Generally covariant quantum information

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GENERALLY COVARIANT QUANTUM INFORMATION

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Abstract

The formalism of covariant quantum theory, introduced by Reisenberger and Rovelli, casts the description of quantum states and evolution into a framework compatible with the principles of general relativity. The leap to this covariant formalism, however, outstripped the standard interpretation used to connect quantum theory to experimental predictions, leaving the predictions of the theory ambiguous. In particular, the absence of a pre-defined time variable or background causal structure resulted in an "order of projections" ambiguity, in which the usual rule for multiple-measurement probabilities (obtained by time-ordered projections) is not defined. Equally troublesome, the probability postulate of Reisenberger and Rovelli fails to reproduce the Born interpretation for the case of simple quantum mechanical systems.

Here, we develop an alternative quantum measurement formalism, based on basic principles of quantum information. After reviewing how this can be done in the context of the traditional formulation of quantum mechanics and noting its implications for the quantum measurement problem, we find that this approach can be generalized to the covariant setting, where it essentially solves the correspondence problems of covariant quantum theory. We show explicit agreement with the Born interpretation of standard quantum mechanics in the context of simple systems. We also demonstrate the origin of the quantum mechanical arrow of time within our framework, and use this to solve the order of projections ambiguity. In addition to compatibility with general covariance, we show that our framework has other attractive and satisfying features — it is fully unitary, realist, and self-contained. The full unitarity of the formalism in the presence of measurements allows us to invoke time-reversal symmetry to obtain new predictions closely related to the quantum Zeno effect.

Chapter 1

Introduction

1.1 Information and Reality

The history of physics stretches back millennia — nearly as soon as written language arrived on the scene, so too did the first attempts at simple, descriptive astronomy [1]. The tendency for humans to seek a basic description of nature continued, so that well before the rise of the Roman empire, the ancient Greeks were already delving deeply into descriptions of matter, motion, optics, and the solar system. By the time of the 16th and 17th century, physics was intimately linked to the scientific revolution taking place in Europe, and has been progressing continually ever since.

By contrast, classical information theory seems to have appeared from out of nowhere in the 1940's [2, 3] — noticeably *later* than such thoroughly modern topics of physics as general relativity and quantum mechanics. Information theory was created as a tool for engineering and communication, but it was quickly apparent that there were some deep connections to fundamental physics — very early in its development, for example, Shannon was told by von Neumann that he should name his information-theoretic quantity *entropy*, since "...your uncertainty function has been used in statistical mechanics under that name..." [4]

There is something curiously fundamental about information — it has an element of universality. Information theory itself does not care what physical form the information takes, and in turn, it may be that everything we know about nature can be cast in terms of pure information. Certainly the laws of physics are represented in terms of strings of information-carrying symbols (i.e. it is possible to learn the laws of physics from reading books and papers), and our sensory perception itself can be described in terms of information carrying channels (discrete electrochemical pulses transmitted by neurons). And indeed, certain information-theoretic ideas of physics such as the holographic principle have led to important dualities in string theory — for example, the Maldacena duality, which demonstrates a formal equivalence between certain string theories in one spacetime and quantum field theories in another. Thus, although information theory was originally built under the assumption that information rests on physics in a chain of logical dependence as follows,

$$\text{laws of physics} \rightarrow \text{matter} \rightarrow \text{information},$$

(as emphasized by Davies) certain thinkers have proposed that a better description of the chain of dependence might instead be

$$\text{laws of physics} \rightarrow \text{information} \rightarrow \text{matter},$$

or perhaps even

$$\text{information} \rightarrow \text{laws of physics} \rightarrow \text{matter [5]}.$$

One motivation for the following work is to probe this line of reasoning through an example — specifically, we wish to express a fundamental rule of physics (the quantum measurement postulate) in an information theoretic framework. We will see that this approach reproduces the standard predictions of ordinary quantum physics easily, but it is also far more powerful than the usual measurement postulate. In particular, the information-theoretic framework has the power to illuminate the concept of measurement in covariant quantum physics — a context in which time loses its special status as an external background variable, and the measurement postulate of familiar quantum mechanics no longer applies.

1.2 Generally Covariant QM, Correspondence, and Internal Consistency

A great deal of work has been done on quantum gravity over the last fifty years, but the theory remains far from complete. This is due, no doubt, to the multiple layers of challenges involved [6]: At one level, it is extremely difficult to obtain experimental input for the theory, due to the enormous magnitude of the Planck mass, relative to typical energies encountered in particle physics. On another level, it is very difficult to perform exact calculations in the smooth-spacetime limit in which we live, due to the complicated nonlinear equations of motion and the difficulty in transitioning from discrete, Planck-scale quantum geometry to a smooth, classical spacetime manifold. And on yet another level, there is ambiguity in the interpretation of the mechanical framework itself, in the very definition of covariant quantum mechanics.

This thesis is concerned with the latter issue. We specifically avoid the first two problems by working primarily with simple systems composed of a few degrees of freedom (whose analysis is already well under control in the established formalism of ordinary Schrödinger picture quantum mechanics), analyzed with the formalism of generally covariant quantum theory. That is, we sidestep gravity completely and focus instead on the formulation of the covariant mechanical framework itself.

A formalism of covariant quantum mechanics does exist [7, 8] (we will review it in chapter 3), but we will see that when it is applied to simple, single-particle systems, it meets with several problems, all related in one way or another to the measurement postulate. Of course, measurement in ordinary quantum mechanics is itself not a completely settled subject; issues of internal consistency remain, with a complete and satisfying explanation consistent with the postulate of unitary evolution remaining surprisingly elusive [9]. However, the basic *rule* for extracting probabilities from quantum mechanics has been known since 1926 with Born's statistical interpretation of the wave function, and it is used in practice every day [10].

In generally covariant quantum mechanics, however, the measurement problem becomes much more serious. In the first place, the generally assumed rule (in fact, the only one in existence prior to the work contained herein) gives probabilities that are different from the established Born rule when applied to simple systems, outside of certain restrictive approximations. Additionally, the formalism came with an ambiguity in the definition of multiple-measurement probabilities — a problem related to the fact that time loses its status as an external, classical parameter in the theory, and thus the theory does not know how to distinguish an "arrow of time." Instead of representing subtle philosophical difficulties in the foundations of the theory, there now appear basic problems of correspondence with established physics, and in obtaining predictions that are free from ambiguity.

This thesis is an attempt to solve these problems using an information theoretic approach to quantum measurement. Chapter 2 will be an introduction to the basic information theoretic tools we need, and a description of standard quantum theory using this approach. We will directly confront the measurement problem and the quantum mechanical arrow of time, defining the latter in a new way which respects the time-reveral symmetry of the formalism. While the formalism is not covariant at this point, in chapter 3 we will use these information-theoretic tools to cast the quantum measurement postulate into a framework which is consistent with a generally covariant setting. Chapter 3 will also contain a demonstration that the interpretational problems of covariant quantum theory are essentially solved using this framework — that is, that the standard Born rule is reproduced exactly (in the relevant context), and that the multiple-measurement ambiguity vanishes. Chapter 4 is devoted to the question of experimental consequences, where we identify an effect we call the time-reversed quantum Zeno effect. Chapter 5 reviews and summarizes some of the general interpretational features of this approach, which are rather novel, and contains our concluding remarks.

Chapter 2

Information Theory, Quantum Mechanics, and Measurement

2.1 Basic Concepts of Information Theory

In terms of quantifying information, we will primarily be interested in a few basic quantities — the entropy, the joint entropy, and the conditional entropy. Each of these can be defined in the context of a classical information source, and a quantum information source. The prototype classical information source is the bit string — a series of 1's and 0's defining a message. If a bit string is n bits long, then there are clearly 2^n possible bit strings that could be sent, *a priori*.

We are interested in quantifying how much information such a message might hold: We might be tempted simply to count the number of bits in the message — that is, out of a space of N possible alternative messages, we take the $\log N$ (to the base 2), guaranteeing that we label a message composed of n bits as containing an amount "n" of information. This was actually proposed in 1928, and is called the "Hartley information content" of a message [11].

The Hartley information lacks a crucial feature, however. Let us imagine, for example, that we are listening to a series of messages emanating from a radio tower, somewhere. We listen for days and weeks and months, and notice a couple of things — each message is a sequence of n bits, but the first bit in any message is *always* a one, and *never* a zero. The Hartley information content of each message is n, but after weeks and weeks of copying down each message, we might decide to simply leave off the first bit, and record only the remaining

$n-1$ bits, since we can always add the additional 1 at a later time, if we wish. Thus, we see that the Hartley information doesn't take into account possible compression schemes, based on knowledge about the statistical nature of the signal, which we might use to successfully store (or transmit) an n-bit message with less than n bits.

Shannon was interested in defining a measure of information content which would express, in effect, the *minimum* average number of bits necessary to express a message, assuming that the probability distribution $p(i)$ for each possible message i was known. It was proven by Shannon in his noiseless coding theorem that this quantity is given by the following expression, called the Shannon entropy [2]:

$$H(p(i)) = -\sum_i \, p(i) \, \log p(i). \tag{2.1}$$

The Shannon entropy tells us, for example, that a given source sends us *no* information, if we know that it is only capable of sending a single message (i.e. if $p(i) = 1$ for one specific i, and $p(j) = 0$ for $j \neq i$ — meaning we already know the message before it is sent). On the other hand, the average information content is maximized if all messages are equally probable, in which case the Shannon entropy would be equal to the Hartley information content of a message.

We can also combine information from different sources. For example, if source A is associated with possible messages i, and source B is associated with possible messages j, then we can treat them together as a single source, AB, with joint probability distribution $p_{AB}(i,j)$, which is related to the single-source distributions by $p_A(i) = \sum_j p_{AB}(i,j)$. The joint entropy for such a combined source, $H(AB)$ is given by the same formula as before, simply summed over all possible pairs of outcomes

$$H(AB) = H(p_{AB}(i,j)) = -\sum_{i,j} \, p_{AB}(i,j) \, \log p_{AB}(i,j). \tag{2.2}$$

Note that the joint entropy satisfies $H(AB) = H(A) + H(B)$ if A and B are independent, uncorrelated sources (we will sometimes use shorthand notation, such that $H(A)$ is short for $H(p_A)$).

We can also introduce another probability distribution, called the conditional probability distribution $p_{A|B}(i|j)$ (or sometimes denoted $p(A|B)$), which is the probability that source A gives outcome i, *given* that source B's outcome is j. It is related to the joint distribution by $p_{A|B}(i|j)\ p_B(j) = p_{AB}(i,j)$. An important result, known as Bayes' rule, is that $p_{A|B}(i|j)\ p_B(j) = p_{B|A}(j|i)\ p_A(i)$.

We can define the amount of entropy in the conditional probability distribution of A, averaged over all potential outcomes of B, via the conditional entropy, $H(A|B)$, as follows:

$$H(A|B) = -\sum_{i,j} p_{AB}(i,j)\ \log p_{A|B}(i|j). \tag{2.3}$$

$H(A|B)$ represents the average minimum number of bits that need to be recieved in order to infer the full message of AB, given that we already know the message from B. The conditional entropy is always related to the joint entropy by the following identity:

$$H(A|B) = H(AB) - H(B) \tag{2.4}$$

These concepts have an analog in quantum mechanics, where instead of probability distributions p_A of potential outcomes, we instead have a density operator ρ_A, which describes the state of a quantum mechanical system, A. Here, instead of the Shannon entropy, the central player is the von Neumann entropy [12], $S(A)$, defined via:

$$S(A) = -\mathrm{Tr}_A\ \rho_A \log \rho_A. \tag{2.5}$$

Here, the trace is taken over an orthonormal basis of states spanning $\mathcal{H}_A$, A's Hilbert space. The von Neumann entropy quantifies the amount of classical uncertainty present in the state ρ_A, and answers a resource question analogous to to the Shannon entropy — $S(A)$ represents the number of quantum bits, *qubits* (superpositions of $|1\rangle$ and $0\rangle$), necessary to build a state encoding the same amount of information as ρ_A [13]. As before, this generalizes immediately to bipartite systems: If we have the joint density operator ρ_{AB} (on the joint Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_B$):

$$S(AB) = -\mathrm{Tr}_{AB}\ \rho_{AB} \log \rho_{AB}. \tag{2.6}$$

The single-component density operators are related to the multi-component density operator by $\rho_A = \mathrm{Tr}_B \rho_{AB}$.

We also have a quantum conditional entropy which satisfies $S(A|B) = S(AB) - S(B)$, just as with the classical conditional entropy. It can be defined using a "conditional amplitude operator" $\rho_{A|B}$ and the equation $S(AB) = -\mathrm{Tr}_{AB}\ \rho_{AB} \log \rho_{A|B}$ in analogy to the classical conditional entropy [14], but $\rho_{A|B}$ is a rather unusual quantity (it is not a density operator), and we will not need it for what follows. In any case, the quantity $S(A|B) = S(AB) - S(B)$ (simply defined in terms of the density operators ρ_{AB} and ρ_B) answers a resource question analogous to the classical case — given control of subsystem B of the combined system AB described by density operator ρ_{AB} (which is unknown to the holder of B), $S(A|B)$ represents the number of qubits that must be transferred from the holder of subsystem A to the holder of subsystem B, in order for the holder of B to infer the full state ρ_{AB} (given any amount of classical communication between the holders of A and B) [15]. Unlike the classical conditional entropy, the quantum conditional entropy can be negative — this is generally an indicator of entangled (nonsperable) states of AB, and a negative conditional entropy indicates the potential to use the shared state ρ_{AB} for quantum communication beyond what is required for the holder of B to determine ρ_{AB}.

2.2 Review of the Quantum Measurement Problem

Ordinary Schrödinger-picture quantum mechanics follows from the following postulates:

1. States: The possible states of a system are represented as vectors ψ in a complex Hilbert space, $\mathcal{H}_0$.

2. Observables: Quantum observables are represented by Hermitian operators on $\mathcal{H}_0$.

3. Evolution: States evolve from one to another in an external, classical time parameter t according to the Schrödinger equation, $i\hbar\partial_t\psi(t) = H_0\psi(t)$.

4. Probability: If we measure observable A of a system, the probability to observe the value λ_i is given by $|\Pi_i|\psi\rangle|^2$, where Π_i is the projector onto the eigenstate $|a_i\rangle$ with eigenvalue λ_i (Born interpretation). If outcome "i" is observed, the system is "collapsed" to the state $|a_i\rangle$ and will subsequently evolve from this state.

One of the most puzzling features of quantum mechanics is the appearance of postulate 4. Postulate 3 seems to indicate that quantum mechanics is deterministic — that is, there is a one-to-one correspondence between states *now*, and states at all times in the future (in keeping with classical mechanics). But postulate 4 seems to contradict this, introducing instead a fundamentally probabilistic feature into the theory. In other words, it is very difficult to see how a fixed unitary evolution operator alone (representing observation of A) can take a *known* superposition $\alpha_1|a_1\rangle + \alpha_2|a_2\rangle + \alpha_3|a_3\rangle + ...$ to precisely one term $|a_n\rangle$, always for *any* values of the α's (which only effect the relative probabilities).

