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Manufacturing Systems Line Balancing using Max-Plus Algebra

By

Ali Fakhri

A Thesis

Submitted to the Faculty of Graduate Studies

through the Industrial Engineering Graduate Program

in Partial Fulfillment of the Requirements for

the Degree of Master of Applied Science

at the University of Windsor

Windsor, Ontario, Canada

@2018 Ali Fakhri

Manufacturing Systems Line Balancing using Max-Plus Algebra

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ABSTRACT

In today's dynamic environment, particularly the manufacturing sector, the necessity of being agile, and flexible is far greater than before. Decision makers should be equipped with effective tools, methods, and information to respond to the market's rapid changes. Modelling a manufacturing system provides unique insight into its behavior and allows simulating all crucial elements that have a role in the system performance.

Max-Plus Algebra is a mathematical tool that can model a Discrete Event Dynamic System in the form of linear equations. Whereas Max-Plus Algebra was introduced after the 1980s, the number of studies regarding this tool and its applications is fewer than regarding Petri Nets, Automata, Markov process, Discrete Even Simulation and Queuing models. Consequently, Max-Plus Algebra needs to be applied and tested in many systems in order to explore hidden aspects of its function and capabilities.

To work effectively; the production/assembly line should be balanced. Line balancing is one of the manufacturing functions that tries to divide work equally across the production flow. Car Headlight Manufacturing Line as a Discrete Manufacturing System is considered which is a combination of manufacturing and assembly lines composed of different stations. Seven system scenarios were modeled and analyzed using Max-Plus to balance the car headlights production line. Key Performance Indicators (KPIs) are used to compare the various scenarios including Cycle Time, Average Deliver Rate, Total Processing Lead Time, Stations' Utilization Rate, Idle Time, Efficiency, and Financial Analysis. FlexSim simulation software is used to validate the Max-Plus models results and its advantages and drawbacks compared with Max-Plus Algebra.

This study is a unique application of Max-Plus Algebra in line balancing of a manufacturing system. Moreover, the problem size of the considered model is at least twice (12 stations) that of previous studies. In the matter of complexity, seven different scenarios are developed through the combination of parallel stations and buffers. Due to that the last scenario is included four parallel stations plus two buffers

Based on the findings, the superiority of scenario 7 compared to other scenarios is proved due to its lowest system delivering first output time (14 seconds), best average delivery rate (24.5 seconds), shortest cycle time (736 seconds), shortest total processing lead time (11,534 seconds), least percentage of idle time (12%), lowest unit cost ($6.9), and highest efficiency (88%). However, Scenario 4 has the best utilization rate at 75%.

DEDICATION

I would like to express my deep gratitude to my wife who helped me and supported my effort in writing this thesis.

ACKNOWLEDGMENTS

Before all, praise goes to God for giving me the strength and knowledge to complete this work. The completion of this dissertation would not have been possible without the support and help of many people, to whom I would like to express my gratitude. On top of the list comes Dr. Hoda ElMaraghy, my supervisor, who provided me with continuous and invaluable support, guidance, and mentorship. I would also like to express my appreciation to the Ph.D. committee members: Dr. Richard Caron and Dr. Waguih ElMaraghy for their valuable comments and suggestions that vastly improved the quality of this thesis. Special thanks also go to my colleagues and mentors at the Intelligent Manufacturing Systems (IMS) Centre at the University of Windsor.

I am also very grateful to my parents who always pushed me forward and supported me, and to my wife who enlightened my life.

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LIST OF ABBREVIATIONS AND NOTATION

This research attempts to use common symbols, signs and abbreviations. A short explanation appears beside all notations.

Symbol	Explanation
DES	Discrete Event System
DEDS	Discrete Event Dynamic System
MPL	Max-Plus Linear
MILP	Mixed Integer Linear Programming
KPIs	Key Performance Indexes
ELCP	Extended Linear Complementary Problem
TEG	Timed Event Graphs
SMPL	Switching Max-Plus Linear
FIFO	First Input, First Output
WIP	Work In Process
CT	Cycle Time
CP	Critical Path
SDR	System Delivery Rate
PLT	Processing Lead Time
UR	Utilization Rate (Stations)
AUR	Average Utilization Rate
ADR	Average Delivery Rate
UC	Unit Cost
LC	Labor Cost
TMC	The Total Equipment Cost
MC	The Cost Per Equipment
LW	Labor Wage Per Hour
GL	Graphic Language
UV	Ultraviolet
HVAC	Heating Ventilation and Air Conditioning
LED	Light-Emitting Diode
PCR	Poly Carbonate Resin

QI	Quality Inspection
CONWIP	Constant Work in Progress
DBR	Drum-Buffer-Rope
MMALs	Mixed-Model Assembly Lines
UR_T	Total Utilization Rate
IDT_i	Idle Time of Station *i*
UR_i	Utilization Rate of station *i*
MPC	Model Predictive Control
X	a set of variables
t_i	The Process Time of Station i
k	The Total Number of Parts
n	The Total Number of Stations
$x_i(k)$	The time instants at which station i start to process part k
$u_i(k)$	The time instants at which incoming part k is fed to the station i
$y_i(k)$	Stands for the time instants at which part k is completed product.
$\mathbb{R}$	The set of reals
$\mathbb{R}_{max}$	The set of $\mathbb{R} \cup \{-\infty\}$
$\oplus$	Maximization
$\otimes$	Addition
$\mathcal{E}$	Minus infinity $(-\infty)$
e	Zero
A^*	Kleene star

CHAPTER ONE
INTRODUCTION

1.1.Motivations

To be sustainable in today's global environment, industries have become more competitive and must minimize their costs. The manufacturing sector is under intense pressure not only to add more variety to its products but also to improve its systems and operations to achieve increased productivity, customer responsiveness, quality, and to minimize manufacturing costs. To manage this situation, new methods have been developed to model, analyze, and control complex manufacturing systems.

A manufacturing system is defined as a method to make a product (output) by considering the interaction of several factors, such as cost, time, equipment, operations, and material (input). Line balancing is a strategy to make production lines operation smooth and flexible; it involves planning a set of operations or designing procedures to fabricate an output in a designated timeframe using the available capacities. In general, mathematical modelling of a manufacturing system is based on physical laws that govern its behaviour.

Max-Plus Algebra as a mathematical tool that has begun to receive heightened attention in the field of manufacturing systems modelling. It is composed of a set of linear equations used to express the event timing dynamics of any deterministic manufacturing system. Therefore, our aim is to model line balancing of manufacturing systems using an efficient mathematical tool like Max-Plus Algebra to simplify and develop the modelling phase and apply the output of the model in the analysis phase.

1.2.System Attributes

A system is defined as an aggregation of objects, either through regular interactions or independently, that perform a function. To analyze a system quantitatively, a set of mathematical means need to be defined or developed. A model is defined as a tool that approximates the behaviour of a system. The processes in a simple system described by Cassandras and Lafortune (2010) is shown in Figure 1.1.

The state of a system is defined as a set of variables X (computable) used to describe the system's conditions throughout the formulation. The modelling process is composed of a set of mathematical relationships including input $u(t)$, the state $x(t)$, and output $y(t)$. The systems can be classified using different attributes (Cassandras & Lafortune, 2010).

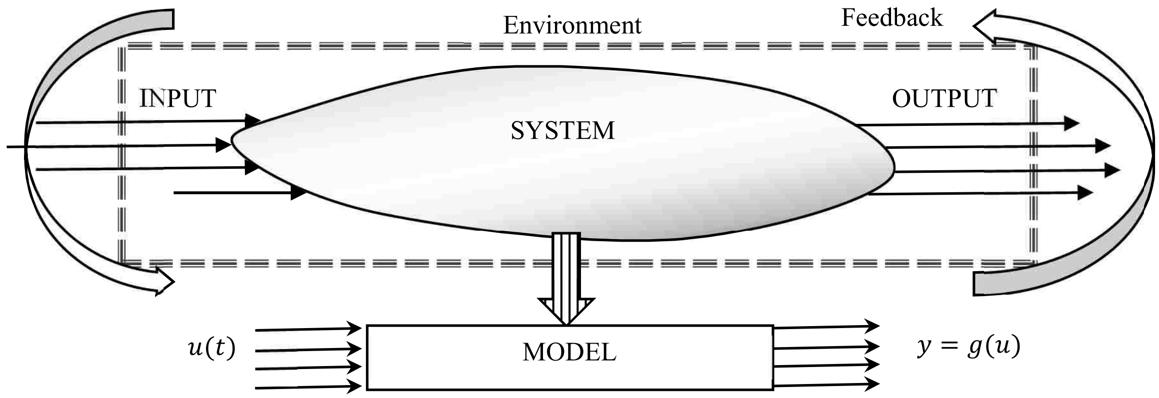


Figure 1.1 A General Modelling Process Structure (Cassandras & Lafortune, 2010)

A system is called dynamic if the output of the system, $y(t)$, is time-dependent. In this case, the system's output $y(t)$ is dependent on the past values of input, $u(t)$. In contrast, a static system is a system in which its output is absolute and non-aligned with the past values of the input.

If the behaviour of the system changes over time, the system is classified as Timed (i.e. Time-Variant), otherwise, Untimed (i.e. Time-Invariant). More precisely, if the same input results in different output during the time, the system is called Time-Variant. Back to Figure 1.1, in Timed system $g(.)$ is dependent on the variable t and is represented as $g(u, t)$.

Based on the nature of this mathematical relationship, a system is classified as either Linear or Nonlinear. If the model behaviour satisfies the additivity, homogeneity, and superposition properties, then the system is called Linear; otherwise, it is Nonlinear.

If a system includes random variables, it is called a Stochastic System. This type of system is defined as a random process in which behaviour changes probabilistically. Consider as an example a dam, the input is rainfall, but building a model of when and how much rain will fall is not feasible. Otherwise, a system is classified as a Deterministic System if the set of input is known and results in a unique output.

The nature of the state space classifies systems differently as Continuous and Discrete. In Discrete systems, the state variables change at a discrete set of points in time. For instance, the state variables such as the number of pieces of equipment in the production line change only when the line faces bottlenecks and requires balancing. In contrast, if the states variables change continuously over time, the system is called Continuous. In the study of water stored behind the dam, the water level is a state variable that is changing continuously over time. The water level rises after a rain and goes down due to evaporations and dam control.

In addition to above-explained conditions and behaviour of the system state's variables, other characters are considered in the literature such as size, scale, logic, language, rules, feedback, stability, function, etc.

1.3.Classifications of Discrete Event System (DES)

Based on the system characteristics, different classifications have been presented in the literature. Discrete Event System (DES) is one of the most common systems that have been studied in academics. Discrete event dynamic system (DEDS) consider events and includes the evolution of the system over the time which is strong to cover many aspects of reality. Discrete Event Dynamic Systems (DEDSs) such as Flexible Manufacturing System (FMS), logistic systems, and traffic control systems are characterized by a set of states X and a set of events E. The set of events cause DEDS to change its state at discrete time instances (Hruz & Zhou, 2007).

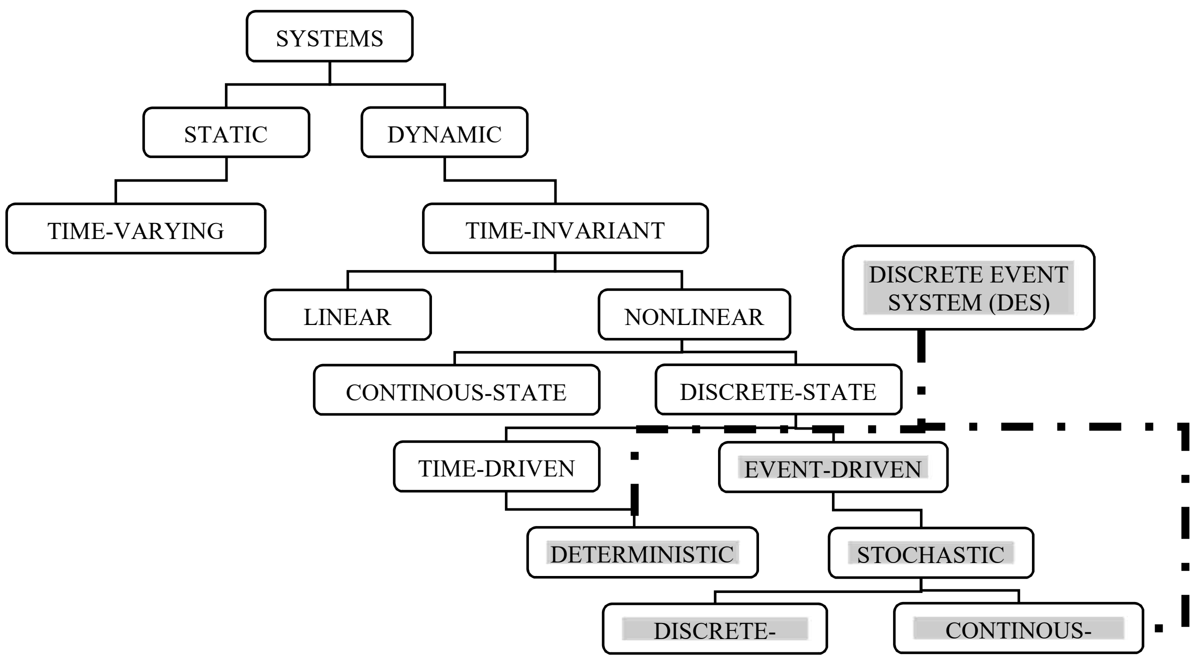


Figure 1.2 Major Systems Classification (Cassanderas & Lafortune, 2010)

There are two types of DEDS: timed and untimed. In untimed system models, the system's evolution is merely viewed as a sequence of states, while in timed models, a sequence of states is assigned to the time instances at which states' transitions take place (Cassandras & Lafortune, 2007). Different modelling methods are used for DEDS analysis such as Petri nets, generalized semi Markov processes, nonlinear programming, automata, computer simulation models and so on. It should be noted that the models used to describe DEDS are nonlinear in the conventional algebra. Recently, a class of DEDS called Max-Plus Linear has been defined (Seybold et al., 2015).

1.4.Discrete Event Systems Modeling Methods (Formal Methods)

DES has been applied to several fields of science and engineering. These fields of research apply different terminologies for DES methods. For example, in the field of industrial engineering, they are known as modelling methods. However, these methods are known as mathematical tools and formal methods in the fields of mathematics and computer science, respectively. Campos et al. (2014) define formal methods as mathematical techniques, often supported by tools, for developing "man-made systems". These methods include Petri nets, Automata and Max-Plus Algebra. In another study, Li and Al-Ahmari (2013) call formal methods major mathematical tools for system development. They classify Automata, State Charts, Petri nets, Graph Theory, Process Algebra, Queueing Networks, and Temporal Logic as formal methods.
Although these methods are known as modelling methods in the fields of electrical, automation and control engineering, there are some other differences in the subdivision of terminologies. For instance, node definition in the control field is similar to merging point, and the rout is the sequence of series stations in the field of industrial engineering. Cassanderas and Lafortune (2010), Heidergott et al. (2006), and Baccelli et al. (1992) provide more information. Some of the most important and famous modelling methods in DES are considered below.

1.4.1. Petri Nets

Petri nets models were developed by C.A Petri in the early 1960s. Petri Net is a weighted bipartite graph (P, T, A, w) where P, T, A and w are a set of places, transitions, arcs, and weighted functions on the arcs respectively. Transitions represent events that may occur, places are the input of the transitions, and finally, arcs define which places are preconditions or post conditions for each transaction. Arcs never connect two places or transactions together. Input and output places of a transition are places from which and to which an arc runs to a transition. Places might possess a discrete number of marks called token, which are fired by transitions as an input and turn to tokens in output places (Cassandras & Lafortune, 2010).

The Petri Net graph is represented in Figure 1.3, then the Petri Net is specified as;

$P = \{p_1, p_2, p_3\}$ $\quad\quad\quad T = \{t_1, t_2, t_3\}$

$A = \{(p_1, t_1), (p_3, t_2), (p_3, t_3), (p_2, t_3)\}$

$w(p_1, t_1) = 1,\ \ w(p_3, t_2) = 1,\ \ w(p_2, t_3) = 1,\ \ w(t_1, p_1) = 1,\ \ w(t_3, p_1) = 1,\ \ w(t_2, p_2) = 1,$ $w(p_3, t_3) = 1, w(t_1, P_3) = 1,$

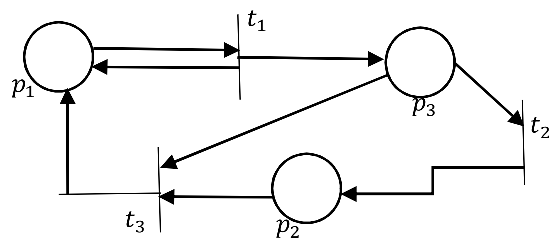


Figure 1.3 Simple Petri Net Model

The two main drawbacks of modelling DEDS with Petri Nets are: 1) graphic representation of complex systems is quite intricate, and 2) Indicating priorities and order is important; however, it is hard to manage when the system is complex.

1.4.2. Automata

DES is comprised of a set of states (X) and events (E) that cause transactions of these states. The event sequences describe the behaviour of a system and the order in which the events arise. However, the time of events' occurrence is unspecified, which associates it to an untimed and logical level of DES abstraction. This kind of DES is modelled by a *language*. If an even E is assumed as the *alphabet*, the sequence of events can configure *strings (words)*. Finally, a language is a set of those strings (Cassandras & Lafortune, 2010).

Automata is a tool that represents a language based on the comprehensible rules. It is comprised of an even set E, X, transition functions, in the initial state X_0 and marked state X_m.

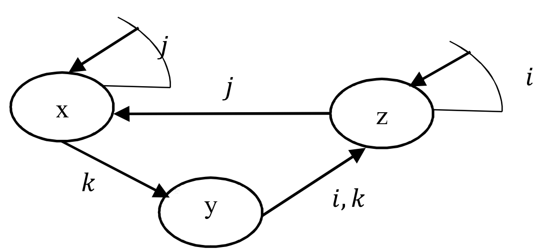


Figure 1.4 Simple Example of Automata Model

Assume $E = \{i, j, k\}$, then the transaction function of $f: X \times E \rightarrow X$ is represented graphically by arcs:

$$f(x,j) = x, \quad f(x,k) = y, \quad f(z,j) = x\ , \quad f(z,i) = i, \quad f(y,i) = f(z,k) = z$$

The notation $f(x,k) = y$ means that if the automaton is in state x, then upon of event k occurrence, the automaton will make a transition to state y .

The main drawback of this way of modelling DES is space explosion, which occurs when a large number of automata are composed. A detailed description of automata can be found in Cassandras and Lafortune (2010) chapter two.

1.4.3. Markov Process

Suppose a sequence of possible events $x_0, x_1, x_2, \ldots, x_k$ observed at times $t_0, t_1, t_2, \ldots, t_k$. Let x_k be the present state of the process at t_k and past history as $\{x_0, x_1, x_2, \ldots, x_{k-1}\}$; then the future states $\{x_{k+1}, x_{k+2}, x_{k+3}, \ldots\}$ is completely independent of the past. Each event relies on the previous event. This is referred to as the memoryless property of the Markove Process. Markov Chain is a stochastic model of the Markove Process in which the future is conditionally independent of past events. In other words, the past and present information is summarized in the present state to probabilistically attain the future. There are two types of Markove Chains called Discrete-Time and Continuous-Time. In the Discrete-Time Markove Chain, the events happen at time instances; however, in Continuous-Time the state transitions occur at time intervals.

1.4.4. Queuing Model

A basic queuing model includes customers who arrive from time to time to the buffers (queues) to get served and eventually leave the system. A queuing model includes three main components: 1) Arrival and service processes (the probability distribution of events), 2) System Capacity such as the capacity of buffers, number of servers, and 3) operating policies and the conditions under which the customers are served, for example, can a customer leave the system before he or she gets served? How many customers get served by each server?

1.4.5. Discrete Events Simulation

Simulation is an imitation of a real-world process/system over time by generating the history of a system. The system behaviour is studied by developing a simulation model. In some cases, the developed model can be solved using mathematical methods, differential calculus, algebraic methods etc. However, in reality, some systems are too complex to yield analytical methods. In these cases, simulation can be used to emulate the behaviour of the system over the time.

1.5.Max- Plus Algebra

Max-Plus Algebra is a new mathematical tool that has been developed in theoretical and practical subjects. For example, Max-Plus has been used in theories such as Set theories, Graph theories, Queuing theories and Computational theories. More details about Max-Plus Algebra can be found in Chapter 2. The relevant literature is expressed in three groups as follows.

Since 1996, De Schutter and his research group they have published 46 articles and books in the field of Max-Plus Algebra and discrete event system such as linear systems modeling (2014), computational techniques (2015), approximation approaches (2011), Model Predictive Control (2001), and practical studies in traffic, transportation, automation and control systems issues (2016).

Gaubert. S (1997) has classified tropical semirings and the family of Max-Plus. He introduced basic techniques to solve Max-Plus linear equation. This is followed by studies in the field of graph theory as well as language theory. Later, Marianne et al. (2001) have declared Convex map that is monotone (preserve the product ordering) and no expansive. They presented the fixed point set when it is nonempty is isomorphic and its dimension strongly connected to components of a critical graph.

Cohen et al. (1998) have summarized the theoretical aspects of Max-Plus achievements through examples. The state space equations, canonical equations, transfer functions, asymptotic behaviour and eigenvalues, stabilization and resource optimization, geometric theories, etc. were discussed as some part of Max-Plus dependent contents.

Goto et al. between 2002 and 2017, have published 19 articles. Their focus is to reduce the number of equations and iterations to make Max-Plus Algebra easier to use. In 2009, Goto and Kasahara proposed an efficient computation method for calculation of Kleene Star (more details are included in section 2.3. Consequently, a High-Speed algorithm is suggested by Goto and Ichige, (2010) to speed up the effect of calculating the Kleene Star for a weighted adjacency matrix. Moreover, they have studied applications of Max-Plus Algebra in other fields such as integer programming (2017), feedback control approach (2010), output constraints (2009), and linear systems (2007).

Adzkiya et al. (2016) have studied Max-Plus Linear (MLP) systems calculation tools. The proposed procedure is based on partitioning the state space and dynamics into regions. The results present finite-state transition systems that either simulate or bi-simulate the original MPL system. Bi-simulation equivalence aims to identify transition systems with the same branching structure, and which thus can simulate each other in a stepwise manner (Baier and Katoen, 2008).

Two optimization problems in Max-Plus Algebra related to the minimization of a product of triangular matrices have been defined by Bouquard et al. (2006). The first problem is a polynomial time optimization algorithm for scheduling a single or two-machine flow shop problem in 2×2 matrices products. The second problem considers the 3×3 matrices, which are shown as NP-hard. They have used a branch-and-bound algorithm to solve it.

Hardouin et al. (2017) have designed an observer-based controller for Max-Plus Linear systems solved in two steps: first, an observer computes the state estimation by using input and output measurements; afterwards these estimates are used to compute the state-feedback control system. This method provides better control than output feedback control, which was common.

Leela-Apiradee et al. (2017) have introduced a closed form of the L-localized solution set of Max-Plus interval linear system and its application to the optimization problem. The feasible set of an optimization problem has been simplified by the number of union set reduction.

Imaev and Judd (2008, 2009) have simplified and reduced the Max-Plus equations. The network connection of blocks was transferred to compose one block that has the same input and output structure. This means, a block defined as a combination of several stations instead of one station. In a block diagram the interconnection of a station with other stations were specified using routing matrices. Furthermore, a new topological method for evaluating the results of the synchronous matrix have been examined (2010). Properties of signal flow graphs (SFGs) over Max-Plus Algebra are investigated, which are referred to as synchronous. Finally, a theory of synchronous a matrix signal flow graph (MSFG), as a pictorial presentation of a set of linear matrix equations, has been addressed in their studies. After proving the hypothesis, a single machine or a factory was used as a sample of the block diagram.

Adzkiya et al. (2016) have developed a software tool called VeriSIMPL2 for Max-Plus Linear (MPL) systems. The proposed procedure is based on partitioning the state space and dynamics into partitioning regions. This open-source software generates finite abstractions of autonomous MPL systems and enables formalization of general properties to be logical. The results present finite-state transition systems that either simulate or bisimulate the original MPL system.

Dealing with stochastic systems have enforced researchers to define uncertain conditions. Using statistical distribution likewise Poisson or Binomial are common in queueing network. Kaise and McEneaney (2015) have applied idempotent methods included Max-Plus basis-expansion approaches as well as curse-of-dimensionality-free methods to stochastic control and games. Curse of dimensionality refers to nonlinear modelling because in this modelling the computational cost and space dimension expanded exponentially. To conquer uncertainty using intervals instead of deterministic variables is extended among Max-Plus studies. In one of the latest studies, Wang and Tao (2016) have proved existence and uniqueness of a strong solution through a polynomial algorithm for such Max-Plus Linear equations. Other than that, Imaev and Judd (2010), Leela-Apiradee et al. (2017), Seybold et al. (2015) and Majdzik et al. (2016) have worked in the domain of stochastic system using Max-Plus. Table 1-1 provides a summary of Max-Plus Algebra theories and relevant research.

Table 1-1 Max-Plus Algebra developed theories

Subjects	References
Systems (Linear, Dynamical, and Stochastic)	Van den Boom and De Schutter (2008); Goncalves et al. (2017); Soudjani et al. (2016); Spacek et al. (1995)
Theories and proofs (Set, Graph, Queuing, and Computational)	Singh et al. (2012); Judd et al. (2011); Goto (2014); Gaubert et al. (2007)
Mathematical Equation and Eigen Function (Operators, Vectors, Eigenvalues, State Space and Partial Differential)	Goto et al. (2017); Baccelli et al. (2016); Shinzawa et al. (2016); Gavalec et al. (2009); Cohen et al. (1998)
Algorithms and Computation (Numerical Methods, Problem Solving, Simulation, and Programming)	Adzkiya et al. (2016); McEneaney (2007); (Leela-Apiradee et al., 2017); Fahim et al. (2017); Imaev and Judd (2008,2009, and 2010)

In this part, the application of Max-Plus Algebra in the manufacturing system is discussed.

Febbraro et al. (1994) have solved scheduling problems by applying Max-Plus Algebra in the manufacturing system. It is assumed that a nominal 'semi-schedule' is available which have composed sequences to be executed by the various machines are fixed. They proved that computational tools coming from Max-Plus Algebra provide an effective way to write the performance indexes in terms of the decision variables representing the binary alternatives.

In addition to developing mathematical theories, Imaev and Judd (2008 & 2009) have developed several modelling approaches in manufacturing systems using Max-Plus Algebra. At first they proposed hierarchical modelling for any deterministic manufacturing system. Then Block Diagram-Based modelling is suggested. A manufacturing system constitutes a network of processing elements. A block with two inputs and two outputs expresses one of the processing elements. A block is defined as a single manufacturing operation, a single machine, a single part, or a factory. Combination of two or more blocks which have same input/output structure, can be seen as a hierarchical model. In such way a huge and complex manufacturing system can be broken into smaller ones (sub-system).

Beyond these studies Imaev and Judd (2010) have defined a block as a 3×3 matrix. Moreover, a class of signal flow graphs (SFGs) was introduced called matrix signal flow graphs (MSFGs). They have called a SFG over Max-Plus Algebra a synchronous SFG, because maximization operation represents synchronization phenomena in discrete event systems. Three topological methods: 1) basic graph reduction rules, 2) return loop method, and 3) basic manufacturing blocks; are applied to evaluate the results of synchronous. Also, machine-based and part-based modelling approaches have been introduced and compared. The part-based approach is preferred for scheduling application; however, machine-based is suggested for buffer allocation. Both approaches give the same result for makespan.

Seleim and ElMaraghy (2014a) developed a method to model manufacturing flow lines using Max-Plus equations. The generated equations can be applied to any size and structure of flow line by considering finite buffers and parallel identical stations. This method is based on the observation that a flow line can be decomposed into different additive features. They suggest that the presented method automatically generates Max-Plus equations and that the use of Max-Plus Algebra can be extended to the large-scale problems. A backflushing control valve is used as a case study where Max-Plus equations are used to model three assembly line configurations. Configurations are composed of five to seven stations and two different number of buffers. To verify the Max-Plus equations, the results have been compared with discrete event simulation and were identical with simulation while the processing time of all stations was deterministic. Compared to discrete event simulation, the developed model ran quicker. The equations have been generated in a few minutes and then using these equations, data analysis only took a few seconds. However, generating and executing discrete event simulation models requires many runs and takes days.

In another study, Seleim and ElMaraghy (2014b) have modelled Mixed-Model Assembly Lines (MMALs) with both closed and open stations by using Max-Plus Algebra. For verification, two numerical examples have been presented. In the first example, an assembly line containing four stations with three different outputs were considered. Three possible product mix variants sequences have been compared and the optimal point for each sequence was founded. The second example studied the effect of changing launching rate of work units of line performance. Their parametric analysis as a method has provided a better understanding for designers. It allows designers to analyze stations sensitivity to the changes in assembly line and their effects on idle time and line length. In addition, decision makers can use the presented analyses to assess whether the improvements can affect the current sequence optimality and if it is needed to increase line capacity by changing launching rate.

Seybold et al. (2015) have introduced a predictive control framework to fault-tolerant control and used Max-Plus Algebra to model battery assembly system. This system is included five stations and two input. The robustness issues which are inevitable in real production systems have been discussed. They show an illustrative example which clearly exhibits the high performance of the proposed approach while all productivity demands are incorporated within the constraints.

Afterward, Majdzik et al. (2016) have represented a framework for fault-tolerant of a battery assembly line (9 stations and considering transportation time). The proposed approach is based on an interval analysis approach, which along with Max-Plus Algebra allows describing uncertain discrete event system such as production line. As a result, the performance of the pilot implementation has validated the recommended strategy for advanced battery assembly system. The recommended approach examined single as well as simultaneous fault concerning processing, transportation and mobile robots.

The proposed solution by Leela-Apiradee et al. (2017) has been applied in product pricing problem that is composed of, 6 parameters (3 products and 3 customers) and 9 constraints. By considering customer purchasing power and cost of transportation an acceptable solution is presented.

Lee et al. (2016) have used Max-Plus Algebra based solution method to compare the performance of three types of WIP-controlled line production systems with constant processing times, such as Kanban, Constant Work in Progress (CONWIP) and Drum-Buffer-Rope (DBR) with a unique analytical approach. A system composed of 6 stations with Poisson arrival and bottleneck placed at the fifth station was considered. To minimize the capacity of the buffer, various models were designed by changing the placement of the bottleneck and the sequence of processing times. They expressed, the actual sojourn times are affected by the buffer capacities and processing time sequence. Table 1-2 illustrates some of the latest research on Max-Plus Algebra.

