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Wrinkling of functionally graded sandwich structures subject to biaxial and in-plane shear loads

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WRINKLING OF FUNCTIONALLY GRADED SANDWICH STRUCTURES SUBJECT TO BIAXIAL AND IN-PLANE SHEAR LOADS

by

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ABSTRACT

Benefits of a functionally graded core increasing wrinkling stability of sandwich panels have been demonstrated in a recent paper [1] where a several-fold increase in the wrinkling stress was observed, without a significant weight penalty, using a stiffer core adjacent to the facings. In the present paper wrinkling is analyzed in case where the facings are subject to biaxial compression and/or in-plane shear loading and the core is arbitrary graded through-the-thickness. Two issues addressed are the effect of biaxial or in-plane shear loads on wrinkling stability of panels with both graded and ungraded core and the verification that functional grading of the core remains an effective tool increasing wrinkling stability under such two-dimensional loads. As follows from the study, biaxial compression and in-plane shear cause a reduction in the wrinkling stress as compared to the case of a uniaxial compression in all grading scenarios. Accordingly, even sandwich panels whose mode of failure under uniaxial compression was global buckling, the loss of strength in the facings or core crimpling may become vulnerable to wrinkling under two-dimensional in-plane loading. It is demonstrated that a functionally graded core with the material distributed to increase the local stiffness at and close to the interface with the facings is effective in preventing wrinkling under arbitrary in-plane loads as compared to an equal weight homogeneous core.

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TABLE OF CONTENTS

SECTION

LIST OF ILLUSTRATIONS

NOMENCLATURE

Symbol	Description
h	Depth of Core affected by wrinkle
h_c	Total height of the core
a	Length of wrinkle wave
FGM	Functionally Graded Material
E_c	Modulus of Elasticity of the Core
G_c	Shear Modulus of the Core
x_1-y_1-z_1	Local Wrinkle Coordinates
x-y-z	Global Coordinates
θ	Angle orientation of the wrinkle on Global axis
σ_x	Stress applied in the x direction
σ_y	Stress applied in the y direction
τ_{xy}	Shear applied in the xy coordinates
σ_1	Stress applied in the 1 direction
σ_2	Stress applied in the 2 direction
τ_{12}	Shear applied in the 12 coordinates
U_f	Total Energy of the Facing
U_c	Total Energy of the Core
$U\sigma$	Total Energy of applied stresses
Π	Total engery of the system
E_1	Modulus of of Elasticity of the facing, long. direction
E_2	Modulus of Elasticity of the facing, transv. direction
G_{12}	Shear Modulus of the facing
ν	Poisson's ratio
V_f	Volume fiber percent of the facing
ρ	Mass density
k_1'	Empirical constant for core mech. properties modeling
k_2'	Empirical constant for core mech. properties modeling
n_1	Empirical constant for core mech. properties modeling

n_2	Empirical constant for core mech. properties modeling
k_1	Ratio y direction to x direction applied stresses
k_2	Ratio of shear to x direction applied stresses

1. INTRODUCTION

Wrinkling in sandwich panels was first considered by Gough et al. [2], Hoff and Mautner [3], Plantema [4] and Allen [5]. Various aspects of static and dynamic wrinkling under uniaxial and biaxial loading have been studied by Vonach and Rammerstorfer [6-8], Gdoutos, et al. [9], Kardomateas [10], Birman and Bert [11], Birman [12], [13], [14], Lim and Bart-Smith [15] and others. Early work on wrinkling in sandwich structures was reflected in review [16]. The papers by Sokolinsky and Frostig [17], Frostig et al. [18], [19], Hohe and Librescu [19, 20] and Phan et al. [21] are examples of work on static and dynamic wrinkling modelling the core by higher-order theories.

Wrinkling represents only one possible mode of failure in sandwich structures that often fail due to the overall buckling or the loss of strength or core crimpling [22]. The vulnerability to wrinkling increases in cases of very thin facings and real-time or residual degradation of the core materials under elevated temperature.

Functionally graded materials (FGM) have existed in nature and biology for thousands years (e.g., bamboo, tendon-to-bone insertion site, dental tissues), but their engineering use dates back only several decades. Grading materials in the thickness direction of the structural component was first considered in Japan in the 1980th as a part of a development of a space plane capable to withstand huge temperatures entering and leaving atmosphere. Since this first attempt, FGM have found extensive applications in engineering utilizing their potential for an improvement of the structure. The list of such improvements includes a reduction of in-plane and transverse through-the-thickness stresses, prevention or reduction of delamination tendencies in laminated or sandwich structures, an improved residual stress distribution, enhanced thermal properties, higher fracture toughness and reduced stress intensity factors [23]. Although a significant part of FGM studies and applications has been concerned with thermal applications (e.g., thermal barrier coatings, ceramic-metal structures subject to high temperature at the exposed ceramic surface, etc.), the effectiveness of these materials under mechanical loading that represents interest in the context of structural design as well as in the studies of human body has also been proven and documented. Numerous reviews of such materials and structures have been published in books dedicated to the subject [24], [25], review articles [26], [27] and conference proceedings [28], [29]. The application of a functionally graded core to increase wrinkling stresses in sandwich panels subject to a one-directional compression was considered in [1] where the linear theory of elasticity solution was obtained for the case of a layered core with various stiffness of the layers. The same paper utilized the energy method expanding the Hoff approach to investigate the advantages of the core with a continuously varying through the thickness stiffness. It was demonstrated that using stiffer layers near the facing-core interface can increase the wrinkling stress by several times under both room and elevated temperature.

The present paper addresses two questions that have not been previously posed:

- The effect of biaxial compression and/or in-plane shear loading on wrinkling stability of sandwich panels with homogeneous and graded cores;
- The effect of core grading on wrinkling stability of sandwich panels undergoing biaxial and/or in-plane shear loading and maximizing the benefits of such grading.

The paper begins with the expansion of the Hoff solution to the case of wrinkling of a sandwich panel with an arbitrary through-the-thickness graded core that is subject to an arbitrary in-plane loading. The orientation of the wrinkling wave, its length and the depth of the affected by wrinkling zone of the core are obtained by a numerical minimization process for prescribed proportions between biaxial compressive and in-plane shear stresses. In the case of biaxial compression and a homogeneous core the solution converges to the previously published study [11]. The second part of the paper employs the linear elasticity theory to obtain an exact solution to the wrinkling problem in the case of an arbitrary in-plane loading and a layer-wise grading of the core. While the previous paper was concerned with a similar problem under one-directional loading [1], biaxial compression and in-plane shear bring peculiarity to the solution that are addressed in the paper. The numerical analysis concentrates on the effect of biaxial compression and in-plane shear on wrinkling in sandwich panels with both homogeneous and continuously graded through-the-thickness cores. The grading cases considered in the examples include linear and quadratic variations of the mass density of a cellular core with the highest mass density at the interface with the facing. The grading via the prescribed mass density variations ensures that the considered core confirms to micromechanics limitations. The local stiffness of the core material is determined from empirical relationships between the mass density and the moduli of elasticity and shear [30]. Numerical examples elucidate the effectiveness of grading in raising the wrinkling load through grading of the core as compared to the equal-weight panels with a homogeneous core. It is demonstrated that replacing a homogeneous core with a linearly or quadratically graded equal-weight core may increase the wrinkling stress combination by several times.

PAPER

I. WRINKLING OF FUNCTIONALLY GRADED SANDWICH STRUCTURES SUBJECT TO BIAXIAL AND IN-PLANE SHEAR LOADS

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2. ANALYSIS

Consider a rectangular sandwich panel, subject to biaxial compression and in-plane shear (Fig. 1). The wrinkling wave (or waves) that would be perpendicular to the orientation of a one-directional compressive load is inclined relative to the orientation of both compressive stresses as shown in Fig. 1. The orientation of the wrinkle in the case of a homogeneous core can be predicted by minimizing the wrinkling stress (e.g., [1]). However, such approach becomes invalid if the stiffness of the pseudo-isotropic core varies through the depth, so that its moduli of elasticity $E_c = E_c(z)$ and shear $G_c = G_c(z)$ depend on the z-coordinate.

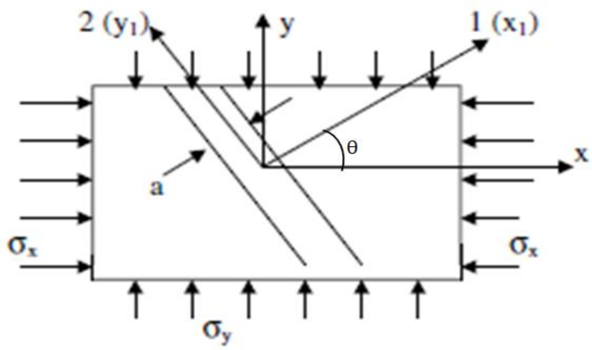


Figure 1 Compressive and Shear Loads

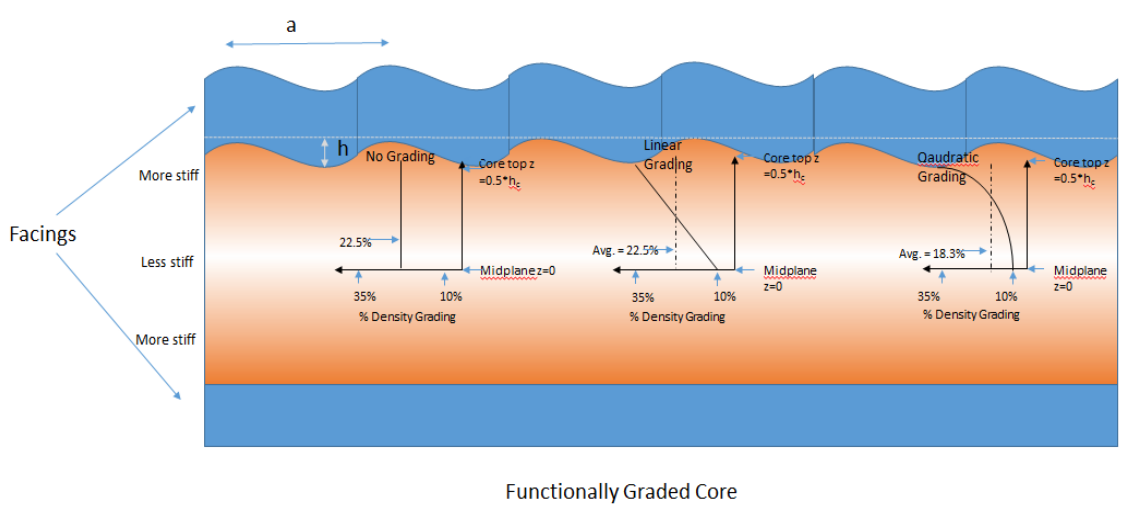


Figure 2 Functionally Graded Core

The analysis of wrinkling is conducted in the system of coordinates where one axis (axis x_1, Fig. 1) is perpendicular to the wrinkling wave, while the orthogonal axis y_1 is oriented along the wave. Accordingly, the applied stresses are transformed into the $x_1 - y_1 - z_1$ coordinate system forming the angle θ with the x-y-z coordinate system:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = L(\theta) \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} c^2\theta & s^2\theta & 2c\theta s\theta \\ s^2\theta & c^2\theta & -2c\theta s\theta \\ -c\theta s\theta & c\theta s\theta & c^2\theta - s^2\theta \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \tag{1}$$

where $c\theta = \cos\theta,\ s\theta = \sin\theta$.

Note that the stresses listed above are the applied stresses and the stresses in individual layers that would be employed in the strength analysis of the facings are different. The latter stresses are determined finding the strains that are uniform through the facing and subsequently specifying the layer-wise stress tensor using the corresponding matrix of reduced stiffness.

The following analysis is conducted by assumption that the core is locally isotropic (pseudo-isotropic), so that the tensor of stiffness is constant at every point of the core, though it varies through the depth reflecting functional grading. This implies that the gradients of the tensor of stiffness through the thickness are negligible at the microscale, while they are present at the macroscale. Such assumption would be acceptable to cellular cores manufactured by standard production methods.

3. WRINKLING UNDER BIAXIAL COMPRESSION AND/OR IN-PLANE SHEAR: ARBITRARY GRADING OF THE CORE

This section represents an extension of the Hoff approach to the case of an arbitrary grading of the sandwich panel core and biaxial and/or in-plane shear loading. In such case the angle of the orientation of the wrinkle is unknown in advance.
The transverse deformation in the core corresponding to a long wrinkling wave with the front perpendicular to the x_1-direction is

$$w = WF(z)\sin\frac{\pi x_1}{a} \tag{2}$$

where a is the length of the wave and $F(z)$ is a function reflecting a decay of through-the-depth displacements of the core from the interface with the facing where $z = 0$ *and* $F(0) = 1$ (the axes z and z_1 are collinear, so that hereafter we do not distinguish between them). Several forms of the function $F(z)$ have been considered in literature. Hoff and Mautner assumed a linear decay of the core displacements from the facing-core interface [3], while Plantema used an exponential decay [4]. A comparison between several available solutions [10] demonstrated that the Hoff method is a good conservative estimate for the wrinkling stress. The same conclusion was reached based on experimental studies in [31]. Accordingly, eqn. (2) can be written as

$$w = W\frac{h-z}{h}\sin\frac{\pi x_1}{a} \tag{3}$$

where h is the depth of the core affected by wrinkling.

The total energy of the sandwich panel with wrinkles includes the contributions of the strain energy of the facings and core and the energy of the applied stresses. The strain energy of the facing per unit width of the wrinkle is dependent only on the stress acting perpendicular to the front of the wave. This is because the wrinkled surface acquires curvature only in the plane $x_1 z$ and the shape of the wrinkle, as given by eq. (3), does not involve and in-plane shear strains or twist. Accordingly,

$$U_f = \frac{D(\theta)}{2}\int_0^a \left(\frac{\partial^2 w_f}{\partial x_1^2}\right)^2 dx_1 \tag{4}$$

where the deflection of the facing, w_f, is available from eqn. (3) by setting $z = 0$ and the bending stiffness of the facing $D(\theta)$ is dependent on the stiffness in the x-y coordinate system and the orientation of the wrinkling wave [11]:

$$D(\theta) = D_{11}(c\theta)^4 + D_{22}(s\theta)^4 + 2(D_{12} + 2D_{66})(s\theta)^2(c\theta)^2 \tag{5}$$

The stiffnesses D_{ij} in (5) are evaluated in the x-y-z coordinate system and they should not be confused for the stiffness tensor in the $x_1 - y_1 - z$ system.

The strain energy of the pseudo-isotropic functionally graded core undergoing deformation according to eqn. (3) is

$$U_c = \frac{1}{2}\int_0^h\int_0^a \frac{\sigma_{cz}^2(z)}{E_c(z)} dx_1 dz + \frac{1}{2}\int_0^h\int_0^a \frac{\tau_{cx_1z}^2(z)}{G_c(z)} dx_1 dz \tag{6}$$

where the stresses in the core are

$$\sigma_{cz} = E_c(z) w,_z \qquad \tau_{cx1z} = G_c(z) w,_{x1} \tag{7}$$

The strain energy of the work of the applied stresses per unit width of the wrinkle is

$$U_\sigma = -\frac{\sigma_1(\theta) t}{2}\int_0^a \left(\frac{\partial w}{\partial x_1}\right)^2 dx_1 \tag{8}$$

where t is the thickness of the facing. The stress σ_1 should be evaluated in terms of the applied stresses in the x-y-z coordinate system. For a prescribed relationship between the applied stresses, the stress characterizing wrinkling is chosen as the axial stress in the x-direction. Other stress components expressed in terms of this stress are

$$\sigma_y = k_1\sigma_x \qquad \tau_{xy} = k_2\sigma_x \tag{9}$$

where the coefficients k_1 and k_2 are known. Accordingly, the stress acting in the direction perpendicular to the wrinkle is

$$\sigma_1 = \sigma_1(\theta) = \sigma_x\left(c^2\theta + k_1 s^2\theta + k_2 c\theta s\theta\right) \tag{10}$$

The wrinkling stress combinations can be specified from the minimum requirement for the total potential energy:

$$\Pi = \Pi(\theta, a, h) = U_f + U_c + U_\sigma \tag{11}$$

The analytical minimization of the energy with respect to the angle of the wrinkle is too cumbersome for practical purposes. One of the possible alternative approaches is to assume that the angle of wrinkle known, determine the critical stress $\sigma_x^{cr}(\theta)$ and subsequently specify the wrinkling stress combination as

$$\sigma_x^{wr} = \min_\theta\left(\sigma_x^{cr}(\theta)\right) \tag{12}$$

For a prescribed angle of orientation of the wrinkling wave and the applied stress combination given in terms of the stress in the x-direction by (9), the strain energy components of the potential energy are

$$U_f = \frac{D(\theta)a}{4}\left(\frac{\pi}{a}\right)^4 W^2$$
$$U_c = \frac{a}{4}\left[\frac{1}{h^2} f_1(h) + \left(\frac{\pi}{a}\right)^2 f_2(h)\right] W^2 \tag{13}$$
$$f_1(h) = \int_0^h E_c(z)\,dz \qquad f_2(h) = \int_0^h G_c(z)\left(\frac{h-z}{h}\right)^2 dz$$

The energy of the applied stresses is given by

$$U_\sigma = -\frac{\sigma_x\left(c^2\theta + k_1 s^2\theta + k_2 c\theta s\theta\right)ta}{4}\left(\frac{\pi}{a}\right)^2 W^2 \tag{14}$$

The critical stress for the prescribed angle can be now evaluated and the wrinkling stress is obtained as

$$\sigma_x^{wr} = \min_\theta\left(\sigma_x^{cr}(\theta)\right) = \min_\theta\left\{\left\{\frac{D(\theta)}{t}\left(\frac{\pi}{a_\theta}\right)^2 + \frac{1}{t}\left[\frac{1}{{h_\theta}^2}\left(\frac{a_\theta}{\pi}\right)^2 f_1(h_\theta) + f_2(h_\theta)\right]\right\}\left(c^2\theta + k_1 s^2\theta + k_2 c\theta s\theta\right)^{-1}\right\}$$

(15)

where a_θ and h_θ are the values of the length of the wrinkling wave and the depth of the core affected by wrinkling, respectively, corresponding to the critical value of the stress σ_x at the prescribed value of the angle θ. These values can be determined numerically or analytically by the minimization of the critical stress with respect to a and h yielding the system of equations

$$\frac{\partial\sigma_x^{cr}(\theta)}{\partial a} = 0 \;\rightarrow\; a_\theta = \pi\sqrt[4]{\frac{D(\theta)h_\theta^2}{f_1(h_\theta)}}$$
$$\frac{\partial\sigma_x^{cr}(\theta)}{\partial h} = 0 \;\rightarrow\; \frac{1}{h_\theta}\sqrt{\frac{D(\theta)}{f_1(h_\theta)}}\left(\frac{\partial f_1(h)}{\partial h}\right)_{h=h_\theta} - \frac{2f_1(h_\theta)}{h_\theta^2}\sqrt{\frac{D(\theta)}{f_1(h_\theta)}} + \left(\frac{\partial f_2(h_\theta)}{\partial h}\right)_{h=h_\theta} = 0 \tag{16}$$

In the case of a homogeneous core and in the absence of the shear stress equation (15) converges to the solution in [11]. In the case of a uniaxial compression of a functionally graded sandwich panel the present solution converges to the result available from [1].

