Master of Information Technology (Research)

Thesis

"Toward a Scientific Taxonomy of Musical Styles"

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Course: IT60

Date: 21st July, 2006

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Abstract

The original aim of the research was to investigate the conceptual dimensions of style in tonal music in order to provide grounds for an objective, measurable categorization of the phenomenon that could be construed as the basis of a scientific taxonomy of musical styles. However, this is a formidable task that surpasses the practical possibilities of the project, which would hence concentrate on creating the tools that would be needed for the following stage.

A review of previous attempts to deal with style in music provided a number of guidelines for the process of dealing with the material. The project intends to avoid the subjectivity of musical analysis concentrating on music observable features. A database of 250 keyboard scores in MusicXML format was built to the purpose of covering the whole span of styles in tonal music, from which it should be possible to extract features to be used in style categorization. Early on, it became apparent that most meaningful pitch-related features are linked to scale degrees, thus essentially depending on functional labeling, requiring the knowledge of the key of the music as a point function.

Different proposed alternatives to determine the key were considered and a method decided upon. Software was written and its effectiveness tested. The method proved successful in determining the instant key with as much precision as feasible. On this basis, it became possible to functionally label scale degrees and chords. This software constitutes the basic tool for the extraction of pitch-related features. As its first use, the software was applied to the score database in order to quantify the usage of scale degrees and chords. The results indisputably showed that tonal music can be characterized by specific proportions in the use of the different scale degrees, whereas the use of chords shows a constant increase in chromaticism.

Part of the material of this work appeared in the Springer-Verlag's 2006 volume of Lecture Notes in Computer Science.

Keywords: style, stylometry, MusicXML, key-determination, algorithm, dot product, scale degree, chord, functional labeling.

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Statement of Authorship

The work contained in this thesis has not been previously submitted to meet requirements of an award at this or any other education institution. To the best of my knowledge, the thesis contains no material previously published or written by another person except where due reference is made.

Signature Date:

Acknowledgement

I wish to thank my Principal Supervisor Dr. Joaquin Sitte for his patience and his open-mindedness; Ray Duplock, for his continued support and help with statistical matters; and very specially, to Raj Singh, who opened BlackBox for me, and devoted a lot of time to solving the problems of input-output in this language, which made programming in it possible for me.

1. Introduction

1.1 Origin of the Project

The long-term motivation for this work is the total inexistence of tools for measuring musical style in an objective, scientific manner. For the outsider it would seem hard to believe that the 20th century had ended without any progress in music attributional studies beyond the mere opinion of the experts.

Apart from the dating of manuscripts or of contemporary copies, the possible recognition of the musical calligraphy and similar devices of history scholars, attribution in music is left to the opinion of musicians, often composers, whose lack of objectivity and scientific attitude is notorious. Often an ancient work of dubious or unknown composer turns up, and the specialists are quick to attribute it to their favorite old master, even if they can only count with the aid of the flimsiest evidence. The process is riddled with emotion and wishful thinking, and understandably has led to a long list of famous misattributions among which probably the most well known are:

U.W. van Wassenaer's "Concerti Armonici" formerly attributed to Handel and later to Pergolesi;

Bernhard Flies' Wiegenlied, for long time attributed to Mozart;

Leopold Mozart's "Toy Symphony" first attributed to Haydn;

Friedrich Witt's "Jena Symphony" first thought to be early Beethoven. In the 1960s, a supposed Fifth Orchestral Suite by J.S. Bach made its way to recordings but has since vanished without a trace.

A more recent example is the inclusion in the Searle official catalog of the works of Liszt, under the number S.715, of a piece for piano and orchestra that for almost a century, had been attributed to its most likely composer, the pianist and composer Sophie Menter. This was done entirely on the basis of the presence of superficial Liszt-like piano mannerisms in the piece – not surprising in a piece by a Liszt's disciple whom he had described as "my only legitimate piano daughter" –, plus pure speculation to "explain" why Menter had asked Tchaikovsky to orchestrate it and conduct it at the premiere without ever revealing that the piece was Liszt's. More importantly, such attribution disregarded the fact that the style of Liszt at the time the work was written, had practically nothing in common with the early Lisztian style the piece supposedly resembles. However, when the work made its first appearance on records in 1982, Maurice Hinson, editor of the American Liszt Society wrote for the liner notes:

> This Concerto in the Hungarian Style contains plenty of Lisztian characteristics and any pianist that had played Liszt will find no difficulty in detecting them and assigning the composition to Liszt [...] Furthermore, listening to the work provides the definitive proof. I played this recording for a professional pianist who was unaware of the story. He exclaimed: "Liszt!" before reaching the opening cadenza. Listening to the complete work confirmed the initial verdict.

This paragraph serves as a perfect example to demonstrate the status of attributional studies in music, in which biased "expert" opinions are averred even if the supporting evidence is lacking. Needless to say, the latest edition of the New Grove (2000) does not list the piece as Liszt's but Menter's.

Understandably, given the lack of authenticity checking, in the history of music there had been a considerable number of deliberate forgeries, in which a composer presented a work of his own, pretending to have found the manuscript of a hitherto unknown work by a master of the past. This was the case with Marius Casadesus' "Adelaide" violin concerto, attributed to Mozart, or the various pieces by Fritz Kreisler that he attributed to Vivaldi, Couperin, Pugnani, Dittersdorf, Francoeur, Stamitz and others.

As a last example, an interesting and rather extreme case of doubtful authorship is the most famous of organ pieces, J.S.Bach's Toccata and Fugue in D minor BWV 565, which music lovers refer to as *The* Toccata and Fugue. Although the public view it as quintessential Bach, there is no manuscript and the oldest extant copies date from Mozart's time. Peter Williams, in his notes for Peter Hurford's recording for Argo stated:

> ...questions now beset it: […] Was it the work of Bach at all? […] Familiar though the work is, it does contain many touches very untypical of Bach and his period. Indeed, some of the most distinctive features are problematic. For example, where else in the music of Bach is there a minor plagal cadence at the close? Is it not odd to have a solo pedal entry in a fugue? What other organ piece begins in octaves? And are such simple effects as the dramatic diminished sevenths really characteristic of its supposed composer?

All these objections are stylistic in nature, and major ones at that. More recently, a strong case has been made for this composition to be originally a solo violin piece, and Bach's authorship has become even more doubtful.

In general, when scholars analyze the style of a composer they refer to one aspect at the time, e.g. their repeated use of particular devices, such as certain sequences, harmonic progressions, falling melodies, rhythmic combinations or formal preferences. Such considerations often offer important insights into the composer style, but it is apparent that observations of this sort are not quantitative in nature.

In "Numerical Methods of Comparing Musical Styles", F.Crane and J. Fiehler state: "A musical style is so complex an organism that common- sense methods can hardly deal with it, except one element at the time" (Crane & Fiehler, 1970). This is true about practically all of the stylistic observations of scholars. Moreover, there is no standardized systematic approach that would allow for comparative studies, such as trying to determine the differences between two composers of similar style like Mozart and Haydn. In this computer age, there is a clear need of a rational tool for dealing with attributional studies and chronological problems in music.

To put it briefly, in musicology there are no objective tools to measure style and their absence creates serious problems for authorship studies.

1.2 Feasibility

There is an a-priori question: Is it possible to characterize musical style in a quantitative way?

In order to answer, the starting point should be the awareness that, when someone is conversant enough with the music of the common practice period, the audition of just a short fragment of an unknown piece – 'short' meaning often less than one minute long – provides grounds to establish with good accuracy the approximate date of composition of the piece, and – with favorable circumstances –, additional details about it such as the nationality of the composer. Undoubtedly, the stylistic elements allowing for such process must be present in the music, even if they have not been explicitly disentangled. At this level, attribution could be viewed as a series of successive approximations. There are large-scale elements that tell the specialist in a matter of seconds which of the major periods of music history the piece being heard belongs to. The following steps are in search for finer details that would narrow down the possibilities.

1.3 The nature of the solution

Assuming it is possible to create a rational tool to measure styles, what would this rational tool be? In the article "Computers and Music" for the first edition of The New Grove, Michael Kassler and Hubert S. Howe (jr.) wrote:

> To appreciate the importance of explicating musical and musicological processes as algorithms, consider that having an algorithm that verified or falsified the statement '*x* is in the style of Beethoven' for any given composition '*x*', would be equivalent to understanding the style of Beethoven so well that one could direct a machine to recognize compositions written in this style. Hence, if one's understanding of this style is insufficient to achieve algorithmic explication, one's knowledge of the style is less certain than it might be. (Kassler & Howe, 1980).

The authors clearly recognized the desirability of the existence of such algorithm, although their position is a bit extreme. In light of the success of literary stylometry, which certainly does not claim to understand the style of the authors whose writing it recognizes, their claim of equivalence between "understanding" and "recognizing" an author's style is unsubstantiated. It is apparent that style understanding is a sufficient but not necessary condition for style recognition. It would be possible to come to a more restricted view of what this algorithm needs to be. It is convenient to start by referring to literary stylometry in order to clarify similarities and differences.

1.4 Literary stylometry

1.4.1 Points of contact with the Music style problem

In contrast with the sad state of affairs in music, literary stylometry has existed and bloomed for some forty years. Popular science magazines and TV programs have reported the most spectacular cases in which a computer program has reliably ratified or rejected an attribution, such as the case of the poem "Shall I die" attributed to Shakespeare, or the solution of the two-century old controversy about the author of The Federalist Papers. Text style analysis is no longer a matter of subjective opinion. Computers have made possible to establish numerical criteria to assign probabilities to particular authors.

David Holmes in "The Evolution of Stylometry in Humanities Scholarship" (1998) has given a concise coverage of forty years of studies in the area. He began stating that at the heart of stylometry "lies an assumption that authors have an unconscious aspect to their style, an aspect which cannot consciously be manipulated but which possesses features which are quantifiable and which may be distinctive."

"The historical development of stylometry", Holmes wrote, "is reflected in the choice of quantifiable features used as authorial discriminators". Those tried have been, successively, word length, sentence length, 'Yule's characteristic K' (a measure of word frequencies based on Zipf's law), all found to be not reliable, until the breakthrough in 1964 by Mosteller and Wallace, who used frequencies of function words – such as conjunctions, prepositions, and articles – an approach that is still valid. Burrows, between 1987 and 1992 established the method that “has now become the standard port-of-call for attributional problems in stylometry” by applying multivariate statistics to the same features, “indicating that the way in which authors use large sets of common function words such as ‘by’, ‘the’, ‘from’, ‘to’, etc, appears to be distinctive. He had tapped into that subconscious usage of words for which, at the lexical level, stylometrists had been searching for effective quantifiable descriptive measures.

Holmes stated that the use of multivariate methods was well established in stylometry, mentioning studies in the 1990s that use cluster analysis, principal components, discriminant analysis and correspondence analysis. Simultaneously, since stylometry can be construed as a problem of pattern recognition, there has been an influx of methods from artificial intelligence, beginning with neural networks in two papers from 1993 and 1994 and genetic algorithms in 1995. He concluded that “the role of artificial intelligence techniques in stylometry seems one of vast potential. They appear to be excellent classifiers and require fewer input variables than standard statistical techniques”. As for the future, he said, “we can expect expansion in the use of automated pattern recognition techniques such as neural networks, to act as tools in the resolution of outstanding authorship disputes”. He also mentioned the then recent introduction of content analysis as a stylometric tool and the exciting prospect of the “transition from lexically based stylometric techniques to syntactically based ones”. In this respect is worth mentioning the contribution of Cynthia Whissell, a proponent of “emotional stylometry” touted as “a new stylometric technique – one which adds some degree of meaning to word-counting analyses” (Whissell, 1997). She argues that

> a combination of stylometric measures with emotional measures provides an improved method of text description which comes closer to representing the complexity of critical commentaries that describe authors’ styles than do techniques which do not quantify emotion.

The technique, pioneered by Osgood (1969), considers two dimensions that explain about 80% of the variance in semantic differential ratios.

It must be apparent that music stylometry faces similar problems and, a priori, a great number of ideas from literary stylometry could be directly applicable to music, beginning with the application of multivariate techniques. The initial assumption by Holmes quoted above, “that authors have an unconscious aspect to their style, an aspect which cannot consciously be manipulated but which possesses features which are quantifiable and which may be distinctive” is at least equally reasonable if not more in music than in language since for a certain musical composer, given the range of available choices, these are freely determined according to personal preferences rather than constrained by semantics.

1.4.2 Differences between music and literature

But the translation of stylometric methods to music is not straightforward. Language and music occur along time, and both seem to consist of phrases and paragraphs. But, in spite of this superficial similarity, music and language are radically diverse. In his insightful article on musical style for the New Grove, Robert Pascall states that language is essentially oriented towards meaning whereas “music is oriented toward relationships rather than meaning” (Pascall, 2001). A language is a system of symbols, words, that stand for objects, actions and qualities referred to as articulated by grammar, whereas musicologists repeatedly have warned that “music is not a language, there is no grammar of music” (Roger Lustig, 1990). “Music cannot be treated as a symbol system. It is unreasonable to inquire about the meaning of music” [...] “The analogy to language is entirely false. Musical syntax does not at all function in the same way as linguistic syntax. Music has no semantics.” (Eliot Handelman, 1990).

In language, the meaning is something lying behind the words, but music is itself the message. If it conveys a meaning, that is none other than the mutual relationships of the sounds. “The pitches and durations that define the style of a composition also constitute its content” (Gustafson, 1986). R. Pascall, in the aforementioned article, states:

> There is no consistent natural meaning in music in relation to natural events, and there is no specific arbitrary meaning as in language. The meaning in music comes from arbitrary order evolved into inherited logic and developed dynamically.

Were a composer asked about the meaning of his music, he could only reply: “I only mean what I am expressing, i.e. music”.

This should have been always clear but it has been muddled by some composers who were so immersed in their subjectivity that really believed their craft somehow managed to transmogrify the landscape surrounding them – or even mundane events in their everyday life during the composition of a work – turning them into music. There are many composers that have indicated a particular spot in some score that ‘represents’ a certain event that took place while at work on the piece. They even believed, against all evidence, that their music can convey to the listeners the ideas that occupied their mind during the composition process. Granted, there are some pieces that are ‘descriptive’ in the sense that the music literally imitates sounds of nature, – cuckoo calls, the roll of thunder, the noise of the wind or air raid sirens –, but those resemblances are extra-musical and to take them as the ‘contents’ of music would amount to mistake mimic for meaning.

Another main difference between music and language is that language is always, necessarily linear – i.e. it consists of a string of successive words, whereas (unless the analysis is limited to music consisting of a single melody such as a flute solo or plainchant) western music, at least after the 9th century, has not less than two dimensions, a horizontal one (melodic) and a vertical one (textural). In literary stylometry, the central issue has always been what words to use as discriminators, or perhaps in what order they are placed, but there have been no doubts that “no potential parameter of style below or above that of the word is equally effective" (Tallentire, cited by Holmes, 1998). On the contrary, in music there is nothing equivalent to the word.

1.5 Musical Style

Let us take a closer look at what is musical style. Pascall points out that the term 'style' "may be used to denote music characteristic of an individual composer, of a period, of a geographical area or centre, or of a society or social function". These characteristics, of course, are the result of the composers' choices. These choices are more alike among those of the same epoch, of the same geographical area, and of those composing for the same social function – such as liturgical or dance music, which is the reason why there is, for example, a style of the classical or baroque period, a style of church music, or a style of Czech music in the Romantic period. Pascall states that the composer

> inherits an usable past and acts by intuitive vision. The product of his vision builds on a stylistic heritage, has a style and import of its own and bequeaths an altered heritage. The stylistic heritage may be seen as general procedures which condition the composer's intuitive choice and invention (Pascall, op.cit)

Epoch is the strongest of these elements, so that for the historian, who groups examples of music according to similarities between them, "a style is a distinguishing and ordering concept, both consistent of and denoting generalities", so much so that "Adler described music history as the history of style" (Pascall).

Interestingly, however, Pascall remarks that personal style is not an important feature in many non-Western musical cultures, in plainchant or in Western folk music. "The relative importance of personal style is a significant and to some extent distinguishing feature of the Western tradition, and it may be seen with notation as part of the process of comparatively fast development in the West" (Pascall, op.cit). Consequently, music stylometry will have to be probably limited to the period of common practice in the West up to the present day.

1.6 Musical Analysis

There is a well-developed tradition of musical analysis in the West dating from the earliest times within the period of common practice, but its area of concern is centered on particular works, which it intends to explain structurally as one would disassemble clockwork to figure out the way it is put together. The question that analysis tries to answer is ‘How does it work’. By means of comparison, its central activity, analysis determines the structural elements and discovers their function (Bent & Pople, 2001) Hence, analysis does not deal with style except by implication, such as the identification of similar structural elements between different works of the same composer or epoch.

Nicholas Cook, in his Guide to Musical Analysis, explains:

> There are a large number of analytical methods, and at first sight they seem very different; but most of them, in fact, ask the same sort of questions. They ask whether it is possible to chop up a piece of music into a series of more-or-less related independent sections. They ask how components of the music relate to each other, and which relationships are more important than others. More specifically, they ask how far these components derive their effect from the context they are in. (Cook, 1987).

In the first five chapters of his book Cook gives an insightful coverage of the most important current analytical methods – traditional methods, Schenkerian, Psychological approaches (Meyer, Reti), Formal approaches (Set-theoretical, Semiotic), and Comparative techniques –, and in the sixth concludes that “the principal types of musical analysis current today do not have any real scientific validity, and we therefore need to rethink what it is that they can tell us about music”. Thus, given that the main preoccupation of analysis is not closely related to our quest, and any of its trends “do not have a sufficiently sound theoretical basis to become a scientific discipline in its own right”

(Cook, op.cit.), we do not need to concern ourselves with musical analysis any more.

1.7 A suggestive analogy

On a more philosophical level, this search could be seen as more concerned with the structure of style as a phenomenon than the markers of style. The idea is: Are there conceptual dimensions to the "style" construct that can be objectively identified? And if so, what are they?

This is the kind of result that has been obtained in an unrelated field that could prove a source of ideas and guidelines for this research, the area of Psychology known as Personality Theory. From olden times, it had been observed that people differ in their predominant desires, characteristic feelings and the way to express them, and they do so in consistent ways across time and situations. The Ancient Greek were the first to notice that personality traits do not occur at random but following patterns, and produced the first taxonomy of personality, the Four Temperaments of Hippocrates, which contemporary research has validated through the Eysenck Personality Inventory. Other researchers in the area have looked for different approaches to the problem. One of particular interest is Raymond Cattell's. Starting from an unabridged dictionary from which a list of 18,000 trait terms was extracted, he reduced it eliminating synonyms and difficult or uncommon words, until he was left with an irreducible set of 171 terms. A group of judges was then asked to rate subjects using this set of words. Their ratings were factor analyzed and clustered, yielding a set of 16 main personality dimensions. Two of his second-order factors coincide with Eysenck's. Cattell wrote that "source traits promise to be the real structural influences underlying personality". "Measuring behaviours in factors [is] the first step in an analytical procedure aiming to discover the structure and function of personality". (Cattell, 1965).

In the 1990s a certain consensus was reached about a five factor model such as Costa and McRae's, the so-called "Big Five" personality variables. This set of five variables (Conscientiousness, Agreeableness, Openness or Intellect, Extraversion or Surgency and Neuroticism or Emotional Stability) includes the same basic two dimensions already mentioned, i.e. extraversion/introversion and emotionality, which suggests through convergent validity (Anastasi) that these two dimensions possess an objective reality.

All this suggests two basic ideas for the music stylometry problem. Firstly, the variety of individual variation in human personalities is at first glance bewildering. However, it takes some methodical application of multivariate statistics to reveal conceptual dimensions underlying the phenomenon and the patterns they create. In a similar way, at first glance, the variety of musical styles may seem bewildering. But there must be objective dimensions in personal music style. It should take the same kind of approach to reveal the pattern underneath. The resemblance of both areas is not coincidental: Individual musical style is largely a reflection of the personality of the creator. Since music creation is so free, so arbitrary, as composers write just what they want, there is little doubt that the main influence in their musical style is their personality.

Cattell, as well as other psychologists, thought that there might be a mapping of the universe of personality to the universe of musical style, and conceived the possibility of devising a test of Personality based on musical preferences. But he was also the one that went ahead and created the test, which came out to the public through the Institute for Personality and Abilities Testing (Cattell, R. and McMichael, R. (1960).) The test included one hundred musical excerpts, which were played for the subjects, who had to choose for each whether Like, Dislike or Indifferent. Cattel and Saunders factor-analyzed the results and reported finding 11 main factors, about which they say:

> Our general hypothesis that these independent dimensions of choice will turn out to be personality and temperament factors rather than patterns of specific musical content or school seems sufficiently sustained. (Cattell & Saunders, 1953)

Cattell found similar results for a test of preferences for paintings. While these findings have to be taken with some caution because of the practical defects of the procedure, it is a first indication of a number of personality factors aligned with the perception of musical styles.

The ultimate goal of this project is to build a method to classify tonal music identifying the main dimensions of style so that every tonal work can be mapped to the region where it belongs according to its parameters. Whatever the variables used to categorize style turn out to be, it is a mandatory result that works of indisputably similar style cluster together. Furthermore, a work very similar to others that are included in the training set will have to cluster with them, thus providing a clear-cut way to assess the success of the method.

Due to the current lack of consensus about the dimensions of style, the problem calls for a method of unsupervised learning, since

> Unsupervised learning considers the case where there are no output values and the learning task is to gain some understanding of the process that generated the data. This type of learning includes density estimation, learning the support of a distribution, clustering and so on (Cristianini & Shauwe-Taylor, 2000).

In this way, the application of such methods to the musical database would offer a first glimpse into the dimensions of musical style.

1.8 Conclusions

J. Rudman, addressing the problems of stylometry, suggested:

> Study style in its totality. Approximately 1,000 style markers have already been isolated. We must strive to identify all of the markers that make up “style” – to map style the way biologists are mapping the gene.
>
> ...The autoradiogram with its multiple markers does not claim infallibility but does claim probabilities approaching certainty...It is important to look at as many of the myriad style markers as possible – some markers will overlap with those of the controls and of the other suspects, but a matching pattern should emerge (Rudman, 1998).

This program should be equally applicable to music. The goal is to characterize as wide a range of musical styles as possible by means of the widest variety of variables that could be derived from the observable elements of the material. The emphasis should be on a comprehensive view of musical style in the manner suggested by Rudman. This approach is basically what has been described as "category analysis" by Bent and Pople, meaning a method that starts with the breaking down of its material into those facets that are constantly present. This would provide, in LaRue's words, 'a set of categories that are satisfactorily distinct'. Each category would then be given a scale of measurement, and this measurement is what would be the critical operation of the analysis. As David Stech observes, "the depth of study required for a musical analysis is determined by the particular goals of the analyst. To draw a few general conclusions concerning a large number of compositions, detailed analysis of each work may not be necessary" (Stech, 1981).

For these reasons, it would be desirable to approach the material with the open mindedness of someone free of cultural bounds. For example, if one of the variables of interest concerns harmonic progressions, something the composer of the classical period was acutely aware of, there is no problem applying the same analysis to pre-baroque or serial works for which harmonic progression was a non-existing concept. We are interested in the parameters of the material, not the features that the composers were conscious about. Hence, it is immaterial if the concept of harmonic progression is anachronistic to the work being considered.

This is the long-term goal that has served as the motivation for this project. The great success achieved by literary stylometry has taken more than forty years and the combined effort of many individuals. In music, nothing had been done yet, and very little practical methodology can be adopted from that field.

It should be necessary to consider all the available aspects of music in which conceivably a composer's style could be distinguishable. Certain composers, such as Finzi, are easily recognized by some inflexions of their melodies; others like Stravinsky, by their peculiar rhythms and absence of melody; others like Delius, by their harmonic language; still others by their dense textures, or violent dynamic changes. Probably, some of these aspects will prove good markers, but it will be unavoidable to start testing them all and submit the results to statistical analyses.

The preceding discussion gives ground to consider that:

- Music contains enough stylistic information to make possible the existence of music stylometry.
- The radical differences between music and language means that both stylometries could only share general methods of research.
- The use of computers for multivariate techniques applied to a suitable set of markers might result in comparable success to literary studies.
- These studies should be based on the observable elements of music, specifically disregarding musical analysis.
- The long-term goal is to identify the main dimensions of the phenomenon of musical style.

During the preparatory work for this project it became gradually clearer that for most of the features of interest that are related to pitch, a prerequisite was the knowledge of the key at each point in the musical excerpt. Therefore, the extraction of features hinges on the determination of key. This considerable additional problem had to be tackled first. In the process, the inadequacy of the MusicXML format for this purpose also became apparent. MusicXML is not conducive to harmonic studies either, as the notes of a single chord are generally spread along several pages of text. It was decided that a new format was required so that the vertical information was presented together in a workable way; the new format was devised and a program was written to convert the files in the database to it.

With the converted database it was possible to calculate accurately the key as a point function. A further problem that could be solved in a pragmatic way was the determination of a criterion to decide that there was no detectable key in the music. Unfortunately, these previous tasks that had to be carried out in order to provide the tools necessary for the extraction of features consumed most of the available time for the project. Consequently, the feature selection and extraction and the application of multivariate statistics had to be left for a further stage and complete this one with a first feature-extraction program that functionally labeled notes and chords, so that its application to the database allowed obtaining basic information about tonal music in general.

Chapter 2 gives a summary coverage of previous attempts to study musical style which furnished valuable ideas or guidelines for this project. Chapter 3 gives a general explanation of the chosen approach and the reasons for the treatment of the material, often in view of previous work. Chapter 4 gives a detailed account of the way the different problems were dealt with. Chapter 5 presents the results of the application of the programs to the database, and Chapter 6 discusses the limitations of the procedure and suggests some ways for improvement. Appendix I is a list of the pieces in the database. Appendix II contains tables with the figures for the frequency of use of scale degrees and chords. Appendix III provides an introduction to musical notation for people not familiar with it. Appendix IV gives a format example for MusicXML. Appendix V is a report on the keys of the fugue themes of the first volume of Das wohltemperirte Klavier extracted from the program output. Finally, Appendix VI contains the scores of pieces referred to repeatedly in this project.

2. Previous attempts to deal with musical style

Constant Lambert (1948) observed that the composers of the Baroque and the Classicism had no interest in developing a personal style. They borrowed from one another, and their craft included a series of standards that allowed the mediocre ones to reach “the honorable level that makes them still listenable”. The interest in the idea of style, and the fact that it is a characteristic feature of individual creators arose during the Romantic period. Consequently, during the 19^{th} and 20^{th} centuries there had been an interest in characterizing the style of particular artists, writers and composers. While in music this idea has never been pursued in a scientific and systematic manner, there have been a number of attempts that had centered on peculiar details of individual creators’ style, typically their consistent preference for some particular choices.

Alfred Sentieri tried to systematize this idea in his PhD dissertation “A method for the specification of style change in music” (1978) where he proposed to measure change in selected style details. He started with a definition: “The commonality, the frequency and the relative occurrence of [characteristic] details make up the information which analysts observe and quantify in order to define style” (Sentieri, 1978). Moreover, his study assumes that “aspects of style can be detected in the order and pattern of the music symbols found on the written page” (p.9). He proposed “a quantitative approach to analysis based on identifying and measuring various details from the works. Specifically, he stated: “The development of style can be measured by specifying rates of growth and decay in the use of various aspects of style”. Following Paisley, who had defined personal style as ‘an individual’s deviations from norms”, he views the composer as a chooser of a reduced number of elements within the potential complete set they belong to: “A musical artist’s preference for certain details […] can therefore be expressed statistically”.

Sentieri applied this method to the study of the measurement of stylistic changes in the sacred vocal works of the Venetian Baroque, namely a group of composers associated with the St. Mark Basilica between 1600 and 1750 –

Gabrielli, Croce, Monteverdi, Cavalli, Lotti and Vivaldi –. In spite of the small number of works considered in the study (between three and five per composer), a number of interesting trends and individual differences were found. For example, from Gabrielli through Vivaldi there is a steady decline in rhythmic variety and an increase in the use of perfect melodic intervals.

Paisley’s (1967) “Encoding Behaviour” had introduced a method for style analysis which included the concept of “minor encoding habits” defined as details “both inconspicuous and ubiquitous, too much in the background of the work to be noticed by a forger or disciple or to be varied consciously for effect by the author himself “(quoted by Sentieri, 1978). They are idiosyncrasies of the artist and not the result of deliberate manipulation of the material. They can be considered as stylistic ‘fingerprints’ and will help to distinguish the work of its composer from other’s. It is apparent that this concept agrees with Holmes’ views on stylometry.

On the same line of thought but much more concretely, David Cope defined “signatures” as “contiguous note patterns which recur in two or more works of a single composer and therefore indicate aspects of that composer’s musical style. The signatures he identified are typically two to five beats (four to ten melodic notes) in length and usually consist of composites of melody, harmony and rhythm. He asserted that signatures typically occur between four and ten times in any given work. Variations often include pitch transposition, interval alteration, rhythm refiguring, and voice exchange” (Cope, 2001). These “signatures” are recognized using pattern-matching processes. Unfortunately, Cope is not specific as to the process of detection he used to produce the collection of Mozart’s signatures presented in his book.

Although many composers have consciously and systematically used some note patterns as signatures – typically BACH, used by many composers beginning with J.S.Bach himself, or Shostakovich’s DSCH which is a transliteration of “D.Sch” into German musical notation –, unconscious use of signatures probably indicate deeply seated musical preferences. It is not possible to make a blanket statement as to the unconscious nature of such signatures, especially since self-quoting is a favorite device of many composers. Many composers have been fond of cryptography, and have had the inclination to include encrypted messages in their music (Elgar, Berg). Thus it is probably irresponsible to assume that all composers use signatures in an unconscious way.

The interest in using computers to systematize the study of style dates from the late 1960s when it first started to seem feasible. Although these attempts have been quite limited in their extent and success, several of them are worth mentioning.

James Gabura presented a paper on Computer Analysis of Musical Style in 1965 in which he tried to find "an objective measure of style" with a view to obtaining an insight into the stylistic differences between the piano sonatas of Haydn, Mozart and Beethoven. After several experiments, he used a training method for separating hyperplanes and was able to differentiate between piano Sonatas of Haydn, Mozart and Beethoven just by considering chord pitch structure (Gabura, 1965).

Arthur Mendel, in "Some preliminary Attempts at Computer-Assisted Style Analysis in Music" (1969) reported one of the first concrete attempts at using a computer for style analysis in music. Their work, based on the analysis of the masses of Josquin Desprez, included the devising of a system, i.e. a language and a compiler for converting musical notation to digital, and the information retrieval program, which was carried out at Princeton University and reported in the article "IML-MIR: A data-Processing System for the Analysis of Music" published in 1967. They entered 1100 pages of the complete works of Josquin. Unfortunately their system was tied to the IBM 7094 which by the time the article was published had been replaced by the IBM 360, and this limited its possible diffusion. The article mentions an interesting attempt (even if it looks like it did not reach significance) to use the computer to determine, through stylistic differences, the authenticity of a section of the *Missa L'homme armé super voces musicales* that exists only in a manuscript written long after Josquin's death. Being a specialist in 16th century, Mendel concludes with a list of the most significant developments during that century, in order to ask the reader how to define them for the computer to assess.

Mendel also inspired a paper from 1974, P. H. Patrick's "Computer Study of a Suspension-Formation in the Masses of Josquin Desprez" which shows that the IML-MIR system was then used as a data source by means of Fortran programs, as well as a 1978 sequel, "A Computer-Assisted Study of Dissonance in the Masses of Josquin Desprez" by P. H. Patrick and Karen Strickler, which presents a Fortran program that took three years to write, to classify dissonances in the masses of Josquin and gather statistical data about them.

Another early attempt at measuring style is reported in the 1979 paper by Fred Hoffstetter (1979) "The Nationalistic Fingerprint in Nineteen-Century Romantic Chamber Music". The author's intention was to assess some statements from the *Cobbett's Cyclopedic Survey of Chamber Music* regarding the way 19th century nationalism was expressed in chamber music. After formulating his hypotheses, Hofstetter wrote:

> An exhaustive search for evidence to support or reject these hypotheses would require consideration of many different kinds of compositional procedures. For example analysis of form, harmonic patterns, textures, articulations, rhythms, and tempi could be used.

Given the limited computing power at the time, Hoffstetter settled for a much more limited scope, "on the basis of counts of melodic intervals in a controlled database consisting of 130 melodies selected from sixteen string quartets". Beyond the obvious limitations of sample size and detail of characterization, and the objectionable choice of data base due mostly to the limitations in size, this study attracted several criticisms from Cook, particularly "how appropriate is intervallic distribution as a stylistic criterion" (Cook, 1987).

There was an even earlier study that merits mentioning, Helen Budge's PhD dissertation "A study of chord frequencies based on the music of representative composers of the eighteenth and nineteenth centuries" (1943). Since computers were not available at the time, the study was based on hand-made harmonic analyses. Representative samples from works of 24 composers in the current repertoire were chosen. In the case of short pieces, the entire work was analyzed, and in the case of multi-movement works, samples were taken from each movement. One or more works for each composer were taken from each of five groups of works (orchestral, chamber, piano, choral, songs), except when the composer was not noted for composing only in some of the groups. A total of some 66,000 chords were counted, establishing the frequency values per composer and period. Her results show a number of very interesting trends and figures, for example, the classical period is the one that shows the least variety of chords; and there is a constant increase of chromaticism with time. Although the frequencies vary, those found in the first ten places are always the Tonic, the Dominant Seventh, the Dominant, the Subdominant, the Superdominant and the Submediant and their inversions (see Appendix III). There are also conclusions for chord usage for individual composers – for example, Wagner shows the lowest use of the tonic chord and Verdi the highest. The composers with the widest chord vocabularies were Beethoven and Mussorgsky and the one with the most limited one was Rossini. Methodologically, she made an important point: She decided to take the music at face value, refusing to classify incomplete chords.

Following Knud Jeppesen, Gustave Fredric Soderlund in “Direct Approach to Counterpoint” gave a detailed account of the stylistic elements found in plainchant and the works of Palestrina and Lassus, explaining which were usual and which not and in what circumstances, often indicating their relative frequencies, for example for root movement statistics.

One interesting project among the early ones was Dorothy Gross’ PhD dissertation, A Set of Computer Programs to Aid in Musical Analysis (1975) in which she developed a package of computer programs to do pattern tracing, thematic analysis, grouping of sonorities and harmonic analysis. Her goal had been to use computers to carry out full analyses but she found that instead, the computer had brought to light the inadequacies of formalized musical theoretical systems. Because of this, considering that musical analysis is full of tedious and time-consuming mechanical chores that could be better left to a computer, she ended limiting her attempt to duplicate “the more routine parts of quantitative analysis”. But, as she explained

> Our one program going beyond routine operations is our harmonic analysis program, which started as a small chord-labeling option and grew into a project in simulating human thought as we realized that the definitions found in textbooks were entirely insufficient for even the analysis of a Haydn’s minuet.

She finally admitted that the program was not up to the task of dealing with music as complex as Chopin’s Etude No.24.

There is one last study that is worth mentioning because of its similarity with this project, in spite of it dealing with a different area of music, a structuralist project described by Cook as follows:

> ...the most significant results naturally come when you use a large number of traits together in order to characterize styles. This is what Alan Lomax and his co-workers did in the Cantometrics project [which] involved the comparison of several thousand songs selected to be as representative as possible of all the world’s cultures.[...] There are thirty-seven different aspects of the music being considered here – or more precisely we should say that it is being evaluated along thirty-seven dimensions. (Cook, 1987)

This list of 37 variables range from the purely musical (Tonal Blend, Melodic Shape, Phrase length), to those pertaining to the performance itself (Tempo, Volume, Rubato, Nasality). More than just a study, Cantometrics was “an attempt to establish universally applicable guidelines for the study of folksong; a way of defining song style for major cultural areas (e.g. India, West Africa); and an approach to a broader understanding of the interrelationship between the song and its function”(Thieme, 2001).

Perhaps because the expectations about what computers could do proved excessive for the limited power of the machines of the time, these kind of studies seem to have died out. In the 1980s, music studies by computer became a province of AI, where they were split between musical analysis and musical synthesis. The latter have no interest for this project and the former were hijacked by those who tried to capitalize on Chomsky's discoveries about grammars and misguidedly apply them to music. Not surprisingly, this approach has turned out fruitless. There have been several attempts at using computers to carry out different forms of musical analysis, such as Schenkerian or Semiotic, which understandably have run into trouble because of the lack of solid ground on which to build.

In 1970 Crane and Fiehler had conceived musical style as characterized by a large number of variables, and as consequence "the style of a work [could be] represented by a point in multidimensional Euclidian space". Consequently, they used cluster analysis to compare musical styles. In view of the importance of their conception and method, it was disappointing that they applied it only to a very small and marginal area, the case of twenty chansons by three composers of the early 15th century. In order to compare their styles, they used 145 variables, 21 of which did not discriminate and had to be discarded.

Following their lead, it is possible to envision a process to develop a scientific taxonomy of musical styles: If every individual style (even those who do not yet exist) could be characterized univocally by a minimum set of *n* numerical variables, each particular set of *n* numbers would be the *n* components of a vector that represented it in the n-dimensional universe of musical style. We have no idea what these dimensions can be. But musical knowledge can easily suggest a large number of observable features. In the aforementioned article, Pascall wrote: "Style manifests itself in characteristic usages of form, texture, harmony, melody, rhythm and ethos". Each of these areas, excluding form and ethos, should offer a number of variables that could be quantified by the appropriate criteria, and their measure carried out in practical terms by software routines. Since it is not clear what parameters would turn out to be the markers of style, it would be necessary to carry out a thorough study of the elements of the musical material in order to determine the appropriate discriminating elements. Applying the measuring software to a large database of musical examples, each of which could be reasonably brief to be said to possess a unique style, would yield a matrix. Using existing statistical tools such as PCA or SOMs, it would be possible to determine from that matrix what are the most significant dimensions of musical style, and possibly a practical way to categorize particular pieces. In this way the musical counterpart of stylometry could be developed.

The studies mentioned above, irrespective of their relative success, have some aspect that points to the future in relation to the use of computers to scientifically characterize musical style, be it in their conception, approach, or method. The main points to keep in mind are:

- The limitations and incompleteness of musical theory prove inadequate as a basis of analysis
- The composer acts as a filter, choosing elements from a larger set on the basis of quantifiable personal preferences
- The creative process entails minor encoding habits that create stylistic signatures of the composer
- An objective measure of style requires the systematic search of parameters to carry out statistical analyses
- The basis of the study must be observable features of the printed score

Despite the horror that musicians feel toward rational explanations of musical mysteries, it has to be recognized that in music as in psychology, E.L.Thorndike's statement holds true: "Whatever exists, exists in some quantity, and can in principle be measured".

3. Overview of the Project

3.1 Guiding ideas from previous research

Most of the studies mentioned in the last Chapter provide ideas that are worth considering for this project, whether in their general conception or views, or in their methods. It is worth mentioning:

The relevance of chord frequencies for style characterization (Budge)
The existence of gradual historical trends (Budge, Sentieri)
The mathematical method suggested for finding keys (Gabura)
The use of element tallies as variables (Soderlund)
Style represented by points in multidimensional space (Crane & Fiehler)
The use of cluster analysis to define styles (Crane & Fiehler, Lomax).
The use of a large number of traits together in order to characterize styles (Cook, Lomax)

3.2 Rationale

Following Rudman's advice to identify all of the markers that make up style, combined with Pascall's assertion about the manifestation of musical style into characteristic usages of texture, harmony, melody, and rhythm, the main dimensions of the style phenomenon will have to be found examining every measurable aspect of music. A tentative list of features to be considered as possible markers could include:

Melody

Proportion of stepwise/leaps
Relative frequencies of intervals
Frequency of use of sequences
Frequency and extension of scale-wise passages
Use of modes
Phrase length
Chromaticism

- Tonal ambiguity
- Lyricism – assuming a reasonable operationalization method

Rhythm

- Variety
- Frequency of use of sequences
- Explicitness

Harmony

- Relative frequencies of chords
- Relative frequencies of root movements
- Harmonic rhythm
- Modulations (methods and frequencies)
- Frequency of use of sequences
- Treatment of dissonances
- Frequency of Standard chord progressions
- Frequency of Non-standard chord progressions
- Frequency and type of cadences

Texture

- Density
- Type (Homophony/Counterpoint)

Dynamics

- Range
- Grain
- Suddenness
- Constancy
- Use of dynamic swells

Tempo

- Constancy
- Extent of use of Rubato

Devices

Extent of use of:

- Alberti bass
- Cascades of repeated chords
- Chromatic scales
- Chromatic base line

Many of these features could be directly measured from the music. Others may require an elaborate procedure to extract the information. An unsupervised learning method and a dimensionality reduction procedure should be able to identify the main dimensions of the phenomenon.

3.3 How to deal with the music

This project deals with inferring metadata from a large database of representative musical pieces. It begins with a number of choices. The first is a philosophical one. It involves the approach to the problem of making sense of the music. In this context, a few common musical terms require a brief explanation.

A tonality or "key" is often identified with the scale of the same name, which consists of playing successively in ascending or descending order all the diatonic notes belonging to the key. Each note in a scale receives a name relative to its position, which is referred to as "scale degree". The most important of these are the Tonic, which is the one that gives its name to the key, the Dominant, which is one fifth above the Tonic, and the Subdominant, which is one fifth below. (For more details, see Appendix III).

The key forms a contextual frame that allows a musician to identify the Tonic after hearing only a few notes. In tonal music, the Tonic shifts from time to time within a single piece, and this change is referred to as "modulation". Sometimes, this change is only brief and the music returns to the original key; in those cases it is common to refer to it as "tonicization", although there is no strict criterion to say when a change of key deserves the name of modulation or only tonicization.

Composers modulate for the sake of variety, and even very short pieces generally modulate. When a modulation occurs, the only tell-tale sign of the change of Tonic is the appearance of additional sharps or flats (or of naturals where the former used to be). If instead the modulation is going to last for a while, normally composers write a double bar line and change the key signature.

Although music theoreticians, unlike mathematicians, have never subjected their system of rules to logical analysis of completeness and absence of internal contradictions, these rules give the impression of being exhaustive and complete. If they turned out to be such, it would be a good idea to take them as a starting point. For this reason, it is illustrative to refer to several studies that tried to follow this path.

Music studies are also full of mechanical, tiresome and error-prone chores, such as transposing, that require little knowledge apart from counting. Consequently, as soon as computers became available, many researchers thought computers would be ideal tools to take care of those tasks that can be accomplished by the mere application of rules, as well as to provide insight into the rules themselves. However, the impression of logic in music rules did not survive scrutiny. In 1968, John Rothgeb pioneered the use of computers to solve the problem of harmonizing the unfigured bass with results less than satisfactory. Years later he summarized them writing, with irony, that "the computer made a significant and well-defined contribution to the study by exposing deficiencies in the theories under investigation and in suggesting further lines of enquiry".

Since then, other researchers had gone through the experience of trying to create a computerized system of analysis based on musical principles and come to the conclusion that the system of music rules does not provide a basis for the design of a rational tool. Gross' conclusion in this respect was that:

> Music analysis with the computer has brought to light the inadequacies of existing music theory in fully describing musical attributes because

> the computer [...] reveals all too clearly the gaps and loopholes in formalized theoretical systems (Gross, 1975).

A few years later, H.J. Maxwell attempted the artificial intelligence approach to identify chords and keys. In his PhD dissertation, "An Artificial Intelligence approach to computer-implemented analysis of harmony in tonal music" he recognized that "there is no clear-cut, non-intuitive method for performing harmonic analysis of tonal music". Claiming the superiority of knowledge-directed intelligent methods over brute-force algorithms, he pointed out that the ability to tell a chord from a non-chord is crucial in building a computer harmonic analysis program. (Maxwell, 1984).

This distinction is a subtle one. In principle, the term “chord” refers to the simultaneous sound of at least three notes – two simultaneous notes are not referred to as a “chord” but as a “harmonic interval” –. Harmonic theory is mostly based on three-note chords called “triads” and four-note chords called seventh chords. Nevertheless, not any simultaneous combination of three or four notes is a “chord”. Only a few of the possible combinations are considered such, and they are those whose notes can be arranged as stacks of thirds i.e. pairs of notes whose theoretical frequencies are related by the ratios 6:5 or 5:4 (see Appendix III). All other simultaneous combinations of notes that often occur in music are considered the accidental result of the movement of the voices. Naturally, this does not mean that they sound any different to true chords to the ears of the listener; the point refers to their lack of a structural role in the music

Maxwell first described the central difficulty saying that "chords define the existence of tonality, but the tonality in turn defines the functions of the chords". These two problems cannot be separated. He developed an expert system based on 55 rules centered on two main issues: "Which vertical sonorities are chords" and "What is the key", while being aware that both problems were not independent, as he explained:

> Once it is decided exactly what notes are in a chord, and what key in which to analyze the chord, finding the function label is a simple matter. But the label given may, in turn, influence what notes, which sonority, should be chosen as 'the chord'. The key is also dependent on the chords that are selected for labeling because its strength depends on the functions that can be assigned to them. This is the very crux of the problem, a symmetrical dependency – that the identity of the key depends on the chord functions, while the chords and their functions are determined by the key (Maxwell, op.cit).