Table 1-2 Max-Plus Algebra Application

Application Group	References
Modelling (Mathematical and Systems)	Imaev and Judd (2007,2008,2009); Seleim and ElMaraghy (2014a,2014b); Majdzik et al. (2017)
Performance and System Optimization, evaluation and Simulation	Bouquard et al. (2005); Seybold et al. (2015); Adzkiya et al. (2016); Hochang et al. (2016); Kersbergen et al. (2016); Han et al. (2017); Leela-Apiradee et al. (2017)
Scheduling, Planning (Real-Time Systems) and Computer Simulation	Febbraro et al. (1994); Yun-Xiang et al. (2016); Singh et al. (2012 & 2013)
Control (Theory, Systems, Feedback, Predictive Robotics and Automation)	Hardouin et al. (2017); Song et al. (1998); Dias et al. (2015, 2016)

Modelling DES can be classified based on industrial applications. Particularly, a number of articles focused on the industrial applications of Max-Plus Algebra have been mentioned below.

A predictive controller model, as well as railway traffic for online traffic management of railway networks with a periodic timetable, have been considered by Kersbergen et al. (2016). The railway system is described by a switching Max-Plus Linear Model. The measurement of running, dwell times and future running times are assumed to be available. The switching Max-Plus linear model for the railway is used to determine optimal dispatching actions, by recasting that problem as a Mixed Integer Linear Programming (MILP) problem.

Heidergott et al. (2006) have applied the foregoing technique for Dutch passenger train network. Consequently, Case (2010) has modelled a simple railway network as a part of the Dutch train transportation system. Two transit stations supposed to have interconnection plus their connection with outside. This model is involved at least for 440 trains.

Han et al. (2016) have applied a Max-Plus Algebra model to develop a general framework for resolving resource utilization conflicts of air traffic system. The developed optimization system considered six different types of model with and without buffer in view of different purposes such as fuel consumption and flight delay. The constraints between input and output variables based on the observation that jet route can be divided into different sub-segments were obtained. The performance of system resources proposed by Max-Plus has been verified through simulations. Table 1-3 represents some of the related researches in different industries.

Table 1-3 Max-Plus Algebra industries usages

Industrial Applications	References
Automation and Control	Van den Boom et al. (2000-2018); Addad et al. (2008-2012); Armstrong et al. (2014), Ahmane et al. (2006); Goto et al. (2010)
Transportation	Kersbergen et al. (2008); Van den boom et al. (2000-2016); Han et al. (2017, 2016); Haddad et al. (2016); Shang et al. (2010); Case J (2010)
Manufacturing	Febbraro et al. (1994); Seleim and ElMaraghy (2014 a, 2014b); Majdzik et al. (2017); Hochang et al. (2016); Imaev et al. (2007,2008,2009); Gorji et al. (2013)

1.6. Advantages and Drawbacks of Max-Plus Algebra

A tool like Max-Plus Algebra certainly has some advantages and drawbacks; these are discussed below.

Advantages:

- The event timing dynamics of any deterministic system can be expressed by a set of linear equation (Imaev & Judd, 2009; Muijsenberg, 2015; Seleim, 2016).
- Provides computational engine to calculate system's quantitative characteristics (Imaev & Judd, 2009).
- Provides a strong framework for both modelling and analysis (Cassandras & Lafortune, 2010).
- The set of possible solutions can be obtained directly under a set of initial conditions (Adzkiya et al., 2016; Imaev & Judd 2009).
- Can study the periodicity of a system in order to characterize the system behaviour (Cassandras & Lafortune, 2010).
- Can be used in the analysis and control of manufacturing systems (Seleim, 2016).
- Can describe the order of the system's events (Wetjens, 2004).
- Researchers have benefited from the guidelines and concepts provided by Max-Plus Algebra (Cohen, 1997).

- Max-Plus is an appropriate method to model a system for short-term and small-sized systems (Di Febbraro et al., 1994).

Drawbacks:

- The model should be defined for a certain level of complexity (Wetjens, 2004).
- Simultaneously, a model designer should deal with several difficulties such as manufacturing structure and entities as well as mathematical formulation and calculate solution by Max-Plus operations over matrices (Seleim, 2016).
- The model structure is not flexible, whereby any changes in the system's structure will be required to develop from the bottom (Muijsenberg, 2015). sometimes minor changes lead to an increase in the number of equations and dimensions of matrices (Seleim, 2016).
- The number of studies involved in this new area of the system theory for DES has remained small compared with other tools (Cohen, 1997).
- There is not any available tool or software to facilitate modelling and analysis process (Seleim, 2016).
- The type of models in the Max-Plus framework is limited to marked graphs and extensions to stochastic DES are not easy (Cassandras & Lafortune, 2010).
- Performance evaluation is not executed in the field of comparison of Max-Plus with other modelling methods, particularly for large instances (Houssin, 2011).
- Some fundamental shortages slow down the progress of the model (Cohen, 1997).

The weakness of DES modelling methods:

- Generally, DES modelling methods lead to nonlinear models.
- Discrete event simulation is usually computationally expensive and does not supply equations needed to analyze and predict systems behaviour (Imaev & Judd 2008). Also, it is time-consuming and can give information only for the given simulated parameters of the system; numerous simulation runs would be required (Seleim, 2016).
- Queueing network and Markov chains are used to evaluate long-term performance characteristics of systems, especially stochastic ones (Imaev & Judd 2008; Seleim, 2016).
- Timed-event graph and directed graphs are subclasses of Petri nets in which each place is limited only to one incoming and one outgoing arc (Imaev & Judd, 2008).
- Simulation is certainly the most common tool for DESs and requires a high level of detail, which leads to an increase in the complexity of the system. Furthermore, simulation is not an appropriate tool to explain the effects of parameters on system behaviour.

1.7. Research Gap Analysis

In this section the relevant literature is considered critically. Recently, Max-Plus Algebra has been expanded among scholars and academics. Since this high interest in the Max-Plus tool is limited to the last two decades, Max-Plus Algebra requires further justifications and verifications.

Since the 1980s, Max-Plus Algebra has broadly been developed in theories. However, few researchers have examined the application of Max-Plus in engineering fields, particularly in manufacturing systems (Leela-Apiradee et al., 2017). In spite of that, the use of Max-Plus Algebra in practical and real-life systems has not been expanded sufficiently. Cohen et al. (1998) declare that the application of Max-Plus did not receive enough attention. This phenomenon is clear by comparison of the number of publication in theories and applications.

Despite progress in Max-Plus theories, developed algorithms and proposed computational methods (Seleim, 2016; Imaev, 2009), there are still difficulties with its calculation steps. Max-Plus is constructed by matrices and numerous equations. Difficulties in calculation have limited users to those who have enough knowledge in mathematics (Bouquard et al., 2006). In addition, due to these complex calculation steps, Max-Plus Algebra is suggested and applied for small-size problems.

To tackle the difficulties of Max-Plus calculation steps, the mathematical software has been used. The majority of researchers have benefited from Matlab and Mathematica as general software. However, this requires sufficient knowledge of language programming. Thus, to solve one issue another issue is raised. Consequtively, Adzkiya et al. (2016) have introduced an open-source software for verification of Max-Plus Linear (MLP) systems.

It is clear that Max-Plus Algebra as a linear mathematical modelling tool has not been expanded to accommodate the stochastic systems (Seleim, 2016). Interval system is a prominent method applied in the majority of suggested stochastic systems. To solve these linear systems, different heuristic models have been developed. Hence, there is a lack of application of a linear mathematical tool such as Max-Plus Algebra in this type of systems.

In light of previous research, Max-Plus Algebra is suggested for small size problems (Seleim, 2016; Leela-Apiradee et al., 2017; Seybold et al., 2015; Majdzik et al., 2016; Bouquard et al., 2006) and hardly ever the model developed in matters of size (Imaev, 2009).

Discrete Event Simulation is the most popular method to model DES. However, the comparison of this method with Max-Plus has hardly been discussed in previous research.

By looking into the application of Max-Plus Algebra in industries, it has been noticed that most of the studies have been done in the fields of transportation, control, and automation. However, it is believed that there have not been enough studies in the field of manufacturing systems.

There have been a few scholars who applied Max-Plus Algebra in flow-line discrete manufacturing systems (Majdzik et al., 2016; Seybold et al., 2015; Seleim, 2016). However, there are other types of manufacturing systems processes and configurations, such as repetitive, continuous, cellular manufacturing, and job shop, that need to be modelled and tested using this mathematical tool.

Behaviour of a manufacturing system are broad and require more studies such as bypassing, disturbance (breakdown and downtime), backtracking/reentrancy (close and open loops, reworks and reschedule), alternative process, sourcing/allocation policies, buffer (converge and diverge) transportation method, etc. (Seybold et al., 2015; Majdzik et al., 2016; Imaev, 2009).

Finally, Max-Plus Algebra has been applied in optimization (Bouquard et al., 2006; Xiaoping et al., 2013), simulation (Becker & Latovetsky, 2011), scheduling (Di Febbraro, 1994; Kubo & Nishinari, 2018), planning (Abbou et al., 2017), performance evaluation (Singh & Judd, 2012) reconfiguration (Zhu et al., 2004), etc.

This study is a unique application of Max-Plus Algebra in line balancing of a manufacturing system. In this research, Line balancing a manufacturing system is modelled using Max-Plus Algebra. To do that seven scenarios are designed to find out the best combination of adding parallel identical stations and bottleneck. To analyze the designed scenarios, some Key Performance Indicators are defined. Finally, FlexSim as simulation software is used to verify the outcome of the Max-Plus model.

1.8.Research Scope

This study Max-Plus Algebra has been used to model the line balancing of a manufacturing system. The considered manufacturing system is a flow line system with no machine failure rate. The manufacturing system is deterministic and discrete flow with no backtracking. This research has provided a couple of tools to be used in this application domain that is explained below.

Firstly, this research provided a general method of line balancing a manufacturing system that can be extended to other types of discrete manufacturing systems. This model will help the manufacturer and industries to find the best answer to balance their manufacturing lines without changing the configuration of their flow line.

Secondly, the developed line balancing included seven scenarios. The first scenario was a simple flow line. Next scenarios have been designed based on previous scenarios by identifying bottleneck and critical path. The scenarios became more complex by adding parallel stations to the bottleneck and adding a finite buffer to the stations that their downstream process is a bottleneck.

Thirdly, several manufacturing Key Performance Indexes (KPIs) have been defined, such as Product Completion Time (Average Delivery Rate by the system in seconds), Total Processing Lead Time, Station's Utilization Rate, Idle Time and Efficiency of the System. Financial Analysis is formulated to compare all scenarios by using a combination of total processing lead time, utilization rate and etc.; these are used to evaluate the different scenarios. These KPIs can be extended to other modelling purposes.

Finally, the scenarios were tested using discrete event simulation tool (Flexim). Same data, parameters, variables and conditions, etc. were applied when simulating the scenarios to make the comparison with results from Max-Plus valid. Both Flexim and Max-Plus Algebra have resulted in the same outcomes. However, Max Plus Algebra was quicker and easier to manage.

1.9.Research Contributions

Firstly, this study represents a practical approach to use Max-Plus Algebra and proves how this method is easy to understand for decision makers with a little background or basic knowledge of mathematics.

Secondly, this research is the first study of the application of Max-Plus Algebra in line balancing of a discrete manufacturing system. This method is applied to different structures/configurations of a manufacturing system such as Series, Merged, Paralleled, Buffered and Combined.

Thirdly, the problem size of the developed model is at least twice that of previous studies (seven stations). The last scenario composed of 12 stations, 4 parallel identical stations and two buffers.

Fourthly, seven scenarios are developed to optimize the line balance of the considered manufacturing system. Several KPIs are defined to analyze and test the designed scenarios.

Finally, discrete event simulation like FlexSim is used to verify and compare the results obtained by using Max-Plus Algebra.

1.10. Thesis statement

The thesis statement of this research could be formulated as follows:

Max-Plus Algebra could be a competitive and fast tool in the field of modelling manufacturing system and analyze its line balancing results.

The competitive term in this study refers to the being easy to use for model designer and easy to change the structure, size and complexity of a system. The fast term refers to the time consumed for modelling process of a system. This research considered the period of time from designing point to analyze the results of modelled system under equal conditions in comparison with discrete event simulation tool like FlexSim. The advantages of Max-Plus Algebra are demonstrated in the Car Headlight manufacturing system as a practical case study.

1.11. Research overview

This research is organized into six chapters as follows. In chapter 2, the basic context in the field of DEDS, the introduction of Max-Plus Algebra and numerical examples have been noted. In chapter 3, the definition of the system elements, variables and equations are discussed. Moreover, the general mathematical formulation by Max-Plus Algebra used in modelling of a manufacturing system for different structures is presented in this section. Chapter 4, includes the application of Max-Plus Algebra to model a manufacturing system for a Car Headlight flow line. The brief explanation of the Car Headlight process, equipment and parts and other related issues for a better understanding of the flow line have been described in this chapter. Seven scenarios are developed by having varied system structures to balance manufacturing line. Max-Plus Algebra as a tool is used and compared with discrete event simulation. At the end of chapter 4, several analyses were carried out based on scenarios output. In chapter 5, a confirmation and comparison are presented using Flexim as a simulation tool. Finally, the conclusion, contribution and research highlights, limitations of the current work, overview and future work are stated in chapter 6.

CHAPTER TWO
BASICS OF MAX-PLUS ALGEBRA AND MATHEMATICAL CONCEPTS

2.1. Introduction

This chapter presents an introduction to Max-Plus Algebras and shows their application in manufacturing systems.

2.2. Max-Plus Algebras

A Max-Plus Algebra is the set $\mathbb{R}_{max} = \mathbb{R} \cup \{-\infty\}$, where $\mathbb{R}$ is the set of real numbers, together with the binary operators $\oplus$ and $\otimes$, where $x \oplus y = max(x, y)$ and $x \otimes y = x + y$. For convenience we use the symbols $e = 0$ and $\varepsilon = -\infty$.

The following properties are satisfied by Max-Plus Algebras.

1. Associativity. For all $x, y, z \in \mathbb{R}_{max}$ we have $x \oplus (y \oplus z) = (x \oplus y) \oplus z$ and $x \otimes (y \otimes z) = (x \otimes y) \otimes z$.
2. Commutativity. For all $x, y \in \mathbb{R}_{max}$ we have $x \oplus y = y \oplus x$ and $x \otimes y = y \otimes x$.
3. Distributivity of $\otimes$ over $\oplus$. For all $x, y, z \in \mathbb{R}_{max}$ we have $x \otimes (y \oplus z) = (x \otimes y) \oplus (x \otimes z)$.
4. Idempotency of $\oplus$. For all $x \in \mathbb{R}_{max}$ we have $x \oplus x = x$.
5. Zero element: For all $x \in \mathbb{R}_{max}$ we have $x \oplus \varepsilon = x$.
6. Unit element: For all $x \in \mathbb{R}_{max}$ we have $x \otimes e = x$.
7. Zero absorption for $\otimes$: For all $x \in \mathbb{R}_{max}$ we have $x \otimes \varepsilon = \varepsilon \otimes x = \varepsilon$.

In future chapters we will need to use exponentiation. We define

$$x^{\otimes^n} = \underbrace{x \otimes x \otimes \ldots \otimes x}_{n\ times} = \underbrace{x + x + \cdots + x}_{n\ times} = nx.$$

2.3. Max-Plus Algebra over Matrices

Let $A, B, C \in \mathbb{R}_{max}^{m \times n}$ and let $[A]_{ij}$ be the i, jth element, that is, the element in row i and column j, of the matrix A. Thus, we have that, if $C = A \oplus B$ then

$$[C]_{ij} = [A \oplus B]_{ij} = \max\big([A]_{ij}, [B]_{ij}\big).$$

The $m \times n$ zero matrix is denoted by $\mathcal{E}_{m\times n}$ and is given by $[\mathcal{E}_{m\times n}]_{ij} = \varepsilon$. Note that $A \oplus \mathcal{E} = A$.

Now we define matrix multiplication. Let $A \in \mathbb{R}_{max}^{m\times p}$ and $B \in \mathbb{R}_{max}^{p\times n}$. If $C = A \otimes B$ then

$$[C]_{ij} = [A \otimes B]_{ij} = \bigoplus_{l=1}^{p} [A]_{il} \otimes [B]_{lj}.$$

The $n \times n$ identity matrix is denoted by E_n and is given by $[E_n]_{ij} = e$, if $i = j$, and $[E_n]_{ij} = \varepsilon$, otherwise. For any matrices $A \in \mathbb{R}_{max}^{m\times n}$ and $B \in \mathbb{R}_{max}^{n\times p}$ we have that $A \otimes E_n = A$ and $E_n \otimes B = B$.

For matrix exponentiation, we have $A^{\otimes^0} = E_n$ and

$$A^{\otimes^n} = \underbrace{A \otimes A \otimes \ldots \otimes A}_{n\ times}.$$

The Kleene star operator on A $\in \mathbb{R}_{max}^{n\times n}$ is denoted by A^* and is given by

$$A^* = A^{\otimes^0} \oplus A^{\otimes^1} \oplus \ldots \oplus A^{\otimes^\infty} = \bigoplus_{n=0}^{\infty} A^{\otimes^n}.$$

Through the rest of the thesis, the $\otimes$ operator will be omitted whenever its use is obvious, thus $a \otimes b \oplus c \otimes d$ will be written as $ab \oplus cd$ or $A^{\otimes^2}$ as A^2.

A set of linear equations in a Max-Plus Algebra, see, for example, Heidergott et al. (2006) and Baccelli et al. (2001), can be written as

$$X = AX \oplus BU \tag{2.1}$$

which has the solution

$$X = A^* BU.$$

2.4. Modelling A Manufacturing System Using Max-Plus Algebra

Using a development is similar to

We now give small example showing the use of Max-Plus Algebra to model a manufacturing system (Seleim, 2016). In Figure 2.1 we see a simple manufacturing system with four stations. Stations 1 and 2 have inputs U_1 and U_2, respectively and there is no specified starting time. Stations 1 and 3 are in series and station 4 is a merged point. The system has no buffer and no parallel stations.

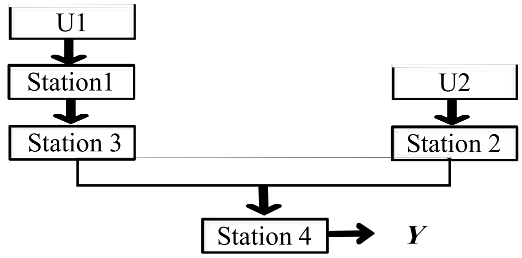


Figure 2.1 A mixed manufacturing system

Let the processing times for stations 1, 2, 3 and 4 be t_1, t_2, t_3, and t_4, respectively. The time instants that raw material for part k is fed to the stations 1 and 2 are $u_1(k)$ and $u_2(k)$ respectively. The starting time of operation for processing part k in stations 1, 2, 3 and 4 are denoted by $x_1(k)$, $x_2(k)$, $x_3(k)$ and $x_4(k)$, respectively. The time instants of the finished part k at the last station is $y(k)$.

The equations for each stations 1 and 2 are

$$x_1(k) = u_1(k) \oplus x_1(k-1)t_1, \text{ and} \quad (2.2)$$

$$x_2(k) = u_2(k) \oplus x_2(k-1)t_2. \quad (2.3)$$

Since station 3 is in series after station 1 we have

$$x_3(k) = x_1(k)t_1 \oplus x_3(k-1)t_3. \quad (2.4)$$

Combining *(2.2)* and *(2.4)* we have

$$x_3(k) = u_1(k)t_1 \oplus x_1(k-1)t_1^2 \oplus x_3(k-1)t_3. \quad (2.5)$$

Since the station 4 is a merging point we use *(2.3)* and *(2.5)* to get

$$x_4(k) = x_3(k)t_3 \oplus x_2(k)t_2 \oplus x_4(k-1)t_4. \quad (2.6)$$

By substituting *(2.3)* and *(2.5)* into *(2.6)* we get

$$x_4(k) = u_1(k)t_1t_3 \oplus x_1(k-1)t_1^2t_3 \oplus x_3(k-1)t_3^2 \oplus u_2(k)\, t_2 \oplus x_2(k-1)t_2^2 \oplus x_4(k-1)t_4. \quad (2.7)$$

Since the arrival time of the finished product k is equal to the starting time of processing at station 4 plus processing time of station 4 we get

$$Y_k = x_4(k)t_4. \quad (2.8)$$

We combine equations *(2.2), (2.4), (2.5) and (2.7)* to get

$$X_k = AX_k \oplus BX_{k-1} \oplus DU_k \quad (2.9)$$

and we can rewrite *(2.8)* to get

$$Y_k = CX_k, \quad (2.10)$$

where

$$X_k = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}_k, U_k = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_k,$$

$$A=\begin{bmatrix}\varepsilon & \varepsilon & \varepsilon & \varepsilon\\ \varepsilon & \varepsilon & \varepsilon & \varepsilon\\ t_1 & \varepsilon & \varepsilon & \varepsilon\\ \varepsilon & t_2 & t_3 & \varepsilon\end{bmatrix},\ \mathrm{B}=\begin{bmatrix}t_1 & \varepsilon & \varepsilon & \varepsilon\\ \varepsilon & t_2 & \varepsilon & \varepsilon\\ \varepsilon & \varepsilon & t_3 & \varepsilon\\ \varepsilon & \varepsilon & \varepsilon & t_4\end{bmatrix},\ D=\begin{bmatrix}e & \varepsilon\\ \varepsilon & e\\ \varepsilon & \varepsilon\\ \varepsilon & \varepsilon\end{bmatrix}$$

and

$$C=[\varepsilon \quad \varepsilon \quad \varepsilon \quad t_4].$$

Since $A^n=\mathcal{E}$ for $n>3$,

$$A^*=\begin{bmatrix}e & \varepsilon & \varepsilon & \varepsilon\\ \varepsilon & e & \varepsilon & \varepsilon\\ \varepsilon & \varepsilon & e & \varepsilon\\ \varepsilon & \varepsilon & \varepsilon & e\end{bmatrix}\oplus\begin{bmatrix}\varepsilon & \varepsilon & \varepsilon & \varepsilon\\ \varepsilon & \varepsilon & \varepsilon & \varepsilon\\ t_1 & \varepsilon & \varepsilon & \varepsilon\\ \varepsilon & t_2 & t_3 & \varepsilon\end{bmatrix}\oplus\begin{bmatrix}\varepsilon & \varepsilon & \varepsilon & \varepsilon\\ \varepsilon & \varepsilon & \varepsilon & \varepsilon\\ \varepsilon & \varepsilon & \varepsilon & \varepsilon\\ t_1t_3 & \varepsilon & \varepsilon & \varepsilon\end{bmatrix}=\begin{bmatrix}e & \varepsilon & \varepsilon & \varepsilon\\ \varepsilon & e & \varepsilon & \varepsilon\\ t_1 & \varepsilon & e & \varepsilon\\ t_1t_3 & t_2 & t_3 & e\end{bmatrix}.$$

We define

$$\hat{A}=A^*B=\begin{bmatrix}e & \varepsilon & \varepsilon & \varepsilon\\ \varepsilon & e & \varepsilon & \varepsilon\\ t_1 & \varepsilon & e & \varepsilon\\ t_1t_3 & t_2 & t_3 & e\end{bmatrix}\begin{bmatrix}t_1 & \varepsilon & \varepsilon & \varepsilon\\ \varepsilon & t_2 & \varepsilon & \varepsilon\\ \varepsilon & \varepsilon & t_3 & \varepsilon\\ \varepsilon & \varepsilon & \varepsilon & t_4\end{bmatrix}=\begin{bmatrix}t_1 & \varepsilon & \varepsilon & \varepsilon\\ \varepsilon & t_2 & \varepsilon & \varepsilon\\ t_1 & \varepsilon & t_3 & \varepsilon\\ t_1t_3 & t_2 & t_3 & t_4\end{bmatrix}\text{ and}$$

$$\hat{B}=A^*D=\begin{bmatrix}\varepsilon & \varepsilon & \varepsilon & \varepsilon\\ \varepsilon & \varepsilon & \varepsilon & \varepsilon\\ t_1 & \varepsilon & \varepsilon & \varepsilon\\ t_1t_3 & t_2 & t_3 & \varepsilon\end{bmatrix}\begin{bmatrix}e & \varepsilon\\ \varepsilon & e\\ \varepsilon & \varepsilon\\ \varepsilon & \varepsilon\end{bmatrix}=\begin{bmatrix}e & \varepsilon\\ \varepsilon & e\\ t_1 & \varepsilon\\ t_1t_3 & t_2\end{bmatrix}.$$

According to Seleim (2016), equation *(2.9)* can be rewritten as:

$$X_k=\hat{A}X_{k-1}\oplus\hat{B}U_k \tag{2.11}$$

where $\hat{A}=A^*B$ and $\hat{B}=A^*D$.

For example, if we set $t_1=2$, $t_2=3$, $t_3=4$, and $t_4=1$ we get

$$A^*=\begin{bmatrix}e & \varepsilon & \varepsilon & \varepsilon\\ \varepsilon & e & \varepsilon & \varepsilon\\ 2 & \varepsilon & e & \varepsilon\\ 6 & 3 & 4 & e\end{bmatrix},\ \hat{A}=\begin{bmatrix}2 & \varepsilon & \varepsilon & \varepsilon\\ \varepsilon & 3 & \varepsilon & \varepsilon\\ 2 & \varepsilon & 4 & \varepsilon\\ 6 & 3 & 4 & 1\end{bmatrix},\text{ and }\hat{B}=\begin{bmatrix}e & \varepsilon\\ \varepsilon & e\\ 2 & \varepsilon\\ 6 & 3\end{bmatrix}$$

giving the main equation of the system

$$X_k=\begin{bmatrix}2 & \varepsilon & \varepsilon & \varepsilon\\ \varepsilon & 3 & \varepsilon & \varepsilon\\ 2 & \varepsilon & 4 & \varepsilon\\ 6 & 3 & 4 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}_{k-1}\oplus\begin{bmatrix}e & \varepsilon\\ \varepsilon & e\\ 2 & \varepsilon\\ 6 & 3\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}_k.$$

2.5. Summary

In this chapter, the basics of Max-Plus Algebra were introduced followed by an example showing the application of modeling manufacturing system with Max-Plus linear equations. In the following chapters, we will use Max-Plus Algebra to model a manufacturing system and a case study will be presented.

CHAPTER THREE

MODELLING OF MANUFACTURING SYSTEM BY MAX-PLUS ALGEBRA

3.1. Introduction

In chapter two, the basic context of Max-Plus Algebra were discussed. In this chapter, variables, state space description, assumptions, and modelling the different structure of a manufacturing system using Max-Plus Algebra are explained.

3.2.General Proposition of a Model using Max-Plus Algebra

3.2.1. Variables

To model a manufacturing system, the definition of entities needs to be clear. Inputs, variables and output of the station i as a member of the manufacturing system are modelled and assumed as below:

K: an event counter, which is the number of parts (jobs),

t_i: denotes processing time of station i,

$U_i(k)$: denotes the time instants at which incoming part k is fed to the station i,

$x_i(k)$: denotes the time instants when station i starts processing part k,

$y(k)$: stands for the time instants at which part k is completed and leaves the line as a finished product.

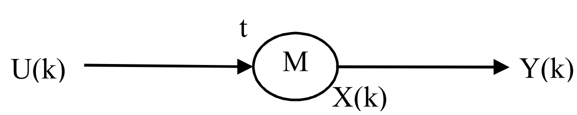


Figure 3.1 A simple structure of a system

3.2.2. State Space Description

In the systems theory, the term state has a precise meaning and it describes behaviour of a system at a time instant in a measurable way. The state space of a system is the set of all possible values (usually denoted by X) (Cassandras & Lafortune, 2010). In this study, corresponding state variables such as U(k), X(k), Y(k), rely on part k which makes the system dynamic by considering events of such system that evolve in time. Two primary conditions are assumed in this system. First, the input (U), such as raw material or part, is available with no arrival time. Second, each station starts processing part $k+1$ when part k leaves the station. As discussed in chapter two, the state equations are represented in several different forms. However, the common form is based on differential equations as below:

$$X(k+1) = \max(U(k+1), X_i(k) + t_i) \text{ or}$$

$$X(k+1) = max(X_{i-1}(k+1) + t_{i-1}, X_i(k) + t_i)$$

3.2.3. Assumptions in the Manufacturing System

The following assumptions are considered in the developed general model; otherwise, it will be noted.

1- Process time of stations is deterministic and fixed.

2- All stations have infinite buffers.

3- First In, First Out (FIFO) operation rule is followed.

4- The process sequence is based on production flow line.

5- The station only processes one part at a time.

6- Stations do not have any failure, downtime or stoppage.

7- The process time includes loading time, operation time, and unloading time.

3.3.Modelling of a Manufacturing System

A manufacturing system can be modelled using different structures, such as splitting, merging, by-passing, back-tracking, batching, re-entrance, etc. Max-Plus Algebra can be used to model those structures to make it simpler and more understandable. In this research, different structures of a manufacturing system, such as Series, Merged, Parallel, Finite Buffer and the combination of those are studied and modelled using Max-Plus Algebra. For more details, we refer the readers to Imaev (2009); Muijsenberg (2015); Adzkiya et al. (2016); Seleim (2016).

3.3.1. Modelling "n" Series Stations

In this structure, stations are laid out consecutively. There is no merged point, parallel station or buffer. Parts are processed by passing through all stations. This structure is shown in Fig. 3.2 can be explained by Max-Plus equations as follows:

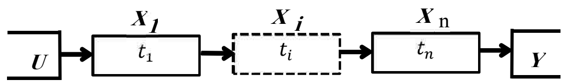


Figure 3.2 n series stations

By following the main equation *(2.8)* the main state equation is:

$$X(k) = \hat{A}X(k-1) \oplus \hat{B}U(k) \tag{3.1}$$

where:

$$\hat{A} = A^* \otimes B = \begin{bmatrix} t_1 & \varepsilon & \cdots & \varepsilon \\ t_1^2 & t_2 & & \varepsilon \\ \vdots & \vdots & \ddots & \vdots \\ t_1^2 t_2 .. t_{n-1} & t_2^2 t_3 .. t_{n-1} & \cdots \; t_{n-1}^2 & t_n \end{bmatrix}$$

$$\text{and } \hat{\mathrm{B}} = \mathrm{A}^* \otimes \mathrm{D} = \begin{bmatrix} \mathrm{e} \\ \mathrm{t}_1 \\ \vdots \\ \mathrm{t}_1 \mathrm{t}_2 .. \mathrm{t}_{n-1} \end{bmatrix}$$

From the main equation it can be deduced that for any station *i*, the state equation for the part *k* is equal to:

$$\mathrm{X}_{i,k} = \mathrm{t}_i \mathrm{X}_{i,k-1} \oplus \mathrm{t}_{i-1}^2 \mathrm{X}_{i-1,k-1} \oplus \mathrm{t}_{i-2}^2 \mathrm{t}_{i-1} \mathrm{X}_{i-2,k-1} \oplus \ldots \oplus \mathrm{t}_1^2 \mathrm{t}_2 \ldots \mathrm{t}_{i-1} \mathrm{X}_{1,k-1} \oplus \mathrm{t}_1 \mathrm{t}_2 \ldots \mathrm{t}_{i-1} \mathrm{U}_k$$

Therefore, the Max-Plus equation can be generated directly by state equation *(3.1)* with any number of stations.

