your computer, open and save the file correctly in the TI-*nspire* CX software, and present the file on the handheld before the allotted time has expired. If you experience any difficulty importing images, be sure to ask for assistance prior to the designated assessment.

## TECH SHEET – CREATING LISTS USING DATA CAPTURE ON AN IMAGE WITH THE TI-*n*spire CX HANDHELD

This tech sheet is designed to provide the user with instruction on how to create lists of data using the data capture feature with the TI-*n*spire CX handheld graphing calculator. Work through the tutorial and complete the associated exercises, noting any additional information on this sheet that you found helpful. This sheet should be completed prior to beginning the accompanying graphing calculator lab activity.

In many mathematical models, the use of real-world data has been restricted due to the fact that the data has been very difficult to obtain. The CX handheld has a feature that will allow data to be collected from the image that is being considered. To obtain accurate information, however, the user should be sure to include the correct scale prior to collecting data from the image. This can be done by adjusting the axes and the scale of the image. Due to a drag and drop feature, the axes can be moved, and rescaled, to a convenient location prior to beginning the data capture. This will allow the user to easily specify known measurements to produce data that accurately describes the model. For example, the user may wish to use the x-axis as ground level or have the y-axis bisect the object or image being analyzed. It is worth being noted that although mathematical models can be analyzed in many ways, the user's prior knowledge of mathematics can often produce models that are less complicated to analyze after making adjustments than those that are analyzed without adjustments being made.

- To begin, turn the calculator on and the home screen will appear. Using the down arrow or the touchpad, scroll to highlight the graphing icon.

- Once the graphing icon has been selected, press the “enter” key and the main graphing screen will appear. Select “My Documents” on the main screen and then select the image file to be opened. The image that will be used in this example is the image of a bridge. At this point the selected image will appear on the screen of the CX handheld as shown in Figure 9.



*Figure 9.* Screen shot of Sydney Harbour Bridge from TI-84 Plus CX Handheld.

- Once the selected image appears on the screen of the CX handheld, select the “menu” key, followed by “7: Points & Lines”, and then “1: Point”. The following screen image, shown in Figure 10, should appear prior to selecting the “1: Point” option.

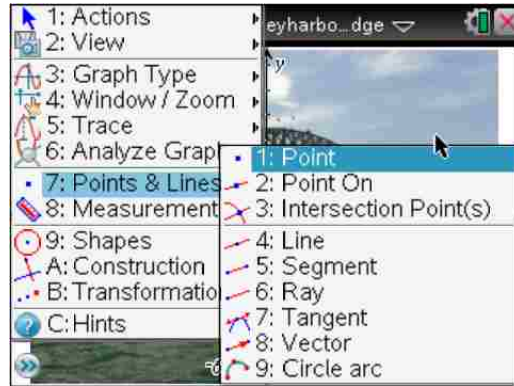


Figure 10. Screen shot of Points & Line Menu from TI-nspire CX Handheld.

- Once the option has been selected, an icon will be attached to the cursor. To attach the point on the image, place the point in the desired location and click to fix the location. Of note is that the point that is being selected for the initial point of the data capture may not be located initially on either the horizontal or vertical axes. The point may be moved after fixing it in the desired location but before variable names are assigned. Once the initial point has been selected, the point must be labeled.
- To label the point with its coordinates, select “menu”, “1: Actions”, and “7: Coordinates and Equations” as shown on the screen below in Figure 11.

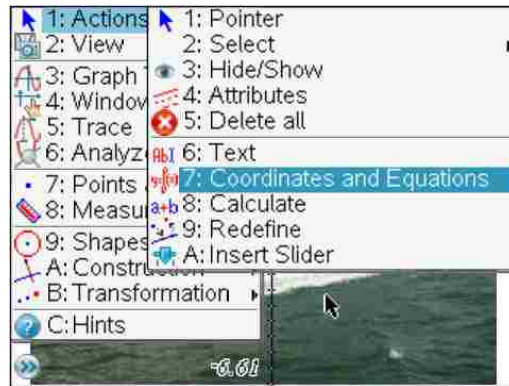


Figure 11. Screen shot of Actions Menu from TI-nspire CX Handheld.

- After selecting this option, coordinates will be associated with the point. For this example, select a point on the outline of the image that is slightly above the negative x-axis. Once the coordinates have been assigned, variable names must be created for the ordered pair so that values generated during data capture can be properly stored. To assign variable names to the ordered pair, use the touch pad in the center of the CX handheld to highlight the ordered pair. Select the “var” key, then “1: Store Var”, and type in the variable name. Repeat the process to assign a variable to the second coordinate. Figure 12 shows the screen image that appears when the variable names are being assigned. Once both of the variable names have been assigned, the data capture of the points can begin. The data for this exercise will be obtained without applying any adjustment or scaling of the axes.

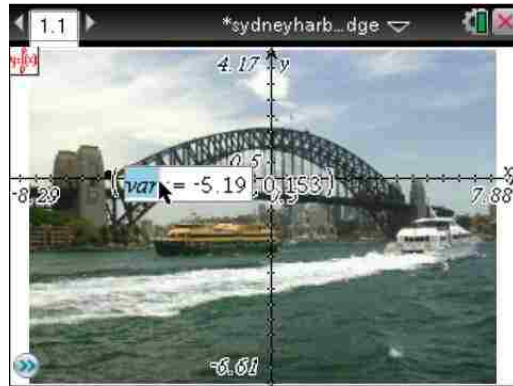


Figure 12. Screen shot of Sydney Harbour Bridge with coordinates from TI-nspire CX Handheld.

- Select the home key on the key pad to return to the main menu screen. Select the “Lists and Spreadsheets” icon by highlighting the icon and pressing the “enter” key. Using the touch pad, highlight a formula cell, which is any cell that is highlighted gray in the row above row 1. The formula cell row is the row that is marked with a diamond as the row number.
- The next step in the process will involve assigning the variable names of the data that will be captured to the columns. The data capture feature will take data in the form of ordered pairs from the image and directly place it in the columns assigned to each of the variables. For each variable that is selected, a new column must be designated to receive the data. To assign the data from the first coordinate to column 1, perform the following key strokes while the appropriate formula cell is highlighted. Select the “menu” key, then “3: Data”, “2: Data Capture”, and finally “1: Automatic”. This sequence requires that a variable name is assigned to the column. The variable names assigned to the ordered pair in this example are “xcoord” and “ycoord”. This process must be



repeated for the second coordinate. The sequence of options for data capture that will bring up the screen that requires the variable name entry is seen in Figure 13.

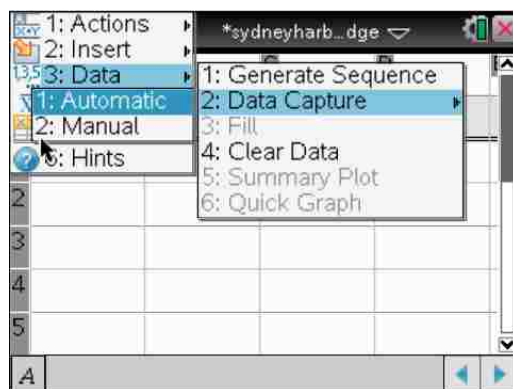


Figure 13. Screen shot of Automatic Data Capture Menu from TI-*n*spire CX Handheld.

- Using the touchpad, return to the document page that contains the image from which the data is to be captured. Highlight the point that was marked earlier that is to be used as the starting point of the data capture procedure. To highlight the initial point, use the touch pad to arrow over to it and then press the center of the touchpad to select it. At this point the hand icon will change to a small set of axes. Using the “ctrl” key simultaneously with the center of the touch pad, the axes icon should change to a closed hand. At this point, use the closed hand icon to trace over the part of the image that will be used in the mathematical model being studied. In this case, trace over the outline of the bridge. As the closed hand icon is moved from one side of the screen to the other, you will see the values of the ordered pair changing on the screen as the movement is occurring. This is an indication that the data capture procedure is

working correctly. The screen capture created during the data capture procedure and the list of data that was generated during the data capture procedure can be seen in Figure 14 and Figure 15, respectively. It is important to note that the data capture feature is a type of free-hand data collection and requires that the part of the image that is being modeled be traced with as much accuracy as possible. It is imperative that great care be taken when tracing the image to provide the most accurate data possible. The data from the data capture procedure is now in list form in the document that contains the image.



Figure 14. Screen shot of Sydney Harbour Bridge during Data Capture from TI-*n*spire CX Handheld.

	A	B	C
	=capture(xcoord,1)	=capture(ycoord,1)	
1	-5.18644	0.152542	
2	-4.88136	0.254237	
3	-4.52542	0.305085	
4	-4.27119	0.355932	
5	-3.91525	0.457627	

Figure 15. Screen shot of Data Capture lists from TI-*n*spire CX Handheld.

### **EXERCISE**

As a check, import the image of the sunset sent to you via e-mail onto your CX handheld and complete the process above by placing the origin in the center of the sunset and capturing all data points that are located above the x-axis. While no two sets of data points from the data capture procedure will be the same, check your results with the solution below to verify that you have accurately completed the process. If you experience any difficulty producing the lists, be sure to ask for assistance prior to beginning the accompanying graphing calculator lab activity.

### SOLUTION

#### TECH SHEET – CREATING LISTS USING DATA CAPTURE ON AN IMAGE WITH THE TI-*n*spire CX HANDHELD

- The figure below shows the image of the sunset with the axes moved to the approximate center of the setting sun. To move the axes, click a spot on the image, select the “ctrl” key and then click once on the center of the touch pad. At this point there will be a closed hand located on the screen. By moving the icon on the image, the location of the axes may be readjusted as seen in Figure 16 below.

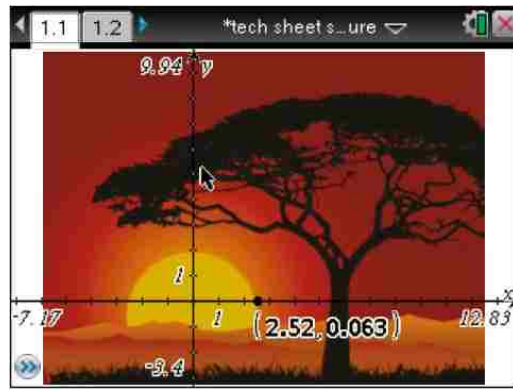


Figure 16. Screen shot of Sunset with initial point labeled for Data Capture from TI-*n*spire CX Handheld.

- Once the axes have been adjusted, place a point on the outline of the sun and collect the required data using the automatic data capture feature in the spreadsheet application. The figure below contains the data generated from the automatic data capture. Since the process may vary slightly due to the nature of the data capture technique, the lists may show slight differences in value. Be sure that the data that was obtained is reasonably similar to the data in Figure 17 below.

	A	B	C
	=capture(xcoord,1) =capture(ycoord,1)		
1	-2.38994	0	
2	-2.20126	0.377358	
3	-2.26415	0.440252	
4	-2.20126	0.691824	
5	-2.07547	0.754717	
C1			

Figure 17. Screen shot of Data Capture lists for Exercise from TI-84 Plus CX Handheld.

## TECH SHEET – REGRESSION ANALYSIS WITH THE TI-*n*spire CX HANDHELD

This tech sheet is designed to provide the user with instruction on how to perform regression analysis with the TI-*n*spire CX handheld graphing calculator. Work through the tutorial and complete the associated exercises, noting any additional information on this sheet that you found helpful. This sheet should be completed prior to beginning the accompanying graphing calculator lab activity.

Analyzing real-world data requires data collection and analysis. To find trends in the data to describe the model under consideration, regression analysis is used. By examining raw data, regression analysis is a useful tool to find an equation to model what is occurring. By finding an appropriate equation to model the data, predictions can be made about future outcomes. An accurate model can provide the user with valuable information about the scenario being analyzed. Once the data can be described by a function, analyses can be performed to determine critical information about the model that describes how the real-world scenario is performing.

- In order to perform regression analysis using the CX handheld, the data being analyzed must appear in a “Lists & Spreadsheets” document. The user may generate data using the data capture feature or import data from a computer. To open an existing file on the CX handheld, highlight “My Documents”, use the touchpad to highlight the file, and then use the “enter” key to select the file. Using the touch pad, select the sheet of the document that contains the list

data. For this example, the data obtained in the tech sheet on automatic data capture will be used. A screen shot of the data is given in Figure 18 below.

	A	B
	=capture(xcoord,1) =capture(ycoord,1)	
1	-2.38994	0
2	-2.20126	0.377358
3	-2.26415	0.440252
4	-2.20126	0.691824
5	-2.07547	0.754717

Figure 18. Screen shot of Data Capture lists for Regression Tech Sheets from TI-nspire CX Handheld.

- Now that the “Lists & Spreadsheets” window has been opened, highlight the formula cell in column A, select the “menu” key, “4: Statistics”, and “1: Stat Calculations”. The resultant screen image appears in Figure 19 below.

	A	B
2	-2.20126	0.377358
3	-2.26415	0.440252
4	-2.20126	0.691824
5	-2.07547	0.754717

Figure 19. Screen shot of Statistics Menu from TI-nspire CX Handheld.

Next, select the type of regression analysis. For this example, the part of the image that the data capture was used on resembles a quadratic function. Therefore, select

option “6: Quadratic Regression”. In order to determine the accuracy of the regression analysis, the correlation coefficient or the coefficient of determination should be reviewed. Once the regression type has been selected, the corresponding regression window will open. In the case of the quadratic regression, fill in the appropriate columns from the lists on which the regression analysis is to be performed. The regression data in the example appears in columns A and B in the list and the regression equation will be saved to  $f1(x)$  on the graphing screen of the document. Saving the regression equation directly to the graphing screen containing the image being analyzed allows for the equation to be graphed in an overlay fashion directly on the image. This allows for a visual check of the regression equation as a model of the data as well as allowing the CX handheld feature of a “quick regression” to be used. This feature will be detailed later in the tech sheet. The resultant screen image for having selected Quadratic Regression and contains the information necessary to perform the regression analysis on the data from the sunset automatic data capture is seen in Figure 20.

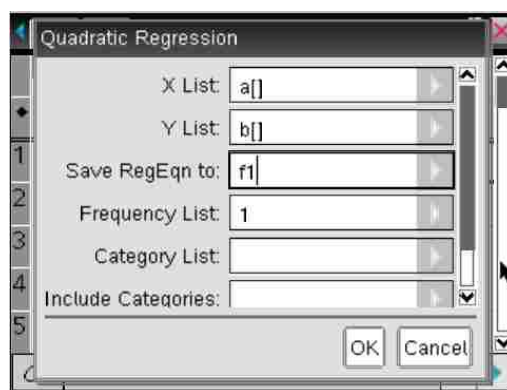


Figure 20. Screen shot of Quadratic Regression Menu from TI-84 Plus CX Handheld.



- With the regression analysis being completed, the regression equation will appear on the graphing screen in  $f1(x)$ , as was coded on the Quadratic Regression window. At the top of the document, select the sheet containing the graphing screen and highlight  $f1(x)$ , being sure to highlight the entire equation and select the “enter” key. The screen image containing the graph of the regression equation and the original image is shown in Figure 21 below.

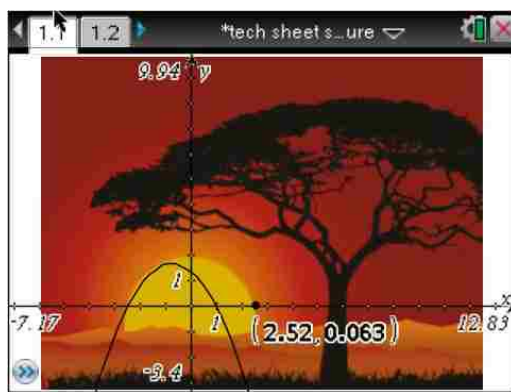


Figure 21. Screen shot of Sunset with Regression Equation Overlay from TI-84 Plus CE Handheld.

- The original task was to find an equation that modeled the portion of the sunset that is above the  $x$ -axis after the axes had been adjusted by placing the origin at the approximate center of the setting sun. As can be seen from the quadratic regression in the screen image above, the equation seems to model the image well only on part of the graph. The quadratic equation produced through the regression analysis appears to need a horizontal shift as well as a horizontal stretch to model the data more accurately. The TI-84 Plus CE has a feature that performs a type of quick regression by allowing the user to grab the graph of the regression equation and move it to create a better fit to the model.

- To use the quick regression, simply select the “ctrl” key followed by touching the center of the touchpad. This will produce the closed-hand icon which allows the graph of the function to be moved to allow for a better fit of the regression equation to the data obtained using the data capture feature. Figure 22 shows the original image with an overlay of the graph of the updated regression equation, which is now available for use in any further analysis of the model.

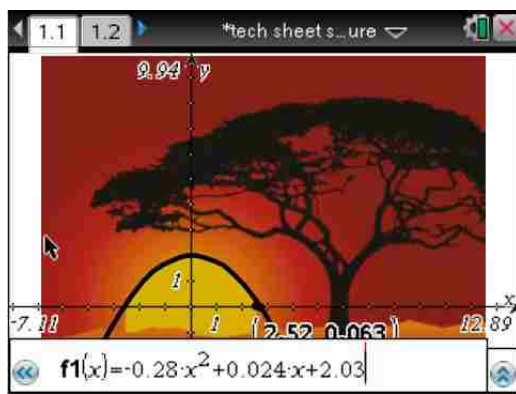


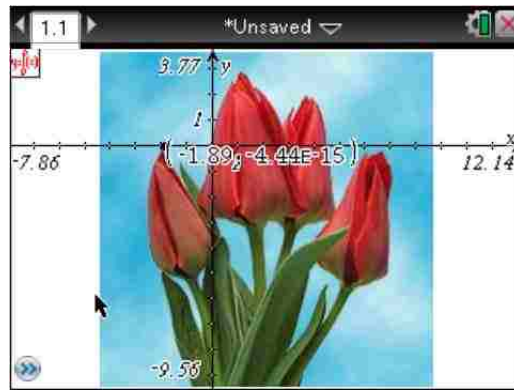
Figure 22. Screen shot of Sunset with Quick Regression Equation Overlay from TI-nspire CX Handheld.

## **EXERCISE**

As a check, import the file sent to you via e-mail containing the image of a flower and the associated data that was obtained by performing an automatic data capture. Using the list data perform an appropriate regression analysis on the portion of the flower that is above the x-axis, write the regression equation to  $f(x)$ , overlay the regression equation onto the image, and perform a quick regression to fine-tune the originally calculated regression equation. If you experience any difficulty producing the appropriate regression equation, be sure to ask for assistance prior to beginning the accompanying graphing calculator lab activity.

SOLUTION  
TECH SHEET – REGRESSION ANALYSIS  
WITH THE TI-*n*spire CX HANDHELD

- The image of the flower that you received with a set of axes superimposed on the image can be seen in Figure 23. The portion of the outline of the tulip that will be analyzed is on the interval  $[-1.89, 1.01]$ .



*Figure 23.* Screen shot of Tulips from TI-*n*spire CX Handheld.

- The file also contains another sheet containing the lists that were generated using the automatic data capture procedure to collect data on the outline of the flower that is on the designated interval. Sinusoidal Regression was chosen to model the data for this solution. The figure below is the final result of performing a sinusoidal regression on the captured data on the given interval. The regression equation below in Figure 24 is the result of a quick regression, or transformation, of the original regression equation.

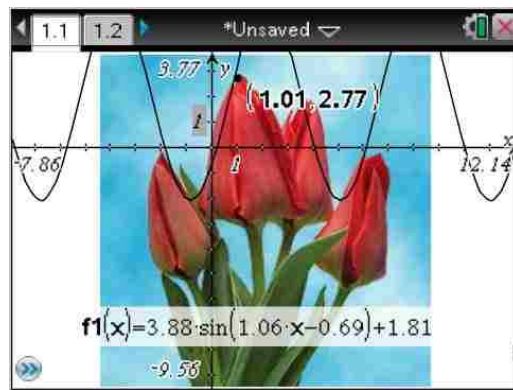


Figure 24. Screen shot of Tulips with Regression Equation Overlay from TI-nspire CX Handheld.

- There are two notes to be made in regards to performing regression analysis using the CX handheld. The first note is in regards to the quick regression feature. This feature on the CX handheld is very useful, although it is only available with the following regression analyses: linear, quadratic, exponential, logarithmic, sinusoidal, and cosinusoidal functions. The second note pertains specifically to sinusoidal and cosinusoidal regression. When performing these types of regression analysis it is important to note that the approximate period should be entered in the “Sinusoidal Regression” dialog box in the “Period” field even though the required field is optional. If the period is not entered, there is a possibility that a “Singular Matrix Error” message will appear.

## TECH SHEET – DATA COLLECTION USING THE VERNIER EASYTEMP TEMPERATURE PROBE WITH THE TI-*n*spire CX HANDHELD

This tech sheet is designed to provide the user with instruction on how to collect and analyze data using the Vernier EasyTemp temperature probe with the TI-*n*spire CX handheld graphing calculator. Work through the tutorial and complete the associated exercises, noting any additional information on this sheet that you found helpful. This sheet should be completed prior to beginning the accompanying graphing calculator lab activity.

