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Neural MRAC based on modified state observer

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NEURAL MRAC

BASED ON

MODIFIED STATE OBSERVER

by

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ABSTRACT

A new model reference adaptive control design method with guaranteed transient performance using neural networks is proposed in this thesis. With this method, stable tracking of a desired trajectory is realized for nonlinear system with uncertainty, and modified state observer structure is designed to enable desired transient performance with large adaptive gain and at the same time avoid high frequency oscillation. The neural network adaption rule is derived using Lyapunov theory, which guarantees stability of error dynamics and boundedness of neural network weights, and a soft switching sliding mode modification is added in order to adjust tracking error.

The proposed method is tested by different theoretical application problems simulations, and also Caterpillar Electro-Hydraulic Test Bench experiments. Satisfying results show the potential of this approach.

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1. INTRODUCTION

Applications of artificial neural networks in the field of control have been developed for decades. Neural networks' universal function approximation property can be useful in solving control problems. Various adaptive control techniques using neural networks were put forward.

At the same time, based on the philosophy of feedback linearization, dynamic inversion is developed for nonlinear control design. In this approach, an co-ordinate transformation is carried out to make the system dynamics take a linear form, then linear design methods could be taken, and based on this method, model reference adaptive control(MRAC) is developed. The drawback of dynamic inversion is its sensitivity to modeling errors and parameter inaccuracies while neural networks technique is able to cancel out the inversion error. The neural networks are trained online using a Lyapunov-based approach.

Though it provides stability, to reduce tracking error, it is required to increase adaption gain, and for conventional mode reference adaptive control it usually leads to oscillation in neural network output, as a result the control signal will oscillate. In many control application scenarios, unwanted oscillation in control signal may eventually lead to failure of the system.

The objective of this thesis is to present an approach using neural network controller based on with modified predictor structure, which prevents high frequency oscillation in high adaption gain, and combines with a soft-switching sliding mode modification, which ideally reduce tracking error. In paper 1, the method is introduced and applied in theoretical application of robot-arm motion and ship steering control; In paper 2, a missile autopilot control problem is taken to show the method's ability in reducing oscillation and tracking error; In paper 3, the method is applied in a Caterpillar Electro-Hydraulic test bench for piston velocity tracking control purpose, and satisfying experimental results show its potential.

PAPER

I. Development and Time Domain Analysis of a New Model Reference Adaptive Controller

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ABSTRACT

A new model reference adaptive control design method using neural networks that guarantees *transient performance* is proposed in this paper. Stable tracking of a desired trajectory can also be achieved for nonlinear systems that operate under uncertainties. A modified state observer structure is designed to enable desired transient performance fast with large adaptive gains and at the same time avoid high frequency oscillations during uncertainty learning. The neural network adaptation rule is derived using Lyapunov theory, which guarantees *stability* of error dynamics and boundedness of neural network weights. An extra term is added in the controller expression using a 'soft switching' sliding mode that can be used to adjust tracking errors. Analytical bounds are derived and simulation results from two representative problems are presented to demonstrate the performance of the developed control technique.

1. INTRODUCTION

The field of artificial neural networks and its application to control systems has seen phenomenal growth in the last two decades. The origin of research on artificial neural networks can be traced back to 1940s [1]. In 1990, a compiled book was published [2] detailing various applications of artificial neural networks. A good survey paper appeared in 1992 [3], which outlined various applications of artificial neural networks to control system design. The main philosophy that is exploited in system theory applications is the universal function approximation property of neural networks [4]. Benefits of using neural networks for control applications include its ability to effectively control nonlinear plants while adapting to unmodeled dynamics and time-varying parameters.

In 1990, a paper by Narendra and Parthasarathy demonstrated the potential and applicability of neural networks for the identification and control of nonlinear dynamical systems [5]. The authors suggested various architectures as well as learning algorithms useful for identification and adaptive control of nonlinear dynamic systems using recurrent neural networks. Since then, Narendra and his co-workers have come up with a variety of useful adaptive control design techniques using neural networks, including applications concerning multiple models [6].

In 1992, Sanner and Slotine [7] developed a direct adaptive tracking control architecture with Gaussian Radial Basis Function (RBF) networks to compensate for plant nonlinearities. The update process also kept the weights of the neural networks bounded. In 1996, Lewis et al. [8] proposed an online neural network that approximated unknown functions and it was used in designing a controller for a robot. Their approach avoided some of the limiting assumptions (like linearized models) of traditional adaptive control techniques. More important, their theoretical development also provided a Lyapunov stability analysis that guaranteed both tracking performance as well as boundedness of weights. However, the applicability of this technique was limited to systems which could be expressed in the "Brunovsky form" [9] and which were affine in the control variable (in state space form). A robust adaptive output feedback controller for SISO systems with bounded disturbance was studied by Aloliwi and Khalil [10]. In a more recent paper, an adaptive output feedback control scheme for the output tracking of a class of nonlinear systems was presented by Seshagiri and Khalil using RBF neural networks [11].

A relatively simpler and popular method of nonlinear control design is the technique of dynamic inversion (e.g. [12, 13, 14]), which is essentially based on the philosophy of feedback linearization [9, 15]. In this approach, an appropriate co-ordinate transformation is carried out to make the system dynamics take a linear form. Linear control design tools are then used to synthesize the controller. A drawback of this approach is its sensitivity to modeling errors and parameter inaccuracies. One way of addressing the problem is to augment the dynamic inversion technique with the H_∞ robust control theory [14]. Important contributions have come from Calise and his co-workers in a number of publications (e.g. [16 - 20]), who have proposed to augment the dynamic inversion technique with neural networks so that the inversion error is cancelled out. The neural networks are trained online using a Lyapunov-based approach (similar to the approach followed in [7] and [8]). This basic idea has been extended to a variety of cases, namely output based control design [19, 20], reconfigurable control design [21] etc. The feasibility and usefulness of this technique has been demonstrated in a number of applications in the field of flight control.

MRAC has been widely applied recently to solve control problems for system with matched unmodeled dynamics [22][23]. With MRAC, it is difficult to achieve a desired(fast) transient performance *and* avoid unwanted high frequency oscillations at the same time when uncertainties are present. It is due to the fact that high gains are required typically to learn the uncertainties online. If neural networks are used to represent uncertainties, this process necessarily results in an uncertainty model showing oscillations during the transient learning period before the weights stabilize. Use of dynamic inversion to cancel the uncertainties during learning then leads to oscillatory control signals which if unchecked could excite the unmodeled high frequency dynamics of the plant and lead to instability. Various NN-based MRAC methods have been recently developed (for example, [24][25]) to address the issue of boundedness of tracking errors. Modification to the adaptive law such as σ-modification [26], e-modification [27] have been introduced. These methods modify the adaptive law by adding a factor depending on the prediction error and ensure the convergence of parameter estimation. Moreover, when close to steady state conditions, the modification term becomes inactive and therefore, the estimation accuracy is guaranteed. In [28], a projection operator was used to modify the adaptive law. Projection operator replaces the common Lipschitz continuous property with an arbitrary many times continuous differentiability, and estimation parameters are proven to be bounded.

Although these developments help improve the robustness of the adaptive control laws, their tracking accuracy can only be shown to be bounded, and the bound depends on the magnitude of disturbances. At the same time, a typical MRAC cannot avoid unwanted oscillations. Recently many methods were developed to solve these two problems. In [29][30], a new MRAC neural networks controller named $\mathcal{L}_1$ adaptive controller is proposed, and the transient performance of both system's input and output signal are characterized with some norms. This adaptive control architecture has a low-pass filter in the feedback loop, and its desired transient performance can be guaranteed by increasing adaption gain and improving the NN approximation, and at the same time, the high frequency oscillation is avoided. In [31], an adaptive control method that allows fast adaptation for systems with slow reference models is given. In this method, in order to allow fast adaptation, the neural network is trained with a high bandwidth state emulator. Low bandwidth control is maintained by a filter to isolate fast emulator dynamics from the control signal. In [32], a novel Kalman-filter version of the *e*-modification [27] is developed. In this method the standard *e*-modification term is interpreted as the gradient of a norm measure of a linear constraint violation, and this linear constraint is then used to develop a Kalman-filter-based *e*-modification. It is shown that this method leads to smaller tracking errors without generating significant oscillations in the system response.

Sliding mode control (SMC) is inherently robust to uncertainties [33][34][35]. In SMC, trajectories are forced to reach a designed sliding surface in a finite time and to stay on the surface for all future time. Dynamics on the sliding surface is independent of matched uncertainties and the sliding surface is designed so as to guarantee the asymptotic stability of control objective. Though it has many advantages, a major drawback of the SMC in applications is control switching along the sliding surface, called 'chatter' thus oscillations are usually unavoidable. Saturation functions with a boundary layer[9] can be used to alleviate chatter but it cannot be eliminated. Also, asymptotic stability inside the boundary layer needs to be separately shown. Recently, there has been some work with higher-order sliding mode controllers to avoid 'chatter'[36]. A soft-switching sliding mode technique has been introduced by Lyshevsky [37][38] to eliminate 'chatter'. By modifying signum function used in a typical SMC to continuous real-analytic function, for example, hyperbolic tangent functions, the soft switching sliding mode controller avoid oscillations and remain asymptotic stable at the same time. In [39], a systematic way to combine adaptive control and SMC for trajectory tracking in presence of parametric uncertainties and uncertain nonlinearities is developed. The sliding mode controller is smoothed with two methods based on the concept of boundary layer [34]. Asymptotic stability of adaptive system in presence of parametric uncertainties are realized, and the drawback of control chattering is reduced significantly. In [40], a modified switching function which provides low-chattering control signal is introduced, and the SMC is combined with a neural network adaptive controller which identifies modeling error online. In [41], by using a similar approaching to SMC, a novel approach that combines NN feed forward controller with continuous robust integral of sign of error (RISE) feedback controller is introduced. In this interesting method, by designing sliding surface using sign of error, a continuous RISE feedback is combined with a NN-based adaptive controller, and it is shown that using Lyapunov theory the tracking error is *asymptotically stable*, while typical NN-based controller formulations can only yield uniformly ultimately bounded (UUB) stability [9], and at the same time, the control is free from oscillations. Experimental results show the potential of this method in reducing tracking errors [42][43].

This paper develops a new neural network MRAC with guaranteed transient performance and asymptotic stability, and at the same time free from unwanted oscillations. Based on MRAC neural networks controller, the neural network observer structure is modified in the manner of [44]. The basic notion is to separate the functions of a controller and observer. That is the controller stabilizes (tracks) a reference and an observer tracks the true system. By having a dynamic observer instead of just calculating the uncertainty as in other MRAC approaches, it is believed that the designer can make the estimation error decay fast with the observer gains. This allows one to use higher learning rates for the adaptation that helps achieve better tracking performance without inducing high-frequency oscillations. At the same time, the modified term is inactive when neural network estimation is ideal, therefore the estimation accuracy is guaranteed. Furthermore, the proposed technique has a sliding mode term to provide asymptotic stability. Since this technique has an observer in the loop, the excellent RISE scheme is not applicable; a soft switching sliding mode term is added to guarantee asymptotic stability. It is proven using Lyapunov method that it ideally leads to asymptotic stability instead of UUB, and at the same time is free from oscillations which is common for typical sliding mode based adaptive controller. In general, the proposed controller enables higher adaptive gain without generating oscillations, provides better transient performance and asymptotic stability at the same time.

Rest of the paper is organized as follows. In Section 2, the system dynamics and the neural networks structure are defined. In Section 3, the new control solution is proposed. Stability proofs for both the observer and state error signals are presented and the guaranteed transient performance is explained in Section 4. Illustrative simulation studies of a robot-arm and a ship steering control problems are carried out in Section 5 and conclusions are drawn in Section 6.

2. PROBLEM DESCRIPTION

Consider the following single input single output (SISO) system.

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \cdots \\ \dot{x}_n = b(u - f(\mathbf{x})) \end{cases} \tag{1}$$

and the system output is defined as

$$y = cx_1 \tag{2}$$

c is a non-zero constant. The initial condition is set to

$$\mathbf{x}(0) = \mathbf{0}\,. \tag{3}$$

The set of equations in (1) can be written in a compact form as

$$\dot{\mathbf{x}}(t) = Ax(t) + B(u - f(x)) \tag{4}$$

where $\mathbf{x} \in \mathbb{R}^n$ is the system state vector, and all states are assumed to be measurable. $u \in \mathbb{R}$ is control signal, A is $n \times n$ system matrix, B is $n \times 1$ vector, $b > 0$, (A, B) is controllable. $f : \mathbb{R}^n \to \mathbb{R}$ is an unknown continuous nonlinear function.

The control objective is to design a neural adaptive controller which ensures output $y(t)$ tracks a desired bounded continuous trajectory $r(t)$, and the system behavior follows a nominal linear time-invariant (LTI) system which is designed through standard methods(for example, through linear quadratic regulator theory[37]), and at the same time guarantee desired transient and steady state performance.

Assume the following NN approximation of $f(\mathbf{x})$ exists

$$f(\mathbf{x}) = \mathbf{W}^T \phi(\mathbf{x}) + \varepsilon(\mathbf{x}) \quad |\varepsilon(\mathbf{x})| < \varepsilon^* \tag{5}$$

where $\phi(\mathbf{x})$ is a set of radial basis functions[34], and each element of $\phi(\mathbf{x})$ is defined as,

$$\phi(y) = \exp(-(y - z)^T (y - z) / \sigma^2) \tag{6}$$

In (6), z is the location of selected center, σ is the 'width'. $\mathbf{W}$ are the ideal network weights, $\varepsilon(\mathbf{x})$ is the network approximation error, ε^* is its uniform bound. Further assume that a compact convex set Ω is known a priori such that

$$\mathbf{W} \in \Omega \tag{7}$$

In order to realize tracking control for this SISO system, the following neural network adaptive controller is developed.

3. CONTROL SOLUTION

The proposed controller is a combination of a linear feedback control, neural network adaptive control and a soft switching sliding mode control. First of all, divide the controller expression into three parts-the linear feedback control $K_1\mathbf{x}$, neural networks adaptive control u_e and soft switching sliding mode control μ

$$u = K_1\mathbf{x} + u_e + \mu \tag{8}$$

where K_1 the closed loop feedback gain, which ensures closed-loop reference dynamics matrix $(A - BK_1)$ is Hurwitz. The linear feedback control ensures stability when there is no uncertainty; the adaptive control is obtained through neural networks observer, and cancels the uncertainty; the soft switching sliding mode control guarantees asymptotic stability in presence of neural networks estimation error, and it is going to be exactly defined later with stability proof.

Substitute (8) into (4), (4) becomes

$$\dot{\mathbf{x}}(t) = A_m\mathbf{x}(t) + B(u_e(t) + \mu - f(\mathbf{x})) \tag{9}$$

where $A_m = A - BK_1$.

Define the following state observer structure,

$$\dot{\hat{\mathbf{x}}}(t) = A_m\hat{\mathbf{x}}(t) + B(u_e(t) + \mu - \hat{f}) - K_2\tilde{\mathbf{x}}(t) \tag{10}$$

where $\hat{\mathbf{x}}(t)$ represents the observer states at time t. The initial conditions for observer are

$$\hat{\mathbf{x}}(0) = \mathbf{0} \tag{11}$$

Since the uncertainty and the true neural network weights are unknown, they are represented as $\hat{\mathbf{W}}^T\phi(\mathbf{x})$ where $\hat{\mathbf{W}}$ represents the estimated neural network weights with a proper weight update law. The observer gain matrix is assumed diagonal for convenience and are given by $K_2 = diag(k_2^1, k_2^2, ..., k_2^n)$. In the observer structure, $\hat{f}$ is assumed to be canceled perfectly by neural networks controller, i.e. $\hat{f} = \hat{\mathbf{W}}^T \phi(\mathbf{x})$.

Define the observer error as

$$\tilde{\mathbf{x}}(t) \equiv \hat{\mathbf{x}}(t) - \mathbf{x}(t) \tag{12}$$

And the adaptive weight update law is defined as follows

$$\dot{\hat{\mathbf{W}}}(t) = \Gamma_c \Pr oj(\hat{\mathbf{W}}(t), \phi(\mathbf{x})\tilde{\mathbf{x}}(t)^T PB) \tag{13}$$

where P is defined by $A_m^T P + PA_m = -Q$, with Q being a positive definite matrix and Γ_c is the learning rate of the neural network. The projection operator property guarantees the boundedness of neural networks weights error

$$\tilde{\mathbf{W}}^T \tilde{\mathbf{W}} \leq W_{\max} \tag{14}$$

where $W_{\max} \equiv \max_{\mathbf{W} \in \Omega} 4\|\mathbf{W}\|^2$, $\tilde{\mathbf{W}} \equiv \hat{\mathbf{W}} - \mathbf{W}$ [28].

Now with neural networks weights, the adaptive control expression becomes

$$u_e = k_g r(t) + \hat{\mathbf{W}}^T \phi(\mathbf{x}) \tag{15}$$

where

$$k_g \equiv \frac{1}{CA_m^{-1}B} \tag{16}$$

is the open loop gain of the reference system.

By subtracting (9) from (10), and using (15), the observer error dynamics is rewritten as

$$\dot{\tilde{\mathbf{x}}}(t) = (A_m - K_2)\tilde{\mathbf{x}} + B(\tilde{\mathbf{W}}^T \phi(\mathbf{x}) - \varepsilon) \tag{17}$$

By using Lyapunov method [9], it will be shown that the neural network estimation error and the observer error are bounded. By introducing the observer gain K_2, the learning process is made smooth and the modified term $K_2\tilde{\mathbf{x}}(t)$ decreases as $\tilde{\mathbf{x}}$ decreases, therefore the learning accuracy is guaranteed. As a result, the modified observer structure allows for high values of adaptation gain without generating high frequency oscillations.

4. STABILITY ANALYSIS

In this section, Lyapunov method is used to prove the boundedness of the observer error dynamics. And in order to assure asymptotic convergence of reference error, the soft-switching sliding mode controller is derived. Details of the proofs are provided in the following subsections.

4.1. OBSERVER ERROR

To get the error bound for neural network observer, consider a Lyapunov function as $V(\tilde{\mathbf{x}}, \tilde{\mathbf{W}}) = \tilde{\mathbf{x}}^T P\tilde{\mathbf{x}} + \Gamma_c^{-1}\tilde{\mathbf{W}}^T\tilde{\mathbf{W}}$, and differentiate $V(.)$ to get

$$\dot{V} = \dot{\tilde{x}}^T P\tilde{x} + \tilde{x}^T P\dot{\tilde{x}} + \Gamma_c^{-1}(\tilde{\mathbf{W}}^T\dot{\tilde{\mathbf{W}}} + \dot{\tilde{\mathbf{W}}}^T\tilde{\mathbf{W}}) \tag{18}$$

substitute the weight update law (13) and observer error dynamics (17), (18) becomes

$$\begin{aligned}\dot{V} &= -\tilde{\mathbf{x}}^T Q\tilde{\mathbf{x}} - 2\tilde{\mathbf{x}}^T K_2 P\tilde{\mathbf{x}} - 2\tilde{\mathbf{x}}^T PB(\tilde{\mathbf{W}}^T\phi(\mathbf{x}) - \varepsilon(\mathbf{x})) \\ &\quad + 2\tilde{\mathbf{W}}^T \Pr oj(\hat{\mathbf{W}}, \phi(\mathbf{x})\tilde{\mathbf{x}}^T PB) \\ &\le 2\|PB\varepsilon\|\|\tilde{\mathbf{x}}\| - [\lambda_{\min}(Q) + 2\lambda_{\min}(K_2P)]\|\tilde{\mathbf{x}}\|^2\end{aligned} \tag{19}$$

where $\lambda_{\min}$ represents the minimum eigenvalue. Therefore $\dot{V} \le 0$ when

$$\|\tilde{\mathbf{x}}(t)\| \ge \frac{2\|Pb\varepsilon^*\|}{\lambda_{\min}(Q) + 2\lambda_{\min}(K_2P)} \tag{20}$$

As a result,

$$V(\tilde{\mathbf{x}}, \tilde{\mathbf{W}}) \le \tilde{\mathbf{x}}^T P \tilde{\mathbf{x}} + \Gamma_c^{-1} W_{\max}$$
$$\le \lambda_{\max}(P) \left(\frac{2\left\| PB\varepsilon^* \right\|}{\lambda_{\min}(Q) + 2\lambda_{\min}(K_2 P)} \right)^2 + \Gamma_c^{-1} W_{\max} \quad (21)$$

and at the same time

$$V(\tilde{\mathbf{x}}, \tilde{\mathbf{W}}) \ge \tilde{\mathbf{x}}^T P \tilde{\mathbf{x}} \ge \lambda_{\min}(P) \left\| \tilde{\mathbf{x}} \right\|^2 \quad (22)$$

(21) and (22) lead to

$$\left\| \tilde{\mathbf{x}}(t) \right\| \le \sqrt{\frac{\left(\lambda_{\max}(P) \left(\frac{2\left\| Pb\varepsilon^* \right\|}{\lambda_{\min}(Q) + 2\lambda_{\min}(K_2 P)} \right)^2 + \Gamma_c^{-1} W_{\max} \right)}{\lambda_{\min}(P)}}$$
$$\equiv \gamma_0 \quad (23)$$

In (23), by increasing adaptation gain Γ_c and observer gain K_2, $\left\| \tilde{\mathbf{x}} \right\|$ can be driven to be as small as possible, therefore, precise uncertainty estimation using online neural networks is guaranteed. Also, with the modification term to smooth out learning, the state observer structure suppresses the high frequency oscillation so that increased adaptation gain and smooth control are possible at the same time.