3.3.2. Modelling "n" Merged Stations

The merged station is a station that receives input from more than one station. Hence, state of part *k* at station *i* is calculated by considering the main state equation as *(3.1)*, where $\hat{\mathrm{A}}$ and $\hat{\mathrm{B}}$ will be:

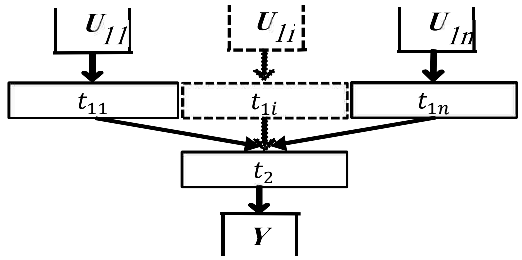


Figure 3.3 "n" merged stations

$$\hat{A} = A^{*} \otimes B = \begin{bmatrix} t_{11} & \varepsilon & & \cdots & \varepsilon \\ \varepsilon & t_{12} & & & \varepsilon \\ \vdots & & \ddots & & \vdots \\ \varepsilon & \varepsilon & & t_{1n} & \varepsilon \\ t_{11}^{2} & t_{12}^{2} & \cdots & t_{1n}^{2} & t_{2} \end{bmatrix}, \, and \;\; \hat{B} = A^{*} \otimes D = \begin{bmatrix} e & \varepsilon & & \cdots & \varepsilon \\ \varepsilon & e & & & \varepsilon \\ \vdots & & \ddots & & \vdots \\ \varepsilon & \varepsilon & & e & \varepsilon \\ t_{11} & t_{12} & \cdots & & t_{1n} \end{bmatrix}$$

3.3.3. Modelling Parallel Identical Stations

In section 3.3.1, modelling n series stations, it has been assumed that there is no parallel station in the system. However, adding a parallel station is one of the ways to reduce idle time, which is a priority for manufacturers. In other words, the bottleneck station gets one or more identical stations, which lead to tasks distribution among those parallel stations. For example, if an identical station is added to station *i*, while this station is processing part *k*, the identical station processes the next part (*k+1).*

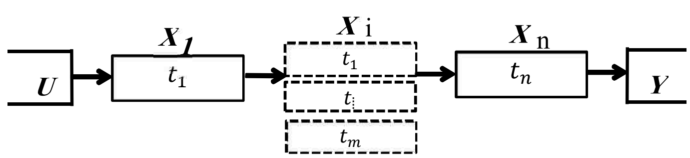


Figure 3.4 Flow line with the parallel identical station

As shown in figure (3.4), *m* parallel stations are added to station *i*. These stations can be assumed to be one station for the purpose of modelling. Therefore, the condition that the station should have finished processing part $k-1$, would be replaced by $k-m$. The conditions for the first and last stations are unchanged. Thus, all variables are similar to the series model, except matrix *B*, which would be represented by two matrices. The first matrix is the same as matrix *B*, but the difference is that all entities in the corresponding column of the parallel station are replaced with ε.

Likewise, $$B_1 = \begin{bmatrix} t_1 & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \cdots\vdots\cdots & t_n \end{bmatrix} \text{ and } B_i = \begin{bmatrix} \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & t_i & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon \end{bmatrix}$$

The main equation will have an extra nominal for each group of parallel stations and evolves to:

$$\boldsymbol{X}_k = \widehat{\boldsymbol{A}}\boldsymbol{X}_{k-1} \oplus \widehat{\boldsymbol{AP}}_{\boldsymbol{i}}\boldsymbol{X}_{k-m} \oplus \widehat{\boldsymbol{B}}U_k \qquad (3.2)$$

3.3.4. Modelling A Station with Finite Buffer

Buffering is one of the ways that manufacturers are using to account for fluctuations and variations in their systems. Buffers can be raw material storage, finished part inventories, and unfinished parts. Assuming the station has a buffer with B parts changes the main state equation as below:

$$\boldsymbol{X}_k = \widehat{\boldsymbol{A}}\boldsymbol{X}_{k-1} \oplus \widehat{\boldsymbol{B}}U_k \oplus \widehat{\boldsymbol{AB}}_{\boldsymbol{i}}\boldsymbol{X}_{k-B-1} \qquad (3.3)$$

In which matrices $\hat{A}$, and $\hat{B}$ are same as before and $\widehat{AB}_i$ is added as below:

$$\widehat{\boldsymbol{AB}}_{\boldsymbol{i}} = \begin{bmatrix} \varepsilon & \dots & \varepsilon & \varepsilon & \dots & \varepsilon \\ \vdots & & \vdots & \vdots & & \vdots \\ & & & \varepsilon & & \\ \varepsilon & \dots & \varepsilon & e & \varepsilon & \\ \vdots & & \vdots & \varepsilon & \vdots & \vdots \\ & & & \vdots & & \\ \varepsilon & \dots & \varepsilon & \varepsilon & \varepsilon & \varepsilon \end{bmatrix}$$

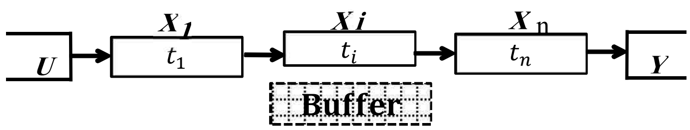


Figure 3.5 Flow line with the buffer station

3.3.5. An Integrated Model (Series, Merged, Parallel Stations and Finite Buffer)

The last structure is the combination of all the previous station models. In the real world, a manufacturing system takes advantage of using both parallel station(s) and buffer(s) to balance the production line. By using the parallel station at a bottleneck and buffers at stations whose upstream process is the bottleneck, the production flow would be smoother. By having all systems structures such as series, merged, parallel station and finite buffer in one system, the main equation is changed as below:

$$\boldsymbol{X}_k = \widehat{\boldsymbol{A}}\boldsymbol{X}_{k-1} \oplus \widehat{\boldsymbol{B}}U_k \oplus \widehat{\boldsymbol{AP}}_{\boldsymbol{i}}\boldsymbol{X}_{k-i} \oplus \widehat{\boldsymbol{AB}}_{\boldsymbol{i}}\boldsymbol{X}_{k-Bi} \qquad (3.4)$$

The matrices' structure should be similar to those matrices that were discussed in previous models.

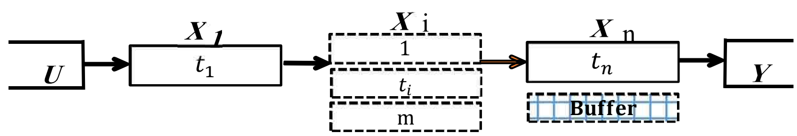


Figure 3.6 An integrated model (Series, Merged, Parallel Stations and, Finite Buffer)

3.4.The General form of Modelling Formulation using Max-Plus Algebra

In this research, a general form of the manufacturing system is composed of three parallel stages each of which has three stations in series, two merged stations, and a series of finishing stations. Figure 3.7 represents the overall view of the considered manufacturing system. To make it easy to understand, the general framework does not include any parallel stations or buffers. In this section, the Max-Plus Algebra equations related to this framework are explained, and more details will be discussed in chapter 4.

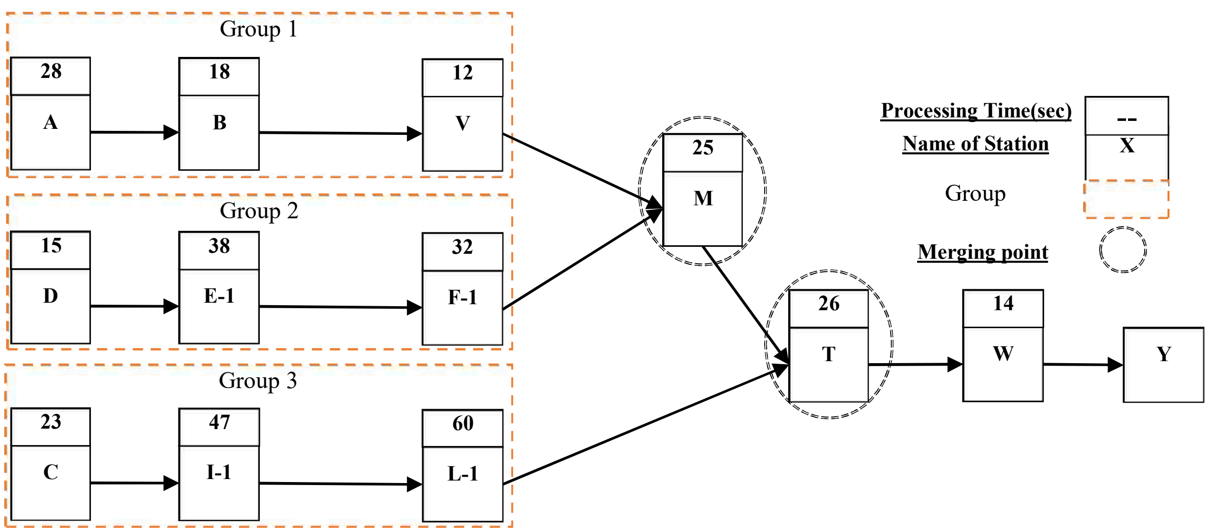


Figure 3.7 The General Structure of modelled manufacturing system

Using Max-Plus Algebra, the system's framework and behaviour can be modelled by the next state equations. Variables, state space and assumptions are the same as the general form explained in section (3.2).

The following equations are applied for each station:

Group 1, stations A, B and, V:

Station A: $$x_A(k) = u_A(k) \oplus x_A(k-1) t_A \quad (3.5)$$

Station B: $x_B(k) = x_A(k)t_A \oplus x_B(k-1)t_B$ *(3.6)*

Expanding equation *(3.5)* into the *(3.6)* would result in the following:

$$x_B(k) = (u_A(k) \oplus x_A(k-1)t_A)t_A \oplus x_B(k-1)t_B x_B(k)$$

$$x_B(k) = u_A(k)t_A \oplus x_A(k-1)t_A^2 \oplus x_B(k-1)t_B \quad (3.7)$$

By following the same procedure, the equation for station *V* is expanded as follows:

Station V: $x_V(k) = x_B(k)t_B \oplus x_V(k-1)t_V =$

$$u_A(k)t_A t_B \oplus x_A(k-1)t_A^2 t_B \oplus x_B(k-1)t_B^2 \oplus x_V(k-1)t_V \quad (3.8)$$

Group 2, stations D, E and, F:

Station D: $x_D(k) = u_D(k) \oplus x_D(k-1)t_D$ *(3.9)*

Station E: $x_E(k) = x_D(k)t_D \oplus x_E(k-1)t_E$ *(3.10)*

in group 2 and 3, the same as group 1, calculations are made by expanding *(3.9)* into *(3.10)*:

$$x_E(k) = u_D(k)t_D \oplus x_D(k-1)t_D^2 \oplus x_E(k-1)t_E \quad (3.11)$$

Station F: $x_F(k) = x_E(k)t_E \oplus x_F(k-1)t_F$ *(3.12)*

(3.12) and *(3.11)*:

$$x_F(k) = u_D(k)t_D t_E \oplus x_D(k-1)t_D^2 t_E \oplus x_E(k-1)t_E^2 \oplus x_F(k-1)t_F \quad (3.13)$$

Merging point M, *(3.8)* and *(3.13)*: $x_M = x_V(k)t_V \oplus x_F(k)t_F \oplus x_M(k-1)t_M x_M =$

$$u_A(k)t_A t_B t_V \oplus x_A(k-1)t_A^2 t_B t_V \oplus x_B(k-1)t_B^2 \oplus x_V(k-1)t_V^2 \oplus u_D(k)t_D t_E t_F \oplus x_D(k-1)t_D^2 t_E t_F \oplus x_E(k-1)t_E^2 t_F \oplus x_F(k-1)t_F^2 \oplus x_M(k-1)t_M \quad (3.14)$$

Group 3, stations C, I and, L:

Station C: $x_C(k) = u_C(k) \oplus x_C(k-1)t_C$ *(3.15)*

Station I: $x_I(k) = x_C(k)t_C \oplus x_I(k-1)t_I$ *(3.16)*

(3.15) and *(3.16)* : $x_I(k) = u_C(k)t_C \oplus x_C(k-1)t_C^2 \oplus x_I(k-1)t_I$ *(3.17)*

Station L: $x_L(k) = x_I(k)t_I \oplus x_L(k-1)t_L$

(3.17) and *(3.16)*: $x_L(k) = u_C(k)t_C t_I \oplus x_C(k-1)t_C^2 t_I \oplus x_I(k-1)t_I^2 \oplus x_L(k-1)t_L$ *(3.18)*

Merging point T $x_T = x_M(k)t_M \oplus x_L(k)t_L \oplus x_T(k-1)t_T$ *(3.19)*

(3.19), (3.14) and *(3.18):*

$$x_T = u_A(k)t_A t_B t_V t_M \oplus x_A(k-1)t_A^2 t_B t_V t_M \oplus x_B(k-1)t_B^2 t_M \oplus x_V(k-1)t_V^2 t_M \oplus$$

$$u_D(k)t_D t_E t_F t_M \oplus x_D(k-1)t_D^2 t_E t_F t_M \oplus x_E(k-1)t_E^2 t_F t_M \oplus x_F(k-1)t_F^2 t_M \oplus$$

$$x_M(k-1)t_M^2 \oplus u_C(k)t_C t_I t_L \oplus x_C(k-1)t_C^2 t_I t_L \oplus x_I(k-1)t_I^2 t_L \oplus x_L(k-1)t_L^2 \oplus x_T(k-1)t_T \quad (3.20)$$

Station W: $x_W(k) = x_T(k)t_T \oplus x_W(k-1)t_W$ *(3.21)*

(3.20) and *(3.21)*:

$$x_W(k) = u_A(k)t_A t_B t_V t_M t_T \oplus x_A(k-1)t_A^2 t_B t_V t_M t_T \oplus x_B(k-1)t_B^2 t_M t_T \oplus x_V(k-1)t_V^2 t_M t_T \oplus$$

$$u_D(k)t_D t_E t_F t_M t_T \oplus x_D(k-1)t_D^2 t_E t_F t_M t_T \oplus x_E(k-1)t_E^2 t_F t_M t_T \oplus x_F(k-1)t_F^2 t_M t_T \oplus$$

$$x_M(k-1)t_M^2 t_T \oplus u_C(k)t_C t_I t_L t_T \oplus x_C(k-1)t_C^2 t_I t_L t_T \oplus x_I(k-1)t_I^2 t_L t_T \oplus x_L(k-1)t_L^2 t_T \oplus x_T(k-1)t_T^2 \oplus x_W(k-1)t_W \quad (3.22)$$

Station Y: $x_Y(k) = x_W(k)t_W \oplus x_Y(k-1)t_Y =$

$$u_A(k)t_A t_B t_V t_M t_T t_W \oplus x_A(k-1)t_A^2 t_B t_V t_M t_T t_W \oplus x_B(k-1)t_B^2 t_M t_T t_W \oplus x_V(k-1)t_V^2 t_M t_T t_W$$

$$\oplus u_D(k)t_D t_E t_F t_M t_T t_W \oplus x_D(k-1)t_D^2 t_E t_F t_M t_T t_W \oplus x_E(k-1)t_E^2 t_F t_M t_T t_W$$

$$\oplus x_F(k-1)t_F^2 t_M t_T t_W \oplus x_M(k-1)t_M^2 t_T t_W \oplus u_C(k)t_C t_I t_L t_T t_W \oplus x_C(k-1)t_C^2 t_I t_L t_T t_W$$

$$\oplus x_I(k-1)t_I^2 t_L t_T t_W \oplus x_L(k-1)t_L^2 t_T t_W \oplus x_T(k-1)t_T^2 t_W \oplus x_W(k-1)t_W^2 \oplus x_Y(k-1)t_Y \quad (3.23)$$

All above equations have described the state of a manufacturing system and can be extracted into the state equation form as:

$$X_k = AX_{k-1} \oplus BX_k \oplus DU_k$$
$$Y_k = CX_k$$

Where:

$$X_K = [x_1, x_2, \dots, x_{13}]^T$$
$$U_K = [u_1, u_2, u_3]^T$$
$$C_K = [\varepsilon, \varepsilon \dots, t_Y]$$

$$A = \begin{bmatrix}
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
t_A & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & t_B & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & t_D & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & t_E & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & t_V & \varepsilon & \varepsilon & t_F & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_C & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_I & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_M & \varepsilon & \varepsilon & t_L & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_T & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_W & \varepsilon
\end{bmatrix}
\quad
B = \begin{bmatrix}
t_A & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & t_B & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & t_V & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & t_D & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & t_E & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_F & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_M & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_C & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_I & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_L & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_T & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_W & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_Y
\end{bmatrix}
\quad
D = \begin{bmatrix}
e & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon \\
\varepsilon & e & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & e \\
\varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon
\end{bmatrix}$$

According to equation *(3.1)*, the implicit equation can be transformed into the main state equation ($X_k = \hat{A}X_{k-1} \oplus \hat{B}U_k$). where, $\widehat{\boldsymbol{A}} = \boldsymbol{A}^*\boldsymbol{B}$, $\widehat{\boldsymbol{B}} = \boldsymbol{A}^*\boldsymbol{D}$ and $\boldsymbol{A}^*$, $\widehat{\boldsymbol{A}}$ and $\widehat{\boldsymbol{B}}$ can be calculated as:

$$\widehat{\boldsymbol{A}} = \begin{bmatrix}
t_A & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
t_A^2 & t_B & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
t_A^2 t_B & t_B^2 & t_V & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & t_D & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & t_E & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_F & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
t_A^2 t_B t_V & t_B^2 t_V & t_V^2 & t_D^2 & t_E^2 & t_F^2 & t_M & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_C & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_C^2 & t_I & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_C^2 t_I & t_I^2 & t_L & \varepsilon & \varepsilon & \varepsilon \\
t_A^2 t_B t_V t_M & t_B^2 t_V t_M & t_V^2 t_M & t_D^2 t_M & t_E^2 t_M & t_F^2 t_M & t_M^2 & t_C^2 t_I t_L & t_I^2 t_L & t_L^2 & t_T & \varepsilon & \varepsilon \\
t_A^2 t_B t_V t_M t_T & t_B^2 t_V t_M t_T & t_V^2 t_M t_T & t_D^2 t_M t_T & t_E^2 t_M t_T & t_F^2 t_M t_T & t_M^2 t_T & t_C^2 t_I t_L t_T & t_I^2 t_L t_T & t_L^2 t_T & t_T^2 & t_W & \varepsilon \\
t_A^2 t_B t_V t_M t_T t_W & t_B^2 t_V t_M t_T t_W & t_V^2 t_M t_T t_W & t_D^2 t_M t_T t_W & t_E^2 t_M t_T t_W & t_F^2 t_M t_T t_W & t_M^2 t_T t_W & t_C^2 t_I t_L t_T t_W & t_I^2 t_L t_T t_W & t_L^2 t_T t_W & t_T^2 t_W & t_W^2 & t_Y
\end{bmatrix}$$

$$
\widehat{\boldsymbol{B}} = \boldsymbol{A}^{*}\boldsymbol{D} = \begin{bmatrix} e & \varepsilon & \varepsilon \\ t_A & \varepsilon & \varepsilon \\ t_At_B & \varepsilon & \varepsilon \\ \varepsilon & \mathrm{e} & \varepsilon \\ \varepsilon & t_D & \varepsilon \\ \varepsilon & t_Dt_E & \varepsilon \\ t_At_Bt_V & t_Dt_Et_F & \varepsilon \\ \varepsilon & \varepsilon & \mathrm{e} \\ \varepsilon & \varepsilon & t_C \\ \varepsilon & \varepsilon & t_Ct_I \\ t_At_Bt_Vt_M & t_Dt_Et_Ft_M & t_Ct_It_L \\ t_At_Bt_Vt_Mt_T & t_Dt_Et_Ft_Mt_T & t_Ct_It_Lt_T \\ t_At_Bt_Vt_Mt_Tt_W & t_Dt_Et_Ft_Mt_Tt_W & t_Ct_It_Lt_Tt_W \end{bmatrix}
$$

$\widehat{\mathbf{A}}$ and $\widehat{\boldsymbol{B}}$ provide the process time of the stations, meaning that all the equations will have a solution, which is the starting time for each part in all stations. All these equations as a part of a dynamic system can be applied in the analysis phase as well as control or optimization. In case of changes to the processing time of station, or even changes to a finished product, as long as the structure of the system does not change, all equations and calculations are similar and can still be used on the structure.

3.5. Summary

In this chapter, the different structures of a manufacturing system were considered. The mathematical formulation in the general structure was presented and matrices based on Max-Plus equations were constructed. In chapter 4, different scenarios of the real and practical case study will be modelled and analyzed.

CHAPTER FOUR

CAR HEADLIGHT MODELING AND LINE BALANCING BY MAX-PLUS ALGEBRA

4.1.Introduction

In the previous chapter, parameters, conditions, assumptions and different structures of the manufacturing systems were explained. In this chapter, the basic structure of a Car Headlight manufacturing system and mathematical model formulation, equations and matrices by Max-Plus Algebra are presented. Car Headlight as a practical manufacturing system, which is a combination of manufacturing and assembly line, is analyzed and balanced by considering several operation scenarios. Finally, the results of this analysis have been demonstrated.

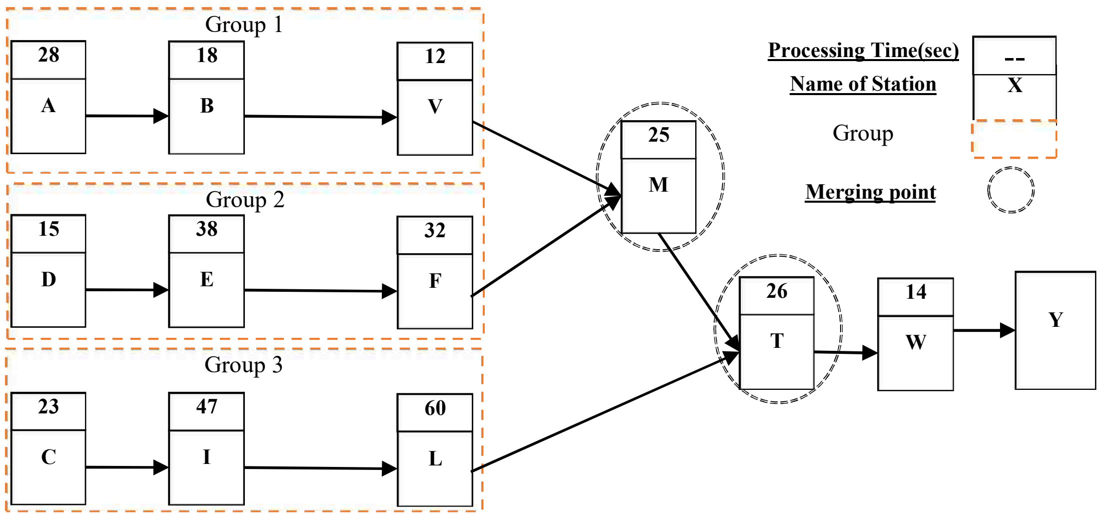
4.2. Car Headlight Manufacturing System

Based on the system classification discussed in the chapters 2 and 3, Car Headlight manufacturing system attributes are described below.

Based on Figure 1.2, the Car Headlight manufacturing system is classified as a dynamic system and due to this fact the output of the system is time-dependent on the past values of input. Since the considered manufacturing system changes over time, the system is recognized as a Time-Variant system. As discussed in previous chapters, Max-Plus Algebra makes the modelled systems linear, hence modelling Car Headlight manufacturing system using Max-Plus Algebra would be a linear system. Additionally, this system is deterministic as there are no random variables or uncertainty conditions. Finally, the Car Headlight manufacturing system is known as a Discrete Event System because the state variables change at discrete set of points over time.

4.2.1. Manufacturing Flow Line

Manufacturing Flow Line is a process of using machines and labour to make goods that are sold to end customers. Generally, the manufacturing line is composed of a sequence of processes. Each process includes the operation(s) that makes a part closer to a finished product. On the other hand, the production line is a broad term including lines that transform the raw material into finished products like car production lines. Specifically, a type of manufacturing or production line in which parts and components are added together in a series of steps with no changes to their features or identifications is called an assembly line. The considered manufacturing system is a Car Headlight flow line, which is a combination of the manufacturing line, for reflector and frames produced by moulding machines, and assembly line for lights, kits, units and other parts joined to the frames. The Car Headlight manufacturing line is shown in Figure 4.1 and an exploded view of the Car Headlight is shown in Figure 4.2.



A	Reflector Moulding Machine	B	Laser Cutting Machine	V	Quality Inspection
D	Middle Frame Moulding Machine	E	Assembly Station (Lights and Bulbs)	F	Assembly Station (Covers, Socket and Kit)
C	Back Frame Moulding Machine	I	Assembly Station (HVAC, Sockets, Bolt and Washer)	L	Assembly Station (Control Unit, LED and Directional Light)
M	Assembly Station (Reflector to Middle Frame)	T	Assembly Station for (Projector Lens and Cover)	W	Final Quality Inspection and Labeling

Figure 4.1 Scenario 1, The General Structure of modelled manufacturing system

The Car Headlight manufacturing line includes plastic moulding, milling, vacuum, heating machines, press, automated and programmed robots, adhesive guns, and fixtures arranged in 12 stations with different tools such as a screwdriver, air compressor, wrench, gauge, printer, and tape machine. In addition, Ultraviolet (UV) coating, lumen adjuster and laser are used for quality control. Some of the machines and stations of the Car Headlight flow line are presented in Figures 4.3 to 4.8.

4.2.2. Components, Parts and Modules

To make driving safer and more convenient, auto manufacturers take advantage of technology by adding/removing components and parts to the basic configuration of a Car Headlight. These additional parts, such as sensors and control kits, make headlight structure more complex, which is beyond of the scope of this research. The list of standard parts is given in Table 4-1.

Table 4-1 List of Car Headlight Parts and Components

1	Lamp cower low beam	13	Screw-Trox-Bolt with
2	Cover high beam/daytime running lights	14	Socket housing
3	Bulb socket, turn insider	15	Repair kit headlight
4	Control unit xenon light	16	Lamp holder for xenon
5	Control unit directional light	17	LED lights
6	Parts set parking-light bulb, halogen	18	Cover
7	Repair kit, HVAC servomotor	19	Seal washer
8	Bulb xenon light with an ignition element	20	Projector lens
9	Bulb yellow/blue	21	Headlight lens
10	Bulb	22	Reflector
11	Long-life Bulb	23	Socket Wire
12	Bulb	24	Housing

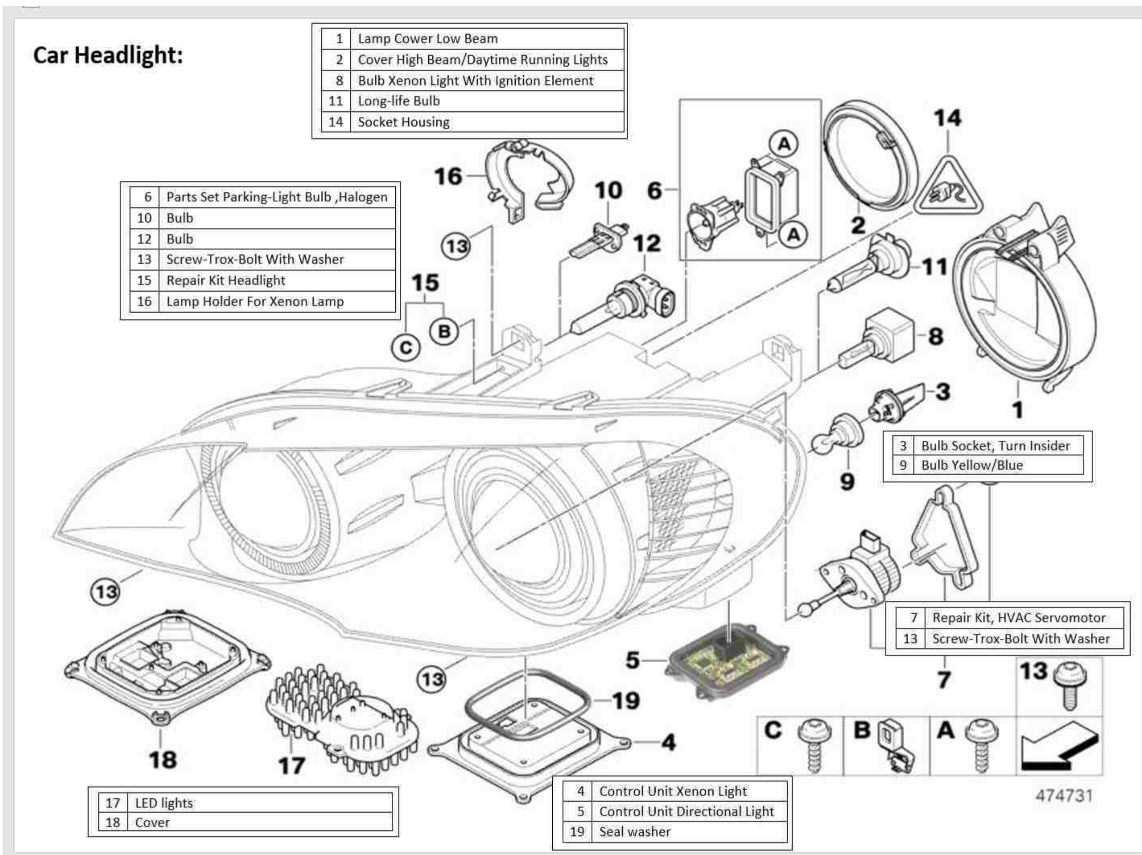


Figure 4.2 Exploded view of Car Headlight (workshop-manuals.com)

4.2.3. Car Headlight Assembly Process Plan Description

As shown in Figure 4.1, the Car Headlight flow line is composed of 12 stations. It includes three moulding machines (e.g. Figure 4.3), which produce frames such as reflector, middle frame and cover. The reflector is the main component of Car Headlight, which is moulded and finished by Stations *A*

(Moulding Injection Machine) and *B* (Laser Cutting Machine), respectively. Raw materials are Poly Carbonate Resin, Plexiglas and Acrylic. They are sucked from a container by vacuum machines to the moulding injection machine (Station *A*).

Figure 4.3 Moulding Injection Machine (Haishi-machinery.com)

The outcome of station A is a moulded part, which is taken by automated robots to Station *B* (Laser Cutting Machine). This station cuts and sands the edges (Figure 4.4). Finally, the reflector is delivered to the Quality Inspection (Station *V*).