A possible improvement to the accuracy of the Hoff method could be achieved using the power-series expression for a decay of through-the-thickness core displacements employing a polynomial expression for the function

$$F(z)=\sum_{i=0}^{i=N} b_i\left(\frac{2z}{h_c}\right)^i,\ 0\le z\le h_c \tag{17}$$

where b_i are coefficients and h_c is the thickness of the core. The series (17) converge to the Hoff method if $b_0=1,\ b_1=-{h_c}/{2h},\ b_j=0\ (j\ge 2)$ where h is an unknown depth of the core affected by wrinkling displacements. Accordingly, the unknowns in case of two-dimensional loading include are the wrinkling stress, length, depth and orientation of the wrinkle and N coefficients b_i. A detailed analysis of the correction introduced by eqn. (17) using a polynomial expression for $F(z)$ instead of a linear function as in the Hoff method is outside the scope of the current investigation.

4. WRINKLING UNDER BIAXIAL COMPRESSION AND/OR IN-PLANE SHEAR: LAYER-WISE GRADING OF THE CORE – ELASTICITY SOLUTION

This section contains an expansion of the previous study to a particular case of a functionally graded core where it consists of layers of material with different mass density (and stiffness) (Fig. 1). As was demonstrated in Ref. 1, concentrating the layers of higher stiffness near the facing-core interface results in a significant increase in the wrinkling stress, without a weight penalty. The advantage of the solution presented in Ref. 1 is the accurate evaluation of the stresses in the core that is accomplished using the linear elasticity theory. Subsequently, these stresses are utilized in the framework of the energy method yielding the wrinkling stress.

The approach developed in Ref. 1 is further expanded in this paper to account for a combined compression and/or in-plane shear applied to the sandwich panel. Therefore, the solution is outlined here only to the extent necessary to account for a more complicated loading, as compared to the case of uniaxial compression considered in the previous study.

The assumptions utilized in the analysis are:

- Linear elasticity theory is applicable to the analysis of both the facing and core;
- Each layer of the core is homogeneous and isotropic;
- The problem is geometrically linear;
- The integrity of bonds between the facing and core and between the layers of the core is not violated;
- Axial displacements in the core in the direction perpendicular to the wrinkling wave are negligible.

In the $x_1 - y_1 - z$ coordinate system the linear elasticity equations for the i-th layer of the core are

$$\begin{aligned} \sigma_{x_1}^{(i)},_{x_1} + \tau_{x_1 z}^{(i)},_{z} &= 0 \\ \tau_{x_1 z}^{(i)},_{x_1} + \sigma_{z}^{(i)},_{z} &= 0 \end{aligned} \qquad (18)$$

Using the elastic constitutive equations and linear strain-displacement relationships where displacements perpendicular to the wrinkle are disregarded (see Ref. 1 for details), the equations of equilibrium (18) become

$$\begin{aligned} & w^{(i)},_{x_1 z} = 0 \\ & Q^{(i)}_{x_1 x_1} w^{(i)},_{zz} + G^{(i)} w^{(i)},_{x_1 x_1} = 0 \end{aligned} \tag{19}$$

where $w^{(i)}$ is the displacement of the i-th layer of the core in the z-direction, $Q^{(i)}_{x_1 x_1}$ is the reduced stiffness of the i-th layer and $G^{(i)}$ is its shear modulus.

The mode shape of the wrinkle in the facing is represented as shown in Eqn. (2) where the function $F(z) = 1$. Contrary to the case of an arbitrary grading of the core, in this solution we do not make an assumption regarding the function $F(z)$, but derive it from the elasticity equations.

The solution of the second (19) is found exactly, while the first equation is satisfied in the integral sense by representing the displacement as a product of two functions:

$$w^{(i)} = X_i(x_1) Z_i(z_i) \tag{20}$$

where the coordinate z_i is counted from the interface between the i-th and (i-1)-th layers (for the layer adjacent to the facing-core interface $z_1 = 0$ at the interface.).

The substitution of (20) into the second equation (19) and the separation of variables enables us to evaluate the functions $X_i(x_1)$ and $Z_i(z_i)$, so that the displacement in the i-th layer can be represented by

$$w^{(i)} = \left(C^{(i)}_1 \sinh \lambda_i z_i + C^{(i)}_2 \cosh \lambda_i z_i \right) W \sin f_i \lambda_i x_1 = Z_i(z_i) W \sin f_i \lambda_i x_1 \tag{21}$$

where

$$f_i = \sqrt{\frac{Q_{x_1x_1}^{(i)}}{G^{(i)}}}, \quad \lambda_i = \frac{\pi}{af_i} \tag{22}$$

The constants of integration $C_j^{(i)},\ j = 1,2$ can be specified from the continuity conditions for the displacements at the facing-core and layer-to-layer interfaces as well as the conditions of continuity of normal and transverse shear strains between the adjacent layers. For example, if wrinkling is symmetric about the middle plane of the sandwich panel, we can obtain the constants of integration from

$$\begin{aligned}
&Z_1(z_1 = 0) = 1 \\
&Z_1(z_1 = t_1) = Z_2(z_2 = 0) \\
&Z_1'(z_1 = t_1) = Z_2'(z_2 = 0) \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&Z_i(z_i = t_i) = Z_{i+1}(z_{i+1} = 0) \\
&Z_i'(z_i = t_i) = Z_{i+1}'(z_{i+1} = 0) \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&Z_N\left(z_N = \frac{t_N}{2}\right) = 0
\end{aligned} \tag{23}$$

where the number of layers from the interface to the middle plane is equal to N , the $N-th$ layer being bisected by the middle plane, $1 < i < N$, the thickness of the $i-th$ layer is denoted by t_i and $(\ldots)' = \frac{d(\ldots)}{dz}$. Modifications of eqns. (23) in cases of antisymmetric wrinkling or wrinkling of only one facing in case of bending load applied to the panel are discussed in Ref. 1.

The potential energy of the sandwich panel with wrinkling can now be evaluated using the stresses in the core determined above by the theory of elasticity approach. The strain energy in the facing is determined by the first equation (13) and the energy of the applied stresses is given by eqn. (14). The strain energy in the core in the case of symmetric wrinkling is

$$U_c = \frac{1}{2}\int_0^l \sum_{i=1}^N \int_{z_i=0}^{z_i=t_i} \frac{\left(\sigma_z^{(i)}\right)^2}{Q_{x_1x_1}^{(i)}} dx_1 dz + \frac{1}{2}\int_0^l \sum_{i=1}^N \int_{z_i=0}^{z_i=t_i} \frac{\left(\tau_{x_1z}^{(i)}\right)^2}{G^{(i)}} dx_1 dz$$

$$= \frac{a}{4} W^2 \sum_{i=1}^N \left\{ Q_{x_1x_1}^{(i)} \int_{z_i=0}^{z_i=t_i} \left[Z_i\left(z_i\right),_z\right]^2 dz + \left(\frac{\pi}{a}\right)^2 G^{(i)} \int_{z_i=0}^{z_i=t_i} \left[Z_i\left(z_i\right)\right]^2 dz \right\} \tag{24}$$

where the upper limit of the integral for the $N-th$ layer should be its half-thickness, i.e. $z_N = \frac{t_N}{2}$.

The critical stress for a prescribed angle of orientation of the wrinkling wave can now be determined as

$$\sigma_x^{cr}\left(\theta\right) = \min_l \left\{ \frac{1}{\pi^2 t}\left(\frac{\pi^4 D\left(\theta\right)}{a^2} + a^2 F \right)\left(c^2\theta + k_1 s^2\theta + k_2 c\theta s\theta\right)^{-1} \right\},$$

$$F = \sum_{i=1}^N \left\{ Q_{11}^{(i)} \int_{z_i=0}^{z_i=t_i} \left[Z_i\left(z_i\right),_z\right]^2 dz + \left(\frac{\pi}{a}\right)^2 G^{(i)} \int_{z_i=0}^{z_i=t_i} \left[Z_i\left(z_i\right)\right]^2 dz \right\} = g_1 + \left(\frac{\pi}{a}\right)^2 g_2 \tag{25}$$

$$g_1 = \sum_{i=1}^N Q_{11}^{(i)} \int_{z_i=0}^{z_i=t_i} \left[Z_i\left(z_i\right),_z\right]^2 dz, \qquad g_2 = \sum_{i=1}^N G^{(i)} \int_{z_i=0}^{z_i=t_i} \left[Z_i\left(z_i\right)\right]^2 dz$$

The minimization of the critical stress yields both the length of the wrinkling wave corresponding to the angle θ and the corresponding critical stress:

$$a_{wr}\left(\theta\right) = \pi \sqrt[4]{\frac{D\left(\theta\right)}{g_1}}, \qquad \sigma_x^{cr}\left(\theta\right) = \frac{1}{t}\left(2\sqrt{D\left(\theta\right) g_1} + g_2\right) \tag{26}$$

The actual wrinkling stress of the sandwich panel can now be specified from eqn. (12).

5. NUMERICAL EXAMPLES

5.1. EXAMPLE INPUTS

The numerical analysis was undertaken for three different facings, i.e. glass/epoxy, E-glass/vinyl ester and graphite epoxy AS 3501 cross-ply facings. The properties of these materials are [32]:

Glass/epoxy:

$E_1 = 38.6GPa,\ E_2 = 8.27GPa,\ G_{12} = 4.14GPa,\ \nu_{12} = 0.26,\ V_f = 0.45$

E-glass/vinyl ester:

$E_1 = 24.4GPa,\ E_2 = 6.87GPa,\ G_{12} = 2.89GPa,\ \nu_{12} = 0.32,\ V_f = 0.30$

AS 3501 graphite/epoxy:

$E_1 = 138.0GPa,\ E_2 = 9.0GPa,\ G_{12} = 6.9GPa,\ \nu_{12} = 0.30,\ V_f = 0.65$

where V_f is the volume fraction of fibers.

Each cross-ply facings was composed of four 0.25-mm thick layers with the outermost 0^o layer at the top and bottom facings. The core material was polyurethane foam. The mass density and modulus of elasticity of solid polyurethane are $\rho = 1200\, {kg}/{m^3}$, $E = 1.6GPa$, respectively [30]. Two core thicknesses were analyzed, i.e. 10mm and 25mm. The mass density of the graded core varied from 10% of the density of solid polyurethane at the middle plane of the sandwich panel to 35% of this density at the interface with the facings. Two grading schemes were compared, representing the mass density of the core by the linear and quadratic functions of the z-coordinate counted from the facing-core interface $(z = 0)$:

Linear grading:

$$\rho_c(z) = \left(0.35 - 0.5\frac{z}{h_c}\right)\rho \qquad (27)$$

Quadratic grading:

$$\rho_c(z) = \left(0.35 - \left(\frac{z}{h_c}\right)^2\right)\rho \tag{28}$$

where h_c is the thickness of the core.

The local stiffness of the foam, i.e. the effective moduli of elasticity (E_c) and shear (G_c), were obtained as functions of the ratio of the local mass density of the core to that of the solid material and the solid core material modulus of elasticity E by the empirical formulae of Gibson and Ashby [30], [33]:

$$\frac{E_c}{E} = k_1'\left(\frac{\rho_c}{\rho}\right)^{n_1}, \qquad \frac{G_c}{E} = k_2'\left(\frac{\rho_c}{\rho}\right)^{n_2} \tag{29}$$

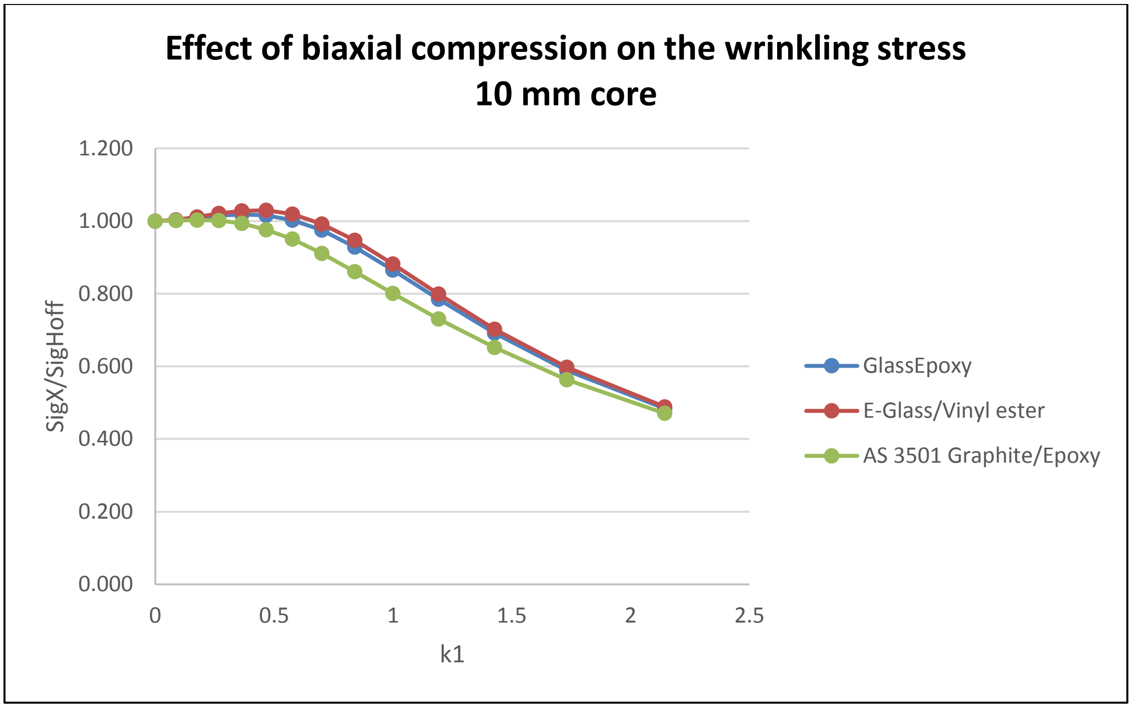
where k_r and n_r $(r = 1,2)$ are empirical constants reported as $k_1' = 1$, $k_2' = 0.4$ and $n_1 = n_2 = 2$.

The following numerical results address two questions. The first set of results (Figs. 3-14) illustrates the effect of biaxial compression and/or in-plane shear on the wrinkling stress specified as the stress σ_x corresponding to wrinkling for given coefficients k_1 and/or k_2 in sandwich panels with linear and quadratic mass density grading. The second set of results (Figs. 15-18) is concerned with the effect of grading on the wrinkling stress for a variety of k_1 and/or k_2. In these results, all stresses are normalized with respect to the uniaxial Hoff wrinkling stress for the sandwich panel with a homogeneous core that has the same total weight as the weight of compared graded core panels. The curves shown in these figures for a homogeneous core panel demonstrate the effect of biaxial compression and/or in-plane shear for a panel without grading, so that these curves can be compared to the results shown for graded core panels in Figs. 3-14.

The results shown in Figs. 3-14 are concerned with the effect of biaxial compression and/or in-plane shear on wrinkling. The wrinkling stresses were normalized with respect to those obtained by the Hoff theory for the sandwich panel of equal weight with the core density of 22.5% and 18.3% of that of solid polyurethane for the cases of linear and quadratic grading, respectively, and subject to uniaxial compression in the x-direction $(k_1 = k_2 = 0)$. Accordingly, the ratio SigX/SigHoff represents the wrinkling value of the applied stress σ_x^{wr} normalized with respect to the wrinkling stress by the classical Hoff solution for the uniaxially compressed panel with a homogeneous core.

5.2. LINEAR GRADED EXAMPLES

The effect of biaxial compression on the wrinkling stress of sandwich panels is demonstrated in Fig. 3 for the case of a linear mass density variation of the core. Predictably, additional compression in the y-direction results in a reduction in the value of σ_x necessary to trigger wrinkling. While the absolute values of the wrinkling stress vary, dependent on the material of the facings, the general trend demonstrate in Fig. 2 is consistent for all facing materials. Such similarity of the curves for various loading combinations for a relative wrinkling stress remains a common feature in all subsequent results. The angle of the orientation of the wrinkling wave for a linear mass density variation is shown in Fig. 4. The wave is parallel to the y-axis, i.e. $\theta = 0$, when the uniaxial loading is applied in the x-direction. As the load increases, the wave rotates and at an equal biaxial compression in both x- and y-directions, $k_1 = 1$, the wave is perpendicular to either the x- or y-axes (in this case there is no difference between axes x and y). Note that the effect of the thickness of the core is minimal, both on the relative wrinkling stress in the x-direction shown in Fig. 3 as well as on the angle of orientation of the wrinkling wave (Fig. 4). In conclusion, the effect of biaxial compression is both reducing the maximum applied compressive stress resulting in wrinkling as well as reorienting the wrinkling wave so that it “rotates” from the position orthogonal to the largest compressive stress assuming the orientation inclined relative to both applied stresses.