4.2. REFERENCE ERROR

Notice that, with adaptive control and linear feedback control alone, or in other words when $\mu = 0$, the controller is able to track reference system, however, with a soft-switching sliding mode controller, the tracking error can be shown asymptotic stable. Define a reference LTI system dynamics as

$$\dot{\mathbf{x}}_r = A_m \mathbf{x}_r + b k_g r \quad (24)$$

By subtracting the reference dynamics (24) from actual system dynamics (9), the tracking error dynamics are expressed as

$$\begin{aligned}\dot{\mathbf{e}} &\equiv \dot{\mathbf{x}}(t) - \dot{\mathbf{x}}_r(t) \\ &= A_m\mathbf{e} + b(\mu + (\hat{\mathbf{W}}^T - \mathbf{W}^T)\phi(\mathbf{x}) - \varepsilon(\mathbf{x}))\end{aligned} \tag{25}$$

Recalling the definition of system dynamics as given in (1), (25) can be written as

$$\begin{cases}\dot{e}_1 = e_2 \\ \dot{e}_2 = e_3 \\ \cdots \\ \dot{e}_n = bK_1\mathbf{e} + b(\mu + \tilde{\mathbf{W}}^T\phi(\mathbf{x}) - \varepsilon(\mathbf{x}))\end{cases} \tag{26}$$

Then, define the sliding surface as

$$s(t) \equiv \sum_{p=0}^{n-1} \lambda_p e_1^{(n-p-1)}(t) \tag{27}$$

where $\lambda_i > 0, i = 0,1,...,n-1$. In most cases we can just take $\lambda_0 = 1$. For example, when n=3, the sliding manifold is $s = e_3 + \lambda_1 e_2 + \lambda_2 e_1$.

With a Lyapunov function $V_s = \frac{1}{2}s^2$, its derivative is given by

$$\begin{aligned}\dot{V}_s = s\dot{s} &= s(\sum_{p=1}^{n-1} \lambda_p e_1^{(n-p)} + e_1^{(n)}) \\ &= s(\sum_{p=1}^{n-1} \lambda_p e_1^{(n-p)} + \dot{e}_n) \\ &= s(\sum_{p=1}^{n-1} \lambda_p e_1^{(n-p)} + bK_1\mathbf{e} + b(\mu + D))\end{aligned} \tag{28}$$

where

$$D \equiv \tilde{\mathbf{W}}^T\phi(\mathbf{x}) - \varepsilon(\mathbf{x}) \tag{29}$$

Recall NN approximation property (5) and weights error boundedness (14), immediately the following bound for D is obtained

$$\begin{aligned}\|D\| &= \left\|\tilde{\mathbf{W}}^T\phi(\mathbf{x}) - \varepsilon(\mathbf{x})\right\| \\ &\le W_{\max} + \varepsilon^* \equiv D^* \end{aligned} \tag{30}$$

Now the soft switching sliding manifold control term is formulated as

$$\mu = -\sum_{p=1}^{n-1}\lambda_p e_1^{(n-p)} / b - K_1\mathbf{e} - \beta\tanh(\alpha s), \tag{31}$$

By substituting (31) into (28), it can be shown that

$$\dot{V}_s = bs(-\beta\tanh(\alpha s) + D) \tag{32}$$

When s>0

$$\begin{aligned}\dot{V}_s &= bs(-\beta\tanh(\alpha s) + D) \\ &\le bs(-\beta\tanh(\alpha s) + D^*)\end{aligned} \tag{33}$$

$\dot{V}_s \ge 0$ only when

$$\tanh(\alpha s) \le D^* / \beta \tag{34}$$

which leads to

$$0 < s \le \frac{1}{2\alpha}\ln\frac{1+\frac{D^*}{\beta}}{1-\frac{D^*}{\beta}} \tag{35}$$

When s<0

$$\begin{aligned}\dot{V}_s &= bs(-\beta\tanh(\alpha s) + D) \\ &\le b(-s)(\beta\tanh(\alpha s) + D^*)\end{aligned} \tag{36}$$

$\dot{V}_s \geq 0$ only when

$$\tanh(\alpha s) > -D^* / \beta \tag{37}$$

which leads to

$$0 > s \geq \frac{1}{2\alpha} \ln \frac{1 - \frac{D^*}{\beta}}{1 + \frac{D^*}{\beta}} \tag{38}$$

From (35) and (38), it can be observed that the bound for the sliding manifold is

$$\|s\| < \frac{1}{2\alpha} \ln \frac{1 + \frac{D^*}{\beta}}{1 - \frac{D^*}{\beta}} \equiv \gamma_1 \tag{39}$$

As long as $\alpha > 0$, $\beta > D^*$, the sliding manifold will remain bounded. Notice that, by increasing α and β the bound of the sliding manifold will converge to 0. The tracking of controller system is asymptotically stable when $\gamma_1 = 0$.

To sum up, with (8), (15), and (31), we can get the final expression for proposed controller as follows

$$u = K_1 \mathbf{x}_r + k_g r(t) + \hat{\mathbf{W}}^T \phi(\mathbf{x}) - \sum_{p=1}^{n-1} \lambda_p e_1^{(n-p)} / b - \beta \tanh(\alpha s) \tag{40}$$

Note that since no discontinuous function is introduced, this controller is *smooth*. Without generating additional oscillations, the controller will drive the tracking error asymptotic to 0.

5. SIMULATION

This section contains two representative applications with which the performance of the proposed controller is analyzed. The first problem is about a robot arm motion and the second one relates to ship steering.

5.1. ROBOT ARM MOTION

Consider a single-link robot arm motion in the presence of friction and disturbance. This problem is described in [45]. The governing equation of motion for the robot arm is given by

$$\ddot{\theta} + \Omega^2 \sin\theta = u + w(t) \tag{37}$$

where θ (radian) is the arm angle u (N/m • kg)is the specific torque(i.e., the torque divided by the moment of inertia), and $\Omega^2 = g/l$, where l is the length of the arm and g is the gravitational acceleration. A value of $\Omega = 4$ is used in simulations. In order to analyze the controller performance under uncertainties, an uncertainty function $w(t)$ is added as a matched unknown disturbance.

Governing equation (37) is converted to a state space form by defining $\mathbf{x} \equiv [\theta, \dot{\theta}]^T$. In terms of $\mathbf{x}$

$$\begin{aligned} \dot{\mathbf{x}} &= A\mathbf{x} + B(u - f) \\ &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\left(u + w - 16\sin x_1\right) \end{aligned} \tag{38}$$

Note that in controller design, the nonlinear term $-16\sin x_1$ is added to w as an uncertainty and ($w - 16\sin x_1$) is estimated by online neural networks. This allows for use of linear system theory to design the nominal controller.

Take $K_1 = \begin{bmatrix} 10 & 10 \end{bmatrix}$ as the closed-loop feedback gain, which provides fast convergence to the arm angle. Take observer gain $K_2 = diag(\begin{bmatrix} 10 & 10 \end{bmatrix})$, learning rate $\Gamma_c = 500$ which is large enough for fast adaptation, $Q = I$, sampling time 0.1 second, finishing time 40 seconds, and $W_{\max} = 25$ as a conservative estimation to the bound of weights. Centers of RBF distribute over the grid: $x_1 = \{-3,-1,1,3\}$ $x_2 = \{-3,-1,1,3\}$, and all widths are set to 1. The RBF centers and widths selected ensures desired sensitivity of neural networks within the working region. The command signal is $r(t) = \sin(t/3)$, and the objective is to make arm angle of actual system tracking the reference linear system output with command as input, which is sinusoid rotation. The reference system is designed as,

$$\begin{aligned} \dot{x}_r &= (A - BK_1)x_r + Bk_g r \\ &= A_m x_r + Bk_g r \\ &= \begin{bmatrix} 0 & 1 \\ -10 & -10 \end{bmatrix} x_r + \begin{bmatrix} 0 \\ 1 \end{bmatrix} k_g r \end{aligned} \tag{39}$$

The sliding manifold is designed as

$$s = e_2 + 10e_1, \tag{40}$$

After tuning, the sliding mode modification parameter is set to

$$\alpha = 5, \beta = 5. \tag{41}$$

Increase in α will increase the controller's sensitivity to tracking error; however as α approaches infinity, the hyperbolic tangent sigmoid function of αs will converge to $\text{sign}(s)$ and becomes equivalent to typical sliding mode control that brings in chatter or oscillations. Further increase β can accelerate the response speed but it results in overshooting. The simulations were carried out for two cases:, i) disturbance $w(t) = 0$ and ii), $w(t) = \cos(10t)$.

For case i), the command input r, actual system output state θ, reference system output state θ_r, and for comparison, the system output θ_M using MRAC with same design parameters except $K_2 = 0, \mu = 0$, are all shown in Figure 1. It can be seen that proposed actual system output overlaps with reference system output perfectly using proposed method, while MRAC does not provide precise tracking. In Figure 2, control signal using proposed method and MRAC is compared, though the difference in control is small, from Figure 3 the tracking error comparison plot shows clearly that proposed method reduced tracking error by more than 80% percent comparing to MRAC.

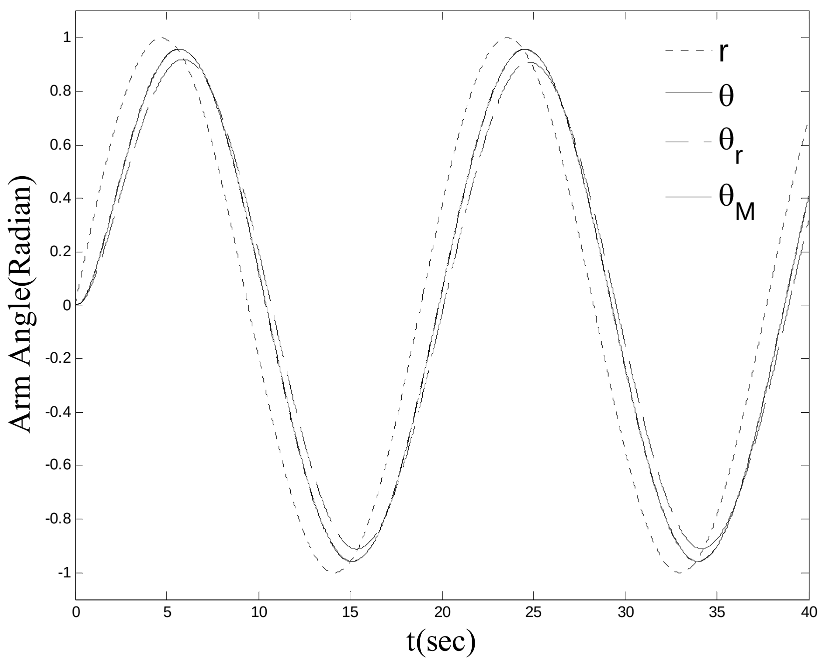

Figure 1. Arm angle, $w(t) = 0$

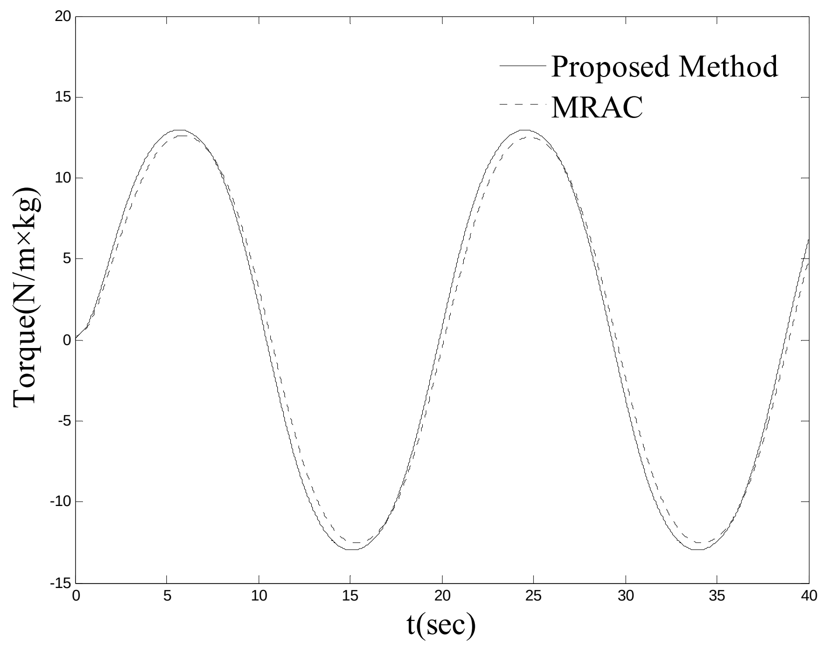

Figure 2. Control signal, $w(t) = 0$

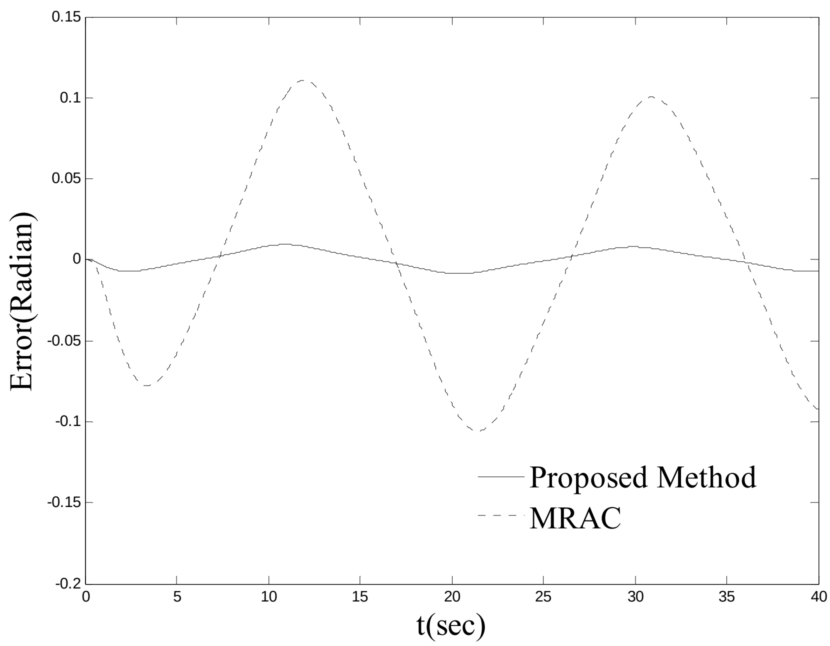

Figure 3. Tracking error, $w(t) = 0$

For case ii), the command input r, actual system output state θ, reference system output state θ_r, and for comparison, the system output θ_M using MRAC with same design parameters except $K_2 = 0, \mu = 0$, are all shown in Figure 4. As it shows, in presence of added disturbance, proposed actual system output still overlaps with reference system output perfectly using proposed method, while MRAC is clearly interfered by disturbance. In Figure 5, control signal using proposed method and MRAC is compared, for both of them the oscillations are due to added disturbance, and the small difference in control leads to significant difference in tracking error, as is shown in Figure 6, proposed method shows its robustness, in presence of additional disturbance, the tracking error is still under 20% of MRAC.

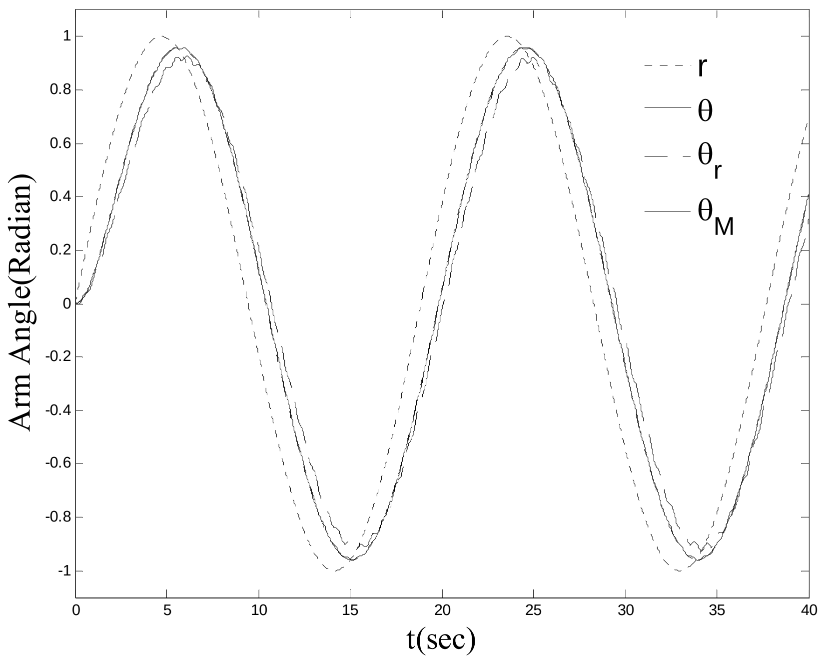

Figure 4. Arm angle, $w(t) = \cos(10t)$

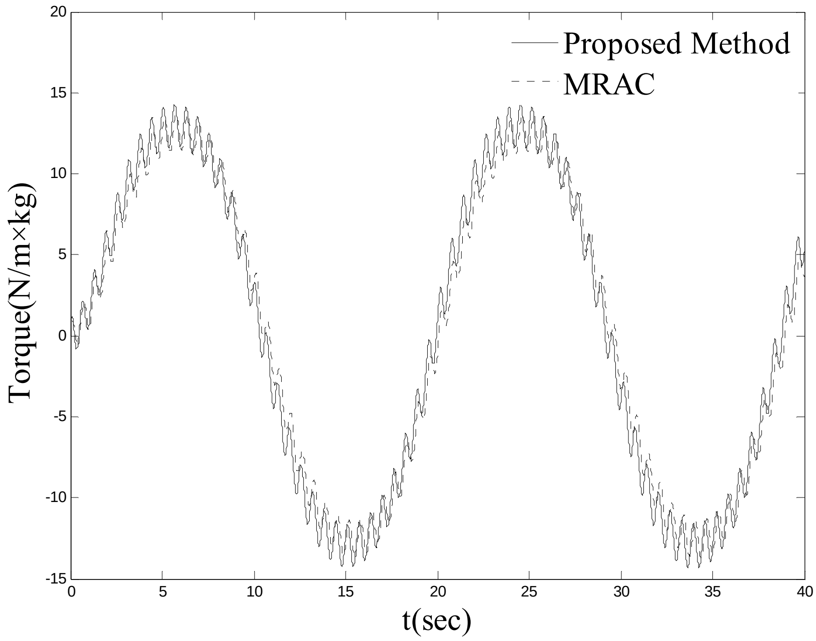

Figure 5. Control signal, $w(t) = \cos(10t)$

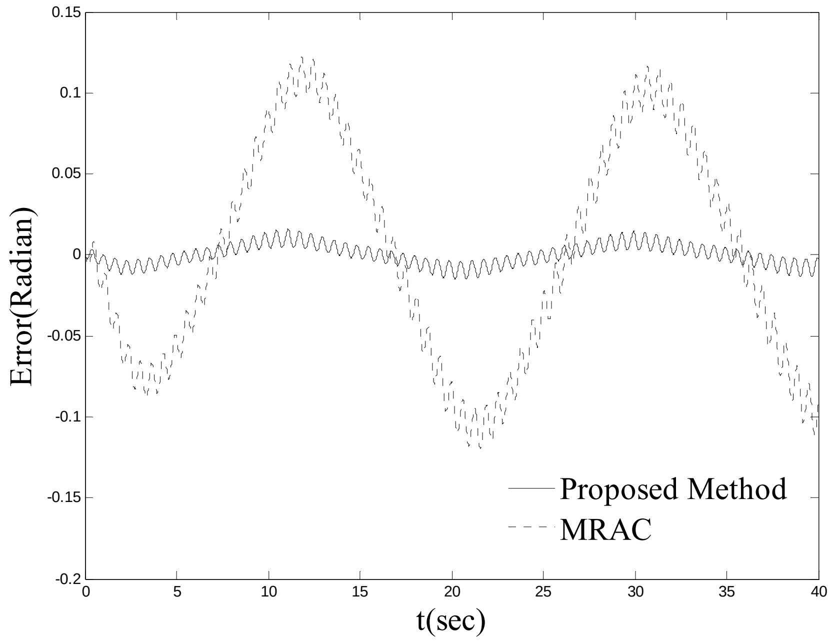

Figure 6. Tracking error, $w(t) = \cos(10t)$

5.2. SHIP STEERING

Use a ship steering example from [46]. The governing equations are,

$$(T_0 + \Delta T)\ddot{\varphi} + \dot{\varphi} = (K_0 + \Delta K)\delta + w \tag{42}$$

where φ is the ship heading angle, δ is the rudder angle, T_0 and K_0 are nominal parameters that are of the ship design velocity, ΔT_0 and ΔK_0 are variety of T_0 and K_0, and w is the model uncertainty parameters and the disturbance uncertainties of the system.

