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Mathematical Modeling and Optimization of Three-echelon Capacitated Supply Chain Network Design

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Mathematical Modeling and Optimization of Three-echelon Capacitated Supply Chain Network Design

by

Sahand Ashtab

A Dissertation

Submitted to the Faculty of Graduate Studies

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Mathematical Modeling and Optimization of Three-echelon Capacitated

Supply Chain Network Design

by

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Declaration of Co-authorship/ Previous Publication

Co-Authorship Declaration

I hereby declare that the key ideas, primary contributions, experimental designs, data analysis and interpretation, in the papers mentioned in the table below, were performed by the author, and supervised by Dr. Richard J. Caron and Dr. Esaignani Selvarajah as co-advisors.

I am aware of the University of Windsor Senate Policy on Authorship and I certify that I have properly acknowledged the contribution of other researchers to my dissertation, and have obtained written permission from each of the co-authors to include the published, accepted and submitted papers in my thesis.

I certify that, with the above qualification, this dissertation, and the research to which it refers, is the product of my own work.

Declaration of Previous Publication

This dissertation includes 3 original papers that have been previously published/accepted for publication in peer reviewed journals and proceedings of refereed conferences as well as 1 submitted paper.

Thesis Chapter	Publication title/full citation	Publication status
Chapter 3	**S. Ashtab**, R. J. Caron, E. Selvarajah, A binary quadratic optimization model for three level supply chain design, **Procedia CIRP** 17 (2014) 635-638	Published
Chapter 4	**S. Ashtab**, R. J. Caron, E. Selvarajah, A characterization of alternate optimal solutions for a supply chain network design model, **INFOR: Information Systems and Operational Research** 53 (2015) 90-93	Accepted
Chapter 5	**S. Ashtab**, R. J. Caron, E. Selvarajah, Increased flexibility in three-echelon multi-capacitated supply chain design model, **International Journal of Operational Research**	Submitted
Chapter 6	**S. Ashtab**, R. J. Caron, E. Selvarajah, A Binary, non-convex, variable-capacitated supply chain model **International Journal of Supply Chain Management** 4 (2015) 25-29	Published

I certify that I have obtained a written permission from the copyright owners to include the published/accepted materials in my dissertation. In particular, while acknowledging the original contributions and ideas made in the paper entitled "A Binary Quadratic Optimization Model for Three Level Supply Chain Design", few changes have been made in this paper before including it in the dissertation. I certify that the above materials describe work completed during my registration as graduate student at the University of Windsor.

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Abstract

This dissertation is concerned with mathematical modeling and optimization of three-echelon capacitated supply chain network design (SCND) with suppliers, distribution centers (DCs) and customer zones. An introduction to SCND is provided followed by a literature review. Being inspired by a real world SCND problem, a Mixed Integer Linear Program (MILP) is developed for a three-echelon SCND. The model takes into account the operational costs of a built DC based on its actual activity level rather than the assumption that a built DC operates at the maximum capacity. The suppliers and customer zones are at fixed known locations and the DCs are picked from a set of potential DC locations. Then, a well-known model on three-echelon multi-capacitated SCND [1] is studied where both suppliers and DCs and their corresponding capacity levels are picked from a set of potential supplier and DC locations and a set of predetermined discrete capacity levels, respectively. We characterize the complete set of alternate optimal solutions in [1]. We extend the model through the addition of a constraint set that eliminates certain undesirable optimal solutions and we show that the extended model requires, essentially, the same computational effort as the original. We then deploy a new set of variables and present a new formulation for three-echelon multi-capacitated SCND. We show that the new formulation is more efficient as it offers lower computational times. We then present two approaches which allow an exponential increase in available capacity levels to facilities. We demonstrate the merits of the exponentially increased flexibility. Inspired by the merits of increased flexibility in capacity assignment, we present a technology-based, variable-capacitated, supply chain design model that is unique in that it allows complete variability in the choice of capacity level. This avoids the need to determine, a priori, a set of potential capacity levels. Another merit of the model is that the built facilities do not have unused capacity. Last, the conclusions and future work are provided.

Dedication

I present this dissertation to my parents.

Acknowledgments

I extend my most sincere thanks to Dr. Richard J. Caron and Dr. Esaignani Selvarajah for their support, leadership and encouragement. Their guidance made the creation of this document a most interesting and enjoyable experience for me. I also offer my thanks to my committee members, Dr. Samir Elhedhli, Dr. Yash Aneja, Dr. Fazle Baki and Dr. Guoqing Zhang for their helpful comments in improving my dissertation. My deepest gratitude is extended to my parents for their encouragement and support. My rewards and accomplishments would not have been achieved without their support.

Contents

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Initialisms

DC	Distribution Center
SCND	Supply Chain Network Design
UFL	Uncapacitated Facility Location
SPLP	Simple Plant Location Problem
CPLP	Capacitated Plant Location Problem
LP	Linear Program
MILP	Mixed Integer Linear Program
CMILP	Cluster Mixed Integer Linear Program
BO	Binary Non-linear Optimization
PWL	Piecewise Linear

Chapter 1

1. Introduction

A supply chain network consists of several linked echelons which work together to fulfill customers demand and maximize the profitability of the supply chain or to minimize its overall costs. Higher profitability for a supply chain can be achieved when the involved echelons do not focus on maximizing their own profit, and instead, work closely together to increase the overall supply chain profitability. Product, fund and information are the major flows that connect the supply chain echelons with each other and it is shown in figure 1.
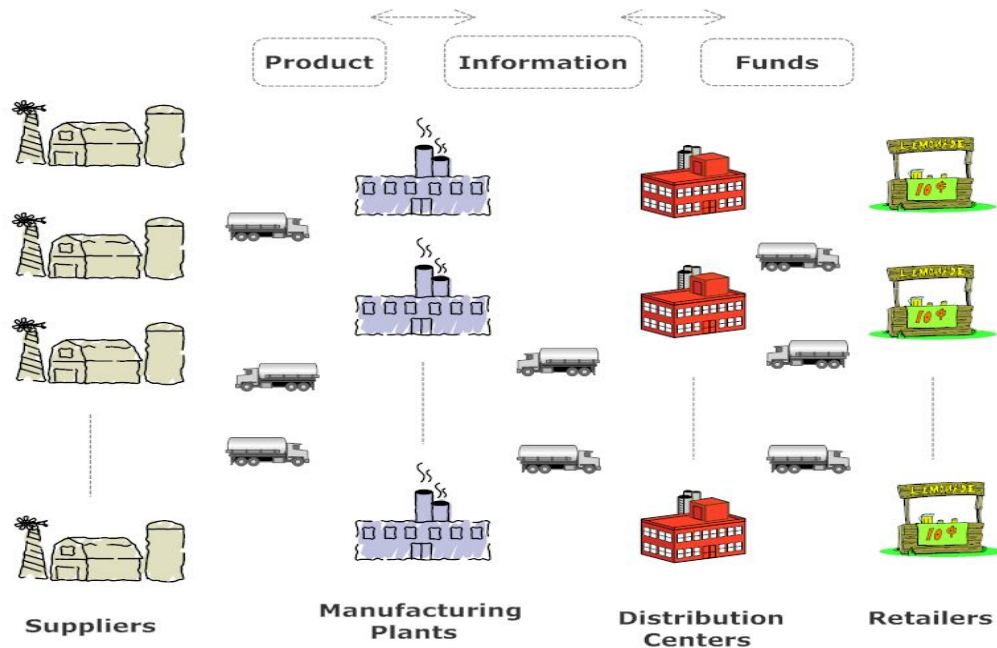
Figure 1: Flows in a supply chain network

The term echelon refers to each level or stage in the supply chain network. For example, a supply chain network with customer zones and distribution centers can be referred to as a two-level or two-echelon supply chain network.

The four echelons in a typical supply chain are suppliers, manufacturing plants, DCs and customer zones. Not all companies have all four echelons in their supply chain. The configuration of a supply chain network depends on the customer's needs and the role each echelon plays to fulfill them. For example, companies which ship their final product directly from the manufacturing plant to the final customer can save funds by not establishing and operating DCs. In year 2007, DELL bypassed its DCs and customer zones (retailers) and sold its products directly to final customers [2].

There are three different decision phases for supply chain management; strategic, planning (tactical) and operational [2]. The first phase is concerned with the long-term and large investments in the supply chain configuration. The optimal supply chain configuration is critical for companies wishing to become or stay competitive in the global market place. The determination of the number, location and the capacity level of the established plants throughout the supply chain are among the most important decisions to be made in this phase. The second phase is concerned with determining a guideline for the supply chain functions over a specific period of time, i.e., quarter of a year. Demand forecasting, inventory planning, determination of the products flow between the supply chain echelons are the important components of the planning phase. In the third phase, the time horizon is a week or day. The decisions in this phase concern the fulfillment of final customer demand. DC operations [3], setting delivery schedules of trucks, handling incoming customer demands and placing replenishment orders are among the daily operational level decisions to be

made. We are concerned with the strategic level decisions to determine the number, location, and capacity of the facilities; and with the tactical decision to determine the flow between the facilities such that the total network cost is minimized and the total demand is fulfilled. The network cost includes building, operational and land costs of establishing DCs and suppliers, and in-bound and out-bound transportation costs.

One of the major trade-offs which influences the supply chain design is between the level of responsiveness to the customers and the efficiency of operating the supply chain. If we want to design and operate a fully responsive supply chain network we have to establish our DCs as close as possible to our customer zones to ensure fast replenishments. This requires the establishment of more and smaller DCs which makes it a costly supply chain network to operate. The relationship between desired response time and number of facilities is given in figure 2 [2]. We see that as the desired response time increases, as expected, the required number of facilities decreases. We also note the decreasing slope of the function. That is, as the response times increases the rate of decrease in the number of required facilities decreases.
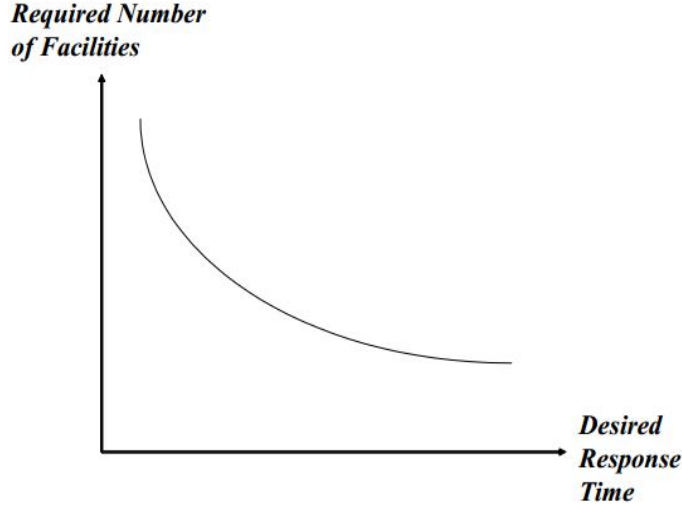
Figure 2: Relationship between desired response time and number of facilities [2].

On the other hand, with centralization policy fewer and larger DCs are built to supply the total demand of the network. This would help to significantly reduce the costs of operating the supply chain network. Due to economies of scale the centralization policy is very economical and efficient. Thus, supply chains operating with centralization policy are referred to as efficient supply chains. For example, operational and administration cost per unit of output in a large DC would be less than a small DC. In summary, the main objective of a responsive supply chain is to supply the network demand as quickly as possible and the main objective of an efficient supply chain is to supply the network demand at the lowest costs. A comparison between a responsive and an efficient supply chain is given in table 1 [2].

	Efficient Supply Chains	Responsive Supply Chains
Primary goal	Minimum cost	Minimum response time
Manufacturing strategy	High utilization	Maintain capacity flexibility
Inventory Strategy	Minimal inventory	Buffer inventory
Lead time	Reduce, not at the expense of costs	Reduce aggressively
Supplier strategy	cost and quality	speed, flexibility and quality

Table 1: Comparison of efficient and responsive supply chains [2].

We can classify the supply chain network design problem into two main types in terms of capacity assignment. The first type of supply chain network design problem is the most basic model developed and is referred to as Simple Plant Location Problem (SPLP) or Uncapacitated Facility Location (UFL) problem [4]. The customer zones are at fixed known locations. DCs are selected from a set of potential DC locations. The objective function is to minimize the summation of the transportation costs and the fixed costs which are associated with establishing DCs. In SPLP it is assumed that if a DC is built it can supply the total demand of the customer zones assigned to it. One major drawback of the SPLP model is that the fixed costs of DCs only depend on their location; however, the size of a DC also has to be taken into account in determining its fixed land and building costs. Another drawback for the SPLP model is that it does not consider the capacity restrictions on built facilities.

The second type of supply chain network design problem is referred to as Capacitated Plant Location Problem (CPLP). CPLP model is the SPLP model with added constraints that enforce capacity restrictions on the built DCs.

