Number Sense Mediated by Mathematics Self-Concept in Impacting Middle School Mathematics Achievement

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NUMBER SENSE MEDIATED BY MATHEMATICS SELF-CONCEPT IN IMPACTING MIDDLE SCHOOL MATHEMATICS ACHIEVEMENT

by

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ABSTRACT

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The purpose of the current study was to extend the research on number sense to the middle school level and to simultaneously consider socioemotional elements related to the construct at this developmental stage. Its genesis was initially rooted in an ongoing and dramatic emphasis by U.S. policymakers, researchers, and educators on improving mathematics achievement in order to compete globally in technology and innovation. Despite debates about optimal curriculum and instruction, tremendous support exists for the construct of number sense. However, middle school research examining the phenomena has been limited to intervention protocols targeting specific skillsets and better measurement of its domains. Concomitantly, educational research has produced ample evidence of the decline in student mathematics motivation over time, and the corresponding literature establishes a link between mathematics self-concept and mathematics achievement, particularly during adolescence.

The Early Childhood Longitudinal Study, Kindergarten Class of 1998-99 provides a sample of 4,425 U.S. eighth graders for the present study, assessed directly and indirectly in cognitive, demographic, and affective domains. Multiple regression analyses confirmed the hypotheses that number sense predicts both mathematics self-concept and mathematics achievement at the middle school level, when controlling for gender, race, socioeconomic status, and special education services. Additionally, a path analysis with Statistical Analysis Software (SAS) and the Sobel test revealed that mathematics self-concept mediates the relationship between number sense and mathematics achievement. This indirect effect, when combined with the direct effect of number sense, results in a significant, medium total effect value of .35 for the model. By incorporating this knowledge regarding the interconnection of these three constructs into mathematics curriculum and instruction, as well as teacher education, the United States can move closer to bringing about equity of opportunity and motivating students to pursue more complex mathematics coursework and subsequently professions.

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LIST OF ABBREVIATIONS

CCSSI: Common Core State Standards Initiative

CCSSM: Common Core State Standards for Mathematics

CST: California Standards Test

ECLS-K: Early Childhood Longitudinal Study, Kindergarten Class of 1998-99

GFI: Goodness of Fit

IRT: Item Response Theory

NAEP: National Assessment of Educational Progress

NCES: National Center for Education Statistics

NCLB: No Child Left Behind

NCTM: National Council of Teachers of Mathematics

OECD: Organization for Economic Cooperation and Development

PGFI: Parsimonious Goodness of Fit

PISA: Program for International Student Assessment

REM: Reciprocal effects model

SAS: Statistical Analysis Software

SDQ: Student Description Questionnaire

SDQ I: Version of SDQ intended for pre-adolescents (grades two through six, ages eight through 12)

SDQ II: Version of SDQ intended for early adolescents in junior high school and high school (grades seven to 11, ages 13 to 17)

SDQ II-S: Short-form version of the SDQ II (51 versus 102 total items)

SDQ III: Version of SDQ intended for late adolescents and adults (ages 16 and up)

SES: Socioeconomic status

SMP: Student Mathematical Practice

SRMR: Standardized Root Mean Square Residual

TIMSS: Trends in International Mathematics and Science Study

Chapter 1: Introduction

From researchers, policymakers, and educators to the President of the United States there is a persistent and strong effort to increase American mathematics achievement in hopes of globally competing in technology and innovation (Bush, 2006; Clinton, 1998; National Research Council [NRC], 2001; National Mathematics Advisory Panel, 2008; Obama, 2011, 2012; Schoenfeld, 2004). Mechanisms of reaching this goal have included intervention to improve high-stakes test performance and earlier access to more advanced curriculum. Implementation, however, has been characterized by heightened bureaucratic strain on school administrators and teachers and subsequent deficiencies in remediation needed for students to realize such goals.

As debates continue over traditional versus reform curriculum (Ben-Chaim, Fey, Fitzgerald, Benedetto, & Miller, 1998; Schoenfeld, 2004) and computational proficiency versus conceptual understanding (Baroody, Bajwa, & Eiland, 2009; Cowan et al., 2011; Wu, 1999), frustrated middle school students are losing motivation (Eccles et al., 1993; Schielack & Seeley, 2010). Despite national debates and reforms that started with Sputnik in the 1950s, facilitated movements like "new math" and "back to basics," and yielded *A Nation at Risk*, Goals 2000, No Child Left Behind, and Race to the Top, the United States does not seem to be gaining an advantage over its competitors (Drew, 2011; Hanushek, Peterson, & Woessmann, 2011; Hennessy, 2002; Peterson, Woessmann, Hanushek, & Lastra-Andón, 2011; Stevenson, et al., 1990). On the contrary, though the National Center for Education Statistics (NCES) reports that National Assessment of Educational Progress (NAEP) average mathematics scores for U.S. eighth graders have increased 21 points since 1990, and that over one third of these students are enrolled in

Algebra I (NCES, 2011), certain data cannot be ignored. For one, there are still a significant number of students not even scoring at the basic level, meaning they do not exhibit an understanding of arithmetic operations, and this figure varies according to race, ranging from 14% for Asian students to 49% for Black students (NCES, 2011). Second, a look at 2009 NAEP questions from the number properties/operations and algebra domains in relation to the 2010 Common Core State Standards for Mathematics (CCSSM) reveals material that is around the fifth and sixth grade levels, respectively (Loveless, 2011). Though we may be moving in the right direction, it appears that a great deal more work must be done.

Concurrent with the call for enhanced mathematics instruction and curriculum is a struggling economy whose reality argues for fiscal cuts wherever possible, and that is subject to the rapidly changing circumstances of globalization. The resulting outcome is often less money for education, fewer jobs for the highly educated, and difficulty in recruiting and maintaining the most effective teachers (Boyd, Grossman, Lankford, Loeb, & Wyckoff, 2009). Some tout the inevitable benefits of a market solution to education, which would allow parent choice to create competition and yield school improvements (Chubb & Moe, 1990), but studies have not provided consistent evidence to support these claims (Barrow & Rouse, 2008; Usher, Kober, Jennings, & Renter, 2011), and some point out that the neoliberal opportunity bargain upon which this system is based has altered the face of human capital and the goals of the American people (Brown, Lauder, & Ashton, 2011).

As societal focus has shifted more towards money and the individual, idealistic calls to view education in the context of social progress seem more pertinent than ever.

Though few individuals will openly challenge Dewey's (1900) emphasis on education for social growth rather than profit, the reality of such a belief system would require that education move in the opposite direction of its current path. The continued emphasis on standards and accountability is not without merit, but the rigid nature of some of its corresponding bureaucratic and capitalistic solutions cannot effectively enable interventions, both academic and social, that many students need. Regardless of which methods or social perspectives best facilitate enhanced education, the United States continues to embrace it as the primary means of realizing the American Dream. From Hirsch's (1996) demand for intellectual capital through nationalized, content-specific standards to Friedman's free market model allowing for choice and privatization (1980), education remains at the forefront of economic and social progress. George W. Bush (2000) spoke of an open nation, where "every citizen has access to the American dream; an America that is educated, so every child has the keys to realize that dream," and President Obama (2011) continues to emphasize education as a means of economic growth in saying, " . . . if we want to win the future – if we want innovation to produce jobs in America and not overseas – then we also have to win the race to educate our kids."

America is not, however, currently outperforming other nations, and mathematics achievement is no exception. The 2009 Program for International Student Assessment (PISA) revealed that, of 34 Organization for Economic Cooperation and Development (OECD) countries, 17 outperformed U.S. fifteen-year-olds in mathematics and that the U.S. mathematics score was "statistically significantly below the OECD average" (OECD, 2010, p. 8). Hanushek, Peterson, and Woessmann (2010), even after narrowing this 2009 PISA analysis to White students and students with at least one college educated parent, found that 12 OECD countries had over double the U.S. percentage of highly accomplished students, and only eight fell below the United States. Using NAEP proficiency guidelines to determine a similar PISA measure, Peterson et al. (2011) determined that U.S. students taking the PISA as 15-year-olds in 2009 ranked 32 among participating countries. And in their comparison of the United States to 33 other OECD countries on the 2009 PISA, Petrilli and Scull (2011) reveal that the United States ranks below the average in mathematics. It is clear that the nation wants and needs to improve mathematics education, but it does not seem that emphasizing standards and accountability is producing the desired result. Before drastically altering curriculum and instruction, a more focused look at possible obstacles in mathematics development is in order.

Statement of Problem and Significance of Study

The last century has seen intense debate around mathematics instruction and curriculum in the aptly named “math wars,” and though the root of this controversy may be social and economic in character, the strain between traditional and reform mathematics is primarily reflected by a preference for basic skills or process. According to Schoenfeld (2004),

> An exclusive focus on basics leaves students without the understandings that enable them to use mathematics effectively. A focus on “process” without attention to skills deprives students of the tools they need for fluid, competent performance. The extremes are untenable. (p. 280-281)

The most promising framework to weave together mathematics perspectives in this debate that pits facts against concept is number sense (Berch, 2005; Greeno, 1991;

Jordan, 2007; Jordan, Kaplan, Olah, & Locuniak, 2006; Lago & DiPerna, 2010), and not surprisingly, the 2008 National Mathematics Advisory Panel's Final Report addressed the need to enhance academic focus on this number sense construct. Numerous studies support the positive effects of number sense in elementary school (Jordan, Glutting, & Ramineni, 2009; Jordan, Glutting, Ramineni, & Watkins, 2010; Moeller, Pixner, Zuber, Kaufmann, & Nuerk, 2011), but at the middle school level, number sense research is primarily limited to intervention protocols targeting specific skillsets and distinct elements of the construct (Cutler, 2001; Gay & Aichele, 1997; Greenes, Schulman, & Spungin, 1993; Markovits & Sowder, 1994; Scott, 1987). Though such efforts are beneficial in their ability to delineate and hone certain elements of number sense, research must first determine how number sense at this juncture contributes to overall mathematics achievement so that curriculum and instruction are developed and employed appropriately.

The longitudinal advantage of early number sense in terms of overall mathematics achievement at the elementary level must extend to older students. Assuming the trajectory continues in adolescence, research should explore how to overcome obstacles associated with insufficient number sense at the middle school level. The present study attempts to highlight a distinct path for number sense, shedding light on how number sense must *continue* to play an important role in mathematics curriculum and instruction as students progress to middle school and into coursework like algebra. Rather than solely working to incorporate more critical thinking at elementary levels, it may be necessary to attend to more basic mathematical thinking as students move on to higher grade levels. Although seemingly intuitive given the hierarchical nature of mathematics learning and the call for curriculum to focus more on number sense, research has not adequately addressed adolescent number sense.

Looking beyond merely curriculum, in 2007 the National Math Panel surveyed over 1,000 Algebra I teachers across the nation and found that 58% of middle school teachers and 65% of high school teachers in the sample noted “working with unmotivated students” as the number one challenge to mathematics instruction (Hoffer, Venkataraman, Hedberg, & Shagle, 2007, p. 32). Accordingly, the 2008 National Mathematics Advisory Panel called for a closer look at mathematics anxiety and the importance of student beliefs regarding effort, both of which contribute to student motivation and competence beliefs. The multidimensional, hierarchical construct of self-concept, and specifically, the contextually richer and more self-reflective construct of mathematics self-concept, is an excellent theoretical framework for analyzing mathematics competence in relation to motivation (Marsh & Craven, 1997, 2006; Marsh, Craven, & Martin, 2006; Marsh & O’Mara, 2008). Mathematics self-concept is a critical component of middle school development, not only because of the increasing complexity of the subject at this level, and the demonstrated declines in motivation during this period (Steinmayr & Spinath, 2007), but also as a result of identity and competency beliefs potentially leading a student to determine that he or she is or is not “a math person” at the young, impressionable age of twelve or thirteen. Given that no studies could be found on specific mathematics content domains contributing to mathematics self-concept in middle school, the mathematics self-concept framework will be used to broaden previous research by highlighting number sense.

Focus on Middle school

In 1928, Wilbur Alden Coit published a research article in *The School Review* proclaiming the following:

> It has been the writer's experience during twenty-five years of teaching that many pupils studying algebra have difficulty with simultaneous equations, both simple and quadratic, because of the inability to add and subtract fractions, to substitute, and to deal with negative numbers. (p. 504)

It seems little has changed over the last 85 years. The global economy's "digital Taylorism" (Brown et al., 2011) reflects the world's increasing reliance on technology and thus the need for a continuing focus on mathematics and science education in America. Middle school mathematics is a logical place to target given that it has traditionally been characterized by a transition from arithmetic to algebra. These elementary to middle school curriculum progressions are often articulated as intertwined paths beginning in elementary school and moving through middle school, yielding ample evidence that stronger foundational skill levels and early mathematics coursework regulate mathematics achievement (Baroody, Lai, & Mix, 2006; Bodovski & Farkas, 2007; Byrnes & Wasik, 2009; Hickey, 2009; Kikas, Peets, Palu, & Afanasjev, 2009; Tolar, Lederberg & Fletcher, 2009). Furthermore, in light of policy directing earlier access to algebra coursework, there has been a more recent emphasis on algebraic thinking and the benefits it yields at very elementary stages of instruction (Carpenter, Levi, Franke, & Zeringue, 2005; Carraher, Schliemann, Brizuela, & Earnest, 2006; Ketterlin-Geller, Jungjohann, Chard, & Baker, 2007). A faulty fix like mandating higher levels of mathematics curriculum in middle school, however, may translate into an inability to progress and succeed in a technology-laden environment that demands increasingly complex mathematical and scientific knowledge.

The Common Core State Standards for Mathematics, or CCSSM (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010), create a framework that expertly reflects the need to attend to number sense, particularly in the operations and algebraic thinking domain represented in kindergarten through fifth grade. This set of expectations focuses on making sense of operations from the onset of mathematics learning, allowing cognitively guided instruction to tap more individualized student thinking prior to the introduction of algorithms (Carpenter, Fennema, Franke, Levi, & Empson, 1999). More broadly, the Student Mathematical Practices (SMPs) comprised in the CCSSM are intended to be the result of student mathematical behaviors, and thus extend beyond what can be answered with basic multiple choice problems. The first SMP, requiring students to make sense of problems and explore ways to reach optimal solutions, provides an excellent example of number sense skills in expecting that students "make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt" (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010, p. 6). The SMPs are present at all stages of mathematical development, and their call for National Council of Teachers of Mathematics (NCTM) process standards (2000) and NRC mathematics proficiency areas (2001) clearly establishes a strong and consistent number sense foundation.

By middle school, the CCSSM detail more traditional algebra curriculum, incorporating objectives around rational numbers, expressions and equations, and functions in addition to topics in geometry, statistics, and probability. The assumption is that previous standards are a continuous part of curriculum and instruction, and in this way, as well as through the SMPs, they reflect the ongoing need for number sense. The current study of eighth graders sets out to show just *how* critical this construct remains to overall mathematics achievement at the middle school level and how this is influenced by motivation. Results will shed light on how the CCSSM may be augmented at the student, classroom, and district level to reach students who have not previously acquired sufficient number sense and to better achieve number sense for new and continuing students.

Theoretical Frameworks

In keeping with a stereotypical American mindset, balance is often undermined in educational reforms. There are ongoing battles over the best way to teach mathematics that mirror the phonics versus whole language debate in the field of literary education (Schoenfeld, 2004). In addition to the arguments about memorization versus conceptual understanding and traditional versus reform curriculum, there is great struggle and variability in views about how to incorporate meaning into learning. This tension includes "informal baggage" like finger counting (Dehaene, 1997), manipulations like "joining to" and "separating from" (Carpenter et al., 1999), and relational thinking as a way to make the learning of arithmetic more meaningful by employing "fundamental properties of numbers and operations to transform mathematical expressions rather than simply calculating an answer following a prescribed sequence of procedures" (Carpenter et al., 2005). Despite the proven advantages of drawing from dual camps of instruction in reading (Pearson, 2004), such measures are not being articulated well in the mathematics classroom.

A comprehensive understanding of number properties and operations will make the connection between arithmetic and algebra more clear to students (Carpenter et al., 2005; Ketterlin-Geller et al., 2007; NRC, 2001), but the idea that associated techniques like cognitively guided instruction (Carpenter et al., 1999) will eventually result in the fluency required for higher-order mathematics has not been adequately researched. For some, the "bogus dichotomies" of these controversies (Wu, 1999) are irrelevant given the reality that skills and concepts go hand in hand (Bass, 2003). Mathematics education would benefit from an overarching framework that can incorporate multiple perspectives and techniques while still realizing a "connected body of mathematical understanding and competencies" (NCTM, 2000 p. 29).

In light of such entrenched and conflicting ideologies, how can research find a way to carefully scrutinize obstacles to mathematics achievement, and even more importantly, determine strategies to improve it? According to NCTM (n.d.), number sense is the ability to "naturally decompose numbers, use particular numbers as referents, solve problems using the relationships among operations and knowledge about the base-ten system, estimate a reasonable result for a problem, and have a disposition to make sense of numbers, problems, and results." The expansion of this definition by McIntosh, Reys, & Reys (1992) will serve as the number sense framework for the present study. Specifically, number sense is determined by a set of key skills (such as relative and absolute magnitude, place value, and addition and subtraction), and beyond having sufficient knowledge of these skills, a person with strong number sense must exhibit facility with and application of numbers and operations. Such tenets allow for the inclusion of skills, no matter how attained, and more pertinently, in ways that reflect conceptual understanding and flexible application, thereby making number sense an ideal conceptual backdrop for the study of mathematics achievement.

Finally, mathematics self-concept, based upon student questionnaire items regarding interest and perceived competence, will draw from the Marsh and Shavelson (1985) multidimensional model of self-concept, specifically looking at the second order mathematics academic factor and tapping both evaluative and affective elements (Marsh, 1990d; Skaalvik & Valås, 1999; Senturk, 2000). Clear support for the link between academic achievement and academic self-concept (Marsh, 1986; Marsh & O'Mara, 2008; Marsh, Parker, & Barnes, 1985; Marsh & Yeung, 1998; Möller, Pohlmann, Köller, & Marsh, 2009), the existence and validity of a mathematics self-concept subscale (Marsh, 1990a, 1993b; Marsh & Shavelson, 1985), and evidence of self-concept's meditational capabilities regarding achievement (Marsh, Walker, & Debus, 1991; Skaalvik & Skaalvik, 2009) provide a solid foundation for reviewing such relationships within the context of number sense.

Design of Study

Data for the study will come from the Early Childhood Longitudinal Study, Kindergarten Class of 1998-99 (ECLS-K), a kindergarten through eighth grade public-use database maintained by NCES. Child-level within-year data for nearly 4,500 eighth grade students across the United States will be analyzed. The cross-sectional study of middle school mathematics performance will draw independent and dependent variables from administrator surveys, cognitive assessments, student questionnaire data, and teacher mathematics academic ratings for students. Studies have revealed the future mathematical benefits of components of early number sense (Jordan, Kaplan, Ramineni, & Locuniak, 2009; Mazzocco, Feigenson, & Halberda, 2011), but little research has determined whether number sense skills have been acquired and are maintained at the middle school level. At the same time, given studies regarding the decline of motivation for middle school students (Marsh, 1989a; Nottelmann, 1987; Steinmayr & Spinath, 2008), it is imperative to assess the role that mathematics self-concept plays in this number sense and mathematics achievement connection. The ECLS-K's broad spectrum of data provides the demographic, cognitive, and affective scope best suited for such analyses.

Structural equation modeling through path analysis will be employed to determine whether and to what extent number sense predicts mathematics achievement and mathematics self-concept in eighth grade. Furthermore, it will test the capability of student mathematics self-concept to mediate the relationship between number sense and mathematics achievement. The covariates of race, gender, socioeconomic status (SES), and special education services will serve as additional exogenous variables, all with direct paths to the predictor (number sense), mediator (mathematics self-concept) and outcome (mathematics achievement) measures.

Research Questions

In order to address the relationships between number sense, mathematics self-concept, and middle school mathematics achievement, the following three research questions and hypotheses were developed:

> **Research Question 1:** Does number sense proficiency in grade eight predict overall mathematics achievement?
>
> **Hypothesis 1:** Higher number sense proficiency predicts higher mathematics achievement when controlling for gender, SES, race, and special education services.
>
> **Research Question 2:** How does number sense proficiency in grade eight relate to mathematics self-concept?
>
> **Hypothesis 2**: Higher number sense proficiency predicts higher mathematics self-concept when controlling for gender, SES, race, and special education services.
>
> **Research Question 3**: Does mathematics self-concept mediate the relationship between number sense proficiency and middle school mathematics achievement?
>
> **Hypothesis 3**: Mathematics self-concept mediates the influence of number sense on higher mathematics achievement when controlling for gender, SES, race, and special education services.

Summary

Perhaps there is no longer a need to dwell on doing away with meaningless computation and memorization and realize that, with enough practice using more intuitive, meaningful models, such processes can happen on their own. Could it be that such methods of critical thinking are very effective, but only if we ensure that they result in some automaticity with number operations? Theoretical debates aside, middle school mathematics performance is at the forefront of federal and state education policy, and curriculum must ensure all students have adequate background knowledge to succeed upon gaining access to more advanced subject matter. Number sense incorporates the multitude of skills and conceptual frameworks critical to the transition from elementary to middle school mathematics, specifically a generalization of arithmetic as algebra becomes more prominent. Early number sense has the potential to impact mathematics growth positively in elementary years, and as society moves to push more and more middle school students into algebra, the current study will show how imperative it is that the evaluation of number sense persists throughout adolescence.

As affective domains are highly susceptible to developmental changes during adolescence, their resulting effect on continued mathematics interest and engagement also must be considered when assessing learning trajectories. In light of number sense's developmental impact, research regarding the beneficial impact of mathematics self-concept on performance needs to look closely at such skills at the middle school level. As these students must reach a new level of cognition and consider several basic ideas at once in order to transfer prior knowledge effectively, self-concept can help determine where motivation may be hindered. The present study moves across disciplines to weave together content and motivation as an improved means of analyzing student mathematics performance. It is not intended simply to confirm the importance of number sense and mathematics self-concept, but rather to explore their connection to one another and in the context of mathematics achievement in ways that inform curriculum and instruction, including pedagogical strategies that can enhance motivation while simultaneously bridging skills gaps. Understanding this relationship will aid states and localities in bringing about equity of opportunity and motivating students to pursue more complex mathematics coursework and subsequently professions.