Maxwell's system proceeded through several stages, first determining consonances and levels of dissonance, and on the basis of these and their metrical placement, telling chords from non-chords. Based on the chosen chords, the tonality was assumed, and the analysis proceeded from beginning to end, “analyzing as long as possible in the currently established key, and only attempting to modulate when a certain threshold of functional weakness is exceeded".

In his dissertation, Maxwell analyzed only three pieces from the French Suites of J. S. Bach. It would have been interesting if he had continued perfecting his system, but he does not seem to have done it. In 1992 he contributed an abridged version of his dissertation to the compilation "Understanding Music with AI: perspectives on music cognition" but there again he referred only to the same pieces.

Maxwell's cross-dependency is a serious drawback to the analytical approach, and in spite of the complicated nature of his system, the results admittedly left a lot of margin for improvement. Music cannot be dealt with as if there was an intrinsic logic to it. The cross-dependency that he contended with questions the very meaning of “functional weakness”.

In the well tempered system, all triads sound the same. Not more than 0.01% of the people have absolute pitch – that is, the ability to label a note in isolation. If an experiment was carried out, for example playing for the subjects an

F major triad on the piano, asking them to try to remember it, then after a minute or so, playing an E major triad, and asking them whether the pitch of the second chord was higher, equal or lower than the first one, we could expect that the overwhelming majority would have no clue about the answer. "Absolute" pitch is, in fact, quite relative, and the memory of sound fades away very quickly. (Parncutt & Levitin, 2000).

What this means is that a chord in itself is meaningless. What makes an impression is the succession of two chords, when one follows while the first one is still clear in the memory. The smallest set of chords that allows harmonizing a melody i.e. provide a tonal frame to it, comprises the Tonic, the Dominant and the Subdominant triads, which are called "principal chords". The key is strongly implied by these chords.

It was Hugo Riemann who gave their roles the haughty name of "functions". In his scheme, there are three functions, the Tonic, the Dominant and the Subdominant; the Submediant triad shares the Tonic function, the Leading tone triad the Dominant function and the Supertonic the Subdominant function, on the basis that each of them share two out of three notes with the respective principal chord while the Mediant triad shares equally on both the Tonic and the Dominant ones, which makes it the most ambiguous of them all. Understandably, the effect of the presence of the chords is what creates the tonal context.

However, any piece of music is populated with "non-functional" notes and chords that merely act as fillers in the guise of "passing" elements or mere ornaments, and there is no criterion that would make possible to separate them from "real" notes. In general, looking at a particular spot in the score it would be possible to take the fillers as functional, which would turn the functional into fillers. In a logical system, this choice would run into trouble in the form of the appearance of eventual inconsistencies, forcing the analyst to retread the labyrinth until finding the turn that went wrong. But in music not even this can be taken for granted.

It is easy to see why this is shaky ground to establish theoretical foundations. Composers do not follow rules and even if rules could be thoroughly established a posteriori, it can be taken for granted that every one of them would have to admit exceptions. Music does not provide an explicit or implicit set of coherent rules. Trying to establish basic ground rules on which all music is based as the foundations of computer musical analysis is a vain hope. Consequently, when trying to identify the key, the lack of rules of reference makes necessary to resort to a non-analytical method such as some sort of statistics.

3.4 Source of the study

Having discarded musical analysis as a method, there is no alternative than centering on the observable elements of the music. The first consequent decision involves what is the best way to "observe" them, what the source of the study will be, and this is another point of contention. Music is a psychoacoustic event, and all of the studies in ethnomusicology are forced to take as their source the recorded sound. The Cantometrics project is a typical example. However, this is a study of western music, all of which exists in written form. Thus, it seems natural to base the study on scores.

It is clear that Beethoven's Eroica does not exist in the way that Michelangelo's David or the Mona Lisa do. We tend to forget this fact because well known music pieces solidify into a "performance tradition" in which the composer has little intervention. We are used for the Funeral March of the Eroica to last 19 minutes as Toscanini used to do it, but Beethoven marked it to last only 12'22". This shows that a recording is not "the work", but just a particular view of it. On the other hand, the score of a piece is clearly not the music either but something like a cook recipe, i.e. a limited and imperfect set of directions to create it. Most of its worst limitations – lack of logic, use of multiple symbols or names for a single object, methodological inconsistencies – stem from its empirical origin as something unplanned that started in the middle ages as simple aids to the memory of the performer and built up with increasing complexity but without central direction. However, such origin means that it is a representation that highlights the relative importance of musical aspects according to the Western tradition. It "reflects an emphasis upon vertical relationships – the most characteristic feature of Western European music – and a disinterest in rhythmic complexity". (P. Hopkins quoted by Cook, 1987). It is nothing short of a miracle that the notation system coalesced essentially into a semi-logarithmic plot of pitches vs time – see Appendix III.

Cook notes that performers supply a great deal of information, which is not actually in the score: "an interpretation in which intervals, rhythms and dynamics are given what seem to the performer to be appropriate values" (Cook, 1996). This is the result of the fact that the composer's instructions are cast in a grid of limited resolution. When he writes down the score, he is capturing to the best of his ability but with unavoidable limitations, a sonic image in his imagination, trusting that the performer skills and instincts, based on the score, will reproduce what his fantasy conceived. The process of creation of a musical work could be seen as a travel from imagination to sound, that includes a necessary leap of faith in the middle step that passes from the composer to the performer.

It would seem perfectly logical to carry out a study like this based on sound recordings, applying to them the appropriate techniques of signal analysis. However, doing so would introduce an unassessable amount of noise, in the form of differences in performance – in the first place, tempo, dynamics and phrasing. It is amazing to compare different renditions of something so carefully written as a piece of music, because quite often two versions of the same piece are hardly identifiable as such. It seems preferable to take the score at face value. It represents, to the best of the composer's craft, the hard image of his music, and it does so to comparable degree for the composers from the seventeenth to the twentieth century. The score, in Cook's words is "convenient and tolerably adequate" (Cook, 1987). Admittedly, it is an image of the music captured with poor resolution, but it can be converted to digital practically free of information noise, and has the advantage that whatever is not written in it can be discarded as not having been provided by the composer. Consequently, this project will consider the scores as its prime material.

Importantly, only observable elements are taken into account. For example, whenever there are only two simultaneous notes, no attempt to “interpret” it as an incomplete triad is acceptable, no matter how obvious it may seem to the analyst. This is what is meant by “taking the score at face value”, which implies reading only what is explicitly in it.

3.5 Music Database

The next problem concerns the musical materials to build the database. Since the aim of this project is to consider as great a variety of styles as feasible, ideally it would be desirable to include in it music of as many composers as possible from throughout the period of common practice, in order to ensure an acceptably wide range of styles. From a practical point of view, it is necessary to limit the range to some reasonable number. The problems of considering a limited range include two uncertainties: First, whether the procedure covers adequately the stylistic range – since it is not known how and if different styles are going to cluster, there is no way to make certain that the samples provided do not omit significant styles –. Second, whether similar styles would cluster together or not. A most reasonable option would be to include samples for as wide a stylistic range as possible, on the one side, and on the other, whenever possible, to include samples of two composers of very similar style. This would show how widespread the different styles turn out and how close similar styles lay.

Individual pieces of music range in duration from a few seconds to several hours. While it is arguable that the style of a short piece is self-consistent, the same cannot be said about multi-movement pieces or extended single movements. Budge (1943) found that the chord counts for different movements of works by Mozart, Schumann and Mendelssohn differed significantly. Moreover, the typical symphonic poem lasts fifteen minutes, along which it goes through all the stages of a story or a number of contrasting moods. In order to consider each piece of data a unary representation of style, it would be necessary to select either very short works or chunks of works that sound self-consistent. The extension of each will vary with the nature of the piece but typically, it could be an excerpt of around one to three minutes, or roughly between 15 to 120 measures, depending on the tempo. Naturally, this is the main reason why form is not among the variables to be considered. Individual styles can be recognized in excerpts too brief for form to be a factor.

All the music in the database needs to be purely instrumental, because setting a text poses a number of constraints on the material which happen to be the same throughout the ages, hence it adds two sources of noise. It is a fact that the orchestral composer is able to choose his sound almost without limitations; therefore, orchestral music is as close to his 'pure' style as possible. A less serious limitation but probably still significant would be to consider only keyboard pieces. It is arguable that such database would cover identical stylistic scope as an orchestral database, although there is a caveat suggested by the fact that many composers whose orchestral style is very distinctive are not nearly as easily recognizable by their keyboard pieces (Prokofiev, for example). Regrettably, as in the case of vocal music, there are constraints imposed by the limitations of the keyboard peculiarities (hand reach, number of simultaneous notes that can appear, the unavoidable decay typical of all percussion instruments, the presence in the score of keyboard devices intended to simulate continuous sounds – repeated chords, tremolos – and the effect of the piano pedal that is often invisible in the score, and more often than not reflects the editor's choices rather than the composer). These constraints may have an obscuring effect on the conclusions. Another major problem would be the absence from the database of crucial composers whose keyboard works are non-existing or non-significant (Berlioz, Wagner, Bruckner, Mahler, Elgar, Sibelius, Stravinsky).

On the other hand, keyboard pieces are polyphonic, have a wide expressive range and are concise (in textbooks is customary to present piano reductions of orchestral examples). The choice of exclusively keyboard pieces has not only the advantage that scores are much simpler than orchestral ones. There is an abundance of short pieces – between 30 seconds and three minutes – in the keyboard literature, leaving no doubt about the uniqueness of mood and style. These reasons are weighty enough to justify a database based purely on piano-harpsichord pieces. Organ pieces will not be considered because the organ techniques are different enough to introduce confounding variables. It was decided that a minimum database should comprise not less than 250 works, and estimated that a good choice would be 10 pieces from 25 composers considered the cornerstones of the keyboard repertoire, spanning 300 years. They were chosen following as much as possible the previous guidelines. Figure 1 shows their names and life spans.

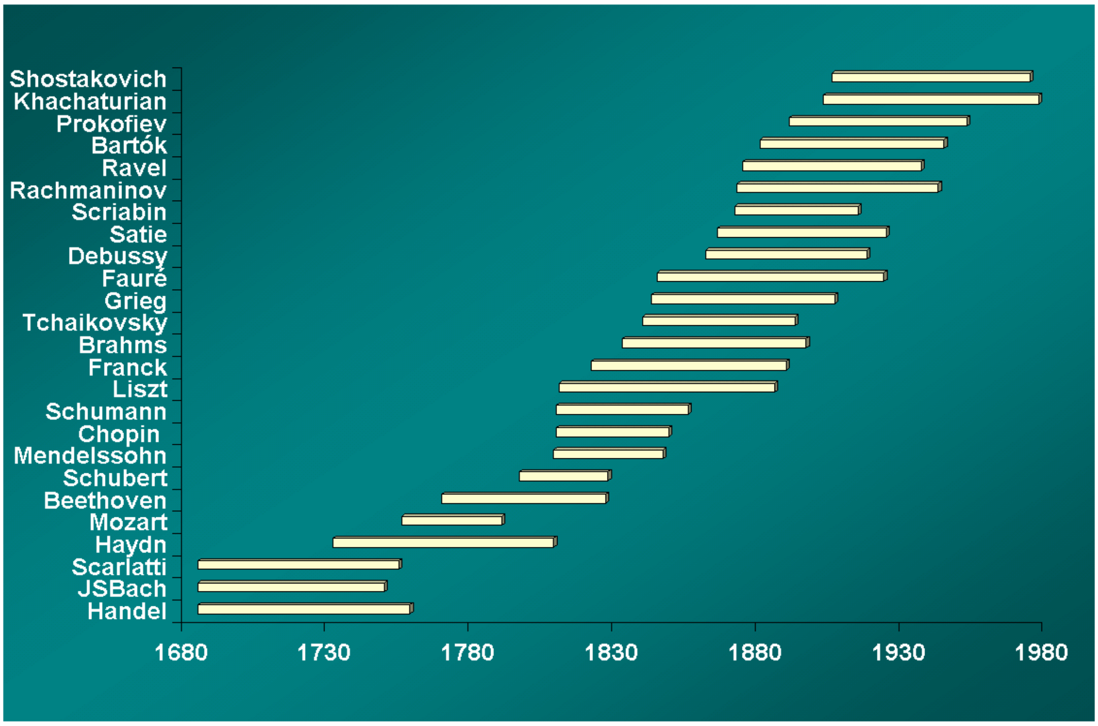


Figure 1 - Life span of the composers in the database

It is apparent that at any given year within the range, at least three composers are represented, with the only exception of the period 1791-1797 (from Mozart's death to Schubert's birth) and in the years since the death of Prokofiev. As far as possible short pieces – one to six pages long – were preferred, and otherwise excerpts that constitute complete sections, such as the exposition in a Sonata movement, making about 30 pages and between 20 and 30 minutes of music by each composer.The pieces were chosen to be as representative of the style as possible. This means that well-known works were preferred in general, although there were special cases were other reasons predominated. Whenever there were pairs of contemporary composers of similar styles, the selections tried to be matched in order to make comparisons easier. For example,

- Bach and Handel's are similar series of keyboard dances such as Sarabande, Gigue, Gavotte, Air, Courante, Allemande, Menuetto;
- Haydn Mozart and Beethoven's are sonata movements or excerpts.
- Wherever possible examples are taken from series or Etudes and Préludes – Chopin, Fauré, Debussy, Scriabin, Rachmaninov's.
- Ravel, Bartók and Khachaturian's Sonatinas are wholly included
- Pieces from Schumann and Tchaikovsky's Album for the Young
- Pieces from Prokofiev and Khachaturian's Music for Children

Some composers' styles vary noticeably along their lifetime. In those cases, the pieces selected were from a single period, e.g. middle Beethoven, late Liszt. The complete list of the database is given in Appendix I.

3.6 Data Format

3.6.1 Digital music standard

Each of the pieces or excerpts needs to be available in digital form. Unfortunately, there is not a standard coding system for digital music notation. A good number of programs are available to carry out different tasks such as typesetting music, playing music through a MIDI port or working as sequencers, ranging from the freeware to the very expensive. Each one sets its own format, and conversion from one to another is generally impossible. Even successful programs that could become de facto standards such as Finale use a format that its creators have for years refused to make public. Consequently, there is no such thing as widely available collections of digital music in the internet. There are some collections, very few and very poor, and it would be hopeless to try to base a study on the available material as it was initially expected. The only widely accepted standard is MIDI, but everybody who works with music notation knows that MIDI does not know about stems, beams, measures, or the difference between G-sharp and A-flat.

Fortunately, in 2001, a format intended for exchange of information and analysis, MusicXML, became available on a royalty-free basis and has already been adopted by a large number of applications, raising the hopes that it would become the long waited universal translator. MusicXML comes in two formats: Part-wise, in which successive measures are contained within each part, and time-wise, where successive musical parts are contained within each measure. Part-wise, the default option, is used by most applications. It is simple to convert from one to the other. Notice that the internal structure of a measure in MusicXML does not vary regardless of the format. Piano pieces are really two-part scores. However, the Part-wise format treats both parts as linked.

The procedure, then, consists on selecting the pieces and excerpts and entering their scores in digital form. The chosen tool for this is the SharpEye2 program, an acceptably accurate Music OCR program that produces its output in MusicXML language. However, there are important musical elements that SharpEye2 currently does not recognize, e.g. arpeggios, tremolos, simultaneous grace notes, and particularly beams that go from one staff to the other or across bar lines. This means that the actual process needs a complication: Scanning the score to SharpEye2, importing the MusicXML file to Finale, making the necessary additions and corrections and exporting it back to MusicXML. This is time-consuming but it ensures that data will be practically error-free.

3.6.2 The problem of key determination

The knowledge of the key at every point in the score is essential information for most of the possible variables of interest that could be derived either from the horizontal or vertical dimensions of music. Traditionally, computer music studies have been performed based on a small number of examples selected or transposed to be in C major or A minor, as well as verified to be non-modulating. This is the easy way to circumvent the problem of identifying the tonic for the purpose of functional labeling, and the only way a “Theme Dictionary” could be compiled. The great majority of music pieces do not comply with these requirements. Even in the early 18th century, music was likely to modulate or at least tonicize within the framework of a few measures. It would be impossible to build a database of non-modulating pieces.

In the music score the key is never explicitly indicated but merely implied. There are several ways to find the tonic by looking at the score, but none of them will work in all cases, and corroboration from different elements in the score is sometimes needed. The problem is a complex one because composers work in the manner of programmers who do not document their programs. Thus, a composer could establish a basic tonal plan for a work, and realize it by using the appropriate chords and modulations. But the tonal plan is not shown on the score. Once this has been completed, decorations and all, the scaffolding is removed. Determining the key entails some degree of analysis, for which the analyst has to do a sort of reverse engineering on the score. He figures out the tonality based on certain elements starting with the key signature and the final chord, and then checks for accidentals that reveal the minor mode, suggest modulations, secondary dominants and the like. This should not be a problem for music from the 18th and early 19th centuries, when composers followed strictly a number of well known conventions. Although there can always be exceptions – such as Jean-Fery Rebel's Cahos –, in the baroque and classical periods, establishing clearly the tonic was the first preoccupation of the composer, and any listener with a modest musical training could sing back the tonic after hearing only a few seconds of music. The analysis of the music of this period could have been argued to be 'objective'. But as we move ahead in time, the key gets progressively more blurred. Beethoven, at the opening of his 9th symphony allowed for thirty seconds of modal haze. That was an intended effect. Matters turned more complicated in the works of Liszt and Wagner. Tonal ambiguity can be purposely created, as in Liszt's "Bagatelle sans tonalité", which means that a key cannot necessarily be objectively found even in music that could otherwise be considered tonal. If the key cannot be unambiguously inferred from the observable elements, there is a potential intrusion of subjectivity, which has to be avoided.

This means that a necessary tool in computer music studies should be a method to determine the key as a point function, including the possible diagnostic that no key is detectable in a passage. If there were an objective mathematical method to find the key of a chunk of music based on its observable features, the immediate identification of scale degrees would greatly enhance its usefulness for analytical purposes.

James Gabura, an undergraduate at the Toronto University in 1965, presented a paper on "Computer Analysis of Musical Style" in which he made several attempts to objectively measure style by means of analyzing parameters such as melodic autocorrelation, chord structure, chord duration, chord type, key and modulation. To this purpose he coded pitch and duration directly from the scores. While describing his analysis of the distribution of pitches, he made the following intriguing remark:

> It was found that the computer could determine automatically the key in which the sample was written by comparing this distribution with arbitrarily assumed distributions for all twenty-four possible keys. By this method it was possible to determine it with perfect accuracy over the range of musical examples tested.

Later on, he added: "Algorithms were devised which can detect key and key change (modulation) within a section or movement".

Finally, in relation to modulation:

> [...]it was observed that with the parameters adequately adjusted, the computer indicated key changes decisively without oscillation between the two keys present (Gabura, 1965).

Unfortunately, he gave no information about the "arbitrarily assumed distributions" or the algorithms devised.

In his reworking of the article for the 1970 compilation "The Computer and Music", he explained it this way:

> For each of the excerpts coded it was possible to determine the key simply on the basis of the pitch-class distribution of the excerpt. To do this the excerpt distribution is matched against a set of *key numbers*, which define the diatonic pitch classes contained in each of the possible 24 keys (Gabura, 1970).

He went on to explain the matching process, giving two alternative mathematical methods, in the first of which the key is assigned to the key index for which the dot product between the distribution of pitches and the key numbers is maximum. But again he did not explain the nature or provenance of the set of key numbers. If there was a set of "key numbers" like he suggested, it would certainly be not 'arbitrarily assumed' but a set that in an essential way represented the tonal system.

More recently, in The Cognitive Foundations of Musical Pitch (1990), Carol Krumhansl followed a similar path in trying to identify the key with her key-profile algorithm. In this method, the piece or except is represented by its “input vector”, a 12-value vector where each value is the total duration in seconds of one of the pitch classes in the piece. The key is represented by a “key-profile”, another 12-value vector representing one particular key. There are 24 key profiles for each of the major and minor keys. The key of the piece is identified as that whose profile has the largest correlation with the input vector. Since the difference between the correlation and the dot product is about normalization, David Temperley (1999) suggested simplifying the algorithm by replacing the former by the latter. In other words, the method is essentially identical to Gabura’s. The problem is how to get the numbers that make the key profiles. Krumhansl’ solution was experimental: A number of subjects “were asked to rate how well a pitch class fit with a prior context establishing a key, such as a cadence or scale” (Krummhansl & Kessler, 1982, quoted by Temperley). All the key profiles are rotated versions of one another. Table 1 shows the values for each pitch-class arrived at by Krumhansl, as listed in her 1990 book (quoted in Temperley, 1999):

Table 1 - Krumhansl-Kessler key profile

I	I♯-II♭	II	II♯-III♭	III	IV	IV♯-V♭	V	V♯-VI♭	VI	VI♯-VII♭	VII
Major											
6.35	2.23	3.48	2.33	4.38	4.09	2.52	5.19	2.39	3.66	2.29	2.88
Minor											
6.33	2.68	3.52	5.38	2.60	3.53	2.54	4.75	3.98	2.69	3.34	3.17

The success of this algorithm was limited. Temperley (1999) suggested a number of ways to improve it, in particular adjusting the key profile. His revised values “were arrived at using a mixture of theoretical reasoning and trial and error”. Temperley’s is a significant improvement over the original method but still leaves a good margin for uncertainty, and he concluded speculating about other factors – psychological, musical – “which appear to play a role in key finding”. In other words, he did not think analysis could be excluded from the key-finding algorithm.

However, more than twenty years before Gabura's paper, Helen Budge (1943) produced the already mentioned thesis, "A Study of Chord Frequencies (Based on the Music of Representative Composers of the 18th and 19th centuries)". The study was undertaken to show the relative frequency of the chords occurring in diatonic harmony, for which the statistical information had been lacking.

Since computers were not available at the time, her study was based on hand-made harmonic analyses. She selected 24 composers from François Couperin to Edward MacDowell, and analyzed a large number of their works – mostly excerpts but including some substantial works in their entirety: Handel's Judas Maccabeus, Mozart's Symphony in G minor, Schumann's Carnaval and Mendelssohn's St.Paul. A total of 65,902 chords were hand-counted of which 11,049 were chromatic. Diatonic chords were classified and tabulated, and their relative frequencies calculated.

As could be expected, she found that chord frequencies vary along time. However, the changes in the figures are much slighter than it could have been expected. For example, the frequency of the Tonic triad goes from 22.89% in early 18th Century to 19.69% in late 19th Century. Along the period of common practice, the frequencies of the main chords do not vary so much that their relative positions could change.

CHORD	FREQUENCY
I	34.37
V^7	14.67
V	11.25
IV	7.74
II	4.52
VI	4.18
II^7	2.00
VII	1.70
III	1.13
VI^7	0.94

Table 2 - Budge's Overall Chord Frequencies

Table 2 shows her overall results for the 10 chords with the largest frequencies as a percentage of the total of diatonic plus chromatic chords. As usual, chords are labeled with Roman numerals, in which I stands for the Tonic, II for the Supertonic, III for the Mediant and so on (see Appendix III). The superscript 7 indicates a seventh chord.

This shows that tonal harmony, whether from the baroque or the post-romanticism, has a strong peak at the tonic, a secondary one at the dominant, a tertiary one at the subdominant. This realization brought home the idea that these frequencies epitomize the tonal system, and the statistics for any tonal piece should match these figures more or less closely. Consequently, they could provide a method to build the unexplained set of key numbers referred to by Gabura.

However, these figures cannot be used directly as they correspond to the frequency of chords rather than pitch classes, and it is these that are needed. It would be necessary to use Table 2 to derive the implied frequencies for all pitch classes. The task would require some reasonable assumptions about the distribution of loose notes.

In addition, the information from Budge is incomplete in the sense that she counted but did not classify the dissonant chords. However, considering that these are a set of roughly fifty chords that account for only 16.76% of the total, the effects of the imprecision of an educated guess cannot have a severe effect on the outcome.

Robert Ottman, in Advanced Harmony gives a list of all the altered chords that "enjoy some degree of usage". In Ottman's nomenclature, capital Roman numerals indicate major and small case Roman, minor. The superscript "^{o}" indicates a diminished triad, and the superscript "^{d}" indicates diminished seventh chords. Eliminating repetitions – for example $i_{\sharp}^{d7}$, iii^{d7}, v^{d7}, and $vi_{\sharp}^{d7}$ are enharmonic inversions of the same chord (see Appendix III) –, there are 52 types of altered chords left. They have been tentatively assigned frequencies of 1% , 0.5 % or 0 as follows:

Borrowed chords (Major only, as in minor they are secondary dominant):

i: 1%

ii^{o}; ii^{o7}: .5%

iv: 1%

Diminished seventh chords:

$i_{\sharp}^{d7}$; $ii_{\sharp}^{d7}$; vii^{d7} : 1%

Neapolitan Sixth:

II♭: 0.75%;

Secondary dominant chords:

I^{-7}; III♭: .5%;

II^7; III^7; VI^7; $VI♭^7$; VII^7: 1%

This leads to the working assumptions about the frequencies of scale degrees listed in Table 3.

Table 3 - Frequencies of scale degrees

Scale Degree	Major mode	minor mode
I	16.80%	18.16%
#I-♭II	0.86%	0.69%
II	12.95%	12.99%
#II-♭III	1.41%	13.34%
III	13.49%	1.07%
IV	11.93%	11.15%
#IV-♭V	1.25%	1.38%
V	20.28%	21.07%
#V-♭VI	1.8%	7.49%
VI	8.04%	1.53%
#VI-♭VII	0.62%	0.92%
VII	10.57%	10.21%

These percentages form a first approximation to a suitable set that could be used in place of Gabura's key numbers and could be construed as the "key pro-file". It should be remembered that

- These numbers are based on a limited statistic – and at that, one that tries to encompass the whole period of common practice. It cannot be assumed that they will work for the range of styles covered in the database.

- Their derivation involved a number of reasonable but speculative assumptions about the frequencies of several chords on which there was no available information.
- The figures for the minor mode were adjusted proportionally in order to equal the norms of the major and minor vectors. This was done to preclude system bias in favor of either mode.

The most natural way to make the set more accurate would be to apply the software to the database, carrying out separate statistics for major and minor (disentangling them from within pieces), counting the notes in both cases and finally producing an improved estimation free of speculation. This work was finally carried out – See Chapter 5.1.

Notice that the frequency of the dominant, both in minor and minor, is greater than the tonic. This, of course, is the result of the fifth degree of the scale being part of both the chords of greatest frequency. As expected, the differences between major and minor are concentrated in the so-called modal degrees.

3.6.3 The algorithm

Let us consider a number of 12-dimensional orthogonal spaces with the same orientation. Let $\boldsymbol{P_0}$ be the space of the 12 pitch classes, so that p_0 corresponds to the note C, p_1 to C#, and so on. Let $\boldsymbol{P_i}$ $(1 \leq i \leq 11)$ be spaces whose dimensions coincide with those of $\boldsymbol{P_0}$ after i cyclical rotations of the axis around the diagonal of the first quadrant.

Given a brief musical non-modulating tonal excerpt, it is possible to measure the accumulated duration of each of the pitch classes in it. The excerpt could then be construed as represented in $\boldsymbol{P_0}$ by a vector **B**, with each component being the accumulated duration of the corresponding pitch class in the excerpt. This vector could be represented in all the 12-dimensional spaces of the pitch classes $\boldsymbol{P_0}$ through $\boldsymbol{P_{11}}$. In this way each or these spaces contains a rotated version of **B**.

Let $\boldsymbol{S}$ be the space of the scale degrees, so that s_0 corresponds to the Tonic, s_1 to the Raised Tonic, s_2 to the Superdominant and so on. Let us accept that two ordered sets of 12 numbers exist that respectively epitomize the major and minor modes, each number representing the ideal amplitudes of the scale degrees: Tonic, Raised tonic, Supertonic, and so on, and let us call these sets "key profiles", following Krumhansl. Each set of 12 numbers could be taken as a the components of two vectors $\mathbf{A_1}$ and $\mathbf{A_2}$, i.e. vectors whose components represent the respective scale degrees for the Major and the Minor scales. Let us now superimpose $\boldsymbol{S}$ with each of $\boldsymbol{P_0}$ through $\boldsymbol{P_{11}}$, so that s_0 is congruent with p_0 , and let us form the 24 sets of dot products $\boldsymbol{A_1 \bullet B}$ and $\boldsymbol{A_2 \bullet B}$; the maximum among the resulting scalars corresponds to the representation of **B** in the space $\boldsymbol{P_j}$. This means that the Tonic of the test piece is $\boldsymbol{j}$ semitones rotated in relation to C, and the key is either major o minor according to which of $\mathbf{A_1}$ or $\mathbf{A_2}$ the maximum dot product corresponded to.

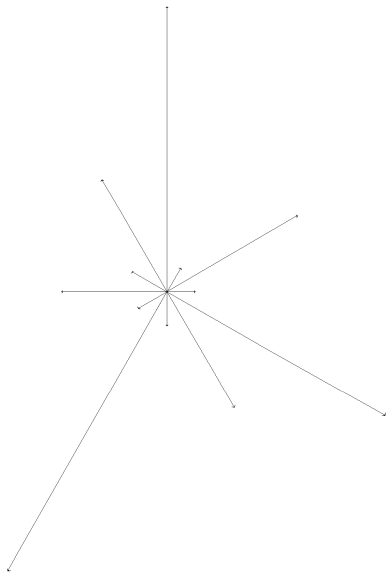
There is a simpler way to visualize this process. The components of each vector could be plotted on a plane, radiating from a point, like the spokes of a wheel separated by 30º angles. The result of doing this with the two key profiles $\mathbf{A_1}$ and $\mathbf{A_2}$ is shown in Figure 2. It is apparent that the shapes of these two wheels are quite different.

If we now consider the expression for the dot product in terms of the rectangular components,

$$\mathbf{A} \bullet \mathbf{B} = \sum_{i=0}^{i=11} A_i B_i \qquad [1]$$

the right-hand side of the equation could be also interpreted as the sum of twelve dot products between pairs of co-linear vectors respectively equal to the rectangular components of A and B:

$$\sum_{i=0}^{i=11} A_i B_i = \sum_{i=0}^{i=11} \mathbf{A_i} \bullet \mathbf{B_i} \qquad [2]$$



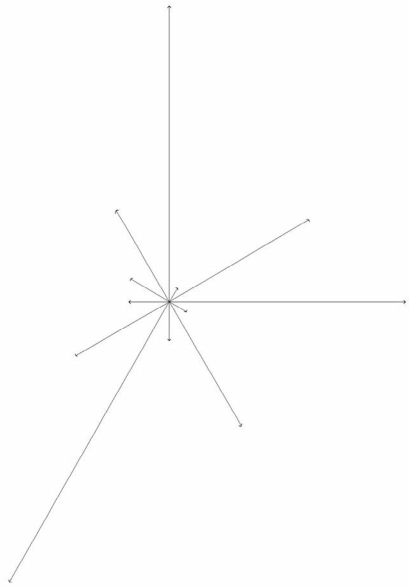


Figure 2 - Major (left) and minor (right) key profiles

The vector **B** of the accumulated durations representing an excerpt could also be portrayed as the spokes of a wheel, as shown in Fig. 4. Thus, the first value for the equation [2], corresponding to the key of C major is obtained overlaying the durations wheel **B** on the major key profile wheel $\mathbf{A_1}$ so that the vector representing C coincides with the Tonic, and calculating the sum of all the dot products between coinciding vectors, that is, adding the products of their respective moduli. Next, the duration's wheel is turned 30º clockwise, which brings the vector representing the note B to coincide with the Tonic. The second value for equation [2], in this position, corresponds to the key of B major. The procedure is repeated turning the wheel each time 30º until the first 12 values for the major keys have been calculated. Next, the same procedure is applied again but this time overlaying the durations wheel **B** with the minor key profile wheel $\mathbf{A_2}$. The result is a set of 24 positive real numbers. The largest number of the set corresponds to the key the excerpt resembles the most.

As an example, let us now consider a small excerpt such as the first fugue theme of J.S. Bach "The Well-Tempered Clavier" (Fig.3). These are just a few notes, and the accumulated durations can be obtained by inspection, or else reference can be made to the output of the program shown in Table 4:

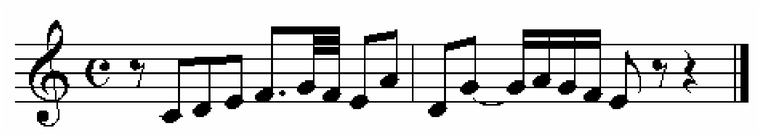


Figure 3 -Theme from Fugue No.1

Table 4 - Accumulated durations for Fugue 1

C	1	4	8.33
C#-Db	0	0	0.00
D	2	8	16.66
D#-Eb	0	0	0.00
E	3	12	25.00
F	3	9	18.75
F#-G	0	0	0.00
G	4	9	18.75
G#-Ab	0	0	0.00
A	2	6	12.50
A#-Bb	0	0	0.00
B	0	0	0.00

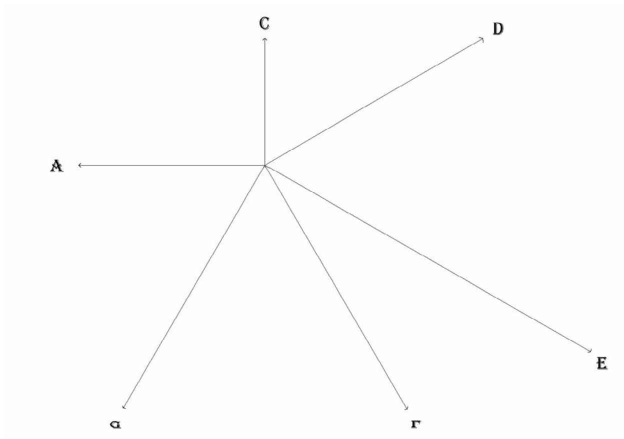


Figure 4 -Accumulated durations wheel

Pitch class	Major	minor
I	16.80	18.16
I# - II♭	0.86	0.69
II	12.95	12.99
II# - III♭	1.41	13.34
III	13.49	1.07
IV	11.93	11.15
IV# - V♭	1.25	1.38
V	20.28	21.07
V# - VI♭	1.80	7.49
VI	8.04	1.53
VI#- VII♭	0.62	0.92
VII	10.57	10.21

In order to calculate the dot product of the accumulated duration's vector and the key profiles, we begin by aligning the Tonic in the Major profile with the note C in the accumulated duration's vector. Using the figures for the revised key profile as they appear in Table 3, repeated here for convenience, the calculation is straightforward. Each component of the key profile is multiplied by the accumulated duration for the note that overlaps it:

C times Tonic	=	8.33 x	16.80	=	139.9440
D times Supertonic	=	16.66 x	12.95	=	215.7470
E times Mediant	=	25.00 x	13.49	=	337.2500
F times Subdominant	=	18.75 x	11.93	=	223.6875
G times Dominant	=	18.75 x	20.28	=	380.2500
A times Submediant	=	12.50 x	8.04	=	100.5000
Total					1397.3785

This result is the dot product for C Major. It only comprises six values because the other six durations are zero. The next step is to repeat the calculation after the accumulated durations wheel has been rotated 30° clockwise, which aligns the Tonic in the major key profile with the pitch class B (although, in this example, the accumulated duration of B is zero), giving the result for the key of B Major. And so on until all 12 possible rotations in major have been carried out. Next, the same process is repeated with the wheel representing the minor mode key profile, carrying out all 12 calculations. For example, for d minor, the pitch D in the duration's wheel is aligned with the tonic minor. The results are:

C times raised submediant	=	8.33 x	0.92	=	7.6636
D times tonic	=	16.66 x	18.16	=	302.5456
E times supertonic	=	25.00 x	12.99	=	324.7500
F times submediant	=	18.75 x	13.34	=	250.1250
G times subdominant	=	18.75 x	11.15	=	209.0625
A times dominant	=	12.50 x	21.07	=	263.3750
Total					1357.5217

In this way, a list of 24 dot products is obtained, one for each of the possible 24 major and minor keys. The program produces the following output:

DOT PRODUCTS	
0	1397.3785
1	387.8772
2	1054.0227
3	1024.0175
4	547.1377
5	1133.0578
6	341.8734
7	1258.0492
8	717.3914
9	773.8360
10	1055.3232
11	531.1676
12	984.3009
13	725.2671
14	713.1223
15	1255.2183
16	428.6243
17	1060.0559
18	564.2982
19	983.4734
20	1032.9143
21	424.7148
22	1357.5217
23	629.6983

Global Key: C Major

Comparing these values, the largest of them turns to be the first one; hence, the found key is C Major as the program output indicates.

The calculation of the key for each of the 24 fugue themes of the first volume of The Well Tempered Clavier has been done as a traditional application. It has to be said that it is not a good application example, as many of the themes are tonally very vague and fail to convey a sense of key. However, the program managed to find the right key in every case. For more details see Appendix V that includes the program output for the 24 Fugue themes.

Since the dot product between two vectors is maximal when they are co-linear, the comparison of these values points to the key whose resemblance to the accumulated duration's vector is the greatest. Comparing visually the graphics of Figs.2 with Fig.3, the resemblance is not impressing, suggesting that the theme is not typically tonal. This is also indicated by the fact that the second most similar key, with a result that is not much smaller, turns out to be d minor, suggesting that this theme, while still resembling C major more than any other key, is not very well shaped in tonal terms. Ultimately, the method should also indicate whether a key is not determinable from the passage. This has to be understood to mean that the music vector **B** is not unambiguously shaped in a tonal way. Notice that even in this case, the nature of the calculation always gives a result for the best alignment for the vector. The criterion to disqualify the choice of key is discussed in the next chapter.

4. Details of the work carried out

4.1 MusicXML

The first task of the software is to parse the MusicXML file, finding the relevant elements and processing them. Thus, it is convenient here to describe the structure of the MusicXML files. The standard includes all of the information available from the score and several additional items. Basically there are two kinds of information included: That pertaining to the sound itself such as pitch, duration or tempo, and that concerning the appearance of symbols in the score such as stem direction, beams, and rest location.

A MusicXML file consists of a header comprising the first 23 lines, followed by the music information proper. The header specifies some basic metadata about the score: Title, composer, instruments involved and MIDI information.

After the header, the specific information begins. Measure 1 includes general data for the whole score. “Attributes” encompasses “divisions” which is the number of fractions of a quarter note required for the score. The idea about “divisions” is to be able to express all the durations by means of integers. In simple cases, this number is one quarter of the maximum common denominator of the durations of the notes in the score. Depending on the rhythmic complexity of the score, this number may become too large to handle. The first page of Beethoven’s Sonata Op.13 is a typical example of this problem. It includes 128ths, a 9-tuplet of 128ths, a 6-tuplet and a 7th-tuplet of 64ths – whose respective denominators are 128, 144, 96 and 112. Thus divisions equals 2016. Currently, MusicXML has a maximum limit value for divisions is 1024, so this example can only be accommodated by approximating some durations.

The specification of the key indicates the position of the tone signature in the circle of fifths – a number from 0 to 7, positive for sharps and negative for flats – followed by the indication of mode. Unfortunately, this last feature is not implemented by any software and it appears as “major” by default. As this is an important bit of information, it requires hand correction for each score whose initial key is minor. The time signature is indicated by means of “beat” (its numerator) and “beat type” (its denominator). This is followed by the specification of clefs for both staves, and finally the tempo indication. See Appendix IV for a full example.

There are a number of alternatives that make for complicated parsing. Every time a new system begins, for measures other than 1, there is an additional line to indicate this fact. Tone-signature changes omit the <divisions> line. Time-signature changes omit <divisions> and <key>; if key and time signatures changed simultaneously within the piece, the only omission would be the <divisions> line.

For measures other than 1, what follows the <measure> tag is the information about each note, typically listing the specification of pitch, duration, and voice, followed by graphical representation information. Pitch is indicated by means of “step” (note name), “alter” (alteration, of magnitude 1 or 2, negative for flat and positive for sharp) and octave (numbered from 0 to 7). Naturally, the note duration is measured in divisions.

There are other variants. Not every note has duration. There are grace notes, for which duration is not specified. For a grace note, the indication <grace/> appears before <pitch>. Grace notes can be counted as notes, but since they carry no duration, a calculation of the durations of each pitch class would automatically disregard them. The same applies to trills, turns, mordents, and generally cadenza-like cascades of grace notes. Tremolos, either written or abbreviated, would be counted correctly. And there are rests, whose duration is specified but have no pitch. Rests can appear simply as <rest/> avoiding the information about placement on the staff.

An important complication involves multiple layers. Quite often, as in a fugue, a single musical part – that is, the music written on a single staff –comprises several voices, which is made apparent in the score by the presence on the staff of simultaneous notes with different durations. For two voices, usually composers set stems up for the notes of one voice and stems down for the other, but there are remarkable inconsistencies, and not infrequently extra voices appear so that there is more than one line with stems in the same direction. In MusicXML there is a “musical counter” to which the duration of each note is added; provision has not been made so that each layer covers the complete measure, as its creator knew that in instrumental music voices appear and disappear any time. Hence the counter does not have to reach the value of the measure duration at its ending. Two voices often coincide in some of their notes, giving rise to cue notes – i.e. notes with double stems, to indicate that they are common to both voices – indicated by the additional line <cue/>.

When the note is part of a chord, the indication <chord/> appears before the <pitch>. While the duration element in a note moves the MusicXML musical counter, a <chord/> element keeps this counter from moving further. For this purpose, the first note in a chord (which is always the one with the lowest pitch) appears as a regular note but each of the others has a ‘chord’ element preceding the ‘pitch’ element. This means that it is not possible to look for the ‘duration’ element in isolation as durations do not have a one-to-one relationship with the notes.

The way it has been defined, the nature of MusicXML is essentially part-wise: When there are multiple layers in a measure, MusicXML lists the notes and rests of the top voice for the whole measure, and then goes back to the beginning and proceeds with the next lower voice and so on. The duration of each note or rest is added to the musical counter. In order to represent parallel musical parts, the counter has to be able to move backwards and forwards, which is done by means of the ‘forward’ and ‘backup’ elements, for which a ‘duration’ is specified.

Different voices may span the whole length of the piece. In a choral piece, where the term “voices” has a concrete meaning, every part lasts from beginning to end. However, in instrumental music, the usual practice is to make “voices” appear and disappear even in the middle of a measure without any further consideration, since it only means a change in the number of fingers of a hand that press keys. This makes it impossible in general to relate the duration of the piece to the accumulated duration of all the notes. To our purposes, in order to estimate accurately the relative importance of the individual pitch classes, all the note durations are added, disregarding the presence or not of the <chord/> element in order to obtain the percentages for each pitch class.

Hence, for a program interested in processing the notes, the basic parsing strategy is to locate a ‘note', then check the next line for either <grace/> and/or <chord/>, or <cue/> or <rest/> (followed by the specification of <display-step> and <display-octave>, or simply <rest/>) or <pitch>. In the last case, the following line is <step>; the optional ‘alter’ that might follow it has to be checked for, then the octave information, and after that, the duration has to be obtained, as long as it is not a grace note.

The MusicXML part-wise format is quite straightforward for any application dealing with melodic information, and many developers have been able to create MusicXML players. However, when the information of interest is harmonic, the format becomes quite awkward. It could be expected that the time-wise format were the answer. Unfortunately this is not the case. Both formats have been implemented using the measure as building block. The part-wise format presents rows of blocks, one on top of another, whereas the time-wise presents piles of blocks, one after another. The inside of each block is the same in both cases, and its part-wise nature is inconvenient for parsing chords.

In order to test the accuracy of the parsing program it was initially made to output the list of notes as well as counting notes, rests, and grace notes.

