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Age by Stage Modeling of Dynamic Heterogeneity

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AGE BY STAGE MODELING OF DYNAMIC HETEROGENEITY

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Mortality modeling has come a long way since the demographer Benjamin Gompertz (1779-1865). We address populations where mortality is structured by the joint effects of age and state and individuals can change state at each age. Dynamic states are the most complex and interesting states to consider and we focus on three categories of states: being married or unmarried, being below or above a particular income threshold, or being in one of four income states. We examine how the transience of our particular states at each age drives the cohort dynamics such as the demographic structure and lifespan inequalities within the cohort.

In each chapter we used two U.S. nationally representative data-sets (the Health and Retirement Survey RAND data-set, and the National Longitudinal Survey of Youth) to statistically estimate the probabilities of survival and transitions between states at each age with regression analysis. These probabilities were incorporated into discrete age and discrete state matrices. We examine age-specific state structure, the average remaining life expectancy, its variance, cohort simulations, dynamic heterogeneity and individual trajectories.

In chapter 2 we find that the survival advantage of being married changed with age. At young ages, it was negligible. At mid to late ages it was considerable, and at late old ages, it was disadvantageous. The probability of staying and becoming married

decreases with age. Married people live longer than unmarried people, the benefit is enhanced for males at mid-ages. At early ages more women entered marriage than men, while at late ages more women exited marriage than men. In contrast to our dynamic model, the results of a model in which state became fixed at some particular age leads to conflicting results among interviews.

In chapter 3 we consider three threshold income levels. We find, consistent with earlier literature, that for most ages the above threshold income state has the highest one-period survival probability at each age for mid-ages to about age 80. The advantage is greatest between those above and below the 1× poverty threshold (1 × the annual official poverty line) when compared to those above and below 2× or 3× poverty. Yet more state switching occurs across the threshold as the income threshold is increased. The largest discrepancy in average remaining life expectancy and its variance occurs at mid-ages. And fewer individuals are in the lower income state between ages 40-60. Our results suggest that dynamic heterogeneity in poverty and the transience of the poverty state is associated with income-related mortality disparities (less transience, especially of the higher income states, more disparities). This chapter extends the literature on individual poverty dynamics and stage-by-age matrix models.

In chapter 4 we again used state-by-age modeling to capture individual entry and exit in dynamic states, and the four income states considered here are: <1×, 1-2×, 2-3×, >3× the poverty threshold. These income states are very relevant since current income inequality research examines the spread of income in various populations but few studies consider how dynamic heterogeneity and probabilities of transitioning in and out of income states at each age influence mortality disparities in cohorts.

We find that for most ages the higher income states have the highest probability of surviving from one year to the next until about age 86 when the order of the income states does not equate to the order of survival advantage. In general, each income state has the highest annual probability of staying in the same state at each age, with the next highest transition being to move to higher income states. The greatest advantage in average remaining life expectancy between consecutive states is for those in $<1\times$ poverty moving to 1-2$\times$ poverty at ages 32-49. The largest discrepancy in average remaining life expectancy and its variance between all states and the $<1\times$ poverty state occurs at mid-ages (40-60). And the fewest individuals are in the lower income states between ages 40-60. Our findings are consistent with results based on other data sets, but we also investigate the dynamic heterogeneity in income state at each age. They reveal that annual stasis probabilities in income state at each age influences the cohort state structure, the dynamic heterogeneity of the cohort, and inequalities or income related mortality disparities at each age.

The dynamic models and analysis used here provide a link between distinct characteristics of individuals in a cohort, such as various state variables and senescence, with the dynamics of age-structured cohorts. This dissertation extends the literature on modeling individuals in a cohort that are undergoing dynamic heterogeneity and stage-by-age matrix models. And it serves as a bridge between stage by age matrix models and other multistate methods.

To my children...

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Table of Contents

List of Figures

List of Tables

CHAPTER 1

Introduction

Populations are heterogeneous, individuals are dynamic and stochastic across their lifetimes, and survival advantages can change with age. I account for these observations with simple age-by-stage matrix models of a cohort of individuals experiencing dynamic heterogeneity over their lifetime (see Table 1.1 for some relevant definitions). Each chapter in this dissertation uses a variation of this model with a different application. In chapter 2, I consider the dynamic state of marital status and compare gender and age-specific differences in marital entry and exit rates. In chapter 3, I consider three distinct poverty thresholds and how the probabilities of crossing the threshold and the resulting survival (of being above and below each threshold) shapes the cohort dynamics. In chapter 4, I again investigate the three income thresholds, but with a multi-stage by age model where individuals can be in one of four income states that have survival, life expectancy and variance ramifications at each age. To estimate probabilities of survival and transitioning across the lifespan I combine data from two U.S. nationally representative data-sets, using the Health and Retirement Survey (HRS) for ages above 50, and the National Longitudinal Survey of Youth 1979 (NLSY79) for ages below 50, for all three chapters.

Both the fusion of the data and the application of the age-by-stage model are unique. My goal is to understand the consequences of changing states on cohort dynamics and individual variability, ultimately connecting individual stochasticity in state to cohort dynamics and heterogeneity. In each chapter I begin by explaining how marital status or income state is associated with mortality, and give a brief intellectual history. I then specify the methods, which are similar across chapters. Next I examine stage and age based patterns of survival and age patterns of state stasis or transitioning. I observe: cohort projections and the change in a cohort's stage distribution with age, the remaining life expectancy at each age and the variance in the remaining years of life for each age. With Markov chain analysis: I also observe simulated cohorts and summary statistics such as first passage times into each state, etc. This exploration leads me to a couple of general conclusions. First, the transience of a state has consequences on mortality disparities between states; the more transient (higher probability of exiting) the state the less disparity in life expectancy. And second, mid-life seems to be the least transient age period for the states viewed here with the greatest survival advantage. For example, those with a higher income will tend to stay in that state and married individuals are most likely to stay married at those ages. Around mid-ages is also the time most individuals are distributed in the higher survival state.

This nuanced perspective of investigating cohort dynamics based on individual stochasticity and dynamic heterogeneity is a new development in the ecological mortality modeling world, but it has some crossover with classic human demography's multi-state life table analysis. My approach achieves the same results but from a slightly different perspective. In the rest of the introduction I present a brief intellectual history of mortality modeling to frame our model, noting that this is the first time I am aware of that similarities between age-by-state matrix modeling and multi-state life table analysis (or increment-decrement life tables) are explored in depth.

1.0.1 Mortality modeling

Mortality and survivorship modeling has come a long way since Gompertz first modeled a homogeneous population experiencing exponential physiological decline with age (Gompertz, 1825). Gompertz was the first to make a mathematical model of age-specific hazard based on his observation that "It is possible that death may be the consequence of two generally co-existing causes; the one, chance, without previous disposition to death or deterioration; the other, a deterioration, or an increased inability to withstand destruction [with age]" (Gompertz, 1825). Since then models of mortality have become more complex, including different functional forms, more parameters and heterogeneity among individuals in populations.

For instance, one approach is to assign individuals in a population different levels of 'frailty' that determine their mortality rate trajectory (Vaupel and Canudas-Romo, 2002; Yashin et al., 2001). The 'frailty' of an individual is thought to be an underlying trait that individuals carry with them throughout their lifetimes, perhaps related to specific genetic or biological characteristics. Other models have incorporated heterogeneity using a 'vitality' measure, that reflects an individual's capacity to deal with external and internal stressors (Anderson et al., 2008; Li and Anderson, 2009; Li et al., 2013; Strehler and Mildvan, 1960; Wagner, 2011). It has also been pointed out that the decline in vitality with age is non-linear, although perhaps starting out linear, and fixed heterogeneity of vitality has been incorporated (Steinsaltz and Evans, 2004; Vaupel and Canudas-Romo, 2002).

Fixed heterogeneity models assume that individuals are born into a certain (health) state which stays with them throughout their lives; that it is an inherent characteristic of the individual (Vaupel and Canudas-Romo, 2002; Wagner, 2011). Dynamic heterogeneity models assume an individual can switch (health) states throughout his or her life and are a relatively new addition to the bio-demography field (Steiner et al., 2012; Tuljapurkar and Steiner, 2010). Having dynamic heterogeneity in the model reflects that human individuals can and do switch between (health) states over the course of their lives. As opposed to fixed heterogeneity where a population has a specific, hard to identify, fixed 'distribution of innate abilities'.

1.0.2 Structured population modeling

There are many ways that a population can be structured. Ecologists have found that stages are important to mortality models since they are often a better indicator of annual survival than age in many organisms (for instance, in sea turtles) (Crouse et al., 1987; Lefkovitch, 1965). However, in humans and other organisms with determinate growth, age is also known to be necessary to characterize rates of death and birth.

Ecologists are accustomed to dynamics of some populations (such as reproduction and mortality dynamics) that depend on either stage or age or both, and have used both stage structure and age structure. For instance, when studying *Pinus sylvestris*, or other forest species, a modified Lefkovitch matrix (discrete-time deterministic matrix model allowing any transition from one stage to another) was used. Then the renewal equation was incorporated to obtain the distribution of the ages and stages of individuals at birth, as well as the stable age and stage distributions (Houllier and Lebreton, 1986; Houllier et al., 1988). In this model individual survival and transition parameters did not vary with age, and individuals could not return to a past stage.

There have been other models (for instance on plant populations) that explicitly structured populations into age and size (as the stage) using matrices (Law, 1983), but without including heterogeneity. New theoretical work focuses on life history analysis for organisms in which rates of birth and death are determined by a combination of age and stage, and stage transition rates vary with age (Steiner et al., 2012; Tuljapurkar and Steiner, 2010). This age-by-stage approach incorporates the phenomenon of 'dynamic heterogeneity' (Tuljapurkar et al., 2009) and is a relatively new addition to mortality modeling theory from an ecological perspective. I make use of this new work and update the model by making the stage transition and survival rates themselves functions of both age and stage.

Classic mathematical demography has always viewed populations from a (single-state) life table perspective, which is age structured, and has developed its own techniques to deal with "transitions between multiple states of existence" (Rogers, 1980). In his work, Rogers (1980) united all the methods of "multiple decrement mortality tables, tables of working life, nuptiality tables, tables of educational life, and multi-regional life tables [as] members of a general class of increment-decrement life tables called multistate life tables". Multistate life table analysis has a history of using matrix notation as well, which in this form is most similar to our age-by-stage matrix models (Rogers, 1980; Schoen, 1988; Willekens et al., 1982). Over time multistate life table analysis has evolved to deal with small panel data and multiple covariates (Land et al., 1994) and a range of data complexities (Willekens and Putter, 2014).

The model used here can address similar questions as those addressed by multi-state (or increment-decrement) life table analysis (Schoen, 1988). However, my model emphasizes the discrete time framework, and the stage by age structure throughout the lifetime, facilitating cohort projections and Markov chain analysis, including the fundamental matrix (Kemeny and Snell, 1976) (which multi-state demography does not use), and the realizations of the process encapsulated by simulated individual lifetime trajectories. In deference to work on multi-state life table analysis, I use the human demography convention and use the term 'state' rather than the more general ecology term 'stage'.

Similiar to multi-state analysis, the model I use can have far-reaching applications, besides poverty status and marital state, states could be defined as any dynamic condition with survival or mortality ramifications, such as socioeconomic status, health state etc. (Tuljapurkar and Boe, 1998), or even environment (Coulson and Tuljapurkar, 2008). The state transition rates have important survival ramifications and lead to interesting questions, for instance, how long can an individual maintain stasis in his/her higher survival state? How does an individual's ability to maintain stasis change with age? If individuals enter the lower survival state how does their ability to exit change with age? And of course, what are the consequences for state-discrepancies in average remaining life expectancy, the variance in life expectancy, the cohort stage structure, and total time spent in each state across the life course?

The questions above are overarching and I address each of them when I consider a specific cohort's dynamics in each chapter. The specific objectives of each chapter are to answer the following: 1) How does the transience of marriage affect cohort dynamics for males and females? 2) How do cohorts differ in dynamic heterogeneity and cohort dynamics (such as variance in average remaining life expectancy) when the binary state (poor or not poor) is defined by different income thresholds: “official” poverty, “near poverty”, and approximately at the median income? 3) When there are four distinct income states individuals can transition into, what are the age and state specific cohort dynamics? This dissertation research bridges the gap between ecological stage by age modeling and mathematical human demography by estimating survival and transition parameters as stage specific functions of age.

Age-specific	Describes a quantity that is a function of age, i.e. changes with age.
Cohort	A group of individuals in a population born at the same time.
Dynamic heterogeneity	When the (health) status of individuals in a population varies and individuals can switch status throughout their lives.
Fixed heterogeneity	When the (health) status of individuals in a population vary and are fixed at birth.
Fundamental matrix	A matrix whose entries inform about the expected number of visits to a transient state. Denoted '**N**', it can be used to determine other properties about the Markov chain. Mathematically it is a series of the age-by-state matrix **L**, raised to the power of x, where x ranges from 0 to infinity and denotes age. In closed form, **N** converges to the inverse of the identity matrix minus **L**
Hazard	Also called instantaneous hazard. The instantaneous risk of dying.
Mortality	Probability of death for one-period.
One-period	Time between age x and $x + 1$; one year.
Stasis	The annual probability of staying in the same state.

Stochastic	When the outcome is drawn from a probability distribution.
Survival	Probability of surviving for one-period. Mathematically, the number of individuals alive at age $x+1$/ number of individuals alive at age x
Survivorship	Cumulative survival; the proportion of a birth cohort surviving at each age. Mathematically, the number of individuals alive at age x/ number of individuals in the birth cohort.
Remaining life expectancy	The years an individual is expected to stay alive in the future given that he or she has survived to a particular age.
Renewal equation	An equation that can be used to calculate the number of offspring entering a population at a certain stage and time. It shows how present births were generated by previous births, i.e. how the population 'renews' itself. Mathematically, the number of births at time t is a function of two components: births to women alive previous to time $t = 0$, and births to women born since $t = 0$.

Transience	Lasting only for a short time. Mathematically, an individual in a ‘transient’ state has a high annual probability of exiting, and low annual probability of stasis.

Table 1.1: Some relevant definitions

CHAPTER 2

An age-by-stage model of mortality with age-specific state transitions: effects of the dynamic state of marital status

2.1 Summary

Mortality modeling has come a long way since Gompertz. We address populations where mortality is structured by the joint effects of age and state and individuals can change state at each age and where we focus on one such state, marital status.

Using empirical data, we estimated the probabilities of survival and transitions between states (married vs unmarried) at each age. These probabilities were incorporated into age-state matrices, and survivorship and state structure at each age were quantified. Utilizing Markov chain analysis, we analytically obtained remaining life expectancy and simulated individual lifetime trajectories.

The survival advantage of being married changed with age. At young ages, it was negligible. At mid to late ages it was considerable, and at late old ages, it was disadvantageous. The probability of staying and becoming married decreases with age. Married people live longer than unmarried people, the benefit is enhanced for males at mid-ages. At early ages more women entered marriage than men, while at

late ages more women exited marriage than men. In contrast to our dynamic model, the results of a model in which state became fixed at some particular age leads to conflicting results depending upon which ages are chosen.

The dynamic model and its analysis serves to provide a link between distinct characteristics of individuals in a cohort, such as various state variables and senescence, with the dynamics of age-structured cohorts. This paper extends the literature on modeling individuals in a cohort undergoing dynamic heterogeneity, using stage-by-age matrix models.

2.2 Background

Human demographers, sociologists, and epidemiologists have long observed that marital status is correlated with mortality (Kaplan and Kronick, 2006; Manzoli et al., 2007; Roelfs et al., 2011; Zheng and Thomas, 2013) often even surpassing gender discrepancies in mortality (Trowbridge, 1994). The trend of married individuals having a decreased rate of all-cause mortality, when compared to unmarried counterparts, is consistent across a wide array of cultures and regions (Va et al., 2011). For instance, in a meta-analysis and meta-regression Shor et al. (2012) analyzed 600 million people from 24 different countries and found that married individuals had a decreased relative hazard as compared to divorcees, with men benefiting more from marital status than women. Most studies also confirm that, when compared to married individuals, all alternative statuses of 'unmarried' (such as never married, divorced, and widowed) have a significantly higher risk of death (Manzoli et al., 2007; Trowbridge, 1994). This trend in higher all-cause mortality for the unmarried (which seems to be increasing over generations) coupled with the increased age of entry into marriage, and increasingly transient nature of marriage (Bureau, 2015; Iwashyna and Christakis, 2003; Lin and Brown, 2012; Liu and Umberson, 2008; Roelfs et al., 2011; Schoen and Weinick, 1993; Trowbridge, 1994) emphasizes the need for modeling that takes into account individual dynamics at each age to understand population trends. Others have noted that the average duration of first marriages and beyond, average age of entry into first marriage and beyond, and other trends in marital status over a lifetime are in flux (Lin and Brown, 2012; Liu and Umberson, 2008; Lundberg, 2012; Robards et al., 2012). Several papers have also noted that focusing on cross sectional data to understand differences in marital status and mortality is not sufficient (Goldman et al., 1995; Johnson et al., 2000; Robards et al., 2012).

To capture marriage transitioning throughout the life course we use two nationally representative data-sets, the National Longitudinal Survey of Youth 1979 (NLSY79) and the Health and Retirement Survey (HRS), and fuse them together to obtain marital status based probabilities of survival and transitioning as functions of age for males and females (Bureau of Labor Statistics, 2012; Moldoff et al., 2014). We then use a state-by-age matrix model to project a birth cohort through all ages, noting the state distribution at each age, and use Markov chain theory to obtain the marital state-specific average remaining life expectancies, and simulated individual trajectories of individuals who may change marital status over their life course.

Our approach is a special case of recent models in ecological mortality modeling and bio-demography that explicitly structure populations by both age and state, including distinct survival and state transition probabilities at each age (see Steiner et al. (2012); Tuljapurkar and Steiner (2010)). The principal distinction of our model is that the probabilities of survival and transitioning at each age are themselves estimated as parametric functions of age. This new development directly intersects with classical demography's multi-state lifetable analysis, which also accounts for individuals entering and exiting different marital statuses throughout their lifetime (Schoen and Weinick, 1993; Willekens et al., 1982; Zeng et al., 2012). In theory our approach overlaps most closely with Rogers (1980) presentation of multi-state life table analysis (or increment-decrement life tables), as he used matrix notation. In practice, there are many statistical tools and packages in existence that will calculate the survival and transition probabilities (often the required input for multi-state life table analysis as well) for many complicated data scenarios (Cai et al., 2010; Willekens and Putter, 2014). Here we choose to directly obtain our probabilities with regression analysis and then apply our age-by-state matrix model to understand the dynamic heterogeneity of marital status. We ask how does the survival advantage of being married change with age? How does the probability of becoming married or staying married change with age? At which ages is marriage more advantageous for males and females? How does the survival advantage appear when we consider marital state as a fixed (rather than dynamic) state?

Our results addressing these questions are in agreement with current literature on marital state and mortality (Clarke, 1995; Schoen and Standish, 2001). Our analysis includes the following steps. (1a) First, we statistically analyze the empirical data to create parametric functions that address the question: how do the probabilities of survival and transitioning between states (married vs unmarried) quantitatively depend upon age for each gender? (1b) We incorporate the age-specific probabilities of survival and state transitions obtained from evaluating these functions at each age into our age-state matrices for each gender. (2) Next, we use these matrices to project a given cohort of newborns forward through life to quantify survivorship to each age and state structure at each age. (3) Then, we also use these matrices in Markov chain analysis in two ways: (a) to analytically obtain the fundamental matrix from which we readily calculated remaining life expectancy at each age and (b) to simulate individual lifetime trajectories from which we calculated first passage times, transitioning, and the total time individuals spend in each state. Lastly we addressed the question: (4) If we had used a model in which state (marital status) became fixed at some particular age rather than being dynamic, how would estimated survivorship to each age differ from what we found in the dynamic model? The model and its analysis serves to provide a link between distinct characteristics of individuals in a cohort, such as various state variables and survival, with the dynamics of age-structured cohorts.

2.3 Methods: Theory

2.3.1 Model theory

We use a discrete time, discrete state, discrete age, Markov chain matrix with two-states at each age. The matrix is similar in structure to a population projection age-state matrix (Tuljapurkar and Steiner, 2010) but here there is no reproduction, which is similar to Steiner et al. (2012). It is also similar to the discrete age-time model of multistate demographic growth (Rogers, 1980) except again we only view a cohort and do not include fecundity (fig.2.1 illustrates the matrix model inputs and outputs). We use this age-by-state matrix, which we denote **L**, to calculate cohort dynamics and individual trajectories, which are steps 2 and 3 above, respectively. Specifically, the matrix is used: (2.1) to project a cohort from birth across the lifetime, which enables tracking of the state-distribution and survivorship of an initial cohort with a particular state distribution from age 0 to a maximum age, as described by the following equation:

$$n(x) = \mathbf{L}^x n(0) \tag{2.1}$$

(2.2) to analyze remaining life expectancy (mean and variance in age at death) and generate individual stochastic trajectories across all ages, where each individual is a realization or sample path of the Markov process. The Markov chain is described by this transition matrix:

$$\mathbf{P} = \left(\begin{array}{c|c} \mathbf{L} & 0 \\ \hline \mathrm{m} & 1 \end{array} \right) \tag{2.2}$$

Here we will define the age-and-state matrix $\mathbf{L}$, and in the appendix (B.1) we show how it can be used in (2.1) and (2.2). L is based on parameters of four functions at each age, $s_1(x)$,$s_2(x)$,$t_{21}(x)$ and $t_{22}(x)$:

> $s_1(x)$ = Probability of survival from age x to age $x+1$ for an unmarried individual. $1 - s_1(x)$ = Probability that an unmarried individual dies between ages x and $x+1$ (one-period state-specific mortality).
>
> $s_2(x)$ = Probability of survival from age x to age $x+1$ for a married individual. $1 - s_2(x)$ = Probability that a married individual dies between ages x and $x+1$ (one-period state-specific mortality).
>
> $t_{21}(x)$ = Probability of becoming married at age $x+1$ for an individual who is unmarried at age x (transitioning from unmarried to married). $1 - t_{21}(x) = t_{11}(x)$ = Probability that an individual who is unmarried at age x will remain unmarried at age $x+1$, conditional on survival.

$t_{22}(x)$ = Probability of staying married at age $x+1$ for a individual who is married at age x (being married and staying married). $1 - t_{22}(x) = t_{12}(x) =$ Probability that an individual who is married at age x will become unmarried by age $x+1$, conditional on survival.

Here x is a 1-year age interval (so age x to $x+1$ is a one year step), although, depending on the model or data, other interval lengths can be used (Keyfitz, 1968). State transition probabilities (transitioning to a new state or staying in a state at the next age) are conditional on survival. When the conditional state transition probabilities are multiplied by the probability of survival, the unconditional transition probabilities results.

The unconditional state transition matrix at each age, x, takes the form:

		State at age x	
		Unmarried	Married
State at age x+1	Unmarried	$s_1(x)t_{11}(x)$	$s_2(x)t_{12}(x)$
	Married	$s_1(x)t_{21}(x)$	$s_2(x)t_{22}(x)$

We denote each unconditional state transition matrix as $\mathbf{Q}(x)$, and each unconditional state transition matrix is inserted into the sub-diagonal of the age matrix at the appropriate column. There are 100 unconditional state transition matrices, each representing a 1-year increment from 0 to 100 years of age. The age-state block matrix has dimensions 101×101 blocks, each block is comprised of a 2×2 $\mathbf{Q}(x)$ matrix, and has the following form:

$$\mathbf{L} = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ \mathbf{Q}(1) & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & \mathbf{Q}(2) & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{Q}(3) & \dots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \mathbf{Q}(98) & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & \mathbf{Q}(99) & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & \mathbf{Q}(100) & 0 \end{bmatrix}$$

The age-by-state block matrix $\mathbf{L}$ has a structure reminiscent of a Leslie matrix from which fecundity has been removed, in that an individual always transitions to the next age at each time step (Kot, 2001; Leslie, 1945). Additionally, since each $\mathbf{Q}(x)$ is a 2×2 unconditional state transition matrix, each '0' in the $\mathbf{L}$ matrix is a 2×2 matrix of zeros. Since there are 2 state classes and 101 age classes, the dimensions of the age-state matrix $\mathbf{L}$ is 202×202. The $\mathbf{L}$ matrix is the age-by-state matrix in fig. 2.1 that can be used to produce cohort projections (see B.1.1) and markov chain analysis. With markov chain analysis we calculate the average remaining life expectancy (by using the fundamental matrix, see B.1.2) and the simulated individual trajectories (see B.1.3).

2.4 Methods: Empirical data

To calculate the age-and-state-specific survival and transition rates, we incorporate empirical data from two datasets, The Health and Retirement Study (HRS) and the National Longitudinal Survey of Youth (NLSY79). The statistical models which are crucial for step 1 and running the matrix model in steps 2 and 3 are derived from logistic regression analysis on both data sets as described in this section.

2.4.1 HRS RAND

The Health and Retirement Study (HRS) is a publicly accessible longitudinal household survey data set for the study of retirement and health among the elderly (individuals over age 50 and their spouses) in the United States. We use the RAND HRS Data files Version O which "are a cleaned, processed, and streamlined collection of variables derived from HRS" (Moldoff et al., 2014). The survey consists of 6 cohorts (A.1) and we use longitudinal data compiled from 11 interview waves that fall approximately around these years: 1992, 1994, 1996, 1998, 2000, 2002, 2004, 2006, 2008, 2010, 2012. As a nationally represented data set of $37,319$ individuals, HRS has over-sampled Hispanics, Blacks, and residents of Florida, and provides weighting variables to make it representative of the community-based (non-institutionalized) population. For our purposes we subset the data to only include individuals between 50 and 95 years old. We include weights in all analysis. Here we classify all individuals who responded "married", "married, spouse absent", and "partnered" as a married individual. Individuals who responded "separated", "divorced", "widowed", or "never married" are classified in the unmarried category. Individuals are pooled into repeated observations for each of their interview responses as explained after the following section.

2.4.2 NLSY1979

The NLSY79 Cohort is a longitudinal project that follows the lives of a sample of American youth born between 1957-64. The cohort originally included 12,686 respondents ages 14-22 when first interviewed in 1979; after two subsamples were dropped, 9,964 respondents remain in the eligible samples (Bureau of Labor Statistics, 2012). We use data available from interview wave 1 (1979 survey year) to interview wave 25 (2012 survey year), this includes one year intervals from 1979-1994, and two-year intervals from 1994-2012. Since we are studying the state of marital status we subset the data to include observations from age 18 (the US minimum age of marriage without parental consent in most states) to age 50. A survey response of "married" comprised the married state. Survey responses of "never married", "separated","divorced", and "widowed" were included in the unmarried category. Retention rates for NLSY79 respondents from 1979 to 1993 exceeds 90 percent. Rates from 1994 until 2000 exceeded 80 percent. Rates from 2002 until 2012 have been in the 70s. (Retention rate is calculated by dividing the number of respondents interviewed by the number of respondents remaining eligible for interview) (Bureau of Labor Statistics, 2012). More detailed information about retention rates can be found at NLSY79's website (`www.nlsinfo.org/content/cohorts/nlsy79/intro-to-the-sample`).

2.4.3 Quantifying the functional dependence of survival and transition probabilities on age: logistic regression

There has been much discussion as to how to calculate transition probabilities for Markov transition models (Islam et al., 2004; Islam and Chowdhury, 2006; Korn and Whittemore, 1979; Lawless and Rad, 2015; Yu et al., 2010). (The latter two sources give a good background on the history of estimating transition probabilities from data and propose methods for higher Markov models). One very practical proposal for calculations of binary markov transition models has been to use logistic regression probabilities (Muenz and Rubinstein, 1985). Since logistic regression is very straightforward and intuitive, especially when we have a time-dependent covariate (age), and since the dependent variable is dichotomous (married or unmarried) we employ it for our analysis. Researchers might look towards Yang et al. (2007) or Fujiwara and Caswell (2002) which are two distinct ways to calculate Markov transition probabilities that can incorporate a range of data complexities. Additionally Willekens and Putter (2014) discusses many different statistical packages in R that are used to estimate transition probabilities for multi-state models.

The NLSY pre-1994 data, NLSY post-1994 data, and HRS data are analyzed separately to obtain probabilities for ages $18-22$, $23-50$, and $51-95$, respectively. The results are later combined to give probabilities of survival and transitioning over our complete age range of $18-95$. Within each data set, we pooled all interview waves except for the last wave, this represents the data at time t. For analysis the pooled observations are weighted based on each data set's weight at observation. All the interview waves except for the first one are then pooled together to represent the data at time $t+i$ (where i varies between 1 and 2 years depending on the data, at the end we standardize our probabilities to be for a one-year period). Instead of tracking individuals longitudinally over several ages, we perform a pooled logistic regression analysis on all observations of all ages from time t to $t+i$, where individuals have a particular age x at time t and age $x+1$ at time $t+1$. This method tracks how marital status changes or stays the same from one interview to the next. If an individual did not respond at that specific point (either the interview at t or $t+i$ or both interviews), that observation is omitted from the analysis. However if that specific individual responded later in the study (a different observation), that observation is included in the analysis. This technique has been called the pooled repeated observation method (PRO) and the analysis a pooled logistic regression (Dagostino et al., 1990). The pooled logistic regression analysis was performed in R and the survey package (Lumley, 2014) with `svyglm` was used to incorporate weights. For HRS we have 362,997 observations between the ages of 50 and 95; $161,522$ observations for males, $201,475$ observations for females. For NLSY79 after pooling we have 289,321 observations between the ages of 18 and 50; $145,960$ males, $143,361$ females. NLSY79 is further seperated into two groups, pre-1994 and post-1994 since in 1994 the survey began interviewing every two years (as opposed to one). After regression coefficients are calculated for all three groups (pre-1994 NLSY, post-1994 NLSY, and HRS) the coefficients were logit transformed to obtain probabilities. Probabilities were then adjusted to reflect a one year period for all three groups (see A.3 for more on this adjustment). Thus we obtained parameter estimates for the functional dependence of survival and transition probabilites on age for each gender. We then evaluated these functions at each age for use in our matrix models.

To check for a correlation of a specific marital status at each age and response rate or missing observation, weighted contingency tables for each age were used and chi-squared tests (Pasek, 2016) were performed in R. Men and women were analyzed separately, and for this analysis deceased individuals were removed from the population (they were not counted as non-response).

2.4.4 Failure time analysis

To address the research question in step 4 we used standard failure time analysis, where age at death is the dependent variable to generate survivorship curves. For HRS data, Kaplan-Meier failure time analysis was performed in R with the 'survival' package (Therneau, 2015). Age at death was used for the time until failure and a censor variable was used to reflect if individuals had a recorded death, were alive, or were missing from the study (Fox, 2001).

2.5 Results

2.5.1 How do the probabilities of survival and transitioning between states (married vs unmarried) quantitatively depend upon age for each gender?

The first research question is answered by evaluating the functions for the dependence of state transition and survival probabilities on age. The data were weighted, pooled, and combined (see appendix fig. B.1) and regression coefficients (found in Tables 2.1, 2.2, 2.3, 2.4, 2.5, and 2.6) were obtained. These coefficients were logit transformed and adjusted to represent a probability over a one-year period, our parameter values.

However, we note that if response rate is not independent of state, that phenomenon could confound our ability to interpret the results. There clearly is an increase in non-response with age (appendix figures B.2a and B.2b), which is expected since most longitudinal studies see an increase in non-response with study progression, but we ask, does response rate depend upon marital status as well? To compare the proportion of response at $x+1$ based on marital status at age x, weighted chi-square statistics were calculated at each age. The chi-square statistic for each age looks very different for males and females, (appendix fig.B.2c and B.2d, respectively). We can easily see that for females from NLSY79 there is no significant association between response rate and marital status. Females from HRS have slightly higher incidence of non-response associated with marital status between the ages of 65 and 75 but no overall pattern emerges. Thus we assume that the response rate for each marital state, for women, should not bias the pooled logistic regression results in any direction. Males on the other hand, seem to experience a pattern of increased non-response for those unmarried at age x when compared to those married at age x, peaking at middle ages. This is important to keep in mind in parameter estimation as the logistic regression will exclude a higher proportion of males who are unmarried due to non-response versus married. However the overall proportion of married to unmarried male observations (in appendix fig.B.1c) will be in agreement with the distribution of individuals in each state as calculated later in the Markov models.

Now focusing on our annual rates, which were used in matrices that project from age x to age $x+1$, we see different age-specific survival based on marital state (the dashed vertical line in fig.2.2 shows the seam between NLSY pre-1994, NLSY post-1994 and HRS; i.e., the age at which we concatenated the probability parameters based on each data-set).

Comparing males and females and married vs unmarried survival probabilities (fig.2.2), several patterns are apparent. At young ages, there is no significant difference in survival between married and unmarried states but there is a significant difference between genders. At older ages there is a significant difference between survival of married and unmarried individuals. The difference is most pronounced for males (Table 2.1) although there is a crossover late in life where married and unmarried males have similar survival (fig. 2.2c). For males, unmarried and married states have the largest discrepancy in one-period survival at age 77, with a 0.016 difference in probability. At age 90 there is a crossover in survival advantage. For females, the greatest advantage in one-period survival occurs for married females at age 79 of an increases 0.009 survival probability. Females also experience a crossover, above which unmarried females have a very slight advantage, at age 92.

Married males and females have pretty much equivalent probabilities of staying married at younger ages (below mid-ages)(fig. 2.2). The other trend is that after mid-ages, married males have a higher probability of staying married than married females (fig. 2.2e). This can be explained easily, since married females at older ages are more likely to become widowed (and thus transition into the 'unmarried' state) than males. Furthermore, unmarried females begin with higher probabilities of becoming married at age x, but then are surpassed by unmarried males' probability of becoming married at the age of 31. After mid-ages unmarried males at age x continue to have higher probabilities than unmarried females, of transitioning into marriage at age $x + 1$ for the rest of their lifespan. The probability of entering into marriage from unmarried at age x is in agreement with Clarke (1995), who notes that in 1989 and 1990 the drop in probability of entry into marriage with age was not as steep for men as women.

Slight discontinuities in the probability curves at the dashed vertical lines reflect the joining of our parameters from different data-sets. We chose not to smooth our probability curves, but rather to focus on the general trend (as discussed above), which in our view is readily apparent.

2.5.2 Projection of a cohort of newborns: survivorship to each age and state structure at each age

The survivorship curves obtained from our model of dynamic heterogeneity, by cohort projection of both the male and female cohorts (fig. 2.3), are consistent with previously published human population survivorship curves for males and females (Bureau, 2015). Thus it is at least possible that dynamic heterogeneity is a process that underlies previously documented survivorship curves.

Before describing the age-pattern of state structure generated by our model, we note the age-pattern of state structure in the empirical data. The percentage of married males increases with age, then levels out and then decreases (see appendix fig. B.1c). Females also have a dome-shaped increase and decrease in the percent married with age, however, in contrast to males the decrease of married women with age happens more quickly and there are more unmarried females at old ages (appendix fig. B.1d). The age-pattern of state structure in the data-sets are similar to the age-pattern of the proportion of married and unmarried individuals in the general US population (for instance, see the United Census Bureau's *Marital Status and Living Arrangements of Adults 18 Years and Over: March 1995*). Thus, we think the statistical models calculated from our data set are likely to be generalizable, as will be our estimates of age-specific survival and transition probabilities for males and females in the general U.S. population.

To examine the age-pattern of state structure generated by our model, we started a cohort of newborns entirely in the unmarried state, and we assumed no marriage occurs until the age of 18. The mathematics of the model lead to an equilibrium state structure for those who are still alive; technically, this is a quasi-stationary distribution, since in the long run everyone will die (Aalen and Gjessing, 2001; Darroch and Seneta, 2013). The quasi-stationary state structure is achieved for a cohort of males, fig.2.3c, and of females, fig.2.3d. Here the equilibrium distribution for married and unmarried males and females is dome-shaped (as the data dictated) with men having a wider 'dome', ie. more married individuals into late old age.

The model projection of the proportion who are married at each age (fig.2.3c and 2.3d) is similar to the age-pattern seen in the empirical data; specifically the surplus of unmarried women in late ages when compared to males is similar to what is observed. To partially explain this observation we can consider the transition parameters for females. There is low probability of marriage entry after 80 and a decreasing probability of staying married (the increasing probability of becoming unmarried) with age. These individual dynamics (probability of transitioning) shape the cohort's demographic structure. In the following sections with the results from the Markov chain analysis we will view average individual dynamics (remaining life expectancy) and the lifetime trajectories that also affect the cohort demographic structure.

2.5.3 Markov chain analysis: the fundamental matrix

2.5.3.0.1 Average individual life expectancy From the fundamental matrix (B.1.2) we calculate the age-specific average remaining life expectancy for married and unmarried individuals. The calculation is conditional on surviving to that age, and also incorporates the probability that an individual might transition t each age from its current married or unmarried state, the dynamic heterogeneity. For married (blue) and unmarried (red) males at age x, the life expectancy difference is more pronounced midlife, and there is less difference in life expectancy at early and late ages (fig. 2.4a). For males, the difference in life expectancy between the two states is more than 1.5 years for the ages 31-68, and more than 1 year for the ages of 26-75 (fig. 2.4b). For females we see the same general trend, that marital status affects average remaining life expectancy the most at middle ages and the least at younger and older ages (fig. 2.4c). However married females age-specific life expectancy is greater than unmarried females for a shorter range than males. The difference in female life expectancy for the two states is greater than 1.5 years for the ages 34-57, and greater than 1 year for the ages of 30-69 (fig. 2.4d). Married men enjoy a maximum advantage in life expectancy of 2.53 years at the age of 45 over unmarried men. Married women have a maximum advantage of life expectancy at age 43 of 2.19 years. We can relate these results back to our probability of transitioning (fig.2.2e and 2.2f) by noting that those in the middle ages also have the highest probability of staying married once married at age x, for both males and females, although more pronounced for males.

2.5.4 Simulation: Individual lifetime trajectories

We simulated individual trajectories for a cohort of 10,000 individuals for males and females (fig. 2.5a and 2.5b). Transitioning into marriage begins at age 18 and that is where the most transitioning (the mode) into marriage occurs for both genders (fig.

2.5c and 2.5d). At early ages more females enter marriage than males and generally fewer individuals are becoming married (not just the first marriage) with age for both genders (fig. 2.5g and 2.5h). The average age of first entry into marriage is 27.5 for males, 25.0 for females with a standard deviation of 9.9 and 7.9, respectively.

The first transition out of marriage for most individuals peaks around age 25 (mode for men, age 24 is the mode for women) and than decreases for later ages (fig.2.5e, 2.5f). The mean for exiting marriage for the first time is age 42.9 (standard deviation of 19) and age 44.4 (standard deviation) of 20.3 for men and women, respectively. We observe that there are two peaks of marriage exit, with a smaller peak occuring about age 70 for both men and women. This old age peak in marriage exit is higher for women than for men, perhaps due to becoming widows at these ages. This observation explains why the females (fig. 2.5j) have a higher number of individuals unmarried at later ages than the males (fig. 2.5i).

We also analyzed the distribution of the total years individuals spend in the married state over their lifetime (fig. 2.6a and 2.6d). This model is stochastic, so on consecutive runs we see slightly different outputs, yet men usually are spending more years of their life married (mean 32.6 years, mode 0 years, n=10,000) than women (mean 30.7 years, mode 0 years, n=10,000), and females have a lower standard deviation of total years spent married (18.0 years) than males (19.3 years). Consequently males in the cohort are spending less time unmarried, on average (38.9 years, mode 30 years), than females (45.8 years, mode 27 years) (fig. 2.6c, 2.6d), and women have a higher standard deviation of total years spent unmarried (19.3 years vs. 16.1 years). Keep in mind the entire cohort has spent a minimum of 18 years at the beginning of life unmarried. Finally we see that the average age at death for males (age 75.4, mode

85) is significantly less than for females (mean age 80.0, mode 87), also explaining why females are spending less time married than males (fig. 2.6e, 2.6f). We can only speculate as to which direction the causation occurs but the stochastic individual trajectories and dynamic heterogeneity give us a unique insight into the backbone of the cohort state structure at each age.

2.5.5 A model in which state (marital status) becomes fixed rather than dynamic

To answer our last research question we look at Kaplan-Meier cumulative survival curves of different waves of HRS interview data. The goal is to see how survivorship (cumulative survival) calculated from age at death for two groups of individuals differs, when marital status is defined at only a single, specific interview wave. The failure time analyses are based on HRS interview waves and range from observations at ages 50 to 100 (fig. 2.7 and 2.8). First, looking at the female failure time curves we see that married women at interview wave 1 had better survivorship than unmarried women after age 65. When marital status at wave 2 is considered, married females have a survivorship advantage after age 62. Married females at wave 3 and 4 again have an advantage after 65, before then there is some crossing over in the curves with similar survivorship for married and unmarried individuals who are in earlier ages. Being married at wave 5 shows a survivorship advantage throughout the lifetime when compared to those unmarried females at that wave. Survivorship based on marital status at waves 6, 7, 8, 9 and 10 are more erratic with more than one crossover. At these waves individual females are older (mostly between 70 and 80). It is difficult to explain the trends based on marital status at a specific wave alone without taking into account what we know about survival and marital state transitioning at each age. For instance, from the one-period survival graph for married and unmarried females (fig. 2.2d) we see that the largest difference in survival based on marital status for females occurs approximately between the ages of 65-85. Ages 50-65 and 85-95 show a similar survival for both states and can explain why mixed groups of different aged individuals at each interview wave time point can yield slightly different survivorship curves based on interview wave (fig. 2.7).

Our survival parameters for males show a greater difference than females in survival based on marital status at each age (fig. 2.2c). A crossover between the two states occurs at age 90, with survival becoming close between the two states at about 86 years and also starting close at 50 years. This helps to explain the trend in failure time curves for males from HRS (fig. 2.8). We see that survivorship is greater for males who were married at interview wave 1, 2, 3, 4, 5 and 6. When survivorship for individuals who are married at interview waves 7, 8, 9 and 10 are considered, we see switching in the advantage of being married versus unmarried. We believe this to be due to the smaller advantage in annual survival at later ages, and the fact that at later interview waves, more HRS individuals are in later ages.

We conclude that failure time analysis based where individuals are classified by their marital status at a single interview wave fails to capture the underlying trend in the survivorship and annual survival based on the dynamic state of marital status.

2.6 Discussion

"Marital status research has emphasized an examination of the impact of *change* in marital status in addition to an assessment of the effects of the marital status classification at baseline" (Johnson et al., 2000). Here we demonstrate a state-by-age matrix model that takes into account the dynamic heterogeneity of marital status, i.e. changing marital status over the life course. Models of this kind are increasingly more common as age patterning of marriage entry and exit is changing over time, yet close to 90% of women are still predicted to become married (Goldstein and Kenney, 2001) with about 40% of first marriages ending in divorce. Understanding the individual heterogeneity in marital status is extremely crucial as, across generations and countries, marital status is still one of the most important indicators of mortality risk as compared to other well established health factors (Sbarra et al., 2012), not to mention implications for the welfare of children (Su et al., 2015). Furthermore Goldstein and Kenney (2001) describes a trend where marriage is more common for women with college degrees than for those without. This phenomenon, of marriage being a privilege of the better educated, also occurs in other countries but varies based on a society's concept of gender roles in marriage, and the degree of economic inequality (Kalmijn, 2013). Discrepancies in marital patterns based on education, economic level, etc. are a source of increasing socioeconomic inequality. Bramlett and Mosher (2002) displays differences in probability of marriage entry (first marriage and second marriage) and exit ('marriage disruption') based on race and ethnicity, family background, the presence of children, duration of marriage, duration of divorce, and marriage cohort (from 4-year intervals from 1950-1984). It would be interesting to apply our state-by-age matrix model to compare marital patterns and cohort dynamics between populations with different education levels, socioeconomic statuses, and race or ethnicity; especially since the above mentioned studies show that there are differences in the probability of marriage entry and exit based on these covariates.

Findings could elucidate the link between the age-structured patterning of transitions and mortality, the cohort dynamics, and the covariates mentioned above.

