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Model Predictive Control of Thermal Dynamics in Building

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Abstract

Energy requirements for heating and cooling of residential, commercial and industrial spaces constitute a major fraction of end-use energy consumption. In the United States, there are almost 81 million buildings, which consume 71% of the national electricity energy usage, 41% of the total national energy usage, and are responsible for 38% of national CO_2 emissions. Heating, Ventilation, and Air Conditioning (HVAC) uses almost 50% of the energy in these buildings. Modern Control schemes are becoming increasingly popular to meet the energy savings requirements for buildings.

This thesis presents a ventilation control strategy for multi-zone VAV air-conditioning systems based on Model Predictive Control (MPC). MPC is one of the most suitable modern control solutions for keeping thermal comfort, in buildings, with minimal energy consumption. This is due, in part, to the fact that it can handle multi-input and multi-output system and constraints as well as added in input or output.

Due to the fact that MPC relies on a dynamic model, the first step in the process is obtaining a thermal dynamic model for the to-be-controlled building. The model is based on three heat transfer processes: conduction, convection and radiation. The model also captures the heat generation inside the building. Finally, the model is written in form, which is more convenient for control design.

The proposed model is then used to design predictive control strategies. MPC can handle a multiple control objective, inherent complexity due to the coupled and multivariable nature of the problem, and the presence of constraints. In MPC, the control input signal is calculated by solving an optimal control problem, where a cost function is minimized over a given horizon. At last, we can compare the energy consumption of the MPC HVAC control scheme with the traditional two position on/off HVAC control scheme.

Chapter 1

Introduction

1.1 Motivation

Throughout the last few decades, people have been using more and more energy in buildings, including lighting and HVAC system. Since buildings account for around 41% of the total energy consumption in the Unites States (Figure 1.1), and contribute more than one-third of greenhouse emissions, the building sector presents significant opportunities for meaningfully impact in the global energy and emissions scenario. In this context, it is important to note that space heating and cooling account for around one-fourth to one-third of the total energy consumption in commercial and residential buildings. This highlights the need for the development of energy efficient HVAC technologies for buildings as a crucial step in the move toward achieving the energy, environmental and sustainability objectives set by governments throughout the world.

The problem of efficiency enhancement of building HVAC systems is inherently multidisciplinary and presents divers opportunities from a research perspective in several different areas of technology such as design, architecture, alternative energy, modeling and control design. In this regard, the opportunities offered by the field

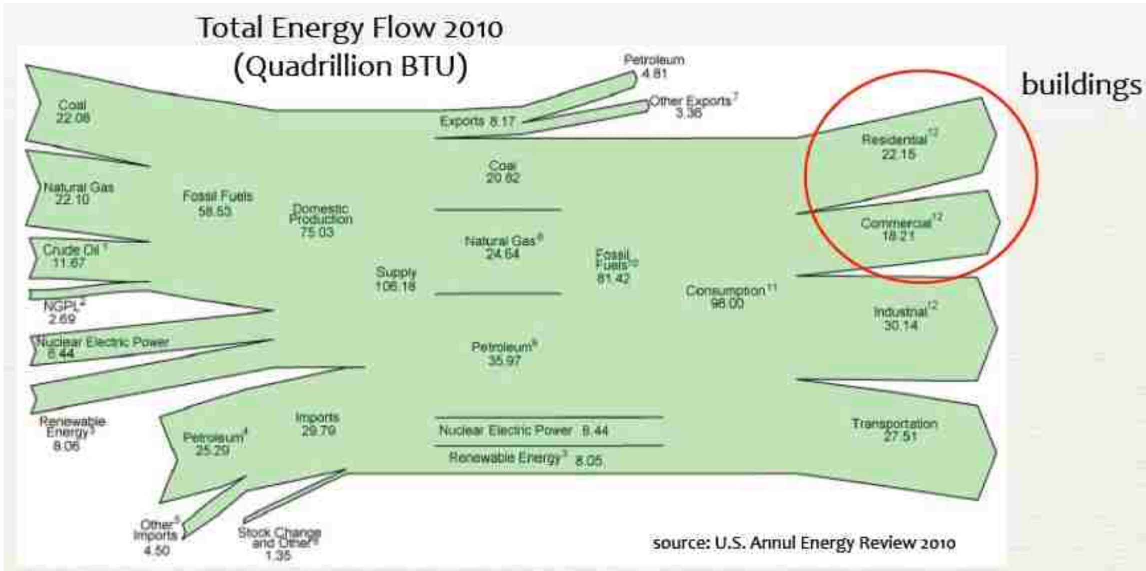


Figure 1.1: Total Energy Flow 2010 (Quadrillion BTU).

of controls engineering are particularly important, because its applicability is not limited to new and upcoming building technologies. Controls also have a significant retrofit potential in the sense that they can be successfully applied to improve both the efficiency and performance of older, existing HVAC systems equipment [1].

In this work, choose MPC as the control scheme, which is quite popular in industry and chemical engineering. The benefit of MPC use in HVAC is the ability to treat constraints on input or output signals in Multi-input, Multi-output (MIMO) systems. This feature is particularly important in building thermal control.

1.2 Research Objective

The primary objective of this research is to minimize the energy usage in buildings while also keeping the building thermally comfortable in occupation periods.

This research objective can be broken down into a set of three distinct objectives, each of them is briefly described below.

1.2.1 Heat Transfer and Modeling Development

Chapter 2 introduces the heat transfer method used in modeling and the heat gain generation inside the building. Due to building thermal model is a continuous dynamical model and not convenient for directly using in MPC process, converts the building model into discrete state space model firstly, and then using the discrete time state space model in the future MPC strategies.

1.2.2 Duct System Design

Based on the different heating or cooling loads in each room, calculate the required air flow rate and dynamic resistance in each room. Design the duct system and compare the constant supply flow rate duct system with the constant pressure duct system.

1.2.3 Traditional On/Off control

Analyze the traditional two position On/Off control in this building dynamic model in different HVAC systems.

1.2.4 Model Predictive Control

At last, converts the building dynamic model to discrete MPC state space model, and defines the cost function. Using optimal control to minimize the cost function, finds the input signal. This chapter also introduces the constraints added in input, saturation constraints and rate limit constraints. In chapter 5, introduces the concept about occupation sequence. Analyze the simulation results and compare with traditional On/Off control.

Chapter 2

Building Model

If a temperature gradient exists, energy will move from a higher-temperature region to a lower-temperature region by either conduction, convection, or radiation.

2.1 Basic Heat Transfer

The following section describes the fundamental law of heat transfer and the network node analysis approach that is used to model the building. Section 2.1 was taken in its entirety from references [2] and [3].

2.1.1 Heat Conduction

In heat transfer, conduction (or heat conduction) is a mode of transfer of energy inside the material of matter, due to a temperature difference. Heat transfer occurs through the wall in a building from the warmer side to the cooler. It is from inside to outside in winter, and from outside to inside in summer. Compared with other heat transfer methods, conduction is the only way energy can be transferred through a solid. It may best be illustrated by the simple, idealized situation shown in Figure 2.1.

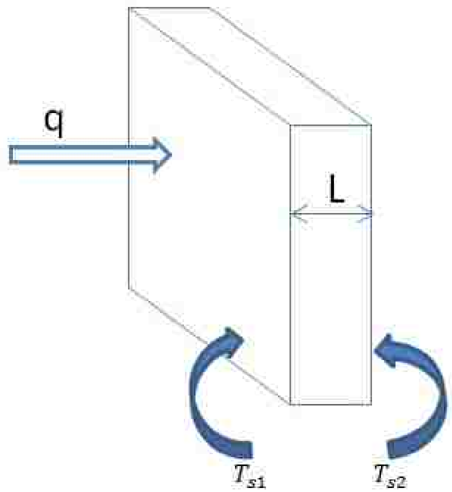


Figure 2.1: Heat Conduction in the Slab.

$$q \propto A_w \frac{T_{s1} - T_{s2}}{L},$$

where $\propto$ means "proportional to", L is the wall thickness, A_w is the area of a plate or walls, T_{s1} and T_{s2} are the temperatures on two sides of the plate or wall, and q is the heat flux rate. However, this relation does not take the wall material into account; if the wall were foam instead of concrete, q would clearly be less. The constant of proportionality is a material property, called thermal conductivity k. Thus,

$$q = k \frac{(T_{s1} - T_{s2})A_w}{L} = \frac{(T_{s1} - T_{s2})}{L/(kA_w)}, \tag{2.1}$$

where k has units of $W/(m \cdot K)$. The denominator $L/(kA_w)$ can be considered the conduction resistance associated with the driving potential $(T_{s1} - T_{s2})$. Equation (2.1) is similar to the electrical current flow through an electrical resistance, $I = (V_1 - V_2)/R$, where $(V_1 - V_2)$ is driving potential, R is electrical resistance, and electrical current I is the rate of flow of charge instead of rate of heat transfer. Thermal resistance has units K/W. The thermal/electrical resistance analogy allows tools used to solve electrical circuits to be used for heat transfer problems. In this project, the thermal/electrical resistance analogy is widely used in the building thermal dynamic model [3].

2.1.2 Heat Convection

Convection is the concerted, collective movement of ensembles of molecules within fluids (i.e. liquids, gases) and rheids. Convection of mass cannot take place in solids, since neither bulk current flows nor significant diffusion can take place in solids.

A simple, idealized situation is shown in Figure 2.2. Let us consider a surface at temperature T_s in contact with a fluid or gas at T_f. Newton's law of cooling expresses the rate of heat transfer from the surface area A_s as

$$q = h_c A_s (T_s - T_f) = \frac{T_s - T_f}{1/h_c A_s}, \tag{2.2}$$

where h_c is the heat transfer coefficient (value shown in Table 1) and has units of $W/(m^2 \cdot K)$. Hence, the convection resistance $1/(h_c A_s)$ has units of K/W. Convection resistance has the same units as conduction resistance. Using this method, analysis of the heat flux transfer in the wall is more convenient. Similar to conduction, the direction of heat transfer is based on the temperature gradient. If $T_f > T_s$, heat transfers from fluid to the surface, and q is written as just $q = h_c A_s (T_s - T_f)$. Resistance is the same, but the sign of the temperature difference is reversed.

For heat transfer to be considered convection, fluid in contact with the surface must be in motion; if not, the mode of heat transfer is conduction. If fluid motion is caused by an external force (e.g., fan, pump, wind), it is forced convection. If fluid motion results from buoying forces caused by the surface being warmer or cooler, it is free (or natural) convection. In general, heat convection that occurs around the wall is free (or natural convection). Hence the heat transfer coefficient h_c should be

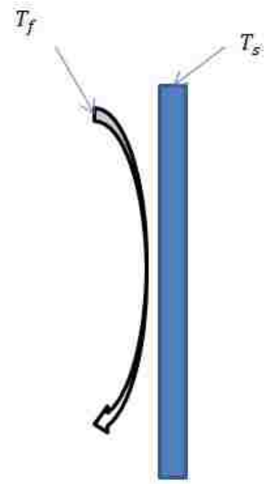


Figure 2.2: Heat Convection.

Table 2.1: Heat Transfer Coefficients by Convection Type

Convection Type	$h_c, W/(m^2 \cdot K)$
Free, gases	2 to 25
Free, liquids	10 to 1000
Forced, gases	25 to 250
Forced, liquids	50 to 20 000
Boiling, condensation	2500 to 20 000

chosen from 2 to 25.

2.1.3 Radiation

Matter emits thermal radiation at its surface when its temperature is above absolute zero. This radiation is in the form of photons of varying frequency. These photons leaving the surface need no medium to transport them, unlike conduction and convection (in which heat transfer occurs through matter). The rate of thermal radiant energy emitted by a surface depends on its absolute temperature and its surface characteristics. A surface that absorbs all radiation incident upon it is called a black surface, and emits energy at the maximum possible rate at a given temperature. The heat emission from a black surface is given by the Stefan-Boltzmann law [2] [3]:

$$q_{emitted,black} = A_s \sigma T_s^4,$$

where $E_b = \sigma T_s^4$ is the blackbody emissive power in W/m^2, T_s is absolute surface temperature (unit is Kelvin (K)), σ equals to $5.67 \times 10^{-8}(W/(m^2 \cdot K^4))$ is the Stefan-Boltzmann constant, A_s is the surface area. If a surface is not black, the emission per unit time per unit area is

$$E = \varepsilon \sigma T_s^4,$$

where E is emissive power, and ε is emissivity, where $0 < \varepsilon < 1$. For a black surface, $\varepsilon = 1$.

Nonblack surfaces do not absorb all incident radiation. The absorbed radiation is

$$q_{absorbed} = \alpha A_s G,$$

where absorptivity α is the fraction of incident radiation absorbed, and irradiation G is the rate of radiant energy incident on a surface per unit area of the receiving surface due to emission and reflection from surrounding surfaces. For a black surface, $\alpha = 1$.

A surface's emissivity and absorptivity are often both functions of the wavelength distribution of photons emitted and absorbed, respectively, by the surface. However, in many cases, it is reasonable to assume that both α and ε are independent of wavelength. If so, $\alpha = \varepsilon$ (a gray surface).

Two surfaces at different temperatures that can "see" each other can exchange energy through radiation. The net change rate depends on the surfaces' (1) relative size, (2) relative orientation and shape, (3) temperatures, and (4) emissivity and absorptivity. However, for a small area A_s in a large enclosure at constant temperature t_{surr}, the irradiation on A_s from the surroundings is the blackbody emissive power of the surroundings $E_{b,surr}$. So, if $t_s > t_{surr}$, net heat loss from gray surface A_s in the radiation exchange with the surroundings at T_{surr} is

$$q_{net} = q_{emitted} - q_{absorbed} = \varepsilon A_s E_{bs} - \alpha A_s E_{b,surr} = \varepsilon A_s, \sigma(t_s^4 - t_{sum}^4) \tag{2.3}$$

where $\alpha = \varepsilon$ for the gray surface. If $t_s < t_{surr}$, the expression for q_{net} is the same with the sign reversed, and q_{net} is the net gain by A_s.

Note that q_{net} can be written as

$$q_{net} = \frac{E_{bs} - E_{b,surr}}{1/(\varepsilon A_s)}.$$

In this form, $E_{bs} - E_{b,surr}$ is analogous to the driving potential in an electric circuit, and $1/(\varepsilon A_s)$ is analogous to electrical resistance. This is a convenient analogy when only radiation is being considered, but if convection and radiation both occur at a surface, convection is described by a driving potential based on the difference in the first power of the temperature, whereas radiation is described by the difference in the fourth power of the temperature. In cases like this, it is often useful to express net radiation as

$$q_{net} = h_r A_s(t_s - t_{surr}) = \frac{t_s - t_{surr}}{1/(h_r A_s)} \tag{2.4}$$

where $h_r = \sigma\varepsilon(t_s^2 + t_{surr}^2)(t_s + t_{surr})$ is often called a radiation heat transfer coefficient. The disadvantage of this form is that h_r depends on t_s, which is often the desired result of the calculation. When $t_{surr} = t_f$ in Equation (2.4), the total heat transfer from a surface by convection and radiation combined is

$$q = q_{rad} + q_{conv} = (t_s - t_f)A_s(h_r + h_c),$$

where the temperature difference $(t_s - t_f)$ is in either kelvins or C°, the difference is the same, either can be used. However, absolute temperatures must be used to calculate h_r (absolute temperatures are $K = C^\circ + 273.15$.). Note that h_c and h_r are always positive, and that the direction of q is determined by the sign of $(t_s - t_f)$.

2.2 Electrical Circuits

A useful concept used in heat transfer is the representation of heat transfer using electrical circuits. A electrical circuit is the representation of the resistance to heat flow as though it were an electric resistor. The heat flow is analogous to the current and the thermal resistance is analogous to the electric resistor.

Each wall consists of several layers of materials with different thicknesses. The thicknesses and characteristics of these materials determine the properties of the walls which affect the behavior of the temperature in the room: the wall thermal capacitances and resistances. Each wall can be represented by an RC network in which capacitors represent thermal capacitances and resistors represent thermal resistance of the wall. For the case where there is also heat transfer through different media, the equivalent resistance is the sum of the resistances of the components that make up the composite. In a similar way, in cases where there are different heat transfer modes, the total resistance is the sum of the resistances of the different modes.

Additionally, heat storing elements (mass) can be represented with capacitors, and the temperature can be replaced by electrical potential. For nodal analysis, the expression which must be satisfied at each node, i, takes the form.

$$\sum_i \frac{\delta T_i}{R_{sum}} + q_i + q_{other} = 0, \tag{2.5}$$

where q_i represents the heat flux from air condition and q_{other} represents heat transfer added to the node by means other than surface conduction, convection, and radiation.

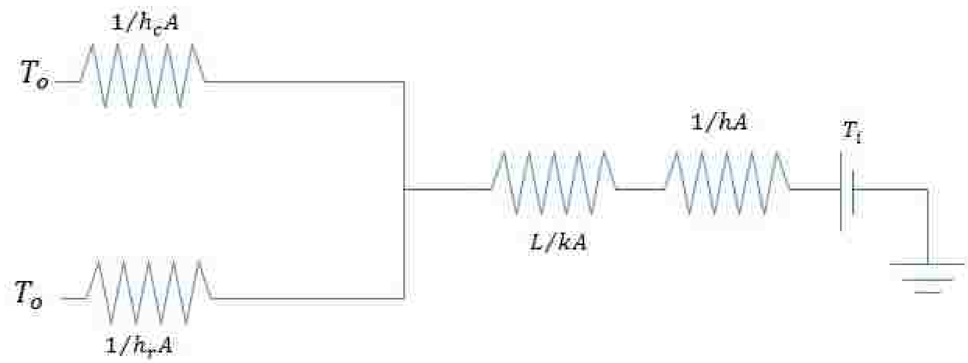


Figure 2.3: Electrical Circuit.

In addition, the internal heat generation also need to be considered. The internal heat generation will be discussed in section 2.3.

For conduction, the resistance is

$$R_{cond} = \frac{L}{kA_s}. \tag{2.6}$$

For convection, the resistance is

$$R_{conv} = \frac{1}{hA_s}. \tag{2.7}$$

For radiation, the resistance is

$$R_{radi} = \frac{1}{\varepsilon A_s}. \tag{2.8}$$

Hence, the value of R_{sum} should be

$$R_{sum} = R_{cond} + R_{conv} + R_{radi},$$

R_{cond}, R_{conv}, and R_{radi} have already been introduced in section 2.1. An example of such network is given in Figure 2.3.

In this project, we separate the heat transfer process through the walls, roofs,

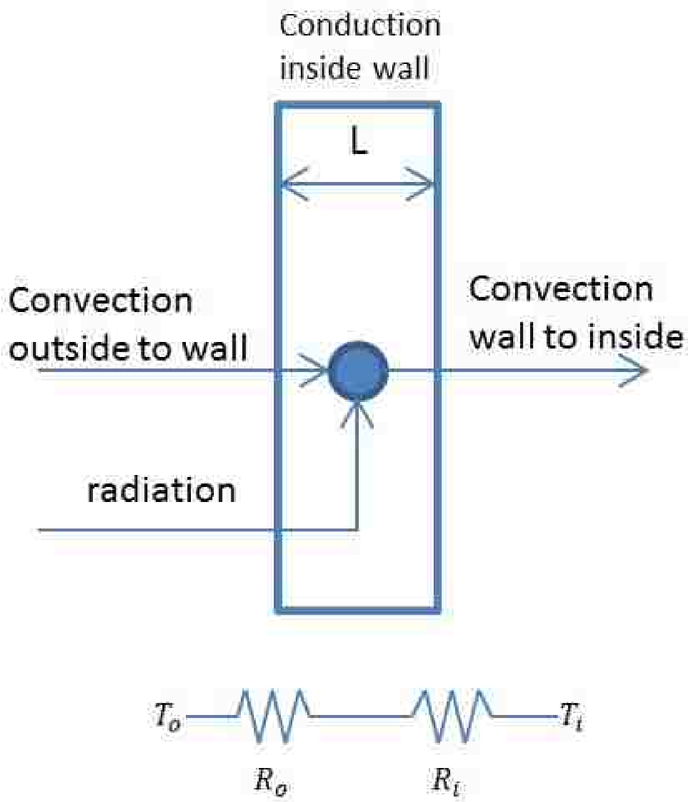


Figure 2.4: Electrical Circuit for the Wall.

floors, and windows into two parts (Figure 2.4). The first part is from outside the room to the middle of the wall, the other part is from the middle of the wall to inside the room. Hence these two parts' resistances are equal to

$$R_o = R_{conv1} + \frac{R_{cond}}{2} + R_{radi} \tag{2.9}$$

and

$$R_i = R_{conv2} + \frac{R_{cond}}{2}. \tag{2.10}$$

2.3 Other Heat Generation

The design cooling load (or heat gain) is the amount of heat energy to be removed from a house by the HVAC equipment to maintain the house at the indoor design temperature when the worst case outdoor design temperature is being experienced. There are two types of cooling loads: sensible cooling load and latent cooling load. This subsection was taken in its entirety from reference [4] .