Chapter 2: Literature Review

The current study was first inspired by teacher input and anecdotes regarding struggles with middle school mathematics instruction. The recent emphasis in policy and curriculum on access to algebra at the middle school level and the resulting changes it has yielded in terms of textbook design and high school curriculum paths heightens the implications of such experiences. The standards movement, No Child Left Behind (NCLB), and the inclusion of an algebra content domain by the National Council of Teachers of Mathematics (NCTM) in 2000 have guided recent literature regarding mathematics achievement toward the topics of algebraic content, thinking, and tracking during adolescence. Number sense, and how early skills in this domain predict mathematics achievement trends, has been a recent focus of elementary mathematics education. At the same time, but typically in psychology communities, research on middle school motivation in mathematics has emphasized longitudinal paths and contextual influences for general mathematics achievement without a focused look at the number sense construct.

This first goal of the present study is to carry the research on number sense's contribution to mathematics achievement over to the middle school level. Additionally, it will explore how mathematics motivation influences this relationship. An overview of current trends and areas of emphasis in middle school mathematics provides the backdrop for the study's rationale. A review of the number sense and mathematics self-concept constructs, as well as key findings for both over time, will provide the theoretical frameworks that support the research questions and proposed model. Weaving the affective domain of mathematics self-concept into research on number sense, a key component of mathematics achievement, will provide a more in-depth examination of adolescent mathematics success.

Middle School Mathematics

The importance of algebra. A great deal of research has been conducted to determine when to start algebra, how to integrate it, what content it demands, standards to improve it, and ways to assess it. This discourse gained significant momentum with the release of *A Nation at Risk* in 1983 and continued to expand with the requirements of NCLB legislation. In fact, algebra coursework was reported for only 16% of 13-year-old U.S. students in 1986 (Perie, Moran, & Lutkus, 2005), but for 2007 eighth graders in the Early Childhood Longitudinal Study, Kindergarten Class of 1998-99 (ECLS-K), 39% were enrolled in Algebra I or a more advanced mathematics course (Walston & McCarroll, 2010).

Increased algebra instruction in middle school is not a simple manner. In 2005, only 6% of U.S. eighth graders who took the National Assessment of Educational Progress (NAEP) were considered “advanced,” (Perie, Grigg, & Dion, 2005). This same percentage was found for advanced U.S. eighth graders who participated in the Trends in International Mathematics and Science Study (TIMSS) in 2007 (Mullis, Martin, & Foy, 2008). Linking 2005 NAEP data to results of the 2007 Program for International Student Assessment (PISA), even narrowing the focus to white students and students with at least one parent holding a college degree, revealed “advanced” student percentages of only 8% and 10%, respectively (Hanushek, Peterson, & Woessmann, 2010). Furthermore, these more advantaged populations were still outperformed internationally. Mathematics scores for White students in the United States were below 24 countries (for which all race/ethnicity groups were included), and U.S. students with at least one degree-holding parent had scores lower than 19 countries (for which all parent education levels were considered). A subsequent analysis of 2007 NAEP data in relation to the 2009 PISA by Hanushek, Peterson, and Woessmann (2011) placed the U.S. 32nd among participating nations and revealed that less than one third of U.S. students were deemed proficient according to PISA and NAEP scores. Though 2011 U.S. NAEP results indicate a rise in “proficient” and above scores for fourth and eighth graders, it is still noteworthy that over 8% of fourth graders and 27% of eighth graders were not at or above the “basic” level, characterized as “partial mastery of prerequisite knowledge and skills that are fundamental for proficient work at each grade” (National Center for Education Statistics [NCES], 2011, p. 2). These percentages indicate that far too many students are not succeeding in mathematics, and that this circumstance is even more pronounced in middle school.

Though significant implementation variations exist at the district, state, and federal level, and in the midst of conflicting research regarding the best way and time to introduce algebra to students, there appears to be strong agreement on one fact:

American students are not performing to their potential in mathematics, and algebra is a significant piece of the puzzle. In a 2010 Brown Center Report on American student learning (Loveless, 2011), public NAEP items from the number properties/operations and algebra content strands were coded according to their corresponding grade level on the 2010 Common Core State Standards for Mathematics (CCSSM). For items from the eighth grade number properties and operations domain, researchers calculated an average grade level of 5.2, indicating that these eighth grade NAEP items fell at about the fifth grade level. In fact, over 90% of these test questions were actually deemed below the eighth grade level according to the CCSSM. Additionally, the average CCSSM grade level for items from the NAEP algebra content domain was only 6.2. Though all students should have the opportunity to access and achieve in higher-level mathematics courses, the issue of readiness for such enrollment merits further attention.

When the National Opinion Research Center surveyed Algebra I teachers in 2007 on what they would like to change, the most frequent response was more emphasis on mastering basic mathematics skills and concepts. Loveless (2008) uses such findings to asserts that requiring ablebra for all middle school students is not an issue of ability, but rather readiness. Misplacement is detrimental to student progress, and to teachers who should not be expected to teach multiple years of mathematics curriculum in eighth grade. Also noteworthy, his cross-sectional study of NAEP data from 2007 revealed no correlation between the percentage of eighth graders taking advanced mathematics classes (Algebra I, Geometry, and Algebra II) and mathematics achievement (indicated by NAEP mathematics scores). Allensworth, Nomi, Montgomery, and Lee (2009) looked at the Chicago policy mandating Algebra I (or higher level coursework) for all ninth graders and found that low-ability students did show the most dramatic increases in higher mathematics course enrollment. However, the failure rates for low-ability and average-ability students almost tripled, and mathematics grades and overall grade point averages declined across all ability groups.

In a longitudinal study of the California Standards Test (CST), Liang, Heckman, and Abedi (2012) illuminated the controversy surrounding the 2008 mandate by the California State Board of Education that all eighth graders take algebra. Not only did they discover a significant decline in students taking Geometry in ninth grade compared to their eighth grade Algebra I enrollment (Geometry being the next logical course for mathematics), they found that the chance of passing the ninth grade Algebra I CST was greater for students who had taken General Mathematics in eighth grade than for those who had taken and failed the Algebra I CST in eighth grade. One method of addressing likely skill gaps has focused on the conceptual understanding accompanying arithmetic operations. Emphasizing number properties and reasoning about strategies, and eventually algorithms, has provided a stronger delineation of how to best address the arithmetic to algebra transition.

Arithmetic to algebra. Students who succeed in arithmetic have a solid grasp of basic numeric concepts and arithmetic operations as well as a strong numerical working memory (Geary, 1994; Geary, Bow-Thomas, & Yao, 1992). Brown and Quinn (2007) assert that “elementary algebra is built on a foundation of fundamental arithmetic concepts” (p. 8), and according to Wu (2001), grasping these foundations is critical to the study of algebra given that “algebra is the generalisation of arithmetic and the first experience in symbolic representation of numbers” (p. 1). Arithmetic is frequently referenced as a prerequisite to algebra success, so it is of particular interest that students often struggle to bridge the gap between arithmetical and algebraic concepts (Linchevski & Herscovics, 1996; Olive & Cağlayan, 2007). At the middle school level, whether students are enrolled in General Mathematics, Pre-algebra, Algebra I, or Geometry, there is a clear shift in curriculum and instruction from arithmetic to algebra.

As far back as the 1920s, studies were conducted to determine mathematical difficulties associated with algebra. With 25 years of experience in algebra instruction, Coit (1928) conducted a study of ten sections across four high schools and reported that students often had trouble with simple and quadratic equations due to problems with fraction operations, substitution, and negative number procedures. Even factoring out careless errors, test results revealed that many students were failing basic mathematics problems. Drills were created and carried out by teachers over a ten-day period, with significant improvements noted, not only at the end of this period, but two-months later with no additional drilling. In this case, addressing computation skills, a component of number sense, was clearly beneficial in bridging the arithmetic to algebra gap.

In another study by Cooke and Fields (1932), the New Stanford Arithmetic Test, the Detroit Advanced Intelligence Test, and the Columbia Research Bureau Algebra Test were administered to algebra students, and results indicated a stronger relationship between arithmetic ability and algebra achievement than between intelligence and algebra achievement. Such findings have continued throughout the years, focusing on a multitude of skills from carrying out spontaneous operations on unknown quantities (Herscovics & Linchevski, 1994) to fraction proficiency (Brown & Quinn, 2007) to computational fluency (Tolar, Lederberg, & Fletcher, 2009). Given the long history of the studied connection between elementary-level mathematics skills and higher-level mathematics coursework like Algebra I, it is obvious that a focus on middle school algebra curriculum relies heavily on specific skillsets. If teachers are not aware of the full range of arithmetic obstacles for students, they may not be able to address significant cognitive stumbling blocks in algebra coursework appropriately.

Ideally, middle school students possess strong conceptual and working-memory skills and are able to make good strategy decisions as they solve problems. According to Geary (1994),

> In all, individuals who are skilled at mathematics reasoning appear to execute basic computations quickly and automatically, are able to keep important information in mind while performing other operations, and have developed schemas to aid in the representation, translation, and solution of mathematical problems. (p. 146)

Research on the advantages of effectively incorporating algebraic thinking into elementary curriculum and instruction (Day & Jones, 1997; Carraher, Schliemann, Brizuela, & Earnest, 2006; Ketterlin-Gellar, Jungjohann, Chard, & Baker, 2007) has provided a means to bridge the arithmetic to algebra transition in one direction, namely, by pushing algebra to lower grade levels. But what happens if students still do not have adequate facility with numbers by the time they reach middle school? Why are there not measures to ensure mastery goals for such elementary level curriculum continue as students move up in grade levels? In essence, why not also push number sense to upper grade levels? The mathematics construct needed to accommodate overlap and progression from elementary to middle school successfully must ensure critical thinking is included early, computational fluency is maintained, and problem solving incorporates both.

Number Sense

Narrow approaches to mathematics instruction, such as those with a dominant focus on either memorization or conceptual understanding, make for memorable and dramatic slogans and reforms, but they also overshadow the art of teaching for authentic learning and understanding. Additionally, they undermine the variations in student skill proclivities, learning styles, and levels of development, and the flexibility that often must accompany effective classroom instruction and management.   When studies focus on specific mathematics content, subsequent curriculum change can neglect other critical skillsets. In order to most effectively improve mathematics education, constructs must include appropriate elements of content and pedagogy. As the National Research Council (NRC) suggests, "The integrated and balanced development of all five strands of mathematical proficiency (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition) should guide the teaching and learning of school mathematics" (2001, p. 11). An ongoing obstacle to school achievement in mathematics is the tendency to focus on one strand at the expense of others. The literature described in this chapter will provide an overview of how a well-defined number sense framework can best inform the study of foundational skills critical to middle school mathematics success.

Why computational fluency isn't enough. The importance of working memory and automaticity, and the ability of their resulting efficiency to influence higher-order thinking and generalizability are critical to mathematics progress, and they are frequently incited in the description of computational fluency. The automization that comes with greater computational fluency can allow for developmental gains as operational memory capacity demands decrease (Bloom, 1986; Case & Bereiter, 1984; Gagne, 1983). When a student expends a large of amount of effort and cognitive assets attending to basic computation, he or she may have little available for more complex problem solving. According to the National Mathematics Advisory Panel (2008), the automaticity that is the product of practice "frees up working memory for more complex aspects of problem solving" (p. 30). Automaticity with simpler concepts allows student to make effective use of derived facts and direct-retrieval procedures, and in turn, solve more complex problems (Geary, 1994). Furthermore, automatic processing makes solving complex problems more efficient and less susceptible to errors (Geary & Burlingham-Dubree, 1989; Geary & Widaman, 1992; Kaye, 1986; Resnick & Ford, 1981). For example, even when poor addition fact processing does not hinder conceptual understanding or procedural knowledge, it may in fact lead to continued inaccuracy in subsequent mathematics tasks (Cumming & Elkins, 1999). Thus, algebra actually relies on computational fluency in the sense that higher-level mathematics courses are facilitated by the goals of computational fluency, as in the case of number combination mastery (Jordan & Levine, 2009; Baroody, Bajwa, & Eiland, 2009).

The automaticity that accompanies computational fluency (Skinner, Fletcher, & Henington, 1996; Skiba, Magnusson, Marston, & Erickson, 1986; Singer-Dudek & Greer, 2005) facilitates the higher-order thinking characteristic of middle school mathematics, which differs from arithmetic by requiring more decision making as problems have multiple operations and methods for deriving solutions (Geary, 1994). Such fluency also may be linked to algorithms as a result of their aim to provide procedural efficiency. Although many educators scoff at the term "algorithm" in light of the potentially oppressive ways it can be employed, Ball et al. (2005) assert that algorithms have practical merit and can contribute to the theoretical development of mathematics, specifically the development of computational fluency. Algorithms serve as guidelines by providing clear steps that may aid in understanding process and meaning when working through a solution. With effective instruction, they allow for generalization that can transition students effectively from arithmetic to algebra. For instance, procedures in carrying out whole number tasks like multiplication are applicable to calculating partial products and adding when working with polynomials (NRC, 2001). Mathematics problems often have multiple approaches, and proficient problem solvers will understand how and when to utilize different algorithms along the way.

In addressing the mathematics education debate that pits basic skills against conceptual understanding and facts against higher-order thinking, Wu (1999) claims such controversies are the product of a general misunderstanding of mathematics, not only in education but in society in general, and that it is critical to understand that, "precision and fluency in the execution of the skills are the requisite vehicles to convey the conceptual understanding" (p. 1). Meticulously comparing student-driven methods to various algorithms, he asserts that skills utilized within these algorithms and in the selection of algorithms are accompanied by deep mathematical comprehension. Bass (2003) also claims that skills and concept go hand in hand, and emphasizes that algorithms need not be taught as rote and meaningless learning procedures. By contrast, an algorithm should be taught via multiple examples and in ways that reveal the mathematical significance of all of its components in reaching a solution. In this way, it provides an "image of developing computational fluency in which basic skill is both strongly present and inseparable from conceptual understanding" (Bass, 2003, p. 322). The flexibility associated with computational fluency indicates that one no longer relies on formal algorithms and can make connections between multiple mathematics concepts (Russell, 2000).

Baroody et al. (2009) address the instructional implications of the controversy surrounding fluency, and in concentrating on students with mathematical difficulties, claim that, "In effect, fluency with the basic number combinations begins with and grows out of number sense" (p. 69). Number sense is also clear in the National Mathematics Advisory Panel's (2008) "General Principles for Learning" across all mathematics content areas, which states that, "conceptual understanding, computational fluency, and problem-solving skills are each essential and mutually reinforcing, influencing performance on such varied tasks as estimation, word problems, and computation" (p. 30).

Even assuming that computational fluency sufficiently incorporates the conceptual understanding reflected in number sense, it is limited in its ability to portray such comprehension in students, and there is no way to pinpoint when a student maximizes both in their developmental processes. For example, Reys and Yang (1998) studied sixth and eighth grade students in Taiwan and discovered that higher scores by students on complex computation problems were not necessarily accompanied by corresponding high scores in the domain of number sense. In practical terms, some students might understand that ¼ is the same as 0.25 and can conceptualize this with a diagram, the size of their hamburger at McDonald's, or the quarters that make up a dollar. But others might simply know the equality from memorizing basic fraction/decimal conversions or because they can quickly divide one by four mentally. Number sense accounts for computational fluency, but also asserts that students engage with numbers and operations in a way that entails flexibility, relevance, and application.

Development of a dynamic and comprehensive framework. Few would challenge Greeno's (1991) characterization of number sense "theoretically as a form of cognitive expertise" (p. 170), nor disagree with Berch's (2005) assertion regarding the complications of operationalizing such a framework. Though these theorists eloquently delineate the justifiably elusive characteristics of number sense, practitioners and policymakers still need a way to gauge the impacts of various interventions, curriculum designs, and assessment strategies. In 2008, the National Mathematics Advisory Panel defined number sense, in simplest and broadest terms, as incorporating "an ability to immediately identify the numerical value associated with small quantities (e.g., 3 pennies), a facility with basic counting skills, and a proficiency in approximating the magnitudes of small numbers of objects and simple numerical operations" (p. 27). Regarding formal instruction, the panel stated that number sense "requires a principled understanding of place value, of how whole numbers can be composed and decomposed, and of the meaning of the basic arithmetic operations of addition, subtraction, multiplication, and division" (p. 27). The NCTM's president's message in 2008 (Fennell) emphasized the importance of number sense in education, touching on "place value, composing and decomposing numbers, understanding how addition, subtraction, multiplication, and division work, acquiring basic facts, and developing fluency with whole-number operations" as well as "how the commutative, associative, and distributive properties work and how they are used in learning basic-fact combinations, adding columns of numbers, and seeing how the multiplication algorithm works" (p. 3). The broad nature of these topics and the evolving character of number sense as students experience more mature curriculum reflect the difficult nature of creating a definitive measure of number sense.

The model best able to address the expansive theoretical underpinnings of number sense while providing a concrete tool for practitioner use and researcher analysis is that developed by McIntosh, Reys, and Reys (1992). It holds the following:

> Number sense refers to a person's general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and to develop useful strategies for handling numbers and operations. It reflects an inclination and an ability to use numbers and quantitative methods as a means of communicating, processing, and interpreting information. It results in an expectation that numbers are useful and that mathematics has a certain regularity. (p. 3)

Acknowledging that the term "number sense" explains the broadening scope of skills that begin with arithmetic and grow to encompass properties, logic, and authentic applications, these authors summarize the complex interplay of numbers, operations, and settings in the following three tenets: "knowledge and facility with numbers, knowledge and facility with operations, and applying knowledge and facility with numbers and operations to computational settings" (p. 4). More effectively than any other number sense framework, theirs transforms Greeno's (1991) metaphor of environments for conceptual domains to measureable form. The adaptability permitted by the interaction of these three components in terms of content and developmental stages is both a detriment and an advantage. While its flexibility limits articulation of the number sense construct, it also allows for a fluid framework over time that incorporates metacognition, growth, and context.

From an empirical standpoint, many studies have employed factor analysis to determine the underlying structure of number sense. Jordan, Kaplan, Olah, and Locuniak (2006), for example, established a two-dimensional structure for their number sense battery, which included number skills in counting, recognition, estimation, patterns, and combinations and that incorporated nonverbal calculations and story problems. The first factor encompassed basic number skills, and the second factor described conventional arithmetic. Lago and DiPerna (2010) also found a two-factor model, one factor focused on number-related skills like counting and identifying numbers, and another pertaining to rapid naming. Though a great deal of the aforementioned McIntosh et al. (1992) framework can be categorized according to these tasks, one area the factors do not adequately reflect is application. This last component of the McIntosh et al. (1992) framework incorporates growth and context, elaborating on the ways in which number sense reveals itself differently as students uniquely engage in mathematical thinking.

Number sense findings. Consistent results have revealed that early number sense is predictive of future mathematics performance in addition to growth rates in mathematics achievement. To date, the majority of longitudinal studies have focused on early elementary years. Jordan, Kaplan, Locuniak, and Ramineni (2007) found that number sense measured in kindergarten, in addition to growth in number sense from kindergarten to first grade, was a “reliable and powerful predictor” of first grade mathematics performance, accounting for 66% of its variance (p. 40, 42). Jordan, Glutting, Ramineni, and Watkins (2010) administered a number sense brief screener to assess student number sense longitudinally and determined that kindergarten results were predictive of third grade mathematics proficiency. Earlier, Jordan, Glutting, and Ramineni's (2009) conducted a longitudinal study of students that measured number sense (including addition and subtraction calculations as well as number magnitude and combinations) from first grade to the end of third grade. Results indicated that number sense over this period strongly predicted mathematics achievement, including applied problem solving.

There are also longitudinal studies that do not directly reference the number sense construct, but that have analyzed components of the framework with similar results. For example, Mazzocco, Feigenson, and Halberda (2011) found that preschool ability to represent numbers and their capacities mentally served as a predictor of mathematics performance at the age of six. Jordan, Kaplan, Ramineni, and Locuniak (2009) analyzed number competencies for children over a three-year period via growth curve modeling and found that higher kindergarten number competence modestly and significantly predicted mathematics performance through third grade as well as mathematics achievement growth as students moved from grade one to three. Looking specifically at place-value skills, Moeller, Pixner, Zuber, Kaufmann, and Nuerk (2011) found first grade skills in this area were predictive of third grade addition tasks. Bodovski and Farkas (2007) used data from the ECLS-K to look at students' mathematics skills as they progressed from kindergarten to third grade and found that students with higher initial skills (measuring topics from basic number and shape to multiplication and division) showed higher mathematics growth than those with lower initial scores.

Longitudinal studies on pre-existing mathematics skills or prior mathematics knowledge at very early elementary school ages are also pertinent given that such skills are typically subsumed by number sense. For example, skills that a kindergarten student might come to school already possessing include number identification, counting, and relative size, all of which contribute to overall number sense. These components of prior knowledge reveal the same empirical advantages as studies referencing the number sense construct or components of it. In their study of Estonian children, for example, Kikas, Peets, Palu, and Afanasjev (2009) found that students with greater pre-mathematics skill levels at the beginning of first grade showed faster growth in mathematics and possessed higher mathematics scores at the conclusion of the study at the end of third grade. Duncan et al. (2007) looked at six longitudinal studies (primarily spanning elementary years), focusing on entry-level cognitive, socioemotional, and attention skills data in relation to subsequent mathematics achievement. They confirmed that entry-level skills in all of these areas determined later achievement, the strongest of which was initial mathematics skills. Finally, Morgan, Farkas, and Wu (2011) discovered that students with mathematics difficulties in kindergarten had the lowest growth trajectories in mathematics from grades one to five, and that those with no initial difficulties experienced the highest growth.