Define $x = [\varphi, \dot{\varphi}]^T$, write governing equation (42) as,

$$\begin{aligned}\dot{\mathbf{x}} &= A\mathbf{x} + B(u - f) \\ &= \begin{bmatrix} 0 & 1 \\ 0 & -1/T_0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ K_0/T_0 \end{bmatrix}(\delta - f)\end{aligned} \tag{43}$$

where

$$f = -\Delta T/(T_0(T_0 + \Delta T))x_2 - (T_0\Delta K - K_0\Delta T)/(T_0(T_0 + \Delta T))\delta - 1/(T_0 + \Delta T)w \tag{41}$$

$$\begin{aligned}&T_0 = 261.73, K_0 = 0.42, \\ &\Delta T = 1/T_0 \times 25\%, \Delta K_0 = -K_0/T_0 \times 25\%\end{aligned} \tag{42}$$

It is assumed that there is no knowledge about f at all, and it is taken completely as unmodeled dynamics, the neural networks will estimate its value online.

For an initial design, the feedback gain $K_1 = [1, 33]$ is taken to result in the reference system have a 5% settling time at approximately 200 seconds. High values of observer gain $K_2 = diag([100 \quad 100])$ are selected to allow quick observer error decay. The learning rate is set at $\Gamma_c = 100$, $Q = I$; a sampling time 0.1 second was taken as appropriate with a final time of 500 seconds, and $W_{\max} = 5$ which serves as a conservative bound for the weights. It should be noted observed that further increase in the learning rate leads to unstable behavior for the MRAC due to oscillations in the control signal. Although the proposed controller does not exhibit such behavior, for the performance comparison to be valid, a higher learning rate is not used. Centers of RBF are distributed over the grid: $x_1 = \{-15, 0, 15\}$ $x_2 = \{-2, 0, 2\}$, and all widths are set to 1.These numbers have been selected a based on the working domain of the system states. The command signal is

$$r(t) = \begin{cases} 0, & 0 \le t \le 5 \sec onds \\ 10, & 5 < t \le 250 \sec onds \end{cases}$$

Controller objective is to make the plant track the reference linear system output with command as input that will result in the ship make a turn of 10 degrees. With the design parameters set at the values in the previous section, the reference system can be calculated as

$$\begin{aligned} \dot{x}_r &= (A - BK_1)x_r + Bk_g r \\ &= A_m x_r + Bk_g r \end{aligned} \tag{44}$$

The sliding manifold is designed as

$$s = e_2 + 10e_1 \tag{45}$$

and the sliding mode modification parameter is,

$$\alpha = 10, \beta = 10 \tag{46}$$

They are tuned in a similar manner as mentioned in previous subsection.

In the simulation, the rudder is simulated by a close-loop servo, which is expressed as,

$$T_e \dot{\delta}_E = K_E \delta_E - \delta \tag{47}$$

where T_E=2.5s is the rudder time constant, K_E=1 is the gain of rudder, and δ is real rudder command, $|\delta| \le 35^\circ$, $|\dot{\delta}_E| \le 3^\circ$, δ_E is the total input control. For the controller design, the servo dynamics is considered unknown. Results from two cases are discussed in the following sections. In the first case there is no disturbance and $w(t) = 0$ and in the second $w(t) = 5e4\sin(0.05t)$.

Histories of the command input, actual system output, reference system output, and for comparison, the system output using MRAC with same design parameters except $K_2 = 0, \mu = 0$, are all shown in Figure 7. It can be seen that both proposed method and MRAC are close to the reference system and both method exhibit good tracking. However, the control signal history in Figure 8. shows clearly that control signal using MRAC is *quite oscillatory* until steady state while proposed method provides smooth control. This is due to the fact that the uncertainty estimation is a part of the observer dynamics in the new technique and it allows for smooth signals and faster uncertainty estimation, and since control is used to cancel the uncertainties, it also implies proposed method's advantage over MRAC in neural networks uncertainty estimation. Furthermore in Figure 9, it can be seen that with proposed method tracking error quickly converges while tracking error of MRAC is not relatively larger in magnitude(though not in absolute value), and is oscillatory.

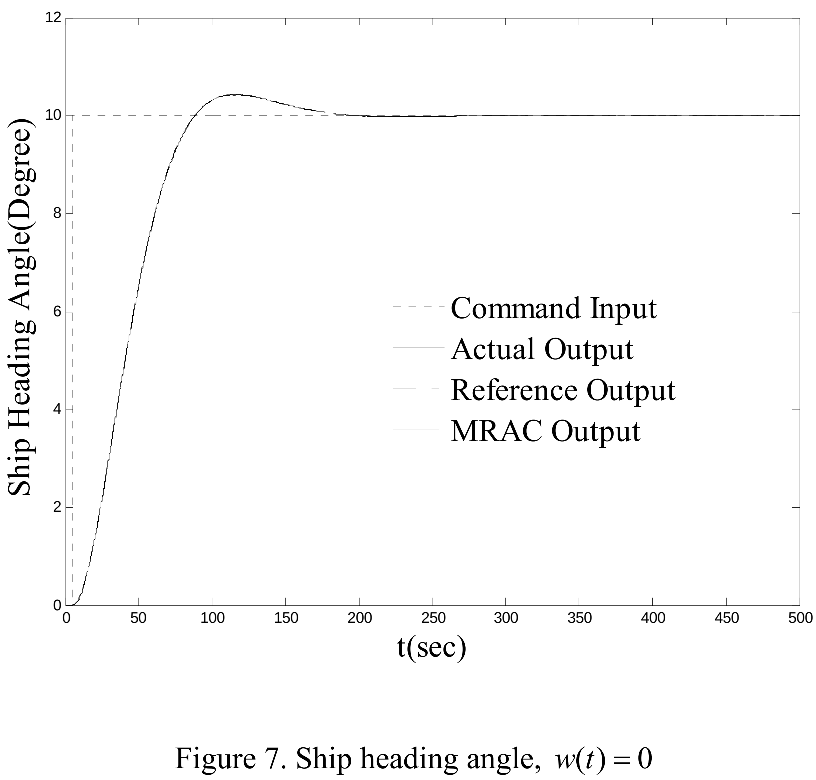

Figure 7. Ship heading angle, $w(t) = 0$

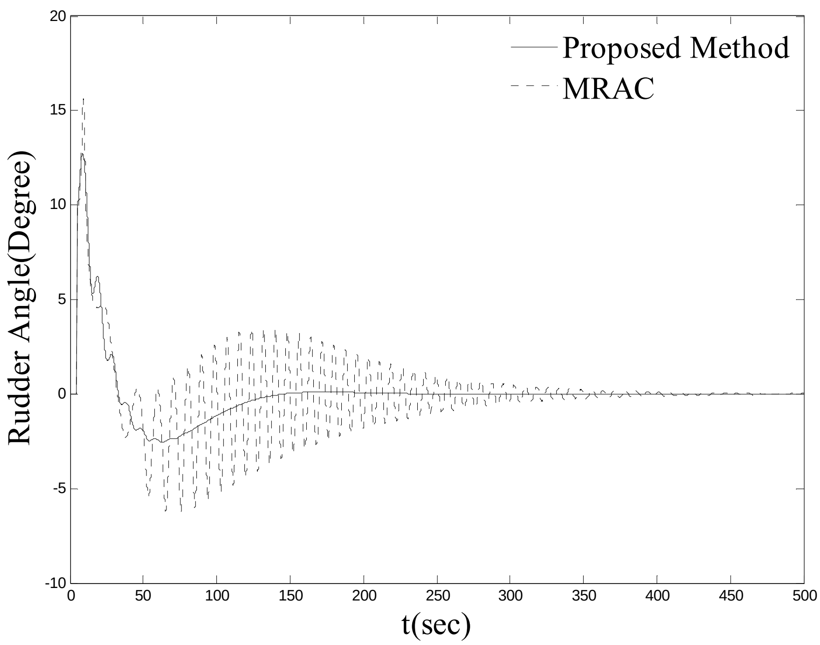

Figure 8. Rudder angle, $w(t) = 0$

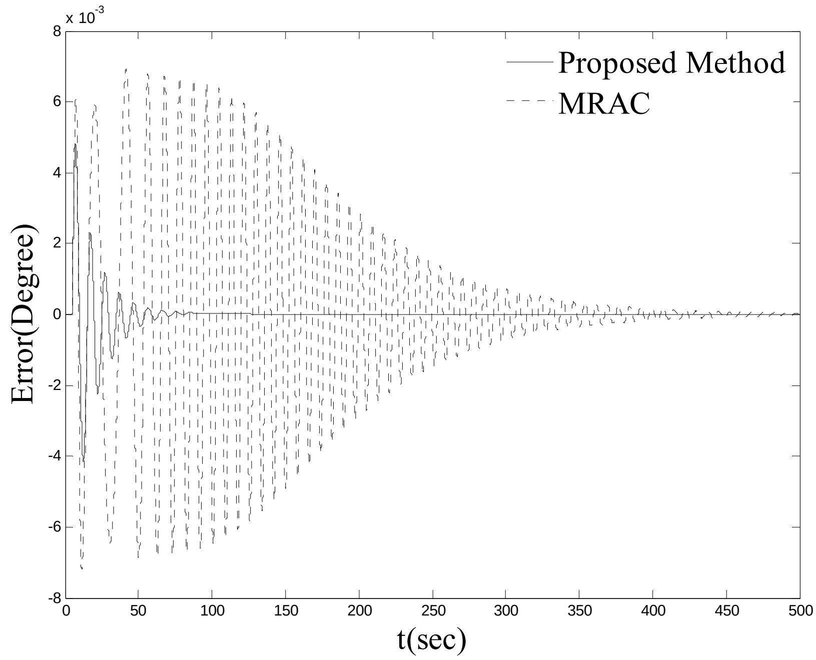

Figure 9. Tracking error, $w(t) = 0$

Histories of the command input, actual system output, reference system output, and for comparison, the system output using MRAC with same design parameters except $K_2 = 0, \mu = 0$, are all shown in Figure 10 for the case with disturbances in the system. As in the previous case, the proposed method and MRAC overlap with reference output. But as is shown in Figure 11, the transient response of the proposed method shows smooth control signals in the presence of disturbances while the MRAC control is oscillatory. In Figure 12, the tracking errors of both methods show similar behavior as the control signals.

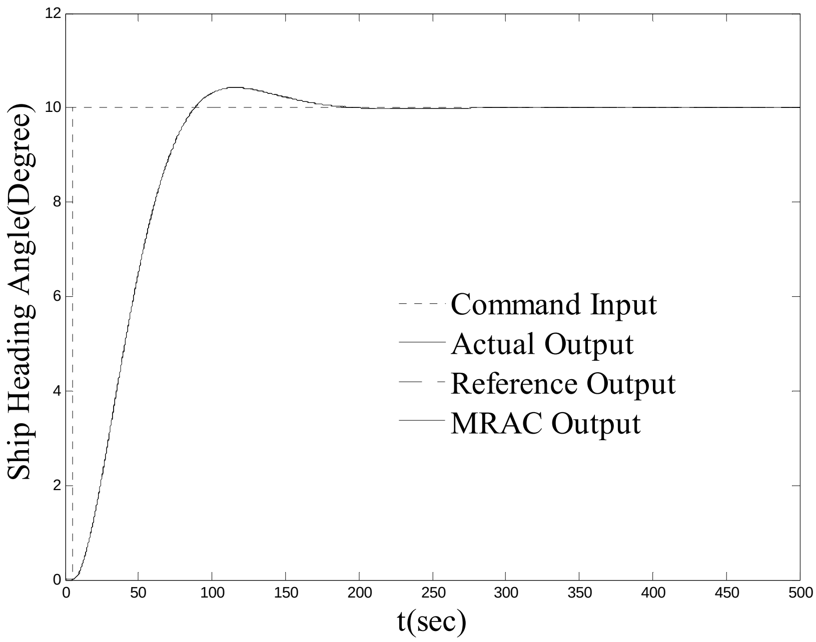

Figure. 10 Ship heading angle, $w(t) = 5e4\sin(0.05t)$

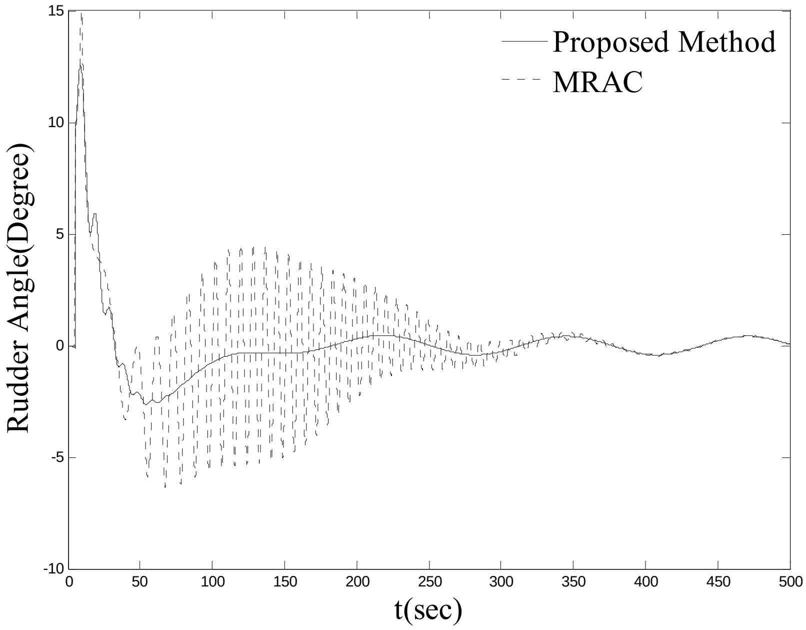

Figure 11. Rudder angle, $w(t) = 5e4\sin(0.05t)$

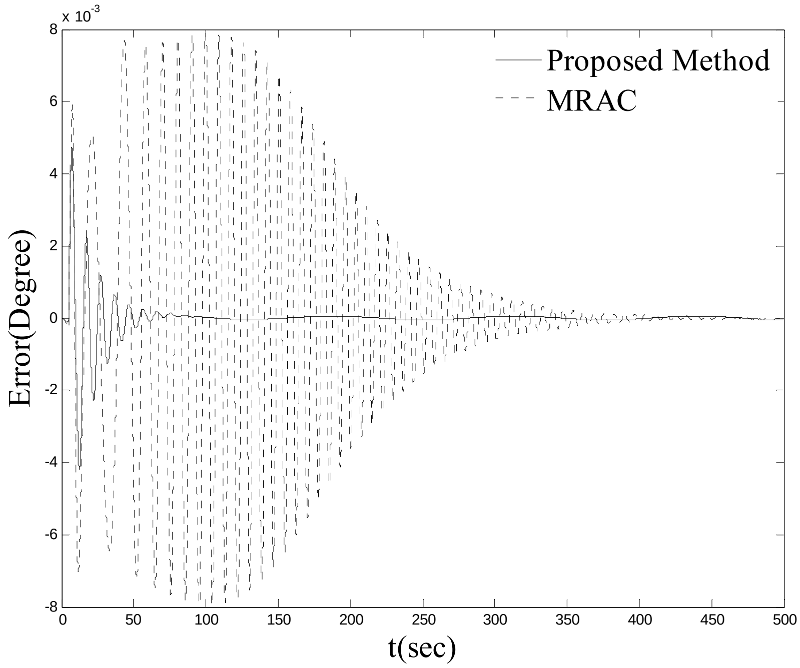

Figure 12. Tracking error $w(t) = 5e4\sin(0.05t)$

6. CONCLUSION

A new robust adaptive control has been derived in this paper. Bounds on transient response error shave been derived. A novel sliding mode term has been added to result in asymptotic stability of the errors instead of the usual upper bounded derivations. Performance of the proposed technique was evaluated with two representative problems and compared with a typical model reference adaptive controller. It is clear from the results that the transient response of the new controller is superior and does not show oscillatory behavior while learning and cancelling out the uncertainties. This fact is crucial in any implementation for two reasons. The first is that the oscillatory signals could lead to excitation of troublesome unmodeled dynamics. Second, it could lead to controller fatigue. From the limited examples, the performance of the proposed controller seems to be robust to these problems that are germane to adaptive controllers. Furthermore, the settling time of the system with the proposed controller seems to be faster than that of the typical model reference adaptive controller.

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II. A New Model Reference Adaptive Controller in Missile Autopilots Design

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ABSTRACT

A new model reference adaptive control design method using neural networks that guarantees *transient performance* is proposed in this paper. Stable tracking of a desired trajectory can also be achieved for nonlinear systems that operate under uncertainties. A modified state observer structure is designed to enable desired transient performance fast with large adaptive gains and at the same time avoid high frequency oscillations during uncertainty learning. The neural network adaptation rule is derived using Lyapunov theory, which guarantees *stability* of error dynamics and boundedness of neural network weights. An extra term is added in the controller expression using a 'soft switching' sliding mode that can be used to adjust tracking errors. Analytical bounds are derived and simulation of the proposed method is presented to solve a missile autopilot design problem.

1. INTRODUCTION

The field of artificial neural networks and its application to control systems has seen phenomenal growth in the last two decades. The origin of research on artificial neural networks can be traced back to 1940s [1]. In 1990, a compiled book was published [2] detailing various applications of artificial neural networks. A good survey paper appeared in 1992 [3], which outlined various applications of artificial neural networks to control system design. The main philosophy that is exploited in system theory applications is the universal function approximation property of neural networks [4]. Benefits of using neural networks for control applications include its ability to effectively control nonlinear plants while adapting to unmodeled dynamics and time-varying parameters.

In 1990, a paper by Narendra and Parthasarathy demonstrated the potential and applicability of neural networks for the identification and control of nonlinear dynamical systems [5]. The authors suggested various architectures as well as learning algorithms useful for identification and adaptive control of nonlinear dynamic systems using recurrent neural networks. Since then, Narendra and his co-workers have come up with a variety of useful adaptive control design techniques using neural networks, including applications concerning multiple models [6].

