

2015

Anisotropic Spacetimes and Black Hole Interiors in Loop Quantum Gravity

Anton Joe

Louisiana State University and Agricultural and Mechanical College

Follow this and additional works at: https://digitalcommons.lsu.edu/gradschool_dissertations



Part of the [Physical Sciences and Mathematics Commons](#)

Recommended Citation

Joe, Anton, "Anisotropic Spacetimes and Black Hole Interiors in Loop Quantum Gravity" (2015). *LSU Doctoral Dissertations*. 3485.
https://digitalcommons.lsu.edu/gradschool_dissertations/3485

This Dissertation is brought to you for free and open access by the Graduate School at LSU Digital Commons. It has been accepted for inclusion in LSU Doctoral Dissertations by an authorized graduate school editor of LSU Digital Commons. For more information, please contact gradetd@lsu.edu.

ANISOTROPIC SPACETIMES AND BLACK HOLE INTERIORS IN LOOP QUANTUM GRAVITY

A Dissertation

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

in

The Department of Physics and Astronomy

by

Anton Joe

M.S., Indian Institute of Technology, Kharagpur, 2012

December 2015

Acknowledgements

First of all, I would like to thank my parents V A Vincent and C O Mary for their unconditional love and encouragement throughout my journey. Without their support, it would not have been possible to make my dreams come true. I thank my sister Elna Merin, brother Britto Joseph and my brother-in-law Ullas T S, for always believing in me, loving me and motivating me in many ways to succeed in all my endeavors.

I would like to whole heartedly thank my advisor Dr. Parampreet Singh, for spending his time, providing guidance and expertise during all stages of my research. I am highly indebted to him for identifying my strengths, molding me to be a better researcher, taking good care and showing immense love and support throughout my journey at LSU. I thoroughly enjoyed the time we spent on numerous academic and other discussions, playing tennis, shopping, travelling and many lunch together. This work would not have been complete without his support.¹

I have greatly benefitted through discussions with many members in the Department of Physics & Astronomy at LSU. First, I thank Dr. Ivan Agullo for spending his time to teach me and Noah Morris aspects of differential geometry. I greatly benefitted from various stimulating discussions with Dr. Jorge Pullin. I extend my thanks to Dr. Miguel Megevand for his collaboration and sharing his expertise in computation and programming. Many thanks to Dr. Peter Diener for his interesting discussions at many occasions. My special thanks to Dr. Javier Olmedo, Brajesh Gupt and Noah Morris for their support and great company in quantum gravity group. Finally, I am grateful to Dr. Ken Schafer for valuable advice during my Ph. D.

I would like to extend my special gratitude to Dr. Naresh Dadhich from India for giving me an opportunity to collaborate with him, discussing very important aspects in gravitational physics which inspired me and greatly benefitted my research work.

My endless thanks to Dr. Gaurav Khanna from University of Massachusetts, Dartmouth, for giving me summer internship in 2011 and guiding me throughout which gave me confidence to pursue PhD in US in the field of LQC.

¹This dissertation was compiled by Dr. Singh, with help from Nikhil Damodaran and Padmapriya, from the body of work carried out during Anton Joe's research as a Ph. D student after his demise.

I wish to thank my Graduate Advisor in IIT, Kharagpur, Dr. Sayan Kar for his guidance and support during my MSc program. My special thanks to Dr. S P Khastgir, Dr. Arghya Taraphder, Dr. Krishna Kumar and Dr. Somnath Bharadwaj from IIT, Kharagpur, for their help at various stages. I would like to thank my high school teacher Mrs. Sunitha and IIT entrance exam coaching teacher Mr. P.C Thomas for believing in me and pushing me to reach my goals.

My hearty thanks to my friend Rahul from UMass, Dartmouth, with whom I enjoyed discussing various topics in physics. I wish to thank my close friends from India, Pankaj, Kingshuk, Sailesh and Swetha Boddu for always being there for me. I would also like to thank all my batch mates in IIT, Kharagpur for helping me in different stages and giving me wonderful company throughout my graduate studies. I thank my friends in LSU, Aswin, Nikhil, Hari and Padmapriya for making me feel Home away from Home. Finally, I would like to thank each and every one who made my journey in LSU so enjoyable.

Table of Contents

| | |
|--|------|
| Acknowledgements | ii |
| List of Tables | vi |
| List of Figures | vii |
| Abstract | viii |
| Chapter 1 Introduction | 1 |
| Chapter 2 Generic bounds on expansion and shear scalars in Kantowski-Sachs spacetime | 11 |
| 2.1 Classical Hamiltonian of Kantowski-Sachs space-time | 11 |
| 2.2 Comparison of different quantization prescriptions | 14 |
| 2.2.1 Constant δ prescription | 17 |
| 2.2.2 An ‘improved dynamics inspired’ prescription | 19 |
| 2.2.3 ‘Improved Dynamics’ prescription | 20 |
| 2.3 Uniqueness of $\bar{\mu}$ prescription | 22 |
| 2.4 Energy density in the ‘improved dynamics’ | 25 |
| 2.5 Discussion | 27 |
| Chapter 3 Emergence of ‘charged’ Nariai and anti-Bertotti-Robinson spacetimes in LQC | 30 |
| 3.1 Higher genus black hole interior: classical aspects | 31 |
| 3.2 Effective loop quantum dynamics | 33 |
| 3.2.1 Kantowski-Sachs spacetime with a positive cosmological constant | 36 |
| 3.2.2 Kantowski-Sachs spacetime with a negative cosmological constant | 38 |
| 3.2.3 Bianchi-III LRS spacetime with a negative cosmological constant | 39 |
| 3.3 Properties of the asymptotic spacetime with a constant p_c | 42 |
| 3.4 Emergent ‘charge’ and cosmological constant in loop quantum cosmology | 45 |
| 3.5 ‘Uncharged’ Nariai spacetime and $\tilde{\Lambda} = 0$ anti-Bertotti-Robinson spacetimes | 49 |

| | |
|--|----|
| 3.6 Discussion | 51 |
| Bibliography | 54 |
| Appendix A: Copyright Permissions | 59 |
| Appendix B: Letter explaining primary authorship | 61 |
| Vita | 62 |

List of Tables

| | | |
|-----|---|----|
| 3.1 | Some features of (anti) Nariai and (anti) Bertotti-Robinson spacetimes. | 45 |
|-----|---|----|

List of Figures

| | | |
|-----|--|----|
| 2.1 | Evolution of p_c for the massless scalar field evolution | 26 |
| 3.1 | Triads for Kantowski-Sachs spacetime with positive cosmological constant. | 37 |
| 3.2 | Behavior of $\cos(c\delta_c)$ is shown in the asymptotic regime where p_c is a constant. | 38 |
| 3.3 | Triads in Kantowski-Sachs model sourced with a negative cosmological constant. | 40 |
| 3.4 | Triads for negative cosmological constant in higher genus black hole spacetime. | 41 |
| 3.5 | Triads for the higher genus black hole interior is shown in the asymptotic regime. | 41 |

Abstract

This thesis deals with understanding quantum gravitational effects in those anisotropic spacetimes which serve as black hole interiors. Two types of spacetime are investigated. Kantowski-Sachs spacetime and Bianchi-III LRS spacetime. The former, in vacuum, is the interior spacetime for Schwarzschild black holes. The latter is the interior for higher genus black holes. These spacetimes are studied in the context of loop quantum cosmology. Using effective dynamics of loop quantum cosmology, the behavior of expansion and shear scalars in different proposed quantizations of the Kantowski-Sachs spacetime with matter is investigated. It is found that out of the various proposed choices, there is only one known prescription which leads to the generic bounded behavior of these scalars. The bounds turn out to be universal and are determined by the underlying quantum geometry. This quantization is analogous to the so called ‘improved dynamics’ in the isotropic loop quantum cosmology, which is also the only one to respect the freedom of the rescaling of the fiducial cell at the level of effective spacetime description. Other proposed quantization prescriptions yield expansion and shear scalars which may not be bounded for certain initial conditions within the validity of effective spacetime description. These prescriptions also have a limitation that the “quantum geometric effects” can occur at an arbitrary scale. We show that the ‘improved dynamics’ of Kantowski-Sachs spacetime turns out to be a unique choice in a general class of possible quantization prescriptions, in the sense of leading to generic bounds on expansion and shear scalars and the associated physics being free from fiducial cell dependence. The behavior of the energy density in the ‘improved dynamics’ reveals some interesting features. Even without considering any details of the dynamical evolution, it is possible to rule out pancake singularities in this spacetime. The energy density is found to be dynamically bounded. These results show that the Planck scale physics of the loop quantized Kantowski-Sachs spacetime has key features common with the loop quantization of isotropic and Bianchi-I spacetimes.

The loop quantum dynamics of Kantowski-Sachs spacetime and the interior of higher genus black hole spacetimes with a cosmological constant has some peculiar features not shared by various other spacetimes in loop quantum cosmology. As in the other cases, though the quantum geometric effects resolve the physical singularity and result in a non-singular bounce, after the bounce a spacetime with small spacetime curvature does not emerge in either the subsequent backward or the forward evolution. Rather, in the asymptotic limit the spacetime manifold is a product of two constant curvature spaces.

Interestingly, though the spacetime curvature of these asymptotic spacetimes is very high, their effective metric is a solution to the Einstein's field equations. Analysis of the components of the Ricci tensor shows that after the singularity resolution, the Kantowski-Sachs spacetime leads to an effective metric which can be interpreted as of the 'charged' Nariai spacetime, while the higher genus black hole interior can similarly be interpreted as anti Bertotti-Robinson spacetime with a cosmological constant. These spacetimes are 'charged' in the sense that the energy momentum tensor that satisfies the Einstein's field equations is formally the same as the one for the uniform electromagnetic field, albeit it has a purely quantum geometric origin. The asymptotic spacetimes also have an emergent cosmological constant which is different in magnitude, and sometimes even its sign, from the cosmological constant in the Kantowski-Sachs and the interior of higher genus black hole metrics. With a fine tuning of the latter cosmological constant, we show that 'uncharged' Nariai, and anti Bertotti-Robinson spacetimes with a vanishing emergent cosmological constant can also be obtained.

Chapter 1

Introduction

Einstein's century old general theory of relativity has been extremely successful in explaining our cosmos and its dynamics. We now know that our universe is expanding, light bends in gravitational field, rotating binary neutron stars loose energy due to gravitational waves, there is a black hole at the center of our galaxy and so on. In most situations thrown out by the cosmos, general relativity is a perfectly adequate theory to explain it, but the theory has one major drawback. It is not compatible with the other main pillar of modern Physics, quantum mechanics. Hence, in phenomena where quantum effects are important along with gravity, physicists hit a road block. This inability to attain a harmony between general relativity and quantum mechanics is one of the deepest conceptual problems in present day physics. The very first solution found for Einstein's equations of general relativity corresponds to spherical non-rotating black hole (Schwarzschild black hole). This solution was singular (a point where the equations break down) at the center of spherical symmetry. Such singularities - where spacetime comes to an abrupt halt was seen in other solutions of Einstein's equations as well. Arguably the most famous of such singularities is the putative big bang singularity at the 'beginning' of our universe. The singularity theorems proved by Geroch, Hawking and Penrose showed that singularities arise naturally in Einstein's theory. On the observational front, there are evidences for existence of black holes (rotating ones, though) and for expansion of the universe (which when traced back leads to a singular point). Hence there is a pressing need to understand black holes, big bang and other such singularities that appear in general relativity. This issue of understanding singularities in general relativity is related to one of the most important conceptual problems in modern day Physics - the dissonance between the quantum theory and general relativity. Quantum field theory - the theory of subatomic particles and their interactions with each other has not yet found a way to incorporate gravity. Similarly, general relativity that governs the large scale evolution of cosmos does not confirm to laws of

quantum mechanics. It is clear that to comprehend this universe better, it is absolutely essential to have a theory which accounts for both gravity as well as quantum mechanics. Our research aims to contribute towards the growing body of work trying to achieve this goal. Specifically, our research revolves around the two puzzles of interior of the black hole and the quantum ‘beginning’ of our universe. It has been long thought that a quantum theory of gravity will provide important insights on these problems.

Loop quantum gravity (LQG) [1] is one of the leading candidates for a theory of quantum gravity. It is a non-perturbative approach to quantizing gravity that maintains the background independence of general relativity. The name arises from the usage of holonomies around loops as basic variables. LQG has already produced a lot of impressive results such as the existence of a minimum area gap and calculation of black hole entropy. Though a full theory of quantum gravity is not yet available, insights on the problem of classical singularities have been gained for various spacetimes in loop quantum cosmology (LQC) in recent years [2]. LQC is a quantization of symmetry reduced spacetimes using techniques of loop quantum gravity (LQG) which is a non-perturbative canonical quantization of gravity based on the Ashtekar variables: the $SU(2)$ connections and the conjugate triads. The elementary variables for the quantization are the holonomies of the connection components, and the fluxes of the triads. The classical Hamiltonian constraint, the only non-trivial constraint left after symmetry reduction in the minisuperspace setting, is expressed in terms of holonomies and fluxes and is quantized. Quantization of various isotropic models in LQC demonstrates the resolution of classical singularities when the spacetime curvature reaches Planck scale. The big bang and big crunch are replaced by a quantum bounce, which first found in the case of the spatially flat isotropic model [3, 4, 5] is tied to the underlying quantum geometry and has been shown to be a robust phenomena through different analytical [6] and numerical investigations [7, 8, 9]. This was a huge improvement over the previously popular Wheeler-de Witt (WDW) approach to quantum cosmology which could not achieve the resolution of singularities in cosmology. One of the key differences of LQC when compared to WDW theory is that the basic configuration variables are holonomies. The existence of a minimum area in LQG implies that the loops around which holonomies are constructed cannot be made to shrink arbitrarily. The corrections to dynamics due to these holonomies make gravity repulsive at scales comparable to the

Planck scale. Thus when evolving the FRW spacetime backwards, before reaching the putative singularity, the quantum corrections stop the contraction and the make time to expand towards further past. Thus instead of the big bang of classical (or WDW) cosmology, the universe undergoes a big bounce. A generalization of these results has been performed for Bianchi models [10, 11, 12, 13, 14, 15, 16, 17], where the quantum Hamiltonian constraint also turns out to be non-singular.

The LQC equations governing the evolution of the universe are typically difference equations that are not too conducive for extracting physics analytically. The intractability of the equations become even more pronounced in anisotropic or inhomogeneous settings. However, under simplifying assumption that the bounce occurs at a high volume (compared to the Planck volume), and that the wavefunction of the universe is highly peaked, one can approximate the difference equations of LQC with a set of differential equations. For sharply peaked states which lead to a macroscopic universe at late times, it is possible to derive an effective spacetime description [18, 19, 20]. Additionally, instead of calculating the expectation values of observables as in LQC, one can apply techniques of classical mechanics to an effective Hamiltonian that incorporates quantum corrections. Due to its tractability and the remarkable agreement with LQC, the effective theory has been widely used in literature. For example, the effective theory was used in calculating the effect of LQC in pre-inflationary dynamics of FRW spacetimes and to prove that strong singularities (points in spacetime beyond which geodesics cannot be extended) do not occur in flat isotropic model. Due to recent progress in numerical techniques in LQC achieved here in LSU, it was possible to test the effective theory for FRW spacetimes for very general states[21, 8]. Introduction of high performance computing techniques to loop quantum cosmology has facilitated the comparison of evolution of widespread wave functions in LQC with that of predictions of effective theory. Effective dynamics has been extremely useful in not only extracting physical predictions, but also to gain insights on the viability of various possible quantizations. In particular it has been shown that for isotropic models there is a unique way of quantization, the so called ‘improved dynamics’ or the $\bar{\mu}$ quantization [5], which results in a consistent ultra-violet and infra-red behavior and is free from the rescalings of the fiducial cell introduced to obtain finite integrations on the non-compact spatial manifold [22, 23]. Note that the fiducial cell which acts like an infra-red regulator is an arbitrary choice

in the quantization procedure. Hence a consistent quantization prescription must yield physical predictions about observables such as expansion and shear scalars independent of the choice of this cell if the spatial topology is non-compact.

The improved dynamics quantization of the isotropic LQC results in a generic bound on the expansion scalar of the geodesics in the effective spacetime and leads to a resolution of all possible strong singularities in the spatially flat model [24, 25]. These results have also been extended to Bianchi models, where $\bar{\mu}$ quantization results in generic bounds on expansion and shear scalars [23, 27, 28, 17], and the resolution of strong singularities in Bianchi-I spacetime [27]. There are other possible ways to quantize isotropic and anisotropic models, such as the earlier quantization of isotropic models in LQC – the μ_o quantization [29, 4] and the lattice refined models [30]. In these quantization prescriptions,¹ quantum gravitational effects can occur at arbitrarily small curvature scales and the expansion and shear scalars are not bounded in general [22, 23].

In the context of the black holes, effective Hamiltonian techniques in LQC can again be employed to gain insights on the Planck scale physics in the interior spacetime. In particular, the Schwarzschild black hole interior corresponds to the vacuum Kantowski-Sachs spacetime. Similarly, Schwarzschild de Sitter and Schwarzschild anti-de Sitter black hole interiors can also be studied in minisuperspace setting using Kantowski-Sachs cosmology with a positive and a negative cosmological constant respectively. Additionally the Bianchi III LRS spacetime which is analogous to Kantowski-Sachs spacetime but with a negative spatial curvature, turns out to be corresponding to the higher genus black hole interior. Using symmetries of these spacetimes, the connection and triad variables simplify and a rigorous loop quantization can be performed which results in a quantum difference equation, and an effective spacetime description. Loop quantization of Kantowski-Sachs spacetimes has been mostly studied for the vacuum case [32, 33, 34, 35, 36, 37, 38], where the quantum Hamiltonian constraint has been found to be non-singular. Ashtekar and Bojowald proposed a quantization of the interior of the

¹Our usage of term “quantization prescriptions” in loop quantization here is different from an earlier work in isotropic LQC [31]. Here different quantum prescriptions refer to the way the area of the loops over which holonomies in the quantum theory are constructed are constrained with respect to the minimum area gap. Whereas in Ref. [31], different quantum prescriptions were used to distinguish the quantum Hamiltonian constraints in the $\bar{\mu}$ quantization of isotropic LQC.

Schwarzschild interior and concluded that the wavefunction of universe can be evolved across the classical central singularity pointing towards singularity resolution [32]. Spherically symmetric spacetimes have been studied in the midisuperspace setting by Campiglia, Gambini, Pullin [37, 35, 36], to quantize Schwarzschild black hole [38] and calculate the Hawking radiation [39]. Though these works provide important insights on the quantization of black holes in LQG, it is to be noted that the quantization prescription used in these works is analogous to the earlier works in isotropic LQC (the μ_o quantization) which was found to yield inconsistent physics. In particular, the loop quantization in these models is carried out such that the loops over which holonomies are considered have edge lengths (labeled by δ_b and δ_c) as constant. As in the case of the μ_o quantization in LQC, the constant δ quantization of Schwarzschild interior has been shown to be dependent on the rescalings of the fiducial length L_o in the x direction of the $\mathbb{R} \times \mathbb{S}^2$ spatial manifold [40, 41, 42]. To overcome these problems, Boehmer and Vandersloot proposed a quantization prescription motivated by the improved dynamics in LQC [40], which we label as $\bar{\mu}$ quantization in Kantowski-Sachs model. In this prescription, δ_b and δ_c depend on triad components in such a way that the effective Hamiltonian constraint respects the freedom in rescaling of length L_o . This prescription has been used to understand the phenomenology of the Schwarzschild interior [43] and has been recently used to loop quantize spherically symmetric spacetimes [42]. It is to be noted that this prescription leads to “quantum gravitational effects” not only in the neighborhood of the physical singularity at the origin, but also at the coordinate singularity at the horizon, which points to the limitation of dealing with Schwarzschild interior in this setting. This problem has been noted earlier, see for eg. Ref. [43] where the problem with the fiducial cell at the horizon in this prescription is noted. However, note that such an issue does not arise in the presence of matter which is the focus of the present manuscript.

In literature, another quantization prescription inspired by the improved dynamics, which we label as the $\bar{\mu}'$ prescription² has been proposed. In this prescription though edge lengths δ_b and δ_c are functions of the triads, problems with fiducial length rescalings persist [41]. These prescriptions have also been analyzed for the von-Neumann stability of the quantum Hamiltonian

²Our labeling of the $\bar{\mu}$ and $\bar{\mu}'$ prescriptions in Kantowski-Sachs spacetime is opposite to that of Ref. [41]. This difference is important to realize to avoid any confusions about the physical implications or the limitations of these prescriptions while relating this work with Ref. [41].

constraints which turn out to be difference equations [30]. It was found that $\bar{\mu}'$ quantization, in contrast to the $\bar{\mu}$ quantization, does not yield a stable evolution. These studies indicate that if we consider fiducial length rescaling issues, $\bar{\mu}$ quantization in the Kantowski-Sachs spacetime is preferred over the constant δ quantization [32] and the $\bar{\mu}'$ quantization prescription [41]. However one may argue that these issues which arise for the non-compact spatial manifold, can be avoided if the topology of the spatial manifold is compact ($\mathbb{S}^1 \times \mathbb{S}^2$).