A very effective way to target intervention and remediation is to study specific mathematics skills in order to improve overall mathematics achievement. For this reason, extensive research exists on various components of number sense. Looking at the whole number and decimal skills of sixth graders in Taiwan, Yang (2005) found that an emphasis on written algorithms hindered number sense development, as students were unable to explain their mathematical reasoning or estimate sufficiently. Star and Rittle-Johnson (2009) looked at the broad implications of a number sense component in relation to problem solving, finding that students with stronger estimation skills were more adept at adopting different strategies. Number sense also has been studied in the context of specific objectives, such as helping to better assess middle school understanding of percentages (Gay & Aichele, 1997), how its relationship with arithmetic skills is mediated by symbolic number ordering in university students (Lyons & Beilock, 2011), and how it is can be interpreted in relation to numerical reasoning (McIver, 2005). Furthermore, it has been proposed as a way to bridge gaps that may be the result of other skill deficiencies, as in the case of studies linking number sense and computational fluency (Griffin, 2004; Griffin, Sarama, & Clements, 2003). In fact, Cowan et al. (2011) used their established link between conceptual knowledge and basic calculation skill to support the number sense implication that "conceptual knowledge explains the relationship between basic calculation proficiency and mathematics achievement" (p. 787).

In light of elementary school findings revealing the potential of number sense to impact future performance and achievement growth, middle school implications for number sense warrant further exploration. Number sense studies that do exist at the middle school level focus on strategies to develop the construct (Cutler, 2001; Greenes, Schulman, & Spungin, 1993; Markovits & Sowder, 1994) or on number sense in relation to other skills, such as computation (Gay & Aichele, 1997; Reys & Yang, 1998).

Number sense at the middle school level has also been studied as a way to improve other mathematics skills. For example, Jordan (2007) emphasized how middle school skills like factoring, rational number computation, and working with algebraic equations rely on calculation fluency, and how the development of number sense can work to overcome weakness in this area.

A broader look at middle school mathematics studies reveals research on specific subsets of number sense as a means of heightening student mathematics achievement, including the positive effects of computation skills (Geary, 1994; McIntosh & Reys, 1997; Reys, Reys, & Hope, 2010; Royer, Tronsky, Chan, Jackson, & Marchant, 1999; Scott, 1987). In their study linking deficient whole number, decimal, and fraction skills to difficulty determining reasonableness of answers in eighth grade Kuwaiti students, Alajmi and Reys (2010) recommended a stronger emphasis on number sense. This emphasis has also been the focus of curriculum studies that proclaim the advantages of material that allows for enhancing student sense-making (Ben-Chaim, Fey, Fitzgerald, Benedetto, & Miller, 1998).

This glimpse into number sense research highlights the importance of the construct, and particularly, its critical influence as students enter school. However, it also reveals the inadequacy of such research beyond the elementary years. If mathematics skills at the kindergarten level are capable of influencing overall mathematics achievement and growth throughout elementary school years, a reasonable deduction is that number sense continues to be a critical component of success in mathematics through the middle school years and beyond, particularly in the context of course selection and tracking which becomes more pronounced in adolescence. For instance, utilizing data from the Colorado Student Assessment Program, Billings (2009) discovered a positive relationship between fifth grade number sense and ninth grade algebra scores, providing evidence of the ability of elementary number sense to predict early high school performance. This use of the number sense framework in the context of mathematics achievement represents a start at moving the longitudinal analysis through the middle school level and reflects the continued influence of number sense at higher levels of curriculum. Even though the present investigation does not assess number sense in ninth grade, others studies on number sense and elementary mathematics achievement growth (Jordan et al., 2010; Moeller et al., 2011) provide a powerful basis for predicting those students with initially higher number sense skills will maintain a mathematical advantage through middle school and high school.

Societal gaps and the importance of number sense. Research reveals that low-income elementary students experience flatter number sense trajectories than their peers and that number sense deficiencies are more prevalent in low-income students (Jordan et al., 2007; Jordan et al., 2006; Jordan, Kaplan, et al., 2009; Jordan & Levine, 2009). A similar pattern is present for students with weak prior mathematics knowledge and pre-existing mathematics difficulties (Kikas et al., 2009; Morgan et al., 2011). This trend is compounded at the middle school level by achievement declines often associated with major alterations in material, pedagogy, expectations, and general structure (Schielack & Seeley, 2010). NAEP research on eighth grade mathematics revealed that mathematics students below the tenth percentile rank who are placed in higher-level classes like Algebra I are more likely to be poor, Black and Hispanic, have parents without the optimal knowledge or resources for academic assistance, be enrolled in urban high-poverty schools, and have less experienced and less qualified teachers (Loveless 2008, 2010). Furthermore, the lower performing Algebra I students in the NAEP study scored roughly seven times lower than their peers in the same courses, with prominent gaps in the content areas of number systems, fractions, and percentages, all of which are key components of number sense. Such societal gaps are also evident in an ECLS-K study that revealed socioeconomic discrepancies in eighth grade algebra enrollment (Walston & McCarroll, 2010). This disparity is even more dramatic when one considers the positive influence that higher-level high school mathematics courses have on students attending four-year institutions following graduation (Schneider, Swanson, & Riegle-Crumb, 1997) and pursuing science, technology, engineering, and mathematics in college (Chen, 2009).

An analysis of NAEP mathematics scores by Johnson and Kritsonis' (2010) reveals an achievement gap that appears to be widening rather than narrowing as it did in the 1970s and 1980s. They assert that minority students, typically placed in lower mathematics tracks relative to their white peers, have less access to resources, and do not have adequately trained and experienced teachers. Additionally, they point out that the 33-point disparity between White and African American students in 1990 increased to 39 points in 2000. Similarly, the Latino/White gap went from 28 to 33 points during this period. And in 2003, on average, African American and Latino students in grades four and eight performed three years behind their White peers in mathematics. Looking more deeply at the NAEP scores, it appears that the gap exists across all mathematics content areas, and tragically, only worsens with rising complexity in the subject. According to Loveless and Coughlan (2004), although the main NAEP has revealed heightened mathematics performance by U.S. students, long-term trend results show an increase ten times smaller. Conducting an item-analysis of computation skills embedded within the content domain "number sense, properties, and operations," their results indicate improvements in scores in the 1980s that actually began to reverse in the 1990s. Though not sufficient to measure number sense, this research does shed light on the achievement gaps in some its major components.

Aside from achievement differences related to race and socioeconomic status, gaps appear to widen based on the foundational knowledge students bring to school and/or acquire at very elementary levels. Students with strong foundational number knowledge in first grade are more inclined to gain advantages from their elementary mathematics learning (Baroody, Lai, & Mix, 2006). Children with mathematics difficulties as well as those at risk for low achievement in the subject of mathematics often experience difficulty with memorization (Baroody et al., 2009). In fact, by the end of their first grade year, students without a mastery of addition facts are at a major disadvantage for further mathematics operations, thereby making them more prone to mathematics difficulties, and subsequently resulting in "a spiral of failure and frustration" (Baroody et al., 2009, p. 69). Jordan, Kaplan et al. (2009) used growth curve modeling to determine the positive influence of early number competence on elementary mathematics achievement. Results from their study of six public schools showed that kindergarten number competence modestly and significantly predicted achievement growth rate from grades one to three. Additionally, they found that number competence development was a modest and significant predictor of third grade mathematics achievement.

Research has clearly shown the need to focus on specific skillsets in preparation for middle school mathematics, to ensure that students begin to think algebraically before reaching this level of schooling, and to consider the advantages of early mathematics knowledge and how this varies according to societal contextual factors. Asserting that number sense is the strongest mathematics construct to study as a predictor of middle school mathematics achievement, as it best incorporates all of these components, the current study seeks to look within the student as well. How students are influenced by perceptions of their own ability may significantly interact with adolescent mathematics achievement, including number sense. An appropriate affective construct must be selected prior to analysis of this relationship.

Mathematics Self-Concept

Middle school is a time of development when changes and disruptions in social affiliations at school with students and teachers as well as parental interactions at home impact adolescent identity, competence, and motivation (Wigfield & Wagner, 2005). During this same period, academic material requires higher-level cognition and introduces new topics that influence these evolving relationships. Evaluating motivation in the context of competence can reveal how student behavior is best energized and directed (Elliot & Dweck, 2005).

Student identities have the potential to impact educational experience and performance deeply, and schools have the capacity to influence student identity development in the ways they provide engaging instruction and facilitate motivation and cooperation (Eccles & Midgley, 1989; Wigfield & Wagner, 2005). Reciprocally, identity formation has the potential to deeply impact student educational experience and performance (Roeser & Lau, 2002). For example, how students perceive their ability to manipulate algebraic equations in middle school can be a strong determinant of their entry-level mathematics course in high school, in turn prescribing the highest level of mathematics they attain before graduating, the subsequent major(s) they pursue in college, and finally the career paths they are eligible to enter. As students absorb perspectives on the value of mathematics and their potential to comprehend it, their propensity to seek additional challenge and pursue more complex coursework can have life-altering consequences. The current study seeks to determine if and how the evolution of competency beliefs and interest in mathematics exerts influence on the contribution of number sense to middle school mathematics success and development.

As it turns out, maturation may even exacerbate identity formation once established. Jacobs, Lanza, Osgood, Eccles, and Wigfield (2002) found a steady decrease in mathematics competency beliefs as children progressed from grade one to twelve, and Gottfried, Marcoulides, Gottfried, Oliver, and Guerin (2007) found a decline in intrinsic mathematics motivation and mathematics achievement from age nine to 17, spanning elementary, middle, and high school. Similarly, research has revealed a drop in self-concept of ability, interest, values, and expectations regarding mathematics during the elementary to middle school transition (Eccles et al., 1989; Eccles & Midgley, 1989; Mac

Iver & Reuman, 1988). In fact, middle school itself can be a general detriment to motivation (Eccles et al., 1993), and mathematics is no exception (Schielack & Seeley, 2010; Watt, 2008). Such early adolescent identity determinations in conjunction with declining views of ability may not only hinder continued effort and success in mathematics, but also have the potential to impact how students develop their sense of purpose and orientation in the world.

There are a variety of factors, biological as well as social, influencing middle school affective domain development. The result these factors exert on motivation can influence performance, goals, academic interest, and conceptions of self, and is therefore critical to understanding how to best instruct and guide students mathematically at this juncture in their schooling. Although there are multiple motivation constructs appropriate for evaluating mathematics, they rely on context and vary according to their emphasis. Based upon the items utilized in the ECLS-K for determining perceived interest and competence in mathematics, self-concept figures to be the best measure of mathematics motivation for the present study. Therefore, an overview of research in this area and the clear advantages of this construct in the context of middle school student development both academically and socioemotionally is provided.

Most individuals define self-concept generally as the way in which a person perceives him/herself, and this idea serves as a large umbrella term for academic research on self-concept as well. As a motivational construct, self-concept has a long history accompanied by extensive studies that have yielded multiple models, subsections, and statistical evaluation methods. Its origin can be traced to William James in 1890 when he first recognized different elements of self-concept by claiming that an individual developed a general self evaluation based on an average of more specific evaluations (Marsh, Xu, & Martin, 2012). The framework utilized in the current study, however, began in 1976 when Shavelson, Hubner, and Stanton characterized it as multifaceted and hierarchical. Their model states that environment (including interactions with others and attributions) contributes to perceptions of oneself and that there are multiple levels of self-concept. Furthermore, such levels become more situation-specific, less stable, and increasingly versatile as they develop from a global concept of self over time.

The Student Description Questionnaire (SDQ) was developed to provide strong empirical support for the multidimensional nature of self-concept (Marsh, Smith, & Barnes, 1983). Subsequent research utilizing this instrument yielded substantial support for the strength of the Shavelson, Hubner, and Stanton multifaceted model (Marsh, 1990a; Marsh, 1993b; Marsh, Craven, & Debus, 1998; Marsh & Hocevar, 1985; Marsh, Parker, & Barnes, 1985; Marsh, Relich, & Smith, 1981,1983; Marsh & Shavelson, 1985; Marsh, Smith, & Barnes, 1983; Marsh, Smith, Barnes, & Butler, 1985). Over time, statistical analyses have resulted in academic and nonacademic subsections of the SDQ and three separate instruments (SDQ I, II, and III) according to age level. Perhaps most significant among these measures is the "Marsh/Shavelson Revision" which presented a separation of academic mathematics and verbal domains as advantageous over a general measure of academic self-concept (Marsh & Shavelson, 1985). The statistical support for these differentiated self-concepts (Marsh, 1986, 1989b) expanded to eight academic areas (Marsh, 1992) and even found support for noncore academic subjects (Marsh, 1990d; Marsh, Byrne, & Shavelson, 1988).

The differentiation of self-concept into academic and nonacademic realms, and the subsequent separation of academic self-concept into different subject areas, was the result of ongoing research, typically based on the use of one or more of the SDQ instruments. The content specificity of academic self-concept continued to be supported by data revealing invariance across gender and age with regard to verbal, mathematics, academic, and even general self-concept measures (Marsh, 1993b). Along with this consistency came an emphasis on the Shavelson, et al. (1976) model's claim of self-concept's differentiability from other constructs, such as academic achievement, and a search for causal relationships among them. The internal/external frame of reference model (Marsh, 1986), for example, allows for self-concept in specific academic areas to be shaped not only by students' comparison of their performance to those around them, but also their own performance in a specific subject relative to other subject areas. This theory was confirmed by a review of multiple studies on mathematics and verbal self-concept revealing a link between mathematics and verbal achievement, but not between mathematics and verbal self-concept (Marsh, 1986, 1990a; Marsh et al., 1988). Academic self-concept's content-specificity has also been strengthened by continued findings confirming the link between subject specific self-concept and corresponding subject achievement (Marsh, 1992, 1993a; Marsh et al., 1988; Marsh, Parker, & Barnes, 1985; Marsh, Smith, Barnes, & Butler, 1983; Marsh, Trautwein, Lüdtke, Köller, & Baumert, 2006; Marsh, Walker, & Debus, 1991; Marsh & Yeung, 1998). Mathematics self-concept as measured by the SDQ II (Marsh, Parker, & Barnes, 1985), intended for younger adolescents in grades seven through 12 and adapted for the ECLS-K by its developers, provides the ideal framework for middle school mathematics motivation given its construct validity and clear link to mathematics achievement.

The present study will assume the reciprocal effects model (REM) for academic self-concept and academic achievement. The REM model was inspired by Marsh's longitudinal findings supporting the theory that academic self-concept predicts academic performance beyond prior achievement (1990b). This view regarding the causal ordering of achievement and self-concept in academics has been strongly supported through studies revealing the cause and effect relationship between academic self-concept and corresponding academic achievement (Guay, Marsh, & Boivin, 2003; Marsh, Byrne, & Yeung, 1999; Marsh & Craven, 2006; Marsh & Martin, 2011; Marsh & O'Mara, 2008, Marsh & Yeung, 1997; Yeung & Lee, 1999). Looking at the relationship of number sense and mathematics achievement being mediated by mathematics self-concept, the REM model appropriately allows for the bi-directionality of paths between all constructs.

Existing research on mathematics self-concept. Mathematics self-concept research has sought to decipher the relationship between a student's perception of self and his/her overall performance in mathematics at various age levels, with a particular emphasis on elementary school years. However, research must concurrently consider the documented decline in mathematics self-concept as students move through adolescence, including findings that such a decline represents a trend across multiple countries (Nagy et al., 2010) and that it may reach its lowest point in ninth grade (Marsh, Parker, & Barnes, 1985) occurring at the conclusion of or immediately following middle school. Although self-concepts may increase again at various times, it is important to note the likely state of mathematics self-concept at the middle school level where changing mathematics coursework has such dramatic implications, as well as to consider the heightened specificity of correlations between subject areas and corresponding subject self-concepts at this developmental stage (Marsh & O'Mara, 2008).

The link between academic self-concept and academic achievement has reinforced the multidimensionality of the self-concept construct and revealed how subject areas become more important as students age. In her review of the extensive research on general and academic self-concept studies in 1984, Byrne noted "unquestionably, a persistent relationship between one's self-concept and his or her academic achievement (p. 440)," and this is nowhere more evident than in studies focused on mathematics. In his testing of the internal/external frame of reference path model, Marsh (1986) found positive and significant path coefficients for all 13 of his analyses on mathematics achievement and mathematics self-concept. This relationship also has been evidenced at multiple age levels via SDQ III studies (Marsh & O'Neill, 1984; Marsh & Shavelson, 1985; Marsh, Trautwein et al., 2006), SDQ II studies (Marsh, Parker, & Barnes, 1985; Marsh & Yeung, 1998; Skaalvik & Skaalvik, 2009; Yeung & Lee, 1999), and SDQ I studies (Marsh, Relich, & Smith, 1981; Marsh & Shavelson, 1985; Marsh, Smith, & Barnes, 1985; Marsh, Smith, Barnes, & Butler, 1983; Skaalvik & Valås, 1999; Senturk, 2000). Likewise, there are established links between mathematics self-concept and coursework enrollment and interest (Marsh, 1989b; Marsh & Yeung, 1997, 1998), and even later educational attainment (Marsh & O'Mara, 2008). Longitudinal studies have further revealed how mathematics self-concept predicts mathematics achievement (Marsh, 1990b; Marsh & O'Mara, 2008; Marsh, Köller, Trautwein, Ludtke, & Baumert, 2005; Marsh & Yeung, 1998). Perhaps most telling is the meta-analysis of longitudinal research on the influence of self-beliefs and academic achievement by Valentine, DuBois, and Cooper (2004). Some 54 of the total 60 studies reviewed reveal self-beliefs, including self-concept, self-efficacy, and self-esteem, to be a significant and positive predictor of later achievement (even after accounting for the impact of previous achievement). Moreover, this finding was consistent for all eleven mathematics self-concept analyses within the study.

Across multiple age levels and over time, it is clear that mathematics self-concept and mathematics achievement are connected. Research findings on mathematics self-concept show the need for more of a focus on middle school and a closer look at specific areas of mathematics achievement. As research has revealed the advantage of looking at mathematics self-concept over general academic self-concept and global self-concept, it seems logical that a deeper analysis of components of mathematics achievement, like number sense, might provide better descriptive results and subsequently more targeted and effective curricular and instructional adaptations and strategies.

Summary

It may be argued that number sense has always provided a strong underpinning for the ongoing research and debate surrounding instructional content and methodology in mathematics. Challenges have involved finding ways to accurately define and assess number sense, and such obstacles are unavoidable given the individuality of student mathematical behaviors in development and practice. Though unable to tangibly assess certain skillsets, the CCSSM detail how poignant number sense is to mathematics development in the way that they strive to define “what students should understand and be able to do in their study of mathematics" (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010, p. 4) rather than providing solely a checklist of discrete skills. Through consistent expectations like the Student Mathematical Practices (SMPs), calling for ongoing sense making and contextualization, these standards have woven number sense into all stages of mathematics development. To date, studies on number sense have sought to verify its importance over less contextualized frameworks, to reveal the multiple ways in which it may be evident to students, and to show patterns in its development through the elementary years. By evaluating number sense beyond the elementary years, research can strengthen the CCSSM's call for SMPs across all grade levels and content areas.

Mathematics self-concept, though showing decline as students mature, predicts mathematics achievement across elementary, middle, and high school. Extending number sense to middle school mathematics achievement must consider the role of mathematics self-concept. In their analysis of propensity, opportunity, and antecedent factors using ECLS-K data for students as they progressed from kindergarten to third grade, Byrnes and Wasik (2009) discovered that the strongest predictors of mathematics achievement were propensity items, or those factors pertaining to student ability or engagement following exposure to various content (Byrnes & Miller, 2007). Although antecedent factors like SES and parent expectations and opportunity factors involving instructional material and strategy contributed to their model of mathematics achievement, the strongest contributors were ratings for motivation and self-regulation and pre-existing mathematics skills at each grade level. This critical contribution of cognitive and affective elements to mathematics achievement at the elementary level provide strong theoretical support for the goals of the present study.

Mathematics skills do not develop in a vacuum, reliant only on sources external to the student, such as curriculum, teacher knowledge, and pedagogical techniques. And mathematics self-concept is clearly susceptible to multiple internal and external sources, including mathematics achievement. In 1984, Reyes looked at the importance of affective variables in mathematics education, specifically in their ability to impact classroom learning environments, and Liang, Heckman, and Abedi (2012) recently reflected on the need to incorporate learning science research like motivation into studies of mathematics achievement in order to enhance classroom engagement. Given the predicted impact both have, the current study will examine their individual as well as interactive impact on middle school mathematics in order to better design and carry out curriculum and instruction.

Chapter 3: Methodology

Given the continuing political and social emphasis on standards and assessment in mathematics education and the national trend to increase and accelerate algebra access, it is imperative to look more specifically at possible skill gaps and psychological determinants of success that are present in middle school. Studies have revealed the future benefits of elements of early number sense (Jordan, Kaplan, Ramineni, & Locuniak, 2009; Mazzocco, Feigenson, & Halberda, 2011), but little research has determined whether number sense skills are mastered and maintained at the middle school level. At the same time, in light of studies regarding the decline of motivation at this developmental stage (Marsh, 1989a; Nottelmann, 1987; Steinmayr & Spinath, 2009), it is desirable to assess the role that mathematics self-concept plays in this number sense and mathematics achievement connection.

Accordingly, the goal of the current study is to look at adolescent mathematics achievement in the context of both skill and motivation with a secondary analysis of the Early Childhood Longitudinal Study, Kindergarten Class of 1998-99 (ECLS-K). This chapter will begin with a description of the dataset and student demographics in the study. A detailed specification of how the data were used to measure number sense, mathematics achievement, and mathematics self-concept will follow, including a preliminary factor analysis of the items used in the ECLS-K to measure mathematics self-concept. Next, a breakdown of the utilization of structural equation modeling via path analysis explains how the present study addresses the research hypotheses that a) higher number sense proficiency predicts higher mathematics achievement, b) higher number sense proficiency predicts higher mathematics self-concept, and c) mathematics self-concept mediates the influence of number sense on higher mathematics achievement for middle school students. Finally, procedures for determining the moderating effects of gender ad race are discussed.