Also strange is the fact that postulate 4 seems to feature the observer in a peculiar way. If quantum mechanics is indeed a fundamental theory, then observers themselves should ultimately be describable as quantum mechanical systems — but postulate 3 is generally used to describe quantum mechanical systems interacting with other quantum mechanical systems. It is only when we can identify one system as an *observer* that we invoke postulate 4 — yet postulate 4 itself does not make it clear how to distinguish observer systems from other systems which do not qualify as an observer. This conceptual problem has been around since the very beginning of quantum mechanics [16].

These reservations do not prevent us from *using* quantum mechanics, of course. It is understood, when we go into the lab and study quantum mechanical systems such as individual atoms, photons, etc., that *we* are the observer, and that postulate 4 need only be invoked when *we* perform a measurement. It then gives us a useful and accurate rule for extracting probabilities from quantum mechanical states. The quantum measurement problem is thus primarily a problem of self-consistency in the basic formulation of the theory (this situation will change, however, as problems in the description of measurement become much more serious in the generally covariant formalism of quantum mechanics, as we will see in chapter 3).

Many attempts have been made to resolve this problem, such that an *effect* identical to postulate 4 can be seen to emerge from quantum mechanics [9] (or in some cases, a modified theory postulating a "physical collapse" [17]) in some limit, and so postulate 4 can be dropped as a defining feature of the theory, or so that it can be understood in a more natural way. The amount of literature on such "interpretations of quantum mechanics" is enormous [18, 19], and it will be impossible to fully review every approach here. Furthermore, the goal of this thesis is not to settle all questions regarding the quantum measurement problem — instead, the goal is to point out in a sufficiently general way what measurement *is*, invoking concepts that will remain useful in the covariant context, where the standard prescription has failed basic requirements of correspondence with established physics. However, this process *will* bear directly on the measurement problem, and so for the purposes of comparison to established ideas we mention here in a very cursory way some of the more commonly discussed descriptions/interpretations of quantum measurement.

2.2.1 The Description of Bohm and de Broglie

Bohmian mechanics [20] takes an unusual approach to the measurement problem — rather than to regard the wave function $\Psi(x) = \langle x|\Psi\rangle$ (describing a particle) as the sole quantity physically relevant to an observation, it postulates instead that particles have *definite* position and velocity, and are merely guided by the wave function or "pilot wave" (and quantities derived from it, such as a "quantum potential"), which also contains information on the probability density of the particle's location. In this way, interference can still take place, since the pilot wave is certainly capable of, for example, propagating through two slits simultaneously, while the associated particle travels through only one (though which-path information is not available to the observer without destroying the interference pattern, as in the standard description of QM).

For our purposes, the important point of this framework is that the uncertainty of particle position is, in essence, a classical uncertainty. The wave function $\Psi(x)$ may be known exactly, and thus have zero associated von Neumann entropy, yet the position of the particle is described by a *classical* probability distribution, which can in principle be associated with a non-zero Shannon entropy (indicating that there is indeed information to be gained about the particle, though there is apparently no information to be gained about the wave function). In this way, a measurement can be described as a process of uncovering the "true" position of the particle, and the associated wave-function collapse as a process of throwing away the pieces of the wave function which can no longer affect subsequent motion of the particle. Thus, a single outcome is expected probabilistically as new information is uncovered.

There are some well-known problems with this picture [9]. First, it appears to give a very special status to the position observable $\hat{x}$ as the defining feature of a particle, while the common intuition seems to be that there is no reason for a bare mechanical formalism to select out certain observables as special. Second, the theory is known to admit very non-classical trajectories, and it has not been obvious how to recover the expected classical behavior. Third, the formalism appears to be difficult to generalize to relativistic quantum field theory. Thus, Bohm-de Broglie mechanics is perhaps best thought of as a reminder that it is possible to construct hidden-variable theories which are consistent with quantum theory.

2.2.2 The Description of Everett

Everett was concerned with precisely the issue we have described — the two apparently contradictory forms of evolution, and the ultimate impossibility of an external observer when considering a closed universe as a whole. Von Neumann had earlier described the measurement of a quantum system as process whereby the observed microscopic system becomes increasingly entangled with the detector, the light used to read the detector, the observer's optic nerve, etc., but had concluded that the point at which one truncates this description and invokes the collapse postulate was "arbitrary to a very large extent." [21]

Everett took this train of thought very seriously, and constructed a "relative state" description of measurement based on the entanglement of the quantum system, Q, with an observer, A [22]. That is, if the system Q is initially described by the state $|Q\rangle = \sum_i \alpha_i |a_i\rangle$, then a measurement by A will lead to the combined system state $|QA\rangle = \sum_i \alpha_i |a_i, i\rangle$. He then noted that each "branch" of the wave function $|a_i, i\rangle$ can be regarded as an independent outcome, which appears to the observer as a collapse. The probabilities in quantum mechanics, then, represent the probability that the observer will end up in a particular branch of the wave function, though all branches are equally real.

This description is important, and in some sense it represents a first step toward our own approach. However, the "many worlds interpretation" is incomplete as a full explanation. Given an arbitrary entangled state of a system and observer, it does not give us a basis of states in which the observer can expect to find himself as one potential description, nor even does it describe why we should expect a branching of experience states at all (one might simply expect that our states of experience are by nature highly entangled with the environment). It mainly serves as a framework for describing what we already know — *given* that we understand probabilistic projective collapse as something that happens in our experience, "many worlds" grants us an interpretation which upholds consistency with the evolution postulate (postulate 3) as a primary concern.

2.2.3 Environmental Decoherence

More recently, people have begun to take advantage of a feature of QM that is surely at the heart of much mystery [23, 9]. That is, a pure state density operator $\rho_{QE} = |QE\rangle\langle QE|$ (i.e. one with zero von Neumann entropy) describing two systems — say, a quantum system Q and an environment E — will reduce to a *mixed* state ρ_Q (one with non-zero von Neumann entropy) if a partial trace is taken over E, if the full state of QE is entangled (that is, if the full state $|QE\rangle \in \mathcal{H}_Q \otimes \mathcal{H}_A$ cannot be expressed as a simple product state $|Q\rangle \otimes |E\rangle$).

It is difficult to overstate the importance of this feature — it is completely non-classical. Classical Shannon entropy satisfies $H(AB) \geq H(A)$ for the joint probability distribution $p_{AB}(i,j)$ and the marginal distribution $p_A(i)$ — this says, in effect, that it never takes *less* classical information (on average) to specify a message from A *and* B than it takes to specify a message from A alone (or from B alone). Quantum mechanics says something else entirely — even if we knew *exactly* the state describing a quantum system and its environment, QE, uncertainty (and thus, hidden information) can still exist in the state describing Q alone.

This feature has been put to use in confronting the measurement problem. The idea is that in any measurement, the environment will inevitably interact with the quantum system Q or the detector D, forming a large entangled state describing the combined system QDE. But we never examine the state of the environment — we are only interested in, at most, the state of the quantum system and detector, QD. Ignoring a piece of the system, e.g. describing QD independent of E, is implemented via the partial trace. Thus, classical uncertainties creep into the description of QD, and we no longer know which state describes the system. These classical uncertainties can then be associated with the different possible outcomes of the experiment — in essence, then, a measurement reveals the "true" state describing QD independent of E (a case in which we expect to recover exactly one of a number of possibilities).

This description, of course, can be made far more precise. Indeed, one property of this formalism is that the orthogonal basis in which ρ_{QD} becomes diagonal (and thus uncertainty in QD is formally equivalent to a classical probability distribution) is dependent on the details of the dynamical environment. Thus, the "experience basis" of QD is a detailed emergent property of the system and its environment. Zurek has proposed a "stability criterion" (a condition on the detector-environment Hamiltonian) for determining this observed basis of potential outcomes.

Since it is based on fundamental principles, this approach generalizes to nearly any interpretation of quantum mechanics. The downside of this approach is that it seems to put a surprisingly large responsibility on the environment — one may wonder if quantum measurement is even possible without an environment, or how much control over the possible alternative outcomes of a detector one actually has. In any case, though, environmental decoherence is certainly a ubiquitous fact of quantum mechanics, whether or not it represents a full solution to the measurement problem.

2.3 Self-Referential Probabilities

In this section, we begin a description of an approach to measurement first suggested by Cerf and Adami [24, 25], in which a model of the observer (and the information available to him) plays a central role. Before discussing quantum mechanics, we demonstrate through an example how these ideas work in the context of classical mechanics — in essence, simply making explicit some assumptions that are ordinarily made implicitly. When we then move to a description of quantum measurement, we will find the same principles at work, though the ordinary assumptions implicitly made in classical mechanics will have to be modified. The payoff will be significant, though, as the approach combines the best aspects of the Everett interpretation and the decoherence approach, without their major disadvantages. Most important for us, however, these ideas will be generalizable to the generally covariant setting in the next chapter, where the ordinary projection postulate fails.

2.3.1 Information-Theoretic Description of Classical Measurement

In classical mechanics, states are described as points in phase space, which, for our purposes we can think of as the space of positions and momenta. Thus, for a single particle living in ordinary 3-D space, to specify its state is to specify six numbers — three coordinates $\vec{q}$, and three momenta $\vec{p}$. Our notation will be considerably more abstract, however — we will simply differentiate distinct points in phase space by assigning distinct labels, α, β, etc. The exact form of our phase space will not be important for the arguments here.

Suppose we have a single classical object (say, a bowling ball B), which we know was prepared in one of two possible states, α_0 or β_0, with equal probability (the subscript indicates the "initial time $t = 0$"). Let us say that the states α_0 and β_0 have the same momentum $\vec{p}_0$, but the position of state β_0 is shifted some significant distance away compared with α_0, i.e. $\vec{q}_0(\beta_0) = \vec{q}_0(\alpha_0) + \vec{x}$.

The mathematical description of this situation is that of a probability distribution σ_B on the bowling ball's phase space. Since there are two equiprobable states, we have $\sigma_B(\alpha_0) = \frac{1}{2}$ and $\sigma_B(\beta_0) = \frac{1}{2}$. For all other points in the ball's phase space, $\sigma_B = 0$. The interpretation is straightforward — we simply do not know which is the *true* state of the ball. The probability distribution represents our position of ignorance, which we can quantify by the nonzero Shannon entropy content of the ball's distribution.

Measurement, in classical mechanics, is now a procedure by which we uncover a system's true state (or in general, reduce our average uncertainty about the state of a system). Much of the time this is regarded as something of a trivial process (i.e. there are no chapters on classical measurement in undergraduate or graduate classical mechanics texts), but let us be a bit more careful. We enlarge our phase space by introducing an observer into the mechanical system — perhaps a human with a finger extended in front of the ball's α-path, but not in front of the ball's β-path. Let us refer to this known initial state of the human observer as γ_0. Thus, if the ball is initially in state α_0, evolution of the system will cause the ball to collide with the human at time $t = 1$, exchanging momentum between the two systems, and leaving the human in a disturbed state $\gamma_1^\star$, with the ball in a disturbed state $\alpha_1^\star$. If the ball was initially in state β_0, then at time $t = 1$ the human's state will be an unchanged γ_1, and the ball will simply remain in its undisturbed, time-evolved state β_1.

In this way, the state of the human observer (call her Alice, or A) becomes correlated with the state of the bowling ball — this is the essence of measurement. Let us focus on the state of the observer alone: Standard Hamiltonian evolution has taken the observer's initial state γ_0 from one of complete certainty, $H(A) = 0$, to a probability distribution σ_A at time $t = 1$. We have that $\sigma_A(\gamma_1) = \frac{1}{2}$ and that $\sigma_A(\gamma_1^\star) = \frac{1}{2}$, with $\sigma_A = 0$ for all other points in the observer's phase space. The entropy of the observer herself has thus increased to $H(A) = \log(2)$.

We now come to the main point of this example — although the mathematical formalism of mechanics does not distinguish between them, the interpretation of the observer's probability distribution σ_A is very different from the interpretation of the ball's probability distribution σ_B. Whereas σ_B represents Alice's ignorance of the state of the ball at time $t = 0$, the distribution σ_A represents *predictive* probabilities for one of two potential outcomes to be realized by the observer at the time $t = 1$. Certainly σ_A will not represent the ignorance of the observer of her own state at the time $t = 1$ — the observer will experience precisely one outcome, and not the other (gaining information in the process).

We denote these distinct interpretations as the "ignorance" interpretation and the "predictive" interpretation of mechanical probabilities. We emphasize again that the laws of mechanics do not distinguish between the two — one probability distribution is mathematically the same in character as another. It is only our knowledge that the probabilities given by σ_A are self-referential that allows us to make the distinction — i.e. we recognize that the observer is obtaining probabilities which describe *her own state*, and this indicates the predictive interpretation of σ_A.

After this measurement, the observer has gained information on the original state of the bowling ball. In this case, she has gained *complete* information. This is expressed via the vanishing conditional entropy after the measurement has taken place: $S(B|A) = S(AB) - S(A) = 0$ — i.e. if we know the state of Alice, then we know the state of the ball. However, Alice *always* knows the state of Alice, so as soon as the measurement has taken place, she will naturally update the state of the whole system to a single point in phase space, rather than to maintain the probability distribution as the description of the state of the system. The probability distribution σ_A describing Alice is simply the theory telling Alice the relative probabilities for each possible way the state might become updated (whereas σ_B describing the ball itself simply represents Alice's ignorance, and thus has no such interpretation by itself).

2.3.2 Information-Theoretic Description of Quantum Measurement

Classically correlated quantum mixtures

The second example is very analogous to the first. We have one of two possible orthogonal preparations of a quantum system Q, ρ_{Q1} and ρ_{Q2}, with respective probabilities p_1 and p_2. Thus an observer (Alice) can describe the quantum state as a density operator $\rho_Q = p_1\rho_{Q1}+p_2\rho_{Q2}$. Alice simply does not know in which orthogonal state the system was prepared — this is the standard ignorance interpretation of the probabilities once again. As before, the entropy of the system is nonzero, and it is again possible to learn something about the "true" state of the system.

By introducing a quantum model of the observer, A, correlations may again be passed on to the observer originally in a definite state $|\text{ready}\rangle$ (exactly as in the previous example). If the system was prepared in state ρ_{Q1}, then the observer evolves to the state $|\text{see } 1\rangle$, and if the system was prepared in state ρ_{Q2}, the observer evolves to the state $|\text{see } 2\rangle$. After observation, the full state of the combined system will be:

$$\rho_{QA} = p_1\rho_{Q1}|\text{see } 1\rangle\langle\text{see } 1| + p_2\rho_{Q2}|\text{see } 2\rangle\langle\text{see } 2|. \tag{2.7}$$

Focusing on the observer Alice alone (by performing a partial trace), we have the diagonal density operator:

$$\rho_A = p_1|\text{see } 1\rangle\langle\text{see } 1| + p_2|\text{see } 2\rangle\langle\text{see } 2|. \tag{2.8}$$

Analogous to the previous classical example, this density operator has non-zero von Neumann entropy — in fact, the von Neumann entropy is identical to that of the classical Shannon entropy of a probability distribution $\{p_1, p_2\}$. It is also self-referential (used by Alice to describe Alice) and thus we have a predictive interpretation of the probabilities p_1 and p_2 — these probabilities do *not* represent the ignorance of the observer after the measurement. They represent the probability that the observer will realize one outcome or the other, gaining information on the original state preparation in the process.

As before, we have that $S(Q|A) = S(QA) - S(A) = 0$ — this is due to the fact that, although we have used quantum mechanics, Alice has become classically correlated with Q. Given the state Alice realizes after the measurement, she will naturally update her description of Q to the appropriate pure state.

