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Composite model representation for Computer Aided Design of Functionally Gradient Materials

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COMPOSITE MODEL REPRESENTATION FOR COMPUTER AIDED DESIGN OF FUNCTIONALLY GRADIENT MATERIALS

by

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ABSTRACT

Functionally Gradient Materials (FGMs) feature smooth transition from one material to another within a single object. FGMs modeling is considered to be one of the new challenges in Computer Aided Design (CAD) area. To overcome this challenge, this thesis presents a composite approach to model FGMs. The input in STL format can be meshed and voxelized in FGMs modeling system. The material composition in each voxel can be generated from multiple different types of control features. And LTI filters including Gaussian Filter and Average Filter are applied to blur default material features in order to generate FGMs inside models. The LTI filtering method gives an effective and controllable approach to distribute material composition in FGMs area. Forbidden zone mapping function is also proposed in this thesis to actively eliminate the forbidden zone in FGMs modeling. Unwanted material composition can be effectively removed from the modeling process while the original material transition trend is preserved. At last, Erosion Function is introduced in the thesis to generate FGMs area between outer portion and inner portion of an object. Material composition contour level map is generated to help the tool path plan for Additive Manufacturing (AM).

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1. INTRODUCTION

Functionally Gradient Materials (FGMs) feature smooth transition from one material to another within a single object, allowing engineers to customize the physical response of different regions of the object by modifying the material composition at each region [1]. FGMs afford the engineer the ability to highly customize the properties of a single object by modifying the object's material composition independently at different regions. For example, a particular region of the object might need an increased Young's modulus, wear resistance, or thermal properties [2]. As shown in Figure 1.1, a pressure vessel is designed to hold high temperature fluids where the material has been tailored to increase thermal resistance while maintaining high strength [3].

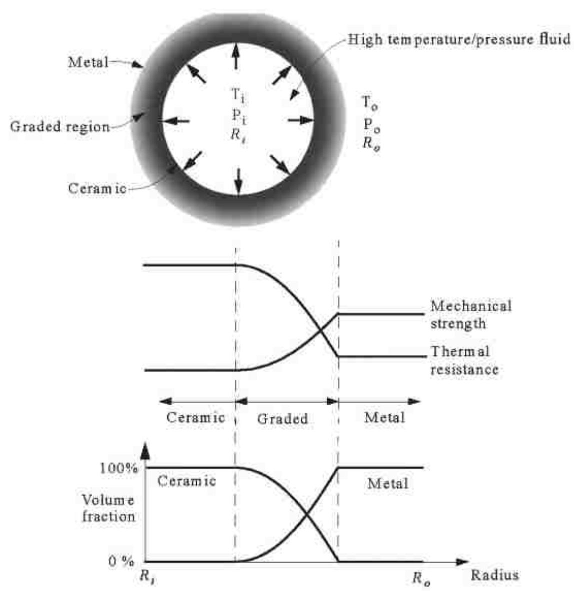


Figure 1.1. Heterogeneous Pressure Vessel [3]

Modeling of FGMs is considered to be one of the challenges in the Computer Aided Design area. One of the limitations is that current CAD software only concerns the geometry information of the desired model, however it does not consider any material information. Additionally, there is no existing software for the users to manipulate material information inside models.

A significant amount of research has been done on modeling of FGMs objects. A variety of models are applied in the manipulation and representation of FGMs material distribution such as volume model, voxel model, control point model, implicit function model, explicit function model, etc. All of the FGMs models can be generalized into two categories, evaluated models and unevaluated models. Evaluated models are discretized representations of an object where each voxel contains material volume fraction information; however unevaluated models are function-based representations of an object where functions are used to calculate the material volume fraction composition [1].

Evaluated models present FGMs objects through dividing the model into small cells. There are mainly two types of methods used in evaluated models: voxel based model and volume mesh based model. In voxel based model, the spatial material distribution is represented in a uniform 3D voxel grid with each voxel indicating the type of material in this unit grid [4]. Volume mesh based model is similar to voxel model. The only difference is that it uses a collection of polyhedrons instead of spatial grids to represent 3D models [5]. Tetrahedron and hexahedrons are the most commonly used cell elements in volume mesh model.

However, unevaluated models represent material distribution of FGMs through analytical mathematical functions instead of spatial subdivision. As a result, unevaluated models are accurate and precise regardless of resolution. There are many FGMs modeling methods based on unevaluated models. Analytical functional representation applies explicit functions to manipulate the material composition in each point inside the FGMs model in Cartesian coordinate [6]. Siu and Tan [7] proposed a control source representation method. Material composition is based on the distance between the control feature and the point. Moreover, multiple control feature model are applied. Multiple control features of different types are used as the material variation references to define material compositions. Liu [8] proposed a local composition control method by using Laplace equation based approach. Biswas [9] realized a field distant method using implicit source profiles to control material composition. Complex topologies of FGMs models can be represented by this method. Gupta and Tandon [10] create a convolution material based method. It takes the shape of models into control feature to generate the material composition by associated method.

This thesis aims to utilize the integration of evaluated and unevaluated models to present a new method to design and represent FGMs with the advantages of the both evaluated and unevaluated models. Multiple control feature based method is used to develop unevaluated material distribution and voxel-based method is used to represent FGMs for the data format. Multiple control feature based method gives accurate calculation of the FGMs distribution and an efficient way to store and represent calculated data. Filtering method is used to develop

FGMs distribution based on filtering distribution functions. A special mapping function, forbidden zone mapping function is introduced to eliminate the forbidden composition area in FGMs modeling. Moreover, Erosion Function is applied to generate FGMs distribution. The methodology discussed in this thesis would give a new vision of computer aided design for FGMs and would be helpful for the entire process of additive manufacturing.

2. REPRESENTATION OF MATERIAL GRADING INFORMATION

Traditional CAD modeling systems only contain geometry information of an object. However, FGMs modeling needs to contain material composition information and grading distribution information besides geometry information. In general, space is defined to model FGMs objects as a product space $T = E^3 \times M^n$, where E^3 represents the 3D geometry space and M^n is n-dimensional material space. Material space V is defined as a subset of M^n :

$$V = \{ v \in M^n \mid \|v\| = \sum_{i=1}^{n} v_i = 1 \; and \; v_i \geq 0\} \quad (1)$$

Each voxel in the object could be described as one voxel $P(x \in E^3, v \in V)$ in T, where x and v denotes the geometric and material points, respectively [11]. By the concept of material space, material composition information in FGMs objects can be generated and represented by our defined mapping from geometry space to material space.

In our approach, the material default composition of FGMs objects is assigned to each reference feature. The material grading information at each voxel is represented by the value calculated by material distribution functions. The material distribution function is defined in the entire Euclidean space E^3 ; however it is only valid for the voxel within the model. At each reference feature, only one material could be assigned. Each material of the object has its own material grading distribution generated from its corresponding reference feature and material distribution function. Single voxel is specified with different material distribution, each material composition at this single voxel is normalized. The process is shown in Figure 2.1.

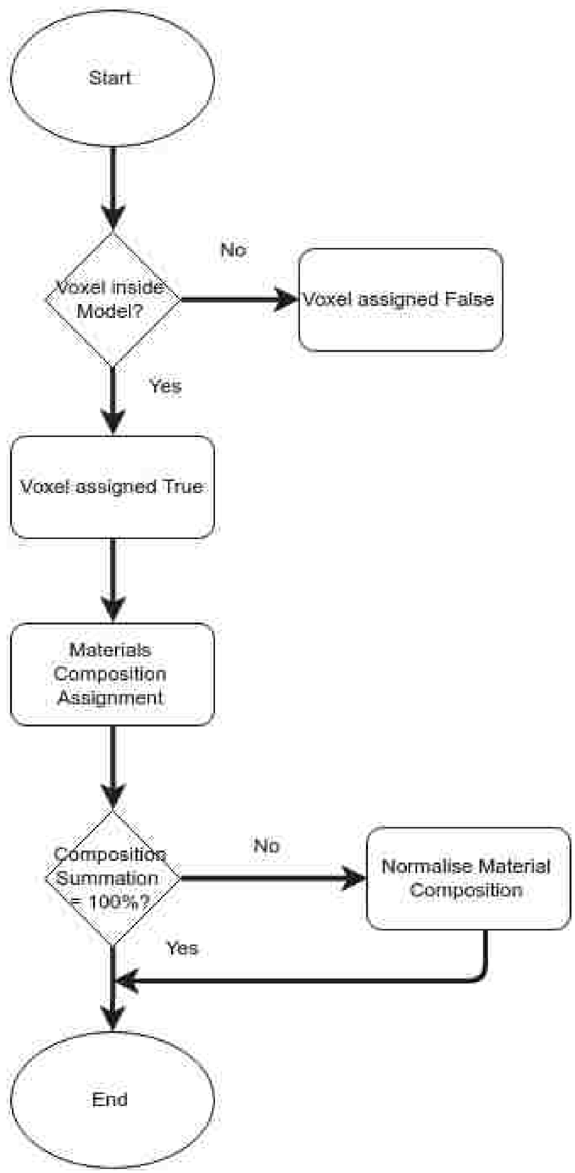


Figure 2.1. Flow diagram of representation of material grading information

3. CONCEPT AND DESIGN PATTERN

The design pattern is depicted in the flowchart as shown in Figure 3.1. At the offset, an STL model is produced from CAD software. The 3D STL model is then sliced into 2D layers along Cartesian axis with the prescribed resolution. First, at each layer, bounding box is identified which is used to contain the whole positions of voxel. Next, the voxel existence in each layer is calculated by Boolean operations of layer contour. Then, default reference features are then selected in the object. And then, FGMs distribution for the model is generated by applying composite methods. Finally, voxel based representation is shown in the last step.

