MATHEMATICAL DISPOSITIONS OF NOVICE UPPER-ELEMENTARY TEACHERS: A PHENOMENOLOGICAL STUDY

by

Leslie Anne Soltis

Liberty University

A Dissertation Presented in Partial Fulfillment
Of the Requirements for the Degree

Doctor of Education

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ABSTRACT

The purpose of this transcendental phenomenological study was to describe the mathematical dispositions of novice upper-elementary teachers by exploring their experiences as teachers of mathematics in the Great Lakes region of the United States. Ball's theory of mathematical knowledge for teaching and Bandura's self-efficacy theory guided this study. The research project sought to answer the central research question: How do novice upper-elementary teachers perceive and describe their experiences teaching mathematics? Data, in the forms of audio diaries, individual interviews, and online focus groups were collected from a purposeful sample of 10 novice upper-elementary teachers. Data analysis followed a systematic procedure that included the 3 core processes of epoche, transcendental phenomenological reduction, and imaginative variation. Three themes emerged from this research: Life Changing Decisions, Connections with Students, and Rethinking Mathematics Class. The findings revealed how novice upper-elementary teachers aspire to put their students first and make a difference in the way their students experienced learning mathematics. Trends in their intentional actions and behaviors indicated a productive disposition. Further research is needed about teachers' collective efficacy and the mathematical dispositions of secondary teachers of mathematics.

Keywords: mathematics education, teaching self-efficacy, self-efficacy beliefs, productive disposition, novice elementary teachers, teacher education

Dedication

This research project is dedicated to my Lord. I hope my work is pleasing in His eyes and glorifies His name.

Acknowledgments

There are several people I wish to recognize, without whom this dissertation would not have been possible. Dr. Fred Milacci first opened my eyes to the realm of qualitative inquiry and its possibilities for my research interests. Dr. Sandra Battige was a servant-leader with her primary focus on my growth and well-being as an educational researcher. Dr. James Zabloski increased the quality of my work with his expertise in transcendental phenomenological research and adherence to Liberty University's high standards. Dr. Corinne Schaeffer provided collegial support and dedicated her time to reading my work. The participants in this study graciously shared their personal experiences of teaching mathematics. And last, but not least, Mr. Robert Soltis stood lovingly by my side every step of the way.

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List of Abbreviations

Association of Mathematics Teachers Educators (AMTE)

Common Core State Standards Initiative (CCSSI)

Council for the Accreditation of Educator Preparation (CAEP)

Elementary Mathematics Specialists (EMS)

Mathematical Knowledge for Teaching (MKT)

National Assessment of Educational Progress (NAEP)

National Council for Accreditation for Teacher Education (NCATE)

National Council of Teachers of Mathematics (NCTM)

National Council on Teacher Quality (NCTQ)

National Research Council (NRC)

Pennsylvania Alternative System of Assessment (PASA)

Qualitative Data Analysis Software (QDAS)

Self-Efficacy for Teaching Mathematics Instrument (SETMI)

CHAPTER ONE: INTRODUCTION

Overview

Discussion of the mathematical knowledge needed for teaching has been abundant in mathematics education literature for almost a decade (Ball, Thames, & Phelps, 2008; Hoover, Mosvold, Ball, & Lai, 2016). The discourse often references elementary teachers, whose deficiencies in mathematical knowledge have been well-documented (Burroughs & Yopp, 2010; Capraro, An, Ma, Rangel-Chavez, & Harbaugh, 2012; Jacobbe, 2011; Kastberg & Morton, 2014; Livy & Vale, 2011; Lo & Luo, 2012; Maher & Muir, 2013). The field of teacher education is starting to acknowledge the importance of an integrated approach to the development of mathematical knowledge and skills, one that considers preservice elementary teachers' disposition toward the learning and teaching of mathematics (Cooke, 2015; Stohlman, Cramer, Moore, & Maiorca, 2015), particularly in undergraduate mathematics content courses for elementary teachers (Zazkis, Leikin, & Jolfaee, 2011). Few studies, however, examine the connection between practice in the field and teachers' mathematical dispositions, meaning their intentional actions and behaviors related to teaching mathematics (Katz & Raths, 1985). This research project addresses a gap in the literature by exploring elementary school teachers' lived experiences as teachers of mathematics to truly understand their mathematical dispositions.

In this chapter, a background on mathematics teacher education research is provided to demonstrate the importance of studying the mathematical dispositions of practicing elementary teachers. A discussion follows regarding how this qualitative inquiry is situated in the researcher's motivation and philosophical assumptions but is also grounded in teacher education literature. Then, the purpose of the research project and ways it can significantly contribute to the knowledge base in mathematics teacher education are presented. Next, the broad research

questions the researcher seeks to answer are introduced. Finally, definitions supported by the literature are listed to avoid any confusion in the meaning of terms used throughout this study.

Background

Dispositions are "neither invisible aspects of a teacher's psyche nor fixed personality traits" (Diez & Murrell, 2010, p. 14), which suggests they can be assessed and developed. Diez and Murrell (2010) described professional dispositions the following way:

[Dispositions] are commitments and habits of thought and action that grow as the teacher learns, acts, and reflects under the guidance of teachers and mentors in a preparation program and in the first years of practice. They are visible in a teacher's decisions and actions over time and especially in the teacher's reflections about the consequences of those decisions and actions. (p. 14)

In the context of mathematics education, the National Research Council (NRC, 2001) recognizes productive disposition as an essential aspect of being mathematically proficient. According to the NRC (2001), productive disposition refers to "the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics" (p. 131).

Historical Context

According to Diez and Murrell (2010), one of the earliest references to disposition in teacher education literature was made by Lilian Katz and James Raths. Teachers who had the requisite skills for teaching but did not use them to meet the needs of learners intrigued Katz and Raths (1985). This discrepancy prompted the researchers to propose "the goals of teacher education programs should include not only the acquisition of knowledge and skills, but also a class of outcomes [they] propose to call <u>dispositions</u>" (Katz & Raths, 1985, p. 1). By the late

1990s, disposition became an integral part of the discourse in teacher education (Diez & Murrell, 2010).

Unlike Katz and Raths' earlier observation, Ball and associates (2008) found elementary teachers did not possess the requisite mathematical knowledge and skills to meet the needs of learners. In their desire to improve the teaching and learning of elementary-level mathematics, the teacher educators sought to identify precisely what teachers needed to do to successfully teach mathematics. Their practice-based theory decomposed the complex task of teaching into everyday practices. Minimally, teachers must be able to apply procedural knowledge accurately and identify student errors. Teaching also requires skills in analyzing errors, communicating conceptual understandings, and unpacking mathematical concepts and ideas for students. Other professional knowledge includes anticipating student interpretations of mathematical tasks, analyzing students' mathematical thinking, and understanding common student conceptions and misconceptions. Ultimately, teachers should be able to design appropriate mathematics instruction and evaluate the effectiveness of instruction (Ball et al., 2008). Since its inception, the theory has substantially informed research on the development and evaluation of teacher education programs (Hoover et al., 2016).

Initially, studies focused on assessing the mathematical understandings of preservice elementary teachers that were deemed necessary to carry out everyday teaching tasks (Thanheiser, Browning et al., 2014). For instance, Maher and Muir (2013) discovered preservice elementary teachers' partial understanding of place value prevents them from being able to fluently multiply multi-digit numbers and adequately unpack the multiplication algorithm to students. In a similar study, Burroughs and Yopp (2010) revealed teachers' perception of repeating decimals as processes rather than as numbers hinders their ability to communicate a

conceptual understanding of the real number system to students. The results of such studies, called *static studies of knowledge* (Thanheiser, Browning et al., 2014, p. 439), have prompted other educational researchers to examine the development of preservice elementary teachers' mathematical knowledge, particularly in mathematics content courses for elementary teachers (Kastberg & Morton, 2014; Livy & Vale, 2011; Zazkis et al., 2011). Research has also shown when preservice elementary teachers are given opportunities to decompose and approximate various teaching practices of elementary school teachers (Grossman, 2011), they begin to think like teachers (Charalambous, Hill, & Ball, 2011), and their mathematical knowledge for teaching increases (Hoover et al., 2016).

Elementary teachers' growth in practices associated with the mathematical knowledge for teaching requires a productive disposition about engaging in such practices (Charalambous et al., 2011). However, research studies have revealed many preservice elementary teachers describe an adversarial relationship with mathematics (Hobden & Mitchell, 2011), hold deeply entrenched negative attitudes toward the teaching and learning of mathematics based on their past experiences (Lutovac & Kaasila, 2014), and often transfer their negative views of mathematics onto the students they eventually teach (Capraro et al., 2012; Sloan, 2010). Since that time, researchers have been investigating ways to incorporate productive dispositions toward mathematics in teacher education programs (Bartell, Webel, Bowen, & Dyson, 2013; Beswick & Muir, 2013; Charalambous et al., 2011; Maasepp & Bobis, 2015; Spitzer, Phelps, Beyers, Johnson, & Sieminski, 2011; Stohlman et al., 2015). The first step may be to explore novice elementary teachers' experiences teaching mathematics with a focus on participants' understanding of their own mathematical dispositions.

Social Context

Despite the extensive research literature on how to teach mathematics, the majority of fourth graders and almost two thirds of eighth graders in the United States are performing below the proficiency level in mathematics (National Assessment of Educational Progress [NAEP], 2015). Some researchers attributed the shortfall in student achievement to environmental factors such as poverty rate, prevalence of single-parent households, income levels, expenditures per pupil, and school district size (Koshal, Koshal, & Gupta 2013). However, Darling-Hammond and Lieberman (2012), two leading figures in the mathematics education reform movement, found the effects of a well-prepared teacher on student achievement were stronger than any social factors. Darling-Hammond (2010) determined a well-prepared mathematics teacher is capable of understanding the development of students' mathematical knowledge, planning lessons in response to state standards and individual student needs, analyzing and assessing student understanding, and reflecting on and revising teaching practices. As a result, much of the mathematics reform movement has emphasized the necessity of effective teacher education programs to initiate genuine reforms in mathematics education (Darling-Hammond & Lieberman, 2012).

To initiate genuine reforms in mathematics education, mathematics teacher educators are starting to conduct self-study research (Leaman & Flanagan, 2013; Marin, 2014; McGlynn-Stewart, 2010). Self-study research is an opportunity to explore, understand, and improve not only one's teaching practices but also the field of mathematics education (Marin, 2014). At the conclusion of the self-study, McGlynn-Stewart (2010) admitted while her undergraduate students faced and overcame their fears of learning and teaching mathematics, she faced her own fears of adequately preparing them. Similarly, through self-study, Leaman and Flanagan (2013) were

forced to reframe and expand their initial assumptions about teacher education. Finally, research suggested teacher educators and preservice elementary teachers embark on parallel journeys (McGlynn-Stewart, 2010) when navigating the learning experiences provided in mathematics content courses (Leaman & Flanagan, 2013; Marin, 2014).

Theoretical Context

Bandura's (1977) self-efficacy theory purports all psychological changes within a person stem from a common cognitive mechanism called perceived self-efficacy. Self-efficacy is defined as "people's beliefs about their capabilities to exercise control over their own level of functioning and over events that affect their lives" (Bandura, 1993, p. 118). Bandura (1986) emphasized self-efficacy is "concerned not with the skills one has but with the judgements of what one can do with whatever skills one possesses" (p. 391). Bandura's definition of self-efficacy is consistent with the original reference of Katz and Raths (1985) to teacher disposition, which is "the probability of actual frequencies with which categories of skills are employed, rather than simply whether or not they have been mastered by the candidate" (p. 8).

In relationship to teaching mathematics at the elementary level, the theory suggests elementary teachers' self-efficacy beliefs influence how they feel, think, motivate themselves, and behave during their early experiences teaching mathematics (Bandura, 1993). This theory is useful to guide the exploration of novice elementary teachers' experiences as teachers of mathematics. Other studies have reported teachers' sense of efficacy predicts their effort level to help correct students' faulty mathematical reasoning (Spitzer et al., 2011), their willingness to work with students rather than refer them to special education (Tschannen-Moran, Woolfolk-Hoy, & Hoy, 1998), their persistence in the face of difficulties (Gresham, 2009), their experimentation with different teaching techniques such as inquiry-based methods (Wilkins,

2008), and their enthusiasm about teaching (Bates, Latham, & Kim, 2011).

Situation to Self

Researchers in the human sciences, of which education is a part, do not pursue research simply for the sake of research (van Manen, 1990). Their motivation typically comes from what Milacci and Kuhne (2014) referred to as an "itch" that stems from an issue or problem experienced in everyday life. I openly disclose to the reader my personal motivation for conducting the study and my philosophical assumptions underlying this research project (Creswell, 2013) because the researcher is considered the human instrument in qualitative research inquiry (Patton, 2015).

My Personal Motivation

Ever since I earned my bachelor's degree in secondary education, my professional life has been devoted to mathematics education. Most of my teaching career has been spent teaching college-level mathematics courses, ranging from remedial algebra to the calculus series. Throughout my experiences in academia, I have observed that mathematics faculty generally classify themselves as either pure mathematicians or mathematics educators. I place myself in the latter group. Neither classification has an extensive background in elementary mathematics education. Despite this shortcoming, mathematics content courses for elementary teachers are typically taught by professors of mathematics rather than by education faculty.

Although I have been teaching college-level mathematics for over 19 years, I now find myself overwhelmed by the awesome responsibility of nurturing and developing productive mathematical dispositions and mathematical content knowledge among preservice elementary school teachers. Additionally, national efforts to increase mathematical proficiency (NRC, 2001) and improve K-12 mathematics teacher education (Association of Mathematics Teachers

Educators [AMTE], 2016) place pressure on me to effectively prepare my elementary education students for the task of teaching a robust mathematics curriculum (Common Core State Standards Initiative [CCSSI], 2016). When I consider how powerful the learning experiences provided in mathematics content courses can be on preservice elementary teachers' mathematical proficiency—and that of their future students—I am aware of my professional obligation to follow through with my students' experiences teaching mathematics in real elementary school settings. Thus, examining the lived experiences of novice elementary mathematics teachers who completed undergraduate mathematics content courses has become the focus of my doctoral research.

My Philosophical Paradigm and Assumptions

Given my background in statistical analysis, I assumed my research project would involve an experiment to determine whether observed differences in a dependent variable (i.e. elementary teachers' mathematical disposition) can be attributed to the independent variable (i.e. some theory-based intervention in a mathematics content course). However, while immersing myself in the literature on my research topic, the words of van Manen (1990) struck a chord with me when he said, "much of educational research tends to pulverize life into minute abstracted fragments and particles that are of little use to practitioners" (p. 7). I felt a more holistic account of novice elementary teachers' experiences with teaching mathematics was essential to truly understand their mathematical dispositions, not one restricted by cause-and-effect relationships and self-report surveys typical of many quantitative research methods (Creswell, 2013). A holistic account entails reporting multiple perspectives, describing the many factors involved in learning and teaching mathematics, and assembling a larger picture that emerges about novice elementary teachers' mathematical dispositions (Creswell, 2013; Patton, 2015).

As a relatively new instructor of mathematics content courses for preservice elementary teachers, I am seeking an in-depth understanding of what it means to be a teacher of elementary-level mathematics, particularly as it pertains to their mathematical disposition. Researchers who identify with social constructionism "seek to understand the world in which they live and work" (Creswell, 2013, p. 24) and acknowledge the human world must be studied differently than the physical world (Patton, 2015). My identification with the assumptions of social-constructionism paradigm—not my prior experiences analyzing quantitative data—has ultimately guided my qualitative inquiry into the mathematical dispositions of novice elementary teachers.

Ontology. My ontological assumption aligns with Thomas's theorem (as cited in Patton, 2015), which states "If men define situations as real, they are real in their consequences" (p. 121). I expected elementary teachers and teacher educators would have different experiences and perceptions about mathematics content courses, all of which were real and deserve attention (Patton, 2015). In my research, I honored the idea of multiple realities (Patton, 2015) by disclosing my biases and explaining how they may affect the research, acknowledging my findings provide only one perspective, and investigating the multiple realities constructed by my participants as well as the implications of those constructions for their lives and interactions with others (Patton, 2015). A phenomenological approach allowed me to describe the different perspectives of elementary school teachers as themes emerged in the findings (Moustakas, 1994).

Epistemology. My epistemological assumption, the belief that knowledge is uncertain and based on fallible human judgment (Schwandt, 2015), led me to believe even though subjective data were gathered from participants, this is how knowledge is known (Creswell, 2013). In this study, I am not claiming to objectively describe reality (Patton, 2015); instead, I engaged in the social construction of multiple realities. I believe the more I got to know my

participants in the context of where they lived and worked, the more I discovered about their mathematical dispositions in the real-world context of everyday teaching (Creswell, 2013). To identify and describe the mathematical dispositions of novice elementary teachers, I relied on their direct quotes as evidence (Creswell, 2013).

Axiology. My axiological assumption, the belief that all research is fundamentally value-laden (Creswell, 2013), compelled me to explain my role as the researcher in the study. Therefore, I openly discussed my own experiences learning and teaching mathematics in an attempt to reveal the values and biases I brought to this study. During data collection and analysis, I bracketed my own perspective in an effort to increase my objectivity as the human instrument in this qualitative inquiry (Moustakas, 1994). To further address any potential biases in information gathered from the field, I used member checking, meaning participants were given an opportunity to review and confirm—or alter—my analyses to accurately reflect their perceptions of their actions and behaviors related to teaching mathematics (Moustakas, 1994).

Problem Statement

Currently, 60% of American students are classified as below proficiency in mathematics upon entering their middle-level years of school (NAEP, 2015). If students are to become proficient in mathematics, then their teachers must be mathematically proficient as well (Darling-Hammond & Lieberman, 2012). According to the NRC (2001), for a teacher to be mathematically proficient, he or she must also have a productive mathematical disposition. Several studies have indicated elementary teachers enter the teaching profession with unproductive mathematical dispositions (Hobden & Mitchell, 2011; Lutovac & Kaasila, 2014). As a result, several researchers in teacher education have focused on identifying instructional activities and methods that create mathematically rich opportunities for preservice elementary

teachers. These preservice activities, such as exploring mathematical content with manipulatives (Lutovac & Kaasila, 2014; Sloan, 2010; Zazkis et al., 2011), integrating mathematics with music (An, Ma, & Capraro; 2011) and art (Zazkis et al., 2011), and writing mathematical autobiographies (Hobden & Mitchell, 2011; Lutovac & Kaasila, 2014), were designed to help preservice teachers develop productive dispositions toward mathematics. However, these studies have provided only a glimpse into preservice teachers' knowledge, skills, and dispositions (Thanheiser, Whitacre, & Roy, 2014) in a university setting (Beswick & Muir, 2013; Feldhaus, 2012; Namukasa, Gadanidis, & Cordy, 2009; Savard, 2014). Much of the literature regarding teachers' mathematical dispositions came from a quantitative perspective in the static, laboratory-like setting of preservice college classrooms. The problem is few studies have examined the mathematical dispositions of elementary teachers after they experienced teaching mathematics on a daily basis.

Purpose Statement

The purpose of this transcendental phenomenological study was to describe the mathematical dispositions of novice upper-elementary teachers by exploring their experiences as teachers of mathematics in the Great Lakes region of the United States. For the purposes of this study, mathematical dispositions referred to trends in participants' intentional actions and behaviors related to teaching elementary-level mathematics content (Katz & Raths, 1985).

Participants were novice teachers of upper-elementary school mathematics who had completed their undergraduate teacher education programs at universities in Pennsylvania and had less than five years' teaching experience. Upper-elementary refers to grades 4 through 6. The theories guiding this study were self-efficacy theory (Bandura, 1977) and the practice-based theory of mathematical knowledge for teaching (Ball et al., 2008) as they related to novice elementary

teachers' experiences in carrying out the everyday tasks associated with teaching elementarylevel mathematics content.

Significance of the Study

Hearing the voices of upper-elementary teachers as they describe their everyday experiences teaching mathematics ultimately provided insight into their dispositions about mathematics. An in-depth understanding of the mathematical dispositions of elementary teachers was significant to the field of mathematics education and addressed a gap in the literature. Teachers, teacher educators, mathematicians, mathematics education reformers, members of the entire school community, and educational researchers might be interested in the research findings.

Practical Significance

Knowledge, attitudes, and beliefs directly influence teachers' instructional practices (Ernest, 1989), which, in turn, impact students' mathematical achievement (Darling-Hammond & Lieberman, 2012). The National Council of Teachers of Mathematics (NCTM, 2016) insists children of all levels can benefit from the use of inquiry-based instructional practices. Research has shown, of the three teacher attributes of knowledge, attitudes, and beliefs, beliefs were the strongest indicator of upper-elementary teachers' willingness to implement such reform-minded practices (Wilkins, 2008). The nature of inquiry-based learning encourages children—and their teachers—to make sense of problems and persevere in solving them, which is one of the Standards for Mathematical Practice described in the CCSSI (2016) that teachers at all levels are expected to develop in their students. This mathematical practice also aligns with emerging conceptualizations of mathematical proficiency, which should be a central goal for schools in the United States (NRC, 2001).

Clark and associates (2014) pointed out to the mathematics education community "If the promise of CCSSM is to be realized, then teacher beliefs related to the importance and necessity for student struggle will need to be realized, particularly with elementary teachers" (p. 271).

Successful implementation of CCSSM, or other conceptually-based mathematics standards, requires balanced attention to teachers' mathematical content knowledge and their beliefs about mathematics and learning mathematics (Campbell et al., 2014). According to Diez and Murrell (2010), teachers' dispositions are visible in their decisions, actions, and reflections. Thus, exploring novice upper-elementary teachers' experiences associated with teaching elementary-level mathematics content can provide greater insight into their mathematical dispositions. A deeper understanding of novice upper-elementary teachers' mathematical dispositions can provide valuable knowledge to enhance the curriculum of both undergraduate and continuing teacher education programs. Thus, it is important for mathematics education researchers to study teachers' mathematical dispositions (Campbell et al., 2014; Clark et al., 2014; Cooke, 2015; Diez & Murrell, 2010; Katz & Raths, 1985).

Empirical Significance

When the National Council for Accreditation of Teacher Education (NCATE) added dispositions to its expectations, changes occurred in teacher education practices and in the direction of educational research (Diez & Murrell, 2010). Mathematics education was no exception. Mathematics teacher educators came to recognize the importance of an integrated approach to the development of mathematical knowledge and skills, one that considered preservice elementary teachers' mathematical dispositions (Cooke, 2015; Stohlman et al., 2015). To develop productive dispositions among preservice elementary teachers, some professors of mathematics education engaged in self-study research, during which they reflected deeply upon

their own teaching practices and underlying beliefs (Leaman & Flanagan, 2013; Marin, 2014; McGlynn-Stewart, 2010). Many experimental studies investigated the effectiveness of specific interventions to develop preservice elementary teachers' self-efficacy beliefs for regulating their own learning of mathematical concepts and mastering the task of teaching mathematics conceptually (Beswick & Muir, 2013; Charalambous, Panaoura, & Philippou, 2009; Maasepp & Bobis, 2015; Namukasa et al., 2009; Savard, 2014). Few studies, however, have examined the connection between mathematical disposition and actual practice in the field. Thus, this study was different from previous studies as it focused on elementary teachers' experiences beyond the university setting.

Theoretical Significance

Mathematical disposition can be also described as a combination of mathematical beliefs and mathematics self-efficacy, both of which were found to be strong predictors of mathematics teaching efficacy (Briley, 2012). Mathematics teaching self-efficacy was a construct predominately studied using quantitative research methods with self-report surveys (Klassen, Tze, Betts, & Gordon, 2011; Moriarity, 2014). Surprisingly, nearly half of recent self-efficacy studies have continued to use "discredited, poorly conceptualized, and flawed measures of teacher self-efficacy" (Klassen et al., 2011, p. 32), which might result in misleading conclusions. Grouws, Howald, and Colangelo (1996) demonstrated the likelihood of reaching misleading conclusions when they said:

Strongly agreeing to the statement 'I am good in mathematics' when mathematics is perceived of as an accurate implementation of known procedures carries quite different implications for success, or willingness to learn . . . than when mathematics is perceived as figuring out relationships and discovering principles. (p. 4)

In addition, the self-efficacy studies chosen by Klassen et al. (2011) in their literature review preceded the implementation of rigorous state standards for mathematics. The adoption of such standards was anticipated to result in a "considerable increase in cognitive demands on mathematical content and mathematical practices expected across Grades 4-8" (Campbell et al., 2014, pg. 455). Therefore, the findings of previous self-efficacy studies may not be generalizable to novice upper-elementary teachers of the current era, the target population of this study. To advance self-efficacy theory, this study employed qualitative research methods to further explore how the mathematical dispositions of practicing teachers compare with attributes that are consistent with the reform ideals of the NCTM (2016).

Research Questions

In light of the purpose of this study, my intent was to answer the central research question (CQ) from the perspective of novice upper-elementary teachers. The four sub-questions (SQ) probed into specific areas I needed to explore (Creswell, 2013) to accomplish this task.

CQ: How do novice upper-elementary teachers perceive and describe their experiences teaching mathematics? Research studies examined preservice elementary teachers' experiences in undergraduate mathematics content and methods courses (Beswick & Muir, 2013; Charalambous et al., 2009; Charalambous et al., 2011; Grossman, 2011; Hart, Oesterle, & Swars, 2013; Kastberg & Morton, 2014; Maasepp & Bobis, 2015; Namukasa et al., 2009; Savard, 2014; Smith, Swars, Smith, Hart, & Haardörfer, 2012; Zazkis et al., 2011). The instructional practices used in these courses changed significantly as a result of Ball's work in clarifying the mathematical knowledge needed for teaching. Research, however, has indicated teachers' professional growth tends to develop over time through a series of gradual changes in their knowledge, beliefs, dispositions, and classroom practices (Goldsmith, Doerr, & Lewis,

2014). Therefore, it was important to explore novice elementary teachers' experiences in actually carrying out—as opposed to simply learning about—the everyday practices associated with teaching elementary-level mathematics content.

SQ1: What do novice upper-elementary teachers' experiences reveal about their mathematical dispositions? Professional dispositions had been an integral part of the discourse in teacher education (Diez & Murrell, 2010), including mathematics teacher education.

Mathematical dispositions cannot be measured solely with Likert-type scales (Katz & Raths, 1985) that are typically found in self-report surveys. Novice elementary teachers may not even recognize or encounter their own belief systems surrounding mathematics (Diez & Murrell, 2010). Engaging participants in guided reflections about their experiences teaching mathematics revealed valuable information about their mathematical dispositions.

SQ2: How do trends in novice upper-elementary teachers' intentional actions and behaviors compare to tendencies associated with a productive mathematical disposition, as defined by the National Research Council? The NRC (2001) defined a productive mathematical disposition as "the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics" (p. 131). Mathematics education researchers recognized the importance of developing productive mathematical dispositions among elementary teachers (Beswick & Muir, 2013; Charalambous et al., 2009; Cooke, 2015; Hart et al., 2013; Kastberg & Morton, 2014; Leaman & Flanagan, 2013; Maasepp & Bobis, 2015; Marin, 2014; McGlynn-Stewart, 2010; Namukasa et al., 2009; Savard, 2014; Stohlman et al., 2015; Wilkins, 2008; Zazkis et al., 2011). A productive disposition was characterized not only as an integral part of mathematical proficiency (Feldhaus, 2012; Gautreau, Kirtman, &

Guillaume, 2011; Kuzle, 2013; NRC, 2001; Siegfried, 2012) but also a mindset of teachers needed to bring about genuine reforms in mathematics education (Campbell et al., 2014; Clark et al., 2014; NCTM, 2016). Given the prominent role productive disposition has in mathematics education, comparing novice elementary teachers' actions and behaviors to those associated with a productive disposition was fitting.

