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Design of a Gravity Compensation System for Human Lower-Limb Rehabilitation

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DESIGN OF A GRAVITY COMPENSATION SYSTEM FOR HUMAN LOWER-LIMB REHABILITATION

by

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Table of Contents

LIST OF TABLES

LIST OF FIGURES

Chapter 1

Chapter 2

Chapter 3

Chapter 4

ABSTRACT

DESIGN OF A GRAVITY COMPENSATION SYSTEM FOR HUMAN LOWER-LIMB REHABILITATION

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The purpose of the research is to analyze a real system that is used to assist patients for lower-limb rehabilitation. The mechanism is based on a rotatable parallelogram linkage structure. Two groups of extension springs are used to suspend the weight of both the system and patients. During application, an adjustable proportion of users' body weight can be suspended.

In an ideal design, equations have been derived for determining the spring constants for any given suspended weight. These equations govern both initial conditions and adjustment conditions. Factors affecting the system parameters as well as adjusting method are discussed. In the actual system, factors that were not previously considered have been taken into consideration. Revised equations are provided with regard to real prototype. 3D models of all necessary parts are created for building prototype.

Chapter1

Introduction

1.1 Concept of Rehabilitation Robotics

Robotics technologies are becoming more prevalent for therapy activities. A field of research dedicated to assisting rehabilitation through robotic devices is called rehabilitation robotics [1]. Rehabilitation robotics can be considered as a combination of engineering robotics and medical rehabilitation. Since the 1990, the research in rehabilitation robotics has steadily increased. While the number of rehabilitation robots has increased only a limited number. These devices have been applied in actual clinical situations.

One kind of these robots is used in the process of recuperating the disabled in standing up, balancing and gait. A positive aspect of using these devices is that the disabled can repeat the process and practice as many times as they wish. Furthermore, such robotics can provide exact measurements of any progress in the rehabilitation.

1.2 Current Lower-limbs Rehabilitation Related Technology

In the last several years [2], the interest in the field has grown exponentially due to growing demand. The number of patients needing physical strength recovery and limb exercise are increasing. Therefore, in the purpose of training either the upper or lower limb body, rehabilitation robotics have been developed. Several current robotic systems for lower-limb rehabilitation will be reviewed [3].

Lokomat lower-limb recovery system [4, 5] (Fig. 1.1(a)) includes an adjustable body weight support system and a treadmill. This device imposes a fixed kinematic gait pattern. This pattern is determined from healthy subjects. This system does not reproduce the appropriate dynamic sensory input that occurs during normal gait. Normally this sensory input is critical for gait rehabilitation. Lokomat employs fixed foot motion, which do not satisfy one or more of the neural inputs associated with normal gait. Currently, this is the most popular system applied clinically.

The LokoHelp (Fig. 1.1(b)) is another device [6] designed for walking therapy after suffering from an injury. This device provides a body weight support system. The whole system is based on a treadmill. User will wear a boot which is suspended on a gait control system. During the training process, the user will be lifted and will be following the gait boot to walk. The LokoHelp allows patients to perform a waking type motion alone, without the need for multiple trained therapists to move their legs. The whole system is powered by a treadmill.

Fig. 1.1(a): Lokomat Lower-limb Recovery System [4, 5]

Fig. 1.1(b): LokoHelp Training Machine [6]

The Gantrainer GT I [7, 8] (Fig. 1.1(c)) is a commercialized lower-limb rehabilitation device. This is a combination of an end-effector based robot and an adjustable body weight system. Two sliding foot plates are secured to the patient’s feet. Suspended ropes and chains enable user to have more freedom of movement than the previous two devices. The device can let users adapt their own moving speed depending on their ability.

The GaitMaster5 (Fig. 1.1(d)) [9] is a gait rehabilitation system with a locomotion interface (LI) for stair climbing or descending developed by University of Tsukuba. The system has two 2-DOF manipulators equipped with footpads. The patient's feet are strapped to sensor-laden pads on motion platforms. At the beginning, the machine controls the movement of the patients, but after several hours, the patient may have already regain some muscle memory. When the patient’s muscle memory improves, the system control will be tweaked to allow more autonomous movement.

Fig. 1.1(c):Gantrainer GT I Lower-limb Rehabilitation Device [7, 8]

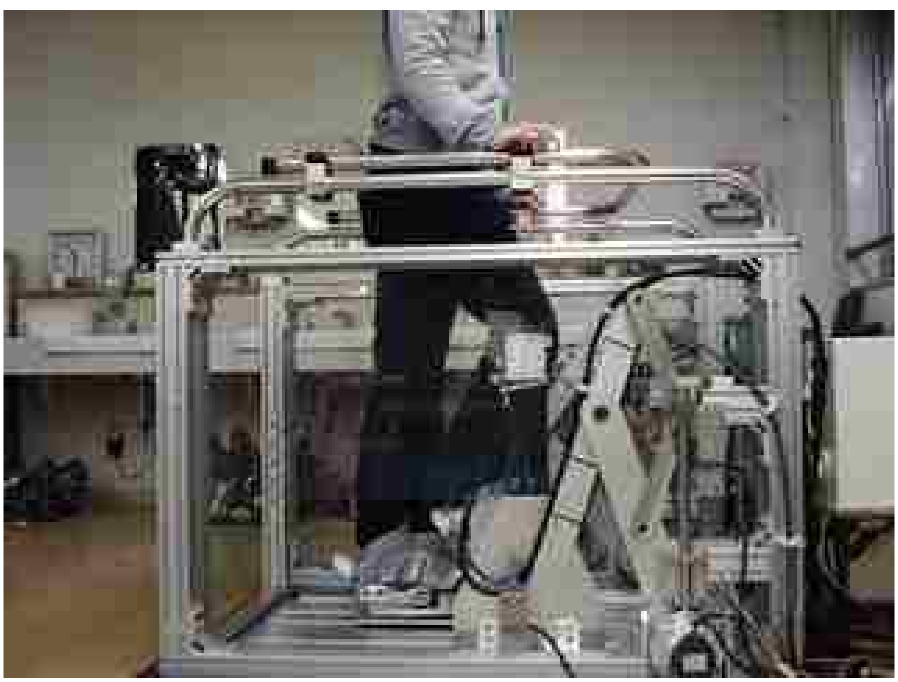

Fig. 1.1(d):GaitMaster5 Gait Recovery Device [9]

Generally lower-limb rehabilitation is based on either treadmill machine or programmable foot plates. When applying treadmill device, users will walk following the adjustable speed of the belt. There is an overhead suspension device holding the user to support their body weight. By using this kind of a treadmill system, people can only train above the treadmill device, since the treadmill is put on the fixed location. They can not practice working like the true scene, which is walking on the real ground without route limitation. The suspension systems can not be adjusted easily while responding the users' body weight. When users are using these devices, they feel like being rigidly hung through the suspension rope. There are not enough buffers to make the training comfortable.

1.3 Review of Prior Design

Previously, a two degree-of-freedom suspension concept has been presented in Cheng's thesis "*Dynamics and control of Two Degree-of-freedom Suspension System with Application to Rehabilitation*"[10]. Based on a parallelogram linkage mechanism, a free moving system in planar working space was created. Since the inertia of the system is inevitable in most of the devices that trying to eliminate the gravity effect, the purpose of the system is to generate gravity compensation. To achieve these, Cheng applied Lagrange's equation to the analysis of the system. He used motion deviation arrive at the link inertia expression. He also discussed how the system responded under a load. A feed-forward adaptive control method was introduced. The error in trajectory was controlled to a value less than 0.1 mm.

Based on his study, Cheng pointed out the idea of applying this system to lower-limb rehabilitation. A general idea of a device which can lift weight of patient was introduced. Based on his ideal concept, the machine can enable users to adjust the suspended weight and keep the force constant no matter how patient moves. He provided that the design was theoretical possible and with strong advantages in simplicity and lightness.

1.4 Design Objective

Gravity elimination has always been a challenge that is encountered in designing a lower-limb rehabilitation system. The first goal is to establish a new concept based on previous two degree-of-freedom suspension system. The new ideal concept will at least have the gravity compensation ability. Different than previous design, this design will focus more on real building purpose, which makes the design more realistic. To find more details about how to calculate and adjust the suspended user's body weight, force equations of different conditions that may occurs in the concept will be generated. Any influence factor that may affect the final result will be considered. Other than an ideal design concept, an actual design model will be created based on practical error factors. New force equations will be updated and the actual relation between all effective parameters and suspended user's body weight will be built.

The further goal is to build a real prototype based on design concept. Establishing 3D models of every detailed part can help to have a better actual view and simulation before preparing for purchasing parts. Then the materials needed in building a prototype will be chosen. The ultimate goal is to test the prototype about its performance, working error and durability. Then improvements can be found for the future design.

Chapter2

Suspension System Concept Model

In previous study [10], the basic concept of a two degree-of-freedom suspension system has been examined. The system aims at eliminating or reducing the weight of the suspended body. By using a spring and linkage structure, some linkage mass can be theoretically compensated. Since the objective of this investigation is to build a lower-limb rehabilitation system, an updated suspension model is needed to eliminate the suspended weight both theoretically and practically.

2.1 Concept Model Establishment

The parallel linkages suspension system is established showed in Fig. 2.1(a). The basis of the system is a parallelogram structure with one side fixed. The structure can rotate through four vertices O_1, O_2, B, E. Beyond the parallelogram structure O_1O_2BE, a long linkage can rotate through vertex B on the parallelogram. The system is tensioned by two extension springs. The primal goal is to let the two extension springs take up the weight that is suspended on the system. The weight that the system can lift depends on the extension of springs and the attachment location that the cable is connected to the links. To make the suspended mass adjustable, the cable connection point to the respective links can be adjusted.

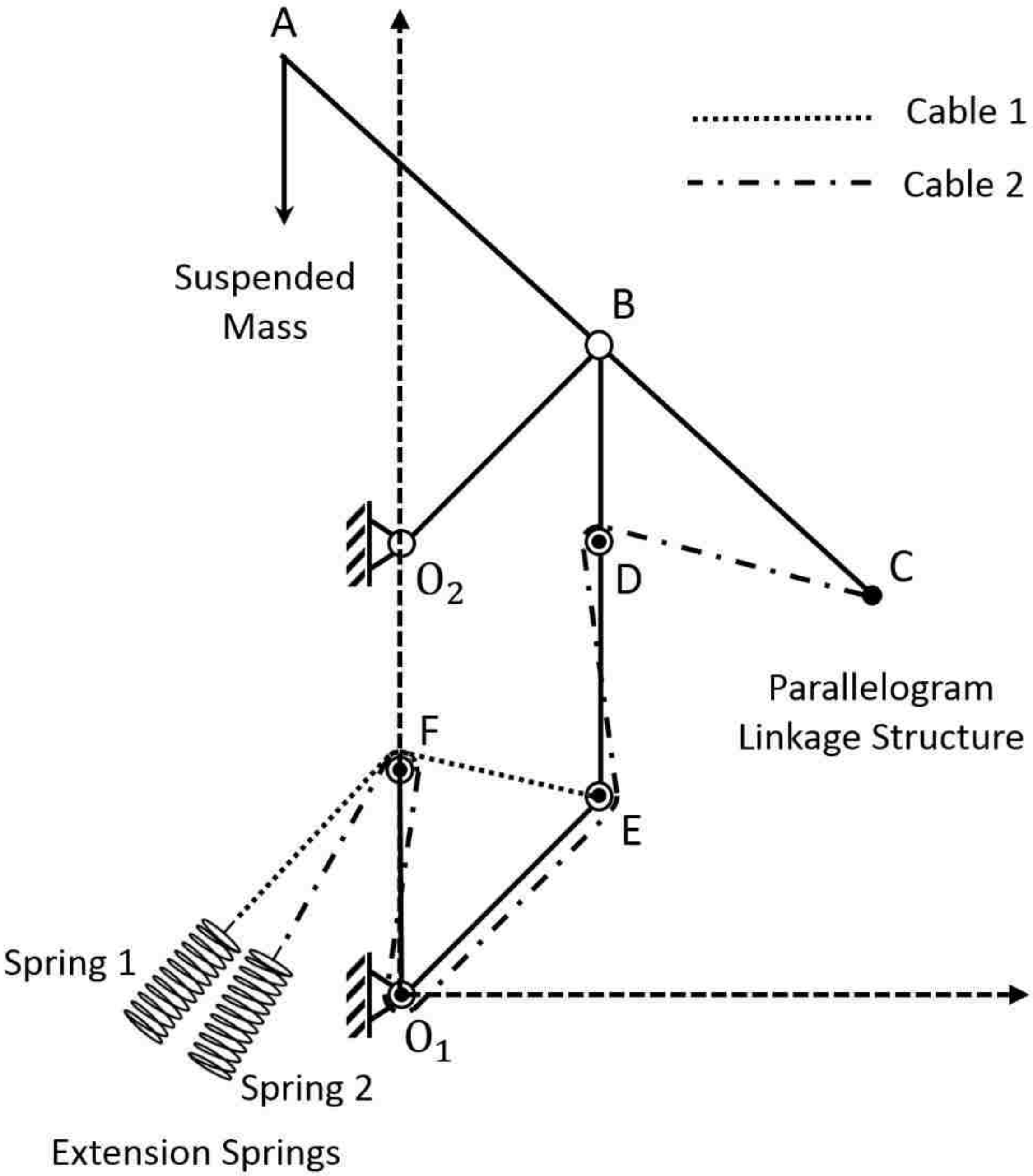

Fig. 2.1(a): Basic Concept of the Suspension System

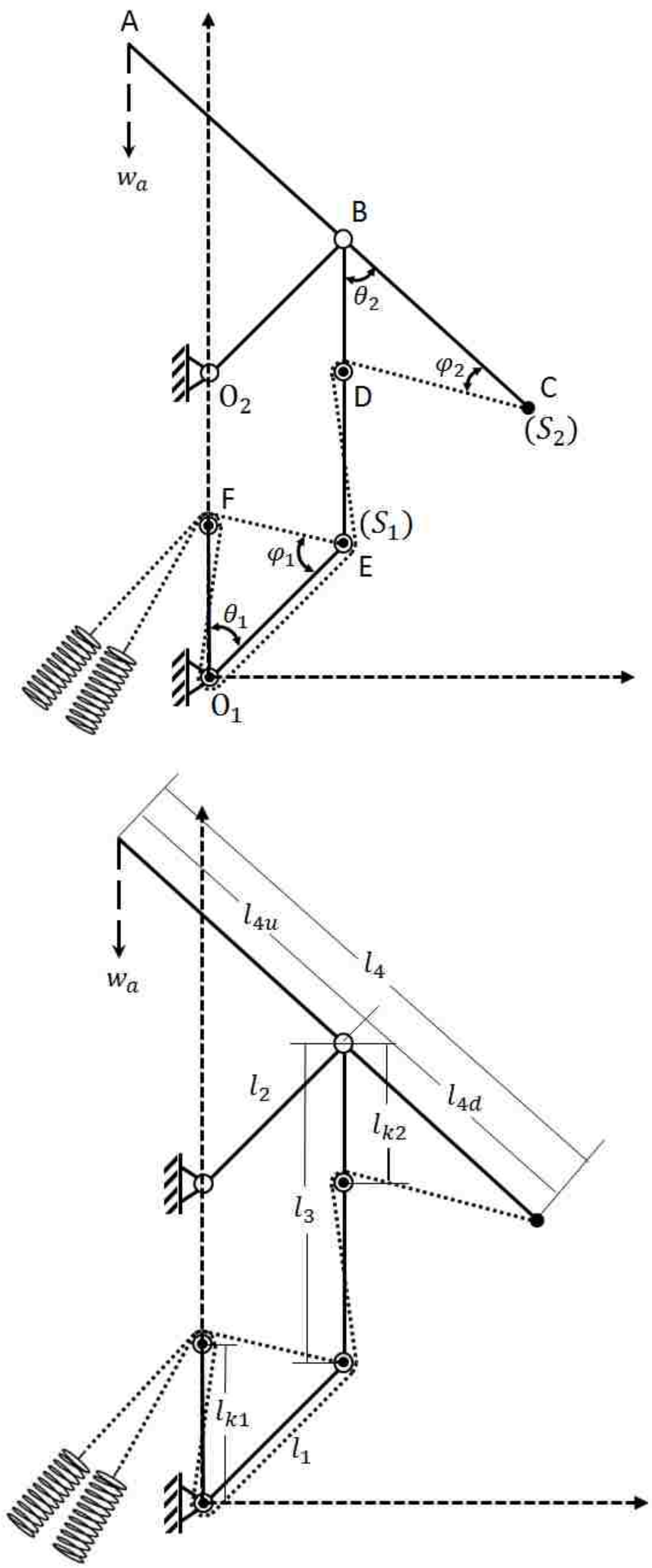

Fig. 2.1(b): Concept of the Suspension System under Initial Conditions

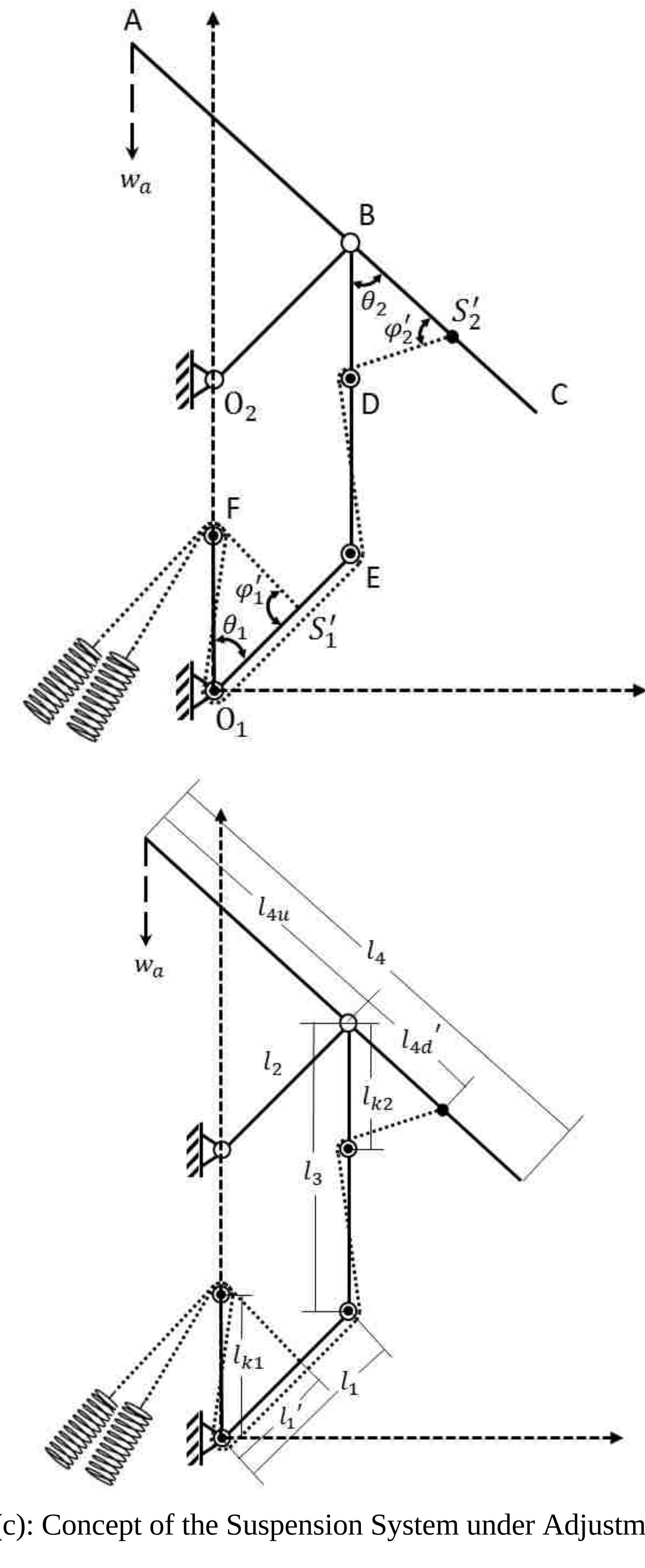

Fig. 2.1(c): Concept of the Suspension System under Adjustment Conditions

As shown in Fig. 2.1(b) and Fig. 2.1(c). This design is basically composed by four main links $\overline{O_1E}$, $\overline{O_2B}$, $\overline{BE}$ and $\overline{AC}$. This four links form a parallelogram suspension system. Points O_1 and O_2 are fixed in this system. This does not mean that O_1 and O_2 can not be moved during the application. Points O_1 and O_2 are just relative fixed in the system showed in Fig. 2.1(b) and Fig. 2.1(c). Another link $\overline{O_1F}$ is fixed at point O_1. Link $\overline{O_1F}$ can not pivot around point O_1. Link $\overline{AC}$ is a solid link but can be made up of different materials.

