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Modeling And Simulation Of a Continious Folding Process Of An Origami Pattern

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MODELING AND SIMULATION OF A CONTINUOUS FOLDING PROCESS OF AN ORIGAMI PATTERN

By

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ABSTRACT

The engineering applications of origami has gathered tremendous attention in recent years. Various aspects of origami have different characteristics based on its application. The shape changing aspect is used in areas where size is a constraint. The structural rigidity aspect is utilized where strength is needed with a minimal increase in weight. When polymer or metal sheets are processed to have origami creases, they exhibit an improvement in mechanical properties. The sheets which create a specific local texture by means of tessellated folds patterns are called folded textured sheets[1]. These sheets are utilized to create fold cores. These light-weight sandwiched structures are heavily used in the aerospace industry, due to its ability to prevent moisture accumulation on the aeronautical structures at higher altitudes.

The objective of the current research is to explore a new method for the continuous production of these folded textured sheets. The method uses a laser etching setup to mark the sheet with the origami pattern. The pattern is then formed by dies and passes through a conveyor system which is specifically arranged like a funnel to complete the final stage of the forming process. A simulation approach is utilized to evaluate the method. Results show the feasibility of the process along with its limitations. The design is made to be feasible for scaling up for large scale manufacture

DEDICATION

*To my father - **Muthukrishnan** and my mother - **Vijayalakshmi**,*

who are my pillars of strength and support

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I sincerely thank my research advisor, Dr. Zbigniew Pasek, for his guidance and support to complete my thesis. I am deeply grateful for his time, weekly meetings and technical comments during my research and study. His patience and trust in my abilities aided me to thoroughly explore the subject and formulate a research topic at my own pace. This has allowed me to grow and gain confidence as a researcher who can truly make a contribution in the future.

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LIST OF ACRONYMS

UD	Uni directional
CP	Crease Patterns
MIO	Modular Isometric Origami
α	Basic Element Angle
ψ	Angle of side of tessellation with surface
γ	Angle between two creases in a tessellation
L	Length of one tessellation of the folded cell with respect to the flat surface
S	Cross Sectional Side of folded cell
H	Height of the folded cell
V	Distance between vertex and side
l	Length of crease to edge of tessellation
θ	Angle of the dies
NP	Non-deterministic Polynomial-time hardness

INTRODUCTION

1.1 History of Origami

The word *origami* taken from the ancient art of paper folding, combines the Japanese roots *ori*, meaning ‘folded’, and *kami*, meaning ‘paper’[2]. Despite the art having existed for centuries, the huge majority of practical applications have come within the past few decades. Rapid growth in computer technology and advancements in mathematical modelling have helped the art to step into new avenues of expression, from analysis to design techniques.

For centuries origami artists have designed in the highly available medium of paper, producing a variety of folding patterns ranging from statuesque representations of physical phenomena to animated creatures capable of motion. Some designs have the ability to be tessellated, enabling surprising expansion and contraction of the structure [3]. The past few decades have brought a heightened interest in origami which has resulted in the increased interest in developing science, technology and computational methods that could be used to solve modern complex problems.

The work of Akira Yoshizawa, of Japan, a creator of origami designs and a writer of books on origami, inspired a modern renaissance of the craft [2]. He invented the process and techniques of wet-folding and set down the initial set of symbols for the standard Yoshizawa-Randlett system that Robert Harbin and Samuel Randlett later improved upon. His work was promoted through the studies of Gershon Legman as published in the seminal books of Robert Harbin's Paper Magic and more so in Secrets of the Origami Masters which revealed the wide world of paper folding in the mid-1960s.

Modern origami has attracted a worldwide following, with ever more intricate designs and new techniques. One of these techniques is 'wet-folding,' the practice of dampening the paper somewhat during folding to allow the finished product to hold shape better. Variations such as modular origami, also known as unit origami, is a process where many origami units are assembled to form an often decorative whole.

Complex origami models normally require thin, strong paper or tissue foil for successful folding; these lightweight materials allow for more layers before the model becomes impractically thick. Modern origami has broken free from the traditional linear construction techniques of the past, and models are now frequently wet-folded or constructed from materials other than paper and foil. With popularity, a new generation of origami creators has experimented with crinkling techniques and smooth-flowing designs used in creating realistic masks, animals, and other traditional artistic themes[2].

Traditionally it was used only as a display of art because of the aesthetics and did not have any practical usage. In 1970, Japanese astrophysicist Koryo Miura invented the now widely used ‘miura fold’. It was proposed to hold a variety of practical applications. Miura utilized this fold for solar arrays, and in 1995 a solar panel array for space satellites with this design was unfolded on the Space Flyer Unit, a Japanese satellite[3]. A folded Miura fold can be packed into a compact shape, its stiffness reflecting only the thickness of the folded material. Folded structures can be opened in one motion by pulling on its opposite ends, and likewise folded by pushing the two ends together[4].

Geometric origami has long been limited by the constructability of its structures. Models based on square and hexagonal grid symmetries are commonly known, and some interesting work has been done with octagons and the occasional pentagon, but very little else. Alex Bateman and Robert Lang have used computer-aided design algorithms to create geometric works based on unusual or irregular polygons [5] by specifying a desired folded structure, then engineering a crease pattern to fit their constraints. It is an avenue of research with a different perspective.

In present times, origami was made popular by American physicist, Robert J. Lang. He is also one of the foremost origami artists and theorists in the world. He is known for his complex and elegant designs, most notably of insects and animals[4]. He has studied the mathematics of origami and used computers to study the theories behind origami. He has made great advances in making real-world applications of origami to engineering problems.

Oribotics [5] is a field of research concerned with the aesthetic, biomechanic, and morphological connections among nature, origami, and robotics. In current research, the focus is on the actuation of fold-programmed materials such as paper and synthetic fabrics. The design of the crease pattern, the precise arrangement of mountain and valley folds, and the way they fold and unfold directly inform mechanical design. Therefore, a key area of current research is focused on the discovery of patterns that have complex expressions that can be actuated repeatedly. This research has resulted in some artistic exhibitions; the largest to date was at the 2010 Ars Electronica Festival in Linz, Austria.

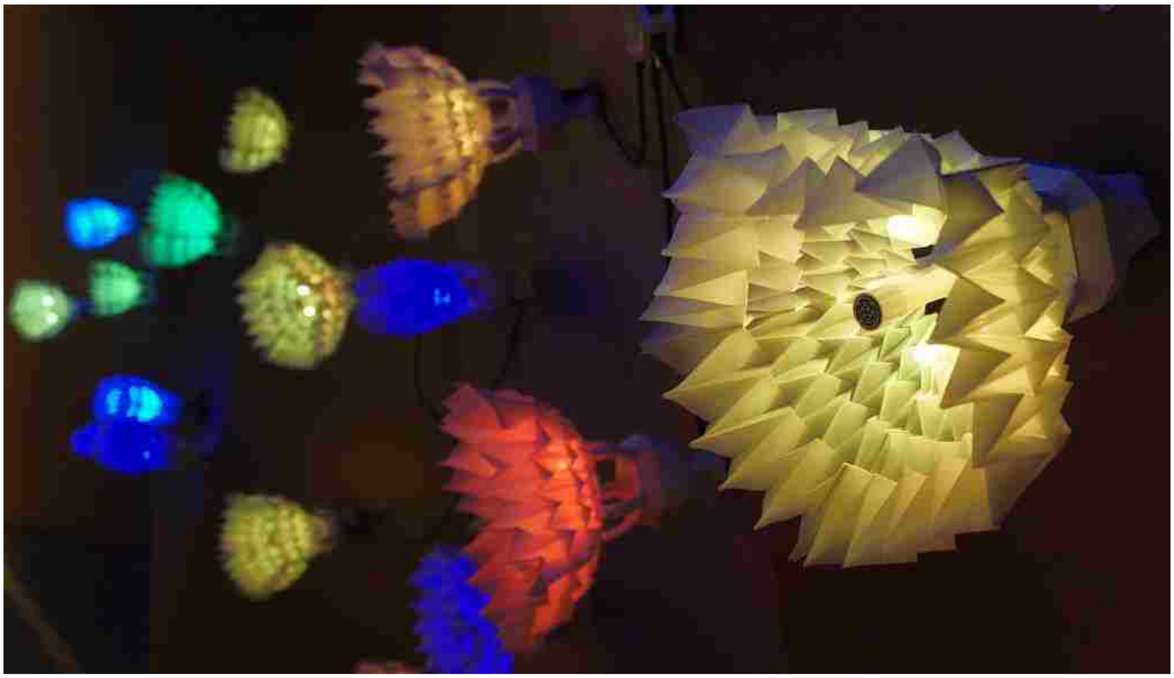


Figure 1 Oribotics at the 2010 Ars Electronica Festival.

Since then a plethora of applications are found by new researchers, who make use of the tools developed by them.

1.2 Modern Uses and Engineering Interest

Origami art has potential for influencing future engineering product design, especially in systems where mass, stowed volume, or cost are to be minimized. This influence may be as indirect as inspiring designers to consider folding in designs or to recognize the possible use of origami approaches in the design of new systems or the analysis of existing systems. But origami also has the potential of being a source of detailed design information. As the potential benefits of origami-based design are becoming more apparent, it is important that resources become available to facilitate the design of origami-based systems. They have a variety of applications from diverse fields ranging from medicine to space. It has been found that origami structures are used by nature in its foldings. This has inspired structures that mimic the patterns of nature.

Various levels of crease characterization exist amongst sheet materials. Paper is a well understood material in the context of creasing. Textiles have become better understood through study into crease proofing and pleating. New manufacturing methods and a recent interest in folded designs of various size scales and materials warrant better characterization of creases in materials beyond paper and textiles. Work in paper and textile folding and the related manufacturing methods are discussed in the following sections. Building on this knowledge helps to characterize the crease properties of non-paper sheet materials, especially polymers and metals, which can expand the possibilities of origami-inspired design [7].

"Origami-based design" is design in which there is a tie to origami and the concepts of folding. A representation of this design domain is shown in Figure 2.

