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Multi-objective optimal battery placement in distribution networks

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MULTI-OBJECTIVE OPTIMAL BATTERY PLACEMENT IN DISTRIBUTION NETWORKS

by

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ABSTRACT

Due to high penetration of renewable energy resources in today's electricity generation, considerable voltage fluctuations are witnessed in power systems. As an attempt to solve this issue, in this study, multi-objective optimal placement and sizing of distribution-level battery storage system is performed using semidefinite programing. Placement of one or multiple battery system is studied under various objectives including the cost, voltage regulation, reactive power dispatch, renewable resource curtailment, and minimum network power losses. Power flow equations are solved in the form of semidefinite constraints and the rank constraint is ignored. Additionally, combination of these objectives to form a multi-objective problem and regularization of the number of battery sites are studied. Finally, simulation results are provided to analyze the proposed formulation.

ACKNOWLEDGMENTS

To my mother who sacrificed her life to make my dreams come true. Her strength, kindness, and courage inspires me everyday. To my father who taught me forgiveness. Whose belief in gender equality encouraged me to move forward. Thank you. In your happiness I find the true joy.

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Special thanks to my friends who made my stay in Rolla enjoyable and held my hand through my journey.

‘‘Live in such way, that your
righteousness and knowledge does
not trouble others. Be calm and master of yourself.’’

Khayyam

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

Page

LIST OF TABLES

1. INTRODUCTION

Growing environmental concerns caused a considerable reduction in the use of fossil fuels in the past decade. Most of U.S. carbon dioxide emission is associated with electricity generation. About 35% of the total U.S. energy-related CO_2 emissions in 2016, was related to emissions of carbon dioxide by the U.S. electric power sector [1]. As an attempt to replace fossil fuels, renewable energy resources including wind, solar, hydropower, and biomass were introduced. Among which solar photovoltaic (PV) technologies are one of the fastest growing. PV uses materials which absorb photons of lights and release electron charges, hence, it is the direct conversion of light into electricity [2]. The basis of a PV system is the PV cells which group together to form a panel or array [3]. These cells are made of different types of semiconductors [3]. Figure 1.1. shows the equivalent circuit of the ideal and practical PV cell [3].

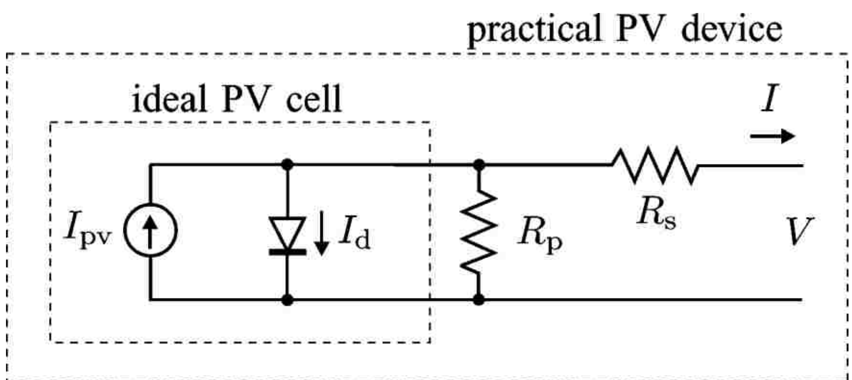

Figure 1.1. Equivalent circuit of the PV cell [3]

In Figure 1.1., I_{pv} is the generated current by the light (directly proportional to the Sun radiation), I_d is the Shockley diode equation, R_s and R_p are the equivalent series and parallel resistance of the array.

A PV system can be either standalone or grid connected [4]. If a standalone application is used, the system must be able to handle power variations from the PVs with a sufficient storage capacity [4]. For a grid-connected application, PV arrays can supply power to both utility grid and local loads. In a conventional PV system, the output is a dc current which highly depends on the solar irradiance, temperature, and voltage at the terminals of this system [5]. In grid-connected mode this dc power is transformed and connected to the grid using a PV inverter which converts the generated dc power to ac power used for ordinary power supply to electric devices [5], [6]. Additional elements are included in PV system configuration depending on local regulations, the converter topology and the modulation used to control it [5]. In general two groups of requirements can be considered when installing PV systems which are performance requirements and legal regulations [5]. A generic grid-connected PV is depicted in Figure 1.2. [5].

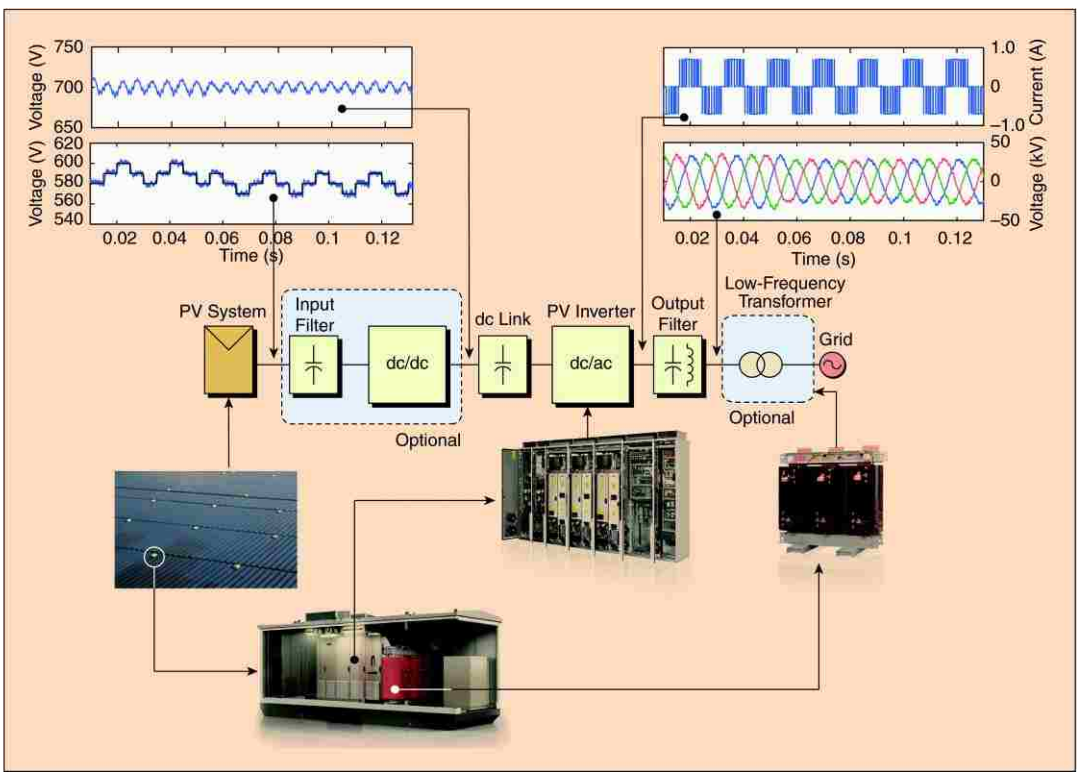

Figure 1.2. A generic grid-connected PV structure [5]

Technical potential of PV systems deployed on rooftops varies in the continental United States. In a report presented by National Renewable Energy Laboratory (NREL), how much energy could be generated by installing PV on all suitable roof areas, is investigated. Figure 1.3. shows that California has the greatest potential to offset electricity use since its rooftop PVs could generate 74% of the electricity sold by the local utilities in 2013.

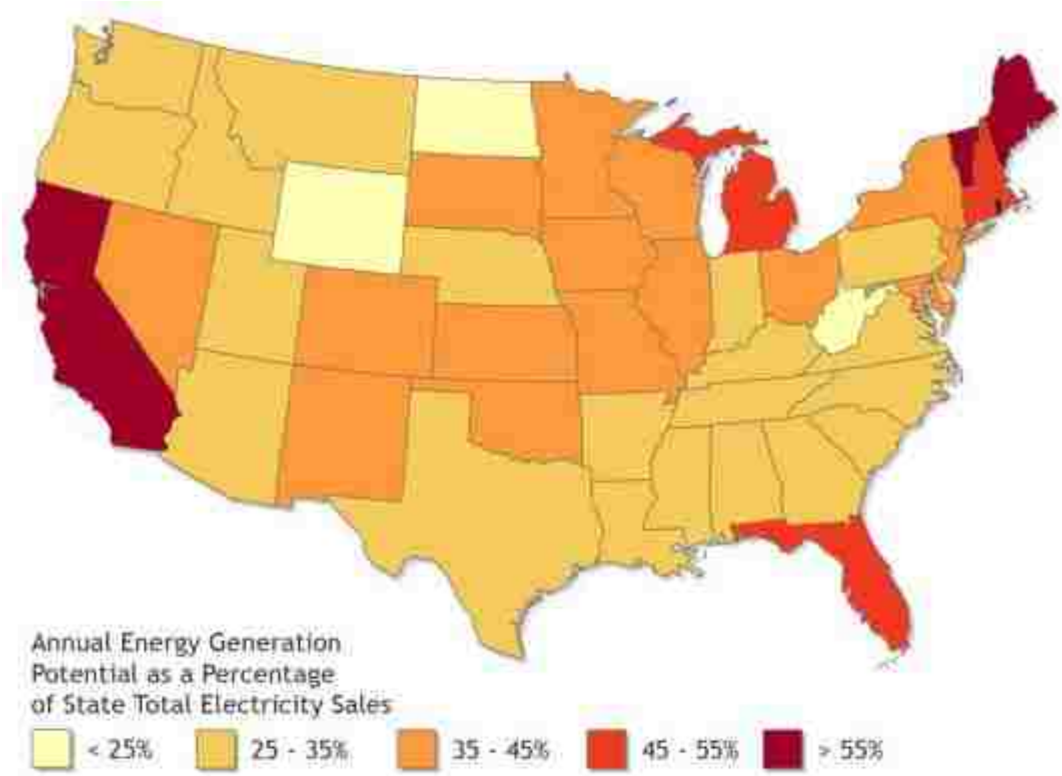

Figure 1.3. Potential rooftop PV annual generation from all buildings. Shown as a percentage of each state's total electricity sales in 2013 [5]

There is also another method to use solar energy which is referred to as concentrating solar power (CSP). CSP plants as shown in Figure 1.4. use mirrors or lenses to concentrate solar thermal energy which will be used to drive traditional steam turbines that produce electricity. However a considerable reduction in price of PVs over the last years resulted in wider application of them in power systems in compare to CSP.

Figure 1.4. CSP Plant

According to a report published by U.S. Department of Energy's NREL, since 2006 U.S. annual electricity generation from solar and wind increased by a factor of 11 [5]. Figure 1.5. illustrates this significant growth in the use of renewable resources.

As the use of Solar Photovoltaics (PV) expanded in distribution networks, concerns about the impact of its voltage fluctuations on the operation of the network also grew. The increasing penetration of commercial and residential PV generations causes load imbalances and reverse power flow in the distribution system. Reverse power flow may result in several undesired conditions including over voltage of the distribution feeder (loss of voltage regulations), increased short circuit currents, and potential protection miscoordination [8]. Furthermore, when the generated power by the distributed resources exceed the load on a feeder line section, voltage may rise on that section [9]. Consequently, a significant increase in voltage forces on-load tap-changers (OLTC) and other voltage control devices such as line voltage regulators to operate continuously. Therefore, their lifespan will be shortened [10]. A research conducted on a system with an assumption of 20%PV penetration such a high level of penetration more than doubles transformer tap changes [11].

Among all the issues mentioned above, loss of voltage regulation, which is discussed in the following section, is the most probable one, therefore, it has received considerable attention in the past years.

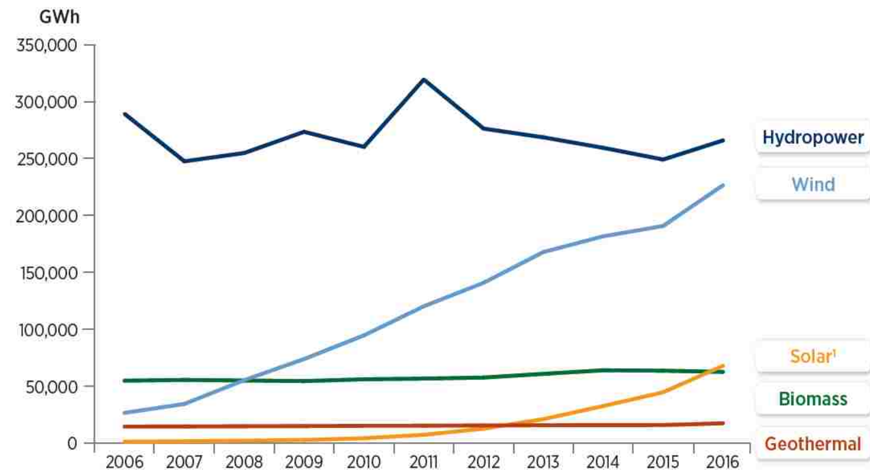

Figure 1.5. U.S. Renewable Electricity Generation by Technology [5].
[1]Includes generation from CSP and grid-connected PV; assumes a 25% capacity factor for CSP and an 18% capacity factor for PV

1.1. VOLTAGE REGULATION METHODS

Voltage regulation in weak and highly penetrated distribution networks has become a challenge for distribution network operators [12]. This can be caused by cloud-induced fluctuations in PV power [11]. In a data presented in [13] four typical days in November 2011 were chosen to indicate four classes of solar radiations. As seen in Figure 1.6., where a positive current shows a reverse power flow back to the substation, the specified feeder can easily have a peak reverse power flow of more than 3MW. This value varies drastically depending on how clouded it is. The data in this Figure is taken from SCADA system of Southern California Edison (SCE) [13].