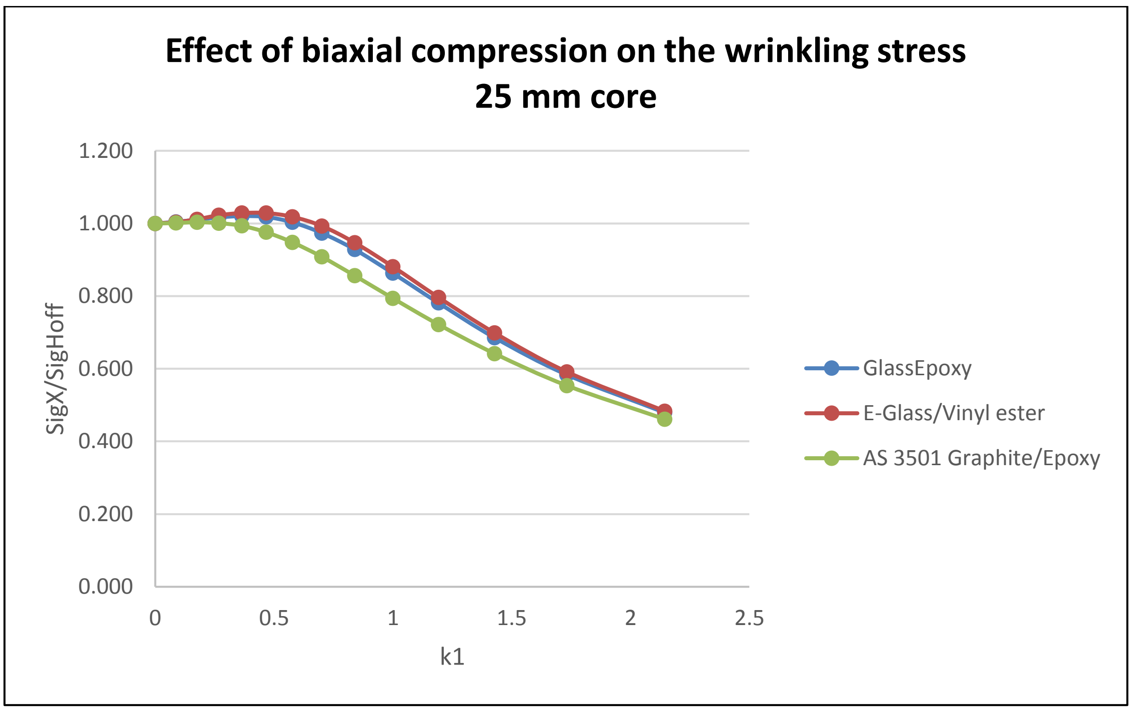
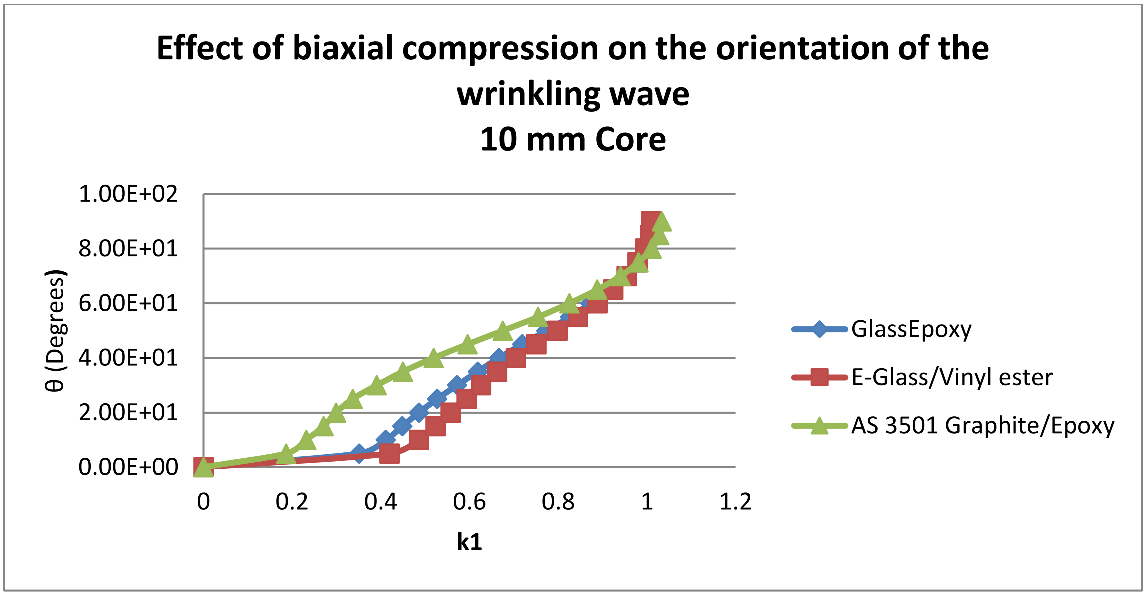


Figure 3 The effect of biaxial compression on the wrinkling stress in cases of 10-mm thick and 25-mm thick cores with a linear mass density grading through the thickness



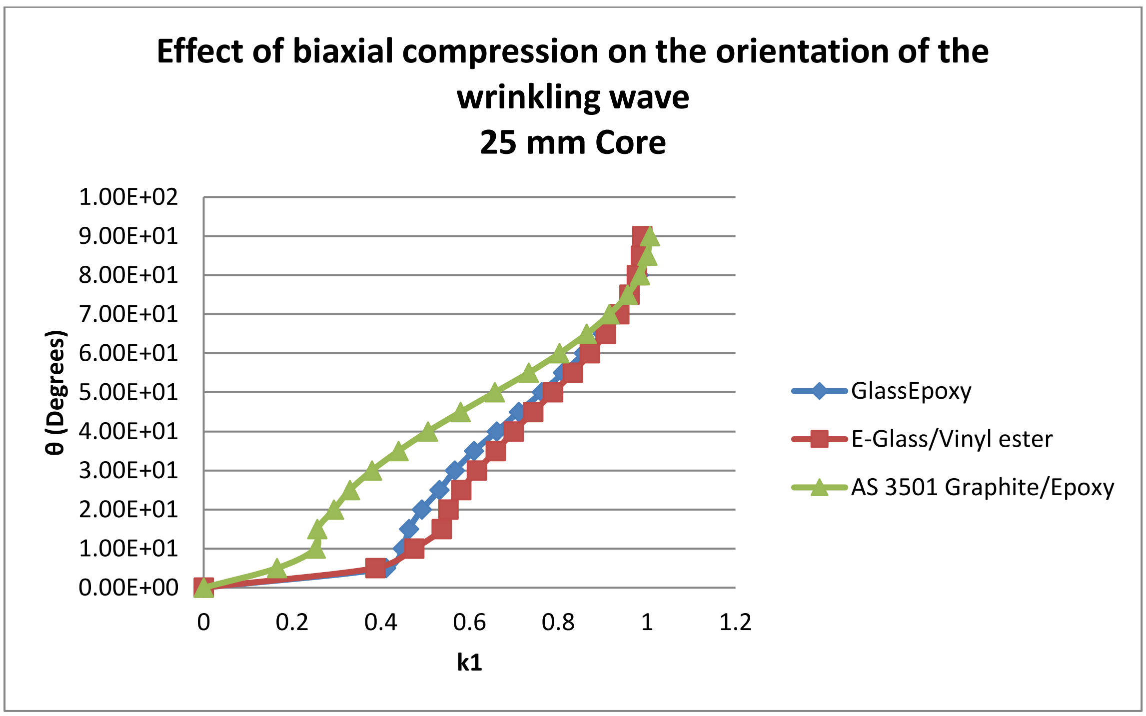
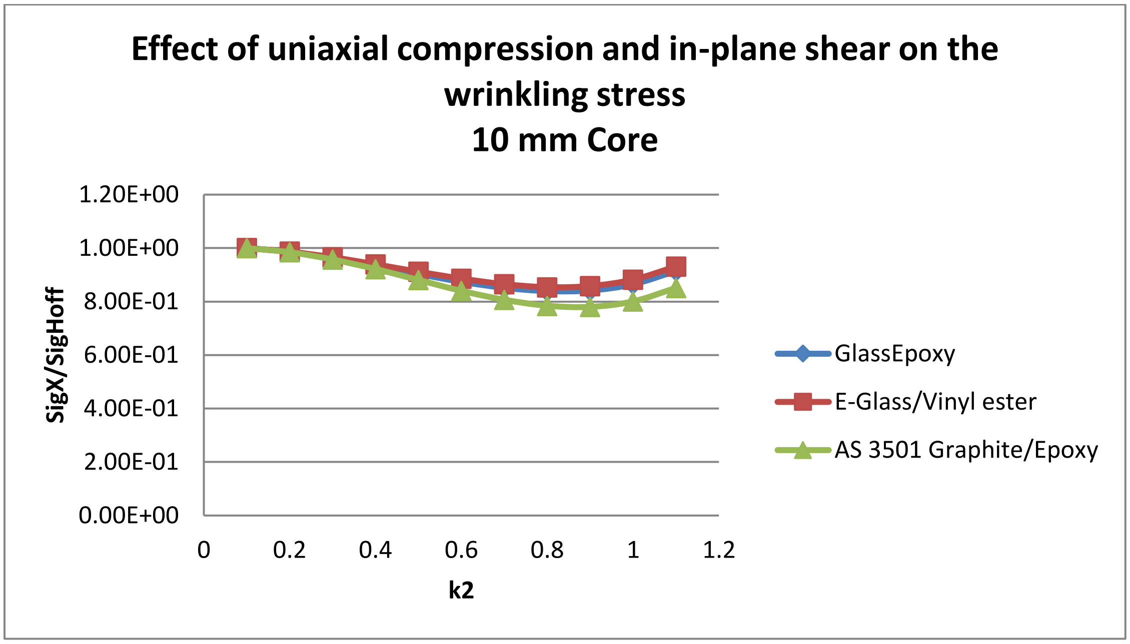


Figure 4 The angle of orientation of the wrinkling wave relative to the y-axis in cases of 10-mm thick and 25-mm thick cores with a linear mass density grading through the thickness

The effect of a combined action of uniaxial compression in the x-direction and in-plane shear on both the wrinkling stress and orientation of the wrinkling wave is shown in Figs. 5 and 6, respectively. While the value of the normalized compressive wrinkling stress remains practically unaffected by the thickness of the core, the effect of in-plane shear is different from that for biaxial compression. As in-plane shear stress increases, the compressive wrinkling stress is reduced, but at larger in-plane shear, this trend is reversed. The orientation of the wrinkling wave changes due to in-plane shear, although it never “rotates” by 90 degrees as in the case of biaxial compression.



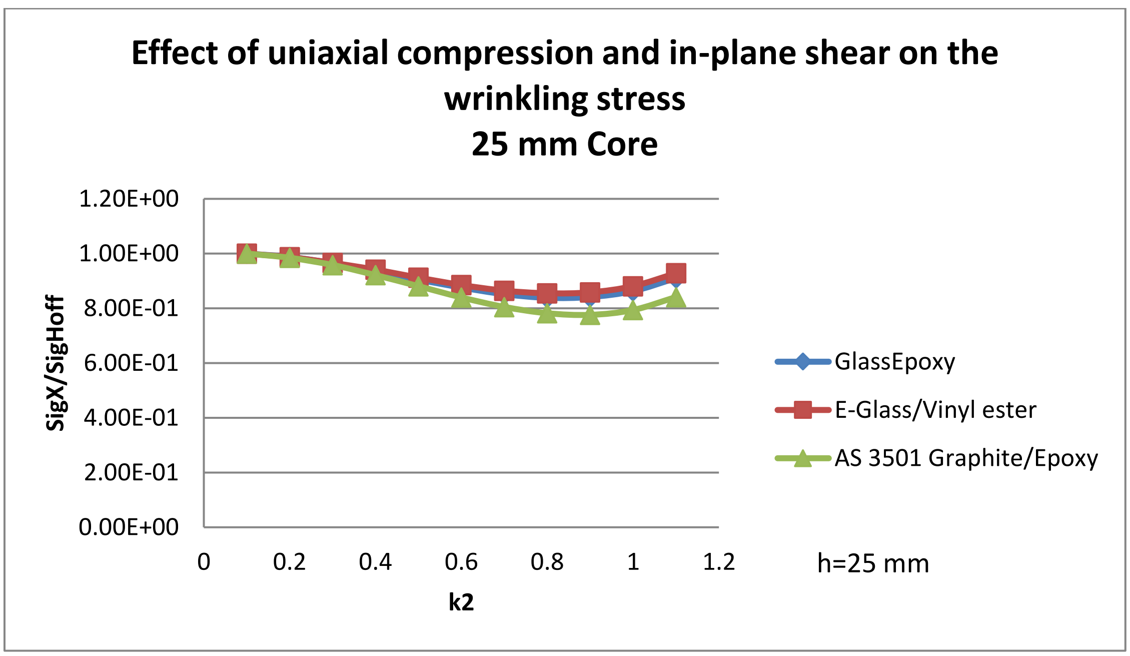
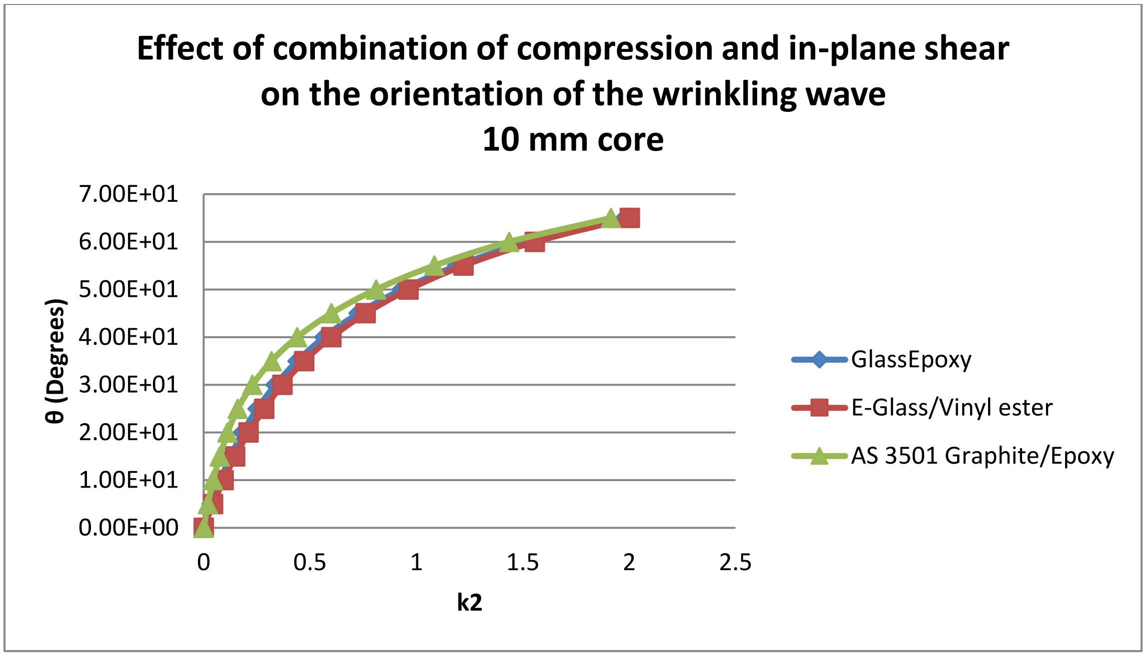


Figure 5 The effect of combined uniaxial compression and in-plane shear on the wrinkling stress in cases of 10-mm thick and 25-mm thick cores with a linear mass density grading through the thickness