CPLP models can be divided into two subgroups: single capacitated supply chain network design model [5, 6] and multi capacitated supply chain network design model [1]. In single capacitated supply chain network design, if a DC is

built there will be a single capacity level available to be assigned to it. In multi capacitated supply chain network design, if a DC is built a capacity level will be chosen from a predetermined set of discrete capacity levels to be assigned to it.

In SPLP and CPLP models, the assignment of customer zones to DCs, or DCs to suppliers determines if the supply chain network design model is single-source or multiple-source. In single-source supply chain network design model any individual customer zone is supplied by only a unique DC. [7] is among the early papers that discusses single-source and multi commodity distribution system. In multi source case, more than one DC can supply an individual customer zone.

Mathematical formulation for single-source model involves use of binary variables. If a customer zone is assigned to a built DC, the binary variable will have the value of one and it will be zero otherwise. However, continuous variables can be used to formulate the mathematical model for multi-source model. Indeed, the computational times for solving the single-source model will be more than the multi-source model due to involvement of binary integer variables.

1.1. Research scope and outline of dissertation

1.1.1. Scope

This dissertation is concerned with mathematical modelling and optimization of three-echelon capacitated supply chain network design with suppliers, DCs and customer zones. The focus is to provide new and improved mathematical models for SCND which captures the following features.

1) Inspired by a real world SCND problem, we provide cost functions for facilities that give an insight to the factors which influence facility-oriented costs.

2) Provision of more flexibility in capacity assignment to the facilities that are built. This would allow for less costly SCND.

3) Improved mathematical formulations that offer lower computational times and reduce SCND costs. Reduced *cost* and *time* are indeed significantly important factors for companies.

1.1.2. Outline of this dissertation

Chapter two provides a literature review on supply chain network design. Chapter three studies a supply chain network design with customer zones at fixed known locations with known demand, and suppliers at known locations with sufficient capacity to supply the network demand. The DCs are picked from a potential set of DC locations. The capacity level for the built DCs is picked from a predetermined set of discrete capacity levels. Our proposed supply chain network design model captures the operational costs of a built DC based on its actual activity level and not assuming by default that built DCs are functioning at their maximum capacity limit.

Chapter four studies a supply chain network design with customer zones at fixed known locations with known demand, and with suppliers and DCs whose location and corresponding capacity levels are picked from a set of potential suppliers and DC locations, and a set of predetermined set of discrete capacity levels, respectively. We extend the model by provision of a set of constraints to

eliminate undesirable alternate optimal solutions.

In chapter five, we provide a new mathematical formulation for our extended model from chapter four. It is done via deployment of new sets of variables. We show that our new model requires less computational efforts and thus is suggested for three-echelon multi-capacitated supply chain network design with characteristics discussed in this chapter. Further more, we present and evaluate the performance of two approaches to exponentially increase the flexibility in capacity assignment to built facilities. This increased flexibility can only lead to less costly supply chain network design. Subsequently, we determine the superior approach.

In chapter six, we present a variable-capacitated, technology-based supply chain network design model. The model is unique in that, it allows complete variability in the choice of capacity level and so avoids the need to determine, a priori, a set of potential capacity levels. Another merit of this model is, there would be no un-used capacity level in a built DC. The building costs and operational costs in a built DC are concave functions of the shipment volume to fully capture economies of scale based on the activity level.

This dissertation includes 3 original papers that have been previously published/accepted for publication in peer reviewed journals and proceedings of refereed conferences. Chapter three includes the published paper entitled "A binary quadratic optimization model for three level supply chain design"[8] with some changes while acknowledging the major ideas and contributions from the paper. Chapter four includes the accepted paper entitled "A characterization of

alternate optimal solutions for a supply chain network design model"[9]. Chapter six includes the published paper entitled "A Binary, non-convex, variable-capacitated supply chain model"[10].

1.2. Engineering motivation of this research

This dissertation is motivated by a real world SCND problem for an Iconic Canadian Company with operations across Canada and, in the United States of America. The company has an existing Canadian network of $2\,976$ customer zones that are supplied by 13 existing DCs that are supplied by 47 suppliers.

1.3. Significance of this research

This dissertation is concerned with the strategic level decisions to determine the number, location, and capacity level of the facilities, i.e., DCs, and with the tactical decision to determine the flow between echelons. These decisions are associated with important and long-term investments. According to the results of a real case study, companies can save up to millions of dollars every year by using effective optimization mathematical models to establish their supply chain design.

1.4. Research gaps and contributions

In the literature, facility oriented costs such as building costs, land costs and operational costs are often incorporated into a single constant; however, each cost element is influenced by different factors. We develop a mixed integer linear mathematical model for a three-echelon multi-capacitated supply chain network design that captures the following features. The model takes into account the operational costs of a facility, i.e., DC, based on the actual activity level of

a DC while taking into account the economies of scale. The building costs depend on the size of the built facility and the land costs depend both on the location and the size. The inclusion of the operational costs of the built DCs is done via addition of continuous variables and constraints. We provide a model simplification for a large scale supply chain design case study which reduces the problem size. We show the effectiveness of our model and the simplification by solving the large scale supply chain design problem using LINGO ®.

We study a well-known model in literature for three-echelon multi-capacitated supply chain network design [1]. We characterize the complete set of alternate optimal solutions in the model presented by Amiri [1]. We extend Amiri's model through the addition of a constraint set that eliminates certain undesirable optimal solutions and we show that the extended model requires, essentially, the same computational effort as the original. Furthermore, we propose an improved mathematical model for three-echelon multi-capacitated supply chain design. We show that the new model can be solved to optimality 58% faster, in average, than the extended model for our test problems.

In the literature, we observe a positive trend in terms of the flexibility in capacity assignment to the built facilities in the supply network. Starting with uncapacitated (UFL) and then extended to single-capacitated and multi-capacitated supply chain designs. This increased flexibility is indeed desirable because increasing the flexibility in capacity assignment can only lead to less costly supply chain designs. In this regard, we propose two approaches to exponentially increase the flexibility in capacity assignment. We show that ex-

ponentially increased flexibility that lead to less costly supply chain design can be obtained in short computational times, i.e., 10 seconds.

The models discussed in literature for capacitated supply chain network design pick a capacity level from a predetermined set of discrete capacity level(s) and assigns it to a built facility, i.e., DC. For example, if the network demand in a supply chain is 2 200 units and the available potential capacity levels for potential DCs are 2 000 and 4 000 units, then the model will be pushed to pick and assign the capacity level of 4 000 to a built DC to cover only 2 200 demand units. In deterministic supply chain design it may not be ideal as the DC may have unused capacity level. A novel model is introduced to the literature with a unique feature that allows complete variability in the choice of capacity levels and so avoids the need to determine, a priori, a set of potential capacity levels. The facility oriented costs are captured by concave functions of shipments to capture economies of scale. It is a concave, binary optimization (BO) problem. A technology-based piecewise linearization (PWL) approach is utilized to solve the model in shorter computational times. As PWL is also a non-linear program, we develop a mixed integer linear program (MILP). It is shown that while MILP model has more variables and constraints than BO and PWL models, as expected, it produces the optimal solution in the shortest computational times. To our best knowledge, our proposed MILP model is the first model which captures technology break points and economies of scale at the same time while allowing complete flexibility in capacity assignment to the built facilities in the supply chain network design.

Before proceeding to the literature review in chapter 2, we provide a table

of notation used throughout this dissertation.

	Indices
i	index for customer zones
j	index for DCs
k	index for suppliers
t	index for the capacity level of DCs
h	index for the capacity level of suppliers
l	index for technology level of a DC
	Decision Variables
x	variable for building a DC
q	variable for building a supplier
y	variable for assignment of customer zones to DCs
u	variable for assignment of a DC to a supplier
	Variables
z	shipment volume from DC to customer zones
w	shipment volume from supplier to DC
f	fixed costs of building a DC
g	fixed costs of building a supplier
c	total shipping costs to fully fulfill customer zone demand
p	pallet demand at a customer zone
b	building costs
v	variable costs
a	slope of the technology-based linear cost function
n	intercept of the technology-based linear cost function
d	distance
	Parameters
ρ	percentage of a pallet from a supplier on an outbound pallet
ω	cost of shipping one pallet one kilometer

Table 2: Notation

Chapter 2

2. Literature Review

Excellent reviews of the published literature on supply chains can be found in [11, 12, 13, 14]. Contributions to supply chain design include solutions to the determination of facility location, DC capacity, inventory levels, transportation planning (routing and scheduling), stochastic demand, etc., as well as combinations of the above. The focus of this dissertation is development and optimization of mathematical models for three-echelon capacitated supply chain network design.

Early works on SPLP model are presented in [15, 16, 17, 18]. More studies on SPLP model can be found in [4, 19, 20, 21, 22, 23, 24]. We begin with a SPLP model formulation. Customer zones and DCs are indexed by i and j, respectively. Let y_{ji} be the fraction of demand shipped from a DC at site j to a customer zone at site i. The variable associated with establishment of a potential DC is denoted by x_j. Its value is 1 if DC is built at site j and otherwise it is 0. Let f_j be the fixed costs of building a DC at site j. Let c_{ji} be the transportation cost per period for supplying all the demand at customer zone i from DC at site j. The mixed integer linear model for SPLP presented in [4] is given below. Note that the y_{ji} are continuous variables and thus the SPLP model presented below is for a multi-source supply chain network design.

$$\text{Minimize} \quad \sum_{j \in J} f_j x_j + \sum_{j \in J} \sum_{i \in I} c_{ji} y_{ji}$$

Subject to:

$$\sum_{j \in J} y_{ji} = 1 \quad \forall i \in I, \tag{1}$$

$$y_{ji} \leq x_j \quad \forall i \in I, j \in J, \tag{2}$$

$$x_j \in \{0, 1\} \quad \forall j \in J, \tag{3}$$

$$y_{ji} \in [0, 1] \quad \forall i \in I, j \in J, \tag{4}$$

The objective function is to minimize the total fixed costs of building DCs and the total transportation costs. Constraint set (1) ensures that the demand at customer zone i is fully served by the assigned DCs. Constraint set (2) ensures that no DC can be assigned to serve a customer zone unless it is built. That x variables are binary is given by constraint set (3) and constraint set (4) defines y variables to be continuous.

A SPLP model with linear inventory costs is studied in [25]. The inventory costs are represented by a linear function of the number of built DCs and embedded directly into the mathematical formulation. In order to incorporate the inventory cost into the design model, the value of slope from inventory cost's linear regression function becomes part of f_j because it reflects a constant increase in the inventory cost for building a new DC.

In [26], a pure integer non-linear program is presented for a SPLP with convex transportation costs. Demand at customer zone i is denoted by p_i. Unlike the previous model where y_{ji} was the fraction of demand supplied to customer zone i from DC j, in the following model, the y_{ji} variable denotes the

flow of products between DC j and customer zone i. The transportation cost for shipping products from DC j to customer zone i is $g(y_{ji})$ which is a convex nonlinear function. The following model is obtained.

$$\text{Minimize} \quad \sum_{j \in J} \sum_{i \in I} g(y_{ji}) + \sum_{j \in J} f_j x_j$$

Subject to:

$$\sum_{j \in J} y_{ji} = p_i \quad \forall i \in I, \tag{5}$$

$$y_{ji} \leq p_i\, x_j \quad \forall i \in I, j \in J, \tag{6}$$

$$x_j \in \{0, 1\} \quad \forall j \in J, \tag{7}$$

$$y_{ji} \geq 0, \quad \text{integer}, \forall i \in I, j \in J \tag{8}$$

Constraint set (5) ensures that the product flow between DC j and customer zone i is equal to the demand at customer zone i. Constraint set (6) ensures that no shipment can be made from DC j to customer zone i unless it is built. Constraint set (7) is to define the x variables to be binary. Constraint set (8) is to enforce the y variables to take positive integer values. Integer requirement on the transported amounts are assumed which enables making an exact linearization of the nonlinear costs. That is, the cost function for each variable y_{ji} in the interval $0 \leq y_{ji} \leq p_i$ is linearized with break points at each integer point. This is done via addition of binary variables and the number of binary variables depends on the values of the demands.

In [27], a non-linear SPLP problem is studied where the total fixed costs of

the DCs in the supply network is non-linear function of the number of built DCs. The cost elements which form the total fixed costs are annual discounted capitalization cost, annual operating and maintenance cost and depreciation. Only the convex case of the non-linear fixed costs is studied. In [28] a branch and bound algorithm is proposed to solve the location problem. A dual based optimization procedure is presented in [29] for three-echelon and in [30] for two-echelon SPLP model. Multi product SPLP model is studied in [31, 32].