Our first goal, which is studied in Chapter 2, deals with the following issue. For all the models studied so far, it has been found that all three prescriptions lead to singularity resolution. Still, little is known about the conditions under which singularity resolution occurs for the arbitrary matter. Hence, various pertinent questions remain unanswered. In particular, which of these quantization prescriptions promises to generically resolve all the strong singularities³ within the validity of the effective spacetime description in LQC? Is it possible that in any of these quantization prescriptions, expansion and shear scalars may not be generically bounded in effective dynamics which disfavor them over others? Are there any other consistent quantization prescriptions for the Kantowski-Sachs model, or is the $\bar{\mu}$ quantization prescription unique as in the isotropic LQC? Finally, what is the fate of energy density if expansion and shear scalar are generically bounded? Note that in the isotropic LQC, and the Bianchi-I model similar questions were raised in Refs. [22, 24, 27], and the answers led to $\bar{\mu}$ quantization as the preferred choice. It turned out to be a unique quantization prescription leading to generic bounds on expansion and shear scalars, which were instrumental in proving the resolution of all strong singularities in the effective spacetime [24, 27].

We answer these questions in the effective spacetime description in LQC for Kantowski-Sachs spacetime with minimally coupled matter. The expansion and shear scalars are tied to the geodesic completeness of the spacetime and are independent of the fiducial length at the classical level. We will be interested in finding the quantization prescription which promises to resolve all possible classical singularities generically. Such a quantization prescription is expected to yield bounded behavior of these scalars. It is also reasonable to expect, due to the underlying Planck scale quantum geometry, that in the bounce regime, depending on the approach to the classical singularity, at least one of the scalars takes Planckian value. We find that in the effective

³For a discussion of the strength of the singularities in LQC, see Ref. [24].

dynamics for constant δ and $\bar{\mu}'$ prescriptions, these scalars are not necessarily bounded above. In the cases where the classical singularities are resolved, it is possible that the expansion and shear scalars in these prescriptions can take arbitrary values in the bounce regime. In contrast, for the $\bar{\mu}$ quantization prescription, we show that the expansion and shear scalars turn out to be generically bounded by universal values in the Planck regime. It is to be noted that in the $\bar{\mu}$ prescription, the bounded behavior of the expansion scalar has been mentioned earlier for the Schwarzschild interior [44].

We find that the behavior of expansion and shear scalars in the $\bar{\mu}$ prescription is similar to the improved dynamics of isotropic and Bianchi-I spacetime in LQC where the universal bounds on expansion and shear scalars were found. Next, we address the important question of the uniqueness of the $\bar{\mu}$ prescription. For this we consider a general ansatz to consider edge lengths δ_b and δ_c as functions of triads, allowing a large class of loop quantization prescriptions in the Kantowski-Sachs spacetime. We find that demanding that the expansion and shear scalars be bounded leads to a unique choice – the $\bar{\mu}$ quantization prescription. In this quantization prescription we also investigate the behavior of the energy density and find that its potential divergence is determined only by the vanishing $g_{\Omega\Omega}$ component of the spacetime metric. This is unlike the behavior in the classical GR, and other quantization prescriptions where divergence in energy density can occur when either of g_{xx} or $g_{\Omega\Omega}$ components vanish. An immediate consequence of this behavior is that the pancake singularities which occur when g_{xx} component of the line element approaches zero, and $g_{\Omega\Omega}$ is finite, are forbidden. It turns out that energy density is bounded dynamically, since $g_{\Omega\Omega}$ never becomes zero and approaches an asymptotic value. This property of $g_{\Omega\Omega}$ was first seen in the case of vacuum Kantowski-Sachs spacetime, and turns out to be true for all perfect fluids [45]. These results show that the $\bar{\mu}$ quantization in the Kantowski-Sachs spacetime is strikingly similar to the $\bar{\mu}$ quantization in the isotropic and Bianchi-I spacetimes. It leads to generic bounds on the expansion and shear scalars and is independent of the rescalings of the fiducial cell.

Our first main result is that the analysis of the expansion and shear scalars for the loop quantized Kantowski-Sachs spacetime reveals that there exists a unique quantization prescription which leads to their universally bounded behavior [46]. In Chapter 3, similar conclusions hold for the higher genus black hole interiors. Using the corresponding effective Hamiltonian

approach for this quantization prescription, singularity avoidance via a quantum bounce due to underlying loop quantum geometric effects in black hole interior spacetimes has been found [40, 41, 47]. These studies noted that the emergent spacetime is “Nariai type” [40, 47]. Further, these “Nariai type” spacetimes were found to be stable under homogeneous perturbations in the case of vacuum [48]. However, the detailed nature of these spacetimes and their relation if any with the known spacetimes in the classical theory was not found. An examination of these spacetimes, which is a goal of Chapter 3, reveals many novel interesting features which so far remain undiscovered in LQC.

The spatial manifold of Kantowski-Sachs spacetime has an $\mathbb{R} \times \mathbb{S}^2$ topology whereas the higher genus black hole interior has the spatial topology of $\mathbb{R} \times \mathbb{H}^2$. Numerically solving the loop quantum dynamics one finds that on one side of the temporal evolution, in the asymptotic limit, the spacetime emergent after the bounce has the same spatial topology, but has a constant radius for the \mathbb{S}^2 (\mathbb{H}^2 in the case of higher genus black hole) part and an exponentially increasing \mathbb{R} part. We thus obtain a spacetime which is a product of two constant curvature spaces. Interestingly, though the emergent spacetime has a high spacetime curvature, yet it turns out to be a solution of the Einstein’s field equations. In the analysis of these spacetimes, the sign of the Ricci tensor components provide important insights. Here we recall that in the classical GR, properties of the sign of the Ricci tensor components have been used to establish dualities between (anti) Nariai and (anti) Bertotti Robinson spacetimes [49]. Analysis of the components of the Ricci tensor reveals that the emergent spacetime in the evolution of Kantowski-Sachs spacetime with positive or negative cosmological constant is a ‘charged’ Nariai spacetime, where as the emergent spacetime in the evolution of higher genus black hole interior with a negative cosmological constant is actually an anti-Bertotti-Robinson spacetime with a cosmological constant [50]. These emergent spacetimes are ‘charged’ in the sense that they are solutions of the classical Einstein’s field equations with a stress energy tensor which formally corresponds to the uniform electromagnetic field. In addition, these spacetimes in the same asymptotic limit after the bounce also have an emergent cosmological constant, different from the one initially chosen to study the dynamics of black hole interiors. We find that the asymptotic emergence of ‘charge’ and cosmological constant that develop after the bounce is purely quantum geometric in origin. The ‘charged’ Nariai and anti-Bertotti-Robinson spacetimes occur in only one side of the tempo-

ral evolution in the Kantowski-Sachs spacetime with positive and negative cosmological constant and in higher genus black hole interior spacetimes with a negative cosmological constant respectively. The higher genus black hole interior with a positive cosmological constant does not yield any of these spacetimes in the asymptotic limit. The emergence of ‘charged’ Nariai and anti-Bertotti-Robinson spacetimes present for the first time examples of time asymmetric evolution in LQC, and indicate the same for the black hole interiors in the loop quantization. However, note that the uncharged Nariai spacetime which is a non-singular spacetime classically [51], can be considered as the maximal Schwarzschild-de Sitter black hole where the cosmological horizon and the black hole horizon of a Schwarzschild-de Sitter black hole coincide [52]. Thus the emergent spacetimes in the above cases in LQC are closely related to the original spacetimes - but are rather special as they are nonsingular and are parameterized by an emergent ‘charge’ and an emergent cosmological constant. Our analysis shows that with a fine tuning of the value of the cosmological constant in the Kantowski-Sachs spacetime, it is possible to obtain ‘uncharged’ Nariai spacetime. However, such a spacetime turns out to be unstable [45]. Similarly, for the higher genus black hole interior, an anti-Bertotti-Robinson spacetime with a vanishing emergent cosmological constant can arise, but it too is unstable.

All the above interpretations of emergent spacetime after the bounce is based on the fact that it is a product of two spaces having constant curvature $R_0^0 = R_1^1 = k_1$ and $R_2^2 = R_3^3 = k_2$, which could be written as $k_1 = \lambda + \alpha_1, k_2 = \lambda + \alpha_2$. Now if we set $\alpha_1 = -\alpha_2 < 0$, it is charged Nariai while for $\alpha_1 = -\alpha_2 > 0$, it is anti-Bertotti-Robinson with a cosmological constant. Note that anti-Bertotti-Robinson spacetime has electric energy density negative. The moot point is simply that what emerges after bounce is a product of two constant curvature spaces which by proper splitting of these constants lead to a nice interpretation as a mixture of Nariai and Bertotti-Robinson spacetimes which are exact solutions of classical Einstein equation. It is remarkable that emergent spacetime is solution of classical equation albeit with a proper choice of constants. This may be an innate characteristic of quantum dynamics of this type of spacetimes and is perhaps reflection of discreteness in spacetime structure.

In summary, our studies of Kantowski-Sachs spacetimes and higher genus black hole interiors in LQC reveals many so far unexplored features of quantum geometry. We show that there is a unique quantization prescription which results in a bounded behavior of expansion and

shear scalars. Thus, limiting many other potential loop quantizations of the Kantowski-Sachs spacetime. It is rather surprising that this quantization results in a highly asymmetric evolution across the bounce. The spacetime after the singularity resolution retains high quantum curvature and can be interpreted as a classical spacetime with an effective ‘charge.’ It is for the first time in literature, one finds such a phenomena resulting from the underlying quantum geometry. In the future research, it will worthwhile to understand this effective charge in more detail. It is tempting to relate this result with ideas of geometrodynamics where many properties matter are envisioned to result from the underlying features of geometry [53]. At this stage, however, this is only a speculation.

Finally, it is important to state that our results are in the caveat of assumption of homogeneity and the validity of effective dynamics. It can be hoped that our results do capture some element of truth of the full quantum gravitational dynamics of these spacetimes, and open a new window to explore the quantum geometric effects in black hole interiors and spacetime beyond the would be central singularities.

Chapter 2

Generic Bounds on Expansion and Shear Scalars in Kantowski-Sachs Spacetime¹

In this Chapter, based on Ref. [46], we study the way loop quantization prescriptions can be restricted by demanding that the expansion and shear scalars have a bounded behavior. This Chapter is organized as follows. In the next section we summarize the Kantowski-Sachs spacetime in terms of Ashtekar variables and obtain the classical equations. In the next section, we introduce the effective Hamiltonian constraint, and derive expressions for expansion and shear scalars for three quantization prescriptions. We discuss the boundedness of these scalars and for completeness also discuss their dependence on fiducial cell. Then we consider a general ansatz and investigate the conditions under which a quantization prescription yields bounded behavior of expansion and shear scalars. This leads us to the uniqueness of the $\bar{\mu}$ quantization prescription. Then the behavior of energy density is discussed, which is followed by a summary of the main results.

2.1 Classical Hamiltonian of Kantowski-Sachs space-time

We consider the Kantowski-Sachs spacetime with a spatial topology of $\mathbb{R} \times \mathbb{S}^2$. Utilizing the symmetries associated with each spatial slice, the symmetry group $\mathbb{R} \times SO(3)$, and after imposing the Gauss constraint, the Ashtekar-Barbero connection and the conjugate (densitized) triad can be expressed in the following form [32]:

$$A_a^i \tau_i dx^a = \tilde{c} \tau_3 dx + \tilde{b} \tau_2 d\theta - \tilde{b} \tau_1 \sin \theta d\phi + \tau_3 \cos \theta d\phi, \quad (2.1)$$

$$\tilde{E}_i^a \tau_i \partial_a = \tilde{p}_c \tau_3 \sin \theta \partial_x + \tilde{p}_b \tau_2 \sin \theta \partial_\theta - \tilde{p}_b \tau_1 \partial_\phi, \quad (2.2)$$

¹Sections 2.1 - 2.5 are reproduced from A. Joe and P. Singh, *Class. Quant. Grav.* 32, 015009 (2015) (Copyright 2015 Institute of Physics Publishing Ltd) [46] by the permission of the Institute of Physics Publishing. See Appendix A for the copyright permission from the publishers.

where $\tau_i = -i\sigma_i/2$, and σ_i are the Pauli spin matrices. The symmetry reduced triad variables are related to the metric components of the line element,²

$$ds^2 = -N(t)^2 dt^2 + g_{xx} dx^2 + g_{\Omega\Omega} (d\theta^2 + \sin^2 \theta d\phi^2). \quad (2.3)$$

as

$$g_{xx} = \frac{\tilde{p}_b^2}{\tilde{p}_c}, \quad \text{and} \quad g_{\Omega\Omega} = |\tilde{p}_c|. \quad (2.4)$$

The modulus sign arises because of two possible triad orientations. Without any loss of generality, we will assume the orientation to be positive throughout this analysis. Since the spatial manifold in Kantowski-Sachs spacetime is non-compact, we have to introduce a fiducial length along the non-compact x direction. Denoting this length be L_o , the symplectic structure is given by

$$\Omega = \frac{L_o}{2G\gamma} \left(2d\tilde{b} \wedge d\tilde{p}_b + d\tilde{c} \wedge d\tilde{p}_c \right). \quad (2.5)$$

Here γ is the Barbero-Immirzi parameter whose value is fixed from the black hole entropy calculations in loop quantum gravity to be 0.2375. Since the fiducial length can be arbitrarily rescaled, the symplectic structure depends on L_o . This dependence can be removed by a rescaling of the symmetry reduced triad and connection components by introducing the triads p_b and p_c , and the connections b and c :

$$p_b = L_o \tilde{p}_b, \quad p_c = \tilde{p}_c, \quad b = \tilde{b}, \quad c = L_o \tilde{c}. \quad (2.6)$$

The non-vanishing Poisson brackets between these new variables are given by,

$$\{b, p_b\} = G\gamma, \quad \{c, p_c\} = 2G\gamma. \quad (2.7)$$

Note that p_b and p_c both have dimensions of length squared, whereas b and c are dimensionless. Also note that c and p_b scale as L_o where as other two variables are independent of the fiducial cell.

In Ashtekar variables, the Hamiltonian constraint for the Kantowski-Sachs spacetime with minimally coupled matter corresponding to an energy density ρ_m can be written as

$$\mathcal{H}_{\text{cl}} = \frac{-N}{2G\gamma^2} \left[2bc\sqrt{p_c} + (b^2 + \gamma^2) \frac{p_b}{\sqrt{p_c}} \right] + N 4\pi p_b \sqrt{p_c} \rho_m, \quad (2.8)$$

²This metric can be expressed as the one for the Schwarzschild interior by choosing $N(t)^2 = \left(\frac{2m}{t} - 1\right)^{-1}$ where m denotes the mass of the black hole, and identifying $g_{xx} = \left(\frac{2m}{t} - 1\right)$ and $g_{\Omega\Omega} = t^2$.

and the physical volume of the fiducial cell is $V = 4\pi p_b \sqrt{p_c}$. In the following, the lapse will be chosen as unity.³ Using the Hamilton's equations, for $N = 1$, the dynamical equations become,

$$\dot{p}_b = -G\gamma \frac{\partial \mathcal{H}_{\text{cl}}}{\partial b} = \frac{1}{\gamma} \left(c\sqrt{p_c} + \frac{bp_b}{\sqrt{p_c}} \right) \quad (2.9)$$

$$\dot{p}_c = -2G\gamma \frac{\partial \mathcal{H}_{\text{cl}}}{\partial c} = \frac{1}{\gamma} 2b\sqrt{p_c} \quad (2.10)$$

$$\dot{b} = G\gamma \frac{\partial \mathcal{H}_{\text{cl}}}{\partial p_b} = \frac{-1}{2\gamma\sqrt{p_c}} (b^2 + \gamma^2) + 4\pi G\gamma\sqrt{p_c} \left(\rho_m + p_b \frac{\partial \rho_m}{\partial p_b} \right) \quad (2.11)$$

$$\dot{c} = 2G\gamma \frac{\partial \mathcal{H}_{\text{cl}}}{\partial p_c} = \frac{-1}{\gamma\sqrt{p_c}} \left(bc - (b^2 + \gamma^2) \frac{p_b}{2p_c} \right) + 8\pi\gamma G p_b \left(\frac{\rho_m}{2\sqrt{p_c}} + \sqrt{p_c} \frac{\partial \rho_m}{\partial p_c} \right). \quad (2.12)$$

The vanishing of the classical Hamiltonian constraint, $\mathcal{H}_{\text{cl}} \approx 0$, yields

$$\frac{2bc}{\gamma^2 p_b} + \frac{b^2}{\gamma^2 p_c} + \frac{1}{p_c} = 8\pi G \rho_m \quad (2.13)$$

which using the expressions for the directional Hubble rates $H_i = \dot{\sqrt{g_{ii}}}/\sqrt{g_{ii}}$ can be written as the Einstein's field equation for the 0 – 0 component:

$$2 \frac{\dot{\sqrt{g_{xx}}}}{\sqrt{g_{xx}}} \frac{\dot{\sqrt{g_{\Omega\Omega}}}}{\sqrt{g_{\Omega\Omega}}} + \left(\frac{\dot{\sqrt{g_{\Omega\Omega}}}}{\sqrt{g_{\Omega\Omega}}} \right)^2 + \frac{1}{g_{\Omega\Omega}} = 8\pi G \rho_m. \quad (2.14)$$

Introducing the expansion θ and the shear σ^2 of the congruence of the cosmological observers

$$\theta = \frac{\dot{V}}{V} = \frac{\dot{p}_b}{p_b} + \frac{\dot{p}_c}{2p_c}. \quad (2.15)$$

and

$$\sigma^2 = \frac{1}{2} \sum_{i=1}^3 \left(H_i - \frac{1}{3}\theta \right)^2 = \frac{1}{3} \left(\frac{\dot{p}_c}{p_c} - \frac{\dot{p}_b}{p_b} \right)^2 \quad (2.16)$$

we can rewrite eq.(2.14) as

$$\frac{\theta^2}{3} - \sigma^2 + \frac{1}{g_{\Omega\Omega}} = 8\pi G \rho_m. \quad (2.17)$$

To investigate if the Kantowski-Sachs spacetime is singular, we consider the expansion and the shear scalars of the geodesics. At a singular region one or more of these diverge. This divergence causes the curvature invariants to blow up. To see this, we can compute the Ricci scalar R , which for the Kantowski-Sachs metric turns out to be

$$R = 2 \frac{\ddot{p}_b}{p_b} + \frac{\ddot{p}_c}{p_c} + \frac{2}{p_c}. \quad (2.18)$$

³To make a connection with the Schwarzschild interior, a convenient choice of lapse is $N = \frac{\gamma\sqrt{p_c}}{b}$ [32]. For studies of the expansion and shear scalars and the phenomenological implications of Kantowski-Sachs spacetime with matter, the choice $N = 1$ is more useful, and is thus considered here.

Using the equations for the expansion and the shear scalar, the Ricci scalar can be expressed as

$$R = 2\dot{\theta} + \frac{4}{3}\theta^2 + 2\sigma^2 + \frac{2}{p_c} . \quad (2.19)$$

Thus, a divergence in θ and σ^2 signals a divergence in the Ricci scalar. For this reason, understanding the behavior of expansion and shear scalars is important to gain insights on not only the properties of the geodesic evolution, but it is also useful to understand the behavior of curvature invariants. The scalars, θ and σ^2 , diverge if either one or both of $\frac{\dot{p}_b}{p_b}$ and $\frac{\dot{p}_c}{p_c}$ diverge. From the Hamilton's equations of motion (3.8) and (3.9), these ratios are,

$$\frac{\dot{p}_b}{p_b} = \frac{1}{\gamma} \left(\frac{c\sqrt{p_c}}{p_b} + \frac{b}{\sqrt{p_c}} \right) \quad (2.20)$$

$$\frac{\dot{p}_c}{p_c} = \frac{2b}{\sqrt{p_c}\gamma} . \quad (2.21)$$

It is clear from equations (2.20) and (2.21) that the expansion and shear scalars diverge as the triad components vanish, and/or the connection components diverge. In the Kantowski-Sachs spacetime with perfect fluid as matter, classical singularities occur at a vanishing volume. The structure of the singularity can be a barrel, cigar, pancake or a point [54]. For all these structures, either p_b or p_c vanish, causing a divergence in θ and σ^2 .⁴

At the above classical singular points, the energy density also diverges. From the vanishing of the Hamiltonian constraint $\mathcal{H}_{cl} \approx 0$, the expression for energy density becomes

$$\rho_m = \frac{1}{8\pi G\gamma^2} \left[\frac{2bc}{p_b} + \frac{b^2 + \gamma^2}{p_c} \right] . \quad (2.22)$$

Thus, if either of p_b or p_c vanishes, ρ_m grows unbounded as the physical volume approaches zero.