In such networks, either the large impedance of distribution networks or the high level of renewable penetration and the resulting power flow excursions cause voltage fluctuations that can be outside of ANSI (American National Standard Institution) boundaries [12]. Such fluctuations can also create voltage flicker or excessive operation of the voltage regulating equipment [11].

Variation of node voltages in micro grids may even cause system instability [11]. An example of such issues is the growing concern with bi-directional power flow in distribution feeders [14], [15]. As a result, network operators have started to install various voltage regulator technologies including on load tap changer (OLTC), fly wheel-based voltage regulators, Ultracapacitors and Energy Storage Systems (ESSs), and solar/wind curtailment [16], [17]. Three of mentioned techniques are described in the following sections.

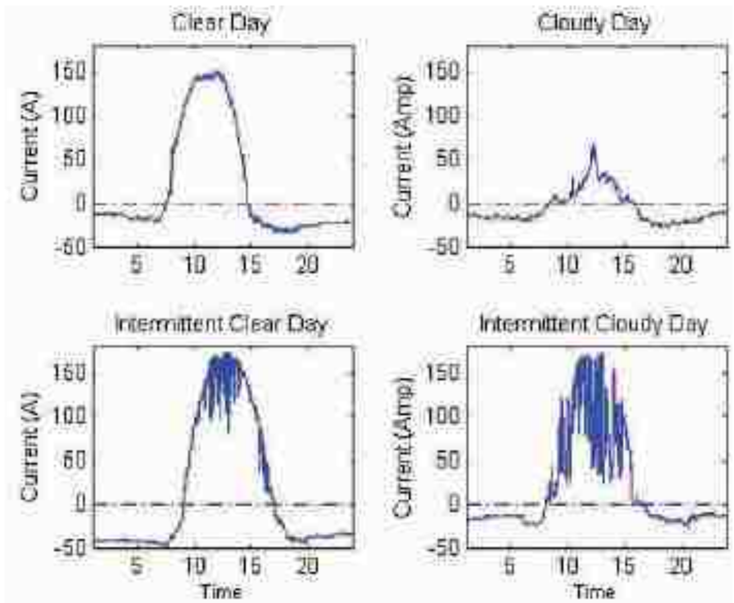

Figure 1.6. Line current measurement at the substation for one of SCE's feeders.
A positive current indicates reverse power flow into the substation and
a negative current shows real power flowing into the feeder [13]

1.1.1. OLTC. Traditionally OLTC, switched capacitors (SC) and step voltage regulators (SVR) have been employed to achieve desired voltage. The most common voltage control technique on the distribution network is to use OLTCs. They use an efficient method to control the voltage by shifting phase angle and adjusting the voltage magnitude [18]. OLTC is an autotransformer which measures the voltage and current, estimates the voltage at a remote point then changes the tap if the voltage exceeds the limits. Typically, each tap provides a range of $\pm\mathbf{10}\%$ of transformer rated voltage with 32 steps. An intentional time delay of 30 to 60 seconds is always implemented in OLTCs to avoid unnecessary tap change operations during the transient voltage fluctuations [18].

A simple radial feeder connected with a Distributed Generation (DG) is illustrated in Figure 1.7. [18]. Where, an OLTC transformer, a local load, a reactive power (Q) compensator, automatic voltage controllers (AVCs), a line drop compensator (LDC) and an energy storage device are also connected to the network [18].

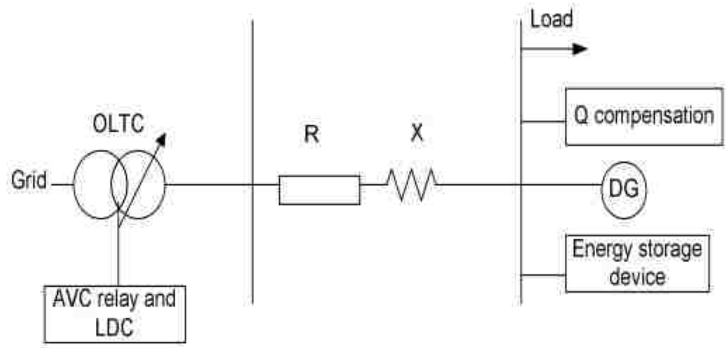

Figure 1.7. Simple radial feeder with connected DG [18]

OLTCs are used at the distribution systems to raise the starting voltage of a feeder so that some point along the feeder has a desired voltage. This strategy is referred to as line drop compensation and it is proportional to the load [19].

Typically low-voltage (LV) networks have off-load tap changing transformers, therefore, a number of studies in the literature analyze applicability of OLTC technology to such networks. In [20] a coordinated control of OLTC with ESS is used in a LV distribution network to solve the voltage rise caused by PV high penetration. Application of OLTC-fitted transformers to LV networks to increase the penetration of domestic-scale PV systems is investigated in [21].

1.1.2. Solar Curtailment. Unpredictable electricity generation of renewable energy resources forced the system operators to utilize less renewable energy than is generated. Term curtailment is used to refer to the use of less wind or solar power than is potentially available [22]. Curtailment can be used in distribution systems when the generation is more than consumption which may cause voltage control issues. In [23] a study was performed on a typical 240-V/75-kVA Canadian suburban distribution feeder with 12 houses with roof-top PV systems. To investigate coordinated active power curtailment of grid connected PV inventers. As seen in Figure 1.8. for about an 11-h period, there is considerably more energy produced by PVs than consumed by the load [23].

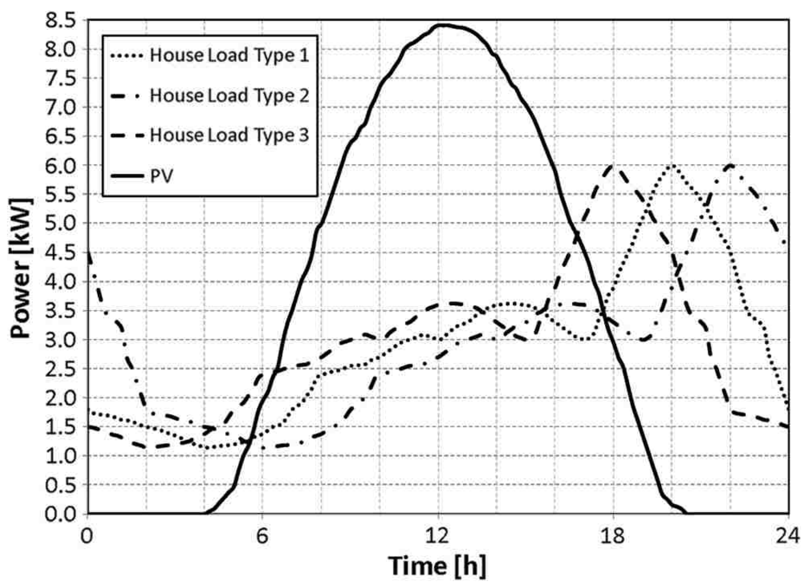

Figure 1.8. Load profile and PV production of houses for a 24-h period [23]

In general, three reasons for curtailment may include network constraints, security, and excessive generation relative to load levels [24]. Curtailment of primarily generators connected to distribution grid levels can also occur due to grid faults and scheduled grid maintenance which is a part of network security category [24]. In [25] the maximum amount of generation that can be connected to a power distribution system is referred to as hosting capacity. This will be defined by the network characteristics such as load requirements and generation unit parameters. The ability to curtail the power generation of certain PV arrays at times when otherwise the hosting capacity would be exceeded, will allow for larger installation of such energy resources.

1.1.3. Ultra Capacitors and Batteries [26]. At night or on a cloudy day when PV array is not functional, to balance power supply and demand a storage unit is used.

ESS for distribution systems is mostly in the form of a Battery ESS (BESS) [27]. The lead acid batteries are the most popular ESSs used in the distribution systems because of their low cost. The energy in BESSs is stored in the chemical form and can be converted into electrical and vice versa by an electrochemical reaction. The battery behavior described by its voltage is written as below [28].

$$V = V_{oc} \pm IR$$

Where V_{oc} is the open circuit voltage and R is the internal resistance and it depends on parameters such as charge and discharge current, temperature …etc. Current I is positive during charge and negative during discharge [28]. However, the unreliable and fluctuate output of Solar Panels deep discharges or overcharges batteries, therefore, it shortens their life spans. Figure 1.9. illustrates a typical charge and discharge characteristic of a lead-acid battery unit. In this Figure, C is in Ampere-hour (Ah) and it is the capacity of the battery storage unit. SoC denotes battery state of charge. The term rest that is shown in Figure 1.9., means that no current is moving through the cells and they are neither being charged or discharged [29].

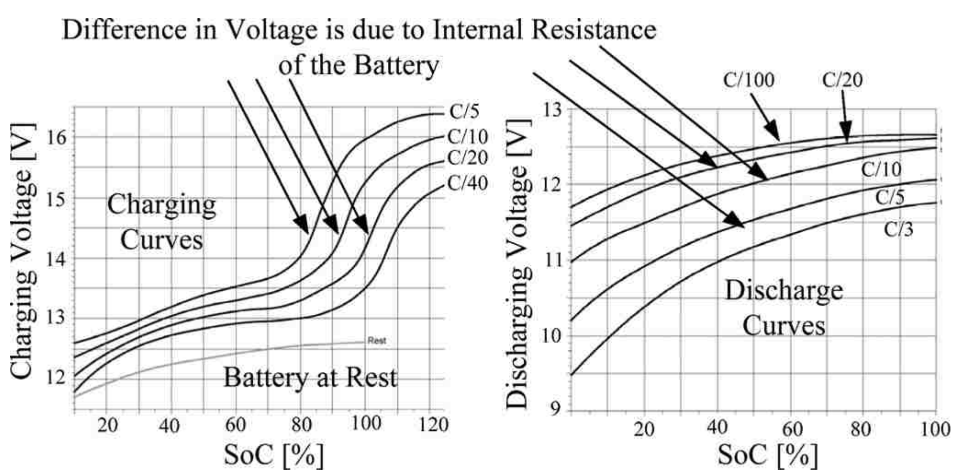

Figure 1.9. Battery state of charge and discharge [29]

Another ESS which can be used in conjunction with BESSs is ultra capacitor. The energy is stored in ultra capacitors by physically separating positive and negative charges unlike batteries which store energy chemically. They have a much longer lifespan in compare to batteries, however, battery's higher energy density allows them to store more energy over a longer period. In [26] both ESSs are employed in a hybrid system to reduce the battery size pack while expanding its life span. Among the mentioned voltage regulation methods BESS offers a promising solution that provides added features such as load-profile planning, reactive power control, and frequency excursion compensation. Figure 1.10. shows stationary BESSs. In this study lead acid batteries are assumed to be used at the proposed distribution system.

Figure 1.10. Stationary battery units. Courtesy: Mitsubishi Electric [30]

1.2. PROBLEM DESCRIPTION

As mentioned in the previous section, employing BESSs is one of the most efficient methods to regulate voltage. It is widely used in the distribution systems today, however, the battery mechanism and its cost caused complexity in the network computations. Battery charge and discharge depend on various factors including temperature and voltage. Therefore, while modeling the battery to solve the power flow for the network these factors must be considered. Furthermore, to optimally install a battery system, the location, power, and energy capacity of the battery system needs to be selected. Consequently, several methods have been proposed to optimally place and size BESS in the distribution systems. The location and size of BESS are derived so that the minimum number of batteries used can regulate the voltage at the maximum number of nodes possible. To achieve this goal, both active and reactive power must be considered in the optimization. The R/X ratio in the distribution systems causes the active power to be an influential factor in the voltage regulation. However, just considering the active power will not utilize the reactive power injection capabilities of the BESSs and hence, will lead to larger sizing than needed. The problem itself can be formulated using several optimization methods also each of those formulations have different solving approaches. In this thesis different OPF formulation approaches alongside possible solution methods are described. Among which Semidefinite Programming (SDP) is chosen to optimally locate the BESS in the distribution system. To solve the SDP problem a software package referred to as CVX in MATLAB is utilized.

In the next section existing methods that are addressed in the literature are presented.

2. OPTIMIZATION

Optimization problem involves choosing a value from a defined set to minimize (or maximize) a real function and computing the value of the function correspondingly. Many engineering problems such as power system operations, include the efficient use of limited resources to meet a defined objective [31]. Therefore, most of these problems can be modeled to an optimization problem of a specified objective function subject to given constraints [31].

There are constrained and unconstrained optimization problems. Most of the constrained problems can be converted to unconstrained ones. Some of major unconstrained optimization approaches used in power system operation are Newton-Raphson optimization, Lagrange multiplier method, and line search.

Optimal Power Flow (OPF) as well as different techniques which are used to solve power system operation problems are reviewed in the following sections [32].

2.1. OPTIMAL POWER FLOW

Power flow (load flow) is a network solution showing current, voltage, active and reactive power at each bus in the system. The relationship between active and reactive power consumption and generation is nonlinear. Thus, the power flow solution requires nonlinear programming (NP) techniques. Since it provides valuable information regarding power system operation, power flow analysis is important for transmission planning. General form of power flow equations for any bus k is shown below.

$$P_k = \sum_{j=1}^{N} |V_k||V_j|(G_{kj}\, cos(\theta_k - \theta_j) + B_{kj}\, sin(\theta_k - \theta_j)) \tag{2.1}$$

$$Q_k = \sum_{j=1}^{N} |V_k||V_j|(G_{kj} \cos(\theta_k - \theta_j) + B_{kj} \sin(\theta_k - \theta_j)) \tag{2.2}$$

Where P_k, Q_k, and θ_k are active power, reactive power and voltage angle of bus k respectively. G_{kj} and B_{kj} as the real and imaginary parts of the admittance matrix element Y_{kj}.