In 1992, Sanner and Slotine [7] developed a direct adaptive tracking control architecture with Gaussian Radial Basis Function (RBF) networks to compensate for plant nonlinearities. The update process also kept the weights of the neural networks bounded. In 1996, Lewis et al. [8] proposed an online neural network that approximated unknown functions and it was used in designing a controller for a robot. Their approach avoided some of the limiting assumptions (like linearized models) of traditional adaptive control techniques. More important, their theoretical development also provided a Lyapunov stability analysis that guaranteed both tracking performance as well as boundedness of weights. However, the applicability of this technique was limited to systems which could be expressed in the "Brunovsky form" [9] and which were affine in the control variable (in state space form). A robust adaptive output feedback controller for SISO systems with bounded disturbance was studied by Aloliwi and Khalil [10]. In a more recent paper, an adaptive output feedback control scheme for the output tracking of a class of nonlinear systems was presented by Seshagiri and Khalil using RBF neural networks [11].

A relatively simpler and popular method of nonlinear control design is the technique of dynamic inversion (e.g. [12, 13, 14]), which is essentially based on the philosophy of feedback linearization [9, 15]. In this approach, an appropriate co-ordinate transformation is carried out to make the system dynamics take a linear form. Linear control design tools are then used to synthesize the controller. A drawback of this approach is its sensitivity to modeling errors and parameter inaccuracies. One way of addressing the problem is to augment the dynamic inversion technique with the H_∞

robust control theory [14]. Important contributions have come from Calise and his co-workers in a number of publications (e.g. [16 - 20]), who have proposed to augment the dynamic inversion technique with neural networks so that the inversion error is cancelled out. The neural networks are trained online using a Lyapunov-based approach (similar to the approach followed in [7] and [8]). This basic idea has been extended to a variety of cases, namely output based control design [19, 20], reconfigurable control design [21] etc. The feasibility and usefulness of this technique has been demonstrated in a number of applications in the field of flight control.

MRAC has been widely applied recently to solve control problems for system with matched unmodeled dynamics [22][23]. With MRAC, it is difficult to achieve a desired(fast) transient performance *and* avoid unwanted high frequency oscillations at the same time when uncertainties are present. It is due to the fact that high gains are required typically to learn the uncertainties online. If neural networks are used to represent uncertainties, this process necessarily results in an uncertainty model showing oscillations during the transient learning period before the weights stabilize. Use of dynamic inversion to cancel the uncertainties during learning then leads to oscillatory control signals which if unchecked could excite the unmodeled high frequency dynamics of the plant and lead to instability. Various NN-based MRAC methods have been recently developed (for example, [24][25]) to address the issue of boundedness of tracking errors. Modification to the adaptive law such as σ-modification [26], e-modification [27] have been introduced. These methods modify the adaptive law by adding a factor depending on the prediction error and ensure the convergence of parameter estimation. Moreover, when close to steady state conditions, the modification term becomes inactive and therefore, the estimation accuracy is guaranteed. In [28], a projection operator was used to modify the adaptive law. Projection operator replaces the common Lipschitz continuous property with an arbitrary many times continuous differentiability, and estimation parameters are proven to be bounded.

Although these developments help improve the robustness of the adaptive control laws, their tracking accuracy can only be shown to be bounded, and the bound depends on the magnitude of disturbances. At the same time, a typical MRAC cannot avoid unwanted oscillations. Recently many methods were developed to solve these two problems. In [29][30], a new MRAC neural networks controller named $\mathcal{L}_1$ adaptive controller is proposed, and the transient performance of both system's input and output signal are characterized with some norms. This adaptive control architecture has a low-pass filter in the feedback loop, and its desired transient performance can be guaranteed by increasing adaption gain and improving the NN approximation, and at the same time, the high frequency oscillation is avoided. In [31], an adaptive control method that allows fast adaptation for systems with slow reference models is given. In this method, in order to allow fast adaptation, the neural network is trained with a high bandwidth state emulator. Low bandwidth control is maintained by a filter to isolate fast emulator dynamics from the control signal. In [32], a novel Kalman-filter version of the *e*-modification [27] is developed. In this method the standard *e*-modification term is interpreted as the gradient of a norm measure of a linear constraint violation, and this linear constraint is then used to develop a Kalman-filter-based *e*-modification. It is shown that this method leads to smaller tracking errors without generating significant oscillations in the system response.

Sliding mode control (SMC) is inherently robust to uncertainties [33][34][35]. In SMC, trajectories are forced to reach a designed sliding surface in a finite time and to stay on the surface for all future time. Dynamics on the sliding surface is independent of matched uncertainties and the sliding surface is designed so as to guarantee the asymptotic stability of control objective. Though it has many advantages, a major drawback of the SMC in applications is control switching along the sliding surface, called 'chatter' thus oscillations are usually unavoidable. Saturation functions with a boundary layer[9] can be used to alleviate chatter but it cannot be eliminated. Also, asymptotic stability inside the boundary layer needs to be separately shown. Recently, there has been some work with higher-order sliding mode controllers to avoid 'chatter'[36]. A soft-switching sliding mode technique has been introduced by Lyshevsky [37][38] to eliminate 'chatter'. By modifying signum function used in a typical SMC to continuous real-analytic function, for example, hyperbolic tangent functions, the soft switching sliding mode controller avoid oscillations and remain asymptotic stable at the same time. In [39], a systematic way to combine adaptive control and SMC for trajectory tracking in presence of parametric uncertainties and uncertain nonlinearities is developed. The sliding mode controller is smoothed with two methods based on the concept of boundary layer [34]. Asymptotic stability of adaptive system in presence of parametric uncertainties are realized, and the drawback of control chattering is reduced significantly. In [40], a modified switching function which provides low-chattering control signal is introduced, and the SMC is combined with a neural network adaptive controller which identifies modeling error online. In [41], by using a similar approaching to SMC, a novel approach that combines NN feed forward controller with continuous robust integral of sign of error (RISE) feedback controller is introduced. In this interesting method, by designing sliding surface using sign of error, a continuous RISE feedback is combined with a NN-based adaptive controller, and it is shown that using Lyapunov theory the tracking error is *asymptotically stable*, while typical NN-based controller formulations can only yield uniformly ultimately bounded (UUB) stability [9], and at the same time, the control is free from oscillations. Experimental results show the potential of this method in reducing tracking errors [42][43].

This paper develops a new neural network MRAC with guaranteed transient performance and asymptotic stability, and at the same time free from unwanted oscillations. Based on MRAC neural networks controller, the neural network observer structure is modified in the manner of [44]. The basic notion is to separate the functions of a controller and observer. That is the controller stabilizes (tracks) a reference and an observer tracks the true system. By having a dynamic observer instead of just calculating the uncertainty as in other MRAC approaches, it is believed that the designer can make the estimation error decay fast with the observer gains. This allows one to use higher learning rates for the adaptation that helps achieve better tracking performance without inducing high-frequency oscillations. At the same time, the modified term is inactive when neural network estimation is ideal, therefore the estimation accuracy is guaranteed. Furthermore, the proposed technique has a sliding mode term to provide asymptotic stability. Since this technique has an observer in the loop, the excellent RISE scheme is not applicable; a soft switching sliding mode term is added to guarantee asymptotic stability. It is proven using Lyapunov method that it ideally leads to asymptotic stability instead of UUB, and at the same time is free from oscillations which is common for typical sliding mode based adaptive controller. In general, the proposed controller enables higher adaptive gain without generating oscillations, provides better transient performance and asymptotic stability at the same time. In this paper, a longitudinal missile dynamics model is studied using proposed method, in this model the reference system is designed using RSLQR method [45], by developing the existing MRAC controller, the proposed method is shown improved transient performance and reduced high frequency oscillations significantly.

Rest of the paper is organized as follows. In Section 2, the system and the neural networks structure is defined. In Section 3, the control solution is proposed. A stability proof of both observer and state error signal is put forward, and the guaranteed transient performance is also explained in Section 4. Simulation studies of a missile autopilot problem were carried out in Section 5 and conclusions are drawn in Section 6.

2. LINEAR LONGITUDINAL DYNAMICS

In this paper, the longitudinal missile dynamics taken from [45] is studied. The governing equations of the missile dynamics are

$$\begin{aligned} \dot{\alpha} &= \frac{Z_\alpha}{V}\alpha + q + \frac{Z_\delta}{V}\delta_e \\ \dot{q} &= M_\alpha \alpha + M_\delta \delta_e \\ \ddot{\delta}_e &= -2\zeta_\alpha \omega_\alpha \dot{\delta}_e - \omega_\alpha(\delta_e - \delta_c) \end{aligned} \tag{1}$$

The states modeled are α , q and δ_e (normal acceleration, pitch rate (radian/s), and elevator fin deflection (radian)), and includes a second order actuator model. The autopilot is designed using the robust servomechanism linear quadratic regulator (RSLQR) approach [45], which incorporates integral control into a LQR state feedback design to build a type 1 controller. This will enable zero steady state error to constant commands, and at the same time LQR controller provides desired stability and robustness. The autopilot design model with matched uncertainty in state space from is

$$\dot{\mathbf{x}} = A\mathbf{x} + B(u - f) \tag{2}$$

with initial condition

$$\mathbf{x}(0) = \mathbf{0} \tag{3}$$

And the output of system is

$$y(t) = C\mathbf{x}(t) \tag{4}$$

In (2), $\mathbf{x} = [\int e_r, \alpha, q, \delta_e, \dot{\delta}_e]^T$, $e_r = y - r$, $A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{Z_\alpha}{V} & 1 & \frac{Z_\delta}{V} & 0 \\ 0 & M_\alpha & 0 & M_\delta & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\omega_\alpha^2 & 2\zeta_\alpha\omega_\alpha \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \omega_\alpha^2 \end{bmatrix}$. In (4), $C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix}$.

$f(\mathbf{x})$ is a matched continuous nonlinear uncertainty function, and it is treated as unknown during controller design.

The control objective is to design a neural adaptive controller which ensures output $y(t)$ tracks a desired bounded continuous trajectory $r(t)$, the system behavior follows a nominal linear time-invariant (LTI) system which is designed through standard methods(for example, through linear quadratic regulator theory[37]), and at the same time guarantees desired transient and steady state performance.

Assume the following NN approximation of $f(x)$ exists

$$f(\mathbf{x}) = \mathbf{W}^T \phi(\mathbf{x}) + \varepsilon(\mathbf{x}) \quad |\varepsilon(\mathbf{x})| < \varepsilon^* \tag{5}$$

where $\phi(\mathbf{x})$ is a set of radial basis functions, and each element of $\phi(\mathbf{x})$ is defined as

$$\phi(x) = \exp(-(y-z)^T (y-z)/\sigma^2) \tag{6}$$

In (6), z is the location of selected center, σ is the 'width'. $\mathbf{W}$ are the ideal network weights, $\varepsilon(\mathbf{x})$ is the network approximation error, ε^* is its uniform bound. Furthermore, assume that a compact convex set Ω is known a priori such that

$$\mathbf{W} \in \Omega \tag{7}$$

In order to realize tracking control for this SISO system, the following neural network adaptive controller is developed.

3. CONTROL SOLUTION

The proposed controller is a combination of a linear feedback control, neural network adaptive control and a soft switching sliding mode control. First of all, divide the controller expression into three parts-the linear feedback control $K_1\mathbf{x}$, neural networks adaptive control u_e and soft switching sliding mode control μ

$$u = K_1\mathbf{x} + u_e + \mu \tag{8}$$

where K_1 the closed loop feedback gain, which ensures closed-loop reference dynamics matrix ($A - BK_1$) is Hurwitz. The linear feedback control term ensures stability when there is no uncertainty or when it is compensated for; the adaptive control part cancels the uncertainty term that is estimated online through a neural network in conjunction with an observer. Note that there is always a residual error in uncertainty calculations with a neural network. The soft switching sliding mode control guarantees asymptotic stability in presence of such errors. By substituting (8) into (2), (2) becomes

$$\dot{\mathbf{x}}(t) = A_m\mathbf{x}(t) + B(u_e(t) - f(\mathbf{x}(t))) \tag{9}$$

where $A_m = A - BK_1$. And define the following state observer structure,

$$\dot{\hat{\mathbf{x}}}(t) = A_m\hat{\mathbf{x}}(t) + B(u_e(t) + \mu - \hat{f}) - K_2\tilde{\mathbf{x}}(t) \tag{10}$$

where $\hat{\mathbf{x}}(t)$ represents the observer states at time t. Since the uncertainty and the true neural network weights are unknown, they are represented as $\hat{\mathbf{W}}^T\phi(\mathbf{x})$ where $\hat{\mathbf{W}}$ represents the estimated neural network weights with a proper weight update law. The observer gain matrix is assumed diagonal for convenience and are given by $K_2 = diag(k_2^1, k_2^2, ..., k_2^n)$. In the observer structure, $\hat{f}$ is assumed to be canceled perfectly by neural networks controller, i.e. $\hat{f} = \hat{\mathbf{W}}^T \phi(\mathbf{x})$. The initial conditions for observer are

$$\hat{\mathbf{x}}(t) = \mathbf{0} \tag{11}$$

The observer error is

$$\tilde{\mathbf{x}}(t) \equiv \hat{\mathbf{x}}(t) - \mathbf{x}(t) \tag{12}$$

And the adaptive weight update law is defined as follows:

$$\dot{\hat{\mathbf{W}}}(t) = \Gamma_c \Pr oj(\hat{\mathbf{W}}(t), \phi(\mathbf{x}(t))\tilde{\mathbf{x}}(t)^T PB) \tag{13}$$

where P is defined by $A_m^T P + PA_m = -Q$, with Q being a positive definite matrix and Γ_c is the learning rate of the neural network. The projection operator property guarantees the boundedness of neural networks weights error

$$\tilde{\mathbf{W}}^T \tilde{\mathbf{W}} \leq W_{\max} \tag{14}$$

where $W_{\max} \equiv \max_{\mathbf{W} \in \Omega} 4\|\mathbf{W}\|^2$, $\tilde{\mathbf{W}} \equiv \hat{\mathbf{W}} - \mathbf{W}$ [28].

Now with neural networks weights, the adaptive control expression becomes

$$u_e = k_g r(t) + \hat{\mathbf{W}}^T \phi(\mathbf{x}) \tag{15}$$

where

$$k_g \equiv \frac{1}{CA_m^{-1}B} \tag{16}$$

is the open loop gain of the reference system. Subtract (9) from (10), and substitute (15), the observer error dynamics is written as,

$$\dot{\tilde{\mathbf{x}}}(t) = (A_m - K_2)\tilde{\mathbf{x}} + B(\tilde{\mathbf{W}}^T \phi(\mathbf{x}) - \varepsilon) \tag{17}$$

By using Lyapunov method [9], it will be shown that the neural network estimation error and the observer error are bounded. By introducing the observer gain K_2, the learning process is smoothed, and the modified term decrease as $\tilde{\mathbf{x}}$ decreases, therefore the learning accuracy is guaranteed. As a result, the modified observer structure enables increasing adaptation gain without generating high frequency oscillations.

4. STABILITY ANALYSIS

In this section, Lyapunov method is used to prove the boundedness of the observer error dynamics. And in order to assure asymptotic convergence of reference error, the soft-switching sliding mode controller is derived. Details of the proofs are provided in the following subsections.

4.1. OBSERVER ERROR

To get the error bound for neural network observer, consider a Lyapunov function as $V(\tilde{\mathbf{x}}, \tilde{\mathbf{W}}) = \tilde{\mathbf{x}}^T P \tilde{\mathbf{x}} + \Gamma_c^{-1} \tilde{\mathbf{W}}^T \tilde{\mathbf{W}}$, and differentiate $V(.)$ to get

$$\dot{V} = \dot{\tilde{x}}^T P \tilde{x} + \tilde{x}^T P \dot{\tilde{x}} + \Gamma_c^{-1}(\tilde{\mathbf{W}}^T \dot{\tilde{\mathbf{W}}} + \dot{\tilde{\mathbf{W}}}^T \tilde{\mathbf{W}}) \tag{18}$$

substitute the weight update law (13) and observer error dynamics (17), (18) becomes

$$\begin{aligned} \dot{V} &= -\tilde{\mathbf{x}}^T Q \tilde{\mathbf{x}} - 2\tilde{\mathbf{x}}^T K_2 P \tilde{\mathbf{x}} - 2\tilde{\mathbf{x}}^T PB(\tilde{\mathbf{W}}^T \phi(\mathbf{x}) - \varepsilon(\mathbf{x})) \\ &\quad + 2\tilde{\mathbf{W}}^T \operatorname{Pr} oj(\hat{\mathbf{W}}, \phi(\mathbf{x})\tilde{\mathbf{x}}^T PB) \\ &\le 2\|PB\varepsilon\|\|\tilde{\mathbf{x}}\| - [\lambda_{\min}(Q) + 2\lambda_{\min}(K_2 P)]\|\tilde{\mathbf{x}}\|^2 \end{aligned} \tag{19}$$

where $\lambda_{\min}$ represents the minimum eigenvalue. Therefore $\dot{V} \le 0$ when

$$\|\tilde{\mathbf{x}}\| \ge \frac{2\|Pb\varepsilon^*\|}{\lambda_{\min}(Q) + 2\lambda_{\min}(K_2 P)} \tag{20}$$

As a result,

$$V(\tilde{\mathbf{x}}, \tilde{\mathbf{W}}) \le \tilde{\mathbf{x}}^T P \tilde{\mathbf{x}} + \Gamma_c^{-1} W_{\max}$$

$$\le \lambda_{\max}(P)\left(\frac{2\left\|PB\varepsilon^*\right\|}{\lambda_{\min}(Q) + 2\lambda_{\min}(K_2 P)}\right)^2 + \Gamma_c^{-1} W_{\max} \tag{21}$$

and at the same time

$$V(\tilde{\mathbf{x}}, \tilde{\mathbf{W}}) \ge \tilde{\mathbf{x}}^T P \tilde{\mathbf{x}} \ge \lambda_{\min}(P)\left\|\tilde{\mathbf{x}}\right\|^2 \tag{22}$$

(21) and (22) lead to

$$\left\|\tilde{\mathbf{x}}\right\| \le \sqrt{\frac{\left(\lambda_{\max}(P)\left(\frac{2\left\|Pb\varepsilon^*\right\|}{\lambda_{\min}(Q) + 2\lambda_{\min}(K_2 P)}\right)^2 + \Gamma_c^{-1} W_{\max}\right)}{\lambda_{\min}(P)}}$$

$$\equiv \gamma_0 \tag{23}$$

In (23), by increasing adaptation gain Γ_c and observer gain K_2, $\left\|\tilde{\mathbf{x}}\right\|$ can be driven to be as small as possible, therefore, precise uncertainty estimation using online neural networks is guaranteed. Also, with the modification term to smooth out learning, the state observer structure suppresses the high frequency oscillation so that increased adaptation gain and smooth control are possible at the same time.

