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Reliability-based design with system reliability and design improvement

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RELIABILITY-BASED DESIGN WITH

SYSTEM RELIABILITY AND DESIGN IMPROVEMENT

by

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ABSTRACT

This thesis focuses on developing a methodology for accurately estimating series system probability of failure. Existing methods for series system based design optimization are not that accurate because they assign reliability to each failure mode; as a result complete system reliability goes down. According to method proposed in this work, the user will assign required system reliability at the start and then optimizer will apportion reliability to every failure mode in order to meet required system reliability level. Detlevson second order upper bounds are used to estimate system probability of failure. Several examples have been shown to verify the results obtained.

Another work done for this thesis is coming up with a new and innovative way to achieve feasible design early. It has been noticed that for practical applications engineers don't have time and resources to achieve optimal design. So to reduce computational effort and achieve reliable design a methodology is described. Several examples were used to verify the results obtained from our method.

Reducing computational expense is of prime importance in the field of reliability-based design. It has been shown that by using our proposed method it is possible to get feasible design early. It may not be the optimal design but it will be feasible and will satisfy reliability requirement.

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1. INTRODUCTION

1.1 BACKGROUND

The objective of this research is to develop a methodology for reliability-based design optimization for series systems and to develop a new strategy for reducing computational expenses. In today's competitive market engineers face new challenges due to ever increasing complexity in design and application of new technology. With the demand of quality and reliability going northwards it has become imperative to improve existing methods for designing new products.

Reliability can be defined as the probability that a product or system performs its intended function under stipulated time for a specific period of time. According to IEEE reliability is defined as "the ability of a system or component to perform its required functions under stated conditions for a specified period of time [1]". Another web definition for reliability in particular for mechanical systems is "Mechanical reliability is the probability that a spare, item, or unit will perform its prescribed duty without failure for a given time when operated correctly in a specified environment [2]". To deal with this ever increasing demand for reliability and quality reaserchers have to look beyond conventional design methods and one of the unconventional methods is reliability-based design which uses statistics and probabilistic engineering for coming up with new ways for design.

In this work the focus is on design optimization. Optimization of the design is essential to maximize benefits and make effective use of resources. The natural goal of a design engineer is to combine reliability considerations and optimization in a single framework for product or system design. Traditionally, design has been based on engineering judgment and experience, usually resulting in conservative designs. Advances in computational methods, resources, and new developments in reliability and optimization theories have opened new possibilities for reliability-based design optimization.

RBDO can provide optimum designs in the presence of uncertainty. Uncertainty is everywhere during a product design process. It is a major challenge to deal with uncertainty; uncertainty can be due to lack of knowledge or variations in the manufacturing process. Statistical methods can be applied for analysis of uncertainty in design. With analysis of uncertainty engineers will be able to manage and reduce the effects of uncertainty by making appropriate decisions, for example, determining optimal design variables during the design process. Through the uncertainty analysis on an existing design, engineers evaluate if the design satisfies all the design requirements in the presence of uncertainty. Specifically, engineers can know whether the design is reliable and safe. If the design does not satisfy reliability requirement, the uncertainty analysis will provide engineers with a useful guidance to improving the design. Therefore, uncertainty analysis is an important component in design under uncertainty.

In deterministic design it is assumed that there is no uncertainty in the design variables and modeling parameters. Therefore, there is no variability in the outputs. However, there exists inherent input and parameter variation that results in output variation. Deterministic optimization typically yields optimal designs that are pushed to the limits of design constraint boundaries resulting in optimal designs that are usually unreliable. Input variation is fully accounted for in reliability-based design optimization (RBDO), which can be a powerful tool in design under uncertainty.

Probabilistic analysis is the art of formulating a mathematical model within which one can ask and get answer to the question: "What is the probability that a structure behaves in a specified way when given that one or more of its material properties or geometric dimensions and properties are of a random nature?"[1] Reliability and optimization are two key elements of engineering design of structural and mechanical systems.

Reliability-based design optimization (RBDO) is a mathematical framework for solving optimization problems in the presence of uncertainty, typically manifested by probabilistic description given in objective or constraint functions. With new models and formulations appearing almost every year, RBDO has emerged as a viable tool for engineering design under uncertainty. Combined with optimization, the model-based design enables engineers to identify design options effectively and automatically. The conventional deterministic optimization design ignores the fact that, in real life, there are many sources of uncertainty, such as manufacturing variations and that leads to quality loss and low reliability. For this reason, incorporating uncertainty in design has received increasing attention and applications, such as those found in automotive, civil, mechanical, and aerospace engineering.

The other reason of uncertainty consideration is that engineering systems have become increasingly sophisticated and that the occurrence of failure events may lead to higher catastrophic consequences. The expectation of higher reliability and lower environmental impact has become imperative to avoid catastrophic loss. Reliability-based design (RBD) is one of the unconventional design methods to meet this expectation.

RBD seeks a design that has a probability of failure less than some acceptable value and, therefore, ensures that failure events be extremely unlikely.

Optimization is a design tool that assists designers automatically to identify the optimal design from a number of possible options, or from an infinite set of options. Optimization design is increasingly applied in industry since it provides engineers a cheap and flexible means to identify optimal designs before physical deployment. In engineering design the focus is to optimize performances of the product while meet all the design requirements.

Design is an iterative process. Designer's intuition, experience are the always needed to come up with a design in almost all fields. Engineers strive to design the best systems. In general a good design means reliable, cost effective and durable systems. Increasingly the modern engineering community is employing optimization as a tool for design. Optimization is used to find optimal designs characterized by lower cost while satisfying performance requirements. Typical engineering examples include minimizing the weight of a cantilever beam while satisfying constraint on maximum stress and allowable deflection, and so on.

The basic paradigm in design optimization is to find a set of design variables that optimizes an objective function while satisfying the performance constraints .The design feasibility in reliability based design is formulated probabilistically such that the probability of the constraint satisfaction (reliability) exceeds the desired limit. The reliability assessment for probabilistic constraints often involves an iterative procedure; traditional optimization designs are pushed to the limits of system failure boundaries, leaving very little or no room for accommodating uncertainties in engineering design. It is therefore important to incorporate uncertainty in engineering design optimization and develop computational techniques that enable engineers to make efficient and reliable decisions. In recent years many probabilistic design methods have been developed and applied in engineering design. The major emphasis in reliability-based design is to achieve high reliability of a design by ensuring the probabilistic constraint satisfaction at desired levels of reliability.

Uncertainty analysis is an important task of a probabilistic design, through which it is possible to understand how much the impact of the uncertainty associated with the system input is on the system output by identifying the probabilistic characteristics of system output. Then perform synthesis (optimization) under uncertainty to achieve design objective by managing and mitigating the effects of uncertainty on system .In spite of the benefits of probabilistic design, one of the most challenging issues for implementing probabilistic design is associated with the intensive computational demand of uncertainty analysis. Design requirements can be transformed into mathematical constraints. The designer is faced with the challenge of coming up with design artifacts which are consistent with design constraints.

Any design set which is in feasible region is acceptable. Competitive pressure continues to force product improvement demands on engineering and product departments. An improved design is the one that continues to comply with the same set requirements but improves the merit function. Being able to optimize a product for desired performance output in the pre-design phase means more time for product innovation and less time to market.

In reliability-based design optimization there is a trade-off between obtaining higher reliability and lowering cost. The first step in RBDO is to characterize important uncertain variables and failure modes. Statistical models are used to find the probability distribution of random variables. In RBDO formulation critical failure modes in deterministic optimization are replaced with constraints on probabilities of failure corresponding to each of the failure mode or with a single constraint on the system probability of failure. Some of the techniques used for reliability analysis are the First Order Reliability Method (FORM) [3], Second Order Reliability Method (SORM) [4].

Traditionally researchers have formulated RBDO as a nested optimization problem also known as double-loop method. The computational expense increases with an increase in random variables and failure modes. To reduce the computational effort, researchers have developed sequential RBDO methods [5]. In these methods the cycles of deterministic optimization and reliability analysis are decoupled and the procedure is repeated until convergence is achieved.

1.2 RESEARCH OBJECTIVES

This thesis investigates and develops new formulations and methodologies for reliability based design optimization (RBDO). The motivation for our work comes from the fact that current work lacks practicality. Engineers working in industry may not have time and resources to go for optimum design. A significant burden is feasibility check of probabilistic constraints. In this work, a new methodology for RBD has been proposed to reduce computational effort for achieving a feasible design using multiple levels of reliability without compromising on numerical accuracy. The main focus is on developing methodologies that are computational less expensive. Traditionally double-loop strategies have been used for estimating series systems probability of failure that increases computational effort. An efficient formulation RBDO for series systems approximating system probability of failure using single-loop algorithm has been proposed in this thesis. Without optimization the two objectives are described below.

1.3 RBDO FOR SERIES SYSTEMS

A mechanical system consists of a number of components; hence there can be multiple failure modes. Instead of taking into account only component failure modes, system reliability is concerned with both component level and system level reliability. In this work only series based systems are considered. In a series based system if one of the components fails then the whole system fails. Second order bounds proposed by Ditlevsen [6] are used to estimate system probability of failure. The objective is to develop an efficient approach to solve design optimization problems that involve series systems.

1.4 DESIGN IMPROVEMENT FOR RBD

Engineers may not have time and resources to search for a truly optimum design. The second objective of this thesis is to identify an improved and feasible design quickly without expensive optimization. The motivation for our work is to come up with an innovative formulation for reducing computational effort without compromising with reliability requirement. In this work multiple levels of reliability have been used to move as quickly as possible to the feasible design solution. Previous work in the field of RBD has shown that after deterministic optimization the reliability is lower than the required reliability. As known that RBD is an iterative process, hence computational expense increases as lot of iterations are used up to achieve feasible design as reliability achieved after deterministic optimization is lower. To reduce computational expense and achieve feasible design a methodology has been proposed in this work. Multiple reliability levels have been used for achieving feasible design quickly. Start deterministic optimization with higher than required reliability level. Use the information obtained from higher than required reliability deterministic optimization for reliability analysis at required reliability level. This way it is possible to push the design solution quickly towards feasible design space.

The design solution obtained may not be a true optimum design but it will be feasible with reliability requirement satisfied. It is observed that a lot of iterations are used up during the end part of optimization without any significant improvement in design solution. To counter this issue the convergence criteria is set as whenever design solution enters feasible region then stop.

1.5 OVERVIEW OF THE THESIS

This thesis is organized as follows: Section 2 consists of the overview of previous work in system reliability design optimization and methodologies used for reducing computational effort. The First Order Reliability Method (FORM), inverse reliability method are also included in this section.

Section 3 gives detailed description of series system reliability design optimization; examples to verify results are also included in this section. A flowchart is developed to effectively illustrate the procedure for our method.

Section 4 deals with work done for reducing computational effort and achieving a feasible solution. Detailed description is given about the mathematical model, and examples are used to show the efficiency of the model.

Section 5 is conclusion section and it includes the summary of research work and future work.

2. BACKGROUND AND PREVOIUS WORK

2.1. INTRODUCTION

In this section reliability-based design and inverse reliability method are discussed. Existing methods are also described. Previous work done in developing reliability-based designs and the work initiatives taken from them for our research are documented.