The idea of OPF was presented in the early 1960s as an extension to the conventional economic dispatch [31]. It is used to determine the state of power system that guarantee affordability, reliability, security, and dependability [31]. In optimal power flow (OPF) values of one or more control variables must be found to optimize (maximize or minimize) a defined objective. It has various applications in power systems including Energy Management Systems (EMS) and transmission planning.

2.1.1. Economic Dispatch. The objective is to minimize the total system cost or generator fuel consumption by determining the output power generation of each unit while satisfying load demand constraints. The fundamental of economic dispatch problem is the knowledge of the fuel cost curve.

A thermal unit system generally consists of a boiler and, the steam turbine and the generator. By combining the input-output characteristic of the boiler and the turbine-generator a convex curve (fuel cost curve) shown in Figure 2.1. will be obtained [32].

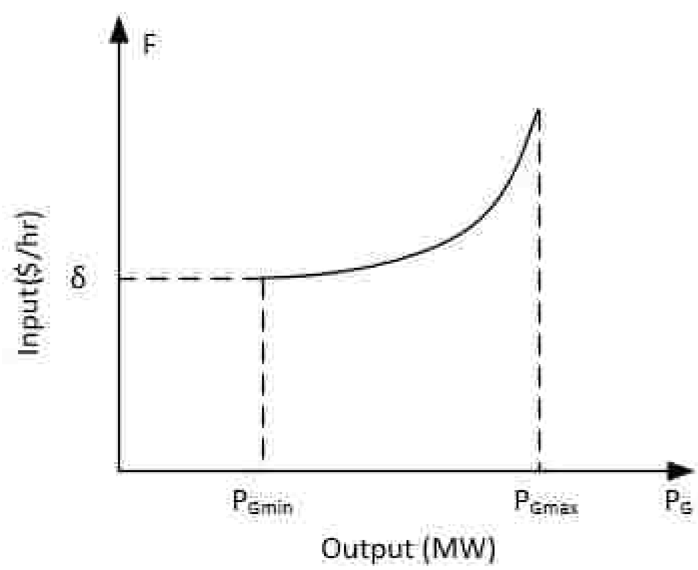

Figure 2.1. Input-output characteristic of the generating unit [32]

Generator characteristics in a practical system including discontinues prohibited zones, ramp rate limits and cost functions, are non-linear [33]. Hence, generally objective function which is the function to be minimized (maximized) based on the fuel cost curve of a generating unit is also nonlinear. The common form of the objective function is presented below.

$$F = \sum_{i=1}^{n}(\alpha_i P_i^2 + \beta_i P_i + \delta) \tag{2.3}$$

Where P_i is the generated power (P_G) at bus i and α_i, β_i,and δ are coefficients of the generating unit function. δ is a constant shown in Figure 2.1. which is the fuel consumption without the power output. This function is followed by equality and inequality constraints which represent the network characteristics. These constraints include network power balance at each node (generation and injection), limitations on all variables, line-flow constraints, etc. Various methods are developed to solve such problems, including Genetic Algorithm (GA) and Dynamic Programing (DP).

2.1.2. Power Loss Minimization. To obtain a better voltage profile and a lower current flow through the lines in power system, power loss minimization is performed alongside cost minimization. This is beneficial in distribution networks where due to low voltage levels there is a major power loss. Generally, two methods can be used to solve this type of optimization problems. First, the slack bus generation minimization which has a linear objective function and therefore is easy to solve. Second, minimization of the summation of power losses on all lines which involves more complex computations. The second approach is more desirable since the first one only minimizes the total power loss in the system whereas sometimes only a specific area of the system is desired [31].

However, due to changing loads on feeders where the load density is high, power loss for a network will not remain minimum for all load cases. Therefore, in [34] reconfiguration of the network and placement of distribution generation (DG) units are suggested. In [35] it is stated that sizing of DGs play an important role in minimizing the losses. It can be observed from a 3D graph presented in this study (Figure 2.2.) that for a particular bus, as the size of DG is increased the losses are decreased to a minimum value and increased beyond the optimal DG size at that location. Therefore it is concluded that given the characteristics of a distribution system, size of a DG can only be as high as consumption within the system boundaries.

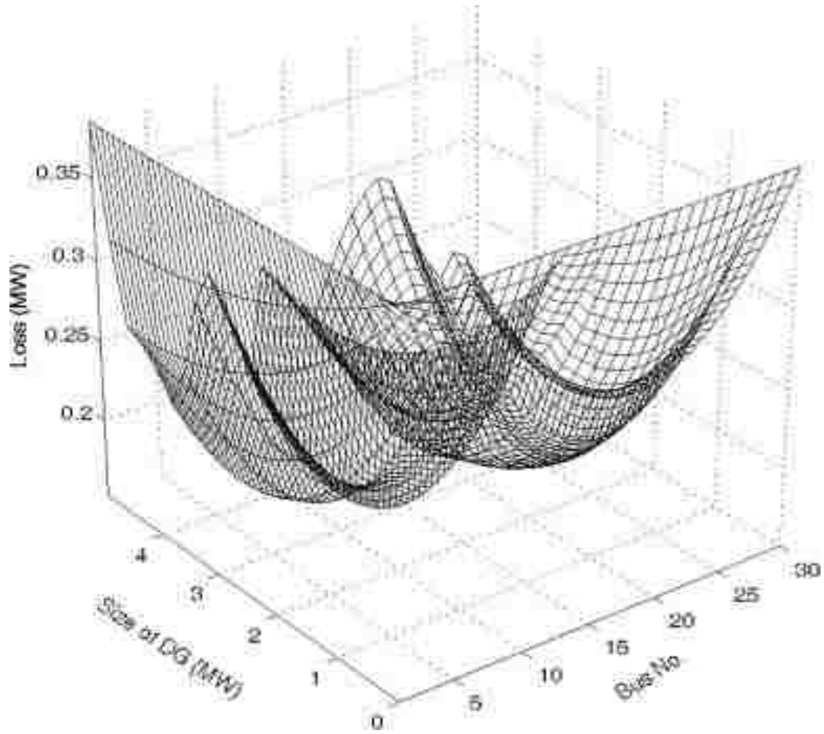

Figure 2.2. Effect of size and location of DG on system loss [35]

2.1.3. Reactive Power (VAR) Planning. This has different objectives including improvement of voltage profile, minimization of system active power losses and determination of optimal VAR compensation dispatch. To achieve these objectives various methods such as transformer tap changing, shunt capacitors and SVCs (switched virtual circuit) have been employed.

Reactive power balance and reactive power economic dispatch are classic VAR dispatch methods. Reactive power balance is calculating voltage balance of the system under the assumption that the generated reactive power by the generator and VAR compensation devices equals load reactive power and system reactive power loss. Also reactive power economic dispatch is minimization of active power loss by determination of reactive power of reactive power sources. This is done by considering the system load demand as a constraint [32].

VAR planning matters the most when additional devices need to be installed to improve voltage profiles in the network while minimizing the cost of the compensations.

These additional devices help balancing the reactive power in the system. Several factors including the transformer tap ratios, bus arrangements, etc., must be considered to localize the best place to install these reactive compensation devices. This can be formulated into an objective function presented below which was introduced in [36].

$$\sum_{i=1}^{m} \alpha_i C_{ci} + q_{ci} C_{bi} \qquad (2.4)$$

Where there are i=1,2,…, m buses, $\alpha_i \in \{0,1\}$ indicates there is a capacitor placed at bus i or not. C_{ci} is the fixed installment cost for each capacitor and C_{bi} is dollar per Mvar cost. q_{ci} is the size of capacitor. This is subject to constraints including the generated reactive power of the system, voltage limitations on bus i, the transformer ratios. The objective function and the constraints are nonlinear since they include mix of discrete (q_{ci}) and continuous (constraints like voltage and transformer ratio) variables. Several methods including Linear programing (LP), Nonlinear Programing (NP), quadratic programming, etc., have been employed to solve such problems.

2.2. CONVENTIONAL METHODS

2.2.1. Linear Programing. Linear programing (LP) method is used when an optimization problem can be expressed by a linear objective function and constraints. A standard form of an optimization problem is shown below [31]:

$$\begin{aligned} &\text{Maximize} && P(x) = c^T x \\ &\text{Subject to} && A(x) \le b \\ &x_j \ge 0 && \forall j \in \{1, n\} \end{aligned} \qquad (2.5)$$

Where A is an $m \times n$ matrix, x is a $n \times 1$ vector, c^T is a $1 \times n$ vector, and b is a $m \times 1$ vector. When writing the constraints, we must distinguish between equality and inequality. The objective function and the constraints are assumed to be continuous and defined on a nonempty subset of $\mathfrak{R}$. Also, the maximization of an objective function $P(x)$ is equal to minimization of - $P(x)$.

LP follows duality. In other words, the original linear problem is referred to as primal and can be converted to its dual:

$$
\begin{aligned}
&\text{Minimize} && Q(y) = b_D y \\
&\text{Subject to} && A_D y \geq c_D^T \\
&y_j \geq 0 && \forall j \in \{1, m\}
\end{aligned}
\tag{2.6}
$$

Where A_D is an $n \times m$ matrix, y is a $m \times 1$ vector, c_D is a $1 \times m$ vector, and b_D is a $n \times 1$ vector. Duality in LP reduces the computation needed to solve multidimensional problems since the old objective function turns into constraints and the primal constraints are converted to objective function in the dual problem.

In most applications when encountering practical problems and seeking their optimal solutions, it is also desirable to know the sequences of a change in the variable. Therefore, it is more convenient not to resolve the problem when a small change occurs to the variables. Sensitivity analysis is a study used to compute such solutions after performing the optimization.

Throughout the years different methods have been presented to solve these problems including graphical method, simplex method, and revised simplex method. However, it must be considered that linearization will always perform poorly away from the operating point and it also neglects losses and couplings between real and reactive power which are important considerations for planning and operations [37].

2.2.2. Nonlinear Programing (NP). Generally power system operation problems are nonlinear and the source of nonlinearity is most often a physical process that cannot be linearized. Therefore, NP solutions can easily handle OPF problems with nonlinear constraints and objective functions. NP problems can be classified into four types including NP Problems with nonlinear objective function and linear constraints, Quadric Programing (QP), Convex Programing, and separable Programing [31].

Quadric problems are often characterized by the following formulation [38].

$$
\begin{aligned}
&\text{Minimize} && f(x) = \frac{1}{2} x^T Q x + c^T x \quad x \in \mathbb{R}^n \\
&\text{Subject to} && Ax \leq b, x \geq 0
\end{aligned}
\tag{2.7}
$$

If Q is a positive semidefinite matrix, then $f(x)$ is a convex function, and if Q is zero the problem will be a LP [38]

There are different techniques to solve NP problems. In first-order methods such as the generalized reduced gradient (GRG), the first step is to choose a search direction in the iterative procedure. This direction is determined by the first partial derivatives of the equations (the reduced gradient). NP methods have global convergence which means regardless of the starting point the convergence can be guaranteed. In compare to LP approaches, this method provides more accurate results [32].

2.3. INTERIOIR POINT METHODS [31].

As mentioned before one of the most popular methods to solve LP problems is Simplex method. However, this method requires long calculations thus it increases the convergence time. The worst-case scenario in Simplex method happens when the solution visits every vertex in the feasible region before reaching the optimal solution. Therefore, to decrease the convergence time Narendra Karmarkar's work on variations of interior point (IP) received much attention in the past decades. Karmarkar's algorithm is very different than simplex method since it rarely visits many extreme points before an optimal point is found. This algorithm stays inside of the feasible region and tries to position a current solution as the "center of the universe'' in finding a better direction for the next move. Although this approach may require more computational time in finding a moving direction, a better direction is achieved resulting in less iterations. For a large problem, IP method requires a fraction of number of iterations the simplex method would require.

As illustrated in Figure 2.3., simplex method seeks the optimal solution from vertex to vertex along the edges of the feasible space whereas IP methods which finds the solution from inside of the feasible space.

Variations of IP method proposed by Karmarkar include projective, affine-scaling and path-following. Projective scaling methods have a major benefit which is their superior worst case running time. Suppose that the size of the problem is defined by the number of bits, N, it required to present the problem in a computer. If the algorithm's running time never exceeds some fixed power of N, the algorithm is said to have a polynomial time. The projective scaling methods have such characteristics. Since Karmarkar's discovery, many variants of IP methods have been proposed of which primal affine method is briefly discussed in the next section.

