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RELIABILITY PREDICTION IN EARLY DESIGN STAGES

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ABSTRACT

In the past, reliability is usually quantified with sufficient information available. This is not only time-consuming and cost-expensive, but also too late for occurred failures and losses. For solving this problem, the objective of this dissertation is to predict product reliability in early design stages with limited information. The current research of early reliability prediction is far from mature. Inspired by methodologies for the detail design stage, this research uses statistics-based and physics-based methodologies by providing general models with quantitative results, which could help design for reliability and decision making during the early design stage. New methodologies which accommodate component dependence, time dependence, and limited information are developed in this research to help early accurate reliability assessment. The component dependence is considered implicitly and automatically without knowing component design details by constructing a strength-stress interference model. The time-dependent reliability analysis is converted into its time-independent counterpart with the use of the extreme value of the system load by simulation. The effect of dependent interval distribution parameters estimated from limited point and interval samples are also considered to obtain more accurate system reliability. Optimization is used to obtain narrower system reliability bounds compared to those from the traditional method with independent component assumption or independent distribution parameter assumption. With new methodologies, it is possible to obtain narrower time-dependent system reliability bounds with limited information during early design stages by considering component dependence and distribution parameter dependence. Examples are provided to demonstrate the proposed methodologies.

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1. INTRODUCTION

1.1 BACKGROUND

Reliability is the probability that a product performs its intended function under specified conditions during a specified period of time [1]. In the past, reliability analysis had been primarily regarded as a study of failures and failure time data of products, meaning a product's reliability could be quantified only after observing field failure data and/or life testing results. This is often too late due to risks and losses that have already occurred. Nowadays, reliability is viewed as an important criterion of product performance. Research indicates that the major product performance and up to 70% of the product cost are determined in early design stages [2]. With the trend of design for reliability in modern industries, reliability analysis as early as in conceptual design stages is imperative.

Progress has been made in reliability prediction during early stages, but many questions still need answers. In the conceptual design stage, reliability information is sparse or may not be available. Thus, it is hard to obtain quantitative reliability results. A series of methodologies in qualitative reliability prediction have been developed by Tumer's and Stone's groups based on function modeling [3-6]. Function modeling is an important stage for generating design concepts during conceptual design. The overall function is created first and is then decomposed into a number of sub-functions. Solutions are sought to realize the sub-functions. Design concepts are then generated from the solutions. The key to high reliability is to make sure that the design concepts generated have sufficient intrinsic reliability. Function modelling based methodologies, which enable the early reliability analysis mainly in a qualitative way after the product functions are determined, are more or less subjective.

Besides qualitative methods discussed above, relative reliability measures are also provided, whose objective is to rank design concepts with quantitative reliability indexes. A good attempt is the development of the Relative Reliability Risk Assessment (R^3I) method [7]. In the conceptual design stage, though limit information is available, quantitative reliability prediction is usually more preferred. Traditional reliability approaches, such as Failure Modes and Effects Analysis (FMEA) and Fault Tree Analysis (FTA), often restrict the information to what is obtained from current product testing data, and they often result in unseasonable results, such as 1 or 0 for reliability. Bayesian approaches [8-11] are proposed to use in early design stages and perform better than the traditional methods because all the information available can be used, no matter if it is old or new, objective or subjective, or point or interval values. However, the application of Bayesian models is sensitive to the appropriate prior distributions.

Due to the lack of computational models during the early design stage, physical-based methods are rarely used. Recently, there was an attempt to extend one of the physical-based reliability strategies, the stress and strength interference theory, to the reliability analysis in conceptual design. The method is called the conceptual stress and conceptual strength interference theory (CSCSIT) [12]. The CSCSIT method is a good attempt to use physics-based methodologies in product early design stage; however, it did not consider the issues of component dependence and time dependence.

From the state-of-the-art, we see that the research on early design reliability methodologies has progressed in spite of the challenges and is gaining more attention.

The methodologies, however, still have their limitations, and the research in reliability prediction during early design stages is far from mature. Even though the challenges are formidable, they undoubtedly provide great opportunities of exploring new ways to deal with reliability in conceptual design.

1.2 RESEARCH OBJECTIVE

The objective of this research is to predict product reliability in early design stages. With the predicted reliability, the research results can help engineers reduce the likelihood of failures to an acceptable level before the test of manufactured products or field deployment. To achieve this objective, four research tasks are performed.

Research task 1 focuses on the survey of reliability prediction in early design stages. This research task intends to answer the questions, such as how far reliability methodologies for early conceptual design have been progressed and what is needed for further research? This research task results in Paper 1.

Research task 2 concentrates on the consideration of component dependence in early reliability prediction. The component dependence is ignored in existing studies and practices. In this task, physics-based reliability methodologies are used. This is a new development because physics-based (structural) reliability methodologies have been rarely applied in conceptual design before, they are widely used in only parameter or detail design stage where computational models are available. This research task produces Paper 2 [13].

Research task 3 focuses on the accommodation of time dependent issue in early reliability design stages. Research task 2 is for time invariant reliability problems. It is extended to time variant problems in research task 3. The goal of this task is to evaluate the time-dependent system reliability for a given period of time in early design stages. This research task produces Paper 3 [14].

Research task 4 concentrates on the effect of dependent interval distribution parameters on reliability prediction. In this task, the distribution parameters are estimated from scarce and point-interval-mixed samples. The distribution parameters are dependent since they are estimated from the same set of data. The dependent relationship leads to more accurate reliability prediction than the traditional independent assumption. This research task produces Paper 4 [15].

The outcomes of above research tasks are expected to enable engineers to understand how dependence affects the reliability prediction in early design stages and how to predict system reliability efficiently with good accuracy. With the accurate system reliability prediction in early design stages, this dissertation will enhance system designs in decision making with respect to system configurations, optimization, lifecycle cost, maintenance, and warranty.

1.3 ORGANIZATION OF DISSERTATION

As discussed in Section 1.2, the four tasks in this study have produced four papers, which constitute this dissertation.

The first paper is entitled “Reliability Methodologies for Conceptual Design: What is Done; What is Needed?” Rather than reviewing the entire body of the literature on reliability methodologies for conceptual design, this work focuses on assessing the feasibility of predicting reliability in the early design stage. In addition to providing the current state-of-the-art of the methodologies, this survey also shows that early reliability consideration provides great opportunities for new research in conceptual design, including accounting for dependent component failures in system reliability prediction, the use of physics-based reliability approaches, and information aggregation for reliability quantification.

The second paper is entitled “System Reliability Analysis with Dependent Component Failures during Early Design Stage – A Feasibility Study”. This work is concerned with the reliability prediction of a new product whose components are independently designed, tested, and manufactured by different suppliers. A system reliability method is developed to predict the reliability of the new product in the early design stage using the component reliabilities provided by component suppliers. The method is based on the strength-stress interference model that takes the dependence between components into consideration, thereby eliminating the assumption of independent component failures. As a result, the predicted system reliability bounds are much narrower than those from the assumption of independent component failures.

The third paper is entitled “Narrower System Reliability Bounds with Incomplete Component Information and Stochastic Process Loading”, which is the extension of time invariant problems in Paper 2 to time-dependent system reliability analysis. The new method can be applied to more common engineering applications because it can answer the question about the system reliability with respect to time; for example, what is the probability that a system can still work without failure after five years? A general model is developed to implicitly and automatically incorporate component dependence. With this general model, system designers do not need to know component resistance distributions (both distribution types and parameters), component failure modes, and other detail information such as dimensions. Simulation is used to obtain the extreme value of the system stochastic process load for a given period of time, and optimization models are established to estimate the system reliability interval. The width of the system reliability interval is then reduced significantly.

The fourth paper is entitled “Effect of Dependent Interval Distribution Parameters on Reliability Prediction”. This study investigates the effect of the dependence of distribution parameters on the accuracy of reliability analysis results. The major approach is numerical simulation and optimization. This study indicates that the independent distribution parameter assumption makes the estimated reliability bounds wider than the true bounds due to interval samples. The reason is that the actual combination of the distribution parameters may not include the entire box-type domain assumed by the independent interval parameter assumption. The results of this study not only reveal the cause of the inaccuracy of the independent distribution parameter assumption, but also demonstrate a need of developing new reliability methods to accommodate dependent distribution parameters.

PAPER

I. RELIABILITY METHODOLOGIES FOR CONCEPTUAL DESIGN: WHAT IS DONE; WHAT IS NEEDED?

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ABSTRACT

Reliability methodologies have been used for a long time in product design, manufacturing, and operation, but how far reliability methodologies for early conceptual design have progressed and what is needed for further research? This paper intends to answer these questions. Reliability methodologies for conceptual design are critical because product reliability is primarily determined in this design stage even though sufficient information is usually lacking. Major performances and vital cost of a product are also determined in the early design stage. The importance and challenges of reliability for conceptual design are therefore emphasized in this paper. Rather than reviewing the entire body of the literature on reliability methodologies for conceptual design, this work focuses on assessing the feasibility of predicting reliability in the early design stage. The assessment is summarized in the following aspects for each methodology: the objective, input (information required), output, assumptions, tools, scope, and nature (quantitative or qualitative). In addition to providing the current state-of-the-art of the methodologies, this survey also shows that early reliability consideration provides great opportunities for new research directions in the conceptual design, including accounting for dependent

component failures and time-dependent issues in system reliability prediction, the use of physics-based reliability approaches, and information aggregation for reliability quantification.

1. INTRODUCTION

Reliability is the ability of a system to perform its intended function without failures, and it is usually quantified by the probability of such ability. The system in the above definition is in a general sense so that the definition of reliability is also applicable to a variety products, assemblies, subsystems, equipment, components, services, and processes. There are two major application areas of reliability methodologies. The first is reliability analysis [1, 2] whose task is to predict and evaluate the reliability. Potential failure modes and their causes are also identified during the reliability analysis. The second is reliability-based design during which optimal design concepts and design variables are determined so that reliability requirements are met with a reduced lifecycle cost [3, 4]. Overall, the focus of reliability methodologies is to eliminate failures and/or reduce the likelihood of failures to an acceptable level.

In the past, reliability analysis was mostly regarded as a passive term since it could be quantified only when field failure data and/or life testing data become available. With the advancement of design methodologies and simulation techniques, reliability is now addressed more upfront in the design stages, even as early as in the conceptual design stage [5-7]. Performing reliability analysis upfront will not only ensure high reliability, robustness, safety and availability, but also reduce product lifecycle cost [8]. It has been well recognized that reliability can be built into products in the design stage and can be maintained throughout production and operation.

Predicting reliability in early design stages, however, is a challenging task due to the following reasons:

(1) Reliability data are scarce. The data include those of failure modes, time to failure, product downtime, and so on. In the early design stage, data may not be available or are limited.

(2) The relationship between components and the system is unclear. As a result, it is difficult to predict the system level reliability even if the information about the component level reliability is available.

(3) Product functions delivered by components and their interfaces could suffer from common-cause failures, shared excessive loading, dependent strength deterioration, and so on. This requires considering dependencies between functions, failure modes, components, and subsystems.

(4) Limited reliability data may come from various sources with different formats. For examples, reliability information of new products can be collected from their parent products; expert opinions could be solicited by designers; information may be obtained from test results of similar components or prototypes. All relevant information needs to be aggregated and processed to make reliability prediction at each milestone of the design project.

The research on early design reliability methodologies has progressed in spite of the challenges and is gaining more attention. The methodologies, however, are far from mature compared to those for detail design (or parameter design). The methodologies are quite different with respect to their scopes, assumptions, information required, and outcomes. The purpose of this review is to investigate how far those methodologies have evolved and provide useful insight that can help better understand and choose the methodologies for specific applications. We also provide suggestions about the future research directions for early design reliability methodologies. The contributions of this work are multifold. (1) As discusses previously, it summarizes the reliability methodologies for conceptual design by treating them as black-boxes so that the methodologies could be better understood. (2) New research directions beyond traditional reliability engineering are given with the focus of physics-based methodologies. (3) Insight from the aspect of mechanical engineering is offered with respect to both what has been done and what is needed for reliability consideration in conceptual design.

The rest of this paper is organized as follows. Section 2 discusses the role of reliability analysis in conceptual design and then examines and reviews existing reliability methodologies in conceptual design. Section 3 reviews efforts made in reliability related methodologies in conceptual design, including sensitivity analysis, uncertainty quantification, and risk analysis. In Section 4 a methodology summary is provided, and future research is heighted in Section 5. Conclusions are made in the last section.

2. RELIABILITY METHODOLOGIES IN CONCEPTUAL DESIGN

A design process is usually divided into four stages: problem definition, conceptual design, embodiment design, and detail design. We herein focus on conceptual design, during which design concepts are generated and selected. In this section we review reliability methodologies that can be used in this design stage.

2.1 RELIABILITY CONSIDERATION IN CONCEPTUAL DESIGN

In the early design stage, in addition to setting up reliability requirements and target [9], other major reliability-related tasks are also conducted, including the following:

- Identify potential failure modes, their causes, and their consequences.
- Estimate the likelihood of the occurrence of failure modes.
- Generate design concepts whose failures could be eliminated or their likelihood could be reduced.
- Evaluate the system reliability or the product-level reliability for each design concept.
- Select the best design concepts with respect to reliability.

Since reliability is related to risk and is also a major driving factor of lifecycle cost, the above activities are usually accompanied by risk analysis [10] and lifecycle cost analysis [11]. The current reliability methodologies handle one or more these tasks as will be reviewed next.

2.2 METHODOLOGIES IN RELIABILITY ENGINEERING

Many methodologies in reliability engineering are commonly used in the conceptual design stage. They include Failure Modes and Effects Analysis (FMEA) [12], Fault Tree Analysis (FTA) [13], Event Tree Analysis (ETA) [14], Root Cause Analysis (RCA) [15], and Reliability Block Diagrams (RDB) [16]. We briefly review these methodologies with a focus on the new development for FMEA.

FMEA is used to identify and prioritize potential failures. Its three major tasks are shown in Table 1. The prioritization of failure modes is determined through the risk priority number (RPN), which is determined by the following three factors: failure occurrence (*O*), effect severity (*S*), and detection difficulty (*D*), all evaluated with a 10-point scale. Eq. (1) shows the RPN. The higher is the RPN of a failure mode, the greater is the risk.

$$\mathrm{RPN} = O \times S \times D \tag{1}$$

Table 1 Three FMEA tasks

Task	**Result**
Identify failures	Failure modes, causes, and effects
Prioritize failures	RPN and the most risky failure modes
Reduce risks	Effective measures to reduce risks

FMEA has been applied widely in industry [17, 18]. It has, however, several shortcomings [19]. The relative importance among *O*, *S*, and *D* is not considered; their different combinations may produce exactly the same RPN, but their hidden risk implications may be totally different; and the three factors are difficult to be precisely evaluated. Besides, FMEA often misses key failures [20]; FMEA is performed too late to affect key decisions [21]; the FMEA process is also tedious [22]; the RPN may not be a good measure of risk [23, 24]. Numerous modifications of FMEA have therefore been made. To overcome the difficulties of assigning risk factors, Wang et al. [25] proposed fuzzy risk priority numbers (FRPNs). Chin et al. [26] used the data envelopment analysis (DEA) to determine the risk priorities of failure modes. In order to resolve the difficulty of incorporating different types of information into the fuzzy RPN, Chin et al. [27] employed a multiple attribute decision analysis with the group-based evidential reasoning (ER) approach. Other fuzzy theory based methods are also reported in [28-30]. While they add quite flexibility to FMEA, fuzzy theory based methods have some limitations due to the use of subjective factors.

One of the remarkable improvements is the scenario-based FMEA [31-33], where a failure scenario is an undesired cause-effect chain of events as shown in Fig. 1 [31]. The expected cost E_C is the product of probability of an event (failure effect) p and the associated failure cost C for a simple failure event; namely, $E_C = pC$. For a failure scenario flow with multiple failure effect events F_i ($i = 1, 2, \ldots, n$), the expected cost of the scenario is given by

$$E_C = p(F_1)\prod_{i=2}^{n} p(F_i | F_{i-1})C \tag{2}$$

where $p(F_i | F_{i-1})$ is the conditional probability of effect F_i given that effect F_{i-1} has occurred, and C is the cost of the failure scenario. If there are m failure scenarios, the total expected cost is given by

$$E_C = \sum_{j=1}^{m} E_{C_j} \tag{3}$$

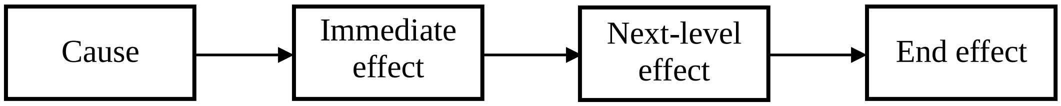

Figure 1 Failure scenario [31]

As discussed above, the failure occurrence (*O*) and effect severity (*S*) in the original FMEA are replaced by the probability and cost, respectively. The detection difficulty (*D*) can also be considered in the scenario-based FMEA using a probability measure and can be included in Eq. (2) [31]. By using probabilities and costs, the scenario-based FMEA provides a consistent basis for risk analysis and decision making with more accurate risk evaluations. Bayesian methods have also been introduced into FMEA (more Bayesian approaches will be discussed in the next subsection). For example, Lee [34] combined Bayesian belief network theory with traditional FMEA and proposed the BN-FMEA method, which models the system failure cause and effect relationships and their uncertain consequences with better precision and consistency. Other FMEA approaches have also been developed, including a simulation method for considering possible combinations of failures automatically [35], an FMEA for lean systems [36], and the assessment of the impact of multiple failure modes [37].

The fault tree analysis (FTA) [38] is another important tool for system reliability. It can be applied for both simple and complex engineering systems; and existing systems and new systems [39, 40]. A tree is constructed downwards, dissecting the system for further detail until the primary events leading to the top event are known. Lee et al. [41] reviewed FTA-related articles published before 1985. Shalev and Tiran [39] proposed a practical operative tool called condition-based fault tree analysis (CBFTA) to improve system reliability. Dynamic FTA (DFTA) [42] is a notable extension to FTA by defining additional gates called dynamic gates to model complex interactions. Some researchers have recently used the fuzzy set theory and evidence theory in FTA analysis [43] to reduce the error from the inaccuracy of primary event data.

A Reliability Block Diagram (RBD) is an inductive methodology to perform system reliability analysis by using a graphical representation [44]. The system structure is usually in series or parallel or their combination. Examples of the extension of the RBD method include the RBD method for repairable multi-state systems [45] and the RBD with general gates [46].

The above traditional reliability methodologies have been widely used in reliability engineering. They are general methods, and most of them can be used in all stages of product design and development, but they have more or less limitations in the application of reliability prediction in the early design stage due to subjective factors involved. There are other methodologies recently proposed that suit the need of conceptual design. Some of them are reviewed in the next subsection.

2.3 BAYESIAN METHODOLOGIES

In the conceptual design stage, reliability information is sparse or may not be available. Traditional statistical approaches restrict the information to what is obtained from current product testing data [47], and they often result in unseasonable results, such as 1 or 0 for reliability. The information may also come from different sources with different formats, for example, from previous similar products and components, expert opinions, experiments, limited physical testing, and simulations. For these cases, many Bayesian approaches perform better than the traditional methods because all the information available can be used, no matter if it is old or new, objective or subjective, or point or interval values.

The Bayes' Theorem is expressed by

$$\pi(\theta \mid y) = \frac{f(y \mid \theta)\pi(\theta)}{\int f(y \mid \theta)\pi(\theta)d\theta} \tag{4}$$

where θ is a parameter vector, y is a data vector, $\pi(\theta)$ is a prior probability density function, and $f(y \mid \theta)$ is the probability density function of the data, referred to as the likelihood when viewed as a function of the parameter vector given the data. The result of integrating the data with prior information in Eq. (4) is the joint posterior distribution $\pi(\theta \mid \mathrm{y})$. Eq. (4) provides significant flexibility for various types of input information mentioned above [48].

Data from previous comparable products under similar conditions of use may be available. As indicated in [47], the application of the Bayesian hierarchical models is reported for the prediction of failure probabilities during early flights of new lunch vehicles, for which sparse or no system level failure data are available. But the "prior" information on comparable products can be used to estimate the reliability of new products. The major approach of doing so is the use of the hierarchical model, where the probability density function of a new space vehicle is based on the prior known parameter from comparable products.

Bayesian methods are also able to integrate lifetime data collected at component, subsystem, and system levels with prior information at any level. A typical Bayesian model for assessing the reliability of such multicomponent systems is discussed in [49]. The model allows pooling of information from similar components and expert opinions.

It can also handle censored data. Several sources of information relevant to estimating system reliability are assumed available, including (1) lifetime data collected at the individual component level, (2) lifetime data collected at the system or subsystem level, (3) expert opinions regarding the reliability of components and subsystems of the current product, and (4) expert opinions regarding the distributions of the lifetimes of similar components. The relationship between the state of the system and those of components is known, and it could be expressed as a series, parallel, or the combination system. Under the assumption that all the component lifetimes are independent, the distribution of the system lifetime is analytically available given the distributions of component lifetimes. The method follows a four-step procedure.

- Step 1: Determine the prior distributions of the distribution parameters of the component distributions. It is the $\pi(\theta)$ term in Eq. (4).
- Step 2: Use the component and system lifetime data, which is y in Eq. (4), to formulate the likelihood $f(\mathrm{y}|\theta)\pi(\theta)$. In this step, both the distributions of component and system lifetimes are incorporated, and the system lifetime distribution is expressed in terms of the distributions of component lifetimes.
- Step 3: Use Eq. (4) to obtain the posterior distribution of the distribution parameters of component lifetimes.
- Step 4: Use Markov chain Monte Carlo (MCMC) to solve Eq. (4) and then obtain the system reliability.

Although the methodology is developed for general reliability analysis, it could be used in the early design stage given its ability of incorporating diverse sources of information at different levels about the system.

There are variants of Bayesian reliability methodologies with different scopes, assumptions, and implementations [50]. For example, the Bayesian model updating approach [51] assesses the reliability of a product throughout all its life cycle stages. For the early design stage, historical data, CAE-induced knowledge, simulation results, and expert opinions are used to formulate a Bayesian model for the reliability index. The model is built in such a way that it can be easily updated when more information is available as the design evolves. If system test data and component data are available, the two kinds of data can be integrated for the system reliability assessment with the Bayesian approach in [52]. A similar work is reported in [53] where three sources of information could be handled, including warranty data that are collected for the product's components that have been released to the market, raw data from test or field, and engineering judgment of the reliability impact due to the planned design changes.

The Bayesian reliability methods have been further expanded with the Bayesian Network (BN). BN is a probabilistic graphical model that represents a set of random variables and their conditional dependencies through a directed acyclic graph (DAG). The BN methodology has become a popular approach applied to assess system reliability [50, 54] of nuclear power systems, military vehicles, and sensors. Martz et al. [55] and Martz and Waller [56] used static Bayesian procedure to estimate the reliability of a complex system. Weber and Jouffe [57, 58] developed dynamic Bayesian networks (DBN) to dynamically model and control the complex manufacturing processes. Hybrid BN is developed to assess reliability aiming for both discrete and continuous variables for real world applications. Langseth et al. [59] summarized the research on the inference of hybrid BN. Hamada et al. [60, 61] presented a Bayesian approach which not only simultaneously combines basic events and independent higher-level failure rate, but also automatically propagates the highest-level data to lower levels in the fault tree.

One of the key factors of using the Bayesian model is to select appropriate prior distributions. Many references are available about choosing prior distributions, including [62-64].

2.4 USE OF HERITAGE DATA

As discussed above, Bayesian methods could incorporate various kinds of data, and there are other specific approaches that could directly use the data from previous products for the reliability analysis of a new product. The parenting process [65] is such an approach. The overall procedure of the method is depicted in Fig. 2.

For each of the failure causes, the method starts from searching for the failure rate λ from the warranty database of the previous products. The failure times are assumed to follow lognormal distributions. Then expert judgment is solicited for the adjustment of the failure rate of the current product with a parenting factor γ, which is also assumed to follow a lognormal distribution. Then the failure rate of the new product is adjusted by $\lambda_{new} = \gamma\lambda$. A relationship matrix between a failure mode and its possible failure causes is also established. From the matrix, the time distribution of a failure mode is obtained by using the failure rates of all the failure causes, $\lambda_{new,i}$, $i = 1, 2, \dots, n$, where n is the number of causes for the failure mode. This method creates a direct link for the reliability between a new product and its parents.

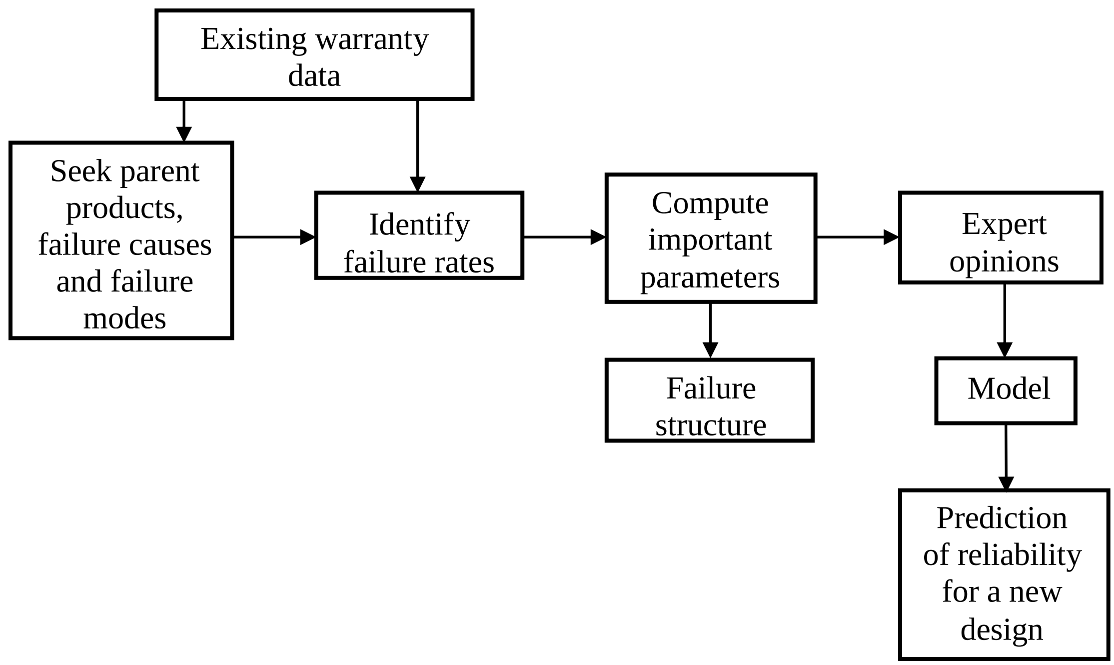

Figure 2 Parenting process [65]

2.5 RELIABILITY METHODS FOR DESIGN CONCEPT EVALUATION

The reliability predictions can be used to compare design concepts with respect to reliability. In many cases, however, it is impossible to obtain quantitative reliability predictions, but the design concepts have to be evaluated, in order to select the best design concepts for the later design stages. For this case, it is desirable to identify relative reliability measures so that the design concepts can be ranked without quantitative reliability indexes. A good attempt is the development of the Relative Reliability Risk Assessment (R^3I) method [66]. It is used with sparse data during conceptual design even though no quantitative data are available for reliability. The steps of R^3I method are shown in Fig. 3.

For each design concept, the method starts from the functional modeling where the complete functions of the product with their input-output flows [67, 68] are created.

Then the Analytic Hierarchy Process (AHP) [69] is used to obtain the priority measures of functions of the design concept with respect to evaluation criteria (attributes). The advantage of this method [66] is that the weights of the attributes are not assigned subjectively; instead, they are evaluated by the entropy method. With both of the priority measures and weights available, the R^3I index of the design concept is computed. This process is repeated for all the design concepts, and finally the design concepts are ranked according to their R^3I indexes. It is noted that reliability could be one of the attributes in the evaluation process, but it may not be necessary.

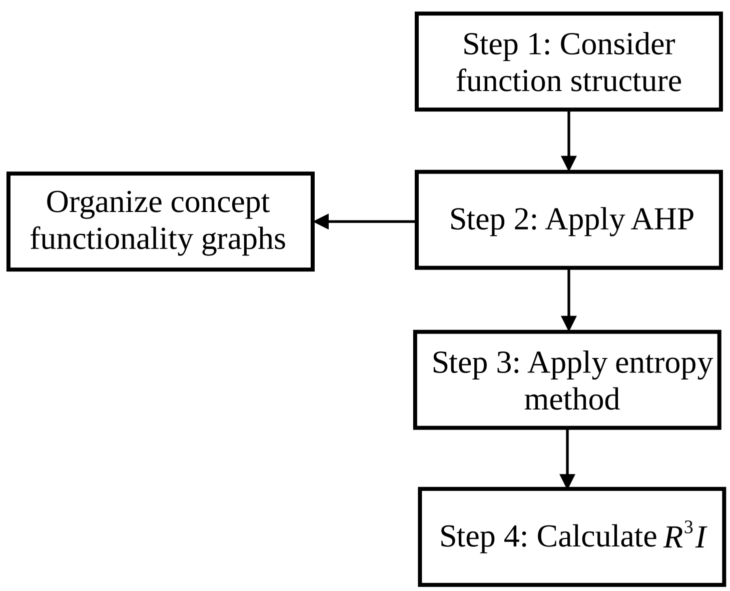

Figure 3 Steps of the R^3I method [66]

2.6 FUNCTION MODELING METHODOLOGIES

Function modeling is an important stage for generating design concepts during conceptual design. The overall function is created first and is then decomposed into a number of sub-functions. Solutions are sought to realize the sub-functions. Design concepts are then generated from the solutions. The key to high reliability is to make sure that the design concepts generated have sufficient intrinsic reliability. Stone and Wood [67] introduced a functional basis in conceptual design as a design language to comprehensively and consistently describe product functions in a function-flow format, and this makes the design in a systematic and repeatable manner. By reconciling and evolving previous efforts, this functional basis is served as the evolved definitions of functional modeling and the taxonomy of engineering design at many scales [70].

A series of methodologies in this area have been developed by Tumer's and Stone's groups. Their Function-Failure Mode Method [71] provides a matrix-based analytical approach to making design decisions in order to avoid potential failures based on the link between functionality and failure modes of components. An elemental function-failure design method (EFDM) [72] was proposed specifically for use in the conceptual design stage, and the advantages of EFDM over traditional FMEA were demonstrated using the Bell 206 rotorcraft data. The latter-developed Function Failure Design Methodology (FFDM) [73, 74] fully allows the FMEA-style failure analysis to be used in the conceptual design. The steps shown in Fig. 4 include: (1) Generate a black-box model to best describe the overall function. (2) Use the function flow of the overall function to identify the most common failure modes for that function. (3) Derive a complete functional model that includes all sub-functions for the overall function. Failure modes identified in the former step are addressed here. If needed, additional sub-functions are added to mitigate the effects of major failure modes. (4) Generate solutions to sub-functions and the overall solutions (design concepts) for the overall function. (5)

Evaluate the design concepts with respect to reliability. During the implementation, the function-failure analysis and the associated knowledge base [67, 70, 71, 75] are called.

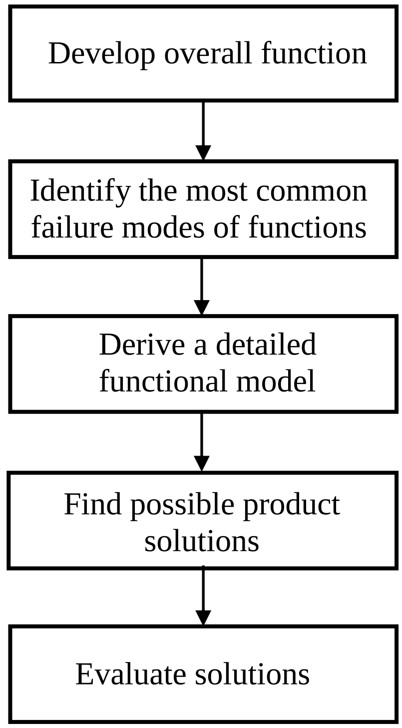

Figure 4 The FFDM procedure [73]

The FFDM is then extended to the Functional Failure Rate Design Method (FFRDM) [76] that can effectively provide recommendations to mitigate failure modes with high likelihood of occurrence. A more robust knowledge base and repository data at Oregon State University are used by FFRDM. More quantitative ways are provided to deal with the same reliability issues; for example, O'Halloran et al. calculated function-flow failure rates (FFFR) using component failure rates [77] and proposed a hierarchical Bayesian model with frequency weighting method [78] toward predicting reliability in the early design, especially during the functional modeling and concept generation. Based on these achievements, they presented the Early Design Reliability Prediction Method (EDRPM) to facilitate decision making in the early design using quantitative reliability results [79]. The steps of the methodology are as follows: (1) Set the reliability goal. (2) Gather component failure rate evidence. (3) Investigate function level distributions. (4) Component elimination using function and component level graphs. (5) Determine final design alternatives.

The functional-failure identification and propagation (FFIP) framework has also been introduced by Tumer's research group for designing reliable complex systems [80, 81].The architecture of FFIP is shown in Fig. 5. The three major modules in the FFIP are the graphical system model, the behavioural simulation, and the functional-failure logic (FFL) reasoner. The FFIP graph-based modelling approach has several advantages. (1) Capture function-configuration-behaviour architecture of a system at an abstract level. (2) Facilitate the assessment of potential functional failures. And (3) generate fault propagation paths through the FFL reasoner, which translates the dynamics of the system into functional failure identifiers.