4.2 Key determination

In the search for features that could be construed as variables for the characterization of style, one essential element is the knowledge of the key. It was necessary to start by implementing software to determine the key by the method of the maximal dot product. In order to do this, it is necessary to collect a large enough number of notes for the calculation to be meaningful. One first way to accomplish this goal would be to calculate over the whole of the piece and extract a global result. This result would be meaningful if pieces did not modulate something that almost never happens. Even simple pieces modulate often and no composer is known to have kept a tally on how long he stayed in the main key, hence the global result in general cannot be expected to coincide with the key the piece is supposed to be in. The majority of pieces begins and ends in the same key, but this does not guarantee the global result. However, the procedure is not difficult to implement.

The parsing program was modified to carry out this calculation using a tri-dimensional array of accumulators, one level per octave, so that all individual notes on the range have their accumulator. The pointer within each level is derived from the note name and the alteration – magnitude and sign – if present. Once the end of the file is reached, the totals are collated in 12 pitch classes and divided by the accumulated duration of all the notes to give percentages for each. These percentages are used as the components of the vector representing the whole piece in the calculation of the dot products.

The first result of applying this program to all the pieces written by Handel, J.S.Bach, Scarlatti, Mozart and Haydn included in the database was, somewhat surprisingly, that the Global key calculation coincided with the expected key in 49 out of 50 pieces, the only exception being the Adagio of Mozart's Fantasy K.457. While this result is encouraging for the method employed, it is of not much further use. What is needed is a tool to determine the key as a point function. Knowing that the method entails collecting a number of notes in order to infer the key, the concept of "point" here becomes fuzzy. Ideally, the piece could be divided in sections and the key of each found. And then each section subdivided further and so on until the process came to an irreducible minimum beyond which there are not enough notes to make a valid calculation.

Within MusicXML it would be really cumbersome to work out an algorithm to implement this method because of the constraints of the format, but there is a simple solution that can be worked out and is close enough to working at the "point" level: Since the natural "unit" of the MusicXML format is the measure, calculating the dot product for each measure is straightforward enough. It entails repeating the calculation done for the global key, extending only to one measure, and do it for each one.

Measure by measure results for Scarlatti's Sonata

Measure	Key
1	G minor
2	G minor
3	B♭ Major
4	B♭ Major
5	B♭ Major
6	B♭ Major
7	B♭ Major
8	E♭ Major
9	B♭ Major
10	G minor
11	B♭ Major
12	G minor
13	G minor
14	G minor
15	G minor
16	G minor

In order to demonstrate the pros and cons of this method, let us consider a brief example, a 16-measure Sonata in G minor by Domenico Scarlatti, which appears in the Dover collection simply as “Erstausgabe”. The score can be found in Appendix VI. As anticipated, the Global key calculation gives the correct result of G minor. Calculating one bar at a time gives quite reasonable results, as the preceding list shows. The Sonata begins in G minor, in measure 3 it modulates to the relative major key, B flat Major, in which it remains until measure 10 or 11 where it returns to G minor.

Apart from the oscillation between B-flat major and G minor in measure 10, the only anomaly is in measure 8. If we examine the notes' durations (quarter note = 8) in the content of that measure, we only find:

C=8	E♭=6	F=6	G=6	A=2	B♭=12

This reveals that there are too few notes and too much tonic (B♭) in that measure. If we remember that the key numbers say that the note with the largest presence is not the tonic but the dominant, it is not surprising that E-flat comes as the key. The five top dot products for the measure show additional detail:

E♭M = 1546.8124

B♭M = 1466.3928

b♭m = 1456.8026

e♭m = 1349.6004

f m = 1218.507

These figures show that B-flat major is trailing E-flat by only 5.2%. This result suggests the convenience of adopting the criterion to analyze as long as possible in the currently established key. Fluctuations in the key prompt to compare the figures. In cases like this where the difference between the newfound key and the previous one is very low, it is reasonable to stay with the previous key. This principle also applies to the oscillation in measure 10. It is apparent that the method yields here a quite acceptable result.

Another brief example, Handel's Sarabande from Suite No.16 – the score can be also be found in Appendix VI –, coincidentally also in G minor, reveals the limitations of applying the method to measures. Whilst the global key is again correctly identified as G minor, the measure-by-measure calculation yields:

Measure by measure results for Handel's Sarabande

Measure	Key
1	G minor
2	G minor
3	B♭ Major
4	F Major
5	C minor
6	F Major
7	C minor
8	D Major
9	G minor
10	D minor
11	D minor
12	D Major
13	B♭ Major
14	E♭ Major
15	B♭ Major
16	B♭ Major
17	C Major
18	D Major
19	G minor
20	G minor
21	C minor
22	G minor
23	G minor
24	G minor

This is a simple piece, and it certainly is not continuously modulating. The problem is due to the measures being too brief. Inspection reveals that measure 2 only contains the G minor triad, measure 4 only the F major triad. Nobody could possibly guess better using only the information for the current measure.

It is apparent that in some cases a measure may contain an adequate number of notes for a consistent calculation of the dot product, but in other cases the scarcity of notes produce unsatisfactory results.

The method requires enough notes for the key to be established, just as any human listener would. As it is not possible, given the nature of music, to ensure that a certain time interval will provide them, the best solution would be to program an adjustable window to perform the calculation. The window would slide forward adding new notes while at the same time dropping the oldest ones; its width could be varied to optimize stability. The result of this process would be a set of 24 apparently continuous functions representing point by point the resemblance of the excerpt to each of the major and minor keys.

The internal structure of a measure in MusicXML does not vary with the format. This is certainly an inconvenience because programming a sliding window out of the information parsed from a measure is not an easy task. By and large, the notes in a keyboard piece come all mixed up. For instance, a chord is read from the bass up but this applies only the notes for the right hand. The part of the chord that belongs in the bottom staff only appear after the top one has been done with, so that ultimately the notes of a particular chord will appear spread along several pages of XML code. In order to perform the calculation it would be very convenient to convert the format to one that was truly time-wise.

The solution would be a program that took input in XML and generated a file with the appropriate format. The logical way to organize this file would be as a list of successive vertical sonorities – a sort of array of time slices of the score. Each slice will be made up of all the different notes present at a certain moment, and it will end at the next note change, i.e. it would be as short in duration as the shortest note present at each particular point.

Once the particulars of this format were chosen, an attempt was made to see whether it would be possible to convert it to MusicXML in a simple way. One measure from Scarlatti's "Erstausgabe" in part-wise format was converted by hand to the new format. This example was chosen for several reasons: It has voices, chords, grace notes, a mordent and a rest.

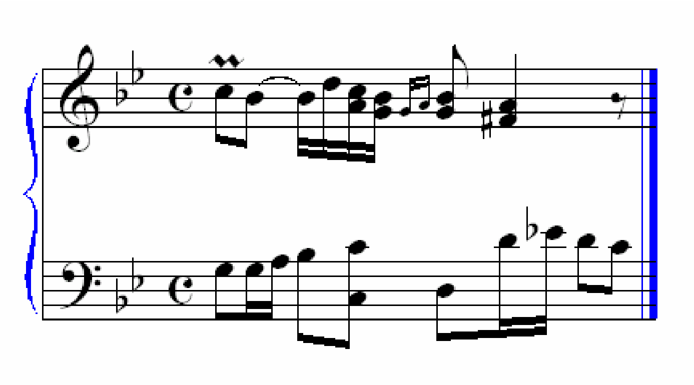


Figure 5 - Second measure from a Scarlatti's Sonata

In a keyboard score like this, the format (part-wise or time-wise) describes the notes beginning by the top staff from left to right up to the end of the measure, then it backs up to the beginning of the measure and then does the same thing for the bottom staff. In this way, the notes of a chord that is partly on each staff, like the C-C-A-C in the example, are listed beginning by the top A in the top staff, as a chord with the C, and some pages later, the bottom C of the bottom staff, as a chord with the top C. As can be seen in the code (see Appendix III), the three chords it contains are scattered along seven pages of XML code.

A format was needed that showed the four notes together, preferably from bottom to top, and did the same advancing in time for every vertical structure. In other words, a format that was truly time-wise in a sort of 'molecular' way rather than in block. Hence the decision to describe the score as if it was made of a succession of vertical "slices", so that the contents of each slice are described from bottom to top, one at a time, from left to right. This example would show a first slice of duration 4 made up of G3 and C5, a second of duration 2 made up of G3 and B♭4; the next, duration 2, comprising A3 and B♭4 and so on.

The proposed slice-wise format has been put together manually by cutting and pasting from the part-wise version, with only one exception. There is one single change that was not made just by sorting out and copying: Since “slices” are vertical structures, there is no use for the <chord/> element, and for this reason this is not used. But instead, a sort of 'dual' entity has to be introduced. Recall that in the existing format, <chord/> is necessary to indicate that a note is simultaneous with another note already described, so that its duration is not counted. It could be construed as a way to indicate that the note that follows extends toward another dimension. Likewise, in the slice-wise format it is necessary to indicate that a note that participates in a slice had already appeared in a previous slice and so it continues into the current one. It could be thus seen as a way to say that the note extends in another dimension. To that purpose the <continuing/> entity was introduced, and the notational elements that do not apply were omitted, leaving only the pitch elements. Considering the inconvenience of the lack of a truly time-wise format in the MusicXML definition, this format has been proposed to the originator of MusicXML as a third alternative. The example appears after the part-wise in Appendix IV. Meanwhile, as for this project there is no advantage in having a MusicXML-compliant format, it was decided that the program would generate a much more compact slice-wise code discussed below.

Since the width of the slice equals that of the shortest note in it, is it easy to see that every flurry of 128ths would create a cataract of slices probably containing one note each. This is a concrete problem not entirely solved in the existing formats which corresponds to the limit in the value of “divisions”. For practical reasons, the decision was taken that any note shorter than a one-eighth of the beat-type would be ignored because typically such short durations mean they are ornamental fluff. For this purpose, a shortest-time limit is derived from the time signature.

The slice-generating program keeps a counter duplicating the MusicXML musical counter, i.e. counting forward the duration of each note and rest, stopping at every chord, incrementing and decrementing for each <forward> and <backup>. It creates an entry (slice) for each note of minimum meaningful duration. The resulting file structure allows for analysis of key by means of a variable window, plus analysis of harmony.

There are three events of interest for the clock: <note>, <backup> and <forward>. The last two simply subtract or add its duration to the clock.

Initially (for each measure) if there are changes in time signature, the values of divisions, beat and beat type are saved, the time slice duration and the shortest-time limit duration for notes not to be disregarded are recalculated. Subsequently, the duration of every note and rest is added to the clock while grace notes are ignored. Whenever a 'backup' or 'forward' is found, its duration is subtracted or added to the clock. Provided that the duration is not less than the lower limit, the note or rest information is placed in as many slices of unit width as required. The array is bi-dimensional – time and layer–. This requires two indexes, *r* for slice pointer, and *s* for layer pointer, so that the pair always points to the first available spot on the array. If there were no <chords/>, *r* and the clock would be identical. But the clock is not incremented when there is a chord, so that the <chord/> element determines that a new layer has to be added, so *r* has to be first decreased and *s* incremented, and then *r* added the chord note duration. Naturally, this process leads to empty spaces in some layers, but this is irrelevant as those spaces are indistinguishable from actual rests.

All this process is carried out for the goal of immediate availability of vertical information. If this were a general-purpose format, all the information would be valuable and would have to be kept. However, there are a number of information items that are of no use to this project, for example octaves, since notes duplicated at different octaves, to every practical harmonic purpose, counts as a single note. Another item that is not considered – even if this would horrify any musician – is chord inversion – see Appendix III. The goal is to identify what notes are present in the slice, and inversions are irrelevant in this respect. The notes in each slice are then cleaned from the octave information and sorted out from lower to higher pitch, a crucial step for the identification of chords.

Sometimes two successive chords, leading originally to different slices, may turn to be identical, for example if there was only a voice exchange between them. Moreover, some 'chords' may turn out not to be such as in the case that it comprised many notes repeated in different octaves. It can even turn out to be a single note, as in the beginning of Mozart's Sonata K.457. For these reasons, once the slices are generated, successive ones that are identical are grouped together and their durations added. This process leaves untouched the structure of the chord plus the duration;

An important clarification has to be made regarding sorting by pitch. Each one of the 12 pitch classes can have three different names – e.g. D# – E♭ – F𝄫 – (except for G#/A♭ which has only two). That is, there are 35 names for 12 pitch classes, which makes a nightmare of the problem of sorting note names by pitch. The solution implemented in the program, (which would work even if it was desired to keep the octave information) is as follows:

First, assign note names the following numbers:

C –▶ 0   D –▶ 6   E –▶ 12   F –▶ 16   G –▶ 22   A –▶ 28   B –▶ 34

Second, make the names in successive octaves the same modulo 37 (i.e. add to these names the octave number times 37).

Third, in order to take care of alterations, add the following to the previous values:

𝄫 –▶ -5   ♭ –▶ -3   ♮ –▶ 0   ♯ –▶ 2   𝄪 –▶ 5

The result (see Table 5) is a set of mostly different numbers for each note name. Only six out of 35 have to be adjusted individually adding or subtracting 1 in each case, which makes for a simple routine. For:

C♯ add 1; D♭ add 1; F𝄫 subtract 1; F♮ subtract 1; G𝄫 subtract 1; and C♭ add 1;

Table 5 - Numerical Code for Note Names

pitch class 1	B♯ = 36	C♮ = 0	D𝄫 = 1
pitch class 2	B𝄪 = 2	C♯ = 3	D♭ = 4
pitch class 3	C𝄪 = 5	D♮ = 6	E𝄫 = 7
pitch class 4	D♯ = 8	E♭ = 9	F𝄫 = 10
pitch class 5	D𝄪 = 11	E♮ = 12	F♭ = 13
pitch class 6	E♯ = 14	F♮ = 15	G𝄫 = 16
pitch class 7	E𝄪 = 17	F♯ = 18	G♭ = 19
pitch class 8	F𝄪 = 21	G♮ = 22	A𝄫 = 23
pitch class 9	G♯ = 24	A♭ = 25	
pitch class 10	G𝄪 = 27	A♮ = 28	B𝄫 = 29
pitch class 11	A♯ = 30	B♭ = 31	C𝄫 =32
pitch class 12	A𝄪 = 33	B♮ = 34	C♭ = 35

As all the numbers are different, it is possible to go back to the original note names once they are sorted out. Only two numbers (20 and 26) from 0 through 36 do not receive an associated note name.

With this information, the program generates a new file that is really the time-wise version of the XML file, adding some additional information in case it is needed: A header with the name of the composer and title of the work.

Then, the typical line representing a C major triad slice is of the form:

0×12×22×~8

where the numbers represent the notes according to the convention; the "×" character (ALT-158) is a separator (better than a space) and the "~" (ALT-126) precedes the duration. All the lines with text information begin with the non-printable tag character (ALT-255) in order to be skipped when necessary. An example to show the structure of the resulting file follows:

 Mikrokosmos 126
 Bela Bartók
 Global Key: C Major (0)
 Measure 1
 divisions 8
 Key signature -1
 Reported key F major
 time signature 2 / 4
 measure duration = 16
 limit duration = 1
0×12×22×~8
0×12×18×28×~8

Measure 2

time signature 3 / 4

measure duration = 24

0×6×22×31×~8

0×6×15×28×~4

0×6×12×22×~4

0×6×15×~4

0×6×12×22×~4

Of course, a slice may have only one note:

25×~128

or just a rest:

~384

It turns out that slices that are different in terms of note names may turn out to be identical once reduced to pitch classes. It would reduce the chance of insufficient information if all contiguous slices were assured to be different. Thus, an auxiliary program was written to condense the slices once they have been generated. It reads the input file and whenever two successive slices are identical, instead of writing it down it adds their durations. This should be straightforward enough were it not for unpredictable presence of info lines – such as change of time signature, which must be passed to the output as they are, but making sure the timing is correct (i.e. the writing is always delayed because the current slice is always being compared with the previous one, which has not been written yet).

A section of a musical excerpt represents, to a variable extent, the key it is in, which could be considered as a constant value surrounded by noise. In this context, a modulation – in particular a direct one – would be equivalent to a step function. A sliding window is a standard means of filtering out noise. Side effects of the window are the narrowing of the measuring range in an amount equal to the window width and the fact that both the amplitude and the delay introduced are proportional to the width, as illustrated in Fig. 6 below. The function f consists of a series of steps, and just underneath, functions f_1 and f_2 represent the results of measuring f by means of two windows of different widths. It is convenient to consider the window as a device whose measuring point lies in the middle of the width. Considering that the initial value output by the window corresponds to the point in abscissas equal to its width, for the results to be correctly timed, the data have to be backdated an amount equal to one-half of the window width. Thus, in Fig. 6 both functions are shown backdated by half the respective widths w_1 and w_2. As can be seen in the graphic, both the amplitudes measured and the delays introduced by the window are proportional to their widths, and the measuring point indicates in all cases the correct timing of the step events.

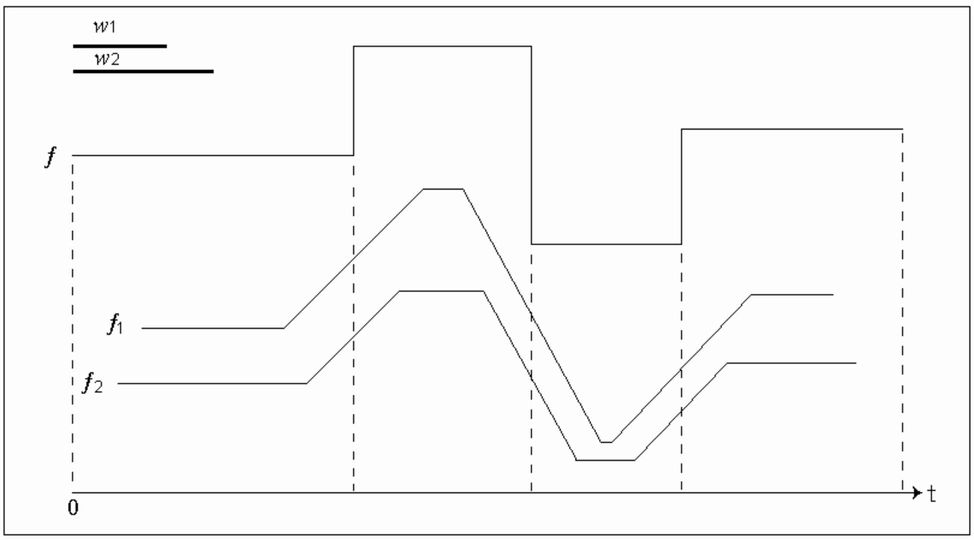


Figure 6 - Effect of changing the window width

A new program implemented a 15-slice wide sliding window in the manner described, in order to determine the key in a precise way. Running it on Handel's Sarabande , the values for the three keys that at some point reach the top, are plotted in Figure 7. The small divisions on the abscissa represent the slices. The vertical lines and the numbers in between correspond to the measures. Notice that the number of slices per measure varies. The curves represent the values of the dot product for the keys of B flat Major (blue), G minor (yellow) and D minor (cyan). For a baseline, two additional curves show the dot products for the keys it resembles the least, E major (purple) and c sharp minor (brown). The score comprises 99 slices. Since the window width is 15 slices, the diagram shows 85 slices (total – width + 1). The first slice represented is number 7, which is the last one in measure 3.

From Fig. 7 it can be observed that the key is B-flat Major up to the beginning of measure 6, G minor until the beginning of measure 10, D minor until early in measure 13, B-flat Major until the beginning of measure 17 and G minor again until the end. Unlike the succession of rapidly changing keys in the measure-by-measure procedure, this analysis makes musical sense, and can be verified on the score (see Appendix VI).

For a more recent example, Fig. 8 shows the results for the Prelude Op.34 No.14 of Dmitri Shostakovich. The program output recognizes six sections and four keys: e flat minor (brown), b flat minor (cyan), G flat major (purple) and E flat major (yellow). The corresponding resemblance functions, done here with a window 20 slices wide, have been plotted together with those for the two most tonally distant keys, C major (blue) and a minor (violet), for the sake of comparison. Again, measures are indicated by vertical lines, and measure numbers have been written near the top.

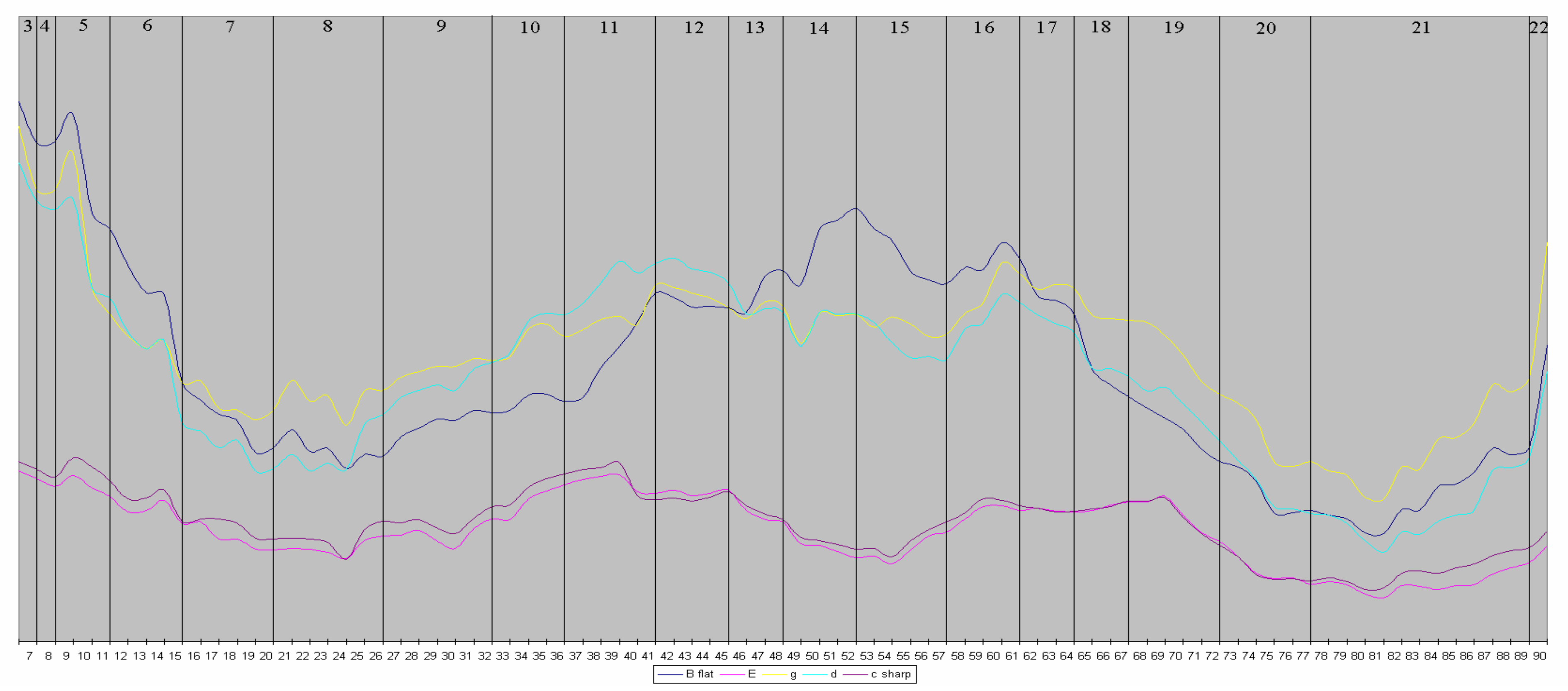


Figure 7- Main keys for Handel's Sarabande

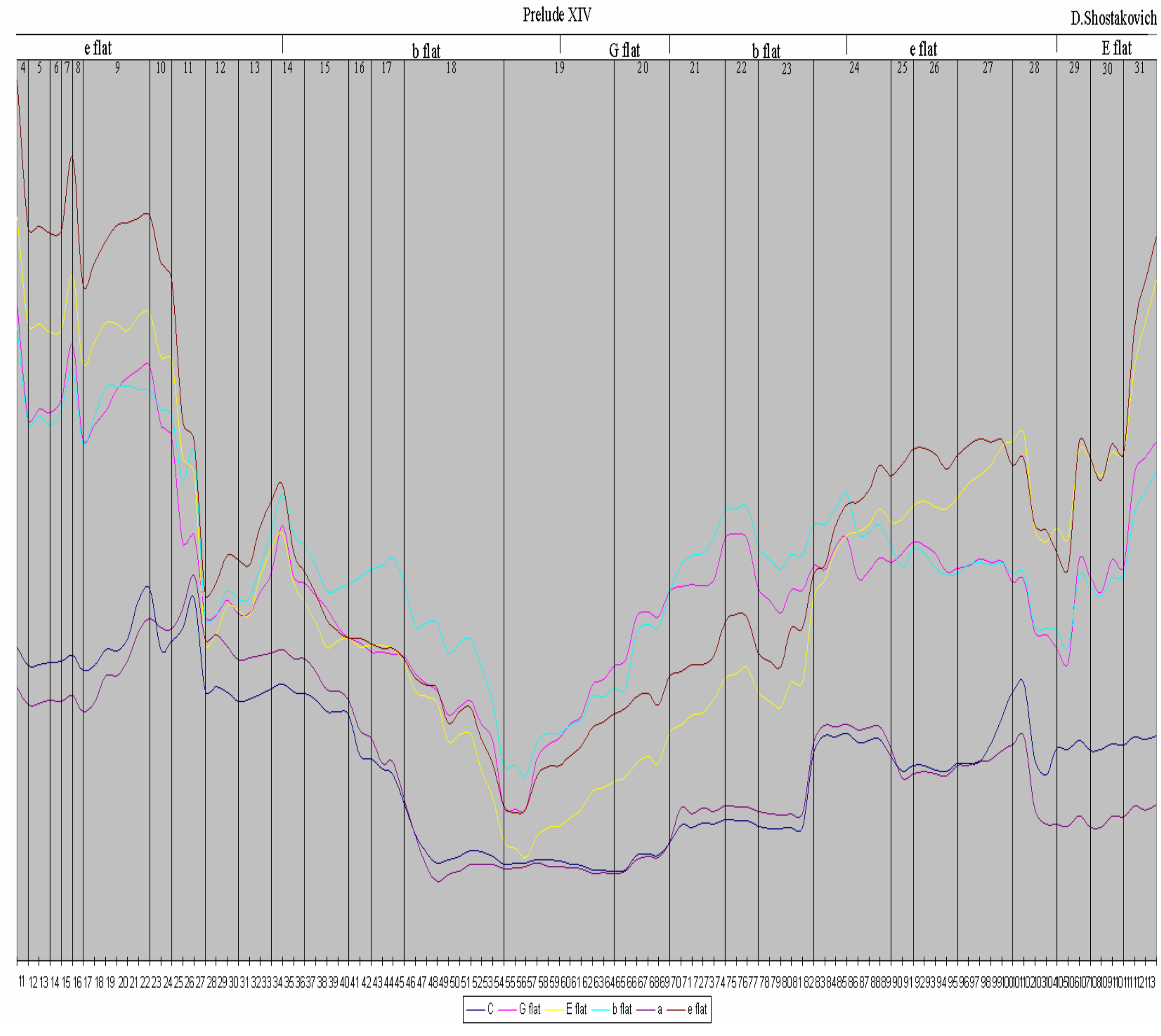


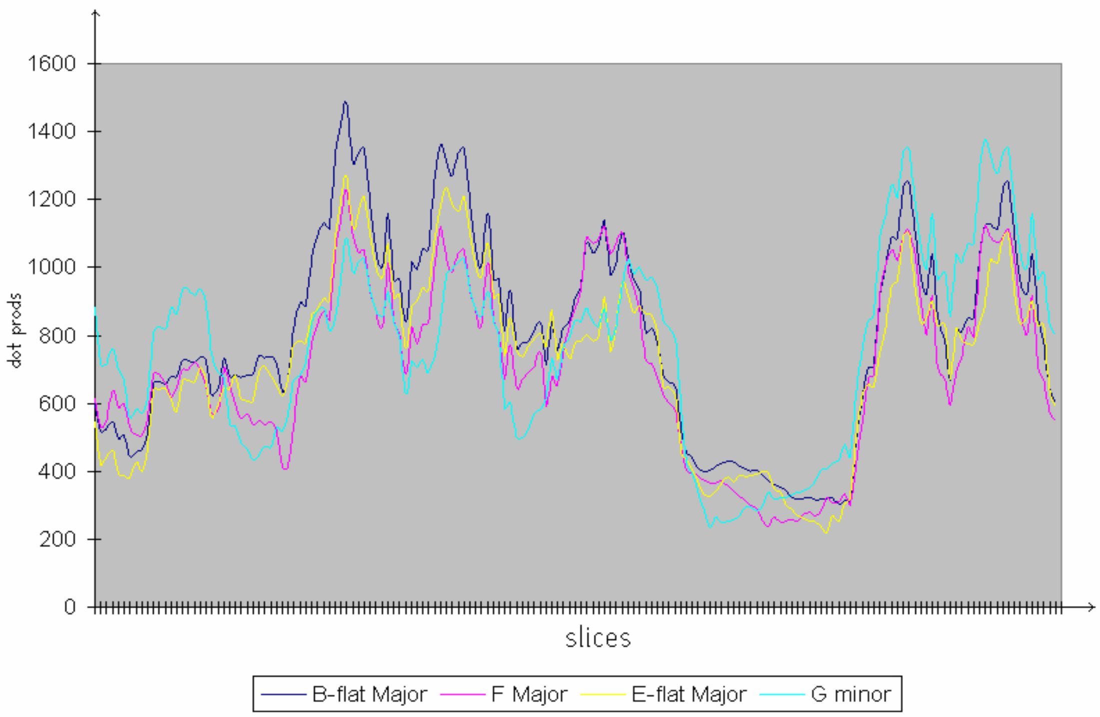
Figure 8 - Main keys for Shostakovich Prelude

Modulation is something whose location can seldom be indicated with total precision. Classicism favored above all, the modulation to the dominant and the method of the pivot chord – that is, a chord that exists in both the original and the new keys. This generally meant that more than one chord could be seen as the pivot and often there was a time interval that could be construed as being in either key. It is the direct modulations that allow accurate localization.

4.3 The width of the sliding window

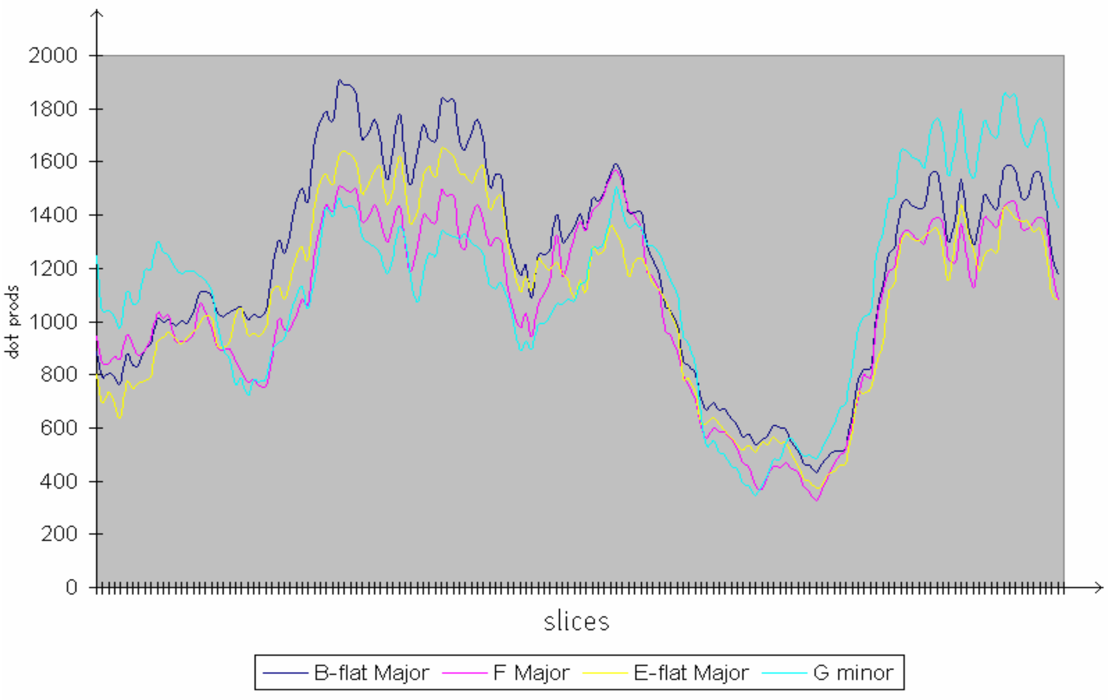
The window width is a variable that determines the filtering characteristics. Too narrow, as exemplified by the measure-by-measure calculation in Handel's Sarabande, brings about spurious results. Too wide, and the integrating effect may make some fugitive key changes (tonicizations) disappear. In order to investigate this issue, the program was run on the Scarlatti Sonata presented earlier, with three different widths: 10, 15 and 20 slices. They are shown in figures 9 to 11 respectively.

Figure 9- Sonata in G, narrow window



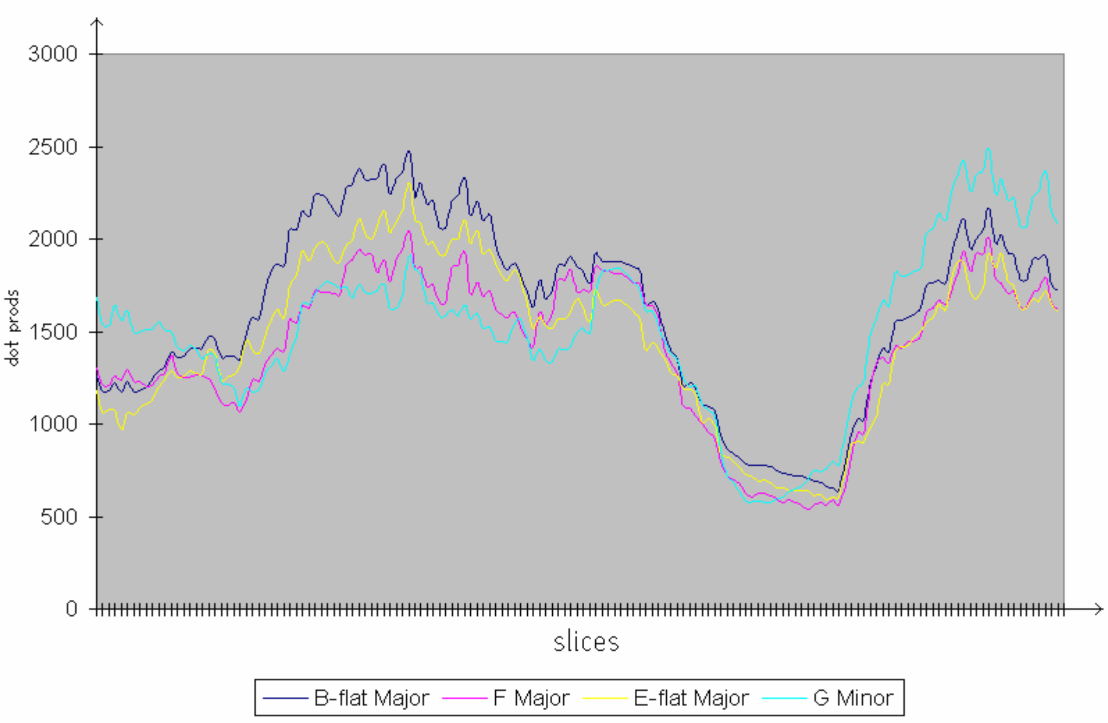
It is easy to notice in Figure 9 how noisy the short window is which allows for a number of spurious "keys" to be identified. The curves corresponding to F Major and

E-flat Major are worthy of attention because they briefly reach top value. Figure 10 - Sonata in G, medium window



Brief key changes in the narrow window can be traced to the presence of a single chord, making apparent that it shows too much fine detail. Figure 10 shows the case of the medium width window:

Figure 11- Sonata in G, wide window



The same keys almost match top value whereas in Figure 11, the wide window, they are close but never get to the top. The wide one is the most stable but it is possible that it rounds things too much, so that some tonicizations could be missed.

The comparison suggests that the medium-size window width seems just right.

However, this is a parameter that cannot be invariable. The most convenient value depends on the piece. The executable version of the program will have a default value of 15 but allowing the user to adjust it. The same applies to the threshold in the number of slices that bring about a key change. A practical default value is 5 but this will also be adjustable.

4.4 Measure of tonalness

There is one other problem that has been briefly mentioned: The dot product calculation unavoidably has a maximum pointing to some key, no matter how non-tonal the material may be, simply because there has to be one dot product that is larger than all the others. But there are cases in which the music does not have a definite key. In order to constitute a general method to determine the key, it is imperative that the program includes the diagnostic that a key is not determinable from the music.

There is a wide margin for speculation about what in the musical fabric are the essential elements that convey the sense of key. As it has been shown by Budge, tonal music has a strong peak in the tonic, a secondary one in the dominant and a tertiary one in the subdominant, which represent respectively some 35%, 25% and 8 % of the chords. This means that out of three chords, one has to be a tonic and two have to belong to this group. So a possible criterion would be to check whether at least these basic proportions are present. But the idea that tonic and dominant should suffice to establish the sense of key finds an immediate counter-example – already mentioned – at the introductory 30 seconds of Beethoven's Ninth Symphony whereby the exclusive presence of both notes leaves the mode undetermined.

Since there is not a harmonic criterion available, it would be more convenient to deal with scale degrees rather than chords. And this means that the dominant should have a presence of about 20%, then the tonic (~ 18%), the mediant (~13 %), the supertonic (~12%) and the subdominant (~11%). Together, these scale degrees represent three out of four notes. Now, the fact that the last three have about the same frequency suggests they are already marginal values, so it is problematic that significant changes in their relative frequencies would have an impact. It would take time-consuming research to put numbers to the boundaries of the sense of key.

A different approach suggests that rather than the relative presence of certain elements, what may be critical is the relative absence of others. What could be the minimum that a piece needs to have in order to sound tonal?

Figure 12 shows the result for the Scarlatti Sonata in G for all 24 keys. Most of them follow a similar general pattern but the values have a large spread. Extreme key values differ in a factor that varies between 4.51 and 2.81.

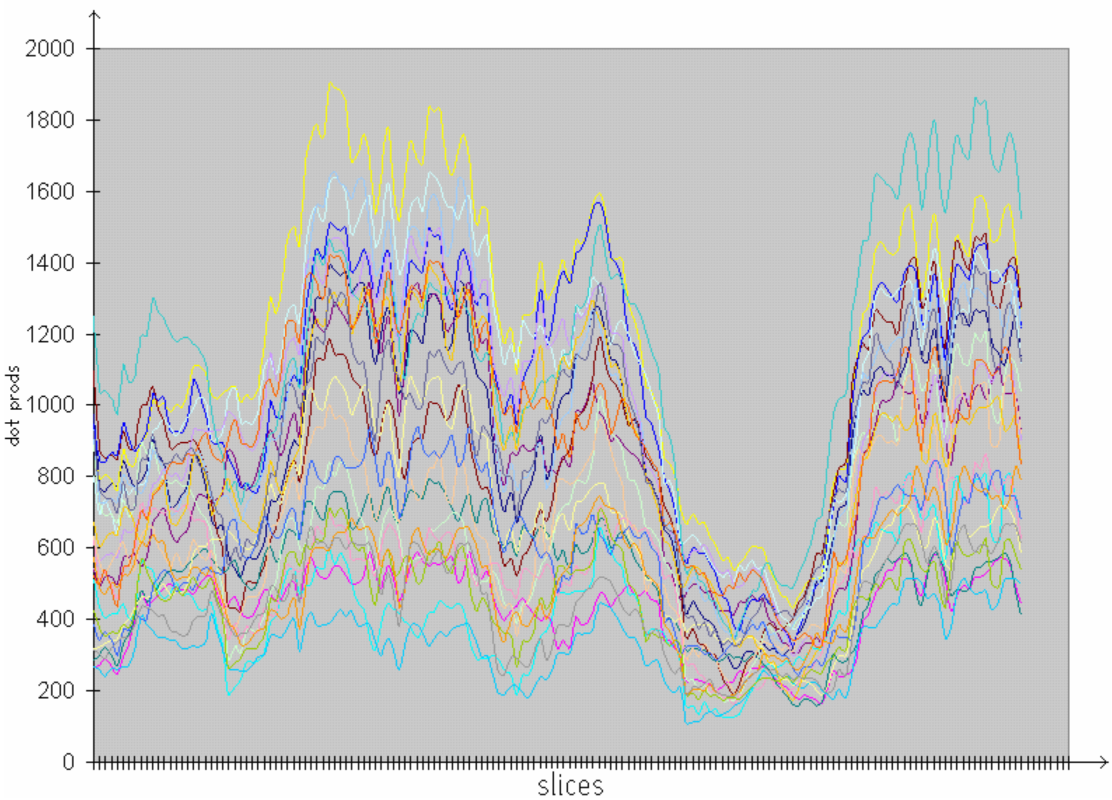


Figure 12 - Range of dot product values

To provide a contrast, consider an imaginary piece where the notes were chosen at random. The calculation of the dot product for different keys would yield very similar figures for them all. In other words, the spread of values in this case would be much lower than in a real piece. What seems to point at a definite key, then, could be a marked disparity between the values of the dot products for different keys.

The preceding consideration is not only a thought experiment, but is amenable to investigation, and for this purpose, a 26th composer was added to the database, Ernest Krenek, whose Op.83 is a collection of 12 short piano pieces in the 12-tone technique. The majority of the listeners feel that 12-tone pieces sound as if the notes had been chosen randomly, which means that those pieces reliably avoid conveying a sense of tonality. It must be remembered that 'randomness' is not an unavoidable feature of the technique but rather a choice of the composers of the Second Viennese School and their followers. However, the nature of the technique itself, by reason of its main rule, – that the twelve pitch-classes have to be sorted in a 'series', and none of them can be repeated before all the others have appeared – is biased toward an even distribution of the 12 pitch classes, as opposed to the tonal system which is based on a hierarchy of the different scale degrees.

In order to verify the contention, a histogram of the accumulated frequencies on the piece was programmed. It was then run on the Scarlatti Erstausgabe Sonata, to provide a tonal reference comparison, as well as on the whole set of Krenek's pieces in the database. The results can be seen in Figures 13 and 14, the first one depicting the Scarlatti's "Erstausgabe" Sonata and the second one, the piece entitled "On the High Mountains", No.10 of Krenek's Op.83.

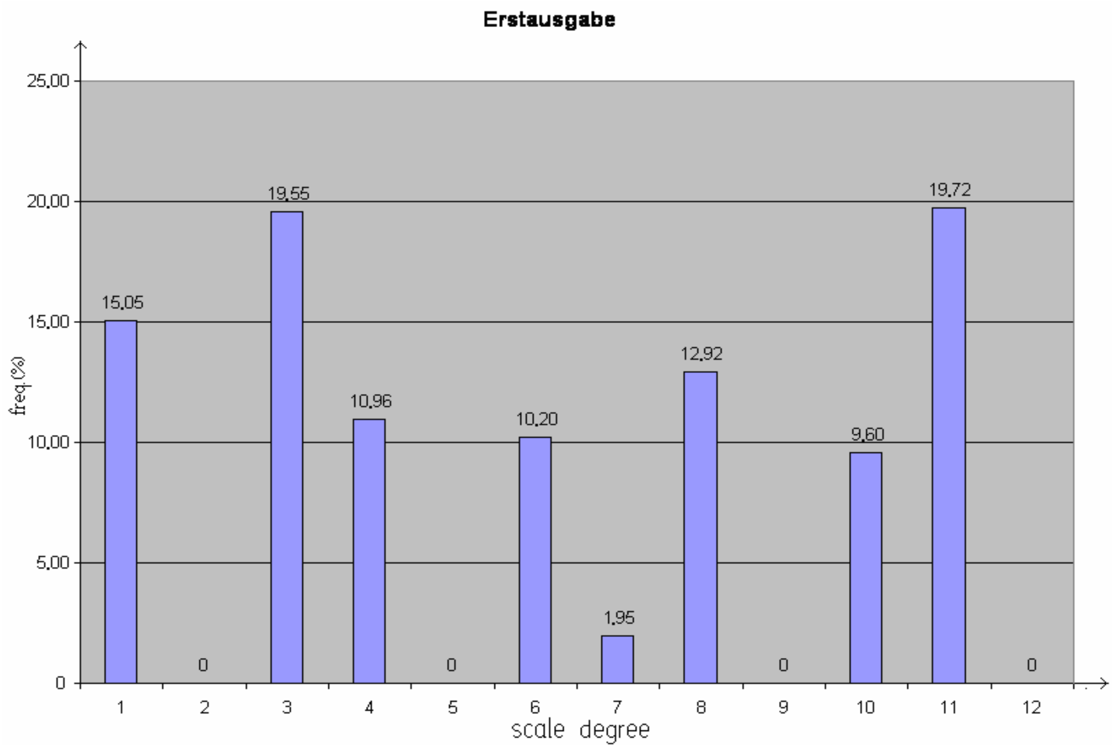


Figure 13 - Scarlatti Frequency Histogram

Notice that in this tonal piece, the frequencies vary from 0 to nearly 20%. The pattern does not reproduce the key profile because, as it modulates, there are two key sets.