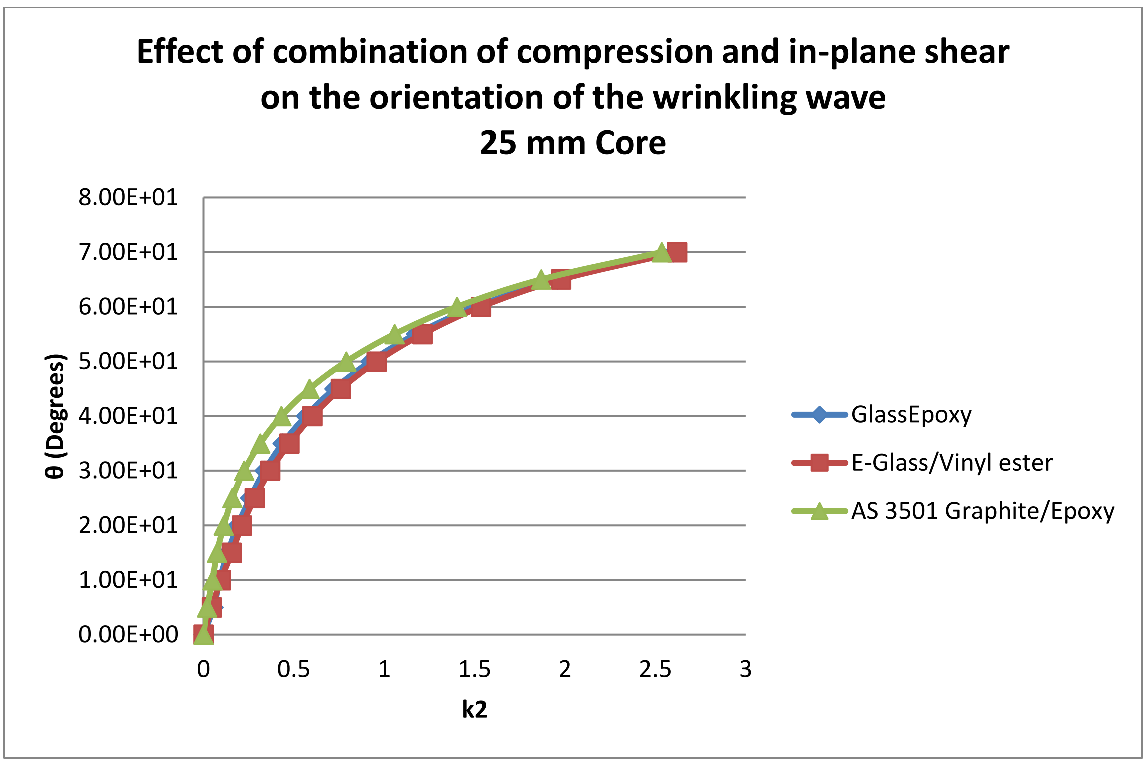
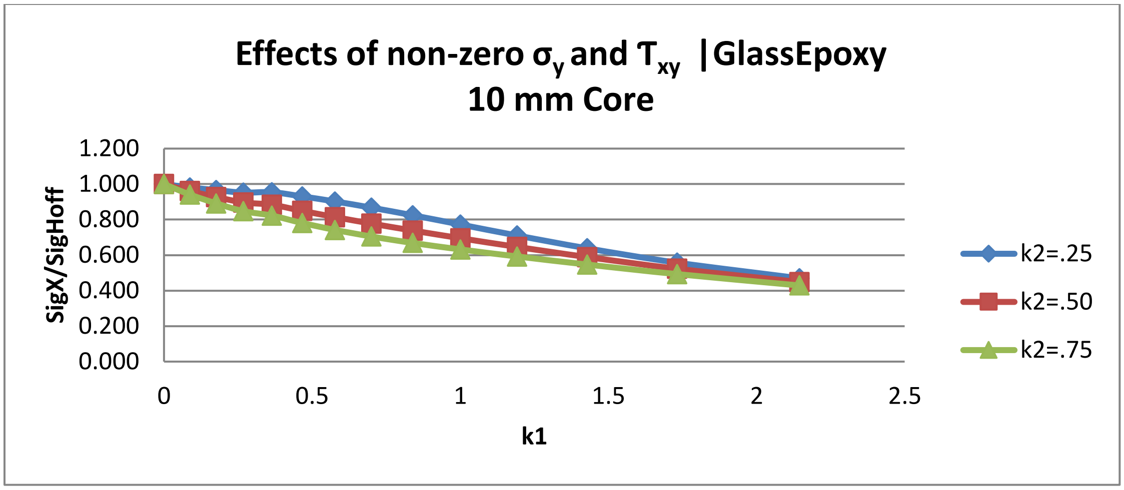
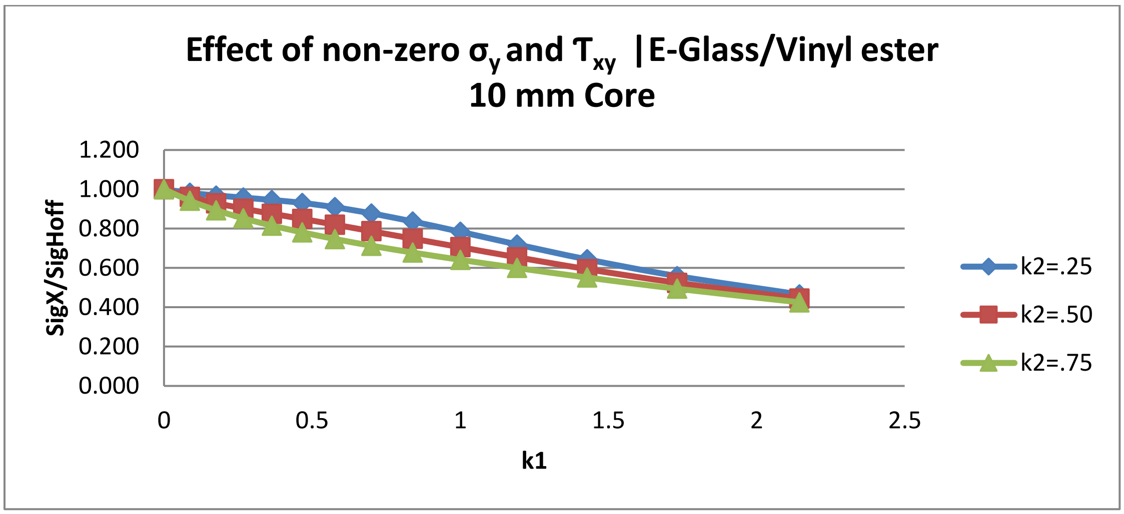


Figure 6 The angle of orientation of the wrinkling wave relative to the y-axis under uniaxial compression and in-plane shear in cases of 10-mm thick and 25-mm thick cores with a linear mass density grading through the thickness





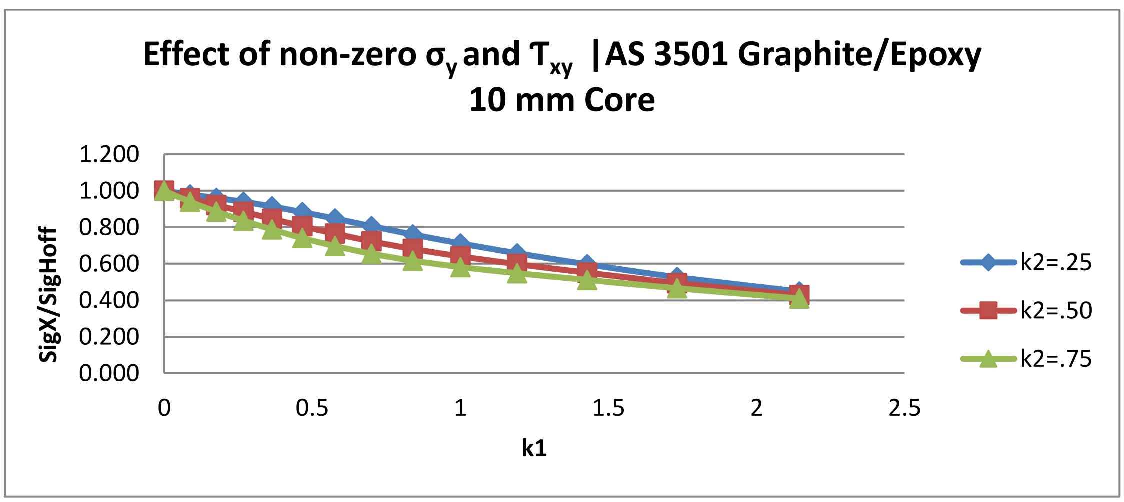


Figure 7 The effect of combined biaxial compression and in-plane shear on the wrinkling stress for panels with various facing materials and 10-mm thick cores with a linear mass density grading through the thickness

The general case of a combination of biaxial compression and in-plane shear is shown in Fig. 7 for 10-mm thick core with a linear mass density grading. An increased in-plane shear noticeably reduces the relative wrinkling stress, particularly if the compressive stress in the y-direction is in range between about one-third to 1.5 times the stress in the x-direction. A thicker core does not qualitatively affect the results compared to the panel from the same materials but with a 10-mm core in Fig. 7, as shown for panels with a 25-mm core in Fig. 8.

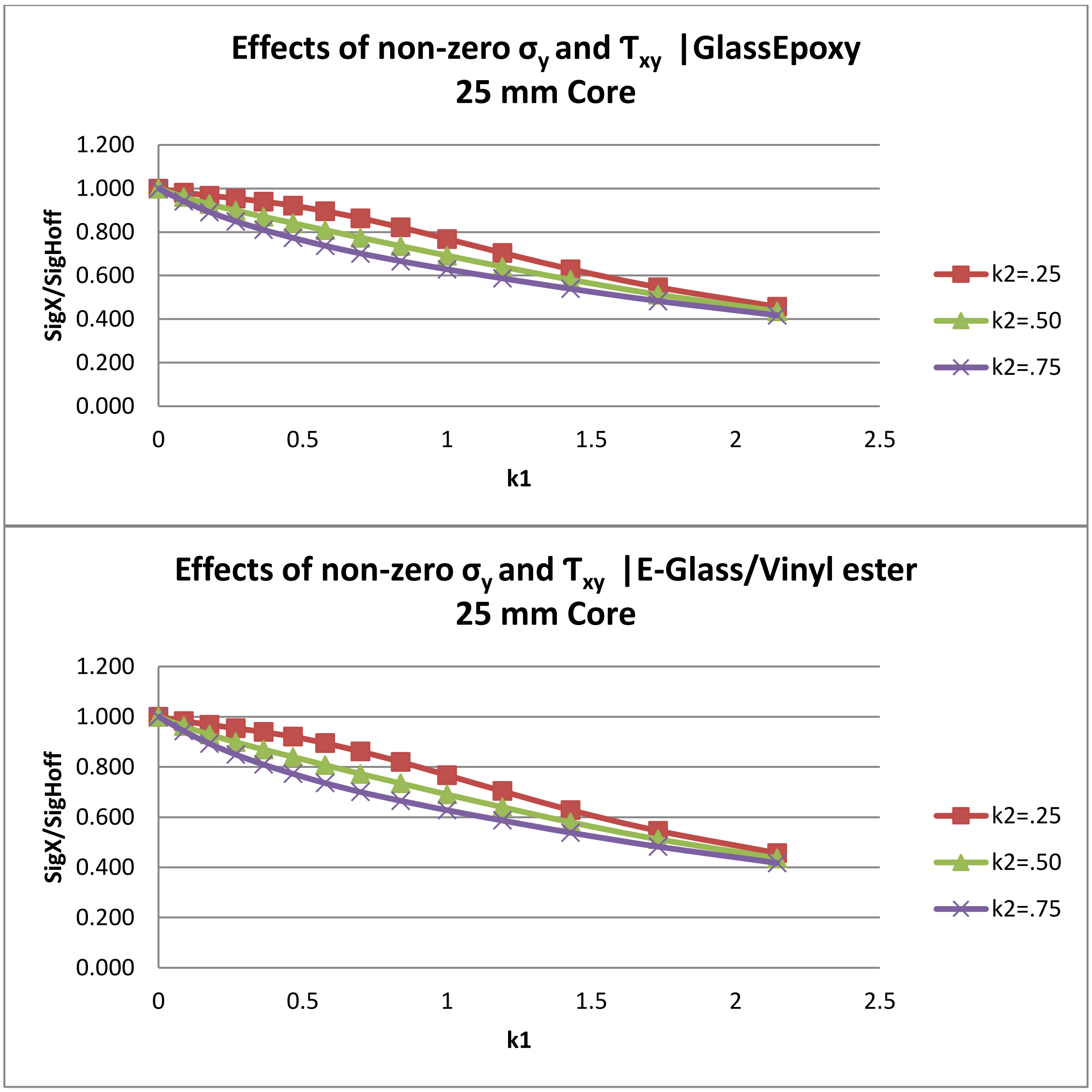


Figure 8 The effect of combined biaxial compression and in-plane shear on the wrinkling stress for panels with various facing materials and 25-mm thick core with a linear mass density grading through the thickness

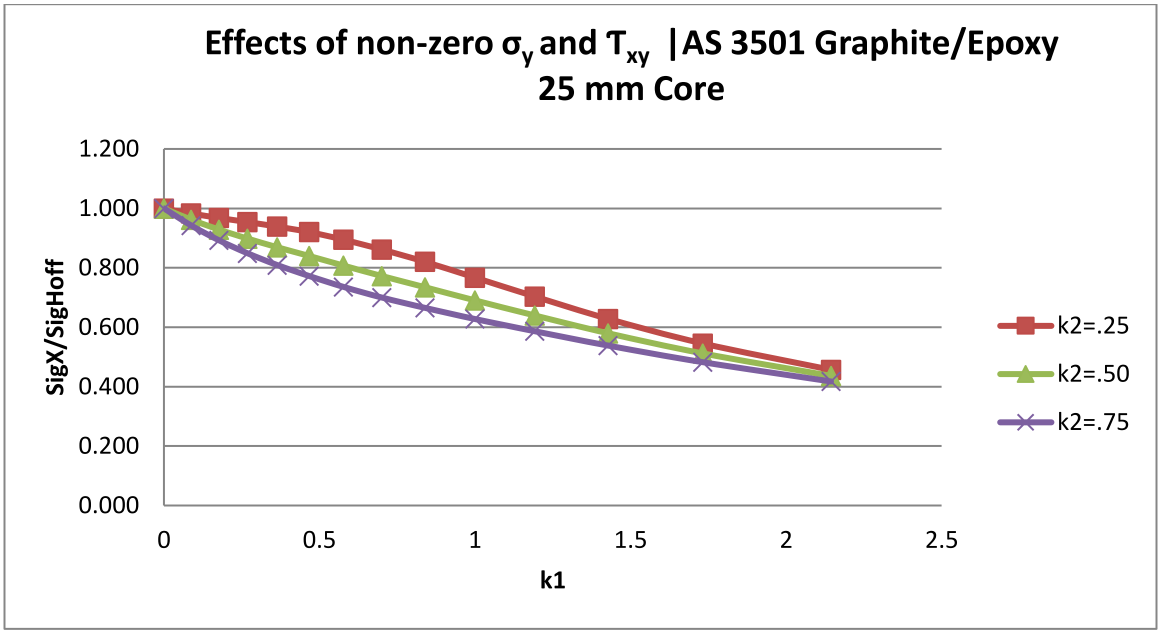


Figure 8 The effect of combined biaxial compression and in-plane shear on the wrinkling stress for panels with various facing materials and 25-mm thick core with a linear mass density grading through the thickness (cont.)

5.3. QUDRATICALLY GRADED EXAMPLES

As follows from the previous study [1] and the analysis in the present paper discussed below, functional grading of the core resulting in a higher stiffness in the regions adjacent to the facings enhances wrinkling stability. The following results (Figs. 9-14) are concerned with the effect of biaxial compression and/or in-plane shear on the wrinkling stress σ_x for sandwich panels with a quadratic through-the-thickness mass density grading of the core. These panels have even higher local core stiffness immediately under the facings than their counterparts with a linear mass density grading. Therefore, their behavior when subject to biaxial and/or in-plane shear loading as compared to the response of their counterparts with a linearly graded core mass density represents an interest for a better insight into a desirable grading design.

As is evident from the following results, the trends associated with biaxial and/or in-plane shear loading are almost identical for panels with linear and quadratic grading. In particular, a comparison of the curves for the same grading materials in Fig. 9

(quadratic grading) to their counterparts in Fig. 3 (linear grading) reveals a very close similarity between the relative wrinkling stresses. Furthermore, the orientations of the wrinkling waves are very close in the cases of a quadratic grading of the core (Fig. 10) and a linearly graded core (Fig. 4). The effect of in-plane shear on the relative wrinkling stress and orientation of the wrinkling wave demonstrated for sandwich panels with a quadratic grading in Figs. 11 and 12 are also similar to those in their linearly graded counterparts (Figs. 5 and 6). Similarity is also observed in the relative wrinkling stress in the panels with quadratically and linearly graded cores subject to a combination of biaxial and in-plane shear loads (Fig. 13 vs. Fig. 7 and Fig. 14 vs. Fig. 8).

It is emphasized that the similarity in the relative wrinkling stresses in quadratic and linear graded core panels observed above does not imply that the absolute values of the wrinkling stresses are close to each other. The quadratic grading results in a higher wrinkling stability of the facings and higher absolute values of the corresponding stresses as follows from the results shown below.

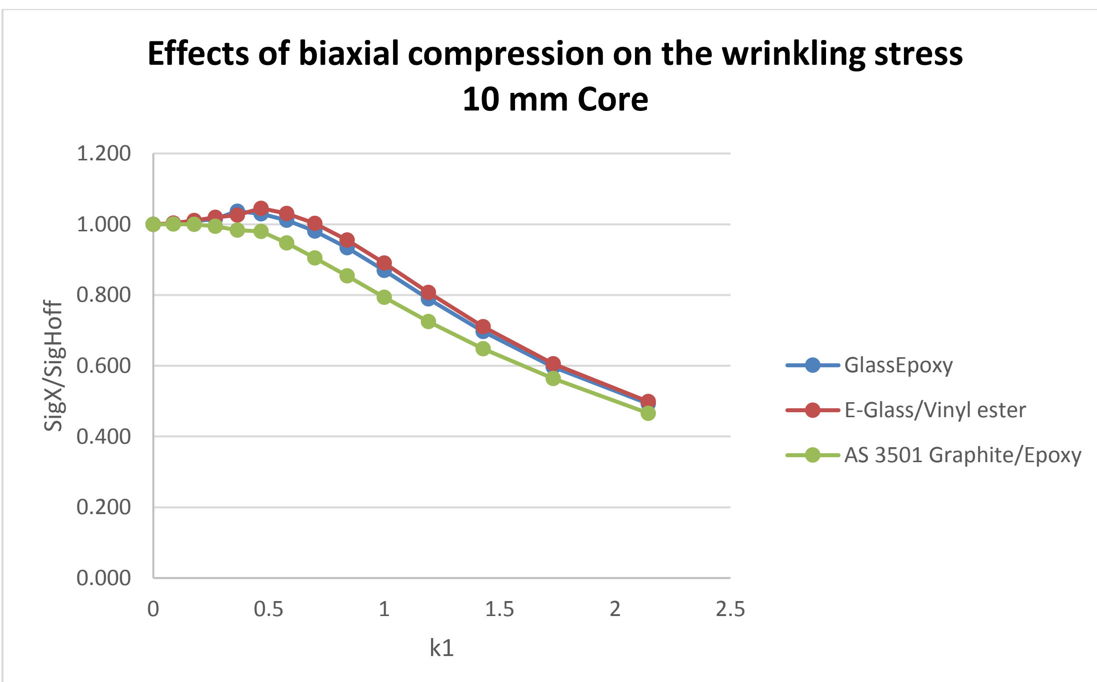


Figure 9 The effect of biaxial compression on the wrinkling stress in cases of 10-mm thick and 25-mm thick cores with a quadratic mass density grading through the thickness

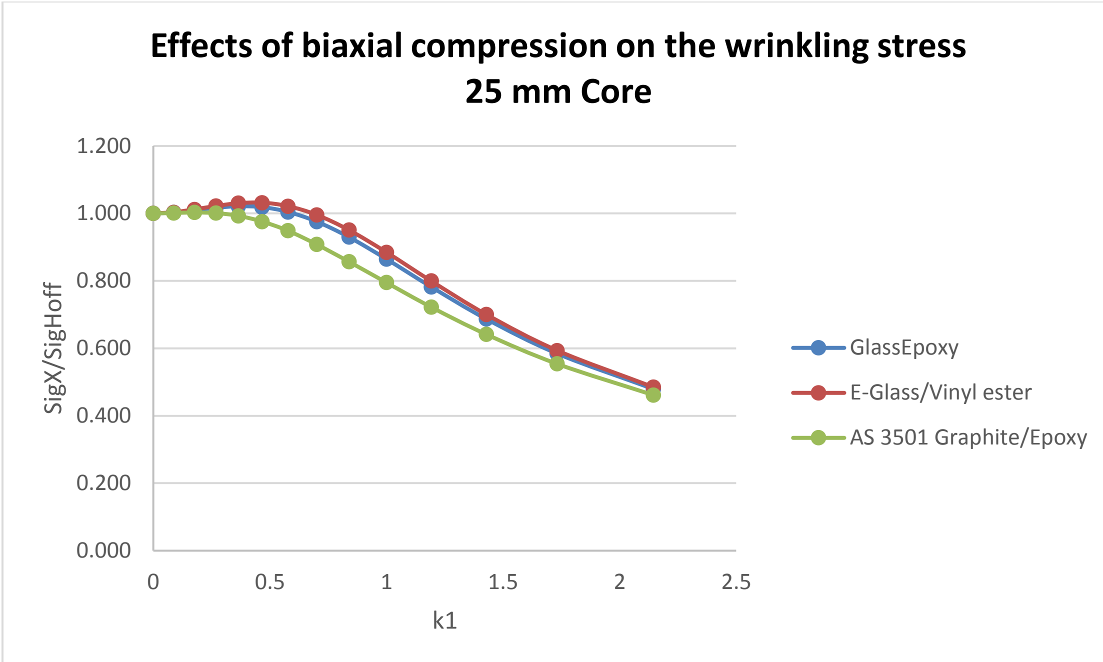


Figure 9 The effect of biaxial compression on the wrinkling stress in cases of 10-mm thick and 25-mm thick cores with a quadratic mass density grading through the thickness (cont.)