There are four pivots O_1, O_2, B and E. There are total 5 pulleys in this system. Points O_1, D and E each have a pulley. Point F has two pulleys and each of the pulleys must be separately rotatable. A spring with spring constant K_1 is initially connected at point E by a cable. The connected point can be adjusted along link $\overline{O_1E}$ (from point E to point O_1). The connected cable goes around one of the pulleys at point F. Another spring with spring constant K_2 is initially connected at point C by steel wire. The connected point can be adjusted along link $\overline{BC}$ (from point C to point B). The second connected cable goes around four pulleys at points F, O_1, E and D. A suspended weight or load is attached at point A. Point S_1 is the original position that cable 1 is initially connected with link $\overline{O_1E}$ at point E. $S_1{}'$ is the moving joint that cable 1 is connected to link $\overline{O_1E}$. Point S_2 is the original position that cable 2 is initially connected to link $\overline{BC}$ at point C. Point $S_2{}'$ is the new position that cable 2 is connected to link $\overline{BC}$. The nomenclature for the equations can be found in Appendix A.

2.2 Spring Constant Determination

The extension spring system is the key part of the design. Two springs can affect the linkage design and the suspended weight. Spring constant describes the spring elasticity and directly determines the suspended load. Force equations are established under two conditions. The initial condition describes the status when the two springs are suspending the linkages at initial points (point E and point C) through their respective cables. This is used to set up the initial suspended load. The adjustment condition describes the status when moving cables attachment locations (Point S_1 and point S_2). This is used to increase or decrease the suspended mass during the whole system working process.

2.2.1 Force equations under initial conditions

From Fig. 2.2(a), the equation for the moment about pivot O_1 is:

$$K_1(l_{s1} - l_{s10})l_1 \sin(\varphi_1)$$

$$= (w_a + m_4 g)l_1 \sin(\theta_1) + \left(\frac{1}{2}m_1 l_1 + \frac{1}{2}m_2 l_2 + m_3 l_1\right) g \sin(\theta_1) \qquad (2.1)$$

In this calculation, point B (shown in Fig. 2.1(b)) is assumed as the mass center of link 4 (shown in Fig. 2.1(b)).

l_{s10} is set as the free-length of spring 1. l_{s1} is spring 1 length after extension. According to the sine rule:

$$l_{s1} - l_{s10} = \frac{l_{k1}\sin(\theta_1)}{\sin(\varphi_1)} \tag{2.2}$$

Equation (2.1) becomes,

$$K_1 \frac{l_{k1}\sin(\theta_1)}{\sin(\varphi_1)} l_1 \sin(\varphi_1)$$

$$= (w_a + m_4 g) l_1 \sin(\theta_1) + \left(\frac{1}{2} m_1 l_1 + \frac{1}{2} m_2 l_2 + m_3 l_1\right) g \sin(\theta_1) \tag{2.3}$$

In the current case, link 1 and link 2 are two opposite and parallel sides of the parallelogram linkages, which means $l_1 = l_2$. Therefore,

$$K_1 l_{k1} = (w_a + m_4 g) + \left(\frac{1}{2} m_1 + \frac{1}{2} m_2 + m_3\right) g \tag{2.4}$$

Then, the expression of spring constant K_1 is:

$$K_1 = \frac{M_1}{l_{k1}} \tag{2.5}$$

M_1 is the total weight, and can be written as:

$$M_1 = (w_a + m_4 g) + \left(\frac{1}{2} m_1 + \frac{1}{2} m_2 + m_3\right) g \tag{2.6}$$

If point B (shown in Fig. 2.1(b)) is not the mass center of link 4, the reversed equation is:

$$K_1 (l_{s1} - l_{s10}) l_1 \sin(\varphi_1)$$

$$= (w_a + m_{4u} g + m_{4d} g) l_1 \sin(\theta_1) + \left(\frac{1}{2} m_1 l_1 + \frac{1}{2} m_2 l_2 + m_3 l_1\right) g \sin(\theta_1)$$

$$- \frac{1}{2} \sin(\theta_2)\,(m_{4u} g l_{4u} + m_{4d} g l_{4d}) \tag{2.7}$$

Combining Equation (2.2),

$$K_1 l_{k1} = (w_a + m_{4u}g + m_{4d}g) l_1 \sin(\theta_1) + \left(\frac{1}{2}m_1 l_1 + \frac{1}{2}m_2 l_2 + m_3 l_1\right) g \sin(\theta_1)$$

$$-\frac{1}{2}(m_{4u} g l_{4u} + m_{4d} g l_{4d}) \frac{\sin(\theta_2)}{\sin(\theta_1)} \tag{2.8}$$

In this case, the weight of the parallelogram linkage is also included in the model. However, one of the primal goals is to compensate for linkage weight. To simplify the calculation, only the case where the mass center of link 4 is located at point B will be discussed.

Based on Fig. 2.2(b), the equation for the moment about pivot B is:

$$w_a l_{4u} \sin(\theta_2) + \frac{1}{2} l_{4u} m_{4u} g \sin(\theta_2) = K_2 (l_{s2} - l_{s20}) \sin(\varphi_2)\, l_{4d} + \frac{1}{2} l_{4d} m_{4d} g \sin(\theta_2) \tag{2.9}$$

l_{s20} is set as the free-length of spring 2. l_{s2} is spring 2 length after extension. According to the sine rule:

$$l_{s2} - l_{s20} = \frac{l_{k2} \sin(\theta_2)}{\sin(\varphi_2)} \tag{2.10}$$

Equation (2.9) becomes,

$$w_a l_{4u} \sin(\theta_2) + \frac{1}{2} l_{4u} m_{4u} g \sin(\theta_2) = K_2 \frac{l_{k2} \sin(\theta_2)}{\sin(\varphi_2)} \sin(\varphi_2)\, l_{4d} + \frac{1}{2} l_{4d} m_{4d} g \sin(\theta_2) \tag{2.11}$$

After simplification,

$$\mathrm{K_2} = \frac{\mathrm{w_a l_{4u}} + \frac{1}{2}\mathrm{l_{4u} m_{4u} g} - \frac{1}{2}\mathrm{l_{4d} m_{4d} g}}{\mathrm{l_{k2}\, l_{4d}}} \tag{2.12}$$

The expression of spring constant $\mathrm{K_2}$ is:

$$K_2 = \frac{M_2}{l_{k2}} \tag{2.13}$$

$\mathrm{M_2}$ is an expression of weight, which can be written as:

$$\mathrm{M_2} = \frac{\mathrm{w_a l_{4u}} + \frac{1}{2}\mathrm{l_{4u} m_{4u} g} - \frac{1}{2}\mathrm{l_{4d} m_{4d} g}}{\mathrm{l_{4d}}} \tag{2.14}$$

If assuming$\mathrm{l_{4u}} = \mathrm{l_{4d}}$ and $\mathrm{m_{4u}}$=$\mathrm{m_{4d}}$,

$$\mathrm{M_2} = \mathrm{w_a}$$

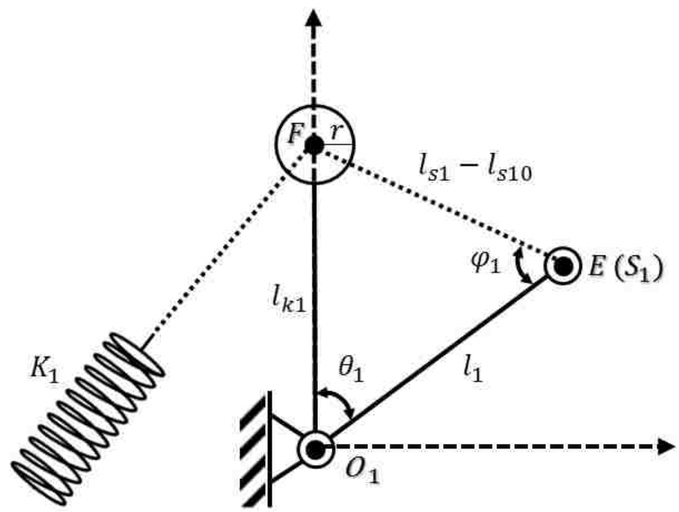

Fig. 2.2(a): Force Diagram for Spring Constant K_1 Calculation under Initial Conditions

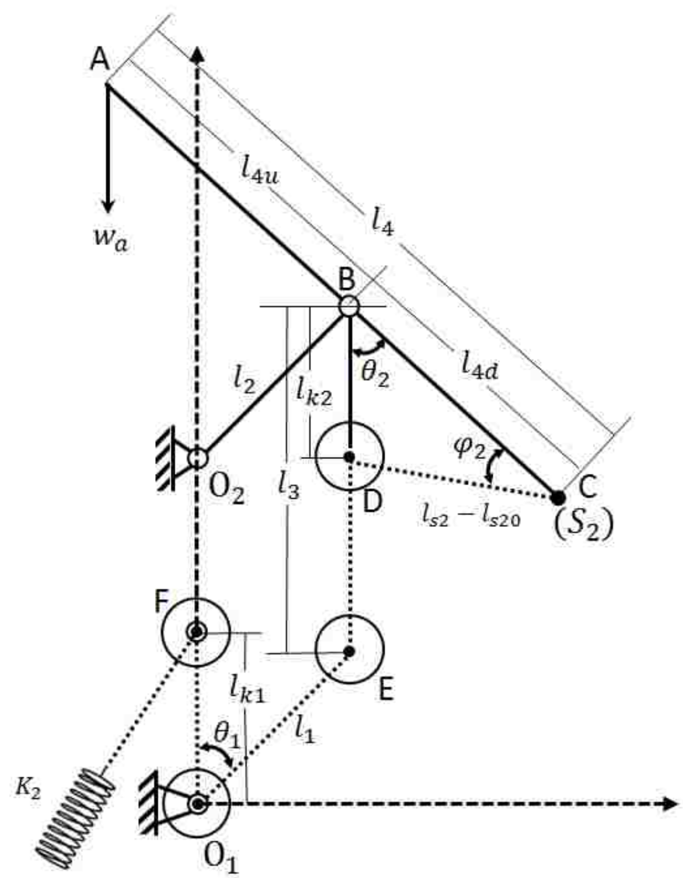

Fig. 2.2(b): Force Diagram for Spring Constant K_2 Calculation under Initial Conditions

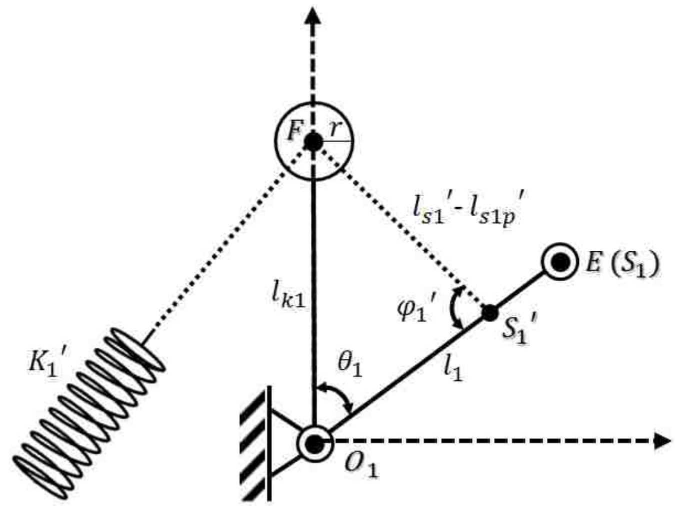

Fig. 2.3(a): Force Diagram for Spring Constant K_1' Calculation under Adjustment Conditions

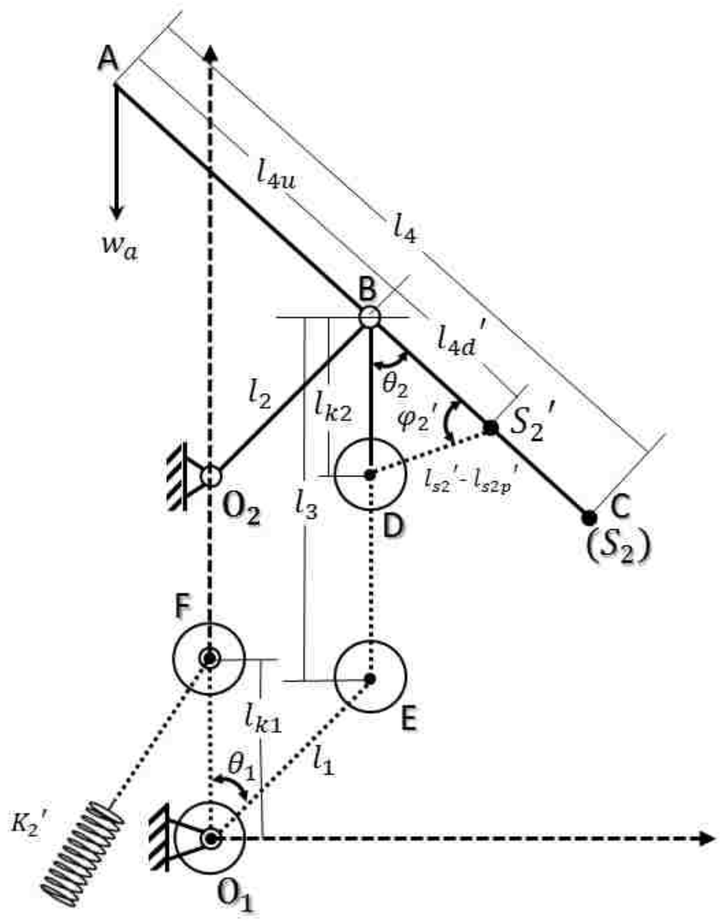

Fig. 2.3(b): Force Diagram for Spring Constant K_2' Calculation under Adjustment Conditions

2.2.2 Force equations under adjustment conditions

From Fig. 2.3(a), the equation for the moment about pivot O_1 is:

$$K_1'(l_{s1}' - l_{s10}')l_1' \sin(\varphi_1')$$

$$= (w_a + m_4 g)l_1 \sin(\theta_1) + \left(\frac{1}{2}m_1 l_1 + \frac{1}{2}m_2 l_2 + m_3 l_1\right) g \sin(\theta_1) \quad (2.15)$$

l_{s10}' is set as the free-length of spring 1. l_{s1}' is spring 1 length after extension. According to the sine rule:

$$l_{s1}' - l_{s10}' = \frac{l_{k1} \sin(\theta_1)}{\sin(\varphi_1')} \quad (2.16)$$

Equation (2.15) becomes,

$$K_1' \frac{l_{k1} \sin(\theta_1)}{\sin(\varphi_1')} l_1' \sin(\varphi_1')$$

$$= (w_a + m_4 g)l_1 \sin(\theta_1) + \left(\frac{1}{2}m_1 l_1 + \frac{1}{2}m_2 l_2 + m_3 l_1\right) g \sin(\theta_1) \quad (2.17)$$

With the previous $l_1 = l_2$ assumption,

$$K_1' l_{k1} = \left[(w_a + m_4 g) + \left(\frac{1}{2}m_1 + \frac{1}{2}m_2 + m_3\right) g\right] \frac{l_1}{l_1'} \quad (2.18)$$

The expression of spring constant K_1' is:

$$K_1' = \frac{M_1}{l_{k1}} \frac{l_1}{l_1'} \quad (2.19)$$

where M_1 is expressed in Equation.(2.6).

Based on Fig. 2.3(b), the equation for the moment about pivot B is:

$$w_a l_{4u} \sin(\theta_2) + \frac{1}{2} l_{4u} m_{4u} g \sin(\theta_2)$$

$$= K_2'(l_{s2}' - l_{s20}') \sin(\varphi_2') \, l_{4d}' + \frac{1}{2} l_{4d} m_{4d} g \sin(\theta_2) \quad (2.20)$$

l_{s20}' is set as the free-length of spring 2. l_{s2}' is spring 2 length after extension. According to the sine rule:

$$l_{s2}' - l_{s20}' = \frac{l_{k2} \sin(\theta_2)}{\sin(\varphi_2')} \quad (2.21)$$

Equation (2.20) becomes,

$$w_a l_{4u} \sin(\theta_2) + \frac{1}{2} l_{4u} m_{4u} g \sin(\theta_2)$$

$$= K_2' \frac{l_{k2} \sin(\theta_2)}{\sin(\varphi_2')} \sin(\varphi_2') \, l_{4d}' + \frac{1}{2} l_{4d} m_{4d} g \sin(\theta_2) \quad (2.22)$$

After simplification, the expression of spring constant K_2' is,

$$K_2' = \frac{w_a l_{4u} + \frac{1}{2} l_{4u} m_{4u} g - \frac{1}{2} l_{4d} m_{4d} g}{l_{k2} \, l_{4d}'} \quad (2.23)$$

If assuming $l_{4u} = l_{4d}$ and m_{4u}=m_{4d},

$$K_2' = \frac{w_a l_{4u}}{l_{k2} \, l_{4d}'} \quad (2.24)$$

2.3 Initial Conditions Assumption

The linkage mass, length and position angle determine what kind of spring to choose with a certain spring constant number. Based on the possible building model and reliable set up, the assumption of linkage initial conditions is set in Table. 2.1.

Type	Nomenclature	Value	Unit
Mass	m_1	6.5	Kg
	m_2	6.5	Kg
	m_3	20	Kg
	m_4	20	Kg
	m_{4u}	10	Kg
	m_{4d}	10	Kg
Length	l_1	0.5	m
	l_2	0.5	m
	l_3	1	m
	l_4	2	m
	l_{4u}	1	m
	l_{4d}	1	m
	l_{k1}	0.5	m
	l_{k2}	0.4	m
Angle	θ_1	45°	Degree
	θ_2	45°	Degree

Table.2.1: Initial Value Table

2.4 Suspended weight calculation

2.4.1Suspended Mass Equations

The ultimate goal is to lifting a certain weight that suspended on top of link 4. A proper spring can be chosen through previous calculation. When determining the suspended mass, the spring constant is a fixed number. A certain initial suspended mass is set at initial conditions (points S_1 and E are coincided, points S_2 and C are coincided (shown in Fig. 2.1(b))). By changing the cable attachment locations ($S_1{}'$ and $S_2{}'$), the suspended mass can be adjusted.

The expression of suspended weight w_a can be derived from Equations (2.5), (2.13), (2.19), (2.24).These equations will either include K_1 or K_2. Once each spring constant is fixed, K_1 will be the same as $K_1{}'$ and K_2 will be the same as $K_2{}'$.

Suspended weight w_a can be written with respect to K_1 as:

$$w_a = \frac{K_1 l_{k1} l_1{}'}{l_1} - \left(\frac{1}{2}m_1 + \frac{1}{2}m_2 + m_3 + m_4\right)g \tag{2.25}$$

When $l_1{}' = l_1$, cable 1 is attached at the initial location (S_1).

Suspended weight w_a can also be written with respect to K_2 as:

$$w_a = \frac{K_2 l_{k2}\, l_{4d}{}' + \frac{1}{2} l_{4d} m_{4d} g - \frac{1}{2} l_{4u} m_{4u} g}{l_{4u}} \tag{2.26}$$

When $l_{4d}{}' = l_{4d}$, cable 2 is attached at the initial location (S_2).

2.4.2 Influence Factors of Suspension Mass

Equations (2.25) and (2.26) shows that the value of spring constant, linkage length and linkage mass can affect the final suspended weight. Once the springs and linkages are specified, the suspension mass only depends on the adjustable length $l_1{}'$ and $l_2{}'$.

Fig. 2.4(a) shows how suspended mass changes through changing linkage length l_{k1}. Except for changing the value of l_{k1}, all the linkage length, mass and position angle are set at the initial condition. In this case, the two groups of cables attachment points (S_1 and S_2) are at the original location. The values of both spring constant K_1 and K_2 are fixed at 2000 N/m.

When l_{k1} is smaller than 0.23m, the lifting mass M_a is no longer positive. Therefore, l_{k1} should always bigger than 0.23m to guarantee the function of this suspension system.

Fig. 2.4(b) shows how suspended mass changes through changing linkage lengthl_{k2}. Except changing the value of l_{k2}, all the linkage length, mass and position angle are set at the initial condition. In this case, the two groups of cables attachment points (S_1 and S_2) are at the original location. The values of both spring constant K_1 and K_2 are fixed at 2000 N/m.