The sensible cooling load refers to the dry bulb temperature of the building and the latent cooling load refers to the wet bulb temperature of the building. For HVAC equipment selection, sensible cooling load must be calculated firstly.

Factors that influence the sensible cooling load:

1. Glass windows or doors
2. Exterior walls
3. Roofs
4. Ceilings under an attic
5. Sunlight striking windows, skylights, or glass doors and heating the room
6. Air infiltration through cracks in the building, doors, and windows
7. People in the building
8. Equipment and appliances operated in the summer
9. Lights

From the list above, the heat generation for items 1 to 4 can be calculated from basic heat transfer theory, which we introduced in section 2.1 and section 2.2.

2.3.1 Sunlight

Solar radiation through vertical and horizontal single glass windows at 60 degrees north are indicated below,

Direction	Time (h)																
	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	Heat Radiation (W/m2)																
N	1	135	87	61	69	80	87	92	93	92	87	80	69	61	87	135	1
NE	2	366	508	471	333	154	92	92	93	92	92	154	333	471	508	366	2
E	1	371	621	718	706	610	447	232	100	232	447	610	706	718	621	371	1
SE	0	149	367	546	658	708	689	605	461	605	689	708	658	546	367	149	0
S	0	22	44	73	212	390	532	619	649	619	532	390	212	73	44	22	0
SW	0	22	42	57	69	82	105	266	461	266	105	82	69	57	42	22	0
W	0	22	42	57	69	80	87	97	100	97	87	80	69	57	42	22	0
NW	0	22	47	57	69	80	87	92	93	92	87	80	69	57	47	22	0
Horizontal	0	51	148	272	395	498	576	625	641	625	576	498	395	272	148	51	0

Figure 2.5: Sunlight Irradiance Rate to The Building from Eight Directions in 24 hours.

$$P_S = x_g A J, \tag{2.11}$$

where x_g is the fenestration structure factor, A is the fenestration area (including frame), and J is the irradiance rate, a chosen value from Figure 2.5.

2.3.2 Heat Gain from People in the Building

Cooling load associated with human occupation is calculated as

$$P_P = n X_T^u, \tag{2.12}$$

where n is the number of indoor people and X_T^u is the average metabolic rate, measured in Watt (W). These factors can be found in the 2009 ASHRAE Handbook. In this work, X_T^u is chosen equal to 130 W, which is the general value for an adult when room dry bulb temperature is 20°C.

2.3.3 Heat Gain from Equipment and Appliances Operated

Cooling load associated with electric equipment is calculated as

$$P_E = 1000 n_1 n_2 \frac{N}{\eta}, \tag{2.13}$$

where n_1 is the installation coefficient, whose value is usually chosen between 0.7 to 0.9, n_2 is the coincidence factor, whose value is usually chosen between 0.57 to 0.8, N represents installed power, η is the motor efficiency.

2.3.4 Heat Gain from Lights

The heat gained from lights in a modern office, residential room, or production area may be of a significant amount. Heat emitted to the room depends on the preferred light level in the room, type of lights and their construction, and location of the light equipment. The preferred light level in a room depends primarily on the type of activity. For a common residential house, the level may be in the range of 150 lux.

Unless special arrangements for local cooling or air outlets through the lighting equipment are used, the electric power of the light is converted into heat emitted to the room. Required electric power to achieve a recommended light level can be estimated as

$$P_L = \frac{b}{\eta_e \eta_r I_s}, \tag{2.14}$$

where P_L is the installed electric power (unit is W/m^2), b is the recommended light level (lux), η_e is the light equipment efficiency (common range is 50% to 80%), η_r is the room lighting efficiency (common range is 0.3 to 0.6), I_s is the emitted light from the source ($lumen/W$). For fluorescent, I_s can be chosen from 50 to 90 ($lumen/W$).

2.4 Building Structure

In this project, we use a normal house's structure in Bethlehem, PA, as the basic model. This residence has 9 rooms (including kitchen, living room, bathrooms and bedrooms). Figure 2.6 shows the simple structure of this residential house, with three floors, and underground basement. Due to the absence of air conditioning in the basement, we do not need to consider it in this case. The first floor is composed of kitchen (room No.8), dining room (room No.7), and living room (room No.6). The second floor has two bedrooms (room No.3 and room No.4) and a bathroom (room No.5). Because the bathroom is very small and the external wall of the bathroom is small, we assume there is no air flow into the bathroom. The third floor has two bedrooms (room No.1 and room No.2). Figure 2.6 shows the basic residence structure.

2.5 Building RC Model

Recall that we have already discussed the electrical circuits principle. Room resistance model can easily be described by the figure shown in Figure 2.7. The room can be thought of as a network of first-order systems, where the nodes are the system states and these represent the room temperature or the temperatures in the wall, floor or ceiling.

Based on the RC method, we can simplify the one-room resistance model (Figure 2.7) and also the whole house structure. Figure 2.8 is the illustration of the building thermal model, based on the building structure (Figure 2.6). In these figures, each room (or zone) has a thermal capacitance, for example $C1$ means the thermal capacitance of room 1 (or zone 1).

2.6 Analytical Model

The inside temperature usually processes with slow dynamics, which can be described by a physical model developed based on the Equation 2.5. Before that, we need to make several assumptions [4].

Since the building is a complex system, a complete theoretical approach is impractical. Therefore the following assumptions are made:

1. Air in the room is fully mixed. The density of the air is assumed to be constant.

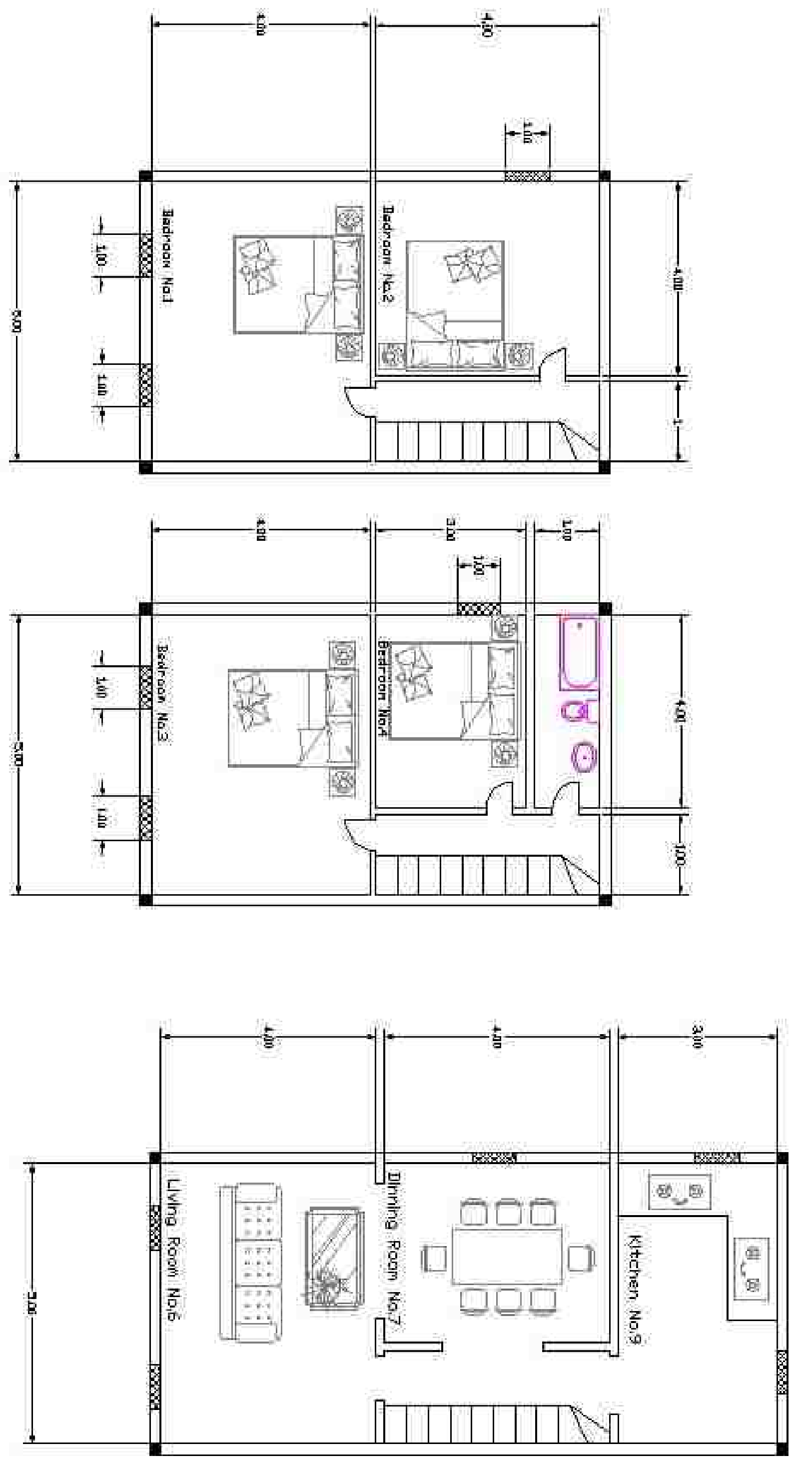


Figure 2.6: Building Structure.

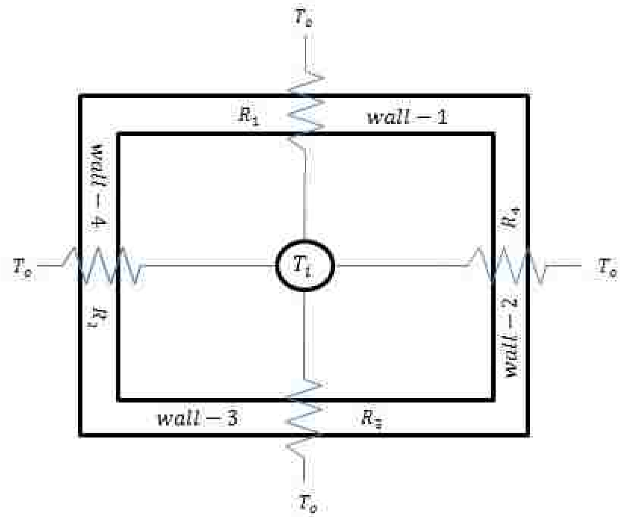


Figure 2.7: Room Resistance Model.

2. The humidity ratio of the room is constant (not influenced by temperature).

3. Each external wall, roof, and glass has the same properties. Each internal structure also has the same properties.

4. Heat gain or loss through usage events, such as door opening and closing, will be considered as negligible.

Therefore, the state variables are the temperatures of the rooms, walls, roofs, floors, and windows. The input variables are the air flow rates into each room. The equation shown below describes each wall temperature sensible heat balance,

$$\frac{dT_w}{dt} = \frac{1}{C_w}\left(\frac{T_o - T_w}{R_{o-w}} + \frac{T_w - T_i}{R_{i-w}}\right), \tag{2.15}$$

where

$$C_w = LA\rho_w C_{pw}, \tag{2.16}$$

where R_{o-w} and R_{i_w} can be found from Equation (2.9) and Equation (2.10), T_o is the outside temperature, T_i is the inside temperature, L is the thickness of the wall, A is

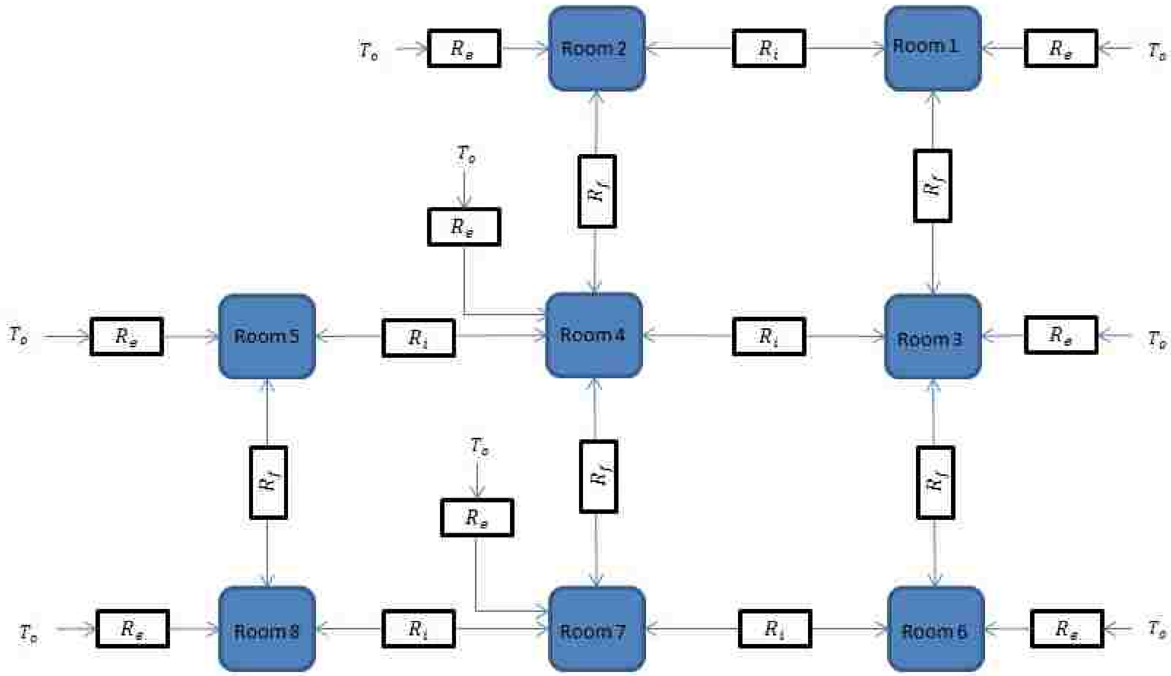


Figure 2.8: Whole Building Resistance Model.

the area of the wall, C_{pw} is the specific heat of the wall, ρ is the density of the wall. Equation (2.15) and Equation (2.16) also can be used for roofs, floors, and windows. Because each room at least has four walls, one roof, and one floor (some rooms in this building model may have windows), we can get the total heat flux to each room by using Equation 2.15, and the equation regarding temperature in each room can be written based on the sensible heat balance as

$$\frac{dT_i}{dt} = \frac{\rho C_a f_i}{C_i}(T_{sup} - T_i) + \sum_{n=1}^{4} \frac{T_{wn} - T_i}{C_i R_{i-wn}} + \frac{T_r - T_i}{C_i R_{i-r}} + \frac{T_f - T_i}{C_i R_{i-f}}, \tag{2.17}$$

where T_i is the temperature of room i, T_{wn} is the temperature of wall n, T_r is the temperature of roof, T_f is the temperature of floor, T_{sup} is temperature of air supply, C_a is the specific heat of air, C_i is the specific heat of room i, f_i is the air supply volume flow rate, R_{i-wn} is the heat resistance of Wall n, R_{i-r} is heat resistance of

Table 2.2: Heat Transfer Coefficients of Building Component

	$\rho(Kg/m^3)$	$k(W/m^2 * K)$	$C(J/Kg * K)$	$Thickness(m)$
External Wall	2200	0.492	800	0.24
Interior Wall	1000	1.02	400	0.18
Roof	1358	0.35	600	0.15
Floor	1358	0.35	600	0.15
Glass	2500	2.5	300	0.05
Air	1.205	$None$	1005	$None$

roof, R_{i-f} is heat resistance of floor.

2.7 Parameters

We have the Equation (2.17) to calculate the room temperatures. Before we simulate the building dynamics, we need to choose physical parameters. Table 2.2 shows typical physical parameter for walls, roofs, windows, and air (all of these value are chosen from the 2009 ASHARE Fundamental Handbook).

Chapter 3

Duct system

3.1 Outside Temperature

The outside temperature is constantly changing. In the morning or at night, the outside temperature is lower than the average temperature of the whole day, and at noon, due to sunshine and other reasons, the temperature is higher. Therefore, in our simulations, we cannot simply use constant temperature throughout the day.

We take Allentown's two-day temperature record as our database (July 5 and July 6, 2012). The temperature data is shown in Table 3.1 and Figure 3.1. Interplation is used to get the temperature at each discrete sampling time (2 minutes or 10 minutes).

3.2 Air Flow Calculations

From section 2.3, we know the heat gain in each room not only comes from heat transfer through wall, floor, and roof due to different temperatures outside and inside, but also comes from human occupancy, lighting, window irradiance, and electric equipment. Hence, we need to add all of them into the heat balance Equation (2.5), and then calculate the required air flow rate in each zone [4].

The required air flow rate for each zone in the HVAC system can be calculated as

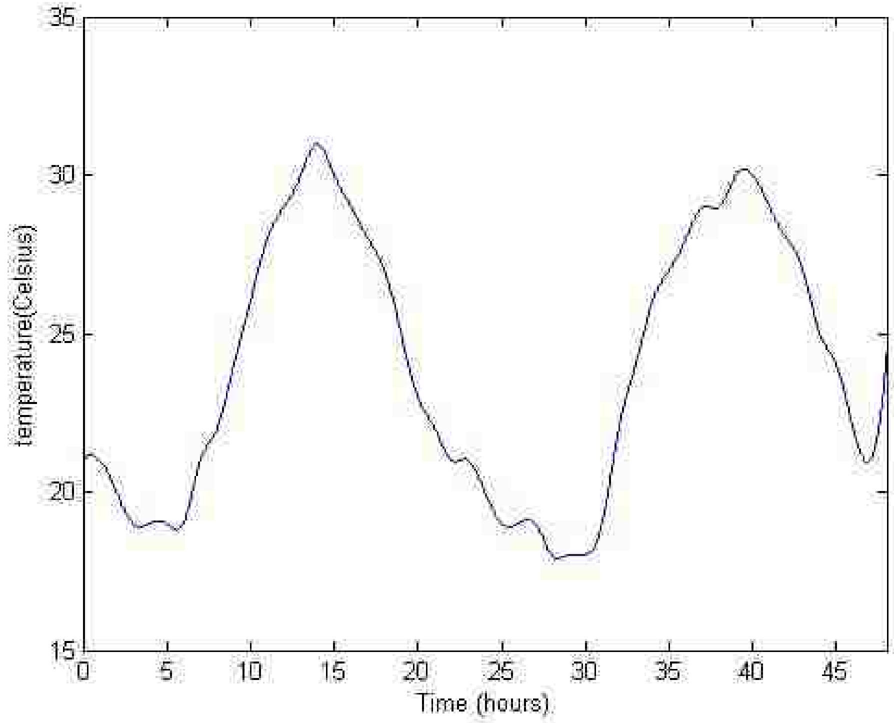


Figure 3.1: Outside Temperature in Two Days, July 5 and July 6 2012.

$$L = \frac{Q}{C_p \rho (T_i - T_o)}, \tag{3.1}$$

where L is the air flow rate (m^3/s), Q is the heat gain (KW), C_p is the specific heat capacity air ($1.005 KJ/Kg \cdot^\circ C$), ρ is density of air 1.2 (Kg/m^3), T_i is the indoor temperature ($^\circ C$), T_o is the supply temperature ($^\circ C$).

Set the indoor temperature $T_i = 20^\circ C$, and choose the maximum outside temperate as $T_o = 31^\circ C$. Calculate each room's required air flow rate.

Take two rooms as examples, the first being the bedroom in the third floor (bedroom 1). From Figure 2.6, we can see that the area for bedroom 1 is $20m^2$ ($A_1 = 20m^2$) and the height is $3m$ ($h = 3m$), and total volume is $60m^3$ ($V_1 = 60m^3$). Set the indoor temperature as $20^\circ C$ and outside temperature as $31^\circ C$. Calculate the maximum heat gain Q in bedroom 1. Another room located on the first floor is the living room. The area is $16m^2$ ($A_6 = 16m^2$), the height is $3m$ ($h = 3m$), and the total volume is

Table 3.1: Outside Temperature in Two Days, July 5 and July 6 2012

Time	$Day1$	$Day2$	Time	$Day1$	$Day2$
12:00 AM	$21°C$	$21°C$	12:00 PM	$28°C$	$28°C$
1:00 AM	$21°C$	$20°C$	1:00 PM	$29°C$	$29°C$
2:00 AM	$21°C$	$19°C$	2:00 PM	$30°C$	$30°C$
3:00 AM	$20°C$	$19°C$	3:00 PM	$31°C$	$31°C$
4:00 AM	$19°C$	$19°C$	4:00 PM	$30°C$	$30°C$
5:00 AM	$19°C$	$18°C$	5:00 PM	$29°C$	$29°C$
6:00 AM	$19°C$	$18°C$	6:00 PM	$28°C$	$28°C$
7:00 AM	$19°C$	$18°C$	7:00 PM	$27°C$	$27°C$
8:00 AM	$21°C$	$19°C$	8:00 PM	$25°C$	$25°C$
9:00 AM	$22°C$	$22°C$	9:00 PM	$25°C$	$25°C$
10:00 AM	$24°C$	$24°C$	10:00 PM	$22°C$	$24°C$
11:00 AM	$26°C$	$26°C$	11:00 PM	$21°C$	$22°C$

$48m^3$ ($V_6 = 48m^3$). Therefore, the total heat gain in each room can be calculated by Equation (3.2).

$$Q = P_L + P_E + P_p + P_s + Q_{rad} + Q_{conv} + Q_{cond}, \tag{3.2}$$

where Q_{cond} is the heat gain from heat conduction (KW), Q_{conv} is the heat gain from heat convection (KW), Q_{rad} is the heat gain from heat radiation (KW). P_L is the heat gain from light (KW), P_E is the heat gain from electrical equipment (KW), P_p is the heat gain from people (KW), P_s is the heat gain from sunlight (KW).