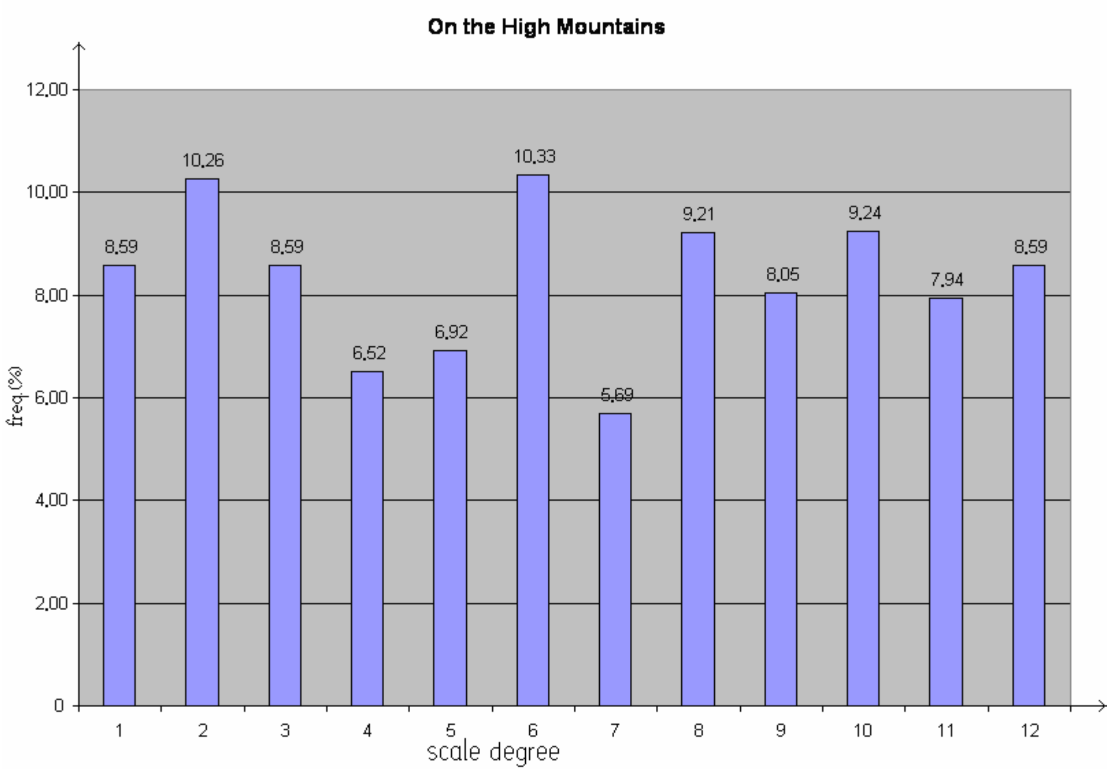


Figure 14 - Krenek Frequency Histogram

Figure 14 shows the histogram for the Krenek's piece. All of the 12 tones are present with a relatively even distribution, with frequencies varying between 5.69 % and 10.33 %. This distribution supports the idea that the dot products values will be also rather even, so that their spread will be relatively limited. In order to investigate this issue, the program was modified to output, for each slice, the six top dot products and their relative deviation to the maximum. To provide a measure of spread, since all these values are smaller than the top value, the average of these deviations was also calculated for each slice.

Since the decay tends to be faster at the beginning, it was possible that consideration of the relative deviation of the second top dot product by itself may provide a more sensitive indication; the program was also modified to provide this information.

Running it on the Erstausgabe Sonata shows a wide separation for the values of the dot products throughout the piece. A typical section of this output is copied below. The first column gives the slice number. Each successive column gives the six top keys in descending order from the top, followed by a number which represents the relative deviation to the maximum, i.e. the difference in value between its dot product and the top value, divided by the top value. The figures show these ratios multiplied by 10,000 so that they can be read as percentages with two decimal figures. That is to say, 1378 means 13.78%. In all the slices for this example, the maximum corresponds to G minor. The next key – G Major or F major depending on the slice –, is between 13.78 % and 16.49% below the value for G minor. The third column correspond to the key that falls in third place, and their values are between 16.55 % and 23.53% below, and so on. The rightmost columns shows the average deviation for the slice, which varies between 19.88 % and 24.23 %. The program calculates also the Global average distance to the top for the whole piece which in this case turns out to be 17.43 %.

Measure 2

16	G minor	(0)	G Major	(1378)	C Major	(2353)	F Major	(2380)	D minor	(2595)	C minor	(2675)	Ave.	2276.2
17	G minor	(0)	G Major	(1699)	F Major	(2011)	Bb Major	(2305)	D minor	(2606)	C Major	(2757)	Ave.	2275.6
18	G minor	(0)	F Major	(1641)	G Major	(1701)	Bb Major	(2175)	C Major	(2555)	D minor	(2788)	Ave.	2172.0
19	G minor	(0)	F Major	(1309)	G Major	(1672)	Bb Major	(2156)	C Major	(2229)	C minor	(2578)	Ave.	1988.8
20	G minor	(0)	G Major	(1580)	F Major	(1655)	Bb Major	(2258)	C Major	(2344)	C minor	(2667)	Ave.	2100.8
21	G minor	(0)	G Major	(1538)	F Major	(1702)	Bb Major	(2251)	C Major	(2592)	D minor	(2767)	Ave.	2170.0
22	G minor	(0)	F Major	(1649)	G Major	(1707)	Bb Major	(2158)	C Major	(2655)	C minor	(2795)	Ave.	2192.8
23	G minor	(0)	G Major	(1558)	F Major	(2331)	Bb Major	(2514)	D Major	(2683)	D minor	(3031)	Ave.	2423.4
24	G minor	(0)	G Major	(1541)	F Major	(2126)	Bb Major	(2350)	D Major	(2782)	C Major	(3074)	Ave.	2374.6

The same process with the Krenek pieces shows a marked contrast. Below there is a typical excerpt from the output for the Op.83 No.8 “Glass Figures”:

Measure 6

30	A minor	(0)	E minor	(48)	A# minor	(184)	B minor	(249)	B Major	(342)	G Major	(345)	Ave.	233.6
31	A# minor	(0)	Bb Major	(34)	G minor	(270)	F Major	(351)	C minor	(383)	D# minor	(491)	Ave.	305.8
32	F Major	(0)	A# minor	(48)	Bb Major	(88)	E minor	(137)	B minor	(194)	C Major	(252)	Ave.	143.8
33	A# minor	(0)	B minor	(153)	F# Major	(420)	F Major	(426)	B Major	(455)	Bb Major	(526)	Ave.	396.0
34	B minor	(0)	B Major	(277)	E minor	(661)	F# Major	(804)	E Major	(963)	D Major	(972)	Ave.	735.4
35	A Major	(0)	B minor	(7)	D minor	(60)	D Major	(285)	F Major	(421)	A minor	(443)	Ave.	243.2

The dot products are very close in value to each other. For these slices the value for the second most important key is between 2.77 % and 0.07 % below the top one. The following keys are between 6.61 % and 0.6 % below the top. Notice also the quick alternation of keys between slices. No two successive calculations point to the same key. The proximity of the results for different keys seemingly indicates the lack of key in the passage.

This is not just a peculiar example. Running the program on the ten Krenek's pieces in the database gives the results shown in Table 6 for the average distance as well as the average of the second dot product:

Piece	Average distance	Second dot prod	Ratio
No.2 Peaceful Mood	10.96167 %	4.4651 %	2.45
No.3 Walking on a Stormy Day	8.86072 %	3.9536 %	2.24
No.4 The Moon Rises	6.93072 %	2.9567 %	2.34
No.5 Little Chessmen	8.66524 %	4.0344 %	2.15
No.6 A boat, slowly sailing	6.92748 %	2.3912 %	2.90
No.8 Glass Figures	6.28932 %	2.7795 %	2.26
No.9 The Sailing boat	7.97247 %	3.7628 %	2.12
No.10 On the High Mountains	11.15006 %	4.6151 %	2.42
No.11 Bells in the fog	6.35129 %	2.5481 %	2.49
No.12 Indian-summer Day	9.23957 %	4.8816 %	1.89
Average	8.33485 %	3.6388 %	2.29

Table 6 - Data from Krenek's pieces

Comparison with a number of pieces from the classical period suggests that *the tonal weakness of the music correlates with the rate of decay of the dot products*. It would perhaps be possible to establish a pragmatic watershed between tonal and non-tonal. For example, considering the average deviation, the boundary could be set at 10% and using that criterion, most of these pieces only show incidental islands of tonality – for example, in No.9 a key is detected only for 7 slices out of 142 and in No.8 none at all whereas in the Scarlatti at no point a key fails to be detected.

If, instead, the criterion was based on the relative deviation of the second dot product, from Table 6 it can be seen that for the Krenek's pieces, this value relates to the average distance by a factor fluctuating between 2.90 and 2.15, with only one falling down to 1.89, and averaging 2.39. On classical period pieces like the Scarlatti and Handel examples, the factor tends to be smaller, in this case 1.78 and 1.97 respectively. Nevertheless, its fluctuations are large whereas the average deviation is more rounded up. In the Handel's case there are 7 slices that are closer to the top by less than 10% against 30 whose second dot products falls within 5% of the top as they come around the points of modulation. Consequently the 10% mark for the average deviation stands as a better measure of "tonalness" than the relative deviation of the second dot prod.

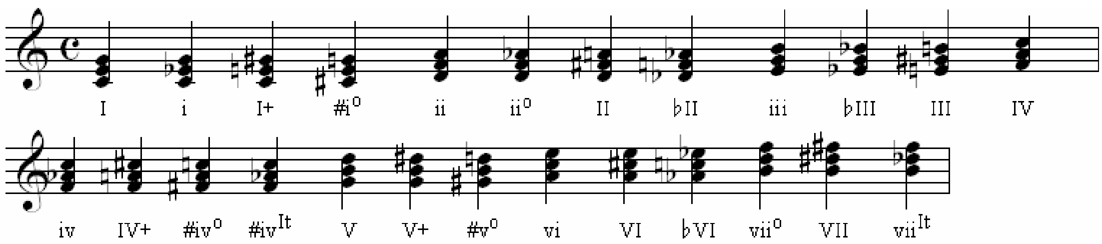
To sum up, the program that calculates the dot product as a point function based on slices and a window about 15 slices wide considered to apply half the window width back, plus the criterion to decide when a key is not detectable in a passage, constitutes a practical tool to find the key at every point in a score. Once in possession of this tool, it is possible to deal with functional labelling of pitch-classes and chords.

4.5 The labelling of chords

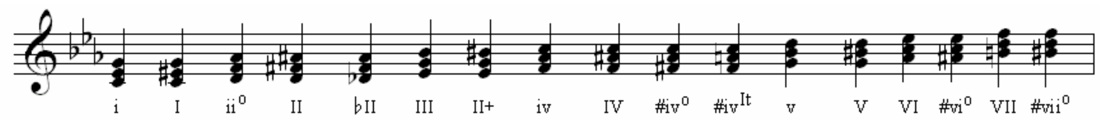
Knowing the key of a passage, the task of labelling explicit chords should be relatively straightforward. It essentially entails a table search routine following proper formatting. The slice format was decided upon for the specific purpose of chord identification, through the elimination of octaves and sorting of the note names to allow for table search. These choices, as has been said, disregard the issue of chord inversions which are largely irrelevant for chord identification.

"Chord" is a musical but not physical concept. Three simultaneous notes is the minimum required to form a chord, but the majority of the possible "vertical sonorities" or combinations of three notes form no recognizable "chord". Harmony is based on the superposition of thirds. Two thirds make a triad, three thirds make a seventh chord. Adding additional thirds, the possibilities are exhausted when the chord reaches the thirteenth because all the diatonic notes have been used. The core of harmony deals with triads and seventh chords, i.e. three and four-note chords. One more third added to a seventh chord would be a ninth above the bass, which could be conceptualized as a seventh chord with an added second. Similarly, the chords of eleventh and thirteenth are popularly known as "added fourth" and "added sixth", and although some five-note chords have a degree of usage, more often than not when one of these three additional thirds appear, some of the lower notes are omitted, which means there are practically no significant numbers of six or seven-note chords. Harmony treatises usually deal with all these chords in one brief chapter. Ottman lists eight ninth chords “which are used with some degree of regularity”, five in major and three in minor, and he goes on to say that “Chords of the eleventh containing a ninth, and chords of the thirteenth containing an additional ninth or eleventh, are comparatively rare” and most cases can be explained as a simpler chord with a non-harmonic tone. No other chord of five or more notes is mentioned as having any degree of usage. Consequently it does not seem worth the complication to try to identify five-note chords and beyond. Thus the chord-labelling program includes tables for 91 chords of three and four notes, for all the possible 30 keys. These chords form four groups, going from I up to VII. They are:

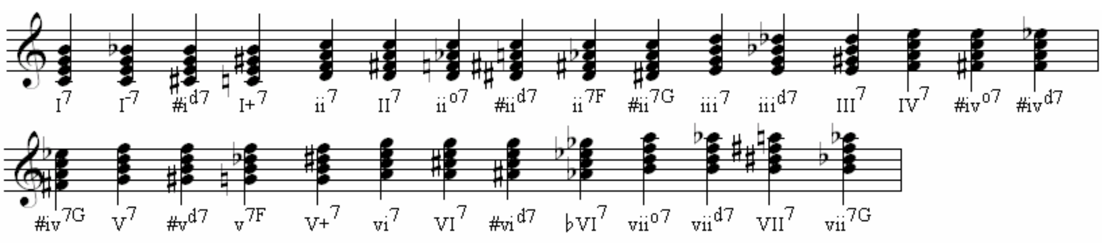
Triads in Major mode:



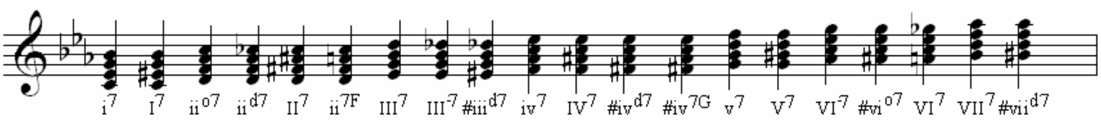
Triads in minor mode:



Seventh chords in Major mode:



and Seventh chords in minor mode:



In Ottman’s nomenclature, capital Roman numerals stand for major chord, small-case Roman for minor chord. A ‘+’ following the numeral indicates augmented chord; a ‘o’ superscript, diminished chord. The seventh chords are indicated by a superscript ‘7’ If the seventh is minor, the superscript is preceded by a ‘-‘, and if it is diminished, by ‘d’. The ones with superscripts ‘It’, ‘G’ and ‘F’ are altered chords belonging to a group called “augmented sixths” and the superscripts mean respectively Italian, German and French sixths – geographical names of unknown origin.

If the piece of music under consideration were a hymn, i.e. a succession of block chords, in the compact format each chord would occupy a slice, with repeated notes omitted and sorted from the bottom up, ready for comparison with the table. The program first identifies the key at the point where the chord lies, and then searches the corresponding chord table for a match. When found it displays the label.

However, music seldom is a succession of chords. Many pieces – examples in the database include J.S.Bach’s Sarabande from the French suite No.1 – do not include a single chord or vertical sonority with more than 2 notes in it. The whole piece is a single-note melody accompanied by a single-note bass line. Many Mozart and Haydn’s pieces follow the same pattern. Of course, the harmony has to be thought of as “implied”, something that is accomplished by means of several devices. The most basic ones are the left hand arpeggiating the chord – i.e. playing them successively rather than simultaneously –, or presenting its notes in a 4-note pattern known as “Alberti bass” (see Mozart’s example on page 86). Other times the melody itself outlines the chords. One way or another, the notes of the implied chord have to be present in the close vicinity of one another.

If this was all, implied chords could be handled by a window. The problem is that together with the implied chord notes there are also many others that do not form part of the chord. They are known as “non-harmonic notes”, and musically they have been classified in several categories – none of them useful to identify them as such. Ornamental as they are purported to be, non-harmonic notes can be considered noise from an informational point of view, and this is precisely their role when trying to identify the harmony. There is nothing objective that would allow telling non-harmonic apart from harmonic notes, and the analyst generally does on the basis of what is expected or usual. For example, there are a number of standard chord progressions. An analyst facing a bunch of notes looks among them for the notes that form an expected chord, and labels as non-harmonic all those who do not belong to it. This problem goes back to the original Maxwell's conundrum: Knowing the harmony it would be possible to tell what notes are harmonic and what not, but we only get to guess the harmony based on the notes.

There is no objective method to determine what notes are harmonic. But the only way to find the implied chords is to gather the notes that imply them. Naturally, gathering notes will collect both harmonic and non-harmonic ones, the grain and the chaff. Observing the problem from the listener's point of view, who usually does not know analysis, his ears have to tell him what the harmony is, and from his point of view all the notes are equally salient. This is a matter that relies on the composer's art. If he saturated the texture with non-harmonic notes, it would be letting them obfuscate the harmonic fabric in the listener's mind. It is clear that the chord notes have to be there in a proportion that allows the listener to hear the harmony, or at least make its presence be felt through in spite of the noise, for example by lasting longer than the passing ornaments. If this is the case, it probably makes sense to collect the loose notes and check if they make up a chord.

The procedure could be similar to the gathering of slices for determining the key. It would entail examining a bunch of notes, then dropping the oldest one, collecting a new one and doing it again. This is possible but it seems not very practical. In the case of finding the key, the idea was to detect something that remained invariable for a significant lapse. A chord, instead, may last a whole measure or more, but often the harmony changes three or four times in a measure. The window should have to be much narrower, since often a single slice carries a complete chord, which does not continue in the next slice. The first idea that this suggests is that the collection should be stopped once a chord has been identified. However, other times a slice may contain a single note or even none at all. This would require a window of variable width, and in that case, the collection would have to be stopped when enough notes – i.e. at least three – have been collected.

A test on a few scores using this logic gave disappointing results and reasons for them. Quite often, as in the Alberti bass, a chord is spanned by four notes, one of which is repeated. In this case, the window would close after the first three have been found, and a new collection would start in the wrong moment. This might either not leave enough notes in the window or collect material belonging to a different chord which would confuse matters as spurious chords would arise, just in the manner that the window lost resolution power at the points of modulation, because chord change, in many ways, is akin to key change. Another similar inconvenience is that often a new chord collection begins at the very end of a measure.

For these reasons the scheme of a variable-width window was abandoned. Although there is no general rule, chords tend to be aligned with beats, especially with downbeats, and generally there is no point in a collection of notes straddling a bar line. A better strategy seemed to be to collect for a fixed interval, typically a beat, and check the contents, then move to the next beat be it whether a chord was identified or not.

The process was implemented in a new program running piggyback with the chord labelling one. The motive of this peculiarity is that after processing the slices in order to calculate the keys, the whole slice-wise file is in memory, already in numerical format. Each slice in memory has its slice number and is associated with the corresponding key. It was practical to start a new process at the end of the previous one instead of saving the file and opening it again with the following program, having to run again all the routines to convert characters to numbers.

This program consists of a loop that repeats for every slice until the end. The first step is reading a slice from memory as well as its number and the key.

The slice duration is added to a counter in order to track the slice position in relation to the beat. If the slice contains anything but a rest, its duration is added to all the items in a buffer corresponding to the notes present, using the note code numbers as the buffer index.

If at this point the collection interval is over or a bar line has been reached, the subsequent step depends on the number of notes present in the buffer. If it is less than three there is nothing to identify, so the buffer is cleared and a new period begins; if it is three or higher, the identification step follows.

In general, the buffer will contain a number of notes of different durations. Although nothing can be assumed in music, it is only reasonable that the notes that are more important – at least for harmonic purposes – are those of longer duration. Hence, the indexes of the buffer will be sorted out in increasing order of duration. With the indexes in this order, a pseudo-slice is put together, and its subsequent process depends on the number of notes. There are five cases: 3, 4, 5, 6 or 7. (If greater than seven, which is unlikely, simply a warning is issued). For any number greater than 4 the program drops the first note in the slice – that is, the one with the shortest duration – and tries successively all the combinations of the remainder. If these are still greater than 4, the first note in the first one is dropped and all the combinations of the remainder tried successively. This means that for a 7-note pseudo-slice there are 98 combinations that will be tried; 70 of them are 3 and 4-note slices that the program will try to identify. Of course, 7 notes is the most that can be expected in a beat, and a trial with all the chromatic notes from C to F# did not find a single chord. But another 7-note slice with all the diatonic notes in C major found 14: V^7; V; vii^o; iii^7; iii; vii^{o7}; ii; I^7; I; IV; ii^7; vi; vi^7; IV^7, which of course is the whole set of triads and seventh chords in the diatonic scale.

Considering the 4-note case, the slice is first re-sorted to put it back in order of increasing note code numbers, and then passed to a sub-procedure that searches in a table for its functional label.

A successful identification ends the processing of the slice. This is again a point of contention since a collection of notes may contain a number of identifiable chords, and it would be possible for the program to present all the found labels. However, at this stage is preferable to automatically choose the longest in duration of those, allowing for possible errors in case that there were several options of equal duration. On the other hand, if there is no match in the table, the chord notes are dropped one by one and the remainder tried to be identified as a triad. If one is found in tables, the process is complete. The chord is assigned to the oldest slice that participated in the pseudo-slice, and the buffer and the collecting interval are cleared.

The harmonic rhythm of a piece cannot be predicted, and as a result a new parameter will have to be set by default but allow the user to vary it. In most cases, a time signature 2 means there is a chord per measure; 3 could mean a chord in each beat or one per measure; 4 usually brings two chords per measure and occasionally one; 6, 9 and 12 are the same as 2, 3, and 4 respectively. Nothing can be said in general about 5 or 7.

The strategy adopted by default is to start with the beat duration and use a time factor equal to the integer division of the numerator of the time signature by 2, or set it to 3 for signature of 9 and to 4 for signature of 4 beats. The time factor multiplied by the duration of the beat determines the collection interval. This should work for the majority of cases but in case the recognition of chords were poor, it could be manually changed.

4.6 The key profile revisited

One last action of the program uses the availability of the key information for each slice to compile information equivalent to that inferred from the data collected by Budge. It is possible now to count the notes – or, rather, their duration – slice by slice, and assign it to either major or minor mode depending of the instant key. In this way, the total duration of each scale degree can be accumulated and an average obtained per piece or per composer, for each mode.

In this way, the numbers collected should have to approach closely those used in the program to calculate the key. The process could be iterated in order to increase their accuracy.

As a first verification of the assertion, the values obtained for the three Baroque composers are shown in Table 7. The norm of the vectors have not been equalized, so that direct comparison of the values between columns can be misleading.

Table 7 - Key profile for the Baroque

Pitch classes	Handel's Averages		Scarlatti's Averages		J.S.Bach's Averages	
	Major	minor	Major	minor	Major	minor
I	19.20	20.18	18.52	18.57	21.21	19.92
I♯ - II♭	0.27	0.12	0.57	0.49	0.51	0.27
II	13.11	12.99	15.91	12.11	12.39	13.75
II♯ - III♭	0.05	15.75	0.07	14.85	0.16	13.89
III	15.49	0.39	13.55	0.64	15.39	0.62
IV	10.78	10.64	12.47	11.74	10.77	13.16
IV♯ - V♭	0.58	0.19	0.80	2.34	0.85	0.37
V	21.28	21.49	20.90	18.87	19.96	18.69
V♯ - VI♭	0.25	5.69	0.13	7.06	0.51	7.19
VI	7.68	1.98	6.97	1.98	6.45	2.13
VI♯ - VII♭	0.20	2.49	0.34	2.29	0.73	1.29
VII	11.08	8.03	9.71	9.01	11.03	8.66

In the face of these values, it is interesting to compare them to the key profile used this far. Assuming these values are not ideal, at some point they might lead to incorrect keys. This, in term, would assign some notes to the wrong modes, having some influence in the note frequencies. Hence, an iterative process of improvement should be possible: Replacing the original set of values for a more accurate one should lead to a yet more accurate set and so on until the result converges to the point where it does not improve any further.

Table 8 - Key Profile Improvement

Pitch classes	Baroque averages		Key profile		Improved set	
	Major	minor	Major	minor	Major	minor
I	19.64	19.56	16.8	18.16	19.64	20.17
I# - II♭	0.45	0.29	0.86	0.69	0.45	0.30
II	13.81	12.95	12.95	12.99	13.81	13.36
II# - III♭	0.09	14.83	1.41	13.34	0.09	15.30
III	14.81	0.55	13.49	1.07	14.81	0.57
IV	11.34	11.84	11.93	11.15	11.34	12.22
IV# - V♭	0.75	0.97	1.25	1.38	0.75	1.00
V	20.71	19.68	20.28	21.07	20.71	20.30
V# - VI♭	0.30	6.65	1.8	7.49	0.30	6.86
VI	7.03	2.03	8.04	1.53	7.03	2.10
VI#- VII♭	0.42	2.02	0.62	0.92	0.42	2.09
VII	10.61	8.57	10.57	10.21	10.61	8.84
norm	38.94	37.75	37.01	37.57	38.94	38.94

Table 8 shows, under "Baroque average", the average of the note frequencies for Handel, J.S.Bach and Scarlatti, and beside them, the key profile used. It is apparent that the numbers are quite close, but it would be difficult to establish that the differences proved significant. Since these numbers are used as components of a vector, it is important that they do not introduce a bias toward one of the modes. Since the norm of the average values was close but not identical, an adjustment was introduced, reducing the magnitude of the 'minor' components with a fixed factor to bring in line with the 'major' components. These adjusted values are listed as "improved set".

In order to investigate the effect of this slight change, the original key profile set was replaced in the programs by the Improved set; the modified versions were applied to Bach's Sarabande and the results compared. It turned out that the modified program identified the same keys exactly or within one slice of the same locations with a single exception: The old version recognized a brief modulation to G minor throughout measure 5. The new version does as well, but only during 4 slices rather than 7 in the old one, which makes it just under the threshold of 5 slices. As a result, the three chords found in this measure changed labels, a significant change for a very slight difference. There were no other noticeable differences.

5. Results

This study was undertaken in order to lay the foundations of a scientific taxonomy of musical styles, based on the application of multivariate techniques to a wide range of features extracted from musical scores. To that purpose, an appropriate database was selected and built. But before long it became apparent that many of the most important features to be studied could not be extracted from the material without the previous development of certain tools, beginning with the determination of key as a point function, a pre-requisite for the functional labeling of chords and notes.

The first task was a parsing program that counted the notes and accumulated their durations, producing as output a histogram either of the accumulated durations or collapsing them in pitch classes.

On the basis of the study by Budge on the frequency of chords, an initial set of key numbers was inferred and used to calculate the global key of a piece by obtaining its dot-product with the vector of the accumulated durations. The excellent agreement of the results with the expected keys showed that the method worked properly, and the program was modified to carry out the calculation measure by measure.

The limitations of this procedure made clear that it was necessary to use a sliding window as a measurement frame instead of measures. The awkwardness of the MusicXML format for this purpose required converting the database files to a true slice-wise format.

Once this was done, the next step was figuring out how to take into account the amount of the delay introduced by the window. The solution proved straightforward and accurate. The following step was to establish a pragmatic criterion to decide when there is no detectable key in the music.

Counting with an effective tool to calculate the key as a point function, it became possible to take on the problem of functional labeling of scale degrees and chords, as this is the basis of the measurement of pitch-related music features that could be considered as possible markers of style.

The set of programs created along this path constitute tools to carry out these functions. This section deals with their application to the files in the database as first examples of the feature-extraction process.

In the process, two derived databases were created, one made up of slice-wise files generated by the program that condenses the slices and another with the addition of the keys and chord labels, whenever they apply, generated by the chord-labeling program. They will be essential tools of use in the follow up to this project.

The materials in the database had been chosen with the intention of reaching a good balance by means of selecting 10 short pieces – or otherwise, excerpts – by each of 25 representative composers in the harpsichord/piano literature. In the Baroque, it was habitual for a composer to create single-page pieces but in later times, although many composers had continued to write brief preludes, significant short pieces were relatively in short supply. As a result, the share of different composers to the database varied ranging from 14 to 47 pages, with an average of 30. A better balance would have been achieved by taking into account the number of notes or else the total duration of each composer's contribution to the database. However, this information was not available a priori (moreover, those two numbers do not keep a constant ratio to one another).

There have been two types of data collection: One about the frequencies of scale degrees, the other about the frequencies of chords. The results of both appear in tables included in Appendix II. They summarize the results per composer and give the general averages.

In the interest of giving each composer's share the corresponding weight, a weighted average was used. In the case of chords, since there are four groups of chords, (triads and sevenths, two for each mode), the straightforward procedure was to use the each composer's total number of chords as weight.

The case of scale degrees is more problematic. Two possible ways to assign weights would be using the number of pages or the number of measures for each piece. None of them is ideal. For example, one page of Brahms contains generally many more notes than one by Haydn. Also, Baroque pieces, in print, are short because they generally include multiple repetitions, something that the romanticism tended to replace by slightly varied reprises of the material.

5.1 Data about scale degrees

Since the programs include the calculation of the total duration of the piece or excerpt, these figures are available for calculating the weights. But what duration should be used? Seconds? Beats? Measures? Do the ephemeral notes of a passage marked “Prestissimo volando” weigh as much as the pensive notes of an “Adagio lamentoso”? Does one measure of a four-voice fugue weigh four times that of a simple melody? In addition to these issues, the ‘duration’ indicated by MusicXML is a misleading figure that depends on the rhythmical complexity of the piece. For example, the duration of one measure of Chopin’s Prelude No.7 is 12 and the total duration of the piece is 1356 whereas those of Prelude No.24 are 3072 and 258987 respectively, a ratio more than twenty times larger than the 4 to 1 relationship of their timings.

Consequently, it was decided that the most equitable weights were the reported durations divided by the measure durations in each case, as this removes the scale factor, so that the reported numbers are as close as possible to a practical measure of the “number of notes”. For each composer the duration of each piece (separately by mode) as well as the measure duration, appears in rows 14, 27 and 28 respectively of the scale degrees tables. Table 9 shows the maximum, minimum, average and standard deviation for the durations, for both major and minor modes:

Table 9 - Duration Statistics

Durations	maximum	minimum	average	std.dev.
Major	243.34	31.16	102.10	49.26
minor	171.95	16.57	66.01	38.86

Somewhat surprisingly, these durations are in good agreement with the number of pages by composer, as Table 10 shows:

	Total Durations	Pages	Ratio
Handel	95.31000	16	5.956576
JSBach	64.57569	14	4.612550
Scarlatti	230.5611	24	9.606713
Haydn	137.5826	28	4.913666
Mozart	137.0010	32	4.281283
Beethoven	167.7302	30	5.591007
Schubert	415.2837	47	8.835823
Schumann	89.06667	14	6.361905
Mendelssohn	151.1215	37	4.084366
Chopin	111.0964	32	3.471762
Liszt	144.2076	24	6.008652
Franck	141.9698	34	4.175582
Brahms	214.2097	37	5.789452
Tchaikovsky	215.6229	34	6.341850
Grieg	232.0600	34	6.825293
Fauré	187.1378	45	4.158617
Debussy	204.8764	40	5.121910
Scriabin	127.9715	28	4.570409
Rachmaninov	177.9056	44	4.043308
Satie	149.2161	24	6.217336
Ravel	245.2205	41	5.980988
Bartók	183.4567	27	6.794691
Prokofiev	147.0435	31	4.743337
Shostakovich	71.5067	15	4.767112
Khachaturian	161.0208	33	4.879419
Totals/Average	4202.7500	765	5.493790

Table 10 - Duration and pages

The average results shown in the tables for scale degrees in Appendix II constitute a second approximation to the key profile. For the sake of comparison, Table 11 shows the initial key profile, and the values for the Improved Sets which repeat the averages from Table 8 (Ch. 4) and Tables 17 and 18 in Appendix II respectively, but the figures shown here have been adjusted in order to equalize the norms as discussed before.

Pitch classes	Key profile		Improved set		New Improved set	
	Major	minor	Major	minor	Major	minor
I	16.80	18.16	18.95	19.46	18.93	19.64
I# - II♭	0.86	0.69	0.43	0.29	1.81	1.11
II	12.95	12.99	13.33	12.89	9.97	10.67
II# - III♭	1.41	13.34	0.09	14.76	1.89	15.63
III	13.49	1.07	14.29	0.55	16.61	1.32
IV	11.93	11.15	10.94	11.79	8.93	9.44
IV# - V♭	1.25	1.38	0.72	0.96	2.29	2.57
V	20.28	21.07	19.98	19.59	21.24	21.27
V# - VI♭	1.80	7.49	0.29	6.62	2.22	7.11
VI	8.04	1.53	6.78	2.03	7.00	2.73
VI#- VII♭	0.62	0.92	0.41	2.02	2.68	3.05
VII	10.57	10.21	10.24	8.53	8.60	7.04
norm	37.01	37.57	37.57	37.57	37.57	37.57

Table 11 - Further improvement in Key Profile

Comparison between this second improved set with the first reflects the differences between the Baroque and the whole of the period of common practice. The main difference is a general increase in the proportion of chromatic notes. Nevertheless, the pillars of the key profile, that is the values for the Dominant, the Tonic and the Mediant, remain practically unchanged.

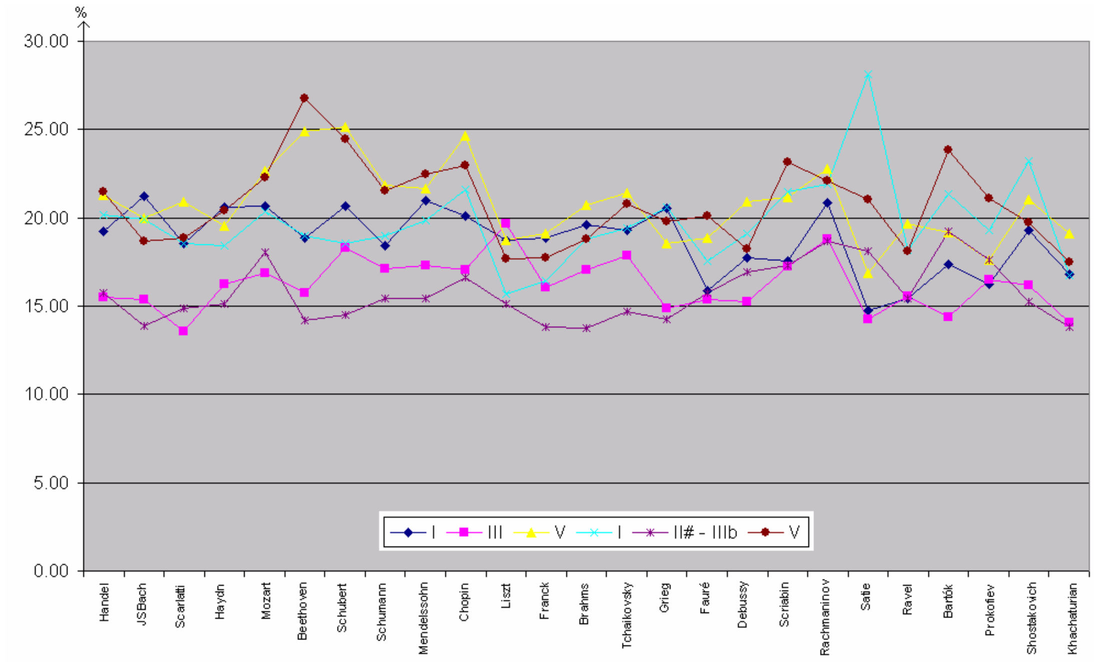


Figure 15 - Scale degrees with highest frequencies

Figure 15 shows the variations of scale degrees I, III, V in major and I, ♭III and V in minor (the three scale degrees with highest frequency) for the 25 composers in approximately chronological order.

The lines seemingly fluctuate but it is apparent that there are no consistent historical upwards or downwards trends. In order to verify this impression, a Two-step cluster – a method that tries to determine the optimum number of clusters – was run on all the scale degrees. Using the Schwartz Bayesian Criterion, the procedure found only a single cluster, which means that along the approximately 300 years covered by the pieces in the database there were no significant differences in the relative scale degree frequencies.

What this means is a reassurance about the early conclusions of this study. Tonal music is essentially characterized by the relative frequency of the scale degrees and while along the period of common practice the change in style has been astounding, nevertheless the frequencies of use of the main scale degrees continue to be within the same narrow limits. This is also the reason why the same key profile can successfully be used along the whole of the period.

Using the Akaike Information Criterion, the procedure was able to separate the composers in two clusters, and it did it in strictly chronological fashion, placing the dividing line between Chopin and (late) Liszt, that is, one cluster that contains all the baroque, classical and early romantic period, and the other including all the post-romantic and modern tonal. The main difference found between the two was an increased use of chromatic notes.

Considering this last result, it was worth comparing the historical behavior of the scale degrees with lowest frequencies. These have been plotted in Fig. 16:

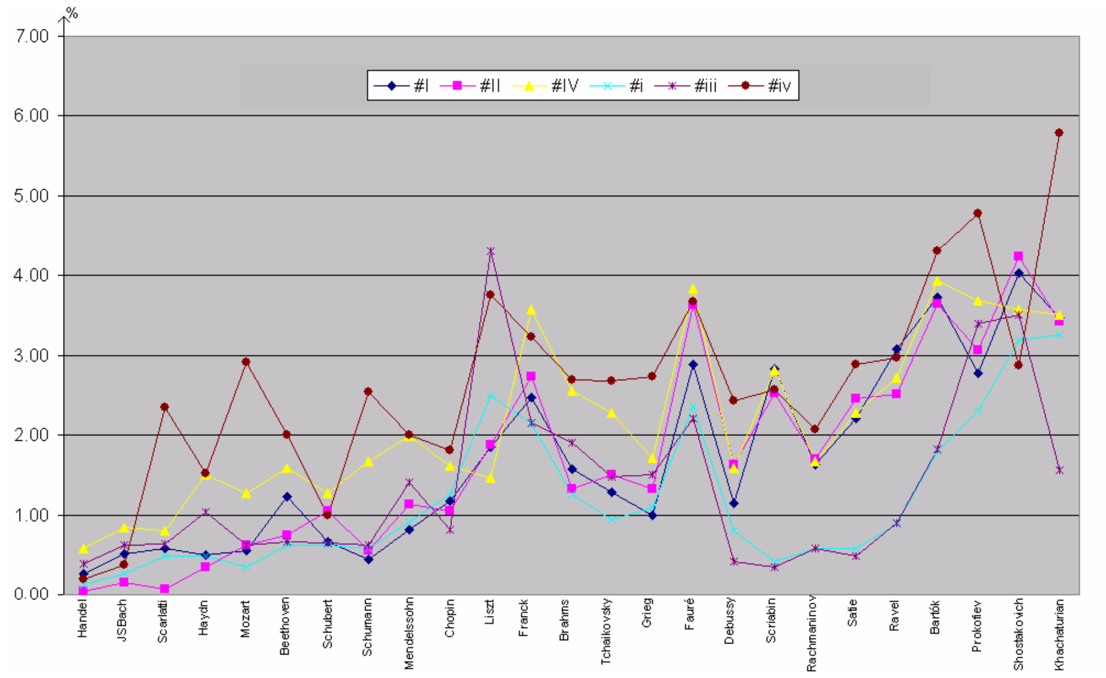


Figure 16 - Scale degrees with lowest frequencies

Unlike Figure 15, – and apart from the slump shown by the raised Tonic and raised Mediant in minor mode between Fauré and Ravel –, these scale degrees show a consistent historical rise, which coincides with one of the key findings of Budge's study.

Table 17 in Appendix II show the frequencies of use of the scale degrees for each of the composers in the database for major mode and Table 18 the same for minor mode. On both cases, the last row, Tot.dur., indicates the weighted average value of the normalized duration of the pieces of each composer in order to calculate the weighted averages of the scale degrees. The last two columns are weighted averages and weighted standard deviations, respectively:

$$\bar{x}_w = \frac{\sum_i w_i . x_i}{\sum_i w_i} \qquad sd_w = \sqrt{\frac{N \cdot \sum_i w_i \cdot (x_i - \bar{x}_w)^2}{(N-1) \cdot \sum_i w_i}}$$

Each of the cells in the first 25 columns of these Tables are averages from ten individual pieces, i.e. for each composer, the values can be considered as ten-excerpt samples. In order to show a measure of the dispersion of the values, Figures 19 to 24 show the same individual curves presented in Fig.15, that is, the average frequency of use of the three main diatonic scale degrees in both modes for the individual composers, but the dots have been replaced by the confidence intervals at 95%. The white horizontal line represents the value of the weighted average. These figures confirm the stability of the frequencies of the diatonic scale degrees.

Figures 25 to 30 also show the confidence intervals at 95% but for the curves presented in Fig.16, that is, the average frequency of use of the chromatic scale degrees with lowest frequency of use. These figures confirm the gradual historical increase in the frequency of chromatic scale degrees, since it is apparent that older values tend to be below the average and reciprocally.

5.2 Data about chords

The frequency of use of chords along the period is investigated as an example of feature extraction and a verification of the effectiveness of the method in comparison with the Budge benchmark figures. It is not to be expected that it would characterize a composer style. However, ratios of chord use and chord sequences are likely candidates to be meaningful features, as seen in 4.1. Tables 19 to 24 for chords show the number of chords obtained by the program for each individual composer and Tables 25 to 30 gives the corresponding percentages. In order to obtain the percentages per composer, the number of chords has been weighted with the duration of the piece, normalized by the duration of the measure for each. Each row corresponds to a chord. There are separate tables for major and minor modes. Table 31 shows the values of diatonic chords grouped as discussed next.

Chords are grouped in order to get an understanding of their relative proportions, and for that purpose, those whose role is equivalent in both modes have been added together, i.e. in the “Grouped” column, ‘I’ in major has been added to ‘i’ in minor, ‘ii’ in major to ‘ii^{o}’ in minor and so on. The purpose of these groupings is to make apparent the relative importance of the chords. But groupings have to be made in a way that makes conceptual sense. For example, chord ‘i’ functions in minor as the tonic chord but in major it is a borrowed chord, the first category of chromatically altered chords. It would be meaningless to add both figures in order to measure the frequency of the diatonic chord ‘i’ because ‘i’ is not diatonic in major. Likewise, chord IV is diatonic in major but in minor it is a secondary dominant, often called V of VII, and so the chord in minor does not represent a diatonic chord and its frequency has not been added to the figure in major.

A different problem appears in minor because of the dual nature of this scale. Although the natural minor scale is the basic ‘minor’ mode, the majority of the music uses the harmonic scale with a raised seventh degree. This scale contains the chords III+, V and $\#vii^{o}$ whereas the natural scale contains instead the chords III, v and VII. Since it is legitimate to consider any of these chords as diatonic in minor, when trying to gauge the relative frequencies of the diatonic chords both variants appear as mutual alternatives, and so their numbers have been added together.

Table 12 – Overall Frequency of Triads

Triad in Major mode		Triad in Minor mode	
I	31.30	i	34.84
ii	3.86	ii^{o}	3.75
iii	5.29	III^{+} plus III	5.51
IV	4.14	iv	5.08
V	7.60	V plus v	10.84
vi	4.12	VI	3.65
vii^{o}	2.61	#vii^{o} plus VII	3.12

The total of diatonic triads plus seventh chords amounts to 82.52% of the total.

For minor mode, natural and harmonic are considered together. The percent results for triads and seventh chords are presented in Tables 12 and 13. The percentages refer separately to the totals per mode in order to compare them:

Table 13 - Overall Frequency of Seventh Chords

Seventh in Major mode		Seventh in Minor mode	
I^{7}	4.59	i^{7}	2.02
ii^{7}	1.57	ii^{o7}	3.25
iii^{7}	1.77	III^{7}	0.37
IV^{7}	1.56	iv^{7}	1.84
V^{7}	6.87	V^{7} plus v^{7}	6.12
vi^{7}	4.98	VI^{7}	3.63
vii^{o7}	1.02	VII^{7}	0.48

These figures are averages over the whole period of common practice. In order to assess the changes in chord frequencies along the historical period, a two-step cluster was run considering all of the chords as variables. Again as in the case of the pitch-classes frequencies, the procedure found a single cluster including all the composers. In the suspicion that perhaps fewer variables would make possible a subdivision, the procedure was repeated including only the main chords, I, IV, V in major and i, iv and V in minor but again the procedure failed to find significant differences between composers. Although the effect does not result significant, Figure 17 showing the plots for the most important chords, I, IV, V and V7 as well as the total of diatonic chords, reveals that there is a general historical decline in the frequency of diatonic chords. This result is consistent with the trend of constantly increasing chromaticism indicated by Budge.