Unlike the SPLP model where the total number of facilities, i.e., DCs, is unknown and will be determined by the model's solution, in p-median problems the total number of built facilities is a constant and given. Studies on p-median problem can be found in [33, 34, 35]. The goal in p-median problem is to locate p facilities in the supply chain network to serve q customer zones such that the summation of weighted distances is minimized. The weight of a customer zone can be determined or represented by its demand. A p-median model presented in [34] is given below.

$$\text{Minimize} \quad \sum_{j \in J} \sum_{i \in I} p_i \, d_{ji} \, y_{ji}$$

Subject to:

$$\sum_{j \in J} y_{ji} = 1 \quad \forall i \in I \tag{9}$$

$$y_{ji} \leq x_j \quad \forall i \in I, j \in J \tag{10}$$

$$\sum_{j \in J} x_j = p \tag{11}$$

$$x_j \in \{0, 1\} \tag{12}$$

$$y_{ji} \in \{0, 1\} \tag{13}$$

Let d_{ji} be the distance from the DC j to customer zone i and p_i be the demand at customer zone i. Constraint set (9) and constraint set (10) are the same as constraint set (1) and (2), respectively. Constraint set (11) sets the total number of built DCs to be p. Constraint sets (12) and (13) are to enforce the x and y variables to be binary.

We next present single-capacitated supply chain network design model [5, 36, 37, 38]. There are potential DC locations in the supply chain network from which some will be built to deliver a product to customer zones with known demands. The DCs have limited capacity and incur a fixed cost if they are built. At each DC site there is only a single potential capacity level available which can be assigned to a built DC. Thus, it is a single-capacitated supply chain design model. The total transportation costs to supply a product to a specific customer zone from each potential DC is also given. The mathematical formulation is as follows. It is a pure integer linear program. Let y_{ji} be 1 if customer zone i is assigned to DC j and be 0 otherwise. Let c_{ji} be the cost of

serving customer zone i by DC j. It is a single source design model. Let x_j be 1 if DC j is built and be 0 otherwise. Let t_j be the capacity level which will be assigned to a built DC at site j. A two-echelon single-capacitated design model is studied in [5] and is given below.

$$\text{Minimize} \quad \sum_{j \in J} f_j \, x_j + \sum_{i \in I} \sum_{j \in J} c_{ji} \, y_{ji}$$

Subject to:

$$\sum_{j \in J} y_{ji} = 1 \quad \forall i \in I \tag{14}$$

$$\sum_{i \in I} p_i \, y_{ji} \le t_j \, x_j \quad \forall j \in J \tag{15}$$

$$y_{ji} \le x_j \quad \forall i \in I, j \in J \tag{16}$$

$$x_j \in \{0, 1\} \tag{17}$$

$$y_{ji} \in \{0, 1\} \tag{18}$$

The objective function minimizes the total fixed costs and transportation costs. Constraint set (14) together with constraint set (18) assign a unique DC to serve a customer zone. Constraint set (15) ensures that if a DC is built it can not ship more than its assigned capacity level. Constraint set (16) ensures that if a DC is not built it will not be assigned to serve any customer zone. Constraint sets (17) and (18) are to enforce the x and y variables to be binary, respectively.

Suppose that for some $\hat{j} \in J$ we have $x_j = 0$. From (15), we get that

$$\sum_{i \in I} p_i \, y_{ji} \leq 0.$$

Since $p_i > 0$, $\forall i \in I$ and since y_{ji} is binary, this implies that $y_{ji} = 0$ $\forall i \in I$ establishing that constraint set (16) is redundant in the above integer program (IP) formulation; however, it is not necessarily redundant in the relaxed linear program (LP) formulation. In fact, having constraint set (16) in the relaxed LP formulation tightens the feasible region, and therefore, may well accelerate the computational times. Below we provide a simple example to show how the addition of constraint set (16) tightens the feasible region. Suppose a supply chain with one DC with potential capacity level of 500 units that will incur a fixed cost of 1 000 if it is built. There is one customer zone with demand of 200 units. The total shipping costs from the DC to the customer zone to fulfill the demand is 1 500. The relaxed LP formulation is provided below. The objective function is to minimize the total fixed and transportation costs and is given by,

$$\text{Minimize} \quad 1\,000\,x_1 + 1\,500\,y_{11}.$$

First set of constraints are to ensure that each customer zone is served by a built DC. In our example, we have one customer zone which has to be served by the only DC and, therefore, we get $y_{11} = 1$. Second set of constraints are to ensure that the shipment from a built DC is not exceeding its capacity level. In our example, we get $200\,y_{11} \leq 500\,x_1$. From $y_{11} = 1$ and $200\,y_{11} \leq 500\,x_1$, we get $x_1 \geq 0.4$. From $x_1 \in [0,1]$, we know that the upper bound for variable

x is one and, thus, we have $0.4 \leq x_1 \leq 1$. Adding constraint set (16), $y_{11} \leq x_1$, while having $y_{11} = 1$, yields $x_1 \geq 1$. From $0.4 \leq x_1 \leq 1$ and $x_1 \geq 1$, we can conclude that $x_1 = 1$. We see that, in this example, addition of the constraint set (16) tightened the bounds of the x variable. This leads to a better mathematical formulation and would indeed accelerate the computational times. We also demonstrate this fact by solving 10 random test problems generated by the procedure given in [1] for a supply chain with 500 customer zones and 15 potential DCs. We use LINGO $^{\circledR}$ 14.0 x64 on a DELL server with 32 G of RAM and two 2.50 GHz CPUs to solve our test problems. The computational times for the model including constraint set (16) are reported in the second column of table 3. The computational times for the model excluding constraint set (16) are reported in the third column of table 3. We observe that, as expected, having constraint set (16) accelerated the computational times.

#	With (Sec)	Without (Sec)
1	3	6
2	2	6
3	2	7
4	2	7
5	2	4
6	2	4
7	2	3
8	2	4
9	2	5
10	2	6

Table 3: Computational times to optimality with and without constraint set (16).

In [6], a computer program is developed for a three-echelon single-capacitated supply chain network design with non-linear inventory costs and multiple products. There is a capacity restriction for each product in each manufacturing plant and built DC. Manufacturers are at known locations and DCs will be

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picked from a set of potential DC sites. The program starts with initiation of number of DCs. Then it estimates the throughput on each available DC given the facility capacity restrictions. Once the DC throughput is established, the DC fixed costs and inventory costs are computed on a per-unit basis. These per unit costs are then added to per-unit transportation and variable costs. The process is iterated until obtaining the best number and size of DCs.

In [7], a multi product supply chain network design is studied which is similar to [6], except that instead of non-linear inventory costs, linear variable costs is included in the objective function.

In [39], an integrated model for supply chains that are operating globally is studied with the focus on how to better implement manufacturing strategies to overcome the challenges associated with intensive global competitions. Given the current built manufacturing plants and the demand pattern of customer zones, the Capacity Expansion Problem [40] is studied which involves determination of times of capacity expansion to supply the network demand.

In the recent works, facility location problem with economies and diseconomies of scale is studied [41]. The economies of scale is to capture the savings that are achieved through increasing the production volume. Diseconomies of scale, however, is to capture the increase in the marginal costs due to facility congestion and over utilization of resources after production level exceeds a certain level.

As we see in the literature, there is a positive trend in increasing the flexibility in capacity assignment to the facilities that are built. Unlike the single-

capacitated supply chain network design problem where there is a single potential capacity level available for a built facility, in multi-capacitated supply chain network design problem there is a predetermined set of potential capacity levels from which one is chosen to be assigned to a built facility. Multi echelon multi capacitated supply chain network design with one commodity and multiple commodities is studied in [1] and [42], respectively. In the next chapter, we study a three-echelon multi-capacitated supply chain network design model inspired by a real world supply chain design problem.

Chapter 3

3. Mixed Integer Linear Program for Three Level Supply Chain Network Design: Large Scale Case Study

3.1. Introduction

In this chapter, we present a mixed integer linear optimization model for multi-capacitated, multi-product, three-echelon supply chain network design including suppliers, DCs, and customer zones. Our model considers DC land, building and variable costs and takes into account economies of scale. In our model, the variable costs are based on the actual activity level of a DC rather than the assumption that the built DCs are operating at the capacity level limit. The inclusion of the real variable costs is via the addition of continuous variables and constraints.

We present a model simplification that helps reduce the problem size. We demonstrate the effectiveness of our model and model simplification through the design of a real-world supply chain with 47 suppliers at fixed locations, 83 potential DCs and 2 976 fixed customer zones [8].

3.2. Problem definition

The delivery units from suppliers to DCs are unique in-bound pallets of supplier provided products; and the delivery unit from DCs to customer zones is an out-bound pallet. Each of the products supplied to the DCs come from a single supplier, though the supplier might have multiple locations. For example, product A might come from a single supplier and that supplier will supply all DCs. That said, there are two complications. First, a single supplier may in

fact be the unique supplier of more than one product. In this case, we simply consider the supplier to be multiple suppliers, one for each product supplied. We assume that each pallet shipped from that supplier has a unique product. Second, a single supplier has multiple locations. We treat this supplier as a single supplier and assume that for a given DC, the closest of the supplier locations is the supplier. Consequently, there are no decisions (variables) involving the suppliers; only the associated transportation costs, which we term the *in bound* transportation costs.

We present a mixed integer linear optimization model whose solution will determine DC selection and size as well as the DC to customer zone assignment. The objective is to minimize transportation costs, land costs, building cost and variable costs of DCs while taking into account economies of scale. The complexity of the supply chain with multi products is reduced by creating a standardized out-bound pallet by averaging the weekly delivery data from an existing large DC. The data is also used to determine a fixed, known standardized pallet demand at each customer zone. We then aggregated the DCs annual transportation costs (fuel, labor, insurance and maintenance), demand and kilometers driven to get a common cost per pallet per kilometer. To determine the DC cost functions we aggregate costs from similar sized DCs in the existing network and determined costs as functions of the number of standardized pallets output from a DC. In terms of pallets per week, DC size is selected as either 500, $2\,000$, $4\,000$, $5\,000$ or $10\,000$ that are suggested by the client company. Thus, it is a three-echelon, multi-product, multi-capacitated

supply chain network design problem.

Once a DC is selected, it will receive products from the nearest location of each of the suppliers. Based on total customer demand of out-bound pallets from a DC we can apply a conversion factor to determine the in-bound pallets required by a DC from each supplier. This conversion factor reflects the DC activity of receiving products from the suppliers and repackaging them for the customers. The delivery cost of the in-bound pallets is included in the model.

3.3. Mathematical formulation

The set of customer zones is indexed by

$$i \in I = \{\, 1, 2, \ldots, m-1, m \,\}.$$

and the set of potential DC locations is indexed by

$$j \in J = \{\, 1, 2, \ldots, n-1, n \,\}.$$

The set of DC capacity levels is indexed by

$$t \in T = \{\, 1, 2, \ldots, \mu-1, \mu \,\}.$$

A set of binary variables select the DCs and their corresponding capacity levels. For all $j \in J$ and $t \in T$, let

$$x_j^t = \begin{cases} 1 & \text{if a DC with capacity level } t \text{ is built at location } j, \\ 0 & \text{otherwise.} \end{cases}$$

Since the variables are binary, the requirement that each built DC is assigned

a single capacity level is captured by the set of inequalities

$$\sum_{t \in T} x_j^t \leq 1, \quad \forall j \in J. \tag{19}$$

The second set of variables assign fractions of a customer zone demand to built DCs. For all $j \in J$ and $i \in I$, let y_{ji} be the fraction of demand at customer zone i served by DC j.