Both the FFDM and FFIP methods help deal with reliability in the conceptual design. The focus of the latter method is slightly different, and it is used to estimate potential faults in a qualitative way. The combination of function, structure, and behavior modeling is used to estimate potential faults and their propagation paths at a highly abstract system concept level before any potentially high-cost design commitments are made. Flow State Logic (FSL) method was also proposed to consider energy, material, and signal (EMS) flow failure propagation in addition to the original failure analysis in FFIP [82]. FFIP can be applied as a reliability-based design tool in the application of the development of a prognostic and health management (PHM) system in the early design stage [83]. A related design stage failure identification framework has also been developed based the function-based failure analysis and dimensional analysis. The framework allows for more detailed behavioral models derived from information available at the configuration level [84]. The FFIP method also has been improved by its extension to continuous flow levels from former discrete ones [85].

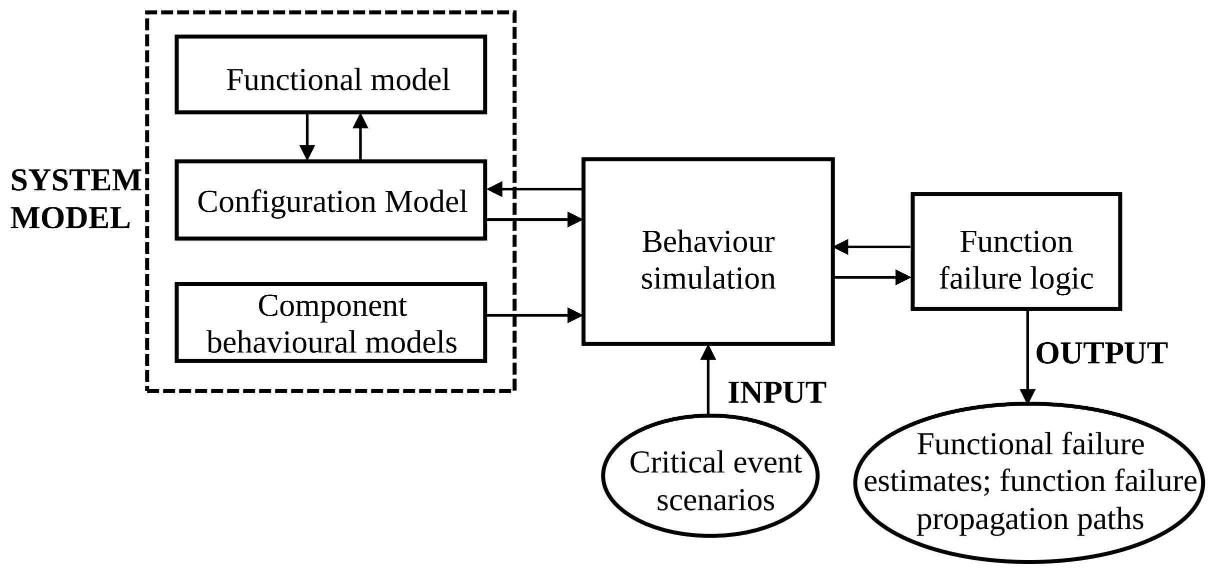

Figure 5 Architecture of the FFIP framework [80]

In sum, function modelling methodologies are based on function-flow format. They provide a functional basis with a more consistent classification scheme and enable the reliability/failure/risk analysis in the early design stage mainly in a qualitative way after the product functions are determined.

2.7 PHYSICS-BASED RELIABILITY METHODOLOGIES

Many of the methodologies reviewed above are statistics based; namely, they depend on statistical data of failure times either from testing or from field. On the contrary, physics-based reliability methodologies [86, 87] predict reliability based on computational models derived from physics. They are widely used in the detail design stage where computational models (called limit-state functions [88, 89]) are available for checking the state of a component or a system. For example, if a limit-state function is defined as the difference between maximum stress and material strength, then a positive limit-state function indicates a failure because the stress is greater than the strength.

Given the distributions of the input variables, the reliability, which is the probability that the limit-state function is negative, can be computed either analytically or numerically. Due to the lack of computational models during the early design stage, physical-based methods are rarely used. Recently, however, there was an attempt [90][90] to extend one of the physical-based reliability strategies, the stress and strength interference theory, to the reliability analysis in conceptual design. The method is called the conceptual stress and conceptual strength interference theory (CSCSIT) [90].

According to the stress and strength interference theory, reliability *R* is calculated by

$$R = \Pr(\text{strength} > \text{stress}) \tag{5}$$

where the distribution of the stress can be estimated from the computational model of the stress with respect to input variables (such as those of dimensions and loadings) whose distributions are available. The CSCSIT method extends the above traditional theory into conceptual stress and conceptual strength interference theory that parametrizes the conceptual design space by introducing reliability related parameters into functional design. Based on CSCSIT, a practical analysis framework is proposed to support functional design for reliability. A conceptual stress *Cste* is assumed to be a linear combination of the EMS (energy-material-signal) parameters, and a conceptual strength *Cstn* is defined as the product of a conceptual safety factor and a percentile value of the conceptual stress. Then the reliability of the *i*-th sub-function is computed by Eq. (6).

$$R_i = \Pr(Cstn_i > Cste_i) \tag{6}$$

And the reliability of the product is then given by

$$R = \Pr(Cstn_1 > Cste_1 \bigcap Cstn_2 > Cste_2 \bigcap \cdots) \tag{7}$$

With the given function model of a design concept, its associated EMS flows, the distributions of the EMS parameters, and safety factors, the method estimates the reliability using Eq. (7). As a result, the reliability of a design concept can be predicted. Furthermore, all the design concepts can be compared with their estimated reliability.

3. OTHER METHODOLOGIES

In this section, we give several examples about other methodologies that could be used for conceptual design. One example is the sensitivity analysis, which explores how sensitive the product reliability would be with respect to specific data sources, expert opinions, failure data of a particular component, and so on. Knowing the sensitivity of information sources, one is able to identify the most important information sources. Then resources can be optimally allocated to collect more information from the important sources. There are multiple methods available for sensitivity analysis, such as the local derivative, normalized derivative, Monte Carlo regression, variance-based, and simplified model fit methods. To reconcile various approaches to performing a sensitivity analysis in conceptual design, Hutcheson and McAdams [91] presented a local sensitivity analysis used for screening a large number of concepts during conceptual design and a global sensitivity analysis performed during the later stages of design.

The other example is the uncertainty quantification in the early design stage. This is a broader topic. Not only probabilistic representations of uncertainty can be used, but non-probabilistic representations of uncertainty can also be used, especially for sparse information. The concept of the multi-stage uncertainty quantification method [92], which was originally developed for model validation, for example, could be modified for uncertainty quantification in conceptual design. Hutcheson et al. [93] proposed a function-based method for addressing uncertainty of engineering systems in the early design stage. By performing function-based sensitivity analysis from previous designs and storing the results, significant knowledge about the sensitivity to design variable uncertainty can be retained and reused. In order to reduce uncertainty due to the lack of knowledge during the design process, Barrientos et al. [94] developed a methodology to model design evolution in concurrent design teams and to help reduce the effects of uncertainty and risk.

The last example is risk related methodologies. Since risk is the product of cost and the probability of failure, and the probability of failure is complementary to reliability, a risk assessment with less mature data during early design phase is needed. The current state of the art in quantitative risk analysis is probabilistic risk analysis [95]. The FFDM method discussed previously can also be used for risk analysis [71, 73]. Based on functional failure data, Mehr and Tumer [96] presented a risk management method named RUBIC-Design. The RUBIC-Design is a numerical and real-time method, which is capable for recognizing the major risk factors and their propagation during the early phases of concurrent and distributed engineering design. The risk in early design (RED) method [97, 98] was proposed based on functions rather than physical components in order to perform risk assessments in the conceptual design phase of a product. RED produces specific detailed preliminary risk assessments based on catalogued historical failure data. A functional failure reasoning methodology [99] was proposed for risk analysis based on the analysis of functional failures and their impact on the overall system functionality during the early design stage. An integrated multi-domain functional failure and propagation analysis approach [100] was presented to help designers understand the interplay between components and thus evaluate the design in an integrated manner in the early design stage.

4. METHODOLOGIES SUMMARY

We have reviewed a number of reliability methodologies for conceptual design. Even though the review is not exhaustive, it indeed demonstrates how the methodologies could help predict the reliability of design concepts and how they help evaluate design concepts with respect to reliability. To better understand, compare, and select the methodologies, we provide a summary of the methodologies in this section. We then point out several important areas that are worthwhile to devote efforts in the future research.

The methodologies reviewed above are quite different in many aspects. To better understand them, and more importantly, to select appropriate methodologies for specific applications, we summarize the methodologies by tabulating them in a consistent way. The summary is given in Table 2 in the appendix. Each row of the table represents one methodology, and its columns include the objective, the input required, the output produced, the assumptions, tools and scope of the methodology, and the nature (quantitative or qualitative) of the methodology. For the ease of presentation, we treat each methodology as a black box as shown in Fig. 6, and it therefore has its input (the information needed by the methodology) and output (the outcome of the methodology). The input may include the following items: product overall function and sub-functions, function flows, system configurations, historical data, expert opinions, distributions of relevant data, and others. The outcome may include the following items: product reliability, relative product reliability, reliability indexes, risk of potential failure modes, and others. Going inside the black box, we can also find the details about each of the methodologies, such as assumptions, major approaches or tools, and the nature of the methodology.

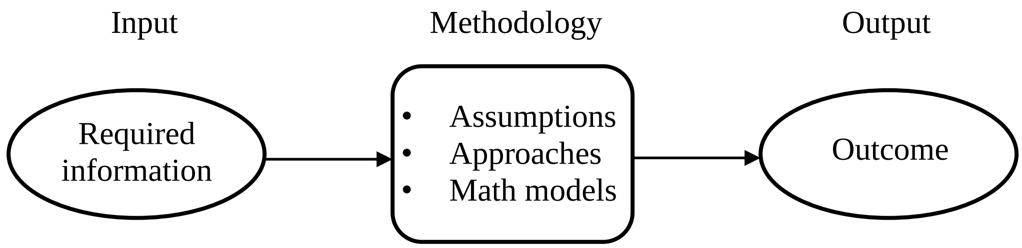

Figure 6 Black box of methodology

5. FUTURE RESEARCH DIRECTIONS

As discussed previously, quantifying reliability is a challenging task in conceptual design; the methodologies are not as mature as those for detail design, such as structural reliability analysis and reliability-based design optimization (RBDO). Even though the challenges are formidable, they undoubtedly provide great opportunities of exploring new ways to deal with reliability in conceptual design. Several thoughts of the future reliability research in this area are discussed in the following subsections.

5.1 NEW SYSTEM RELIABILITY PREDICTION METHODS FOR DEPENDENT COMPONENTS

Predicting the reliability of a new product during conceptual design is essentially performing a system reliability analysis because the product is typically consists of a number of components. Knowing all the component reliabilities is not sufficient to predict the system reliability since the states of the components may be statistically dependent. For example, the transmission system consists of 24 major mechanical components, whose states are dependent because of shared stochastic loading and operation environment. Even though the component suppliers could provide the individual component reliabilities, the designers of the system could not accurately estimate the system reliability unless component failures are assumed independent. Such an assumption is used in many current system reliability methodologies.

The independent assumption may product large errors. For the above transmission system, which is series system, if all the components had a relatively high reliability of $R = 0.9999$, then the system reliability would be $R_S = 0.9999^{24} = 0.9976$. This low product reliability could make the system designers eliminate the design concept. In reality, such a transmission system is commonly used, and its reliability is much higher than the calculated reliability. This example indicates that the independent component assumption may lead to erroneous decisions in design concept selection.

One solution is using system reliability bounds. The well-known equation for a series system is

$$R_1 R_2 R_3 \cdots \leq R_S \leq \min\{R_1, R_2, R_3, \cdots\} \tag{8}$$

where R_i is the reliability of the *i*-th component, and R_S is the system reliability. For the above transmission system, the system reliability bounds are given by $0.9976 \leq R_S \leq 0.9999$, which covers the true system reliability. But the interval is too wide and may make decision making difficult for concept selection.

New system reliability methodologies are therefore needed for conceptual design. There are several potential ways to address this problem. First, the width of the system reliability bounds in Eq. (8) could be narrowed. Reducing the width of system reliability bounds requires some information about dependence between components states. New methodologies should accommodate all the information available to the designers of the product, such as the stochastic load acting on the new product, strength-stress interference of a component, and the range of a possible factor of safety that is commonly used in the design of a component. The use of such information will promote the consideration of dependent components. Then optimization could be used to search for the maximum and minimum system reliabilities, and the two extreme values should form a narrower system reliability bound given that more information is used.

The other possible way is to obtain a more accurate point estimate of system reliability, instead of a bound. Doing so requires knowing details of component design from suppliers, and some information may be proprietary and may therefore not be available to the designers of the new product. New methodologies for component and system reliability analyses should be developed for both component designers and system designers. From the perspective from component designers, rather than providing single-valued component reliability to the designers of a new product, more information could be supplied. The additional information should be adequate so that the designers of the new product could use it to accurately predict the system reliability; and the proprietary details of the component, such as key parameters of the component, material properties, and manufacturing tolerances, should also be protected from being revealed. On the other hand, from the perspective of the designers of the new product, they could use the additional component reliability information to rebuild the limit-state functions of all the components without knowing all the details of the component designs. As a result, the dependence of component states could be considered, and thus an accurate system reliability prediction can be obtained.

5.2 THE USE OF PHYSICS-BASED RELIABILITY METHODOLOGIES

As discussed in Sec. 2.7, physics-based (structural) reliability methodologies have been rarely applied in conceptual design, but they are widely used in the parameter or detail design stage where computational models are available. The computational models, also called limit-state functions, are derived from physics theories and can be readily used to predict the working or failure states of components and systems, thereby the component reliability and system reliability. For the *i*-th component or failure mode of a product, the limit-state function is defined by

$$Y_i = g_i(\mathbf{X}) \tag{9}$$

where $\mathbf{X}$ is a vector of random variables, whose joint distribution function is known, Y is a response (state) variable, and $g_i(\cdot)$ could be an explicit function or a black-box model. If $Y_i = g(\mathbf{X}_i) > 0$ indicates that a failure will not occur, then the reliability is given by

$$R_i = \Pr\{g_i(\mathbf{X}) > 0\} \tag{10}$$

R can then be evaluated by structural reliability analysis.

During the conceptual design stage, more and more simulations are used, especially for design concept evaluation. It is therefore highly desirable to develop methodologies that could integrate the physics-based reliability methodologies and traditional conceptual design methodologies in the following aspects.

(1) Integrate component reliabilities estimated by physics-based methodologies as shown in Eq. (10) and reliabilities estimated by reliability engineering methodologies such as those based on field data, statistics, or experiments.

(2) Assess the dependencies of all the components in a product system for the system reliability, which is determined by component reliabilities estimated by statistical methods and those estimated by physics-based methods. This relies on the methodologies discussed in Sec. 5.1. Use the factor of safety in conceptual design. The factor of safety is the ratio of resistance to load and is required to be greater than 1. It tells how much stronger a component or system than it usually needs to be for intended loads. As discussed in Sec. 2.7, the factor of safety is used in the stress and strength interference theory [90], which can be further developed with more involvement of physics models. One potential way is to identify the equivalence between reliability and the factor of safety [101] during the conceptual design. Once a factor of safety is determined by the

designers, the reliability will also be known. This will greatly enhance the reliability-based conceptual design because engineers are more familiar with the concept of the factor of safety.

(3) Employ multidisciplinary design methodologies. The product design usually involves multidisciplinary teams, such as those responsible for mechanical, electrical, material, and operational aspects of the design. Multidisciplinary design optimization (MDO) [102] is a system design methodology that can effectively handle the coupling between multidisciplinary teams and components. MDO has been successfully applied in detail design stages, especially in the aircraft design. Its use in conceptual design has also been reported [103, 104]. Reliability capability [105-109] has also been added to the MDO in the detail design stage. If the methodologies are extended to the conceptual design stage, the reliability of complex engineering systems could be greatly enhanced. The research in this area will rely on not only what has been discussed in Secs. 5.1 and 5.2, but also the following areas: efficient analysis for uncertainty propagation from one discipline to other disciplines, management of coupling state variables, and so on.

5.3 APPLICATIONS IN MECHANICAL ENGINEERING

Many of the reliability methodologies assume constant failure rates or exponential distributions for component lifetimes. Constant failure rates are commonly seen in electronic components, but they may not be applicable for most mechanical components. Although constant failure rates make computations easy, assuming a constant failure rate may result in a high risk as indicated in [110]. More tractable computational methods are required for dealing with non-constant failure rates and general distributions, such as normal, lognormal, Weibull, and extreme value distributions. Dealing with truncated random variables is also worthwhile to devote further research efforts since they are commonly encountered in mechanical engineering applications. For example, random dimensions and clearances of mechanisms are truncated because they vary within their tolerance limits [111]. For a wind turbine or hydrokinetic turbine, when the wind or river velocity reaches a cut-out velocity, the system will be shut down for a safety reason. The velocity is therefore a truncated random variable [112].

5.4 ACCOMMODATION OF TIME- AND SPACE- DEPENDENT UNCERTAINTY

As discussed in Sec. 5.2, it is desirable to integrate statistics-based reliability methodologies and physics-based reliability methodologies. The former methodologies usually handle time-related information, such as the time to failure. They can predict reliability for a given period of time; in other words, the result is the time-dependent reliability. The majority of the latter methodologies, however, are only based time-independent limit-state functions as indicated in Eq. (10), and the predicted reliability does not change over time. In reality, many limit-state functions are functions of time; for example, the motion errors of a robot are different at different time instants. In addition, some of the input variables of limit-state functions are time-dependent stochastic processes, such as the river velocity considered in the hydrokinetic turbine design and ocean wave loads on marine structures. With the consideration of the time factor, the reliability for a period of time $[0, T]$ is then defined by

$$R = \Pr\{g(\mathbf{X}(t), t) > 0, \text{ for all } t \in [0, T]\} \tag{11}$$

The input variables $\mathbf{X}(t)$ are stochastic processes, and they usually nonstationary in mechanical engineering applications.

Time-dependent physics-based reliability has recently received increasing attention, and some of relative methodologies [113-122] could be used in conceptual design. One of the research tasks is to estimate product reliability for a period of $[0, T]$ given component reliabilities from statistics-based and physics-based methodologies. For example, if the reliability of Component 1 is estimated by a statistics-based approach with $R_1 = \Pr\{t > T\}$ and that of Component 2 is estimated by a physics-based approach with $R_2 = \Pr\{g(\mathbf{X}(t), t) > 0, \text{ for all } t \in [0, T]\}$, what is the reliability of the product which consists of the two components? Since the sates of the two components are usually statistically dependent, this research task will rely on what has been discussed in Sec. 5.1.

5.5 INFORMATION AGGREGATION

As reviewed in Sec. 2.3, Bayesian approaches can handle various information, more methodologies are desired to integrate data with different structures and from different sources, such as the following:

• Full distributions: Sufficient information is available for many standard mechanical components, such as gears and shafts, about their manufacturing impressions, strengths, and usage cycles, and so on. The associated complete probability distributions are therefore available.

• Distributions with uncertain parameters: The distribution types of some variables are known, but the distribution parameters, are imprecise due to limited knowledge. For example, it is well-known that the fatigue life of some structures follows a lognormal distribution or Birnbaum–Saunders distribution. But the parameters that define the distribution are uncertain.

• Interval variables: A parameter may be estimated between lower and upper bounds. The tolerances of the dimensions of a mechanical component are specified as intervals. Expert opinions are also sometimes expressed in the form of intervals.

• Multilevel data: We may have partial information at component, subsystem, and system levels. This situation can also occur for a single component when its life is tested by several testing approaches, such as fail/pass, censored, and aging.

• Time-dependent parameters: Uncertainties may change over time. For example, the random strength of a component degrades over time, and loadings vary randomly over time.

The other related issue is how to integrate probabilistic information (distributions) and non-probabilistic (intervals). Extensive research has been conducted in statistics about how to deal with interval samples [123]. The statistical approaches to interval data could be introduced into the reliability quantification with other types of data.

5.6 DECISION MAKING UNDER VARIOUS UNCERTAINTIES

During conceptual design, there are many decisions to be made, such as how to determine solutions to realize product functions and sub-functions, how to combine such solutions, and how to select the best design concepts. Reliability is one of the most important considerations in decision making. The research questions to be answered include the following.

(1) How to incorporate reliability requirements in design concept evaluation? The concept evaluation assesses relative strengths and weaknesses of design concepts with respect to customer needs and engineering criteria. There are many concept selection methodologies, such as the decisions matrix [124] and Pugh's method [125]. Reliability

can be incorporated in the evaluation criteria. The criteria may include the initial reliability, the time-dependent reliability (the reliability over a period of time), the possible reliability range (lower and upper bounds), and the associated cost to achieve the expected reliability.

(2) How to compare and select design concepts with the lack of quantitative reliability estimations? As discussed in the previous subsection, different types of uncertainty may result in the predicted reliability in different formats, such as point estimates and interval estimates. Reliable methodologies are needed to assist decision making on selecting design concepts with the interval reliability estimates.

(3) How to optimally allocate limited resources to ensure accurate reliability assessment? With limited information for reliability assessment in the early design stage, the resolution of reliability prediction may not be good enough for decision making. For example, if the predicted reliability bounds of two design concepts are too wide, we may not be able to distinguish one from the other in term of their reliabilities. In this case, more information should be collected, and this poses a question of how to effectively allocate resources for collecting more information. Sensitivity analysis is therefore required, and it allows designers to understand how sensitive the product reliability prediction will be with respect to specific data resources, expert opinions, failure data of particular components, and so on. By evaluating the sensitivity indexes of these input sources, designers will be are able to identify the most important input sources. When more data are required, then designers can collect additional information from the identified important input sources.

6. CONCLUSIONS

Conceptual design is the most crucial stage in product design. Considering reliability in this design stage has a much greater impact on product performance and quality than doing so in latter design stages. It can not only help generate design concepts with high intrinsic reliability but also help evaluate and select the best design concepts with respect to reliability.

As indicated by this study, the current reliability methodologies for conceptual design are much less mature than their counterpart in detailed parameter design stage; the major obstacle is the lack of information in the early design stage. It is the reason that there exist a variety of reliability methodologies with different capabilities, application scopes, and required information. This study shows that considering reliability upfront in the conceptual design is feasible.

The majority of reliability methodologies provide qualitative results, and they are used mainly for failure mode and cause identification, failure effect analysis, risk assessment, and action decisions for eliminating failures or reducing their likelihood of occurrences. Methodologies originated reliability engineering including Failure Mode and Effects Analysis, Fault Tree Analysis, and Event Tree Analysis. Many improvements have been made for these methodologies so that they could provide quantitative or semi-quantitative results. The methodologies developed in the area of engineering design focus on introducing reliability into the functional modeling where potential reliability issues are addressed for solutions that realize sub-functions and the overall product function.

There are also many quantitative reliability methodologies, which can be used to estimate the reliability of a component or system and can therefore provide useful information for decision making on design conception selection. Since sufficient information may not be available, these methodologies employ some assumptions in order to produce quantitative results. Typical assumptions are as follows: The components of a new product have independent states (the failure of one component will not affect those of other components), prior distributions are available when a Bayesian approach is use, and the component or system life follows a specific distribution, such as an exponential distribution with a constant failure rate.

The challenges of considering reliability in conceptual design also provide great opportunities for future research in this area. The key topic is to accurately predict product reliability in the early design stage to better assist decision making. This requires new methodologies in aggregating reliability information with multilevel and multi-format uncertainties, dealing with dependent components, integrating statistics-based and physics-based approaches, better modeling the reliability of mechanical components, and performing uncertainty propagation for a multidisciplinary system.

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APPENDIX

Table 1 Summary of reliability methodologies for conceptual design

Methodology	Objective	Input	Output
Scenario-based FMEA [31]	• Evaluate risk events	• Cost of failure • Probability of failure • Immediate effect • Next-level effect • End effect	• Expected cost and risk of each failure
	Assumptions/tools/scope		**Nature**
	• Known cost of failure • Known relationship among events • Total probability theory is used		Quantitative
Fault Tree Analysis [41]	**Objective**	**Input**	**Output**
	• Show compliance with reliability requirements • Guide the resource redeployment	• Already-identified undesirable events • Probabilities of basic events	• Combinations of events that cause system failure • Probability of system failure • Minimal cut sets • Importance rankings of contributors to system failure
	Assumptions/tools/scope		**Nature**
	• Graphical representation • Boolean operations • Independent basic events • Binary states of events		Qualitative/ quantitative
Event Tree Analysis [126]	**Objective**	**Input**	**Output**
	• Define potential accident sequences • Enable probability assessment of success/failure	• Initial undesired events • Accident consequences • System success criteria	• Combinations of events that cause system failure • Probability of system success/failur e
	Assumptions/tools/scope		**Nature**
	• Anticipated operating pathways • Independent basic events • Binary states of events		Qualitative/ quantitative

Table 1 Summary of reliability methodologies for conceptual design (cont.)

Methodology	**Objective**	**Input**	**Output**
Root Cause Analysis [15]	• Identify what, how and why events happened • Prevent recurrence	• Initial undesired events • Causal factors	• Root cause map • Future recommenda-tions
	Assumptions/tools/scope		**Nature**
	• Collected data about the event is complete • Causal factor knowledge is accurate		Qualitative
Reliability Block Diagram [127]	**Objective**	**Input**	**Output**
	• Analyze system reliability	• Component structure • Component reliabilities	• System reliability
	Assumptions/tools/scope		**Nature**
	• Known component reliability • Relationship between system and components		Quantitative
ISFA [100] (Integrated System Failure Analysis)	**Objective**	**Input**	**Output**
	• Identify failure propagation paths in the early design stage	• System configuration • Input-output flows of components	• Failure propagation paths • Failure impact on the system
	Assumptions/tools/scope		**Nature**
	• Behavioral simulation • Event sequence diagram • Advanced modeling languages		Qualitative
Use of heritage and other relevant data [53]	**Objective**	**Input**	**Output**
	• Predict reliability of a new product	• Warranty data, field data, and test data of relevant existing products • Design changes of the new product	• Reliability bounds w.r.t. time of the new product
	Assumptions/tools/scope		**Nature**
	• Repairable product • As-bad-as-old repair • Constant failure rate • Power Law process for the intensity function • Bayesian method		Quantitative
Parenting process [65]	**Objective**	**Input**	**Output**
	• Predict reliability of a new product	• Parent products • Warranty data of parent products • Failure mode and failure cause relationship of parent products	• Probability of failure of new product

Table 1 Summary of reliability methodologies for conceptual design (cont.)

	Assumptions/tools/scope		**Nature**
	• Expert estimates of the change in failure rates • Lognormal distribution of the change and the failure rate of parent products • No change in failure mode and failure cause relationship in the new product		Quantitative
Early Design Reliability Prediction [78, 79]	**Objective**	**Input**	**Output**
	• Assist decision making in functional modeling stages • Select concepts by required system reliability	• Functional modeling • Solutions to sub-functions • Component failure rates	• Design alternatives that meet reliability target
	Assumptions/tools/scope		**Nature**
	• Time-independent and normally distributed failure rates • Known standard deviations at component level • Hierarchical Bayesian model		Quantitative
CSCSIT [90] (Conceptual Stress and Conceptual Strength Interference Theory)	**Objective**	**Input**	**Output**
	• Evaluate reliability in the early stage • Identify the weak spots of the function structure • Analyze the sensitivity of reliability	• Function structures with energy, material and signal (EMS) flow paths • Probability distribution of EMS parameters • Similar functions from existing designs	• Conceptual stress • Conceptual strength • System reliability
	Assumptions/tools/scope		**Nature**
	• Linear limit-state function • Known EMS parameters of their distributions • Known parameters such as factor of safety		Quantitative
Fuzzy Reliability method [5]	**Objective**	**Input**	**Output**
	• Quantify imprecision and uncertainty in early reliability and risk analysis	• Failure rate for all components • Subsystem reliability	• System reliability • Average cost of system operation
	Assumptions/tools/scope		**Nature**
	• One mission of finite duration • Independent subsystems connected in series. • Perfect failure detection and switching among redundant components • Binary status of all components, subsystems, and system • Exponential life-distributions for all components		Quantitative

Table 1 Summary of reliability methodologies for conceptual design (cont.)

Methodology	**Objective**	**Input**	**Output**
Reliability Prediction Models [6]	• Predict system reliability • Predict average mission cost	• Simulation modeling inputs • System-level inputs • Subsystem-level inputs	• Mission reliability • Average cost of system operation
	Assumptions/tools/scope		**Nature**
	• Triangular distribution of the failure rates for the components in a subsystem • Simulation-optimization tool		Quantitative
FFDM [73] (Function Failure Design Method)	**Objective**	**Input**	**Output**
	• Predict likely failure modes • Improve product designs in the early stage	• Failure knowledge from previous products • Product functionality • Concept generator	• Product design concepts • Design recommenda-tions
	Assumptions/tools/scope		**Nature**
	• Knowledge from previous accident study • Overall function of each black-box • Filter matrix		Qualitative
FFRDM [76, 77] (Functional Failure Rate Design Method)	**Objective**	**Input**	**Output**
	• Mitigate failure modes • Predict system reliability analysis in the functional design stage	• Repository Data from two data sources: NPRD-95 and FMD-97	• Design recommenda-tions • Reduced likelihood of failure
	Assumptions/tools/scope		**Nature**
	• Function-flow fails in a specific failure mode • Knowledge from previous failure modes study		Quantitative /qualitative
FFIP [81, 82] (Functional Failure Identification and Propagation)	**Objective**	**Input**	**Output**
	• Evaluate and assess the potential of system functional failures	• Critical event scenarios • Documented historical data	• Functional failure • Functional failure propagation paths
	Assumptions/tools/scope		**Nature**
	• Reliable and complete functional basis • Function-failure logic reasoner that allows reasoning at a functional level • Combine hierarchical system models with behavioral simulation and qualitative reasoning		Qualitative

Table 1 Summary of reliability methodologies for conceptual design (cont.)

Methodology	**Objective**	**Input**	**Output**
Bayesian reliability model using multiple sources of information [49]	• Estimate system reliability	• Component lifetime data • System lifetime data • Expert data regarding the current product and similar products	• System lifetime distribution • System reliability
	Assumptions/tools/scope		**Nature**
	• Independent component lifetimes • Known system-component structure (series, parallel, and mixture)		Quantitative
Hierarchical model [47]	**Objective**	**Input**	**Output**
	• Estimate reliability of complex systems	• System available data • Prior judgment • Engineering experience	• System reliability
	Assumptions/tools/scope		**Nature**
	• Well-understood relationship between system and its components • Specific distribution of probability density function based on prior known parameter		Quantitative
Relative reliability risk assessment [66]	**Objective**	**Input**	**Output**
	• Predict reliability in the early stage	• Function structure	• Relative reliability risk index • Concept functionality graph
	Assumptions/tools/scope		**Nature**
	• Reasonable relative rating • Reasonable distributed weights for the functions		Quantitative

II. SYSTEM RELIABILITY ANALYSIS WITH DEPENDENT COMPONENT FAILURES DURING EARLY DESIGN STAGE – A FEASIBILITY STUDY

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ABSTRACT

It is desirable to predict product reliability accurately in the early design stage, but the lack of information usually leads to the use of independent component failure assumption. This assumption makes the system reliability prediction much easier, but may produce large errors since component failures are usually dependent after the components are put into use within a mechanical system. The bounds of the system reliability can be estimated, but are usually wide. The wide reliability bounds make it difficult to make decisions in evaluating and selecting design concepts, during the early design stage. This work demonstrates the feasibility of considering dependent component failures during the early design stage with a new methodology that makes the system reliability bounds much narrower. The following situation is addressed: the reliability of each component and the distribution of its load are known, but the dependence between component failures is unknown. With a physics-based approach, an optimization model is established so that narrow bounds of the system reliability can be generated. Three examples demonstrate that it is possible to produce narrower system reliability bounds than the traditional reliability bounds, thereby better assisting decision making during the early design stage.

1. INTRODUCTION

There are four design stages in a design process, including problem definition, conceptual design, embodiment design, and detail design [1]. The early design stage includes problem definition and conceptual design. During the problem definition stage, the problem and working criteria/goals are defined, information such as voice of customer is gathered, and functional modeling is performed [2]. During the conceptual design stage, design concepts are generated, analyzed, and selected [3]. In this work, we consider reliability in the conceptual design stage. Reliability is the ability of a product to perform its intended function without failure, and it is usually quantified by the probability of such ability [4]. In the past, reliability issues were usually addressed when field failure data and/or life testing data became available. This treatment is too late because losses have already occurred. It is therefore necessary to perform reliability analysis in the early design stage. Considering reliability upfront will not only ensure high reliability, robustness, safety, and availability, but also reduce risk and product lifecycle cost [5]. Specifically, predicting system reliability helps decision making in the early design stage [6]. For example, after several design concepts are generated, the best design concept(s) should be selected. In many cases, the product reliability is a major decision factor for keeping or eliminating design concepts. Reliable decision making relies on the accurate system reliability prediction.