As a kind of reality test, the program was run with the test for tonalness disabled, on a piece by Bartók not included in the database, the ostensibly dissonant Perpetuum Mobile (No.135 in the Mikrokosmos collection). The program identified a single chord in the whole piece, a Tonic chord in D major at the first beat in measure 10, somehow resulting form the collection process. This was a reassuring verification that the collection process does not go about finding chords where there is none.

Table 14 summarizes the number of diatonic chords found in the database, categorized in triads and seventh chords:

Diatonic chords	Triads	Sevenths	Total
Major mode	8905	3376	12281
Minor mode	6242	1656	7898
Total	15147	5032	20179

Table 14- Number of diatonic chords

These chords represent 82.51 of the 24,455 chords found.

Each column in Tables 19 to 24 in Appendix II show the number of chords found in the total for the ten pieces of each the composers in the database. An additional column at the right end shows the number of diatonic chords grouped in the way that was discussed above as well as their relative percentage. In each case, the last row shows the total number of chords for the corresponding mode, and in the case of the tables for minor mode, below the total for minor, the overall total is also shown.

Table 31 shows the percentages for the grouped diatonic chords, to the effect of making an easier comparison between them. From these tables, the main diatonic chords – that is I + i, IV + iv, V + v + V, and V7 + v7 + V7 – have been plotted in Fig. 17 as well as the total percentage of diatonic chords. It is apparent that all the curves, with the possible exception of the Tonic, show a declining historical frequency

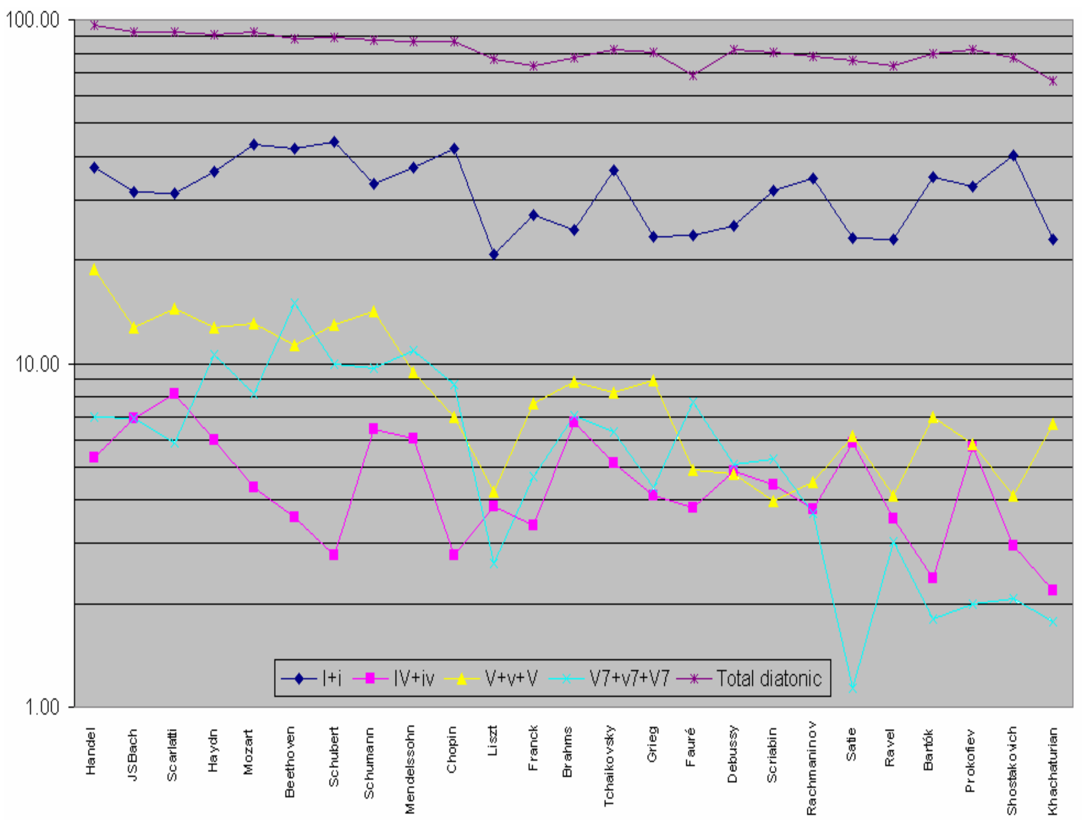


Figure 37 - Percentage of use of the main diatonic chords

6. Discussion

6.1 Comparison with Budge's results

Budge's study is a benchmark for the results of this study and it is worth comparing to the figures that have been obtained. In order to do so, in the first place it is necessary to limit the scope to those composers of the same periods.

Budge's data comprised 65,902 chords. In comparison, in this database, if the data is limited to the same historical period, the program has labeled 15,568. If it were possible to group the chords as she had, counting all the diatonic triads and seventh chords, the data could be compared directly. However, this would take some guessing as to her choices. Obviously, as she reported the figures for all the diatonic chords without differentiating modes, she had chosen to add together chords in major and minor that have the same degree as root. The reasonable assumption has been made that in her grouping she considered the chords III+, VII and v7 in minor as chromatic, and for this motive they have not been considered for this count. With these assumptions, the groupings are shown in the column "Grouped-as" which is followed by another column with Budge's figures for the sake of comparison.

Budge found that out of 65,902 chords, 54,853 (83.74%) were diatonic (64.89% triads and 18.85% sevenths) and 11,049 (16.76%) were chromatic. In general, the results obtained by the program are very close to hers. Globally, diatonic chords represented 83.86% (63.61% triads and 19.73% sevenths). Using her data as expected frequencies, a goodness-of-fit test would give the probability that this distribution is indeed the same as hers, but whether the test reaches significance or not depends essentially on the assumptions made above, which cannot be verified. Table 15 shows the individual results for triads and sevenths for comparison purposes:

Triads			Sevenths		
Chords	Results	Budge’s	Chords	Results	Budge’s
I	35.22	34.37	I^7	2.65	0.07
II	3.94	4.52	II^7	2.14	2.00
III	2.78	1.13	III^7	1.13	0.04
IV	4.78	7.74	IV^7	1.52	0.32
V	10.97	11.25	V^7	8.06	14.67
VI	3.60	4.18	VI^7	3.71	0.94
VII	2.60	1.7	VII^7	0.76	0.81
Totals	63.89	64.89	Totals	19.97	18.85

Table 15 - Comparison with Budge's results

There are some important differences between both databases. First, Budge’s data included eleven composers that are not in this study’s database: François Couperin, Rameau, Gluck, K.P.E.Bach, Cherubini, Weber, Rossini, Wagner, Verdi, Mussorgsky, MacDowell. The database, for the same period, includes Franck and Grieg whom Budge did not consider. Second, Budge included vocal and orchestral works whereas the database comprises keyboard pieces only. These two confounding variables make difficult to attribute the differences.

Based on the different sizes of the samples and repertoires, the resulting figures are really close. However, they are much more so in some cases than in others. The agreement is much better in the case of the triads than the seventh chords. In the former, there is good agreement about I, II and V, a significant disparity about chord III and a secondary one in chord IV. As to the latter, some startling disagreements appear in the case of I7, III7 and VI7 whereas V7 shows an unexpectedly low figure.

There are two or three more sources of possible differences. In the first place, Budge stated unequivocally: “Chords were taken at their face value, and no extra unwritten notes were assumed for them”. This suggests that in most if not all cases in which the chord-labeling program collects the notes to form a chord, she would not have counted it. As a result, not only the count but the frequencies of chords may be affected for it is not known whether the frequencies of block chords are the same as those of implicit chords, or whether these proportions change for different chords – for example, a tonic chord may well be more likely to appear explicitly than a Mediant chord. With hindsight, it would have been possible to keep separate counts for explicit and collected chords thus eliminating this uncertainty.

Secondly, Budge recognized all the kinds of non-harmonic notes, but also considered tempo in her labeling. In her Dissertation, she shows as example a passage of a Haydn's Sonata and states:

> Several notes are regarded as non-harmonic because the speed does not permit them to be recognized as chord tones. If this piece were to be played *Adagio* instead of *Allegretto* the fifth beat of measure 1 would be identified as a IV^6_4 chord, the second beat of measure 3 as a V_6 chord, the fifth beat of measure 3 as a V^4_3 chord, etc

In other words, using her musical sense, she disregarded many chords that the program would have counted. Again, these discriminated chords are more likely to be uncommon chords, and as a result, her results would differ from the findings of this study.

Thirdly, she considered the problem of modulation and stated her position in relation to it: She recognized a modulation when a V^7 – I progression occurred in the new key but not those changes introduce by secondary dominants, and "pivotal chords were classified as the property of the key to whose cadence pattern they belonged rather than the preceding key". The results of these choices would be a considerable number of differences between her classification and the chord-labeling program's.

6.2 Limitations of the programs

The note-parsing program does not present any conceptual problem. The method for finding the key is undeniably correct and, as has been discussed, works with sufficient precision for most practical purposes.

It is worth noting, as a general comparison between figures 15 and 16, that the scale degrees in Fig.15 are diatonic whereas those in Fig.16 are chromatic. The first ones are the ones who define the shape of the key profiles, which has been supported by a few trials with a key profile in which the chromatic degrees had been replaced by zeroes, which found essentially the same keys. Although more extensive comparisons would be necessary, conceptually the stability of the key profile is compatible with the historical increase in chromaticism because of the relative low weight of the chromatic scale degrees.

There are two further topics that need to be discussed: The chord collection procedure and the test for tonalness.

6.2.1 The collection procedure

In order to check how the collection process works, and especially when it can malfunction, the next example presents the results of running it on the first 8 measures of Mozart's Sonata K.545, with time factor manually adjusted to 2, on account of the harmonic rhythm in the piece:

Figure 48 - Beginning of Mozart's Sonata K.545

The keys and chords have been written under the score in the locations where the program found them to occur. The excerpt is found to be in C major except for the second half of measure 5 which is considered to be in F major. There are no block chords in measures 1 through 4, but the program picks the implied harmony correctly; the same could be said about measures six and eight. The only problematic spot is measure five. It is based enough on chord IV for the program to consider it a tonicization to F major. Most analysts would agree that this “modulation” should not have been found. But how is chord iii detected?

The program output reveals the details (for the slice format, refer to p.60). Each line lists the slice number followed by the slice itself. The asterisk indicates the slice corresponds to the beginning of a measure. The ‘7’ after slice 39 tells the number of notes collected in the buffer, and then, still under the same number 39 – the pseudo-slice put together followed by the corresponding key. In this case k = 0 means the key is C major. The following line shows the resulting pseudo-slice after dumping the extra notes and the label found for it:

```
33   2  15  28  4 *
34   2  15  34  2
35   2  0  15  2
36   1  6  2
37   1  12  2
38   1  15  2
39   1  22  2
7
39    7  0  6  12  34  22  28  15  k =  0
3  12  22  34 Triad Label: iii
```

Slice 33 is the first in measure 5. There are no chords in the first half of the measure. Three 2-note slices are followed by 4 single-note slices. The collection puts together a 7-note pseudo-slice. Those notes form the ascending scale of A minor over an F in the bass. Except for the quarter F in the bottom staff and the eighth initial A of the scale, all the other notes are 16ths: C, D, E, B and G. Hence, the program keeps them in ascending order; in the search for chords, the program starts to dump them beginning by C. Unfortunately, this means that the expected chord – F, A, C – will not be found. The next note to be dropped is D. At that moment, the only existing triad among the combinations of the remaining notes – E, F, G, A and B – is E, G, B, i.e. the iii chord in C major. Notice also that the identified chord on the second half of the measure, I of F major, is the expected IV of C major.

Exactly the same problem affects the first half of measure 7, although in this case one wonders whether it would not be too much to ask of anybody to identify an implicit chord as the dominant at a moment when the explicit notes include the tonic and the subdominant.

This example brings about a number of considerations:

In the first place, successful as it is, the strategy of collecting notes could be further refined with a more sophisticated method to choose among the possible chords in the buffer other than just successively dropping the notes of shortest durations. In the search for improvements it would be necessary to resist the temptation of using a table of chord progressions. Such table would violate the central idea in this project to always find the information through objective methods and rejecting musical analysis. Alternatively, perhaps scale-wise passages should deserve a special treatment in substitution of the collection/dropping strategy.

The simultaneous consideration of this example and the comparison with Budge's results brings the concern whether the wrong identification of chord 'iii' where IV should have been found might be a systematic rather than random occurrence, as it would contribute to explain both the elevated figures for the former and the reduced numbers of the latter.

Further, the brief tonicization in measure 5 should have been preferably omitted. The program listing reveals it only occupies 8 slices. Either a widening of the window or of the threshold in the minimum number of slices to recognize a change of key would eliminate this and similar brief modulations. In this particular case it would also lead to a welcome increase in the number of recognized IV chords, again raising the doubt whether the effect could be systematic.

One possible improvement could take advantage of the knowledge of the key. If instead of dropping notes with whatever criterion, the strategy were to list all the possible chords in the buffer and pick the most likely one according to the obtained chord statistics, undoubtedly it would not mislabel any chords for having dropped a note as it presently occurs, and the choice certainly would be at least arguable. Besides, this approach would simplify the chord labelling program.

To sum up, given the good agreement with Budge's benchmark figures, the collection process is effective. It is worth refining, though, and a comparison of alternative methods, applied to the same pieces, should make apparent what is the best strategy.

6.2.2 The test for tonalness

As noted in section 5.4, it was found that the tonal weakness of the music correlates with the within-slice rate of decay of the dot products. The chord-labelling program embodies an empiric test for determining tonalness: it decides that the music has no detectable key whenever, for a period of not less than five consecutive slices, the decay of the first six dot-products is so slow that the average deviation from the top is less than 10%. Generally tonal pieces are unaffected by this check because the decay is faster. However, occasionally the condition is satisfied when the windows straddles a point of modulation. In this condition, the values of the dot products for the previous and closely related keys are diminishing whilst those of the new key are increasing. Consequently, the set of values falls within much narrower limits than usual. If this diminution continues for long enough – i.e. more than five slices –, a band of slices would be marked as "not detectable key", thus preventing chords to be labelled in that section.

Inasmuch as the database comprises tonal pieces, when running the programs to classify chords from pieces of the database, the detector of non-tonal sections was turned off. But a proportion of the chords must have been found during those sections where the values of the dot products are unstable and perhaps lead to misleading passing keys and consequently, mislabelled chords. One case in which the detector was purposely left on concerns the example Shostakovich's Prelude depicted in Fig.8. The curves clearly suggest a possible moment of indefinite tonality in bars 11/12. However, the program did not detect a no-key area with a window width of 20 slices.

The effect might not be limited to the areas of non-detectable key. Since they may intrude in the middle of a section of a certain tonality, the finding or labelling of some of the chords in the marginal areas may be affected as well.

With the benefit of hindsight, it would have been preferable to have a lower count of chords than a proportion of potentially mislabelled ones. In order to have a check on the magnitude of this effect, a second run was done on the works of Liszt included in the database, this time with the detector on. The choice of Liszt is due to the marginal tonality of several of his pieces included in the database, all of which belong to his late years. Table 16 shows for each of Liszt's pieces, the number of slices marked as 'no detectable key' (n.d.k) in relation to the total; the number of chords found with the detector 'on' and 'off' – hence unreliably classified –; and the list of chords that were found in those sections when the detector was off. From 2350 slices, 480 (20.43%) were found to have no detectable key. Out of 495 chords for Liszt, 61 (12.32%) were found in the sections of no detectable key and would not have been counted. Of the 61 chords, 38 were diatonic and 23 (37.3%) chromatic. If all these chords had been excluded, the proportion of chromatic chords for Liszt would have dropped from 23% to 21%.

Table 16 - Effect of tonalness test on Liszt's works

Liszt's pieces	n.d.k slices	out of	no detectable key		chords found in non detectable key sections when the detector is off
			det.off	det.on	
4 Pieces - 1	67	190	55	45	vi vi7(2) #vi^{o}
4 Pieces - 2	71	200	59	49	#vi^{d7}(2) V iii II^{7} IV^{7} ii^{o}
4 Pieces - 3	8	79	24	23	vii^{o} vii^{d7} ♭III I^{7}
4 Pieces - 4	-	75	27	27	-
Abschied	60	216	73	50	iii(8) vi(6) ii(2) iii^{7}(2) V
En Rêve	63	261	31	22	III IV ♭III I ♭II #vii^{o} I
Lugubre G. I	35	518	63	63	-
RW-Venice	25	154	34	32	♭III ♭II
Unstern	67	338	62	55	I+ iii v^{7F}(3) ♭III
Wiegenlied	84	319	94	78	#iv^{o}(4) iii vii^{d7}(4) iv ii^{o7} III

In other words, the effect of the criterion for disregarding the key selected by the maximum dot product has a quite significant impact on the results. It would be easy to change the program to give the results for those sections with a warning that the classification of the chord is probably doubtful because of the uncertain tonality, but before a decision is taken in this respect, it would be necessary to carefully consider a good number of "excluded" chords and analyze whether the chord classification is correct or should be disregarded. This is one of the refinements that should be part of the next stage.

6.3 Conclusions

This project has been based on no assumptions or subjective criteria, trying to come to results that are objective. Software implementing a mathematical method to find the key as a point function in a piece of music has been developed, as well as a criterion to decide when there is not a detectable key in a passage; the method works reliably, and provides the basis for functional labeling of scale degrees and chords. It also has provided a new insight into tonal music, revealing that throughout the period of common practice, tonal music makes use of scale degrees in fixed proportions that vary between close limits; and that the use of diatonic chords steadily declines along the years, thus verifying Budge's conclusion that there is a constantly increasing chromaticism along time. The note collection procedure used in order to label chords should be further refined, although the agreement with Budge's data suggests that the figures obtained are not far from the mark.

These results constitute the basic tools that are necessary to continue this research in order to measure observable features which, through the use of the appropriate statistical tools, should lead to find what are the main dimensions of the musical style phenomenon, thus laying the foundations of a scientific taxonomy of tonal music style.

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Appendix I **-** *Listing of the database*

The following is the list of compositions included in the database of this study.
The numbers of score pages are indicated between brackets.

Handel: Suites (16)

Suite No.5: Air and Variations (3)
Suite No.7: Passacaille (3)
Suite No.10: Air (1)
Allegro (1)
Suite No.11: Sarabande (2)
Suite No.13: Allemande (1)
Courante (1)
Suite No.14: Menuetto (2)
Aria (1)
Suite No.16: Gigue (1)

J.S. Bach: French Suites (14)

Suite No.1: Sarabande (1)
Suite No.2: Air (1)
Menuet (1)
Suite No.3: Anglaise (1)
Gigue (2)
Suite No.4: Allemande (2)
Courante (2)
Suite No.5: Gavotte (1)
Bourrée (2)
Suite No.6: Polonaise (1)

Scarlatti (24)

Sonata in g (Erstausgabe) (1)

Sonata in g K.30 (5)

Sonata in a K.149 (2)

Sonata in C K.159 (2)

Sonata in h K.197 (2)

Sonata in e K.291 (3)

Sonata in E K.380 (4)

Sonata in D K.415 (2)

Sonata in G K.431 (1)

Sonata in D K.534 (2)

J. Haydn: Sonatas (28)

No.48: Adagio (3)

No.49: Scherzando (3)

Menuet (1)

No.50: Largo e sostenuto (1)

Finale (3)

No.51: Allegro moderato (5)

Adagio (3)

Finale (1)

No.52: Allegro con brio (4)

Adagio (4)

Mozart: Sonatas (32)

No.9 K.311: Andante con espressione (4)

No.10 K.330: Allegro moderato (exposition only) (3)

No.11 K.331: Andante grazioso (excerpt) (1)

Menuetto (4)

Rondo (4)

(No.14) K.457:Adagio from Fantasy (3)

No.16 K.545: Allegro (4)

Andante (3)

Rondo (2)

No.18 K.576: Adagio (4)

Beethoven: Sonatas (30)

Op.13: Grave (4)

Adagio cantabile (3)

Op.27 No.2: Adagio sostenuto (3)

Allegretto (1)

Presto agitato (excerpt) (3)

Op.31 No.2: Largo, Allegro (excerpt) (3)

Allegretto (excerpt) (2)

Op.53: Allegro con brio (excerpt) (4)

Op.57: Allegro assai (excerpt) (3)

Andante con moto (4)

Schubert (47)

Impromptus:

Op.90 No.1 (9)

Op.90 No.2 (excerpt) (6)

Op.90 No.3 (7)

Op.90 No.4 (excerpt) (6)

Op.142 No.2 (5)

Moment musicaux:

Op.94 No.1 (4)

Op.94 No.2 (4)

Op.94 No.3 (2)

Op.94 No.5 (2)

Op.94 No.6 (2)

Schumann (14)

Childhood scenes Op.15:

From foreign peoples (1)

The suppliant child (1)

Dreaming (1)

Almost too serious (2)

The scarecrow (2)

Child falling asleep (1)

Album for the Young Op.68:

The merry peasant (1)

Spring Song (2)

The first loss (1)

Italian sailor’s song (2)

Mendelssohn: (37)

Songs without words

Op.19 No.1 (4)

Op.30 No.6 (2)

Op.38 No.6 (5)

Op.53 No.3 (6)

Op.62 No.5 (2)

Op.62 No.6 (4)

Op.67 No.2 (4)

Op.67 No.4 (5)

Op.85 No.1 (2)

Op.102 No.1 (3)

Chopin (32)

Preludes Op.28

No.4 (1)

No.6 (1)

No.7 (7)

No.20 (1)

No.24 (4)

Etudes Op.10

No.3 (excpt) (1)

No.5 (4)

No.12 (5)

Etudes Op.25

No.9 (2)

No.12 (6)

Liszt (24)

Wiegenlied (3)

Four little piano pieces:

1 (2)

2 (2)

3 (1)

4 (1)

La lugubre gondola I (4)

En Rêve (2)

Richard Wagner – Venice (2)

Abschied (2)

Unstern (5)

Tchaikovsky (34)

The Seasons Op.37:

No.6 Barcarolle (5)

No.7 Song of the Reaper (3)

No.9 Hunter's Song (4)

No.10 Autumn song (3)

No.11 Troika (5)

No.12 Christmas (7)

Album for the Young Op.39:

No.6 Winter morning (2)

No.17 Ancient Neapolitan Song (1)

No.19 The witch (2)

No.20 Dreaming (2)

Brahms (37)

Acht Klavierstücke Op.76:

No.3 Intermezzo (2)

No.5 Capriccio (5)

No.8 Capriccio (4)

Rhapsody Op.79 No.2 (7)

Fantasien Op.116:

No.2 Intermezzo (3)

No.5 Intermezzo (2)

Drei Intermezzi Op.117 No.1 (3)

Sechs Klavierstücke Op.118:

No.1 Intermezzo (2)

No.2 Intermezzo (4)

No.3 Ballade (5)

Franck (34)

Danse lente (2)

Prelude, coral & fugue:

Prelude (6)

Choral (2)

Transition (2)

Fugue (excerpt) (4)

Coda (2)

Prelude, aria & final:

Prelude (excerpt) (5)

Aria (6)

Finale (2 excerpts) (5)

Grieg (34)

Funeral March for Rikard Nordraak (5)

Elegiac melodies:

Letzter Frühling Op.34 No.2 (2)

Lyric pieces:

Shepherd boy Op.54 No.1 (2)

Peasant March Op.54 No.2 (4)

March of the Trolls Op.54 No.3 (excerpt) (4)

Notturno Op.54 No.4 (4)

Bell ringing Op.54 No.6 (2)

Illusion Op.57 No.3 (4)

Home sickness Op.57 No.6 (4)

Phantom Op.62 No.5 (3)

Fauré (45)

Impromptus:

No.1 Op.25 (excpt) (5)

No.2 Op.31 (excpt) (5)

No.3 Op.34 (excpt) (6)

No.4 Op.91 (excpt) (5)

No.5 Op.102 (7)

Préludes op.103

No.1 (4)

No.4 (3)

No.6 (3)

No.7 (4)

No.9 (3)

Scriabin (28)

Etudes:

Op.2 No.1 (2)

Op.8 No.3 (4)

Op.8 No.11 (3)

Op.8 No.12 (5)

Op.42 No.5 (6)

Preludes:

Op.9 No.1 (2)

Op.11 No.14 (2)

Op.11 No.24 (2)

Op.17 No.4 (1)

Op.22 No.1 (1)

Rachmaninov (44)

Elegy Op.3 No.1 (6)

Romance Op.10 No.6 (2)

Moment musical Op.16 No.5 (4)

Preludes:

Op.23 No.5 (6)

Op.23 No.10 (2)

Op.32 No.10 (5)

Op.32 No.12 (4)

Etudes-tableaux:

Op.33 No.2 (4)

Op.33 No.7 (4)

Op.39 No. 2 (7)

Debussy (40)

Arabesque No.1 (5)

Rêverie (5)

Suite Bergamasque: Clair de lune (6)

Children’s Corner: Doctor Gradus ad Parnassum (5)

The little shepherd (2)

Golliwog’s Cakewalk (5)

Pour le Piano: Sarabande (3)

Preludes Book 1: Des pas sur la neige (2)

La Cathédrale engloutie (5)

La fille aux cheveux de lin (2)

Satie (24)

1ère Gymnopedie (4)

1ère Sarabande (3)

1ère Gnosienne (2)

4iême Gnosienne (2)

5iême Gnosienne (2)

6iême Gnosienne (2)

Air du Grand Prieur (3)

Prélude de la Porte Héroïque du Ciel (2)

Pieces froides 3 (3)

Véritables Préludes Flasques 2 (1)

Ravel (41)

Pavane pour une infante défunte (4)

Menuet antique (excerpt) (3)

Prélude (2)

Sonatina:

1 (4)

2 (2)

3 (6)

Mirroirs:

Oiseaux tristes (4)

Alborada del gracioso (excerpt) (5)

Gaspard de la Nuit:

Ondine (excerpt) (6)

Scarbo (excerpt) (5)

Bartok (27)

Sonatina:

1 (2)

2 (1)

3 (4)

Allegro barbaro (6)

Suite Op.14: No.4 (2)

Mikrokosmos

No.126 (2)

No.130 (2)

No.142 (4)

No.150 (3)

Roumanian folk dances: No.4 (1)

Prokofiev (31)

Etude Op.2 No.4 (5)

March Op.3 No.3 (2)

Montagues and Capulets (5)

Prelude Op.12 No.6 ‘Harp’ (5)

Visions Fugitives Op.22:

No.1 (1)

No.5 (1)

No.10 (2)

Music for children Op.65

March of the Grasshoppers (2)

Moonlit Meadows (2)

Toccata (6) (excerpt)

Shostakovich (15)

Préludes Op. 34:

No.1 in C (1)

No.3 in G (2)

No.5 in D (2)

No.7 in A (1)

No.8 in f sharp (1)

No.12 in g sharp (3)

No.13 in F sharp (1)

No.14 in e flat (1)

No.17 in A flat (1)

No.23 in F (2)

Khachaturian (33)

Toccata (excerpt) (5)

Sonatina:

1 (4)

2 (3)

3 (excerpt) (8)

Music for children I:

1 (1)

3 (1)

7 (2)

10 (4)

Music for children II:

5 (3)

7 (2)

Krenek (14)

Twelve short piano pieces in the 12-tone technique Op.83:

No. 2 Peaceful Mood (1)

No. 3 Walking on a Stormy Day (2)

No. 4 The Moon Rises (1)

No. 5 Little Chessmen (1)

No. 6 A Boat, Slowly Sailing (2)

No. 8 Glass Figures (1)

No. 9 The Sailing Boat, Reflected in the Pond (1)

No. 10 On the High Mountains (2)

No. 11 Bells in the Fog (1)

No. 12 Indian-Summer Day (2)

Total: 779 pages

Average: 29.96 per composer

Appendix II - Tables

Major	Handel	JSBach	Scarlatti	Haydn	Mozart	Beethoven	Schubert	Schumann	Mendelssohn
I	19.20	21.21	18.52	20.59	20.64	18.88	20.68	18.41	20.99
I# - IIb	0.27	0.51	0.57	0.50	0.55	1.23	0.66	0.45	0.81
II	13.11	12.39	15.91	11.53	11.16	11.32	9.57	11.54	10.00
II# - IIIb	0.05	0.16	0.07	0.34	0.63	0.74	1.05	0.55	1.14
III	15.49	15.39	13.55	16.23	16.87	15.73	18.27	17.12	17.33
IV	10.78	10.77	12.47	13.08	10.52	9.62	8.47	8.44	9.05
IV# - Vb	0.58	0.85	0.80	1.51	1.28	1.59	1.27	1.67	1.98
V	21.28	19.96	20.90	19.56	22.65	24.92	25.14	21.82	21.66
V# - VIb	0.25	0.51	0.13	0.69	1.03	0.81	1.29	1.83	1.78
VI	7.68	6.45	6.97	6.03	5.67	4.31	4.82	6.67	5.09
VI# - VIIb	0.20	0.73	0.34	0.89	0.62	2.09	1.31	1.02	1.70
VII	11.08	11.03	9.71	9.00	8.34	8.70	7.39	10.44	8.41
Tot.dur.M	54.24	31.16	66.83	96.92	106.74	88.18	243.34	54.71	67.51

Major	Chopin	Liszt	Franck	Brahms	Tchaikovsky	Grieg	Fauré	Debussy	Scriabin
I	20.09	18.73	18.83	19.62	19.29	20.55	15.90	17.73	17.53
I# - IIb	1.17	1.85	2.48	1.58	1.28	1.00	2.88	1.15	2.83
II	10.09	7.01	8.36	9.33	8.46	10.21	8.97	11.45	6.41
II# - IIIb	1.05	1.88	2.73	1.33	1.50	1.33	3.63	1.62	2.52
III	17.07	19.64	16.09	17.07	17.86	14.86	15.37	15.24	17.23
IV	9.12	7.66	8.31	7.82	8.17	9.13	7.52	8.12	6.68
IV# - Vb	1.62	1.46	3.58	2.55	2.28	1.71	3.84	1.59	2.81
V	24.64	18.75	19.11	20.70	21.41	18.54	18.85	20.91	21.17
V# - VIb	0.78	2.85	2.95	1.65	2.41	1.97	3.44	2.43	2.31
VI	6.18	8.11	7.12	7.80	6.82	9.90	6.75	8.90	7.06
VI# - VIIb	1.27	3.30	3.19	2.26	2.63	1.41	4.32	2.61	3.60
VII	6.82	8.65	7.21	8.24	7.82	9.32	8.43	8.16	9.79
Tot.dur.M	54.04	73.15	59.86	99.73	128.03	136.52	139.65	173.00	67.37

Major	Rachm.	Satie	Ravel	Bartók	Prokofiev	Shostakovich	Khachaturian	W. Aver.	W. S.D
I	20.88	14.74	15.43	17.35	16.25	19.26	16.81	18.51	2.00
I# - IIb	1.63	2.21	3.07	3.73	2.77	4.03	3.46	1.77	1.13
II	6.31	11.38	9.21	9.31	7.77	8.04	8.56	9.75	1.90
II# - IIIb	1.70	2.46	2.51	3.65	3.07	4.25	3.42	1.85	1.14
III	18.82	14.26	15.56	14.36	16.52	16.17	14.08	16.24	1.54
IV	5.95	9.29	8.51	8.52	8.72	6.27	8.67	8.74	1.50
IV# - Vb	1.68	2.28	2.72	3.94	3.69	3.58	3.51	2.24	1.00
V	22.81	16.89	19.67	19.15	17.60	21.02	19.10	20.77	2.34
V# - VIb	3.17	2.13	2.76	3.57	3.42	2.89	3.58	2.17	1.03
VI	5.71	10.35	6.83	6.10	7.65	5.12	5.95	6.85	1.59
VI# - VIIb	2.93	5.02	5.09	3.44	2.81	2.28	4.73	2.62	1.48
VII	8.35	8.92	8.45	6.67	9.55	6.98	7.93	8.41	1.01
Tot.dur.M	102.7	115.1	190.8	138.3	104.78	54.93	104.96	102.1	49.26

Table 17 - Average Percentage of Use of Scale Degrees in Major

Minor	Handel	JSBach	Scarlatti	Haydn	Mozart	Beethoven	Schubert	Schumann	Mendelssohn
I	20.18	19.92	18.57	18.39	20.34	19.01	18.55	19.01	19.88
I# - Iib	0.12	0.27	0.49	0.48	0.34	0.62	0.62	0.58	0.91
II	12.99	13.75	12.11	12.58	12.47	10.48	12.21	11.24	9.93
II# - IIIb	15.75	13.89	14.85	15.14	18.05	14.21	14.47	15.43	15.45
III	0.39	0.62	0.64	1.03	0.63	0.66	0.65	0.63	1.41
IV	10.64	13.16	11.74	11.35	8.73	8.74	10.00	10.63	9.09
IV# - Vb	0.19	0.37	2.34	1.52	2.91	2.00	1.00	2.54	2.01
V	21.49	18.69	18.87	20.41	22.29	26.74	24.44	21.51	22.44
V# - VIb	5.69	7.19	7.06	6.78	3.83	6.02	7.60	7.70	6.07
VI	1.98	2.13	1.98	1.56	1.63	1.32	1.01	1.14	2.59
VI# - VIIb	2.49	1.29	2.29	1.55	1.99	1.24	1.77	3.27	4.34
VII	8.03	8.66	9.01	9.14	6.73	8.72	7.62	6.28	5.83
Tot.dur.m.	41.07	33.41	163.73	40.66	30.26	79.56	171.95	34.36	83.61

Minor	Chopin	Liszt	Franck	Brahms	Tchaikovsky	Grieg	Fauré	Debussy	Scriabin
I	21.60	15.71	16.42	18.80	19.44	20.63	17.55	19.13	21.47
I# - Iib	1.26	2.49	2.14	1.25	0.93	1.09	2.35	0.81	0.41
II	10.35	7.94	9.58	10.54	10.07	10.47	7.88	10.03	8.91
II# - IIIb	16.60	15.14	13.83	13.73	14.69	14.25	15.76	16.90	17.31
III	0.82	4.31	2.16	1.91	1.48	1.50	2.21	0.41	0.35
IV	8.20	6.87	9.87	9.39	9.89	8.81	9.45	10.87	7.97
IV# - Vb	1.81	3.76	3.22	2.70	2.67	2.73	3.67	2.42	2.57
V	22.95	17.68	17.72	18.77	20.80	19.81	20.10	18.21	23.17
V# - VIb	7.48	7.07	8.33	8.59	7.02	6.64	6.19	7.25	7.64
VI	2.50	4.40	3.93	3.86	3.55	3.41	3.97	4.27	1.44
VI# - VIIb	1.74	4.56	4.42	3.91	3.20	4.50	3.88	5.57	3.26
VII	4.62	9.93	8.26	6.44	6.19	6.10	6.81	4.08	5.45
Tot.dur.m.	57.06	71.06	82.11	114.48	87.59	95.54	47.49	31.88	60.61

Minor	Rachm.	Satie	Ravel	Bartók	Prokofiev	Shostakovich	Khachaturian	W. Aver.	W.S.D.
I	21.94	28.13	18.15	21.32	19.29	23.19	16.77	19.32	2.09
I# - Iib	0.58	0.59	0.88	1.80	2.30	3.20	3.25	1.10	0.80
II	9.90	6.87	9.85	7.19	8.08	9.07	12.59	10.49	1.67
II# - IIIb	18.65	18.09	15.41	19.26	17.61	15.25	13.81	15.37	1.54
III	0.58	0.48	0.90	1.82	3.40	3.51	1.57	1.30	0.97
IV	8.47	8.02	10.31	4.79	5.71	6.26	7.08	9.28	1.72
IV# - Vb	2.07	2.88	2.97	4.31	4.77	2.87	5.78	2.53	1.15
V	22.11	21.05	18.13	23.86	21.12	19.71	17.51	20.92	2.59
V# - VIb	5.52	6.53	9.15	5.08	5.37	6.52	7.49	6.99	1.08
VI	2.55	2.81	4.83	1.23	2.90	3.17	4.53	2.68	1.21
VI# - VIIb	2.13	2.50	2.64	3.22	3.87	2.80	3.36	3.00	1.14
VII	5.42	1.99	6.68	6.01	5.40	4.29	6.08	6.92	1.67
Tot.dur.m.	75.21	34.08	54.42	45.18	42.26	16.57	56.06	66.01	38.86