In many mathematical models, the use of real-world data has been restricted due to the fact that the data has been very difficult to obtain. For this experiment, the temperature of a liquid will be collect, recorded, and analyzed using a temperature probe that attaches directly to the CX handheld. The CX handheld has built-in software that is compatible with the temperature probe. This Tech Sheet will familiarize the user with the features of the data collection software as it relates to the temperature probe.

- To begin, turn the calculator on and the home screen will appear. Using the down arrow or the touchpad, scroll down to highlight the “Add Vernier DataQuest” icon.
- Select the “enter” key and a document will open in the DataQuest application format as shown in the following screen image in Figure 25.

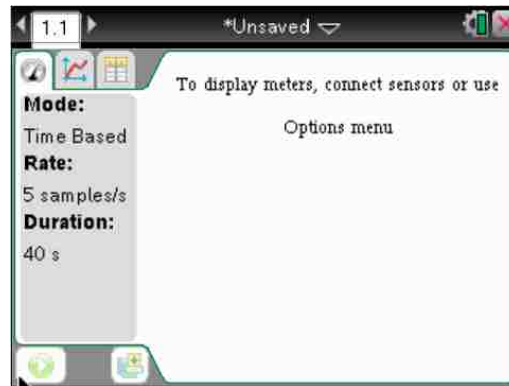


Figure 25. Screen shot of DataQuest home screen from TI-nspire CX Handheld.

- Once the screen above appears, attach the temperature probe to the CX handheld and wait for the current temperature to appear. Once the temperature appears, the probe is ready to begin the data collection process.
- For this experiment data will be collected by taking a glass of water at room temperature, filling it with ice, and collecting temperature data as the water cools. The probe is to be placed initially into the cup without ice. Once the probe has been inserted, add ice and begin collecting data.
- To prepare for the data collection select the “menu” key, followed by “1: Experiment”, and “8: Collection Setup”. This series of keystrokes is shown in the screen image below in Figure 26.

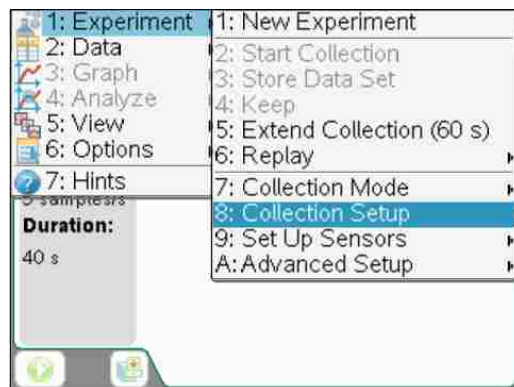


Figure 26. Screen shot of Experiment Menu from TI-nspire CX Handheld.

- After the series of keystrokes above has been completed, select the “enter” key to enter parameters for the collection setup. For this example, data collection should be coded to take place every second for 300 seconds.
- Once the glass of water and ice have been set up, insert the probe into the water, add ice, and select the “menu” key, “1: Experiment”, and “2: Start Collection”. Since the data collection was coded to run for 300 seconds, the process will stop on its own. If, however, the experiment needs to be stopped for any reason, select the “menu” key, “1: Experiment”, and “2: Stop Collection”.
- When the experimental data collection with the temperature probe is complete, the CX handheld screen should contain a plot of the raw data that was collected. In addition, by selecting the “Table View” icon, the data from the collection will be presented in list form.
- The figure below contains the data generated from the temperature probe’s automatic data capture feature in “Graph View”. Since the process may vary



slightly due to the nature of the amount of water, initial water temperature, and amount of ice added, the graphs may show slight differences in value. Be sure that the data that was obtained is reasonably similar to the data in the graph below in Figure 27.

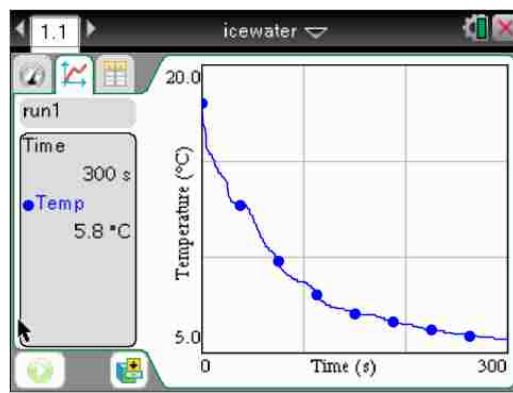


Figure 27. Screen shot of Ice Water Data Collection plot from TI-84 Plus CX Handheld.

- If you experience any difficulty performing the data collection procedure with the temperature probe, be sure to ask for assistance prior to beginning the accompanying graphing calculator lab activity.

EXPLORING THE RELATIONSHIP BETWEEN  
AVERAGE AND INSTANTANEOUS RATES OF CHANGE  
WITH THE TI-*n*spire CX HANDHELD

In this activity, the concepts of average and instantaneous rates of change will be analyzed in an effort to show the connection between the two rates. The concepts will be explored with graphical, numerical, and algebraic techniques in an effort to develop a way of calculating an instantaneous rate of change exactly, as opposed to calculating it through estimation techniques.

The relationship between average and instantaneous rate of change will be analyzed using data collected with the TI-*n*spire CX handheld. The data will result from placing a temperature probe in boiling water, until the temperature stabilizes, and data capture will begin when the temperature probe is removed and placed directly into a cup of ice water. The data will be captured using a temperature probe attachment to the CX handheld that collects the data and places it in a “Lists and Spreadsheets” document. Follow the procedures below carefully, completing the required tasks and preparing any tables and plots that are required. Once the activity has been completed, be sure to download all requested lists and graphs into the copy of this document that was sent to you via e-mail. The algebraic computations must be entered directly into the document as well using some type of equation editor.

The lab activity is divided into two separate parts. Part I is a graphical and numerical exploration and must be submitted prior to receiving directions for Part II. Part II is an algebraic analysis of the concepts that were discovered through the graphical and numerical exploration that was conducted in Part I.

## PART I

- To begin the activity, heat the water until it boils. At that point, place the temperature probe directly into the water until the temperature stabilizes. At the point that the temperature has stabilized, carefully remove the probe and place it directly into the cup of ice water. Data collection should begin at the instant that the probe is removed from the boiling water. Once activated, the temperature probe will automatically begin to collect the data and place it in a “Table View” format in a document on the CX handheld. Once the data has been collected, insert the list of temperature vs. time in a table below.
- In addition, a plot of temperature vs. time is created. Insert the plot of temperature vs. time below.
- Now that the data has been collected and plotted, it will be analyzed using the CX handheld. Using the raw data, calculate the average rate of change between 0 and 20 seconds, 5 and 20 seconds, 10 and 20 seconds, 15 and 20 seconds, and 19 and 20 seconds and record your results in a table below. Once the average rate of change computations have been completed, find the equations of each of the five lines containing the sets of points described above. Overlay the five lines on the plot with the experimental data and insert the plot below.
- Describe the relationship between the lines as each time period being considered gets closer to the 20 second mark. From a geometric stand point, what type of lines do most, or all, of these lines appear to be in relation to the data at the 20 second mark? As the time intervals become smaller and smaller,

the appearance of the lines changes. Describe this graphical change and make a prediction about what type of line would exist at the 20 second mark if the exact value of the instantaneous rate of change could be calculated using this method.

- Perform an exponential regression on the data. Using this equation, examine the values for the average rate of change between 19.1 and 20 seconds, 19.01 and 20 seconds, 19.001 and 20 seconds, 19.0001 and 20 seconds, and 19.00001 and 20 seconds. Place these results in a chart below. What do you notice about these values as the interval between the two times becomes smaller and smaller? Do the values approach a particular value? If so, what value is it and what is it an estimate of?
- Is it possible to calculate this value exactly using the formula for average rate of change? Describe the process that would be used and note any difficulties that may arise.

## **PART II**

Part I of this activity was a graphical and numerical exploration of the relationship between average and instantaneous rates of change. Through the exploratory data analysis, it was shown that the instantaneous rate of change at a point can be approximated using the formula for average rate of change on increasingly smaller intervals containing the point in question. Although a very accurate approximation of instantaneous rate of change can be found using numerical methods, it is important to derive a technique that produces an exact value, which is the purpose of this part of the activity.

- Using your knowledge of average and instantaneous rates of change describe the iterative process that can be used to more accurately predict the instantaneous rate of change at a point. Specifically, describe what is being changed in the process and the manner in which it is being changed.
- Based on the response above, create a concise and accurate verbal model to describe how to calculate the instantaneous rate of change of a function,  $f(x)$ , on the interval  $[x, x+h]$ , where “ $x$ ” is a fixed point. This verbal model will be used to develop a formula for instantaneous rate of change.
- Translate the verbal model used above into mathematical symbols, being sure to use the concept of average rate of change on the interval  $[x, x+h]$  as well as the notion of a limit when creating the formula.
- This formula is the formula for the derivative of the function  $f(x)$  at the point “ $x$ ”. To verify the accuracy of the formula above, use the notion of the derivative to evaluate the instantaneous rate of change of the model in Part I at

20 seconds. Use the regression equation found in Part I of this activity when calculating the instantaneous rate of change of the temperature 20 seconds.

Use the CX handheld to compute the derivative. Does your model appear to accurately calculate the instantaneous rate of change at 20 seconds? If not, where do you think that the formula is flawed? Place the results of your computations below, followed by the analysis of your formula's accuracy.

SOLUTIONS  
EXPLORING THE RELATIONSHIP BETWEEN  
AVERAGE AND INSTANTANEOUS RATES OF CHANGE  
WITH THE TI-*nspire* CX HANDHELD

In this activity, the concepts of average and instantaneous rates of change will be analyzed in an effort to show the connection between the two rates. The concepts will be explored with graphical, numerical, and algebraic techniques in an effort to develop a way of calculating an instantaneous rate of change exactly, as opposed to calculating it through estimation techniques.

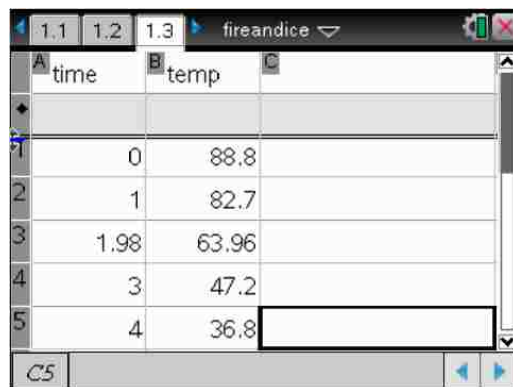
The relationship between average and instantaneous rate of change will be analyzed using data collected with the TI-*nspire* CX handheld. The data will result from placing a temperature probe in boiling water, until the temperature stabilizes, and data capture will begin when the temperature probe is removed and placed directly into a cup of ice water. The data will be captured using a temperature probe attachment to the handheld that collects the data and places it in a “Lists and Spreadsheets” document. Follow the procedures below carefully, completing the required tasks and preparing any tables and plots that are required. Once the activity has been completed, be sure to download all requested lists and graphs into the copy of this document that was sent to you via e-mail. The algebraic computations must be entered directly into the document as well using some type of equation editor.

The lab activity is divided into two separate parts. Part I is a graphical and numerical exploration and must be submitted prior to receiving directions for Part II. Part

II is an algebraic analysis of the concepts that were discovered through the graphical and numerical exploration that was conducted in Part I.

### **PART I**

- To begin the activity, heat the water until it boils. At that point, place the temperature probe directly into the water until the temperature stabilizes. At the point that the temperature has stabilized, carefully remove the probe and place it directly into the cup of ice water. Data collection should begin at the instant that the probe is removed from the boiling water. Once activated, the temperature probe will automatically begin to collect the data and place it in a “Table View” format in a document on the CX handheld. Once the data has been collected, insert the list of temperature vs. time in Figure 28 below.



The screenshot shows a TI-84 Plus CX handheld calculator in 'Table View' mode. The screen displays a table with two columns: 'time' and 'temp'. The data points are as follows:

	A time	B temp	C
1	0	88.8	
2	1	82.7	
3	1.98	63.96	
4	3	47.2	
5	4	36.8	

The calculator interface includes a top menu bar with options 1.1, 1.2, and 1.3, and a title bar that reads 'fireandice'. The bottom of the screen shows the 'C5' label and navigation arrows.

Figure 28. Screen shot of Data Collection from Exploring the Relationship Between Average and Instantaneous Rates of Change from TI-84 Plus CX Handheld.



- In addition, a plot of temperature vs. time is created. Insert the plot of temperature vs. time below (Figure 29).

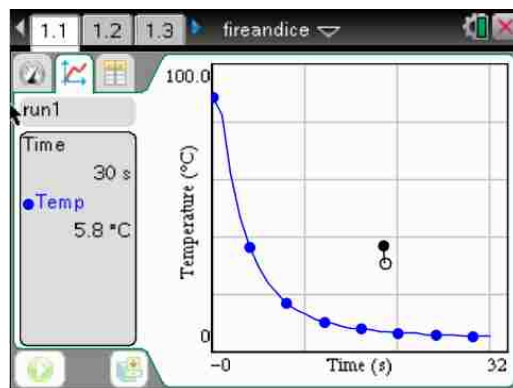


Figure 29. Screen shot of Plot of Temperature vs. Time from Exploring the Relationship Between Average and Instantaneous Rates of Change from TI-Nspire CX Handheld.

- Now that the data has been collected and plotted, it will be analyzed using the CX handheld. Using the raw data, calculate the average rate of change between 0 and 20 seconds, 5 and 20 seconds, 10 and 20 seconds, 15 and 20 seconds, and 19 and 20 seconds and record your results in a table below. Once the average rate of change computations have been completed, find the equations of each of the five lines containing the sets of points described above and place the results in Table 2. Overlay the five lines on the plot with the experimental data and insert the plot below (Figure 30).

Table 2  
Equations of Lines Through Experimental Data Points

Interval	Average Rate of Change on Interval	Equation of line passing through endpoints of the interval
(0, 89.9) and (20, 7.1)	-4.135	$y = -4.135x + 89.0$
(5, 29.5) and (20, 7.1)	-1.490	$y = -1.490x + 36.9$
(10, 13.3) and (20, 7.1)	-0.620	$y = -0.620x + 19.5$
(15, 8.8) and (20, 7.1)	-0.340	$y = -0.340x + 13.9$
(19, 7.4) and (20, 7.1)	-0.300	$y = -0.300x + 13.1$

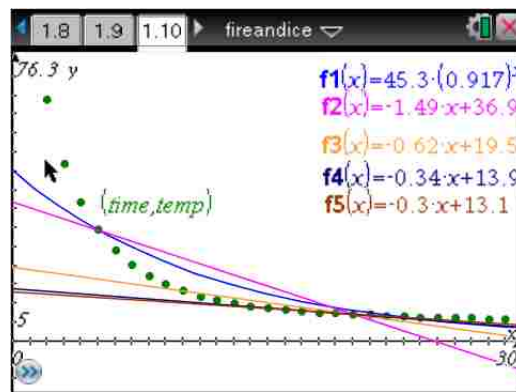


Figure 30. Screen shot of Data Collection from Exploring the Relationship Between Average and Instantaneous Rates of Change with equations of lines through experimental data points from TI-nspire CX Handheld.

- Describe the relationship between the lines as each time period being considered gets closer to the 20 second mark. From a geometric stand point, what type of lines do most, or all, of these lines appear to be in relation to the data at the 20 second mark? As the time intervals become smaller and smaller, the appearance of the lines changes. Describe this graphical change and make a prediction about what type of line would exist at the 20 second mark if the

exact value of the instantaneous rate of change could be calculated using this method.

From a geometric standpoint, the lines that appear are secant lines to the curve that would be formed by the data. As the distance between the chosen time and the 20 second mark gets smaller and smaller, the lines may appear to be tangent to the 20 second mark. Ultimately, the line representing the instantaneous rate of change at 20 seconds will be a tangent line to that point.

- Perform an exponential regression on the data. Using this equation, examine the values for the average rate of change between 19.1 and 20 seconds, 19.01 and 20 seconds, 19.001 and 20 seconds, 19.0001 and 20 seconds, and 19.00001 and 20 seconds. Place these results in a chart below. What do you notice about these values as the interval between the two times becomes smaller and smaller? Do the values approach a particular value? If so, what value is it and what is it an estimate of?

As the interval between the two times becomes smaller and smaller, the average rates of change approaches a particular value,  $-0.7328$ . This value is an estimate of the instantaneous rate of change at the 20 second mark using the exponential regression equation.

Table 3  
Average Rate of Change on Interval Using Exponential Regression Equation

Interval	Average Rate of Change on Interval Using Exponential Regression Equation $y = 45.1842(.916971)^x$
(19.1, 8.7497) and (20, 8.093)	-.7297
(19.01, 8.8182) and (20, 8.093)	-.7325
(19.001, 8.8251) and (20, 8.093)	-.7328
(19.0001, 8.8258) and (20, 8.093)	-.7329
(19.00001, 8.8258) and (20, 8.093)	-.7328

- Is it possible to calculate this value exactly using the formula for average rate of change? Describe the process that would be used and note any difficulties that may arise.

It is not possible to calculate the instantaneous rate of change for any point using only the formula for average rate of change. The computations break down due to the fact that two distinct points are needed to calculate the average rate of change, however only one point is available when considering the instantaneous rate of change.

## PART II

Part I of this activity was a graphical and numerical exploration of the relationship between average and instantaneous rates of change. Through the exploratory data analysis, it was shown that the instantaneous rate of change at a point can be approximated using the formula for average rate of change on increasingly smaller intervals containing the point in question. Although a very accurate approximation of instantaneous rate of change can be found using numerical methods, it is important to derive a technique that produces an exact value, which is the purpose of this part of the activity.

- Using your knowledge of average and instantaneous rates of change describe the iterative process that can be used to more accurately predict the instantaneous rate of change at a point. Specifically, describe what is being changed in the process and the manner in which it is being changed.

In order to more accurately estimate the instantaneous rate of change at a point, the average rate of change must be calculated as the interval between the two points becomes smaller and smaller. In other words, to estimate the instantaneous rate of change using the average rate of change, the average rate of change is to be examined as the length of the interval between the two points approaches zero.

- Based on the response above, create a concise and accurate verbal model to describe how to calculate the instantaneous rate of change of a function,  $f(x)$ ,

on the interval  $[x, x+h]$ , where “ $x$ ” is a fixed point. This verbal model will be used to develop a formula for instantaneous rate of change.

The instantaneous rate of change of  $f(x)$  at the point  $x$  is equal to the limit of the average rate of change on the interval  $[x, x+h]$  as  $h$  approaches zero.

- Translate the verbal model used above into mathematical symbols, being sure to use the concept of average rate of change on the interval  $[x, x+h]$  as well as the notion of a limit when creating the formula.

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

- This formula is the formula for the derivative of the function  $f(x)$  at the point “ $x$ ”. To verify the accuracy of the formula above, use the notion of the derivative to evaluate the instantaneous rate of change of the model in Part I at 20 seconds. Use the regression equation found in Part I of this activity when calculating the instantaneous rate of change of the temperature 20 seconds. Use the CX handheld to compute the derivative. Does your model appear to accurately calculate the instantaneous rate of change at 20 seconds? If not, where do you think that the formula is flawed? Place the results of your computations in Figure 31, followed by the analysis of your formula’s accuracy.

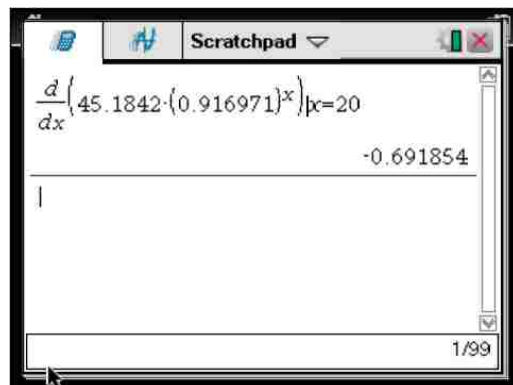


Figure 31. Screen shot of calculation of the derivative of the regression equation for comparison to the average rates of change from Exploring the Relationship Between Average and Instantaneous Rates of Change from TI-*n*spire CX Handheld.

The computation of the derivative using the CX handheld provides the most accurate calculation of the instantaneous rate of change of temperature at the 20 second mark based on the given regression equation. The values that appear in the chart above are only used to estimate the instantaneous rate of change by examining the average rates of change over increasingly smaller time intervals.

## EXPLORING THE RELATIONSHIP BETWEEN DERIVATIVES AND INTEGRALS WITH THE TI-*n*spire CX HANDHELD

In this activity, the derivative and the integral will be explored in an effort to show the connection between the two concepts. The concepts will be explored with graphical, numerical, and algebraic techniques in an effort to develop the powerful relationship between them and illustrate the foundation of their relationship with one another.

The relationship between the derivative and the integral will be explored in two distinct parts. Part I of the activity will consist of graphical, numerical, and algebraic exploration to highlight the relationship between the derivative and the integral, while Part II of the activity will focus on applications of the results found in Part I of the activity. Part II will involve using real-world data that will be collected and analyzed with the TI-*n*spire CX handheld. Part II will require calculation of the area of a setting sun in an imported image file. The data will be captured using the CX handheld feature that collects the data and places it in a “Lists and Spreadsheets” document. Follow the procedures below carefully, completing the required tasks and preparing any tables and plots that are required. Once the activity has been completed, be sure to download all requested lists and plots into the copy of this document that was sent to you via e-mail. The algebraic computations must be entered directly into the document as well using an equation editor.