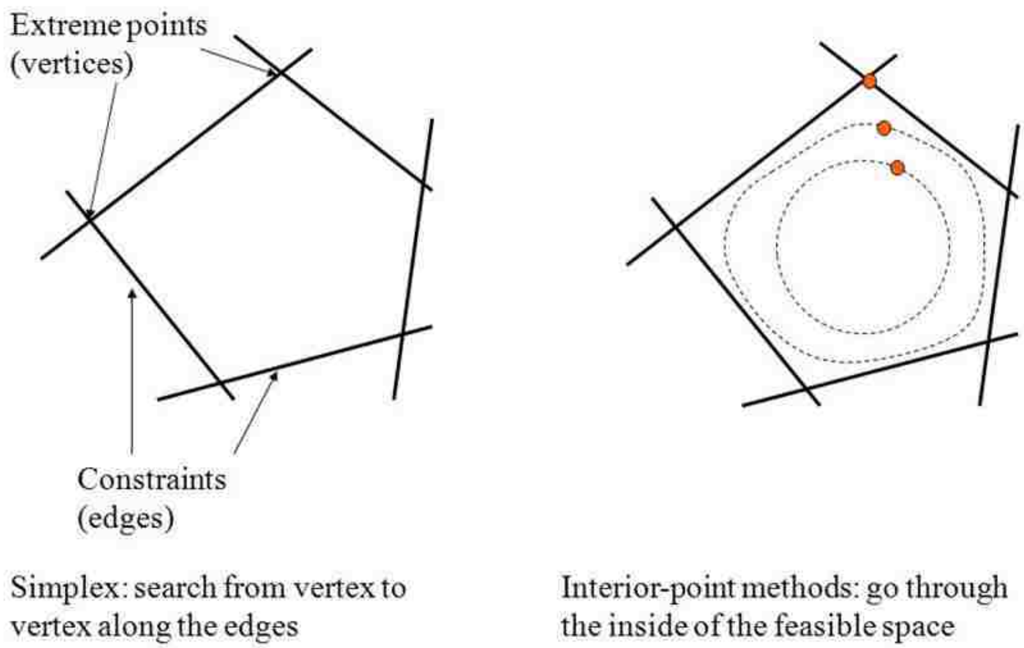

Figure 2.3. Simplex versus IP method [39]

In 1984 Karmarkar introduced first efficient practical, polynomial-time interior point method for LP. In this algorithm, each step must lie in the null space of A which is in parallel with the feasible space. In other words, there is a set of feasible solutions $(x^0, x^1, \ldots)$ that must satisfy $Ax_i = b$. In IP method, the feasible point is moved to the center of the feasible space via a transformation. After computing the new direction, the interior point is moved back to its original space. This direction is called the Projected Gradient Direction or p^k and the projection matrix P is introduced below [38].

$$P = I - A^T(AA^T)^{-1}A \tag{2.8}$$

Where a vector v will be transformed into $Pv = p$ and p will be in the null space of A. There is also another transformation required to center the iterative which needs the scaling to show that the iterative is equidistant from all constraint boundaries in the transformed feasible space [38]. This transformation is done using $x^k = e$, where $e = [1\ 1\ ...\ 1]^T$ [38]. The steps of the method are summarized below [38]:

1. Let $K = 0$
2. Let $D = diag(x^k)$
3. Compute $\hat{A} = AD$, $\hat{c} = Dc$
4. Compute $\hat{P}$ from $\hat{P} = I - \hat{A}^T(\hat{A}\hat{A}^T)^{-1}\hat{A}$
5. Set $p^k = \hat{P}\hat{c}$
6. Set $\theta = -min_j p_j^k$. The factor θ is used to determine the maximum step length that can be taken before exiting the feasible region.
7. Compute $\hat{x}^{k+1} = e + \frac{\alpha}{\theta}\, p^k$
8. Compute $x^{k+1} = D\hat{x}^{k+1}$
9. If $\|x^{k+1} - x^k\| < \varepsilon$, then done, Else set $k = k + 1$ and go step 2.

2.4. DYNAMIC PROGRAMMING

Dynamic programing was developed in 1950s through the work of Richard Bellman [31]. It can solve nonconvex, non-continuous, and nondifferentiable functions. DP can be considered as a transformation multiple vector decision process to a series of single vector decision processes [31].

In this method a large complex problem can be divided into a set of smaller simpler sub-problems. Each sub-problem is solved individually and the solution is saved, therefore next time the same sub-problem occurs the system uses the stored solution. Generally, these subproblems are easier to solve than the actual problem.

Dynamic Programing (DP) is commonly utilized to solved optimization problems. By combining the solution of sub-problems DP finds the best way to solve these problems. It is suitable for solving optimization problems that involve generation schedule in power systems, where energy management and power balance can be considered simultaneously [40]. In [41] by discretizing possible levels of energy in each battery system to a step size of E_{stp}, the problem is divided into sub-problems. After solving each sub-problem graph below showing all possible transitions is obtained.

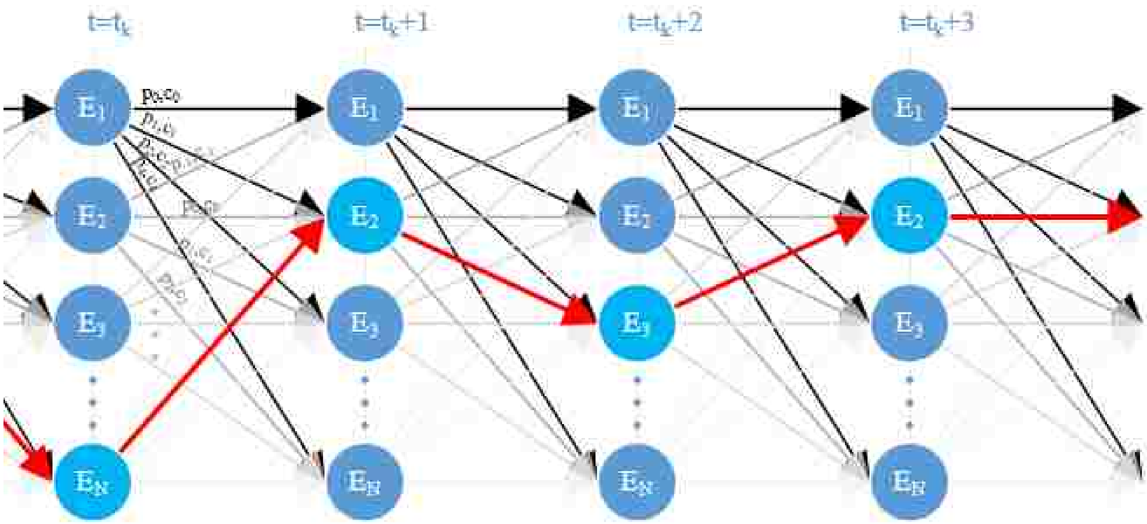

Figure 2.4. Dynamic programing graph of the economic dispatch problem [41]

In Figure 2.4. nodes are possible energy levels in battery resources at time t_k. In DP method the dimension of the problem is reduced by its ability to maintain the solution's feasibility, Hence, it requires less computational burden in compare to LP [40].

In DP method the complexity increases drastically with the number of constraints. It comes to a point that even more than two constraints can be difficult to solve.

2.5. GENETIC ALGORITHM [31]

Genetic algorithms (GA) stem from both natural biological genetics and modern computer science. They are referred to as stochastic search methods that originate from Darwinian thinking of natural selection and natural genetics. GA operates on a population of individuals, each of which is a potential solution to a given problem. This population is chosen randomly and lies in the feasible solution space.

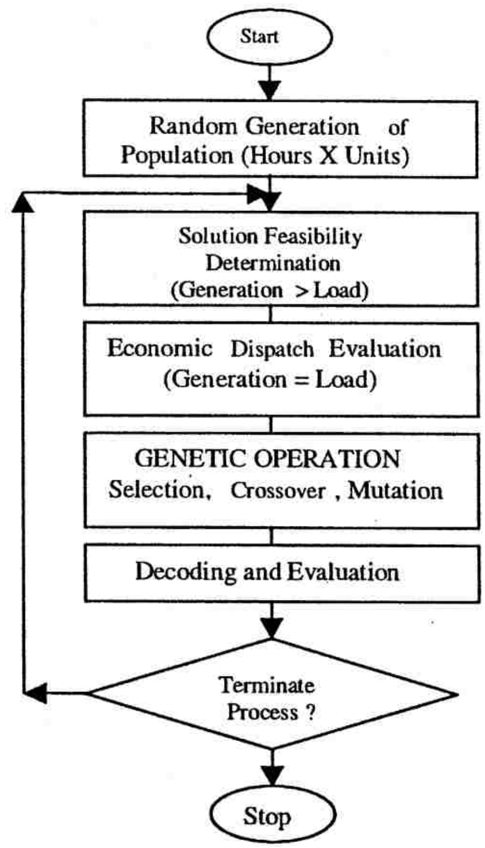

Figure 2.5. Flowchart of the GA approach [42]

There are various operators used in GA to perform different stages of an optimization process. These operators ensure that integrity or fitness of new generation is continuously improved at each stage of optimization problem. These include production operator, mutation operator, and crossover operator. The production operator generates copies of any individual that passes the fitness test of the goal function and otherwise eliminates them from the solution space. The mutation operator helps finding a global extrema by randomly exploring the solution space. This action is done by flipping the bits of selected candidates from the population. The crossover operator is responsible of finding better performing offspring by recombining individuals within the generation. GA can be applied to power systems in different areas including:

1. Expansion or structural planning
2. Operation planning
3. Generation/transmission and distribution operation
4. Power flow and harmonic analysis

Genetic algorithms can be used in unit commitment problems which can be considered as a part of operation planning application. Generally, the unit commitment (UC) problem involves determining the optimal set of generating unit within the next one to seven days.

In [42] application of GA for the solution of UC problem is demonstrated by the means of the flowchart illustrated in Figure 2.5. According to [43] for largescale problems the execution time of first-generation generations increases significantly and the solution quality decreases.

2.6. CONVEX OPTIMIZATION [37]

A function f is convex if for any two points within its range, x and y, the line between x and y lies above or on the function graph in a Euclidean space (a vector space) of at least two dimensions. In other words, any points on the straight line between $(x, f(x))$ and $(y, f(y))$ is greater than or equal to the value of f at the corresponding point between x and y as illustrated in Figure 2.6. The optimization problem is convex if the objective function and the constraints are convex.

Before the development of Convex Programing, LP was the most popular optimization method and many OPF problems were modeled based on linear power flow approximations. However, for more accuracy NLP algorithms were utilized for nonconvex models.

Semidefinite programing (SDP), and second-order cone programing (SOCP) were formed as convex generalization of LP. Since then many researches involving the new

SOCP and SDP power flow approximations have been conducted, implementing these methods in different contexts.

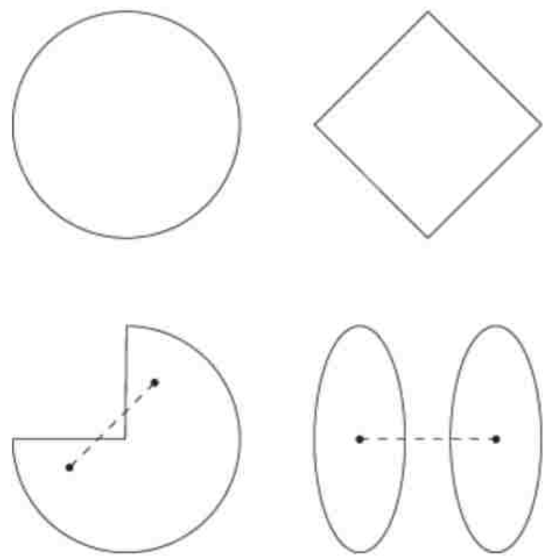

Figure 2.6. Convex (top) and nonconvex (bottom) sets [37]

2.6.1. Second-Order Cone Programing. Generally, a subset $\boldsymbol{C}$ of a vector V is a cone if for each $\boldsymbol{x} \in \boldsymbol{C}$ and a nonnegative scalar, $\boldsymbol{\alpha x} \in \boldsymbol{C}$. The cone $\boldsymbol{C}$ is convex if $\boldsymbol{\alpha x} + \boldsymbol{\beta y}$ belongs to $\boldsymbol{C}$, for any positive scalars $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and any $\boldsymbol{x}$, $\boldsymbol{y}$ in $\boldsymbol{C}$ [44]. The cone is convex it satisfies the convex function description.

Second-order cone (SOC) or ice cream cone or Lorenz is described as the set below:

$$\{ (y, t) \in \mathbb{R}^{n+1} : \|y\| \leq t\} \tag{2.9}$$

Since if $\|y\| \leq t$, then $\|\alpha y\| \leq \alpha t$ for any $\alpha \geq 0$, it satisfies the definition of a cone. SOC shown in Figure 2.7. , is also referred to as quadric cone since it is defined by a quadric inequality.

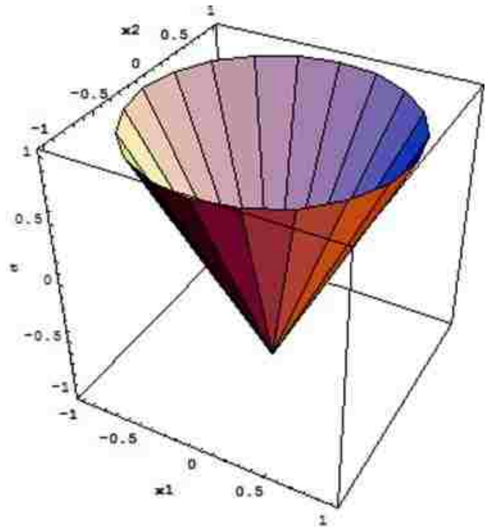

Figure 2.7. A second-order cone in R^3 [45]

Using the convex function definition, triangle inequality and homogeneity of the two-norm, convexity of a second-order cone can be checked. (LMI)

$$\|\alpha y_1 + (1-\alpha) y_2\| \leq \alpha \|y_1\| + (1-\alpha)\|y_2\| \\ \leq \alpha t_1 + (1-\alpha) t_2 \qquad (2.10)$$

Where (y_1, t_1) and (y_2, t_2) are in SOC and $\alpha \in [0,1]$. It can be concluded from the equation above that $(\alpha y_1 + (1-\alpha) y_2, \alpha t_1 + (1-\alpha) t_2)$ is also in the SOC therefore it is convex. If we assume $y = Ax + b$ and $t = c^T x + d$, the standard form of an SOCP shown below will be obtained.

$$minimize\ k^T x \\ \text{subject to } \|A_i x + b_i\| \leq c_i^T x + d_i \qquad (2.11)$$

OPF formulations can be obtained using SOCP method. In [46] this method is employed to improve the economic efficiency of VSC (voltage source converter) type AC-DC grids. Furthermore, distribution system reconfiguration is modeled in [47] using convex programing including SOCP.