Pure Quantum States

We now reach the final case, where the distinction we have been making becomes most important, and the fundamental difference between classical and quantum measurement becomes manifest.

We start with a system (say a qubit), Q, in a *pure* quantum state $|Q\rangle = \alpha|0\rangle + \beta|1\rangle$. Note that there is no ignorance here whatsoever — the observer, Alice, already knows the "true" state precisely, i.e. the entropy of Q is zero. Nevertheless, our observer with this knowledge can perform a "measurement" on this system in the $\{|0\rangle, |1\rangle\}$ basis using the same unitary interaction as in the previous example. It should already be clear that this is a fundamentally different undertaking than those described above — the observer is seeking to "uncover" no pre-existing information about Q (since the state of Q is already known with full precision), but by reason of more than 75 years of traditional, continuous usage, we are unfortunately stuck with the same word, "measurement," to describe both scenarios.

We can see how this is done by introducing the observer system, Alice, who is initially in a known state $|\text{ready}\rangle$. The initial state is thus given by:

$$|QA\rangle = (\alpha|0\rangle + \beta|1\rangle)\,|\text{ready}\rangle \tag{2.9}$$

Now a unitary entangling interaction occurs[1], which performs Alice's measurement. The

[1] The measurement operation used here is $\widetilde{M} = \sum_i \Pi_i \otimes L_{iR}$, where the Π_i are the projection operators on $\mathcal{H}_Q$ for the $\{|0\rangle, |1\rangle\}$ basis states, and L_{iR} is defined on $\mathcal{H}_A$ by $L_{iR}|\text{ready}\rangle = |\text{see } i\rangle$. A fully unitary operator which implements this can be written, for example, as $M = \sum_i \Pi_i \otimes L_{iR} + [\sum_i \Pi_i \otimes L_{Ri}]\left[\sum_j \Pi_j \otimes \Pi_{\text{see}j}\right] + [\mathbf{1} \otimes \sum_i \Pi_{\text{see }i}] - [\sum_i \Pi_i \otimes \Pi_{\text{see }i}]$, where $L_{Ri} = L_{iR}^\dagger$, and $\Pi_{\text{see }i}$ is the projector onto the state $|\text{see } i\rangle$

state has now become:

$$|QA\rangle = \alpha|0\rangle|\text{see } 0\rangle + \beta|1\rangle|\text{see } 1\rangle. \tag{2.10}$$

Now when we focus on the observer (performing the partial trace), we find that she is again described by a diagonal density operator:

$$\rho_A = |\alpha|^2 \,|\text{see } 0\rangle\langle\text{see } 0| + |\beta|^2 \,|\text{see } 1\rangle\langle\text{see } 1|. \tag{2.11}$$

Note that something interesting has happened — we have obtained again a mixed state (i.e. a state of positive entropy) describing Alice, *even though she is not correlated with any pre-existing information.* The form of this observer density operator ρ_A is nevertheless indistinguishable from the form of ρ_A obtained in the previous example (where we established a correlations with $S(Q)$ bits of classical information), and again we must conclude that since it is self-referential in nature (Alice uses it to describe Alice), it cannot represent the observer's ignorance, but must represent probabilities for one of two possible alternatives to be realized by Alice — with probability $p_0 = |\alpha|^2$ Alice sees outcome 0, and with probability $p_1 = |\beta|^2$ she sees outcome 1. Note that this train of logic originates already at the level of classical mechanics — once we have defined the concept of information held by quantum systems via the von Neumann entropy in analogy to classical mechanics and the Shannon entropy, the connection between self-referential probabilities and *prediction* forces us into a situation in which we must interpret quantum mechanics as a probabilistic theory, and the realization of one outcome from a set of possibilities (as described by postulate 4) by quantum mechanical observers is guaranteed.

Lest the reader believe that he has stumbled across some form of circular reasoning, we note that the above does not constitute a "derivation" of the Born interpretation (postulate 4 of standard quantum mechanics, described above). The form of the final probability assignments is determined by the definition of the von Neumann entropy and conditional entropy, which allows us to quantify the amount of information held by the observer — thus, assuming the standard form of the von Neumann entropy is in a sense equivalent to the Born interpretation, in terms of calculating measurement probabilities in this description. Nevertheless, this information-theoretic description has the important property that is *unitary*, whereas the standard measurement postulate is not. It is the unitarity of this description that will find essential in the next chapter, and in chapter 4. We are happy to regard the von Neumann entropy itself as a fundamental ingredient — in essence, the definition of quantum information.

The Basis Problem

We should point out here that there is something called the "basis problem" in decoherence frameworks such as this [26, 27]. The argument is that density operators such as ρ_A have many possible representations — for example, we could express ρ_A appearing in equation 2.11 in the following way:

$$\rho_A = \frac{1}{2}|\mathrm{X}\rangle\langle\mathrm{X}| + \frac{1}{2}|\mathrm{Y}\rangle\langle\mathrm{Y}|. \tag{2.12}$$

Where the generally non-orthogonal states $|X\rangle$ and $|Y\rangle$ are given by:

$$|\mathrm{X}\rangle = |\alpha||\text{see } 0\rangle + |\beta||\text{see } 1\rangle \tag{2.13}$$

$$|\mathrm{Y}\rangle = |\alpha||\text{see } 0\rangle - |\beta||\text{see } 1\rangle \tag{2.14}$$

The question naturally arises — which basis represents the *experience basis* of Alice? That is, which basis in the one in which she can expect to see one particular outcome? This can be answered by the extension of a recent argument due to Zurek [28], which we repeat here, and which can be extended to Hilbert spaces of any dimension. Any general *unitary* entangling operation between Q and A as above, which takes $(\alpha|\nu\rangle + \beta|\omega\rangle)|A_0\rangle \Rightarrow \alpha|\nu\rangle|A_\nu\rangle + \beta|\omega\rangle|A_\omega\rangle$

implies the following:

$$\langle QA|QA\rangle_{\text{before}} - \langle QA|QA\rangle_{\text{after}} = 0 \tag{2.15}$$

$$\langle Q|Q\rangle_{\text{before}} - \langle QA|QA\rangle_{\text{after}} = 0 \tag{2.16}$$

$$2 \text{ Re } \alpha^*\beta\langle\nu|\omega\rangle(1 - \langle A_\nu|A_\omega\rangle) = 0. \tag{2.17}$$

This must hold for arbitrary phases, so we have:

$$\langle\nu|\omega\rangle(1 - \langle A_\nu|A_\omega\rangle) = 0. \tag{2.18}$$

This implies one of two possibilities — either the "measured" states $|\nu\rangle$, $|\omega\rangle$ are orthogonal, or else the states of the observer A, $|A_\nu\rangle$, $|A_\omega\rangle$ must be identical, up to phase (in which case no information at all has been transferred to A). Since this operation is supposed to represent a measurement (the state of the observer changing, based on the possible states of the observed system), we are forced to arrive at the first conclusion. This argument can then be extended to the case that both systems are observers — i.e. if Bob then "measures" the state of Alice by asking her the outcome of her measurement on Q, for any information to be transferred, we require that the possible alternatives $|A_\nu\rangle$, $|A_\omega\rangle$ are required to be orthogonal if any information is to be transferred to Bob.

We are thus led to the conclusion that for information to be transferred between observers in this unitary picture, the alternative measurement states of the observers *must be orthogonal*. This is important for us, because there is a *unique* diagonal, orthonormal representation of any density operator (up to degenerate cases which are of measure zero in any reasonable state space) [12]. In this representation, the diagonal elements p_i of ρ_A, if interpreted as elements of a probability distribution, give a Shannon entropy identical to the von Neumann entropy of A, i.e. $H(p_i) = S(A)$. Thus, in the description above, we expect Alice to realize exactly one alternative of the orthogonal states $|\text{see } 0\rangle$, $|\text{see } 1\rangle$, with the respective probabilities $p_0 = |\alpha|^2$ and $p_1 = |\beta|^2$.

One might object to this proposal on the following grounds: Suppose in an experiment (performed by Alice with Q a qubit for this example), the final state is *not* given by a perfectly entangled state:

$$|QA\rangle = \alpha|0,0\rangle + \beta|1,1\rangle \tag{2.19}$$

but in keeping with inefficient, realistic detectors, is instead given by:

$$|QA\rangle = \alpha|0,0\rangle + \gamma|1,0\rangle + \delta|1,1\rangle \tag{2.20}$$

(we use shorthand notation $|0\rangle$ for $|\text{see } 0\rangle$, etc. here). That is, there is an amplitude that Alice does not see Q's transition to the $|1\rangle$-state, since the detector is not completely reliable. In this case, ρ_A describing Alice is no longer diagonal in the $\{|0\rangle, |1\rangle\}$ basis she set out to observe, yet Alice certainly expects to see one of these two outcomes, not a superposition of them.

This may at first appear to be a problem, since we do not perceive our experience basis to be dependent on the efficiency of the detectors we use. However, what is failing here is just our oversimplification of the observer-detector as a single system, A. In reality, the detector is a separate system D which first becomes entangled with the quantum system Q, and then becomes entangled with the observer system, A. Thus to handle this issue we must expand the description of the observed state further — the state of QDA representing a less than perfect measurement is now expressed as:

$$|QDA\rangle = \alpha|0,0,0\rangle + \gamma|1,0,0\rangle + \delta|1,1,1\rangle. \tag{2.21}$$

Now the reduced density operator describing the observer Alice is again diagonal in the $\{|0\rangle, |1\rangle\}$ basis she set out to measure, and the probabilities again have their standard interpretation. Note that we assumed again perfect entanglement between D and A, but this is now a perfectly reasonable assumption, since these are macroscopically distinguishable states — i.e. there is never any real inefficiency in the detector's ability to communicate detection to the observer. If the detector goes "click," indicating a detection of the $|1\rangle$ state, we are safe in assuming that an attentive observer will always be able to distinguish this detector state from the absence of a click. Of course there are many more steps analogous to this in a still more complete description of the detector and observer, but the central point is that as long as the *final* communication between detector and observer can be regarded as reliable, then no basis problem exists in this formalism. In the idealized limit of perfectly efficient detectors, however, we may safely bypass this additional step and consider the detector and observer to be a single system — we will often do this in what follows, suppressing the observer-detector distinction unless a detailed description of the detection interaction requires us to abandon the assumption of perfectly efficient detectors.

Multiple Observers and Collapse

It is in what follows that standard quantum mechanics gets into trouble, postulating a collapse — a kind of non-unitary "updating" of the state of Q analogous to what we did in the first two examples, where only classical correlation existed. As we have emphasized, there is a key difference in the third example (measurement on a pure state), however — knowledge of the outcome of Alice's measurement does not contain any additional information on the "true state" of Q, since the state was already known prior to the measurement. In fact, not only was the state known prior the measurement, but we no longer have $S(Q|A) = 0$ after the measurement has been performed (as we did in the first two examples). I.E. the full description of Q is *not* determined entirely by knowledge of Alice's measurement outcome — this is a surprising conclusion, because it stands in stark contrast with the standard measurement postulate of quantum mechanics.

What, then, happens to the state of a system after a measurement? Fundamentally, the answer is "nothing at all." After the entanglement process above, the full state of the system remains given by equation 2.10, while the interpretation of equation 2.11 is simultaneously "one outcome or another" due to its self-referential nature. This proposition confronts the key conceptual difficulty of the measurement problem — how can we reconcile purely unitary evolution with the apparent "wave function collapse" that is observed each time we perform a measurement on a pure quantum system?

The concept that nothing happens to the full QA wave function at observation, while at the same time the observer A sees "one outcome or another" may appear at first to be contradictory. This is, after all, why the measurement problem has remained so elusive for so many years. Nevertheless, we can *check* the consistency of this prescription, and find that it does indeed reproduce the standard predictions of the orthodox quantum measurement postulate — we do this by introducing additional observers, Bob and Charlie.

The purpose of Bob will be to perform another, subsequent observation on Q, and the purpose of Charlie will be to talk to Alice and Bob and to compare the results of their measurements. Thus, we can model Bob by another single qubit (just as we modeled Alice), and we can model Charlie by a two-qubit system (one to record Alice's measurement and one for Bob). Let us first examine the case that Bob measures Q in the same basis as Alice — that is, the $\{|0\rangle, |1\rangle\}$ basis. After Bob's measurement interaction, the state of the combined system becomes:

$$|QAB\rangle = \alpha|0\rangle|A \text{ sees } 0\rangle|B \text{ sees } 0\rangle + \beta|1\rangle|A \text{ sees } 1\rangle|B \text{ sees } 1\rangle \tag{2.22}$$

Now we introduce Charlie, with basis $\{|0_A 0_B\rangle, |1_A 0_B\rangle, |0_A 1_B\rangle, |1_A 1_B\rangle\}$, who interacts unitarily with Alice and Bob, to generate the final state:

$$|QABC\rangle = \alpha|0\rangle|A \text{ sees } 0\rangle|B \text{ sees } 0\rangle|0_A 0_B\rangle + \beta|1\rangle|A \text{ sees } 1\rangle|B \text{ sees } 1\rangle|1_A 1_B\rangle \tag{2.23}$$

We can now examine the reduced density operators describing observers A, B, and C, in order to extract the predictions of the theory:

$$\rho_A = |\alpha|^2 \, |A \text{ sees } 0\rangle\langle A \text{ sees } 0| + |\beta|^2 \, |A \text{ sees } 1\rangle\langle A \text{ sees } 1| \tag{2.24}$$

$$\rho_B = |\alpha|^2 \, |B \text{ sees } 0\rangle\langle B \text{ sees } 0| + |\beta|^2 \, |B \text{ sees } 1\rangle\langle B \text{ sees } 1| \tag{2.25}$$

$$\rho_C = |\alpha|^2 \, |0_A 0_B\rangle\langle 0_A 0_B| + |\beta|^2 \, |1_A 1_B\rangle\langle 1_A 1_B| \tag{2.26}$$

The interpretation of these density operators according to our self-referential probability rule is that both Alice and Bob independently see outcome 0 with probability $|\alpha|^2$, and outcome 1 with probability $|\beta|^2$. The interpretation of ρ_C, however, is that the results of Alice and Bob's measurements will always be in agreement. This is precisely what the original non-unitary collapse postulate of standard quantum measurement theory was designed to achieve. We have seen here that when the full description of the *system and the observers* is taken into account (and the self-referential nature of observer density operators is noted), then the theory reproduces what we know while remaining fully unitary.