## PART I

- In order to explore the relationship between the derivative and the integral, an analysis of area under a curve will be done. For each of the functions listed below, calculate the area, using known geometric formulas, on the interval  $[0, x]$  and note the results in the chart below. Computations to derive the area formulas on  $[0, x]$  should be placed below Table 4 and be labeled clearly.

Table 4  
Template for Area on  $[0, X]$

FUNCTION	AREA ON $[0, X]$
$f(x) = 5$	
$g(x) = 3x$	
$h(x) = 5x + 2$	

- Examine the results of the area computations on the interval  $[0, x]$  for each of the functions in the chart above. Note the relationship between the area of the function on  $[0, x]$  and the original function itself. Create a verbal model to describe this relationship.
- Given that  $\int_0^x f(t)dt$  is the area under  $f(x)$  on the interval  $[0, x]$ , use the verbal model above to develop a formula expressing the relationship between  $f(x)$  and  $\int_0^x f(t)dt$  and the remainder of the functions being considered. This relationship is one of the most powerful relationships in the study of calculus.
- To expand on this discovery, use the functions listed in the chart above and compute the area for each, using known geometric formulas, on the interval

$[x_1, x_2]$  and note the results in the chart below. Computations to derive the area formulas on  $[x_1, x_2]$  should be placed below Table 5 and be labeled clearly.

Table 5  
Template for Area on  $[x_1, x_2]$

FUNCTION	AREA ON $[x_1, x_2]$
$f(x) = 5$	
$g(x) = 3x$	
$h(x) = 5x + 2$	

- In an effort to present the results on the interval  $[x_1, x_2]$  more clearly, let the area formulas for each of the functions under consideration be represented by  $F(x)$ ,  $G(x)$ , and  $H(x)$ , respectively. With this in mind, express each of the area calculations on  $[x_1, x_2]$  located in the chart above, in terms of  $F(x_1)$ ,  $F(x_2)$ ,  $G(x_1)$ ,  $G(x_2)$ ,  $H(x_1)$ , and  $H(x_2)$  and note the results in Table 6.

Table 6  
Template for Area on  $[x_1, x_2]$  in terms of  $F(x_1)$ ,  $F(x_2)$ ,  $G(x_1)$ ,  $G(x_2)$ ,  $H(x_1)$ , and  $H(x_2)$

FUNCTION	AREA ON $[x_1, x_2]$ in terms of $F(x_1)$ , $F(x_2)$ , $G(x_1)$ , $G(x_2)$ , $H(x_1)$ , and $H(x_2)$
$f(x) = 5$	
$g(x) = 3x$	
$h(x) = 5x + 2$	

- Using the notation of calculus, remember that the area under  $f(x)$  on the interval  $[x_1, x_2]$  is given by  $\int_{x_1}^{x_2} f(t)dt$ . Using this representation of the area

on  $[x_1, x_2]$ , and the results of the revised area calculations in the chart above, give a description of the relationship that has been discovered.

- In conclusion, the two relationships that have been discovered in this activity are the basis of one of the most powerful theorems in mathematics. While this activity has been designed to highlight the relationships, it is by no means intended to be a formal proof of the theorem. Rather, it was created to allow self-discovery of the concept through exploration.

## **PART II**

Part I of this activity was intended to develop, through exploration, the concepts described by the Fundamental Theorem of Calculus, Parts I and II. The Fundamental Theorem of Calculus, Part I states that:

**If  $f$  is continuous on  $[a, b]$ , then the function  $F(x) = \int_a^x f(t) dt$  has a derivative at every point  $x$  in  $[a, b]$ , and  $\frac{dF}{dx} = \frac{d}{dx} \int_a^x f(t) dt = f(x)$ .**

The exploration in Part I of the activity that focused on calculating area on the interval  $[0, x]$  highlighted the fact that integration and differentiation are inverse processes of each other. The additional computations done in Part I of the activity that focused on calculating area on the interval  $[x_1, x_2]$  highlighted the results of the Fundamental Theorem of Calculus, Part II that states:

**If  $f$  is continuous on  $[a, b]$ , and if  $F$  is any anti-derivative of  $f$  on  $[a, b]$ , then**

$$\int_a^b f(x) dx = F(b) - F(a).$$

The implications of the Fundamental Theorem of Calculus, Part I and Part II allow area computations to be done without requiring that areas be represented by known geometric shapes or be computed by approximation techniques such the Rectangular Approximation Method (RAM), the Trapezoidal Rule, Simpson's Rule, or any other approximation method. Using the results of this theorem, areas are able to be calculated exactly, given the function that models the data in question.

To explore the powerful results of the Fundamental Theorem of Calculus, consider the task of calculating the surface area of a sunset. In this activity an image of a sunset will be modeled to calculate the area of the sunset as it appears in the image.

- To begin, download the image provided onto the CX handheld as described in the accompanying Tech Sheet. Position the image on the set of axes and find a function to model the data. (Hint: In order to calculate the area of the sunset, find the area of the portion above the x-axis, find the area of the portion below the x-axis, and then add the two areas together to estimate the total area.)

Insert the image of the sunset, with axes included, in the space below. Include the regression equation information on the screen image as well.

- Once the outline of the sunset has been modeled using regression analysis, use the Fundamental Theorem of Calculus to develop an expression that could be used to calculate the surface area of the sunset and include the results below.
- While the definite integral is able to be calculated by hand very easily for many functions, there are also many situations where the integral must be calculated using technology. To compute an area using the definite integral evaluation feature on the CX handheld complete the following series of key strokes:
  - Turn on the TI-*n*spire CX handheld, select “A: Calculate” from the home screen under “Scratchpad”. Once this option has been selected, select the “catalog” key, which is represented with an icon of an open book. Once this key has been selected, highlight “2”, select “Calculus” and then “Numerical Integral” followed by

the “enter” key. The screen image of this process is located in Figure 32 below.

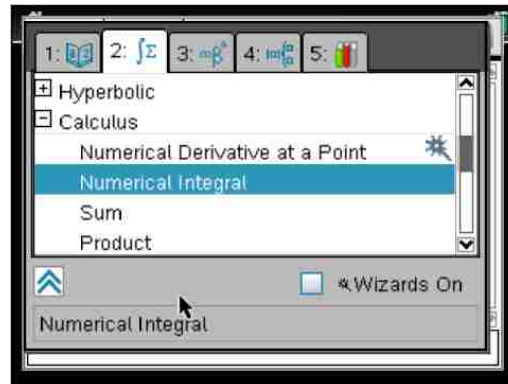


Figure 32. Screen shot of Numerical Integration screen from the TI-*n*spire CX Handheld.

- Once the “enter” key has been selected, a screen will appear with a template for the set-up of a definite integral calculation. Enter the information for the desired calculations and compute the definite integral that describes the area of the image of the sunset. Include a screen capture of any computations done on the CX handheld in the space below.
- While the activity in Part II does not require that particularly complicated tasks be done to complete the analysis, it serves as a basic model for how to analyze real world data. As a closing activity, provide a summary of the process that was used to analyze the image of the sunset to find its area.

SOLUTION  
EXPLORING THE RELATIONSHIP BETWEEN  
DERIVATIVES AND INTEGRALS WITH THE TI-*n*spire CX HANDHELD

In this activity, the derivative and the integral will be explored in an effort to show the connection between the two concepts. The concepts will be explored with graphical, numerical, and algebraic techniques in an effort to develop the powerful relationship between them and illustrate the foundation of their relationship with one another.

The relationship between the derivative and the integral will be explored in two distinct parts. Part I of the activity will consist of graphical, numerical, and algebraic exploration to highlight the relationship between the derivative and the integral, while Part II of the activity will focus on applications of the results found in Part I of the activity. Part II will involve using real-world data that will be collected and analyzed with the TI-*n*spire CX handheld. Part II will require calculation of the area of a setting sun in an imported image file. The data will be captured using the CX handheld feature that collects the data and places it in a “Lists and Spreadsheets” document. Follow the procedures below carefully, completing the required tasks and preparing any tables and plots that are required. Once the activity has been completed, be sure to download all requested lists and plots into the copy of this document that was sent to you via e-mail. The algebraic computations must be entered directly into the document as well using an equation editor.

## PART I

- In order to explore the relationship between the derivative and the integral, an analysis of area under a curve will be done. For each of the functions listed below, calculate the area, using known geometric formulas, on the interval  $[0, x]$  and note the results in the chart below. Computations to derive the area formulas on  $[0, x]$  should be placed below Table 7 and labeled clearly.

Table 7  
Area on  $[0, X]$

FUNCTION	AREA ON $[0, X]$
$f(x) = 5$	$5x$
$g(x) = 3x$	$(3/2)x^2$
$h(x) = 5x + 2$	$(5/2)x^2 + 2x$

- Examine the results of the area computations on the interval  $[0, x]$  for each of the functions in the chart above. Note the relationship between the area of the function on  $[0, x]$  and the original function itself. Create a verbal model to describe this relationship.

The original function is the derivative of the area function.

- Given that  $\int_0^x f(t)dt$  is the area under  $f(x)$  on the interval  $[0, x]$ , use the verbal model above to develop a formula expressing the relationship between  $f(x)$  and  $\int_0^x f(t)dt$  and the remainder of the functions being considered. This relationship is one of the most powerful relationships in the study of calculus.



$$f(x) = \frac{d}{dx} \int_0^x f(t) dt$$

- To expand on this discovery, use the functions listed in the chart above and compute the area for each, using known geometric formulas, on the interval  $[x_1, x_2]$  and note the results in the chart below. Computations to derive the area formulas on  $[x_1, x_2]$  should be placed below Table 8 and be labeled clearly.

Table 8  
Area on  $[x_1, x_2]$

FUNCTION	AREA ON $[x_1, x_2]$
$f(x) = 5$	$5x_2 - 5x_1$
$g(x) = 3x$	$(3/2)x_2^2 - (3/2)x_1^2$
$h(x) = 5x + 2$	$(5/2)x_2^2 + 2x_2 - (5/2)x_1^2 - 2x_1$

- In an effort to present the results on the interval  $[x_1, x_2]$  more clearly, let the area formulas for each of the functions under consideration be represented by  $F(x)$ ,  $G(x)$ , and  $H(x)$ , respectively. With this in mind, express each of the area calculations on  $[x_1, x_2]$ , located in the chart above, in terms of  $F(x_1)$ ,  $F(x_2)$ ,  $G(x_1)$ ,  $G(x_2)$ ,  $H(x_1)$ , and  $H(x_2)$  and note the results in Table 9.

Table 9  
 Area on  $[x_1, x_2]$  in terms of  $F(x_1)$ ,  $F(x_2)$ ,  $G(x_1)$ ,  $G(x_2)$ ,  $H(x_1)$ , and  $H(x_2)$

FUNCTION	AREA ON $[x_1, x_2]$ in terms of $F(x_1)$ , $F(x_2)$ , $G(x_1)$ , $G(x_2)$ , $H(x_1)$ , and $H(x_2)$
$f(x) = 5$	$F(x_2) - F(x_1)$
$g(x) = 3x$	$G(x_2) - G(x_1)$
$h(x) = 5x + 2$	$H(x_2) - H(x_1)$

- Using the notation of calculus, remember that the area under  $f(x)$  on the interval  $[x_1, x_2]$  is given by  $\int_{x_1}^{x_2} f(t)dt$ . Using this representation of the area on  $[x_1, x_2]$ , and the results of the revised area calculations in the chart above, give a description of the relationship that has been discovered.

$$\int_{x_1}^{x_2} f(t)dt = F(x_2) - F(x_1)$$

- In conclusion, the two relationships that have been discovered in this activity are the basis of one of the most powerful theorems in mathematics. While this activity has been designed to highlight the relationships, it is by no means intended to be a formal proof of the theorem. Rather, it was created to allow self-discovery of the concept through exploration.

## **PART II**

Part I of this activity was intended to develop, through exploration, the concepts described by the Fundamental Theorem of Calculus, Parts I and II. The Fundamental Theorem of Calculus, Part I states that:

**If  $f$  is continuous on  $[a, b]$ , then the function  $F(x) = \int_a^x f(t) dt$  has a derivative at every point  $x$  in  $[a, b]$ , and  $\frac{dF}{dx} = \frac{d}{dx} \int_a^x f(t) dt = f(x)$ .**

The exploration in Part I of the activity that focused on calculating area on the interval  $[0, x]$  highlighted the fact that integration and differentiation are inverse processes of each other. The additional computations done in Part I of the activity that focused on calculating area on the interval  $[x_1, x_2]$  highlighted the results of the Fundamental Theorem of Calculus, Part II that states:

**If  $f$  is continuous on  $[a, b]$ , and if  $F$  is any anti-derivative of  $f$  on  $[a, b]$ , then**

$$\int_a^b f(x) dx = F(b) - F(a).$$

The implications of the Fundamental Theorem of Calculus, Part I and Part II allow area computations to be done without requiring that areas be represented by known geometric shapes or be computed by approximation techniques such the Rectangular Approximation Method (RAM), the Trapezoidal Rule, Simpson's Rule, or any other approximation method. Using the results of this theorem, areas are able to be calculated exactly, given the function that models the data in question.

To explore the powerful results of the Fundamental Theorem of Calculus, consider the task of calculating the surface area of a sunset. In this activity an image of a sunset will be modeled to calculate the area of the sunset.

- To begin, download the image provided onto the CX handheld as described in the accompanying Tech Sheet. Position the image on the set of axes and find a function to model the data. (Hint: In order to calculate the area of the sunset, find the area of the portion above the x-axis, find the area of the portion below the x-axis, and then add the two areas together to estimate the total area.) Insert the image of the sunset, with axes included, in the space below. Include the regression equation information on the screen image as well.

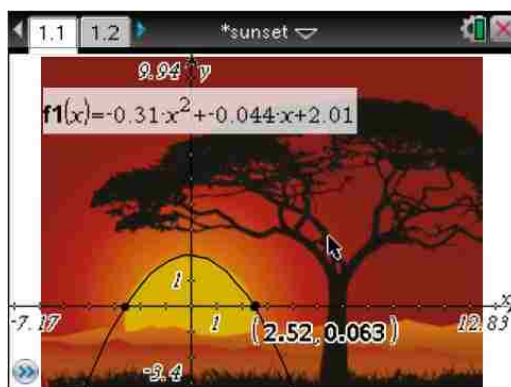


Figure 33. Image of Sunset with Regression Equation from Exploring the Relationship Between Derivatives and Integrals from TI-nspire CX Handheld.

- Once the outline of the sunset has been modeled using regression analysis, use the Fundamental Theorem of Calculus to develop an expression that could be used to calculate the surface area of the sunset and include the results below.

The original function describing the outline of the upper portion of the sunset is given by:

$$f(x) = -0.31x^2 - 0.044x + 2.01.$$

The area function that will be used to find the area of the image of the sunset above the x-axis is given by:

$$F(x) = -\frac{0.31x^3}{3} - \frac{0.044x^2}{2} + 2.01x + C$$

The original function describing the outline of the lower portion of the sunset is given by:

$$g(x) = 0.148x^3 + 0.789x^2 + 1.377x - 0.338$$

The area function that will be used to find the area of the image of the sunset below the x-axis is given by:

$$G(x) = \frac{0.148x^4}{4} + \frac{0.789x^3}{3} + \frac{1.377x^2}{2} - 0.338x + C$$

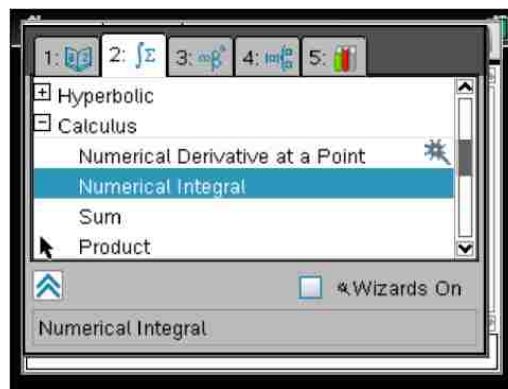
Therefore, the area of the entire sunset, as it appears in the image, is given by the following equation:

Area Of Sunset Image =

$$\int_{-2.62}^{2.48} (-0.31x^2 - 0.044x + 2.01)dx + \int_{-2.62}^{2.48} (0.148x^3 + 0.789x^2 + 1.377x - 0.338)dx$$

$$\text{Area of Sunset Image} = (F(2.46)-F(-2.62))+(G(2.48)-G(-2.62))$$

- While the definite integral is able to be calculated by hand very easily for many functions, there are also many situations where the integral must be calculated using technology. To compute an area using the definite integral evaluation feature on the CX handheld complete the following series of key strokes:
  - Turn on the TI-*n*spire CX handheld, select “A: Calculate” from the home screen under “Scratchpad”. Once this option has been selected, select the “catalog” key, which is represented with an icon of an open book. Once this key has been selected, highlight “2”, select “Calculus” and then “Numerical Integral” followed by the “enter” key. The screen image of this process is located in Figure 34.



*Figure 34.* Screen shot of Numerical Integration Menu from Exploring the Relationship Between Derivatives and Integrals from TI-*n*spire CX Handheld.

- Once the “enter” key has been selected, a screen will appear with a template for the set-up of a definite integral calculation. Enter the information for the desired calculations and compute the definite integral that describes the area of

the image of the sunset. Include a screen capture of any computations done on the CX handheld in the space below (Figure 35).

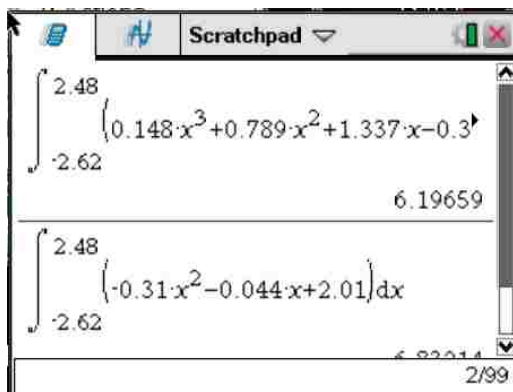


Figure 35. Numerical Integration of Regression Equations from Exploring the Relationship Between Derivatives and Integrals from the TI-*n*spire CX Handheld.

The final area of the image of the sunset is 13.0287 units squared.

- While the activity in Part II does not require that particularly complicated tasks be done to complete the analysis, it serves as a basic model for how to analyze real world data. As a closing activity, provide a summary of the process that was used to analyze the image of the sunset to find its area as it appeared in the image.

STUDENT RESULTS COMPONENT  
EXPLORING THE RELATIONSHIP BETWEEN  
AVERAGE AND INSTANTANEOUS RATES OF CHANGE  
WITH THE TI-*n*spire CX HANDHELD

The graphing calculator lab Exploring the Relationship Between Average and Instantaneous Rates of Change has been administered annually over a several year period of time. During those years, student performance was assessed to verify that the target concepts were being acquired. This graphing calculator lab activity was administered to 31 students just beginning the study of Calculus during the late Spring of 2012. Each of the students received Part I of the graphing calculator lab activity and corresponding Tech Sheets and was instructed to read through the instructions and complete a preliminary attempt at the activity independently. The objective of this activity was for students to discover the relationship between average and instantaneous rate of change. More specifically, students were to discover the formula for the derivative using graphical, numerical, and algebraic methods of analysis and to reinforce the concepts through the use of multiple representations of the data in consideration. As students were working through the activity, they were encouraged to create a list of the topics that they had difficulty with and to score themselves in each of the competency areas. In general, the students appear to have scored themselves higher than the instructor, but there was no pattern that emerged in the data. While the student scoring seemed inconsistent in comparison to the instructor's scoring, the students provided adequate written assessments of what they did and did not know how to do without assistance. They also had realistic views on how their past experiences in mathematics and associated technology impacted their ability to complete the activity successfully. Students received an overall grade on the activity as well



as subscores in the following areas: competency in graphical analysis, numerical analysis, and algebraic analysis as well as competency in graphing calculator technology, and conceptual understanding of the relationship between average and instantaneous rate of change (the derivative). The accompanying rubric in Table 10 was used to assess student performance both by the instructor and the students as outlined in the Unit Lesson Plan presented earlier.

The data represented in the figures (Figures 36-45) below indicates that as a student's graphical competency increases, so does their overall performance on the graphing calculator lab activity. Likewise, there appear to be similar trends in the areas of numerical, algebraic, and technological competency as well as overall conceptual understanding. When the instructor raw score is compared to student GPA and final unweighted student averages in Geometry, Algebra II, and Pre Calculus, there appear to be no trends in the data indicating a relationship between any of the categories and the instructor raw score.