2.6.2. Semidefinite Programing. In semidefinite programming (SDP), a positive semidefinite matrix is chosen to optimize a linear function that is subject to linear constraints. This type of optimization is similar to linear programming where the vector of the variables is replaced with a symmetrical matrix and nonnegative constraints with positive semidefinite ones [48]. Such constraints are nonsmooth and nonlinear, but convex so SDPs are convex optimization problems [49]. SDP problems like LP follow duality. Below is the most common standard formulation of SDP [49]

$$\begin{array}{ll} \text{Maximize} & c^T x \\ \text{Subject to,} & F(x) \geq 0 \end{array} \qquad (2.12)$$

Where $\mathrm{F(x)} \triangleq F_0 + \sum_{i=1}^{m} x_i F_i$, and the problem data are the vector $c \in \mathbb{R}^m$ and m+1 symmetric matrices $F_0, \ldots, F_m \in \mathbb{R}^{n \times n}$. Also, the inequality sign in $F(x) \geq 0$ indicates that F(x) is positive semidefinite [49]**.**

SDP can be considered as an extension of LP where the inequalities are replaced by matrix inequalities or the first orthant is replaced by the cone of positive semidefinite matrices [49].

2.7. RELAXATIONS [37]

Almost all formulations of power system optimization problems are nonconvex. These problems traditionally have been solved using linearization. However more accurate methods including convex relaxations now exist. Nonconvex problems can be approximated with conex relaxations. Consider blew optimization

$$\begin{array}{ll} \text{Minimize} & f(x) \\ \text{Subject to} & x \in X \end{array}$$

This can be converted to the equation below

$$\text{Minimize} \quad f(x)$$

Subject to $x \in Y$

The equation above is a relaxation if $X \subseteq Y$ (i.e., for any $x \in X, x \in Y$). This definition is illustrated in the Figure 2.8. This means that the minimum objective of a relaxation is less than or equal to the original objective. They also provide bounds on the true optima. If $X \subset Y$. Then by constructration,

$$\min_{x \in Y} f(x) \leq \min_{x \in X} f(x) \leq f(\acute{x}), \acute{x} \in X \quad (2.13)$$

A relaxed optimum and a feasible solution gives a two sided bound on the optimal objective.

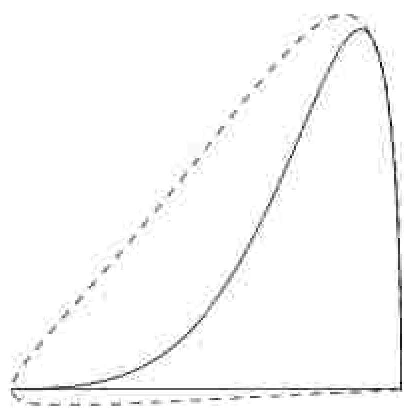

Figure 2.8. A convex relaxation (dashed) of a nonconvex set (solid) [37]

3. LITERATURE REVIEW

3.1. VOLTAGE REGULATIONS

In this section different existing methods of voltage regulations, other than use of BESS, in the literature are reviewed. In [50], an optimum method to combine active and reactive power based on R/X ratio has been applied to achieve a good voltage regulation for each node. As an attempt to control OLTC positions, to calculate Distributed Energy Resources (DER) active power and minimize overall energy system costs, [51] used a mixed-integer linear programing algorithm. In [52] a multi-period AC OPF technique for evaluating network capacity for accommodating variable DG is proposed and voltage control of transformers and voltage regulators are also embedded with the formulation. This method is coded in AIMMS optimization modeling environment and solved using a NP solver. It is reported in [53] that using a four-port DC/DC converter which is suitable for renewable energy harvesting applications, maximum power point tracking was achieved while maintaining a regulated output voltage. In [54] the optimal coordination of switched capacitors and tap-changing transformers in a radial distribution system is considered and the voltage constraints are included in the formulation. The optimization problem is approximated by a constrained discrete quadric method and two algorithms are presented to solve the approximation [54]. First one is randomized algorithm that would not guarantee optimality and the second one is a deterministic algorithm [54]. A DP method for solving reactive power/voltage control problem in a distribution system is presented in [55]. The considered constraints in the study are maximum allowable switching operations in a day for under load tap changer and each capacitor and the voltage limit on the feeder, the secondary bus voltage is limited [55].

3.2. BATTERY PLACEMENT SIZING AND ALLOCATION

Considering energy balance as a fundamental factor for transient stability of a micro grid, [56] has developed an energy function to allocate BESSs. In order to minimize costs associated with upgrades and network losses, [57] applies an optimal allocation of ESSs for load management. In [58] a multi-objective genetic algorithm is presented to determine size, location, and OPF of BESS units in a grid. To incorporate depreciation costs and [59] used an estimation of battery life expectancy in a study on a single typical household with a rooftop PV installation in Belgium. As a continuation of previous work in [58] and [59], [60] analyzes the impact of location of the battery in the feeder. The BESS model proposed in that study also includes a three-phase inverted with bidirectional active and reactive power. An optimization is performed using DP in [61] with a focus on optimal scheduling of grid connected PV systems with BESS. Placement of BESSs to meet voltage regulation requirements in conjunction with smart PV inverters is investigated in [62] This method uses simulated annealing approach in conjunction with a set of rule-based placement heuristics that speed up convergence [62].

In this paper, the goal is to utilize a method capable of containing the power flow equations as a constraint so that the battery sizing is performed by considering both active and reactive capacity of the BESS to achieve voltage regulation as well as other objectives.

3.3. SDP

In general, including the non-convex power flow equations as an optimization constraint is a technical challenge. In such optimization problems, to reduce the computational burden convexification of the problem has been used [63]. In [64] a suboptimal approach of sequential convex programming was proposed. Semi-Definite Programing (SDP) relaxation is a promising convexification approach [17], [65]–[68]. In this approach, the non-convex rank constraint is eliminated after the problem is converted to a SDP relaxation. The challenge in derivation of the SDP relaxation is meeting the optimality under the rank one condition [65]. In particular, if the network is radial or is resistive, this method is very effective [69]. Recently, applications of SDP have been investigated for mesh networks in addition to the radial distribution networks [66]. Details on accuracy and feasibility of SDP is studied in [70]. In [71], semidefinite programing was deployed to optimize the placement and sizing of a BESS. To this end, a sensitivity matrix was introduced which contained the voltage sensitivity of each bus to the power injected at other buses. Using this matrix, first the main network was divided into clusters. The clustering is performed based on the sensitivity and the number of clusters is the number of individual BESSs to be used. However, this method uses the sensitivity matrix as a linear entity and hence, this matrix will not present the nonlinear behavior of the system if multiple BESSs are installed. Hence, the results might be sub-optimal. A proposed multi-iteration SDP is utilized in this study instead of the sensitivity matrix method of [71] to solve this issue.

4. SIMULATIONS

4.1. AN INTRODUCTION TO CVX [72]

CVX is a Matlab-based modeling language which is designed to solve convex optimization problems including SDP, SOCP. It supports a number of standard problems such as linear and quadratic programs (LPs/QPs), second-order cone programs (SOCPs), and semidefinite programs (SDPs). It uses a particular approach to convex optimization called disciplined convex programming (DCP), which is proposed by Michael Grant, Stephen Boyd, and Yinyu Ye. DCP implements a set of rules referred to as the DCP ruleset that are sufficient but not necessary for convexity. Hence, it is possible to write codes that violate this set but are convex in fact. Three supported disciplined convex programs followed by three possible constraints are written as below.

- A minimization problem, which includes convex objective function and zero or more constraints.
- A maximization problem, which includes concave objective function and zero or more constraints.
- A feasibility problem, which includes one or more constraints and no objective.
- An equality constraint, made using ==, where both sides are affine.
- A less-than inequality constraint, indicated with <=, where the left side is convex and the right side is concave.
- A greater-than inequality constraint, indicated with >=, where the left side is concave and the right side is convex.

This program provides special modes for two specific problem cases including SDP mode and geometric (GP) mode. In SDP mode the constraints are typical expressed using linear matrix inequality (LMI). Various solvers are supported in CVX to solve different types of programming which are listed in the table below.

Power functions and p-norms are converted using a method described in [73]. This method is presented as a linear approximation for conic quadric problems and it uses

Schur complex. This approach is exact as long as $p = p_n/p_d$ is rational. Consider $x^p \leq y, p = 2$ as an example, which can be represented with exactly one 2x2 LMI:

$$x^2 \leq y \;\Leftrightarrow \begin{bmatrix} y & x \\ x & 1 \end{bmatrix} \geq 0 \tag{4.1}$$

The base CVX function library supports both common Matlab functions such as sum, trace, max, and min and new functions such as matrix fractional function (matrix_frac(x,Y)) which imposes constraint that Y is symmetric and positive definite.

4.2. DISTRIBUTION SYSTEM

In this study a real distribution system in Paradise Hill California is used. An interconnection map provided by San Diego Gas & Electric Company (SDG&E) showing transmission system, and substation area of SDG&E's distribution system is demonstrated in Figure 4.1.

This map is drawn in Microsoft Visio (Figure 4.2.) to better picture the existing buses and loads. This will later simplify modeling of the distribution system.

The Visio drawing presented in Figure 4.2. is consisted of 31 buses and 89 loads. Based on the map three locations are considered to place the batteries. Bus voltage profiles in four cases including the system before placing batteries, after placement of one battery, two batteries, and three batteries are studied and compared. In the next section the problem formulation is explained.

4.3. PROBLEM FORMULATION

In this study, lower-case and upper-case letters denote a vector and a matrix, respectively. $\Im(\cdot)$ and $\Re(\cdot)$ indicate the imaginary and real parts of the variables, respectively. $[A]_{ij}$ is the ij-th element of A. a^{T*}, a^T, and $a*$ denote the complex-conjugate transpose, transpose, and complex-conjugate of a. All zero and one matrices of appropriate dimensions are denoted by 0 and 1. Diag(a) returns a matrix A where $[A]_{ii} = [a]_i$. I is the unity matrix. Tr(A) is the trace, $\lambda_{max}(A)$ is the largest singular value. $|\cdot|$ is the absolute value, $\|a\|_1 = \sum_j \left|[a]_j\right|$ is the linear norm, and $\|a\|_2^2 = a^{T*}a$ is the Euclidean norm of a. Additionally, e_k, $k \in \{1, \ldots, \#N\}$ is the basis of $R^{\#N}$. Also, $EE_{k,k} = [E_{k,k}, 0; 0, E_{k,k}]$ and $EE_{k,w} = [(e_k - e_w)(e_k - e_w)^T, 0; 0, (e_k - e_w)(e_k - e_w)^T]$.

Element-wise (Schur) product of the two matrices is $a \circ b = diag(a)b$. Additionally, Schur complement of the block A of the matrix M = [A, B; B^T, C] is defined as S = C − $B^TA^{-1}B$. S is positive semidefinite if A and M are both positive semidefinite.