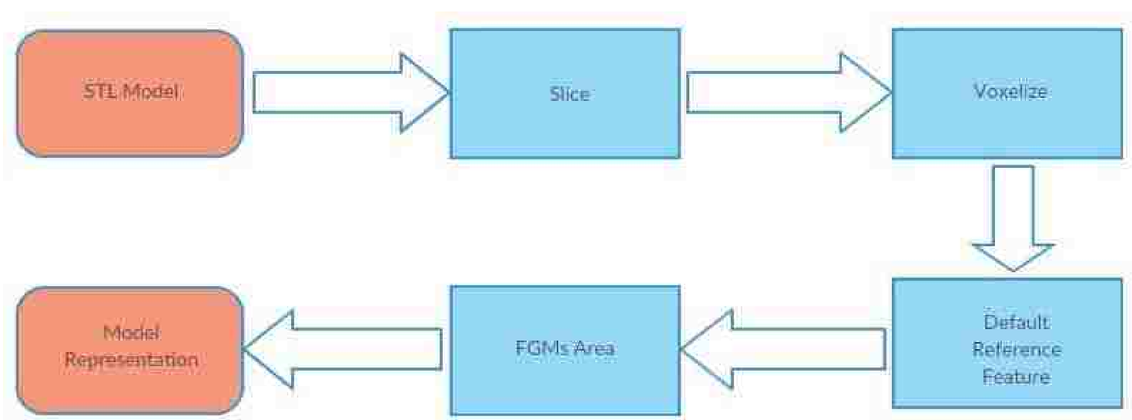


Figure 3.1. FGMs modeling design pattern

The first task is to model the geometry. Our method is using commercial CAD software to define the physical boundary of the object. The geometric data is exported from CAD software to FGMs modeling system using STL format.

Vertex and triangle information are retrieved by analyzing STL file. Using vertex and triangle information, layer contour information is generated by the intersection of layer plane and triangle edges. The bounding box of the layer is identified to store voxels as shown in Figure 3.2.

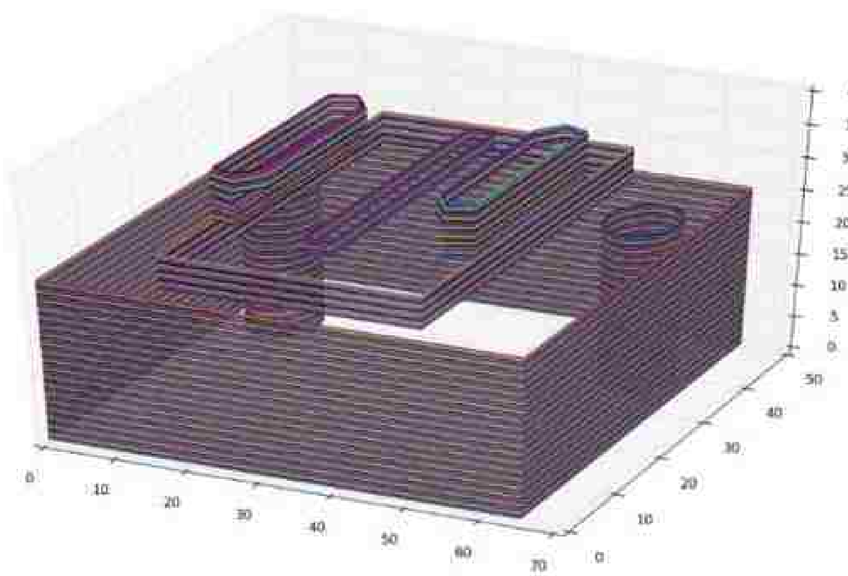


Figure 3.2. Layer contours of the object

The existence of voxel is determined by the binary operations of object contours. Binary value is assigned to each voxel according to whether it is included in odd times of contour or even times of contour. If the voxel is included in odd times of contour, true value would be assigned and if the voxel is included in even times of contour, false value would be assigned as shown in Figure 3.3.

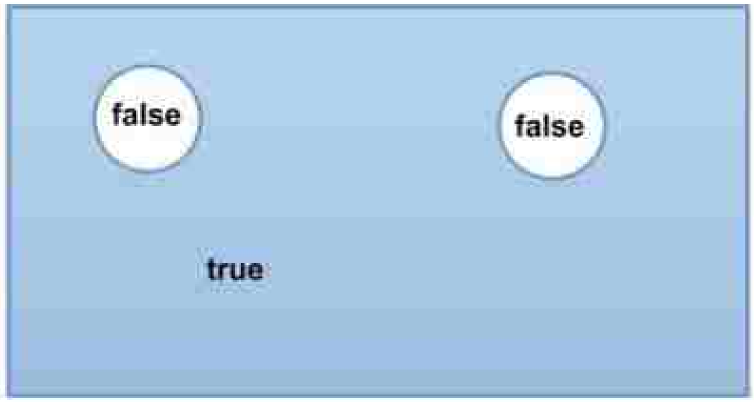


Figure 3.3. Binary representation of model layer

Models have one of three directions of layer orientations to be sliced which are x, y, z directions in Cartesian coordinate. Voxels in each layer are generated by utilizing the layer contour and Boolean value information. The computation efforts and representation accuracy depend on resolutions of voxels. Since the geometric model is three dimensional and resolution is one dimensional parameter, the computational time complexity could be defined as function of resolution:

$$T = CR^3 \quad (2)$$

Where T is computational time complexity, C is constant and R is resolution. The higher the resolution is, the more computation efforts will be and the more accurate for FGMs representation. Figure 3.4 shows the relationship of the computation of voxelization time versus resolution in FGMs modeling system.

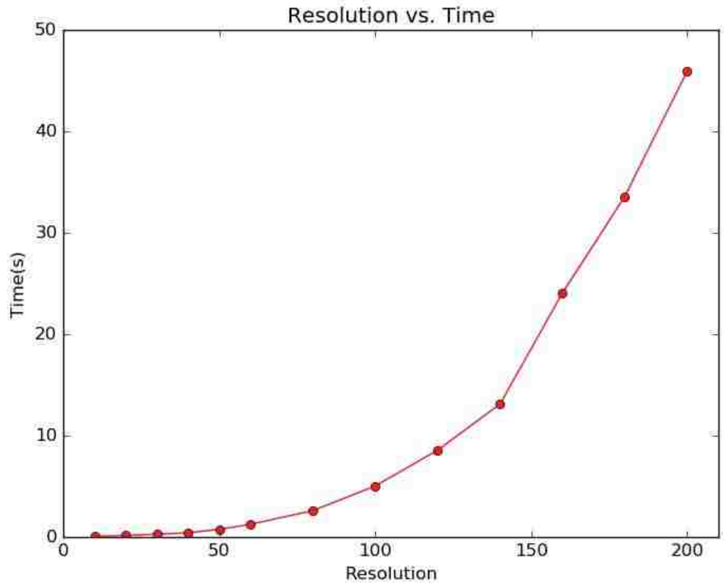


Figure 3.4. Computation of voxelization time vs. resolution in FGMs modeling system

4. CONTROL SOURCE MODELING

4.1. FGMS DISTRIBUTION ALONG AXIS

Many FGMs models have their gradients along slicing axis, so that each of the single layers would have the same material composition as shown in Figure 4.1:

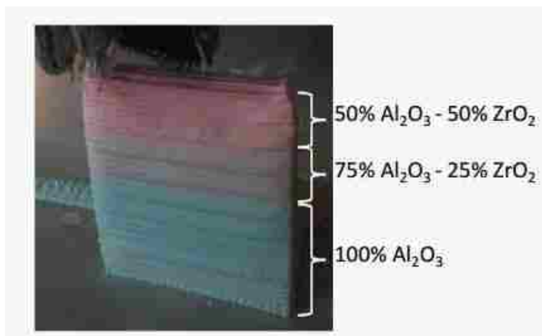


Figure 4.1. Ceramic gradient materials [12]