2.2. INVERSE RELIABILITY ANALYSIS

Inverse reliability methods are becoming popular nowadays. The evaluation of the percentile value of the performance function is an inverse reliability problem. The mathematical expression for inverse reliability formulation can be expressed as below

$$P(g(\mathbf{X}) < g^r) = R \tag{1}$$

It states that the probability that the performance function $g(\mathbf{X})$ is less than the r-percentile value g^r is equation to R . $\mathbf{X}=\{X_1, X_2...X_n\}$ is a vector of independent random variables. If the probability p is known, then the reliability index β is given by

$$\beta = \left| \Phi^{-1}(p) \right| \tag{2}$$

where Φ is the standard normal cumulative distribution function. The space that contains the original random variables $\mathbf{X}=\{X_1, X_2...X_n\}$ is called X-space. To make the shape of integrand $f_{\mathbf{X}}(\mathbf{x})$ regular, all the random variables $\mathbf{X}=\{X_1, X_2...X_n\}$ are transformed to a standard normal space where the transformed random variables $\mathbf{U}=\{U_1,U_2...U_n\}$ follow the standard normal distribution. The transformed space is called as U-space. The transformation from **X** to **U** space is based on the condition that the *cdf's* remain the same after transformation. Reliability index β is a distance and is always non-negative. The MPP is a tangent point of the circle with radius β and the performance function $g'(X)=g(X)-g^r=0$ and also a point that has the minimum value of performance function $g(X)=0$ on circle. Figure 2.1 shows the feasible region. The MPP is the shortest distance between origin and performance function curve $g(\mathbf{U})=0$

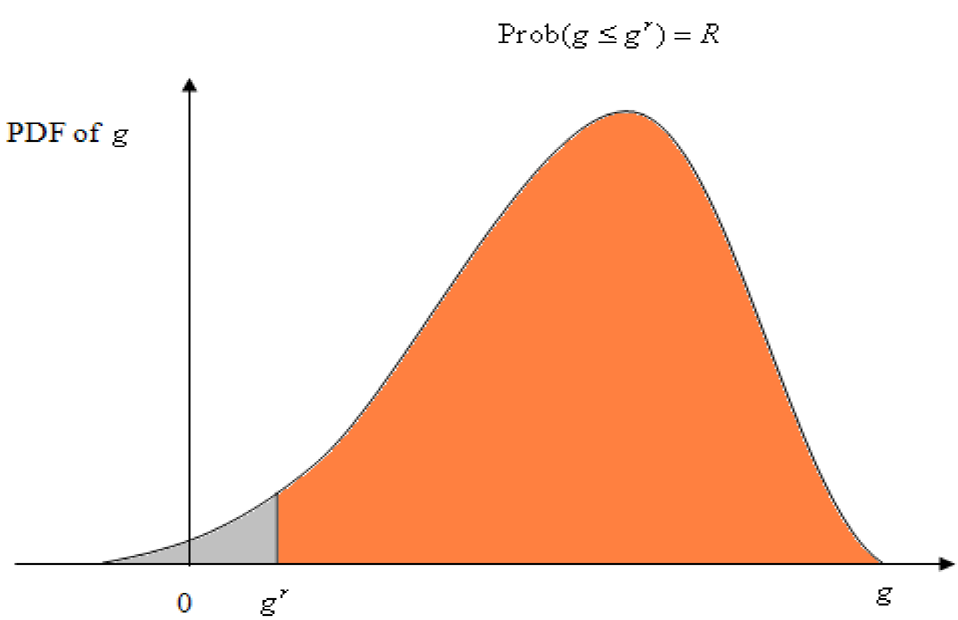

Figure 2.1 Feasible region

The mathematical model for MPP search can be stated as find the MPP that minimizes the performance function $g(\mathbf{U})$ such that MPP remains on the surface of β circle

$$\min_{\mathbf{u}} \; g(\mathbf{u}) \qquad (3)$$

$$\text{Subject to: } \beta = \left\| \Phi^{-1}(p) \right\|$$

2.3. RELIABILITY BASED DESIGN OPTIMIZATION (RBDO)

In a deterministic design optimization the design is pushed to the limits of the design constraints. The resulting design maybe subjected to a high chance of failure due to no consideration given to uncertainties. Uncertainties are present everywhere and hence need to be taken into account. Reliability-based design is a methodology which takes into account this problem. Reliability-based design optimization deals with obtaining a reliable design. There is a trade-off between reliability and low cost. The important step in RBDO is to quantify various random variables and failure modes.

2.4. PREVIOUS WORK FOR DESIGN IMPROVEMENT

2.4.1 Enriched Performance Measure Approach. The enriched performance measure approach for reliability-based design optimization was proposed by Byeng D. Youn, Kyung K. Choi and Liu Du [7] to improve numerical efficiency by reducing calculations in reliability-based design optimization. A new enhanced hybrid mean value method is described in their work. As known that deterministic design is not reliable design but it is quite close to the feasible design. The idea is to efficiently move the design to deterministic optimum design and then move it back to feasible region to obtain optimum design which satisfies reliability requirements. According to the author the numerical efficiency can be improved by an efficient feasibility check for probabilistic constraints. The authors have used a new concept called as design closeness in their work, according to them a lot of iterations can be saved by utilizing information from previous design obtained. $\mathbf{X}$ is the vector of random design variables. The design closeness is defined as

$$\Delta d^{(k)} = \left\| \sqrt{\sum (\mathbf{X})^{-1}} [d^{(k)} - d^{(k-1)} \right\|_{L_2} \leq \varepsilon_d \quad (4)$$

$$\Delta x^{*(k-1)} = \left\| \sqrt{\sum (\mathbf{X})^{-1}} [x^{*(k-1)} - x^{*(k-2)} \right\|_{L_2} \leq \varepsilon_d \quad (5)$$

Where $d^{(k)}$ and $d^{(k-1)}$ are the designs at $k-$th and $(k-1)$th iterations and $x^{*(k)}$ and $x^{*(k-1)}$ are the Most Probable Points (MPP) at the $(k-1)$th and $(k-2)$th iterations. The design closeness leads to MPP closeness hence early convergence can be achieved. Inverse reliability analysis method is used for the Most Probable Point (MPP) estimation.

2.4.2. Single Loop Approach for Reliability-based Design. A single loop method for reliability-based design [8] has been proposed in which double loop optimization is collapsed into an equivalent single loop optimization problem by imposing the Karush-Kuhn-Tucker optimality conditions of the reliability loop as equivalent deterministic equality constraints of the design optimization loop. Therefore it eliminates the repeated MPP search for reliability assessment by converting the probabilistic optimization problem into an equivalent deterministic optimization problem.

The single loop approach is computationally efficient and accurate, and the number of required function evaluations is comparable to deterministic optimization. The reliability- based optimization problem is expressed as follows

$$\min_{\mathbf{d},\mu_{\mathbf{X}}} f(\mathbf{d}, \boldsymbol{\mu}_{\mathbf{X}}, \boldsymbol{\mu}_{\mathbf{P}})$$

$$\text{Subject to: } g^{R}_{i}(\mathbf{d}, \mathbf{X}, \mathbf{P}) \geq 0, i = 1,2,...n \qquad (8)$$

$$\mathbf{d}^{L} \leq \mathbf{d} \leq \mathbf{d}^{U}, \boldsymbol{\mu}_{\mathbf{X}}^{L} \leq \boldsymbol{\mu}_{\mathbf{X}} \leq \boldsymbol{\mu}_{\mathbf{X}}^{U}$$

where $\mathbf{d}$ is the vector of deterministic design variables, **X** is the vector of random design variables and **P** is the vector of random design parameters, n is the number of constraints and f( ) is the objective function. The vectors **X** and **P** are evaluated at the MPP and the mean objective function is minimized subject to constraints which are evaluated in X-space. It is discussed in this method that it uses KKT optimality conditions of the reliability loops as equality constraints of the design optimization loop in order to relate the $\mathbf{d}, \boldsymbol{\mu}_{\mathbf{X}}, \boldsymbol{\mu}_{\mathbf{P}}$ and $\mathbf{d}, \mathbf{X}, \mathbf{P}$ vectors, where $\boldsymbol{\mu}_{\mathbf{X}}$ is the vector of mean of random design variables and $\boldsymbol{\mu}_{\mathbf{P}}$ is the vector or random design parameters .

The initial point and target reliability index is given in the start. The initial point is taken as the mean of variables and at this point the normalized gradient vector is calculated. If the design vector is changed as compared to previous iteration then the normalized gradient vector is updated to calculate value of design vectors which are then used for constraint evaluation and if design vector is not updated then current gradient vector is used to calculate design vectors for each constraint which are then used to evaluate the constraints.

If non-normal variables are encountered then it is necessary to transform the non-normal distribution variables into normal distribution variables. The main advantage which comes out of single loop approach as compared to double loop is that computation cost reduction for repeated reliability loops. It solves an equivalent single-loop deterministic optimization problem as compared to performing nested design optimization and reliability loops. If the efficiency of single loop approach is compared with decoupling approach then single loop is more efficient as it doesn't solve successive deterministic and reliability optimization problems. As discussed in this approach the one more advantage this approach has is that it doesn't update the constraint gradients unnecessarily and the gradient are updated only if the design mean values have changed and hence improving efficiency.

2.4.3. Sequential Optimization and Reliability Assessment. It is an efficient probabilistic design approach for design optimizations that involve probabilistic constraints. In this approach a single loop method [9] is developed to decouple uncertainty analysis and optimization analysis. It involves an efficient inverse MPP search method. Moving on to algorithm for this method there is no information about the MPP for the first cycle to counter this problem they are set as mean of random design variables and random parameters. The optimization problem according to sequential optimization and reliability assessment is expressed as follows:

$$\text{Minimize: } f(\mathbf{d}, \boldsymbol{\mu}_{\mathbf{X}}) \quad (9)$$

$$DV = \{\mathbf{d}, \boldsymbol{\mu}_{\mathbf{X}}\} \quad (10)$$

$$\text{Subject to: } g_i(\mathbf{d}, \mathbf{X}) \geq 0, i = 1, 2, \ldots m$$

where **d** is the vector of deterministic design variables and **X** vector of random variables , $\boldsymbol{\mu}_{\mathbf{X}}$ is the vector of mean of random design variables and g( ) is the constraint function. Steps taken to improve the efficiency are firstly to reduce the computational effort the MPP obtained from last cycle is taken as the starting point for the MPP search in next cycle as MPP's obtained from two consecutive cycles are close. Secondly the starting point of the optimization of one cycle is taken as the optimum point of the previous cycle. Thirdly if it is noticed that after one cycle of optimization the design variables have not changed significantly as compared to previous cycle then it is not wise to search for the MPP again for the current probabilistic constraint in the following probabilistic assessment. The convergence criterion for SORA [9] is if the objective function between two consecutive cycles are infinitesimally small or all the reliability requirements are satisfied.

SORA is different from the double loop method as it employs the strategy of sequential single loops for optimization and reliability assessment, which separates the reliability assessment from the optimization loop. Percentile formulation for the probabilistic constraints is used in SORA instead of the reliability formulation to avoid evaluating the actual reliabilities. Major advantage with using sequential cycles is that it reduces the total number of reliability analyses and hence reduces computational cost. Due to these measures a series of equivalent deterministic optimization problems is formulated which helps in identifying optimum solution quickly. As discussed earlier that probabilistic constraints are evaluated at their MPP hence there is no need for repeated reliability assessment within each optimization cycle. It leads to improved efficiency as compared to other methods.