SQ3: How do novice upper-elementary teachers describe their self-efficacy beliefs of teaching mathematics? Self-efficacy theory stated novice teachers' perceptions of their capabilities to carry out the everyday tasks of teaching elementary-level mathematics influence how they feel, think, motivate themselves, and behave in elementary school settings (Bandura, 1993). Furthermore, the choices teachers make based on their mathematics teaching self-efficacy result in the cultivation of different competencies, interests, and social networks that can significantly alter their life courses (Bandura, 1993). Thus, an in-depth description of the lived experiences of novice elementary teachers included their self-efficacy beliefs of teaching mathematics.

SQ4: What factors do novice upper-elementary teachers identify as influencing their experiences teaching mathematics? Novice teachers are unfinished products (Feiman-Nemser, 2012b). The contexts of teaching—prior education, initial training, school, profession, community, and society—can help create and affirm, or even challenge, novice elementary teachers' commitments as teachers of mathematics (Bauml, 2015; Diez & Murrell, 2010; Feiman-Nemser, 2012a; Fulton, Yoon, & Lee, 2005). In a transcendental phenomenological study, all presuppositions are put aside to allow for a fresh perspective on the factors novice elementary teachers believe have influenced their experiences teaching mathematics (Moustakas, 1994).

Definitions

For the purposes of this study, the following terms were operationally defined.

- 1. *Induction programs* "Professional support and additional training within the first years of practice" (Loewenberg-Ball et al., 2008, p. 5xii).
- 2. *Mathematical dispositions* Trends in a person's intentional actions and behaviors related to teaching or learning mathematics (Katz & Raths, 1985)
- 3. *Mathematical knowledge for teaching (MKT)* "The mathematical knowledge needed to carry out the work of teaching mathematics . . . [and] by 'teaching' we mean everything that teachers must do to support the learning of their students" (Ball et al., 2008, p. 395).
- 4. *Mathematics self-efficacy* A "situational or problem-specific assessment of an individual's confidence in his or her ability to successfully perform or accomplish a particular [mathematical] task or problem" (Hackett & Betz, 1989, p. 262).
- 5. *Mathematics teaching self-efficacy* Teachers' beliefs in their capability to organize and execute courses of action required to successfully teach elementary mathematics content (Tschannen-Moran et al., 1998).
- 6. *Novice teacher* Beginning teacher who has less than five years' teaching experience (Ingersoll, 2012).
- 7. Productive mathematical disposition "The tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics" (NRC, 2001, p. 131).
- 8. *Self-efficacy* "People's beliefs about their capabilities to exercise control over their own level of functioning and over events that affect their lives" (Bandura, 1993, p. 118).

Summary

For students to become proficient in mathematics, they need mathematically proficient teachers (Darling-Hammond & Lieberman, 2012). Productive disposition is an essential component of mathematical proficiency (NRC, 2001), but studies have shown elementary teachers begin their teaching careers with unproductive mathematical dispositions (Hobden & Mitchell, 2011; Lutovac & Kaasila, 2014). This shortcoming has prompted many researchers to investigate ways to develop productive dispositions about mathematics in teacher education programs (Bartell et al., 2013; Beswick & Muir, 2013; Charalambous et al., 2011; Hobden & Mitchell, 2011; Lutovac & Kaasila, 2014; Maasepp & Bobis, 2015; Spitzer et al., 2011; Stohlman et al., 2015; Zazkis et al., 2011). Few researchers, however, have critically examined elementary teachers' mathematical dispositions after they have experienced teaching the subject on a daily basis. To begin to address this gap in the literature, this research project provided a rich, deep, and thick description of upper-elementary teachers' experiences teaching mathematics at the onset of their careers.

CHAPTER TWO: LITERATURE REVIEW

Overview

Chapter Two provides a context for the current research project that is grounded in the literature to demonstrate the importance of studying the mathematical dispositions of upper-elementary teachers. First, I discuss the conceptual framework that guides my investigation into the lived experiences of novice elementary school teachers. Then, I provide a synthesis of the related literature on national trends in mathematics education, the mathematical proficiency of preservice elementary teachers, the development of productive mathematical dispositions in mathematics content courses designed for elementary teachers, and support systems necessary to sustain productive mathematical dispositions among practicing teachers. Most importantly, I identify gaps in the literature that can be addressed by the study.

Theoretical Framework

The practiced-based theory of mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008) and self-efficacy theory (Bandura, 1977) guided the development of the research problem. To illustrate their influence, the framework begins with descriptions of the origin and constructs of each theory, followed by a discussion of how each theory has informed the literature on the mathematical dispositions of elementary school teachers. Finally, I present how the research project relates to the theories and may potentially advance them.

Practice-Based Theory of Mathematical Knowledge for Teaching

Historical context. What do teachers need to know to teach mathematics? Surprisingly, in the context of subject-matter content, educational researchers did not seek answers to this question until the mid-1980s (Ball et al., 2008; Shulman, 1986). Shulman (1986) was the first researcher to ask: "How does the successful college student transform his or her expertise in the

subject matter into a form that high school students can comprehend?" (p. 8). Their ground-breaking work prompted Shulman (1986) to suggest there was special content knowledge unique to teaching, which he termed, pedagogical content knowledge. Such knowledge included "the ways of representing and formulating the subject that make it comprehensible to others . . . [and an] understanding of what makes the learning of specific topics easy or difficult" (Shulman, 1986, p. 9). Shulman asserted instructional practices provided in content areas would need to improve dramatically to meet the standards of understanding.

Origin of theory. Two decades later, Ball et al. (2008) observed the idea of pedagogical content knowledge had greatly influenced research on teaching and teacher education, but its promise to improve teaching and learning had not been realized. They attributed this shortcoming to the term's lack of theoretical development, clear definition, and empirical foundation. Ball et al., out of the University of Michigan, posed a more focused, subject-specific question: "What do teachers need to do in teaching mathematics—by virtue of being responsible for the teaching and learning of content—and how does this work demand mathematical reasoning, insight, understanding, and skill?" (p. 395). The researchers focused their qualitative inquiry on the everyday tasks associated with teaching elementary-level mathematics. Their work resulted in the practice-based theory of mathematical knowledge for teaching (MKT) and extended the seminal work of Shulman (1986) as illustrated in Figure 1. MKT is the first of two theories framing this study.

Structure of MKT. The six domains of the mathematical knowledge for teaching framework are (a) common content knowledge, (b) specialized content knowledge, (c) horizon content knowledge, (d) knowledge of content and students, (e) knowledge of content and teaching, and (f) knowledge of content and curriculum (Ball et al., 2008). Due to the limited

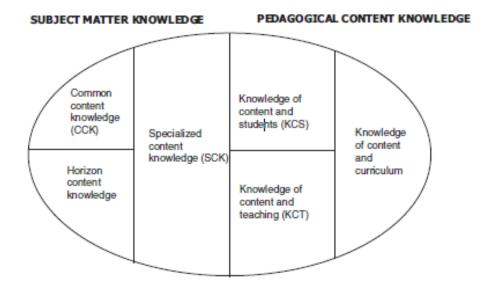


Figure 1. Domains of mathematical knowledge for teaching. From "Content Knowledge for Teaching: What Makes It Special?" by D. L. Ball, M. H. Thames, and G. Phelps, 2008, Journal for Teacher Education, 59, p. 403. Copyright 2008 by Sage.

scope of this project, the domains of horizon content knowledge and knowledge of content and curriculum are not discussed. A detailed discussion of the other four domains follows.

Common content knowledge. Ball et al. (2008) described common content knowledge as "the mathematical knowledge and skill used in settings other than teaching" (p. 399). Minimally speaking, teachers must understand the mathematical topics in the student curriculum. Teachers who possess common content knowledge are capable of identifying student errors, engaging in error analyses, applying procedural knowledge, and communicating conceptual understandings to students (Ball et al., 2008). These skills are essential in planning and carrying out effective instruction.

Specialized content knowledge. Ball et al. (2008) characterized specialized content knowledge as "mathematical knowledge and skill unique to teaching" (p. 400). This body of knowledge is exclusive to teachers and enables them to unpack or decompose mathematical concepts and ideas, so they are visible and learnable by students (Ball et al., 2008). The task of

unpacking mathematical concepts to students demands deep mathematical understanding and reasoning skills, which means teachers must have mathematical knowledge that extends beyond what is being taught to students.

Knowledge of content and students. Ball et al. (2008) posited "many demands of teaching require knowledge at the intersection of content and students" (p. 401). When teachers have knowledge of mathematics and students, they can anticipate student interpretations of mathematical tasks, analyzing students' mathematical thinking, and understanding common student conceptions and misconceptions (Ball et al., 2008). This knowledge base is instrumental in providing developmentally appropriate mathematical tasks and individualized remediation.

Knowledge of content and teaching. According to Ball et al. (2008), the last domain of MKT combines knowing about mathematics and knowing about pedagogy. Knowledge of mathematics and teaching is evident when teachers can design and evaluate appropriate mathematics instruction (Ball et al., 2008). This type of knowledge informs the instructional decisions made before, during, and after a mathematical lesson takes place.

To further illustrate the four types of knowledge, a scenario involving a seventh-grade mathematical concept (Common Core State Standards Initiative, 2016) is provided. Most readers who are not teachers can calculate 5 - (-2) = 7 by applying a rote procedure that changes subtraction to addition of the opposite. This is common content knowledge, which is needed, but not enough, to teach the concept of subtracting integers (Ball et al., 2008). When teachers choose an appropriate model to explain why subtraction of an integer is the same as addition of its opposite, they demonstrate knowledge of content and teaching. In a conceptual model, black chips can represent positive numbers and red chips represent negative numbers. Teachers may start the lesson with the example (-5) - (-2) because students can easily "take

away" two red chips from the five red chips, leaving three red chips, or -3. This decision also represents teachers' knowledge of content and teaching. Then, teachers may strategically present another example, such as 5 - (-2), in which students will experience cognitive conflict trying to take away two red chips from five black chips. Anticipating students' interpretations of the task at hand shows knowledge of content and students. Teachers can prompt students to represent the number five in a different way (i.e. five black chips and two pairs of opposite color chips) that allows them to take away two red chips, leaving the original five black chips plus the two additional black chips. Providing a visual representation that facilitates students' ability to understand subtraction of a number as adding its opposite requires specialized content knowledge, which is knowledge specific to teachers (Ball et al., 2008).

Alignment with national standards. The CCSSI (2016) calls for key shifts in mathematics education, which are reflected in trends among teacher education and professional development programs. First, mathematics teachers are asked to "significantly narrow and deepen the way time and energy are spent in the classroom . . . [to] help students gain strong foundations in mathematical knowledge" (CCSSI, 2016). Teacher educators are asked to do the same in mathematics content courses to help preservice teachers gain strong foundations in mathematical knowledge for teaching (Long, DeTemple, & Millman, 2015). Second, the national standards are designed around coherent progressions from grade to grade (CCSSI, 2016), just as the four domains of the mathematical knowledge for teaching are fully developed in a natural progression from preservice to novice to master teacher. Third, CCSSI (2016) demands rigor, which refers to "deep, authentic command of mathematical concepts, not making mathematics harder or introducing topics at earlier grades" (Rigor section, para. 1). Following a similar progression, rigor can be promoted in initial teacher education programs (Shulman, 1986)

and continuing professional development opportunities.

Impact on research. The mathematical knowledge for teaching framework has substantially informed research on the design and evaluation of teacher education and professional development programs (Hoover, Mosvold, Ball, & Lai, 2016). Its clear definition and empirical foundation allow for the decomposition of the complex practice of teaching into four specific domains (Ball et al., 2008). Armed with this information, teacher educators are better equipped to help elementary teachers learn to attend to the essential elements of teaching mathematics (Grossman, 2011). When preservice elementary teachers are afforded opportunities to learn mathematics in ways that are intertwined with teaching, their mathematical knowledge needed for teaching increases (Hoover et al., 2016). A later section addresses research studies on the associated learning experiences and their mixed findings.

The practice-based theory of mathematical knowledge for teaching has also informed research into the characteristics of elementary teachers that need remediation before they can master the everyday tasks demanded for teaching. For instance, many preservice elementary teachers have well-documented misconceptions about elementary-level mathematics content (Burroughs & Yopp, 2010; Jacobbe, 2011; Kastberg & Morton, 2014; Livy, Muir, & Maher, 2012), are inept at solving mathematical problems (Capraro, An, Ma, Rangel-Chavez, & Harbaugh, 2012), hold deeply entrenched negative attitudes toward the teaching and learning of mathematics (Lutovac & Kaasila, 2014), and do not typically identify themselves as mathematics teachers. These are but a few of the obstacles established in the literature that preservice elementary teachers need to overcome to become effective teachers of mathematics.

Serious issues such as these may hinder the adequate development of preservice elementary teachers' MKT by the end of a four-year program. Goldsmith, Doerr, and Lewis

(2014) studied teacher education reform and found teachers' professional growth tends to develop over time through a series of gradual changes in their knowledge, beliefs, dispositions, and classroom practices. An important angle Hoover and colleagues (2016) believed needs further research is: "how to distinguish between the mathematical knowledge that is essential to know before assuming sole responsibility for classroom instruction and the knowledge that can be safely left to later professional development" (p. 20). Their stance calls for an investigation into not only a realistic time frame to sufficiently develop the four domains of MKT among elementary teachers, but also mathematical dispositions associated with the continued pursuit of such knowledge. This study provides a rich, thick description of the lived experiences of elementary teachers at the onset of their teaching careers to address these issues.

Relationship to research project. The qualitative inquiry into the lived experiences of novice elementary teachers relates to the practice-based theory of mathematical knowledge for teaching at many levels. The study's purposeful sample consisted of participants who, as preservice teachers in undergraduate mathematics content courses, engaged in activities intended to prepare them to teach elementary-level mathematical concepts for conceptual understanding. Novice elementary teachers' reflections about their shared experiences in such courses shed some light on instructional practices and learning experiences that promoted—or inhibited—the development of their mathematical knowledge needed for teaching and, more important, productive mathematical dispositions. The research questions of this study explored the various facets that define elementary teachers' dispositions about mathematics and learning mathematics. The subsequent interview questions allowed for the everyday tasks associated with teaching mathematics to enter the conversation. The theory of MKT deals with the everyday tasks and demands of teaching (Ball et al., 2008).

Self-Efficacy Theory

Origins of theory. Bandura (1977) developed self-efficacy theory, the second theory framing this study, to reconcile an apparent disconnect between theory and practice in the field of behavioral change. In the treatment of fearful and avoidant behavior, mastery performances seemed to play a more prominent role in the acquisition and retention of new behavior patterns than the widely accepted notion of cognitive processes (Bandura, 1977). The Stanford University psychology professor presented a unifying theoretical framework that was able to explain and predict changes in behavior achieved by using different methods of treatment (Bandura, 1977).

Bandura (1977) assigned self-efficacy a central role in the integrative theoretical framework. The theorist proposed a common cognitive mechanism, called perceived self-efficacy, brought about all psychological changes regardless of the treatment method used. Expectations of personal efficacy, or self-efficacy beliefs, stem from the person's processing of information from four principal sources: (a) mastery experiences, (b) vicarious experiences, (c) social persuasion, and (d) physiological and emotional states (Bandura, 1977). Bandura considered mastery experiences to be the most powerful source.

Construct of self-efficacy. Self-efficacy is defined as "people's beliefs about their capabilities to exercise control over their own level of functioning and over events that affect their lives" (Bandura, 1993, p. 118). Bandura (1986) clarified that self-efficacy is "concerned not with the skills one has but with the judgements of what one can do with whatever skills one possesses" (Bandura, 1986, p. 391). Perceived self-efficacy influences how people feel, think, motivate themselves, and behave through four major processes: (a) cognitive, (b) motivational, (c) affective, and (d) selection (Bandura, 1993).

Cognitive processes. When people regard ability as an acquirable skill rather than an inherent capacity, they seek opportunities to expand their competencies, learn from their mistakes, set challenging goals, and use analytic strategies efficiently (Bandura, 1993). Upper-elementary teachers who view mathematical ability as learnable rather than innate may be more willing to engage in challenging mathematical tasks to achieve it. In the pursuit of acquiring mathematical skills, these teachers may become aware of their own learning or thinking processes.

Motivational processes. People's self-efficacy beliefs contribute to their motivation by determining their goals, effort level, perseverance in the face of difficulties, and resilience to failures (Bandura, 1993). Motivational processes are associated with grit, which is understood to mean the "strength of mind or spirit characterized by unyielding courage in the face of hardship or danger" (Merriam-Webster, 2015). Upper-elementary teachers' willingness to experiment with innovative teaching methods, which can potentially expose their misconceptions and weaknesses in mathematics, may depend upon whether they believe learning mathematics is a worthwhile and achievable endeavor in the first place.

Affective processes. People's perceived coping self-efficacy regulates their avoidance behavior and anxiety levels (Bandura, 1993). Research statistics indicate elementary teachers with high levels of mathematics anxiety tend to allocate more time for seatwork, devote less time to conceptual meaning and problem solving, and spend 50% less time teaching mathematics than elementary teachers with low levels of mathematics anxiety (Sloan, 2010). Upper-elementary teachers who were provided opportunities to decompose and approximate various teaching practices of elementary school teachers (Grossman, 2011) as preservice teachers may exhibit lower levels of mathematics anxiety and increased mathematics teaching self-efficacy as

practicing teachers in their own classrooms.

Selection processes. The choices people make based on their personal efficacy beliefs result in the cultivation of different competencies, interests, and social networks that can significantly alter their life courses (Bandura, 1993). Kajander (2010) discovered preservice teachers, after learning of their weaknesses and participating in an intervention, expressed an interest—as opposed to an apprehension—in developing their conceptual knowledge in mathematics. In addition, many preservice teachers became less concerned with traditional teaching practices and more interested in practices to promote deep learning and problem solving. However, the preservice teachers felt they would be more successful in their development if they were given more time to hone their skills and had been placed with cooperating teachers who shared their ideas (Kajander, 2010). Professional development of this nature may only be sustainable when novice upper-elementary teachers are integrated into a school culture that embraces continuous professional growth of all teachers.

Impact on research. Since the mid-1970s, self-efficacy theory has informed educational research at multiple levels. The hierarchal nature of its application is best explained by its principal theorist, Albert Bandura (1993):

Students' beliefs in their efficacy to regulate their own learning and to master academic activities determine their aspirations, level of motivation, and academic accomplishments. Teachers' beliefs in their personal efficacy to motivate and promote learning affect the types of learning environments they create and the level of academic progress their students achieve. Faculties' beliefs in their collective instructional efficacy contribute significantly to their schools' level of academic achievement. (p. 117)

A similar hierarchy can apply to teacher education programs in higher education settings, where

the students are preservice elementary school teachers, the teachers are teacher educators, and faculties are mathematics and education faculty.

Self-efficacy theory has informed mathematics education research, particularly in studies involving preservice elementary school teachers. This occurrence is not surprising considering the theory's roots in the treatment of fearful and avoidant behavior (Bandura, 1977) and the literature's documentation that many elementary education majors enter their preparation programs with high levels of mathematics anxiety (Sloan, 2010), negative attitudes toward mathematics (Hobden & Mitchell, 2011; Lutovac & Kaasila, 2014), and weaknesses in mathematical conceptual knowledge (Burroughs & Yopp, 2010; Jacobbe, 2011; Kastberg & Morton, 2014; Livy et al., 2012; Thanheiser, Whitaker et al., 2014).

The relationships among anxiety, attitudes, and knowledge in the context of mathematics have been the topics of many research studies (Evans, 2011; Hadley & Dorward, 2011; Jamil, Downer, & Pianta, 2012; Swars, Smith, Smith, & Hart, 2009), and their findings have been consistent with self-efficacy theory. For instance, research studies have shown that gains in preservice teachers' mathematical conceptual knowledge are associated with lower levels of mathematics anxiety (Evans, 2011) and greater confidence in their mathematics teaching ability (Bates, Latham & Kim, 2011; Briley, 2012) regardless of their actual mathematical performance (Bates et al., 2011). In turn, preservice teachers with lower levels of mathematics anxiety are more confident in their own skills and abilities to teach and learn mathematics effectively (Haciomeroglu, 2013; Hadley & Dorward, 2011; Jamil et al., 2012).

Despite being confident in their own skills and abilities to teach and learn mathematics, many preservice teachers do not feel confident in their ability to influence their future students' learning of mathematics (Bates et al., 2011). This finding contradicts other studies that have

shown when practicing teachers possessed strong beliefs in their ability to learn and teach mathematics, student achievement tends to improve (Hadley & Dorward, 2011; Zerpa, Kajander, & Van Barneveld, 2009). Bates et al. (2011) suggested the need for additional studies in this area, specifically longitudinal studies designed to study new teachers' beliefs in their teaching abilities after gaining real experience in the classroom.

Recent mathematics education reforms have called for less emphasis on memorization, procedural knowledge, and rote calculations, and more emphasis on understanding, conceptual knowledge, and problem solving (National Research Council, 2001). Zerpa et al. (2009) suggested such reform efforts should begin in teacher education programs because preservice teachers are more likely to embrace and demonstrate conceptual change during this time of their lives. In response, many teacher-education programs have incorporated various teaching practices aimed at helping young children develop conceptual understanding into their mathematics methods and content courses for preservice elementary teachers. The instructional methods used by the instructors of these courses are often based on the principles of social constructivism. The impact of these learning experiences on preservice teachers' mathematical dispositions are reviewed in the next section.

Relationship to research project. According to Tschannen-Moran, Woolkfolk-Hoy and Hoy (1998), "the optimism of young teachers may be somewhat tarnished when they are confronted with the realities and complexities of the teaching task" (p.232). Therefore, self-efficacy theory is directly connected to the fourth research sub-question of this transcendental phenomenological study, which searched for factors upper-elementary teachers believe have influenced their experiences teaching mathematics near the onset of their careers. Additionally, recognizing every new teacher who enters an elementary school classroom does so with different

perceptions about mathematics and the teaching and learning of mathematics is important. The second and third research sub-questions sought to understand how participants' mathematical dispositions compare with a productive mathematical disposition, which is a "habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy" (NRC, 2009, p. 116) and their self-efficacy beliefs about teaching mathematics, respectively.

In addition, a rich description of the lived experiences of novice upper-elementary teachers requires both textural descriptions of *what* they experienced during the early stages of teaching mathematics and structural descriptions of *how* they experienced it (Creswell, 2013; Moustakas, 1994). Self-efficacy theory may contribute to complete textural descriptions and meaningful structural descriptions. Thus, the data analysis methods recommended by Moustakas (1994) involve examining elementary teachers' perceptions of their self-efficacy to carry out the tasks that are clearly defined by Ball and associates (2008) in the mathematical knowledge for teaching framework.

Related Literature

Elementary teachers are currently expected to teach demanding mathematics curriculum that calls for conceptual understanding, procedural fluency, and problem solving (CCSSI, 2016) in an effort to increase the mathematical proficiency of all students in the United States (NRC, 2001). To accomplish this feat, the mathematical preparation of preservice elementary teachers involves a "complex combination of changing beliefs and improving knowledge" (Smith, Swars, Smith, Hart, & Haardörfer, 2012, p. 339). This review of related literature focuses on the development of preservice elementary teachers' mathematical knowledge base and belief systems, particularly in the context of mathematics content courses designed for elementary

teachers. Their continued professional growth as practicing teachers relies on strong networks of supports (Loewenberg-Ball et al., 2008), which are also reviewed.

Mathematical Proficiency

Emphasis in mathematics education. In its influential study of research in K-8 mathematics education, the NRC (2001) recommended mathematical proficiency should be a central goal for schools in the United States. The term mathematical proficiency captures the 16-member panel's comprehensive definition of what it means for anyone to learn mathematics successfully (NRC, 2001). By *anyone*, the panel is not just referring to school-aged children; the reference to anyone has implications for preservice teachers and practicing teachers of mathematics. If children are to become increasingly proficient in mathematics, then their teachers must also be mathematically proficient (Darling-Hammond & Lieberman, 2012).

Mathematical proficiency encompasses a coherent body of knowledge, skills, abilities, and beliefs that can be categorized into five interdependent strands: (a) conceptual understanding, (b) procedural fluency, (c) strategic competence, (d) adaptive reasoning, and (e) productive disposition (NRC, 2001). A rope is often used to represent the intertwined nature of the strands of mathematical proficiency (see Figure 2). The strand particularly relevant to this study is productive disposition, which refers to "the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics" (NRC, 2001, p. 131).

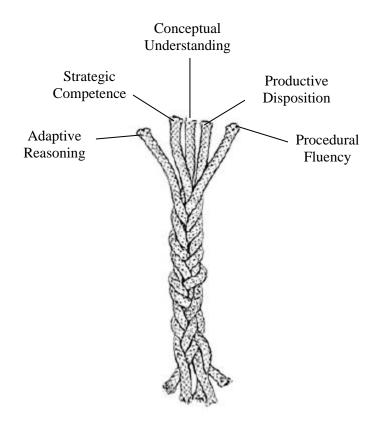


Figure 2. Intertwined strands of mathematical proficiency. From Adding it up: Helping children learn mathematics by NRC, 2001, p. 117. Copyright 2001 by National Academy of Sciences.

Treatment of the affective strand. Although the NRC (2001) advocated the "integrated, balanced treatment of all strands of mathematical proficiency at every point in teaching and learning" (p. 9), mathematics education researchers recognized a productive disposition is essential for the other four cognitive strands to function and develop properly (Feldhaus, 2012; Gautreau, Kirtman, & Guillaume, 2011; Kuzle, 2013; Siegfried, 2012). For instance, when faced with a nonroutine mathematical task, people with unproductive disposition make no attempt to solve the problem, or give up quickly, because they do not see themselves as capable of thinking mathematically or envision success with diligent work. On the contrary, people with productive disposition can employ—and thereby strengthen—one or more of the

other four strands of mathematical proficiency to help them think about and engage in a nonroutine mathematical task.

The diverse committee (NRC, 2001) also stipulated in its report that "students should not be thought of as having proficiency when one or more strands are underdeveloped" (p. 135). The four cognitive strands of mathematical proficiency (i.e. conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning) can be readily evaluated using conventional pencil-paper assessments, but the evaluation of the affective strand remains somewhat elusive. Therefore, Siegfried (2012) argued mathematics teachers do not fully assess their students' mathematical proficiency. Even though students use their productive disposition to help them solve mathematical problems and build new knowledge, their dispositions are usually not visible in their written work. In this regard, Siegfried considered disposition to be the "hidden strand" of mathematical proficiency. A similar argument holds for mathematics teacher educators and their students, preservice elementary teachers.

Although instruments have been designed to measure elementary teachers' complex beliefs systems surrounding mathematics (McGee & Wang, 2014), mathematical dispositions cannot be measured solely with Likert-type scales (Katz & Raths, 1985). Some researchers observed studies of mathematics teaching self-efficacy rely predominately on quantitative research methods that employ such instruments (Klassen, Tze, Betts, & Gordon, 2011; Moriarity, 2014). Therefore, more mathematics education researchers need to act as human instruments and listen carefully to the stories of elementary teachers as they describe their lived experiences learning and teaching mathematics. With this knowledge base, teacher educators can better understand preservice teachers' dispositions toward mathematics and target the development of productive dispositions in their instructional practices. When teacher educators attend to the

development of mathematical dispositions rather than focus exclusively on the acquisition of mathematics skills and knowledge, teacher education becomes enriched (Cooke, 2015; Katz & Raths, 1985; Wilkins, 2008).

