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An investigation of the influence of the 2007-2009 recession on the day of the week effect for the S&P 500 and its sectors

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AN INVESTIGATION OF THE INFLUENCE OF THE 2007-2009 RECESSION ON

THE DAY OF THE WEEK EFFECT FOR THE S&P 500 AND ITS SECTORS

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ABSTRACT

Several studies have shown that the mean returns and the volatility structure of stock markets change seasonally or by day of the week. For instance, some authors found out that Monday returns are lower compared to Friday returns or that volatility on Wednesdays are lower compared to the rest of the week. Other researchers showed that these effects have changed after certain periods of economic stress. This led to the question, whether the day of the week effects in returns and volatility are in the US stock market and if patterns have changed from pre-recession through the 2007-2009 recession into the post-recession period. Therefore, a study investigating returns from February 2005 to January 2018 for the S&P 500 and its ten sectors was conducted. To investigate any changes, the data set was separated into three distinct periods. The mean returns were modeled to follow an autoregressive process, while an EGARCH formulation was selected as the appropriate model for the volatility. To estimate the effects, three different approaches were used. Results show that there is only a small day of the week effect in mean returns, and that Wednesdays differ from the rest of the week. However, almost every sector indicate day of the week effects in volatility, where volatility was highest on Tuesdays in pre-recession period, whereas results differ from sector to sector for post-recession period. The findings for the post-recession period are consistent across every approach, whereas the results for the recession period are different depending on the model used in the analysis.

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1 INTRODUCTION

In an efficient market, one would expect that the mean returns should be the same for each day of the week (French, 1980). However, some researchers have shown evidence of a “day of the week effect” that contradicts this assumption. This effect describes the fact that mean returns vary depending on the day of the week. Researchers like Cross (1973) and French (1980) showed that, for instance, S&P 500 Monday returns are lower compared to the rest of the week. Other studies given by Rodriguez (2012), Choudry (2000) and Seif et al. (2015) indicate that this effect also holds for international markets. More recent studies, such as those of Berument and Kiymaz (2001) and Rodriguez (2012), have shown that this pattern does not only restrict to mean returns, but can also affect volatility, thus implying that some days of the week are more volatile than others. The main objective of the research presented in this theses is to investigate the presence of the day of the week effect in recent S&P 500 data using several approaches, but more importantly, to determine if the nature of this effect has changed across the most recent recession. The recession may have triggered a change in the trading practices, or the manner in which good or bad news is perceived, and these may have affected any day of the week effects that were present prior to the recession. It is also interesting to determine if any change in “trading practices” have persisted through the post-recession period. If such changes are present, whether these changes apply to all sectors of the S&P 500 index will be of added interest.

When modeling a time series such as stock returns, with returns often calculated as the difference in the logarithm of consecutive prices (hence sometimes called log-returns), an autoregressive moving average (ARMA) model is assumed. Usually, returns are not highly autocorrelated so that the orders of the process are small. The standard ARMA model assumes that the error terms are white noise meaning that the error terms have zero mean, equal and finite variance, and are uncorrelated. While this is, in general, true of stock return data, the errors are not conditionally homoscedastic as is generally true in most empirical time series modeled as an ARMA process. It is well-known that volatility appears in clusters. There are periods with high volatility and periods with low volatility indicating that error terms are conditionally heteroscedastic. That is, the variance of the error terms, when conditioned on the past observations are not all equal. In other words, the error terms will tend to exhibit higher volatility if the most recent past was highly volatile, and will have low volatility if the reverse is true.

To overcome this problem, Engle (1982) introduced the autoregressive conditional heteroskedastic (ARCH) model, which later was expanded to a generalized form by Bollerslev (1986). Both the ARCH model as well as the generalized autoregressive conditional heteroskedastic (GARCH) formulation allows the conditional variance to vary over time such that volatility clusters can be accommodated. In a GARCH(p,q) model, the volatility is described by the equation

$$a_t = \sigma_t \varepsilon_t, \quad \sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i a_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 , \tag{1.1}$$

where ε_t is an independent and identically distributed series with zero mean and variance one, with parameters $\alpha_0 > 0$, $\alpha_i \geq 0$ and $\beta_j \geq 0$ for all *i* and *j* to ensure positive variance.

As one can see, the volatility at time t is influenced by the square of the past returns $a_{t-i}, i = 1, 2, \ldots, p$. This is the ARCH part of the model. In addition, volatility at time t is influenced by past volatilities through the terms in $\sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$. This represents the GARCH portion. While GARCH models have shown their utility in modeling conditional heteroscedasticity in time series data, it treats positive and negative shocks symmetrically. That is, it assumes that the effect of positive news (positive shocks) is the same as that due to negative shocks (negative news). However, it is widely known that bad news have a larger influence than good news (Chang, 2010). To remedy this drawback of GARCH models, Nelson (1991) introduced the exponential GARCH (EGARCH) model which allows the shocks to have different influence depending on their sign. Altogether, it models the volatility using the formulation:

$$\begin{aligned} a_t &= \sigma_t \varepsilon_t, \\ \ln(\sigma_t) &= \alpha_0 + \frac{1 + \beta_1 B + \ldots + \beta_{s-1} B^{s-1}}{1 - \alpha_1 B - \ldots - \alpha_m B^m} g(\varepsilon_{t-1}) \\ g(\varepsilon_t) &= \theta \varepsilon_t + \gamma\left(|\varepsilon_t| - E(|\varepsilon_t|)\right) \end{aligned} \tag{1.2}$$

where θ, γ, α_i and β_j are constants, B is the back shift operator meaning $B^r g(\varepsilon_t) = g(\varepsilon_{t-r})$ and both $1 + \beta_1 B + \ldots + \beta_{s-1} B^{s-1}$ and $1 - \alpha_1 B - \ldots - \alpha_m B^m$ are polynomials with zeros outside the unit circle and no common factors. The asymmetry effect is indicated by θ.

Several authors have indicated that asymmetric models like the EGARCH models fit stock return data better than other GARCH type models (Awartani and Corradi, 2005). Du (2017) mentions this as well. Therefore, EGARCH model was selected as the appropriate model to carry out the investigation on the day of the week effects and changes to these effects from pre-recession to recession period, to post-recession. Modeling returns as a pure EGARCH process, however, ignores a possible non-zero mean for the returns as well as autocorrelation between adjacent values. The traditional approach for resolving such issues is to use an autoregressive (AR) component to model the returns and a GARCH type model to explain the behavior of the residuals of the AR model. Financial time series analysts usually refer to the AR component as the “mean” model and the GARCH (in our case the EGARCH) component as the volatility model. Usually, the “mean” is modeled as AR(k) process, where the order k is determined in an initial step. The main purpose of this study is to investigate whether the day of the week effect has changed over time. This question is based on studies of Hui (2005) as well as Akbalik and Ozkan (2017), who showed that the day of the week effect diminished after certain periods in some markets. Berand Kiymaz (2001) showed that the day of the week effect changed after the crisis in 1987 for the S&P 500. The data used in this study consists of returns computed from S&P 500 index data and from its ten sectors, observed from February 2005 through January 2018. Each of these time series is divided into the three investigated periods pre- recession, recession and post- recession. Since the crisis lasted from December 2007 to June 2009 (nber.org), the breaking points for the periods are set to be December 3rd 2007 and June 30th 2009.

The time period studies here is the most recent with respect to similar studies on the day of the week effect on S&P 500 returns. While it uses most of the data utilized by Liu (2015) in her investigation, this study supplements that data with more recent observations.

Three different approaches of estimating the effects are carried out. The first approach estimates the mean and the volatility model jointly. It uses dummy variables to model the effect of Tuesday through Friday with Monday held as the reference. It also introduces dummy variables to identify the recession and post-recession periods, with the pre- recession period taken as the reference period. Interactions between these dummy variables are also introduced. In this approach, however, the dummy variables only influence the intercept of both the "mean" part and the volatility component of the model. Therefore, the second approach is introduced. It is based on the formulation that expresses the squared volatility as an ARMA process. In this second approach, the residuals are extracted from the estimated mean model and the interactions between dummies and lagged residuals are added to the estimated model. The third approach is based on the study by Berument and Kiymaz (2001). In this approach, the data is divided into three distinct sub periods: pre- recession, recession and post- recession. After that, the day of the week effect for both mean and volatility is estimated separately for each sub period by introducing dummy variables to both models. In this approach, dummy variables identifying the period as well as interaction effects are redundant. Therefore, the models in this approach are less complex compared to the other two approaches. Details of these three approaches are given in the Methodology Section in Section 4.

The rest of the thesis is as follows. Section 2 gives a short literature overview, Section 3 introduces the volatility modeling using ARCH, GARCH and EGARCH models, Section 4 describes the data, methodology and the results. Section 5 concludes the results.

2 LITERATURE REVIEW

Considering the process generating stock return one can think of two possible approaches. On the one hand, the underlying process can be assumed to be operational continuously over calendar time. Since markets are usually open Monday to Friday, this would imply that Monday returns- representing a three day investment- are three times larger than returns on the rest of the week. On the other hand, when generating stock returns in trading time, returns reflect a one day investment and should therefore be equal among all days (French, 1980).

However, empirical studies show evidence that neither of the above hypotheses are true. Cross (1973) analyzed daily stock returns on the S&P 500 from 1953 until 1970. He found evidences that mean returns on Mondays are lower than mean returns on Fridays which is consistent from year to year. French (1980), who studied S&P 500 returns in the period 1953- 1977, found similar patterns. Other researches like Rodriguez (2012), Choudry (2000) and Seif et al. (2015) who investigated 6 Latin American stock markets, 6 different Asian markets and 9 emerging markets, respectively, found a negative effect on Monday returns, showing that this phenomenon is not limited to specific markets.

This widely known phenomenon is called day of the week effect and implies that average returns differ throughout the week depending on the day (Apolinario et al., 2006). More recent studies also investigated a day of the week effect on volatility.

A lot of research has been done to investigate this effect in different markets. As already mentioned, Cross (1973) and French (1980) found that Monday returns are lower compared to Friday returns for the S&P 500. Berument and Kiymaz (2001) support these results and conclude that S&P 500 returns in the period 1973-1997 are highest for Wednesday and lowest for Mondays. In addition to that, they found evidence for a day of the week effect on volatility where the highest and lowest ones are on Friday and Wednesday respectively. Dividing their observed data into pre-1987 and post-1987 subsets, they found that during the first period volatility was highest on Tuesday, whereas volatility after 1987 was highest on Friday, indicating that patterns change after some time, but the day of the week phenomena do not disappear.

Kiymaz and Berument (2003) found similar results for the markets in Canada, Germany, Japan, the United Kingdom and the United States. They found, for instance, highest volatility on Monday for Germany and Japan, whereas Mondays show lowest volatility for Canada. Although the effects differ from country to country, there is clear evidence for day of the week effects in both returns and volatility.

These results are not only valid in developed countries but also for emerging markets. Choudhry (2000) investigated seven different Asian markets and found significant day of the week effects on returns and volatility. However, the effects differ depending on the market. For instance, in only three markets the mean returns on Monday and Tuesday are different from each other. Moreover, the results indicate that the volatility in six out of the seven markets increases on Mondays.

Seif et al. (2015) found clear evidence for a day of the week effect for returns in most of the investigated nine advanced emerging markets, including Brazil, Mexico and Turkey. In general, Mondays tend to have negative and less than average whereas Fridays tend to have positive or above average returns. However, they didn’t find any evidence for day of the week effect in volatility.

Rodriguez (2012) examines six Latin American markets and all of them show day of the week effects in both returns, except for Mexico, and volatility. Although the results differ, there is a trend for Mondays to have lower returns and higher volatility, whereas Fridays have higher returns and lower volatility.

The described pattern holds for other markets as well. Dyl and Maberly (1986), for instance, showed that returns for S&P 500 futures differ depending on the day. Harvey and Huang (1991) and Ederington and Lee (1993) found high volatilities in Friday's returns for different foreign exchange futures and foreign exchange markets. These findings underline that the day of the week effect for both returns and volatility are not limited to any specific asset class.

Since the day of the week effect was known by traders for a long time, one would expect them to diminish in markets striving for efficiency. Indeed, Akbalik and Ozkan (2017) investigated stock market returns of five fragile countries in the post subprime crisis period 2009-2015 and found that only the Indonesian market showed evidence for day of the week effects on returns. These results are in line with Hui (2005) who found that during the Asian crisis in 1997 the day of the week effect disappeared for some markets.

However, an international study by Dicle and Levendis (2014), including 37631 stocks traded on 51 stock exchanges in 33 countries shows that there are still day of the week effects. In contrast to most of the papers, they don't only use market level data, but they also investigate the day of the week effect for individual stocks. Their findings are quite interesting as they show that the effect has disappeared in about the half of the world's equity market, but, they are still present for individual stocks meaning that the effect cannot be detected at market level. More precisely, a day of the week effect exists in only 24 of the 51 markets for Mondays and 32 for Fridays. However, breaking the analysis down to individual stocks they found day of the week effects for most of them, even if there is no effect for the overall market. Since the amount of markets with day of the week effect reduces when using value weighted instead of equally weighted market, they conclude that the effect is especially present for small stocks.

All the discussed papers show evidence for a day of the week effect in volatility and returns, although it seems to diminish. This may lead to the question, what gives rise to this phenomena.

French (1980) found, by comparing holiday and non-holiday returns that negative Monday returns are due to the weekend and not a general market closing matter. This may be caused by the fact that most companies publish bad news at or right before weekends (Dicle and Levendis, 2014). Berument and Kiymaz (2001) argue that this may cause high volatility on Fridays since weekend expectations are already priced on Friday.

Ederington and Lee (1993) proved that the day of the week effect for volatility is mainly caused by major macroeconomic announcements. Harvey and Huang (1991) support this argument and state that in fact most of the macroeconomic news are released on either Thursday or Friday which leads to an increase in volatility during these two days. However, their study is based on futures prices so this might be different in other markets. Low volatility on Wednesday can be due to the fact that it is the middle of the week and therefore provides the most time to react to new information (Berument and Kiymaz, 2001).

The so called spill-over effect may be another possible explanation. This hypothesis states that day of the week effects may be caused by preceding effects in other markets (Dicle and Levendis, 2014). Indeed, Choudhry (2000) found evidence for spill-over effects for the markets in Thailand and Malaysia from the Japanese market. Dicle and Levendis (2014) found similar results for some of their observed markets.

Related to this explanation is the one given by Aggarwal and Rivoli (1989). Their findings of a Tuesday effect in four Asian markets indicate that the day of the week effect may differ in different time zones. Since the markets are 13 hours ahead of American time, the Monday effect in the US- meaning low returns on Mondays- may shift to a Tuesday effect in the Asian market. Seif et al. (2015) couldn't support this hypothesis for their investigated countries including Malaysia and Taiwan.

Campbell and Hentschel (1992) state that periods with high volatility could cause changes in returns. This hypothesis is supported by French and Roll (1986), whereas Seif et al. (2015) couldn't find any evidence for their observed markets.

Since informed traders are not willing to trade in volatile periods, high volatility coincides with low trading volume (Foster and Viswanathan, 1990). Dicle and Levendis (2014) call this the impact of liquidity. Indeed, they found evidence that this is a reason for the day of the week effect. Kiymaz and Berument (2003) support these results since, for instance, high volatility coincides with low volume on Fridays in the US stock market.

Additional possible reasons for the day of the week effect are given in Dicle and Levendis (2014). The settlement issue states that the gap between settlement and trading can lead to advantages for traders at the end of the week compared to those trading in the first day of the week. Differences in trading small and large stocks can lead to biased estimates when using market indices, thus resulting in what they call the non-trading problem. Indeed, their investigations of more than 50 markets indicate that the day of the week effect is mainly explainable by size, spill-over, liquidity and non-trading effects.

Finally, it is worth to note that the presence of the day of the week effect may be useful for some traders. Although it is not profitable to buy the portfolio every Monday and sell it on Friday compared to holding the stock (French, 1980), traders can still make advantages on the knowledge of the effect. Knowing that Monday returns are negative, one could delay planned purchases until Monday and planned sales for Friday (French, 1980). Moreover, if investors know certain patterns in volatility they may adjust their portfolio according to the estimated risk for specific assets. They also can use their knowledge of predicted volatility for their valuation or for speculative purposes (Berument and Kiymaz, 2001).

3 VOLATILITY MODELS

3.1 INTRODUCTION TO VOLATILITY

In general, stock market and asset return prices fluctuate throughout the day. This effect is called volatility which is a measure for the dispersion of prices and returns. Most often, it is calculated by taking the standard derivation of the returns. However, most commonly available stock market data provides closing prices and the above approach to computing volatility is not feasible. With daily data, stock market volatility cannot be observed directly and is inferred by looking at the fluctuation of daily returns. According to Tsay (2013), there are three general types of volatility measures. The first and the usual type is volatility defined as the conditional standard deviation of daily returns. Second measure assumes that stock prices follow a specific model or price formulas such as the Black-Scholes-formula. With such a formulation, one can use the observed prices to calculate the so called implied volatility. This, however, is model dependent and may therefore not be accurate. The third type is realized volatility. Intraday returns, which nowadays are easy to get, can be used to calculate the daily volatility. In this study, the first type of volatility measure is employed.

Most of the time volatility is measured using daily returns. The square-root-of time rule allows a rescaling of the estimates. To calculate volatility for T days one can multiply the daily volatility by the square root of T to get an estimate for the T-day volatility. Since there are usually 252 trading days in the United States, multiplying by $\sqrt{252}$ gives an estimate for annual volatility (Tsay, 2013).

There are some features which are very common for most assets and markets. For certain periods, volatility is large and for other periods it is low (Tsay, 2002). This phenomenon is known as volatility clustering. Reider (2009) states that during most of the crises, including Great Depression, attacks in New York in 2001, or the bankruptcy of WorldCom in 2002, volatility rose sharply and after some time it dropped down. Moreover, volatility jumps very seldom indicating a sustained behavior. Third, volatility is stationary which means that it does not grow infinitely large but stays within a certain range (Tsay, 2002). Additionally, volatility responds differently to bad and good news. Decreases in prices have a higher effect on volatility changes than increasing prices. This is called the leverage effect (Tsay, 2002).