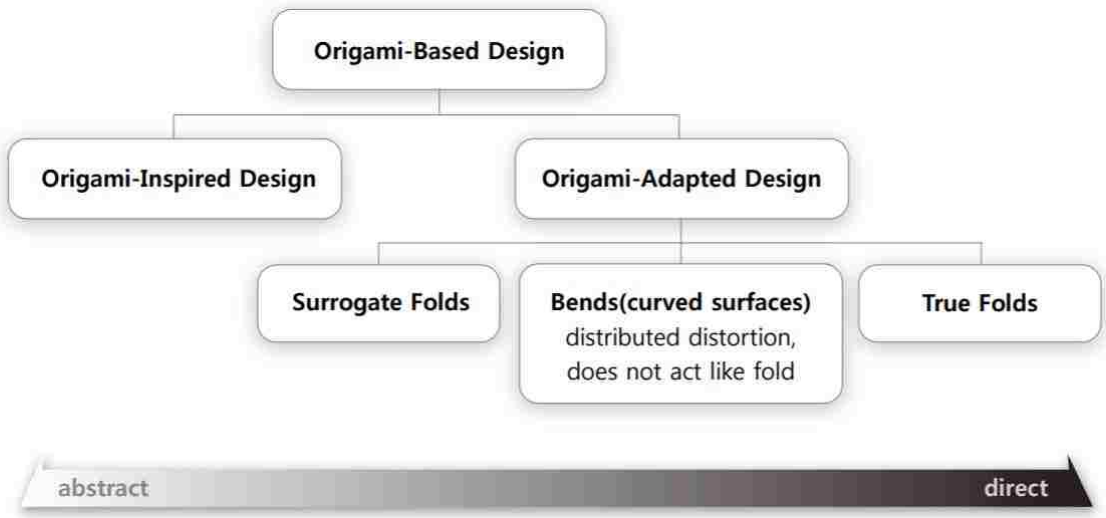


Figure 2 The tree mapping of origami-based design with more abstract ideas found on the left side and more direct ideas on the right (reproduced from [8])

"Origami-based design" is subdivided into two branches: "origami-inspired design" and "origami-adapted design" [8]. "Origami-inspired design" is more abstract and there is a loose link between the emergent design and the origami. Things that are not origami, but remind one of origami, such as DNA and protein folding [9] and assembled static architecture, can be found in this branch. On the other hand, "origami-adapted design" draws more directly from origami and the relationship between classic origami models and the emergent design is more apparent.

Smart materials may depend on their structural geometry. Buckling and crumpling of thin walled materials, especially used in lightweight and deployable structures, is commonly considered as failure. It appears that these "failure" patterns are common in natural structures and that they have their specific functions - for growth, deployment or stiffening making thin walls better resistant to loading. Experimental modeling of natural origami-patterns by crushing (excessive - and fast - axial loading) or by twisting helps analyzing the underlying geometric / physical rules and leads to novel designs for technical self-deployable structures, for arrays of high stiffness and finally for smart mechanisms including elastic and auxetic properties and anticlastic shapes in materials that initially are inextensible, non-elastic and unable to be curved in the third dimension.

From a technical point of view the natural origami-patterns are not necessarily optimal designs, which can be translated directly into a technical device. As natural smart materials may be a compromise – for example between optimal stiffness versus minimal energy expenditure – the inspired designer should do his own physical experiments on the basis of the modeling experiments and on artificial shapes and not content himself with mere "similarity" to natural patterns, otherwise known as biomorphism. Biomimetics requires the transfer of function [10].

Simple folding-based designs are prevalent in the packaging and filtration industries and design concepts have been created in a variety of other applications. Products such as the eyeglass telescope, solar arrays, car covers, automotive airbags, medical stents, temporary shelters, kayaks, crash boxes, athletic shoes, eatery and appliance drawers present evidence that foldable solutions are viable in engineering design. Use of origami folding techniques in structural DNA nanotechnology provides an easier and faster way to construct DNA nanostructures with various shapes. Several experiments proved that DNA origami nanostructures possess abilities to enhance efficacies of chemotherapy, reduce adverse side-effects, and even circumvent drug resistance [11].

Folded patterns can address complex situations, as observed in nature. Hornbeam leaves have a flexible configuration that reduces effective area in the wind yet have sufficient rigidity to support its own weight. Further the leaf is efficiently stowed before emerging from the bud [12].

Origami-based structural tessellations are found to facilitate great versatility in system function and properties through kinematic folding. It is shown that kinematic folding rules and flat-foldable tessellated arrays collectively provide novel solutions to the long-standing challenges of conventional, electronically-steered acoustic beamformers. Using ordered planar tessellations, investigators have shown that origami-inspired structural/material system design may empower favorable characteristics due to folding-induced changes of the topology. Such folding of two-dimensional planar structures introduces three-dimensional functionality and alters fundamental properties like stiffness and Poisson's ratio when the system is used for structural purposes [13].

Sandwich structures are good candidates for the aeronautical industry as they provide a good stiffness/mass ratio (figure 3). Aramid paper honeycomb cores are one of the best candidates. Composite sandwich structures with cellular cores are used in numerous lightweight applications e.g. in aerospace, automotive, marine or civil engineering. In this context, honeycomb core structures made from aluminium or Nomex® aramid paper have a top ranking when it comes to weight-specific mechanical properties [14].

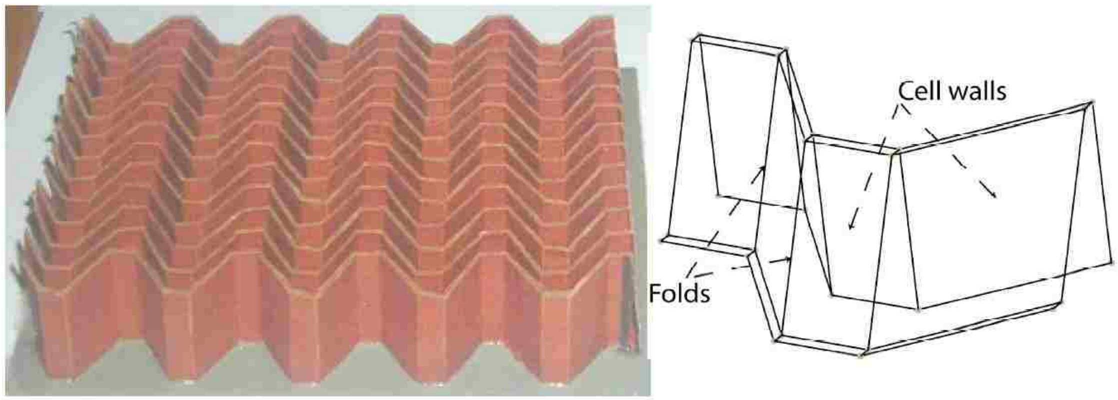


Figure 3 Folded core and nominal unit cell

The scientific community is interested in the ability of these patterns to minimize the storage area of large structures. This is done without any considerable loss to the mechanical integrity of the structure. In certain cases, the structural integrity is enhanced by leveraging the patterns used in the folds.

As the usefulness of folded design has been realized, tools have been developed to aid in crease pattern generation. Commercial CAD programs such as SolidWorks® incorporate sheet metal tools into their design capabilities and Mathematica® can simulate paper folding.

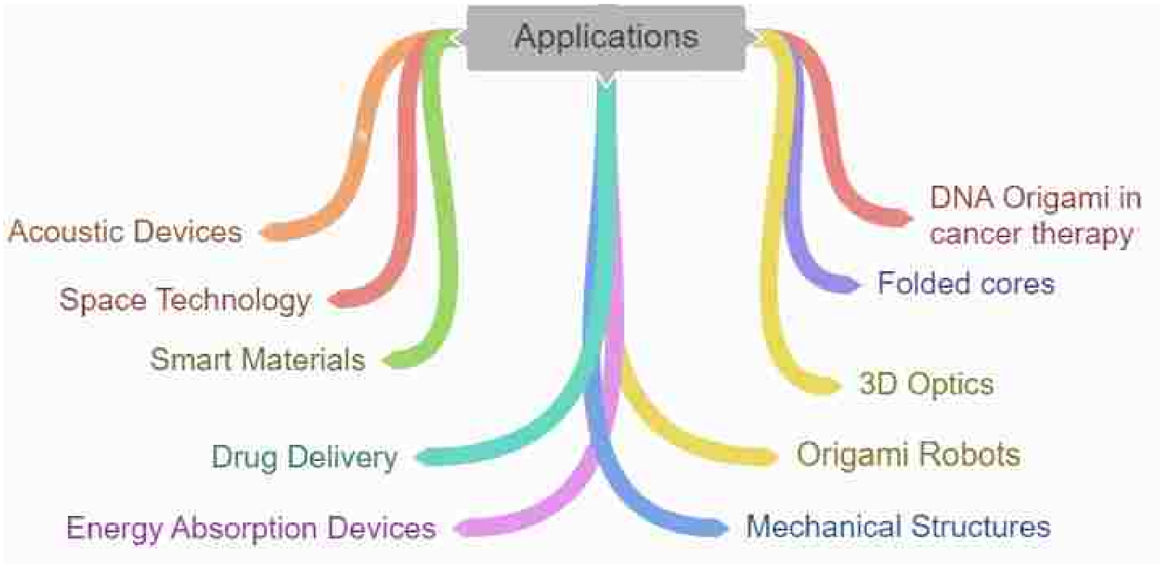


Figure 4 Various application of Origami

As origami design is highly innovative and has seldom been employed in engineering practice, it has the potential to create game-changing products (figure 4). Addressing considerations specific to origami-adapted designs has the potential of facilitating better outcomes. Although the art of product development benefits from intuition and experience, it is often improved when a technical approach is taken.

1.3 Manufacturing challenges

Utilization of origami patterns on industrial materials involves many challenges which do not occur in conventional folding. This is because of the thickness and variation in mechanical properties of the materials.

Origami is able to achieve a high level of kinematic complexity. Origami designs can realize great advantages in that they 1) are fabricated from a flat sheet of material, 2) have low friction joints, 3) have a low material volume and subsequently have a low mass, 4) require only one manufacturing technique (folding), 5) often have great spanning abilities in their configurations ranging from compact to expanded configurations, 6) can have higher area moment of inertias than similar curved surfaces rendering them less likely to need structural reinforcement , 7) can have controlled buckling, 8) often have synchronous deployment that requires few actuation and constraint points, 9) can often be tessellated and 10) can have negative Poisson's ratios.[8]

Materials that have origami-like creases have the ability to both fold and unfold. Hence a material that tends to bend at a previous fold is a material that has origami-like creases. Some materials do not crease (e.g. sheet metal) because they do not exhibit decreased stiffness along the fold. Creases develop when the bending stress is greater than the elastic limit of the material . The formation of a crease resets the elastic memory to a non-zero angle; the harder the crease is pressed, the greater the residual angle [7].

The main manufacturing step is folding the base material into a three-dimensional structure (Fig. 1a). The folding technique allows different types of unit cell geometries. The main application for foldcores is the usage as structural sandwich cores, for which they are predestined by featuring high strength and stiffness to weight ratios. Furthermore, their mechanical properties can be adjusted to the application by varying the unit cell geometry and the base material. Beyond their mechanical properties, foldcores feature also multifunctional aspects. Good thermal insulation and acoustic damping are two characteristics which they share with other sandwich core materials like honeycombs. An advantage to honeycomb cores is the open cellular design, which allows ventilation through the open foldcore channels[15].