Table 10  
EXPLORING THE RELATIONSHIP BETWEEN  
AVERAGE AND INSTANTANEOUS RATES OF CHANGE  
WITH THE TI-*n*spire CX HANDHELD

Scoring Rubric				
	0	1	2	3
Graphical Competency	Student is unable to begin to describe the transformation of the secant lines on the interval $[x, x+h]$ as $h$ approaches zero without assistance.	Student is able to successfully describe the transformation of the secant lines on the interval $[x, x+h]$ as $h$ approaches zero with major assistance.	Student is able to successfully describe the transformation of the secant lines on the interval $[x, x+h]$ as $h$ approaches zero with minor assistance.	Student is able to successfully describe the transformation of the secant lines on the interval $[x, x+h]$ as $h$ approaches zero without assistance.
Numerical Competency	Student is unable to begin to successfully calculate the equations of the secant lines without assistance.	Student is able to successfully calculate the equations of the secant lines with major assistance.	Student is able to successfully calculate the equations of the secant lines with minor assistance.	Student is able to successfully calculate the equations of the secant lines without assistance.
	Student is unable to begin to successfully use iterative methods to estimate the instantaneous rate of change at the target value without assistance.	Student is able to successfully use iterative methods to estimate the instantaneous rate of change at the target value with major assistance.	Student is able to successfully use iterative methods to estimate the instantaneous rate of change at the target value with minor assistance.	Student is able to successfully use iterative methods to estimate the instantaneous rate of change at the target value without assistance.

Algebraic Competency	Student is unable to begin to successfully create verbal and symbolic models to describe the process of calculating instantaneous rate of change at a target value without assistance.	Student is able to successfully create verbal and symbolic models to describe the process of calculating instantaneous rate of change at a target value with major assistance.	Student is able to successfully create verbal and symbolic models to describe the process of calculating instantaneous rate of change at a target value with minor assistance.	Student is able to successfully create verbal and symbolic models to describe the process of calculating instantaneous rate of change at a target value without assistance.
Technological Competency	<p>Student is unable to begin to complete Tech Sheets on Graphing Functions, Data Collection using the Temperature Probe, and Regression Analysis without assistance.</p> <p>Student is unable to begin to successfully complete computations on the graphing calculator without assistance.</p>	<p>Student is able to successfully complete Tech Sheets on Graphing Functions, Data Collection using the Temperature Probe, and Regression Analysis with major assistance.</p> <p>Student is able to successfully complete computations on the graphing calculator with major assistance.</p>	<p>Student is able to successfully complete Tech Sheets on Graphing Functions, Data Collection using the Temperature Probe, and Regression Analysis with minor assistance.</p> <p>Student is able to successfully complete computations on the graphing calculator with minor assistance.</p>	<p>Student is able to successfully complete Tech Sheets on Graphing Functions, Data Collection using the Temperature Probe, and Regression Analysis without assistance.</p> <p>Student is able to successfully complete computations on the graphing calculator without assistance.</p>

Conceptual Understanding	Student is unable to begin to successfully express the relationship between average and instantaneous rates of change by developing the formula for the derivative without assistance.	Student is able to successfully express the relationship between average and instantaneous rates of change by developing the formula for the derivative with major assistance.	Student is able to successfully express the relationship between average and instantaneous rates of change by developing the formula for the derivative with minor assistance.	Student is able to successfully express the relationship between average and instantaneous rates of change by developing the formula for the derivative without assistance.
Graphical Competency				
Numerical Competency				
Algebraic Competency				
Technological Competency				
Conceptual Understanding				
TOTAL (out of 15 points)				

As discussed earlier, the following figures (Figures 36-45) represent data obtained in the Spring 2012 administration of the Average vs. Instantaneous Rate of Change graphing calculator lab activity. The first set of figures shows comparisons between the overall instructor score on the lab activity versus overall student score, each of the category subscores, GPA, and final averages in Geometry, Algebra II and Pre Calculus.

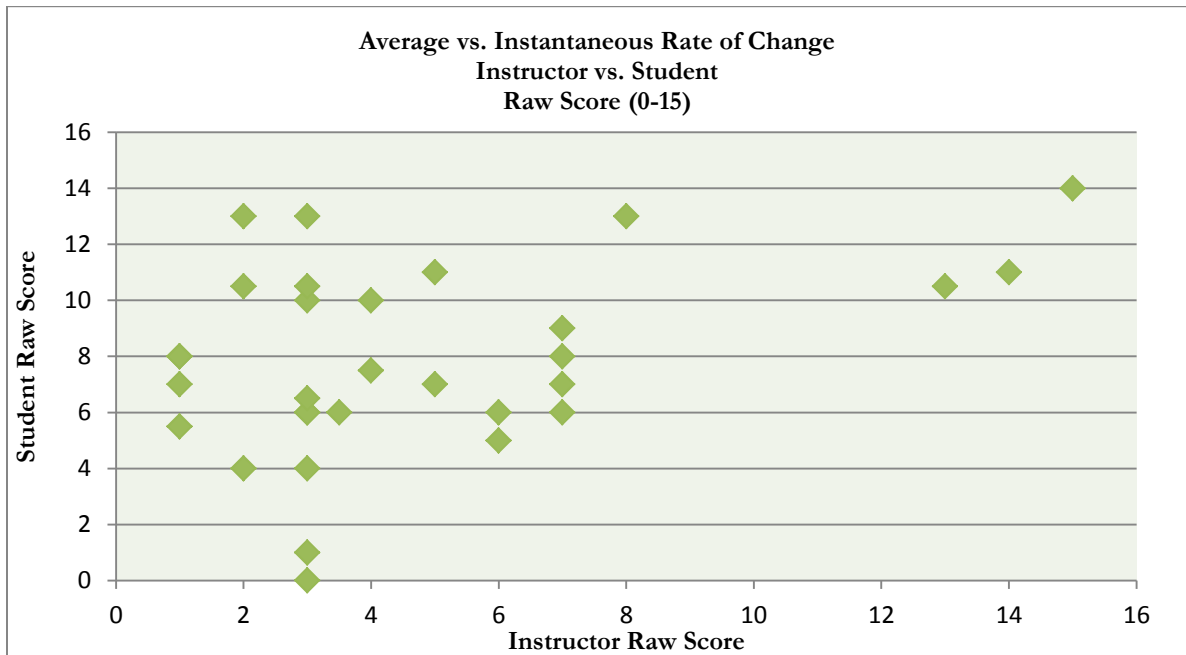


Figure 36. Instructor vs. Student Raw Score from EXPLORING THE RELATIONSHIP BETWEEN AVERAGE AND INSTANTANEOUS RATES OF CHANGE.

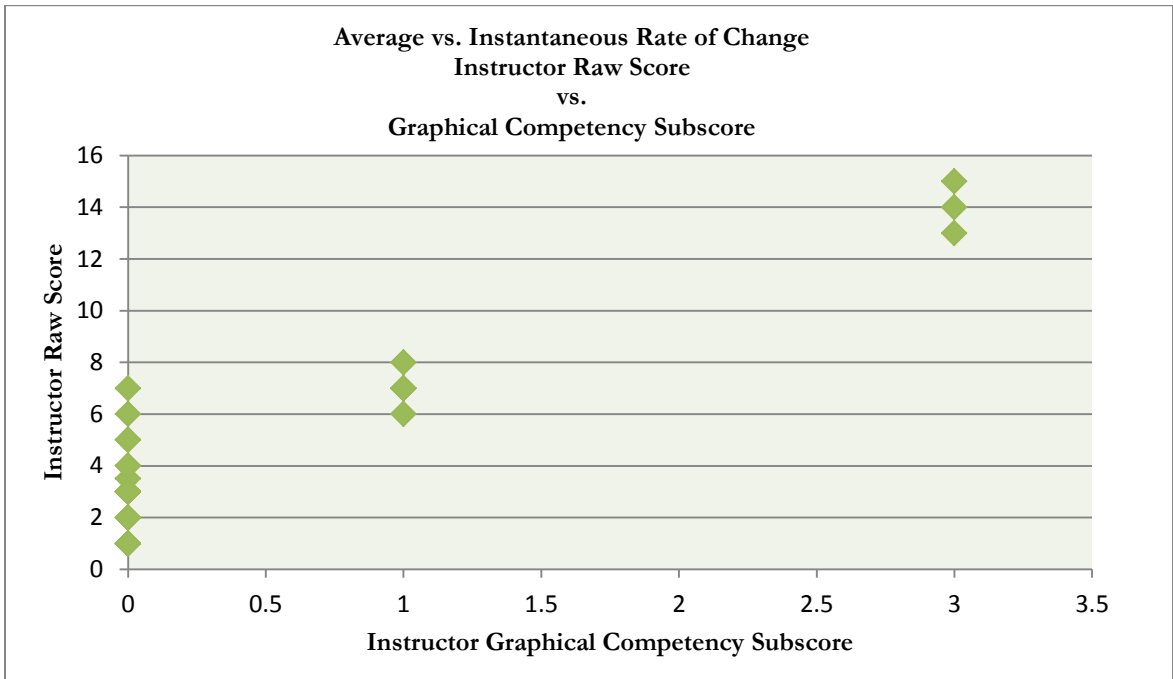


Figure 37. Instructor Raw Score vs. Graphical Competency Subscore from EXPLORING THE RELATIONSHIP BETWEEN AVERAGE AND INSTANTANEOUS RATES OF CHANGE.

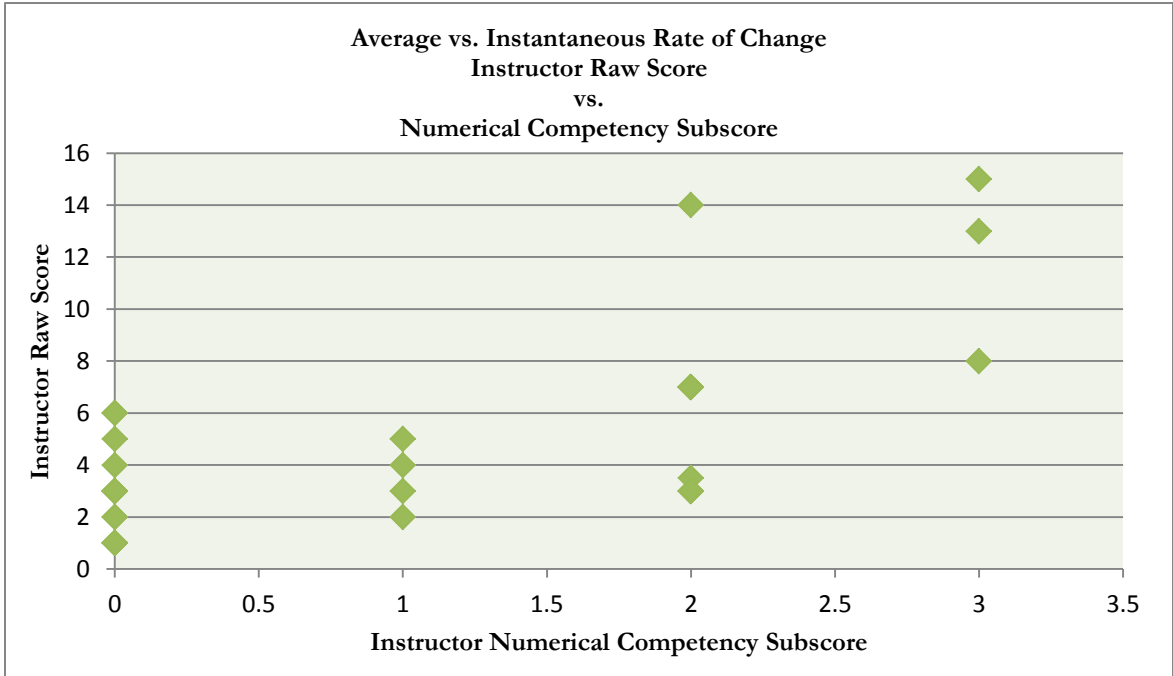


Figure 38. Instructor Raw Score vs. Numerical Competency Subscore from EXPLORING THE RELATIONSHIP BETWEEN AVERAGE AND INSTANTANEOUS RATES OF CHANGE.

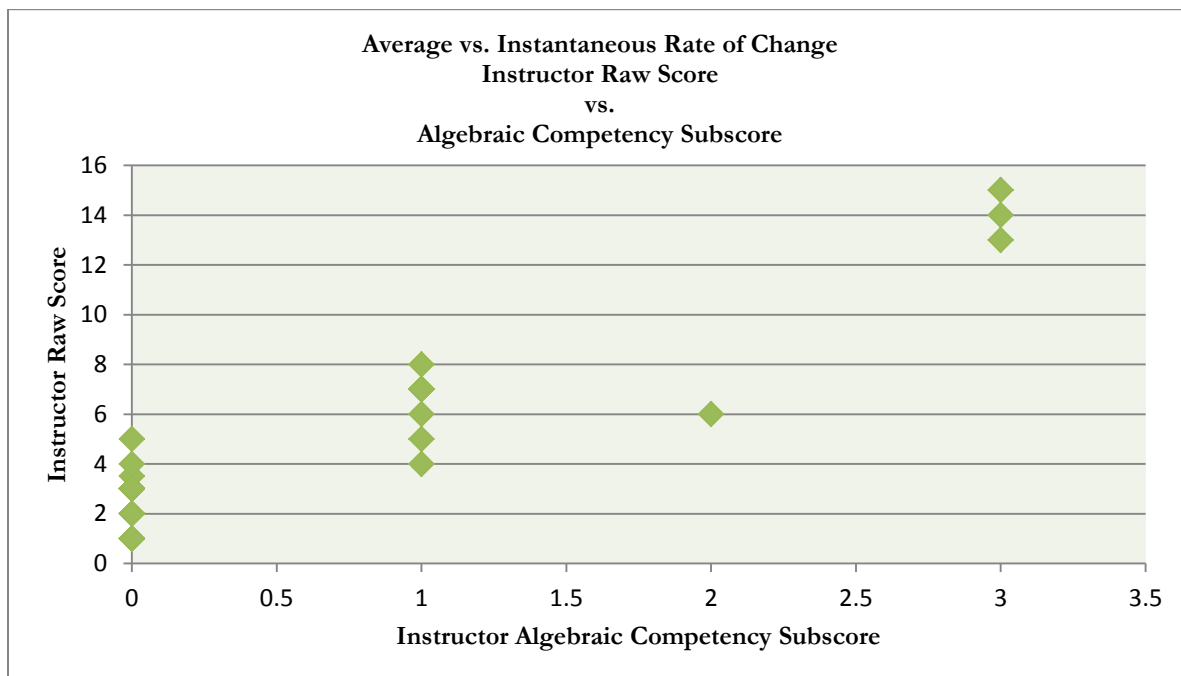


Figure 39. Instructor Raw Score vs. Algebraic Competency Subscore from EXPLORING THE RELATIONSHIP BETWEEN AVERAGE AND INSTANTANEOUS RATES OF CHANGE.

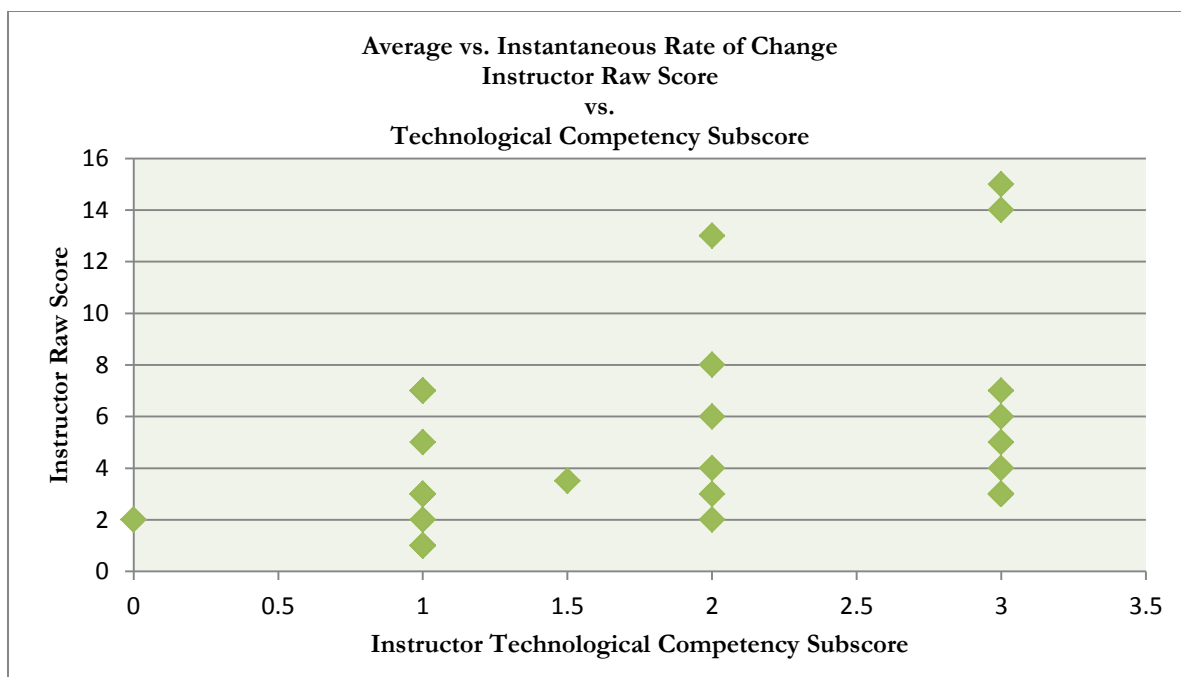


Figure 40. Instructor Raw Score vs. Technological Competency Subscore from EXPLORING THE RELATIONSHIP BETWEEN AVERAGE AND INSTANTANEOUS RATES OF CHANGE.

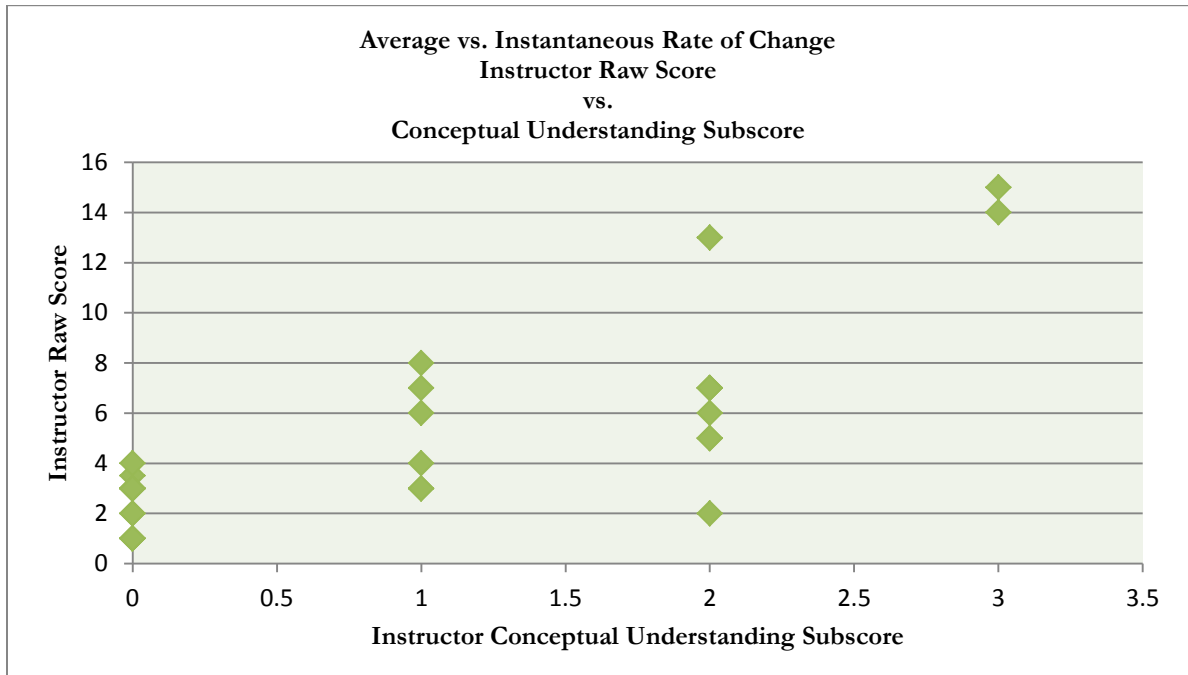


Figure 41. Instructor Raw Score vs. Conceptual Understanding Subscore from EXPLORING THE RELATIONSHIP BETWEEN AVERAGE AND INSTANTANEOUS RATES OF CHANGE.