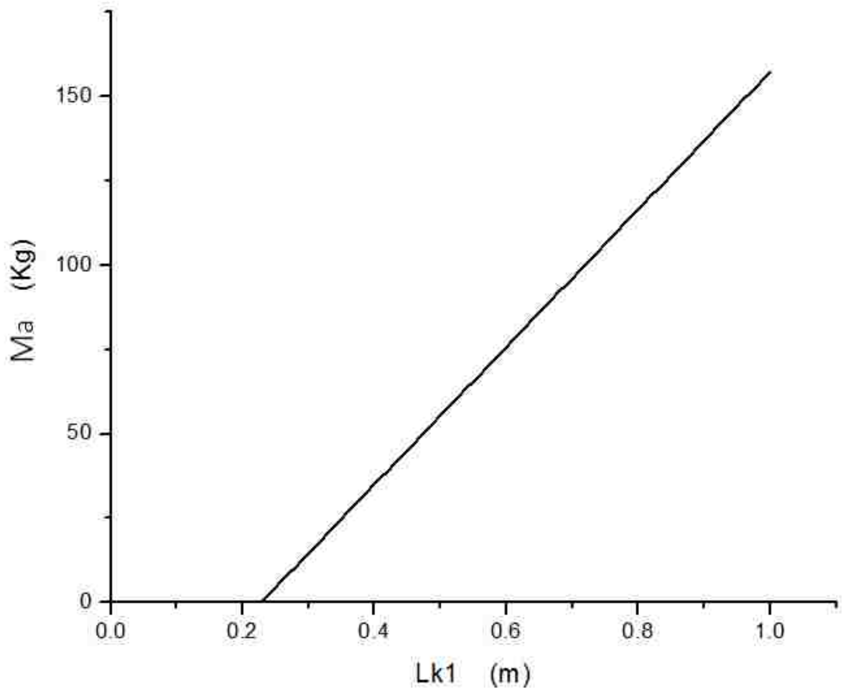

Fig. 2.4(a): Relation between Suspended Mass M_a and linkage length l_{k1}

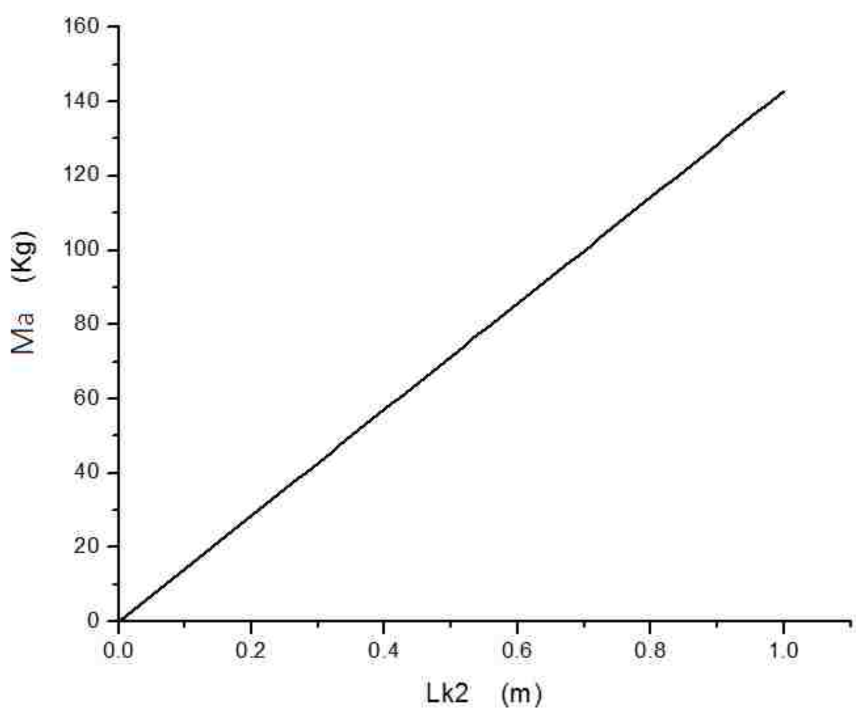

Fig. 2.4(b): Relation between Suspended Mass M_a and linkage length l_{k2}

Fig. 2.4(c) shows how suspended mass changes through changing spring constant K_1 and spring constant K_2. The linkage length, mass and position angle are all fixed at the initial value. In this case, the two groups of cables attachment points (S_1 and S_2) are at the original location. It's obvious that when spring constant K_1 is getting close to 1000N/m, the suspended mass is almost zero. Therefore, K_1 has to be larger than 1000N/m. When determining the two spring constant values, proper springs with the same suspended mass can be chosen following Fig. 2.4(c). In the initial assumption, spring 1 and spring 2 shares the same constant value, which is approximately the cross point of the two lines in the figure.

Fig. 2.4 (a, b, c) indicate that when fixing the other parameters, the suspended mass varies linearly from the value of linkage length l_{k1}, l_{k2} and spring constant. Once the linkage parameters need to be changed, relevant values can be directly adjusted following Fig. 2.4 (a, b, c). When point B is assumed as the mass center of link 4, Equation (2.26) shows that the weight of link 4 will not affect the suspended mass. However, Equation (2.25) shows that the changed value of linkage weight will directly affect suspended mass. As a result, every time the linkage weight is changed, either linkage length l_{k1} or spring constant K_1 can be updated according to Equation (2.25), while other parameters like linkage length l_{k2} and spring constant K_2 can still remain the same value. In this design model, the situation that all the factors having linear relation with the suspended mass makes the set up easy.

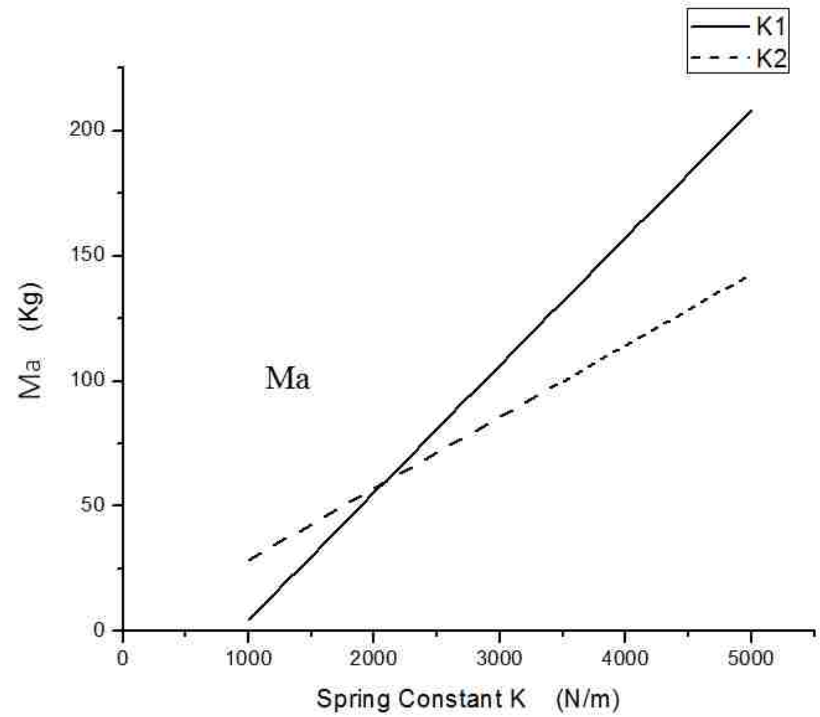

Fig. 2.4(c): Relation between Suspended Mass M_a and K_1, K_2

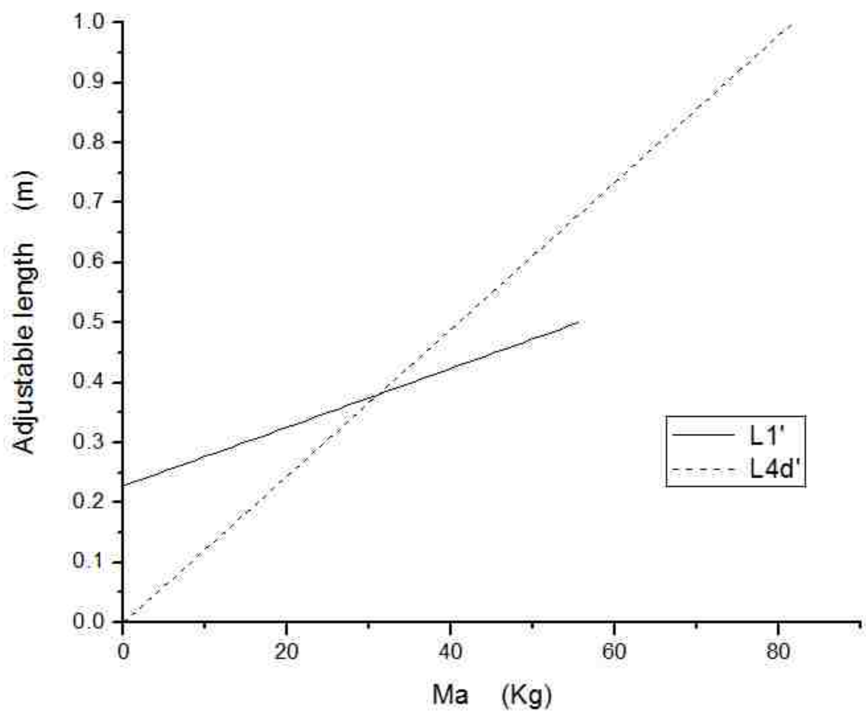

Fig. 2.5: Suspended Mass Adjusting Reference Graph

Fig. 2.5 illustrates how to adjust the suspended mass through this reference graph. Adjusting$l_1{}'$ and $l_{4d}{}'$ (shown in Fig. 2.1(c)) by moving the cable attachment points $S_1{}'$ and $S_2{}'$ is one way to control how much mass to suspend. Since both $l_1{}'$ and $l_{4d}{}'$ will affect the lifting mass, they need to be controlled together. The good result is the suspended mass will change linearly when adjusting both $l_1{}'$ and $l_{4d}{}'$. However, the slopes of the two lines are different as shown in Fig. 2.5. To fix this slope difference, an adjustment equation can be applied with a constant slope number which depends on different initial conditions. Fig. 2.5 shows that there is limit to the adjustable length $l_1{}'$. When $l_1{}'$ is smaller than 0.228, M_a goes negative. The range of $l_1{}'$ is from 0.228m to 0.5m.

Chapter 3

System Analysis for Actual Design

3.1 Actual Application Analyses

In Chapter 2, the establishment of a two-degree of freedom suspension system model is described and calculations to control the suspended mass are generated. In these calculations, an ideal model has been used to simplify the calculations and find easiest solutions. However, some assumptions in ideal case can not be fully realized in an actual design. Therefore, the effect of ideal conditions will be analyzed and calculations for the actual design will be described.

In an actual application, pulleys are necessary when applying a cable transmission system. Each point of O_1, E and D will have one pulley installed. Point F will have two pulleys (cables from spring 1 and spring 2 will go through this location). The pulley diameter will have an effect on the force equations. Point B is assumed as the mass center and length center of link 4. In an actual application, link 4 will be composed of two links $\overline{AB}$ and $\overline{BC}$.

In the following paragraph, the equations are rederived by taking pulley diameter into consideration. Then, the new calculation of suspended mass in actual design will be derived. The nomenclature for the equations can be found in Appendix A.

3.2 Force Equations for Actual Conditions

3.2.1 Actual Spring Constant Determination

From Fig. 3.1(a), the equation for the moment about pivot O_1 is:

$$K_{1p}\left(l_{s1p} - l_{s10p}\right)l_1 \sin(\varphi_1 + \Delta\varphi_1)$$

$$= (w_a + m_4 g)l_1 \sin(\theta_1) + \left(\frac{1}{2}m_1 l_1 + \frac{1}{2}m_2 l_2 + m_3 l_1\right) g \sin(\theta_1) \tag{3.1}$$

In this calculation, point B (shown in Fig. 2.1(b)) is still assumed as the mass center of link 4 (shown in Fig. 2.1(b)).

l_{s1p} is set as the free-length of spring 1. l_{s10p} is spring 1 length after extension. According to the sine rule:

$$l_{s1p} - l_{s10p} = \frac{(l_{k1} + r')\sin(\theta_1)}{\sin(\varphi_1 + \Delta\varphi_1)} \tag{3.2}$$

Equation (3.1) becomes,

$$K_{1p}\frac{(l_{k1} + r')\sin(\theta_1)}{\sin(\varphi_1 + \Delta\varphi_1)} l_1 \sin(\varphi_1 + \Delta\varphi_1)$$

$$= (w_a + m_4 g)l_1 \sin(\theta_1) + \left(\frac{1}{2}m_1 l_1 + \frac{1}{2}m_2 l_2 + m_3 l_1\right) g \sin(\theta_1) \tag{3.3}$$

In the current case, $l_1 = l_2$. Therefore,

$$K_{1p}(l_{k1} + r') = (w_a + m_4 g) + \left(\frac{1}{2}m_1 + \frac{1}{2}m_2 + m_3\right) g \tag{3.4}$$

Then, the expression of spring constant K_{1p} is:

$$K_{1p} = \frac{M_1}{(l_{k1} + r')} \tag{3.5}$$

M_1 is an expression of weight. r' and $\Delta\varphi_1$ can be expressed by position angle φ_1, θ_1 and pulley diameter r. They can be written as:

$$M_1 = (w_a + m_4 g) + \left(\frac{1}{2}m_1 + \frac{1}{2}m_2 + m_3\right)g \tag{3.6}$$

$$r' = \frac{r}{\sin(\pi - \theta_1 - \varphi_1 - \Delta\varphi_1)} \tag{3.7}$$

$$\Delta\varphi_1 = \arcsin\left(\frac{r\sin(\varphi_1)}{l_{k1}\sin(\theta_1)}\right) \tag{3.8}$$

Based on Fig. 3.1(b), the equation for the moment about pivot B is:

$$w_a l_{4u}\sin(\theta_2) + \frac{1}{2}l_{4u}m_{4u}g\sin(\theta_2)$$

$$= K_{2p}(l_{s2p} - l_{s20p})\sin(\varphi_2 - \Delta\varphi_2)\,l_{4d} + \frac{1}{2}l_{4d}m_{4d}g\sin(\theta_2) \tag{3.9}$$

l_{s20p} is set as the free-length of spring 2. l_{s2p} is spring 2 length after extension. According to the sine rule:

$$l_{s2p} - l_{s20p} = \frac{(l_{k2} - r^*)\sin(\theta_2)}{\sin(\varphi_2 - \Delta\varphi_2)} \tag{3.10}$$

Equation (3.9) becomes,

$$w_a l_{4u}\sin(\theta_2) + \frac{1}{2} l_{4u} m_{4u} g \sin(\theta_2)$$

$$= K_{2p} \frac{(l_{k2} - r^*)\sin(\theta_2)}{\sin(\varphi_2 - \Delta\varphi_2)} \sin(\varphi_2 - \Delta\varphi_2)\, l_{4d} + \frac{1}{2} l_{4d} m_{4d} g \sin(\theta_2) \qquad (3.11)$$

After simplification,

$$K_{2p} = \frac{w_a l_{4u} + \frac{1}{2} l_{4u} m_{4u} g - \frac{1}{2} l_{4d} m_{4d} g}{(l_{k2} - r^*)\, l_{4d}} \qquad (3.12)$$

The expression of spring constant K_{2p} is:

$$K_{2p} = \frac{M_2}{l_{k2} - r^*} \qquad (3.13)$$

M_2 is an expression which includes linkage length and mass. r^* and $\Delta\varphi_2$ can be expressed by position angle φ_2, θ_2 and pulley diameter r. They can be written as:

$$M_2 = \frac{w_a l_{4u} + \frac{1}{2} l_{4u} m_{4u} g - \frac{1}{2} l_{4d} m_{4d} g}{l_{4d}} \qquad (3.14)$$

$$r^* = \frac{r}{\sin(\theta_2 + \varphi_2 - \Delta\varphi_2)} \qquad (3.15)$$

$$\Delta\varphi_2 = \arcsin\left(\frac{r \sin(\theta_2 + \varphi_2)}{l_{4d}\sin(\theta_2)}\right) \qquad (3.16)$$

If assuming $l_{4u} = l_{4d}$ and m_{4u}=m_{4d},

$$M_2 = w_a$$

After taking the pulley diameter into consideration, the spring 2 initial length l_{s20p} will be different from the spring 2 length l_{s20} that without considering the pulley diameter. This will affect the initial length set up every time the linkage position angle or the cable 2 attachment location is changed. The total change in the spring 2 initial length is:

$$l_{s20p} - l_{s20} = \frac{\pi + \Delta\varphi_2 - \theta_2 - \varphi_2 + \arcsin\left(\frac{2r}{l_3 - l_{k2}}\right)}{2\pi} \cdot 2\pi r + 2\sqrt{\frac{(l_3 - l_{k2})^2}{4} - r^2}$$

$$+ \frac{\theta_1 + \arcsin\left(\frac{2r}{l_3 - l_{k2}}\right)}{2\pi} \cdot 2\pi r + l_1 + \frac{\pi + \arcsin\left(\frac{2r}{l_{k1}}\right) - \theta_1}{2\pi} \cdot 2\pi r + 2\sqrt{\frac{{l_{k1}}^2}{4} - r^2}$$

$$+ \pi r - l_3 + l_{k2} - l_1 - l_{k1} \tag{3.17}$$

After simplification,

$$l_{s20p} - l_{s20} = \left[2\pi + \Delta\varphi_2 - \theta_2 - \varphi_2 + 2\arcsin\left(\frac{2r}{l_3 - l_{k2}}\right) + \arcsin\left(\frac{2r}{l_{k1}}\right)\right] \cdot r$$

$$+ 2\sqrt{\frac{(l_3 - l_{k2})^2}{4} - r^2} + 2\sqrt{\frac{{l_{k1}}^2}{4} - r^2} + \pi r - l_3 + l_{k2} - l_{k1} \tag{3.18}$$

Equation(3.18)shows how much spring 2 initial length changes after taking pulley diameter into consideration.

Then, the calculations for adjustment conditions are shown in the following paragraph.

From Fig. 3.1(c), the equation for the moment about pivot O_1 is:

$$K_{1p}{}'\left(l_{s1p}{}' - l_{s10p}{}'\right)l_1{}' \sin(\varphi_1{}' + \Delta\varphi_1{}')$$

$$= (w_a + m_4 g)l_1 \sin(\theta_1) + \left(\frac{1}{2}m_1 l_1 + \frac{1}{2}m_2 l_2 + m_3 l_1\right) g \sin(\theta_1) \tag{3.19}$$

In this calculation, point B (shown in Fig. 2.1(b)) is still assumed as the mass center of link 4 (shown in Fig. 2.1(b)).

$l_{s1p}{}'$ is set as the free-length of spring 1. $l_{s10p}{}'$ is spring 1 length after extension. According to the sine rule:

$$l_{s1p}{}' - l_{s10p}{}' = \frac{(l_{k1} + r'')\sin(\theta_1)}{\sin(\varphi_1{}' + \Delta\varphi_1{}')} \tag{3.20}$$

Equation (3.19) becomes,

$$K_{1p}{}' \frac{(l_{k1} + r'')\sin(\theta_1)}{\sin(\varphi_1{}' + \Delta\varphi_1{}')} l_1{}' \sin(\varphi_1{}' + \Delta\varphi_1{}')$$

$$= (w_a + m_4 g)l_1 \sin(\theta_1) + \left(\frac{1}{2}m_1 l_1 + \frac{1}{2}m_2 l_2 + m_3 l_1\right) g \sin(\theta_1) \tag{3.21}$$

In the current case, $l_1 = l_2$. Therefore,

$$K_{1p}{}'(l_{k1} + r'') = \left[(w_a + m_4 g) + \left(\frac{1}{2}m_1 + \frac{1}{2}m_2 + m_3\right) g\right] \frac{l_1}{l_1{}'} \tag{3.22}$$

Then, the expression of spring constant $K_{1p}{}'$ is:

$$K_{1p}{}' = \frac{M_1}{(l_{k1} + r'')} \frac{l_1}{l_1{}'} \tag{3.23}$$

M_1 is an expression of weight. r'' and $\Delta\varphi_1{}'$ can be expressed by position angle $\varphi_1{}'$, θ_1 and pulley diameter r. They can be written as:

$$M_1 = (w_a + m_4 g) + \left(\frac{1}{2}m_1 + \frac{1}{2}m_2 + m_3\right)g \tag{3.24}$$

$$r'' = \frac{r}{\sin(\pi - \theta_1 - \varphi_1{}' - \Delta\varphi_1{}')} \tag{3.25}$$

$$\Delta\varphi_1{}' = \arcsin\left(\frac{r\sin(\varphi_1{}')}{l_{k1}\sin(\theta_1)}\right) \tag{3.26}$$

Based on Fig. 3.1(d), the equation for the moment about pivot B is:

$$w_a l_{4u}\sin(\theta_2) + \frac{1}{2}l_{4u}m_{4u}g\sin(\theta_2) = K_{2p}{}'(l_{s2p}{}' - l_{s20p}{}')\sin(\varphi_2{}' - \Delta\varphi_2{}')\,l_{4d}{}' + \frac{1}{2}l_{4d}m_{4d}g\sin(\theta_2) \tag{3.27}$$

$l_{s20p}{}'$ is set as the free-length of spring 2. $l_{s2p}{}'$ is spring 2 length after extension. According to the sine rule:

$$l_{s2p}{}' - l_{s20p}{}' = \frac{(l_{k2} - r^{**})\sin(\theta_2)}{\sin(\varphi_2{}' - \Delta\varphi_2{}')} \tag{3.28}$$

Equation (3.27) becomes,

$$w_a l_{4u}\sin(\theta_2) + \frac{1}{2}l_{4u}m_{4u}g\sin(\theta_2) = K_{2p}{}'\frac{(l_{k2} - r^{**})\sin(\theta_2)}{\sin(\varphi_2{}' - \Delta\varphi_2{}')}\sin(\varphi_2{}' - \Delta\varphi_2{}')\,l_{4d}{}' + \frac{1}{2}l_{4d}m_{4d}g\sin(\theta_2) \tag{3.29}$$

After simplification,

$$K_{2p}' = \frac{w_a l_{4u} + \frac{1}{2} l_{4u} m_{4u} g - \frac{1}{2} l_{4d} m_{4d} g}{(l_{k2} - r^{**})\, l_{4d}'} \tag{3.30}$$

The expression of spring constant K_{2p}' is:

$$K_{2p}' = \frac{M_2^{*}}{(l_{k2} - r^{**})\, l_{4d}'} \tag{3.31}$$

M_2^{*} is an expression includes linkage length and mass. $r^{**}$and $\Delta\varphi_2'$ can be expressed by position angle φ_2', $\theta_2$and pulley diameter r. They can be written as:

$$M_2^{*} = w_a l_{4u} + \frac{1}{2} l_{4u} m_{4u} g - \frac{1}{2} l_{4d} m_{4d} g \tag{3.32}$$

$$r^{**} = \frac{r}{\sin(\theta_2' + \varphi_2' - \Delta\varphi_2')} \tag{3.33}$$

$$\Delta\varphi_2' = \arcsin(\frac{r \sin(\theta_2 + \varphi_2')}{l_{4d}' \sin(\theta_2)}) \tag{3.34}$$