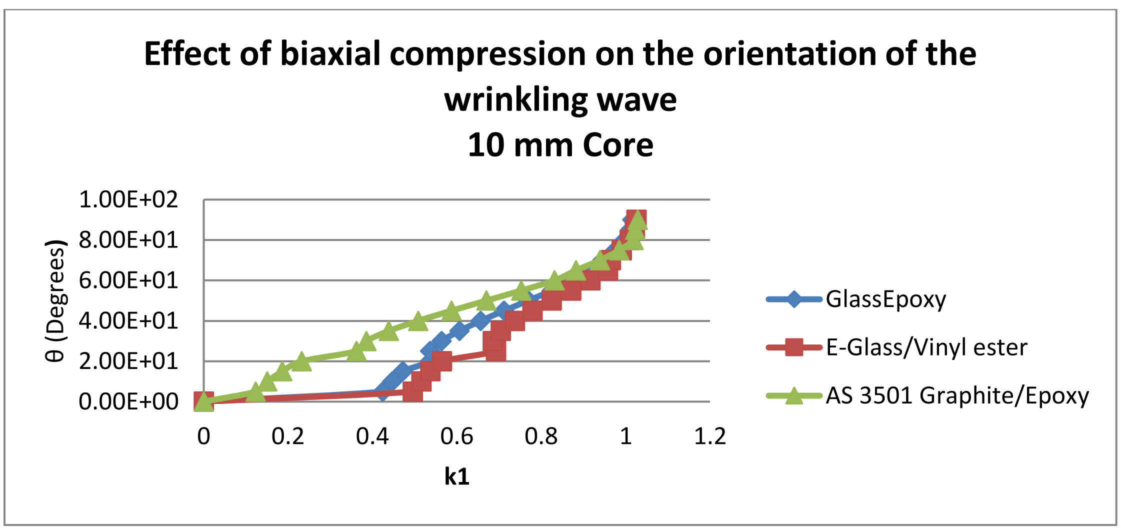


Figure 10 The angle of orientation of the wrinkling wave relative to the y-axis in cases of 10-mm thick and 25-mm thick cores with a quadratic mass density grading through the thickness

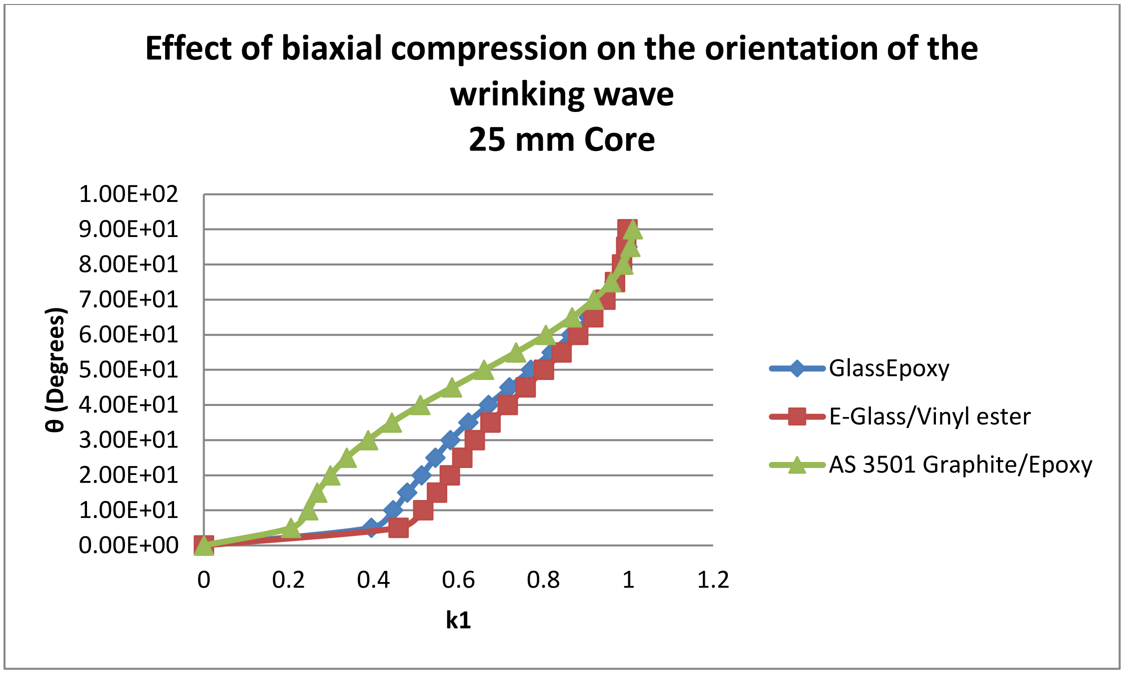


Figure 10 The angle of orientation of the wrinkling wave relative to the y-axis in cases of 10-mm thick and 25-mm thick cores with a quadratic mass density grading through the thickness (cont.)

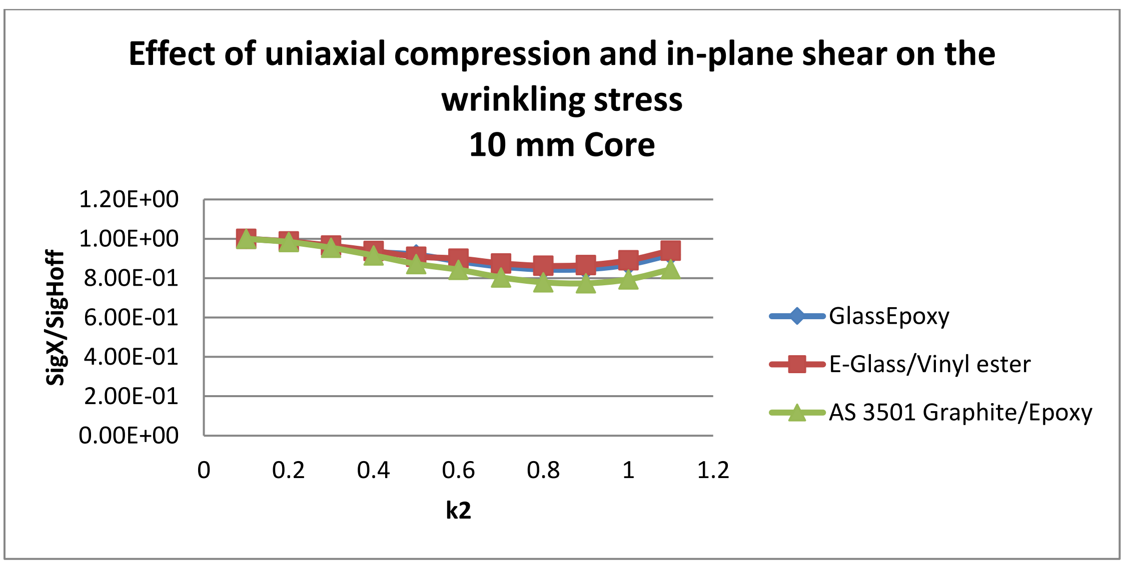


Figure 11 The effect of uniaxial compression and in-plane shear on the wrinkling stress in cases of 10-mm thick and 25-mm thick cores with a quadratic mass density grading through the thickness

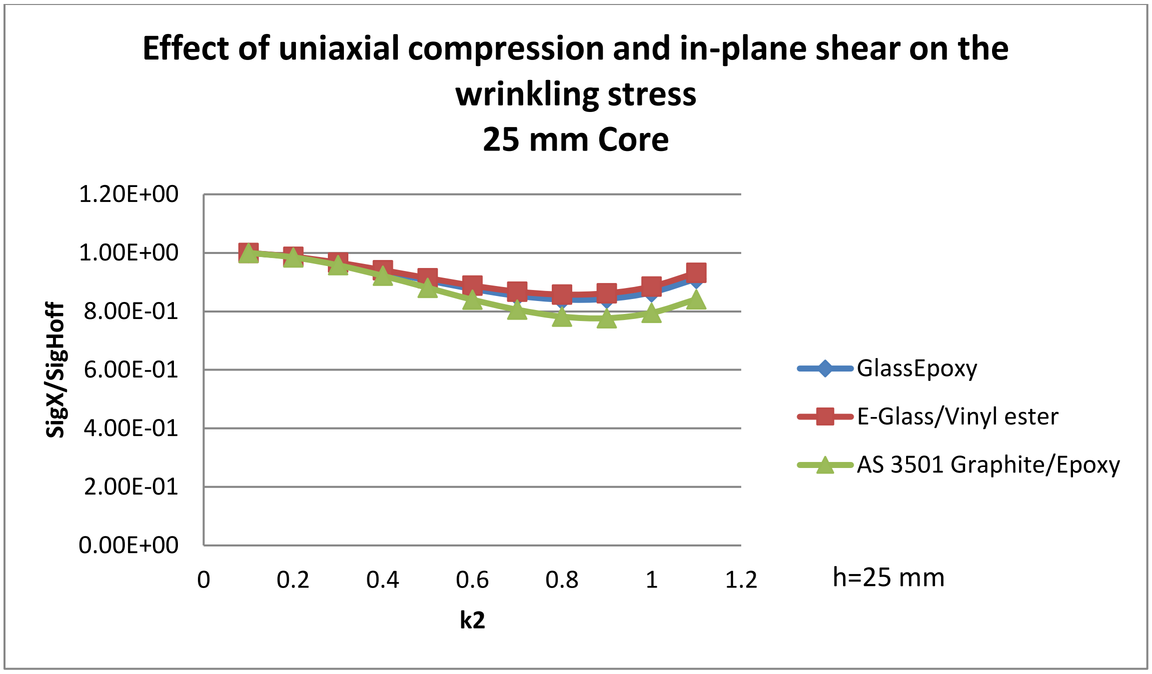


Figure 11 The effect of uniaxial compression and in-plane shear on the wrinkling stress in cases of 10-mm thick and 25-mm thick cores with a quadratic mass density grading through the thickness (cont.)

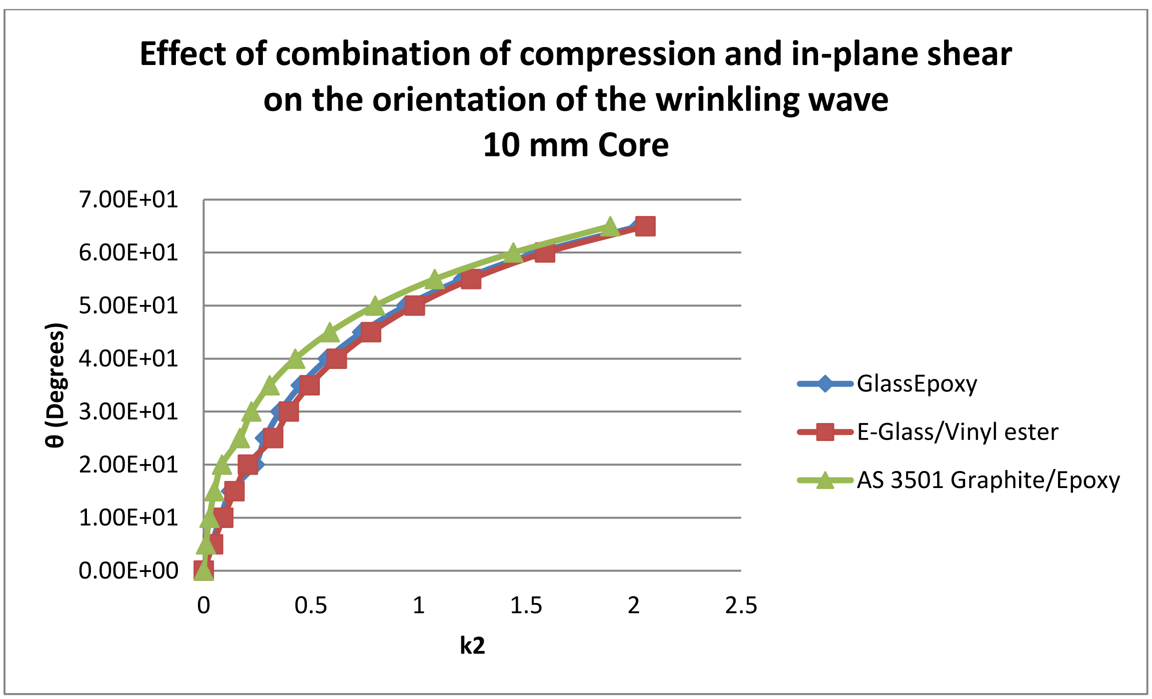


Figure 12 The angle of orientation of the wrinkling wave relative to the y-axis under uniaxial compression and in-plane shear in cases of 10-mm thick and 25-mm thick cores with a quadratic mass density grading through the thickness

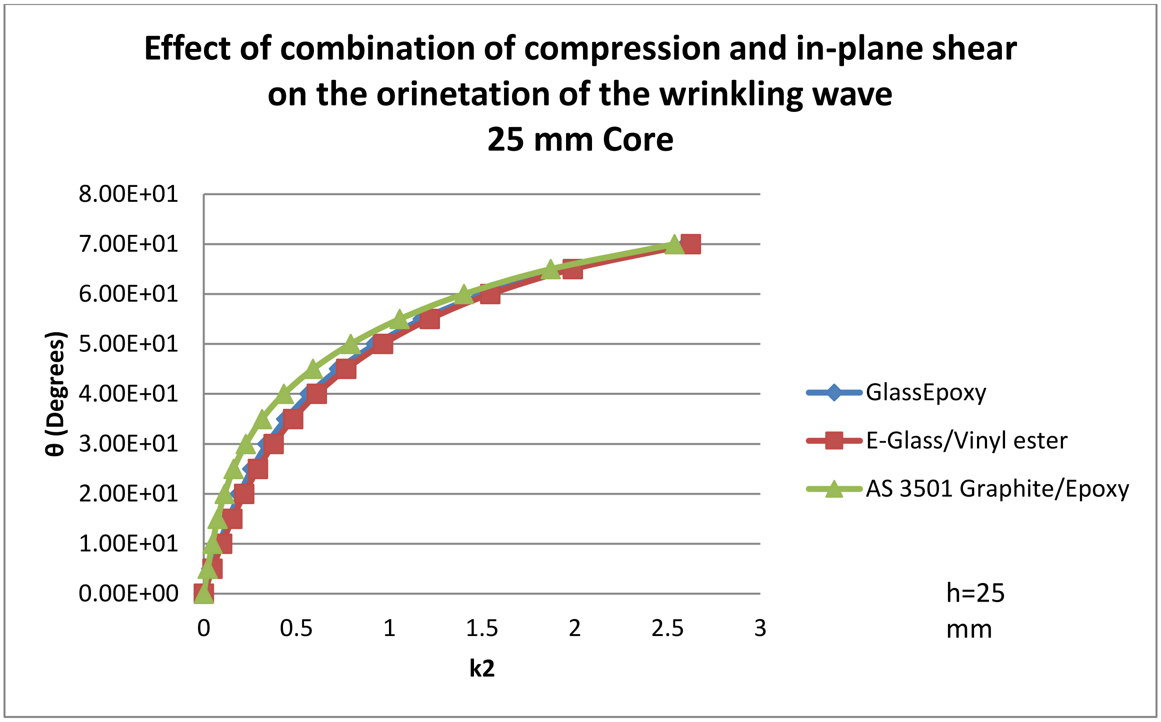
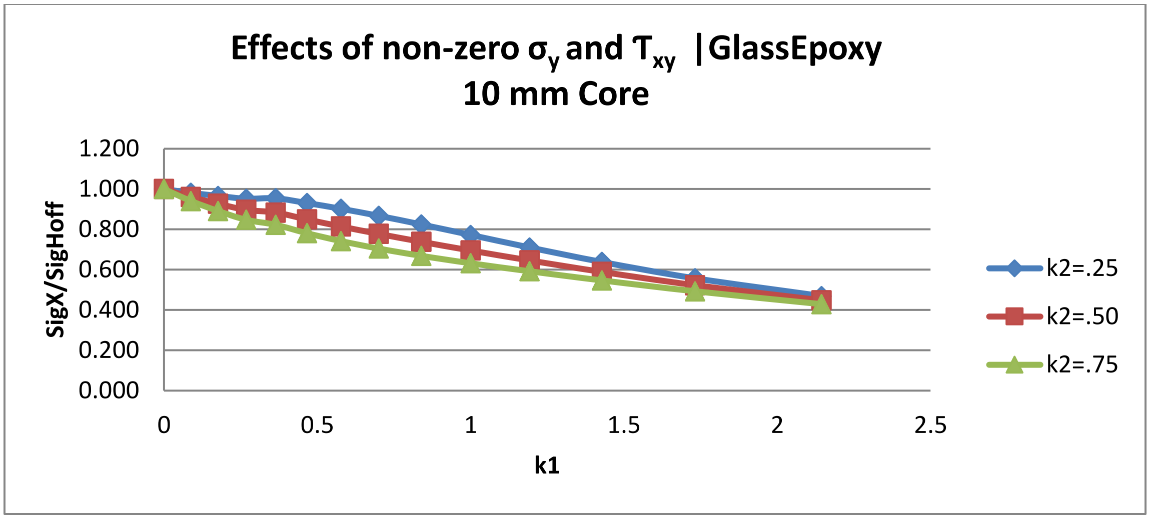
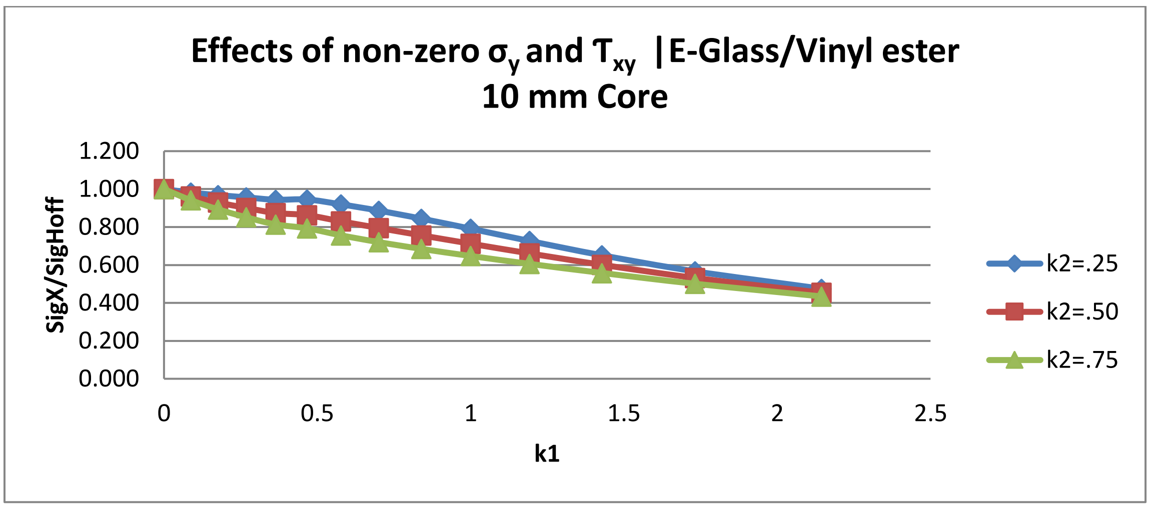


Figure 12 The angle of orientation of the wrinkling wave relative to the y-axis under uniaxial compression and in-plane shear in cases of 10-mm thick and 25-mm thick cores with a quadratic mass density grading through the thickness (cont.)