Mathematical Proficiency of Elementary Teachers

Research on the five strands. Recent research literature has suggested all five strands of mathematical proficiency are underdeveloped among preservice and practicing elementary teachers. Numerous researchers have provided evidence that elementary teachers are lacking in conceptual understanding (Burroughs & Yopp, 2010; Kastberg & Morton, 2014), procedural fluency (Livy & Vale, 2011; Maher & Muir, 2013), strategic competence (Capraro et al., 2012; Lo & Luo, 2012), adaptive reasoning (Jacobbe, 2011; Livy et al., 2012), and productive disposition (Hobden & Mitchell, 2011; Lutovac & Kaasila, 2014; Sloan, 2010). At the same time, research findings have illuminated the interdependent relationships among the five strands of mathematical proficiency (NRC, 2001).

Conceptual understanding. Conceptual understanding refers to "an integrated and functional grasp of mathematical ideas" (NRC, 2001, p. 118). In other words, people with conceptual understanding take ownership of their mathematical knowledge. They can organize seemingly isolated facts and methods into a coherent whole and assimilating (or accommodating) new mathematical ideas into their sophisticated schema. According to the NRC (2001), "a significant indicator of conceptual understanding is being able to represent mathematical situations in different ways and to know how different representations can be useful for different purposes" (p. 119).

Burroughs and Yopp (2010) used a case study to investigate preservice elementary teachers' conceptions about the repeating decimal .999.... Participants wrestled with—or

completely avoided—making sense of the mathematical statement .999... = 1, which suggested preservice teachers do not understand the relationships between whole numbers, fractions, and repeating decimals. Semi-structured interviews revealed participants perceived repeating decimals as processes developed through long division, not as objects such as numbers on the number line. The researchers found it interesting that preservice teachers' misconceptions about the real number system were formed in early grades, not advanced algebra or calculus courses taken in high school. For this reason, Burroughs and Yopp urged instructors of mathematics content courses for elementary teachers to provide opportunities for preservice elementary teachers to compare multiple representations of numbers that include repeating decimals, such as $\frac{1}{3} + \frac{2}{3} = 0.333 \dots + 0.666 \dots = 0.3 + 0.6 + 0.03 + 0.06 + 0.003 + 0.006 + \dots = 0.9 + 0.09 + 0.09 + \dots = 0.999 \dots = 1$, so they might form conceptions of repeating decimals as objects that are acted upon, as opposed to results of processes.

Procedural fluency. Procedural fluency includes "knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently" (NRC, 2001, p. 121). Contrary to popular belief, the NRC (2001) stated that procedural fluency supports, rather than clashes with, conceptual understanding. The council asserted that when students do not have procedural fluency, they devote too much time and cognitive energy working out results have recalled or easily calculated. The increased demand on their cognitive load interferes with their ability to understand and connect important mathematical ideas (NRC, 2001).

In their mixed-method study, Maher and Muir (2013) discovered preservice elementary teachers could not fluently perform the multiplication algorithm with multi-digit numbers or adequately explain how the algorithm worked. Their analysis of participants' responses revealed

when the preservice elementary teachers first learned mathematical algorithms for the addition, subtraction, multiplication, and division of whole numbers, they did so without a thorough understanding of place value (Maher & Muir, 2013). The preservice teachers in their study were in their final year of teacher training, so the researchers were deeply concerned about their findings. In a similar study on ratio and measurement knowledge, Livy and Vale (2011) learned it was not uncommon for first-year preservice elementary teachers to invent incorrect procedures to solve ratio problems. Moreover, the participants were not able to think about the reasonableness of absurd answers they had obtained when incorrectly solving measurement problems. Teacher educators, however, could turn preservice teachers' errors into positive learning experiences by providing opportunities for them to share and correct common misconceptions about mathematical concepts (Livy & Vale, 2011).

Strategic competence. Strategic competence is defined as "the ability to formulate mathematical problems, represent them, and solve them" (NRC, 2001, p. 124). When faced with a nonroutine problem, people with strategic competence can construct a mental model of the situation, employ a variety of problem-solving strategies, and distinguish which strategies are better suited for solving specific types of problems. Strategic competence aligns with Polya's (1945) proposed four steps in mathematical problem solving: (a) understand the problem, (b) devise a plan, (c) carry out the plan, and (d) look back. These problem-solving steps have been accepted worldwide.

Using a grounded theory design, Capraro and associates (2012) explored the problemsolving strategies preservice teachers tended to use when solving an open-ended puzzle problem as well as the explanations they would provide to prospective middle-school students. The researchers were able to make several interesting discoveries when they asked participants to solve a puzzle with 720 possible outcomes—only four of which were correct. First, preservice teachers unanimously employed the guess-and-check strategy, and the majority carried out the strategy in the literal sense. Most participants haphazardly guessed until they arrived at one correct solution and then stopped, while a select few approached the problem systematically and were able to consider multiple solutions. Capraro et al. (2012) cautioned preservice teachers' consistent misuse of the guess-and-check strategy exposes them to repeated failures at finding correct solutions, which can negatively influence their anxiety level, self-efficacy beliefs, and motivation for engaging in mathematical tasks.

In addition to unproductive mathematical disposition, preservice teachers' misuse of the guess-and-check strategy—if left unchecked—may negatively impact their teaching behaviors and future students' mathematical proficiency (Campbell et al., 2014; Capraro et al., 2012). Another disturbing discovery was no preservice teachers could provide an explanation of the solution at the level of mathematical thinking needed to completely solve the puzzle (Capraro et al., 2012). To prepare preservice teachers to effectively teach problem solving, teacher educators can model a variety of different problem-solving strategies and incorporate more adaptive reasoning with the guess-and-check strategy, as this appears to be preservice elementary teachers' "go-to" strategy for approaching mathematical problems and teaching problem solving (Capraro et al., 2012).

Adaptive reasoning. Adaptive reasoning concerns "the capacity to think logically about the relationships among concepts and situations" (NRC, 2001, p. 129). Adaptive reasoning is based not only on formal proof and deductive reasoning but also on intuition, informal explanations, patterns, analogies, and metaphors (NRC, 2001). People with adaptive reasoning can consider alternative ways to solve mathematical problems, justify their methods and

conclusions, and clearly explain what they are doing to others. Students with underdeveloped adaptive reasoning are obsessed with solving a mathematical problem the "right" way and obtaining the correct answer; they believe methods and answers could be easily verified by either asking the teacher or checking answers printed in the back of their textbook.

Livy et al. (2012) examined preservice elementary teachers' capacity to think logically about the relationship between area and perimeter. As part of their comparative case study, 222 preservice elementary teachers were asked to respond to a fourth-grade student's claim that as the perimeter of a rectangle increases, so does its area. Of the participants, 72% believed the student's incorrect claim was true, and only 5% of the others were able to provide a convincing argument to show the students' claim was false. The majority of preservice elementary teachers believed there was a direct relationship between perimeter and area (Livy et al., 2012), a finding consistent with a similar study conducted by Ma (1999) about 13 years earlier. The researchers—being teacher educators in the school of education at a university—were perplexed as to why this misguided belief prevailed in light of the fact the preservice teachers were given (a) similar problems on tutorials and practice tests, (b) opportunities to explore the relationship between area and perimeter using an interactive website, and (c) open access to their class notes and textbook to complete the task. (Livy et al., 2012).

Productive disposition. According to the NRC (2001), "mathematical proficiency goes beyond being able to understand, compute, solve, and reason. It includes a disposition toward mathematics that is personal" (p. 133). Recall productive disposition refers to "the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics" (NRC, 2001, p. 131). Research findings on the development of productive

dispositions among preservice elementary teachers during their preparation programs are reviewed in a later section.

Raising the bar in teacher education. The Council for the Accreditation of Educator Preparation (CAEP, 2013) and the National Council on Teacher Quality (NCTQ, 2014) have encouraged states to re-examine their recruitment criteria in an effort to improve the pool of teacher candidates. Both councils (CAEP, 2013; NCTQ, 2014) recommend teacher education programs screen potential candidates for academic proficiency by requiring a minimum 3.0 grade point average and performance in the top half of the college-bound population on a national normed achievement test such as the ACT, SAT, or GRE. While mathematical proficiency at the high school level should be a condition for admission into teacher education programs (Greenberg & Walsh, 2008), it is not enough for producing quality elementary teacher candidates (Ronfeldt, Schwartz, & Jacob, 2014). Ronfeldt and associates (2014) expressed concern that some teacher education programs might falsely conclude that an increased investment in recruitment standards warrants a reduced investment in teacher training.

Many qualitative researchers (Burroughs & Yopp, 2010; Capraro et al., 2012; Kastberg & Morton, 2014; Livy et al., 2012; Livy & Vale, 2011; Maher & Muir, 2013) offered rich, deep, and thick descriptions and analyses of preservice elementary teachers' underdeveloped strands of mathematical proficiency. Such contributions to the knowledge base have been invaluable to the investment in mathematics teacher training. More recently, Kastberg and Morton (2014) challenged researchers to identify instructional activities and methods that create mathematically rich opportunities for preservice elementary teachers to learn content that is essential for their future work as teachers.

Research has shown taking more general-audience mathematics courses has little or no

effect on adequately training elementary teachers to teach mathematics (Jonker, 2012; Maher & Muir, 2013; Smith et al., 2012). A qualitative inquiry conducted by Burroughs and Yopp (2010) revealed instruction in high school algebra and introductory calculus courses can negatively contribute to preservice elementary teachers' understanding of repeating decimals, a concept that is part of the elementary mathematics curriculum (CCSSI, 2016). Therefore, in addition to raising recruitment standards, the NCTQ also recommends a 3:1 framework for most teacher education programs with three mathematics content courses that specifically address elementary and middle-school topics and one mathematics methods course that directly aligns with these content courses (Greenberg & Walsh, 2008).

Mathematics Content Courses for Elementary Teachers

Course rationale. Thirty years ago, Shulman (1986) envisioned special sections of courses in content areas for teachers after realizing "instruction in the liberal arts and content areas have to improve dramatically to meet the standards of understanding required for teaching" (p. 13). The NCTQ reported university mathematicians who led the charge that "elementary teacher candidates need a rigorous program of study that returns them to the topics they encountered in elementary and middle-school grades, but which is by no means remedial" (Greenberg & Walsh, 2008, p. 4). The National Council for Accreditation for Teacher Education (NCATE, 2008) concurs by requiring teacher education programs to include a mathematical component designed to prepare preservice elementary teachers to teach elementary school mathematics.

Trends to adopt conceptually-based mathematics curricula have impacted the educational system in the United States and countries around the world. For instance, the CCSSI (2016) redefined the mathematics curricula in 42 states, including the states that comprise the setting of

this study. The initiative calls for three key shifts in mathematics education. First, fewer topics should be covered in greater depth to help students build solid foundations in mathematics.

Second, coherence in the progression of major topics should be emphasized so mathematics is no longer viewed as "a list of disconnected topics, tricks, or mnemonics" (CCSSI, 2016). And third, equal attention should be given to conceptual understanding, procedural skills and fluency, and application of mathematical knowledge. Teacher education programs have responded by supplementing their mathematics methods courses with mathematics content courses designed to prepare preservice elementary teachers to teach for understanding.

In mathematics content courses for elementary teachers, preservice elementary teachers can develop an in-depth understanding of elementary mathematics concepts that will increase their confidence and capability in teaching mathematics (Hart, Oesterle, & Swars, 2013; Kastberg & Morton, 2014). Based on their qualitative inquiry, Zazkis, Leikin, and Jolfaee (2011) reported most preservice teachers felt they understood mathematical ideas for the first time in their lives. Additionally, these preservice teachers, having been taught a variety of ways to approach mathematical tasks, felt empowered to field students' questions more effectively. Preservice elementary teachers' view of the subject of mathematics was extended as a direct result of their learning experiences in such courses. Although the participants repeatedly voiced their belief that mathematics today is different, the researchers maintained "it is not the mathematics that has changed, but rather the preservice teacher's view of mathematics" (Zazkis et al., 2011, p. 18).

Course structure. The course content offers an overview of the fundamental concepts of elementary and middle-school mathematics. Typical topics include number systems, operations, patterns, number theory, algebra, measurement, geometry, probability, and data analysis

(Greenberg & Walsh, 2008; Zazkis et al., 2011). Authors of course textbooks strive to impart the mathematical reasoning skills, deep conceptual understandings, and positive attitudes about learning and teaching mathematics that are necessary to become effective teachers of mathematics in elementary and middle schools (Long et al., 2015).

Curricula of mathematics content courses for elementary teachers often reflect an inquiry-based approach to learning (Beckmann, 2013; Long et al., 2015), which is rooted in constructivism. Inquiry-based learning is the vision of reform-based mathematics education (CCSSI, 2016; National Council of Teachers of Mathematics [NCTM], 1991). According to constructivist theories, preservice teachers learn the mathematical knowledge needed for teaching best when they actively engage with the subject matter; it is impossible for such knowledge to be transferred passively from teacher educators to preservice teachers (von Glasersfeld, 1989). Mathematics teacher educators who embrace constructivist theories (a) organize student-centered activities that support independent learning, (b) facilitate group discussions that promote mathematical discourse, and (c) engage preservice teachers in meaning-making tasks (Doruk, 2014).

Course instructors. Mathematics content courses are generally taught by mathematics faculty, some of whom have no formal training in education. Many educational researchers are concerned mathematics faculty are ill equipped to prepare preservice elementary teachers to teach elementary-level mathematics concepts (Greenberg & Walsh, 2008; Hart et al., 2013; Smith et al., 2012). For instance, Hart et al. (2013) conducted a study in which mathematics professors from various universities confessed their inability to relate to elementary school contexts, their uncertainty about what mathematical knowledge preservice teachers needed, and their discomfort using student-centered methods. Other researchers have revealed well-

intentioned educators often exaggerate their use of reform-minded practices (Allen, 2011; Gill & Boote, 2012) and endorse traditional approaches in the teaching and learning of mathematics.

Research has shown teacher educators typically learn to teach about teaching in isolation (Lovin et al., 2012). Mathematics teacher educators are no exception, as evidenced by the increased number of self-studies in mathematics (Leaman & Flanagan, 2013; Marin, 2014). Self-study research is an opportunity to explore, understand, and improve one's teaching practice and the field of mathematics education simultaneously (Marin, 2014).

Leaman and Flanagan (2013) employed self-study methodology to bridge the theory-intopractice gap, which personally haunted them as teacher educators. The researchers bravely conducted a self-study of their first experience using the pedagogy of authentic role-playing as situated learning, a technique that "allows preservice teachers to suspend the lesson in order to gain access to the complex, critical thinking used by effective teachers in their moment-tomoment practice" (p. 46). After engaging in this process, the mathematics teacher educators were able to reframe and expand their three initial assumptions about teacher education (Leaman & Flanagan, 2013). First, modeling did help bridge the theory-into-practice gap, but more so when the professor modeled vulnerability rather than correctness. Second, situated learning contexts do improve learning because of the co-creation of learning experiences, not the context itself. Third, "pressing the pause button" did allow the professors to make teacher-thinking explicit—not because of their expertise—but because of their timing, improvisation, or genuine desire to know. The two transformed teacher educators "embraced the vulnerable task of learning to teach the K-12 student all over again in front of adult learners" (Leaman & Flanagan, 2013, p. 59). Their journey into teacher education exemplified the ideals of constructivism and professional learning.

McGlynn-Stewart (2010) shared a personal story regarding the courageous decision to take on a new mathematics course devoted exclusively to the teaching and learning of mathematics using a student-centered, problem-based approach. The teacher educator was troubled by undergraduate students' negative attitudes toward mathematics and their willingness to experiment with different teaching strategies in other content areas, except mathematics. Like Leaman and Flanagan (2013), the researcher used self-study methodology to discover how preservice elementary teachers might effectively relearn mathematics and apply their learning in a concurrent elementary school placement (McGlynn-Stewart, 2010). The part-time, nontenured teacher educator admitted that in doing so, she jeopardized her own perceived efficacy of teaching mathematics, the positive and trust relationships she had previously built with her students, and the likelihood of receiving favorable evaluations from students. At the conclusion of the self-study one year later, McGlynn-Stewart (2010) realized while her undergraduate students faced and overcame their fear of learning and teaching mathematics, she also faced her own fears of teaching them. Other mathematics education researchers observed teacher educators and preservice elementary teachers embark on parallel journeys (McGlynn-Stewart, 2010) when navigating the learning experiences provided in mathematics content courses (Leaman & Flanagan, 2013; Marin, 2014).

Preservice Elementary Teacher Beliefs

Although preservice teachers tend to be receptive to changes in the field of education (Zerpa et al., 2009), change can be difficult. Even changes for the better may initially have a negative impact on teachers' personal efficacy (Tschannen-Moran et al., 1998). To add to the gravity of the situation, self-efficacy beliefs might be enduring (Tschannen-Moran et al., 1998). Consequently, numerous researchers have investigated ways to develop preservice elementary

teachers' self-efficacy beliefs for regulating their own learning of mathematical concepts and mastering the task of teaching mathematics conceptually (Bartell, Webel, Bowen, & Dyson, 2013; Beswick & Muir, 2013; Charalambous, Hill, & Ball, 2011; Spitzer, Phelps, Beyers, Johnson, & Sieminski, 2011; Stohlman, Cramer, Moore, & Maiorca, 2015).

Development of productive mathematical disposition. Researchers have found preservice elementary teachers hold traditional beliefs about the nature, teaching, and learning of mathematics when they begin and complete their teacher education programs (Dede & Karakus, 2014). Traditional beliefs mean they view the subject of mathematics as a static body of knowledge based purely on arbitrary rules. When teachers view mathematics this way, they tend to use traditional teaching methods that emphasize drill-and-practice and memorization (Charalambous, Panaoura, & Philippou, 2009). Teachers with productive disposition, on the other hand, view mathematics as being a connected set of concepts that make sense, rather than a laundry list of rules to be memorized without any practical use in the real world (NRC, 2001). When teachers see sense in mathematics, they tend to use innovative teaching methods that are more student centered and inquiry based (Campbell et al., 2014). Therefore, the development of productive disposition among preservice elementary teachers has become an important goal of mathematics content courses for elementary teachers (Long et al., 2015).

Charalambous et al. (2009) wondered whether a teacher education program grounded in the history of mathematics could change preservice elementary teachers' epistemological beliefs about mathematics. Their findings suggested learning mathematics in a historical context can present mathematics as a growing body of knowledge that addresses evolving human needs, which can change preservice teachers' Platonic view of mathematics (Charalambous et al., 2009). Ironically, Charalambous et al. also discovered preservice teachers developed more

negative attitudes toward mathematics with the historical approach. Interviews identified preservice teachers' difficulties with the content, insufficient time to absorb concepts, examination anxiety, inability to apply what they were learning to teaching elementary children, and past negative experiences in mathematics classes as factors contributing to the observed negative effect. The researchers were critical of other researchers in mathematics education, who they claim only report successful practices, failing to mention what does not work in teacher education programs (Charalambous et al., 2009).

Hart and associates (2013) examined preservice teachers' perspectives about their experiences in mathematics content courses for elementary teachers. The participants reported an alarming negative affect that included increased anxiety, decreased efficacy, and uncaring instructors. Their study was significant because these common themes about teacher development emerged among participants from a variety of settings (Hart et al., 2013), which made its findings more generalizable (Patton, 2015) than other studies that relied solely on one setting (Beswick & Muir, 2013; Namukasa, Gadanidis, & Cordy, 2009; Savard, 2014).

Other researchers proposed teacher education programs should be grounded in the core practices of teaching (Charalambous et al., 2011; Grossman, 2011). Grossman (2011) called these *approximations of practice*, which provide opportunities for preservice teachers to begin thinking and acting like teachers. In the past, such opportunities were reserved for field experiences. Although there has been a trend to increase the number of field experiences, Grossman warned "in the buzz and complexity of classroom life, it is virtually impossible to 'pause' interactions to provide [specific and targeted] feedback" (p. 2840). For this reason, simulations of certain teaching practices have been incorporated into many mathematics content and methods courses.

The mathematical knowledge for teaching framework (Ball et al., 2008) provides the necessary language and structure to represent and decompose complex teaching practices (Grossman, 2011). For instance, previous studies documented preservice teachers rely on memorized procedures that they are unable to explain conceptually to others (Maher & Muir, 2013; Thanheiser, Browning et al., 2014). According to the mathematical knowledge for teaching framework, these preservice teachers lack knowledge of content and students (Ball et al., 2008). This shortcoming prompted Charalambous et al. (2011) to conduct a qualitative inquiry into how preservice teachers might learn to provide instructional explanations during a two-course sequence. The trio concluded the practice of providing instructional explanations was learnable by preservice teachers given certain factors. Apparently, teachers' growth in this practice was associated with their subject-matter knowledge, active and deliberate reflection, development of alternative images of teaching, and productive disposition about engaging in the practice (Charalambous et al., 2011). The researchers acknowledged they explored preservice teachers' performance in simulated environments but asserted "decisions that teachers make in such artificial environments are good indicators of teachers' potential performance in similar real-classroom settings" (p. 461).

Maasepp and Bobis (2015) explored factors contributing to the effectiveness of educational interventions in mathematics content courses that were designed to nurture positive mathematical beliefs among preservice elementary teachers. The teacher educators concluded the most influential factor was the instructor's ability to build positive rapport with preservice elementary teachers and create a safe learning environment that fosters conceptual understanding of elementary-level mathematics content. According to Maasepp and Bobis, only instructors who can "help break down the stereotyped images of mathematics, mathematics teachers, and

mathematics pedagogy" (p. 15) should be selected to teach mathematics content courses.

Sources of self-efficacy beliefs. Several experimental studies in mathematics teacher education have employed interventions encompassing the four principal sources of self-efficacy beliefs that were originally proposed by Bandura (1977), which include mastery experiences, vicarious experiences, social persuasion, and physiological and emotional states. For instance, when Namukasa et al. (2009) approached teacher change in therapeutic terms by incorporating eight biweekly mathematics therapy sessions in a methods course, they discovered preservice teachers can reexperience mathematics, feel positive about learning it, and be motivated to consider alternative teaching practices. Beswick and Muir (2013) challenged preservice teachers' existing views on traditional teaching practices by using video excerpts of exemplary teaching practices in conjunction with guided discussions. The pair learned preservice teachers struggled to identify evidence of students' understanding or "observable teaching actions likely to contribute to it" (p. 19) when not explicitly directed to look for such behaviors. Savard (2014) investigated transitioning from elementary school student to university student to teacher by providing opportunities for small groups of preservice teachers to design a counting activity, rehearse it in front of the class, receive feedback from their peers, and write reflections on their performances. The educator claimed, "having them do what teachers do helped them put theory into action and made them realize how it works and why it works" (p. 369).

The results of studies demonstrate preservice teachers' deep processing of information generated from mastery experiences, vicarious experiences, social persuasion, and physiological and emotional states—not a superficial exposure to these sources—brings about actual changes in self-efficacy beliefs (Bandura, 1977; Tschannen-Moran et al., 1998). Whether these learning experiences bring about lasting changes in preservice elementary teachers' beliefs that continue

into their teaching careers needs further research (Briley, 2012).

Status of research. Klassen and associates (2011) analyzed 218 teacher-efficacy research studies conducted in the last 12 years to see how researchers responded to previous suggestions provided in the influential and comprehensive review by Tschannen-Moran and associates (1998). Their analysis revealed the following points. (a) Most studies continue to use quantitative approaches and collect data from teachers at one point in time using self-report surveys; (b) Most researchers do not examine how teachers' self-efficacy beliefs form, develop, and change over time; (c) Only 4% of studies that indicated a specific curriculum area examined mathematics teaching self-efficacy; (d) Middle-level teachers remain underrepresented in self-efficacy research; and (e) Almost one half of the studies continue to use "discredited, poorly conceptualized, and flawed measures" (p. 36) of teachers' self-efficacy, which may result in misleading conclusions (Klassen et al., 2011).

In designing this study, I considered the issues raised by Klassen et al. (2011) regarding the status of teacher efficacy research. The qualitative approach in this study addresses the issue that no significant change in the proportions of qualitative and mixed methods research has been reported since the previous 12-year period. In addition, the conceptual framework of self-efficacy theory and the mathematical knowledge for teaching framework allows for a greater level of specificity in both context (i.e. fourth- through sixth-grade novice teachers) and subject matter (i.e. conceptually-based, elementary-level mathematics). Finally, the study examines elementary teachers' self-efficacy beliefs during the early stages of teaching.

Supporting Novice Elementary Teachers

As this review of the literature has shown, mathematics education researchers have extensively studied the deficiencies in mathematical knowledge among preservice elementary

teachers as well as factors that contribute to the development of their mathematical knowledge and belief systems in mathematics content courses designed for elementary teachers. Some researchers have called for additional research into the initial knowledge that is needed for teaching mathematics (Hoover et al., 2016; Kastberg & Morton, 2014), while others have questioned whether positive outcomes can be sustained in school environments (Bates et al., 2011; Briley, 2012). Almost 50 years ago, Robert Schaeffer (1967), the dean of Columbia University's Teachers College, made this statement about the importance of supporting novice teachers:

It is trivial to argue about the degree of knowledge necessary to begin teaching, while we ignore the crucial question of how teachers can continue to learn throughout their careers. The real problem about the substantive knowledge possessed by new teachers is not its initial quantity but the fact that the school environment makes so few provisions for its steady expansion. (p. 14)

With proper support systems, novice teachers can develop the skills and habits of mind that are necessary to build a solid foundation for future teaching success (Chong, Loh, & Mak; 2014). On the other hand, when proper support systems are not available, between 40% and 50% of new teachers in the United States leave the profession within their first five years of service (Ingersoll, 2012).

The transition from teacher preparation to the initial years of teaching can be difficult for many elementary teachers. In the landmark book, *Teachers as Learners*, Feiman-Nemser (2012b) suggested new teachers have two jobs: teaching and learning to teach. Even though novice teachers are often viewed as unfinished products (Feiman-Nemser, 2012b), most schools assign them the same responsibilities as teachers with 20 years of experience and unrealistically

expect them to perform at equivalent levels (Teague & Swan, 2013). Novice teachers also have additional burdens of larger class sizes, more students with special needs or behavioral issues, extracurricular duties, and fewer educational resources in their classrooms (Feiman-Nemser, 2012a). Novice teachers often feel pressured to adhere to mandated curriculum guides and rigid pacing calendars, which can result in perceived professional dilemmas about meeting their students' learning needs, marginalizing other subject areas, and respecting administrators' expectations (Bauml, 2015). These unrealistic expectations, additional burdens, and added pressures undermine the continued professional growth and confidence levels of novice elementary teachers.

Induction programs. Feiman-Nemser (2012a) examined the evolving role of teacher induction programs considering recent standards-based reforms in education. The distinguished teacher educator and scholar concluded induction is no longer viewed as a temporary bridge designed to ease new teachers' entry into their roles; educational leaders now view induction as "a process of incorporating new teachers into collaborative professional learning communities" (p. 12). When induction is viewed as cultural transformation rather than temporary support, the benefits include continuous learning by all teachers, shared responsibility for teaching and learning, quality learning environments for students, increased student achievement, and rewarding career paths for teachers (Feiman-Nemser, 2012a).