Figure 4.4 Laser Cutting Machine (masscuttingsystems.com)

Table 4-2 Process Plan Car Headlight Manufacturing Line

Station	Code	Part No	Part Name	Operation Description	Workstation	Total time	Precedence
A	10	22	Reflector	Poly Carbonate Resin, Plexiglas and Acrylic are injected into Molding Machin. The robot takes out two reflectors and places them on the conveyor.	A1	28	-
B	15	22	Reflector (Cutting sharp edge and Finishing)	Operator loads reflector. Extra parts are cut and removed. Unloading by the robot.	A2	18	10
V	20	-	Quality Inspection	Quality tests: checking edges, holes, shape, laser wave	P1	12	15
D	30	25	Middle Frame	The chemical mix is sucked in and cast	A3	15	
E	40	6,8,10,11,12,	Set Parking-Light Bulb, Halogen, Xenon Light Ignition, Long-life Bulb	Parts picked up from the shelves. screws used to fasten on middle frame.	A4	38	30
F	50	1,2,3,9,13,15,16,	Lamp Cower Low Beam (cap), Cover High Beam/Daytime Running Lights, Lamp holder, Bulb Socket, Turn Insider, Headlight Repair Kit	Parts in this station are installed with a universal wrench to middle frame equipped by lights. Dust and scrap are cleaned by an air compressor	A5	32	40
M	60		Screws	Finished reflector and middle frame with all parts is moved by conveyor through the station. All parts are visually checked by the operator	A6	25	20, 50
C	70	24	Headlight Housing(back frame)	Back frame is cast in molding machine	A7	23	
I	80	7,13,23	Repair Kit, HVAC Servomotor, Screw-Trox-Bolt With Washer, Socket Wire, Socket Housing	The repair kit is supported by Trox-bolt, screwed to frame. Back cover is picked up and is screwed at each corner. A socket wire is connected	A8	47	70
L	90	4,5,17,18,19	Xenon Control Unit, Directional Light, washer, LED lights & Cover	A seal washer is placed under the frame and closed by screw Washer. LED and its cover is closed on the back frame.	A9	60	80
T	100	20, 21	Projector Lens, Headlight Lens (Front cover)	Unfinished Headlight, Projector Lens, and headlight lens are picked up and placed on the heating machine and glued	A10	26	60, 90
W	110	-	Quality inspection, Labeling(sticker)	approved function of products, box on pallet filled with finished products	P2	14	100

Middle frame and Cover are moulded at Stations *D* and *C* respectively. Light Set, Kits, Holders, and Socket are connected to Middle Frame in the Assembly Stations *E* and *F*.

In the first merging assembly station (*M*), the approved reflector (Stations *A*, *B* and *V*) is screwed to the output sub-assembly of Station *F (*Figure 4.5). which Is Middle Frame with Lights, Bulbs and Kit.

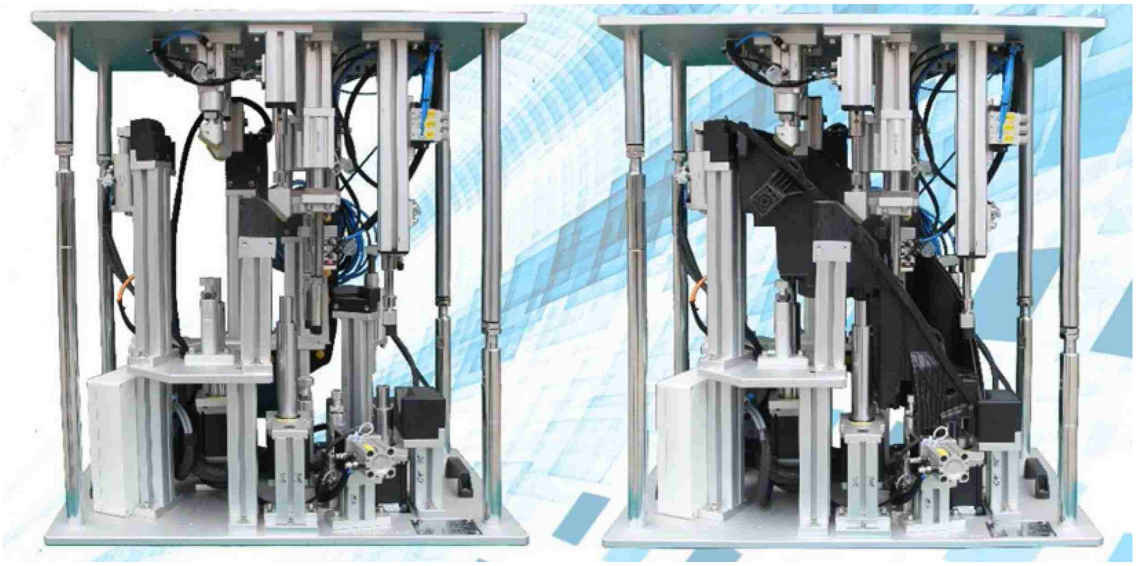


Figure 4.5 Reflector Sub-Assembly Station into Lamp Seat (eooeintl.com)

Parts and modules such as Wires, Control Unit, Sockets and Lights (Figure 4.6) are installed into the moulded Back Frame coming from station C in Stations *I* and *L*.

In the second merging assembly Station *T*, the output from station *M*, which were Reflector and Middle Frame is connected to the Back Frame assembled with lights, HVAC, Kits, Sockets, etc.

Figure 4.6 Projector Lens and Headlight Lens Holder Sub-Assembly Station (eooeintl.com)

Finally, Projector Lens and Headlight Lens (Front cover) are stacked with adhesive under pressure (heat and glue) to the unfinished headlight (Figure 4.7).

Figure 4.7 Plasma Spray and Adhesive Gluing (eooeintl.com)

At the end of line, by running several tests in Quality Inspection station such as Air Tightness (Figure 4.8), Lumen adjusting, Ultraviolet and lights Direction tests, a finished headlight comes out with a label and is ready to ship at Station W. Completed process plan and the rest of stations are provided in Appendices 3 and 4 respectively.

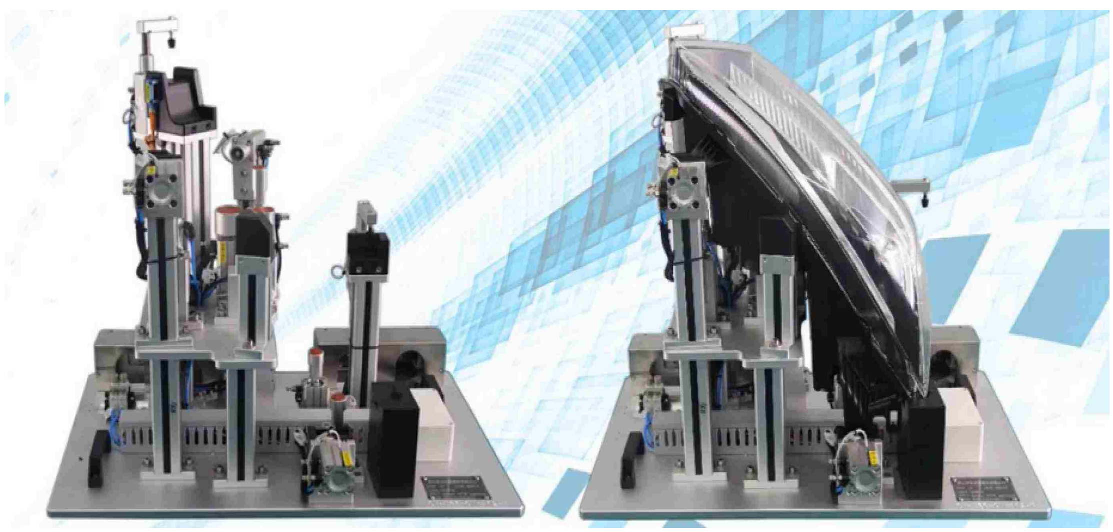


Figure 4.8 Air Tightness Test (Quality Inspection) (eooeintl.com)

4.3.Line Balancing Scenarios

Manufacturing line is composed of different stations. Each station performs at a specific rate/speed. Line balancing is one of the manufacturing functions that try to divide works equally across the production flow. The advantages of line balancing are productivity improvement, and reductions in Work in Process (WIP), labour idle time, bottlenecks, etc. Line balancing can be achieved in different ways such as combining a couple of stations into one station, and adding parallel stations and/or buffers. In this research, seven scenarios are proposed and tested to balance the considered manufacturing line.

4.3.1. Scenario 1 - No Buffer or Parallel Identical Station

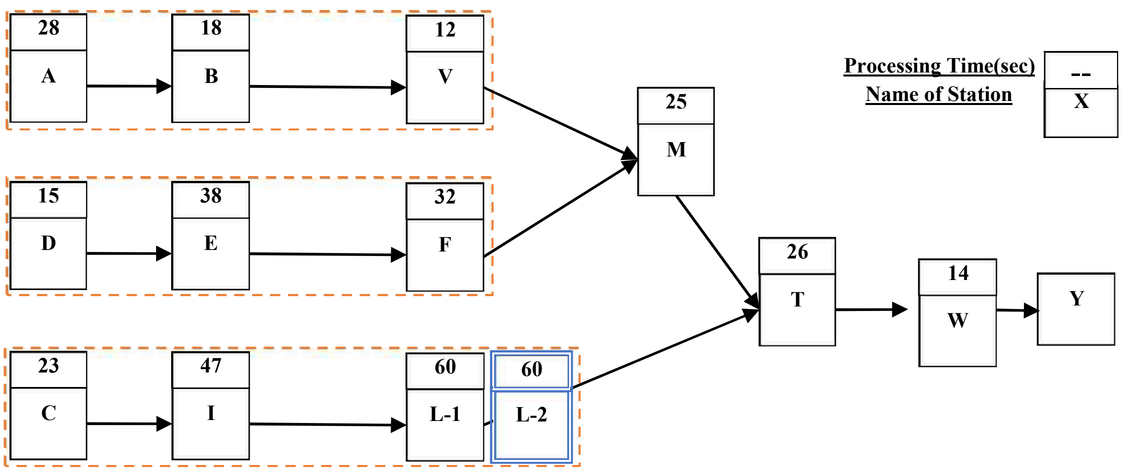
The first scenario is a simple flow line as shown in figure 4.1. There is no buffer or parallel station. The main Max-Plus Algebra equation of this scenario is the same as equation 2.8 and is as below:

$$\boldsymbol{X_k = \hat{A}X_{k-1} \oplus \hat{B}U_k}$$

Numerical matrices for $\hat{A}$ and $\hat{B}$ are provided in Appendix 2.

4.3.2. Scenario 2 - Parallel Stations Used at Bottleneck (Station L)

In this scenario, the bottleneck station is detected and an identical parallel station is added to the bottleneck station in the manufacturing line. Station L is the bottleneck and a parallel identical station at station L (L-1 and L-2) is added (Figure 4.9,).



A	Reflector Moulding Machine	B	Laser Cutting Machine	V	Quality Inspection
D	Middle Frame Moulding Machine	E	Assembly Station (Lights and Bulbs)	F	Assembly Station (Covers, Socket and Kit)
C	Back Frame Moulding Machine	I	Assembly Station (HVAC, Sockets, Bolt and Washer)	L-1, L-2	Assembly Station (Control Unit, LED and Directional Light)
M	Assembly Station (Reflector to Middle Frame)	T	Assembly Station for (Projector Lens and Cover)	W	Final Quality Inspection and Labeling

Figure 4.9 The Structure of Scenario 2, Parallel Identical Station

The main equation is as follows. Matrix $\hat{B}$ has to deal with k parts (the part that is leaving station i is unchanged). The only change is to Matrix $\hat{A}$, which represents upcoming parts as k-1 in regards to the corresponding column of the parallel station. Therefore, the 10th column of the matrix $\hat{A}$, which is representative of Station L, is replaced by ε, which is minus infinity (-∞) in Max-Plus Algebra. Finally, an additional matrix ($\widehat{AP_L}$), related to those parallel stations, is inserted and multiplied by the vector X_{k-2}. $\widehat{AP_L}$ and indicates the 10th column of $\hat{A}$.

$$\boldsymbol{X_k = \hat{A}X_{k-1} \oplus \hat{B}U_k \oplus \widehat{AP_L}X_{k-2}}$$

$$\hat{A} = \begin{bmatrix}
t_A & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
t_A^2 & t_B & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
t_A^2 t_B & t_B^2 & t_V & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & t_D & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & t_E & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_F & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
t_A^2 t_B t_V & t_B^2 t_V & t_V^2 & t_D^2 & t_E^2 & t_F^2 & t_M & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_C & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_C^2 & t_I & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_C^2 t_I & t_I^2 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
t_A^2 t_B t_V t_M & t_B^2 t_V t_M & t_V^2 t_M & t_D^2 t_M & t_E^2 t_M & t_F^2 t_M & t_M^2 & t_C^2 t_I t_L & t_I^2 t_L & \varepsilon & t_T & \varepsilon & \varepsilon \\
t_A^2 t_B t_V t_M t_T & t_B^2 t_V t_M t_T & t_V^2 t_M t_T & t_D^2 t_M t_T & t_E^2 t_M t_T & t_F^2 t_M t_T & t_M^2 t_T & t_C^2 t_I t_L t_T & t_I^2 t_L t_T & \varepsilon & t_T^2 & t_W & \varepsilon \\
t_A^2 t_B t_V t_M t_T t_W & t_B^2 t_V t_M t_T t_W & t_V^2 t_M t_T t_W & t_D^2 t_M t_T t_W & t_E^2 t_M t_T t_W & t_F^2 t_M t_T t_W & t_M^2 t_T t_W & t_C^2 t_I t_L t_T t_W & t_I^2 t_L t_T t_W & \varepsilon & t_T^2 t_W & t_W^2 & t_Y
\end{bmatrix}$$

$$\widehat{AP_L} = \begin{bmatrix}
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
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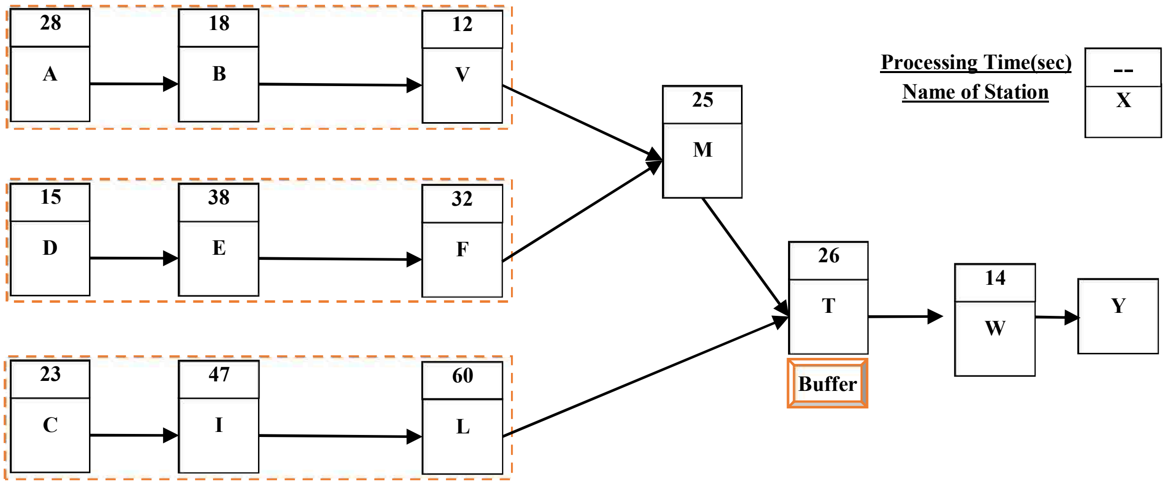
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_L & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_L^2 & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_L^2 t_T & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_L^2 t_T t_W & \varepsilon & \varepsilon & \varepsilon
\end{bmatrix}$$

4.3.3. Scenario 3 - Buffer is added at Station T

The manufacturing system uses a buffer to avoid variation in the manufacturing process. Having a buffer ensures manufacturers that there are enough supplies to run the manufacturing line smoothly without interruption for shortage of parts. Generally, there are three kind of buffers, such as raw material, finished product and unfinished product (WIP). By buffering raw material and finished products, manufacturers guard against fluctuations in the supply chain. Unfinished product inventory is usually placed at the station whose upstream stations frequently break down or have limited capacity. In other words, by adding a buffer to those stations, the operators will not experience any idle time during production. Based on the result of Scenario 1, Station L is the bottleneck. Hence, a buffer of 10 parts is added to station T, which is placed right after the bottleneck (Figure 4.10).



A	Reflector Moulding Machine	B	Laser Cutting Machine	V	Quality Inspection
D	Middle Frame Molding Machine	E	Assembly Station (Lights and Bulbs)	F	Assembly Station (Covers, Socket and Kit)
C	Back Frame Molding Machine	I	Assembly Station (HVAC, Sockets, Bolt and Washer)	L	Assembly Station (Control Unit, LED and Directional Light)
M	Assembly Station (Reflector to Middle Frame)	T	Assembly Station (Projector Lens and Cover)	W	Final Quality Inspection and Labeling

Figure 4.10 Structure of Scenario 3, Buffer at Station T

Matrices $\hat{A}$, $\hat{B}$ are unchanged. However, an additional matrix, $\widehat{AB_T}$ with the same dimension as matrix $\hat{A}$, is inserted. Matrix $\widehat{AB_T}$ is multiplied by vector X_{k-b}, where (b=10) is the buffer size.

$$\boldsymbol{X_k = \hat{A}X_{k-1} \oplus \hat{B}U_k \oplus \widehat{AB_T}X_{k-10}}$$

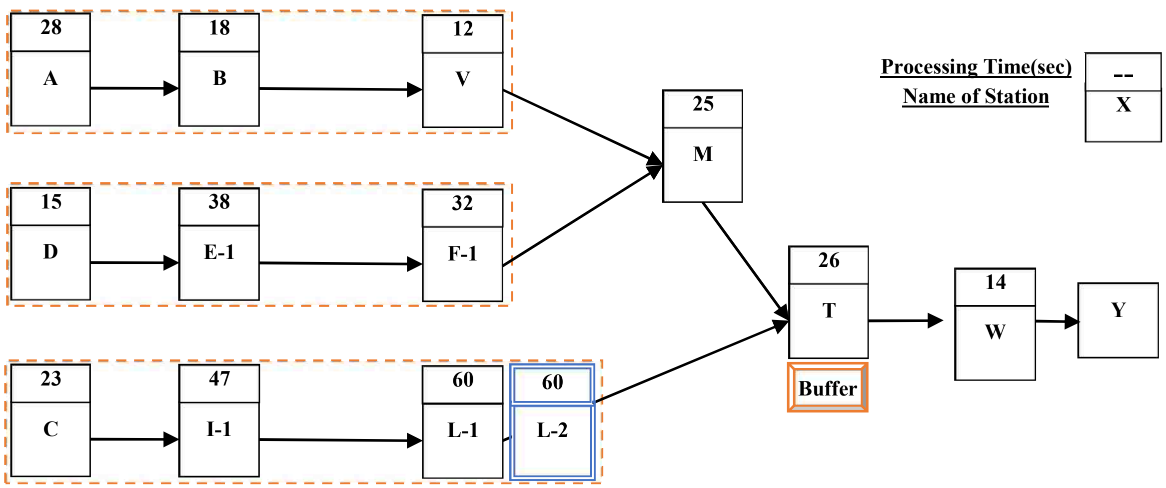
$$\widehat{AB_T} = \begin{bmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & e & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & e & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_L \oplus t_M & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & (t_L \oplus t_M)t_T & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & (t_L \oplus t_M)t_T t_W & \varepsilon & \varepsilon \end{bmatrix}$$

4.3.4. Scenario 4 - Buffer and Parallel Station

Scenario 4 is the combination of Scenarios 2 and 3 (Figure 4.11). In other words, an identical parallel station and buffer are added to Station L and T respectively. The main Max-Plus equation for this scenario is as follows:

$$\boldsymbol{X_k = \hat{A}X_{k-1} \oplus \hat{B}U_k \oplus \widehat{AP}_L X_{k-2} \oplus \widehat{AB}_T X_{k-10}}$$

The structure of matrices $\boldsymbol{\hat{A}}$, $\boldsymbol{\hat{B}, \widehat{AP}_L, and\ \widehat{AB}_T}$ are the same as those were demonstrated in scenarios 2 and 3.

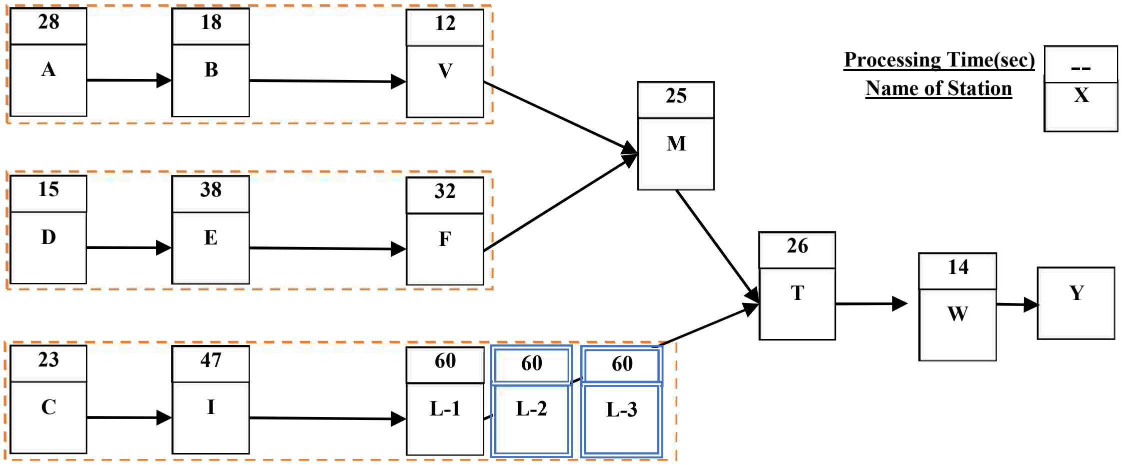


A	Reflector Moulding Machine	B	Laser Cutting Machine	V	Quality Inspection
D	Middle Frame Moulding Machine	E	Assembly Station (Lights and Bulbs)	F	Assembly Station (Covers, Socket and Kit)
C	Back Frame Moulding Machine	I	Assembly Station (HVAC, Sockets, Bolt and Washer)	L-1, L-2	Assembly Station (Control Unit, LED and Directional Light)
M	Assembly Station (Reflector to Middle Frame)	T	Assembly Station (Projector Lens and Cover)	W	Final Quality Inspection and Labeling

Figure 4.11 Structure of Scenario 4, Combination of Parallel Identical Station and Buffer

4.3.5. Scenario 5 - Three Parallel Stations at Bottleneck

Adding a parallel station to the bottleneck will not always lead to improving efficiency and reduction in idle time. This scenario is an extension of the second scenario, by having three parallel stations at the bottleneck (Figure 4.12). The main Max-Plus equation and matrices are same as the scenario 2, with a difference at the index of X changed to k-3.



A	Reflector Moulding Machine	B	Laser Cutting Machine	V	Quality Inspection
D	Middle Frame Moulding Machine	E	Assembly Station (Lights and Bulbs)	F	Assembly Station (Covers, Socket and Kit)
C	Back Frame Moulding Machine	I	Assembly Station (HVAC, Sockets, Bolt and Washer)	L-1, 2,3	Assembly Station (Control Unit, LED and Directional Light)
M	Assembly Station (Reflector to Middle Frame)	T	Assembly Station (Projector Lens and Cover)	W	Final Quality Inspection and Labeling

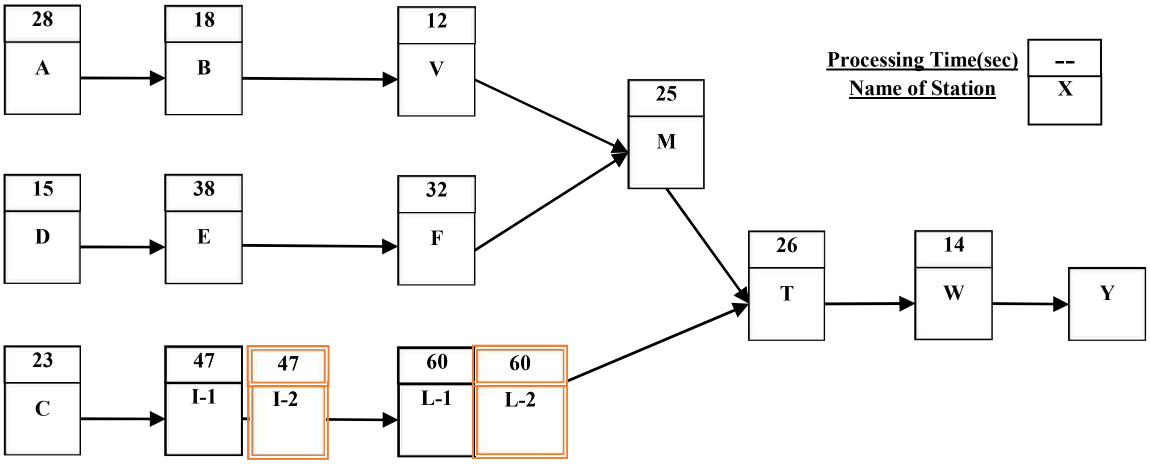
Figure 4.12 Structure of Scenario 5 Three Parallel Identical Stations

$$\boldsymbol{X_k = \hat{A}X_{k-1} \oplus \hat{B}U_k \oplus \widehat{AP}_L X_{k-3}}$$

This scenario is chosen to show that having parallel stations at a bottleneck is beneficial; otherwise, it cannot improve efficiency parameters.

4.3.6. Scenario 6 - Parallel Stations in Two Different Stations

By reconsidering Scenario 2, Station I is also found to be a bottleneck. Therefore, two parallel stations are added in Stations I and L (Figure 4.13).



A	Reflector Moulding Machine	B	Laser Cutting Machine	V	Quality Inspection
D	Middle Frame Moulding Machine	E	Assembly Station (Lights and Bulbs)	F	Assembly Station (Covers, Socket and Kit)
C	Back Frame Moulding Machine	I-1, I-2	Assembly Station (HVAC, Sockets, Bolt and Washer)	L-1, L-2	Assembly Station (Control Unit, LED and Directional Light)
M	Assembly Station (Reflector to Middle Frame)	T	Assembly Station (Projector Lens and Cover)	W	Final Quality Inspection and Labeling

Figure 4.13 Structure of Scenario 6 Two Parallel Identical Station at I and L

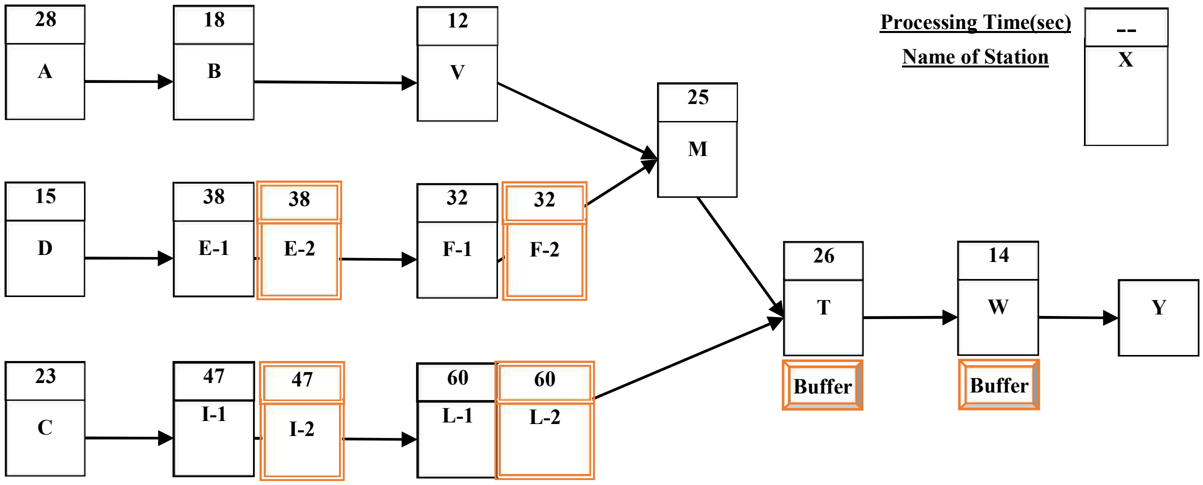
Similar to Scenario 2, matrix $\hat{B}$ is unchanged. However, two columns of the matrix $\hat{A}$, which correspond to the parallel stations, are changed. In this scenario, these columns are columns 9 and 10, which represent Stations I and L. Finally $\widehat{AP}_L$ should be constructed by two equivalent columns for Stations I and L as below:

$$\boldsymbol{X_k = \hat{A}X_{k-1} \oplus \hat{B}U_k \oplus \widehat{AP}_{IL}X_{k-2}}$$

$$\widehat{AP}_L = \begin{bmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_I & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_I^2 & t_L & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_I^2 t_L & t_L^2 & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_I^2 t_L t_T & t_L^2 t_T & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_I^2 t_L t_T t_W & t_L^2 t_T t_V & \varepsilon & \varepsilon & \varepsilon \end{bmatrix}$$

4.3.7. Scenario 7 – Combination of Four Parallel Stations and Two Buffer

This scenario is the extension of Scenarios 6 and 3. Identical stations are added to Stations E, F, I, L and five parts in buffers of Stations T and W (Figure 4.14).



A	Reflector Moulding Machine	B	Laser Cutting Machine	V	Quality Inspection
D	Middle Frame Moulding Machine	E-1, E-2	Assembly Station (Lights and Bulbs)	F-1, F-2	Assembly Station (Covers, Socket and Kit)
C	Back Frame Moulding Machine	I-1, I-2	Assembly Station (HVAC, Sockets, Bolt and Washer)	L-1, L-2	Assembly Station (Control Unit, LED and Directional Light)
M	Assembly Station (Reflector to Middle Frame)	T	Assembly Station (Projector Lens and Cover)	W	Final Quality Inspection and Labeling

Figure 4.14 Structure of Scenario 7 the Combination of Four Parallel Identical Station and Two Buffers

Each parallel identical station or buffer participates in the main Max-Plus equation of this scenario as below by an extra matrix:

$$\boldsymbol{X_k = \hat{A}X_{k-1} \oplus \hat{B}U_k \oplus \widehat{AP_E}X_{k-2} \oplus \widehat{AP_F}X_{k-2} \oplus \widehat{AP_I}X_{k-2} \oplus \widehat{AP_L}X_{k-2} \oplus \widehat{AB_T}X_{k-5} \oplus \widehat{AB_W}X_{k-5}}$$

The above equation can be simplified by combining $\widehat{\mathrm{AP}}$ matrices which are multiplied by the same vector index. This state can be applied to $\widehat{\mathrm{AB}}$ as well. Thus:

$$\boldsymbol{X_k = \hat{A}X_{k-1} \oplus \hat{B}U_k \oplus \widehat{AP_{EFIL}}X_{k-2} \oplus \widehat{AB_{TW}}X_{k-5}}$$

For those parallel stations, $\boldsymbol{\hat{A}}$ receives ε in the correspondent columns, instead of $\boldsymbol{\widehat{AP_{EFIL}}}$ including those stations.

$$\widehat{AP_{EFIL}} = \begin{bmatrix}
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & t_E & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_F & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & t_E^2 & t_F^2 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_I & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_I^2 & t_L & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & t_E^2 t_M & t_F^2 t_M & \varepsilon & \varepsilon & \varepsilon & t_I^2 t_L & t_L^2 & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & t_E^2 t_M t_T & t_F^2 t_M t_T & \varepsilon & \varepsilon & \varepsilon & t_I^2 t_L t_T & t_L^2 t_T & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & t_E^2 t_M t_T t_W & t_F^2 t_M t_T t_W & \varepsilon & \varepsilon & \varepsilon & t_I^2 t_L t_T t_W & t_L^2 t_T t_W & \varepsilon & \varepsilon & \varepsilon
\end{bmatrix}$$

Also, the $\widehat{AB_{TW}}$ the structure covers two stations (T and W) and index of X is k-5.

$$\widehat{AB_{T,W}} = \begin{bmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & e & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & e & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_L \oplus t_M & e & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & (t_L \oplus t_M)t_T & t_T & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & (t_L \oplus t_M)t_T t_W & t_T t_W & \varepsilon \end{bmatrix}$$

4.4. Analysis and Results

In the previous section, seven scenarios were introduced to consider different combinations of the flow line, parallel stations, and buffers. These scenarios can be extended/modified into any kind of manufacturing flow line. Manufacturers can use those scenarios to simulate and optimize their manufacturing lines and also to find out how well the line is balanced. To do that, they need to study the manufacturing line performance to find out which stations are bottlenecks and which stations require a buffer. Parallel stations can be added to a bottleneck, and buffers can be allocated to stations whose upstream stations are a bottleneck.