Although methodologies exist for early reliability prediction [7-9], predicting reliability early is still a challenging task due to various reasons. Herein, we focus on one of the most important reasons – the lack of dependence information between component failures. Nowadays it is a common practice for a product (or system) to have its components designed and manufactured from different companies (suppliers). These components are individually and independently designed, tested, and manufactured. The reliability of each component may be known to the designers of a new product. When the components are assembled into a system for operation, they are dependent, and the dependent relationship needs to be considered for obtaining the system reliability. The dependence comes from the following reasons: components operate under the same environment, they are subjected to the same load, they deform dependently due to geometric constraints, and the output of one component is the input to other components, and vice versa.

Lacking dependent component states poses a challenge for the early product design because it is difficult to define the exact dependent relationship of components due to the limited information available to the designers of the new product. Even if the designers could acquire the reliability of each component from the supplier who designed and manufactured the component, they do not have access to all the details that are necessary for the system reliability prediction, such as the material properties, geometry, and critical parameters of the component. As a result, the joint probability density of the states of all the components is not available in general.

For the above reasons, approximations to the system reliability are usually used. The commonly used reliability engineering methods are based on the assumption of independent component failures [10-12] on the condition that component reliabilities are given. The independent component state assumption makes the system reliability analysis much easier, but may produce large errors and may therefore lead to erroneous decisions for design concept evaluation and selection. Besides, Park et al. [13] demonstrated that the error due to ignoring dependence can be negligible for a highly reliable system. The conclusion is verified by various conditions. But for design concepts that may not have high reliability, considering component dependence is still necessary for concept evaluation and selection with respect to reliability.

Efforts have been made to improve the accuracy of system reliability by considering component dependence. Humphreys and Jenkins [14] reviewed and summarized the development of techniques of dealing with dependent component failures before 1991. Zhang and Horigome [15] proposed a method to predict system reliability by considering both dependent component failures and time-varying failure rates under several assumptions about system states and time-varying failure and repair rates. This study is suitable for system and component failures due to a cumulative shock-damage process. Pozsgai and Neher [16] summarized approaches to the reliability of mechanical systems with the dependence consideration, such as common-mode failures, load-sharing, and functional dependence. Neil et al. [17] developed hybrid Bayesian Networks (BNs) to model dependable systems with a new iterative algorithm, which combines dynamic discretization with propagation algorithms to realize inference in hybrid BNs. This model uses several assumptions; for example, the repair time is negligible. Marriott and Bate [18] considered dependent failures of nuclear submarines. Their method is based on the unified partial model (UPM), which provides a way to assess the effects of dependent failures on a system in an auditable manner. The method, however, may not be applicable for early designs due to the limited information available for the input of the UPM model. Recently, Youn et al. [19], Nguyen et al. [20], and Wang et al. [21] presented system reliability analysis models for problems where all the component parameters are known.

In summary, it may not be easy to apply these methodologies in the early design stage because of limited information about component dependence.

The alternative way is to estimate the bounds of the system reliability. For instance, for a series system, with the inclusion-exclusion principle [22], the system reliability analysis involves the joint probabilities associated with the components of the system. When the component states are dependent, it is difficult to calculate the probabilities of the intersections for a large number of components; thus system reliability bounds $\left[R_S^{\min}, R_S^{\max}\right]$ are of interest, where $R_S^{\min}$ and $R_S^{\max}$ are the minimum and maximum system reliabilities, respectively. The analysis may require the marginal component probabilities, $\Pr(C_i)$ for component C_i, and the joint probabilities of small sets of components, for example, bicomponent probabilities $\Pr(C_i C_j)$ for components *i* and *j*; tricomponent probabilities $\Pr(C_i C_j C_k)$ for components *i*, *j*, and *k*; and so on. Even the bicomponent joint probability $\Pr(C_i C_j)$, however, still needs knowing the joint probability of C_i and C_j. Without using joint probabilities, Boole [23] derived an inequality equation to calculate the system probability bounds for series systems with only the unicomponent probabilities $\Pr(C_i)$, namely, component reliabilities. The bounds produced, however, may be too wide for practical use, as will be discussed in the next section.

In the area of structural reliability which is based on computational models derived from physics principles, narrower system reliability bounds could be produced because joint probabilities are computationally available [24]. The first-order approximation method for system reliability analysis proposed by Hohenbichler and

Rackwitz [25] produces narrow system reliability bounds. The method is efficient, but cannot be used in conceptual design because it requires all detailed information about components, such as component limit-state functions, which may not be available during conceptual design. Kounias [26], Hunter [27], and Ditlevsen [28] also developed methodologies for series systems with both unicomponent probabilities $\Pr(C_i)$ and bicomponent probabilities $\Pr(C_i C_j)$. Zhang [29] generalized the methodologies with high order joint probabilities, such as tricomponent and quadricomponent probabilities. These methods still have some drawbacks. The system reliability bounds have the order-dependency problem, meaning that different orders of components may result in different system reliability bounds. The computational demand is also intensive since all the possible ordering alternatives need to be considered. Song and Kiureghian [30] later used linear programming (LP) to address some of these drawbacks. The LP method has no restrictions on component ordering and can incorporate incomplete component probabilities and inequality constraints on component probabilities. Its efficiency deteriorates as the dimension of the problem increases because the size of the problem expands exponentially with respect to the number of components. Ramachandran [31] reviewed and summarized progresses made on structural reliability bounds before 2004. Recently, Domyancic and Millwater [32] summarized and compared different computational methods such as first order bounds, Ditlevsen bounds, KAT lower bound, and LP bounds and demonstrated the applications in series systems. However, as the computational models may not be available during the early design stage, these methods could hardly be applied for the system reliability analysis of a new product.

The purpose of this work is to explore possible ways to accurately and efficiently produce narrow system reliability bounds during the early design stage using a physics-based method with limited information. We demonstrate the feasibility for the following situation: component reliabilities are provided to the designers of a new product from individual suppliers, and the system designers know the load, to which the new product is subjected. We also assume that a component has only one major failure mode that is related to the strength of the component. With a physics-based approach, we establish an optimization model to produce narrower bounds of the product (system) reliability, which will better assist the decision making process in the early design stage.

We review the methodologies of system reliability modeling in Section 2. We then present the proposed system reliability analysis in Section 3, followed by three examples in Section 4. More discussions on the uncertainty in input variables are provided in Section 5. Conclusions and future work are given in Section 6.

2. REVIEW OF SYSTEM RELIABILITY MODELING

There are three typical types of systems, including series systems, parallel systems, and mixed systems. Herein we focus on series systems. The proposed methodology in this work can be extended to the other two types of systems.

A series system consists of components in series as shown in Fig. 1. The failure of one component can result in the failure of the entire system. This type of system is also referred to as a weakest link system.

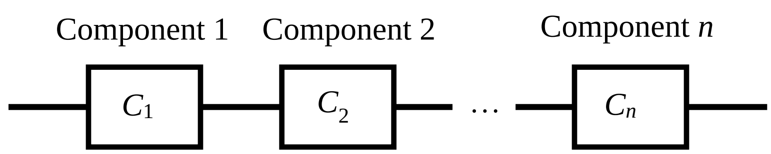

Figure 1 Series system

We denote the components by $C_1, C_2, ..., C_n$. Correspondingly, their reliabilities are denoted by $R_1, R_2, ..., R_n$. If the states of the components are assumed to be independent, the system reliability is

$$R_S = \prod_{i=1}^{n} R_i \tag{1}$$

The direct use of the above method with the independent component assumption may not be applicable to many mechanical systems. For example, the speed reducer system shown in Fig. 2 consists of one motor, one belt, one drum, two couplings, three shafts, four gears, four keys, and eight bearings, with a total of 24 components. For a simple demonstration, assume the reliability of each component is $R = 0.9999$ or the probability of failure is $p_f = 10^{-4}$, then the system reliability is $R_S = 0.9999^{24} = 0.9976$ according to Eq. (1), or the probability of system failure is $p_{f,S} = 1 - R_S = 2.4 \times 10^{-3}$. The calculated probability of system failure is so high that the design would be rejected for any practical applications. In reality, however, given the high component reliability 0.9999, the actual system reliability of the speed reducer system should be much higher than the calculated value 0.9976. The reason is that the states of the components are dependent because all the components share the common load in this speed reducer system.

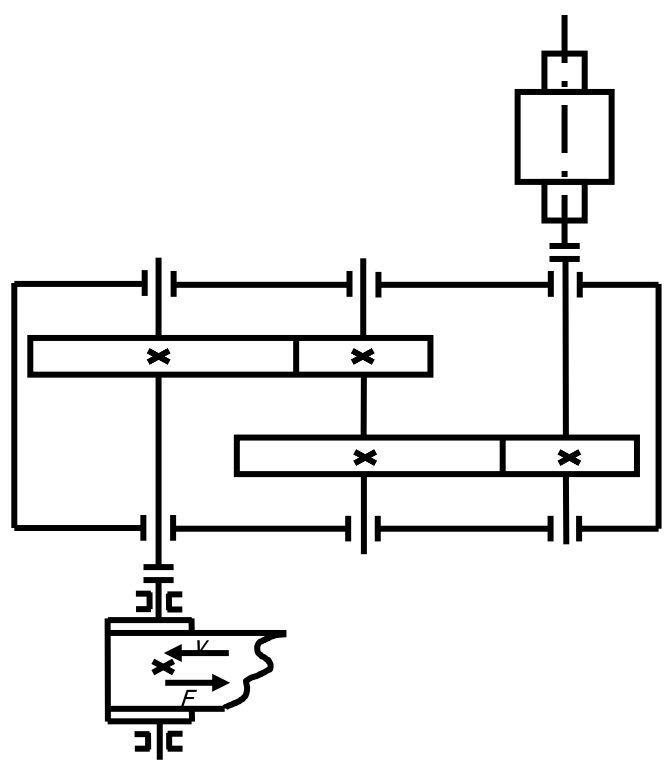

Figure 2 A speed reducer system

On the other hand, without considering the dependence, the design could be extremely conservative. For instance, if the required system reliability of the speed reducer in Fig. 2 is $R_S = 0.999$ and the reliability of each component is the same, then the required component reliability should be at least $\sqrt[24]{0.999} = 0.999958$, or the probability of failure of each component should be less than or equal to $p_f = 4.1687 \times 10^{-5}$. For the aforementioned reason of dependent components, the actual required maximum component reliability should be much lower than 0.999958, or the actual required minimum probability of component failure should be much larger than $p_f = 4.1687 \times 10^{-5}$.

Since it is difficult to obtain the system reliability without knowing the dependence between component failures, the bounds of the system reliability are usually used. The upper bound is given by [33]

$$R_S \le \min\{R_i\},\ i = 1, \dots, n \tag{2}$$

The component dependence could be positive or negative. If a failure of one component leads to an increased tendency for other components to fail, the dependence is positive, and vice versa. For most mechanical systems, the dependence is positive [34], and we therefore consider only positive dependence. For positive dependence, the lower bound of the system reliability is given by [33]

$$\prod_{i=1}^{n} R_i \le R_S,\ i = 1, \dots, n \tag{3}$$

Therefore

$$\prod_{i=1}^{n} R_i \le R_S \le \min\{R_i\},\ i = 1, \dots, n \tag{4}$$

Or the bounds of the probability of system failure are

$$\max\left\{p_{f,i}\right\} \le p_{f,S} \le 1-\prod_{i=1}^{n}(1-p_{f,i}),\ i=1,2,\ ...,n \tag{5}$$

where $p_{f,S}$ is the probability of system failure, which is equal to $1-R_S$; $p_{f,i}$ is the probability of component failure and $p_{f,i}=1-R_i$. In Eq. (4), estimating the reliability bounds requires only knowing component reliabilities, but the width or the distance between the lower and upper bounds is usually too large. Take the above speed reducer system as an example. If the component reliability is 0.9999, the system reliability bounds are then $0.9976 \le R_S \le 0.9999$, or the bounds of the probability of system failure are $10^{-4} \le p_{f,S} \le 2.4\times10^{-3}$.

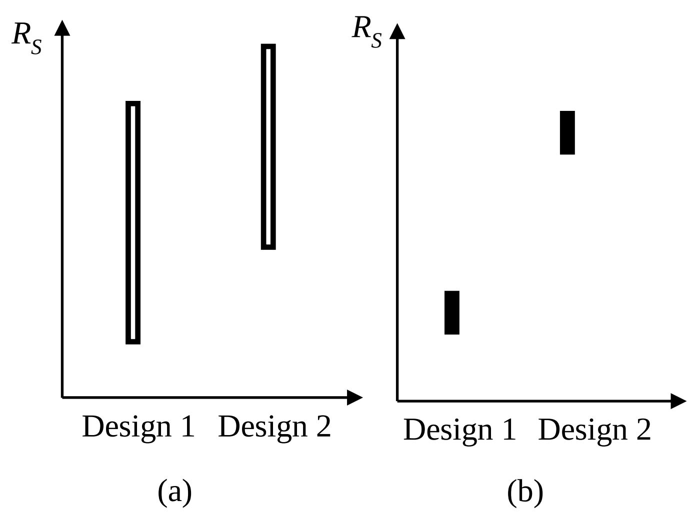

Figure 3 System reliability bounds of two designs

The wide gap between the lower and upper bounds makes decision making extremely difficult. For example, during the early design stage, if the bounds of the system reliability of two design concepts are as shown in Fig. 3 (a), designers will not be able to differentiate one design from the other with respect to reliability because the two bounds are so wide and they overlap with each other. If the bounds of the system reliability of two design concepts were narrower as shown in Fig. 3 (b), designers would easily differentiate one design from the other and will conclude that design 2 is more reliable than design 1.

To address the above problem, we propose a physics-based approach that produces narrower bounds for the system reliability.

3. SYSTEM RELIABILITY ANALYSIS WITH DEPENDENT COMPONENTS

The objective of this work is to explore a possible way to produce narrower bounds of system reliability in order to assist decision making in the early design stage. To show the feasibility, we focus on problems where the failure of a system can be predicted using the physics-based stress-strength interference model. The overview of the proposed method is discussed in the next subsection followed by details in the subsequent subsection.

3.1 OVERVIEW OF THE PROPOSED METHOD

As mentioned previously, we focus on series systems. The components of the system may be designed, manufactured, and tested independently by different companies or suppliers. The reliability analysis of the components is the responsibility of the suppliers. The reliability of each component of a new product is available to the system designers, who are responsible to predict the system reliability. The system designers may also have knowledge about the factors of safety that the suppliers may have used in their component designs. In addition to component reliabilities, the system designers may also have other information, such as the load to which the system is subjected. The system designers, however, do not have access to all the detailed information (usually proprietary) about the component designs, such as the analysis models and material properties, e.g., the distributions of the strengths of the components.

With the above information available, we develop a system reliability prediction methodology based on the stress-strength interference model. Instead of providing a single-valued system reliability, the proposed method produces system reliability bounds, which are much narrower than those from the traditional method discussed in Section 2.

The task of the proposed method is then to search for the maximum and minimum system reliabilities, and this is accomplished by establishing an optimization model for the system reliability bounds. The objective of the optimization model is the system reliability, the design variables are unknown distribution parameters of components, and the constraints are those related to component reliabilities and factors of safety of the components.

The above assumptions, along with other assumptions we use in this work, are summarized as follows:

- The new product is a series system. The reason we select series systems is that they are commonly encountered in mechanical applications, such as the speed reducer in Fig. 2. The proposed method can be extended to parallel systems and mix systems.
- Each physical component has only one major failure mode related to the strength of the component. If a physical component has multiple failure modes, to use the proposed method, one can treat each failure mode as a single component. For example, if there are two physical components, each having two failure modes, then there are four components from the viewpoint of system analysis.
- The load and strength of each component are independent. This assumption holds for many problems where material strengths do not depend on the load applied to the component.
- The system designers of the new product know the load, to which the new product is subjected. The examples of the system load include the output torque of the speed reducer in Fig. 2, the wind velocity or water velocity of a wind turbine

or hydrokinetic turbine, the force acting on the slider of a crank-slider mechanism. The system designers also know the distribution types of the strengths of the components, but the distribution parameters of the strengths are unknown.

- Component reliabilities are provided by component suppliers to the system designers of the new product.

3.2 SYSTEM RELIABILITY MODEL

We start from the models for the case with general distributions and then present the models for a special case with normal distributions.

3.2.1 General Optimization Model. In order to obtain the system reliability bound with dependent components, the designers of the new product need to ask component suppliers to provide component reliabilities. The limit-state function of the *i*-th component is defined by

$$Y_i = S_{Ste,i} - S_{Stn,i} \tag{6}$$

where $S_{Ste,i}$ is the stress in the component, $S_{Stn,i}$ is the strength of the component, and $-Y_i$ or $S_{Stn,i} - S_{Ste,i}$ is the design margin. $S_{Ste,i}$ is determined by the component load $w_i L$ or a function of $w_i L$. Substituting $S_{Ste,i}$ with $w_i L$ in Eq. (6), we could rewrite the limit-state function as

$$Y_i = w_i L - S_{R,i} \tag{7}$$

where $S_{R,i}$ is the general resistance of the component to the load. $S_{R,i}$ is in general a function of the component strength $S_{Stn,i}$ and other parameters, such as the dimension variables of the component. The information about some of the parameters may be proprietary to the component supplier. As will be discussed later, the proposed method does not require the designers of the new product to know such proprietary information.

For the system to which component i belongs, L is the total load to the system, and w_i indicates the fraction of the load that component i shares, and w_i is a constant. If the load acting on the component is equal to the load acting on the system, $w_i = 1$; if the load acting on the component is less than the load acting on the system, $w_i < 1$. w_i can be determined by the simplified free-body diagram of component i as shown in Fig. 4, where L_i is the load applied to the component. Note that Fig. 4 is only a schematic diagram, which shows how the system load is shared by components, and it is not a real free-body diagram. Also note that L_i is the resultant force acting on the component and could produce point forces, distributed forces, bending moments, and torques that exert on the component.

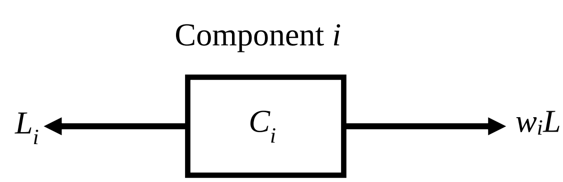

Figure 4 Simplified free-body diagram of component i

The reliability and probability of failure of component i are given by

$$R_i = \Pr\{Y_i < 0\} \tag{8}$$

and

$$p_{f,i} = \Pr\{Y_i > 0\} \tag{9}$$

We assume that the component resistance $S_{R,i}$ and the load to the system L are independent. Let the probability density functions (pdf) of the component load and resistance be $f_{L_i}(l)$ and $f_{S_{R,i}}(s)$, respectively, and let their joint pdf be $f_{L_i,S_{R,i}}(l,s)$. Then the component reliability is calculated by

$$R_i = \Pr\{Y_i < 0\} = \iint_{w_i L < S_{R,i}} f_{L_i,S_{R,i}}(l,s)dlds \tag{10}$$

Given all the component limit-state functions, the safe condition of the system is determined by the intersection $\{Y_1 < 0 \bigcap Y_2 < 0 \bigcap ... \bigcap Y_n < 0\}$, or $\{Y_1 < 0, Y_2 < 0, ..., Y_n < 0\}$. Then the system reliability is given by

$$R_S = \Pr(Y_1 < 0,\ Y_2 < 0, ...,\ Y_n < 0) = \Pr(\mathbf{Y} < 0) \tag{11}$$

where $\mathbf{Y} = (Y_1,\ Y_2,\ ...,\ Y_n)$. Using the joint pdf $f_{\mathbf{Y}}(\mathbf{y})$ of $\mathbf{Y} = (Y_1,\ Y_2,\ ...,\ Y_n)$, we have

$$R_S = \Pr(\mathbf{Y} < 0) = \int f_{\mathbf{Y}}(\mathbf{y})\,\mathrm{d}\,\mathbf{y} \tag{12}$$

If the distributions of the loads and resistances of all the components are available, $f_{\mathbf{Y}}(\mathbf{y})$ will also be available, and the system reliability can then be obtainable by Eq. (12). As discussed previously, for the system designers of the new product, however, the distribution parameters of component resistances are unknown. We denote $\mathbf{d} = (\mathbf{d}_1, \mathbf{d}_2, \ldots, \mathbf{d}_n)$ for the distribution parameters of component resistances, where $\mathbf{d}_i$ contains the distribution parameters of the resistance of component i. For example, if the resistance of component i is normally distributed, then $\mathbf{d}_i = (\mu_{S_{R,i}}, \sigma_{S_{R,i}})$, where μ and σ denote the mean and standard deviation, respectively. Some of the parameters in $\mathbf{d}$ are proprietary to the component suppliers. Without knowing the distributions of the component resistances, the designers of the new product will not be able to obtain an exact system reliability prediction. As mentioned previously, the proposed method uses all the information available to the designers of the new product to produce narrow bounds of the system reliability with the assumption that the distribution types of the component resistances are known while the distribution parameters are unknown.

The system reliability bounds are found by solving for the minimum and maximum system reliabilities through using optimization models. We now discuss such optimization models, including their design variables, objective functions, and constraint functions.

The design variables are those of unknown distribution parameters of the component resistances, denoted by $\mathbf{d}$. For example, if the component resistances follow normal distributions, the design variables will be means and standard deviations $\mathbf{d} = (\mathbf{d}_1, \mathbf{d}_2, ..., \mathbf{d}_n) = (\mu_{S_{R,1}}, \sigma_{S_{R,1}}, \mu_{S_{R,2}}, \sigma_{S_{R,2}}, ..., \mu_{S_{R,n}}, \sigma_{S_{R,n}})$.

The objective function is the system reliability given in Eq. (12). It is a function of known distribution parameters of the system load $\mathbf{p}_L$, and unknown design variables $\mathbf{d}$. The objective function is denoted by $R_S(\mathbf{d};\mathbf{p}_L)$. Maximizing $R_S(\mathbf{d};\mathbf{p}_L)$ produces the maximum system reliability $R_S^{\max}$ while minimizing $R_S(\mathbf{d};\mathbf{p}_L)$ produces the minimum system reliability $R_S^{\min}$.

There are multiple constraint functions. The reliability of a component gives an equality constraint according to Eq. (10), and there are therefore n equality constraints, as shown below.

$$h_i(\mathbf{d};\mathbf{p}_L) = \iint\limits_{w_i L < S_{R,i}} f_{L_i, S_{R,i}}(l,s)\,dlds = R_i,\ i = 1, 2, ..., n \tag{13}$$

Although the designers of the new product may not know the actual factors of safety used by component designers from the suppliers, they have good knowledge about the range of the factors of safety of the components. Denote the lower and upper bounds of the factors of safety by $n_{s,i}^{\min}$ and $n_{s,i}^{\max}$, respectively, we have the following inequality constraints.

$$n_{s,i}^{\min} \le n_{s,i}(\mathbf{d};\mathbf{p}_L) \le n_{s,i}^{\max} \tag{14}$$

There are therefore totally $2n$ inequality constraints given by

$$g_i(\mathbf{d};\mathbf{p}_L) = n_{s,i}^{\min} - n_{s,i}(\mathbf{d};\mathbf{p}_L) \le 0,\ i = 1, 2, \ldots, n \tag{15}$$

and

$$g_{i+n}(\mathbf{d};\mathbf{p}_L) = n_{s,i}(\mathbf{d};\mathbf{p}_L) - n_{s,i}^{\max} \le 0,\ i = 1, 2, \ldots, n \tag{16}$$

In addition, the designers may also have good knowledge about the coefficients of variation, which are the ratios of standard deviations to means of component resistances. Denote a coefficient of variation by c, and its lower and upper bounds by $c_i^{\min}$ and $c_i^{\max}$, respectively. From $c_i^{\min} \le c_i(\mathbf{d};\mathbf{p}_L) \le c_i^{\max}$, we have other $2n$ inequality constraints.

$$g_{i+2n}(\mathbf{d};\mathbf{p}_L) = c_i^{\min} - c_i(\mathbf{d};\mathbf{p}_L) \le 0,\ i = 1, 2, \ldots, n \tag{17}$$

and

$$g_{i+3n}(\mathbf{d};\mathbf{p}_L) = c_i(\mathbf{d};\mathbf{p}_L) - c_i^{\max} \le 0,\ i = 1, 2, \ldots, n \tag{18}$$

Next, we construct the optimization models. The optimization model for the minimum system reliability is based on the objective function as shown in Eq. (12) and the constraint functions that are listed in Eqs. (13) - (18). The optimization model for the minimum system reliability is then given by

$$\begin{cases} \min_{\mathbf{d}} \; R_S(\mathbf{d};\mathbf{p}_L) \\ \text{subject to} \\ h_i(\mathbf{d};\mathbf{p}_L) = \iint_{w_i L < S_{R,i}} f_{L_i,S_{R,i}}(l,s)\,dlds = R_i, \; i = 1,2,...,n \\ g_i(\mathbf{d};\mathbf{p}_L) = n_{s,i}^{\min} - n_{s,i}(\mathbf{d};\mathbf{p}_L) \le 0, \\ g_{i+n}(\mathbf{d};\mathbf{p}_L) = n_{s,i}(\mathbf{d};\mathbf{p}_L) - n_{s,i}^{\max} \le 0, \\ g_{i+2n}(\mathbf{d};\mathbf{p}_L) = c_i^{\min} - c_i(\mathbf{d};\mathbf{p}_L) \le 0, \\ g_{i+3n}(\mathbf{d};\mathbf{p}_L) = c_i(\mathbf{d};\mathbf{p}_L) - c_i^{\max} \le 0, \end{cases} \tag{19}$$

For the maximum system reliability, we just change the first line of the optimization model in Eq. (19) from $\min_{\mathbf{d}} R_S(\mathbf{d};\mathbf{p}_L)$ to $\max_{\mathbf{d}} R_S(\mathbf{d};\mathbf{p}_L)$. The two optimization models will produce the minimum and maximum system reliabilities, thereby the system reliability bounds.

3.2.2 Optimization Model for Normal Distributions. After having presented the general case, we now discuss a special case where all random variables are normally distributed. Suppose $S_{R,i}$ and L follow normal distributions $S_{R,i} \sim N(\mu_{S_{R,i}}, \sigma^2_{S_{R,i}})$ and $L \sim N(\mu_L, \sigma_L^2)$, respectively. From Eq. (7), the mean and standard deviation of Y_i are

$$\mu_i = w_i \mu_L - \mu_{S_{R,i}} \tag{20}$$

$$\sigma_i = \sqrt{(w_i \sigma_L)^2 + \sigma^2_{S_{R,i}}} \tag{21}$$

The reliability of component i is then calculated by

$$R_i = \Pr(Y_i < 0) = \Phi\left(-\frac{\mu_{Y_i}}{\sigma_{Y_i}}\right) = \Phi\left(-\frac{w_i\mu_L - \mu_{S_{R,i}}}{\sqrt{(w_i\sigma_L)^2 + \sigma^2_{S_{R,i}}}}\right), \; i = 1,2,...,n \tag{22}$$

where Φ is the cumulative distribution function of a standard normal variable. It can be shown that every linear combination of $(Y_1, Y_2, ..., Y_n)$ is normally distributed if the resistances $S_{R,i}$ $(i = 1, 2, \ldots, n)$ and load L are independently and normally distributed. As a result, vector $\mathbf{Y} = (Y_1,\ Y_2,\ ...,\ Y_n)$ follows a multivariate normal distribution denoted by $N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, where the mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$ are given by

$$\boldsymbol{\mu} = (\mu_1, \mu_2, ..., \mu_n) \tag{23}$$

and

$$\boldsymbol{\Sigma} = \begin{pmatrix} \mathrm{cov}_{11} & \cdots & \mathrm{cov}_{1n} \\ \vdots & \ddots & \vdots \\ \mathrm{cov}_{n1} & \cdots & \mathrm{cov}_{nn} \end{pmatrix} \tag{24}$$

where

$$\mathrm{cov}_{ij} = \begin{cases} \sigma_i^2 & i = j \\ \mathrm{cov}(Y_i, Y_j) & i \neq j \end{cases} \tag{25}$$

From Eq. (7), we can derive the covariance between Y_i and Y_j , and it is given by

$$\mathrm{cov}_{ij} = \mathrm{cov}(Y_i, Y_j) = \mathrm{cov}\left[w_i L - S_{R,i}, w_j L - S_{R,j} \right] = w_i w_j \sigma_L^2 \tag{26}$$

Thus, the covariance matrix $\boldsymbol{\Sigma}$ in Eq. (24) is rewritten as

$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_{Y_1}^2 & \cdots & w_1 w_n \sigma_L^2 \\ \vdots & \ddots & \vdots \\ w_n w_1 \sigma_L^2 & \cdots & \sigma_{Y_n}^2 \end{pmatrix} \tag{27}$$

After $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ are obtained, the pdf of $\mathbf{Y}$ is fully defined by

$$f(y_1, y_2, ..., y_n) = \frac{1}{(2\pi)^{\frac{n}{2}} \left| \boldsymbol{\Sigma} \right|^{\frac{1}{2}}} \exp\left\{ -\frac{1}{2} (\mathbf{y} - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\mathbf{y} - \boldsymbol{\mu}) \right\} \tag{28}$$

The system reliability is then obtained by integrating the probability density function using Eq. (12).

For the system designers of the new product, however, the distribution parameters of component resistances, for example, the means $\mu_{S_{R,i}}$ and standard deviations $\sigma_{S_{R,i}}$ ($i = 1, 2, \ldots, n$) of normal distribution are unknown. As a result, the complete information that defines the mean vector $\boldsymbol{\mu}$ and the covariance matrix $\boldsymbol{\Sigma}$ in Eq. (28) are not available to the designers. Thus, the exact system reliability cannot be obtained.

Narrow system reliability bounds can be found with the proposed optimization model. For this case, the design variables become $\mathbf{d} = (\mathbf{d}_1, \mathbf{d}_2, ..., \mathbf{d}_n)$ $= (\mu_{S_{R,1}}, \sigma_{S_{R,1}}, \mu_{S_{R,2}}, \sigma_{S_{R,2}}, ..., \mu_{S_{R,n}}, \sigma_{S_{R,n}})$ as discussed previously, and the distribution parameters of the system load become $\mathbf{p}_L = (\mu_L, \sigma_L)$. The constraint functions associated with component reliabilities, according to Eq. (22), are given by

$$h_i(\mathbf{d}; \mu_L, \sigma_L) = \Phi\left(-\frac{w_i \mu_L - \mu_{S_{R,i}}}{\sqrt{\sigma_{S_{R,i}}^2 + (w_i \sigma_L)^2}} \right) = R_i, \; i = 1, 2, ..., n \tag{29}$$

And we have totally $2n$ inequality constraints according to the range of factors of safety $n_{s,i}$.

$$g_i(\mathbf{d}; \mu_L, \sigma_L) = n_{s,i}^{\min} - \frac{\mu_{S_{R,i}}}{w_i \mu_L} \le 0, \; i = 1, 2, \ldots, n \tag{30}$$

and

$$g_{i+n}(\mathbf{d}; \mu_L, \sigma_L) = \frac{\mu_{S_{R,i}}}{w_i \mu_L} - n_{s,i}^{\max} \le 0, \; i = 1, 2, \ldots, n \tag{31}$$

In addition, we have other $2n$ inequality constraints according to the ranges of the coefficients of variation c_i of the unknown distributions.

$$g_{i+2n}(\mathbf{d};\mu_L,\sigma_L)=c_i^{\min}-\frac{\sigma_{S_{R,i}}}{\mu_{S_{R,i}}}\le 0,\ i=1,2,\ldots,n \tag{32}$$

and

$$g_{i+3n}(\mathbf{d};\mu_L,\sigma_L)=\frac{\sigma_{S_{R,i}}}{\mu_{S_{R,i}}}-c_i^{\max}\le 0,\ i=1,2,\ldots,n \tag{33}$$

The optimization model for the minimum system reliability is then given by

$$\begin{cases} \min\limits_{\mathbf{d}}\ R_S(\mathbf{d};\mu_L,\sigma_L) \\ \text{subject to} \\ h_i(\mathbf{d};\mu_L,\sigma_L)=\Phi\left(-\dfrac{w_i\mu_L-\mu_{S_{R,i}}}{\sqrt{\sigma_{S_{R,i}}^2+(w_i\sigma_L)^2}}\right)=R_i,\ i=1,2,...,n \\ g_i(\mathbf{d};\mu_L,\sigma_L)=n_{s,i}^{\min}-\dfrac{\mu_{S_{R,i}}}{w_i\mu_L}\le 0, \\ g_{i+n}(\mathbf{d};\mu_L,\sigma_L)=\dfrac{\mu_{S_{R,i}}}{w_i\mu_L}-n_{s,i}^{\max}\le 0, \\ g_{i+2n}(\mathbf{d};\mu_L,\sigma_L)=c_i^{\min}-\dfrac{\sigma_{S_{R,i}}}{\mu_{S_{R,i}}}\le 0, \\ g_{i+3n}(\mathbf{d};\mu_L,\sigma_L)=\dfrac{\sigma_{S_{R,i}}}{\mu_{S_{R,i}}}-c_i^{\max}\le 0, \end{cases} \tag{34}$$

For the maximum system reliability, we just change the first line of the optimization model in Eq. (34) from $\min R_S(\mathbf{d};\mathbf{p}_L)$ to $\max R_S(\mathbf{d};\mathbf{p}_L)$.