This prescription and correspondence is entirely general — it does not depend on our systems being qubits, or on the basis in which A and B perform their measurements. To see this, let Q be an arbitrary system, whose initial state is given by:

$$|Q\rangle = \sum_i \alpha_i |a_i\rangle. \tag{2.27}$$

Let our model of the first observer, A, be spanned by states $|i\rangle$ that are in one-to-one correspondence with the basis states $|a_i\rangle$. A measurement interaction leads to the following entangled state:

$$|QA\rangle = \sum_i \alpha_i |a_i, i\rangle. \tag{2.28}$$

Now we introduce B, who measures in another basis $|b_j\rangle$. Our model of B is spanned by the states $|j\rangle$, which are in one-to-one correspondence with the basis states $|b_j\rangle$ of Q. The overlap between these two bases is given by the unitary matrix $U_{ij} = \langle b_j|a_i\rangle$. Thus, when B becomes entangled with Q during his measurement interaction, we arrive at the entangled state:

$$|QAB\rangle = \sum_{i,j} \alpha_i U_{ij} |b_j, i, j\rangle \tag{2.29}$$

As before, we can introduce a third observer C, which we use as a simple device to obtain joint probabilities. Our model for C is spanned by the states $|ij\rangle$. His interaction with A and B leads to the final state:

$$|QABC\rangle = \sum_{i,j} \alpha_i U_{ij} |b_j, i, j, ij\rangle \tag{2.30}$$

Focusing now on the self-referential observer density operators, we find:

$$\rho_A = \sum_i |\alpha_i|^2 \, |i\rangle\langle i| \tag{2.31}$$

$$\rho_B = \sum_{i,j} |\alpha_i|^2 \, |U_{ij}|^2 \, |j\rangle\langle j| \tag{2.32}$$

$$\rho_C = \sum_{i,j} |\alpha_i|^2 \, |U_{ij}|^2 \, |ij\rangle\langle ij| \tag{2.33}$$

and these give rise to the following probabilities:

$$p_A(i) = |\alpha_i|^2 \tag{2.34}$$

$$p_B(j) = \sum_i |\alpha_i|^2 \, |U_{ij}|^2 \tag{2.35}$$

$$p_C(ij) = |\alpha_i|^2 \, |U_{ij}|^2 \tag{2.36}$$

Note that p_A and p_B describe exactly the individual probabilities associated with the ordinary measurement postulate of a collapse during the A-measurement and subsequent B-measurement, while p_C describes exactly the standard joint probabilities for the sequence of the two measurements.

From these, we can also define conditional probabilities. For example, given outcome i from observer A, what is the probability that observer B will see outcome j? This is generally given by the formula $p(j|i) = \frac{p(ij)}{p(i)}$, and in the specific case above, takes the form $p(j|i) = |U_{ij}|^2$. Again the standard probabilistic, predictive interpretation of quantum theory is recovered.

2.4 Origin of the Measurement Problem

The previous example demonstrates that our two propositions are entirely consistent, namely:

1. The self-referential probabilities obtained from the observer density operators represent the predictive probability for the realization of *one outcome or another*.

2. No "updating" (via reduction or collapse) takes place when a measurement is performed on a pure state, since the state is already known completely prior to the measurement. The full state merely becomes entangled with the observers via unitary evolution.

In fact, we see that when a model of the observer is taken into account, these propositions lead precisely to the standard probabilistic interpretation of quantum theory, while the theory itself retains full unitarity.

From this perspective, it is instructive to examine the key assumption which historically led to the measurement problem to begin with. It is essentially the abandonment of the observer from the analysis of measurement — not surprisingly, this is due to classical intuition. In classical mechanics, as in our first example, we are used to the following situation: A classical system C is described by a distribution σ_C with non-zero entropy $H(C)$. The act of bringing in an observer A to make an ideal measurement means that we obtain $S(C|A) = S(CA) - S(A) = 0$. I.E. given the result of the observer's measurement, we obtain all information possible about the state of C. This builds the following classical shorthand intuition: we can skip the analysis of the observer, and simply imagine that an act of "ideal observation" exists, which reduces the state of C to one of zero entropy. Again, this is fully justified by the fact that $S(C|A) = 0$ can be achieved after an idealized, perfect classical measurement.

The measurement problem arises when we try to apply this intuition to an already pure quantum system. When a measurement has been performed on a pure state of a quantum system Q by the observer A, we no longer have that $S(Q|A) = 0$ — in fact, the conditional entropy typically is non-zero and negative. This is entirely different from the classical case. Nevertheless, buried in the standard formalism of projective measurement is the assumption that a "perfect measurement" projects Q to a single, new, pure state with zero entropy which is known completely if only we know the outcome of the measurement — *exactly* as in classical mechanics, but without the same information-theoretic justification. For example, Dirac [16] makes this explict as he writes with his own emphasis:

> *When a maximum observation is made on a system, its subsequent state is completely determined by the result of the observation and is independent of its previous state.*

Yet, it is clear that this classical intuition (which suppresses the description of the observer) causes us to miss something essential, namely unitarity and internal consistency. The price we pay for adopting this classical intuition (based on an information-theoretic condition that does not hold in QM) is known as the quantum measurement problem.

2.5 The Quantum Mechanical Arrow of Time

Projective measurement in standard quantum mechanics is not a unitary process — that is, time reversal is not a good symmetry of the theory when the measurement postulate is invoked. Any unitary evolution described by the evolution operator U takes an arbitrary (mixed or pure) state ρ to the state $U\rho U^\dagger$, leaving the von Neumann entropy constant — that is, $S(\rho) = S(U\rho U^\dagger)$.

Projective measurement does not have this property. If we have a complete set of orthogonal projectors, Π_i, which represent the possible outcomes of a measurement, then a projective measurement takes the state ρ to the state $\rho' = \sum_i \Pi_i \rho \Pi_i$. This does not leave the entropy invariant — in fact, it has been proved that $S(\rho') \geq S(\rho)$ [12]. This is not a statistically true statement like the second law of thermodynamics — it is fundamentally true for *all* projective measurements on all quantum states (in the absence of the additional information on the measurement outcome). This means that the formalism of projective measurement selects a preferred time direction, sometimes referred to as the quantum mechanical arrow of time.

In the previous section, we described a way (originally described by Cerf and Adami) to reproduce the effects of the measurement postulate by including a quantum description of the observer into the mechanical framework, while retaining purely unitary evolution. How, then, is the quantum mechanical arrow of time emergent?

Consider, in the unitary framework of the previous sections, a series of N measurements (performed by N observers) on the initial state of Q, taken to be $|Q\rangle = \sum_i \alpha_i |a_i\rangle$. Extending the argument leading to equations 2.31 and 2.32, the reduced density operator describing the N^{th} observer takes the following form:

$$\rho_N = \sum_{i,j...k,l} |\alpha_i|^2 |U_{ij}^{(2)}|^2 ... |U_{kl}^{(N)}|^2 |l\rangle\langle l|. \tag{2.37}$$

Defining the conditional probabilities $p_n(ij) = |U_{ij}^{(n)}|^2$, this implies the following probability relation:

$$p_N(l) = \sum_{i,j...k} p_1(i) p_2(ij) ... p_N(kl). \tag{2.38}$$

Now, due to the fact that the entropy of the n^{th} observer $S(A_n)$ is equal to the classical Shannon entropy of the n^{th} observer's classical probability distribution, $H(p_n(i))$, this implies that the entropy of subsequent observers is generally at least as great as the entropy of the preceding observers — at least as far as this description holds, where each observer is performing the analog of a projective measurement, defined by the measurement operator $\widetilde{M}^n = \sum_i \Pi_i^Q \otimes L_i^{A_n}$, where $L_i^{A_n}$ is defined on the n^{th} observer state space by $L_i^{A_n}|\text{ready}\rangle =$

$|i\rangle$ (as described in the footnote in section 2.3.2, this can easily be implemented within a fully unitary measurement operator M). For this type of measurement, it is also true that $S(Q) = S(N)$, where N represents the last observer to perform a measurement.

Thus, the quantum mechanical arrow of time is emergent in this fully unitary framework, when we look explicitly at the entropy of the observers, or at the entropy of Q itself. There is nothing fundamentally special about the "forward" direction of time that we observe — the sequence of observations represented by the unitary operator $...M^3M^2M^1$ can in principle be "undone" by the (also unitary) operator $M^{1\dagger}M^{2\dagger}M^{3\dagger}....$ We will return to this property in chapter 4, where it will allow us to apply time reversal arguments to a phenomenon associated with quantum measurement — the quantum Zeno effect.

Chapter 3

Covariant Quantum Mechanics and Information

Having discussed the way in which quantum measurement can be regarded as information held by an observer, after a unitary entangling interaction has occurred, we are ready to discuss another quantum mechanical framework in which it has been difficult to formulate a successful measurement postulate.

The framework is that of generally covariant quantum mechanics, which has been developed primarily for use in quantum gravity. Quantum gravity challenges our understanding of nature in a fundamental way; the issue is not so much that the equations are difficult to solve (though this is certainly also true), but that it forces us to come to grips with a new language for mechanics — general covariance — in which all variables are treated on an equal footing.

General covariance is often regarded as synonymous with "invariance of the equations of motion under arbitrary spacetime diffeomorphism," but its implications are much further reaching (though exactly how far these implications go has caused a great deal of confusion [29]). Here, we regard spacetime diffeomorphism invariance as a natural expression (in the particular case of field theory) of a more fundamental re-formulation of mechanics, in which dynamics is no longer about evolution in an external time parameter, and the distinction between "independent" observables (such as time), and "dependent" observables (such as the position of a particle) are washed away. Generally covariant formulations of mechanics (both classical and quantum) can be applied to any system [8], while full spacetime diffeomorphism invariance itself makes sense only in the context of field theories (or similar).

In this chapter, we review the covariant notion of partial and complete observables, and then review the postulates of covariant quantum mechanics, as proposed by Rovelli. We then come to the central point of this thesis — that the measurement postulate in this framework has failed to reproduce the familiar and well-established predictions of ordinary non-covariant quantum mechanics, even in the simple context of few degrees of freedom, quite apart from field theory and quantum gravity. Thus, we will primarily be concerned with simple mechanical systems, which we will be analyzing from the point of view of generally covariant quantum mechanics. We will then see that the information-theoretic framework for measurement introduced in the previous chapter survives in this context, and brings the predictions of the covariant formulation back into correspondence with known physics.

3.1 Partial and Complete Observables

A major step in the formulation of covariant quantum mechanics (pointed out explicitly by Rovelli [30]) is to realize that there are two distinct notions of observability. We can measure individual quantities, such as the length of a spring, the time displayed by a clock, etc. — in general, these quantities are not predictable. There is no theory of physics that tells us "what time it is," yet there is a quantity corresponding to the reading of a clock which can be observed.

The other notion of observability has to do with predictive power, and is thus directly related to the mechanical treatment of dynamics. Dynamical predictions have to do with the correlations between the first kind of observables — it is from these kinds of observations (the correlations themselves) that we construct mechanical theories.

This line of reasoning led Rovelli to define two types of observables that play a role in our theories:

Partial observables: Partial observables are physical quantities which can be measured by an appropriate measuring apparatus and procedure. The space of partial observables for a particular physical system is called its *extended configuration space*, which we denote as $\mathcal{M}$. [30]

Complete observables: Complete observables are relations between physical quantities that can be predicted from the knowledge of the state of a system.

Covariant language forces a careful distinction between "partial observables," which are simply experimentally accessible quantities, and "complete observables," which are quantities that represent the conjunction of two or more partial observables, and can be predicted from the physical state of a system. For example, the position of a classical harmonic oscillator, q, is a partial observable. The time, t, is also a partial observable. These are both experimentally accessible quantities. Note, however, that a series of results of q-measurements alone cannot be used to determine the current and future state of the oscillator, nor can a series of t-readings from a clock. However, a series of measurements of $q(t)$, which simultaneously combine a q-measurement with a t-measurement, *can* be used to reconstruct the current and future states of the oscillator (represented by a curve in the $q - t$ plane, i.e. a curve in $\mathcal{M}$) — thus, $q(t)$ represents a *complete* observable.

These concepts underlie general covariance in a direct way. In the context of general relativity, Einstein was led to diffeomorphism-invariant equations of motion by noting that coordinate points on the spacetime manifold have no objective meaning [31] — in this language, they represent neither partial nor complete observables [8]. Instead, Einstein noted that all predictions are made through the comparison of physical, dynamical quantities to other physical, dynamical quantities — a concept he referred to as "spacetime coincidences." In the present context, the points in $\mathcal{M}$ represent possible spacetime coincidences a given system might predict. That is, a point in $\mathcal{M}$ represents a simultaneous determination of all physically measurable quantities appearing in the theory. The principle of general covariance, then, is the statement that there is no universally preferred coordinatization of $\mathcal{M}$ — different coordinatizations of $\mathcal{M}$ must lead to the same physical predictions. In simple mechanical systems involving few degrees of freedom, where "x" and "t" represent partial observables in a particular coordinatization of $\mathcal{M}$, general covariance implies that a theory must be able to express predictions independent of the identification of a preferred time coordinate on $\mathcal{M}$ (though the equations of motion may indeed look different in different coordinate systems). In more sophisticated systems like full general relativity or quantum gravity, where "x" and "t" are not even partial observables, general covariance then implies diffeomorphism invariance of the equations of motion on the spacetime manifold. Due to the difficulty in dealing with quantum gravity and diffeomorphism-invariant field theories in general, we will restrict ourselves to to the first type of general covariance, where the equations of motion may not be invariant, but physical predictions must nevertheless be independent of coordinate choice.

Whereas the distinction between these two notions of observability is typically glossed over in ordinary treatments of quantum mechanics, generally covariant physics forces us to pay attention — much confusion has resulted from the failure to distinguish between these notions of observability, leading to contradictory arguments over what quantities are observable in quantum gravity. Using this distinctions, however, we can define the Hilbert spaces on which covariant quantum theory will be based:

Kinematical Hilbert space: The space of L^2 functions on the extended configuration space, $\mathcal{M}$, with respect to an appropriate specified measure. We denote the kinematical Hilbert space as $\mathcal{K}$.

Physical Projector: The physical projector, P, takes arbitrary states in $\mathcal{K}$ into solutions to the equations of motion, the space of which is denoted by $\mathcal{H}$. That is:

$$P: \mathcal{K} \rightarrow \mathcal{H} \tag{3.1}$$

$$P: \psi^{\mathcal{K}}(x) \mapsto \psi^{\mathcal{H}}(x) \tag{3.2}$$

$$\psi^{\mathcal{H}}(x) = \int_{\mathcal{M}} dx' \; W(x;x')\psi^{\mathcal{K}}(x') \tag{3.3}$$

Where $W(x;x')$ is the propagator for the theory, which is generally determined by the Hamiltonian [7].

Physical Hilbert space: This is the space of solutions to the equations of motion on $\mathcal{M}$, which generally have support over all of $\mathcal{M}$ (i.e. these states generally do not have compact support). We denote this space as $\mathcal{H}$ (as above). If each solution $\psi^{\mathcal{H}}$ in $\mathcal{H}$ can be obtained by projecting a state $\psi^{\mathcal{K}}$ of $\mathcal{K}$, then $\mathcal{H}$ can be made into a Hilbert space with the following inner product:

$$\langle \psi^{\mathcal{H}} | \phi^{\mathcal{H}} \rangle = \langle \psi^{\mathcal{K}} | P | \phi^{\mathcal{K}} \rangle \tag{3.4}$$

$$\langle \psi^{\mathcal{K}} | P | \phi^{\mathcal{K}} \rangle = \int_{\mathcal{M}} dx \; dx' \; \psi^{*\mathcal{K}}(x) W(x; x') \phi^{\mathcal{K}}(x') \tag{3.5}$$

We also have the property that $\langle x | P^{\dagger} P | x' \rangle = \langle x | P | x' \rangle = W(x; x')$ [8]. Typically, there are many kinematical states which project to the same physical state.

The postulates of generally covariant quantum mechanics then define the connection between the concept of partial and complete observables and the kinematical and physical Hilbert spaces, $\mathcal{K}$ and $\mathcal{H}$.