4.2. REFERENCE ERROR

Note that with adaptive control and linear feedback control alone (with $\mu = 0$), the controller is able to track reference system but with bounded tracking errors. However, with a soft-switching sliding mode controller, the tracking error can be shown to be asymptotic stable. Define a reference LTI system dynamics as

$$\dot{\mathbf{x}}_r = A_m \mathbf{x}_r + b k_g r \tag{24}$$

By subtracting the reference dynamics (24) from actual system dynamics (9), the tracking error dynamics are expressed as

$$\begin{aligned}\dot{\mathbf{e}} &\equiv \dot{\mathbf{x}}(t) - \dot{\mathbf{x}}_r(t) \\ &= A_m\mathbf{e} + b(\mu + (\hat{\mathbf{W}}^T - \mathbf{W}^T)\phi(\mathbf{x}) - \varepsilon(\mathbf{x}))\end{aligned} \tag{25}$$

Recalling the definition of system dynamics as given in (1), (25) can be written as

$$\begin{cases}\dot{e}_1 = e_2 \\ \dot{e}_2 = \dfrac{Z_\alpha}{V}e_2 + e_3 + \dfrac{Z_\delta}{V}e_4 \\ \dot{e}_3 = M_\alpha e_2 + M_\delta e_4 \\ \dot{e}_4 = e_5 \\ \dot{e}_5 = -\omega_\alpha^2 e_4 + 2\zeta_\alpha\omega_\alpha e_5 + bK_1\mathbf{e} + b(\mu + \tilde{\mathbf{W}}^T\phi(\mathbf{x}) - \varepsilon(\mathbf{x}))\end{cases} \tag{26}$$

Then, define the sliding surface as

$$s = e_5 + \begin{bmatrix}\lambda_2 & \lambda_3 & \lambda_4\end{bmatrix} e_r \tag{27}$$

where $e_r = \begin{bmatrix}e_2 & e_3 & e_4\end{bmatrix}^T$ and $\begin{bmatrix}\lambda_2 & \lambda_3 & \lambda_4\end{bmatrix}$ is selected in such a way that

$$A_r = \begin{bmatrix} Z_\alpha / V & 1 & Z_\alpha / V \\ M_\alpha & 0 & M_\delta \\ -\lambda_2 & -\lambda_3 & -\lambda_4 \end{bmatrix} \tag{28}$$

is Hurwitz. Therefore, when *s=0*,

$$\begin{aligned}\dot{e}_r &= \begin{bmatrix} Z_\alpha / V & 1 & Z_\alpha / V \\ M_\alpha & 0 & M_\delta \\ 0 & 0 & 0 \end{bmatrix} e_r + \begin{bmatrix}0\\0\\1\end{bmatrix} e_5 \\ &= A_r e_r\end{aligned} \tag{29}$$

is asymptotic stable. Take differential to the sliding surface, the following relationship is obtained,

$$\begin{aligned}\dot{s} &= \dot{e}_5 + \begin{bmatrix}\lambda_2 & \lambda_3 & \lambda_4\end{bmatrix}\dot{e}_r \\ &= \dot{e}_5 + \begin{bmatrix}\lambda_2 & \lambda_3 & \lambda_4\end{bmatrix}\left(\begin{bmatrix} Z_\alpha / V & 1 & Z_\alpha / V \\ M_\alpha & 0 & M_\delta \\ 0 & 0 & 0 \end{bmatrix} e_r + \begin{bmatrix}0\\0\\1\end{bmatrix} e_5\right) \\ &= \Sigma + bK_1\mathbf{e} + b(\mu + \tilde{\mathbf{W}}^T\phi(\mathbf{x}) - \varepsilon(\mathbf{x})) \\ &= \Sigma + bK_1\mathbf{e} + b(\mu + D) \end{aligned} \tag{30}$$

where

$$\Sigma = -\omega_\alpha^2 e_4 + 2\zeta_\alpha\omega_\alpha e_5 + \begin{bmatrix}\lambda_2 & \lambda_3 & \lambda_4\end{bmatrix}\left(\begin{bmatrix} Z_\alpha / V & 1 & Z_\alpha / V \\ M_\alpha & 0 & M_\delta \\ 0 & 0 & 0 \end{bmatrix} e_r + \begin{bmatrix}0\\0\\1\end{bmatrix} e_5\right) \tag{31}$$

$$D = \tilde{\mathbf{W}}^T\phi(\mathbf{x}) - \varepsilon(\mathbf{x}) \tag{32}$$

Take Lyapunov function $V_s = \frac{1}{2}s^2$, take its derivative,

$$\begin{aligned}\dot{V}_s &= s\dot{s} \\ &= s\left(\Sigma + bK_1\mathbf{e} + b(\mu + D)\right)\end{aligned} \tag{33}$$

Recall NN approximation property (5) and weights error bound (14), immediately the following bound for *D* is obtained

$$\begin{aligned}\|D\| &= \left\|\tilde{\mathbf{W}}^T\phi(\mathbf{x}) - \varepsilon(\mathbf{x})\right\| \\ &\le W_{\max} + \varepsilon^* \equiv D^*\end{aligned} \tag{34}$$

Now take the soft switching sliding mode modification term in this manner

$$\mu = -K_1\mathbf{e} + \frac{1}{b}\left(-\Sigma + \beta \tan\mathrm{sig}(\alpha s)\right), \tag{35}$$

substitute (35) into (30), (30) becomes

$$\dot{V}_s = bs(-\beta\tanh(\alpha s) + D) \tag{36}$$

As long as $\alpha > 0$, $\beta > D^*$, the sliding manifold will remain bounded. By increasing α and β , the bound of the sliding manifold will converge to 0. The tracking of controller system is asymptotically stable when $\gamma_1 = 0$.

To sum up, with (8), (15), and (35), the final expression for proposed controller is finally obtained as

$$\begin{aligned} u &= K_1 \mathbf{x}_r + k_g r(t) + \hat{\mathbf{W}}^T \phi(\mathbf{x}, u) \\ &\quad - K_1 \mathbf{e} + \frac{1}{b}\left(-\Sigma + \beta \tan \mathbf{sig}(\alpha s)\right) \end{aligned} \tag{37}$$

Note that since no discontinuous function is introduced, this controller is *smooth*. Without generating additional oscillations, the controller will drive the tracking error asymptotic to 0.

5. SIMULATION

Recall the close loop system description (8), and with numerical parameters provided in [45], using RSLQR method, the reference system can be obtained.

$$B = [0 \quad 0 \quad 0 \quad 0 \quad 4624]^T \tag{38}$$

$$K_1 = [0.0681, 0.0099, -0.7994, 2.9394, 0.0101] \tag{39}$$

$$A_m = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & -1.75 & -1.55e3 & 0 & 0 \\ 0 & -4.12e-2 & 0 & -1.51e2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 3.15e2 & -4.57 & 3.471e3 & -1.82e4 & -1.28e2 \end{bmatrix} \tag{40}$$

Take observer gain $K_2 = 100$, learning rate $\Gamma_c = 100$, $Q = I$. Notice that, here further increase learning rate will lead to unstable behavior for MRAC due to the oscillation in control signal, but for proposed will work better, in order for comparing higher learning rate is not taken. The sliding mode defined by

$\lambda_2 = 16.85$, $\lambda_2 = 2.31$, $\lambda_3 = -22.2$, on the selected sliding surface the tracing error will converge to 0. α=0.01, β=1 is tuned sliding mode parameters, further increasing α will increase the sensitivity to tracking error, while as α approaches infinite the soft switching sliding mode will be equivalent to typical sliding mode, therefore generates chatter; further increasing β accelerate rise time but brings in overshooting. The sampling time is set to 0.01 second, and $W_{\max} = 5$ as a conservative estimation to the bound of weights. The following functions are used as basis function

$$\phi = \begin{bmatrix} \phi_1(x_1), \phi_1(x_2), \phi_1(x_3), \phi_1(x_4), \phi_1(x_5), \\ x_1, x_2, x_3, x_4, x_5 \end{bmatrix}^T \tag{41}$$

where $\phi_i(x_j) = \exp(-x_j^2)$. And the centers of RBFs are all set to 0, and widths are all 1. The radial basis function ensures desired sensitivity for neural networks to approximate error.

Apply the proposed controller (37), and the results is compared to MRAC with identical design parameters except for modified observer gain $K_2 = 0$, and sliding mode controller $\mu = 0$. Three different cases are tested:

Case A

$$f = K_1\mathbf{x} + 0.1\alpha + 0.5\alpha q \tag{42}$$

In this case the linear feedback control is canceled by unmodeled dynamics, and beyond that additional nonlinear disturbance is added.

The command signal is

$$r(t) = \begin{cases} 0, & 0 \le t \le 0.1 \sec onds \\ 3^\circ, & 0.1 < t \le 10 \sec onds \end{cases} \tag{43}$$

Under the command signal the missile is going to make a turning of 3 degrees in AoA. The command signal, actual system AoA trajectory and reference system AoA trajectory are shown in Figure 1. It can be seen that precise tracking of the reference system is realized. Also the uncertainty estimation is perfect according to Figure 2. Figure 3 is the comparison of AoA tracking error between proposed method and MRAC. For this case, the tracking error magnitude of proposed method is much smaller than MRAC during transient. For both methods tracking error converges. In Figure 4, control signal of both methods are compared. It can be seen that proposed method completely gets rid of the high-frequency oscillation which is shown in MRAC. At the same time, because control is used to cancel uncertainty which is estimated by neural networks, reducing oscillations in control signal implies that the neural network estimation is also smooth.

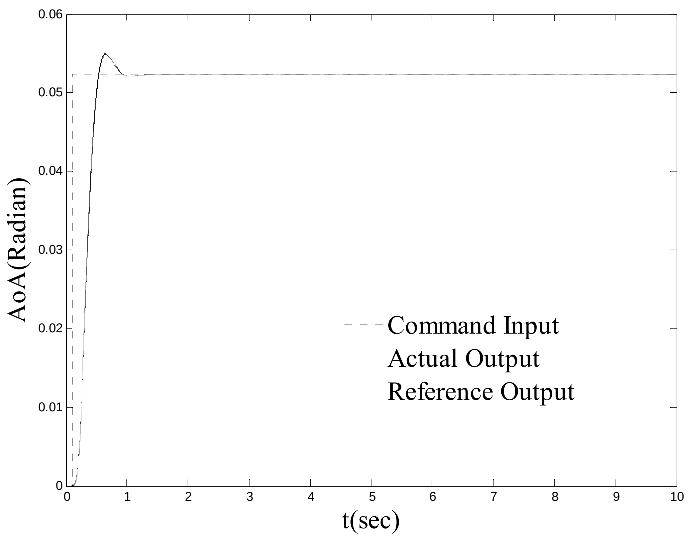

Figure 1. AoA trajectory, $f = K_1\mathbf{x} + 0.1\alpha + 0.5\alpha q$

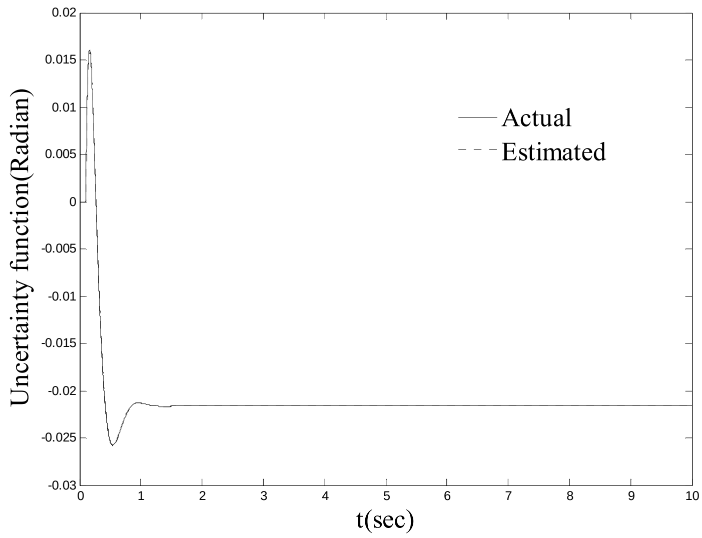

Figure 2. Uncertainty estimation, $f = K_1\mathbf{x} + 0.1\alpha + 0.5\alpha q$

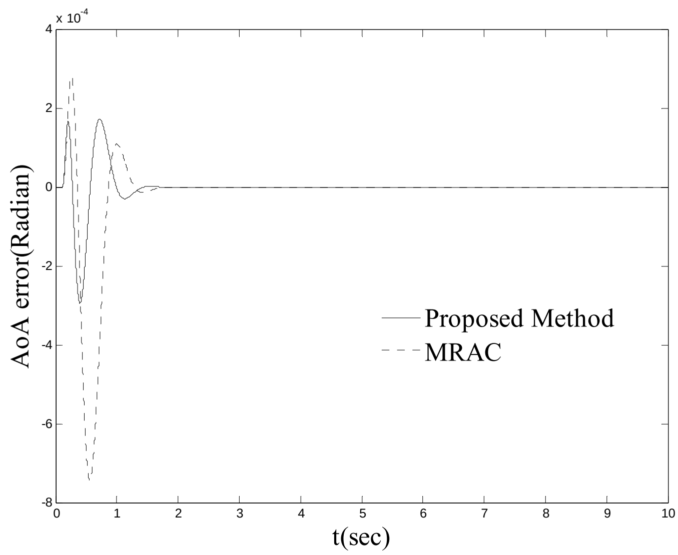

Figure 3. Tracking error history, $f = K_1\mathbf{x} + 0.1\alpha + 0.5\alpha q$

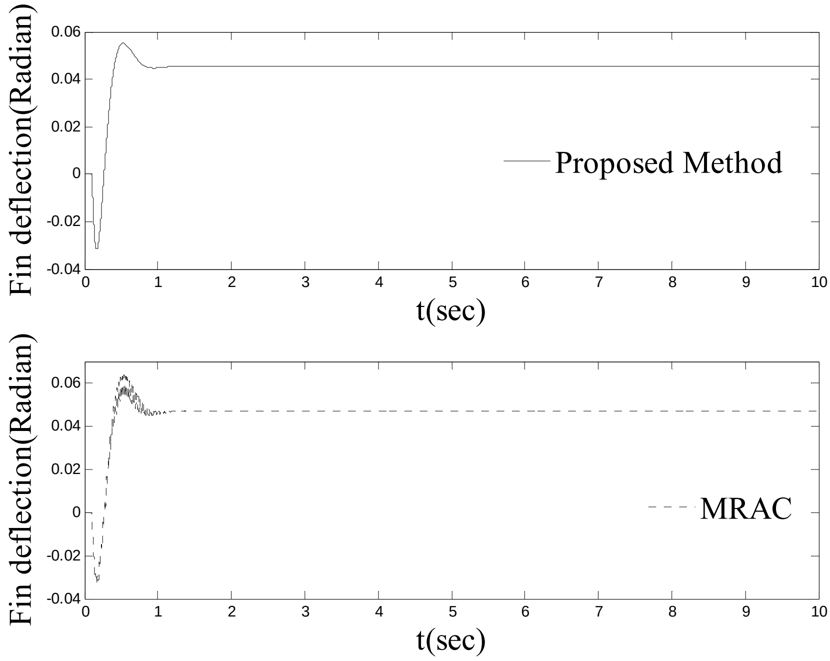

Figure 4. Control history, $f = K_1\mathbf{x} + 0.1\alpha + 0.5\alpha q$

Case B

$$f = K_1\mathbf{x} + 0.1\alpha + 0.5\alpha q + 0.1\sin(t) \tag{44}$$

Now comparing to case A, a time variant function is added as additional disturbance to test the robustness of proposed method.

The command signal is

$$r(t) = \begin{cases} 0, & 0 \le t \le 0.1 \sec onds \\ 3^\circ, & 0.1 < t \le 10 \sec onds \end{cases} \tag{45}$$

The command signal, actual system AoA trajectory and reference system AoA trajectory are shown in Figure 5. The tracking is still very precise under added disturbance. The uncertainty estimation is shown in Figure 6, and the estimation is fast and accurate. Figure 7 is the comparison of AoA tracking error between proposed method and MRAC. Comparing to previous case, proposed method keeps almost same performance when additional disturbance is added, while MRAC's tracking error significantly increases. Control signal of both methods are compared in Figure 8, and like previous case proposed method shows its advantage over MRAC in control smoothness.

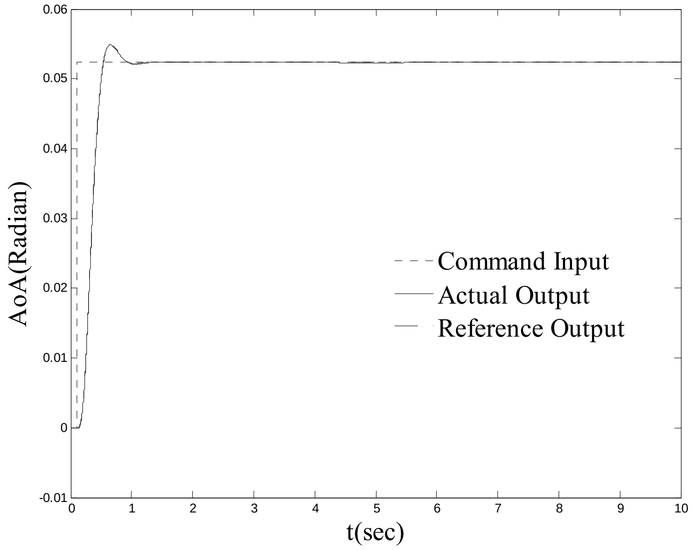

Figure 5. AoA trajectory, $f = K_1\mathbf{x} + 0.1\alpha + 0.5\alpha q + 0.1\sin(t)$

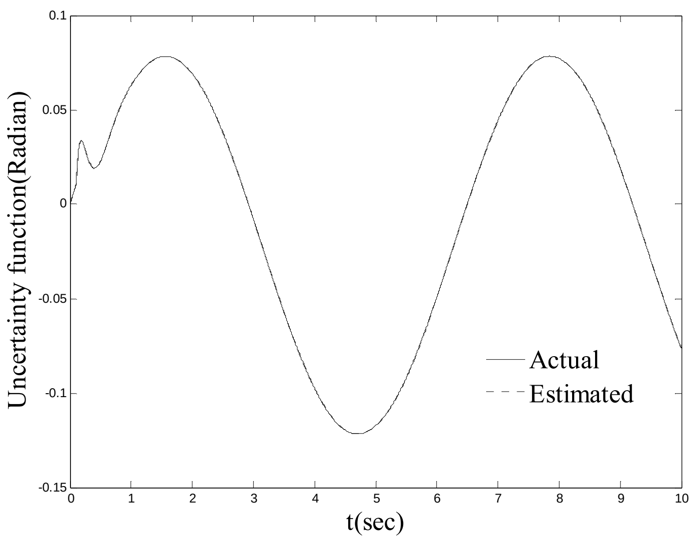

Figure 6. Uncertainty estimation, $f = K_1\mathbf{x} + 0.1\alpha + 0.5\alpha q + 0.1\sin(t)$

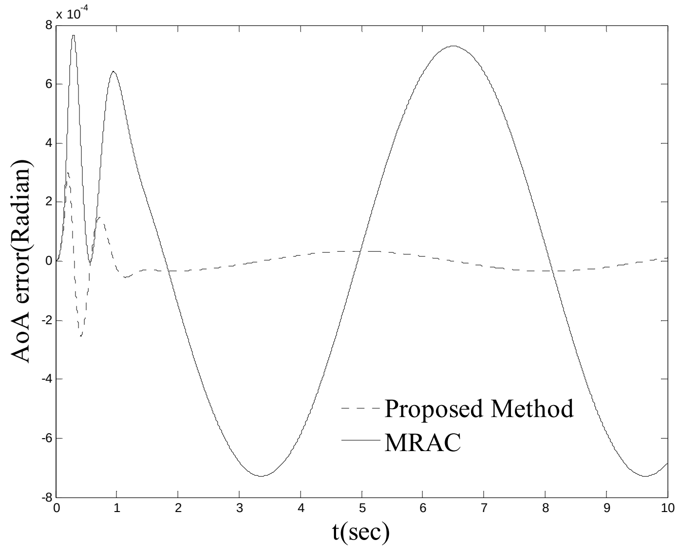

Figure 7. Tracking error history, $f = K_1\mathbf{x} + 0.1\alpha + 0.5\alpha q + 0.1\sin(t)$

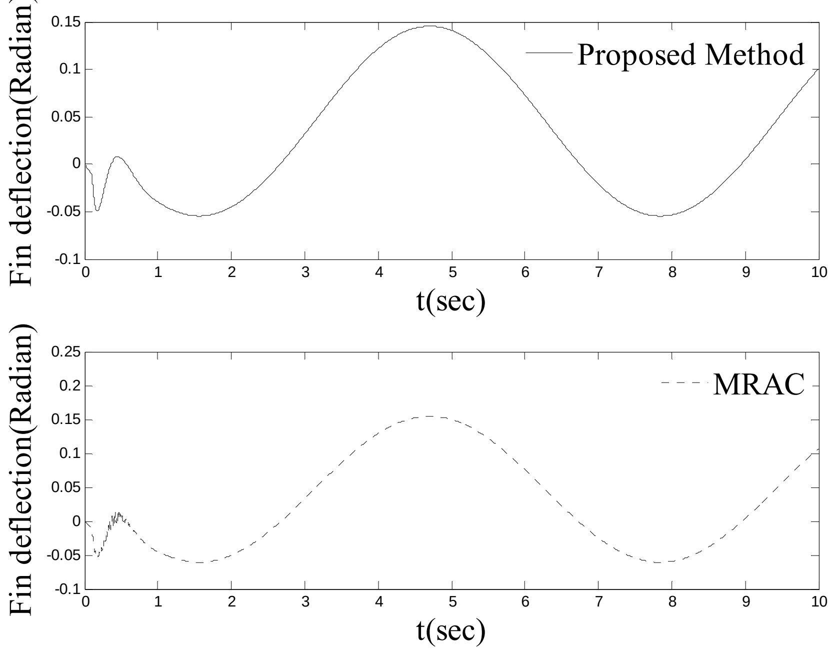

Figure 8. Control history, $f = K_1\mathbf{x} + 0.1\alpha + 0.5\alpha q + 0.1\sin(t)$

Case C

To further verify the method, take

$$f = K_1\mathbf{x} + 0.1\alpha + 0.5\alpha q + 0.1\sin(t) \tag{46}$$

same as case B.