If assuming $l_{4u} = l_{4d}$ and m_{4u}=m_{4d},

$M_2^{*} = w_a l_{4u}$

Similar in a, there will be a difference between spring 2 initial length l_{s20}' and l_{s20p}'. The total difference of spring 2 initial length during adjustment between ideal conditions and actual conditions is:

$$l_{s20p}{}' - l_{s20}{}' = \frac{\pi + \Delta\varphi_2{}' - \theta_2 - \varphi_2{}' + \arcsin\left(\frac{2r}{l_3 - l_{k2}}\right)}{2\pi} \cdot 2\pi r + 2\sqrt{\frac{(l_3 - l_{k2})^2}{4} - r^2}$$

$$+ \frac{\theta_1 + \arcsin\left(\frac{2r}{l_3 - l_{k2}}\right)}{2\pi} \cdot 2\pi r + l_1 + \frac{\pi + \arcsin\left(\frac{2r}{l_{k1}}\right) - \theta_1}{2\pi} \cdot 2\pi r$$

$$+ 2\sqrt{\frac{{l_{k1}}^2}{4} - r^2} + \pi r - l_3 + l_{k2} - l_1 - l_{k1} \qquad (3.35)$$

After simplification,

$$\mathrm{l_{s20p}}' - \mathrm{l_{s20}}' = \left[2\pi + \Delta\varphi_2{}' - \theta_2 - \varphi_2{}' + 2\arcsin\left(\frac{2r}{l_3 - l_{k2}}\right) + \arcsin\left(\frac{2r}{l_{k1}}\right)\right] \cdot r$$

$$+ 2\sqrt{\frac{(l_3 - l_{k2})^2}{4} - r^2} + 2\sqrt{\frac{{l_{k1}}^2}{4} - r^2} + \pi r - l_3 + l_{k2} - l_{k1} \qquad (3.36)$$

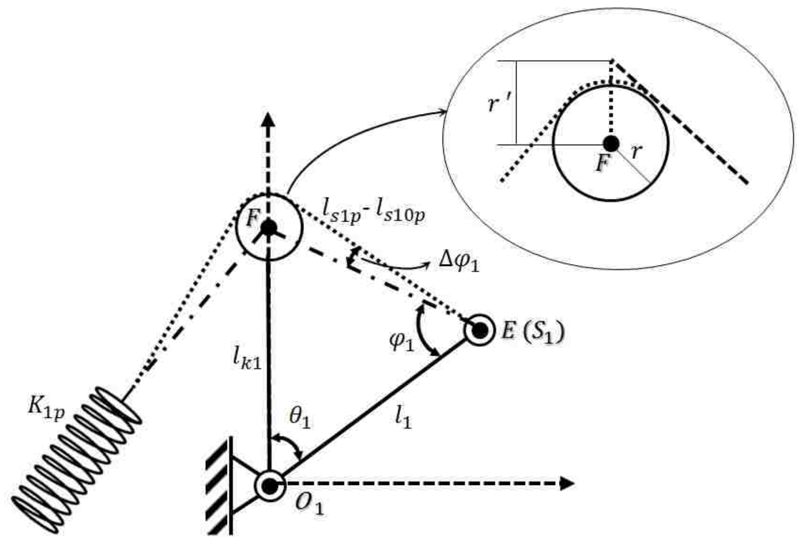

Fig. 3.1(a): Force Diagram for Spring Constant K_{1p} Calculation under Initial Conditions for an Actual Design

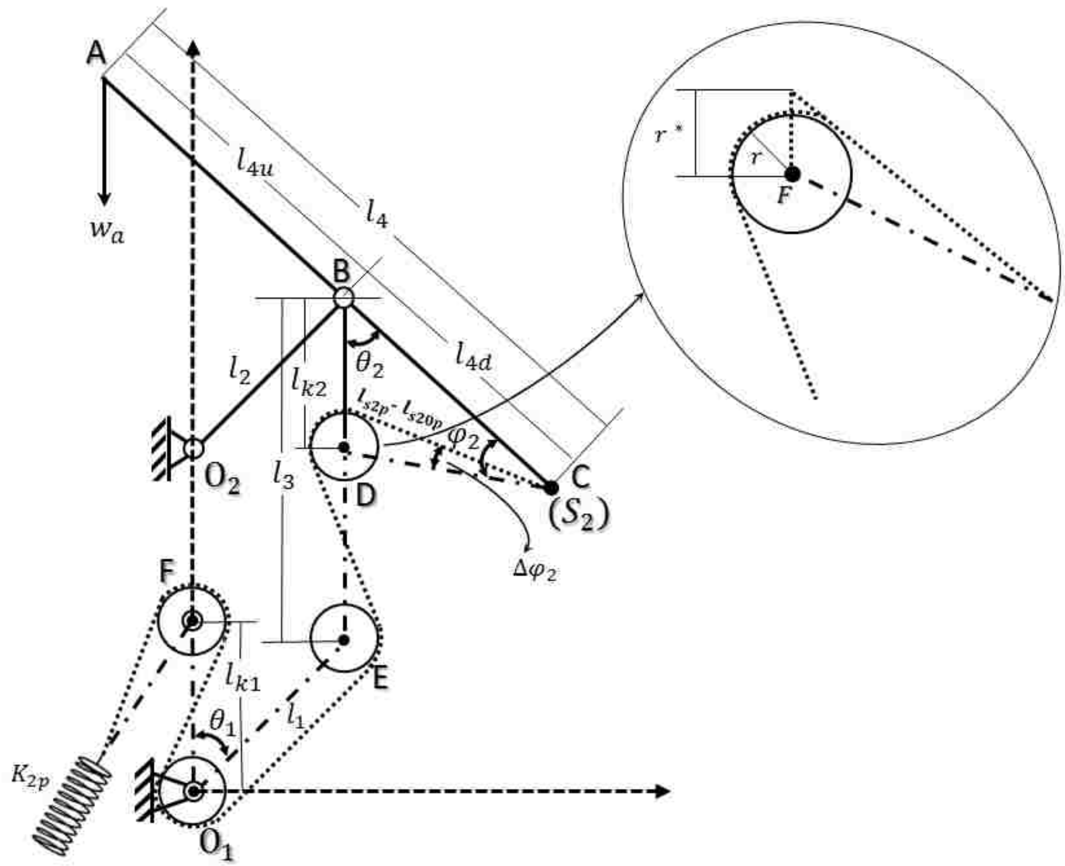

Fig. 3.1(b): Force Diagram for Spring Constant K_{2p} Calculation under Initial Conditions for an Actual Design

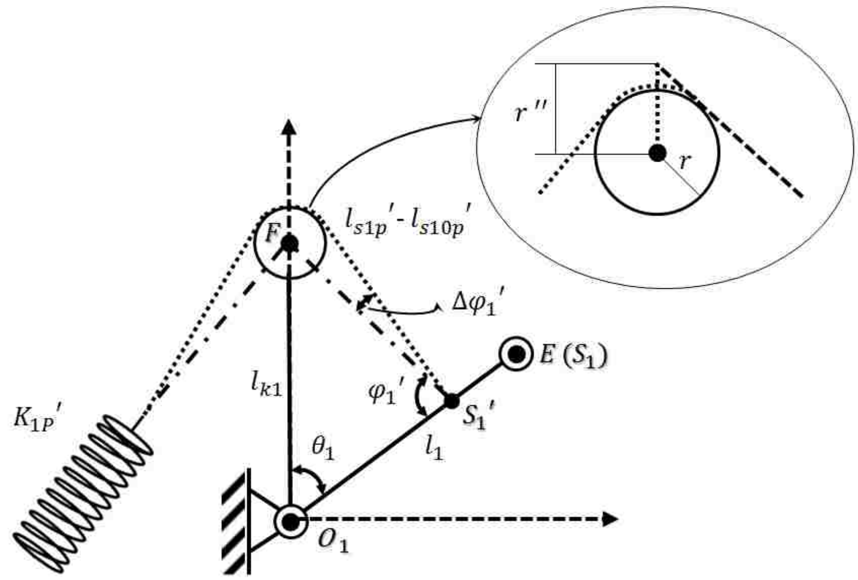

Fig. 3.1(c): Force Diagram for Spring Constant $K_{1p}{}'$ Calculation under Adjustment Conditions for an Actual Design

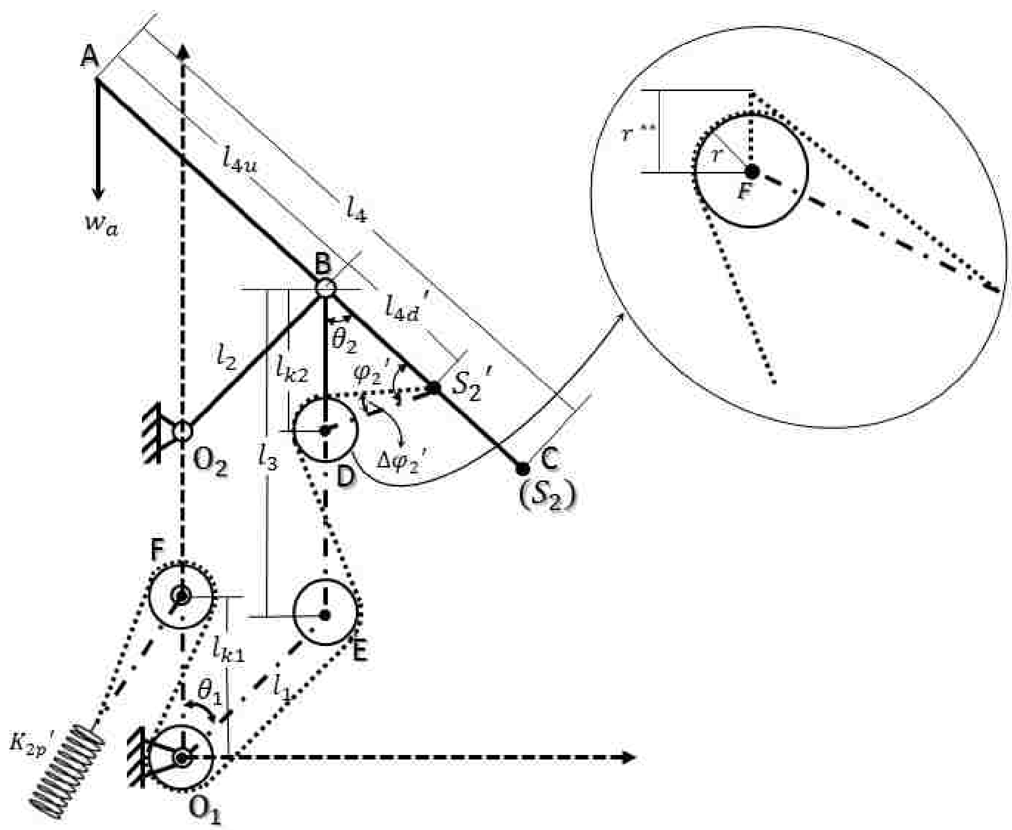

Fig. 3.1(d): Force Diagram for Spring Constant $K_{2p}{}'$ Calculation under Adjustment Conditions for an Actual Design

3.2.2 Actual Suspended Weight Calculation

The suspended weight equation will change once the pulley diameter is taken into consideration. Similar to ideal situations, the spring constant is a fixed number when calculating the suspended weight.

The expression of suspended weight w_a can be derived from Equations(3.5), (3.13), (3.23), (3.31). These equations will either include K_{p1} or K_{2p}. Once each spring constant is fixed, K_{1p} will be the same as $K_{1p}{}'$ and K_{2p} will be the same as $K_{2p}{}'$.

Suspended weight w_a can be written with respect to K_{1p} as:

$$w_a = \frac{K_p(l_{k1} + r'')l_1{}'}{l_1} - \left(\frac{1}{2}m_1 + \frac{1}{2}m_2 + m_3 + m_4\right)g \tag{3.37}$$

Where,

$$r'' = \frac{r}{\sin(\pi - \theta_1 - \varphi_1{}' - \Delta\varphi_1{}')} \tag{3.38}$$

$$\Delta\varphi_1{}' = \arcsin\left(\frac{r\sin(\varphi_1{}')}{l_{k1}\sin(\theta_1)}\right) \tag{3.39}$$

$$\varphi_1{}' = \sin^{-1}\left(\frac{l_{k1}\sin(\theta_1)}{\sqrt{{l_{k1}}^2 + {l_1}'^2 - 2l_{k1}l_1{}'\cos(\theta_1)}}\right) \tag{3.40}$$

When $l_1{}' = l_1$, cable 1 is attached at the initial location (S_1).

Suspended weight w_a can also be written with respect to K_{2p} as:

$$w_a = \frac{K_2(l_{k2} - r^{**})\, l_{4d}' + \frac{1}{2} l_{4d} m_{4d} g - \frac{1}{2} l_{4u} m_{4u} g}{l_{4u}} \tag{3.41}$$

Where,

$$r^{**} = \frac{r}{\sin(\theta_2 + \varphi_2' - \Delta\varphi_2')} \tag{3.42}$$

$$\Delta\varphi_2' = \arcsin\left(\frac{r \sin(\theta_2 + \varphi_2')}{l_{4d}' \sin(\theta_2)}\right) \tag{3.43}$$

$$\varphi_2' = \sin^{-1}\left(\frac{l_{k2} \sin(\theta_2)}{\sqrt{l_{k2}^2 + l_{4d}'^2 - 2 l_{k2} l_{4d}' \cos(\theta_2)}}\right) \tag{3.44}$$

When $l_{4d}' = l_{4d}$, cable 2 is attached at the initial location (S_2).

3.2.3 Influence Factors in Actual Suspension Mass

In chapter 2, the relationship between the spring constant, linkage length and linkage mass affect the final suspended weight in ideal design has been derived. In the actual case, Equations $(3.37 - 3.44)$ show that the suspended mass will also be affected by the linkage position angle θ_1, θ_2 and the pulley diameter r.

When analyzing these factor effects, the default pulley diameter is 0.035m, θ_1 is 45° and θ_2 is 45°. Other parameters will follow Table. 2.1

Fig. 3.2(a) shows how suspended mass changes through changing link 1 position angle θ_1. Except for changing the value of θ_1, all the linkage length, mass and position angle are set at the initial condition. In this case, the two groups of cables attachment points (S_1 and S_2) are at the original location. The value of both K_1 and K_2 are fixed at 2000 N/m.

When θ_1 is increasing from 0 to 90°, the lifting mass M_a varies around 65Kg. However, this variation is nonlinear. It will be hard to keep exactly the suspended weight when link 1 position angle has been changed.

Fig. 3.2(b) shows the comparison of Ma versus θ_1 with different pulley diameters. It's obvious that the larger the pulley diameter is, the greater is the variation in the curve on Fig. 3.2(b). That means the error is getting bigger when pulley diameter r is increasing. When pulley diameter r equals to zero, the curve is a straight line with zero slope, which means that Ma and θ_1 are independent. This matches the result of the ideal design condition.

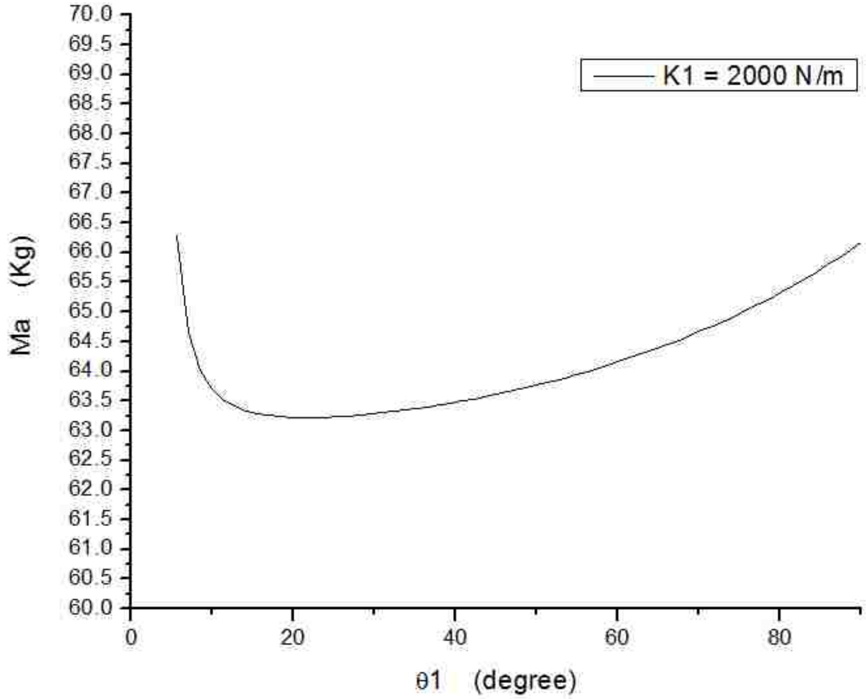

Fig. 3.2(a): Relation between Suspended Mass Ma and link 1 Position Angle θ_1 under Actual Conditions with Spring Constant $K_1 = 2000 N/m$

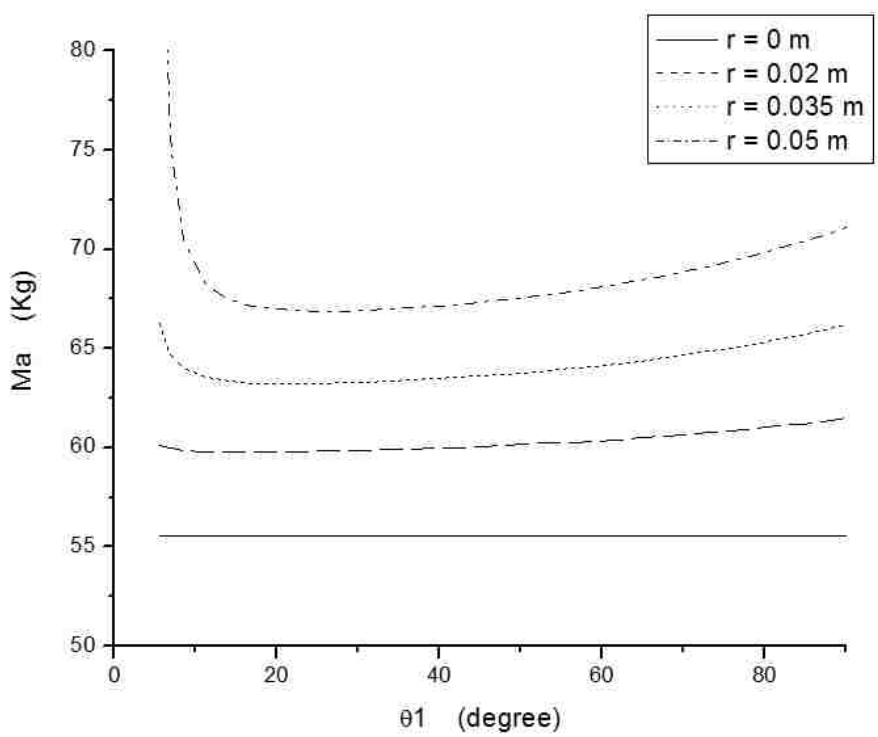

Fig. 3.2(b): Comparison of Relations between Suspended Mass Ma and link 1 Position Angle θ_1 with different pulley diameters

Fig. 3.2(c) shows how suspended mass changes through changing link 1 position angle θ_1 with different spring constant K_1 . Similar to the conditions of Fig. 3.2(a), except that three spring constant values are given to make the comparison.

When K_1 is increasing, the suspended mass M_a is increasing too. The increasing rate of M_a is following the same ratio as K_1 increasing rate. This on the other hand proves that the suspended mass will increase linearly by increasing the spring constant K_1 even in actual design with pulley diameter considered. Therefore, K_1 value is changed as the new initial condition, the suspended weight will be easy to determine.

Fig. 3.2(d) shows how suspended mass changes through changing link 4 position angle θ_2. Except changing the value of θ_2, all the linkage length, mass and position angle are set as the initial condition. In this case, the two groups of cables attachment points (S_1 and S_2) are at the original location. The values of both K_1 and K_2 are fixed to be 2000 N/m.

When θ_1 is increasing from 0 to 90°. The lifting mass M_a is changing in a large range from 13Kg to 53Kg. After 17°, this M_a-θ_1 curve becomes stable and the value of M_a gradually equal to 52Kg. Since this changing is still nonlinear, it will be hard to keep exactly the suspended weight when link 2 position angle has been changed.