3.2 Generally Covariant Quantum Mechanics

Covariant quantum theory (as proposed by Rovelli) can now be formulated according to the following postulates [8]:

1. States: Kinematical states encode information on the outcome of measurements of experimentally accessible quantities (partial observables), and are represented as vectors in the Hilbert space $\mathcal{K}$. Physical states encode information on the dynamical predictions of the theory — they are solutions to the equations of motion, $H\Psi = 0$ (the Wheeler-DeWitt equation, where H is known as the relativistic Hamiltonian, or the Hamiltonian constraint, which appears in the canonical formulation of generally covariant systems). These states are vectors in the Hilbert space $\mathcal{H}$.

2. Observables: Partial observables are represented by Hermitian operators on $\mathcal{K}$. A given Hermitian operator on $\mathcal{K}$ also represents a complete observable if it additionally

commutes with the relativistic Hamiltonian, H. States of $\mathcal{K}$ are spanned by eigenstates of a complete set of partial observables. States of $\mathcal{H}$ are spanned by eigenstates of a complete set of complete observables.

3. Probability: If the initial kinematical state can be described as $|\phi^{\mathcal{K}}\rangle$, then the probability of observing the state $|\psi^{\mathcal{K}}\rangle$ is given by the square of the *physical* inner product, i.e.:

$$\begin{aligned} \mathcal{P}_{\phi\to\psi} &= \left|\langle\psi^{\mathcal{H}}|\phi^{\mathcal{H}}\rangle\right|^2 && (3.6)\\ &= \left|\langle\psi^{\mathcal{K}}|P|\phi^{\mathcal{K}}\rangle\right|^2 && (3.7)\\ &= \left|\int_{\mathcal{M}} dx\ dx'\ \psi^{*\mathcal{K}}(x)W(x;x')\phi^{\mathcal{K}}(x')\right|^2 && (3.8)\end{aligned}$$

After the measurement of a particular combination of partial observables corresponding to the state $|\psi\rangle$, the system is now described by the physical state $|\psi^{\mathcal{H}}\rangle = P|\psi^{\mathcal{K}}\rangle$. We denote this as the RR (for Reisenberger-Rovelli) probability postulate.

Note that the "evolution" postulate of ordinary, non-covariant quantum mechanics has in a sense become absorbed into the "states" postulate here, and the notion of observable has simultaneously been refined to accommodate this fact — in this way, explicit evolution in terms of an external, classical time variable is avoided as a fundamental ingredient of the theory, as required by generally covariant theories. Covariant quantum theory is also designed to encompass traditional QM — it is merely designed to be more general. To establish a bit of familiarity for this formalism, we examine a familiar physical system, quantized in this manner:

Single nonrelativistic particle: The extended configuration space $\mathcal{M}$ for a single particle confined to the real line is $\mathbb{R}^2$, which differs from the ordinary nonrelativistic configuration space ($\mathbb{R}$) because time must be included as a partial observable. This is due to the fact that dynamical predictions generally require us to make some form of physical time measurement, and measurable quantities must correspond to Hermitian operators on $\mathcal{M}$. Note that we do not any more treat time as a "special" external quantity or background — it is merely part of one particularly useful coordinatization of $\mathcal{M}$, which we will continue to use.

The Kinematical state space $\mathcal{K}$ is thus $L^2(\mathbb{R}^2)$, and is spanned by common eigenstates of $\hat{x}$ and $\hat{t}$ (representing a complete set of partial observables), $|x,t\rangle$. As is well known, this space has "non-physical" properties, such as the absence of a ground state, due to the fact that the spectrum of $\hat{t}$ is the real line. Note, however, that in this formalism, $\mathcal{K}$ is merely a space on which experimentally accessible quantities are defined as Hermitian operators — it does not contain information on which "physical" states are allowed into the theory.

The relativistic Hamiltonian takes the form $H = H_0 - i\hbar\frac{\partial}{\partial t}$ (where H_0 is the ordinary nonrelativistic Hamiltonian $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)$), and the equations of motion are $H\Psi(x,t) = 0$. Solutions to these equations of motion constitute the physical state space, $\mathcal{H}$.

The complete observables, now, are represented by the Heisenberg position operators $\hat{x}(t)$ — the eigenstates of $\hat{x}(t)$ do not live in $\mathcal{K}$, they live in $\mathcal{H}$. This is due to the fact that $\hat{x}(t)$ is related to $\hat{x}$ by $\hat{x}(t) = \exp\frac{(itH_0)}{\hbar}\ \hat{x}\ \exp\frac{(-itH_0)}{\hbar}$, and thus a state (call it $|x;t\rangle$ — *note the semicolon instead of the comma!*) satisfying $\hat{x}(t)|x;t\rangle = x|x;t\rangle$ and $\hat{x}(0)|x;0\rangle = x|x;0\rangle$ implies that $|x;t\rangle = \exp\frac{(itH_0)}{\hbar}|x;0\rangle$ — these states carry dynamical predictions, rather than simply the labels of points on $\mathcal{M}$.

The connection to ordinary Schrödinger-picture formalism is this: If at time $t = 0$ we describe an L^2 function $\Psi(x) = \langle x,0|\Psi\rangle$, then the time-evolved solution to the Schrödinger equation $\Psi(x,t)$ is given by $\langle x;t|\Psi\rangle = \exp\frac{(-itH_0)}{\hbar}\Psi(x)$.

A more general and realistic measurement of x and t determines a localized state on $\mathcal{K}$, but is generally smeared over $\mathcal{M} = \mathbb{R}^2$, not just over $\mathbb{R}$ — for example, if an $\hat{x}$-measurement is localized around the point x_0 with precision a and a $\hat{t}$-measurement is localized around the point t_0 with precision b, then (depending on how the measurement is actually performed), we might have a $\mathcal{K}$-state description $\phi^{\mathcal{K}}(x,t) = \exp\left[\frac{-(x-x_0)^2}{a^2} + \frac{-(t-t_0)^2}{b^2}\right]/(2\pi ab)$ (which treats time fundamentally on the same footing as space, taking into account the fact that time measurements have a finite resolution, just as well as space measurements).

This then projects to a full physical state in $\mathcal{H}$, which represents the dynamics of the particle *after* such a localizing measurement has been performed:

$$\phi^{\mathcal{H}}(x,t) = \langle x;t|\phi^{\mathcal{K}}\rangle = \int dx'\ dt'\ W(x,t;x',t')\phi^{\mathcal{K}}(x',t') \tag{3.9}$$

where, for the free Schrödinger equation, the propagator is known to take the form $W(x,t;x',t') = \left(\frac{2\pi m}{i\hbar(t-t')}\right)^{1/2} \exp\left(\frac{m(x-x')^2}{2i\hbar(t-t')}\right)$ [7].

The attractive quality of this formalism is, of course, its generality. If instead of single-particle quantum mechanics we wished to study a system which truly requires a covariant treatment, for example minisuperspace quantum cosmology with a scalar field, the picture would remain much the same. The extended configuration space would again be $\mathbb{R}^2$, which can be coordinatized by the variables Ω (related to the cosmological expansion parameter a by $a = e^{\Omega}$), and ϕ (the average value of the scalar field). Kinematical states (representing possible measurement outcomes or "quantum coincidences") would again be $L^2(\mathbb{R}^2)$, while the Hamiltonian takes the form

$$H = \hbar^2 e^{-3\Omega}\left(\frac{1}{24}\frac{\partial^2}{\partial\Omega^2} - \frac{1}{2}\frac{\partial^2}{\partial\phi^2}\right) - 6ke^{\Omega} + e^{3\Omega}V(\phi),$$

from which $\mathcal{H}$ and the propagator are constructed [32]. Note that no time variable *at all* appears in such a theory — there is not even an obvious variable that is *analogous* to time (though often one variable is arbitrarily chosen to "be" the time, in order to force the theory into a formalism somewhat resembling standard quantum mechanics, and different choices of the time variable tend to lead to unitarily inequivalent theories) [32]. Nevertheless, the interpretation in terms of covariant quantum theory is essentially identical to that of single-particle quantum mechanics — our measurements from earth localize a $\mathcal{K}$-state $\Psi^{\mathcal{K}}(\Omega, \phi)$, which we then project to a full physical state $\Psi^{\mathcal{H}}(\Omega, \phi)$ which solves $H\Psi(\Omega, \phi) = 0$ in order to get the dynamical predictions of the theory. Although we have dynamics and dynamical predictions, time is nowhere to be seen — it is no longer a fundamental player in the formalism.

Though tremendously general, this formalism has had one critically weak link. The RR measurement postulate (postulate 3) of covariant quantum theory (namely $\mathcal{P}_{\phi\to\psi} = \left|\langle\psi^{\mathcal{K}}|P|\phi^{\mathcal{K}}\rangle\right|^2$) does not reproduce the measurement postulate of orthodox quantum mechanics, though it is clearly constructed in analogy to the standard Born interpretation. This was realized immediately by Reisenberger and Rovelli [7]. To obtain agreement with standard QM in the case of single measurements requires the use of a small-region approximation, which assumes that $\psi^{\mathcal{K}}$ has support in a sufficiently small region of the extended configuration space. For multiple measurements, it originally appeared to require a pre-defined time variable to order the measurements, though Hellmann, Mondragon, Perez, and Rovelli [33] have recently argued that any series of measurements can be treated mathematically within the formalism as a single measurement, for the purposes of making predictions. We will elaborate upon these points further in the section on correspondence, where we will argue that the Reisenberger-Rovelli postulate cannot be a *general* probability postulate.

3.3 Generally Covariant Quantum Information

3.3.1 The Covariant Partial Trace

Recently, we have proposed a quantum information theoretic interpretation of measurement within covariant quantum theory that appears to solve the above difficulties [34]. The idea can be thought of as an extension of the description of standard quantum measurement given by Cerf and Adami, which we reviewed in chapter 2. That is, we include a model of the observer in any experimental scenario, and obtain probabilities by looking at the information held by the observer's reduced density operator — generally speaking, due to the properties of entanglement, quantum theory predicts that a measurement will result in an initially pure observer state evolving into a mixed one, and thus the predictions of quantum mechanics are inherently probabilistic.

To invoke these ideas in the context of covariant quantum mechanics, we clearly need a partial trace. This is simple on the kinematical Hilbert space $\mathcal{K}$, which has the usual L^2 tensor product form, but it is not so obvious on the physical Hilbert space $\mathcal{H}$ in which states cannot generally be expressed as a tensor product of subsystem states.

To begin, we define a covariant system analogous to Q by specifying its extended configuration space $\mathcal{M}_Q$ and a relativistic Hamiltonian H_Q. To include an observer system analogous to A, we enlarge the configuration space via the Cartesian product, i.e. $\mathcal{M} = \mathcal{M}_Q \times \mathcal{M}_A$, and define a new Hamiltonian H for the combined system. In what follows, let x represent coordinates of $\mathcal{M}_Q$, and let y represent coordinates of $\mathcal{M}_A$ — i.e. we express a general point on $\mathcal{M}$ with the pair, $\{x, y\}$.

By analogy with Cerf and Adami, we wish the partial trace to express local information held by a specific subsystem, but not by using a preferred time variable. Instead, we select a generic region of interest $\mathcal{S}$ somewhere in $\mathcal{M}$. In covariant theories like general relativity, there is in general no diffeomorphism-invariant meaning of a general region on the *spacetime* manifold, since spacetime coordinates represent neither partial nor complete observables — in the present case, however, we are working directly on the space of experimentally observable quantities $\mathcal{M}$, thus our regions $\mathcal{S}$ have a definite meaning in terms of experimental data. In the limit of the Schrödinger picture, the region $\mathcal{S}$ corresponds to a constant time slice of space, but in general $\mathcal{S}$ may be chosen freely, provided a few conditions are met: We require that the physical state $\phi^{\mathcal{H}}$ under consideration can be expressed via the projection of a $\mathcal{K}$ state $\phi^{\mathcal{K}}$ with support in $\mathcal{S}$ — this requirement simply means that the physical state of the system can be reconstructed from local experimental data (also, this local state is normalized with respect to $\mathcal{H}$, i.e. we require $\langle \phi^{\mathcal{K}} | P | \phi^{\mathcal{K}} \rangle = 1$). Next, we require that for all points $\{x, y\}$ and $\{x', y'\}$ in $\mathcal{S}$, the propagator takes the form $W(x, y; x', y') = W_Q(x; x')\delta(y - y')$, where $W_Q(x; x')$ is obtained from the free Hamiltonian H_Q. This expresses the fact that we are considering a region $\mathcal{S}$ where no interactions between system Q and A are taking place (e.g. before or after the measurement interaction has taken place) and the evolution of system A is trivial. That is, if a measurement has been performed, the measuring system A does not forget about the outcome anywhere in $\mathcal{S}$ — it merely holds the relevant information. Such a region $\mathcal{S}$ satisfying these conditions is a generalized replacement for the concept of a constant-time slice (which satisfy these requirements in standard QM).

When these conditions are met, we can approximate the full Wheeler-DeWitt equation $H\Psi(x, y) = 0$ as $H_Q\Psi(x) = 0$ in our region of interest, and thus the physical state space $\mathcal{H}$ is locally indistinguishable from $\mathcal{H}_Q \otimes L^2(\mathcal{M}_A) \equiv \mathcal{H}_Q \otimes \mathcal{K}_A$ within $\mathcal{S}$. This effective tensor product form of the state space within $\mathcal{S}$ is the main trick which allows us to use the information-theoretic ideas from chapter 2.

We define a new projector, P_Q, from $\mathcal{K}_Q$ to $\mathcal{H}_Q$, so that we can now express our state as a physical density operator via $\rho = P_Q|\phi^{\mathcal{K}}\rangle\langle\phi^{\mathcal{K}}|P_Q^\dagger$ (valid only in the region $\mathcal{S}$). Now express $|\phi^{\mathcal{K}}\rangle$ via a Schmidt decomposition as $\sum_i \lambda_i|\phi_i^{\mathcal{K}_Q}\rangle|\phi_i^{\mathcal{K}_A}\rangle$. P_Q operates only on $\mathcal{K}_Q$, so we can take a partial trace over $\mathcal{K}_Q$ and using the cyclic property of the trace, the properties of the projector, and the physical inner product ($\langle x|P^\dagger P|x'\rangle = W(x; x')$), we obtain a reduced density operator on $\mathcal{K}_A \equiv L^2(\mathcal{M}_A)$:

$$\rho_A = \sum_{i,j} \int_{\mathcal{M}_Q} dx\; dx'\; \lambda_i\lambda_j\phi_i^{\mathcal{K}_Q}(x)W_Q(x; x')\phi_j^{*\mathcal{K}_Q}(x')|\phi_i^{\mathcal{K}_A}\rangle\langle\phi_j^{\mathcal{K}_A}|. \tag{3.10}$$

This is now a reduced density operator on $\mathcal{K}_A$, containing the physically relevant information locally available to the observer A within $\mathcal{S}$. Note the range of integration is contained entirely within $\mathcal{S}$ due to the support of the L^2 functions $\phi^{\mathcal{K}}$. This covariant definition immediately reduces to the ordinary definition of the partial trace when the region of interest $\mathcal{S}$ is a constant time slice as in the Cerf-Adami picture, but it is clearly more general — $\phi^{\mathcal{K}}$ may be smeared in any number of ways over a non-zero time interval (if there is a time variable), provided that the observer system A is making no transitions in $\mathcal{S}$. It also remains meaningful for fully covariant systems having no pre-defined time variable at all.