Data Set and Participants

The National Center for Educational Statistics (NCES) sponsors the ECLS-K. Because of its focus on middle school, the present study will draw information from the Kindergarten Class of 1998-1999, which has completed data collection through the eighth grade. The full sample ECLS-K was selected over the National Assessment of Educational Progress (NAEP) because of its socioemotional measures and public accessibility. Drawing from public and private schools and diverse racial/ethnic and socioeconomic (SES) communities, the data include cognitive measures from student direct assessment. In addition, they provide social, emotional, and physical measures as well as home and school characteristics according to parent, teacher, student, and administrator input (Tourangeau, Nord, Lê, Sorongon, & Najarian, 2009).

The ECLS-K database, which uses a multistage probability sample design, initially included a nationally representative group of 21,260 students enrolled in 944 kindergarten programs from the 1998 to 1999 academic year. Although the sample was freshened to maintain the best possible representativeness over time, the current study is not representative of the entire population as it includes only eighth grade students who completed a student questionnaire and whose mathematics teachers provided academic rating scores. The cross-sectional data analyzed were gathered during the seventh and final round of collection during the spring of 2007. Individual records are present for

9,725 students at this time, for which 4,450 students possessed mathematics skill ratings provided by their mathematics teachers (the other half of the sample having science teacher ratings instead). Of the 4,450 students with this mathematics achievement score, 4,383 students possessed a perceived interest and competence in mathematics score based on a completed student description questionnaire. The 67 students without this, representing 1.5% of the sample, were removed listwise. Finally, of the 4,383 students, 138 did not have a number sense rating provided through a highest proficiency mastered score. These 138, or 3.1% of the sample, were also removed listwise. The final sample size included 4,245 students. According to Tabachnick & Fidell (2007), the removal of less than 5% of randomly missing data in a large data set is likely not serious, and different techniques of attending to the issue will yield comparable outcomes. Using this rule of thumb, the 4.6% removed in the present study posed no serious concern. No gender values were missing for the updated sample. For students without SES data (422, or 9.5%), the sample mean was used. Finally, in the case of undocumented racial category (4, or less than 1%), students were categorized as White for analyses.

Demographics for the sample are presented in Table 1. Males and females were equally represented. Over half of the participants were White (66.4%). Only a small percentage of students in the sample (8.8%) received special education services.

Table 1

Frequencies and Percentages for the Demographic Variables (N = 4,245)

Variables	Frequency	Percentage
Gender		
Males	2125	50.1
Females	2120	49.9
Race		
White	2818	66.4
Black	422	9.9
Hispanic	708	16.7
Asian	297	7.0
Special education status		
Did not receive special education services	3872	91.2
Received special education services	373	8.8

Note: The White race category includes Native American, Alaska Native, and more than one race, not Hispanic. The Asian race category includes Pacific Islander.

Student Demographic Variables

The gender variable was coded for male and female students. Race/ethnicity was dummy coded according to the following categories: White (including Native American, Alaska Native, and more than one race, not Hispanic), Black, Hispanic (specified and unspecified), and Asian (including Pacific Islander). The Native American, Alaska Native, and more than one race, not Hispanic categories were added to the White group in the coding process due to their disproportionately small size, thereby improving statistical power and preventing spurious results. The variable for special education status was coded according to whether students did or did not receive special education services, as indicated by school coordinators. SES was not changed from its ECLS-K format as a multi-measure composite standardized and centered at the mean, and it ranged from -4.75 to 2.67 with a mean score of .12 and standard deviation of .80. These values were determined according to student household information provided through parent interviews, including parental education levels, occupations, and income (Tourangeau, Nord et al., 2009).

Study Variables

Number sense. The process and content frameworks for the eighth grade ECLS-K mathematics content domain assessments were based upon those employed for NAEP data from 1997 to 2007. These frameworks reflect current and proposed curriculum by the education community, including the National Council of Teachers of Mathematics (NCTM). As such, the mathematics portion incorporates: "number sense, properties, and operations; measurement; geometry and spatial sense; data analysis, statistics, and probability; and pattern, algebra, and functions" and addresses concept, procedure, and problem solving (Tourangeau, Nord et al., 2009, p. 2-4).

Item Response Theory, or IRT, (Lord, 1980) was utilized in calculating scores, which allows for an estimation of true scores based upon those items answered by reviewing patterns of responses, item difficulty, likelihood of guessing, and discriminatory capability. This method helps overcome the difficulty of administering the full test at each round. IRT assumes that the probability of correctly answering an item is dependent on at least one test-item characteristic as well as the test taker's ability with regard to the construct being assessed. For the ECLS-K direct child cognitive assessments, initial routing tests were followed by the administration of different second-stage test booklets. Routing test items in conjunction with core items represented in all second-stage tests provided a common scale, which in turn, allowed for the student's estimated score under the assumption that all test items were answered (Najarian, Pollack, & Sorongon, 2009; Tourangeau, Lê, Nord, & Sorongon, 2009; Tourangeau, Nord et al., 2009).

Nine proficiency probability scores (shown in Table 2) were calculated in mathematics based on clusters of questions from the mathematics assessment, each representing a hierarchical set of skills. The nine content clusters, each composed of four questions with similar difficulty and content, were incorporated at several instances over the mathematics assessment. The corresponding proficiency levels assume a Guttman model (Guttman, 1950), therefore implying that a student who passes a specific level has mastered all levels below it, and that not passing a given level reflects non-mastery of all levels above it. The mathematics test items were not answered for all proficiency levels in eighth grade, rather only for levels seven through nine. Subsequently, IRT procedures provided probability estimates for levels one through six. In order to master a proficiency level, students were required to correctly answer three or all four of the problems for that level. Given that missing data were not random (due to changing assessment difficulty with grade and the fact that all students were not tested in each proficiency level at each round), imputation methods were sometimes utilized, taking into account response patterns, and when necessary, responses to the entire battery of mathematics items (Najarian et al., 2009; Tourangeau, Lê et al., 2009; Tourangeau, Nord et al., 2009).

Table 2

ECLS-K Mathematics Proficiency Levels

Level 1: Number and shape	identifying some one-digit numerals, recognizing geometric shapes, and one-to-one counting of up to 10 objects
Level 2: Relative size	reading all single-digit numerals, counting beyond 10, recognizing a sequence of patterns, and using nonstandard units of length to compare objects
Level 3: Ordinality, sequence	reading two-digit numerals, recognizing the next number in a sequence, identifying the ordinal position of an object, and solving a simple word problem
Level 4: Addition/subtraction	solving simple addition and subtraction problems
Level 5: Multiplication/division	solving simple multiplication and division problems and recognizing more complex number patterns
Level 6: Place value	demonstrating understanding of place value in integers to the hundreds place
Level 7: Rate and measurement	using knowledge of measurement and rate to solve word problems
Level 8: Fractions	demonstrating understanding of the concept of fractional parts
Level 9: Area and volume	solving word problems involving area and volume, including change of units of measurement

(Tourangeau, Nord, Lê, Sorongon, & Najarian, 2009, p. 3-10, 3-11)

An in-depth look at problems within each cluster (requiring restricted access) would likely be applicable to all three sections of the McIntosh, Reys, and Reys (1992) model depicted in Figure 1 that follows. Looking solely at the subject matter documented for each level, and in some cases the tasks to be performed, levels one, two, and three can be found in "knowledge and facility with numbers," levels four, five, and six in "knowledge and facility with operations," and levels seven, eight, and nine in "applying knowledge and facility with numbers and operations to computational settings" (McIntosh et al., 1992). However, to maximize the potential inclusion of number sense items, levels 6 and 7 were collapsed into one level and recoded as 6. Additionally, levels 8 and 9 were collapsed and recoded as 7. Accordingly, highest proficiency level mastered was recoded so that all scores of 6 and 7 were recoded as 6, and all scores of 8 and 9 were recoded as 7. The subsequent scale with a maximum score of 7 instead of 9 was intended to heighten the construct validity of the number sense variable by increasing the likely occurrence of number properties and operations problems. The hierarchical nature of the number sense framework and the IRT methods used in the ECLS-K to calculate probability proficiency levels make the variable for highest proficiency level mastered an excellent indicator of the independent, endogenous variable number sense in the study.

1 Knowledge of and facility with NUMBERS.	1.1 Sense of orderliness of numbers	1.1.1 Place value 1.1.2 Relationship between number types 1.1.3 Ordering numbers within and among number types
	1.2 Multiple representations for numbers	1.2.1 Graphical/symbolic 1.2.2 Equivalent numerical forms (including decomposition/recomposition) 1.2.3 Comparison to benchmarks
	1.3 Sense of relative and absolute magnitude of numbers	1.3.1 Comparing to physical referent 1.3.2 Comparing to mathematical referent
	1.4 System of benchmarks	1.4.1 Mathematical 1.4.2 Personal
2. Knowledge of facility with OPERATIONS.	2.1 Understanding the effect of operations	2.1.1 Operating on whole numbers 2.1.2 Operating on fractions/decimals
	2.2 Understanding mathematical properties	2.2.1 Commutativity 2.2.2 Associativity 2.2.3 Distributivity 2.2.4 Identities 2.2.5 Inverses
	2.3 Understanding the relationship between operations	2.3.1 Addition/Multiplication 2.3.2 Subtraction/Division 2.3.3 Addition/Subtraction 2.3.4 Multiplication/Division
3. Applying knowledge of and facility with numbers and operations to COMPUTATIONAL SETTINGS.	3.1 Understanding the relationship between problem context and the necessary computation	3.1.1 Recognize data as exact or approximate 3.1.2 Awareness that solutions may be exact or approximate
	3.2 Awareness that multiple strategies exist	3.2.1 Ability to create and/or invent strategies. 3.2.2 Ability to apply different strategies 3.2.3 Ability to select an efficient strategy
	3.3 Inclination to utilize an efficient representation and/or method	3.3.1 Facility with various methods (mental, calculator, paper/pencil) 3.3.2 Facility choosing efficient number(s)
	3.4 Inclination to review data and result for sensibility	3.4.1 Recognize reasonableness of data 3.4.2 Recognize reasonableness of calculation

Figure 1. McIntosh, Reys, & Reys number sense model (1992, p. 4)

Reliability for the ECLS-K highest proficiency level mastered score could not be gauged via traditional task replication and subsequent alpha coefficient or split-half reliability calculations as the score was not calculated based upon replicated items nor variance of repeated ability estimates. However, as Najarian et al. (2009) point out, there is a way to attend to the overall purpose of reliability measurement, which is to ascertain the consistency of a measurement under distinct circumstances. For the ECSL-K eighth grade sample, multiple strategies were implemented to account for the large sample size and inability to directly obtain data for all test items. The overlap of scores found by reviewing actual item response data and data obtained through IRT ability estimates and item parameters, which was 61% for exact agreement and 98% for agreement that was off by 1 or less, provides evidence of sufficient reliability for the highest proficiency level mastered rating (Najaraian et al., 2009, p. 4-22).

Mathematics achievement. As noted previously, this study only includes students with a mathematics teacher academic rating scale present. By the time participants had reached eighth grade, a large number of schools departmentalized their subject matter. The ECLS-K database includes English, mathematics, and science academic ratings. The sample size of the participants was reduced for the current study because mathematics teachers scored only half of the students (with science teachers providing scores for the remaining students). In the spring, teachers rated items in reference to the student as 5 for "outstanding," 4 for "very good," 3 for "good," 2 for "fair," and 1 for "poor." Eighth grade mathematics academic teacher rating scores were based on an average of these ratings for the following seven items, listed in order of ascending difficulty:

Uses Calculator to Solve Problems
Uses Computer to Complete Mathematics Assignments
Applies Mathematical Concepts to Real World
Talks about Reasoning in Solving a Problem
Uses Representations to Model Mathematical Ideas
Explains Reasoning in Solving a Problem in Writing
Conducts Proofs or Demonstrates Mathematical Reasoning
(Tourangeau, Nord, Lê, Sorongon, & Najarian, 2009, p. 3-32)

There are both pluses and minuses to using the teacher mathematics academic rating scale as a measure of mathematics achievement for the study. The primary disadvantage of this type of scoring is that it may be limited by subjective interpretations and bias in student-teacher relationships. The ECLS-K manual points out that teacher ratings "overlap and augment the information gathered through the direct cognitive assessment battery," but also sheds light on their capability to measure process rather than simply product, such as how students "express their ideas, solve mathematical problems, or investigate scientific phenomena" (Tourangeau, Nord et al., 2009, p. 3-29). Furthermore, teachers have the advantage of more accurately assessing students who may have undocumented learning disabilities and/or test anxiety, and of considering academic performance over a longer period of time. Lastly, unlike the mathematics IRT scale score (from which the proficiency levels were developed), teacher academic rating values do not have any direct overlap with the probability proficiency scores, and thereby the highest proficiency level mastered. This variable independence prevents collinearity and allows for a stronger analysis of number sense in relation to mathematics achievement.

ECLS-K reliability of the student skill measure provided by teacher academic ratings was calculated through IRT analysis utilizing a partial credit model, in accordance with Muraki (1992). This model considers item rating patterns in determining item difficulty and the corresponding placement of students on an interval scale. For mathematics, the generalized partial credit technique revealed the reliability of teacher academic rating scores estimating student mathematics ability to be quite high at .95 (Tourangeau, Nord et al., 2009, p. 3-30).

Mathematics self-concept. The mathematical self-concept measure for the study came from the ECLS-K Student Description Questionnaire (SDQ), developed from the SDQ II developed by Marsh, Parker, and Barnes (1985). The SDQ II was designed in an effort to evaluate the multi-dimensional, hierarchical structure of self-concept for young adolescents in grades seven through 12. The mathematics scale from Marsh's SDQ II included 10 items intended to measure "ability, enjoyment, and interest in mathematics and reasoning" (Marsh, 1990c, p. 2). A shorter version of the SDQ II (the SDQ II-S) was developed in 2002 by Ellis, Marsh, and Richards, containing 51 of the original 102 items. The reliability of the four items measuring mathematics self-concept on the SDQ II-S was .89 (Marsh, Ellis, Parada, Richards, & Heubeck, 2005). The reliability of the four items taken from the SDQ II for the ECLS-K dataset was also moderately high at .89. Students answered, "not at all true," "a little bit true," "mostly true," or "very true" for each item (1 corresponding to "not at all true" and 4 corresponding to "very true") (Tourangeau, Nord et al., 2009). The items rated were as follows: (1) "Math is one of my best subjects," (2) "I get good grades in math," (3) "I like math," and (4) "I enjoy doing work in math" (NCES, 2010). The average score for these items was used to represent "Perceived Interest/Competence – Math" which in turn represented the mathematics self-concept variable for the present study.

Procedure

Structural equation modeling (SEM) represents a “family of related procedures” as opposed to a specific statistical method (Kline, 2011). This technique brings together exploratory factor analysis and multiple regression analyses (Tabachnick & Fidell, 2007). When model evaluation involves effects for observed variables (also referred to as indicators, or manifest variables), path analysis (the earliest SEM technique established) is appropriate. The path model provides a structural representation of observed variables, and this structural model reflects causal hypotheses. Disturbances in the model indicate the inference of probabilistic causality. In this sense, the disturbances are representative of outcome variable causes not present in the model. Path coeffiicients provide direct effect estimates. Indirect, or mediator, effects communicate additional causal effects onto the outcome variable from the input variable (Kline, 2011). For the present path analysis, direct effects are provided from number sense to mathematics self-concept and mathematics achievement. The indirect effect of mathematics self-concept augments the indicated impact of number sense on mathematics achievement.

Statistical Analysis Software, SAS, version 9.2, was employed for the current study model, specifically, the Covariance Analysis of Linear Structure Equations procedure using maximum-likelihood estimation. The extent to which number sense predicts mathematics self-concept and mathematics achievement in eighth grade was determined. Accordingly, the proposed structural model (Figure 2) predicts that the exogenous variable number sense influences the endogenous variables of mathematics self-concept and mathematics achievement. In addition to these direct effects, the model tests whether or not a student’s mathematics self-concept mediates the relationship between number sense and mathematics achievement, representing its indirect influence on mathematics achievement. The covariates (gender, race, SES, and special education services) serve as additional exogenous predictors, all with direct paths to the predictor (number sense), mediator (mathematics self-concept) and outcome (mathematics achievement) variables. D1, D2, and D3 are measurement error terms (disturbances) associated with number sense, mathematics self-concept, and mathematics achievement. The model is recursive in that it assumes uncorrelated disturbances with unidirectional causal effects (Kline, 2011).

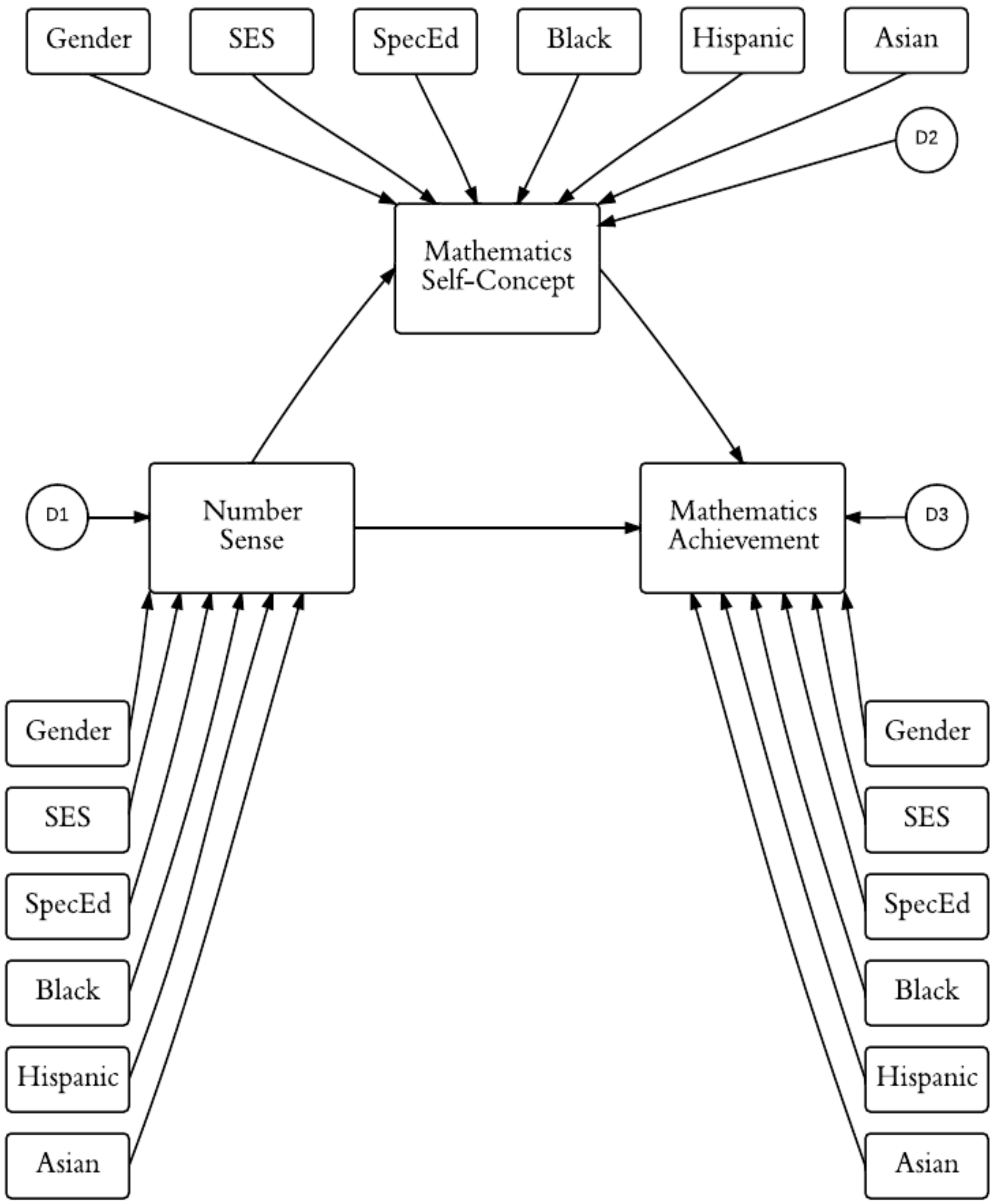

Figure 2. Proposed structural model for path analysis

Direct effect path coefficients represent change in the outcome variable based on a one-unit change in a specified predictor variable when all other variables are held constant. Indirect paths exist when such direct effect paths are compounded by other present variables. In the current study, there are two direct effects on the endogenous variable mathematics achievement, one coming from the exogenous variable number sense and one from the other endogenous variable mathematics self-concept. The indirect, or mediator, effect occurs because mathematics self-concept functions both as a predictor and criterion (Kline, 2011). The direct effect path coefficients were calculated and deemed significant from number sense to mathematics achievement and from number sense to mathematics self-concept to address the first two research hypotheses. In order to attend to the third hypothesis predicting that mathematics self-concept mediates the relationship between number sense and mathematics achievement, the indirect path determined allows for the calculation of the total of direct and indirect effects of number sense on mathematics achievement. This total effects calculation provides a more comprehensive analysis of the impact of number sense on adolescent mathematics achievement.

Testing for mediation was initially performed in accordance with procedures suggested by Baron and Kenny (1986). Given the large sample size of the regression models, the complex sample design of the ECLS-K, and to ensure the external validity of the findings, the Sobel test (Sobel, 1982) was used to assess statistical significance of mediation rather than bootstrapping standard errors by weighting repeated resamples. The cross-sectional weight C7CPTM0 provided in the ECSL-K user manual was selected to address various limitations of data collection and reporting, such as nonresponse bias and differential probabilities of selection at various stages of sampling (Tourangue, Lê et al., 2009, Tourangeau, Nord et al., 2009). This specific weight was intended for analyses involving direct and indirect student and teacher data. The appropriate nesting variables (in accordance with the Taylor series method employed by SAS) for computing standard errors for the C7CPTM0 full sample weight are C7CPTMST and C7CPTMPS, also found and discussed in the ECLS-K user manual and methodology report (Tourangeau, Nord et al., 2009; Tourangue, Lê et al., 2009). Design effects for stratification and clusters were incorporated for student level data to correct for the assumption by SAS of simple random sampling.