Using Equation (3.1) and (3.2), we can get the required air flow rate L in each room, shown in Table 3.2.

3.3 Duct Design

3.3.1 Duct Design Procedure

Duct design for residential or even commercial and industrial buildings should consider space air diffusion, noise levels, air distribution system, duct heat gains and

Table 3.2: Required Air Flow Rate

Room	KW	Air Flow (m^3/h)	Air Flow (m^3/s)	CFM	$Design\ Value$
No.1-Bedroom 1	1740	538	0.1443	306	300
No.2-Bedroom 2	1200	358	0.0995	211	250
No.3-Bedroom 3	1400	418	0.1161	246	250
No.4-Bedroom 4	750	224	0.0622	132	200
No.6-Living room 6	1800	538	0.1493	316	350
No.7-Dining room 7	1000	298	0.0829	176	200
No.8-Kitchen 8	1200	358	0.0995	211	250

losses, balancing, and fire and smoke control. If there are deficiencies in the duct system, it may cause the system to operate incorrectly or increase the expense to own and operate. Poor design or lack of system sealing can produce inadequate airflow rates at each terminal [3] [4].

In the HVAC duct system design, calculating dynamic resistance to duct system is also necessary, as well as calculating a required air flow rate for each zone .

The general procedure for HVAC duct system design should be as follows:

1. Arrange supply and return outlets to provide proper distribution of air in each space. Adjust calculated air quantities for duct heat gains.

2. Select outlet sizes from manufactures' data.

3. Sketch the duct system, connecting supply outlets and return intakes with AHU (air-handling units/air conditioners).

4. Divide the system into sections and number each section.

5. Size ducts by the selected design method. Usually, there are three methods: Equal-Friction Method, Static Regain Method, and T-Method. In this project, the Equal-Friction method has been used as the design method.

6. Lay out the system in detail.

7. Resize duct sections to approximately balance pressures at each junction.

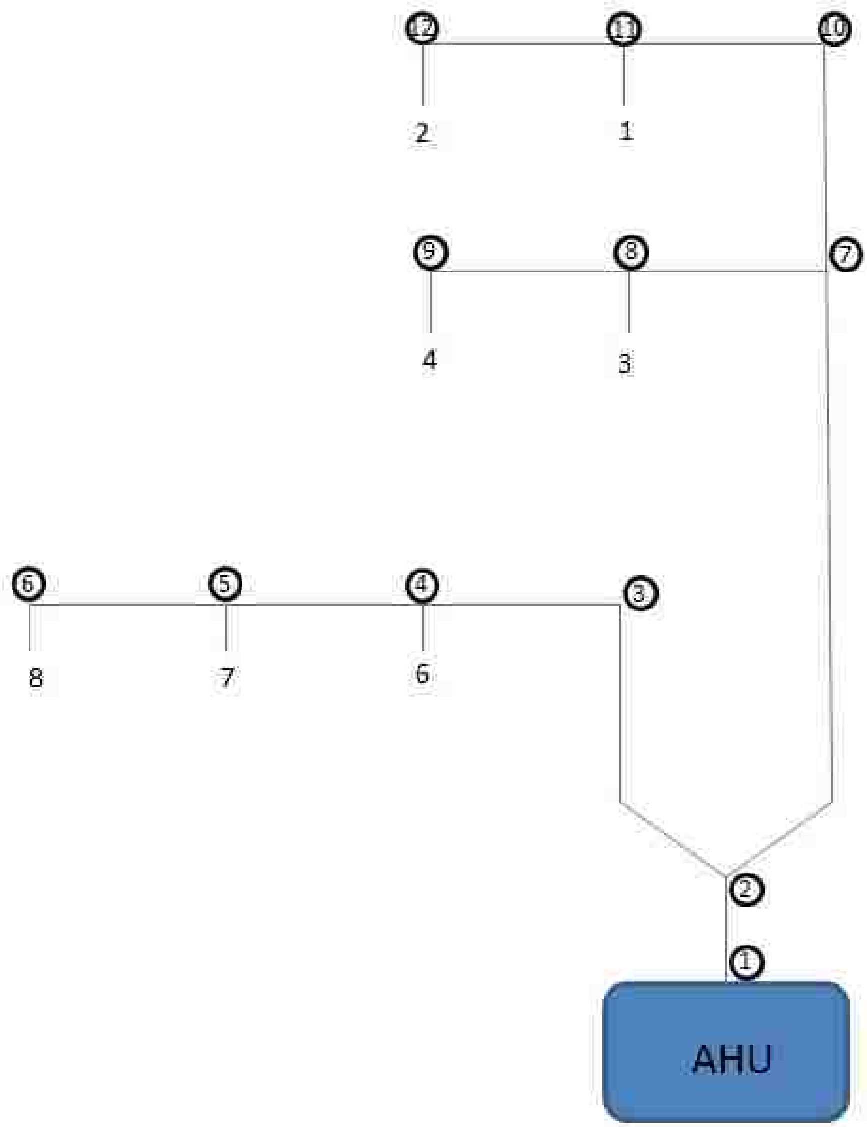


Figure 3.2: Duct Sketch.

3.3.2 Duct Pressure Drop

Duct system losses are the irreversible transformation of mechanical energy into heat. The two main types of losses are friction losses, dynamic losses, and thermal gravity effect [4].

Friction Losses

Friction losses are due to fluid viscosity and result from momentum exchange between molecules or between individual particles of adjacent fluid layers moving at different velocities. Friction losses occur along the entire duct length.

For fluid flows in conduits, the friction loss can be calculate by the Darcy equation:

$$\Delta P_f = \frac{1000fL}{D_h} \times \frac{\rho V^2}{2}, \tag{3.3}$$

where ΔP_f is the friction losses in terms of total pressure (Pa), f is the friction factor (dimensionless), L is the duct length (m), D_h is the hydraulic diameter (m). V velocity (m/s), ρ is the density (kg/m^3).

and the friction factor is given by

$$\frac{1}{\sqrt{f}} = -2log(\frac{\varepsilon}{3.7D_h} + \frac{2.51}{R_e\sqrt{f}}), \tag{3.4}$$

where ε is the material absolute roughness factor, R_e is the Reynolds number. Based on Darcy Equation (3.3) and Equation (3.4), the fluid resistance caused by friction in round ducts can be determined from the friction chart (Figure 3.3).

For a rectangular duct, the relationship between rectangular and round ducts is used to determine size equivalency based on equal flow, resistance, and length. This relationship equation was developed by Huebscher in 1948,

$$D_e = \frac{1.30(ab)^{0.625}}{(a+b)^{0.250}}, \tag{3.5}$$

where D_e is the circular equivalent of rectangular duct for equal length fluid resistance and air flow rate, a is the length, one side of duct (m). b is the length, adjacent side of duct (m).

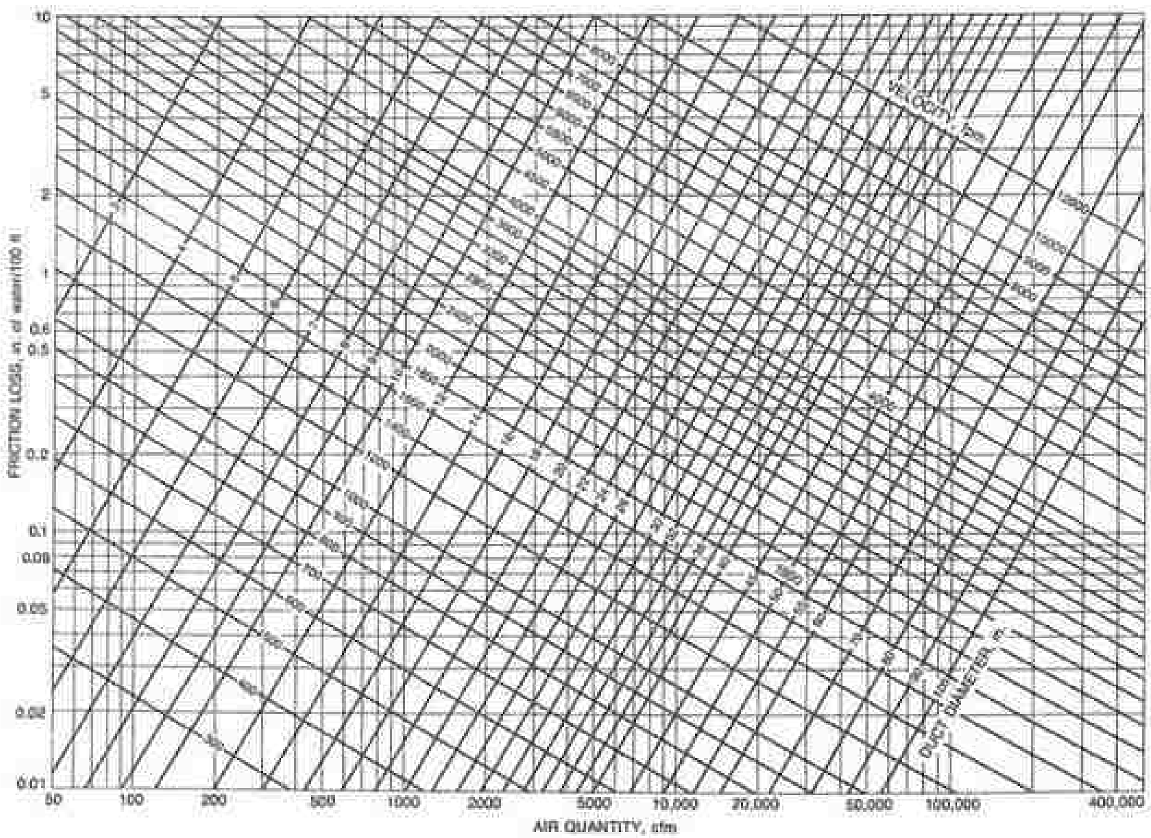


Figure 3.3: Friction Chart, the 1997 ASHRAE Handbook.

Dynamic Losses

Dynamic losses result from flow disturbances caused by duct mounted equipment and fittings that change the airflow path's direction and area [4]. The dimensionless coefficient C is used for fluid resistance. The fluid resistance coefficient represents the ratio of total pressure loss to velocity pressure at the referenced cross section

$$C = \frac{\Delta P_j}{\rho(V^2/2)} = \frac{\Delta P_j}{P_v}, \tag{3.6}$$

where C is the local loss coefficient (dimensionless), ΔP_j is the total pressure loss (Pa), ρ is the density (Kg/m^3), V is the velocity (m/s), P_v is the velocity pressure (Pa).

The local loss coefficients can be based on the duct mounted equipment and fittings (entries, exits, elbows, transition, and junctions), and can be found in manufacturer's database.

Thermal gravity effect

The pressure change in the duct is caused by friction, fittings, equipment and also the thermal gravity effect [4]. From Bernoulli equation

$$\frac{\rho_1 V_1^2}{2} + P_1 + g\rho_1 Z_1 = \frac{\rho_2 V_2^2}{2} + P_2 + g\rho_2 Z_2 + \Delta P_{t,1-2}, \tag{3.7}$$

where, V is the streamline velocity (m/s), P is the absolute pressure (Pa), ρ is the density (Kg/m^3), g is the acceleration caused by gravity (m/s^2), Z is the elevation (m). $\Delta P_{t,1-2}$ is the total pressure loss caused by friction and dynamic losses between section 1 and section 2 (Pa).

Atmospheric pressure at any elevation expressed in terms of the atmospheric pressure P_a at the same elevation is given by

$$P_{z1} = P_a - g\rho_a z_1$$

$$P_{z2} = P_a - g\rho_a z_2$$

From Equation (3.7), the thermal gravity effect for each non horizontal duct with a density other than that of ambient air is determined by the following equation:

$$\Delta P_{se} = g(\rho_a - \rho)(z_2 - z_1), \tag{3.8}$$

where ΔP_{se} is the thermal gravity effect (Pa), z_1 and z_2 are the elevation from datum in direction of air flow (m), ρ_a is the density of ambient air (Kg/m^3), ρ is the density of air or gas within duct (Kg/m^3), g equal to 9.81 (m/s^2) (gravitational acceleration).

Hence, the pressure drop in each part of the duct system is shown in Table 3.3,

Table 3.3: Pressure Loss Calculation in Each Duct Section

Section	Air Flow(m^3/s)	Duct Size (mm)	V (m/s)	Length(m)	Pressure Drop
1-2	0.85	500×400	4.25	1	1.0174
2-3	0.38	320×320	3.71	3	5.0498
3-4	0.38	320×320	3.71	2	3.3076
4-5	0.21	250×250	3.84	4	3.2516
5-6	0.12	200×200	3	3	2.6756
2-7	0.47	500×250	3.76	6	6.8672
7-8	0.21	250×250	3.84	2	5.9158
8-9	0.0944	200×160	2.95	4	2.2665
7-10	0.26	320×250	3.25	3	2.7409
10-11	0.26	320×250	3.25	2	2.8174
11-12	0.12	200×200	3	4	3.4756
$Branch_1$	0.17	320×160	3.32	1	3.0010
$Branch_2$	0.12	200×200	3	1	2.7181
$Branch_3$	0.12	200×200	3	1	2.7181
$Branch_4$	0.0944	200×160	2.95	1	1.8326
$Branch_6$	0.17	320×160	3.32	1	3.9756
$Branch_7$	0.0944	200×160	2.95	1	3.1651
$Branch_8$	0.21	200×200	3	1	2.1781

based on the duct sketch Figure 3.2.

3.3.3 Duct RC network

The unit for pressure is Pa, and the dimensionless unit is $ML^{-1}T^{-2}$. The unit for resistance is Ω, and the dimensionless unit is $L^2MT^3I^2$, where, M is mass (Kg), T is time ($seconds$), L is length (m), and I is electric current (A).

Hence, using the pressure drop in each duct part, we can transform them to RC network by multiplying $L^3T^5I^2$ ($L^2MT^3I^2/ML^{-1}T^{-2}$) with the pressure drop. Consider the air pressure drop in each branch as electric resistance, and the power coming from AHU as the voltage, and the air flow in each branch as the electric current.

The damper is composed of a valve or plate that stops or regulates the flow of air inside each branch (Figure 3.5). It is also used to cut off air flow to an unused room, or regulate it for room-by-room temperature. In general, the volume control damper operation can be manual or automatic. In this project, model predictive control will be used for damper . Hence, each of the dampers in branch terminals are automatic dampers, operated by electric or pneumatic motors, in turn controlled by a thermostat or building automation system. More details will be shown in next chapters [4].

In each branch, if the volume control damper is partially closed, the resistance in the terminal section is increased. If the duct system is a constant current HVAC system (air flow rate is a constant value in HVAC system), the total "Voltage" is changed due to the resistance increased, and the air flow rate through this damper will decrease, however, other terminal's air flow rate will increase, hence the total air flow rate will maintain in a constant value. If the duct system is a constant pressure HVAC system (Pressure is a constant value in HVAC system), the total "Voltage" is not changed, and the air flow rate through this damper will decrease due to the resistance increased, the total air flow rate will also decrease due to the total resistance increased. In these ways, a damper can modulate the flow of conditioned air in order to keep the indoor temperature in a comfortable range.

Table 3.4 shows the resistance in each duct section. Based on the resistance in each section, an air flow simulation Matlab program was built to analyze air flow in each branch and terminal. It can simulate the air flow to each terminal in the duct, when the volume control dampers are in different positions.

At last, the duct system resistance can be illustrated by the simple, idealized situation shown in Figure 3.5. AHU supply the air flow into the duct system. It can be the constant current HVAC system or constant pressure HVAC system. If the dampers are totally open, the air flow rate into each room is shown in Table 3.5.

Table 3.4: Duct Section Pressure Drop and Resistance

Section	Pressure Drop	Resistance	Section	Pressure Drop	Resistance
1-2	1.0174	1	10-11	2.8174	2.8
2-3	5.0498	5	11-12	3.4756	3.4
3-4	3.3076	3.3	$Branch_1$	3.0010	3
4-5	3.2516	3.2	$Branch_2$	2.7181	2.7
5-6	2.6756	2.6	$Branch_3$	2.7181	2.7
2-7	6.8672	6.8	$Branch_4$	1.8326	1.8
7-8	5.9158	5.9	$Branch_6$	3.9756	3.9
8-9	2.2665	2.2	$Branch_7$	3.1651	3.1
7-10	2.7409	2.7	$Branch_8$	2.1781	2.1

Table 3.5: Zones Air Flow Rate

Section	Design Airflow	Actual Air flow
Bedroom 1	300	292
Bedroom 2	250	238
Bedroom 3	250	258
Bedroom 4	200	231
Living room 6	350	335
Dinning room 7	200	179
Kitchen 8	250	265
Total	1800	1800

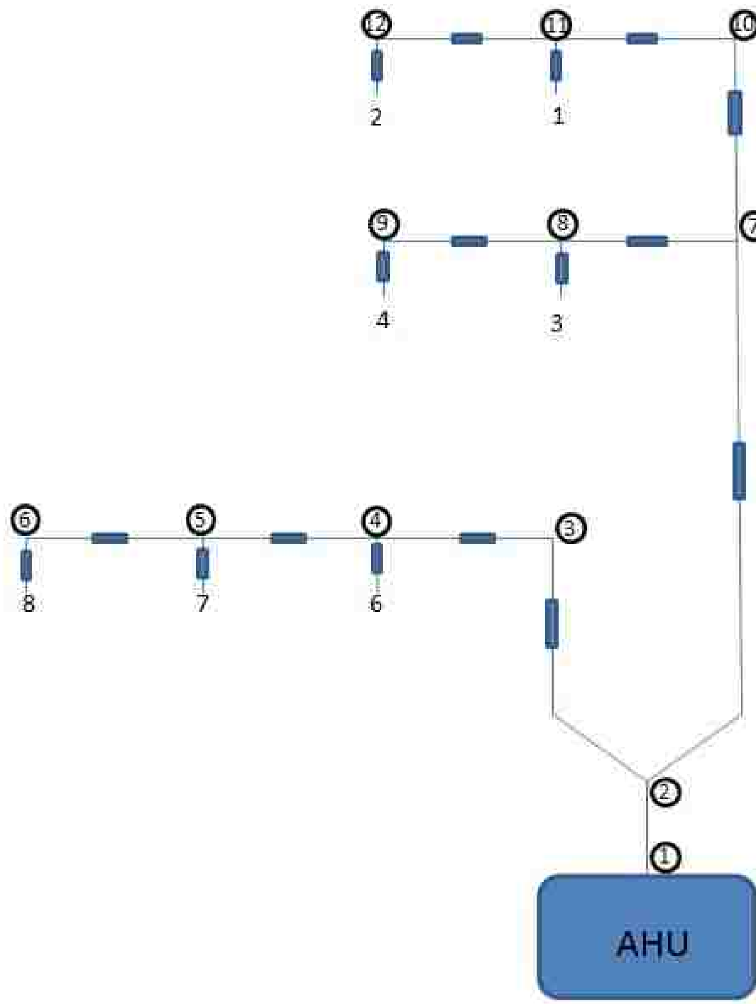


Figure 3.4: Duct System with Resistance.

3.3.4 Building Thermal Analysis

Before we design the controller for the air conditioning system, analysis of the temperature inside the building without air conditioning is necessary. Due to the heat transfer from outside and heat generation which comes from lighting, human activity, electric equipment and sunlight radiation through window, the inside temperature will change constantly. Assuming there is no air conditioning, the indoor temperature for each room will change only due to heat transfer and heat generation inside the building.