2.5 PREVIOUS WORK FOR SYSTEM RELIBILITY

2.5.1 A Single Loop Approach for System Reliability-Based Design. A new methodology for series system based reliability optimization has been proposed by Zissimos P. Mourelatos, jinghong Liang and Efstratios Nikolaidis [10]. The basic idea is to provide system reliability and the optimizer will apportion reliability for each constraint accordingly to satisfy system reliability. The MPP's are approximated using Krush-Kuhn-Tucker optimality conditions at each iteration. The reliability index for each failure mode is included in design variable set and this set is updated after every iteration. An active set strategy is used to identify critical failure modes and the failure probability for non-critical failure modes is assumed to be zero.

2.5.2 Reliability-Based Optimal Design of Series Structural Systems. Sorensen, J., and Enevoldsen, I. [11] developed a methodology for approximating series system reliability. The reliability index of failure mode of each component of series system will be replaced by a function that denotes the minimum of the corresponding limit-state function. Two types of problems have been addressed. First is minimizing the cost of the system depending upon reliability requirement and second one is maximizing reliability subject to cost and design constraints. It involves reformulation of above stated problems into semi-infinite deterministic optimization problem that are solved in conjunction with reliability calculations. Semi-infinite optimization algorithm and reliability method to calculate system reliability can be used independently of each other. The deterministic optimization and reliability assessment is completely decoupled.

2.5.3. Bayesian Network for System Reliability Assessment. Bayesian methods have not been used to estimate failure probability in mechanical and civil systems. A new methodology has been proposed for structural system reliability assessment [12]. Multiple failure events and interactions between failure modes are included in the Bayesian network. Branch and bound method has been used to incorporate only critical failure modes for calculating system failure probability. The use of Bayesian networks with branch and bound method improves the efficiency and it has been shown that this methodology can be applied to large structural systems.

To incorporate the interaction among various failure modes all the input random variables are used as root nodes in the Bayesian network. To account for multiple failure modes, a new Bayesian network has been constructed which accounts for conditional probability of failure modes. It is important to include the effect of probability of failure of one mode on the whole system. Joint PDF will be used for correlated failure modes. The Bayesian network advantage is that it allows backward propagation to update the probabilistic information of any node. When new information is available on any node then the failure probability of whole system can be updated.

3. RELIABILITY-BASED DESIGN FOR SERIES SYSTEM

3.1 INTRODUCTION

The objective of this section is to discuss an innovative approach for accurately estimating series system reliability. The term system reliability refers to the reliability analysis of the overall engineering system which may fail under one or more multiple failure modes, as opposed to the term component reliability, which refers to a single failure mode. There are many components in an engineering system and hence there can be multiple failure modes. So instead of having a single failure mode as is the case in component reliability analysis, system reliability analysis takes into account both component level and system level estimates. This approach identifies the critical failure modes that contribute most to the overall system reliability. The design process will include both component level and system level reliability analysis. The results of system reliability analysis provides insight into which failure mode contributes more towards system reliability and what is the probability of failure with current input variables.

The objective of our work is to develop a robust, flexible approach for solving reliability based optimal design problems for series based systems. The problems are formulated to minimizing the cost of the design subject to system reliability constraints. It is not possible to solve this problem exactly and only an approximation can be made.

Several approaches [12, 13, 14] have been developed to solve the reliability-based optimization for series systems. Most of the methods employ reliability analysis by the FORM (first order reliability method) [6] as an integrated part of the optimization cycle.

A decoupled approach to reach optimal design within specified reliability requirements has been proposed in this thesis.

The design of mechanical systems face some system reliability issues such as the effect of low reliability of one component on the response of another component , the effect of low reliability of one component on the operating limit of the whole system and the effect of low reliability of one component on the cost of the system. Uncertainty and optimization are also major concerns in a design. Uncertainty from randomness in load, materials must be considered in design to ensure safety and reliability. RBDO provides safer and more efficient designs than deterministic design optimization because it explicitly accounts for uncertainty using probability theory. As a result RBDO is being used as an effective design tool for automotive, aerospace and engineering structures.

Reliability affects system design, specifications, and unreliability of a mechanical system can be very costly and catastrophic. Mechanical systems are assembly of components made by different manufacturers; each maker has its own design criteria and tolerances, hence it is very important to set up reliability level for the whole system considering the reliability of each and every component.

Recently some series system reliability-based design optimization methods have been proposed. In our work a single-loop approach for series system RBDO has been proposed which allows for an optimal apportionment of the reliability of a series system among its failure modes (constraints) .The proposed algorithm ensures overall system reliability rather than an arbitrary reliability for each failure mode as is the case with component RBDO methods.

3.2 STRATEGY

The system reliability analysis consists of two basic steps. The first one is formulating the problem in a probabilistic context and the second step is using computational methods to calculate component and system reliability. Identification of relevant random variables is very important. Critical response parameters and their limits should be identified. In real engineering applications for complicated systems with multiple components or multiple failure mechanisms, system reliability needs to be evaluated. In our work reliability bounds [15] are used for the system probability of failure. A system might contain a number of components and each component will have different reliability. In a series system if one of the component fails then the whole system fails hence it is very important to measure the probability of failure of each component; Once it is obtained, system probability of failure is computed.

In our work the system reliability is provided and limit state constraints are specified. The optimizer will apportion the reliability of each constraint to satisfy the overall system reliability and hence an optimal solution within the specified limits will be achieved. Only series system is used in this work. Sequential cycles of reliability analysis and deterministic optimization are used. It is noted that component reliability is high but when all the components are considered then the system reliability goes down for series system, This can lead to high chance of failure. So to improve system reliability a methodology to accurately estimate system probability of failure has been proposed. The design process begins with the definition of component and system reliabilities to be achieved. The reliability design process will then iterate until those requirements are met.

In a series systems reliability approach the optimizer determines the optimal values of the maximum allowable failure probabilities of all failure modes. The user specifies a system reliability level and the optimizer allocates optimally the specified system reliability among its failure modes. A target reliability index β is needed for each constraint (failure mode). However, the optimizer must determine the failure probability of each failure mode by apportioning the system probability of failure among all failure modes. A natural way to do this is to include all β **(**reliability index**)** into the design variable set. The active constraint set is updated at each iteration during the optimization process. The proposed algorithm ensures overall system reliability rather than using an arbitrary reliability for each failure mode, as is the case with the conventional methods. Thus, the user can directly control system reliability by specifying an acceptable system reliability instead of deciding arbitrarily on a minimum reliability level for each failure mode.

3.3 PROCEDURE

The system probability of failure is computed by the union of individual failure events [15]. It is very difficult to compute the probability of the union of failure events after individual failure probabilities are obtained. Let's suppose there are three failure events A, B & C ,then the probability of the union of three failure events can be expressed as

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(AB) - P(BC) - P(AC) + P(ABC) \quad (1)$$

The joint probabilities of events A & B , B & C and A & C are respectively P (AB) ,P (BC) and P (AC). From previous it is known that it is very hard to compute the joint probability of more than two events at one time, Several approximation bounds have been proposed such as Cornell's first-order bounds [6]

$$\max_{1\le i\le n} P(E_i) \le P(\bigcup_{i=1}^{n} E_i) \le \sum_{i=1}^{n} P(E_i) \quad (2)$$

$\mathrm{P}(E_{\mathrm{i}})$ is the probability of failure for the i^{th} failure mode and *n* is the total number of failure modes. To accurately estimate the probability of failure, Ditlevson [6] proposed second order bounds

$$P_F^{all} \le \sum_{i=1}^{k} P_i - \sum_{i=1}^{k} \max P_{ij} \quad (3)$$

where P_i is the probability for the $i-$th event and P_{ij} is the joint probability of the $i-$th and $j-$th events. Ranking the individual events in order of decreasing probability will give tightest bounds according to the above stated bounds given by Ditlevsen [6].

A target reliability index $\beta_i = \Phi^{-1}(P_{F_i})$ is needed for each constraint (failure mode). The reliabilities for failure modes has been included in design variable set however the optimizer must determine the failure probability of each failure mode by apportioning the system probability of failure. Design variables **d** are initialized and the distributions of random parameters and variables. Upper and lower limits are assigned to the reliability index values for constraints .In each iteration, the optimizer determines each P_{F_i} and the corresponding target reliability index $\beta_i = \Phi^{-1}(P_{F_i})$ is calculated, simultaneously it should also be made sure that $P_F^{all} \le \sum_{i=1}^{k} P_i - \sum_{i=1}^{k} \max P_{ij}$ where P_{ij} is the joint probability between $i-$th and $j-$th mode ,i.e. the system probability of failure should be less than a specified probability of failure P_{sys} .

In this work sequential cycles of deterministic optimization and reliability analysis are used. In the first cycle solve the deterministic optimization model which is given below

$$\begin{aligned}
&\text{min: } f(\mathbf{d}, \boldsymbol{\mu}_{\mathbf{X},} \boldsymbol{\mu}_{\mathbf{P}}) \\
&\text{Design Variable: } DV = \{\mathbf{d}, \boldsymbol{\mu}_{\mathbf{X}}, \boldsymbol{\mu}_{\mathbf{P}}, \boldsymbol{\beta}_i\} \\
&\text{Subject to: } g_i(\mathbf{d}, \mathbf{X}, \mathbf{P}) \ge 0, i = 1, 2, \ldots m
\end{aligned} \tag{4}$$

where $\mathbf{d}$ is the vector of random design variables, $\boldsymbol{\mu}_{\mathbf{X}}$ is the vector of mean of design variables , $\boldsymbol{\mu}_{\mathbf{P}}$ is the vector of mean of random parameters and $g_i(\mathbf{d}, \mathbf{X}, \mathbf{P})$ is the limit state function of the $i-$th constraint. The objective function is evaluated at the $(\mathbf{d}, \boldsymbol{\mu}_{\mathbf{X},} \boldsymbol{\mu}_{\mathbf{P}})$ mean point and the constraints are evaluated at **d, X, P.**

The individual failure probabilities are estimated through a first order approximation to the limit state in a space that has been obtained through an approximate equivalent normal transformation of the basic random variables. The mathematical representation of the Ditlevsen [6] bounds shown above is a bound on the union operation only and it is not a true bound on the system reliability. Bivariate normal integral [16] is used to calculate the joint failure probability of two limit states.

$$\Phi(-\beta_i, -\beta_j; \rho_{ij}) = \frac{1}{2\pi(1-\rho^2)} \int_{-\infty}^{\beta_i} \int_{-\infty}^{\beta_j} \varphi(x, y, \rho_{ij})\, dxdy \tag{5}$$

where $\Phi(,;\rho)$ is the bivariate normal cumulative distribution function (CDF) and $\varphi(,;\rho)$ is the probability density function (PDF) of a bivariate normal vector with zero means, unit variances, and a correlation coefficient ρ given by

$$\Phi(-\beta_i, -\beta_j; \rho) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left[-\frac{1}{2} \frac{\beta_1^2 + \beta_2^2 - 2\rho\beta_1\beta_2}{1-\rho^2} \right] \tag{6}$$