2.2 Comparison of different quantization prescriptions

Due to the underlying quantum geometry, the loop quantization of the classical Hamiltonian of the Kantowski-Sachs spacetime yields a difference equation [32]. The difference equation arises

⁴Note that for the vacuum Kantowski-Sachs spacetime, the expansion and shear scalars are ill defined at the horizon because of the coordinate singularity. However, $\theta^2/3 - \sigma^2$ is regular at the horizon, and can be used to understand the behavior of the curvature invariants. As an example, in this case, the Kretschmann scalar at the horizon can be written as $K_{t=2m} = 12(\theta^2/3 - \sigma^2)^2$, which being finite shows that the singularity at $t = 2m$ is not physical.

due to non-local nature of the field strength of the connection in the quantum Hamiltonian constraint which is expressed in terms of holonomies of connection components over closed loops. The action of the holonomy operators on the triad states is discrete, leading to a discrete quantum Hamiltonian constraint which is non-singular.⁵ The resulting quantum dynamics can be captured using an effective Hamiltonian constraint derived using the geometrical formulation of quantum mechanics [55]. Here one treats the Hilbert space as a quantum phase space and seeks an embedding of the finite dimensional classical phase space into it. For the isotropic and homogeneous models in LQC, such a suitable embedding has been found using sharply peaked states which probe volumes larger than the Planck volume [19, 20]. For these models, the dynamics from the quantum difference equation and the effective Hamiltonian turn out to be in an excellent agreement for states which correspond to a classical macroscopic universe at late times. Recent numerical investigations show that the departures between the effective spacetime description and the quantum dynamics are negligible unless one consider states which correspond to highly quantum spacetimes, such as states which are widely spread or are highly squeezed and non-Gaussian, or those which do not lead to a classical universe at late times [8, 9]. Though the effective Hamiltonian constraint has not been derived for the anisotropic spacetimes in LQC using the above embedding approach, an expression for it has been obtained by replacing b with $\frac{\sin b\delta_b}{\delta_b}$ and c with $\frac{\sin c\delta_c}{\delta_c}$ in (2.8), where δ_b and δ_c are the edge lengths of the holonomies [41, 40]. Following this procedure for the case of the loop quantization of the vacuum Bianchi-I spacetime, the resulting effective Hamiltonian dynamics turns out to be in excellent agreement with the underlying quantum evolution [56]. In the following we will assume that the effective Hamiltonian constraint for the Kantowski-Sachs spacetime as obtained from the above polymerization of the connection components, and assume it to be valid for all values of triads. For a general choice of δ_b and δ_c , the effective Hamiltonian constraint for the Kantowski-Sachs model with matter is given as [41, 40]:

⁵In principle, there can also be inverse triad modifications in the quantum Hamiltonian constraint. However, such modifications can not be consistently defined for spatially non-compact manifolds since they depend on the fiducial length. For this reason, we do not consider inverse triad modifications in this analysis. However, this problem does not arise if the spatial topology is compact, and conclusions reached in this manuscript remain unaffected in this case. It is also possible to get rid of terms depending on inverse triad using a suitable choice of lapse.

$$\mathcal{H} = \frac{-N}{2G\gamma^2} \left[2 \frac{\sin(b\delta_b)}{\delta_b} \frac{\sin(c\delta_c)}{\delta_c} \sqrt{p_c} + \left(\frac{\sin^2(b\delta_b)}{\delta_b^2} + \gamma^2 \right) \frac{p_b}{\sqrt{p_c}} \right] + N4\pi p_b \sqrt{p_c} \rho_m. \quad (2.23)$$

Note that (2.23) goes to the classical Hamiltonian (2.8) in the limit $\delta_b \rightarrow 0$ and $\delta_c \rightarrow 0$. However, due to the existence of minimum area gap in LQG, in the quantum theory, one shrinks the loops to the minimum finite area. Different choices of the way holonomy loops are constructed and shrunk lead to different δ_b and δ_c , and different properties of the quantum Hamiltonian constraint. We will identify these choices as different prescriptions to quantize the theory, which lead to different functional forms of δ_b and δ_c in the polymerization of the connection, and hence result in different effective Hamiltonian constraints. This is analogous to the situation in the quantization of isotropic spacetimes in LQC, where the older quantization was based on constant δ (the so called μ_o quantization [29, 4]), and improved quantization is based on a δ which is function of isotropic triad $\delta \propto 1/\sqrt{p}$ (the so called $\bar{\mu}$ quantization [5]). As in the isotropic case, the physics obtained from the theory is dependent on these holonomy edge lengths and hence they have to be chosen carefully. This can be further seen by noting that $\sin(b\delta_b)$ and $\sin(c\delta_c)$ in (2.23) can be expanded in infinite series as $b\delta_b - \frac{b^3\delta_b^3}{3!} + \dots$ and $c\delta_c - \frac{c^3\delta_c^3}{3!} + \dots$. Hence it is required that $b\delta_b$ and $c\delta_c$ should be independent of fiducial length. Else different terms of the expansion will have different powers of L_o and any calculation based on this Hamiltonian will yield results which are sensitive to the choice of L_o . Of the possible choices of holonomy edge lengths that can be motivated, we have to choose the one that gives a mathematically consistent theory which renders the physical scalars such as expansion and shear scalars independent of the choice of fiducial length, as in classical GR. There are three proposed prescriptions in LQC literature for the choice of holonomy edge-lengths in the Kantowski-Sachs model: the constant δ [32], the $\bar{\mu}$ (or the ‘improved dynamics’) prescription [40], and the $\bar{\mu}'$ (inspired from the improved dynamics) quantization prescriptions [41]. Due to their similarities with the notation of the isotropic model, we will label the effective Hamiltonian constraint for constant δ with μ_o . The effective Hamiltonians for ‘improved dynamics’ inspired prescription will be labeled by $\bar{\mu}'$, and that of ‘improved dynamics’ prescription with $\bar{\mu}$.

2.2.1 Constant δ prescription

The simplest choice of δ 's is to choose them as constant. The resulting effective Hamiltonian constraint then corresponds to the loop quantization of Kantowski-Sachs spacetime where the holonomy considered over the loop in $x - \theta$ plane, and the loop in the $\theta - \phi$ plane has minimum area with respect to the fiducial metric fixed by the minimum area eigenvalue Δ in LQG: $\Delta = 4\sqrt{3}\pi\gamma l_{\text{Pl}}^2$. In the quantization of the Schwarzschild interior proposed in Ref. [32], the δ 's were chosen equal⁶ $\delta_b = \delta_c = 4\sqrt{3}$. Loop quantization with constant δ_b and δ_c is also considered in various other works on the loop quantization of black hole spacetimes [39, 35, 36], and is analogous to the μ_o quantization in the isotropic LQC [29, 4]. Here we will assume the same prescription in the presence of matter. The resulting effective Hamiltonian constraint for $N = 1$ with minimally coupled matter is:

$$\mathcal{H}_{\mu_0} = \frac{-1}{2G\gamma^2} \left[2 \frac{\sin(b\delta_b)}{\delta_b} \frac{\sin(c\delta_c)}{\delta_c} \sqrt{p_c} + \left(\frac{\sin^2(b\delta_b)}{\delta_b^2} + \gamma^2 \right) \frac{p_b}{\sqrt{p_c}} \right] + 4\pi p_b \sqrt{p_c} \rho_m. \quad (2.24)$$

Using the Hamilton's equations, the equations of motion for the triads are

$$\dot{p}_b = -G\gamma \frac{\partial \mathcal{H}_{\mu_0}}{\partial b} = \frac{1}{\gamma} \left(\cos(b\delta_b) \frac{\sin(c\delta_c)}{\delta_c} \sqrt{p_c} + \frac{\sin(b\delta_b) \cos(b\delta_b)}{\delta_b} \frac{p_b}{\sqrt{p_c}} \right), \quad (2.25)$$

$$\dot{p}_c = -2G\gamma \frac{\partial \mathcal{H}_{\mu_0}}{\partial c} = \frac{2}{\gamma} \cos(c\delta_c) \frac{\sin(b\delta_b)}{\delta_b} \sqrt{p_c}. \quad (2.26)$$

From these one can find the expressions for expansion⁷ and shear scalars for Kantowski-Sachs spacetime with matter as follows,

$$\theta = \frac{1}{\gamma} \left(\frac{\sqrt{p_c} \cos(b\delta_b) \sin(c\delta_c)}{p_b \delta_c} + \frac{\sin(b\delta_b)}{\sqrt{p_c} \delta_b} (\cos(b\delta_b) + \cos(c\delta_c)) \right) \quad (2.27)$$

$$\sigma^2 = \frac{1}{3\gamma^2} \left((2 \cos(c\delta_c) - \cos(b\delta_b)) \frac{\sin(b\delta_b)}{\delta_b \sqrt{p_c}} - \frac{\cos(b\delta_b) \sin(c\delta_c)}{\delta_c} \frac{\sqrt{p_c}}{p_b} \right)^2. \quad (2.28)$$

It is clear from the above expressions that the expansion and shear scalars are unbounded and blow up as p_b or p_c approach zero, precisely as in the classical Kantowski-Sachs spacetime if the effective spacetime description is assumed to be valid for all values of triads. Note that

⁶Since [32] was using an area gap of $\Delta = 2\sqrt{3}\pi\gamma l_{\text{Pl}}^2$, the corresponding holonomy edge lengths were $2\sqrt{3}$. For $\Delta = 4\sqrt{3}\pi\gamma$, edge lengths should be $4\sqrt{3}$.

⁷The expressions for θ in three prescriptions studied in this section were also obtained for the Schwarzschild interior in Ref.[44], however no physical implications were studied except for noticing the bounded behavior in the case of $\bar{\mu}$ prescription.

the effective spacetime description is expected to breakdown in the regime when the volume of the spacetime is less than Planck volume [8]. Hence, in this quantization prescription there are no generic bounds on the expansion and shear scalars within the expected validity of effective dynamics. Even if one considers a specific matter model which results in a singularity resolution and a bounce of the mean volume, the dependence of θ and σ^2 on the triads shows that these scalars may not necessarily take Planckian values in the bounce regime. The spacetime curvature in the bounce regime can in principle be extremely small in this effective dynamics. Note that the maximum value of expansion (2.27) and shear scalars (2.28) depends on the values of p_b and p_c . Since the values of triads at the bounce can be made arbitrarily large or small by the choice of initial conditions and the matter content, the maximum values of expansion and shear scalars, reached near the bounce, can hence take arbitrary values. This problem is analogous to the dependence of energy density at the bounce on the momentum of the scalar field or the triad in the μ_o quantization of isotropic LQC. There too by choosing different initial conditions it is possible to obtain “quantum bounce” at arbitrarily small spacetime curvature.

Let us now consider the issue of fiducial cell dependence for this prescription. Since $\delta_b = \delta_c = 4\sqrt{3}$, they are independent of the rescaling under the fiducial length L_o . However, since c is proportional to L_o , therefore $c\delta_c$ depends on the fiducial length L_o . Due to this reason, the resulting physics from the effective Hamiltonian constraint (2.24), in particular the expressions for expansion and shear scalars, unlike in the classical theory, are not independent of the fiducial length rescaling. Again this problem of constant δ prescription in the Kantowski-Sachs spacetime is analogous to the one for the μ_o quantization of the isotropic LQC, where the resulting physical predictions such as the scale at which the quantum bounce occurs and the infra-red behavior depend on the fiducial volume of the fiducial cell [4, 22]. This problem is tied to the dependence of the expansion and triad scalars in this quantization prescription on triads as discussed above. Since p_b can be rescaled arbitrarily by rescaling L_o , the curvature scale in the bounce regime inevitably depends on the fiducial length L_o and hence can take arbitrary values.

In conclusion, we find that constant δ quantization prescription does not provide a generic bounded behavior of expansion and shear scalars. Further, it is possible to obtain “quantum gravitational effects,” originating from the trigonometric functions in eq.(2.24), at any arbitrary scale.

2.2.2 An ‘improved dynamics inspired’ prescription

For the isotropic models in LQC, the problems with constant δ (i.e. μ_o) quantization were overcome in the improved dynamics (the $\bar{\mu}$ quantization) [5], where $\bar{\mu}$ is related to the isotropic triad as $\bar{\mu} = \Delta/\sqrt{p}$ [5]. This quantization turns out to be independent of the various problems of the μ_o quantization, and is also the unique prescription for the quantization of isotropic models in which physical predictions are free of the dependence on the fiducial cell in the effective spacetime description [22]. Motivated by the success of $\bar{\mu}$ quantization, a different prescription for the choice of δ_b and δ_c for Kantowski-Sachs model has been considered [41], where

$$\delta_b = \sqrt{\frac{\Delta}{p_b}}, \quad \text{and} \quad \delta_c = \sqrt{\frac{\Delta}{p_c}}. \quad (2.29)$$

We note that this choice for δ 's is also motivated from the lattice refinement scheme [30]. The effective Hamiltonian constraint for this quantization becomes:

$$\mathcal{H}_{\bar{\mu}'} = \frac{-1}{2G\gamma^2\Delta} \left[2 \sin(b\delta_b) \sin(c\delta_c) p_c \sqrt{p_b} + (\sin^2(b\delta_b) p_b + \gamma^2 \Delta) \frac{p_b}{\sqrt{p_c}} \right] + 4\pi p_b \sqrt{p_c} \rho_m. \quad (2.30)$$

As we noted above, for the effective Hamiltonian constraint to yield a consistent physics, the argument of trigonometric functions should be independent of the fiducial length. However since b is independent of L_o and p_b is proportional to L_o , $b\delta_b = b\sqrt{\frac{\Delta}{p_b}}$ depends on fiducial length. Similarly $c\delta_c$ also depends on the fiducial length. This clearly shows that this quantization is unsuitable for Kantowski-Sachs spacetime because the resulting physical implications will be sensitive to the fiducial length L_o .

The equations of motion for the triads in this quantization are

$$\dot{p}_b = -G\gamma \frac{\partial \mathcal{H}_{\bar{\mu}'}}{\partial b} = \frac{\cos(b\delta_b)}{\gamma\sqrt{\Delta}} \left(p_c \sin(c\delta_c) + p_b \sqrt{\frac{p_b}{p_c}} \sin(b\delta_b) \right) \quad (2.31)$$

$$\dot{p}_c = -2G\gamma \frac{\partial \mathcal{H}_{\bar{\mu}'}}{\partial c} = \frac{2}{\gamma\sqrt{\Delta}} \sqrt{p_b p_c} \sin(b\delta_b) \cos(c\delta_c), \quad (2.32)$$

using which the expansion and shear scalars turn out to be as follows:

$$\theta = \frac{1}{\gamma\sqrt{\Delta}} \left[\frac{p_c}{p_b} \cos(b\delta_b) \sin(c\delta_c) + \sqrt{\frac{p_b}{p_c}} \sin(b\delta_b) (\cos(b\delta_b) + \cos(c\delta_c)) \right], \quad (2.33)$$

$$\sigma^2 = \frac{1}{3\gamma^2\Delta} \left[\frac{p_c}{p_b} \cos(b\delta_b) \sin(c\delta_c) + \sqrt{\frac{p_b}{p_c}} \sin(b\delta_b) (\cos(b\delta_b) - 2\cos(c\delta_c)) \right]^2. \quad (2.34)$$

We see that the $\bar{\mu}'$ quantization has the same problem as the constant δ quantization as far as the divergence of θ and σ^2 is concerned. These scalars can potentially diverge for $p_b \rightarrow 0$, $p_b \rightarrow \infty$,

$p_c \rightarrow 0$ or $p_c \rightarrow \infty$. As in the constant δ quantization prescription, even if the singularities are resolved, the curvature scale associated with singularity resolution can be arbitrarily small and depends on the initial conditions. Also remembering that it has spurious dependency on the fiducial length we are led to the conclusion that $\bar{\mu}'$ quantization is not apt for Kantowski-Sachs spacetime. The results that constant δ and $\bar{\mu}'$ quantizations do not yield necessarily consistent physics is in accordance with a similar study in FRW model in LQC [22]. As remarked earlier, problems of this prescription have also been noted in the context of the von-Neumann stability analysis of the resulting quantum Hamiltonian constraint [30].⁸

2.2.3 ‘Improved Dynamics’ prescription

The improved dynamics prescription is based on noting that the field strength of the Ashtekar-Barbero connection should be computed by considering holonomies around the loop whose minimum area with respect to the physical metric is fixed by the minimum area eigenvalue (Δ) in LQG. This is in contrast to the constant δ prescription where the minimum area with respect to the fiducial metric was fixed with respect to the underlying quantum geometry. In this scheme we obtain the holonomy edge lengths as [40]:

$$\delta_b = \sqrt{\frac{\Delta}{p_c}}, \quad \delta_c = \frac{\sqrt{\Delta p_c}}{p_b}. \quad (2.35)$$

Now the effective Hamiltonian (2.23) becomes,

$$\mathcal{H}_{\bar{\mu}} = \frac{-p_b \sqrt{p_c}}{2G\gamma^2 \Delta} \left[2 \sin(b\delta_b) \sin(c\delta_c) + \sin^2(b\delta_b) + \frac{\gamma^2 \Delta}{p_c} \right] + 4\pi p_b \sqrt{p_c} \rho_m. \quad (2.36)$$

Before we proceed further, we note an important property of this effective Hamiltonian not shared by \mathcal{H}_{μ_o} and $\mathcal{H}_{\bar{\mu}'}$. Due to the scaling properties of b, c, p_b and p_c , $b\delta_b$ and $c\delta_c$ are invariant under the change of the fiducial length L_o . Thus $\sin(b\delta_b)$ and $\sin(c\delta_c)$ are independent of fiducial length. Due to this reason, we expect that the physical predictions concerning scalars such as expansion and shear scalars will be independent of L_o in this prescription, as in the classical theory.

⁸For different prescriptions, the problems in the effective dynamics and the numerical instability of the quantum difference equation in the corresponding quantization run in parallel. See Ref. [7] for a discussion of these issues in different quantizations in LQC.

The evolution equations for triads and cotriads turn out to be as follows:

$$\dot{p}_b = -G\gamma \frac{\partial \mathcal{H}_{\bar{\mu}}}{\partial b} = \frac{p_b \cos(b\delta_b)}{\gamma\sqrt{\Delta}} (\sin(c\delta_c) + \sin(b\delta_b)), \quad (2.37)$$

$$\dot{p}_c = -2G\gamma \frac{\partial \mathcal{H}_{\bar{\mu}}}{\partial c} = \frac{2p_c}{\gamma\sqrt{\Delta}} \sin(b\delta_b) \cos(c\delta_c) \quad (2.38)$$

$$(2.39)$$

Using (2.15), (3.14) and (3.15), we obtain the following expression for the expansion scalar,

$$\theta = \frac{1}{\gamma\sqrt{\Delta}} (\sin(b\delta_b) \cos(c\delta_c) + \cos(b\delta_b) \sin(c\delta_c) + \sin(b\delta_b) \cos(b\delta_b)). \quad (2.40)$$

Unlike the case of \mathcal{H}_{μ_o} and $\mathcal{H}_{\bar{\mu}'}$, the expansion scalar turns out to be independent of the fiducial length L_o , and is generically bounded above by a universal value:

$$|\theta| \leq \frac{3}{2\gamma\sqrt{\Delta}} \approx \frac{2.78}{l_{\text{Pl}}}. \quad (2.41)$$

Similarly for the shear scalar, using (2.16), (3.14) and (3.15), we get

$$\sigma^2 = \frac{1}{3\gamma^2\Delta} (2 \sin(b\delta_b) \cos(c\delta_c) - \cos(b\delta_b) (\sin(c\delta_c) + \sin(b\delta_b)))^2. \quad (2.42)$$

As for the expansion scalar, σ^2 turns out to be independent of L_o and has a universal maximum:

$$\sigma^2 \leq \frac{5.76}{l_{\text{Pl}}^2}. \quad (2.43)$$

Hence both shear and expansion scalars are bounded above in this quantization prescription of the Kantowski-Sachs spacetime. Unlike constant δ and $\bar{\mu}'$ quantization prescriptions, the expansion and shear scalars take Planckian values in the bounce regime and curvature scale associated with singularity resolution does not depend on the initial conditions. Note that for the improved dynamics prescription, similar properties of expansion and shear scalar were earlier found for the isotropic model [24] and the Bianchi models [23, 27, 28, 17]. In the isotropic and Bianchi-I model, using the boundedness properties of expansion and shear scalars it was found that strong singularities are generically resolved in the effective spacetime description [24, 27].⁹ Above results provide a strong evidence that strong singularities may be generically absent in this quantization of Kantowski-Sachs spacetime.

⁹These results have also been extended to the effective description of the hybrid quantization of Gowdy models [57].

2.3 Uniqueness of $\bar{\mu}$ prescription

In the previous section, we found that of the three proposed quantization prescriptions for the Kantowski-Sachs spacetime in LQC, only the the $\bar{\mu}$ effective Hamiltonian leads to consistent physics and results in generic bounds on expansion and shear scalars. In this section we pose the question whether $\bar{\mu}$ quantization is the only possible choice for which the expansion and shear scalars are generically bounded singularity resolution in the Kantowski-Sachs spacetime? A similar question was posed in the isotropic models in LQC, where the answer turned out to be positive [22, 23]. We will see that in the Kantowski-Sachs spacetime, under the assumption that δ_b and δ_c have a general form given in eq.(2.47), the answer also turns to be in an affirmative in the effective spacetime description.