Additionally, much research on marital status and mortality focuses on exploring the causality of the relationship between marital status and health. Williams and Umberson (2004) finds the short-terms strains of marital dissolution to be an indicator of self-assessed health, Zheng and Thomas (2013) found that married individuals are more likely to overestimate their health status, and Iwashyna and Christakis (2003) found that married individuals consistently received better health care. Traditional reasons for the robust disparity between mortality of married and unmarried individuals includes: a marriage protective effect explained by: marriage encouraging healthy behaviors and reducing risky ones, reciprocal care-giving, improved economics of pooling resources, increased social integration reducing stress (Lillard and Panis, 1996), or a marriage selective effect whereby healthier or more economically independent individuals are more likely to get married (Goldman et al., 1995). Either direction of causality being the case, patterns of individuals entry into and exit out of different marital statuses varies with different covariates, and has important implications on overall cohort demographic structure and survivorship. Comparisons in individual trajectory patterns clarifies disparities between gender (as seen in our study), ethnicity, cohort periods, and country mortality rates.

By first answering how marital status, gender and age affect annual survival and the probability of changing marital status at each age, we explicitly observed the dynamic nature of marital status and the ramifications of marital status on survival at each age. The trends in survival that our model indicate are consistent with other reported findings in marriage and mortality research that “married persons have significantly lower mortality than unmarried persons, [a]... result that is established for both men and women, but is observed to be greater for men" (Lillard and Panis, 1996). The 'crossover', where in late life the trend of married individuals doing better than unmarried individuals seems to reverse, can be explained in a few ways. First, it could reflect the increased probability of being in the unmarried state (widowed) at later ages. This also could explain why in late life the difference in female married and unmarried survival probability is less than the male difference in probability (this explanation is also reasonable when taking into account the decreased probability of staying married and becoming married at late ages). Second, as explained in Steinsaltz and Evans (2004), in many mortality models there is the phenomenon of the weaker individuals dying off leaving the more robust individuals behind in any state. Thus a high proportion of weaker unmarried individuals experienced their earlier mortality (represented by the married survival curve outperforming the unmarried survival curve), leaving unmarried survivors in the cohort whom are more robust with higher survival on average than the married survivors at late ages.

The statistical models of the effect of age and state on annual survival and transitions generated a curve of survivorship at each age that is consistent with what is commonly seen for human survivorship, suggesting that it is at least possible that dynamic heterogeneity is a widespread phenomenon underlying human cohort dynamics.

Our simulations and probabilities of survival and transitioning showed that at earlier ages, more women are entering marriage than men, yet more women are also exiting marriage. Both men and women are entering marriage at a decreased rate with age over their lifetimes. And both men and women are exiting marriages with a local maximum around 25 years and 70 years (average about age 43). On average, men are spending more time in the state of being married than women, with women having much greater variation in their greater time spent unmarried.

As seen in the simulations, generally, at earlier ages more women become married than men. However between 1970 and 1984, the percent of 20-24 year old individuals never married increased (Goldscheider and Waite, 1986). This would mean if we would compare generations, the distributions would be shifting. Clarke (1995) actually shows the peak in first marriages in 1970 occurs at 20-24 years but in 1990 occurs in the age range of 25-29 years for both men and women. However, Clarke was looking at 4 year age intervals with 15-19 included, at 1-year age intervals they would have found that 18 and 19 years give peaks in marital rates as we have found. Schoen and Weinick (1993) and Schoen and Standish (2001) also show trends in first marriages by four-year age intervals for different birth cohorts and the general pattern and approximate values are in agreement with our distributions of first entry into marriage (they find that the peak of the distribution goes down with birth cohort, but the general shape of the curve stays the same). The same papers also examine trends in divorce up to age 50 and find the same general pattern, divorce declining with age. Our findings that fewer marriages occur with age for both genders also agrees with Clarke (1995) finding that most remarriages occur in the 20-24 year interval as compared to other 4-year age intervals up to 64 for both 1989 and 1990 marital rates, for both males and females (20-24 is the youngest age interval they view for this group).

Average remaining life expectancy for married and unmarried individuals also varied by gender, with married men having a greater increase in life expectancy for a longer period of lifetime than married women when compared to their unmarried counterparts. We expected that incorporating the dynamic nature of the married state into life expectancy calculations would differ from published work that does not incorporate the dynamic heterogeneity of marital status at each age. One study, comparing life expectancy between married and unmarried individuals with different educational attainments at the age of 55, Brown (2014), found differences of about 3.2 years for males and 1.7 years for females. Our analysis predicts less of a difference in life expectancy of married and unmarried individuals at age 55 than that paper; we found a 1.9 difference for males and 1.4 for females. We think this makes sense because our estimate is based on the assumption that an individual in the married state at age x, could have been in either state at any prior or future age. Work that does not take into account transitioning in and out of marital status assumes a person remains in the married or unmarried state from the time-point of the study for the rest of their lives, and thus results in more drastic differences in life expectancy based on current status. In their classic paper, Willekens et al. (1982) used multi-state life tables to look at life expectancy based on marital state at age 20 for Belgian women (with no assumption that they stay in that state for the rest of their life course). They found the difference between married and never-married, widowed, and divorced to be 0.15, 1.98, and 0.41 years of life expectancy, respectively. Here we found that at age 20, married and unmarried women (consisting of a high proportion of never married, few divorcees, and fewer widows) had about a 0.20 year difference in life expectancy. These results seem to be in agreement.

Lastly we emphasize how analysis of age-specific survivorship that classifies individuals for a lifetime by their marital status at one particular point in time provides a different result than our analysis, which incorporates the dynamic nature of marital status. The fixed marital status approach groups the individuals by their status at a given interview and analyzes the age at death of each group; using classic failure time analysis, a survivorship curve is created for each group. At each interview wave, the grouping of individuals is based solely on the marital status at that particular time. For example, grouping the individuals by their marital status at interview waves 1, 2, 3, 4, 5, and 6, indicated that married males have higher survivorship than unmarried males throughout the lifetime, the details differing among the interview waves. However, grouping the individuals by their marital status at interview waves 7, 8, 9, and 10, indicated that there are some ages where unmarried males have higher survivorship than married males. Similar results with even more differences among interview waves were found for females. Thus neglecting the transience of marital status provides limited results; results are limited to making conclusions about individuals classified for the totality of their lives based on their marital status at only a specific time point. For example classifying individuals by their martial status at age 30 does not help us to understand how their marital status at age 70 influences mortality. For that we need a dynamic model. Without knowing the transition and survival probabilities at each age it would have been difficult to interpret the changing survivorship advantage of the married state at each interview wave.

Extensions of our models include generalizing to a case with more than two states, which can easily be accommodated in the age-state matrix. For instance, this can be done by looking at the subtle different patterning between marriage, cohabitation, divorce, and widowed marital states, as is often done with multi-state life table techniques.

We hope that future research on dynamic states will incorporate some of our techniques to elucidate a nuanced perspective on cohort dynamics. As briefly discussed, comparative studies might be especially useful to show how individual age-dependence in transition probabilities and mortality influence cohort dynamics. Our model also allows exploration of how individuals entering and exiting any of several transient states throughout their lifetime affects the cohort dynamics. The state-by-age matrix approach has far-reaching implications for any population or cohort where individuals undergo dynamic heterogeneity with consequences on survival.

Table 2.1: HRS logistic regression: Dependent Variable = Survival at time t+1

Predictor	**Estimate**	Std. Error	t value	$Pr(>\|t\|)$
Intercept (Male, Unmarried)	7.685	0.255	30.165	<2e-16 ***
Slope (Male, Unmarried)	-0.075	0.003	-22.771	<2e-16 ***
Δ Intercept (Married)	2.011	0.310	6.484	8.98e-11***
Δ Intercept (Female)	1.240	0.319	3.890	1.00e-04 ***
Δ Slope (Married)	-0.022	0.004	-5.477	4.33e-08***
Δ Slope (Female)	-0.009	0.004	-2.233	0.026 **
Δ Intercept (Married, Female)	-0.498	0.450	-1.105	0.269
Δ Slope (Married, Female)	0.006	0.006	0.980	0.327

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table 2.2: HRS logistic regression: Dependent Variable = Marital status at time t+1

Predictor	**Estimate**	Std. Error	t value	$Pr(>\|t\|)$
Intercept (Male, Unmarried)	0.440	0.309	1.426	0.154
Slope (Male, Unmarried)	-0.048	0.005	-10.219	<2e-16***
Δ Intercept (Married)	5.782	0.386	14.991	<2e-16***
Δ Intercept (Female)	1.250	0.439	2.848	0.004 ***
Δ Slope (Married)	0.006	0.006	0.973	0.330
Δ Slope (Female)	-0.038	0.007	-5.575	2.47e-08***
Δ Intercept (Married, Female)	-0.360	0.525	-0.688	0.491
Δ Slope (Married, Female)	0.014	0.008	1.766	0.077 *

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table 2.3: NLSY79-pre1994 logistic regression: Dependent Variable = Survival at time t+1

Predictor	**Estimate**	Std. Error	t value	$Pr(>\|t\|)$
Intercept (Male, Unmarried)	8.690	0.770	11.283	< 2e-16***
Slope (Male, Unmarried)	-0.101	0.029	-3.439	0.000585***
Δ Intercept (Married)	-2.911	2.324	-1.252	0.210
Δ Intercept (Female)	1.104	1.416	0.780	0.435
Δ Slope (Married)	0.158	0.086	1.826	0.0677*
Δ Slope (Female)	-0.009	0.054	-0.175	0.861
Δ Intercept (Married, Female)	-0.399	3.039	-0.131	0.895
Δ Slope (Married, Female)	0.006	0.113	0.055	0.956

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table 2.4: NLSY79-pre1994 logistic regression: Dependent Variable = Marital status at time t+1

Predictor	**Estimate**	Std. Error	t value	$Pr(>\|t\|)$
Intercept (Male, Unmarried)	-2.974	0.103	-28.671	< 2e-16***
Slope (Male, Unmarried)	0.028	0.004	6.844	7.75e-12***
Δ Intercept (Married)	3.718	0.257	14.461	< 2e-16***
Δ Intercept (Female)	1.211	0.146	8.291	< 2e-16***
Δ Slope (Married)	0.049	0.009	5.099	3.43e-07***
Δ Slope (Female)	-0.041	0.005	-6.927	4.31e-12***
Δ Intercept (Married, Female)	-1.192	0.333	-3.579	0.000345***
Δ Slope (Married, Female)	0.040	0.012	3.188	0.001433***

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table 2.5: NLSY79-post1994 logistic regression: Dependent Variable = Survival at time t+1

Predictor	**Estimate**	Std. Error	t value	$Pr(>\|t\|)$
Intercept (Male, Unmarried)	7.324	0.868	8.437	< 2e-16***
Slope (Male, Unmarried)	-0.066	0.020	-3.168	0.001***
Δ Intercept (Married)	0.541	1.646	0.329	0.742
Δ Intercept (Female)	2.055	1.293	1.589	0.112
Δ Slope (Married)	0.016	0.039	0.428	0.668
Δ Slope (Female)	-0.043	0.030	-1.426	0.153
Δ Intercept (Married, Female)	0.911	2.492	0.366	0.714
Δ Slope (Married, Female)	-0.018	0.058	-0.310	0.756

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table 2.6: NLSY79-post1994 logistic regression: Dependent Variable = Marital status at time t+1

Predictor	**Estimate**	Std. Error	t value	$Pr(>\|t\|)$
Intercept (Male, Unmarried)	0.408	0.266	1.535	0.12469
Slope (Male, Unmarried)	-0.068	0.006	-9.911	< 2e-16***
Δ Intercept (Married)	1.292	0.396	3.256	0.00113***
Δ Intercept (Female)	0.405	0.389	1.041	0.29787
Δ Slope (Married)	0.099	0.010	9.825	< 2e-16***
Δ Slope (Female)	-0.010	0.010	-1.083	0.27868
Δ Intercept (Married, Female)	-0.418	0.557	-0.750	0.45318
Δ Slope (Married, Female)	0.009	0.014	0.639	0.52310

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

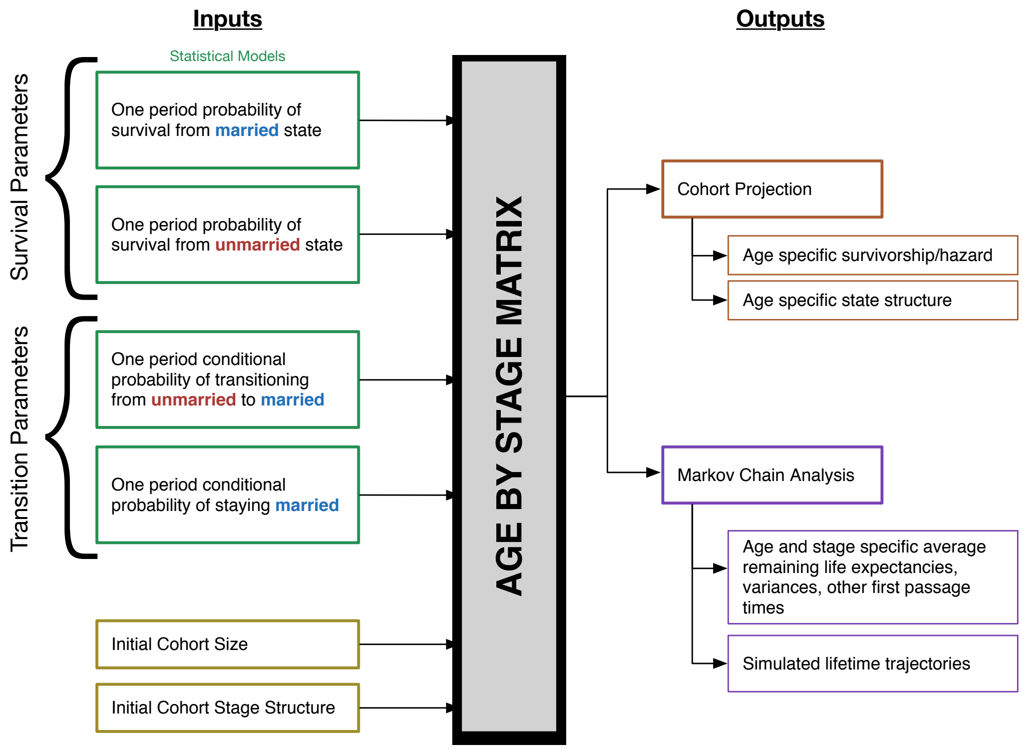


Figure 2.1: Flowchart of dynamic age-by-state model. Inputs include statistical models of the probability parameter values plus initial size and demographic structure. Outputs can be derived from cohort projections and markov chain analysis. (Flowchart made in *Omnigraffle*)

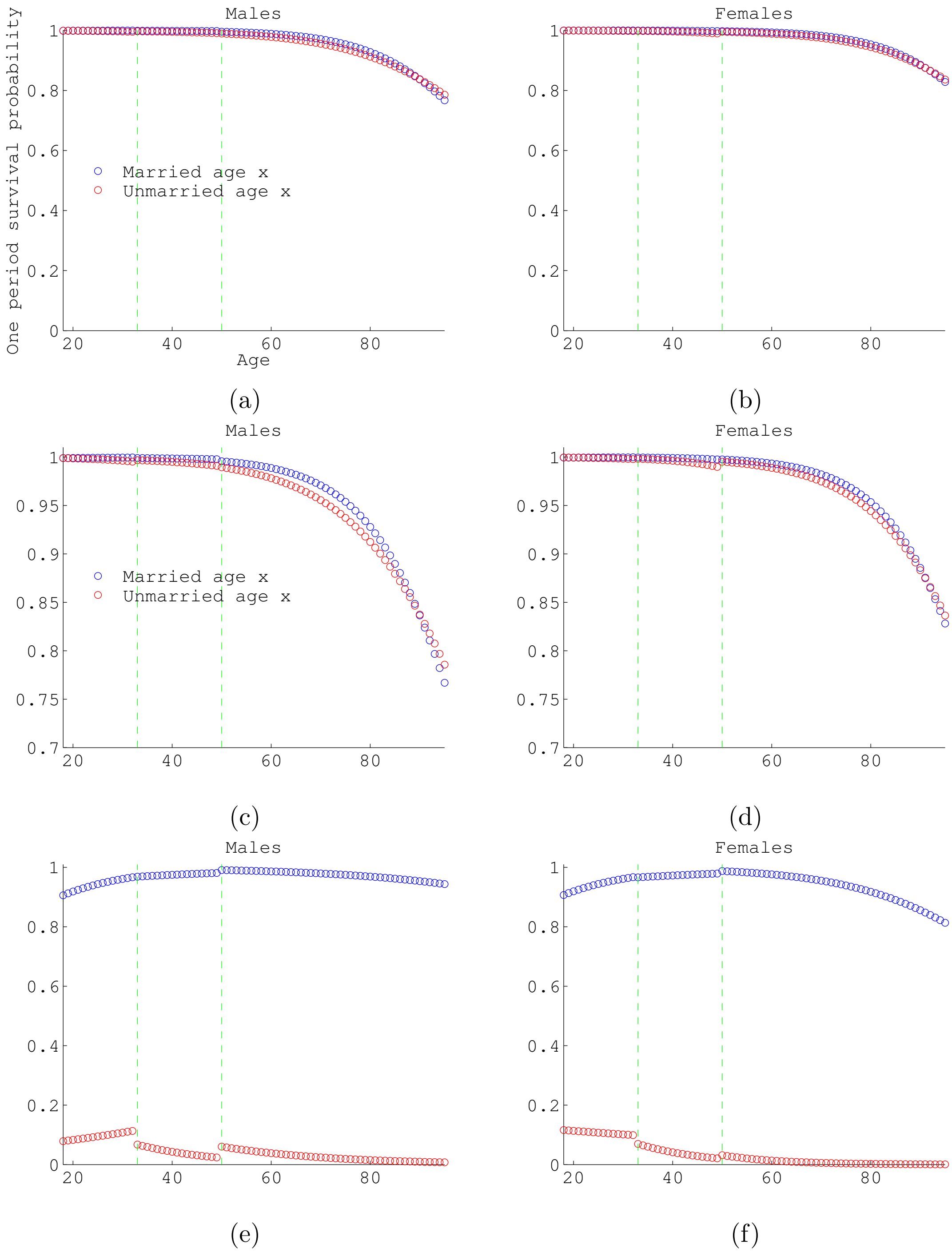


Figure 2.2: Annual survival (number of individuals alive at age $x+1$/ number of individuals alive at age x) and conditional transition probabilities (given survival). Annual survival probability for (a, c) males and (b, d) females based on marital status at age x (blue = married, red = unmarried). Conditional probability for (e) males and (f) females who are unmarried at age x to become married (red lines) and for those who are married at age x to remain so (blue lines). Logit transformed coefficients from NLSY79 pre-1994, NLSY post-1994 and HRS are combined. The dashed vertical lines represents the ages that the data sets are joined and the x-axis spans from ages 18-95.

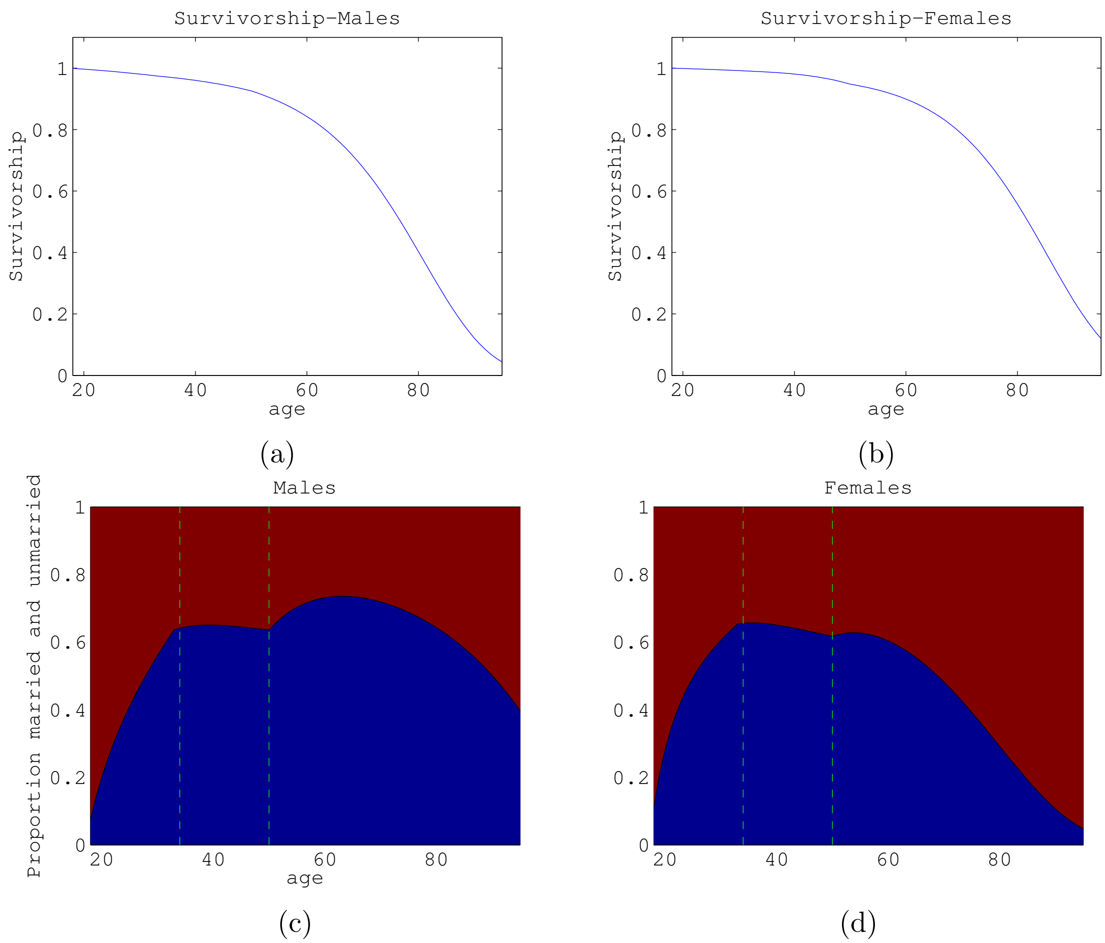


Figure 2.3: Cohort dynamics. From cohort projection of 1,000,000 individuals in the unmarried state until age 18. Survivorship (number of individuals alive at age x/ number of individuals in the birth cohort) for males (2.3a) and females (2.3b). The area plots (2.3c and 2.3d) show the proportion of individuals in the married (blue) and unmarried states (red) at age x (spanning from 18-95). The green vertical lines in panels (c) and (d) are the seams between data-sets utilized: NLSY79 pre-1994, NLSY post-1994, and HRS (at age 33 and 50).

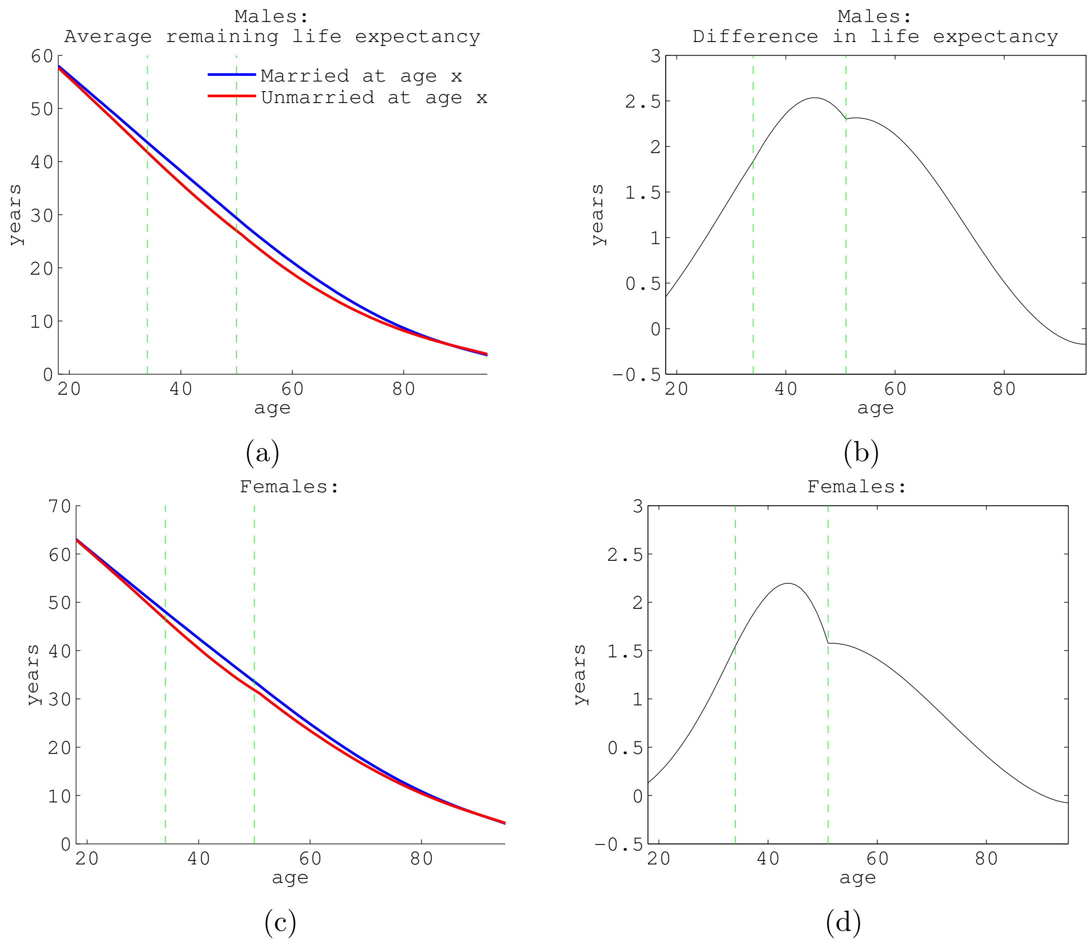
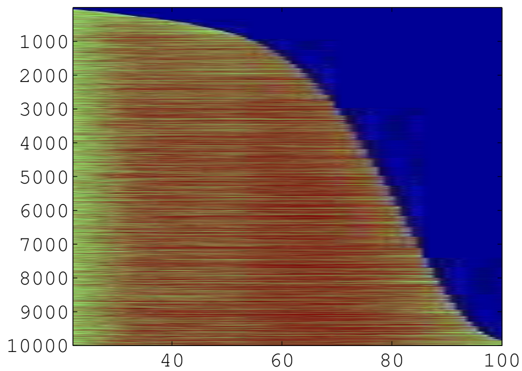
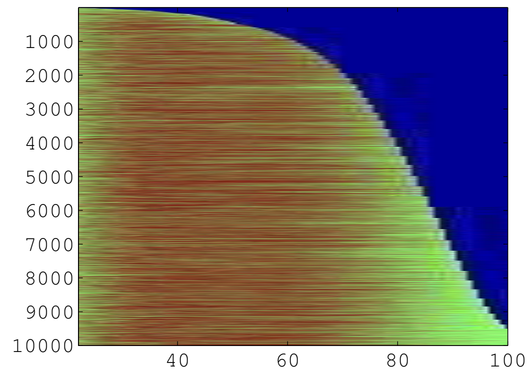


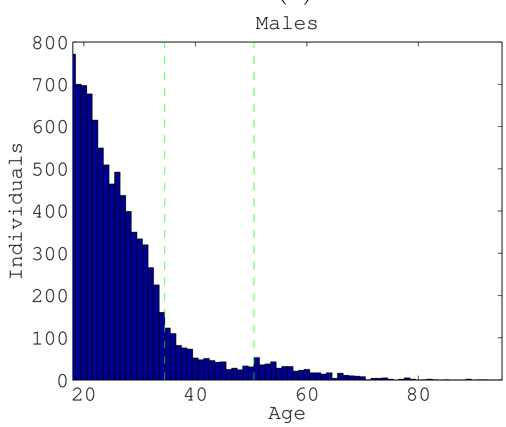
Figure 2.4: The average remaining life expectancy, conditional on having survived to a specific age, for males (2.4a) and females (2.4c). This average remaining life expectancy calculation takes into account the dynamic nature of marital state at all the subsequent ages. (2.4b) and (2.4d) show the difference in average remaining life expectancy between those married and unmarried at age x, for males and females, respectively. Every panel's x-axis spans from age 18 to 95 and the green vertical lines in each panel are the seams between data-sets utilized: NLSY79 pre-1994, NLSY post-1994, and HRS (at age 33 and 50).


1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
40
60
80
100

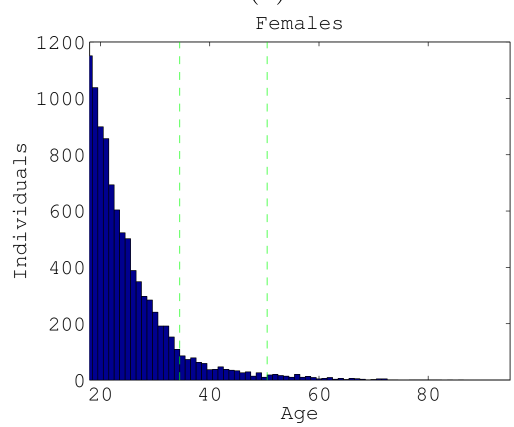
(a)


1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
40
60
80
100

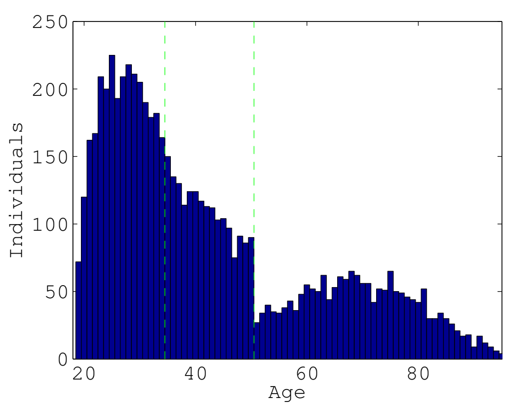
(b)


Males
Individuals
Age
0
100
200
300
400
500
600
700
800
20
40
60
80

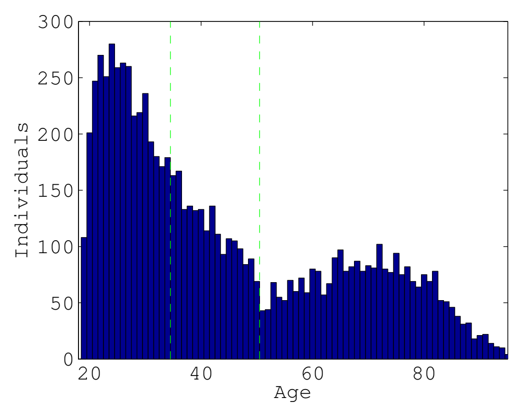
(c)


Females
Individuals
Age
0
200
400
600
800
1000
1200
20
40
60
80

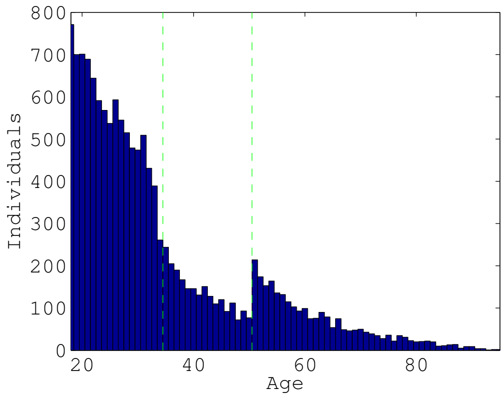
(d)


Individuals
Age
0
50
100
150
200
250
20
40
60
80

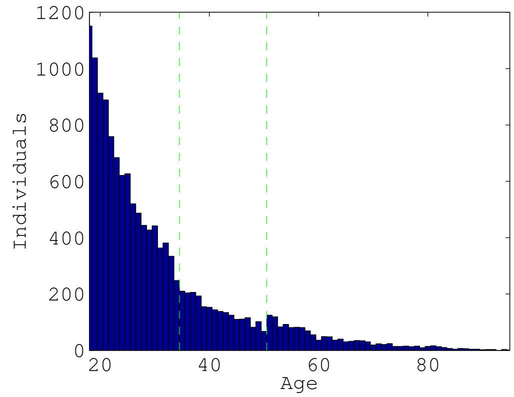
(e)


Individuals
Age
0
50
100
150
200
250
300
20
40
60
80

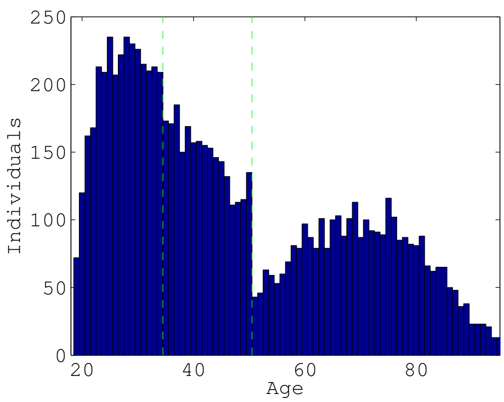
(f)



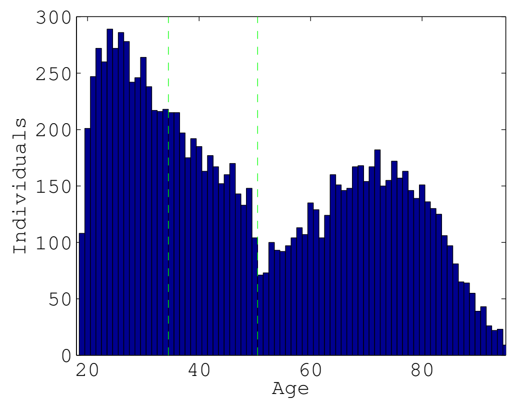
(g)



(h)

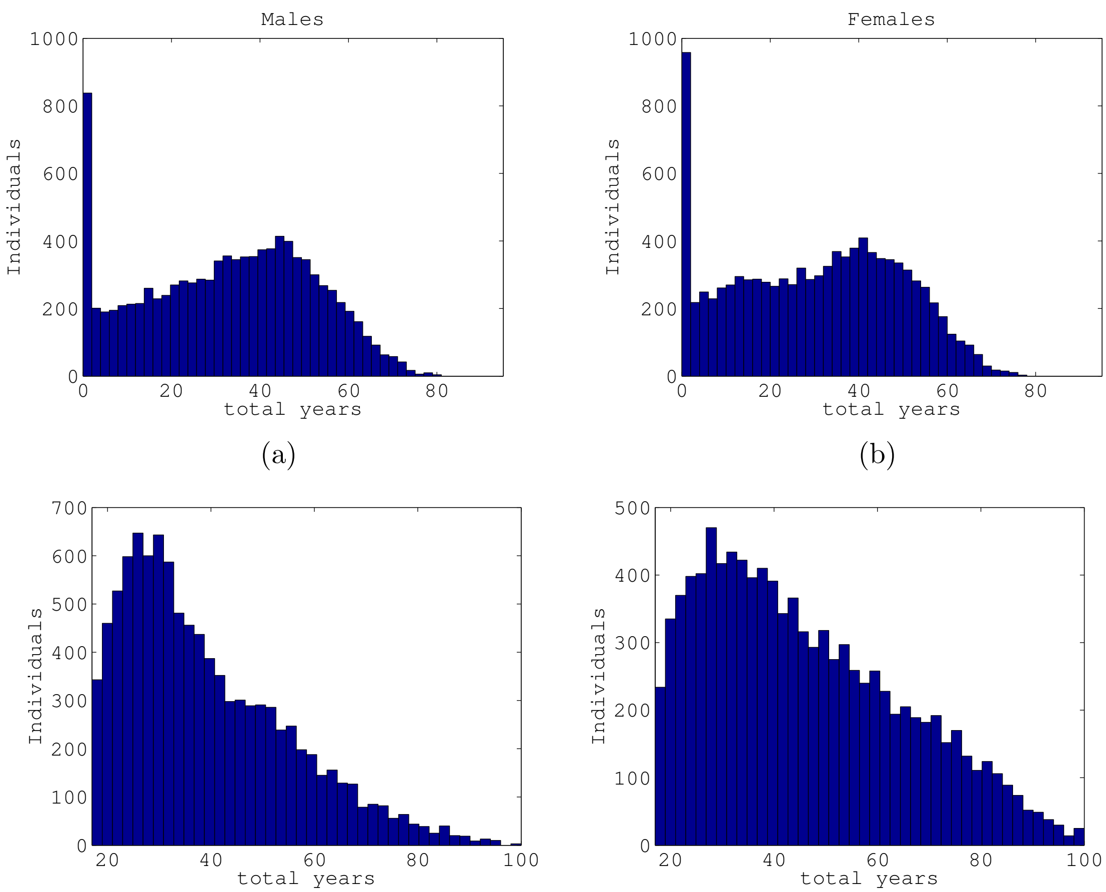


(i)



(j)

Figure 2.5: **Cohort simulations of 10,000 individuals lifetime trajectories for ages 18-95.** 2.5a and 2.5b depict the state transitions for each of 10,000 simulated individuals arranged in order of lifespan (which is equivalent to the survivorship curves). Individuals start unmarried (green), enter and exit marriage (red) at different ages throughout their lifetime and then they die (blue). First passage times: 2.5c and 2.5d depict the distribution of the age at which individuals transition from unmarried to married for the first time, for males and females respectively. 2.5e and 2.5f depict the distribution of the age at which individuals transition from married to unmarried for the first time for males and females, respectively. Age of transitions: 2.5i and 2.5j depict the distribution of the ages at which individuals transition into marriage (including not only the first marriage, but all subsequent marriage as well) and similarly, 2.5g and 2.5h depict the distribution of the ages individuals transition out of marriage (not only for the first time), for males and females respectively. The green vertical lines in panels (c)-(j) are the seams between data-sets utilized: NLSY79 pre-1994, NLSY post-1994, and HRS (at age 33 and 50).


Males
Females
Individuals
total years
(a)
(b)
(c)
(d)

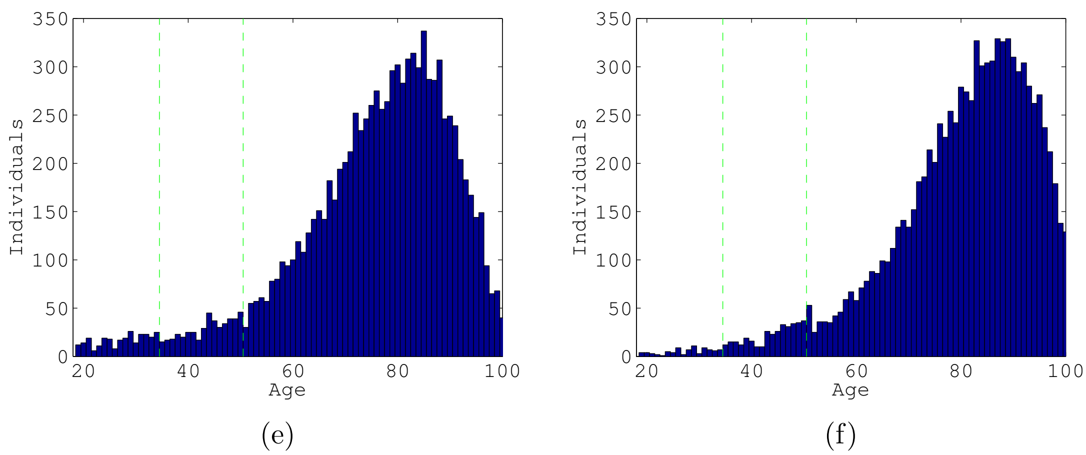
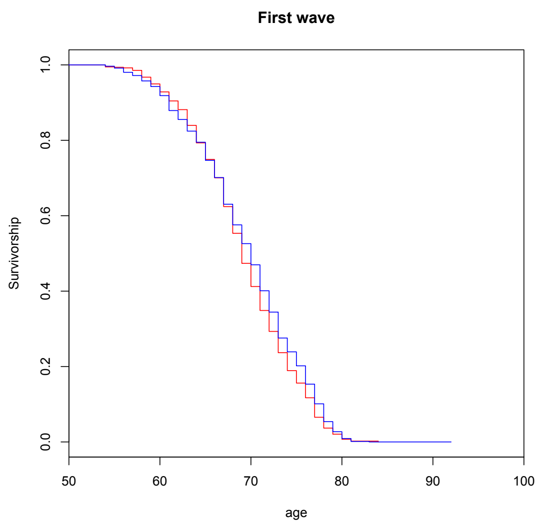
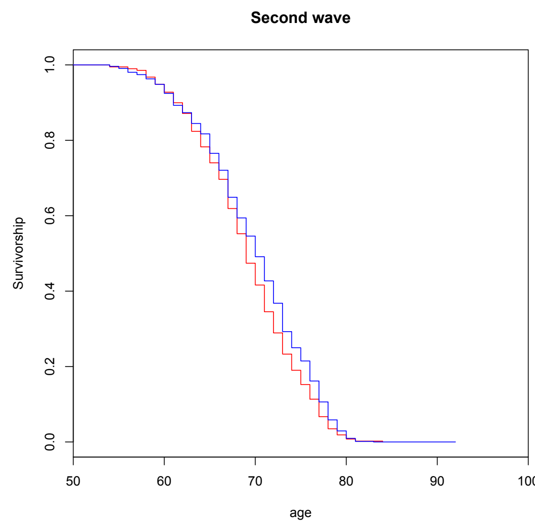


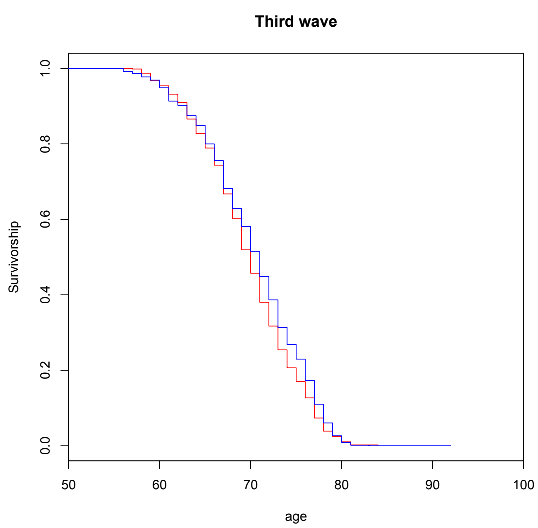
Figure 2.6: The distribution of the total years an individual spends in the married (2.6a,2.6b) and unmarried states (2.6c,2.6d), for males and females respectively. Male individuals spent a mean of 32.58 years married (standard deviation of 19.31 years, mode 0 years) and 38.95 years unmarried (standard deviation of 16.13 years, mode 30 years). Females spent a mean of 30.73 years married (standard deviation 18.00 years, mode 0 years), and 45.82 years unmarried with a standard deviation of 19.35 years and mode of 27 years. The means incorporate the model requirement that the first 18 years of life are spent unmarried, and means also incorporate individuals who were never married over their life course. The lifespan, or age at death is seen for males (2.6e) and females (2.6f). The mean age at death is 75.39 for males and 80.01 for females, mode of age at death is 85 and 87, and the standard deviation is 15.38 and 14.00 for males and females, respectively. The green vertical lines in (e) and (f) are the seams between data-sets utilized: NLSY79 pre-1994, NLSY post-1994, and HRS (at age 33 and 50).