The joint probability of two limit state functions needs to be estimated in order to use these bounds. As shown in Figure 3.1 The angle between the two limit state functions provide the information about the correlation between the two failure modes. The mathematical representation of the correlation coefficient [6] is

$$\rho_{ij} = \sum_{r=1}^{m} \alpha_{ir} \alpha_{jr} = \cos v_{ij} \tag{7}$$

where α_i is the normalized gradient of the *i*-th constraint, and α_j is the normalized gradient of the *j*-th constraint. The initial point $\mathbf{d}^0, \mathbf{X}^0, \mathbf{P}^0$ that is needed to evaluate the constraints is taken equal to $(\mathbf{d}, \boldsymbol{\mu}_\mathbf{X}^\mathbf{0}, \boldsymbol{\mu}_\mathbf{P}^\mathbf{0})$, at this point the initial normalized gradient vector α for the i^{th} constraint is taken equal to

$$\alpha_i^0 = \frac{\nabla g_i(\mathbf{d}^0, \mathbf{X}^0, \mathbf{P}^0)}{\left\| \nabla g_i(\mathbf{d}^0, \mathbf{X}^0, \mathbf{P}^0) \right\|} \tag{8}$$

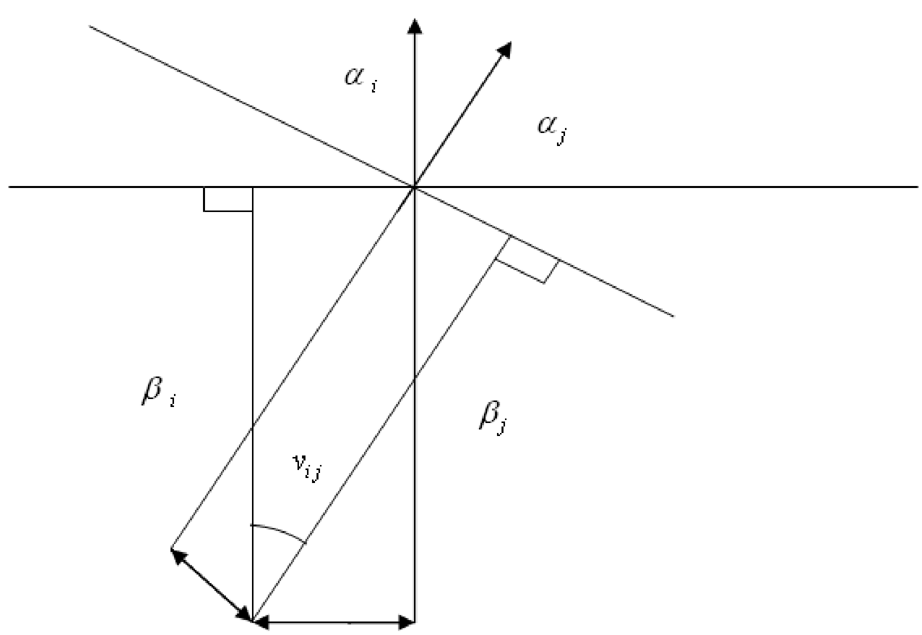

Figure 3.1 Joint probability of two failure modes

Now there is enough information to use Ditlevsen [6] bounds. The convergence criterion is set as if the probability of failure obtained from second order bounds is greater than required system reliability then go back to deterministic optimization. The MPP information obtained in the last cycle is used to modify the probabilistic constraints. Then reliability analysis is done for the MPP's obtained from deterministic optimization. This whole cycle will continue until convergence is achieved.

3.4 FLOWCHART FOR SERIES SYSTEM RBDO

The flowchart for our work is shown in Figure 3.2 . First of all initialize the design variables and provide initial reliability level for probabilistic constraints. The idea is to include reliability levels for constraints in design set so that reliability for constraints can be controlled by optimizer to achieve the required reliability requirement. Figure 3.3 below shows flowchart for proposed work.

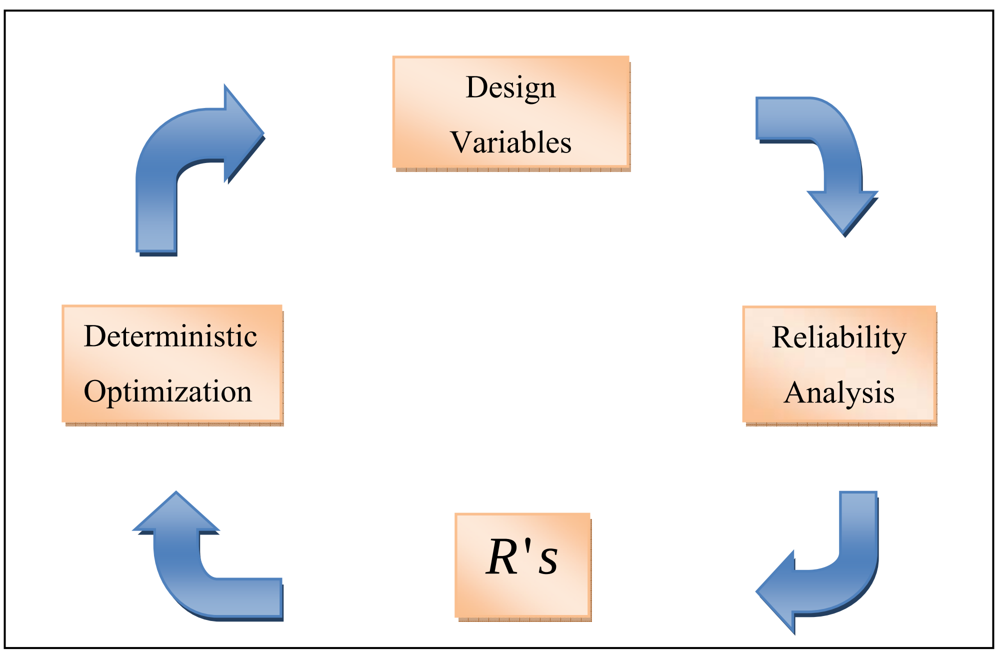

Figure 3.2 Basic framework for reliability-based design of series system

The deterministic optimization will provide design variable information to the reliability analysis and reliability analysis will provide MPP information to formulate probabilistic constraints for next cycle. The formulation of single-loop optimization problems is as follows

$$\min_{\mathbf{d},\boldsymbol{\mu}_{\mathbf{X}},\beta} f(\mathbf{d},\boldsymbol{\mu}_{\mathbf{X}},\boldsymbol{\mu}_{\mathbf{P}}) \tag{9}$$

$$\text{Subject to: } g(\mathbf{d},\mathbf{X},\mathbf{P}) \geq 0$$

In order to use ditlevsen [6] second order bounds approximation for system failure probability information from reliability analysis is needed. The failure probability of limit state constraints is obtained from reliability analysis. It is very difficult to calculate the joint failure probability of more than two failure events so ditlevsen [6] bounds are used to approximate failure probability if there are number of failure modes as shown in vehicle crash worthiness test [17] example later in the text. In this particular design problem there are nine constraints or failure modes and the system reliability is set. Once probability of failure for individual failure modes is calculated the correlation coefficient of failure modes needs to be evaluated. It is possible to calculate joint failure probability by using upper bivariate normal integral [16] for which α_i values should be known, which are obtained from reliability analysis. The first cycle is represented as K=1 in the flowchart. By solving bivariate normal integral [16] the joint failure probability of multiple failure modes is calculated. At this time Ditlevsen [6] second order bounds can be used to estimate system failure probability.

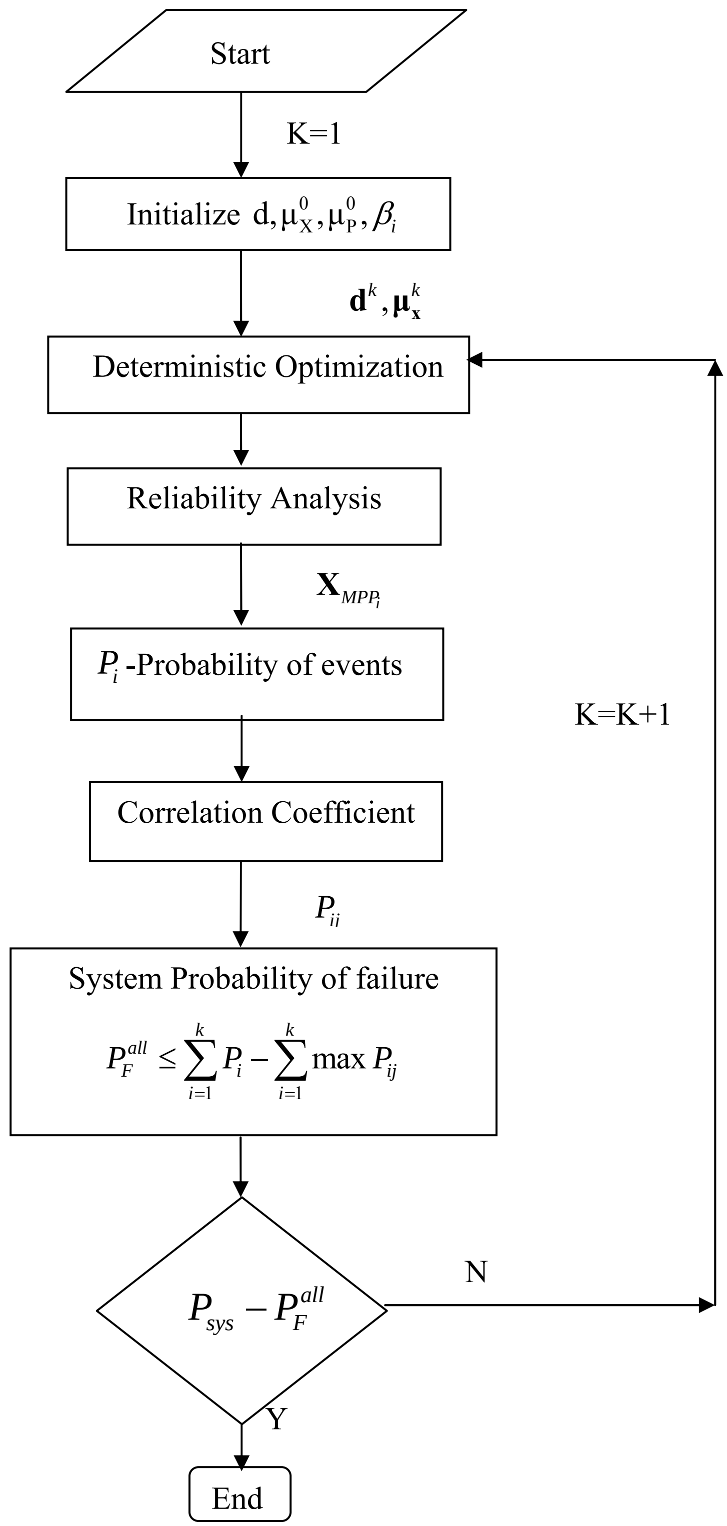

Figure 3.3 Flowchart for RBD of series system

This method is efficient for reliability based optimization of series based system. Some examples are shown later which will prove that optimum solution can be achieved for series based design without any major increase in computational expense as compared to other methods without compromising reliability requirement.