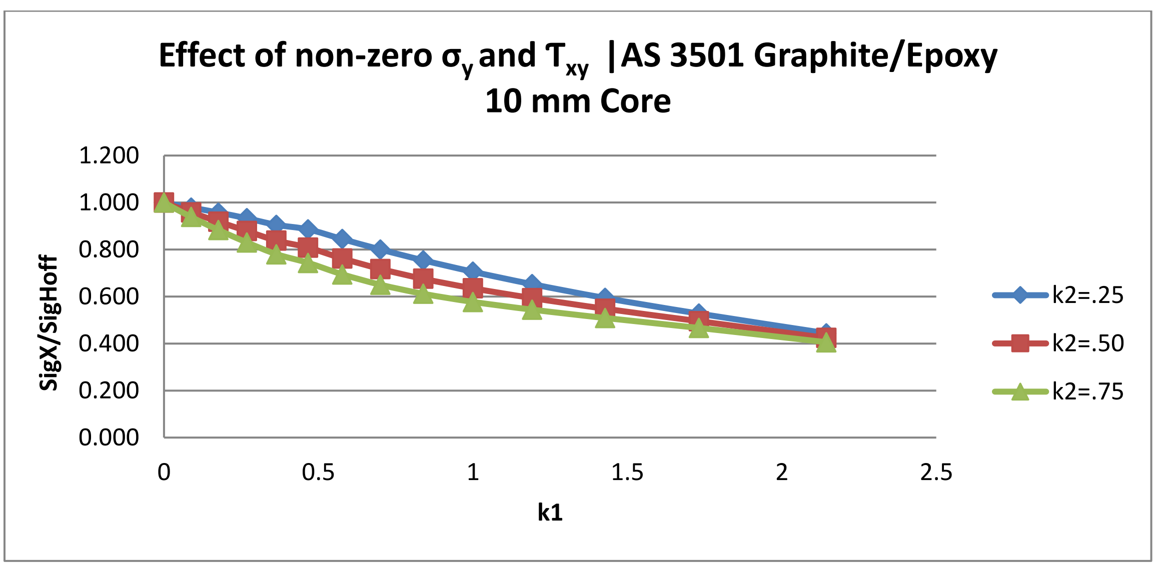
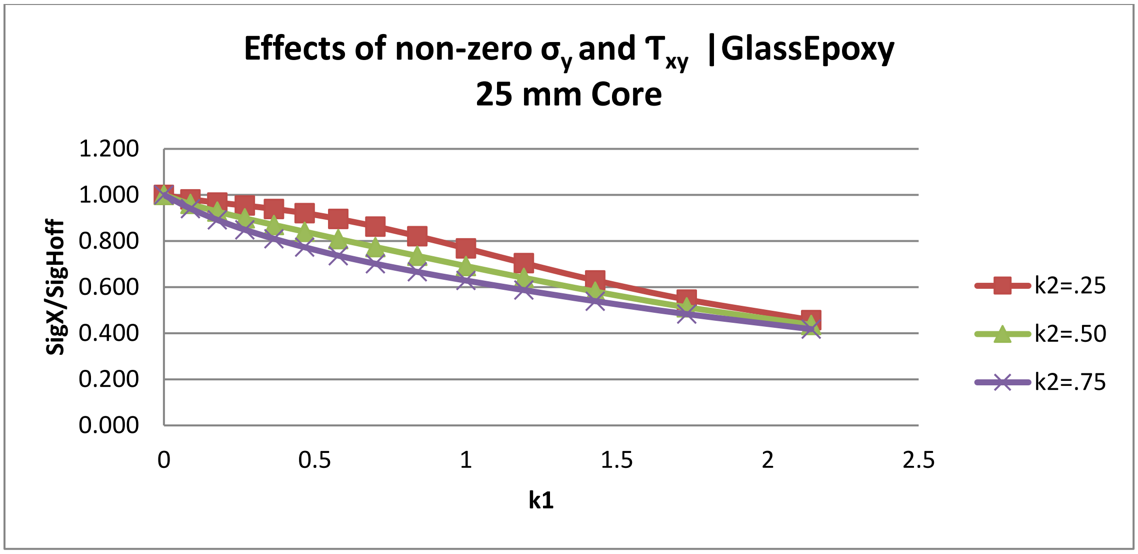
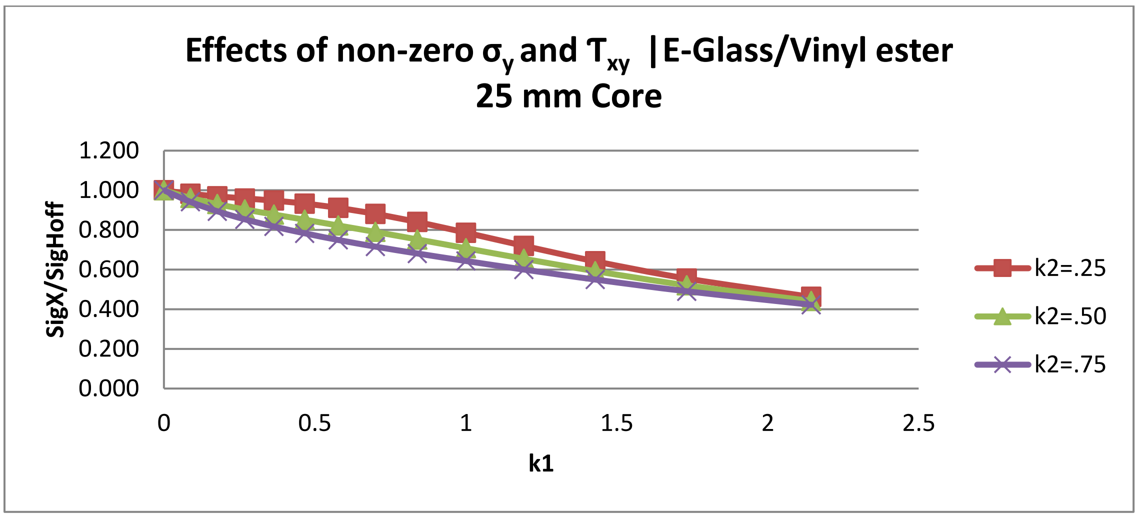


Figure 13 The effect of combined biaxial compression and in-plane shear on the wrinkling stress for panels with various facing materials and a quadratic mass density grading of a 10-mm core through the thickness





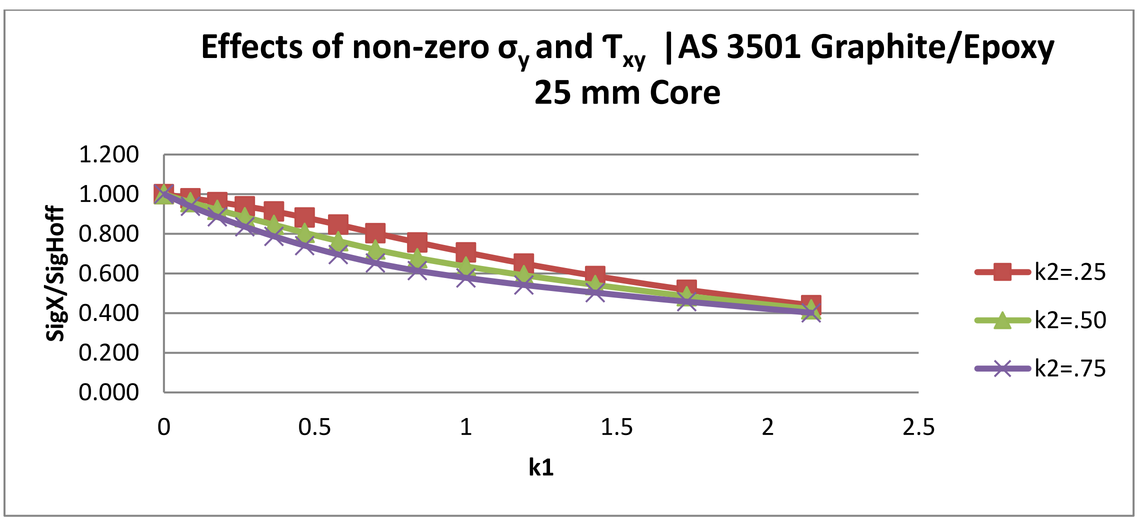


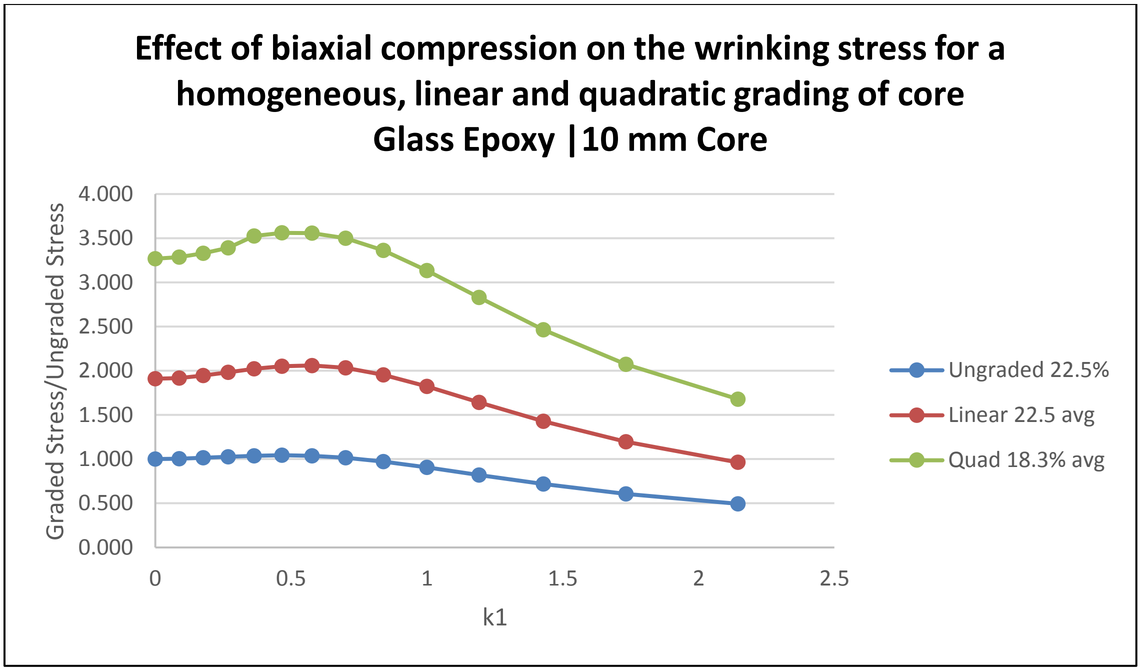
Figure 14 The effect of combined biaxial compression and in-plane shear on the wrinkling stress for panels with various facing materials and a quadratic mass density grading of a 25-mm core through the thickness

5.4. COMPARISON

5.4.1. Biaxial Loading. While the results shown above demonstrate the effect of biaxial compression and/or in-plane shear on the wrinkling stress of sandwich plates with a graded mass density of the core, it is useful to more clearly elucidate the effect of grading. This is reflected in the following figures where the stress ratio along the vertical axis represents a ratio of the stress acting in the x-direction and corresponding to wrinkling ("graded stress") to the wrinkling stress for the panel with a homogeneous core of equal weight under uniaxial compression ("ungraded stress").

As follows from the analysis of Fig. 15, grading is a highly efficient tool for increasing the wrinkling stress of sandwich panels. Even using the core of the same weight, the wrinkling stress can be nearly doubled in the case of a linear distribution of mass density through the thickness and increased by 3.5 to 4 times compared to a homogeneous core using a quadratic mass density distribution. Such spectacular improvements are in agreement with the observations made for uniaxially compressed panels with layer-wise mass density distribution in Ref. 1. Biaxial compression generally results in smaller wrinkling stress value along the x-direction if compression in the y-direction exceeds σ_x $(k_1 > 1)$. Predictably, the same grading schemes become less effective in a thicker 25-mm core, but the advantage of grading is still evident (Fig. 16).

The benefits of grading are also present under a combined uniaxial compression and in-plane shear as is demonstrated in Figs. 17 and 18 for panels with 10-mm and 25-mm thick cores. Contrary to the case of biaxial compression where the wrinkling stress in the x-direction abruptly decreases as soon as it is exceeded by the stress in the y-direction, in-plane shear does not significantly affect the wrinkling value of σ_x within the entire range of shear stresses considered here. Similar to the case of biaxial compression, the effectiveness of grading of the core is reduced in a thicker core.



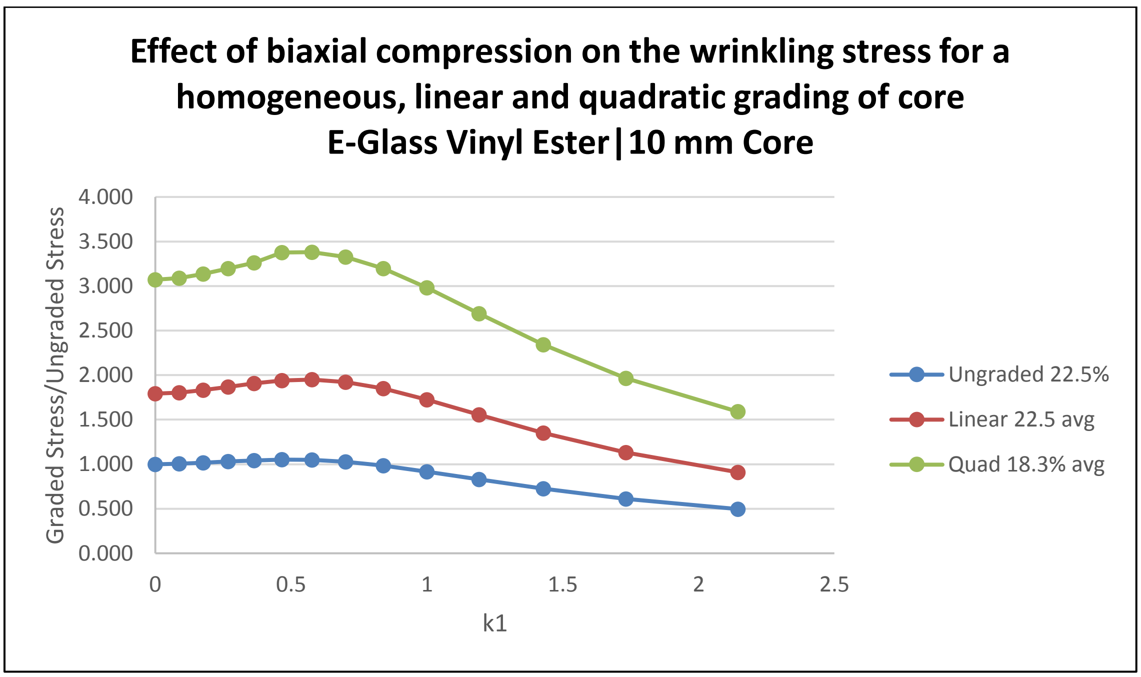
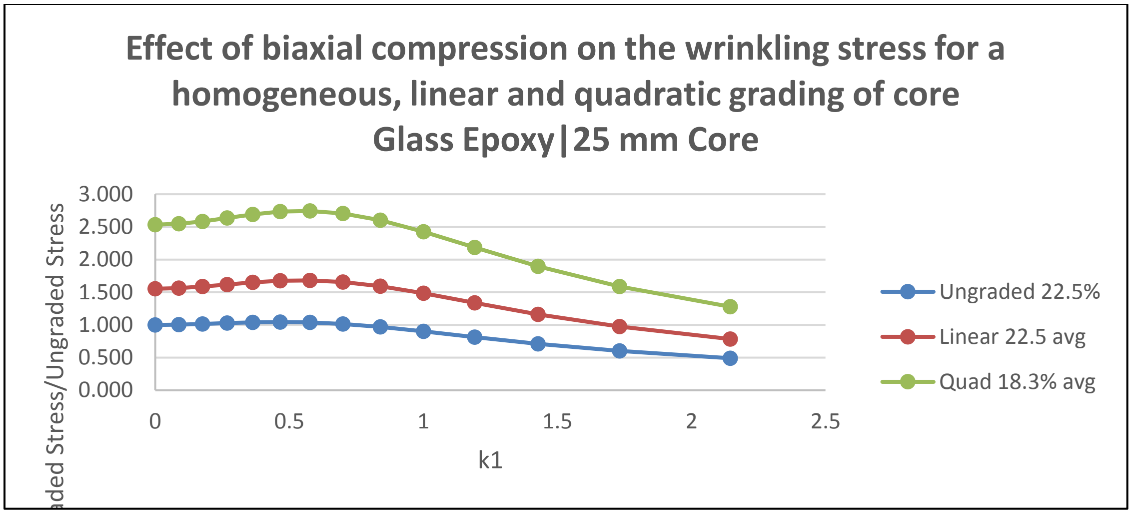
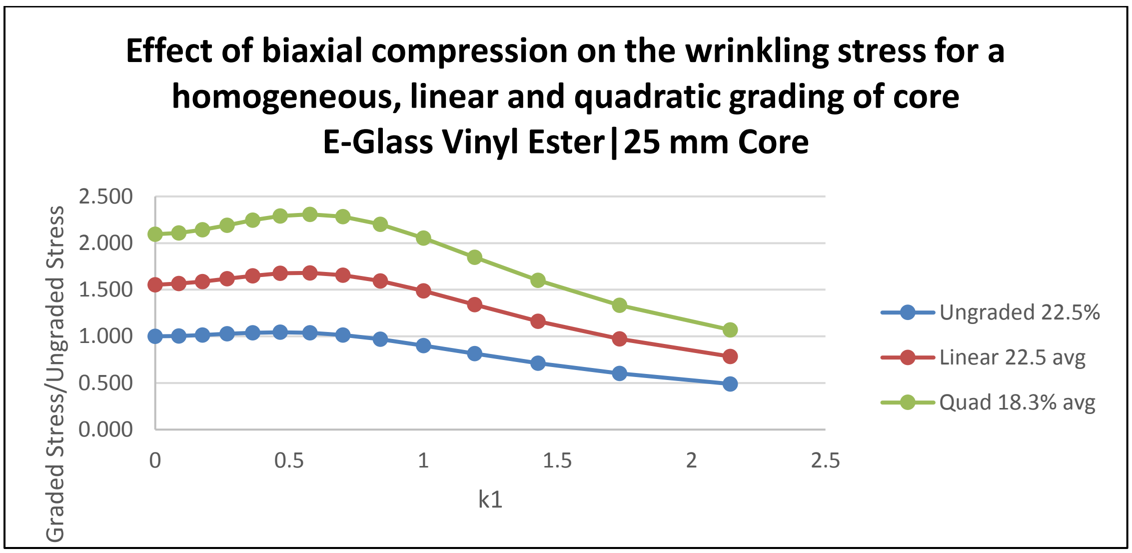