Virtual testing using dynamic finite element simulations is an efficient way to investigate the mechanical behaviour of small- and large-scale structures reducing time- and cost-expensive prototype tests. Furthermore, numerical models allow for efficient parameter studies or optimizations [16].

One major disparity is that thin materials are assumed to have low thickness. This enables models to be established based on zero- thickness approximations. Conversely, engineering materials like plastic sheets and so on have a significant thickness and come across self-intersection issues even when the fold is done along a single vertex. Another discrepancy is that the creases of thin materials act as hinges due to reduction in stiffness, whereas thicker materials do not undergo such a noticeable variation in stiffness.

The simplest way to eliminate such difficulties is to manufacture using thin sheets as workable material. The challenges on using thin sheets is mitigated by selecting materials that have an acceptable degree of stiffness.

Optimization methods that constitute usage of FEA (finite-element analysis) to distribute mechanical properties for initially flat structures to determine optimal crease patterns so that the desired motions can be achieved are utilized.

1.4 Area of Focus

In the research environment of the ground and air transportation industry constant work is conducted on developing lighter, cheaper and mechanically beneficial structures and materials. Especially in the aerospace industry, twin-skinned sandwich structures with a thick lightweight cellular core are well-known and widely used because of their excellent weight-specific stiffness and strength properties. The research on new cellular sandwich core structures typically involves a large amount of specimen manufacture and testing to investigate the influence of certain core cell geometry parameters on the mechanical properties, since especially here a relatively large design space is provided from classical honeycomb cores to innovative folded core structures. This is both a time- and cost-intensive procedure.

Nowadays, numerical simulations based on the finite element (FE) method have become a standard tool in the development process of the aircraft industry – from the material level over the component level up to the full aircraft. Therefore, it is reasonable to use this technique also for the characterization of cellular sandwich core structures. Instead of manufacturing expensive prototypes of innovative core structures, FE models on a parametric basis are generated and dynamic simulations of compressive and shear tests are performed in order to evaluate the cell wall deformation behaviour and the effective mechanical properties of different core geometries. This method of virtual testing, which can be combined with a core geometry optimisation for certain requirements, can be a very time- and cost-efficient approach [16].

Folded cores or more commonly known as sandwiched structures comprise of a lightweight cellular core which is sandwiched between two firm faces. Numerous applications are found in the aerospace industry because of its paramount weight-saving design goals. In this relation, honeycomb structured cores made of aluminum or Nomex paper is the most commonly used core type today due to their excellent weight-specific mechanical properties. However, honeycomb cores are known to suffer from an undesirable moisture accumulation problem whereby the condensed moisture is trapped inside the sealed hexagon cells leading to deterioration of the mechanical performance over time [17]

Folded cores contrived by folding sheet material into a 3D structure according to origami patterns, have been found to be resistant to moisture accumulation problems because of open channels in such structures. Moreover, they allow for customized mechanical properties with a wide range of feasible configurations. Therefore, they turn up as a promising alternative to regular honeycomb cores and have seen a rise in research interest from the aerospace industry in recent years. For example, in the transnational project CELPACT, the fabrication cost and impact performance of three different advanced cellular core concepts, i.e. folded core, a selected laser melted lattice core were evaluated and compared [18]. Besides, the aircraft manufacturer Airbus presented a sandwich fuselage concept VeSCo that incorporates folded cores as a sandwich core material [19] and has made a 4.5 m^2 test assembly consisting of approximate 165,000 creases [20].

Although manufacturing of patterns and mechanical testing remain regular procedures, finite element analysis (FEA) method, is the most regularly used tool that is time- and cost-efficient, and has been commonly used in the development of new composite structures.

Additionally, FEA simulations can provide analysis details such as the stress/strain data for each cross-section which are usually difficult to be obtained experimentally. As a result, a number of numerical studies of folded-core sandwich structures, such as virtual in- and out-of-plane quasi-static compression and shear tests [14], [16], [19], [21], [22], low and high-velocity impact simulations [23]–[25], residual bending strength simulations after impact [26] and macro- and multi-scale modelling [21], [14], [22]–[24], are available in the literature. However, most folded cores used in research works are made of two simple Miura -based unit cell geometries with zigzag and chevron shapes [27]. So far, only partial production methods for origami folds is available. No literature has been found by the author on any explicit production methods of the origami folds.

1.5 Problem Statement

As seen from the above discussion the need for a continuous manufacturing process is the major focus of this thesis. As a solution to this problem this research proposes a manufacturing method that will aid in the production process. The main objective of this research is to:

1. Utilize origami structures as folded cores by mass production of desired pattern.

2. Explore a new method for the production of the origami pattern
3. Provide continuous folding of origami patterns for the chosen sheet material
4. Determine the parameters governing the folding process in the case of machine folding

1.6 Approach

The material to be formed into origami sheet is found after evaluating the list of available materials through various filters. The mathematical model for the parameters governing miura-ori pattern is determined. The mathematical properties for the folding of the sheet is also derived. The mathematics is used as the basis for building the CAD model. The CAD model of the various stages is explained and a final complete layout of the manufacturing process is presented. A finite element analysis of the forming process is evaluated under various different boundary conditions and the results are explained. The various limitations of the layout are presented along with the possible future work.

2 LITERATURE REVIEW -A COMPARISON AND DISCUSSION OF CREASE PATTERNS MANUFACTURING TECHNIQUES IN ORIGAMI.

2.1 Idea Map

The entire list of literature utilized is categorized in Table1 based on four major areas of focus.

Table 1 Idea Map

Paper Title, Author and Year of Publishing	**Pattern Parameters**	**Fold cores**	**Simulation**	**Industry Applications**
(K.Miura,1985) Method of Packaging and Deployment of Large Membranes in Space	✓			
(Elsayed and Basily,2004) A Continuous Folding Process for Sheet Materials	✓			✓
(Herrmann et al., 2005) Sandwich Structures Technology In Commercial Aviation-Present Applications and Future Trends		✓		✓
(Nguyen et al.,2005) Simulation of impact on sandwich structures			✓	
(Demaine and O'Rourke, 2007) Geometric folding algorithms - linkages, origami, polyhedra	✓			
(Heimbs et al., 2007) Experimental and Numerical Analysis of Composite Folded Sandwich Core Structures Under Compression	✓		✓	
(Heimbs,2008) Virtual Testing of Sandwich Core Structures with LS-DYNA	✓		✓	
(Heimbs,2009) Virtual testing of sandwich core structures using dynamic finite element simulations				

(Heimbs et al., 2010) Sandwich structures with textile-reinforced composite foldcores under impact loads		✓		
(Tachi, 2011) Rigid-Foldable Thick Origami	✓			✓
(Baranger et al., 2011) Modelling of the Behaviour of Aramid Folded Cores Up to Global Crushing: Modelling of Folded Cores Under Compression		✓		
(Kresling, 2012) Origami-structures in nature: lessons in designing “smart” materials	✓			
(Nishiyama, 2012) Miura Folding: Applying Origami to Space Exploration	✓			
(Francis et al., 2013) Origami-like creases in sheet materials for compliant mechanism design	✓			
(Heimbs, 2013) Foldcore Sandwich Structures and Their Impact Behaviour: An Overview		✓		
(Francis et al., 2014) From Crease Pattern to Product: Considerations to Engineering Origami-Adapted Designs	✓			
(Qattawi et al., 2014) Design Analysis for Origami-Based Folded Sheet Metal Parts		✓	✓	
(Song et al., 2014) Origami lithium-ion batteries				✓
(Tolman et al, 2014) Material selection for elastic energy absorption in origami-inspired compliant corrugations			✓	
(Wood, 2014) The challenge of manufacturing between Macro and Micro: classic ways of folding paper into dynamic shapes--origami, pop-up books--inspire methods to engineer millimeter-scale machines				✓
(Zhou et al., 2014) Mechanical properties of Miura-based folded cores under quasi-static loads	✓	✓	✓	
(Lebée,2015) From Folds to Structures, a Review	✓			

(Brown Et al.,2016) Origami acoustics: using principles of folding structural acoustics for simple and large focusing of sound energy				✓
(Eidini, 2016) Zigzag-base folded sheet cellular mechanical metamaterials		✓		
(Fuchi et al., 2016) Design Optimization Challenges of Origami-Based Mechanisms With Sequenced Folding		✓		
(Klett et al., 2016) Designing Technical Tessellations		✓	✓	
(Turner et al, 2016) A review of origami applications in mechanical engineering		✓		
(Udomprasert et al.) DNA origami applications in cancer therapy				✓
(Crampton, 2017) Considering Manufacturing in the Design of Thick-Panel Origami Mechanisms	✓		✓	
(Heimbs et al., 2009) Sandwich Panels with Cellular Cores Made of Folded Composite Material: Mechanical Behaviour And Impact Performance		✓	✓	
(Heimbs et al., 2006) Numerical Determination of the Nonlinear Effective Mechanical Properties of Folded Core Structures for Aircraft Sandwich Panels		✓	✓	
(Heimbs et al., 2009) Sandwich Structures with Folded Core: Mechanical Modeling and Impact Simulations		✓	✓	
(Klaus,2009) Residual Strength Simulations of Sandwich Panels After Impact		✓	✓	
(Qattawi, 2012) Extending Origami Technique to Fold Forming of Sheet Metal Products		✓		
(Schenk et al.,2014) Novel Stacked Folded Cores for Blast-Resistant Sandwich Panels		✓		

The core topics are elaborated as follows:

Part 1: Pattern Parameters –contains a brief outline of the literature on the characteristics of the origami patterns

Part 2: Fold cores – Gives a brief review of the research done on the properties of various folded cores

Part 3: Cites publications that use simulation software as validations for improvements in both manufacturing and service industries.

2.2 Pattern Parameters

Space missions which require ultra-low mass, have a necessity for erectable structures to be packaged for deployment. The isometric condition of the structure must be held unchanged for the entire process. The process should also have a fundamental repetitive aspect. Miura provides a means of folding by using Bi-axial shortening of a plane into developable double corrugation (DDC) surfaces as shown in figure 5.[28] The method of folding is along two mutually perpendicular directions at the same time and in a uniform way. This method of folding forms the basis of the study for this thesis.