The determination of model fit in SEM is based on model design and variable relationships being analyzed, and techniques for measuring it may be highly sensitive to the size and type of sample data. The Chi-Square statistic, intended to evaluate how different the sample is from the fitted covariance matrices (Hu & Bentler, 1999), and its degrees of freedom and *p* value were reported as per Kline (2011). The Goodness of Fit, or GFI, was also reported to show the proportion of variance accounted for by the estimated population covariance (Tabachnick & Fidell, 2007). Because both of these statistics have shown an upward bias with large sample sizes (Barrett, 2007; Hooper, Coughlan, & Mullen, 2008; Miles & Shevlin, 2007), supplementary model fit indices were provided. First, the standardized Root Mean Square Residual (SRMR) was selected over the Root Mean Square to account for varying ranges of variable scores. Additionally, the Parsimonious Goodness of Fit (PGFI) statistic was reported, which adjusts the GFI to account for loss in degrees of freedom (Mulaik, 2007; Mulaik et al., 1989).

A framework extensively studied for verbal and mathematics self-concepts is the internal/external (I/E) frame of reference model (Marsh, 1986) which asserts that academic self-concept in various subjects, including mathematics, is subject to both internal and external comparison processes. Externally, a student develops his academic self-concept by comparing him/herself to other students in the same area of academics and according to achievement measures like grades and test scores. Internally, a student compares his/her performance and ability in one subject to that of other subjects. Though the I/E frame of reference model is not evaluated as part of the present study, findings regarding its validity are pertinent to moderation analyses according to gender and race. Specifically, Marsh, Hau, Artelt, Baumert, and Peschar (2006), in their study of 25 countries using the PISA 2000 database, demonstrated the cross-cultural generalizability of the I/E framework. Likewise, Möller, Pohlmann, Köller, and Marsh (2009), in their meta-analytical path analysis of the I/E model, confirmed generalizability of the positive path from mathematics achievement to mathematics self-concept across gender, nation, and age.

Simultaneous group analysis procedures were conducted to determine whether gender and race moderated the relationships between number sense, mathematics self-concept, and mathematics achievement. This was carried out by first estimating the model separately for males and females, and then White, Black, Hispanic, and Asian student groups. Upon determining which of these models fit the data well, further analysis continued regarding how the path estimates varied according to demographic.

The next chapter will detail the findings from the path analysis and will provide descriptive statistics for the sample. The last chapter will then discuss these findings in light of the theoretical frameworks chosen for the analysis and in relation to previous related research. Limitations of the data will be addressed as well as research implications for the future. Finally, how the results can be utilized at the student, classroom, district, and state levels as well as what this means in the context of the CCSSM will be discussed.

Chapter 4: Results

As students enter and progress through middle school, they experience multiple and profound life changes, and the present study's goal is to address some of these by integrating the content and socioemotional characteristics of mathematics education at this time. A path analysis tested the predictive capabilities of number sense in terms of mathematics achievement and mathematics self-concept, and then whether mathematics self-concept serves as an intervening variable for the path from number sense to mathematics achievement.

The following portion of the present study provides a description of study variable values, details preliminary procedures conducted to prepare the data for analysis, and finally relays how each research question was answered and what results followed. The Early Childhood Longitudinal Study, Kindergarten Class of 1998-1999 (ECLS-K) implemented a complex sample design. Therefore, the weighted figures provided were found by applying appropriate sample weights and design variables to account for stratified and cluster sampling.

Descriptive Statistics for Study Variables

Means and standard deviations for number sense, mathematics self-concept, and mathematics achievement are summarized in Table 3.

Table 3

Descriptive Statistics for Study Variables (N = 4,250)

Variable	Range	Unweighted		Weighted	
		M	*SD*	*M*	*SD*
Number Sense	3.00 - 7.00	6.29	.70	6.24	.02
Mathematics Self-Concept	1.00 - 4.00	2.62	.89	2.60	.02
Mathematics Achievement	1.14 - 4.94	3.09	.94	3.01	.03

The number sense scores for the eighth grade were based on seven proficiency probability scores, corresponding to the following content areas: (1) number and shape, (2) relative size, (3) ordinality and sequence, (4) addition and subtraction, (5) multiplication and division, (6) place value, and (7) fractions. The scores for the current study ranged from 3 to 7, indicating that the eighth graders tested had at a minimum mastered number and shape, relative size, and ordinality and sequence. These levels required identifying and reading one-digit and two-digit numerals, counting beyond 10, recognizing geometric shapes, pattern sequences and ordinal positions of objects, determining the next number in a sequence, comparing items with nonstandard measurement units, and performing basic word problems. At the other end of the spectrum, students scoring a 7 additionally exhibited dexterity in addition, subtraction, multiplication, and division problems as well as place value to the hundredth place and working with fractional parts. A minimum score of 3 is no surprise given that students, by the time they reach middle school, are likely able to recognize and count numbers and understand them in terms of size and order. A high score of 7 is also reasonable given that mathematics curriculum prior to eighth grade includes mathematics skills from all seven levels. An unweighted mean score of 6.29 (weighted at 6.24) indicates that the average eighth grade student in the sample has not mastered fractions nor area and volume, but has mastered levels corresponding to number and shape, relative size, ordinality and sequence, addition and subtraction, multiplication and division, and place value.

Mathematics self-concept had a value range of 1 to 4, 1 indicating a consistent response of "not at all true," and 4 reflecting a consistent response of "very true" regarding questions about interest and perceived skill in the subject of mathematics. The unweighted mathematics self-concept mean was 2.62 (weighted at 2.60). This value is reasonable, indicating a slightly above average rating of mathematics self-concept for the sample of eighth graders.

Mathematics achievement scores ranged from 1.14 to 4.94, and the unweighted mean measured 3.09 (with a weighted mean of 3.01). Based on teacher responses to seven difficulty-related questions about each student's mathematics skills, the range reflects ratings at both ends of the spectrum, with 1 representing "poor" and 5 representing "outstanding." The mean is a bit above average, falling in between "good" and "very good."

Preliminary Analyses

Factor analysis. The four mathematics items in the Student Description Questionnaire (SDQ) administered for the ECLS-K were not identical to the four questions in the shorter version of the SDQ II, the SDQ II-S (Marsh, Ellis, Parada, &

Richards, & Heubeck, 2005). Moreover, the current study's sample did not include all eighth graders but only those with mathematics academic rating scale scores. For these reasons, a confirmatory factor analysis using oblique rotation with version 20 of the Statistical Package for the Social Sciences (SPSS) was first performed for the four mathematics self-concept items.

An analysis of the eigenvalues confirmed a total of one factor with an eigenvalue greater than 1 (see Table 4). This single factor accounted for about 75.16% of the variance. An examination of the scree plot suggested that only one factor be retained (see Figure 3). The factor loadings by item are presented in Table 5.

Table 4

Total Variance Explained, ECLS-K Mathematics Self-Concept Items

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	3.006	75.157	75.157	3.006	75.157	75.157
2	.552	13.795	88.952			
3	.274	6.845	95.797			
4	.168	4.203	100.000			

Extraction Method: Principal Component Analysis.

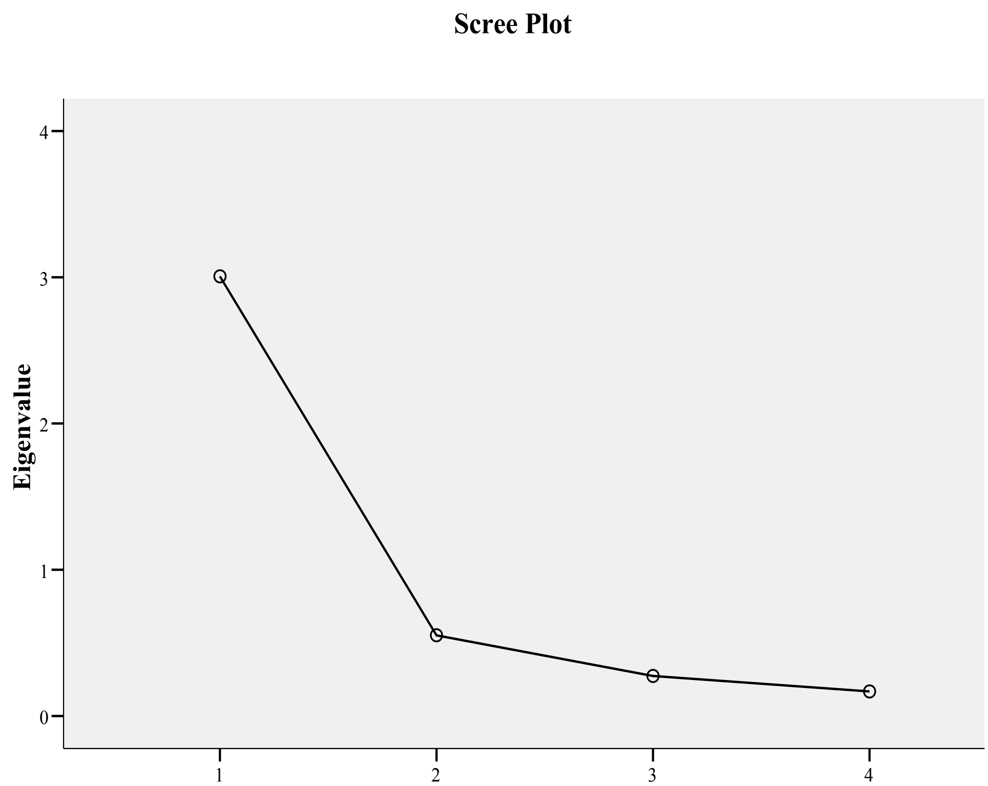

Figure 3. Scree plot, ECLS-K mathematics self-concept items

Table 5

Component Matrix, Mathematics Self-Concept Items

	Component
	1
Math is one of my best subjects	.896
I get good grades in math	.787
I like	.910
I enjoy doing work in math	.870

Extraction Method: Principal Component Analysis.
1 component extracted.

The four items fit best on one factor, which supports their combination into a single scale for the mathematics self-concept construct. The coefficient alpha for the combined scale was .89, and as shown in Table 3, the weighted mean and standard deviation for mathematics self-concept were 2.60 and .02, respectively. The factor analysis confirmed the dimensional structure of the SDQ items used to gauge interest and competence in mathematics.

Test for mediation. Steps in establishing mediation according to Kenny (2012), Kenny and Baron (1986), and Judd and Kenny (1981) were followed in preparation for testing the model. Results of preliminary regression analyses are presented in Table 6. They confirm that number sense is a predictor of mathematics achievement ($\beta = .47$, $p < .0001$) and mathematics self-concept ($\beta = .24$, $p < .0001$). These values, in addition to the absence of high modification indices in the structural equation modeling software used, indicated that neither multicollinearity nor correlated error were an issue in the model. The precision of the path coefficients was not weakened by independent variable correlation.

A regression with mathematics achievement as the criterion variable and both number sense and mathematics self-concept as the input variables was also conducted to show that the mediator variable does in fact impact the outcome variable. In this case, controlling the input variable number sense accounts for the possibility of both the mediator mathematics self-concept and outcome mathematics achievement being influenced by number sense. Regression results in Table 6 indicate a significant standardized coefficient of .40, $p < .0001$. The last step testing for mediation implies establishing complete mediation, which would indicate that the impact of number sense on mathematics achievement is 0 after accounting for mathematics self-concept. However, since the current study does not assert that mathematics self-concept completely mediates the relationship between number sense and mathematics, this was not conducted. Partial mediation is indicated by the affirmation of steps one through three which is in accordance with the hypotheses.

Table 6

Multiple Linear Regression Results in Testing for Mediation

Path	*B*	*β*	*SE*	*t*
Number Sense to Mathematics Achievement	.62	.47	.02	32.70***
Number Sense to Mathematics Self-Concept	.30	.24	.02	15.10***
Mathematics Self-Concept to Mathematics Achievement (Controlling for Number Sense)	.53	.40	.02	28.51***

Note. ***p < .001

Path Analysis Findings

Several goodness-of-fit indices were utilized to evaluate the model. As revealed in Table 7, the Chi-square statistic was .00 (with 0 degrees of freedom and a p value < .001), and the Goodness of Fit Index (GFI) was 1.00. Both of these values indicate perfect fit as a result of model saturation, and their bias due to large sample size was no surprise (Bentler & Bonnet, 1980). To further test model fit, the Standardized Root Mean Square Residual (SRMR) and Parsimonious Good-ness of Fit (PGFI) were calculated, both with a .00 value as shown in Table 7. As the model fit the data well, the study hypotheses were evaluated vis-à-vis the proposed structural model findings.

Table 7

Fit Indices for the Path Analysis

Index	Value
Chi-Square	.00
Goodness of Fit Index (GFI)	1.00
Standardized root mean square residual (SRMR)	.00
Parsimonious Goodness of Fit (PGFI)	.00

First hypothesis. It was hypothesized that, after controlling for gender, socioeconomic status (SES), special education services, and race, that number sense would significantly predict mathematics achievement. As shown in Figure 4 and Table 8, number sense significantly and positively predicted mathematics achievement (β = .27, $p < .001$). Thus, higher number sense scores for eighth graders were associated with higher mathematics achievement values. Specifically, each one-unit increase in number sense corresponded to a .27-unit increase in mathematics achievement. The first hypothesis was supported.

Second hypothesis. It was hypothesized that, after controlling for gender, SES, special education services, and race, number sense would significantly predict mathematics self-concept. As shown in Figure 4 and Table 8, number sense significantly and positively predicted mathematics self-concept (β = .25, $p < .001$). This pattern indicates that higher number sense scores were associated with higher mathematics self-concept scores. Specifically, each one-unit increase in number sense corresponded to a .25-unit increase in mathematics self-concept, which had a range in value from 1 to 4. The data supported the second hypothesis as well.

Third hypothesis. It was hypothesized that, after controlling for SES, gender, special education services, and race, that mathematics self-concept would mediate the effect of number sense on mathematics achievement. The Sobel test, which assesses the statistical significance of indirect effects, was used to test this hypothesis. The findings reveal that the indirect effect of number sense on mathematics achievement ($\beta = .08$) was statistically significant ($z = 11.99$, $p < .001$). Therefore, mathematics self-concept mediated the effect of number sense on mathematics achievement. The third hypothesis was supported.

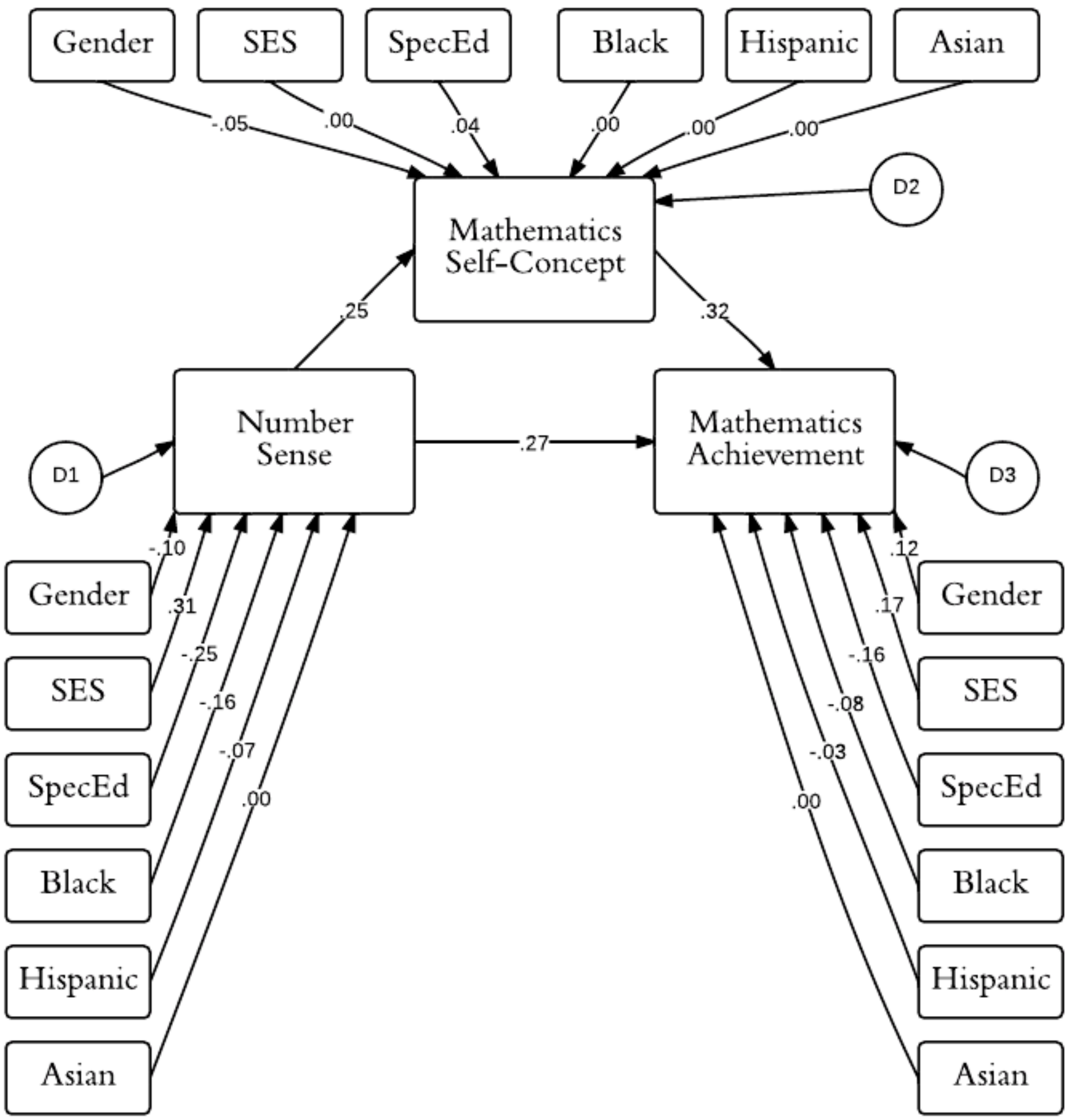

Figure 4. A path model for number sense, mathematics self-concept, and mathematics achievement (standardized coefficients)

Table 8

Path Coefficients for the Path Analysis

Path	*B*	*β*	*SE*	*t*
Number Sense to:				
Mathematics Self-Concept	.32	.25	.02	14.13 ***
Mathematics Achievement	.36	.27	.01	18.53 ***
Mathematics Self-Concept to:				
Mathematics Achievement	.33	.32	.01	24.98 ***
Gender to:				
Number Sense	-.14	-.10	.01	-7.12 ***
Mathematics Self-Concept	-.10	-.05	.02	-3.36 **
Mathematics Achievement	.23	.12	.01	9.65 ***
SES to:				
Number Sense	.27	.31	.01	21.17 ***
Mathematics Self-Concept	-.02	-.02	.02	-1.13
Mathematics Achievement	.19	.17	.01	11.51 ***
Special Education Services to:				
Number Sense	-.55	-.25	.01	-18.09 ***
Mathematics Self-Concept	.11	.04	.02	2.41 *
Mathematics Achievement	-.46	-.16	.01	-12.07 ***
Black vs. White to:				
Number Sense	-.31	-.16	.01	-11.04 ***
Mathematics Self-Concept	.01	.00	.02	0.16
Mathematics Achievement	-.20	-.08	.01	-5.84 ***
Hispanic vs. White to:				
Number Sense	-.12	-.07	.02	-4.43 ***
Mathematics Self-Concept	-.08	-.03	.02	-1.95 *
Mathematics Achievement	-.08	-.03	.01	-2.31 *
Asian vs. White to:				
Number Sense	.08	.02	.01	1.44
Mathematics Self-Concept	.10	.02	.02	1.38
Mathematics Achievement	.10	.02	.01	1.63

*** $p < .001$

Direct, indirect, and total effects. Direct effects for the model are represented previously in Figure 4 and Table 8 by standardized path coefficients (regression coefficients). Using Cohen (1988) standards, the effect size is small at .10, medium at .30, and large at .50. Since gender was coded as 0 for males and 1 for females, Figure 4 and Table 8 indicate that being female corresponded to a small reduction in number sense (β = -.10) but a small increase in mathematics achievement (β = .12). SES had a medium positive effect on number sense (β = .31) as well as a small positive effect on mathematics achievement (β = .17). Students not receiving special education services were coded as 0, and those receiving such services were coded as 1. The data reveal that receiving special education services was associated with a small negative effect on number sense (β = -.25) and mathematics achievement (β = -.16). In terms of race, Black students were the only group to indicate a negative relationship with number sense (small at -.16) in comparison to White students, and no racial categories show direct effects for mathematics self-concept or mathematics achievement.

Shrout and Bolger (2002) recommend gauging effect size for mediation via the previously referenced Cohen (1988) thresholds. However, Kenny (2012) suggests that these values be squared since indirect effect is the product of two effect sizes, yielding the following figures: .01 for small, .09 for medium, and .25 for large. It is also noteworthy that Cohen himself pointed out the importance of perpective in such interpretations when he urged effect size measures be interpreted in the context of population variability (Cohen, 1994). For the present study, the indirect effect size is found by multiplying the direct effect paths from number sense to mathematics self-concept (.25) and mathematics self-concept to mathematics achievement (.32). The calculation yields an indirect, mediator effect measure of .08, which may be interpreted as small to medium in the current study.

It is important to remember that, in the current study, number sense was not presented as the sole predictor of mathematics achievement, nor was mathematics self-concept suggested as a complete mediator of their relationship. As mentioned in the methods section, the comprehensive assessment of mathematics for eighth graders in the study also includes measurement, geometry and spatial sense, data analysis, statistics, probability, patterns, algebra, and functions (Tourangeau, Nord et al., 2009). Even this formulation does not account for other contributing achievement factors, such as practice and parental influence. Cast in this light, direct effects from number sense to mathematics achievement of .27 and from number sense to mathematics self-concept of .25 more dramatically reflect how critical number sense is to the much broader measure of mathematics achievement at the middle school level. It also indicates how crucial an indirect effect value of .08 represents for mathematics self-concept.