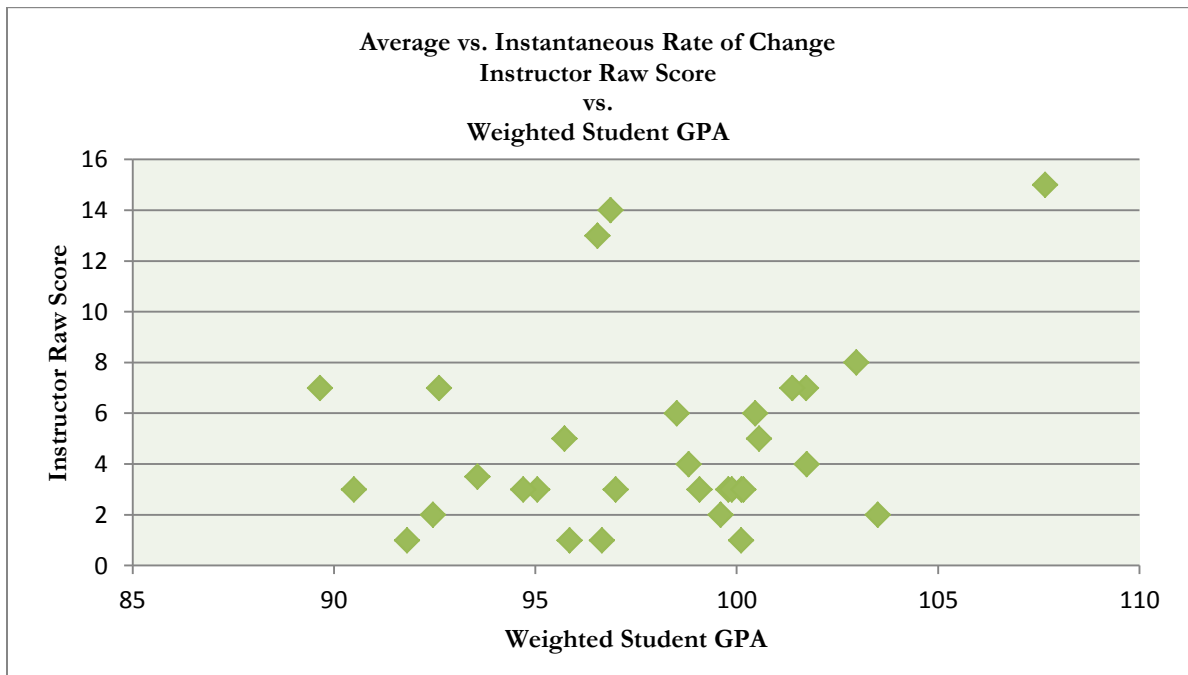


Figure 42. Instructor Raw Score vs. Weighted Student GPA from EXPLORING THE RELATIONSHIP BETWEEN AVERAGE AND INSTANTANEOUS RATES OF CHANGE.



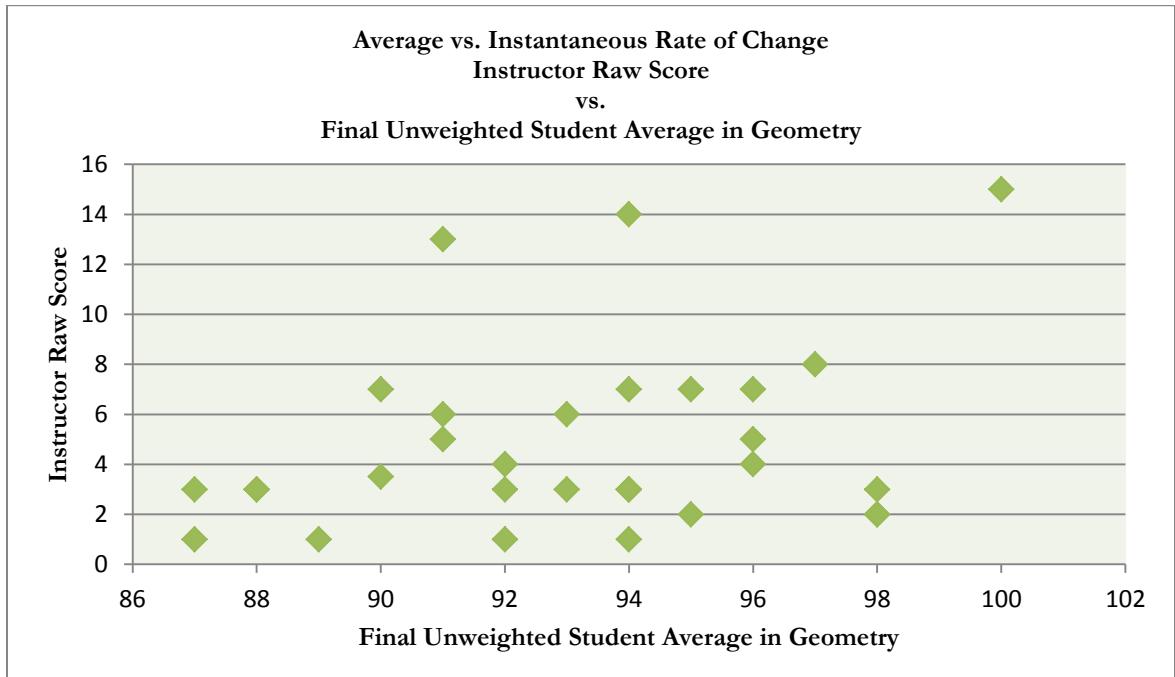


Figure 43. Instructor Raw Score vs. Final Student Unweighted Average in Geometry from EXPLORING THE RELATIONSHIP BETWEEN AVERAGE AND INSTANTANEOUS RATES OF CHANGE.

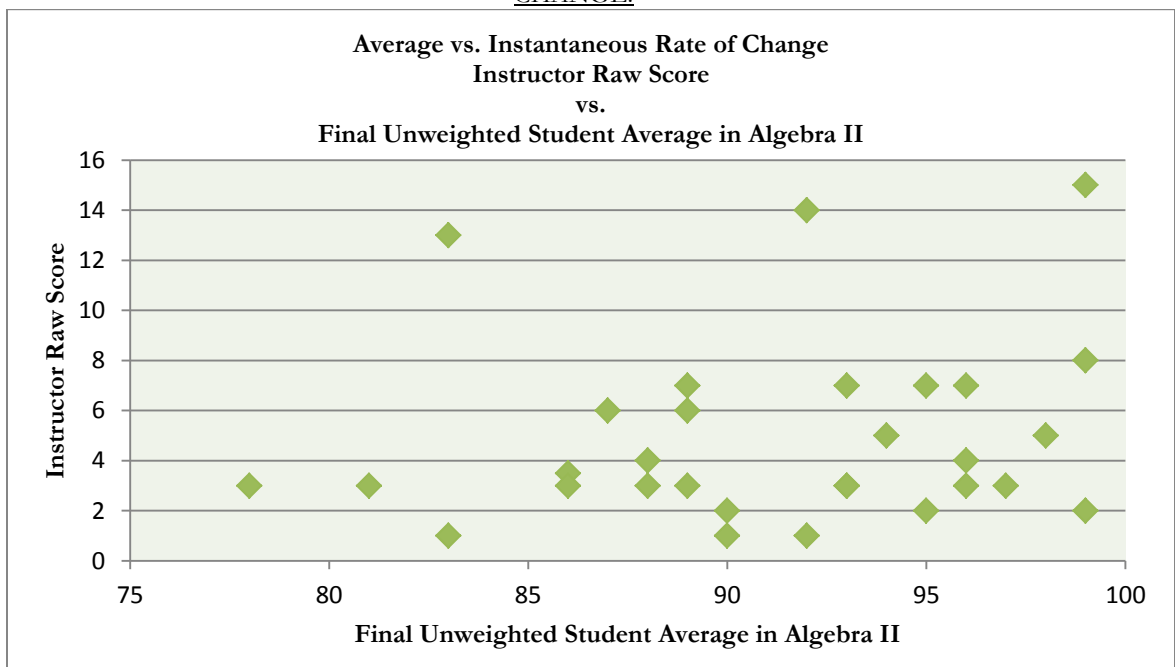
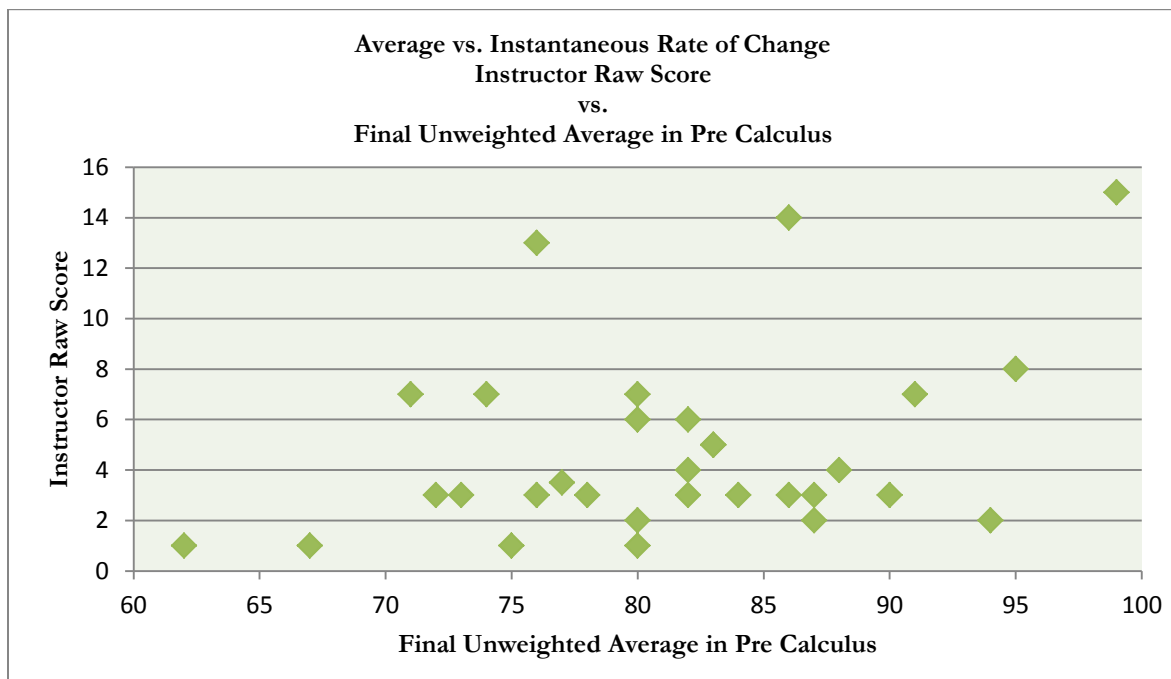


Figure 44. Instructor Raw Score vs. Final Unweighted Average in Algebra II from EXPLORING THE RELATIONSHIP BETWEEN AVERAGE AND INSTANTANEOUS RATES OF CHANGE.



*Figure 45.* Instructor Raw Score vs. Final Unweighted Average in Pre Calculus from EXPLORING THE RELATIONSHIP BETWEEN AVERAGE AND INSTANTANEOUS RATES OF CHANGE.

STUDENT RESULTS COMPONENT  
EXPLORING THE RELATIONSHIP BETWEEN  
DERIVATIVES AND INTEGRALS WITH THE TI-*n*spire CX HANDHELD

The graphing calculator lab Exploring the Relationship Between Derivatives and Integrals has been administered annually over a several year period of time. During those years, student performance was assessed to verify that the target concepts were being acquired. This graphing calculator lab activity was administered to 22 students studying Calculus during the late Spring of 2012. In addition, this graphing calculator lab activity was administered to two different levels of Calculus classes. One of the classes is an accelerated Calculus class with 7 students and the other is a Calculus class with 15 students. Each of the classes received the graphing calculator lab activity and corresponding Tech Sheets and was instructed to read through the instructions and complete a preliminary attempt at the activity independently. The objective of this activity was for students to discover the relationship between the derivative and the integral. More specifically, students were to discover the Fundamental Theorem of Calculus, Parts I and II, and use the Theorem to solve an applied problem with the use of technology. The activity uses graphical, numerical, and algebraic activities to aid in the discovery process, and to reinforce the concepts through the use of multiple representations of the data in consideration. Students received an overall grade on the activity as well as subscores in the following areas: competency in graphical analysis, numerical analysis, and algebraic analysis as well as competency in graphing calculator technology, and conceptual understanding of the Fundamental Theorem of Calculus. As students were working through the activity, they were encouraged to create a list of the topics that they had difficulty with and to score

themselves in each of the competency areas. In general, the students appear to have scored themselves higher than the instructor, but there was no pattern that emerged in the data. While the student scoring seemed inconsistent in comparison to the instructor's scoring, the students provided adequate written assessments of what they did and did not know how to do without assistance. They also had realistic views on how their past experiences in mathematics and associated technology impacted their ability to complete the activity successfully. Students received an overall grade on the activity as well as subscores in the following areas: competency in graphical analysis, numerical analysis, and algebraic analysis as well as competency in graphing calculator technology, and conceptual understanding of the relationship between average and instantaneous rate of change (the derivative). The accompanying rubric (Table 11) was used to assess student performance both by the instructor and the students as outlined in the Unit Lesson Plan presented earlier.

The data represented in the figures (Figures 46-56) below indicates that as a student's graphical competency increases, so does their overall performance on the graphing calculator lab activity. Likewise, there appear to be similar trends in the areas of numerical, algebraic, and technological competency as well as overall conceptual understanding. When the instructor raw score is compared to student GPA and final unweighted student averages in Geometry, Algebra II, Pre Calculus, and Calculus there appear to be no trends in the data indicating a relationship between any of the categories and the instructor raw score.

Table 11  
EXPLORING THE RELATIONSHIP BETWEEN  
DERIVATIVES AND INTEGRALS WITH THE TI-*n*spire CX HANDHELD

Scoring Rubric				
	0	1	2	3
Graphical Competency	Student is unable to begin calculation of the area of known geometric figures on the intervals $[0,x]$ and $[x_1, x_2]$ without assistance.	Student is able to successfully calculate the area of known geometric figures on the intervals $[0,x]$ and $[x_1, x_2]$ with major assistance.	Student is able to successfully calculate the area of known geometric figures on the intervals $[0,x]$ and $[x_1, x_2]$ with minor assistance.	Student is able to successfully calculate the area of known geometric figures on the intervals $[0,x]$ and $[x_1, x_2]$ without assistance.
Numerical Competency	Student is unable to successfully calculate the area of the setting sun using the Fundamental Theorem of Calculus, Part II without assistance.	Student is able to successfully calculate the area of the setting sun using the Fundamental Theorem of Calculus, Part II with major assistance.	Student is able to successfully calculate the area of the setting sun using the Fundamental Theorem of Calculus, Part II with minor assistance.	Student is able to successfully calculate the area of the setting sun using the Fundamental Theorem of Calculus, Part II without assistance.

Algebraic Competency	Student is unable to begin to successfully express area formulas on the interval $[x_1, x_2]$ in terms of $F(x_1)$ , $F(x_2)$ , $G(x_1)$ , $G(x_2)$ , $H(x_1)$ , and $H(x_2)$ without assistance.	Student is able to successfully express area formulas on the interval $[x_1, x_2]$ in terms of $F(x_1)$ , $F(x_2)$ , $G(x_1)$ , $G(x_2)$ , $H(x_1)$ , and $H(x_2)$ with major assistance.	Student is able to successfully express area formulas on the interval $[x_1, x_2]$ in terms of $F(x_1)$ , $F(x_2)$ , $G(x_1)$ , $G(x_2)$ , $H(x_1)$ , and $H(x_2)$ with minor assistance.	Student is able to successfully express area formulas on the interval $[x_1, x_2]$ in terms of $F(x_1)$ , $F(x_2)$ , $G(x_1)$ , $G(x_2)$ , $H(x_1)$ , and $H(x_2)$ without assistance.
Technological Competency	<p>Student is unable to begin to complete Tech Sheets on Importing Images, Creating Lists using the Data Capture feature, and Regression Analysis without assistance.</p> <p>Student is unable to begin to successfully complete computations on the graphing calculator without assistance.</p>	<p>Student is able to successfully complete Tech Sheets on Importing Images, Creating Lists using the Data Capture feature, and Regression Analysis with major assistance.</p> <p>Student is able to successfully complete computations on the graphing calculator with major assistance.</p>	<p>Student is able to successfully complete Tech Sheets on Importing Images, Creating Lists using the Data Capture feature, and Regression Analysis with minor assistance.</p> <p>Student is able to successfully complete computations on the graphing calculator with minor assistance.</p>	<p>Student is able to successfully complete Tech Sheets on Importing Images, Creating Lists using the Data Capture feature, and Regression Analysis without assistance.</p> <p>Student is able to successfully complete computations on the graphing calculator without assistance.</p>

Conceptual Understanding	Student is unable to begin to successfully express the relationship between the derivative and the integral, in particular, discover the Fundamental Theorem of Calculus, Parts I and II, without assistance.	Student is able to successfully express the relationship between the derivative and the integral, in particular, discover the Fundamental Theorem of Calculus, Parts I and II, with major assistance.	Student is able to successfully express the relationship between the derivative and the integral, in particular, discover the Fundamental Theorem of Calculus, Parts I and II, with minor assistance.	Student is able to successfully express the relationship between the derivative and the integral, in particular, discover the Fundamental Theorem of Calculus, Parts I and II, without assistance.
Graphical Competency				
Numerical Competency				
Algebraic Competency				
Technological Competency				
Conceptual Understanding				
TOTAL (out of 15 points)				

The following figures (Figures 46-56) represent data obtained in the Spring 2012 administration of the Exploring the Relationship Between Derivatives and Integrals graphing calculator lab activity. The first set of figures shows comparisons between the overall instructor score on the lab activity versus overall student score, each of the category subscores, GPA, and final averages in Geometry, Algebra II, Pre Calculus, and Calculus.

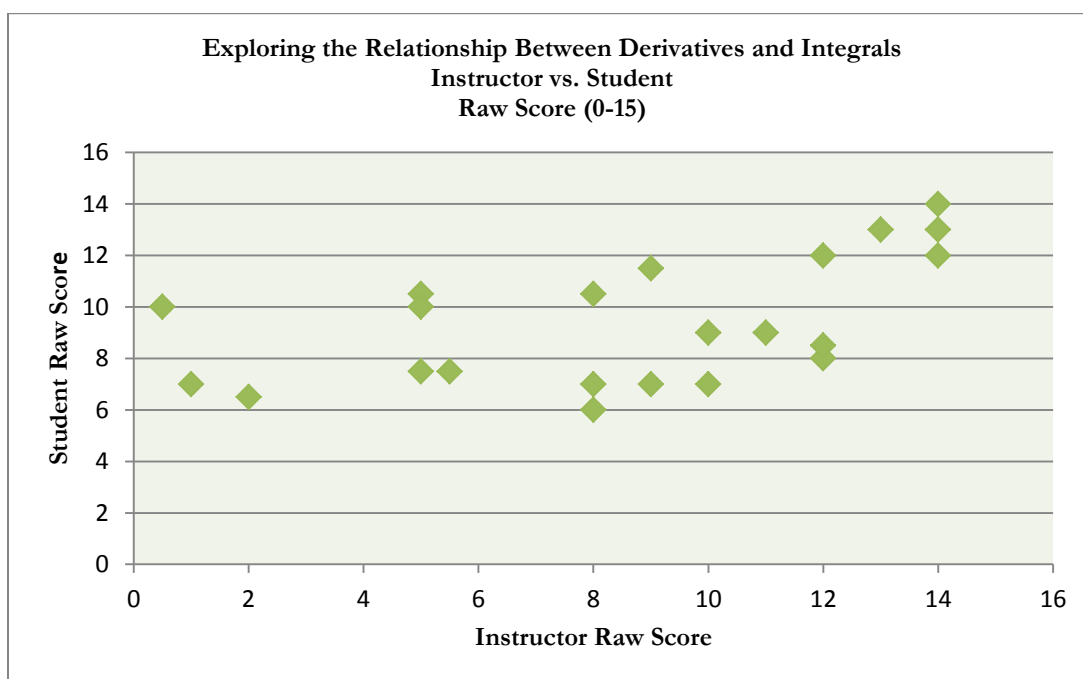


Figure 46. Instructor vs. Student Raw Score from EXPLORING THE RELATIONSHIP BETWEEN DERIVATIVES AND INTEGRALS WITH THE TI-*n*spire CX HANDHELD.



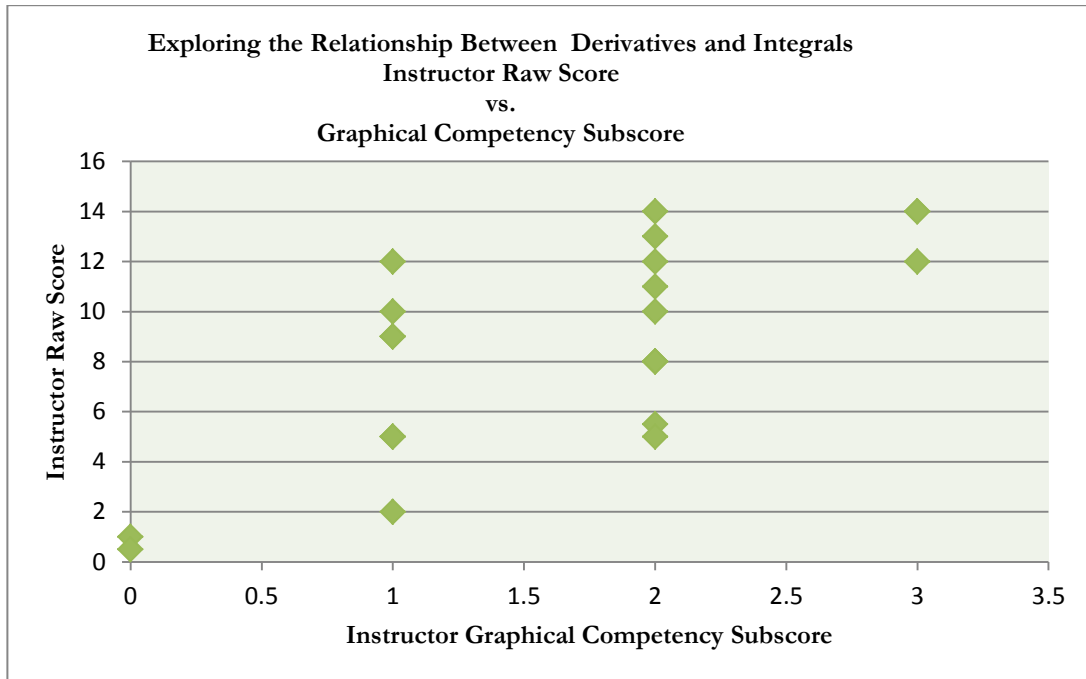


Figure 47. Instructor Raw Score vs. Graphical Competency Subscore from EXPLORING THE RELATIONSHIP BETWEEN DERIVATIVES AND INTEGRALS WITH THE TI-*mspire* CX HANDHELD.