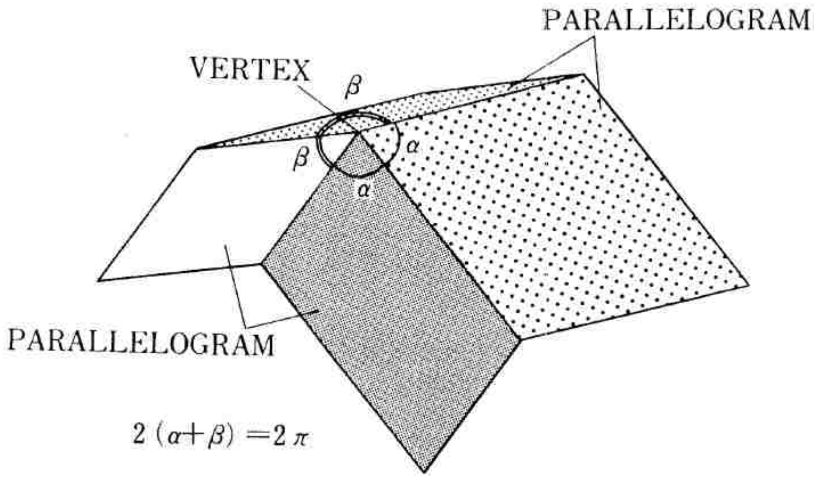


Figure 5 Fundamental region of Developable Double Corrugation Surface

The paper titled ‘A Continuous Folding Process for Sheet Materials’ presents a new sheet folding technology for different patterns using continuous manufacturing techniques. A novel approach is developed for the continuous folding process where sheet material is progressively folded in two dimensions, through a set of rollers, followed by a configured roller for the final folding in the third dimension.[29] The final roller can be designed for longitudinal folding, cross-folding and angular folding to produce the desired folded pattern. Miura-ori patterns produced by this method are tested against honeycomb structures and a comparison is shown in its mechanical characteristics.

The folded textured sheets form part of ongoing research into the properties and applications of textured sheets. By introducing a “local” texture (such as corrugations, dimples, folds, etc.) to otherwise isotropic thin-walled sheets, the “global” mechanical properties of the sheets can be favorably modified. The “local” texture has no clearly defined scale, but lies somewhere between the material and the structural level and in effect forms a microstructure. The texture patterns in folded textured sheets are inspired by origami folding, as the resulting sheets need not necessarily be developable. The texture consists of distinct fold lines, and it is therefore better to speak of polygonal faceted surfaces. Figure 6 shows the example sheets used in this paper: the Miura sheet. (The models are made of standard printing paper, and the parallelograms in both sheets have sides of 15 mm and acute angles of 60°. The Miura sheet is folded from a single flat sheet of paper) The first obvious property of the folded sheet is the ability to undergo relatively large deformations by virtue of the folds opening and closing. Moreover, the fold patterns enable the sheet to locally expand and contract—and thereby change their global Gaussian curvature—without any stretching at material level [1].

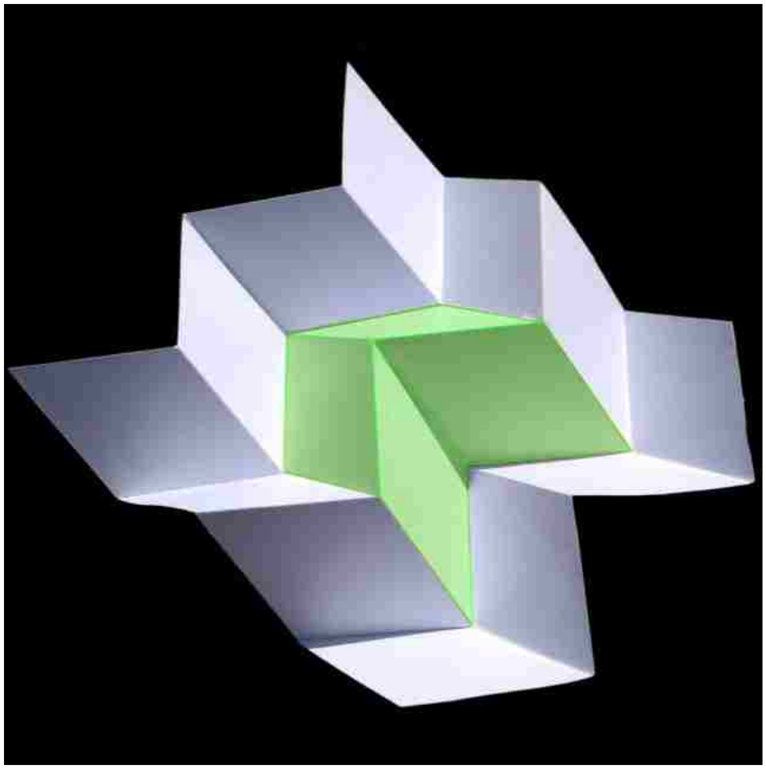


Figure 6 Photographs of the Miura sheet. The model is made of standard printing paper, and the parallelograms have sides of 15mm and an acute angle of 60◦ . The Miura sheet is folded from a single flat sheet of paper.

With the potential of parametric models and the ability to cover the stiffness and strength of the respective folded core structures with an acceptable degree of accuracy, a geometry optimization can be performed in order to identify a folded core geometry with optimized mechanical properties with a minimum density. Using this numerical approach, this can be done much more efficiently than conducting experimental test series. This paper only covers the flatwise compression behavior but this investigation can easily be extended to the transverse shear properties and the behavior of foldcore sandwich panels under various impact conditions [19].

2.3 Fold Cores

The publication titled 'Sandwich Structures Technology In Commercial Aviation' gives a brief overview of sandwich application history in general and present composite sandwich structures at Airbus along with a list of current R&D developments for sandwich structures [17]. The potential for sandwich cores mainly lies in its weight effectiveness and continuous stiffness distribution. The possibility of functional integration like thermal insulation together with its excellent damping properties make it the most suitable for aviation.

The challenges lie in the development of such sandwich structures. Cost effective, in service non-destructive testing (NDT) methods for complex sandwich structures need further development. Newer methods for load introduction and reinforcement concepts along with verified simulation methods and tools for simulating damage behaviour of complex sandwich structure are required.

Innovative sandwich core structures can be produced by folding composite prepreg sheets to three-dimensional zigzag structures. The foldcores in this study refer to [14] are made from aramid and carbon woven fabrics with epoxy resin (figure 7). This paper describes the cell walls' mechanical behaviour under flatwise compression as well as low and high velocity impact loads. Their weight-specific compression properties are even higher than those of Nomex® honeycomb cores of similar density. The dual-core configuration with two layers of foldcore and an aramid composite middle layer showed the potential of a two-phase energy absorption behavior, which can be tailored by the choice of the constituents and the variation of the geometrical configuration. Dynamic finite element simulations have shown to be an efficient tool in the development of such innovative structures that exhibit a large design flexibility in terms of possible core geometries and materials [14].

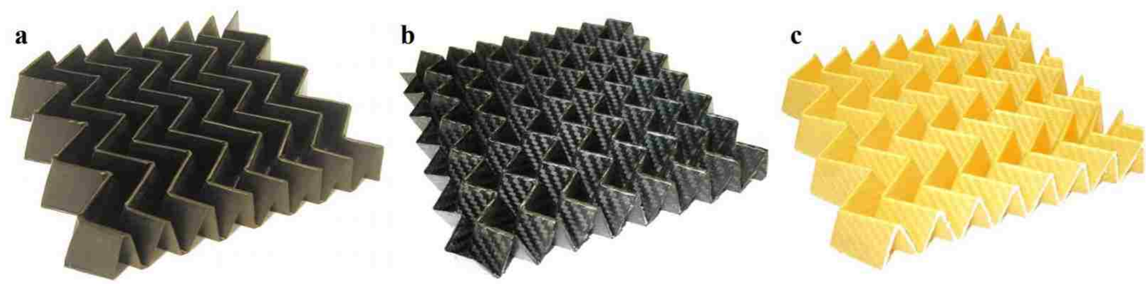


Figure 7 Various Composite foldcore structures made of a) UD carbon fiber laminate, b) woven carbon fabric and c) woven aramid fabric

The folding pattern is a simple zigzag geometry, based on the unit cell geometry in Fig. 8. Four configurations with different materials and geometries were investigated in this study, which are referred to as type A, B, C and D. The global densities vary from 103 to 119 kg/m³. All details are given in the overview table in Fig. 9. The aramid foldcore type A and the carbon foldcore type B are based on the same geometry

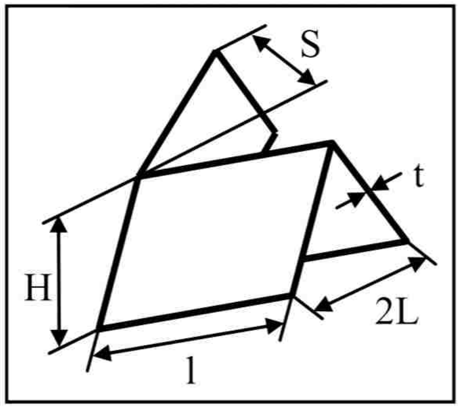


Figure 8 Unit Cell Geometry for the composite structures

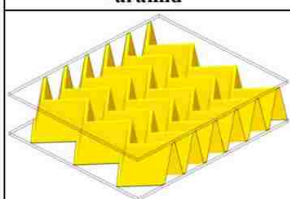
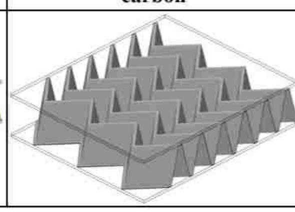
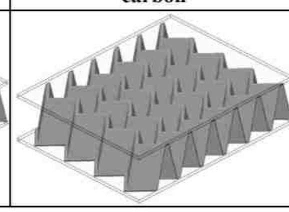
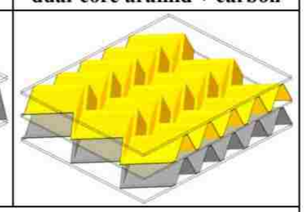
***Type A*:** **aramid**	***Type B*:** **carbon**	***Type C*:** **carbon**	***Type D*:** **dual-core aramid + carbon**
			
Core geometry:			
l=29 mm, L=10 mm, S=20 mm		l=18 mm,L=7.5 mm,S=15.5 mm	l=29 mm, L=10 mm, S=20 mm
Core height H:			
28 mm			14 mm (AF) / 14 mm (CF)
Cell wall material:			
Krempel AGBD2008 prepreg (twill weave fabric, 2 plies)	Krempel KGBD2008 prepreg (twill weave fabric, 2 plies)		Krempel AGBD2008 (AF) / Krempel KGBD2008 (CF)
Cell wall thickness t:			
0.40 mm	0.48 mm	0.50 mm	0.40 mm (AF) / 0.52 mm (CF)
Core density ρ:			
103 kg/m³	113 kg/m³	114 kg/m³	109 kg/m³ (AF)/119 kg/m³ (CF)

Figure 9 Different foldcore configurations investigated

The mechanical behavior of composite sandwich structures with textile-reinforced composite foldcores, which are produced by folding prepreg sheets to three-dimensional zigzag structures, is evaluated under compression, shear and impact loads. While foldcores made of woven aramid fibers are characterized by a rather ductile behavior, carbon foldcores with their brittle nature absorb energy by crushing, showing extremely high `velocity impact loads tends to be very localised. In addition to regular single-core sandwich structures, a dual-core configuration with two foldcores is also investigated, showing the potential of a two-phase energy absorption behaviour. In addition to experimental testing, finite element models for impact simulations have been developed. Despite the high degree of complexity of the models due to the various skin and core failure modes that have to be covered, the results correlate well with test data, allowing for efficient parameter studies or detailed evaluations of damage patterns and energy absorption mechanisms [23]

The graph (figure 10) shows the difference due to the material used in the fold cored ranging from aramid fibres, carbon fabric and a dual core combination of carbon and aramid.