First wave
Survivorship
1.0
0.8
0.6
0.4
0.2
0.0
50
60
70
80
90
100
age

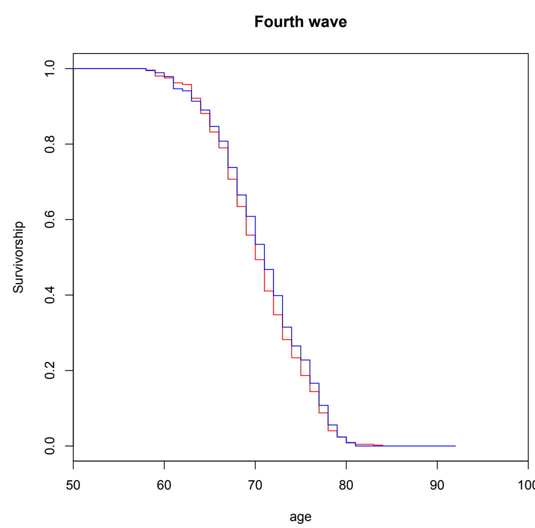
(a)


Second wave
Survivorship
1.0
0.8
0.6
0.4
0.2
0.0
50
60
70
80
90
100
age

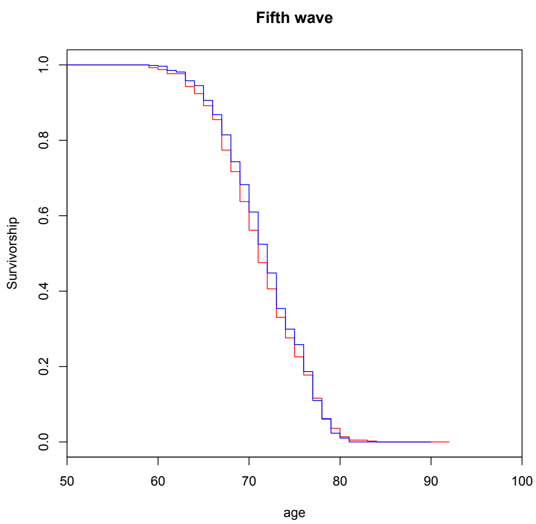
(b)


Third wave
Survivorship
1.0
0.8
0.6
0.4
0.2
0.0
50
60
70
80
90
100
age

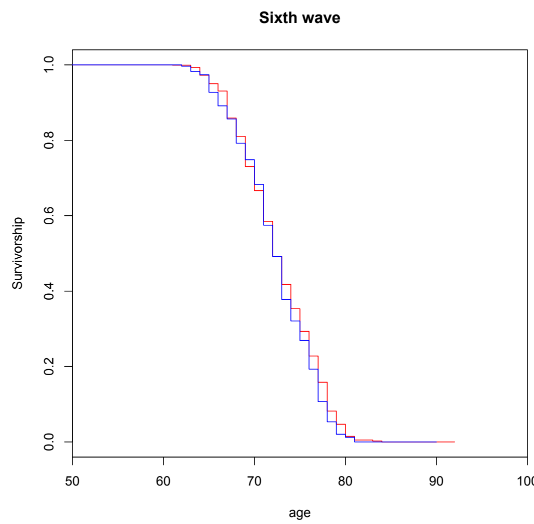
(c)


Fourth wave
Survivorship
1.0
0.8
0.6
0.4
0.2
0.0
50
60
70
80
90
100
age

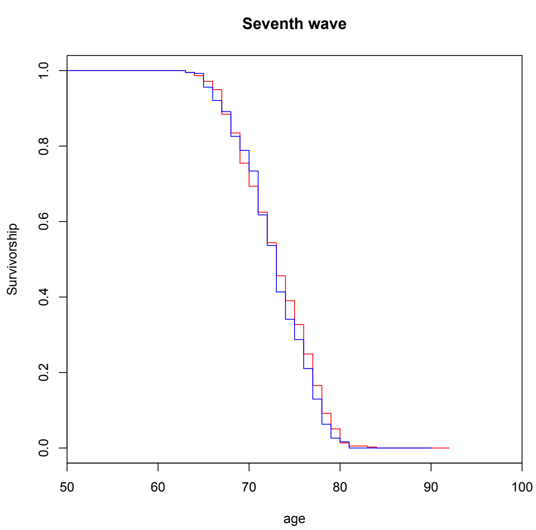
(d)


Fifth wave
Survivorship
0.0
0.2
0.4
0.6
0.8
1.0
50
60
70
80
90
100
age

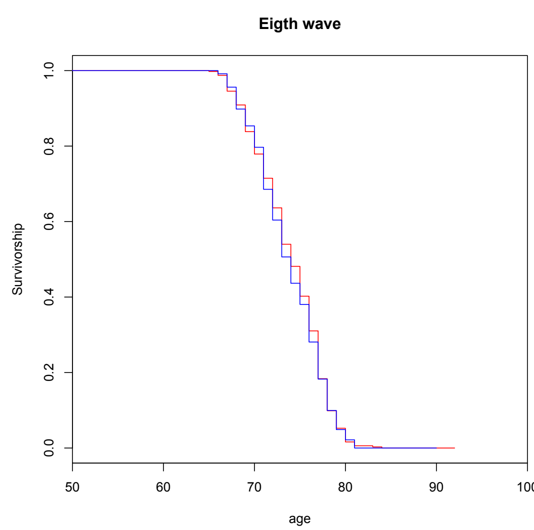
(e)


Sixth wave
Survivorship
0.0
0.2
0.4
0.6
0.8
1.0
50
60
70
80
90
100
age

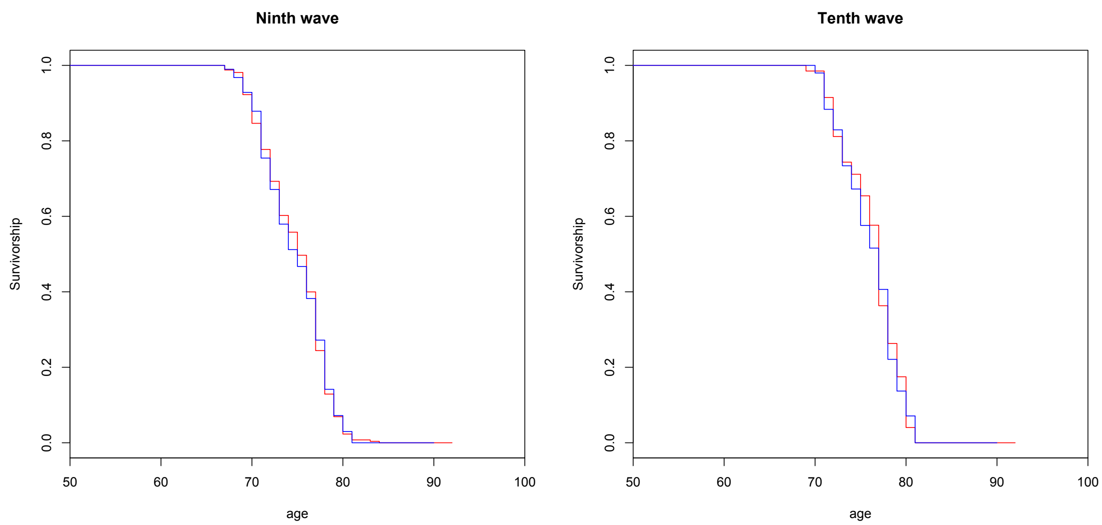
(f)


Seventh wave
Survivorship
0.0
0.2
0.4
0.6
0.8
1.0
50
60
70
80
90
100
age

(g)

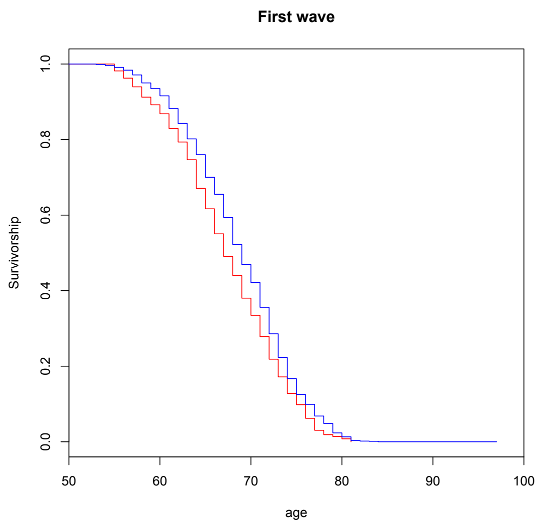

Eigth wave
Survivorship
0.0
0.2
0.4
0.6
0.8
1.0
50
60
70
80
90
100
age

(h)

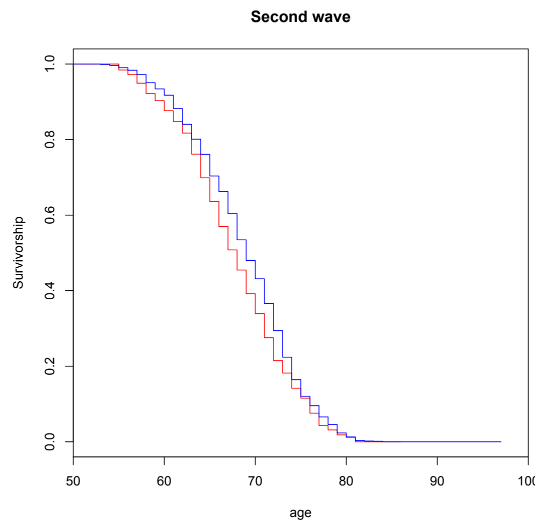


(i) (j)

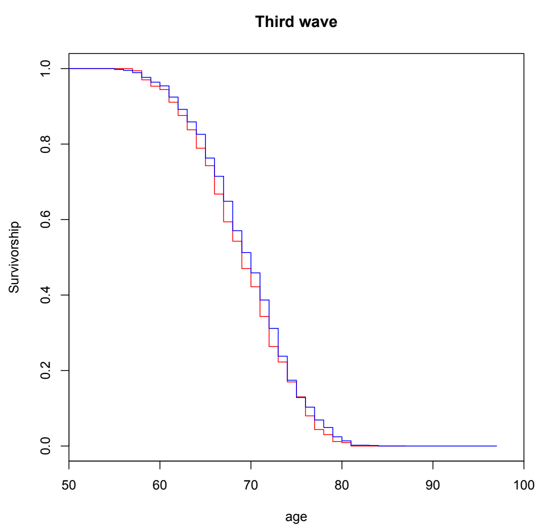
Figure 2.7: **How marital status (blue=married, red=unmarried) at specific HRS interview waves affects FEMALE survivorship (no. alive at age x/no. in birth cohort)**. A series of Kaplan Meier failure time analysis were used to obtain survivorship estimates of female individuals in HRS. For each respective analysis, individuals were classified by their marital status at a particular interview wave (a) first, (b) second, (c) third, (d) fourth, (e) fifth, (f) sixth, (g) seventh, (h) eighth, (i) ninth, (j) tenth. Weighted failure time analysis was performed in R.


First wave
Survivorship
0.0
0.2
0.4
0.6
0.8
1.0
50
60
70
80
90
100
age

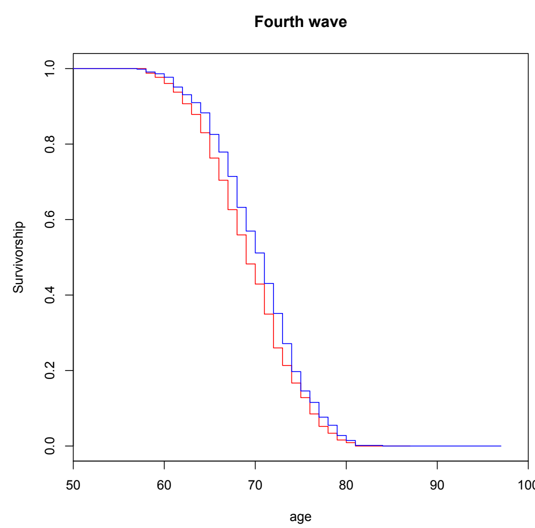
(a)


Second wave
Survivorship
0.0
0.2
0.4
0.6
0.8
1.0
50
60
70
80
90
100
age

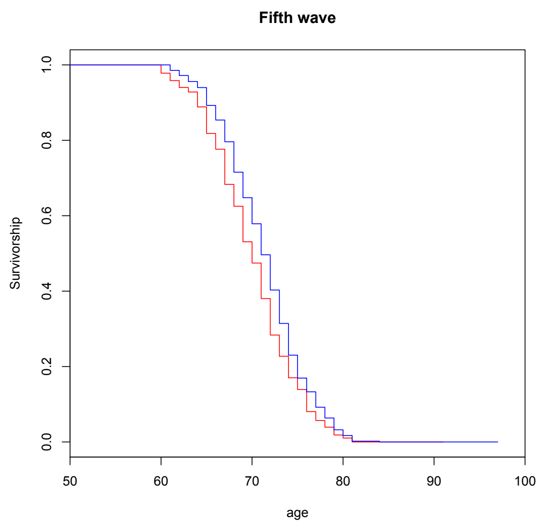
(b)


Third wave
Survivorship
0.0
0.2
0.4
0.6
0.8
1.0
50
60
70
80
90
100
age

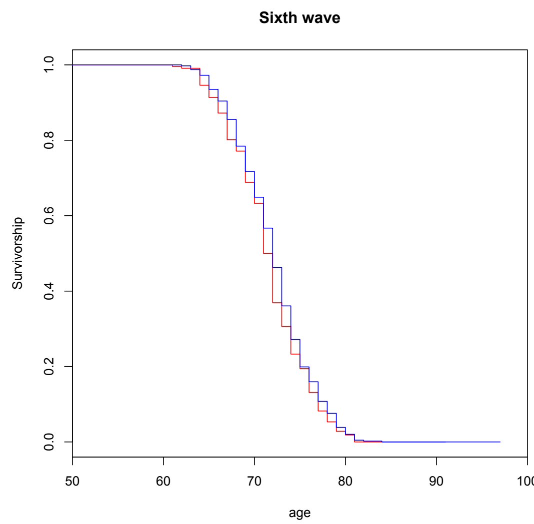
(c)


Fourth wave
Survivorship
0.0
0.2
0.4
0.6
0.8
1.0
50
60
70
80
90
100
age

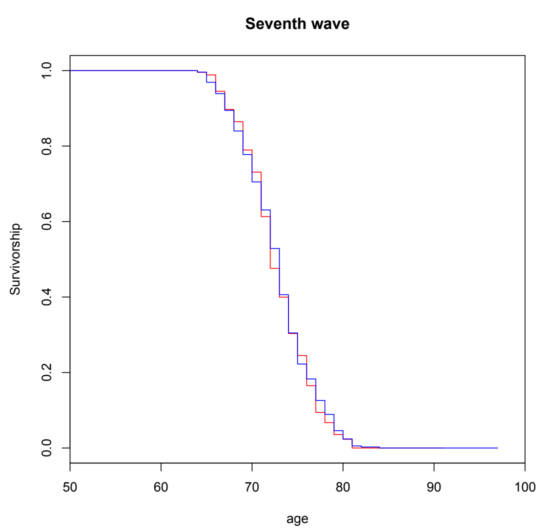
(d)


Fifth wave
Survivorship
0.0
0.2
0.4
0.6
0.8
1.0
50
60
70
80
90
100
age

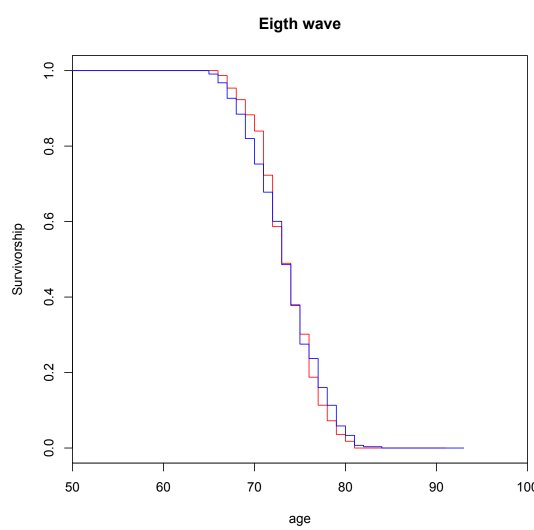
(e)


Sixth wave
Survivorship
0.0
0.2
0.4
0.6
0.8
1.0
50
60
70
80
90
100
age

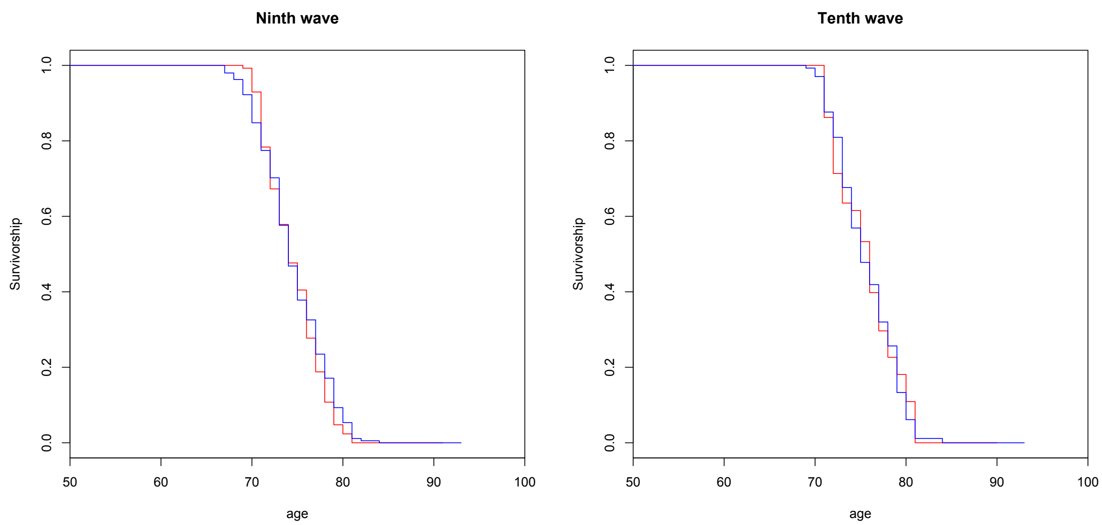
(f)


Seventh wave
Survivorship
0.0
0.2
0.4
0.6
0.8
1.0
50
60
70
80
90
100
age

(g)


Eigth wave
Survivorship
0.0
0.2
0.4
0.6
0.8
1.0
50
60
70
80
90
100
age

(h)



(i) (j)

Figure 2.8: **How marital status (blue=married, red=unmarried) at specific HRS interview waves affects MALE survivorship (no. alive at age x/no. in birth cohort)**. A series of Kaplan Meier failure time analysis were used to obtain survivorship estimates of male individuals in HRS. For each respective analysis, individuals were classified by their marital status at a particular interview wave (a) first, (b) second, (c) third, (d) fourth, (e) fifth, (f) sixth, (g) seventh, (h) eighth, (i) ninth, (j) tenth. Weighted failure time analysis was performed in R.

CHAPTER 3

Poverty dynamics, poverty thresholds and mortality: an age-stage Markovian model

3.1 Summary

Recent studies have examined the risk of poverty throughout the life course, but few have considered how transitioning in and out of poverty, or other income states, shapes the dynamic heterogeneity and mortality disparities of a cohort at each age.

Here we use state-by-age modeling to capture individual heterogeneity in crossing one of three different poverty thresholds (defined as 1×, 2× or 3× the "official" poverty threshold) at each age. We examine age-specific state structure, the remaining life expectancy, its variance, and cohort simulations for those above and below each threshold.

Survival and transitioning probabilities are statistically estimated by regression analyses of data from the Health and Retirement Survey RAND data-set, and the National Longitudinal Survey of Youth. Using the results of these regression analyses, we parameterize discrete state, discrete age matrix models.

We found that individuals above all three thresholds have higher annual survival than those in poverty, especially for mid-ages to about age 80. The advantage is

greatest when we classify individuals based on 1× the “official” poverty threshold. The greatest discrepancy in average remaining life expectancy and its variance between those above and in poverty occurs at mid-ages for all three thresholds. And fewer individuals are in poverty between ages 40-60 for all three thresholds. Our findings are consistent with results based on other data sets, but also suggest that dynamic heterogeneity in poverty and the transience of the poverty state is associated with income-related mortality disparities (less transience, especially of those above poverty, more disparities).

This paper applies the approach of age by stage matrix models to human demography and individual poverty dynamics. In so doing we extend the literature on individual poverty dynamics across the life course.

3.2 Background

In 2014, 14.8% of the U.S. population lived below the poverty threshold (DeNavas-Walt and Proctor, 2015). In that year, the official poverty threshold for a family of four was an annual income of $24,008. If a family's annual income falls below a threshold, all the individuals in the family are considered below the threshold as well. It is widely accepted that those in poverty have higher mortality risk then those above poverty (Marmot, 2002). Yet how many of these people stay below the official poverty threshold the next year? As the ‘official’ poverty threshold is set very low, it is also known that negative effects of relatively low income are also seen for individuals “near” poverty, variously defined as 1.25, 1.5, and 2× ‘official’ poverty, all the way up to the median income level, which is approximately 3× the ‘official’ poverty threshold. For this reason we define 3 possible “poverty” thresholds, 1×, 2×, and $3\times$ the 'official' poverty threshold set by the U.S. Census Bureau as the poverty level. We compare annual survival, remaining life expectancy, and entry and exit of individuals above and below each poverty threshold at each age. We also investigate how three cohorts, each with one of the three specified poverty thresholds experience dynamic heterogeneity, that is, how the demographic structure of the population varies as individuals cross in and out of poverty at each age.

It is well documented that income levels are dynamic; thus being "in poverty" is also dynamic Bane and Ellwood (1985). Of those classified as poor in 2009, for example, 26.9% were classified as not being poor in 2010 and 35.4% were classified as not being poor in 2011. Of those not poor in 2009, 4.1% were poor in 2010 and 5.4% did become poor in 2011 (Edwards, 2014). Other income levels are dynamic as well, such as $2\times$poverty threshold ($48,016 for a family of four in 2014) and $3\times$poverty threshold ($72,024 for a family of four) and have important health (Hokayem and Heggeness, 2014a), and policy ramifications as well (Cellini et al., 2008).

Our objective is to answer the following four questions: 1) How often in their life course do individuals cross above and below each threshold? 2) How does being above or below a threshold affect the probability of survival from one age to the next? 3) How does being above and below a particular threshold income level change the expected fate of a cohort, such as the remaining life expectancy and variance in remaining life expectancy? 4) How many total years are spent above and below each threshold during an individual's life?

These questions are examined by using a state-by-age matrix model to analyze empirical data from the National Longitudinal Survey of Youth 1979 (NLSY79) and the Health and Retirement Survey (HRS)(Bureau of Labor Statistics, 2012; Moldoff et al., 2014). In this model demographic fates of individuals depend upon both stage (income status) and age. From NLSY79 and HRS we are able to estimate one-year survival probabilities and one-year transition probabilities across three different income thresholds, for ages 22-95. Our results are consistent with current literature on poverty entry and exit rates (such as Cellini et al. (2008); DeNavas-Walt and Proctor (2015); Edwards (2014)) and near poverty entry and exit rates (Hokayem and Heggeness, 2014b). Our method of using state-by-age models (with two states, above poverty and in poverty for each of the three defined poverty thresholds) leads us directly to the variance in average remaining life expectancy at each age and the heterogeneity in income state. Matrix age-by stage models have been used in many contexts (for instance to analyze plants, such as a perennial shrub (Caswell, 2012), animals, for instance whales (Caswell, 2009), humans (Caswell, 2015), and in epidimiological analysis, for instance to analyze rubella (Metcalf et al., 2012))and are useful for connecting individual stochasticity in life path to overall cohort dynamics. Other methods that use the combination of age and stage to predict demographic fates, include multistate life table approaches (as reviewed by Willekens and Putter (2014)) or classic increment-decrement life table techniques (Schoen, 1988)) can address similar questions. In deference to work on multi-state life table analysis, we use the human demography convention and use the term 'state' rather than the more general ecology term 'stage'. Our approach serves as a bridge between age-by-stage matrix models and poverty dynamics over the life-course.

3.2.1 Dynamic heterogeneity in income level and mortality risk

The association of poverty with poor health is well-documented, including different theories of mechanisms and pathways of causation (Adler et al., 1994; Benzeval and Judge, 2001; Cellini et al., 2008; McDonough et al., 2010). Cellini et al. (2008) also summarizes different modeling approaches for poverty dynamics (for example the tabulation or count method, life table method, bivariate hazard rate method, multivariate hazard rate (or spell based) method, components-of variance method, and some less used multivariate methods). Regardless of technique used, the association between mortality risk and poverty status is apparent, but what about an association at higher income levels? Rehkopf et al. (2008) looked at different income levels and their associated mortality risk for individuals in the United States between the ages of 18-77 and found that the greatest mortality risk is for the "population whose family income is below the median (equal to \$20,190 in 1991, 3.2 times the poverty level)". In other words mortality risk decreased as income increased until near the median income level, while above this level there was no significant change in mortality risk with income increase. Thus there are income related disparities in mortality risk up to the median income.

The "near poverty" threshold, usually defined officially as 1.25$\times$ the poverty threshold (although sometimes defined as 1.3$\times$, 1.5$\times$, or 2$\times$ the poverty threshold) has also received attention (Hokayem and Heggeness, 2014a,b). Individuals with incomes just above the poverty threshold have characteristics quite similar to those "in poverty" in terms of assistance program participation rates. Furthermore transitions into and out of "near poverty" are frequent (Hokayem and Heggeness, 2014b).

Most studies have not looked at the full range of age-specific annual survival and transition rates, despite general trends across the life cycle. For instance, we know "[Health] Disparities are smallest during childhood, adolescence, and early adulthood and greatest in middle age, becoming weaker again in older populations" (Adler and Rehkopf, 2008). In order to better address health disparities it is useful to know the age-specific dynamics and associated mortality risk of being below 1×, 2×, and 3× the "official" poverty threshold; thus we investigate each of these poverty thresholds separately. A better understanding of individual income dynamics enables a clearer identification of those most at risk.

3.2.2 Age-by-stage matrix model

We construct three matrix models, one for each of three poverty thresholds (1×, 2× and 3× the "official" poverty threshold); each matrix is a discrete time, discrete state, discrete age, Markov chain matrix with two-income states, above or below the chosen threshold, at each age. The matrix is similar in structure to the Tuljapurkar and Steiner (2010) population projection age-stage matrix, but here, as in Steiner et al. (2012), there is no reproduction. It is also similar in concept, to the model of multi-state mathematical demography presented in Rogers (1980). Matrix methods have since become more widespread as the data necessary to construct the have become more available and insights they can provide are expanding. Here we use our age-by-stage models to trace dynamic heterogeneity in a cohort, that is, individual state switching (above and below poverty) and cohort heterogeneity (variance in state structure) at each age (also termed individual stochasticity since individual life course trajectories are stochastic and differ even between identical individuals

(Caswell, 2009)). As is conventional with matrix methods (Caswell, 2012), we also use the age-by-stage matrices to construct fundamental matrices from Markov Chain theory to determine remaining life expectancy for individuals in a given state at a given age and to determine expected remaining years in each income state (for theory see C.1.2).

We address the following research questions:

For each of the three threshold income levels (1×, 2×, and 3× the "official" poverty level) we ask:

- How does annual survival probability change with age for those above and below a particular poverty threshold? What are the age-specific entry and exit probabilities for the two states (above and below a particular threshold)?

- How does state-structure (proportion above and below the threshold) change with age?

- How does remaining life expectancy and the expectation of remaining life below the specified income threshold change with age? What is the variance in remaining life expectancy for those below and above the income threshold?

- How does the poverty status and survival of simulated individuals change across their lifetimes, where the the age-specific probabilities of survival and state transitions of the model are used to assign fates to individuals in a simulated birth cohort. Specifically we ask, at what ages do individuals transition below and above a particular threshold? And how long do simulated individuals spend in poverty during their lifetime?

Our state-by-age model answers our research questions by emphasizing age and state structured cohort dynamics for the three different thresholds. We examine how age-specific individual stochasticity affects overall cohort dynamics.

3.3 Methods: Theory

The age-by-state matrix, $\mathbf{L}$, calculates cohort dynamics and individual trajectories. Specifically, $\mathbf{L}$ is a square matrix whose dimension equals the number of states times the number of ages classes (note: a maximum age must be set). $\mathbf{L}$ is used in the following two ways (equations): Equation (3.1) is used to project a vector n that represents the number of individuals in each state at each age from birth across the lifetime (Caswell, 2001), which enables tracking of the state-distribution and survivorship of an initial cohort (with its initial state distribution, $n(0)$) at each age x:

$$n(x) = \mathbf{L}^x n(0) \tag{3.1}$$

where the matrix $\mathbf{L}$ is raised to the xth power at each age x. Equation (3.2) is used to analyze remaining life expectancy (mean and variance in age at death) and generate individual stochastic trajectories across all ages, where an individual is a realization or sample path of a Markov process. The Markov chain is described by this matrix (Keyfitz and Caswell, 2005):

$$\mathbf{P} = \left(\begin{array}{c|c} \mathbf{L} & 0 \\ \hline \mathrm{m} & 1 \end{array}\right) \tag{3.2}$$

Here we will define the age-and-state matrix $\mathbf{L}$, and in the appendix (C.1) we show how it can be used in (3.1) and (3.2). $\mathbf{L}$ has dimensions of age-by-stage and each element is copmposed of the product of two of the following four functions at each age, $s_1(x)$,$s_2(x)$,$t_{12}(x)$ and $t_{22}(x)$:

$s_1(x)$ = Probability that an individual whose income is below a threshold survives between ages x and $x+1$ (one-period state-specific survival).

$s_2(x)$ = Probability that an individual whose income is above a threshold survives between ages x and $x+1$ (one-period state-specific survival).

$t_{21}(x)$ = Conditional probability of exiting poverty before age $x+1$ for an individual who is in poverty at age x. $(1-t_{21}(x)) = t_{11}(x)$ = probability that an individual whose income is in poverty (below the income threshold) at age x will remain in poverty at age $x+1$, conditional on survival.

$t_{22}(x)$ = Conditional probability of staying above poverty at age $x+1$ for an individual who is above poverty (the particular threshold) at age x. $(1-t_{22}(x)) = t_{12}(x)$ = probability that an individual whose income is above poverty at age x will enter by age $x+1$, conditional on survival.

Here x is a 1-year age interval (so age x to $x+1$ is a one-year step), although other interval lengths can be used (Keyfitz, 1968). State transition probabilities (transitioning to a new state or staying in a state at the next age) are conditional on survival; 'state' is being below or above a particular income level. Subscripts follow standard convention of row, i, and then columns, j, and transitions are j to i (i.e. t_{12} is transitioning from state 2 to 1, richer to poorer). Also note that every probability has a complement; the complement to the annual state specific survival is the annual state specific mortality, i.e. the probability of dying in one year.

Multiplying the survival probabilities by the conditional state transition probabilities results in a matrix for each age, $\mathbf{Q}(x)$ which takes the following form:

		State at age x	
		< income threshold	> income threshold
State at age x+1	< income threshold	$s_1(x)t_{11}(x)$	$s_2(x)t_{12}(x)$
	> income threshold	$s_1(x)t_{21}(x)$	$s_2(x)t_{22}(x)$

Table 3.1: Structure of unconditional state transition matrices, Q(x)

We denote each survival-weighted state transition matrix, also denoted the unconditional state transition matrix as $\mathbf{Q}(x)$, and each unconditional state transition matrix is inserted into the sub-diagonal of the age matrix at the appropriate column. In our implementation of the model, we set a maximum age of 100 and thus there are 100 unconditional state transition matrices, each representing a 1-year increment from 0 to 100 years of age. The age-state block matrix has dimensions 101×101 blocks, each block comprises a 2×2 matrix, and has the following form (where the 0's represent 2×2 matrices comprised entirely of zeros):

$$\mathbf{L} = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ \mathbf{Q}(1) & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & \mathbf{Q}(2) & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{Q}(3) & \dots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \mathbf{Q}(98) & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & \mathbf{Q}(99) & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & \mathbf{Q}(100) & 0 \end{bmatrix}$$

The age-state block matrix $\mathbf{L}$ has a structure reminiscent of a Leslie matrix from which fecundity has been removed, in that an individual always transitions to the next age at each time step (Keyfitz and Caswell, 2005; Kot, 2001; Leslie, 1945). Since there are 2 state classes and 101 age classes, the dimensions of the age-state matrix **L** is 202$\times$202. The **L** matrix can be used to produce cohort projections (C.1.1) and Markov chain analysis. With Markov chain analysis we calculate the average remaining life expectancy, average remaining life in poverty or below an income threshold, and the variance (by using the fundamental matrix, as explained in C.1.2) and the simulated individual trajectories (see C.1.3).

3.4 Methods: Empirical data

3.4.1 HRS RAND

The Health and Retirement Study (HRS) is a publicly accessible longitudinal household survey data set for the study of retirement and health among individuals over age 50 and their spouses in the United States. We use the RAND HRS Data files Version O which "are a cleaned, processed, and streamlined collection of variables derived from HRS" (Moldoff et al., 2014). The survey consists of 6 cohorts and we use longitudinal data compiled from 6 of the 11 interview waves that fall approximately around these years: 2002, 2004, 2006, 2008, 2010, 2012. As a nationally representative data set of $37,319$ individuals, HRS has over-sampled Hispanics, Blacks, and residents of Florida, and provides weighting variables to make it representative of the community-based (non-institutionalized) population. For our purposes we subset the data to only include individuals between 50 and 95 years old. We use survey sample weight in all analyses. And the income threshold that defines poverty is from the United States Census Bureau (thresholds vary by year, size of family, and number of children). Individuals are pooled into repeated observations for each of their interview responses as explained after the following section.

3.4.2 NLSY1979

The NLSY79 Cohort is a longitudinal project that follows the lives of a sample of American youth born between 1957-64. The cohort originally included 12,686 respondents aged 14-22 when first interviewed in 1979; after two sub-samples were dropped, 9,964 respondents remain in the eligible samples Bureau of Labor Statistics (2012). We use data available from interview wave 1 (1979 survey year) to interview wave 25 (2012 survey year), which includes one-year intervals from 1979-1994, and two-year intervals from 1994-2012. Since we are studying the state of poverty we subset the data to include observations from age 22 (when individuals enter the work-force post college) to age 50. Retention rates for NLSY79 respondents from 1979 to 1993 exceeded 90 percent. Rates from 1994 until 2000 exceeded 80 percent. Rates from 2002 until 2012 were in the 70s. (Retention rate is calculated by dividing the number of respondents interviewed by the number of respondents remaining eligible for interview) (Bureau of Labor Statistics, 2012). More detailed information about retention rates can be found at NLSY79's website. Poverty rates are based on annual poverty income guidelines by the U.S. Department of Health and Family Services (which are also based on family size). We recognize that the threshold varies minimally between the U.S. Department of Health and Family Services and the U.S. Census Bureau, but previous research comparing slight differences in poverty thresholds shows the effect to be insignificant (Cellini et al., 2008).

3.4.3 Quantifying the functional dependence of survival and transition probabilities on age: logistic regression

There has been much discussion as to how to calculate transition probabilities for Markov transition models (Islam et al., 2004; Islam and Chowdhury, 2006; Korn and Whittemore, 1979; Lawless and Rad, 2015; Yu et al., 2010). (The latter two sources give a good background on the history of estimating transition probabilities from data and propose methods for higher Markov models). One very practical proposal for calculations of binary Markov transition models has been to use logistic regression probabilities (Muenz and Rubinstein, 1985). Since logistic regression is very straightforward and intuitive, especially when we have a time-dependent covariate (age), we employ it for our analysis. Researchers might look towards Yang et al. (2007) or Fujiwara and Caswell (2002) which are two distinct ways to calculate Markov transition probabilities that can incorporate a range of data complexities. The Willekens and Putter (2014) review is also a good resource as to the vast array of statistical packages in R that can be used to estimate transition probabilities for demographic multi-state models. Logistic regression models are often used in the poverty context to find determinants of poverty (Haughton and Khandker, 2009). With logistic regression covariates can also be incorporated into the state-by-age model, although here we only include age and current state as our independent variables.

The HRS data, NLSY79 pre-1994, and post-1994, are analyzed separately for ages 22-33, 34-50, and 51-95, respectively, and the results are later combined to give probabilities of survival and transitioning over our complete age range of $22 - 95$. Within each data-set, interviews are pooled together except for the last wave; this represents the data at time t. All the interview waves except for the first one are then pooled together to represent the data at time $t+i$. (To find i for each data-set we average the interval in months between each successive interview wave. For instance, for HRS the interval between interview years is normally distributed about 24 months, for NLSY79 pre-1994 the interval is normally distributed about 12 months, and for NLSY post-1994 there is a normal distribution in interval length about 24 months. After the regression we adjust our probabilities to be over a one year period, i.e. t to $t+1$, see appendix for more on this adjustment). Instead of tracking individuals longitudinally over several ages, we perform a pooled logistic regression analysis on all observations of all ages from time t to $t+i$, where individuals have a particular age x at time t and age $x+i$ at time $t+i$. If an individual did not respond at that specific point (either the interview at t or $t+1$ or both interviews), that observation is omitted from the analysis. However if that specific individual responded later in the study (a different observation), that observation is included in the analysis. This technique has been called the pooled repeated observation method (PRO) and the analysis called a pooled logistic regression (Dagostino et al., 1990). The pooled logistic regression analysis was performed in R and the survey package Lumley (2014) with `svyglm` was used to incorporate weights. For HRS we have 186,585 observations between the ages of 50 and 95. For NLSY79 after pooling we have 183,238 observations between the ages of 22 and 50. For analysis the pooled observations are weighted based on each data set's weight at observation.

For each income threshold and data-set we ran regression analysis to obtain estimates for the functional dependence of survival and transition probabilities on age and state. The regression coefficients were logit transformed into probability values. Since the actual time between interview waves i varied (see A.2) and we needed to calculate annual rates from the probabilities, we developed a protocol for converting the probabilities to annual (12 mo) rates (as explained in A.3). We then evaluated these functions at each age for use in our matrix models.

To check for an association of income level and response rate at each age, the difference in percent of non-response was calculated for each age. Three income level thresholds (1×, 2× and 3× the income level that "officially" defines "poverty") were used to define binary states (above and below each threshold) and separate analyses were run for each. For this analysis deceased individuals were removed from the population (they were not counted as non-response).

In our main analysis we investigated three income levels to classify the individuals into "states" in order to see if the impact of income level on mortality depended upon where the line between rich and poor was drawn. Also we were interested in whether the dynamics of changing state were similar if the line was drawn at a very low income level versus a higher income level.

3.5 Results

3.5.1 How do the probabilities of survival and transitioning between income states depend upon age and current state?

We answered the first research question by performing pooled logistic regression for individuals classified into two states at each age; below and above 1×, 2×, and 3× the poverty threshold (fig.C.1). Since the pooled logistic regression approach omits missing data, we first addressed the issue of whether non-response was itself associated with poverty status. In C.2 we show the proportion of missing observations at age $x + i$ based on state at age x, and the difference in non-response between each state. There is slightly lower response by those in the lower income states for all three thresholds. The difference does not seem to be dependent on age so we assume that for our analysis the impact on the results are minimal.

For ages 50 and above (the HRS data) there is a significant difference between survival of those above and below the 1×, 2× and 3× the "official" poverty thresholds (fig. 3.1). The richer survived better than the poorer group no matter where the threshold was set. The maximum observed difference in annual survival between the two groups (at any age) was 2.3% for the 1×poverty threshold, and 1.9% for both the 2× and 3× poverty threshold.

In terms of absolute difference, those above 1× poverty have their maximum annual survival advantage between ages 75-79 of 0.022. Those above 2× poverty have the maximum advantage also in the late 70s with a maximum difference of 0.018, and those above 3× poverty have maximum advantage at age 76 of 0.018. For the 1× poverty threshold there is a crossover at age 91 where those below threshold have a slight survival advantage. For the 2× and 3× poverty threshold the crossover is at age 88. At younger ages (22-50 from NLSY79) there is less disparity in survival probability between states (except at ages 45-50 for the 1× poverty threshold).

Individuals above 1× poverty income tend to stay there throughout their life course. Those below 1× poverty income have the greatest probability of exiting their low income state at younger ages (fig. 3.2). Those below 2× and 3× poverty income threshold also have decreasing probability of exiting their income state, and the one-period probability declines with age. Those above 2× poverty have a slight increasing probability of remaining above 2× poverty until mid-ages, where the probability of remaining above 2× poverty declines with age. Those above 3× poverty have a more dramatic increase in the probability of remaining above 3× poverty until midages as well, and then a very sharp decrease of the probability of remaining above 3× poverty with each age.

Discontinuity in the curve (at the dashed vertical lines) is a seam perhaps representing period effects, i.e. the joining of NLSY79 (ages 22-50 between 1979-2012) to HRS (ages 50-95 between 2002-2012). The age-structure of interviews in each data-set are different, as can be seen in fig.A.1; NLSY79 mostly comprises a single classic cohort where all the individuals are aging together across time, whereas HRS comprises several cohorts, including a new cohort entering the data-set in 2004 and another new cohort in 2010. Regardless, our adjustments to the probabilities, and the data's large sample size create a general age-specific trend consistent with previous literature.

3.5.2 Cohort distributions

We want to know the state structure (the relative proportion of richer and poorer individuals) at each age for any initial cohort. We note that each $\mathbf{Q}(x)$ represents the joint demographic processes of survival to the next age and transitions among income levels during that age interval. For example $\mathbf{Q}(1)$ takes any initial cohort from age 0 to age 1 and $\mathbf{Q}(2)$ takes the cohort from age 1 to age 2. The cumulative demographic processes experienced by a cohort from age 0 through age 2 is given by the product of the two, the matrix product $\mathbf{Q}(2)\mathbf{Q}(1)$. We note that matrix multiplication is written from right to left and that matrix multiplication is not commutative. Thus the cumulative demographic processes experienced by a cohort from age 0 to age x is given by the matrix product

$$\mathbf{Qcum}(x) = \mathbf{Q}(x)\mathbf{Q}(x-1)\mathbf{Q}(x-2)...\mathbf{Q}(3)\mathbf{Q}(2)\mathbf{Q}(1) \tag{3.3}$$

We observed that the state structure at each age appears to converge to the dominant eigenvector of the cumulative matrix product for any initial cohort, after a certain age. We are not sure of the generality of this observed result. As expected, and despite underlying state switching, throughout all ages there is an almost constant percentage of individuals below 1× poverty (about 12%, fig. 3.3). The percentage below 2× and 3× poverty is slightly U-shaped with increasing percentages in the lower income states after age 55. (This correlates with the increased probability of exiting the higher income state after mid-ages). The demographic structure defined by the dominant eigenvectors of $\mathbf{Qcum}(x)$, the cumulative age-specific unconditional state transition matrix, is similar to the demographic structure seen in the empirical data (C.1).

We cannot refer to this as a 'stable state' distribution since as the cohort is being projected across time it is also changing age, and the relative number of individuals above and below the threshold changes at each age. However if the cohort would be theoretically stuck at an age for a long period of time, the dominant eigenvector of the cumulative matrices, $\mathbf{Qcum}(x)$, represents the stable state distribution that would be approached. More precisely, if we picked an age 'z' and assumed that all $\mathbf{Q}(x) = \mathbf{Q}(z)$, then the quasi-stable (Seneta, 1981) distribution would be the dominant eigenvector of $\mathbf{Q}(z)$. The subdominant eigenvalue of cumulative conditional transition matrix (with survival not included) represents how quickly a cohort converges to the quasi-stable distribution, and we find that convergence happens very quickly for all three thresholds. In other words, even if the initial state distribution varies, we observed that a cohort will quickly (by age 30) converge into the quasi-stable distribution represented by $\mathbf{Qcum}(x)$ the dominant eigenvector of the cumulative state matrices.

3.5.3 The fundamental matrix: life expectancy and variances

We want to know how many years individuals of a given age, in a given income level, are expected to remain alive. We also want to know how many of their remaining years will be spent in poverty. This information is encapsulated in the "fundamental matrix", which is a matrix that is essentially the summation of the powers of L across all ages. Each element of the fundamental matrix is the mean number of visits to either poverty or above poverty (depending on the index) conditional on survival to a particular age and state. Analysis of the fundamental matrix provides age-by-state-specific average remaining life expectancy, and expected remaining years below an income threshold (fig.3.4). Those above 'poverty' at age x have less expected remaining years in 'poverty' than those already in 'poverty' at age x, for every defined threshold (fig.3.4 dashed lines). As age increases, for the higher than $1\times$ poverty income state, the average proportion of life that can be expected to be in the below $1\times$ poverty state decreases (blue dotted lines). This is generally true for those above $2\times$ poverty as well; however, from about ages 60 to 80 the expected proportion of those living below $2\times$ poverty increases slightly (and increases much more for those above $3\times$ poverty). Regardless of initial stage, there are fewer expected years of remaining life for those below a $1\times$ poverty threshold than below a $2\times$ or $3\times$-poverty threshold.

The difference in average remaining life expectancy between the income states is greatest (for all thresholds) at mid-ages. The peak difference is 1.44 years, 1.35 years, and 1.01 years for those above and below 1×, 2× and 3× poverty, respectively. Those above and below a 1× poverty threshold have an average life expectancy difference of over 1/2 year from ages 34-81, for those above and below 2× poverty this occurs between ages 34-78, and for 3× poverty ages 37-77.