Table 18 – Average Percentage of Use of Scale Degrees in Minor

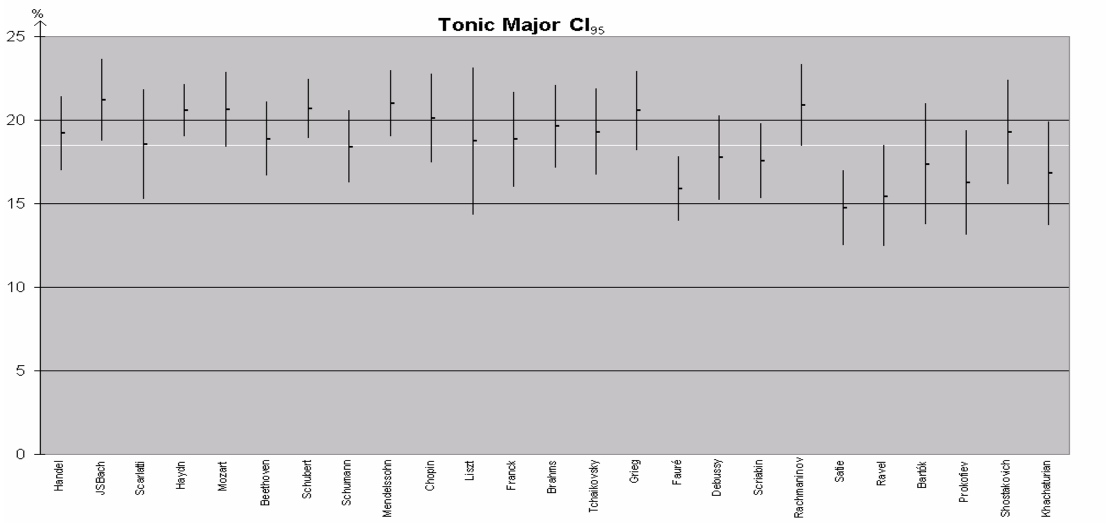


Figure 59 - Confidence Intervals for Tonic Major

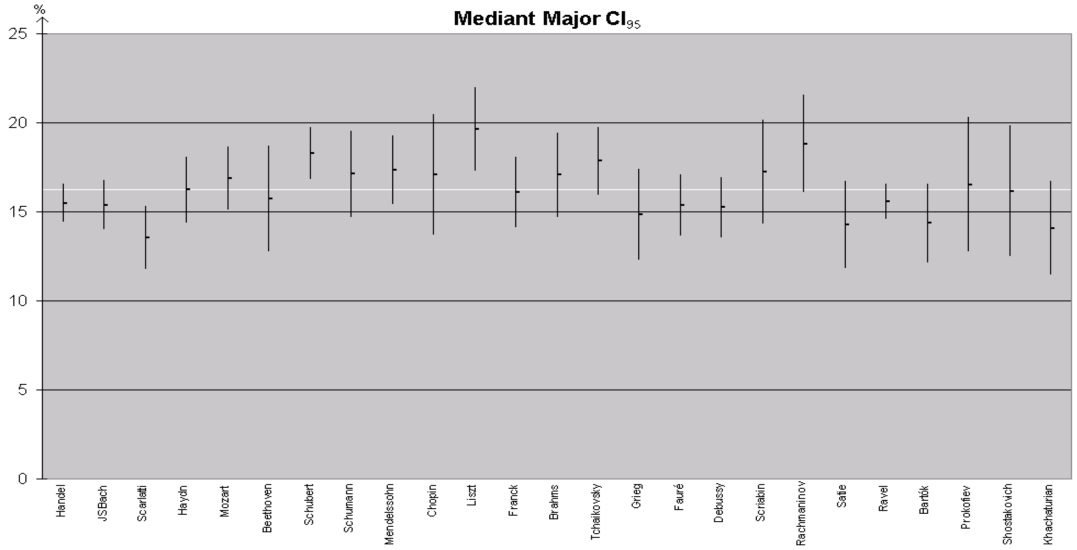


Figure 20 - Confidence Intervals for Mediant Major

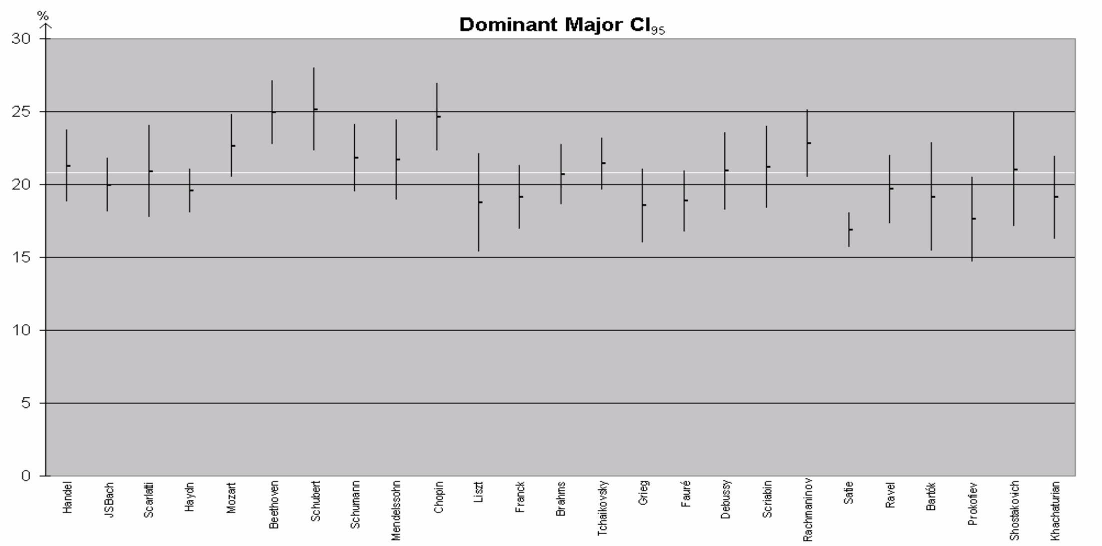


Figure 61 - Confidence Intervals for Dominant Major

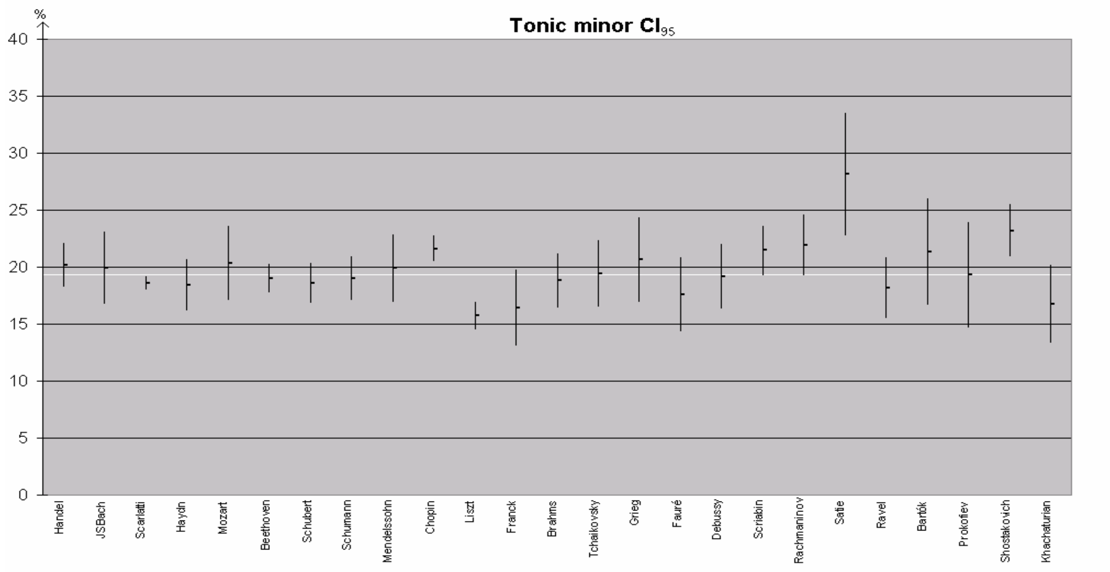


Figure 72 - Confidence Intervals for Tonic minor

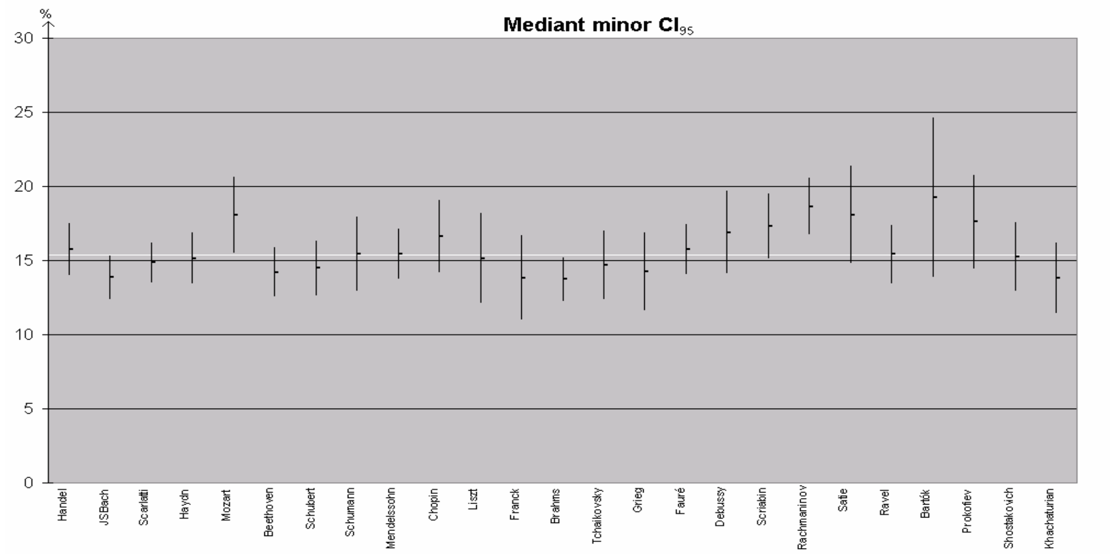


Figure 83 - Confidence Intervals for Mediant minor

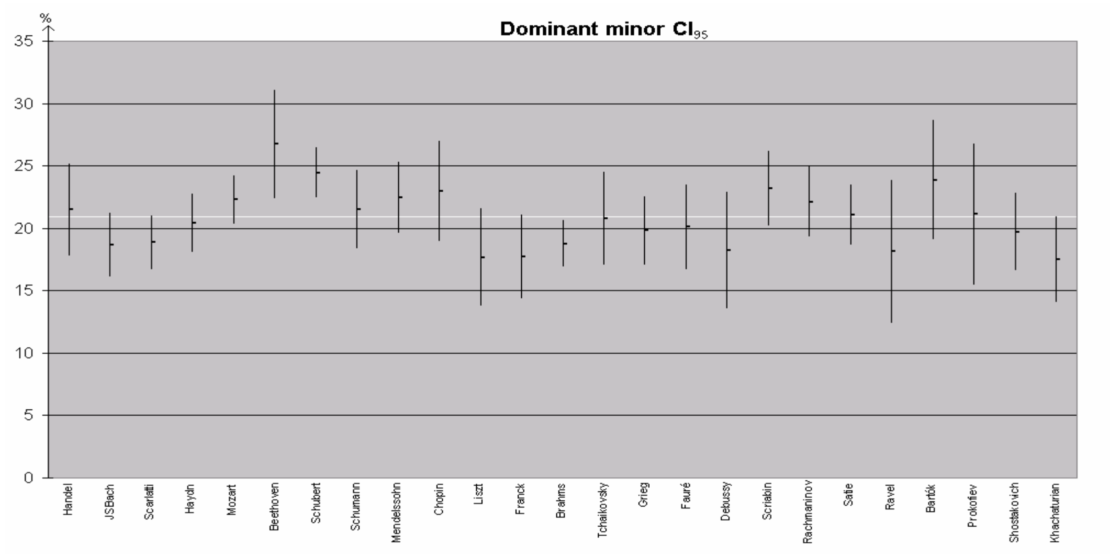


Figure 94 - Confidence Intervals for Dominant minor

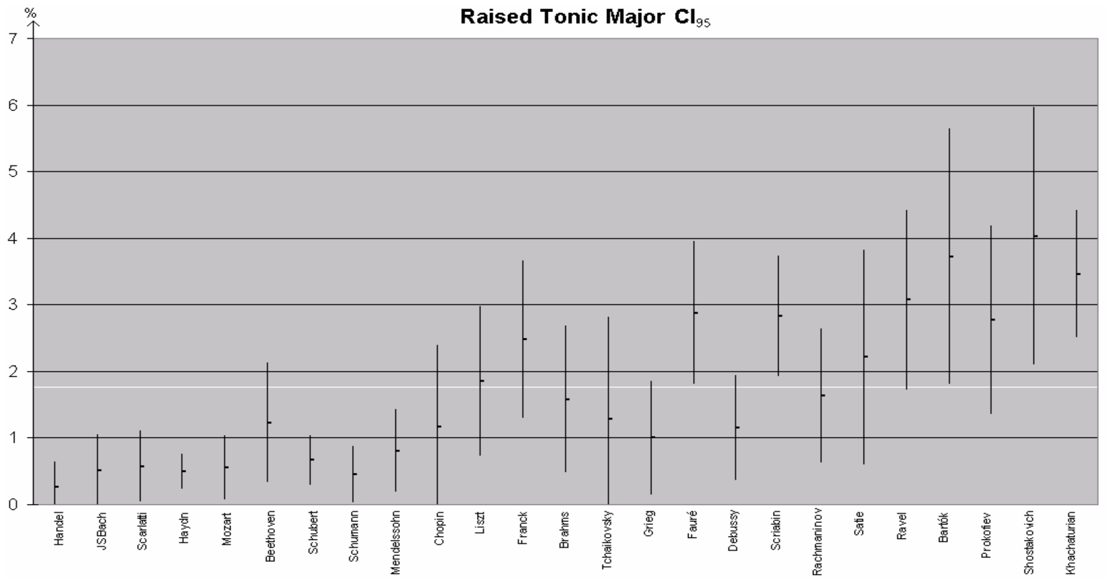


Figure 25 - Confidence Intervals for Raised Tonic Major

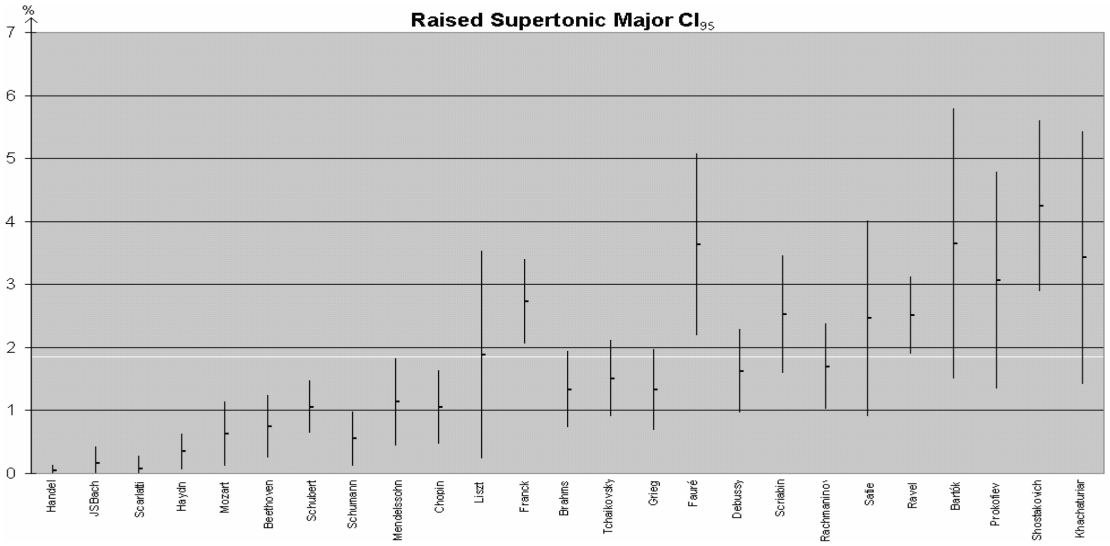


Figure 106 - Confidence Intervals for Raised Supertonic Major

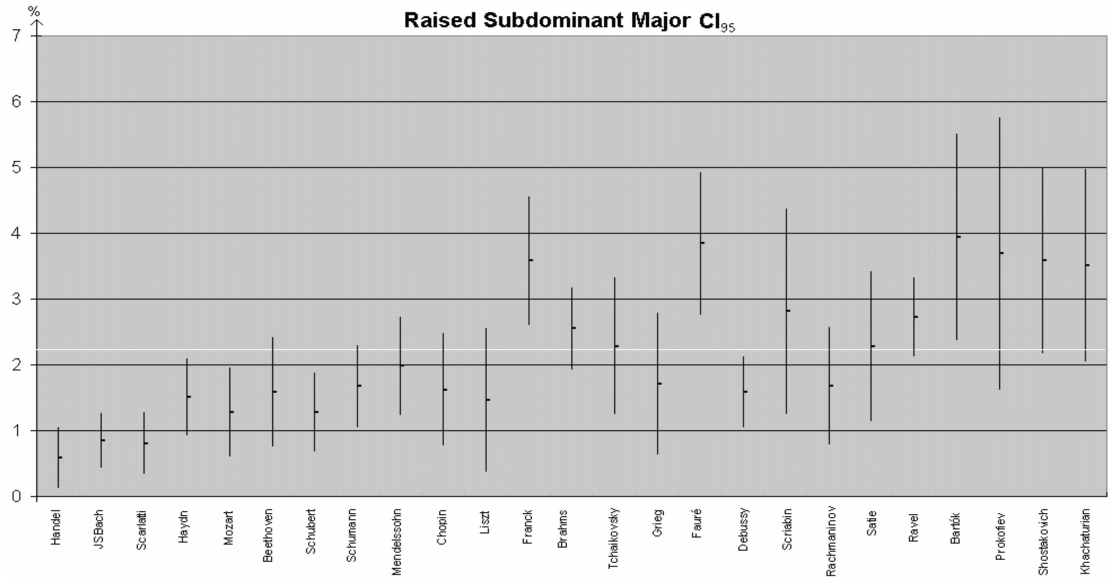


Figure 117 - Confidence Intervals for Raised Subdominant Major

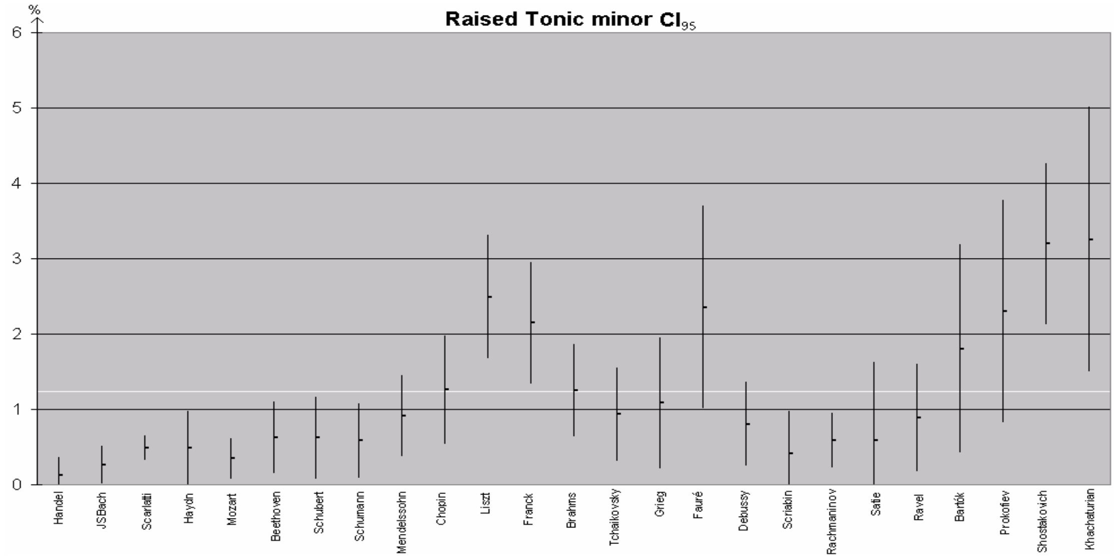


Figure 128 - Confidence Intervals for Raised Tonic minor

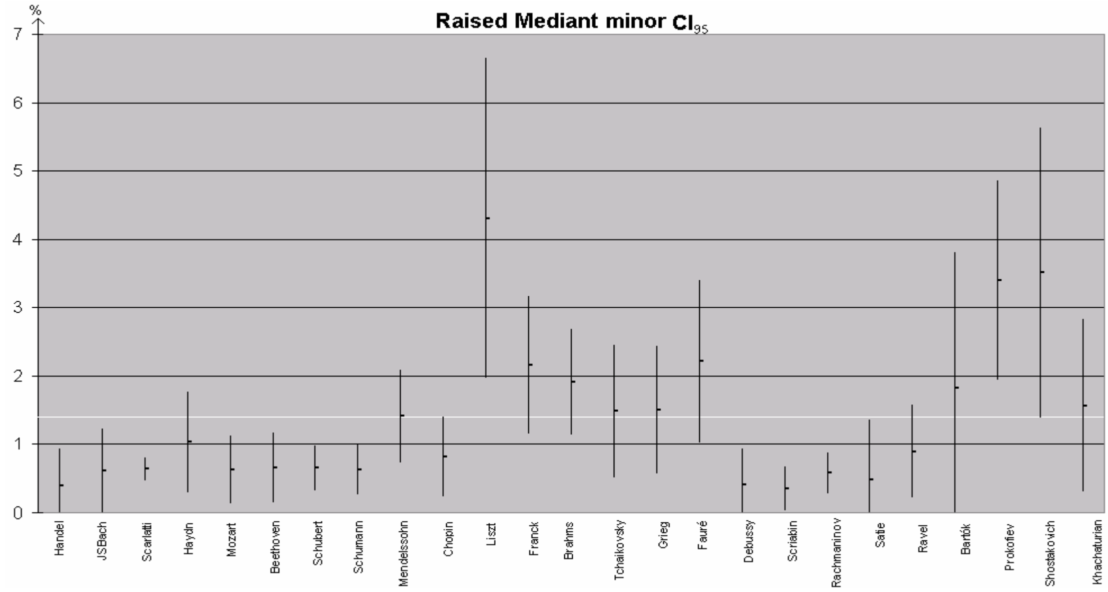


Figure 29 - Confidence Intervals for Raised mediant minor

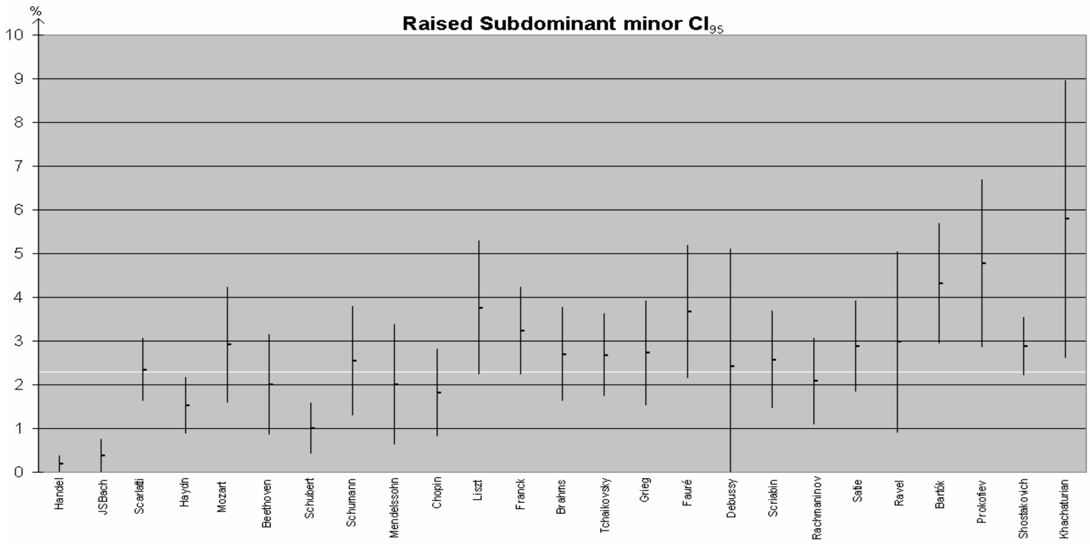


Figure 30- Confidence Intervals for Raised Subdominant minor

Major	Handel	JSBach	Scarlatti	Haydn	Mozart	Beethoven	Schubert	Schumann	Mendelssohn
I	163	98	129	216	381	246	759	115	204
i	0	1	0	1	3	1	12	3	2
I+	0	1	0	2	6	1	2	4	0
#iº	2	0	8	2	1	4	10	3	2
ii	23	14	23	30	55	19	34	12	13
iiº	0	1	0	0	3	1	13	6	2
II	2	2	0	2	2	1	18	7	1
bII	0	0	0	0	0	0	0	0	1
iii	7	9	8	13	11	11	56	7	12
bIII	0	0	0	0	0	2	8	0	0
III	2	1	0	1	2	0	10	8	3
IV	25	20	20	41	46	26	33	22	32
iv	0	0	0	1	1	2	19	1	4
IV+	0	0	2	0	3	0	2	0	0
#ivº	1	6	0	4	8	4	2	0	8
#iv It	0	0	0	0	4	0	0	0	0
V	77	44	51	75	108	41	185	46	29
V+	0	0	0	1	0	0	2	0	0
#vº	0	0	0	1	0	0	2	0	1
vi	14	9	7	22	42	6	51	17	8
VI	0	2	0	2	2	2	1	2	0
bVI	0	0	0	0	0	0	1	0	1
viiº	21	28	36	31	39	11	18	7	23
VII	0	0	0	0	0	1	0	1	0
vii It	0	0	0	0	0	0	0	0	0
I7	26	9	9	19	27	8	18	11	23
I -7	2	4	0	15	3	14	37	6	10
#i d7	1	0	0	1	0	3	0	0	1
I+ -7	0	0	0	1	0	0	0	0	0
ii7	7	3	8	8	3	4	19	3	4
II7	2	0	5	6	1	6	7	4	3
iiº7	0	1	0	0	0	0	5	0	5
#ii d7	1	0	0	0	0	3	0	0	1
ii7F	0	0	0	0	0	0	0	0	0
#ii7G	0	0	0	0	0	2	0	0	0
iii7	22	8	9	5	5	3	23	8	8
iii d7	0	1	0	1	1	5	1	0	0
III7	0	0	0	0	1	2	3	5	1
IV7	7	5	7	9	3	4	6	5	6
#ivº7	0	0	0	2	0	0	2	2	1
#iv d7	0	1	0	1	4	1	8	4	4
#iv 7G	0	0	0	0	0	0	2	0	0
V7	35	14	21	64	74	94	168	39	65
#v d7	0	0	0	0	1	4	0	0	2
v 7F	0	0	0	1	0	0	0	0	0
V+7	0	0	0	0	0	0	0	0	0
vi7	12	7	19	5	32	6	13	11	15
VI7	0	0	0	1	1	1	5	1	1
#vi d7	0	0	0	0	0	3	1	0	1
bVI -7	0	0	0	0	2	0	0	0	0
viiº7	2	1	15	7	6	0	6	3	1
vii d7	0	0	1	0	1	4	13	0	3
VII7	0	0	0	0	0	0	0	0	0
vii7G	0	0	0	0	0	0	0	0	0
Total M	454	290	378	591	882	546	1575	363	501

Table 19 - Chords in Major mode, part 1

Major	Chopin	Liszt	Franck	Brahms	Tchaikovsky	Grieg	Fauré	Debussy	Scriabin	Rachmaninov
I	161	71	102	185	281	184	176	277	90	189
i	1	3	4	20	6	6	21	28	9	31
I+	3	2	0	1	3	2	21	2	10	14
#iº	2	2	5	6	3	4	12	2	5	10
ii	16	24	5	9	29	27	14	61	13	10
iiº	0	1	3	5	12	2	14	6	7	2
II	1	4	2	9	2	19	11	6	3	1
bII	0	0	2	0	0	5	6	2	0	0
iii	12	44	19	24	30	45	28	68	46	69
bIII	0	0	2	3	0	3	18	12	2	2
III	1	0	2	3	2	6	15	10	1	11
IV	8	14	9	39	35	33	33	45	9	20
iv	1	0	11	8	1	13	12	17	3	33
IV+	0	1	0	1	3	0	4	1	1	0
#ivº	0	2	4	7	2	7	7	7	2	3
#iv It	1	0	0	0	0	4	1	0	1	0
V	25	6	18	43	57	55	38	43	15	25
V+	0	3	0	0	0	2	15	7	5	3
#vº	0	0	1	3	0	0	3	3	0	2
vi	4	12	16	30	42	48	20	58	25	26
VI	0	1	3	0	2	4	10	2	1	1
bVI	0	0	1	2	0	6	9	1	0	3
viiº	3	5	6	18	9	7	29	5	5	12
VII	0	0	3	1	9	3	3	5	2	0
vii It	0	0	1	0	6	0	0	0	0	0
I7	12	18	16	38	21	43	34	46	30	55
I -7	13	11	7	29	34	8	45	29	15	27
#i d7	0	1	4	6	2	0	2	1	3	3
I+ -7	0	0	0	1	0	0	1	0	0	0
ii7	3	3	2	13	2	36	7	41	5	8
II7	6	1	3	13	1	4	7	9	2	1
iiº7	0	2	3	5	4	1	9	8	2	8
#ii d7	1	0	4	0	7	0	1	0	2	0
ii7F	1	0	1	1	12	0	1	0	0	0
#ii 7G	0	0	0	0	0	0	0	0	0	0
iii7	3	9	4	14	7	21	12	23	9	8
iii d7	1	2	5	2	3	0	4	0	0	5
III7	0	0	1	3	0	2	9	4	0	3
IV7	12	8	3	6	4	37	21	11	2	8
#ivº7	0	3	11	1	3	9	7	11	4	13
#iv d7	1	0	0	5	3	2	3	0	0	1
#iv 7G	0	0	0	0	8	0	0	0	0	0
V7	50	11	10	50	47	31	69	62	25	29
#v d7	0	1	3	0	2	3	2	1	2	3
v7F	1	0	0	2	2	0	1	0	6	0
V+7	0	2	1	0	0	0	5	0	3	2
vi7	36	26	17	51	47	57	73	110	32	41
VI7	2	3	2	4	2	3	7	0	6	5
#vi d7	4	1	1	1	1	0	2	1	2	0
bVI -7	1	0	0	0	0	0	4	1	1	0
viiº7	0	1	2	3	9	24	8	20	1	5
vii d7	0	4	3	8	14	2	1	2	3	2
VII7	0	0	0	2	0	1	3	3	5	0
vii 7G	0	0	0	0	4	2	0	0	1	0
Total M	386	302	322	675	773	771	858	1051	416	694

Table 20 - Chords in Major mode, part 2

Major	Satie	Ravel	Bartók	Prokofiev	Shostakovich	Khachaturian	Totals	Percent	Grouped	Grouped %
I	74	186	136	104	97	107	4731	31.31	7987	32.66
i	17	41	23	16	20	34	303	2.01		
I+	3	10	9	4	1	2	103	0.68		
#iº	9	36	20	3	1	19	171	1.13		
ii	47	38	19	23	8	18	584	3.86	934	3.82
iiº	4	12	5	6	2	6	113	0.75		
II	13	4	6	5	2	5	128	0.85		
bII	5	5	1	2	4	5	38	0.25		
iii	51	87	52	34	17	39	799	5.29	1314	5.37
bIII	12	7	3	1	5	5	85	0.56		
III	10	15	7	1	2	4	117	0.77		
IV	23	26	16	31	8	11	625	4.14	1100	4.50
iv	11	9	7	8	3	24	189	1.25		
IV+	2	2	2	0	1	2	27	0.18		
#ivº	2	6	12	14	1	10	119	0.79		
#iv It	0	0	2	1	2	0	16	0.11		
V	43	23	36	30	10	26	1149	7.60	2162	8.84
V+	1	1	3	9	1	2	55	0.36		
#vº	0	5	1	0	1	6	29	0.19		
vi	51	51	11	31	7	15	623	4.12	964	3.94
VI	8	15	8	4	3	3	76	0.50		
bVI	6	6	3	8	3	3	53	0.35		
viiº	3	18	24	16	10	10	394	2.61	686	2.81
VII	8	2	2	1	2	3	46	0.30		
vii It	0	0	2	3	0	1	13	0.09		
I7	37	61	24	51	9	49	694	4.59	883.00	3.61
I -7	22	42	3	5	2	18	401	2.65		
#i d7	1	4	0	0	0	0	33	0.22		
I+ -7	0	0	0	1	0	1	5	0.03		
ii7	14	9	16	8	3	8	237	1.57	541.00	2.21
II7	8	5	2	2	0	1	99	0.66		
iiº7	4	6	2	1	1	18	85	0.56		
#ii d7	0	1	0	0	0	1	22	0.15		
ii7F	0	0	0	0	0	0	16	0.11		
#ii7G	0	0	0	0	0	0	2	0.01		
iii7	15	27	6	8	7	3	267	1.77	302.00	1.23
iii d7	1	8	0	0	0	0	40	0.26		
III7	6	10	0	0	0	0	50	0.33		
IV7	26	23	5	4	4	8	234	1.55	406.00	1.66
#ivº7	1	10	2	1	1	2	86	0.57		
#iv d7	0	0	0	1	0	0	39	0.26		
#iv 7G	0	0	0	0	0	0	10	0.07		
V7	8	32	12	13	7	8	1038	6.87	1610.00	6.58
#v d7	0	2	4	3	0	0	33	0.22		
v7F	0	1	0	1	1	3	19	0.13		
V+7	0	2	0	1	1	0	17	0.11		
vi7	25	42	40	8	9	18	752	4.98	1091.00	4.46
VI7	7	6	7	0	3	4	71	0.47		
#vid7	0	0	0	0	1	2	21	0.14		
bVI-7	0	0	0	1	0	0	10	0.07		
viiº7	7	16	5	2	0	10	154	1.02	199.00	0.81
vii d7	0	0	4	1	0	0	66	0.44		
VII7	1	0	0	1	0	0	16	0.11		
vii 7G	0	0	0	0	0	0	7	0.05		
Total M	586	912	542	468	260	514	15110	100.00	8905.00	82.51

Table 21 - Chords in Major mode, part 3

Minor	Handel	JSBach	Scarlatti	Haydn	Mozart	Beethoven	Schubert	Schumann	Mendelssohn
i	102	75	119	92	118	225	336	67	227
I	1	2	8	1	1	8	17	3	10
iiº	5	25	21	12	7	35	56	9	17
II	0	0	3	0	0	2	0	1	3
bII	0	0	1	1	1	4	7	2	2
III	4	1	3	1	4	2	10	5	24
III+	4	17	21	11	8	7	34	3	6
iv	13	18	45	10	4	14	36	13	38
IV	0	3	7	2	3	1	13	3	8
#ivº	0	1	3	1	1	3	0	1	2
#iv It	0	0	0	1	0	3	5	0	1
v	8	3	8	4	7	2	8	3	22
V	49	23	56	29	35	83	131	28	59
VI	5	7	10	7	2	20	35	7	17
#viº	2	2	6	2	5	5	12	0	8
VII	4	2	0	3	5	5	4	2	7
#viiº	9	14	28	12	10	6	10	0	9
i7	5	4	7	0	4	0	15	2	17
I7	0	2	1	3	0	1	2	1	3
iiº7	15	11	8	5	1	7	30	4	24
ii d7	0	0	0	0	0	0	2	0	5
II7	1	0	3	0	0	0	1	0	1
ii7F	0	0	0	0	2	0	1	0	1
III7	2	3	1	2	0	0	1	1	0
III -7	0	0	0	0	0	0	0	0	0
#iii d7	0	0	0	0	0	0	1	0	12
iv7	4	9	5	4	2	5	6	2	15
IV7	0	2	0	2	2	9	1	0	9
#iv d7	2	0	2	1	5	12	8	0	2
#iv 7G	0	0	0	0	10	5	10	0	1
v7	0	1	0	2	3	0	0	0	2
V7	15	23	26	24	17	74	79	14	60
VI7	8	3	14	4	0	20	16	8	15
#viº7	1	1	2	5	4	3	4	1	21
VI -7	0	0	0	0	0	1	0	0	0
VII7	0	0	2	3	2	1	12	2	6
#vii d7	3	7	8	15	4	10	12	0	9
Total m	262	259	418	259	267	573	915	182	663
Total	716	549	796	850	1149	1119	2490	545	1164

Table 22 - Chords in Minor mode, part 1

Minor	Chopin	Liszt	Franck	Brahms	Tchaikovsky	Grieg	Fauré	Debussy	Scriabin
i	190	32	105	164	217	121	89	46	175
I	7	14	5	25	3	16	4	2	1
iiº	31	5	14	25	7	11	11	10	5
II	1	1	5	10	4	1	0	2	2
bII	6	4	2	5	1	16	4	1	0
III	4	11	9	18	10	19	9	8	7
III+	10	17	23	23	32	24	10	6	26
iv	15	5	17	57	35	21	10	15	28
IV	0	2	14	16	18	19	5	11	1
#ivº	3	6	6	7	0	0	7	1	0
#iv It	2	1	2	2	0	1	0	0	4
v	7	6	16	17	2	12	5	6	3
V	26	9	25	67	53	50	12	12	15
VI	15	6	30	33	10	28	19	12	14
#viº	3	9	9	23	13	10	3	4	1
VII	0	0	10	16	5	8	4	12	4
#viiº	3	8	9	11	5	9	3	1	1
i7	7	11	11	3	17	11	2	14	14
I7	2	0	2	10	4	3	1	1	1
iiº7	21	0	8	28	35	18	10	2	16
ii d7	1	6	3	1	0	0	0	0	0
II7	3	2	0	10	6	11	0	0	2
ii7F	1	0	2	5	2	5	4	0	7
III7	0	9	4	3	1	0	0	1	3
III -7	0	2	4	6	0	0	2	0	0
#iii d7	1	1	2	2	0	0	2	0	2
iv7	8	5	9	14	10	16	6	20	8
IV7	3	5	5	9	17	7	4	10	0
#iv d7	4	3	6	10	6	4	1	2	3
#iv7G	1	0	3	2	10	5	0	0	8
v7	0	0	0	4	4	8	1	1	2
V7	22	2	26	47	35	18	17	2	17
VI7	15	3	20	53	15	28	8	6	24
#viº7	17	2	15	15	3	24	9	14	14
VI -7	1	1	1	1	0	0	5	2	0
VII7	1	0	2	5	1	2	1	1	3
#vii d7	14	5	23	7	6	9	1	0	5
Total m	445	193	447	754	587	535	269	225	416
Total	831	495	769	1429	1360	1306	1127	1276	832

Table 23 - Chord in Minor mode, part 2

Minor	Rachmaninov	Satie	Ravel	Bartók	Prokofiev	Shostakovich	Khachaturian	Totals	Percent
i	217	111	87	114	126	39	62	3256	34.84
I	4	2	9	1	3	1	3	151	1.62
iiº	17	2	6	3	10	2	4	350	3.75
II	3	3	2	0	2	0	0	45	0.48
bII	5	0	2	0	3	1	7	75	0.80
III	6	2	1	2	6	0	1	167	1.79
III+	19	0	17	14	9	3	4	348	3.72
iv	24	24	16	1	9	2	5	475	5.08
IV	1	2	8	2	2	6	6	153	1.64
#ivº	6	0	0	0	1	0	0	49	0.52
#iv It	2	0	2	0	3	1	0	30	0.32
v	7	5	13	2	3	1	6	176	1.88
V	21	1	13	12	8	3	17	837	8.96
VI	12	1	20	7	7	2	15	341	3.65
#viº	5	0	7	0	6	0	17	152	1.63
VII	5	0	8	1	1	2	4	112	1.20
#viiº	9	0	2	6	3	2	10	180	1.93
i7	9	12	9	3	11	1	0	189	2.02
I7	6	0	0	0	0	0	0	43	0.46
iiº7	16	19	17	1	0	4	4	304	3.25
ii d7	0	0	1	1	0	0	1	21	0.22
II7	3	0	0	0	2	0	6	51	0.55
ii7F	0	0	0	1	0	0	2	33	0.35
III7	1	1	0	0	0	0	2	35	0.37
III -7	1	0	0	0	0	1	1	17	0.18
#iii d7	0	0	0	0	0	0	0	23	0.25
iv7	10	1	6	2	1	0	4	172	1.84
IV7	2	1	1	0	0	0	7	96	1.03
#iv d7	5	0	1	0	0	0	2	79	0.85
#iv 7G	0	0	3	0	0	0	0	58	0.62
v7	0	0	2	0	0	0	2	32	0.34
V7	14	1	2	1	1	0	3	540	5.78
VI7	30	3	14	2	14	4	12	339	3.63
#viº7	14	19	11	0	1	3	10	213	2.28
VI -7	0	0	0	0	0	0	1	13	0.14
VII7	0	0	0	0	0	0	1	45	0.48
#viid7	5	0	0	1	1	0	0	145	1.55
Totalm	479	210	280	177	233	78	219	9345	100.00
Total	1173	796	1192	719	701	338	733	24455	100.00

Table 24 - Chords in Minor mode, part 3

Major	Handel	JSBach	Scarlatti	Haydn	Mozart	Beethoven	Schubert	Schumann	Mendelssohn
I	22.77	17.85	16.21	25.41	33.16	21.98	30.48	21.10	17.53
i	0.00	0.18	0.00	0.12	0.26	0.09	0.48	0.55	0.17
I+	0.00	0.18	0.00	0.24	0.52	0.09	0.08	0.73	0.00
#iº	0.28	0.00	1.01	0.24	0.09	0.36	0.40	0.55	0.17
ii	3.21	2.55	2.89	3.53	4.79	1.70	1.37	2.20	1.12
iiº	0.00	0.18	0.00	0.00	0.26	0.09	0.52	1.10	0.17
II	0.28	0.36	0.00	0.24	0.17	0.09	0.72	1.28	0.09
bII	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.09
iii	0.98	1.64	1.01	1.53	0.96	0.98	2.25	1.28	1.03
bIII	0.00	0.00	0.00	0.00	0.00	0.18	0.32	0.00	0.00
III	0.28	0.18	0.00	0.12	0.17	0.00	0.40	1.47	0.26
IV	3.49	3.64	2.51	4.82	4.00	2.32	1.33	4.04	2.75
iv	0.00	0.00	0.00	0.12	0.09	0.18	0.76	0.18	0.34
IV+	0.00	0.00	0.25	0.00	0.26	0.00	0.08	0.00	0.00
#ivº	0.14	1.09	0.00	0.47	0.70	0.36	0.08	0.00	0.69
#iv ^{It}	0.00	0.00	0.00	0.00	0.35	0.00	0.00	0.00	0.00
V	10.75	8.01	6.41	8.82	9.40	3.66	7.43	8.44	2.49
V+	0.00	0.00	0.00	0.12	0.00	0.00	0.08	0.00	0.00
#vº	0.00	0.00	0.00	0.12	0.00	0.00	0.08	0.00	0.09
vi	1.96	1.64	0.88	2.59	3.66	0.54	2.05	3.12	0.69
VI	0.00	0.36	0.00	0.24	0.17	0.18	0.04	0.37	0.00
bVI	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.09
viiº	2.93	5.10	4.52	3.65	3.39	0.98	0.72	1.28	1.98
VII	0.00	0.00	0.00	0.00	0.00	0.09	0.00	0.18	0.00
vii ^{It}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
I7	3.63	1.64	1.13	2.24	2.35	0.71	0.72	2.02	1.98
I -7	0.28	0.73	0.00	1.76	0.26	1.25	1.49	1.10	0.86
#i ^{d7}	0.14	0.00	0.00	0.12	0.00	0.27	0.00	0.00	0.09
I+ -7	0.00	0.00	0.00	0.12	0.00	0.00	0.00	0.00	0.00
ii7	0.98	0.55	1.01	0.94	0.26	0.36	0.76	0.55	0.34
II7	0.28	0.00	0.63	0.71	0.09	0.54	0.28	0.73	0.26
iiº7	0.00	0.18	0.00	0.00	0.00	0.00	0.20	0.00	0.43
#ii ^{d7}	0.14	0.00	0.00	0.00	0.00	0.27	0.00	0.00	0.09
ii ^{7F}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
#ii ^{7G}	0.00	0.00	0.00	0.00	0.00	0.18	0.00	0.00	0.00
iii7	3.07	1.46	1.13	0.59	0.44	0.27	0.92	1.47	0.69
iii ^{d7}	0.00	0.18	0.00	0.12	0.09	0.45	0.04	0.00	0.00
III7	0.00	0.00	0.00	0.00	0.09	0.18	0.12	0.92	0.09
IV7	0.98	0.91	0.88	1.06	0.26	0.36	0.24	0.92	0.52
#ivº7	0.00	0.00	0.00	0.24	0.00	0.00	0.08	0.37	0.09
#iv ^{d7}	0.00	0.18	0.00	0.12	0.35	0.09	0.32	0.73	0.34
#iv ^{7G}	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00	0.00
V7	4.89	2.55	2.64	7.53	6.44	8.40	6.75	7.16	5.58
#v ^{d}**7**	0.00	0.00	0.00	0.00	0.09	0.36	0.00	0.00	0.17
v ^{7F}	0.00	0.00	0.00	0.12	0.00	0.00	0.00	0.00	0.00
V+7	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
vi7	1.68	1.28	2.39	0.59	2.79	0.54	0.52	2.02	1.29
VI7	0.00	0.00	0.00	0.12	0.09	0.09	0.20	0.18	0.09
#vi ^{d7}	0.00	0.00	0.00	0.00	0.00	0.27	0.04	0.00	0.09
bVI -7	0.00	0.00	0.00	0.00	0.17	0.00	0.00	0.00	0.00
viiº7	0.28	0.18	1.88	0.82	0.52	0.00	0.24	0.55	0.09
vii ^{d7}	0.00	0.00	0.13	0.00	0.09	0.36	0.52	0.00	0.26
VII7	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
vii ^{7G}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Total M	454	290	378	591	882	546	1575	363	501

Table 25 - Chords in Major mode in percentage, part 1

Major	Chopin	Liszt	Franck	Brahms	Tchaikovsky	Grieg	Fauré	Debussy	Scriabin
I	19.37	14.34	13.26	12.93	20.66	14.09	15.62	21.66	10.82
i	0.12	0.61	0.52	1.40	0.44	0.46	1.86	2.19	1.08
I+	0.36	0.40	0.00	0.07	0.22	0.15	1.86	0.16	1.20
#iº	0.24	0.40	0.65	0.42	0.22	0.31	1.06	0.16	0.60
ii	1.93	4.85	0.65	0.63	2.13	2.07	1.24	4.77	1.56
iiº	0.00	0.20	0.39	0.35	0.88	0.15	1.24	0.47	0.84
II	0.12	0.81	0.26	0.63	0.15	1.45	0.98	0.47	0.36
bII	0.00	0.00	0.26	0.00	0.00	0.38	0.53	0.16	0.00
iii	1.44	8.89	2.47	1.68	2.21	3.45	2.48	5.32	5.53
bIII	0.00	0.00	0.26	0.21	0.00	0.23	1.60	0.94	0.24
III	0.12	0.00	0.26	0.21	0.15	0.46	1.33	0.78	0.12
IV	0.96	2.83	1.17	2.73	2.57	2.53	2.93	3.52	1.08
iv	0.12	0.00	1.43	0.56	0.07	1.00	1.06	1.33	0.36
IV+	0.00	0.20	0.00	0.07	0.22	0.00	0.35	0.08	0.12
#ivº	0.00	0.40	0.52	0.49	0.15	0.54	0.62	0.55	0.24
#iv ^{It}	0.12	0.00	0.00	0.00	0.00	0.31	0.09	0.00	0.12
V	3.01	1.21	2.34	3.00	4.19	4.21	3.37	3.36	1.80
V+	0.00	0.61	0.00	0.00	0.00	0.15	1.33	0.55	0.60
#vº	0.00	0.00	0.13	0.21	0.00	0.00	0.27	0.23	0.00
vi	0.48	2.42	2.08	2.10	3.09	3.68	1.77	4.53	3.00
VI	0.00	0.20	0.39	0.00	0.15	0.31	0.89	0.16	0.12
bVI	0.00	0.00	0.13	0.14	0.00	0.46	0.80	0.08	0.00
viiº	0.36	1.01	0.78	1.26	0.66	0.54	2.57	0.39	0.60
VII	0.00	0.00	0.39	0.07	0.66	0.23	0.27	0.39	0.24
vii ^{It}	0.00	0.00	0.13	0.00	0.44	0.00	0.00	0.00	0.00
I7	1.44	3.64	2.08	2.66	1.54	3.29	3.02	3.60	3.61
I -7	1.56	2.22	0.91	2.03	2.50	0.61	3.99	2.27	1.80
#i ^{d7}	0.00	0.20	0.52	0.42	0.15	0.00	0.18	0.08	0.36
I+ -7	0.00	0.00	0.00	0.07	0.00	0.00	0.09	0.00	0.00
ii7	0.36	0.61	0.26	0.91	0.15	2.76	0.62	3.21	0.60
II7	0.72	0.20	0.39	0.91	0.07	0.31	0.62	0.70	0.24
iiº7	0.00	0.40	0.39	0.35	0.29	0.08	0.80	0.63	0.24
#ii ^{d7}	0.12	0.00	0.52	0.00	0.51	0.00	0.09	0.00	0.24
ii ^{7F}	0.12	0.00	0.13	0.07	0.88	0.00	0.09	0.00	0.00
#ii ^{7G}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
iii7	0.36	1.82	0.52	0.98	0.51	1.61	1.06	1.80	1.08
iii ^{d7}	0.12	0.40	0.65	0.14	0.22	0.00	0.35	0.00	0.00
III7	0.00	0.00	0.13	0.21	0.00	0.15	0.80	0.31	0.00
IV7	1.44	1.62	0.39	0.42	0.29	2.83	1.86	0.86	0.24
#ivº7	0.00	0.61	1.43	0.07	0.22	0.69	0.62	0.86	0.48
#iv ^{d7}	0.12	0.00	0.00	0.35	0.22	0.15	0.27	0.00	0.00
#iv ^{7G}	0.00	0.00	0.00	0.00	0.59	0.00	0.00	0.00	0.00
V7	6.02	2.22	1.30	3.49	3.46	2.37	6.12	4.85	3.00
#v ^{d}7	0.00	0.20	0.39	0.00	0.15	0.23	0.18	0.08	0.24
v ^{7F}	0.12	0.00	0.00	0.14	0.15	0.00	0.09	0.00	0.72
V+7	0.00	0.40	0.13	0.00	0.00	0.00	0.44	0.00	0.36
vi7	4.33	5.25	2.21	3.56	3.46	4.36	6.48	8.60	3.85
VI7	0.24	0.61	0.26	0.28	0.15	0.23	0.62	0.00	0.72
#vi ^{d7}	0.48	0.20	0.13	0.07	0.07	0.00	0.18	0.08	0.24
bVI -7	0.12	0.00	0.00	0.00	0.00	0.00	0.35	0.08	0.12
viiº7	0.00	0.20	0.26	0.21	0.66	1.84	0.71	1.56	0.12
vii ^{d7}	0.00	0.81	0.39	0.56	1.03	0.15	0.09	0.16	0.36
VII7	0.00	0.00	0.00	0.14	0.00	0.08	0.27	0.23	0.60
vii ^{7G}	0.00	0.00	0.00	0.00	0.29	0.15	0.00	0.00	0.12
Total M	386	302	322	675	773	771	858	1051	416