To ensure that every customer zone demand is fully served by built DCs, we have the equalities

$$\sum_{j \in J} y_{ji} = 1, \quad \forall i \in I. \tag{20}$$

The goal is to determine the location and capacity level of DCs to be built as well as the customer zone to DC assignment so that the supply network is optimized. We use weekly cost (in dollars) as the optimization criterion. Let f^t to be the weekly fixed costs associated with building a DC with capacity b^t. We assume that these costs do not depend on location. Let ℓ_j^t be the weekly fixed costs associated with the purchase of the land required for a DC to be built at location j with capacity b^t. These costs are dependent on location. The fixed costs for the DCs that are built are denoted by

$$C_1(x) = \sum_{t \in T} f^t \sum_{j \in J} x_j^t \ + \ \sum_{t \in T} \sum_{j \in J} \ell_j^t \, x_j^t. \tag{21}$$

Let v^t be the weekly variable costs of running a distribution with capacity b^t. These variable costs include items such as utilities, municipal taxes and labour. We assume that the variable costs do not depend on location, but only on level of activity, i.e., the number of pallets shipped. In our model, the variable costs

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are based on the actual activity level in a built DC rather than the assumption that built DCs are working at capacity level limit. To capture this feature, we introduce a new set of continuous variables. Let z_j^t be the shipment made from DC j operating at capacity level t. The variable costs of DCs is presented by the following function

$$C_2(z) = \sum_{t \in T} v^t \sum_{j \in J} z_j^t. \tag{22}$$

The planning period is a week and so we define the demand from customer zone i to be p_i pallets per week and the available output capacity for a DC with capacity index t to be b^t pallets per week. That the shipment volume from a built DC does not exceed its capacity level is captured by the inequalities

$$z_j^t \leq b^t x_j^t, \quad \forall j \in J, t \in T. \tag{23}$$

To link the shipment volume variables with the assignment variables, and also ensure that the shipment volume from a distribution center equals the output of a distribution center, we have the set of equalities,

$$\sum_{t \in T} z_j^t = \sum_{i \in I} p_i \, y_{ji}, \quad \forall j \in J. \tag{24}$$

The transportation costs are the final element in our objective function. We first deal with the *out-bound* transportation costs, that is, the costs of shipping from the DCs to the customer zones. Let d_{ij} be the distance, in kilometers, from the j-th potential distribution location to the i-th customer zone; and let ω be the cost to transport one pallet a distance of one kilometer. It is assumed that ω is constant for all truck types, regardless of load and street route. The

out-bound transportation costs are

$$C_3(y) = \omega \sum_{j \in J} \sum_{i \in I} p_i \, d_{ij} \, y_{ji}. \tag{25}$$

Since each supplier ships a unique set of products in a variety of truck sizes and pallet sizes, we allocate to each a unique "cost per pallet-kilometer". The set of suppliers is indexed by

$$k \in K = \{\, 1, 2, \ldots, \nu - 1, \nu \,\}.$$

Let ω_k be the cost per pallet-kilometer of shipping from supplier k. Let $\tilde{d}_{jk}$ be the number of kilometers from the DC at location j to supplier k. In the case when a supplier has multiple locations, $\tilde{d}_{jk}$ is the distance from the DC location to the nearest supplier location.

One challenge is that the pallets from each of the suppliers differ; and they all differ from the pallets shipped from a DC to a customer zone. What is needed is a conversion factor at the DC to represent the activity of receiving pallets from the suppliers and creating the pallets to ship to customer zones. Let's refer to these as out-bound pallets.

We have assumed that all out-bound pallets are uniform in size and content. Let ρ_k be the percentage of a pallet from supplier k on an out-bound pallet. Thus, $p_i \rho_k$ is the number of pallets from supplier k shipped to the DC that serves customer zone i required to meet the weekly needs of that customer zone. Definition of ρ_k also captures the repackaging process in a distribution center with cross docking operations.

The weekly demand from supplier k to supply all the customer zones is

$$\rho_k \sum_{i \in I} p_i.$$

This total demand quantity is shipped from the supplier to the built DCs. The number of in-bound pallets from supplier k to a DC built at location j depends on the customer zones assigned to that DC and is given by

$$\rho_k \sum_{i \in I} p_i y_{ji}.$$

The total in-bound transportation cost from all suppliers can be written as

$$C_4(y) = \sum_{k \in K} \omega_k \rho_k \sum_{j \in J} \tilde{d}_{jk} \sum_{i \in I} p_i y_{ji}. \tag{26}$$

The objective function is assembled from (21), (22), (25) and (26) and is given by

$$C(x, y) = C_1(x) + C_2(z) + C_3(y) + C_4(y) \tag{27}$$

The optimization model is assembled from (19), (20), (23), (24) and (27). It is a Mixed Integer Linear Program (MILP). The explicit presentation of the model is given below.

$$\text{Minimize} \quad C(x, y) = C_1(x) + C_2(z) + C_3(y) + C_4(y)$$

subject to:

$$\sum_{t \in T} x_j^t \leq 1, \quad \forall j \in J$$

$$\sum_{j \in J} y_{ji} = 1, \quad \forall i \in I$$

$$z_j^t \leq b^t x_j^t, \quad \forall j \in J, t \in T$$

$$\sum_{t \in T} z_j^t = \sum_{i \in I} p_i \, y_{ji}, \quad \forall j \in J.$$

$$x_j^t \in \{0,1\} \quad \forall j \in J, \ t \in T.$$

$$y_{ji} \geq 0 \quad \forall j \in J, \ i \in I.$$

$$z_j^t \geq 0, \quad \forall j \in J, \ t \in T.$$

3.4. Case study

There are $2\,976$ customer zones, 83 potential DC locations, 5 potential capacity levels for DC and 47 suppliers. The number of variables, constraints and the computational results of the MILP model are reported in table 4. We use LINGO $^\circledR$ 14.0 x64 on a DELL server with 32 G of RAM and two 2.50 GHz CPUs. We see that the MILP model with $247\,838$ variables and $3\,557$ constraints is solved to optimality after 9 hours using LINGO $^\circledR$.

	MILP
# Binary Variables	415
# Continuous Variables	$247\,423$
# Constraints	$3\,557$
# Iterations	$8\,372\,520$
Time (hour)	9
Objective Function Value	$3\,628\,145$
Status	Optimal

Table 4: Number of variables, constraints and the computational results of the MILP model

We propose a model simplification which is to cluster the customer zones. The customer zones are clustered according to the first two characters of their postal code creating 133 clusters.

The mixed integer linear program remains unchanged except that I is replaced by I^{cl}, the set of customer zone clusters, and d_{ij} is replaced by d_{ij}^{cl} the distance from the DC at location j to the center of cluster $i \in I^{cl}$. Let p_i be the demand at cluster i.

Subsequently, $C_3^{cl}(y)$ and $C_4^{cl}(y)$ denote the out-bound and in-bound transportation costs, respectively. This gives us a Cluster Mixed Integer Linear Program (CMILP). For clarity, we give an explicit statement of CMILP model.

$$\text{Minimize} \quad C(x, y) = C_1(x) + C_2(z) + C_3^{cl}(y) + C_4^{cl}(y)$$

Subject to :

$$\sum_{t \in T} x_j^t \leq 1, \quad \forall j \in J$$

$$\sum_{j \in J} y_{ji} = 1, \quad \forall i \in I^{cl}$$

$$z_j^t \leq b^t x_j^t, \quad \forall j \in J, t \in T$$

$$\sum_{t \in T} z_j^t = \sum_{i \in I^{cl}} p_i \, y_{ji}, \quad \forall j \in J.$$

$$x_j^t \in \{0, 1\} \quad \forall j \in J, \ t \in T.$$

$$y_{ji} \geq 0 \quad \forall j \in J, \ i \in I^{cl}.$$

$$z_j^t \geq 0, \quad \forall j \in J, \ t \in T.$$

where

$$C_3^{cl}(y) = \omega \sum_{j \in J} \sum_{i \in I^{cl}} p_i \, d_{ji}^{cl} \, y_{ji},$$

and

$$C_4^{cl}(y) = \sum_{k \in K} \omega_k \rho_k \sum_{j \in J} \tilde{d}_{kj} \sum_{i \in I^{cl}} p_i \, y_{ji}.$$

In table 5, the number of variables, constraints and the computational results of the CMILP model is given. The model has a total number of 11 869 variables and 714 constraints and is solved to optimality in around 10 minutes.

The selected DCs and their corresponding capacity level for MILP and CMILP models are reported in table 6 given below.

	CMILP
# Binary Variables	415
# Continuous Variables	11 454
# Constraints	714
# Iterations	1 076 455
Time (Hour)	0.16
Objective Value	4 285 733
Status	Optimal

Table 5: Number of variables, constraints and the computational results of the CMILP model

Capacity	MILP	CMILP
500	*	*
2,500	*	#63
4,000	*	*
5,000	*	*
10,000	#50, #55	#31, #49

Table 6: Selected DCs (Identification number and capacity)

We note that the optimal solution obtained from CMILP may not be optimal for the MILP. Therefore, we input the solution of CMILP in terms of DC selection into the MILP model. This would provide MILP objective function value for the CMILP which is provided in table 7.

	MILP Objective Function value
MILP	3 628 145
CMILP	3 730 578

Table 7: MILP objective function values

We suggest CMILP model for a supply chain design with characteristics discussed in this chapter. Reason is, it produced a supply chain network design which is 2.82% more costly than the design cost obtained from MILP but was obtained in 98.2% less computational times, i.e., around 10 minutes.

3.6. Impact

Our model was integrated with the company's routing and transportation software. The company created a new *Network Analyst* management position and is using the model to inform current decisions on DC expansion. The

model has continuing importance as the company continues to grow and expand throughout the world.

3.7. Discussions and conclusion

Being inspired by a real world supply chain network design problem, a Mixed Integer Linear Program (MILP) for a three-echelon, multi-capacitated supply chain network design is developed. There are suppliers, DCs and customer zones in the supply network. The capacity level of built DCs are picked from a predetermined set of discrete capacity levels. There are multi products in the supply chain under review. A common out-bound pallet shipped from DCs to customer zones is defined. A conversion rate was used to quantify the number of in-bound pallets based on their average existence in an out-bound pallet.

Our model captures building, land and variables costs of built DCs. The inclusion of the variable costs was via addition of continuous variables and constraints to the model. We used the model to solve a real world supply chain network design problem. We see that the MILP model with $247\,838$ variables and $3\,557$ constraints was solved to optimality after 9 hours using LINGO $^{®}$. We proposed a model simplification which was to cluster the customer zones. The customer zones were clustered according to the first two characters of their postal code creating 133 clusters. The model was presented as Cluster Mixed Integer Linear Program (CMILP) and was solved in around 10 minutes.

We input the solution of CMILP in terms of DC selection into MILP model to obtain MILP objective function value as well as the non-cluster assignment of customer zones to built DCs. We suggest CMILP model for a large scale supply

chain design with characteristics discussed in this chapter. Reason is, CMILP produced a supply chain network design which was 2.82% more costly than the design cost obtained from MILP, but was obtained in 98.2% less computational times, i.e., 10 minutes.

Chapter 4

4. A Characterization of Alternate Optimal Solutions for a Supply Chain Network Design Model

4.1. Introduction

In chapter 3, we studied a three-echelon supply chain network design where the suppliers were at fixed known locations with sufficient capacity to serve the network demand. The DCs were picked from a set of potential DC locations. The capacity level of a built DC was selected from a predetermined set of discrete capacity levels. The proposed model captured the operational costs of the built DCs based on their actual activity level rather than assuming that built DCs are operating at their capacity level limit.

The supply chain under consideration in this chapter has a given finite set of customer zones with known demands, and a finite set of potential facility (supplier and DC) locations. If a facility is built at a potential location, the capacity level is selected from a finite set of capacity levels. The design problem is to choose the locations and capacity levels at which the facilities are to be built; and to determine the flow of goods from suppliers to DCs to customers in order to minimize costs. Specifically, we characterize the complete set of alternate optimal solutions in the well-known model presented by Amiri [1]. We extend Amiri's model through the addition of a constraint set that eliminates certain undesirable optimal solutions and we show that the extended model requires, essentially, the same computational effort as the original.

We start with the model presented by Amiri [1] for three-echelon, multi-capacitated supply chain network design with suppliers, DCs and customer zones. In some cases, we change Amiri's notations to be consistent throughout this dissertation.

The set of customer zones is indexed by i, the potential DC locations by j, the potential supplier locations by k, the DC capacity levels by t and the supplier capacity levels by h. Let p_i be the demand from customer zone i during the planning horizon. It is possible that there are many customers within a customer zone so that p_i is the aggregate demand. While demand is stochastic, historical data can be used to forecast a fixed demand for a specific planning horizon.

Let b_j^t be the capacity of a DC at location j built at level t and e_k^h be the capacity of a supplier at location k built at level h. Let u_{jk}^t be the fraction of the total demand of a DC at location j with capacity level t that is delivered from a supplier at location k and let y_{ij} be the fraction of the total demand of customer zone i that is delivered from a DC at location j. A DC with capacity level t is built at location j if and only if $x_j^t = 1$ and $q_k^h = 1$ if and only if a supplier with capacity level h is built at location k. The binary variables x and q determine the number, location and capacity of the facilities while the real-valued variables u and y determine the flow of goods.