The NCTM (2016) has taken the position that induction programs play a crucial role in the development of the next generation of teachers. Induction programs are not to be confused with mentoring (Loewenberg-Ball et al., 2008). A comprehensive induction program for novice teachers presented by the National Commission on Teaching and America's Future has four main goals: (a) building and deepening teacher knowledge; (b) integrating novice teachers into a

school culture that embraces continuous professional growth of all teachers; (c) supporting the constant development of the teaching community in the school; and (d) encouraging professional dialogue that articulates the goals, values, and best practices of the community (Fulton, Yoon, & Lee, 2005).

Mathematics specialists. The National Mathematics Advisory Panel (2008) noted increasing all elementary teachers' mathematical knowledge for teaching was a "problem of huge scale" (p. 14), so it recommended the use of elementary mathematics specialists as a more practical alternative. Elementary mathematics specialists (EMS) are "teachers, teacher leaders, or coaches who are responsible for supporting effective mathematics instruction and student learning at the classroom, school, district, or state levels" (Association of Mathematics Teacher Educators [AMTE], 2017, p. 1). Many experts in mathematics education believe every elementary school should have access to an elementary mathematics specialist (AMTE, 2017; NCTM, 2016; NRC, 2009). Polly, Mraz, and Algozzine (2013) observed the role of EMS professionals has shifted from teaching children to improving the instructional practices of adults. Subsequently, elementary mathematics specialists' roles have been redefined as content experts, promoters of reflective practice, professional development facilitators, and supporters of schoolwide learning communities (Polly et al., 2013).

Summary

What is currently known about the development of elementary teachers' dispositions toward mathematics has been examined mainly from the perspective of preservice teachers during their teacher education programs. Research studies have focused on ways to increase the mathematics teaching self-efficacy of preservice elementary teachers in mathematics content courses because "the first few years of teacher development are critical to the long-term

development of teaching efficacy" (Smith et al., 2012, p. 338). To achieve this goal, researchers have studied how preservice teachers learn the core practices of teaching, as outlined in the mathematical knowledge for teaching framework. Although the results of such studies are encouraging, research is lacking on how these learning experiences provide transferable mathematical knowledge and lasting self-efficacy beliefs for novice teachers in elementary school settings. In this age of accountability, school environments may not be the fertile soil necessary for newly planted seeds to grow.

CHAPTER THREE: METHODS

Overview

The intent of the study is to gain a deeper understanding of the phenomenon of mathematical disposition by studying the lived experiences of elementary teachers. In this chapter, I explain why the phenomenon should be studied using a transcendental phenomenological approach in the setting. Then, I describe my sampling techniques, data collection process, philosophical assumptions, strategies for both analyzing data and establishing trustworthiness, and actions to ensure ethical behavior throughout the research project.

Design

This qualitative study used a transcendental phenomenological design. Phenomenology is both a research method and a philosophy of qualitative researchers (Dowling, 2007). The transcendental phenomenological approach fit the research problem because the phenomenological reduction process was consistent with my epistemological assumptions about how knowledge is derived. I shared Moustakas's (1994) sentiment that "the most crucial learnings [in my life] have come from lonely separation from the natural world, from immersions and self-dialogues and from transcendental places of imagination and reflection" (p. 41).

Qualitative research was appropriate to study the complex mathematical beliefs of novice upper-elementary teachers because their voices were rarely heard in mathematics teaching self-efficacy research (Creswell, 2013; Klassen, Tze, Betts, & Gordon, 2011). Additionally, the core strategies of qualitative research were congruent with my philosophical assumptions about the nature and acquisition of knowledge (Gall, Gall, & Borg, 2007). I believed the best way to gain a deep understanding of elementary teachers' mathematical dispositions was by talking directly with the teachers out in the field and allowing them to freely tell their stories about teaching

mathematics (Creswell, 2013). By analyzing elementary teachers' responses from audio diaries, individual interviews, and focus-group discussions, I was able to provide a holistic account of their mathematical dispositions that extended beyond the techniques used in quantitative research (Creswell, 2013). In accordance with the constructivist tradition, qualitative inquiry is oriented toward an openness to whatever themes emerge in the findings, as opposed to being restricted by predetermined research hypotheses that were based on an explicit theoretical framework (Patton, 2015).

A phenomenological approach was applicable in the search for common meaning for novice elementary teachers in their lived experiences teaching mathematics. Phenomenology allowed me to study several teachers who had shared the experiences of (a) teaching elementary mathematics content for conceptual understanding, and (b) completing mathematics content courses for elementary teachers as part of their teacher education (Creswell, 2013). Phenomenological research allowed me to develop a composite description of the essence of the mathematical disposition of all elementary teachers and understand the meaning they ascribed to being teachers of mathematics (Moustakas, 1994; van Manen, 1990).

I chose a transcendental approach as opposed to a hermeneutical approach, to describe the essential aspects of the mathematical disposition of novice elementary teachers from their perspectives (Creswell, 2013; Dowling, 2007). Transcendental means "in which everything is perceived freshly as if for the first time" (Moustakas, 1994, p. 34). My intent was to bracket or set aside my personal experiences as a mathematics teacher educator and allow for a fresh perspective on elementary teachers' perceptions of what it means to teach mathematics (Moustakas, 1994). The hermeneutical approach presents no formal procedure to reduce bias (Creswell, 2013).

The phenomenon to be examined was the mathematical disposition of novice elementary teachers. Mathematical disposition refers to trends in a teacher's intentional actions and behaviors related to teaching or learning mathematics (Katz & Raths, 1985). Novice elementary teachers may not even recognize their own strong dispositions toward mathematics (Diez & Murrell, 2010). Therefore, it was important to engage them in meaningful discussions about their experiences as teachers of mathematics.

Research Questions

Given that the purpose of the study was to explore the lived experiences of novice upperelementary teachers and describe their mathematical dispositions, the following questions were presented:

CQ: How do novice upper-elementary teachers perceive and describe their experiences teaching mathematics?

SQ1: What do novice upper-elementary teachers' experiences reveal about their mathematical dispositions?

SQ2: How do trends in novice upper-elementary teachers' intentional actions and behaviors compare to tendencies associated with a productive mathematical disposition, as defined by the National Research Council?

SQ3: How do novice upper-elementary teachers describe their self-efficacy beliefs of teaching mathematics?

SQ4: What factors do novice upper-elementary teachers identify as influencing their experiences teaching mathematics?

Setting

The research project was set in the Great Lakes region of the United States. The

participants were alumni of six universities in western Pennsylvania. Table 1 provides information about the selected universities, for which pseudonyms were assigned to protect the identities of participants. An interesting finding was most of these alumni obtained teaching positions in western Pennsylvania and surrounding areas. The locations of the study took place in public libraries close to where participants live or work. Pennsylvania and Ohio implemented the Common Core Standards for Mathematics in the 2013-14 school year (Common Core State Standards Initiative [CCSSI], 2016), which meant the stated mathematics curriculum experienced by participants as well at its phase of implementation were similar (van Manen, 1990).

Participants

The participants for this study were selected from alumni of six universities in western Pennsylvania using purposeful-sampling strategies (Patton, 2015). As an instructor at a state university and member of a state-level professional organization, the researcher had a positive rapport with several education faculty of state institutions. With the assistance of colleagues, I compiled a comprehensive list of graduates who met the initial qualifications of the participant group.

The participant group was comprised of elementary teachers who earned undergraduate degrees in education from accredited universities in Pennsylvania. This decision ensured participants had common experiences in their mathematics education preparation (van Manen, 1990). Additionally, the participant group included novice teachers who currently taught elementary grades 4 through 6 in schools that adopted the Common Core State Standards for Mathematics. This decision ensured participants shared experiences (van Manen, 1990) in carrying out elementary-level mathematics curriculum that emphasized the development of

Table 1
University/College Characteristics

Characteristic	Valleyview	Mountain- view	Lakeview	Hillview	Riverview	Rockview
Enrollment (UG)	4,300	10,500	5,050	900	4,300	7,600
Student-Faculty Ratio	16:01	18:01	16:01	10:01	18:01	22:01
Student Population						
Female	62%	57%	34%	44%	66%	59%
Male	38%	43%	66%	56%	34%	41%
White	82%	65%	77%	76%	86%	85%
Minority	17%	28%	13%	19%	14%	14%
International	1%	7%	10%	5%	<1%	1%

conceptual understandings and productive dispositions toward mathematics (CCSSI, 2016; National Research Council [NRC], 2009). The intended sample size was between 12 and 15 participants or until data saturation occurred (Creswell, 2103). While all participants in the study experienced the phenomenon and shared similar experiences, they differed vastly in age, gender, certification, school setting, years' experience, self-efficacy beliefs, and teaching philosophy. This maximum variation sample allowed the researcher to capture all relevant aspects of their lived experiences teaching mathematics (Moustakas, 1994).

Procedures

My first step in the process was to secure approval from Liberty University's Institutional Review Board (see Appendix A for IRB approval). Upon approval, I reached out to a fellow professor who specialized in writing assessment for feedback on the readability of the audio diary prompt, interview questions, and focus-group prompts (see Appendix B for expert request letter). When no changes were needed, a sample of two novice elementary teachers was

gathered from a list of university alumni to conduct a brief pilot study. These peers of potential participants were asked to provide feedback on the clarity of the audio diary instructions, interview questions, and focus-group prompts (see Appendix C for elementary teacher request letter). After incorporating feedback from the pilot study, I contacted the directors of graduate records at state universities through email and requested lists of elementary teachers who met the qualifications of the study (see Appendix D for request email). Then, I mailed invitation letters to approximately 100 qualified participants who taught in Ohio or Pennsylvania, introducing myself and beginning to build collegial relationships (see Appendix E for invitation letter). A few days later, I mailed informed consent letters share the purpose of the study and expectations for participation (see Appendix F for participant consent form). After receiving insufficient results, I contacted several superintendents for assistance in identifying potential participants in their school districts.

After signed consent letters were received, all respondents completed McGee and Wang's (2014) Self-Efficacy for Teaching Mathematics Instrument (see Appendix G for survey). I contacted the authors through email to obtain their permission to use the instrument (see Appendix H for SETMI approval). I used this survey to identify participants who had different self-efficacy beliefs. McGee and Wang (2014) developed the SETMI in response to the need for a better measurement of mathematics teacher self-efficacy, one that directly aligns with Bandura's ideas on self-efficacy and also considers the complex belief systems and content knowledge of elementary mathematics teachers. The authors claimed, "The SETMI is a valid and reliable measure of two aspects of self-efficacy: pedagogy in mathematics and teaching mathematics content" (p. 400). Validity of survey content was established with related literature, previously published instruments, and consultation with elementary mathematics experts.

Reliability of survey responses was verified using Cronbach's alpha.

After survey results were collected, I used purposeful-sampling techniques to obtain a maximum variation sample. To ensure the study could be replicated, I reported data regarding age, gender, type of certification, grade level, years of experience, and school district location (see Table 2). Next, I contacted the selected participants to assign pseudonyms and to schedule individual interview sessions. Then, I emailed participants regarding instructions on how to maintain their audio diaries (see Appendix I for audio diary script) and participate in an online focus-group discussion (see Appendix J for online focus-group instructions). Upon the completion of the audio diaries, I conducted individual interview sessions using an interview guide (see Appendix K for interview questions). During the interview sessions, I wrote observation notes to record a detailed description of each experience. I personally transcribed all audio recordings of audio diaries and interviews to hear the voices of my participants (J. Zabloski, personal communication, January 9, 2016).

Based on the data collected from audio diaries and interview sessions, I formed groups of participants with diverse perspectives on mathematics education and scheduled online focus-group sessions. The decision to highlight diversity in group interviews was based on Patton's (2015) assertion "a little bit of argument can go a long way towards teasing out what lies beneath 'opinions' and can allow both focus-group facilitators and participants to clarify their own and others' perspectives" (p. 477). I emailed participants regarding an invitation (see Appendix L), which provided information needed to register for their online focus-group discussion (see Appendix M for online focus-group prompts). The day before their scheduled group session, I sent an email to remind each group member of the meeting (see Appendix N). The transcripts of focus-group responses were instantly downloaded. Upon the completion of data collection, I

Table 2

Participant Overview

Participant	Age	Gender	Certification(s)	Grade Level/ Subject	Years in Education	School Location
Annie	32	Female	Early Childhood, Elementary K-5	4th/All Subjects	6	Rural
Barbara	27	Female	Early Childhood, Elementary	5th/Math and Science	5	Suburban
Carly	25	Female	Early Childhood, Special Education	3rd and 4th/Reading and Math Learning Support	3	Suburban
Derek	24	Male	Secondary Math 7-12	6th/Math	1	Suburban
Emma	48	Female	Elementary, Math and Science	5th/Math	4	Rural
Faith	32	Female	Elementary K-6, Special Education N-12, Middle- Level Math	4th/Math	5	Rural
George	28	Male	Early Childhood, Elementary	4th/All Subjects	3	Rural
Helen	36	Female	Early Childhood, Elementary	4th/All Subjects	2	Rural
Isabella	29	Female	Elementary K-6	4th/All Subjects 5th-	5	Rural
John	24	Male	Elementary K-6, Special Education N-8	8th/Social Studies, Reading, Math	2	Suburban

compensated all participants. As data were gathered from each participant, I analyzed them using constant comparison and the qualitative data analysis software program, *Atlas.ti* (see http://www.atlasti.com).

The Researcher's Role

Throughout my professional career, I had always seen myself as a mathematician and an educator. I pursued a bachelor's degree in secondary education with a specialization in mathematics only to find out years later that I was only three credits shy of a dual degree in education and mathematics. With a bachelor's degree in hand, I accepted a position teaching middle-school mathematics. In this setting, I first became interested in curriculum development, as I observed the negative effects that I believed ability grouping had on my students' attitudes toward mathematics. Three years later, my master's degree and extensive background in mathematics would land me a position with the mathematics department of a small, private liberal arts college.

In this position, I taught mostly mathematics survey courses designed to foster productive mathematical dispositions among non-mathematics majors. To accomplish this fete, I created thematic units so my college students would begin to appreciate the usefulness of mathematics in everyday life and view themselves as doers of mathematics. Many of my colleagues who were pure mathematicians felt teaching such courses was beneath them, whereas I enjoyed teaching these courses from a social-constructivist perspective.

Many preservice elementary teachers taking these problem-solving courses saw me as more of an educator than a mathematician. Word travels fast on the college campuses about which professors to take for certain courses, so I had plenty of opportunities to work with the student population of elementary education majors. While working with these preservice

teachers, I could not help but notice their high anxiety levels, lack of confidence, and mathematical deficiencies. I became intrigued with this population who had a passion for teaching children but seemed lacking in the confidence and skills necessary to give their future students a solid foundation in mathematics.

Preservice elementary teachers' passion for teaching young children ignited my passion for teaching preservice teachers. As I reflected upon my doctoral work until this point, I noticed I have been researching how to improve mathematics education since day one. My focus on mathematics education led me to my current teaching position with the mathematics department at a state university with a long history of training teachers in Pennsylvania. At this university, all preservice teachers were required to take two mathematical reasoning courses for elementary teachers as part of their general education requirement. The inclusion of such courses in teacher education curricula reflected a reform-oriented approach, which was a perspective that I also shared. Being a novice instructor of these courses myself, I was motivated to explore how the learning experiences provided in these types of courses equipped elementary teachers with productive dispositions toward mathematics and the mathematical knowledge they needed for teaching. Although I was a faculty member of one of the degree-conferring institutions from which the pool of the graduates was generated, the participants of the study did not include my former students for ethical reasons.

Data Collection

Data triangulation is defined as "the use of a variety of data sources in a study" (Patton, 2015, p. 316). In the study, I achieved data triangulation by combining three different kinds of data. First, participants maintained a reflective audio diary that spontaneously captured the rawness of their emotions and documented their behaviors while helping children learn

mathematical concepts. Second, I conducted individual interviews. Third, participants engaged in online focus-group interviews to elicit their shared experiences in mathematics courses for elementary teachers and their perceptions of how these experiences prepared them to teach mathematics.

I had a specific strategy behind following this specific sequence for data collection. In my description of the mathematical dispositions of novice elementary teachers, like Katz and Raths (1985), I was specifically interested in examining their habits of mind as opposed to their mindless habits. Habits of mind are patterns of actions that were intentionally chosen by the teacher in particular contexts and at particular times (Katz & Raths, 1985). Reflective audio diaries elicited participants' habits of mind and provided rich information for probing questions in individual interviews, the second method of data collection. I used the data gathered from audio diaries and individual interviews to form three groups of elementary teachers with diverse perspectives on mathematics education (Patton, 2015), who then interacted in real-time focus-group discussions.

Audio Diaries

Participants maintained audio diaries using an electronic recording device. For most of their lives, the current generation of novice elementary teachers was surrounded by tools of the digital age such as computers, the Internet, cell phones, and social media. Equipped with the skills and mindsets of "digital natives" (Prensky, 2001), these teachers embraced using technology to instantaneously document their early experiences teaching mathematics. During a 3-week period, participants recorded at least three audio diary entries each week (see Appendix I for audio diary script). The content of audio diary entries included their responses to three prompts: (a) What did you intend to do in your mathematics lesson today?, (b) What did you

actually do in today's mathematics lesson?, and (c) If you could change anything about today's lesson what would you change? I asked participants to submit files via email on a weekly basis. Some audio files contained sensitive information, so when received, each file was coded using the participant's pseudonym and stored on a password-protected computer.

Audio diaries were used primarily for individuals to talk about themselves rather than as an opportunity for talking about others—unless such talk was needed to describe one's own experiences (Noyes, 2004). Participants were encouraged to describe their experiences from "the inside . . . almost like a state of mind: the feelings, the mood, the emotions" (van Manen, 1990, p. 64). The purpose of using reflective audio diaries was two-fold. First, audio diaries were an innovative way to collect data about participants' dispositions toward mathematics and self-efficacy beliefs in developing pedagogy in mathematics and teaching mathematics content (Creswell, 2013; Noyes, 2004; van Manen, 1990). Second, the use of audio diaries encouraged participants to reflect regularly upon their intentions and actions, which could help them become more conscious of their own beliefs, values, and dispositions (Diez & Murrell, 2010).

Interviews

Participants completed interviews in private study rooms at local public libraries after normal school hours. Interviews are the primary form of data collection in a phenomenology (Creswell, 2013), which made them appropriate for the study. Individual interviews provided elementary teachers the opportunity to explain their experiences as teachers of mathematics, as well as experiences that shaped them as teachers of mathematics. The format was open-ended, semi-structured, and face-to-face with a duration of between one and two hours (Moustakas, 1994).

Individual interview questions. During each audio-recorded interview session, I

followed an interview guide containing questions that were based on my research questions and related literature (Moustakas, 1994; van Manen, 1990). The following questions guided the individual interviews.

- 1. Please describe a typical day teaching.
- 2. Let's think back to your K-12 mathematics education experiences. What memorable experiences stand out to you?
- How would you compare studying mathematics and studying other subjects in school?
 (Grouws, Howald, & Colangelo, 1996)
- 4. How do you feel about yourself as a doer and learner of mathematics?
- 5. Who was your best mathematics teacher? What traits made him/her a good mathematics teacher?
- 6. Let's move on to when you were training to become a teacher. What specific experiences helped prepare you to teach elementary-level mathematics? What useful things did you take away from your mathematics courses, mathematics methods course, and field experiences?
- 7. Let's turn to your teaching practices. What features of mathematics instruction do you value and use regularly in your mathematics instruction?
- 8. What personal experiences have shaped the way you teach mathematics to your elementary students?
- 9. Please tell me about one of your successful mathematics lessons. What contributed to its success? What feelings were generated by this experience?
- 10. Please tell me about one of your not-so-successful mathematics lessons. What thoughts were generated following this experience? What can you do differently the next time you

teach this lesson?

- 11. Please describe one of your students who is good at mathematics. What reasons do you give for the student being good at mathematics? (Feldhaus, 2012)
- 12. Describe how confident you feel in your ability to carry out the everyday tasks of teaching elementary-level mathematics.
- 13. What challenges have you experienced during the early stages of your career teaching elementary-level mathematics?
- 14. What types of support have you experienced as a new teacher of elementary-level mathematics?
- 15. Have you shared all that you feel is significant with reference to teaching mathematics?

 Prior to each interview, I found a quiet place to review and set aside my presuppositions regarding elementary teachers, which helped me be more receptive to the participants' experiences teaching mathematics (Moustakas, 1994). Throughout the interview, I wrote many reflective notes (Moustakas, 1994). Follow-up interviews were deemed unnecessary based upon

the themes that emerged during the data analysis process.

Question 1 was an ice breaker intended to provide participants with a comfort level about their ability to respond to interview questions (Rubin & Rubin, 2012). The main interview questions followed the participants' experiences related to mathematics in chronological order from past to present (Rubin & Rubin, 2012). Questions 2, 5, 7, 9, and 10 asked for lived-experience descriptions of learning and teaching mathematics from the perspectives of participants to understand their mathematical dispositions (van Manen, 1990). Questions 3, 4, 6, and 11 were presented to shed light on how participants' dispositions compare to tendencies associated with a productive disposition. Question 12 and the probes in Questions 9 and 10

explored participants' mathematics teaching self-efficacy beliefs. Questions 8, 13, and 14 addressed factors that participants believed influenced their experiences teaching mathematics. Question 15 provided an opportunity for participants to have the last word in sharing their stories.

Online Focus-Group Interviews

I collected the third form of data from online, synchronous, text-based, focus-group interviews using the qualitative platform *itracks Chat* (see http://www.itracks.com). Three separate focus groups of five members each were created to ensure all participants had an opportunity to contribute efficiently (Krueger & Casey, 2014). Participants logged onto the secure platform for their assigned 1-hour time slot using the registration code that I had provided them via email (see Appendix J for online focus-group instructions).

Focus-group interview prompts. I facilitated the group interviews with the following four prompts.

- 1. To begin the discussion, let's consider the best mathematics course you have experienced in your K-12 education. What specific aspects of the course made it positive for you?
- Next, let's look back on your teacher education program. Please tell me about your
 preservice mathematics content courses. Provide as much detail as you think is necessary
 to give a clear idea of the courses.
- 3. In what ways have your feelings about learning mathematics changed as a result of participating in these courses? In what ways did your preservice mathematics content courses prepare you for success as a teacher of mathematics?
- 4. Finally, let's turn to your current experiences teaching mathematics. Describe your

experiences in carrying out the everyday tasks of teaching elementary-level mathematics (i.e. identifying student errors, engaging in error analyses, applying procedural knowledge, communicating conceptual understandings, unpacking mathematical concepts, anticipating student interpretations of mathematical tasks, analyzing students' mathematical thinking, understanding common student conceptions and misconceptions, and designing and evaluating appropriate mathematics instruction). Provide an example so we can understand how you think about helping students learn mathematics.

Each group interview began by having all participants type their initial responses to the first prompt. After participants posted their initial responses, they were able to read the responses of other group members. Then, in real time, participants had about 15 minutes to engage in a conversation with other participants and the researcher. This interaction was an opportunity for everyone to ask clarifying questions, make suggestions, exchange ideas, and expand their original answers. Conversations for the remaining prompts flowed in a similar manner.

The first prompt engaged participants in a conversation about their perceptions of the ideal environment for learning mathematics, which shed light on factors that influenced their experiences teaching mathematics. The second prompt asked participants to describe features of their preservice mathematics content courses that stood out to them (Patton, 2015). Listening to their perceptions of their learning experiences in such courses helped me understand the ways they saw sense in mathematics, which is one facet of a productive disposition. The third prompt required participants to reflect upon the state of their own mathematical knowledge and beliefs, which revealed gradual changes in their mathematical dispositions. The last prompt encouraged participants to contemplate their ability to carry out the everyday tasks of teaching mathematics

that were outlined in the mathematical knowledge for teaching framework (Ball, Thames, & Phelps, 2008).

Focus groups were appropriate in the current study for two main reasons. First, the open discussion format led to stronger understandings of diverse perspectives among the participants (Patton, 2015). And second, some participants might have perceived participating in online focus groups as less threatening than speaking one-on-one with the researcher, which might result in their increased willingness to share their true experiences teaching mathematics (Creswell, 2013).

Data Analysis

I used a modified version of the Stevick-Colaizzi-Keen method recommended by Moustakas (1994) to analyze data in this transcendental phenomenology. This systematic procedure provided detailed data analysis steps for less experienced qualitative researchers to follow (Creswell, 2013). According to Moustakas, three core processes facilitate the discovery of new knowledge in human science research: (a) epoche, (b) transcendental phenomenological reduction, and (c) imaginative variation.

Epoche was the first step in phenomenological data analysis. Epoche is a Greek word meaning to suspend judgment (Schwandt, 2015). In the epoche process, the researcher makes his or her preconceived thoughts, judgments, and biases transparent, and then puts them aside to be more receptive to the views reported by the participants (Moustakas, 1994). I carried out the epoche through reflective meditation, which involved letting my preconceptions and prejudgments enter and leave my mind freely until I experienced a sense of closure (Moustakas, 1994). As I reflectively meditated, I wrote down and reviewed the prejudgments in an attempt to disconnect myself from them. Although Moustakas (1994) admitted perfect epoche is rarely

achieved, the expert still believed the effort and work involved in the process significantly reduce the influence of preconceived thoughts, judgments, and biases on the research inquiry.

Transcendental phenomenological reduction is the second step in phenomenological data analysis. This process "involves a prereflective description of things just as they appear and a reduction to what is horizonal and thematic" (Moustakas, 1994, p. 91). To accomplish this task, I uploaded all verbatim transcripts and written responses into the Qualitative Data Analysis Software (QDAS) *Atlas.ti*. Beginning with the first participant, I listed every statement that was significant for describing her mathematical disposition, initially giving each statement an equal value (Moustakas, 1994). At the same time, I carefully recorded my thought processes in a journal. I repeated this process for each participant until I achieved data saturation (Creswell, 2013). From the list of significant statements, I identified all nonrepetitive and nonoverlapping statements, which are called *horizons* or meaning units of the experience (Moustakas, 1994.

Next, I examined these significant statements and clustered them into themes (Creswell, 2013; Moustakas, 1994). Finally, I synthesized the statements and themes to write textural descriptions of "what" the novice elementary teachers experienced teaching mathematics.

Imaginative variation is the third step in phenomenological data analysis. During the imaginative variation process, the researcher sought possible meanings by considering different perspectives, positions, roles, and functions (Moustakas, 1994). From the textural descriptions I obtained through transcendental phenomenological reduction, imaginative variation led me to write structural descriptions of *how* the mathematical dispositions of elementary teachers came to be in the context of time, space, causality, relation to self, or relation to others (Moustakas, 1994). Finally, I uncovered the essence of novice upper-elementary teachers' mathematical dispositions as a whole (Moustakas, 1994).

Trustworthiness

Within the qualitative community, there are many different perspectives regarding the validation and evaluation of qualitative inquiry (Creswell, 2013; Patton, 2015). Creswell (2013) claimed the criteria described by Yvonna Lincoln and Egon Guba are widely used in the judgment of qualitative reports. Lincoln and Guba (as cited in Schwandt, 2015) coined the term "trustworthiness" and established the following criteria for judging the goodness of qualitative inquiry: credibility, dependability, confirmability, and transferability.