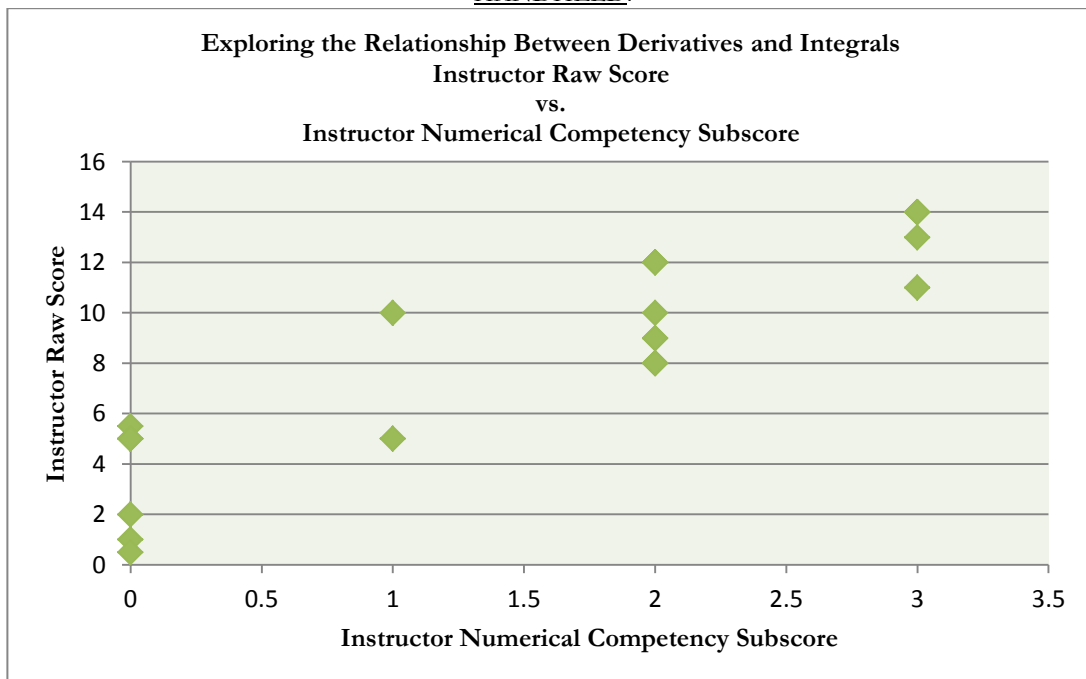


Figure 48. Instructor Raw Score vs. Numerical Competency Subscore from EXPLORING THE RELATIONSHIP BETWEEN DERIVATIVES AND INTEGRALS WITH THE TI-*mspire* CX HANDHELD.

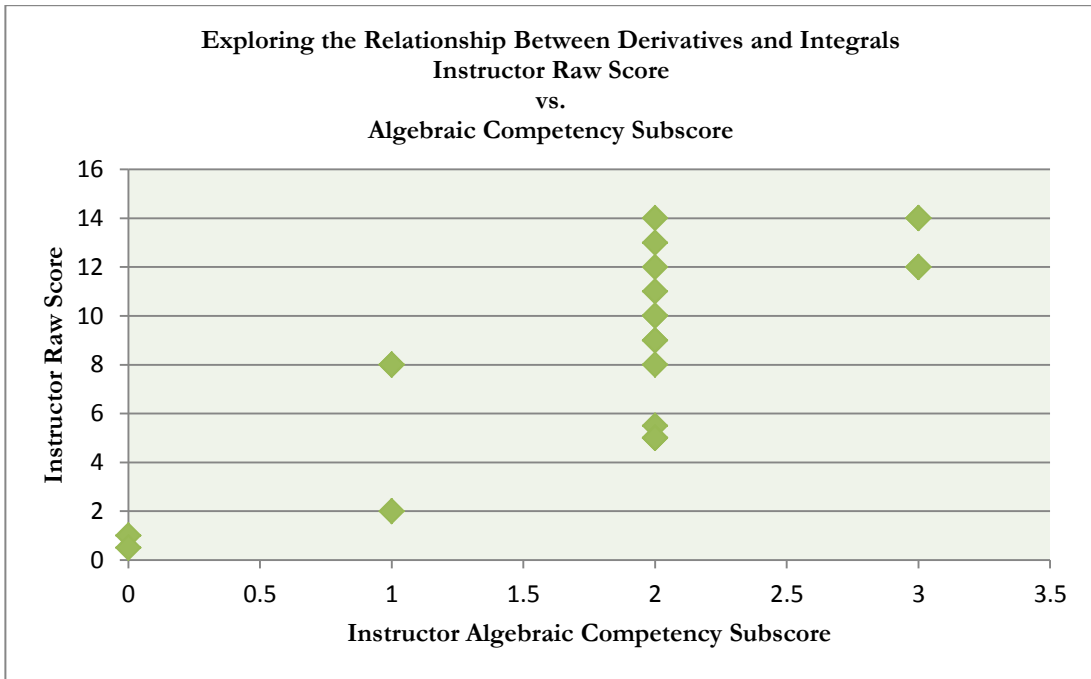


Figure 49. Instructor Raw Score vs. Algebraic Competency Subscore from EXPLORING THE RELATIONSHIP BETWEEN DERIVATIVES AND INTEGRALS WITH THE TI-*n*spire CX HANDHELD.

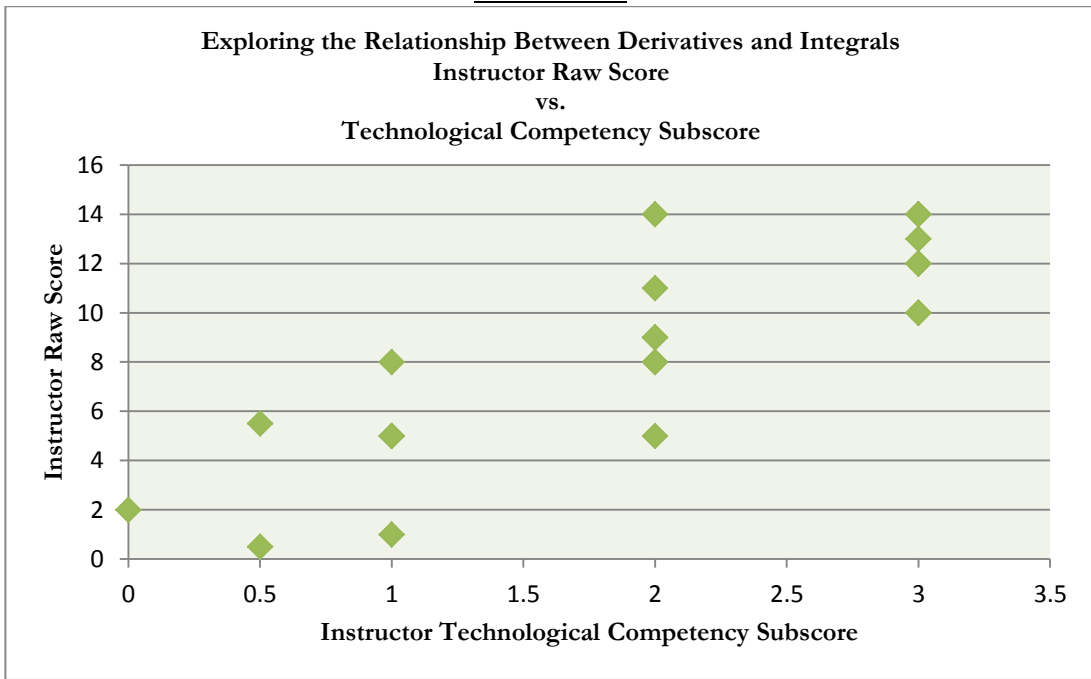


Figure 50. Instructor Raw Score vs. Technological Competency Subscore from EXPLORING THE RELATIONSHIP BETWEEN DERIVATIVES AND INTEGRALS WITH THE TI-*n*spire CX HANDHELD.

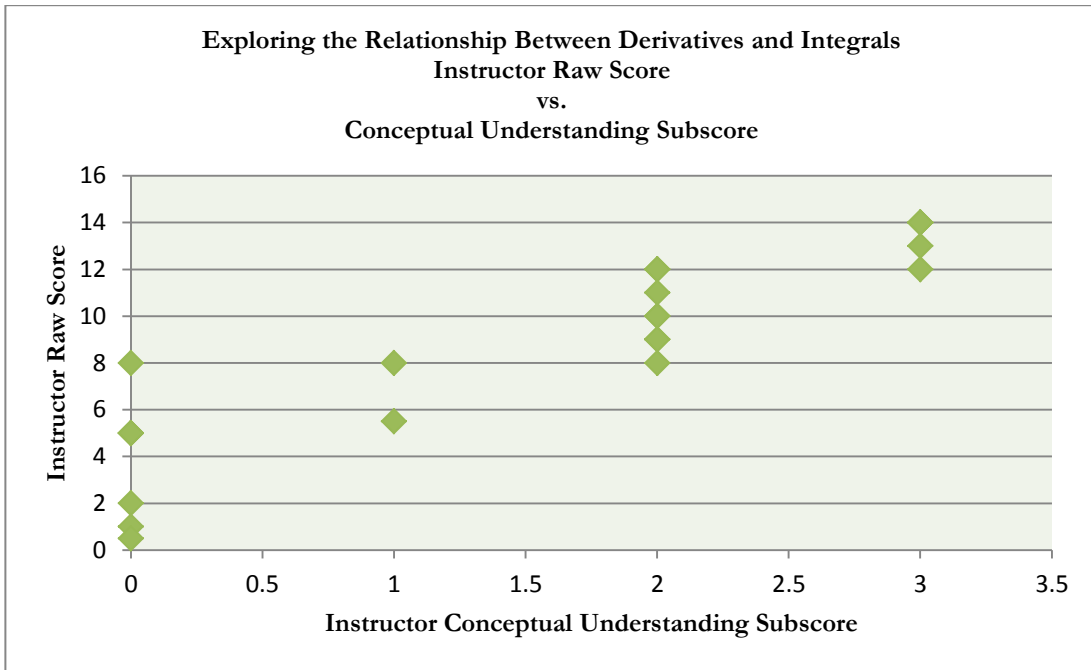


Figure 51. Instructor Raw Score vs. Conceptual Understanding Subscore from EXPLORING THE RELATIONSHIP BETWEEN DERIVATIVES AND INTEGRALS WITH THE TI-*n*spire CX HANDHELD.

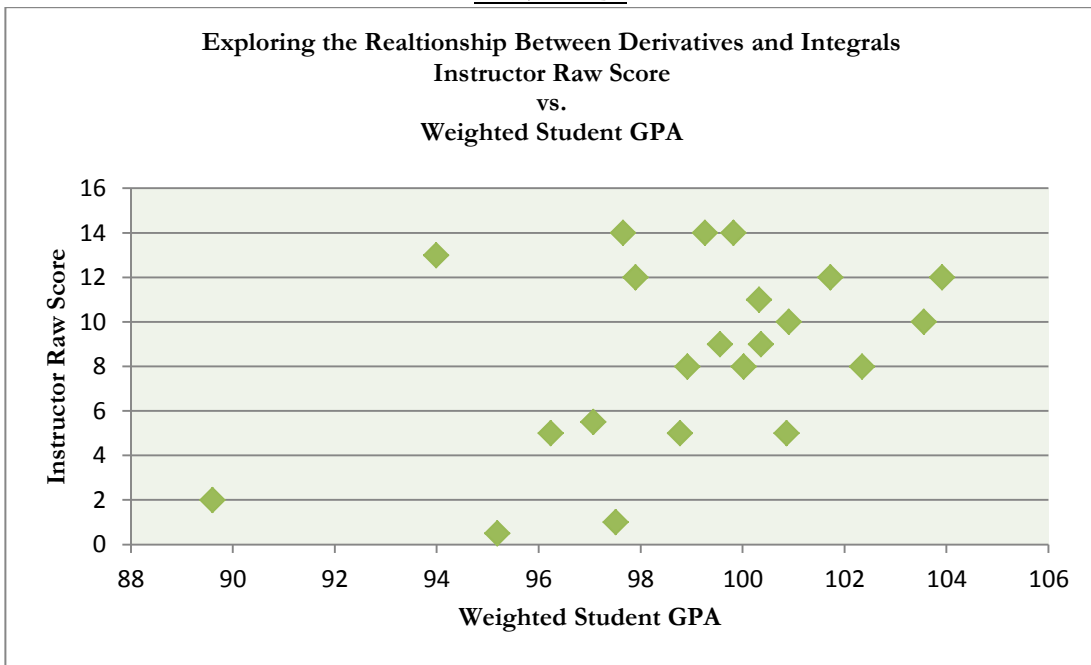


Figure 52. Instructor Raw Score vs. Weighted Student GPA from EXPLORING THE RELATIONSHIP BETWEEN DERIVATIVES AND INTEGRALS WITH THE TI-*n*spire CX HANDHELD.

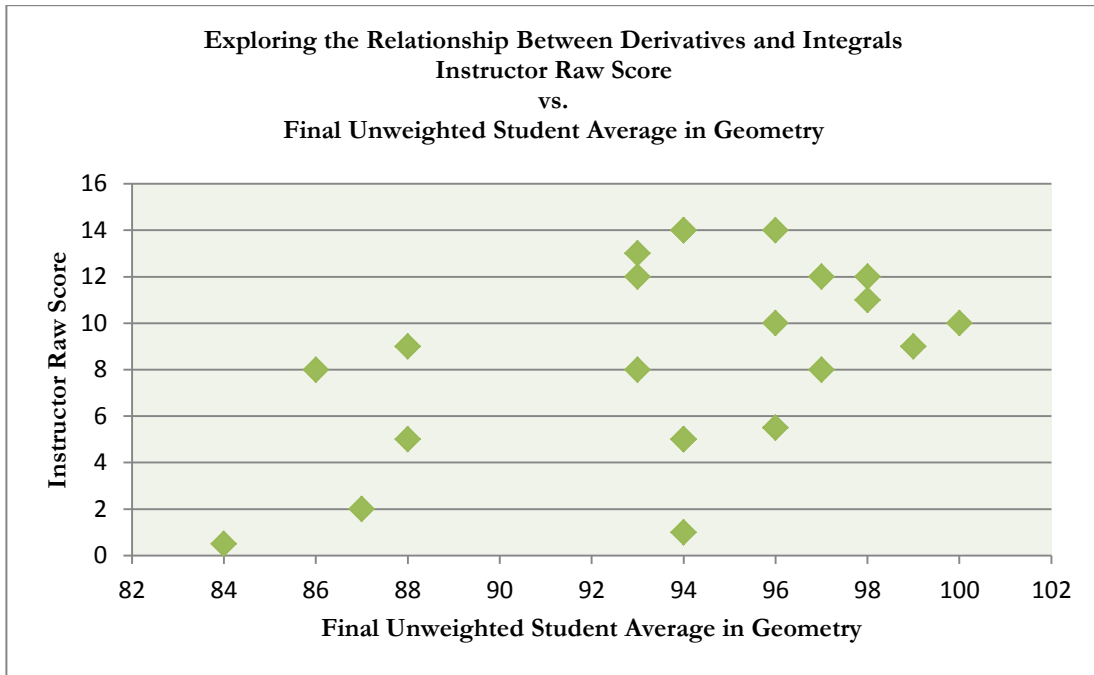


Figure 53. Instructor Raw Score vs. Final Unweighted Student Average in Geometry from EXPLORING THE RELATIONSHIP BETWEEN DERIVATIVES AND INTEGRALS WITH THE TI-*n*spire CX HANDHELD.

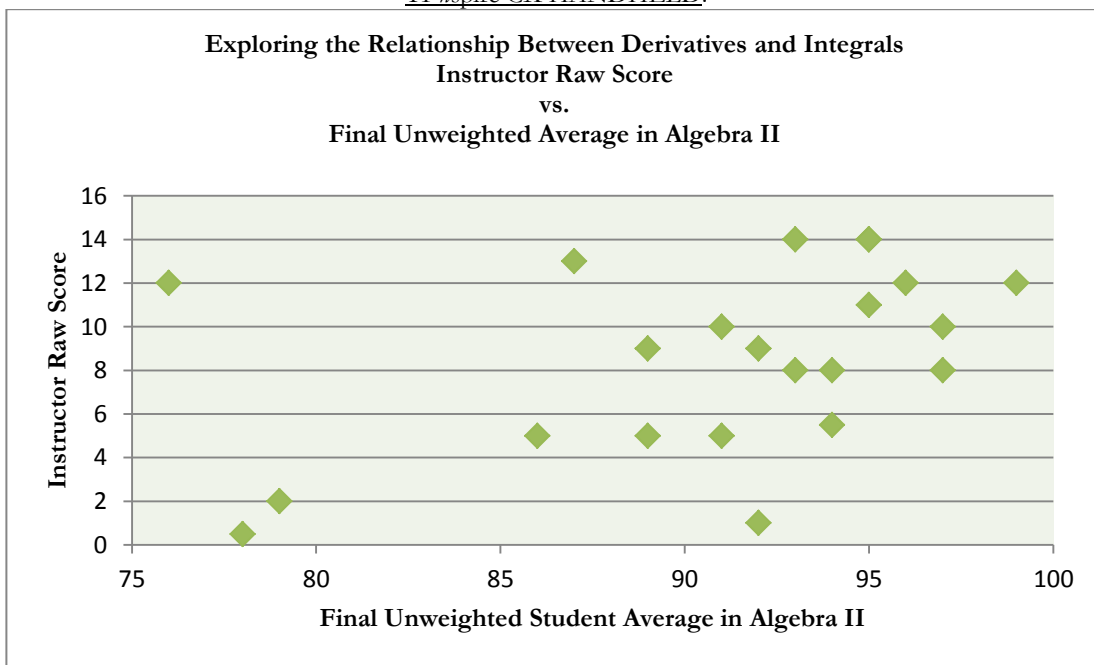
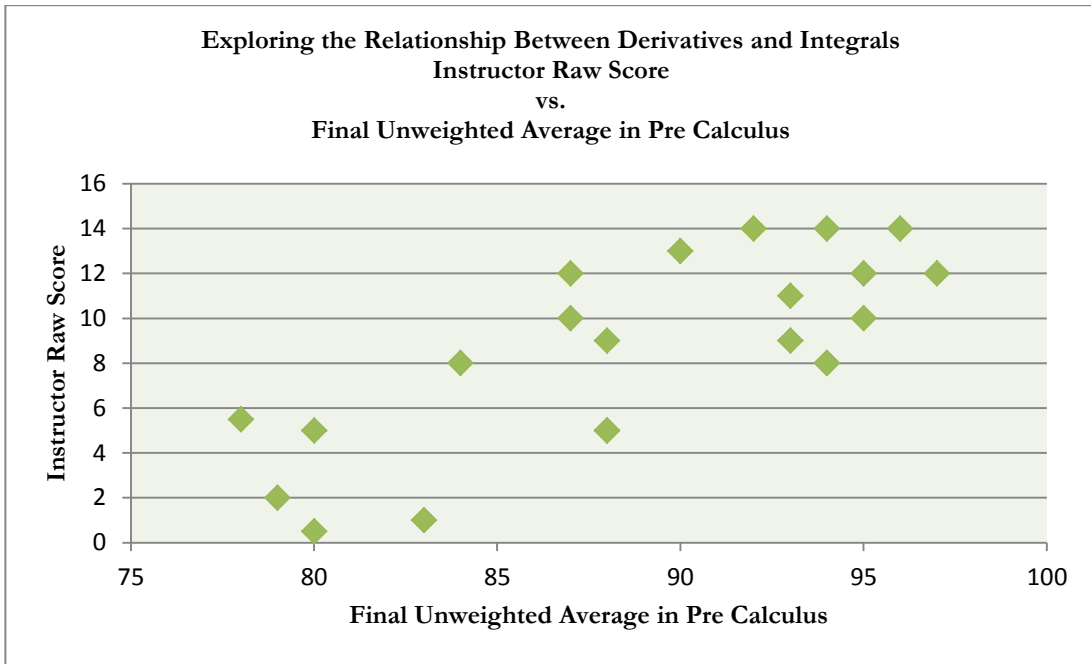
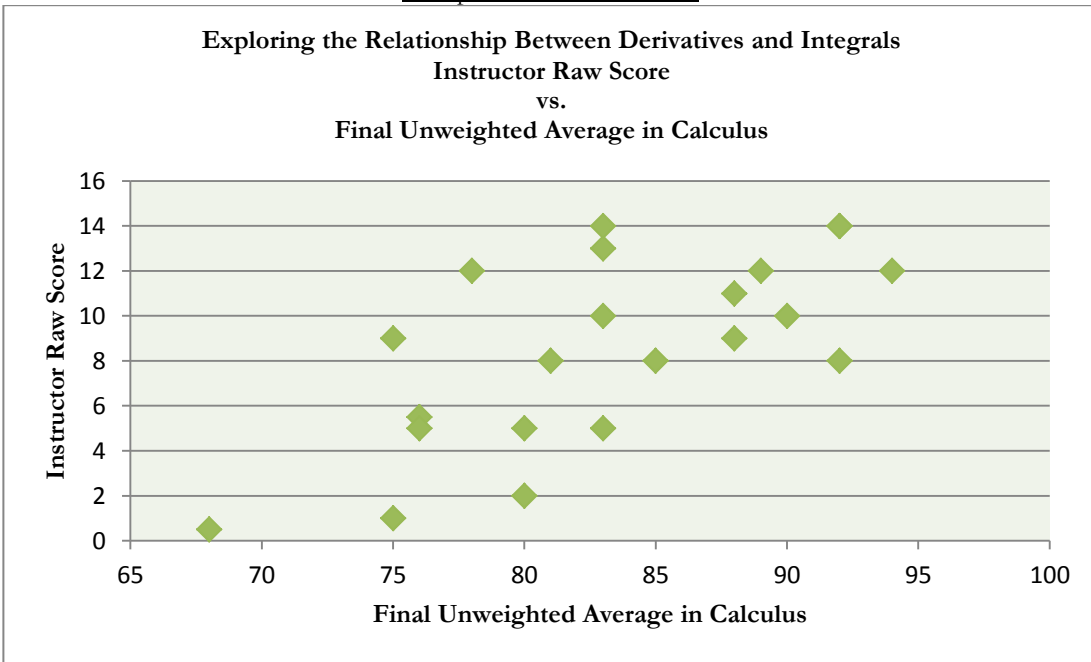


Figure 54. Instructor Raw Score vs. Final Unweighted Student Average in Algebra II from EXPLORING THE RELATIONSHIP BETWEEN DERIVATIVES AND INTEGRALS WITH THE TI-*n*spire CX HANDHELD.