3.5 EXAMPLES

3.5.1 Cantilever Beam Example. A cantilever beam [9] is shown in the Figure 3.4 below; the objective is to minimize the cross-sectional area of the beam. The objective function can be written as

$$\text{Minimize: } (\mathbf{d}) = bh \tag{10}$$

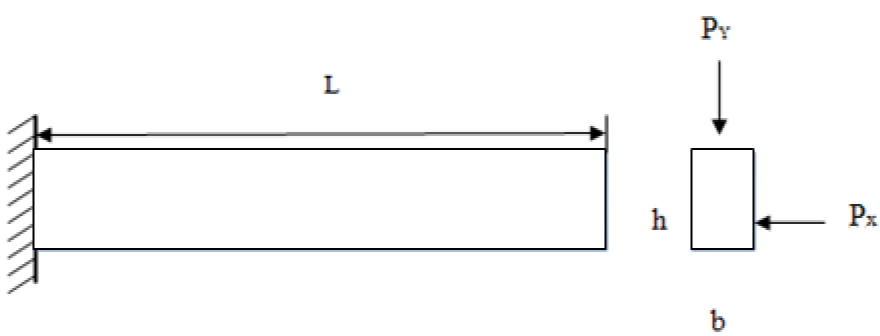

Figure 3.4 Cantilever beam example

There can be four possible failure modes for this rectangular beam under external load P the first constraint is tip displacement cannot exceed the allowable value D_0 , the second constraint is that stress should not exceed the maximum yield stress S . Maximum deformation should be less then T_0, last constraint is that shear stress should be less then maximum shear stress limit. Mathematically these four failure modes can be represented as

$$g_1 = \frac{4L^3}{E}\sqrt{\left(\left(\frac{P_X}{b^3h}\right)^2 + \left(\frac{P_Y}{bh^3}\right)^2\right)} - D_0 \le 0 \tag{11}$$

$$g_2 = \frac{6L}{bh}\left(\frac{P_X}{b} + \frac{P_Y}{h}\right) - S \le 0 \tag{12}$$

$$g_3 = \frac{P_X L}{bhE} - T_0 \le 0 \tag{13}$$

$$g_4 = \frac{3V}{2bh} - M_0 \le 0 \tag{14}$$

b and h are the respective breadth and height of the beam and these are our design variables. P_x and P_Y are the external forces and L =100 in is the length of the beam. E (29e6 psi) is the material modulus of elasticity. Table 3.1 shows distribution of variables. The breadth and length of the beam have dimension bounds also. Dimension bounds are given below

$$\left.\begin{array}{l} 1 \le b \le 10 \\ 1 \le h \le 20 \end{array}\right\}$$

Table 3.1 Distribution of design variables

Variable	Mean	Standard deviation	Distribution
P_X	500lb	100lb	Normal
P_Y	1000lb	100lb	Normal
E	29e6psi	3e6psi	Normal
S	40000psi	2000psi	Normal

The required probability of failure of the system is 0.0027 that means system reliability is 99.9973. There are four constraints; the required system reliability will be given at the start and the optimizer will distribute failure probability for each constraint to get optimized solution for beam problem with satisfied reliability. The reliability index for each constraint is included in the design variable set. An upper limit and a lower limit is assigned to reliability index for each constraint. The active constraint set is updated after each iteration. The results obtained are comparable to results obtained from other methods. The cantilever beam is to be designed for minimum cross section.

Results:

The results given in Table 3.2 show that system reliability analysis result is comparable with optimum result obtained from other methods. The target system reliability is satisfied and the optimum value for cross-section obtained is 9.48 in^2.

$\beta_1, \beta_2, \beta_3, \beta_4$ are also design variables, The optimal values of component reliabilities are 0.9987, 0.999 , 0.9989, 0.9992 respectively.

Table 3.2 Comparison of results

Method	Objective value	b	h
Single-Loop Approach	9.5202	2.6093	3.6126
System Reliability	9.48	2.502	3.7888

3.5.2 Two-Bar Example. A two bar structural problem [18] is used as an example in our work. The objective is to minimize the volume V . For minimizing the volume two values i.e. diameter d and height H of the bar needs to be optimized. The mathematical representation of volume V is

$$Total\,volume\;\; V \;=\; 2\pi Td\sqrt{B^2+H^2} \tag{15}$$

There are two constraints for this problem:

$$\sigma = F\sqrt{B^2+H^2}\,/\,2\pi THd \tag{16}$$

$$\sigma_{crit} = \pi^2 E(T^2+d^2)/8(B^2+H^2) \tag{17}$$

where σ is the normal stress and σ_{crit} is the critical buckling stress. Given constant parameters are:

$$\sigma_{\max} = 400 N / mm^2$$

$$\text{External Force} = 150\ kn$$

$$E = 210\, N / mm^2$$

$$B = 750\, mm$$

$$T = 2.5\, mm$$

where $\sigma_{\max}$ is the normal stress limit, E is the elastic modulus, B is the width of structure, T is the thickness of the structure. First constraint is that normal stress should be less than normal stress limit $\sigma_{\max}$ and second constraint is that buckling stress value should be less then critical buckling stress limit

$$\sigma \le \sigma_{\max} \tag{18}$$

$$\sigma \le \sigma_{crit} \tag{19}$$

The two-bar structure design is a very typical design problem that reflects the situation in most of the real world problems where the technical efficiency (the normal stress and the buckling stress constraints) and the economical efficiency (the volume objective) are conflicting. For this particular problem we need d and H

$$d = x_1 \text{ mm}$$

$$H = x_2 \text{ mm}$$

where d is the nominal diameter of the cross-section, H the height of the two bar structure. The design variables d & H have design limits. There is an upper and lower limit for these variables and optimum solution for minimizing the volume should be in between these two limits to satisfy all the constraints. Bounds on the design variables are

$$20\,mm \le d \le 80\,mm, \qquad 200\,mm \le H \le 1000\,mm$$

The desired system reliability is 99.9970. There are two constraints for this design problem. Each constraint will have its own reliability requirement. The reliability index for each constraint is included in the design set and an upper and lower limit is assigned to each constraint. The reliability index limit for both constraints is set as 2.5 as lower limit and 3.5 as upper limit. Ditlevson[6] second order bounds are used to calculate system reliability. Table 3.3 shows result obtained from methodology proposed in this work.

Table 3.3 Results

Method	Objective value	d	H
System Reliability Analysis	567930 mm^3	37.72 mm	596.60 mm

The results show that system reliability analysis result. The target system reliability is satisfied and the optimum value for volume obtained is 5.6793e5. The optimal dimensions of the column in this case are d = 37.72 mm and H = 596.60 mm.

3.5.3 Vehicle Crash Worthiness Test. A vehicle crashworthiness [10] study is performed under a variety of side impact constraints. Reliability based design optimization of vehicle crashworthiness has gained considerable attention recently due to uncertainties in structural design variables, material properties and operating conditions, for this reason these properties and variables are very important in automotive vehicle side impact studies. One of the major safety requirements for a vehicle is to qualify the vehicle side impact test.

The performance of the dummy in side impact, in terms of head injury criterion (HIC), chest V∗C (viscous criterion) values and rib deflections (upper, middle and lower) must meet European Enhanced Vehicle-Safety Committee (EEVC) requirements. The finite element model of the vehicle used in this study and the moving deformable barrier are shown in Figure 3.5. The velocity of B-Pillar at middle point and the velocity of front door at B-Pillar are considered. In side impact design, the increase of gage design variables tends to improve the dummy performance. However, the vehicle weight is simultaneously increased, which is undesirable. For this reason, an optimization problem is formed by minimizing the vehicle weight subject to a number of safety constraints on the dummy according to the EEVC procedure. They include HIC, abdomen load, rib deflection or V ∗ C, and pubic symphysis force.

A total of seven random variables and four random parameters are used. The seven random variables (x1~x7) represent dimensions of some vehicle structural parts including thickness of B-Pillar (inner and reinforcement), thickness of floor side, Thickness of cross member, thickness of door beam, thickness of door belt line reinforcement and thickness of roof rail. The four random parameters include the material of B-Pillar (inner) 8x and floor side (inner) 9x as well as the barrier height 10x and barrier hitting position 11x. Table 3.4 shows sequentially the description of the random variables and parameters and their lower and upper bounds. The objective is to reduce the vehicle weight.

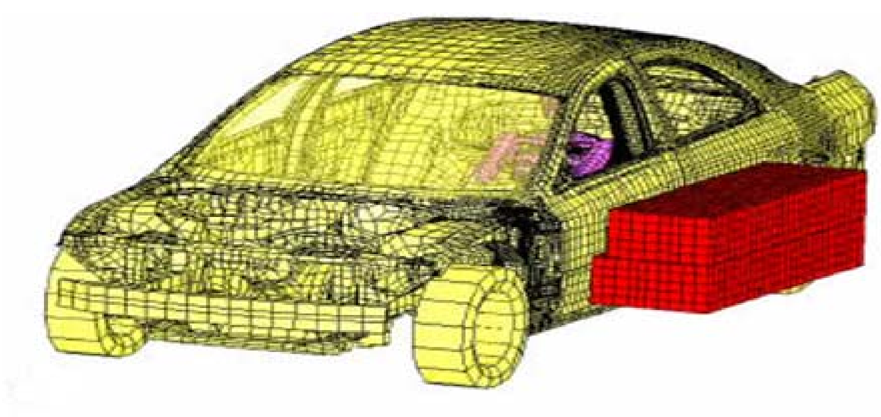

Figure 3.5 Vehicle crashworthiness test

The weight is represented as:

$$\text{weight} = 1.98 + 4.90x_1 + 6.67x_2 + 6.98x_3 + 4.01x_4 + 1.78x_5 + 2.73x_7 \tag{20}$$

Objective is to minimize weight:

Minimize weight (d)

Subject to: $P(abdomen\,load \le 1.0kN) \ge P_s$

$P(upper\,/\,middle\,/\,lower\,VC \le 0.32\,mls) \ge P_s$

$P(upper\,/\,middle\,/\,lower\,rib deflection \le 32\,mm) \ge P_s$

$P(public\;\;symphysis\;\;force, F \le 4.0kN) \ge P_s$

$P(velocity\,of\;\;B - pillar\,at\,middle\,po\text{int} \le 9.9\,mm\,/\,ms) \ge P_s$

$P(velocity\,of\;\;front\,door\,at\,B\text{-}pillar \le 15.7mm\,/\,ms) \ge P_s$

$d^{\mathrm{L}} \le d \le d^{\mathrm{U}}, d \in R^9$ and $X \in R^{11}$

Constraints are represented as:

$$Load_{\text{Abdomen}} = 1.16 - 0.3717 x_2 x_4 - 0.00931 x_2 x_{10} - 0.484 x_3 x_9 + 0.01343 x_6 x_{10} \quad (21)$$

$$\begin{aligned} Deflection_{\text{rib\_u}} &= 28.98 + 3.818 x_3 - 4.2 x_1 x_2 + 0.0207 x_5 x_{10} + 6.63 x_6 x_9 \\ &\quad - 7.7 x_7 x_8 + 0.32 x_9 x_{10} \end{aligned} \quad (22)$$

$$\begin{aligned} Deflection_{\text{rib\_m}} &= 33.86 + 2.95 x_3 + 0.1792 x_{10} - 5.057 x_1 x_2 - 11.0 x_2 x_8 \\ &\quad - 0.0215 x_5 x_{10} - 9.98 x_7 x_8 + 22.0 x_8 x_9 \end{aligned} \quad (23)$$

$$Deflection_{\text{rib\_1}} = 46.36 - 9.92 x_2 - 12.9 x_1 x_8 + 0.1107 x_3 x_{10}$$