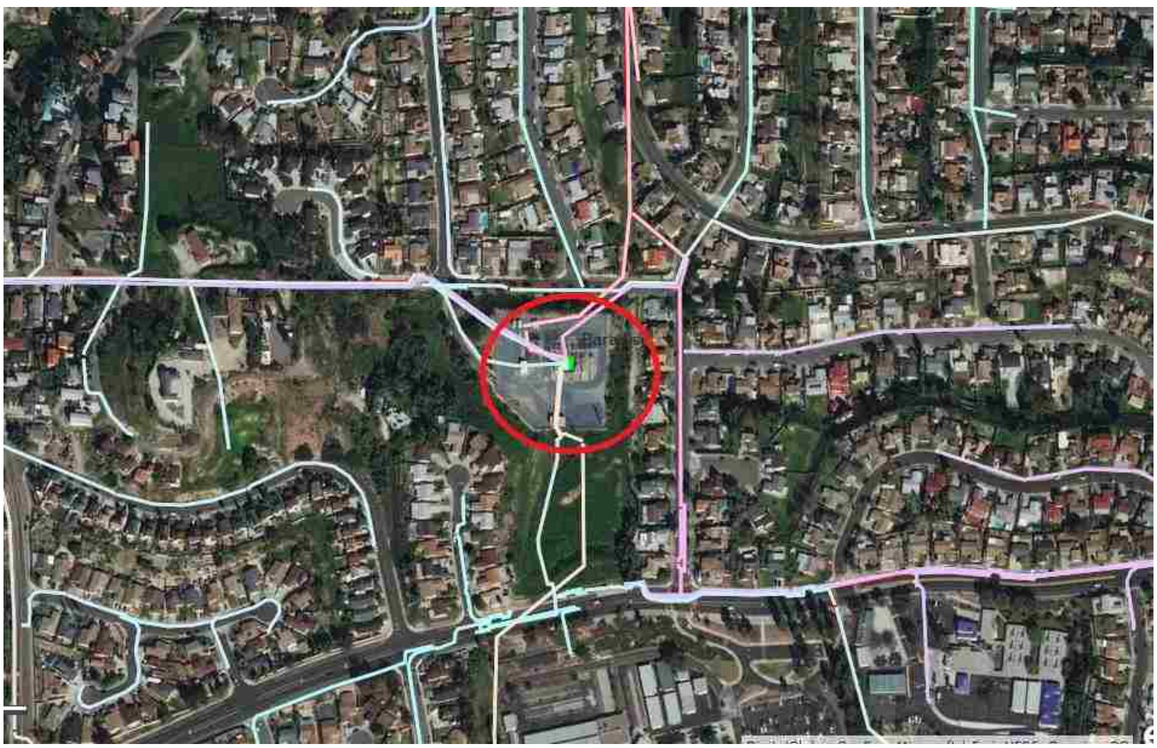

Figure 4.1. SDG&E's Interconnection Map, Paradise Substation [74]

4.3.1. SDP Relaxation. Power system is often modeled as below [68]

Minimize $\sum f(p)$ (4.2a)

Subject to $i = Yv$ (4.2b)

$i_k^* v_k = \sum s^g - s^d$ (4.2c)

$$p^{min} \leq p^{g} \leq p^{max} \tag{4.2d}$$

$$q^{min} \leq q^{g} \leq q^{max} \tag{4.2e}$$

$$p_{k,w}^{t} \leq p_{k,w}^{t-max}, \forall k, w \in N \tag{4.2f}$$

$$S_{k,w}^{t} \leq S_{k,w}^{t-max}, \forall k, w \in N \tag{4.2g}$$

$$|v_k - v_w| \leq v_{k,w}^{drop-max} \tag{4.2h}$$

Where $f(p)$ can be any semidefinite representative function and n ∈ N is the set of nodes within the distribution system. (4.2c) is the power balance equation where $s^g = p^g + jq^g$ and is the generated complex power and $s^d = p^d + jq^d$ is the demand complex power. p^g and q^g are the generated power and reactive power respectively. Similarly, p^d and q^d are the load or demand active and reactive power at this node. p^g, p^d, q^g, and q^d represent the vectors of the generated and consumed active and reactive power throughout the distribution system, respectively. If a bus does not have each of these entities, then a value of zero is considered for the corresponding vector elements. p^{min}, q^{min}, p^{max}, and q^{max} indicate the minimum and maximum limitations on active and reactive power dispatch levels of each node. $p_{k,w}^{t}$ and $S_{k,w}^{t}$ are the active and complex power flowing between buses k, and w, which have nominal limits of $p_{k,w}^{t-max}$ and $S_{k,w}^{t-max}$. Additionally, (4.2h) denotes the voltage drop on the line between nodes k and w.

To turn (4.2) into a SDP relaxation, composing additional equations is required. Injected power into node k can be written as [65]

$$P_k = \Re\{V_k I_k^*\} = \Re\{V^* e_k e_k^* I\} = \Re\{V^* Y_k V\} \tag{4.4}$$

The equation above is useful in composing $v^T e_k e_k^T Y^* v^*$ which is related to node k. Trace function is used in converting the power flow problem to an SDP problem. Particularly the rotational property of the trace function is needed (i.e. $v^T e_k e_k^T Y^* v^* = Tr(v^T e_k e_k^T Y^* v^*) = Tr(e_k e_k^T Y^* v^* v^T)$). This will lead to extract a new variable $V = \bar{v}\bar{v}^T$. The admittance matrix can be written as

$$\bar{Y}_{k,w} = \frac{1}{2}\begin{bmatrix} (Y_{k,w} + Y_{k,w}^T)^* & J(Y_{k,w} - Y_{k,w}^T)^* \\ J(Y_{k,w} - Y_{k,w}^T)^* & (Y_{k,w} + Y_{k,w}^T)^* \end{bmatrix}$$

$$Y_{k,w} = \left(y_{k,w}^C + y_{k,w}\right)e_k e_k^T - y_{k,w} e_k e_w^T \tag{4.5}$$

Where $Y_{k,k} = e_k e_k^T Y$, and $y_{k,w} = [Y]_{k,w}$, and $y_{k,w}^C$ is taken from the π-model of the line between buses k and w, and it is the admittance representing the shunt element. Now the symmetric matrices can be written as

$$Y_{k,w}^{\Re} = \Re(\bar{Y}_{k,w}) \tag{4.6}$$

$$Y_{k,w}^{\Im} = \Im\left(\bar{Y}_{k,w}\right) \tag{4.7}$$

(4.6) and (4.7) are used to write the equations below which will be employed to compose several objective functions for the optimization.

$$p_k^g + p_w^d = Tr\left(Y_{k,k}^{\Re} V\right),\ q_k^g - q_k^d = Tr(Y_{k,k}^{\Im} V) \tag{4.8}$$

$$p_{k,w} = Tr\left(Y_{k,k}^{\Re} V\right),\ q_{k,w} = Tr(Y_{k,w}^{\Im} V) \tag{4.9}$$

$$p_{k,w}^{line} = p_{k,w} + p_{w,k},\ q_{k,w}^{line} = q_{k,w} + q_{w,k} \tag{4.10}$$

$$\left|v_q\right|^2 = Tr(E e_k e_k^T V),\ |v_k - v_w|^2 = Tr(E e_k e_w^T V) \tag{4.11}$$

Finally, the OPF problem in (4.2) can be converted to a convex SDP relaxation as

Minimize $\quad V(V, p^g, q^g)$ (4.12a)

Subject to $\quad \forall q, w \in N$

$$p_k^g - p_k^d = Tr\left(Y_{k,k}^{\Re} V\right), q_k^g - q_k^d = Tr(Y_{k,k}^{\Im} V) \tag{4.12b}$$

$$p_k^{min} \le p_k^g \le p_k^{max}, \tag{4.12c}$$

$$q_k^{min} \le q_k^g \le q_k^{max} \tag{4.12d}$$

$$\{v_k^{min}\}^2 \le Tr(E e_k e_k^T V) \le \{v_k^{max}\}^2 \tag{4.12e}$$

$$\begin{bmatrix} (S_{k,w}^{t-max})^2 & Tr\left(Y_{k,w}^{\Re} V\right) & Tr(Y_{k,w}^{\Im} V) \\ Tr(Y_{k,w}^{\Re} V) & 1 & 0 \\ Tr(Y_{k,w}^{\Im} V) & 0 & 1 \end{bmatrix} \ge 0 \tag{4.12f}$$

$$-p_{k,w}^{t-max} \leq Tr\left(Y_{k,w}^{\Re} V\right) \leq p_{k,w}^{t-max} \tag{4.12g}$$

$$Tr(E e_k e_w^T V) \leq \{v_{k,w}^{drop-max}\}^2 \tag{4.12h}$$

$$V \geq 0 \tag{4.12i}$$

$$Tr\left(E_{ref}^{\Re} V\right) = 1, Tr\left(E_{ref}^{\Im} V\right) = 0 \tag{4.12j}$$

$\sum f(p)$ in (4.2) is replaced by a function where v is the vector of voltages within the distribution system. $V = \bar{v}\bar{v}^T$ was eliminated from (4.1) and replaced by (4.12i), since it is a non-convex constraint with a rank of one (Rk(v)=1). (4.12b) indicates power balance equations. Limitations on BESS active and reactive power are enforced by (4.12c) and (4.12d). Based on the gird requirements (such as ANSI C84.1- 2011 standard), (4.12e) sets boundaries on the square Euclidean norm of the voltage of bus k. The complex power passing through the line between buses k and w is controlled by (4.12f). Similarly, the square Euclidean norm of total voltage and active power flowing in the line between buses k and w and voltage drop on this line is controlled by (4.12g) and (4.12h), respectively.

The voltage of the slack bus or the reference bus of the distribution system should also be controlled. It is necessary since if the constraint related to this bus which is presented in (4.12j) is eliminated, the optimization will converge to a wrong feasible point. $E_{ref}^{\Re}$= [$E_{1,1}$, 0; 0, 0] and $E_{ref}^{\Im}$ = [0, 0; 0, $E_{1,1}$] extract the real and imaginary parts of the reference bus voltage, respectively.

4.3.2. Optimized Voltage Extraction. When the optimization problem (4.12) reaches a feasible point, V rank will be one and generated active and reactive powers values will be obtained. However, in the previous section to avoid a non-convex constraint, $\boldsymbol{V = \bar{v}\bar{v}^T}$ was replaced with a semidefinite constraint (4.12i), therefore, extraction of v value is required. To do so, one might simply take the first column of V as the solution although this approach is not accurate. To calculate the rank of V small singular values generated as a result of numerical errors must be eliminated. Singular value decomposition can be utilized to eliminate these errors by generating (4.13)

$$\mathrm{V} = \mathrm{U} \sum W^{T*} \tag{4.13}$$

Where U and W indicate the orthogonal basis of singular values. $\sum = [\sigma_i]_{ii}$ is a diagonal matrix containing singular values and σ_i is the i-th singular value of it. Commonly Σ is sorted from the largest singular value to the smallest. If $\sigma_2 < \varepsilon^2\sigma_1$ in (4.12) it can be concluded that the rank 1 condition is satisfied (where ε is an arbitrary clamping assumption such as 1%). Then $\bar{v}$ can be calculated using $\bar{v} = \sqrt{\sigma_1}u_1$ where u_1 is the vector associated with the σ_1,. Finally, v can be extracted as $v = [v]_i = [\bar{v}]_i + J[\bar{v}]_{(N+i)}$ which is the complex voltage vector for the underlying system.

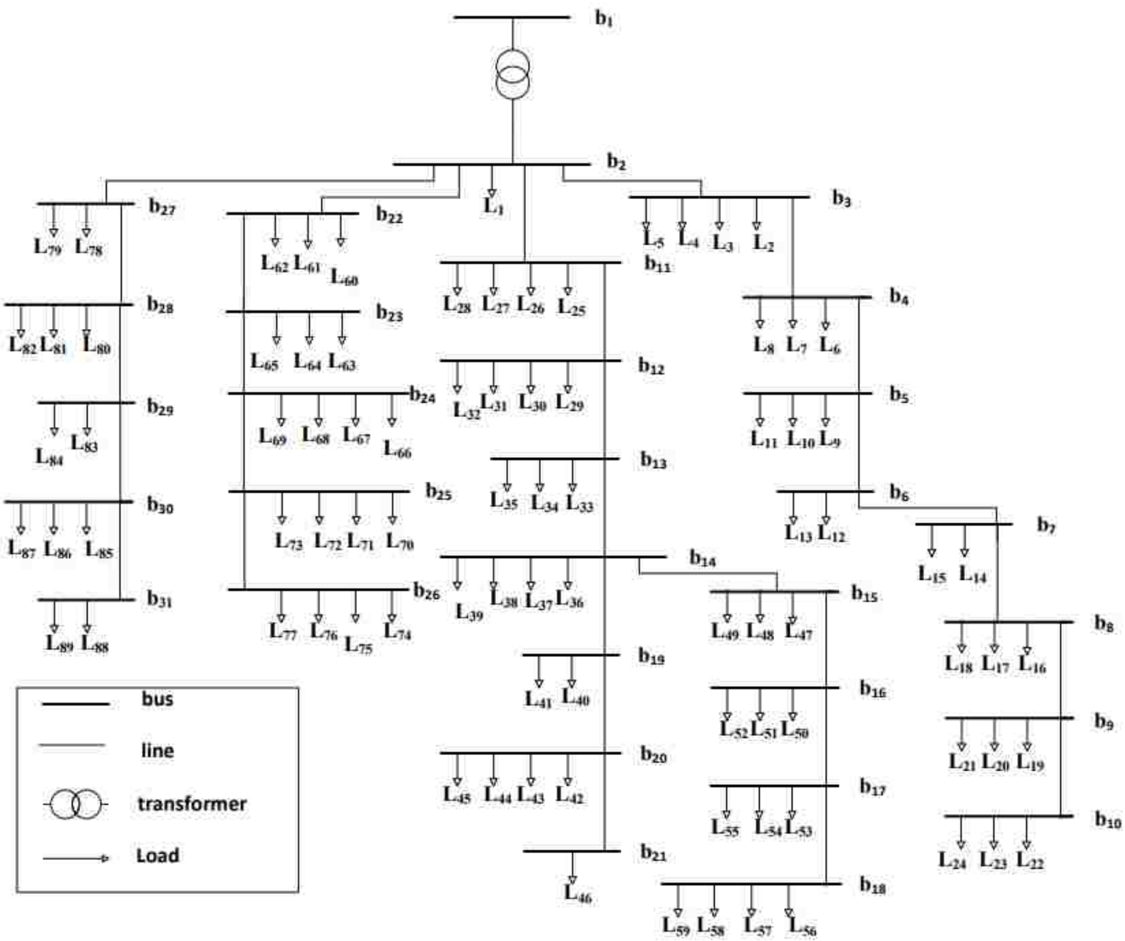

Figure 4.2. Paradise Distribution System

4.3.3. Multi-Objective Dispatch. Using the method introduced in the last sections various objective functions can be convexified. In this section various objective functions will be combined. Each of them will be defined as an auxiliary variable O_i where o = $[O]_i$ is the vector of auxiliary variables deployed to form various objective functions.