Credibility

Credibility refers to the extent to which my findings accurately represent the participants' views of their life experiences (Schwandt, 2015). The credibility of this qualitative inquiry depended heavily on my ability to gather rich information (van Manen, 1990) and engage in systematic and conscientious analysis (Patton, 2015). The strategies used to achieve credibility were a pilot study, triangulation, and member checking. By conducting a pilot study, I refined my interview questions, gained experience in interviewing, and adapted my data collection procedures, all of which increased my chances of obtaining high-quality data (Creswell, 2013).

Secondly, I employed triangulation, which involved the "use of multiple data sources, multiple investigators, and multiple theoretical perspectives" (Schwandt, 2015, p. 307).

Triangulating among three different data sources provided corroborating evidence for the themes and explanations I gleaned from the transcripts (Creswell, 2013; van Manen, 1990). This procedure helped me to uncover the meaning elementary teachers ascribe to mathematics and learning mathematics as I examined the phenomenon from different vantage points (Schwandt, 2015). Finally, I used member checking, whereby I solicited feedback from participants on the credibility of my findings and interpretations (Creswell, 2013). Participants were given an

opportunity to review and confirm—or alter—my written account (Schwandt, 2015) to ensure it accurately reflected their perceptions of their experiences related to mathematics (Moustakas, 1994). According to Lincoln and Guba (1985), member checking is "the most critical technique for establishing credibility" (p. 314).

Dependability and Confirmability

Dependability and confirmability are the naturalistic equivalents for reliability and objectivity in quantitative research (Lincoln & Guba, 1985). While both qualitative terms are associated with consistency, dependability focuses on the process, whereas confirmability is concerned with the product (Patton, 2015). As the human instrument in this qualitative inquiry, I had the responsibility to ensure my processes were logical, traceable, and documented and my findings and interpretations were noticeably linked to the data (Schwandt, 2015). To achieve dependability and confirmability, I maintained an audit trail, which is "a systematically maintained documentation system" (Schwandt, 2015, p. 10). As part of the audit trail, I engaged in memoing, which is "an analytic procedure for explaining or elaborating on the coded categories that a fieldworker develops in analyzing data" (Schwandt, 2015, p. 196). Leaving an audit trail allowed my research consultant and other committee members to critically examine whether I used dependable procedures and generated confirmable findings (Patton, 2015; Schwandt, 2015).

Transferability

Transferability deals with the researcher's responsibility to provide enough information about the study so readers can establish the similarity between the study and other studies to determine whether its findings might be transferred (Schwandt, 2015). The term is analogous with external validity in quantitative studies (Patton, 2015). I provided a rich, thick description

of the participants, setting, and analyses to allow readers to make informed decisions regarding the transferability of my findings (Creswell, 2013).

Ethical Considerations

I adhered to high standards of ethical behavior throughout the entire research process, following the advice of the Apostle Paul to "in everything set them an example by doing what is good" (Titus 2:7, New International Version). Prior to conducting the study, I obtained IRB approval, invited only participants with whom I have no relationship, collected voluntary consent forms from participants, and reminded participants of the voluntary nature of the study and their right to withdraw from the study at any time. While collecting data, I protected the identities of participants by assigning pseudonyms (Creswell, 2013), treated participants as coresearchers (Moustakas, 1994), refrained from sharing personal information during interviews, and compensated participants for their time and effort (Creswell, 2013). While analyzing data, I kept an open mind (Creswell, 2013) and removed any misconceptions by allowing participants to review and confirm or alter data to accurately reflect their perceptions of experiences teaching mathematics (Moustakas, 1994). At the conclusion, I tried to alleviate any anxieties that the participants might have about the research (Gall et al., 2007) and securely stored their data using a password-locked computer and locked desk. Three years after the study, I will destroy all research records by erasing electronic records from password-locked computers and shredding completed surveys and consent forms.

Summary

I explored participants' experiences teaching mathematics by analyzing audio diaries, individual interviews, and focus-group responses. In doing so, I described novice upper-elementary teachers' experiences with teaching mathematics and how these experiences helped

shape their mathematical dispositions. I achieved trustworthiness in my research project by incorporating a pilot study, data triangulation, member checking, an audit trail, and a rich thick description of the study.

CHAPTER FOUR: FINDINGS

Overview

This transcendental phenomenological study explores the experiences of novice upperelementary teachers to uncover the essence of their mathematical dispositions. The chapter begins with detailed descriptions of the participants to allow their stories to be heard in the context of the research questions. Data from audio diaries, individual interviews, and focusgroup interviews are presented in the form of themes using transcendental phenomenological reduction, as discussed in Chapter Three. The chapter concludes with responses to the research questions of this study.

Participants

Ten novice teachers participated in this study. The purposeful-sampling plan sought to select participants from a comprehensive list of qualifying graduates who represented the education alumni demographics from state universities in Pennsylvania. Although a total of 106 potential participants were identified, only 10 volunteered to participate. The participants in the study taught in various school settings and possessed different characteristics concerning age, gender, certifications held, background, years of experience, self-efficacy beliefs, and teaching philosophy. Three of the participants were male and seven were female. Their ages ranged from 24 to 48 years. All participants were Caucasian. Pseudonyms were used to protect the identities of participants. The 10 participants represented a variety of upper-elementary novice teachers who graduated from six different colleges or universities in Pennsylvania.

Annie

Annie was a self-assured fourth-grade teacher with almost six years of experience in a self-contained classroom setting. Even though mathematics was her least favorite subject to

learn in school, it became her favorite to teach. Annie expressed her confidence as a mathematics teacher grew partly as a result of her own struggles in learning mathematics:

I do struggle with teaching reading and I think a large part of that is because it comes naturally to me and I don't know how to break it down for my students to understand, where I am able to do that with math because I had to break it down for myself to understand.

Annie's steady effort in learning—and relearning—mathematics would shape the way she taught mathematics to her students. She stated, "My goal is to reach those kids who were like me. . . to make them understand the relationship of how place value and numbers and all that work."

Annie possessed a productive mathematical disposition and took on the responsibility of empowering her students to make sense of mathematics. She revealed her belief, "I learned that a lot more does come from letting them figure it out as opposed to me just telling, telling, telling," The extra time spent on this type of instruction solidified mathematical concepts and procedures for all students, which allowed Annie to spend less time reteaching content and preparing for state assessments. She declared to her students, "I feel like I have taught you well enough and I hope you can apply it to the test." Thus, Annie's commitment to use an inquiry-based approach to teaching mathematics shaped her experiences as it increased her confidence in her teaching abilities.

Annie's daily mathematics time was very dynamic. She divided her students into flexible groups based on the results of their unit preassessments. Groups typically rotated through five different learning centers: (a) maintaining old skills, (b) mastering mathematics facts, (c) engaging in problem-solving tasks, (d) independently practicing new skills, or (e) participating in small-group instruction. Annie felt the intensive small-group instruction contributed the most to

her effectiveness as a teacher of mathematics:

Since I meet with each group daily, I am able to get a very clear picture of each student's understanding. I take notes and observe where a student is struggling or excelling, and I use that information to guide my instruction. I rarely move on if a student hasn't demonstrated understanding of the concept.

During small-group instruction, Annie expected her students to draw mathematical models and engage in discourse in the discovery of important connections in mathematics. She expressed excitement in seeing the "light bulbs" going off, particularly among her low-level students.

Annie could hardly wait to share these successes with her colleagues in their collaborative effort to eliminate whole-group instruction.

Barbara

Barbara was a feisty fifth-grade science teacher who began teaching mathematics in her fifth year. She felt as though she had been teaching mathematics for the past five years because mathematics students would regularly come to her for help. Over the years, Barbara gained confidence in her ability to offer struggling students a slightly different approach that made more sense than the way they were originally taught. She appreciated the natural connection between mathematics and science, which shaped her teaching practices. Barbara stated, "I really like teaching science and math together because I use the math that we're learning in fifth grade.

Any chance I can, I show them how that relates to what we're doing in science in fifth grade."

Perceiving mathematics as useful and worthwhile in life, Barbara wanted her students to say things like, "Hey mom, when I go home and we go shopping, let me figure out the sale price for you!"

When Barbara was in school, she felt she excelled in mathematics with minimal effort.

She compared her middle- and high-school learning experiences to those in the "classic Charlie Brown classroom [where] the teacher talks at students who are not engaged in the task."

Barbara's lukewarm perception of her own mathematics classes inspired her to make the learning experiences of her students more engaging. Barbara tried to follow the advice of her college professors to make learning mathematics hands on and fun.

To accommodate the needs of the high population of special education students placed in her classes, Barbara implemented small-group mathematics instruction with flexible grouping and learning centers. She sensed some colleagues disapproved of her nontraditional approach. Ironically, the most experienced fifth-grade teacher in the building had no special needs students. Nevertheless, Barbara believed she did a better job at reaching her students than her two colleagues who depended on whole-group instruction.

Barbara acknowledged she was teaching mathematics in a different way than the way than she had been taught. As a first-year teacher of mathematics, she felt more comfortable with the mathematics curriculum but admitted, "There's definitely a lot to learn . . . I think that it will be that way for a long time." Barbara wanted to find that balance in what she described as "We're going to keep doing it this way and let you really understand the math, but if you don't get it, then we're going to go old school and you're just going to learn the basic rules." She articulated her professional dilemma, "I want [my students] to understand why, why, but [they're] not ready to understand why."

Barbara described her experiences in teaching mathematics as enjoyable, yet frustrating. She believed her students were not adequately prepared to learn fifth-grade mathematics, which prevented her from doing her job effectively. Her students' inability to master their basic skills regardless of the amount of repetition also contributed to Barbara's frustration in teaching math.

Carly

Carly was a bubbly learning support teacher for third- and fourth-grade students in writing, reading, and mathematics. She worked with 3 fourth graders who were pulled out of regular classrooms for their mathematics instruction. One student received supplementary one-on-one instruction, while the other two students took part in a 25-minute interactive lesson that incorporated lots of manipulatives, visual aids, explorations, and skill practice. Carly wanted her students to understand the basics, build up to applications, and dabble in challenging questions, all while experiencing a little bit of success.

Despite being a struggling mathematics learner when she was in elementary school, Carly developed a productive disposition toward mathematics. Her educational experiences revealed her outlook as a learner and teacher of mathematics. Carly recalled feeling anxious when her learning support came to an abrupt stop in seventh grade. She appreciated her ninth-grade mathematics teacher for not allowing her to fall through the cracks by pushing "that productive struggle... opened the door to problem solving not just in math, but in real life." While in college, Carly enjoyed learning different ways to teach mathematical concepts from a more meaningful perspective than what she had experienced in school. She expressed in the focus-group interview that her professor "threw multiple approaches and strategies at us so you could hardly keep up." Upon entering the teaching profession, Carly felt unprepared to teach elementary-level mathematics and cherished having a mathematics coach in her building for the first two years of her career. She was the only participant in the study to receive this type of personalized support, which she described as phenomenal.

As a third-year mathematics teacher, Carly expressed her confidence level "goes up and down." She felt confident in her knowledge of fourth-grade content and her ability to "deliver

instruction that was more meaningful than just worksheet after worksheet." Carly felt very confident in her ability to provide supplementary instruction, which involved reteaching mathematical concepts and demonstrating different strategies for problem solving. She believed her supplementary instruction was "super, super successful." Carly admitted her confidence got a "little shaky" when trying to provide a similar path to success for her other two students. She expressed concern that they could dabble into a little bit of everything but not necessarily master anything.

Carly described many positive experiences while teaching a hands-on unit on fractions. She clearly valued reflection based on the painstaking way that she maintained her audio diary. Carly's diary exceeded the required number of entries, and her typical reflection lasted several minutes longer than the other participants in the study. She even kept a handwritten log that she used as a guide to accurately record everything that happened during each lesson. Carly seemed to enjoy talking about her teaching experiences, often chuckling and imitating student talk. As a coresearcher in this study, she expressed her appreciation for this "really neat" opportunity to reflect upon her mathematics experiences throughout the last few years.

Derek

Derek was an intense young man who taught sixth-grade mathematics at a middle school. He was the only participant in the study with a degree in secondary education. Having taken a "boat load" of mathematics courses, Derek believed he was better equipped to answer commonly asked student questions, such as "Why do we need this?" or "Where am I ever going to see this again?" In his audio diary, Derek described his crucial role in developing productive mathematical dispositions among students in the district:

So, it all trickles down to if the students can put in the work to build on the memorization

or build in numerical fluency between numbers and how they interact with one another with the four math operations . . . I see that with a few misconceptions on some of the math operations, it can build great trouble or difficulty for them to enjoy math. So, they really have to put in the work—and that's both the teachers in the elementary and middle school—to help out the high school teachers so that students don't get discouraged by math.

As someone certified to teach upper-level mathematics, Derek admitted explaining "easy" sixth-grade content was more difficult than he anticipated. He also acknowledged needing improvement in motivating students with low interest in mathematics. At the same time, Derek questioned whether the role of motivator should be placed solely on him and not the students. He expressed feeling angry or mad when he was unsuccessful in getting his students to be attentive and participate in class. As a first-year teacher, Derek felt he simply needed more time to expand his toolbox with different techniques and fun activities. He enjoyed collaborating with the other mathematics teachers in his building on a daily basis.

Derek appreciated structure as a former scholar athlete, military veteran, and school soccer coach. For this reason, his classroom was very structured and routine. Derek explained he went through the same agenda with all four mathematics groups, which always contained a bell ringer, estimation activity, whole-group lesson, and some type of conclusion. Perceiving the middle-school years as a "weird" period for everyone, Derek felt compelled to teach the soft skills of self-discipline, organization, and working with others.

Derek's experiences teaching mathematics were sometimes influenced by his emotional state. He expressed receiving a lot of "blank stares" from students while teaching a lesson negatively affected his overall mood, which carried over to his next class. Reflecting on a lesson

about unit rates and ratios, Derek explained his growing disenchantment with teaching:

Um, today was fairly monotonous. Ah, reviewing material is kind of not too fantastic for me, especially when it bugs me when students don't know it. I know it's difficult for me because I mean, I have obviously worked with [rates] for a very—a decent amount of time and the recall for me is quite simple compared to a student that just learned it. So, it makes me a little angry, but also kind of boring for me to cover material that we already talked about.

Despite experiencing the woes of being a first-year teacher, Derek remained optimistic about his future in mathematics education.

Emma

Emma was a mature fifth-grade teacher who taught mathematics in an impoverished school with no learning support. Although teaching was not her first career choice, Emma felt teaching mathematics became her calling. She walked away from a successful business career on the west coast to return home, raise her family, and pursue a degree in education. As a fifth-year teacher, Emma admitted having some reservations about the field of education. Emma sensed many families in her district did not really value education, and she expressed frustration in trying to motivate her fifth graders to even want to learn mathematics.

Emma was a business-minded professional, and her entrepreneurial spirit influenced her experiences in teaching mathematics. Reflecting on her experiences related to mathematics, Emma felt she has come full circle. She explained in the interview:

Where before, I really didn't feel like I was adept at [math]. And just that persistence, and trying to look at it—look at in a whole bunch of different ways, and looking things up on the internet, and that evolved into that I like it, and I feel like I am better at it, and I

can do it now. So, that was definitely—that was an evolution.

During her teacher education program, Emma appreciated being introduced to a "way better" approach for learning mathematical concepts than just memorizing a bunch of algorithms. She realized the professor was trying to prepare the class for teaching mathematics in the Common Core era. While Emma remained open-minded to this shift in mathematics education, she recalled many "strong-willed" classmates in her cohort voiced their resistance.

Emma wanted to share her personal success story with struggling students to encourage their perseverance in mathematics. She expressed her enjoyment in the challenge of figuring out mathematical problems, as well as her frustration and disappointment when students did not get excited by that same challenge. Coming from the business world, Emma felt pressured to deliver on student growth pertaining to her defined student learning outcomes. At the same time, she managed to maintain a sense of humor when things did not go exactly as planned. To better achieve her goals, Emma regularly evaluated the effectiveness of her teaching practices and tried to make appropriate adjustments based on educational research. Being a lifelong learner, she was committed to professional learning and incorporating evidence-based teaching strategies into her classroom.

Faith

Faith was a compassionate fourth-grade mathematics teacher with a dual degree in early childhood and special education plus a certificate to teach middle-level mathematics. Her teaching career began as a high school learning support and life skills teacher. Faith remembered feeling nervous when she was first asked to move down to the elementary school and teach fourth-grade mathematics. She described her predicament as, "I thought I can do math, I enjoy math, but can I teach math?" Faith felt having to teach her own nephew that year significantly

affected her teaching experiences. She explained in the interview:

Math was not his strong suit. I think that was probably one of the best things for me because, you know, there is a special place [in your heart] for nephews. . .. So, it just really helped me make sure that I was explaining [concepts] in three different levels to the kids, make sure I had activities that reached all of them, and even [leveled] homework. . .. I don't want him to feel like he's not understanding something or being left behind.

Faith wanted all students to appreciate mathematics and understand its usefulness in their lives.

Faith expressed her belief that students should be taught only by teachers who enjoyed mathematics. She realized a teacher's beliefs about mathematics—positive or negative—would shape the beliefs of receptive students. Faith expressed confidence in her ability to positively influence her fourth graders:

Some kids will just come in [fourth grade] and just say, 'I don't like math. It's just not my thing.' And I'll say to them, 'Tell me at the end of the year how you feel.'... And I've had a lot of kids say, 'I like math' and that's always my goal.

Faith also felt "very confident" in her ability to carry out the everyday tasks of teaching math. Knowing that students tend to moan and groan about the measurement unit, she wanted to try a more experiential approach. Reflecting on the lesson, Faith remembered sharing in her students' excitement and said, "They were having fun in mathematics class and they were laughing and just enjoying it and . . . they were experimenting on their own in their groups and trying to come up with [other conversions]." Faith also expressed success in building the confidence of this year's students who wanted nothing to do with fractions and had it in their heads that it was going to be hard. She attributed many of her successes in the classroom to the

guidance that she solicited from the other mathematics teachers in her building.

George

George was a straight-talking fourth-grade teacher with almost three years of experience in a self-contained classroom setting. George expressed mathematics was always his favorite subject in school because it came easy to him. In other subject areas, he recalled struggling more often, feeling afraid to ask questions, and studying less. George maintained his personal interest in learning mathematics even though his teachers used just straight lecture format. As a teacher, George took issue with relying solely on a traditional approach because he wanted his students' learning experiences to be more hands on and interactive. After introducing learning centers in his third year, George believed he still needed to get better at creating different centers to keep students engaged throughout the year and prevent boredom.

George described his entry into the teaching profession as a "culture shock." He quickly realized how the Common Core State Standards Initiative (CCSSI) changed the way mathematics was being taught. Reflecting on his teacher education program, George regretted not learning how manipulatives could be used in the classroom. Consequently, he described his undergraduate preparation to deliver the mathematics curriculum as "more like throw you in, sink or swim type of thing." As a first-year teacher, George remembered having to teach himself the new strategies just before teaching the lessons to his students. He developed an appreciation for the new way of teaching mathematics because it made sense to him.

This year, George felt a lot more confident in his ability to carry out the everyday tasks of teaching mathematics. In the interview, he described how he was able to help his students overcome their initial fear of multiplying two-digit numbers:

When we first start introducing it, [the students] are usually freaked out and they think

they can't do it. It's too hard. But, by the time we get through it, a couple days with it, and just to see the light bulbs go off and them get excited about it and want to try more than just two-by-twos, saying how easy it is after they understand that they have to put in their place holders and add their um partial products. They start to enjoy it, and it is fun for me to see them get excited about it.

George felt confident in his ability to analyze students' ways of thinking, which allowed him to not only show his students new ways to solve mathematical problems but also learn new ways from them. However, his greatest challenge was his students' unwillingness to tackle mathematics problems on their own. George refused to tell his students what to do to solve problems because he wanted them to develop productive mathematical dispositions. He expected his fourth-grade students to use the strategies that they were taught as opposed to being "spoon fed" everything.

Helen

Helen was an outgoing fourth-grade teacher who had earned her degree in elementary education almost 15 years ago. Although the memories of her teacher education program had faded over the years, she was able to recall spending lots of time writing comprehensive lesson plans. As a second-year teacher in a self-contained classroom, Helen painted a clear picture of how completely unrealistic this planning process was in her actual professional life:

For everything that you have to teach within the course of one year of time . . . you're not spending a whole lot of time on one specific item . . . We had to have all the stuff in one lesson plan, and it would take hours to do one lesson plan and then present it. And it's really like, what can you get done in 15 minutes' time, so you know what you're doing for the next day? Because that's just math. You have to have your reading plans, your

guided reading centers set up. You have to have your mathematics centers set up. You have to have your social studies ready to go, and science ready to go. Your morning work ready to go.

Although her confidence in teaching mathematics increased with experience, Helen admitted needing a "little extra help" with introducing certain topics. In these instances, she felt she could go to the "guys" at Khan Academy or Math Antics for ideas.

Despite her initial feelings of unpreparedness, Helen expressed she enjoyed teaching mathematics. In a light-hearted interview filled with laughter, she shared her experiences. Helen arrived at her face-to-face interview wearing a Hawaiian costume from a school celebration. At one point, she sang a jingle that she used to remind her students of "our three special steps" in the standard algorithm for multiplication. On several occasions, Helen excitedly grabbed a sheet of notebook paper to illustrate how different mathematical models worked. To emphasize the importance of fourth graders knowing their mathematics facts, she stated amusingly, "I can't even say math facts enough! Am I saying it enough? Can I say math facts 15 more times?"

As a result of her K-16 learning experiences in mathematics education, Helen had developed a productive mathematical disposition. She described being transformed from a struggling student who hated fractions in elementary school to a successful college student whose diligence impressed the mathematics professor. Helen realized when she could check her own work, learning mathematics became easy. She wanted her fourth graders to make use of this worthwhile approach, but felt they were too lazy to follow through with the process. Helen struggled with getting her students—even the gifted ones—to value learning mathematics. She believed gifted students equated having to work hard with not being smart. Helen tried to encourage her gifted students' continued growth by saying, "Well, you've got to do things you

don't know how to do in order to get better."

Isabella

Isabella was an energetic fourth-grade teacher who had been teaching mathematics and science for almost two years. She spent the first two years of her teaching career working in learning support, first in fifth grade and then in grades K to 3. Isabella then changed school districts, where she taught kindergarten before moving up to fourth grade. She felt she developed more teaching strategies as a result of working with various grade levels.

Isabella described her first year of teaching mathematics as a learning curve that was "very hard" and "kind of crazy." When faced with teaching the concept of volume for the first time, she recalled thinking, "let me remember what that is." In addition, Isabella remembered getting "really worked up" prior to teaching difficult topics. She believed students could sense her lack of confidence, which changed the learning atmosphere. To make matters worse, Isabella struggled to find appropriate resources until the district adopted a formal mathematics curriculum the next year. Reflecting on the obstacles she overcame as a first-year teacher, she concluded triumphantly:

It was hard, but you know, I figured it out and worked my tail off to kind of work through that, and I think that helped me, you know? It helped me to really own and really learn on my own, you know?

At first, Isabella relied exclusively on whole-group instruction and independent practice in her mathematics classes. After developing an awareness of the vast differences in her students' academic levels and home lives, she decided to incorporate other learning experiences. Isabella confidently expressed in the focus-group interview, "I've trialed and errored lots of things. So, at this point, I know centers are a good idea." She spent the first 30 minutes of class

teaching a whole-group lesson that focused on the daily target and followed an "I do-we do-you do" process. For the next hour, Isabella split the class into four learning centers: (a) small-group instruction, (b) computerized enrichment, (c) skill-based practice, and (d) problem solving.

Being a former athlete, Isabella creatively incorporated a sports theme into her mathematics classes. She reinforced teamwork by having students with mixed ability levels work together in their team huddles to answer two key questions. Isabella then rewarded productive teams with cheerleader pom poms. Exit tickets served not only as a way for Isabella to know if her students mastered the daily target but also an opportunity for her students to monitor their own learning. She clarified, "They tell me if they're . . . a rookie player, starter, or MVP." Additionally, Isabella shared, "We work on collaboration and perseverance in kind of going with the sports theme and also applying it to the classroom." Reflecting on her successful mathematics students, she declared, "Hard work pays off, you know, and practice."

John

John was a reluctant teacher of mathematics who accepted a teaching position at a small, private school two years ago. He planned on teaching mostly social studies and reading until a mixed mathematics class for fifth and sixth graders was dumped on his lap. John recalled receiving the mathematics curriculum with "not a lot of direction as far as where we're heading, what you need to accomplish in a year." He sensed there might be lower expectations at his school compared to public schools.

John never felt mathematics was one of his strong suits, yet he conveyed an appreciation for the hard work associated with learning mathematics. During the interview, he expressed deep respect for his algebra teacher who held each student accountable as well as his dissatisfaction with his geometry teacher who allowed students to "goof off" and easily pass. At

the college level, John felt his mathematics professors were either too intelligent to teach basic concepts or they were out of touch with how to teach. He believed the professors graded on a curve because the class average was so low.

Upon entering the teaching profession, John explained his reservations about teaching mathematics:

That first week I was a nervous wreck because I didn't know where to start, where to end.

... So, a lot of it was just kind of finding out how to teach math and really not having, I feel like, a strong understanding of how to do it.

He also questioned his knowledge of the mathematical content and confessed, "I was really nervous because I had middle-school math and I thought, wow, you know, this is a little more advanced. Am I really ready for this?" For the first half of the school year, John was further challenged with behavioral issues and stated, "I was dealing with that more so than teaching." He recalled a humbling experience when his principal observed one of his lessons. John felt he "screwed up" because the former mathematics teacher intervened when a high-level learner could not understand the concept. In the hopes of doing a better job, John began jotting down mental notes of what went well, what didn't really work, and important reminders for next year. Unlike other participants in the study, he was reluctant to seek help from his colleagues.

Despite these initial challenges, John clearly understood the value of connecting mathematics skills to everyday life. He stated confidently in the interview, "Pretty much every avenue you go down—subject-wise in school, work-wise—you're going to need to understand even the basic, minimal concepts of what's going on in math." John tried to instill this belief in his students. The second-year teacher appreciated working through problem with students because it helped him realize mathematics could be taught in different ways. John seemed

comfortable with expressing he was still growing as a teacher of mathematics.

Results

The results of this study were found by analyzing the data gathered from a survey, audio diary, individual interview, and focus-group interview. Data saturation was achieved when the participants began to repeat many of the same experiences in their individual and focus-group interviews. The audio diary supported the themes that emerged from the interviews, as did the survey after an in-depth examination.

While analyzing the data, I bracketed personal thoughts, judgments, and biases to be more receptive to the views reported by the participants (Moustakas, 1994). To begin the coding process, all transcripts from audio diaries, individual interviews, and focus-group interviews were uploaded into Atlas.ti. I read the transcripts of each participant at least three times. Preliminary coding categories were found by searching for words, phrases, patterns of behavior, ways of thinking, and important events that repeated or stood out (Bogdan & Biklen, 2016). Then, I assigned these codes to the data units using the Atlas.ti software. Each code was reconsidered equally according to its significance in describing the mathematical dispositions of novice upper-elementary teachers. When significant codes were established, I clustered them into themes (Creswell, 2013; Moustakas, 1994). Textural descriptions of what the novice upperelementary teachers experienced teaching mathematics were written. From the textural descriptions, I searched for possible meanings to write structural descriptions of how the mathematical dispositions of elementary teachers came to be in the context of time, space, causality, relation to self, or relation to others (Moustakas, 1994). With the reduction process, three themes emerged from this research: Life Changing Decisions, Connections with Students, and Rethinking Mathematics Class.