The conditions we have placed on the region $\mathcal{S}$ in order for this partial trace to be defined are the most immediate ones which allow us to use the reduced von Neumann entropy to quantify information in a way completely analogous to standard quantum mechanics on a $t =$ constant slice of space. It is possible that one or more of our conditions may be relaxed, but it also may be the case that it simply does not make sense to define quantum information in a region that fails to satisfy these criteria (for example, if one tries to use a region which does not support any $\mathcal{K}$-states for a given $\mathcal{H}$-state, the theory might appear to lack a unitary evolution, when it is really one's choice of region which is simply inadequate — this effect has been demonstrated in certain toy models resembling quantum gravity, in slightly different language [35]).

3.3.2 A New Measurement Postulate

With this partial trace we can implement the idea of Cerf and Adami covariantly, without any preferred coordinates on $\mathcal{M}$, and without any external time variable. Within a given region $\mathcal{S}$, an observer A will be described by a mixture on his kinematical state space $\mathcal{K}_A$, even if we know the full physical state in $\mathcal{H}$ is pure. The probabilities and experience basis can be extracted exactly as we have described in chapter 2 — by finding the unique diagonal, orthogonal basis of ρ_A obtained from the covariant partial trace. The entries along the diagonal representation of ρ_A represent the respective probabilities for the observer to realize the associated basis state. Note that although the Schmidt decomposition of a given state $\phi^{\mathcal{K}}$ in $\mathcal{K}_Q \otimes \mathcal{K}_A$ defines a basis in which the reduced density operator ρ_A is diagonal when the partial trace is over $\mathcal{K}_Q$, the covariant partial trace does not necessarily leave ρ_A diagonal in the Schmidt basis of a given state of $\mathcal{K}_Q \otimes \mathcal{K}_A$.

3.4 Correspondence and the Schrödinger Picture Limit

In this section, we analyze explicitly the case of single-particle detection, in order to test correspondence with the well-established Born rule of standard quantum mechanics. Since we will be trying to establish explicit correspondence, we will make use of the standard coordinates $\{X, T\}$ (rather than the abstract set of coordinates x that we have been using) as in standard Schrödinger-picture quantum mechanics. This may have a tendency to hide the coordinate-invariant nature of the formalism, but we will be able to show disagreement between the predictions of the RR postulate and our covariant quantum information approach (CQI), and agreement of CQI with standard quantum mechanics.

3.4.1 Single Particle Measurement and the Born Interpretation

It is important to realize that our covariant quantum information framework predicts *different* probabilities than does the Reisenberger-Rovelli postulate. This is because entanglement is a necessary feature of measurement in our covariant information approach, whereas the Reisenberger-Rovelli probability postulate assigns probabilities between any two kinematical states. Thus, there are many situations in which the Reisenberger-Rovelli postulate assigns a probability but our framework does not — for example, whenever we ask about the probability that a system is found in state $\psi^{\mathcal{K}}$, without specifying a quantum system to perform the measurement and hold information on its outcome.

Equally important is that this shift in interpretation can lead to different probabilities for the same physical question. To see this explicitly, we analyze here single-particle dynamics, where we can make contact with the well-known Born interpretation as a correspondence rule that must be reproduced in the appropriate limit.

It was immediately clear to Reisenberger and Rovelli that there was a difficulty with their original probability postulate [7, 36, 33, 37]. Here we consider the system described in their original paper [7] — a single particle satisfying the Schrödinger equation, and a simple two-state detector designed to activate if the particle passes through spacetime region $\mathcal{R}$. The detector is prepared in state $|0\rangle$, and transitions to state $|1\rangle$ if the particle interacts with the detector in $\mathcal{R}$. The interaction Hamiltonian can be written as $H_{int}(x) = \alpha V(x)(|1\rangle\langle 0| + |0\rangle\langle 1|)$, where the potential $V(x)$ is constant in $\mathcal{R}$ and zero elsewhere. The potential is assumed to be sufficiently weak so that first order perturbation theory can be used, and thus transitions from the state $|1\rangle$ back to the state $|0\rangle$ can be ignored. In the following analysis, the coordinates $x = \{X, T\}$ are used (that is, $f(x)$ is shorthand for a function $f(X, T)$ of the usual space and time coordinates).

To ask the question "what is the probability that the detector is activated?" in the standard Born-rule interpretation, we need an initial state $\Psi_0(X)$ at some initial time T_0 and we must specify some later time T after the interaction to look at the form of the wave function. Reisenberger and Rovelli showed that at late time T the combined system has evolved to a state of the form:

$$\begin{aligned}\Psi_T(X) &= \int dX'\; W(X,T;X',T_0)\Psi_0(X',T_0)\;|0\rangle + \\ &\frac{\alpha}{i\hbar}\int_{\mathcal{R}} dX'\; dT'\; W(X,T;X',T')V(X',T')\Psi(X',T')\;|1\rangle, \end{aligned} \tag{3.11}$$

where W is the free propagator and $\Psi(X, T) = \Psi(x) = \int dx'\; W(x, x')\Psi_0(x')$ is the freely propagated initial state. From here, the Born rule tells us that the probability that the detector was activated is the square of the second term integrated over X at the constant time T. With some manipulation and use of the properties of the Schrödinger propagator, it can be shown that this probability takes the following form:

$$\mathcal{P}_{\mathrm{Born}} = \frac{\alpha^2}{\hbar^2}\int_{\mathcal{R}} dx \int_{\mathcal{R}} dx' \Psi^*(x)W(x;x')\Psi(x'). \tag{3.12}$$

Now, for this scenario, the postulate of Reisenberger and Rovelli for covariant quantum theory is that:

$$\mathcal{P}_{\mathrm{R-R}} = |\langle\mathcal{R}|P|\Psi_0^{\mathcal{K}}\rangle|^2 \tag{3.13}$$

$$= \left|\int dx \int dx'\; \mathcal{R}^*(x)W(x;x')\Psi_0^{\mathcal{K}}(x')\right|^2. \tag{3.14}$$

Here, $\mathcal{R}(x)$ is a normalized, constant function in $\mathcal{R}$, and zero elsewhere. $\Psi_0^{\mathcal{K}}$ is the initial state prepared on a constant-time slice as above (the superscript $\mathcal{K}$ merely serving to emphasize that such states defined at a constant time are, in fact, $\mathcal{K}$-states). Using the definition of $\Psi(x)$ above, we can re-write this as follows:

$$\mathcal{P}_{\mathrm{R-R}} = \left| \int dx\, \mathcal{R}^*(x)\Psi(x) \right|^2 \tag{3.15}$$

$$= \int dx \int dx'\ \Psi^*(x)\mathcal{R}^*(x)\mathcal{R}(x')\Psi(x') \tag{3.16}$$

$$= \int_{\mathcal{R}} dx \int_{\mathcal{R}} dx'\ \Psi^*(x)\Psi(x'). \tag{3.17}$$

This form of $\mathcal{P}_{\mathrm{R-R}}$ is similar to the form of $\mathcal{P}_{\mathrm{Born}}$ shown in equation (3.12), except for the missing factor of the propagator in the integrand. If $\mathcal{R}$ is taken to be sufficiently small, then these two results can be made to agree up to a constant factor related to the description of the measurement apparatus. A similar small-$\mathcal{R}$ limit has been taken in all previous presentations of RR covariant quantum theory. However, it is easy to construct examples outside of this limit where disagreement immediately arises. For example, take $\mathcal{R}$ to be the union of two points at $x = a$ and $x = b$ on a single constant-time slice (so that $W(a;b) \approx 0$) — then we have that $\mathcal{P}_{\mathrm{B}orn} \propto |\Psi(a)|^2 + |\Psi(b)|^2$, but that $\mathcal{P}_{\mathrm{R-R}} \propto |\Psi(a)|^2 + |\Psi(b)|^2 + \Psi^*(a)\Psi(b) + \Psi^*(b)\Psi(a)$. If a and b are sufficiently separated, then the probabilities $\mathcal{P}_{\mathrm{B}orn}$ and $\mathcal{P}_{\mathrm{R-R}}$ cannot be made to be proportional functions of a and b.

This illustrates an extremely important point — if the original Reisenberger-Rovelli probability postulate fails to reproduce what we already know to be true in standard atomic-scale quantum mechanics, it can certainly not be taken for granted in the unexplored Planck-scale or entire universe-scale regimes, for which it was designed to be used in the first place.

Our covariant quantum information approach, by contrast, has no difficulty here. Recall that our CQI framework requires us to define a region of the extended configuration space, $\mathcal{S}$, in order to obtain probabilities. If we take the same measurement apparatus defined above, and if $\mathcal{S}$ is taken to be a constant-time slice of space, then the Born result is reproduced identically. However, we are by no means *required* to take $\mathcal{S}$ to be a constant-time slice — one main motivation for this approach to physics was in fact to do away with the preferred status of the time variable in the construction of the theory.

To see this explicitly in this example, note that to the future of the interaction region $\mathcal{R}$, the detector is evolving trivially, thus the physical Hilbert space $\mathcal{H}$ is indistinguishable from $\mathcal{H}_Q \otimes \mathbb{C}^2$ in this region[1]. Thus all regions to the future of $\mathcal{R}$ are equivalent for the purposes of calculating the transition probability, so long as they are large enough to support a $\mathcal{K}$ state $\Psi^{\mathcal{K}}(x)$ that projects to the relevant physical states on $\mathcal{H}$. Using the covariant partial trace, the detector's reduced density operator can be expressed as follows:

$$\rho_A = \left[\frac{\alpha^2}{\hbar^2} \langle\phi|\phi\rangle^{\mathcal{H}_Q} \; |1\rangle\langle 1| + \langle\psi|\psi\rangle^{\mathcal{H}_Q} \; |0\rangle\langle 0| \right] \tag{3.18}$$

where $\phi(x)$ and $\psi(x)$ are defined as follows:

$$\begin{aligned} \phi(x) &= \int_{\mathcal{R}} dx' \int_{\mathcal{M}_Q} dx'' W(x;x') V(x') W(x',x'') \Psi_0(x'') \\ \psi(x) &= \int_{\mathcal{M}_Q} dx' \; W(x;x') \Psi_0(x'), \end{aligned} \tag{3.19}$$

which represent unnormalized states in $\mathcal{H}_Q$. The probability for the detector to be activated is thus given by:

$$\mathcal{P}_{\mathrm{CQI}} = \frac{\alpha^2}{\hbar^2} \langle\phi|\phi\rangle^{\mathcal{H}_Q}, \tag{3.20}$$

which depends on the physical inner product in $\mathcal{H}_Q$ rather than on the preferred status of any time coordinate. However, because of the isomorphism between $\mathcal{H}_Q$ and $L^2(\mathbb{R})$ on a constant time slice [8], we can immediately see that this probability reduces to that obtained by the Born interpretation above, which *does* make use of a preferred time coordinate (but

[1] Actually, by the argument of chapter 2, we take the system to be $\mathcal{H}_Q \otimes \mathbb{C}^2 \otimes \mathbb{C}^2$ in this region, making a distinction between the detector D and the observer A, which become entangled to the future of $\mathcal{R}$. We then take the covariant partial trace over both $\mathcal{H}_Q$ and D. We are suppressing the explicit mention of the additional detector system in the calculation to avoid distraction based on this finer point of the formalism, which is required to do away with off-diagonal elements in the reduced density operator in the $|0\rangle, |1\rangle$ basis.

is well-established experimentally). Using the above definitions of $\phi(x)$ and $\Psi(x)$, we have:

$$
\begin{aligned}
\mathcal{P}_{CQI} &= \frac{\alpha^2}{\hbar^2}\langle\phi|\phi\rangle^{\mathcal{H}_Q} && (3.21)\\
&= \frac{\alpha^2}{\hbar^2}\langle\phi^{\mathcal{K}}_{t=T}|P|\phi^{\mathcal{K}}_{t=T}\rangle^{\mathcal{K}_Q} && (3.22)\\
&= \frac{\alpha^2}{\hbar^2}\int dX\ \phi^*(X,T)\phi(X,T) && (3.23)\\
&= \int dX\ \left|\frac{\alpha}{i\hbar}\int_{\mathcal{R}} dX'\ dT'\ W(X,T;X',T')V(X',T')\Psi(X',T')\right|^2 && (3.24)\\
&= \frac{\alpha^2}{\hbar^2}\int_{\mathcal{R}} dx \int_{\mathcal{R}} dx'\Psi^*(x)W(x;x')\Psi(x'), && (3.25)
\end{aligned}
$$

which, though defined in a coordinate-independent, covariant manner by equation (3.20), reduces to exact agreement with the Born rule of equation (3.12)[2]. The entanglement properties of measurement and use of the covariant partial trace are responsible for the discrepancy with the Reisenberger-Rovelli postulate, and agreement with standard QM. In our CQI framework, entanglement is *essential* for obtaining probabilities, for it is only through entanglement that a partial trace leads a quantum observer into a mixed state. This is not true of the R-R postulate, which assigns probabilities between any two states in $\mathcal{K}$.

3.4.2 Time Ordering and Multiple-Measurement Probabilities

Another difficulty associated with the Reisenberger-Rovelli probability postulate is that there is no preferred time variable or causal structure available to order multiple measurements. For example, suppose we wish to measure the state $|\phi\rangle$, and also the state $|\xi\rangle$. What is the

[2] There is a potentially confusing point here and it is important that it not be misunderstood. Equation (3.20) is obtained from a generic region $\mathcal{S}$ to the future of the measurement interaction region $\mathcal{R}$, and does not depend upon the identification of any specific time variable. However, in evaluating equation (3.20), we have made use of a constant-time slice on $\mathcal{M}_Q$. The latter is merely a calculational convenience offered by the specific form of the Schrödinger propagator for the purposes of evaluating the physical inner product — it introduces no interpretation as a "probability at time T," which is precisely what we are attempting to avoid. The resulting probability is valid in any sufficiently large region $\mathcal{S}$ to the future of the measurement interaction, including those regions which might not even contain a constant-T surface themselves. The thing that is covariant about this treatment is that equation (3.20) is obtainable from a generic region $\mathcal{S}$ and does not depend on the identification of any time variable. The means by which a given physical inner product might be evaluated, however, is not something that we necessarily need to banish "T" from — we are free, of course, to take advantage of any convenient mathematical properties a given propagator might have in a particular coordinate system.

probability that the system is measured in *both* of these states? In general, if the projector onto $|\phi\rangle$, Π_ϕ, does not commute with the projector onto $|\xi\rangle$, Π_ξ, then we need to distinguish between $|\Pi_\phi\Pi_\xi|\Psi\rangle|^2$ and $|\Pi_\xi\Pi_\phi|\Psi\rangle|^2$, which will be two distinctly different probabilities. In standard QM, this distinction is easy — we simply order the projections according to the time variable t (or order them according to the causal structure on a fixed background spacetime). However, a fully covariant, background-free formalism does not have such constructs — the theory must not be required to reference an external time variable, or a preferred time coordinate of $\mathcal{M}$ in order to make predictions (likewise, nonperturbative quantum gravity must not be required to reference a fixed background causal structure to make predictions).