The objective function is to minimize the sum of the transportation costs and the fixed costs of establishing suppliers and DCs. Let $\bar{c}_{jk}$ be the cost of shipping one unit of demand to a DC at location j from a supplier at location k

and let c_{ij} be the cost of shipping one unit of demand to customer zone i from a DC at location j. The in-bound and out-bound transportation costs are given by

$$T_1(u) = \sum_{t,j,k} \bar{c}_{jk}\, b_j^t\, u_{jk}^t \qquad \text{and} \qquad T_2(y) = \sum_{i,j} c_{ij}\, p_i\, y_{ij},$$

respectively. The fixed costs of establishing and operating the DCs and suppliers over the planning horizon are

$$F_1(x) = \sum_{j,t} f_j^t\, x_j^t \quad \text{and} \quad F_2(q) = \sum_{k,h} g_k^h\, q_k^h,$$

respectively, where f_j^t is the fixed cost of opening and operating DC at location j with capacity level t and g_k^h is the fixed cost of opening and operating supplier at location k with capacity level h. The objective function is a summation of the four cost elements and is given by

$$Z_0(u, y, x, q) = T_1(u) + T_2(y) + F_1(x) + F_2(q).$$

The constraints

$$\sum_j y_{ij} = 1, \quad \forall\, i, \tag{28}$$

ensure that the demand at each customer zone is covered by built DCs while constraints

$$\sum_i p_i\, y_{ij} \leq \sum_t b_j^t\, x_j^t, \quad \forall\, j, \tag{29}$$

ensure that the capacity level at each DC is sufficient to meet out-bound shipments. That each DC and each supplier is assigned a single capacity level is

ensured by

$$\sum_t x_j^t \leq 1, \quad \forall j, \tag{30}$$

and

$$\sum_h q_k^h \leq 1, \quad \forall k, \tag{31}$$

respectively. The set of constraints

$$\sum_i p_i \, y_{ij} \leq \sum_{k,t} b_j^t \, u_{jk}^t, \quad \forall j, \tag{32}$$

ensure that the total out-bound shipment from a DC is not greater than the total in-bound shipment from suppliers to that DC. As we are minimizing the transportation costs in the objective function, it is expected that the in-bound shipment to a DC be equal to the out-bound shipment from that DC. The inequalities

$$\sum_{j,t} b_j^t \, u_{jk}^t \leq \sum_h e_k^h \, q_k^h, \quad \forall k, \tag{33}$$

ensure that the total in-bound shipment from a built supplier to the DCs is not greater than the chosen capacity level of that supplier. The final sets of constraints are

$$y_{ij} \geq 0, \ \forall \, i, \ j, \tag{34}$$

and

$$u_{jk}^t \geq 0, \ \forall \, k, \ j, \ t, \tag{35}$$

which, together with the unstated constraints that x_j^t and q_k^h are binary, give

Amiri's model which is to

$$\textbf{AM0:} \qquad \min_{x,q\text{binary}} \{ Z_0(u, y, x, q) \mid (28) - (35) \}.$$

Amiri presented a Lagrangian based solution algorithm to solve **AM0** and his numerical experience showed that the model can produce excellent solutions in reasonable computational times. Consequently, his model and solution method provides an effective and efficient tool for supply chain design.

In the next section, we will show that, theoretically, Amiri's model could produce results that are undesirable, that is, that are inconsistent with the model definition. There are two types of potential inconsistency. The first type has shipments from a supplier to an unselected DC-capacity level combination. The second type has fractional variables assigned values greater than unity.

4.3. The set of alternate optimal solutions

We characterize the complete set of alternate optimal solutions to **AM0** with the change of variables

$$w_{jk} = \sum_t b_j^t u_{jk}^t \quad \forall j \in J, k \in K, \tag{36}$$

where

$$w_{jk} \geq 0 \quad \forall j \in J, k \in K. \tag{37}$$

We now replace $T_1(u)$ with

$$T_1(w) = \sum_{j,k} \bar{c}_{jk} w_{jk}$$

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giving the new expression for the objective

$$Z_1(w, y, x, q) = T_1(w) + T_2(y) + F_1(x) + F_2(q).$$

We next replace constraints (32) and (33) with

$$\sum_i p_i \, y_{ij} \leq \sum_k w_{jk}, \quad \forall j, \tag{38}$$

and

$$\sum_j w_{jk} \leq \sum_h e_k^h \, q_k^h, \quad \forall k, \tag{39}$$

respectively. Model **AM0** is equivalent to

AM1: $\min_{x, q \text{binary}} \{ Z_1(w, y, x, q) \mid (28) - (31), (34), (37), (38), (39) \}$

Given the w_{jk} from an optimal solution to **AM1**, the entire set of optimal solutions to **AM0** is the complete solution set to (36) and (37).

Example 1. Consider a supply chain with one supplier ($k \in \{1\}$) having a single ($h \in \{1\}$) available capacity level of $e_1^1 = 6\,000$. Suppose also that there is a single one DC ($j \in \{1\}$) with three ($t \in \{1, 2, 3\}$) available capacity levels of $b_1^1 = 1\,000$, $b_1^2 = 3\,000$ and $b_1^3 = 5\,000$. The three customer zones ($i \in \{1, 2, 3\}$) have demands $p_1 = 1\,500$, $p_2 = 1\,500$ and $p_3 = 1\,000$.

We examine the constraints in **AM1**. From constraint (28) we get $y_{11} = 1$, $y_{21} = 1$, and $y_{31} = 1$. Thus, (34) is satisfied. From (29) we have $4\,000 \leq 1\,000\,x_1^1 + 3\,000\,x_1^2 + 5\,000\,x_1^3$. Combining this with (30) and the fact that x is binary, yields $x_1^1 = 0$, $x_1^2 = 0$ and $x_1^3 = 1$. From (38) we have $w_{11} \geq 4\,000$ so that

(37) is redundant. From (39) we have $w_{11} \leq 6\,000\,q_1^1$. Since q_1^1 is binary, we have that $q_1^1 = 1$ since $q_1^1 = 0$ implies that $w_{11} \leq 0$ contradicting $w_{11} \geq 4\,000$. Thus, combining, we get $4\,000 \leq w_{11} \leq 6\,000$ for feasibility. An optimal solution will have $w_{11} = 4\,000$.

The result, is that any non-negative solution to

$$4\,000 = 1\,000\,u_{11}^1 + 3\,000\,u_{11}^2 + 5\,000\,u_{11}^3$$

is feasible and optimal for **AM0**. Let's examine a few of the possible optimal solutions to **AM0**. Consider, $u_{11}^1 = 4$, $u_{11}^2 = 0$ and $u_{11}^3 = 0$. In this case, u_{11}^1 is not fractional. Next, consider $u_{11}^1 = 1$, $u_{11}^2 = 1$ and $u_{11}^3 = 0$. This is inconsistent with $x_1^1 = 0$ and $x_1^2 = 0$. For example, it has a shipment from supplier 1 being delivered to a DC with capacity level 1 $(u_{11}^1 = 1)$, yet this DC-capacity level combination is not selected $(x_1^1 = 0)$. For a third example, consider $u_{11}^1 = 0$, $u_{11}^2 = 0$ and $u_{11}^3 = .8$. This solution is optimal for **AM0** and is consistent, i.e., desirable.

In the next section we introduce a set of constraints that give an extension to Amiri's model eliminating unwanted, inconsistent alternate optimal solutions.

4.4. The extension

We extend **AM0** with the addition of the constraints

$$\sum_k u_{jk}^t \leq x_j^t, \quad \forall j, t \tag{40}$$

giving the new model

Extended Model: $\quad \min\limits_{x,q\text{binary}} \{\, Z_0(u, y, x, q) \mid (28) - (35), (40)\,\}.$

Since the u variables are non-negative and since the x variables are binary, these new constraints, that explicitly link the u and x variables, remove the alternate optimal solutions that either have shipments delivered to an non-existent DC-capacity level combination or that have non-fractional values for u. That the y variables do not take on non-fractional values is guaranteed by constraints (28) and (34).

Applied to Example 1, the unique optimal solution to the **Extended Model** is $u_{11}^1 = 0$, $u_{11}^2 = 0$ and $u_{11}^3 = .8$.

We now test the impact of this additional set of constraints on computational times. To do so, we create six instances of the largest model instance solved by Amiri, that is, we have 500 customer zones, 30 potential DC locations and 20 potential supplier locations. We create the examples using the method described by Amiri. We solve the **AM0** model and the **Extended Model** using LINGO $^®$ 14.0 x64 on a DELL server with two 2500 MHz CPUs. The times taken to reach optimality for the **AM0** ranged from 56 to 185 seconds with an average of 122. The times taken to reach optimality for the **Extended Model** ranged from 53 to 277 seconds with an average of about 121. Both models produce identical solutions with, essentially, the same computational effort.

We characterized the complete set of alternate optimal solutions in Amiri's model for three-echelon, multi-capacitated supply chain design; and we demonstrated that the set may well contain undesirable solutions, that is, solutions that are inconsistent with model definition. We extended Amiri's model by addition of a set of constraints to eliminate certain undesirable optimal solutions. We recommend that the extended model be adopted to ensure that all algorithms when applied to any problem instance produce only desirable optimal solutions.

Chapter 5

5. Increased Flexibility in Three-echelon Multi-capacitated Supply Chain Network Design

5.1. Introduction

In chapter 4, we see that the three-echelon, multi-capacitated supply chain network design challenge is to determine the numbers, locations and capacity levels of suppliers and DCs; as well as the product flow from suppliers to DCs and then to customer zones in order to meet customer zones demand at minimum cost. Mathematical models for multi-capacitated supply chain network design provide a finite set of capacity levels from which to choose; and include variables and constraints to ensure the selection of a single capacity level for each facility to be built.

By eliminating the constraints that enforce a single capacity selection, we allow for the selection of several capacity levels for a single supplier or DC. If such a selection occurs, the supplier or DC is built with size equal to the sum of the selected capacity levels. This gives an exponential increase in the number of available capacity levels. The increased flexibility allows for less costly supply chain network designs. We present numerical results that demonstrate improved solutions, that is, lower cost solutions, with lower computational effort.

In subsection 5.2, we create a new version of the model presented in chapter 4, by using the change of variables introduced in [43]. This new version has fewer continuous variables and constraints. We compare the performance of the two models according to the computational times required to solve a set

of test problems. We establish that the new model is superior with respect to the computational times required to solve a set of test problems. We then provide a further modification that gives an exponential increase to the number of available capacity levels by the elimination of the constraints that enforce the selection of a single capacity. The advantage, of course, is that an increase in the number of available capacity levels allows for less costly supply chain designs. The mathematical model is presented in subsection 5.3.

While the extension gives an exponential increase in the number of available capacity levels, the number of available capacity levels is still finite. Thus, we could achieve the same increased flexibility in capacity assignment in a more straightforward way by providing the larger set of capacity levels while keeping the constraints which enforce a single capacity selection. The model is presented in subsection 5.4 and we validate the modification with numerical evidence.

5.2. The mathematical formulation

The three-echelon multi-capacitated supply chain model considered in this paper is the extended Amiri model given by

$$\min_{x,q\text{binary}} \{ Z_0(u,y,x,q) \mid (28)-(35),(40) \}. \tag{41}$$

We refer to this as Model (41), or, simply, M41.

The model can be reduced in both the number of continuous variables and constraints using the change of variables

$$w_{jk} = \sum_t b_j^t u_{jk}^t \quad \forall j,k, \tag{42}$$

with the constraints

$$w_{jk} \geq 0 \quad \forall\, j, k. \tag{43}$$

This change of variables was introduced in [43, 9]. The w_{jk} variables give the total shipment volume from supplier k to DC j. As a result of this change of variables, $T_1(u)$ is replaced with

$$T_1(w) = \sum_{j,k} \bar{c}_{jk} w_{jk},$$

$Z = Z(u, y, x, q)$ is replaced with

$$Z_w = Z(w, y, x, q) = T_1(w) + T_2(y) + F_1(x) + F_2(q) \tag{44}$$

and (32), (33), and (40) are replaced with

$$\sum_i p_i\, y_{ij} \leq \sum_k w_{jk}, \quad \forall j, \tag{45}$$

$$\sum_j w_{jk} \leq \sum_h e_k^h\, q_k^h, \quad \forall k, \tag{46}$$

and

$$\sum_k w_{jk} \leq \sum_t b_j^t\, x_j^t \quad \forall j. \tag{47}$$

respectively.

The modified model is to

$$\min_{x,q\text{binary}} \{ (44) \mid (28), (29), (30), (31), (34), (43), (45), (46), (47) \}. \tag{48}$$

We refer to this as Model (48), or, simply, M48.

While M48 has the same number of binary variables as M41, it has fewer

continuous variables and constraints than M41.

For example, if there are 500 customer zones, 30 DCs and 20 suppliers, then M41 has 18 000 continuous variables, 250 binary variables and 780 constraints; while M48 has 15 600 continuous variables, 250 binary variables and 660 constraints.