There are n equality constraint functions, which may cause numerical difficulties in solving the optimization models. We could improve the optimization models by eliminating some of the design variables using the equality constraints. This will not only reduce the scale of the optimization but also improve the robustness of the solution process [35]. An equality constraint imposes a functional relationship on design variables, and design variables $\mu_{S_{R,i}}$ can then be substituted with remaining design variables. From Eq. (22), we obtain

$$\mu_{S_{R,i}} = w_i \mu_L + \Phi^{-1}(R_i)\sqrt{\sigma_{S_{R,i}}^2 + (w_i \sigma_L)^2} \tag{35}$$

Thus, design variables $\mu_{S_{R,i}}$ and all the equality constraints are eliminated. Plugging Eq. (35) into Eq. (34) yields

$$\begin{cases} \min\limits_{\mathbf{d}} \; R_S(\mathbf{d}; \mu_L, \sigma_L) \\ \text{subject to} \\ g_i(\mathbf{d}; \mu_L, \sigma_L) = n_{s,i}^{\min} - \dfrac{w_i \mu_L + \Phi^{-1}(R_i)\sqrt{\sigma_{S_{R,i}}^2 + (w_i \sigma_L)^2}}{w_i \mu_L} \le 0, \; i = 1, 2, \; \ldots, \; n \\ g_{i+n}(\mathbf{d}; \mu_L, \sigma_L) = \dfrac{w_i \mu_L + \Phi^{-1}(R_i)\sqrt{\sigma_{S_{R,i}}^2 + (w_i \sigma_L)^2}\,{}_i}{w_i \mu_L} - n_{s,i}^{\max} \le 0, \\ g_{i+2n}(\mathbf{d}; \mu_L, \sigma_L) = c_i^{\min} - \dfrac{\sigma_{S_{R,i}}}{w_i \mu_L + \Phi^{-1}(R_i)\sqrt{\sigma_{S_{R,i}}^2 + (w_i \sigma_L)^2}} \le 0, \\ g_{i+3n}(\mathbf{d}; \mu_L, \sigma_L) = \dfrac{\sigma_{S_{R,i}}}{w_i \mu_L + \Phi^{-1}(R_i)\sqrt{\sigma_{S_{R,i}}^2 + (w_i \sigma_L)^2}} - c_i^{\max} \le 0, \end{cases} \tag{36}$$

The new vector of the design variables in Eq. (36) is $\mathbf{d} = \boldsymbol{\sigma}_{S_R} = (\sigma_{S_{R,1}}, \sigma_{S_{R,2}}, \ldots, \sigma_{S_{R,n}})$. The bounds of $\sigma_{S_{R,i}}$ can be determined by plugging Eq. (30) and Eq. (31) into Eq. (29), respectively.

$$\sigma_{S_{R,i}}^{\min} = \sqrt{\left(\frac{\left(n_{s,i}^{\min} - 1\right) w_i \mu_L}{\Phi^{-1}(R_i)}\right)^2 - \left(w_i \sigma_L\right)^2} \tag{37}$$

$$\sigma_{S_{R,i}}^{\max} = \sqrt{\left(\frac{\left(n_{s,i}^{\max} - 1\right) w_i \mu_L}{\Phi^{-1}(R_i)}\right)^2 - \left(w_i \sigma_L\right)^2} \tag{38}$$

The predicted system reliability bounds cover the true value if the true design point, which produces the true system reliability, falls into the feasible region defined by the constraint functions. It is therefore important to carefully select the parameters for the constraint functions. The designers of the new product could select these parameters based on their experiences, their knowledge about component design, and design standards in their specific areas.

4. NUMERICAL EXAMPLES

In this section we provide three examples for three cases: (1) a system consists of different components with the same load, (2) a system consists of identical components with the same load, and (3) a system consists of different components with different loads. In the third example, we also demonstrate the superiority of the proposed method in early design decision making over that of the traditional method. Since the reliability is high, to easily show the accuracy of the results, we use the probability of failure.

4.1 EXAMPLE 1: THREE DIFFERENT COMPONENTS SHARING THE SAME LOAD

A new design consists of three different components, supplied by three different companies, as shown in Fig. 5. They are subjected to the same load L. The resistances of the three components are known to the component designers, and their distributions are $S_1 \sim N(3500, 350^2)$ kN, $S_2 \sim N(3200, 260^2)$ kN, and $S_3 \sim N(4000, 400^2)$ kN. The three random variables are independent. The load L is known to both component designers and system designers of the new product. The distribution of the load is $L \sim N(2000, 200^2)$ kN. The probabilities of failure of the components obtained from the component designs are therefore $p_{f,1} = 9.920 \times 10^{-5}$, $p_{f,2} = 1.2696 \times 10^{-4}$, and $p_{f,3} = 3.87 \times 10^{-6}$ according to Eq. (22). The information about the component reliability is provided to the system designers of the new product. In addition, the system designers of the new product are confident that the factors of safety of the three components are between 1.5 and 2.5 and that the coefficients of variation of component resistances are between 0.08 and 0.2. The information available to the system designers of the new design is summarized in Table 1.

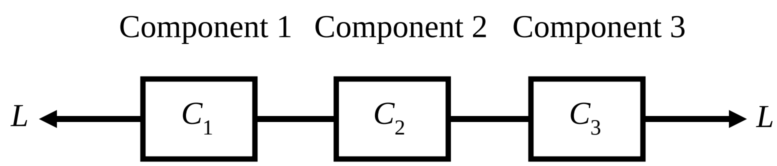

Figure 5 Three different components sharing same load

Table 1 Information available to the designers of the new product

Known information	Value
Probability of component failure $p_{f,1}$	9.920×10^{-5}
Probability of component failure $p_{f,2}$	1.2696×10^{-4}
Probability of component failure $p_{f,3}$	3.87×10^{-6}
Distribution of system load L	$N(2000, 200^2)$ kN
Factor of safety for component 1 $n_{s,1}$	$[1.5, 2.5]$
Factor of safety for component 2 $n_{s,2}$	$[1.5, 2.5]$
Factor of safety for component 3 $n_{s,3}$	$[1.5, 2.5]$
Coefficient of variation of resistance of component 1 c_1	$[0.08, 0.20]$
Coefficient of variation of resistance of component 2 c_2	$[0.08, 0.20]$
Coefficient of variation of resistance of component 3 c_3	$[0.08, 0.20]$

For the system designers of the new product, the task is to estimate the system reliability of the new product using the information in Table 1. The simplified free-body diagrams of the three components are the same. Fig. 6 shows the simplified free-body diagram of component 1.

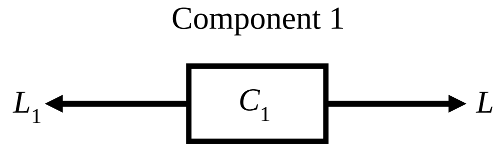

Figure 6 Simplified free-body diagram of component 1

The three components are subjected to the same load L, and their limit-state functions are therefore given by

$$\begin{cases} Y_1 = L - S_1 \\ Y_2 = L - S_2 \\ Y_3 = L - S_3 \end{cases} \tag{39}$$

Thus, the system reliability of the new product is

$$R_S = \Pr(Y_1 < 0,\ Y_2 < 0,\ Y_3 < 0) = \int_{-\infty}^{0} f(\mathbf{y})\mathrm{d}\mathbf{y} \tag{40}$$

where $\mathbf{Y} = (Y_1,\ Y_2,\ Y_3) \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. From Eq. (35), the means of component resistance $\mu_{S_i},\ i = 1,2,3$, are given by

$$\mu_{S_i} = \mu_L + \Phi^{-1}(R_i)\sqrt{\sigma_{S_i}^2 + (\sigma_L)^2} \tag{41}$$

The covariance between any two limit-state functions is $\mathrm{cov}(Y_i, Y_j) = \sigma_L^2$ according to Eq. (26), and the covariance matrix $\boldsymbol{\Sigma}$ is then given by

$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_{Y_1}^2 & \sigma_L^2 & \sigma_L^2 \\ \sigma_L^2 & \sigma_{Y_2}^2 & \sigma_L^2 \\ \sigma_L^2 & \sigma_L^2 & \sigma_{Y_3}^2 \end{pmatrix} \tag{42}$$

The design variables are $\mathbf{d} = (\sigma_{S_1}, \sigma_{S_2}, \sigma_{S_3})$. Thus, the optimization model is created using Eq. (36).

For the maximum system reliability, we just change the first line of the optimization model in Eq. (43) from $\min_{\mathbf{d}} R_S(\mathbf{d}; \mu_L, \sigma_L)$ to $\max_{\mathbf{d}} R_S(\mathbf{d}; \mu_L, \sigma_L)$. Table 2 shows the bounds of the probabilities of system failure obtained from the traditional method and the proposed method. The results indicate that the proposed method produces much narrower bounds than those from the traditional method. The two bounds are also plotted in Fig. 7.

$$\begin{cases} \min\limits_{\mathbf{d}} \ R_S(\mathbf{d};\mu_L,\sigma_L) \\ \text{subject to} \\ g_i(\mathbf{d};\mu_L,\sigma_L) = 1.5 - \dfrac{\mu_L + \Phi^{-1}(R_i)\sqrt{\sigma_{S_i}^2 + (\sigma_L)^2}}{2000} \le 0, \qquad i = 1,2,3 \\ g_{i+3}(\mathbf{d};\mu_L,\sigma_L) = \dfrac{\mu_L + \Phi^{-1}(R_i)\sqrt{\sigma_{S_i}^2 + (\sigma_L)^2}}{2000} - 2.5 \le 0, \\ g_{i+6}(\mathbf{d};\mu_L,\sigma_L) = 0.08 - \dfrac{\sigma_{S_i}}{\mu_L + \Phi^{-1}(R_i)\sqrt{\sigma_{S_i}^2 + (\sigma_L)^2}} \le 0, \\ g_{i+9}(\mathbf{d};\mu_L,\sigma_L) = \dfrac{\sigma_{S_i}}{\mu_L + \Phi^{-1}(R_i)\sqrt{\sigma_{S_i}^2 + (\sigma_L)^2}} - 0.20 \le 0, \end{cases} \tag{43}$$

Table 2 System reliability analysis results

Methods	**Bounds of** $p_{f,s}$	**Interval width**
Traditional method	$[1.2696,\ 2.3002]\times 10^{-4}$	1.0306×10^{-4}
Proposed method	$[2.2891,\ 2.30]\times 10^{-4}$	0.0109×10^{-4}
Exact	2.2950×10^{-4}	

The true value of the probability of system failure is also provided in both Table 2 and Fig. 7, and it is calculated as if all the distributions of S_1, S_2, S_3, and L were known. Note that in reality, both component designers and system designers only know some of the distributions. Even though the exact value may never be known, we use it to verify the accuracy of the proposed method. As indicated by the results, the probability bounds from the proposed method do contain the exact value. To easily show the accuracy, we also use the percentage errors of the lower and upper bounds of the probabilities of system failure relative to the true value. The errors of the traditional and proposed methods are $[-44.68\%, 0.23\%]$ and $[-0.26\%, 0.22\%]$, respectively. They are also shown in Fig. 7.

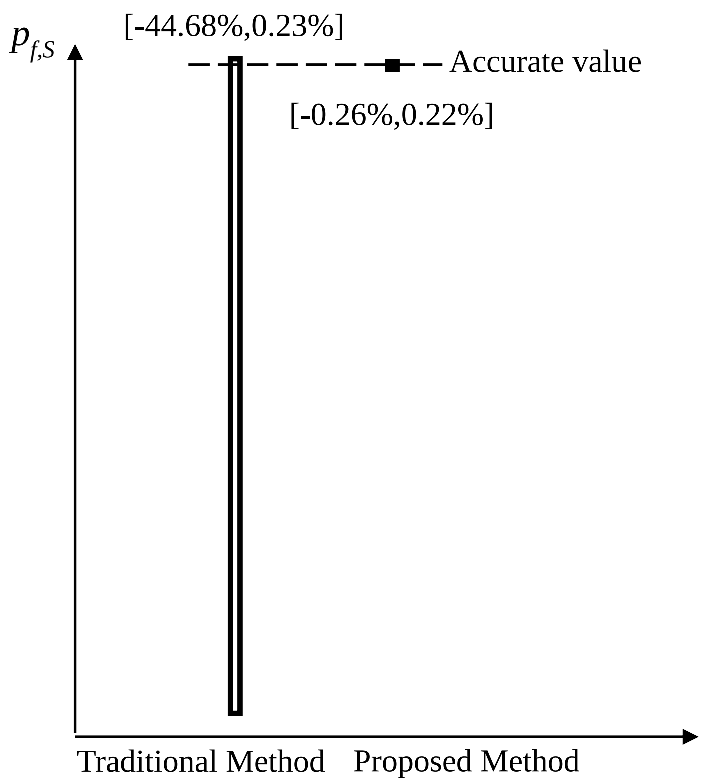

Figure 7 Bounds of probabilities of system failure

4.2 EXAMPLE 2: THREE IDENTICAL COMPONENTS SHARING THE SAME LOAD

The system configuration is the same as that in Example 1. The three components are also subjected to the same load L. But the three components are identical here. The component resistance is known to the component designers, and its distribution is $S \sim N(4000, 130^2)$ kN. The load L is known to both component designers and system designers, and its distribution is $L \sim N(2400, 450^2)$ kN. The probability of failure of the component obtained from the component supplier is $p_f = 3.1789 \times 10^{-4}$ and is provided to the system designers. In addition, the system designers estimate that the factors of safety of the component are between 1.5 and 2.2 and that the coefficient of variation of component resistance is between 0.03 and 0.15. The information available to the system designers of the new design is summarized in Table 3.

Table 3 Information available to the designers of the new product

Known information	**Value**
Probability of component failure $p_{f,1}$	3.1789×10^{-4}
Probability of component failure $p_{f,2}$	3.1789×10^{-4}
Probability of component failure $p_{f,3}$	3.1789×10^{-4}
Distribution of system load L	$N(2400, 450^2)$ kN
Factor of safety for component 1 $n_{s,1}$	$[1.5, 2.2]$
Factor of safety for component 2 $n_{s,2}$	$[1.5, 2.2]$
Factor of safety for component 3 $n_{s,3}$	$[1.5, 2.2]$
Coefficient of variation of resistance of component 1 c_1	$[0.03, 0.15]$
Coefficient of variation of resistance of component 2 c_2	$[0.03, 0.15]$
Coefficient of variation of resistance of component 3 c_3	$[0.03, 0.15]$

For the system designers of the new product, the task is to estimate the system reliability of the new product using the information in Table 3. The simplified free-body diagrams of the three components are the same as that in Example 1, as shown in Fig. 6.

The component limit-state functions are $Y_1 = Y_2 = Y_3 = L - S$ according to Eq. (39). Plugging their limit-state functions into Eqs. (40) through Eq. (43), we obtain the optimization model as follows.

$$\begin{cases} \min_{\mathbf{d}} \; R_S(\mathbf{d};\mu_L,\sigma_L) \\ \text{subject to} \\ g_i(\mathbf{d};\mu_L,\sigma_L) = 1.5 - \dfrac{\mu_L + \Phi^{-1}(R_i)\sqrt{\sigma_{S_i}^2 + (\sigma_L)^2}}{2400} \le 0, \qquad i = 1,2,3 \\ g_{i+3}(\mathbf{d};\mu_L,\sigma_L) = \dfrac{\mu_L + \Phi^{-1}(R_i)\sqrt{\sigma_{S_i}^2 + (\sigma_L)^2}}{2400} - 2.2 \le 0, \\ g_{i+6}(\mathbf{d};\mu_L,\sigma_L) = 0.03 - \dfrac{\sigma_{S_i}}{\mu_L + \Phi^{-1}(R_i)\sqrt{\sigma_{S_i}^2 + (\sigma_L)^2}} \le 0, \\ g_{i+9}(\mathbf{d};\mu_L,\sigma_L) = \dfrac{\sigma_{S_i}}{\mu_L + \Phi^{-1}(R_i)\sqrt{\sigma_{S_i}^2 + (\sigma_L)^2}} - 0.15 \le 0, \end{cases} \tag{44}$$

For the maximum system reliability, we just change the first line of the optimization model in Eq. (44) from $\min_{\mathbf{d}} \; R_S(\mathbf{d};\mu_L,\sigma_L)$ to $\max_{\mathbf{d}} \; R_S(\mathbf{d};\mu_L,\sigma_L)$. Table 4 shows the bounds of the probabilities of system failure obtained from the traditional method and the proposed method. The results also indicate that the proposed method produces narrower bounds than those from the traditional method. The two bounds are also plotted in Fig. 8.

The exact (true) value of the probability of system failure is also provided in both Table 4 and Fig. 8. The exact value is calculated as if the distributions of S and L were known. As indicated by the results, the bounds of the probability of system failure from the proposed method do contain the exact value of the probability of system failure. The relative errors of the two methods are $[-48.10\%, 55.65\%]$ and $[-3.52\%, 54.64\%]$ as shown in Fig. 8.

Table 4 System reliability analysis results

Methods	Bounds of $p_{f,s}$	Interval width
Traditional method	$[3.1789,\ 9.5337]\times10^{-4}$	6.3548×10^{-4}
Proposed method	$[5.9094, 9.4721]\times10^{-4}$	3.5627×10^{-4}
Exact	6.1252×10^{-4}	

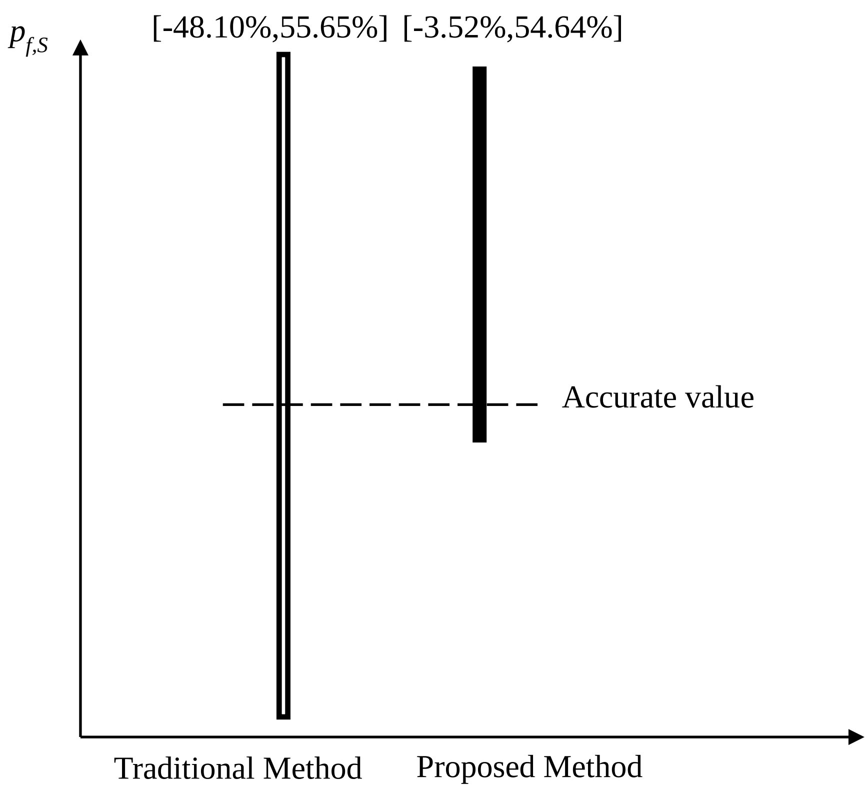

Figure 8 Bounds of probabilities of system failure

4.3 EXAMPLE 3: TWO DIFFERENT COMPONENTS SHARING DIFFERENT LOADS

Two design concepts for a hoisting device with a load L are generated. They are shown in Fig. 9. Cables 1 and 2 are used in design concept 1 while cables 3 and 4 are used in design concept 2. All the cables are supplied by different companies. Both reliability and cost are two major factors for choosing one design concept between the two. The cost of design concept 2 is estimated 20% cheaper than that of design concept 1 because the components in design concept 2 are cheaper. The distribution of the weight of the block $L \sim N(1500, 160^2)$ kN is known to the system designers of the new hoisting device. The resistances of the two cables used in design concept 1 are only known to the component designers, and they are independently distributed with $S_1 \sim N(1200, 100^2)$ kN and $S_2 \sim N(2500, 250^2)$ kN. Using the distributions, the component designers estimate the probabilities of failure of the two cables are $p_{f,1} = 2.2078 \times 10^{-4}$ and $p_{f,2} = 3.7709 \times 10^{-4}$, and the results are provided to the system designers of the new product.

For design concept 2, the slope is $\theta = 30^\circ$, and the coefficient of kinetic friction between the block and surface is $\mu_R = 0.2$; they are known to system designers. The resistances of the two cables are only known to the component designers, and their distributions are $S_3 \sim N(600, 65^2)$ kN and $S_4 \sim N(1220, 140^2)$ kN . The two random variables are independent. The probabilities of failure of the two cables obtained from the component design are $p_{f,3} = 1.9475 \times 10^{-4}$ and $p_{f,4} = 2.5523 \times 10^{-4}$, and they are also provided to the system designers of the new product. In addition, for both concepts of the new product, the system designers estimate that the factors of safety of all the cables are between 1.5 and 2.5 and that the coefficients of variation of component resistances are between 0.08 and 0.2. The information available to the system designers of the two design concepts is summarized in Tables 5 and 6, respectively.

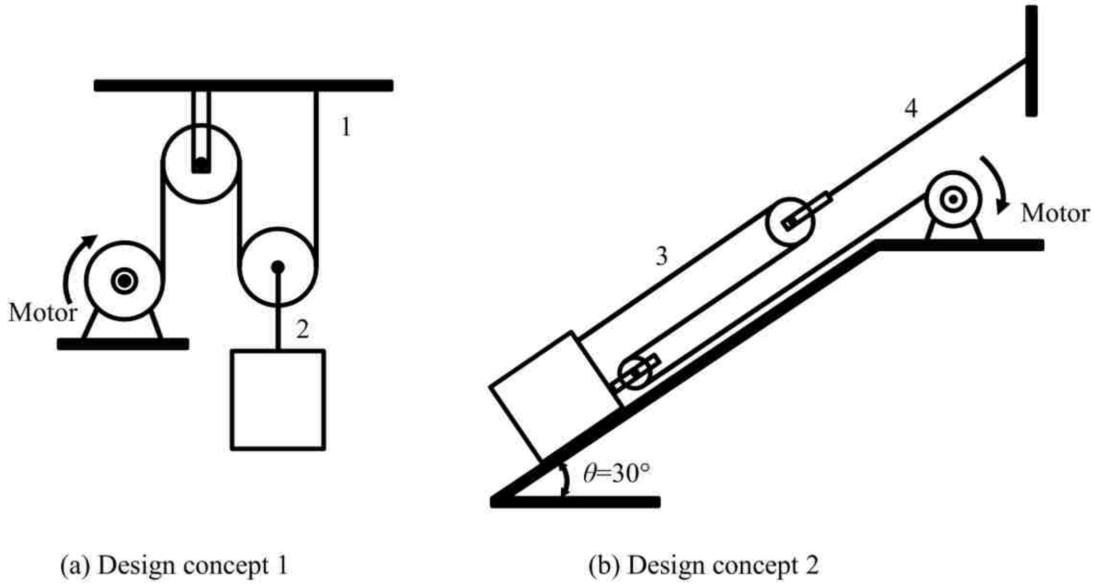

Figure 9 Two components sharing different loads

Table 5 Information available for design concept 1

Known information	Value
Probability of component failure $p_{f,1}$	2.2078×10^{-4}
Probability of component failure $p_{f,2}$	3.7709×10^{-4}
Distribution of system load L	$N(1500,160^2)$ kN
Factor of safety for component 1 $n_{s,1}$	$[1.5, 2.5]$
Factor of safety for component 2 $n_{s,2}$	$[1.5, 2.5]$
Coefficient of variation of resistance of component 1 c_1	$[0.08, 0.20]$
Coefficient of variation of resistance of component 2 c_2	$[0.08, 0.20]$

The simplified free-body diagram of design concept 1 is shown in Fig. 10.

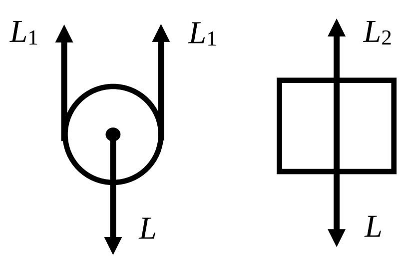

Figure 10 Simplified free-body diagrams of design concept 1

Table 6 Information available for design concept 2

Known information	**Value**
Probability of component failure $p_{f,3}$	1.9475×10^{-4}
Probability of component failure $p_{f,4}$	2.5523×10^{-4}
Distribution of system load L	$N(1500,160^2)$ kN
Factor of safety for component 1 $n_{s,3}$	$[1.5, 2.5]$
Factor of safety for component 2 $n_{s,4}$	$[1.5, 2.5]$
Coefficients of variation of resistance of component 1 c_3	$[0.08, 0.20]$
Coefficients of variation of resistance of component 2 c_4	$[0.08, 0.20]$
Slope	30°
Coefficient of friction	$\mu_R = 0.2$

We have

$$\begin{cases} L_1 = 0.5L \\ L_2 = L \end{cases} \tag{45}$$

The limit-state functions of the two cables in concept 1 are given by

$$\begin{cases} Y_1 = 0.5L - S_1 \\ Y_2 = L - S_2 \end{cases} \tag{46}$$

The simplified free-body diagram of design concept 2 is shown in Fig. 11.

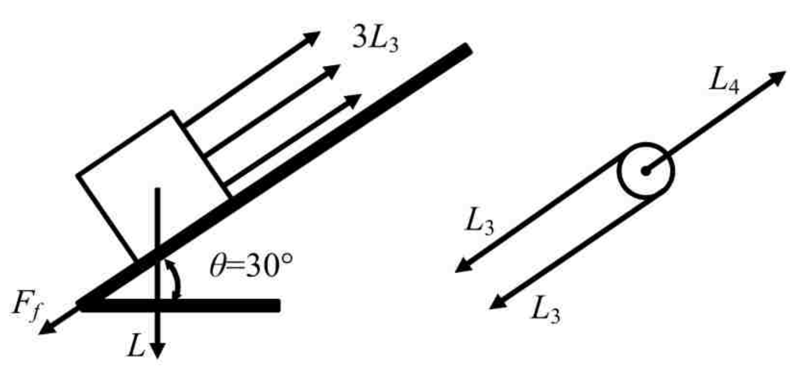

Figure 11 Simplified free-body diagrams of design concept 2

Based on force equilibrium, we obtain

$$\begin{cases} L_3 = \dfrac{L(\sin\theta + \mu_R \cos\theta)}{3} \\ L_4 = \dfrac{2L(\sin\theta + \mu_R \cos\theta)}{3} \end{cases} \tag{47}$$

The limit-state functions of the two cables are then given by

$$\begin{cases} Y_3 = \dfrac{L(\sin\theta + \mu_R \cos\theta)}{3} - S_3 \\ Y_4 = \dfrac{2L(\sin\theta + \mu_R \cos\theta)}{3} - S_4 \end{cases} \tag{48}$$

The general limit-state function of the four cables for both design concepts is therefore

$$Y_i = w_i L - S_i \tag{49}$$

where $i = 1, 2, 3, 4$.

The system reliability of design concept 1 is then given by

$$R_{S_1} = \Pr(Y_1 < 0,\ Y_2 < 0) = \int_{-\infty}^{0} f(\mathbf{y}_1)\mathrm{d}\mathbf{y}_1 \tag{50}$$

where $\mathbf{Y}_1 = (Y_1,\ Y_2) \sim N(\boldsymbol{\mu}_1, \boldsymbol{\Sigma}_1)$. The mean function of component resistance μ_{S_i} is given by

$$\begin{cases} \mu_{S_1} = w_1\mu_L + \Phi^{-1}(R_1)\sqrt{\sigma_{S_1}^2 + (w_1\sigma_L)^2} \\ \mu_{S_2} = w_2\mu_L + \Phi^{-1}(R_2)\sqrt{\sigma_{S_2}^2 + (w_2\sigma_L)^2} \end{cases} \tag{51}$$

The covariance between the two limit-state functions is $\mathrm{cov}(Y_1, Y_2) = w_1 w_2 \sigma_L^2$ according to Eq. (26), and the covariance matrix $\boldsymbol{\Sigma}_1$ is then given by

$$\boldsymbol{\Sigma}_1 = \begin{pmatrix} \sigma_{Y_1}^2 & w_1 w_2 \sigma_L^2 \\ w_1 w_2 \sigma_L^2 & \sigma_{Y_2}^2 \end{pmatrix} \tag{52}$$

The design variables are $\mathbf{d}_1 = (\sigma_{S_1}, \sigma_{S_2})$. Thus, the optimization model of concept 1 is created using Eq. (36).

$$\begin{cases} \min_{\mathbf{d}_1} \ R_{S_1}(\mathbf{d}_1; \mu_L, \sigma_L) \\ \text{subject to} \\ g_1(\mathbf{d}_1; \mu_L, \sigma_L) = 1.5 - \dfrac{w_1\mu_L + \Phi^{-1}(R_1)\sqrt{\sigma_{S_1}^2 + (w_1\sigma_L)^2}}{1500 w_1} \le 0 \\ g_2(\mathbf{d}_1; \mu_L, \sigma_L) = 1.5 - \dfrac{w_2\mu_L + \Phi^{-1}(R_2)\sqrt{\sigma_{S_2}^2 + (w_2\sigma_L)^2}}{1500 w_2} \le 0 \\ g_3(\mathbf{d}_1; \mu_L, \sigma_L) = \dfrac{w_1\mu_L + \Phi^{-1}(R_1)\sqrt{\sigma_{S_1}^2 + (w_1\sigma_L)^2}}{1500 w_1} - 2.5 \le 0 \\ g_4(\mathbf{d}_1; \mu_L, \sigma_L) = \dfrac{w_2\mu_L + \Phi^{-1}(R_2)\sqrt{\sigma_{S_2}^2 + (w_2\sigma_L)^2}}{1500 w_2} - 2.5 \le 0 \\ g_5(\mathbf{d}_1; \mu_L, \sigma_L) = 0.08 - \dfrac{\sigma_{S_1}}{w_1\mu_L + \Phi^{-1}(R_1)\sqrt{\sigma_{S_1}^2 + (w_1\sigma_L)^2}} \le 0 \\ g_6(\mathbf{d}_1; \mu_L, \sigma_L) = 0.08 - \dfrac{\sigma_{S_2}}{w_2\mu_L + \Phi^{-1}(R_2)\sqrt{\sigma_{S_2}^2 + (w_2\sigma_L)^2}} \le 0 \\ g_7(\mathbf{d}_1; \mu_L, \sigma_L) = \dfrac{\sigma_{S_1}}{w_1\mu_L + \Phi^{-1}(R_1)\sqrt{\sigma_{S_1}^2 + (w_1\sigma_L)^2}} - 0.20 \le 0 \\ g_8(\mathbf{d}_1; \mu_L, \sigma_L) = \dfrac{\sigma_{S_2}}{w_2\mu_L + \Phi^{-1}(R_2)\sqrt{\sigma_{S_2}^2 + (w_2\sigma_L)^2}} - 0.20 \le 0 \end{cases} \tag{53}$$

where $w_1 = 0.5$ and $w_2 = 1$ from Eq. (45). For the maximum system reliability, we just change the first line of the optimization model in Eq. (53) from $\min_{\mathbf{d}_1} \ R_{S_1}(\mathbf{d}_1; \mu_L, \sigma_L)$ to $\max_{\mathbf{d}_1} \ R_{S_1}(\mathbf{d}_1; \mu_L, \sigma_L)$.

For design concept 2, the optimization model is similar to that in Eq. (53) with the following modifications: (1) change design variables from $\mathbf{d}_1 = (\sigma_{S_1}, \sigma_{S_2})$ to $\mathbf{d}_2 = (\sigma_{S_3}, \sigma_{S_4})$, (2) change component reliabilities from R_1 and R_2 to

R_3 and R_4, and (3) change w_1 and w_2 to w_3 and w_4, where $w_3 = (\sin\theta + \mu'\cos\theta)/3$ and $w_4 = 2(\sin\theta + \mu'\cos\theta)/3$ according to Eq. (47).

Table 7 shows the bounds of the probabilities of system failure obtained from the traditional method and the proposed method for the two design concepts. The results not only indicate that the proposed method produces much narrower bounds for the probabilities of system failure than those from the traditional method, but also demonstrate the feasibility of the proposed method to assist the system designers to select a better concept with respect to reliability. The bounds of the two concepts are plotted in Fig. 12. It shows that design concept 2 is more reliable than design concept 1. This is because the probability of system failure of design concept 2 is lower than that of design concept 1 using proposed method. It is hard, however, to make decisions using the traditional method as the bounds for the probabilities of system failure of the two design concepts are wide and overlap as shown in Fig. 12. Thus, with the new system reliability analysis, the system designers may select design concept 2 because it has higher reliability and lower cost.