And change the command signal to sinusoid function

$$r(t) = 3^{\circ}\sin(t) \tag{47}$$

Now the command signal, actual system AoA trajectory and reference system AoA trajectory are shown in Figure 9. It is shown that precise tracking of reference system under a sinusoid input is realized. The uncertainty estimation is shown in Figure 10, it can be seen that the neural networks perfectly estimate the unmodeled dynamics. Figure 11 is the comparison of AoA tracking error between proposed method and MRAC. It is shown that proposed method has better transient and steady state tracking performance comparing to MRAC. Control signal of both methods are compared in Figure 12, and again the proposed method removed oscillations comparing to MRAC.

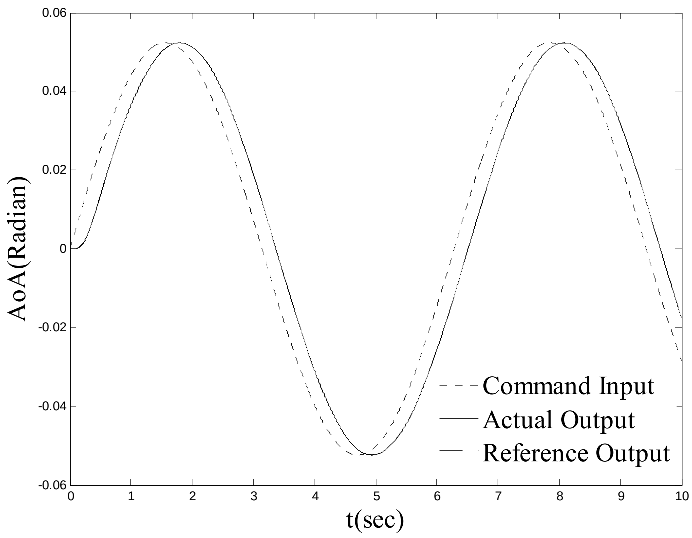

Figure 9. AoA trajectory, $f = K_1\mathbf{x} + 0.1\alpha + 0.5\alpha q + 0.1\sin(t)$

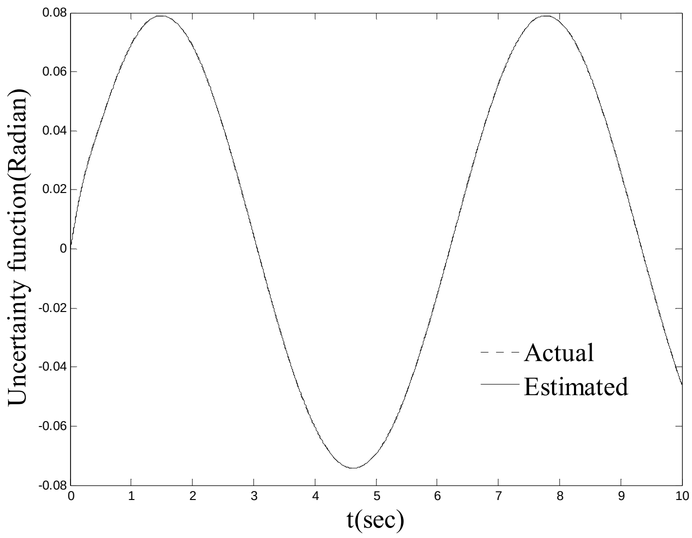

Figure 10. Uncertainty estimation, $f = K_1\mathbf{x} + 0.1\alpha + 0.5\alpha q + 0.1\sin(t)$

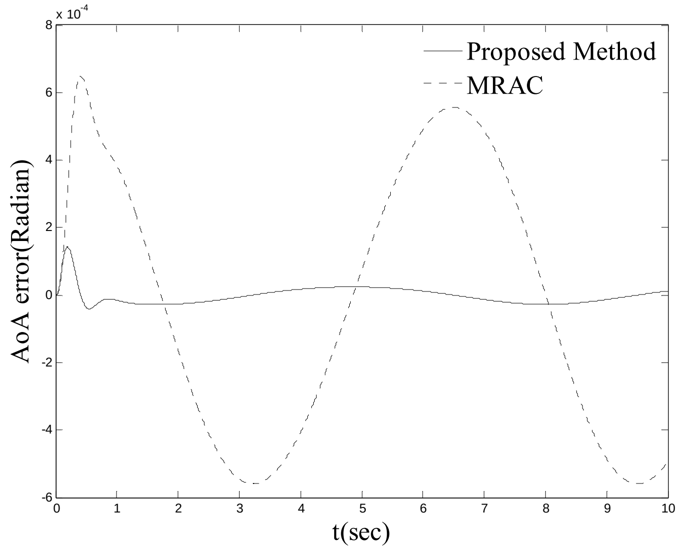

Figure 11. Tracking error history, $f = K_1\mathbf{x} + 0.1\alpha + 0.5\alpha q + 0.1\sin(t)$

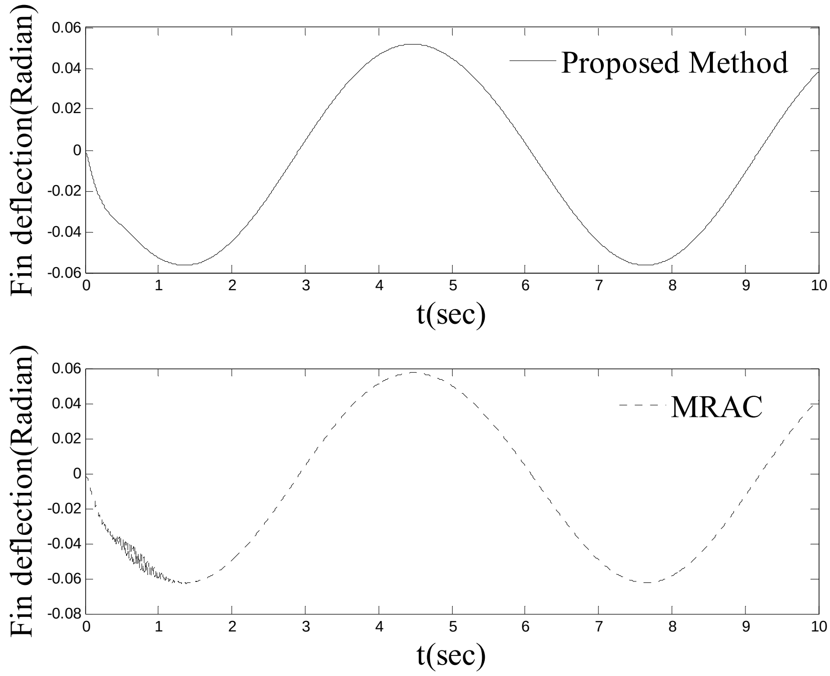

Figure 12. Control history, $f = K_1\mathbf{x} + 0.1\alpha + 0.5\alpha q + 0.1\sin(t)$

6. CONCLUSION

A new robust adaptive control has been derived in this paper. Bounds on transient response error shave been derived. A novel sliding mode term has been added to result in asymptotic stability of the errors instead of the usual upper bounded derivations. Performance of the proposed technique was evaluated with a missile autopilot problem and compared with a typical model reference adaptive controller. It is clear from the results that the transient response of the new controller is superior and does not show oscillatory behavior while learning and cancelling out the uncertainties. This fact is crucial in any implementation for two reasons. The first is that the oscillatory signals could lead to excitation of troublesome unmodeled dynamics. Second, it could lead to controller fatigue. From the missile autopilot example, the performance of the proposed controller seems to be robust and smooth.

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III. Electro-Hydraulic Piston Control

Using a New Model Reference Adaptive Controller

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ABSTRACT

A new model reference adaptive control design method using neural networks that guarantees *transient performance* is proposed in this paper. Stable tracking of a desired trajectory can also be achieved for nonlinear systems that operate under uncertainties. A modified state observer structure is designed to enable desired transient performance fast with large adaptive gains and at the same time avoid high frequency oscillations during uncertainty learning. The neural network adaptation rule is derived using Lyapunov theory, which guarantees *stability* of error dynamics and boundedness of neural network weights. An extra term is added in the controller expression using a ‘soft switching’ sliding mode that can be used to adjust tracking errors. The method is applied to control the velocity of an electro-hydraulic piston, and experimental results show that desired performance is achieved with smooth control effort.

1. INTRODUCTION

Applications of artificial neural networks (NN) in the field of control have been developed for decades. Various applications of neural networks in control system designed were outlined in [1]. In [2], it is claimed that neural networks' universal function approximation property can be useful in solving control problems. Narendra and Parthasarathy provided stability proof for the first time, and demonstrated the potential of neural networks in identification and control in nonlinear systems [3]. In 1992, Sanner and Slotine [4] developed a directive tracking control method with Gaussian radial basis function (RBF) networks for feedback control of nonlinearity. From then on, various adaptive control techniques using neural networks were put forward.

Based on the philosophy of feedback linearization [5]-[6], dynamic inversion [7]-[9] is developed for nonlinear control design. In this approach, an co-ordinate transformation is carried out to make the system dynamics take a linear form, then linear design methods could be taken, and based on this method, combining adaptive learning technique, model reference adaptive control (MRAC) is developed. The drawback of dynamic inversion is its sensitivity to modeling errors and parameter inaccuracies. Calise et al. proposed to introduce neural networks to dynamics inversion technique in order to cancel out the inversion error [10]-[13]. The neural networks are trained online using a Lyapunov-based approach, similar to the approach followed in [4] [14].

MRAC has been widely applied recently in solving control problems for system with matched unmodeled dynamics [15][16]. For conventional MRAC, it is hard to achieve desired transient performance and avoid unwanted high frequency oscillations at the same time. Various NN-based MRAC methods were developed (for example, [17]-

[18]), some modifications to the adaptive law were also introduced for better transient performance. Modification to the adaptive law such σ-modification [19], e-modification [20] are introduced. These methods modify the adaptive law by adding factor depending on the prediction error, which ensures the convergence of parameter estimation, moreover, when close to steady state conditions, the modification term becomes inactive and therefore the estimation accuracy is guaranteed. In [21], a projection operator is developed to modify the adaptive law. Projection operator replaces the common Lipschitz continuous property with arbitrary many times continuous differentiability, and estimation parameters are proven to be bounded.

However, although these developments to the adaptive law can be employed to improve robustness, tracking accuracy can only be shown bounded, and the bound depends on the disturbances itself. At the same time, typical MRAC cannot avoid unwanted oscillations. Recently many methods were developed to solve these two problems. In [22][23], a new MRAC neural networks controller named $\mathcal{L}_1$ adaptive controller is proposed, and transient performance of both system's input and output signal are characterized. This adaptive control architecture has a low-pass filter in the feedback loop, and its desired transient performance can be guaranteed by increasing adaption gain and improving NN approximation, and at the same time, high frequency oscillation is avoided. In [24], an adaptive control method that allows fast adaptation for systems with slow reference models is given. In this method, in order to allow fast adaptation, the neural network is trained with a high bandwidth state emulator. Low bandwidth control is maintained by a filter to isolate fast emulator dynamics from control signal. In [25], a novel Kalman-filter version of the e-modification [20] is developed. In this method standard e-modification term is interpreted as the gradient of a norm measure of a linear constraint violation, and this linear constraint is then used to develop a Kalman-filter-based e-modification. It is shown that this method leads to smaller tracking error without generating significant oscillation in the system response.

At the same time, because of its simplicity, adaptation to disturbance and guaranteed transient performance, sliding mode controller (SMC) is also often used in adaptive control [26][27][28]. In SMC, trajectories are forced to reach a designed sliding surface in finite time and to stay on the surface for all future time. Dynamics on the sliding surface is independent of matched uncertainties and the sliding surface is designed so as to guarantee the asymptotic stability of control objective. Though it has many advantages, an outstanding drawback of SMC in application is when control switching signs along the sliding surface oscillations are usually unavoidable. A soft-switching sliding mode technique has been introduced by Lychevsky [29][30]. By modifying sign function used in typical SMC to continuous real-analytic function, for example, tanh and erf, the soft switching sliding mode controller avoid oscillations and remain asymptotic stable at the same time. In [31], a systematic way to combine adaptive control and SMC for trajectory tracking in presence of parametric uncertainties and uncertain nonlinearities is developed. The sliding mode controller is smoothed with two methods based on the concept of boundary layer [27]. Asymptotic stability of adaptive system in presence of parametric uncertainties is realized, and the drawback of control chattering is reduced significantly. In [32], a modified switching function which provides low-chattering control signal is introduced, and the SMC is combined with a neural network adaptive controller which identifies modeling error online. In [33], by using a similar approaching to SMC, a novel approaching which combines NN feedforward controller with continuous robust integral of sign of error (RISE) feedback controller is introduced, in this method, by designing sliding surface using sign of error, a continuous RISE feedback is combined with NN-based adaptive controller, and it is shown that using Lyapunov theory the tracking error is asymptotically stable, while typical NN-based controller can only yield uniformly ultimately bounded (UUB) stability, and at the same time, the control is free from oscillations, experimental results show the method's potential in reducing tracking error [34][35].

In recent years, electronic control of hydraulic systems has been explored extensively, however, most research focused on controlling precise actuator using sophisticated servo-valves and high-precision instrumentation, which can be prohibitively expensive, and not suit for industrial hydraulic machinery environment. In [36], a practical control algorithm is presented and tested for use on this kind of electro-hydraulic machinery. Electro-hydraulic systems have been widely used in the industry. Hydraulic systems are capable of produce large force/torque at high speeds while maintaining a high power-to-size ratio. However, significant nonlinearities, such as dead-band, saturation, hysteresis, and nonlinear gain in hydraulic systems make it difficult to design high performance force/position tracking controllers. To cope with the nonlinearities, nonlinear control methods are typically used. Feedback linearization method is used [37-39]. However this method requires accurate system model which is difficult to achieve for electro-hydraulic system. Sliding mode control has also been used in electro-hydraulic system control [40-42]. However, the performance of sliding mode control is complicated by the choice of dead band. If the dead band is too small, chattering may occur; if the dead band is too large, it may deteriorate the tracking performance. In addition to the nonlinearities, there also exist significant uncertainties in the hydraulic system. Parameters may change with time due to different operating conditions, temperature, and/or wearing of hydraulic components. To deal with the uncertainties and the nonlinearities, several nonlinear adaptive control methodologies have been proposed. A nonlinear adaptive control based on back stepping is applied to the electro-hydraulic system force control in [43]. An adaptive sliding mode control technique is proposed in [44]. A nonlinear adaptive robust control scheme is proposed in [45] for single-rod hydraulic actuator motion control.

This paper develops a new neural network MRAC with guaranteed transient performance and asymptotic stability, and at the same time free from unwanted oscillations. Based on MRAC neural networks controller, the neural network observer structure is modified in the manner of [46]. In this modification, instead of introducing any additional filters, by adding a factor of observer error in the neural network observer structure, it is shown that high frequency oscillations are avoided, and as a result this new method enables further increasing adaptive gain, which leads to better tracking performance. At the same time, the modified term is inactive when neural network estimation is ideal, therefore the estimation accuracy is guaranteed. Moreover, in order to get better transient performance and stability, a soft-switching sliding mode modification [29] is combined with neural network adaptive controller, it is proven using Lyapunov method that it ideally leads to asymptotic stability instead of UUB, and at the same time is free from oscillations which is common for typical sliding mode adaptive controller. In general, the proposed controller enables higher adaptive gain without generating oscillations, provides better transient performance and asymptotic stability at the same time. This method to a Caterpillar electro-hydraulic test bench [36] for velocity tracking control and the results shows satisfactory tracking performance is achieved with smooth control.

Rest of the paper is organized as follows. In Section 1, the system and the neural networks structure is defined. In Section 2, the control solution is proposed. A stability proof of both observer and state error signal is put forward, and the guaranteed transient performance is also explained in Section 3. Section 4 includes description of the electro-hydraulic piston system and results and analysis of a series of experiments using the control algorithm.

2. PROBLEM DESCRIPTION

Consider the following single input single output (SISO) system.

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \cdots \\ \dot{x}_n = b(u - f(\mathbf{x})) \end{cases} \tag{1}$$

and the system output is defined as

$$y = cx_1 \tag{2}$$

c is a non-zero constant. The initial condition is set to

$$\mathbf{x}(0) = \mathbf{0}\,. \tag{3}$$

The set of equations in (1) can be written in a compact form as

$$\dot{\mathbf{x}}(t) = Ax(t) + B(u - f(\mathbf{x})) \tag{4}$$

where $\mathbf{x} \in \mathbb{R}^n$ is the system state vector, and all states are assumed to be measurable. $u \in \mathbb{R}$ is control signal, A is $n \times n$ system matrix, B is $n \times 1$ vector, $b > 0$, (A, B) is controllable. $f : \mathbb{R}^n \to \mathbb{R}$ is an unknown continuous nonlinear function.

The control objective is to design a neural adaptive controller which ensures output $y(t)$ tracks a desired bounded continuous trajectory $r(t)$, and the system behavior follows a nominal linear time-invariant (LTI) system which is designed through standard methods(for example, through linear quadratic regulator theory[29]), and at the same time guarantee desired transient and steady state performance in the presence of uncertainties.

Assume the following NN approximation of $f(\mathbf{x})$ exists

$$f(\mathbf{x}) = \mathbf{W}^T \phi(\mathbf{x}) + \varepsilon(\mathbf{x}) \quad |\varepsilon(\mathbf{x})| < \varepsilon^* \tag{5}$$

where $\phi(\mathbf{x})$ is a set of radial basis functions, and each element of $\phi(\mathbf{x})$ is defined as

$$\phi(y) = \exp(-(y - z)^T (y - z) / \sigma^2) \tag{6}$$

In (6), z is the location of selected center, σ is the 'width'. $\mathbf{W}$ are the ideal network weights, $\varepsilon(\mathbf{x})$ is the network approximation error, ε^* is its uniform bound. Further assume that a compact convex set Ω is known a priori such that

$$\mathbf{W} \in \Omega \tag{7}$$

In order to realize tracking control for this SISO system, the following neural network adaptive controller is developed.

3. CONTROL SOLUTION

The proposed controller is a combination of a linear feedback control, neural network adaptive control and a soft switching sliding mode control. First of all, divide the controller expression into three parts-the linear feedback control $K_1\mathbf{x}$, neural networks adaptive control u_e and soft switching sliding mode control μ

$$u = K_1\mathbf{x} + u_e + \mu \tag{8}$$

where K_1 the closed loop feedback gain, which ensures closed-loop reference dynamics matrix $A_m = A - BK_1$ is Hurwitz. The linear feedback control ensures stability when there is no uncertainty; the adaptive control is obtained through neural networks observer, and cancels the uncertainty; the soft switching sliding mode control guarantees asymptotic stability in presence of neural networks estimation error, and it is going to be exactly defined later with stability proof.

Substitute (8) into (4), (4) becomes

$$\dot{\mathbf{x}}(t) = A_m\mathbf{x}(t) + B(u_e(t) + \mu - f(\mathbf{x})) \tag{9}$$

And define the following state observer structure,

$$\dot{\hat{\mathbf{x}}}(t) = A_m\hat{\mathbf{x}}(t) + B(u_e(t) + \mu - \hat{f}) - K_2\tilde{\mathbf{x}}(t) \tag{10}$$

where $\hat{\mathbf{x}}(t)$ represents the observer states at time t. The initial conditions for observer are

$$\hat{\mathbf{x}}(0) = \mathbf{0} \tag{11}$$

Since the uncertainty and the true neural network weights are unknown, they are represented as $\hat{\mathbf{W}}^T\phi(\mathbf{x})$ where $\hat{\mathbf{W}}$ represents the estimated neural network weights with a proper weight update law. The observer gain matrix is assumed diagonal for convenience and is expressed as $K_2 = diag(k_2^1, k_2^2, ..., k_2^n)$. In the observer structure, $\hat{f}$ is assumed to be canceled perfectly by neural networks controller, i.e. $\hat{f} = \hat{\mathbf{W}}^T \phi(\mathbf{x})$.