Along the slicing axis, the material distribution is defined as linear functions as shown in Figure 4.2:

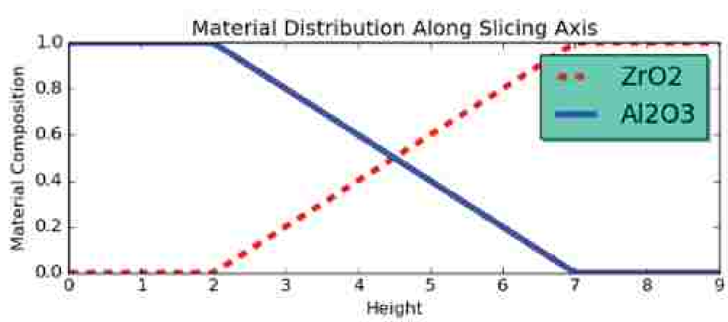


Figure 4.2. ZrO_2 and Al_2O_3 composition diagram along vertical direction

In our approach, the control sources are a pair of bottom and top surface, front and back surface, left and right surface. Using the linear distribution function in slicing direction, material composition can be easily assigned which is developed from either pair of the surface. Since FGMs are developed along slicing axis, every voxel in the contour layer of the same height will be assigned the same composition value. Therefore, calculating each composition value for each voxel is easy to be implemented. Examples of models are shown in Figure 4.3.

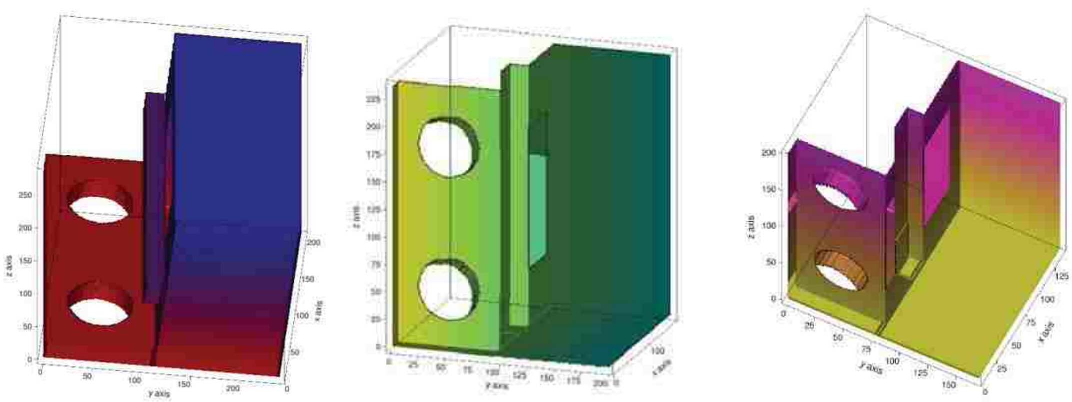


Figure 4.3. Linear Material Distribution for FGMs along x, y, z axis

4.2. MULTIPLE CONTROL SOURCES WITH DIFFERENT TYPES

Control source (S) is defined to be origin of material distribution. Each control source is assigned with one type of material, and each voxel in the model is influenced by the control source. In each voxel, materials have a set of composition weight values (C) controlled by the distance (d) from the voxel point to the material control source. Several types of control sources are defined. For our approach, the three control sources used in the FGM modeling system are planar, point and line.

The material composition weight for each material at one voxel is function of $\frac{1}{d}$. There are two regions inside the model for material distribution. One voxel inside the model can fall into only one of the following regions according to the distance with each material control source:

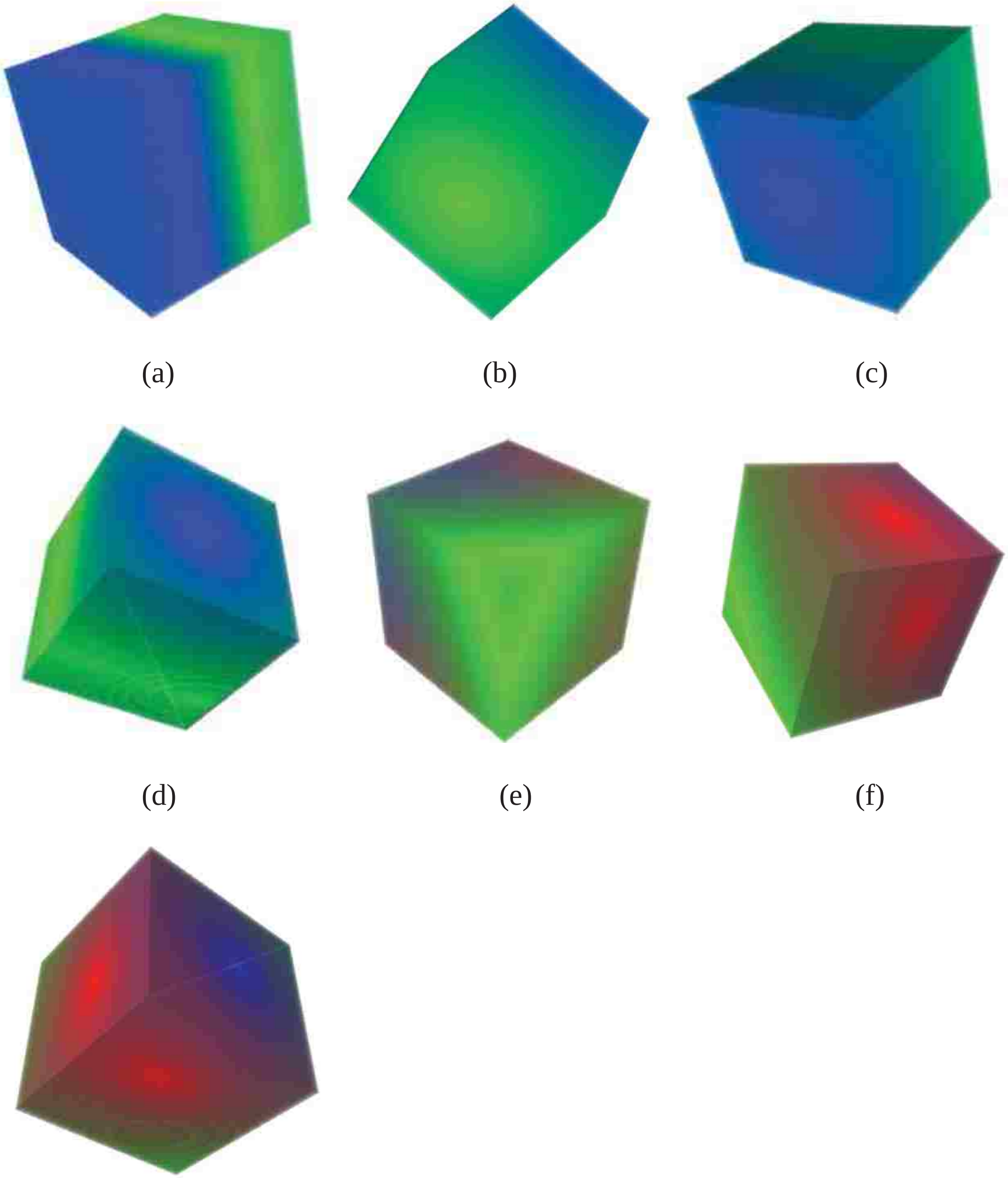
A. Single material region: Only one material describes the region due to the voxel inside the corresponding material control source (m). The distance between the material control source and voxel thus is equal to zero. Material composition for each material is $\mathrm{C_i} = 1$ for $\mathrm{i} = \mathrm{m}$ and $\mathrm{C_i} = 0$ for $\mathrm{i} \neq \mathrm{m}$.

B. Multiple material region: None of the distance from control source to voxel is equal to zero.

$$C_i = \frac{\frac{1}{d_i}}{\frac{1}{d_1} + \frac{1}{d_2} + \cdots + \frac{1}{d_{i-1}} + \frac{1}{d_i} + \frac{1}{d_{i+1}} + \cdots + \frac{1}{d_n}} \tag{3}$$

Where n is the number of material control source. This equation can ensure the summation of control source weight is equal to 100 percent.

As shown in Figure 4.4 and 4.5, the material distribution is generated by planar, point and line source. Each control source is with one kind of material indicated by one color. The material composition can be generated by the same type of control sources or different types of control sources based on the distance between the control source and voxel.



(g)

Figure 4.4. FGMs modeling using different control sources in cubic (a) two planar control sources (b) (c) Two point control sources. (d) Two different types of control sources: one planar and one point control source (e) (f) (g) Three different types of control sources: one planar, one point and one line control source.