*Figure 55.* Instructor Raw Score vs. Final Unweighted Average in Pre Calculus from EXPLORING THE RELATIONSHIP BETWEEN DERIVATIVES AND INTEGRALS WITH THE TI-*n*spire CX HANDHELD.



*Figure 56.* Instructor Raw Score vs. Final Unweighted Average in Calculus from EXPLORING THE RELATIONSHIP BETWEEN DERIVATIVES AND INTEGRALS WITH THE TI-*n*spire CX HANDHELD.

STUDENT RESULTS COMPONENT  
DISCUSSION OF OVERALL RESULTS

The data presented in Figures 36 through 56 has provided evidence for continued research in the area of teaching Calculus using multiple representations of mathematical models. In an effort to diversify the sample, the activities will be presented to various levels of students taking introductory-level Calculus courses during the 2012-2013 school year.

Perhaps the most significant outcome from the graphing calculator lab activities was the student and teacher informal discussion sessions that occurred during the learning process. Students were able to provide clear and accurate descriptions of what they knew and did not know. While many students felt the activities were challenging, they also shared that they felt a true sense of accomplishment when they had completed the activities. They were proud of the effort that they put forth and felt that they truly understood the target concepts. Students also commented that the exploratory activities presented them with multiple representations of mathematical models, which they found to be helpful in making connections to discover the target concepts.

Presenting mathematical concepts using exploratory activities that incorporate multiple representations of data for mathematical models with the TI-*n*spire CX Handheld as a computational tool has provided the students sampled with a strong conceptual understanding of the derivative and its relationship to the integral. After completing the activities, students were able to effectively communicate both orally and in writing about the derivative and the integral.

Due to the success of the exploratory graphing calculator lab activities in regards to student understanding of target concepts, similar activities are currently in the design stages for other major concepts in Calculus. Through the use of graphical, numerical, and algebraic representations of mathematical models, it is the hope that student understanding of mathematics will continue to improve.

## ADDITIONAL QUARTERLY ASSESSMENTS

In addition to the exploratory graphing calculator lab activities presented thus far, students in each of the classes, Calculus and Advanced Calculus, were also administered quarterly assessments in order to determine if the use of the graphing calculator increased performance in mathematics. Each of the classes was given two open-ended questions to complete on a quarterly basis. The questions covered the same concept, but one question was to be done without the use of a graphing calculator, and the second question allowed the use of a graphing calculator. The assessments are provided on the following pages.

Following the presentation of the quarterly assessment open-ended questions are tables containing the results of the open-ended questions (Tables 12-15). The tables present student performance on each of the quarterly assessments, both with and without the use of a graphing calculator. In addition, plots were created to highlight trends in student performance. The plots are contained in Figures 57-64. Upon examination of the data from the quarterly assessments two major trends appear: First and foremost, students, in general, received higher scores on questions involving the use of the graphing calculator than on similar questions not allowing the use of a graphing calculator. In addition, students' performance on the tasks, both with and without the use of a graphing calculator, increased as the year went on.

Based on the evidence presented herein, the data appear to suggest that increased student performance in mathematics may be correlated to the use of a graphing calculator in mathematics instruction. Due to the trends that were observed, continued assessments are planned for the upcoming semester to gather more data to support the hypothesis. It is



encouraging to see not only an increase in student performance in mathematics, but also an increased amount of enthusiasm from the students as they feel more confident in their abilities to problem-solve in mathematics. It is the intention that continued research will lead to the exact cause of the increase improvement in performance in mathematics seen in the data that is presented.

Calculus 2012  
Open-Ended Questions - No Calculator

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Fill in the blanks to complete the statement.

- 1) The graph of  $y = 0.1 - |x| - 9$  can be obtained from the graph of  $y = |x|$  by reflecting across the   ?  -axis, shrinking vertically by a factor of   ?  , and then shifting vertically   ?   units in the   ?   direction. 1) \_\_\_\_\_

Solve the problem.

- 2) Suppose that the average monthly low temperatures for a small town are shown in the table. 2) \_\_\_\_\_

Month	1	2	3	4	5	6	7	8	9	10	11	12
Temperature (°F)	19	27	38	45	57	62	65	58	51	41	33	25

Model this data using  $f(x) = a \sin(b(x - c)) + d$ .

Find the derivative of the function at the specified point.

- 3)  $f(x) = \frac{8}{x+2}$  at  $x = 0$  3) \_\_\_\_\_

Compute the average of the RRAM and LRAM approximations to estimate the area between the graph of the function and the x-axis over the given interval using the indicated number of subintervals. (The function is non-negative on the given interval).

- 4)  $f(x) = 10 + 3x - x^2$ ;  $[0, 4]$ ; 4 subintervals 4) \_\_\_\_\_

**Calculus 2012**  
**Open-Ended Questions - Calculator Use Permitted**

**SHORT ANSWER.** Write the word or phrase that best completes each statement or answers the question.

**Fill in the blanks to complete the statement.**

- 1) The graph of  $y = 0.8|-x| - 2$  can be obtained from the graph of  $y = |x|$  by reflecting across the   -axis, shrinking vertically by a factor of   , and then shifting vertically    units in the    direction. 1) \_\_\_\_\_

**Solve the problem.**

- 2) Suppose that the average monthly low temperatures for a small town are shown in the table. 2) \_\_\_\_\_

Month	1	2	3	4	5	6	7	8	9	10	11	12
Temperature (°F)	19	27	38	43	57	62	65	58	51	41	33	25

Model this data using  $f(x) = a \sin(b(x - c)) + d$ .

**Find the derivative of the function at the specified point.**

- 3)  $f(x) = \frac{8}{x+2}$  at  $x=0$  3) \_\_\_\_\_

**Compute the average of the RRAM and LRAM approximations to estimate the area between the graph of the function and the x-axis over the given interval using the indicated number of subintervals. (The function is non-negative on the given interval).**

- 4)  $f(x) = 14 + 5x - x^2$ ;  $[0, 4]$ ; 4 subintervals 4) \_\_\_\_\_

**Advanced Calculus 2012**  
**Open-Ended Questions - No Calculator**

**SHORT ANSWER.** Write the word or phrase that best completes each statement or answers the question.

Find the limit of  $f(x)$  as (a)  $x \rightarrow -\infty$ , (b)  $x \rightarrow \infty$ , (c)  $x \rightarrow 0^-$ , and (d)  $x \rightarrow 0^+$ .

$$1) f(x) = \begin{cases} \frac{x-6}{x-3}, & x \leq 0 \\ \frac{1}{x^2}, & x > 0 \end{cases} \quad 1) \underline{\hspace{2cm}}$$

**Solve the problem.**

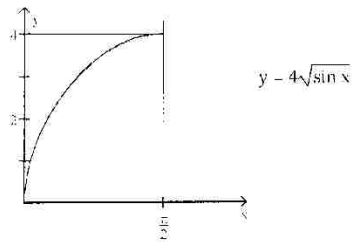
- 2) Find the number of units that must be produced and sold in order to yield the maximum profit, given the following equations for revenue and cost:  
 $R(x) = 7x$   
 $C(x) = 0.001x^2 + 0.9x + 20$ . 2)

**Find the area enclosed by the given curves.**

- 3) Find the area of the region in the first quadrant bounded on the left by the  $y$ -axis, below by the line  $y = \frac{1}{3}x$ , above left by  $y = x + 4$ , and above right by  $y = -x^2 + 10$ . 3)

**Find the volume of the solid generated by revolving the shaded region about the given axis.**

- 4) About the  $x$ -axis. 4)



**Advanced Calculus 2012**  
**Open-Ended Questions - Calculator Use Permitted**

**SHORT ANSWER.** Write the word or phrase that best completes each statement or answers the question.

Find the limit of  $f(x)$  as (a)  $x \rightarrow -\infty$ , (b)  $x \rightarrow \infty$ , (c)  $x \rightarrow 0^-$ , and (d)  $x \rightarrow 0^+$ .

$$1) f(x) = \begin{cases} \frac{x-9}{x-3}, & x \leq 0 \\ \frac{1}{x^2}, & x > 0 \end{cases}$$

1) \_\_\_\_\_

**Solve the problem.**

2) Find the number of units that must be produced and sold in order to yield the maximum profit, given the following equations for revenue and cost:

$$R(x) = 6x$$

$$C(x) = 0.01x^2 + 0.5x + 80.$$

2) \_\_\_\_\_

**Find the area enclosed by the given curves.**

3) Find the area of the region in the first quadrant bounded on the left by the  $y$ -axis, below by the line  $y = \frac{1}{3}x$ , above left by  $y = x + 4$ , and above right by  $y = -x^2 + 10$ .

3) \_\_\_\_\_

**Find the volume of the solid generated by revolving the shaded region about the given axis.**

4) About the  $x$ -axis

4) \_\_\_\_\_

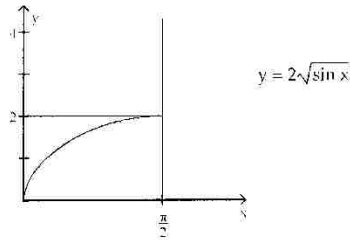


Table 12  
 CALCULUS 2012  
 OPEN-ENDED QUESTIONS – NO CALCULATOR

Student ID	Student #	Question 1	Question 2	Question 3	Question 4
132801	1	1	1	2	3
121036	2	2	2	3	4
132326	3	1	2	2	3
121107	4	1	2	3	3
132215	5	1	2	4	4
994503	6	0	1	2	3
132150	7	1	3	3	4
132173	8	2	2	3	3
994451	9	1	2	2	3
121251	10	1	2	3	3
994175	11	1	1	2	4
132271	12	1	2	2	3
994910	13	0	2	4	4
121019	14	2	2	3	4
132034	15	2	4	3	3
132183	16	1	2	2	3
121076	17	3	3	4	3
132171	18	2	3	3	4
994408	19	2	2	3	3
132126	20	1	2	2	4
132091	21	2	2	2	3
994121	22	1	2	3	3
132122	23	1	2	3	4
121180	24	2	4	3	3
994429	25	0	2	3	3
121345	26	1	2	4	4
121199	27	2	2	3	4
132104	28	2	3	3	3
121219	29	1	2	2	3
132286	30	2	1	3	3
132113	31	3	3	2	3

Table 13  
 CALCULUS 2012  
 OPEN-ENDED QUESTIONS – CALCULATOR USE PERMITTED

Student ID	Student #	Question 1	Question 2	Question 3	Question 4
132801	1	0	1	3	3
121036	2	0	2	4	4
132326	3	1	3	3	4
121107	4	1	2	3	3
132215	5	1	3	4	4
994503	6	0	3	3	3
132150	7	1	3	4	4
132173	8	2	3	4	3
994451	9	2	3	3	4
121251	10	1	2	2	4
994175	11	2	1	3	4
132271	12	2	3	2	4
994910	13	1	2	4	4
121019	14	3	3	3	4
132034	15	2	4	3	4
132183	16	2	3	3	3
121076	17	3	3	4	4
132171	18	3	3	3	4
994408	19	2	3	4	4
132126	20	2	2	4	4
132091	21	2	3	3	4
994121	22	2	1	3	3
132122	23	2	2	3	4
121180	24	3	4	4	4
994429	25	2	3	3	4
121345	26	2	3	4	4
121199	27	3	2	3	4
132104	28	3	3	4	3
121219	29	2	2	3	3
132286	30	2	1	3	4
132113	31	3	3	4	4

Table 14  
 ADVANCED CALCULUS 2012  
 OPEN-ENDED QUESTIONS – NO CALCULATOR

Student ID	Student #	Question 1	Question 2	Question 3	Question 4
121290	1	0	1	3	3
121055	2	2	2	3	3
121152	3	1	2	3	4
121159	4	1	2	3	4
121273	5	0	1	3	3
994437	6	2	2	3	4
121206	7	2	3	3	4
121013	8	1	2	3	4
121023	9	4	4	4	4
121033	10	2	3	4	4
121040	11	1	3	3	4
121265	12	1	2	3	4
121059	13	2	2	2	3
121065	14	1	1	2	3
123004	15	0	1	2	3
123005	16	1	2	2	2
121077	17	2	3	3	3
121087	18	2	2	2	3
121095	19	2	2	3	4
121099	20	2	3	3	3
121118	21	1	2	2	3
121254	22	1	3	4	3



Table 15  
 ADVANCED CALCULUS 2012  
 OPEN-ENDED QUESTIONS – CALCULATOR USE PERMITTED

Student ID	Student #	Question 1	Question 2	Question 3	Question 4
121290	1	1	3	4	4
121055	2	2	2	3	3
121152	3	1	2	4	4
121159	4	2	2	4	4
121273	5	1	2	3	4
994437	6	2	3	4	4
121206	7	2	3	4	4
121013	8	2	3	3	4
121023	9	4	4	4	4
121033	10	3	3	4	4
121040	11	2	3	3	4
121265	12	1	3	4	4
121059	13	2	3	3	3
121065	14	2	2	3	4
123004	15	0	1	3	3
123005	16	1	3	2	3
121077	17	3	4	4	4
121087	18	2	3	2	3
121095	19	3	3	4	4
121099	20	2	3	3	3
121118	21	2	4	4	4
121254	22	1	3	4	4

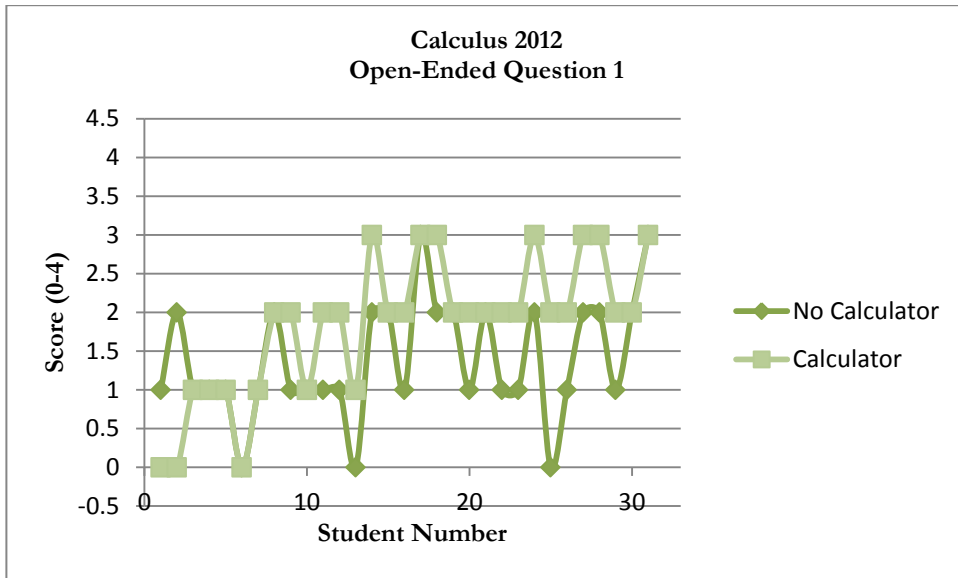


Figure 57. Calculus 2012 Student Scores on Open-Ended Question 1 With and With Out Calculator Usage.

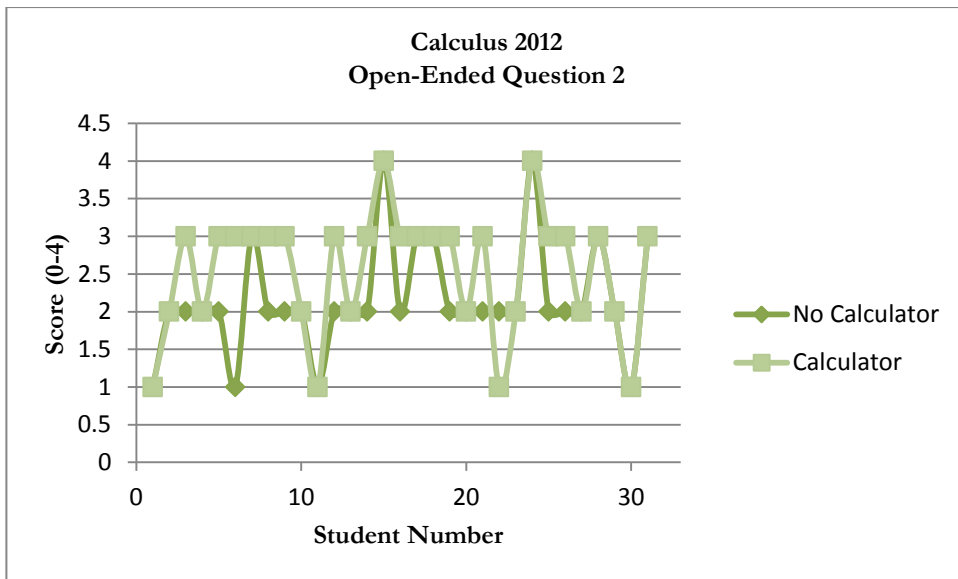


Figure 58. Calculus 2012 Student Scores on Open-Ended Question 2 With and With Out Calculator Usage.

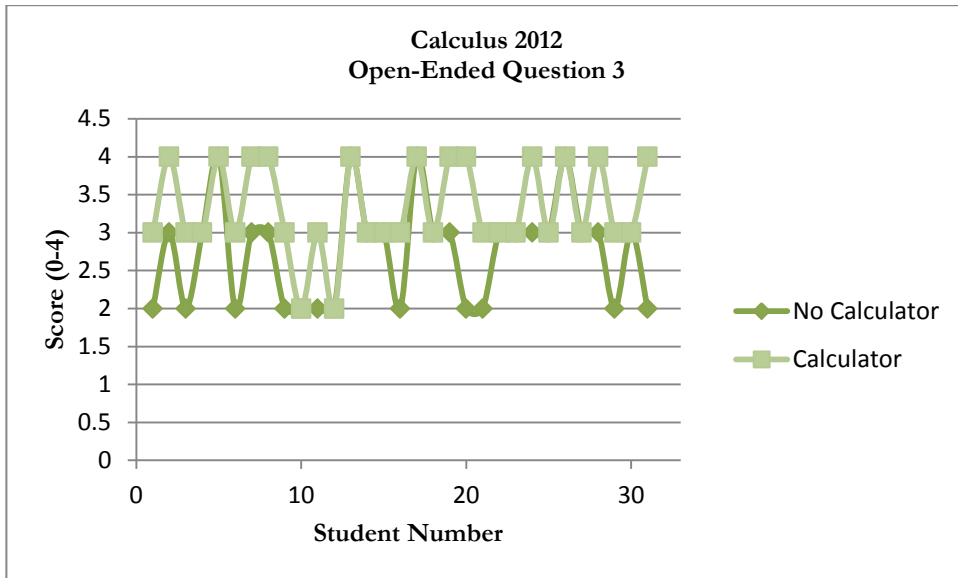


Figure 59. Calculus 2012 Student Scores on Open-Ended Question 3 With and With Out Calculator Usage.