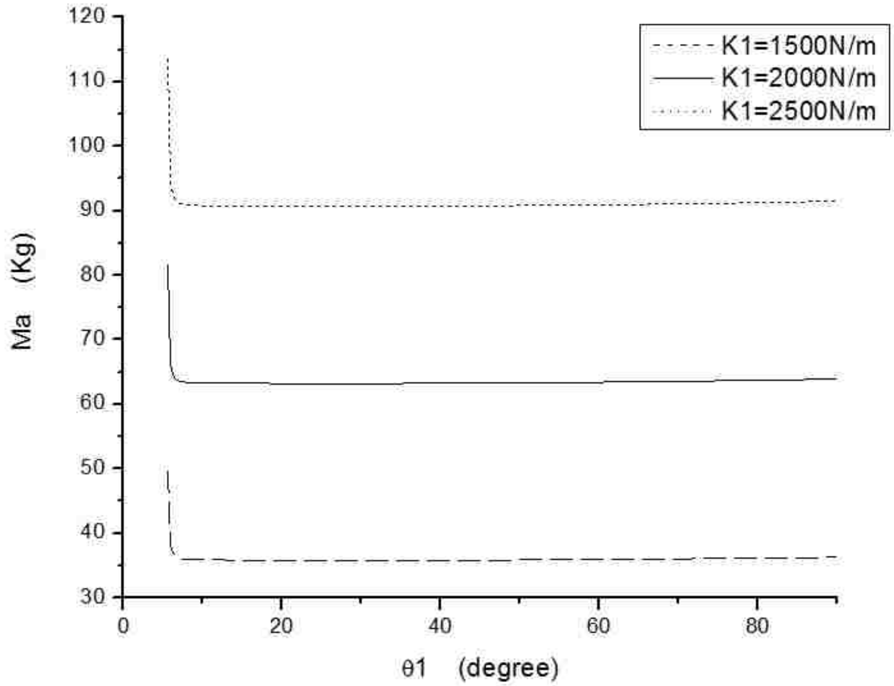

Fig. 3.2(c): Relation between Suspended Mass Ma and Link 1 Position Angle θ_1 under Actual Condition with Different Spring Constant K_1

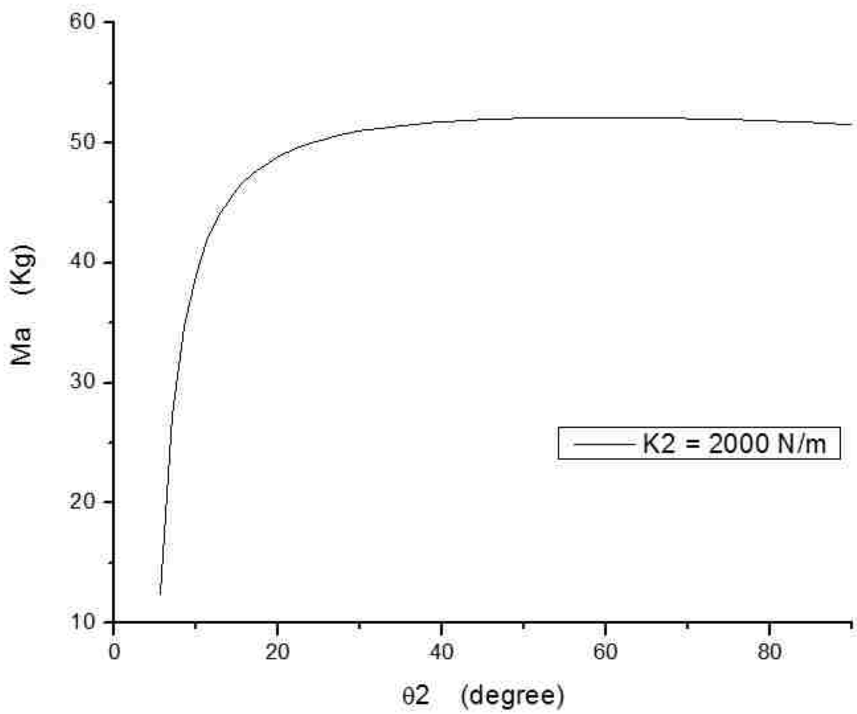

Fig. 3.2(d): Relation between Suspended Mass Ma and Link 4 Position Angle θ_2 under Actual Conditions with Spring Constant $K_2 = 2000\text{N/m}$

Fig. 3.2(e) shows the comparison of Ma versus θ_2 with different pulley diameters. It's obvious that the larger the pulley diameter is, the greater the variation in the curve on Fig. 3.2(e) is. That means the error is getting bigger when pulley diameter r is increasing. When pulley diameter r equals to zero, the curve is a straight line with zero slope, which means that Ma and θ_2 are independent. This also matches the result of the ideal design condition.

Fig. 3.2(f) shows how suspended mass changes through changing link 4 position angle θ_2 with different spring constant number K_2 . Similar to the condition of Fig. 3.2(d), except that three spring constant values are given to make the comparison.

When K_2 is increasing, the suspended mass M_a is increasing too. The increasing rate of M_a is following the same ratio as K_2 increasing rate. This on the other hand proves that the suspended mass will increase linearly by increasing the spring constant K_2 even in actual design with pulley diameter considered. Therefore, if K_2 value is changed as the new initial condition, the suspended weight will be easy to determine.

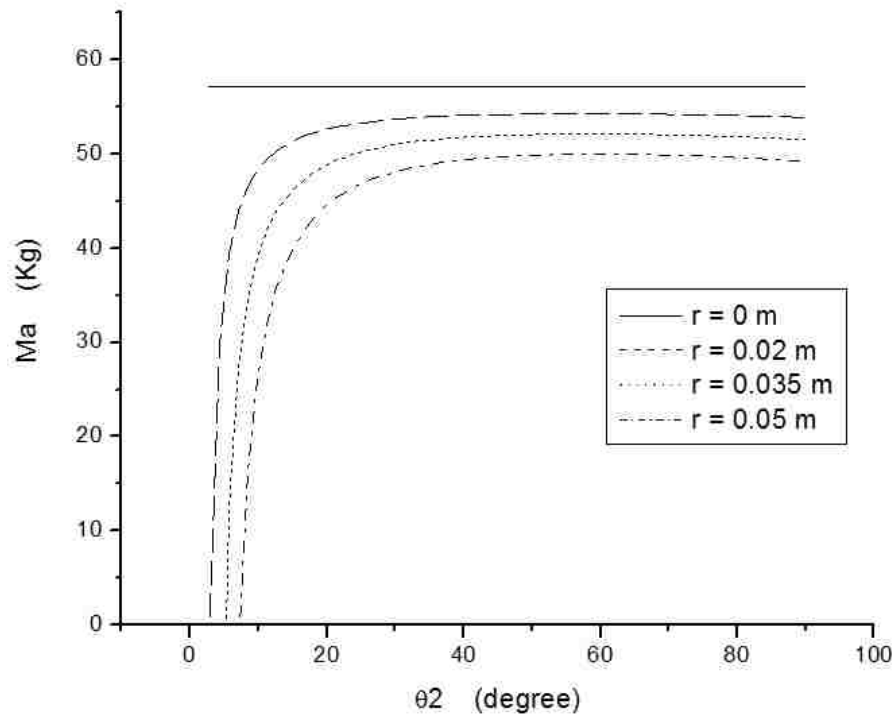

Fig. 3.2(e): Comparison of Relations between Suspended Mass Ma and link 4 Position Angle θ_2 with different pulley diameters

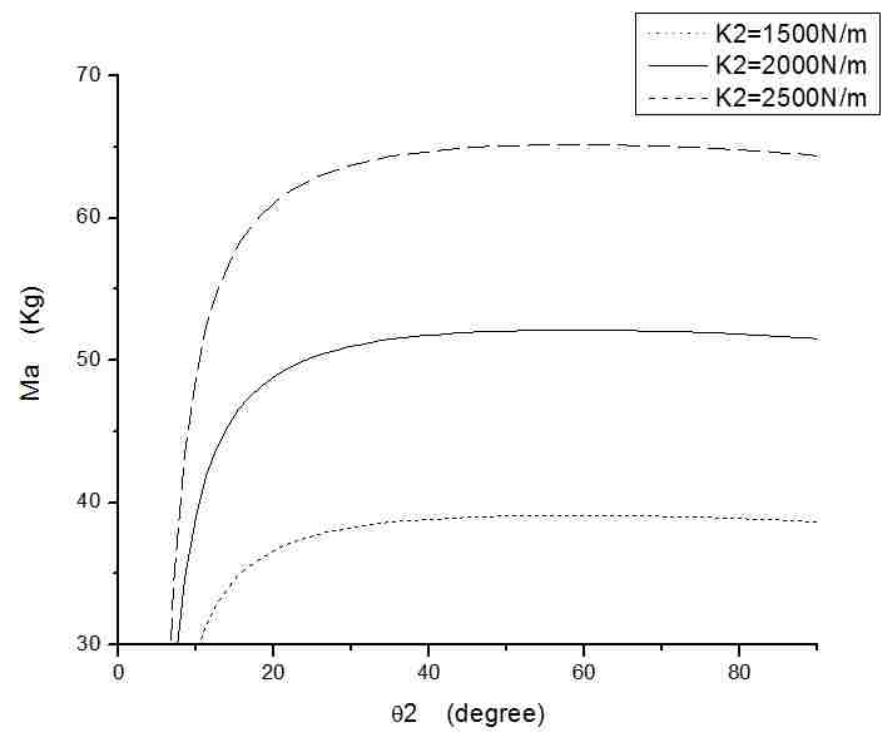

Fig. 3.2(f): Relation between Suspended Mass Ma and Link 4 Position Angle θ_2 under Actual Condition with Different Spring Constant K_2

Fig. 3.3(a) shows how suspended mass act through changing pulley diameter following the Equation (3.37). Other than changing the value of r, all the linkage length, mass and position angle are set as the initial condition. In this case, the two groups of cables attachment points (S_1 and S_2) are at the original location. The value of both K_1 and K_2 are fixed to be 1500 N/m.

When pulley diameter is increasing from 0.01m to 0.1m, the lifting mass has approximately linear increase. If pulley diameter is changed as the initial condition, changing of suspended mass can be regarded linearly. But this linear approximation can cause small error.

Fig. 3.3(b) shows how suspended mass act through changing pulley diameter following the Equation (3.41). Other than changing the value of r, all the linkage length, mass and position angle are set as the initial condition. In this case, the two groups of cables attachment points (S_1 and S_2) are at the original location. The values of both K_1 and K_2 are fixed to be 1500 N/m.

When pulley diameter is increasing from 0.01m to 0.1m, the lifting mass has approximately linear decrease. If pulley diameter is changed as the initial condition, the changing of suspended mass can be regarded linearly. But similar to Fig. 3.3(a), this linear approximation can also cause small error.

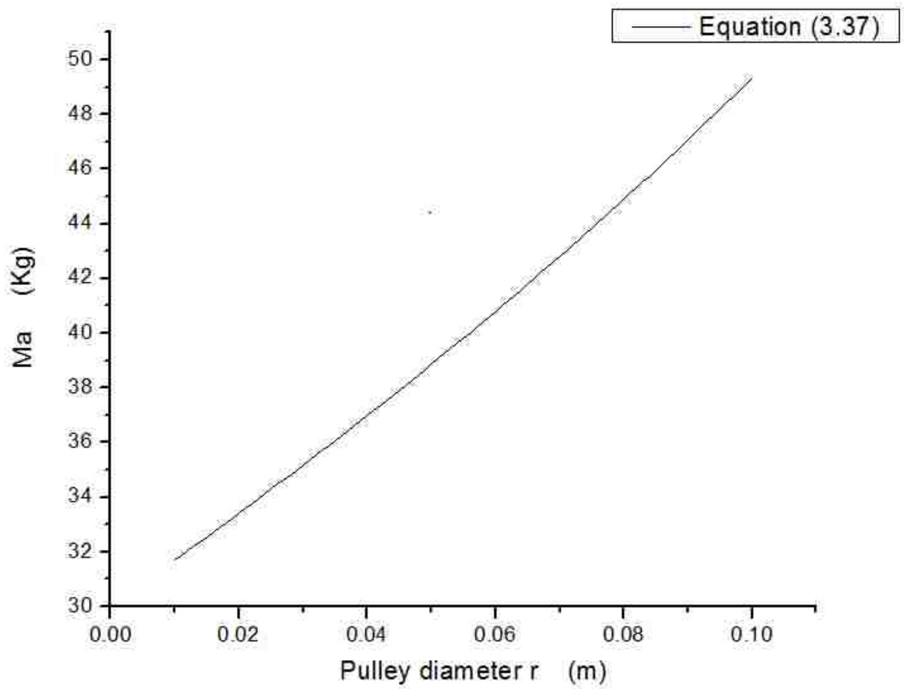

Fig. 3.3(a): Plot of Relation between Suspended Mass Ma and Pulley Diameter r following The Calculation in Equation (3.37)

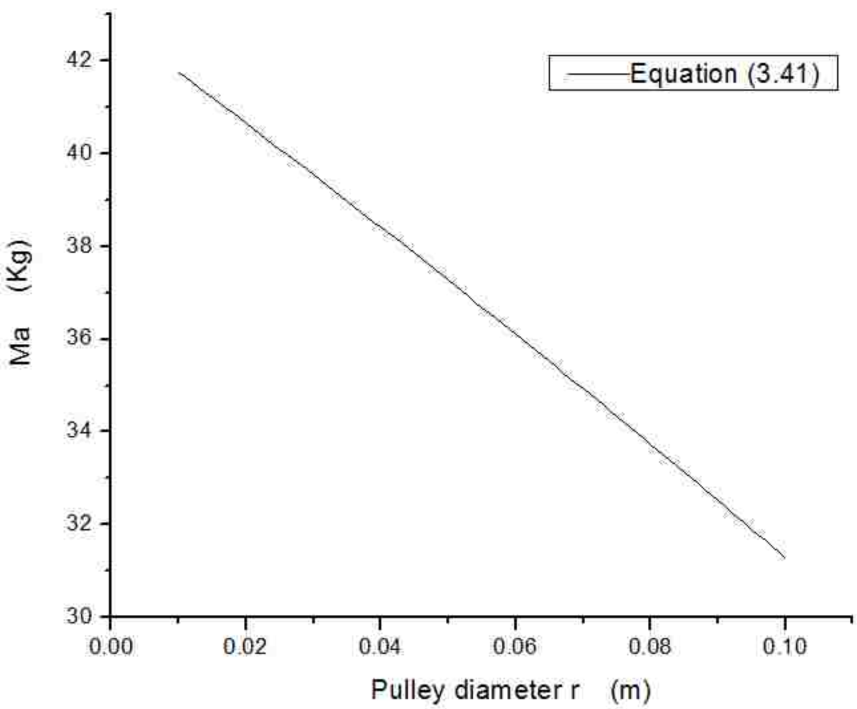

Fig. 3.3(b): Plot of Relation between Suspended Mass Ma and Pulley Diameter r following The Calculation in Equation (3.41)

Fig. 3.3(c) shows the comparison of different lifting mass Ma changing trend with two Equations (3.37) and (3.41). Two curves go in different trend when increasing the pulley diameter. The best pulley diameter setting to reduce the error is to find the value of r at the intersection of two curves. In this case ($\mathrm{K_1 = K_2 = 1500N/m}$), the intersection value of r is 0.045m. In the initial condition setting ($\mathrm{K_1 = K_2 = 2000N/m}$), the best r should be 0.035m.

Fig. 3.4(a) shows the comparison of how suspended mass Ma changes by adjusting $\mathrm{l_1}'$ between actual and ideal conditions. The actual condition curve in the figure is approximately a straight line. The Ma value in actual conditions is greater than the value in ideal case with the same $\mathrm{l_1}'$ value. The Ma value difference between two conditions is getting larger with the increasing of $\mathrm{l_1}'$. The range of adjustable length $\mathrm{l_1}'$ is from 0.208m to 0.5m.

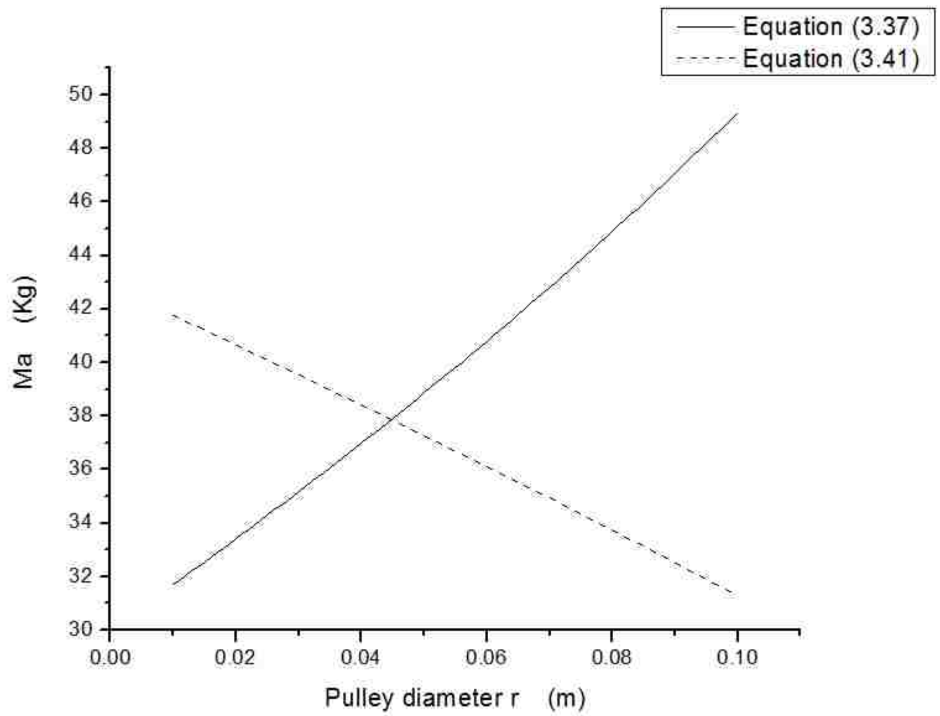

Fig. 3.3(c): Comparison between Relation of Suspended Mass Ma and Pulley Diameter r following The Calculation in Equation (3.37) and Equation (3.41)

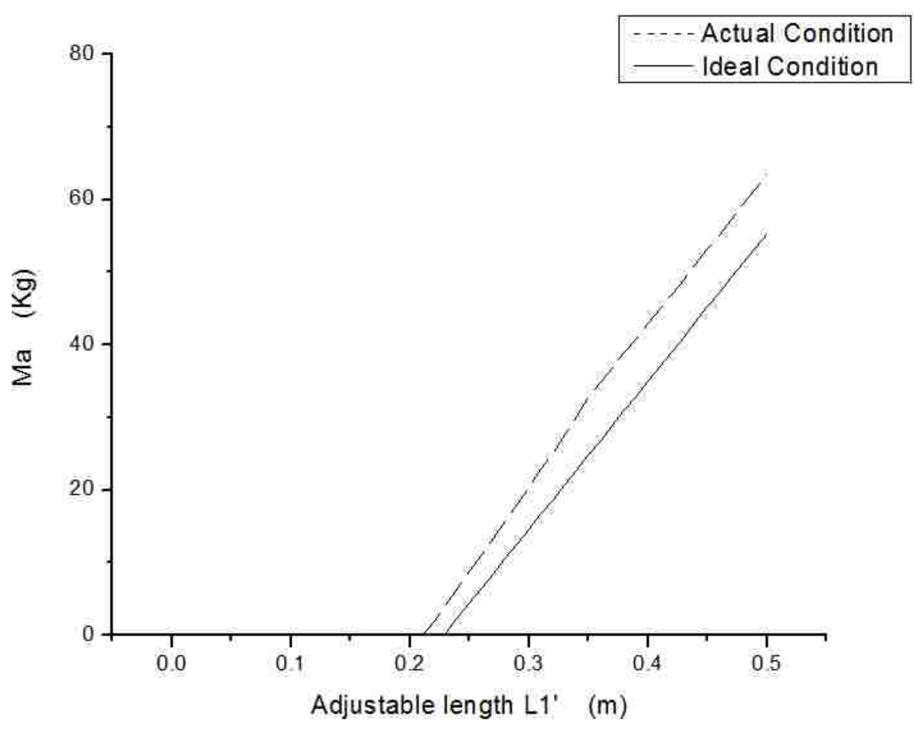

Fig. 3.4(a): Comparison of Suspended Mass Changing between Ideal Conditions and Actual Conditions when Adjusting l_1'

Fig. 3.4(b) shows the comparison of how suspended mass Ma changes by adjusting l_{4d}' between actual and ideal conditions. The actual condition curve in the figure is also approximately a straight line. The Ma value in actual condition is smaller than the value in ideal case with the same l_{4d}' value. The Ma value difference between two conditions is getting larger with the increasing of l_{4d}'. The range of adjustable length l_{4d}' is same in both conditions.

Fig. 3.4(c) illustrates how to adjust the suspended mass through this reference graph in real situation. As it has been described in the paragraph under Fig. 2.5, adjusting l_1' and l_{4d}' by moving the cables attachment point S_1' and S_2' is the way to control how much mass to lift. It is still good for us since the suspended mass is changing approximately linearly when adjusting both l_1' andl_{4d}' in this actual case. On the other hand, the slope difference between two curves still needs to be fixed. Similar to ideal conditions, an adjustment equation can be applied with a constant slope number which depends on different initial conditions.