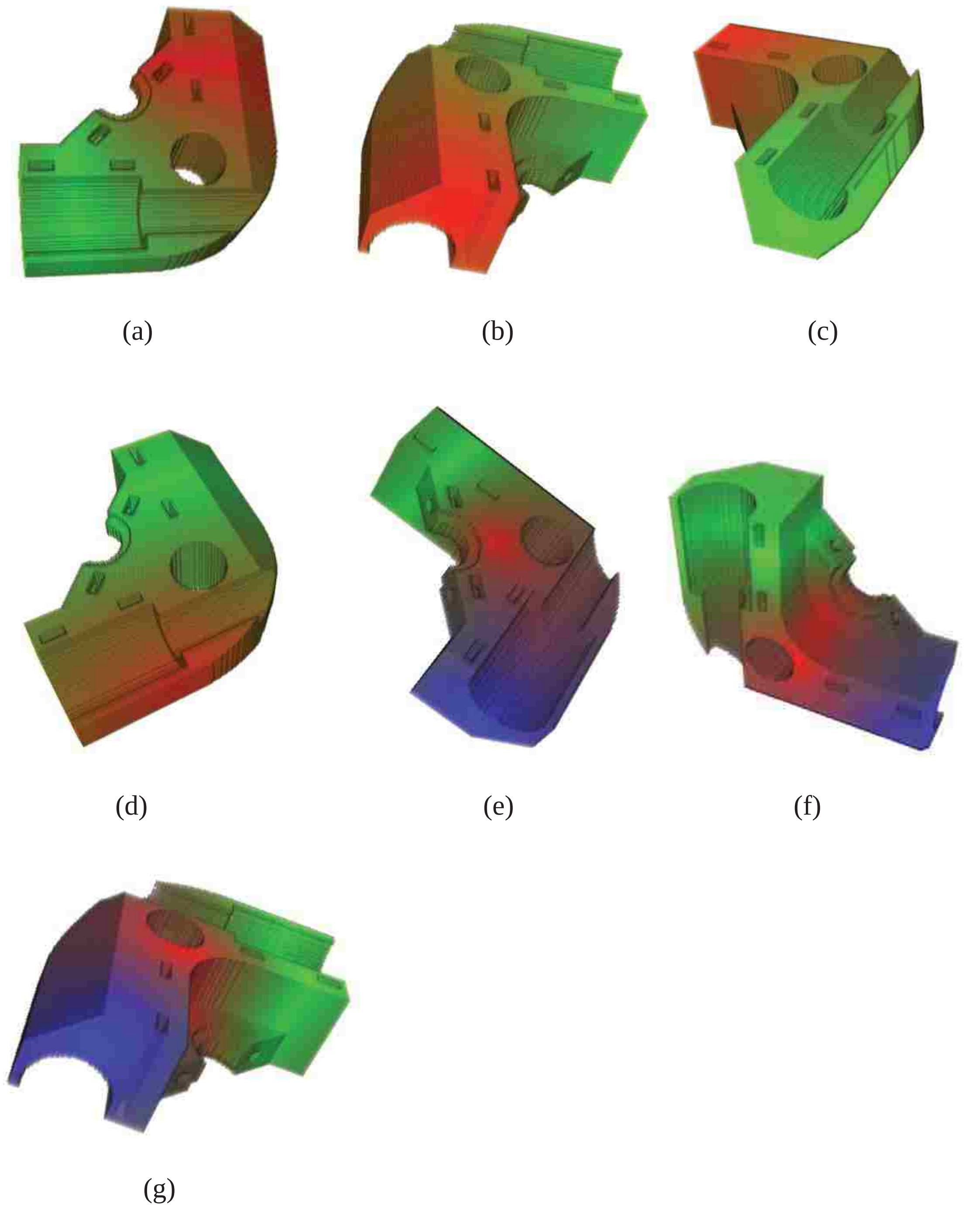


Figure 4.5. FGMs modeling using different control sources in drucken (a) two planar control sources (b) (c) Two point control sources. (d) Two different types of control sources: one planar and one point control source. (e) (f) (g) Three different types of control sources: one planar, one point and one line control source.

The material composition is calculated using the above equations if the voxel falls in either of the two conditions, so that the method in nature is unevaluated due to the accurate calculation of material composition function and independence of resolution. However, voxel is the material composition carrier and is the final result for rendering and representing the model. Although the voxel based representation is evaluated since voxels are separated in three dimensional space, material composition in voxel is derived from distribution function, thus the accuracy for each voxel is guaranteed.

5. FGMs DESIGN WITH LTI FILTER

In this section, the process of filtering method used to generate FGMs model is discussed. Filtering is the most basic operation in computer vision. In the broadest sense, the term “filtering” is a function of values in the vicinity of a given pixel to determine its final output value [13].

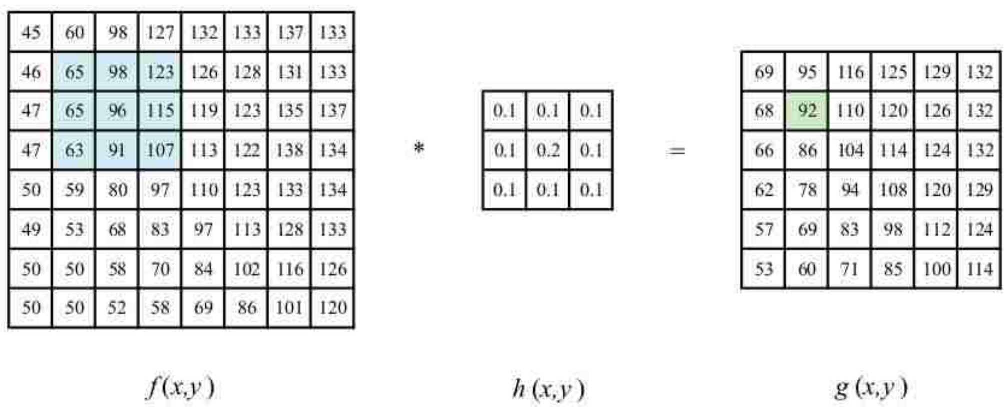


Figure 5.1. Two dimensional image filtering process [13]

The mathematical general filtering function can be defined as [13]:

$$g = f * h \tag{4}$$

Where g and f are new and original pixel value matrix, and h is filtering kernel. As shown in Figure 5.1, each pixel value in green multiplies the corresponding value in filter matrix to generate the target pixel value in green in the new pixel value matrix.

Filtering method is first introduced into FGMs modeling in this thesis. Applying filtering in FGMs modeling would increase the flexibility of FGMs modeling. And the FGMs area can be controlled by the size of filtering kernel and parameters of filters. In the thesis, two linear translation-invariant (LTI) filters, Gaussian Filter and Average Filter are used to generate FGMs area.

5.1. MULTIPLE CONTROL SOURCES WITH DIFFERENT TYPES

Two dimensional Gaussian function is shown in Figure 5.2. Central point has the highest value when compared with other points in the neighboring region.

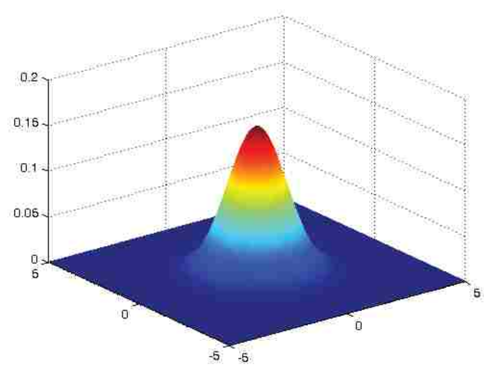


Figure 5.2. Two dimensional Gaussian function distribution

In this thesis, Gaussian equation is three dimensional (3D) since the model is three dimensional. The weight of center voxel and the weights of neighboring voxel can be calculated in the following Gaussian equation:

$$f(x,y,z) = \frac{1}{(2\pi)^{\frac{3}{2}}\sigma^2} e^{-\frac{x^2+y^2+z^2}{3\sigma^2}} \tag{5}$$

Where σ is blur factor and e is euler number. $x^2 + y^2 + z^2$ is the squared distance from the center voxel. When the distance increases, the weight for the corresponding voxel will decrease. Given a filter kernel size, the convolution matrix can be generated by the distribution value of the above Gaussian equation. In theory, the Gaussian distribution is non-zero throughout and would require a convolution kernel of infinitely large size, however kernel element values can be set to zero which are more than about three standard deviations from the mean in practice, so that the kernel can be truncated to the given kernel size. A two dimensional integer-valued convolution kernel that approximates Gaussian function with σ of 1.0 is shown in Figure 5.3.