The cost of active power generation is one of the most common objective functions. To convert this function to SDP form, Schur complement is used as

$$\text{Minimize} \quad O_1 \tag{4.11a}$$

$$\text{Subject to} \quad (4.12)\text{b} - (4.12\text{j}), \forall q \in N_D, O_1 = \sum_k C_k$$

$$\begin{bmatrix} C_k - c_{1k} p_k^g - c_{0_k} + K & c_{2k} p_k^g \\ c_{2k} p_k^g & 1 \end{bmatrix} \geq 0 \tag{4.11b}$$

N_D represents the set of nodes with fuel consuming generators. K is a large constant to keep $C_k - c_{1k} p_k^g - c_{0_k} > 0$, otherwise, conditions of the Scur complement will not be satisfied.

As stated in the previous sections, voltage regulations play an important role in the performance of the power system equipment. Therefore, it must be considered while solving the OPF problem. Below in the voltage regulation equations where the voltage values are regulated to a predefined $v_k^{ref} = 1\ p.u$ and b_1 is the slack bus.

$$\text{Minimize} \quad O_2 \tag{4.12a}$$

$$\text{Subject to} \quad (4.10\text{b}) - (4.10\text{j}), \forall k \in N \setminus \{b_1\}$$

$$(v_k^{ref})^2 - O_2 \leq Tr\left(EE_{k,k} V\right) \leq \left(v_k^{ref}\right)^2 + O_2 \tag{4.12b}$$

If the target is to minimize the total losses over the distribution lines, using (4.10) losses can be calculated. Note that no lower boundary is required since the active power loss is always positive.

$$\text{Minimize} \quad O_3 \tag{4.13a}$$

Subject to $\quad (4.10\text{b}) - (4.10\text{j}), \forall k, w \in N$

$$Tr\left(Y_{k,w}^{\Re} V\right) + Tr(Y_{w,k}^{\Re} V) \leq O_3 \quad (4.13\text{b})$$

Minimizing renewable resource curtailment can also be modeled when a cost function is associated with the total curtailed power $\hat{p}_k^g - p_k^g$ where $\hat{p}_k^g$ is expected generation and p_k^g is the dispatched generation level. Hence, a second order curtailment penalty function, where N_R is the set of nodes with renewable energy resources, can be considered as

Minimize $\quad O_4 \quad (4.14\text{a})$

Subject to $\quad (4.10\text{b}) - (4.10\text{j}), \forall k \in N_R, O_4 = \sum_k C_k$

$$\begin{bmatrix} C_k - c_{3k}(\hat{p}_k^g - p_k^g) + K & c_{4k}(\hat{p}_k^g - p_k^g) \\ c_{4k}(\hat{p}_k^g - p_k^g) & 1 \end{bmatrix} \geq 0 \quad (4.14\text{b})$$

5. RESULTS

5.1. CASE STUDY

In this section four different cases mentioned before will be discussed. In Case #1 bus voltages and active powers are evaluated without the presence of batteries. Case #2 investigates voltage profiles and generated powers with the presence of one battery. In Case #3 observed changes when two batteries are placed in the distribution system, are discussed. Case #4 placement of all three batteries is compared to best results of each Case study. Note that the system is evaluated during peak hours at night when PVs are off and batteries are at the maximum discharge level.

5.1.1. Case #1. Bus Voltage profiles without any batteries in place are investigated in this scenario to use as a reference for the next case studies. Bus voltage values are presented in Table 5.1 and Figure 5.1. They are later compared with other scenarios to insure voltage improvement after placing the batteries.

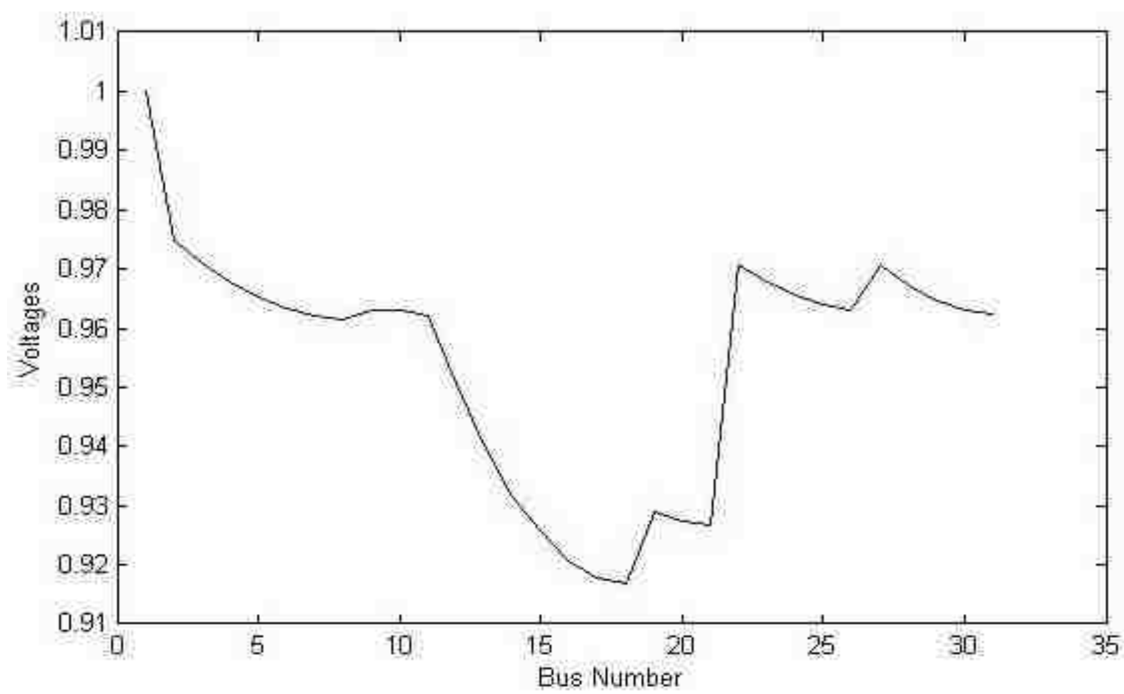

Figure 5.1. Bus voltages without the presence of batteries

Table 5.1. Bus voltages without the presence of batteries

Bus Number	Voltage	Bus Number	Voltage	Bus Number	Voltage
2	0.9747	12	0.9507	22	0.9708
3	0.9709	13	0.9401	23	0.9678
4	0.9677	14	0.9314	24	0.9655
5	0.9653	15	0.9257	25	0.9638
6	0.9633	16	0.9206	26	0.9629
7	0.962	17	0.9177	27	0.9706
8	0.9616	18	0.9169	28	0.9672
9	0.9632	19	0.9289	29	0.9648
10	0.9629	20	0.9273	30	0.9631
11	0.9621	21	0.9266	31	0.9622

5.1.2. Case #2. As shown in 1.10. stationary batteries take a considerable space therefore according to the existing plan of Paradise Hill neighborhood, three buses including bus 2 (inside the substation), 14, and 25 are chosen for placement of the batteries. Bus 1 is considered as a slack bus. After observing the system changes with different values of power, maximum generated power of batteries is assumed to be 400 kW. Figure 5.2. illustrates the bus voltages after placement of a single battery on each of the mentioned buses separately. As seen in this figure, placing the battery on Bus #14 provides the highest voltage values in compare to other two candidate buses. In this case if placing only one battery is desired, the best location would be on bus #14.

Small voltage reduction levels can be observed in some buses, however the overall voltage profiles are improved when placing the battery on bus #14. Also, the generated power of battery on bus #14 is 300 kW which is very close to the predefined maximum generated power. The lowest voltage profiles would occur after placing the battery on bus #25.

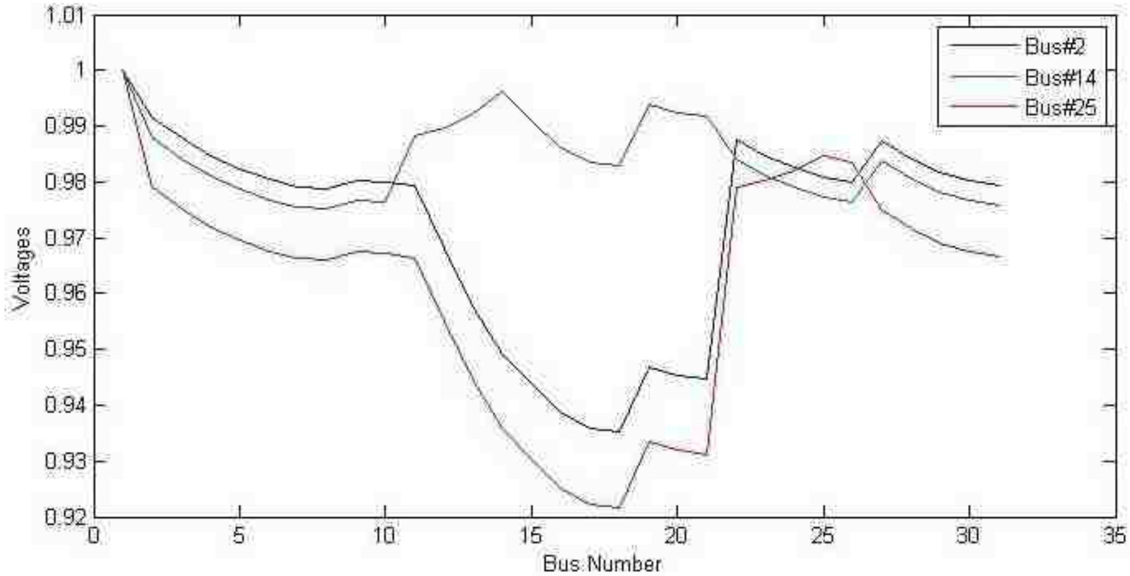

Figure 5.2. Bus voltages after placing one battery

Voltage values provided in Table 5.2. indicates a significant voltage improvement as a result of placing one battery at bus #14.

Table 5.2. Bus voltages after placement of one battery

Bus Number	Voltage	Bus Number	Voltage	Bus Number	Voltage
2	0.988	12	0.9899	22	0.9842
3	0.9842	13	0.9921	23	0.9812
4	0.9812	14	0.9961	24	0.979
5	0.9788	15	0.9909	25	0.9773
6	0.9768	16	0.9862	26	0.9764
7	0.9756	17	0.9835	27	0.9839
8	0.9752	18	0.9828	28	0.9806
9	0.9767	19	0.9938	29	0.9782
10	0.9765	20	0.9924	30	0.9766
11	0.9883	21	0.9917	31	0.9758

5.1.3. Case #3. Now two batteries are placed in three arrangements including buses 2 and 14, 2 and 25, and 14 and 25 (Figure 5.3.). As expected, the most proper arrangement is bus 14 and 25, since these buses have the highest generated power in compare to bus #2 at maximum discharge. At this point it is obvious that placing the battery on bus #2 is not efficient due to the little improvement that is makes in compare to other locations.

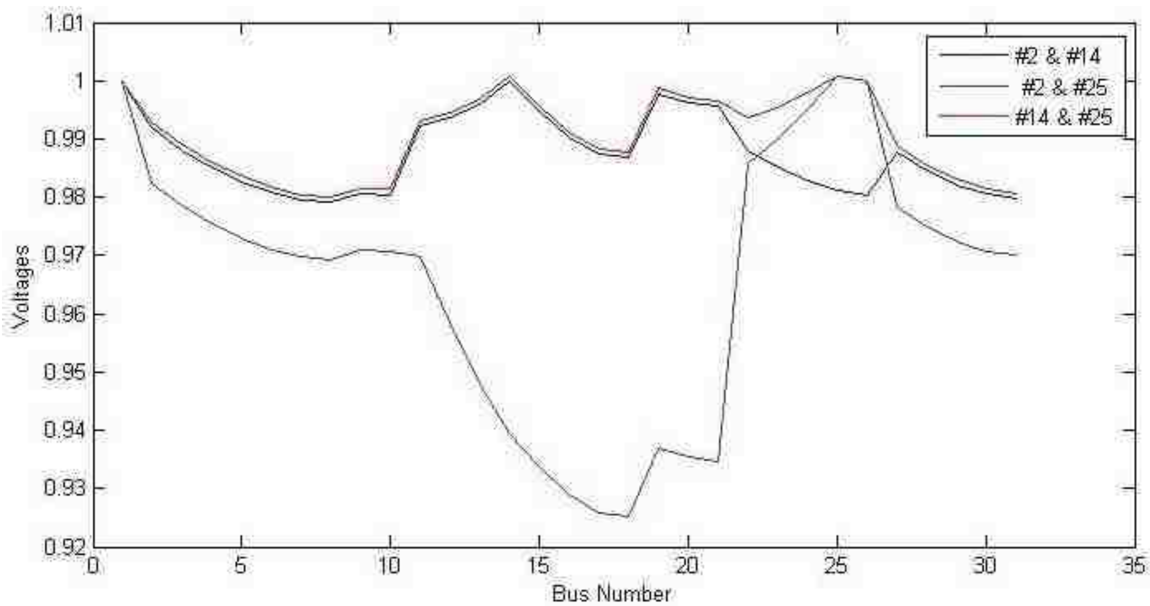

Figure 5.3. Bus voltages after placing two batteries

5.1.4. Case #4. To achieve the accurate results, placing all three batteries is compared to the best results of each scenario. Also to better observe the results, system voltage profiles before placing batteries is included in the graph. Figure 5.4. shows that due to small power generated by placing the battery on bus #2, placing batteries in all three candidate locations has the same result as placing only 2 on bus #14 and #25. Furthermore, this study significantly improved the voltage profiles during peak hours by placing only two batteries in the entire proposed distribution system.