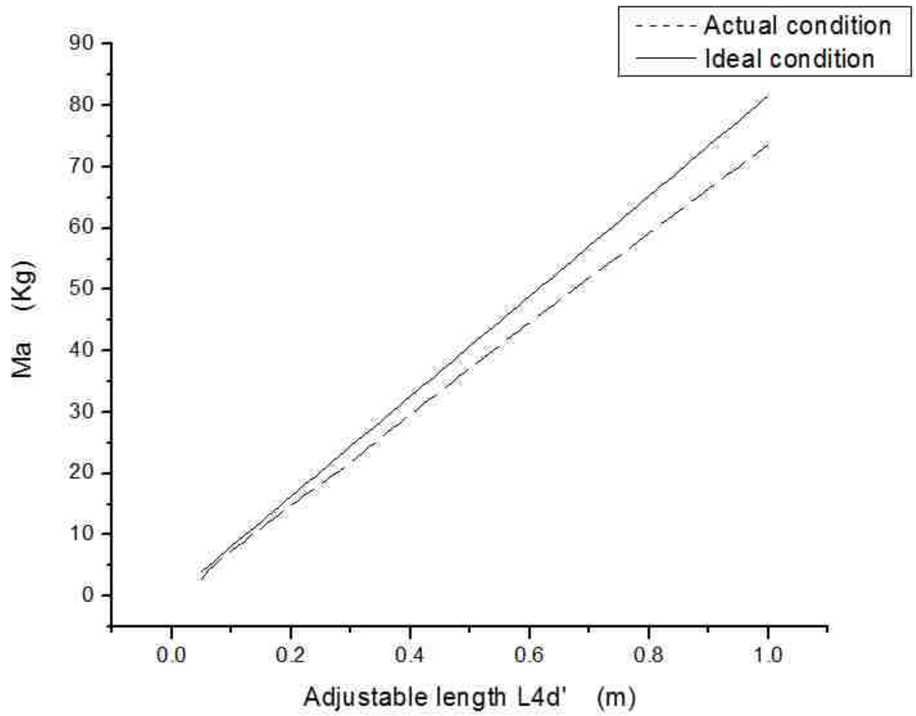

Fig. 3.4(b): Comparison of Suspended Mass Changing between Ideal Conditions and Actual Conditions when Adjusting l_{4d}'

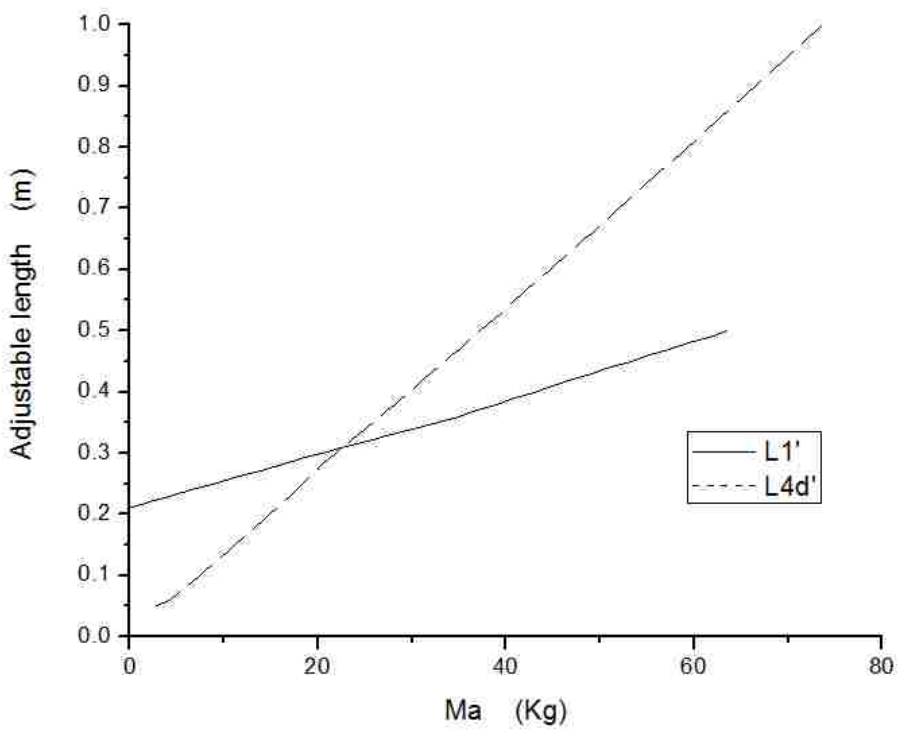

Fig. 3.4(c): Suspended Mass Adjusting Reference Graph in Actual Conditions

Chapter 4

Building Prototype

In previous theoretical design chapters, a two degree-of-freedom spring suspension system model has been established for lower-limb rehabilitation. Equations have been established to set the initial linkage length, mass, spring constant number and pulley diameter. With these parameters information, a prototype can be generated and material can be decided. In this chapter, a 3D model is established at the beginning. The specifications of the linkage dimension, position angle, spring constant number, pulley diameter are derived from previous calculations. Next, a bill of material list will be generated depending on the engineering specifications. Then an assembling plan is created to finish the prototype.

4.1 Establishment of 3D model

The theoretical design gives us a parallelogram linkage suspension system. Next step is to turn the ideal theory into real 3D model. In this section, 3D models of every component of the suspension system are established. All the following 3D models are designed using SolidWorks.

When building the real 3D model, a stable base is necessary to hold the parallelogram linkage structure. This base is also required to be movable since one purpose of using this system is to practice walking. In the model, the based is composed by two main parts showed in Fig. 4.1(a) and Fig. 4.1(b). The top point of base 1 is O_2 point in theoretical design.

Since the base will hold the whole mechanical system on one side, the system has chance to loss balance. Two of the extension linkage shown in Fig. 4.1(c) is designed to be connected with base 2. These two extensions linkages will have a 20 degree angle instead of being parallel. This will make the whole system more stable as well as protecting the user who stands on the area that surround by base 2 and two extension linkages.

Model in Fig. 4.1(d) represents link k1 in theoretical design. It is composed by one steel tube and two metal pieces. This tube is fixed together with base 2 on the bottom. The gap close to the bottom will be designed as O_1 point in theoretical design. This linkage can not rotate at any situation and it will move together with base structure. The top point of two metal pieces is connected with the top point of base 1. Therefore, base 1, base 2 and link k1 form a triangle structure which will stabilize the whole system.

Fig. 4.1(e), Fig. 4.1(f), Fig. 4.1(g) and Fig. 4.1(h) show the linkage model of links 1, 2, 3, 4d and 4u. The model in Fig. 4.1(e) represents link 1 and 2 in theoretical design. This model is composed of one steel tube and two metal pieces. All these linkages can rotate on the connected joint point, except the joint between link 4d and 4u. Link 4d and 4u will be fixed as one tube once they are connected.

Each spring is divided into 4 parallel springs. Therefore, the spring constant number will be 1/4 of the original one. Since the spring has a large constant number of 2000N/m as well as a large load is rare, a 500N/m-constant number large load spring is much easier to find. Fig. 4.1(i) shows this 4-spring system. Two of 4-spring systems are fixed on base 2.

Fig. 4.1(j) and Fig. 4.1(k) are models of pulley and bottom wheel. There are total five pulleys fixed on linkages. One of the pulleys is designed for spring 1 steel wire pathway. The other four pulleys are used for spring 2 steel wire pathway. There are total six wheels put on the bottom. They are used to help moving the machine.

Fig. 4.1(l) is the whole system 3D model after assembling the components.

Fig. 4.2 is the simulation 3D model in working situation. Two groups of steel wires will connected the adjustable points on linkages and the spring systems. The user wearing special vest is suspended through rope that connected a hook. The hook is assembled on top of linkage 4u. The user can practice walking in the area that surrounded by extension linkages. By adjusting the steel wire connection point on linkage, the suspended mass will vary in a certain range, which decides how much weigh users need to support by themselves.