Before we introduce the mechanism to increase variability, we want to establish that M48 is superior to M41. We generate 10 problem instances, using the procedure in [1], for a supply chain with 500 customer zones, 30 potential DCs and 20 potential suppliers. Throughout, we refer to these as instances 1 to 10. The solver is LINGO $^{\circledR}$ 14.0 x64 run on a DELL server with two 2500 MHz CPUs. In fact, we solve the models with the parametric objective functions

$$Z^p = T_1(u) + T_2(y) + (\, p * (F_1(x) + F_2(q)\,) \quad \text{and} \tag{49}$$

$$Z^p_W = T_1(w) + T_2(y) + (\, p * (F_1(x) + F_2(q)\,), \tag{50}$$

where the parameter p takes on each of the fourteen values in the set

$$\{0.01, 0.5, 1, 2, 3, 4, \ldots, 10, 100, 10000\}.$$

As p increases, the relative fixed costs of the suppliers and DCs increase leading to solutions with fewer facilities, referred to as consolidation. We chose the smaller and larger p values to see the effect. The p values from 1 to 10 balance the fixed costs against transportation costs which, in our examples, are an order of magnitude different.

The results are given in table 8. For each of the 14 parameter values, we

solve 10 problem instances. All problems were solved to optimality and, since the optimal solution is unique, both models produce the same solution for a given parameter value. We are concerned only with the solution times. The first column is the value of the parameter p. The mean μ and standard deviation σ of the solution time, in seconds, over the ten instances is given for each model. The last column in the table reports the per cent reduction in the average time using M48 . For example, when $p = 0.01$, the average computational times is 38% faster using M48. Over all 140 instances, the average savings in computational times is 58% which establishes the superiority of M48 over M41.

We note that for very large or very small values of p, the computational times are lower as the two costs are out of balance, that is, one dominates the other and drives the algorithm. For both models, the longest running times are when $p = 9$, a parameter value we will use in what comes next. Based on this experiment, we conclude that M48 is superior to M41 and it is used to introduce the method for increased variability.

5.3. Increased variability

In M48 there are $|T|$ choices for the capacity levels for each DC. The removal of constraint set (30) will allow the selection of several capacity levels for each built DC. Rather than the interpretation that several DCs with different capacities would be built at the same location, our interpretation is that we would build a single DC with the combined capacity of all of the selected capacities. Consequently, the removal of (30) gives an exponential increase in the number of available DC capacity levels from $|T|$ to $(2^{|T|} - 1)$, excluding the

49

	M41		M48		Δ time
p	$\mu_{(M41)}$	$\sigma_{(M41)}$	$\mu_{(M48)}$	$\sigma_{(M48)}$	$(\mu_{(M41)} - \mu_{(M48)})/\mu_{(M41)}$
0.01	20.4	11.98	12.7	3.92	0.38
0.5	101.6	32.01	35.8	10.15	0.65
1	138.2	81.48	62.6	47.81	0.55
2	210.6	144.89	96.7	69.12	0.54
3	291.1	169.06	133.9	123.95	0.54
4	337.4	215.41	144.5	78.36	0.57
5	430.3	241.10	141.8	88.93	0.67
6	630.2	345.75	192.9	57.58	0.69
7	641.1	359.56	256.5	167.09	0.60
8	773.8	218.42	265.6	137.99	0.66
9	821.3	390.45	341.5	164.45	0.58
10	782.6	252.10	322.1	181.93	0.59
100	220.5	131.77	118.2	119.42	0.58
10000	53.1	32.01	24.7	15.06	0.53
					Average = 0.58

Table 8: Average μ and deviation σ of computational times over instances 1 to 10.

choice of building a DC with null capacity. Similarly, eliminating (31) gives an exponential increase in the number of supplier capacity levels. This results in

$$\min_{x, q \text{binary}} \{ (44) \mid (28), (29), (34), (43), (45), (46), (47) \}. \tag{51}$$

We refer to it as M51, a model with an exponential increase in the number of capacity levels leading, potentially, to less costly supply chain network designs.

We now compare the relative merits of the increased variability in M51 over M48. We generated 10 additional instances. The instances labeled 11 to 20 have 500 customer zones, 15 potential DCs and 10 potential suppliers. Table 9 summarizes the problem instances.

Instance Numbers	Customer Zones	Potential DCs	Potential suppliers
1 - 10	500	30	20
11 - 20	500	15	10

Table 9: Summary of problem instances

In table 8, we saw that the highest average running times correspond to the parameter value $p = 9$. Consequently, we set $p = 9$ and use the objective function

$$Z_W^{(9)} = T_1(w) + T_2(y) + (9 * (F_1(x) + F_2(q)))$$ (52)

in M51 and in all further numerical results.

Table 10 summarizes the comparison of M48 and M51 based on their performance in the solution of instances $1 - 20$. LINGO $^{®}$ produce an optimal solution for M48 in computational times ranging from a minimum of 15 seconds to a maximum of 631 seconds while for M51 the times range from 78 to 13 425 seconds. On average, M51 required about 21 times the effort required for M48 to reach optimality; however, we see that it is in the cost of solution refinement. As expected, the supply chain design cost (objective function) for M51 is lower than for M48. This is the benefit of the increased flexibility.

	Optimal Objective		Time (seconds)	
#	**M48**	**M51**	**M48**	**M51**
1	256,786	209,727	402	3,305
2	231,563	208,846	279	4,648
3	241,245	212,286	148	5,643
4	252,810	230,834	156	10,659
5	233,245	218,242	296	13,425
6	240,252	223,252	203	12,974
7	244,967	225,500	631	3,450
8	239,956	222,189	570	4,899
9	229,362	207,535	192	4,458
10	234,206	219,046	372	8,344
11	316,095	222,964	18	99
12	244,523	217,219	21	337
13	236,140	212,372	15	271
14	316,098	222,965	21	78
15	230,876	202,460	27	105
16	207,924	199,880	30	475
17	242,737	229,614	25	200
18	230,255	221,516	22	722
19	242,462	215,552	12	218
20	232,518	205,132	20	136

Table 10: Comparison of optimal cost and solution times for M48 and M51

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We rerun LINGO ® on M51 with a stopping time of 10 seconds, about two-thirds of the minimum time required by M48. We reported the best objective function value obtained by LINGO ® on M51 in table 11. In all cases, the best objective function value for M51 obtained in 10 seconds was smaller than the optimal for M48.

#	M51 Opt Objective	M51 10 s Objective	Percentage Optimality Gap
1	209,727	213,653	1.8
2	208,846	213,471	2.2
3	212,286	217,553	2.4
4	230,834	235,687	2.1
5	218,242	223,903	2.5
6	223,252	233,287	4.3
7	225,500	228,908	1.5
8	222,189	226,608	2.0
9	207,535	210,754	1.5
10	219,046	228,432	4.1
11	222,964	225,216	1.0
12	217,219	219,295	0.9
13	212,372	214,146	0.8
14	222,965	225,216	1.0
15	202,460	205,333	1.4
16	199,880	204,454	2.2
17	229,614	231,232	0.7
18	221,516	22,3941	1.1
19	215,552	217,573	0.9
20	205,132	206,528	0.7
			Average = 1.7

Table 11: Comparison of optimal and 10-second design cost for M55

Considering the results from tables 10 and 11, we see that in at most two-thirds of the solution time, i.e. 10 seconds, M51 produced an improved supply chain design with an average decrease in cost of about 9.5% and with an average optimality gap of 1.7%. We establish that the increased flexibility in capacity assignment offered by M51 is desirable as it leads to less costly supply chain design and can be achieved in short computational times, i.e., 10 seconds. We see that M51 produces high quality solutions very quickly.

5.4. Increased variability vs larger capacity set

An alternative to removal of the constraint sets (30) and (31) to gain the exponential increase in flexibility in capacity assignment in the DCs and suppliers is to simply make the complete set of capacities in the original M48. We denote by ML48 the version of M48 that has the complete set of $(2^{|T|} - 1)$ DC

and $(2^{|H|} - 1)$ supplier capacity levels. While M51 and ML48 now have the same level of flexibility, ML48 has more variables and constraints. In table 12 we show the various problem sizes for the models and instances.

	Variables			Constraints		
Instances	M48	M51	ML48	M48	M51	ML48
1 - 10	15 850	15 850	17 150	660	610	660
11 - 20	7 775	7 775	8 425	580	555	580

Table 12: Instance sizes for the models

We now compare the performance of M51 and ML48. We first compare their performance based on the time to reach optimality for instances 11-20. Indeed, the objective function values are the same and we are only concerned with the computational times.

The results are reported in table 13. As we see, it takes about 11 times longer for ML48 to reach optimality than M51.

	Time (seconds)	
#	ML48	M51
11	557	99
12	3,465	337
13	2,127	271
14	527	78
15	2,250	105
16	2,034	475
17	1,551	200
18	8,222	722
19	4,534	218
20	1,458	136

Table 13: Comparison of ML48 and M51 : Optimality (instances 11-20)

We next compare the performance of M51 and ML48 based on the time to reach optimality for instances 1-10. In fact, for this set of test problems, ML48 could not reach optimality within 6 hours which is already far more than the computational times required by M51. We report the results in table 14. For

ML48, we report the best objective function value and the lower bound obtained

after solving the instances for 6 hours.

#	M51		ML48		
	Optimal Obj.	Time (Sec)	Best Obj.	Lower Bound	Time(Sec)
1	209,727	3,305	209,825	201,533	21,600
2	208,846	4,648	208,846	201,090	21,600
3	212,286	5,643	212,474	202,116	21,600
4	230,834	10,659	230,834	224,127	21,600
5	218,242	13,425	218,242	212,009	21,600
6	223,252	12,974	223,456	219,203	21,600
7	225,500	3,450	225,500	221,698	21,600
8	222,189	4,899	222,189	218,694	21,600
9	207,535	4,458	207,824	199,260	21,600
10	219,046	8,344	219,046	212,616	21,600

Table 14: Comparison of M51 and ML48 : Optimality (instances 1-10)

Results from tables 13 and 14 demonstrate the superiority of M51 over ML48

in reaching optimality. We next compare the performance of M51 and ML48

when solved for 10 seconds. The results are given in table 15. We see that, M51

offers about 3.2% less costly design than ML48 when solved for 10 seconds.

	Objective Function	
#	M51	ML48
1	213,653	223,078
2	213,471	217,507
3	217,553	220,735
4	235,687	245,388
5	223,903	230,035
6	233,287	251,152
7	228,908	239,840
8	226,608	237,126
9	210,754	220,498
10	228,432	234,576
11	225,216	225,404
12	219,295	219,769
13	214,146	221,043
14	225,216	226,198
15	205,333	214,404
16	204,454	217,526
7	231,232	238,197
18	223941	227,702
19	217,573	220,553
20	206,528	219,125

Table 15: Comparison of M51 and ML48 : 10 Seconds (instances 1-20)

Overall, tables 10 and 11 demonstrate that M51 leads to less costly supply chain designs which can be achieved in 10 seconds. While M51 and ML48 offer the same flexibility in capacity assignment, we observe that M51 outperforms ML48 when solved to optimality according to the results from tables 13 and 14. Not only that, M51 could offer more savings in supply chain design costs than ML48 when solved for 10 seconds according to the results from table 15. Thus, we conclude that, for three-echelon multi capacitated supply chain network design, good quality solutions can be obtained by using M51 in short computational times and, therefore, it is suggested.

5.5. Conclusion

We first provided an extension to a well-known model from literature [1], called M41, for three-echelon multi-capacitated supply chain design network

design. This model eliminated undesirable alternate optimal solutions that the original model could admit to.

We then presented M48 which is a reformulation of M41 with fewer continuous variables and constraints. Extensive computational experiments demonstrated that M48 is superior to M41 because it can produce the same solution as M41 in 58% less computational times on average.

We then presented two approaches which allow exponential increase in available capacity levels to DCs and the suppliers. Indeed, the increased flexibility in capacity assignment can only lead to less costly supply chain designs. The first approach was to eliminate a set of constraints with binary variables which enforce the selection of a single capacity level. By doing so, we allowed for the selection of several capacity levels for a single DC or supplier. This gave an exponential increase in the number of available capacity levels. While this extension provides an exponential increase, the number of available capacity levels is still finite. The second approach was to expand the set of capacity levels by adding more potential capacity levels to the set while keeping the constraints with binary variables which enforce the selection of a single capacity level. This approach provided the same flexibility as the first approach but led to the increase in number of binary variables which is not desirable. We first validated both approaches with numerical examples and showed that, as expected, the increased flexibility in capacity assignment leads to less costly supply chain network design. We then showed that the exponentially increased flexibility via elimination of a set of constraints (M51) performs better than the

approach where the number of binary variables were increased while the set of constraints were kept (ML48). The computational results showed that when both M51 and ML48 were solved to optimality, M51 required less computational times than ML48. Furthermore, we observed that when both were solved for 10 seconds, M51 produced less costly supply chain network design than ML48. Therefore, we suggest M51 for three-echelon multi-capacitated supply chain network design.

Chapter 6

6. A Binary, Non-convex, Variable-capacitated Supply Chain Design Model

6.1. Introduction

In chapters 3 and 4, we studied three-echelon multi-capacitated supply chain network design where the capacity levels of built DCs and the suppliers are picked from a predetermined set of potential discrete capacity levels. In chapter 5, we proposed two approaches to exponentially increase the flexibility in capacity assignment and compared the performance of both approaches.