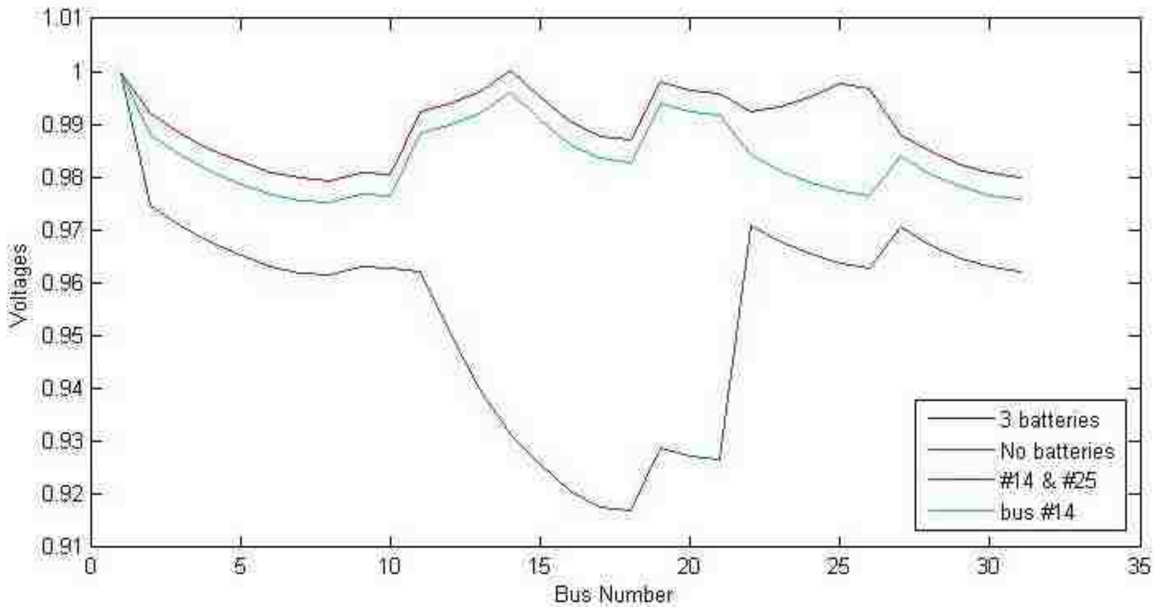

Figure 5.4. Bus voltages after placing three batteries. Best results of each case is also included

Table 5.3. illustrates a significant increase in voltage profiles when adding two batteries at bus 14 and 25. Results show that most of the bus voltage values are close to 1.

Table 5.3. Voltage profiles after adding two batteries to the system

Bus Number	Voltage	Bus Number	Voltage	Bus Number	Voltage
2	0.9921	12	0.994	22	0.9924
3	0.9884	13	0.9962	23	0.9934
4	0.9853	14	1.0003	24	0.9953
5	0.983	15	0.9951	25	0.9977
6	0.981	16	0.9904	26	0.9968
7	0.9798	17	0.9877	27	0.9881
8	0.9794	18	0.987	28	0.9848
9	0.9809	19	0.998	29	0.9824
10	0.9807	20	0.9966	30	0.9808
11	0.9925	21	0.9959	31	0.9799

All of the cases above were investigated during the peak hours at night and under an assumption that the solar panels are turned off. However, the impact of batteries on the distribution system with the presence of PVs also needs to be considered. Figure 5.5. shows the presence of rooftop solar panels on some of the loads (houses).

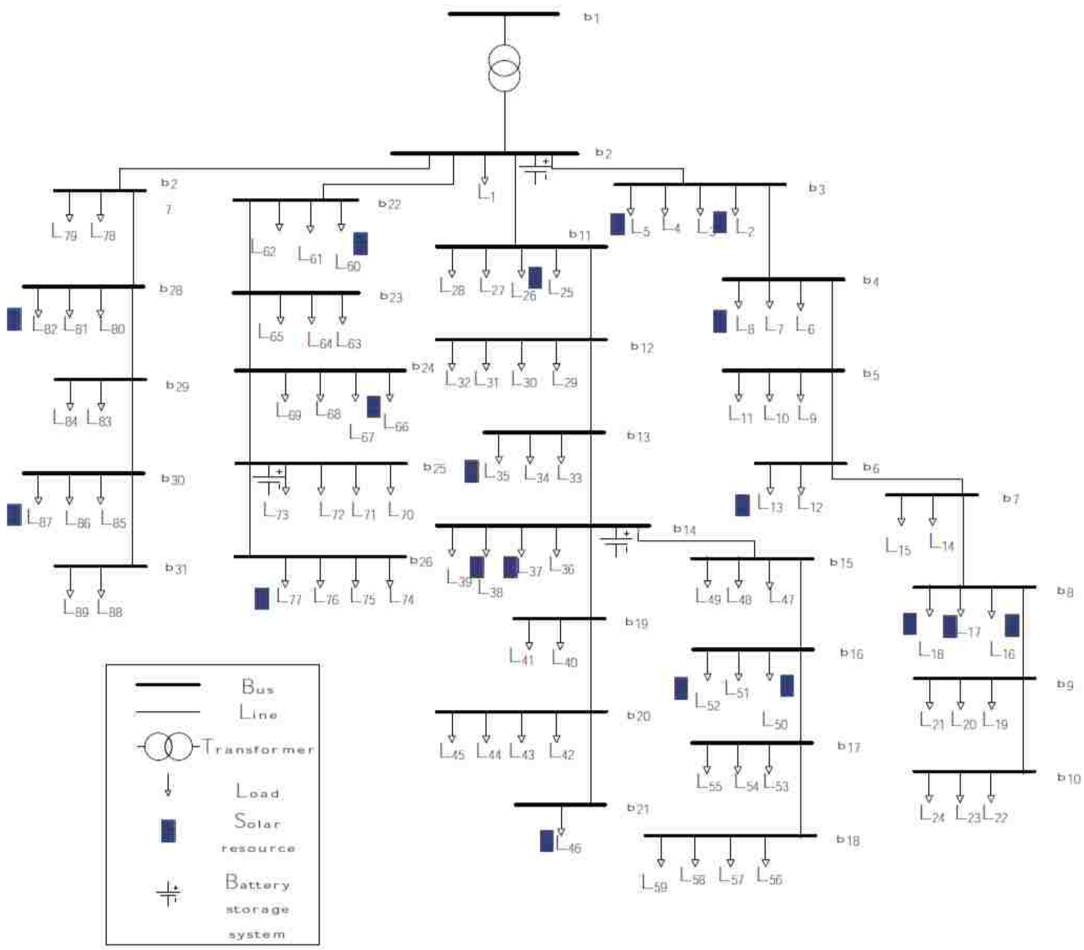

Figure 5.5. Distribution system with the presence of solar panels

Similar to Case #1, to better evaluate the results, system voltages at noon without any batteries in place are shown in Figure 5.6. This is compared to Case #1 to obtain the maximum voltage changes between two buses. As it can be observed in Figure 5.6. The maximum bus voltage difference between these two conditions in almost 0.03 P.U.

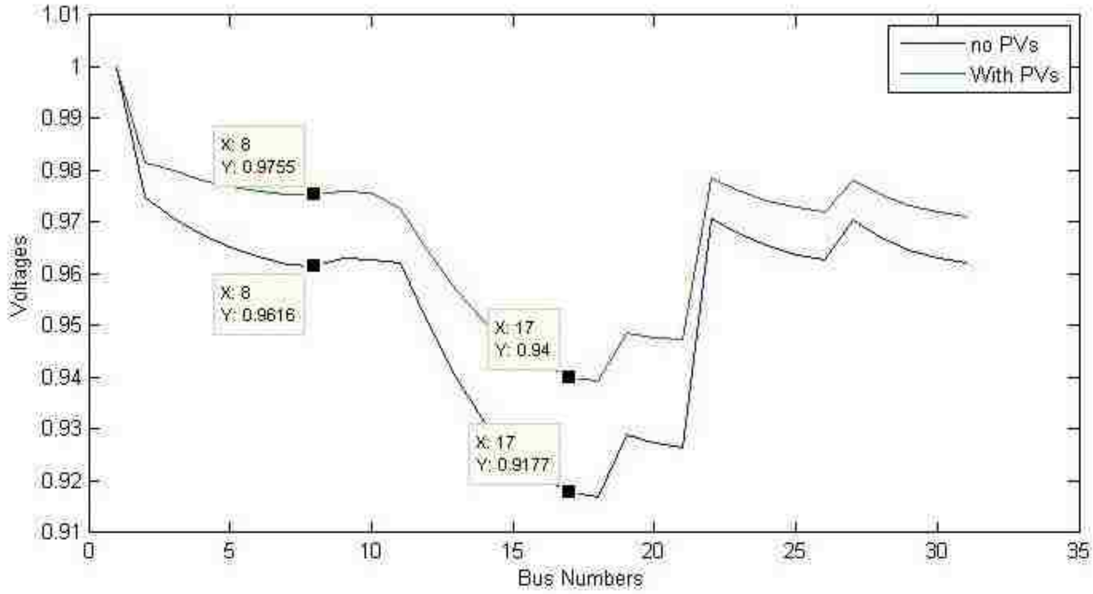

Figure 5.6. Bus voltages with and without the presence of PVs

To discover the improvement in voltage differences as a result of placing two batteries at bus 14 and 25, this arrangement of batteries is shown both during peak hours and at noon (with and without the presence of PVs). This is illustrated in Figure 5.7.

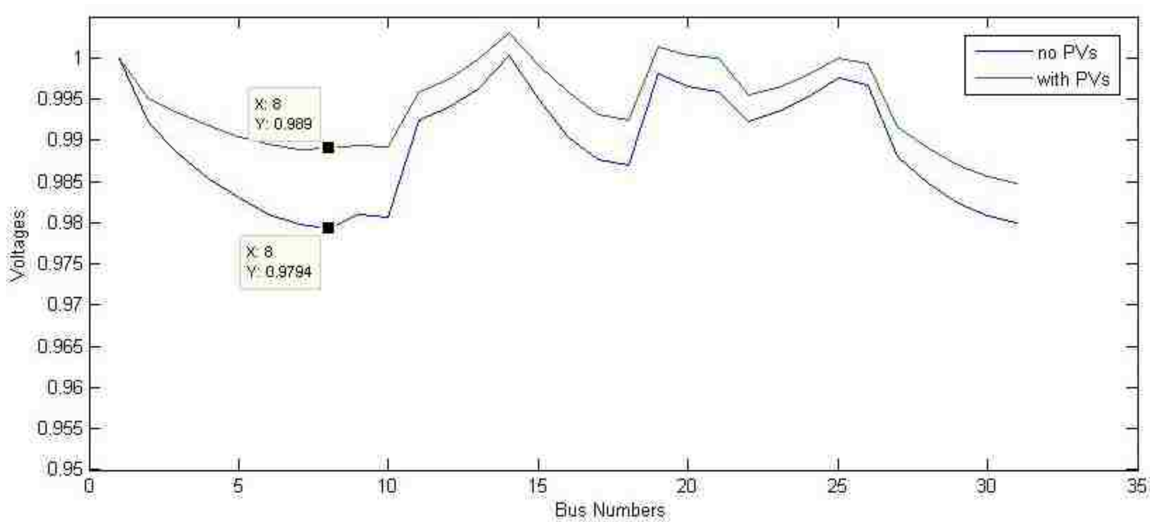

Figure 5.7. Bus voltages after placing batteries on buses 14 and 25.

As it can be seen in Figure 5.7. the difference between the bus voltages is reduced from 0.03 P.U. to 0.01 P.U. This shows the enhancement in voltage profiles caused by the battery placement.

5.2. CONCLUSION

Placement of batteries significantly improves the voltage profiles in the distribution systems. Utilities may use this method to control the voltage and power of the systems with high penetration of PVs. Furthermore, by storing the energy that was derived from PVs and releasing it when needed in the network, batteries allow the use of all existing resources. However, in most cases the excessive generated power by PVs are either fed back into the system which causes voltage imbalances or avoided by curtailment.

5.3. FUTURE WORK

Electric vehicles and transportation systems demand large charging powers and hence, grids with high number of electrified transportation systems need to cope with the newly added load profile by these systems. The combination of resource intermittencies and large loads induced by electric vehicles demand a method for coordination between the charging patterns of the vehicles and the energy management controller of the microgrid.

Large loads such as electric vehicles and autonomous transportation systems can induce large stresses over the grid. To mitigate this problem, methods for dynamic coordination of electric vehicles and the microgrid controller is needed. Battery placement and dynamic allocation of charging stations for microgrids with high penetration of renewable resources and electrified transportation systems can be investigated as a multi-objective optimization problem. To incorporate power quality objectives, this optimization needs to consider voltage equations within the power flow problem. However, this will lead to a non-convex problem which may be solved by the algorithm presented in this thesis.

Sizing of the stationary batteries can also be investigated using a distributed optimization. In this method a day is divided into six time periods and each of them are considered to be a cluster. Distributed optimization can then be used to solve the problem.

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