3.2 MODELING RETURNS AND VOLATILITY

The main idea of volatility studies is based on the fact that returns are barely correlated, but dependent (Tsay, 2002). Figure 3.1 shows the autocorrelation function (ACF) of the S&P 500 returns from February 2005 to January 2018 as well as the ACF for the absolute returns. The upper panel doesn't show (with some exceptions) any significant correlations. However, the ACF of the absolute returns clearly indicates dependencies among returns. This dependency is modeled by volatility models, some of which are explained below (based on the discussion found in Tsay (2013) and Tsay (2002)).

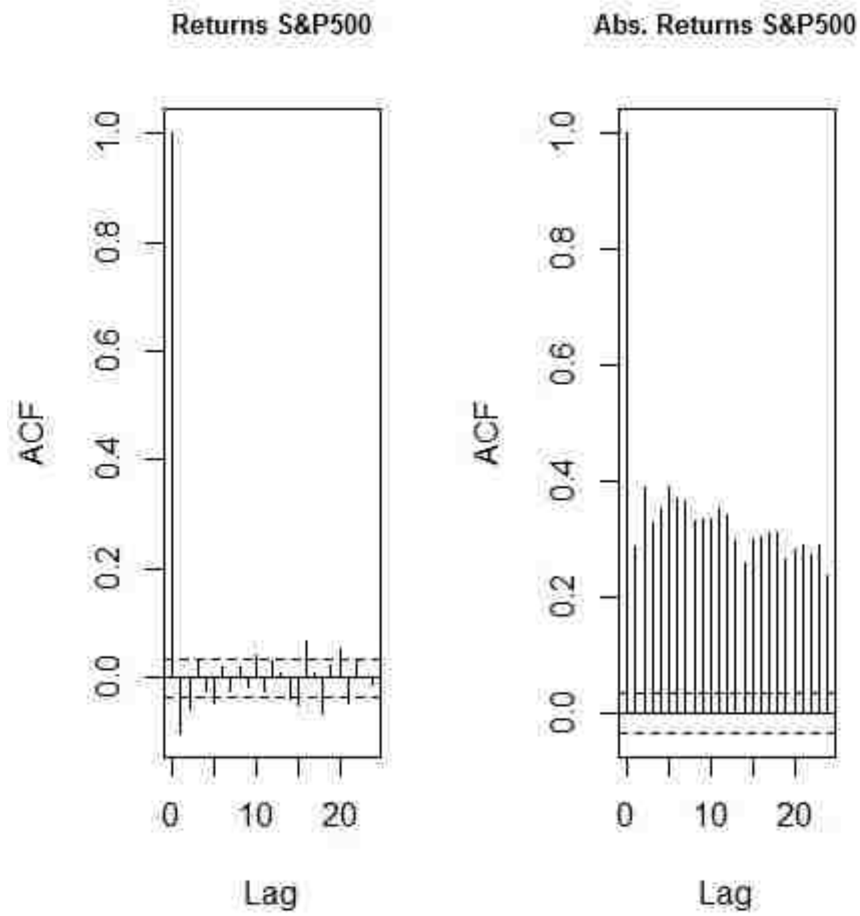

Figure 3.1: ACF of the S&P 500 returns and absolute returns from 2005-2018

Of special interest when investigating returns and volatility are the conditional mean μ_t and conditional variance σ_t. Letting r_t denote the daily return on day *t*, the conditional mean and conditional variance are defined as:

$$\mu_t = E\left(r_t \middle| F_{t-1}\right), \quad \sigma_t^2 = Var\left(r_t \middle| F_{t-1}\right) , \tag{3.1}$$

where F_{t-1} is the information set available at time $t-1$, often containing all linear functions of past returns (Tsay, 2002). Since in practice most of the returns are not highly correlated, the equation for μ_t need not be very complex. For instance, it can follow a stationary ARMA(p,q) model, that is

$$r_t = \mu_t + a_t,\ \mu_t = \phi_0 + \sum_{i=1}^{p} \phi_i r_{t-i} + \sum_{i=1}^{q} \theta_i a_{t-i} \ , \tag{3.2}$$

where p and q are non-negative integer. Additional variables may be included in this equation to test for dependencies. For instance, by adding dummy variables according to the day it is possible to investigate day of the week effects in returns (Tsay, 2002). Inserting equation (3.2) into (3.1) equation gives

$$\sigma_t^2 = Var\left(r_t \middle| F_{t-1}\right) = Var\left(a_t \middle| F_{t-1}\right) \ , \tag{3.3}$$

because μ_t is a function of the passed values and therefore known at time $t-1$.

In a standard linear regression model the error term a_t is assumed to be homoscedastic which means that its volatility is constant. However, in practice the volatility is not constant which is called heteroscedasticity (Reider, 2009). The following models try to capture these conditional heteroscedasticities.

3.3 ARCH MODEL AND ITS PROPERTIES

The simplest among all these models is an autoregressive conditional heteroscedasticity model, abbreviated as ARCH model, which was first introduced by Engle (1982). It is based on the idea that the mean corrected returns are uncorrelated and that it is linearly dependent on its own squared lags. More precisely, given an observed time series $\{a_t : t = 1,\ 2,\ ...\}$, an ARCH(m) model is given by

$$a_t = \sigma_t \varepsilon_t, \quad \sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i a_{t-i}^2 \ , \tag{3.4}$$

where ε_t is an independent and identically distributed series and with zero mean and variance one. Most of the time, ε_t is assumed to be normally or student-t distributed (Tsay, 2002). Moreover, to ensure positive variance, $\alpha_0 > 0$ and $\alpha_i \geq 0$ for all *i*.

To study some properties of an ARCH model, assume $m = 1$ and consider the ARCH(1) model. First, the condition $\alpha_1 < 1$ ensures the variance to be stationary (Reider, 2009). This is easy to see by defining $v_t = a_t^2 - \sigma_t^2$ and rewriting equation (3.4) as

$$\begin{aligned} \sigma_t^2 &= \alpha_0 + \alpha_1 a_{t-1}^2 \\ a_t^2 - v_t &= \alpha_0 + \alpha_1 a_{t-1}^2 \\ a_t^2 &= \alpha_0 + \alpha_1 a_{t-1}^2 + v_t \ . \end{aligned}$$

Thus, an ARCH(1) process can be written as an AR(1) process of a_t^2 which is stationary when $\alpha_1 < 1$. Second, since

$$E(a_t) = E\left(E\left(a_t \middle| F_{t-1}\right)\right) = E\left(E\left(\sigma_t\varepsilon_t \middle| F_{t-1}\right)\right) = E\left(\sigma_t E\left(\varepsilon_t \middle| F_{t-1}\right)\right) = 0 \ ,$$

the unconditional mean is zero (Tsay, 2002). Moreover, the unconditional variance is calculated using (Tsay, 2002)

$$\begin{aligned} Var(a_t) &= E\left(a_t^2\right) = E\left(E\left(a_t^2 \middle| F_{t-1}\right)\right) = E\left(E\left(\sigma_t^2\varepsilon_t^2 \middle| F_{t-1}\right)\right) = E\left(\sigma_t^2 E\left(\varepsilon_t^2 \middle| F_{t-1}\right)\right) \\ &= E\left(\sigma_t^2\right)E\left(\varepsilon_t^2 \middle| F_{t-1}\right) = E\left(\alpha_0 + \alpha_1 a_{t-1}^2\right)Var\left(\varepsilon_t^2 \middle| F_{t-1}\right) = \alpha_0 + \alpha_1 E\left(a_{t-1}^2\right). \end{aligned}$$

Since we assume a_t to be a stationary process, $Var(a_t) = E\left(a_t^2\right) = E\left(a_{t-1}^2\right) = Var\left(a_{t-1}\right)$ and therefore the unconditional variance of a_t is given by

$$Var(a_t) = \frac{\alpha_0}{1-\alpha_1} \ .$$

Moreover, the definition of σ_t^2 leads to a kurtosis larger than 3 indicating fatter tails compared to the normal distribution. This is easily seen from the following derivation

$$\begin{aligned} Kurt(a_t) &= \frac{E\left(a_t^4\right)}{\left(E\left[a_t^2\right]\right)^2} \\ &= \frac{E\left(\sigma_t^4\right)E\left(\varepsilon_t^4\right)}{\left(E\left[\sigma_t^2\right]\right)^2\left(E\left[\varepsilon_t^2\right]\right)^2} \\ &= 3\frac{E\left(\sigma_t^4\right)}{\left(E\left[\sigma_t^2\right]\right)^2} > 3, \end{aligned}$$

where the last step follows from Jensen's inequality with the convex function $f(x)=x^2$ and hence $E(\sigma_t^4) > (E[\sigma_t^2])^2$ (Reider, 2009). This property that numerically large shocks are more likely than a Gaussian distribution would suggest is in line with empirical studies (Tsay, 2002).

Equation (3.4) shows the great advantage of an ARCH model. After a large shock has occurred, indicated by large values for at least one value in the set $\{a_{t-i}^2\}_{i=1,\ldots,m}$, the conditional variance tends to be large as well. This well-known phenomenon of volatility clustering is represented well as large shocks tend to be followed by large shocks (Tsay, 2002). However, this model has some weaknesses. First, it does not reflect the leverage effect since negative and positive shocks have the same sign and amount. Second, ensuring stationarity and finite moments leads to strict restrictions on the estimated variables (Tsay, 2002). Third, to ensure long persistency of the effect of large shocks the order of an ARCH model has to be very high, otherwise volatility will drop down too fast after a crisis. Efforts to rectify this problem led to the generalized ARCH model, abbreviated by GARCH, which was first introduced by Bollerslev (1986).

3.4 GARCH MODEL AND ITS PROPERTIES

In contrast to ARCH models, the volatility in a GARCH model is not only a function of the past shocks, but also of the past volatility. Formally, in a GARCH(m,s) model the time series $\{a_t : t=1,\ 2,\ \ldots\}$ is expressed as

$$a_t = \sigma_t \varepsilon_t, \quad \sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i a_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2 \ , \tag{3.5}$$

where ε_t is an independent and identically distributed series with zero mean and variance one, with $\alpha_0 > 0$, $\alpha_i \geq 0$ and $\beta_j \geq 0$ for all *i* and *j* to ensure positive variance. Moreover, $\sum_{i=1}^{\max(m,s)} \left(\alpha_i + \beta_j\right) < 1$ where $\alpha_i = 0$ for $i > m$ and $\alpha_j = 0$ for $j > s$. This constraint can be seen by rewriting equation (3.5) as an ARMA process. Defining again $v_t = a_t^2 - \sigma_t^2$ and substituting it in Equation (3.6) one obtains

$$\begin{aligned} \sigma_t^2 &= \alpha_0 + \sum_{i=1}^{m} \alpha_i a_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2 \\ a_t^2 - v_t &= \alpha_0 + \sum_{i=1}^{m} \alpha_i a_{t-i}^2 + \sum_{j=1}^{s} \beta_j \left(a_{t-j}^2 - v_{t-j}\right) \\ a_t^2 &= \alpha_0 + \sum_{i=1}^{\max(m,s)} \left(\alpha_i + \beta_i\right) a_{t-i}^2 + v_t - \sum_{j=1}^{s} \beta_j v_{t-j} \ , \end{aligned}$$

indicating an ARMA process on $\left\{a_t^2\right\}$ (Tsay, 2002). The autoregressive polynomial associated with this ARMA process is $\phi(z) = 1 - \sum_{i=1}^{\max(m,s)} \left(\alpha_i + \beta_i\right) z^i$. It is well-known that an ARMA process cannot have a stationary solution if the autoregressive polynomial has roots equal to one. Since $\phi(1) = 0$ implies $\sum_{i=1}^{\max(m,s)} \left(\alpha_i + \beta_i\right) = 1,$ we have the restriction $\sum_{i=1}^{\max(m,s)} \left(\alpha_i + \beta_i\right) \neq 1.$ Assuming this restriction, the unconditional variance can be calculated as

$$E\left(a_t^2\right) = \frac{\alpha_0}{1 - \sum_{i=1}^{\max(m,s)} \left(\alpha_i + \beta_i\right)} \quad . \tag{3.6}$$

Clearly, we also need the restriction $\sum_{i=1}^{\max(m,s)} \left(\alpha_i + \beta_i\right) < 1$ for the unconditional variance to be positive. Setting $m = s = 1$, which is the most commonly used GARCH formulation, allows one to clearly observe some properties of GARCH models. Similar to ARCH models, GARCH models capture volatility clusters since large values for $\left\{a_{t-i}^2\right\}_{i=1,\ldots,m}$ and $\left\{\sigma_{t-i}^2\right\}_{i=1,\ldots,m}$ indicate large forecasts for the current volatility. Moreover, GARCH(1,1) model produces heavier tails than the normal distribution. Additionally, less parameters are necessary to ensure a slower decay in volatility and therefore the model is in general simpler than an ARCH model which needs higher orders to propagate the high volatilities of the past over a longer period. (Tsay, 2013). Another way of describing and understanding a GARCH(1,1) model is that it reflects volatility as weighted value of unconditional variance of returns, residuals and conditional variance of last period where the weights are $1 - \alpha_1 - \beta_1$, α_1 and β_1 which sum up to one. To see this, use equation (3.6), as suggested in Reider (2009) and replace α_0 to get

$$\begin{aligned} \sigma_t^2 &= \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \\ &= (1 - \alpha_1 - \beta_1) E\left[\sigma^2\right] + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad . \end{aligned}$$

Another way of writing a GARCH model is to express it as ARCH(∞) model. This can be seen easily in the GARCH case by iteratively substituting (3.5) as follows:

$$
\begin{aligned}
\sigma_t^2 &= \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \\
&= \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \left(\alpha_0 + \alpha_1 a_{t-2}^2 + \beta_1 \sigma_{t-2}^2 \right) \\
&= \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \alpha_0 + \alpha_1 \beta_1 a_{t-2}^2 + \beta_1^2 \left(\alpha_0 + \alpha_1 a_{t-3}^2 + \beta_1 \sigma_{t-3}^2 \right) \\
&\vdots \\
&= \alpha_0 \sum_{i=0}^{\infty} \beta_1^i + \alpha_1 \sum_{i=0}^{\infty} \beta_1^i a_{t-1-i}^2 \\
&= \frac{\alpha_0}{1-\beta_1} + \alpha_1 \sum_{i=1}^{\infty} \beta_1^i a_{t-1-i}^2 .
\end{aligned}
$$

Therefore the conditional variance can be expressed as weighted sum of past squared residuals. The decreasing weights indicate that past information becomes more and more irrelevant (Reider, 2009).

In practical applications, the estimated model is used to get an idea about future volatility. Reider (2009) showed how to use the estimated parameters to forecast future conditional volatility as follows:

$$
\begin{aligned}
\sigma_t^2 &= \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \\
\hat{\sigma}_{t+1}^2 &= \alpha_0 + \alpha_1 E\left(a_t^2 \middle| F_{t-1} \right) + \beta_1 \sigma_t^2 \\
&= \alpha_0 + \alpha_1 \sigma_t^2 + \beta_1 \sigma_t^2 \\
&= \alpha_0 + \left(\alpha_1 + \beta_1 \right) \sigma_t^2 \\
&= \sigma^2 + \left(\alpha_1 + \beta_1 \right) \left(\sigma_t^2 - \sigma^2 \right),
\end{aligned}
$$

and

$$\begin{aligned}\hat{\sigma}^2_{t+2} &= \alpha_0 + \alpha_1 E\left(a^2_{t+1} \middle| F_{t-1}\right) + \beta_1 E\left(\sigma^2_{t+1} \middle| F_{t-1}\right) \\ &= \alpha_0 + \left(\alpha_1 + \beta_1\right)\hat{\sigma}^2_{t+1} \\ &= \sigma^2 + \left(\alpha_1 + \beta_1\right)\left(\hat{\sigma}^2_{t+1} - \sigma^2\right) \\ &= \sigma^2 + \left(\alpha_1 + \beta_1\right)^2\left(\sigma^2_t - \sigma^2\right),\end{aligned}$$

where equation (3.6) is used to replace the parameter α_0 . Repeating this procedure, the *h*-step forecast is given by

$$\begin{aligned}\hat{\sigma}^2_{t+h} &= \alpha_0 + \left(\alpha_1 + \beta_1\right)\hat{\sigma}^2_{t+h-1} \\ &= \alpha_0 + \left(\alpha_1 + \beta_1\right)\left(\hat{\sigma}^2_{t+h-1} - \sigma^2\right) \\ &= \sigma^2 + \left(\alpha_1 + \beta_1\right)^h\left(\sigma^2_t - \sigma^2\right) \quad .\end{aligned}$$

This equation points out that an increasing horizon will lead to forecast values approaching the unconditional variance, that is $\hat{\sigma}^2_{t+h} \to \sigma^2$ as $h \to \infty$. Moreover, it indicates that $\alpha_1 + \beta_1$ gives an idea on the speed with which the conditional volatility decreases over time. The half-life volatility K, expressing the time period until the volatility is halved after a crisis, can be calculated by using $\left(\alpha_1 + \beta_1\right)^K = 0.5$ or equivalently $K = \frac{\ln(0.5)}{\ln(\alpha_1 + \beta_1)}$ (Reider, 2009).

As discussed earlier, the GARCH model is useful for capturing volatility clusters. However, it is still not able to catch the leverage effect. Indeed, since the conditional volatility is a function of its squared forecasts (past conditional volatilities) and squared shocks, the reaction on negative and positive price movements remain the same. Therefore, a model in which not only the amount of a_{t-1}, but also its sign influence the conditional volatility must be employed. The exponential GARCH model, abbreviated EGARCH, is one such model.