We start with the effective LQC Hamiltonian (2.23), where the holonomy edge lengths δ_b and δ_c are any general functions of the triads. Then the Hamilton's equations lead to the following expressions for shear and expansion scalars.

$$\theta = \frac{1}{\gamma} \left(\frac{\sqrt{p_c} \cos(b\delta_b(p_b, p_c)) \sin(c\delta_c(p_b, p_c))}{p_b \delta_c(p_b, p_c)} + \frac{\sin(b\delta_b(p_b, p_c))}{\sqrt{p_c} \delta_b(p_b, p_c)} (\cos(b\delta_b(p_b, p_c)) + \cos(c\delta_c(p_b, p_c))) \right) \quad (2.44)$$

$$\begin{aligned} \sigma^2 &= \frac{1}{3\gamma^2} \left[(2 \cos(c\delta_c(p_b, p_c)) - \cos(b\delta_b(p_b, p_c))) \frac{\sin(b\delta_b(p_b, p_c))}{\delta_b(p_b, p_c) \sqrt{p_c}} \right. \\ &\quad \left. - \frac{\cos(b\delta_b(p_b, p_c)) \sin(c\delta_c(p_b, p_c)) \sqrt{p_c}}{\delta_c(p_b, p_c) p_b} \right]^2. \end{aligned} \quad (2.45)$$

We now find what general choices of $\delta_b(p_b, p_c), \delta_c(p_b, p_c)$ yield a bound on expansion and shear scalars. These scalars become unbounded when either an inverse power of a triad blows up as that triad tends to zero or when a positive power of triad blows up as that triad tend to infinity. In eqs. (2.44) and (2.45), the trigonometric factors are always bounded and hence the terms that will decide the boundedness of the expansion and shear scalars are

$$T_b = \frac{1}{\sqrt{p_c} \delta_b(p_b, p_c)} \quad \text{and} \quad T_c = \frac{\sqrt{p_c}}{p_b \delta_c(p_b, p_c)}. \quad (2.46)$$

Then the task at hand reduces to finding general functions of triads which when chosen as the holonomy edge lengths, give an upper bound on T_c and T_b . To this end we make an assumption

that δ_b and δ_c are functions of p_b and p_c such that one can express their inverses as

$$\delta_b^{-1} = \sum B_{ij} p_b^{m_i} p_c^{n_j}, \quad \delta_c^{-1} = \sum C_{ij} p_b^{m_i} p_c^{n_j}, \quad (2.47)$$

where $m_i, n_j \in \mathbb{R}$. This ansatz includes all the three choices of δ_b and δ_c discussed in Sec. III, but is more general. Using (2.47), one can write (2.46) as

$$T_c = \sum C_{ij} p_b^{m_i-1} p_c^{n_j+1/2}, \quad (2.48)$$

$$T_b = \sum B_{ij} p_b^{m_i} p_c^{n_j-1/2}. \quad (2.49)$$

We now require that if θ and σ^2 have to be bounded then T_c and T_b should not diverge as triads tend to zero or infinity. This is possible only if m_i and n_j in (2.48) and (2.49) satisfy certain constraints. We find that these constraints only allow $\delta_b \propto (p_c)^{-1/2}$ and $\delta_c \propto p_c^{1/2}/p_b$, the same as in the $\bar{\mu}$ quantization (3.13).

First let us take a closer look at (2.48) from which we wish to obtain constraints on δ_c . Keeping p_c as nondiverging and nonvanishing, one can obtain bounds on values of m_i , the powers of p_b with nonzero coefficients. As $p_b \rightarrow 0$, for each term in T_c to be nondiverging, they should all have a non-negative power of p_b . Thus, for any nonzero C_{ij} , $m_i \geq 1$. Also, as $p_b \rightarrow \infty$, any positive power of p_b diverges. Hence for T_c to be bounded, for any nonzero C_{ij} , $m_i \leq 1$. Therefore, the only possible value for m_i that leaves T_c bounded for $p_b \rightarrow 0$ and $p_b \rightarrow \infty$ is $m_i = 1$. Similarly, to find the allowed values for n_j , we study the behavior of T_c as p_c goes to zero and infinity for a finite nonzero value of p_b . It is clear that positive powers of p_c will result in a divergence of T_c as $p_c \rightarrow \infty$ where as negative powers will result in a divergence when $p_c \rightarrow 0$. This implies that the only choice of n_j that leaves T_c bounded for the whole range of p_c is $n_j = -1/2$. Finally, we consider the case of both the triads simultaneously approaching one of the extreme values - zero or infinity. For $m_i = 1$ and $n_j = -1/2$, from (2.48) it can be seen that T_c is independent of triads i.e, it is just a constant. Hence for both the triads simultaneously approaching an extreme value, T_c remains bounded. For any other choice of m_i or n_j , T_c can diverge, causing a divergence in the expansion and shear scalars.

Repeating the same analysis, for T_b in (2.49), it can be seen that the only values of m_i and n_j which keep T_b bounded for the whole domain of p_b and p_c are $m_i = 0$ and $p_c = 1/2$. Thus from (2.47) it can be seen that the only choice of δ 's which keeps θ and σ^2 bounded throughout

the entire domain of triads correspond to

$$\delta_c \propto \frac{\sqrt{p_c}}{p_b}, \quad \delta_b \propto \frac{1}{\sqrt{p_c}}. \quad (2.50)$$

These are precisely the functional dependencies of the holonomy edge lengths on these triads in the ‘improved dynamics’ prescription. (2.36). Thus, for the general ansatz (2.47) we find that the only possible choices of δ_b and δ_c which result in bounded expansion and shear scalars for the geodesics in the effective dynamics correspond to $\bar{\mu}$ prescription. It is important to stress that we found the uniqueness of $\bar{\mu}$ quantization prescription by only demanding that the expansion and shear scalars be bounded, and our argument is not tied to requirements based on fiducial cell rescaling freedom or to the topology of the spatial manifold. But, it is rather interesting that the prescription which results in generic bounds on scalars is the one which is also free from the freedom under rescalings of the fiducial cell. It is straightforward to see that requiring $b\delta_b$ and $c\delta_c$ to be independent of fiducial length L_o , and assuming that δ_b and δ_c are constructed from the triads p_b and p_c , one is led to the $\bar{\mu}$ prescription.

In the above analysis we have seen that by requiring that the expansion and shear scalars be always bounded, one can find the exact dependence of δ_b and δ_c on the triads. The same functional forms of δ_b and δ_c can be obtained from an independent physical motivation. Note that holonomy corrections in the effective Hamiltonian arise from the field strength of the connection components b and c , where one has to take the holonomies around closed loops with edge lengths determined by δ_b and δ_c . To compute the field strength, the loops over which the holonomies are considered are shrunk to the minimum area eigenvalue in LQG. One could in principle form loops from holonomies with constant edge lengths δ_b , δ_c or as in the $\bar{\mu}'$ scheme, where $\delta_b = \sqrt{\frac{\Delta}{p_b}}$ and $\delta_c = \sqrt{\frac{\Delta}{p_c}}$. But loops with such edge lengths do not have physical area matching the minimum area gap from LQG. The constant δ quantization takes the holonomy loops to have constant fiducial area, but not the physical area. However, fiducial area is not independent of rescaling of fiducial length and thus is not a physical quantity. In this quantization, a loop with edges of length δ_b along θ and ϕ directions will have a physical area $\delta_b^2 p_c$.¹⁰ This area is clearly dependent of the triad and can even vanish as $p_c \rightarrow 0$, thus becoming smaller than the minimum area eigenvalue of LQG. Similarly, in $\bar{\mu}'$ quantization, the area of a loop with edge δ_b each along

¹⁰It is straightforward to see that the same conclusion is reached if the loop is in $x - \theta$ plane.

θ and ϕ directions will be $\frac{\Delta p_c}{p_b}$. Once again this area is not constant and can go below the minimum area gap of LQG if p_c/p_b becomes less than unity. In contrast the loops constructed in the improved dynamics with $\delta_b = \sqrt{\Delta/p_c}$ and $\delta_c = \sqrt{\Delta p_c}/p_b$ in $x - \theta$ and $\theta - \phi$ planes have a physical area Δ , which is same as the minimum area gap. Thus, this argument further supports the improved dynamics or the $\bar{\mu}$ prescription for the Kantowski-Sachs spacetime.

2.4 Energy density in the ‘improved dynamics’

We have so far seen that out of various possible quantization prescriptions, the $\bar{\mu}$ prescription for the Kantowski-Sachs spacetime is the only one which results in bounded expansion and shear scalars for all the values of triads. Also, the resulting physics turns out to be independent of the rescalings under fiducial length. In this sense, this is the preferred choice for the loop quantization in the Kantowski-Sachs model. We now investigate the issue of the boundedness of the energy density in this prescription. It will be useful to recall some features of classical singularities in this context. In classical GR, approach to singularities in the Kantowski-Sachs spacetime is accompanied by a divergence in the energy density for perfect fluids when the volume vanishes [54]. The nature of the singularity – whether it is isotropic or anisotropic depends on the equation of state of matter. Apart from the isotropic or the point like singularity, cigar, pancake and barrel singularities can also form in the classical Kantowski-Sachs spacetime. For the point singularity both g_{xx} and $g_{\Omega\Omega}$ vanish, for the cigar singularity $g_{xx} \rightarrow \infty$ and $g_{\Omega\Omega} \rightarrow 0$, for the barrel singularity g_{xx} approaches a finite value and $g_{\Omega\Omega} \rightarrow 0$, and for the pancake singularity g_{xx} vanishes and $g_{\Omega\Omega}$ approaches a finite value. In terms of the triad components, for point, cigar and barrel singularities both p_b and p_c vanish. However, the pancake singularity occurs at a finite value of p_c , with p_b vanishing.

We now investigate whether the energy density is bounded in the effective spacetime description of the $\bar{\mu}$ quantization. The energy density can be obtained from the Hamiltonian constraint $H_{\bar{\mu}} \approx 0$ as

$$\rho_{\bar{\mu}} = \frac{1}{8\pi G\gamma^2\Delta} \left[2\sin(b\delta_b)\sin(c\delta_c) + \sin^2(b\delta_b) + \frac{\gamma^2\Delta}{p_c} \right]. \quad (2.51)$$

It is clear that this energy density is bounded for all values of triads and cotriads except when $p_c \rightarrow 0$. Especially, we note that even if the triad p_b is vanishing, the energy density is bounded as far as p_c is nonzero. Since a pancake singularity is attained when p_c remains finite, we can

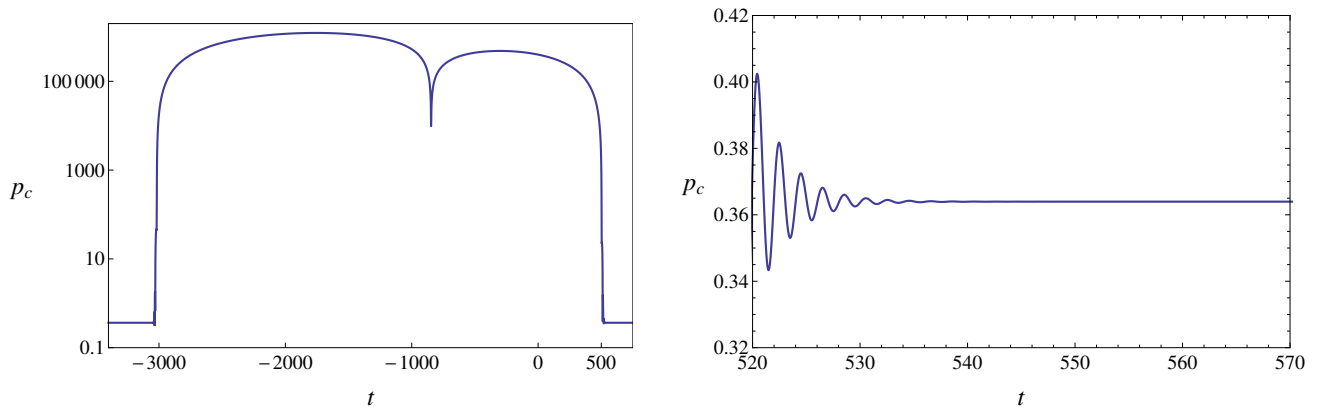


Figure 2.1: Evolution of p_c for the massless scalar field evolution

already conclude that such a singularity is absent in the effective description of the Kantowski-Sachs spacetime for the $\bar{\mu}$ quantization.¹¹

Let us now return to the properties of the energy density in general, and understand its behavior for the generic singularities. The energy density in $\bar{\mu}$ approach will be bounded if p_c does not vanish. In the non-singular evolution, one expects that the dynamics results in a non-zero value of p_c . The pertinent question is whether in effective dynamics this happens to be true. Numerical analysis of the Hamilton's equations shows that the answer turns out to be positive. The first evidence of this behavior of p_c was reported in the vacuum Kantowski-Sachs case, where it was found that due to holonomy corrections, p_c (as well as p_b) undergo non-singular evolution, and p_c never approaches zero throughout the evolution [40]. It was found that p_c approaches an asymptotic non-zero value after classical singularity is avoided. Detailed numerical analysis of effective Hamiltonian constraint (2.36) for different types of matter fields shows that a similar behavior occurs for p_c in general [45]. An example of this phenomena is shown in Fig. 2.1, where we plot the behavior of p_c versus proper time for the case of massless scalar field in a typical numerical simulation. Giving the initial date at $t = 0$ we numerically solve the Hamilton's equations for the effective Hamiltonian constraint (2.36). The initial conditions are $p_b(0) = 5 \times 10^5$, $b(0) = -0.1$, $p_c(0) = 4 \times 10^5$, $c(0) = 0.16$ (all in Planck units). Initial value of energy density is obtained by solving the Hamiltonian constraint. During the past and future

¹¹In contrast, this is not true in the constant δ and the 'improved dynamics inspired' quantizations discussed earlier. For these prescriptions, the expression of energy density contains inverse power of p_b as well as p_c in the expression for energy density. Thus, allowing all kinds of singularities.

evolution, the physical volume does not go to zero when the classical singularity is approached, but instead bounces. The triad p_c never goes to zero in the entire evolution, but asymptotes towards a constant value. A similar plot is obtained for the vacuum case, where it was shown that some cycles of classical phases appear before p_c reaches Planck regime [41]. These results, and also of Ref. [40], confirm that dynamically p_c is always bounded away from zero. Hence, we conclude that the energy density (2.51) is always bounded in the loop quantization of the Kantowski-Sachs spacetime.

2.5 Discussion

Classical Kantowski-Sachs spacetime is singular for generic matter choices, which calls upon a quantum gravitational treatment to see if the singularity persists. A good understanding about the geodesic completeness of a spacetime can be obtained via expansion and shear scalars. Any divergence in these scalars indicates presence of a singularity. Since singularity denotes break down of the theory which is used to describe spacetime, it is hoped that the right theory of quantum gravity will resolve these singularities in general. A quantum theory of spacetime should pass various consistency tests. If the spatial manifold is non-compact, then the expansion and shear scalars must be independent of the choice of the fiducial cell. If the singularities are indeed resolved, then the curvature scale associated with singularity resolution should not be arbitrary. Due to quantization ambiguities, various prescriptions can exist for quantization of a spacetime. Is it possible that a particular prescription is favored over others? This question was earlier posed in the isotropic [22] and Bianchi-I spacetime in LQC [23], where it was found that $\bar{\mu}$ quantization prescription in contrast to other quantization prescriptions leads to generic bounded behavior of expansion and shear scalars, and physical predictions free from the rescaling under fiducial cell. The goal of this analysis was to answer this question in the loop quantization of Kantowski-Sachs spacetime assuming the validity of effective spacetime description for minimally coupled matter.

Previous works on loop quantization of Kantowski-Sachs spacetime have been mostly devoted to study the vacuum case, for which the expansion scalar has been partially studied earlier [44]. Little details about the physics of singularity resolution for generic matter were so far available. Three quantization prescriptions were proposed in the literature. Of these, only one was shown

to be preferred in the sense that the effective Hamiltonian does not depend on the rescalings of the fiducial length. This quantization prescription (denoted by $\bar{\mu}$) is the analog of the improved dynamics in isotropic LQC [5]. The other two quantization prescriptions, denoted by μ_o and $\bar{\mu}'$ lead to resolution of singularities in the vacuum case, but were known to be problematic under rescalings of the fiducial cell. Unlike $\bar{\mu}$ prescription, these also yield quantum difference equations which are von-Neumann unstable [30]. We obtained the expansion and shear scalars using the effective dynamics in each of these prescriptions and found that except the case of $\bar{\mu}$ quantization, in both the other choices these scalars are not necessarily bounded in the effective spacetime. Thus it is possible that a strong curvature singularity may not get resolved for μ_o and $\bar{\mu}'$ prescriptions for some choices of matter depending on the initial conditions in effective dynamics. Even if the singularities are resolved, we found that the associated curvature scale is arbitrary. In contrast, the $\bar{\mu}$ quantization leads to universal bounds on the expansion and shear scalars which are dictated by the underlying Planckian geometry for Kantowski-Sachs spacetime with matter. These bounds point towards a generic resolution of singularities in this prescription. Analysis of the behavior of energy density in $\bar{\mu}$ prescription reveals that it is dynamically bounded because p_c is bounded from below. It turns out that this is a generic feature of all types of perfect fluids, whose details will be reported in a future work [45]. It is interesting to note that without solving dynamical equations, it is possible to rule out pancake singularities in the $\bar{\mu}$ prescription. The bounded behavior of expansion and shear scalars and energy density is a strong indication that curvature singularities may be generically resolved in the $\bar{\mu}$ quantization prescription of the Kantowski-Sachs spacetime with matter, as in the case of isotropic and Bianchi-I model [24, 26, 27].

To investigate whether there is another quantization prescription which gives a bounded behavior of expansion and shear scalars, we considered a general ansatz of the edge lengths of the holonomies. It turns out that $\bar{\mu}$ quantization is a unique choice for which the expansion and shear scalars are bounded. For any other prescription, expansion and shear scalars can be unbounded in the effective dynamics. It is remarkable that the demand that these scalars are bounded also chooses the prescription which is free from the rescalings of the fiducial cell. This property is shared by the $\bar{\mu}$ quantization in the isotropic and Bianchi-I spacetime in LQC [22, 23]. All these similarities between the $\bar{\mu}$ quantization of the isotropic, Bianchi-I and Kantowski-Sachs

spacetimes bring out a harmonious and robust picture of the loop quantization.

Finally, it is important to stress that though this analysis provides further insights on the loop quantization of Kantowski-Sachs spacetime, singling out the $\bar{\mu}$ prescription on various grounds, more work is needed to rigorously formulate the $\bar{\mu}$ prescription in the quantum theory. It is known that for the Schwarzschild interior, the $\bar{\mu}$ quantization results in quantum gravitational effects at the event horizon where the spacetime curvature in the classical theory can be very small [40]. The existence of these effects is tied to the choice of the coordinates which lead to the classical coordinate singularity at the horizon. Not distinguishing it from the curvature singularity, quantum geometric effects resulting from the holonomies of the connection components thus become significant at the horizon resolving even the coordinate singularity. Note that this coordinate artifact does not arise in the Kantowski-Sachs spacetime in presence of matter. These issues will be closely examined in the $\bar{\mu}$ quantization of the Schwarzschild interior [58]. Further, it has been reported that the Kantowski-Sachs vacuum spacetime in the $\bar{\mu}$ prescription leads to the Nariai-like spacetime after the bounce in the asymptotic approach [40].¹²¹³ It turns out that this feature is more general, which reveals some subtle properties of the effective spacetime in LQC [50]. A deeper understanding of these issues is required to gain further insights on the details of the physics of singularity resolution in the Kantowski-Sachs spacetime in LQC.

¹²It is important to make a distinction here with the classical Nariai spacetime, since in the asymptotic approach to Nariai-like spacetime, the spacetime is quantum.

¹³Before the Nariai-like phase is asymptotically approached, the spacetime gives birth to baby blackhole spacetimes [41].

Chapter 3

Emergence of ‘Charged’ Nariai and Anti-Bertotti-Robinson Spacetimes in LQC¹

In the previous chapter, we summarized the way a unique quantization prescription for the Kantowski-Sachs spacetimes emerges in LQC. The goal of this chapter, based on Ref. [50], is to understand the nature of the spacetime beyond the classical singularity in this quantization prescription. We study Kantowski-Sachs spacetimes with cosmological constant, which serve as the interior of the Schwarzschild-DeSitter black holes, and Bianchi-III LRS spacetimes with a negative cosmological constant which serve as the interior of higher genus black holes. We show that in these spacetimes, the evolution is highly time asymmetric and results in a spacetime which is a product of two constant curvature in the evolution in one branch of the singularity resolution. Interestingly, such a spacetime is a solution of general relativity with an effective ‘charge’. In the loop quantum evolution of Kantowski-Sachs spacetime beyond the classical singularity one obtains a ‘charged’ Nariai spacetime. And, in the loop quantum evolution Bianchi-III LRS spacetime, one obtains the anti-Bertotti-Robinson spacetime. The emergence of this ‘charge’ is purely quantum geometric in nature.

This chapter is organized as follows. In Sec. I, we summarize some of the main features of the classical theory for the higher genus black hole interior in terms of the Ashtekar variables and the way the symmetry reduced triads are related to the metric components in these models. This is based on the similar analysis for the Kantowski-Sachs spacetime in the previous chapter. In the same section, we introduce the classical Hamiltonian in Ashtekar variables and derive the classical Hamilton’s equations which result in a singular evolution. Note that in the classical

¹Sections 3.1 - 3.6 are reproduced from N. Dadhich, A. Joe and P. Singh, *Class. Quant. Grav.* 32, 185006 (2015) (Copyright 2015 Institute of Physics Publishing Ltd) [50] by the permission of the Institute of Physics Publishing. See Appendix A for the copyright permission from the publishers.

evolution of the Kantowski-Sachs or the higher genus black hole interior, there is no emergence of the classical Nariai or anti-Bertotti-Robinson spacetimes. The loop quantum evolution derived from the effective Hamiltonian constraints is discussed in Sec. II, which is non-singular. Here after deriving the modified Hamilton's equations, we first discuss the boundedness of expansion and shear scalars, and then study the numerical solutions in different cases. In section III we consider the properties of the asymptotic spacetime emerging in the loop quantum evolution and find them as the 'charged' Nariai spacetime and the anti Bertotti-Robinson spacetime of classical GR. These spacetimes have an emergent 'charge' and an emergent cosmological constant, both arising from the loop quantum geometry of the spacetime. The values of these quantum geometric 'charge' and cosmological constant are computed in Sec. IV. We then consider the fine tuned case where there is no 'charge' in the spacetime emerging from the loop quantum model of Kantowski-Sachs spacetime and a vanishing emergent cosmological constant in the loop quantum model of higher genus black hole interior in Sec. V. We summarize with a discussion of the results in Sec. VI.