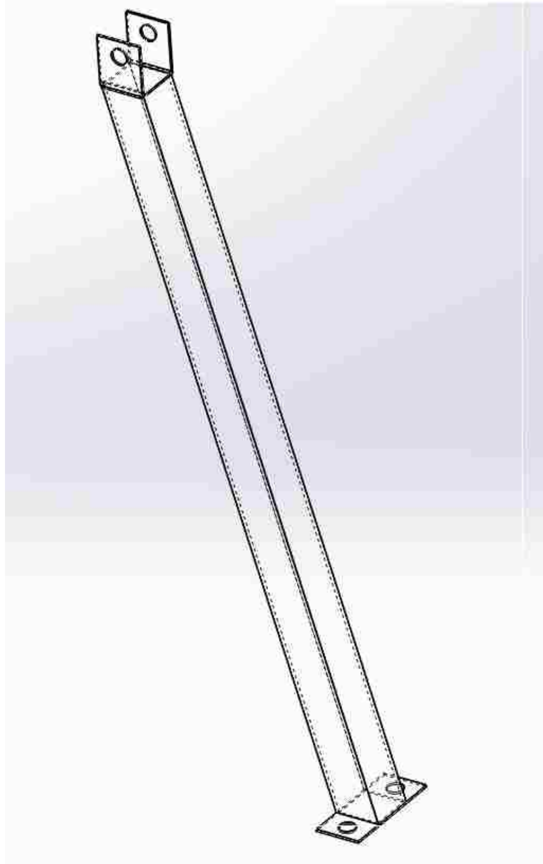

Fig. 4.1(a): Base 1

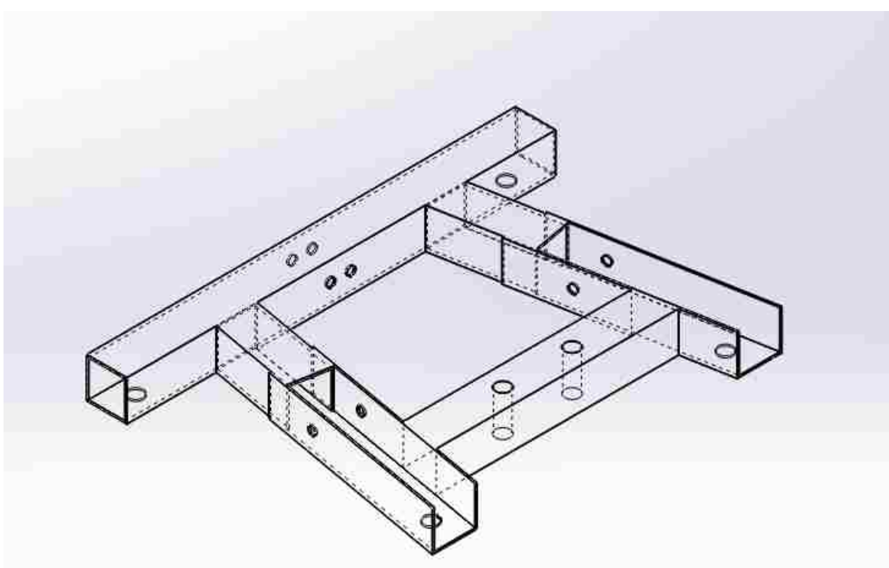

Fig. 4.1(b): Base 2

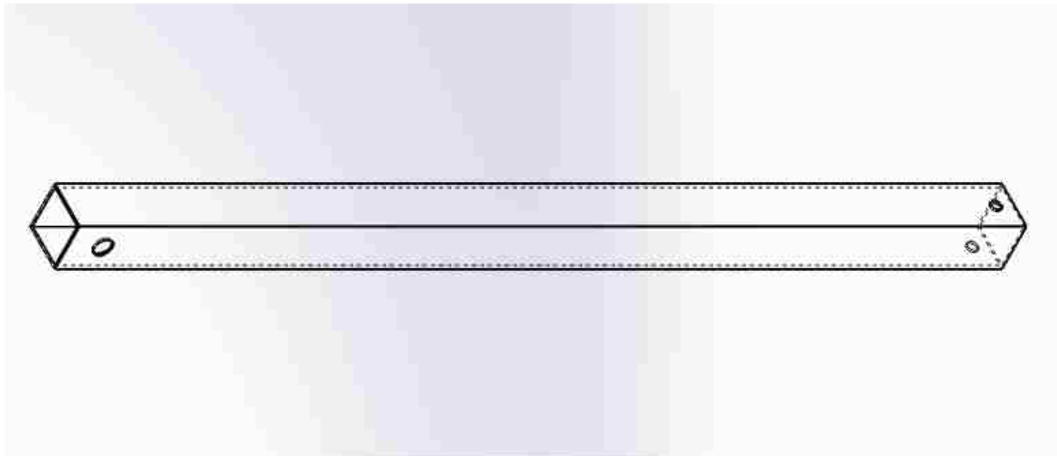

Fig. 4.1(c): Base Extension Linkage

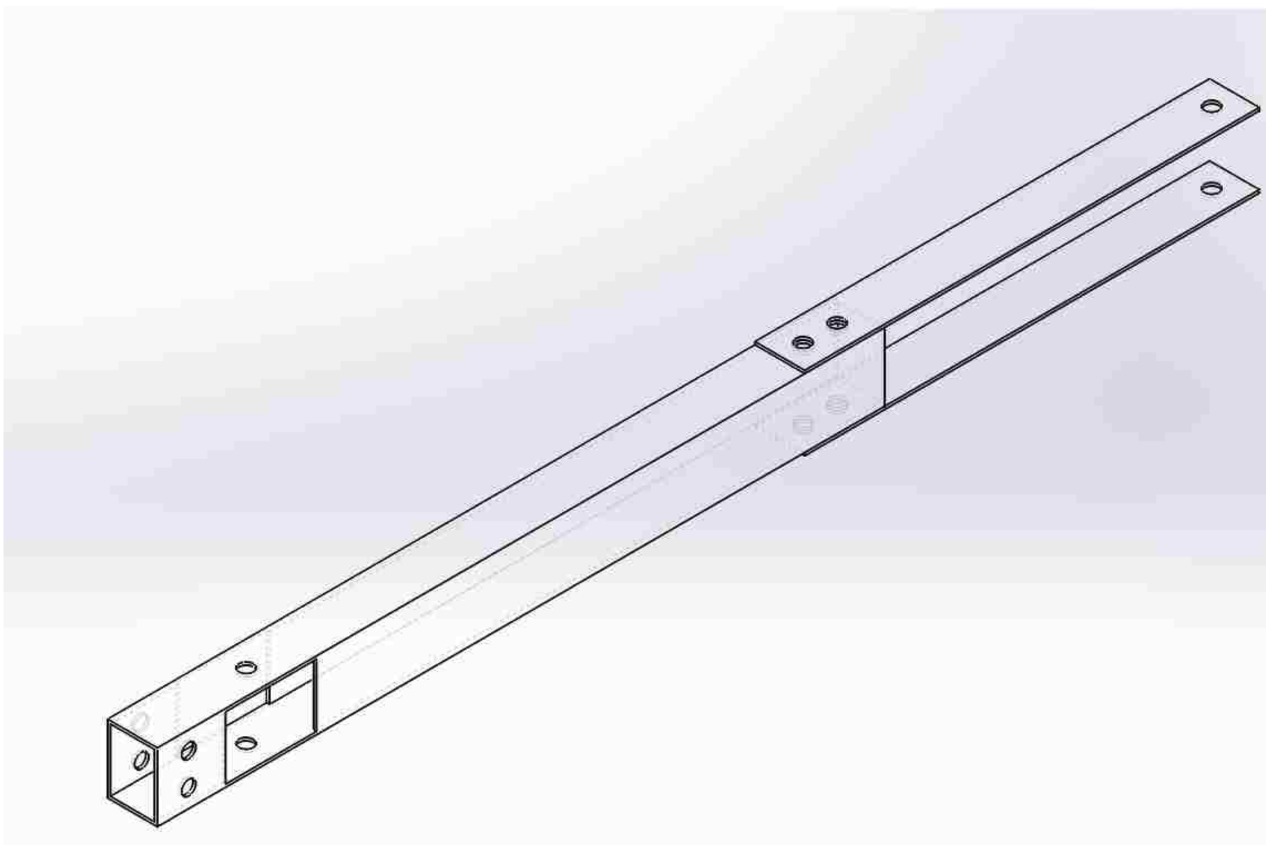

Fig. 4.1(d): Link k1 (l_{k1}) 3D Model

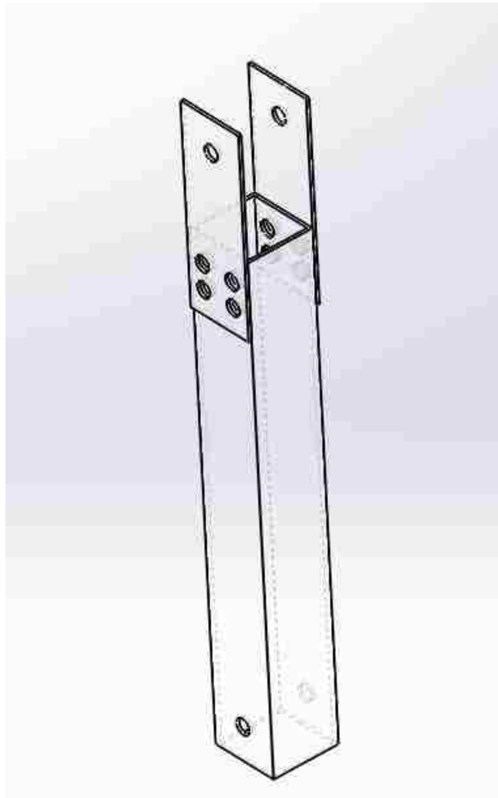

Fig. 4.1(e): Links 1 and 2 (l_1 and l_2) 3D Model

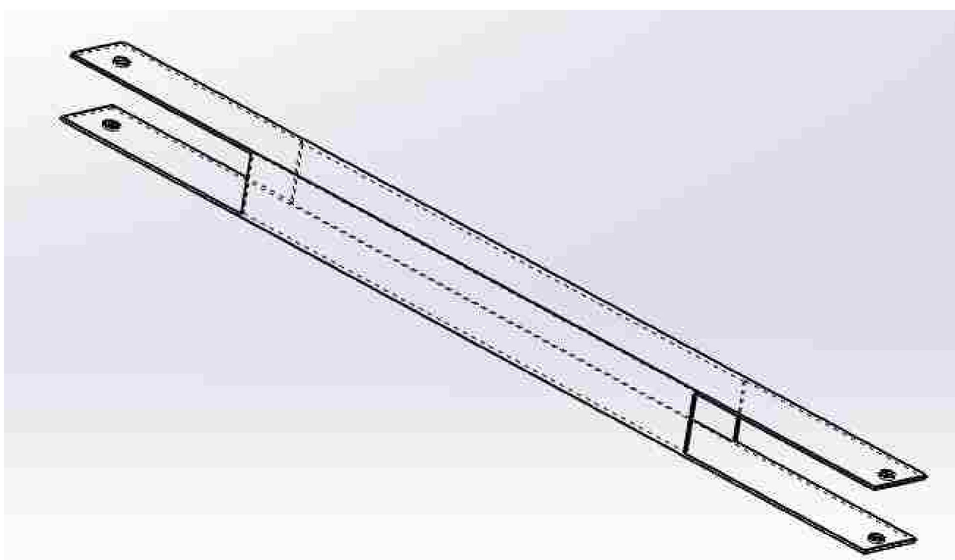

Fig. 4.1(f): Link 3 (l_3) 3D Model

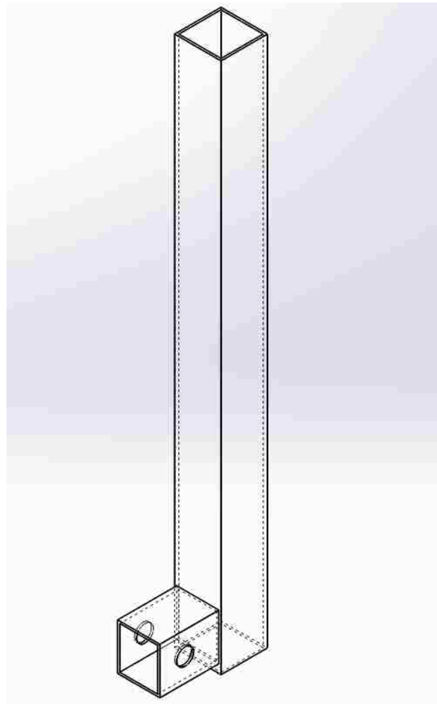

Fig. 4.1(g): Link 4d (l_{4d}) 3D Model

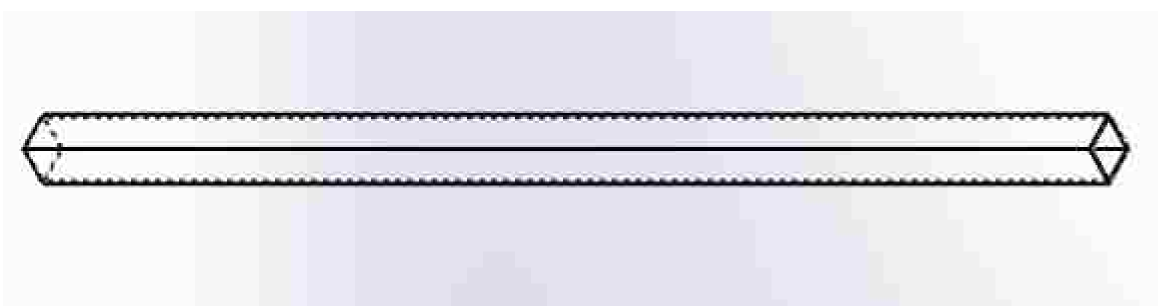

Fig. 4.1(h): Link 4u (l_{4u}) 3D Model

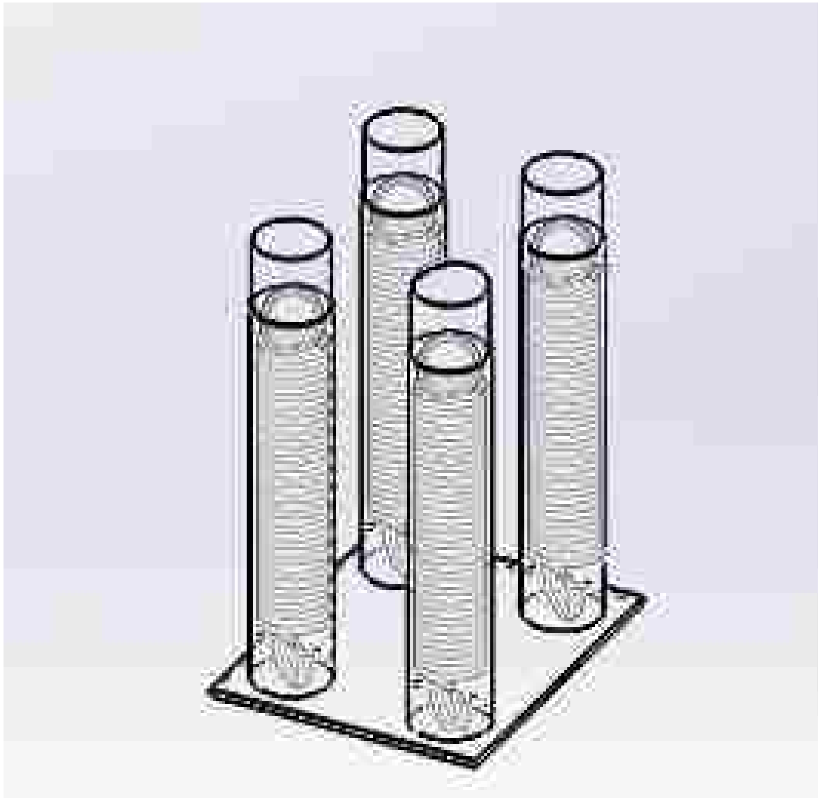

Fig. 4.1(i): Spring System Simplified Model

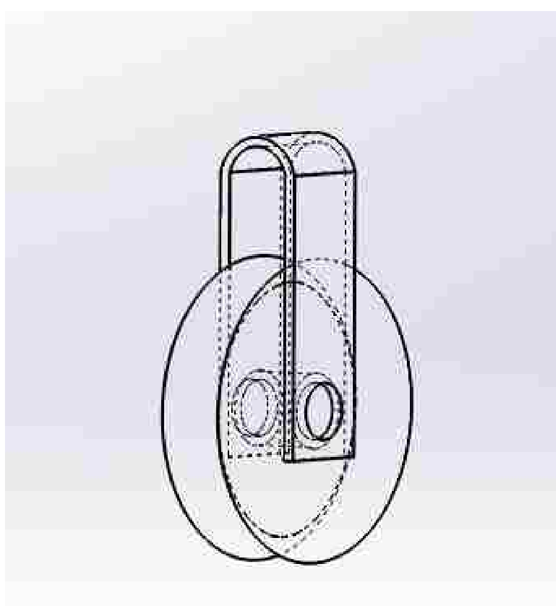

Fig. 4.1(j): Pulley Model

Fig. 4.1(k): Bottom Wheel Model

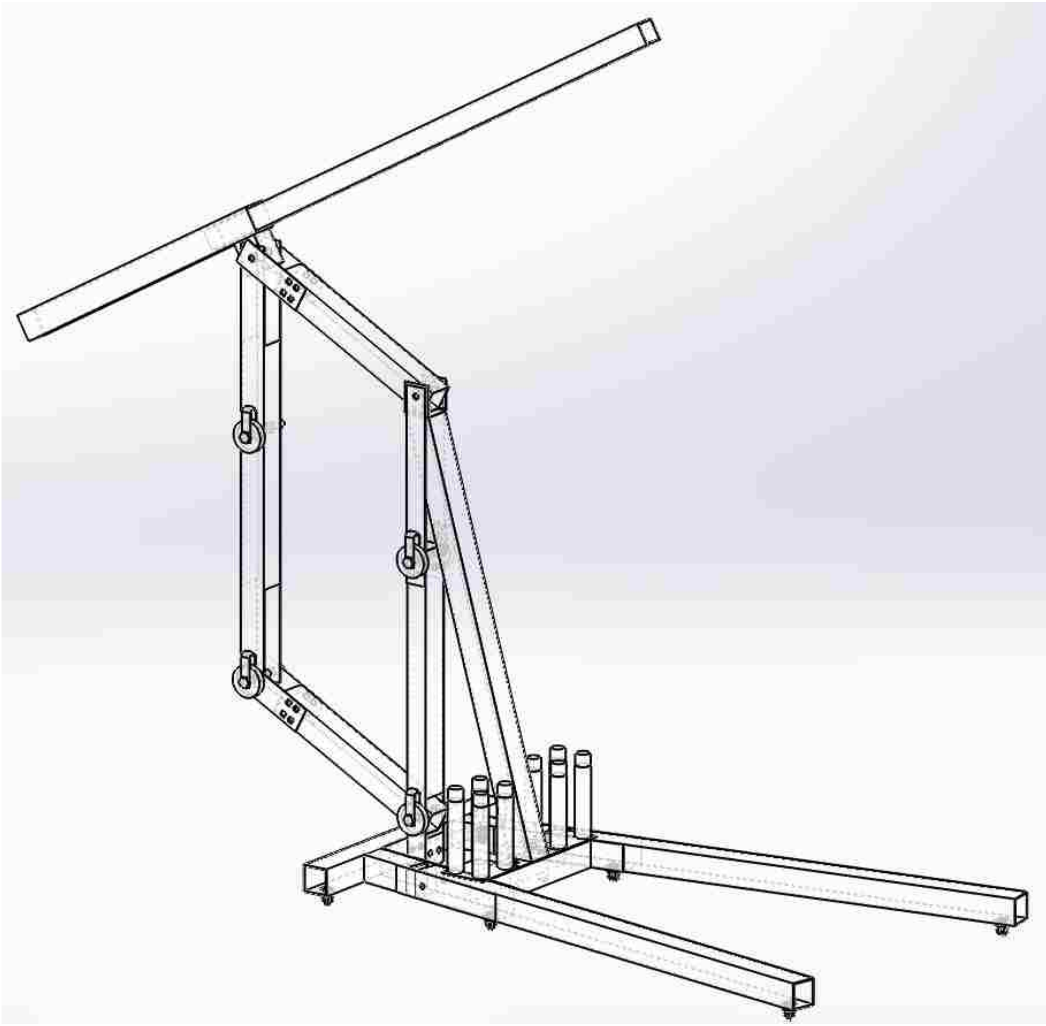

Fig. 4.1(l): Whole System 3D Model

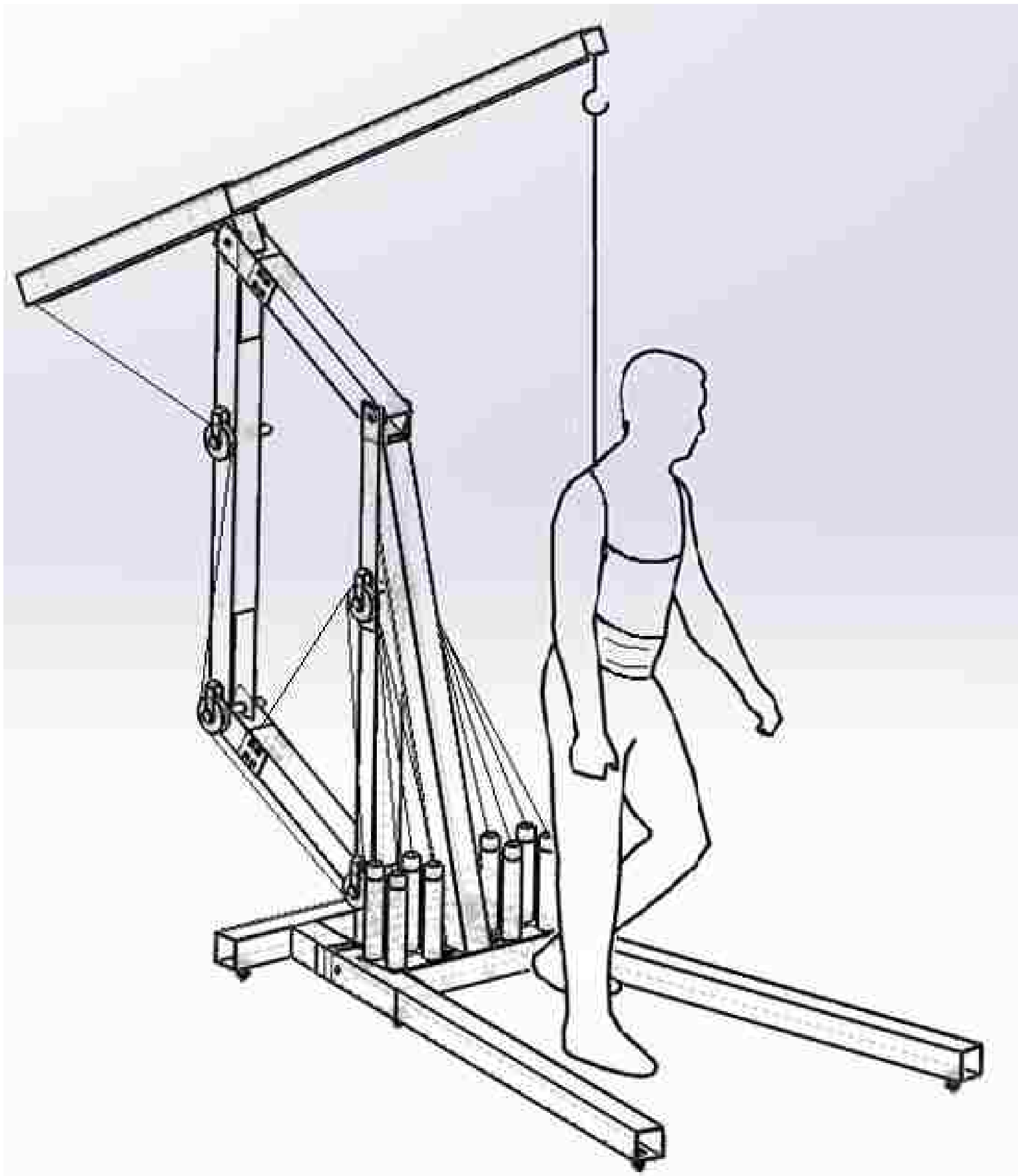

Fig. 4.2: System Working Simulation Model

4.2 Bill of Materials

Component Description	Quantity	Part Number	Supplier
1 Ton Capacity Foldable Crane	1	69512	Pittsburgh Automotive
Extension Spring 1.500OD, 0.162 302/304 stainless steel, 20.000 overall length	8	823	W.B Jones Spring
Heavy-Duty Wall/Ceiling Mount Pulley	4	348562	Lowe's
M12 × 1.75 × 150 Metric Hex Bolt Grade 8.8. Coarse thread. Steel - zinc plated	5	MCS12150CPP109	Northeast Fasteners
M12 × 1.75 × 90 Metric Hex Bolt Grade 10.9. Coarse thread. Steel - zinc plated	15	MCS1290CPZ	Northeast Fasteners
9/16 - 12 Hex Nut Grade 5 / Coarse (USS) Thread / Zinc Finish	20	HN916C5	Northeast Fasteners
Stainless Steel Square Tubing T-304 2.75" × 2.75" × 40"	1	N/A	Stored
Stainless Steel Square Tubing T-304 2.5" × 2.75" × 45"	1	N/A	Stored
Stainless Steel Square Tubing T-304 3" × 2.25" × 35"	1	N/A	Stored
Stainless Steel Square Tubing T-304 2.5" × 2.5" × 50"	1	N/A	Stored
Stainless Steel Piece 304 2.25" × 0.125" × 40"	1	N/A	Stored

Stainless Steel Piece 304 2.75" × 0.125" × 35"	1	N/A	Stored
Stainless Steel Wire	1	42250	Steel And Wire

Table.4.1: Bill of Material List

Table.4.1 shows significant items need to be purchased in the construction of the suspension weight system. Short description, quantity needed, part number and supplier are listed in the table. Some stocked minor materials (screws, rubber bands, etc.) are not listed.

Each of the stainless steel square tubes and pieces has its own engineering drawings. The drawings can be found in Appendix B.

Chapter 5

Conclusion

A gravity compensation system model for human lower-limb rehabilitation has been established. In an ideal concept model, force equations have successfully been proven that all the influence factors (linkage mass, linkage length and spring constant numbers) have a linear relationship with the suspended weight. Furthermore, the linkage position angle in the parallelogram structure does not have any effect on the suspended weight. This result completely simplifies the system initial setting process as well as the suspended weight adjustment process. When considering conditions for the actual model, introducing the pulley diameter into the calculations makes the relationship between the influence factors and the suspended weight nonlinear. The plot result shows that linkage position angles do have a nonlinear effect on the suspended weight. On the other hand, the influence errors will approaching zero when the position angle is in a certain range. Within this range, those influence factors have an approximate linear relationship with the suspended mass. In both ideal conditions and actual conditions, the adjustable cable attachment point will have approximately linear effect on the suspended weight. This result satisfies the design of the linkage system.

In the future, based on the established models and the bill of materials, a prototype will be built and experiments can then be tested for accuracy, functionality and durability.

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APPENDIX A

Table. A.1: Table of Nomenclature

w_a Weight of suspended mass

m_1 Mass of link $\overline{O_1E}$

m_2 Mass of link $\overline{O_2B}$

m_3 Mass of link $\overline{BE}$

m_4 Mass of link $\overline{AC}$

m_{4u} Mass of link $\overline{AB}$

m_{4d} Mass of link $\overline{BC}$

K_1 Spring 1 constant

K_2 Spring 2 constant

l_2 The linkage length between points O_2 and B

l_3 The linkage length between points B and E

l_4 The linkage length between points A and C

l_{4u} The linkage length between points A and B

l_{4d} The linkage length between points B and C

$l_1{}'$ The length between points O_1 and $S_1{}'$

$l_{4d}{}'$ The length between points B and $S_2{}'$

l_{k1} The length between points O_1 and F

l_{k2} The length between points B and D

l_{s10} The free length of spring1 in the condition that the cable 1 is connected at point E

l_{s1} The length of spring1 after applying force in the condition that the cable 1 is connected at point E

$l_{s10}{}'$ The free length of spring1 in the condition that the cable 1 is connected at moving point $S_1{}'$

$l_{s1}{}'$ The length of spring1 after applying force in the condition that the cable 1 is connected at moving point $S_1{}'$ and the pulley diameter is not under consideration

l_{s20} The free length of spring 2 in the condition that the cable 2 is connected at point C

l_{s2} The length of spring 2 after applying force in the condition that the cable 2 is connected at point C

$l_{s20}{}'$ The free length of spring 2 in the condition that the cable 2 is connected at moving point $S_2{}'$

$l_{s2}{}'$ The length of spring 2 after applying force in the condition that the cable 2 is connected at moving point $S_2{}'$

θ_1 Position angle of link $\overline{O_1E}$ and link $\overline{O_1F}$

θ_2 Position angle of link $\overline{BE}$ and link $\overline{BC}$

φ_1 Angle of Spring 1 steel wire and link $\overline{O_1E}$ in initial situation (S_1 and E are coincided)

φ_2 Angle of Spring 2 steel wire and link $\overline{BC}$ in initial situation (S_2 and C are coincided)

$\varphi_1{}'$ Angle of Spring 1 steel wire and link $\overline{O_1E}$ when connecting at moving point $S_1{}'$

$\varphi_2{}'$ Angle of Spring 2 steel wire and link $\overline{BC}$ when connecting at moving point $S_2{}'$

New nomenclature for the equations is provided below:

r Pulley diameter

l_1 The linkage length between point O_1 and E

l_{s10p} The free length of spring 1 in the condition that the cable 1 is connected at point E and the pulley diameter is considered

l_{s1p} The length of spring 1 after applying force in the condition that the cable 1 is connected at point E and the pulley diameter is considered

$l_{s10p}{}'$ The free length of spring 1 in the condition that the cable 1 is connected at moving point $S_1{}'$ and the pulley diameter is considered

$l_{s1p}{}'$ The length of spring 1 after applying force in the condition that the cable 1 is connected at moving point $S_1{}'$ and the pulley diameter is considered

l_{s20p} The free length of spring 2 in the condition that the cable 2 is connected at point C and the pulley diameter is considered

l_{s2p} The length of spring 2 after applying force in the condition that the cable 2 is connected at point C and the pulley diameter is considered

$l_{s20p}{}'$ The free length of spring 2 in the condition that the cable 2 is connected at moving point $S_2{}'$ and the pulley diameter is considered

$l_{s2p}{}'$ The length of spring 2 after applying force in the condition that the cable 2 is connected at moving point $S_2{}'$ and the pulley diameter is considered

$\Delta\varphi_1$ Angle between considering pulley diameter situation and ignoring pulley diameter situation that cable 1 attached to link $\overline{O_1E}$ at point E under initial situation (points S_1 and E are coincided)

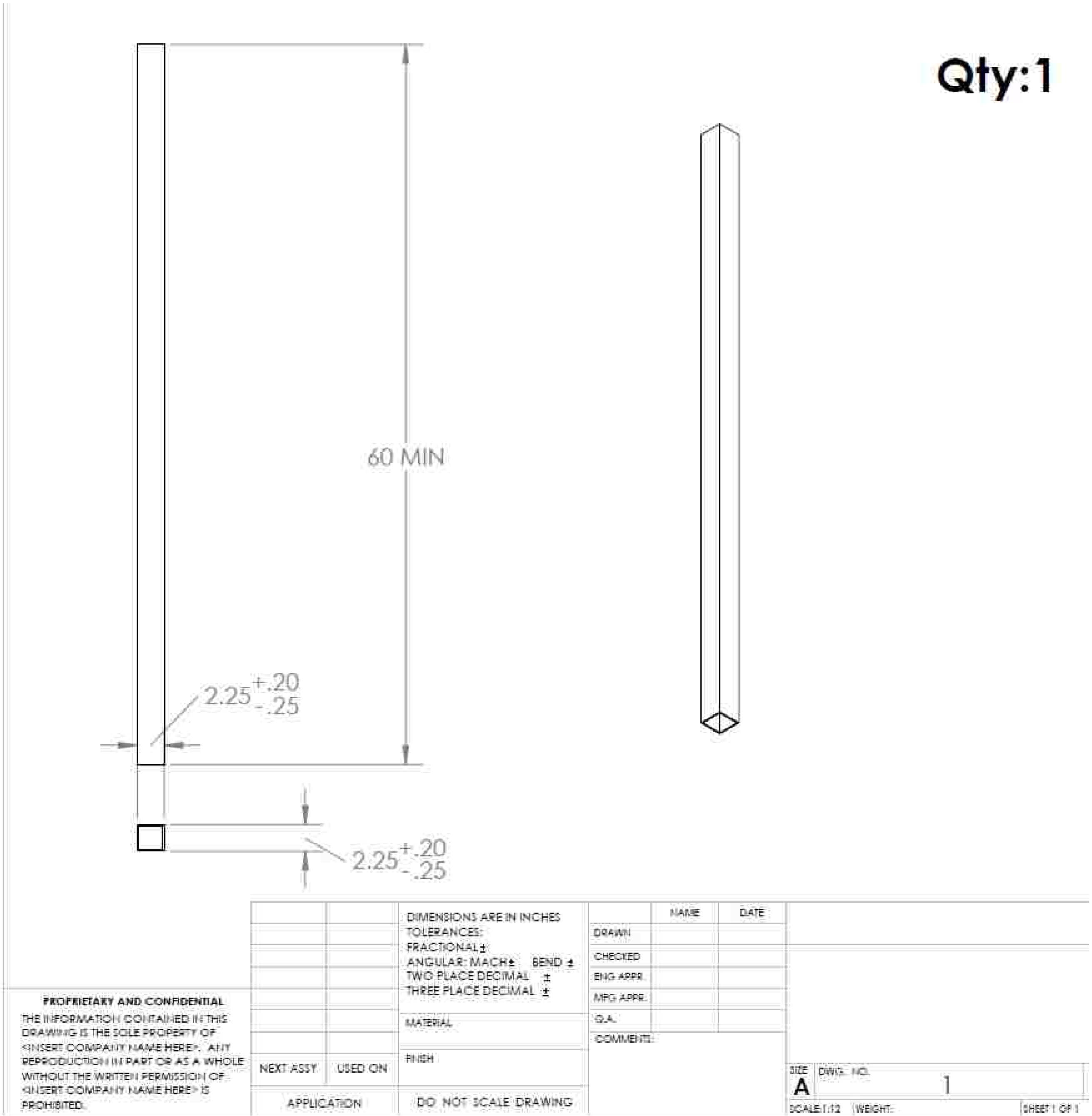
$\Delta\varphi_2$ Angle between considering pulley diameter situation and ignoring pulley diameter situation that cable 2 attached to link $\overline{BC}$ at point C under initial situation (points S_2 and C are coincided)