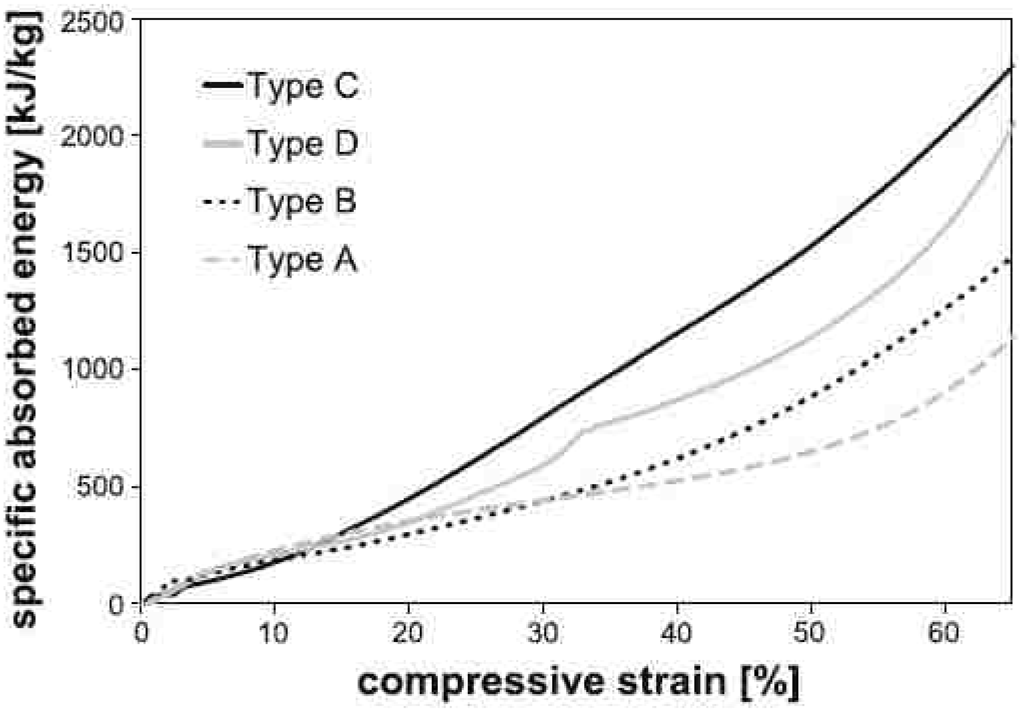


Figure 10 Comparison of specific energy absorption of four different foldcore types.

2.4 Simulation based Authentication

The approach of virtual material testing as a promising alternative to extensive experimental testing. It uses detailed finite element models of the cellular core in combination with virtual material testing. A variation of geometric or constitutive parameters can easily be performed in order to optimize the structure's mechanical properties.[21] The numerical determination of the effective properties of sandwich core structures by virtual testing turned out to be a promising method, which could be adopted for drop test simulations.

Virtual testing simulations of fold core structures shows that not only numerous modelling parameters and an appropriate choice of material model for the cell wall paper influence the simulation results, also the consideration of imperfections was essential[19]. The results of virtual compression test simulations were found to be consistent with experimental data. Only the post-failure behavior of the core structures could not be represented satisfactorily with the respective material models used. With the potential of parametric models and the ability to cover the stiffness and strength of the respective folded core structures with an acceptable degree of accuracy, a geometry optimization can be performed in order to identify a folded core geometry with optimized mechanical properties with a minimum density. Using this numerical approach, this can be done much more efficiently than conducting experimental test series

Two different approaches to account for imperfections in the FE-models were investigated: One way is to include the actual imperfections in the model. Global geometric imperfections can be modelled by randomly distorting the foldcore geometry prior to meshing (Fig.11.a). Local imperfections like uneven cell walls can be represented by randomly modifying all node's coordinates (‘node-shaking’, Fig. 11.b). Both features were included in the parametric models based on random numbers. This is a reasonable approach towards reality, but still does not account for all possible imperfections.

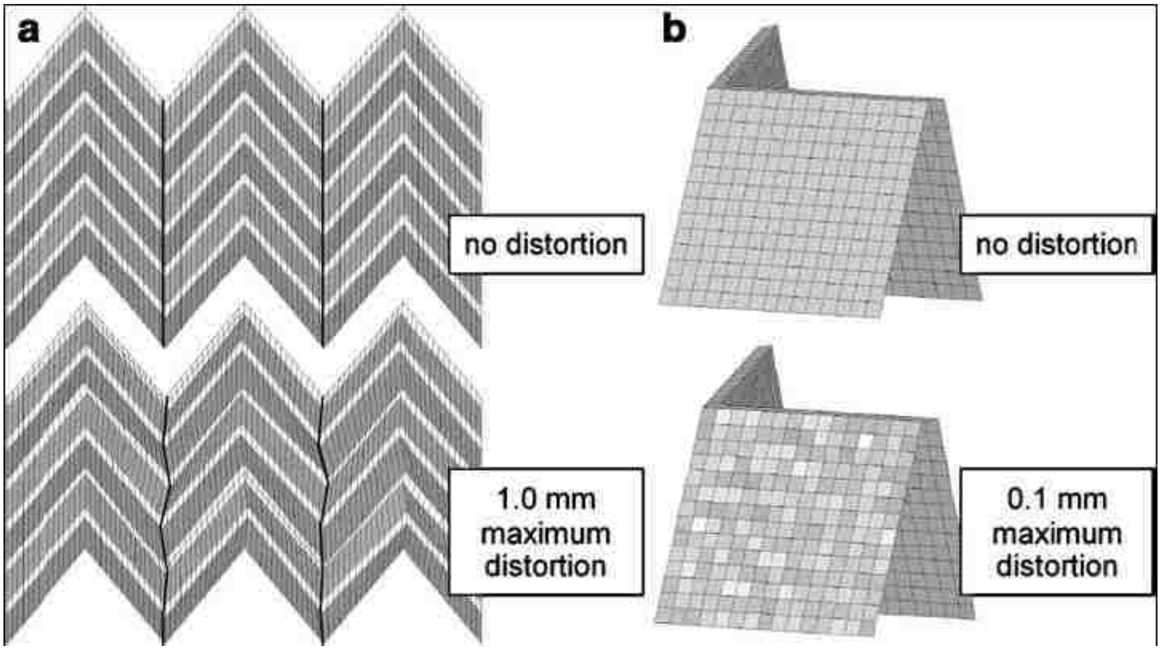


Figure 11 Imperfections through a geometry distortion and b node-shaking

The other way is to keep the ideal mesh and reduce the cell wall properties in such a way that the effective structural behaviour matches experimental data. This inverse approach requires basic experimental data of the folded core. These data are the target values, while the cell wall's thickness as well as stiffness and strength are defined as parameters. Within a parameter identification loop with an optimisation software, compression and shear test simulations are performed in order to determine the set of parameters with the best correlation to the target values. Hereby, the lack of imperfections in the FE-model is compensated by the use of cell wall properties that are on purpose lower than in reality.[19]

The method of virtual testing of cellular sandwich core structures using dynamic finite element simulations as an efficient alternative to experimental test series was presented in this paper.[16] This study included different core structures – hexagonal and expanded honeycomb cores as well as folded cores of different geometries and materials – all showing a different mechanical behaviour under compression and shear loads. The respective material modelling in LS-DYNA, the methods to incorporate imperfections in the model and the influence of simulation parameters like the mesh size were addressed. The results of virtual test in simulations showed a good correlation to experimental results with respect to cell wall deformation mechanisms and stress– strain relationships (figure12). Therefore, these models cannot only be used for the complete characterization of the mechanical behaviour under compression, tension and shear loading in all in-plane and out of- plane directions, which is usually not done experimentally. They also allow for a detailed investigation of cell wall deformation patterns and failure modes to get a better understanding of the structural behaviour. It is difficult to obtain data using solely experimental observations.

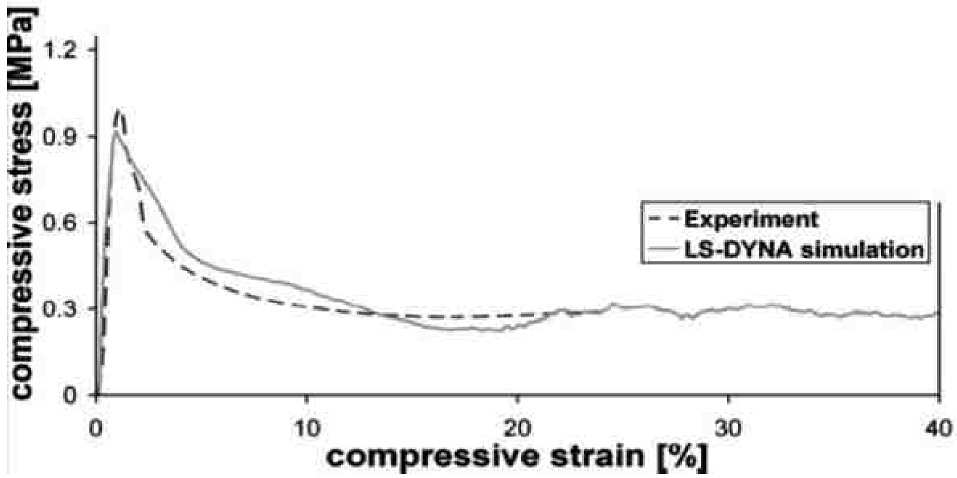


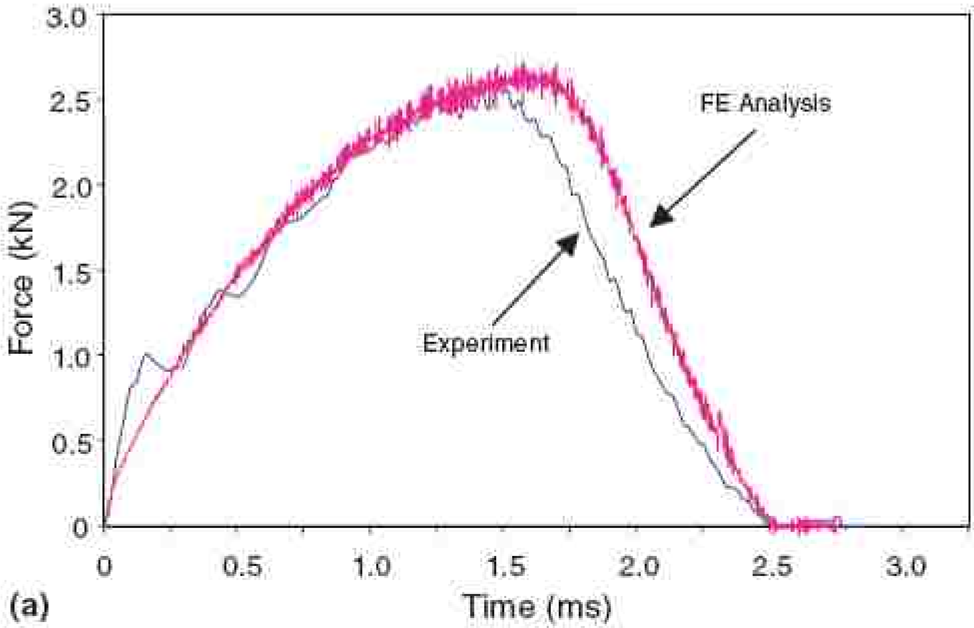
Figure 12 Kevlar foldcore cell wall deformation and stress–strain curve for flatwise compression (reproduced from [16])