Figure 15 The effect of biaxial compression on the wrinkling stress in sandwich panels with a homogeneous ("ungraded") core and with the linear and quadratic variations of the mass density of 10-mm core through the thickness. The weight of the core in all three panels is equal





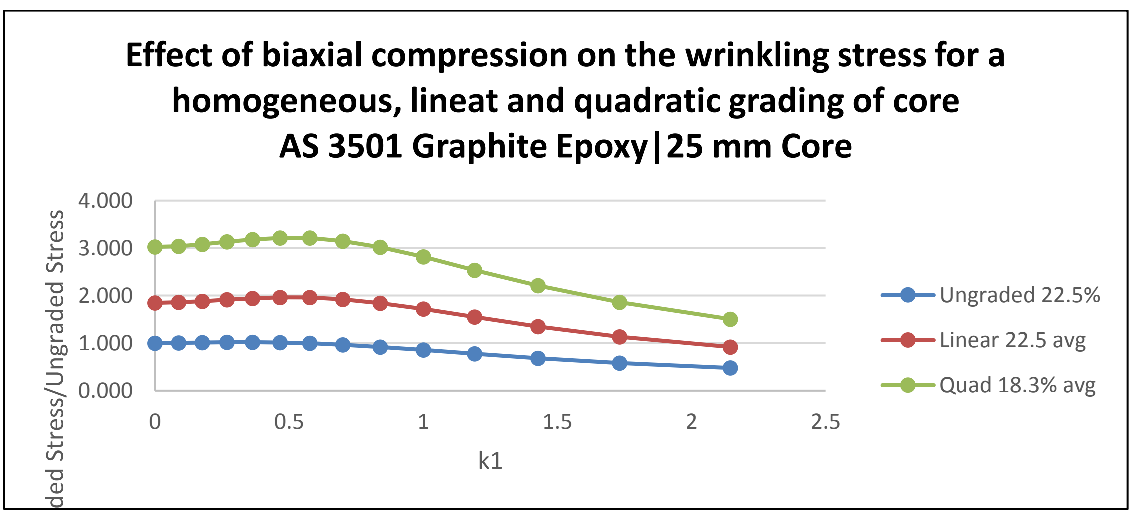
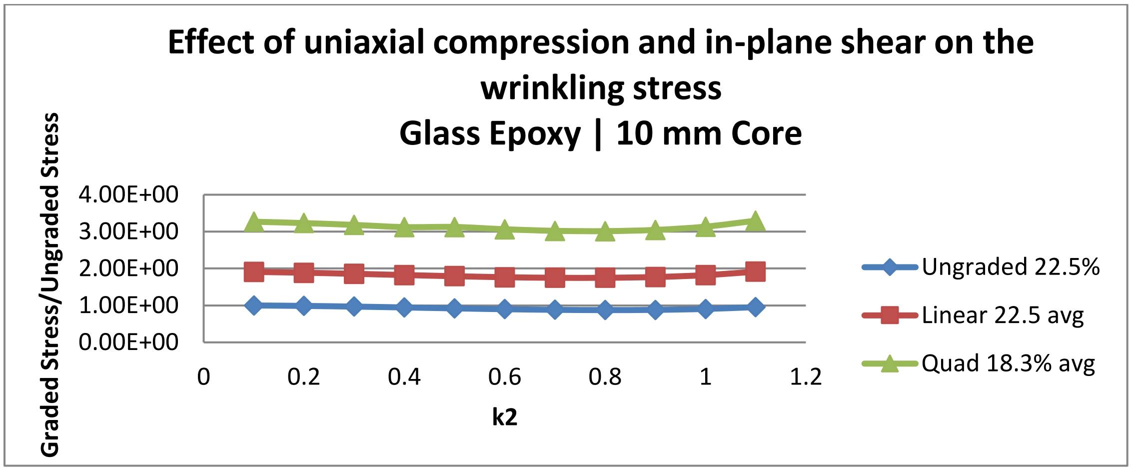
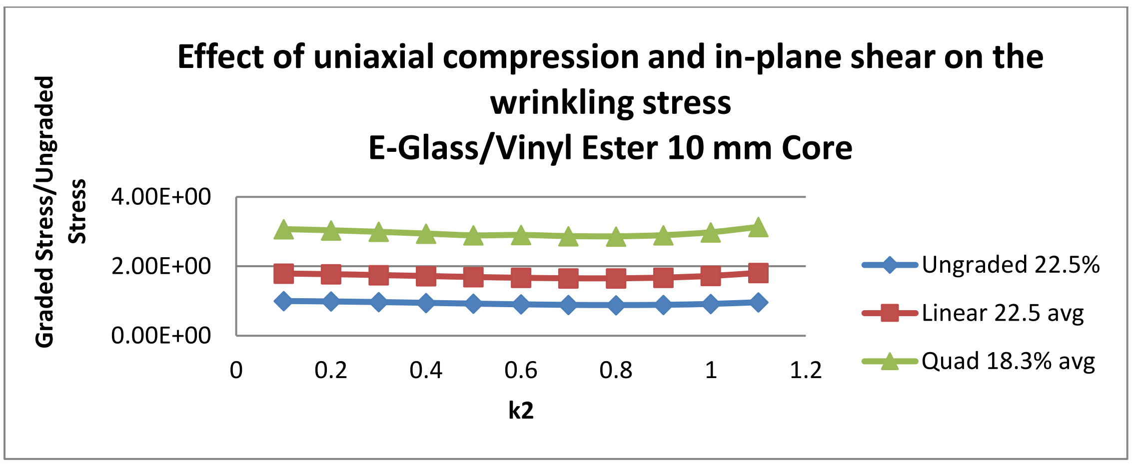


Figure 16 The effect of biaxial compression on the wrinkling stress in sandwich panels with a homogeneous ("ungraded") core and with the linear and quadratic variations of the mass density of 25-mm core through the thickness. The weight of the core in all three panels is equal

5.4.2. Shear Loading





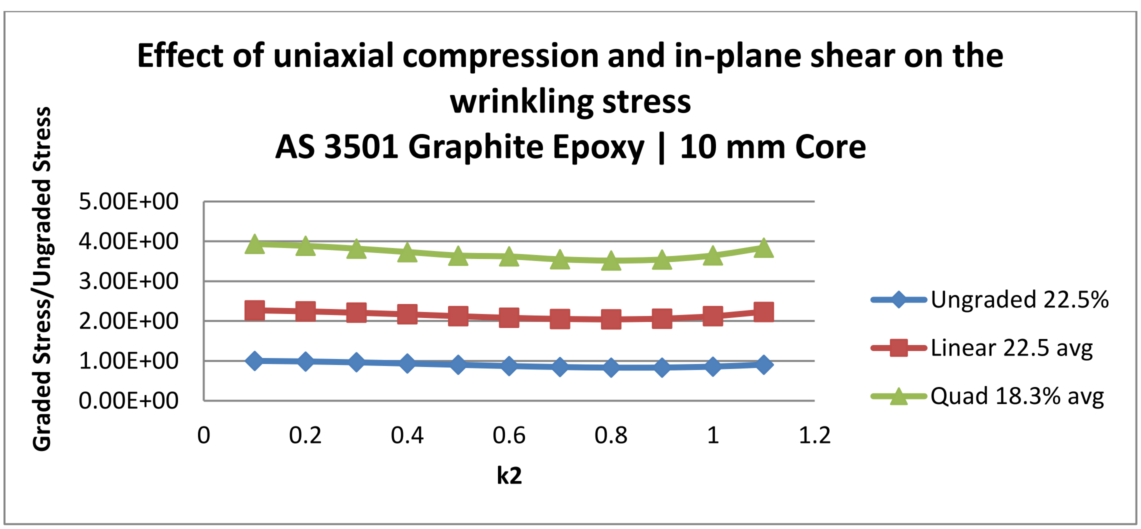


Figure 17 The effect of in-plane shear on the wrinkling stress in sandwich panels with a homogeneous ("ungraded") core and with the linear and quadratic variations of the mass density of 10-mm core through the thickness. The weight of the core in all three panels is equal

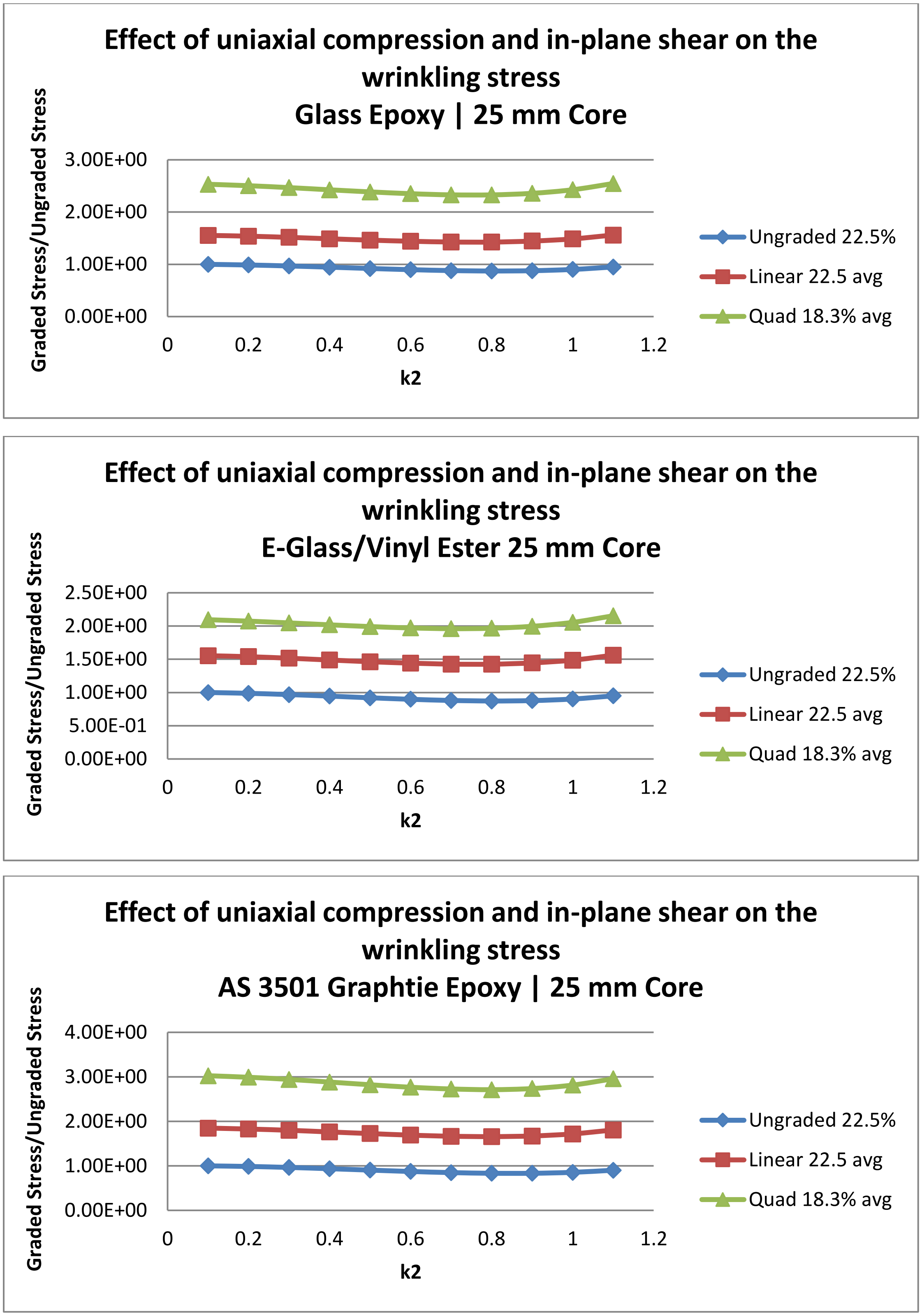


Figure 18 The effect of in-plane shear on the wrinkling stress in sandwich panels with a homogeneous ("ungraded") core and with the linear and quadratic variations of the mass density of 25-mm core through the thickness. The weight of the core in all three panels is equal

5.4.3. Combined Biaxial and Shear Loading. Finally, we demonstrate the effect of a combined biaxial and in-plane shear loading on the wrinkling stress in the panels with homogeneous, linearly and quadratic graded cores. In particular, the nondimensional wrinkling stresses are presented for sandwich panels with glass/epoxy facings and homogeneous, linearly and quadratically graded 10-mm cores in Fig. 19. Three graphs in Fig. 19 correspond to different in-plane shear loads (different values of the coefficient k_2). Remarkably, the difference between these graphs is minimal reflecting a relatively small effect of in-plane shear on the wrinkling stress of homogeneous and graded panels. However, the effect of compression in the y-direction that has already been observed in the previously considered cases is observed in Fig. 19 as well. The advantages of grading the core using a higher mass density and accordingly, higher stiffness material at and in the vicinity to the interface with the facings is preserved at all combinations of biaxial and in-plane shear loads. Similar conclusions follow from all cases considered. As an example, the results in Fig. 20 are presented for sandwich panels with AS 3501 graphite/epoxy facings and homogeneous, linearly and quadratically graded 25-mm thick cores.