Figure 3.6 shows 4 representative rooms' indoor temperatures without air condi-

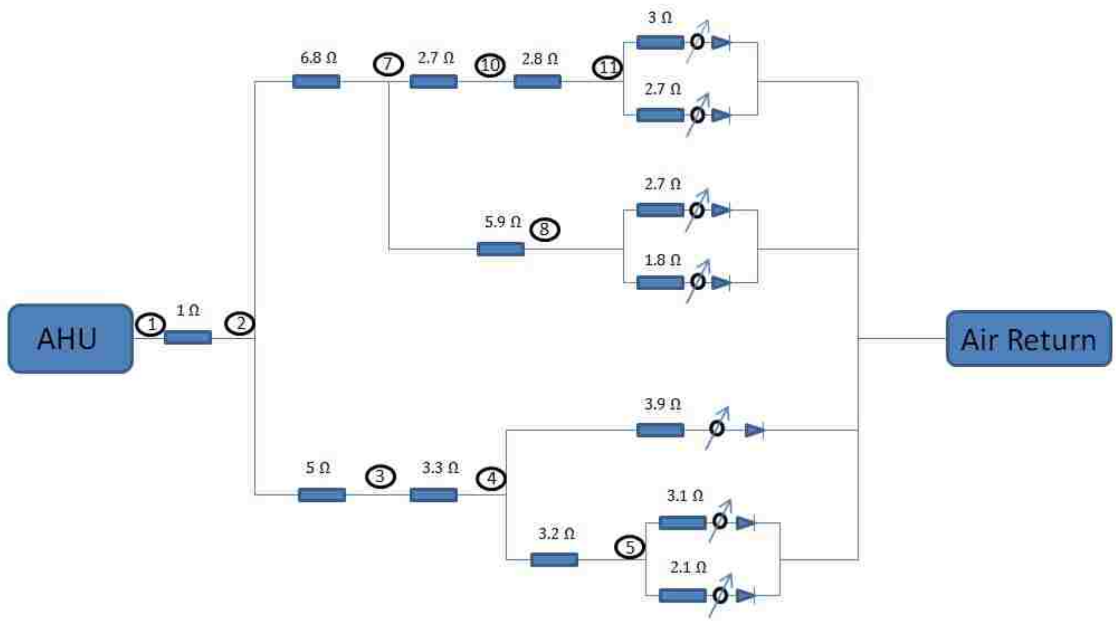


Figure 3.5: RC Duct System with Damper 1.

tioning. They are bedroom No.1 (3rd floor), bedroom No.3 (2nd floor), living room No.6 (1st floor), and kitchen No.8 (1st floor). Among these 4 rooms, only bedroom No.1 and kitchen No.8 have external roofs, which can absorb radiation energy. From Figure 3.6, we can see the indoor temperature of bedroom No.1 is higher than bedroom No.3. This is due to the external roof absorbing more heat flux from sunlight radiation. Kitchen No.8 also has an external roof, and its temperature is higher than living room No.6, even though they are in the same floor.

3.3.5 Dampers Analysis

Dampers are essential to the HVAC system. There are a flow meters inside the HVAC dampers, which can monitor air flow, temperature and humidity throughout homes and buildings. Some dampers are adjustable metal plates installed inside of a duct to close off or control the volume of heating and air conditioning in a specified room or area. These dampers can be motorized or manual.

Dampers can either shut off or allow air flow as desired and help to maintain

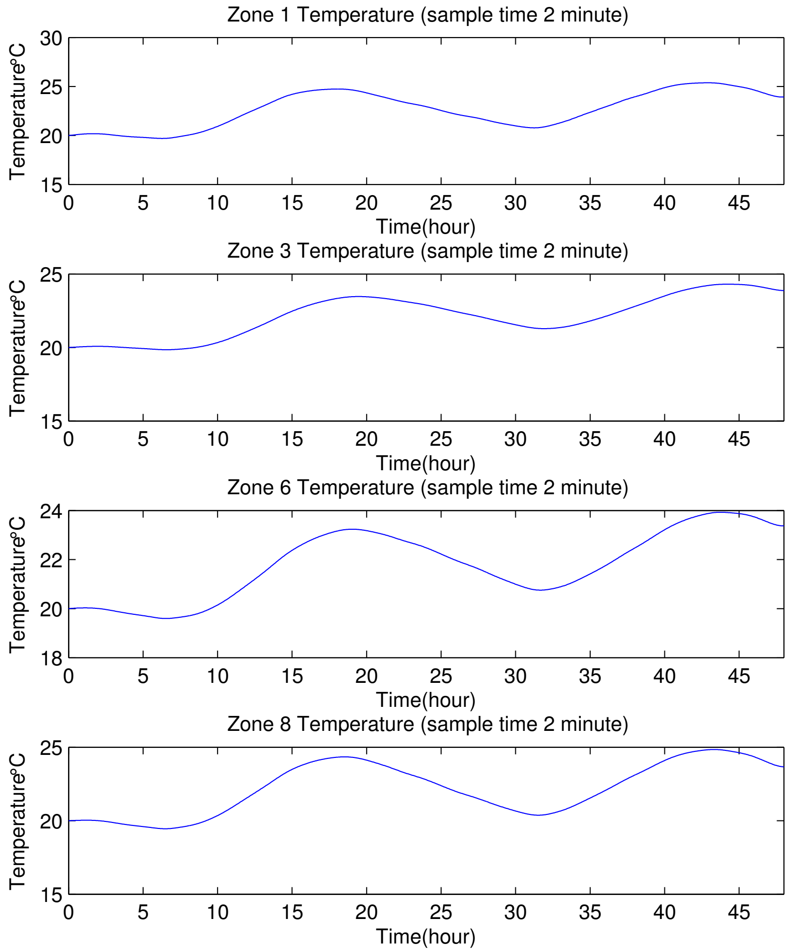


Figure 3.6: Bedroom No.1, Bedroom No.3, Living Room No.6, and Kitchen No.8's Temperature without AC in Two Days

comfortable temperatures for people and business equipment. Multi-level homes and businesses benefit by having dampers in the HVAC system to provide more comfortable living or working areas. Zone and bypass controllers control the dampers by managing air movement through the air ducts. A zone controller controls the temperature and ventilation specifications in one or more zones or space by operating one or more zone dampers.

In this research project, the duct system is modeled in a RC network (Figure 3.5). In each terminal, there is a damper to control the airflow rate (DT1 to DT4, DT6

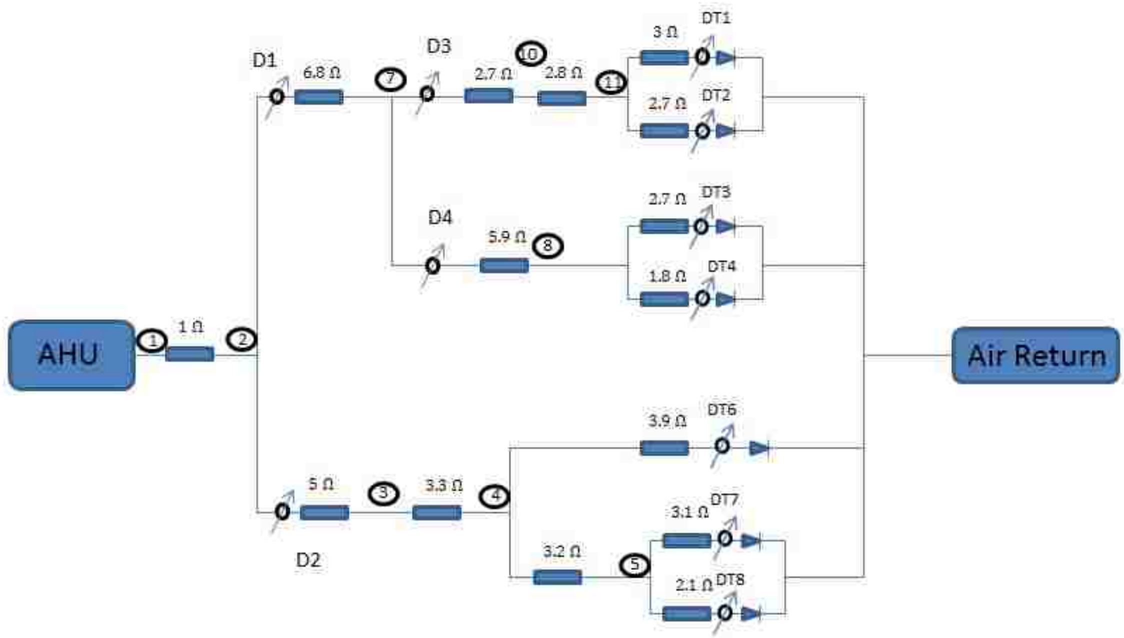
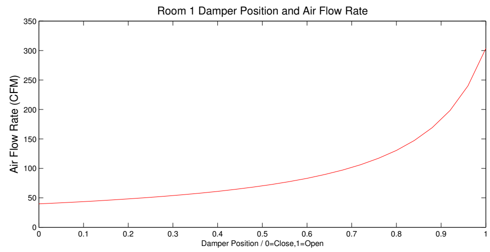


Figure 3.7: RC Duct System with Damper 2.

to DT8). Actually, duct system also needs dampers in each point of junction. From point 2 to 7, there is a damper (Damper 1) that controls the sum of second floor and third floor air flow rate. From point 2 to 3 (or 4), the damper (Damper 2) controls the air flow rate to first floor. In point 7, there should also be another two dampers (Damper 3 and Damper 4), which control the air flow rate in the second floor and the third floor respectively. Hence, the new duct system plan is shown in Figure 3.7, with all of the dampers.

For these seven terminal dampers, plots figures based on damper position and air flow rate in Figure 3.8. We assume when the damper is totally open, the resistance for the damper shown in Figure 3.7, and when the damper is totally closed, we use 25 as a parameter times the original damper resistance value as the closed the damper resistance value. If we assume the total air supply volume rate from the AHU cannot change (constant current system), the total air flow rate is a constant value. Table 3.6 shows the air flow rate for dampers totally open or closed. Table 3.8 shows the air flow rate for dampers partially open or closed.



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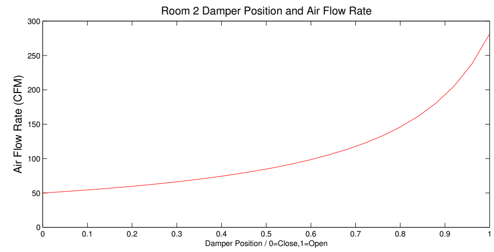

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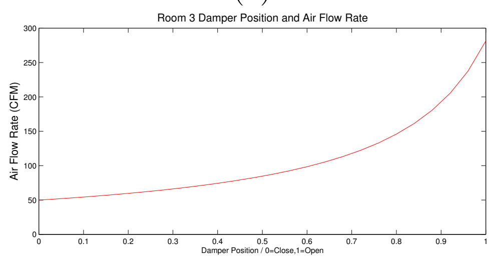

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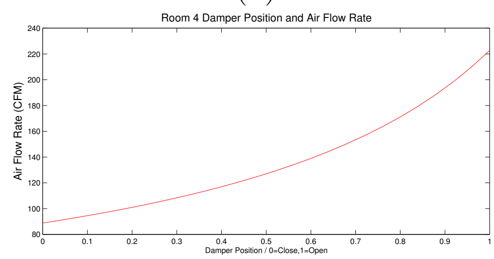

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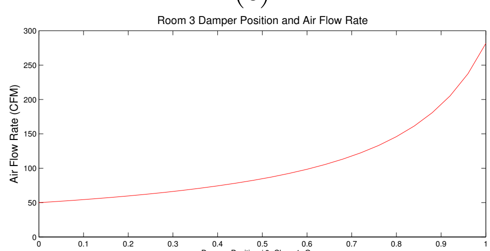

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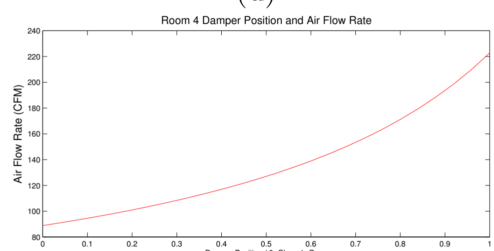

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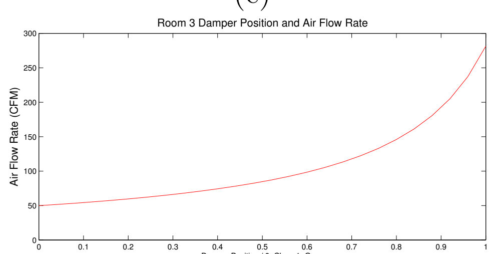

(c)

Figure 3.8: (a) Room No.1 Damper Air Flow Rate with Damper Position. b) Room No.2 Damper Air Flow Rate, Damper Position, and Resistance. (c) Room No.3 Damper Air Flow Rate, Damper Position, and Resistance. (d) Room No.4 Damper Air Flow Rate, Damper Position, and Resistance. (e) Room No.6 Damper Air Flow Rate, Damper Position, and Resistance. (f) Room No.7 Damper Air Flow Rate, Damper Position, and Resistance. (g) Room No.8 Damper Air Flow Rate, Damper Position, and Resistance.

Table 3.6: 1. Constant Current System, Air Flow Rate (Unit: CFM) with Damper Position / 1=Open, 0=Close

Damper		Flow		Flow		Flow		Flow		Flow		Flow
Terminal Damper1	1	292	0	22	1	312	1	311	0	32	0	35
Terminal Damper2	1	238	1	452	1	254	1	253	0	26	0	28
Terminal Damper3	1	258	1	278	0	18	1	274	1	427	0	18
Terminal Damper4	1	231	1	249	1	419	1	246	1	383	0	16
Terminal Damper6	1	335	1	343	1	342	0	20	1	399	1	731
Terminal Damper7	1	179	1	183	1	183	1	279	1	214	1	391
Terminal Damper8	1	265	1	271	1	270	1	412	1	315	1	577
Total Air Flow		1800		1800		1800		1800		1800		1800

Table 3.7: 2. Constant Current System, Air Flow Rate (Unit: CFM) with Damper Position / 1=Open, 0=Close

Damper		Flow		Flow		Flow		Flow		Flow
Terminal Damper1	0.8	96	1	306	1	306	0.8	124	0.8	136
Terminal Damper2	1	393	1	249	1	249	0.8	101	0.8	111
Terminal Damper3	1	272	0.8	82	1	270	1	367	0.8	80
Terminal Damper4	1	244	1	369	1	242	1	329	0.8	72
Terminal Damper6	1	340	1	340	0.8	95	1	376	1	601
Terminal Damper7	1	182	1	182	1	255	1	201	1	322
Terminal Damper8	1	269	1	268	1	377	1	297	1	475
Total Air Flow		1800		1800		1800		1800		1800

Table 3.8: 1. Constant Pressure System, Air Flow Rate (Unit: CFM) with Damper Position / 1=Open, 0=Close

Damper		Flow		Flow		Flow		Flow		Flow		Flow
Terminal Damper1	1	292	0	21	1	307	1	295	0	27	0	17
Terminal Damper2	1	238	1	444	1	250	1	240	0	22	0	14
Terminal Damper3	1	258	1	272	0	18	1	260	1	369	0	9
Terminal Damper4	1	231	1	244	1	412	1	233	1	331	0	8
Terminal Damper6	1	335	1	336	1	336	0	19	1	345	1	370
Terminal Damper7	1	179	1	180	1	180	1	265	1	184	1	198
Terminal Damper8	1	265	1	266	1	265	1	391	1	272	1	292
Total Air Flow		1800		1762		1776		1708		1554		912

Table 3.9: 2. Constant Pressure System, Air Flow Rate (Unit: CFM) with Damper Position / 1=Open, 0=Close

Damper		Flow		Flow		Flow		Flow		Flow
Terminal Damper1	0.8	95	1	303	1	294	0.8	113	0.8	82
Terminal Damper2	1	388	1	246	1	240	0.8	92	0.8	67
Terminal Damper3	1	268	0.8	81	1	260	1	333	0.8	48
Terminal Damper4	1	241	1	365	1	233	1	298	0.8	43
Terminal Damper6	1	336	1	336	0.8	91	1	342	1	363
Terminal Damper7	1	180	1	180	1	245	1	183	1	194
Terminal Damper8	1	265	1	265	1	362	1	270	1	287
Total Air Flow		1776		1778		1729		1633		1088

If we assume the pressure of the AHU is cannot change (constant pressure system), the total air flow changes due to the resistance change in the duct system. Table 3.8 shows the air flow rate for dampers totally open or closed. Table 3.9 shows the air flow rate for dampers partially open or closed.

Both of these systems will be used in traditional On/Off control and MPC schemes.

Chapter 4

Traditional HVAC Control Method

HVAC control systems use different control modes to accomplish their purpose, energy saving or best thermal comfortable. Control modes in HVAC commercial applications include two-position On/Off control, proportional control, proportional-integral control, proportional-integral-derivative control, fuzzy control, and MPC. In this chapter, two-position On/Off control will be introduce [5] .

4.1 Two-Position On-Off Control

Two-position control is widely used in simple HVAC systems to start and stop electric motors on unit heaters, fan coil units, and refrigeration machines. In two-position On/Off control, two values of the controlled variable (usually equated with on and off) determine the position of the final control element. Between these values is a zone called the "differential gap" or "differential" in which the controller cannot initiate an action of the final control element. As the controlled variable reaches one of the two values, the final control element assumes the position that corresponds to the demands of the controller, and remains there until the controlled variable changes to the other value. The final control element moves to the other position and remains there until the controlled variable returns to the other limit (Figure 4.1).

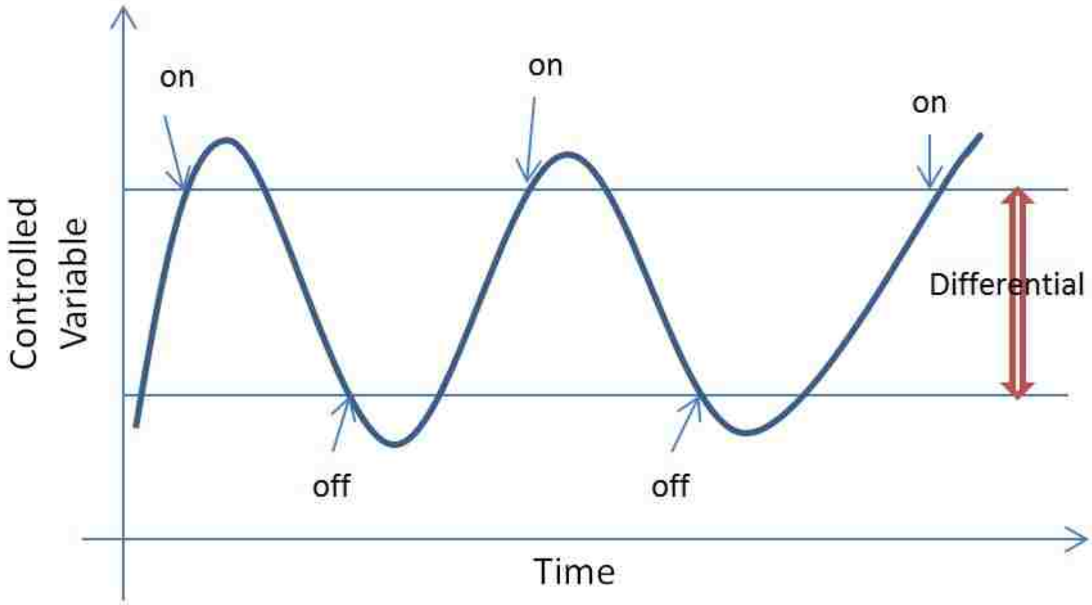

Figure 4.1: Two-Position On/Off Control Scheme.

In this case study, the controller is set to open a cooling valve, when the zone temperature reaches $20°C$, and to close the valve when the temperature drops to $18°C$. The difference between these two temperatures ($2°C$) is referred to as differential gap. The controlled variable fluctuates between these two temperature values as shown in Figure 4.1. Because the two-position On/Off control method is simple and inexpensive, it is used extensively for both industrial and commercial control.

In the HVAC system, the thermostat gives the signal to the AHU to let it turn on or turn off. In summer, due to the outside temperature being almost always higher than the inside temperature, and to save energy, only the cooling system is allowed to work. In winter, however, only the heating system is allowed to work.