Define the observer error as

$$\tilde{\mathbf{x}}(t) \equiv \hat{\mathbf{x}}(t) - \mathbf{x}(t) \tag{12}$$

The adaptive weight update law is defined as follows[28]:

$$\dot{\hat{\mathbf{W}}}(t) = \Gamma_c \, \mathrm{Pr}\, oj(\hat{\mathbf{W}}(t), \phi(\mathbf{x})\tilde{\mathbf{x}}(t)^T PB) \tag{13}$$

where P is defined by $A_m^T P + PA_m = -Q$, with Q being a positive definite matrix and Γ_c is the learning rate of the neural network. The projection operator property guarantees the boundedness of neural networks weights error

$$\tilde{\mathbf{W}}^T \tilde{\mathbf{W}} \le W_{\max} \tag{14}$$

where $W_{\max} \equiv \max_{\mathbf{W} \in \Omega} 4\|\mathbf{W}\|^2$, $\tilde{\mathbf{W}} \equiv \hat{\mathbf{W}} - \mathbf{W}$ [21].

Now with neural networks weights, the adaptive control expression becomes

$$u_e = k_g r(t) + \hat{\mathbf{W}}^T \phi(\mathbf{x}) \tag{15}$$

where

$$k_g \equiv \frac{1}{CA_m^{-1}B} \tag{16}$$

is the open loop gain of the reference system.

Subtract (9) from (10), and substitute (15), the observer error dynamics is obtained as,

$$\dot{\tilde{\mathbf{x}}}(t) = (A_m - K_2)\tilde{\mathbf{x}} + B(\tilde{\mathbf{W}}^T \phi(\mathbf{x}) - \varepsilon) \tag{17}$$

By using Lyapunov method [9], it will be shown that the neural network estimation error and the observer error are bounded. By introducing the observer gain K_2, the learning process is smoothed, and the modified term decrease as $\tilde{\mathbf{x}}$ decreases, therefore the learning accuracy is guaranteed. As a result, the modified observer structure enables increasing adaptation gain without generating high frequency oscillations.

4. STABILITY ANALYSIS

In this section, Lyapunov method is used to prove the boundedness of the observer error dynamics. And in order to assure asymptotic convergence of reference error, the soft-switching sliding mode controller is derived. Details of the proofs are provided in the following subsections.

4.1. OBSERVER ERROR

To get the error bound for neural network observer, consider a Lyapunov function as $V(\tilde{\mathbf{x}}, \tilde{\mathbf{W}}) = \tilde{\mathbf{x}}^T P \tilde{\mathbf{x}} + \Gamma_c^{-1} \tilde{\mathbf{W}}^T \tilde{\mathbf{W}}$, and differentiate $V(.)$ to get

$$\dot{V} = \dot{\tilde{x}}^T P \tilde{x} + \tilde{x}^T P \dot{\tilde{x}} + \Gamma_c^{-1} (\tilde{\mathbf{W}}^T \dot{\tilde{\mathbf{W}}} + \dot{\tilde{\mathbf{W}}}^T \tilde{\mathbf{W}}) \tag{18}$$

substitute the weight update law (13) and observer dynamics (17), (18) becomes

$$\begin{aligned} \dot{V} &= -\tilde{\mathbf{x}}^T Q \tilde{\mathbf{x}} - 2\tilde{\mathbf{x}}^T K_2 P \tilde{\mathbf{x}} - 2\tilde{\mathbf{x}}^T PB(\tilde{\mathbf{W}}^T \phi(\mathbf{x}) - \varepsilon(\mathbf{x})) \\ &\quad + 2\tilde{\mathbf{W}}^T \operatorname{Pr}oj(\hat{\mathbf{W}}, \phi(\mathbf{x})\tilde{\mathbf{x}}^T PB) \\ &\le 2\|PB\varepsilon\|\|\tilde{\mathbf{x}}\| - [\lambda_{\min}(Q) + 2\lambda_{\min}(K_2 P)]\|\tilde{\mathbf{x}}\|^2 \end{aligned} \tag{19}$$

therefore $\dot{V} \le 0$ when

$$\|\tilde{\mathbf{x}}\| \ge \frac{2\|Pb\varepsilon^*\|}{\lambda_{\min}(Q) + 2\lambda_{\min}(K_2 P)} \tag{20}$$

As a result,

$$V(\tilde{\mathbf{x}}, \tilde{\mathbf{W}}) \le \tilde{\mathbf{x}}^T P \tilde{\mathbf{x}} + \Gamma_c^{-1} W_{\max}$$
$$\le \lambda_{\max}(P) \left(\frac{2\|PB\varepsilon^*\|}{\lambda_{\min}(Q) + 2\lambda_{\min}(K_2 P)} \right)^2 + \Gamma_c^{-1} W_{\max} \tag{21}$$

and at the same time

$$V(\tilde{\mathbf{x}}, \tilde{\mathbf{W}}) \ge \tilde{\mathbf{x}}^T P \tilde{\mathbf{x}} \ge \lambda_{\min}(P) \|\tilde{\mathbf{x}}\|^2 \tag{22}$$

(21) and (22) lead to

$$\|\tilde{\mathbf{x}}\| \le \sqrt{\frac{\left(\lambda_{\max}(P) \left(\frac{2\|Pb\varepsilon^*\|}{\lambda_{\min}(Q) + 2\lambda_{\min}(K_2 P)} \right)^2 + \Gamma_c^{-1} W_{\max} \right)}{\lambda_{\min}(P)}}$$
$$\equiv \gamma_0 \tag{23}$$

In (23), by increasing adaptation gain Γ_c and observer gain K_2, $\|\tilde{\mathbf{x}}\|$ can be driven to be as small as possible, therefore, precise uncertainty estimation using online neural networks is guaranteed. Also, with the modification term to smooth out learning, the state observer structure suppresses the high frequency oscillation so that increased adaptation gain and smooth control are possible at the same time.

4.2. REFERENCE ERROR

Note that with adaptive control and linear feedback control alone (with $\mu = 0$), the controller is able to track reference system but with bounded tracking errors. However, with a soft-switching sliding mode controller, the tracking error can be shown to be asymptotic stable. Define a reference LTI system dynamics as

$$\dot{\mathbf{x}}_r = A_m \mathbf{x}_r + b k_g r \tag{24}$$

where $u_r \equiv \mathbf{W}^T \phi(\mathbf{x}_r)$ is the reference controller, which cancels uncertainty.

By subtracting the reference dynamics (24) from actual system dynamics (9), the tracking error dynamics are expressed as

$$\dot{\mathbf{e}} \equiv \dot{\mathbf{x}}(t) - \dot{\mathbf{x}}_r(t)$$
$$= A_m\mathbf{e} + b(\mu + (\hat{\mathbf{W}}^T - \mathbf{W}^T)\phi(\mathbf{x}) - \varepsilon(\mathbf{x})) \tag{25}$$

Recalling the definition of system dynamics as given in (1), (25) can be written as

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = e_3 \\ \ldots \\ \dot{e}_n = bK_1\mathbf{e} + b(\mu + \tilde{\mathbf{W}}^T\phi(\mathbf{x}) - \varepsilon(\mathbf{x})) \end{cases} \tag{26}$$

Then, define the sliding surface as

$$s \equiv \sum_{p=0}^{n-1} \lambda_p e_1^{(n-p-1)} \tag{27}$$

where $\lambda_i > 0, i = 0,1,...,n-1$. In most cases, the designer can just set $\lambda_0 = 1$. For example, when n=3, the sliding manifold is $s = e_3 + \lambda_1 e_2 + \lambda_2 e_1$.

With a Lyapunov function $V_s = \frac{1}{2}s^2$, its derivative is given by

$$\dot{V}_s = s\dot{s} = s(\sum_{p=1}^{n-1} \lambda_p e_1^{(n-p)} + e_1^{(n)})$$
$$= s(\sum_{p=1}^{n-1} \lambda_p e_1^{(n-p)} + \dot{e}_n)$$
$$= s(\sum_{p=1}^{n-1} \lambda_p e_1^{(n-p)} + bK_1\mathbf{e} + b(\mu + D)) \tag{28}$$

where

$$D \equiv \tilde{\mathbf{W}}^T\phi(\mathbf{x}) - \varepsilon(\mathbf{x}) \tag{29}$$

Recall NN approximation property (5) and weights error boundedness (14), immediately we have the following bound for D

$$\begin{aligned}\|D\| &= \left\|\tilde{\mathbf{W}}^T \phi(\mathbf{x}) - \varepsilon(\mathbf{x})\right\| \\ &\le W_{\max} + \varepsilon^* \equiv D^*\end{aligned} \tag{30}$$

Now the soft switching sliding manifold control term is formulated as

$$\mu = -\sum_{p=1}^{n-1} \lambda_p e_1^{(n-p)} / b - K_1 \mathbf{e} - \beta \tanh(\alpha s)\,, \tag{31}$$

By substituting (31) into (28), it can be shown that

$$\dot{V}_s = bs(-\beta \tanh(\alpha s) + D) \tag{32}$$

When s>0

$$\begin{aligned}\dot{V}_s &= bs(-\beta \tanh(\alpha s) + D) \\ &\le bs(-\beta \tanh(\alpha s) + D^*)\end{aligned} \tag{33}$$

$\dot{V}_s \ge 0$ only when

$$\tanh(\alpha s) \le D^* / \beta \tag{34}$$

which leads to

$$0 < s \le \frac{1}{2\alpha} \ln \frac{1 + \frac{D^*}{\beta}}{1 - \frac{D^*}{\beta}} \tag{35}$$

When s<0

$$\begin{aligned}\dot{V}_s &= bs(-\beta \tanh(\alpha s) + D) \\ &\le b(-s)(\beta \tanh(\alpha s) + D^*)\end{aligned} \tag{36}$$

$\dot{V}_s \geq 0$ only when

$$\tanh(\alpha s) > -D^* / \beta \tag{37}$$

which leads to

$$0 > s \geq \frac{1}{2\alpha} \ln \frac{1 - \frac{D^*}{\beta}}{1 + \frac{D^*}{\beta}} \tag{38}$$

From (35) and (38), it can be observed that the bound for the sliding manifold is

$$\|s\| < \frac{1}{2\alpha} \ln \frac{1 + \frac{D^*}{\beta}}{1 - \frac{D^*}{\beta}} \equiv \gamma_1 \tag{39}$$

As long as $\alpha > 0$, $\beta > D^*$, the sliding manifold will remain bounded. By increasing α and β , the bound of the sliding manifold will converge to 0. The tracking of controller system is asymptotically stable when $\gamma_1 = 0$.

To sum up, with (8), (15), and (31), we can get the final expression for proposed controller as follows

$$u = K_1 \mathbf{x}_r + k_g r(t) + \hat{\mathbf{W}}^T \phi(\mathbf{x}) - \sum_{p=1}^{n-1} \lambda_p e_1^{(n-p)} / b - \beta \tanh(\alpha s) \tag{40}$$

Note that since no discontinuous function is introduced, this controller is *smooth*. Without generating additional oscillations, the controller will drive the tracking error asymptotic to 0.

5. EXPERIMENTAL RESULTS

The test bed for the control method is a Caterpillar Electro-Hydraulic Test Bench (Figure 1.), which was a gift from Caterpillar to Missouri University of Science and Technology as part of a laboratory dedicated to electro-hydraulics and mechatronics.

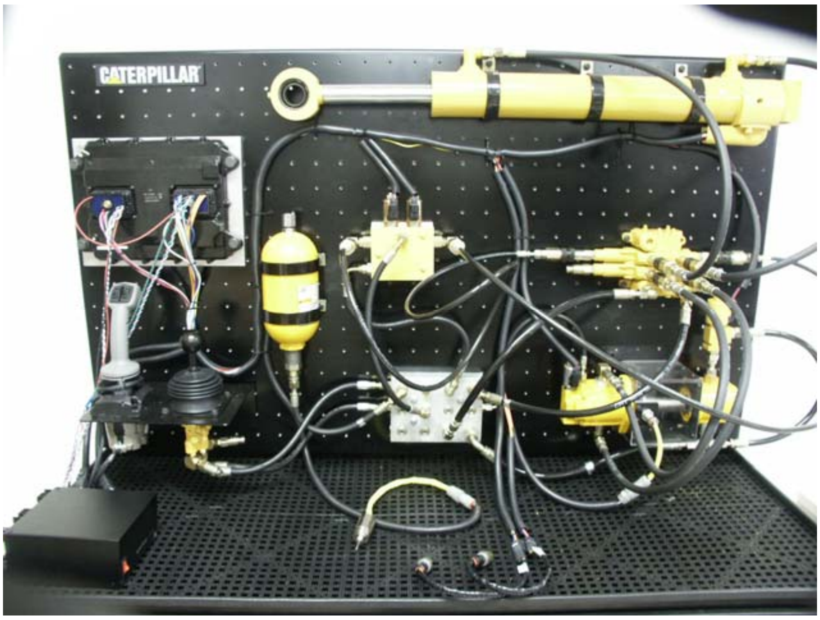

Figure 1. Caterpillar Electro-Hydraulic Test Bench

The test bench consists of the electro-hydraulic valves and piston, with five distinct physical components which affect the system operation and dynamics: control electronics, pilot solenoid valve, spool valve, hydraulic cylinder, and sensors. Additionally, specialized computer hardware and software interfaces with the control electronics and sensors, providing for real-time computerized control. The complete system is shown in diagram form in Figure 2.

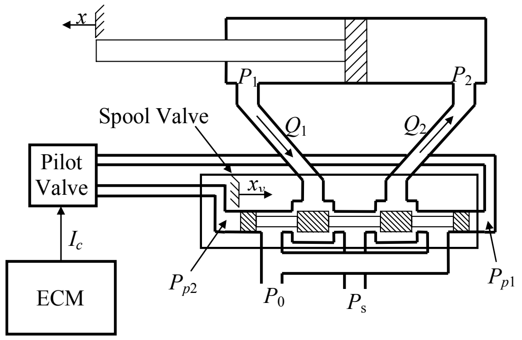

Figure 2. Electro-Hydraulic System Diagram

The electro-hydraulic system considered in this study cannot be well described by a linear, time-invariant model, since the nonlinear characteristics such as friction, dead band, and nonlinear valve gains cannot be neglected. The system includes pressure sensors that measure the pressures in each chamber of the piston and an encoder that measures the piston displacement. In this system, the spool valve is contained in a sealed housing with no integrated sensor; therefore it is impossible to measure its position either in real-time or offline. Additionally, it is subject to significant and unpredictable stiction effects and flow forces, so its position cannot be accurately predicted based solely on the control input and measured states.

The input current which feeds into the pilot valve will decide the direct input into forward and reverse valve, the relationship between them is,

$$\begin{cases} I_{cf} = I_c + I_{cf0}, & I_{cr} = 0, & \text{if } I_c > 0 \\ I_{cf} = 0, & I_{cr} = I_c + I_{cf0}, & \text{if } I_c < 0 \\ I_{cf} = 0 & I_{cr} = 0, & \text{if } I_c = 0 \end{cases} \tag{41}$$

where I_c is the input current, I_{cf} is input to forward valve, I_{cr} is input to reverse valve, $I_{cf0} = I_{cr0} = 0.4$ A is the estimated dead band value of each valve.

From previous work [36], a simple input-output model for the piston response was developed based on experimental data. In order to remove noise from encoder, a low-pass filter is utilized for obtaining filtered position signal. The filtered piston position is numerically differentiated to calculate the (approximate) piston velocity. It is done online so a first order backwards finite difference is used. The relationship between velocity and encoder position output is

$$v = \dot{x} = -5x + 5x_e \tag{42}$$

where v is estimated velocity, x is filtered position, x_e is encoder position output.

As a result, instead of using a high order system, in the following experiments, a simple linear model with matched uncertainties is used,

$$\dot{v} = \frac{1}{m}(-Bv + b(I_c - f(x, v, P_1, P_2, I_c)) \tag{43}$$

where x (mm) is the displacement of the piston, v (mm/sec) is piston velocity, $B = 2$ (kg/sec) is the estimated value of the viscous friction coefficient, $b = 1$ is the estimated value of the control gain. P_1 (kPa) and P_2 (kPa) are the measured pressure from each chamber, f is the unknown nonlinear dynamics. $m = 3.85$ kg is the measured piston mass. The sample period is 0.01 sec.

With the feedback control gain K_1, the closed loop reference velocity dynamics are

$$\dot{v}_r = \frac{1}{m}(-(B + bK_1)v_r + bk_g r) = \frac{1}{m}(-B_m v_r + bk_g r) \tag{44}$$

The desired reference linear system should be realizable, and given that satisfied, as fast as possible. In order to obtain a suitable center value for B_m, a series of open loop experiments are conducted. The results are shown in Table 1.

It can be seen that the test bench is highly nonlinear and asymmetric; the forward direction saturates at $I_c > 0.2$ A and the reverse direction saturates at $I_c > 0.3$ A. The settling time varies from 4.45 to 5.9 sec. As a result, a realizable and desired reference performance is taken as $K_1 = 1$, $B_m = 3$, $k_g = 3$. Therefore closed loop reference system is modeled as

$$\dot{v}_r = \frac{1}{3.85}(-3v_r + 3r) \tag{45}$$

Table 1. Open Loop Experiments Results

I_c (A)	Steady state velocity (mm/s)	5% settling time (s)
0.04	9.9	4.15
−0.04	−10.5	4.22
0.07	17.6	4.26
−0.07	−18.5	4.33
0.1	21.7	5.75
−0.1	−24.5	5.72
0.2	22.5	5.7
−0.2	−26	5.74
0.3	22.7	5.8
−0.3	−27.2	5.9
0.4	22.9	5.8
−0.4	−27.5	5.9

From Table 1, the average experimental settling time is 5.27 sec, and the 5% settling time of reference system is 3.85 seconds, providing a 27% decrease in the 5% settling time. Experiments show that, the maximum forward acceleration is 11.2 mm/s^2, and the maximum reverse acceleration is 13.1 mm/s^2.

The control law is

$$I_c = K_1 v + k_g r + \hat{W}^T \phi - K_1 e - \beta \tanh(\alpha s) \tag{46}$$

The radial basis function $\phi \in \mathbb{R}^{12}$ used for neural network structure is

$$\phi = \left[\phi_1(v), \phi_2(x), \phi_3(P_1), \phi_4(P_2), \phi_5(I_c), 1\right]^T \tag{47}$$

where

$$\phi_1(v)=\left[e^{-(v-z_1)^2/\sigma_1^2}, e^{-(v-z_2)^2/\sigma_1^2}, e^{-(v-z_3)^2/\sigma_1^2}\right]^T$$

$$\phi_2(x)=e^{-(x-z_4)^2/\sigma_2^2}$$

$$\phi_3(P_1)=\left[e^{-(P_1-z_5)^2/\sigma_3^2}, e^{-(P_1-z_6)^2/\sigma_3^2}, e^{-(P_1-z_7)^2/\sigma_3^2}\right]^T$$

$$\phi_4(P_2)=\left[e^{-(P_2-z_8)^2/\sigma_4^2}, e^{-(P_2-z_9)^2/\sigma_4^2}, e^{-(P_2-z_{10})^2/\sigma_6^2}\right]^T$$

$$\phi_5(I_c)=e^{-(I_{c1}-z_{11})^2/\sigma_7^2}$$

Here $z_1=0$ mm/s, $z_2=15$ mm/s, $z_3=-15$ mm/s, $z_4=0$ mm/s, $z_5=z_8=0$ kPa, $z_6=z_9=40$ kPa, $z_7=z_{10}=40$ kPa, $z_{11}=0$ A, $\sigma_1=1$, $\sigma_2=20$, $\sigma_3=\sigma_4=\sigma_5=\sigma_6=2$, and $\sigma_7=0.05$. The centers and widths of the RBF are selected so that the neural network can estimate uncertainty over the entire working region of the system with similar sensitivity. Notice that, as long as f is a continuous function of x, v, P_1, P_2, and I_c, the neural network approximation assumption (3) is valid.