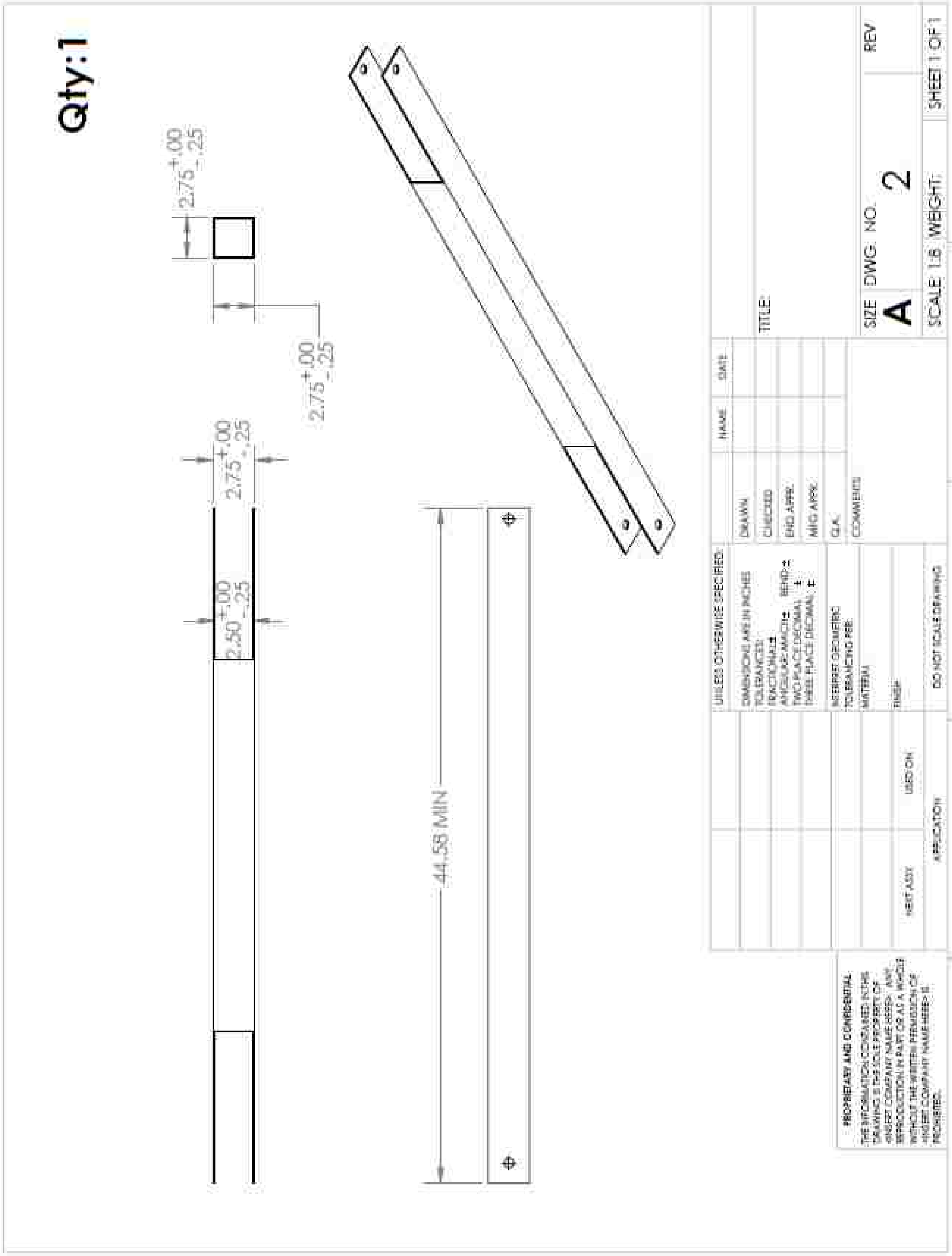
$\Delta\varphi_1{}'$ Angle between considering pulley diameter situation and ignoring pulley diameter situation that cable 1 attached to link $\overline{O_1E}$ at point $S_1{}'$

$\Delta\varphi_2{}'$ Angle between considering pulley diameter situation and ignoring pulley diameter situation that cable 2 attached to link $\overline{BC}$ at point $S_2{}'$

APPENDIX B

Steel Tubes Engineering Drawings (Unit: Inch)

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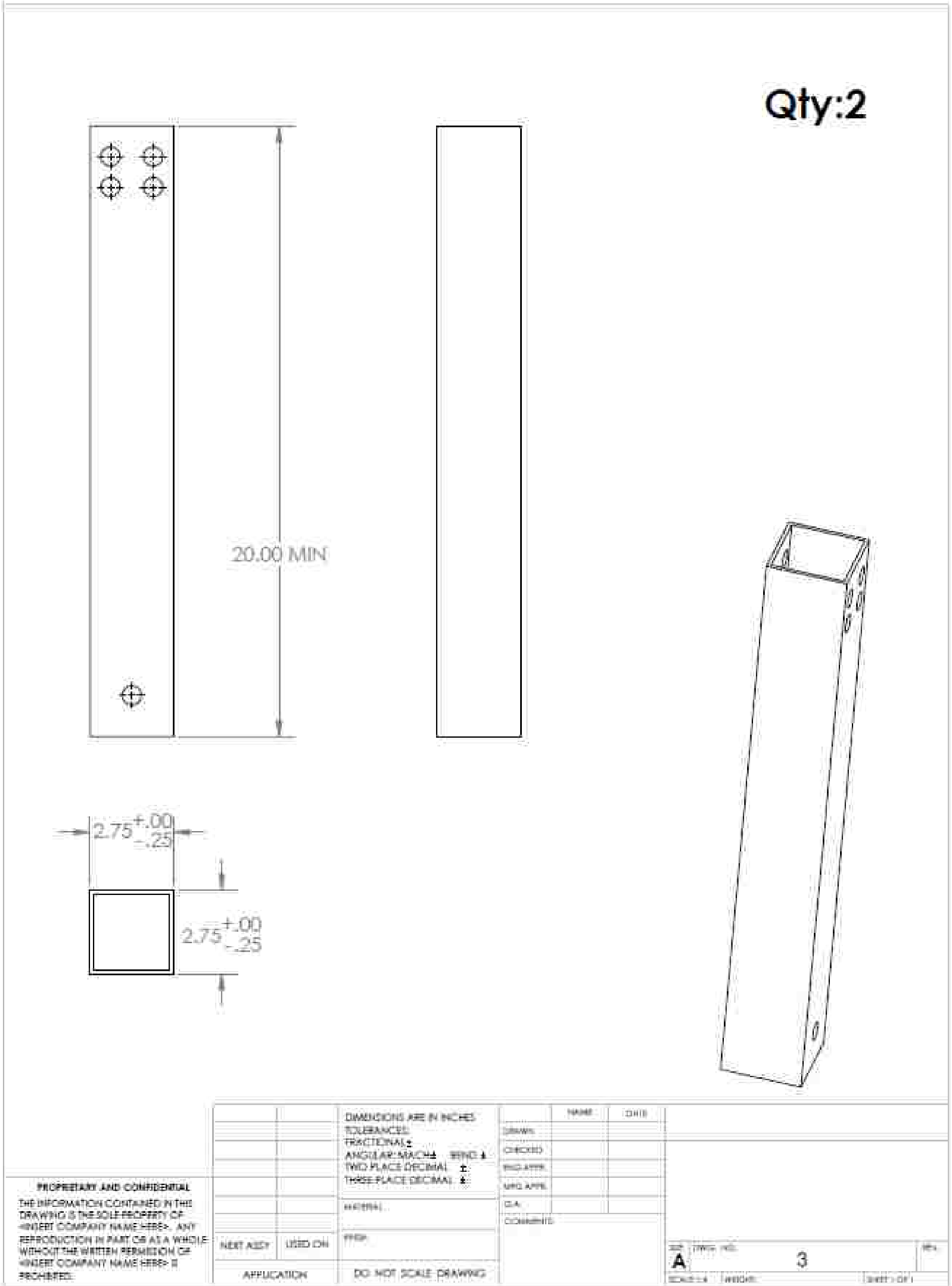
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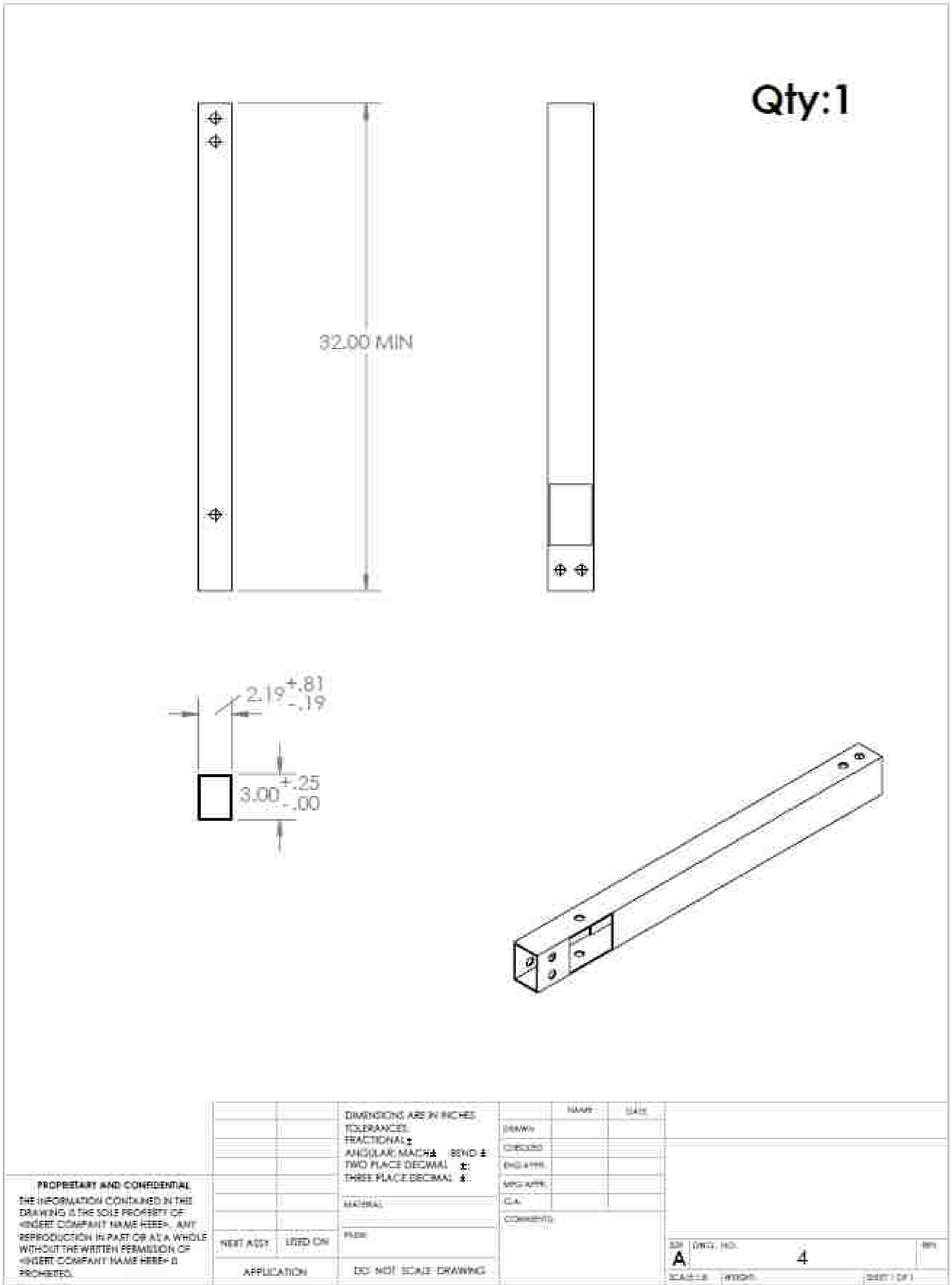
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3.00 +.25 -.00
DIMENSIONS ARE IN INCHES
TOLERANCES:
FRACTIONAL±
ANGULAR: MACH± BEND ±
TWO PLACE DECIMAL ±
THREE PLACE DECIMAL ±
NAME
DATE
DRAWN
CHECKED
ENG APPR.
MFG APPR.
Q.A.
COMMENTS:
MATERIAL
FINISH
NEXT ASSY
USED ON
APPLICATION
DO NOT SCALE DRAWING
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THE INFORMATION CONTAINED IN THIS DRAWING IS THE SOLE PROPERTY OF <INSERT COMPANY NAME HERE>. ANY REPRODUCTION IN PART OR AS A WHOLE WITHOUT THE WRITTEN PERMISSION OF <INSERT COMPANY NAME HERE> IS PROHIBITED.
SIZE
A
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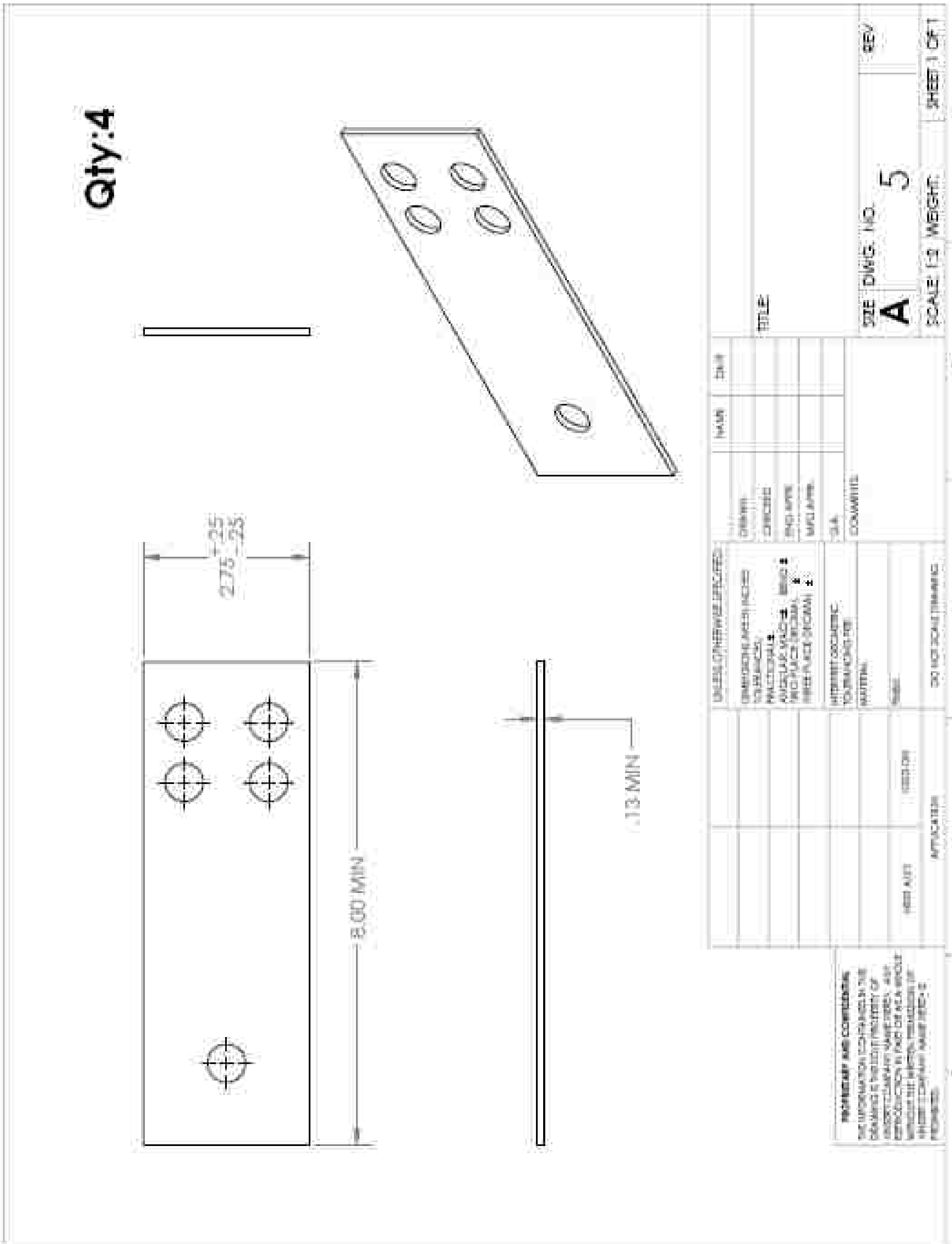
4
REV
SCALE:
WEIGHT:
SHEET 1 OF 1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SIZE
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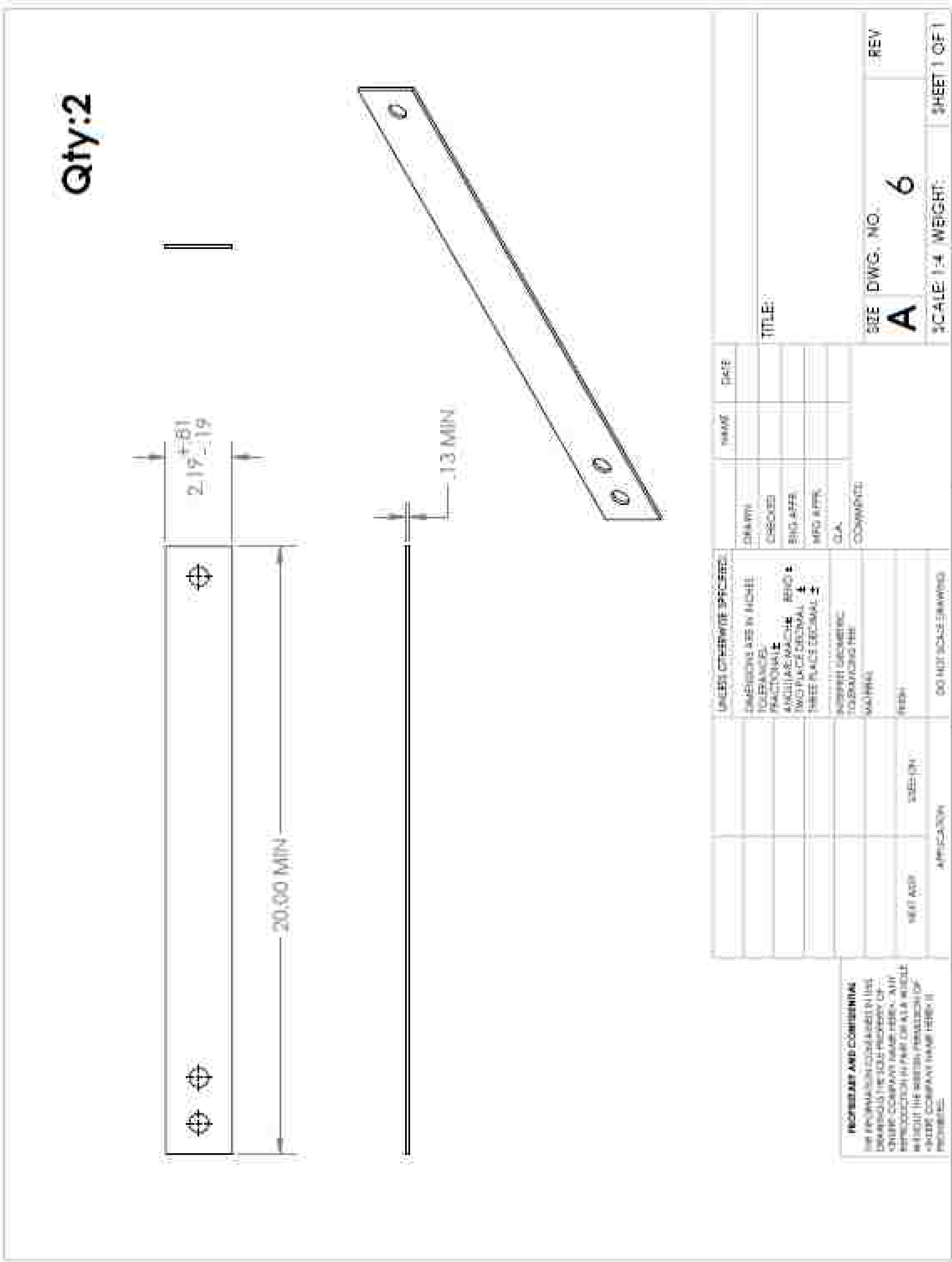
DWG. NO.
5
REV
SCALE 1:2
WEIGHT:
SHEET 1 OF 1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2.19 +.81 -.19
.13 MIN
UNLESS OTHERWISE SPECIFIED
NAME
DATE
DRAWN
CHECKED
ENG APPR.
MFG APPR.
Q.A.
COMMENTS:
TITLE:
SIZE
A
DWG. NO.
6
REV
SCALE: 1:4
WEIGHT:
SHEET 1 OF 1
NEXT ASSY
USED ON
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VITA

The author was born in Heilongjiang Province, China on February 27th, 1990 to Lianke He and Yuping Wang. He attended Shanghai Jiao Tong University (SJTU), Shanghai, China, in September, 2008. He earned his Bachelor of Science degree in Mechanical Engineering from SJTU in June, 2012. Then he attended Lehigh University pursuing a Master of Science degree supervised by Professor Meng-Sang Chew. The author was especially interested in research area related with robotics, system controls and high-precision manufacturing design. During his study at Lehigh University, he also served as a Teaching Assistant in the Department of Mechanical Engineering and Mechanics.