3.5 EGARCH MODEL

The exponential GARCH(m,s) model was first introduced by Nelson (1991) and is given by

$$\begin{aligned} a_t &= \sigma_t \varepsilon_t, \\ \ln\left(\sigma_t^2\right) &= \alpha_0 + \frac{1+\beta_1 B+\ldots+\beta_{s-1}B^{s-1}}{1-\alpha_1 B-\ldots-\alpha_m B^m}\, g\left(\varepsilon_{t-1}\right) \\ g\left(\varepsilon_t\right) &= \theta\varepsilon_t + \gamma\left(\left|\varepsilon_t\right| - E\left(\left|\varepsilon_t\right|\right)\right), \end{aligned} \tag{3.7}$$

where θ, γ, α_i and β_j are constants, B is the back shift operator meaning $B^r g\left(\varepsilon_t\right) = g\left(\varepsilon_{t-r}\right)$ and both $1+\beta_1 B+\ldots+\beta_{s-1}B^{s-1}$ and $1-\alpha_1 B-\ldots-\alpha_m B^m$ are polynomials with zeros outside the unit circle and no common factors. Note that $g\left(\varepsilon_t\right)$ has zero mean and constant unconditional variance and is therefore a white noise process. Moreover, $E\left(\left|\varepsilon_t\right|\right) = \sqrt{\frac{2}{\pi}}$ if $\varepsilon_t \sim N\left(0,1\right)$. Hence, a_t follows an ARMA(m,s) process. However, there are some differences compared to a simple GARCH model.

First, the use of logarithms relax constraints for the parameters such that they can be any real numbers instead of being restricted to be positive (Tsay, 2013). The second great advantage of the EGARCH models is its capability to capture the leverage effect. To see this, it is helpful to rewrite (Tsay, 2013)

$$
\begin{aligned}
g(\varepsilon_t) &= \theta\varepsilon_t + \gamma\left(|\varepsilon_t| - E\left(|\varepsilon_t|\right)\right) \\
&= \begin{cases} (\theta+\gamma)\varepsilon_t - \gamma E\left(|\varepsilon_t|\right) & if\ \varepsilon_t \geq 0 \\ (\theta-\gamma)\varepsilon_t - \gamma E\left(|\varepsilon_t|\right) & if\ \varepsilon_t < 0 \end{cases}.
\end{aligned} \tag{3.8}
$$

Equation (3.8) shows a way to interpret the parameters. On the one hand, θ determines the sign effect such that negative shocks may have different influence than positive ones. On the other hand, γ reflects the size effect and determines, as done by ARCH and GARCH models, the magnitude of the shock. In general, since negative news tend to increase volatility more than good news, θ is expected to be negative while γ should be positive (Tsay, 2013).

It is easier to see the useful features of the EGARCH model by assuming $m = s = 1$. Then equation (3.7) becomes, as shown by Tsay (2013),

$$
\begin{aligned}
(1-\alpha_1 B)\ln\left(\sigma_t^2\right) &= (1-\alpha_1)\alpha_0 + g(\varepsilon_{t-1}) \\
\ln\left(\sigma_t^2\right) &= \alpha_0 - \alpha_0\alpha_1 + \alpha_1\ln\left(\sigma_{t-1}^2\right) + g(\varepsilon_{t-1}) \\
\ln\left(\sigma_t^2\right) &= \alpha_0 - \alpha_0\alpha_1 + \alpha_1\ln\left(\sigma_{t-1}^2\right) + \theta\varepsilon_{t-1} + \gamma\left(|\varepsilon_{t-1}| - E\left(|\varepsilon_{t-1}|\right)\right) \\
\ln\left(\sigma_t^2\right) &= \begin{cases} const + (\theta+\gamma)\varepsilon_{t-1} & if\ \varepsilon_{t-1} \geq 0 \\ const + (\theta-\gamma)\varepsilon_{t-1} & if\ \varepsilon_{t-1} < 0 \end{cases},
\end{aligned}
$$

where $const = \alpha_0 - \alpha_0\alpha_1 + \alpha_1\ln\left(\sigma_{t-1}^2\right) - \gamma E\left(|\varepsilon_{t-1}|\right)$. Now taking exponential on both sides one obtains

$$\sigma_t^2 = \begin{cases} \exp(const)\cdot\exp\left[\left(\theta+\gamma\right)\varepsilon_{t-1}\right] & if\ \varepsilon_{t-1} \geq 0 \\ \exp(const)\cdot\exp\left[\left(\theta-\gamma\right)\varepsilon_{t-1}\right] & if\ \varepsilon_{t-1} < 0 \end{cases}$$

$$\sigma_t^2 = \begin{cases} \exp(const)\cdot\exp\left[\left(\theta+\gamma\right)\frac{a_{t-1}}{\sigma_{t-1}}\right] & if\ a_{t-1} \geq 0 \\ \exp(const)\cdot\exp\left[\left(\theta-\gamma\right)\frac{a_{t-1}}{\sigma_{t-1}}\right] & if\ a_{t-1} < 0 \end{cases}, \tag{3.9}$$

indicating the ability of the EGARCH model to capture of the leverage effect when $\theta \neq 0$.

Another useful and often used form of the EGARCH(1,1) model can be obtained by letting $\beta_1 = \alpha_1$ and $\omega = \alpha_0 + \alpha_0\alpha_1$. Then (3.7) becomes

$$\ln\left(\sigma_t^2\right) = \omega + \beta_1 \ln\left(\sigma_{t-1}^2\right) + g\left(\varepsilon_{t-1}\right)\ , \tag{3.10}$$

such that the model reduces to an AR(1) process (Reider, 2009).

3.6 NEWS IMPACT CURVES

Assuming $\varepsilon_t \sim N\left(0,1\right)$ and using the same definitions as above, with $\beta_1 = \alpha_1$ and $\omega = \alpha_0 + \alpha_0\alpha_1$, and substituting in equation (3.9) one obtains

$$\sigma_t^2 = \begin{cases} \exp\left(\omega + \beta_1 \ln\left(\sigma_{t-1}^2\right) - \gamma\sqrt{\frac{2}{\pi}}\right) \cdot \exp\left[\left(\theta+\gamma\right)\frac{a_{t-1}}{\sigma_{t-1}}\right] & if\ a_{t-1} \geq 0 \\ \exp\left(\omega + \beta_1 \ln\left(\sigma_{t-1}^2\right) - \gamma\sqrt{\frac{2}{\pi}}\right) \cdot \exp\left[\left(\theta-\gamma\right)\frac{a_{t-1}}{\sigma_{t-1}}\right] & if\ a_{t-1} < 0 \end{cases}$$

$$\sigma_t^2 = \begin{cases} \left(\sigma_{t-1}\right)^{2\beta_1} \cdot \exp\left(\omega - \gamma\sqrt{\frac{2}{\pi}}\right) \cdot \exp\left[\left(\theta+\gamma\right)\frac{a_{t-1}}{\sigma_{t-1}}\right] & if\ a_{t-1} \geq 0 \\ \left(\sigma_{t-1}\right)^{2\beta_1} \cdot \exp\left(\omega - \gamma\sqrt{\frac{2}{\pi}}\right) \cdot \exp\left[\left(\theta-\gamma\right)\frac{a_{t-1}}{\sigma_{t-1}}\right] & if\ a_{t-1} < 0 \end{cases} \quad . \tag{3.11}$$

By replacing the conditional variance at time *t-1* by the unconditional variance, we get the equation for the so called News impact curve, abbreviated by NIC, (Engle and Ng, 1993), given by

$$\sigma_t^2 = \begin{cases} \left(\sigma\right)^{2\beta_1} \cdot \exp\left(\omega - \gamma\sqrt{\frac{2}{\pi}}\right) \cdot \exp\left[\left(\theta+\gamma\right)\frac{a_{t-1}}{\sigma}\right] & \text{if } a_{t-1} \geq 0 \\ \left(\sigma\right)^{2\beta_1} \cdot \exp\left(\omega - \gamma\sqrt{\frac{2}{\pi}}\right) \cdot \exp\left[\left(\theta-\gamma\right)\frac{a_{t-1}}{\sigma}\right] & \text{if } a_{t-1} < 0 \end{cases} \quad . \tag{3.12}$$

This function connects the influence of past shocks to current volatility and defines a measure on the influence of shocks to volatility estimates (Engle and Ng, 1993). To study the behavior of the news impact curve of an EGARCH model it is useful to compare it with that of a GARCH model. In the GARCH model expressed as

$$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma^2 \tag{3.13}$$

one sees that the conditional volatility is a quadratic function of the past shocks where it reaches the minimum when $a_{t-1} = 0$. The latter fact, which holds for EGARCH model as well if $\gamma + \theta > 0$, is very reasonable since no shocks or no news should be considered to have the least impact on volatility. In the EGARCH model, however, the volatility is increasing exponentially with shocks and the impact is different for positive and negative shocks. Figure 3.2 retrieved from Engle and Ng (1993), compares the News Impact curves for GARCH(1,1) and EGARCH(1,1) models, where for the latter one the conditions $\theta < 0$ and $\gamma + \theta > 0$ are assumed. It shows that both functions are minimized at $a_{t-1} = 0$ (note that $\varepsilon_t = a_t$ and $h_t = \sigma_t^2$). Moreover, it illustrates the fact that the GARCH model cannot capture the leverage effect since the NIC graph is symmetric. In contrast, the EGARCH model is asymmetric where bad news tend to have bigger influence than good news, assuming $\theta < 0$. Additionally, since the exponential function increases faster than a polynomial function, in the EGARCH model the conditional variance grows faster with the attendance of big news, no matter if they are good or bad (Engle and Ng, 1993).

However, Engle and Ng (1993) used the Japan stock market to estimate volatility based on an EGARCH model and found that the unconditional variance implied by the model is greater than the unconditional variance of the squared residuals which, mathematically, can never happen. To see this, we can write

$$a_t^2 = \sigma_t^2 + \left(a_t^2 - \sigma_t^2\right).$$

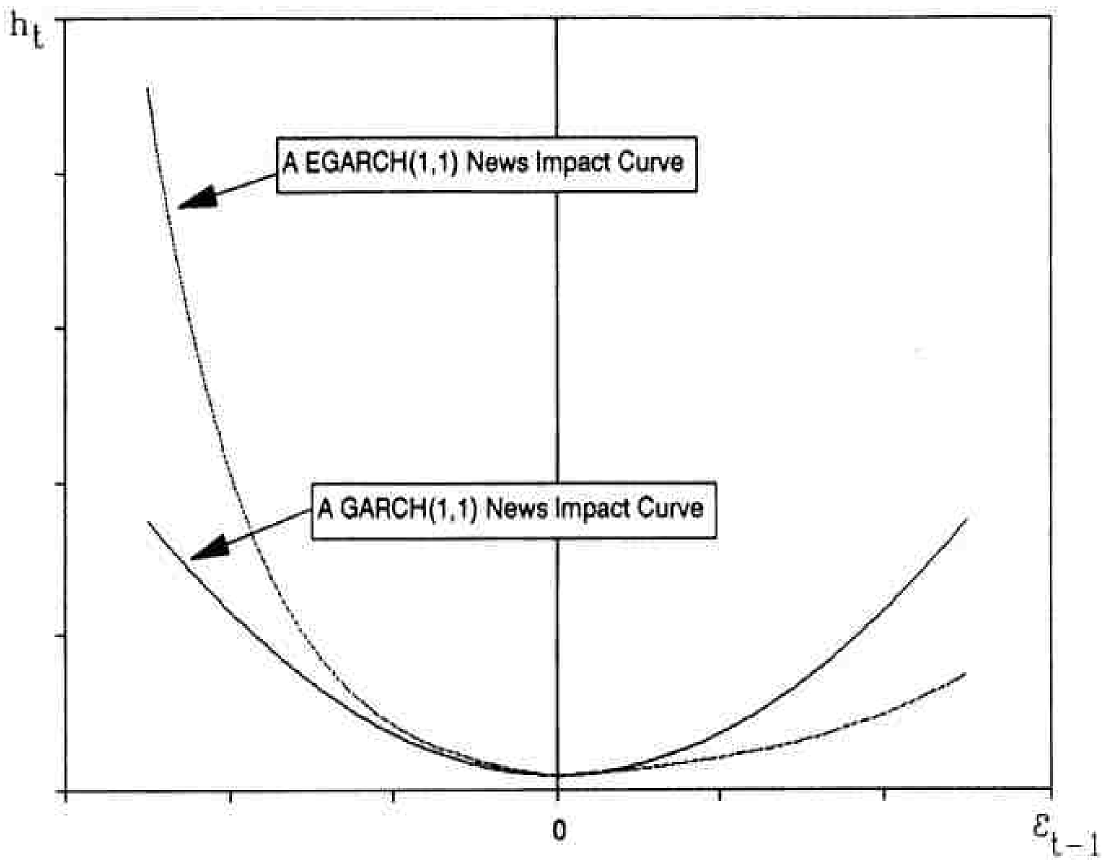

Figure 3.2: News Impact Curves for GARCH(1,1) and EGARCH(1,1) models, retrieved from Engle and Ng (1993)

Using $\sigma_t^2 = E\left(a_t^2 \middle| F_{t-1}\right)$ shows that the terms on the right hand side are uncorrelated since the vectors $\sigma_t^2 = E\left(a_t^2 \middle| F_{t-1}\right)$ and $\left(a_t^2 - E\left(a_t^2 \middle| F_{t-1}\right)\right)$ are orthogonal. Hence it follows

$$Var\left(a_t^2\right) = Var\left(\sigma_t^2\right) + Var\left(a_t^2 - \sigma_t^2\right) \geq Var\left(\sigma_t^2\right) ,$$

which does not hold for their observed data. Therefore, the EGARCH model may not be appropriate for some assets or markets (Engle and NG, 1993). Several authors, however, have found through empirical studies that EGARCH models fit stock return data quite well when compared to other GARCH type models (Awartani and Corradi, 2005; Du, 2017).

4 EMPIRICAL STUDY

4.1 DATA

The data consists of prices for the S&P 500 (SP) and ten different sectors starting from February 14th 2005 and ending January 12th 2018. The ten sectors are Consumer Discretionary (CD), Consumer Staples (CS), Energy (En), Financials (Fin), Health Care (HC), Industrial (Ind), Information Technology (IT), Materials (Mat), Telecommunication Services (TS) and Utilities (Ut). The S&P 500 is one of the world's largest stock markets, including 500 leading US companies and representing almost 80% of the market capitalization (us.spindices.com)

All data was downloaded from us.spindices.com. This webpage includes net returns, total net returns and price returns for all sectors. The first two account for dividends, where the last item ignores all dividends. In this study, price returns where used for the calculation. It also includes a Real Estate Sector. However, due to missing data for the years 2005-2007, this sector is not included in the investigation. Table 4.1, retrieved from eresearch.fidelity.com, shows the distribution of the different industries over the different sectors.

The daily returns are calculated by using the standard formula for log returns, given by

$$R_t = \ln\left(\frac{P_t}{P_{t-1}}\right).$$

Table 4.1: Sector Breakdowns for the S&P 500

Sector	Industry
Consumer Discretionary	Automobile Components, Automobiles, Distributors , Diversified Consumer Services , Hotels, Restaurants & Leisure , Household Durables , Internet & Catalog Retail , Leisure Products , Media , Multiline Industry, Specialty Retail , Textile, Apparel & Luxury Goods
Consumer Staples	Beverages , Food & Staples Retailing, Food Products, Household Products, Personal Products, Tobacco
Energy	Energy Equipment & Services, Oil, Gas & Consumable Fuels
Financials	Banks, Capital Markets, Consumer Finance, Diversified Financial Services, Insurance, Mortgage REIT, Thrifts & Mortgage Finance
Health Care	Biotechnology, Health Care Equipment & Supplies, Health Care Providers & Services, Health Care Technology, Life Sciences Tools & Services, Pharmaceuticals
Industrials	Aerospace & Defense, Air Freight & Logistics, Airlines, Building Products, Commercial Services & Supplies, Construction & Engineering, Electrical Equipment, Industrial Conglomerates, Machinery, Marine, Professional Services, Road & Rail, Trading Companies & Distributors, Transportation Infrastructure
Information Technology	Communications Equipment, Electronic Equipment, Instruments & Components, IT Services, Internet Software & Services, Semiconductors & Semiconductor Equipment, Software, Technology Hardware, Storage & Peripherals
Materials	Chemicals, Construction Materials, Containers & Packaging, Metals & Mining, Paper & Forest Products
Telecommunication Services	Diversified Telecommunication Services, Wireless Telecommunication Services
Utilities	Electric Utilities, Gas Utilities, Independent Power and Renewable Electricity Producers, Multi-Utilities, Water Utilities

Since the return for the first day cannot be calculated, there are 3251 daily returns available to estimate the parameters. The data includes 607 Mondays, 666 Tuesdays, 670 Wednesdays, 656 Thursdays and 652 Fridays. To investigate if the day of the week effect in both mean return and volatility has changed after the subprime crisis, the data is divided into three periods: the pre-recession, the recession and the post-recession period. Officially, the crisis started in December 2007 and ended in June 2009 (nber.org). Therefore, the hypothesized “structural” break points in this study are set to be December 3rd 2007 and June 30th 2009. The observed data consists of 704 days for the pre-recession, 397 days of the recession and 2150 days for the post- recession period. The distribution of the days in each of the sub periods are summarized in Table 4.2. Clearly, the recession period consists of less data points compared to the other periods, whereas the distribution for the day of the week is nearly uniform within each period.

Table 4.2: Summary of investigated data from February 2005 to January 2018

	Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**	**Total**
Pre	129	144	145	143	143	704
Rec	76	81	82	79	79	397
Post	402	441	443	434	430	2150
Total	607	666	670	656	652	3251

Figure 4.1 and Figure 4.2 show plots of the returns for the S&P 500 and each of the sectors, respectively. These plots clearly display the heteroskedastic behavior of the returns. Moreover, the volatility clusters indicating different periods of high and low volatility are revealed. Especially the recession period, indicated in dotted red lines in

Figure 4.1, from December 2007 to June 2009 defines a time where the volatility was very high.