(24)

$$\begin{aligned} VC_{\text{upper}} &= 0.261 - 0.0159 x_1 x_2 - 0.188 x_1 x_8 - 0.019 x_2 x_7 + 0.0144 x_3 x_5 + 0.0008757 x_5 x_{10} \\ &\quad + 0.08045 x_6 x_9 + 0.00139 x_8 x_{11} + 0.00001575 x_{10} x_{11} \end{aligned} \quad (25)$$

$$\begin{aligned} VC_{\text{upper}} &= 0.214 + 0.00817 x_5 - 0.131 x_1 x_8 - 0.0704 x_1 x_9 + 0.03099 x_2 x_6 - 0.018 x_2 x_7 \\ &\quad + 0.0208 x_3 x_8 + 0.121 x_3 x_9 - 0.00364 x_5 x_6 + 0.007715 x_5 x_{10} - 0.0005354 x_6 x_{10} \\ &\quad + 0.00121 x_8 x_{11} \end{aligned} \quad (26)$$

$$VC_{\text{lower}} = 0.74 - 0.061 x_2 - 0.163 x_3 x_8 + 0.001232 x_3 x_{10} - 0.166 x_7 x_9 + 0.227 x_2^2 \quad (27)$$

$$Force_{\text{Public}} = 4.72 - 0.5 x_4 - 0.19 x_2 x_3 - 0.0122 x_4 x_{10} + 0.009325 x_6 x_{10} + 0.000191 x_{11}^2 \quad (28)$$

$$Velocity_{\text{B\_Pillar}} = 10.58 - 0.674 x_1 x_2 - 1.95 x_2 x_8 - 0.02054 x_3 x_{10} - 0.0198 x_4 x_{10} + 0.028 x_6 x_{10} \quad (29)$$

$$Velocity_{\text{Door}} = 16.45 - 0.489 x_3 x_7 - 0.843 x_5 x_6 + 0.0432 x_9 x_{10} - 0.0556 x_9 x_{11} - 0.000786 x_{11}^2 \quad (30)$$

Table 3.4 Properties of design and random parameters of vehicle side impact model

Random Variable	Std Dev.	Distr. type	d_l	d^L	d	d^U
1 (B-pillar inner)	0.03	Normal	1	0.50	1.00	1.50
2 (B-pillar reinforce)	0.03	Normal	2	0.50	1.00	1.50
3 (Floor side inner)	0.03	Normal	3	0.50	1.00	1.50
4 (Cross member)	0.03	Normal	4	0.50	1.00	1.50
5 (Door beam)	0.03	Normal	5	0.50	1.00	1.50
6 (Door belt line)	0.03	Normal	6	0.50	1.00	1.50
7 (Roof rail)	0.03	Normal	7	0.50	1.00	1.50
8 (Mat. B-pillar inner)	0.006	Normal	8	0.192	0.30	0.345
9 (Mat. Floor side inner)	0.006	Normal	9	0.192	0.30	0.345
10 (Barrier height)	10.0	Normal	10^{th} and 11^{th} random variables are not regarded as design variables			
11 (Barrier hitting)	10.0	Normal				

Table 3.5 shows the results obtained from methodology proposed in this work.

Table 3.5 Results for vehicle crashworthiness test

Method	Weight	d_1	d_2	d_3	d_4	d_5	d_7
System Reliability Analysis	24.06	0.5	1.2601	0.5	1.2006	0.875	0.5

Table 3.5 shows result obtained from our proposed method for series system reliability based design optimization. The objective is to minimize the cost for vehicle crashworthiness test.

4. DESIGN IMPROVEMENT FOR RELIABILITY-BASED DESIGN

4.1 INTRODUCTION

Engineering systems consists of a large number of variables; it is up to engineers to utilize their engineering knowledge, judgment and experience to specify values to these variables that will lead to design of an effective engineering system. A design task might be for a small component like designing a rectangular bar which will have small number of design variables or it can be complex design task with more than 100 design variables, in that case even a skilled designer is unable to take into account all of the variables simultaneously. Engineers therefore go for design optimization technique to improve the performance of system, increase reliability and reduce cost. Design optimization involves application of numerical algorithms and techniques. Design optimization is increasingly deployed by engineers in industry today as this provides means to identify optimal design before physical production starts. Due to global competitive market, industries are forced to improve their quality of design and at the same time advances in computing power have given an added advantage to engineers to explore alternative design optimization paths. This leads to cost effective designs and increases the confidence level in the design.

During last ten years numerous efforts have been made to develop efficient reliability based design optimization [13, 19, 20]. The reliability optimization process is an iterative process and depending upon the complexity of design problem the number of iterations varies. A reliability level is set at the start of design process and in the end a design which will satisfy all the constraints and reliability level will be obtained. If the number of iterations is large then computational cost will increase. To address this problem a new method has been proposed to reduce the number of iterations and hence reduce computational time and cost. To obtain an optimum solution the optimizer calls the objective and constraint function repeatedly. First order reliability method (FORM) [3] is used for the MPP search, two nested loops are used in reliability based optimization, the inner loop is reliability analysis loop and the outer one is optimization loop. The number of function calls depends upon the number of variables and limit state functions/constraints and if these are large then number of function calls will go up.

The motivation for coming up with a new method for reducing computational time is that experience has shown that engineers working in industry don't have that much time and resources to go for truly optimum design. Achieving the optimal design by reliability based design optimization is a computationally expensive procedure which requires a lot of iterations and as the number of random variables increases and problem becomes more complex the computational time increases. Engineers are satisfied by something close to optimum but not true optimum point. It is seen that during the end of convergence a lot of iterations are used up without any significant improvement in the design values obtained. In our work it is shown later that convergence can be achieved in 2-3 iterations and reliability requirement will be met at the end, but it may not be a true optimum point but close to optimum so that design will be safe and computation expense can be reduced.

Existing RBD methods such as Diagonal direction method, Hybrid Mean Value method have proven to be inefficient and instable, other specialized methods are mostly gradient-based, but there is no guarantee of convergence. Some of the existing RBD methods have convergence difficulties with non-concave and non-convex problems. It is our goal in this work to solve any kind of performance functions. The need for this work arises from the fact that for complex design problems it is not affordable to search for truly optimum design. So there is need to find ways to reduce computational time. The other major issue is that even if optimum solution is obtained it still might not be true optimum design because the distributions of random variables may not be accurate. In our work an innovative methodology has been developed to reach feasible design solution. It might not be true optimum design but it will satisfy reliability requirement and reduce number of iterations.

4.2 STRATEGY

The idea is to conduct deterministic design optimization followed by reliability analysis. Deterministic design is not a very good design but information obtained from deterministic optimization can be worked upon to reduce computational effort. There will always be trade-off between optimum design and computational time and from methodology proposed in this work the final design will be bit conservative design but this is a practical need for industries today to get reliable design at the earliest. The idea is to use two reliability levels, one is the required reliability and other is higher than required reliability. Recall that the reliability constraint is defined as

$$P\left\{g_i(\mathbf{d},\mathbf{X},\mathbf{P})\leq 0\right\}\geq R_{i,}\, i=1,2,...,n_G \tag{1}$$

In the above equation $g_i(\mathbf{d},\mathbf{X},\mathbf{P})$ is the constraint function, R_i is the required reliability and n_G is the number of constraints.

The idea is to not go for true optimum point but to get a near optimum point quickly and to achieve this is to use higher reliability. An initial starting point design point is given and then the numerical optimizer computes the objective function and constraint functions. The required reliability is also given at the start. Numerical optimizer evaluates the objective function and the constraint functions and depending on the evaluation the optimizer will generate a search direction. The convergence criteria is set such that the optimizer can compare the design values obtained from previous iteration and check whether the solution converges or not. If the solution doesn't converge then the optimizer will generate a new design point. This procedure will repeat until convergence is achieved.

It is observed that for a design problem with multiple constraints, some constraints are never active and their reliabilities are very high; these constraints may not be affecting the design optimization values but they dominate the design process. To counter this problem and to reduce computational effort, first conduct reliability assessment at a higher-than-desired reliability. By using the higher-than-desired reliability level it is possible to push the design point quickly towards the feasible region. In a probabilistic design the required reliability is often higher than the reliability achieved by deterministic design. The feasible region of a probabilistic design is narrower than a deterministic design.

4.3 PROCEDURE

The proposed design improvement method has two levels of reliability. The first level is that of required reliability and the other level is the higher-than-required reliability. In this work sequential RBD method is used.

Figure 4.1 shows graphically sequential cycles for deterministic optimization and reliability analysis loops. In each cycle, at first solve an equivalent deterministic optimization problem, which is formulated by the information of the MPPs obtained in the last cycle. R_{high} is the higher-than-desired reliability and R_{des} is the desired reliability.

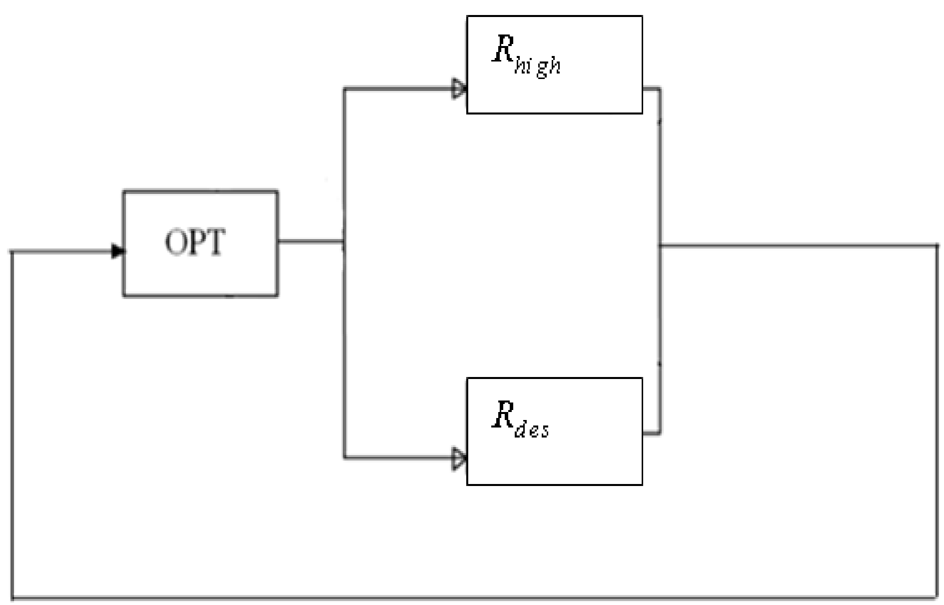

Figure 4.1 Sequential cycles of deterministic optimization & reliability assessment

Once the design solution is updated, then perform reliability assessment to identify the new MPPs and to check if all the reliability requirements are satisfied.

The current MPP information is used to formulate new constraint functions for the deterministic optimization in the next cycle in which the constraint boundary will be shifted to the feasible region. Using this strategy, the reliability of constraints improves progressively and the solution to a probabilistic design can be found within a few cycles, and the need for searching MPPs can be reduced significantly. The idea is to move the design solution quickly to its optimum in order to reduce the need for locating MPP's by using the higher-than-desired reliability at the start, to reduce number of iterations. Two reliability levels have been used for design improvement. In each cycle first solve an equivalent deterministic optimization problem which is formulated from the information of the MPP's obtained from the last cycle. The optimization model is shown below

$$\text{Minimize: } f(\mathbf{d}, \boldsymbol{\mu}_{\mathbf{X}}, \boldsymbol{\mu}_{\mathbf{P}})$$

$$\text{DV}=(\mathbf{d}, \boldsymbol{\mu}_{\mathbf{X}}, \boldsymbol{\mu}_{\mathbf{P}}) \quad (2)$$

$$\text{Subject to: } g_i(\mathbf{d}, \mathbf{X}, \mathbf{P}) \leq 0$$

Below is a list of steps of the design improvement method.