In this section, the defined scenarios are tested and analyzed to find out the best way to balance the Car Headlight manufacturing line effectively. The scenarios are tested for 30 finished Car Headlights using Matlab to solve the corresponding Max-Plus Algebra models. The considered manufacturing systems assumptions were mentioned in section 3.2.3.

To analyze the results, Key Performance Indicators (KPIs) are defined and applied including Cycle Time, Average Delivery Rate, Total Processing Lead Time, Stations' Utilization Rate, Idle Time and System Efficiency. Finally, a Financial Analysis is carried out and the results are discussed to evaluate these scenarios.

4.4.1. Cycle Time (CT) and Average System Delivery Rate

Cycle Time (CT) is defined as a period of time to make a product(s), such as a part, set, pack, bundle or batch in a manufacturing system. Cycle time is different than the process time which only includes the time instance at a particular station to process one part. However, Cycle Time (CT) accounts for the total time taken for part(s) to be processed at entire manufacturing line from first to the last stations. The System Delivery Rate measures the capability of a system to deliver a finished part over the time. Average Delivery Rate (ADR) of a system is defined as the average time taken from the start to the end of the manufacturing/assembly process.

To reduce Cycle Time of the system, it is crucial to recognize which station(s) is a bottleneck by finding out the critical path. In a manufacturing system, a bottleneck is a machine/process that slows down or reduces the capacity of the production line due to its long process time and/or limited capacity. The group of stations that the bottleneck belongs to is called Critical Path (CP), which has the longest processing time in total. All other machines/processes must wait for the bottleneck to complete its process. Therefore, the bottleneck can definitely impact system key performance indicators. Moreover, the critical path (CP) reorganization is necessary to calculate the required number of parallel stations, and to find out which station(s) requires a buffer.

"When would all 30 parts be completed in the manufacturing/assembly system?" is the critical question that should be answered by system designers while modelling and examining results of different scenarios. The Cycle Time for 30 parts in all seven scenarios are demonstrated in Figure 4.15. The shortest CT (736 seconds) belongs to the scenario 7 and the longest (1910 second) relates to the scenario 1. Principally, additional stations or buffers should be assigned only to a bottleneck station on and critical path; otherwise, not only can the system's indicators not be improved but also some system parameters will be negatively affected. For instance, Scenario 5 has one more station, and its corresponding cost, in comparison to Scenario 2, whereas CT is equal for both scenarios.

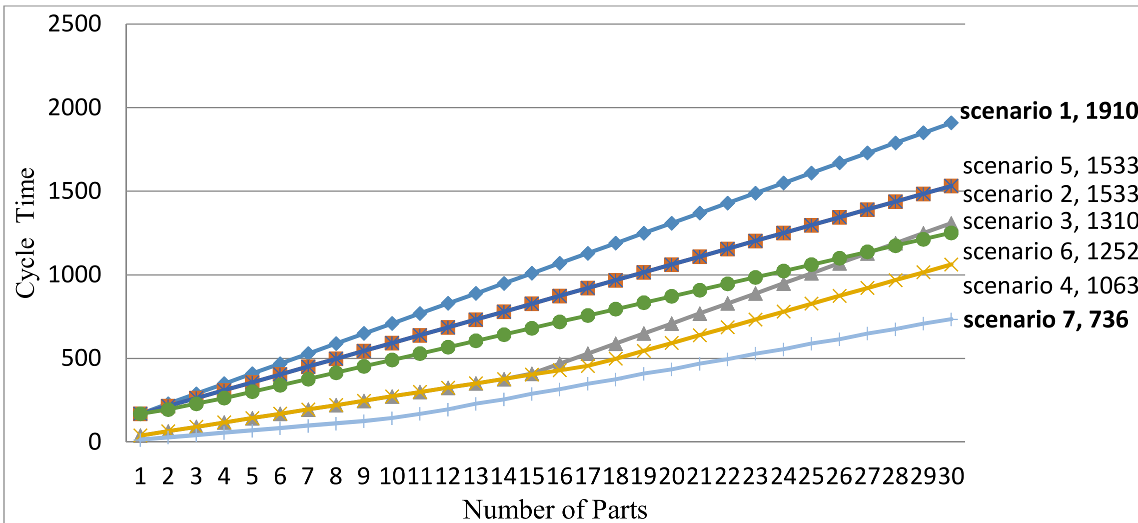


Figure 4.15 Cycle Time of batch (30 parts) in different scenarios

According to Table 4-3, the first and seventh scenarios have the most and least Average Delivery Rate for 30 parts with 63.7 and 24.5 seconds, respectively. This proves the beneficial impact of adding the required number of parallel stations at bottlenecks and buffers at stations whose upstream is the bottleneck.

Based on the first scenario outcomes, station *L* is identified as a bottleneck. As for Scenario 2, by adding one parallel station to station *L*, Average Delivery Rate (ADR) is reduced to 51.1 seconds and the bottleneck is moved to station *I*. Before adding more identical stations, buffer allocation is tested in the third scenario. Station *T* is recognized as requiring a buffer due to its upstream process, station *L,* which is a bottleneck. As the third scenario indicates, the buffer has a significant impact on ADR reduction (43.7 seconds compared with the scenario 1; 63.7 seconds).

Once the effect of the buffer is proved, the combination of adding a parallel station to the bottleneck which is station *L*, and allocating a buffer to station *T* is tested. Based on the fourth scenario, the ADR is lowered down to 35.4 seconds.

Table 4-3 General Findings of modelling system scenarios

Scenario No.	**The First Output**	**Average Delivery Rate**	**Bottleneck**	**Critical Path Stations**	**No. Stations**	**No. of Parallel Station**	**Stations with Buffer**
1	170s	63.7	L	CILTW	12	-	-
2	170s	51.1	I	CILTW	13	L	-
3	40s	43.7	L	CILTW	12	-	T
4	40s	35.4	I	CILTW	13	L	T
5	170s	51.1	I	CILTW	14	L(2)	-
6	170s	41.7	E	DEFMTW	14	I,L	-
7	14s	24.5	L	CILTW	16	E (2), F (2), I(2), L(2)	T, W

In scenario 5, adding one more identical station to station *L* is assessed. In this scenario, the buffer is not considered to enable us to evaluate the impact of just adding more parallel stations. Scenario 5 shows ADR, and delivering time of the first output are increased substantially to 51.1 and 170 seconds respectively. For all five scenarios, the critical path is the same chain of stations C, I, L, T, and W (CILTW). By comparing these five scenarios, it can be concluded that adding buffer(s) has a significant impact on the reduction of first output time.

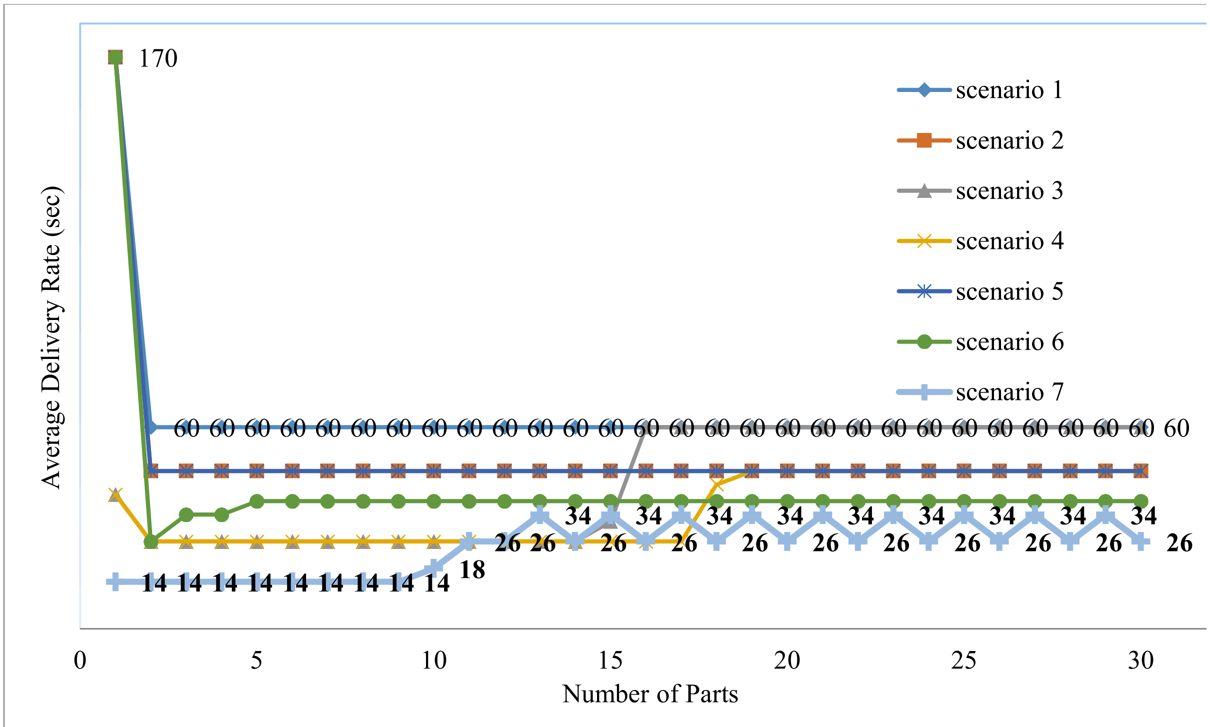


Figure 4.16 Capability of System to Deliver a Finished Part in Seconds

In Scenario 6, one parallel station is added to stations *L* and *I* each, which are recognized as a bottleneck. Subsequently, the bottleneck is moved to station *E* and ADR is reduced to 41.7 seconds, which is lower than previous scenarios. Also, the critical path is altered to become DEFMTW. As shown in Figure 4.16, the first headlight output is delivered at the 40th second in Scenario 6. However, buffer allocation alone will not lead to having a substantial reduction in ADR. Based on Scenario 4, it is clear that the combination of parallel station and buffer is more effective in the reduction of ADR.

Hence, by evaluating all previous scenarios, Scenario 7 is designed. In this scenario, parallel stations are added to stations *E*, *F*, *I*, and *L*; also buffers of five parts are added to each of stations *T* and *W*. The results are superior. The significant reductions in ADR of 62% from 63.7 to 24.5 seconds and the first output delivery decrease from 170 to the 14th second prove that the appropriate combination of parallel stations and buffers will result in more balanced production line flow in the manufacturing system.

Approximately, after the 20th part, when the effect of buffers is gone, the Delivery Rate of scenarios would be constant and dependent to the minimum processing time (30 seconds) of the manufacturing line (station L). In other words, by adding just enough number of parallel stations, the capability of the system delivery rate is doubled (Deliver a part every 30 seconds in scenario 7 in contrast deliver a part every 60 seconds in scenario 1). The bottleneck returned to station L and the critical path became CILTW.

4.4.2. Total Processing Lead Time

The processing /production lead time of station i, (PLT_i) is the time between the initiation and completion of a process at the station. Therefore, the total processing time of station *i* is calculated as follows:

$$PLT_i = Completion\ time\ of\ Last\ Part\ at\ Station\ i - Starting\ Time\ of\ First\ Part\ at\ Station\ i$$

based on Max-Plus definition $PLT_i = X_i(k) + t_i - X_i(1)$ where,

k: is the total number of parts,

n: is the total number of stations $i = 1, \dots, n$,

t_i: is the process time of station *i,*

$X_i(k)$: is the time instants when station *i* starts to process part *k*,

The total processing lead time of the system (PLT_T) is defined as the sum of all stations' processing lead times.

$$PLT_T = \sum_{i=1}^{n} PLT_i$$

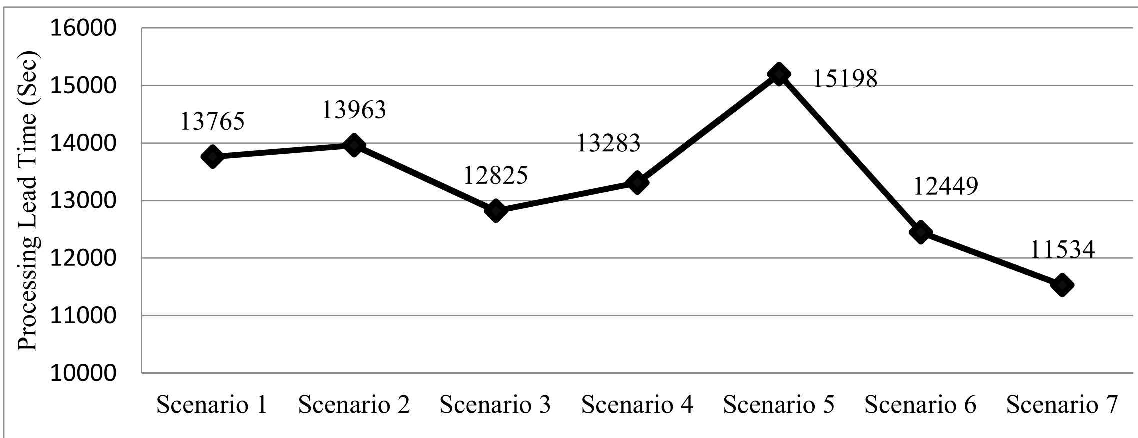


Figure 4.17 Total Processing Lead Time of Scenarios (Sec) for 30 parts

As shown in Figure 4.17, Scenario 5 has the longest PLT, which is 15,198 seconds. It proves that having more parallel stations will not always improve the performance of the system. Parallel stations are required to be added to the bottleneck station. Furthermore, Scenario 7 has the shortest PLT, 11,534 seconds. This predicates that adding parallel stations to the bottlenecks and buffers to the stations whose upstream stations are bottlenecks will lead to balancing the manufacturing system efficiently.

4.4.3. Stations Utilization Rate (UR)

Utilization Rate of station i (UR_i) measures the proportion of the available time at which station i is being used. Utilization rate compares the operation time of the station to the available time from the start to the end of process.

$$UR_i = \frac{Total\ Processing\ Lead\ Time\ of\ Station\ i}{Total\ Processing\ Lead\ Time\ of\ System}$$

$$UR_i = \frac{X_i(k) + t_i - X_i(1)}{X_n(k) + t_n}$$

In addition, the Utilization Rate of the system (UR_T) is defined as the average of the utilization rate of all stations.

$$UR_T = \frac{\sum_{i=1}^{n} UR_i}{n}$$

Figure 4.18 displays scenarios' Average Stations Utilization Rate. Scenario 4 results in the best average stations utilization rate with 75% and the lowest rate belongs to the simple flow line without buffer or parallel station (scenario 1), being 60%. By way of explanation, adding a parallel station and a buffer results in the average stations utilization rate of the system is improved by 15%.

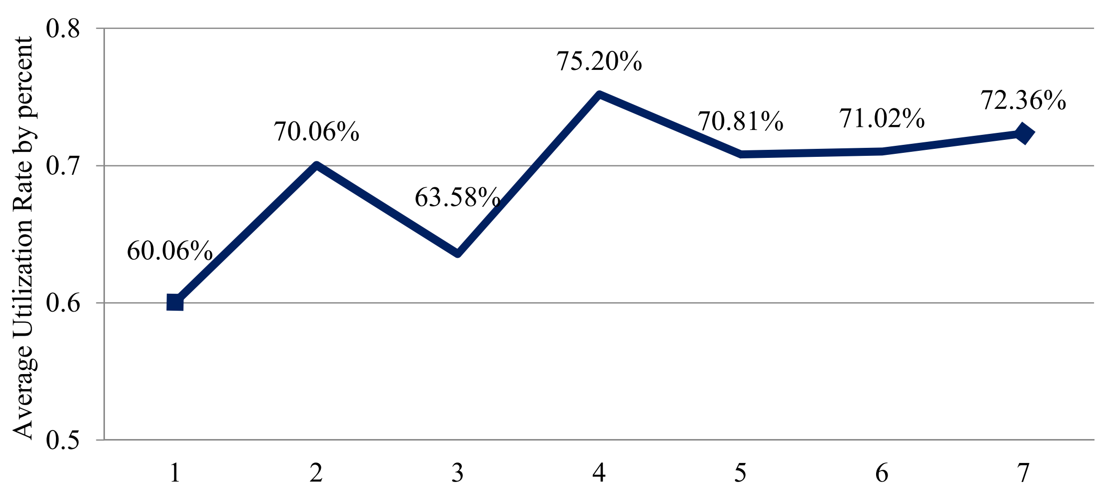


Figure 4.18 Average Stations Utilization Rate of different scenarios

Buffers support utilization rate by preventing delay or idle time in the early times as well as having a constant and smooth flow line. In contrast, the analysis shows that having more parallel stations cannot guarantee the improvement in utilization rate. In spite of all the previous analyses, Scenario 4 has the best average stations utilization rate even in contradiction of having fewer stations and buffers than Scenarios 6 and 7.

4.4.4. Idle Time and Efficiency of the System

No manufacturing system runs with 100% efficiency. There is some unproductive time when either operators or machines will not to be used. Idle time is any period of time at which the production is not engaged. Idle time is inevitable, but manufacturers try to minimize its impact on the system. Idle time (IDT) is calculated by the subtraction of actual Processing Lead Time (PLT) of a system from available time to produce *k* parts.

If the actual time to produce *k* part at station *i* is defined as:

$$RT_i = k \times t_i$$

Then, actual PLT for *k* parts in the system will be:

$$RT = \sum_{i=1}^{n} RT_i$$

Thus, the idle time of station *i* (IDT_i) and the system (IDT) are calculated as follows:

$$IDT_i = PLT_i - RT_i$$

$$IDT = \sum_{i}^{n} IDT_i,$$

The efficiency of a system is defined as the productive time of available time and is calculated as:

$$System\ Efficiency = 1 - Idle\ Time\ of\ a\ System$$

Station i takes $k \times t_i$ to produce *k* units. For example, station *A* takes $30 \times 28 = 840$ seconds, station B takes $30 \times 18 = 540$ seconds to makes 30 parts and so on. To extend it to the entire production line, 10,140 seconds are required to produce 30 parts. Idle time is in contrast with efficiency. The higher idle time, the lower efficiency. As declared in Figure 4.19, Scenarios 7 and 5 have the best and the least efficiency at 81 and 67 percent, respectively.

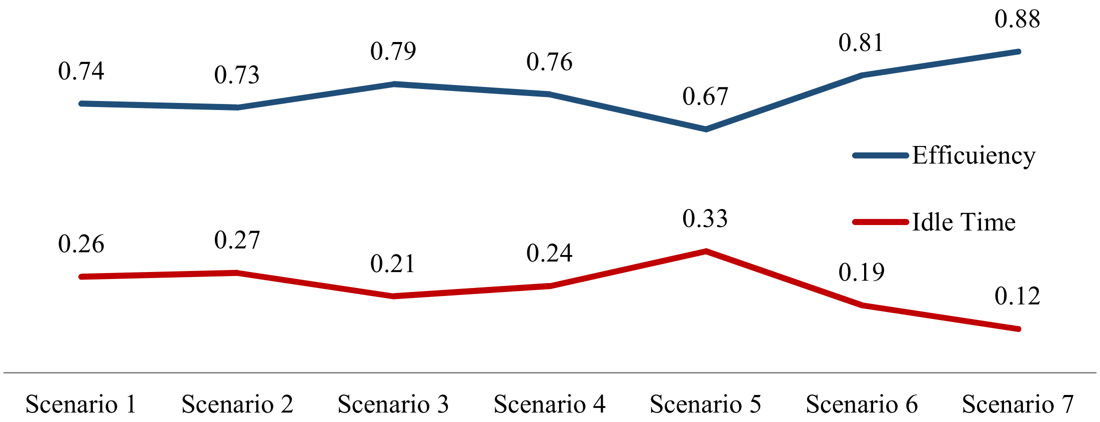


Figure 4.19 Efficiency and Idle Time of Scenarios

4.4.5. Financial Analysis

The manufacturer requires finding in an efficient way to utilize the equipment and operators. To measure the appropriate number of stations and labours, managers take advantage of different financial KPIs. These KPIs help the investors to have a deeper understanding of business. In this research, Manufacturing Cost Per Unit (UC) is used to analyze how much a company's process is efficient and how much overhead costs must be paid to make a finished part.

To calculate the Unit Cost (UC), two vital costs are considered: fixed cost and variable cost. Fixed cost relates to machines and other hardware (MC). The more equipment, the higher cost of maintenance, investment, etc. Furthermore, Unit Labor Cost (ULC) is considered a variable cost, which measures the efficiency of labour. This KPI is defined as the average cost of labour per part and calculated as total hours cost over the total units.

Therefore, the projected unit cost (UC) of the system for d amount of market demand is the summation of total equipment cost, and the total labour cost over market demand and is calculated as below:

$$Cost\ per\ Unit = \frac{Total\ Cost\ of\ Labor + Total\ Cost\ of\ \ Equipment}{Market\ Demand}$$

$$TC = \frac{TMC + LC}{d}$$

Which TMC and LC are calculated as:

$$TMC = MC \times n$$

$$LC = LW \times PLT$$

Where

TMC: is the total equipment cost

MC: is the cost per equipment

n: is the total number of stations

LC: Total Labor Cost

LW: Labor Wage per labour hour

PLT: Total Processing Lead Time to make *d* amount of demand

To analyze the projected financial costs of any proposed scenario financially, these assumptions are considered:

- ❖ Market demand is 300,000 parts per year,
- ❖ Average annual equipment cost of each station is $10,000 and is nonrefundable,
- ❖ Average Labour wage is $60 per hour.

As discussed earlier, the number of stations is modified in each scenario. According to the table (4-4), Scenario 7 has the highest cost of equipment caused by the number of stations. The cost of labour depends on the unit process time. Scenario 5 has the longest unit process time, 8.44 minutes, which leads to the highest labour cost of over two million dollars. In contrast, the seventh scenario with the least unit process time has the lowest labour cost of $1,889,666 and unit labour cost of $6.8.

Table 4-4 Financial Analysis

Scenario No.	***1***	***2***	***3***	***4***	***5***	***6***	***7***
Process time (Unit/Minute)	*7.64*	*7.75*	*7.12*	*7.37*	*8.44*	*6.91*	*6.40*
No Of Station	*12*	*13*	*12*	*13*	*14*	*14*	*16*
Cost Of Station	*120,000*	*130,000*	*120,000*	*130,000*	*140,000*	*140,000*	*160,000*
Cost Of Labor	2,294,166	2,327,167	2,137,500	2,213,833	2,533,000	2,074,833	1,922,333
Cost per Part	*8.0472*	*8.1905*	*7.525*	*7.8127*	*8.91*	*7.3827*	*6.9411*

Since the total cost is the combination of equipment cost and labour cost, any changes to the equipment cost and labour wage may vary the results. However, as long as the proportion of labour cost is higher than the cost of stations, it will play a crucial role in financial results.

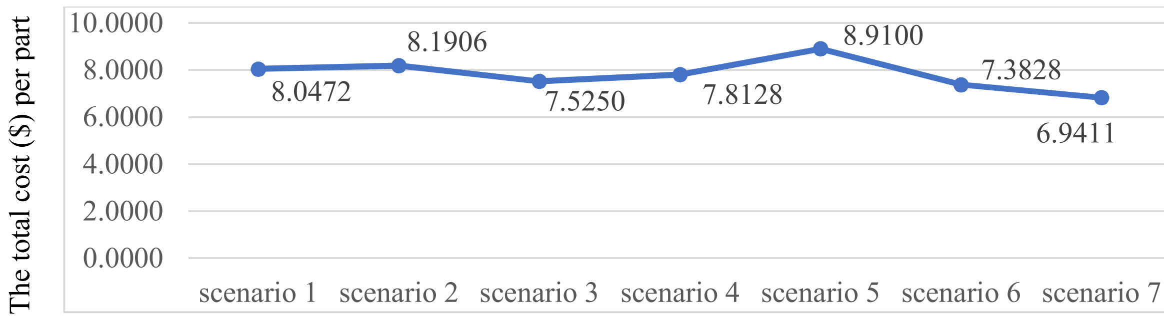


Figure 4.20 The Cost of Finished Part in Scenario ($)

Lastly, evaluating all financial KPIs (Figure 4.20), Scenario 7 has the lowest cost per part ($6.94) and was chosen by decision makers. However, by having any changes, the optimal financial solution will be changed. For instance, if the cost of stations increases from $10k to $100k, the best scenario would be Scenario 4.

4.5.Summary

The manufacturing system is composed of different stations that operate at a specific speed/rate. To avoid labour idleness, reductions in manufacturing costs and improve efficiency, manufacturers try to balance the manufacturing system and divide tasks into equal portions. Line balancing can be done in different ways, such as merging a couple of low processing time stations into one station, adding parallel stations to bottleneck stations with long processing time, and/or adding buffers to stations following the bottleneck stations. In this chapter, different manufacturing scenarios have been modelled and analyzed using Max-Plus Algebra. Car Headlight assembly is chosen as the case study to evaluate the scenarios with different structures.

To balance the system, seven scenarios have been introduced. These scenarios consider different combinations of series stations, parallel stations and buffers. Adding parallel stations and buffers help system designers to improve KPIs. It is crucial to determine which station(s) is a bottleneck and which group of stations represent a critical path (CP) which requires the buffer, and how many parallel stations are required to achieve more balanced production flow. Several manufacturing KPIs such as Cycle Time (CT), Average Delivery Rate, Total Processing Lead Time (PLT), Average Stations Utilization Rate, Idle Time (IDT) and Efficiency were applied. Unit product Cost (UC) was calculated and used as a financial KPI to analyze the system financially.

In conclusion, Scenarios 3 and 4 represent buffer allocation reducing the first output cycle time substantially by 130 seconds. Additionally, in Scenario 7, by having two buffers, the first output comes after the 14^{th} second. Scenario 5 indicates that only adding a parallel station to a bottleneck will enhance system KPIs. Based on the sixth scenario, adding adequate parallel stations lowers the Average Delivery Rate of the entire process substantially.

In addition to these, the combination of parallel stations and buffers lead to the greatest reduction in ADR as well as delivering first output. Scenario 7 is a complete structure, which includes both parallel stations and buffer. The superiority of scenario 7 to other scenarios is proved and it was chosen due to its lowest system delivering first output time (14 seconds), shortest total processing lead time (11,534 seconds), least percentage of idle time (12%), lowest unit cost ($6.9), and highest efficiency (88%). However, Scenario 4 has the best Average Stations Utilization Rate at 75%.

CHAPTER FIVE

DISCRETE EVENT SIMULATION (FLEXSIM) COMPARISON BY MAX-PLUS ALGEBRA

5.1. Introduction

In the previous chapter, seven different system configuration scenarios were modelled and analyzed using Max-Plus Algebra. In this chapter, the same seven scenarios are simulated by FlexSim to compare the performance of modelling the manufacturing system scenarios using Discrete Events Simulation tools versus using the Max-Plus Algebra method. Also, it has been attempted to show how modelling with the simulation method is useful and in what kind of conditions it will help decision makers. The advantages and drawbacks of these two methods are compared at the end of this chapter. It should be noted that similar assumptions, variables and parameters are considered for both methods. The laptop specifications used for modelling the designed DES are as follows:

Windows 10 Enterprise; Processor: Intel® Core™ i7-6700 HQ CPU @ 2.6 GHz; RAM: 16 GB; HDD: 1TB; and Graphics: NVIDIA GeForce GTX 960. Several pieces of software have been used for simulating the discrete manufacturing system scenarios. MATLAB version 8.5 and FlexSim version 18.0.2 that was released on March 5, 2015 and 2018 were applied to solve Max-Plus equations and simulate DES models, respectively.

5.2. FlexSim as a Discrete Event Simulation tool

FlexSim is one of the common Discrete Event Simulation Software packages founded in 1993 and developed by FlexSim software products, Inc. For more than three decades of providing a 3D object-oriented simulation engine, it has been used in many industries and services from manufacturing, transportation, and logistic to the oil industry and mining.

Cai (2015) declared that FlexSim is user-friendly, flexible, Open Graphic Language (Open GL) and analytically accurate software. Because of its accuracy and sensitivity of data running, complex models take a long time and much modelling effort to set and run.

5.3. Analysis

To demonstrate modelling with FlexSim, some results are presented in detail. The software developer believed users' needs are varied. Sometimes, FlexSim is used to generate data and sometimes to make one particular decision (FlexSim.com, n.d.). Therefore, in the considered version, FlexSim provides analysis through the Dashboard charting system as well as the possibility to export data to other tools such as Excel and Tableau. One drawback of graphical outputs is its ambiguity and inflexibility. The available version of the software does not allow customizing and manipulating figures efficiently based on the user's interests. Therefore, the Data Export module is inserted in FlexSim to work with data in a flexible and efficient way. The dashboard charting system has two different parts: statistics collector and calculator table. In this Section, the main scheme of each scenarios' layout is presented. In addition, Gantt charts are used to extract the simulation data such as cycle time, delivery rate, average stations utilization rate, system efficiency and idle time. By using Gantt charts, we would be able to observe how smooth the manufacturing line is; and how often the stations need to wait for downstream to finish their process and pass the unfinished part. In the following, the related outputs and results obtained by FlexSim for the previously used seven scenarios tested for 30 parts are demonstrated.

5.3.1. Scenario 1- No Buffer or Parallel Identical Station

Figure 5.1 represents the simulation layout of the first scenario. As it can be seen, the total simulation run time is 1910 seconds that is the same as cycle time obtained by Max-Plus. Tables 5-1 and 5-2 summarize the results obtained by using FlexSim and Max-Plus. Both models report the same KPIs output such as Average Stations Utilization Rate (60.06%), Total Processing Lead Time (13765 sec) and System Efficiency (74%) and System Idle time (26%).