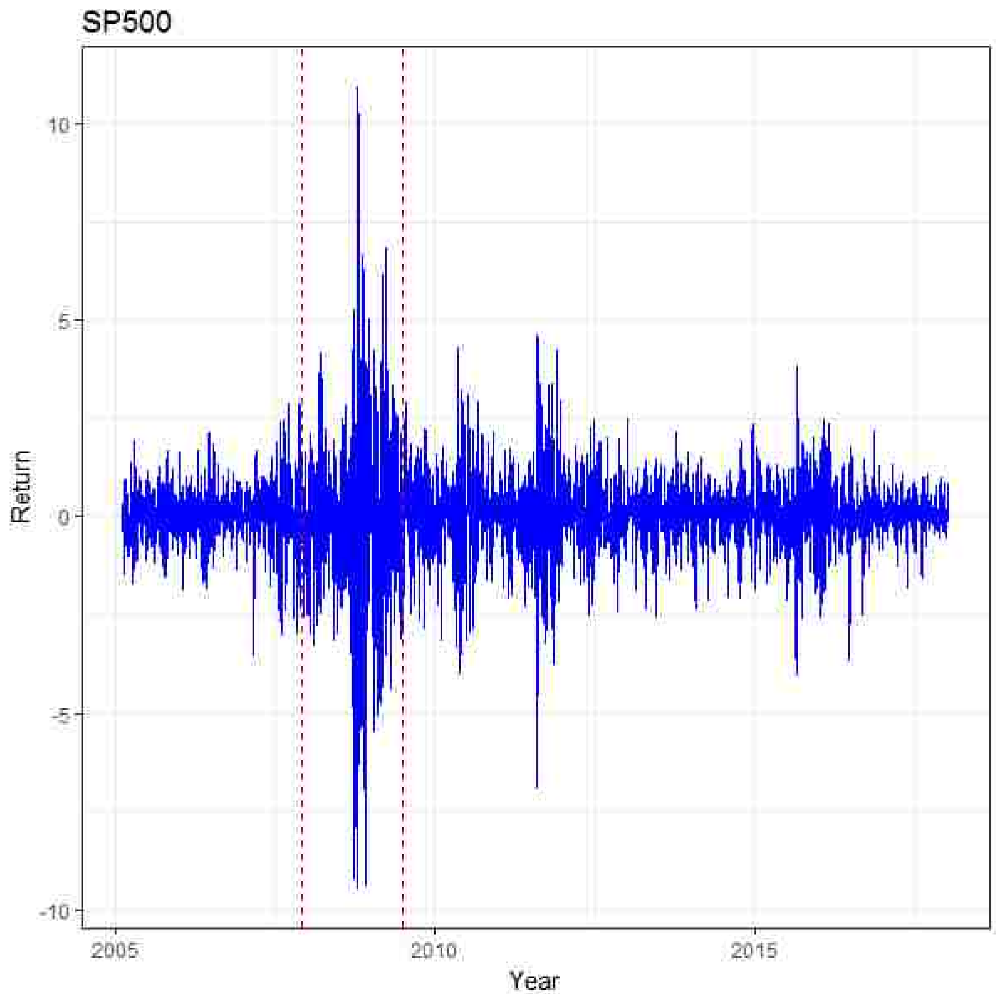

Figure 4.1: S&P 500 Returns from February 2005 to January 2018

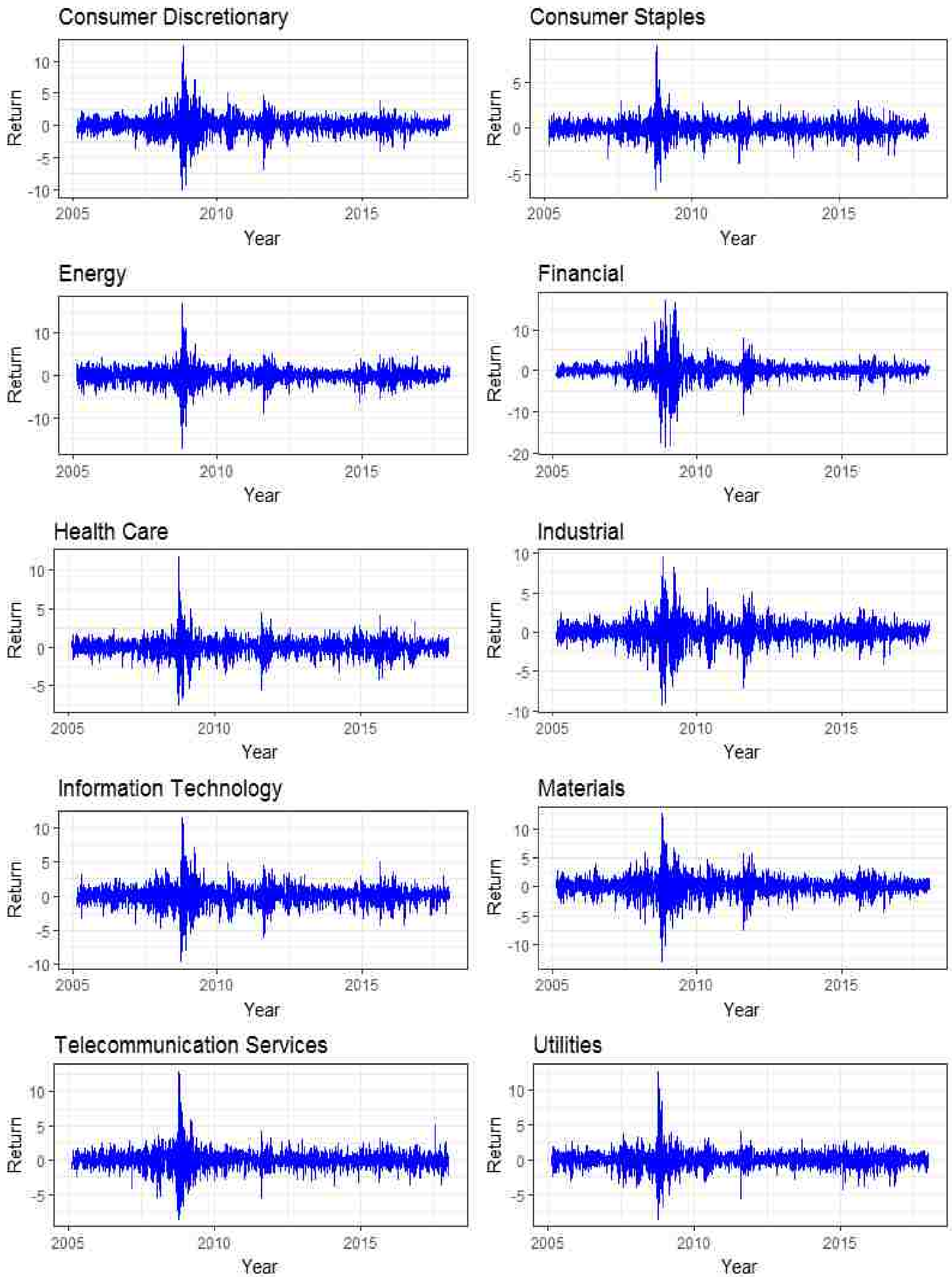

Figure 4.2: Sector returns from February 2005 to January 2018

Table 4.3 shows the overall mean return for each investigated sector and the S&P 500. It also includes the calculated mean returns for each period, separated by the day of the week. To avoid decimals, the returns were multiplied by 100.

The table indicates some differences in mean returns. Considering the S&P 500, the highest mean return is found on Tuesday and the lowest is on Monday. However, for the Utilities sector Monday shows the highest mean returns where Wednesday has the lowest. Although this might indicate a small day of the week effect in returns, the differences are quite small. The highest difference between maximum and minimum mean returns is 0.22 for the Financial Sector, whereas all the other differences are between 0.07 and 0.15. Moreover, the mean returns between pre- and post-recession periods are similar for all sectors. The differences lie between 0.07 and -0.06 for the Consumer Discretionary sector and the Energy sector respectively. This suggests that mean returns do not differ in periods before and after crises. However, the mean returns drop down heavily during the recession period. For instance, the financial sector mean return dropped from 0.01 to -0.24 over the recession period. For all other sectors, mean returns became negative as well. Compared to the pre-recession period, returns dropped between 0.09 and 0.24 indicating a recession effect on the mean return, at least for some of the investigated sectors. These effects are also visible when comparing the day of the week returns for pre- and recession periods.

Similar to Table 4.3, Table 4.4 shows the standard deviation of the returns of each sector multiplied by 100. The squares of the standard deviations can be regarded as the unconditional variances of returns, but obtained without correcting for any autocorrelation between successive returns. Figures for the complete observational period as well as for each of the three periods under study and the day of the week are given. The Financial sector can be considered as an outlier among all other sectors, with a difference of 0.78 between the lowest and highest standard deviations. This coincides with the fact that the Financial sector shows the highest overall volatility. Among the remaining sectors, the differences between highest and lowest volatility range from 0.09 for the Consumer Staples sector to 0.44 for the Energy sector. Similar to mean returns, mean volatility for the pre- and post- recession periods do not differ. However, volatility almost doubles or triples during the recession period, where it even quadruples for the Financial sector. Concerning the day of the week, there are differences for all periods and sectors. In the pre-recession period, the majority of the sectors have highest volatility on Tuesdays and lowest on Mondays. This pattern changes to recession period, where Monday became the most and Friday the least volatile day of the week for all sectors. In the post-recession period, there is no clear pattern such that effect for each day of the week differs from sector to sector.

In summary, Table 4.3 and Table 4.4 indicate that volatility and returns changed during the recession period compared to pre- and post-recession periods. In general, mean returns were lower and volatility was higher in the recession period compared to pre-and post-recession. Moreover, the differences suggest a day of the week effect. However, since the differences are small, it has to be investigated if these effects are significant and if they have changed over the periods.

Table 4.3: Mean of investigated data in 10^{-2} units

	Overall	Mo	Tu	We	Th	Fr
CD	0.03	-0.02	0.08	0.02	0.07	0.02
CD Pre	0.00	0.02	-0.05	0.11	- 0.01	- 0.05
CD Rec	-0.10	-0.41	0.29	- 0.36	0.05	- 0.10
CD Post	0.07	0.04	0.08	0.07	0.11	0.07
CS	0.03	0.03	0.07	- 0.00	0.04	0.00
CS Pre	0.03	0.06	-0.02	0.12	0.03	- 0.02
CS Rec	-0.06	-0.15	0.16	- 0.28	0.06	- 0.10
CS Post	0.04	0.05	0.08	0.01	0.03	0.03
En	0.02	-0.05	0.09	0.02	- 0.00	0.04
En Pre	0.08	0.03	0.01	0.15	0.01	0.19
En Rec	-0.10	-0.14	0.10	- 0.28	- 0.18	- 0.02
En Post	0.02	-0.06	0.11	0.03	0.02	- 0.01
Fin	0.01	-0.13	0.09	0.07	0.00	- 0.01
Fin Pre	0.00	-0.01	-0.01	0.12	- 0.09	- 0.00
Fin Rec	-0.24	-1.29	0.46	- 0.19	- 0.28	0.04
Fin Post	0.05	0.04	0.05	0.10	0.09	- 0.02
HC	0.03	-0.01	0.10	0.03	0.03	0.00
HC Pre	0.03	-0.00	-0.01	0.14	0.02	- 0.01
HC Rec	-0.08	-0.27	0.26	- 0.35	0.01	- 0.06
HC Post	0.06	0.03	0.11	0.07	0.04	0.02
Ind	0.03	-0.01	0.06	0.01	0.04	0.03
Ind Pre	0.03	0.07	-0.01	0.15	- 0.04	- 0.00
Ind Rec	-0.16	-0.38	0.18	- 0.45	- 0.09	- 0.06
Ind Post	0.06	0.03	0.06	0.05	0.10	0.05
IT	0.04	0.03	0.09	0.09	0.02	- 0.03
IT Pre	0.04	0.04	0.04	0.20	- 0.01	- 0.10
IT Rec	-0.09	-0.43	0.16	- 0.22	0.08	- 0.03
IT Post	0.07	0.11	0.09	0.11	0.02	- 0.00
Mat	0.02	-0.08	0.05	0.06	0.01	0.07
Mat Pre	0.05	-0.02	-0.03	0.19	- 0.03	0.13
Mat Rec	-0.13	-0.54	0.09	- 0.28	- 0.09	0.16
Mat Post	0.04	-0.02	0.06	0.09	0.05	0.04
TS	0.01	0.05	0.03	- 0.07	0.05	- 0.01
TS Pre	0.04	-0.04	-0.01	0.16	0.03	0.07
TS Rec	-0.11	-0.02	0.06	- 0.56	0.11	- 0.13
TS Post	0.02	0.10	0.04	- 0.06	0.05	- 0.02
Ut	0.02	0.08	0.01	- 0.07	0.03	0.04
Ut Pre	0.05	0.14	0.05	0.06	- 0.04	0.07
Ut Rec	-0.11	-0.00	-0.01	- 0.42	- 0.01	- 0.06
Ut Post	0.03	0.08	0.01	- 0.05	0.06	0.04
SP	0.03	-0.02	0.08	0.03	0.03	0.01
SP Pre	0.03	0.03	-0.01	0.14	- 0.02	- 0.00
SP Rec	-0.12	-0.42	0.21	- 0.33	- 0.04	- 0.03
SP Post	0.05	0.05	0.08	0.06	0.05	0.02

Table 4.4: Standard Deviation of investigated data in 10^{-2} units

	Overall	Monday	Tuesday	We	Thursday	Friday
CD	1.32	1.41	1.39	1.30	1.35	1.15
CD Pre	0.88	0.78	1.01	0.88	0.90	0.82
CD Rec	2.67	2.99	2.93	2.63	2.65	2.06
CD Post	1.03	1.05	1.00	1.00	1.10	1.01
CS	0.85	0.89	0.85	0.83	0.89	0.79
CS Pre	0.62	0.56	0.72	0.59	0.62	0.58
CS Rec	1.57	1.79	1.66	1.41	1.59	1.35
CS Post	0.72	0.69	0.64	0.74	0.78	0.72
En	1.72	1.92	1.65	1.78	1.78	1.48
En Pre	1.48	1.37	1.58	1.49	1.51	1.42
En Rec	3.28	3.97	3.01	3.36	3.45	2.54
En Post	1.34	1.42	1.29	1.41	1.37	1.22
Fin	2.07	2.38	2.29	1.98	2.04	1.60
Fin Pre	1.02	0.96	1.13	1.04	1.00	0.97
Fin Rec	4.86	5.66	5.63	4.55	4.71	3.33
Fin Post	1.34	1.41	1.29	1.34	1.40	1.25
HC	1.03	1.13	1.00	0.99	1.10	0.94
HC Pre	0.69	0.63	0.75	0.69	0.72	0.66
HC Rec	1.84	2.27	1.75	1.55	2.02	1.48
HC Post	0.91	0.90	0.88	0.92	0.97	0.89
Ind	1.31	1.36	1.35	1.27	1.39	1.17
Ind Pre	0.83	0.67	0.88	0.88	0.86	0.85
Ind Rec	2.52	2.77	2.71	2.33	2.69	2.01
Ind Post	1.09	1.09	1.07	1.07	1.17	1.04
IT	1.30	1.36	1.34	1.28	1.30	1.20
IT Pre	0.97	0.78	1.01	1.02	1.02	1.00
IT Rec	2.44	2.83	2.73	2.33	2.33	1.88
IT Post	1.07	1.03	1.02	1.07	1.12	1.11
Mat	1.55	1.73	1.56	1.58	1.58	1.30
Mat Pre	1.16	1.05	1.26	1.21	1.20	1.04
Mat Rec	2.99	3.65	3.01	3.06	2.88	2.25
Mat Post	1.24	1.28	1.23	1.25	1.34	1.13
TS	1.23	1.34	1.31	1.26	1.22	0.97
TS Pre	0.91	0.80	0.94	1.02	0.90	0.83
TS Rec	2.48	3.05	2.75	2.46	2.24	1.71
TS Post	0.94	0.87	0.97	0.95	1.04	0.83
Ut	1.13	1.23	1.11	1.12	1.13	1.05
Ut Pre	0.95	0.96	0.99	0.92	0.96	0.94
Ut Rec	2.12	2.56	2.10	1.94	2.09	1.90
Ut Post	0.89	0.86	0.86	0.95	0.92	0.85
SP	1.19	1.33	1.23	1.16	1.22	1.01
SP Pre	0.78	0.65	0.87	0.79	0.78	0.76
SP Rec	2.41	2.92	2.60	2.27	2.40	1.75
SP Post	0.94	0.96	0.90	0.93	1.01	0.89

4.2 METHODOLOGY

As stated and summarized in Section 2, many studies were done to investigate the day of the week effect on returns. More recent studies also investigated a day of the week effect in volatility. Some of them showed that the effect has diminished after certain periods. Akbalik and Ozkan (2017), who analyzed returns for the five fragile countries Brazil, India, Indonesia, South Africa and Turkey, as well as Hui (2005), who analyzed a sample of Asian markets such as Hong Kong, Taiwan and Korea, show that there is no day of the week effect in returns after the subprime crisis 2009 and Asian crisis in 1997 respectively. These results lead to the question of whether the day of the week effect in volatility for the American stock market has disappeared as well in the years after the crisis or if there has been any changes in the effects across the pre-recession, recession and post- recession periods.

There are several theories on how to find break points and structural changes for variances. Sensier and van Dijk (2004) propose a model where they include indicator functions to both return and volatility equations and find the break point by minimizing the squared residuals. However, they do not use a volatility model such as GARCH. Other methods on finding structural breaks are given in Smith (2008).

In this study, however, the break points are assumed to be known. Officially, the subprime crisis started in December 2007 and ended in June 2009 (nber.org). Therefore, the hypothesized "structural" break points are set to be December 3rd 2007 and June 30th 2009. This is done similarly in Du (2017).

The three periods before start of recession, recession itself and post-recession are described by dummy variables, which we will term the period dummies. To investigate the day of the week effect, additional dummy variables, which will be termed day dummies, were also created. These dummy variables, as well as terms representing the interaction between period and day dummies, are included on both the mean and the volatility equations, where volatility is modeled to follow an EGARCH(1,1) model. The so called "mean model," is actually the portion of the complete model that explain the behavior of the returns while the EGARCH part models the volatility of the residual of the mean model. Moreover, the mean model contains an AR(k) part to capture any autoregressive behavior associated with returns.