Step 1: The first step is to conduct a deterministic optimization. For the first cycle start with an initial design point to carry out deterministic optimization. Two reliability levels have been used in this work R_{des} and R_{high}. R_{des} is the desired reliability level and R_{high} is the higher-than-desired reliability level. R_{des} and R_{high} are initialized at the start. It is noted that the deterministic design may not be the optimum design; in order to lead optimization to its optimal point as quickly as possible something should be done. To push design set towards feasible design space perform reliability assessment with R_{high} as known that using higher reliability will lead to feasible region quickly. This is done for the first iteration only, and from the second iteration both reliability levels are used depending upon the convergence. The deterministic optimization is conducted with R_{high} only as this will save time to reach the feasible region.

Step 2: After the step 1 perform reliability analysis at R_{des}. The reason to do this is to check if the reliability constraints are satisfied. If the MPP's are in feasible region, then convergence is achieved; and if not, conduct optimization with MPP's obtained from R_{high}.The whole procedure repeats till convergence is achieved. The mathematical model of reliability analysis in next cycle at R_{des} is given as

$$\text{Minimize } g(\mathbf{u}) \text{ Subject to: } \|u\| = \Phi^{-1}(R_{des}) \tag{4}$$

If the value of a constraint function at MPP is less than zero then the design is in the feasible region and if it is greater than zero then the convergence is not achieved and design is still in the failure region.

Step 3: The next step is to perform reliability assessment with R_{high} to obtain the MPP. The reason to do reliability assessment at R_{high} arises from the fact that the focus is on reaching the feasible design space in less iterations, and by previous work it is known that it is not possible to reach optimal point after first iteration. So to save time start off with the higher-than-desired reliability to push the design solution quickly towards feasible space. The MPP's of all the reliability constraints are identified. The mathematical model for the MPP search at R_{high} is

$$\text{Minimize } g(\mathbf{u}) \qquad (3)$$

$$\text{Subject to: } \|u\| = \Phi^{-1}(R_{high})$$

Finding the MPP is a minimization problem, which usually involves an iterative search process.

4.4 FLOWCHART

The flowchart is shown in Figure 4.2. Start deterministic optimization with the higher-than-desired reliability (R_{high}) and perform reliability assessment with R_{high} as the need is to search feasible region quickly. For the second iteration perform deterministic optimization with the MPP's obtained from at R_{high}; and using the information from deterministic optimization perform reliability assessment at R_{des}. If g 's are feasible i.e. if design is in feasible region then convergence is achieved unless

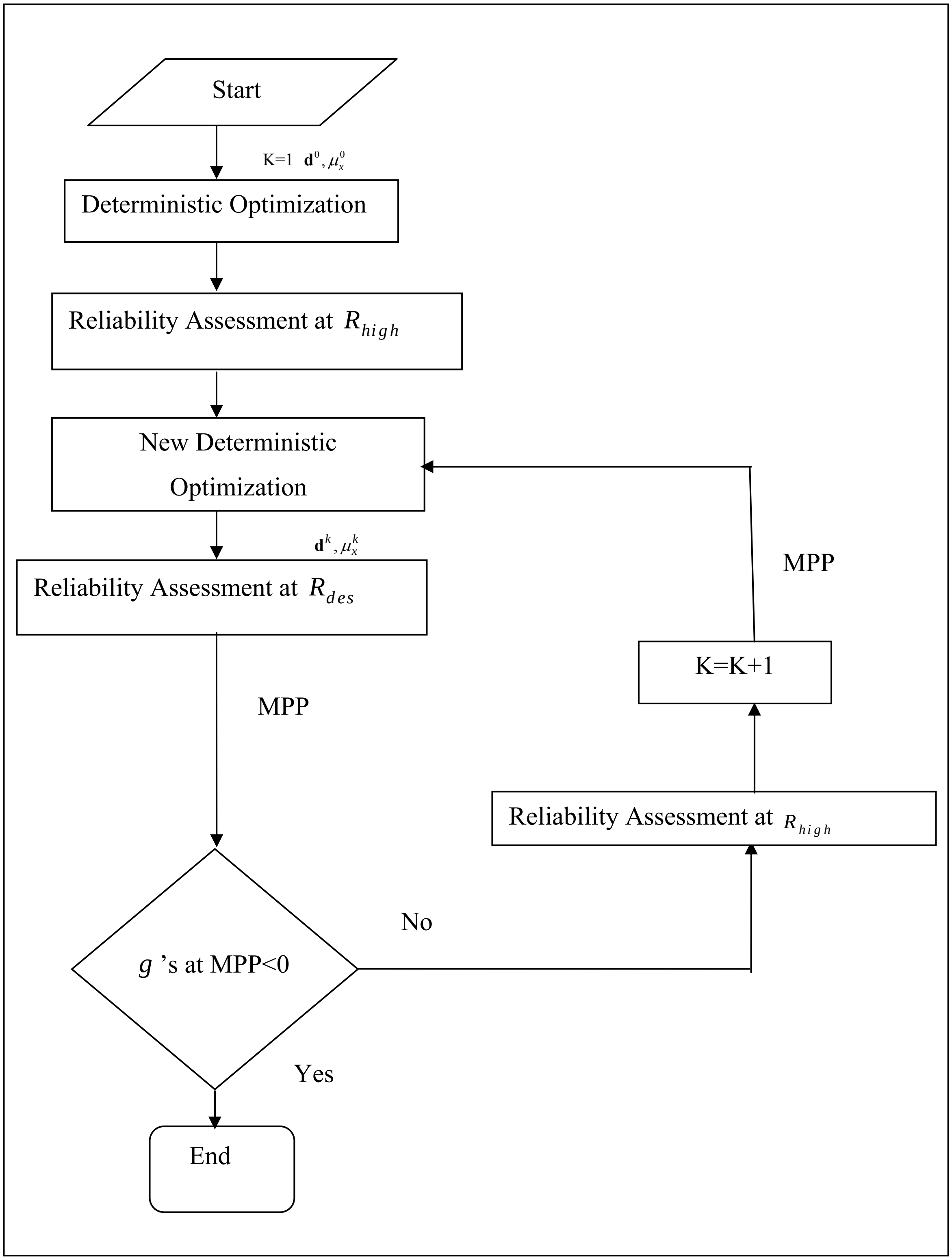

Figure 4.2 Flowchart for design improvement for RBD

perform reliability analysis with R_{high}; for next iteration use data from reliability analysis at R_{high} for deterministic optimization, and whole procedure repeats again till convergence is reached. For the first cycle there is no information about the MPP's so they are set as the mean of random design variables.

It is shown later in this thesis that this method is efficient in getting near optimum solution at required reliability level in two iterations only. Even with more random variables and reliability constraints it is possible to reach feasible solution in two-three iterations and another important thing is the number of function calls is less as higher than required reliability is employed at the start so that feasible region can be reached quickly and in process less reliability calculations are used.

4.5 EXAMPLES

Below are some of the examples used for verifying the numerical efficiency of our work as compared to work done by other people.

4.5.1 Cantilever Beam Problem. A cantilever beam [8] is shown in Figure 4.3. The objective is to minimize the cross-sectional area of the beam. The objective function is

$$\text{Minimize:} (\mathbf{d}) = bh \tag{5}$$

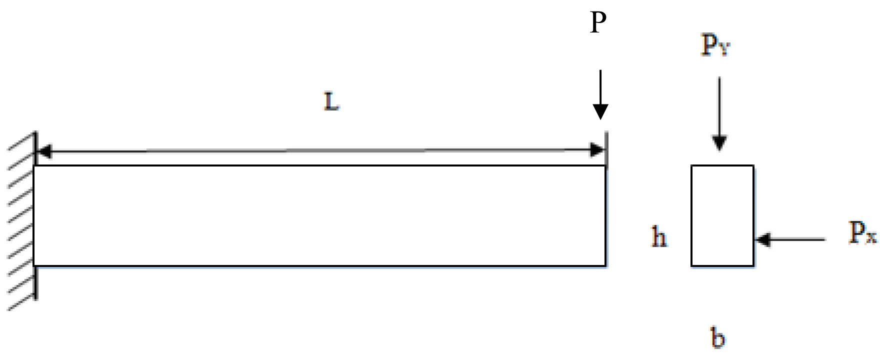

Figure 4.3 Cantilever beam problem

There are two failure modes for this rectangular beam under external load *P*. The first failure mode is that the tip displacement exceeds the allowable value D_0 , and the second failure mode is that the stress is greater than the- maximum yield stress *S*. The limit-state functions associated with these two failure modes can be represented as

$$g_1 = \frac{4L^3}{E}\sqrt{\left(\left(\frac{P_X}{b^3h}\right)^2 + \left(\frac{P_Y}{bh^3}\right)^2\right)} - D_0 \le 0 \tag{6}$$

$$g_2 = \frac{6L}{bh}\left(\frac{P_X}{b} + \frac{P_Y}{h}\right) - S \le 0 \tag{7}$$

b and h are the respective breadth and height of the beam and these are our design variables. P_X and P_Y are the external forces and L =100 in is the length of the beam.

E (29e6psi) is the material modulus of elasticity. The breadth and length of the beam have dimension bounds also. The dimension bounds are given below.

$$\left.\begin{array}{l} 1 \le b \le 10 \\ 1 \le h \le 20 \end{array}\right\}$$

The desired reliability is 0.999. To obtain a design solution as quickly as possible use a higher-than-desired reliability level R_{high} 0.9999. Perform deterministic optimization (DO) with R_{high} and with information from DO perform reliability analysis at R_{high} as is is known fact that it's not possible to obtain a feasible solution after the 1st step itself and reliability is low after 1st step so use higher reliability in order to move the design towards feasible solution quickly.

After reliability assessment at R_{high} in the first cycle, information to formulate a new deterministic optimization model for the new cycle is obtained. From the second cycle onwards reliability analysis will be done with R_{des} i.e. 0.999 at first. If the design is feasible then convergence is achieved; and if it still not feasible, perform reliability analysis with the higher-than-desired reliability R_{high} . The MPP's from R_{high} is then used to formulate a new deterministic optimization model for the next iteration. The results are shown in Table 4.3. It is seen that with our work it is possible to obtain feasible solution in 2 iterations only.

Table 4.1 Distribution of design variables

Variable	Mean	Standard deviation	Distribution
P_X	500lb	100lb	Normal

Table 4.1 Distribution of design variables (cont.)

P_Y	1000lb	100lb	Normal
E	29e6psi	3e6psi	Normal
S	40000psi	2000psi	Normal

Table 4.2 shows the convergence history for beam problem. As stated earlier focus in this work is to push the design solution quickly towards feasible design space, and if $g's < 0$ then convergence is achieved. From Table 4.2 it can be seen that in first iteration design is still not feasible as g_1 and g_2 are less than zero, the feasible design is reached in second iteration as $g's < 0$ and the objective value is 9.9335.