Table 5-1 FlexSim Data Transferred into The Excel for Scenario 1

Station	Start Time	End Time	Lead Time	Part	Process Time(Sec)	Efficient Time(Sec)	Idle Time(Sec)	Utilization Rate
A	0	840	840	30	28	840	1070	43.98%
B	28	858	830	30	18	540	1370	43.46%
V	46	870	824	30	12	360	1550	43.14%
D`	0	450	450	30	15	450	1460	23.56%
E	15	1155	1140	30	38	1140	770	59.69%
F	53	1187	1134	30	32	960	950	59.37%
M	85	1212	1127	30	25	750	1160	59.01%
C	0	690	690	30	23	690	1220	36.13%
I	23	1433	1410	30	47	1410	500	73.82%
L	70	1870	1800	30	60	1800	110	94.24%
T	130	1896	1766	30	26	780	1130	92.46%
W	156	1910	1754	30	14	420	1490	91.83%

The First Output (Sec)	170	**Cycle Time (Sec)**	1910	**Average Stations Utilization Rate**	60.06%
Average Delivery Rate (Sec)	63.7	**Total Processing Lead Time (Sec)**	13765	**System Efficiency**	0.74
				System Idle Time	0.26

Table 5-2 KPIs Outputs for Scenario 1 Obtained by Max-Plus Model

The First Output(Sec)	170	**Cycle Time (Sec)**	1910	**Average Station Utilization Rate**	60.06%
Average Delivery Rate (Sec)	63.7	**Total Processing Lead Time (Sec)**	13765	**System Efficiency**	0.74
				System Idle Time	0.26

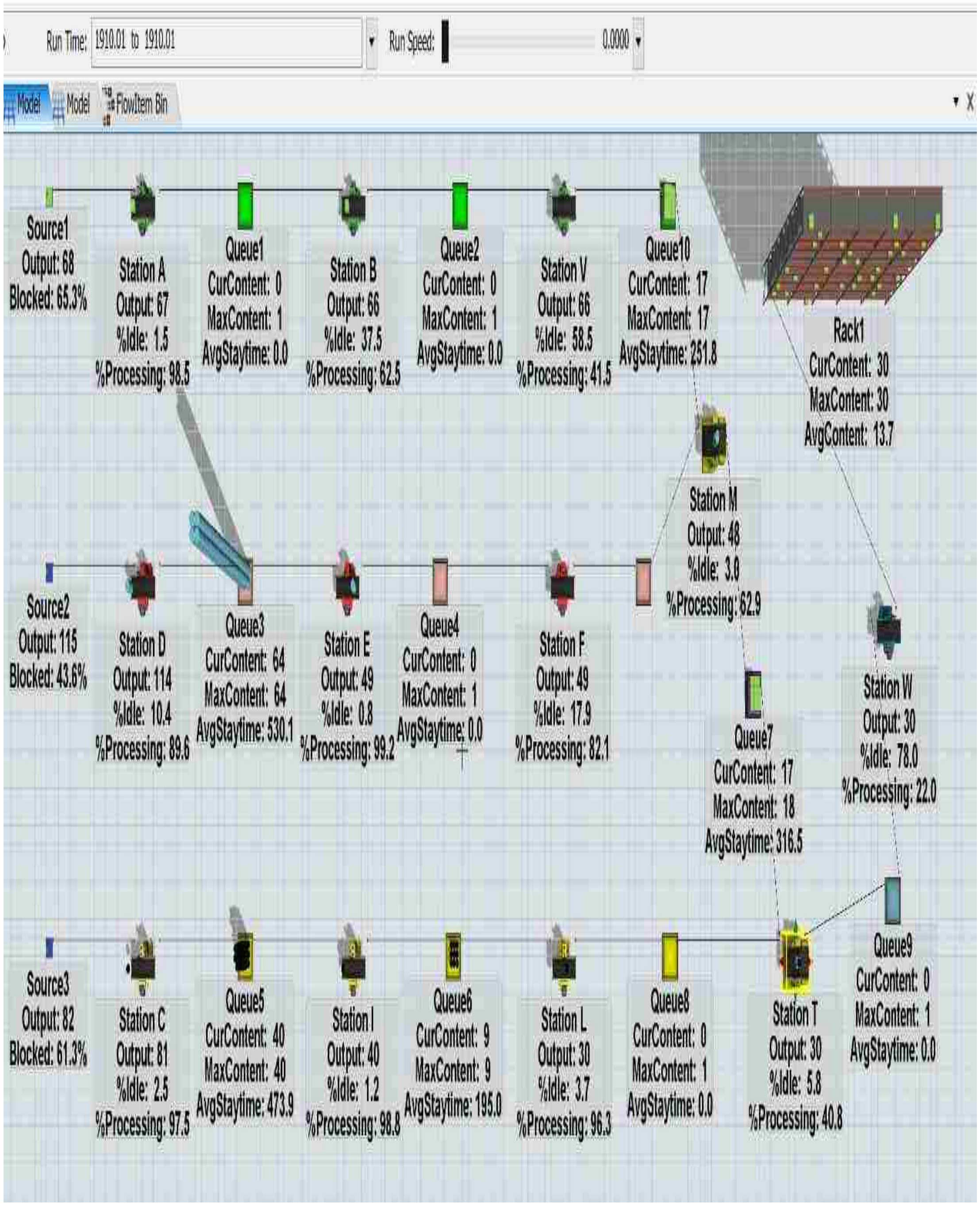


Figure 5.1 Final Results for Scenario 1

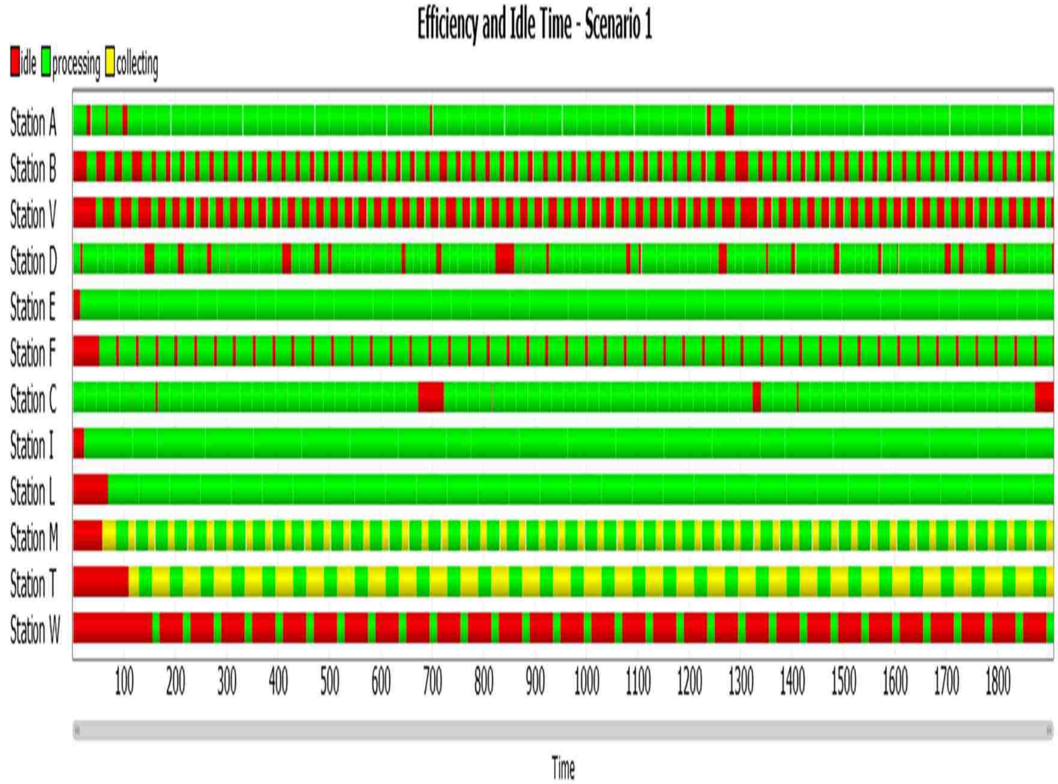


Figure 5.2 FlexSim Gantt Chart for Scenario 1

Scenario 2- Two Parallel Stations at Stations L

As it is shown in Figure 5.3, the total simulation run time is the same as cycle time obtained by Max-Plus,1533 sec. The simulation model has verified KPIs output of Max-Plus model by obtaining 70.06% as Average Stations Utilization Rate, 13963 seconds for Total Processing Lead Time, 73% and 27% as System Efficiency and Idle Time, respectively More details are represented in Tables 5-3 and 5-4.

Table 5-3 FlexSim Data Transferred into The Excel for Scenario 2

Station	Start Time	End Time	Lead Time	Part	Process Time(Sec)	Efficient Time(Sec)	Idle Time(Sec)	Utilization Rate
A	0	840	840	30	28	840	693	54.79%
B	28	858	830	30	18	540	993	54.14%
V	46	870	824	30	12	360	1173	53.75%
D`	0	450	450	30	15	450	1083	29.35%
E	15	1155	1140	30	38	1140	393	74.36%
F	53	1187	1134	30	32	960	573	73.97%
M	85	1212	1127	30	25	750	783	73.52%
C	0	690	690	30	23	690	843	45.01%
I	23	1433	1410	30	47	1410	123	91.98%
L1	70	1446	1376	15	60	900	633	89.76%
L2	117	1493	1376	15	60	900	633	89.76%
T	130	1519	1389	30	26	780	753	90.61%
W	156	1533	1377	30	14	420	1113	89.82%

The First Output (Sec)	170	**Cycle Time (Sec)**	1533	**Average Stations Utilization Rate**	70.06%
Average Delivery Rate (Sec)	51.7	**Total Processing Lead Time (Sec)**	13963	**System Efficiency**	0.73
				System Idle Time	0.27

Table 5-4 KPIs Outputs for Scenario 2 Obtained By Max-Plus Model

The First Output(Sec)	170	**Cycle Time (Sec)**	1533	**Average Stations Utilization Rate**	70.06%
Average Delivery Rate (Sec)	51.7	**Total Processing Lead Time (Sec)**	13963	**System Efficiency**	0.73
				System Idle Time	0.27

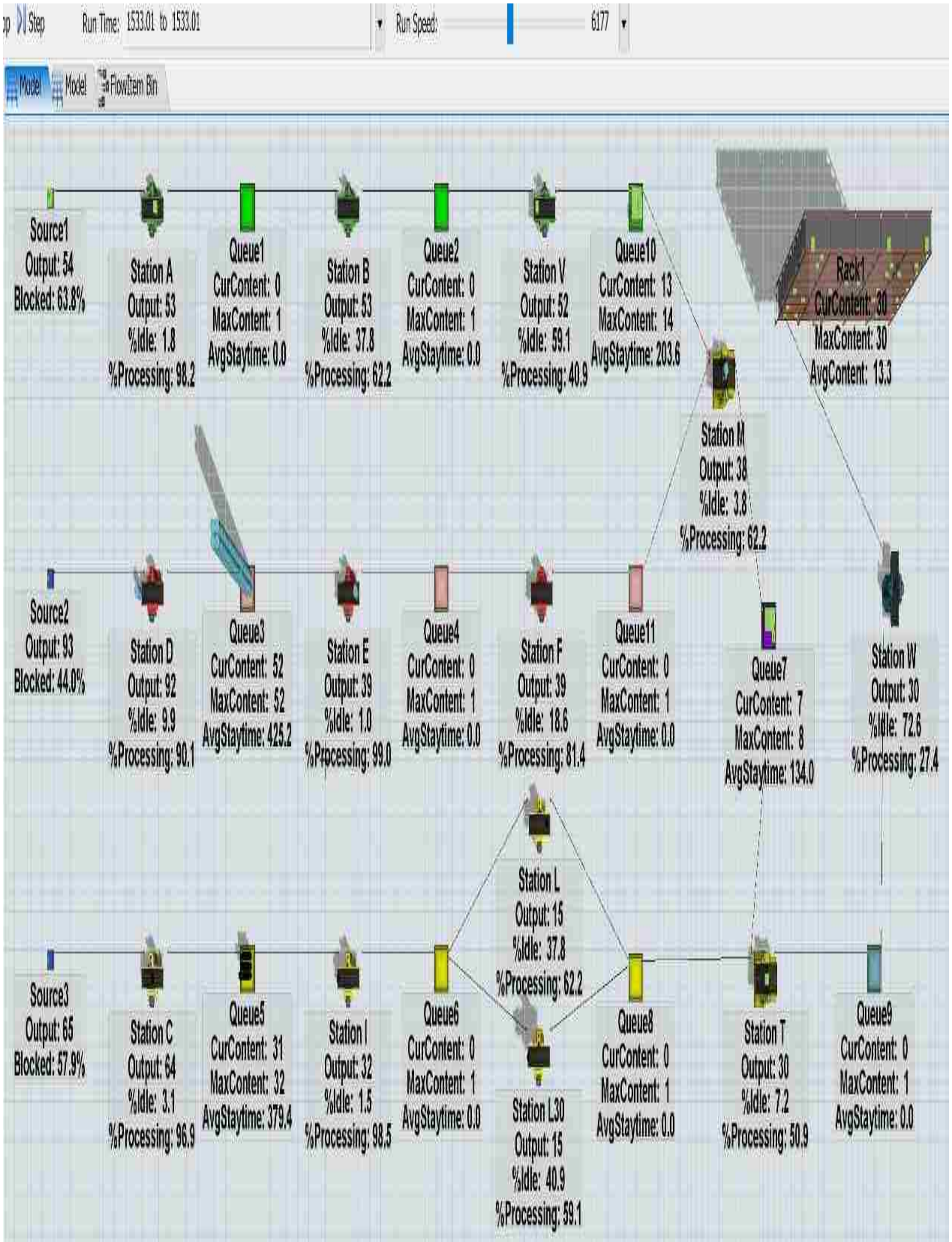


Figure 5.3 Final Results for Scenario 2, Two Parallel Stations at L

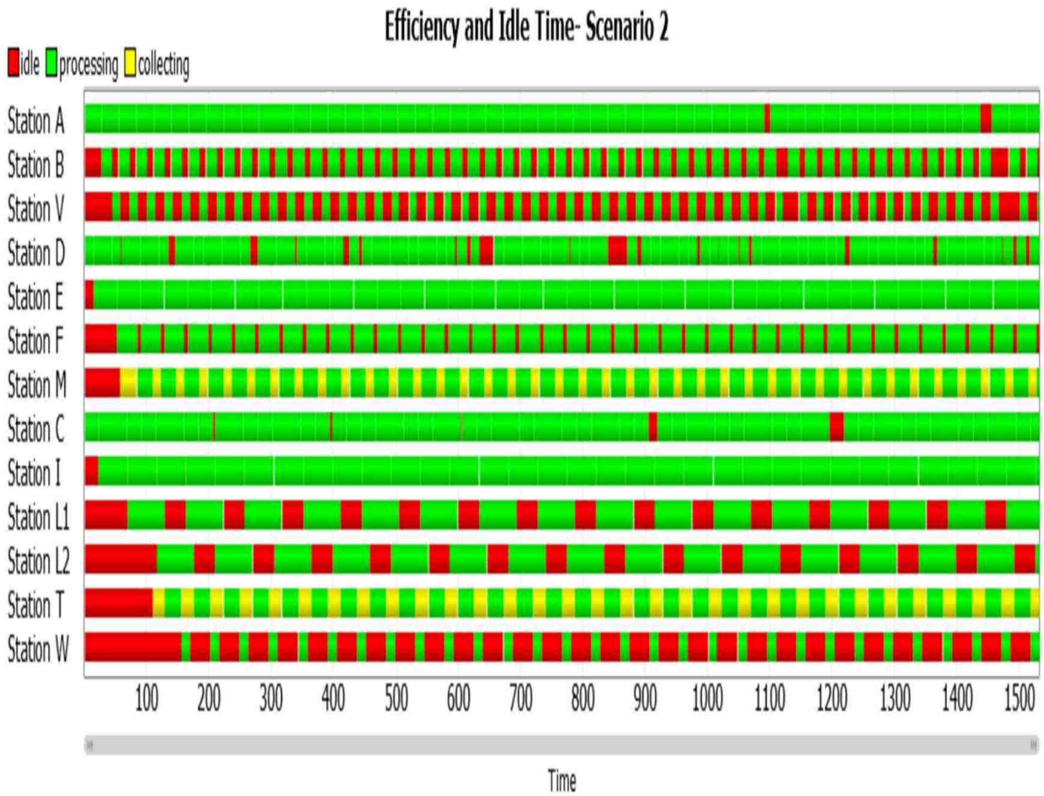


Figure 5.4 FlexSim Gantt Chart for Scenario 2, Two Parallel Stations at L

5.3.2. Scenario 3- Buffer at Station T

According to Figure 5.5, the total simulation run time is the same as cycle time obtained by Max-Plus,1310 sec. The simulation output and KPIs results are shown in Table 5-5. By comparing these results with what have been obtained with the Max-Plus model (Table 5-6), it can be confirmed that both models result the same outcomes such as Average Stations Utilization Rate 0f 63.58%, Total Processing Lead Time of 12825 seconds, System Efficiency of 79%, and System Idle time of 21%.

Table 5-5 FlexSim Data Transferred into The Excel for Scenario 3

Station	Start Time	End Time	Lead Time	Part	Process Time(Sec)	Efficient Time(Sec)	Idle Time(Sec)	Utilization Rate
A	0	840	840	30	28	840	1030	46.41%
B	28	858	830	30	18	540	1330	45.86%
V	46	870	824	30	12	360	1510	45.52%
D`	0	450	450	30	15	450	1420	24.86%
E	15	1155	1140	30	38	1140	730	62.98%
F	53	1187	1134	30	32	960	910	62.65%
M	85	1212	1127	30	25	750	1120	62.27%
C	0	690	690	30	23	690	1180	38.12%
I	23	1433	1410	30	47	1410	460	77.90%
L	70	1870	1800	30	60	1800	70	99.45%
T	0	1296	1296	30	26	780	1090	98.93%
W	26	1310	1284	30	14	420	1450	98.02%

The First Output (Sec)	40	**Cycle Time (Sec)**	1310	**Average Stations Utilization Rate**	63.58%
Average Delivery Rate (Sec)	43.7	**Total Processing Lead Time (Sec)**	12825	**System Efficiency**	0.79
				System Idle Time	0.21

Table 5-6 KPIs Outputs for Scenario 3 Obtained by Max-Plus Model

The First Output (Sec)	40	**Cycle Time (Sec)**	1310	**Average Stations Utilization Rate**	63.58%
Average Delivery Rate (Sec)	43.7	**Total Processing Lead Time (Sec)**	12825	**System Efficiency**	0.79
				System Idle Time	0.21

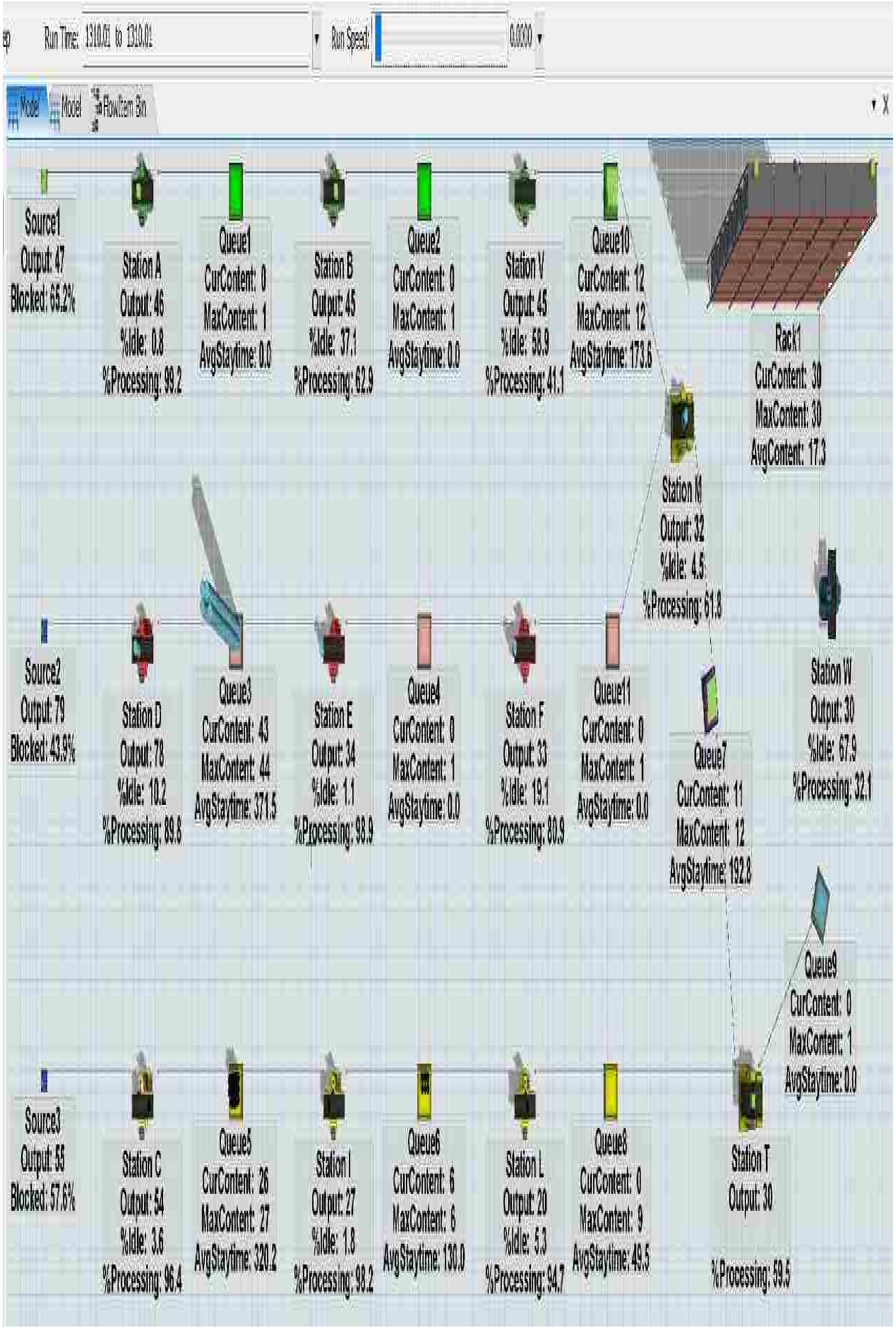


Figure 5.5 Final Results for Scenario 3, Buffer at Station T

Figure 5.6 FlexSim Gantt Chart for Scenario 3, Buffer at Station T

5.3.3. Scenario 4- Two Parallel Stations at Station L and Buffer at Station T

As it is shown in Figure 5.5, the total simulation run time is the same as cycle time obtained by Max-Plus,1533 seconds. Table 5-7 and Table 5-8 are summarized the results obtained using FlexSim and Max-Plus. Both models output and KPIs are the same.

Table 5-7 FlexSim Data Transferred into The Excel for Scenario 4

Station	Start Time	End Time	Lead Time	Part	Process Time(Sec)	Efficient Time(Sec)	Idle Time(Sec)	Utilization Rate
A	0	840	840	30	28	840	653	56.26%
B	28	858	830	30	18	540	953	55.59%
V	46	870	824	30	12	360	1133	55.19%
D`	0	450	450	30	15	450	1043	30.14%
E	15	1155	1140	30	38	1140	353	76.36%
F	53	1187	1134	30	32	960	533	75.95%
M	58	1212	1154	30	25	750	743	75.49%
C	0	690	690	30	23	690	803	46.22%
I	23	1433	1410	30	47	1410	83	94.44%
L1	70	1446	1376	15	60	900	593	92.16%
L2	117	1493	1376	15	60	900	593	92.16%
T	0	1049	1049	30	26	780	713	98.68%
W	26	1063	1037	30	14	420	1073	97.55%
The First Output (Sec)		40	Cycle Time (Sec)		1063	Average Stations Utilization Rate		75.20%
Average Delivery Rate (Sec)		35.4	Total Processing Lead Time (Sec)		13283	System Efficiency		0.76
						System Idle Time		0.24

Table 5-8 KPIs Outputs for Scenario 4 Obtained By Max-Plus Model

The First Output (Sec)	40	Cycle Time (Sec)	1063	Average Stations Utilization Rate	75.20%
Average Delivery Rate (Sec)	35.4	Total Processing Lead Time (Sec)	13283	System Efficiency	0.76
				System Idle Time	0.24

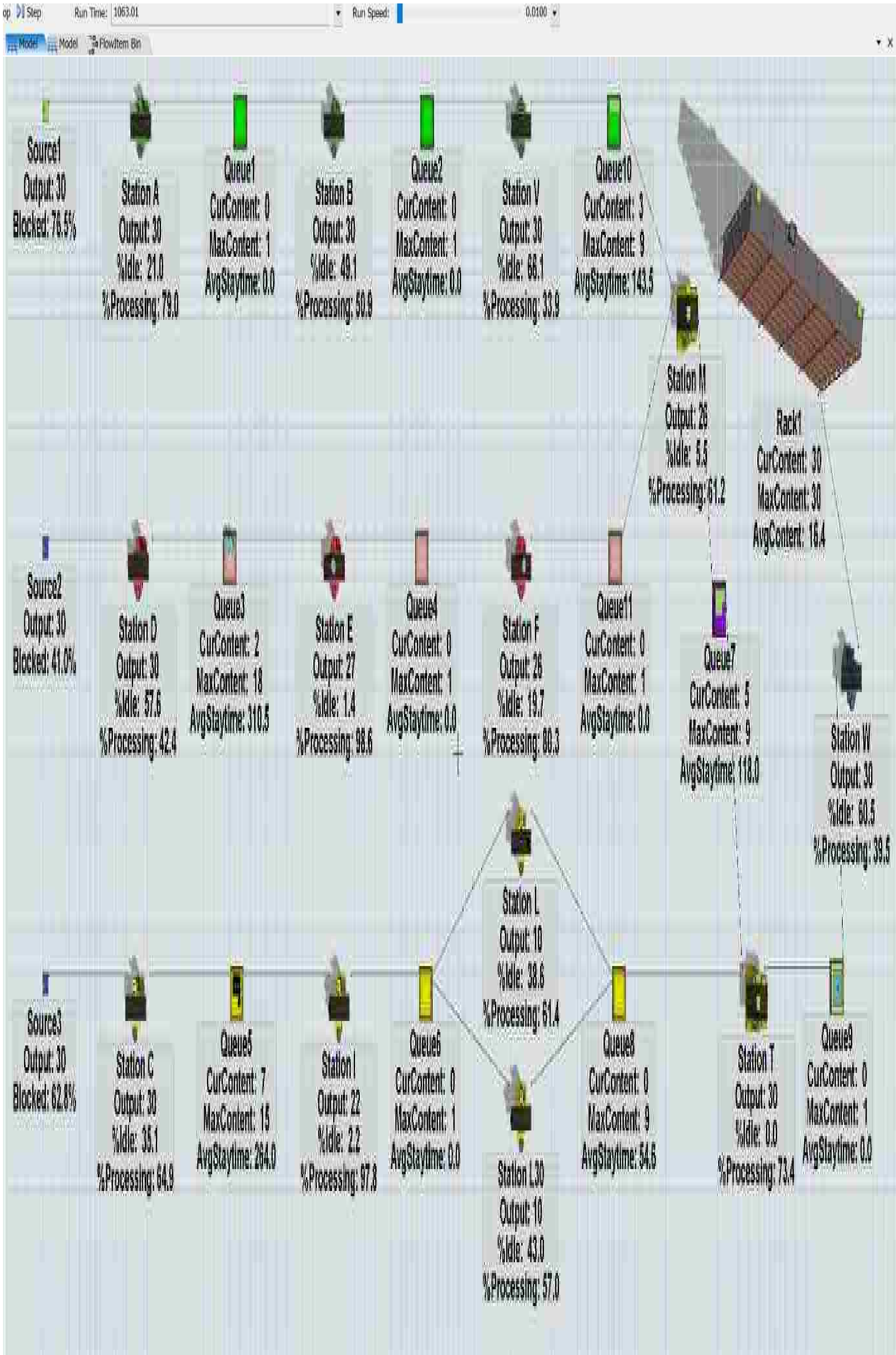


Figure 5.7 Final Results for Scenario 4, Two Parallel Station at L and Buffer T

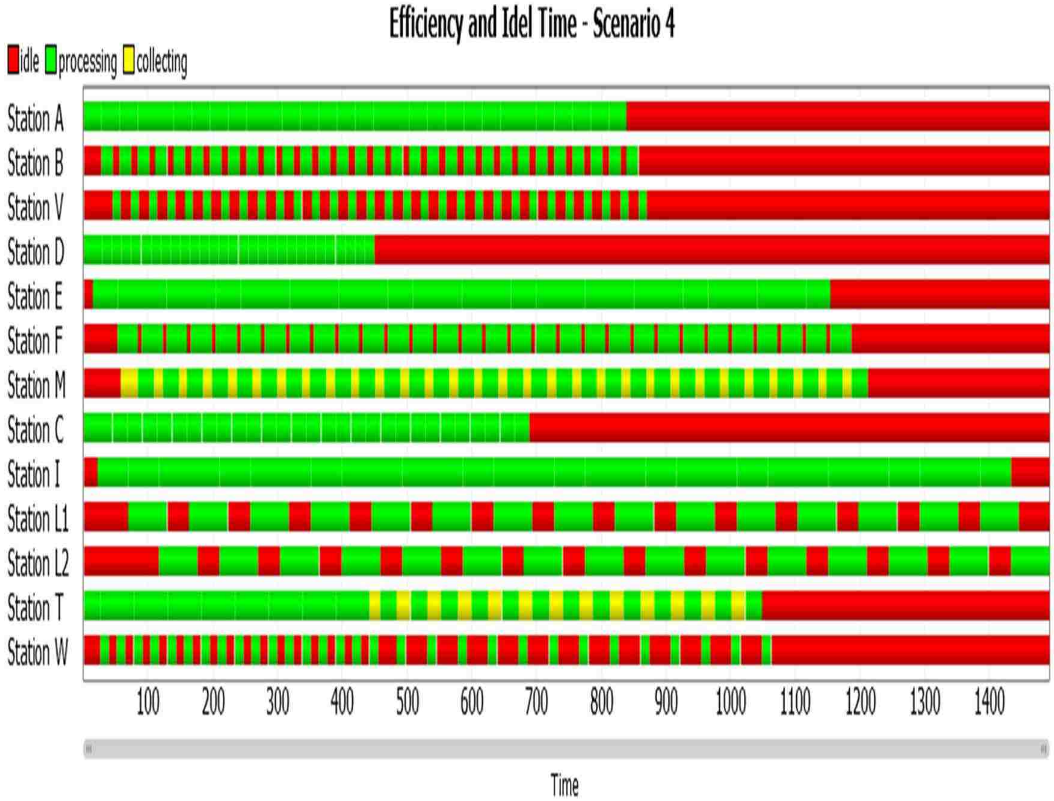


Figure 5.8 FlexSim Gantt Chart for Scenario 4, Two Parallel Station at L and Buffer T

5.3.4. Scenario 5- Three Parallel Stations at L

In this scenario, simulation KPIs output are slightly different than the Max-Plus model. As it has been discussed in section 4.3.5, having more parallel stations will not always improve the performance of the system. Parallel stations are required to be added to the bottleneck. As it is shown in Figure 5.10, station L2 is redundant; and the workload is divided between stations L1 and L2. However, the Max-Plus model has utilized all three stations of L1, L2, and L3. That's why the Total Processing Lead Time of simulation model is 13963 seconds which is 8% less than Max-Plus Total Processing Lead Time, 15198 seconds. In contrast, Average Stations Utilization Rate of the Max-Plus model is 70.81% which is 0.7% more than simulation model, 70.10%. The Max-Plus model has better results as of System Efficiency and Idle Time. Max-Plus Model's System Efficiency is 67% that is 7% more than System Efficiency of Simulation model, 60%; also, Max-Plus Model's Idle Time is 33% that is 7% less than Idle Time of simulation model (40%). The rest KPIs such as Cycle Time (1533 sec) and Average Delivery Rate (51.1%) are equal.