Analysis of the fundamental matrix also facilitates quantifying the variance and coefficient of variance in average remaining life expectancy at each age. Variance in a population represents the range of possibilities individuals can experience in terms of lifespan, and in terms of time in income states. We can see that for all ages (except for a few in late old age), those in the lesser income state have the greatest variance in their average remaining life expectancy (fig. 3.5). The difference in variance is most pronounced in the mid-40s. There is a maximum difference in variance of 37.14 years for those above and below 1× poverty, 33.19 years for the 2× poverty states, and 24.42 years for the 3×-poverty states. The difference between the variance for both income states is above 15 years for those between the ages of 33-57, 33-58, and 37-55 for 1×, 2× and 3× poverty thresholds, respectively. Variance scales with the mean in general. To look at variability independent of the mean, we also examined the coefficient of variation. In general, the coefficient of variation increases with age, especially more so for those in the lower income state.

3.5.4 Simulation: Individual lifetime trajectories

For our simulation of a cohort of 10,000 individuals, (fig. 3.6), we can clearly observe the age patterning and dynamic heterogeneity over the life course. We see an abundance of individuals in the below income threshold state at the end of their life, and more individuals in the above income state between the ages of 40 and 60 for all three thresholds. Most individuals enter the lower income states at early ages (fig. 3.7), and there is a slight peak again at age 60 for those below 2× and 3× the poverty threshold. Individuals also exit the lower income state at early ages, and the amount of individuals exiting levels out until age 70. Note that "exiting" as it is counted here can be the result of death as well.

Individuals spent a mean of 3.92 years (standard deviation of 4.50 years, mode 0 years) below 1× poverty (fig. 3.7). A mean of 11.64 years was spent below 2× the poverty threshold (standard deviation of 8.70, mode of 0 years). And a mean of 20.42 years was spent below 3× the poverty threshold (standard deviation of 11.4841, mode of 16 years).

3.5.5 Limitations

Our model is in a discrete age, binary discrete state, discrete time Markov chain framework. The probability of remaining alive and of transitioning between richer and poorer states by the next age is dependent only upon the current age and current income level of an individual. The past history of an individual is not considered in the model, even though mortality rates and poverty transition rates have been found to be related not only to current state but also to past history in other studies (Bane and Ellwood, 1985). Half of those who end poverty spells return to poverty in the next four years (Stevens, 1999). It is this relationship that our analysis is based on. One can consider this a theory of poverty entry and exit that has no duration dependence, a 'neutral theory' where whether individuals enter or exit poverty at each age depends only upon their current state of income and not on any intrinsic traits. Poverty risk is equally spread throughout a cohort such that at a given age for a given income level the same probability rules apply to all individuals. Our simulations give insight into the heterogeneity among individual life trajectories that results.

3.6 Discussion

The fundamental causes of poverty and the pathways through which higher mortality results are under ongoing investigation (Osowole et al., 2012; Rehkopf et al., 2010; Rogers, 1992; Sorlie et al., 1995; Tuljapurkar and Boe, 1998). Moreover, the relationship between income and mortality is complicated by the transience of the income state. Benzeval and Judge (2001) was able to control for initial health status and found that there is indeed a causal relationship between low income and poor health. In the same study they distinguished between persistent poverty and occasional episodes. All studies that have done likewise found that long term poverty or 'near poverty' is a greater indicator for poor health than 'episodic' poverty (Edwards, 2014; Hokayem and Heggeness, 2014b). Neilson (2008) found that in Chile, at some point between 1996-2001, 30 percent of the population had income under the poverty line (Neilson et al., 2008). Under closer analysis only 9.2 percent were under the poverty line for that entire period, the remainder had experienced transient/episodic poverty. They further discovered that when the poverty line is increased, chronic poverty increases systematically while transient poverty levels out (in their case at $2\times$ the Chilean poverty threshold).Backlund et al. (1996) and Rehkopf et al. (2008) also found that as income increases past a threshold, the health benefits associated with increased income diminishes, perhaps at the median income level.

All of these observations suggest that the transience of the poverty state has important ramifications for health outcomes, mortality risk and population dynamics. And indeed, the transience of poverty has received much attention, especially due to policy implications (Benzeval and Judge, 2001; Edwards, 2014; Hokayem and Heggeness, 2014b; McDonough et al., 2010; Sacker et al., 2007).

Here, our state-by-age model captures individual heterogeneity in entering and exiting either 1×, 2× or 3× the official poverty threshold at each age. Being above and below 1× the poverty threshold has important ramification for families, such as which government sponsored programs can be utilized, although 'near poverty', defined as up to 2× the poverty threshold, is recognized as a state with health consequences as well. 3× poverty is roughly similar to the U.S. median income, above which their are minimal income-related disparities in health. Our approach adds to prior work in that we observe the transience of these income thresholds at each age across the life course, and then consider the resulting disparities in one-period survival, average remaining life expectancy, and variance of remaining life expectancy.

We find that the higher income state has the highest annual survival probability from mid-ages to about age 80, with a crossover in late old age. The advantage is greatest between those above and below 1×-poverty when compared to those above and below 2× or 3× poverty. However the age patterning for each threshold is quite similar. Those above 1× poverty have a constant high probability of staying above 1×-poverty at all ages. Those above 2× poverty have the highest probability of staying above threshold at midages, and those above 3× poverty have a similiar pattern but with a steeper decline after mid-ages. For those in the lower income states, the annual probability of exiting those states declines with age. Those below

1× poverty have the highest probability of exiting at each age, but the sharpest decline with age. Although Willekens and Putter (2014) did not investigate age-specific transition rates, the annual rates are roughly similar in magnitude for exiting poverty and above poverty, and for exiting 2× poverty and above 2× poverty. Our transition rates are also consistent with poverty entry and exit rates mentioned in Edwards (2014) (although our rates are age-specific). Our poverty entry rates are also consistent with Rank and Hirchl (2001) who used the Panel Study of Income Dynamics (PSID) and found that "individuals within the sample face a significant risk of poverty at some point during their adult lives, particularly during the early (20-40) and later (60-80) stages of adulthood". This is what we found in our simulations, across the life-course, more individuals are in the higher income state between the ages of 40 and 60. In another study using the PSID, Rank et al. (2014) also found, like our simulations, that from age 60 to 90, entry into income poverty (and asset poverty) decreases (they discuss policy implications).

Our results are also in approximate agreement with Cellini's review of the dynamics of poverty in the U.S., "that those experiencing poverty had a roughly 1 in 3 chance of leaving poverty in any given year"(Cellini et al., 2008). In that review they also discuss some of the demographics underlying poverty exit and entry rates such as race, household size, sex of household head, and education.

Our cohort state distributions at each age shows that the number of individuals in poverty, (below the 1× poverty threshold) is almost constant with age, perhaps since an almost equal number of people exit and enter 1× poverty after age 33 (as shown in our simulations). The 2× and 3× poverty threshold projected cohorts have similar state-distributions at each age; a decrease in individuals in the lower income state until mid-ages and an increase thereafter, steeper for the cohort with the 3× poverty threshold.

We found that the greatest difference in average remaining life expectancy based on state at age x occurs near ages 40-60 (in agreement with Adler and Rehkopf (2008)), regardless of which threshold is considered. Although the magnitude of the difference is greater for those above and below the 1× poverty threshold, the age pattern in life-expectancy is similar for the other thresholds as well. Other literature points out that the inequality between life expectancies based on income quartile is increasing over time (the years 2000-2010) (Chetty et al., 2016). This observation, coupled with the higher rate of transitioning for those below 1× poverty might point to focusing on those below the 2× poverty threshold to decrease income-related mortality or health discrepancies.

For all three lower income states, the largest discrepancy in variance of average remaining life expectancy occurs from the mid-30's to late 50s. Individuals below the income thresholds have the highest variance, meaning they have a greater range of possibilities in life trajectory. Those above threshold have less variance, meaning individuals will more consistently reach their higher average remaining life expectancy.

Our results point to a phenomenon at mid-ages, that an individual in the higher (above 2× or 3× poverty) income state is less likely to enter the lower income state. And the higher the probability of stasis in the higher income state, the fewer individuals there are in the lower income states. However, there is higher inequality in life-expectancy, and variance at mid-ages (since at mid-ages the higher income state is less transient).

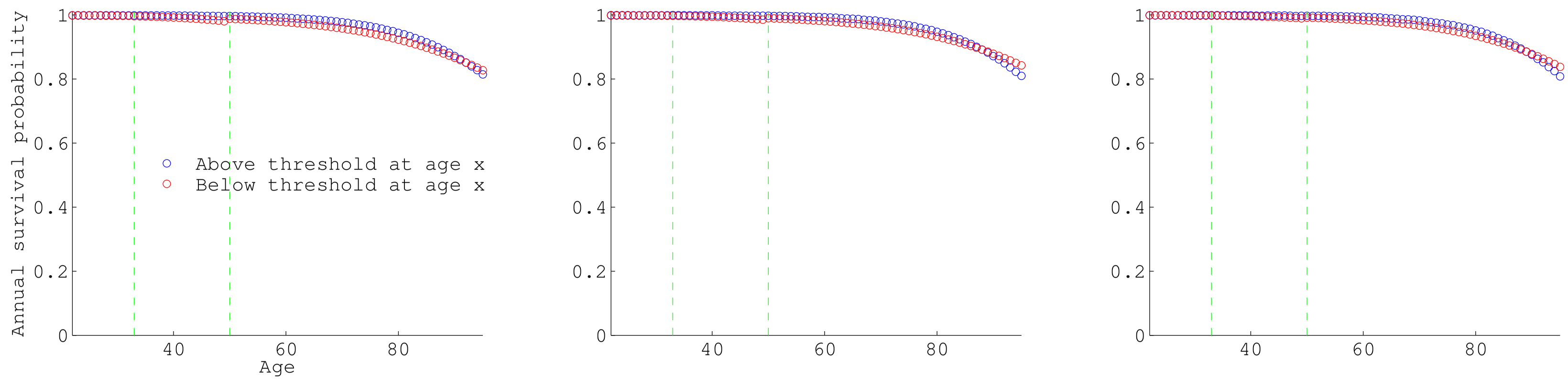
Our simulation findings are in agreement with "The most consistent finding in the literature... that the probability of entering poverty is much higher in young adulthood than in other stages of life" (Cellini et al., 2008). Rank and Hirschl (2015) (with PSID data) finds the occurrence of poverty is fairly widespread, between the ages of 25 and 60 they find 61.8 percent of the population will experience at least one year of relative poverty. They found that "a predominate pattern is that individuals are often likely to experience one or two years of poverty, and then rise out of poverty, with perhaps an additional spell down the road." When they looked at age groups they also found that those between the ages of 45 and 54 experienced the least incidence of poverty as opposed to the 25-34, 35-44 and 55-64 groups.

The dynamics of the poverty state is important; the less transient the higher income state, the larger the discrepancy in age-specific average remaining life expectancy and the variance. At young ages, where there is the highest probability of exiting the lower income state, there is a smaller difference in the average remaining life expectancy. Although more people are in poverty at young ages, they have more possibilities across their lifespan to change income state, thus there is less discrepancy in remaining life expectancy between income states.

We agree with Gillespie et al. (2014), that focusing on mid-ages to decrease income-related health disparities could help decrease lifespan inequality. Sandoval et al. (2009) combined period effects with looking at age classes to find fewer individuals enter poverty in their 40s and 50s as compared to other age classes, and found that the risk of poverty has been increasing over time (from 1970, 1980 to 1990), even for the low risk age-classes.

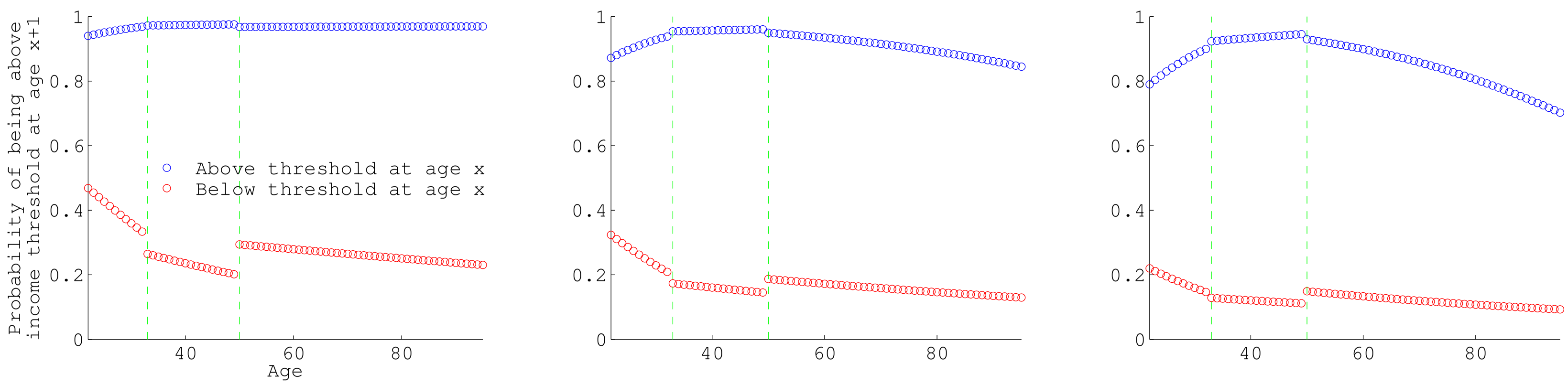
Further expansions of our model include examining how much of the changing poverty relationships with age are due to selection out of the cohort due to death. Additionally, comparative studies of age-specific poverty dynamics would yield interesting insight into different age-patterning across countries, regions and even data-sets (Chetty et al., 2016; McDonough et al., 2010; Sacker et al., 2007). Examining period effects in our simulations (for instance, as Sandoval et al. (2009) did between 1968 and 2000 and found that the life-course risk of poverty is increasing, especially in the 1990s) could allow the model to examine the effects of periods of economic turmoil. Adding an inter-generational element would be interesting as well, since there is an association between children's and parent's income (Chetty et al., 2014). this could be done for instance by using a Markov chain with reqards framework as Caswell (2015) has done. And we could infinitely extend out matrix model dimensions to include additional states with hyperstate matrix models (Roth and Caswell, 2016), for instance adding a state to reflect whether an individual has been in poverty in the past decade.

Our state-by-age-structured modeling of individuals undergoing stochastic entry and exit from 1×, 2× and 3× poverty yields a nuanced perspective of dynamic heterogeneity across the life course. We directly relate annual individual transience to state-specific disparities in life expectancy and variance. In doing so we extend the literature on individual poverty dynamics and stage-by-age matrix models. Our results suggest that dynamic heterogeneity in poverty and the transience of the poverty state is associated with income-related mortality disparities.



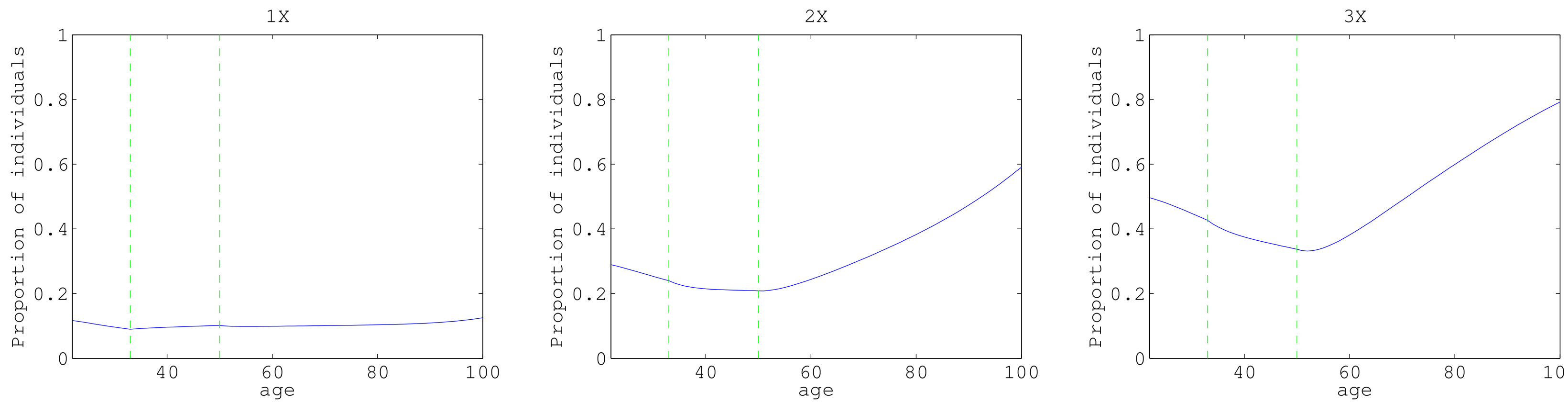
(a) (b) (c)

Figure 3.1: **Age-specific annual survival probabilities.** Annual survival (the number alive at age $x+1$/ the number alive at x) for those above (red) and below (blue) the poverty threshold at age x. a, b and c differ in threshold income used to define poverty, 1×, 2×, and 3× the 'official' poverty income level, respectively. The green vertical lines in each panel are the seams between data-sets utilized: NLSY79 pre-1994, NLSY79 post-1994 and HRS (at age 33 and 50).



(a) (b) (c)

Figure 3.2: **Age-specific conditional probabilities of being above the poverty threshold.** Annual probability of being above the poverty threshold at age $x + 1$ for those below (red) or above (blue) the poverty threshold at age x, conditional on survival. a, b, and c, differ in the threshold income used to define poverty, 1×, 2×, and 3× the 'official' poverty income level, respectively. The green vertical lines in each panel are the seams between data-sets utilized: NLSY79 pre-1994, NLSY79 post-1994 and HRS (at age 33 and 50).



(a) (b) (c)

Figure 3.3: **Predicted proportion of individuals that are below the poverty threshold** Predicted proportion of individuals of age x that are below the poverty threshold according to the dominant eigenvector of the cumulative $\mathbf{Q}(x)$ matrix $\mathbf{Qcum}(x)$, the cumulative unconditional state transition matrix for age x. a, b, and c, differ in the threshold income used to define poverty, $1\times$, $2\times$, and $3\times$ the 'official' poverty income level, respectively. The green vertical lines in each panel are the seams between data-sets utilized: NLSY79 pre-1994, NLSY79 post-1994 and HRS (at age 33 and 50).

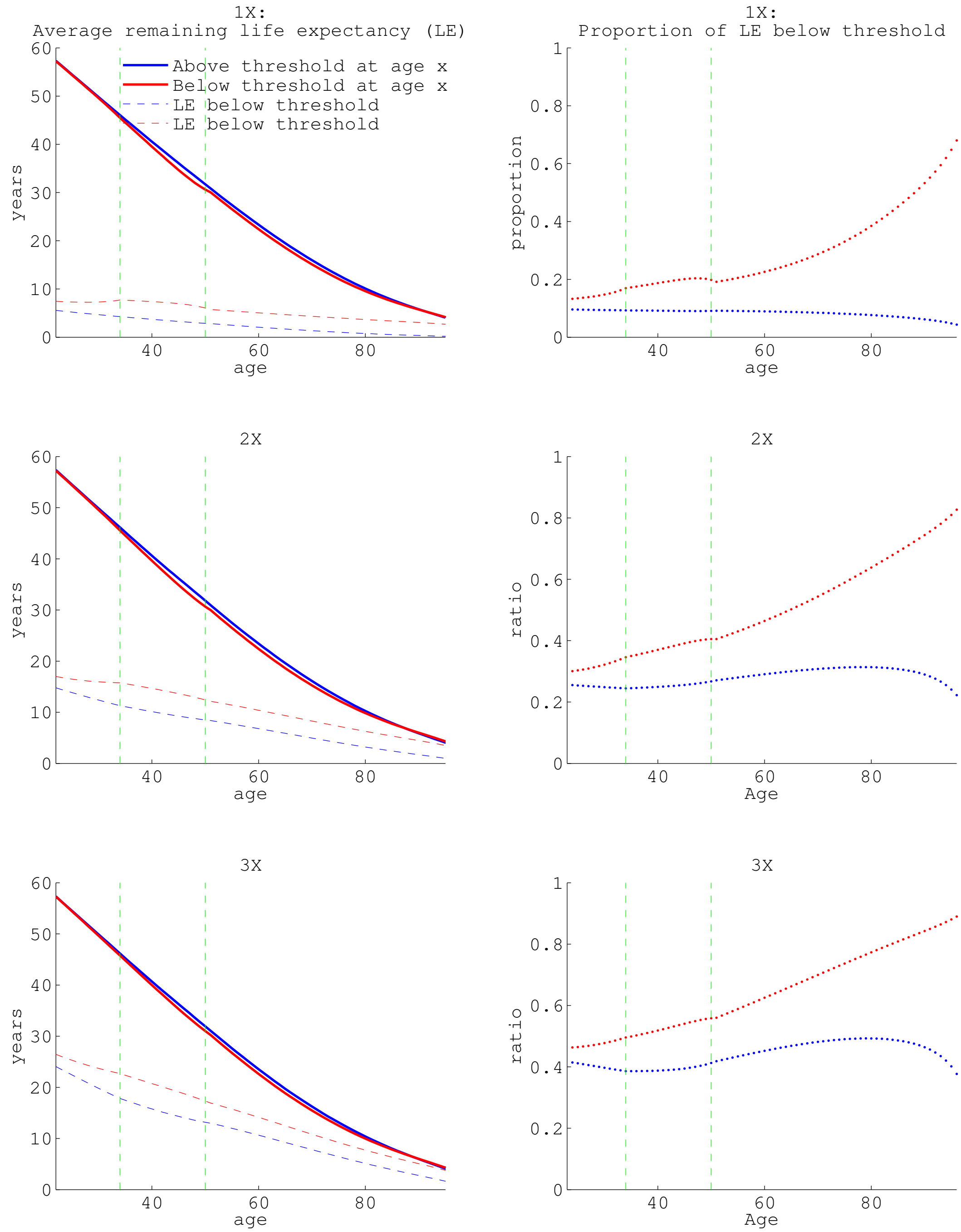


Figure 3.4: **The average remaining life expectancies and remaining years in poverty.** Average remaining life expectancies (solid lines) and expected years in poverty (dashed lines) conditional on surviving to age x based on state at age x (red= below threshold, blue= above threshold). a, c, and e, differ in the threshold income used to define poverty, $1\times$, $2\times$, and $3\times$ the 'official' poverty income level, respectively. The ratio of the average remaining life in poverty (or below $2\times$ poverty or $3\times$ the 'official' poverty threshold) to total average remaining life are graphed in panels (b), (d) and (f). In other words, this is the proportion of average remaining life below a specified threshold. All values are calculated from the fundamental matrix (see C.1.2). The dashed green vertical lines in each panel are the seams between data-sets utilized: NLSY79 pre-1994, NLSY79 post-1994 and HRS (at age 33 and 50).

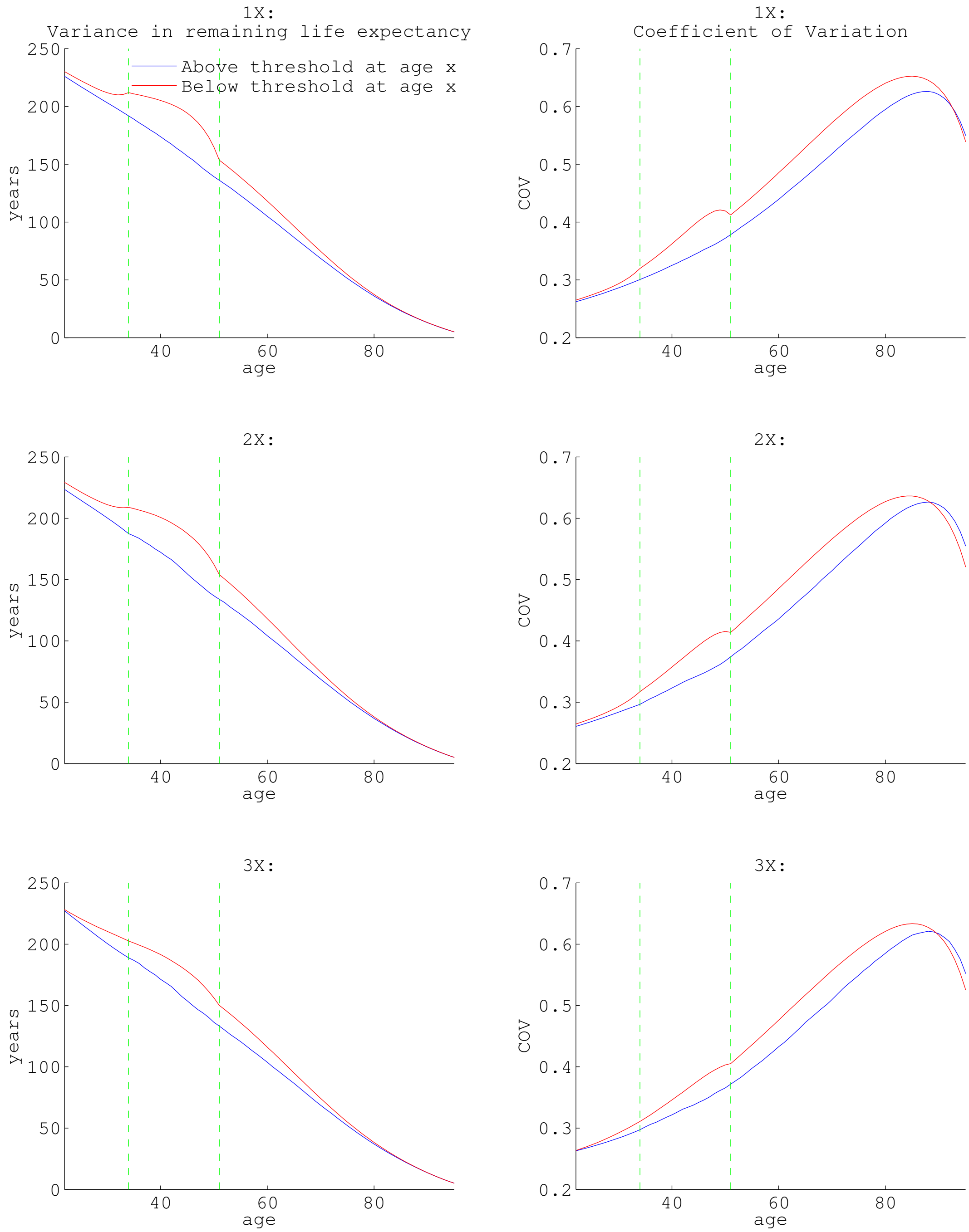


Figure 3.5: **The variance of remaining life expectancy, conditional on surviving to a specific age.** a, c, and e depict the variance of remaining life expectancy conditional on surviving to age x based on state at age x (red= below threshold, blue= above threshold). The age-specific coefficient of variation (the standard deviation divided by the mean) for state (red=below threshold, blue=above threshold) at age x are graphed in panels (b), (d) and (f). The dashed green vertical lines in each panels are the seams between data-sets utilized: NLSY79 pre-1994, NLSY79 post-1994 and HRS (at age 33 and 50).

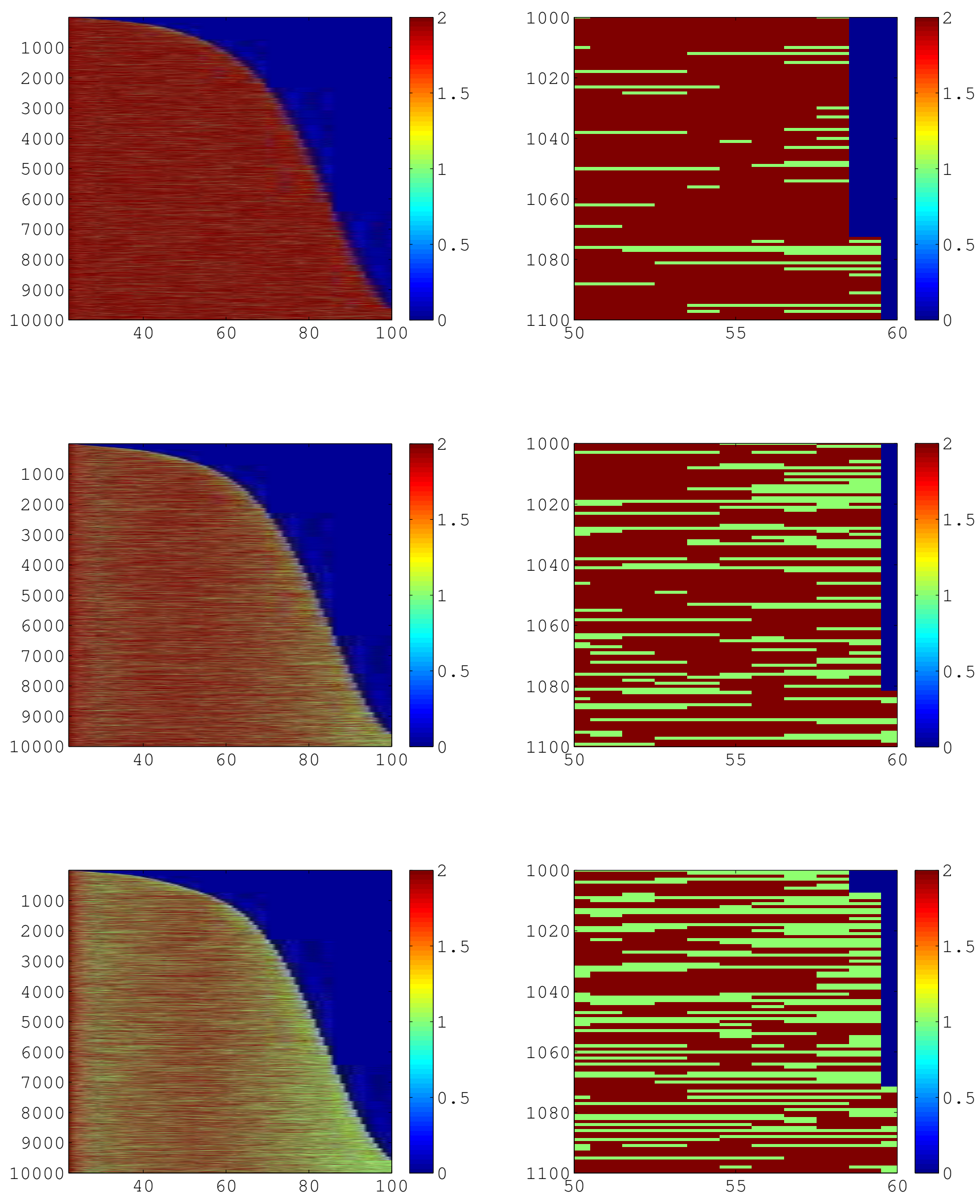


Figure 3.6: **Cohort simulations of 10,000 individuals lifetime trajectories.** The right column is a 'snapshot' of 100 individuals between the ages of 50 and 60 with mortality around 60. Red is the above income threshold state, green is below the income threshold, and blue is death. The first column is the entire cohort and is in the shape of a survivorship curve for all three rows. The first row of panels has a threshold at 1× poverty income threshold, second row is 2× and third row is 3× poverty income threshold.

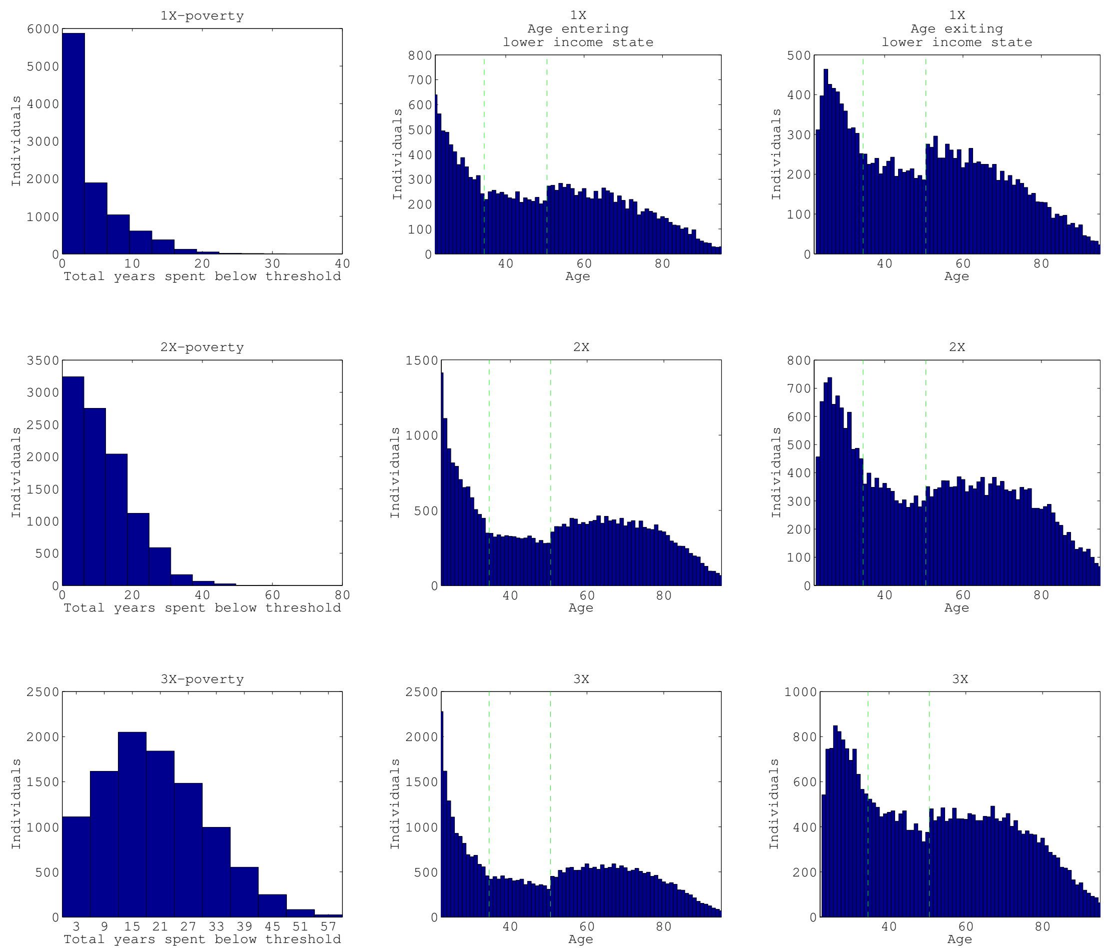


Figure 3.7: **Cohort simulations of 10,000 individuals: Distributions of residence times and ages of transitions** The initial cohort is the same for each row but each row is a separate simulation with a particular income threshold in place. Distributions of the years an individual spends in the lower income state are depicted in panels a, d, and g. The panels in the second column are distributions of ages of entry into the lower income state. The panels in the third column are distributions of ages of exit from the lower income state. The first row of panels has a threshold at 1× poverty income threshold, second row is 2× and third row is 3× poverty income threshold. The dashed green vertical lines in panels in the last two columns are the seams between data-sets utilized: NLSY79 pre-1994, NLSY79 post-1994 and HRS (at age 33 and 50).

CHAPTER 4

Four income states, stasis and transitioning: the resulting dynamic heterogeneity and income-mortality inequality with an age-stage Markovian model

4.1 Summary

Current income inequality research examines the spread of income in various populations but few consider how dynamic heterogeneity and probabilities of transitioning in and out of poverty and other income states at each age influence mortality disparities in cohorts.

Here we use state-by-age modeling to capture individual entry and exit in four income categories: $<1\times$, 1-2$\times$, 2-3$\times$, $>3\times$ the poverty threshold at each age. We examine age-specific state structure, the remaining life expectancy, its variance, and cohort simulations for each income category.

Survival and transition probabilities are statistically estimated by regression analyses of two U.S. nationally representative data-sets: the Health and Retirement Survey RAND data-set, and the National Longitudinal Survey of Youth. Using the re-

sults of these regression analyses, we parameterize discrete state, discrete age matrix models.

We find that for most ages the higher income states have the highest probability of surviving from one year to the next until about age 86 when the order of the income states does not equate to the order of survival advantage. In general, each income state has the highest annual probability of staying in the same state at each age, with the next highest transition being to move to higher income states. The greatest difference in average remaining life expectancy between consecutive states is for those in 1-2× and 2-3× poverty at ages 32-49. The largest discrepancy in average remaining life expectancy and its variance among all states and the <1× poverty state occurs at mid-ages (40-60). And the fewest individuals are in the lowest income category between ages 40-60. Our findings are consistent with results based on other data sets, but our results include a novel analyis not previously explored: an across the life-course investigation of the dynamic heterogeneity in income state at each age. We found that annual stasis probabilities in income state at each age influences: the cohort state-structure, the dynamic heterogeneity of the cohort, and inequalities or income related mortality disparities at each age.

This paper extends the literature on individual income state dynamics and stage-by-age matrix models.

4.2 Background

There is a significant relationship between income distributions and mortality rates and life expectancy (Backlund et al., 1996). Investigating income distributions in countries with ongoing economic development is of particular interest to research on health science and public policy (Preston, 1975). However, few comparison studies between or within countries also consider individual transience through various income levels, rather they focus on summary statistics of income levels or income inequality (Marmot, 2002) such as: country or regional poverty rates, Gini coefficients, or the percentage of the national income received by a particular percentile of the population. Here we present a state-by-age matrix model with four different income categories to investigate a cohort's dynamic heterogeneity in income status at ages 22-95. We examine the probabilities that individuals transition into and out of these four income states: 0-1×poverty, 1-2×poverty, 2-3×poverty and above 3×poverty. We combine analyses based on two U.S. nationally representative data-sets, the Health and Retirement Survey (HRS) and the Nationally Longitudinal Survey of Youth (NLSY79) to estimate annual state-specific survival and income-state entry and exit probabilities (Bureau of Labor Statistics, 2012). These probabilities are utilized in the matrix model for cohort projections and simulations to examine: cohort state structure, average remaining life expectancy, the variance of life expectancy at each age, average remaining years in each income state for each age, average duration in each income state, and ages where individuals are entering and exiting each income state.

Our lowest income state, poverty (below the 'official' poverty level), has the most well-documented mortality consequences (Benzeval and Judge, 2001). The poverty threshold is updated annually and consistently used across many research studies, making it a very relevant income state to investigate. It is also a widespread phenomenon: the 'official' poverty rate was 13.5 percent, based on the U.S. Census Bureaus 2015 estimates (updated annually to account for inflation). That year, an estimated 43.1 million Americans lived in poverty according to the official measure

(Proctor et al., 2016). For one single person to be in poverty that year they had to have an annual income under $12,082 (the poverty threshold is based on year and family size). The next income state, 1-2×poverty, sometimes referred to as 'near poverty' has characteristics quite similar to poverty. Transitions into and out of both low income states, poverty and near poverty, are frequent (Hokayem and Heggeness, 2014b). The third income state, 2-3× poverty, is the last income state under the median income level (about 3.2×the poverty threshold). As we move past the median income level mortality discrepancies decrease, in other words, above the median income level there is "no significant change in mortality risk with income increase" (Rehkopf et al., 2008). Thus the highest income state we consider are those above 3×poverty which is approximately the median income.

We construct a matrix model that is a discrete time, discrete state, discrete age, Markov chain matrix with the four-income state categories at each age. The matrix is similar in structure to Tuljapurkar and Steiner (2010)'s population projection age-stage matrix, but here, as in Steiner et al. (2012), there is no reproduction. It is also similar in concept to the model of multi-state mathematical demography presented in Rogers (1980). Matrix methods provide distinct analytic formulas for estimating similar entities as other multi-state methods, while providing additional insights.

We use regression analaysis of the empirical data and our matrix model to answer the following four questions: 1) What is the probability of entering or exiting an income state from one age to the next? 2) What is the effect of income category on annual survival probability each age? 3) What is the average remaining life expectancy and variance in life expectancy at each age for each income category? 4) In our Markovian framework, what is the average duration and net years spent in each income category?

The state-by-age model answers our research questions by emphasizing age and state structured cohort dynamics for the four income categories. We compare and contrast the rates and dynamics (answers to the research questions above) of each income state, emphasizing age-specific inequalities among states. The objective is to determine the role dynamic heterogeneity of four income states has on mortality disparities and extend the literature on multi-state analysis.

4.3 Methods: Theory

The age-by-state matrix, L, can be used to project cohort dynamics and to simulate individual trajectories. The matrix dimensions equals the number of states (in this case four) times the number of ages (note: a maximum age must be set). It is used in equation (4.1) to project a vector N that represents the number of individuals in each state at each age from birth across the lifetime, which enables tracking of the state-distribution and survivorship of an initial cohort at each age x, as described by the following equation:

$$N(x) = \mathbf{L}^x N(0) \tag{4.1}$$

where the matrix $\mathbf{L}$ is raised to the xth power at each age x. Markov chain theory utilizes $\mathbf{L}$ per equation (4.2) to analyze remaining life expectancy (mean and variance in age at death) and generate individual stochastic trajectories across all ages, where an individual is a realization or sample path of a Markov process. The Markov chain is described by this matrix:

$$\mathbf{P} = \left(\begin{array}{c|c} \mathbf{L} & 0 \\ \hline \text{m} & 1 \end{array}\right) \tag{4.2}$$

Here we will define the age-and-state matrix **L**, and in the appendix (C.1) we show how it can be used in (4.1) and (4.2). **L** has dimensions of age-by-stage and uses survival functions and transition functions for each income state that are functions of age. There are four survival functions $s_j(x)$, one for each income state ($j = 1, 2, 3, 4$), which provides the probability that an individual whose income is in one of the four income categories survives from ages x to $x + 1$ (*i.e.* the annual state-specific survival). There are 16 functions for the probability of transitioning state $t_{ij}(x)$, where $i = 1, 2, 3, 4$ and $j = 1, 2, 3, 4$ and $t_{ij}(x)$ equals the probability for an individual in income state j at age x to enter income state i before age $x + 1$ conditional on survival. i can equal j, a special case where the 'transition' probability is the annual probability of stasis.

State transition probabilities ($T(x)$) (transitioning to a new state or staying in a state at the next age) are defined as conditional on survival (separating survival from state transitions conditional on survival helps to elucidate different components of the biological processes occurring from one age to the next). Here a 'state' is an income category, 0-1× 'official' poverty, 1-2× 'official' poverty, 2-3× 'official' poverty or above 3× 'official' poverty. Note that the complement to the annual state specific survival is the annual state-specific mortality, *i.e.* the probability of dying in one year.

Multiplying the survival probabilities by the conditional state transition probabilities results in a matrix for each age, $\mathbf{Q}(x)$ which takes the following form:

		State at age x			
		<1×poverty	1-2×poverty	2-3×poverty	>3×poverty
State at age x+1	<1×poverty	$s_1(x)t_{11}(x)$	$s_2(x)t_{12}(x)$	$s_3(x)t_{13}(x)$	$s_4(x)t_{14}(x)$
	1-2×poverty	$s_1(x)t_{21}(x)$	$s_2(x)t_{22}(x)$	$s_3(x)t_{23}(x)$	$s_4(x)t_{24}(x)$
	2-3×poverty	$s_1(x)t_{31}(x)$	$s_2(x)t_{32}(x)$	$s_3(x)t_{33}(x)$	$s_4(x)t_{34}(x)$
	>3×poverty	$s_1(x)t_{41}(x)$	$s_2(x)t_{42}(x)$	$s_3(x)t_{43}(x)$	$s_4(x)t_{44}(x)$

Table 4.1: Structure of four-state transition matrix, Q(x)

Each unconditional state transition matrix, $\mathbf{Q}(x)$, is inserted into the sub-diagonal of the age matrix at the appropriate column. In our implementation of the model, we set a maximum age of 100 and thus there are 100 state transition matrices, each representing a 1-year increment from 0 to 100 years of age. The age-state block matrix has dimensions 101×101 blocks, each block is comprised of a 4×4 matrix, and has the following form (where the 0's represent 4×4 matrices comprised entirely of zeros):

$$\mathbf{L} = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ \mathbf{Q}(1) & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & \mathbf{Q}(2) & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{Q}(3) & \dots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \mathbf{Q}(98) & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & \mathbf{Q}(99) & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & \mathbf{Q}(100) & 0 \end{bmatrix}$$

The age-state block matrix $\mathbf{L}$ looks like a standard Leslie matrix from which fecundity has been removed, in that an individual always transitions to the next age at each time step (Keyfitz and Caswell, 2005; Kot, 2001; Leslie, 1945). Since there are 4 state classes and 101 age classes, the dimensions of the age-state matrix $\mathbf{L}$ is 404×404. The $\mathbf{L}$ matrix can be used to produce cohort projections (see C.1.1) and Markov chain analysis. With Markov chain analysis we calculate the average remaining life expectancy, average remaining life in an income state, and the variance (by using the fundamental matrix, see C.1.2) and the simulated individual trajectories (see C.1.3).