To summarize, the returns are modeled using the formulation

$$\begin{aligned} R_t &= \lambda_0 + \lambda_1 D_1 + \lambda_2 D_2 + \lambda_3 D_3 + \lambda_4 D_4 + \lambda_5 D_5 + \lambda_6 D_6 \\ &+ \lambda_7 D_1 D_5 + \lambda_8 D_2 D_5 + \lambda_9 D_3 D_5 + \lambda_{10} D_4 D_5 \\ &+ \lambda_{11} D_1 D_6 + \lambda_{12} D_2 D_6 + \lambda_{13} D_3 D_6 + \lambda_{14} D_4 D_6 + \sum_{i=1}^{k} \phi_i R_{t-i} + a_t \end{aligned} \tag{4.1}$$

and the residual term a_t is assumed to follow the modified EGARCH model

$$\begin{aligned} a_t &= \sigma_t \varepsilon_t , \\ \ln\left(\sigma_t^2\right) &= \omega_0 + \omega_1 D_1 + \omega_2 D_2 + \omega_3 D_3 + \omega_4 D_4 + \omega_5 D_5 + \omega_6 D_6 \\ &+ \omega_7 D_1 D_5 + \omega_8 D_2 D_5 + \omega_9 D_3 D_5 + \omega_{10} D_4 D_5 \\ &+ \omega_{11} D_1 D_6 + \omega_{12} D_2 D_6 + \omega_{13} D_3 D_6 + \omega_{14} D_4 D_6 \\ &+ \gamma_1 \ln\left(\sigma_{t-1}^2\right) + \alpha_1 g\left(\varepsilon_{t-1}\right), \\ g\left(\varepsilon_{t-1}\right) &= \delta a_t + \left(\left|a_t\right| - E\left(\left|a_t\right|\right)\right), \end{aligned} \tag{4.2}$$

where D_1, D_2, D_3 and D_4 are the dummy variables for Tuesday, Wednesday Thursday and Friday respectively and D_5 and D_6 are dummy variables representing the recession period from December 3rd 2007 until June 30th and the following post-recession period, respectively. Hence, the coefficients in the terms with one or more dummy variables are the adjustments one should make to the corresponding coefficients without the dummies, with the latter representing the coefficients for Mondays in the pre-recession period. The formulation in (4.2) is based on the EGARCH(1,1) formula given in the SAS AUTOREG procedure which was used in this study (support.sas.com). In contrast to what has been expressed in Equation (3.10) in Section 3.5, the function $g\left(\varepsilon_{t-1}\right)$ has a coefficient and is defined differently to the formulation in (3.7). However, defining $\beta_1 = \gamma_1,\ \theta = \alpha_1\delta$ and $\gamma = \alpha_1$ leads to the form given in (3.10). Hence, the model in (4.2) is equivalent to

$$\begin{aligned}
\ln\left(\sigma_t^2\right) &= \omega_0 + \omega_1 D_1 + \omega_2 D_2 + \omega_3 D_3 + \omega_4 D_4 + \omega_5 D_5 + \omega_6 D_6 \\
&\quad + \omega_7 D_1 D_5 + \omega_8 D_2 D_5 + \omega_9 D_3 D_5 + \omega_{10} D_4 D_5 \\
&\quad + \omega_{11} D_1 D_6 + \omega_{12} D_2 D_6 + \omega_{13} D_3 D_6 + \omega_{14} D_4 D_6 \\
&\quad + \beta_1 \ln\left(\sigma_{t-1}^2\right) + g\left(\varepsilon_{t-1}\right) \\
&= a + b \ln\left(\sigma_{t-1}^2\right) + g\left(\varepsilon_{t-1}\right), \\
g\left(\varepsilon_{t-1}\right) &= \theta a_t + \gamma\left(\left|a_t\right| - E\left(\left|a_t\right|\right)\right).
\end{aligned} \tag{4.3}$$

The representations of the EGARCH employed in the following are based on Section 3.5 formulation instead of the corresponding version given in SAS documentation. Therefore, the definitions given above are assumed for all following models.

From now on, the parameters *a* and *b* given by

$$
\begin{aligned}
a &= \omega_0 + \omega_1 D_1 + \omega_2 D_2 + \omega_3 D_3 + \omega_4 D_4 + \omega_5 D_5 + \omega_6 D_6 \\
&\quad + \omega_7 D_1 D_5 + \omega_8 D_2 D_5 + \omega_9 D_3 D_5 + \omega_{10} D_4 D_5 \\
&\quad + \omega_{11} D_1 D_6 + \omega_{12} D_2 D_6 + \omega_{13} D_3 D_6 + \omega_{14} D_4 D_6, \\
b &= \beta_1,
\end{aligned}
$$

will be referred to as intercept and slope parameters respectively. Note that the model in (4.1) and (4.3) is a generalization of the model utilized by Berument and Kiymaz (2001) to model day of the week effects. They, however, used a GARCH formulation for the volatility model and did not use dummies for the periods.

Taking expectation on both sides of equation (4.3) yield to

$$
\begin{aligned}
E\left[\ln\left(\sigma_t^2\right)\right] &= \omega_0 + \omega_1 D_1 + \omega_2 D_2 + \omega_3 D_3 + \omega_4 D_4 + \omega_5 D_5 + \omega_6 D_6 \\
&\quad + \omega_7 D_1 D_5 + \omega_8 D_2 D_5 + \omega_9 D_3 D_5 + \omega_{10} D_4 D_5 \\
&\quad + \omega_{11} D_1 D_6 + \omega_{12} D_2 D_6 + \omega_{13} D_3 D_6 + \omega_{14} D_4 D_6 \\
&\quad + \beta_1 E\left[\ln\left(\sigma_{t-1}^2\right)\right] + E\left[g\left(\varepsilon_{t-1}\right)\right] \\
&= a + bE\left[\ln\left(\sigma_{t-1}^2\right)\right] + E\left[g\left(\varepsilon_{t-1}\right)\right],
\end{aligned}
\tag{4.4}
$$

where

$$
\begin{aligned}
a &= \omega_0 + \omega_1 D_1 + \omega_2 D_2 + \omega_3 D_3 + \omega_4 D_4 + \omega_5 D_5 + \omega_6 D_6 \\
&\quad + \omega_7 D_1 D_5 + \omega_8 D_2 D_5 + \omega_9 D_3 D_5 + \omega_{10} D_4 D_5 \\
&\quad + \omega_{11} D_1 D_6 + \omega_{12} D_2 D_6 + \omega_{13} D_3 D_6 + \omega_{14} D_4 D_6, \\
b &= \beta_1.
\end{aligned}
$$

Using $E\left[g\left(\varepsilon_{t-1}\right)\right] = 0$ and assuming $E\left[\ln\left(\sigma_t^2\right)\right] = E\left[\ln\left(\sigma_{t-1}^2\right)\right]$, the unconditional variance can then be estimated by

$$E\left[\ln\left(\sigma^2\right)\right] = \frac{a}{1-b}$$
$$\sigma^2 \approx e^{\frac{a}{1-b}} \tag{4.5}$$

We propose three approaches to determine if the day of the week effects on returns and volatility has changed across the three periods under study.

The first technique, based on the model described in equations (4.1) and (4.3), is an adaptation of the method used by Berument and Kiymaz (2001). In this model, however, the dummy variables only influence the intercept a of the EGARCH formulation, and do not consider any changes in the slope b due to day of the week or period. Therefore, a second approach was carried out to factor in any influence of these factors on the slope b. The third approach is based on part of the investigations done by Berument and Kiymaz (2001) where the data is divided into different sub periods and the day of the week effect is calculated separately for each period. This approach allows the parameters of the function g in equation (3.7) to change from period to period.

For all approaches, the autoregressive order was estimated in an initial step. To estimate the most appropriate order k, the model described in (4.1) and (4.3) was used, excluding all dummy variables. The dummies were eliminated because most of them in the means part of the model were insignificant and because the presence of dummies in the volatility part of the model did not result in any added benefit in preliminary runs. Hence, the representation used is the simple autoregressive model with the error terms following an EGARCH(1,1) process:

$$R_t = \lambda_0 + \sum_{i=1}^{k} \phi_i R_{t-i} + a_t ,$$
$$a_t = \sigma_t \varepsilon_t ,$$
$$\ln\left(\sigma_t^2\right) = \omega_0 + \beta_1 \ln\left(\sigma_{t-1}^2\right) + g\left(\varepsilon_{t-1}\right).$$

Then all orders between zero and five were examined and the one with the lowest AICC value is chosen to be the most appropriate one. This procedure is employed in a similar way in Rodriguez (2012), who found orders between 2 and 4 to fit the best for his data. Therefore, the highest considered order was set to be 5.

Having determined the most appropriate order for the autoregressive part, the three different approaches, which will be described in the following, were used to find the day of the week effect, and how these effects changed across the three periods, for returns and volatility.

4.2.1 Modeling Approach I. As already mentioned, the first approach is based on the model in (4.1) and (4.3). However, to reduce the probability of an over estimated model, the full model was reduced by a model selection technique.

In a first step, several variable selection procedures were applied to the S&P 500 data. Since this is the broadest and most representative data used in this study, the best selection procedure determined by using this data was then applied to all other sectors as well. The considered methods included a two-step procedure where in a first step the mean model was reduced, assuming an EGARCH model without any dummies for the error term. In the second step, the resulting mean model was fixed and then the same variable reduction method was carried out for the volatility model that includes the dummies. Another procedure utilized was to reduce the full model, including all dummies in both mean and volatility model. A further approach was to add one variable after the other. To compare all these techniques, the AICC values were compared to decide which model fits the best. However, since the AICC values often were close to one another, not only the final values were considered, but also the progression of these values throughout the model selection procedure. Some of the models produced very unstable AICC estimates meaning that the values sometimes increased when removing an insignificant variable.

Taking everything into consideration, the following selection procedure was chosen. The AR(k) part which was determined in a previous step was included in all models considered. The full model described in (4.1) and (4.3) was first fitted. Then, holding the volatility model fixed as in (4.3), the mean model (4.1) was reduced step by step, while keeping the AR(k) part intact. The least significant variable, that is the one with the highest p-value, was removed from the regression at each step. Whenever two or more variables had similar p-values, where similar was defined having a difference smaller than or equal to 0.05, all variables were removed successively and the one which lowered the AICC the most was chosen to be removed. These steps were repeated until all of the remaining variables in the mean model were significant at the 5% level.

4.2.2 Modeling Approach II. As mentioned earlier the second approach is based on the idea that the hypothesized day of the week effects and structural breaks not only change the intercept in the volatility model, but also the slope. Therefore, the calculation of the volatility equation given in (4.3) was done differently in the second approach.

The basic idea of this approach is to get an estimate for the residuals $\hat{a}_t$ of the mean model obtained from Approach I and apply a regression analysis on it. Hence, the model

$$\begin{aligned} R_t &= \lambda_0 + \lambda_1 D_1 + \lambda_2 D_2 + \lambda_3 D_3 + \lambda_4 D_4 + \lambda_5 D_5 + \lambda_6 D_6 \\ &+ \lambda_7 D_1 D_5 + \lambda_8 D_2 D_5 + \lambda_9 D_3 D_5 + \lambda_{10} D_4 D_5 \\ &+ \lambda_{11} D_1 D_6 + \lambda_{12} D_2 D_6 + \lambda_{13} D_3 D_6 + \lambda_{14} D_4 D_6 + \sum_{i=1}^{k} \phi_i R_{t-i} + a_t \end{aligned} \tag{4.6}$$

$$\begin{aligned} a_t &= \sigma_t \varepsilon_t , \\ \ln\left(\sigma_t^2\right) &= \omega_0 + \beta_1 \ln\left(\sigma_{t-1}^2\right) + g\left(\varepsilon_{t-1}\right), \end{aligned} \tag{4.7}$$

was estimated, where only the significant variables from Approach I were included in equation (4.6). Then, the residuals $\hat{a}_t$ were extracted.

Note that the square of these residuals can be regarded as estimates of the conditional volatilities $\sigma_t^2 = E\left(a_t^2 \middle| F_{t-1}\right)$. Therefore, the logarithms of the squared values of $\hat{a}_t$ and their lags can be used to estimate the parameters in equation (3.10) through a simple regression analysis. To consider the effects of the dummies to the slope, however, the day of the week and period dummies as well as their interactions with the lagged residuals should be added to the regression equation. Hence the model in (4.3) becomes

$$\begin{aligned}\ln\left(a_t^2\right) &= \omega_0 + \omega_1 D_1 + \omega_2 D_2 + \omega_3 D_3 + \omega_4 D_4 + \omega_5 D_5 + \omega_6 D_6 \\ &+\omega_7 D_1 D_5 + \omega_8 D_2 D_5 + \omega_9 D_3 D_5 + \omega_{10} D_4 D_5 \\ &+\omega_{11} D_1 D_6 + \omega_{12} D_2 D_6 + \omega_{13} D_3 D_6 + \omega_{14} D_4 D_6 + \beta_0 \ln\left(a_{t-1}^2\right) \\ &+\beta_1 D_1 \ln\left(a_{t-1}^2\right) + \beta_2 D_2 \ln\left(a_{t-1}^2\right) + \beta_3 D_3 \ln\left(a_{t-1}^2\right) + \beta_4 D_4 \ln\left(a_{t-1}^2\right) \\ &+\beta_5 D_5 \ln\left(a_{t-1}^2\right) + \beta_6 D_6 \ln\left(a_{t-1}^2\right) + \beta_7 D_1 D_5 \ln\left(a_{t-1}^2\right) + \beta_8 D_2 D_5 \ln\left(a_{t-1}^2\right) \\ &+\beta_9 D_3 D_5 \ln\left(a_{t-1}^2\right) + \beta_{10} D_4 D_5 \ln\left(a_{t-1}^2\right) + \beta_{11} D_1 D_6 \ln\left(a_{t-1}^2\right) \\ &+\beta_{12} D_2 D_6 \ln\left(a_{t-1}^2\right) + \beta_{13} D_3 D_6 \ln\left(a_{t-1}^2\right) + \beta_{14} D_4 D_6 \ln\left(a_{t-1}^2\right) + \varepsilon_t \\ &= a + b \ln\left(a_{t-1}^2\right) + \varepsilon_t ,\end{aligned} \tag{4.8}$$

where the dummies are defined as before and $\ln\left(a_t^2\right)$, $\ln\left(a_{t-1}^2\right)$ are the logarithm of the squared residuals of the estimated mean model observed at time *t* and *t-1* respectively. Note that in this case the intercept and the slope parameters are given by

$$\begin{aligned} a &= \omega_0 + \omega_1 D_1 + \omega_2 D_2 + \omega_3 D_3 + \omega_4 D_4 + \omega_5 D_5 + \omega_6 D_6 \\ &+\omega_7 D_1 D_5 + \omega_8 D_2 D_5 + \omega_9 D_3 D_5 + \omega_{10} D_4 D_5 \\ &+\omega_{11} D_1 D_6 + \omega_{12} D_2 D_6 + \omega_{13} D_3 D_6 + \omega_{14} D_4 D_6 , \\ b &= \beta_0 + \beta_1 D_1 + \beta_2 D_2 + \beta_3 D_3 + \beta_4 D_4 + \beta_5 D_5 + \beta_6 D_6 \\ &+\beta_7 D_1 D_5 + \beta_8 D_2 D_5 + \beta_9 D_3 D_5 + \beta_{10} D_4 D_5 \\ &+\beta_{11} D_1 D_6 + \beta_{12} D_2 D_6 + \beta_{13} D_3 D_6 + \beta_{14} D_4 D_6 , \end{aligned}$$

allowing the dummy variables to effect both the mean and the slope parameters. Substituting a_t^2 and a_{t-1}^2 in place of σ_t^2 and σ_{t-1}^2 in Equation (4.7), however, results in introducing a bias term to equation (4.8). This is because while $\sigma_t^2 = E\left(a_t^2 \middle| F_{t-1}\right)$, $\ln(\sigma_t^2) \neq E\left(\ln(a_t^2) \middle| F_{t-1}\right)$. Thus, Approach II leads to biased estimates for the intercept *a*.

This can be seen by expressing $\sigma_t^2 = \frac{a_t^2}{\varepsilon_t^2}$ and substituting this into the EGARCH formulation in (3.10)

$$\begin{aligned} \ln\left(\frac{a_t^2}{\varepsilon_t^2}\right) &= \omega + \beta_1 \ln\left(\frac{a_{t-1}^2}{\varepsilon_{t-1}^2}\right) + g\left(\varepsilon_{t-1}\right) \\ \ln\left(a_t^2\right) - \ln\left(\varepsilon_t^2\right) &= \omega + \beta_1\left(\ln\left(a_{t-1}^2\right) - \ln\left(\varepsilon_{t-1}^2\right)\right) + g\left(\varepsilon_{t-1}\right) \\ \ln\left(a_t^2\right) &= \omega + \beta_1 \ln\left(a_{t-1}^2\right) - \beta_1 \ln\left(\varepsilon_{t-1}^2\right) + \ln\left(\varepsilon_t^2\right) + g\left(\varepsilon_{t-1}\right) \\ &= \omega + \underbrace{\ln\left(\varepsilon_t^2\right) - \beta_1 \ln\left(\varepsilon_{t-1}^2\right)}_{z} + \beta_1 \ln\left(a_t^2\right) + g\left(\varepsilon_{t-1}\right). \end{aligned}$$

Hence, the estimated intercept *a* is biased by *z*. To correct for the bias when estimating the unconditional volatility, the expected value of ln(ε_t^2) has to be calculated. Since $\{\varepsilon_t\}$ forms an independent and identically distributed set of random variables, we have the equality $E\left(\ln(\varepsilon_t^2)\right) = E\left(\ln(\varepsilon_{t-1}^2)\right)$, and therefore it follows that

$$\begin{aligned} E(z) &= E\left(\ln\left(\varepsilon_t^2\right)\right) - \beta_1 E\left(\ln\left(\varepsilon_t^2\right)\right) \\ &= \left(1-\beta_1\right) E\left(\ln\left(\varepsilon_t^2\right)\right). \end{aligned}$$

Now, assuming $\varepsilon_t \sim N(0,1)$, it follows that $\varepsilon_t^2 \sim \chi_1^2$. Following the derivations in Pav (2015), the expected value of a logarithm of a chi-square distribution can be computed using the formula:

$$E\left(\ln\left(\varepsilon_t^2\right)\right) = \ln(2) + \psi\left(\frac{\upsilon}{2}\right)$$
$$= \ln(2) + \psi\left(\frac{1}{2}\right)$$
$$= \ln(2) + \left(-\gamma - 2\ln(2)\right),$$

where $\psi(\cdot)$ is the Psi-function, υ are the degrees of freedom of the Chi-square distribution and $\gamma \approx 0.57722$ being the Euler's constant. The last step follows from Olver et al. (2010). Now, to correct for the bias in estimating the intercept, the term $(1-\beta_1)\left(\log(2) + \left(-\gamma - 2\ln(2)\right)\right)$ has to be subtracted from the estimate of a from the regression. Hence, the intercept and the slope in equation (4.5) become

$$\begin{aligned}
a = {} & \omega_0 + \omega_1 D_1 + \omega_2 D_2 + \omega_3 D_3 + \omega_4 D_4 + \omega_5 D_5 + \omega_6 D_6 \\
& + \omega_7 D_1 D_5 + \omega_8 D_2 D_5 + \omega_9 D_3 D_5 + \omega_{10} D_4 D_5 \\
& + \omega_{11} D_1 D_6 + \omega_{12} D_2 D_6 + \omega_{13} D_3 D_6 + \omega_{14} D_4 D_6 \\
& - (1-b)\left(\ln(2) + \left(-\gamma - 2\ln(2)\right)\right), \\
b = {} & \beta_0 + \beta_1 D_1 + \beta_2 D_2 + \beta_3 D_3 + \beta_4 D_4 + \beta_5 D_5 + \beta_6 D_6 \\
& + \beta_7 D_1 D_5 + \beta_8 D_2 D_5 + \beta_9 D_3 D_5 + \beta_{10} D_4 D_5 \\
& + \beta_{11} D_1 D_6 + \beta_{12} D_2 D_6 + \beta_{13} D_3 D_6 + \beta_{14} D_4 D_6 .
\end{aligned}$$

Similar to the derivations in (4.4) and (4.5), the unconditional variance can then be estimated by

$$E\left[\ln\left(\sigma^2\right)\right] = \frac{a}{1-b}$$
$$\sigma^2 \approx e^{\frac{a}{1-b}}.$$

The model given in Equation (4.8) was fitted to the logarithm of the squared residuals obtained by subtracting the estimated mean part of the model, using SAS REG procedure and the model selection function BACKWARD, where the least significant variables were removed one after the other, to find the most appropriate model.