Table 7 System failure analysis results of the two concepts for the new system

Concepts	**Methods**	**Bounds of** $p_{f,s}$	**Interval width**	**Exact value**
Concept 1	Traditional method	$[3.7709,\ 5.9779]\times10^{-4}$	2.2070×10^{-4}	5.9498×10^{-4}
	Proposed method	$[5.8877,\ 5.9769]\times10^{-4}$	0.0892×10^{-4}	
Concept 2	Traditional method	$[2.5523,\ 4.4993]\times10^{-4}$	1.9470×10^{-4}	4.4931×10^{-4}
	Proposed method	$[4.4354,\ 4.4987]\times10^{-4}$	0.0633×10^{-4}	

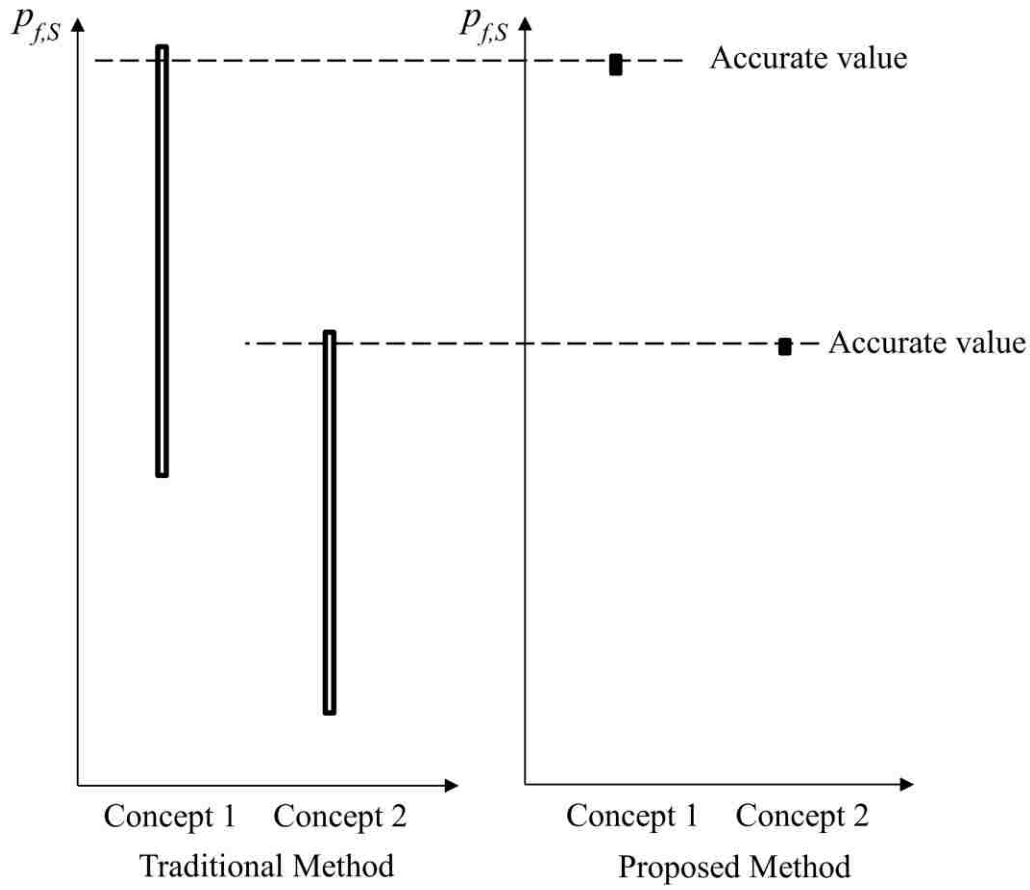

Figure 12 Bounds of probabilities of system failure

The exact (true) value of the probability of system failure of each concept is also provided in Table 7. The exact value of design concept 1 is calculated as if all the distributions of S_1, S_2, and L were known; the exact value of design concept 2 is calculated as if all the distributions, S_3, S_4, and L, were known. As indicated by the results, the bounds of the probabilities of system failure using proposed method do contain the exact values of the probabilities of system failure.

4.4 SUMMARY OF THE EXAMPLES

The proposed method has produced narrow system reliability bounds where the true system reliability resides. The examples also demonstrate the effect of dependent component states on system reliability. In Examples 1 and 3, the true probabilities of system failure are close to the upper bounds of the probabilities of system failure that are from independent component assumption. This means that the effect of the dependency is not significant. For Example 1, the coefficients of correlation between component 1 and 2, 2 and 3, and 1 and 3 are 0.3025, 0.2727, and 0.2219, receptively. For Example 3, the coefficients of correlation between component 1 and 2 of concept 1, and component 3 and 4 of concept 2 are 0.3367 and 0.2207, receptively. These small coefficients of correlation indicate weak component dependency. Even so, it is risky for the designers of a new product to make decisions by treating components as independent states, because they may not know the weak dependency in advance during the conceptual design stage.

The result of Example 2 clearly shows the significant impact of dependent components on system reliability because the true probability of system failure is far away from the upper bound that is produced from the assumption of independent components. The coefficients of correlation between the three components are all 0.9230, which indicates the strong correlation between the components.

5. DISCUSSIONS ABOUT THE UNCERTAINTY IN INPUT VARIABLES

The uncertainty in input variables will also affect the accuracy of reliability analysis [36, 37]. The proposed method can actually accommodate the uncertainty in some of its input variables, including the component factors of safety and coefficients of variation of component strengths. The system designers know neither their nominal values nor the uncertainty associated with these input variables. By treating the unknown variables as either design variables or constraints in the system reliability model in Eqs. (19) and (34), the proposed method can identify the likely values of the input variables corresponding to the minimum and maximum system reliabilities.

The uncertainty in other input variables is not considered in the proposed system reliability model. They include component probabilities of failure, the distribution of system load, and the types of component strength distributions. The uncertainty in these input variables may be in different forms due to different reasons. For example, if the samples for the system load are not sufficient, there might be several possible candidate distributions, and the distribution parameters themselves might also be random variables [37]. In an extreme case, if the data are too scarce, the load may be described by only an interval [38]. The component reliabilities may also be intervals because component suppliers may report percentage errors for their component reliabilities.

The proposed system reliability model in Eq. (19) can then be modified to account for the uncertainty in input variables. If several candidate distributions are possible for random input variables, the methodology for imprecise random variables [37] can be incorporated. If the uncertainty in the dependence between input variables has to be considered, the Bayesian approach [36] may be applied. If the uncertainty is in the form of intervals, denoted by $\mathbf{y}$, the system reliability in Eq. (19) can be modified as

$$\begin{cases} \min_{\mathbf{d}} \min_{\mathbf{y}} R_S(\mathbf{d}, \mathbf{y}; \mathbf{p}_L) \\ \text{subject to} \\ h_i(\mathbf{d}, \mathbf{y}; \mathbf{p}_L) = \iint_{w_i L < S_{R,i}} f_{L_i, S_{R,i}}(l, s) dl ds = R_i, \; i = 1, 2, ..., n \\ g_i(\mathbf{d}, \mathbf{y}; \mathbf{p}_L) = \min_{\mathbf{y}} n_{s,i}^{\min} - n_{s,i}(\mathbf{d}, \mathbf{y}; \mathbf{p}_L) \le 0, \\ g_{i+n}(\mathbf{d}, \mathbf{y}; \mathbf{p}_L) = n_{s,i}(\mathbf{d}, \mathbf{y}; \mathbf{p}_L) - \max_{\mathbf{y}} n_{s,i}^{\max} \le 0, \\ g_{i+2n}(\mathbf{d}, \mathbf{y}; \mathbf{p}_L) = c_i^{\min} - c_i(\mathbf{d}; \mathbf{p}_L) \le 0, \\ g_{i+3n}(\mathbf{d}, \mathbf{y}; \mathbf{p}_L) = c_i(\mathbf{d}; \mathbf{p}_L) - c_i^{\max} \le 0, \end{cases} \tag{54}$$

In the above model, one more loop is added for identifying the extreme values with respect to interval variables. Due to the uncertainty in the input variables, the system reliability bounds produced will be wider, and the computational cost will also be higher. Efficient numerical algorithms are needed to solve the optimization model.

6. CONCLUSIONS

This work is concerned with the reliability prediction of a new product whose components are independently designed, tested, and manufactured by different suppliers. A system reliability method is developed to predict the reliability of the new product in the early design stage using the component reliabilities provided by component suppliers. The method is based on the strength-stress interference model that takes the dependence between components into consideration, thereby eliminating the assumption of independent component failures. As a result, the predicted system reliability bounds are much narrower than those from the assumption of independent component failures. This study has shown the feasibility of considering dependent component failures for predicting system reliability bounds in early design stage. The proposed method provides reliability predictions for decision making on eliminating or keeping design concepts during the conceptual design stage. It is useful if a concept selection method, for example, the Pugh Chart method, requires all design concepts be ranked with respect to performance criteria, including reliability. For some situations, however, designers of the new product are only interested in if the reliability requirement could be satisfied. Then the proposed method is not necessary once the minimum reliability (the lower bound) from the independent component assumption in Eq. (5) reaches the reliability target.

The proposed method is applicable for time invariant reliability problems. It can be extended to time variant problems in the future work. Time-dependent reliability could be addressed by considering time-dependent component stresses and strengths. The major research task is to obtain the autocorrelation function of the unknown stochastic processes of generalized component strengths. The ultimate goal is to evaluate the time-dependent system reliability for a given period of time.

As discussed in Sec. 5, uncertainty may also exist in the input variables required by the proposed system reliability method. The future work will be the development of computational methods that can efficiently solve the optimization models with the extra loop that accommodates the uncertainty in input variables.

This work assumes each component has only one failure mode. For a component with multiple failure modes, the component designers may use multiple limit-state functions to evaluate the reliability of the component. Although the component reliability may be reported to the designers of the new product, they however know neither the failure modes nor the limit-state functions of the component. A possible way to deal with this problem is to model the multiple failure modes of the component using a single equivalent limit-state function that can represent the limit-state functions of the multiple failure modes. Then the optimization models proposed in this work could be applied.

The proposed method is applied to series systems. Its application to parallel systems and mix systems is also a possible research task in the future work. Our future work will also deal with situations where a new product is subjected to multiple forces.

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III. NARROWER SYSTEM RELIABILITY BOUNDS WITH INCOMPLETE COMPONENT INFORMATION AND STOCHASTIC PROCESS LOADING

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ABSTRACT

Incomplete component information may lead to wide bounds for system reliability prediction, making decisions difficult in the system design stage. The missing information is often the component dependence, which is a crucial source for the exact system reliability estimation. Component dependence exists due to the shared environment and operating conditions. But it is difficult for system designers to model component dependence because they may have limited information about component design details if outside suppliers designed and manufactured the components. This research intends to produce narrow system reliability bounds with a new way for system designers to consider the component dependence implicitly and automatically without knowing component design details. The proposed method is applicable for a wide range of applications where the time-dependent system stochastic load is shared by components of the system. Simulation is used to obtain the extreme value of the system load for a given period of time, and optimization is employed to estimate the system reliability bounds, which are narrower than those from the traditional method with independent component assumption and completely dependent component assumption. Examples are provided to demonstrate the proposed method.

Keywords: System reliability; Incomplete information; Time-dependent loading; Optimization

1. INTRODUCTION

System reliability is the probability that a system performs its intended function within a given period of time under specified conditions [1]. The task of system reliability analysis is to obtain such a probability. Accurately predicting system reliability is challenging due to limited information. Without complete information, assumptions are usually made and may lead to a large error in the system reliability prediction. For example, system reliability could be estimated within an interval determined by its minimum and maximum bounds [2-4]. When the width of the bounds is too large, it will be difficult to make system level decisions, such as the selection of design concepts, lifecycle cost assessment, warranty policy, and maintenance planning.

The complete information of exact system reliability estimation includes not only the system configuration and component reliabilities, but also the statistical relationship (dependence) between components. Such dependence exists once the components are assembled and are in operation in a system. For example, components may be operated in the same environment (e.g. temperature and humidity), they may share the same load (e.g. pressure and power), they may deform dependently due to geometric constraints, and the output of one component may be the input of others.

Knowing component dependence information, however, requires the details of component designs, such as dimensions of a component and its material properties. Such detail information is seldom available to system designers. One of the major reasons is due to outsourced components. It is a practical business mode for a product (system) to have its components ordered from suppliers, who design, test, and manufacture the components individually and independently. The detail design information of components is usually proprietary to component suppliers. For example, the compressors and condensers of a refrigeration system may be supplied by other companies. When these components are assembled together in the refrigeration system, their states are statistically dependent. It is, however, not easy for refrigeration system designers to accommodate component dependence because they do not know the detail design information of the components – the information contains commercial and confidential data that belong to the component suppliers, who may be reluctant to share the information with the system designers.

If the joint probability density of all the component states is not available, the system reliability could be estimated with its bounds $\left[R_S^{\min}, R_S^{\max}\right]$. $R_S^{\min}$ is the minimum system reliability and $R_S^{\max}$ is the maximum system reliability. To assess the reliability bounds, marginal component probabilities and joint probabilities are usually needed. The marginal component probability is the component probability, such as $\Pr(C_a)$ for component a. The joint probability is for at least two components, for example, for components *a* and *b*, the joint probability is $\Pr(C_a C_b)$; for components *a*, *b*, and *c*, the joint probability is $\Pr(C_a C_b C_c)$. With no joint probabilities involved, Boole [5] proposed an equation for series systems to estimate the bounds of system probability with the only information of component probabilities. This method will be elaborated in Section 2. Although using only component probabilities is easy, the obtained reliability bounds may be too wide for practical applications.

Efforts have been made to reduce the width of reliability bounds, especially in structural reliability engineering. With the models developed from physics principles, the joint probabilities become available [6] and thus lead to narrower system reliability intervals. Hohenbichler and Rackwitz [7] employed the first-order approximation to narrow the bounds of system reliability. In their method, components' detail information such as limit-state functions was required. Kounias [8], Hunter [9], and Ditlevsen [2] also obtained narrower reliability bounds for series systems. Their methods require both component probabilities $\Pr(C_a)$ and bicomponent probabilities $\Pr(C_a C_b)$. Zhang [10] proposed a general methodology by incorporating high order joint probabilities such as $\Pr(C_a C_b C_c)$ and $\Pr(C_a C_b C_c C_d)$.

Some of the above methods have the order-dependency problem, which means the results of system reliability dependent on the order of components. Besides, the computations are expensive because every possible ordering alternative needs to be considered. In order to deal with these drawbacks, Song and Kiureghian [11] developed a linear programming (LP) methodology, which not only has no component ordering restrictions but also could incorporate inequality constraints as well as incomplete component probabilities. However, the problem size expanded exponentially with the increasing of the number of components, which dramatically deteriorates the efficiency of the LP method. Ramachandran [4] reviewed the techniques for narrower bounds in structural reliability published before 2004. Domyancic and Millwater [12] summarized multiple popular computational methods for series systems, such as the first order bounds, Ditlevsen bounds, and LP bounds. All of the above methods require the detail information of components and are not applicable for systems whose component details are unknown to system designers.

To address the above problem, Cheng and Du [13] performed a feasibility study and demonstrated the possibility of producing narrower system reliability bounds using a physics-based method. In their method, component reliabilities and distribution types of component resistances were provided by component suppliers. With the limited information from component suppliers, along with the knowledge of the system load and other information estimated by system designers, narrower system reliability bounds were produced. This method, however, is limited to only time-independent problems, for which the system reliability is constant and does not change over time.

In this work, we extend the aforementioned time-independent method [13] to time-dependent system reliability analysis for systems that are subject to a time-dependent stochastic system load. The new method can be applied to more common engineering applications because it can answer the question about the system reliability with respect to time; for example, what is the probability that a system can still work without failure after five years? A general model is developed to implicitly and automatically incorporate component dependence. With this general model, system designers do not need to know component resistance distributions (both distribution types and parameters), component failure modes, and other detail information such as dimensions. Simulation is used to obtain the extreme value of the system stochastic process load for a given period of time, and optimization models are established to estimate the system reliability interval. The width of the system reliability interval is then reduced significantly.

Note that although there are many existing methods for incomplete information for reliability analysis, the new method is different from the existing ones. For example, the existing methods deal with reliability analysis with incomplete information in input variables of a limit-state function, such as a small number of samples [14], interval samples [15, 16], input variables in the form of both finite samples and probability distributions [17], input random variables with only their means and covariance matrix [18, 19], and other formats of incomplete information of input variables, including marginal distributions, partial joint distributions, bounds, and higher moments [20]. In existing methods, all component limit-state functions are known; but in the new method, component limit-state functions are unknown and are assumed by system designers. In existing methods, partial information about all input variables is known; but in the new method no information is available for many input variables (such as dimensions and structure of a component).

In Section 2, the system reliability modeling is reviewed. In Section 3, the new system reliability methodology with dependent components and time-dependent loading is elaborated. Following that, two examples are presented in Section 4, and conclusions are given in Section 5.

2. REVIEW OF SYSTEM RELIABILITY MODELING

From a statistical point of view, reliability at time instant *t* is estimated using the time to failure T by

$$R(t) = \Pr(T > t) \quad (1)$$

Reliability can also be estimated by a physics-based method with a limit-state function $g(\cdot)$. A limit-state function specifies a condition of a component, beyond which the component no longer fulfills its intended function [21]. $g(\cdot) > 0$ indicates that the component is able to function properly, and then the reliability of the component is calculated by

$$R = \Pr(g(\mathbf{X}) > 0) \quad (2)$$

where **X** is a vector of random variables. The above reliability does not change over time because the limit-state function is time independent.

When the limit-state function is given by $G = g(\mathbf{X},\ \mathbf{Y}(\tau))$, where **Y** is a vector of time-dependent stochastic process, which varies with time τ, the reliability will be time dependent. It is calculated by

$$R(t) = \Pr\left\{g(\mathbf{X},\ \mathbf{Y}(\tau)) > 0,\ \text{for all } \tau \le t\right\} \quad (3)$$

Component designers can use Eq. (2) or (3) to compute component reliabilities if the limit-state functions are known. After the component reliabilities are known, system designers perform system reliability analysis.

Three types of systems are commonly encountered and they are series systems, parallel systems, and mix systems. In this work we focus on series systems since they are most commonly used in mechanical applications. As shown in Fig. 1, in a series system, if one component fails, the entire system will fail. For instance, a speed reducer system is a series system, which consists of components, such as a belt, drums, shafts, gears, keys, and bearings. If at least one of these components fails, the speed reducer will not work.

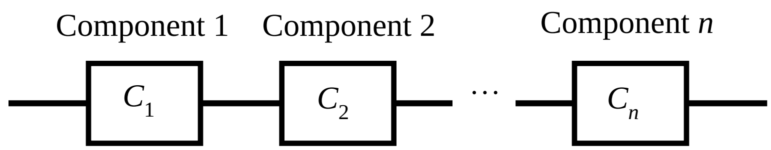

Figure 1 Series system

Suppose the reliabilities of components $C_1,\ C_2,\ ...,\ C_n$ are $R_1(t),\ R_2(t),\ ...,\ R_n(t)$, respectively. Knowing all the component reliabilities is not sufficient for the accurate system reliability prediction. As discussed previously, the information about component dependence is also required. When such information is not available due to outsourced components, a precise system reliability prediction will not be possible. However, if the component states are assumed to be independent, the system reliability is given by

$$R_S(t) = \prod_{i=1}^{n} R_i(t) \tag{4}$$

The above independence assumption gives the worst-case system reliability and may result in large errors because of strong component dependence in many mechanical systems. To this end, the best-case system reliability may also be considered. It is the minimal component reliability among the reliabilities of all components. It is obtained from the assumption that all component states are completely dependent. The system reliability bounds are then given by [22]

$$\prod_{i=1}^{n} R_i(t) \le R_S(t) \le \min\{R_i(t)\},\ i = 1, 2, ..., n \tag{5}$$

The interval of the probability of system failure is given by

$$\max\left\{p_{fi}(t)\right\} \le p_{fS}(t) \le 1-\prod_{i=1}^{n}(1-p_{fi}(t)),\ i=1,2,...,n \tag{6}$$

where $p_{fS}(t)$ is the probability of system failure in the period of time $[0,t]$ and is equal to $1-R_S(t)$; $p_{fi}(t)$ is the probability of component failure and is equal to $1-R_i(t)$. As discussed previously, although Eqs. (5) and (6) are simple to use, the width of the system reliability interval is usually too wide, and the lower bound is too conservative.

Our previous study [13] demonstrates the feasibility of producing narrower system reliability bounds for systems with time-independent loads. The previous study is limited to time-independent problems, and we extend it to time-dependent problems in this work. Details are discussed in Section 3.

3. SYSTEM RELIABILITY ANALYSIS WITH DEPENDENT COMPONENTS AND TIME-DEPENDENT LOADING

The objective of this work is to accurately predict the reliability of systems whose components are designed and manufactured independently by outside suppliers. This study focuses on systems that are subject to time-dependent stochastic loading. The accuracy is achieved with narrower system reliability bounds by incorporating the component failure dependence.

3.1 OVERVIEW

The assumptions of this work are summarized below.

(1) The new product is a series system. Series systems are widely used in mechanical applications. For example, a speed reducer is a series system, which consists of gears, shafts, bearing, and other components. If one component fails, the system will not function properly. The same principle of the new method is also applicable for parallel systems or mix systems with parallel and series subsystems. Details will be discussed in Sec. 3.2.

(2) Component failures are due to excessive load. In other words, if the load of a component is greater than its resistance, the component fails. Both the load and resistance here are in general sense. For example, if the stress of a component is greater than the yield strength, a failure occurs; if the deflection of a component is greater than the allowable deflection, a failure occurs. The general load and resistance can therefore be stresses and strengths, or demand and capacity, respectively.

(3) The load and resistance of a component are statistically independent. The assumption comes from the fact that the material strength is usually independent from component structures and load. For special cases when the assumption does not hold, the

predicted reliability bounds may or may not cover the true reliability. The new method may not be applicable for the special cases.

(4) Component reliabilities are provided by component designers to system designers. If some of the component suppliers are not able to provide their component reliability information, the system designers may request relevant information from the supplier and perform necessary testing. Then they can estimate the component reliability or its range.

(5) System designers may or may not know the distribution types of component resistances, and they do not know the distribution parameters of component resistances. Recall that the component resistance is in a general sense and that component details may be embedded in the component resistance. Without knowing the component details, it may not be possible for system designers obtain the distribution parameters of component resistances. Estimating the unknown distribution parameters is the key issue that this study addresses.

(6) The system load is known to system designers. It is a time-dependent stochastic process. The component load can be obtained through a system-level analysis, such as the use of free-body diagrams. This analysis will be discussed in Sec.3.2. If the system load is time independent, the method in Ref. [13] can be directly used.

(7) System designers may also have other knowledge about component design, such as the range of the factor of safety of a component. To obtain the range, system designers may consult with design handbooks and manuals, rely on their own design experience, or request such information from the component supplier.

The proposed method has three tasks at the system level. The first task is to reconstruct component limit-state functions using the stress-strength interference model. The component limit-state functions are time dependent because of the stochastic process of the system load. The component loads are functions of the system load. This task deals with unknown information about component design.

The second task is to find the distribution of the extreme value of the system load. The purpose of this task is to convert the time-dependent component limit-state functions into their time-invariant counterparts, and the conversion requires the extreme system load. Simulation is used to obtain the samples of time-dependent system load, and saddlepoint approximation (SPA) is used to estimate the cumulative distribution function (CDF) of the extreme system load.

The last task is to establish system reliability optimization models. The objective of the optimization is to find the probability of system failure. The design variables are unknown parameters of the distribution of general component resistances. Note that component design details may be embedded in the general component resistances, but are not required to be found. This safeguards the proprietary information of component suppliers. The constraint functions are those such as component reliabilities and factors of safety of the components. If no knowledge is available about the distribution types of component resistances, system designers may assume the types, for example, a Weibull distribution. The Weibull distribution is selected for two reasons. First, the Weibull distribution is used widely in industry. The Weibull analysis is a standard tool in commercial software for data analysis, and engineers are familiar with the distribution. Second, the Weibull distribution is capable of modeling many types of sample data and could cover a number of distributions by changing its distribution parameters. For example, if the shape parameter is less than 1, the distribution is close to an exponential distribution, which can describe the early failures or infant mortality; if the shape parameter is equal to 2, the distribution becomes a Rayleigh distribution, which indicates the risk of wear-out failure increases steadily over the product’s lifetime; if the shape parameter is between 3 and 4, the distribution is approximate to a normal distribution, which can model rapid wear-out failures during the final period of product life; if the shape parameter is greater than 10, the distribution is similar to an extreme value distribution, which can also model the final period of product life [23, 24]. With an unknown distribution type, assuming a distribution type may affect the system reliability bounds reduction. The Weibull distribution is the first choice due to above reasons. The other way is to assume a number of possible distribution types, and then find the extreme values from the results of all the assumed distribution types.

A flowchart of the proposed methodology is shown in Fig. 2.

3.2 CONSTRUCTION OF COMPONENT LIMIT-STATE FUNCTIONS

The objective of this task is to reconstruct component limit-state functions, which provide an effective way to deal with incomplete information about component design. Let the system load be $L(\tau)$, where $\tau \in [0,t]$, and component load be $w_i L(\tau)$ for component i. Constant w_i indicates the load that component i shares. w_i can be determined by a system analysis, such as a force analysis by a simplified free-body diagram of component i in Fig. 3, where $L_i(\tau)$ is the load of the component and is equal to $w_i L(\tau)$.

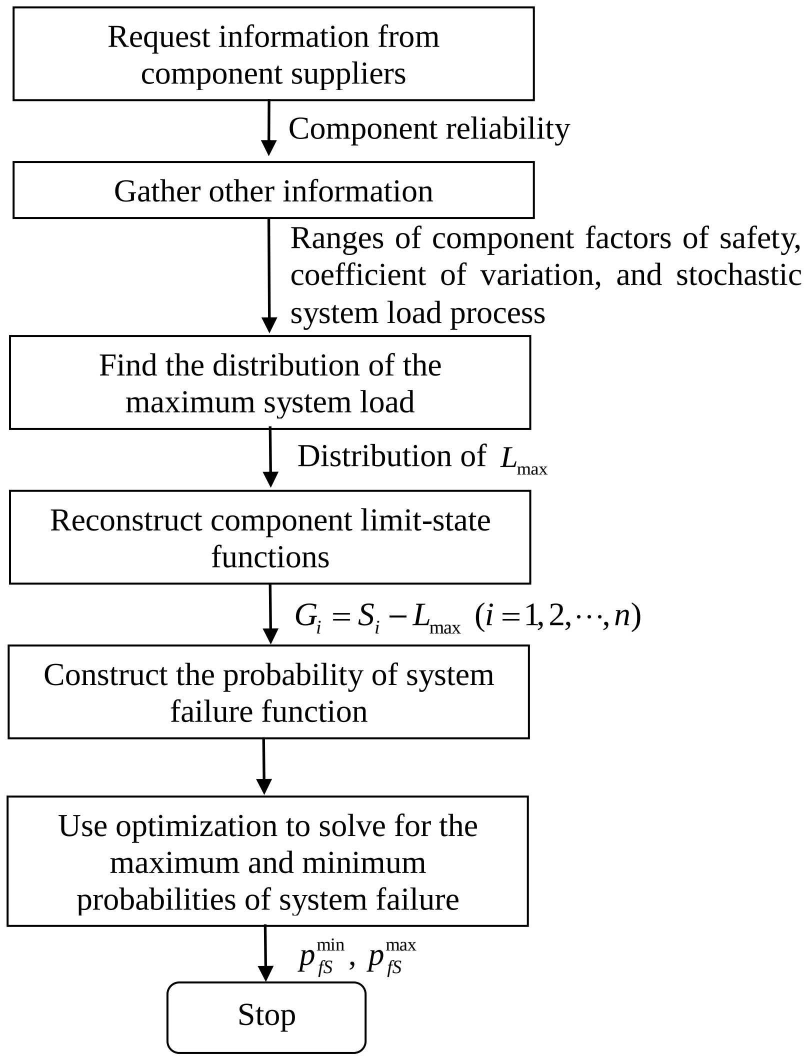

Figure 2 Flowchart of the proposed methodology

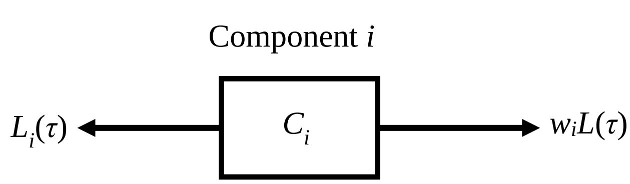

Figure 3 Simplified free-body diagram of component i

Using the strength-stress interference theory, system designers reconstruct the component limit-state function as follows:

$$G_i' = S_i' - w_i L(\tau),\ \ \tau \le t \tag{7}$$

where S_i' is the allowable component resistance. Dividing by w_i, Eq. (7) is rewritten as

$$G_i = S_i - L(\tau),\ \ \tau \le t \tag{8}$$

where $S_i = S_i'/w_i$, and $G_i = G_i'/w_i$. S_i is the general resistance of the component. It is usually a function of component details, such as the actual material strength and component dimensions, which may be proprietary to the component supplier. Such proprietary details do not explicitly appear in Eq. (8), and consequently, the proprietary information is safeguarded.

In this work, the equation $G_i = S_i - L$ is a general representation of the limit-state function of a component. S_i is the general resistance of the component and L is the system load. G_i is therefore a linear function of L.

Eq. (8) can represent an actual component limit-state function that is not linear with respect to L. If the load is in a nonlinear form $h(L)$, then the actual component limit-state function established by a component supplier is

$$G_i = S_i - h(L) \tag{9}$$

We can solve for L by letting $S_i - h(L) = 0$ and then express L as a function of S_i given by

$$L = W(S_i) \tag{10}$$

Then the limit-state function G_i can be modified as

$$G_i' = W(S_i) - L \tag{11}$$

where $W(S_i)$ is the new general resistance. Then we obtain a new component limit-state function with a linear form of L. This new component limit-state function on the component supplier side is consistent with the one in Eq. (8) assumed by system designers.

With the assumption that the load $L(\tau)$ and resistance S_i are independent in Eq. (8), the probability of component failure is then given by

$$\begin{aligned} p_{fi}(t) &= \Pr\left\{S_i < L(\tau),\ \text{for any } \tau \in [0,t]\right\} \\ &= \Pr\left\{S_i < L_{\max}\right\} \\ &= \int_0^1 F_{S_i}(l)dF_{L_{\max}}(l) \end{aligned} \quad (12)$$

where $F_{S_i}(\cdot)$ is the CDF of S_i, and $F_{L_{\max}}(\cdot)$ is the CDF of the maximum system load $L_{\max}$. If system designers knew the distribution of S_i, they could use Eq. (12) to reproduce the same component reliability as the one supplied by component designers.

For a series system, with all the component limit-state functions available, the system failure region is determined by the union $\{G_1 < 0 \bigcup G_2 < 0 \bigcup ... \bigcup G_n < 0\}$. Then the probability of system failure is given by

$$\begin{aligned} p_{fS}(t) &= \Pr\left\{\bigcup_{i=1}^{n} S_i < L_{\max}\right\} \\ &= \Pr\left\{S_{\min} < L_{\max}\right\} \\ &= \int_0^1 F_{S_{\min}}(l)dF_{L_{\max}}(l) \end{aligned} \quad (13)$$

where $S_{\min}$ is the minimum general resistance of all the components. The CDF of $S_{\min}$ is

$$\begin{aligned} F_{S_{\min}}(s) &= \Pr(S_{\min} < s) \\ &= 1 - \Pr(S_{\min} \geq s) \\ &= 1 - \prod_{i=1}^{n} \Pr(S_i \geq s) \\ &= 1 - \prod_{i=1}^{n} \left[1 - F_{S_i}(s)\right] \end{aligned} \tag{14}$$

Then the probability of series system failure is given by

$$p_{fS}(t) = \int_0^1 \left\{1 - \prod_{i=1}^{n} \left[1 - F_{S_i}(l)\right]\right\} dF_{L_{\max}}(l) \tag{15}$$

Although we focus on series systems in this work, the methodologies could be extended to other system configurations. For a parallel system, given all the component limit-state functions, the system failure region is determined by the union $\{G_1 < 0 \bigcap G_2 < 0 \bigcap ... \bigcap G_n < 0\}$. Then the probability of system failure is given by

$$\begin{aligned} p_{fS}(t) &= \Pr\left\{\bigcap_{i=1}^{n} S_i < L_{\max}\right\} \\ &= \Pr\left\{S_{\max} < L_{\max}\right\} \\ &= \int_0^1 F_{S_{\max}}(l) dF_{L_{\max}}(l) \end{aligned} \tag{16}$$

where $S_{\max}$ is the maximum general resistance of all the components. The CDF of $S_{\max}$ is

$$F_{S_{\max}}(s) = \Pr(S_{\max} < s) = \prod_{i=1}^{n} F_{S_i}(s) \tag{17}$$

Then the probability of system failure is given by

$$p_{fS}(t) = \int_0^1 \prod_{i=1}^{n} F_{S_i}(l) dF_{L_{\max}}(l) \tag{18}$$

For a mix system, which is a combination of series and parallel subsystems, the equations for series subsystems in Eq. (15) and those for parallel subsystems in Eq. (18) can also be combined. For example, for a system in Fig. 4, we can first use Eq. (18) to obtain the probability of failure of the parallel subsystem with components C_2 and C_3. Then the parallel subsystem could be viewed as a component C in series with component C_1. Then we can use Eq. (15) to obtain the probability of system failure.