4.2 One Sensor in Building

At first, we assume there is only one thermal sensor in the building to measure the inside temperature. Usually it is installed on the first floor. Hence, we install thermal

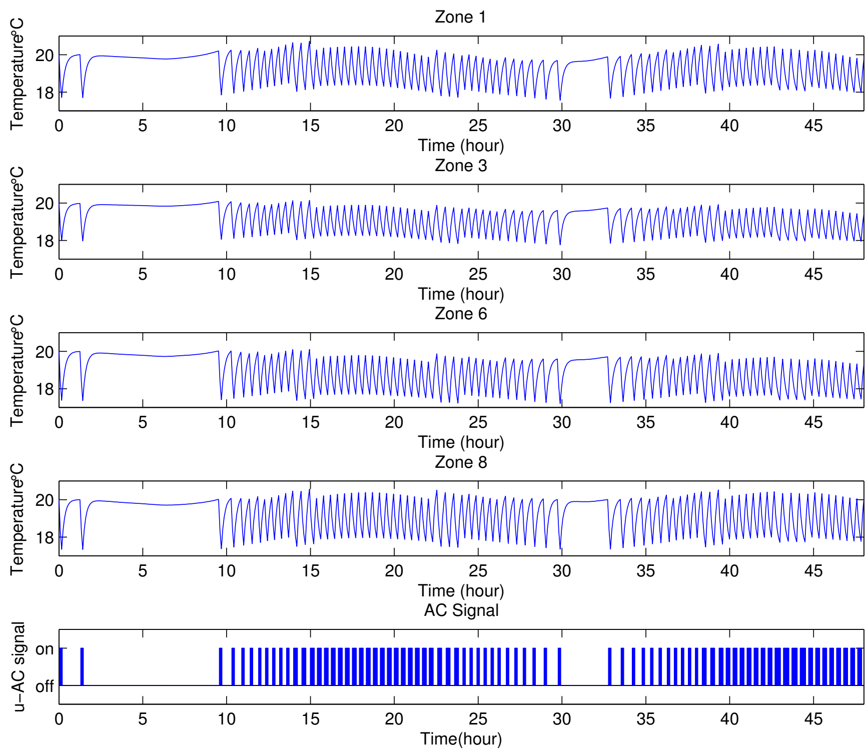

Figure 4.2: Temperature of Bedroom No.1, Bedroom No.3, Living Room No.6, and Kitchen No.8 and AC Signal for Two-Position On/Off Control Using just One Sensor. Differential Gap between 18°C and 20°C.

sensor in living room No.6. The sampling time is 2 minutes, which means that every 2 minutes the temperature sensor measures indoor temperature. The temperature is compared with the upper and lower boundaries to take the decision of turning the AHU on or off.

From Figure 4.2 we can see that the upper boundary is 20°C and the lower boundary is 18°C. During the first day, because the outside temperature from 2:00 AM to 9:00 AM is around 20°C, the inside temperature can remain in a comfortable range even turn with the air conditioning off for a couple of hours. When the outside temperature is around 25°C, it takes about $8 \sim 10$ minutes to reduce the inside temperature from 20°C to 18°C. When the cooling system turns off, the temperature

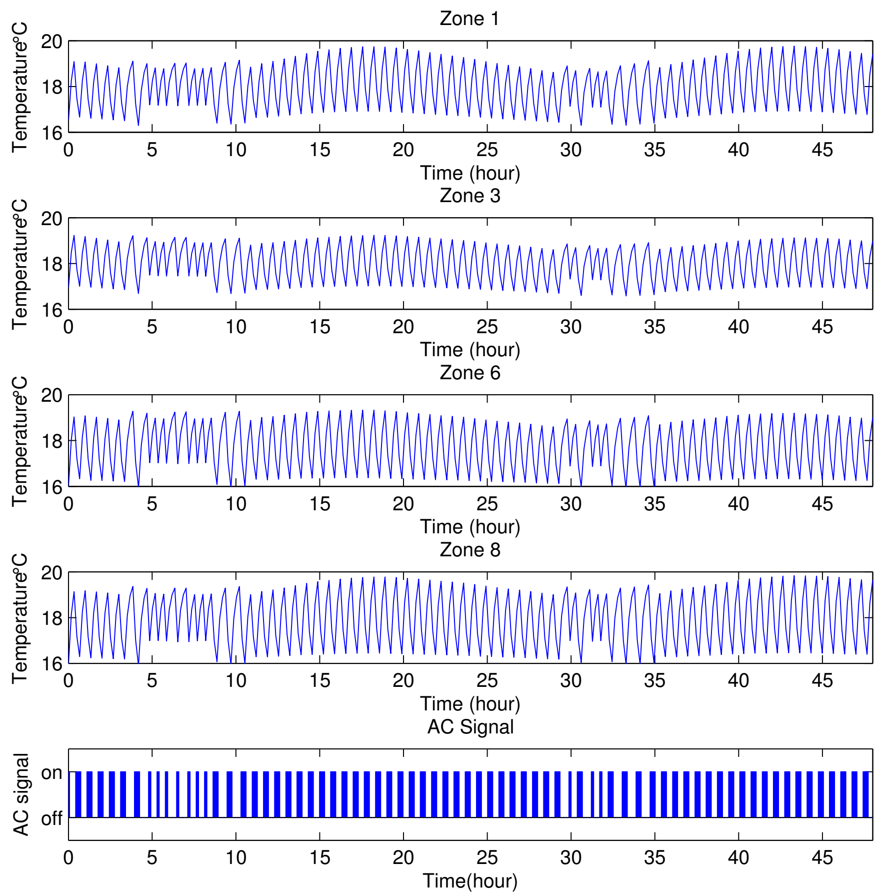

Figure 4.3: Temperature of Bedroom No.1, Bedroom No.3, Living Room No.6, and Kitchen No.8 and AC Signal for Two-Position On/Off Control Using just One Sensor. Differential Gap between $17°C$ and $19°C$.

increases from $18°C$ to $20°C$, taking about $15 \sim 20$ minutes. However, at noon the outside temperature is almost $30°C$. It takes about $12 \sim 14$ minutes to cool down and the temperature remains within the comfort range for $12 \sim 14$ minutes after turning off the cooling system.

In addition, if we set the upper boundary at $19°C$ and the lower boundary at $17°C$, and we want to keep the indoor temperature in this range, the AHU will turn on and off more frequently than in the first case. When the outside temperature is not very high, it takes about 15 minutes to cool the room from $19°C$ to $17°C$, when the cooling system turns off, the temperature changes from $17°C$ to $19°C$ in about 18 minutes. However, at noon the outside temperature is almost $30°C$. It takes about 20 minutes to cool down and the temperature remains within the comfort range for 10 minutes after turning off the cooling system (Figure 4.2).

From Figure 4.2 and Figure 4.3, we can see that living room No.6 temperature can be kept within the demand range very well. However, in other rooms, there are several temperature overshoots. That is because we only have one thermal sensor in the living room. In most residential buildings, people may want to feel comfortable and save energy usage. Therefore, installing more than one thermal sensor in residential buildings may improve both comfort and efficiency. If possible, we can install thermal sensors in each room. Each room's thermal sensor can measure inside temperature and control the damper individually at every sampling time.

4.3 Multiple Sensors in Building

As we mentioned before, there is a damper to control the air flow rate in each branch terminal. Hence, if we close or open several dampers while the AHU is working, the air flow distributed to each terminal will change. This simulation is run on Matlab program, and can calculate each terminal's air flow based on the seven terminal dampers' positions and four junction dampers' positions (See section 3.5 (Dampers Analysis)). In this case study, we need to consider two kinds of HVAC duct systems. The first one is a constant current HVAC system, when the air flow rate is kept at a constant value (1800 CFM) during AHU operation. The second one is a constant pressure HVAC system, where the pressure across the AHU is kept at a constant value and the total air flow rate changes as function of the resistance in the duct system.

We assume that if there are 3 or more rooms that need cooling air, the AHU should be turned on. Otherwise, the AHU should be turned off.

4.3.1 Constant Current HVAC System

For a constant current HVAC system, the total air flow rate will be maintained at 1800 CFM during AHU operation. The upper and lower boundaries are set at $20°C$ and $18°C$. The indoor temperature for the bedroom No.1, bedroom No.3, living room No.6, and kitchen No.8 are shown in Figure 4.4, Figure 4.5, Figure 4.6, and Figure 4.7. Figure 4.8 shows the total air flow rate during two days. From these figure, we can see that the temperature overshoots is large in the lower boundary. There are several temperature overshoots in the upper boundary for Bedroom No.1 and Kitchen No.8. That is because both of these two room has external roof, which will absorb more heat flux from sun radiation. The total air flow rate keeps in 1800 CFM when AHU operation (Figure 4.8).

4.3.2 Constant Pressure HVAC System

For a constant pressure HVAC system, the pressure across the AHU will be maintained constant during AHU operation. The upper and lower boundaries are set at $20°C$ and $18°C$. The indoor temperature for the bedroom No.1, bedroom No.3, living room No.6, and kitchen No.8 are shown be in Figure 4.9, Figure 4.10, Figure 4.11, and Figure 4.12. Figure 4.13 shows the total air flow rate during two days. In this constant pressure HVAC system, the overshoot is better than in constant current HVAC system and the total air flow rate is not always 1800 CFM (Figure 4.13).

4.3.3 Conclusion

From these ten figures, we can see that the overshoot is large in some situations. That is because: 1) the outside temperature changes with time, so the heat transfer flux is not a constant; 2) the opening and closing of one damper influence other branch, and hence the air flow for each terminal changes based on the different duct

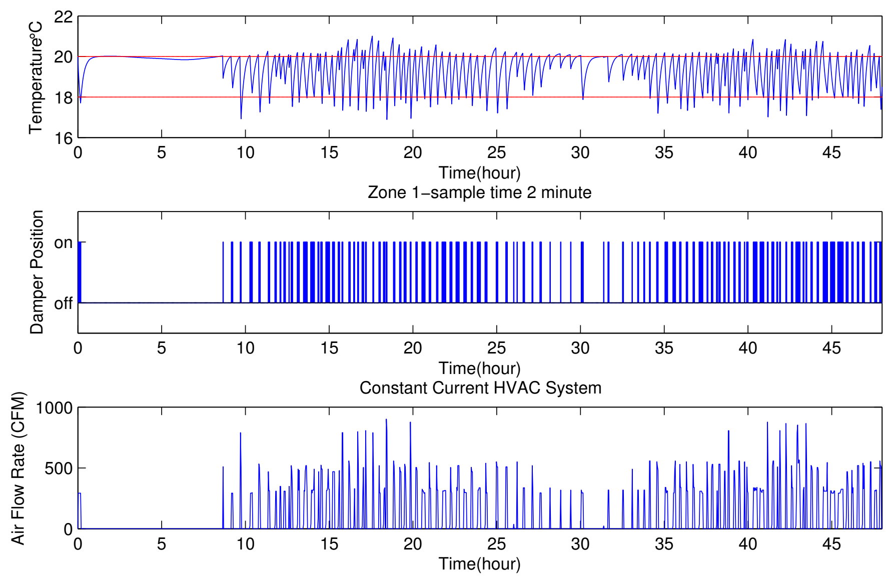

Figure 4.4: Bedroom No.1 Temperature, Damper Position and Air Flow Rate for Two-Position On/Off-Control Using Multiple Sensors in the Constant Current HAVC System. Differential Gap between $18°C$ and $20°C$.

damper positions. The constant pressure HVAC system has a better performance than the constant current HVAC system. The overshoots in the constant pressure HVAC system are smaller than those in the constant current HVAC system.

From the point of view of energy consumption, a heat pump or cooling system has a high transient power consumption when it is turned on, and then it usually uses lower amounts of power at steady state. Hence, in a two-position control system, the air conditioning system is turned on and off very frequently (almost once every 15 minutes) with the subsequent increase in power consumption and possible equipment damage. Control schemes need to balance the efficiency of turning the HVAC system on and off frequently with the added energy consumption and physical fatigue of frequent switching. This tradeoff can be taken into account by the Model Predictive Control technique, which picks control actions for the air condition that minimize a cost function, subject to the room thermal dynamics and constraints on the indoor temperature, air flow and flow rate gradient [6].

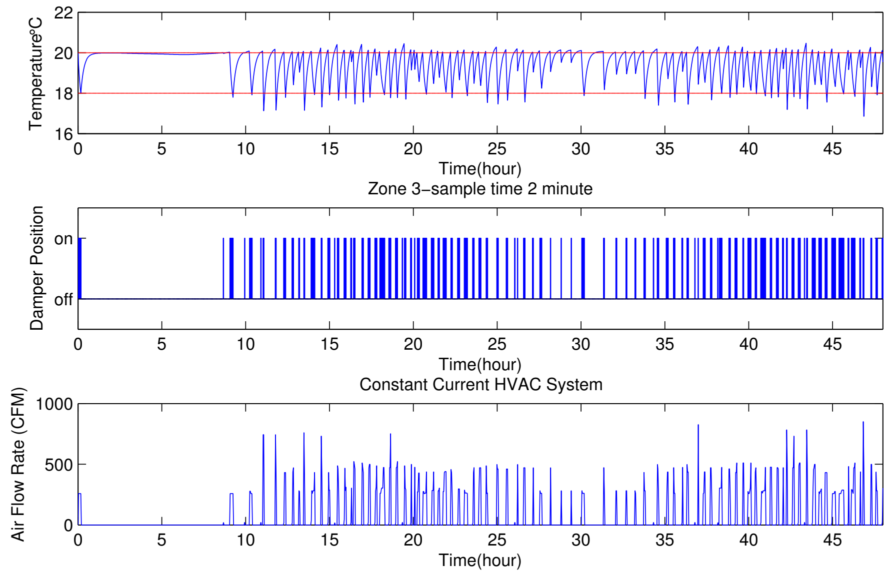

Figure 4.5: Bedroom No.3 Temperature, Damper Position and Air Flow Rate for Two-Position On/Off-Control Using Multiple Sensors in the Constant Current HAVC System. Differential Gap between $18°C$ and $20°C$.

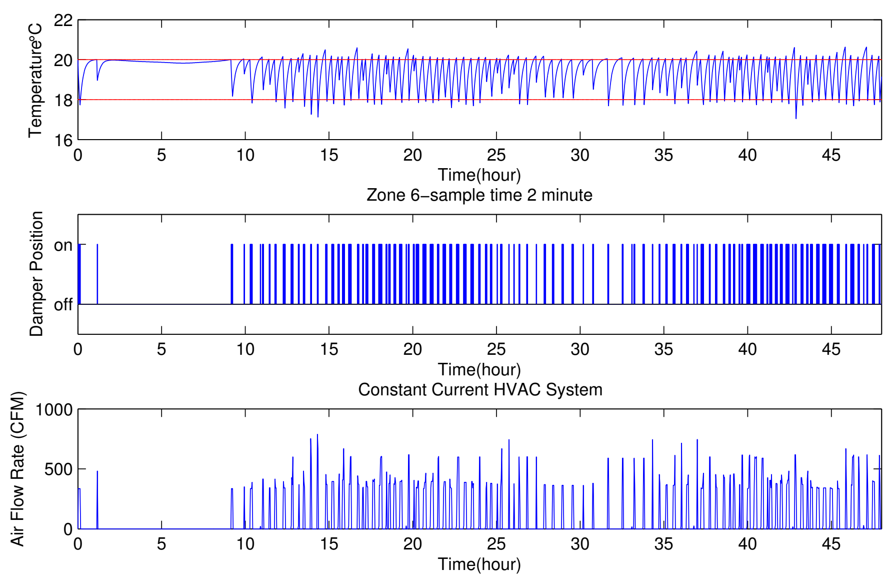

Figure 4.6: Living Room No.6 Temperature, Damper Position and Air Flow Rate for Two-Position On/Off-Control Using Multiple Sensors in the Constant Current HAVC System. Differential Gap between $18°C$ and $20°C$.

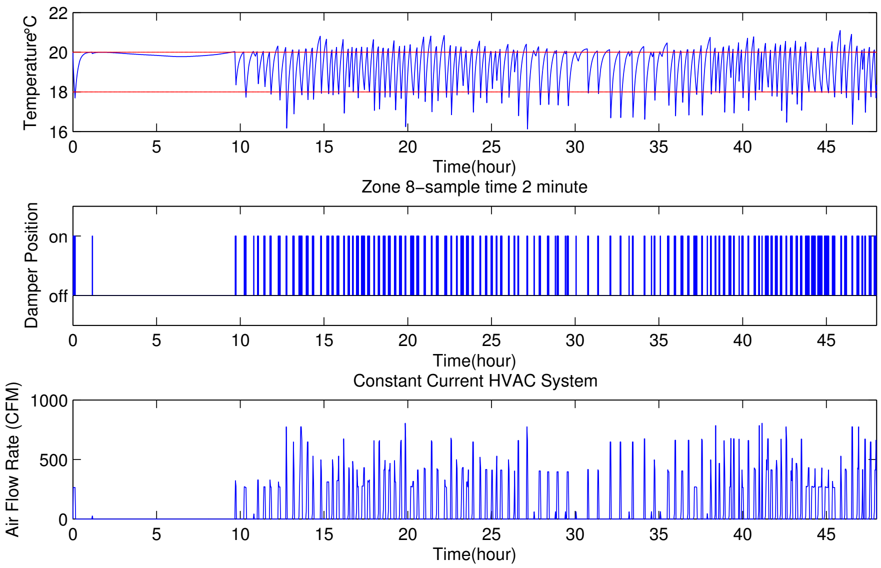

Figure 4.7: Kitchen No.8 Temperature, Damper Position and Air Flow Rate forTwo-Position On/Off-Control Using Multiple Sensors in the Constant Current HAVC System. Differential Gap between 18°C and 20°C.

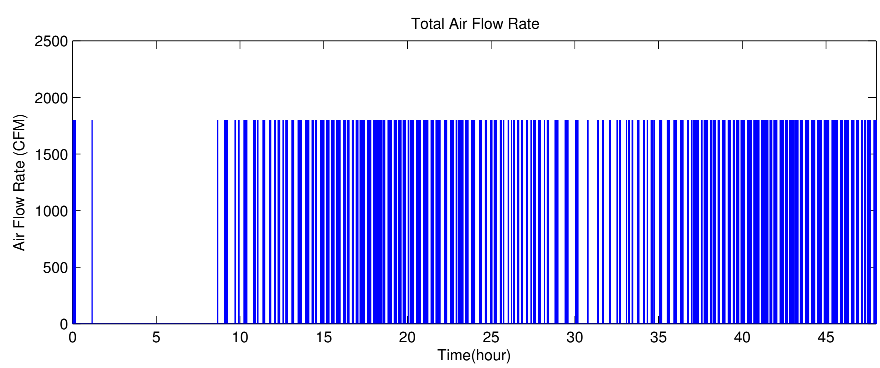

Figure 4.8: Two-Position On/Off-Control Using Multiple Sensors in the Constant Current HAVC System. Total Air Flow Rate

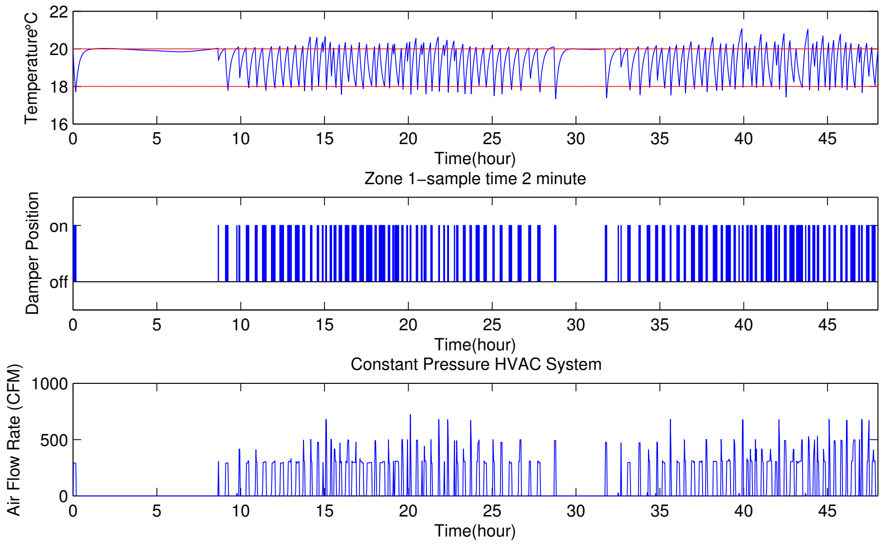

Figure 4.9: Bedroom No.1 Temperature, Damper Position and Air Flow Rate for Two-Position On/Off-Control Using Multiple Sensors in the Constant Pressure HAVC System. Differential Gap between 18°C and 20°C.

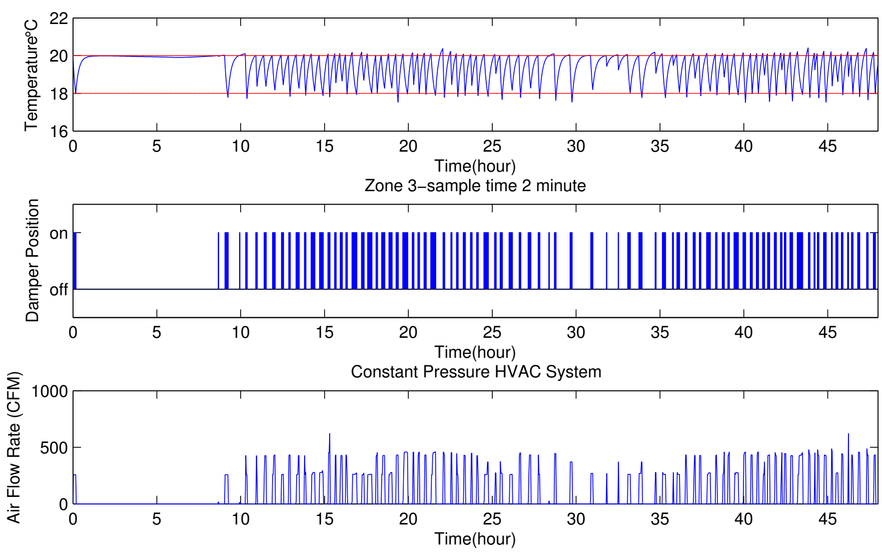

Figure 4.10: Bedroom No.3 Temperature, Damper Position and Air Flow Rate for Two-Position On/Off-Control Using Multiple Sensors in the Constant Pressure HAVC System. Differential Gap between 18°C and 20°C.