Table 4.2 Convergence history

Iteration	g_1	g_2	Objective value
1	0.3292	0.4859	7.8235
2	-0.0607	-0.1063	9.9335

This method is effective in reducing the computational expense as shown above by reducing the number of function calls. Results are compared with SORA as it is an efficient method for reliability-based design and in recent years many researchers have compared their work with SORA. The objective value is a bit higher than SORA, but the focus is not search for the optimal solution, the objective is to reduce computational time and feasible solution is what is desired.. As long as reliability is met design will be safe. Table 4.3 shows the comparison of results with SORA

Table 4.3 Comparison of results

Method	Objective value	Function evaluations	# of iterations
SORA	9.5794	893	4
New method	9.9335	485	2

4.5.2 Minimize Volume of Two-Bar Structure. A two-bar structure design problem is used in this work as another example [18]. The two-bar structure design reflects the situation in most of the real world problems where the technical efficiency (the normal stress and the buckling stress constraints) and the economical efficiency (the volume objective) are conflicting.

The objective is to minimize the volume of a two-bar structure. Mathematical equation for objective function is

$$\text{Minimize: } V = 2\pi Td\sqrt{B^2 + H^2} \tag{8}$$

T is the thickness of the cross-section and is 2.5mm, the width of the structure B =750mm, external force F =150kn and the elastic modulus E =210,000 N/mm^2, normal stress limit = 400 N/mm^2. The two constraints are that the normal stress should be less than normal stress limit and that the buckling stress should be less than that critical buckling stress limit.

$$\sigma = F\sqrt{B^2 + H^2} / 2\pi THd \tag{9}$$

$$\sigma_{\mathrm{crit}} = \pi^2 E(T^2 + d^2)/8(B^2 + H^2) \tag{10}$$

where σ is the normal stress, σ_{crit} is the critical buckling stress.

$$\sigma \le \sigma_{\max} \tag{11}$$

$$\sigma \le \sigma_{crit} \tag{12}$$

The variables are normally distributed. The bounds on the design variables are

$$20mm \le d \le 80, 200mm \le H \le 1000mm$$

The distribution of design variables is given below in Table 4.4 and Table 4.5 shows convergence history.

Table 4.4 Distribution of design variables

Variable	Mean	Standard deviation	Distribution
E	210,000N/mm ^{2}	1E3N/mm ^{2}	Normal
F	150kN	7.5kN	Normal

Table 4.5 Convergence history

Iteration	g_1	g_2	Objective value
1	1.0112	0.0859	525000
2	-0.1017	-0.0063	580500

The results are shown in Table 4.6.

Table 4.6 Comparison of results

Method	Objective value	Opt. point	# of iterations
SORA	580690	34.8712,750	3
New method	580500	34.8902,750	2

The results show the efficiency of new method to solve non liner problems in less time and within reliability limits. The required reliability level for this problem is 0.999 and to reach feasible region earlier higher than required reliability is employed at the start to it is taken as 0.9999 ,and use this higher than required reliability if convergence is not achieved from required reliability level to push the design solution in feasible region.

The results show that convergence is achieved after 2nd iteration only while SORA takes 3 iterations to reach optimal solution. The results from two methods are quite close so efficiency is good too. The design obtained is feasible as seen from Table 4.5 that g_1 and g_2 are less than zero after second iteration. This satisfies the reliability requirement and hence is can be said that it is a feasible design.

4.5.3 Minimize the Weight of a Symmetric Three-Bar Truss. A slightly more complex design problem is three-bar truss [21] as shown in Figure 4.4 below. A force P is acting on the structure and a design has to be developed which can support this force. It should satisfy a number of constraints such as member buckling, failure by deflection at node 4 and failure by resonance when natural frequency of the truss structure is below a given threshold. The structure is statically indeterminate. The structure is symmetric so

$A_1 = A_3$

A_1 = cross sectional area of material 1 & 3

A_2 = cross sectional area of member 2

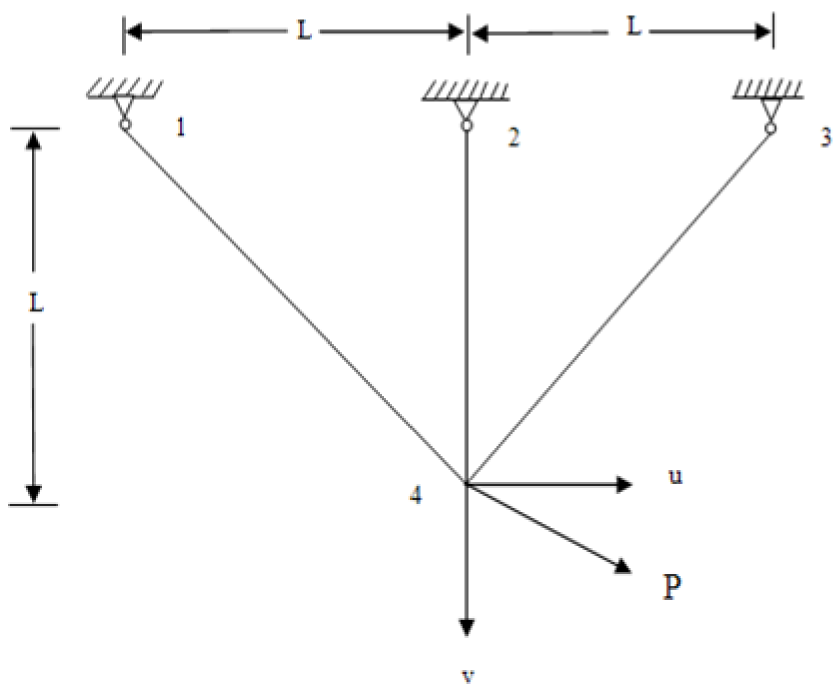

Figure 4.4 Symmetric three-bar truss

The design variables for this problem will be A_1 and A_2, the objective is to minimize the total volume of the structure which is given below

$$\text{volume} = L(2\sqrt{2}A_1 + A_2) \tag{13}$$

A number of constraints are considered for this design problem. Horizontal and vertical displacements at node 4 can be described as

$$u = \frac{\sqrt{2}LP_u}{A_1E} \tag{14}$$

$$v = \frac{\sqrt{2}LP_v}{(A_1 + \sqrt{2}A_2)E} \tag{15}$$

where E is the elasticity modulus and P_u and P_v are the horizontal are vertical components of the load P

$$P_u = P\cos\theta \tag{16}$$

$$P_v = P\sin\theta \tag{17}$$

Stresses acting on members 1, 2 and 3 under load P can be computed as

$$\sigma_1 = \frac{1}{\sqrt{2}}\left[\frac{P_u}{A_1} + \frac{P_v}{(A_1 + \sqrt{2}A_2)}\right] \tag{18}$$

$$\sigma_2 = \frac{\sqrt{2}\,P_v}{(A_1 + \sqrt{2}A_2)} \tag{19}$$

$$\sigma_3 = \frac{1}{\sqrt{2}}\left[\frac{P_v}{(A_1 + \sqrt{2}A_2)} - \frac{P_u}{A_1}\right] \tag{20}$$

Horizontal and vertical deflection constraints are also given in the problem, the criteria is that these deflections should be less then specified limits Δ_u and Δ_v respectively.

The horizontal deflection constraint is:

$$\frac{\sqrt{2}\,l\,P_u}{A_1 E} \leq \Delta_u \tag{21}$$

The vertical deflection constraint is:

$$\frac{\sqrt{2}\,l\,P_v}{E(A_1 + \sqrt{2A_2})} \leq \Delta_V \tag{22}$$

Some structures support machinery in motion and dynamic loads. These structures vibrate with a certain frequency known as natural frequency. There can be a number of modes of vibration and each mode of vibration has its own frequency. According to physics resonance is the tendency of a system to oscillate at larger amplitude at some frequencies than at others. These are known as the system's resonance frequencies (or resonant frequencies). At a resonant frequency the frequency of oscillation does not change with changing amplitude. Therefore it has to be kept in mind while designing structures that no frequency should be close to frequency of operating machinery. The natural frequency of the structure should be less than a specified frequency ω_0 Hertz. The eigenvalue corresponding to a frequency of ω_0 Hertz is given as $(2\pi\omega_0)^2$. The lowest eigenvalue ξ for the structure should be less than $(2\pi\omega_0)^2$. The frequency constraint can be written as

$$\frac{3EA_1}{\rho L^2(4A_1 + \sqrt{2}A_2)} \geq (2\pi\omega_0)^2 \tag{23}$$

A_1 & A_2 must be non-negative

Table 4.7 is the design data for three bar truss problem from Jasbir S. Arora [21].

Table 4.7 Design data for the three-bar truss

Allowable stress	Members 1 and 3, $\sigma_{1a} = \sigma_{3a} = 5000\ psi$ Member 2, $\sigma_{2a} = 20000\ psi$
Allowable displacements	$u_a = 0.005\, in$ $v_a = 0.005\, in$

Table 4.7 Design data for the three-bar truss (cont.)

Modulus of elasticity	$E = (1.00\text{E}+07)$ psi
Weight density	(1.00E-01) lb / in^3
Constant	$\beta = 1.0$
Lower limit on design	(0.1, 0.1, 0.1) in^2
Upper limit on design	(100, 100, 100) in^2
Starting point	(1, 1, 1) in^2
Lower frequency limit	2500 Hz
Load P	40000 lb
Angle θ	45

Result: Using the data given above values for design variables A_1 & A_2 are obtained.

The result is obtained in 3 iterations. The desired reliability is set as 0.999 and higher then desired reliability is 0.9999. Result is given in table given below. Table 4.7 shows results obtained from our method. This example is from Jasbir S Arora[22] , this example is not compared with any other method as this has not been used before for reliability-based design. Table 4.8 shows the convergence history and Table 4.9 shows the results obtained using proposed methodology.

Table 4.8 Convergence history

Iteration	g_1	g_2	Objective value
1	0.592	1.8217	15.1432
2	-0.0057	-0.2167	23.5823

Table 4.9 Result

Method	Cost	$A_1 = A_2$	A_3	# of iterations
New Method	23.5823	7.6467	1.9517	3

The objective for this problem was to minimize the cost subject to some constraints. The minimized cost obtained is 23.5823. It proves that our work is efficient in reducing computational effort and on the other hand it satisfies reliability requirement too. The design obtained is feasible as it can be seen from Table 4.8 that the value of g_1 and g_2 is less than zero at second iteration. So reliability is satisfied and feasible design solution is obtained.

5. CONCLUSION

This thesis presents two new methodologies for reliability-based design. The first work is system reliability-based design optimization and the second work is design improvement for reliability-based design. Examples have been given to show the efficiency of the proposed methods.

The approach for system reliability-based design optimization uses the second order upper probability bounds for accurately estimating probability of failure of a series system. A system might have number of components and each component might have different reliability requirements. In order to achieve the desired system reliability it shown in this work that if reliability requirements for various failure modes are included in design set. This way the desired reliability for the complete system can be achieved.

Another methodology proposed in this work is the design improvement for reliability-based design. Existing methods are not efficient and are computationally expensive. To reduce computational effort a new methodology has been proposed for design improvement for reliability-based design that uses both the desired and higher-than-required reliability levels. By using information obtained from reliability assessment at the higher-than-desired reliability level it is possible to push the design solution quickly towards the feasible design space. The method described in this thesis will give a quick feasible design that satisfies the reliability requirements and is close to the optimum solution.

Possible future work can be accurately estimating the higher-than-desired reliability for design improvement method, so that numerical efficiency increases.

Another future research is to extend the first approach to system reliability-based design to parallel systems, as in this work the focus is on series system only. Try out example with greater number of random variables and increased complexity in order to prove the efficiency of methodology described in this work.

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VITA

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