Using the STEPWISE or FORWARD model selection in the SAS procedure was considered initially. However, the techniques didn't improve results based to their AICC values. Therefore the BACKWARD method was used to keep consistent with the model selection technique used in Approach I.

4.2.3 Modeling Approach III. Another way of investigating the day of the week effect and its changes over different periods is carried out in Berument and Kiymaz (2001). They divided their observed data corresponding to different sub periods and evaluated their model for each of these separately. Therefore, for the third approach the data was separated into three distinct sub periods: pre-recession, recession and post-recession period. Using this method, the dummy variables for the recession and all interaction effects are redundant and the mean and the volatility model reduce to

$$R_t = \lambda_0 + \lambda_1 D_1 + \lambda_2 D_2 + \lambda_3 D_3 + \lambda_4 D_4 + \sum_{i=1}^{k} \phi_i R_{t-i} + a_t \qquad (4.9)$$

and

$$\begin{aligned} a_t &= \sigma_t \varepsilon_t, \\ \ln\left(\sigma_t^2\right) &= \omega_0 + \omega_1 D_1 + \omega_2 D_2 + \omega_3 D_3 + \omega_4 D_4 + \gamma_1 \ln\left(\sigma_{t-1}^2\right) + g\left(\varepsilon_{t-1}\right), \qquad (4.10) \\ g\left(\varepsilon_{t-1}\right) &= \theta a_t + \gamma\left(\left|a_t\right| - E\left(\left|a_t\right|\right)\right), \end{aligned}$$

where D_1, D_2, D_3 and D_4 are the dummy variables for Tuesday, Wednesday Thursday and Friday. Note that this is again based on the formulas in Section 3.5 rather than the SAS AUTOREG formulation.

The above model was applied to the three sub periods. Similar to the other approaches, a model selection was used to determine significant parameters. In a first step, the mean model (4.9) was reduced by holding the volatility model fixed and removing the least significant variable. In this manner, one variable after the other was eliminated until all left variables were significant at the 5% level. Note that the order k was determined in the initial step and that this parameter was not considered when removing variables. In a second step, the determined mean model was fixed and the volatility model was reduced. Similarly, the least significant variables were deleted one after the other until all estimates were significant at the 5% level.

4.3 RESULTS

For reasons of clarity, only the final results from the model building process are reported in this section. Results for the investigation of the S&P 500 for all three approaches as well as the SAS code can be found in Appendix B.

Table 4.5 shows the results for the initial step where the autoregressive order is determined. Only the S&P 500 and the Consumer Staples, Financial and Information Technology sectors exhibit autoregressive patterns with order one, whereas the remaining seven sectors don’t show any effects due to lag returns. Although this is in contrast to Rodriguez (2012), who found orders between 2 and 4 as appropriate for the markets he investigated, it is in line with what is described in Section 3.1, namely that returns are usually not highly correlated. At the same time, it coincides with the findings in Figure 3.1 showing the autocorrelation functions for the S&P 500, where only the first lag is significant.

Table 4.5: Autoregressive order results

AR Order	SP500	CD	CS	En	Fin	HC	Ind	IT	Mat	TS	Ut
k	1	0	1	0	1	0	1	0	0	0	0

After estimating the autoregressive orders for each investigated sector, the day of the week effect for both mean returns and volatility were estimated for all three approaches. The results are described in the following.

4.3.1 Results Approach I. Table 4.6 shows the day of the week effect on mean returns, where Table 4.8 shows the estimates for the intercept and the slope of the volatility model, each separated in the three investigated periods. The tables are based on the estimates for the corresponding model described in (4.3). To improve comparability, the effect for the dummies are added up to show the effect for each day and each period. For instance, the Wednesday post-recession effect is calculated as the sum of the intercept term and the Dummy estimates for Wednesday, post-recession and its interaction. Thereby, insignificant parameter estimates are set equal zero.

Clearly, Table 4.6 indicates that there is only a slight day of the week effect. In fact, four sectors, namely Energy, Health care, Industrial and Materials, don’t show any effects. For these market sectors, the mean returns from Monday to Friday and from pre-recession to recession and post-recession periods are not significantly different from one another. This is surprising, since one would expect at least an effect over the recession period. The S&P 500 and the Financial, Information Technology as well as Consumer Staples sectors, however, indicate a negative effect during the recession period, but this effect only holds for Wednesdays and Fridays in the Consumer Staples sector. The negative mean returns are not unexpected, because as these parameter estimates indicate, one would expect the mean returns to drop down during recession period. More precisely, the mean returns dropped between 0.22 for the S&P 500 and the Information Technology sector and 0.47 in the financial sector. Moreover, on Wednesdays and Fridays during the recession period, the return for the Consumer Staples sector fell by 0.25 and 0.33 respectively. In fact, most of the effects are due to the period and not because of the day of the week. Having a closer look, only Wednesdays and Fridays show any day of the week effects. To distinguish between a period and a day of the week effect, the latter one is given in bold letters. For the Financial and the Consumer Discretionary sector, Wednesday returns are 0.09 higher while the latter sector also show a Friday effect in the post-recession period. Moreover, Wednesdays tend to have lower returns for the Telecommunication Services sector as well as the Utilities sector during the post-recession period.

Summarizing, the day of the week effect for the returns are quite small. Only Wednesdays and Fridays show significant differences compared to other days. These effects vary during different periods indicating small changes in the day of the week during pre-, post- and recession periods.

Table 4.6: Mean Effects in 10^{-2} units using Approach I

		SP	CD	CS	En	Fin	HC	Ind	IT	Mat	TS	Ut
Mo	Pre	0.04	-	0.03	-	-	-	-	-	-	-	0.04
	Rec	-0.18	-	0.03	-	-0.47	-	-	-0.22	-	-	0.04
	Post	0.04	-	0.03	-	-	-	-	-	-	-	0.04
Tu	Pre	0.04	-	0.03	-	-	-	-	-	-	-	0.04
	Rec	-0.18	-	0.03	-	-0.47	-	-	-0.22	-	-	0.04
	Post	0.04	-	0.03	-	-	-	-	-	-	-	0.04
We	Pre	0.04	**0.09**	0.03	-	**0.09**	-	-	-	-	-	0.04
	Rec	-0.18	**0.09**	**-0.22**	-	**-0.38**	-	-	-0.22	-	-	0.04
	Post	0.04	**0.09**	0.03	-	**0.09**	-	-	-	-	**-0.11**	**-0.07**
Th	Pre	0.04	-	0.03	-	-	-	-	-	-	-	0.04
	Rec	-0.18	-	0.03	-	-0.47	-	-	-0.22	-	-	0.04
	Post	0.04	-	0.03	-	-	-	-	-	-	-	0.04
Fr	Pre	0.04	-	0.03	-	-	-	-	-	-	-	0.04
	Rec	-0.18	-	**-0.30**	-	-0.47	-	-	-0.22	-	-	0.04
	Post	0.04	**0.14**	0.03	-	-	-	-	-	-	-	0.04

The negative mean returns during the recession period seem to be quite obvious. However, there is also the possibility of an oversimplification. Indeed, considering Figure 4.3 which shows the centered moving average logarithmic prices for the S&P 500 with window size 20, indicates that the S&P 500 dropped down during part of the recession period, displayed between observations 705 and 1101, but then increased over the rest of the period designated as the recession. This ‘v’- shape pattern indicates negative mean returns only for the first few month of the recession. Therefore, the expected pattern of negative returns in the recession period may only hold for the first half of the recession period, but not for the second one. When averaged over the whole recession period, the negative returns seem to have dominated the positive returns, thereby yielding an overall negative value for the mean return.

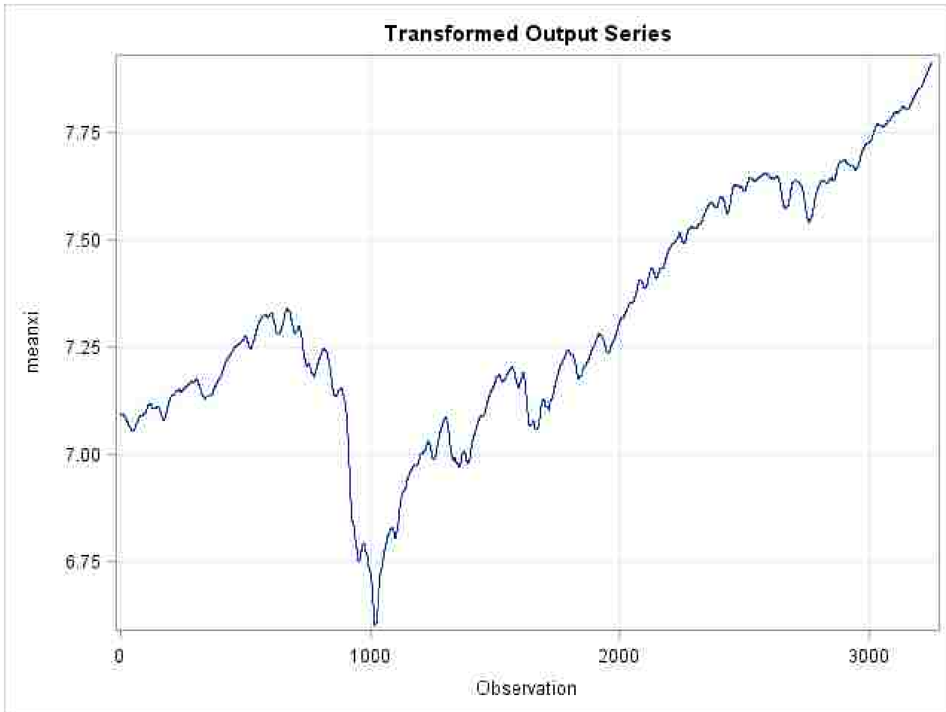

Figure 4.3: Centered Moving Average returns for S&P 500

Table 4.8 indicates both a day of the week effect and a change in the effects between the different periods with regard to volatility. Only the Materials sector, where only Tuesday and recession period is significantly different, shows miner effects, whereas all other sectors show significant differences in volatility depending on the day and the period.

For the pre-recession period, Tuesday shows highest volatility for the S&P 500 and all its ten investigates sectors. This is interesting since other studies found Monday to have the highest volatility; see for example Rodriguez (2012). However, it is in line with the findings in Table 4.4 showing that the unconditional standard deviation is high for Tuesday.

At the same time, most of the sectors have the lowest volatility on Mondays or Wednesdays during the pre-recession period. For seven out of the eleven investigated sectors there is no difference between Monday and Wednesday volatility and for five sectors there is no difference between Monday, Wednesday, and Friday volatilities. Only the Health Care, Information Technology, Telecommunication Services and Utilities sectors show a difference between Monday, Friday and Wednesday volatilities. The S&P 500 index and the Consumer Staples sector show differences between Monday and Wednesday volatilities. Simply put, the majority of the sectors only show Thursday and Tuesday effects on volatility during the pre-recession period, with volatility associate with Tuesdays shown as having the highest volatility compared to the rest of the week.

Investigations for the recession period show different results. During this period, Monday shows the highest volatility for most of the sectors which is in line with the results in Section 4.1. In contrast, Information Technology and Utilities sectors show highest volatility on Wednesday and Friday respectively, and the Material sector indicates lower volatility on Tuesday during the recession period. According to Dicle and Levendis (2014), this high Monday volatility may be due to the fact that a lot of bad news are released over the weekend. While this hypothesis was not supported by pre-recession data, during the recession periods traders may be more sensitive and reactive to bad news which is manifested through high volatility on Mondays. The Information Technology sector is an exception, showing the lowest volatility on Mondays, but this may be due to the nature of the commodity being traded. For the remaining sectors, the lowest volatility during the recession period is mainly seen on Wednesdays. This result is supported by the hypothesis given by Berument and Kiymaz (2001) who stated that the middle of the week provides most time to react on new information and therefore has lower volatility.

For the post-recession period starting in July 2009, the day with the highest volatility differs from sector to sector. Thursday shows highest volatility for the S&P 500 index and the Consumer Staples, Consumer Discretionary, and Financial sectors. Tuesday has the highest volatility for Health Care, Industries, Materials and Telecommunication Service sectors while Wednesday shows the most volatility for the Information Technology and Utilities sectors. Monday has the highest volatility for the Energy sector. However, with some exceptions, the lowest volatility exists, similar to the pre-recession period, on Mondays or Wednesdays for the majority of the sectors. Indeed, in six sectors, volatility on Mondays and Wednesdays are not significantly different from each other and are lower compared to the rest of the week. Additionally, in five sectors there is no difference between Monday, Wednesday and Friday volatility. Moreover, the Industrial sector indicates lowest volatility for Monday only. Out of the remaining sectors, Energy, Health Care and Telecommunication Services show lowest volatility on Fridays, Utilities for Thursday and Materials doesn’t show any effect except for Tuesdays. Table 4.7 summarizes the results and gives an overview of the days with highest and lowest volatility for each investigated sector.

The fact that Mondays, Wednesdays and partly Fridays don’t show any difference in pre- and post- recession periods indicates some similarities between these periods. Although they do exhibit some similarities, the post-recession period seems to be more volatile for these days, as indicated by higher estimates. Tuesdays are less volatile in the recession and the post-recession period, but Tuesdays show highest volatility in the pre-recession for all investigated sectors.

Table 4.7: Lowest and highest volatility for each sector using Approach I

	SP500	CD	CS	En	Fin	HC	Ind	IT	Mat	TS	Ut
Pre high	Tu	Tu	Tu	Tu	Tu	Tu	Tu	Tu	Tu	Tu	Tu
Pre low	Mo, We	Mo, We	Mo, We, Fr	Mo, We, Fr	Mo, We, Fr	Mo	Mo, We, Fr	Mo	Mo, We, Th, Fr	Mo, Th	Mo
Rec high	Mo	Mo	Mo	Mo	Mo	Mo	Mo	We	Mo, We, Th, Fr	Mo	Fr
Rec low	We	We	Tu	We	We, Fr	We	We	Mo	Tu	We	We
Post high	Th	Th	Th	Mo, We	Th	Tu, Th	Tu	We	Tu	Tu	We
Post low	Mo, We, Fr	Mo, We	Mo, We, Fr	Fr	Mo, We, Fr	Fr	Mo, We, Fr	Mo	Mo, We, Th, Fr	Fr	Th

Interestingly, volatility did not increase for all days of the week in the recession period. In fact, volatility decreased for seven sectors on Wednesdays, five sectors on Thursdays and three sectors on Friday. Only Monday volatility increased for all sectors, where Tuesday volatility decreased for all of them.

After the recession, except for Mondays, volatility did not drop for all day of the week. In fact, the volatility even increased in six sectors on Tuesdays, seven on Wednesdays, five on Thursdays and four on Fridays. Comparing pre- and post-recession periods shows that, in general, the volatility was higher after the crisis. Except for

Tuesdays, the majority of the sectors show higher volatility. In fact, seven sectors are more volatile on Mondays, Wednesdays, Thursdays and Fridays.

The fact that during the recession period the volatility did not increase for every day of the week, but increased for some of them after the recession indicates changes in the day of the week effect. This can be caused by an increase in uncertainty over the recession period, which lasted until the post-recession period. Having experienced a lot of losses, investors could have been more cautious and reacted faster after certain news to prevent possible losses. However, there is also the possibility of modeling problems. The recession period consists of only 397 data points such that the estimates might be not accurate enough and the effects are postponed to the post-recession period.

Summarizing, the investigation indicate day of the week effects in volatility. In general, Mondays and Wednesdays have the lowest volatility during non-recession periods, whereas Wednesdays show lowest volatility in the recession period. While Monday volatility is lowest before and after recession, volatility sharply increased during the recession period such that Monday became the most volatile day. A reason for this might be the tendency to release bad news over weekends. In the pre-recession period Tuesday show highest volatility, whereas this is different for each sector in the post-recession period. This also indicates that there has been a change in the day of the week effect after the recession period. It is worth to note that a two-way procedure, where the volatility model was reduced by removing the least significant variable in a second step, led to similar results. Except the Energy and Utility sector, there were only slight differences in the estimates itself and hence in the day of the week effect in volatility.