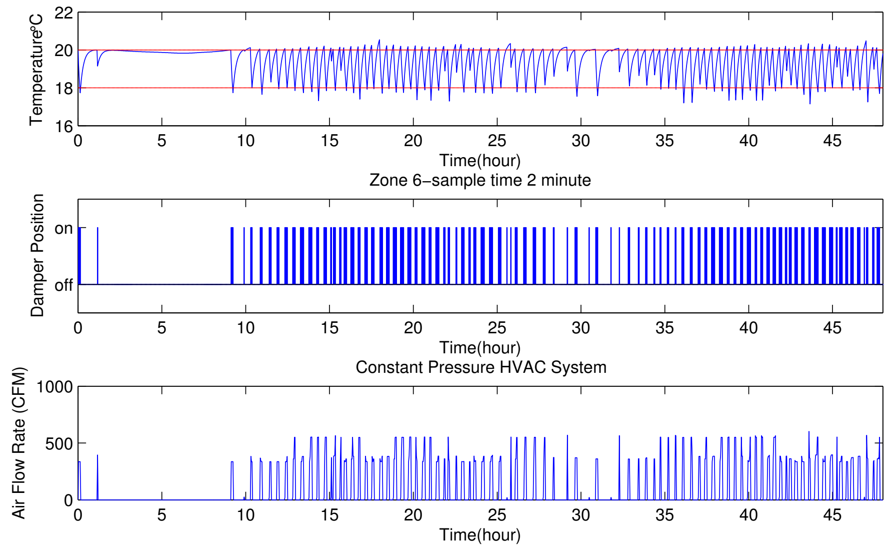

Figure 4.11: Living Room No.6 Temperature, Damper Position and Air Flow Rate for Two-Position On/Off-Control Using Multiple Sensors in the Constant Pressure HAVC System. Differential Gap between 18°C and 20°C.

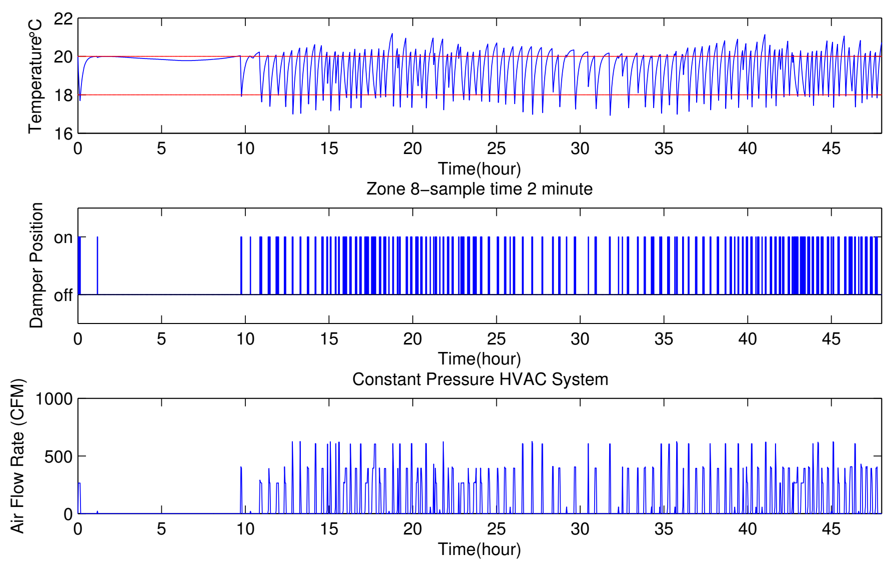

Figure 4.12: Kitchen No.8 Temperature, Damper Position and Air Flow Rate for Two-Position On/Off-Control Using Multiple Sensors in the Constant Pressure HAVC System. Differential Gap between 18°C and 20°C.

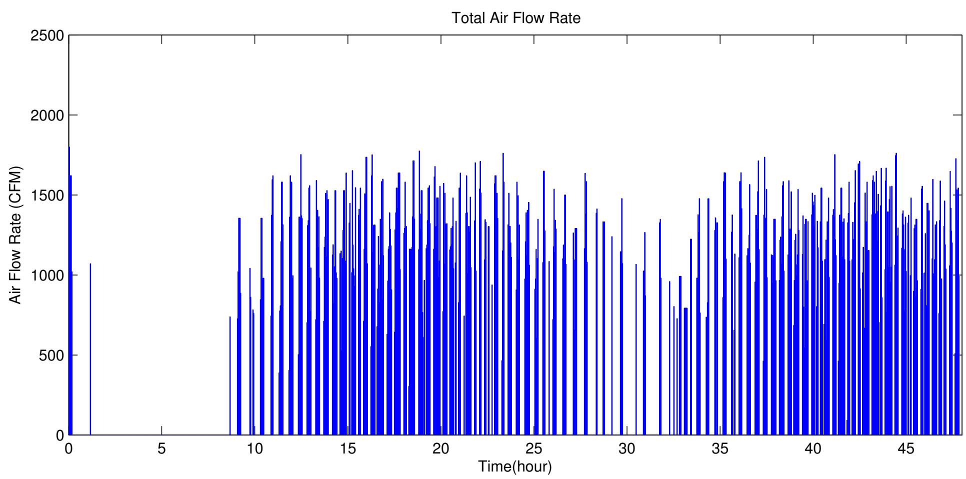

Figure 4.13: Two-Position On/Off-Control Using Multiple Sensors in the Constant Pressure HAVC System. Total Air Flow Rate

Chapter 5

Model Predictive Control

Model Predictive Control (MPC) schemes have become increasingly popular for a variety of processes over the last two decades. This can be attributed to MPC's ability to handle constrained multivariable problems in an optimal control environment, and the fact that they are intuitively tunable. Therefore, MPC has been widely applied in chemical and other related industries. A building HVAC system is a particularly suitable candidate for the application of predictive control methodologies, because of multiple control objectives, inherent complexity due to coupled and multivariable nature of the problem, and presence of constraints [7].

5.1 Overview

MPC is a form of control in which control action at the current time is obtained by solving a finite-time horizon, closed-loop, optimal control problem, using the current state of the plant as the initial state. In MPC, the control input is calculated by solving an optimal control problem (minimization of a cost function) over a given horizon. However, only the first element of the sample of the input sequence is applied to the system. At the next instant, a new optimization is performed based on current measurements. Predictive control has been successfully used in many applications. In particular, for heating and cooling systems, different formulations of cost functions and constraints have been proposed to minimize the consumption energy or to guarantee a desired comfort temperature.

Most research efforts have focused on optimal operation on variable air volume (VAV) systems, which could optimally control air flow rate and air temperature set points.

In this case, we extend the MPC method to our HVAC system, which pertains to control of flow rate in the duct system in order to meet the various control objectives.

5.2 State Space Model for Predictive Control Design

For convenience, we convert the heat balance Equation (2.15) and Equation (2.17) into a state space model [8],

$$\begin{aligned} \dot{x} &= A_c x + B_c u + B_o T_o(k) \\ y &= C_c x \end{aligned} \tag{5.1}$$

where u is the manipulated variable or input variable, y is the process output, $\dot{x}$ is the partial derivative of x, which is the state variable vector with assumed dimension n. In this work, x is defined as

$$x = [T_1\ T_2\ T_3\ T_4\ T_5\ T_6\ T_7\ T_8\ T_9\ T_{wall} \cdots\ T_{roof} \cdots\ T_{floor} \cdots\ T_{window} \cdots\ T_o]$$

where the first 9 items represent the temperatures for each room, T_{wall}, T_{roof}, T_{floor}, and T_{window} are the wall temperatures, roof temperatures, floor temperatures, and window temperatures using Equation (2.15), and T_o is the outside temperature. The total number of items in x is 61. In Equation (5.1), A_c, B_c, C_c, and B_o are the state space matrix, based on the heat balance equation (Equation (2.15) and Equation (2.17)) and building structure. Matrix A_c dimension is 61×61, Matrix B_c dimension is 61×9, Matrix C_c dimension is 9×61, Matrix B_o dimension is 61×1.

The manipulated input signal is u, mark it as u_i, which represents the cooling or heating load to zone i. The input equation can be expressed as

$$u_i = f_i(T_{sup} - T_i) \tag{5.2}$$

and

$$f_i = \frac{u_i}{(T_{sup} - T_i)} \tag{5.3}$$

where f_i is the air flow rate for room i, T_{sup} is the temperature of the air supply, T_i is the temperature of room i.

Because the house thermal dynamic model continuous in time, the model should be temporally discretized. In particular, by employing a Zero-Order Hold with sample time T_s (10 minute or other time choose, based on maximum frequency control of system), a discrete version of the model is obtained, i.e.,

$$\begin{aligned} x(k+1) &= A_d x(k) + B_d u(k) + B_{od} T_o(k) \\ y(k) &= C_d x(k) \end{aligned} \tag{5.4}$$

where matrixes A_d, B_d, B_{od}, and C_d have the same dimensions as their continuous-time counterparts. They are the discrete time matrixes of A_c, B_c, B_o, and C_c

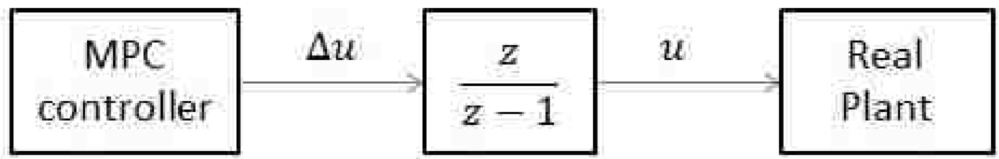

Figure 5.1: MPC Controller and Plant.

5.3 State Variable Choices

Most of the material presented in this section is based on [8] and [9].

As usual, our plant model (5.4) expresses the plant state x in terms of the values of the input u. However, the cost function penalizes changes Δu in the input, in addition to the input values u themselves. Figure 5.1 shows the real controller producing the signal u, which is passed to the real plant. It also shows the MPC controller producing the signal Δu, which is passed to the MPC plant.

There are several ways to change the signal in a state-space model of the MPC. All of them involve augmenting the state vector. One of these ways is to define the state vector in the following procedure. For simplicity, first assume our building thermal dynamic model is a regular MIMO system. Because, our building modeling has 9 rooms, hence the MIMO system should has 9 inputs (heating or cooling load u) and 9 outputs (inside temperature y). It can be described as

$$\begin{aligned} x_m(k+1) &= A_m x_m(k) + B_m x_m(k) + B_{mo} T_o(k) \\ y(k) &= C_m x_m(k) \end{aligned} \tag{5.5}$$

As we mentioned before, u is the manipulated variable or input variable, represented by Equation (5.2), y is the process output (the temperature for each room), and x_m is the state variable vector with assumed dimension n. In this case study, the dimension of x_m is 61 (including room temperatures, wall temperatures, roof temperatures, floor temperatures and window temperatures).

Taking a difference operation on both sides of Equation (5.5), we obtain

$$x_m(k+1)-x_m(k) = A_m(x_m(k)-x_m(k-1))+B_m(u(k)-u(k-1))+B_{mo}(T_o(k)-T_o(k-1)) \tag{5.6}$$

where $T_o(k)$ is the outside temperature at time k. $\Delta T_o(k)$ is very small, therefore we ignore the influence of ΔT_o. $\Delta x_m(k) = x_m(k) - x_m(k-1)$ is the change in the state at time k. Let us denote the difference of the state variable by

$$\begin{aligned} \Delta x_m(k+1) &= x_m(k+1) - x_m(k) \\ \Delta x_m(k) &= x_m(k) - x_m(k-1) \end{aligned} \tag{5.7}$$

and the difference of the control variable by

$$\Delta u(k) = u(k) - u(k-1) \tag{5.8}$$

Equation (5.7) and Equation (5.8) are the increments of the variables $x_m(k)$ and $u(k)$. With this transformation, the state-space equation can be expressed as

$$\Delta x_m(k+1) = A_m \Delta x_m(k) + B_m \Delta u(k). \tag{5.9}$$

Note that the input to this state-space model is $\Delta u(k)$. The next step is to connect Δx_m to the output $y(k)$. To do so, we design the new state augment variable vector as

$$x(k) = [\Delta x_m(k)^T \; y(k)]^T \tag{5.10}$$

Note that

$$y(k+1)-y(k) = C_m(x_m(k+1)-x_m(k)) = C_m\Delta x_m(k+1) = C_mA_m\Delta x_m(k)+C_mB_m\Delta u(k) \tag{5.11}$$

Putting together Equation (5.9) with Equation (5.11) leads to the following state-space model

$$\begin{aligned}
\overbrace{\begin{bmatrix} \Delta x_m(k+1) \\ y(k+1) \end{bmatrix}}^{x(k+1)} &= \overbrace{\begin{bmatrix} A_m & 0_m^T \\ C_mA_m & I \end{bmatrix}}^{A} \overbrace{\begin{bmatrix} \Delta x_m \\ y(k) \end{bmatrix}}^{x(k)} + \overbrace{\begin{bmatrix} B_m \\ C_mB_m \end{bmatrix}}^{B} \Delta u(k) \\
y(k) &= \overbrace{\begin{bmatrix} 0_m & I \end{bmatrix}}^{C} \begin{bmatrix} \Delta x_m \\ y(k) \end{bmatrix}
\end{aligned} \tag{5.12}$$

where, $0_m = \overbrace{\begin{bmatrix} 0 \; 0 \; 0 \; \ldots \; 0 \end{bmatrix}}^{n}$. The triplet (A, B, C) is called the augmented model, which will be used in the design of predictive control.

This state-space representation has been used in the literature of predictive control. It is also used in the Matlab Model Predictive Control Toolbox.

5.4 Prediction of State and Output Variables

We assume that at the sampling instant k_i, $k_i > 0$, the state variable vector $x(k_i)$ is available through measurement. The state $x(k_i)$ provides the current plant information. The future control trajectory is denoted by

$$\Delta u(k_i), \Delta u(k_i+1), ..., \Delta u(k_i+N_c-1)$$

where N_c is called the control horizon, which determines the number of parameters used to capture the future control trajectory. With the given information $x(k_i)$, the future state variables are predicted for N_p number of samples, where N_p is called the prediction horizon. We denote the future state variables as

$$x(k_i+1|k_i), x(k_i+2|k_i), ..., x(k_i+m|k_i), ..., x(k_i+N_p|k_i)$$

where $x(k_i+m|k_i)$ is the predicted state variable at k_i+m with given current plant information $x(k_i)$. The control horizon N_c is chosen to be smaller than (or equal to) the prediction horizon N_p [10].

Based on the state-space model (A, B, C), the future state variables are calculated sequentially using the set of future control parameters:

$$\begin{aligned}
x(k_i+1|k_i) &= Ax(k_i) + B\Delta u(k_i) \\
x(k_i+2|k_i) &= Ax(k_i+1|K_i) + B\Delta u(k_i+1) \\
&= A^2x(k_i) + AB\Delta u(k_i) + B\Delta u(k_i+1) \\
&\vdots \\
x(k_i+N_p|k_i) &= A^{N_p}x(k_i) + A^{N_p-1}B\Delta u(k_i) + A^{N_p-2}B\Delta u(k_i+1) + \\
&\quad ... + A^{N_p-N_c}B\Delta u(k_i+N_c-1)
\end{aligned} \tag{5.13}$$

From the predicted state variables, the predicted output variables are by substitution

$$
\begin{aligned}
y(k_i+1|k_i) &= CAx(k_i) + CB\Delta u(k_i) \\
y(k_i+2|k_i) &= CAx(k_i+1|K_i) + CB\Delta u(k_i+1) \\
&= CA^2x(k_i) + CAB\Delta u(k_i) + CB\Delta u(k_i+1) \\
&\vdots \\
y(k_i+N_p|k_i) &= CA^{N_p}x(k_i) + CA^{N_p-1}B\Delta u(k_i) + CA^{N_p-2}B\Delta u(k_i+1) + \\
&\quad ... + CA^{N_p-N_c}B\Delta u(k_i+N_c-1) \qquad (5.14)
\end{aligned}
$$

Note that all predicted variables are formulated in terms of the current sate variable information $x(k_i)$ and the future control changes $\Delta u(k_i+j)$, where $j = 0, 1, ...N_c-1$.

We define the vectors

$$Y = [y(k_i+1|k_i)\ y(k_i+2|k_i)\ y(k_i+3|k_i)\ ...\ y(k_i+N_p|k_i)]^T,$$

$$U = [u(k_i)\ u(k_i+1)\ u(k_i+2)\ ...\ u(k_i+N_c-1)]^T,$$

and

$$\Delta U = [\Delta u(k_i)\ \Delta u(k_i+1)\ \Delta u(k_i+2)\ ...\ \Delta u(k_i+N_c-1)]^T.$$

We collect Equation (5.13) and Equation (5.14) together in a compact matrix form as:

$$Y = Fx(k_i) + \Phi\Delta U \qquad (5.15)$$

Where

$$F = \begin{bmatrix} CA \\ CA^2 \\ CA^3 \\ \vdots \\ CA^{N_p} \end{bmatrix}, \phi = \begin{bmatrix} CB & 0 & 0 & \cdots & 0 \\ CAB & CB & 0 & \cdots & 0 \\ CA^2B & CAB & CB & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ CA^{N_p-1}B & CA^{N_p-2}B & CA^{N_p-3}B & \cdots & CA^{N_p-N_c}B \end{bmatrix}$$

5.5 Cost Function

As we mentioned before, the most important step in the design of a predictive control algorithm is the specification of a suitable cost function which is to be minimized. For this HVAC system, the cost function is proposed as

$$J = \sum_{i=1}^{N_c} \alpha_i u(k+i|k)^2 + \gamma \sum_{i=1}^{N_p} [y_{ref}(k+i|k) - y(k+i|k)]^2 + \psi \sum_{i=1}^{N_c} [u(k+i|k) - u(k+i-1|k)]^2 \tag{5.16}$$

The following nomenclature is used in the above equation, y_{ref} is the reference signal for output i, α_i is the weight corresponding to u in the energy term, γ penalty associated with regulation error term, ψ penalty associated with slew rate term.

The first term in Equation (5.16) seeks the minimization of the energy consumption over the control horizon. The second term uses to minimize the prediction error. The last term penalizes the change rates for the system inputs over the control horizon and therefore prevents from abrupt changes in the actuation signals.

We write the data vector that contains the set-point temperatures as

$$Y_{ref}^T = \overbrace{\begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}}^{N_p} y(k_i)$$

and rewrite the cost function J that reflects the control objective as

$$\begin{aligned} J &= U^T \alpha U + (Y_{ref} - Y)^T \gamma (Y_{ref} - Y) + \Delta U^T \Psi \Delta U \\ J &= U^T \alpha U + (Y_{ref} - Fx(k_i) - \Phi \Delta U)^T \gamma (Y_{ref} - Fx(k_i) - \Phi \Delta U) + \Delta U^T \Psi \Delta U \end{aligned} \tag{5.17}$$

where Ψ is a diagonal matrix in the form that $\Psi = r_w I_{N_c \times N_c}$ ($r_w \geqslant 0$), where r_w is used as a tuning parameter for the desired closed-loop performance. I is an identity matrix, with $N_c \times N_c$ dimensions.

To find the optimal ΔU that will minimize J, by using Equation (5.17), cost function J is expressed as:

$$\begin{aligned} J = (U + \Delta U)^T \alpha (U + \Delta U) + (Y_{ref} - Fx(k_i))^T \gamma (Y_{ref} - Fx(k_i) \\ -2 \Delta U^T \Phi^T \gamma (Y_{ref} - Fx(k_i)) + \Delta U^T (\Phi^T \gamma \Phi + \Psi) \Delta U \end{aligned} \tag{5.18}$$

The first derivation of the cost function J is

$$\frac{\partial J}{\partial \Delta U} = -2\Phi^T (Y_{ref} - Fx(k_i))\gamma + 2(\Phi^T \gamma \Phi + \Psi)\Delta U + 2\alpha(U + \Delta U)$$

The necessary condition of the minimum J is obtained as

$$\frac{\partial J}{\partial \Delta U} = 0$$

From which we find the optimal solution for the control signal as

$$\Delta U = (\Phi^T \gamma \Phi + \Psi + \alpha)^{-1} [\Phi^T (Y_{ref} - Fx(k_i))\gamma - \alpha U(k_i - 1)] \tag{5.19}$$

5.6 Constraints

As we mentioned before, one of the important advantages of MPC over other control methodologies is that is has the ability to satisfy constraints on the states, inputs, and even outputs. In HVAC systems only constraints on the inputs are required. Constraint on outputs are only necessary in some cases. However, constraints on states are not deemed necessary. Usually, the most useful constraints using in HVAC system are saturation and rate limit.