Recently, Hellmann, Mondragon, Perez, and Rovelli have argued that this time ordering problem can be avoided by interpreting all predictions within the RR postulate strictly as single-measurement probabilities [33, 37]. That is, instead of constructing Π_ϕ and Π_ξ separately, we enlarge the system to include at least one measuring apparatus that determines for example whether or not the state ϕ was observed. We then take the projector onto the state $|\xi\rangle$ as well as the projector onto the state $|\phi = yes\rangle$ of the detector (call it Ω_ϕ). Then the single projector $\Pi_\xi\Omega_\phi$ on this larger, combined system gives an effective multiple-measurement probability $|\Pi_\xi\Omega_\phi|\Psi\rangle|^2$ according to the original probability postulate, but makes use of only a single non-unitary collapse and so evades the multiple-measurement probability problem. Note that in the case of N measurements, this approach makes use of $N-1$ measurement devices to remove the ordering ambiguity.

This approach is similar to our own, which makes use of N observers for N measurements. Note that in our strategy, no external time ordering is required because we simply specify a given region $\mathcal{S}$, and ask if a given measurement apparatus A, B, etc. is in a mixed state within this region. If so, we compute probabilities exactly as before, for each measurement apparatus. Because this formalism reduces identically to that of Cerf and Adami when we study simple Schrödinger-equation systems and constant-time slices, all the effective collapse features of the CA formalism are retained. No collapse postulate is required, and thus no special time variable needs to be selected to order projections for multiple measurements.

The CQI formalism does, however, allow a different kind of time ordering to emerge in the context of multiple measurements. We can pick out a series of regions $\mathcal{S}$, $\mathcal{S}'$, $\mathcal{S}''$, etc. and order them according to the von Neumann entropy of the observed system Q (having made use of the covariant partial trace). In the case of systems described by the Schrödinger equation, the regions corresponding to constant-time slices, and all measurement interactions corresponding to ideal projective measurements of Q, this ordering corresponds exactly to the ordering of projective collapse by an external t-variable, since it is known that projective measurements can only increase the entropy of a system [12] (the arrow of time as discussed in section 2.5).

We can describe this prescription for an arrow of time in a more general and coordinate-independent terms: We have two measurement systems whose configurations are described by $\mathcal{M}_A$ and $\mathcal{M}_B$, while the system under observation is described on configuration space $\mathcal{M}_Q$.

The description of a covariant collapse consists in specifying three regions of the extended configuration space, $\mathcal{S}$, $\mathcal{S}'$, and $\mathcal{S}''$. With respect to the covariant partial trace, the reduced density operators ρ_A and ρ_B are pure (they have zero entropy) when expressed in the region $\mathcal{S}$. In region $\mathcal{S}'$, ρ_B is pure, but ρ_A can be expressed as some mixed state $\rho_A = \sum_i |c_i|^2 |\phi_i^{\mathcal{K}_A}\rangle\langle\phi_i^{\mathcal{K}_A}|$. In region $\mathcal{S}''$, we have that:

$$\rho_A = \sum_i |c_i|^2 |\phi_i^{\mathcal{K}_A}\rangle\langle\phi_i^{\mathcal{K}_A}| \tag{3.26}$$

$$\rho_B = \sum_{i,j} |c_i|^2 |U_{ij}|^2 |\phi_j^{\mathcal{K}_B}\rangle\langle\phi_j^{\mathcal{K}_B}| \tag{3.27}$$

where each detector state $\phi_i^{\mathcal{K}}$ reflects a given state (say $|R_i\rangle$) of the system Q as before, and $U_{ij} = \langle R_j|P_Q|R_i\rangle$ are the transition amplitudes between these. This structure exactly mimics the entanglement and entropy structure leading to the effective collapse in the Cerf-Adami picture, and if we compare the entropy of these reduced density operators to the Shannon entropy of a classical distribution, we are led to the classical probabilities $p_A(i) = |c_i|^2$ and $p_B(j) = \sum_i |c_i|^2 |U_{ij}|^2$. Thus we have identified a general type of collapse in the covariant formalism which reduces immediately to the standard Schrödinger picture form in the appropriate limit (when $\mathcal{S}$, $\mathcal{S}'$, $\mathcal{S}''$ are constant-time slices), but whose general form requires no classical background causal structure or preferred configuration variable to play any special role. As before, an entropic time ordering is contained in the observer systems themselves.

The fact that it is possible to define an "arrow of time" in this way without the general need for a specific time *variable* is interesting. It remains to be seen how closely this type of entropic time ordering resembles other notions of time in generally covariant systems such as quantum cosmology (which will have to be extended to contain explicit "observer systems" in order to invoke our probability postulate).

To sum up, Hellmann, Mondragon, Perez, and Rovelli seek to solve the time-ordering problem by postponing a non-unitary collapse until the end of the analysis, while we *never* make use of projective collapse within our CQI approach. In addition, we resolve the Born correspondence problem in the case of single measurements, and define an entropic quantum mechanical arrow of time independent of any coordinate on $\mathcal{M}$ or background causal structure. We do not interpret quantum probabilities as probabilities for the full physical state $\Psi^{\mathcal{H}}$ of the universe to make any kind of jump to a new physical state — the probabilities in covariant quantum information simply represent the information held by the reduced state of an entangled, localized observer (as obtained by our covariant partial trace). In essence, our CQI picture realizes perfect correspondence with standard QM in the appropriate limit and removes the most serious objections to covariant quantum theory as proposed by Reisenberger and Rovelli.

3.4.3 Nonlocal Correlations and Collapse

Not surprisingly, a formalism that dispenses with collapse will have implications for nonlocality in the EPR scenario [38]. Quickly setting up the scenario, we have Alice located at spatial point a, and Bob at point b. An entangled quantum state is prepared, $\alpha|00\rangle + \beta|11\rangle$, and Alice is given control of the first qubit, and Bob the second. For simplicity here, we take the qubits to be abstract two-state systems that transform trivially under Lorentz transformations, i.e. the two-state system should not be thought of as a spinor in 3-space.

Suppose now that Alice performs a measurement on her qubit in the $\{|0\rangle, |1\rangle\}$ basis. The standard explanation is that the full, nonlocal state has *instantly* been collapsed to $|0\rangle|0\rangle$ or to $|1\rangle|1\rangle$. Worse still, if Bob performs a measurement at an instant that is spacelike separated from Alice's measurement, there are frames of reference in which the order of collapse changes. Some involve Alice collapsing the state, and Bob performing a measurement on a collapsed state, and some involve Bob collapsing the state. This state of affairs has caused concern among many who notice the seeming contradiction with special relativity. Happily for relativists, a no-communication theorem ensures that the effects of this collapse cannot be used to send superluminal information between Alice and Bob [39].

This is not the end of the story, though. What did we mean when we said the state was collapsed *instantly* by Alice? Instantly in which frame of reference? One immediate and seemingly reasonable answer is Alice's frame of rest [40]. However, suppose a third observer, Charlie, were to pass by Alice with some velocity exactly as she performs her measurement. If the non-local collapse is instant in Alice's frame, it cannot be instant in Charlie's frame, though he observed the measurement outcome at precisely the same moment as Alice. The addition of such relativity considerations to the mix causes thinking along these lines to be thorny enough that clever experiments have actually been performed to observe the speed of collapse of separated, entangled pairs [41] (incidentally, setting lower bounds of order 10^4c on the speed of quantum information).

In our CQI picture, the description is clarified. The wave function does not undergo a collapse, instantaneous or otherwise. Probabilities are interpreted as the classical uncertainty of the observer's reduced state, which becomes a mixture via local entangling interactions with the EPR pair — we do *not* have probabilities for any kind of non-local quantum operation, and require no propagation of any effect. Alice and Charlie do not disagree on the way in which the state of the universe is collapsed, since there are simply no nonlocal effects happening at all. The "speed of collapse" is an undefined concept in this picture. Alice and Charlie are both described by mixtures to the future of Alice's measurement event, and Bob is described by a mixture to the future of his own measurement event — these are entirely local statements, in which there is absolutely no contradiction.

The addition of relativity simply means that there are more spacelike surfaces (related by Lorentz transformations) on which we can express the full physical state in $\mathcal{H}$ as the projection of a kinematical state. On some of these surfaces, Alice has performed a measurement (and is described by a mixture) but Bob has not. On others, Bob has performed a measurement (and is described by a mixture), but Alice has not. However, on *any* constant-time slice in which both Alice and Bob could have compared notes via transmission of their measurement results, the $\mathcal{K}$-state describes Alice as entangled with Bob:

$$|Q_1AQ_2B\rangle = \alpha|0\rangle|\text{Alice sees }0\rangle|0\rangle|\text{Bob sees }0\rangle + \beta|1\rangle|\text{Alice sees }1\rangle|1\rangle|\text{Bob sees }1\rangle, \quad (3.28)$$

Calculating the reduced density operators for Alice and Bob, standard results are obtained. Individually for Alice and Bob, we have:

$$\begin{aligned}\rho_A &= |\alpha|^2|\text{Alice sees }0\rangle\langle\text{Alice sees }0| + |\beta|^2|\text{Alice sees }1\rangle\langle\text{Alice sees }1| &(3.29)\\ \rho_B &= |\alpha|^2|\text{Bob sees }0\rangle\langle\text{Bob sees }0| + |\beta|^2|\text{Bob sees }1\rangle\langle\text{Bob sees }1|, &(3.30)\end{aligned}$$

which returns the standard probabilities. Considered together, we have:

$$\begin{aligned}\rho_{AB} &= |\alpha|^2|\text{Alice sees }0\rangle\langle\text{Alice sees }0||\text{Bob sees }0\rangle\langle\text{Bob sees }0|\\ &+ |\beta|^2|\text{Alice sees }1\rangle\langle\text{Alice sees }1||\text{Bob sees }1\rangle\langle\text{Bob sees }1|, \quad (3.31)\end{aligned}$$

which reproduces the standard correlations (which can also be seen via the vanishing conditional entropy: $S(A|B) = S(AB) - S(B) = 0$ — i.e. if we know the results of Bob's experiment, then we know the results of Alice's experiment, and vice versa). The no-communication theorem still applies as well as ever, as any local unitary operation applied to Alice alone will have no effect on the information available to Bob, due to the partial trace.

Of course, this discussion of the EPR scenario introduces no new experimentally measurable results — it is the same familiar story, merely told in a different language. The novelty is simply that the conceptual baggage created by the concept of instantaneous, non-local collapse can be discarded entirely in favor of local statements about the state of the observers, and shared information that exists on certain constant-time slices. All operations described by all observers are entirely local from start to finish, and thus Lorentz transformations imply no strange, superluminal propagation of quantum information for observers in differing states of motion.

Chapter 4

Experimental Prediction: Time-Reversed Quantum Zeno Effect

Our approach to quantum theory emphasizes the fact that measurement and collapse are merely the information theoretic consequences of a system becoming entangled with an observer, and that all evolution is fundamentally unitary. A consequence of this fact is that time-reversal becomes a useful symmetry to invoke, even in the presence of quantum measurements.

To see this feature in action, we will analyze a well-known effect of quantum measurement — the quantum Zeno effect [42]. Ordinarily, the non-unitary formalism of projective measurement is invoked to demonstrate the surprising fact that *merely observing* a quantum system can effectively slow its evolution. It has also been shown that clever use of projective measurement can also be used to accelerate the evolution of a system, in what has been termed the quantum anti-Zeno effect [43] (here, we take the term "quantum Zeno effect" to be more general than the particular application to decaying atomic states — the context in which it was originally demonstrated).

However, it should be clear that projective measurement corresponds to entanglement with an observer. Hence, both the quantum Zeno effect and the quantum anti-Zeno effect may be obtained as the result of entangling a quantum system with an ancilla, as we will soon show. We would further like to point out that the time reversal of these effects also represents a way to manipulate the evolution of a quantum system. That is, *disentangling* a quantum system from an ancilla can be used to accelerate (or slow) its evolution. We refer to this as the time-reversed quantum Zeno effect. Not surprisingly, a combination of Zeno effect and time-reversed Zeno effect cancel completely, leaving ordinary time evolution. An important distinction to understand, however, is that the time-reversed Zeno effect is *not* the same as the anti-Zeno effect — both the Zeno and anti-Zeno effect result from entanglement, whereas the time-reversed Zeno effect will result from disentanglement, all of which we will now demonstrate.

Let us first analyze the quantum Zeno effect within this framework (here for convenience and familiarity we revert to non-covariant language, where evolution in an external time variable is assumed). We consider a system composed of three subsystems, Q, A, and B. Q will be the time-dependent system under study, while A (for "Alice," or "ancilla") represents a system we will entangle with Q. Then B (Bob), represents the macroscopic observer, modeled quantum mechanically. To keep our demonstration as simple as possible, all systems will be modeled by a single qubit, and we take Q to be the only system with non-trivial time evolution in the absence of interactions. Quantum measurements of Q by Bob will take place at $t = 2\epsilon$, but we will see that the result of this measurement will depend additionally on entanglement between Q and A, which we may choose to generate earlier at time $t = \epsilon$.

Specifically, we take Q to evolve independently as $|Q(t)\rangle = \cos(\omega t)|0\rangle + i\sin(\omega t)|1\rangle$, as is well-known to occur when the energy eigenstates of Q are $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$. The systems A and B are taken to be in the state $|0\rangle$ at $t = 0$.

In the absence of intermediate entanglement, the situation is straightforward. At time $t = 2\epsilon$ (with ϵ a small time interval compared to ω^{-1}), Bob interacts with Q by means of a CNOT gate (effectively an ideal measurement), producing the entangled state $|QB(2\epsilon)\rangle = \cos(\omega 2\epsilon)|0\rangle|0\rangle + i\sin(\omega 2\epsilon)|1\rangle|1\rangle$. At this time, the reduced density operator describing Bob is $\rho_B = \cos^2(\omega 2\epsilon)\,|0\rangle\langle 0| + \sin^2(\omega 2\epsilon)\,|1\rangle\langle 1|$, and so the probability for Bob to detect Q in the state $|1\rangle$ is thus $\sin^2(\omega 2\epsilon) \approx 4\omega^2\epsilon^2$.

Now we analyze the situation again, with the introduction of an intermediate CNOT operation at $t = \epsilon$, entangling Q with A before Bob's measurement interaction at $t = 2\epsilon$. Thus at $t = \epsilon$, we have $|QA(\epsilon)\rangle = \cos(\omega\epsilon)|0\rangle|0\rangle + i\sin(\omega\epsilon)|1\rangle|1\rangle$, while Bob remains in the state $|0\rangle$. At time $t = 2\epsilon$, though, (just before we apply the CNOT operation to entangle Bob with QA), this state has evolved to:

$$
\begin{aligned}
|QA(2\epsilon)\rangle &= \cos(\omega\epsilon)\left[\cos(\omega\epsilon)|0\rangle + i\sin(\omega\epsilon)|1\rangle\right]|0\rangle \\
&+ i\sin(\omega\epsilon)\left[\cos(\omega\epsilon)|1\rangle - i\sin(\omega\epsilon)|0\rangle\right]|1\rangle. \qquad (4.1)
\end{aligned}
$$

Now applying the CNOT between Q and B, representing Bob's measurement, and rearranging gives us the state from which we can extract predictions:

$$
\begin{aligned}
|QAB(2\epsilon)\rangle &= \cos^2(\omega\epsilon)|0\rangle|0\rangle|0\rangle + \sin^2(\omega\epsilon)|0\rangle|1\rangle|0\rangle \\
&+ i\cos(\omega\epsilon)\sin(\omega\epsilon)|1\rangle(|0\rangle + |1\rangle)|1\rangle. \qquad (4.2)
\end{aligned}
$$

Performing a partial trace over Q and A gives us the reduced density operator describing Bob alone:

$$
\rho_B = \left[\cos^4(\omega\epsilon) + \sin^4(\omega\epsilon)\right]|0\rangle\langle 0| + 2\cos^2(\omega\epsilon)\sin^2(\omega\epsilon)|1\rangle\langle 1|. \qquad (4.3)
$$

The probability that Bob finds Q to have transitioned from the initial $|0\rangle$ state to the $|1\rangle$ state at time $t = 2\epsilon$ is thus $2\cos^2(\omega\epsilon)\sin^2(\omega\epsilon) \approx 2\omega^2\epsilon^2$ — *half* of its value in the case of no intermediate entanglement. This is the quantum Zeno effect on simply evolving qubits (which can, of course, be iterated an arbitrary number of times). Note that this is *not* a result of non-unitary projections, or of altering the single particle evolution of Q — we have merely introduced an intermediate entangling CNOT operation (which is a fixed operator with no time dependence itself), and nothing else. Quantum evolution has effectively been traded for quantum information.