Total effects are "the sum of all direct and indirect effects of one variable on another" (Kline, 2011, p.167). For the current path analysis, the direct effect of number sense on mathematics achievement was represented by a path coefficient of .27. The indirect effect of this path by way of mathematics self-concept was found to be .08. Total effects for the model are therefore represented by .35, the sum of .27 and .08. This interpretation of standardized effects as path coefficients indicates that a one standard deviation increase in eighth grade number sense will positively impact student mathematics achievement by .35 standard deviations. This outcome is the result of all suggested direct and indirect causal relationships between number sense and mathematics achievement. For the current study, this includes the direct causal link between number sense and mathematics achievement and the indirect causal link between number sense and mathematics achievement through mathematics self-concept.

Moderating effects. Moderation (or interaction) effects were evaluated in the model for the race and gender. The full model was therefore estimated separately for males and females, and then for each racial group in comparison to the other racial categories. This process was intended to determine whether the demographic variables of gender and race uniquely affect the strength of the causal relationship between number sense, mathematics self-concept, and mathematics achievement and to test the external validity of the model (Kenny, 2011). Table 9 shows the direct, indirect, and total effects for each gender and racial category.

Table 9

Standardized effects on Mathematics Achievement by Gender and Race

Variable	Total Effect	Direct Effect	Indirect Effect
Females			
Mathematics Self-Concept	.31	.31	
Number Sense	.44	.37	.08
Males			
Mathematics Self-Concept	.33	.33	
Number Sense	.30	.21	.08
Whites			
Mathematics Self-Concept	.31	.31	
Number Sense	.35	.28	.07
Blacks			
Mathematics Self-Concept	.30	.30	
Number Sense	.15	.09	.06
Hispanics			
Mathematics Self-Concept	.34	.34	
Number Sense	.42	.31	.11
Asians			
Mathematics Self-Concept	.30	.30	
Number Sense	.46	.37	.09

Note. $p < .001$ for all values.

Gender. The results for the simultaneous group analyses for gender are depicted in Figure A1 and summarized in Tables A1 and A2 in Appendix A. Gender did moderate the relationships between number sense, mathematics self-concept, and mathematics achievement as both models fit the data well. Table 9 shows that the total effect of the model was .30 for males and .44 for females. The indirect effect of mathematics self-concept is the same for both females and males (.08). Looking more closely at the direct effects, it appears the influence of number sense on mathematics achievement is stronger for females than males (.37 for females, .21 for males). In the case of gender, therefore, model variation is likely attributable to differences in the direct effect of number sense on mathematics achievement.

Race. In the midst of developmental changes during adolescence and contributing forces like parental and teacher relationships, many students may find race, whether directly or indirectly, influences their educational progress. According to Graham and Hudley (2005), this is truly unique for each individual, and how a person views their ethnic minority's competence significantly impacts how they view their personal abilities and the ways in which they seek achievement. Specifically, ethnic identity can serve a protective role: "When adolescents of color are strongly identified with their ethnic group, they are more motivated to achieve and have a greater repertoire of skills to ward off threats to their competence" (Graham & Hudley, 2005, p. 406). In many cases, it is impossible to disentangle ethnic factors from SES, which has the ability to directly impact a student's well being. SES can impact health, access to resources, and subsequently, overall academic readiness (Brooks-Gunn, Linver, & Fauth, 2005). To explore how race might impact the relationship between number sense, mathematics self-concept, and mathematics achievement, the model was evaluated separately for each group for comparison. All results are summarized in Tables A3 and A4.

Whites versus Blacks. The results for the simultaneous group analyses of Whites versus Blacks are depicted in Figure A2. Strong model fit for both racial categories reveals that race (Whites versus Blacks) did moderate the relationships between number sense, mathematics self-concept, and mathematics achievement. The model total effect was .35 for White students and .15 for Black students. The indirect effects were similar, but the direct path from number sense to mathematics achievement was only .09 for Black students and .28 for White students. Black students did show a medium direct effect for mathematics self-concept on mathematics achievement that was similar to the White students (.30 compared to .31). So mathematics self-concept impacts achievement for Black students, but not as much by way of number sense, and number sense is much less influential for Black students compared to White students in terms of overall mathematics achievement.

Whites versus Hispanics. The results for the simultaneous group analyses for Whites versus Hispanics are depicted in Figure A3. Strong model fit for both racial categories reveals that race (Whites versus Hispanics) did moderate the relationships between number sense, mathematics self-concept, and mathematics achievement. As Table 9 shows, the model total effect was .35 for White students and .42 for Hispanic students, and this was attributable to both direct and indirect effects. Whereas the indirect effect for White students was .07, this value was .11 for Hispanic students. The direct effect from number sense to mathematics achievement was also higher for Hispanic students (.31) than White students (.28).

Whites versus Asians. The results for the simultaneous group analyses for Whites versus Asians are depicted in Figure A4. Strong model fit for both racial categories reveals that race (Whites versus Asians) did moderate the relationships between number sense, mathematics self-concept, and mathematics achievement. As Table 9 shows, the model total effect was .35 for White students and .46 for Asian students. The indirect effects are similar, but the direct path from number sense to mathematics achievement was .37 for Asian students in comparison to .28 for White students.

Blacks versus Hispanics. The results for the simultaneous group analyses for Blacks versus Hispanics are depicted in Figure A5. Strong model fit for both racial categories reveals that race (Blacks versus Hispanics) did moderate the relationships between number sense, mathematics self-concept, and mathematics achievement. As Table 9 shows, the model total effect was only .15 for Black students and .42 for Hispanic students, and both indirect and direct effects impacted this difference. Whereas the indirect effect for Black students was only .06, this value was .11 for Hispanic students. Likewise, the direct effect from number sense to mathematics achievement was higher for Hispanic students (.31) than Black students (.09).

Blacks versus Asians. The results for the simultaneous group analyses for Blacks versus Asians are depicted in Figure A6. Strong model fit for both racial categories reveals that race (Blacks versus Asians) did moderate the relationships between number sense, mathematics self-concept, and mathematics achievement. As Table 9 shows, the model total effect was only .15 for Black students and .46 for Asian students. The indirect effects were somewhat smaller for Black students (.06 compared to .09), but the direct path from number sense to mathematics achievement was substantially different, being .37 for Asian students and only .09 for Black students.

Hispanics versus Asians. The results for the simultaneous group analyses for Blacks versus Hispanics are depicted in Figure A7. Strong model fit for both racial categories reveals that race (Hispanics versus Asians) did moderate the relationships between number sense, mathematics self-concept, and mathematics achievement. However, a closer look at model effects indicates that the group differences are minimal. As Table 9 shows, the model total effect was .42 for Hispanic students and .46 for Asian students. Indirect and direct effects were also relatively similar, though the indirect affect of mathematics self-concept appears to be somewhat stronger for Hispanic students (.11) than Asian students (.09). In terms of the direct effects of number sense on mathematics achievement, Asian students were higher than Hispanic students (with standardized path coefficients of .37 and .31, respectively).

Summary

The goal of the path analysis was to determine the predictive extent of number sense on mathematics self-concept and mathematics achievement for eighth graders, and additionally, to explore the meditational capability of mathematics self-concept on the path from number sense to mathematics achievement. Direct and indirect effects for the structural equation model confirm all hypotheses. Number sense positively and significantly predicts both mathematics self-concept and mathematics achievement. Additionally, the relationship between number sense and mathematics achievement is clearly mediated by mathematics self-concept.

Considering the broad context and content of mathematics achievement, all of the findings are noteworthy. First, they highlight just how critical number sense remains in middle school. Whereas previous studies have shown the positive impact of number sense on elementary mathematics performance and growth (Jordan, Glutting, & Ramineni, 2009; Jordan, Glutting, Ramineni, & Watkins, 2010; Jordan, Kaplan, Locuniak, & Ramineni; 2007), a total effect value of .35 at the middle school level indicates how this construct must remain a key element of curriculum as students transition from arithmetic to algebra. In terms of mathematics self-concept, the results indicate that number sense is a strong component of the link between this affective domain and overall mathematics performance. This narrows the focus of previous findings of the cause and effect relationship between mathematics self-theories and mathematics self-concept (Valentine, DuBois, & Cooper, 2004) to number sense. Finally, results of the present study provide a first look at number sense in the context of middle school motivation and indicate that this relationship is carried over to a certain extent in how number sense contributes to mathematics achievement at this academic stage.

Chapter 5: Discussion

The present study sought to determine to what degree number sense, or the ability to flexibly apply knowledge of numbers and operations, contributes to both the academic and affective transformation of students at the middle school level. This task was carried out through a path analysis using cross-sectional data from the Early Childhood Longitudinal Study, Kindergarten Class of 1998-1999 (ECLS-K). Results suggest that number sense influences mathematics self-concept and mathematics achievement in eighth grade. This chapter will detail the significance of these findings in educational research and practical classroom application, review limitations posed by the data and analyses employed, and discuss how the results can be elaborated in future research.

Rationale for the Present Study

Forming one's identity is the "defining life task of adolescence – the question of who one is, who one belongs with, what one is good at, and where one is going in the future" (Roeser & Lau, 2002, p. 93). This process is nowhere more evident than in middle school where students typically reveal a decrease in both values and expectancies, possibly due to a disconnect between changing instructional techniques and their developmental desire for independence, competence, and relevance (Eccles & Midgley, 1989). Unfortunately, there is strong evidence that students at this stage of maturation experience lower engagement and performance in mathematics at the same time that they exhibit a decreased appreciation for its value as a subject (Anderman & Maehr, 1994; Pajares & Graham, 1999; Wigfield, Eccles, MacIver, Reuman, & Midgley, 1991). Academically, this juncture represents a stage of critical mathematics development as students transition from material focused more on arithmetic to that involving complex topics like algebra (Geary, 1994; Ketterlin-Geller, Jungjohann, Chard, & Baker, 2007; Schielack & Seeley, 2010).

In an atmosphere of intense accountability and in the midst of an unprecedented emphasis on mathematics and science, educational policy is striving to create consistent and effective learning experiences. The Common Core State Standards Initiative (CCSSI) represents an effort in the United States to shape curriculum and instruction in a way that maximizes student potential and opportunity. At each grade level of the Common Core State Standards for Mathematics (CCSSM), Standards for Mathematical Practice (SMPs) provide a framework for instruction that emphasizes meaningful mathematical behaviors, such as modeling operations in relevant contexts, thinking both concretely and abstractly about mathematics, and dedicating oneself to finding accurate solutions (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). Underlying this framework is a persistent call for number sense, one that changes in scope and application as students progress from elementary to middle to high school.

How the Present Study Adds to the Literature

There is ample empirical support for the advantages of early number sense in student mathematical growth, particularly at the elementary level (Baroody, Lai, & Mix, 2006; Jordan, Glutting, & Ramineni, 2009; Jordan, Glutting, Ramineni, & Watkins, 2010; Jordan, Kaplan, Locuniak, & Ramineni, 2007). Education research has also demonstrated how elements of number sense, such as place value and computational fluency, contribute positively to elementary mathematics achievement (Byrnes & Wasik, 2009; Cowan, et al., 2011; Duncan, et al., 2007; Kikas, Peets, Palu, & Afanasjev, 2009; LeFevre, et al., 2010; Mazzocco, Feigenson, & Halberda, 2011; Moeller, Pixner, Zuber, Kaufmann, & Nuerk, 2011). In secondary and higher education, the emphasis has been on particular skills within number sense, such as fraction or arithmetic abilities, as they relate to more advanced coursework (Brown & Quinn, 2007; Geary, Saults, Liu, & Hoard, 2000; Geary, Liu, Chen, Saults, & Hoard, 1999; Lyons & Beilock, 2011; Tolar, Lederberg, & Fletcher, 2009). At the middle school level, the literature primarily addresses studying gaps by looking at number sense components (Alajmi & Reys, 2010; Ben-Chaim, Fey, Fitzgerald, Benedetto, and Miller, 1998; Gay & Aichele, 1997; Herscovics & Linchevski, 1994; Stacey & MacGregor, 1997; Royer, Tronsky, Chan, Jackson, & Marchant, 1999).

What is missing from this discourse is the issue of how number sense plays an active part in middle school mathematics achievement. Just as the CCSSM do at these grade levels, educators, parents, and the public assume that number sense is providing a foundation for more complex mathematics topics. But as many teachers still administer benchmark tests and attend to curriculum in keeping with No Child Left Behind guidelines, a focus on grade-level standard requirements may leave little room for incorporating number sense into instruction. As a result, necessary provisions for students who have not excelled in this construct over the years may be neglected. A primary goal of the current study is to encourage a renewed emphasis on number sense due to its potential for enhancing differentiated instruction and progression into more advanced mathematics coursework while still following SMPs and striving for those content elements delineated by the CCSSM.

Research in educational psychology has provided strong evidence for the relationship between affective domains and mathematics achievement. Specifically, mathematics self-concept has been linked to mathematics development in elementary, middle, and high school (Möller, Pohlmann, Köller, & Marsh, 2009). However, this research has not looked at more specific elements of mathematics in this context. Exploring the strength of the relationships between mathematics self-concept and skill areas like number sense allows for a better pinpointing of developmental obstacles. In turn, intervention can be more optimally tailored for student improvement and performance.

The present study adds to the literature on middle school mathematics by interconnecting content and socioemotional factors in hopes of better informing application of the CCSSM, now adopted by 45 of the 50 United States. The importance of affective domains in the study of mathematics achievement is not a new notion, and in fact, the significant motivational declines documented as children move through adolescence may be more extreme for mathematics than other subjects (Gottfried, Marcoulides, Gottfried, Oliver, & Guerin, 2007). Not surprisingly, there is evidence that interest in mathematics drops from childhood to adulthood (Gottfried, Fleming, & Gottfried, 2001), and this waning interest is stronger in adolescence (Fredericks & Eccles, 2002; Watt, 2008). As a motivational construct, mathematics self-concept has been supported as a contributing factor in as well as an outcome of mathematics achievement (Marsh, 1990b, 2007; Marsh, Byrne, & Yeung, 1999; Marsh & Craven, 2006; Marsh & O'Mara, 2008; Marsh & Yeung, 1997), and an additional objective of the present study was to explore this dynamic by narrowing mathematics achievement to number sense as one of its key elements. In exploring the link between number sense and adolescent mathematics self-concept, a focused look at the way this pattern empowers overall mathematics achievement provides a more well-rounded view of how curriculum and instruction can shape mathematical development.

Summary of Methods and Findings

Data for the study was drawn from the last round of data collection for the ECLS-K, occurring during the spring of students' eighth grade year. The large sample size and extensive data allowed for cognitive, demographic, and affective measures to be incorporated into the study design. Single-indicator measures within the ECLS-K database with strong psychometric characteristics were identified for the number sense, mathematics self-concept, and mathematics achievement constructs (Tourangue, Nord, Lê, Sorongon, & Najarian, 2009). The path analysis technique, the oldest form of structural equation modeling, was best suited for studying the relationship between the single observed variables. The five steps identified by Kline (2011) for conducting a structural equation model are: specification, identification, measure selection and data collection, estimation, respecification, and reporting the results. These guidelines will frame the summary description of the current study.

Specification began with the inclusion of the primary exogenous predictor variable number sense and the endogenous outcome variables of mathematics self-concept and mathematics achievement. As an indicator of number sense, highest proficiency level mastered detailed skill level based on clusters of questions including number sense items. The perceived interest and competence in mathematics score, derived from Marsh's Student Descriptive Questionnaire II, was utilized as a measure of mathematics self-concept. Finally, the mathematics teacher academic rating scale was chosen to represent mathematics achievement. Directionality presumed causal effects from number sense to both mathematics achievement and mathematics self-concept, and from mathematics self-concept to mathematics achievement. The paths from number sense to mathematics achievement and mathematics self-concept corresponded to the first two research hypotheses stating that number sense predicts both mathematics achievement and mathematics self-concept. With the additional path from mathematics self-concept to mathematics achievement, the third research hypothesis is represented, asserting that mathematics self-concept mediates the path from number sense to mathematics achievement.

Identification of the recursive model implied unidirectional causal effects and uncorrelated disturbances, as represented in the proposed model diagram from chapter 3 (Figure 2, p. 59). Estimation was conducted by evaluating the Comparative Fit Index, the Root Mean Squared Error of Approximation, and the Standardized Root Mean Square Residual in accordance with Kline (2011). Since each of these measures was deemed sufficient, no respecification was necessary. Consequently, the initial proposed model supported all three research hypotheses.

The first research question asked whether number sense predicts mathematics achievement. Analysis revealed that number sense scores positively and significantly contributed to teacher academic ratings in mathematics for eighth graders in the study.

Whereas previous findings have involved elementary growth patterns following initial number sense ratings, the results of the current study provide strong evidence for the continued contribution of number sense as students move into higher-level coursework. Having obtained positive results for the first research question (with a path coefficient of .27), the present study succeeded in empirically extending the beneficial role of number sense in mathematics achievement to middle school.

For the second research question, number sense was significantly and positively linked to mathematics self-concept for participants. The confirmatory results reveal how number sense, as a specific and critical component of mathematics achievement, predicts mathematics self-concept at the middle school level. The strength of this path coefficient (.25) indicates that number sense may influence mathematics self-concept more than other content domains within mathematics achievement (i.e. geometry and spatial sense, data analysis, statistics, and probability, and algebra and functions).

Finally, a model of mathematics self-concept mediating the impact of number sense on mathematics achievement was verified in accordance with the third research question. This finding provides support for the indirect effects of number sense on mathematics achievement through mathematics self-concept. The total effect coefficient of .35 (combining the direct effect of .27 with the indirect effect of .08) indicates that number sense and mathematics self-concept play a substantial role in mathematics success for eighth graders, and that their interaction and mutual influence must be taken into consideration when studying middle school mathematics.

Moderation analyses revealed that the interrelationship of number sense, mathematics self-concept, and mathematics achievement in the present study was mediated by gender and race. Such findings indicate that the results of the path analysis must also be considered in the context of student demographics. Namely, it appears that middle school mathematics achievement is tied more strongly to number sense for females than males. Specifically, the direct path from number sense to mathematics achievement was only .21 for males but .37 for females. However, the path from mathematics self-concept to mathematics achievement is similar for both males and females (.33 and .31 respectively), and this is in keeping with the Marsh and Yeung study (1998) revealing similar path coefficients for prior mathematics self-concept and follow-up mathematics performance for boys and girls. It is likely that the main reason for the disparity in total effects by gender is due to the high direct effect of number sense on mathematics achievement for females.

In terms of race, variations in the model are more complex, especially considering that special education services and socioeconomic status were included as covariates. Though all racial groups show positive, significant direct effects for number sense on mathematics achievement, and indirect effects on this relationship through mathematics self-concept, there are variations in degree across groups. First, White students reflect the total effects average for the model of .35. However, it is noteworthy that the paths comprising this figure are not necessarily similar across all model paths. For example, the direct path from number sense to mathematics achievement for White students is .28, just a bit above the model average of .27. For mathematics self-concept to mathematics achievement, the direct effect of .31 is also close to the model average of .32. However, the direct effect from number sense to mathematics self-concept is .31 compared to the model path of .25. It seems the relationship between number sense and mathematics self-concept indirectly lowers the total model effects (.07 versus .08).

Hispanic students seem to be most influenced by indirect effects (.11 versus the model average of .08) compared to all other racial groups, and this is due to the elevated direct effects of number sense on their mathematics self-concept and mathematics self-concept on their mathematics achievement. In conjunction with an above-average direct effect value for the path from number sense to mathematics achievement (.31 versus .27), the Hispanic group shows the second highest total effects value of .42.

Asian students reveal the largest total effect calculation (.46) for the model. Since the path from mathematics self-concept to mathematics achievement is the model average of .30 for this racial group, their heightened total effect is primarily due to their elevated path from number sense to mathematics achievement (.37 compared to the model average of .27). However, the group also shows a larger path coefficient from number sense to mathematics self-concept (.30 versus the model average of .25), which also increases the indirect effect value of number sense on mathematics achievement via mathematics self-concept to .09 (just above the model average of .08). Though the majority of their higher total effect figure is due to number sense directly, it is still augmented by an above-average indirect effect.

The model applied to Black students reflects weaker, though still positive and significant, model effects. This lower value (.15 versus the model's .35) does not appear to be substantially related to the influence of mathematics self-concept on mathematics achievement (which is .30 compared to .32 for the model). However, the path from number sense to mathematics self-concept is only .20 compared to the model's path average of .25, and this lowers the indirect value of number sense on mathematics achievement via mathematics self-concept to .06 (versus .08 for the model). Additionally, and perhaps most noteworthy, the path coefficient for number sense to mathematics achievement for Black students is only .09 compared to the model average of .27, and with a *t*-value of 1.79, isn't considered significant.

Limitations

The first round of data collected for the ECLS-K was representative of the U.S. population, but longitudinal analyses were complicated by student mobility, and therefore it was not possible to maintain this level of generalizability throughout the eight-year period. The present study looked solely at students for whom a mathematics teacher provided a skills rating and who themselves completed the student descriptive questionnaire. Because of this constraint, results are indicative of a large sample of eighth graders at one point in time and not necessarily from randomly selected demographics. The use of ECLS-K sample weight and design effect variables, however, does strengthen the accuracy of interpreting results in relation to the broader population by somewhat compensating for nonresponse and selection bias.

Measurement of the variables selected to represent mathematics constructs, in this case number sense, mathematics self-concept, and mathematics achievement, is not without flaws. For number sense, the proficiency level mastery score is based on questions not selected by the researcher, and because of this constraint, it is not possible to confirm that all elements of the McIntosh, Reys, and Reys (1992) framework were addressed. Furthermore, as the CCSSM reveal, number sense is a piece of many different problem types, varies in scope as students develop, and is often represented in mathematical behavior not captured by standardized tests. Ideally, a more comprehensive and broadly measureable scale would be used to represent the number sense variable.