4.4 Methods: Empirical data

4.4.1 HRS RAND

The Health and Retirement Study (HRS) is a publicly accessible longitudinal household survey data set for the study of retirement and health among individuals over age 50 and their spouses in the United States. We use the RAND HRS Data files Version O which "are a cleaned, processed, and streamlined collection of variables derived from HRS" (Moldoff et al., 2014). The survey consists of 6 cohorts and we use longitudinal data compiled from 6 of the 11 interview waves that fall approximately around these years: 2002, 2004, 2006, 2008, 2010, 2012. As a nationally representative data set of $37,319$ individuals, HRS has over-sampled Hispanics, Blacks, and residents of Florida, and provides weighting variables to make it representative of the community-based (non-institutionalized) population. For our purposes we subset the data to only include individuals between 50 and 95 years old. We use survey sample weight in all analyses. And the income threshold that defines poverty is from the United States Census Bureau (thresholds vary by year, size of family, and number of children). Individuals are pooled into repeated observations for each of their interview responses as explained after the following section.

4.4.2 NLSY1979

The NLSY79 Cohort is a longitudinal project that follows the lives of a sample of American youth born between 1957-64. The cohort originally included $12,686$ respondents ages 14-22 when first interviewed in 1979; after two sub-samples were dropped, 9,964 respondents remain in the eligible samples (Bureau of Labor Statistics, 2012). We use data available from interview wave 1 (1979 survey year) to interview wave 25 (2012 survey year), which includes one-year intervals from 1979-1994, and two-year intervals from 1994-2012. Since we are studying income state we subset the data to include observations from age 22 (when individuals enter the work-force post college) to age 50. Retention rates for NLSY79 respondents from 1979 to 1993 exceed 90 percent. Rates from 1994 until 2000 exceed 80 percent. Rates from 2002 until 2012 were in the 70s. (Retention rate is calculated by dividing the number of respondents interviewed by the number of respondents remaining eligible for interview) (Bureau of Labor Statistics, 2012). More detailed information about retention rates can be found at NLSY79's website. Poverty rates are based on annual poverty income guidelines by the U.S. Department of Health and Family Services (which are also based on family size). We recognize that the threshold varies minimally between the U.S. Department of Health and Family Services and the U.S. Census Bureau, but previous research comparing slight differences in poverty thresholds shows the effect to be insignificant (Cellini et al., 2008).

4.4.3 Quantifying the functional dependence of survival probabilities on age: logistic regression

There has been much discussion as to how to calculate transition probabilities for Markov transition models (Islam et al., 2004; Islam and Chowdhury, 2006; Korn and Whittemore, 1979; Lawless and Rad, 2015; Yu et al., 2010). Willekens and Putter (2014) review is a good resource as to the vast array of statistical packages in R that can be used to estimate transition probabilities for demographic multi-state models. One very practical proposal for calculations of binary Markov transition models has been to use logistic regression probabilities (Muenz and Rubinstein, 1985). Since logistic regression is very straightforward and intuitive, especially when we have a time-dependent covariate (age), we employ it to calculate our survival probabilities.

4.4.4 Quantifying the functional dependence of transition probabilities on age: multinomial logistic regression

For calculating the entry and exit probabilities for each income category we considered three options: multiple logistic regression, ordinal logistic regression, and multinomial logistic regression. Multiple logistic regression and multinomial logistic regression are very similar in output, except that multiple logistic regression probabilities might not sum exactly to 1. Ordinal logistic regression or the proportional odds model is usually used when the response is ordinal, for instance looking at survey responses of ‘strongly disagree’ to ‘strongly agree’, etc. One additional condition for using ordinal logistic regression (and why it is sometimes called a proportional odds model) is that the proportional odds assumption must be met; in short, this means the log of the proportion of individuals in each cumulative group should change by the same amount for each group (Venables and Ripley, 2002). We check this by taking the logarithm of the odds of each cumulative group (<1×poverty, of those in both <1×poverty and 1-2×poverty, of those in <1×poverty, 1-2×poverty and 2-3×poverty) and find that an arithmetic sequence is not formed, thus the proportional odds assumption is not met. We therefore use the multinomial logistic regression to calculate transition probabilities into and out of each income state. (We note that for the HRS data we ran ordinal logistic, multiple logistic, and multinomial logistic regression and find the results to be mostly consistent).

4.4.5 Pooling the data, regressions, and combining data-set probabilities

The NLSY79 pre-1994, NLSY79 post-1994 and HRS data for ages 22-33, 34-50, and 51-95, respectively, are analyzed separately and the results are later combined to give probabilities of survival and transitioning over our complete age range of $22-95$. Within each data-set, interview waves are pooled together except for the last wave, this represents the data at time t. All the interview waves except for the first one are then pooled together to represent the data at time $t+i$. (To find i for each data-set we average the interval in months between each successive interview wave. For instance, for HRS the interval between interview years is normally distributed about 24 months, for NLSY79 pre-1994 the interval is normally distributed about 12 months, and for NLSY post-1994 there is a normal distribution in interval length about 24 months. After performing regressions we adjust our probabilities to be over a one year period, *i.e.* t to $t+1$, see appendix for more on this adjustment).

Instead of tracking individuals longitudinally over several ages, we perform a pooled logistic/multinomial regression analysis on all observations of all ages from time t to $t+i$, where individuals have a particular age x at time t and age $x+i$ at time $t+i$. If an individual did not respond at that specific point (either the interview at t or $t+i$ or both interviews), that observation was omitted from the analysis. However if that specific individual responded later in the study (a different observation), that observation was included in the analysis. This technique has been called the pooled repeated observation method (PRO) (Dagostino et al., 1990). Logistic regression analysis was performed in R and the survey package (Lumley, 2014) with `svyglm` was used to incorporate weights. Multinomial logistic regression was also performed in R but with the use of the `multinom` function and the maximum likelihood and multinomial packages, (Henningsen and Toomet, 2011; Ripley and Venables, 2016) respectively. For HRS we have $186,585$ observations between the ages of 50 and 95. For NLSY79 after pooling we have $183,238$ observations between the ages of 22 and 50. For analysis the pooled observations are weighted based on each data sets weight at observation.

To change regression outputs into probabilities we logit transform the coefficients from the logistic regression. To transform the coefficients from the multinomial regression we first calculate probabilities of entering the lowest income category (<1× 'official' poverty) from each of the four income states. We use the fact that the probabilities of being in a certain state j, at time t and entering each of the four income states at $t+1$ must sum to one (since R output sets the coefficients to 0 for the first class, in this case <1×poverty, conditional on survival). So for instance, the probability of starting in state '1' and remaining in state '1' would equal $\frac{1}{(1+exp(\beta_{21})+exp(\beta_{31})+exp(\beta_{41}))}$ where β_{i1} are the regression coefficients for being in state 1 and entering state i (Ripley and Venables, 2016). Likewise, the probability of being in state '2' and entering state '1' would be $\frac{1}{(1+exp(\beta_{22})+exp(\beta_{32})+exp(\beta_{42}))}$. Probabilities for entering states other than <1×poverty are calculated with a normalized exponential function, for instance, the probability of entering state '2' from any state j can be calculated with $\frac{\beta_{2j}}{(1+\sum_{j=2}^{4} exp(\beta_{2j}))}$.

We then adjusted each probability so that it represented a one-year probability, for age x to $x+1$ and concatenated probabilities obtained from each data-set, see A.3 for how this was done for the transition probabilities. For the survival probabilities our calculation here differs from previous work where there were only 2 states (when we mathematically considered how a death observed after two years from a given starting state could have come about by different pathways in and out of both states during the two year period, and then we estimated survival probabilities by taking the roots of quadratic equations that reflected these different pathways as described in A.3). The mathematics for 4 states implied the existence of many more roots so we were constrained to follow a simpler procedure. For a given starting state we estimated annual survival probabilities by taking the square root of the approximately two-year survival probabilities from the regression analysis. We then evaluated these functions at each age for use in our matrix models.

In our main analysis we investigate the impact of income level on mortality where state switching may occur at each age and we investigate how the dynamics of changing state influences the cohort dynamics.

4.5 Results

4.5.1 Annual probabilities of survival and transitioning for each income state

Below mid-ages there is no significant difference in annual survival among the four income states (tables 4.4 and 4.6); however, there is a significant difference above mid-ages (table 4.2). In fig 4.1 the age-patterned survival advantage of each income state is displayed, and, as expected, the lowest income state has the lowest annual survival probability for all ages except for a slight crossover at age 94. The above 3×poverty category has the greatest survival advantage over the other income states at most ages with a maximum advantage at age 79. The 1-2×poverty category has an increasing advantage in annual survival over the <1×poverty category for all ages, whereas 2-3×poverty and >3×poverty categories have an increasing advantage at mid-ages to ages 79, and then the advantage decreases for late old ages. The 2-3×poverty category has a crossover with 1-2×poverty category at age 86, whereas >3×poverty has a crossover with 1-2×poverty at age 89, consequently, 1-2×poverty has the greatest survival advantage over all the income states at late old ages (past age 89).

Calculated transition probabilities for each income state reveal that the most likely income state an individual will be in at age $x + 1$, is the income state they were previously in at age x (fig.4.2); *i.e.*, the probability of stasis is greater than the probability of transitioning for all states. That being said, those above 3×poverty have the greatest annual probability of stasis, followed by those below 1×poverty. We realize that the >3×poverty state includes the most individuals as well (since it is about the median income level), so to consider how the proportion of the cohort in each income state influences transitions we view the transition probabilities for relative income status (we created quartiles based on income below 25%, 25-50%, 50-75%, and 75-100%) for ages greater than 50. Even though there is about the same proportion of people in each income quartile, the annual stasis and transition probabilities at each age still vary based on income quartile (D.3). Since there is not as great a difference in annual survival among income quartiles (D.2) (thus switching states has less effect on annual mortality, especially for the two highest quartiles), we prefer considering individual dynamics in entering and exiting our particular absolute income states.

We see from fig. 4.2 that both the 1-2×poverty state and 2-3×poverty state have greater probabilities of transitioning to a higher income category than to a lower income category for all ages, especially during early ages. Where individuals <1×poverty are most likely to transition to a higher income category at early ages, individuals >3×poverty are most likely to transition to a lower income category at early and late ages.

Discontinuities at the dashed vertical lines represent the joining of probabilities from NLSY pre-1994, NLSY post-1994, and HRS. The distribution of the pooled data can be seen in fig. D.1 and one can keep in mind that the data-sets are structured differently; with new cohorts being added to HRS in 2004 and 2010 while NLSY79 follows the same aging cohort through each interview year. The data-sets also cover different time periods; NLSY79 covers 1979-2012, HRS 2002-2012. Thus there may be a contribution of period effects (due to different time periods with different economic environments for example) as well. The trend in probabilities are considered and in further sections when cohort projections and simulations are performed we see the cohort dynamics are consistent with the dynamics of the empirical data.

4.5.2 Cohort income-state distribution at each age

To perform cohort projections the survival and transition probabilities are utilized in the state-by-age matrix and the state structure of the cohort at each age is considered. First we note that the state-distribution from the projection (fig. 4.3) matches the state distribution from the empirical data (fig. D.1). This means it is at least possible that individual income-state switching is an underlying process that drives the dynamics of the empirical data. We also note that age range 40-60 has the greatest proportion of individuals in higher income states (fig. 4.3). Although the proportion of individuals in the <1×poverty category seems to stay the same with age, the other income states fluctuate much more. From early ages to mid-ages the proportion in each state is greatest for higher income states and least for lower income states, respectively. However, at mid-ages to old ages there is an increasing proportion of individuals in the 1-2×poverty state, surpassing those in the 2-3×poverty state at age 52 and those in the >3×poverty state at age 86.

4.5.3 Outputs of the fundamental matrix

We are interested in how the average remaining life expectancy at each age varies by income category at each age. From ages 40-70 all three of the higher income states have their greatest advantage in life expectancy over the <1×poverty state (fig. 4.4). This shows that even being one income state higher than <1×poverty, for instance the 1-2×poverty state, confers a higher average remaining life expectancy of

1/2 year for ages 37-50, with a maximum advantage of 0.8 years. 2-3×poverty has a life expectancy advantage of over 1/2 year from ages 33-77, and >3×poverty has a higher life expectancy by 1/2 year for ages 30-82, with a 2.0 year maximum difference over those in <1×poverty. When looking at differences in consecutive income states, the biggest consecutive change in life expectancy is moving from <1×poverty to 1-2×poverty from ages 32-49 with a maximum at age 45. From ages 22-30 the greatest consecutive difference in life expectancy is between the 1-2×poverty state and the 2-3×poverty state. For ages greater than 50 the greatest consecutive difference in life expectancy is between the 2-3×poverty and >3×poverty income states.

The variance in life expectancy reflects the diversity in individual trajectories and where they end. Being in the <1×poverty category at any age, is associated with the highest variance. The income states have the greatest discrepancies in variance in the middle of the life course (fig. 4.4). The largest consecutive difference in variance is between <1×poverty to 1-2×poverty at ages 32-51 (with a maximum difference in variance of 19.4 years) followed by the difference between 1-2×poverty and 2-3×poverty at the same ages. After age 50 the consecutive difference in variance follows the order of the income states with the difference decreasing with age; *i.e.*, the difference in variance between >3×poverty and 2-3×poverty after age 50 is the greatest, followed by the difference in variance between 2-3×poverty and 1-2×poverty, and 1-2×poverty and <1×poverty, and all the discrepancies decrease with age.

If an individual is in a specific income state at age x, how much of their remaining life can be expected to be spent in each income state? If someone is in the <1×poverty state at age 22, 7.7 years is expected (on average) in the same state and on average they can look forward to 29.7 years in the >3×poverty state (fig.

4.5). Those $>3\times$poverty at age 22 can expect to spend 32.8 years in the same state, however they can expect to spend about 5.2 years $<1\times$poverty as well. Since this model is dynamic, the average remaining time in each state takes into account possible stage switching at every age. Someone switching into the $>3\times$poverty state will be raising their life expectancy and gaining an expected number of years in each state that looks like fig. 4.5 (d). Someone switching into 1-2$\times$poverty will have the most expected years in the same state after age 60, although previous to that age most expected years left would be in the $>3\times$poverty state. Someone entering or remaining in 2-3$\times$poverty after age 74 has the greatest expected years in the same state. The expected years in each state sums to the total average remaining life expectancy for each state at age x. One can note that on average, until after age 60, regardless of current state, individuals can expect most of their years in the $>3\times$poverty state, *i.e.*, above the median income.

4.5.4 Simulations of individual trajectories

From the cohort dynamics, such as the state structure at each age, we viewed the overall proportion of individuals in each state; however, that does not inform us about individual trajectories and state switching. Therefore, we ran a simulation of 10,000 individuals with the annual survival and transition probabilities for each age and state as defined above. Fig. 4.6 gives insight into the dynamic heterogeneity of individuals in the cohort over their lifetimes. Individuals are arranged in order of death, with black ('0') representing the absorbing state of death. First we notice, as expected, that living individuals are in the shape of a standard human survivorship curve. Second, most individuals are above 3$\times$poverty from age 40 to 60 and most heterogeneity occurs at early and late ages. After age 60 there is much heterogeneity, with an increasing abundance of individuals in the 1-2×poverty state (state '2', yellow) especially among those who decease around and past age 90 (see fig. 4.7). In fig. 4.7 we also compiled simulated individuals with ages of death at age 50, 60, 70, 80, and 90 and looked at each groups state structure at the last 5 years of their life. For those who died at age 50 we see a greater proportion of individuals in the $<1\times$poverty state. However we do not see a similar pattern of state structure for those who died at age 60. For the individuals with deaths at the later ages (70, 80 and 90), they all have an increasing proportion in the 1-2×poverty state at the end of life.

To investigate summaries of passage times we examine the distributions of ages at which individuals first switch to higher and lower income states. Most switching occurs at young ages with a leveling out after age 50 (fig 4.8). Viewing each income state separately we notice a similar pattern, except for entering $<1\times$poverty where switching decreases more steeply past age 50 (perhaps because individuals in this state are dying more quickly), while switching into the other states declines similarly past age 70. The mean age at death is age 76, the mode is 87, and the standard deviation is 13.9 years.

To investigate how long individuals are remaining in a given state during consecutive ages we examine distributions of individual duration's or 'spells' in each state (but only after age 22). The maximum duration in $<1\times$poverty state, having at least spent one year in this state, is 27 years, the mean duration time is 3.25 years, the median is 2 years and the mode is 1 year (fig. 4.9). The maximum year duration in 1-2×poverty state is also 27 (for this simulation), the mean duration time is 2.69 years, the median is 2 years and the mode is 1 year. The maximum year duration in 2-3×poverty state is 21 years, the mean duration time is 2.3 years, the median is 2 years and the mode is again 1 year. Lastly, the maximum years above 3×poverty state is 68 years, the mean duration time is 8.22 years, the median is 5 years, and the mode is 2 years. Note that in our Markovian framework, only present state is considered when determining future state (individuals do not carry their history). Thus probabilities of transitioning are equally spread to all individuals of the same state yet heterogeneity in state still results.

Additionally we ask how long over their lifetimes (after the age of 22) are simulated individuals spending in each income state? On average, the net years over a lifetime that individuals are spending in the <1×poverty state is 3.85 years, mode of 0 years, median of 2 years, and standard deviation of 4.62 years (fig. 4.10). For 1-2×poverty, individuals spend a mean of 5.67 years, mode of 0 years, median of 5 years, and standard deviation of 5.09 years in this income state. For 2-3×poverty individuals spend a mean of 5.32 years, mode of 0 years, median of 4 years, and standard deviation of 4.29 years in this income state. Lastly, the net average years spent above 3×poverty is 29.96 years, mode of 31 years, median of 30 years, and standard deviation of 12.29 years.

For our discussion section we will generally refer to the <1×poverty state as simply 'poverty', the 1-2×poverty state as 'near poverty', 2-3×poverty as 'near median income state', and above 3×poverty as the 'highest income state'.

4.6 Discussion

"The influence of economic conditions on mortality has been recognized at least since biblical times" (Preston, 1975). Much literature discusses the causality of the relationship noting that "The fewer goods and services are provided publicly by the community, the more important individual income is for health" (Marmot, 2002). The same research article discusses that "Income is related to health in three ways: through the gross national product of countries, the income of individuals, and the income inequalities among rich nations and among geographic areas"(Marmot, 2002). Our analysis considers how the income of individuals, and opportunities to move up and down in income state affects the cohort state structure, life expectancy and heterogeneity at each age. In our Markovian framework, every individual has equal probability of transitioning and survival based on their state and age, independent of any intrinsic differences among them. Thus 'inequality' presents itself in our model as lower annual probabilities of moving up in income state, which occurs the most for our poverty state at mid-ages. These ages with the greatest 'inequality' (*i.e.* disparity due to income) are the ages where the higher income states have the highest survival and life expectancy advantage. Yet, although inequality in this sense (of having greater annual probability of stasis in the lowest income and highest income states) is greatest at mid-ages, our simulations and analysis of state-structure at each age shows that mid-ages (about ages 40-60) is also the time period the greatest number of individuals are in the highest income state. When the entire cohort is experiencing the most dynamic heterogeneity, at early and late ages, there is less disparity in survival and life expectancy. Our finding that the fewest individuals are in the lower income states at mid-ages is also in agreement with Rank and Hirschl (2015) that those between the ages of 45 and 54 experienced the least incidence of poverty as opposed to the 25-34, 35-44 and 55-64 groups.

Our findings are also in agreement with Backlund et al. (1996), “The income-mortality gradient was much smaller in the elderly than in the working age population”, perhaps because the working age population in higher income states are able to maintain stability of income. Our results also concur in our finding that stasis in higher income states, has more of an effect on the demographic state structure than exit probabilities of individuals in lower income states (perhaps since there are also fewer individuals in lower income states). Greater annual stasis for individuals in the higher income state effectively renders the demographic state structure to be dominated by higher income state individuals.

Our simulation findings are also in agreement with “The most consistent finding in the literature... that the probability of entering poverty is much higher in young adulthood than in other stages of life” Cellini et al. (2008). We find this to be true for every income state except poverty itself, where in young adulthood individuals in poverty have the greatest probability of rising up in income state.

Our life course approach to looking at life expectancy advantages for each income state showed that the advantage of moving up in consecutive states changes with age. At young ages (22-30) those moving from the near poverty state to the near median income state get the largest improvement in life expectancy from a consecutive change in income state. However the greatest improvement over the life course occurs from ages 32-49 when switching from poverty to near poverty. Though near poverty and poverty have similar characteristics, at these ages there is maximal discrepancy between these income states in life expectancy. For the rest of the life course (ages 50-95) improvements in life expectancy between consecutive state are increasingly more modest with age.

Our estimates of durations and net years spent in each income state are consistent with Stevens (1999) who investigated the persistence of poverty (which he defines as 1.25× 'official' poverty) over multiple spells. In his model he accounts for those with a history of poverty when predicting future poverty spells; he found the mean poverty spell to be 2.7 years with total poverty throughout the 10-year cross section he observes to be 4.0 years on average. Our calculated average duration or 'spell' in (1×)poverty (across the life course) was 3.2 years, near poverty (1-2×poverty) had an average spell of 2.7 years, near median income (2-3×poverty) and the highest income state (>3×poverty) had average duration's of 2.3 and 8.2 years, respectively. While the average net time (from ages 22-95) in poverty was 3.8 years, in near poverty it was 5.66 years, in the near median income state and the highest income state it was 5.3 and 29.9 years, respectively. In a sense, our model is quite optimistic, for most ages, every income state has highest expected remaining life in the highest income state till old ages.

One advantage of utilizing four income state is that we were able to investigate the age-patterning in probability of transitioning for the two intermediate states, where individuals can potentially move up or down in income state. We found that for both the near poverty and near median income states, there is generally a greater probability of moving to a higher income category, and the probability of stasis increases after age 50. The annual probability of stasis in the near poverty state increases more drastically after age 50, perhaps explaining why there is the highest proportion of individuals in this state at old ages (after age 80).

The 'crossover', where in late life the near poverty state has the greatest survival advantage can be explained in a few ways. First, it could reflect the increased probability of being in the near poverty state in later ages. Second, as explained in Steinsaltz and Evans (2004), in many mortality models there is the phenomena of the weaker individuals dying off leaving the more robust individuals behind in any state. Thus a high proportion of weaker individuals in the near poverty state experienced their earlier mortality, leaving survivors in the near poverty state whom are more robust with higher survival on average than those in the other income states at late ages.

A related and interesting expansion of our model is to examine how much of the changing income state relationships with age are due to selection out of the cohort due to death. Additionally, comparative studies of age-specific income state dynamics across countries with different economic developments would yield interesting insight into age-specific income inequalities and transitioning probabilities between countries (Chetty et al., 2016; McDonough et al., 2010; Sacker et al., 2007).

Using regression analysis of the data-sets and our state-by-age-matrix model we were able to investigate the relationship between individual entry and exit rates into four income states, state and age based survival, cohort dynamic heterogeneity, cohort state structure, and individual trajectories. In so doing we answered our above stated research questions and found that individual annual income state transition probabilities and survival rates could be the underlying driving force that determines cohort dynamics such as income-related mortality disparities, and demographic structure. Our results suggest that dynamic heterogeneity in income state and annual stasis probabilities of income states is associated with income-related mortality inequality.

Table 4.2: HRS logistic regression: Dependent Variable = Survival at time $t+i$

Predictor	**Estimate**	Std. Error	t value	$Pr(>\|t\|)$
Intercept (Below 1× poverty)	7.311	0.353	20.717	<2e-16 ***
Slope (Below 1× poverty)	-0.068	0.004	-14.727	<2e-16 ***
Δ Intercept (1-2× poverty)	0.365	0.469	0.778	0.436
Δ Intercept (2-3× poverty)	1.517	0.519	2.923	0.003***
Δ Intercept (Above 3× poverty)	3.303	0.425	7.772	7.80e-15***
Δ Slope (1-2× poverty)	-0.002	0.006	-0.411	0.681
Δ Slope (2-3× poverty)	-0.015	0.006	-2.343	0.019**
Δ Slope (Above 3× poverty)	-0.035	0.005	-6.361	2.01e-10***

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table 4.3: HRS multinomial logistic regression: Dependent Variable = Income status at time $t + i$

Predictor	**Estimate**	Std. Error	t value	$Pr(>\|t\|)$
1-2× poverty at $t + 1$				
Intercept(Below 1× poverty time t)	-0.741	0.131	-5.620	1.90e-08***
Slope(Below 1× poverty)	0.000	0.001	0.221	0.824
Δ Intercept (1-2× poverty)	-0.480	0.182	-2.628	0.008***
Δ Intercept (2-3× poverty)	-1.080	0.239	-4.518	6.21e-06***
Δ Intercept (Above 3× poverty)	-1.557	0.213	-7.286	3.18e-13***
Δ Slope (1-2× poverty)	0.035	0.002	13.183	0.00***
Δ Slope (2-3× poverty)	0.045	0.003	12.573	0.000***
Δ Slope (Above 3× poverty)	0.047	0.003	14.615	0.000***
2-3× poverty at $t + 1$				
Intercept(Below 1× poverty time t)	-1.050	0.211	-4.959	7.08e-07***
Slope(Below 1× poverty)	-0.012	0.003	-3.836	0.000***
Δ Intercept (1-2× poverty)	0.319	0.259	1.230	0.218
Δ Intercept (2-3× poverty)	-0.467	0.284	-1.639	.101
Δ Intercept (Above 3× poverty)	-0.044	0.264	-0.168	.865
Δ Slope (1-2× poverty)	0.025	0.003	6.568	5.09e-11***
Δ Slope (2-3× poverty)	0.062	0.004	14.208	0.000***
Δ Slope (Above 3× poverty)	0.051	0.004	12.692	0.000***
Above 3× poverty at $t + 1$				
Intercept(Below 1× poverty time t)	0.099	0.193	0.514	6.07e-01***
Slope(Below 1× poverty)	-0.025	0.003	-8.565	0.000***
Δ Intercept (1-2× poverty)	-0.397	0.246	-1.614	0.106
Δ Intercept (2-3× poverty)	-0.202	0.272	-0.743	.457
Δ Intercept (Above 3× poverty)	3.557	0.242	14.674	0.000***
Δ Slope (1-2× poverty)	0.031	0.003	8.436	0.000***
Δ Slope (2-3× poverty)	0.052	0.004	12.411	0.000***
Δ Slope (Above 3× poverty)	0.022	0.003	5.826	5.65e-09***

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table 4.4: NLSY79 pre-1994 logistic regression: Dependent Variable = Survival at time $t + i$

Predictor	**Estimate**	Std. Error	t value	$Pr(>\|t\|)$
Intercept (Below 1× poverty)	7.433	1.400	5.307	1.12e-07 ***
Slope (Below 1× poverty)	-0.045	0.051	-0.890	0.373 ***
Δ Intercept (1-2× poverty)	-0.634	2.299	-0.276	0.782
Δ Intercept (2-3× poverty)	5.019	2.912	1.724	0.084*
Δ Intercept (Above 3× poverty)	-1.328	2.335	-0.569	0.569
Δ Slope (1-2× poverty)	0.030	0.085	0.355	0.722
Δ Slope (2-3× poverty)	-0.146	0.102	-1.432	0.152
Δ Slope (Above 3× poverty)	0.081	0.085	0.945	0.344

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table 4.5: NLSY79 pre-1994 multinomial logistic regression: Dependent Variable = Income status at time $t + i$

Predictor	**Estimate**	Std. Error	t value	$Pr(>\|t\|)$
1-2× poverty at $t + 1$				
Intercept(Below 1× poverty time t)	-0.050	0.168	-0.299	0.764
Slope(Below 1× poverty)	-0.026	0.006	-4.291	1.77e-05
Δ Intercept (1-2× poverty)	0.424	0.240	1.766	0.077*
Δ Intercept (2-3× poverty)	0.071	0.344	0.207	0.835
Δ Intercept (Above 3× poverty)	1.022	0.377	2.708	0.006***
Δ Slope (1-2× poverty)	0.057	0.008	6.472	9.61e-11***
Δ Slope (2-3× poverty)	0.079	0.012	6.190	6.01e-10***
Δ Slope (Above 3× poverty)	0.021	0.014	1.493	0.135
2-3× poverty at $t + 1$				
Intercept(Below 1× poverty time t)	0.950	0.281	3.380	0.000***
Slope(Below 1× poverty)	-0.109	0.010	-10.226	0.000***
Δ Intercept (1-2× poverty)	-0.622	0.339	-1.832	0.066*
Δ Intercept (2-3× poverty)	-0.763	0.397	-1.919	0.054*
Δ Intercept (Above 3× poverty)	-1.133	0.412	-2.747	0.005***
Δ Slope (1-2× poverty)	0.114	0.012	8.966	0.000***
Δ Slope (2-3× poverty)	0.195	0.014	13.025	0.000***
Δ Slope (Above 3× poverty)	0.186	0.015	11.923	0.000***
Above 3× poverty at $t + 1$				
Intercept(Below 1× poverty time t)	1.028	0.288	3.565	3.63e-04***
Slope(Below 1× poverty)	-0.114	0.011	-10.395	0.000***
Δ Intercept (1-2× poverty)	1.174	0.366	3.202	0.001***
Δ Intercept (2-3× poverty)	0.091	0.406	0.224	0.822
Δ Intercept (Above 3× poverty)	-0.801	0.405	-1.977	0.047**
Δ Slope (1-2× poverty)	0.025	0.013	1.839	0.065*
Δ Slope (2-3× poverty)	0.149	0.015	9.750	0.000***
Δ Slope (Above 3× poverty)	0.251	0.015	16.373	0.000***

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table 4.6: NLSY79 post-1994 logistic regression: Dependent Variable = Survival at time $t + i$

Predictor	**Estimate**	Std. Error	t value	$Pr(>\|t\|)$
Intercept (Below 1× poverty)	8.527	0.998	8.540	<2e-16 ***
Slope (Below 1× poverty)	-0.104	0.023	-4.490	7.13e-06 ***
Δ Intercept (1-2× poverty)	0.747	1.512	0.495	0.621
Δ Intercept (2-3× poverty)	0.510	2.248	0.227	0.821
Δ Intercept (Above 3× poverty)	-0.041	1.502	-0.027	0.978
Δ Slope (1-2× poverty)	-0.002	0.035	-0.085	0.933
Δ Slope (2-3× poverty)	0.024	0.053	0.454	0.650
Δ Slope (Above 3× poverty)	0.038	0.035	1.093	0.274

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table 4.7: NLSY79 post-1994 multinomial logistic regression: Dependent Variable = Poverty status at time $t+i$

Predictor	**Estimate**	Std. Error	t value	$Pr(>\|t\|)$
1-2× poverty at $t+1$				
Intercept(Below 1× poverty time t)	-0.157	0.214	-0.738	0.460
Slope(Below 1× poverty)	-0.019	0.005	-3.666	2.45e-04***
Δ Intercept (1-2× poverty)	2.026	0.302	6.700	2.08e11***
Δ Intercept (2-3× poverty)	1.677	0.411	4.077	4.55e-05***
Δ Intercept (Above 3× poverty)	0.080	0.399	0.202	0.839
Δ Slope (1-2× poverty)	-0.002	0.007	-0.340	0.733
Δ Slope (2-3× poverty)	0.007	0.010	0.735	0.461
Δ Slope (Above 3× poverty)	0.029	0.009	3.075	0.002***
2-3× poverty at $t+1$				
Intercept(Below 1× poverty time t)	-0.514	0.327	-1.573	0.115
Slope(Below 1× poverty)	-0.037	0.008	-4.626	3.71e-06***
Δ Intercept (1-2× poverty)	1.520	0.404	3.763	1.67e-04***
Δ Intercept (2-3× poverty)	3.850	0.457	8.412	0.000***
Δ Intercept (Above 3× poverty)	2.375	0.435	5.451	5.00e-08***
Δ Slope (1-2× poverty)	0.021	0.009	2.179	0.029
Δ Slope (2-3× poverty)	0.007	0.011	0.656	0.511
Δ Slope (Above 3× poverty)	0.025	0.010	2.408	0.016**
Above 3× poverty at $t+1$				
Intercept(Below 1× poverty time t)	-0.602	0.288	-2.089	0.036
Slope(Below 1× poverty)	-0.027	0.007	-3.881	1.03e-04***
Δ Intercept (1-2× poverty)	1.012	0.398	2.544	0.010
Δ Intercept (2-3× poverty)	3.647	0.433	8.419	0.000
Δ Intercept (Above 3× poverty)	3.723	0.388	9.577	0.000
Δ Slope (1-2× poverty)	0.011	0.009	1.158	0.246
Δ Slope (2-3× poverty)	0.000	0.010	-0.003	0.997
Δ Slope (Above 3× poverty)	0.045	0.009	4.765	1.88e-06***

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

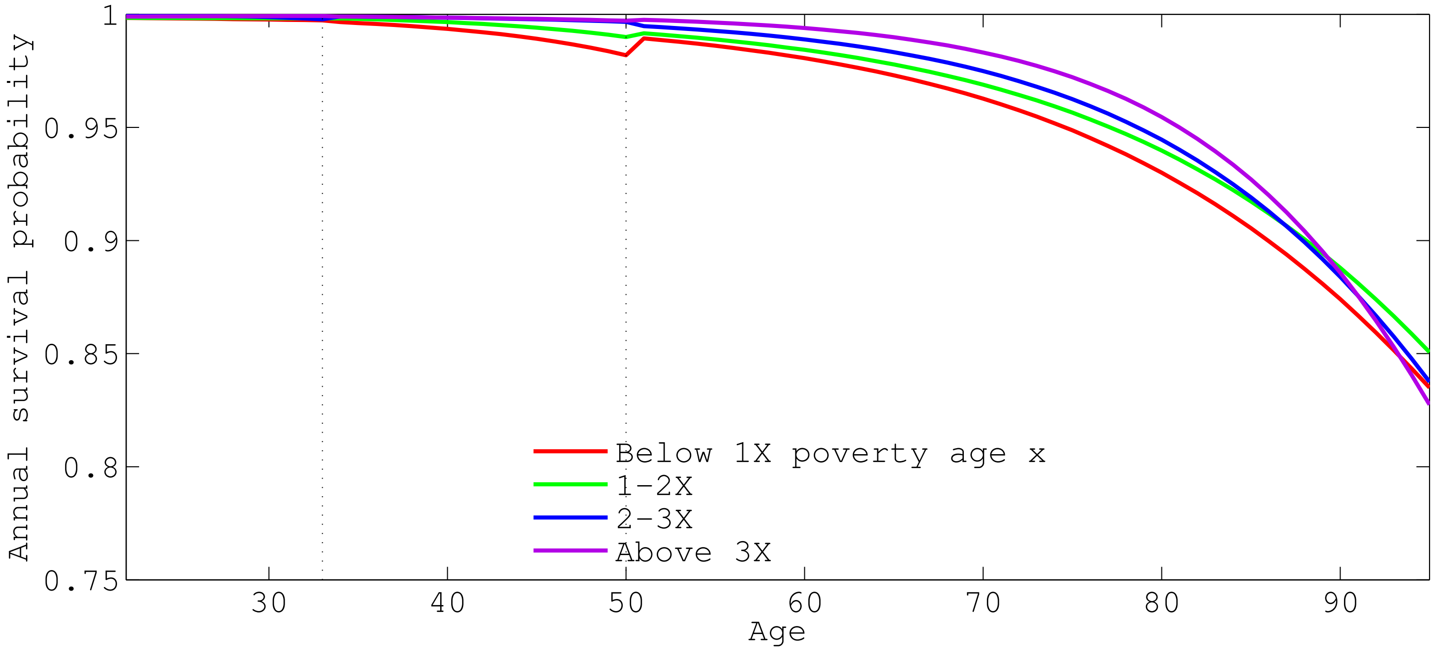


Figure 4.1: Age-specific annual survival probabilities (the number alive at age $x+1$/number alive at age x by income category at age x (below 1× 'offical' poverty = red, 1-2× 'offical' poverty = green, 2-3× 'offical' poverty = blue, >3× 'offical' poverty = purple). The dashed vertical lines represent the seam survival probability values calcualted from NLSY79 pre-1994, NLSY79 post-1994 and HRS (see tables 4.2, 4.4, and 4.6) are joined (at ages 33 and 50).

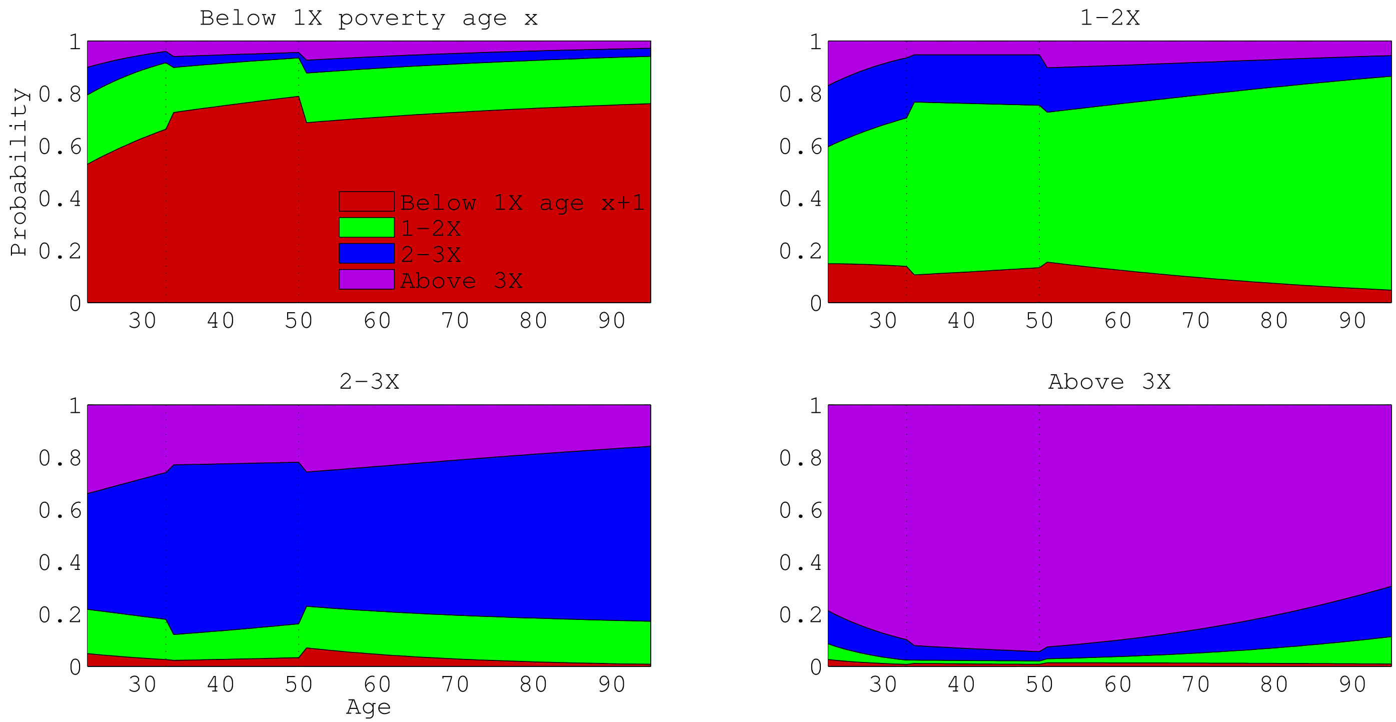
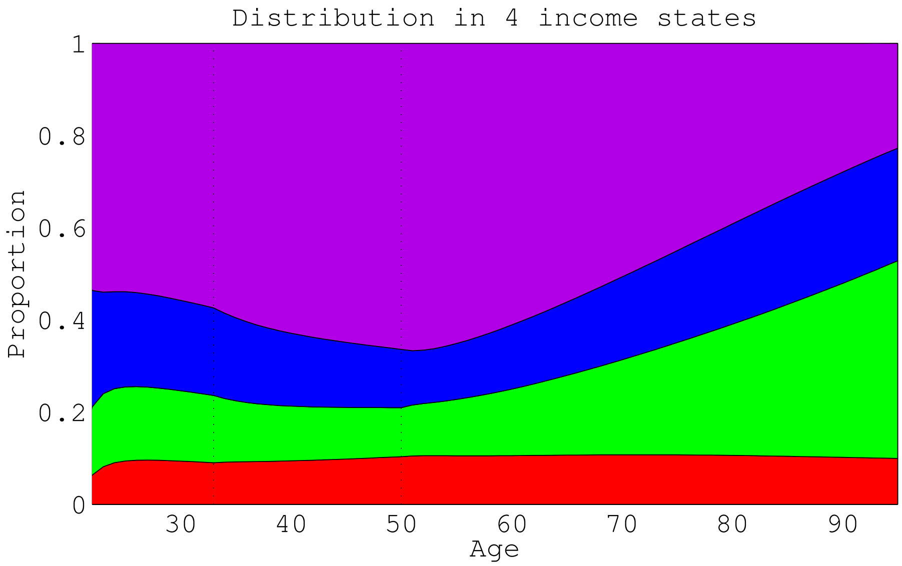
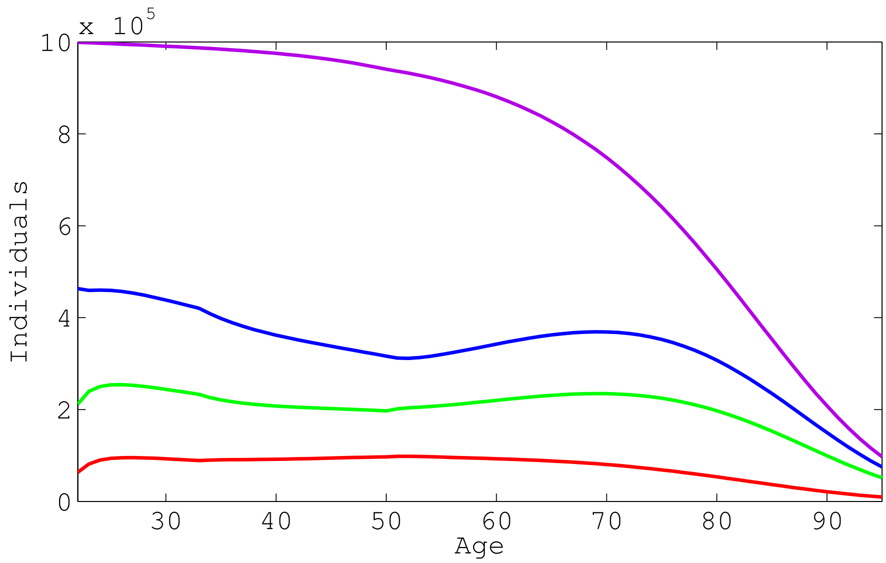


Figure 4.2: Age-specific conditional transition probabilities. One year transition probabilities into future income categories, at age $x+1$ (<1× 'offical' poverty = red, 1-2× 'offical' poverty = green, 2-3× 'offical' poverty = blue, >3× 'offical' poverty = purple, respectively) for those in a given income category at age x, (panel a, b, c, and d for the same four income categories, respectively, conditional on survival (they sum to 1 at each age). In each panel the widest area plot (the greatest probability) corresponds to the probability of staying in the current state at $x+1$. The dashed vertical lines represent the seam where transition probability values calculated from NLSY79 pre-1994, NLSY79 post-1994 and HRS (see tables 4.3, 4.5, and 4.7) are joined (at ages 33 and 50).