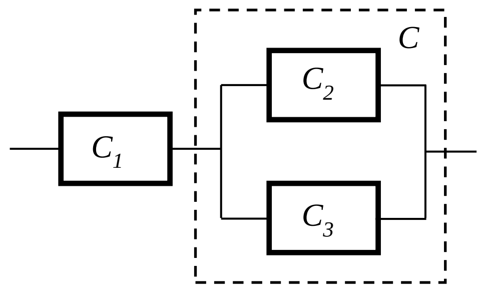

Figure 4 A mix system

Note that no matter how complex the component limit-state functions are and no matter how many failure modes a component may have, system designers reconstruct only one component limit-state function as shown in Eq. (8), which is the difference between the general resistance and load. The reconstructed limit-state function is linear with respect to the two random variables. As a result, the computation is very efficient. The proposed method does not call any original component limit-state functions, and no complex analyses, such as finite element analysis, are needed. The optimization process only requires evaluating Eq. (15) or Eq. (18), which involves a simple integral.

For any system, if the CDF of $L_{\max}$ and the CDFs of S_i ($i = 1, 2, \cdots, n$) were available, p_{fS} would then be obtained. As discussed previously, system designers only know the system load and they do not know the distributions of component resistances.

Thus for a series system, both $F_{L_{\max}}(l)$ and $F_{S_i}(s)$ are unknown in Eq. (15). As will be discussed in Section 3.3, simulation is used to obtain $F_{L_{\max}}(l)$. And as will be shown in Section 3.4, optimization is used to deal with the unknown CDF $F_{S_i}(s)$.

3.3 DISTRIBUTION OF THE EXTREME SYSTEM LOADING $L_{\max}$

The objective of this task is to find the CDF of the extreme system load $L_{\max}$ for a given period of time $[0,t]$. Time-dependent stochastic process loading is commonly encountered. For example, a ship is subjected to stochastic wave loading that varies over time, a hydrokinetic turbine blade is subjected to a time-variant river flow loading, and a wind turbine is subjected to time-dependent wind loading. System designers first draw samples of $L_{\max}$ by simulation and then find the CDF of $L_{\max}$ by saddlepoint approximation.

Finding the distribution of the extreme value of a general stochastic process is a challenging task [25]. Even for a commonly used Gaussian process, there is no analytical form for such a distribution. A general stochastic process loading $L(\tau)$ can be approximated by the Karhunen-Loève (K-L) expansion [26-28]. After $L(\tau)$ is expanded with respect to a number of random variables, samples are generated for the random variables, leading to trajectories (sample realizations) of $L(\tau)$. For each trajectory, its maximum value is found. Then the samples of $L_{\max}$ are available.

Now we discuss a special case where $L(\tau)$ is a Gaussian process. The expansion optimal linear estimation method (EOLE) is applied to generate samples. EOLE is a special case of the K-L expansion [29] for a Gaussian process.

Suppose $L(\tau) \sim \text{GP}(\mu_L(\tau), \sigma_L(\tau), \rho_L(\tau_1, \tau_2))$, where GP stands for a Gaussian process, $\mu_L(\tau)$ is the mean function, $\sigma_L(\tau)$ is the standard deviation function, and $\rho_L(\tau_1, \tau_2)$ is the function of the autocorrelation coefficient. After discretizing $[0, t]$ into m points $[\tau_i]_{i=1,2,\cdots,m}$, $L(\tau)$ is expanded as [30]

$$L(\tau) = \mu_L(\tau) + \sigma_L(\tau) \sum_{i=1}^{p} \frac{U_i}{\sqrt{\eta_i}} \boldsymbol{\varphi}_i^T \boldsymbol{\rho}_L(\tau) \tag{19}$$

where η_i and $\boldsymbol{\varphi}_i^T$ are the eigenvalues and eigenvectors of the correlation matrix $\boldsymbol{\rho}$ with element $\rho_{ij} = \rho(\tau_i, \tau_j)$, $i, j = 1, 2, ..., m$. $\boldsymbol{\rho}_L(\tau) = \left[\rho(\tau, \tau_1), ..., \rho(\tau, \tau_m)\right]^T$, and $p \le m$ is the number of terms of the expansion. U_i $(i = 1, 2, \cdots p \le m)$ are independent standard normal random variables. Then the random samples of U_i are generated to reproduce sample trajectories of $L(\tau)$. After j simulations, j trajectories as well as their maximum values are obtained. Therefore, samples of $L_{\max}$ are available. With these samples, SPA is used to estimate the CDF of $L_{\max}$.

SPA is easy to use [31, 32] and accurate [33, 34] for the CDF approximation. The CDF estimation relies on the cumulant generating function (CGF). The power expansion of the CGF of $L_{\max}$ is given by [31]

$$K_{L_{\max}}(\xi) = \sum_{i=1}^{r} k_i \frac{\xi^i}{i!} \tag{20}$$

where k_i is i-th cumulant. In this work, we use the first four cumulants, which are given by [35]

$$\begin{cases} k_1 = \dfrac{s_1}{n} \\ k_2 = \dfrac{ns_2 - s_1^2}{n(n-1)} \\ k_3 = \dfrac{2s_1^3 - 3ns_1s_2 + n^2 s_3}{n(n-1)(n-2)} \\ k_4 = \dfrac{-6s_1^4 + 12ns_1^2 s_2 - 3n(n-1)s_2^2}{n(n-1)(n-2)(n-3)} + \dfrac{-4n(n+1)s_1s_3 + n^2(n+1)s_4}{n(n-1)(n-2)(n-3)} \end{cases} \tag{21}$$

where $s_r\,(r = 1, 2, 3, 4)$ is the sum of the r-th power of the samples. It is obtained by $s_r = \sum_{i=1}^{n} l_i^r$.

To obtain the CDF of $L_{\max}$, we must find the saddlepoint ξ_s, which is the solution to the equation

$$K'_{L_{\max}}(\xi) = l_{\max} \tag{22}$$

where $K'_{L_{\max}}(\cdot)$ is the first derivative of the CGF. After we obtain ξ_s, the CDF of $L_{\max}$ is approximated by

$$F_{L_{\max}}(l) = \Pr(L_{\max} \le l) = \Phi(z) + \phi(z)(\frac{1}{z} - \frac{1}{v}) \tag{23}$$

where

$$z = \operatorname{sign}(\xi_s)\left\{2\left[\xi_s l - K_{L_{\max}}(\xi_s)\right]\right\}^{1/2} \tag{24}$$

$$v = \xi_s[K''_{L_{\max}}(\xi_s)]^{1/2} \tag{25}$$

where $\operatorname{sign}(\xi_s) = +1, -1,$ or 0, if the saddlepoint ξ_s is positive, negative, or zero. $K''_{L_{\max}}(\cdot)$ is the second derivative of the CGF.

Plugging $F_{L_{\max}}(l)$ into Eqs. (12) and (15), component reliabilities and system reliability with respect to time could be calculated if the CDFs of general component resistances were known. In Section 3.4, optimization models are developed to obtain the probability of system failure bounds with unknown general component resistances.

3.4 OPTIMIZATION MODEL OF SYSTEM RELIABILITY

The goal of this task is to obtain narrower bounds of the probability of system failure, which are found by using optimization models. In this work, we use optimization merely as a numerical solver to find possible extreme values of the probability of system failure because it is easy to incorporate all information available by treating it as constraints in the optimization model.

In our proposed optimization model, the design variables are the unknown distribution parameters of the general component resistances, denoted by $\mathbf{d}$. Note that the system designers may or may not know the distribution types of the general component resistances. Thus, they may assume the distribution types. For example, if system designers know that the general component resistances follow normal distributions, the design variables in the optimization model will be means and standard deviations $\mathbf{d} = (\mathbf{d}_1, ..., \mathbf{d}_n) = (\mu_{S_1}, \sigma_{S_1}, ..., \mu_{S_n}, \sigma_{S_n})$. If system designers do not know the distribution types, they may use two parameter Weibull distributions, and then the design variables become shape parameters and scale parameters $\mathbf{d} = (\mathbf{d}_1, ..., \mathbf{d}_n) = (k_{S_1}, \lambda_{S_1}, ..., k_{S_n}, \lambda_{S_n})$.

The objective function of the optimization model is the probability of system failure in Eq. (15). It is denoted by $p_{fS}(\mathbf{d}; L_{\max})$ and is a function of known system stochastic process extreme load $L_{\max}$ obtained by simulation in Section 3.3 and unknown design variables $\mathbf{d}$. Maximizing $p_{fS}(\mathbf{d}; L_{\max})$ produces the maximum probability of system failure $p_{fS}^{\max}$ while minimizing $p_{fS}(\mathbf{d}; L_{\max})$ produces the minimum probability of system failure $p_{fS}^{\min}$.

Multiple constraint functions are included in the optimization model. A probability of component failure gives an equality constraint. From Eq. (12), n equality constraints are obtained by

$$h_i(\mathbf{d}; L_{\max}) = \int_0^1 F_{S_i}(l) dF_{L_{\max}}(l) = p_{fi},\ i = 1, 2, ..., n \tag{26}$$

Although the components' actual factors of safety used by suppliers may not be provided, system designers may estimate their ranges. The factor of safety is defined as a ratio of average resistance to average load [36] ($n_{si} = \mu_{S_i} / \mu_L$). We use $n_{si}^{\min}$ and $n_{si}^{\max}$ to represent the minimum and maximum of the factors of safety, respectively. From $n_{si}^{\min} \le n_{si}(\mathbf{d}; L_{\max}) \le n_{si}^{\max}$, $2n$ inequality constraints are obtained by

$$g_i(\mathbf{d}; L_{\max}) = n_{si}^{\min} - n_{si}(\mathbf{d}; L_{\max}) \le 0,\ i = 1, 2, \ldots, n \tag{27}$$

$$g_{i+n}(\mathbf{d}; L_{\max}) = n_{si}(\mathbf{d}; L_{\max}) - n_{si}^{\max} \le 0,\ i = 1, 2, \ldots, n \tag{28}$$

Besides, the coefficient of variation, which is defined as the ratio of standard deviation to the mean of component resistance ($c_i = \sigma_{S_i} / \mu_{S_i}$), may also be estimated by system designers. We use $c_i^{\min}$ and $c_i^{\max}$ to represent the minimum and maximum of the coefficient of variation, respectively. From $c_i^{\min} \le c_i(\mathbf{d}; L_{\max}) \le c_i^{\max}$, other $2n$ inequality constraints are obtained by

$$g_{i+2n}(\mathbf{d}; L_{\max}) = c_i^{\min} - c_i(\mathbf{d}; L_{\max}) \le 0,\ i = 1, 2, \ldots, n \tag{29}$$

$$g_{i+3n}(\mathbf{d}; L_{\max}) = c_i(\mathbf{d}; L_{\max}) - c_i^{\max} \le 0,\ i = 1, 2, \ldots, n \tag{30}$$

Then the optimization model for the minimum probability of system failure is

$$\begin{cases} \min\limits_{\mathbf{d}} \ p_{fS}(\mathbf{d}; L_{\max}) \\ \text{subject to} \\ h_i(\mathbf{d}; L_{\max}) = \int_0^1 F_{S_i}(l) dF_{L_{\max}}(l) = p_{fi}, \ i = 1, 2, ..., n \\ g_i(\mathbf{d}; L_{\max}) = n_{si}^{\min} - n_{si}(\mathbf{d}; L_{\max}) \le 0 \\ g_{i+n}(\mathbf{d}; L_{\max}) = n_{si}(\mathbf{d}; L_{\max}) - n_{si}^{\max} \le 0 \\ g_{i+2n}(\mathbf{d}; L_{\max}) = c_i^{\min} - c_i(\mathbf{d}; L_{\max}) \le 0 \\ g_{i+3n}(\mathbf{d}; L_{\max}) = c_i(\mathbf{d}; L_{\max}) - c_i^{\max} \le 0 \end{cases} \tag{31}$$

For the maximum probability of system failure, we just change the objective function from $\min\limits_{\mathbf{d}} p_{fS}(\mathbf{d}; L_{\max})$ to $\max\limits_{\mathbf{d}} p_{fS}(\mathbf{d}; L_{\max})$ in Eq. (31). The two optimization models will produce the bounds of probability of system failure. The predicted probability of system failure bounds cover the exact probability of system failure if the exact design point, falls into the feasible region defined by the constraint functions. Therefore, system designers should carefully select the parameters (e.g. factors of safety and the coefficient of variation) for the constraint functions based on their experience, their expertise about component design, and the design standards in their specific areas. For example, the most important constraints are those on component factors of safety. If system designers know the specific area of the component design, they can consult with the design handbooks and manuals in that area and obtain the range of the component factor of safety. They may also rely on their own design experience to estimate the range of the component factor of safety. If it is difficult to estimate such a range, they may request such information from the component supplier. In some cases, this is possible because the component supplier only provides the range of the component factor of safety, not the exact value. In other cases, providing such information is mandatory for the component supplier. If it is impossible for system designers to come up with a narrow range for the factor of safety, they may loosen the range at the cost of having wider system reliability bounds.

4. EXAMPLES

Two examples are provided in this section. One is for a system with identical components and the other is for a system with different components.

4.1 EXAMPLE 1: A SYSTEM CONSISTS OF IDENTICAL COMPONENTS

As shown in Fig. 5, five identical components provided by a supplier are subjected to a same stochastic process load $L(\tau)$. The distribution of the component resistance is $S \sim N(4000, 130^2)$ kN. Component designers know the distribution type and parameters of the component resistance, while the system designers only know the distribution type. Both the component designers and system designers know the system load $L(\tau)$, which is a Gaussian process with $L \sim \text{GP}(2500, 350^2, \rho_L(\tau_1, \tau_2))$ kN, in which $\rho_L(\tau_1, \tau_2) = \exp(-(\tau_2 - \tau_1)^2 / \zeta^2)$, $\zeta = 0.5$.

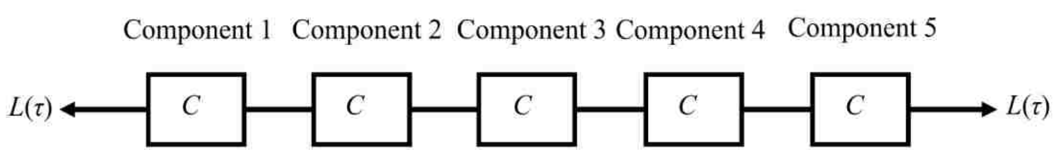

Figure 5 Five identical components sharing same load

Component designers can use a physics-based reliability approach to construct the limit-state function as

$$G = S - L(\tau), \quad \tau \in [0, t] \tag{32}$$

As shown in Fig. 6, the CDFs of the maximum load $L_{\max}$ with different periods of time $[0,1]$ yr, $[0,2]$ yr,..., $[0,12]$ yr are obtained by EOLE and SPA as discussed in

Section 3.3. The first curve is the CDF of $L_{\max}$ for time period $[0,1]$ yr, and the last curve is the CDF of $L_{\max}$ for time period $[0,12]$ yr. Since $L_{\max}$ is the maximum value of load $L(\tau)$ for a certain period of time, it is therefore a non-decreasing function of the duration of the period of time. For example, $L_{\max}$ in five years is always greater than or equal to that in two years. It is the reason that the CDF curves of $L_{\max}$ shift to the right as the time interval increases.

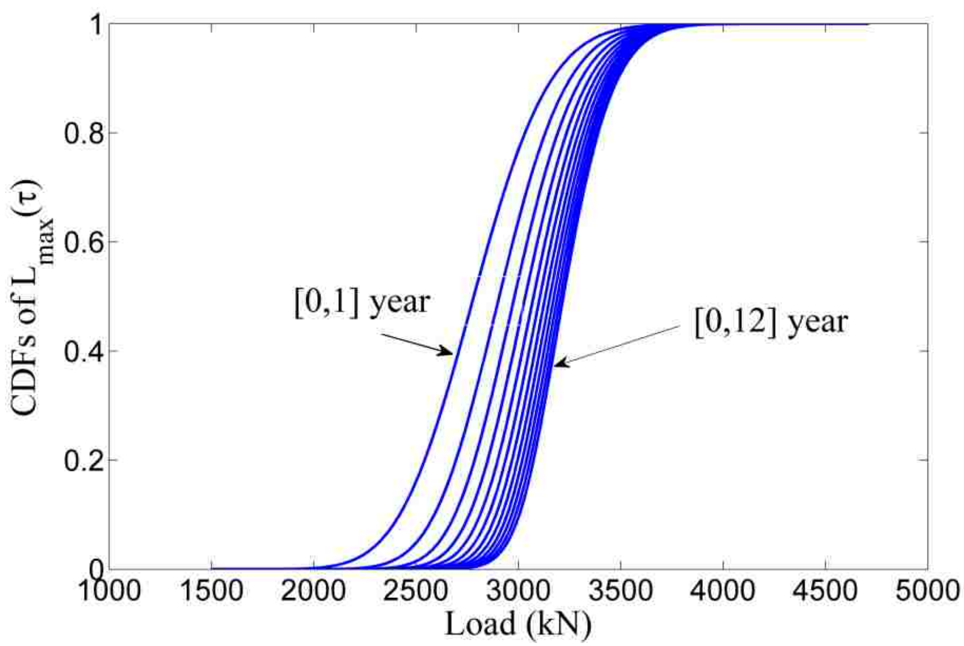

Figure 6 CDFs of maximum load $L_{\max}$

According to Eq. (12), with the CDFs of $L_{\max}$ and the distribution of the component resistance S available, component designers calculate the probability of component failure p_{fC}, which is provided to system designers as shown in Table 1 and Fig. 7. Since the five components are identical, their probabilities of failure are the same.

Along with other information estimated by system designers, all the information available to the system designers is summarized in Table 2.

Table 1 Probability of component failure with respect to time

$[0,t]$ (yr)	$[0,1]$	$[0,2]$	$[0,3]$	$[0,4]$	$[0,5]$	$[0,6]$
$p_{fC}\,(10^{-4})$	1.7236	2.8982	4.1139	5.5065	6.6119	7.8197
$[0,t]$ (yr)	$[0,7]$	$[0,8]$	$[0,9]$	$[0,10]$	$[0,11]$	$[0,12]$
$p_{fC}\,(10^{-4})$	9.1277	10.267	11.419	12.570	13.580	14.808

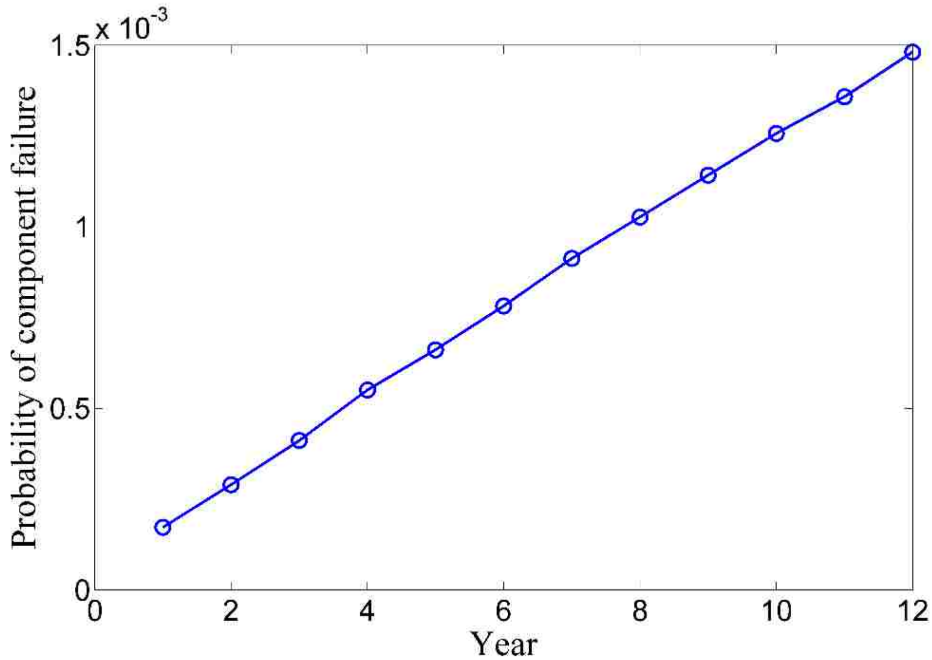

Figure 7 Probability of component failure with respect to time

For the system designers, the task is to assess the probability of system failure using the information in Table 2. In this example, the system designers know the distribution type of the component resistance, which is a normal distribution. Yet, they do not know the distribution parameters. System designers assume that the component resistance follow normal distribution $S_R \sim N(a,b^2)$ kN. There are therefore two design variables, which are the mean a and the standard deviation b.

Table 2 Information available to the system designers

Variables	**Values**
Probability of component failure p_{fC}	Table 1
Distribution type of component resistance	Normal distribution
Factor of safety of component n_s	$[1.5, 2.2]$
Coefficient of variation of component resistance c	$[0.025, 0.12]$
Distribution of system load L	$\mathrm{GP}(\mu_L, \sigma_L^2, \rho)$, $\mu_L = 2500$ kN, $\sigma_L = 350$ kN

System designers reconstruct the limit-state function of the component as

$$G = S_R - L_{\max} \tag{33}$$

According to Eq. (15), the objective function, namely, the probability of system failure is

$$p_{fS} = \int_0^1 \left\{1 - \left[1 - \Phi\left((l-a)/b\right)\right]^5\right\} dF_{L_{\max}}(l) \tag{34}$$

Then the optimization model for the minimum probability of system failure is shown in Eq. (35).

$$\begin{cases} \min\limits_{\mathbf{d}} \ p_{fS}(\mathbf{d}; L_{\max}) \\ \text{subject to} \\ h(\mathbf{d}; L_{\max}) = \int_0^1 \Phi\left((l-a)/b\right) dF_{L_{\max}}(l) = p_{fC} \\ g_1(\mathbf{d}; L_{\max}) = 1.5 - a/\mu_L \le 0 \\ g_2(\mathbf{d}; L_{\max}) = a/\mu_L - 2.2 \le 0 \\ g_3(\mathbf{d}; L_{\max}) = 0.025 - b/a \le 0 \\ g_4(\mathbf{d}; L_{\max}) = b/a - 0.12 \le 0 \end{cases} \tag{35}$$

For the maximum probability of system failure, system designers just change the objective function from $\min_{\mathbf{d}} p_{fS}(\mathbf{d}; L_{\max})$ to $\max_{\mathbf{d}} p_{fS}(\mathbf{d}; L_{\max})$ in Eq. (35). The trust-region-reflective algorithm is used to find the minimum and maximum probability of system failure. Both Table 3 and Fig. 8 show the results from the proposed method and the results from traditional method in Eq. (6). The exact value is also calculated by assuming that all the information (the distributions of S and $L(\tau)$) were known to system designers. Both methods indicate an increasing trend of the probability of system failure with respect to time. They also show that the bounds from the proposed method are much narrower than those from the traditional method. In fact, the average reduction of the reliability bound width is about 74%. The exact value is also contained in the bounds of the probability of system failure from the proposed method. Therefore, the accuracy is improved by applying the proposed method.

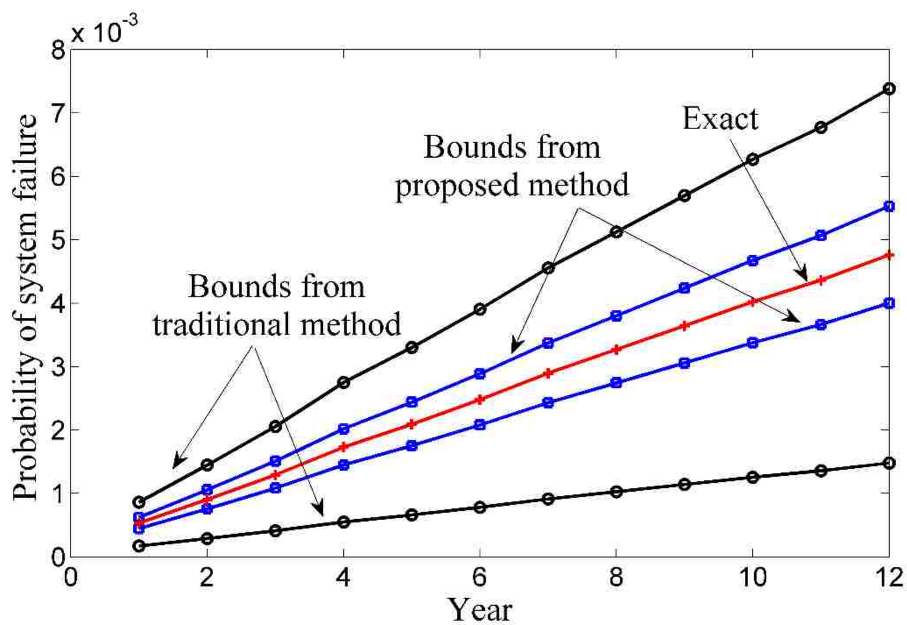

Figure 8 Bounds contrast from traditional and proposed methods

Table 3 Bounds contrast of probability of system failure

$[0,t]$ (years)	Traditional method Bounds of p_{fS} (10^{-4})	Proposed method Bounds of p_{fS} (10^{-4})	Exact (10^{-4})	Reduction of bound width
$[0,1]$	[1.7236, 8.6151]	[4.4691, 6.2438]	5.3343	74.25%
$[0,2]$	[2.8982, 14.483]	[7.5773, 10.57]	9.0424	74.17%
$[0,3]$	[4.1139, 20.553]	[10.833, 15.086]	12.923	74.13%
$[0,4]$	[5.5065, 27.502]	[14.502, 20.185]	17.295	74.16%
$[0,5]$	[6.6119, 33.016]	[17.534, 24.368]	20.906	74.12%
$[0,6]$	[7.8197, 39.037]	[20.8, 28.88]	24.793	74.12%
$[0,7]$	[9.1277, 45.555]	[24.311, 33.733]	28.97	74.14%
$[0,8]$	[10.267, 51.232]	[27.434, 38.032]	32.685	74.13%
$[0,9]$	[11.419, 56.965]	[30.583, 42.38]	36.443	74.1%
$[0,10]$	[12.57, 62.691]	[33.754, 46.736]	40.212	74.1%
$[0,11]$	[13.58, 67.715]	[36.639, 50.668]	43.636	74.09%
$[0,12]$	[14.808, 73.823]	[39.993, 55.278]	47.62	74.1%

4.2 EXAMPLE 2: A SYSTEM CONSISTS OF DIFFERENT COMPONENTS

The system configuration is shown in Fig. 9. A stochastic process load $L(\tau)$ is applied to a steel beam, which is fixed by four bolts on the ground. The four bolts are identical. The beam and bolts are supplied by two independent companies. Both the component designers and system designers know the system load $L(\tau)$, which is a Gaussian process with $L \sim \text{GP}(1800, 200^2, \rho_L(\tau_1, \tau_2))$ kN, in which $\rho_L(\tau_1, \tau_2) = \exp(-(\tau_2 - \tau_1)^2 / \zeta^2)$, $\zeta = 0.5$. The force analysis indicates that only the beam and the two bolts on the right are affected by the load $L(\tau)$. The distribution types and parameters of component resistances are only known to the component designers.

Figure 9 System configuration

The designers of the beam consider excessive bending stress and excessive deflection as two failure modes. The information available to the beam designers is summarized in Table 4.

Table 4 Information available to the beam designers

Variables	**Value**
Yield stress distribution	$S_{y1} \sim \text{In}N(1537, 2.53) \times 10^{-2}$ kPa
Elastic deflection distribution	$\delta_{y1} \sim \text{In}N(-5.2, 3.68) \times 10^{-2}$ m
Modulus of elasticity E	1.5×10^{8} kN/m^2
Length h	$h \sim N(3, 0.002)$ m
Width a	$a \sim N(0.2, 0.0005)$ m
Thickness b	$b \sim N(0.25, 0.0001)$ m
Distribution of system load L	$\text{GP}(\mu_L, \sigma_L^2, \rho)$, $\mu_L = 1800$ kN, $\sigma_L = 200$ kN

With a physics-based approach, the designers of the beam construct the limit-state functions as follows:

$$\begin{cases} G_{1,1} = \delta_{y1} - \dfrac{4h^3}{Eba^3} L(\tau) \\ G_{1,2} = S_{y1} - \dfrac{6h}{a^2\,\mathrm{b}} L(\tau) \end{cases} \quad \tau \in [0, t] \tag{36}$$

where $G_{1,1} < 0$ indicates an excessive deflection, and $G_{1,2} < 0$ indicates an excessive bending stress. Thus, the probability of failure is obtained by $p_{f1} = \Pr(G_{1,1} < 0 \bigcup G_{1,2} < 0)$.

Then with the CDFs of the maximum load $L_{\max}$ obtained by EOLE and SPA in Fig. 10, using Monte-Carlo simulation (MCS) [37], the designers of the beam calculate the probability of beam failure, which is provided to system designers as shown in Table 5.

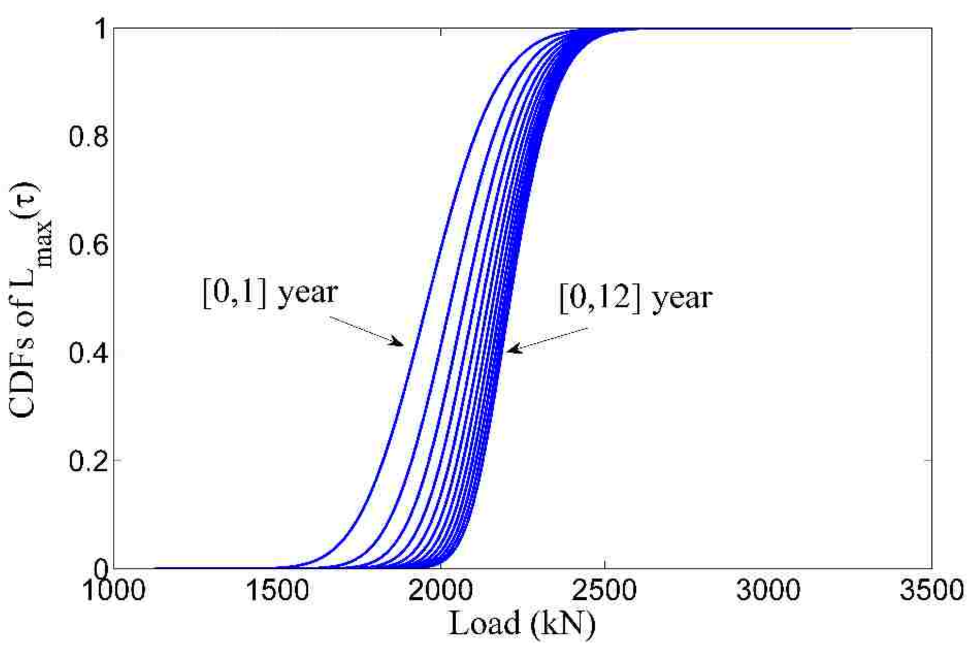

Figure 10 CDFs of maximum load $L_{\max}$

Table 5 Probability of beam failure with respect to time

$[0,t]$ (yr)	$[0,1]$	$[0,2]$	$[0,3]$	$[0,4]$	$[0,5]$	$[0,6]$
$p_{f1}(10^{-4})$	5.61	9.66	13.65	17.94	21.92	26.01
$[0,t]$ (yr)	$[0,7]$	$[0,8]$	$[0,9]$	$[0,10]$	$[0,11]$	$[0,12]$
$p_{f1}(10^{-4})$	30.12	34.09	38.06	42.1	46	49.88

Similarly, the designers of the bolts consider excessive bearing stress as the failure mode. The information available to the bolt designers is summarized in Table 6.

Table 6 Information available to the bolt designers

Variables	**Value**
Yield stress distribution	$S_{y2} \sim \text{In}N(1563, 1.59)\times 10^{-2}$ kPa
Radius r	$r \sim N(0.02, 0.0001)$ m
Distance from bolt to beam d	$d \sim N(0.5, 0.0015)$ m
Distribution of system load L	$\text{GP}(\mu_L, \sigma_L^2, \rho)$, $\mu_L = 1800$ kN , $\sigma_L = 200$ kN

The designers of the bolt construct the limit-state function as

$$G_2 = S_{y2} - \frac{h}{2d\pi r^2} L(\tau) \ \ \tau \in [0,t] \tag{37}$$

where $G_2 < 0$ represents an excessive stress in the bolt. Thus, the probability of failure is obtained by $p_{f2} = \Pr(G_2 < 0)$. Then with the CDFs of the maximum load $L_{\max}$ available in Fig. 10, bolt designers can use MCS to calculate the probability of bolt failure, which is also provided to system designers as shown in Table 7. The probabilities of failure of both beam and bolt are also shown in Fig. 11.