The learning rate for the adaptive controller is selected as $\Gamma=100$, the observer gain is $K_2=100$, and the sliding surface is $s=e=v-v_r$ since the modeled system is first order. The soft switching sliding mode control parameters are α=200 , β=100 . Increasing Γ causes larger overshoot, while decrease it increase error bound; increasing K_2 increases error bound, while decrease it increases overshoot. The parameters are tuned in order to decrease the steady state error bound and obtain the best possible transient performance. Increasing α will increase the feedback controller's sensitivity to tracking error, and increasing β can increase the controller's response speed, but when it is too large there will be significant overshoot. Neural network weights are updated by the adaptive law (10), with $P=1$.

The system diagram is shown in Figure 3.

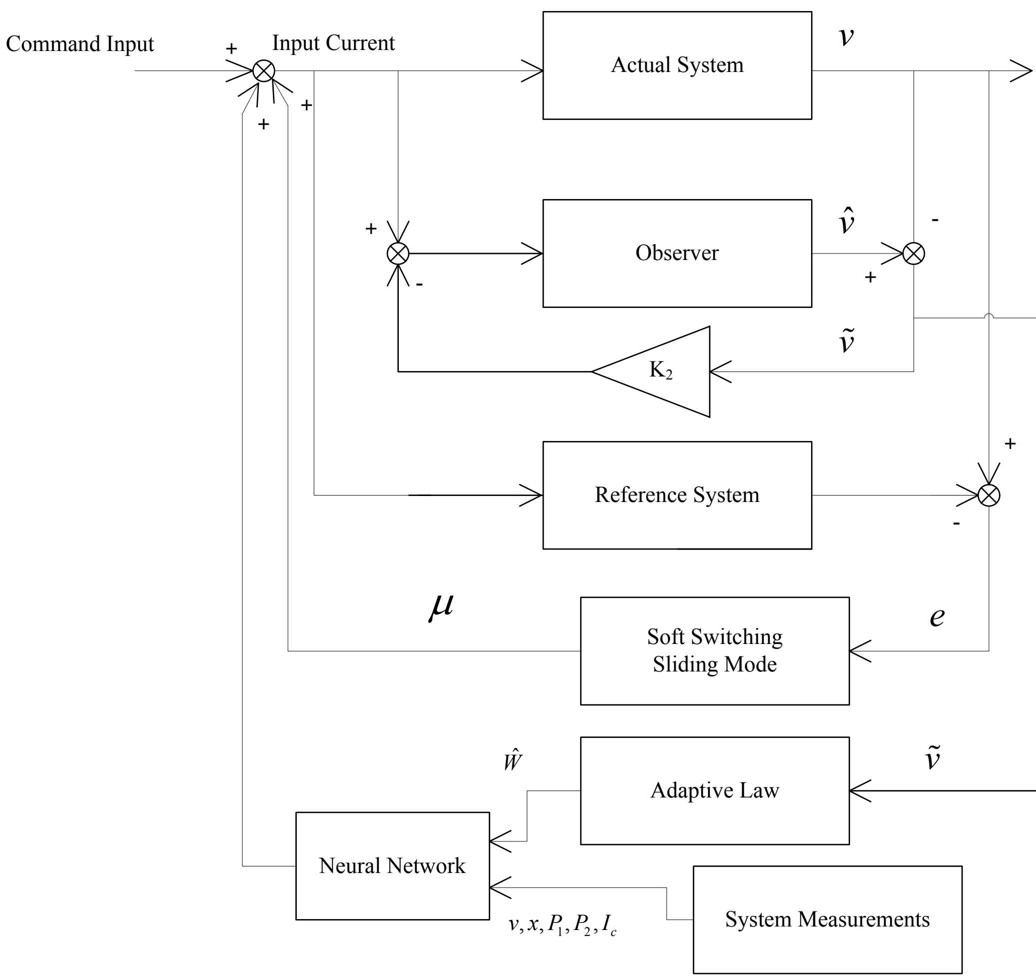

Figure 3. Control System Block Diagram

To verify the feasibility of this controller, a series of different command inputs are tested.

Case 1: constant velocity signal

Take the command input as

$$r = 18\,\mathrm{sign}(\sin(2\pi / 32)t) \text{ mm/s} \tag{48}$$

Results are shown in Figure 4. The velocity trajectory, control history and a velocity tracking error history is given.

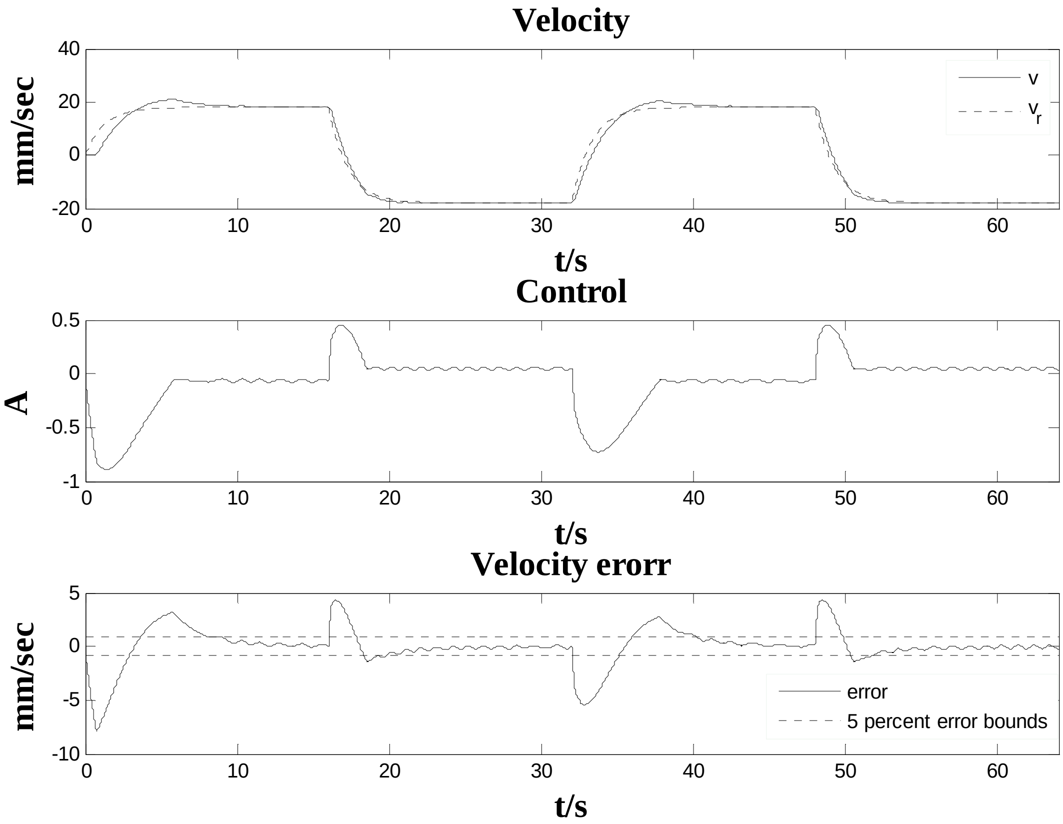

Figure 4. Experimental results for constant reference

There is an initial inevitable delay (approximately 0.8 sec) for each experiment, due to the flow filling process of the test bench. At 0.8 sec, the error reaches peak value of 7.9 mm/sec, after that the controller keeps the velocity error bounded during both steady stage and transient, the closed-loop system velocity tracks the reference system. Disregarding the first step, the 5% settling time for second, third and forth steps are 3.8, 8.25, and 3.7 sec respectively. The difference is due to the nonlinearity and asymmetry of the system. For reverse direction, the settling time is very close to reference model, i.e. 3.85 sec. During the transient stage, the tracking error increases up to a peak value of 4.3 mm/sec for forward direction and 5.4 mm/sec for reverse direction. After the transient, with the adaptive controller, tracking error quickly decreased down to 5% percent bound, i.e. 0.9 mm/sec. During steady stage, the neural network controller takes care of the disturbance and keeps error bounded under 5%.

Case 2: sinusoidal velocity signal

To test the controller stability under time varying conditions, the following reference signal is considered

$$r = 15\sin(t) \text{ mm/s} \tag{49}$$

The results are shown in Figure 5.

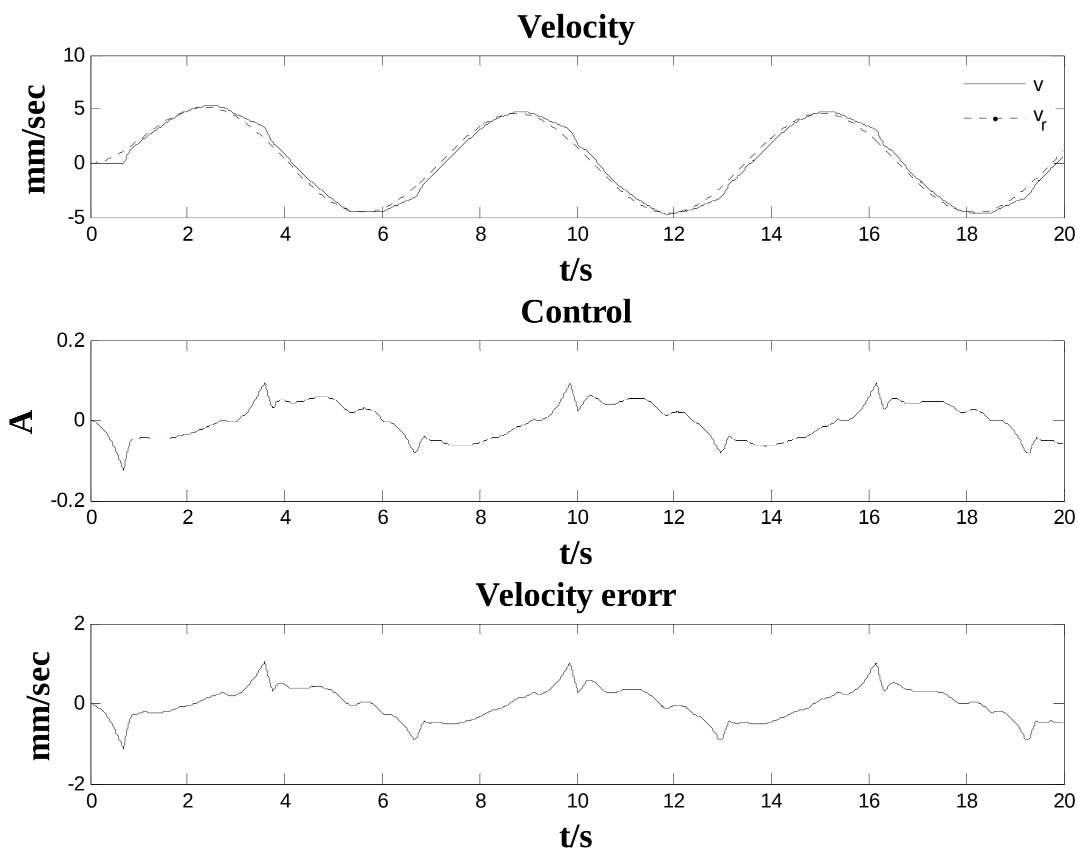

Figure 5. Experimental results for $r = 15\sin(t)$ mm/s

It can be seen that the actual system tracks the reference. The output tracks the desired trajectory when velocity switches direction. Disregard the initial error, peak value of velocity tracking error is 1.0 mm/sec for forward direction and 0.9 mm/sec for reverse direction. The closed-loop velocity is 2.6% higher than reference velocity; the phase delay is under 0.5 degree. At the same time, the control signal is smooth without any high frequency oscillations, as evidenced by a Fast Fourier Transform that showed significant energy only at the 1 rad/s and its multiples, and 90% of energy concentrated under 25 rad/s.

To further verify the system performance, the following reference signal is considered

$$r = 15\sin(5t)\ \text{mm/s} \tag{50}$$

The results are shown in Figure 6. As the results show, compared to low frequency results, the tracking error and the phase delay both increase. Disregard the initial error, peak value of velocity tracking error is 1.4 mm/sec for forward direction and 1.2 mm/sec for reverse direction. The closed-loop velocity is 8.6% higher than reference velocity; the phase delay is 3.4 degree. As frequency continues to increase, the controller will meet the limit which prevents precise tracking. The control signal is still prevented from high frequency oscillations, and a Fast Fourier Transform shows significant energy only at the 5 rad/s and its multiples 10 rad/s, more than 90% of energy concentrated under 55.6 rad/s.

By testing other frequencies, a bode diagram of the frequency properties of the controller system is obtained. Figure 7 shows the steady state magnitude of closed-loop system vs reference system, and the phase delay of actual system comparing to the reference.

As the Figure 7 illustrates, with the proposed neural network controller, when sinusoid signal is input, the actual system is able to track reference system. The cutoff frequency is 13 rad/sec. Due to the limitation of acceleration, as frequency increases, the peak velocity have to decrease, as a result, when applying frequencies even higher, the piston velocity is too small (<0.5mm/s) for effective control. As a conclusion, under working frequency below 13 rad/sec, proposed controller is getting a satisfying result for both magnitudes and phase tracking.

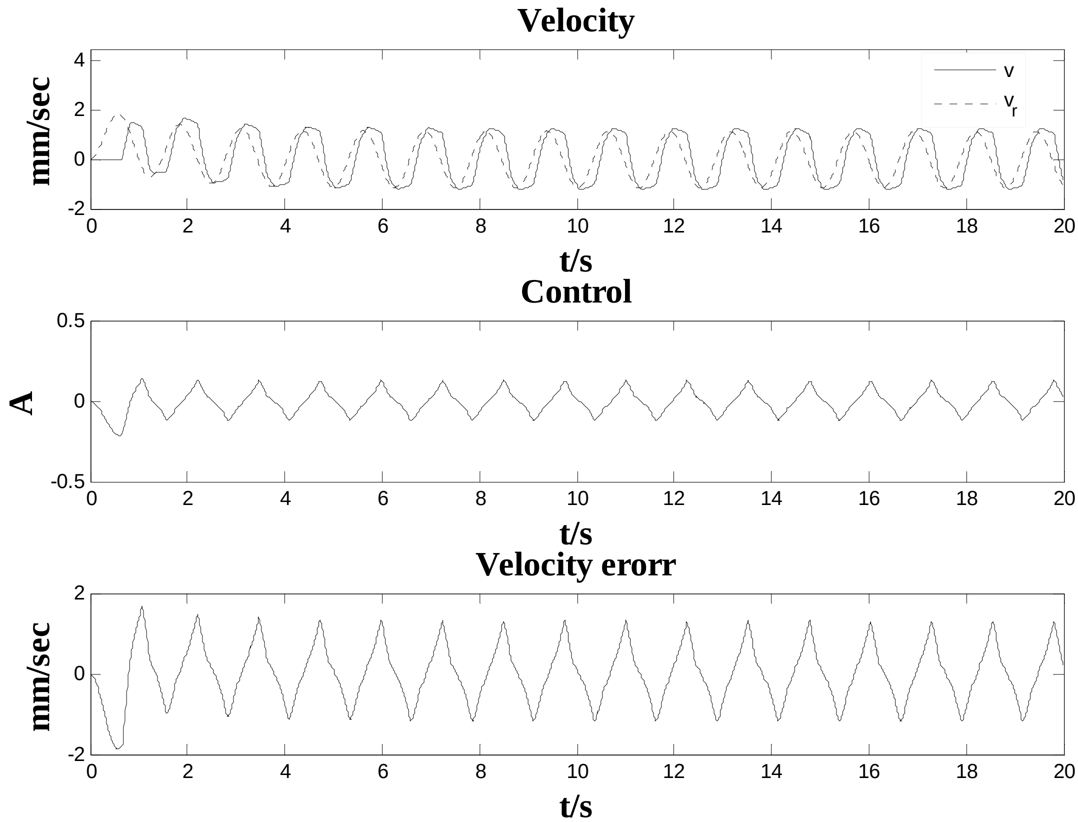

Figure 6. Experimental results for $r = 15\sin(5t)$ mm/s

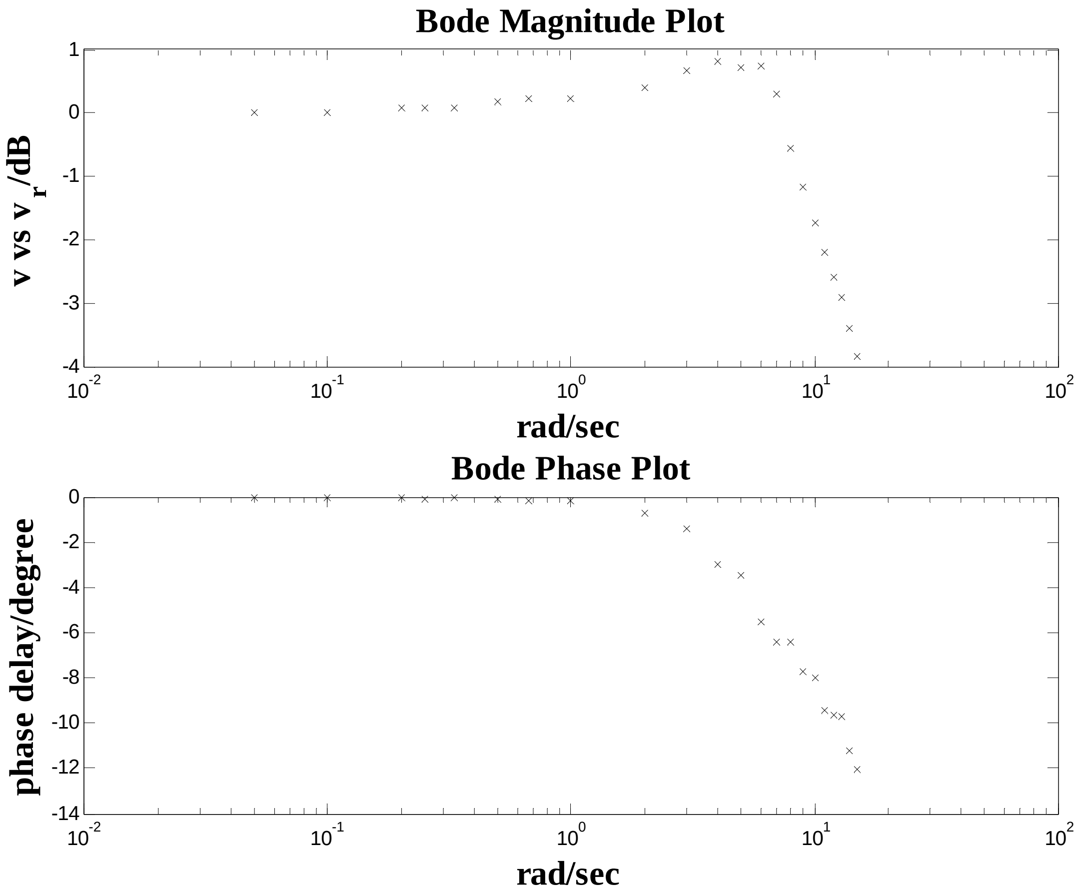

Figure 7. Closed-loop frequency response

6. SUMMARY AND CONCLUSIONS

A new robust adaptive control has been derived in this paper. Bounds on transient response error shave been derived. A novel sliding mode term has been added to result in asymptotic stability of the errors instead of the usual upper bounded derivations. The transient response of the new controller is guaranteed and it reduces oscillatory behavior while learning and cancelling out the uncertainties. The controller is designed and applied in a Caterpillar Electro-Hydraulic Test Bench, for velocity tracking objective. Experimental results show that precise tracking of the reference model system is realized with adaptive controller for different cases. The potential of this technique is with modified state observer structure, it is possible to choose large adaptive gain, suppress high frequency oscillations, and achieve asymptotic stability at the same time.

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SECTION

2. CONCLUSIONS

An new neural network controller based on modified predictor structure is developed in this thesis. The new method combines model reference adaptive control with soft switching sliding control, and prevents high frequency oscillation in high adaption gain. The new method provides asymptotic tracking, and is better in both transient and steady stage performance comparing to traditional MRAC, and it is shown through simulation and experimental results. In paper 1, two theoretical models are considered, one is robot-arm motion control, and the other is ship steering control; In paper 2, a missile autopilot control design problem is studied, with comparison to MRAC, it is show that the method reduces oscillation and tracking error at the same time; In paper 3, the method is applied in a Caterpillar Electro-Hydraulic test bench, in order to precisely control velocity of a piston, and satisfying experimental results are provided.

VITA

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