Although utilizing teacher academic ratings for mathematics achievement is an excellent way to incorporate mathematical thinking and behavior not necessarily reflected in multiple-choice questions, teachers are human, and as such, susceptible to personal bias and subjectivity. Moreover, standards for curriculum at the eighth grade level vary immensely across the United States. Whereas some schools offer multiple levels of mathematics curriculum in eighth grade, including General Mathematics, Pre-algebra, Algebra I, and Geometry, others may only teach one curriculum, and in both circumstances, tracking may or may not be applicable. This coursework factor potentially impacts the way that teachers interpret academic skills for their students.

The ECLS-K mathematics self-concept scale has strong empirical support in the literature (Ellis, Marsh, & Richards, 2002; Marsh, 1993b; Marsh, Ellis, Parada, Richards, & Heubeck, 2005), as implemented for the Department of Education (Najarian, Pollack, & Sorongon, 2009; Tourangeau, Lê, Nord, & Sorongon, 2009; Tourangeau, Nord et al., 2009), and as revealed by the factor analysis performed in the current study. However, the context of this mathematics self-concept is not explored beyond its broad relation to number sense and mathematic achievement. Students may be affected differently by variables beyond their control, such as academic setting, range of mathematics topics being covered, or instructional format. Further study of this affective domain according to such environmental factors might provide even more informative data regarding how it influences students.

As is the case in all causal-modeling analyses, the path analysis was constructed based upon hypotheses proposed by the researcher. Though a strong theoretical basis supports each component of the model, specification results in effect priority based upon researcher hypotheses. This model assumes the well-established reciprocal link between academic self-concept and achievement (Marsh, 1990b; Wigfield, Eccles, & Pintrich, 1996). However, directionality for the model is confounded by the introduction of number sense, which is a distinct element of mathematics achievement, and which has never been studied in relation to mathematics self-concept. Neither the path from number sense to mathematics self-concept, nor the path from number sense to mathematics achievement is reciprocated in the model analyses. Rather, previous reciprocal findings were utilized to predict that number sense, related to mathematics achievement through content and skill, partially predicts mathematics self-concept, and that mathematics self-concept partially predicts mathematics achievement. However, since number sense clearly plays a part in mathematics achievement, it is likely that it influences mathematics self-concept in a similar manner.

Future Research Implications

The current study depicts number sense as a predictor of mathematics self-concept and mathematics achievement, controlling for gender, special education services, race, and socioeconomic status. More environmental details, such as those pertaining to curriculum, pedagogy, and overall classroom and school climate, might provide insight into how the specific aspects of number sense, mathematics self-concept, and mathematics achievement impact one another. Additionally, informative factors potentially influencing the study variables often occur outside of the classroom, as in the case of parental involvement and pressure and the amount of practice and real-life exposure students have to mathematical ideas. When teachers encounter students with underdeveloped number sense skills, descriptive data on the context of deficiencies will better direct their methods of intervention and how they approach instruction. The present study provides evidence of number sense and mathematics self-concept as contributing factors in the myriad of details surrounding middle school mathematics education, and ongoing achievement research must explore details pertaining to both constructs, including how the two may be related contextually.

The number sense construct would benefit from more practical specification, providing deeper awareness of obstacles hindering success. For example, a student might only struggle with ratio and proportion in a word problem, but not when directed to find equivalent fractions in completing a proportion. This circumstance could be remedied with more context-specific application in the classroom. Some students may need to witness a recipe ingredient adaptation or construct a reduced scale diagram of a room in their home to truly comprehend the problem. For another student, the issue might be less about context and more about operations. While grasping the physical representation and appreciating the concepts behind comparing different quantities and measurements, students might get lost in operations like multiplication and division that are required to solve the problem accurately and completely. With a closer look at the context of number sense, whether dealing with numbers, operations, or application, it might be possible to determine which factors are most influential on a student's mathematics progress.

The results of the current study indicate the need for a sharper research focus on mathematics self-concept as well as number sense, possibly providing information on if and when students are more influenced by affective elements. For example, could mathematics self-concept be more of an issue in the classroom than at home? Might it correspond to individual versus group environments? Is it susceptible to influence through pedagogy or classroom structure, such as direct versus cooperative instruction? Likewise, how are various applications of number sense a part of these dynamics and do these patterns fluctuate across different content objectives? Even though such determinations will vary by student, understanding the underlying structure of mathematics self-concept from this perspective might inform more effective curriculum and instruction design.

A more detailed look at the key study variables of number sense and mathematics self-concept must also include further exploration of the reciprocal effects model for mathematics self-concept and mathematics achievement. Number sense as well as other content domains should be analyzed with mathematics self-concept in an effort to determine whether causal links at this level are also reciprocal. This claim is supported by a recent kindergarten study by Ivrendi (2011) that found behavioral self-regulation, already linked to motivational constructs (Ning & Downing, 2010; Ommundson, Haugen, & Lund, 2005; Zimmerman, 1989, 1994; Zimmerman & Martinez-Pons, 1990), is a significant predictor of number sense. Additional studies would benefit not only from a more detailed and context-specific analysis of number sense and mathematics self-concept, but also consideration of the directionality of their interaction with one another.

Finally, moderation results indicating differences in the model according gender and race should be used as a basis for further research into number sense as it impacts both mathematics self-concept and mathematics achievements demographically. For instance, why does number sense have a stronger relationship with mathematics achievement for females than males in middle school? Males have shown an advantage in number sense skills at the elementary level, and this has given them an edge over time as well, even though growth trajectories remain similar (Jordan et al., 2007; Jordan, Kaplan, Olah, & Locuniak, 2006). An analysis of NAEP data from 1990 to 2003 determined that gender gaps favoring males in mathematics achievement were highest in measurement and number and operations in grade eight (McGraw, Lubienski, & Stutchens, 2006). Likewise, though males have not shown a growth advantage in mathematics self-concept, they do maintain a higher level of mathematics self-concept than females that continues at the middle school level (Nagy et al., 2010). It's feasible that initially and consistently lower levels of both number sense and mathematics self-concept for females makes the reliance of mathematics achievement on number sense, or perhaps their valuing of the number sense content domain, that much stronger for them compared to male students.

In terms of race, results of the present study provide extensive research options for the number sense construct according to race as well as how number sense influences mathematics self-concept and mathematics achievement for different racial groups. Furthermore, this line of study must determine whether these relationships are similar, less, or more significant in middle school than at other developmental stages. For instance, it is imperative to more deeply examine why this relationship is so much stronger for Hispanic and Asian students. Previous literature indicates that Hispanic students may enter elementary school with lower mathematics proficiency skills, and that the White-Hispanic achievement gap may go down in magnitude but that it continues to exist (Borman, Stringfield, & Rachuba, 2000; Reardon & Galindo, 2007, 2009). Considering these circumstances in light of the clear link between mathematics self-concept and mathematics achievement for students and the connection between early number sense and mathematics achievement growth, it is possible that this initial disadvantage may be related to their beliefs about mathematics ability and their progress in this subject in middle school. However, similar findings have shown that Black students also may enter with lower mathematics skill levels (Borman, Stringfield, & Rachuba, 2000; Cheadle, 2008), and yet their number sense is less heavily tied to mathematics self-concept and mathematics achievement. Such discrepancies may require a deeper look at contextual variables like school quality, community and family background, teacher racial bias, or stereotype threat for example.

Practical Implications

The findings from the current study have important implications for multiple groups of individuals and organizations across the United States. It includes the state policymakers shaping the curriculum standards and pedagogical strategies employed by districts. It includes superintendents, principals, and mathematics department heads administering such guidelines. It includes the teachers attempting to reach every student in their classroom in the most effective ways, regardless of background and skill. And it includes the students transitioning from curriculum focused more on arithmetic to that more heavily concerned with complex algebraic topics. These students are young adolescents who have been told they can achieve the American Dream through education, and who may or may not realize just how critical this year of their mathematical progress is to their future education and career paths.

The CCSSI has provided mathematics standards adopted by the majority of the United States through the CCSSM, and their purpose is to ensure consistency as well as authentic and comprehensive curriculum on par with international performance and intended to provide all students with the opportunity to succeed. These standards, however, are not intended to provide a rigid road map for content, nor do they assume that alternative and additional subject matter should be precluded from curriculum. Furthermore, the CCSSM do not attempt to tell teachers how to instruct in every setting nor how to attend to the needs of each individual student. They speak to how students should think about and engage in mathematics.

To provide a CCSSM example and how results of the current study might work to aid multiple education participants, consider standard 5 for the eighth grade Expressions and Equations domain. Within the cluster "Understand the connections between proportional relationships, lines, and linear equations," this standard calls for the following: "Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships in different ways" (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010, p. 54). Mathematics achievement in this area might be gauged by accurately calculating gas mileage based on a hypothetical visit to the local Exxon Mobile and then plotting a line representing this mileage rate through slope and points corresponding to gallons used and miles traversed. Number sense comes into play on multiple levels. First, the student must be able to grasp a proportional relationship, which means he or she must understand what a ratio represents, how the numerator and denominator are related, and how this can be applied to not just gas mileage, but recipe calculations or discounts at a retail sale. Going deeper, the student must understand how to multiply and divide and how to apply such operations when working with proportions. If a student is not comfortable with any of these basic prerequisites, all components of number sense, they may not only be unable to construct and apply the proportion based on mileage read and gasoline dispensed, but also be intimidated by the more complex operations that correspond to this practical life application. In this sense, their lack of mathematical foundational skills may be even further hindered by their insecurity surrounding the larger problem at hand. So, even if they were capable of learning how to calculate proportions and subsequently attending to the graphing aspect of the problem, they might be less inclined to engage in this process due to their belief that they are not good at mathematics because they are not good at multiplication and division.

A great deal of responsibility for effectively working with students experiencing such learning predicaments falls on teachers at the classroom level. There is, without a doubt, a wealth of supplemental materials available for mathematics instruction, whether through textbook publishing companies, online teacher forums, professional expertise within schools or districts, or education resource software. However, none of these aids are in the classroom and interacting with a student at any given time. Nor can they account for extenuating circumstances like family background and influence, personal expectations and values, teacher knowledge and experience, or administrative support. So how can teachers use the results of the present study in practice? There is no formula, just as there is no prescribed plan provided by the CCSSM on how to instruct or attend to student needs. However, whereas previous research has emphasized key components of mathematics content in relation to mathematics success distinctly from student motivation as it pertains to mathematics achievement, the present study wishes to make teachers aware of the diversity and flexibility implied by curriculum and required in instruction by integrating these areas of study.

When a middle school student is hesitant to participate in a lesson on interest rates, teachers are faced with the larger task of exploring the nature of this reluctance. Is it because the student does not understand the purpose of interest rates? Is it because they have not mastered decimals and percentages, and have therefore shut down in the face of related material? Does it go as far back as fractions, multiplication, or division? Moreover, are any or all of these circumstances compounded by the student's low mathematics self-concept as it relates to their skill deficiencies and/or in accordance with their gender or race? The scope of the current study cannot account for funding issues, large class sizes, disparate skill levels within a classroom, or teacher capacity to cope with so many issues simultaneously. Without question, the established mediating impact of mathematics self-concept in the relationship between number sense and mathematics achievement complicates the already rigorous and vital role of educators. Hopefully, however, the results will enable mathematics teachers to more prosperously relate and respond to their students.

The present study's confirmation of the strong connection between number sense, mathematics self-concept, and mathematics achievement at the middle school level serves to enhance the ways in which schools employ the standards. It reveals the need to emphasize consistently the number sense represented by SMPs at every grade level, whether or not corresponding operations are explicitly documented in the relevant CCSSM. Moreover, it highlights some important elements of instruction not represented solely by mathematical content. The results should guide states as they develop measures of accountability, districts as they purchase and employ textbooks and technology, schools and teachers as they structure and carry out classroom operations, and students as they practically apply their mathematical knowledge and seek fresh mathematical challenges.

Conclusion

Reviewing advances and setbacks in the implementation of various curriculum and instruction methodologies, the CCSSM authors have now provided a comprehensive framework for student progress that is rooted in number sense at all levels of mathematics instruction. The present study set out to better inform this framework by determining that mathematics self-concept has a partial mediating effect on the positive relationship between number sense and mathematics achievement in eighth grade. Put differently, the influence of number sense on mathematics achievement varies depending on the level of a student's mathematics self-concept. These results amplify the established literature linking both number sense and mathematics self-concept to mathematics achievement.

By providing insight into how instruction must attend to not only content and teaching style, but also internal and external factors influencing student engagement and success.

The number sense construct has been recognized, researched, and implemented in instruction for decades, but accountability for it has always been vague due to its flexible interpretation and pervasive role in mathematics curriculum. The impetus for the present study was the conjecture that the attention placed on number sense, both in new curriculum and remediation, attenuates around the middle school level. Results confirm that the important role that number sense plays in mathematics achievement throughout elementary school remains critical as students mature into adolescence. In light of the clear need for continued number sense emphasis, the general decline in academic motivation as students move from childhood to adolescence (Eccles & Midgley, 1989; Schielack & Seeley, 2010) cannot be ignored. Findings from the current study reveal that the impact of mathematics self-concept in adolescence is specifically linked to number sense, and in fact, that this relationship heightens the manner in which number sense influences mathematics achievement.

The field of education has provided strong evidence in support of instruction and curriculum that considers a broad spectrum of learning preferences and capabilities (Bransford, Brown, & Cocking, 2000; Driscoll, 2005; Eisner, 2002), so why has it not implemented this sort of diversity in analyzing a construct as broad as mathematics achievement? It is imperative that the evaluation of the advantages of number sense in mathematics achievement trajectories and intervention strategies not only place more emphasis on middle school students, but that it be strengthened by the incorporation of affective domains like mathematics self-concept, already known to enhance mathematics achievement (Valentine, DuBois, & Cooper, 2004).

The present study has highlighted the benefits of analyzing education in an interdisciplinary fashion, in this case, integrating mathematics education and educational psychology in order to better address a critical juncture in student academic development. Confirmation of the direct impact of number sense on early adolescent mathematics success in conjunction with the indirect influence experienced through mathematics self-concept should thus inform construction and measurement of future curriculum and instruction across the United States and presumably beyond.

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APPENDIX A

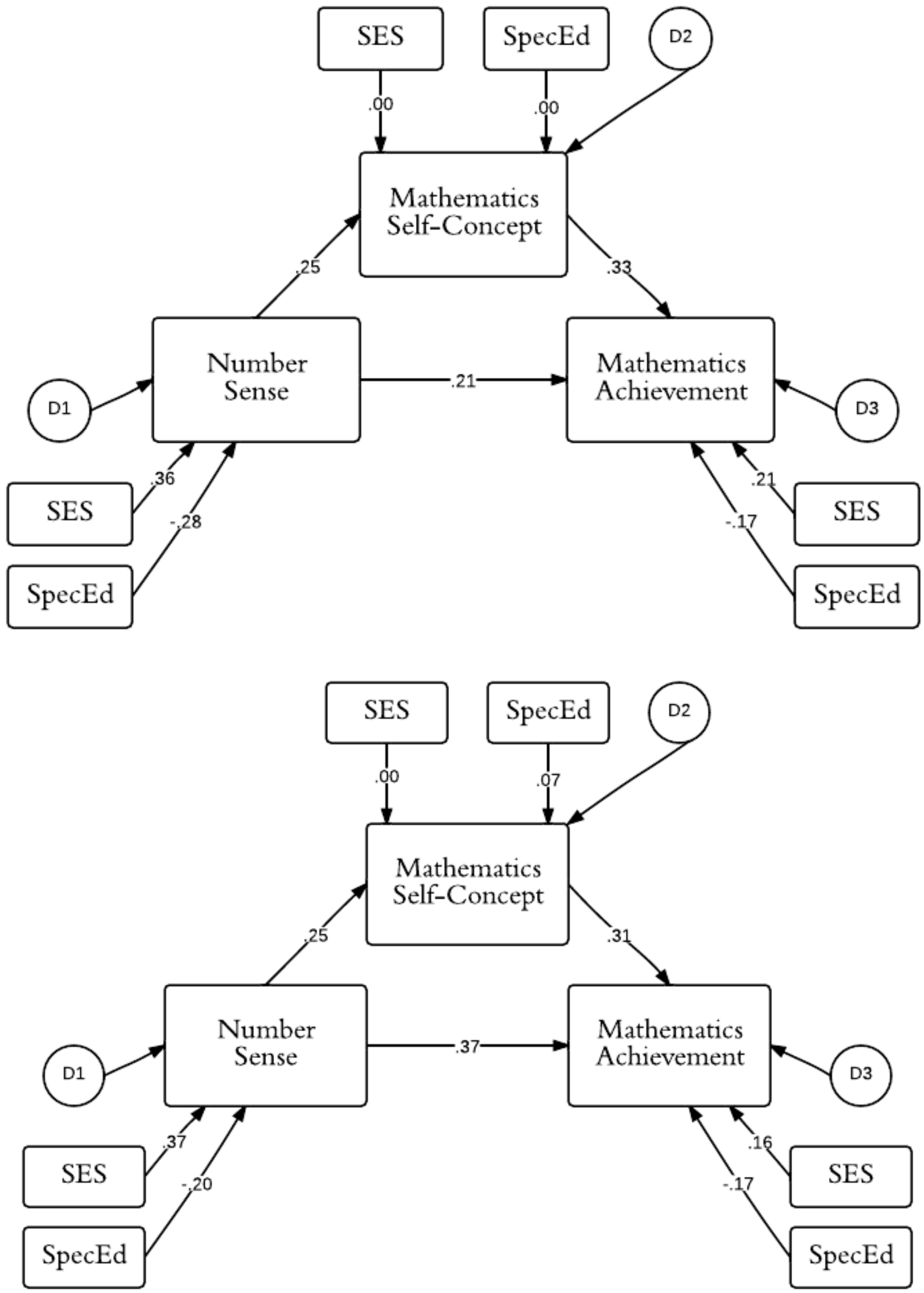

Figure A1. Standardized path coefficients for males (top) and females (bottom).

Table A1

Fit Indices for Model According to Gender

Index	Value
Female:	
Comparative fit index (CFI)	1.00
Root mean squared error (RMSEA)	.00
Standardized root mean square residual (SRMR)	.00
Male:	
Comparative fit index (CFI)	1.00
Root mean squared error (RMSEA)	.00
Standardized root mean square residual (SRMR)	.00

Table A2

Path Coefficients for Model According to Gender

Path	*B*	*SE*	*β*	*t*
Female:				
Number Sense to Mathematics Self-Concept	.33	.02	.25	10.25***
Number Sense to Mathematics Achievement	.50	.02	.37	19.08***
Mathematics Self-Concept to Mathematics Achievement	.32	.02	.31	17.83***
Male:				
Number Sense to Mathematics Self-Concept	.31	.02	.25	10.22***
Number Sense to Mathematics Achievement	.28	.02	.21	10.02***
Mathematics Self-Concept to Mathematics Achievement	.35	.02	.33	17.80***

*** $p < .001$

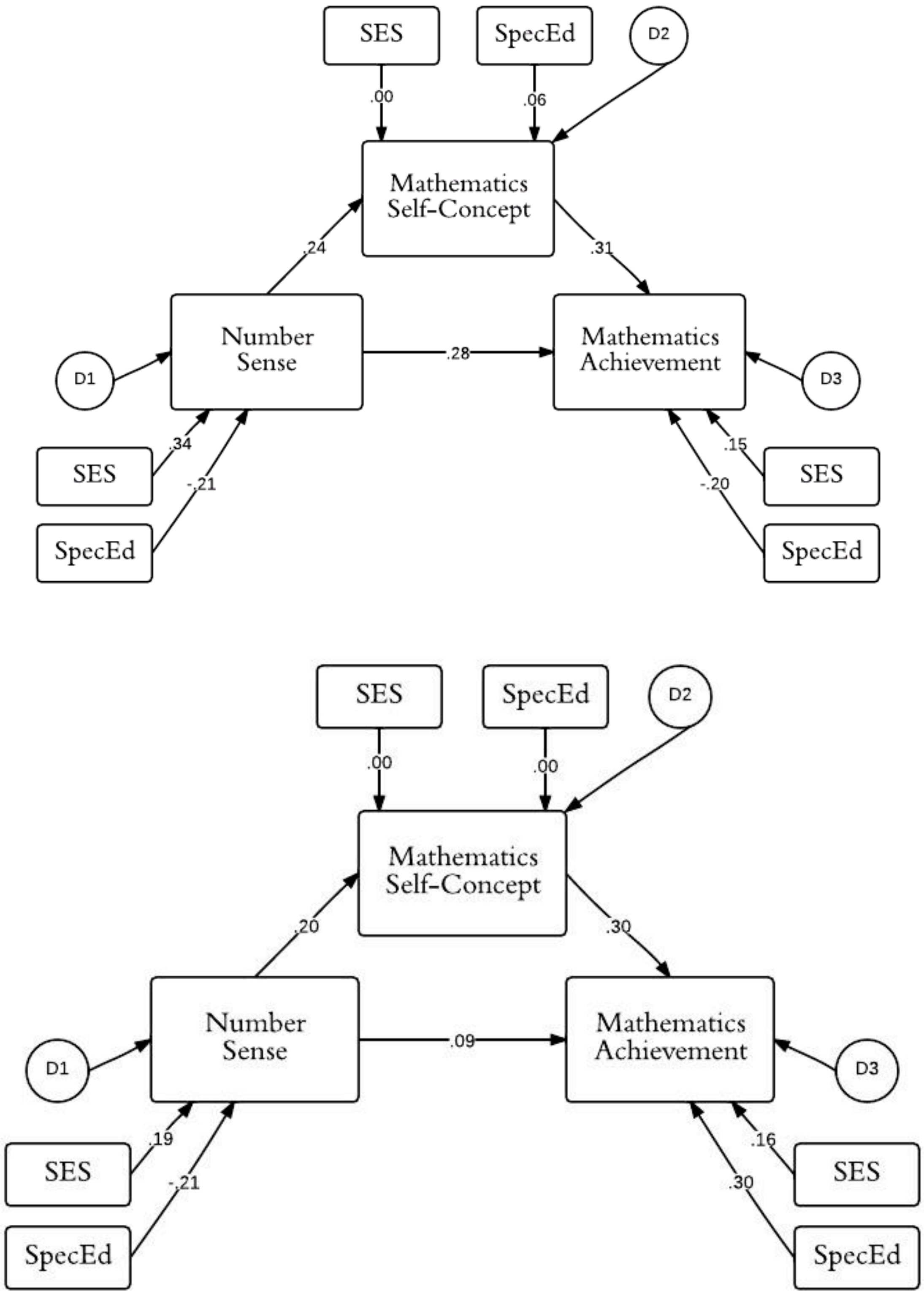

Figure A2. Standardized path coefficients for Whites (top) and Blacks (bottom).