Table 4.8: Intercept and Slope Estimates using Approach I

		SP	CD	CS	En	Fin	HC	Ind	IT	Mat	TS	Ut
Mo	Pre	-0.93	-0.96	-1.12	-0.60	-0.57	-1.01	-0.65	-1.17	-0.41	-1.03	-0.70
	Rec	0.27	0.43	0.63	0.13	0.38	-0.05	0.35	-0.69	0.07	0.06	-0.07
	Post	-0.54	-0.96	-0.81	-0.08	-0.29	-0.44	-0.27	-0.76	-0.41	-0.56	-0.33
Tu	Pre	0.22	0.38	0.43	0.17	0.27	-0.11	0.18	-0.00	0.52	-0.02	-0.10
	Rec	-0.39	-0.46	-1.56	-0.22	-0.00	-0.39	-0.39	-0.51	-0.30	-0.41	-0.29
	Post	-0.36	-0.47	-0.66	-0.27	-0.16	-0.36	-0.05	-0.67	-0.25	-0.44	-0.28
We	Pre	-0.93	-0.96	-1.12	-0.60	-0.57	-0.53	-0.65	-0.48	-0.41	-0.51	-0.29
	Rec	-0.99	-0.85	-0.58	-0.66	-0.77	-0.85	-0.90	0.01	0.07	-0.82	-0.60
	Post	-0.54	-0.96	-0.81	-0.08	-0.29	-0.44	-0.27	-0.07	-0.41	-0.52	0.08
Th	Pre	-0.49	-0.41	-0.61	-0.10	-0.32	-0.20	-0.01	-0.77	-0.41	-1.03	-0.22
	Rec	-0.44	-0.62	-0.52	-0.24	-0.07	-0.24	-0.14	-0.28	0.07	-0.62	-0.39
	Post	-0.10	-0.41	-0.30	-0.32	-0.04	-0.36	-0.17	-0.36	-0.41	-0.56	-0.42
Fr	Pre	-0.42	-0.49	-1.12	-0.60	-0.57	-0.48	-0.65	-0.70	-0.41	-0.47	-0.20
	Rec	-0.44	-0.34	-0.87	-0.58	-0.77	-0.45	-0.32	-0.22	0.07	-0.72	0.43
	Post	-0.54	-0.49	-0.81	-0.69	-0.29	-0.59	-0.27	-0.72	-0.41	-0.87	-0.35
b		0.96	0.95	0.94	0.98	0.97	0.95	0.97	0.93	0.98	0.94	0.97

Figure 4.4 shows the estimated expected unconditional variance for the S&P 500 and the sectors Consumer Staples, Financial and Materials. For each sector, the estimates are separated into the different periods and the day of the week. Note that the ordinate axis has a logarithmic scale with base 10.

The graphs clearly indicate periods of high and low volatility as well as changes in the day of the week effect. This pattern holds for every displayed sector. However, the estimates of the unconditional variance seem to be very high and very unstable. A possible reason for this is that Approach I includes a lot of dummies and therefore is not very flexible leading to inaccurate estimates.

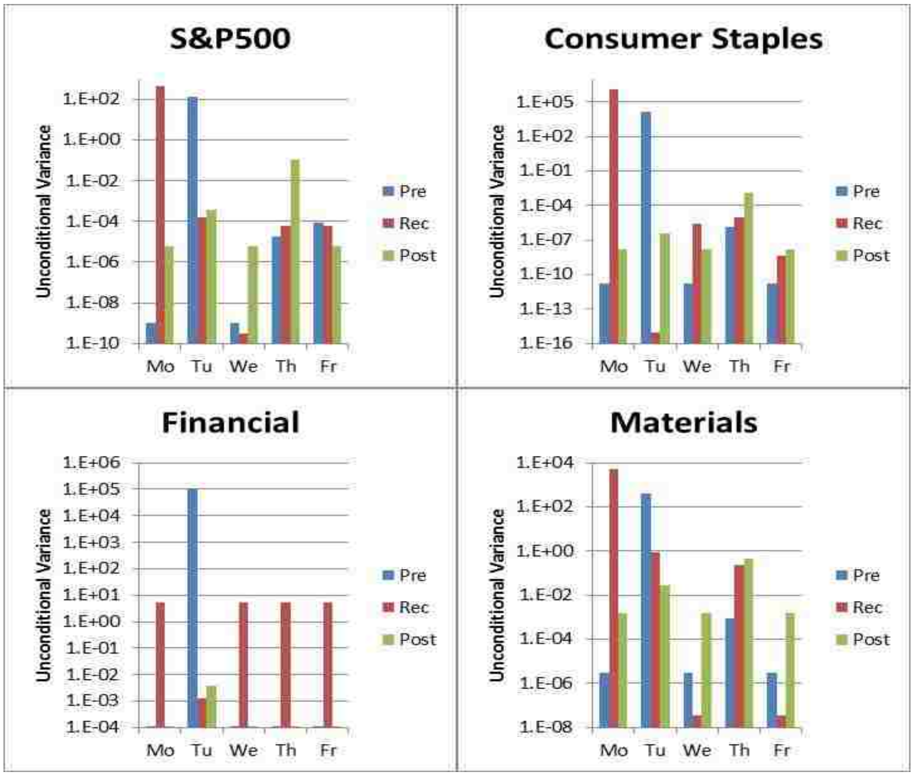

Figure 4.4: Expected unconditional variance for S&P 500 and sectors Consumer Staples, Financial and Materials using Approach I

4.3.2 Results Approach II. Table 4.9 and Table 4.10 show the effects when using the second approach that is allowing the dummies to change the slope of the regression line. The first table summarizes the effects on the intercept, where the second one indicates effects on the slope. Similar to what has been done before, day of the week effects are summarized for each period to improve comparability, meaning that the estimates are added up according to day and period. Insignificant estimates are set equal zero. As was mentioned in Section 4.2.2, the adopted estimation strategy results in a bias estimate for the intercept term *a* which has to be corrected as indicated in that section. This was carried out to obtain the reported results.

Considering Table 4.9 and Table 4.10, it can be seen that effects reduce when using Approach II. Apparently, when allowing the dummies to affect the slope, less estimates are significant and the overall day of the week effect decreases. Only the slope of the sectors Consumer Staples and Materials seems to be unaffected by the dummies.

Table 4.9: Intercept estimates using Approach II

		SP	**CD**	**CS**	**En**	**Fin**	**HC**	**Ind**	**IT**	**Mat**	**TS**	**Ut**
Mo	Pre	-10.2	-9.8	-9.4	-8.3	-8.4	-10.3	-9.8	-9.0	-8.5	-9.9	-8.7
	Rec	-7.3	-7.6	-8.2	-6.4	-5.5	-8.8	-7.9	-7.3	-6.9	-7.4	-7.6
	Post	-8.9	-8.3	-9.4	-8.4	-8.4	-9.9	-8.5	-9.0	-8.5	-8.9	-8.7
Tu	Pre	-9.9	-9.8	-9.0	-8.3	-8.4	-9.3	-9.8	-9.0	-8.5	-9.6	-8.7
	Rec	-7.1	-7.6	-7.8	-8.1	-5.4	-7.8	-7.9	-7.3	-6.9	-7.2	-7.6
	Post	-8.6	-8.3	-9.5	-8.4	-8.3	-8.9	-8.5	-9.0	-8.5	-8.6	-8.7
We	Pre	-10.2	-9.8	-9.4	-8.3	-8.4	-10.3	-9.8	-9.0	-8.5	-9.2	-8.7
	Rec	-7.3	-7.6	-8.2	-6.4	-5.5	-8.8	-7.9	-7.3	-6.9	-6.9	-7.6
	Post	-8.9	-8.3	-9.4	-8.4	-8.4	-9.9	-8.5	-9.0	-8.5	-8.8	-8.7
Th	Pre	-10.2	-9.8	-9.4	-8.3	-8.4	-10.3	-9.8	-9.0	-8.5	-9.8	-8.7
	Rec	-7.3	-7.6	-8.2	-6.4	-5.4	-8.8	-7.9	-7.3	-6.9	-7.5	-7.6
	Post	-8.9	-8.3	-9.4	-8.4	-8.3	-8.6	-8.5	-9.0	-8.5	-8.9	-8.7
Fr	Pre	-10.2	-9.8	-9.4	-8.5	-8.4	-9.2	-9.8	-9.0	-8.5	-9.9	-6.9
	Rec	-7.3	-7.6	-8.2	-6.6	-5.5	-7.7	-7.9	-7.4	-6.9	-7.6	-5.7
	Post	-8.9	-8.3	-9.4	-8.6	-7.3	-8.8	-8.5	-9.0	-8.5	-8.9	-9.2

Similar to the results described above, the majority of the sectors don’t show any differences between Monday, Wednesday and Friday volatility, indicated by similar estimates for both intercept and slope parameters. This is true for six sectors in the recession and the post-recession and for seven sectors in the pre-recession period. In fact, the sectors Consumer Discretionary, Industrials, Materials and Information Technology don’t show any day of the week effect, where this only holds in the pre-and post-recession period for the latter one. However, every sector show differences in volatility during the investigated periods indicating different volatility depending on the time.

Table 4.10: Slope estimates using Approach II

		SP	CD	CS	En	Fin	HC	Ind	IT	Mat	TS	Ut
Mo	Pre	-	-	0.08	-	0.15	-	-	0.06	0.07	-	0.09
	Rec	0.10	-	0.08	0.13	0.15	-	-	0.06	0.07	-	0.09
	Post	0.12	0.13	0.08	0.06	0.10	-	0.11	0.06	0.07	0.08	0.09
Tu	Pre	-	-	0.08	-	0.12	0.08	-	0.06	0.07	-	0.09
	Rec	0.10	-	0.08	-0.12	0.12	0.08	-	0.06	0.07	0.09	0.09
	Post	0.12	0.13	0.08	0.06	0.08	0.08	0.11	0.06	0.07	0.08	0.09
We	Pre	-	-	0.08	-	0.15	-	-	0.06	0.07	-	0.09
	Rec	0.10	-	0.08	0.13	0.15	-	-	0.06	0.07	0.10	0.09
	Post	0.12	0.13	0.08	0.06	0.10	-	0.11	0.06	0.07	0.08	0.09
Th	Pre	-	-	0.08	-	0.12	-	-	0.06	0.07	-0.03	0.09
	Rec	0.10	-	0.08	0.13	0.12	-	-	0.06	0.07	0.07	0.09
	Post	0.12	0.13	0.08	0.06	0.08	0.12	0.11	0.06	0.07	0.05	0.09
Fr	Pre	-	-	0.08	-	0.15	0.10	-	0.06	0.07	-	0.30
	Rec	0.10	-	0.08	0.13	0.15	0.10	-	0.12	0.07	0.14	0.30
	Post	0.12	0.13	0.08	0.06	0.21	0.10	0.11	0.06	0.07	0.08	0.06

Figure 4.5 shows the estimated unconditional variance for the S&P 500 and the sectors Consumer Staples, Financial and Materials. For each sector, the estimates are separated into the different periods and the day of the week. One can see that unconditional variance changes from day to day and from period to period. Especially the recession period was very volatile. Except the Materials sector, each one show differences in the day of the week variance depending on the period. Moreover, the post-recession period was more volatile than the pre-recession period which coincides with the findings in Approach I.

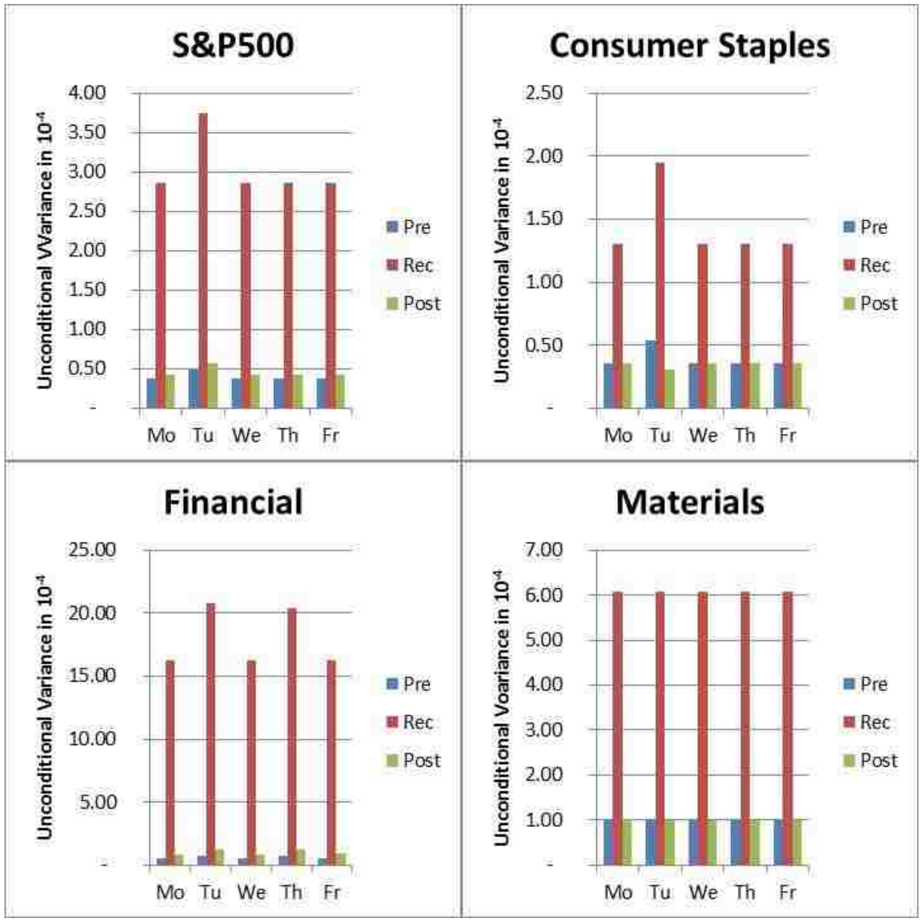

Figure 4.5: Expected unconditional variance for S&P 500 and sectors Consumer Staples, Financial and Materials using Approach II

4.3.3 Results Approach III. The results for the mean and the volatility model using the third approach are summarized in Table 4.11 and Table 4.12, respectively. Similar to what has been done before, the estimates for the mean model are multiplied by 100. Moreover, instead of listing the parameters itself, the tables summarize the effects for each day of the week for every period.

Similar to the first approach, Table 4.11, where day of the week effects are given in bold letters to distinguish from period effects, indicates only a small day of the week effect. In fact, throughout the whole period, only seven sectors show significant day of the week effects. In the pre- recession period, the S&P 500 as well as the sectors Consumer Discretionary, Financial, Health Care, Information Technology and Industrial show a Wednesday effect. This is in line with the findings in the first approach, where only a Wednesday and Friday effect was found for the mean returns. Moreover, the sectors Telecommunication Services and Utilities tend to have lower returns compared to the rest of the week in the post-recession period, which again coincides with the finding in Approach I. However, in contrast to the first approach, the sector Consumer Staples- like all other sectors- don't show any effect during the recession period.

Table 4.11: Mean effects in 10^{-2} units using Approach III

		SP	CD	CS	En	Fin	HC	Ind	IT	Mat	TS	Ut
Mo	Pre	-	-	-	-	-	-	-	-	-	-	0.07
	Rec	-	-	-	-	-0.44	-	-	-	-	-	-
	Post	0.04	0.05	0.03	-	0.06	0.04	-	0.05	-	-	-
Tu	Pre	-	-	-	-	-	-	-	-	-	-	0.07
	Rec	-	-	-	-	-0.44	-	-	-	-	-	-
	Post	0.04	0.05	0.03	-	0.06	0.04	-	0.05	-	-	-
We	Pre	**0.14**	**0.19**	-	-	-	**0.14**	**0.17**	**0.18**	-	-	0.07
	Rec	-	-	-	-	-0.44	-	-	-	-	-	-
	Post	0.04	0.05	0.03	-	0.06	0.04	-	0.05	-	**-0.11**	**-0.10**
Th	Pre	-	-	-	-	-	-	-	-	-	-	0.07
	Rec	-	-	-	-	-0.44	-	-	-	-	-	-
	Post	0.04	0.05	0.03	-	0.06	0.04	-	0.05	-	-	-
Fr	Pre	-	-	-	-	-	-	-	-	-	-	0.07
	Rec	-	-	-	-	-0.44	-	-	-	-	-	-
	Post	0.04	0.05	0.03	-	0.06	0.04	-	0.05	-	-	-

Summarized, investigating the day of the week effect for mean returns for each period separately indicates only a Wednesday effect for pre- and post-recession period, which is similar to the results using Approach I. Moreover, the effect seems to change from time to time since during pre-recession Wednesday returns were higher, whereas they were lower during the post-recession compared to the rest of the week. However, this effect does not hold for every investigated sector.

Table 4.12 shows the volatility effects for each day of the week and period. It clearly indicates day of the week effects for the pre- and post-recession, but only minor effects for the recession period.

For the pre-recession period, volatility on Tuesdays and Thursdays are higher for nine and two sectors respectively. In fact, only the sectors Energy and Utilities don’t show a Tuesday effect in volatility. This coincides with what was found in Approach I when Tuesday shows highest volatility for every sector and Monday, Wednesday and Friday had similar volatility for most of the sectors.

The findings for the recession period differ from that. Except the Financial and Health care sectors, there is no day of the week effect in volatility. For the other two, Wednesday seems to be less volatile supporting the hypothesis given by Berument and Kiymaz (2001) stating that the mid of the week gives more time to react to news. These findings are in contrast to the ones using the first approach when Monday volatility was highest.

In terms of the post-recession, the effects differ from sector to sector. Only the sectors Health Care and Industrial don’t show any day of the week effects in volatility. For the other sectors, one sector shows a Tuesday, six sectors show a Wednesday, four sector show a Thursday and one shows a Friday effect. Despite some differences, the overall picture of having different results for every sector is again in line with the findings in Approach I.

Summarizing, all of the sectors show a day of the week effect in volatility. In the pre-recession period, there is mainly a Tuesday effect such that the volatility increases that day of the week. During the recession period, this effect almost diminished and only two sectors show lower volatility on Wednesdays. The results for the post-recession period differ from sector to sector. This, on the same time, indicates that the effects has changed over time.