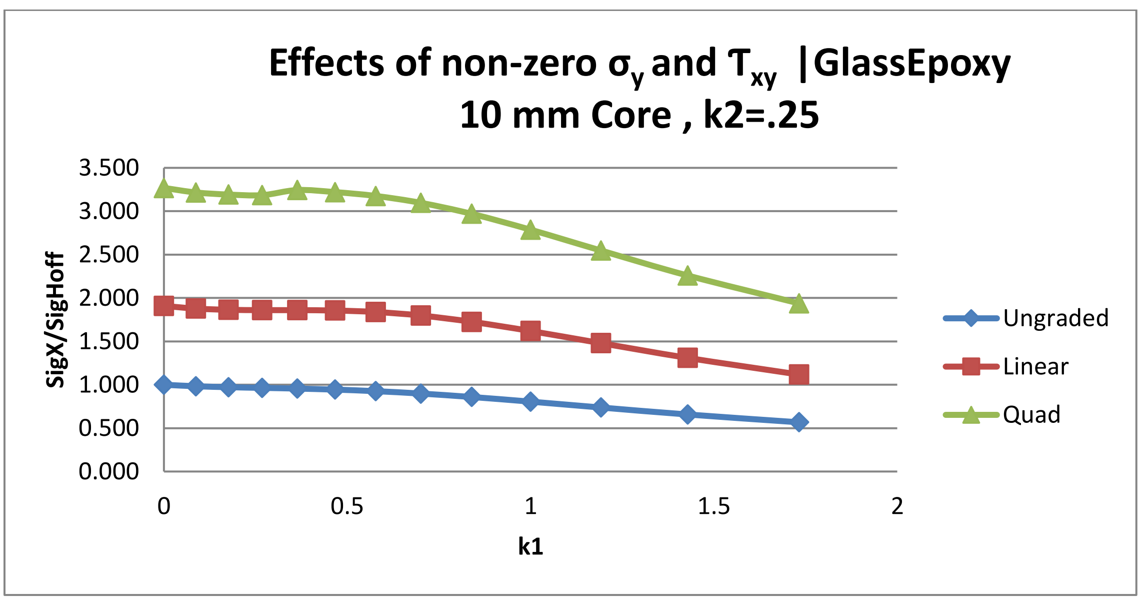
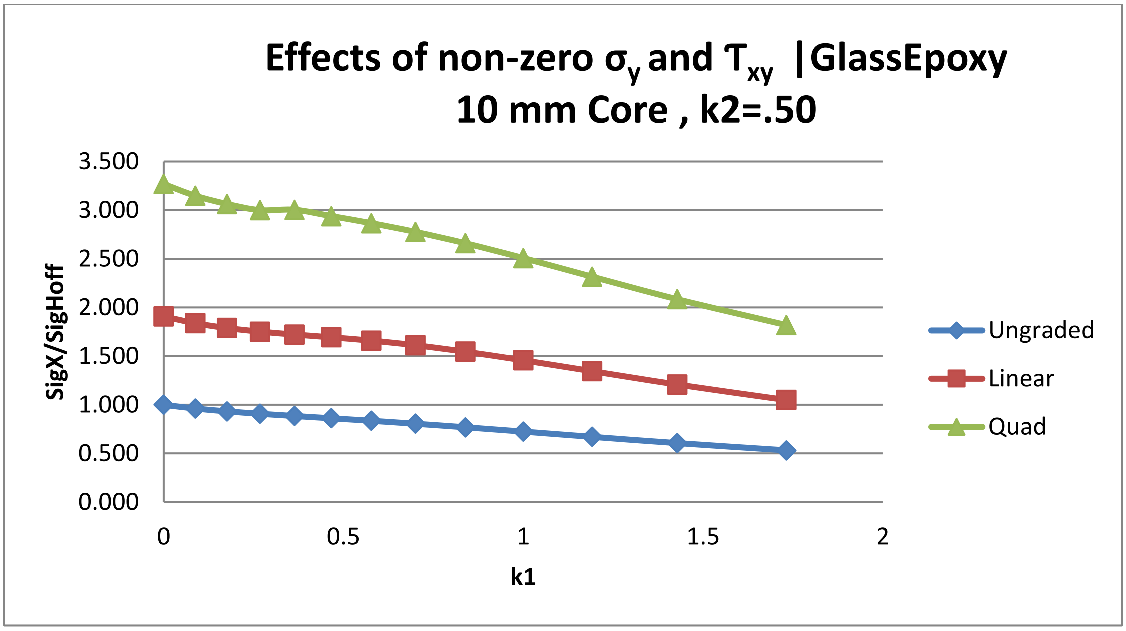


Figure 19 Effect of grading and a combination of biaxial and in-plane shear loading on the wrinkling stress in sandwich panels with glass/epoxy facings and homogeneous, linearly graded and quadratically graded 10-mm thick cores



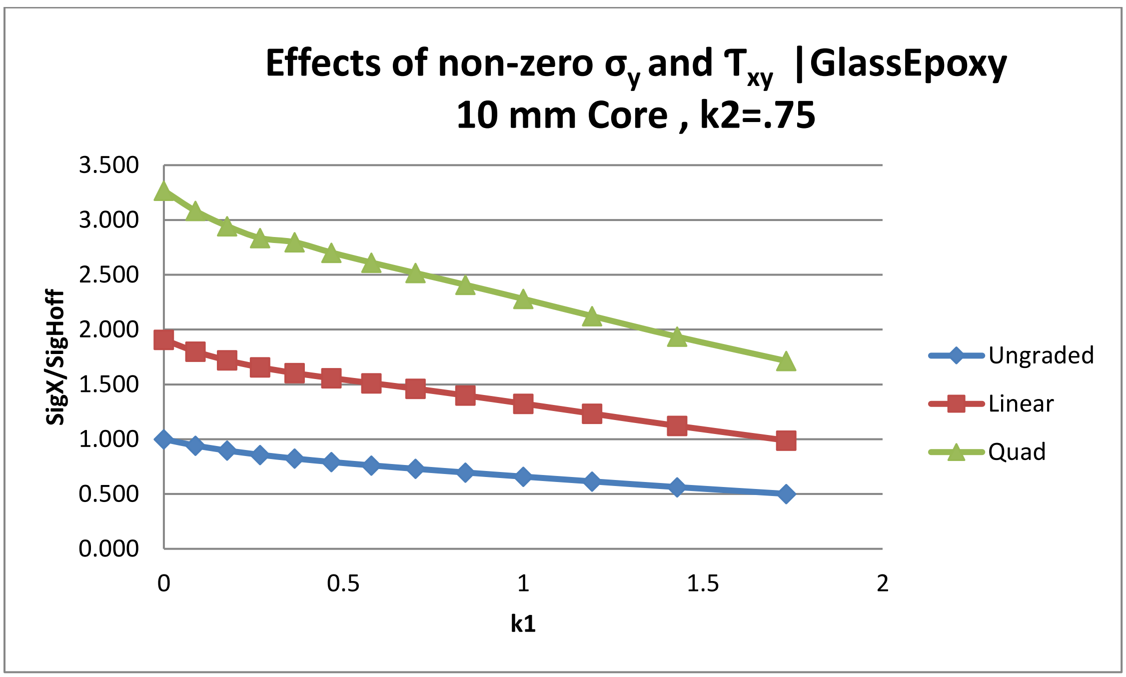


Figure 19 Effect of grading and a combination of biaxial and in-plane shear loading on the wrinkling stress in sandwich panels with glass/epoxy facings and homogeneous, linearly graded and quadratically graded 10-mm thick cores (cont.)

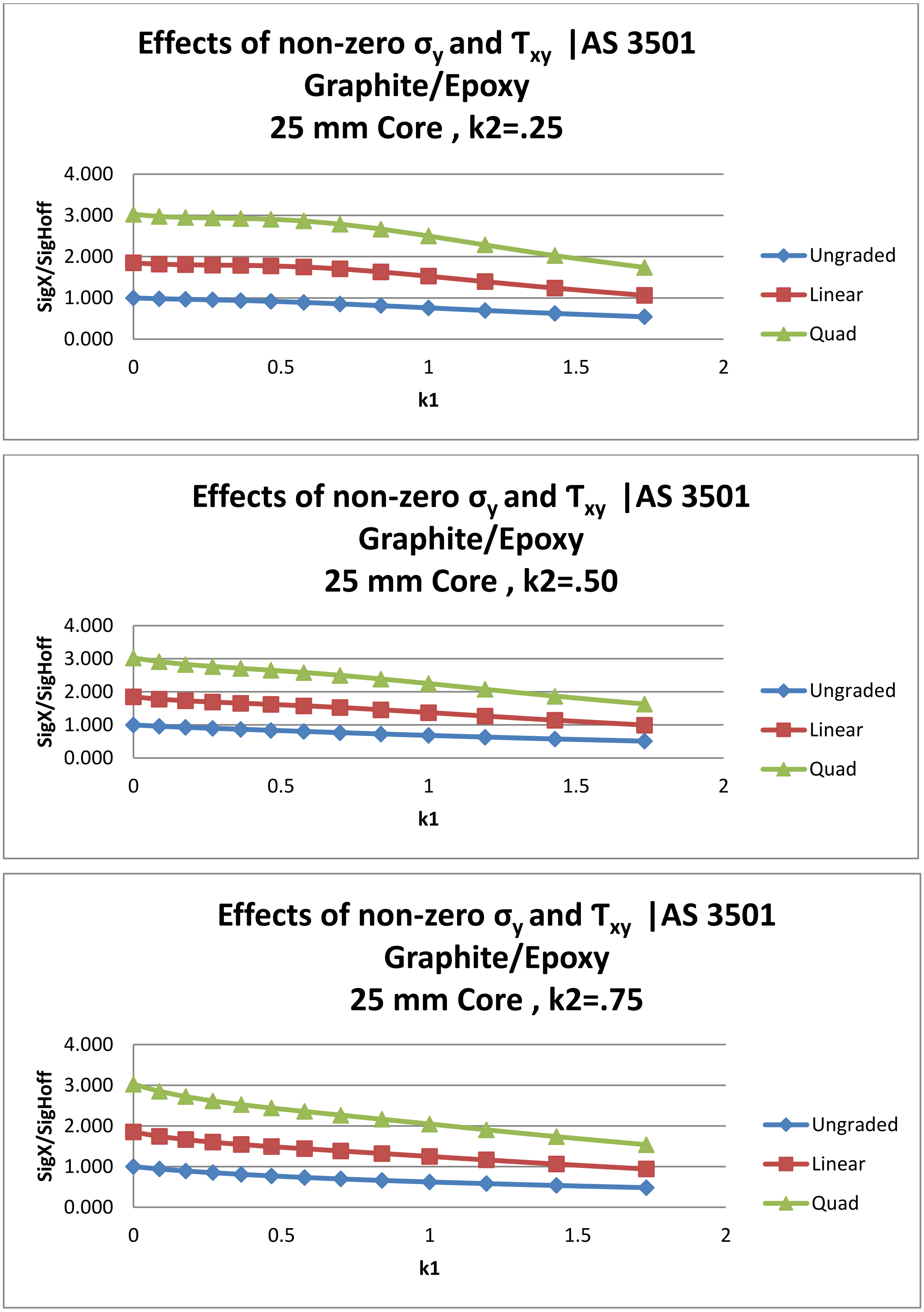


Figure 20 Effect of grading and a combination of biaxial and in-plane shear loading on the wrinkling stress in sandwich panels with AS 3501 graphite/epoxy facings and homogeneous, linearly graded and quadratically graded 25-mm thick cores

6. CONCLUSION

The paper elucidates the two aspects of wrinkling of sandwich panels, i.e. functional grading of the core and the effect of biaxial compression and/or in-plane shear on wrinkling. The core can be arbitrary graded in the thickness direction, as long as such grading complies with micromechanics and manufacturability requirements. Contrary to the previous study of wrinkling in uniaxially compressed sandwich panels that illustrated the advantages of a layered core (Ref. 1), the grading schemes considered in numerical examples are concerned with polymeric cell cores with continuous linear or quadratic variation of the mass density through the thickness.

As follows from the analysis, using a functionally graded core can significantly increase the wrinkling stress as compared to the panel with an equal-weight homogeneous core. The positive effect of functional grading of the core remains valid for the entire range of feasible biaxial compression and/or in-plane shear loads. The effect is more pronounced in thinner cores and in the cores with a higher mass density (and stiffness) of the core material at the interface and in the immediate vicinity to the affected facing (e.g., quadratic grading of the mass density of the polyurethane foam core is more beneficial than linear grading). It should be noted, however, that an excessive reduction of mass density of the core near the middle plane of the panel may be limited by the strength and integrity considerations.

The effect of biaxial compression and in-plane shear on the wrinkling stress defined as the stress σ_x resulting in wrinkling under a prescribed stress combination is demonstrated for both graded and ungraded cores. The presence of a significant compressive stress in the y-direction exceeding σ_x results in a reduced wrinkling value of σ_x. The in-plane shear also affects the wrinkling stress in the x-direction, though this effect is less pronounced compared to that of biaxial compression. If σ_x is larger than the compressive stress in the y-direction and/or in-plane shear stress, it may slightly increase compared to the case of uniaxial compression.

It should be noted that wrinkling represents one of the modes of failure in sandwich panels that is usually realized only in the cases of very thin facings or a compliant core. Other modes of failure that should be examined include overall buckling, the loss of strength in the facings, core shear instability (crimpling) and face dimpling in case of honeycomb cores. Accordingly, the advantages of the improvement in the wrinkling stability should be weighed against the limitations introduced by other failure modes. Nevertheless, a possible improvement of the wrinkling stability under arbitrary in-plane loads that can be achieved without incurring any weight penalty, as demonstrated in this article, represents a potential advantage for sandwich structures with functionally graded cores.

REFERENCES

[[1] Birman, V., and Vo, N., 2017, "Wrinkling in Sandwich Structures with a Functionally Graded Core," Journal of Applied Mechanics, Transactions ASME, 84(2).

[2] Gough, G. S., Elam, C. F., and deBruyne, N. A., 1940, "The stabilization of a thin sheet by a continuous support medium," Journal of the Royal Aeronautical Society 44, pp. 12-43.

[3] Hoff, N. J., and Mautner, S. E., 1945, "The buckling of sandwich-type panels," Journal of Aeronautical Sciences, 12, pp. 285-297.

[4] Plantema, F. J., 1966, Sandwich Construction, Pergamon Press, New York.

[5] Allen, H. G., 1969, "Analysis and Design of Structural Sandwich Panels," Pergamon Press, Oxford.

[6] Vonach, W. K., and Rammerstorfer, F. G., 2001, "A general approach to the wrinkling instability of sandwich plates," Structural Engineering and Mechanics, 12(4), pp. 363-376.

[7] Vonach, W. K., and Rammerstorfer, F. G., 2000, "Wrinkling of thick orthotropic sandwich plates under general loading conditions," Archive of Applied Mechanics, 70(5), pp. 338-348.

[8] Vonach, W. K., and Rammerstorfer, F. G., 2000, "Effects of in-plane core stiffness on the wrinkling behavior of thick sandwiches," Acta Mechanica, 141(1), pp. 1-10.

[9] Gdoutos, E. E., Daniel, I. M., and Wang, K. A., 2003, "Compression facing wrinkling of composite sandwich structures," Mechanics of Materials, 35(3-6), pp. 511-522.

[10] Kardomateas, G. A., 2005, "Wrinkling of wide sandwich panels/beams with orthotropic phases by an elasticity approach," Journal of Applied Mechanics, Transactions ASME, 72(6), pp. 818-825.

[11] Birman, V., and Bert, C. W., 2004, "Wrinkling of composite-facing sandwich panels under biaxial loading," Journal of Sandwich Structures and Materials, 6(3), pp. 217-237.

[12] Birman, V., 2004, "Thermomechanical wrinkling in composite sandwich structures," AIAA Journal, 42(7), pp. 1474-1479.

[13] Birman, V., 2005, "Thermally induced bending and wrinkling in large aspect ratio sandwich panels," Composites Part A: Applied Science and Manufacturing, 36(10 SPEC. ISS.), pp. 1412-1420.

[14] Birman, V., 2004, "Dynamic wrinkling in sandwich beams," Composites Part B: Engineering, 35, pp. 665-672.

[15] Lim, J. Y., and Bart-Smith, H., 2015, "An analytical model for the face wrinkling failure prediction of metallic corrugated core sandwich columns in dynamic compression," International Journal of Mechanical Sciences, 92, pp. 290-303.

[16] Noor, A. K., Burton, W. S., and Bert, C. W., 1996, "Computational models for sandwich panels and shells," Applied Mechanics Reviews, 49(3), pp. 155-199.

[17] Sokolinsky, V., and Frostig, Y., 2000, "Branching behavior in the nonlinear response of sandwich panels with a transversely flexible core," International Journal of Solids and Structures, 37(40), pp. 5745-5772.

[18] Frostig, Y., 2011, "On wrinkling of a sandwich panel with a compliant core and self-equilibrated loads," Journal of Sandwich Structures and Materials, 13(6), pp. 663-679.

[19] Hohe, J., Librescu, L., and Oh, S. Y., 2006, "Dynamic buckling of flat and curved sandwich panels with transversely compressible core," Composite Structures, 74(1), pp. 10-24.

[20] Hohe, J., and Librescu, L., 2008, "Recent results on the effect of the transverse core compressibility on the static and dynamic response of sandwich structures," Composites Part B: Engineering, 39(1), pp. 108-119.

[21] Phan, C. N., Bailey, N. W., Kardomateas, G. A., and Battley, M. A., 2012, "Wrinkling of sandwich wide panels/beams based on the extended high-order sandwich panel theory: Formulation, comparison with elasticity and experiments," Archive of Applied Mechanics, 82(10-11), pp. 1585-1599.

[22] 2005, "Fire degradation, failure prediction and qualification methods for fiber composites," US Coast Guard, Ship Structure Committee, Washington, DC.

[23] Birman, V., Keil, T., Hosder, S., 2013, "Functionally Graded Materials in Engineering," Structural Interfaces and Attachments in Biology, S. Thomopoulos, Birman, V., Genin, G.M., ed., Springer, New York, pp. 19-41.

[24] Suresh, S., and Mortensen, A., 1998, Fundamentals of functionally graded materials : processing and thermomechanical behaviour of graded metals and metal-ceramic composites, IOM Communications Ltd, London.

[25] Miyamoto, Y., 1999, Functionally Graded Materials : Design, Processing and Applications, Kluwer Academic Publishers, Boston.

[26] Birman, V., and Byrd, L. W., 2007, "Modeling and Analysis of Functionally Graded Materials and Structures," Applied Mechanics Reviews, 60, pp. 195-216.

[27] Birman, V., 2014, "Functionally Graded Materials and Structures," Encyclopedia of Themal Stresses, R. B. Hetnarski, ed., Springer, Dordrecht, pp. 1858-1864.

[28] Paulino, G. H., 2008, Multiscale and Functionally Graded Materials : Proceedings of the International Conference FGM IX, Oahu Island, Hawaii, 15-18 October 2006, American Institute of Physics, Melville, N.Y.

[29] Kawasaki, A., Niino, M., and Kumakawa, A., 2008, Multiscale, multifunctional and functionally graded materials : selected, peer reviewed papers from the 10th International Symposium on MM & FGMs, 22nd-25th September 2008, Sendai, Japan, Trans Tech, Stafa-Zuriich, Switzerland ; United Kingdom ; Enfield, NH.

[30] Gibson, L. J., and Ashby, M. F., 1997, Cellular solids, structure and properties, Cambridge University Press, Cambridge.

[31] Tuwair, H., Volz, J., ElGawady, M. A., Chandrashekhara, K., and Birman, V., 2016, "Modeling and analysis of GFRP bridge deck panels filled with polyurethane foam," Journal of Bridge Engineering pp. 04016012-04016011 through 04016012-04016014.

[32] Gibson, R. F., 2007, Principles of composite material mechanics, CRC Press, Boca Raton.

[33] Gibson, L. J., and Ashby, M. F., 1982, "Mechanics of three-dimensional cellular materials," Proceedings of The Royal Society of London, Series A: Mathematical and Physical Sciences, 382(1782), pp. 43-59.

SECTION

2. CONCLUSION

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VITA

Harold Costa, obtained his Bachelor of Science in Mechanical Engineering from St. Martin's University in 2012 and Certificate in Engineering Mechanics in 2015 from Missouri University of Science and Technology. In July 2017 he received his MS degree in Aerospace Engineering also from Missouri University of Science and Technology. He has worked as a Liaison MRB Engineer where he designed and analyzed small and large aircraft repairs/ alterations for the Boeing Company in WA supporting 737 Final Assembly. He has also worked as Manufacturing Engineer team Lead and Special Projects focal in 787 program and Engineering intern at Fluke Biomedical.