(a)



(b)

Figure 4.3: Proportion of individuals in each income category from cohort projection of 100,000 individuals. (a) is the proportion (number in each state/total number alive at each age x) in each state. (b) is the total number of individuals alive in each state at each age. Colors correspond to income categories at each age (red $<1\times$poverty, green 1-2$\times$poverty, blue 2-3$\times$poverty, purple $>3\times$poverty)

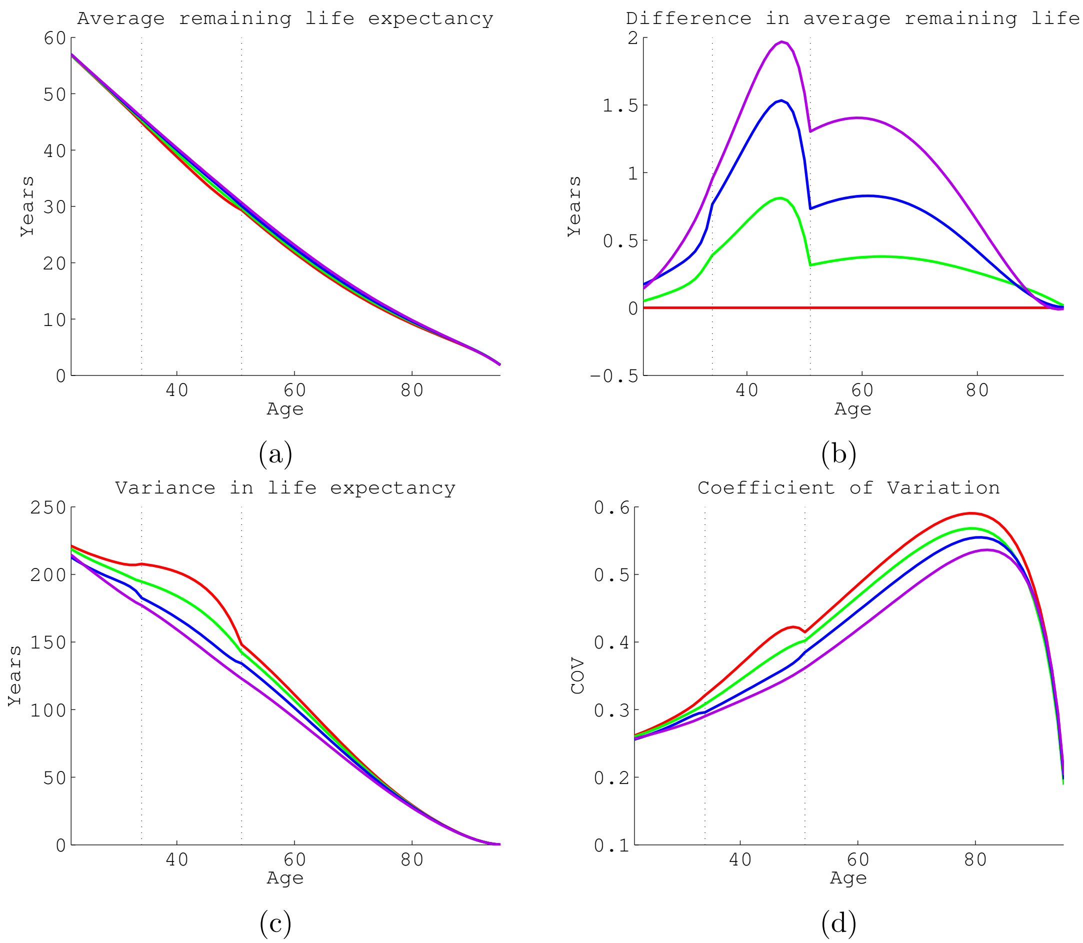


Figure 4.4: The average remaining life expectancy and its variance. (a) average remaining life expectancy, conditional on surviving to age x, based on income state at each age x (red $<1\times$poverty, green 1-2$\times$poverty, blue 2-3$\times$poverty, purple $>3\times$poverty, respectively). (b) difference in average remaining life expectancy between each state and the $<1\times$poverty state. (c) variance in average remaining life expectancy. Variance in life expectancy corresponds to the spread of individual lifetime trajectories. (d) Coefficient of Variation (ratio of the standard deviation to the mean).

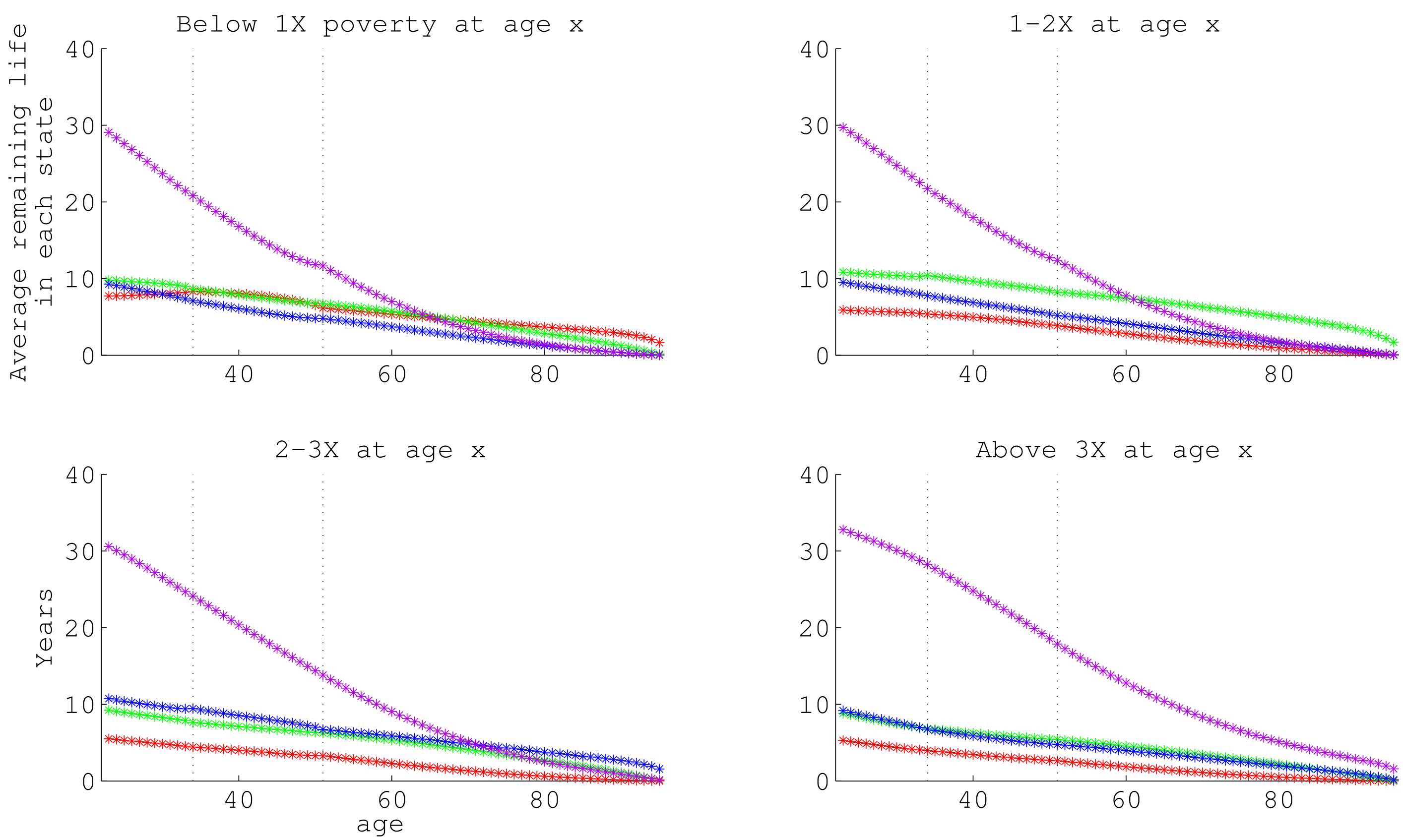


Figure 4.5: The average remaining number of years in each income category. a, b, c, and d are respectively the four possible current income states (<1×poverty, 1-2×poverty, 2-3×poverty, and >3×poverty at age x, respectively). For each current income state at each age x, the average remaining number of years in each of the four income levels is indicated by the four lines (red, green, blue and purple, respectively).

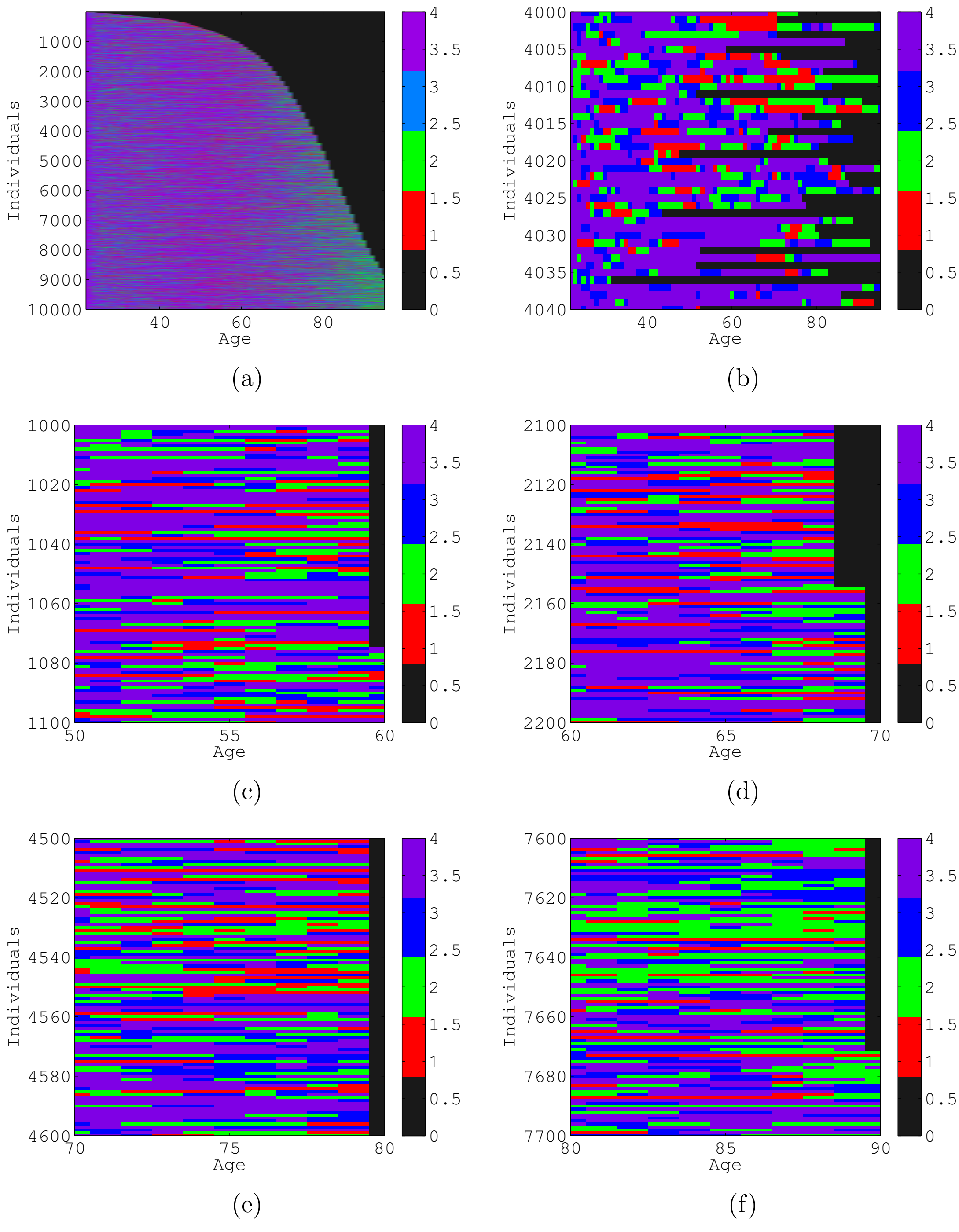


Figure 4.6: Cohort simulation of 10,000 individuals lifetime trajectories. (a) state transitions for each of 10,000 simulated individuals, all starting in the $>3\times$poverty state, arranged in order of lifespan. The state at each age is color-coded: red $<1\times$poverty, green 1-2$\times$poverty, blue 2-3$\times$poverty, purple $>3\times$poverty, and black is death. (b) snapshot of a random sample of individuals between the ages of 22-95, not ordered by lifespan. (c), (d), (e), and (f) snapshots of 100 individuals 10 years prior to their death at 60, 70, 80, and 90, respectively.

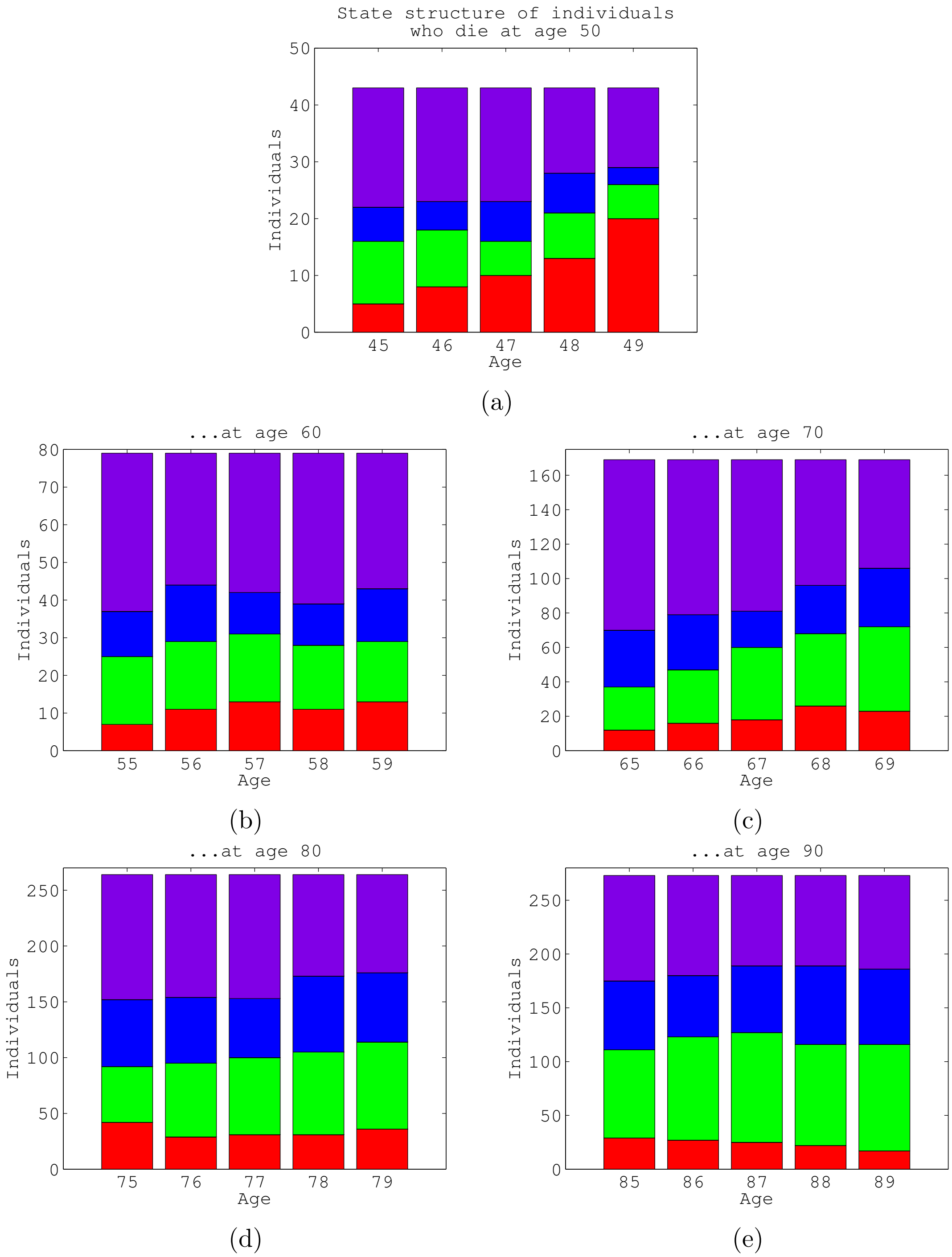
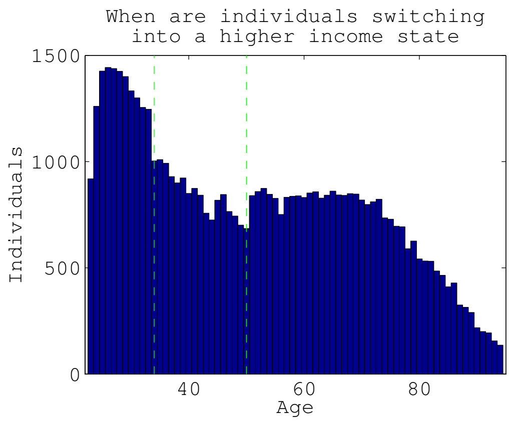
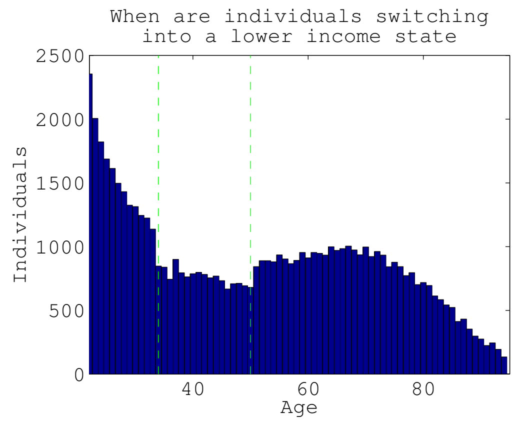


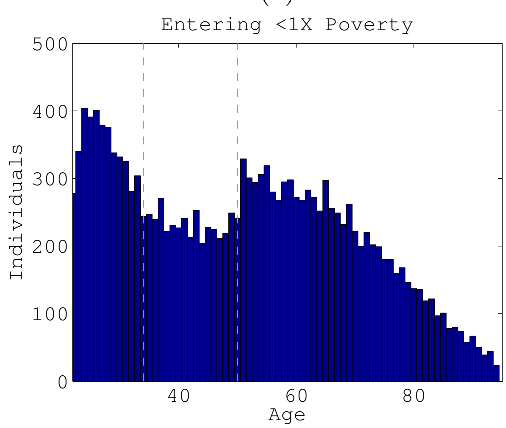
Figure 4.7: State distribution of simulated individuals 5 years prior to their death. Simulated individuals grouped by those who experienced death at ages 50, 60, 70, 80, or 90. (Red is <1×poverty, green is 1-2×poverty, blue is 2-3×poverty, and purple is >3×poverty). (a) The 43 simulated individuals (out of 10,000) who died between ages 49-50. (b) The 79 people who died before age 60. (c) The 169 people who died before age 70. (d) The 264 people who died before age 80. (e) The 273 people who died before age 80.


When are individuals switching
into a higher income state
Individuals
Age
0
500
1000
1500
40
60
80

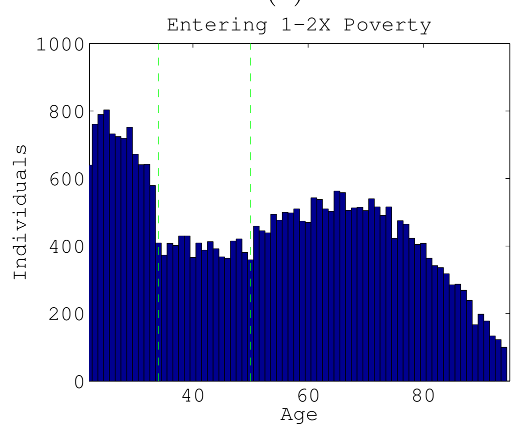
(a)


When are individuals switching
into a lower income state
Individuals
Age
0
500
1000
1500
2000
2500
40
60
80

(b)


Entering <1X Poverty
Individuals
Age
0
100
200
300
400
500
40
60
80

(c)


Entering 1-2X Poverty
Individuals
Age
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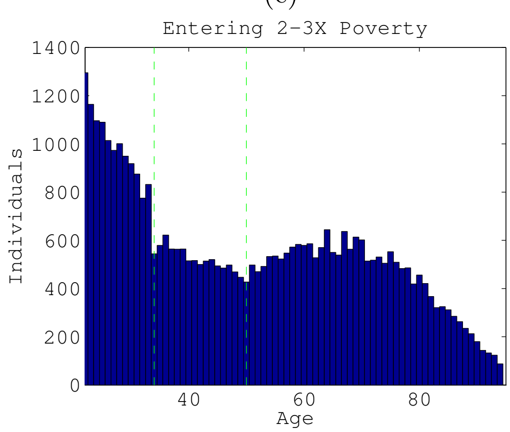
80

(d)


Entering 2-3X Poverty
Individuals
Age
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400
600
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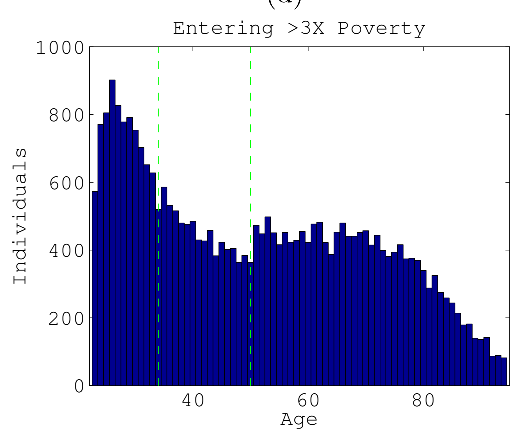
1000
1200
1400
40
60
80

(e)


Entering >3X Poverty
Individuals
Age
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200
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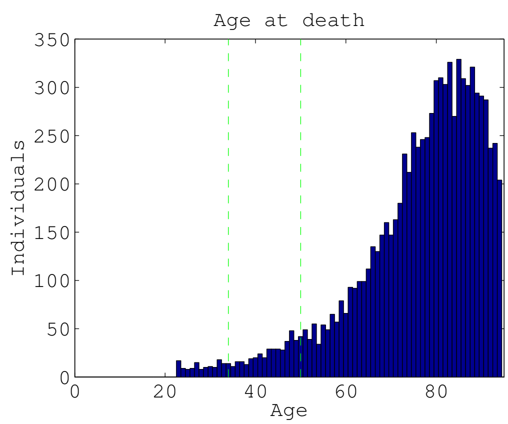
80

(f)



(g)

Figure 4.8: Distributions of income state switching from simulations. (a) Distribution of the ages individuals switch into a lower income state. (b) Distribution of the ages individuals switch into a higher income state. (c), (d), (e), and (f): Distributions of the ages individuals switch into the $<1\times$poverty, 1-2$\times$poverty, 2-3$\times$poverty, and $>3\times$poverty state, respectively. (g) Distribution of the age at death.

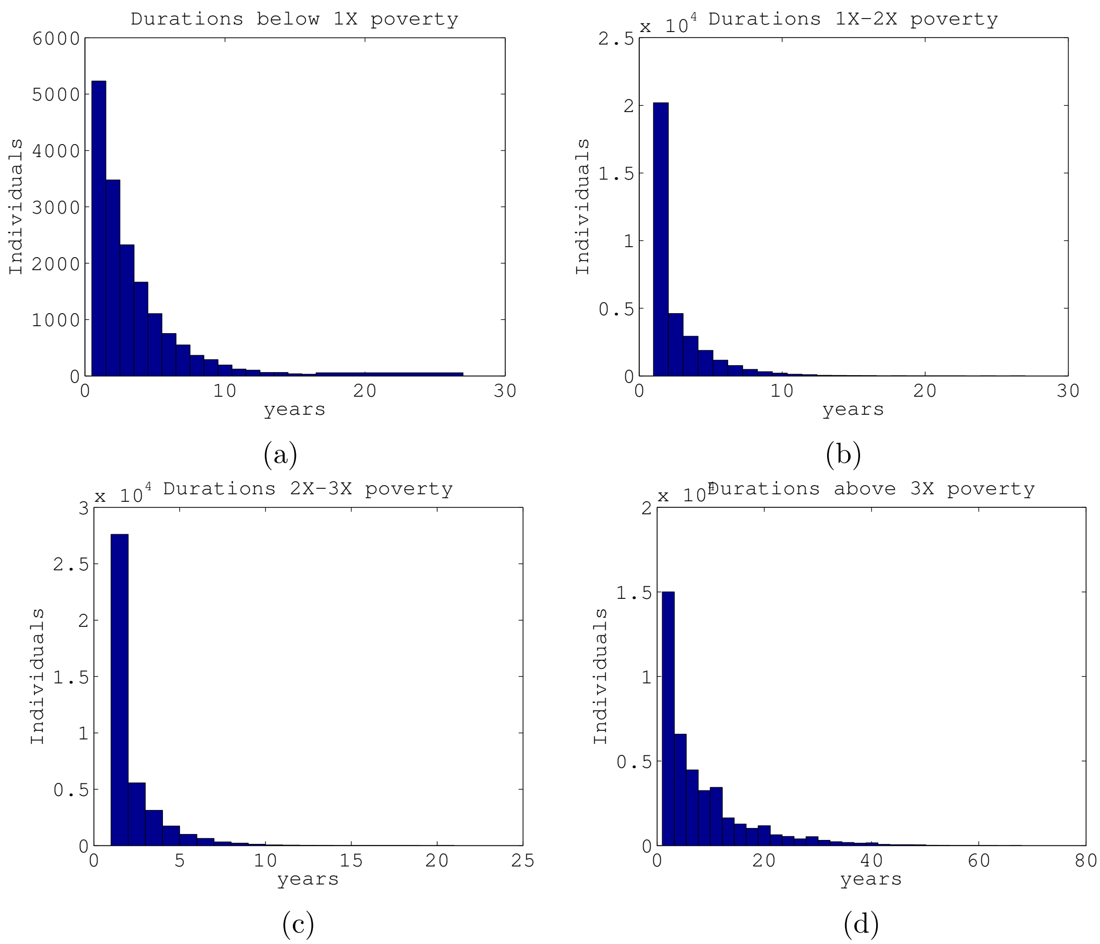
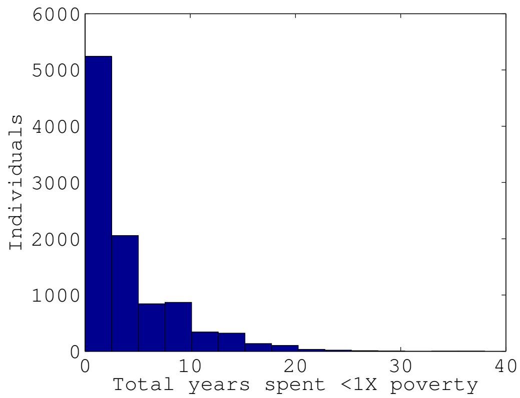
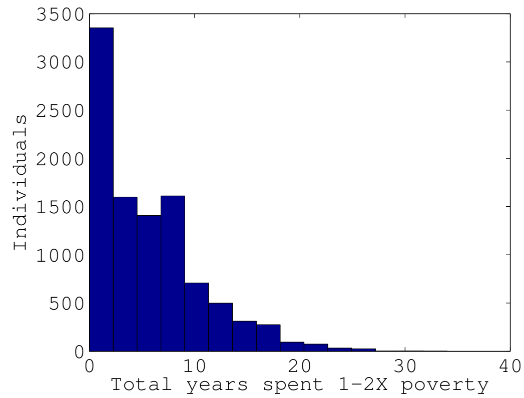


Figure 4.9: Distributions of continuous duration or 'spells' in each income state. The distributions of continuous years spent in (a) <1×poverty, (b) 1-2×poverty, (c) 2-3×poverty, and (d) >3×poverty state, having at least spent 1 year in each respective state.



(a)



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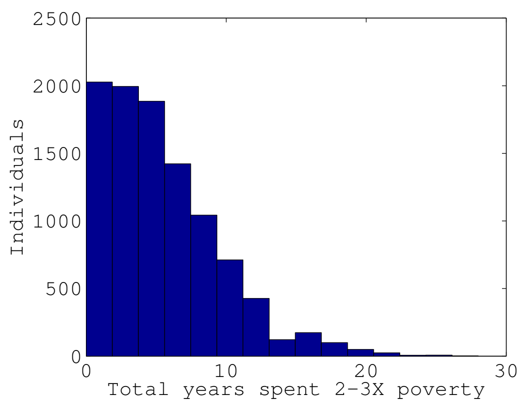
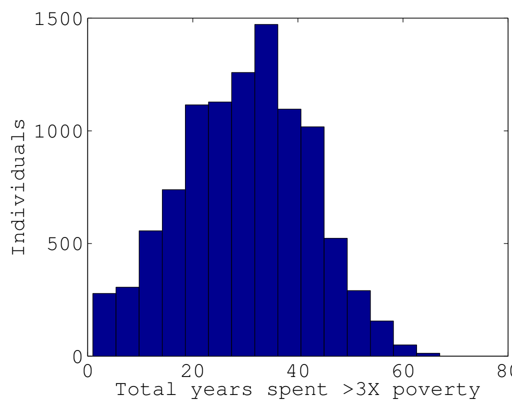


(c)



(d)

Figure 4.10: Distributions of net years spent in each income state. The distribution of total years over individuals life-courses spent in each income state: (a) <1×poverty, (b) 1-2×poverty, (c) 2-3×poverty, and (d) >3×poverty state.

CHAPTER 5

An integrative measure of dynamic heterogeneity

5.1 Entropy

Entropy describes the rate of diversification of trajectories conditional on survival and as in Tuljapurkar and Steiner (2010), can be estimated as:

$$H = -\sum_{i=1}^{K}\sum_{j=1}^{K} \pi_i \boldsymbol{\psi_{ij}} \log \boldsymbol{\psi_{ij}} \tag{5.1}$$

We calculate entropy, H, for each age. K is the number of possible states (2 for chapters two and three, 4 for chapter four). $\boldsymbol{\psi}$ is the cumulative stage transition matrix conditional on survival (similar to cumulative $\mathbf{Q}$ matrices but without being weighted by the probability of survival). And π is the stable stage distribution which equals the dominant eigenvector of $\boldsymbol{\psi}$. Since for each case the greatest value of entropy is $\log(K)$, we can normalize H by dividing by $\log(K)$.

Tuljapurkar and Steiner (2010) investigated the entropy, the quantitative measures of dynamic heterogeneity, across 21 different species and found that there was no correlation between entropy and mean age at death for the species. If we consider our results in the previous chapters, this finding is intuitive. In each chapter the same empirical data were used, and similar pooled cohorts were analyzed to create simulated cohorts experiencing different types of dynamic heterogeneity. The extent to which age-specific dynamic heterogeneity occurred depended upon which *dynamic state* was under analysis. For instance in chapter 3 we considered exactly the same cohort but did separate analysis for each of three different poverty thresholds, each defining a binary state (poor vs not poor). All three analyses had the same average age of death but cohorts exhibited different age-patterns of dynamic heterogeneity (fig.5.2 (where we use the same measure of entropy , H, as Tuljapurkar and Steiner (2010) and Tuljapurkar et al. (2009) except that our H is a function of age).

In the analysis of marital status (ch. 2) we considered the dynamic heterogeneity of entering and exiting marriage (the dynamic state) throughout the life course and compared males and females (fixed states), we found the well-documented gender discrepancy in average age at death. Yet this is despite the fact that males on average spend more of their lifetime in the state with the higher survival advantage, the married state (although the survival advantage is not greater than annual survival of married or unmarried females). The entropy value differs for men and women, with women's dynamic heterogeneity decreasing sharply at the end of the lifespan (as many become unmarried)(fig.5.1). In all chapters we found a crossover in survival advantage at the end of life where more robust individuals in the generally lower survival (unmarried or lower income) states became more abundant. In the analysis of multiple income states (ch. 4) our simulations and entropy at each age (fig. 5.3) show the greatest dynamic heterogeneity in income state towards the end of life, for those who successfully make it to age 90 for instance, and the near poverty state (1-2× poverty) had the greatest survival advantage at these late ages.

In all previous analysis we found that individuals were most abundant in the states with the greatest one-period survival during periods of greatest discrepancy among states. For instance, we found the greatest discrepancy in average remaining life expectancy to occur during mid-ages, and the greatest proportion of individuals were in the higher survival state at these ages as well.

All of these observations must be interpretted in light of the disclaimer that the state-by-age models and analysis techniques create mechanistic models; models that do not capture all of the complexity that determine lifespan or even variance in lifespan. Although the models presented here closely consider age-patterns of entry and exit rates for select states, they do not allow for determining causation, or environmental reasons for specific transition patterns. Furthermore there are infinite internal and external dynamic states with survival ramifications that individuals may be going through on time scales much smaller than a year, not to mention 'quasi-dynamic' states, such as education (where higher educational levels can be achieved at any age, but one cannot return to previous lower education levels), with documented survival ramifications (Kunst and Mackenbach, 1994). Then, of course, there are fixed states such as region of birth that have documented survival ramifications as well (Krieger et al., 2003).

However, the approach here proved to be a powerful tool in examining inequalities or disparities of survival, life expectancy, and variance of life expectancy between particular dynamic states. One definition of health disparities is "[mortality] differences that occur by gender, race or ethnicity, education or income, disability, geographic location, or sexual orientation" and Adler and Rehkopf (2008) makes a point of further differentiating social disparities and biological differences (they also mention there are over 11 different definitions of health disparities, as there is not yet consensus in the literature). Either way, investigating mortality disparities between dynamic states throughout the life course is a novel addition to literature that aims to understand and limit disparities and inequalities in mortality.

For instance, perhaps mid-ages have the most income-state based mortality discrepancies because there are fewer government programs available at those ages to help those in lower income states, or great disparities exist because of government programs already in place for younger and older aged lower income state individuals (such as Social Security and Medicare) simply because they are more abundant at those ages (Sandoval et al., 2009), and many changes in age-specific discrepancies across the life-course can be due to low income individuals already having experienced mortality at young ages (also pointed out by Adler and Rehkopf (2008)).

Dynamic states are the most complex and interesting states to consider, and this dissertation is in no sense exhaustive on the topic. We were able to qualitatively view the dynamic heterogeneity at each age by visualizing the simulated cohort dynamics as it entered and exited each state and quantitatively view the dynamic heterogeneity with histograms of transitioning summaries and entropy calculations presented in this chapter. We compared how the same cohorts experienced different dynamic heterogeneity based on differently defined dynamic states. While we do not know the underlying mechanisms causing specific transition probabilities at each age, we do know that the transitioning and survival rates can drive the demographic structure and inequalities within the cohort.

5.2 Conclusion

Furthermore, we were able to answer our research questions and directly relate individual annual survival and transitioning probabilities to cohort state structure, life expectancy, dynamic heterogeneity, and summary switching ages for each chapter. As discussed in previous chapters, combining analyses of data from two data sets, NLSY79 and HRS to estimate our probabilities for the state-by-age model yielded results consistent with earlier literature.

In our analysis of marital status (ch. 2) we asked how does the survival advantage of being married change with age? How does the probability of becoming married or staying married change with age? At which ages is marriage more advantageous for males and females? How does the survival advantageous appear when we consider marital state as a fixed (rather than dynamic) state?

We found that at young ages the survival advantage in married state was negligible. At mid to late ages it was considerable, and at late old ages, it was disadvantageous. The probability of staying and becoming married decreases with age. Married people live longer than unmarried people, and the benefit is enhanced for males at mid-ages. At early ages more women entered marriage than men, while at late ages more women exited marriage than men and women had higher variance in total years unmarried. The results of a model in which state became fixed at some particular age lead to conflicting results, in contrast to our dynamic model.

In our analyses of binary poverty states we considered three distinct income thresholds as poverty thresholds (ch. 3) and we asked: How often in their life course do individuals cross above and below each threshold? What are the ramifications of being above or below a threshold in terms of probability of survival between one age and the next? How does being above and below a particular threshold income level change the expected fate of a cohort, such as the remaining life expectancy and variance in remaining life expectancy? Does dynamic heterogeneity and the transience across income thresholds affect the cohort dynamics?

We found that the number of individuals in poverty, (below the $1\times$ poverty threshold) is almost constant with age, perhaps since an almost equal number of people exit and enter poverty after age 33. We also found that for those below each threshold, the one-period probability of exiting those states declines with age and those below $1\times$ poverty had the highest annual probability of exiting at each age, but the sharpest decline with age. In general, as threshold increased, so did the number of individuals switching states. For most ages individuals in the higher income state had the greatest probability of surviving from one year to the next for mid-ages to about age 80. The advantage is greatest between those above and below the $1\times$ poverty threshold when compared to those above and below $2\times$ or $3\times$ poverty. The highest income-state inequality in life-expectancy, and variance occurred at mid-ages (since at mid-ages the income state is less transient) and fewer individuals are in the below threshold income states between ages 40-60. Individuals below the income thresholds have the highest variance, meaning they have a greater range of possibilities in life trajectory. Those above threshold have less variance, meaning individuals will more consistently reach their higher average remaining life expectancy. We found that dynamic heterogeneity in income state and the transience of income thresholds is associated with income-related mortality disparities (less transience, especially of the higher income states, more disparities). As noted above we can not establish the direction of causality but we agree with Gillespie et al. (2014), that focusing on mid-ages to decrease income-related health disparities could help decrease lifespan inequality.

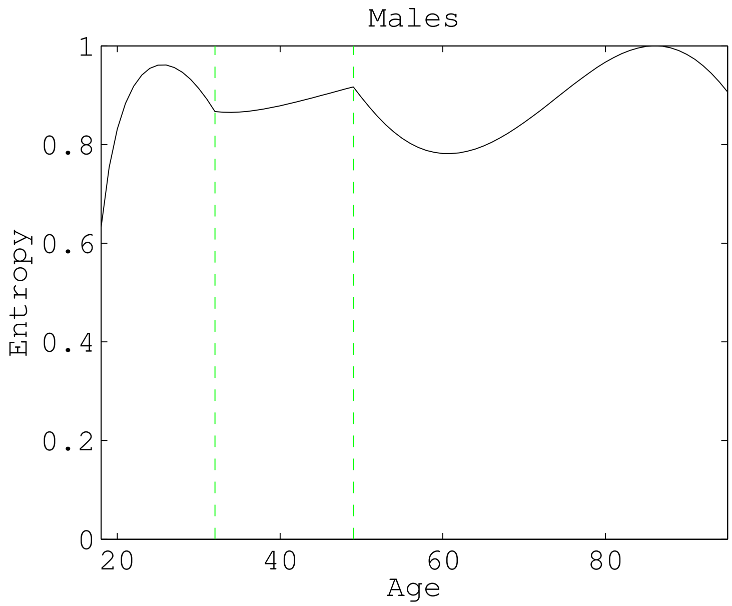
In our analyses of muliple income states (ch. 4) we considered four distinct income categories and asked the following research questions: What is the probability of entering or exiting an income state from one age to the next? What are the annual survival ramifications of being in one of four particular income states at each age? What is the average remaining life expectancy and variance in life expectancy at each age for each income state? In our Markovian framework, what is the average duration and net years spent in each income state? And how do all these results differ between income states?

In general we found that each income state had the highest annual probability of staying in the same state at each age, with the next highest transition being to move to higher income states. For most ages the highest income states had the highest probability of surviving from one year to the next until about age 86 when the order of the income states did not equate to the order of survival advantage. The greatest advantage in average remaining life expectancy between consecutive states is for those moving from $<1\times$ to 1-2$\times$ poverty at ages 32-49. The largest discrepancy in average remaining life expectancy and its variance between all states and the poverty state occurred at mid-ages (40-60). And the fewest individuals were in the lower income states between these ages. Our calculated average duration or 'spell' in 1$\times$poverty (across the life course) was 3.2 years, 1-2$\times$poverty had an average spell of 2.7 years, 2-3$\times$poverty and $>3\times$poverty had average durations of 2.3 and 8.2 years, respectively. While the average net time (from ages 22-95) in 1$\times$poverty was

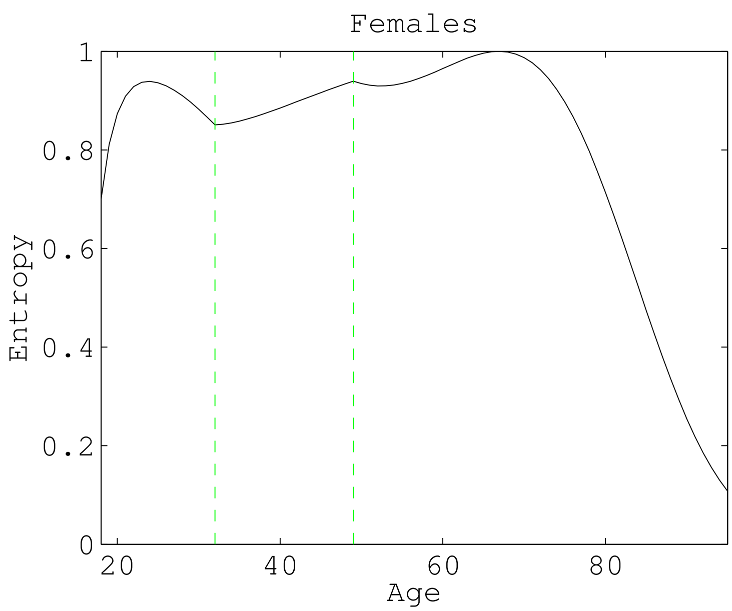
3.8 years, in 1-2$\times$poverty 5.66 years, in 2-3$\times$poverty and $>3\times$poverty 5.3 and 29.9 years, respectively.

In conclusion, we find that over the life course, dynamic heterogeneity is lowest (in the particular dynamic states we investigated) at mid-ages. And the greatest dynamic state-based mortality inequalities and discrepancies in life expectancy occur at these same ages. Perhaps because these are the ages of the 'working age population', who, for various reasons outside the scope of our model (see Backlund et al. (1996)), have greater rates of annual stasis in the more advantageous state. We also found that despite out Markovian framework, which did not incorporate individual histories or fixed states (such as innate biological factors, race or ethnic group, etc.), we found (dynamic) state based inequality, disparities and heterogeneity in our cohorts. We were able to directly relate these inequalities to our annual survival and transition probabilities, suggesting that these probabilities are possible underlying forces of inequality of lifespan.

Using regression analysis of NLSY and HRS and our state-by-age-matrix model we were able to investigate the relationship between individual annual entry and exit rates into particular dynamic states and age based annual survival, cohort dynamic heterogeneity, cohort state structure, and individual trajectories. In so doing, we answered our above stated research questions and extended the literature on multistate methods of analysis.

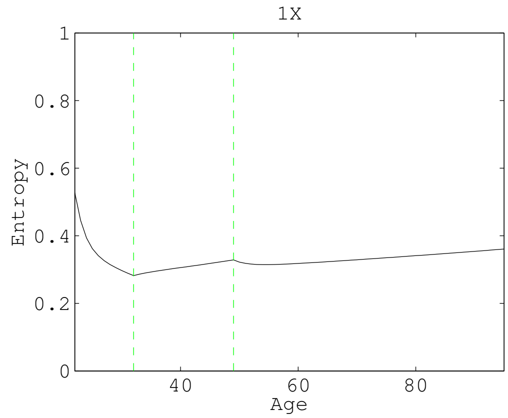


(a)

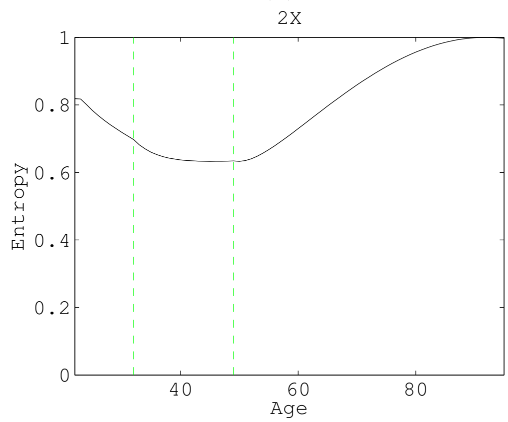


(b)

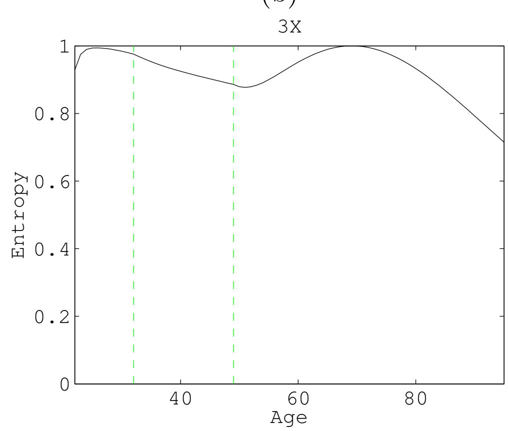
Figure 5.1: Entropy for cohorts classified by marital status and gender: married and unmarried (a) males and (b) females at each age. Entropy calculation is normalized and conditional on survival (see equation 5.1).