3.1 Higher genus black hole interior: classical aspects

In this section, we summarize the classical Hamiltonian constraint for the higher genus black hole interior [47]. This analysis is parallel to the one for the Kantowski-Sachs spacetime which we discussed in the previous chapter. In this case, the Ashtekar-Barbero connection and the conjugate triads simplify to [47]

$$A_a^i \tau_i dx^a = \underline{c} \tau_3 dx + \underline{b} \tau_2 d\theta - \underline{b} \tau_1 \sinh \theta d\phi + \tau_3 \cosh \theta d\phi, \quad (3.1)$$

$$\underline{E}_i^a \tau_i \partial_a = \underline{p}_c \tau_3 \sinh \theta \partial_x + \underline{p}_b \tau_2 \sinh \theta \partial_\theta - \underline{p}_b \tau_1 \partial_\phi. \quad (3.2)$$

Here $\tau_i = -i\sigma_i/2$, and σ_i are the Pauli spin matrices. The symplectic structure is determined by

$$\Omega = \frac{L_o}{2G\gamma} (2d\underline{b} \wedge d\underline{p}_b + d\underline{c} \wedge d\underline{p}_c), \quad (3.3)$$

where L_o is a fiducial scale. It is convenient to work with rescaled triads and connections,

$$p_b = L_o \underline{p}_b, \quad p_c = \underline{p}_c, \quad b = \underline{b}, \quad c = L_o \underline{c} \quad (3.4)$$

which are independent of any change to rescaling of the fiducial cell. These variables satisfy the following Poisson brackets:

$$\{b, p_b\} = G\gamma, \quad \{c, p_c\} = 2G\gamma, \quad (3.5)$$

where $\gamma \approx 0.2375$ is the Barbero-Immirzi parameter whose value is set from the black hole thermodynamics in LQG.

The physical volume of the fiducial cell for the higher genus black hole interior, differs from that of the Kantowski-Sachs spacetime by an overall factor and is given by² $V = 2\pi(\cosh(\theta_0) - 1)p_b\sqrt{p_c}$. Further, the metric of the spacetime in this case is similar to the Kantowski-Sachs spacetime albeit which is spatially open, and is same as of the Bianchi-III LRS spacetime. Thus, as the Schwarzschild case, the higher genus black hole interior can also be studied using a homogeneous anisotropic spacetime metric. Due to this reason, in the following discussion the higher genus black hole interior spacetime in our analysis will also be referred to as Bianchi-III LRS spacetime.

The gravitational part of the classical Hamiltonian for the interior of higher genus black holes in Ashtekar variables turns out to be [47]

$$\mathcal{H}_{HG}^{(g)} = \frac{-N'}{2G\gamma^2} \left[2bc\sqrt{p_c} + (b^2 - \gamma^2) \frac{p_b}{\sqrt{p_c}} \right] \quad (3.6)$$

where we have absorbed the factor $(\cosh(\theta_0) - 1)/2$ in to the lapse N' .

Our goal is to study the dynamics of Kantowski-Sachs and higher genus black hole interior spacetimes in the presence of cosmological constant. Since the forms of the gravitational Hamiltonians for the Kantowski-Sachs as discussed in the previous chapter and the Bianchi-III LRS spacetime (higher genus black hole interior) are very similar, we can write them together in the following form by setting N and N' to unity, and add a term corresponding to the cosmological constant. We obtain the classical Hamiltonian as:

$$\mathcal{H}_{cl} = \frac{-1}{2G\gamma^2} \left[2bc\sqrt{p_c} + (b^2 + k\gamma^2) \frac{p_b}{\sqrt{p_c}} \right] + 4\pi p_b\sqrt{p_c} \rho_\Lambda \quad (3.7)$$

where $k = 1$ for the Kantowski-Sachs spacetime and $k = -1$ for the higher genus black hole interior, and $\rho_\Lambda = \Lambda/8\pi G$ with Λ allowed to have both signs.

²In higher genus black hole interior spacetimes the area of constant $t - x$ surface is given by $\int_0^{2\pi} \int_0^{\theta_0} \sinh(\theta) d\theta d\phi = 2\pi(\cosh(\theta_0) - 1)$.

Using the Hamilton's equations, the classical equations of motion for the symmetry reduced connection and triads turn out to be,

$$\dot{p}_b = -G\gamma \frac{\partial \mathcal{H}}{\partial b} = \frac{1}{\gamma} \left(c\sqrt{p_c} + \frac{bp_b}{\sqrt{p_c}} \right) \quad (3.8)$$

$$\dot{p}_c = -2G\gamma \frac{\partial \mathcal{H}}{\partial c} = \frac{1}{\gamma} 2b\sqrt{p_c} \quad (3.9)$$

$$\dot{b} = G\gamma \frac{\partial \mathcal{H}}{\partial p_b} = \frac{-1}{2\gamma\sqrt{p_c}} (b^2 + k\gamma^2) + 4\pi G\gamma\sqrt{p_c}\rho_\Lambda \quad (3.10)$$

$$\dot{c} = 2G\gamma \frac{\partial \mathcal{H}}{\partial p_c} = \frac{-1}{\gamma\sqrt{p_c}} \left(bc - (b^2 + k\gamma^2) \frac{p_b}{2p_c} \right) + 4\pi\gamma G p_b \frac{\rho_\Lambda}{\sqrt{p_c}} \quad (3.11)$$

where the 'dot' refers to the derivative with respect to proper time.

In the classical theory, unless the matter violates weak energy condition, evolution determined by the above equations generically leads to a singularity. For matter satisfying weak energy condition, there are two special but highly fine tuned cases which lead to a singularity free spacetime. These two cases are allowed by demanding that $b = 0$ at all times for: (i) the positive cosmological constant for $k = 1$, and (ii) the negative cosmological constant for $k = -1$. The first case leads to the classical uncharged Nariai spacetime which is topologically $dS_2 \times \mathbb{S}^2$, which is discussed in Ref. [41]. It is straightforward to see from the above equations that the second case leads to the uncharged anti Nariai spacetime which has a topology $AdS_2 \times \mathbb{H}^2$. These spacetimes are non-singular in the classical theory [51]. In general, when $b = 0$ is not assumed, for arbitrary values of cosmological constant, the evolution is singular. The same is true if instead of ρ_Λ one chooses energy density sourced with perfect fluids or matter fields obeying weak energy condition. In the backward or the forward evolution from a finite volume at a small spacetime curvature, the behavior of the triads and connections is such that the spacetime curvature becomes infinite in a finite time when the physical volume approaches zero.

3.2 Effective loop quantum dynamics

In the loop quantization, the elementary variables are the holonomies of the connection and the fluxes of the corresponding triads. In the case of the homogeneous spacetimes, fluxes turn out to be proportional to triads, thus the key modification, in terms of the variables, from the classical to the quantum theory appears in the usage of holonomies, which are the trigonometric functions of the connection components. The Hamiltonian constraint expressed in terms of holonomies

and then quantized results in a quantum difference equation in the triad representation with discreteness determined by the minimum area in LQG. The physical states obtained as the solutions of the quantum Hamiltonian constraint exhibit non-singular evolution, a result which is a direct manifestation of the underlying quantum geometry. Interestingly, the underlying quantum dynamics can be captured using an effective Hamiltonian constraint. As discussed in the previous chapter, the effective dynamics, derived from the Hamilton's equations, has been shown to capture the quantum evolution to an excellent degree of accuracy in different models at all the scales. In the following, we assume the validity of the effective spacetime description for the Kantowski-Sachs and higher genus black hole interior spacetimes.

In the effective spacetime description, the LQC Hamiltonian for Kantowski-Sachs and the higher genus black hole spacetime can be written as [40, 47]:

$$\mathcal{H} = \frac{-1}{2G\gamma^2} \left[2 \frac{\sin(b\delta_b)}{\delta_b} \frac{\sin(c\delta_c)}{\delta_c} \sqrt{p_c} + \left(\frac{\sin^2(b\delta_b)}{\delta_b^2} + k\gamma^2 \right) \frac{p_b}{\sqrt{p_c}} \right] + 4\pi p_b \sqrt{p_c} \rho_\Lambda, \quad (3.12)$$

where $k = +1$ and -1 for the Kantowski-Sachs spacetime and the Bianchi-III LRS/higher genus black hole spacetime. Here δ_b and δ_c are the functions of triads, whose exact form is dictated by the loop quantization. For the quantization corresponding to the improved dynamics prescription in LQC [5], these are given by

$$\delta_b = \sqrt{\Delta} \frac{1}{p_c^{1/2}}, \quad \delta_c = \sqrt{\Delta} \frac{p_c^{1/2}}{p_b} \quad (3.13)$$

where $\Delta = 4\sqrt{3}\pi\gamma l_{\text{Pl}}^2$ corresponds to the minimum area eigenvalue in LQG. Recently, this quantization prescription has been shown to be the unique choice which leads to physical predictions independent of the choice of the fiducial length L_o , and yield universal bounds on expansion and shear scalars for the geodesics in the effective spacetime description in the Kantowski-Sachs model for arbitrary matter [46]. These properties, which are not shared by other possible quantization prescriptions, also hold true for the Bianchi-III LRS spacetimes as shown later in this section.

To obtain the effective dynamics, we first obtain the Hamilton's equations from (3.12) which are then numerically solved. The resulting Hamilton's equations are:

$$\dot{p}_b = \frac{p_b \cos(b\delta_b)}{\gamma\sqrt{\Delta}} (\sin(c\delta_c) + \sin(b\delta_b)), \quad (3.14)$$

$$\dot{p}_c = \frac{2p_c}{\gamma\sqrt{\Delta}} \sin(b\delta_b) \cos(c\delta_c) \quad (3.15)$$

$$\dot{b} = \frac{cp_c}{p_b\gamma\sqrt{\Delta}} \sin(b\delta_b) \cos(c\delta_c) \quad (3.16)$$

$$\dot{c} = \frac{1}{\gamma\sqrt{\Delta}} \left(\cos(b\delta_b) (\sin(b\delta_b) + \sin(c\delta_c)) \frac{bp_b}{p_c} - c \sin(b\delta_b) \cos(c\delta_c) \right) + \frac{\gamma k p_b}{p_c^{3/2}}. \quad (3.17)$$

where the ‘dot’ refers to the derivative with respect to the proper time τ , and we have used the Hamiltonian constraint $\mathcal{H} \approx 0$ to simplify the equations.

In a similar way, modified Hamilton’s equations can be derived for arbitrary matter energy density. Unlike the classical theory, the solutions from these equations are non-singular. Various properties of singularity resolution for the vacuum case were studied in Refs. [32, 40]. The case of the massless scalar field for $k = 1$ is studied in Ref. [41], and the case of $k = -1$ was earlier investigated in Ref. [47]. General properties of Kantowski-Sachs model and the issues of singularities are discussed in [46], and details of the evolution for different types of matter are studied in Ref. [45]. In particular, there are no divergences in expansion and shear scalars in the Kantowski-Sachs model, thus pointing towards the nonsingular nature of the spacetime [46]. This result can be easily generalized to the higher genus black hole interior as well, so it is worthwhile to discuss it further.

The expansion of a geodesic congruence can be written in terms of triads as

$$\theta = \frac{\dot{V}}{V} = \frac{\dot{p}_b}{p_b} + \frac{\dot{p}_c}{2p_c}. \quad (3.18)$$

where the derivative is with respect to proper time. Thus for the expansion scalar to be bounded, $\frac{\dot{p}_b}{p_b}$ and $\frac{\dot{p}_c}{p_c}$ have to be bounded. From eqs.(3.14) and (3.15), we note that the relative rate of change of triads with respect to proper time is the same for Kantowski-Sachs spacetime and the higher genus black hole interior (the difference due to the sign of curvature affects only the \dot{c} equation). Using these equations, the expansion scalar can be obtained as

$$\theta = \frac{1}{\gamma\sqrt{\Delta}} (\sin(b\delta_b) \cos(c\delta_c) + \sin(c\delta_c) \cos(b\delta_b) + \sin(b\delta_b) \cos(b\delta_b)) \quad (3.19)$$

The dependence of expansion scalar on phase space variables is only through bounded functions. Thus there is a maxima that expansion scalar can reach, corresponding to saturation of the

trigonometric terms. Thus the expansion scalar in the Kantowski-Sachs or the higher genus black hole interior has a universal bound given as

$$|\theta| \leq \frac{3}{2\gamma\sqrt{\Delta}} \approx \frac{2.78}{l_{pl}}, \quad (3.20)$$

where we have used $\gamma \approx 0.2375$. The shear scalar which signifies the anisotropy seen by an observer following a geodesic congruence in Kantowski-Sachs or the higher genus black hole interior spacetimes can be written as

$$\sigma^2 = \frac{1}{2} \sum_{i=1}^3 \left(H_i - \frac{1}{3}\theta \right)^2 = \frac{1}{3} \left(\frac{\dot{p}_c}{p_b} - \frac{\dot{p}_b}{p_b} \right)^2, \quad (3.21)$$

where $H_i = \dot{a}_i/a_i$ are the directional Hubble rates. Using eqs.(3.14) and (3.15), we obtain

$$\sigma^2 = \frac{1}{3\gamma^2\Delta} (2 \sin(b\delta_b) \cos(c\delta_c) - \cos(b\delta_b) (\sin(c\delta_c) + \sin(b\delta_b)))^2. \quad (3.22)$$

As the expansion scalar, the shear scalar is also bounded with a universal bound [46]

$$|\sigma|^2 \leq \frac{5.76}{l_{pl}^2}. \quad (3.23)$$

The boundedness of both expansion and shear scalars point towards the geodesic completeness of the loop quantum model of Kantowski-Sachs and the higher genus black hole interior in the improved dynamics prescription (3.13). These results are in synergy with similar results in isotropic and Bianchi models in LQC [24, 23, 26, 27, 28, 59].

3.2.1 Kantowski-Sachs spacetime with a positive cosmological constant

We now discuss the results from the numerical simulations performed for positive cosmological constant using the effective dynamics of Kantowski-Sachs model. As discussed earlier, in this case the classical dynamics in general leads to a singularity where p_b and p_c vanish, and the spacetime curvature diverges. In LQC, the evolution is strikingly different. Starting from a large value of triad components and a small spacetime curvature, we find that p_b and p_c undergo bounces due to quantum gravitational modifications in the effective dynamics. The triads have different asymptotic behaviors as $t \rightarrow \pm\infty$. In all the numerical simulations that we carried out with a value of cosmological constant not greater than 0.1 in Planck units, it was observed that p_c expands exponentially in the asymptotic regime, after few bounces, in one of the directions in

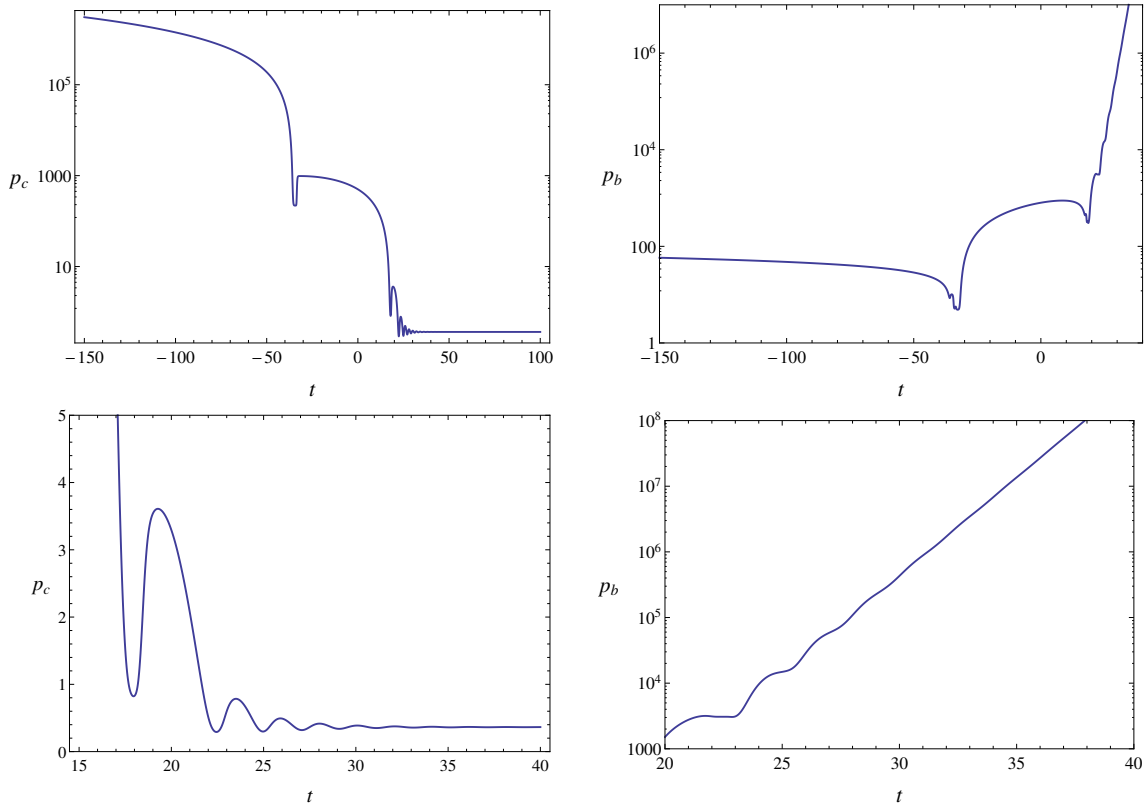


Figure 3.1: Triads for Kantowski-Sachs spacetime with positive cosmological constant.

time whereas it reaches a constant value in the other³. This behavior is shown in the top left plot of Fig. 3.1. In this figure, initial conditions are chosen at $t = 0$ as $p_b(0) = 8 \times 10^2$, $p_c(0) = 5 \times 10^2$, $b(0) = -0.15$, $\rho_\Lambda(0) = 10^{-7}$. (All values in the numerical simulations are in Planck units). Initial value of connection component c is obtained from the vanishing of the Hamiltonian constraint. The top plots show the behavior in the forward and the backward evolution, whereas the bottom plots show the zoomed in behavior of p_b and p_c in the future evolution for positive time. In the forward as well as the backward evolution, the triad p_b undergoes exponential expansion in proper time in the asymptotic regimes. The rate of exponential expansion for p_b in both of the asymptotic regimes turns out to be different. Interestingly, in the asymptotic region in which p_c is exponentially expanding, p_b has the same rate of expansion as p_c and this rate approaches a constant value. In this regime, the holonomy corrections are negligible and the spacetime behaves as a classical spacetime with a small spacetime curvature which is a

³For close to Planckian values of cosmological constant this may not be true. For rather large values of Λ , say greater than 0.1 (in Planck units), one finds de Sitter expansion after quantum bounce in both the directions of time.

solution of the classical Hamilton's equations for the Kantowski-Sachs model, eqs.(3.8-3.11) with a positive cosmological constant. The exponential behavior of the triads shows that the classical spacetime in this regime is a de Sitter spacetime. The other side of the temporal evolution, i.e. when for positive values of time, leads to a spacetime not having small spacetime curvature even long after the bounce. In this region, shown in the large t range of bottom plots in Fig. 3.1, while p_c takes a constant value in the asymptotic regime, p_b grows exponentially. The holonomy corrections are large, denoting that the quantum effects are very significant in this regime. This is evident from the left plot in Fig. 3.2, which shows that $\cos(c\delta_c)$ approaches zero, and hence $|\sin(c\delta_c)|$ is unity. The left plot shows the case for the positive cosmological constant, and the right plot shows for the negative cosmological constant. The initial conditions correspond respectively to those in Fig. 3.1. In the same regime, b is also finite and non-vanishing, and takes a constant value. It turns out that this asymptotic regime is *not* a solution of the classical Hamilton's equations of the Kantowski-Sachs model (eqs.(3.8-3.11)), which do not allow p_c to be a constant when b is non-vanishing. A more detailed characterization of this region, will be carried out in the next section. As we will show, the loop quantum spacetime in this regime is a product of constant curvature spaces with an effective metric which is a solution of Einstein field equations for a 'charged' Nariai spacetime.