Table 7 Probability of bolt failure with respect to time

$[0,t]$ (yr)	$[0,1]$	$[0,2]$	$[0,3]$	$[0,4]$	$[0,5]$	$[0,6]$
$p_{f2}(10^{-4})$	4.2	7.64	11.23	14.44	17.82	21.31
$[0,t]$ (yr)	$[0,7]$	$[0,8]$	$[0,9]$	$[0,10]$	$[0,11]$	$[0,12]$
$p_{f2}(10^{-4})$	25.12	28.53	31.74	35.17	38.55	42.07

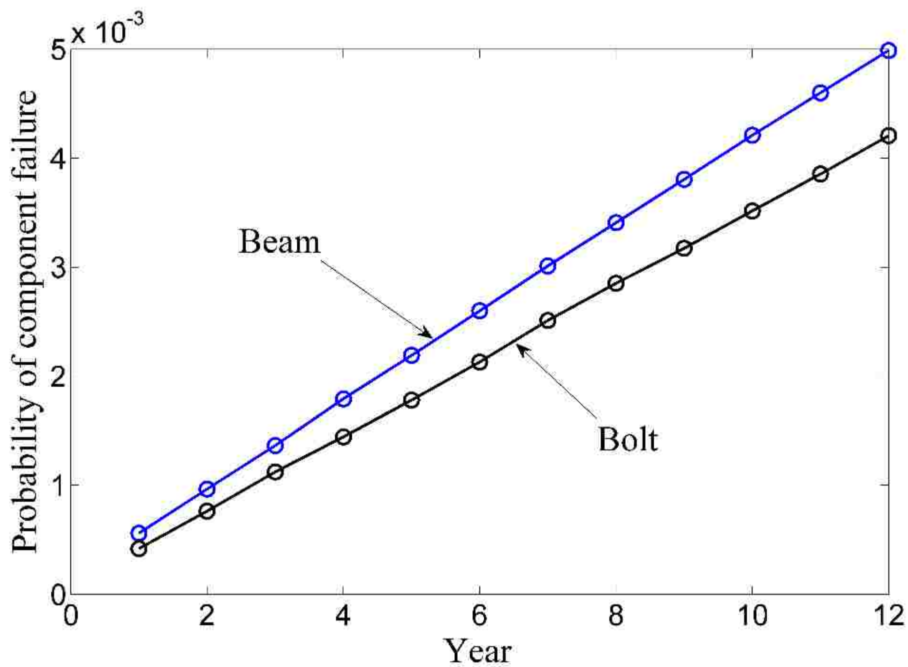

Figure 11 Probabilities of component failure with respect to time

Note that at the component design level, component reliability is calculated with all details, such as the dimensions and material properties. These details appear in the component limit-state functions in Eqs. (36) and (37).

At the system design level, although system designers have no access to the above design details, with the information available to them as shown in Table 8, they reconstruct the limit-state functions of the beam and bolt as follows:

$$\begin{cases} G_1 = S_1 - L_{\max} \\ G_2 = S_2 - L_{\max} \end{cases} \tag{38}$$

where G_1 is for the beam and G_2 is for a bolt.

Table 8 Information available to the system designers

Variables		**Values**
Beam	Probability of failure p_{f1}	Table 5
	Factor of safety n_{s1}	$[1.0, 2.0]$
	Coefficient of variation of resistance c_1	$[0.01, 0.05]$
Bolt	Probability of failure p_{f2}	Table 7
	Factor of safety n_{s2}	$[1.0, 2.0]$
	Coefficient of variation of resistance c_2	$[0.01, 0.05]$
Distribution of system load L		$\mathrm{GP}(\mu_L, \sigma_L^2, \rho)$, $\mu_L = 1800$ kN, $\sigma_L = 200$ kN

Although there are two failure modes or two limit-state functions for the beam, system designers need just one limit-state function, which is G_1 in Eq. (38). Note that no component design details are shown in Eq. (38). Without these design details such as distributions of material strengths and component dimensions, system designers decide to use two-parameter Weibull distributions for the general component resistances. The distributions are denoted by $S_1 \sim \mathrm{WB}(k_{S_1}, \lambda_{S_1})$ and $S_2 \sim \mathrm{WB}(k_{S_2}, \lambda_{S_2})$.

The probability density function of a two-parameter Weibull distribution is

$$f(x; \lambda_{S_i}, k_{S_i}) = \begin{cases} \dfrac{k_{S_i}}{\lambda_{S_i}} \left(\dfrac{x}{\lambda_{S_i}} \right)^{k_{S_i}-1} e^{-(x/\lambda_{S_i})^{k_{S_i}}} & x \geq 0 \\ 0 & x < 0 \end{cases} \tag{39}$$

where k_{S_i} is the shape parameter, and λ_{S_i} is the scale parameter. Then the design variables are $\mathbf{d} = (k_{S_1}, \lambda_{S_1}, k_{S_2}, \lambda_{S_2})$. The mean and standard deviation of the Weibull distribution are calculated by

$$\mu_i = \lambda_{S_i} \Gamma(1 + 1 / k_{S_i}) \tag{40}$$

$$\sigma_i = \lambda_{S_i} \sqrt{\left[\Gamma(1 + 2 / k_{S_i}) - \Gamma^2(1 + 1 / k_{S_i}) \right]} \tag{41}$$

With Eq. (15) as the objective function, the optimization model for the minimum probability of system failure is

$$\begin{cases} \min\limits_{\mathbf{d}} \ p_{fS}(\mathbf{d}; L_{\max}) \\ \text{subject to} \\ h_i(\mathbf{d}; L_{\max}) = \int_0^1 F_{S_i}(l) dF_{L_{\max}}(l) = p_{fi}, \ i = 1, 2 \\ g_i(\mathbf{d}; L_{\max}) = 1.0 - \dfrac{\mu_i}{\mu_L} \le 0 \\ g_{i+2}(\mathbf{d}; L_{\max}) = \dfrac{\mu_i}{\mu_L} - 2.0 \le 0 \\ g_{i+4}(\mathbf{d}; L_{\max}) = 0.01 - \dfrac{\sigma_i}{\mu_i} \le 0 \\ g_{i+6}(\mathbf{d}; L_{\max}) = \dfrac{\sigma_i}{\mu_i} - 0.05 \le 0 \end{cases} \tag{42}$$

For the maximum probability of system failure, system designers just change the objective function from $\min\limits_{\mathbf{d}} \ p_{fS}(\mathbf{d}; L_{\max})$ to $\max\limits_{\mathbf{d}} \ p_{fS}(\mathbf{d}; L_{\max})$ in Eq. (42). The trust-region-reflective algorithm is used to find the minimum and maximum probability of system failure. Both Table 9 and Fig. 12 show the results from our proposed method and the results from traditional method in Eq. (6). The exact value is also calculated by assuming that all the information, such as the components' failure modes and the distributions of S_{yi}, δ_{y1}, $L(\tau)$, and dimension parameters, were known to system designers. Both methods indicate an increasing trend of the probability of system failure with respect to time. They also show that the bounds from the proposed method are much narrower than those from the traditional method. The average reduction of the reliability bound width is about 82%. In addition, the exact value is also contained in the bounds of the probability of system failure from the proposed method.

Table 9 Bounds contrast of probability of system failure

$[0,t]$ (years)	Traditional method Bounds of p_{fS} (10^{-4})	Proposed method Bounds of p_{fS} (10^{-4})	Exact (10^{-4})	Reduction of bound width
[0,1]	[5.61, 14.004]	[7.8003, 8.865]	7.87	87.32%
[0,2]	[9.66, 24.919]	[13.529, 15.73]	13.94	85.58%
[0,3]	[13.65, 36.067]	[19.28, 22.716]	19.98	84.67%
[0,4]	[17.94, 46.747]	[24.807, 29.609]	25.93	83.33%
[0,5]	[21.92, 57.45]	[30.227, 36.483]	31.97	82.39%
[0,6]	[26.01, 68.474]	[35.735, 43.545]	37.95	81.61%
[0,7]	[30.12, 80.146]	[41.488, 50.963]	44.1	81.06%
[0,8]	[34.09, 90.874]	[46.792, 57.901]	49.95	80.44%
[0,9]	[38.06, 101.2]	[51.83, 64.477]	55.56	79.97%
[0,10]	[42.1, 112.02]	[56.94, 71.421]	61.47	79.29%
[0,11]	[46, 122.6]	[62.36, 78.153]	67.19	79.38%
[0,12]	[49.88, 133.42]	[68.893, 85.097]	73.02	80.6%

In reality, even for a standard component, such as the bolt in this example, the component supplier may still be unwilling to share its proprietary information to the system designers, for instance, the distributions of the yield strength and modulus of elasticity of the material. This kind of information could reveal detailed technologies, key manufacturing processes, and cost. It could then adversely affect the component supplier's competitive advantage. As shown in this example, the proposed method can help system designers predict the system reliability without knowing all details that are only available to component designers.

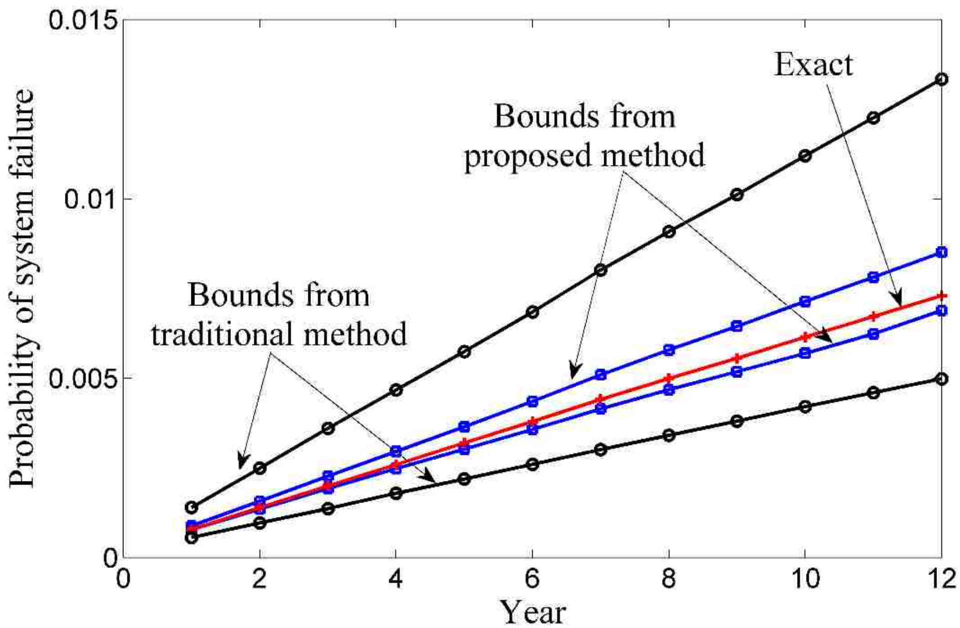

Figure 12 Bounds contrast from traditional and proposed methods

5. CONCLUSIONS

This work demonstrates the feasibility of producing narrower system reliability bounds with incomplete component design information when the system is subjected to stochastic process loading. The new method enables system designers to integrate component reliabilities supplied from component designers with other information available to system designers, such as the statistics of the system load and ranges of component factors of safety. With the integrated information, system designers reconstruct component limit-state functions that do not require proprietary component design details. System designers then use optimization to search for unknown parameters of general component resistance distributions and obtain narrower bounds of system reliability. The analysis process is simplified by converting the time-dependent reliability analysis into its time-independent counterpart with the use of the extreme value of the system load.

Note that if suppliers could provide their total or partial testing data of components, system designers can use the data to calibrate the parameters of the distribution of the general component resistance. This can then reduce the size of design variables of the proposed optimization model and will further narrow the system reliability bounds. Our future research is therefore to develop methodologies to calibrate the distribution parameters. Our other future work will be the full development of the concept proposed in this paper and its applications to more complex systems.

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IV. EFFECT OF DEPENDENT INTERVAL DISTRIBUTION PARAMETERS ON RELIABILITY PREDICTION

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ABSTRACT

Distributions of input variables of a limit-state function are required for reliability analysis. The distribution parameters are commonly estimated using samples. If some of the samples are in the form of intervals, the estimated distribution parameters may also be given in intervals. Traditional reliability methodologies assume that interval distribution parameters are independent, but as shown in this study, the parameters are actually dependent since they are estimated from the same set of samples. This study investigates the effect of the dependence of distribution parameters on the accuracy of reliability analysis results. The major approach is numerical simulation and optimization. This study indicates that the independent distribution parameter assumption makes the estimated reliability bounds wider than the true bounds due to interval samples. The reason is that the actual combination of the distribution parameters may not include the entire box-type domain assumed by the independent interval parameter assumption. The results of this study not only reveal the cause of the inaccuracy of the independent distribution parameter assumption, but also demonstrate a need of developing new reliability methods to accommodate dependent distribution parameters.

1. INTRODUCTION

Uncertainty is the major factor with which reliability analysis deals. It is the difference between the present state of knowledge and the complete knowledge [1]. Uncertainty is usually classified into two types, aleatory uncertainty and epistemic uncertainty. Aleatory uncertainty describes the inherent variability associated with a physical system or environment. It comes from inherent randomness and irreducible variability in nature. Epistemic uncertainty, on the other hand, is due to the lack of knowledge about a physical system or environment. It could be reducible by acquiring more knowledge [2].

Reliability is the probability that a system or component performs its intended function within a given period of time under specified conditions [3]. Reliability analysis is important in engineering applications given the catastrophic consequences when a failure occurs, and uncertainty should be considered in reliability analysis [4]. The aleatory uncertainty is commonly modeled by random variables with probability distributions, which are usually estimated from samples. This kind of uncertainty is induced by variations such as those in temperature, material properties, user operations, and manufacturing imprecision. Take a beam as an example, the aleatory uncertainty exists in the beam dimensions, external forces, and material properties, which can be modeled as random variables with specific distributions if sufficient information available. In real applications, however, we may not get precise and complete information due to limitations of testing conditions and instrumentation, as well as experimental uncertainty. Sometimes, the information may be from judgement and experience. In those cases, samples may be bounded within intervals [5-7]. As a result, epistemic uncertainty arises.

The traditional reliability methodologies, such as first order reliability method (FORM) and second order reliability method (SORM) [8], require great amount of information to construct precise distributions of the input variables for a limit-state function, which predicts the state of a component or system, either in a work condition or a failure condition. As mentioned previously, the distributions of the input variables are often obtained from samples. If some of the samples are intervals, the distribution parameters, such as means and standard deviations, are also intervals. This means that the random input variables with aleatory uncertainty also have epistemic uncertainty in their distribution parameters. The latter uncertainty is therefore called the second order uncertainty because it is on the top of the former uncertainty [9-11].

Although there are situations where some of input variables are not random variables, but also intervals [12-16], in this study, we focus on only the second order uncertainty. In other words, the scope of this study is the reliability analysis involving random input variables with interval distribution parameters. Interval samples lead to interval distribution parameters. Researchers have studied the distribution parameter uncertainty. Kiureghian [17] introduced an index of reliability based on minimizing a penalty function and developed methods for quantifying the uncertainty in the measure of safety arising from the imperfect state of knowledge of distribution parameters. Elishakoff and Colombi [18], and Zhu and Elishakoff [19] proposed methods to tackle parameter uncertainty when scarce knowledge was present on acoustic excitation parameters. Qiu, et al. [20] combined classical reliability theory and interval theory to obtain the system failure probability bounds from the statistical parameter intervals of the basic variables. Jiang, et al. [21] developed a hybrid reliability model based on monotonic analysis for random variables with interval distribution parameters. Sankararaman and Mahadevan [22] proposed a computational methodology based on Bayesian approach to quantify the individual contributions of variability and distribution parameter uncertainty in a random variable. Xie, et al. [11] developed a single-loop optimization model, which combines both probability analysis loop and interval analysis loop, to calculate the reliability bounds with second order uncertainty.

The above-mentioned methodologies, however, treat the intervals of distribution parameters independent. In fact, the parameters of a distribution are dependent because they are estimated from the same set of samples. The independent parameter assumption may make the estimated reliability bounds wider than the true bounds. The purpose of this study is to reveal the effect of dependent distribution parameters on the accuracy of reliability analysis.

The organization of this paper is as follows. Section 2 reviews existing methods for estimating the distribution parameters of a random variable with mixed point and interval samples. Section 3 discusses a likelihood-based approach to estimate the distribution parameters with mixed point and interval samples; it also presents the investigation of how dependent interval distribution parameters affect the accuracy of reliability prediction. Such effect is demonstrated by two examples in Section 4. Section 5 provides conclusions and the research needs for developing new reliability methods that can accommodate dependent interval distribution parameters.

2. REVIEW OF LIKELIHOOD-BASED DISTRIBUTION PARAMETER ESTIMATION

Samples are used to estimate distribution parameters. Traditional statistical methods assume that samples are given in the form of points, and the likelihood-based approach is normally used to estimate distribution parameters. The likelihood is defined as a quantity proportional to the probability density function (PDF) of the observed data [23, 24]. If the samples of a random variable *X* are ($x_1, x_2, \cdots, x_m$), the likelihood function is defined by

$$L(\boldsymbol{p}) \propto \prod_{i=1}^{m} f(x_i | \boldsymbol{p}) \tag{1}$$

where $f(x_i | \boldsymbol{p})$ is the PDF of *X* at x_i with distribution parameters ***p***.

Then the maximum likelihood estimation (MLE) is used to estimate parameters ***p***. The estimator $\hat{\boldsymbol{p}}$ is obtained by maximizing the likelihood function as follows:

$$\hat{\boldsymbol{p}} = \arg\max_{p \in \boldsymbol{p}} \prod_{i=1}^{m} f(x_i | \boldsymbol{p}) \tag{2}$$

In engineering applications, it is also possible that some of the samples are in the form of intervals. For a random variable *X* with interval samples ($y_1, y_2, \cdots, y_n$), where $y_i \in [\underline{y}_i, \overline{y}_i]$, $i = 1, 2, \cdots, n$, Gentleman and Geyer [25] constructed the following likelihood function using the cumulative distribution function (CDF) of *X*:

$$L(\boldsymbol{p}) \propto \prod_{i=1}^{n} \int_{\underline{y}_i}^{\overline{y}_i} f(y_i | \boldsymbol{p}) = \prod_{i=1}^{n} \left[F(\overline{y}_i | \boldsymbol{p}) - F(\underline{y}_i | \boldsymbol{p}) \right] \tag{3}$$

where $F(\overline{y}_i | \boldsymbol{p})$ is the CDF at the upper bound of interval sample $\overline{y}_i$ with distribution parameters ***p*** and $F(\underline{y}_i | \boldsymbol{p})$ is the CDF at the lower bound of interval sample $\underline{y}_i$ with distribution parameters ***p***.

Sankararaman and Mahadevan [26] modified this formulation to include both point data ($x_1, x_2, \cdots, x_m$) and interval data ($y_1, y_2, \cdots, y_n$) in the likelihood function as follows:

$$L(\boldsymbol{p}) \propto \left[\prod_{i=1}^{m} f(x_i | \boldsymbol{p})\right]\left[\prod_{i=1}^{n}\left[F(\overline{y}_i | \boldsymbol{p}) - F(\underline{y}_i | \boldsymbol{p})\right]\right] \tag{4}$$

This likelihood function is constructed using the PDF of point data and the CDF of interval data. Then the maximum likelihood estimate of $\boldsymbol{p}$ can be obtained by maximizing Eq. (4).

Instead of maximizing Eq. (4), the Bayes' theory is used as follows [26]

$$f_p(\boldsymbol{p}) = \frac{L(\boldsymbol{p})}{\int L(\boldsymbol{p}) d\boldsymbol{p}} \tag{5}$$

in which $f_p(\boldsymbol{p})$ is the joint PDF of parameters $\boldsymbol{p}$. With the joint PDF $f_p(\boldsymbol{p})$, the marginal PDF of each distribution parameter can be obtained. Also, the PDF of the random variable X is then given by

$$f_X(x) = \int f_X(x | \boldsymbol{p}) f_p(\boldsymbol{p}) d\boldsymbol{p} \tag{6}$$

The above methods have the advantage of getting precise distributions even though some samples are intervals, therefore hiding the epistemic uncertainty and making reliability analysis easier. This treatment, however, produces only a single reliability prediction although the interval-type of epistemic uncertainty exists. In Section 3, we will discuss a likelihood-based approach to the intervals of distribution parameters from the mixed point and interval samples and then investigate the effects of dependent distribution parameters on reliability analysis.

3. EFFECT OF DEPENDENT INTERVAL DISTRIBUTION PARAMETERS ON RELIABILITY PREDICTION

In this section, we at first use the maximum likelihood approach to obtain the interval distribution parameters from point and interval samples. Instead of calculating the full likelihood [26], we estimate the lower and upper bounds of distribution parameters by using the interval samples. We then show the dependence of the distribution parameters. Finally, we discuss how the dependent interval distribution parameters affect the reliability analysis result.

3.1 ESTIMATION OF DISTRIBUTION PARAMETERS

In this subsection, a likelihood-based approach is used to estimate the bounds of distribution parameters of a random variable *X* with the mixed point and interval samples.

The samples of a random variable *X* are given by both point data ($x_1, x_2, \cdots, x_m$) and interval data ($y_1, y_2, \cdots, y_n$), where $y_i \in [\underline{y}_i, \overline{y}_i]$. According to Eq. (1), the likelihood function of random variable *X* with point and interval data is defined by

$$L(\boldsymbol{p}) \propto \left[\prod_{i=1}^{m} f(x_i | \boldsymbol{p}) \right] \left[\prod_{i=1}^{n} f(y_i | \boldsymbol{p}) \right] \tag{7}$$

where $f(x_i | \boldsymbol{p})$ is the PDF of point data x_i, and $f(y_j | \boldsymbol{p})$ is the PDF of interval data y_i given distribution parameter *p*.

Using Eq. (2), we obtain the distribution parameter estimator $\hat{\boldsymbol{p}}$ from the maximum likelihood function by

$$\hat{\boldsymbol{p}} = \arg\max \left[\prod_{i=1}^{m} f(x_i | \boldsymbol{p}) \right] \left[\prod_{i=1}^{n} f(y_i | \boldsymbol{p}) \right] \tag{8}$$

With the point and interval samples of random variable X available, the bounds of distribution parameter $[\underline{\boldsymbol{p}}, \overline{\boldsymbol{p}}]$ can be obtained.

We take a normal distribution as an example to illustrate this methodology. Suppose a random load F_0 follows a normal distribution $F_0 \sim N(\mu_{F_0}, \sigma_{F_0}^2)$ kN. The samples of the load are given by points ($x_1, x_2, \cdots, x_m$) and intervals ($y_1, y_2, \cdots, y_n$), $y_i \in [\underline{y}_i, \overline{y}_i]$. The mean of the load is calculated by

$$\mu_{F_0} = \frac{1}{m+n}(\sum_{i=1}^{m} x_i + \sum_{i=1}^{n} y_i) \tag{9}$$

Since Eq. (9) is a linear function, the bounds of mean $\left[\underline{\mu}_{F_0},\ \overline{\mu}_{F_0}\right]$ are obtained by

$$\begin{cases} \underline{\mu}_{F_0} = \dfrac{1}{m+n}(\sum_{i=1}^{m} x_i + \sum_{i=1}^{n} \underline{y}_i) \\ \overline{\mu}_{F_0} = \dfrac{1}{m+n}(\sum_{i=1}^{m} x_i + \sum_{i=1}^{n} \overline{y}_i) \end{cases} \tag{10}$$

The standard deviation of the load is calculated by

$$\sigma_{F_0} = \sqrt{\frac{1}{m+n-1}\left(\sum_{i=1}^{m}(x_i - \mu_{F_0})^2 + \sum_{i=1}^{n}(y_i - \mu_{F_0})^2\right)} \tag{11}$$

Since Eq. (11) is a nonlinear function, optimization is used to obtain the minimum standard deviation $\underline{\sigma}_{F_0}$ as follows:

$$\begin{cases} \min\limits_{\mathbf{y}}\ \sigma_{F_0}(y_1, y_2, \cdots, y_n) \\ \text{Subject to} \\ \underline{y}_i \le y_i \le \overline{y}_i,\ i = 1, 2, \cdots, n \end{cases} \tag{12}$$

Changing $\min\limits_{\mathbf{y}}\ \sigma_{F_0}$ to $\max\limits_{\mathbf{y}}\ \sigma_{F_0}$, we also obtain the maximum standard deviation $\overline{\sigma}_{F_0}$. Then the interval distribution parameters $\mu_{F_0} \in \left[\underline{\mu}_{F_0},\ \overline{\mu}_{F_0}\right]$ and $\sigma_{F_0} \in \left[\underline{\sigma}_{F_0},\ \overline{\sigma}_{F_0}\right]$ are available. In subsection 3.2, we will show that the interval distribution parameters are dependent.

3.2 DEPENDENCE BETWEEN DISTRIBUTION PARAMETERS

Theoretically, the distribution parameters are dependent because they are estimated from the same set of samples. Note that this dependence is unlike the statistical dependence between random variables. The latter can be reflected by the joint distribution between two random variables [27]. We will continue to use the example in Sec. 3.1 to reveal the dependent relationship between distribution parameters of a random variable. The method we use is numerical simulation.

As discussed previously, the load F_0 follows a normal distribution $F_0 \sim N(\mu_{F_0}, \sigma_{F_0}^2)$ kN . The samples of the load are $(x_1, x_2, x_3, x_4) =$ (40.486, 31.252, 29.648, 36.285) kN and $(y_1, y_2, \cdots, y_6) =$ ([23.816, 24.788], [24.78, 25.791], [31.765, 33.061], [29.755, 30.969], [39.815, 41.44], [35.797, 37.259]) kN. Using Eqs. (10) and (12), the bounds of mean and standard deviation are calculated with intervals $\mu_{F_0} \in \left[\underline{\mu}_{F_0},\ \overline{\mu}_{F_0}\right]$ =[32.34, 33.098] kN and $\sigma_{F_0} \in \left[\underline{\sigma}_{F_0},\ \overline{\sigma}_{F_0}\right]$ =[5.3582, 6.0849] kN, respectively.

If we do not consider the dependence between the two distribution parameters, the actual values of the two parameters vary in a box defined by $\mu_{F_0} \in \left[\underline{\mu}_{F_0},\ \overline{\mu}_{F_0}\right]$ =[32.34, 33.098] kN and $\sigma_{F_0} \in \left[\underline{\sigma}_{F_0},\ \overline{\sigma}_{F_0}\right]$ =[5.3582, 6.0849] kN. The box is plotted in Fig. 1.

Since the actual distribution parameters are constrained with the box, the reliability prediction will also reside within an interval. The width of the reliability determines the accuracy of the reliability prediction and the amount of epistemic uncertainty in the prediction, which of course depends on the size of the box of the distribution parameters. The area of the box of the distribution parameters may be too large since the actual points of $(\mu_{F_0}, \sigma_{F_0})$ may not occupy the entire area of the box. As a result, the bounds of the reliability prediction using the box constraint will be wider than the actual bounds. Since $(\mu_{F_0}, \sigma_{F_0})$ may not occupy the entire box, they must be constrained by other shape, instead of a box. In other words, the distribution parameters are dependent.

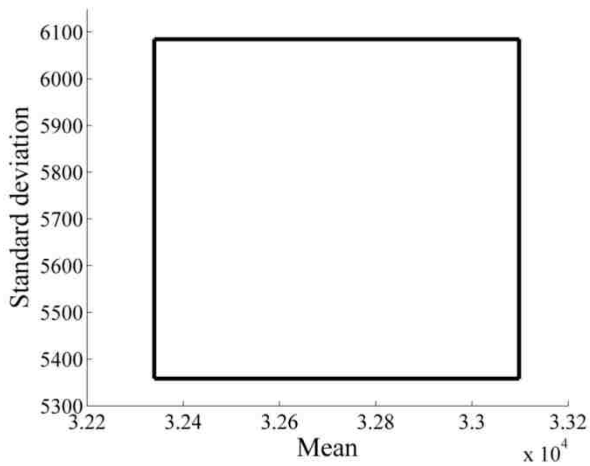


Figure 1 Box domain of mean and standard deviation

To study the dependence between distribution parameters, we perform experiments by random sampling. The simulation sample size is set to 10^5. This size is chosen because it is not only good enough to reveal the dependence relationship, but also suitable for efficient computations. The four point samples are constant while the six interval samples are randomly simulated. The actual values of each of the interval samples are drawn within its intervals. Totally 10^5 sets of samples are obtained, and the same number of means and standards are calculated. The results are shown in Fig. 2. For comparison, the bounds of mean [32.34, 33.098] kN and standard deviation [5.3582, 6.0849] kN by Eqs. (10) and (12) are also plotted in Fig. 2. It is seen that the actual domain of the distribution parameters is smaller than the box-type hyperrectangular determined by the lower and upper bounds of the mean and standard deviation using the independent distribution assumption. The simulation indicates that the actual points of $(\mu_{F_0}, \sigma_{F_0})$ do not appear at the four corners of the box.

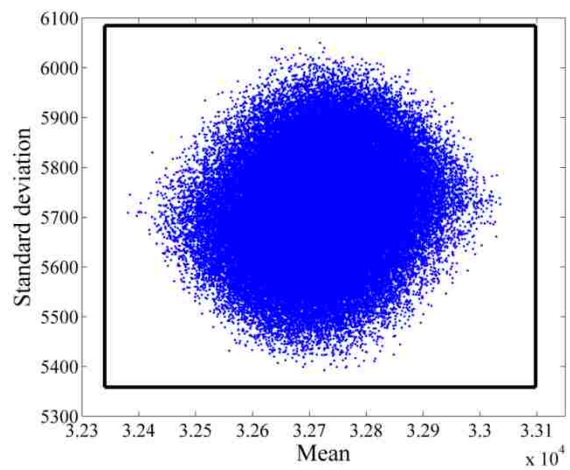


Figure 2 Domain of mean and standard deviation

To investigate how the pattern of distribution parameter relationship changes with respect to the number of interval samples, we also vary the number of interval samples from one to nine while keep the sample size (total number of point samples and interval samples) as ten. The results are shown in Fig. 3. Fig. 3(a) shows the dependent distribution parameter relationship for one interval and nine points; correspondingly, Fig.

3(h) shows the dependent distribution parameter relationship for nine intervals and one point.

Although no clear patterns could be identified, the results clearly indicate that the actual domains of the distribution parameters are smaller than the box-type domains. In Sec 3.3, we will discuss reliability analysis with interval samples.

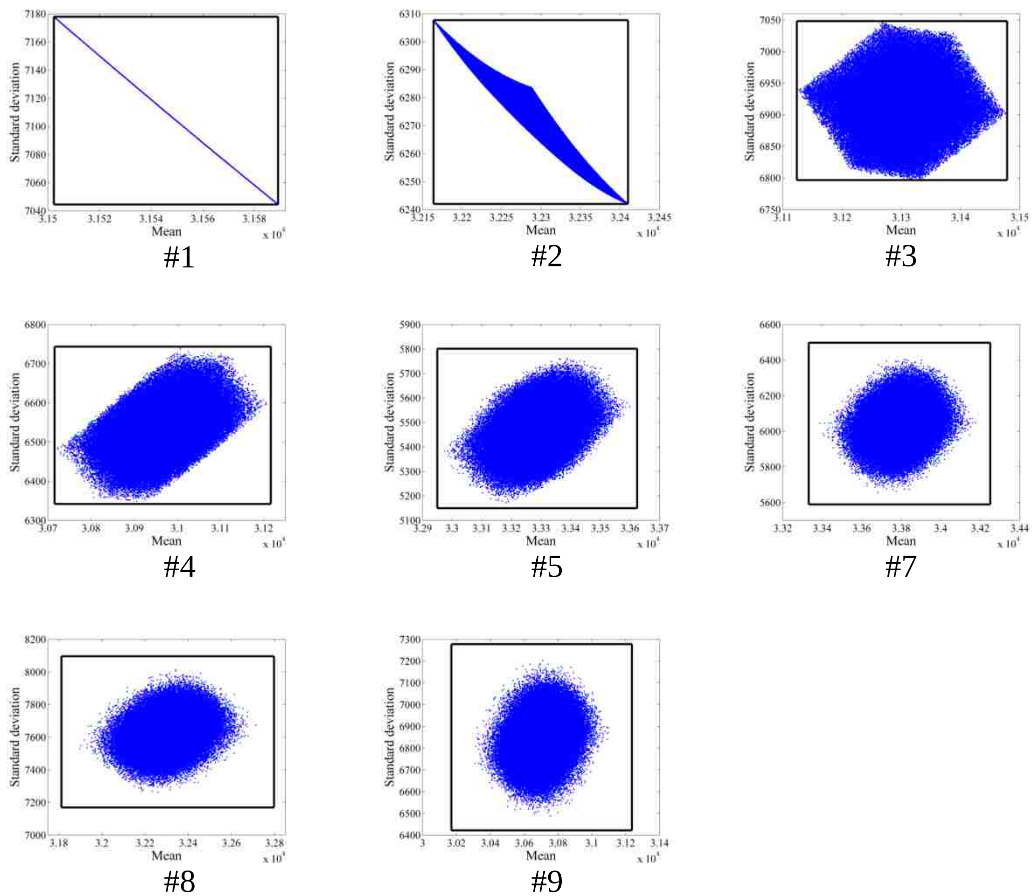


Figure 3 Dependent relationships between distribution parameters with different number of intervals

3.3 RELIABILITY ANALYSIS WITH INTERVAL DISTRIBUTION PARAMETERS

In order to demonstrate the impact of dependent distribution parameters, herein we discuss two methods for reliability analysis. The first method is the traditional reliability analysis that uses the bounds of the distribution parameters directly without accounting for the dependence between the distributions parameters. The second method uses the raw sample data of input random variables, including both point and interval samples. Both methods will produce interval reliability because of interval samples. For engineering applications, we always prefer narrower bounds of reliability prediction or a smaller width of the reliability interval. As we will see, the two methods will produce different reliability bounds, and the latter method will generate narrower reliability bounds and is therefore more preferable.