In this chapter, a new mathematical model is presented [10] for three-echelon capacitated supply chain design and it is unique in that, it allows complete variability in the choice of capacity level and so avoids the need to determine, a priori, a set of potential capacity levels. Furthermore, it avoids having excess and unused capacity in a built facility, i.e., DC.

The supply chain design consists of suppliers, DCs and customer zones where each supplier provides a unique set of goods from known, possibly multiple, locations and where each customer zone has a fixed, known demand so that it exhibits the features of the supply chain of an existing company that operates across Canada and in the United States of America (47 suppliers, 83 potential DC locations and 2, 976 customer zones). A mathematical model is presented whose solution determines the location and capacity level of DCs and assigns customer zones to the selected DCs.

In this chapter, we assume that the operational or variable costs of a built DC is a concave function of its activity level due to economies of scale. This makes the model a Binary, Non-convex, Optimization problem (BO). We first present a piecewise linear (PWL) approximation to the concave cost functions that captures the concept of "technology break-points". We then present a mixed integer linear program (MILP) via addition of continuous variables and constraints which also captures technology break points. The solution of the MILP model determines the location, capacity level and technology level of DCs and assigns customer zones to the selected DCs.

More advanced technology levels in a DC such as full-automated operating system would require larger amount of initial investment than full-manual operating system; however, the cost per processed item or pallet would be lower in full-automatic. An example would be comparing a basic robot that lifts heavy loads and transfers them on to the conveyor with getting the job done manually. This job used to be or can be fulfilled by few workers who manually do the lifting and transferring. Deploying a basic robot would require an initial one-time investment which, for example, could last over a 20-year time horizon with low daily operating costs. In contrast, having two workers doing the job manually would require a low initial investment but indeed a very large annual operating cost which are their wages and would occur every year.

The decision variables for the supply chain design problem will be the selection of DC location, DC capacities and customer zone to DC assignment. The DC locations are chosen from a predetermined set of potential DC locations but

the capacity level will be determined by the total demand from the assigned customer zones. Consequently, the model allows true variability in capacity selection for built DCs. In addition, there will be no excess capacity in a built DC and therefore, the operational costs will not include costs for unused capacity.

The supply chain design is driven by the objective of minimizing the inbound transportation costs, out-bound transportation costs, fixed DC set up costs, building costs, and operational costs. The transportation costs are linear functions of the number of in-bound and out-bound pallets shipped. The building and operational costs are modeled to include economies of scale. Consequently, these cost functions are concave, which, together with the fixed DC set up cost, result in the establishment of fewer DC with larger capacity levels. This feature supports the consolidation policy in supply chain design.

The BO model is developed and demonstrated in subsection 6.2. To capture the concept of "technology break-points" a piecewise linearization (PWL) of the BO model is given in subsection 6.3. The MILP model is provided in subsection 6.4. Numerical examples abstracted from the model company are given to demonstrate the effectiveness of LINGO $^{\circledR}$ to solve the supply chain design models.

6.2. Model formulation

The customer zones, potential DC locations, and suppliers are indexed by $i \in I$, $j \in J$, and $k \in K$, respectively. The binary variables x_j indicate if a DC is to be built at location j. Binary variable y_{ji} is 1 if customer zone i is supplied by DC j, and 0 otherwise. Since y_{ji} is binary, the following set of constraint

would ensure that every customer zone is fully served by a single built DC,

$$\sum_{j \in j} y_{ji} = 1, \quad \forall i \in I.$$

Let p_i be the out-bound pallet demand at customer zone i. Since x_j is binary,

$$\sum_{i \in I} p_i \, y_{ji} - M x_j \leq 0, \quad \forall j \in J,$$

ensures that, for each DC, the total demand from the customer zones assigned to that DC does not exceed the maximum allowable capacity level M, where, for example, M is set as the total network demand.

Clearly, the model allows complete variability, from 0 to M, in the selection of the capacity level for each DC. In fact, the model sets the capacity level of a DC to the total demand from its assigned customer zones.

Let d_{ji} be the distance in kilometers from DC j to customer zone i. If w is the cost of shipping one out-bound pallet one kilometer, then the total out-bound transportation cost is

$$T_{out}(y) = w \sum_{j \in J} \sum_{i \in I} d_{ji} \, p_i \, y_{ji}.$$

Let $\hat{d}_{kj}$ be the distance in kilometers from the nearest location of supplier k to DC j, and let w_k be the cost to ship one in-bound pallet from supplier k one kilometer. Let ρ_k be the percentage of the generic out-bound pallet that is provided by supplier k. Then $(\rho_k p_i)$ is the number of in-bound pallets from supplier s required to assemble the out-bound pallets delivered to customer zone

r. The total in-bound transportation cost is

$$T_{in}(y) = \sum_{k \in K} w_k \sum_{j \in J} \sum_{i \in I} \hat{d}_{kj}(\rho_k p_i) \, y_{ji} = \sum_{k \in K} w_k \rho_k \sum_{j \in J} \hat{d}_{kj} \sum_{i \in I} p_i \, y_{ji}.$$

To capture economies of scale the operational (variable) costs are modeled with the nonlinear function

$$V(y) = \sum_{j \in J} \alpha \left(\sum_{i \in I} p_i \, y_{ji} \right)^{\beta}$$

where $\alpha > 0$ and $0 < \beta < 1$. Similarly, the building costs with

$$B(y) = \sum_{j \in J} \gamma \left(\sum_{i \in I} p_i \, y_{ji} \right)^{\delta}$$

where $\gamma > 0$ and $0 < \delta < 1$. By using α and β for all operational costs and γ and δ for all building costs, the assumption is that costs are independent of location. It would be a simple matter to subscript the cost function parameters in order to account for location and this would not change the complexity of the model. In practice, the cost parameters can be approximated using regression with historical cost data.

In order to produce designs with fewer DCs, the fixed DC set up cost

$$F(x) = f \sum_{j \in J} x_j,$$

where f is a positive parameter, is included in the objective function.

Combining the above, results in the following Binary, Non-linear Optimiza-

tion model (**BO**) with linear constraints and a concave objective function.

$$\text{Minimize} \quad f(x, y) = T_{out}(y) + T_{in}(y) + V(y) + B(y) + F(x) \quad (\textbf{BO})$$

$$\text{Subject to:} \quad \sum_{j \in J} y_{ji} = 1, \quad \forall i \in I,$$

$$\sum_{i \in I} p_i\, y_{ji} - M\, x_j \leq 0, \quad \forall j \in J,$$

$$x_j \in \{\, 0, 1 \,\}, \quad \forall j \in J,$$

$$y_{ji} \in \{\, 0, 1 \,\}, \quad \forall j \in J, \forall i \in I.$$

The model is tested on nine examples abstracted from real data available from the model company and described in table 16. The cost function parameters used are $\alpha = 256.03$, $\beta = 0.7706$, $\gamma = 519.18$, $\delta = 0.5978$ and $f = 50\,000$. The maximum capacity level is set at $M = 10\,000$.

The global solver in LINGO $^{\circledR}$ 14 is run on a 64-bit DELL PC with two 2.50 GHz threads (cores) and with 32 GBs of RAM. All problems are solved to optimality and the results in table 17 show the solution time in seconds, the indexes of the selected DCs and the corresponding capacity levels, and the optimal objective function value. The excessive time required to solve examples 6 - 8 is one motivation to consider a piece-wise linearization of the concave objective function. Another motivation is that the break-points in the piece-wise linearization can capture technology break-points. While the cost functions are linear, the slope of the linear function decreases at the break-points corresponding to a decreased cost per pallet with a higher level of technology.

Ex.	Total Network Pallet Demand	Number of Customer Zones	Number of Potential Distribution Centers
1	1107	11	10
2	1663	10	5
3	725	8	5
4	592	9	3
5	523	6	3
6	19 924	18	14
7	16 726	15	12
8	27 019	10	7
9	29 362	8	5

Table 16: Description of the test problems.

The global solver in LINGO $^{\circledR}$ 14 is run on a 64-bit DELL PC with two 2.50 GHz threads (cores) and with 32 GBs of RAM. All problems are solved to optimality and the results in table 17 show the solution time in seconds, the indexes of the selected DCs and the corresponding capacity levels, and the optimal objective function value.

Ex.	Time (sec.)	Selected DC IDs	Capacity of Selected DCs	Optimal $f(x, y)$
1	29	1	1 107	824 690
2	3	1	1 663	1 065 610
3	1	1	725	337 481
4	1	3	592	259 650
5	1	2	523	342 340
6	145 011	1, 8, 13	5 934, 4 013, 9 977	12 012 173
7	985	1, 12	6 756, 9 970	10 183 400
8	414	2, 4, 7	9 208, 7 845, 9 966	17 609 700
9	31	2, 3, 4, 5	1 777, 8 233, 9 937, 9 415	19 489 200

Table 17: Solution statistics for the BO model.

The excessive time required to solve examples 6 and 7 is one motivation to consider a piece-wise linearization of the concave objective function. Another motivation is that the break-points in the piece-wise linearization can capture technology break-points. While the cost functions are linear, the slope of the linear function decreases at the break-points corresponding to a decreased cost

per pallet with a higher level of technology.

In column 4, we see the tailored capacity assignment to the built DCs. That is the merit of the variable-capacitated supply chain design. There is no un-used capacity in a built DC. Furthermore, there is no need to have a predetermined set of discrete capacity levels, and thus, our proposed model allows for complete flexibility in capacity assignment.

6.3. Piece-wise linearization

The concave parts of the objective function are replaced with piece-wise linearizations. That is, $f(x, y)$ is replaced by

$$f_{pwl}(x, y) = T_{out}(y) + T_{in}(y) + V_{pwl}(y) + B_{pwl}(y) + F(x)$$

where $V_{pwl}(y)$ is obtained as follows. $(B_{pwl}(y)$ is obtained in an analogous way.) Define the break-points Δ_ι, $\iota = 0, 1, 2, 3, 4, 5$, where

$$0 = \Delta_0 < \Delta_1 < \Delta_2 < \Delta_3 < \Delta_4 < \Delta_5 = M$$

and denote the function values at the break-points by

$$V_\iota = \alpha(\Delta_\iota)^\beta.$$

Define, for $j \in J$,

$$\hat{y}_j = \sum_{i \in I} Y_{ji} p_i.$$

Then

$$V_{pwl}(\hat{y}_j) = \begin{cases} V_0 + (\hat{y}_j - \Delta_0)\left(\frac{V_1 - V_0}{\Delta_1 - \Delta_0}\right), & \text{if } \Delta_0 \leq \hat{y}_j \leq \Delta_1, \\[2em] V_1 + (\hat{y}_j - \Delta_1)\left(\frac{V_2 - V_1}{\Delta_2 - \Delta_1}\right), & \text{if } \Delta_1 \leq \hat{y}_j \leq \Delta_2, \\[2em] V_2 + (\hat{y}_j - \Delta_2)\left(\frac{V_3 - V_2}{\Delta_3 - \Delta_2}\right), & \text{if } \Delta_2 \leq \hat{y}_j \leq \Delta_3, \\[2em] V_3 + (\hat{y}_j - \Delta_3)\left(\frac{V_4 - V_3}{\Delta_4 - \Delta_3}\right), & \text{if } \Delta_3 \leq \hat{y}_j \leq \Delta_4, \\[2em] V_4 + (\hat{y}_j - \Delta_4)\left(\frac{V_5 - V_4}{\Delta_5 - \Delta_4}\right), & \text{if } \Delta_4 \leq \hat{y}_j \leq \Delta_5 \end{cases}$$

and

$$V_{pwl}(y) = \sum_{j \in J} V_{pwl}(\hat{y}_j).$$

The five break-points $\Delta_1 = 750$, $\Delta_2 = 1650$, $\Delta_3 = 2900$, $\Delta_4 = 5000$ and $\Delta_5 = 10000$, are motivated by the set of five discrete capacity levels used by the model company. The model is also tested using the five evenly distributed break-points $\Delta_1 = 2000$, $\Delta_2 = 4000$, $\Delta_3 = 6000$, and $\Delta_4 = 8000$ and $\Delta_5 = 10000$.

6.4. MILP model for technology-based variable-capacitated SCND

We note that the piece-wise linear program is also a non-linear program. That is because the piece-wise objective function consists of *if-then* statements which equal *either-or* statements [44].