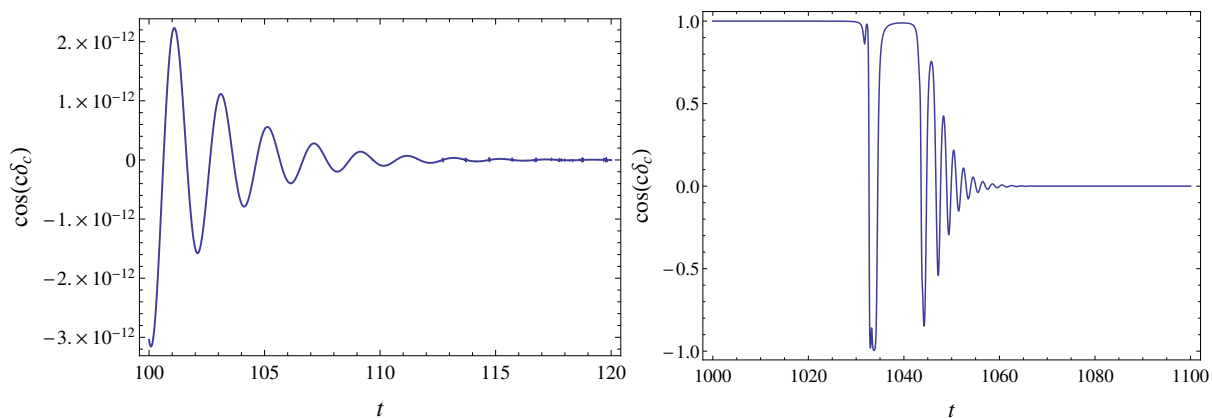


Figure 3.2: Behavior of $\cos(c\delta_c)$ is shown in the asymptotic regime where p_c is a constant.

3.2.2 Kantowski-Sachs spacetime with a negative cosmological constant

In the presence of the negative cosmological constant, the mean volume in the Kantowski-Sachs spacetime undergoes a recollapse at large scales in the classical theory when the expansion is

halted by the negative energy density pertaining to the cosmological constant. Due to the recollapse, the classical Kantowski-Sachs spacetime with a negative cosmological constant in general encounters a past as well as a future singularity with triads vanishing in a finite time. As a result of the quantum gravitational effects originating from the holonomy modifications in the effective Hamiltonian constraint, the classical singularity is avoided in the loop quantum dynamics, generically, as is in the case for the positive cosmological constant. In contrast to the latter case, the effective dynamics in LQC involves several cycles of bounces and recollapses. Thus, the evolution is cyclic. The behavior of the triads in each cycle is such that one triad grows whereas the other decreases. The physical volume oscillates between fixed maxima and minima thanks to the fixed potential due to the negative cosmological constant. An example of a typical evolution is shown in Fig. 3.3, where for the considered initial conditions, p_c increases through multiple bounces in the backward evolution whereas p_b decreases. The initial conditions are chosen at $t = 0$ as $p_b(0) = 8 \times 10^5$, $p_c(0) = 8 \times 10^5$, $b(0) = -0.05$, $\rho_\Lambda(0) = -10^{-8}$ (in Planck units). The behavior of the triads in this regime is such that the mean volume undergoes periodic cycles of expansion and contraction. In contrast, for the evolution in positive time, the series of bounce and recollapses damps down and p_c approaches a constant value in the asymptotic regime. In the same regime, p_b grows exponentially. As in the case of the positive cosmological constant, holonomy corrections are significant in this regime, as can be seen in the right plot in Fig. 3.2. The regime where p_c approaches a constant value asymptotically is not a solution of the classical Hamilton's equations for the Kantowski-Sachs spacetime. Further details of this asymptotic solution are discussed in the next section.

3.2.3 Bianchi-III LRS spacetime with a negative cosmological constant

The evolution of triads in the Bianchi-III LRS spacetime/higher genus black hole interior with a negative cosmological constant is cyclic in nature, similar to the case earlier discussed for the Kantowski-Sachs spacetime. An earlier study of this spacetime in LQC was performed in Ref. [47], where its non-singular properties were first noted. It turns out that to carry out the numerical simulations in this case, it is more convenient to work with a lapse $N = 1/p_b\sqrt{p_c}$.

For any given choice of initial conditions, on one side of the temporal evolution, the triad p_c undergoes several bounces and recollapses until it approaches an asymptotic value. In this

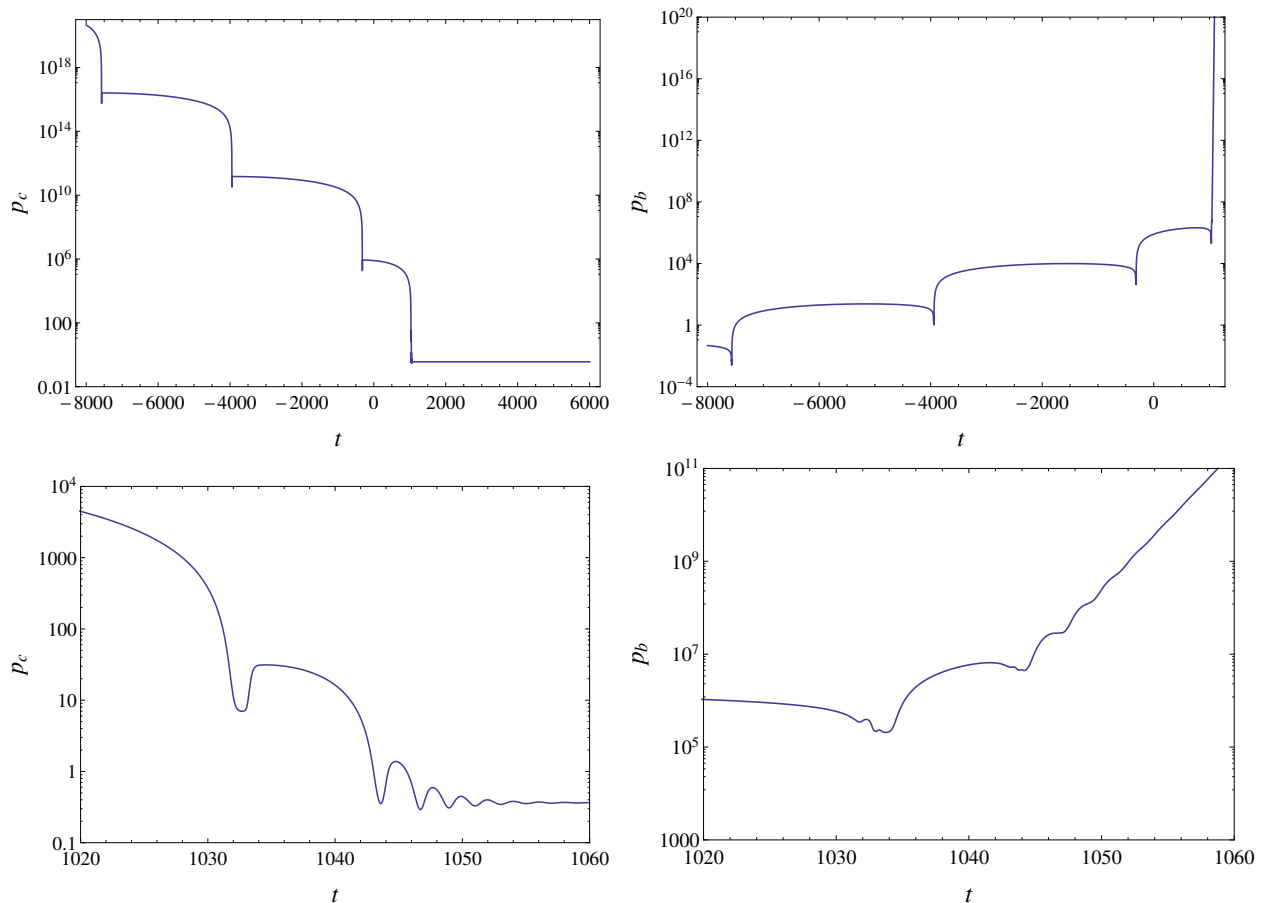


Figure 3.3: Triads in Kantowski-Sachs model sourced with a negative cosmological constant.

regime, p_b increases linearly in coordinate time for lapse $N = 1/p_b\sqrt{p_c}$. One finds that p_b grows exponentially in this regime with respect to the proper time, whereas p_c attains a constant value. For the initial conditions in Fig. 3.4, we show that such an asymptotic region occurs in positive time (see also Fig. 3.5). The lapse for the simulations of this case is chosen to be $N = 1/p_b\sqrt{p_c}$. The initial conditions are chosen at $t = 0$ as $p_b(0) = 8 \times 10^5$, $p_c(0) = 8 \times 10^5$, $b(0) = 0.05$, $\rho_\Lambda(0) = -10^{-8}$ (in Planck units). Note that in contrast to the Kantowski-Sachs spacetime with a negative cosmological constant, the transitions are not as smooth which can be seen in the zoomed version of p_b and p_c in the above figure. In the negative time, p_b decreases whereas p_c increases after several cycles of bounces and recollapse as is shown in Fig. 3.4. For large positive time, the mean volume of the spacetime increases in each cycle of loop quantum bounce and the classical recollapse. Finally, we note that in the asymptotic regime where p_c attains a constant value, holonomy corrections are significant similar to the case of asymptotic spacetime in the loop quantum model of Kantowski-Sachs spacetime shown in Fig. 3.2. As in the case of the

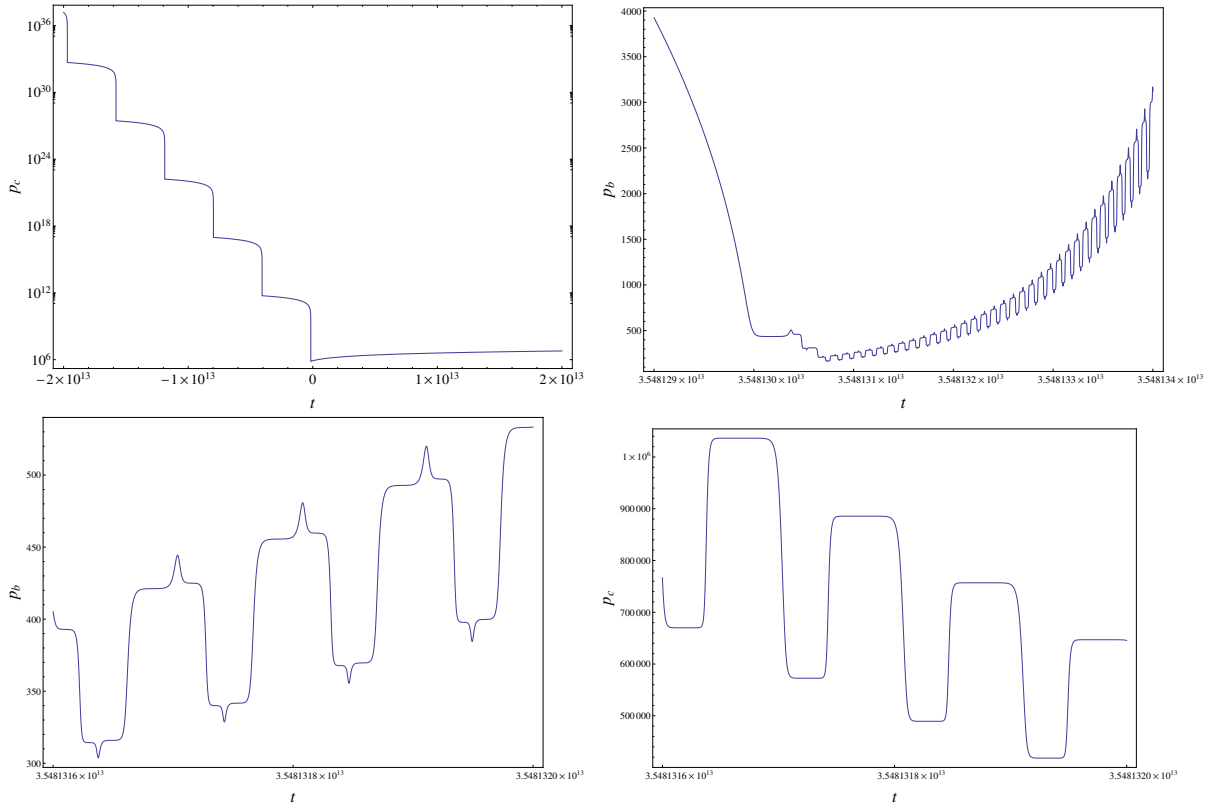


Figure 3.4: Triads for negative cosmological constant in higher genus black hole spacetime.

Kantowski-Sachs spacetime, this regime is not a solution of the classical Hamilton's equations for the Bianchi-III LRS model with a negative cosmological constant. In the following section we discuss further properties of this asymptotic regime.

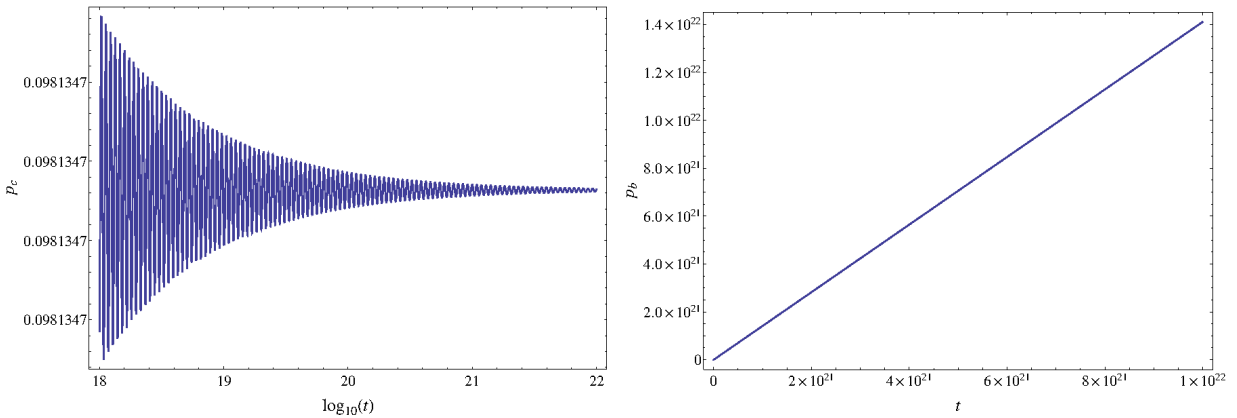


Figure 3.5: Triads for the higher genus black hole interior is shown in the asymptotic regime.

3.3 Properties of the asymptotic spacetime with a constant p_c

The main conclusion from the numerical simulations of the Kantowski-Sachs spacetimes with a positive and a negative cosmological constant, and the Bianchi-III LRS spacetime (or the higher genus black hole spacetime) with a negative cosmological constant is that the spacetime in one side of the temporal evolution after the singularity resolution has a constant value of triad p_c . Let us look at some of the key features of this asymptotic regime. As was illustrated in Fig. 3.2, in this regime $\cos(c\delta_c) = 0$. Thus, $c\delta_c$ takes a constant value, and eq.(3.15) implies that $\dot{p}_c = 0$. Further, using eq.(3.16) we find that b is a constant in proper time for this asymptotic regime in Kantowski-Sachs model as well as the higher genus black hole interior. Since $\delta_b = \sqrt{\Delta/p_c}$, in this regime $b\delta_b$ is also a constant. Using the constancy of $b\delta_b$ and $c\delta_c$ in (3.14) one finds that $\frac{\dot{p}_b}{p_b}$ is a constant. That is, p_b expands exponentially in proper time, both for the Kantowski-Sachs and higher genus black hole spacetimes. For the case of the higher genus black hole interior, where numerical simulations were carried with lapse $N = 1/p_b\sqrt{p_c}$, p_b is a constant with respect to the coordinate time in the asymptotic regime. Note that since $\delta_c = \sqrt{\Delta}p_c^{1/2}/p_b$, for $c\delta_c$ to be a constant, $\frac{c}{p_b}$ has to be a constant. Hence c also exhibits an exponential behavior with respect to proper time. In the simulations discussed in the previous section, one finds that both c and p_b expand exponentially in such a way that $\frac{c}{p_b}$ is a constant in the asymptotic regime.

Since the triads are related to the metric components, knowing the asymptotic behavior of the triads allows us to find the asymptotic behavior of the metric components. The radius of the 2-sphere part, $g_{\theta\theta} = p_c$, hence has the same asymptotic value as the triad p_c . Setting this asymptotic value to be R_0^2 , we have asymptotically

$$g_{\theta\theta}(\tau) = R_0^2, \tag{3.24}$$

a constant value, where τ is the proper time. Also, it is obvious that in the asymptotic region, $g_{\phi\phi} = R_0^2 \sin^2 \theta$ for the Kantowski-Sachs spacetime and $g_{\phi\phi} = R_0^2 \sinh^2 \theta$ for the Bianchi III LRS spacetime.

In the asymptotic regime, while the triad p_c is constant, triad p_b is expanding exponentially. Let us first consider the case when one reaches this asymptotic region in the forward evolution. Then, asymptotically $p_b = p_b^{(0)} e^{\alpha\tau}$, where α is a positive constant. The coefficient $p_b^{(0)}$ is a positive constant which formally has the following meaning. If the exponential expansion began

at some proper time τ_0 , with the initial value of p_b as $p_b^{(i)}$, then, $p_b^{(0)} = \frac{p_b^{(i)}}{e^{\alpha\tau_0}}$. Note that this is not the initial value of p_b where the numerical evolution was started with at $\tau = 0$. Since p_b scales linearly with the fiducial length, $p_b^{(0)}$ is also a fiducial cell dependent quantity. If the asymptotic region with the exponential behavior of p_b had been in the backward evolution, then α would have been a negative quantity and p_b would increase exponentially as $\tau \rightarrow -\infty$. Either way,

$$g_{xx}(\tau) = \frac{p_b^2}{L_0^2 p_c} = \frac{(p_b^{(0)})^2 e^{2\alpha\tau}}{L_0^2 R_0^2}. \quad (3.25)$$

Note that since $p_b^{(0)}$ depends on the fiducial length linearly, g_{xx} is independent of the fiducial cell. Once we have the metric components from (3.24) and (3.25), we can analyze the properties of these asymptotic spacetimes and ask if they satisfy the Einstein equations for some $T_{\mu\nu}$. We do this separately for the Kantowski-Sachs spacetime and the Bianchi III LRS spacetime in the following subsections.

We can now obtain the effective metric of the asymptotic spacetime for the Kantowski-Sachs case with a positive and a negative cosmological constant, and the higher genus black hole interior with a negative cosmological constant. Using equations (3.24) and (3.25), the line element of the emergent asymptotic spacetime in the LQC evolution of Kantowski-Sachs spacetime can be written as

$$ds^2 = -d\tau^2 + \frac{(p_b^{(0)})^2 e^{2\alpha\tau}}{L_0^2 R_0^2} dx^2 + R_0^2 (d\theta^2 + \sin^2 \theta d\phi^2). \quad (3.26)$$

Since $\tau \rightarrow \infty$ in the asymptotic region, it is permitted to substitute $e^{\alpha\tau}$ with $\cosh(\alpha\tau)$. Additionally, using the redefinition $x \rightarrow \frac{p_b^0}{L_0 R_0} x$, the line element can be written as

$$ds^2 = -d\tau^2 + \cosh^2(\alpha\tau) dx^2 + R_0^2 (d\theta^2 + \sin^2 \theta d\phi^2). \quad (3.27)$$

The above line element (3.27) is formally similar to the Nariai metric written in its homogeneous form [51, 52, 40], with the difference that for the Nariai metric $\alpha = \frac{1}{R_0}$. This formal similarity had led the earlier authors [40] to call the asymptotic spacetime obtained as Nariai-type spacetime. In classical GR, for $\alpha \neq 1/R_0$, the spacetime metric corresponds to a charged Nariai spacetime with a uniform electromagnetic field. Thus, the effective metric obtained above formally corresponds to a charged Nariai solution of the Einstein's theory. Such spacetimes are often discussed in literature [60, 61, 62] in static coordinates. One can go to those coordinates using the

transformations $k\tau \rightarrow \sinh(k\tau)$, $R = i\tau$ and $T = ix$. The line element (3.27) becomes

$$ds^2 = -(1 - k^2 R^2) dT^2 + \frac{dR^2}{1 - k^2 R^2} + R_0^2 (d\theta^2 + \sin^2 \theta d\phi^2). \quad (3.28)$$

One can do yet another coordinate transformation $k^2 = \frac{k_0}{R_0^2}$ and $\sin^2 \chi = 1 - k^2 R^2$, to write the static line element in the form used in the references [60, 61, 62],

$$ds^2 = \frac{R_0^2}{k_0} (-\sin^2(\chi) d\tau^2 + d\chi^2) + R_0^2 (d\theta^2 + \sin^2(\theta) d\phi^2). \quad (3.29)$$

Thus we see that the asymptotic spacetime we obtain in the evolution of Kantowski-Sachs spacetime corresponds to a classical charged Nariai spacetime. However, the quantum spacetime is different from these classical charged spacetimes in the sense that the ‘charge’ in LQC evolution of Kantowski-Sachs spacetime is purely of quantum geometric origin and is not electromagnetic.

Similarly, one can write the line element for the asymptotic constant p_c spacetime obtained in the LQC evolution of Bianchi III LRS spacetime (or higher genus black hole interior) with negative cosmological constant using (3.24) and (3.25) together with $e^{\alpha\tau} \rightarrow \cosh(\alpha\tau)$ and $x \rightarrow \frac{p_b^0}{L_0 R_0} x$ as,

$$ds^2 = -d\tau^2 + \cosh^2(\alpha\tau) dx^2 + R_0^2 (d\theta^2 + \sinh^2 \theta d\phi^2). \quad (3.30)$$

Following the coordinate transformations used to reach from (3.27) to (3.29), one can write the line element (3.30) in the static coordinates as

$$ds^2 = \frac{R_0^2}{k_0} (-\sin^2(\chi) d\tau^2 + d\chi^2) + R_0^2 (d\theta^2 + \sinh^2 \theta d\phi^2). \quad (3.31)$$

This metric corresponds the anti-Bertotti-Robinson spacetime, which in classical GR is an electrovacuum solution. As in the case of emergent ‘charged’ Nariai spacetime discussed above, even though classically these spacetimes are solutions of Einstein equations with matter as uniform electromagnetic field in the loop quantum case the energy momentum tensor in these emergent spacetime originates from the quantum geometry and is not electromagnetic in origin.