Table 26 - Chords in Major mode in percentage, part 2

Major	Rachmaninov	Satie	Ravel	Bartók	Prokofiev	Shostakovich	Khachaturian	Averages	Grouped
I	16.11	9.30	15.60	18.92	14.84	28.70	14.60	19.34	32.65
i	2.64	2.14	3.44	3.20	2.28	5.92	4.64	1.24	
I+	1.19	0.38	0.84	1.25	0.57	0.30	0.27	0.42	
#iº	0.85	1.13	3.02	2.78	0.43	0.30	2.59	0.70	
ii	0.85	5.90	3.19	2.64	3.28	2.37	2.46	2.39	3.82
iiº	0.17	0.50	1.01	0.70	0.86	0.59	0.82	0.46	
II	0.09	1.63	0.34	0.83	0.71	0.59	0.68	0.52	
bII	0.00	0.63	0.42	0.14	0.29	1.18	0.68	0.16	
iii	5.88	6.41	7.30	7.23	4.85	5.03	5.32	3.27	3.95
bIII	0.17	1.51	0.59	0.42	0.14	1.48	0.68	0.35	
III	0.94	1.26	1.26	0.97	0.14	0.59	0.55	0.48	
IV	1.71	2.89	2.18	2.23	4.42	2.37	1.50	2.56	4.51
iv	2.81	1.38	0.76	0.97	1.14	0.89	3.27	0.77	
IV+	0.00	0.25	0.17	0.28	0.00	0.30	0.27	0.11	
#ivº	0.26	0.25	0.50	1.67	2.00	0.30	1.36	0.49	
#iv ^{It}	0.00	0.00	0.00	0.28	0.14	0.59	0.00	0.07	
V	2.13	5.40	1.93	5.01	4.28	2.96	3.55	4.70	8.84
V+	0.26	0.13	0.08	0.42	1.28	0.30	0.27	0.22	
#vº	0.17	0.00	0.42	0.14	0.00	0.30	0.82	0.12	
vi	2.22	6.41	4.28	1.53	4.42	2.07	2.05	2.55	3.94
VI	0.09	1.01	1.26	1.11	0.57	0.89	0.41	0.31	
bVI	0.26	0.75	0.50	0.42	1.14	0.89	0.41	0.22	
viiº	1.02	0.38	1.51	3.34	2.28	2.96	1.36	1.61	2.35
VII	0.00	1.01	0.17	0.28	0.14	0.59	0.41	0.19	
vii ^{It}	0.00	0.00	0.00	0.28	0.43	0.00	0.14	0.05	
I7	4.69	4.65	5.12	3.34	7.28	2.66	6.68	2.84	3.61
I -7	2.30	2.76	3.52	0.42	0.71	0.59	2.46	1.64	
#i ^{d7}	0.26	0.13	0.34	0.00	0.00	0.00	0.00	0.13	
I+ -7	0.00	0.00	0.00	0.00	0.14	0.00	0.14	0.02	
ii7	0.68	1.76	0.76	2.23	1.14	0.89	1.09	0.97	2.21
II7	0.09	1.01	0.42	0.28	0.29	0.00	0.14	0.40	
iiº7	0.68	0.50	0.50	0.28	0.14	0.30	2.46	0.35	
#ii ^{d7}	0.00	0.00	0.08	0.00	0.00	0.00	0.14	0.09	
ii ^{7F}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	
#ii ^{7G}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	
iii7	0.68	1.88	2.27	0.83	1.14	2.07	0.41	1.09	1.23
iii ^{d7}	0.43	0.13	0.67	0.00	0.00	0.00	0.00	0.16	
III7	0.26	0.75	0.84	0.00	0.00	0.00	0.00	0.20	
IV7	0.68	3.27	1.93	0.70	0.57	1.18	1.09	0.96	1.66
#ivº7	1.11	0.13	0.84	0.28	0.14	0.30	0.27	0.35	
#iv ^{d7}	0.09	0.00	0.00	0.00	0.14	0.00	0.00	0.16	
#iv ^{7G}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	
V7	2.47	1.01	2.68	1.67	1.85	2.07	1.09	4.24	6.45
#v ^{d}7	0.26	0.00	0.17	0.56	0.43	0.00	0.00	0.13	
v ^{7F}	0.00	0.00	0.08	0.00	0.14	0.30	0.41	0.08	
V+7	0.17	0.00	0.17	0.00	0.14	0.30	0.00	0.07	
vi7	3.50	3.14	3.52	5.56	1.14	2.66	2.46	3.07	4.46
VI7	0.43	0.88	0.50	0.97	0.00	0.89	0.55	0.29	
#vi ^{d7}	0.00	0.00	0.00	0.00	0.00	0.30	0.27	0.09	
bVI -7	0.00	0.00	0.00	0.00	0.14	0.00	0.00	0.04	
viiº7	0.43	0.88	1.34	0.70	0.29	0.00	1.36	0.63	0.81
vii ^{d7}	0.17	0.00	0.00	0.56	0.14	0.00	0.00	0.27	
VII7	0.00	0.13	0.00	0.00	0.14	0.00	0.00	0.07	
vii ^{7G}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	
Total M	694	586	912	542	468	260	514	15110	36.41

Table 27 - Chords in Major mode in percentage, part 3

Minor	Handel	JSBach	Scarlatti	Haydn	Mozart	Beethoven	Schubert	Schumann	Mendelssohn
i	14.25	13.66	14.95	10.82	10.27	20.11	13.49	12.29	19.50
I	0.14	0.36	1.01	0.12	0.09	0.71	0.68	0.55	0.86
iiº	0.70	4.55	2.64	1.41	0.61	3.13	2.25	1.65	1.46
II	0.00	0.00	0.38	0.00	0.00	0.18	0.00	0.18	0.26
bII	0.00	0.00	0.13	0.12	0.09	0.36	0.28	0.37	0.17
III	0.56	0.18	0.38	0.12	0.35	0.18	0.40	0.92	2.06
III+	0.56	3.10	2.64	1.29	0.70	0.63	1.37	0.55	0.52
iv	1.82	3.28	5.65	1.18	0.35	1.25	1.45	2.39	3.26
IV	0.00	0.55	0.88	0.24	0.26	0.09	0.52	0.55	0.69
#ivº	0.00	0.18	0.38	0.12	0.09	0.27	0.00	0.18	0.17
#iv ^{It}	0.00	0.00	0.00	0.12	0.00	0.27	0.20	0.00	0.09
v	1.12	0.55	1.01	0.47	0.61	0.18	0.32	0.55	1.89
V	6.84	4.19	7.04	3.41	3.05	7.42	5.26	5.14	5.07
VI	0.70	1.28	1.26	0.82	0.17	1.79	1.41	1.28	1.46
#viº	0.28	0.36	0.75	0.24	0.44	0.45	0.48	0.00	0.69
VII	0.56	0.36	0.00	0.35	0.44	0.45	0.16	0.37	0.60
#viiº	1.26	2.55	3.52	1.41	0.87	0.54	0.40	0.00	0.77
i7	0.70	0.73	0.88	0.00	0.35	0.00	0.60	0.37	1.46
I7	0.00	0.36	0.13	0.35	0.00	0.09	0.08	0.18	0.26
iiº7	2.09	2.00	1.01	0.59	0.09	0.63	1.20	0.73	2.06
ii ^{d7}	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00	0.43
II7	0.14	0.00	0.38	0.00	0.00	0.00	0.04	0.00	0.09
ii^{7F}	0.00	0.00	0.00	0.00	0.17	0.00	0.04	0.00	0.09
III7	0.28	0.55	0.13	0.24	0.00	0.00	0.04	0.18	0.00
III -7	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
#iii ^{d7}	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00	1.03
iv7	0.56	1.64	0.63	0.47	0.17	0.45	0.24	0.37	1.29
IV7	0.00	0.36	0.00	0.24	0.17	0.80	0.04	0.00	0.77
#iv ^{d7}	0.28	0.00	0.25	0.12	0.44	1.07	0.32	0.00	0.17
#iv^{7G}	0.00	0.00	0.00	0.00	0.87	0.45	0.40	0.00	0.09
v7	0.00	0.18	0.00	0.24	0.26	0.00	0.00	0.00	0.17
V7	2.09	4.19	3.27	2.82	1.48	6.61	3.17	2.57	5.15
VI7	1.12	0.55	1.76	0.47	0.00	1.79	0.64	1.47	1.29
#viº7	0.14	0.18	0.25	0.59	0.35	0.27	0.16	0.18	1.80
VI -7	0.00	0.00	0.00	0.00	0.00	0.09	0.00	0.00	0.00
VII7	0.00	0.00	0.25	0.35	0.17	0.09	0.48	0.37	0.52
#vii ^{d7}	0.42	1.28	1.01	1.76	0.35	0.89	0.48	0.00	0.77
Total m	262	259	418	259	267	573	915	182	663
Total	716	549	796	850	1149	1119	2490	545	1164

Table 28 - Chords in Minor mode in percentage, part 1

Minor	Chopin	Liszt	Franck	Brahms	Tchaikovsky	Grieg	Fauré	Debussy	Scriabin
i	22.86	6.46	13.65	11.46	15.96	9.26	7.90	3.60	21.03
I	0.84	2.83	0.65	1.75	0.22	1.23	0.35	0.16	0.12
iiº	3.73	1.01	1.82	1.75	0.51	0.84	0.98	0.78	0.60
II	0.12	0.20	0.65	0.70	0.29	0.08	0.00	0.16	0.24
bII	0.72	0.81	0.26	0.35	0.07	1.23	0.35	0.08	0.00
III	0.48	2.22	1.17	1.26	0.74	1.45	0.80	0.63	0.84
III+	1.20	3.43	2.99	1.75	2.35	1.84	0.89	0.55	3.13
iv	1.81	1.01	2.21	3.98	2.57	1.61	0.89	1.33	3.37
IV	0.00	0.40	1.82	1.12	1.32	1.45	0.44	0.86	0.12
#ivº	0.36	1.21	0.78	0.49	0.00	0.00	0.62	0.08	0.00
#iv ^{It}	0.24	0.20	0.26	0.14	0.00	0.08	0.00	0.00	0.48
v	0.84	1.21	2.08	1.19	0.15	0.92	0.44	0.47	0.36
V	3.13	1.82	3.25	4.68	3.90	3.83	1.06	0.94	1.80
VI	1.81	1.21	3.90	2.31	0.74	2.14	1.69	0.94	1.68
#viº	0.36	1.82	1.17	1.61	0.96	0.77	0.27	0.31	0.12
VII	0.00	0.00	1.30	1.12	0.37	0.61	0.35	0.94	0.48
#viiº	0.36	1.62	1.17	0.77	0.37	0.69	0.27	0.08	0.12
i7	0.84	2.22	1.43	0.21	1.25	0.84	0.18	1.09	1.68
I7	0.24	0.00	0.26	0.70	0.29	0.23	0.09	0.08	0.12
iiº7	2.53	0.00	1.04	1.96	2.57	1.38	0.89	0.16	1.92
ii ^{d7}	0.12	1.21	0.39	0.07	0.00	0.00	0.00	0.00	0.00
II7	0.36	0.40	0.00	0.70	0.44	0.84	0.00	0.00	0.24
ii^{7F}	0.12	0.00	0.26	0.35	0.15	0.38	0.35	0.00	0.84
III7	0.00	1.82	0.52	0.21	0.07	0.00	0.00	0.08	0.36
III -7	0.00	0.40	0.52	0.42	0.00	0.00	0.18	0.00	0.00
#iii ^{d7}	0.12	0.20	0.26	0.14	0.00	0.00	0.18	0.00	0.24
iv7	0.96	1.01	1.17	0.98	0.74	1.23	0.53	1.56	0.96
IV7	0.36	1.01	0.65	0.63	1.25	0.54	0.35	0.78	0.00
#iv ^{d7}	0.48	0.61	0.78	0.70	0.44	0.31	0.09	0.16	0.36
#iv^{7G}	0.12	0.00	0.39	0.14	0.74	0.38	0.00	0.00	0.96
v7	0.00	0.00	0.00	0.28	0.29	0.61	0.09	0.08	0.24
V7	2.65	0.40	3.38	3.28	2.57	1.38	1.51	0.16	2.04
VI7	1.81	0.61	2.60	3.70	1.10	2.14	0.71	0.47	2.88
#viº7	2.05	0.40	1.95	1.05	0.22	1.84	0.80	1.09	1.68
VI -7	0.12	0.20	0.13	0.07	0.00	0.00	0.44	0.16	0.00
VII7	0.12	0.00	0.26	0.35	0.07	0.15	0.09	0.08	0.36
#vii ^{d7}	1.68	1.01	2.99	0.49	0.44	0.69	0.09	0.00	0.60
Total m	445	193	447	754	587	535	269	225	416
Total	831	495	769	1429	1360	1306	1127	1276	832

Table 29 - Chords in Minor mode in percentage, part 2

Minor	Rachmaninov	Satie	Ravel	Bartók	Prokofiev	Shostakovich	Khachaturian	Averages	Grouped
i	18.50	13.94	7.30	15.86	17.97	11.54	8.46	13.31	
I	0.34	0.25	0.76	0.14	0.43	0.30	0.41	0.62	
iiº	1.45	0.25	0.50	0.42	1.43	0.59	0.55	1.43	
II	0.26	0.38	0.17	0.00	0.29	0.00	0.00	0.18	
bII	0.43	0.00	0.17	0.00	0.43	0.30	0.95	0.31	
III	0.51	0.25	0.08	0.28	0.86	0.00	0.14	0.68	
III+	1.62	0.00	1.43	1.95	1.28	0.89	0.55	1.44	
iv	2.05	3.02	1.34	0.14	1.28	0.59	0.68	1.95	
IV	0.09	0.25	0.67	0.28	0.29	1.78	0.82	0.63	
#ivº	0.51	0.00	0.00	0.00	0.14	0.00	0.00	0.20	
#iv ^{It}	0.17	0.00	0.17	0.00	0.43	0.30	0.00	0.12	
v	0.60	0.63	1.09	0.28	0.43	0.30	0.82	0.72	
V	1.79	0.13	1.09	1.67	1.14	0.89	2.32	3.42	
VI	1.02	0.13	1.68	0.97	1.00	0.59	2.05	1.39	
#viº	0.43	0.00	0.59	0.00	0.86	0.00	2.32	0.62	
VII	0.43	0.00	0.67	0.14	0.14	0.59	0.55	0.46	
#viiº	0.77	0.00	0.17	0.83	0.43	0.59	1.36	0.74	
i7	0.77	1.51	0.76	0.42	1.57	0.30	0.00	0.77	
I7	0.51	0.00	0.00	0.00	0.00	0.00	0.00	0.18	
iiº7	1.36	2.39	1.43	0.14	0.00	1.18	0.55	1.24	
ii ^{d7}	0.00	0.00	0.08	0.14	0.00	0.00	0.14	0.09	
II7	0.26	0.00	0.00	0.00	0.29	0.00	0.82	0.21	
ii ^{7F}	0.00	0.00	0.00	0.14	0.00	0.00	0.27	0.13	
III7	0.09	0.13	0.00	0.00	0.00	0.00	0.27	0.14	
III -7	0.09	0.00	0.00	0.00	0.00	0.30	0.14	0.07	
#iii ^{d7}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.09	
iv7	0.85	0.13	0.50	0.28	0.14	0.00	0.55	0.70	
IV7	0.17	0.13	0.08	0.00	0.00	0.00	0.95	0.39	
#iv ^{d7}	0.43	0.00	0.08	0.00	0.00	0.00	0.27	0.32	
#iv ^{7G}	0.00	0.00	0.25	0.00	0.00	0.00	0.00	0.24	
v7	0.00	0.00	0.17	0.00	0.00	0.00	0.27	0.13	
V7	1.19	0.13	0.17	0.14	0.14	0.00	0.41	2.21	
VI7	2.56	0.38	1.17	0.28	2.00	1.18	1.64	1.39	
#viº7	1.19	2.39	0.92	0.00	0.14	0.89	1.36	0.87	
VI -7	0.00	0.00	0.00	0.00	0.00	0.00	0.14	0.05	
VII7	0.00	0.00	0.00	0.00	0.00	0.00	0.14	0.18	
#vii ^{d7}	0.43	0.00	0.00	0.14	0.14	0.00	0.00	0.59	
Total m	479	210	280	177	233	78	219	9345	25.54
Total	1173	796	1192	719	701	338	733	24455	61.95

Table 30 - Chords in Minor mode in percentage, part 3

Grouped	Handel	JSBach	Scarlatti	Haydn	Mozart	Beethoven	Schubert	Schumann	Mendelssohn
I + i	37.01	31.51	31.16	36.24	43.43	42.09	43.98	33.39	37.03
ii + ii^{o}	3.91	7.10	5.53	4.94	5.40	4.83	3.61	3.85	2.58
iii + III + III^{+}	2.09	4.92	4.02	2.94	2.00	1.79	4.02	2.75	3.61
IV + iv	5.31	6.92	8.17	6.00	4.35	3.57	2.77	6.42	6.01
V + v + V	18.72	12.75	14.45	12.71	13.05	11.26	13.01	14.13	9.45
vi + VI	2.65	2.91	2.14	3.41	3.83	2.32	3.45	4.40	2.15
viio + VII + #vii^{o}	4.75	8.01	8.04	5.41	4.70	1.97	1.29	1.65	3.35
I7 + i7	4.33	2.37	2.01	2.24	2.70	0.71	1.33	2.39	3.44
ii7 + ii^{o}7	3.07	2.55	2.01	1.53	0.35	0.98	1.97	1.28	2.41
iii7 + III7	3.35	2.00	1.26	0.82	0.44	0.27	0.96	1.65	0.69
IV7 + iv7	1.54	2.55	1.51	1.53	0.44	0.80	0.48	1.28	1.80
V7 + v7 + V7	6.98	6.92	5.90	10.59	8.18	15.01	9.92	9.72	10.91
vi7 + VI7	2.79	1.82	4.15	1.06	2.79	2.32	1.16	3.49	2.58
vii^{o}7 + VII7	0.28	0.18	2.14	1.18	0.70	0.09	0.72	0.92	0.60
Total percent	96.79	92.53	92.46	90.59	92.34	88.03	88.67	87.34	86.60

Grouped	Chopin	Liszt	Franck	Brahms	Tchaikovsky	Grieg	Fauré	Debussy	Scriabin
I + i	42.24	20.81	26.92	24.39	36.62	23.35	23.51	25.25	31.85
ii + ii^{o}	5.66	5.86	2.47	2.38	2.65	2.91	2.22	5.55	2.16
iii + III + III^{+}	3.13	14.55	6.63	4.68	5.29	6.74	4.17	6.49	9.50
IV + iv	2.77	3.84	3.38	6.71	5.15	4.13	3.82	4.85	4.45
V + v + V	6.98	4.24	7.67	8.87	8.24	8.96	4.88	4.77	3.97
vi + VI	2.29	3.64	5.98	4.40	3.82	5.82	3.46	5.47	4.69
viio + VII + #vii^{o}	0.72	2.63	3.25	3.14	1.40	1.84	3.19	1.41	1.20
I7 + i7	2.29	5.86	3.51	2.87	2.79	4.13	3.19	4.69	5.29
ii7 + ii^{o}7	2.89	0.61	1.30	2.87	2.72	4.13	1.51	3.36	2.52
iii7 + III7	0.36	3.64	1.04	1.19	0.59	1.61	1.06	1.88	1.44
IV7 + iv7	2.41	2.63	1.56	1.40	1.03	4.06	2.40	2.42	1.20
V7 + v7 + V7	8.66	2.63	4.68	7.06	6.32	4.36	7.72	5.08	5.29
vi7 + VI7	6.14	5.86	4.81	7.27	4.56	6.51	7.19	9.07	6.73
vii^{o}7 + VII7	0.12	0.20	0.52	0.56	0.74	1.99	0.80	1.64	0.48
Total percent	86.64	76.97	73.73	77.78	81.91	80.55	69.12	81.94	80.77

Grouped	Rachmaninov	Satie	Ravel	Bartók	Prokofiev	Shostakovich	Khachaturian	Averages
I + i	34.61	23.24	22.90	34.77	32.81	40.24	23.06	32.65
ii + ii^{o}	2.30	6.16	3.69	3.06	4.71	2.96	3.00	3.82
iii + III + III^{+}	8.01	6.66	8.81	9.46	6.99	5.92	6.00	5.38
IV + iv	3.75	5.90	3.52	2.36	5.71	2.96	2.18	4.51
V + v + V	4.52	6.16	4.11	6.95	5.85	4.14	6.68	8.84
vi + VI	3.24	6.53	5.96	2.50	5.42	2.66	4.09	3.94
viio + VII + #vii^{o}	2.22	0.38	2.35	4.31	2.85	4.14	3.27	2.80
I7 + i7	5.46	6.16	5.87	3.76	8.84	2.96	6.68	3.61
ii7 + ii^{o}7	2.05	4.15	2.18	2.36	1.14	2.07	1.64	2.21
iii7 + III7	0.77	2.01	2.27	0.83	1.14	2.07	0.68	1.23
IV7 + iv7	1.53	3.39	2.43	0.97	0.71	1.18	1.64	1.66
V7 + v7 + V7	3.67	1.13	3.02	1.81	2.00	2.07	1.77	6.58
vi7 + VI7	6.05	3.52	4.70	5.84	3.14	3.85	4.09	4.46
vii^{o}7 + VII7	0.43	0.88	1.34	0.70	0.29	0.00	1.50	0.81
Total percent	78.60	76.26	73.15	79.69	81.60	77.22	66.30	82.52

Table 31 - Percentages for grouped diatonic chords

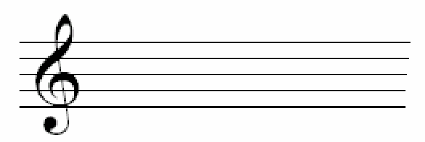
Appendix III - Musical notation basics

This project deals with music. Very little could be said about it without resorting frequently to musical terminology. This chapter introduces the necessary concepts. If you are already familiar enough with musical notation, chords and functions, you can skip it and proceed to Chapter 3.

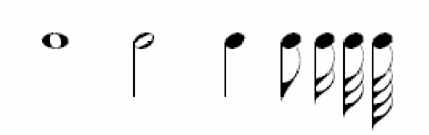
Musical notation is not the music. Instead, it is a set of instructions for the player to perform it. As such, it gives the guidelines, as best as the composer can, to create the “work” although it does so with a detail level limited by the resolution of the notation system.

Such system is the historical result of an agglomeration of elements from different origins. In spite of its forbidding hieroglyphs-hanging-from-a-fence appearance it essentially is a semi-logarithmic plot of frequencies versus time as we shall see, even if the time scale is grainy rather than continuous.

Music is written on the staff, a set of five horizontal lines, used to provide the pitch reference frame as well as the time axis. Notes are placed on the lines of the staff and on the spaces in between, progressing in time from left to right.



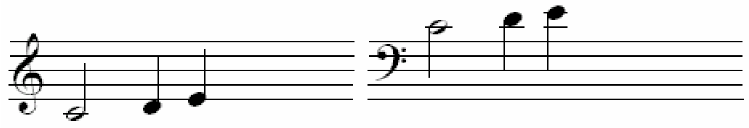
A note represents a sound. Its frequency or pitch is indicated by its vertical position on the staff, and its duration by the kind of symbol that represents it. The note symbols are the following:



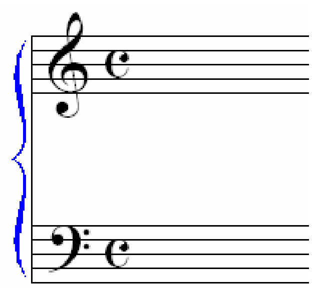
Each symbol lasts twice as long as the next one to its right. In the USA they are called respectively whole note, half, quarter, eighth, sixteenth, thirty-second and sixty-fourth. (The corresponding British terms are semibreve, minim, crotchet, quaver, semiquaver, demisemiquaver and hemi demisemiquaver). The pitch of the note corresponds to the position of the note head on the staff. And there are the corresponding rests, used to represent silence:

The staff is a vertically movable reference, to suit voices or instruments in their best range. The shifting is done by changing the symbol appearing on the left end called “clef”. The one shown above is the “treble clef”, the most common of a set of seven. The next in importance is the "bass clef", used to represent notes much lower in pitch.

The next two figures represent the very same notes using the Treble clef and the Bass clef.



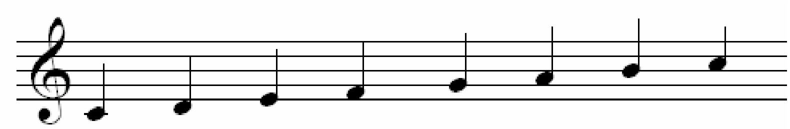
Notice that the note on the first line below the staff in Treble clef is the same as the note on the first line above the staff in Bass clef. This suggests that each of these staves would work as the natural extension of the other, and so they could be combined to make what is called the Grand Staff, in which notes too high for the lower staff go to the top staff and vice-versa.



One single staff normally suffices to write music for a single voice or an instrument like the flute of the violin, but piano or harp music, because of the larger range of the instrument, requires the Grand Staff.

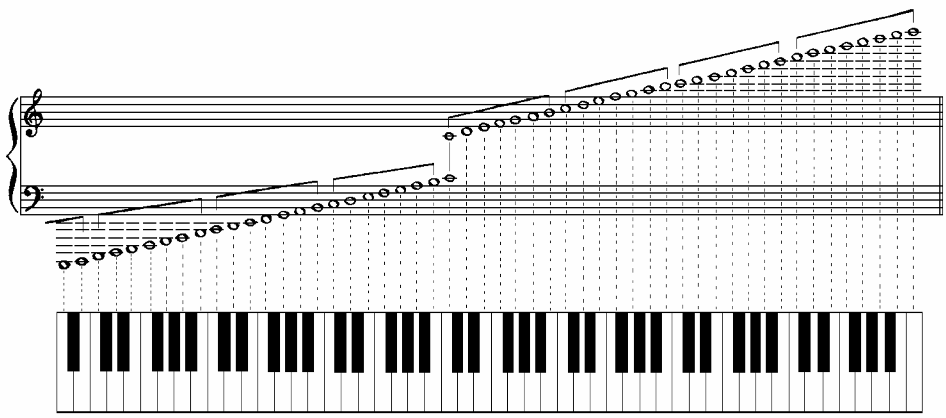
The staff and the keyboard

The five-line staff is the result of an evolution that started in the 10^{th} century as a gross indication of pitch represented by notes being above or below a single line. Since at those times the musical scale only consisted of seven notes, called 'diatonic', which correspond to the white keys in a keyboard or the strings in an Irish harp, it was natural to assign each possible position on the staff to them. This is the scale of C major:



In a keyboard, frequencies increase from left to right. The corresponding notes on the staff increase in frequency from bottom to top; this happens in a logarithmic fashion – i.e. the frequency of each note can be worked out by multiplying the previous one by a constant factor.

Here is the relation between the white piano keys and their musical representation:



The audio range corresponding to the notes of a piano keyboard is divided in eight sub-ranges called octaves (marked here with a bracket) each of which span pitches of double frequency than the one to its left. The white keys in the piano – seven per octave –, are named with the first seven letters of the alphabet, A through G, and these names are repeated for each of the octaves.

Looking at a keyboard provides an insight that the staff actually hides. If you look at a C major scale above, it seems that all the notes are equal and they are equally spaced. But the view of the keyboard reveals that the scale is not homogeneous and its structure is uneven. Effectively, there are white keys and black keys, but the latter are not occupying all the positions in between the former. There are places where there is no black key. These are missing between E and F and between B and C. The reason for this anomaly is that the pitch distance between E and F and between B and C is much smaller than between any other pairs of successive notes. Wherever there is a black key in between it is said that the pitch distance between the white keys is a “tone”, and where there is not, it is a “diatonic semitone”. The black keys, called “chromatic notes”, are in terms of frequency, just in the middle of the diatonic tones, and their distance to each of the neighbouring white notes is called a “chromatic semitone”.

Alterations

As the white notes occupy all the available positions in the staff, there is no room to place the notes corresponding to the black keys. The appearance of chromatic notes posed the problem of how to notate them since there was no space left on the staff between the diatonic notes. With hindsight, the solution adopted was not the best as it has caused an amount of confusion that cannot possibly be assessed. It consisted of labelling the black key placed *at the right of a white key* with the *same* letter name of that white key with the addition of the word “sharp”. This is indicated on the staff by writing the note as if it was that white key and placing *at its left* the symbol #. Unfortunately this notational convention gives the impression that F# was a sort of 'coloured' variant of F rather than another note in its own right, and this idea has had pervasive influence in the minds of musicians.

At this point all the notes had been named already, but for historical reasons, there was one more complication left: the black key placed *at the left of a white key* came to be considered – and named – the "flat" version of that key, which is notated on the staff as if it was the same white note but preceded by the symbol ♭. In this way, any black key – for example, the one that sits between G and A in the keyboard – can appear in the staff in two different forms: as G preceded by a sharp or as an A preceded by a flat.

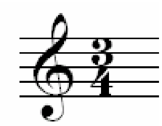
As if this was not enough, the symbols ♭ and ♯ got to be viewed as operators which respectively lower or raise by a semitone the pitch of the note to which they are applied. Thus, the operation of 'sharping' or 'flatting' can be applied to the white keys between which there is no black key. In this way C-flat became another name for B, B-sharp another name for C and so on. Worse still, these operators can be applied twice, thus raising or lowering the note in one full tone, with the combined operator indicated by the symbols for double flat (𝄫) and double sharp (𝄪). The result is that every note in the keyboard can be referred to by three different names and represented on the staff by three different symbols – for example G, F-double sharp and A-double flat. The only exception is G sharp/ A flat that has only those two names. All four symbols – ♯, ♭, 𝄪, 𝄫 – are called generically "alterations" and there is one more symbol called "natural" (♮) that cancels the effect of any of the others.

So far we have dealt with the way in which the variable "pitch", – largely playing the role of the variable in the ordinate axis –, is coded in the score. The abscissa represents time. The basic coding has already been mentioned: The kind of note symbol used indicates its relative duration, and a succession of sounds in time is represented by a horizontal succession of notes. But how does this relate to actual time?

As a rule, sounds are not played at a uniform level, but some notes are stressed or "accented". Owing to the fact that major accents tend to recur at equal intervals, the score is divided in "measures" by means of vertical lines. Measures correspond to equal time lapses. If all measures in a score were of the same graphical size, time would be represented linearly. However, this does not usually happen, although it is recommended as good practice to space the notes according to their duration.

Time signatures

How long does a measure last? The score normally specifies the size of measures at the beginning. Right after the clef, a fraction appears, for example:



This number is called “time signature” and gives precisely the information about the duration of the measure. Recalling that the notes are called with names that indicate the denominator of fractions – such as 8^{th} or 64^{th} –, what this number is telling is that a measure comprises in this case, **three quarters**, that is, the measure duration is three beats, each equal to a quarter note (or any equivalent combination of notes) after which the measure ends, a vertical bar-line appears, and a new measure begins.

There is a variety of time signatures. The denominator can be any power of 2, up to 64; the numerator can be any integer but the most common are 2, 3, 4, 6, 9, 12, and occasionally 5 and 7.

In any case, the fact that measures have equal duration simplifies the problem of relating the duration of notes to actual time. Habitually, measures are numbered, so checking the end of the score to look for the numeration of the last measure we can find out how many measures it contains. This number, multiplied by the denominator of the fraction, tells us how many notes of the kind specified by the denominator of the fraction are there in the whole piece.

Tempo

At the time of Bach, composers did not indicate the speed at which a piece had to be played. (Indeed, they seldom bothered to specify even what the specific intended instrument was). But later, scores started to show, at the beginning, a word such as “Adagio”, “Moderato” or “Allegro”. This is called the “tempo”, an Italian word that conveys the general idea as to how fast or slow the piece has to be played. Since this was too vague, from about the year 1800 some composers began to write more precise speed indications in the form of metronome indications specifying the number of beats per minute, e.g.

Adagio ♩ = 40

In this example, the symbol for quarter note equaled to the number 40 means that the piece has to be played slow enough for 40 quarter notes to last a whole minute. This is about as slow as music usually can be. The normal range of “tempi” goes up to over 200 beats per minute.

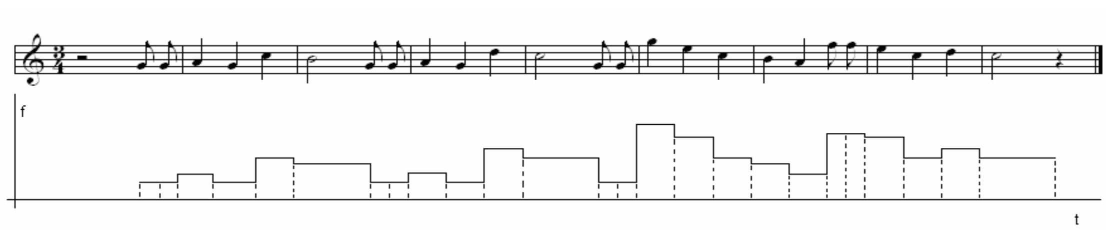
Notice that there is no direct translation of note duration into time. A piece written entirely on half notes at quite a fast tempo such as ♩ = 200 would sound identical to the same piece written entirely in eights at a slow ♩ = 50 tempo. It is just a tradition to write slow pieces in long notes and vice versa.

The ‘tempo’ is all that was needed to translate a score into time. We know the number of notes that the score contains, and we know how many of them are needed to last one minute, so the calculation is straightforward:

$$Duration(s) = \frac{60}{Tempo} * Number\ of\ notes$$

Equivalence between the score and a pitch/time diagram

Now that we know the meaning of the different elements in the score, a brief example will show how close a score is to a true semi-logarithmic graphic of pitch versus time. The next figure shows the melody of “Happy Birthday”, followed by the equivalent diagram.



It is apparent the near perfect agreement of the notes in the score with the corresponding segments of the diagram: Each eighth note is placed above a short interval, each quarter above one that lasts twice as much, each half note over one four times longer; and the bar lines coincide with the corresponding note changes. The time scale depends of the tempo. If this was set at 90 quarter notes per minute, each measure would last 2 seconds and the total duration would be 16 seconds.

Scale degrees

Given that the scale, major or minor, has a certain structure of tones and semitones, it has become a custom to refer to each step or ‘degree’ with a generic name, so that different scale steps can be referred to in relation to its relative position, without reference to particular notes.

Historically, the Tonic (I) was the note that gave name to the scale, and the second in importance was the Dominant (V), which in some of the modes was one fifth above. The next scale degree to receive a name was the one in the middle of these two, which logically was called the Mediant (III). Likewise, it is possible to go from the Tonic to the Subdominant (IV) by descending a fifth, so the note in the middle in this direction was called the Submediant (VI). The step immediately above the Tonic just describes where it is, i.e. Supertonic (II), meaning over the Tonic. The name for the seventh step, depends on the mode: whether it is a tone or a semitone away from the next Tonic, is called the Subtonic or Leading Tone (VII) respectively.

Intervals

The distances between notes have been given absurd names. Imagine that to measure the *distance* from C to E, you started by pointing to a C key while counting "one", then to D saying "two", and finally to E saying "three". This would be right for counting the *keys* but not the *distances* between them. However, this is why that distance is called a "third". Thus, the interval names were assigned by counting how many keys were there from one to the other *including both ends,* and for this reason they are all wrong. In this way the distance from one key to the adjacent one is called a "second". Being equal to one (tone or semitone), it would make sense that it was called a "first". Likewise, the interval between any note and the next one that receives the same name, that is, after you advance up or down to the seventh note in a row, is called an "octave", a term derived from the Latin word for "eight".

Intervals are divided in categories that are also misleading. There is one group comprising the unison (that is, the null interval, the distance from one note to itself), the fourth, the fifth and the octave, which are called "perfect", the rationale being that no matter what note you start from, they are of the same size. This is quite untrue when you compare the sizes of fifths and notice that the distance from B up to F comprises three tones whereas from E up to A or from G up to C it is two tones plus one semitone. Probably, the explanation is that at the time these names were adopted, the interval from F to B was absolutely forbidden both in harmony or melody, and called "diabolus in musica". So, if the existence of this interval could be disregarded, they could well say that all fourths and fifths were equal.

The remaining intervals occur in two variants, minor and major. In this way there are:

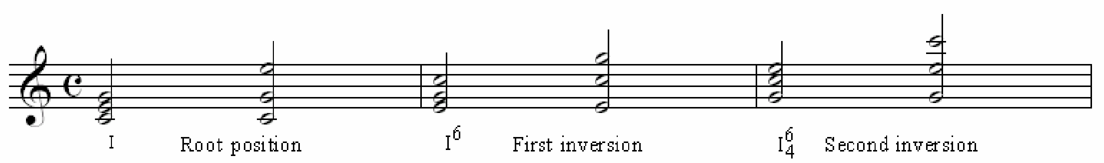
minor second = one semitone
major second = one tone
minor third = one tone plus one semitone
major third = two tones
minor sixth = four tones
major sixth = four tones plus one semitone
minor seventh = five tones
major seventh = five tones plus one semitone

Inversions

Rameau observed that music theory became simpler if the interval from any note to another of the same name going up or down were considered equivalent. This is called the theory of the "interval inversions". For example, to go from C to A you can go up a major sixth or down a minor third. So, it is said that a major sixth inverted is a minor third.

In this way, every major interval inverts into a minor one and vice-versa, and every perfect interval inverts into another perfect one. Due to the incorrect counting technique used to name the intervals, it happens that each interval and its inversion always add up to nine.

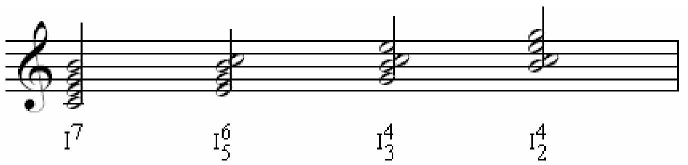
The main consequence of the equivalence of inverted intervals is that a number of different chords were identified to be inverted forms of another. Harmony is based on the superposition of thirds. For example, the major tonic triad (I) comprises Tonic – Mediant – Dominant. The tonic note is the "root"; the mediant is a major third above; and the dominant is a minor third above the mediant or a perfect fifth above the root; when the chord has the tonic at the bass, it is said to be in "root position". Since octaves are irrelevant in harmonic terms, the chord is identifiably the same if the first note over the root is the dominant whilst the mediant is placed in the octave above, as the figure shows:



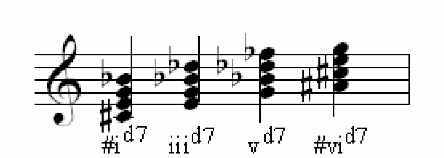
Consider now the chord Mediant – Dominant – Tonic. The structure of this chord is different: From the bass note to the next the distance is a minor third, and from this to the top, a perfect fourth, or a sixth over the bass – hence the name of Sixth chord (I^6)However, as the notes that make the chord are the same as before, only that the lower interval – a major third – has been inverted to a minor sixth, this chord is recognized to be the same, only that it is said to be in "third" position or first inversion.

For the same reason, the structure of the chord Dominant – Tonic – Mediant comprises a perfect fourth and a major third (or a major sixth over the bass) but it is still considered the Tonic triad, this time in "fifth" position or second inversion, as the fifth has been inverted to a fourth.

The same applies to 4-note chords. The figure shows the four positions of the C Major Tonic Seventh chord as well as the symbols used to indicate them:

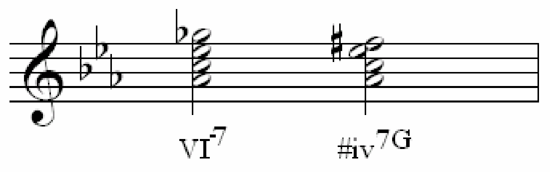


Even when the notes that make them up are the same, the structure of these four chords is clearly different. But this is not the case with the diminished-seventh chords. For example:



These four chords not only comprise the same pitch classes but also they have exactly the same structure – they are the superposition of three minor sevenths so that no ear, however educated, is able to tell which is which.

Even worse is the case of the so-called “German sixth” chord, which is enharmonic – i.e. it coincides, key by key – with a major seventh:



This means that in music two identical objects can be considered different by reasons of spelling. In this project, since it has been decided to take observable features at face value, chords are what they are, and no consideration is given to their possible hidden nature. Consequently, a chord is identified by the notes it is made of, whatever the inversion.

Keys and Key signatures

The succession of all the white keys from C up to the following C is referred to as the scale of C major, meaning that C is the Tonic and Major is the kind of scale. Now, suppose that we want to play a Major scale beginning in G. If we check that we reproduce the same tone-semitone structure of the Major scale, it turns out that we have to play all the white keys except for one, F, which has to be replaced by F#. If we do the same beginning on D, again most of the keys that have to be played are white, but this time there are two exceptions, F# replacing F as before, and C# replacing C. In this way there are seven possible major scales, and each one takes one more sharp than the preceding one until we get to the point where all seven white keys have been replaced by sharped ones. This is the list of alterations corresponding to them:

G Major	F#
D Major	F# - C#
A Major	F# - C# - G#
E Major	F# - C# - G# - D#
B Major	F# - C# - G# - D# - A#
F# Major	F# - C# - G# - D# - A# - E#
C# Major	F# - C# - G# - D# - A# - E# - B#

If we now do the same but beginning on F, we find that we have to play only white keys except for B that has to be replaced by B flat. Likewise, if we begin on B-flat we have to play white keys except for E-flat. In this way we can continue again adding flats until all seven keys have been flatted. Here is the list of the corresponding alterations:

F Major	B♭
B♭ Major	B♭ – E♭
E♭ Major	B♭ – E♭ – A♭
A♭ Major	B♭ – E♭ – A♭ – D♭
D♭ Major	B♭ – E♭ – A♭ – D♭ – G♭
G♭ Major	B♭ – E♭ – A♭ – D♭ – G♭ – C♭
C♭ Major	B♭ – E♭ – A♭ – D♭ – G♭ – C♭ – F♭

This looks like there are fifteen possible major scales, one without alterations, seven with flats and seven with sharps. But obviously this is not possible since there are only twelve notes per octave. What happens is that if we play them on the keyboard, three of these scales coincide, key by key, with each other. They are called “enharmonic” meaning that they are identical except only in the names of the notes which are played on the same keys:

G♭ Major (6♭) coincides with F# Major (6#)

B Major (5#) coincides with C♭ Major (7♭)

D♭ Major (5♭) coincides with C# Major (7#)

Exactly the same thing happens with the minor scales. The scale played on the white notes is A minor. This is the list of keys and the corresponding alterations:

E minor	F#
B minor	F# - C#
F# minor	F# - C# - G#
C# minor	F# - C# - G# - D#
G# minor	F# - C# - G# - D# - A#
D# minor	F# - C# - G# - D# - A# - E#
A# minor	F# - C# - G# - D# - A# - E# - B#

D minor	B♭
G minor	B♭ – E♭
C minor	B♭ – E♭ – A♭
F minor	B♭ – E♭ – A♭ – D♭
B♭ minor	B♭ – E♭ – A♭ – D♭ – G♭
E♭ minor	B♭ – E♭ – A♭ – D♭ – G♭ – C♭
A♭ minor	B♭ – E♭ – A♭ – D♭ – G♭ – C♭ – F♭

Enharmonic keys:

E♭ minor (6♭) coincides with D# minor (6#)

G# minor (5#) coincides with A♭ minor (7♭)

B♭ minor (5♭) coincides with A# minor (7#)

For centuries music did not use to be typeset like text, but instead pages were engraved on metal sheets and symbols punched on it. Imagine that a piece is written in C♭ Major – as usually are Harp pieces – which takes seven flats. Practically every note head would require a flat in front of it, which created a lot of extra work for engravers. In order to save so much punching, it was decided that whatever the set of flats or sharps was required by the key of the piece, it was a better solution to punch them at the beginning of each system, to remind the performer that they applied to all the corresponding note names. This set of sharps or flats is called the "key signature" since for each key, as noted above, there is a particular combination of them. The key signature has the advantage of informing what the initial key of the piece probably is.

Looking at the list of alterations of the scales above you may notice that sharps and flats appear in a particular order:

Sharps order: —► F – C – G – D – A – E – B ◄—Flats order

This list comes in handy to remember the key signature that corresponds to each key.

Pitch and frequency

The Pythagoreans were the discoverers of the relationship between the length of a string and its pitch, and they established the basis of the tuning of the musical notes in the West, deriving it from the first six natural harmonics. Let us derive the frequencies for the natural scale in the octave below middle-C:

Every time a frequency is multiplied by 2, the pitch goes up an octave. Giving C the frequency 264 Hz, the next C corresponds to 528 Hz.

Multiplying 264 times 3 we obtain 792 corresponding to the G belonging to the following octave. Dividing it by 2 we get 396 Hz for G.

Multiplying 264 times 5 = 1320 Hz the corresponding note is E two octaves above. Dividing it by 4 we get the frequency for E, 330 Hz.

Now starting from G = 396 Hz, in the same manner we obtain D = 297 Hz and B = 495 Hz.

We can also consider that C is a fifth above F, which gives F = 352 Hz, and then A = 440 Hz, which is the adopted international pitch reference standard.