$\frac{1}{273}$

1	4	7	4	1
4	16	26	16	4
7	26	41	26	7
4	16	26	16	4
1	4	7	4	1

Figure 5.3. Two dimensional Gaussian kernel with approximation of σ of 1.0 [13]

This matrix is applied to the original voxel model. Each new voxel value can be calculated by the weighted matrix multiplying neighboring voxel value. Center voxel is the highest weight and the neighbor pixels use weights which are proportional to the distance. In the process of modeling, default materials reference feature in voxel models is first selected as shown in Figure 5.4(a) and Figure 5.5(a), then the filter parameters is acquired to defined Gaussian Filter. The kernel matrix is applied to each voxel in the object for obtaining the FGMs model by multiplying center and its neighboring voxels with kernel matrix.

Since the kernel (A) size is constant, circumstance will happen when part of the kernel multiplies voxel value outside the model, which would result in incorrect filtering result near the boundary of model. Filtering mask in the process is then introduced to solve this problem. Using the Binary set (B) of model, the mask matrix can be decided. Using the mask matrix, a new submatrix of filter (S) can be generated according to the true value in mask matrix position as shown in equation (6):

$$S_{mn} = A_{ij} \; if \; B_{ij} = True \tag{6}$$

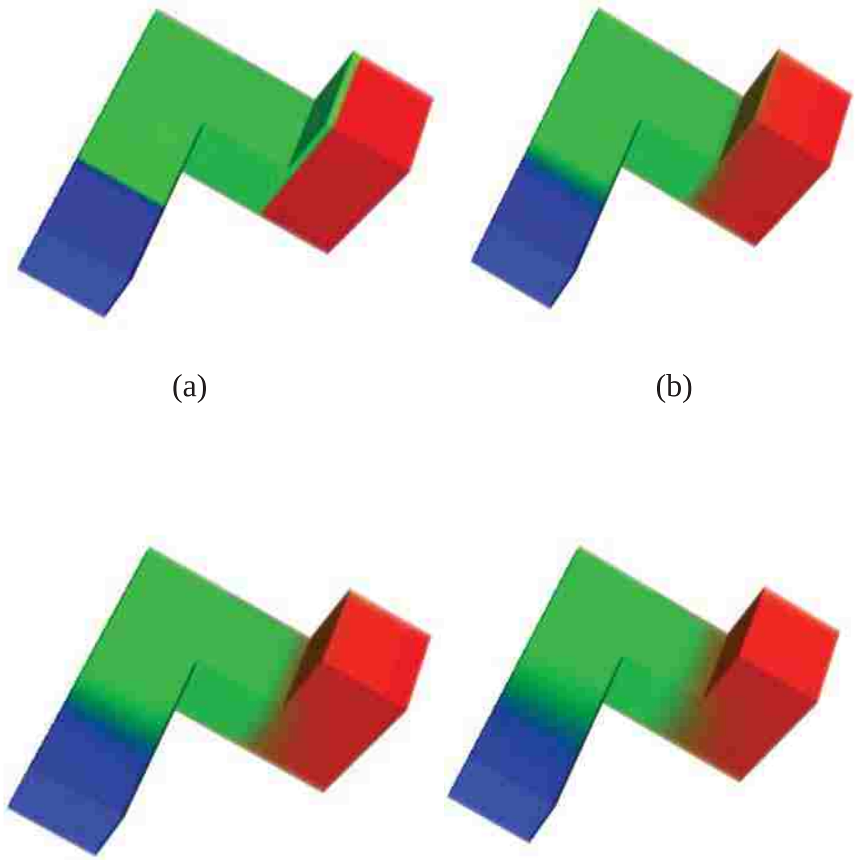
The weight value in each cell of the new submatrix ($\widehat{S}$) will be adjusted by the summation of the old submatrix (S) as shown in the equation below. Applying the new submatrix as a filter kernel to the near boundaries of model would give accurate blended value for the voxel near model boundaries.

$$\hat{S}_{mn} = \frac{s_{mn}}{\sum_{i=0} \sum_{j=0} s_{ij}} \tag{7}$$

The purpose of the Gaussian filtering operation is to blur the clear bound of materials to generate a FGMs area among them. The size of FGMs area can be controlled by defining the size of the filter kernel, which solves the problem in traditional control source FGMs modeling the sizes of FGMs area are difficult to control. Since Gaussian filter is LTI, it has separability in nature. In other words, the filter can be applied each dimension. Given the filter size to be n, using filter separability, the computation time will remain linear time ($O(n)$) instead of cubic time$\left(O(n^3)\right)$. In this way, the computation time can be significantly reduced. Gaussian filtering is an appropriate solution for FGMs modeling to control FGMs generation area.

The filtering process would move the filtering kernel to every voxel inside the model. Only the voxel with neighboring voxels of different material compositions will be modified and the voxel with same material neighboring voxel will remain the same.

The method is an unevaluated method, since the Gaussian filtering function is a consecutive function. However, it is implemented in an evaluated way since the model representation in our system is voxel based method and precision will have an influence on the material composition. Individual points are taken from the Gaussian function and can be formed to be filtering matrix. Although from the representation standpoint, approximation is inevitable, in theory, the filtering method is accurate and independent of resolution. Integration of both evaluated and unevaluated methods will not only be an executable way of model representation, but also an accurate way to describe material distribution.



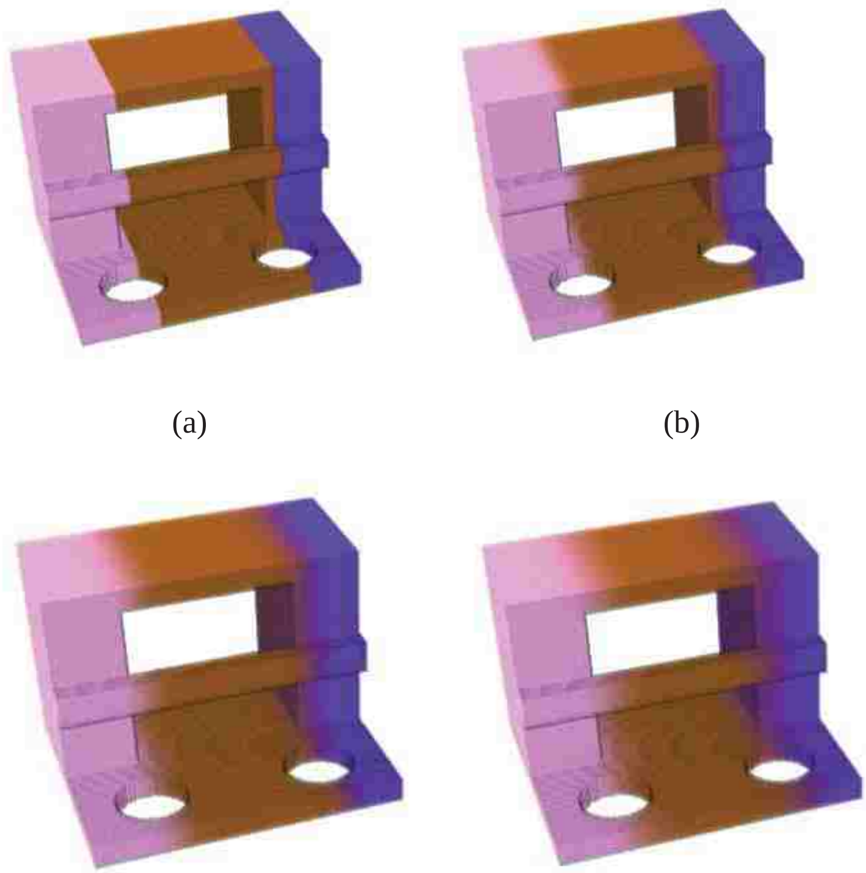
(a) (b)

(c) (d)

Figure 5.4. (a) Original default material reference feature defined, voxel resolution at $100 \times 100 \times 68$ (b) Gaussian filtering with $\sigma = 5$ and kernel size $= 15$ (c) Gaussian filtering with $\sigma = 5$ and kernel size $= 25$ (d) Gaussian filtering with $\sigma = 15$ and kernel size $= 25$

5.2. AVERAGE FILTER

Second LTI used is Average Filter. Average filtering kernel is much simpler than Gaussian filtering kernel. Each value in the filtering matrix is of equal value as shown in Figure 5.6. The summation of all the elements in the matrix is one. The neighboring voxels will have more influences on the center voxel, thus will result in a more sparsely distributed FGMs areas.