Saturation

The saturation constrains the input signal u, which represents heating or cooling load to the building heat balance. In general, each room has a maximum value, hence each component of u should have an upper boundary, u_{max}, which is dependent on the AHU power and the duct size. The lower boundary, u_{min}, is dependent on selected room fresh air requirements. We do not consider this constraint in our project, and we set the lower boundary equal to zero. Therefore,

$$0 \leq u(k+i|k) \leq u_{max}$$

Rate Limit

The rate limit constrains the rate of change of the input signal u. Abrupt changes in the control inputs can damage the mechanical components (like pumps, damper, and compressors) and must be avoided. Therefore, we set

$$\Delta u_{min} \leq \Delta u \leq \Delta u_{max}$$

5.7 Quadratic Programming and Anti-Windup Compensators

Quadratic Programming

In general, MPC can handle an optimization problem with constraints, and we can use Quadratic Programming (QP) to solve, it is a special type of mathematical optimization problem, where a quadratic function of several variables is optimized subject to linear constraints on these variables. The general QP problem can be expressed as

$$\min_{a} = \frac{1}{2}\Theta^2\Phi\Theta + q^T\Theta \tag{5.20}$$

subject to

$$A\Theta = b$$

and

$$G\Theta < h$$

Approaches to solving the QP problem include the Active Set methods [9] and the more recent Interior Point methods [12] [13].

Anti Windup Compensator

Compare with the QP solver, Anti windup compensator has many advantages. Anti windup compensator do not need to rewrite the cost function and constraints.

The building HVAC and air flow system is a complex system, which is subject to actuator saturation. When the input is at saturation, the actual model input is different from the input solved by MPC. Our goal is not to redesign a MIMO controller or transfer the cost function to other forms, but to design an anti-windup compensator that works with the MPC controller to keep it well-behaved and avoid undesirable oscillations when saturation is present. The anti-windup compensator must in addition leave the nominal close-loop unmodified when no saturation is present.

In this case, we need to add two kinds of compensators in the controller, one is magnitude saturation compensator and another one should be rate saturation compensator.

The magnitude saturation function Δu is defined as:

$$\Delta u = sat_{u_{min}}^{u_{max}} \Delta u_c = \begin{cases} \Delta u_{max} & if\ \Delta u \geq \Delta u_{max} \\ \Delta u_c & if\ \Delta u_{min} < \Delta u_c < \Delta u_{max} \\ \Delta u_{min} & if\ \Delta u_c \leq \Delta u_{min} \end{cases} \tag{5.21}$$

And the rate saturation function Δu is defined as:

$$u = sat_{u_{min}}^{u_{max}} u_c = \begin{cases} u_{max} & if\ u \geq u_{max} \\ u_c & if\ u_{min} < u_c < u_{max} \\ u_{min} & if\ u_c \leq u_{min} \end{cases} \tag{5.22}$$

Where u_{min} , u_{max} and $\Delta u_{min}, \Delta u_{max}$ are the maximum and minimum saturation limits. Hence, the dead-zone function can be defined as $Dz(u) = u - u_c$ and $\Delta Dz(u) = \Delta u - \Delta u_c$. Take the magnitude saturation as example design anti-windup compensator [11].

The anti-windup augmentation to the controller can be written as:

$$\begin{aligned} \dot{x}_{aw}(k) &= A_{aw}x_{aw} + B_{aw}Dz(u) + \beta\eta \\ s &= C_{aw}x_{aw} + D_{aw}Dz(u) \\ \eta &= -cx_{aw} - A_{aw}x_{aw} - B_{aw}Dz(u) \end{aligned} \tag{5.23}$$

where c is positive constant and $\beta = 1$ if the system is unsaturated, otherwise $\beta = 0$.

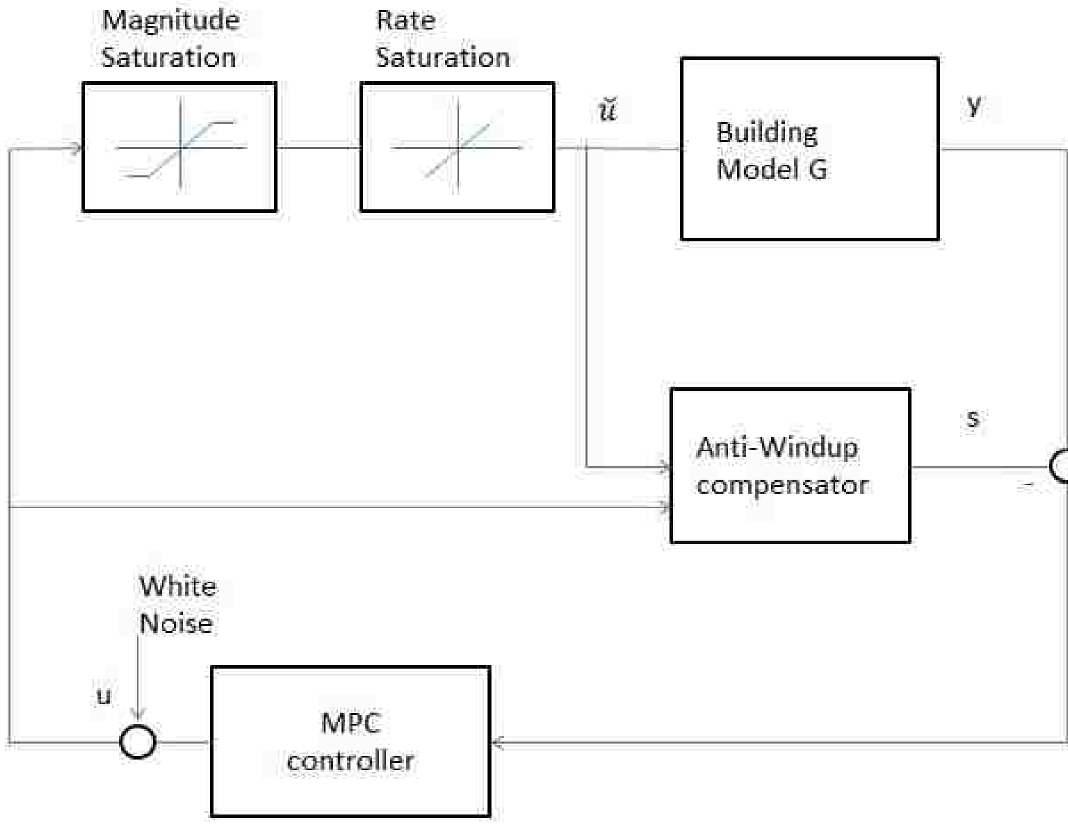

Figure 5.2: Anti-windup Scheme.

By choosing A_{aw}, B_{aw}, C_{aw} and D_{aw} equal to the matrix of Equation (5.12), we will guarantee that x_{aw} will converge to zero fast and smoothly ($\dot{x}_{aw} = -cx_{aw}$) when unsaturated, and so will s. The structure of the anti-windup compensator is shown in Figure 5.2.

5.8 Predictive Scheme

The control scheme is shown in Figure 5.3. The zone temperature T_i is cooled down to its set point T_r by adjusting the supply air flow rate f_i according to the cooling load Q_i. The supply air temperature T_s should have a lower temperature than the zone, when cooling is required. The supply air flow rate f_i is controlled by the damper position inside the VAV box or just a single damper for simplicity.

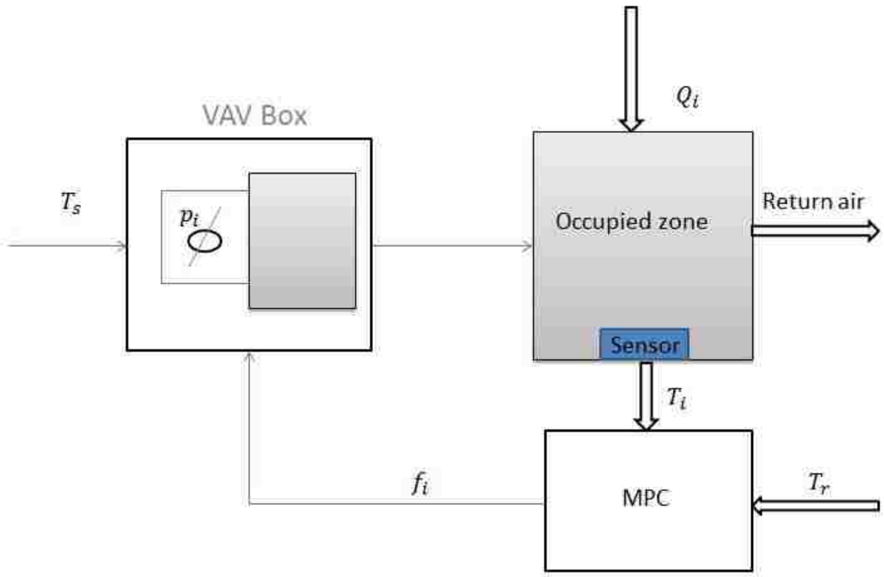

Figure 5.3: Illustration of Predictive Control Scheme.

In general, the VAV box takes f_i as input and p_i as output, which is the control signal for the damper position. The zone temperature produces set points for the air flow according to occupancies' thermal requirement and the zone cooling load, while the VAV damper tracks the set point by adjusting the damper position of the VAV box. The VAV damper consists of a damper with its driving motor and a flow meter for measuring the supply air flow rate.

Chapter 6

MPC Evaluation

The MPC control objective is to minimize the energy consumption and also to satisfy the thermal comfort. As mentioned before, the thermal comfort should be defined only during the occupation periods. Hence, we need to denote one factor, which can represent the future occupation over the prediction horizon for each room. Usually, there should be a sensor to measure the number of people who occupy each room. Therefore, we decide the future occupation sequence by normal sense. Like from 9:00 PM to 6:00 PM, no one will be using the kitchen. Dinner room is only used during breakfast, lunch, and dinner. Bedroom is used between 6:00 PM and 7:00 AM, or other time periods. Intuitively, each element of this vector is defined as [14]

$$\delta_i(k+j) = \left\{ \begin{matrix} 1 & k+j \in Occupation \\ 0 & k+j \in Inoccupation \end{matrix} \right\} \tag{6.1}$$

6.1 Simulation Results

6.1.1 Constant Current HVAC System

In a constant current HVAC system, system assume the AHU supply air flow rate is unchangeable. When the air conditioning is turned on, the air flow is maintained at

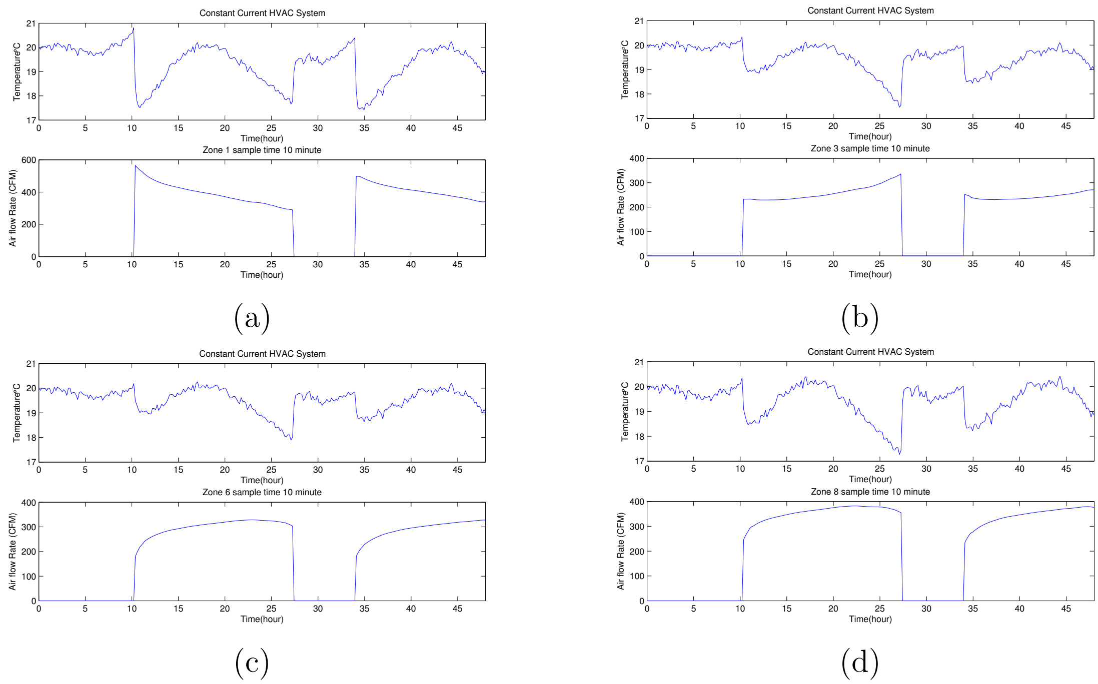

Figure 6.1: MPC Using in the Constant Current HVAC System in Two Days. (a) Temperature (up) and Air Flow Rate (bottom) for Bedroom No.1. (b) Temperature (up) and Air Flow Rate (bottom) for Bedroom No.3. (c) Temperature (up) and Air Flow Rate (bottom) for Living Room No.6. (d) Temperature (up) and Air Flow Rate (bottom) for Kitchen No.8.

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a constant value. We calculated the required AHU air supply rate at 1800 CFM in Chapter 3. Hence, the sum of the air flow rates in each zone should always be equal to 1800 CFM during the AHU operation.

In addition, we do not want the AHU turned on during the whole day. During some time periods the outside temperature is colder than the inside temperature. Therefore, the inside temperature can be kept in a comfortable range even with the air conditioning turned off.

We first use the MPC controller to calculate at each time period the required air flow rates. If the sum of the required air flow rates in each rooms is higher than 400 CFM, we turn on the air condition; otherwise, we turn it off. The plots in Figure 6.1 show the inside temperatures and air flow rates for bedroom No. 1, bedroom No. 3, living room No. 6 and kitchen No. 8. In this situation, the sum of the air flow rates

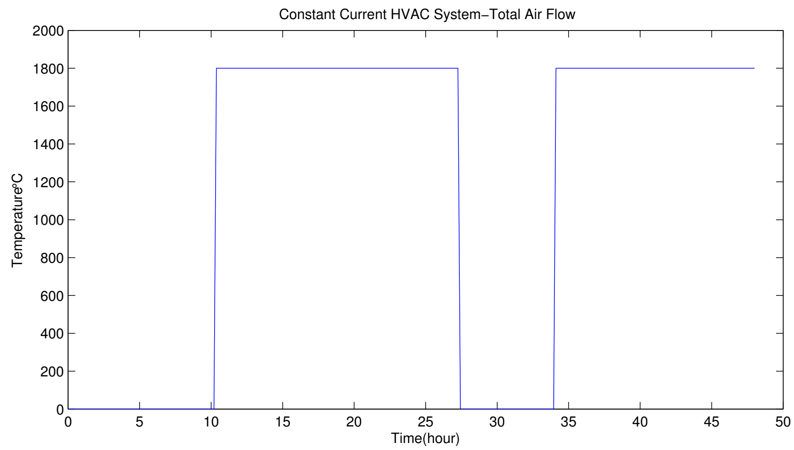


Figure 6.2: Total Air Flow Rate for MPC Using in the Constant Current HVAC System in Two Days.

in each zone is always kept at a constant value (1800 CFM) (Figure 6.2).

In this simulation, the reference temperature is $20°C$ (T_{ref}), the result cannot kept in $20°C$ very well. Even we change the weight α and γ, the simulation results are not change too much. That is because the sum of the air flow rates is always kept at 1800 CFM. Hence, if the total required air flow rate in the building is below 1800 CFM. The extra air flow rate also go into each room, that is the reason why the inside temperate can be $18°C$ or even below $18°C$ in some time periods.

6.1.2 Constant Pressure HVAC System

In this case, we assume the AHU supply pressure is constant. Hence the air flow into each zone is only calculated by the MPC controller. As in the constant current HVAC system, the total air flow rate cannot larger than 1800 CFM, if the total required air flow rate in each rooms is larger than 1800 CFM. The air flow rate into each room will be the value shows in Table 3.5. Otherwise, HVAC duct system will distribute the required air flow rate to each rooms, which calculated by MPC controller. Based on the air flow rates go into each zone, calculating the damper positions based on the RC duct system model Figure 3.7 and Figure 3.8. At last, if the flow rate is below

400 CFM, we turn off the air condition to save energy.

α_i is the weight corresponding to u in the energy term, γ uses to minimize the prediction error. If we want to save the energy as possible as we can, choose a large α_i value and small γ value. If we want to have the best comfortable temperature, choose a small α_i value and large γ value. If we want both energy saving and comfortable temperature, we can choose the same value for α_i and γ.

$\alpha_i = 1$**;** $\gamma_i = 0.1$

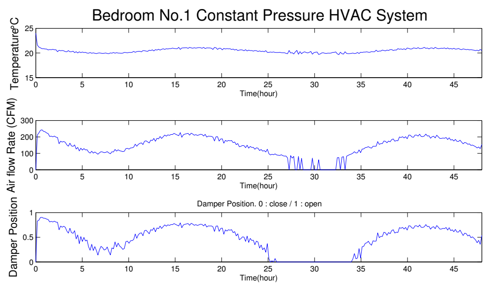
In this case $\alpha_i = 1$; $\gamma_i = 0.1$, it just means we consider energy saving more than the comfortable feeling. Figure 6.3 is showing the inside temperature and air flow rate for bedroom No. 1, bedroom No. 2, living room No. 6 and kitchen No. 8. Because choosing the weight matrix, the minimum energy input were use during the simulation. There are some tracking error exist. In this situation, the total sum of the air flow rates in each zones shows in Figure 6.4.

$\alpha_i = 1$**;** $\gamma_i = 1$

In this case $\alpha_i = 1$; $\gamma_i = 11$, it just means we want both energy saving and comfortable feeling. Figure 6.5 is showing the inside temperature and air flow rate for bedroom No. 1, bedroom No. 2, living room No. 6 and kitchen No. 8. Comparing with the Figure 6.3, the tracking error is reduced and the energy is not increase so much. In this situation, the total sum of the air flow rates in each zones shows in Figure 6.6.

$\alpha_i = 0.1$**;** $\gamma_i = 1$

In this case $\alpha_i = 0.1$; $\gamma_i = 1$, it just means we consider the comfortable feeling firstly and do not care about the energy consumption. Figure 6.7 is showing the inside temperature and air flow rate for bedroom No. 1, bedroom No. 2, living room No. 6 and kitchen No. 8. Compare with the Figure 6.5, there is nearly no tracking errors,



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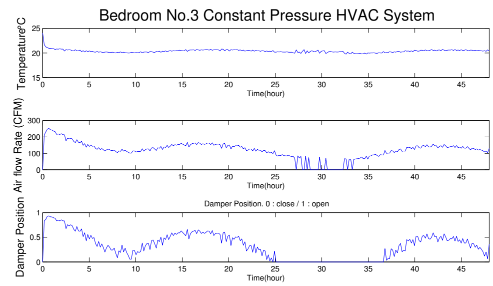

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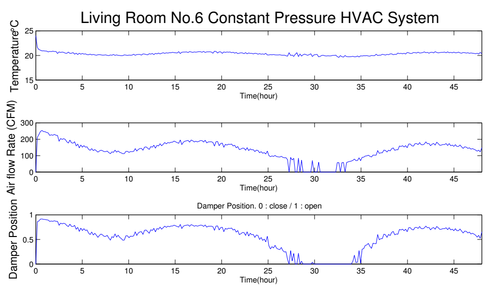

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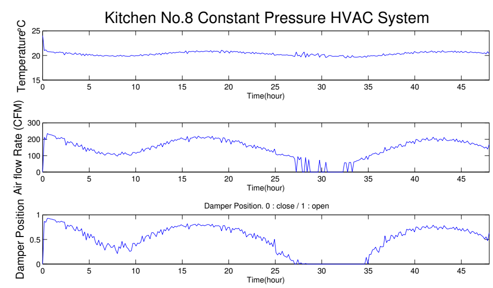

(d)

Figure 6.3: $\alpha_i = 1$; $\gamma_i = 0.1$, MPC Using in the Constant Pressure HVAC System in Two Days. (a) Temperature (up), Air Flow Rate (middle), and Damper Position (Bottom) for Bedroom No.1. (b) Temperature (up), Air Flow Rate (middle), and Damper Position (Bottom) for Bedroom No.3. (c) Temperature (up), Air Flow Rate (middle), and Damper Position (Bottom) for Living Room No.6. (d) Temperature (up), Air Flow Rate (middle), and Damper Position (Bottom) for Kitchen No.8.