Table 5-9 FlexSim Data Transferred into The Excel for Scenario 5

Station	Start Time	End Time	Lead Time	Part	Process Time(Sec)	Efficient Time(Sec)	Idle Time(Sec)	Utilization Rate
A	0	840	840	30	28	840	653	54.79%
B	28	858	830	30	18	540	953	54.14%
V	46	870	824	30	12	360	1133	53.75%
D`	0	450	450	30	15	450	1043	29.35%
E	15	1155	1140	30	38	1140	353	74.36%
F	53	1187	1134	30	32	960	533	73.97%
M	85	1212	1154	30	25	750	743	73.52%
C	0	690	690	30	23	690	803	45.01%
I	23	1433	1410	30	47	1410	83	73.52%
L1	70	1446	1376	15	60	900	593	45.01%
L2	0	0	0	0	0	0	0	0
L3	117	1493	1376	15	60	900	593	89.76%
T	0	1049	1049	30	26	780	713	90.61%
W	26	1063	1037	30	14	420	1073	89.76%

The First Output (Sec)	170	**Cycle Time (Sec)**	1533	**Average Stations Utilization Rate**	70.10%
Average Delivery Rate (Sec)	51.1	**Total Processing Lead Time (Sec)**	13963	**System Efficiency**	0.60
				System Idle Time	0.40

Table 5-10 KPIs Outputs for Scenario 5 Obtained By Max-Plus Model

The First Output (Sec)	170	**Cycle Time (Sec)**	1533	**Average Stations Utilization Rate**	70.81%
Average Delivery Rate (Sec)	51.1	**Total Processing Lead Time (Sec)**	15198	**System Efficiency**	0.67
				System Idle Time	0.33

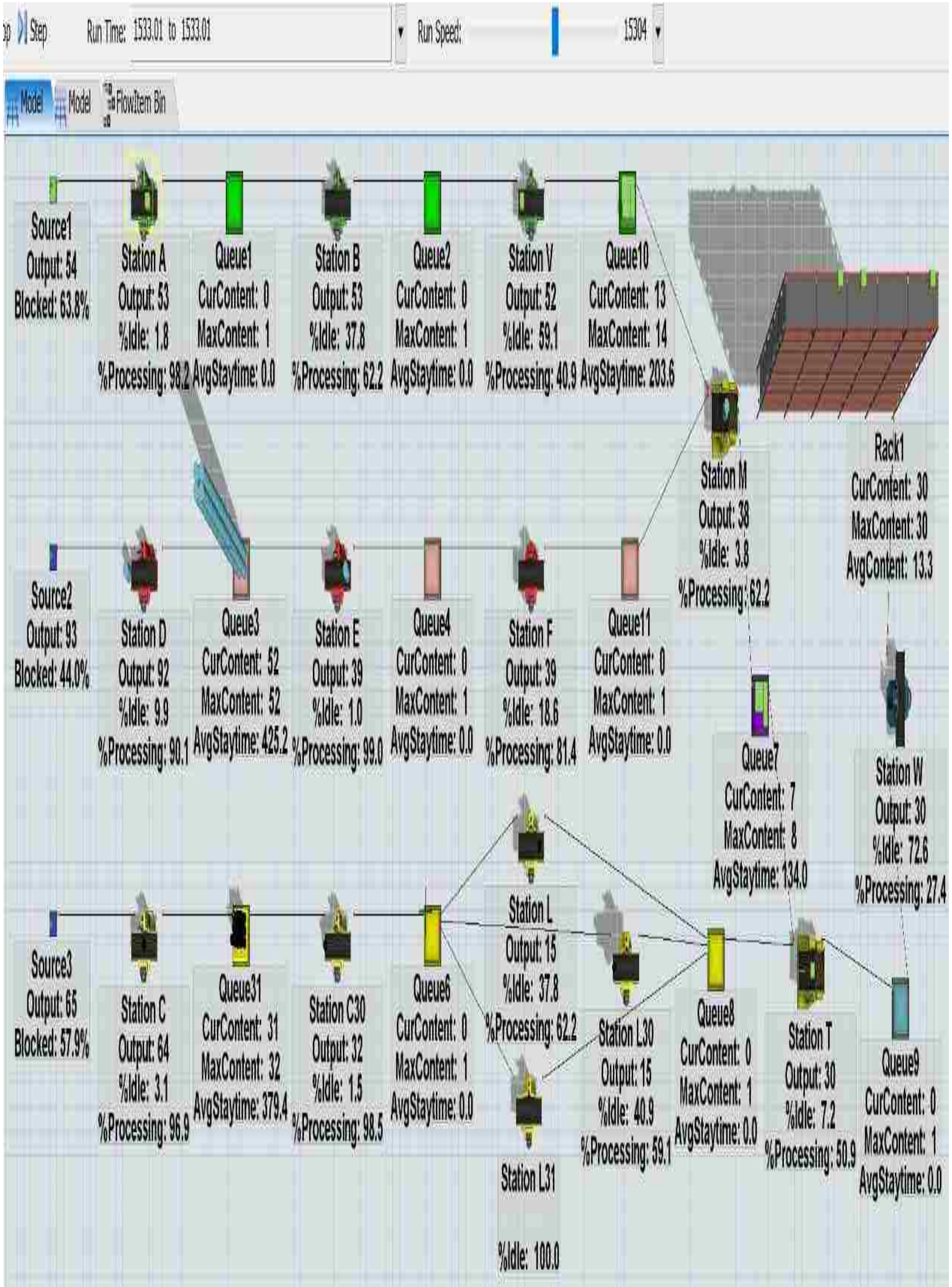


Figure 5.9 FlexSim Cycle Time for Scenario 5, Three parallel stations at L

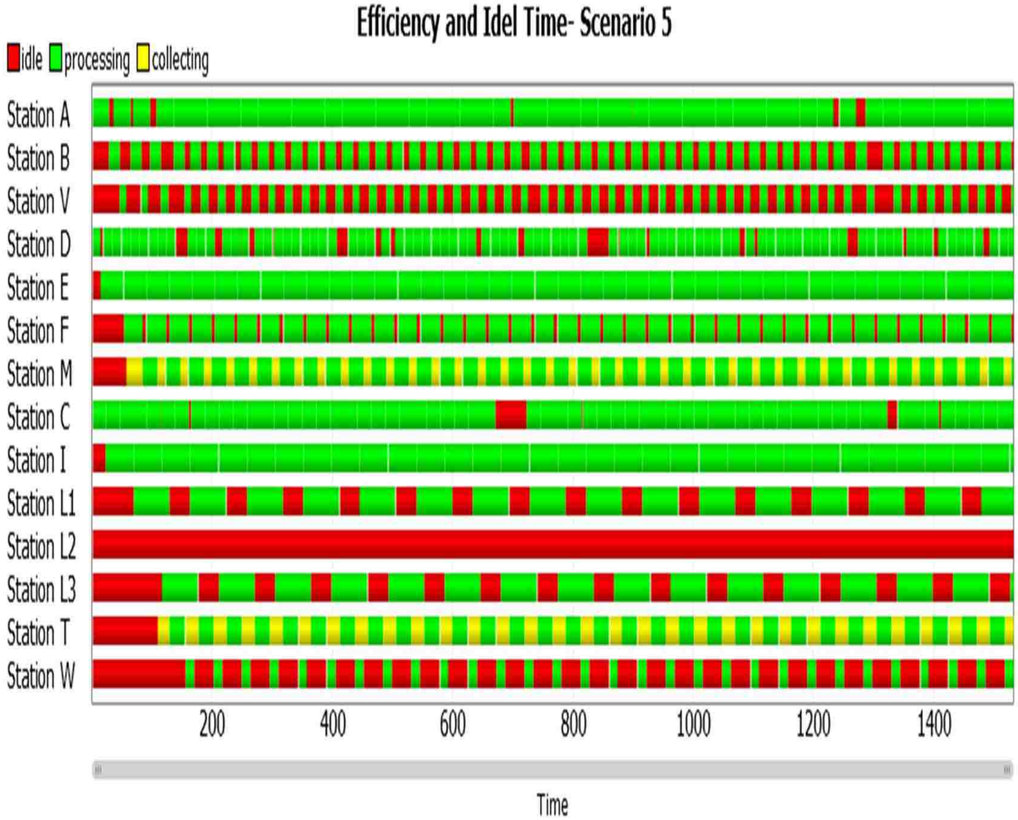


Figure 5.10 FlexSim Gantt Chart for Scenario 5, Three Parallel Stations at L

5.3.5. Scenario 6- Two Parallel Stations at I and L

As it is shown in Figure 5.11, the total simulation run time is the same as cycle time obtained by Max-Plus,1252 seconds. Table 5-11 and Table 5-12 are summarized the results obtained using FlexSim and Max-Plus. Both models result in same KPIs such as Total Processing Lead Time of 12449 seconds, 71% Average Stations Utilization Rate, 81% and 29% system efficiency and idle time.

Table 5-11 FlexSim Data Transferred into The Excel for Scenario 6

Station	Start Time	End Time	Lead Time (Sec)	Part	Process Time(Sec)	Efficient Time(Sec)	Idle Time(Sec)	Utilization Rate
A	0	840	840	30	28	840	412	67.09%
B	28	858	830	30	18	540	712	66.29%
V	46	870	824	30	12	360	892	65.81%
D`	0	450	450	30	15	450	802	35.94%
E	15	1155	1140	30	38	1140	112	91.05%
F	53	1187	1134	30	32	960	292	90.58%
M	85	1212	1127	30	25	750	502	90.02%
C	0	690	690	30	23	690	562	55.11%
I1	23	728	705	15	47	705	547	56.31%
I2	46	751	705	15	47	705	547	56.31%
L1	70	970	900	15	60	900	352	71.88%
L2	93	993	900	15	60	900	352	71.88%
T	130	1238	1108	30	26	780	472	88.50%
W	156	1252	1096	30	14	420	832	87.54%

The First Output (Sec)	170	**Cycle Time (Sec)**	1252	**Average Stations Utilization Rate**	71.02%
Average Delivery Rate (Sec)	41.7	**Total Processing Time (Sec)**	12449	**System Efficiency**	0.81
				System Idle Time	0.19

Table 5-12 KPIs Outputs for Scenario 6 Obtained by Max-Plus Model

The First Output (Sec)	170	**Cycle Time (Sec)**	1252	**Average Stations Utilization Rate**	71.02%
Average Delivery Rate (Sec)	41.7	**Total Processing Time (Sec)**	12449	**System Efficiency**	0.81
				System Idle Time	0.19

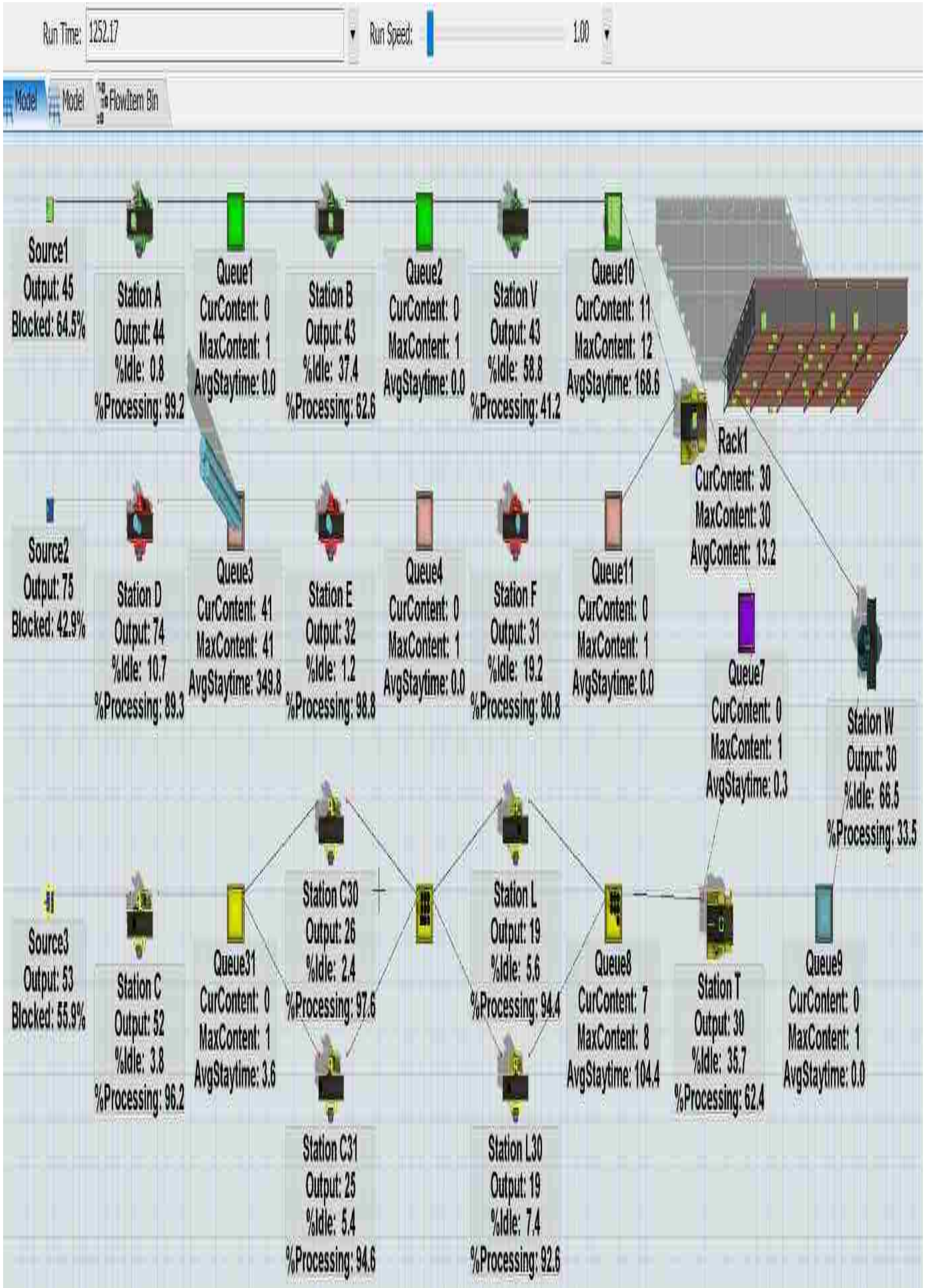


Figure 5.11 FlexSim cycle time for Scenario 6, Two Parallel Stations at I and L

Figure 5.12 FlexSim Gantt Chart for Scenario 6, Two Parallel Stations at I and L

5.3.6. Scenario 7, Four Parallel Stations at E, F, I, and L Plus Two Buffers at T and W

The total simulation run time, 736 seconds is the same as the cycle time obtained by Max-Plus. According to Tables 5-11 and 5-12 the KPIs output obtained using FlexSim and Max-Plus are the same with the Total Processing time of 11534 seconds, 72.36% Average Stations Utilization Rate, 88% and 12% system efficiency and idle time.

Table 5-13 FlexSim Data Transferred into The Excel for Scenario 7

Station	Start Time	End Time	Lead Time (Sec)	Part	Process Time(Sec)	Efficient Time(Sec)	Idle Time(Sec)	Utilization Rate
A	0	840	840	30	28	840	153	84.59%
B	28	858	830	30	18	540	453	83.59%
V	46	870	824	30	12	360	633	82.98%
D`	0	450	450	30	15	450	543	45.32%
E1	15	585	570	15	38	570	423	57.40%
E2	30	600	570	15	38	570	423	57.40%
F1	53	617	564	15	32	480	513	56.80%
F2	68	632	564	15	32	480	513	56.80%
M	85	895	810	30	25	750	243	81.57%
C	0	690	690	30	23	690	303	69.49%

I1	23	728	705	15	47	705	288	71.00%
I2	46	751	705	15	47	705	288	71.00%
L1	70	970	900	15	60	900	93	90.63%
L2	93	993	900	15	60	900	93	90.63%
T	0	876	876	30	26	780	213	88.22%
W	0	736	736	30	14	420	573	74.12%

The First Output (Sec)	14	**Cycle Time (Sec)**	736	**Average Stations Utilization Rate**	72.36%
Average Delivery Rate (Sec)	24.5	**Total Processing Time (Sec)**	11534	**System Efficiency**	0.88
				System Idle Time	0.12

Table 5-14 KPIs Outputs for Scenario 7 Obtained by Max-Plus Model

The First Output (Sec)	14	**Cycle Time (Sec)**	736	**Average Stations Utilization Rate**	72.36%
Average Delivery Rate (Sec)	24.5	**Total Processing Time (Sec)**	11534	**System Efficiency**	0.88
				System Idle Time	0.12

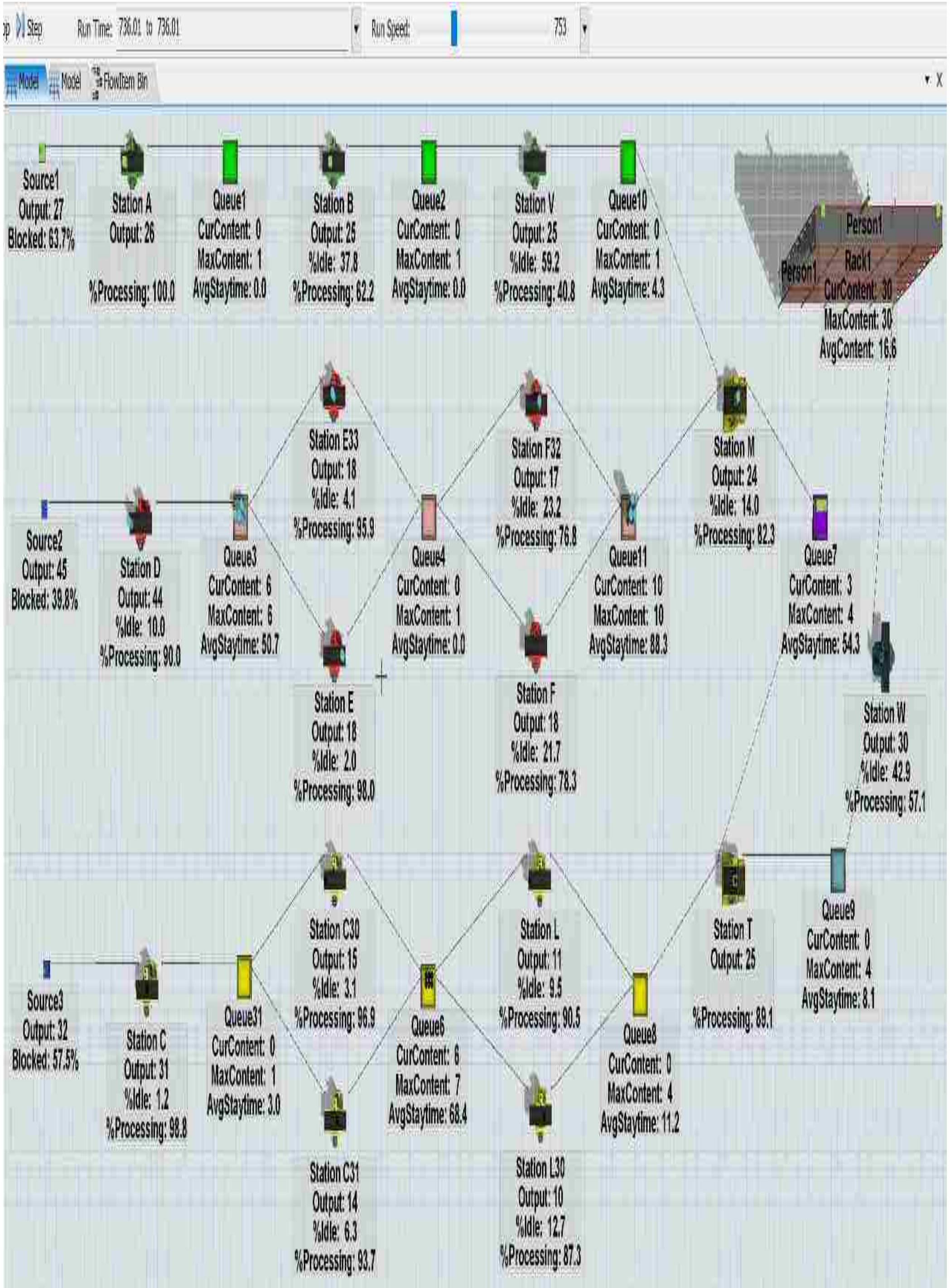


Figure 5.13 FlexSim cycle time for Scenario 7, Four Parallel Station at E, F, I, and L Plus Two Buffers at T and W

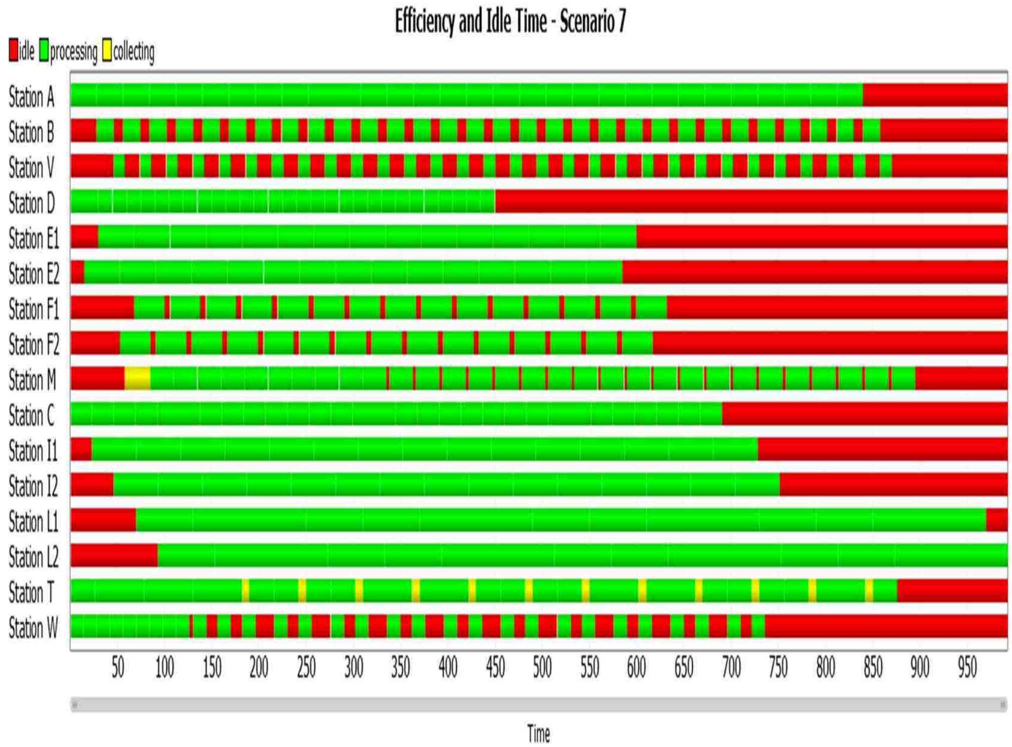


Figure 5.14 FlexSim Gantt Chart for Scenario 7, Four Parallel Station at E, F, I, and L Plus Two Buffers at T and W

5.3.8. Collecting Time

Flexsim measures collecting time at merging point such as Station M and T as a part of processing time. The collecting time is the total time since a merging station receives a part of sub-assembly and waits for the rest of parts to start processing. The results of collecting times at merged stations T and M is summarized in Table 5-15. As the outcomes prove how the combination of adding parallel stations at bottleneck and buffers have reduced the total collecting time significantly. Scenario 1 as a simple flow line without any parallel stations and buffer results in the most collecting time of 1671.1 seconds. As the paralle1 station is added to station T, scenario 2, the total collecting time falls to 1164.01. The third scenario, confirms the benefits of adding buffer by lowering the total collecting time to 712 seconds. The combinations of parallel stations and buffers are tested through scenarios 5 to 7. As scenario 5 indicates, only adding parallel stations will not end to a reduction in collecting time. The trend of reduction in cycle time through scenarios 4, 6, and 7 affirms that adding an adequate number of parallel stations and buffers would decrease total collecting time significantly from 673 seconds to 123 seconds.

Table 5-15 Total collecting time (sec) at stations M and T

Scenario	Station M	Station T	Total Collecting Time
Scenario 1	651	1020.1	1671.1
Scenario 2	521	643.01	1164.01
Scenario 3	443	530.01	712
Scenario 4	404	269	673
Scenario 5	521	643.01	1164.01
Scenario 6	404	24	428
Scenario 7	27	96	123

5.4. Comparison of Max-Plus Algebra with FlexSim Results

The same computer was used to run both models. Max-Plus Algebra was designed in Matlab and for discrete event simulation FlexSim was applied. In the following, some similarities, advantages and drawbacks of these two methods are discussed.

Firstly, Max-plus Algebra finds the results quicker compared to FlexSim. The time required to run scenarios using the Max-Plus Algebra and Simulation method is provided in Table 5-16. Although several preparation steps such as constructing models, setting up stations ports are required to model a simulation, FlexSim provides a button to customize run speed and run time. Hence, users can adjust the run speed to process the model faster. This feature is useful for complex and large size problems. In contrast, Matlab does not have this advantage. As the problem size increases, the number of matrices increases. This leads to the significant growth in model size and complexity that results in an increase in run time. Therefore, as mentioned in the gap analysis (1.7), not having a specialized application software for the Max-Plus Algebra method is one of the drawbacks.

Table 5-16 The required time needed to solve scenarios using Max-Plus Algebra

Scenario (i)	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Max-Plus Algebra Time (sec)	90	280	200	350	480	450	600
Simulation (Sec)	1910	1533	1310	1063	1533	1252	736
Delta	4.71%	18.26%	15.27%	32.93%	31.31%	35.94%	81.52%

Secondly, graphically FlexSim is designed for simulation. Resources, tasks, and conveyors are prepared for all types of systems, which provides a variety of views of a designed system. Definitely, FlexSim as an object-oriented software provides incredible animation to users.

Thirdly, model preparation and any changes in model specifications, such as adding buffers and parallel stations, are easier and faster in Matlab for Max-Plus; however, FlexSim can be remodelled and be changed graphically.

Finally, in the data entry phase, Max-Plus uses the elementary information of a manufacturing system to design, develop or analyze a model. However, Discrete event simulation software requires a great deal of information to draw, model and run a system model.

5.5. Summary

In this chapter, some outcomes obtained by FlexSim as a confirmation of results using Max-Plus are presented. A comparison of using Max-Plus Algebra and FlexSim for modelling and analysis phase demonstrates both methods are useful and have advantages and drawbacks such as time, size of the model, data availability, access to software, and requirement of sufficient knowledge about mathematics and modelling.

CHAPTER SIX

DISCUSSION AND CONCLUSIONS

6.1. Discussion and Overview

Today's global competitive environment has motivated industries to find solutions and alternatives to minimize their manufacturing costs, add more variety to their products and improve their systems and operations to achieve increased productivity, customer responsiveness, and high quality. To manage this situation, manufacturers have tried to use new methods to model, analyze, and control their manufacturing systems.

There have been different tools for modelling discrete event systems such as Petri Net, Markov Chains, Queuing Theory, Discrete Event Simulation, Automata, Supervisory Control, and Max-Plus Algebra.

Discrete Event Simulation is the most popular tool to model discrete event systems over time by generating the history of a system. This method provides a graphical view of systems for the users. However, running a system model repeatedly using a simulation method might take a long time particularly for large problems. Additionally, any changes to the developed simulation model require a great deal of effort.

Max-Plus Algebra as a mathematical tool is composed of a set of linear equations used to express the event timing dynamics of any deterministic manufacturing system. Since the introduction of Max-Plus Algebra in the late 1980s, many researchers have tried to apply this mathematical tool in different fields. By looking into the application of Max-Plus Algebra in industries, the majority of the studies have applied this method in the fields of transportation, control, and automation. However, there have not been enough studies in the field of manufacturing systems.

One of the critical subjects in manufacturing system optimization is line balancing. Line balancing is a strategy to make production lines running constant and flexible; it involves planning a set of operations or designing procedures to fabricate an output in a designated timeframe using the available capacities. This research, to the best of the author’s knowledge, for the first time a model for line balancing a discrete flow line manufacturing system using Max-Plus Algebra is developed.

In the developed model, station process times (load time, operation time, and unloading time) are assumed to be deterministic. Stations have infinite buffers and the system receives parts one at the time. Parts are required to be processed according to the production process sequence. No failure, downtime, stoppage or back tracking is assumed for the stations.

A practical manufacturing case study- the manufacture and assembly of Car Headlights is considered. This manufacturing line is composed of 12 stations. Seven configuration scenarios have been designed to model different structures of the manufacturing system, such as series, merged, paralleled, buffered and combined configurations. Scenarios have been developed based on previous scenarios after identifying bottlenecks and critical paths. The scenarios have become more complex by adding parallel station(s) to the bottleneck station(s) and adding a finite buffer to the stations when their downstream process is a bottleneck.

The first scenario is a simple flow line. In the second scenario, the bottleneck station *L* is recognized. Therefore, the parallel identical station is added at station *L*. The third scenario is designed to test adding a buffer to the line. Hence, a buffer of ten parts is added right after the bottleneck at station *T*. The fourth scenario is the combination of scenarios 2 and 3 by having an identical parallel station and buffer at station *L* and *T*, respectively. The fifth scenario is an extension of the second scenario by having three parallel stations at the bottleneck. By reconsidering the second scenario, the station *I* is recognized as the second bottleneck. Therefore, two different parallel stations are added to the sixth scenario at Stations *I* and *L*. This seventh scenario is the extension of scenarios 6 and 3. Identical parallel stations are added to stations *E, F, I, L* and five parts have been allocated to the buffer of stations *T* and *W*.

To test and compare the developed scenarios, several manufacturing Key Performance Indices (KPIs) such as Cycle Time, Average System Delivery Rate, Total Processing Lead Time, Station's Utilization Rate, Idle Time and Efficiency of the System have been defined and used. Additionally, a Financial Analysis is formulated and conducted to compare all scenarios by unit cost using a combination of total processing lead time, utilization rate, number of stations etc.

Based on the scenario 5 results, only adding an adequate number of parallel stations to the bottleneck will lead to improving efficiency and reduction in idle time. Accordingly, scenario 7 is determined to be the best system structure/configuration, which includes both parallel stations and buffers. The superiority of scenario 7 compared to other scenarios is evident due to its lowest time to deliver first output (14 seconds), shortest total processing lead time (11,534 seconds), least percentage of idle time (12%), lowest unit cost per part ($6.9), and highest efficiency (88%). However, scenario 4 has the best Utilization Rate at 75%.

Finally, the scenarios have been tested using a discrete event simulation tool (Flexim). The same data, parameters, variables and conditions, etc. were applied when simulating the scenarios to make the comparison valid. Both Flexim and Max-Plus Algebra have resulted in the same results. However, Max-Plus Algebra was quicker and easier to use and manage.

6.2. Novelties and Contributions

The conducted research fills some gaps in previous research in the modelling phase as well as the analysis phase. First, this study presents a practical approach to use Max-Plus Algebra. The method is easy to understand for decision makers with little background or basic knowledge of mathematics.