An explicit finite element-based simulation tool has been developed to predict the damage within sandwich structures subjected to low velocity impact (figure 12). The tool, Sandmesh, is capable of automatically generating three-dimensional shell models of both honeycomb and folded structure cores, as well as applying the necessary controls for solution generation [25]. Sandmesh was validated via an experimental test program in which honeycomb sandwich panels were tested for impact resistance and damage. Results showed that for low velocity impact, Sandmesh was capable of accurately predicting both the size and depth of the permanent indentation, as well as providing excellent correlation with the force-time histories.

Dynamic finite element simulations have shown to be an efficient tool in the development of such innovative structures that exhibit a large design flexibility in terms of possible core geometries and materials.

The above methods, which represents the partially folded sheet as a pin-jointed bar framework, enables a nice transition from a purely kinematic to a stiffness matrix approach, and provides insight into the salient behavior without the expense of a full series of experiments. It captures the important behavior of the example sheets, and indicates that the dominant mechanics are a result of the geometry rather than the exact material properties.

Sandwich cores were subjected to compressive forces until buckling occurred as shown in Figure 13.



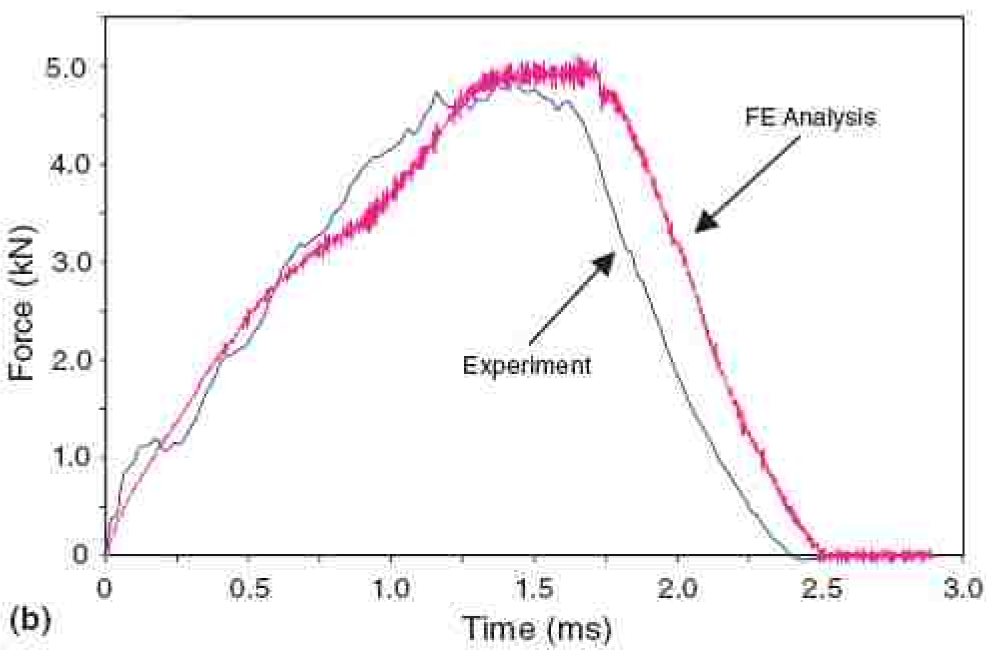


Figure 13 Force-time histories for a 25 mm impactor at (a) 1 J and (b) 15 J (reproduced from [25])

2.5 Manufacturing Method

A novel technique for continuous sheet material folding is developed in which the sheet material is progressively folded in the two dimensions normal to feeding direction through a set of rollers, followed by a configured roller for the final folding in the third dimension, the two dimensional pre-folded sheet transforms into the final three dimensional folded shape, as it passes through the configured main roller. This method is used to form miura-ori patterns.

Miura-ori patter is generated through a sequence of folding steps (figure 14). This pattern can also be produced by folding different sheet materials, such as aluminum, copper, stainless steel, Kraft paper, composites and plastics

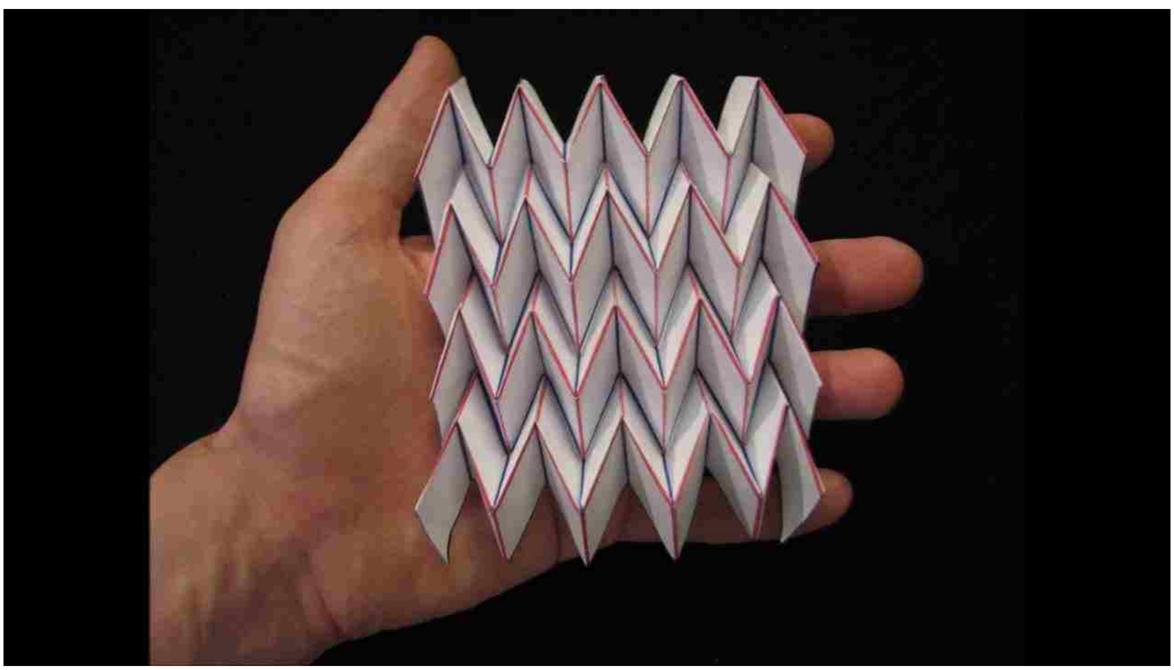


Figure 14 a Chevron pattern, made of paper

It should be emphasized that folding technology for sheet material, is one of the most efficient shape and structural forming processes. Other production methods such as stretch-drawing, forging, pressing and forming may produce costly three-dimensional patterns that appear similar, but the mechanical properties of the resultant patterns are significantly different. This is particularly so in the case of folding thin sheet materials where variations in sheet thickness and/or mechanical properties are unacceptable. Additionally, the developed continuous sheet folding technique is capable of producing very intricate structures that can be economically produced at high-speed from rolled stock through the use of a single manufacturing process, namely folding.

Given a pair of points on a surface, measurement of the length of the shortest path that can be drawn on that surface connecting them is called the intrinsic geometry of the two points. Folding preserves the intrinsic geometry, since any curve drawn on a sheet will have the same length before and after folding. This holds as long as the sheet thickness is relatively small. Of course, thicker sheets will experience deformation and stretching when the bending is significant

Folded patterns also have one or more elementary flat surface, each of which has a specific geometrical shape that forms the basic building elements of the folded pattern. Additionally, a combination or multiplication of these elementary flat surfaces of a specific geometrical shape constitutes the basic building cell of a folded pattern, as it is repeated in two dimensions, creating the three-dimensional folded shape.

Creation of three-dimensional folded Miura structures from a sheet tessellated with the basic building cell is achieved by inducing a permanent edge bending in particular directions between these four polyhedron elements and along all the edges of these four basic elements. This generates the basic building block, which is a three-dimensional structure with block length, block width and block height as shown in Figure 15. Repetition of the building block in the X and Y directions forms the entire folded Miura structure shown in Figure 16.