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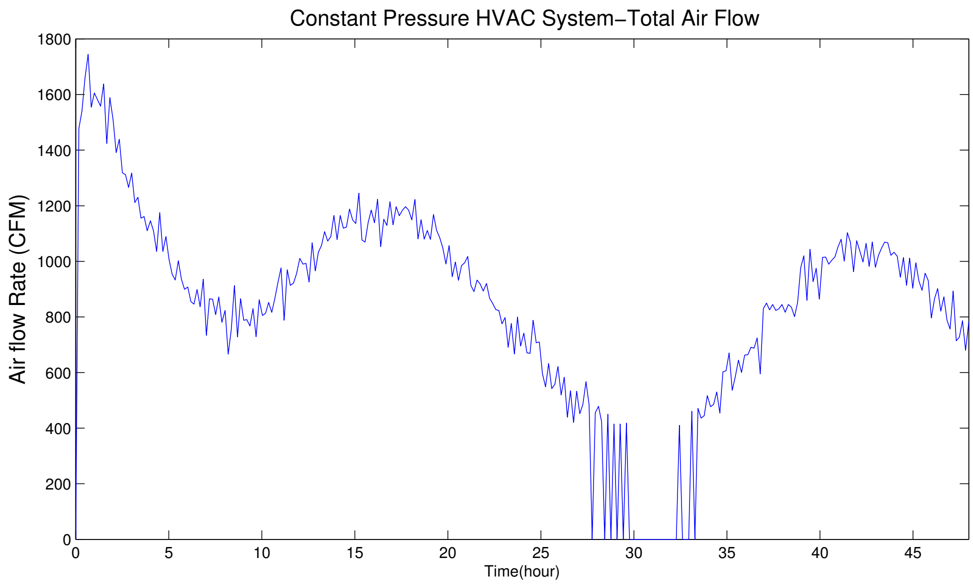


Figure 6.4: Total Air Flow Rate for MPC Using in the Constant Pressure HVAC System in Two Days. $\alpha_i = 1$; $\gamma_i = 0.1$

but the energy consumption increase so much. In this situation, the total sum of the air flow rates shows in Figure 6.8.

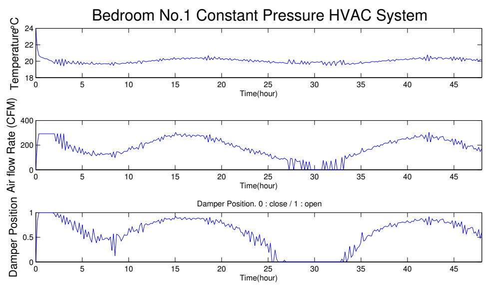
In order to save energy, we turn off AHU, when total required air flow is below 400 CFM. We also can design the boundary as 600 CFM, it will save more energy, however, the thermal comfortable will worse.

6.2 Energy Consumption

The controllable input of the building thermal model is the heat flux (see Equation 6.2). We consider the heat flux generated from AHU as energy consumption. For this case, the total energy consumption formulation is

$$Energy = Airflow * \rho_{air} * C_{p_{air}} \tag{6.2}$$

Figure 6.5 and Table 6.1 provide a comparison of the average energy consumption by different control schemes. In addition, we also need to compare the cost function



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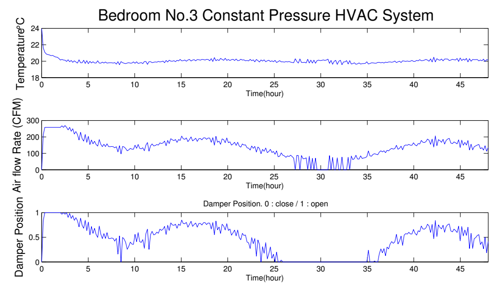

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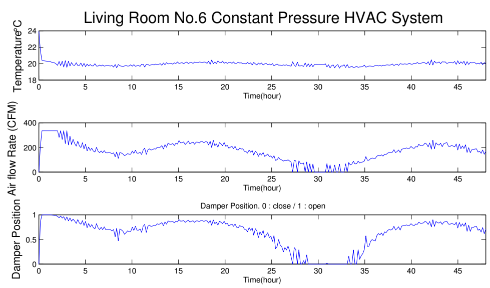

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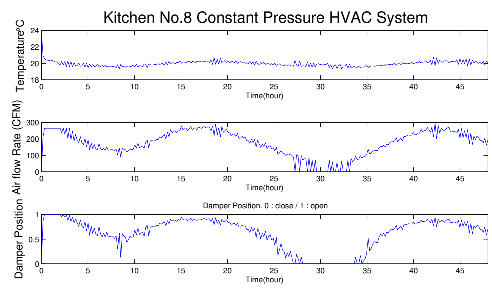

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Figure 6.5: $\alpha_i = 1$; $\gamma_i = 1$, MPC Using in the Constant Pressure HVAC System in Two Days. (a) Temperature (up), Air Flow Rate (middle), and Damper Position (Bottom) for Bedroom No.1. (b) Temperature (up), Air Flow Rate (middle), and Damper Position (Bottom) for Bedroom No.3. (c) Temperature (up), Air Flow Rate (middle), and Damper Position (Bottom) for Living Room No.6. (d) Temperature (up), Air Flow Rate (middle), and Damper Position (Bottom) for Kitchen No.8.

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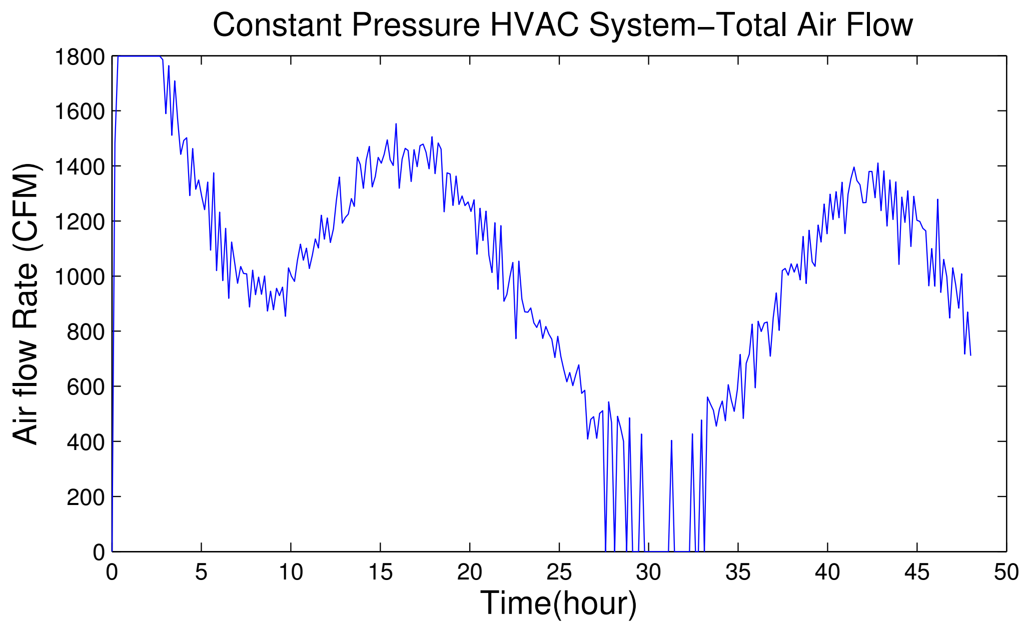
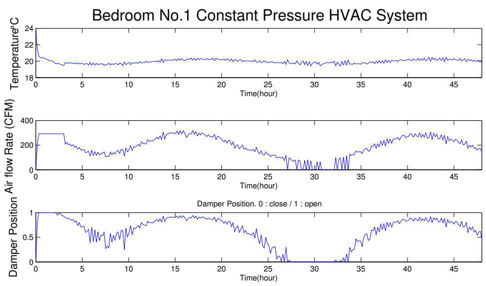


Figure 6.6: Total Air Flow Rate for MPC Using in the Constant Pressure HVAC System in Two Days. $\alpha_i = 1$; $\gamma_i = 1$

value,

$$J = \sum_{i=1}^{N_c} \alpha_i u(k+i|k)^2 + \gamma \sum_{i=1}^{N_p} [y_{ref}(k+i|k) - y(k+i|k)]^2 \tag{6.3}$$

Table 6.2 provide a comparison of the cost function value in different control schemes. It is evident that the MPC case 1 scheme and the Single Sensor two position On/Off are the most energy consuming methods. This is because the air flow rates are constant (1800 CFM), when the AHU is on. However, since MPC case 1 does not need to switch position (On/Off) frequently, it saves energy in the transients. In summary, we can see that the On/Off control method a lot more energy than others. However, due to the cheap price of On/Off control, it is still widely used in HVAC systems. The MPC case 2 is the most efficient, it can save energy and have the best performance in cost function, however it may need several advance controller in the HVAC system, which raises the equipment costs compare the two position on/off control. In conclusion, we need find a trade off between the energy consumption and



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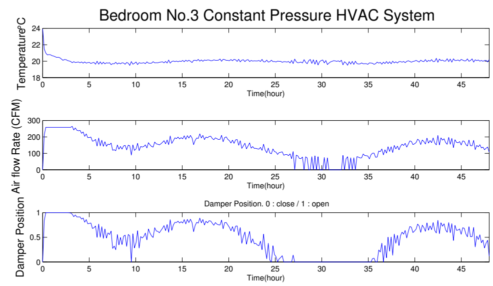

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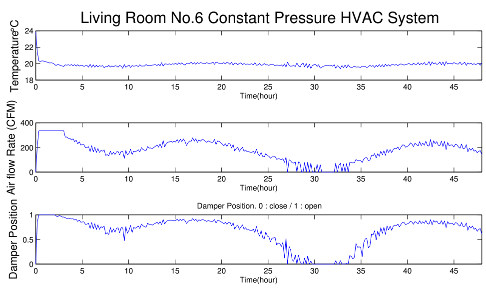

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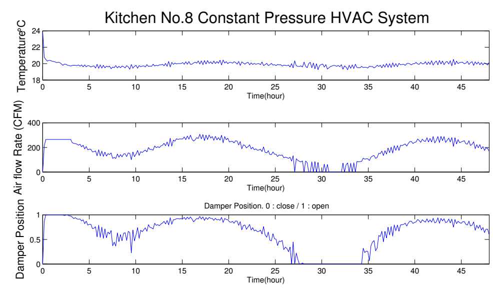

(d)

Figure 6.7: $\alpha_i = 0.1$; $\gamma_i = 1$ MPC Using in the Constant Pressure HVAC System in Two Days. (a) Temperature (up), Air Flow Rate (middle), and Damper Position (Bottom) for Bedroom No.1. (b) Temperature (up), Air Flow Rate (middle), and Damper Position (Bottom) for Bedroom No.3. (c) Temperature (up), Air Flow Rate (middle), and Damper Position (Bottom) for Living Room No.6. (d) Temperature (up), Air Flow Rate (middle), and Damper Position (Bottom) for Kitchen No.8.

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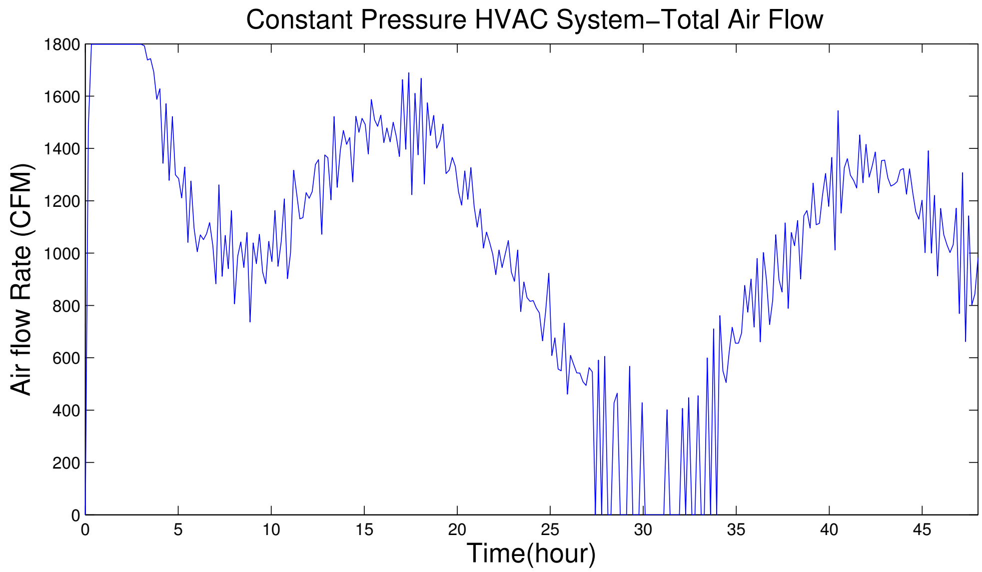

Figure 6.8: Total Air Flow Rate for MPC Using in the Constant Pressure HVAC System in Two Days. $\alpha_i = 0.1$; $\gamma_i = 1$

equipment cost.

Table 6.1: Energy Consumption Compare in Five Control Schemes in Two Days

Control Scheme	Flow Volume (m^3)	Energy Consumption (KJ)
Single Sensor On/Off Control	51378.9	62221
Multiple Sensor On/Off Control in Case 1	54844.7	66418
Multiple Sensor On/Off Control in Case 2	39776.7	48171
MPC in Case 1	63203.2	76540
MPC in Case 2	38779.9	46963

Case 1: Constant Current HVAC System
Case 2: Constant Pressure HVAC System
$\alpha_i = 1$; $\gamma_i = 1$

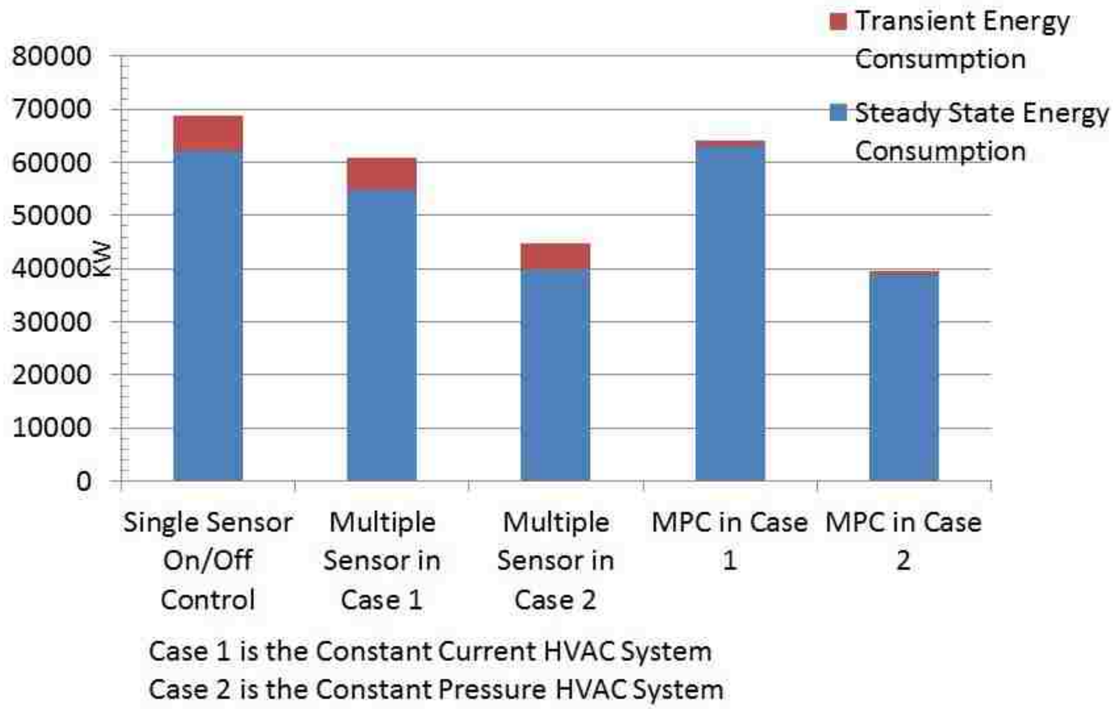

Figure 6.9: Energy Consumption Compare in Five Control Schemes in Two Days.

Table 6.2: Average Cost Function J Value Compare in Five Control Schemes in Two Days

Control Scheme	α_i=0.1, γ=1	α_i=1, γ=0.1	α_i=1, γ=1
Single Sensor On/Off Control	7.2570	1.6126	8.0632
Multiple Sensor On/Off Control in Case 1	5.7872	1.8443	6.9377
Multiple Sensor On/Off Control in Case 2	5.5437	1.6421	6.5326
MPC in Case 1	16.5564	1.7783	16.6039
MPC in Case 2	0.9578	1.2119	1.9500

Case 1: Constant Current HVAC System
Case 2: Constant Pressure HVAC System

Chapter 7

Conclusion and Future Works

7.1 Summary

Chapter 1 illustrates the problems of building HVAC systems in the 21st century. Saving energy is a global issue. Modern control schemes can be applied energy saving in building HVAC systems.

Chapter 2 is a general introduction to the building model based on heat transfer and other heat generation source.

Chapter 3 talks about the duct system design and analysis. The duct system model has been used in this HVAC research to calculate air flow.

In chapter 4, the traditional two position On/Off control is introduced. This control scheme based on a single sensor and multiple sensors is analyzed, based on constant current HVAC systems and constant pressure HVAC systems.

In chapter 5, MPC control modeling and design was discussed. The advantage of MPC control schemes were presented. The control task was posed as an optimization problem with constraints.

In chapter 6, a comparison was made between the traditional On/Off control and MPC scheme. It is found that MPC can save more energy and provide higher comfort.

7.2 Conclusion

In order to use the MPC scheme for the HVAC system in a building, a state space thermal model for the building was obtained. A state space model was obtained from an RC (resistor-capacitor) network which simulates the thermal capacitance and the thermal resistance of walls, roof, and windows (See Chapter 2). Based on the state space model obtained, a MPC cooling strategy was designed.

Simulation results for the five control schemes (single sensor two-position On/Off control, multiple sensor two-position On/Off control in constant current HVAC system, multiple sensor two-position On/Off control in constant pressure HVAC system, MPC scheme in constant current HVAC system, and MPC scheme in constant pressure HVAC system) were compared. Finally, the MPC scheme was found better than the two position On/Off control, not only in terms of energy consumption, but also in terms of control performance. Moreover, MPC does not need to switch the AHU on and off frequently. Therefore it can not only save energy (transient power part) but also minimize equipment damage.

Bibliography

[1] Morosan P.D, Bourdais R., Dumur D., Buisson J. "A dynamic horizon distributed predictive control approach for temperature regulation in multi-zone buildings," *18th Mediterranean Conference on Control and Automation*, June 23-25, 2010.

[2] Bergman T.L., Lavine A.S., Incropera F.P., and DeWitt D.P., *Fundamentals of Heat and Mass Transfer*, John Wiley and Sons, Inc., 2007.

[3] American Society of Heating, Refrigeration and Air Conditioning Engineers, "ASHRAE Fundamentals SI Edition 2009," ASHRAE, 2009.

[4] McQuiston F.C, Parker J.D., Spitler J.D. *Heating Ventilation, and Air Conditioning*, John Wiley and Sons, Inc., 2005.

[5] *Engineering Manual of Automatic Control for Commercial Buildings SI Edition*, Honeywell Inc., 1997.

[6] Aswani A., Master N., Taneja J., Culler D., and Tomlin C., "Reducing Transient and Steady State Electricity Consumption in HVAC Using Learning-Based Model-Predictive Control," *Proceedings of The IEEE*, Vol. 100, No.1, January 2012.

[7] Camacho E.F., Alba C.B., *Model Predictive Control*, Springer, 2nd edition, 2007.

[8] Wang L., *Model Predictive Control System Design and Implementation Using MATLAB*, Springer, 2009

[9] Maciejowski J.M., *Predictive Control with Constraints* , Prentice Hall, 2000

[10] Hazyuk I., Ghiaus C., and Penhouet D., "Optimal Temperature Control of Intermittently Heated Buildings Using Model Predictive Control: Part II-Control Algorithm," *Building and Enviromment*, vol. 51, 2012, p388-394.

[11] Schuster E. *et al.*, "Antiwindup Scheme for Plasma Shape Control with Rate andMagnitude Actuation Constraints in the DIII-D Tokamak," in *Proceeding of the 2003 IEEE International Conference on Decision and Control*, 2003.

[12] Nesterov J.E. and Nemirovsky A.S., *Interior Point Polynomial Methods in Convx Programming: Theory and Applications*, SIAM, Philadelphia, 1994.

[13] Wright S.J., *Primal-Dual Interior-Point Methods*, SIAM, Philadelphia, 1997.

[14] Morosan, P.D., Bourdais, R., Dumur D., and Buisson, J, "Distributed model predictive control based on Benders' decomposition applied to multisource multizone building temperature regulation," *49th IEEE Conference on Decision and Control.*, December 2010, p3914-4919.

Vita

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