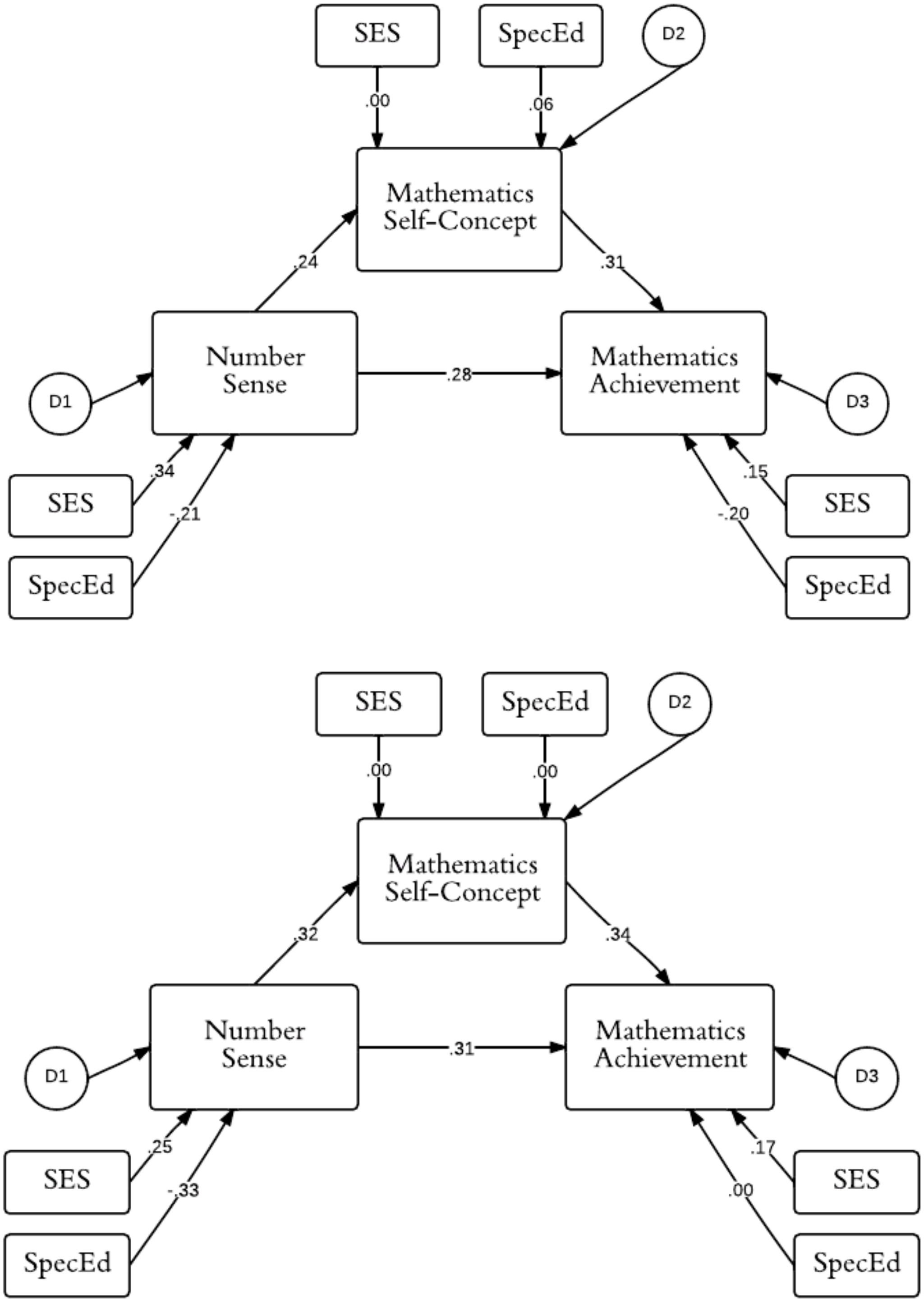

Figure A3. Standardized path coefficients for Whites (top) and Hispanics (bottom).

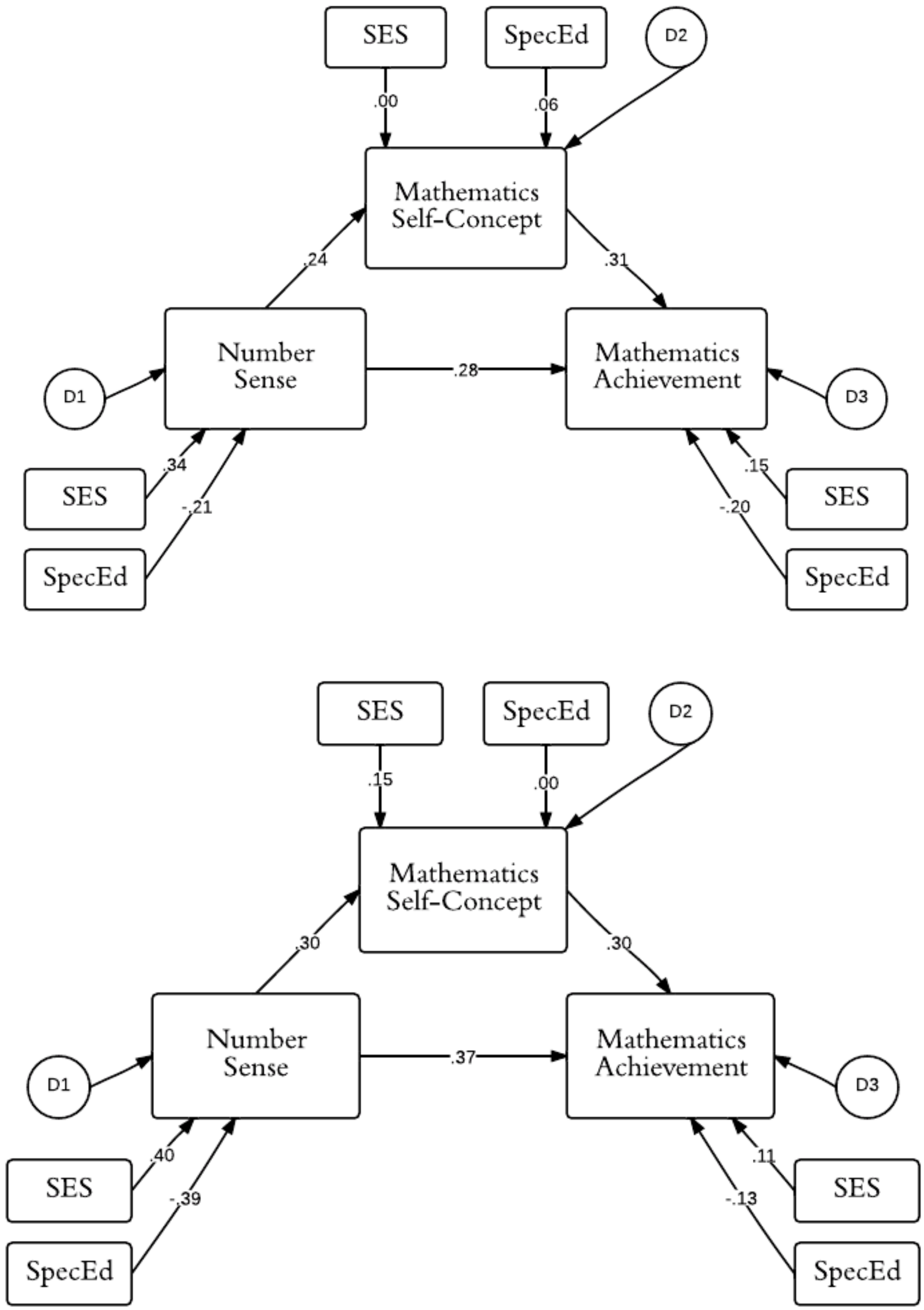

Figure A4. Standardized path coefficients for Whites (top) and Asians (bottom).

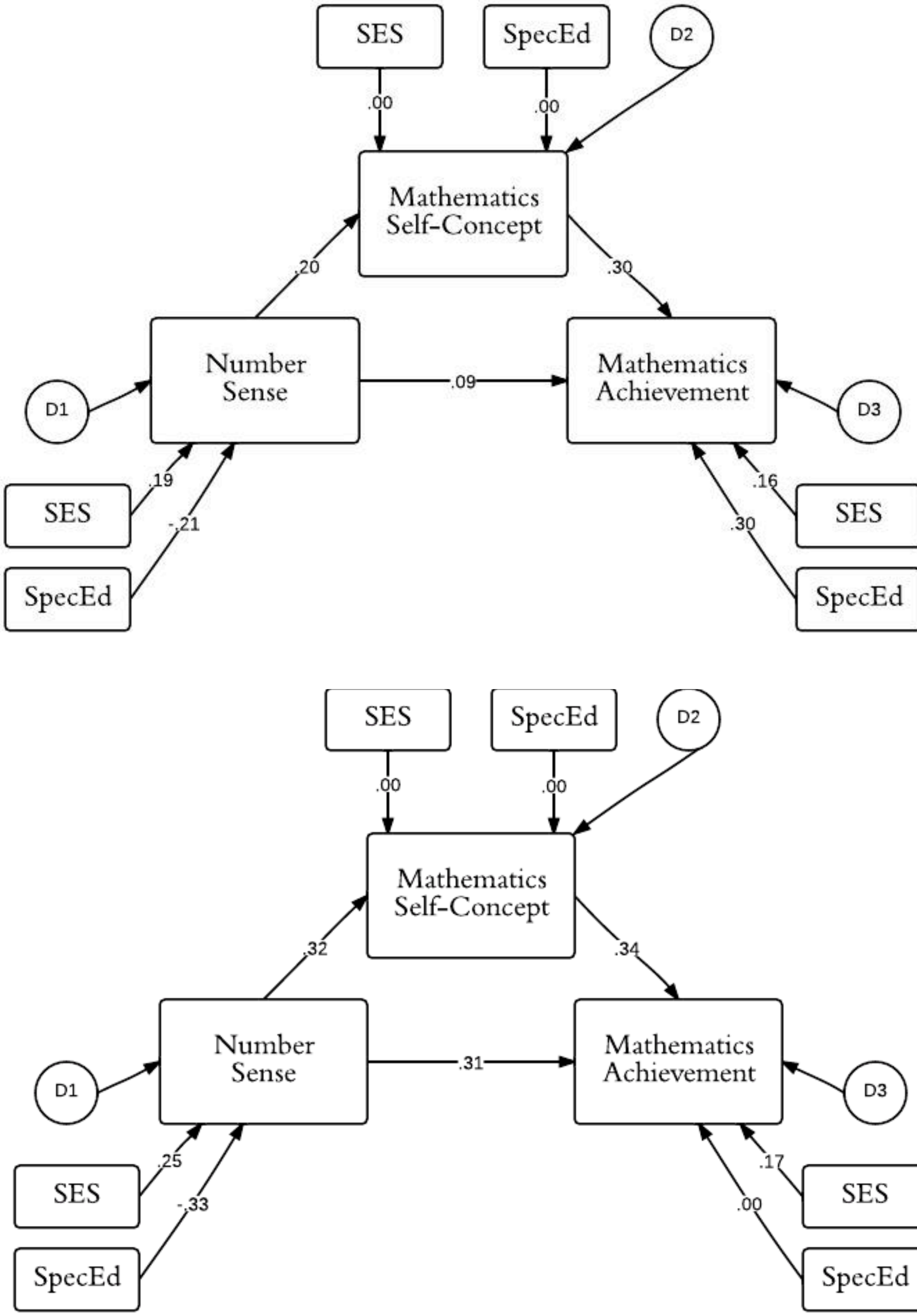

Figure A5. Standardized path coefficients for Blacks (top) and Hispanics (bottom).

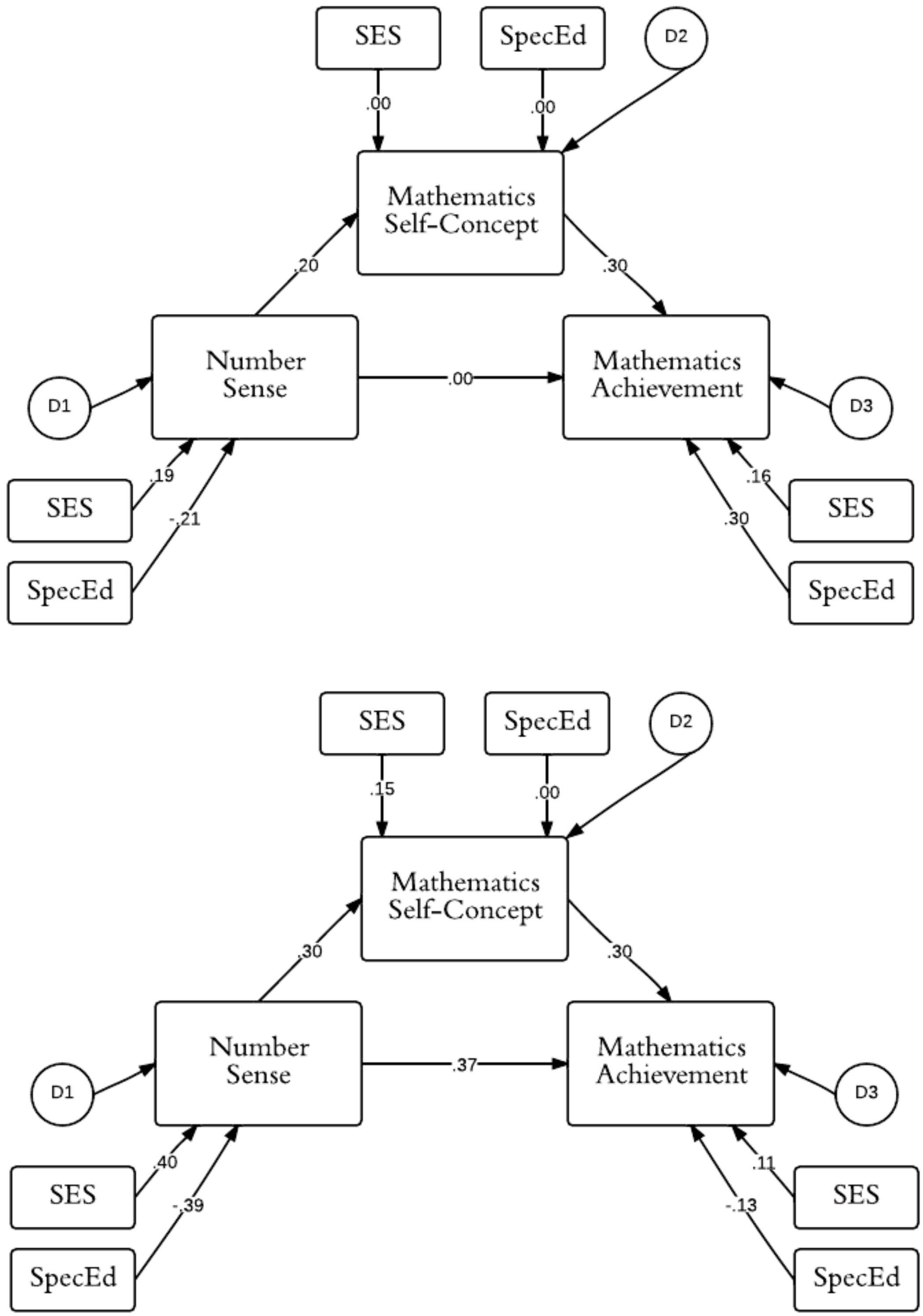

Figure A6. Standardized path coefficients for Blacks (top) and Asians (bottom).

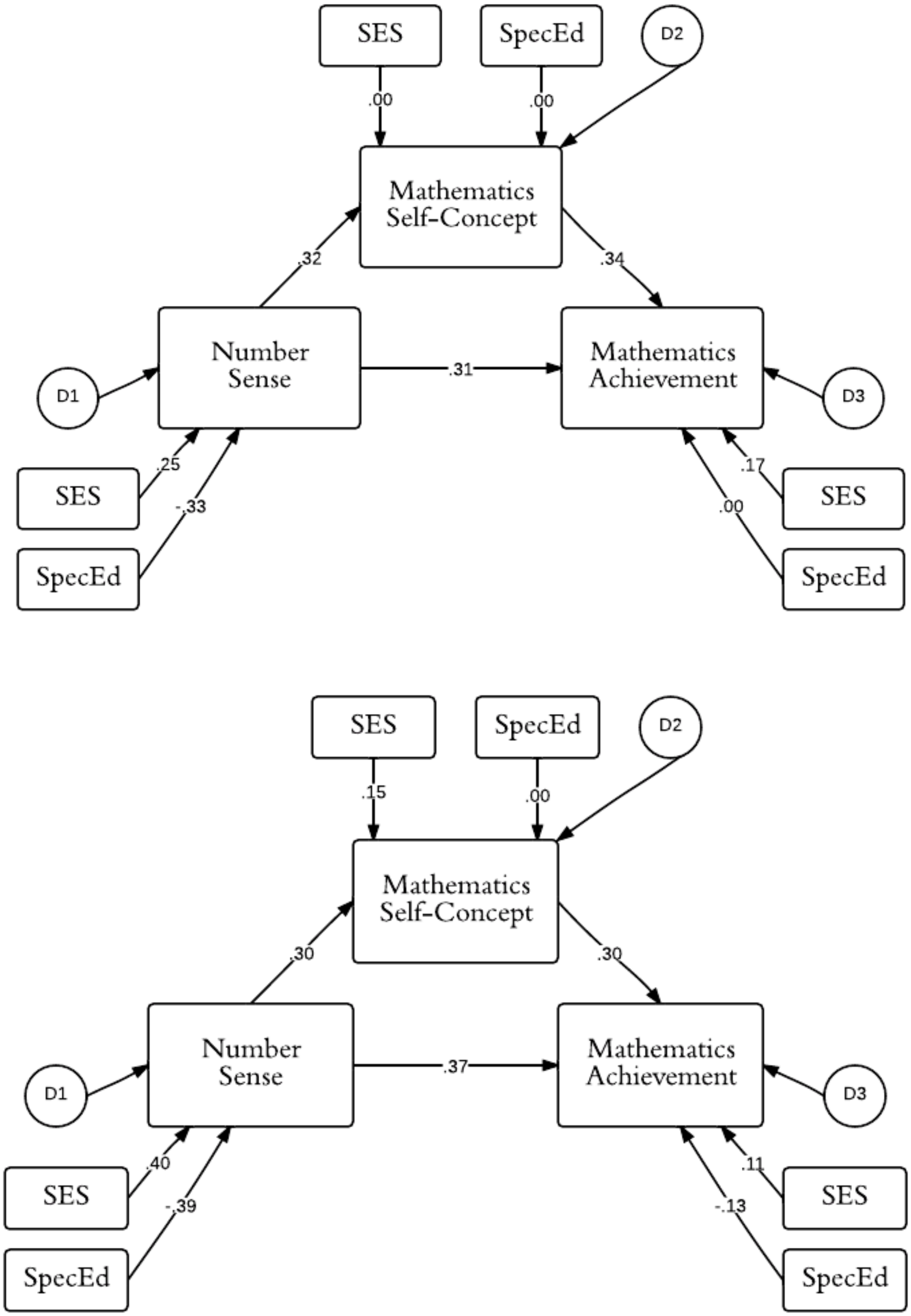

Figure A7. Standardized path coefficients for Hispanics (top) and Asians (bottom).

Table A3

Fit Indices for Model According to Race

Index	Value
White:	
Chi-Square	1.00
Goodness of Fit Index (GFI)	.00
Standardized root mean square residual (SRMR)	.00
Parsimonious Goodness of Fit (PGFI)	.00
Black:	
Chi-Square	1.00
Goodness of Fit Index (GFI)	.00
Standardized root mean square residual (SRMR)	.00
Parsimonious Goodness of Fit (PGFI)	.00
Hispanic:	
Chi-Square	1.00
Goodness of Fit Index (GFI)	.00
Standardized root mean square residual (SRMR)	.00
Parsimonious Goodness of Fit (PGFI)	.00
Asian:	
Chi-Square	1.00
Goodness of Fit Index (GFI)	.00
Standardized root mean square residual (SRMR)	.00
Parsimonious Goodness of Fit (PGFI)	.00

Table A4

Path Coefficients for Model According to Race

Path	*B*	*SE*	*β*	*t*
White:				
Number Sense to Mathematics Self-Concept	.31	.02	.24	11.56***
Number Sense to Mathematics Achievement	.37	.01	.28	15.99***
Mathematics Self-Concept to Mathematics Achievement	.33	.01	.31	19.94***
Black:				
Number Sense to Mathematics Self-Concept	.30	.05	.20	3.62***
Number Sense to Mathematics Achievement	.12	.05	.09	1.79
Mathematics Self-Concept to Mathematics Achievement	.27	.04	.30	6.72***
Hispanic:				
Number Sense to Mathematics Self-Concept	.41	.04	.32	7.94***
Number Sense to Mathematics Achievement	.41	.04	.31	8.38***
Mathematics Self-Concept to Mathematics Achievement	.36	.03	.34	10.45***
Asian:				
Number Sense to Mathematics Self-Concept	.35	.07	.30	4.23***
Number Sense to Mathematics Achievement	.50	.06	.37	6.42***
Mathematics Self-Concept to Mathematics Achievement	.34	.05	.30	5.88***

*** $p < .001$