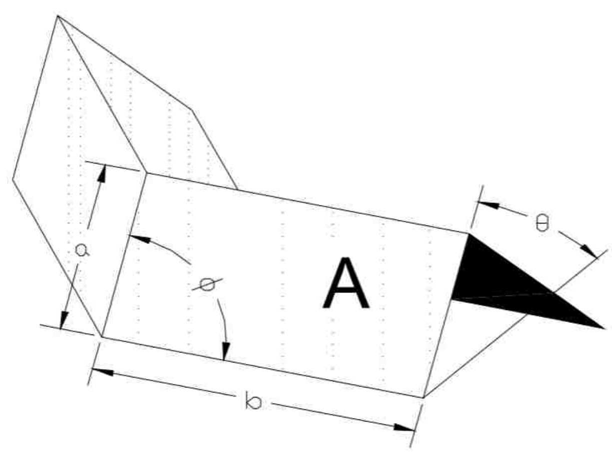


Figure 15 Basic Building Block of Tessellated Miura Pattern Sheet

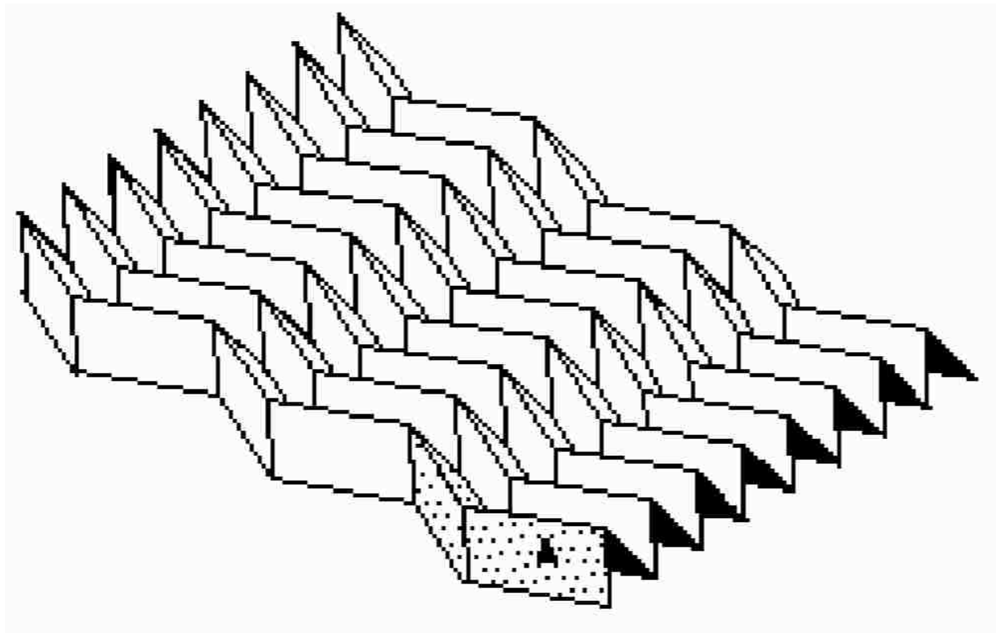


Figure 16 Three Dimensional Miura-ori Structure

In this pre-folding technique, Figure 16, the first set of linear folding rollers, has only one mating Vee grooving, that creates a linear fold positioned at the center of the sheet. As the material advances further through the second set of rollers, which is fitted with three V-grooves, the sheet material will be registered by the initial central linear fold, previously formed by the first set of rollers, but indented with further linear folds, one on each side of the central fold. Similarly, the third set of rollers, which has five grooves, will result in adding two further linear folds to the material, as the process proceeds, further longitudinal folds are made in an arithmetical sequence of 1-3-5-7-9 until the desired amount of sheet contraction is achieved in the normal direction of the sheet advance[29].

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2.6 Summary

It is shown from the literature that not many studies have been completed on the manufacturing methods for the textured sheets. The various mechanical properties of the structures have been shown. Also, only one of the studies found regarding manufacturing of these folded cores include a complete method for their production. Thus, this thesis focuses on a new production method for the folded textured sheets using a simulation approach.

3 THE PROPOSED METHOD FOR OPTIMIZING PROCESS FLOW

3.1 The Process Flow for Industrial Scale

The process flow takes into consideration the various steps involved in the complete manufacturing process and the reviews done to improve it. The four stages start from material input section to the preprocessing stage to the final forming stages as shown in figure 17.

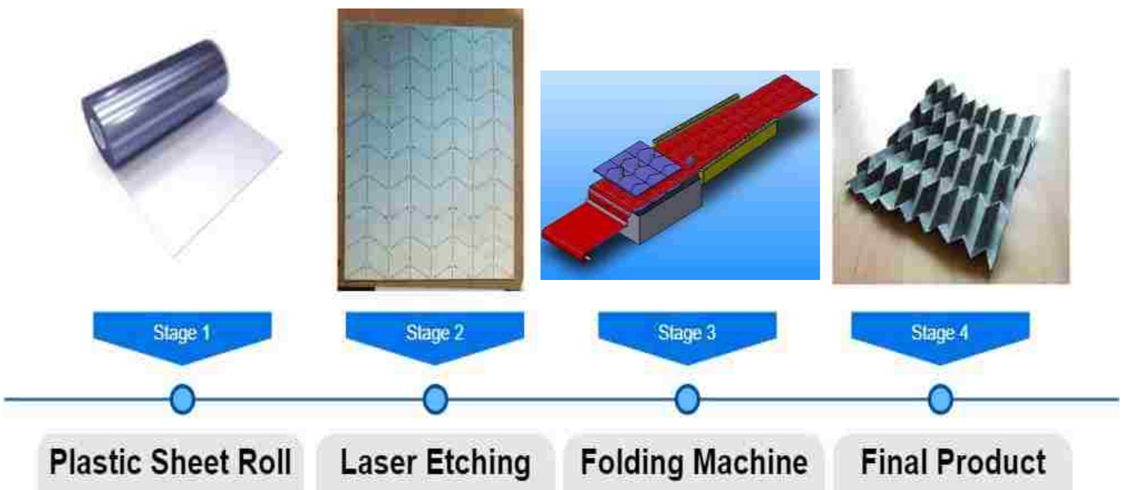


Figure 17 Production process Flowchart for the Miura-ori Pattern

3.1.1 Major Manufacturing Considerations

The various considerations necessary to complete the manufacturing process is as follows:

1. Continuous output of folded textured sheets for any specified length is a requirement when considering manufacturing on a mass scale

2. Continuous input of the given material sheet
3. Sequential process to prevent lag time after each stage.

3.1.2 Material Selection

The selection of materials is an ordered process by which engineers can systematically and rapidly eliminate unsuitable materials and identify the one or a small number of materials which are the most suitable.

When the objectives and constraints can be expressed as well-defined limits on material properties or as material indices, systematic selection by analysis is possible. When constraints are qualitative, solutions are synthesized by exploring other products with similar features, identifying the materials and processes used to make them. When alternatives are sought for an existing material and little further is known, the method of similarity – seeking material with attributes that match those of the target material – is a way forward. And many good ideas surface when browsing. Combining the methods gives more information, clearer insight, and more confidence in the solutions.

The Material Index (M) for a panel (flat plate, loaded in bending) stiffness, length, width specified, thickness:

$$M = \frac{E^{\frac{1}{3}}}{\rho} \tag{3.1}$$

Where,

E = Young's modulus for tension, the flexural modulus for bending or buckling;

ρ = density

A computer aided selection is done using a software called CES Edupack (2017).

A custom database is generated to eliminate unnecessary material categories like Ceramics and glasses, Elastomers, Natural and foam type materials. The selection is done within material libraries like Polymers, Metals, Composites and Alloys as shown in figure 18.

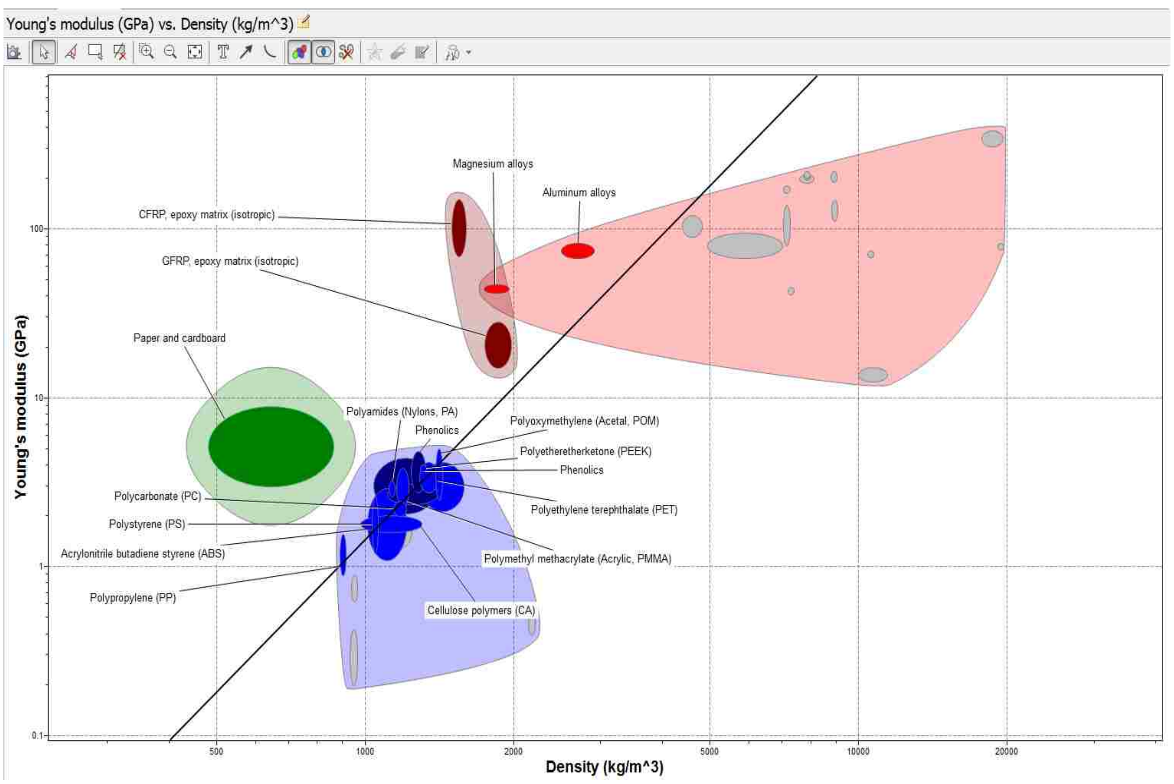


Figure 18 An X-Y plot is created between The Young's Modulus (GPa) vs. Density (kg/m^3)

The line for material index is drawn by giving the slope value as 3. This is taken from the equation for material index. The list of materials based on index value is tabulated. The materials range from carbon fiber reinforced plastic (CFRP) to a variety of Polymers as shown in Table 2. From the list of various possible materials, the material for simulation was chosen based on factors like availability and material properties of the polymer.

Table 2 List of Materials within slope value 3

Name of Polymer	Close to Index Value 3
CFRP, epoxy matrix (isotropic)	0.00301
Paper and cardboard	0.00269
Magnesium alloys	0.00192
Aluminum alloys	0.00156
GFRP, epoxy matrix (isotropic)	0.00147
Polyamides (Nylons, PA)	0.00126
Phenolics	0.0012
Polymethyl methacrylate	0.0012
Polyetheretherketone (PEEK)	0.0012
Polypropylene (PP)	0.0012
Polystyrene (PS)	0.00117
Polyethylene terephthalate (PET)	0.00116
Epoxies	0.00112
Polycarbonate (PC)	0.00112
Acrylonitrile butadiene styrene	0.00111
Polyoxymethylene (Acetal, POM)	0.0011
Cellulose polymers (CA)	0.00108
Polyvinylchloride (tpPVC)	0.00108

For designing a prototype, the initial considerations for the material was chosen to be Polycarbonate (PC). The decision was based on the ease of availability of sheets and low cost of the material.