In this subsection, we model the piece-wise linear program as mixed integer linear program while capturing technology break-points and economies of scale. Let l be the set of technology levels where each technology level is associated with a capacity level range for a built DC. For example, $l = 1$ represents full-

manual technology and captures the capacity level range of $[1, 750]$ shipment units. Let x_j^l be a binary variable which is 1 when DC j is operating with technology level l and can operate in the capacity range associated with technology l, and be 0 otherwise. We note that the assignment of capacity to a built DC is variable because depending on the selected technology, the capacity level of the built DC can be picked from the capacity level interval. Let y_{ji} be a binary variable that is 1 when demand at customer zone i is served by DC j, and 0 otherwise.

The following set of constraints ensure that each DC j will be operating with a unique technology level l if it is built,

$$\sum_{l \in L} x_j^l \le 1 \quad \forall\, j \in J.$$

The following set of constraints ensure that each customer zone i will be served by a single built DC j,

$$\sum_{j \in J} y_{ji} = 1 \quad \forall\, i \in I.$$

The technology level with which a built DC is operating determines the capacity level range for that DC. To ensure that the activity level of a built DC j operating with technology level l is bound by the lower bound and the upper bound of the capacity level range associated with that technology level, we have

$$z_j^l \le x_j^l * \text{UpBound}_l \quad \forall\, j \in J,\ l \in L,$$

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and

$$x_j^l * \text{LowBound}_l \leq z_j^l \quad \forall \, j \in J, \, l \in L,$$

where,

$$z_j^l \geq 0,$$

and,

$$\sum_{l \in L} z_j^l = \sum_{i \in I} p_i * y_{ji} \quad \forall \, j \in J.$$

The out-bound transportation costs are given by,

$$T_{out}(y) = w \sum_{j \in J} \sum_{i \in I} k_{ji} \, p_i \, y_{ji}.$$

And the following captures the in-bound transportation costs,

$$T_{in}(y) = \sum_{s \in S} w_s \sum_{j \in J} \sum_{i \in I} \hat{k}_{sj}(\rho_s p_i) \, y_{ji} = \sum_{s \in S} w_s \rho_s \sum_{j \in J} \hat{k}_{sj} \sum_{i \in I} p_i \, y_{ji}.$$

The fixed set up cost of a DC is captured by,

$$F(x) = f \sum_{j \in J} \sum_{l \in L} x_j^l.$$

Where f is a positive parameter.

The building and operating costs of a DC operating with technology level l are captured by the intercept and the slope of the linear cost function, respectively. The intercept of the linear cost function for technology level l is denoted by n^l and the slope of the linear cost is denoted by a^l. As the technology level l advances, the building costs, n^l, increase and the operating costs per item or pallet, a^l, decreases.

The building costs of a DC operating with technology level l are given by,

$$B(x) = \sum_{j \in J} \sum_{l \in L} x_j^l \, n^l$$

The operating costs of a DC working with technology level l are captured by,

$$V(z) = \sum_{j \in J} \sum_{l \in L} z_j^l \, a^l.$$

Assembling the constraints and the objective function components, a Mixed Integer Linear Program (MILP) for technology-based, variable-capacitated supply chain network design is given below.

Minimize $\quad f(x, y) = T_{out}(y) + T_{in}(y) + V(z) + B(x) + F(x) \quad$ **MILP**

Subject to:

$$\sum_{l \in L} x_j^l \leq 1 \quad \forall \, j \in J.$$

$$\sum_{j \in J} y_{ji} = 1 \quad \forall \, i \in I.$$

$$z_j^l \leq x_j^l * \text{UpBound}_l \quad \forall \, j \in J, \, l \in L,$$

$$x_j^l * \text{LowBound}_l \leq z_j^l \quad \forall \, j \in J, \, l \in L,$$

$$\sum_{l \in L} z_j^l = \sum_{i \in I} p_i * y_{ji} \quad \forall \, j \in J,$$

$$z_j^l \geq 0 \quad \forall j \in J, \, l \in L,$$

$$x_j^l \in \{0, 1\}, \quad \forall j \in J, \, l \in L$$

$$y_{ji} \in \{0, 1\}, \quad \forall \, j \in J, i \in I.$$

The MILP has more continuous variables and constraints than the BO and

PWL models. The BO model, and the PWL with both sets of break-points, and the MILP model, produce identical solutions, i.e., they select the same DCs with the same capacity level and the same assignment of customer zones to built DCs. What is of interest is the time required to find the solution. The computational times (in seconds) and the number of variables and constraints for BO, PWL and MILP models are summarized in table 18.

#	# of Variables			# of Constraints			Computational Times (Sec)			
	BO	PWL (Both)	MILP	BO	PWL (Both)	MILP	BO	PWL (Evenly)	PWL (Selected)	MILP
1	120	120	210	21	21	131	29	1	2	1
2	55	55	100	15	15	70	3	1	1	1
3	45	45	90	13	13	68	1	1	1	1
4	30	30	57	12	12	45	1	1	1	1
5	21	21	48	9	9	42	1	1	1	1
6	266	266	392	32	32	186	145 011	868	3 266	1
7	192	192	300	27	27	159	985	286	151	1
8	77	77	140	17	17	94	414	186	142	1
9	45	45	90	13	13	68	31	21	6	1

Table 18: Number of variables, constraints and computational times for BO, PWL and BIP

In terms of solution times, the PWL model is superior to the BO model. Except for examples 1 and 6, the PWL model with selected break-points is superior to that with evenly distributed break-points. However, the number and values of the break-points should be determined by balancing the desire for fewer breakpoints against a better piecewise linear approximation and by actual technology improvement levels. We observe that MILP model has more variables and constraints than BO and PWL models. As expected, MILP model outperforms BO, and PWL with evenly distributed and selected break-points in computational times. All models yield the same optimal solution and MILP is suggested as it offers low computational times.

Last, we attempt to solve the full network problem for the model company's full supply chain network design using the MILP. LINGO $^{®}$ solved the MILP model to optimality after 5.7 hours and produced $f(x, y) = 4\,399\,267$. The problem has 11 869 variables and 1 129 constraints when solved using the MILP model.

6.5. Conclusion

A Binary, Nonlinear, Concave Optimization model (BO) is developed for three-echelon, variable-capacitated supply chain design with complete variability in capacity selection for the distributions centers and with cost functions that captured economies of scale. A piece-wise linearization (PWL) of the objective function with evenly distributed and selected break-points is introduced to both improve solution times and to capture technology break-points (related to economies of scale). PWL offered lower computational times than BO while producing the same optimal solution because there is less trade off involved in the PWL in terms of economies of scale. Even so, PWL is also a non-linear program. Therefore, a mixed integer linear program (MILP) is developed via addition of continuous variables and constraints which captures technology break points.

MILP, BO and PWL with evenly distributed and selected break-points all yield the same optimal solution but in different computational times. While MILP has more variables and constraints than the BO and PWL models, as expected, it required the lowest computational times, i.e., 1 second, to reach optimality, for our test problems.

Being confident in the performance of the MILP model, we applied it to the model company's full supply chain design. LINGO $^{\circledR}$ solved the MILP model to optimality after 5.7 hours and produced $f(x, y) = 4\,399\,267$. The problem has $11\,869$ variables and $1\,129$ constraints when solved using the MILP model. The MILP model is suggested for technology-based, variable-capacitated, supply chain network design.

Chapter 7

7. Conclusions and Future Work

7.1. Conclusions

The first chapter of this dissertation provided an introduction to supply chain network design followed by a literature review in chapter 2.

In chapter 3, we provided a mixed integer linear optimization model for three-echelon multi-capacitated supply chain design network. The suppliers are at known locations with sufficient capacity to supply the network demand. The DCs are picked from a potential set of DC locations. The corresponding capacity level of the built DCs are picked from a predetermined set of potential discrete capacity levels. The customer zones have known demand and are at known locations. The model captures the operational costs of a facility, i.e., DC, based on the actual activity level of a DC while taking into account the economies of scale. The inclusion of the variable costs was via addition of continuous variables and constraints to the model. Furthermore, the building costs depend on the size of the built facility and the land costs depend on both the location and the size. We applied the model to solve a real world supply chain network design problem with $247\,838$ variables and $3\,557$ constraints. The model was solved to optimality after 9 hours using LINGO ®. We also proposed a model simplification which was to cluster the customer zones. The customer zones were clustered according to the first two characters of their postal code creating 133 clusters. The model was presented as Cluster Mixed Integer Linear Program (CMILP). Both MILP and CMILP models have the same number of binary

variables because binary variables are associated with establishment of DCs and it is unchanged in both models. However; the CMILP has fewer continuous variables than MILP, reduced from 247 423 to 11 454. The CMILP model could be solved to optimality in around 10 minutes. We input the solution of CMILP model in terms of DC selection into MILP to obtain MILP objective function value and non-cluster assignment of customer zones to built DCs. We observed that CMILP model produced a supply chain design which was 2.82% more costly than the design cost obtained from MILP but was obtained in 99.2% less computational times, i.e., 10 minutes. We suggest CMILP model to be used for a supply chain design problem with characteristics discussed in this chapter.

In chapter 4, we studied a well-known model from literature for three-echelon multi-capacitated supply chain network design by Amiri [1]. The location of suppliers and DCs are both picked from a set of potential DC locations and supplier locations. The capacity level of built DCs and suppliers are picked from a predetermined set of potential capacity levels. We characterized the complete set of alternate optimal solutions in Amiri model. We extended Amiri's model through the addition of a constraint set that eliminated certain undesirable optimal solutions and we showed that the extended model required, essentially, the same computational effort as the original.

In chapter 5, we deployed a new set of variables and presented a new formulation for three-echelon multi-capacitated supply chain network design. The new model has fewer continuous variables and constraints than the extended model while capturing the same model characteristics. It is shown that, in aver-

age, the new model can obtain optimal solutions 58% faster than the extended model, which makes it superior. We then presented two approaches which allow exponential increase in available capacity levels to DCs and the suppliers. Indeed, the increased flexibility in capacity assignment can only lead to less costly supply chain designs. The first approach is to eliminate a set of constraints with binary variables that enforce a single capacity selection. By doing so, we allow for the selection of several capacity levels for a single DC or supplier. If such a selection occurs, the DC or supplier is built with size equal to the sum of the selected capacity levels. This gives an exponential increase in the number of available capacity levels. While this extension provides an exponential increase, the number of available capacity levels is still finite. The second approach is to expand the set of capacity levels by adding more potential capacity levels to the set while keeping the constraints which enforce the selection of a single capacity level. This approach provides the same flexibility as the first approach but leads to the increase in number of binary variables and constraints. We validated both approaches with numerical examples. We then showed that the increased flexibility in capacity assignment is desirable because it leads to less costly supply chain network design. Furthermore, we showed that the exponentially increased flexibility done via the first approach outperforms the second approach. We note that, the models discussed in the fifth chapter all have the same objective function. This is to be able to evaluate the supply chain design cost produced by solving the models and then choosing the model which can yield less costly supply chain design in short computational times. We showed

that our proposed reformulated model which offers fewer continuous variables and constraints and also allows for exponential increase in capacity levels can produce less costly supply chain design in short computational times, i.e., 10 seconds, and thus is suggested.

In chapters 3 and 4, the capacity level of a built facility was picked from a predetermined set of capacity levels. In chapter 5, we presented a model which allowed for exponential increase. In chapter 6, we presented a technology-based, variable-capacitated, supply chain design model, and it is unique in that, it allows complete variability in the choice of capacity level and so avoids the need to determine, a priori, a set of potential capacity levels like in multi-capacitated supply chain model. Another merit of the model is that the built facilities will not have unused capacity. Facility oriented costs such as building costs and operational costs are concave functions of the shipment volume in a DC to fully capture economies of scale based on the activity level. It is a binary, concave optimization problem (BO). A piece-wise linearization (PWL) based on technology break points is utilized to solve the problem in lower computational times. Even so, PWL is also a non-linear program. Therefore, a mixed integer linear program (MILP) is developed via addition of continuous variables and constraints which also captures technology break points. As expected, the MILP yields optimal supply chain design for technology-based, variable-capacitated supply chain design in short computational times and thus is suggested.

Stochastic Programming: Demand forecast methods can help the analysts to predict the future demand for a given time period in their supply chain network and plan for the network design accordingly. However, it is worthwhile to study the impact of cost uncertainty in the variable-capacitated supply chain network design which can be caused by fluctuations in the dollar worth or gas price and can indeed impact the network design.

Variable-capacitated supply chain design integrated with inventory: Distribution centers usually hold certain inventory levels to deal with the uncertainties associated with the demand. This would require more capacity level and should be taken into consideration.

Development of exact algorithms or efficient techniques for solving large-scale supply chain network design problems faster: In chapter 3, we see that, for example, clustering the customer zones reduced the problem size and good quality solutions could be obtained in short computational times. Filtering technique is another approach which can help reduce the number of variables.

Different time-periods are not considered in this dissertation for supply chain design planning. Dynamic variable-capacitated supply chain network design with different time periods is another future work for this dissertation.

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