(a) (b)

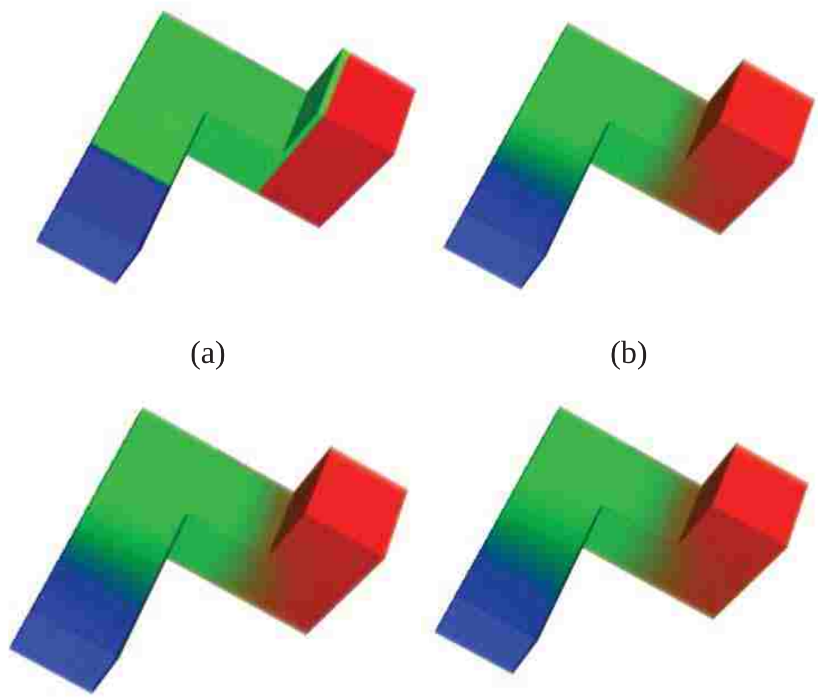
(c) (d)

Figure 5.5. (a) Original default material reference feature defined, voxel resolution at $100 \times 144 \times 122$ (b) Gaussian filtering with $\sigma = 5$ and kernel size $= 11$ (c) Gaussian filtering with $\sigma = 5$ and kernel size $= 21$(d) Gaussian filtering with $\sigma = 11$ and kernel size $= 21$

$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

Figure 5.6. Example of average filter with size of 3×3

Average Filter is applied as the same strategies of Gaussian Filter on to the model object. Default materials are first defined in the model, and later, Average Filter will be applied on every voxel to recalculate the material composition.



(a) (b)

(c) (d)

Figure 5.7. (a) Original default model, voxel resolution at $100 \times 100 \times 68$ (b) Average filtering with kernel size = 5(c) Average filtering with kernel size = 15 (d) Average filtering with kernel size = 25

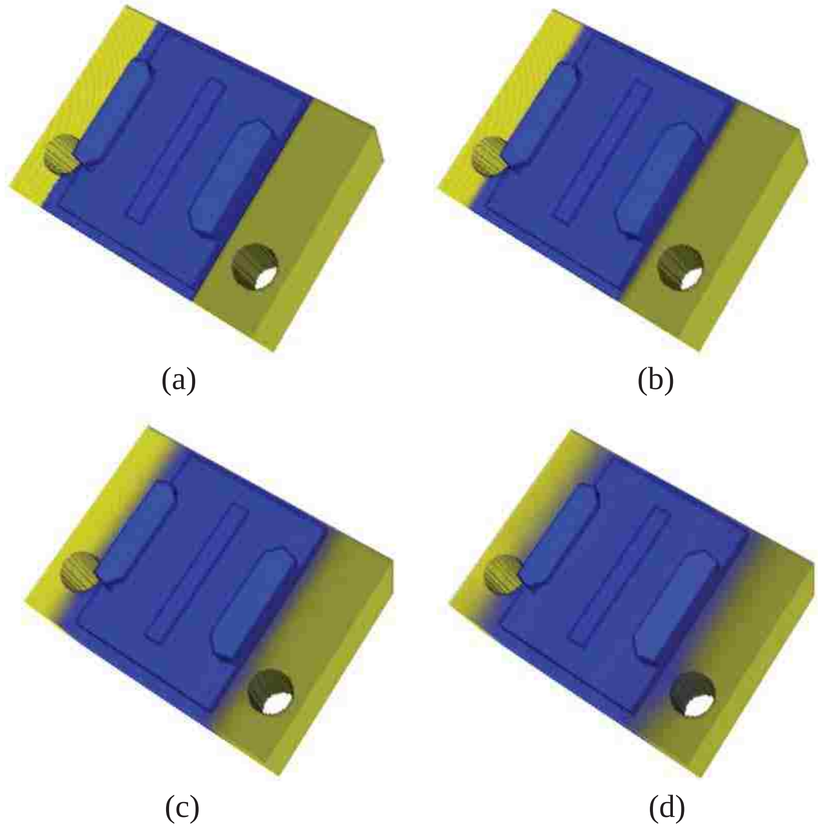


Figure 5.8. (a) Original default model, voxel resolution at 100 × 175 × 135 (b) Average filtering with kernel size = 5 (c) Average filtering with kernel size = 15 (d) Average filtering with kernel size = 25

6. FORBIDDEN ZONE MAPPING

In FGMs fabrication, forbidden zone problems occur frequently. Certain amounts of material compositions are avoided due to the unwanted property it preserves. For example, Fe-Cr Binary Alloy Phase Diagrams is shown in Figure 6.1:

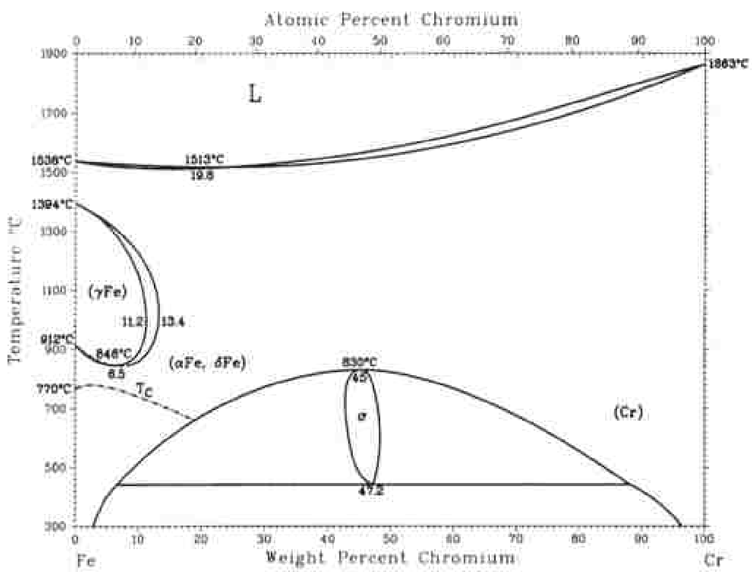


Figure 6.1. Fe-Cr Binary Alloy Phase Diagrams [14]

High-chromium ferritic stainless steels possess excellent strength at high temperatures, good corrosion resistance, and high resistance to stress corrosion cracking. However, in Fe - Cr alloys with high chromium content, the formation of the phase causes drastic deterioration in mechanical properties such as ductility and impact toughness, which is well-known as σ embrittlement [15]. In this example, phase occurs when the weight percent of Chromium between 42.7% and 48.2% at the temperature between 4500C and 8300C.

σ phase is one kind of forbidden zones. In actual manufacturing process, σ phase needs to be avoided in order to get rid of unwanted mechanical properties from FGMs. So that, in the modeling phase, the forbidden zone needs to be eliminated from FGMs modeling process. In order to eliminate the forbidden zone, the mapping function from original material composition to new material composition is established first. The mapping function should not only get rid of the forbidden material composition but also keep the original material distribution trend the same. So that a linear piecewise function is proposed to solve the problem.