Second, in this research, Max-Plus Algebra is used to balance the flow of a manufacturing system that has not been covered in previous studies. To achieve this, different scenarios are developed by adding parallel stations and finite buffer. Bottlenecks have found and then the critical path has been assessed. Then by adding a parallel identical station to the bottleneck, it has been tried to decrease the cycle time. On the other hand, to cut idle time, finite buffers are allocated to the next station after the bottleneck to keep the flow line smooth and constant.

Third, the size of the considered manufacturing system is larger compared to the other studies. As discussed in the gap analysis, most studies used small size problems, while the last scenario in this research is developed for 16 single and parallel identical stations. However, it is possible to model a system with more stations and complexities. Furthermore, the modelled scenarios ran for 30 parts while, there is no limitation for the number of parts in Max-Plus Algebra.

Fourth, in the analysis phase of this research studied several manufacturing Key Performance Indices (KPIs) defined to evaluate the results of developed scenarios, such as Product Completion Time (Average Delivery Rate by system in second), Total Processing Lead Time, Station's Utilization Rate, Idle Time and Efficiency of the System, and Financial analysis.

Fifth, in contrast to most of the previous research, which has illustrated numerical examples, the modelled system is a practical and real example of a manufacturing system. In order to do this, as a case study, the Car Headlight manufacturing system is considered.

Finally, the output of this thesis is several functional codes that result from Matlab and can be generalized to all similar systems. The codes are enabled to generate equations and require the least information of a system, which is competitive compared to Discrete Event Simulation or other similar tools. Furthermore, the method of using modelling output and defined KPIs can apply to similar manufacturing systems.

6.3. Limitations

As it is discussed in previous chapters, there are some unanswered questions and uncovered subjects in the field of Max-Plus Algebra. The number of publications compared with other modelling tools are limited. Different types of manufacturing systems such as continuous manufacturing system, or the behaviour of manufacturing systems such as stochastic processing time, have not been covered appropriately. Additionally, having conditions in the system structure such as reworks and defects have not been studied sufficiently.

Furthermore, there are some limitations to use Max-Plus Algebra. A model designer should know about Max-Plus and how a system can be modelled, simulated and analyzed. Formulating equations and afterward construct matrices followed by lots of calculations are not possible without good knowledge of mathematics and algebra. Particularly by developing a system in the aspect of size and entities relations, require lots of programming techniques. Also by changing the model structure all calculations for equations and matrices should be re-done.

6.4. Recommendations for Future Studies

Max-Plus Algebra is a strong tool in the field of modelling manufacturing systems. Particularly, its application has not been covered enough in previous studies. The following recommendations provide the main direction for future work.

- Developing special software or module to tackle calculation difficulties of Max-Plus equations and constructing matrices.
- Comparison (advantages and drawbacks) of Max-Plus Algebra application with other mathematical tools such as Automata, Markov chain, Petri nets, Discrete Event Simulation and Queuing theory to model practical systems.
- Expanding an innovative algorithm to simplify Max-Plus modelling steps.
- Using Max-Plus Algebra in practical manufacturing systems with stochastic behaviours such as random variables and statistic distribution.
- Applying Max-Plus Algebra for different type of manufacturing systems that have not been covered enough, such as job shop, cellular, continuous, and especially those manufacturing systems that have closed loops like backtracking and reentrancy.
- Adding new features to the manufacturing systems, for example breakdown and downtime, set-up time, alternative process, reworks and reschedule, sourcing/allocation policies, etc.
- Modelling systems larger than previous studies with hierarchical and block diagram algorithms and comparing these with regular modelling methods.

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APPENDIX ONE

MATLAB CODE

Referred to chapter 2,3 and 4.

```
TotalStation=13;
Part=20;
FST=3;
c=0;
PST=10;
BST=11;
NBuf=10;

AT=zeros(TotalStation,FST);
AU=sym('AU%d%d',[TotalStation,1]);
BU=zeros(TotalStation,1);
BufU=sym('AU%d%d',[TotalStation,1]);
Uk=zeros(FST,Part);
Uk1=zeros(FST,1);
IST=zeros(FST,Part);
Y=sym('Y%d%d',[TotalStation,Part+NBuf]);
Y1=sym('Y1%d%d',[TotalStation,1]);

for k=1 : Part
    if k==1
        %first part
        Y(BST,1)=0;
        Y1(BST,1)=T(BST,1);
        for i=BST+1 : TotalStation
            Y(i,1)=Y1(i-1,1);
            Y1(i,1)=Y1(i-1,1)+T(i,1);
        end
        for i=1 : BST-1
            Y(i,1)=0;
            Y1(i,1)=0;
        end
        Y;
        Y1;
        for i=BST : TotalStation
            Y1(i,1)=Y(i,1);
        end
        Y1;
    else
   %second part until last part of buffer
   if  k <= NBuf && k >1
       k;
       Y1;
       BufU(BST,1)=Y1(BST,1)+T(BST);
       for j=BST+1 : size(Buf,1)
```

```
            c=max(BufU(j-1,1)+T(j-1),T(j,1)+Y1(j ,1));
            BufU(j,1)=c;
        end
        BufU;
        for i=1 : BST-1
            BufU(i,1)=0;
        end
        for i=1 : size(B,1)
            Y(i,k)=BufU(i,1);
        end
        BufU;
        for i=1 : size(B,1)
           Y1(i,1)=Y(i,k);
        end
        Y;
        Y1;
    else

    if k==NBuf+1
        for i=1 : FST
            Uk1(i,1)=Uk(i,1);
        end
        for i=1 : size(B,1)
            for j=1 : size(Uk1,2)
                c=B(i,1)+Uk1(1,j);
                for v=1 : size(B,2)
                    c=max(c,B(i,v)+Uk1(v,j));
                end
                BU(i,j)=c;
            end
        end
        %make BU(i,k) to BU(i,k-NBuf) to fix the dimensions and start
from
        %1
        %for i=1 : size(B,1)
            %Y(i,k)=BU(i,k-NBuf);
        %end
        for i=1:BST-1
            Y(i,k)=BU(i,1);
        end
        Y(BST,k)=max(Y1(BST+1,1),BU(BST,k-NBuf))
        for i=BST+1 : size(B,1)
            Y(i-1,k)+T(i-1,1);
            Y1(i,1);
            BU(i,k-NBuf);
            Y(i,k)=max(Y1(i,1),max(Y(i-1,k)+T(i-1,1),BU(i,k-NBuf)))
        end

        for i=1 : FST
```

```
        IST(i,k-NBuf)=Uk1(i,k-NBuf);
    end

else
    for i=1 : size(B,1)
        Y1(i,1)=Y(i,k-1);
    end
    Y1
    for i=1 : size(A,1)
        for j=1 : size(Y1,2)
            c=A(i,1)+Y1(1,j);
            for v=1 : size(A,2)
                c=max(c,A(i,v)+Y1(v,j));
            end
            AU(i,j)=c;
        end
    end
    AU;

    for i=1 : FST
        Uk(i,k-NBuf)=IST(i,k-NBuf-1)+T(i,1);
    end
    Uk;
    for i=1 : FST
        Uk1(i,1)=Uk(i,k-NBuf);
    end
    Uk1;
    for i=1 : size(B,1)
        for j=1 : size(Uk1,2)
            c=B(i,1)+Uk1(1,j);
            for v=1 : size(B,2)
                c=max(c,B(i,v)+Uk1(v,j));
            end
            BU(i,j)=c;
        end
    end

    if k > NBuf+1
        for i=1 : size(A,1)
            for j=1 : size(Y1,2)
                c=Buf(i,1)+Y1(1,j);
                for v=1 : size(A,2)
                    c=max(c,Buf(i,v)+Y1(v,j));
                end
                BufU(i,j)=c;
            end
        end
        BufU
    end
```

```
        for i=1 : size(AU,1)
            for j=1 : size(AU,2)
               % if k > NBuf+1
                    %AT(i,j)=max(BufU(i,j),max(AU(i,j),BU(i,j)));
                %else
                    AT(i,j)=max(AU(i,j),BU(i,j));
                %end
            end
        end
        AT;
        for i=1 : size(B,1)
            Y(i,k)=AT(i,1);
        end


   end
   end
    end
    Y
end
```

APPENDIX TWO

NUMERICAL MATRICES FOR STUDIED SCENARIOS

The Elementary Matrices A, B, D, $\hat{A}$ and $\hat{B}$ (Referred to chapter 3 and 4)

$$A=\begin{bmatrix}
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
t_A & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & t_B & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & t_D & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & t_E & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & t_V & \varepsilon & \varepsilon & t_F & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_C & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_I & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_M & \varepsilon & \varepsilon & t_L & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_T & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_W & \varepsilon
\end{bmatrix}
B=\begin{bmatrix}
t_A & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & t_B & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & t_V & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & t_D & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & t_E & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_F & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_M & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_C & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_I & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_L & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_T & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_W & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_Y
\end{bmatrix}
D=\begin{bmatrix}
e & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon \\
\varepsilon & e & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & e \\
\varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon
\end{bmatrix}$$

$$A=\begin{bmatrix}
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
28 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & 18 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & 15 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & 38 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & 12 & \varepsilon & \varepsilon & 32 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 23 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 47 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 25 & \varepsilon & \varepsilon & 60 & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 26 & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 14 & \varepsilon
\end{bmatrix}
B=\begin{bmatrix}
28 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & 18 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & 12 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & 15 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & 38 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 32 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 25 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 23 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 47 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 60 & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 26 & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 14 & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 0
\end{bmatrix}
D=\begin{bmatrix}
0 & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon \\
\varepsilon & 0 & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & 0 \\
\varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon
\end{bmatrix}$$

$$
\widehat{\boldsymbol{A}}=\boldsymbol{A}^{*}\boldsymbol{B}
\begin{bmatrix}
t_A & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
t_A^2 & t_B & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
t_A^2 t_B & t_B^2 & t_V & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & t_D & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & {t^2}_D & t_E & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & t_D^2 t_E & t_E^2 & t_F & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
t_A^2 t_B t_V & t_B^2 t_V & t_V^2 & t_D^2 t_E t_F & t_E^2 t_F & t_F^2 & t_M & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_C & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_C^2 & t_I & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & t_C^2 t_I & t_I^2 & t_L & \varepsilon & \varepsilon & \varepsilon \\
t_A^2 t_B t_V t_M & t_B^2 t_V t_M & t_V^2 t_M & t_D^2 t_E t_F t_M & t_E^2 t_F t_M & t_F^2 t_M & t_M^2 & t_C^2 t_I t_L & t_I^2 t_L & t_L^2 & t_T & \varepsilon & \varepsilon \\
t_A^2 t_B t_V t_M t_T & t_B^2 t_V t_M t_T & t_V^2 t_M t_T & t_D^2 t_E t_F t_M t_T & t_E^2 t_F t_M t_T & t_F^2 t_M t_T & t_M^2 t_T & t_C^2 t_I t_L t_T & t_I^2 t_L t_T & t_L^2 t_T & t_T^2 & t_W & \varepsilon \\
t_A^2 t_B t_V t_M t_T t_W & t_B^2 t_V t_M t_T t_W & t_V^2 t_M t_T t_W & t_D^2 t_E t_F t_M t_T t_W & t_E^2 t_F t_M t_T t_W & t_F^2 t_M t_T t_W & t_M^2 t_T t_W & t_C^2 t_I t_L t_T t_W & t_I^2 t_L t_T t_W & t_L^2 t_T t_W & t_T^2 t_W & t_W^2 & t_Y
\end{bmatrix}
$$

$$
\widehat{\boldsymbol{A}}=\boldsymbol{A}^{*}\boldsymbol{B}
\begin{bmatrix}
28 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
56 & 18 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
74 & 36 & 12 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & 15 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & 30 & 38 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & 68 & 76 & 32 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
86 & 48 & 24 & 100 & 108 & 64 & 25 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 23 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 46 & 47 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 93 & 94 & 60 & \varepsilon & \varepsilon & \varepsilon \\
111 & 73 & 49 & 125 & 133 & 89 & 50 & 153 & 154 & 120 & 26 & \varepsilon & \varepsilon \\
137 & 99 & 75 & 151 & 159 & 110 & 76 & 179 & 180 & 146 & 52 & 14 & \varepsilon \\
151 & 113 & 89 & 165 & 173 & 124 & 90 & 193 & 194 & 160 & 66 & 28 & 0
\end{bmatrix}
$$

$$\widehat{\boldsymbol{B}} = \boldsymbol{A}^*\boldsymbol{D} = \begin{bmatrix} e & \varepsilon & \varepsilon \\ t_A & \varepsilon & \varepsilon \\ t_At_B & \varepsilon & \varepsilon \\ \varepsilon & \mathrm{e} & \varepsilon \\ \varepsilon & t_D & \varepsilon \\ \varepsilon & t_Dt_E & \varepsilon \\ t_At_Bt_V & t_Dt_Et_F & \varepsilon \\ \varepsilon & \varepsilon & \mathrm{e} \\ \varepsilon & \varepsilon & t_C \\ \varepsilon & \varepsilon & t_Ct_I \\ t_At_Bt_Vt_M & t_Dt_Et_Ft_M & t_Ct_It_L \\ t_At_Bt_Vt_Mt_T & t_Dt_Et_Ft_Mt_T & t_Ct_It_Lt_T \\ t_At_Bt_Vt_Mt_Tt_W & t_Dt_Et_Ft_Mt_Tt_W & t_Ct_It_Lt_Tt_W \end{bmatrix}$$

$$\widehat{\boldsymbol{B}} = \boldsymbol{A}^*\boldsymbol{D} = \begin{bmatrix} 0 & \varepsilon & \varepsilon \\ 28 & \varepsilon & \varepsilon \\ 46 & \varepsilon & \varepsilon \\ \varepsilon & 0 & \varepsilon \\ \varepsilon & 15 & \varepsilon \\ \varepsilon & 53 & \varepsilon \\ 58 & 85 & \varepsilon \\ \varepsilon & \varepsilon & 0 \\ \varepsilon & \varepsilon & 23 \\ \varepsilon & \varepsilon & 70 \\ 83 & 110 & 130 \\ 108 & 136 & 156 \\ 122 & 150 & 170 \end{bmatrix}$$

APPENDIX THREE

PROCESS PLAN FOR AUTOMOTIVE HEADLIGHT ASSEMBLY (REFERRED TO SECTION 4.2.3)

Process Plan Automotive headlight assembly line												
Station	Code	Part No (Is based on exploded view)	Part Name	Operation Description	Workstation (machine)	Setup time (second)	Process time (second)	Unloading time (second)	Total time	Equipment &Tools	Transfer or Material Handler	Precedence
A	10	22	Reflector	Poly Carbonate Resin, Plexiglas and Acrylic are sucked from pack to Injection Molding Machin. Robot takes out reflector from press and place it on conveyor.	A1	0	23	5	28	Press, molding machine, Vacuum and Robot	Pipe & conveyo r	-
B	15	22	Reflector (Cutting sharp edge and Finishing)	Operator loads reflector then cuts and removes extra parts. robot picks it up and puts it on conveyor	A2	5	8	5	18	Milling Machine and Robot	conveyo r	10
V	20	-	Quality Inspection	physical and appearance test such as checking edges , holes and shape of reflector and laser wave	P1	0	12	0	12	Manual by worker		15
D	30	25	Middle Frame	Chemical Material is sucked and casted in Frame	A3	0	10	5	15	Press, molding	(Buffer: Cart)	
E	40	6,8,10,11, 12,	Set Parking-Light Bulb ,Halogen , Bulb Yellow/Blue, Xenon Light With Ignition Element, Long-life Bulb, Two Bulb,	parking light bulb, long life bulb and other parts like halogen pick up from the shelves and boxes. screws are taken to fast parts on middle frame.	A4	5	28	5	38	Screw driver , Air Compress or, ,Wrench and Gauge	(Buffer: Cart)	30

F	50	1,2,3,9,13 ,15,16,	Lamp Cower Low Beam (cap), Cover High Beam/Daytime Running Lights, Lamp holder for xenon lamp, Bulb Socket, Turn Insider, Headlight Repair Kit	Mentioned Part installed on previous output. Parts with their designed teeth –retainer and screws are placed and installed to the frame, all cover with universal wrench are fasted. Dust and scrap clean by air compressor	A5	5	22	5	32	Wrench and Gauge, Air Compressor, Screw driver	(Buffer: Cart)	40
M	60		Screws,	Finished reflector and middle frame with all parts moved by conveyor through the station. All parts visually checked operator screwed closed these two module together	A6	5		5	25			20, 50
C	70	24	Headlight Housing(back frame)	Back frame is casted in molding machine	A7	5	23	5	23	Screw driver and Air Compressor	Conveyor	
I	80	7,13,23	Repair Kit, HVAC Servomotor ,Screw-Trox-Bolt With Washer, Socket Wire, Socket Housing	Repair kit is supported by Trox-bolt with washer and screwed to the back frame. Back cover pick up from cart and is screwed with at each corners. Operator gets a piece of socket wire from box and connect it to socket housing Placement by robot	A8	5	37	5	47	Wrench, Screw driver and Air Compressor	conveyor	70
L	90	4,5,17,18, 19	Control Unit Xenon Light+ Control Unit Directional Light+ Seal washer ,LED lights ,LED Cover	Unit with a seal washer uses under the frame and closed by screw Washer is placed in housing hole and screws are used to tight the back frame. LED and its cover closed on Back frame.	A9	5	14	5	60	Screw driver, lumen adjusting,	conveyor	80

T	100	20, 21	Projector Lens, Headlight Lens (Front cover)	Unfinished Headlight, Projector Lens, and headlight lens are picked up from conveyor and shelves. placed on machine whole edge heats and glues (adhesive substance) with machine and finally to parts pressed together	A1 0	5	16	5	26	Robot, lumen adjusting, Heating machine, and Glue gun, press	conveyo r	60, 90
W	110	-	Quality inspection, Labeling(sticker)	approved function of products , Box on pallet filled with finished products	P2	5	13	5	14	UV coating and Laser, Printer, Tape Machine	(Buffer: Cart)	100

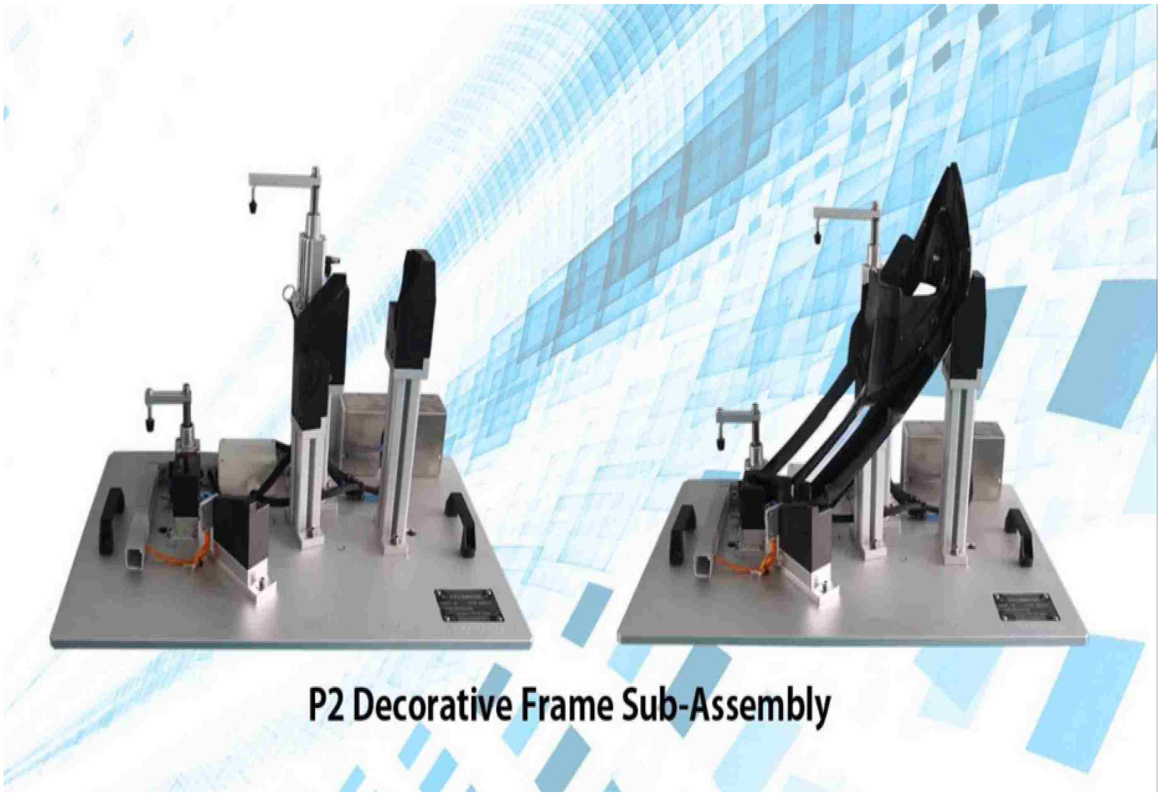
- Work station codes are used based on manufacturer assembly line system and the list is showed below.
- Operation 20 is the precedence of four independent operations namely 30, 40, 50, 60. The operations can be combined as one operation or considered separately. The same condition is supposed for operation code 80,90, 100, 110 and 120.

APPENDIX FOUR

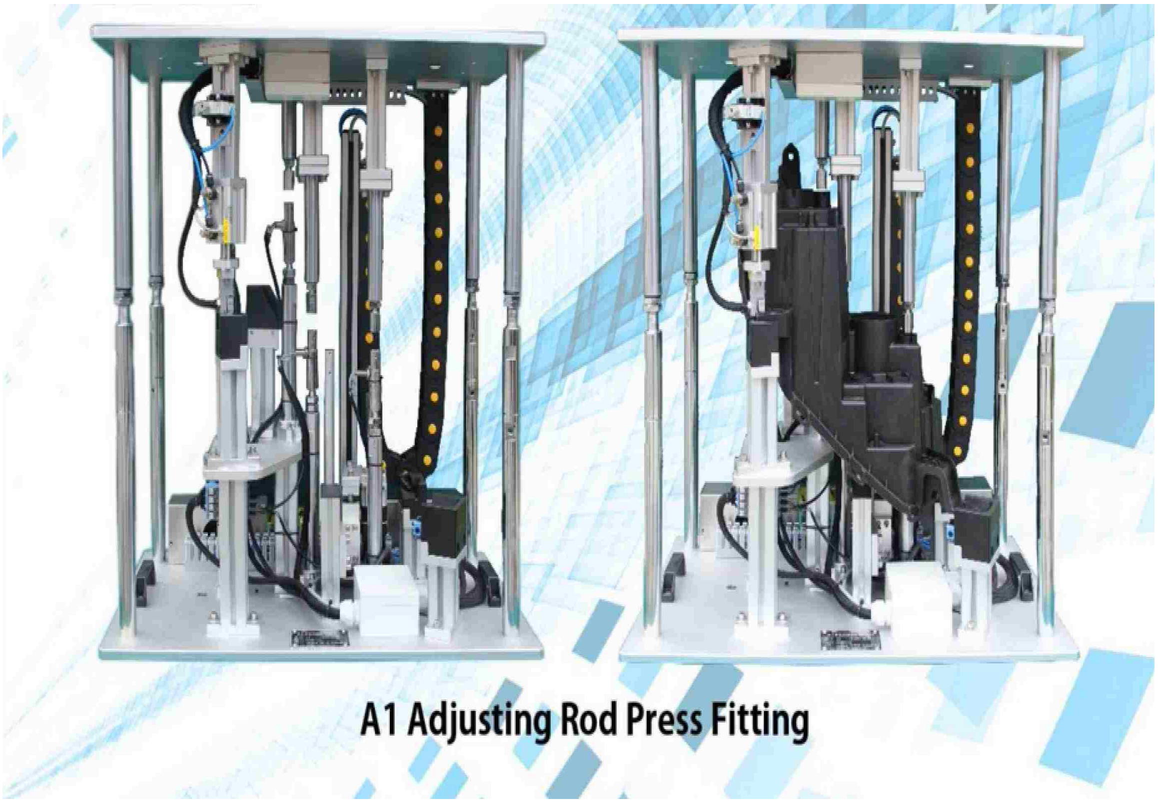
CAR HEADLIGHT MANUFACTURING STATIONS

Referred to section 4.2.3

	Name	Description (automotive frame sub-assembly line- eooeintl.com)
1	A1	Adjusting rod press fitting
2	A2	Reflector adjustment press fitting
3	P1	Decorative Frame sub-assembly
4	P2	Decorative Frame sub-assembly
5	P3	Decorative Frame sub-assembly
6	A3	Wiring harness collating & motor installation
7	A4	Lens and lens holder sub-assembly
8	A5	Install reflector sub-assembly into lamp seat
9	A6	Install decorative frame sub-assembly into the lamp seat
10	A7	Vibration & electrostatic precipitation
11	A8	Plasma spray and adhesive gluing
12	A9	lamp seat and lamp seat housing press fitting
13	A10	Install the lamp housing screws
14	A11	Full-automatic beam adjusting
15	A12	Air tightness test


P2 Decorative Frame Sub-Assembly


P3 Decorative Frame Sub-Assembly

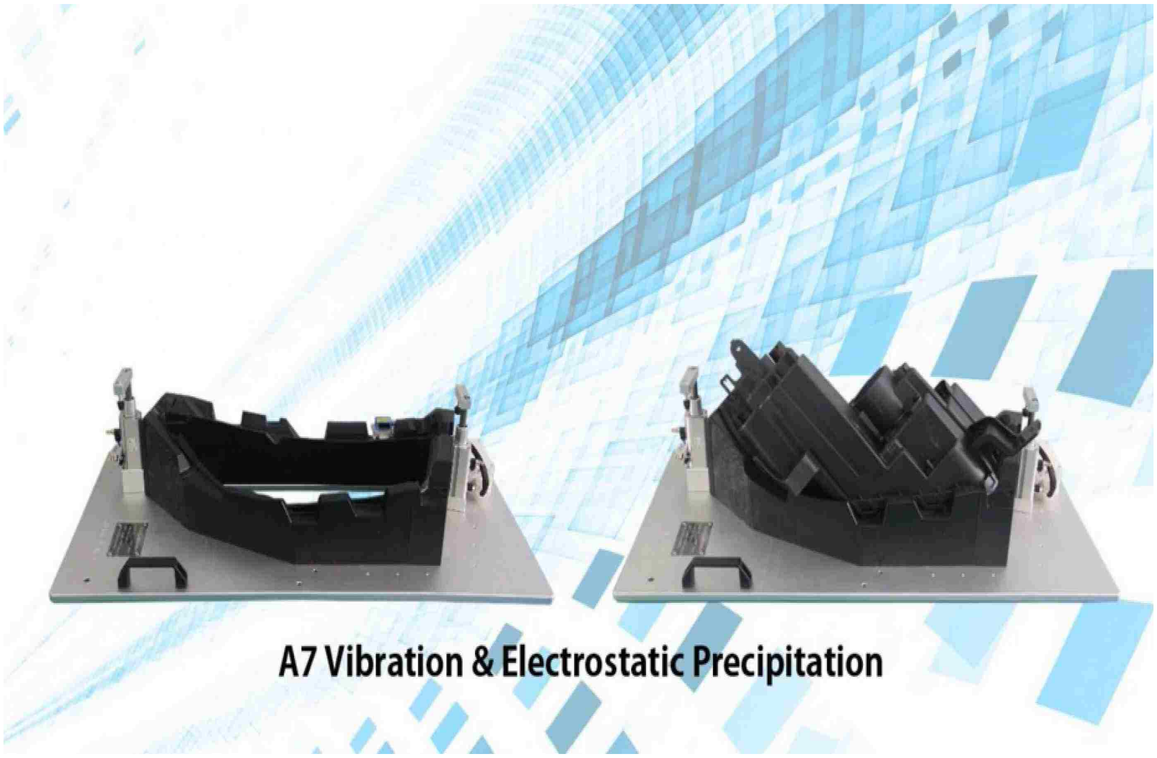


A1 Adjusting Rod Press Fitting

A2 Reflector Adjustment Press Fitting


A3 Wiring Harness Collating & Motor Installation

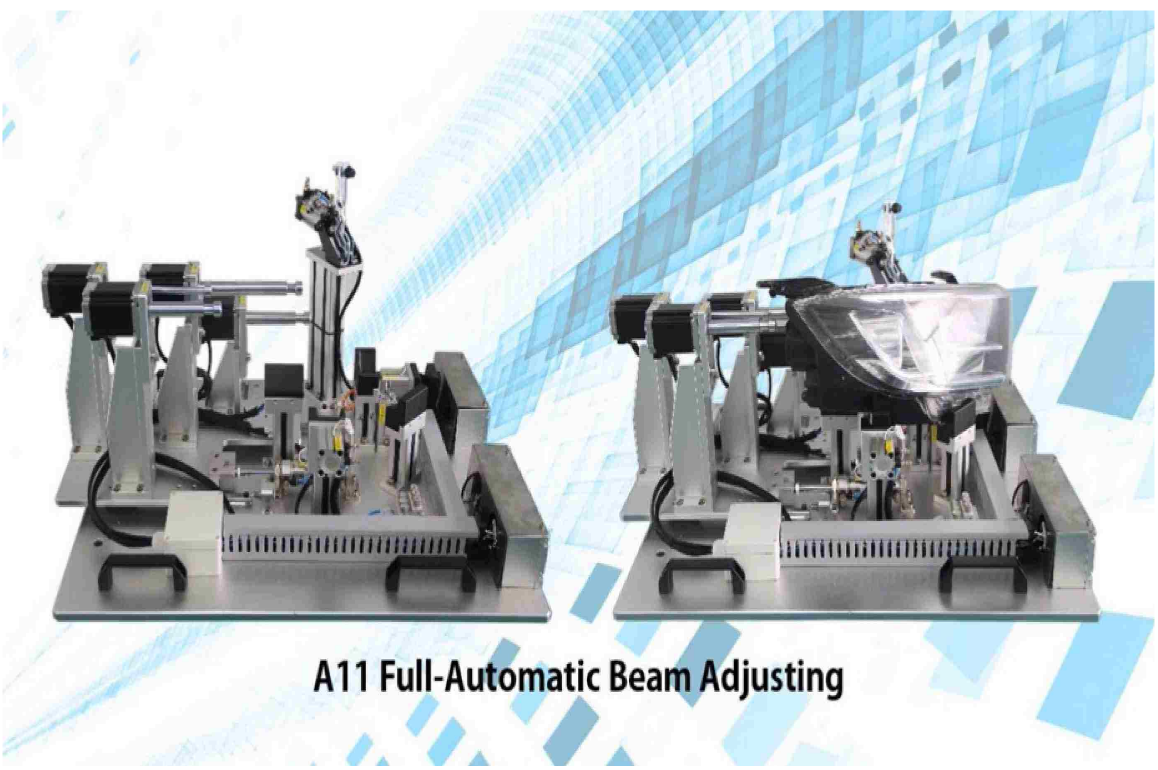

A6 Install Decorative Frame Sub-Assembly Into The Lamp Seat



A7 Vibration & Electrostatic Precipitation

A9 Lamp Seat and Lamp Housing Press Fitting

A10 Install The Lamp Housing Screws



A11 Full-Automatic Beam Adjusting

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