Let the limit-state function be

$$G = g(\mathbf{X}) \tag{13}$$

If a failure occurs when $G < 0$, the probability of failure is given by

$$p_f = \Pr(g(\mathbf{X}) < 0) \tag{14}$$

Let the intervals of distribution parameters of X be $\mathbf{p} \in [\underline{\mathbf{p}}, \overline{\mathbf{p}}]$. Since the probability of failure p_f depends on the distributions of X, as well as p, it is also a function of p; namely, $p_f = p_f(\mathbf{p})$. As a result, the probability of failure is also an interval and $p_f \in [\underline{p}_f, \overline{p}_f]$. Next, we discuss how to obtain the bounds of probability of failure p_f.

The traditional method uses the distribution parameter intervals $\mathbf{p} \in [\underline{\mathbf{p}}, \overline{\mathbf{p}}]$ directly. The minimum probability of failure $\underline{p}_f$ could be obtained by minimizing p_f with respect to $\mathbf{p}$. The optimization model is given by

$$\begin{cases} \min\limits_{\mathbf{p}} \ p_f(\mathbf{p}) \\ \text{subject to} \\ \underline{\mathbf{p}} \le \mathbf{p} \le \overline{\mathbf{p}} \end{cases} \tag{15}$$

For the maximum probability of failure $\overline{p}_f$, the first line of the optimization model in Eq. (15) is changed from $\min\limits_{\mathbf{p}} \ p_f(\mathbf{p})$ to $\max\limits_{\mathbf{p}} \ p_f(\mathbf{p})$.

The other reliability analysis method uses the raw samples including interval samples directly. The minimum probability of failure $\underline{p}_f$ is obtained by minimizing p_f with respect to the interval samples $\mathbf{y} = (y_1, y_2, \cdots, y_n),\ y_i \in [\underline{y}_i, \overline{y}_i],\ i = 1, 2, \cdots, n$. The optimization model is given by

$$\begin{cases} \min\limits_{\mathbf{y}} \ p_f(y_1, y_2, \cdots, y_n) \\ \text{Subject to} \\ \underline{y}_i \le y_i \le \overline{y}_i,\ i = 1, 2, \cdots, n \end{cases} \tag{16}$$

For the maximum probability of failure $\overline{p}_f$, the first line of the optimization model in Eq. (16) is changed from $\min\limits_{\mathbf{y}} \ p_f(\mathbf{y})$ to $\max\limits_{\mathbf{y}} \ p_f(\mathbf{y})$.

Note that in the traditional method, the distribution parameters are assumed independent within box-type constraints. The method with raw data accounts for dependent distribution parameters automatically. As discussed previously, the box-type domain of interval distribution parameters in the former method is larger than and also covers that in the latter method. Roughly speaking, the feasible region of the optimization in the former method is larger than and covers that in the latter method. As a result, the bounds of the probability of failure of the former method are wider than those of the latter method. In Sec. 4, we will demonstrate this with examples.

4. EXAMPLES

In this section, we use three examples to demonstrate the effect of dependent interval distribution parameters on the reliability prediction. The probability of failure bounds from the traditional method and the method using raw data are compared.

4.1 EXAMPLE 1

As shown in Fig. 4, a resultant force $T_k = kF_0$, $k = 1, 2, 3$, is applied at the end of a beam. There are three cases. Case 1 ($k = 1$) has only one force F_0, Case 2 ($k = 2$) has two identical and independent force F_0, and Case 3 ($k = 3$) has three identical and independent F_0. The samples of the force F_0 are obtained through experiments. The ten samples include four points (x_1, x_2, x_3, x_4) and six intervals $(y_1, y_2, y_3, y_4, y_5, y_6)$. The samples are given in Table 1. The distribution F_0 is normal, and the yield strength of the beam is kS_y $(k = 1, 2, 3)$ for the three cases. All the information available is summarized in Table 2.

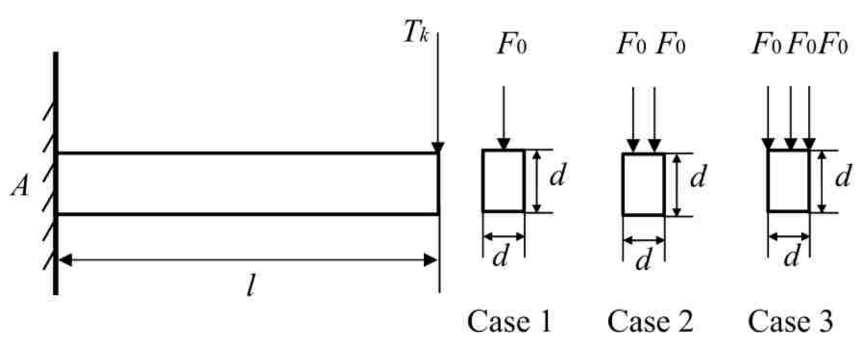


Figure 4 A bending stress applied on a beam

Table 1 Experimental samples of load F_0

Samples	Values (×10^4) N
Points	4.0486, 3.1252, 2.9648, 3.6285
Intervals	[2.3816, 2.4788], [2.478, 2.5791], [3.1765, 3.3061], [2.9755, 3.0969], [3.9815, 4.144], [3.5797, 3.7259]

Table 2 Information available to the beam designers

Variables	Value
Yield stress distribution S_y	$S_y \sim N(70,\ 5^2)$ MPa
Samples of load F_0	Table 1
Distribution type of F_0	Normal distribution
Length l	1.8 m
Width d	0.2 m
Thickness d	0.2 m
Coefficient k	$k = i$ for case i

Excessive bending stress is considered as a failure mode. With a physics-based approach, a limit-state function is constructed.

$$G = kS_y - \frac{T_k(6l)}{d^3} \tag{17}$$

where l is the beam length, and d is the beam width and thickness. $G < 0$ indicates a failure.

Using the samples of F_0 in Table 1; and Eqs. (10) and (12), we obtain the bounds of the mean and standard deviation of F_0 as shown in Table 3.

Table 3 Estimation of distribution parameters of load F_0

	Mean (×10^4 N)	Std (×10^3 N)
F_0	[3.234, 3.3098]	[5.3582, 6.0849]

The limit-state function in Eq. (17) is linear and also follows a normal distribution. The probability of failure is then given by

$$p_f = 1 - \Phi\left(\frac{\mu_{S_y}(d^3k/6l) - \mu_{T_k}}{\sqrt{\left(\sigma_{S_y}(d^3k/6l)\right)^2 + \sigma_{T_k}^2}} \right) \tag{18}$$

Eq. (18) is a monotonic function. With the independent distribution parameter assumption in the traditional method, the minimum probability of failure $\underline{p}_f$ occurs when the denominator is minimum and numerator is maximum in function Φ, while the maximum probability of failure $\overline{p}_f$ occurs when the denominator is maximum and numerator is minimum in function Φ. Therefore, with the independent distribution parameter assumption, the bounds of the probability of failure are

$$\begin{cases} \underline{p}_f = 1 - \Phi\left(\dfrac{\mu_{S_y}(d^3k/6l) - \underline{\mu}_{T_k}}{\sqrt{\left(\sigma_{S_y}(d^3k/6l)\right)^2 + \underline{\sigma}_{T_k}^2}} \right) \\ \overline{p}_f = 1 - \Phi\left(\dfrac{\mu_{S_y}(d^3k/6l) - \overline{\mu}_{T_k}}{\sqrt{\left(\sigma_{S_y}(d^3k/6l)\right)^2 + \overline{\sigma}_{T_k}^2}} \right) \end{cases} \tag{19}$$

Note that the distribution of the resultant force T_k is changing with the three cases. For Case 1, the distribution is $T_1 \sim N(\mu_{F_0}, \sigma_{F_0}^2)$; for Case 2, the distribution is $T_2 \sim N(2\mu_{F_0}, 2\sigma_{F_0}^2)$; and for Case 3, the distribution is $T_3 \sim N(3\mu_{F_0}, 3\sigma_{F_0}^2)$.

After obtaining the reliability prediction from the traditional method with the independent distribution parameter assumption, we now discuss the method with the raw data. The minimum probability of failure is obtained by

$$\begin{cases} \min_{\mathbf{y}} \ p_f(y_1, y_2, \cdots, y_6) \\ \text{Subject to} \\ \underline{y}_i \le y_i \le \overline{y}_i, \ i = 1, 2, \cdots, 6 \end{cases} \tag{20}$$

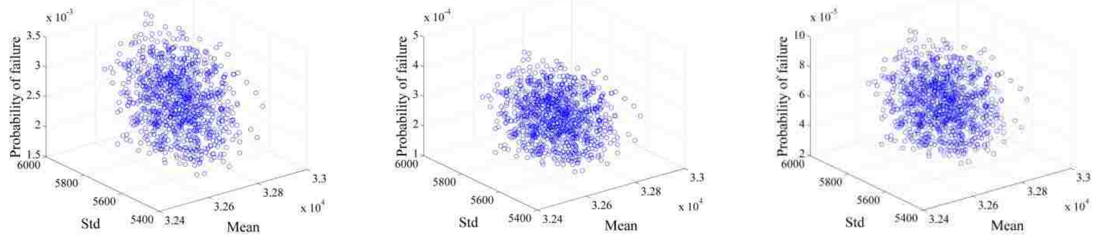
For the maximum probability of failure $\overline{p}_f$, the first line of the optimization model in Eq. (20) is changed from $\min_{\mathbf{y}} \ p_f(y_1, y_2, \cdots, y_6)$ to $\max_{\mathbf{y}} \ p_f(y_1, y_2, \cdots, y_6)$. As seen in Eq. (20), the six intervals are used as the constraints to calculate the probability of failure.

Table 4 shows the bounds of the probabilities of failure obtained from the traditional method with the independent distribution parameter assumption and the method with raw data. 10^3 probabilities of failure from MCS are also plotted in Fig.4.

Table 4 Probability of failure

Case	**Traditional Method**	**Method with Raw Data**	**Percentage Reduction**
1 ($\times 10^{-3}$)	[1.3698, 4.2353]	[1.6231, 3.6823]	28.14%
2 ($\times 10^{-4}$)	[1.1541, 4.7762]	[1.4578, 3.9247]	31.9%
3 ($\times 10^{-5}$)	[2.6348, 11.95]	[3.4132, 9.5485]	34.14%

The results indicate that the method with raw data produces narrower bounds of the probability of failure than those from the traditional method with the independent distribution parameter assumption. The average reduction of the bound width from the former method is about 31%. For this problem with a linear limit-state function, the solution to the probability of failure in Eq. (18) is exact, and the bounds of the probability of failure obtained from the method with raw data are the true bounds. The independent distribution parameter assumption produces wider bounds, which therefore contain higher amount of epistemic uncertainty in the predicted probability of failure.



(a) Case 1 (b) Case 2 (c) Case 3

Figure 5 Probability of failure by numerical simulation

Table 5 and Fig. 6 show the probabilities of failure with numbers of interval samples from one to nine for Case 1. The dependent relationship between the mean and standard deviation with the increasing number of intervals has been shown previously in Fig. 2. The results also indicate that the method with raw data produces narrower bounds than those from the traditional method with the independent distribution parameter assumption.

Table 5 Probability of beam failure with increasing intervals

No.	**Traditional Method ($\times 10^{-3}$)**	**Method with Raw Data ($\times 10^{-3}$)**	**Percentage Reduction**
1	[5.2811, 6.0594]	[5.4492, 5.8773]	44.99%
2	[3.3381, 3.9311]	[3.5556, 3.6959]	76.34%
3	[3.7033, 5.2522]	[3.8922, 5.0117]	27.72%
4	[2.0009, 3.6572]	[2.1036, 3.4861]	16.53%
5	[1.4415, 4.0443]	[1.6156, 3.6735]	20.93%
6	[1.3698, 4.2353]	[1.6231, 3.6823]	28.14%
7	[2.8716, 9.2988]	[3.4344, 8.08]	27.72%
8	[6.5081, 16.166]	[7.443, 14.497]	26.96%
9	[1.7244, 5.7932]	[2.0786, 4.9756]	28.8%

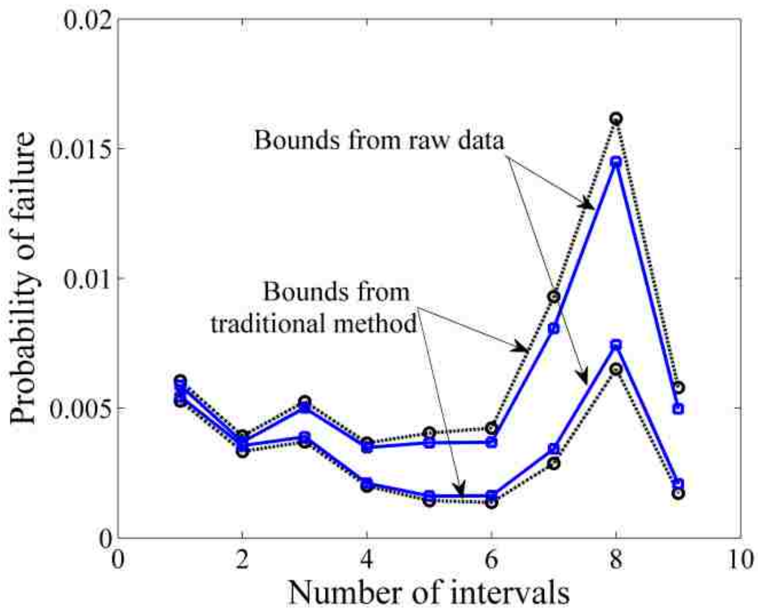


Figure 6 Probability of failure with respect to the number of interval samples

4.2 EXAMPLE 2

This example is modified from Case 3 in example 1. The samples of F_0 have already been given in Table 1. The samples of the yield stress S_y are given in Table 6. S_y is normally distributed and is independent of F_0. All the information available is summarized in Table 7.

Using the samples of S_y in Table 6; and Eqs. (10) and (12), we obtain the bounds of the mean and standard deviation of S_y as shown in Table 8.

Table 6 Experimental samples of strength S_y

Samples	**Values (×10^7) Pa**
Points	6.4254, 7.5463, 6.9363, 6.5101
Intervals	[8.0101, 8.337], [6.1905, 6.4431], [7.2541, 7.5502], [6.8651, 7.1453], [7.6006, 7.9108], [7.5166, 7.8234]

Table 7 Information available

Variables	**Values**
Samples of yield stress S_y	Table 6
Samples of load F_0	Table 1
Distribution type of S_y	Normal distribution
Distribution type of F_0	Normal distribution
Length *l*	1.8 m
Width *d*	0.2 m
Thickness *d*	0.2 m

Table 8 Estimation of distribution parameters of S_y

	Mean (×10^7 Pa)	**Std (×10^6 Pa)**
S_y	[7.0855, 7.2628]	[5.5012, 7.2305]

Using the traditional method, we obtain the bounds of probability of failure as follows:

$$\begin{cases} \underline{p}_f = 1 - \Phi\left(\dfrac{\overline{\mu}_{S_y}(d^3/2l) - \underline{\mu}_T}{\sqrt{\left(\underline{\sigma}_{S_y}(d^3/2l)\right)^2 + \underline{\sigma}_T^2}} \right) \\ \overline{p}_f = 1 - \Phi\left(\dfrac{\underline{\mu}_{S_y}(d^3/2l) - \overline{\mu}_T}{\sqrt{\left(\overline{\sigma}_{S_y}(d^3/2l)\right)^2 + \overline{\sigma}_T^2}} \right) \end{cases} \tag{21}$$

Note that the distribution of the resultant load $T = F_0 + F_0 + F_0$ is $T \sim N(3\mu_{F_0}, 3\sigma_{F_0}^2)$.

After obtaining the reliability prediction from the traditional method with the independent distribution parameter assumption, we discuss the method with the raw data. The dependent relationship between the mean and standard deviation of the yield strength S_y is shown in Fig. 7.

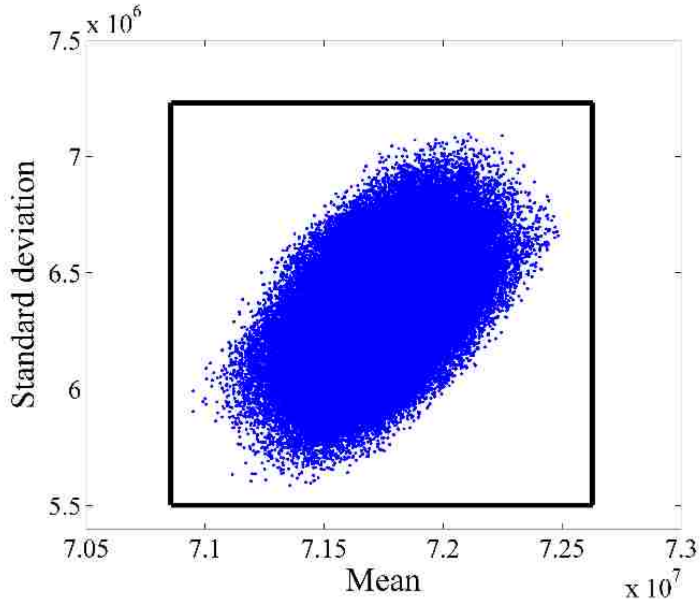


Figure 7 Dependent distribution parameters of S_y

The minimum probability of failure is obtained by

$$\begin{cases} \min\limits_{\mathbf{z},\mathbf{y}} \; p_f(y_1,\cdots,y_6;z_1,\cdots,z_6) \\ \text{Subject to} \\ \underline{y}_i \le y_i \le \overline{y}_i;\; i\text{=}1,2,\cdots,6 \\ \underline{z}_j \le z_j \le \overline{z}_j;\; j\text{=}1,2,\cdots,6 \end{cases} \tag{22}$$

where $y_i \ (i=1,2,\cdots,6)$ are interval samples of F_0, and $z_j \ (j=1,2,\cdots,6)$ are interval samples of S_y. For the maximum probability of failure $\overline{p}_f$, the first line of the optimization model in Eq. (22) is changed to $\max\limits_{\mathbf{z},\mathbf{y}} \; p_f(y_1,\cdots,y_6;z_1,\cdots,z_6)$. The twelve intervals are used as the constraints to calculate the probability of failure.

Table 9 and Fig. 8 show the bounds of the probabilities of failure obtained from the traditional method with the independent distribution parameter assumption and the method with raw data. The results indicate that the method with raw data produces much narrower bounds of the probability of failure than those from the traditional method with the independent distribution parameter assumption. The reduction of the bound width is about 89%.

Table 9 Estimation of the probability of beam failure

	Traditional Method (10^{-4})	**Method with Raw Data (10^{-4})**	**Percentage Reduction**
p_f	[0.1369, 12.359]	[1.2702, 2.5917]	89.19%

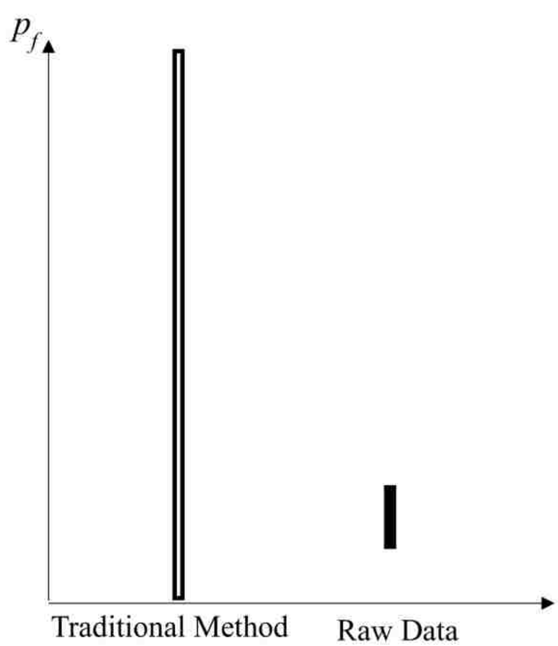


Figure 8 Bounds of probability of failure

4.3 EXAMPLE 3

This problem is the modification of the example given in Ref. [28]. As shown in Fig. 9, a load p is uniformly distributed on a simply supported beam, whose length, width, and height are l, b, and h, respectively. The beam dimensions are in Table 10. The samples of force p and Young's modulus E from experimentations are given in

Table 11. All the information available to beam designers is shown in Table 12. All random variables are independently distributed with lognormal distributions.

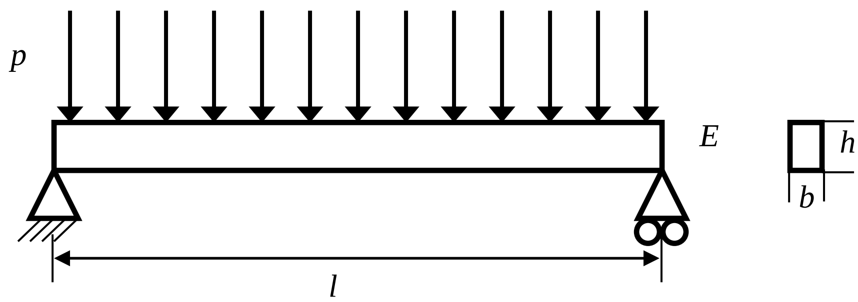


Figure 9 A load applied to a simply supported beam

Table 10 Beam dimensions

Variables	**Mean** μ	**Std** σ	**Distribution type**
Length l	5 m	50 mm	Lognormal
Width b	0.15 m	7.5 mm	Lognormal
Height h	0.3 m	15 mm	Lognormal

Table 11 Experimental samples of E and p

Variables	**Samples**	**Values**
E ($\times 10^{10}$) Pa	Points	3.5243, 3.0626, 2.9824, 3.3142
	Intervals	[2.6608, 2.7694], [2.709, 2.8196], [3.0582, 3.1831], [2.9577, 3.0785], [3.4608, 3.602], [3.2599, 3.3929]
p ($\times 10^{4}$) N/m	Points	1.1798, 1.0215, 0.994, 1.1077
	Intervals	[0.8843, 0.9204], [0.9008, 0.9376], [1.0205, 1.0622], [0.9861, 1.0263], [1.1585, 1.2058], [1.0897, 1.1341]

Table 12 Available information to designers

Variables	**Values**
Beam dimensions	Table 11
Samples of Young's modulus E	Table 12
Samples of load p	Table 12
Distribution type of E	lognormal
Distribution type of p	lognormal
Deflection threshold δ	16 mm

Excessive deflection is considered as a failure mode. With a physics-based approach, a limit-state function is constructed as

$$G = \delta - \frac{5pl^4}{32Ebh^3} \tag{23}$$

where $G < 0$ indicates a failure.

Using the samples of E and p in Table 11; and Eqs. (10) and (12), we obtain the bounds of the means and standard deviations of E and p as shown in Table 13.

Table 13 Means and standard deviations of E and p

Variables	**Mean**	**Std**
E (×10^{10}) Pa	[3.099, 3.1729]	[0.2516, 0.3226]
p (×10^4) N/m	[1.0343, 1.0589]	[0.0866, 0.1104]

The limit-state function in Eq. (23) is nonlinear. The second term can be expressed as V, whose log expression can be transferred into linear function as follows:

$$\ln(V) = \ln\left(\frac{5pl^4}{32Ebh^3}\right) = \ln\left(\frac{5}{32}\right) + \ln(p) + 4\ln(l) - \ln(E) - \ln(b) - 3\ln(h) \tag{24}$$

For lognormal variables, the distribution parameters are

$$\lambda_V = \ln\left(\frac{5}{32}\right) + \lambda_p + 4\lambda_l - \lambda_E - \lambda_b - 3\lambda_h \tag{25}$$

$$k_V = \sqrt{k_p^2 + (4k_l)^2 + k_E^2 + k_b^2 + (3k_h)^2} \tag{26}$$

where λ is the scale parameter, and k is the location parameter of a lognormal distribution. Therefore, V follows a lognormal distribution $V \sim LN(\lambda_V, k_V)$. For a given deflection threshold δ, the probability of failure is

$$p_f = \Pr(G < 0) = \Phi\left(\frac{\lambda_V - \log(\delta)}{k_V}\right) \tag{27}$$

The scale λ and location k can be calculated from the mean μ and standard deviation σ of a lognormal distribution by

$$\lambda = \ln(\mu) - \frac{1}{2}\ln\left[\left(\frac{\sigma}{\mu}\right)^2 + 1\right] \tag{28}$$

$$k = \sqrt{\ln\left[\left(\frac{\sigma}{\mu}\right)^2 + 1\right]} \tag{29}$$

With Tables 10 and 13 available, and using Eqs. (28) and (29), the scale λ and location k of all random variables are calculated in Table 14. For Young's modulus E and load p, the distribution parameters are intervals due to their interval means and standard deviations.

Table 14 Distribution parameters of variables

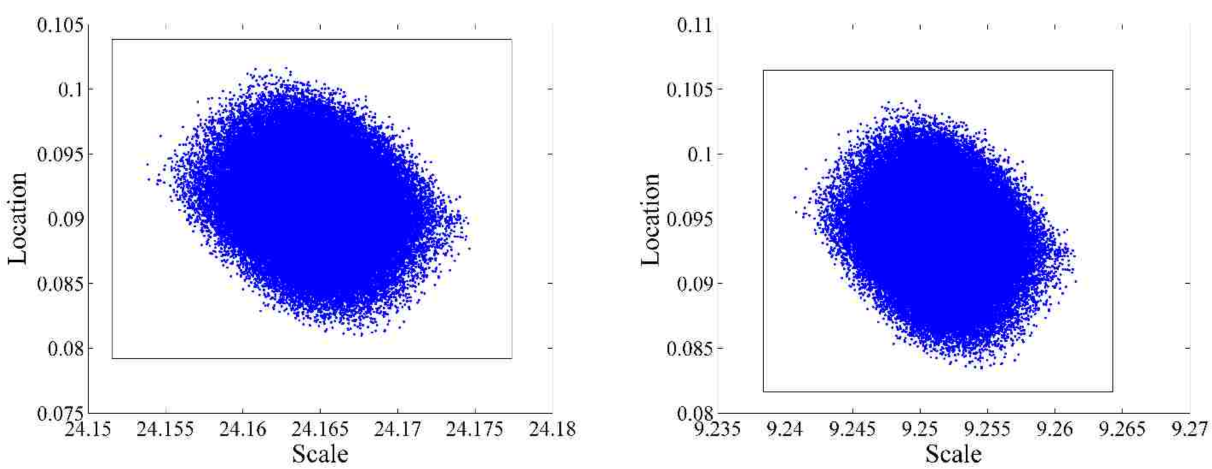
Variables	**scale** λ	**location** k
Length l	1.6094	0.01
Width b	-1.8984	0.05
Height h	-1.2052	0.05
Young's modulus E	[24.151, 24.177]	[0.0792, 0.1038]
Load p	[9.2384, 9.2643]	[0.0816, 0.1065]

For the traditional method, we obtain the bounds of probability of failure as follows:

$$\begin{cases} \underline{p}_f = \Phi\left(\dfrac{\underline{\lambda}_V - \log(\delta)}{\overline{k}_V} \right) \\ \overline{p}_f = \Phi\left(\dfrac{\overline{\lambda}_V - \log(\delta)}{\underline{k}_V} \right) \end{cases} \tag{30}$$

With Table 14 available, the bounds $[\underline{\lambda}_V, \overline{\lambda}_V]$ and $[\underline{k}_V, \overline{k}_V]$ can be calculated using Eqs. (25) and (26). Therefore, the bounds of $[\underline{p}_f, \overline{p}_f]$ are obtained.

After obtaining the probability of failure from the traditional method with the independent distribution parameter assumption, we discuss the method with the raw data. The dependent relationship between the scales and locations of the Young's modulus E and load p from numerical simulation are shown in Fig. 10.



(a) Young's modulus E (b) Load p

Figure 10 Dependent distribution parameters of E and p

The minimum probability of failure is obtained by

$$\begin{cases} \min\limits_{\mathbf{w},\mathbf{v}} \; p_f(w_1,\cdots,w_6;v_1,\cdots,v_6) \\ \text{Subject to} \\ \underline{w}_i \le w_i \le \overline{w}_i;\; i=1,2,\cdots,6 \\ \underline{v}_j \le v_j \le \overline{v}_j;\; j=1,2,\cdots,6 \end{cases} \tag{31}$$

where w_i $(i=1,2,\cdots,6)$ are interval samples of E, and v_j $(j=1,2,\cdots,6)$ are interval samples of p. For the maximum probability of failure $\overline{p}_f$, the first line of the optimization model in Eq. (22) is changed to $\min\limits_{\mathbf{w},\mathbf{v}} p_f(w_1,\cdots,w_6;v_1,\cdots,v_6)$. The twelve interval samples are used as the constraints to calculate the probability of failure.

Table 15 and Fig. 11 show the bounds of the probabilities of failure obtained from the traditional method with the independent distribution parameter assumption and the method with raw data. The results indicate that the method with raw data produces much narrower bounds of the probability of failure than those from the traditional method. The reduction of the bound width is about 75%.

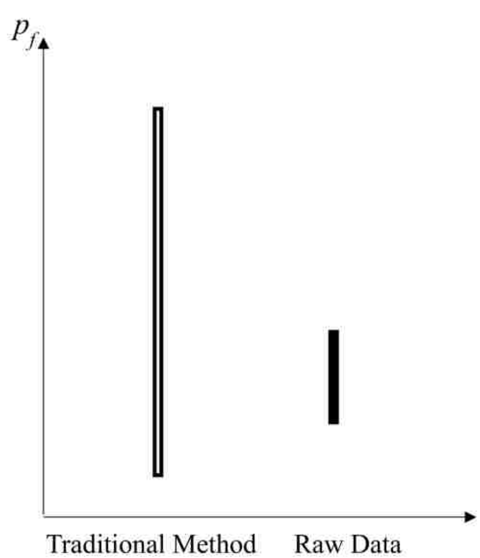


Figure 11 Bounds of probability of failure

Table 15 Estimation of the probability of failure

	Traditional Method (10^{-4})	**Method with Raw Data (10^{-4})**	**Percentage Reduction**
p_f	[1.82, 14.562]	[4.0749, 7.2246]	75.28%

5. CONCLUSIONS

When interval samples exist, the distribution parameters of a random input variable are also intervals. The distribution parameters are dependent because they are estimated from the same set of samples. If the dependence is not considered, the domain of the distribution parameters is a box-shaped hyper rectangular, which is determined by the lower and upper bounds of each distribution parameters. This study shows that the actual domain of the distribution parameters is not a hyper rectangular and that the pattern depends on the number of interval samples. This study also finds that the actual domain is enclosed by and is smaller than the box-shaped hyper rectangular domain. Besides, the ignorance of distribution parameter dependence may also result in wider reliability bounds than the true ones, making decision-making difficult.

In many situations, however, raw point and interval samples are proprietary and may not be available to reliability engineers and design engineers who know only the simple bounds of distribution parameters. As a result, they could only assume that the distribution parameters are independent, leading to the box-shaped hyper rectangular of distribution parameters. One future task is how to report distributions and their parameters to reliability engineers and design engineers so that the dependence of the distribution parameters can be presented without giving the raw samples, for example, a mathematical expression can be found to represent the oval-shaped domain of dependent distribution parameters. The other research issue is to develop efficient reliability methods for problems having input random variables with dependent distribution parameters.

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SECTION

2. CONCLUSIONS

Conceptual design is the most crucial stage in product design. Considering reliability in this design stage has a much greater impact on product performance, quality, and reliability than doing so in latter design stages. It can not only help generate design concepts with high intrinsic reliability but also help evaluate and select the best design concepts with respect to reliability. The current reliability methodologies for conceptual design, however, are much less mature than their counterpart in detailed parameter design stage; the major obstacle is the lack of information in the early design stage. The challenges of considering reliability in conceptual design also provide great opportunities for future research in this area. In this work, some new methodologies are proposed to deal with component dependence, time dependence, and distribution parameter dependence. With the proposed approaches, narrow reliability bounds are achieved, making decision-making easier.

A system reliability method is developed to predict the reliability of the new product in the early design stage using the component reliabilities provided by component suppliers. The method is based on the strength-stress interference model that takes the dependence between components into consideration, thereby eliminating the assumption of independent component failures. As a result, the predicted system reliability bounds are much narrower than those from the assumption of independent component failures. The method is also extended to time-dependent problems. The analysis process is simplified by converting the time-dependent reliability analysis into its time-independent counterpart with the use of the extreme value of the system load.

This study has shown the feasibility of considering dependent component failures and time dependent stochastic loading process for predicting system reliability bounds in the early design stage.

The other challenge of reliability prediction in early design stages is limited information in different formats. The distribution parameters needed for reliability analysis are usually estimated with mixed point and interval samples in early design stages. The distribution parameters are dependent because they are estimated from the same set of samples. If the dependence is not considered, the domain of the distribution parameters is box-shaped hyper rectangular, which is determined by the lower and upper bounds of distribution parameters. This study finds that the actual domain is enclosed by and is smaller than the box-shaped hyper rectangular domain. Besides, the ignorance of distribution parameter dependence will also result in wider reliability bounds than the true ones, making decision-making difficult.

The future work will be the improvement and applications of the proposed methodologies to more complex systems such as mixed systems with multi-loading. How to integrate scarce data with different structures and from different sources is also a potential future task. Another future work is to develop a decision making strategy under various uncertainties in early design stages.

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VITA

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