As a consequence of avoiding the non-unitarity inherent in the projective measurement formalism, we can identify a new effect simply by invoking time reversal symmetry. That is, we start with an entangled QA state and then *disentangle* A from Q (e.g. by applying another CNOT operation), with the end result that Q's effective evolution has been accelerated. To see this explicitly, we start with the following entangled state as a function of time:

$$|QA(t)\rangle = \alpha \left[\cos(\omega t)|0\rangle + i \sin(\omega t)|1\rangle\right]|0\rangle + \beta \left[\cos(\omega t)|1\rangle - i \sin(\omega t)|0\rangle\right]|1\rangle. \tag{4.4}$$

If Bob were to perform measurements on many copies of this system (and assuming that $|\alpha|^2 > |\beta|^2$), he would find that Q oscillates from states of maximum probability in the $|0\rangle$-state at time $0, \frac{\pi}{\omega}, \frac{2\pi}{\omega}, ...$ to states of maximum probability in the $|1\rangle$-state at times $\frac{\pi}{2\omega}, \frac{3\pi}{2\omega}, ...$ (though such probabilities never reach unity at any time).

However, let us apply a CNOT operation between Q and A before Bob makes his measurement. We apply the CNOT at time $t = 0$, which has the effect of disentangling A from Q. The now-separable state as a function of time can be described by:

$$|QA(t)\rangle = \left\{\left[\alpha \cos(\omega t) - \beta i \sin(\omega t)\right]|0\rangle + \left[\alpha i \sin(\omega t) + \beta \cos(\omega t)\right]|1\rangle\right\}|0\rangle. \tag{4.5}$$

After the disentangling CNOT, the solution continues to oscillate with frequency ω between states of maximum probability in $|0\rangle$ and states of maximum probability in $|1\rangle$, but the peaks and troughs will no longer generally occur at $t = 0, \frac{\pi}{2\omega}, \frac{\pi}{\omega}, \frac{3\pi}{2\omega}, \frac{2\pi}{\omega}, ...$ In the case that $\alpha = \cos(\theta)$ and $\beta = -i \sin(\theta)$ for any small value of θ, we have exactly the time-reversal of the Zeno effect shown above, and we find that evolution has been advanced precisely by an amount $\delta t = \frac{\theta}{\omega}$, such that we can express the solution:

$$|Q(t)\rangle = \cos(\omega(t + \delta t))|0\rangle + i \sin(\omega(t + \delta t))|1\rangle \tag{4.6}$$

Notice that this works also for negative values of θ, in which case we have effectively time-reversed the *anti*-Zeno effect (i.e. we have effectively slowed the evolution by disentanglement).

Thus we see that through measurement-like interactions (simple CNOT operations in our analysis, which are ubiquitous in quantum computing, and routinely implemented in the context of laser-trapped atoms [44]), the quantum Zeno effect essentially represents a trade of evolution for entanglement. But in the absence of projective collapse in the formalism, the logic can be reversed and we can also trade entanglement for evolution.

Chapter 5

Discussion: General Features of the CQI Framework

The CQI framework which we have constructed sits at the intersection of covariant quantum mechanics (and by extension quantum gravity/cosmology), quantum information, and quantum measurement, and it bears directly on the quantum mechanical arrow of time, and realism in quantum theory. Here, we recapitulate the main features of the formalism (except for the section on realism, which has some additional detail), point out possible future directions of research, and draw our conclusions.

5.1 Covariance

By construction, our formalism is compatible with generally covariant physics — that is, the notions of state, observable, and evolution are compatable with generally relativistic physics (though we are free to apply them to simple systems as well). The theory is formulated directly on the space of physically accessible quantities $\mathcal{M}$ (the points of which correspond to the "spacetime coincidences" described by Einstein in the context of classical GR). No parameterization of the states on $\mathcal{M}$ is pre-selected to play any special role to formulate the theory or to obtain predictions from it (though it is clear that in many cases the dynamics under investigation may make one choice of coordinates far more *convenient* than another, as is the case with single-particle Schrödinger dynamics). Although we have exclusively treated simple systems as examples, and generally covariant field theories (such as quantum gravity) will typically be much more difficult to control and compute with, the fundamental mechanical framework and interpretation of probabilities is identical.

5.2 Correspondence

Most importantly, the theory reproduces our known physics exactly, in the context of simple, single-particle models. Due to the fact that the covariant partial trace reduces immediately to the standard definition when the region of interest S corresponds to a constant time slice of space, the formalism of Cerf and Adami is guaranteed to emerge in the limit of standard Schrödinger equation dynamics and the usual (though unphysical) assumption of arbitrarily good clocks. The original objections to the RR picture of covariant quantum theory based on its failure to correspond with standard QM predictions are completely resolved in the context of CQI.

5.3 Unitarity

Because there is no collapse or reduction of the full state vector upon observation, and it instead treats measurement as entanglement with the observer, all evolution in the theory is unitary. Additionally, no uncontrollable environment is required to be invoked in order to induce decoherence and classical probabilities. This means that time reversal is a good symmetry, even in the context of measurement, which suggests a new class of measurement-related phenomena (e.g. the time-reversed Zeno effect).

Also, the requirements for the covariant partial trace to be defined in a particular region $\mathcal{S}$ may explain apparent non-unitarity which arises in certain quantum gravity-related models [35]. If a given t-coordinate is chosen poorly on $\mathcal{M}$, such that some $t =$ constant slices (each corresponding to a seperate region $\mathcal{S}$) do not support a $\mathcal{K}$-state which projects to the full physical state in $\mathcal{H}$, there is no reason to expect "time evolution" in this time coordinate to appear unitary — it will not make sense to talk about "conservation of probability" on such slices. Nevertheless, the theory may remain consistent and well-defined.

5.4 Realism

Although the aim of covariant quantum information is primarily to remove ambiguities and to obtain correspondence with the well-established predictions of standard quantum mechanics in the appropriate limit, it also inherits a realist picture from the Cerf-Adami description of measurement [45]. It is remarkable that a well-known interpretational issue in the foundations of quantum mechanics is forced to the surface by our desire to express the concept of measurement covariantly — and even more remarkable, that a realist interpretation of the quantum state function Ψ should naturally emerge in the process.

Our definition of quantum realism is the following: We describe a quantum measurement formalism as *realist* if at each individual time (or in the covariant language, each sufficiently large region of $\mathcal{M}$), all observers can correctly obtain all physical predictions about a quantum system from the same state function Ψ. This definition allows for the possibility that the universe is described by a single quantum state in $\mathcal{H}$, independent of our knowledge of it. It is straightforward to show that the standard description of projective measurement is not a realist formalism, according to this definition [1].

The prototype scenario to demonstrate this has been called "the observer observed" by Rovelli [8, 46]. Here we start with a quantum system Q in a superposition state $|Q\rangle = \alpha|0\rangle + \beta|1\rangle$, and two observers A (Alice) and B (Bob). At time t_1, Bob performs a measurement of Q, in the $|0\rangle, |1\rangle$ basis.

We now ask the question, "what is the state of the system?" As described by Bob (using the standard formalism of projective measurement), the state of Q has been collapsed and will either be described by $|0\rangle$ or $|1\rangle$, depending on the outcome of his measurement. Alice, on the other hand, describes the system differently — no collapse has taken place, but the system has grown into an entangled state that encompasses Bob as well as Q, $|QB\rangle = \alpha|0\rangle|\text{Bob sees }0\rangle + \beta|1\rangle|\text{Bob sees }1\rangle$.

[1] Note that this conception of realism is distinct from that of "Einstein realism" or "local realism" which appear to be fundamentally at odds with observation, since the experimental violation of Bell's inequalities. Here, we are primarily interested in whether or not there exists a quantum description of reality with which all observers can simultaneously agree and obtain ideal predictive power.

This necessary disagreement on the state used to describe the system Q has led many thinkers to the conclusion that quantum states have no independent meaning outside of their use by a particular observer [47, 46]. According to this school of thought, quantum mechanical states are to be thought of a kind of book-keeping device which may be different for each separate observer — a clear retreat from a realist conception of the state function. And indeed, this is a very reasonable conclusion if one takes the process of measurement to physically correspond to a projection onto states of Q. However, we need not do this. Remarkably, the principle that takes us out of this picture is precisely the same principle we used to remove the time ordering ambiguity for generally covariant quantum mechanics, and to obtain correspondence with the Born rule. *Predictions must be obtained from a quantum model of the observer.*

When we do this, we find that we have a new description of the observer observed. Both Alice and Bob can agree that the initial state (a $\mathcal{K}$-state on hypersurface $t = t_0$) of the system is:

$$|QAB\rangle = (\alpha|0\rangle + \beta|1\rangle)|\text{Alice ready}\rangle|\text{Bob ready}\rangle. \tag{5.1}$$

On hypersurface $t = t_1$, after Bob performs his measurement, the system is described by:

$$|QAB\rangle = (\alpha|0\rangle|\text{Bob sees }0\rangle + \beta|1\rangle|\text{Bob sees }1\rangle)|\text{Alice ready}\rangle. \tag{5.2}$$

Predictions are obtained by analyzing Alice and Bob. On this $t = t_1$ slice, we have for Alice that

$$\rho_A = |\text{Alice ready}\rangle\langle\text{Alice ready}| \tag{5.3}$$

indicating that no outcome observed by Alice is yet probabilistic. On the other hand, on the same hypersurface:

$$\rho_B = |\alpha|^2 |\text{Bob sees } 0\rangle\langle\text{Bob sees } 0| + |\beta|^2 |\text{Bob sees } 1\rangle\langle\text{Bob sees } 1| \tag{5.4}$$

(the unique diagonal, orthonormal representation of ρ_B) indicates that the outcome of Bob's experiment will be $|0\rangle$ or $|1\rangle$ with probabilities $|\alpha|^2$ and $|\beta|^2$, respectively.

Now suppose that at $t = t_2$, Alice discusses with Bob the results of his experiment or she performs her own measurement on Q. The state on this hypersurface is:

$$|QAB\rangle = \alpha|0\rangle|\text{Bob sees } 0\rangle|\text{Alice sees } 0\rangle + \beta|1\rangle|\text{Bob sees } 1\rangle|\text{Alice sees } 1\rangle. \tag{5.5}$$

Like Bob, Alice will see outcomes $|0\rangle$ and $|1\rangle$ with respective probabilities $|\alpha|^2$ and $|\beta|^2$, but importantly, the *conditional entropy* between Alice and Bob on this hypersurface is zero: $S(A|B) = S(AB) - S(B) = 0$ — i.e. the outcome of Bob's experiment (which was performed at t_1) completely determines the state of Alice at time t_2.

Thus, all predictions (including those usually ascribed to a collapse) are obtained from a *single* physical state in $\mathcal{H}$, analyzed on different regions (in this case, constant time slices) of $\mathcal{M}$. At no point were Alice and Bob forced to disagree on the physical state describing their situation, yet they are both able to make all of the standard predictions about what they will see. Surprisingly, in the pursuit of consistently incorporating general covariance into a formalism of quantum measurement, we have been led directly into a working realist interpretation of quantum theory. This paints an intuitive picture of quantum reality — that a single physical state Ψ describes the universe; the past, the present, and the future. We may not have complete knowledge of Ψ (and in fact observers may typically disagree about the state of the system simply due to lack of information, just as they do in classical mechanics), but all probabilistic predictions for all observers at all times are ultimately, in principle, obtainable from it.

5.5 Self-Contained

Before we conclude, we would like to emphasize one more feature of covariant quantum information. That is, it is completely self-contained. By this, we mean that the theory does not require an external, classical observer in order to properly interpret probabilities. In fact, the theory requires that we avoid such constructs — probabilistic predictions are not made in CQI unless the observer is included as part of the full description of the system.

This seems to suggest a natural application with quantum cosmology, where the meaning of quantum probabilities is controversial for exactly this reason — to whom do probabilities apply? We would like to suggest that some simple cosmological models (e.g. the simplest minisuperspace cosmologies) simply do not have a consistent probabilistic interpretation, because they do not include enough degrees of freedom to model a quantum observer, and do not model an entangling interaction required to define a measurement. We defer a detailed analysis of this proposal to future work, but we note that it seems to mesh perfectly with historic insights into the character of quantum gravity and quantum cosmology. In particular, DeWitt has noted already in 1967 that quantum gravity appears to have a surprisingly economical character, in the sense that it seems to say very little about time, geometry, etc. unless some kind of physical measuring system is introduced directly into the theory to measure time or geometry [48].

Our proposal merely takes this insight a step further — a fully consistent, generally covariant quantum theory says nothing about *probabilities* unless a quantum subsystem (playing the role of the observer) is introduced to become entangled with the degrees of freedom being studied. Once this is done, however, interpretational problems go away — the probabilities are *not* the probability that the entire universe will collapse to some specific state. The probabilities simply refer to the mixed state of the observer alone, which is in principle the best description of the observer quantum theory can offer.

5.6 Conclusions

The picture painted by covariant quantum information is most remarkable in the way it ties together seemingly unrelated issues in a single, coherent framework. Who would have expected that making a quantum measurement formalism compatible with general covariance would immediately result in a formalism that is also automatically realist, unitary, local, and which defines a quantum mechanical arrow of time independent of any pre-existing causal structure or time coordinate? Furthermore, the theory suggests a new class of readily measurable quantum mechanical effects which are obtainable simply by invoking time-reversal symmetry in the presence of standard, measurement-related phenomena (the first example of these being the time-reversed quantum Zeno effect).

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Vita

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- J. C. Baez, S. J. Olson, *Uncertainty in measurements of distance.*
  Class. Quant. Grav. 19:L121-L126, 2002
- R. Gambini, S. J. Olson, J. Pullin, *Unified model of loop quantum gravity and matter.*
  Gen. Rel. Grav. 38:593-598, 2006
- S. J. Olson, J. P. Dowling, *Information and measurement in generally covariant quantum theory.*
  `arXiv:quant-ph/0701200v3`
- M. Han, S. J. Olson, J. P. Dowling, *Generating entangled photons from the vacuum by accelerated measurements: quantum information theory meets the Unruh-Davies effect.*
  `arXiv:0705.1350v3`
- S. J. Olson, J. P. Dowling, *Probability, unitarity, and realism in generally covariant quantum information.*
  `arXiv:0708.3535v2`, submitted to Class. Quant. Grav.