(a)



(b)



(c)

Figure 5.2: Entropy for cohorts classified by income thresholds of either (a) 1×, (b) 2×, or (c) 3× the poverty threshold at each age. Entropy calculation is normalized (divided by $\log 2$ for each age) and conditional on survival (see equation 5.1).

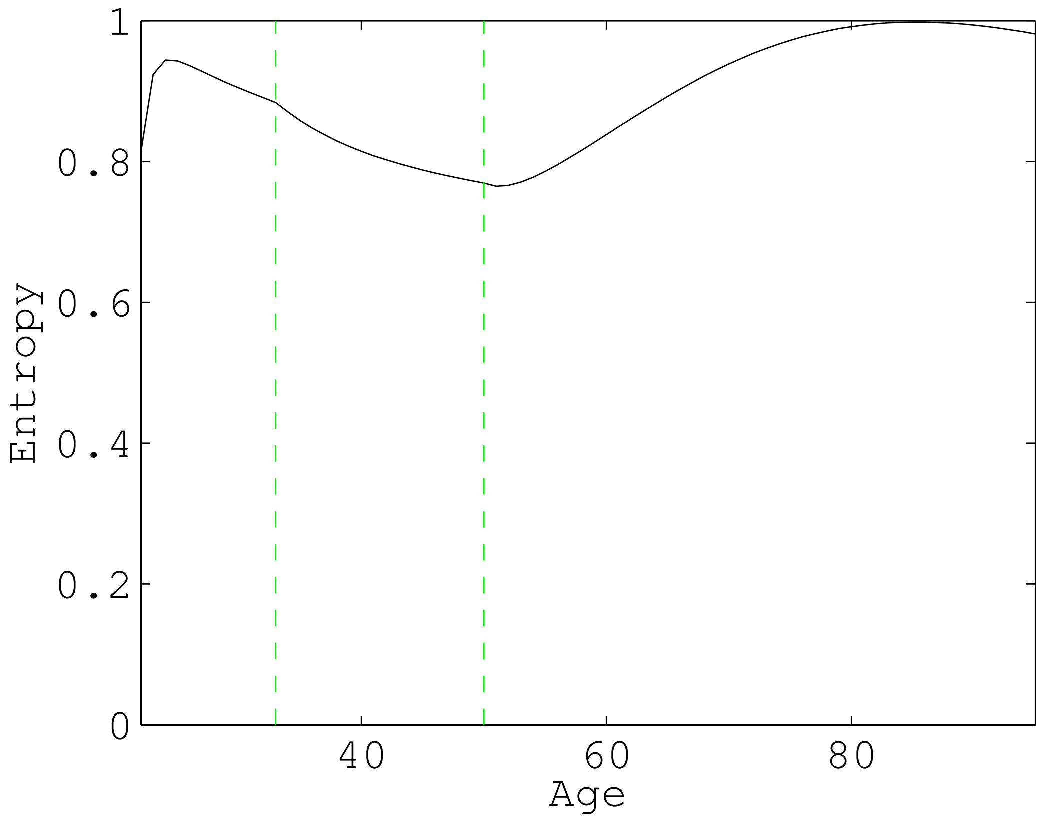


Figure 5.3: Entropy for cohorts classified by income category, <1×poverty, 1-2×poverty, 2-3×poverty, and >3×poverty. Entropy calculations is normalized (divided by log(4) for each age) and conditional on survival (see equation 5.1).

APPENDIX A

A.1 HRS cohorts and variables

The 6 cohorts include: Initial HRS cohort, born 1931 to 1941, first interviewed in 1992 and then for the rest of the survey. AHEAD cohort, born before 1924, and first interviewed in 1993 and subsequently 1995, 1998, and then for the remainder of the survey. Children of Depression (CODA) cohort, born 1924 to 1930, first interviewed in 1998 and subsequently for the rest of the survey. War Baby (WB) cohort, born 1942 to 1947. This cohort was also first interviewed in 1998 and subsequently every two years. Early Baby Boomer (EBB) cohort, born 1948 to 1953. This cohort was first interviewed in 2004. Mid Baby Boomer (MBB) cohort, born 1954 to 1959. This cohort was first interviewed in 2010. Individuals are classified as alive or dead based on the RAND HRS `rwiwstat` variable. Individual age at each wave are calculated from the age at the end of their interview, `rwagey_e`.

A.2 NLSY79 variables

Interviews were performed every year from 1979 to 1994, and then every two years from 1994-2012. At the first interview in 1979 respondents were between the ages of

14 and 22. The variable `rni` at each interview year was used to establish whether the respondent was deceased. The age at interview variable, `ageatint`, was used to represent age at end of interview.

A.3 One-period probability adjustments

The time between interviews for HRS is normally distributed about 23.8 months (fig.A.2). NLSY is bi-normally distributed about 12.4 and 24.1 months (A.2), since after 1994 interviews were performed every 2 years as opposed to every one year. For HRS, NLSY-pre-1994, and NLSY-post-1994 we calculate regression parameters (specified in each chapters tables). The parameters are then logit transformed to calculate probability of survival between t and $t+i$, and probability of transitioning between t and $t+i$. We then adjust i to be 12 months, a one year period.

To calculate our one-period transition parameter values, $t(x)_{21}$ and $t(x)_{22}$, from the logit transformed regression coefficients we use the following equation:

$$\begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix}^{12/i}$$

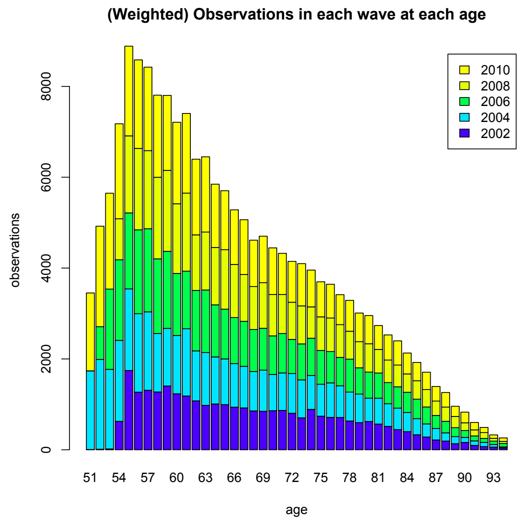
Where r_{21} and r_{22} are the logit transformed regression parameters and $r_{11} = 1 - r_{21}$ and $r_{12} = 1 - r_{22}$. i is either 12.4, 24.1, or 23.8 depending on the data-set. Note, every variable is a function of age, x, but we leave the x out here for clarity.

To calculate our one-year survival parameter values, $s(x)_1$ and $s(x)_2$, from the logit transformed regression coefficients, c_1 and c_2, we estimate with the following equations:

$$c_1 = s_1^2 t_{11} + s_1 s_2 t_{21}$$

$$c_2 = s_2^2 t_{22} + s_2 s_1 t_{12}$$

In this case, we estimate that i = 2 and use this formula for NLSY-post-1994 and HRS. We use the transition probabilities, t_{11} etc., calculated from above. Here c_1, the logit transformed regression coefficient, is the probability of survival between t and $t + i \approx t + 2$, for those below threshold. And c_2 is the same for those above income threshold. Every variable is a function of age x. We estimate that for NLSY-pre-1994 $c_1 = s_1$.



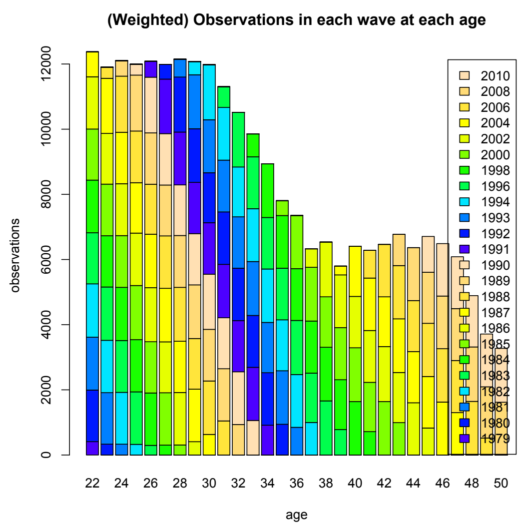


Figure A.1: **Age distribution/structure of each interview wave.** For HRS (a) and NSLY79 (b).

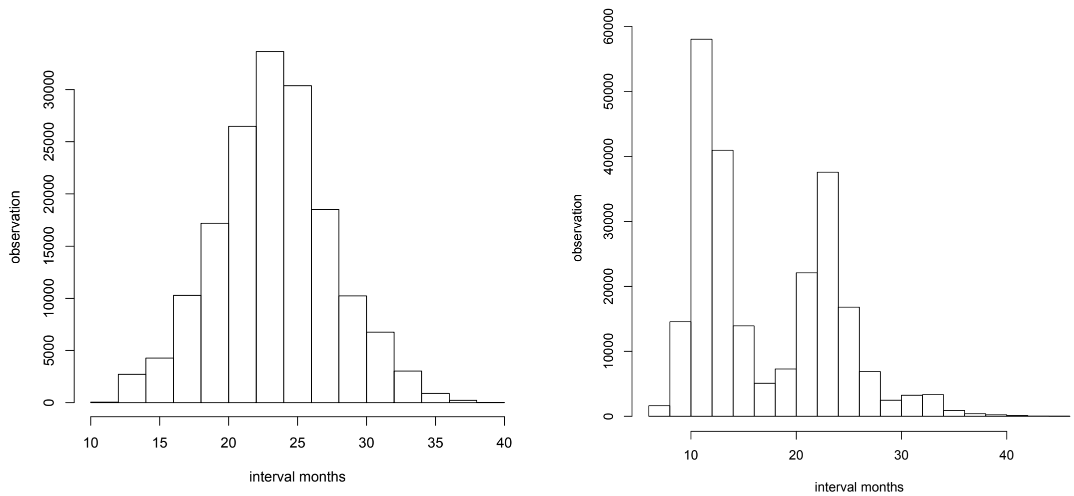


Figure A.2: **Difference in months between interviews for HRS (a) and NSLY79 (b).** HRS is normally distributed about 23.8 months. NLSY is binormally distrbuted about 12.4 and 24.1 months (pre and post-1994 interviews).

APPENDIX B

Ch.2

B.1 Ch.2 Model Theory

B.1.1 Projecting a cohort of newborns from birth onward

To see what happens to a cohort over time we must pass a cohort through the population projection matrix $\mathbf{L}$. We start with a column vector for the population at time 0, $n(0)$, where lets say $1,000,000$ individuals are born into the unmarried state (the first row represents the married state).

$$n(0) = \begin{bmatrix} 0 \\ 1,000,000 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

At the first time step we pass our newborn cohort through the matrix by multiplying $\mathbf{L}n(0)$, and then we get a new population, $n(1)$, which we then again pass through the matrix in a loop for all 100 time steps. In other words:

$$n(x+1) = \mathbf{L}n(x)$$

Which in a non-recursive form is synonymous to using **L** in equation (2.1). The sum of the column vector $n(x)$, $\sum_{i=1}^{202} n(x_i)$, represents the total number of individuals of the cohort who are alive at age x. In each $n(x)$ there are only two nonzero elements: the first value (always on the odd row index) represents the population at age x that is unmarried, $u(x) = n(x_{2k-1})$. The second value (always on the even row index) represents the population at age x that is married, $m(x) = n(x_{2k})$, here $k \epsilon [1, 2, 3..., 200, 101]$. Note that the vector $n(x)$ has dimensions 202x1 and that there are 101 time steps. Survivorship to a particular age is defined as the total population at age x divided by the total initial cohort ($n(0)$, here set at 1,000,000 individuals) (Fox, 2001).

$$\text{Survivorship to age } x = S(x) = \frac{\sum_{i=1}^{202} n(x_i)}{\sum_{i=1}^{202} n(0_i)}$$

The sum of the column vector $n(x)$ can also be implemented mathematically by simply multiplying $n(x)$ by a sum vector (a vector of 1s, sometimes denoted by **j**) of equal size. Hazard at a particular age is the instantaneous risk of mortality, the negative of the slope of the log(survivorship) (the rate of decrease in log(survivorship)), and can also be thought of as the instantaneous risk of dying. It can be calculated in a continuous time framework by (Venables and Ripley, 2002):

$$h = \log(-\log(S(x)))$$

Here we use the same equation even though we are working in a discrete time framework. These survivorship and hazard values are age specific rates for the entire cohort, comprised of both unmarried and married individuals of age x. We are also interested in the proportion of unmarried individuals to married at each age, $(u(x)/m(x))$, and the difference in average remaining life expectancy for married and unmarried at each age. In the next subsections we will show how the model can be used to look at remaining life expectancy for married and unmarried at each age and the variability among individual trajectories of members of a cohort. All of these outputs are dependent on the input survival and transition parameter values, $s_1(x)$, $s_2(x)$, $t_{21}(x)$, $t_{22}(x)$.

B.1.2 Markov chain analysis and the fundamental matrix: the average life expectancy

In the previous section the matrix **L** was used to project a cohort from age 0 to 100, this allows us to examine how the probabilities from the statistical models determine the demographic structure of the cohort. **L** can also be used in Markov chain analysis to calculate the fundamental matrix and the average individual life expectancy, which we will discuss here. Furthermore, stochastic individual life path trajectories where individuals enter and exit transient states and ultimately an absorbing state (death) can be calculated as well, and that will be discussed in the next section.

At age 0, remaining life expectancy is simply called life expectancy (the expected age at death for newborns), at every other age remaining life expectancy is the life expectancy conditional on having reached that age (the number of years an individual is expected to stay alive in the future, given that he/she survived to a particular age). For example, the remaining life expectancy at age 65 (which is conditional on survival to 65) is of special interest to the social security administration and to groups calculating health care spending for the elderly (Lubitz et al., 2003). We can use our **L** matrix to calculate the fundamental matrix (**N**) and then calculate the average individual remaining life expectancy values and related terms at each age (Caswell, 2001).

$$\text{The fundamental matrix} = \mathbf{N} = \mathbf{I} + \mathbf{L} + \mathbf{L}^2 + \ldots = \sum_{x=0}^{\infty} \mathbf{L}^x$$
$$= (\mathbf{I} - \mathbf{L})^{-1} \tag{B.1}$$

$$\text{Life expectancy} = \mathbf{j}'\mathbf{N} \tag{B.2}$$

$$\text{Variance of life expectancy} = \mathbf{j}'(2\mathbf{N}^2 - \mathbf{N}) - \mathbf{j}'(\mathbf{N}) \circ \mathbf{j}'(\mathbf{N}) \tag{B.3}$$

$$\text{Standard deviation of life expectancy} = (\text{Variance of life expectancy})^{0.5} \tag{B.4}$$

Where $\mathbf{I}$ is an identity matrix with the same dimensions as $\mathbf{L}$. We see in eq. A.1 that the fundamental matrix is essentially a series that converges to the inverse of the identity matrix minus $\mathbf{L}$. $\mathbf{j}$ is a column vector of 1s that gives the sum of each column when its transpose is multiplied by a matrix. ($\circ$ represent the Hadamard Product which is element-wise multiplication of $\mathbf{j}'\mathbf{N}$ and $\mathbf{j}'\mathbf{N}$). The fundamental matrix informs about the expected number of visits to a transient state; each element in the series that defines $\mathbf{N}$ represents a cohort experiencing transitions at a specific age defined by $\mathbf{L}$ to the x power. The sum of the series across all possible ages (till death) is the fundamental matrix. Next we generate stochastic individual life path trajectories, as opposed to looking at the average trajectory values.

B.1.3 Markov chain analysis: Generating stochastic individual life-paths, realizations

The matrix $\mathbf{L}$ with dimensions 202×202 describes part of an individual's movement through a Markov chain but an extra absorbing state, death, must be added to complete the description (Caswell, 2001). Therefore, as in equation (2.2) one more column and row is added on to $\mathbf{L}$ which creates $\mathbf{P}$, which is column stochastic with dimensions 203×203. Here we show $\mathbf{P}$ with 2×2 state transition matrices $\mathbf{Q}(x)$ as above so note that for $\mathbf{L}$ each $0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ so the indexes of $\mathbf{Q}$ represent the number of state matrices at each age (age blocks) and each $m(x)$ is a 1×2 row vector with $m(101) = \begin{bmatrix} 1 & 1 \end{bmatrix}$.

$$\mathbf{P} = \left(\begin{array}{cccccccc|c} 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 \\ \mathbf{Q}(1) & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{Q}(2) & 0 & \dots & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{Q}(3) & \dots & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \mathbf{Q}(98) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & \mathbf{Q}(99) & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & \mathbf{Q}(100) & 0 & 0 \\ \hline m(1) & m(2) & m(3) & \dots & m(98) & m(99) & m(100) & m(101) & 1 \end{array}\right)$$

Column 102 is the death absorbing state. Each element in M, which is row 102, is the probability of death for that age and state:

$$m(x) = \begin{bmatrix} 1 - s_1(x) & 1 - s_2(x) \end{bmatrix},$$

Where $1 - s_1(x)$, as above, represents the probability of dying between x and $x + 1$ if unmarried at x, and $1 - s_2(x)$ is the same but if married at x. Since $\mathbf{P}$ is an absorbing Markov chain it can be divided into two sets (Caswell, 2001), a set of absorbing states, α, and a set of transient states, τ. The transient set is age-specific (for each x), since P is of state-by-age form:

$$\tau = \{2, 1\}_x, \qquad x\epsilon\{1, 2, ...101\}$$
$$\alpha = \{0\}$$

There is a pathway from each of the states in τ to α, since an individual can die at any state and age. The transient states are married, '2' and unmarried, '1'. We give quantitative values to the states to enable later calculations. Age itself can also be viewed as a transient state and in that case there would be 202 transient states since for 101 different ages ($\{0, 1, 2...100\}$) an individual transitions (or is born) into either 2, 1, or the absorbing state 0. Let y be a column vector of $i\epsilon[1, 203]$ representing the probability distribution of states where $0 \le y_i \le 1$ and $\sum_i y_i = 1$. So the column vector $y(0)$ represents an individual's probability at age 0 of being in state 0, 1, or 2. Then

$$y(x+1) = \mathbf{P}y(x)$$

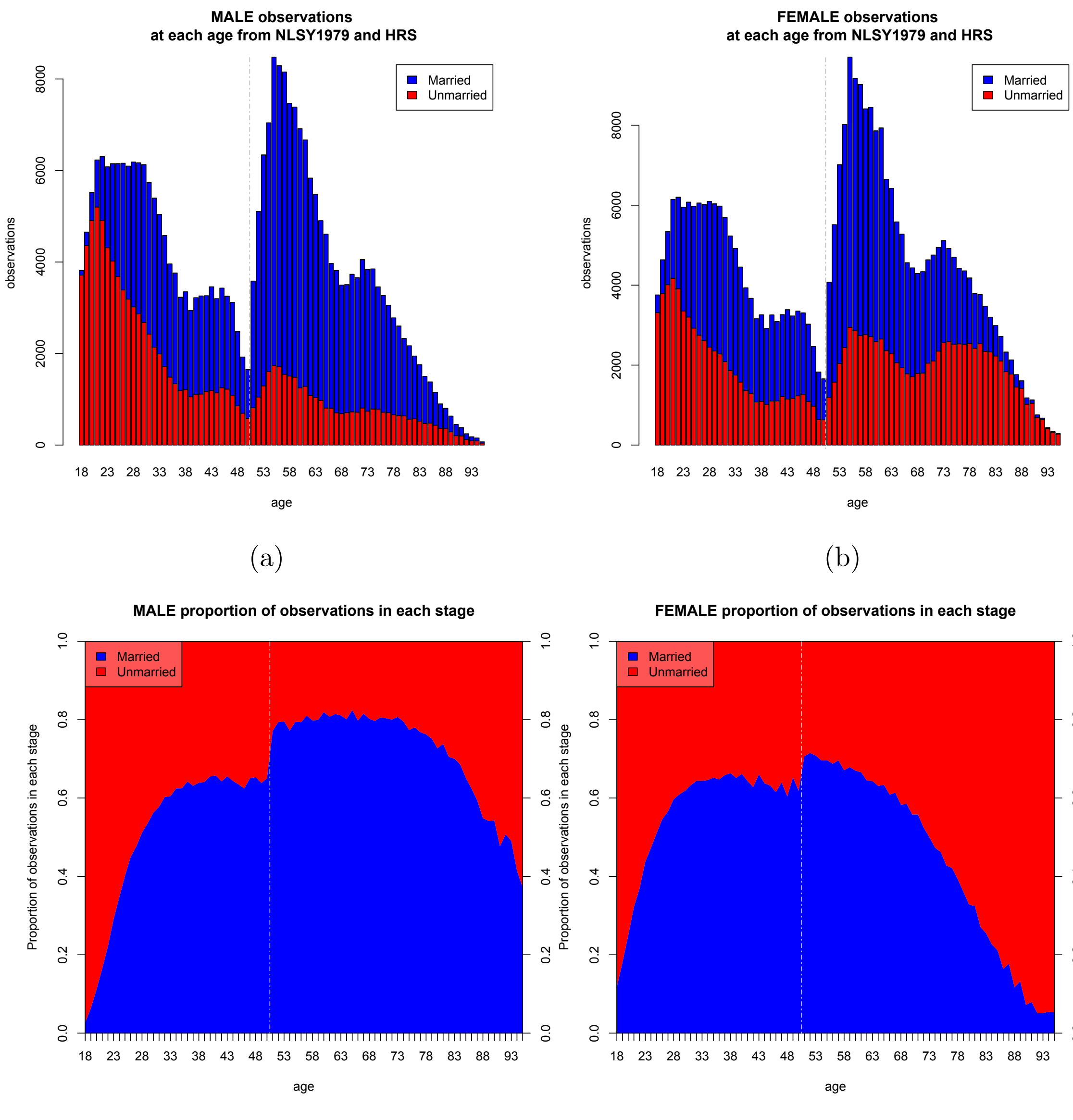
where

$$\mathbf{P}^x = \left(\begin{array}{c|c} \mathbf{L}^x & 0 \\ \hline m\sum_{n=0}^{x-1} \mathbf{L}^n & 1 \end{array} \right)$$

Since it is possible to reach the state 0, death, from every state; τ guarantees that the dominant eigenvalue of $\mathbf{L}$ is strictly less than one so $\lim_{x\to\infty}(\mathbf{L}^x) = 0$ (Caswell, 2001). This means that every individual will enter the absorbing state i.e. eventually die. The Markov chain approach gives rise to the generation of individual life paths based on the probabilities in the $\mathbf{L}$ matrix. The individual life paths are stochastic and we can observe different types of variability among individual trajectories. For instance, variability in age at death, age at first state transition, etc.

B.2 Ch.2 Appendix Figures

See fig. B.1 for the age-specific demographic structure for both data sets. See fig. B.2 for the the response rate based on marital status at time t for both data sets.



(a) (b) (c) (d)

Figure B.1: **Distribution of married/unmarried observations across the ages, for males and females.**(a), (b) weighted repeated pooled observations for NLSY79 and HRS, combined at the dashed grey line (between age 50 and 51). (c), (d) the weighted percent of married and unmarried pooled observations at each age (the dashed line represents the joining of the NLSY79 and HRS dataset). (a), (c) and (b), (d) depict the male and female observations, respectively.

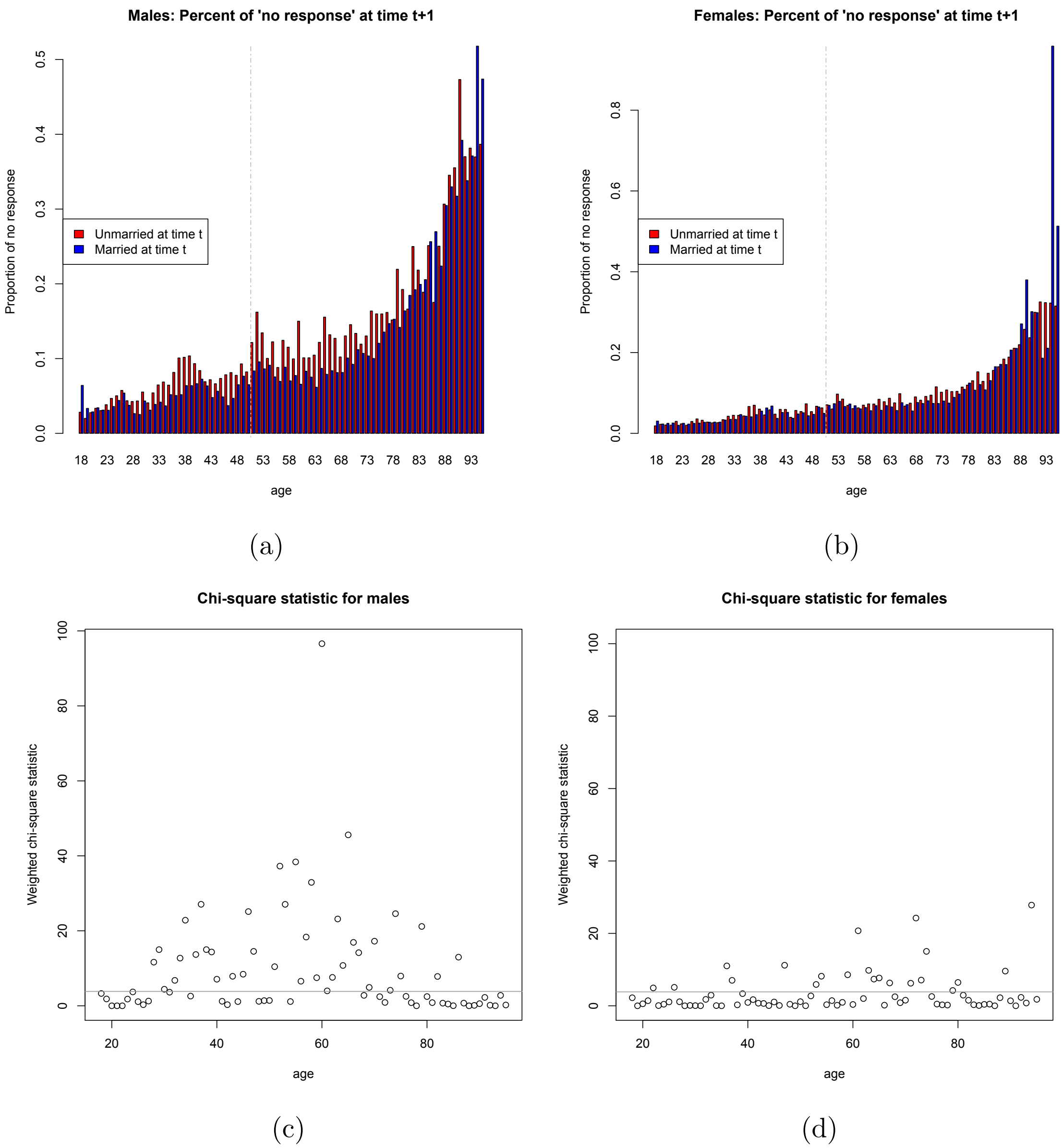


Figure B.2: **The effect of age, marital status, and gender on response rate.** (a), (b): weighted no response (i.e. missing data) at age $t+1$, for a specific state (married: blue, or unmarried: red) at age t. Confirmed deaths are excluded from observations. Panel B.2a are the males and panel B.2b are the females. The vertical dashed grey line represents where NLSY79 and HRS datasets are joined. (c), (d): respective weighted chi-square statistics for the test of significance between married and unmarried (at time t) and response (at time $t+1$). For both (B.2c) the males, and (B.2d) the females, the grey horizontal line represents the chi-square statistic at 0.05 significance. (That is a chi-square value of 3.841 for a test with 1 degree of freedom). Points below the line indicate no significant difference of response rate between martial states at time t.

APPENDIX C

Ch. 3

C.1 Ch.3 Model Theory

C.1.1 Projecting a cohort of newborns from birth onward

To see what happens to a cohort with a specific poverty threshold ($1\times$, $2\times$ or $3\times$ 'official' poverty) we must pass a cohort through the population projection matrix $\mathbf{L}$. For clarity we will refer to the two states as 'below or in poverty' and 'above poverty' although this technique is done three times for the $1\times$, $2\times$ and $3\times$ thresholds. We start with a column vector for the population at time 0, $n(0)$, where lets say $1,000,000$ individuals are born into the above poverty state (the first row represents the below poverty state).

$$n(0) = \begin{bmatrix} 0 \\ 1,000,000 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

At the first time step we pass our newborn cohort through the matrix by multiplying $\mathbf{L}n(0)$, and then we get a new population, $n(1)$, which we then again pass through the matrix in a loop for all 100 time steps. In other words:

$$n(x+1) = \mathbf{L}n(x)$$

Which in a non-recursive form is synonymous to using $\mathbf{L}$ in equation (3.1). The sum of the column vector $n(x)$, $\sum_{i=1}^{202} n(x_i)$, represents the total number of individuals of the cohort who are alive at age x. In each $n(x)$ there are only two nonzero elements: the first value (always on the odd row index) represents the population at age x that is below poverty, $a(x) = n(x_{2k-1})$. The second value (always on the even row index) represents the population at age x that is above poverty, $b(x) = n(x_{2k})$, here $k\epsilon[1, 2, 3..., 200, 101]$. Note that the vector $n(x)$ has dimensions 202x1 and that there are 101 time steps. Survivorship to a particular age is defined as the total population at age x divided by the total initial cohort ($n(0)$, here set at 1,000,000 individuals) (Fox, 2001).

$$\text{Survivorship to age } x = S(x) = \frac{\sum_{i=1}^{202} n(x_i)}{\sum_{i=1}^{202} n(0_i)}$$

The sum of the column vector $n(x)$ can also be implemented mathematically by simply multiplying $n(x)$ by a sum vector (a vector of 1s, sometimes denoted by $\mathbf{j}$) of equal size. Hazard at a particular age is the instantaneous risk of mortality, the negative of the slope of the log(survivorship) (the rate of decrease in log(survivorship)), and can also be thought of as the instantaneous risk of dying. It can be calculated in a continuous time framework by (Venables and Ripley, 2002):

$$h = \log(-\log(S(x)))$$

Here we use the same equation even though we are working in a discrete time framework. These survivorship and hazard values are age specific rates for the entire cohort, comprised of both below income threshold and above income threshold individuals of age x. We are also interested in the proportion of individuals below the income threshold at each age, and the difference in average remaining life expectancy, for instance for above poverty and below poverty individuals at each age. In the next subsections we will show how the model can be used to look at remaining life expectancy for each income state, and the variability among individual trajectories of members of a cohort. All of these outputs are dependent on the input survival and transition parameter values, $s_1(x)$, $s_2(x)$, $t_{21}(x)$, $t_{22}(x)$.

C.1.2 Markov chain analysis and the fundamental matrix: the average life expectancy

In the previous section the matrix **L** was used to project a cohort from age 0 to 100, this allows us to examine how the probabilities from the statistical models determine the demographic structure of the cohort. **L** can also be used in Markov chain analysis to calculate the fundamental matrix (Kemeny and Snell, 1976) and the average individual life expectancy in each state, which we will discuss here. Furthermore, stochastic individual life path trajectories where individuals enter and exit transient states and ultimately an absorbing state (death) can be calculated as well, and that will be discussed in the next section.

At age 0, remaining life expectancy is simply called life expectancy (the expected age at death for newborns), at every other age remaining life expectancy is the life expectancy conditional on having reached that age (the number of years an individual is expected to stay alive in the future, given that he/she survived to a particular age). For example, the remaining life expectancy at age 65 (which is conditional on survival to 65) is of special interest to the social security administration and to groups calculating health care spending for the elderly (Lubitz et al., 2003). We can use our **L** matrix to calculate the fundamental matrix (**N**) and then calculate the average individual remaining life expectancy values and related terms at each age (Caswell, 2001; Kemeny and Snell, 1976).

$$\text{The fundamental matrix} = \mathbf{N} = \mathbf{I} + \mathbf{L} + \mathbf{L}^2 + \ldots = \sum_{x=0}^{\infty} \mathbf{L}^x$$

$$= (\mathbf{I} - \mathbf{L})^{-1} \tag{C.1}$$

$$\text{Life expectancy} = \mathbf{j}'\mathbf{N} \tag{C.2}$$

$$\text{Variance of life expectancy} = \mathbf{j}'(2\mathbf{N}^2 - \mathbf{N}) - \mathbf{j}'(\mathbf{N}) \circ \mathbf{j}'(\mathbf{N}) \tag{C.3}$$

$$\text{Standard deviation of life expectancy} = (\text{Variance of life expectancy})^{0.5} \tag{C.4}$$

Where **I** is an identity matrix with the same dimensions as **L** and **j** is a column vector of 1s that gives the sum of each column when its transpose is multiplied by a matrix. ($\circ$ represent the Hadamard Product which is element-wise multiplication of $\mathbf{j}'\mathbf{N}$ and $\mathbf{j}'\mathbf{N}$). The fundamental matrix is essentially a series that converges to the inverse of the identity matrix minus **L** (eq. C.1). The fundamental matrix informs about the expected number of visits to a transient state. The sum of all of its columns yields the average remaining life expectancy conditional on survival for each age. Next we generate stochastic individual life path trajectories, as opposed to looking at the average trajectory values.

C.1.3 Markov chain analysis: Generating stochastic individual life-paths, realizations

The matrix $\mathbf{L}$ with dimensions 202×202 describes part of an individual's movement through a Markov chain but an extra absorbing state, death, must be added to complete the description (Caswell, 2001). Therefore, as in equation (3.2) one more column and row is added on to $\mathbf{L}$ which creates $\mathbf{P}$, which is column stochastic with dimensions 203 × 203. Here we show $\mathbf{P}$ with 2 × 2 state transition matrices $\mathbf{Q}(x)$ as above so note that for $\mathbf{L}$ each $0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ so the indexes of $\mathbf{Q}$ represent the number of state matrices at each age (age blocks) and each $m(x)$ is a 1×2 row vector with $m(101) = \begin{bmatrix} 1 & 1 \end{bmatrix}$.

$$\mathbf{P} = \left(\begin{array}{cccccccc|c}
0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 \\
\mathbf{Q}(1) & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 \\
0 & \mathbf{Q}(2) & 0 & \dots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \mathbf{Q}(3) & \dots & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \dots & \mathbf{Q}(98) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \dots & 0 & \mathbf{Q}(99) & 0 & 0 & 0 \\
0 & 0 & 0 & \dots & 0 & 0 & \mathbf{Q}(100) & 0 & 0 \\
\hline
m(1) & m(2) & m(3) & \dots & m(98) & m(99) & m(100) & m(101) & 1
\end{array}\right)$$

Column 102 is the death absorbing state. Each element in M, which is row 102, is the probability of death for that age and state:

$$m(x) = \begin{bmatrix} 1 - s_1(x) & 1 - s_2(x) \end{bmatrix}$$

Where $1 - s_1(x)$ represents the below income threshold state, one year probability of mortality at age x, and $1 - s_2(x)$ represents the one period probability of mortality for those in the higher income state at age x. Since $\mathbf{P}$ is an absorbing Markov chain it can be divided into two sets (Caswell, 2001), a set of absorbing states, α, and a set of transient states, τ. The transient set is age-specific (for each x), since $\mathbf{P}$ is of state-by-age form:

$$\tau = \{2, 1\}_x, \qquad x \epsilon \{1, 2, ...101\}$$
$$\alpha = \{0\}$$

There is a pathway from each of the states in τ to α, since an individual can die at any state and age. The transient states are above poverty, '2' and below poverty, '1'. We give quantitative values to the states to enable later calculations. Age itself can also be viewed as a transient state and in that case there would be 202 transient states since for 101 different ages ($\{0, 1, 2...100\}$) an individual transitions (or is born) into either 2, 1, or the absorbing state 0. Let y be a column vector of $i \epsilon [1, 203]$ representing the probability distribution of states where $0 \leq y_i \leq 1$ and $\sum_i y_i = 1$. So the column vector $y(0)$ represents an individual's probability at age 0 of being in state 0, 1, or 2. Then

$$y(x+1) = \mathbf{P} y(x)$$

where

$$\mathbf{P}^x = \left(\begin{array}{c|c} L^x & 0 \\ \hline m \sum_{n=0}^{x-1} \mathbf{L}^n & 1 \end{array} \right)$$

Since it is possible to reach the state 0, death, from every state; τ guarantees that the dominant eigenvalue of $\mathbf{L}$ is strictly less than one so $\lim_{x \to \infty}(L^x) = 0$ (Caswell, 2001). This means that every individual will enter the absorbing state i.e. eventually die. The Markov chain approach gives rise to the generation of individual life paths based on the probabilities in the **L** matrix. The individual life paths are stochastic and we can observe different types of variability among individual trajectories. For instance, variability in age at death, age at first state transition, duration in a state, etc.

C.2 Ch.3 appendix tables

Regression coefficients

Table C.1: HRS logistic regression for each poverty threshold: Dependent Variable = Survival at time t+1

Predictor	Estimate	Std. Error	t value	$Pr(>\|t\|)$
Intercept (Below 1× poverty)	**7.311**	**0.353**	**20.717**	**<2e-16 *****
Slope (Below 1× poverty)	**-0.068**	**0.004**	**-14.727**	**<2e-16 *****
Δ Intercept (Above 1× poverty)	**2.470**	**0.389**	**6.349**	**2.17e-10*****
Δ Slope (Above 1× poverty)	**-0.026**	**0.005**	**5.229**	**1.71e-07*****
Intercept (Below 2× poverty)	**7.460**	**0.233**	**32.017**	**<2e-16 *****
Slope (Below 2× poverty)	**-0.069**	**0.003**	**-22.920**	**<2e-16 *****
Δ Intercept (Above 2× poverty)	**2.804**	**0.306**	**9.166**	**<2e-16 *****
Δ Slope (Above 2× poverty)	**-0.031**	**0.004**	**-7.986**	**1.41e-15*****
Intercept (Below 3× poverty)	**7.920**	**0.200**	**39.621**	**<2e-16 *****
Slope (Below 3× poverty)	**-0.074**	**0.002**	**-28.719**	**<2e-16 *****
Δ Intercept (Above 3× poverty)	**2.695**	**0.310**	**8.694**	**<2e-16*****
Δ Slope (Above 3× poverty)	**-0.030**	**0.004**	**-7.454**	**9.16e-14*****

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table C.2: HRS logistic regression: Dependent Variable = Income status at time t+1

Predictor	Estimate	Std. Error	t value	$Pr(>\|t\|)$
Intercept (Below 1× poverty)	**0.354**	**0.187**	**1.893**	**0.058***
Slope (Below 1× poverty)	**-0.007**	**0.003**	**-2.879**	**0.004*****
Δ Intercept (Above 1× poverty)	**2.418**	**0.235**	**10.307**	**<2e-16*****
Δ Slope (Above 1× poverty)	**0.009**	**0.003**	**2.774**	**0.005*****
Intercept (Below 2× poverty)	**-0.108**	**0.125**	**-0.861**	**0.389**
Slope (Below 2× poverty)	**-0.012**	**0.002**	**-6.784**	**1.17e-11*****
Δ Intercept (Above 2× poverty)	**3.935**	**0.161**	**24.483**	**<2e-16*****
Δ Slope (Above 2× poverty)	**-0.017**	**0.002**	**-7.567**	**3.86e-14*****
Intercept (Below 3× poverty)	**-0.211**	**0.110**	**-1.913**	**0.056***
Slope (Below 3× poverty)	**-0.016**	**0.002**	**10.194**	**<2e-16*****
Δ Intercept (Above 3× poverty)	**4.257**	**0.147**	**29.003**	**<2e-16*****
Δ Slope (Above 3× poverty)	**-0.025**	**0.002**	**-11.987**	**<2e-16*****

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table C.3: NLSY79-pre-1994 logistic regression for each poverty threshold: Dependent Variable = Survival at time t+1

Predictor	Estimate	Std. Error	t value	$Pr(>\lvert t\rvert)$
Intercept (Below 1× poverty)	**7.434**	**1.400**	**5.307**	**1.12e-07 *****
Slope (Below 1× poverty)	**-0.046**	**0.051**	**-0.890**	**0.373**
Δ Intercept (Above 1× poverty)	**0.107**	**1.840**	**0.058**	**0.954**
Δ Slope (Above 1× poverty)	**0.022**	**0.067**	**0.320**	**0.749**
Intercept (Below 2× poverty)	**7.084**	**1.181**	**5.997**	**2.01e-09 *****
Slope (Below 2× poverty)	**-0.028**	**0.044**	**-0.661**	**0.508**
Δ Intercept (Above 2× poverty)	**1.015**	**1.937**	**0.524**	**0.600**
Δ Slope (Above 2× poverty)	**-0.009**	**0.070**	**-0.127**	**0.899**
Intercept (Below 3× poverty)	**8.389**	**1.120**	**7.484**	**7.25e-14 *****
Slope (Below 3× poverty)	**-0.067**	**0.040**	**-1.660**	**0.096***
Δ Intercept (Above 3× poverty)	**-2.284**	**2.179**	**-1.048**	**0.294**
Δ Slope (Above 3× poverty)	**0.103**	**0.080**	**1.290**	**0.197**

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table C.4: NLSY79-pre-1994 logistic regression for each poverty threshold: Dependent Variable = Income status at time t+1

Predictor	Estimate	Std. Error	t value	$Pr(>\|t\|)$
Intercept (Below 1× poverty)	**1.220**	**0.188**	**6.462**	**1.04e-10 *****
Slope (Below 1× poverty)	**-0.056**	**0.007**	**-8.027**	**1.01e-15 *****
Δ Intercept (Above 1× poverty)	**-0.032**	**0.255**	**-0.128**	**0.898**
Δ Slope (Above 1× poverty)	**0.124**	**0.009**	**12.982**	**< 2e-16 *****
Intercept (Below 2× poverty)	**0.664**	**0.137**	**4.850**	**1.24e-06*****
Slope (Below 2× poverty)	**-0.059**	**0.005**	**-11.515**	**< 2e-16*****
Δ Intercept (Above 2× poverty)	**-0.606**	**0.199**	**-3.043**	**0.00234*****
Δ Slope (Above 2× poverty)	**0.139**	**0.007**	**18.640**	**<2e-16*****
Intercept (Below 3× poverty)	**-0.107**	**0.122**	**-0.881**	**0.378**
Slope (Below 3× poverty)	**-0.049**	**0.004**	**-10.720**	**<2e-16*****
Δ Intercept (Above 3× poverty)	**-0.597**	**0.188**	**-3.180**	**0.001*****
Δ Slope (Above 3× poverty)	**0.135**	**0.006**	**19.457**	**<2e-16*****

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table C.5: NLSY79-post-1994 logistic regression for each poverty threshold: Dependent Variable = Survival at time t+1

Predictor	**Estimate**	**Std. Error**	**t value**	$Pr(>\|t\|)$
Intercept (Below 1× poverty)	**8.527**	**0.999**	**8.539**	**<2e-16 *** **
Slope (Below 1× poverty)	**-0.105**	**0.023**	**-4.490**	**7.14e-06 *** **
Δ Intercept (Above 1× poverty)	**0.203**	**1.261**	**0.161**	**0.872**
Δ Slope (Above 1× poverty)	**0.028**	**0.029**	**0.941**	**0.347**
Intercept (Below 2× poverty)	**8.986**	**0.749**	**11.995**	**<2e-16 *** **
Slope (Below 2× poverty)	**-0.108**	**0.017**	**-6.240**	**4.41e-10 *** **
Δ Intercept (Above 2× poverty)	**-0.389**	**1.232**	**-0.316**	**0.752**
Δ Slope (Above 2× poverty)	**0.039**	**0.028**	**1.371**	**0.170**
Intercept (Below 3× poverty)	**9.265**	**0.717**	**12.911**	**<2e-16 *** **
Slope (Below 3× poverty)	**-0.107**	**0.016**	**-6.414**	**1.43e-10*** **
Δ Intercept (Above 3× poverty)	**-0.779**	**1.332**	**-0.585**	**0.559**
Δ Slope (Above 3× poverty)	**0.041**	**0.031**	**1.315**	**0.188**