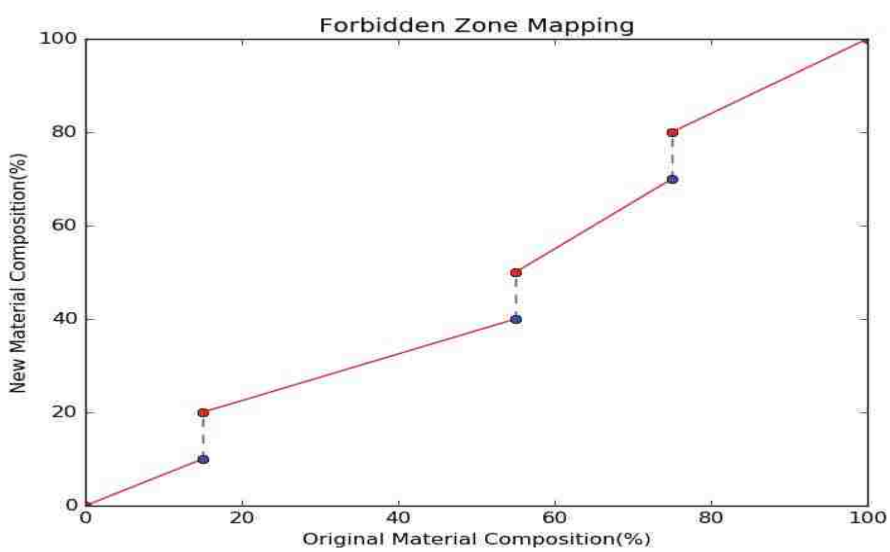


Figure 6.2. Forbidden zone mapping function from original material composition to new material composition

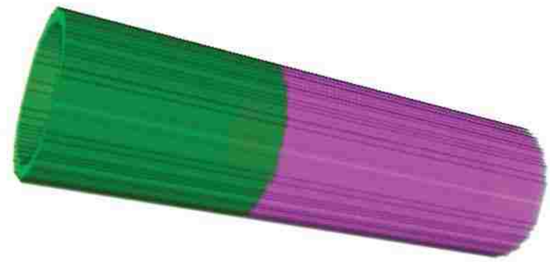
Suppose there are n forbidden zones, each one is $[a_i, b_i]$ and $c_i = \frac{a_i + b_i}{2}, 0 < i \leq n$). Each two consecutive forbidden zones define one piecewise linear function and the overall equation should be:

$$f(x) = \begin{cases} \frac{a_1}{c_1} & 0 \leqslant x \leqslant c_1 \\ \frac{a_i - b_{i-1}}{c_i - c_{i-1}}(x - c_i) + a_i & c_{i-1} < x \leqslant c_i \quad 2 \leqslant i \leqslant n \\ \frac{1 - b_n}{1 - c_n}(x - c_n) + b_n & c_n < x \leqslant 1 \end{cases} \tag{8}$$

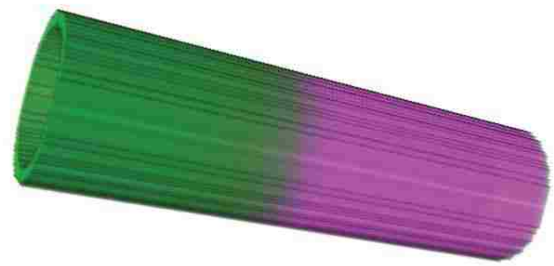
In equation (8), x is the composition of original material and $f(x)$ is the new mapped material composition. In the linear forbidden zone mapping function, as shown in Figure 6.2, the forbidden zone for material composition is [10%, 20%], [40%, 50%] and [70%, 80%]. Respectively, using the mapping function, material composition is mapped to new material composition without having the interference from the forbidden material composition.

The forbidden zone elimination function is applied in FGMs modeling. The material composition of every voxel is taken as the input, using equation (8), and the original material composition is mapped to new material composition.

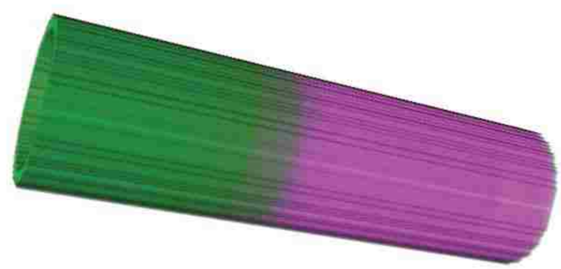
The tube is first voxelized in the FGMs modeling system as shown in Figure 6.3(a), two default material, material A(Purple) and material B(Green) are defined. Average Filter is applied to the model to generate FGMs area as shown in Figure 6.3(b). By defining the composition of two separate forbidden zone[34%, 47%], [74%, 83%] for material A, the forbidden zone mapping function is applied to the model, and redistribute the FGMs area according to the equation (8).



(a)



(b)



(c)

Figure 6.3. (a) Original tube, voxel resolution at $150 \times 40 \times 40$ (b) Average filter size of 31 applied. (c) Eliminating forbidden zone: $34\% - 47\%$, $74\% - 83\%$

By applying forbidden zone mapping function, unwanted material composition can be effectively eliminated and the material distribution will redistribute to generate smooth functionally gradient material. Two clear material boundaries are shown in Figure 6.3(c). At these two boundaries, material A will jump from 34% to 47% and 74% to 84%. Apart from these two boundaries, FGMs are smoothly distributed inside the model with only desired material composition.

7. EROSION FGMS

Some of the FGMs have their material grading from outer surface to inner point. As shown in Figure 7.1, the outer portion of the ring is made of a fatigue resistance material represented in blue, the inner portion of the ring is high-intensity material indicated in red, and there are FGMs between the outer and inner portions which are indicated in purple color [15].

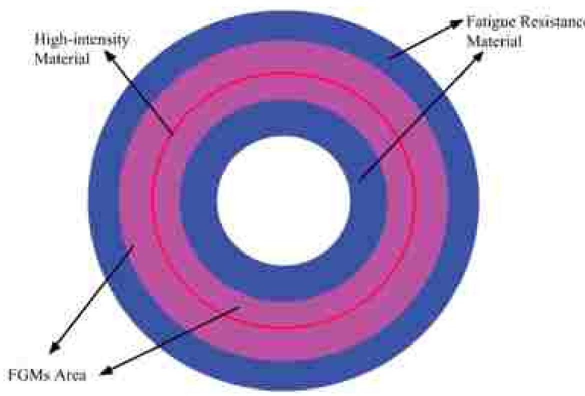


Figure 7.1. FGMs example for a ring

The material distribution along the radius is shown in Figure 7.2. Blue solid line is material composition of high-intensity material and red dash line is material composition of fatigue resistance material.

FGMs are generated in the ring by using Erosion Function. Erosion is one of the fundamental operations in morphological image processing. Since it only needs to calculate the erosion contour number based on binary set of the model, binary erosion is the kind of erosion chosen in our method.

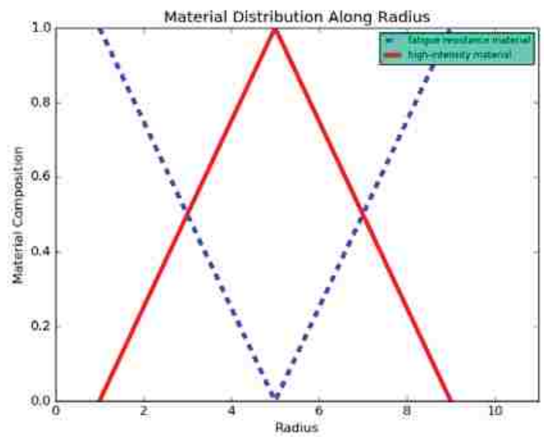


Figure 7.2. Material composition distribution along radius in ring

In our method, the erosion operator takes just the binary set of model to be eroded. Structure elements, also known as kernel, to be 3 × 3 matrix, of which all the elements in it are 1, is shown in Figure 7.3:

1	1	1
1	1	1
1	1	1

Figure 7.3. Structure elements of 3 × 3 matrix

The mathematical definition of erosion for binary images is as follows:

Suppose that X is the set of Euclidean coordinates corresponding to the input binary image, and that K is the set of coordinates for the structuring element. Let Kx denote the translation of K so that its origin is at x. Then the erosion of X by K is simply the set of all points x such that Kx is a subset of X. [13]

To calculate the erosion of a binary input model by this kernel, every binary value of the voxel is considered regardless of the existence in the model. The 3 × 3 matrix kernel is superimposed on every central and neighboring voxels of the model. In this operation, the filtering kernel multiplies the corresponding voxels. If for every voxel in the kernel, the corresponding voxel value in the binary set is 1, then, the voxel value remains 1. If any of the corresponding voxel in binary set is 0, however, the voxel value is then changed to 0.

As shown in Figure 7.4, the result of this operation is to eliminate the border points since the border points are not completely surrounded by 1. In this way, this operation fulfills the goal of shrinking the model.

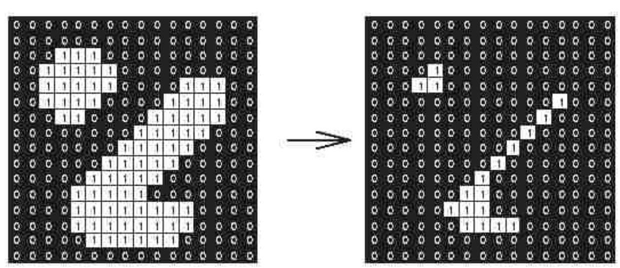


Figure 7.4. Two dimensional example for erosion operation to shrink the model [13]

In our method, the binary set of the voxel of model data is needed. The binary set defines each voxel to be 1 if the voxel is inside the model, otherwise 0 if the voxel outside the model. By utilizing the erosion function, the original binary voxel set (V (original)) is first shrunk to new binary voxel set (V (shrunk)). Then V (original) is subtracted from V (shrunk). The result of this operation will give the outer contour of the modeling object. Conducting this in a loop, whole contour information inside the model can be obtained. Finally, using the binary set of the contour information, material composition is assigned according to the contour index. Examples are shown in Figure 7.5, in gear wheel, cubic and liver model, high density material composition is increasing from outer portion to inner portion.