‘Charged’ Nariai and anti-Bertotti-Robinson spacetimes that emerge in LQC evolution of black hole interiors are product of constant curvature spaces. ‘Charged’ Nariai spacetime has a topology of $dS^2 \times \mathbb{S}^2$ where as the anti Bertotti-Robinson spacetime has the $dS^2 \times \mathbb{H}^2$ topology. Thus, both are product manifolds with each of the two manifolds ($t-x$ manifold whose curvature may be denoted by k_+ and $\theta - \phi$ manifold whose curvature may be denoted by k_-) having

constant curvature. For a discussion of these product of constant curvature manifolds in classical general relativity we refer the reader to [49].⁴ For classical charged Nariai spacetime (as well as for the quantum ones with the non electric or magnetic charge), both the $t - x$ manifold and $\theta - \phi$ manifold have positive curvatures. The special case of both these curvatures being equal $k_+ = k_- > 0$ corresponds to the ‘uncharged’ Nariai spacetime (or just Nariai spacetime as it is commonly referred to in the literature). For anti-Bertotti-Robinson spacetime, k_+ is positive where as k_- is negative. The special case of $k_+ = -k_-$ corresponds to a spacetime with no cosmological constant and is a conformally flat spacetime. There are two other spacetimes (anti-Nariai and Bertotti-Robinson spacetimes) for different choices of the sign of k_+ and k_- as tabulated below.

Table 3.1: Some features of (anti) Nariai and (anti) Bertotti-Robinson spacetimes.

| Type | Topology | R_t^t | R_2^2 | $\tilde{\Lambda}$ |
|------------------------|--------------------|---------|---------|-------------------|
| Nariai | $dS_2 \times S^2$ | +ve | +ve | +ve |
| Anti-Nariai | $AdS_2 \times H^2$ | -ve | -ve | -ve |
| Bertotti-Robinson | $AdS_2 \times S^2$ | -ve | +ve | any sign |
| Anti-Bertotti-Robinson | $dS_2 \times H^2$ | +ve | -ve | any sign |

In the present context, the necessary condition to respect positivity of energy is $k_- > k_+$. This is never true in anti-Bertotti-Robinson spacetimes as $k_- < 0$ and $k_+ > 0$, and hence they always violate positive energy conditions. Even though negative energy solutions of Einstein equations are deemed unphysical, it is plausible that in scenarios where quantum geometric effects are important, the energy density may be allowed to be negative. Such a scenario where a modified gravity theory gives rise to negative energy density has been observed earlier in the context of brane world models [64, 65].

3.4 Emergent ‘charge’ and cosmological constant in loop quantum cosmology

The charged Nariai and anti Bertotti-Robinson solutions in classical general relativity have electromagnetic field as its matter content. The asymptotic spacetimes obtained in the loop

⁴Nariai and Bertotti-Robinson spacetimes have also been shown to be solutions of modified Einsteinian gravity in presence of one loop quantum corrections to the stress energy tensor, see for eg. Ref.[63].

quantum evolution of Kantowski-Sachs and Bianchi III LRS spacetimes are identical to these classical solutions as far as the metric and curvature components are concerned. Hence from a computational point of view determining the energy momentum tensor of these emergent spacetimes follow that of the corresponding general relativistic spacetime. However one must be cautious in interpreting the nature of this energy momentum tensor. Specifically, one should not attribute the energy momentum tensor to the existence of a uniform electromagnetic field as one would do in the classical geometrodynamical setting. The ‘field’ and ‘charge’ arising in loop quantum evolution of black hole interiors is not electromagnetic but quantum geometric. It is plausible that these quantum spacetimes are sourced by some energy momentum tensor of quantum gravitational origin but similar to that of the uniform electromagnetic field in its effect on curvature. This is the sense in which we use the word ‘charge’ or ‘field’ to describe the matter content of emergent spacetimes. The situation is similar to the case of the bulk/brane models, where the Weyl curvature of the bulk induces a ‘charge’ on the brane [64]. Further studies are required to exactly pin point the nature of this ‘charge’ and to determine if this charge is identical to the electromagnetic charge in geometrodynamics. For now, we adopt the techniques of geometrodynamics [53] for computational purposes, relegating the interpretation of the ‘charge’ to future works. In the following subsection we summarize the calculation of energy momentum tensor and charge in a classical setting, which then will be extended to the calculation of non electric (or magnetic) charge in the quantum spacetimes obtained in the evolution of black hole interiors.

Let us first consider the classical aspects of charged Nariai and anti-Bertotti-Robinson spacetimes. In classical geometrodynamics, with a vanishing cosmological constant, for a spacetime to admit a uniform electromagnetic field, the Ricci tensor components must satisfy $R_0^0 = R_1^1 = -R_2^2 = -R_3^3$. For a spacetime satisfying above conditions, the electromagnetic field tensor that satisfies the Einstein equations

$$R_j^i - \frac{R}{2}g_j^i = \frac{1}{4\pi} \left(F^{ik}F_{jk} - \frac{F^{kl}F_{kl}}{4}g_j^i \right) \quad (3.32)$$

can be found uniquely from the Ricci tensor [53]. This result was extended to the case in the presence of a non-zero cosmological constant by Bertotti [66].

The electromagnetic field tensor in charged Nariai spacetime (3.29) is [62, 60]

$$F = q \sin \theta d\theta \wedge d\phi \quad (3.33)$$

when the field is purely magnetic and

$$F = \frac{q}{k_0} \sin \chi d\tau \wedge d\chi \quad (3.34)$$

when field is purely electric. Here q is the electric or magnetic charge. In geometrodynamics, q is purely geometric in origin and has nothing to do with the quantized charge, of say an electron [53]. The electromagnetic field is constant in spacetime and hence one might expect that there is no localized charge. This is indeed true, and one can see that the 4-current vanishes as $F^{ik};_k = 0$. Thus the field can be visualized as created by effective charges at the boundary of the spacetime [49]. The energy momentum of the field is [60]

$$\text{diag}(T_j^i) = \left(\frac{-q^2}{8\pi R_0^4}, \frac{-q^2}{8\pi R_0^4}, \frac{q^2}{8\pi R_0^4}, \frac{q^2}{8\pi R_0^4} \right). \quad (3.35)$$

Since the above energy momentum tensor is traceless, the cosmological constant in the charged Nariai spacetime is determined by the Ricci scalar. The energy momentum tensor and cosmological constant of anti-Bertotti-Robinson spacetime can also be found in the same way. The only difference for anti Bertotti-Robinson spacetime from the above calculation is that one should use $\sinh \theta$ instead of $\sin \theta$ in (3.33) and $-q^2$ instead of q^2 in (3.35).

The emergent loop quantum spacetimes obtained in Sec. III have the same curvature and Einstein tensors as the one discussed in the classical theory above. The difference however is that for these quantum spacetimes the energy momentum tensor in (3.35) is purely of quantum geometric origin. Thus the electric or magnetic charge q in (3.35) should be substituted with an emergent ‘charge’ \tilde{q} which is neither electric nor magnetic. Similarly the cosmological constant is also of quantum geometric origin and we refer to it as $\tilde{\Lambda}$. The emergent ‘charge’ \tilde{q} and $\tilde{\Lambda}$ is related to k_0 and R_0 in (3.29) as in the classical case [60]

$$\tilde{q}^2 = \frac{k - k_0}{2} R_0^2 = \frac{k - \alpha^2 R_0^2}{2} R_0^2. \quad (3.36)$$

and

$$\tilde{\Lambda} = \frac{k + k_0}{2R_0^2} = \frac{k + \alpha^2 R_0^2}{2R_0^2}. \quad (3.37)$$

where $k = +1$ for the ‘charged’ Nariai spacetime and $k = -1$ for the anti-Bertotti-Robinson spacetime. In the latter case, since the electric energy density is negative, the corresponding

emergent charge satisfies $\tilde{q}^2 < 0$. Note that this interpretation is based on the appropriate choice of splitting of the constant curvatures k_+ and k_- giving rise to a mixture of Nariai and (anti) Bertotti-Robinson spacetimes.

Using the asymptotic values of α^2 and R_0^2 obtained numerically from the LQC evolution in Kantowski-Sachs and the higher genus black hole interior, the values of emergent ‘charge’ \tilde{q} and emergent cosmological constant $\tilde{\Lambda}$ can be computed. For the Kantowski-Sachs case, with Λ in eq.(3.12) zero, the the emergent ‘charge’ and cosmological constant of the ‘charged’ Nariai spacetime in geometrized units evaluates to $\tilde{q}^2 = 0.151$ and $\tilde{\Lambda} = 1.610$. The emergent ‘charge’ and cosmological constant change by less than 0.1% of these vacuum values for small magnitudes of cosmological constant ($|\rho_\Lambda| < 10^{-6}$). For higher values of $|\rho_\Lambda|$, the change in these emergent quantities from the values in the vacuum case are more appreciable. It was found that both the emergent ‘charge’ and the cosmological constant varies monotonically with ρ_Λ , such that \tilde{q}^2 decreases and $\tilde{\Lambda}$ increases with increasing ρ_Λ . For example, for $\rho_\Lambda = -10^{-2}$, we find $\tilde{q}^2 = 0.164$ and $\tilde{\Lambda} = 1.487$ in geometrized units. The same quantities turn out to be 0.139 and 1.734 respectively for $\rho_\Lambda = 10^{-2}$. Further work is required to find the precise dependence of these quantities on ρ_Λ . Also note that these quantities depend on the choice of Δ as any change in the holonomy edge length will result in a change in asymptotic values of p_c and $\frac{\dot{p}_b}{p_b}$.

We have already seen that the electric energy density developed in the anti-Bertotti-Robinson spacetime obtained in the evolution of Bianchi III LRS/higher genus black hole interior is negative. It turns out that the emergent cosmological constant is also negative for these spacetimes. Once again, the values of these emergent quantities depend on ρ_Λ . Unlike the case of Kantowski-Sachs spacetime, constant p_c asymptotic regime occurs only if the cosmological constant is negative for the Bianchi III LRS spacetime in LQC evolution. In this case, \tilde{q}^2 decreases with increasing (decreasing magnitude) ρ_Λ whereas $\tilde{\Lambda}$ increases. For example, for $\rho_\Lambda = -1 \times 10^{-8}$, we get $\tilde{q}^2 = -0.050$ and $\tilde{\Lambda} = -4.997$ in geometrized units, whereas for $\rho_\Lambda = -1 \times 10^{-2}$, $\tilde{q}^2 = -0.053$ and $\tilde{\Lambda} = -4.647$. The dependence of both the emergent quantities were seen to be approximately linear with ρ_Λ , as in the case for emergent cosmological constant in LQC evolution of Kantowski-Sachs model.

In summary, the emergent asymptotic spacetimes in the loop quantum evolution of Kantowski-Sachs spacetime and Bianchi III LRS spacetime are a product of constant curvature spaces

whose matter content is parameterized by an emergent ‘charge’ and cosmological constant, both of which are of quantum gravitational origin. The emergent charge in these spacetimes should not be a priori identified with electric or magnetic charge but is a result of quantum geometry. This is similar to the emergence of a ‘tidal charge’ in the brane world scenario discussed in [64] arising from the projection of the Weyl tensor of a 5 dimensional brane world model on to a four dimensional brane.

3.5 ‘Uncharged’ Nariai spacetime and $\tilde{\Lambda} = 0$ anti-Bertotti-Robinson spacetimes

A natural question to be posed in the light of last section is if loop quantum evolution of Kantowski-Sachs spacetime can lead to ‘uncharged’ Nariai spacetime which corresponds to the special case of $k_+ = k_-$ i.e, a spacetime where all the diagonal components of R_j^i are equal. Similarly one could also ask if the special anti Bertotti-Robinson solution with $k_+ = -k_-$ (which corresponds to $\tilde{\Lambda} = 0$) may emerge from LQC evolution of Bianchi III LRS spacetime/higher genus black hole interior.

Let us rewrite the loop quantum Hamiltonian constraint (3.12) for the Kantowski-Sachs spacetime and for the Bianchi III LRS/higher genus black hole interior spacetime as

$$\sin^2(b\delta_b) + 2 \sin(b\delta_b) \sin(c\delta_c) + \beta = 0 \quad (3.38)$$

where $\beta = \gamma^2 \Delta (k/p_c - \Lambda)$ with $k = 1$ for Kantowski-Sachs spacetime, and $k = -1$ for the higher genus black hole. Any solution with $\beta < -3$ is unphysical as it does not solve the Hamiltonian constraint.

We now check what additional conditions should be satisfied for a constant p_c spacetime to exist. Since $\cos(c\delta_c) = 0$ for derivative of p_c to vanish, $\sin(c\delta_c) = \pm 1$ in the constant p_c regime. Hence, the Hamiltonian constraint for the constant p_c regime is a quadratic equation in $\sin(b\delta_b)$,

$$\sin^2(b\delta_b) \pm 2 \sin(b\delta_b) + \beta = 0 \quad (3.39)$$

Existence of a real solution requires $\beta \leq 1$. Hence β is constrained to lie between -3 and 1. It is convenient to introduce $x = \sqrt{1 - \beta}$ which varies from 0 to 2, in terms of which α^2 and $1/p_c$ become:

$$\alpha^2 = \frac{x^2}{\gamma^2 \Delta} (2x - x^2), \quad \text{and} \quad \frac{1}{p_c} = \Lambda + \frac{k(1 - x^2)}{\gamma^2 \Delta}. \quad (3.40)$$

For the curvature components to satisfy $k_+ = k_-$ or $k_+ = -k_-$, or equivalently $\alpha^2 = \pm \frac{1}{p_c}$, the following equation needs to be satisfied

$$x^4 - 2x^3 + k(-x^2 + 1 + \gamma^2 \Delta \Lambda) = 0 . \quad (3.41)$$

The function $f(x) = x^4 - 2x^3 - kx^2$ with $x \in [0, 2]$ has a range $[-4, 0]$ for $k = 1$ and a range $[0, 4]$ for $k = -1$. Hence, the plausible range of cosmological constant is

$$\frac{-1}{\gamma^2 \Delta} \leq \Lambda \leq \frac{3}{\gamma^2 \Delta} \quad (3.42)$$

for the existence of both ‘uncharged’ Nariai spacetime as well as anti Bertotti-Robinson spacetime with $\tilde{\Lambda} = 0$. So, for a Λ lying in the above range, one may find a suitable value of x which will yield a spacetime with equal magnitudes for all the non-vanishing Ricci components.

Another condition which has to be satisfied for any constant p_c spacetime is that $c\delta_c$ should be a constant. This is satisfied when c/p_b is a constant, which yields (using (3.14) and (3.17))

$$1 - x + \cos\left(\frac{\gamma^2 \Delta \Lambda + 1 - x^2}{x\sqrt{2x - x^2}}\right) = 0. \quad (3.43)$$

Since we need to solve equations (3.41) and (3.43) simultaneously, we obtain

$$1 - x + \cos(x\sqrt{2x - x^2}) = 0. \quad (3.44)$$

One of the solutions of the above equation is at $x = 2$, which corresponds to $\alpha = 0$ and such a spacetime will have constant p_b , not an exponentially expanding p_b . In fact such a spacetime will have all the triads and cotriads constant. Since we are interested in obtaining an ‘uncharged’ Nariai spacetime or anti-Bertotti-Robinson spacetime with a vanishing cosmological constant, this solution will not be considered. The second root of (3.44) was numerically found to be $x = 1.31646$. Note that this solution is true for both the Kantowski-Sachs and the higher genus black hole interior spacetimes. Knowing x , one can find the asymptotic values of p_c and α^2 as $p_c = 0.18697$ and $\alpha^2 = 5.34842$ in Planck units.

For the above values of asymptotic p_c and α , one can obtain an ‘uncharged’ Nariai solution in the LQC evolution of Kantowski-Sachs spacetime or anti Bertotti-Robinson spacetime in the LQC evolution of Bianchi III LRS spacetime. The cosmological constant needed for obtaining such a Nariai spacetime turns out to be 7.86251 in Planck units. This is a huge value, especially since the corresponding energy density is around three fourth of the critical energy density of

isotropic loop quantum cosmology. For Bianchi III LRS spacetime, the cosmological constant needed to obtain an anti-Bertotti-Robinson spacetime without emergent cosmological constant turned out to be -2.83432 in Planck units.

For these special spacetimes to emerge naturally from the loop quantum evolution of black hole interiors, the corresponding constant p_c regime should be stable, which turns out to be not the case [45]. The stability was tested by choosing initial conditions from the exact solution, such that the Hamiltonian constraint is satisfied, and then using LQC equations of motion to evolve the spacetime. It was found that the ‘uncharged’ Nariai spacetime evolves to a deSitter spacetime in both past and future evolution and thus is not stable. This does not come as surprise given the high value of positive cosmological constant, for which generic initial conditions lead to de Sitter spacetime in both the forward and backward evolution of Kantowski-Sachs universe in LQC. The anti-Bertotti-Robinson solution with vanishing emergent cosmological constant was also found to be unstable. It evolved in to an anti-Bertotti-Robinson solution with a smaller constant p_c (and thus a nonzero emergent cosmological constant) on one side where as p_c kept on increasing after each recollapse on the other side. In summary we did not find a stable emergent ‘uncharged’ Nariai spacetime or anti-Bertotti-Robinson spacetime with vanishing emergent cosmological constant in the loop quantum evolution of Kantowski-Sachs or Bianchi III LRS spacetimes. Thus whenever a constant p_c regime emerges from the quantum evolution of these spacetimes, there is an associated emergent ‘charge’ and an emergent cosmological constant. In the presented scenario spacetime which is a product of constant curvature spaces with equal magnitudes for all the non-vanishing Ricci tensor components seems to be disfavored in the LQC evolution of black hole interiors.

3.6 Discussion

Loop quantum evolution of Schwarzschild and higher genus black hole interiors were studied in a minisuperspace setting using their isometries to Kantowski-Sachs spacetime (in [40]) and Bianchi III LRS spacetime (in [47]) respectively. These studies found that the black hole interior spacetime undergoes a quantum bounce and the classical central singularity is resolved, similar to the singularity resolution in other loop quantum cosmological models. However, the post bounce evolution in these black hole interior models has the surprising feature that it asymptotes towards

a spacetime where the triad p_c which is the radius of the two-sphere part attains a constant value where as the triad p_b undergoes an exponential expansion. Also, these spacetimes were found to have non-negligible holonomy corrections asymptotically after the bounce, i.e, the quantum geometric effects do not fade away after the bounce. Due to its similarity with the Nariai spacetime which also has a constant $g_{\theta\theta}$ and an exponentially increasing g_{xx} , these asymptotic regions were termed ‘Nariai type’ spacetimes in previous works [40, 47]. In this work, we re-examined the loop quantum evolution of black hole interior spacetimes in the presence of cosmological constant to study the asymptotic spacetime in detail assuming the validity of the effective spacetime description in LQC.

We find that the asymptotic constant p_c spacetime obtained in the effective loop quantum evolution of Kantowski-Sachs spacetime can be interpreted as a ‘charged’ Nariai solution of classical GR. The asymptotic solution obtained in the evolution of Bianchi III LRS spacetime with a negative cosmological constant (or the higher genus black hole interior) can be similarly interpreted as an anti-Bertotti-Robinson spacetime, again a solution to the Einstein equations. Both these solutions are product spacetimes of constant curvature manifolds [49] with $R_0^0 = R_1^1 = \pm R_2^2 = \pm R_3^3$. The constancy of Ricci components was verified numerically for the asymptotic spacetimes emergent in LQC evolution of black hole interiors. These curvatures were found to be Planckian even though the effective metric is a solution of the Einstein equations. Thus the emergent spacetime has the peculiar nature of being isometric to a classical spacetime while the quantum gravity effects (via the holonomy corrections of LQC) are large. It is also noteworthy that classical ‘charged’ Nariai and anti-Bertotti-Robinson spacetimes are nonsingular and thus the geodesics in black hole interiors can be extended to infinity in the loop quantum evolution. Another striking feature of these spacetimes is the existence of an emergent ‘charge’ and cosmological constant - both of quantum geometric in origin. One could fine tune the asymptotic p_c and $\frac{\dot{p}_b}{p_b}$ to obtain ‘uncharged’ Nariai spacetime or anti-Bertotti-Robinson spacetime with vanishing cosmological constant. However these fine tuned spacetimes turn out to be unstable and evolve to more generic ‘charged’ Nariai spacetime or anti-Bertotti-Robinson spacetime with cosmological constant. Thus emergence of the quantum geometric ‘charge’ and cosmological constant seems to be inevitable in the LQC evolution of black hole interior spacetimes. The basic property of emergent spacetime is that it is a product of two spaces of unequal

constant curvatures. The interpretation of these constant curvature spaces in LQC as ‘charged’ Nariai and anti-Bertotti-Robinson spacetimes comes about by a proper splitting of the two curvature constants of product spaces. This is indeed remarkable that emergent spacetime after the bounce is classical GR solution.