Table 4.12: Intercept and Slope estimates using Approach III

		SP	CD	CS	En	Fin	HC	Ind	IT	Mat	TS	Ut
Mo	Pre	-0.54	-0.56	-0.85	-0.70	-0.68	-0.43	-0.53	-0.74	-1.34	-0.68	-0.39
	Rec	-0.13	-0.13	-0.27	-0.19	-	-0.25	-0.13	-0.15	-0.18	-	-0.38
	Post	-0.51	-0.41	-0.65	-0.14	-0.36	-0.43	-0.33	-0.70	-0.22	-0.33	-0.26
Tu	Pre	**0.06**	**-0.05**	**-0.20**	-0.70	**-0.26**	**0.02**	**-0.10**	**-0.33**	**0.06**	**-0.11**	-0.39
	Rec	-0.13	-0.13	-0.27	-0.19	-	-0.25	-0.13	-0.15	-0.18	-	-0.38
	Post	**-0.33**	-0.41	-0.65	-0.14	-0.36	-0.43	-0.33	-0.70	-0.22	-0.33	-0.26
We	Pre	-0.54	-0.56	-0.85	-0.70	-0.68	-0.43	-0.53	-0.74	-1.34	-0.68	-0.39
	Rec	-0.13	-0.13	-0.27	-0.19	**-0.37**	**-0.90**	-0.13	-0.15	-0.18	-	-0.38
	Post	-0.51	-0.41	**-0.30**	-0.14	**-0.16**	-0.43	-0.33	**-0.53**	**-0.06**	**-0.73**	**-0.01**
Th	Pre	-0.54	-0.56	-0.85	-0.70	-0.68	**-0.10**	-0.53	-0.74	**-0.97**	-0.68	-0.39
	Rec	-0.13	-0.13	-0.27	-0.19	-	-0.25	-0.13	-0.15	-0.18	-	-0.38
	Post	**-0.26**	**-0.22**	**-0.46**	-0.14	-0.36	-0.43	-0.33	**-0.51**	-0.22	-0.33	-0.26
Fr	Pre	-0.54	-0.56	-0.85	-0.70	-0.68	-0.43	-0.53	-0.74	-1.34	-0.68	-0.39
	Rec	-0.13	-0.13	-0.27	-0.19	-	-0.25	-0.13	-0.15	-0.18	-	**0.07**
	Post	-0.51	-0.41	-0.65	**-0.32**	-0.36	-0.43	-0.33	-0.70	-0.22	-0.33	-0.26
b	Pre	0.96	0.95	0.93	0.92	0.94	0.97	0.95	0.93	0.89	0.94	0.96
	Rec	0.98	0.98	0.97	0.97	0.99	0.95	0.98	0.98	0.98	0.98	0.97
	Post	0.96	0.96	0.95	0.98	0.96	0.95	0.96	0.93	0.98	0.96	0.98

There are two possible reasons for the difference day of the week effect during the recession period. First, when considering sub periods, the sample size could be too small to detect any day of the week effects. Indeed,

Table 4.2 indicates that this period consists of only 397 days such that each day is only represented by about 80 data points. In contrast to that, having less dummy variables and removing one variable after the other lead to more flexible estimates. Therefore, the third approach could be more accurate compared to the first one.

The news impact curves in Figure 4.6 show the resulting effects using Approach III for the S&P 500 as well as the sectors Financial, Consumer Staples and Materials. They show the day of the week in volatility, if any, as well as the volatility for the rest of the week (ROW) for each period. Note that for some of the estimates the condition $\gamma + \theta > 0$ is not fulfilled and therefore the curve is not minimal at $a_{t-1} = 0$. Also note that curves for the same period are parallel since the dummies only influence the intercept terms.

All news impact curves illustrate the sharp increase in volatility during the recession period. Especially the Financial sector shows a huge recession effect. Moreover, for all four represented sectors the volatility after the recession is larger compared to the pre-recession period. This indicates that the uncertainty which came up during the recession period remained for a long time and is still present. Additionally, the news impact curves show the asymmetric behavior of good and bad news. For all displayed days and periods, volatility increase more with the appearance of bad news. Thus, the chosen EGARCH model should be preferred to a simple GARCH model to capture the leverage effect.

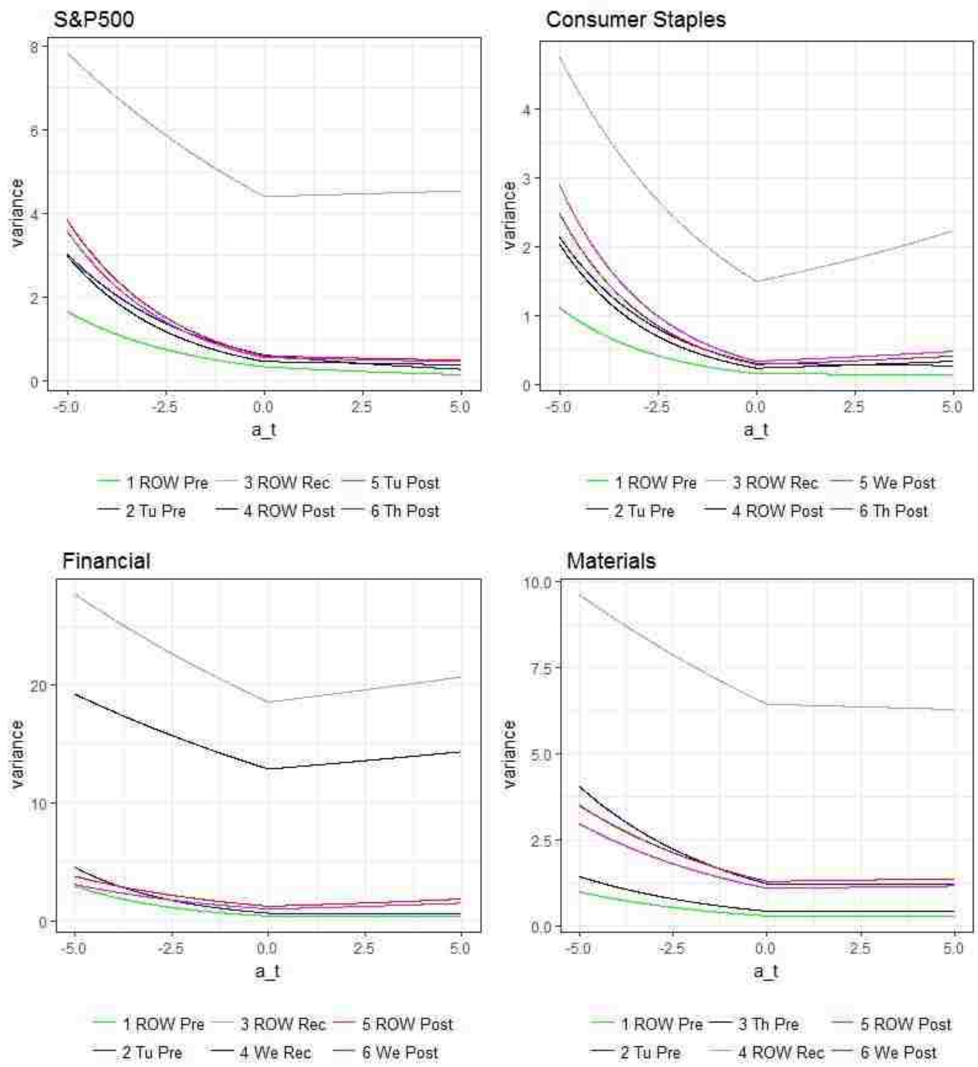

Figure 4.6: News Impact Curves for S&P 500 and sectors Consumer Staples, Financial and Materials

5 CONCLUSION

The goal of this study was to investigate the day of the week effect for returns and volatility. Thereby, the main focus was given to the question if these effects, if present, have changed from pre-recession to recession to post-recession periods. The S&P 500, one of the largest stock market indices in the world, and its ten sectors were investigated. The data consisted of daily returns from February 2005 to January 2018 and was divided into three distinct periods, where the breaking points were December 3rd 2007 and June 30th 2009 according to official dates for the subprime crisis. Since other researchers found out that asymmetric models like an EGARCH model fits the volatility better compared to other GARCH type models, volatility was measured as an EGARCH(1,1) model. Moreover, the mean was modeled to follow an AR(k) model. For the investigation, three different approaches were used. For the first approach, dummy variables for the day of the week and the periods were introduced in both the mean and volatility model. Since this approach only changed the intercept of the regression, the second approach took into consideration that the slope can change due to the dummies. Therefore, the residuals of the mean model were extracted and interaction with the lagged residuals were added. The third approach divided the data into three distinct periods and each one was estimated separately for any day of the week effects.

Investigating the day of the week effect in mean returns show similar results for Approach I and III. Altogether, there is only a small effect for all three periods. In fact, only Wednesdays and Fridays seem to have different returns compared to the rest of the week, where the latter one was only found when using the first approach. Interestingly, Wednesdays had higher returns in the pre-recession period, where this effect changed to lower returns in the post-recession period for the sectors Telecommunication Services and Utilities indicating changes in the effects. However, all in all the day of the week effect in mean returns seems to diminish and to play only a minor role in today's US stock market.

In contrast to that, all three approaches give evidence for a day of the week effect in volatility. For the first and third approach, Tuesdays show highest volatility in the pre-recession period, whereas this varies from sector to sector in the post-recession period. The results for the recession period, however, are not so conclusive. On the one hand, the first and second approach indicates evidence for day of the week effects in volatility. Especially Monday volatility was very high supporting the hypotheses that bad news are mainly released over the weekend. On the other hand, the third approach doesn't show any, with two exceptions, day of the week effects. Since these differences could be due to a small sample size- supporting results from Approach I and II- or because of more flexible estimates- supporting Approach III- further investigations should be done to be able to decide which model is more appropriate and hence which results are more accurate. Moreover, visualizing the results of the third approach using News Impact Curves indicate a sharp increase of volatility during the recession period. Additionally, they also show the asymmetric behavior when volatility increases more with the occurrence of bad news.

In short, this study indicates the presence of a day of the week effect in mean returns- in a moderate way- and volatility. Moreover, the effects seem to change from time to time making it difficult to include when estimating current and future volatility. Nevertheless, it can be useful for both hedging and speculative purposes. Knowing that volatility is higher on certain days allows traders to hedge against the higher uncertainty, where on the same time there is a chance to make profit for risk seeking investors (see Berument and Kiymaz, 2001). However, further research has to be done to answer the questions: Do other markets have similar pattern? What leads to the change of the day of the week effect? Why are the effects different depending on the observed market?

APPENDIX

PARAMETER ESTIMATES FOR THE S&P 500 FOR ALL THREE APPROACHES AND THE CORRSPONDING SAS CODE

Table A.1: Parameter Estimates S&P 500 Approach I

Parameter Estimates					
Variable	**DF**	**Estimate**	**Standard Error**	**t Value**	**Approx Pr > |t|**
Intercept	1	0.000363	0.000134	2.71	0.0067
rt1	1	-0.0676	0.0188	-3.6	0.0003
d5	1	-0.002117	0.000912	-2.32	0.0203
EARCH0	1	-0.9297	0.1023	-9.09	<.0001
EARCH1	1	0.1483	0.0153	9.7	<.0001
EGARCH1	1	0.955	0.004276	223.34	<.0001
THETA	1	-1.1505	0.1344	-8.56	<.0001
HET1	1	1.1494	0.1371	8.39	<.0001
HET2	1	0.2941	0.1869	1.57	0.1157
HET3	1	0.4388	0.1222	3.59	0.0003
HET4	1	0.5061	0.1627	3.11	0.0019
HET5	1	1.2026	0.1626	7.4	<.0001
HET6	1	0.3875	0.0996	3.89	0.0001
HET7	1	-1.8159	0.2724	-6.67	<.0001
HET8	1	-1.2592	0.3259	-3.86	0.0001
HET9	1	-1.147	0.2473	-4.64	<.0001
HET10	1	-1.219	0.2966	-4.11	<.0001
HET11	1	-0.9623	0.1621	-5.94	<.0001
HET12	1	-0.2113	0.2074	-1.02	0.3083
HET13	1	-0.1611	0.1446	-1.11	0.2654
HET14	1	-0.5033	0.1789	-2.81	0.0049

Table A.2: Parameter Estimates Approach II

Variable	**Parameter Estimate**	**Standard Error**	**Type II SS**	**F Value**	**Pr > F**
Intercept	-11.449	0.09696	87215	13943.2	<.0001
d1	0.24145	0.10877	30.8236	4.93	0.0265
d5	3.00576	0.48805	237.253	37.93	<.0001
d6	1.46162	0.2608	196.458	31.41	<.0001
d5a	0.10462	0.04928	28.1865	4.51	0.0338
d6a	0.11875	0.02102	199.598	31.91	<.0001

Table A.3: Parameter Estimates Approach III-Pre Recession Period.
Note: HET1=D1

Parameter Estimates					
Variable	**DF**	**Estimate**	**Standard Error**	**t Value**	**Approx Pr > |t|**
Intercept	1	-0.0002	0.00029	-0.67	0.5042
d2	1	0.00137	0.00067	2.03	0.0419
EARCH0	1	-0.5449	0.1004	-5.43	<.0001
EARCH1	1	0.0608	0.0292	2.08	0.0376
EGARCH1	1	0.957	0.00996	96.06	<.0001
THETA	1	-3.0475	1.5338	-1.99	0.0469
HET1	1	0.6047	0.1441	4.2	<.0001

Table A.4: Parameter Estimates Approach III- Recession Period.

Parameter Estimates					
Variable	**DF**	**Estimate**	**Standard Error**	**t Value**	**Approx Pr > |t|**
Intercept	1	-0.0016	0.00084	-1.96	0.0503
rt1	1	-0.1597	0.0604	-2.65	0.0082
EARCH0	1	-0.1323	0.0624	-2.12	0.034
EARCH1	1	0.1463	0.0501	2.92	0.0035
EGARCH1	1	0.983	0.00798	123.16	<.0001
THETA	1	-0.8991	0.4176	-2.15	0.0313

Table A.5: Parameter Estimates Approach III-Pre Recession Period.
Note: HET1=D1 and HET2 =D3

Parameter Estimates					
Variable	**DF**	**Estimate**	**Standard Error**	**t Value**	**Approx Pr > |t|**
Intercept	1	0.0004	0.00015	2.6	0.0094
rt1	1	-0.0468	0.0218	-2.14	0.032
EARCH0	1	-0.5126	0.0494	-10.37	<.0001
EARCH1	1	0.1564	0.0165	9.48	<.0001
EGARCH1	1	0.9554	0.00455	209.88	<.0001
THETA	1	-1.2256	0.1444	-8.49	<.0001
HET1	1	0.1797	0.0824	2.18	0.0292
HET2	1	0.2499	0.0765	3.27	0.0011

```
/*This SAS Code calculates the day of the week effect on mean returns and volatility.
The returns and the dummy variables were determined in an Excel file*/
data SP;
input d1 d2 d3 d4 d5 d6 rt;
d15=d1*d5; d25=d2*d5; d35=d3*d5; d45=d4*d5;
d16=d1*d6; d26=d2*d6; d36=d3*d6; d46=d4*d6;
rt1=lag1(rt);rt2=lag2(rt);rt3=lag3(rt);rt4=lag4(rt);rt4=lag4(rt);rt5=lag5(rt);
datalines;
datalines;
/*...*/

/*---------Initial Step--------------*/
/*Check for autoregressive order k*/
/*The following model was calculated for orders from 0 to 5. Therfore, this model was
estimated six times, where each time the last variable weas removed.*/
proc autoreg data=SP;
model rt= rt1 rt2 rt3 rt4 rt5 / garch=(p=1,q=1,type=exp) method=ml;
run;


/*---------Approach I --------------*/
/*The following model was reduced stepwise. While holding the hetero part fixed, the model part
was reduced by removing the least significant variable until all estimates were significant at
0.05*/

proc autoreg data=SP;
model rt= rt1 d1 d2 d3 d4 d5 d6 d15 d25 d35 d45 d16 d26 d36 d46 / garch=(p=1,q=1,type=exp)
method=ml;
hetero d1 d2 d3 d4 d5 d6 d15 d25 d35 d45 d16 d26 d36 d46 /coef=unrest;
run;


/*---------Approach II --------------*/
/*Calculate Residuals. Note that in Approach I, the only significant variable was D5.
Therefore, the mean model when calculating residuals only includes D5.*/
proc autoreg data=SP;
model rt= rt1 d5 / garch=(p=1,q=1,type=exp) method=ml;
hetero /coef=unrest;
output out = residuals residual=res;
run;

/*Calculate Interactions*/
data residuals;
        set residuals;
        a_t=log(res*res);
        a_t1=log(lag(res)*lag(res));
        d1a=d1*a_t1;
        d2a=d2*a_t1;
        d3a=d3*a_t1;
        d4a=d4*a_t1;
        d5a=d5*a_t1;
```

```
        d6a=d6*a_t1;
        d15a=d15*a_t1;
        d25a=d25*a_t1;
        d35a=d35*a_t1;
        d45a=d45*a_t1;
        d16a=d16*a_t1;
        d26a=d26*a_t1;
        d36a=d36*a_t1;
        d46a=d46*a_t1;
run;

/*Calculate Volatility Model*/
proc reg data=residuals plots=diagnostics(stats=(default aic));
model a_t = d1 d2 d3 d4 d5 d6 d15 d25 d35 d45 d16 d26 d36 d46 a_t1 d1a d2a d3a d4a d5a d6a
d15a d25a d35a d45a d16a d26a d36a d46a /selection=backward sls=0.05;
run;


/*---------Approach III --------------*/
/*pre-recession*/
data pre_rec;
set SP;
if (d5=1) OR (d6=1) then delete;
run;

proc autoreg data=pre_rec;
model rt= d2 / garch=(p=1,q=1,type=exp) method=ml;
hetero d1 /coef=unrest;
run;

/*recession*/
data rec;
set SP;
if d5=1 ;
run;

proc autoreg data=rec;
model rt= rt1 / garch=(p=1,q=1,type=exp) method=ml;
hetero /coef=unrest;
run;

/*post-recession*/
data post_rec;
set SP;
if d6=1;
run;

proc autoreg data=post_rec;
model rt= rt1 / garch=(p=1,q=1,type=exp) method=ml;
hetero d1 d3 /coef=unrest;
run;
```

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