At first glance, the results of the survey appeared to contradict the way several participants described their self-efficacy beliefs about teaching mathematics. According to the scale created by McGee and Wang (2014), the survey suggested three teachers (2 fourth-grade and 1 fifth-grade) had low self-efficacy scores for teaching mathematics content. All three participants, however, expressed high self-efficacy beliefs during their interviews. When the scores were recalculated using only grade-level specific content, as outlined in the Common Core State Standards Mathematics (CCSSI, 2016), all three teachers received above-average scores. Another inconsistency appeared in Carly's low efficacy score for pedagogy in mathematics that she first reported on the survey and the high self-efficacy beliefs that she later described in her audio diary and interviews. At the end of her interview, Carly expressed her appreciation for being a coresearcher in the study because it helped her to better understand her beliefs about teaching mathematics. The inconsistency in her data may be explained by Carly's professional growth as a teacher of mathematics. Therefore, after closer examination, the modified data from the survey corroborated the themes that emerged.

In addition to data triangulation, I used the strategy of member checking to establish credibility of the findings and interpretation (Creswell, 2013; Lincoln & Guba, 1985). All participants were given an opportunity to review and confirm—or alter—my written account (Schwandt, 2015). One participant provided clarification of the cohort in her teacher education to truthfully characterize her perception of its members. The participants agreed the deep, rich description accurately reflected their perceptions of their experiences related to mathematics (Moustakas, 1994).

Theme Development

In the transcripts of the audio diary reflections, individual interviews, and focus-group interviews, I found significant statements about how the novice upper-elementary teachers were experiencing teaching mathematics. These significant statements were treated equally and became my preliminary coding categories, which are shown in Table 3. Then, I worked to develop a list of nonrepetitive codes that were essential for describing the mathematical dispositions of novice upper-elementary teachers. Finally, I grouped these essential codes into themes, as demonstrated in Table 4.

Table 3

Preliminary Coding Categories

Accountability	Empathy	Patience
Alternate explanations	Evolution	Perseverance
Anger	Extra time spent on math	Professional learning
Anticipates student errors	Failing algebra	Promoting joy of learning
Autonomy	Flexible grouping	Receiving bad evaluation
Behavior management	Fostering self-regulation	Reflection
Building student confidence	Frustration	Scaffolding
Engaging students	Getting through curriculum	Seeing light bulbs
Checking for understanding	Getting to know students	Self-efficacy beliefs
Collaboration with colleagues	Growth mindset	Shift from whole group
Cooperative learning	Hands-on activities	Showing applications
Data-driven decision making	Learning centers	State assessments
Desire for change	Making sense of mathematics	Student growth
Differentiation	Manipulatives	Support systems
Disapproval from colleagues	Mathematical connections	Teachers' views of students
Discovery learning	Motivation	Technology integration
Discourse	Multiple ways to solve tasks	Unpreparedness
Dislike of lecture style	Opposition to memorization	

Table 4

Organization of Codes

Life Changing Decisions	Connections with Students	Rethinking Mathematics Class
Accountability	Accountability	Autonomy
Autonomy	Building student confidence	Desire for change
Development of MKT	Checking for understanding	Collaboration with colleagues
Empathy	Differentiation	Discourse
Evolution	Discourse	Discovery learning
Growth mindset	Empathy	Extra time spent on math
Overcoming obstacles	Engaging students	Flexible grouping
Perseverance	Flexible grouping	Frustration
Teachers' perceptions of support	Fostering self-regulation	Growth mindset
Teachers' view of mathematics	Frustration	Hands-on activities
Shift toward student-centered approach	Getting to know students	Learning centers
	Growth mindset	Making sense of mathematics
	Making sense of mathematics	Motivation
	Motivation	Overcoming obstacles
	Overcoming obstacles	Patience
	Patience	Perseverance
	Perseverance	Promoting joy of learning
	Reflection	Seeking change
	Teachers' views of students	Shift toward student-centered approach
	Shift toward student-centered approach	Showing applications
	Understanding student thinking	Teachers' perception of support
		Teachers' views of mathematics

Life changing decisions. This first theme arose after I recognized many participants reached a point in their experiences with mathematics when they had to make a crucial decision. They decided how they were going to personally respond to a practical dilemma. These decisions had far-reaching consequences regarding the development of their mathematical dispositions and teaching practices. The individual interviews, focus-group interviews, and audio diaries revealed these participants made Life Changing Decisions during their ninth-grade algebra classes, teacher education programs, or first year of teaching mathematics.

Annie recalled feeling totally lost in her eighth-grade algebra class and said in the interview, "I felt the teacher had expected us to understand as she modeled it and then she moved on." For the first time in her life, Annie received a D on her report card and admitted, "It was pretty devasting to me." Reflecting on the experience of repeating algebra as a freshman, she acknowledged, "I needed that." Later in the focus-group interview, Annie identified this repeated course as one of her best mathematics classes:

The teacher I had modeled each problem in a step by step process and explained the reason for the step. Knowing why I was completing each step was very important to me.

. . I went into the class feeling like I would never understand algebra. I left that class with a new confidence. It's definitely an experience that has shaped my teaching.

Making sense of mathematical procedures was something that Annie valued as a mathematics learner. This idea continued to the university level in her teacher education program. Annie recalled working with base-two numbers in her mathematics content courses and shared in the interview, "It was confusing, but it made me better understand how base-ten numbers worked . . . There's really a reason and pattern to it . . . It wasn't until I started those courses that that kind of clicked for me." Her perception of how she was going to teach elementary-level mathematics

content was transformed because she finally understood the meaning behind standard algorithms. Throughout her audio diary reflections, Annie's M.O. was helping her students make sense of mathematical concepts involving fractions. Her decision to maintain a positive outlook while repeating algebra would eventually lead to mathematics being her favorite subject to teach.

Carly remembered feeling anxious and overwhelmed about learning mathematics after being mainstreamed in junior high school. For the next two years, she basically treaded water during mathematics class. When Carly's ninth-grade algebra teacher, Mrs. Prindle (pseudonym) offered to work with struggling students after school, she decided to take her up on the offer. Carly felt she had developed a strong work ethic as a result of those experiences. In the interview, she identified Mrs. Prindle as her best mathematics teacher:

She was difficult as far as she was a tough teacher, but she had that um now I guess what we would call that growth mindset that she was applying. . .. She pushed that productive struggle and didn't allow you to just kind of say 'Oh, I don't know' or 'I'm not good at it.' [Her response] was always, 'What else can you do? I'm not telling you yet. What else can you do? Think about another way.' And it opened up that door to problem solving not just in math, but in real life.

At the beginning of every school year, Carly would explain to her own students that "productive struggle is good." She seemed to emulate Mrs. Prindle by encouraging a growth mindset in Henry (pseudonym), one of her fourth-grade students who was working at the first-grade level:

I let him battle with [my itinerant student] on fourth-grade place value. And he sees that, 'Oh my gosh! I've never been up to par with my peers and now I'm starting to do that.'.

. . I now have him on mixed addition, subtraction, multiplication, and he's never seen multiplication before.

Carly described her experiences teaching a complete unit on fractions in her audio diary. Her small-group instruction incorporated discovery learning and colored fraction tiles. Rather than give direct instruction to her two learning support students, she facilitated "exploration phases" that sparked meaningful discussions about mathematical concepts and vocabulary. Carly's decision to persevere in constructing knowledge during those after-school study sessions resulted in her having a teaching philosophy that strongly aligned with social constructivism.

In Helen's teacher education program, prospective teachers were required to take a physical science course. When no course fit into her schedule, she decided to substitute the more challenging physics course. Helen acknowledged, "I was in way over my head! I had never even taken trigonometry." Compared to her classmates, she realized she needed to "work extra hard" just to maintain a C. As a result of this experience, Helen felt she was able to empathize with her fourth graders who lacked basic skills to solve more complex problems. As a result, she intentionally set aside time during the day for these students to work on their missing skills. On the flip side, learning mathematics came relatively easy to Derek. He expressed anger and recognized the need to control his emotions when his students could not understand certain mathematical concepts that were second nature to him. When his sixth graders remained confused about approaching word problems, Derek expressed his frustration in his audio diary reflection:

Oh, I don't know how many times I can say this until I am blue in the face . . . when I tell them to read the problem carefully, it seems like it doesn't connect. Um, I try to model it each time I do a problem, so I don't know what else I can possibly do.

If Helen had decided to take the less challenging route in her teacher education program, she might not have been as effective in putting herself in her students' shoes.

Unlike the strong-willed prospective teachers in her cohort, Emma decided to keep an open mind in her mathematics methods course. She realized later her professor was trying to prepare the class for teaching mathematics in the Common Core era. A little later in her teaching career, Emma fully embraced the ideals of the initiative:

I really like [Common Core] because they're not teaching [students] a certain set of procedures that you have to go through. [Students] can come up with the answer however they want . . . they just have to be able to explain how they have arrived at the answer. I love that . . . I do very little whole-group instruction now. . .. There really is no reason for me to be up there and just saying, "Okay, first you do this, and then you do this."

Emma's decision to keep an open mind about a new approach to learning mathematics helped her break away from the traditional, teacher-centered approach in her own classroom.

As a first-year teacher, Faith identified her greatest challenge as pacing the curriculum without losing struggling learners. Her nephew happened to be one of those struggling learners that critical year. Faith decided to slow down the pace because she wanted her nephew to enjoy studying mathematics and not feel left behind. Contrary to her intuition, she discovered slowing down the pace was key to her success in getting through the curriculum. Faith shared in the interview, "As backward as it seemed to me, if I slowed it down a little bit for them, they would end up getting the hang of it quicker than working through frustration all the time." Moreover, this decision allowed her to devote the month of May to fun lessons that served as building blocks for fifth grade.

Although this was Barbara's first year teaching mathematics, she had been helping her homeroom students solve their mathematics problems for the previous four years. She

explained, "I know how they were taught, but that doesn't make sense to them." Rather than conform to the other fifth-grade teachers' approach of whole-group instruction, Barbara decided to implement small-group instruction and independent learning centers. She believed when a teaching strategy did not make sense to a child, the teacher should try pulling out manipulatives, providing another example, or approaching the mathematical topic from a "totally different way." Barbara sensed her colleagues disapproved of the nontraditional way that she ran her classroom, but she emphatically replied, "I kind of just don't care." She based her courageous decision on her belief in putting her students' needs first. The development of these participants' dispositions toward mathematics was personal. If they had decided to respond differently to these practical dilemmas, then their experiences in teaching mathematics might have been drastically different.

Connections with students. All teachers communicated they felt ineffective when they first began teaching mathematics. In their repeated attempts to reach students through their instruction, they realized they needed a better understanding of their students. From the individual interviews, focus-group interviews, and audio diary reflections emerged the second theme, Connections with Students.

Many of the participants recognized to become effective teachers of mathematics, they would need to move beyond the boundaries of whole-group instruction. In his second year of teaching, John sensed students were growing tired of him just drilling the information. Derek's audio diary reflections revealed he was already getting bored with the carrying out whole-group instruction after only one year. John and Derek wanted to incorporate more hands on, fun activities into their classrooms, but they expressed having limited resources as newer teachers. During her first four years, Annie recalled having lots of lessons where she was standing at the

board basically teaching one student. In her interview, she openly shared, "I'll be honest. At first, I didn't enjoy teaching mathematics because I didn't feel like I was doing anything."

Emma added:

I would go over concepts very quickly and then, all of a sudden, expect them to um do a problem or something and they had no idea how to even start it. And I would be so like frustrated, like wait a minute. . .. I went over it, you know?

Annie and Emma realized if they wanted to be effective teachers of mathematics, they needed to connect with their students. When asked to describe one of her not-so-successful lessons, Carly affirmed:

My two kids had had their previous learning support teacher for three years, so they already had built that kind of connection. And it wasn't just one lesson. It was like the first three weeks of lessons because they were pretty much, I mean, mute. . .. So, any kind of mathematics lesson, I couldn't gauge where they were. . .. And so, that was super, super frustrating to me just because I got shrug if shoulders. I got shaking of the head. I got just looking at you, just staring. And it's like, how am I going to figure out where they are if I can't get them to communicate with me what they know?

The participants in this study recognized in order to connect with their students there would need to be a shift from a teacher-centered classroom toward one that was more student centered.

George made the statement:

I found in my first few years of teaching that it is interesting to see how students think about solving the problems. When I was able to figure out their way of thinking, it allowed me to show them new ways to solve the problems. I was also taught new ways by students in the classroom as well.

No matter how discouraging or frustrating it might be, the teachers knew sometimes they would need to reteach fundamental mathematics skills before they could even begin their intended lessons. Reflecting upon her lesson about adding fractions with uncommon denominators, Barbara shared:

Um, we're still working on, obviously, learning our multiplication facts. And I think for them, before we can go too much further, they just have to know how to multiply. And that's really hard. I mean, it's a huge problem for some of my kids because I'm like, that's not what I'm supposed be teaching you right now, but we can't go anywhere unless you can multiply.

Emma spoke about her "most frustrating" lesson that was supposed to address her student-learning objective of dividing fractions. After sighing deeply, she said:

So, instead of dividing fractions, we sort of went back to and ended up sort of reviewing multiplying of fractions and what you do. Um, some students were getting very high um numbers that they had to simplify. And so, we also had to sort of go back and um look through our divisibility rules so that we could simplify the fractions.

Two alternative teaching strategies that emerged from the interviews were small-group instruction and flexible grouping. As the participants began to grow as teachers, many of them wanted to incorporate other learning experiences into their classrooms. Isabella shared confidently, "I've trialed and errored lots of things. So, at this point, I know centers are a good idea." In her interview, she provided the following justification:

I just value centers so much because I get to really work directly with five, four, six students, you know? And I can really see—okay, well here's where you're making this error, or let's walk through this strategy. So, you know, having that center time, that

direct instruction, I think is super, super important. Um, and I get to know my kids better, too, you know, which is nice. Um, and then also during the center time, they get to use technology. They get to practice just that skill, you know? And then, they do apply it with their problem solving. So, it's just kind of hitting all those components of math.

Annie added:

I think the key to my mathematics instruction is small group. Since I meet with each group daily, I am able to get a very clear picture of each student's understanding. I take notes and observe where is student is struggling or excelling and I use that information to guide my instruction.

Many of the participants expressed flexible grouping helped them better connect with their students. Flexible grouping allowed these teachers to pinpoint differentiated instruction based on the mathematical topic. It was important to the teachers that their students understood they were not stuck in their group. Barbara explained:

You might be in this group this week because you really get this multiplication problem, but next week we move on to division and you struggle. So, you're going to move from this group to this group, and that's okay.

Emma shared her student pep talk, "You might not get it this year but keep trying because eventually you will. It's not like something you say, 'Oh, I'm not good at math. I'm never going to get this' because eventually you will get it."

The teachers acknowledged any shift in pedagogy would not be possible without the support of others. Annie credited her next-door teacher for helping her understand the value of small-group instruction. Her veteran colleague researched this evidence-based best practice in

mathematics education, and together, they incorporated learning centers into their classrooms. When Barbara decided to abandon the whole-group approach, she cherished the support that she received from her principal. She shared in the interview, "My principal is so supportive she put her kid in my class . . . she likes where I'm going and she's onboard with that. So, I'm going to continue with that." For her first two years, Carly appreciated having a phenomenal mathematics coach who provided scaffolding whenever she needed it. With renewed confidence, she realized those same strategies could be applied to her next teaching setting. Faith appreciated teaching in a supportive environment where caring teachers were willing to discuss ways to reach struggling students. In the interview, she shared:

And still now, going into my fourth year, I'm constantly emailing them, 'Hey, what are you doing with this?' Or I'll go down and see them and say, 'Alright, this is what I have' or 'This kid's doing this, and I've done everything I can. What do you suggest?' . . . And you know, they'll come down to me and say, 'Hey, you had this kid before. What works?

George expressed he finally understood how to incorporate manipulatives into his classroom after working with Helen, his fourth-grade teaching partner. Helen appreciated her awesome principal who supported flexible grouping as a way to bridge the achievement gap in the fourth grade. She explained in the interview:

The biggest challenge, I think, is that you have the student who does not have any kind of number sense at all whatsoever and this student who's on sixth-grade mathematics and um teaching both of them. That's probably the biggest, hardest challenge that there is.

Faith, Emma and Isabella indicated they enjoyed attending professional conferences with her colleagues and learning different teaching strategies. Faith shared, "We went to a conference

this year, the three of us together, so that we could sit together and talk about different things. It was like, 'Wow! That makes sense! We need to try this'." The participants in this study recognized professional growth does not occur in a vacuum. The teachers also recognized they should be teaching their children first, and the mathematics curriculum second.

Rethinking mathematics class. During the individual interviews, focus-group interviews, and audio diary reflections, it became clear to me these teachers wanted their students' experiences in learning mathematics to be different than what they had experienced in school. Although getting through the curriculum in preparation of state assessments and the next grade level was important to the teachers, they recognized learning mathematics should be a meaningful, interactive, and enjoyable journey. As new teachers of mathematics, they spoke about their essential responsibilities in making this happen.

One important responsibility of these teachers was to help students make sense of mathematical concepts and procedures. When they were in school, the teachers remembered they were expected to memorize standard algorithms without understanding the meaning behind the steps. Moreover, it was engrained in them as students that there was only one right way to solve a problem. When the CCSSI was brought up during the focus-group interview, Emma was the first to share, "I think it is way better. I was given a lot of algorithms to memorize without the 'why' behind it. . .. Sometimes, I feel like I am only one who actually likes common core math." Derek agreed with her last point and added, "I like how common core mathematics tries to show how mathematics overlaps itself and that there are different ways of looking at a select problem." Helen chimed in, "I often get complaints [from parents], 'Why can't they just do it the old-fashioned way?" She believed learning different ways to multiply two-digit numbers developed a deeper number sense among her students. Annie made this statement in her audio

diary, "I am going to make sure that they have understood this concept before I move on to showing them the shortcut way."

These teachers felt their primary responsibility was to engage students in the process of learning mathematics. Barbara, Carly, George, and John expressed when they were in school, their mathematics classes were rather monotonous. When asked in the interview to reflect upon her learning experiences in fifth grade, Barbara laughed and said:

It wasn't very memorable for me. It was just another day of mathematics class. . .. I want [my students] to be hands on and I don't want it to be like my elementary experiences where I just sat there. I want them to remember my mathematics class and remember what we did.

John added:

I just remember a lot of kind of drill on the board. This is how it's done. This is how we do it. We didn't have a whole lot of manipulatives and things we could work on with hands on. It was just pencil and paper, workbooks.

Carly remembered the routine as "Do 25 problems that night for homework, bring it in, we're going to quickly go over the answers, do the next lesson, and repeat." Thinking about her own teaching approach, she stated, "I'm confident in the fact that I can deliver instruction that's a little bit more meaningful than just worksheet after worksheet after worksheet." George had to laugh to himself when he made this statement:

[My teachers] had their slides that were already saved, and they would slide their paper down throughout [the lesson] as they were telling us what to do. Um, a few teachers would actually work out the problems as they were doing it every time and clean it off, but mostly it was just straight lecture format. I'd try to follow along and keep up with

them. So, I think that's why I'm trying to do more hands-on things to get the students out of their seats, up to the board more so that they can show me what they're understanding versus not knowing until they take the test.

Annie, Carly, Faith, and Isabella indicated their use of hands-on, discovery-based lessons created an atmosphere conducive to learning for students. In her audio diary, Carly reflected upon how she introduced the idea of equivalent fractions to her learning support students:

I never just jump into this. I start with an explore phase . . . so, the way that I do my explore phase is I give them a fraction. So, I gave them a fraction of um one half to start with. . . . Then I said, 'I want to see if you can match everything that fits underneath that one-half bar.' I said, 'It has to fit perfectly underneath that one half.' I said, 'So, you find however many different fraction bars that will fit underneath that one half.' I tell them that they all have to be the same color. . . . And I give them about five minutes to do that. So, um, you know, as they're working, you can see them trying different things, um some getting frustrated. So, in the end, they finished, and they had found, for the most part—but, well, one little girl found all but one, and then my boy found all of them—what was underneath that one half.

After illustrating how to convert mixed numbers into improper fractions, Annie explained her quick-thinking plan for the next part of the lesson:

So, I wanted to know, could they take that same concept and reverse it and um now take an improper fraction and make it a mixed number? So, I just told them I was going to challenge them, and I wrote an improper fraction on the board and I said, 'Can you use the same thing you just learned and try to make this a mixed number?' And so, the students were like, 'What? I don't know how to do that.' And then a couple of kids were

like, 'Oh, I think I know!' So, it was really cool to see them try to figure it out and then um I think that in every group that I met, there was at least one student who could figure it out. So, I had that student model um what they did, their thought processes, and had them explain.

She concluded in her interview, "I learned that a lot more does come from letting them figure it out as opposed to me just telling, telling, telling." Faith explained how she turned the dreaded lesson on metric and standard conversions into a light-hearted class:

It was one of the goofiest lessons because . . . I went around and dropped [a milliliter of water] on their head. . .. They absolutely loved it and they related to it! So, conversion from day one was simple for them just because they enjoyed the first introductory lesson and they were hands on with moving water from, you know, pints and cups and gallons. . . . It was one of those exciting moments for me because they were having fun in mathematics class and they were laughing and just enjoying it . . . and they were experimenting on their own in their groups and trying to come up with stuff because we only have certain conversions that we need to know.

Above all, she expressed her belief that it was essential for students to have teachers who truly enjoyed mathematics. Faith hoped her love of mathematics would rub off on her students.

Reflecting upon her introductory lesson to the coordinate plane, Isabella shared enthusiastically:

I just loved it because it was so interactive! I put tape for the coordinate grid, and they just loved it because I gave them like all these random characters and told them like to put it at (2, 0). And they were like, 'Yeah!' They just loved it! I used a smartboard . . . so they had to go on up and like actually physically move it and tap it, and they're like emojis. So, they just had a lot of fun with it and they're really good at it. So, it makes it

more fun, and they feel confident. . . . I was all fired up!

Annie, Barbara, Carly, Helen, and Isabella also spoke about how they encouraged their students to become self-regulated learners. In the focus-group interview, Barbara shared, "Self-analysis is a big factor in my classroom. Students seek out their errors. As part of our growth mindset, students understand they can actually learn a lot from their errors." Annie affirmed, "I realize that a lot more does come from letting them . . . find out where those mistakes are and then correct them from that." Carly spoke about using "I can" statements as a way for her students identify what skills they were working on and which they had mastered. Isabella talked about using exit tickets as a way to have her students monitor their own learning of the daily target. She explained, "They tell me if they're . . . a rookie player, starter, or MVP." Helen maintained a motivational chart in her classroom that allowed students to set personal goals and monitor their progress in achieving mathematics fact fluency. Emma summed up in the focus-group interview, "For me, student engagement is the most important aspect to make the experience positive. That is, students who are excited to learn and trying to learn, even if they are having trouble understanding the content."

Research Question Responses

Central question. How do novice upper-elementary teachers perceive and describe their experiences teaching mathematics? All participants had their own story to tell about their early experiences teaching mathematics. Although their personal stories revealed what was significant to them as individuals, the essence of what it meant to be a new teacher of mathematics was also uncovered. In their new roles as teachers of mathematics, they all aspired to put their students first and make a difference in the way their students would experience learning mathematics.

Initial disconnect with students. The teachers soon realized in order to successfully

teach mathematics they would need a better understanding of their students' thinking processes and ability levels. During their interviews, Annie, Carly, and Emma spoke about the initial disconnect they sensed between themselves and their students. Annie shared, "I do recall feeling clueless in my first years because I didn't understand how my students were thinking." For the first three weeks of school, Carly recalled getting shrugs of shoulders, shaking of heads, and blank stares from her students and questioned, "How am I going to figure out where they are if I can't get them to communicate with me what they know?" Emma acknowledged overestimating the knowledge of her students at first and said, "I would go over concepts very quickly and then, all of a sudden, expect them to um do a problem or something and they had no idea how to even start it."

Shift toward student-centered approach. As they began to grow as teachers, they recognized the value in moving beyond the traditional, teacher-centered approach toward one that was more student centered. During the focus-group interview, George offered, "I get frustrated when teachers say there is only one way to solve a problem . . . I found in my first few years of teaching that it is interesting to see how students think about solving the problems."

Annie stated, "I think the key to my mathematics instruction is small group. I take notes and observe where the student is struggling or excelling, and I use that information to guide my instruction." After several trials and errors, Isabella revealed:

I just value centers so much. . . . I get to really work directly with five, four, six students. . . . I get to know my kids better . . . they get to use technology. They get to practice just that skill . . . and then, they do apply it with their problem solving. So, it's just kind of hitting all those components of math.

The teachers stressed the importance of being flexible when dividing their students into small

groups. Barbara said hypothetically, "You might be in this group this week because you really get this multiplication problem, but next week we move on to division and you struggle." In Helen's class, a student could experience both enrichment and remediation:

I have a girl that is amazing at long division and multiplication and does not understand fractions. So, she didn't go to enrichment for fractions because she just bombed the topic pretest. So, it was nice that . . . they had that confidence in like, 'Oh, yeah. I'm in enrichment.'

Although differentiation was a catch phrase they had picked up in their training, the teachers were developing a concrete understanding of what differentiated instruction truly entailed.

Dichotomy between then and now. The teachers planned to make their students' learning experiences more meaningful, interactive, and enjoyable than what they had experienced in school. As opposed to carrying out the memorized steps of standard algorithms, Annie, Barbara, Emma, Faith, George, and Helen's students were experiencing light bulbs going off above their heads. Carly's students no longer seemed like wallflowers during mathematics class because they were actively constructing mathematical knowledge using colored fraction tiles during small-group instruction. In Annie, Barbara, Carly, Derek, Emma, Faith, George, Helen, and Isabella's classes, the students were not expected to sit in their seats and try to keep up with the daily instruction; they were either rotating through learning centers in their flexible groups or participating in a variety of activities. Barbara sacrificed and fought hard to prevent her mathematics classes from becoming the "classic Charlie Brown classroom [where] the teacher talks at students who are not engaged in the task." Rather than search for that right way to solve a problem, the students of Annie, Barbara, Emma, George, Helen, and John were encouraged to try different problem-solving strategies and explain their reasoning. The teachers,

who tolerated just another day of mathematics class when they were school, reflected on how their students enjoyed learning mathematics through hands-on activities and games.

Uphill battle. The journey to make a difference in their classrooms was not without its disappointments. Barbara, Carly, and Helen became frustrated when they were unable to teach their intended lessons. Barbara explained, "Before we can go too much further, they just have to know how to multiply. And that's really hard." Carly shared, "It's been very frustrating um, to be able to see such success with the skill and such limited success with the application of it." Helen, an avid proponent of mathematics facts, explained, "It's just—they can't do a long division problem if they can't divide, then multiply those numbers, and then subtract those numbers if they don't have their mathematics facts." Additionally, Derek and Emma grew weary of having to constantly review previously taught skills. Derek disclosed in his audio diary, "Um, today was fairly monotonous. Ah, reviewing material is kind of not too fantastic for me especially when—it bugs me when students don't know it." After sighing deeply, Emma shared, "So, instead of dividing fractions, we sort of went back to and ended up sort of reviewing multiplying of fractions and what you do." The teachers recognized sometimes they needed to take a step backward before they could move their students forward.

Sub-question 1. What do novice upper-elementary teachers' experiences reveal about their mathematical dispositions? Many of the participants made life changing decisions in their experiences with mathematics. The decisions they made revealed tendencies in their intentional actions and behaviors related to teaching mathematics.