The emergence of a quantum geometric ‘charge’ in the LQC evolution of black hole interior spacetimes is in parallel to a similar result in the brane world scenario [64]. There, the projection of the Weyl tensor of a 5 dimensional bulk spacetime on to a four dimensional brane gave rise to Weyl or ‘tidal’ charge (and not an electric charge) which was taken to be negative on physical considerations. The metric however had exactly the same form as that of Reissner-Nordstrom solution with $Q^2 \rightarrow -Q^2$. The parallel between the current work and the findings of [64] are striking. In both cases, a theory of modified general relativity gives rise to an emergent ‘charge’ which mimics an electric or magnetic charge in the way it affects the geometry of spacetime, but which has its origin not in electromagnetism but in the modifications to general relativity. Thus this emergent ‘charge’ is purely gravitational in nature. However it seems that we are finding evidence that attempts to capture quantum corrections to general relativity through such a ‘charge’. Much further work and analysis is required to further nail its actual character and significance.

Bibliography

- [1] A. Ashtekar and J. Lewandowski, *Class. Quant. Grav.* **21**, R53 (2004) [gr-qc/0404018].
- [2] A. Ashtekar and P. Singh, *Class. Quant. Grav.* **28**, 213001 (2011) [arXiv:1108.0893 [gr-qc]].
- [3] A. Ashtekar, T. Pawłowski and P. Singh, *Phys. Rev. Lett.* **96** 141301 (2006),
- [4] A. Ashtekar, T. Pawłowski and P. Singh, *Phys. Rev. D* **73**, 124038 (2006) [gr-qc/0604013].
- [5] A. Ashtekar, T. Pawłowski and P. Singh, *Phys. Rev. D* **74**, 084003 (2006) [gr-qc/0607039].
- [6] A. Ashtekar, A. Corichi, P. Singh, *Phys. Rev. D* **77**, 024046 (2008) [arXiv:0710.3565 [gr-qc]].
- [7] P. Singh, *Class. Quant. Grav.* **29**, 244002 (2012) [arXiv:1208.5456 [gr-qc]]
- [8] P. Diener, B. Gupt and P. Singh, *Class. Quant. Grav.* **31**, 105015 (2014) [arXiv:1402.6613 [gr-qc]].
- [9] P. Diener, B. Gupt, M. Megevand and P. Singh, “Numerical evolution of squeezed and non-Gaussian states in loop quantum cosmology,” arXiv:1406.1486 [gr-qc]
- [10] D. W. Chiou, *Phys. Rev. D* **75**, 024029 (2007) [arXiv:0609029[gr-qc]].
- [11] L. Szulc, *Phys. Rev. D* **78**, 064035 (2008) [arXiv:0803.3559 [gr-qc]].
- [12] A. Ashtekar and E. Wilson-Ewing, *Phys. Rev. D* **79** 083535 (2009) [arXiv:0903.3397 [gr-qc]].
- [13] M. Martin-Benito, G. A. Mena Marugan and T. Pawłowski, *Phys. Rev. D* **78**, 064008 (2008) [arXiv:0804.3157 [gr-qc]].
- [14] M. Martin-Benito, G. A. Mena Marugan and T. Pawłowski, *Phys. Rev. D* **80** 084038 (2009) [arXiv:0906.3751 [gr-qc]].
- [15] A. Ashtekar and E. Wilson-Ewing, *Phys. Rev. D* **80**, 123532 (2009) [arXiv:0910.1278 [gr-qc]].

- [16] E. Wilson-Ewing, Phys. Rev. D **82**, 043508 (2010) [arXiv:1005.5565 [gr-qc]].
- [17] P. Singh and E. Wilson-Ewing, Class. Quant. Grav. **31**, 035010 (2014) [arXiv:1310.6728 [gr-qc]]
- [18] J. Willis, On the low energy ramifications and a mathematical extension of loop quantum gravity. Ph.D. Dissertation, The Pennsylvania State University (2004).
- [19] V. Taveras, Phys. Rev. **D78** 064072 (2008) [arXiv:0807.3325[gr-qc]]
- [20] P. Singh, V. Taveras, A note on the effective equations in loop quantum cosmology (To appear)
- [21] P. Diener, B. Gupt and P. Singh, Class. Quant. Grav. **31**, 025013 (2014) [arXiv:1310.4795 [gr-qc]].
- [22] A. Corichi and P. Singh, Phys. Rev. D **78**, 024034 (2008) [arXiv:0805.0136 [gr-qc]].
- [23] A. Corichi and P. Singh, Phys. Rev. D **80**, 044024 (2009) [arXiv:0905.4949 [gr-qc]].
- [24] P. Singh, Class. Quant. Grav. **26**, 125005 (2009) [arXiv:0901.2750 [gr-qc]].
- [25] For resolution of various types of singularities in spatially curved isotropic models in LQC, see Ref. [26].
- [26] P. Singh and F. Vidotto, Phys. Rev. D **83**, 064027 (2011) [arXiv:1012.1307 [gr-qc]].
- [27] P. Singh, Phys. Rev. D **85**, 104011 (2012) [arXiv:1112.6391 [gr-qc]]
- [28] B. Gupt and P. Singh, Phys. Rev. D **85**, 044011 (2012) [arXiv:1109.6636 [gr-qc]].
- [29] A. Ashtekar, M. Bojowald and J. Lewandowski, Adv. Theor. Math. Phys. **7**, 233 (2003) [gr-qc/0304074].
- [30] M. Bojowald, D. Cartin and G. Khanna, Phys. Rev. D **76**, 064018 (2007) [arXiv:0704.1137 [gr-qc]].
- [31] G. A. Mena Marugan, J. Olmedo and T. Pawłowski, Phys. Rev. D **84**, 064012 (2011) [arXiv:1108.0829 [gr-qc]]

- [32] A. Ashtekar and M. Bojowald, *Class. Quant. Grav.* **23**, 391 (2006) [gr-qc/0509075].
- [33] L. Modesto, *Class. Quant. Grav.* **23**, 5587 (2006) [gr-qc/0509078].
- [34] L. Modesto, *Phys. Rev. D* **70**, 124009 (2004); *Int. J. Theor. Phys.* **45** 2235 (2006).
- [35] M. Campiglia, R. Gambini and J. Pullin, *AIP Conf. Proc.* **977**, 52 (2008) [arXiv:0712.0817 [gr-qc]].
- [36] R. Gambini and J. Pullin, *Phys. Rev. Lett.* **101**, 161301 (2008) [arXiv:0805.1187 [gr-qc]].
- [37] M. Campiglia, R. Gambini and J. Pullin, *Class. Quant. Grav.* **24**, 3649 (2007) [gr-qc/0703135]
- [38] R. Gambini and J. Pullin, *Phys. Rev. Lett.* **110**, 211301 (2013) [arXiv:1302.5265 [gr-qc]].
- [39] R. Gambini and J. Pullin, *Class. Quant. Grav.* **31**, 115003 (2014) [arXiv:1312.3595 [gr-qc]]
- [40] C. G. Boehmer and K. Vandersloot, *Phys. Rev. D* **76**, 104030 (2007) [arXiv:0709.2129 [gr-qc]].
- [41] D. W. Chiou, *Phys. Rev. D* **78**, 044019 (2008) [arXiv:0803.3659 [gr-qc]].
- [42] D. -W. Chiou, W. -T. Ni and A. Tang, “Loop quantization of spherically symmetric midisuperspaces and loop quantum geometry of the maximally extended Schwarzschild spacetime,” arXiv:1212.1265
- [43] D. W. Chiou, *Phys. Rev. D* **78**, 064040 (2008) [arXiv:0807.0665 [gr-qc]]
- [44] J. Cortez, W. Cuervo, H. A. Morales-Tecotl, *AIP Conf. Proc.* **1548** (2013) 167 (2013)
- [45] A. Joe, P. Singh, A phenomenological investigation of Kantowski-Sachs spacetime in LQC (To appear).
- [46] A. Joe, P. Singh, *Class. Quant. Grav.* **32**, no. 1, 015009 (2015) [arXiv:1407.2428 [gr-qc]]
- [47] J. Brannlund, S. Kloster and A. DeBenedictis, *Phys. Rev. D* **79**, 084023 (2009)
- [48] C. G. Boehmer and K. Vandersloot, *Phys. Rev. D* **78**, 067501 (2008)

- [49] N. Dadhich, “On product space-time with 2 sphere of constant curvature,” gr-qc/0003026.
- [50] N. Dadhich, A. Joe, P. Singh, *Class. Quant. Grav.* **32**, no. 18, 185006 (2015) [arXiv:1505.05727 [gr-qc]].
- [51] N. Dadhich, “Nariai metric is the first example of the singularity free model,” gr-qc/0106023.
- [52] R. Bousso, *Phys. Rev. D* **60**, 063503 (1999) [hep-th/9902183].
- [53] C. W. Misner and J. A. Wheeler, *Annals Phys.* **2**, 525 (1957).
- [54] C. B. Collins, *J. Math. Phys.* **18**, 2116 (1977).
- [55] T. Schilling, *Geometry of quantum mechanics*, Ph. D Dissertation, The Pennsylvania State University (1996); A. Ashtekar, T. Schilling, *Geometrical Formulation of Quantum Mechanics*, gr-qc/9706069.
- [56] M. Martin-Benito, G. A. M. Marugan and T. Pawłowski, *Phys. Rev. D* **80**, 084038 (2009) [arXiv:0906.3751 [gr-qc]].
- [57] P. Tarrío, M. F. Mendez and G. A. M. Marugan, *Phys. Rev. D* **88**, 084050 (2013) [arXiv:1310.1290 [gr-qc]].
- [58] A. Corichi, P. Singh, “Loop quantization of the Schwarzschild interior: $\bar{\mu}$ prescription” (In preparation).
- [59] P. Singh, E. Wilson-Ewing, *Class. Quant. Grav.* **31**, 035010 (2014)
- [60] O. J. C. Dias and J. P. S. Lemos, *Phys. Rev. D* **68**, 104010 (2003)
- [61] V. Cardoso, O. J. C. Dias and J. P. S. Lemos, *Phys. Rev. D* **70**, 024002 (2004)
- [62] R. Bousso, *Phys. Rev. D* **55**, 3614 (1997)
- [63] L. Kofman, V. Sahni, A. Starobinsky, *Sov.Phys.JETP* **58**, 1090 (1983); L. Kofman, V. Sahni, *Phys. Lett.* **B127**, 197 (1983); V. Sahni, L. Kofman, *Phys. Lett.* **A117** 275 (1986).
- [64] N. Dadhich, R. Maartens, P. Papadopoulos and V. Rezanian, *Phys. Lett. B* **487**, 1 (2000)

[65] N. Dadhich, Phys. Lett. B **492**, 357 (2000)

[66] B. Bertotti, Phys. Rev. **116**, 1331 (1959).

Appendix A: Copyright Permissions

Date: Fri, 13 Nov 2015 09:40:45 +0000

From: Permissions <permissions@iop.org>

To: Parampreet Singh <psingh@phys.lsu.edu>

Subject: Re: Request for copyright permission for use in Thesis

Dear Parampreet Singh

Thank you for your email and for taking the time to seek this permission.

I am sorry to hear your sad news.

When the authors transferred the copyright in your article to IOP, we granted back to the authors certain rights, including the right to include the Accepted Manuscript of the article within any thesis or dissertation.

Please include citation details and for online use, a link to the Version of Record.

The only restriction is that if, at a later date, the thesis were to be published commercially, further permission would be required.

Please let me know if you have any further questions.

Many thanks.

Kind regards,

Gemma

Gemma Alaway

Rights & Permissions Adviser

IOP Publishing

Temple Circus, Temple Way, Bristol
BS1 6HG, UK

Direct line +44 (0)117 930 1146

iopublishing.org

From: Parampreet Singh <psingh@phys.lsu.edu>

To: permissions@iop.org,

Date: 12/11/2015 18:25

Subject: Request for copyright permission for use in Thesis

Dear Sir/Madam,

We need to obtain copyright permission for usage of the article communicated by me and published in Classical and Quantum Gravity. It is to be used in my graduate student Anton Joe's PhD thesis. Anton Joe is an author of the article. Unfortunately, the student has passed away and I have to apply for this copyright permission myself. I was given permission for a paper earlier. This email is regarding another paper.

The article is published in Classical and Quantum Gravity in 2015 in Volume 32, page 185006 with the title:

Title: Emergence of the product of constant curvature spaces in loop quantum cosmology

Authors: Naresh Dadhich, Anton Joe and Parampreet Singh

The title of the Anton's thesis is "Anisotropic spacetimes and black hole interiors in loop quantum gravity"

I request you to grant us permission for the above paper for usage in Anton Joe's PhD thesis. Many parts of the paper will be used as it is in the thesis.

Regards,
Parampreet Singh

Parampreet Singh
Asst. Professor
Department of Physics & Astronomy
255 Nicholson Hall, Louisiana State University
Baton Rouge, LA 70803 USA

Date: Thu, 25 Jun 2015 09:59:13 +0100
From: Permissions <permissions@iop.org>
To: "Parampreet Singh" <psingh@phys.lsu.edu>
Subject: Re: Request for copyright permission for use in Thesis
Dear Parampreet Singh

Thank you for your email and for taking the time to seek this permission.
When you transferred the copyright in your article to IOP, we granted back to you certain rights, including the right to include the article within any thesis or dissertation.
Please include citation details and for online use, a link to the Version of Record.
The only restriction is that if, at a later date, the thesis were to be published commercially, further permission would be required.
Please let me know if you have any further questions.
Kind regards,
Zora Catterick
Publishing Assistant
IOP Publishing

Please note: We do not usually provide signed permission forms as a separate attachment. Please print this email and provide it to your publisher as proof of permission.

From: "Parampreet Singh" <psingh@phys.lsu.edu>
To: permissions@iop.org
Date: 23/06/2015 19:47
Subject: Request for copyright permission for use in Thesis

Dear Sir/Madam,
We need to obtain copyright permission for usage of the article written by me and my graduate student Anton Joe in his PhD thesis. Unfortunately, the student has passed away and I have to apply for this copyright permission myself.
The article is published in Classical and Quantum Gravity in 2015 in Volume 32 page 035009 with the title:
Title: Kantowski-Sachs spacetime in loop quantum cosmology: bounds on expansion and shear scalars and the viability of quantization prescriptions
Authors: Anton Joe and Parampreet Singh
The title of the Anton's thesis is "Anisotropic spacetimes and black hole interiors in loop quantum gravity"
I request you to grant us permission for the above paper for usage in Anton Joe's PhD thesis. Many parts of the paper will be used as it is in the thesis.
Regards,
Parampreet Singh
Parampreet Singh
Asst. Professor
Department of Physics & Astronomy
255 Nicholson Hall, Louisiana State University
Baton Rouge, LA 70803 USA
Ph: 225 578 7513, Fax: 225 578 5855

I am applying for permission for:
CLASSICAL & QUANTUM GRAVITY (02649381)

A article extract
year of publication: 2016
volume number: 32
issue number: 1
title of article: Kantowski-Sachs spacetime in loop quantum cosmology: bounds on expansion and shear scalars and the viability of quantization prescriptions
author(s): Anton Joe, Parampreet Singh
number of words: 5000
additional information: The copyright is required to use in the PhD thesis of Anton Joe.

To be used in:

A Book, Journal, Magazine or Academic Paper...

The Thesis details are
type of document: Thesis
publication title: Anisotropic spacetimes and black hole interiors in loop quantum gravity

Please respond directly to the person making the request. They have not received your company name or contact details.

Contact details are

Name: Anton Joe
Email: psingh@phys.lsu.edu
Address: 202 Nicholson Hall, Department of Physics
Louisiana State University
Town /City: Baton Rouge
Post Code: 70803

Help us improve PLSclear: [5 quick questions](#)

This email (and attachments) are confidential and intended for the addressee(s) only. If you are not the intended recipient please notify the sender, delete any copies and do not take action in reliance on it. Any views expressed are the author's and do not represent those of IOP, except where specifically stated. IOP takes reasonable precautions to protect against viruses but accepts no responsibility for loss or damage arising from virus infection. For the protection of IOP's systems and staff emails are scanned automatically.

IOP Publishing Limited Registered in England under Registration No 467514. Registered Office: Temple Circus, Bristol BS1 6HG England Vat No GB 461 6000 84.

Please consider the environment before printing this email

Appendix B: Letter explaining primary authorship

To Whom It May Concern

Sections 3.1-3.6 of this dissertation are based on the published work N. Dadhich, A. Joe, P. Singh, “Emergence of the product of constant curvature spaces in loop quantum cosmology,” *Classical and Quantum Gravity* **32**(18), 185006. As the major professor of Anton Joe, I certify that even though Anton Joe is not listed as a first author in this published work, portions of this article which are reproduced in this dissertation are portions of which he was the primary author on the project.

Sincerely,

Dr. Parampreet Singh

Asst. Professor,

Department of Physics and Astronomy,

Louisiana State University

Vita

Anton Joe was born on January 13th, 1990 into the family of V. A Vincent and C. O Mary in Thrissur, which is the Cultural capital of Kerala in India. He was the best-loved brother to his elder sister Elna Merin and younger brother Britto Joseph.

Anton graduated from Vivekodayam Boys Higher Secondary School, Thrissur, Kerala with an aggregate score of 94 percent in his 12th grade. He was interested in Physics from a very young age which was evident from his talks about time space theories, black holes and about travelling in time to his siblings, when he was in high school. He was always fascinated to discuss about Physics to his friends and family during his school days.

Anton wrote Joint Entrance Examination (JEE) organized by the Indian Institutes of Technology (IIT) and secured an All India Rank (AIR) of 2624 and was ranked within the top 1 percent of students who appeared for the exam. He secured 59th position in National Science Olympiad, conducted by Science Olympiad Foundation in 2007.

His strong basics in Science coupled with his phenomenal interest in Physics led him to pursue his Master of Science under the integrated program in Indian Institute of Technology, Kharagpur, India in 2007. He graduated with a CGPA of 8.1/10 in 2012. He was awarded the Kishore Vaigyanik Protsahan Yojana (KVPY) scholarship, funded by the Department of Science and Technology, Government of India, for his MSc integrated program.

Anton did a number of projects and internships in various Research Institutes while studying in IIT. Some of his projects include:

1. Numerical Loop quantum cosmology under the guidance of Prof. Gaurav Khanna, University of Dartmouth, Massachesseuts, USA (Summer 2011)
2. Comparison of post-Newtonian template families for gravitational wave from inspiralling compact binaries - under the guidance of Prof. Bala R Iyer, Raman Research Institute, Bangalore. (Summer 2010)
3. Matched filtering for parameter estimation of gravitational wave from inspiralling compact binaries in Newtonian approximation - under the guidance of Prof. Archana Pai, Indian Institute of Science Education and Research (IISER), Trivandrum, India. (December 2009)

4. Critical gravitational collapse - under the guidance of Prof. Ghanashyam Date, The Institute of Mathematical Science (IMSc), Chennai, India. (Summer 2009)

He developed a strong interest in the field of Loop quantum Cosmology while at IIT, which motivated him to pursue Ph.D in the US. He scored 990/990 in Physics GRE. Anton enrolled in the doctoral degree program in the Physics and astronomy department at Louisiana State University in August 2012. He worked under the guidance of Dr.Parampreet Singh in the field of Loop quantum Cosmology. He got a perfect CGPA of 4, during his Ph.D program.

His publications and conference presentations include:

1. Joe, A., & Singh, P. (2015). Kantowski-Sachs spacetime in loop quantum cosmology: bounds on expansion and shear scalars and the viability of quantization prescriptions.” *Classical and Quantum Gravity*, **32**(1), 015009.
2. Dadhich, N., Joe, A., and Singh, P. (2015). “Emergence of the product of constant curvature spaces in loop quantum cosmology,” *Classical and Quantum Gravity* **32**(18), 185006.
3. Emergence of constant curvature spacetimes with an effective charge and cosmological constant in loop quantum cosmology, American Physical Society April Meeting 2015, Baltimore, April 2015.
4. Effective dynamics of Kantowski-Sachs spacetime in loop quantum cosmology, American Physical Society April Meeting 2014, Savannah, April 2014
5. Characteristics of Kantowski-Sachs spacetime in loop quantum cosmology”, 7th Gulf Coast Gravity Meeting, Oxford (MS), April 2013

He also worked as a Teaching Assistant for undergraduate physics laboratory and graduate level courses on Classical Mechanics and Electrodynamics in LSU department of Physics and Astronomy from August 2012 to December 2014.

Anton was a calm, generous and selfless person. He was highly knowledgeable yet very humble. He had a great passion for research and he absolutely enjoyed working in Loop quantum Cosmology. In words of his Ph. D supervisor: “Anton was an outstanding and a very dedicated researcher. He was a student a PhD advisor always dreams for.” Apart from academics, reading books and solving puzzles were his favorite pastime right from his childhood. He had also won many chess tournaments in his hometown.

He loved watching cricket and reading interesting questions and answers from Quora during his spare time. His other interests were cooking, playing cricket, soccer and tennis, gyming and travelling. He was a great son, caring brother, brilliant student, loving friend, above all, a wonderful human being! His life will always be an inspiration to all of us.

Anton passed away in a drowning accident on April 26th, 2015. Anton might have left this planet too soon but he is not really lost to us. Whenever people work with utmost dedication, Anton is in their midst. Wherever people are working sincerely towards their research in physics, he is there rejoicing. Whoever values others time, believes in simplicity and truth, keeps the spirit of Anton alive and well.