The composition contour later can be used in designing the tool path for FGMs additive manufacturing. The material composition isoline directs the tool path for same material composition which will facilitate the tool path generation.

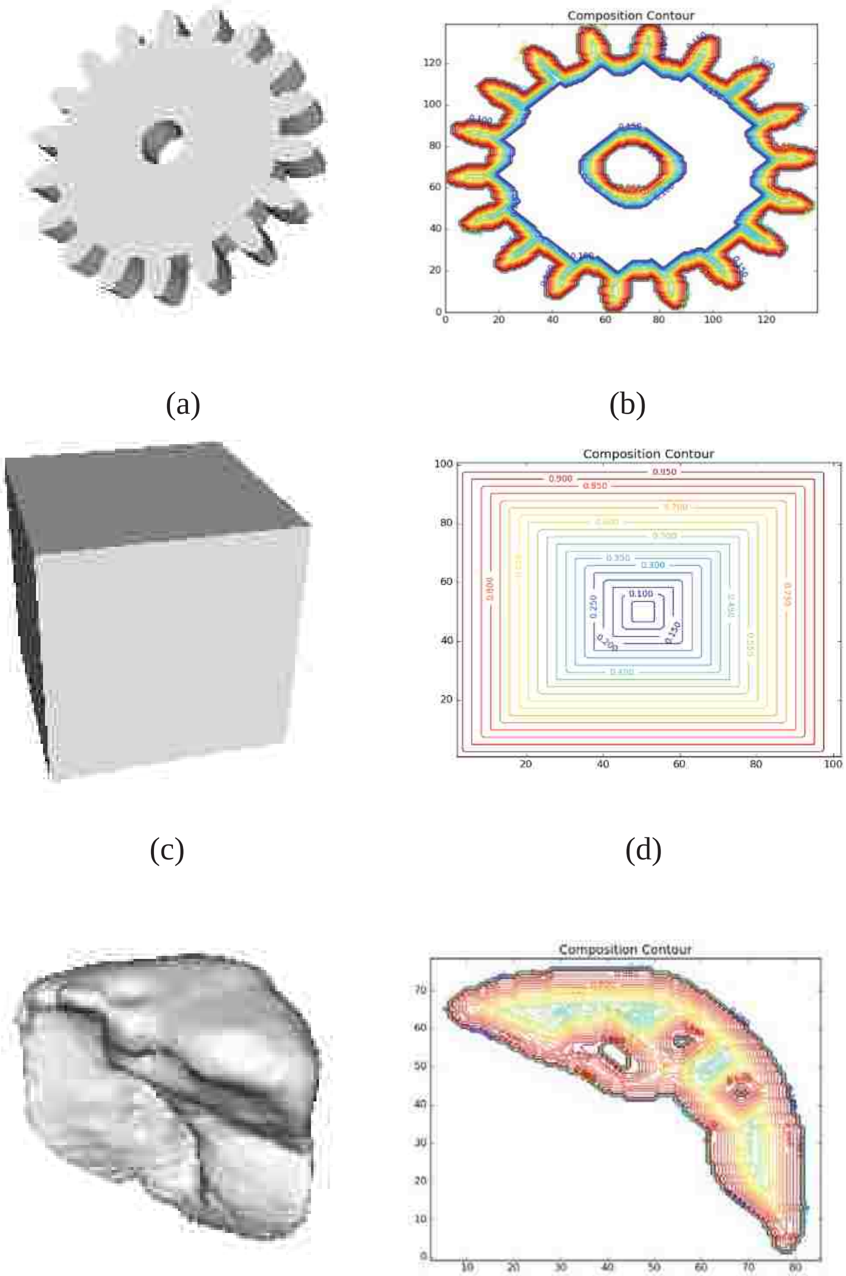


Figure 7.5. (a) Original STL file of Gear wheel (b) Composition contour of one layer for high intensity material at voxel resolution 206 × 206 × 32(c) Original STL file of Cubic (d) Composition contour of one layer for high density material at voxel resolution 150 × 150 × 150 (e) Original STL file of liver (f) Composition contour of one layer for high intensity material at voxel resolution 100 × 80 × 84

8. CONCLUSION

In this thesis, a new approach with the composite methods utilizing evaluated and unevaluated models to manipulate and represent FGMs is presented. Control sources are features introduced to develop FGMs. Different types of control sources can be integrated to develop functionally gradient materials based on distance from control sources. Control sources increase the flexibility of FGMs modeling with the voxel dataset structure. Moreover, although voxels in nature are discrete points in model which will provide inaccurate representation of FGMs, the control source based method to generate voxel information is evaluated, so that FGMs represented in this method can give an accurate representation of FGMs model.

Linear translation-invariant (LTI) filters method is first introduced in FGMs modeling. Applying image filtering method in FGMs modeling in nature improves controlling ability of FGMs areas. And image filtering method improves the computation efficiency since three dimensional filters is implemented using separate one dimensional filters.

Forbidden zone mapping function is first introduced in FGMs modeling. Previous FGMs modeling study does not include the actual additive manufacturing process in which forbidden composition of materials may exist. Using the method of zone mapping function improves the credibility of FGMs modeling while it eliminates the potential risk areas in the phase diagram of materials.

Finally, the voxel representation of FGMs using Erosion function is introduced. Material composition variation between outer and inner portion can be easily implemented and material composition contour can be retrieved to later facilitate tool path generation in AM process.

BIBLIOGRAPHY

[1] Garland, A., Mocko, G., & Fadel, G., "Challenges in Designing and Manufacturing Fully Optimized Functional Gradient Material Objects." *ASME 2014 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*. American Society of Mechanical Engineers, 2014.

[2] Sobczak, J. J., & Drenchev, L., "Metallic functionally graded materials: a specific class of advanced composites." *Journal of Materials Science & Technology* 29.4 (2013): 297-316.

[3] Shin, K. H., Natu, H., Dutta, D., & Mazumder, J., "A method for the design and fabrication of heterogeneous objects." *Materials & Design* 24, no. 5 (2003): 339-353.

[4] Chen, M., & Tucker, J. V., "Constructive volume geometry." In *Computer Graphics Forum*, vol. 19, no. 4, pp. 281-293. Blackwell Publishers, 2000.

[5] Jackson, T. R.,"Analysis of functionally graded material object representation methods." PhD diss., Massachusetts Institute of Technology, 2000.

[6] Zhu, F., "Visualized CAD modeling and layered manufacturing modeling for components made of a multiphase perfect material." PhD diss., University of Hong Kong, 2004.

[7] Siu, Y.K., & Tan, S.T., "'Source-based'heterogeneous solid modeling." *Computer-Aided Design* 34, no. 1 (2002): 41-55.

[8] Liu, H., Maekawa, T., Patrikalakis, N.M., Sachs, E.M., & Cho, W., "Methods for feature-based design of heterogeneous solids."*Computer-Aided Design* 36.12 (2004): 1141-1159.

[9] Biswas, A., Shapiro, V., & Tsukanov, I., "Heterogeneous material modeling with distance fields." *Computer Aided Geometric Design* 21, no. 3 (2004): 215-242.

[10] Gupta, V., & Tandon, P., "Heterogeneous object modeling with material convolution surfaces." *Computer-Aided Design* 62 (2015): 236-247.

[11] Dutta, D., "An approach to modeling & representation of heterogeneous objects." *Ann Arbor* 6 (1998): 48109-2125.

[12] Leu, M. C., Tang, L., Deuser, B., Landers, R. G., Hilmas, G. E., Zhang, S., & Watts, J., "Freeze-form extrusion fabrication of composite structures." In *Proceedings of the Solid Freeform Fabrication Symposium. Austin, TX*, pp. 111-124. 2011.

[13] Szeliski, R., *Computer vision: algorithms and applications*. Springer Science & Business Media, 2010.

[14] Henry, S.D., Moosbrugger, C., Anton, G.J., Sanders, B.R., Hrivnak, N., Terman, C., Kinson, J., Muldoon, K., & Scott Jr, W.W., *ASM handbook*. Vol. 21. Materials Park, OH, USA: ASM international, 2001.

[15] Wang, F., Chen, K. Z., Feng, X. Y., & Feng, X. A., "Developing a manufacturing technology for the components made of a multiphase perfect material." *The International Journal of Advanced Manufacturing Technology* 40, no. 7-8 (2009): 837-846.

[16] Samanta, K., & Koc, B., "Feature-based design and material blending for free-form heterogeneous object modeling." *Computer-Aided Design* 37, no. 3 (2005): 287-305.

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