Decisions made as algebra students. Annie's decision to repeat algebra revealed she valued making sense of mathematical procedures and believed all students can learn mathematics. Throughout her audio diary, she tended to use student-centered approaches, in

which all her students—even the lowest level—discovered procedures involving fractions and mixed numbers. When Carly decided to persevere in learning algebra during voluntary afterschool study sessions, her teaching philosophy of social constructivism started to develop. Her audio diary clearly documented her inclination to have her students experience both "exploration phases" and productive cognitive disturbances while learning about fractions.

Decisions made as preservice teachers. Emma's decision to keep an open mind about Common Core mathematics during her methods course revealed not only her growth mindset but also her belief that students should be given opportunities to become good problem solvers. Therefore, she was committed to professional learning and incorporating evidence-based teaching strategies into her classroom. In her interview, Emma provided the following example: "We always do these posters. . .. That [curriculum summit] helped me with this strategy of doing this group work, and making a poster and explaining your work, and um how everybody comes up with different strategies." Helen's decision to take a college-level physics course without any prior knowledge of trigonometry showed that she could relate to not feeling up to par in mathematics class. She became an empathetic teacher who intentionally set aside time each day for her fourth-grade students to work on their missing skills.

Decisions made as first-year teachers. Barbara's decision not to conform to the traditional, teacher-centered approach that her colleagues relied on revealed her strong belief in putting her students' needs first. In her interview, she acknowledged helping her homeroom students with their mathematics problems for the four years prior to becoming the fifth-grade mathematics teacher. When Faith decided she would not allow her nephew to be one of those students who fell through the cracks in mathematics education, she found herself obligated to use differentiated instruction. She explained in the interview, "So, it just really helped me make sure

that I was explaining [concepts] in three different levels to the kids, make sure I had activities that reached all of them, and even [leveled] homework." The features of mathematics instruction the teachers valued and used regularly in their classrooms were consequences of these crucial decisions.

Sub-question 2. How do trends in novice upper-elementary teachers' intentional actions and behaviors compare to tendencies associated with a productive mathematical disposition, as defined by the National Research Council? A production disposition is the affective strand of mathematical proficiency. The term refers to "the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics" (NRC, 2001, p. 131).

Seeing sense in mathematics. Trends in the teachers' intentional actions and behaviors were uncovered in the data. These trends revealed novice upper-elementary teachers tend to believe mathematics is understandable. Their stories about when mathematics just clicked for either them or their students confirmed the teachers viewed mathematical ability as expandable, not fixed.

Perceiving mathematics as useful and worthwhile. The novice upper-elementary teachers were committed to creating student-centered learning environments in which no child would be left behind. Their commitment entailed interacting with every student, developing different learning centers, and assessing individual student needs for flexible grouping. The actions of these teachers demonstrated they tended to perceive learning mathematics as useful and worthwhile.

Believing in steady effort. The novice upper-elementary teachers' unwillingness to

spoon feed their students mathematical content reflected their belief that steady effort was necessary for learning mathematics. They understood perseverance was an essential part of the learning process. The way that Annie, Carly, and Emma persevered in their own "evolution" from struggling mathematics learners to efficacious teachers of mathematics was evidence that steady effort pays off.

Self-efficacy in mathematics. Many of the teachers communicated they did not see themselves as effective doers and learners of mathematics until after they began teaching it.

Mathematics was Annie's least favorite subject to learn but became her favorite subject to teach.

Carly made the statement:

I like myself as a teacher with it, better than a learner of it because I'm taking it in more. I mean, when you are responsible then for relaying the information, I think you learn a lot more about how you would've perceived it back when you were in school. And how you take it in and using that to help you change your teaching.

Emma stated, "I really never thought of myself as a math person until I started teaching."

Sub-question 3. How do novice upper-elementary teachers describe their self-efficacy beliefs of teaching mathematics? All the teachers acknowledged the journey to becoming an effective teacher of mathematics was a bumpy one. The teachers believed they were achieving success in certain aspects of teaching mathematics. At the same time, they acknowledged there were other aspects in need of improvement.

Beliefs about content knowledge. Annie, Barbara, Faith, Isabella, and John made similar statements about having to master the mathematical concepts in their grade levels. Annie said, "I feel like I've gotten to the point where I understand [subtracting mixed numbers] more because I think my struggle in the beginning was, I didn't fully understand what the process was." Even

though Barbara considered herself "good" at math, she recognized she needed to be "more comfortable" with the content in the Common Core era. She clarified, "The way they're being taught is not the way I was taught." Faith admitted not feeling "scared" to ask a colleague, "Alright, show me again because it's not something that I've done for a while." Isabella made the statement, "I was trained [grades] K to 6, you know? It wasn't, okay . . . you're going to college for fifth-grade math. I think at first, it was kind of like, 'Whoa! Wow, like volume. Wait. Let me remember what that is." John openly shared:

I was really nervous because I had middle-school mathematics and I thought, 'Wow, you know, this is a little more advanced. Am I really ready for this?' And so, at first, I was hesitant, but after I started the second year of doing it . . . like a switch goes off and I know, I can recall, you know, what the procedures are.

Beliefs about teaching practices. Annie, Barbara, and Faith believed they were capable of providing alternate explanations when their students were confused. Annie shared:

I have this one boy um, who one day he's got it and the next day he's lost. . .. By questioning him, it's like I know where his—I know what he's thinking. . .. I know how to get him back to where he needs to be.

Barbara said, "I definitely see things [during small-group instruction] that I'm like, that didn't go so well. Let's take a different approach." Faith shared with confidence, "I knew from the getgo, for my lower-performing kids, that I needed to take time and break it down and then I could get them to where I needed to, and it helped a lot to get them moving." Many of the teachers believed that they were capable of providing differentiated instruction through learning centers and flexible grouping.

Beliefs about students. Barbara, Carly, Faith, and Helen made similar statements about

their challenges in reaching students who are working at such different grade levels. Helen, one of the fourth-grade teachers in the study, summed it up nicely, "You have the student who does not have any kind of number sense at all whatsoever and this student who's on sixth-grade mathematics and um, teaching both of them. That's probably the biggest, hardest challenge that there is."

Barbara, Derek, and John believed they were capable of helping their students value learning mathematics. Barbara shared excitedly, "They could finally figure out the sales [discount] because they always wondered!" Reflecting upon a hands-on statistics activity that involved projectile motion, Derek happily recorded in his audio diary, "So, it was nice to see that even though they don't know physics, they're essentially talking about physics." When John's students questioned why they had to learn about estimation, he immediately responded, "I brought up my brother, his professional career. That's what he does for a living! He estimates on a multimillion [dollar] level. It's his job to be good at this."

Beliefs about student efficacy. Carly, Faith, and George felt they were capable of getting their students to believe they could do well in mathematics. Carly felt a great sense of accomplishment when "her one little girl" who typically works at the second- to third-grade level showed a fourth-grade student how to add fractions. Faith asserted," I think seeing what all we had to do with fractions is probably what scared them the most . . . but it was that first day or two of convincing them you know what you're doing, like we can do this." George said enthusiastically:

I always like when we get to the two-by-two multiplication just because they have no idea about it before they get to us, and it's just so nice to see the light bulb go off once they realize that they can actually do it.

Beliefs about motivation. Derek, Emma, Helen, and George indicated they sometimes struggled with motivating students who showed low interest in learning mathematics. Derek shared woefully,

When we taught LCM and GCF, they would keep getting them mixed up, and they—I could tell that they just didn't want to know what's the difference between the two. And that lesson bummed me out because I was like, 'What did I do wrong?'

Emma said bluntly, "I don't feel confident with that lower um level learner who is not wanting to be there at all and tells you that on a daily basis." Helen admitted, "I think that's what I struggle with the most is like, how do I get them to want to do well?" George added, "I try to coax them into um trying something on their own so I can see what their thinking is to make—to guide them in the right direction from there. Um, they're just not willing to do this." Annie, Derek, and John identified behavior management as one of their greatest challenges.

Sub-question 4. What factors do novice upper-elementary teachers identify as influencing their experiences teaching mathematics? During the interviews, focus-group interviews, and audio diary reflections, the participants identified several factors that they perceived as influencing their experiences teaching mathematics. The identified factors were either experiential or environmental.

Experiential influences. The teachers felt their personal experiences as learners of mathematics significantly influenced their approaches to teaching. Annie stated in the interview, "My goal is to reach those kids who were like me . . . make them understand the relationship of how place value and numbers and all that work." Her audio dairy reflections showed how she was adamant about her students understanding specific concepts before using shortcuts. Barbara appreciated finally seeing a practical use for mathematical skills during her high school science

classes. She wanted her students to experience that same math-science connection much earlier:

That's why I really like teaching science and mathematics together because I use the mathematics that we're learning in fifth grade and any chance I can, I show them how that relates to what they're doing in science in fifth grade.

John communicated a similar sentiment in his interview:

That's probably how I was—how I still am as a learner. If you can relate it to me or show me a concrete kind of idea, I can learn it. . .. What am I doing this for? Not just, here it is, um, good luck. You'll probably never use this again. Like, I want to know . . . I try to incorporate that as much as I can.

Derek tried to follow in his favorite teacher's footsteps and shared enthusiastically, "He was structured and organized . . . I'm like, I want to be just like that!" In his interview, he indicated he typically followed the same agenda, which included a bell ringer, the lesson, and some type of conclusion. In his audio diary, Derek jokingly referred to his hands-on statistics activity as "a little bit of controlled chaos." During her interview, Helen described her epiphany about learning mathematics:

There was something that clicked when I realized I could check my own work to make sure every single problem was correct. It was like . . . why didn't I know how to do this for so long and now I know how to do it?

She sensed her mathematics professor was impressed by her diligence when responding to each test question. Helen explained how this experience influenced her expectations for her students' work, "I encourage my students to read each question carefully, highlight important words, show their work on a separate piece of paper, and check their answers using inverse operations or substitute for the variable and rework the problem." Not liking the lecture style all the time,

George shared what he envisioned for his classroom, "Just being able to see how to get students up and moving, how to get them into centers, which is something I've introduced more this year." Isabella felt good about her early experiences learning mathematics because they were engaging and fun. She became a fun-loving teacher who creatively incorporated a sports theme into her mathematics classes. Being a former athlete, Isabella explained its underlying significance, "We work on like collaboration and . . . perseverance in kind of going with the sports theme and applying it to the classroom."

Environmental influences. The teachers believed the school environment contributed to their experiences teaching mathematics. They appreciated having additional time for mathematics built into the school schedule. It would make the time-consuming, student-centered approaches possible. Emma saw her students for double periods and shared, "That does make it nice because I can do a lot of different things during that time." The small school where John taught was the exception, following a conventional 7-period school day and sometimes combined different grade levels into one mathematics class. This might explain why John felt reviewing mathematics concepts was a "giant" component of his instruction, leaving him no time for more student-centered approaches. Derek's perception that middle school was kind of a weird period for everyone influenced his teaching experiences. In addition to mathematical skills, he felt obligated to teach his sixth graders soft skills such as organization, working with others, and maintaining a positive outlook. Derek's audio diary reflection revealed how he approached working with this unique age group:

I do my instructions [for the hands-on activity] beforehand. I had to be very explicit . . . during my time of explaining the directions. I was very stern and . . . had to be very clear. Um, in this age group, it seems like you have to have those moments to kind of flip

the switch and be able to go from relaxed and joking to okay we need to get to work, you need to listen, and do this.

Many of the teachers thought they benefitted from experiencing a support system that extended beyond being assigned a mentor teacher their first year. The growth of these teachers of mathematics was influenced by their personal experiences in learning mathematics and the school environments in which they taught.

Summary

The purpose of this transcendental phenomenological study was to describe the lived experiences of novice upper-elementary teachers to understand their mathematical dispositions. Ten teachers volunteered to share their experiences as both learners and teachers of mathematics. The participants differed in age, years in education, certifications, teaching assignments, school settings, and self-efficacy beliefs. Data from audio diaries, individual interviews, and online focus groups were used to answer the central research question: How do novice upperelementary teachers perceive and describe their experiences teaching mathematics? Three themes emerged from this research: Life Changing Decisions, Connections with Students, and Rethinking Mathematics Class. The findings indicated the features of instruction that novice upper-elementary teachers valued and used regularly in their classrooms were shaped by their experiences as both learners and teachers of mathematics. They aspired to put their students first and make a difference in the way their students would experience learning mathematics. The novice upper-elementary teachers recognized these aspirations called for student-centered approaches to teaching mathematics. They understood differentiated instruction meant taking a step backward might be required before they could move their students forward.

CHAPTER FIVE: CONCLUSION

Overview

Much of the literature regarding teachers' mathematical dispositions comes from a quantitative perspective with an emphasis on preservice elementary teachers in a university setting. This transcendental phenomenological study builds upon that literature by examining the lived experiences of novice upper-elementary teachers and providing a deep, rich, and thick description of their mathematical dispositions. The chapter begins with a summary of the findings in context of the research questions. A discussion follows in which the findings of the study are reviewed in light of the relevant literature and theoretical framework. The theoretical, empirical, and practical implications of the study are addressed. The study's limitations, within and beyond the researcher's control, are outlined. The chapter concludes with recommendations for future research.

Summary of Findings

This research project sought to answer the central research question: How do novice upper-elementary teachers perceive and describe their experiences teaching mathematics? Three themes emerged from the research: *Life Changing Decisions, Connections with Students*, and *Rethinking Mathematics Class*. The essence of what it means to be a new mathematics teacher was uncovered from the audio diary reflections, individual interviews, and focus-group interviews. Essentially, the teachers realized they needed a better understanding of their students in order to become effective teachers of mathematics. They learned the traditional, teacher-centered approach did not lend itself to this type of understanding. The maturing teachers turned more toward student-centered approaches such as small-group instruction, discovery-based learning, hands-on activities, learning centers, and flexible grouping. They went to great lengths

to make their students' learning experiences more meaningful, interactive, and enjoyable than what they had originally experienced in school. At times, the novice teachers experienced setbacks due to their students' unpreparedness to learn grade-level content.

The first sub-question examined what novice upper-elementary teachers' experiences revealed about their mathematical dispositions. For many participants, their intentional actions and behaviors related to mathematics could be pinned down to a specific point in time when they made a life changing decision. Annie and Carly struggled to make sense of procedures while in algebra class. Emma and Helen were placed outside of their comfort zones during their teacher education programs. Barbara and Faith found themselves personally accountable for student learning as first-year teachers of mathematics. How the participants decided to respond in these situations would have some bearing on the features of mathematics instruction they tended to value and use regularly in their classes.

The second sub-question focused on how trends in novice upper-elementary teachers' intentional actions and behaviors compare to tendencies associated with a productive mathematical disposition, as defined by the National Research Council (2001). The teachers repeatedly spoke about helping all their students make sense of mathematical concepts and procedures, which revealed they tended to see sense in mathematics. Their commitment to transform the way their students would experience learning mathematics showed they tended to perceive it as useful and worthwhile. The teachers made similar statements about how perseverance played an important role in the learning of mathematics, which served as evidence of their belief that steady effort pays off. An interesting finding was many of the teachers tended to see themselves as effective doers and learners of mathematics after they had begun teaching mathematics.

The third sub-question probed into how novice upper-elementary teachers describe their self-efficacy beliefs of teaching mathematics. They characterized their budding self-efficacy beliefs as a "learning curve." The teachers recognized right away they did not truly understand the mathematical concepts in their grade levels. This was also true for teachers who believed they were always good at mathematics. The teachers realized they needed a deeper understanding of the mathematics curriculum. The novice upper-elementary teachers felt they were achieving success in several facets of teaching mathematics. They believed they could help students value learning mathematics, get students to believe they could do well in mathematics, provide alternative explanations for confused students, and differentiate their instruction. At the same time, the teachers felt there were other aspects of teaching mathematics in need of improvement. They struggled with motivating students who showed low interest in learning mathematics. A similar area of concern for them was behavior management. The teachers were further challenged in reaching students who were working at such different grade levels.

The fourth sub-question delved into factors the novice upper-elementary teachers perceived as influencing their experiences teaching mathematics. The teachers spoke about how their personal experiences as learners influenced their teaching practices. The majority took measures to ensure their students' learning experiences were different. Other teachers felt the school environment contributed to their experiences teaching mathematics. Having additional time built into the schedule for mathematics made student-centered approaches possible.

Working with adolescents required teaching soft skills along with mathematical skills. Feeling supported by their colleagues and administrators allowed the teachers to grow professionally.

Discussion

The purpose of this transcendental phenomenological study was to describe the mathematical dispositions of novice upper-elementary teachers by exploring their experiences as teachers of mathematics. The study's findings illuminated the way dispositions toward mathematics were intertwined. The practiced-based theory of mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008) and self-efficacy theory (Bandura, 1977) guided this research.

Theoretical Perspective

In their practiced-based theory of mathematical knowledge for teaching, Ball et al. (2008) identified the everyday tasks of teaching mathematics and the types of knowledge that were essential for carrying out this work. Ball and associates proposed teachers need common content knowledge, which is an understanding of the mathematical topics in the student curriculum. Annie, Faith, Helen, Isabella, and John acknowledged their common content knowledge was a little rusty when they began teaching. Annie admitted struggling with teaching the subtraction of mixed numbers because she did not fully understand the process. John and Helen noted their common content knowledge was strengthened after teaching the topic a second time. Faith added she often consulted with her colleagues to refresh her knowledge of unpracticed topics. Isabella did not view this lack of knowledge as a major setback in her career because she believed elementary teachers could not be sufficiently trained to master every topic in all subject areas for grades K through 6. Derek, on the other hand, possessed extensive common content knowledge from "taking a boat load of mathematics courses" in his teacher education program. Much to his surprise, explaining easy sixth-grade content was more difficult than he had anticipated. Derek's experiences corroborate previous research that taking more generalaudience mathematics courses has little or no effect on adequately training teachers to teach mathematics (Jonker, 2012; Maher & Muir, 2013; Smith, Swars, Smith, Hart, & Haardörfer, 2012).

Teachers also need specialized content knowledge, which makes them capable of unpacking or decomposing mathematical concepts and ideas, so they are visible to students (Ball et al., 2008). Annie, Carly, Emma, and Faith remembered practicing these skills during their teacher education programs, whereas other participants mentioned having to learn them on the job. The teachers in this study used this knowledge to help their students make sense of mathematical concepts and procedures. Their specialized content knowledge varied from topic to topic and often dictated how they would present their lessons. When this knowledge was abundant, the teachers tended to incorporate manipulatives, draw visual representations, and introduce multiple ways to solve problems. When it was lacking, they tended to revert to rote procedures such as the *keep-change-flip* method for dividing fractions.

Ball et al. (2008) described teachers with knowledge of content and students as capable of anticipating student interpretations of tasks, analyzing student thinking, and understanding common student conceptions and misconceptions. Acquiring this type of knowledge was very important to the participants in this study because they felt incapable of reaching their students at first. Annie, Barbara, Carly, Emma, George, Helen, and Isabella believed they developed a real understanding of their students by interacting with them daily during a small-group instruction. Their knowledge of content and teaching naturally flowed from those student-centered experiences. As the teachers gained an understanding of their content and students, they felt more capable of designing and evaluating appropriate mathematics instruction (Ball et al., 2008) to meet their students' needs. The participants in this study recognized it would have been

impossible to learn everything about teaching mathematics during their teacher education programs.

In sharp contrast to the theory of the mathematical knowledge for teaching, Bandura's (1986) theory focused on people's judgements of what they can do with whatever skills they possessed. Self-efficacy theory posits people can bring about change in themselves and their situations through their own efforts (Bandura, 1977). The difficulties or setbacks endured in their pursuits of learning or teaching mathematics "serve a useful purpose in teaching that success usually requires sustained effort" (Bandura, 1989, p. 1179). Their personal efficacy beliefs influenced their cognitive, motivation, selection, and affective processes (Bandura, 1993). Annie described how she worked to make sense of mathematical procedures, became interested in knowing her students, teamed up with her colleague to implement learning centers, and came to love teaching mathematics. Barbara discussed her resolve to use small-group instruction as a way to meet her students' needs and make their learning experiences more memorable and engaging than hers were. Carly spoke about cultivating her ability to provide meaningful, handson instruction by joining forces with the phenomenal mathematics coach in her building. Derek mentioned his determination to push through his failed attempts to motivate his students and expand his toolbox with different techniques and fun activities. Emma discussed how she came full circle in her mathematics education, developed an interest in student-centered approaches to learning, and continued to learn different teaching strategies. Isabella and John spoke about trying different teaching strategies and learning from their mistakes. These snippets from the participants' stories support the significant role self-efficacy beliefs play in the development of upper-elementary teachers' dispositions toward mathematics.

Empirical Perspective

Much of the recent literature in mathematics education emphasized the importance of productive dispositions. Numerous researchers have explored ways to develop productive mathematical dispositions among preservice elementary teachers during their teacher education programs (An, Ma, & Capraro, 2011; Bartell, Webel, Bowen, & Dyson, 2013; Beswick & Muir, 2013; Charalambous, Hill, & Ball, 2011; Dede & Karakas, 2014; Hobden & Mitchell, 2011; Lutovac & Kaasila, 2014; Maasepp & Bobis, 2015; Savard, 2014; Sloan, 2010; Spitzer et al., 2011; Stohlman et al., 2015; Zazkis, Leikin, & Jolfaee, 2011). Many participants in this study expressed their view of mathematics expanded as a direct result of the learning experiences in their mathematics content and methods courses. This finding corroborates previous research conducted by Zazkis et al. (2011). Annie, Carly, and Emma reported seeing the sense behind standard algorithms for the first time. Helen described her epiphany about learning mathematics when she realized she could check her own work by using inverse operations. Barbara felt mathematics had changed since she was in school. All participants appreciated learning there was not one right way to solve a mathematical problem.

This study extends previous research because it examines the disposition of elementary teachers after they have gained experience teaching mathematics daily. The findings diverge from previous research that found elementary teachers continue to enter the teaching profession with unproductive mathematical dispositions (Dede & Karakus, 2014; Hobden & Mitchell, 2011; Lutovac & Kaasila, 2014; Sloan, 2010). The intentional actions and behaviors of the novice upper-elementary teachers in this study aligned with the tendencies associated with a productive disposition. The fact that many participants did not see themselves as "mathematics people" until after they started teaching mathematics is particularly significant. The gradual changes in

their knowledge, beliefs, dispositions, and classroom practices that were discovered confirm previous research by Goldsmith, Doerr, and Lewis (2014).

The third theme uncovered in this research, Rethinking Mathematics Class, contributes to the field of mathematics education. The shared vision of the participants validates what the National Council of Teachers of Mathematics (2016) deemed as necessary to bring about genuine reforms in mathematics education. While productive disposition has been described as an integral part of mathematical proficiency (National Research Council [NRC], 2001), it has also been considered the mindset that elementary teachers must have in order for the promise of the Common Core State Standards for Mathematics to be realized (Clark et al., 2014; NCTM, 2016). Moreover, McKinney (2018) found teacher mindset has a positive impact on student selfefficacy and performance in mathematics. All the participants in this study were adamant about making the learning experiences of their students different than what they had experienced in school. The novice teachers of mathematics perceived their essential responsibilities to be helping students make sense of mathematical concepts and procedures, promoting different problem-solving strategies, providing real-world applications, incorporating inquiry-based activities, engaging students in the learning process, encouraging productive struggle, building student confidence, fostering self-regulated learners, and trying to make learning mathematics enjoyable. The findings of this study suggest novice upper-elementary teachers with a productive disposition are equipped to bring about the much-needed reforms in mathematics education.

Most self-efficacy studies continue to use quantitative approaches and collect data at one point in time using self-report surveys (Klassen, Tze, Betts, & Gordon, 2011; McGann, 2019; Moriarity, 2014). This qualitative study extends self-efficacy research by exploring the lived

experiences of novice upper-elementary teachers. Listening to participants describe their experiences in both learning and teaching mathematics, the researcher gained an in-depth understanding of how their mathematics teaching self-efficacy beliefs formed, developed, and changed in the early stages of their teaching careers. All participants in this study acknowledged the learning curve associated with becoming an effective teacher of mathematics. This finding corroborates the work of Feiman-Nemser (2012b) who suggested new teachers really have two jobs: teaching and learning to teach. The novice teachers in this study added a third job: getting to know your students.

Implications

The findings of this study reveal how novice upper-elementary teachers' professional growth develops over time through a series of gradual changes in their knowledge, beliefs, and teaching practices. This study has significant theoretical, empirical, and practical implications for the field of education. Novice teachers, mentor teachers, school administrators, teacher educators, educational researchers, and educational leaders can use the results of this study.

Theoretical Implications

This study combines Ball and associates' theory of mathematical knowledge for teaching (Ball et al., 2008) and Bandura's self-efficacy theory (Bandura, 1977) to guide its investigation into how novice upper-elementary teachers describe their mathematical dispositions. The study revealed several gaps in teachers' mathematical knowledge for teaching. Participants felt they were not adequately prepared to understand all mathematical topics in the curriculum, unpack every mathematical concept and idea, analyze student thinking, and carry out authentic differentiated instruction. The study also revealed the teachers' sustained efforts to bring about changes in themselves and their mathematics classrooms. Regardless of their knowledge, the

participants expressed their determination to understand their students and make a difference in the way they experienced learning mathematics.

The findings of this study shed new light on Ball and associates' (2008) theory of mathematical knowledge for teaching by distinguishing between the mathematical knowledge that is essential before the teacher becomes solely responsible for classroom instruction and the knowledge that can be safely left for the teacher to acquire through their own efforts (Hoover, Mosvold, Ball, & Lai, 2016; Kastberg & Morton, 2014). The participants in this study felt minor gaps in their common content knowledge were easily resolved after teaching the topic a time or two. Deficiencies in specialized content knowledge, however, required significantly more time and often altered the way participants approached teaching mathematical concepts or ideas. If these gaps were left unchecked, memorization could replace understanding as the predominate way of students learn mathematics (Charalambous, Panaoura, & Philippou, 2009). The participants' experiences suggest knowledge of content and students generally precedes knowledge of content and teaching. For them, differentiation was merely a buzzword in education until they gained an understanding of their students and their individual needs.

Other novice teachers can benefit from reading the participants' journeys and reflecting on the mathematical knowledge and personal efficacy beliefs that are needed to become effective teachers of mathematics. School administrators can use this information to provide timely and differentiated professional learning opportunities for novice teachers of mathematics. Finally, mathematics teacher educators can use this information to guide their approach in preparing preservice teachers who are capable of reaching students through their mathematics instruction.

Empirical Implications

McGee and Wang (2014) developed their Self-Efficacy for Teaching Mathematics

Instrument to measure elementary teachers' complex belief systems surrounding mathematics. The validity of survey content was established, and the reliability of survey responses was verified. This study used this self-report survey to identify participants who had different self-efficacy beliefs. Carly reported a low efficacy score for pedagogy in mathematics. Her score ranked in the 12th percentile, assuming an approximately normal distribution of scores.

Behind every statistic is a person's story. Carly's story was heard in this qualitative study. The findings revealed this low self-reported score was inconsistent with the high self-efficacy beliefs that she later described in her audio diary and interviews. In her audio diary, Carly described her experiences teaching a complete unit on fractions. The third-year teacher's techniques were sophisticated. Throughout the unit, Carly incorporated small-group instruction, colored fraction tiles, facilitated exploration phases, cognitive disturbances, mathematical discourse, foreshadowing of content, a Hershey candy bar application, technology, and learning centers. In the interview, she described some of her successful teaching moments. Carly felt a great sense of accomplishment when her one little girl who typically works at the second- to third-grade level showed a fourth-grade student how to add fractions. She also talked about making positive strides with her other PASA "kiddo" by building his confidence in mathematics and closing the significant gap in his achievement. Other researchers in mathematics education may benefit from listening to and reflecting upon stories like Carly's.