Thus, the frequencies for the octave below middle C are:

Note frequencies	Ratio	Interval
C = 264		
	9/8	tone
D = 297		
	10/9	tone
E = 330		
	16/15	semitone
F = 352		
	9/8	tone
G = 396		
	10/9	tone
A = 440		
	9/8	tone
B = 495		
	16/15	semitone
C = 528		

With these frequencies for the diatonic scale of C, the mutual relationships between the frequencies of the notes are as simple as could be. If all the music was written in C major or A minor, this "just" tuning would be the best available. Multiplying these ratios to make combined intervals, we obtain:

Perfect Octave: $\frac{9}{8}\times\frac{10}{9}\times\frac{16}{15}\times\frac{9}{8}\times\frac{10}{9}\times\frac{9}{8}\times\frac{16}{15}=2$

Major third: $\frac{9}{8}\times\frac{10}{9}=\frac{5}{4}$

Minor third: $\frac{9}{8}\times\frac{16}{15}=\frac{6}{5}$

Perfect Fifth: $\frac{5}{4}\times\frac{6}{5}=\frac{3}{2}$

Perfect Fourth: $\frac{5}{4}\times\frac{16}{15}=\frac{4}{3}$

which are the values assigned by the Pythagoreans, whose idea was that the simplest relations between pitches are the most pleasing.

But notice the frequency ratios between successive notes. The size of the semitone is larger than half tone. Besides, there are two sizes of "tone", namely 9/8 and 10/9. One consequence of this difference is that the "perfect fifth" between D and A is smaller than the other fifths in 80/81.

This calculation does not provide the frequencies for the so far non-existing "black keys" of the keyboard, but it is easy to see how they would appear. Suppose that one jumps a fifth from C to G, and then a fifth from G to D and so on until we come to B, a further jump of a fifth would go somewhere between F and G, where the "black key" F# should be. And from this, the next jump of a fifth would go to C#, then to G#, D#, A#, F and finally C again, closing what is called the “circle of fifths”.

For things to be right it should be possible to go in the opposite direction, descending by fifths until the circle closes. This procedure would define the “black keys” as flats: C down to F, then B flat, E flat, A flat, D flat, G flat, B, E, A, D, G and C.

In other words, twelve jumps of a fifth would finally arrive to a note with the same name as the initial one; which means that a distance of 12 perfect fifths should be equivalent to 7 octaves. However, it is not. Seven octaves amount to a factor $2^7 = 128$, while 12 perfect fifths equal:

$$\left(\frac{3}{2}\right)^{12} = 129.746337890625$$

The small difference between these numbers is called “Pythagorean comma”. Another problem is that the calculated frequencies for the sharp and flat notes are close but do not coincide, so that a just tuning keyboard should have duplicated black keys, one for flat one for sharp. And so far we have limited ourselves to C Major. The numbers would change every time a modulation came along.

These problems proved unsolvable, while the differences in frequencies often are so slight that even a good trained ear could not notice them. This is the reason why in late 16th century the pragmatic solution adopted was the “equal temperament” that consists of dividing the octave evenly in twelve identical semitones equal to $\sqrt[12]{2} = 1.059463...$ In this practical system flats and sharp coincide but all the intervals except for the octave are slightly out of tune, and the beatings can be heard. These are the adopted practical approximate frequencies for the octave below middle C:

C = 261.63
D = 293.66
E = 329.63
F = 349.23
G = 392.00
A = 440
B = 493.88

With these values, the sizes of the main composite intervals result:

Perfect Octave: 2
Major Third: 1.2599 instead of 1.25
Minor Third: 1.1892 instead of 1.2
Perfect Fifth: 1.4983 instead of 1.5
Perfect Fourth: 1.3348 instead of 1.333…

Appendix IV - Format example

Second measure of Scarlatti’s “Erstausgabe” Sonata

MusicXML Part-wise version:

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		</pitch>
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		<voice>1</voice>


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		<notations>
			<ornaments>
				<inverted-mordent place
						ment="above"/>
			</ornaments>
		</notations>
	</note>
	<note>
		<pitch>
			<step>B</step>
			<alter>-1</alter>
			<octave>4</octave>
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	<note>
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    <note>
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        <stem>down</stem>
        <staff>1</staff>
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    <beam number="2">end</beam>
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    <voice>1</voice>
    <type>16th</type>
    <stem>up</stem>
    <staff>1</staff>
```

```
		<beam number="1">end</beam>
		<beam number="2">end</beam>
	</note>
	<note>
		<pitch>
			<step>G</step>
			<octave>4</octave>
		</pitch>
		<duration>4</duration>
		<voice>1</voice>
		<type>eighth</type>
		<stem>up</stem>
		<staff>1</staff>
	</note>
	<note>
		<chord/>
		<pitch>
			<step>B</step>
			<alter>-1</alter>
			<octave>4</octave>
		</pitch>
		<duration>4</duration>
		<voice>1</voice>
		<type>eighth</type>
		<stem>up</stem>
		<staff>1</staff>
	</note>
	<note>
		<pitch>
			<step>F</step>
			<alter>1</alter>
			<octave>4</octave>
		</pitch>
		<duration>8</duration>
		<voice>1</voice>
		<type>quarter</type>
		<accidental>sharp</accidental>
		<stem>up</stem>
		<staff>1</staff>
	</note>
	<note>
		<chord/>
		<pitch>
			<step>A</step>
			<octave>4</octave>
		</pitch>
		<duration>8</duration>
		<voice>1</voice>
		<type>quarter</type>
		<stem>up</stem>
```

```
        <staff>1</staff>
</note>
<note>
        <rest/>
        <duration>4</duration>
        <voice>1</voice>
        <type>eighth</type>
        <staff>1</staff>
</note>
<backup>
        <duration>32</duration>
</backup>
<note>
        <pitch>
                <step>G</step>
                <octave>3</octave>
        </pitch>
        <duration>4</duration>
        <voice>2</voice>
        <type>eighth</type>
        <stem>down</stem>
        <staff>2</staff>
        <beam number="1">begin</beam>
</note>
<note>
        <pitch>
                <step>G</step>
                <octave>3</octave>
        </pitch>
        <duration>2</duration>
        <voice>2</voice>
        <type>16th</type>
        <stem>down</stem>
        <staff>2</staff>
        <beam number="1">continue</beam>
        <beam number="2">begin</beam>
</note>
<note>
        <pitch>
                <step>A</step>
                <octave>3</octave>
        </pitch>
        <duration>2</duration>
        <voice>2</voice>
        <type>16th</type>
        <stem>down</stem>
        <staff>2</staff>
        <beam number="1">end</beam>
        <beam number="2">end</beam>
</note>
```

```
<note>
	<pitch>
		<step>B</step>
		<alter>-1</alter>
		<octave>3</octave>
	</pitch>
	<duration>4</duration>
	<voice>2</voice>
	<type>eighth</type>
	<stem>down</stem>
	<staff>2</staff>
	<beam number="1">begin</beam>
</note>
<note>
	<pitch>
		<step>C</step>
		<octave>3</octave>
	</pitch>
	<duration>4</duration>
	<voice>2</voice>
	<type>eighth</type>
	<stem>down</stem>
	<staff>2</staff>
	<beam number="1">end</beam>
</note>
<note>
	<chord/>
	<pitch>
		<step>C</step>
		<octave>4</octave>
	</pitch>
	<duration>4</duration>
	<voice>2</voice>
	<type>eighth</type>
	<stem>down</stem>
	<staff>2</staff>
</note>
<note>
	<pitch>
		<step>D</step>
		<octave>3</octave>
	</pitch>
	<duration>4</duration>
	<voice>2</voice>
	<type>eighth</type>
	<stem>down</stem>
	<staff>2</staff>
	<beam number="1">begin</beam>
</note>
<note>
```

```
		<pitch>
			<step>D</step>
			<octave>4</octave>
		</pitch>
		<duration>2</duration>
		<voice>2</voice>
		<type>16th</type>
		<stem>down</stem>
		<staff>2</staff>
		<beam number="1">continue</beam>
		<beam number="2">begin</beam>
	</note>
	<note>
		<pitch>
			<step>E</step>
			<alter>-1</alter>
			<octave>4</octave>
		</pitch>
		<duration>2</duration>
		<voice>2</voice>
		<type>16th</type>
		<accidental>flat</accidental>
		<stem>down</stem>
		<staff>2</staff>
		<beam number="1">end</beam>
		<beam number="2">end</beam>
	</note>
	<note>
		<pitch>
			<step>D</step>
			<octave>4</octave>
		</pitch>
		<duration>4</duration>
		<voice>2</voice>
		<type>eighth</type>
		<stem>down</stem>
		<staff>2</staff>
		<beam number="1">begin</beam>
	</note>
	<note>
		<pitch>
			<step>C</step>
			<octave>4</octave>
		</pitch>
		<duration>4</duration>
		<voice>2</voice>
		<type>eighth</type>
		<stem>down</stem>
		<staff>2</staff>
		<beam number="1">end</beam>
```

```
            </note>
        </measure>
```

Proposed Slice-wise version:

```
<measure number="2">
    <slice>
        <duration>4</duration>
        <note>
            <pitch>
                <step>G</step>
                <octave>3</octave>
            </pitch>
            <voice>2</voice>
            <type>eighth</type>
            <stem>down</stem>
            <staff>2</staff>
            <beam number="1">begin</beam>
        </note>
        <note>
            <pitch>
                <step>C</step>
                <octave>5</octave>
            </pitch>
            <voice>1</voice>
            <type>eighth</type>
            <stem>down</stem>
            <staff>1</staff>
            <beam number="1">begin</beam>
            <notations>
                <ornaments>
                    <inverted-mordent placement="above"/>
                </ornaments>
            </notations>
        </note>
    </slice>
    <slice>
        <duration>2</duration>
        <note>
            <pitch>
                <step>G</step>
                <octave>3</octave>
            </pitch>
            <voice>2</voice>
            <type>16th</type>
            <stem>down</stem>
            <staff>2</staff>
            <beam number="1">continue</beam>
            <beam number="2">begin</beam>
```

```xml
		</note>
		<note>
			<pitch>
				<step>B</step>
				<alter>-1</alter>
				<octave>4</octave>
			</pitch>
			<tie type="start"/>
			<voice>1</voice>
			<type>eighth</type>
			<stem>down</stem>
			<staff>1</staff>
			<beam number="1">end</beam>
			<notations>
				<tied type="start"/>
			</notations>
		</note>
	</slice>
	<slice>
		<duration>2</duration>
		<note>
			<pitch>
				<step>A</step>
				<octave>3</octave>
			</pitch>
			<voice>2</voice>
			<type>16th</type>
			<stem>down</stem>
			<staff>2</staff>
			<beam number="1">end</beam>
			<beam number="2">end</beam>
		</note>
		<note>
			<continuing/>
			<pitch>
				<step>B</step>
				<alter>-1</alter>
				<octave>4</octave>
			</pitch>
			<staff>1</staff>
		</note>
	</slice>
	<slice>
		<duration>2</duration>
		<note>
			<pitch>
				<step>B</step>
				<alter>-1</alter>
				<octave>3</octave>
			</pitch>
```

```
                <voice>2</voice>
                <type>eighth</type>
                <stem>down</stem>
                <staff>2</staff>
                <beam number="1">begin</beam>
            </note>
            <note>
                <pitch>
                    <step>B</step>
                    <alter>-1</alter>
                    <octave>4</octave>
                </pitch>
                <tie type="stop"/>
                <voice>1</voice>
                <type>16th</type>
                <stem>down</stem>
                <staff>1</staff>
                <beam number="1">begin</beam>
                <beam number="2">begin</beam>
                <notations>
                    <tied type="stop"/>
                </notations>
            </note>
        </slice>
        <slice>
            <duration>2</duration>
            <note>
                <continuing/>
                <pitch>
                    <step>B</step>
                    <alter>-1</alter>
                    <octave>3</octave>
                </pitch>
                <staff>2</staff>
            </note>
            <note>
                <pitch>
                    <step>D</step>
                    <octave>5</octave>
                </pitch>
                <voice>1</voice>
                <type>16th</type>
                <stem>down</stem>
                <staff>1</staff>
                <beam number="1">continue</beam>
                <beam number="2">continue</beam>
            </note>
        </slice>
        <slice>
            <duration>2</duration>
```

```
		<note>
			<pitch>
				<step>C</step>
				<octave>3</octave>
			</pitch>
			<voice>2</voice>
			<type>eighth</type>
			<stem>down</stem>
			<staff>2</staff>
			<beam number="1">end</beam>
		</note>
		<note>
			<pitch>
				<step>C</step>
				<octave>4</octave>
			</pitch>
			<voice>2</voice>
			<type>eighth</type>
			<stem>down</stem>
			<staff>2</staff>
		</note>
		<note>
			<pitch>
				<step>A</step>
				<octave>4</octave>
			</pitch>
			<voice>1</voice>
			<type>16th</type>
			<stem>down</stem>
			<staff>1</staff>
			<beam number="1">continue</beam>
			<beam number="2">continue</beam>
		</note>
		<note>
			<pitch>
				<step>C</step>
				<octave>5</octave>
			</pitch>
			<voice>1</voice>
			<type>16th</type>
			<stem>down</stem>
			<staff>1</staff>
		</note>
	</slice>
	<slice>
		<duration>2</duration>
		<note>
			<continuing/>
			<pitch>
				<step>C</step>
```

```
                <octave>3</octave>
            </pitch>
            <staff>2</staff>
        </note>
        <note>
            <continuing/>
            <pitch>
                <step>C</step>
                <octave>4</octave>
            </pitch>
            <staff>2</staff>
        </note>
        <note>
            <pitch>
                <step>G</step>
                <octave>4</octave>
            </pitch>
            <voice>1</voice>
            <type>16th</type>
            <stem>down</stem>
            <staff>1</staff>
            <beam number="1">end</beam>
            <beam number="2">end</beam>
        </note>
        <note>
            <pitch>
                <step>B</step>
                <alter>-1</alter>
                <octave>4</octave>
            </pitch>
            <voice>1</voice>
            <type>16th</type>
            <stem>down</stem>
            <staff>1</staff>
        </note>
    </slice>
    <slice>
        <grace/>
        <note>
            <pitch>
                <step>G</step>
                <octave>4</octave>
            </pitch>
            <voice>1</voice>
            <type>16th</type>
            <stem>up</stem>
            <staff>1</staff>
            <beam number="1">begin</beam>
            <beam number="2">begin</beam>
        </note>
```

```
</slice>
<slice>
	<grace/>
	<note>
		<pitch>
			<step>A</step>
			<octave>4</octave>
		</pitch>
		<voice>1</voice>
		<type>16th</type>
		<stem>up</stem>
		<staff>1</staff>
		<beam number="1">end</beam>
		<beam number="2">end</beam>
	</note>
</slice>
<slice>
	<duration>4</duration>
	<note>
		<pitch>
			<step>D</step>
			<octave>3</octave>
		</pitch>
		<voice>2</voice>
		<type>eighth</type>
		<stem>down</stem>
		<staff>2</staff>
		<beam number="1">begin</beam>
	</note>
	<note>
		<pitch>
			<step>G</step>
			<octave>4</octave>
		</pitch>
		<voice>1</voice>
		<type>eighth</type>
		<stem>up</stem>
		<staff>1</staff>
	</note>
	<note>
		<pitch>
			<step>B</step>
			<alter>-1</alter>
			<octave>4</octave>
		</pitch>
		<voice>1</voice>
		<type>eighth</type>
		<stem>up</stem>
		<staff>1</staff>
	</note>
```

```
</slice>
<slice>
        <duration>2</duration>
        <note>
                <pitch>
                        <step>D</step>
                        <octave>4</octave>
                </pitch>
                <voice>2</voice>
                <type>16th</type>
                <stem>down</stem>
                <staff>2</staff>
                <beam number="1">continue</beam>
                <beam number="2">begin</beam>
        </note>
        <note>
                <pitch>
                        <step>F</step>
                        <alter>1</alter>
                        <octave>4</octave>
                </pitch>
                <voice>1</voice>
                <type>quarter</type>
                <accidental>sharp</accidental>
                <stem>up</stem>
                <staff>1</staff>
        </note>
        <note>
                <pitch>
                        <step>A</step>
                        <octave>4</octave>
                </pitch>
                <voice>1</voice>
                <type>quarter</type>
                <stem>up</stem>
                <staff>1</staff>
        </note>
</slice>
<slice>
        <duration>2</duration>
        <note>
                <pitch>
                        <step>E</step>
                        <alter>-1</alter>
                        <octave>4</octave>
                </pitch>
                <voice>2</voice>
                <type>16th</type>
                <accidental>flat</accidental>
                <stem>down</stem>
```

```
				<staff>2</staff>
				<beam number="1">end</beam>
				<beam number="2">end</beam>
			</note>
			<note>
				<continuing/>
				<pitch>
					<step>F</step>
					<alter>1</alter>
					<octave>4</octave>
				</pitch>
				<staff>1</staff>
			</note>
			<note>
				<continuing/>
				<pitch>
					<step>A</step>
					<octave>4</octave>
				</pitch>
				<staff>1</staff>
			</note>
		</slice>
		<slice>
			<duration>4</duration>
			<note>
				<pitch>
					<step>D</step>
					<octave>4</octave>
				</pitch>
				<voice>2</voice>
				<type>eighth</type>
				<stem>down</stem>
				<staff>2</staff>
				<beam number="1">begin</beam>
			</note>
			<note>
				<continuing/>
				<pitch>
					<step>F</step>
					<alter>1</alter>
					<octave>4</octave>
				</pitch>
				<staff>1</staff>
			</note>
			<note>
				<continuing/>
				<pitch>
					<step>A</step>
					<octave>4</octave>
				</pitch>
```

```
				<staff>1</staff>
			</note>
		</slice>
		<slice>
			<duration>4</duration>
			<note>
				<pitch>
					<step>C</step>
					<octave>4</octave>
				</pitch>
				<voice>2</voice>
				<type>eighth</type>
				<stem>down</stem>
				<staff>2</staff>
				<beam number="1">end</beam>
			</note>
			<note>
				<rest>
					<display-step>A</display-step>
					<display-octave>4</display-octave>
				</rest>
				<voice>1</voice>
				<type>eighth</type>
				<staff>1</staff>
			</note>
		</slice>
</measure>
```


Compact code used in this study

```
 Measure 2
0×22×~4
22×31×~2
28×31×~2
31×~2
6×31×~2
0×28×~2
0×22×31×~2
6×22×31×~4
6×18×28×~2
9×18×28×~2
6×18×28×~4
0×~4
```

Appendix V – Application example

The 24 Fugue themes from J.S. Bach “Das wohltemperirte Klavier”, Book 1.

Since the time Longuet-Higgins tried his key-determining algorithm on the fugue themes of Bach’s 24 fugue themes, it has become a tradition to attempt the same set and compare results with others, as if it was a bookmark of algorithm performance. This is not justifiable on logic grounds. The fugue themes are more or less brief melodies that happen to be in all 24 keys. They are not a good set to compare performances for several reasons: a) They are too brief, which makes the results unreliable. b) They are riddled with chromatic notes, so that several of them would present a serious challenge to a musician who would be keen on identifying their key by ear. c) Some even modulate, which should confuse algorithms that simply tried to identify key rather than determine modulation points.

Because of their brevity, the key of the themes has been determined on global terms, without the use of the sliding window, and without using the criterion to assess whether the program conclusion is invalidated by other keys being too close. The program is able to identify correctly all the keys, but of course this is no proof that the themes actually convey the correspondent sense of key.

What follows is an extract from the program log in this task:

```
compiling    34064   19358256
_________________________
Fugue 1 in C major
File Opening Succeeded
Number of lines:  246
Longest line:  38
_________________________
__
Total steps:  15
Total alterations:  0
Total grace notes:  0
Total duration:  48
_________________________

_________RESULTS________
_
C      : 1    4    8.33
C#-Db:  0    0    0.0
D      : 2    8    16.66
D#-Eb:  0    0    0.0
E      : 3    12    25.0
F      : 3    9    18.75
F#-Gb:  0    0    0.0
G      : 4    9    18.75
G#-Ab:  0    0    0.0
A      : 2    6    12.5
A#-Bb:  0    0    0.0
B      : 0    0    0.0
_________________________

____DOT PRODUCTS____
 0    1387.7338
 1    379.8823
 2    1039.5737
 3    970.5272
 4    549.7954999999999
 5    1128.7872
 6    292.0439
 7    1300.0663
 8    655.927
 9    778.0507
 10    1093.4411
 11    461.1675
 12    1013.0659
 13    743.8423
 14    669.4737
 15    1275.418
 16    391.4464
 17    1058.5916
 18    550.5281
 19    957.5627999999999
 20    991.2402
 21    410.9478
 22    1383.8057
 23    553.0775
______________________
 Global Key:  C Major
_________________________
Fugue 2 in C minor
File Opening Succeeded
Number of lines:  306
Longest line:  38
_________________________
Total steps:  20
Total alterations:  3
Total grace notes:  0
```

Total duration: 32
____

_________RESULTS________
_
C : 6 9 28.12
C#-Db: 0 0 0.0
D : 2 4 12.5
D#-Eb: 1 2 6.25
E : 0 0 0.0
F : 2 2 6.25
F#-Gb: 0 0 0.0
G : 4 6 18.75
G#-Ab: 2 6 18.75
A : 0 0 0.0
A#-Bb: 0 0 0.0
B : 3 3
9.369999999999999
____

____DOT PRODUCTS____
0 1245.2655
1 500.5305
2 885.7776
3 519.1187
4 1137.0782
5 1058.7507
6 536.7497999999999
7 1061.0148
8 614.4828
9 1026.544
10 567.4567
11 884.2269
12 1419.0328
13 549.5644
14 679.3724000000001
15 913.3677
16 778.6757999999999
17 1012.9693
18 540.7479
19 1262.1376
20 785.2627
21 632.14
22 683.9084
23 741.821
____
Global Key: C minor
____
Fugue 3 in C# major
File Opening Succeeded
Number of lines: 287
Longest line: 38
____
Total steps: 17
Total alterations: 17
Total grace notes: 0
Total duration: 28
____

____DOT PRODUCTS____
0 431.7558
1 999.9927
2 740.4894
3 709.6803000000001
4 1200.4839
5 326.2623
6 1312.5819
7 599.6172
8 832.5954
9 869.7590999999999
10 511.1526
11 1499.6142
12 741.5961000000001
13 674.0517
14 1186.668
15 449.6772
16 1070.7144
17 450.534
18 1136.1168
19 1036.5138
20 404.0169
21 1096.9539
22 501.7278000000001
23 1247.4294
____
Global Key: C# Major
____
Fugue 4 in C# minor
File Opening Succeeded
Number of lines: 157
Longest line: 38
____
Total steps: 8
Total alterations: 5
Total grace notes: 0
Total duration: 17
____

_________RESULTS________
_
C : 1 2 11.76
C#-Db: 2 5 29.41
D : 0 0 0.0
D#-Eb: 2 5 29.41
E : 3 5 29.41
F : 0 0 0.0
F#-Gb: 0 0 0.0
G : 0 0 0.0
G#-Ab: 0 0 0.0
A : 0 0 0.0
A#-Bb: 0 0 0.0
B : 0 0 0.0
____

____DOT PRODUCTS____
0 699.0041
1 1102.5769
2 557.6235
3 1080.282
4 1135.2285
5 471.1639
6 871.8230000000001
7 614.3526000000001
8 1086.8095
9 668.6335999999999
10 697.8380000000001
11 1051.6606
12 662.3596
13 737.4241000000001
14 938.2468
15 850.4811999999999
16 1193.925
17 455.5147
18 808.1224999999999
19 789.3975999999999
20 871.3878999999999
21 677.8928999999999
22 631.3582
23 1382.8895
____
Global Key: C# minor
____
Fugue 5 in D major
File Opening Succeeded
Number of lines: 370
Longest line: 45
____
Total steps: 24
Total alterations: 6
Total grace notes: 0
Total duration: 264
____

____DOT PRODUCTS____
0 1023.7403
1 819.8462
2 659.8876
3 1266.0994
4 313.9784
5 1257.2099
6 570.1423
7 939.5063
8 986.8001
9 508.3619
10 1532.1284
11 337.2558
12 667.11
13 1077.9708
14 454.9647
15 1231.0129
16 472.5151
17 1062.6446
18 1074.4893
19 383.1319999999999
20 1243.5582
21 541.0425
22 1248.1653
23 699.3630000000001
____
Global Key: D Major
____
Fugue 6 in d minor
File Opening Succeeded
Number of lines: 295
Longest line: 38
____
Total steps: 20
Total alterations: 2
Total grace notes: 0
Total duration: 34
____

_________RESULTS________
_
C : 0 0 0.0
C#-Db: 1 1 2.94
D : 4 5 14.7
D#-Eb: 0 0 0.0
E : 4 6 17.64
F : 4 5 14.7
F#-Gb: 0 0 0.0
G : 4 8 23.52
G#-Ab: 0 0 0.0
A : 2 5 14.7
A#-Bb: 1 4 11.76
B : 0 0 0.0
____

____DOT PRODUCTS____
0 1192.023
1 449.0556
2 1090.9164
3 875.1498
4 609.0504
5 1073.0118
6 460.9038
7 1189.1418
8 578.3567999999999
9 945.4451999999999
10 1105.6164
11 465.3138
12 939.9473999999999
13 783.5982000000001
14 743.2614
15 1016.946
16 532.6692
17 1172.178

18 572.9177999999999
19 874.6793999999999
20 900.7866
21 593.8506
22 1378.9776
23 486.1878

____

Global Key: D minor

____

Fugue 7 in E flat Major
File Opening Succeeded
Number of lines: 365
Longest line: 38

____

Total steps: 24
Total alterations: 10
Total grace notes: 0
Total duration: 32

____

____DOT PRODUCTS____
0 1013.0039
1 484.1782
2 1204.7339
3 413.0811
4 1154.5312
5 835.4259
6 653.5329999999999
7 1132.7802
8 413.1284
9 1276.4174
10 547.3614
11 907.8178
12 1257.1391
13 472.6537999999999
14 994.5592999999999
15 726.2567
16 780.8886999999999
17 1143.3969
18 445.7885
19 1231.5749
20 549.581
21 935.7631
22 858.4834000000001
23 601.9146

____

Global Key: Eb Major

____

Fugue 8 in e flat minor
File Opening Succeeded
Number of lines: 224
Longest line: 38

____

Total steps: 14
Total alterations: 13
Total grace notes: 0
Total duration: 22

____

____DOT PRODUCTS____
0 262.5493
1 1127.7575
2 1002.4371
3 454.1149
4 1378.9013
5 279.2603
6 1185.9027
7 525.4309999999999
8 859.7421999999999
9 1419.84
10 297.7394
11 1242.3167
12 810.0284
13 538.893
14 1153.2259
15 351.8681
16 1436.5609
17 703.7407999999999
18 634.6800999999999
19 913.15
20 458.2534999999999
21 1598.7898
22 397.1712
23 1001.6383

____

Global Key: D# minor

____

Fugue 9 in E Major
File Opening Succeeded
Number of lines: 451
Longest line: 38

____

Total steps: 30
Total alterations: 18
Total grace notes: 0
Total duration: 35

____

_________RESULTS________
_
C : 0 0 0.0
C#-Db: 2 2 5.71
D : 0 0 0.0
D#-Eb: 6 6 17.14
E : 8 9 25.71
F : 0 0 0.0
F#-Gb: 6 9 25.71
G : 0 0 0.0
G#-Ab: 4 5 14.28
A : 2 2 5.71
A#-Bb: 0 0 0.0
B : 2 2 5.71

____

____DOT PRODUCTS____
0 570.8913
1 1330.2775
2 360.9522999999999
3 1157.0827
4 744.2576000000001
5 677.533
6 974.6881000000001
7 423.0652
8 1374.6507
9 550.2866
10 938.9376000000001
11 932.3660000000001
12 458.6385
13 1076.4431
14 606.9857000000001
15 890.9534000000001
16 1061.4195
17 447.7603
18 1035.6017
19 514.4063000000001
20 1105.4977
21 918.1369
22 541.7769000000001
23 1339.38

____

Global Key: E Major

____

Fugue 10 in e minor
File Opening Succeeded
Number of lines: 380
Longest line: 38

____

Total steps: 26
Total alterations: 8
Total grace notes: 0
Total duration: 28

____

_________RESULTS________
_
C : 1 1 3.57
C#-Db: 2 2 7.14
D : 2 3 10.71
D#-Eb: 2 2 7.14
E : 9 9 32.14
F : 0 0 0.0
F#-Gb: 2 2 7.14
G : 3 3 10.71
G#-Ab: 0 0 0.0
A : 0 0 0.0
A#-Bb: 2 2 7.14
B : 3 4 14.28

____

____DOT PRODUCTS____
0 1046.0437
1 1048.3312
2 580.7495
3 1131.6429
4 575.6459
5 1017.1734
6 641.8565
7 746.2325
8 1125.2288
9 636.6481
10 1011.7532
11 473.6829
12 690.0894
13 1153.6896
14 489.7106
15 1105.4117
16 885.6858999999999
17 744.2569
18 680.6596999999999
19 638.5632999999999
20 1254.3548
21 621.8282
22 869.4196000000001
23 863.3303000000001

____

Global Key: E minor

____

Fugue 11 in F Major
File Opening Succeeded
Number of lines: 306
Longest line: 38

____

Total steps: 21
Total alterations: 4
Total grace notes: 0
Total duration: 28

____

_________RESULTS________
_
C : 5 9 32.14
C#-Db: 0 0 0.0
D : 1 2 7.14
D#-Eb: 0 0 0.0
E : 1 1 3.57
F : 2 2 7.14
F#-Gb: 0 0 0.0
G : 4 4 14.28
G#-Ab: 0 0 0.0
A : 4 4 14.28
A#-Bb: 4 6 21.42
B : 0 0 0.0

____

____DOT PRODUCTS____
0 1207.553
1 349.7986

2 1289.285
3 498.066
4 951.9385
5 987.4269999999999
6 483.3242
7 1480.2177
8 350.3432
9 1083.0756
10 686.7671
11 667.1926999999999
12 1159.3928
13 480.8489
14 1068.9469
15 984.3533
16 450.6138
17 1288.8498
18 387.323
19 1281.0914
20 734.0323
21 684.8878000000001
22 1026.7712
23 449.8887999999999

____

Global Key: F Major

____

Fugue 12 in f minor
File Opening Succeeded
Number of lines: 260
Longest line: 38

____

Total steps: 17
Total alterations: 6
Total grace notes: 0
Total duration: 54

____

_________RESULTS_________

C : 3 10 18.51
C#-Db: 1 4 7.4
D : 0 0 0.0
D#-Eb: 0 0 0.0
E : 1 4 7.4
F : 3 7 12.96
F#-Gb: 0 0 0.0
G : 2 9 16.66
G#-Ab: 3 7 12.96
A : 1 4 7.4
A#-Bb: 2 5 9.25
B : 1 4 7.4

____

____DOT PRODUCTS____
0 1086.3292
1 579.4731
2 904.3587000000001
3 682.7191
4 993.3237000000001
5 800.1134999999999
6 716.5724000000001
7 1137.8624
8 694.3607000000001
9 908.3388000000001
10 627.2969000000001
11 901.2287
12 1041.1317
13 581.4768
14 920.2139
15 903.716
16 750.3198
17 820.6839000000001
18 667.3967
19 1240.9308
20 792.2705000000001
21 655.1305000000001
22 847.8674000000001
23 772.8620000000002

____

Global Key: F minor

____

Fugue 13 in F# Major
File Opening Succeeded
Number of lines: 611
Longest line: 38

____

Total steps: 42
Total alterations: 38
Total grace notes: 0
Total duration: 128

____

_________RESULTS_________

C : 1 2 1.56
C#-Db: 10 41 32.03
D : 0 0 0.0
D#-Eb: 6 18 14.06
E : 0 0 0.0
F : 7 16 12.5
F#-Gb: 7 18 14.06
G : 0 0 0.0
G#-Ab: 4 12 9.369999999999999
A : 0 0 0.0
A#-Bb: 3 10 7.81
B : 4 11 8.59

____

____DOT PRODUCTS____
0 351.7478
1 1207.5368
2 654.2045000000001
3 823.9404000000001
4 1037.9842
5 394.1757
6 1462.3155
7 452.9876
8 908.9499999999999
9 734.2640999999999
10 660.8964
11 1346.9894
12 567.124
13 950.2331
14 1181.6185
15 365.592
16 1131.381
17 436.2907
18 1263.9382
19 795.1271999999999
20 551.5387999999999
21 1069.8968
22 509.6795000000001
23 1175.5802

____

Global Key: F# Major

____

Fugue 14 in f# minor
File Opening Succeeded
Number of lines: 307
Longest line: 38

____

Total steps: 20
Total alterations: 12
Total grace notes: 0
Total duration: 38

____

_________RESULTS_________

C : 1 1 2.63
C#-Db: 2 4 10.52
D : 0 0 0.0
D#-Eb: 0 0 0.0
E : 0 0 0.0
F : 0 0 0.0
F#-Gb: 2 6 15.78
G : 0 0 0.0
G#-Ab: 4 8 21.05
A : 4 10 26.31
A#-Bb: 3 3 7.89
B : 4 6 15.78

____

____DOT PRODUCTS____
0 503.7445
1 1000.1967
2 518.1398
3 1166.6076
4 683.9626999999999
5 785.687
6 1079.3318
7 604.6878999999999
8 1196.5041
9 515.1845999999999
10 1044.1989
11 935.7392
12 423.8143
13 944.1229999999998
14 712.8643000000001
15 890.0810999999999
16 894.4363
17 616.6811
18 1326.2317
19 573.8566
20 902.0823999999999
21 832.8135
22 770.1309999999999
23 1108.8847

____

Global Key: F# minor

____

Fugue 15 in G Major
File Opening Succeeded
Number of lines: 418
Longest line: 38

____

Total steps: 31
Total alterations: 3
Total grace notes: 0
Total duration: 50

____

_________RESULTS_________

C : 3 6 12.0
C#-Db: 0 0 0.0
D : 3 5 10.0
D#-Eb: 0 0 0.0
E : 2 5 10.0
F : 0 0 0.0
F#-Gb: 3 5 10.0
G : 7 10 20.0
G#-Ab: 0 0 0.0
A : 8 12 24.0
A#-Bb: 0 0 0.0
B : 5 7 14.0

____

____DOT PRODUCTS____
0 1237.56
1 641.5799999999999
2 745.0600000000001
3 1022.58
4 462.28
5 1363.7
6 433.06
7 1043.42
8 903.6600000000001

9 574.86
10 1264.42
11 345.82
12 924.54
13 968.7400000000001
14 458.58
15 1193.88
16 486.02
17 1104.5
18 818.64
19 635.26
20 1285.84
21 393.28
22 1122.38
23 608.34

Global Key: G Major

Fugue 16 in g minor
File Opening Succeeded
Number of lines: 325
Longest line: 38

Total steps: 22
Total alterations: 7
Total grace notes: 0
Total duration: 46

_________RESULTS_________

C : 1 2 4.34
C#-Db: 1 2 4.34
D : 2 6 13.04
D#-Eb: 1 2 4.34
E : 1 2 4.34
F : 1 2 4.34
F#-Gb: 1 4 8.69
G : 6 14 30.43
G#-Ab: 0 0 0.0
A : 4 4 8.69
A#-Bb: 4 8 17.39
B : 0 0 0.0

____DOT PRODUCTS____
0 1069.102
1 610.8900000000001
2 1047.369
3 573.9367999999999
4 800.7060000000001
5 1131.4035
6 646.7573
7 1016.6763
8 454.9
9 1116.0623
10 1004.4889
11 559.6850999999999
12 1095.6992
13 867.2071
14 766.8443
15 645.8686999999999
16 669.4538
17 1343.0729
18 559.8625
19 865.7231
20 860.423
21 775.0056
22 1070.7572
23 474.0826

Global Key: G minor

Fugue 17 in A flat Major
File Opening Succeeded
Number of lines: 386
Longest line: 38

Total steps: 26
Total alterations: 15
Total grace notes: 0
Total duration: 48

_________RESULTS_________

C : 3 4 8.33
C#-Db: 3 4 8.33
D : 0 0 0.0
D#-Eb: 5 9 18.75
E : 0 0 0.0
F : 5 12 25.0
F#-Gb: 0 0 0.0
G : 3 10 20.83
G#-Ab: 5 7 14.58
A : 0 0 0.0
A#-Bb: 2 2 4.16
B : 0 0 0.0

____DOT PRODUCTS____
0 908.4839000000001
1 617.8543999999999
2 1074.7226
3 377.3009
4 1313.7001
5 549.5724999999999
6 848.3094000000001
7 965.8707999999999
8 544.5489
9 1248.3364
10 405.7009
11 1181.5916
12 1271.1086
13 373.0873999999999
14 1135.5445
15 523.5983
16 1001.335
17 785.5106999999999
18 591.7094000000001
19 1269.6141
20 531.7014
21 967.6319
22 742.6410999999999
23 804.5176

Global Key: Ab Major

Fugue 18 in g sharp minor
File Opening Succeeded
Number of lines: 248
Longest line: 41

Total steps: 15
Total alterations: 14
Total grace notes: 0
Total duration: 32

_________RESULTS_________

C : 0 0 0.0
C#-Db: 0 0 0.0
D : 1 2 6.25
D#-Eb: 2 6 18.75
E : 0 0 0.0
F : 0 0 0.0
F#-Gb: 1 2 6.25
G : 1 2 6.25
G#-Ab: 5 11 34.37
A : 0 0 0.0
A#-Bb: 4 7 21.87
B : 1 2 6.25

____DOT PRODUCTS____
0 386.1436
1 1024.6273
2 771.3448999999999
3 605.0999999999999
4 1351.4431
5 494.3311
6 1076.4622
7 443.9072
8 931.2333
9 1323.2197
10 438.5129
11 1190.6709
12 870.8174
13 609.7686
14 808.9226
15 460.8378
16 1444.0049
17 759.4272000000001
18 716.746
19 909.091
20 509.6452
21 1375.8367
22 424.4239
23 1109.4787

Global Key: G# minor

Fugue 19 in A Major
File Opening Succeeded
Number of lines: 282
Longest line: 38

Total steps: 16
Total alterations: 6
Total grace notes: 0
Total duration: 16

_________RESULTS_________

C : 0 0 0.0
C#-Db: 3 3 18.75
D : 3 3 18.75
D#-Eb: 1 1 6.25
E : 3 3 18.75
F : 0 0 0.0
F#-Gb: 1 1 6.25
G : 0 0 0.0
G#-Ab: 1 1 6.25
A : 3 3 18.75
A#-Bb: 0 0 0.0
B : 1 1 6.25

____DOT PRODUCTS____
0 759.375
1 848.625
2 607.3125
3 1383.75
4 541.75
5 963.5625
6 762.3125
7 673.4375
8 1055.375
9 456.3125
10 1292.875
11 693.3125
12 482.0625
13 1046.5625
14 580.625
15 1103.375
16 750.5

17 790.0
18 1147.5625
19 446.9375
20 917.6875
21 557.1875
22 1121.5625
23 1055.9375

Global Key: A Major

Fugue 20 in a minor
File Opening Succeeded
Number of lines: 435
Longest line: 38

Total steps: 31
Total alterations: 2
Total grace notes: 0
Total duration: 48

__________RESULTS_________

C : 8 11 22.91
C#-Db: 0 0 0.0
D : 5 7 14.58
D#-Eb: 0 0 0.0
E : 4 7 14.58
F : 1 2 4.16
F#-Gb: 0 0 0.0
G : 0 0 0.0
G#-Ab: 2 3 6.25
A : 5 10 20.83
A#-Bb: 0 0 0.0
B : 6 8 16.66

____DOT PRODUCTS____
0 1188.7093
1 571.0899999999999
2 829.5437999999999
3 1136.524
4 567.7225999999999
5 1158.0283
6 407.7512
7 1137.9175
8 968.3519
9 534.9545999999999
10 1027.2432
11 507.1522000000001
12 851.6869
13 781.2910000000001
14 582.6380000000001
15 1450.1047
16 514.1482999999999
17 893.4397
18 789.8871
19 813.5535
20 1090.7981
21 431.1784
22 1069.3216
23 728.9526999999999

Global Key: A minor

Fugue 21 in B flat Major
File Opening Succeeded
Number of lines: 526
Longest line: 38

Total steps: 38
Total alterations: 11
Total grace notes: 0
Total duration: 48

__________RESULTS_________

C : 9 11 22.91
C#-Db: 0 0 0.0
D : 8 11 22.91
D#-Eb: 5 6 12.5
E : 0 0 0.0
F : 3 5 10.41
F#-Gb: 0 0 0.0
G : 3 4 8.33
G#-Ab: 0 0 0.0
A : 4 4 8.33
A#-Bb: 6 7 14.58
B : 0 0 0.0

____DOT PRODUCTS____
0 1066.5683
1 422.8038
2 1361.9646
3 492.7567999999999
4 972.1167000000002
5 1015.2041
6 517.6434999999999
7 1219.2349
8 324.7004
9 1197.7178
10 739.2981
11 704.9796
12 1237.5481
13 554.6033
14 1001.6403
15 870.7672
16 577.9271
17 1340.3169
18 434.4055
19 1040.7492
20 588.5589000000001
21 879.6765
22 1033.4401
23 437.3669

Global Key: Bb Major

Fugue 22 in b flat minor
File Opening Succeeded
Number of lines: 175
Longest line: 38

Total steps: 10
Total alterations: 6
Total grace notes: 0
Total duration: 12

__________RESULTS_________

C : 1 1 8.33
C#-Db: 2 2 16.66
D : 0 0 0.0
D#-Eb: 2 2 16.66
E : 0 0 0.0
F : 3 4 33.33
F#-Gb: 1 1 8.33
G : 0 0 0.0
G#-Ab: 0 0 0.0
A : 0 0 0.0
A#-Bb: 1 2 16.66
B : 0 0 0.0

____DOT PRODUCTS____
0 575.5441999999999
1 845.0128
2 1297.0172
3 423.9328
4 1111.6356
5 282.4765
6 1211.7843
7 995.6139999999999
8 485.1492
9 1133.752
10 392.6915
11 1280.3785
12 852.6077
13 562.372
14 1593.0737
15 431.9032
16 854.677
17 666.8557
18 786.4208
19 1158.4646
20 354.8636
21 1261.4532
22 756.8371999999999
23 717.4713

Global Key: A# minor

Fugue 23 in B Major
File Opening Succeeded
Number of lines: 443
Longest line: 38

Total steps: 29
Total alterations: 23
Total grace notes: 0
Total duration: 68

__________RESULTS_________

C : 0 0 0.0
C#-Db: 10 18 26.47
D : 0 0 0.0
D#-Eb: 9 13 19.11
E : 1 4 5.88
F : 0 0 0.0
F#-Gb: 1 4 5.88
G : 0 0 0.0
G#-Ab: 1 2 2.94
A : 0 0 0.0
A#-Bb: 2 6 8.82
B : 5 21 30.88

____DOT PRODUCTS____
0 490.893
1 1439.713
2 477.934
3 998.6016
4 873.0382999999998
5 729.6764999999999
6 1274.63
7 326.2234
8 1244.4904
9 736.5925
10 660.9944999999999
11 962.1694000000001
12 616.1438000000001
13 1164.8371
14 867.5114999999998
15 585.1505
16 1350.5927
17 443.2292
18 1083.1238
19 502.9838
20 1068.3861
21 989.3969
22 432.6304999999999
23 1051.9825

Global Key: B Major

Fugue 24 in b minor
File Opening Succeeded
Number of lines: 489
Longest line: 38

Total steps: 34
Total alterations: 19
Total grace notes: 0
Total duration: 108

_________RESULTS_________

C : 2 8 7.4
C#-Db: 2 8 7.4
D : 2 8 7.4
D#-Eb: 1 4 3.7
E : 1 4 3.7
F : 1 4 3.7
F#-Gb: 6 34 31.48
G : 1 4 3.7
G#-Ab: 7 8 7.4
A : 7 10 9.25
A#-Bb: 1 4 3.7
B : 3 12 11.11

____DOT PRODUCTS____

0	666.0622000000001
1	1199.7005
2	616.0977
3	884.7392
4	649.5418
5	922.1496000000001
6	1112.7292
7	657.1602
8	967.4325
9	560.3699
10	1129.4568
11	845.4302
12	657.1195
13	1240.7045
14	734.1379000000001
15	763.4431
16	704.5428000000001
17	767.6599000000001
18	1232.1361
19	605.9894999999999
20	934.8383000000001
21	959.3580000000001
22	670.3034
23	881.6722000000001

Global Key: B minor

Appendix VI - Scores

Sonata in G minor ‘Erstausgabe’ (D.Scarlatti)
Sarabande from Suite No.16 (Handel)
Prelude Op.34 No.14 (D. Shostakovich)