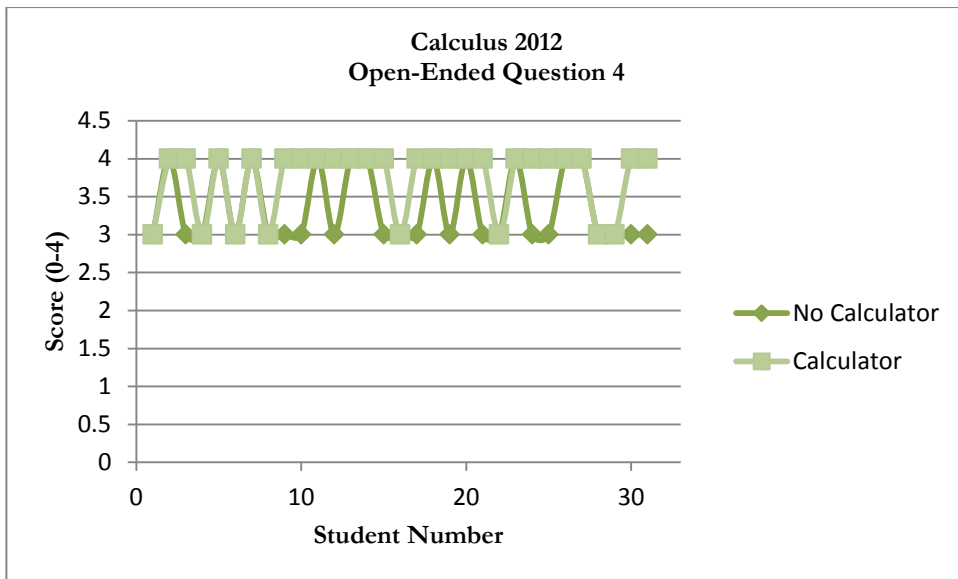


Figure 60. Calculus 2012 Student Scores on Open-Ended Question 4 With and With Out Calculator Usage.

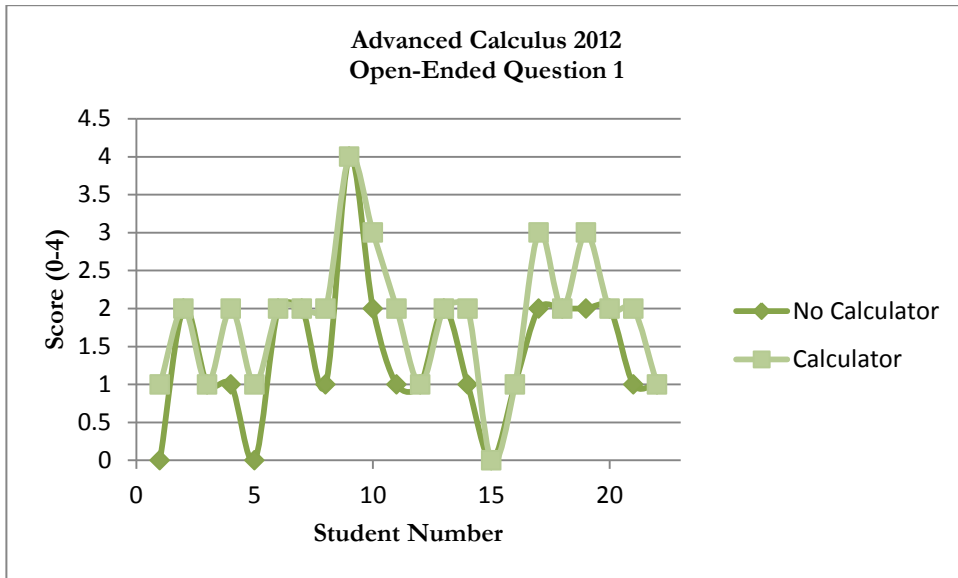


Figure 61. Advanced Calculus 2012 Student Scores on Open-Ended Question 1 With and With Out Calculator Usage.

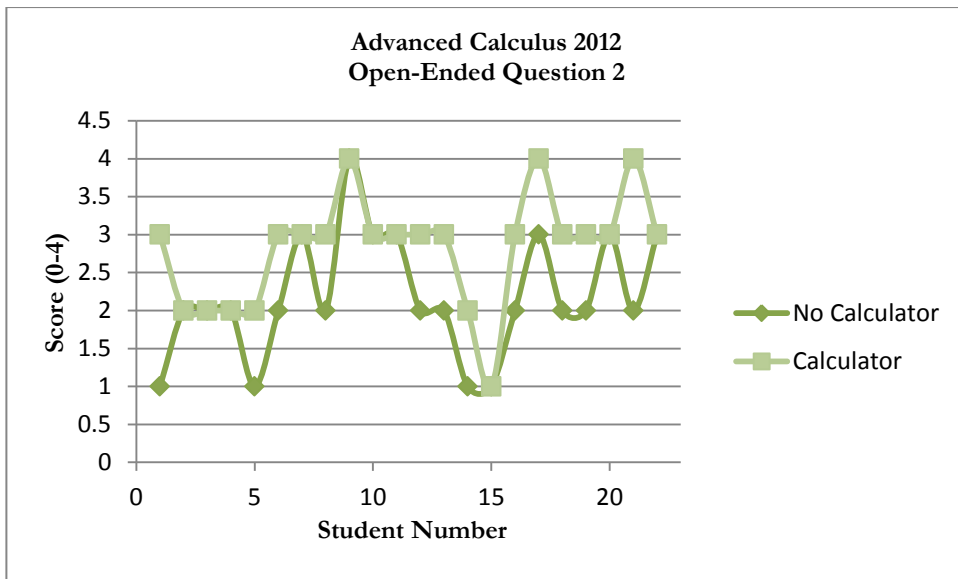


Figure 62. Advanced Calculus 2012 Student Scores on Open-Ended Question 2 With and With Out Calculator Usage.

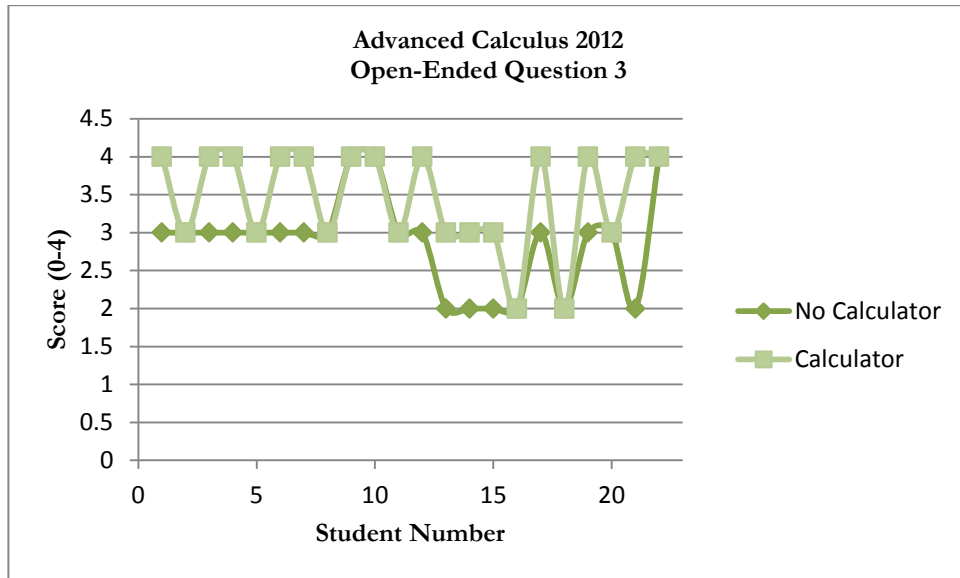


Figure 63. Advanced Calculus 2012 Student Scores on Open-Ended Question 3 With and With Out Calculator Usage.

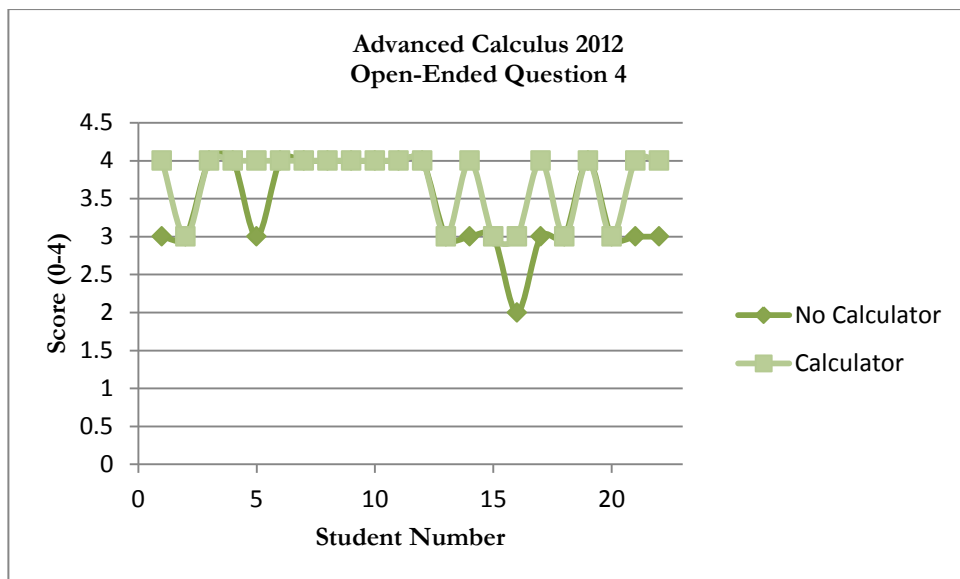


Figure 64. Advanced Calculus 2012 Student Scores on Open-Ended Question 4 With and With Out Calculator Usage.

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APPENDIX A  
 Results for  
**EXPLORING THE RELATIONSHIP BETWEEN AVERAGE AND  
 INSTANTANEOUS RATES OF CHANGE WITH THE TI-*n*spire CX  
 HANDHELD**

Course/Lab	Student ID	GPA	Algebra 1	Algebra 2	Geometry	Pre Calc
1	132081	91.81		74	89	62
1	121036	93.57	100	86	90	77
1	132326	99.80		93	92	84
1	121107	92.46	96	90	98	80
1	132215	99.08		89	94	90
1	994503	101.73		95	94	91
1	132150	102.97		99	97	95
1	132173	95.05		81	88	76
1	994451	89.66	85	89	90	80
1	121251	92.61	95	93	96	71
1	994175	90.50	83	78	87	72
1	132271	100.56	95	94	91	83
1	994910	101.38		96	95	74
1	121019	96.66	92	92	87	67
1	132034	100.12		96	93	86
1	132183	99.89		88	94	82
1	121076	94.70	90	93	94	78
1	132171	97.00		86	88	73
1	994408	95.73		98	96	83
1	132126	101.74		96	96	88
1	132091	95.86		83	92	75
1	994121	107.66		99	100	99
1	132122	96.55		83	91	76
1	121180	99.61	98	95	95	87
1	994429	100.17		97	98	87
1	121345	100.46	94	89	93	80
1	121199	100.12	96	90	94	80
1	132104	98.52		87	91	82
1	121219	96.87	83	92	94	86
1	132286	103.50		99	98	94
1	132113	98.81		88	92	82

APPENDIX A  
 Results for  
**EXPLORING THE RELATIONSHIP BETWEEN AVERAGE AND  
 INSTANTANEOUS RATES OF CHANGE WITH THE TI-*n*spire CX  
 HANDHELD**

Student ID	Graphical Competency (S)	Numerical Competency (S)	Algebraic Competency (S)	Technological Competency (S)	Conceptual Understanding (S)
132081	2	1	2	2	1
121036	1	2	1	1	1
132326	0	0	0	1	0
121107	0	3	0	1	0
132215	3	3	2	3	2
994503	1	1	2	2	2
132150	3	3	3	2	2
132173	1	1	1	1	0
994451	1	1	2	1	1
121251	1	1	2	2	1
994175	1	1	1	2	1
132271	0	1	2	2	2
994910	2	2	2	1	2
121019	0	1.5	1	2	1
132034	1	1	1	2	1
132183	1	1.5	1	2	1
121076	2	2.5	2	1.5	2
132171	0	0	0	0	0
994408	2	2.5	2	2.5	2
132126	2	1.5	2	2.5	2
132091	2	1	2	2	1
994121	3	3	3	3	2
132122	3	2.5	2	2	1
121180	2	3	2	3	3
994429	2	2	2	2.5	2
121345	1	0	1	2	1
121199	0	1.5	2	2.5	1
132104	1	1	2	1	1
121219	1	3	1	3	3
132286	2	2	2	2.5	2
132113	2	1.5	2	1	1



APPENDIX A  
 Results for  
**EXPLORING THE RELATIONSHIP BETWEEN AVERAGE AND  
 INSTANTANEOUS RATES OF CHANGE WITH THE TI-*n*spire CX  
 HANDHELD**

Student ID	Graphical Competency (I)	Numerical Competency (I)	Algebraic Competency (I)	Technological Competency (I)	Conceptual Understanding (I)
132081	0	0	0	1	0
121036	0	2	0	1.5	0
132326	0	2	0	1	0
121107	0	1	0	1	0
132215	0	1	0	1	1
994503	1	2	1	1	2
132150	1	3	1	2	1
132173	0	2	0	1	0
994451	1	2	1	1	2
121251	1	2	1	1	2
994175	0	2	0	1	0
132271	0	1	1	1	2
994910	0	2	1	3	1
121019	0	0	0	1	0
132034	0	0	0	3	0
132183	0	0	0	3	0
121076	0	0	0	3	0
132171	0	0	0	3	0
994408	0	0	0	3	2
132126	0	0	1	2	1
132091	0	0	0	1	0
994121	3	3	3	3	3
132122	3	3	3	2	2
121180	0	0	0	0	2
994429	0	0	0	2	1
121345	1	0	1	3	1
121199	0	0	0	1	0
132104	0	0	2	2	2
121219	3	2	3	3	3
132286	0	0	0	2	0
132113	0	1	0	3	0

APPENDIX A  
 Results for  
**EXPLORING THE RELATIONSHIP BETWEEN AVERAGE AND  
 INSTANTANEOUS RATES OF CHANGE WITH THE TI-*n*spire CX  
 HANDHELD**

Student ID	Instructor Raw Score	Student Raw Score	Instructor Percent Correct	Student Percent Correct
132081	1	8	6.67	53.33
121036	3.5	6	23.33	40.00
132326	3	1	20.00	6.67
121107	2	4	13.33	26.67
132215	3	13	20.00	86.67
994503	7	8	46.67	53.33
132150	8	13	53.33	86.67
132173	3	4	20.00	26.67
994451	7	6	46.67	40.00
121251	7	7	46.67	46.67
994175	3	6	20.00	40.00
132271	5	7	33.33	46.67
994910	7	9	46.67	60.00
121019	1	5.5	6.67	36.67
132034	3	6	20.00	40.00
132183	3	6.5	20.00	43.33
121076	3	10	20.00	66.67
132171	3	0	20.00	0.00
994408	5	11	33.33	73.33
132126	4	10	26.67	66.67
132091	1	8	6.67	53.33
994121	15	14	100.00	93.33
132122	13	10.5	86.67	70.00
121180	2	13	13.33	86.67
994429	3	10.5	20.00	70.00
121345	6	5	40.00	33.33
121199	1	7	6.67	46.67
132104	6	6	40.00	40.00
121219	14	11	93.33	73.33
132286	2	10.5	13.33	70.00
132113	4	7.5	26.67	50.00

APPENDIX B  
Results for  
**EXPLORING THE RELATIONSHIP BETWEEN DERIVATIVES AND  
INTEGRALS WITH THE TI-*n*spire CX HANDHELD**

Course/Lab	Student ID	GPA	Algebra 1	Algebra 2	Geometry	Pre Calc	Calculus
2	121290	93.99	95	87	93	90	83
2	121055	99.26		95	94	96	92
2	121152	103.55		97	100	95	90
2	121159	97.65	95	93	94	94	92
2	121273	97.90		76	93	87	78
2	994437	99.56		89	88	93	88
2	121206	100.02	98	94	93	94	92
2	121013	102.34		97	97	94	85
2	121023	97.07		94	96	78	76
2	121033	100.91		91	96	87	83
2	121040	101.73		99	98	95	89
2	121265	97.51		92	94	83	75
2	121059	95.19		78	84	80	68
2	121065	103.92		96	97	97	94
2	123004	98.91		93	86	84	81
2	123005	96.23	94	86	88	80	76
2	121077	100.86		91	94	88	80
2	121087	100.36		92	99	88	75
2	121095	100.32		95	98	93	88
2	121099	98.77		89	94	88	83
2	121118	99.82		95	96	92	83
2	121254	89.60		79	87	79	80

APPENDIX B  
Results for  
**EXPLORING THE RELATIONSHIP BETWEEN DERIVATIVES AND  
INTEGRALS WITH THE TI-*nspire* CX HANDHELD**

Student ID	Graphical Competency (S)	Numerical Competency (S)	Algebraic Competency (S)	Technological Competency (S)	Conceptual Understanding (S)
121290	3	3	2	3	2
121055	3	3	2	3	3
121152	2	3	2	1	1
121159	2	3	2	3	2
121273	2	3	2	3	2
994437	2	3	2	2.5	2
121206	2	3	1	2.5	2
121013	1	1	1	2	1
121023	2	2	1	1.5	1
121033	1	1	1	2	2
121040	2	1	1	2	2
121265	2	1	1	2	1
121059	2	2	2	2	2
121065	2	2	1	1.5	2
123004	2	1	1	2	1
123005	2	2	1	1.5	1
121077	2	2	2	2.5	2
121087	2	1	1	1	2
121095	2	1	2	2	2
121099	2	2	1	2	3
121118	2	3	2	3	3
121254	2	1	2	0.5	1

APPENDIX B  
Results for  
**EXPLORING THE RELATIONSHIP BETWEEN DERIVATIVES AND  
INTEGRALS WITH THE TI-*nspire* CX HANDHELD**

Student ID	Graphical Competency (I)	Numerical Competency (I)	Algebraic Competency (I)	Technological Competency (I)	Conceptual Understanding (I)
121290	2	3	2	3	3
121055	3	3	2	3	3
121152	2	1	2	3	2
121159	3	3	3	2	3
121273	3	2	2	3	2
994437	1	2	2	2	2
121206	2	2	1	1	2
121013	2	2	1	2	1
121023	2	0	2	0.5	1
121033	1	2	2	3	2
121040	1	2	3	3	3
121265	0	0	0	1	0
121059	0	0	0	0.5	0
121065	2	2	3	3	2
123004	2	2	2	2	0
123005	1	0	2	2	0
121077	2	0	2	1	0
121087	1	2	2	2	2
121095	2	3	2	2	2
121099	1	1	2	1	0
121118	2	3	3	3	3
121254	1	0	1	0	0

APPENDIX B  
Results for  
**EXPLORING THE RELATIONSHIP BETWEEN DERIVATIVES AND  
INTEGRALS WITH THE TI-*nspire* CX HANDHELD**

Student ID	Instructor Raw Score	Student Raw Score	Instructor Percent Correct	Student Percent Correct
121290	13	13	86.67	86.67
121055	14	14	93.33	93.33
121152	10	9	66.67	60.00
121159	14	12	93.33	80.00
121273	12	12	80.00	80.00
994437	9	11.5	60.00	76.67
121206	8	10.5	53.33	70.00
121013	8	6	53.33	40.00
121023	5.5	7.5	36.67	50.00
121033	10	7	66.67	46.67
121040	12	8	80.00	53.33
121265	1	7	6.67	46.67
121059	0.5	10	3.33	66.67
121065	12	8.5	80.00	56.67
123004	8	7	53.33	46.67
123005	5	7.5	33.33	50.00
121077	5	10.5	33.33	70.00
121087	9	7	60.00	46.67
121095	11	9	73.33	60.00
121099	5	10	33.33	66.67
121118	14	13	93.33	86.67
121254	2	6.5	13.33	43.33

## VITA

Kellie Kristine Keiser was born in Toms River, New Jersey on June 10, 1969 to Flossie and Richard Claxton. Mrs. Keiser graduated from Island Heights Elementary School in Island Heights, New Jersey and later graduated second in her class from Central Regional High School in Bayville, New Jersey. Mrs. Keiser went on to attend Lehigh University and graduated in June of 1991 receiving a Bachelor of Arts degree with a double major in Mathematics and Spanish. After graduation, Mrs. Keiser was employed by Educational Testing Service in Princeton, New Jersey from 1992-1998 as a Statistical Analyst and later as a Program Administrator in the Center for the Assessment of Educational Progress (CAEP) and was a part of the Reporting Group for the National Assessment of Educational Progress (NAEP). In September of 1999 her career took a different direction when she became a Teacher of Mathematics at the Academy of Allied Health & Science, a Blue Ribbon School located in Neptune, NJ. After three years of teaching at the Academy, Mrs. Keiser returned to her high school alma matter and is still employed by the district as the Department Chairperson of Mathematics and Science while teaching Honors Pre Calculus, Honors Calculus, and Advanced Placement Calculus courses.