Practical Implications

The NCTM (2016) has taken the position that induction programs play a significant role in the development of the next generation of teachers. The findings of this study highlight the importance of having a supportive environment not only for the transition into the teaching profession but also for the continued growth as teacher of mathematics. The participants felt

their teacher education program had not fully prepared them to teach mathematics. They often turned to other mathematics teachers in their building for suggestions with presenting an unfamiliar topic or reaching a struggling student. For the first two years of her career, Carly felt fortunate to have a mathematics coach who provided scaffolding whenever she needed it. After a few years, the novice teachers found themselves collaborating with veteran teachers to improve their teaching practices. Emma, Faith, George, Isabella, and John felt energized after attending professional conferences. George and Helen shared their principals were very supportive of their efforts to try flexible grouping to close the achievement gap in fourth grade. Barbara felt her principal respected her as the fifth-grade mathematics and science teacher and trusted her decision to implement small-group instruction into her mathematics classes. Her self-efficacy beliefs in this departmentalized setting validate the work of Haley (2018). The participants in this study believed any shift toward a more student-centered approach would not have been possible without the support of others.

The distinguished teacher educator and scholar, Feiman-Nemser (2012a) examined the evolving role of induction programs considering recent standards-based reforms in education. She concluded induction should be viewed as "a process of incorporating new teachers into collaborative professional learning communities" (p. 12) rather than as a temporary bridge designed to ease new teachers' entry into their roles. Educational leaders can use this information to justify implementing comprehensive induction programs for novice elementary teachers.

Delimitations and Limitations

Delimitations are the purposeful decisions made by the researcher that make the study feasible but limit the generalizability of its results (Joyner, Rouse, & Glatthorn, 2013). For the

purpose of this study, the participants were bound by elementary teachers who had successfully fulfilled all their education degree requirements at higher institutions in Pennsylvania. These elementary teachers were in their first five years of teaching a conceptually-based mathematics curriculum. The rationale for selecting such elementary teachers was to ensure participants had shared experiences (van Manen, 1990) not only in training but also in teaching elementary mathematics content for conceptual understanding. The participants were further bound by fourth- through sixth-grade mathematics teachers. The rationale for delimiting the elementary teachers to grades 4 to 6 was to focus on their affective experiences while trying to conceptually teach the notoriously difficult concepts involving fractions (van Steenbrugge, Lesage, Valcke, & Desoete, 2014). Although this decision resulted in a deep, rich, and thick description of upperelementary teachers' beliefs about mathematics teaching and learning, the results may not be generalizable to seventh- and eighth-grade teachers or early childhood elementary teachers. Finally, the setting was bound by the states of Ohio, Pennsylvania, and New York because they fully implemented the Common Core State Standards during the 2013-14 school year (CCSSI, 2016). The experiences of elementary teachers may be different in states that implemented the national standards during other school years or decided not to adopt the national standards at all.

Limitations are factors that the researcher has little or no control over that may affect the application and interpretation of the results of a study (Joyner et al., 2013). Although the sample of 10 participants represented varying ages, certifications held, teaching assignments, and years in education, there was less diversity in their ethnicity and school location. The experiences of non-Caucasian teachers and those teaching in urban school settings were not explored and may be different. Other limitations of the study were related to its design and analysis. Using a qualitative approach, the researcher assumed all participants were truthful in their responses to

individual interview questions and participated in audio diary reflections and focus-group discussions to the best of their abilities (Baron, n.d.). Despite the researcher's efforts to put aside any preconceived thoughts, judgments, and biases to be more receptive to the views reported by the participants, perfect epoche was most likely not achieved (Moustakas, 1994). An inherent limitation when analyzing data in a transcendental phenomenology was articulated by Moustakas (1994):

The essences of any experiences are never totally exhausted. The fundamental textural-structural synthesis represents the essences at a particular time and place from the vantage point of an individual researcher following an exhaustive imaginative and reflective study of the phenomenon. (p. 100)

In this study, data collection occurred in the latter half of the school year when the teachers were either preparing for state assessments or wrapping up their curricula. Their teaching experiences at the beginning of the school year may have provided a different perspective.

Recommendations for Future Research

The findings of this study suggest several directions for future research. Many of the participants attributed their mathematics teaching efficacy to the support they received from colleagues and administrators. Case studies should examine how faculties' beliefs in their collective efficacy contribute to their school's level of mathematical achievement (Bandura, 1993). Even though the participants experienced lectures, drill and practice, and memorization in their high school mathematics classes, they wanted their students' learning experiences to be interactive, engaging, and meaningful. Given this shared experience, the belief systems of secondary mathematics teachers should be examined. Phenomenological studies could explore the dispositions of middle- and high-school mathematics teachers.

Recommendations for future research are based on the delimitations and limitations of this study. This study focused on upper-elementary teachers who were using the Common Core State Standards for Mathematics in the schools in New York, Ohio, and Pennsylvania. Future research may target other grade levels, standards, and geographic areas. Teachers' experiences may vary based on the mathematical content, rigor of curriculum, and cultural features. The participants in this study were all Caucasian and taught in either suburban or rural schools. Additional research is needed regarding the mathematical dispositions of minority elementary teachers in urban schools. The participants completed their audio diary reflections and individual interviews near the end of the school year. This study could be replicated closer to the beginning of the school year before the teachers formed relationships with their students. The findings of this study suggest the teachers may feel less efficacious in their teaching ability until they get to know their students.

Summary

Since the late 1990s, disposition has been an integral part of the discourse in teacher education (Dietz & Murrell, 2010). Researchers in teacher education have studied ways to help preservice elementary teachers develop a productive mathematical disposition. Productive disposition has been characterized as an integral part of mathematical proficiency (NRC, 2001) and the mindset of teachers needed to bring about genuine reforms in mathematics education (Campbell et al., 2014; Clark et al., 2014; NCTM, 2016). Much of what is known about teachers' mathematical dispositions came from a quantitative perspective with an emphasis on preservice teachers. This study was different because it used a qualitative approach to examine the mathematical dispositions of elementary teachers at the onset of their careers. Listening to the voices of novice upper-elementary teachers as they described their everyday experiences

teaching mathematics provided greater insight into their mathematical dispositions. Using transcendental phenomenological reduction, three themes emerged from this research: *Life Changing Decisions, Connections with Students*, and *Rethinking Mathematics Class*.

Regardless of their initial knowledge base, the novice upper-elementary teachers were determined to understand their students and make a difference in the way they experienced learning mathematics. Trends in their intentional actions and behaviors were indicative of productive disposition. The fact that many of the participants did not consider themselves to be mathematics people until after they started teaching mathematics was a significant take-away from this study. Their experiences may suggest a realistic progression and approximate time frame for elementary teachers to sufficiently develop the four domains of the mathematical knowledge for teaching (Ball et al., 2008). The findings of this study show the type of school environment that is needed not only for a smooth transition into the teaching profession but also for continued growth as teacher of mathematics. A logical next step might be to examine how teachers' beliefs in their collective efficacy contribute to their school's level of mathematical achievement.

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APPENDICES

Appendix A: Liberty University's IRB Approval

LIBERTY UNIVERSITY. INSTITUTIONAL REVIEW BOARD

September 20, 2017

Leslie Soltis

IRB Approval 2971.092017: Mathematical Dispositions of Novice Upper-Elementary Teachers: A Phenomenological Study

Dear Leslie Soltis,

We are pleased to inform you that your study has been approved by the Liberty University IRB. This approval is extended to you for one year from the date provided above with your protocol number. If data collection proceeds past one year, or if you make changes in the methodology as it pertains to human subjects, you must submit an appropriate update form to the IRB. The forms for these cases were attached to your approval email.

Thank you for your cooperation with the IRB, and we wish you well with your research project.

Sincerely,

G. Michele Baker, MA, CIP Administrative Chair of Institutional Research The Graduate School



Liberty University | Training Champions for Christ since 1971

Appendix B: Letter Requesting Expert Feedback on Interview Questions

Dear Professor,

I am a graduate student in the School of Education at Liberty University who is

conducting research in partial fulfillment of my doctoral degree in curriculum and instruction.

My research project is titled Mathematical dispositions of novice upper-elementary teachers: A

phenomenological study. The purpose of my research is to describe the mathematical

dispositions of novice upper-level elementary teachers by exploring their experiences as teachers

of mathematics in western Pennsylvania.

I am writing this request to ask if you would be available to review the audio diary

prompt, interview questions, and focus-group prompts that I have developed for use in my study.

It will take approximately one hour of your time to review my wording and provide any

suggestions on ways to improve their readability. Thank you for considering my request.

Respectfully,

Leslie Soltis

Doctoral Candidate

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Appendix C: Letter Requesting Elementary Teacher Participation in Pilot Study

Dear Elementary Teacher,

I am a graduate student in the School of Education at Liberty University. As part of the

requirements of my doctoral degree in curriculum and instruction, I am conducting research on

elementary teachers. The purpose of my research is to describe the mathematical dispositions of

novice upper-level elementary teachers by exploring their experiences as teachers of

mathematics in western Pennsylvania.

I am writing this letter to ask if you would be available to participate in a brief pilot study

to ensure that my audio diary instructions, interview questions, and focus-group prompts are

clearly understandable to elementary teachers. It will take less than one hour of your time to

review my wording and provide any feedback on any words or phrases that you found to be

vague or confusing. Thank you for considering my request. I look forward to hearing from you.

Respectfully,

Leslie Soltis

Doctoral Candidate

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Appendix D: Letter Requesting List of Qualifying Participants

Dear Director of Graduate Records,

I am a graduate student in the School of Education at Liberty University who is

conducting research in partial fulfillment of my doctoral degree in curriculum and instruction.

My research project is titled Mathematical dispositions of novice upper-elementary teachers: A

phenomenological study. This study will explore the mathematical dispositions of elementary

teachers in light of recent changes in the mathematics curricula adopted by elementary schools

and teacher education programs.

I am writing to request a list of potential participants who graduated from your institution

and are currently teaching fourth through sixth grades with less than five years' teaching

experience. It would be helpful if you could provide their home mailing addresses so that I can

contact them about participating in my study. I attached a copy of the consent form for you to

review before you make your decision. Thank you for considering my request. I look forward to

hearing from you.

Respectfully,

Leslie Soltis

Doctoral Candidate

Appendix E: Letter Inviting Qualified Participants



Dear Teacher:

As a graduate student in the School of Education at Liberty University, I am conducting research as part of the requirements for a doctoral degree. The purpose of my research is to explore new teachers' experiences as teachers of mathematics and gain an in-depth understanding of their mathematical dispositions. I am writing to invite you to participate in my study.

If you are fourth through sixth grade teacher, have less than five years' teaching experience, currently teach mathematics in New York, Ohio, or Pennsylvania, and are willing to participate, then you will be asked to take a mathematics teaching self-efficacy survey, maintain an audio diary, and participate in an individual interview and online focus-group interview. It should take approximately three hours for you to complete the procedures listed. Your name, school, and demographics will be requested as part of your participation, but the information will remain confidential.

To participate, please complete and return the attached consent document to me in the self-addressed, stamped envelope. The consent document contains additional information about my research.

If you choose to participate, you will receive a \$100 stipend.

Sincerely,

Leslie Soltis Doctoral Candidate

Appendix F: Participant Consent Form

CONSENT FORM

Mathematical Dispositions of Novice Upper-Elementary Teachers:

A Phenomenological Study

Leslie Soltis

Liberty University

School of Education: Curriculum and Instruction

You are invited to be in a research study of upper-elementary teachers' experiences as teachers of mathematics. You were selected as a possible participant based on your recent entry into the teaching profession and undergraduate training experiences. Please read this form and ask any questions you may have before agreeing to be in the study.

Leslie Soltis, a doctoral candidate in the School of Education at Liberty University, is conducting this study.

Background Information: The purpose of this study is to explore the experiences of novice upper-elementary teachers as teachers of mathematics and describe the tendencies in their actions and behaviors related to teaching elementary-level mathematics.

Procedures: If you agree to be in this study, I would ask you to do the following things:

- 1. Complete the Self-Efficacy for Teaching Mathematics Instrument detailing the overall confidence elementary teachers have for teaching mathematics content. The survey should take approximately 10 minutes.
- 2. Maintain an audio diary by reflecting upon your experiences teaching mathematics at least three times per week for a 3-week period.
- 3. Participate in an interview in which you will explain orally your experiences as a learner and teacher of mathematics. The interview will last about an hour and be audio recorded for transcription purposes.
- 4. Participate in a 1-hour online focus-group interview in which you will explain, in writing with your colleagues, the experiences that have shaped you as a teacher of mathematics.
- 5. Review my written account of your experiences and provide feedback on its accuracy, which should take less than 30 minutes.

Risks and Benefits of Participation: The risks involved in this study are minimal, which means they are equal to the risks you would encounter in everyday life. There are no direct benefits to participating in this study. Benefits to society include contributing to educational research and improving mathematics education.

Compensation: At the conclusion of the study, you will receive a \$100 stipend for your time and effort. If you choose not to complete the study, you will be compensated proportionate to the amount of time involved in the study.

Confidentiality: The records of this study will be kept private. In any sort of report that I might publish, I will not include any information that will make it possible to identify a subject. Research records will be stored securely, and only the researcher will have access to the records. I may share the data I collect from you for use in future research studies or with other researchers; if I share the data that I collect about you, I will remove any information that could identify you before I share the data. I will assign pseudonyms and conduct interviews in study rooms at a local library to protect participant privacy. Data will be stored on a password-locked computer and may be used in future presentations. After three years, all electronic records will be deleted. Interviews will be recorded and transcribed, audio diaries will be transcribed, and focus-group interviews will be downloaded. Files will be stored on a password-locked computer for three years and then erased. Only the researcher and faculty advisor will have access to these files. I cannot assure participants that other members of the focus group will not share what was discussed with persons outside of the group.

Voluntary Nature of the Study: Participation in this study is voluntary. Your decision whether or not to participate will not affect your current or future relations with Liberty University or Edinboro/Indiana University of Pennsylvania. If you decide to participate, you are free to not answer any question or withdraw at any time without affecting those relationships.

How to Withdraw from the Study: If you choose to withdraw from the study, please contact the researcher at the email address/phone number included in the next paragraph. Should you choose to withdraw, data collected from you, apart from focus-group data, will be destroyed immediately and will not be included in this study. Focus-group data will not be destroyed but your contributions to the focus group will not be included in the study if you choose to withdraw.

Contacts and Questions: The researcher conducting this study is Leslie Soltis. You may ask

	f you have questions later, you are encouraged to contact her at				
or	You may also contact the researcher's faculty advisor, Dr.If you have any questions or concerns regarding this				
Sandra Battige at	. If you have any questions or concerns regarding this				
	someone other than the researcher, you are encouraged to				
contact the Institutional Review Board, 1971 University Blvd, Green Hall Suite 1887,					
Lynchburg, VA 24515 or email at irb@liberty.edu.					
Please notify the researcher if	you would like a copy of this information for your records.				
	read and understood the above information. I have asked swers. I consent to participate in the study.				
•	PARTICIPATE UNLESS IRB APPROVAL INFORMATION ATES HAS BEEN ADDED TO THIS DOCUMENT.				
☐ The researcher has my perm	nission to audio-record me as part of my participation in the study.				
Signature:	Date:				

Date:

Signature of Investigator:

Appendix G: Self-Efficacy for Teaching Mathematics Instrument (SETMI)

Removed to comply with copyright.

Appendix H: Approval to Use SEMTI



College of Education Department of Curriculum and Instruction 828-262-2270 FAX 828-262-3686

You have my permission to use the Self Efficacy for Teaching Mathematics Instrument in your research. Please reference the validity information and scoring guide when publishing your findings.

Sincerely,

Jennifer R. McGee, Ed.D. College of Education Appalachian State University 151 College St. Boone, NC 28608 Phone: (828) 262-2270

Phone: (828) 262-2270 Fax: (828) 262-2686

Appendix I: Script for Audio Diary

Audio Diary

To explore your experiences teaching mathematics and to capture your thoughts, beliefs, and emotions in real time, please maintain an audio diary using the following guidelines:

- Throughout the week, you will record at least three audio diary entries using an electronic recording device.
- o There are no time restrictions on the length of your audio diary entries.
- Respond to the given prompts:
 - What did you intend to do in your mathematics lesson today?
 - What did you actually do in today's mathematics lesson?
 - If you could change anything about today's lesson what would you change?
- Talk about <u>you</u>; only talk about others when it is needed to describe your own experiences.
- Describe your experiences as you lived through them, paying attention to the feelings, the mood, the emotions, etc. (Note: There is no need to provide explanations for your state of mind.)
- At the end of each week, you will submit your audio files via email to lsoltis@liberty.edu. You will do this for three consecutive weeks.

Appendix J: Instructions for Participating in Online Focus Groups

Participant Instructions

Thank you for agreeing to participate in an online chat discussion. Below are some basic instructions on how to be a participant in an *itracks Chat* focus group.

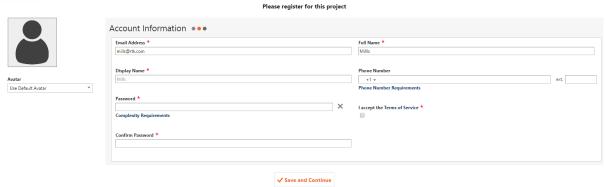
YOUR EMAIL INVITATION •••

Prior to the group start date you will receive an email with information on how to register for the group and instructions for the group. Please read the instructions fully as they will inform you of the length of the project, project start time and tech support information.

A registration link will be present in your invitation. This registration link allows you to set up your login details for the project. Your email address and Project ID are also included as these will be part of the login process after registration.

REGISTRATION AND LOGIN •••

When clicking on the link from your email you will be taken to the registration page and asked to register for the project. It is at this point you will need to create your password. You will also be required to agree to the Terms of Service. In some cases, you may also have profile questions to fill in.



Creating your Password

Your password can be anything that you would like it to be. It is best if you remember this password as it will be the password that you need to enter when logging in additional times. There are some complexity requirements here. They include: **Passwords must be at least 8 characters long. Passwords must contain at least one upper case and one lower case letter.**

When a password meets the requirements on the checkbox to the left tick mark will appear on the password area.



Terms of Service

The terms of service must be agreed to in order to register. To read the Terms of Service, click on the link in the account information area to accept the Terms of Service, Check the 'I accept the Terms of Service' checkbox.



Profile Information

In some cases, there will be additional profile information for you to fill in on this page. In some cases, these fields will also be required to register/begin the group. Required fields are denoted with a red star.

PASSWORD RETRIEVAL

If you forget your password, you can easily retrieve it off of the login page. The login page will appear instead of the registration page when you click on the registration link once you have registered. To retrieve your password, click on the 'I Can't Access My Account' link. Then you will be taken to a page asking for your email address and Project ID. (The project ID can be found in your invite).



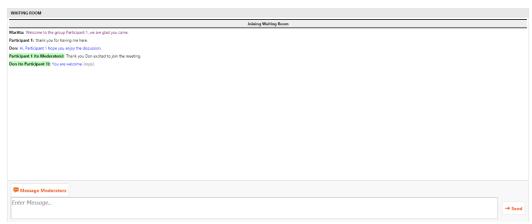
Once the details are entered, click the **Submit** button to send the password reset link to your email. Click the password reset link to create a new password to access the group with.

ARRIVING EARLY

If you register prior to 30 minutes before your group start time, you will be taken to a landing page indicating that you have no activities to view at this time. You can log out and log back in 30 minutes before your group start time.

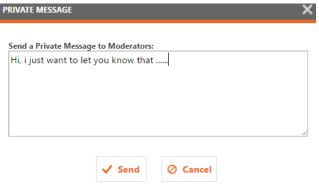
IN THE FOCUS GROUP •••

If you log in 30 minutes or less prior to the time of your group, you will automatically be taken into the Waiting Room of the focus group. In the waiting room, you will be greeted by tech support and will be able to see the chat of others that may be in the group already.



To chat into the group yourself, type your text into the textbox at the bottom of the screen where it says *Enter Message*... Hit the **Send** button to send your text into the chat area.

To private message the moderator, click the Message Moderator button at the bottom of the screen.



You may notice a few different colors of text. Blue text is from the moderator, purple text is from technical support staff and the black text is from other participants. If you see a green highlight around the name in a message this indicates private messages sent to you or by you. Click the 'reply' link after a private message to reply back to private messages.

Once the group time arrives, the moderator will bring the people they select into the Main Room to begin the discussion.

HOW TO ANSWER QUESTIONS ● ●

Moderator text / questions will appear in blue in the chat area. To answer most questions, you will simply need to type your answer into the textbox below the chat area and hit send to send the answer into the discussion.

The moderator may also ask single choice and multiple-choice questions. In these cases, make your selection and click the **Submit** button when you are done.



VISUALS AND LINKS •••

If the moderator decides to show pictures, videos or web links these will appear in an area that will automatically appear to the right of the chat area. The chat area will shrink so you can focus more on the visuals or links the moderator would like to show.

TECH SUPPORT

Tech support can be contacted via the **Message your Moderator** button in the group, by clicking on the **Live Chat** link in the upper right corner of your screen, by email at via phone at .

Appendix K: Individual Interview Questions

- 1. Please describe a typical day teaching.
- 2. Let's think back to your K-12 mathematics education experiences. What memorable experiences stand out to you?
- 3. How would you compare studying mathematics and studying other subjects in school? (Grouws, Howald, & Colangelo, 1996)
- 4. How do you feel about yourself as a doer and learner of mathematics?
- 5. Who was your best mathematics teacher? What traits made him/her a good mathematics teacher?
- 6. Let's move on to when you were training to become a teacher. What specific experiences helped prepare you to teach elementary-level mathematics? What useful things did you take away from your mathematics courses, mathematics methods course, and field experiences?
- 7. Let's turn to your teaching practices. What features of mathematics instruction do you value and use regularly in your mathematics instruction?
- 8. What personal experiences have shaped the way you teach mathematics to your elementary students?
- 9. Please tell me about one of your successful mathematics lessons. What contributed to its success? What feelings were generated by this experience?
- 10. Please tell me about one of your not-so-successful mathematics lessons. What thoughts were generated following this experience? What can you do differently the next time you teach this lesson?

- 11. Please describe one of your students who is good at mathematics. What reasons do you give for the student being good at mathematics? (Feldhaus, 2012)
- 12. Describe how confident you feel in your ability to carry out the everyday tasks of teaching elementary-level mathematics.
- 13. What challenges have you experienced during the early stages of your career teaching elementary-level mathematics?
- 14. What types of support have you experienced as a new teacher of elementary-level mathematics?
- 15. Have you shared all that you feel is significant with reference to teaching mathematics?

Appendix L: Email Invitation for Online Focus Groups

Dear [FULLNAME]

Thank you for agreeing to participate in the upcoming Chat focus group on [STARTDATE].

Registration and Login

You can register for this study here [LOGINURL].

Registration creates your password and creates your user in the research project. You will need this password to log into the research at the time of your focus group.

If you have already registered, click on the registration/login link above and enter in the following:

Email Address: [USERNAME]

Password: (You will be asked to create a password when you log in for the first time.)

Project ID: [PROJECTSUPPORTID]

If you have forgotten your password you can retrieve it by clicking on the "I can't access my account" link on the login page and entering in your email address and Project ID. Upon doing this, you will be sent a password reset link to your email inbox.

About this Study

This focus group is set to start at **[STARTDATE]** and is scheduled for one hour. Please be sure to allocate enough time in your schedule to accommodate the entire session. Once all participants log in, your group Moderator will begin the Chat. Please respond to the questions the moderator posts first and then feel free to reply to other participant's responses.

Thank you and I look forward to your participation!

Leslie Soltis

Need Further Assistance?

If you have any questions or problems entering the discussion, please email or call Technical Support toll free at and select option 2, Monday through Friday. During the weekend please call

Appendix M: Online Focus-Group Prompts

- 1. To begin the discussion, let's consider the best mathematics course you have experienced in your K-12 education. What specific aspects of the course made it positive for you?
- Next, let's look back on your teacher education program. Please tell me about your
 preservice mathematics content courses. Provide as much detail as you think is necessary
 to give a clear idea of the courses.
- 3. In what ways have your feelings about learning mathematics changed as a result of participating in these courses? In what ways did your preservice mathematics content courses prepare you for success as a teacher of mathematics?
- 4. Finally, let's turn to your current experiences teaching mathematics. Describe your experiences in carrying out the everyday tasks of teaching elementary-level mathematics (i.e. identifying student errors, engaging in error analyses, applying procedural knowledge, communicating conceptual understandings, unpacking mathematical concepts, anticipating student interpretations of mathematical tasks, analyzing students' mathematical thinking, understanding common student conceptions and misconceptions, and designing and evaluating appropriate mathematics instruction). Provide an example so we can understand how you think about helping students learn mathematics.

Appendix N: Email Reminding Participants of Online Focus Groups

Dear [FULLNAME]:

This is just a friendly reminder that the *itracks Chat* you agreed to participate in takes place tomorrow. You may start logging in 30 minutes prior to your scheduled meeting time.

I have reattached your original invitation below so that your login information is right at your fingertips.

I look forward to hearing your thoughts on our discussion topics!

Leslie Soltis

Your original invitation is below

Thank you for agreeing to participate in our *itracks Chat*. Detailed in this email are some general instructions followed by specific information needed to access the group. If you have any questions or problems entering the discussion, please reply to this email, or call Technical Support toll free at and select option 2 Monday through Friday.

Below you'll find your Display Name, Email Address, Project ID and the Login Page URL. You must enter your Email Address and Project ID exactly as shown to access the group. Before the Chat starts, I strongly encourage you take a few minutes to test the link below to ensure the login procedure works correctly for you. When you log in for the first time, you will be asked to create a password. Be sure to write it down so that you do not forget it, because from that point onward you will be prompted to enter your password to log in.

The **group is scheduled for one hour**, so please be sure to allocate enough time in your schedule to accommodate the entire session. Once all participants log in, your group Moderator will begin the Chat. Please respond to the questions the moderator posts first and then feel free to reply to other participant's responses.

Thank you and I look forward to your participation!

Leslie Soltis

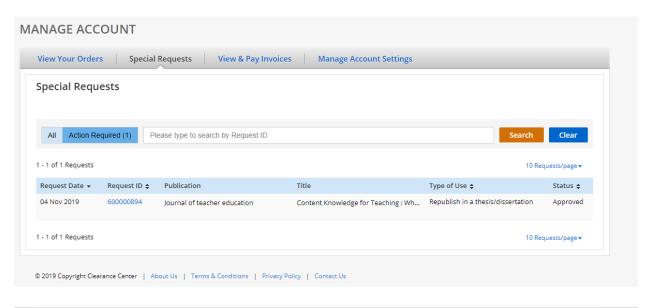
Project Date & Time: **STARTDATE**] Email Address: [**USERNAME**]

Password: (You will be asked to create a password when you log in for the first time.)

Project ID: [PROJECTSUPPORTID]
Login Page URL: [LOGINURL]

If you have any questions regarding this, please feel free to reply to this email.

Appendix O: Permissions from Copyright Holders





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