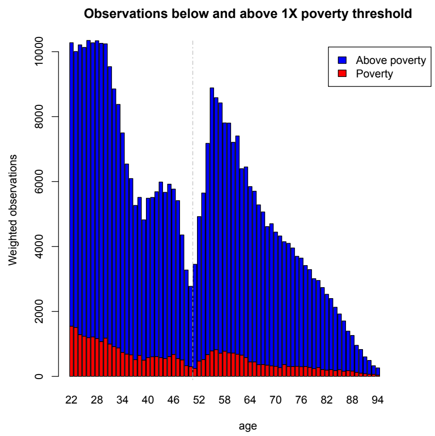
$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

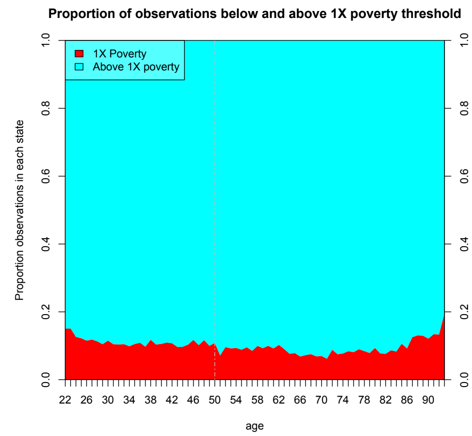
Table C.6: NLSY79-post-1994 logistic regression for each poverty threshold: Dependent Variable = Income status at time t+1

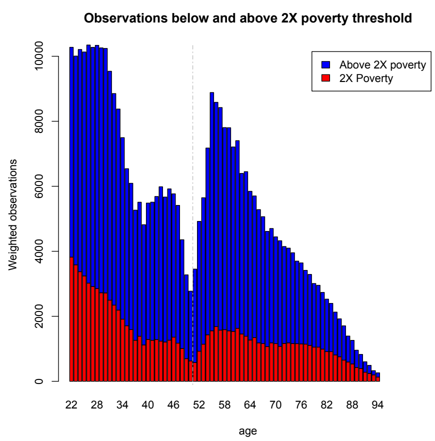
Predictor	**Estimate**	**Std. Error**	**t value**	$Pr(>\|t\|)$
Intercept (Below 1× poverty)	**0.650**	**0.236**	**2.751**	**0.00595 *****
Slope (Below 1× poverty)	**-0.024**	**0.005**	**-4.244**	**2.20e-05 *****
Δ Intercept (Above 1× poverty)	**2.153**	**0.306**	**7.029**	**2.11e-12 *****
Δ Slope (Above 1× poverty)	**0.030**	**0.007**	**4.037**	**5.42e-05 *****
Intercept (Below 2× poverty)	**-0.329**	**0.173**	**-1.903**	**0.057***
Slope (Below 2× poverty)	**-0.013**	**0.004**	**-3.229**	**0.001*****
Δ Intercept (Above 2× poverty)	**2.451**	**0.245**	**9.998**	**<2e-16*****
Δ Slope (Above 2× poverty)	**0.022**	**0.006**	**3.701**	**0.000215*****
Intercept (Below 3× poverty)	**-0.876**	**0.147**	**-5.935**	**2.95e-09*****
Slope (Below 3× poverty)	**-0.009**	**0.003**	**-2.588**	**0.009*****
Δ Intercept (Above 3× poverty)	**1.935**	**0.221**	**8.751**	**<2e-16*****
Δ Slope (Above 3× poverty)	**0.032**	**0.005**	**5.889**	**3.91e-09*****

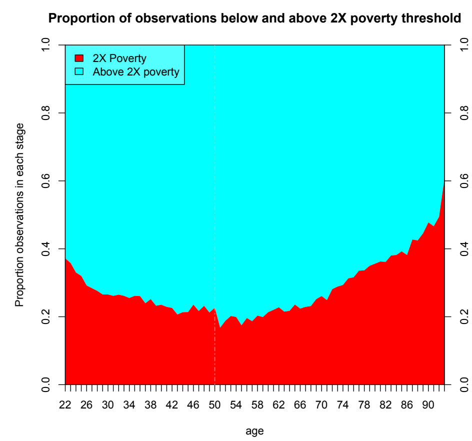
$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

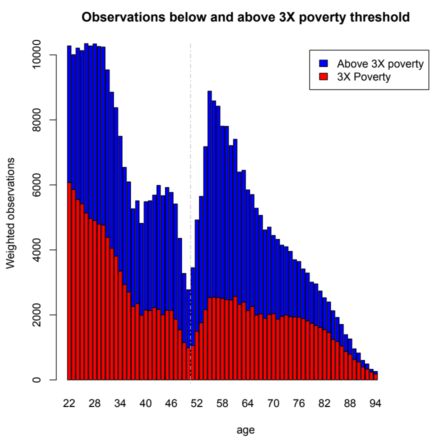
C.3 Ch.3 appendix figures











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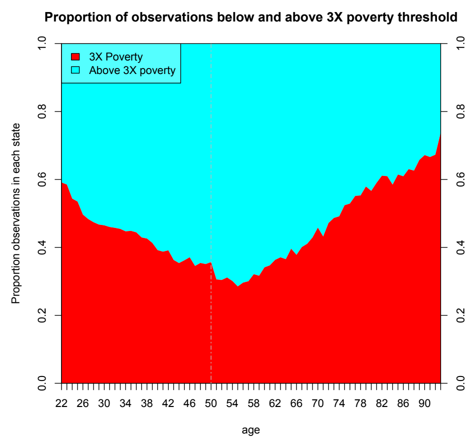
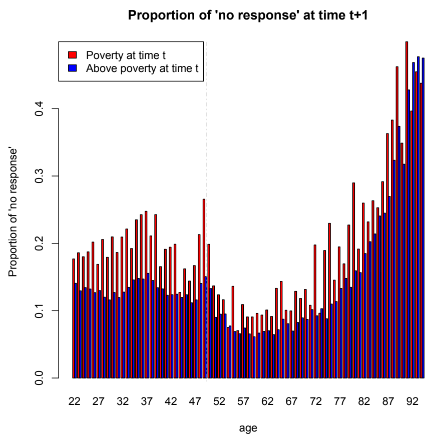
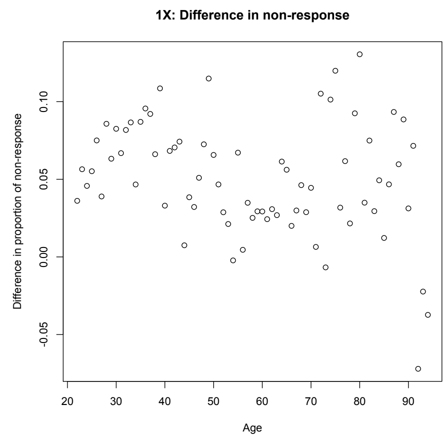
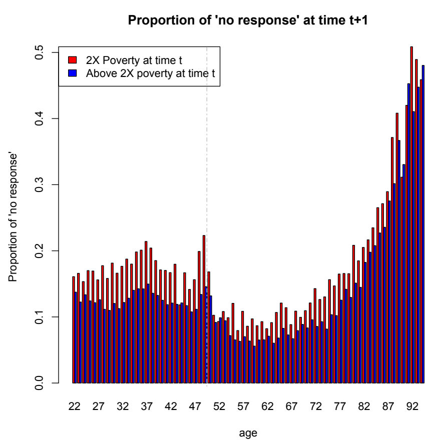


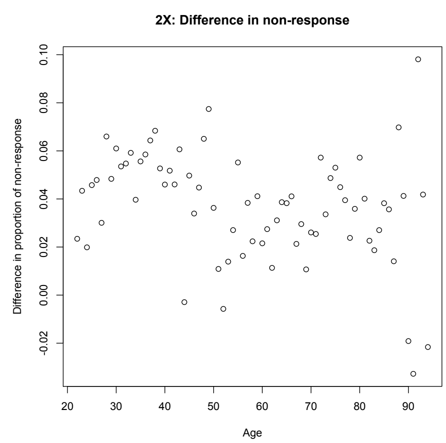
(f)

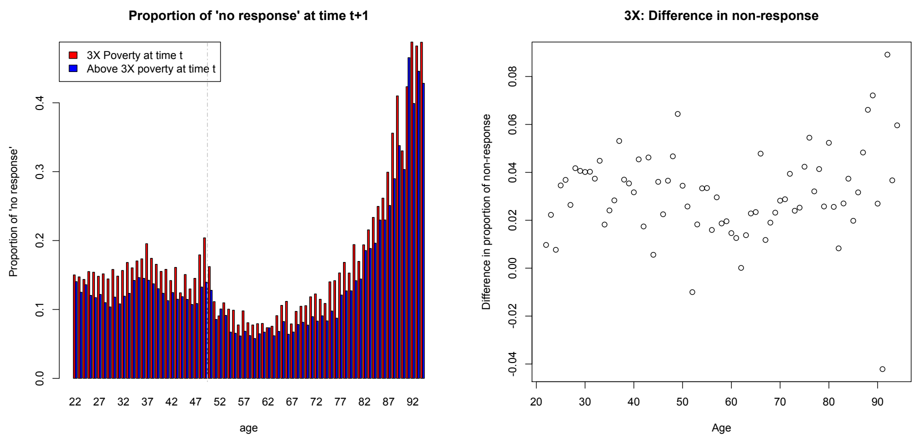
Figure C.1: **Pooled data from HRS and NLSY79.** The dashed vertical line in each figure depicts where NLSY79 and HRS data are joined (at age 50). a, c, e: Weighted observations in each state at each age. b, d, f: Proportion of individuals in each state at each age. Each row examines the same population but differs in the threshold used to classify individuals into different states; a and b: below and above the standard poverty threshold; c and d: below and above 2× poverty threshold; e and f: below and above 3× poverty threshold which is close to national median income levels.


Proportion of 'no response' at time t+1
Poverty at time t
Above poverty at time t
Proportion of 'no response'
age


1X: Difference in non-response
Difference in proportion of non-response
Age


Proportion of 'no response' at time t+1
2X Poverty at time t
Above 2X poverty at time t
Proportion of 'no response'
age


2X: Difference in non-response
Difference in proportion of non-response
Age



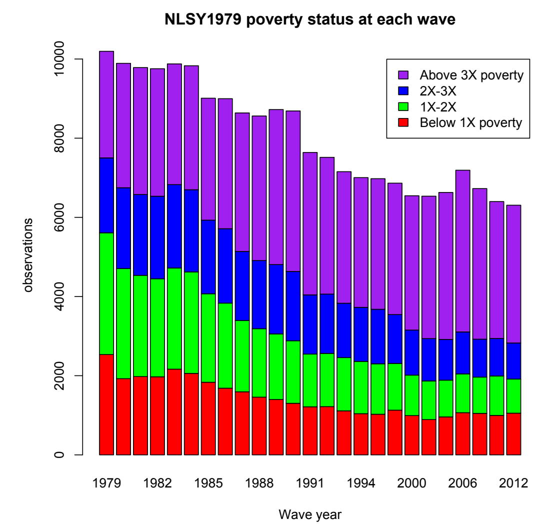
(e) (f)

Figure C.2: **The distribution of non-response based on state and age.** All panels are derived from HRS and NLSY79 data-sets. a, c, and e: The proportion of non response at age $x + 1$ based on being below an income threshold (red) or above an income threshold (blue) at age x. Income thresholds are defined as 1×, 2×, or 3× the 'official' poverty threshold, respectively. b, d, and e: Difference in non-response between the income categories (separated by 1×, 2×, or 3× poverty threshold, respectively) at each age.

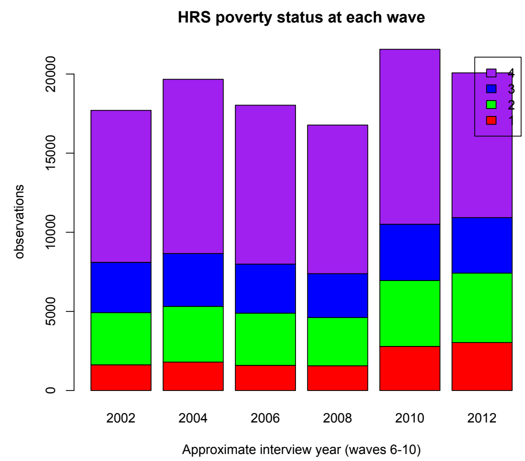
APPENDIX D

Ch. 4

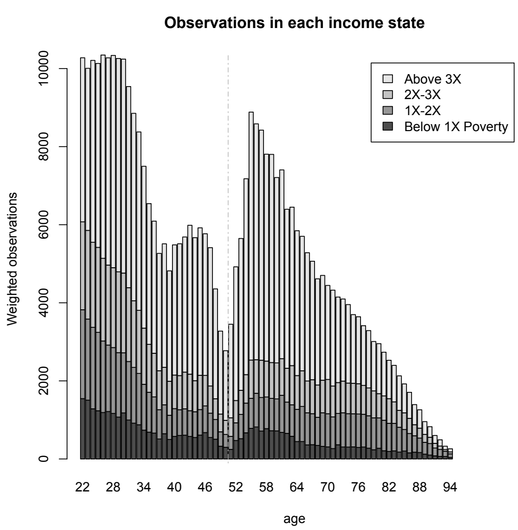
D.1 Ch.4 appendix figures



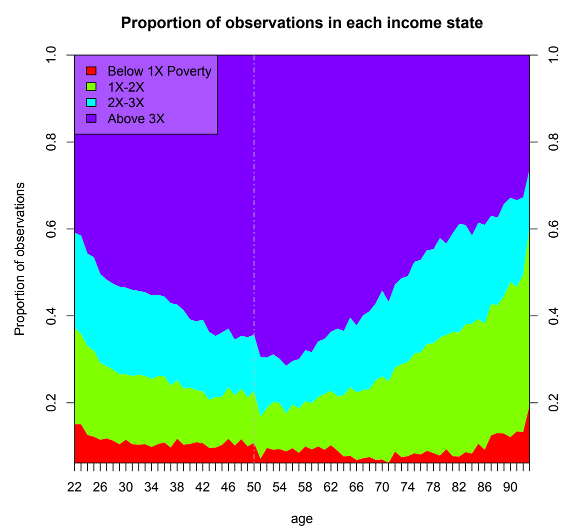
(a)



(b)



(c)



(d)

Figure D.1: Pooled data from HRS and NLSY79. The dashed vertical line in each figure donates where NLSY79 and HRS data are joined (at age 50). (a) NLSY79 data for each interview year, (b) HRS data at each interview year. (c) weighted observations in each income state at each age. (d) proportion of individuals in each income state.

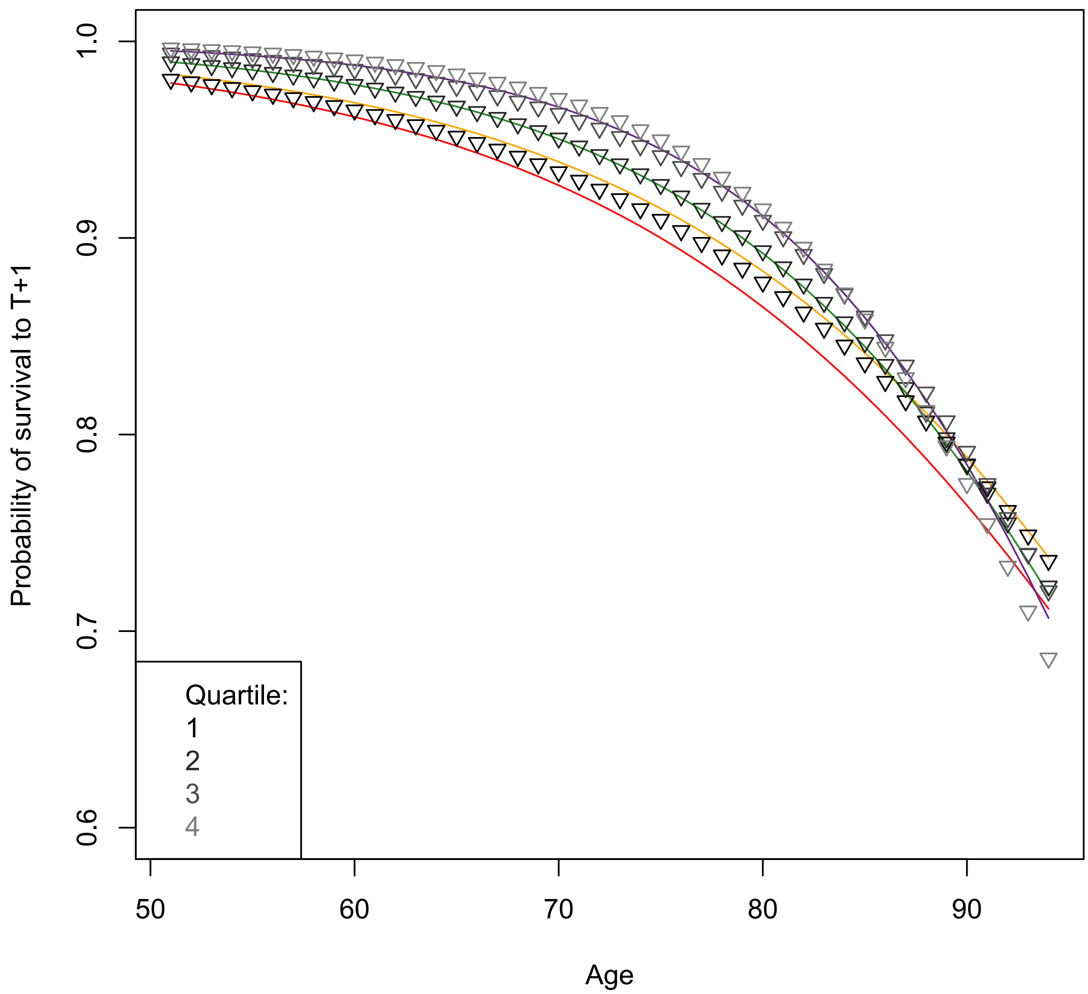


Figure D.2: Annual survival by income quartile. Quartiles are based on relative income levels (0-25% income, 25-50%, 50-75%, and 75-100%) for the pooled HRS population. Quartile 1 is 0-1.8×poverty, Quartile 2 is 1.8-3.3×poverty, Quartile 3 is 3.3-5.82×poverty and Quartile 4 is 5.82-6035.2×poverty (the maximum income level). Quartiles are represented by triangles, for comparison the background colored lines are the income states used in our main analysis (red <1×poverty, yellow 1-2×poverty, green 2-3×poverty, blue >3×poverty)

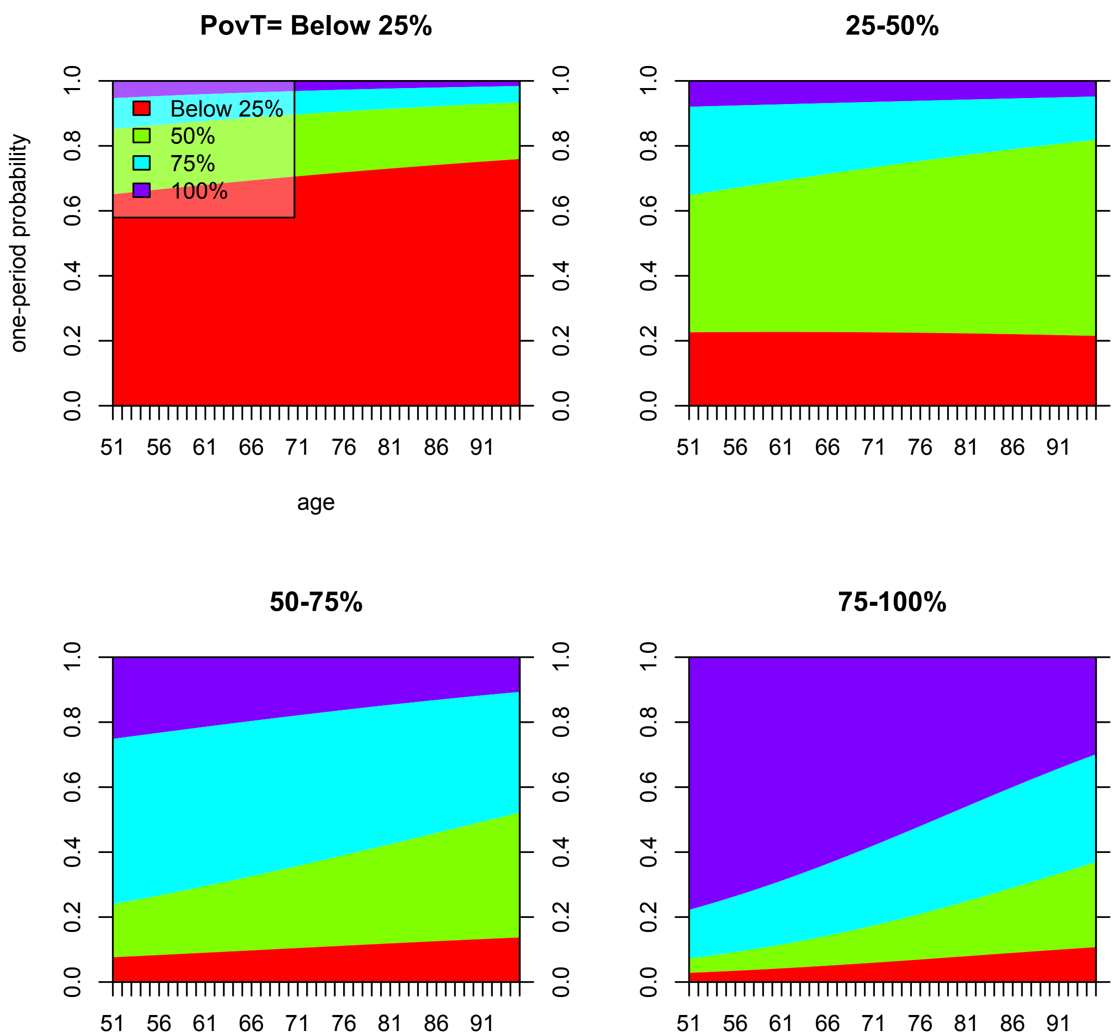


Figure D.3: Annual transition probabilities when separated into quartiles for ages 50 to 95. Quartiles are based on relative income levels (0-25% income, 25-50%, 50-75%, and 75-100%) for the pooled HRS population. Quartile 1 is 0-1.8×poverty, Quartile 2 is 1.8-3.3×poverty, Quartile 3 is 3.3-5.82×poverty and Quartile 4 is 5.82-6035.2×poverty.

Bibliography

Aalen, O. O., Gjessing, H. K., 2001. Understanding the Shape of the Hazard Rate : A Process Point of View. Statistical Science 16 (1), 1–22.

Adler, N., Boyce, T., Chesney, M., Cohen, S., Folkman, S., Kahn, R., Syme, S., 1994. Socioeconomic Status and Health: The Challenge of the Gradient.

Adler, N. E., Rehkopf, D. H., 2008. U.S. Disparities in Health: Descriptions, Causes, and Mechanisms. Annual Review of Public Health 29 (1), 235–252.
URL http://www.annualreviews.org/doi/abs/10.1146/annurev.publhealth.29.020907.090852

Anderson, J. J., Gildea, M. C., Williams, D. W., Li, T., jan 2008. Linking Growth, Survival, and Heterogeneity Through Vitality. The American Naturalist 171 (1), E20–43.

Backlund, E., Sorlie, P. D., Johnson, N. J., 1996. The Shape of the Relationship between Income and Mortality in the United States. Evidence from the National Longitudinal Mortality Study. Ann Epidemiol 6 (1), 12–20.
URL http://www.ncbi.nlm.nih.gov/pubmed/8680619{\%}5Cnhttp://www.annalsofepidemiology.org/article/1047-2797(95)00090-9/abstract

Bane, M., Ellwood, D. T., 1985. Slipping Into and Out of Poverty: The Dynamics of Spells. The Journal of Human Resources 21 (1).

Benzeval, M., Judge, K., 2001. Income and Health: The Time Dimension. Social Science and Medicine 52 (9), 1371–1390.

Bramlett, M. D., Mosher, W. D., 2002. Cohabitation, Marriage, Divorce, and Remarriage in the United States. Vital and health statistics. Series 23, Data from the National Survey of Family Growth (22), 1–93.

Brown, D. C. U. o. M., 2014. Life Expectancy Differentials by Marital Status, Individuals' Own Education and Spousal Education in the United States. Population Association of America, 2014 Annual Meeting.

Bureau, U. C., 2015. Decennial Censuses, 1950 to 1990, and Current Population Survey, March and Annual Social and Economic Supplements, 1993 to 2015. URL `http://www.census.gov/hhes/families/data/marital.html`

Bureau of Labor Statistics, U. D. o. L., 2012. National Longitudinal Survey of Youth 1979 cohort, 1979-2010 (rounds 1-24). Produced and distributed by the Center for Human Resource Research (The Ohio State University), Columbus, OH.

Cai, L., Hayward, M. D., Saito, Y., Lubitz, J., Hagedorn, A., Crimmins, E., 2010. Estimation of Multi-state Life Table Functions and their Variability from Complex Survey Data Using the SPACE Program. Demographic Research 22 (6), 1–23.

Caswell, H., 2001. Matrix Population Models, 2nd Edition. Sinauer Associates, Inc, Sunderland, Massachusetts.

Caswell, H., 2009. Stage, Age and Individual Stochasticity in Demography. Oikos 118 (12), 1763–1782.

Caswell, H., 2012. Matrix Models and Sensitivity Analysis of Populations Classified by Age and Stage: A Vec-Permutation Matrix Approach. Theoretical Ecology 5, 403–417.

Caswell, H., 2015. Lifetime Reproduction and the Second Demographic Transition: Stochasticity and Individual Variation. Demographic Research 33 (20), 561–588.

Cellini, S. R., McKernan, S.-M., Ratcliffe, C., 2008. The Dynamics of Poverty in the United States: A Review of Data, Methods, and Findings. Policy Analysis and Management 27 (3), 577–605.

Chetty, R., Hendren, N., Kline, P., Saez, E., 2014. Where is the Land of Opportunity? The Geography of Intergenerational Mobility in the United States. the Quarterly Journal of Economics 129 (4), 1553–1623.

Chetty, R., Stepner, M., Abraham, S., Lin, S., Scuderi, B., Turner, N., Bergeron, A., Cutler, D., 2016. The Association Between Income and Life Expectancy in the United States, 2001-2014. JAMA: Clinical Review & Education 315 (16), 1750–1766.

Clarke, S. C., 1995. Advance Report of Final Marriage Statistics, 1989 and 1990. National Center for Health Statistics 43 (12).

Coulson, T., Tuljapurkar, S., 2008. The Dynamics of a Quantitative Trait in an Age-Structured Population Living in a Variable Environment. The American Naturalist 172 (5), 599–612.

Crouse, D. T., Crowder, L. B., Caswell, H. A. L., 1987. A Stage-Based Population Model for Loggerhead Sea Turtles and Implications for Conservation. Ecology 68 (5), 1412–1423.

Dagostino, R. B., Lee, M.-l., Belanger, A. J., Kannel, W. B., 1990. Relation of Pooled Logistic Regression to Time Dependent Cox Regression Analysis: The Framingham Heart Study. Statistics in Medicine 9 (May), 1501–1515.

Darroch, J. N., Seneta, E., 2013. On Quasi-Stationary Distribution in Absorbing Discrete-Time Finite Markov Chains. Journal of Applied Probability 2 (1), 88–100.

DeNavas-Walt, C., Proctor, B. D., 2015. Income and Poverty in the United States: 2014. Current Population Reports, U.S. Census Bureau, P60–252.
URL `https://www.census.gov/content/dam/Census/library/publications/2014/demo/p60-249.pdf`

Edwards, A. N., 2014. Dynamics of Economic Well-Being: Poverty, 2009-2011. Current Population Reports, U.S. Census Bureau (January), 70–137.

Fox, G. A., 2001. Failure Time Analysis: Studying Time to Events and Rates at Which Events Occur. Oxford University Press, Inc, New York.

Fujiwara, M., Caswell, H., 2002. Estimating Population Projection Matrices from Multi-Stage Mark-Recapture Data. Ecology 83 (12), 3257–3265.

Gillespie, D. O. S., Trotter, M. V., Tuljapurkar, S. D., 2014. Divergence in Age Patterns of Mortality Change Drives International Divergence in Lifespan Inequality. Demography 51 (1), 1003–17.
URL `http://www.ncbi.nlm.nih.gov/pubmed/24756909`

Goldman, N., Korenman, S., Weinstein, R., 1995. Marital Status and Health Among the Elderly. Soc. Sci. Med. 40 (12), 1717–1730.

Goldscheider, F. K., Waite, L. J., 1986. Sex Differences in the Entry Into Marriage. American Journal of Sociology 92 (1), 91.

Goldstein, J. R., Kenney, C. T., 2001. Marriage Delayed or Marriage Forgone? New Cohort Forecasts of First Marriage for U.S. Women. American Sociological Review 66 (4), 506–519.

Gompertz, B., 1825. On the Nature of the Function Expressive of the Law of Human Mortality, and on a New Mode of Determining the Value of Life Contingencies. Royal Society Publishing 115, 513–583.

Haughton, J., Khandker, S. R., 2009. Chapter 14. Using Regression. Handbook on Poverty and Inequality, 273–292.

Henningsen, A., Toomet, O., 2011. maxLik: A Package for Maximum Likelihood Estimation in R. Computational Statistics 26 (3).

Hokayem, C., Heggeness, M. L., 2014a. Living in Near Poverty in the United States: 1966-2012. Current Population Reports, U.S. Census Bureau P60-248 (May).

Hokayem, C. U. C. B., Heggeness, M. L. N. I. o. H., 2014b. Factors Influencing Transitions Into and Out of Near Poverty: 2004-2012.

Houllier, F., Lebreton, J.-D., 1986. A Renewal-Equation Approach to the Dynamics of Stage-Grouped Populations. Matematical Biosciences 79, 185–197.

Houllier, F., Lebreton, J.-D., Pontier, D., 1988. Sampling Properties of the Asymptotic Behavior of Age- or Stage-Grouped Population Models. Mathematical Biosciences 95, 161–177.

Islam, M. A., Arabia, S., Chowdhury, R. I., 2004. A Three State Markov Model for Analyzing Covariate Dependence. International Journal of Statistical Sciences 3 (i), 241–249.

Islam, M. A., Chowdhury, R. I., 2006. A Higher Order Markov Model for Analyzing Covariate Dependence. Applied Mathematical Modelling 30, 477–488.

Iwashyna, T. J., Christakis, N. A., 2003. Marriage, Widowhood, and Health-Care Use. Social Science & Medicine 57, 2137–2147.

Johnson, N. J., Backlund, E., Sorlie, P. D., Loveless, C. A., 2000. Marital Status and Mortality: The National Longitudinal Mortality Study. Ann Epidemiol 10, 224–238.

Kalmijn, M., 2013. The Educational Gradient in Marriage: A Comparison of 25 European Countries. Demography 50, 1499–1520.

Kaplan, R. M., Kronick, R. G., 2006. Marital Status and Longevity in the United States Population. Journal of Epidemiology and Community Health 60 (9), 760–765.

Kemeny, J. G., Snell, J. L., 1976. Finite Markov Chains. Springer-Verlag.

Keyfitz, N., Caswell, H., 2005. Applied Mathematical Demography, 3rd Edition. Springer-Verlag New York.

Keyfitz, N. U. o. C., 1968. Introduction to the Mathematics of Populations. Addison-Wesley Publishing Company Inc.

Korn, E., Whittemore, A., 1979. Methods for Analyzing Panel Studies of Acute Health Effects of Air Pollution. Biometrics 35 (4), 795–802.

Kot, M., 2001. Elements of Mathematical Ecology. Cambridge University Press, New York.

Krieger, N., Chen, J. T., Waterman, P. D., Rehkopf, D. H., Subramanian, S. V., 2003. Race/Ethnicity, Gender, and Monitoring Socioeconomic Gradients in Health: A Comparison of Area-Based Socioeconomic Measures - The Public Health Disparities Geocoding Project. American Journal of Public Health 93 (10), 1655–1671.

Kunst, A. E., Mackenbach, J. P., 1994. The Size of Mortality Differences Associated with Educational Level in Nine Industrialized Countries. American Journal of Public Health 84 (6), 932–937.

Land, K. C., Guralnik, J. M., Blazer, D. G., 1994. Estimating Increment-Decrement Life Tables with Multiple Covariates from Panel Data: The Case of Active Life Expectancy. Demography 31 (2), 297–319.

Law, R., 1983. A Model for the Dynamics of a Plant Population Containing Individuals Classified by Age and Size. Ecological Society of America 64 (2), 224–230.

Lawless, J. F., Rad, N. N., 2015. Estimation and Assessment of Markov Multistate Models with Intermittent Observations on Individuals. Lifetime Data Analysis 21 (2), 160–179.

Lefkovitch, A. L. P., 1965. The Study of Population Growth in Organisms Grouped by Stages. Biometrics 21 (1), 1–18.

Leslie, P., 1945. On the Use of Matrices in Certain Population Mathematics. Biometrika 33 (3), 183–212.

Li, T., Anderson, J. J., sep 2009. The Vitality Model: A Way to Understand Population Survival and Demographic Heterogeneity. Theoretical Population Biology 76 (2), 118–31.

Li, T., Yang, Y. C., Anderson, J. J., 2013. Mortality Increase in Late-Middle and Early-Old Age: Heterogeneity in Death Processes as a New Explanation. Demography 50 (5), 1563–91.

Lillard, L. a., Panis, C. W., 1996. Marital Status and Mortality: The Role of Health. Demography 33 (3), 313–327.

Lin, I. F., Brown, S. L., 2012. Unmarried Boomers Confront Old Age: A National Portrait. Gerontologist 52 (2), 153–165.

Liu, H., Umberson, D., 2008. The Times They Are a Changin': Marital Status and Health Differentials from 1972-2003. J Health Soc Behav 49 (3), 239–253.

Lubitz, J., Cai, L., Kramarow, E., Lentzner, H., sep 2003. Health, Life Expectancy, and Health Care Spending Among the Elderly. The New England Journal of Medicine 349 (11), 1048–55.

Lumley, T., 2014. Survey: Analysis of Complex Survey Samples. R Package Version 3.30.

Lundberg, S., 2012. Personality and Marital Surplus. IZA Journal of Labor Economics 1 (1), 3.

Manzoli, L., Villari, P., Pirone, G. M., Boccia, A., 2007. Marital Status and Mortality in the Elderly : A Systematic Review and Meta-Analysis. Social Science & Medicine 64, 77–94.

Marmot, M., 2002. The Influence of Income on Health: Views of an Epidemiologist. Health Affairs 21 (2), 31–46.

McDonough, P., Worts, D., Sacker, A., jan 2010. Socioeconomic Inequalities in Health Dynamics: A Comparison of Britain and the United States. Social Science & Medicine 70 (2), 251–60.
URL http://www.ncbi.nlm.nih.gov/pubmed/19857919

Metcalf, C., Lessler, J., Klepac, P., Morice, A., Grenfell, B., Bjornstad, O., 2012. Structured Models of Infectious Disease: Inference with Discrete Data. Theoretical Population Biology 82 (4), 275–282.

Moldoff, M., Chien, S., Campbell, N., Hayden, O., Hurd, M., Main, R., Meijer, E., Martin, C., Carroll, J., Rohwedder, S., Clair, P. S., 2014. RAND HRS Data Documentation, Version O.

Muenz, R., Rubinstein, V., 1985. Markov Models for Covariate Dependence of Binary Sequences. Biometrics 41 (1), 91–101.

Neilson, C., Contreras, D., Cooper, R., Hermann, J., 2008. The Dynamics of Poverty in Chile. Journal of Latin American Studies 40 (02), 251–273.
URL http://journals.cambridge.org/abstract{_}S0022216X08003982

Osowole, O., Ugbechie, R., Uba, E., 2012. On the Identification of Core Determinants of Poverty: A Logistic Regression Approach. Mathematical Theory and Modeling 2 (10), 45–53.

Pasek, J., 2016. Weighting and Weighted Statistics. R Package Version 0.85.

Preston, S. H., 1975. The Changing Relation between Mortality and Level of Economic Development. Population Studies 29 (2), 231–248.

Proctor, B. D., Semega, J. L., Kollar, M. A., 2016. Income and Poverty in the United States : 2015. Tech. Rep. September.

Rank, M., Hadley, H., Williams, J., 2014. A Life Course Approach to Understanding Poverty Among Older American Adults. Fam Soc 91 (4), 337–341.

Rank, M., Hirchl, T., 2001. Poverty Across the Life Cycle: Evidence from the PSID. Journal of Policy Analysis and Management 20 (4).

Rank, M. R., Hirschl, T. A., 2015. The Likelihood of Experiencing Relative Poverty over the Life Course. PLoS ONE 10 (7), 1–11.

Rehkopf, D. H., Berkman, L. F., Coull, B., Krieger, N., 2008. The Non-Linear Risk of Mortality by Income Level in a Healthy Population: US National Health and Nutrition Examination Survey Mortality Follow-Up Cohort, 1988-2001. BMC Public Health 8, 383.
URL http://www.pubmedcentral.nih.gov/articlerender.fcgi?artid=2587469{\&}tool=pmcentrez{\&}rendertype=abstract

Rehkopf, D. H., Jencks, C., Glymour, M. M., 2010. The Association of Earnings with Health in Middle Age: Do Self-Reported Earnings for the Previous Year Tell the Whole Story? Social science & medicine 45 (3), 431–9.
URL http://www.ncbi.nlm.nih.gov/pubmed/20580858

Ripley, B., Venables, R., 2016. nnet Package for R.

Robards, J., Evandrou, M., Falkingham, J., Vlachantoni, A., 2012. Marital Status, Health and Mortality. Maturitas 73 (4), 295–299.

Roelfs, D. J., Shor, E., Kalish, R., Yogev, T., 2011. The Rising Relative Risk of Mortality for Singles: Meta-Analysis and Meta-Regression. American Journal of Epidemiology (34).

Rogers, A., 1980. Introduction to Multistate Mathematical Demography. Environment and Planning A 12 (5), 489–498.

Rogers, R. G., 1992. Living and Dying in the U.S.A.: Sociodemographic Determinants of Death Among Blacks and Whites. Demography 29 (2), 287–303.

Roth, G., Caswell, H., 2016. Hyperstate Matrix Models: Extending Demographic State Spaces to Higher Dimensions. Methods in Ecology and Evolution 7, 1438–1450.

Sacker, A., Wiggins, R. D., Bartley, M., McDonough, P., 2007. Self-Rated Health Trajectories in the United States and the United Kingdom: A Comparative Study. American journal of public health 97 (5), 812–8.
URL http://www.pubmedcentral.nih.gov/articlerender.fcgi?artid=1854880{\&}tool=pmcentrez{\&}rendertype=abstract

Sandoval, D. A., Rank, M. R., Hirschl, T. A., 2009. The Increasing Risk of Poverty Across the American Life Course. Demography 46 (4), 717–737.

Sbarra, D., Hasselmo, K., Nojopranoto, W., 2012. Divorce and Death: A Case Study for Health Psychology. Soc Personal Psychol Compass 6 (12), 905–919.

Schoen, R., 1988. Practical Uses of Multistate Population Models. Annual Review of Scoiology 14, 341–361.

Schoen, R., Standish, N., 2001. The Retrenchment of Marriage : Results from Marital Status Life Tables for the United States, 1995. Population and Development Review 27 (3), 553–563.

Schoen, R., Weinick, R. M., 1993. The Slowing Metabolism of Marriage: Figures from 1988 U. S. Marital Status Life Tables. Demography 30 (4), 737–746.

Seneta, E., 1981. Non-negative Matrices and Markov Chains. Springer New York.

Shor, E., Roelfs, D. J., Bugyi, P., Schwartz, J., 2012. Meta-analysis of Marital Dissolution and Mortality: Reevaluating the Intersection of Gender and Age. Soc Sci Med 75 (1).

Sorlie, P. D., Backlund, E., Keller, J. B., 1995. US Mortality by Economic, Demographic, and Social Characteristics: The National Longitudinal Mortality Study. American Journal of Public Health 85 (7), 949–956.

Steiner, U. K., Tuljapurkar, S., Coulson, T., Horvitz, C., 2012. Trading Stages: Life Expectancies in Structured Populations. Experimental Gerontology 47 (10), 773–81.

Steinsaltz, D., Evans, S. N., 2004. Markov Mortality Models: Implications of Quasistationarity and Varying Initial Distributions. Theoretical Population Biology 65 (4), 319–37.

Stevens, A. H., 1999. Climbing out of Poverty, Falling Back in: Measuring the Persistence of Poverty Over Multiple Spells. The Journal of Human Resources 34 (3), 557–588.
URL `http://www.jstor.org/stable/146380`

Strehler, B., Mildvan, A., 1960. General Theory of Mortality and Aging. Science 132 (3418), 14–21.

Su, J. H., Dunifon, R., Sassler, S., 2015. Better for Baby? The Retreat From Mid-Pregnancy Marriage and Implications for Parenting and Child Well-being. Demography 52 (4), 1167–1194.

Therneau, T. M., 2015. Survival Analysis. R package version 2.38-3.

Trowbridge, C. L., 1994. Mortality Rates by Marital Status. Transactions of Society of Actuaries 46.

Tuljapurkar, S., Boe, C., 1998. Mortality Change and Forecasting: How Much and How Little Do We Know? North American Actuarial Journal 2 (4), 13–47.

Tuljapurkar, S., Steiner, U. K., 2010. Dynamic Heterogeneity and Life Histories. Annals of the New York Academy of Sciences 1204, 65–72.

Tuljapurkar, S., Steiner, U. K., Orzack, S. H., 2009. Dynamic Heterogeneity in Life Histories. Ecology Letters 12 (1), 93–106.

Va, P., Yang, W.-s., Nechuta, S., Chow, W.-h., Cai, H., Yang, G., Gao, S., Gao, Y.-t., Zheng, W., Shu, X.-o., Xiang, Y.-b., 2011. Marital Status and Mortality among Middle Age and Elderly Men and Women in Urban Shanghai. PLoS ONE 6 (11).

Vaupel, J. W., Canudas-Romo, V., 2002. Decomposing Demographic Change Into Direct Vs. Compositional Components. Demographic Research 7, 1–14.

Venables, W., Ripley, B., 2002. Modern Applied Statistics with S, 4th Edition. Springer-Verlag, New York.

Wagner, P., 2011. Vitality Heterogeneity in the Strehler-Mildvan Theory of Mortality. Max Planck Institute for Demographic Research, Working Paper 49 (0), 0–28.

Willekens, F., Putter, H., 2014. Software for Multistate Analysis. Demographic Research 31 (1), 381–420.

Willekens, F. J., Shah, I., Shah, J. M., Ramachandran, P., 1982. Multi-state Analysis of Marital Status Life Tables: Theory and Application. Population Studies 36 (1), 129–144.

Williams, K., Umberson, D., 2004. Marital Status, Marital Transitions, and Health: A Gendered Life Course Perspective. Journal of Health and Social Behavior 45 (1), 81–98.

Yang, X., Shoptaw, S., Nie, K., Liu, J., Belin, T. R., 2007. Markov Transition Models for Binary Repeated Measures with Ignorable and Nonignorable Missing Values. Statistical Methods in Medical Research 16, 347–364.

Yashin, A. I., Iachine, I. A., Begun, A. Z., Vaupel, J. W., 2001. Hidden Frailty: Myths and Reality. Department of Statistics and Demography, Odense University.

Yu, L., Griffith, W. S., Tyas, S. L., Snowdon, A., Kryscio, R. J., 2010. A Nonstationary Markov Transition Model for Computing the Relative Risk of Dementia Before Death. Statistics in Medicine 29 (November 2009), 639–648.

Zeng, Y., Morgan, S. P., Wang, Z., Gu, D., Yang, C., 2012. A Multistate Life Table Analysis of Union Regimes in the Unites States: Trends and Racial Differentials, 1970-2002. Population Research Policy Review 31 (2), 130–134.

Zheng, H., Thomas, P. A., 2013. Marital Status, Self-Rated Health, and Mortality: Overestimation of Health or Diminishing Protection of Marriage? Journal of Health and Social Behavior 54 (1), 128–143.