For the sake of this thesis, initial consideration is more focused on forming origami sheets utilizing polymers as the feeder material. Subsequent iteration can be altered to accommodate various types of materials without changing the core design of the prototype model.

The various properties of polycarbonate are taken into consideration and material values for the boundary conditions are taken for the simulation stage. It is always observed that the yield strength is highest in compression, followed by tension, with the lowest value measured in shear as shown in Fig. 19.

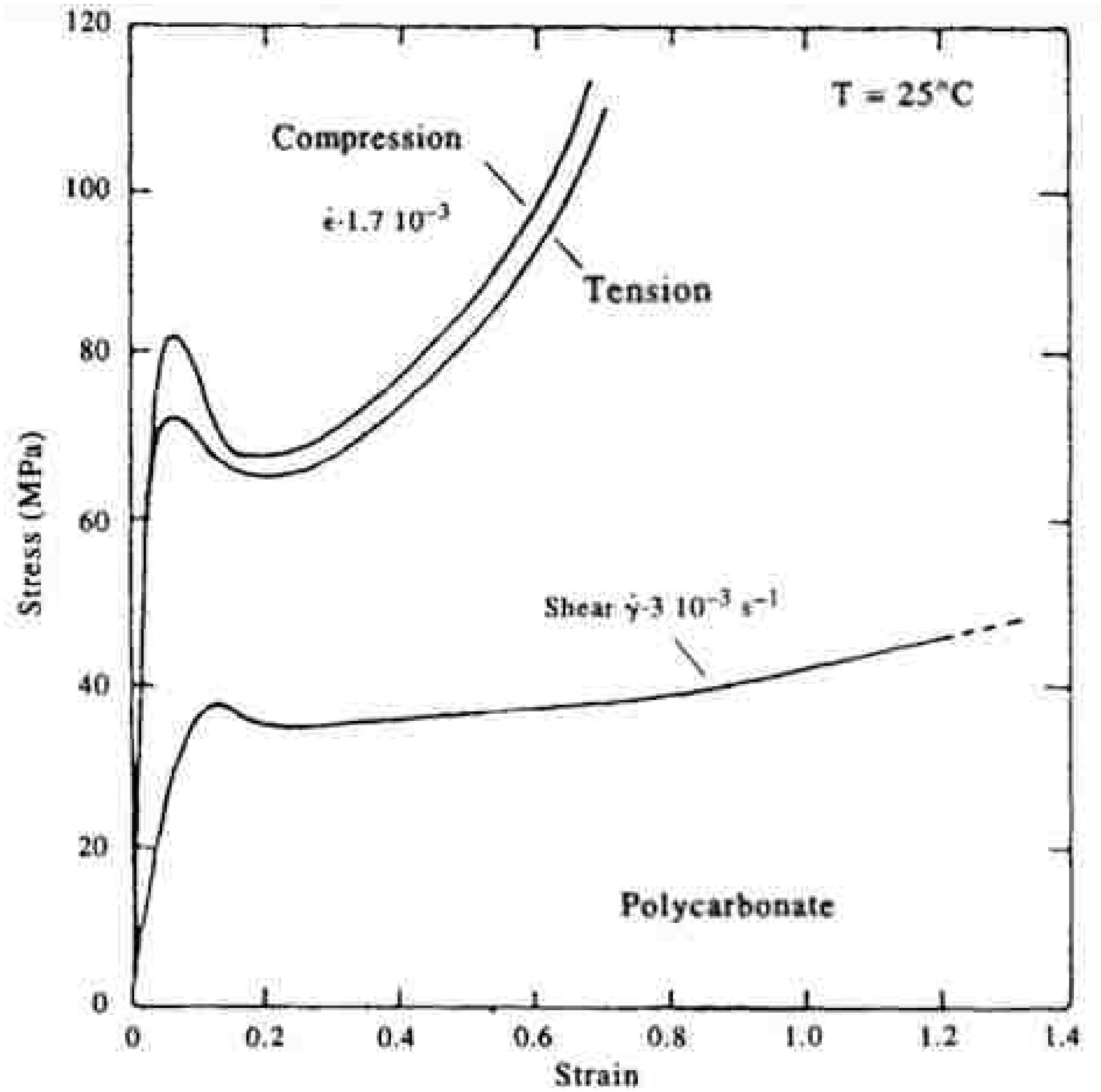


Figure 19 True-stress vs true-strain behaviour of polycarbonate (PC) under three modes of deformation: compression, tension and shear. Note the upper yield, followed by work hardening in tension and compression, but almost non-existent in shear (reproduced from [30])

The yielding behavior in polymers is also very sensitive to the test conditions. It is known that both the temperature and pressure (hydrostatic stress) play a role in determining the yield stress. To explain temperature effect, it is first necessary to define an important physical property of amorphous polymers: the glass transition temperature. The glass transition temperature, T_g, is the point at which the polymer transitions from a rigid material into a leathery material. Below T_g, the polymer is said to be glassy, near T_g, it becomes leathery, and at temperatures far above T_g, the material will turn rubbery.

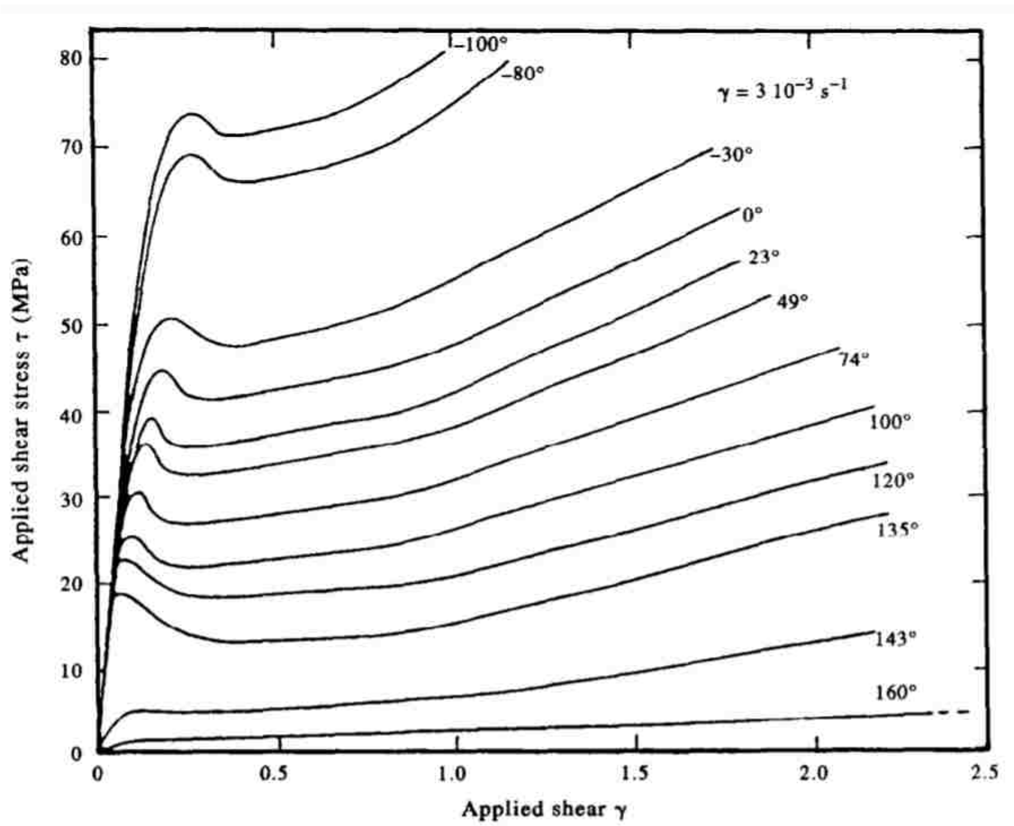


Figure 20 Temperature dependence of yield behaviour of PC in simple shear at constant shear strain rate. Test temperature ranges from - 100°C to + 160°C (Tg = 140°C) (reproduced from [30])

At temperatures below T_g (140^o C), the yield stress drops significantly with increasing temperature, though the general shape of the curve does not change [30]. Above the glass transition temperature, however, there is no clear yield point and necking and strain softening do not occur. In the opposite direction of the temperature scale, yield strength increases. Figure 20 shows a plot of $T_{low} \ll T_g(\approx 140^o C) \ll T_{high}$, with the change of behaviour from an elasto-plastic below T_g, to a rubbery viscoelastic above T_g. The underlying changes with temperature in intermolecular interactions are revealed to some extent.

The thickness of the sheets to be used vary between 0.5 mm to 2 mm. This thickness is chosen based on the stiffness and workability of the sheet.

The width of each sheet roll is taken to be 45 cm. This dimension is taken as point of reference and all other dimensions are calculated from this.

Figure 21 Sample of a Polycarbonate sheet roll [31].

3.1.3 Scope and Limitations

This section lists the assumptions and limitations of the proposed production model.

3.1.3.1 Parameters

1. The material is taken to be a Polycarbonate sheet of uniform thickness
2. The thickness of the sheet is taken as 1mm for simulation purposes.
3. The width of each sheet roll is taken as 45mm

3.1.3.2 Limitations

This research provides an alternate approach to manufacture folded cores with materials like paper and polymers. It is a numerical approach with focus on simulation. This model has to go through further iteration by practical experimentation with a variety of materials until the results are solidified.

3.2 A Mathematical Model for Miura-Ori Fold.

The crease patterns are considered as a single cell from an array of repeating cells that form the pattern. Hence the geometries of a single fold are taken into condition and then extrapolated for the whole pattern. For engineering purposes, designing the shape of a rigidly foldable unit cell is a means to find the material that fulfills certain functions and requirements, which is also closely related to the cell geometry. The real aim is to identify geometric influences on distinct properties such as mechanical performance, density, feasibility, efficiency, cost and a list of application dependent secondary functionality such as drainage ability, optical properties, or visible surface area. To engineer cellular structures that can be folded rigidly to close specifications, bottom-up design strategies has been explored.

3.2.1 Characteristics governing the crease pattern

One way to describe an origami model is by its crease pattern (CP)—the collection of lines (viewed on the unfolded square) where the paper gets creased (plastically deformed into a non-smooth kink) in the final model. Folding along all of these crease lines is often the first step in practically folding a model, and the CP is the basis for algorithms that analyze or design origami [33]. Creases come in two varieties: “mountains” which protrude upwards and “valleys” which protruding downwards (see Fig. 22). For flat origami designs, the CP must satisfy several local properties at each vertex, such as having a zero alternating sum of angles[34]. For our purposes, a CP is a geometric graph drawn within a square. There are many possible ways of measuring the similarity between two graphs, depending on the intended application and the generality of the class of graphs considered. Origami crease patterns belong to the class of geometric graphs in which the locations of the vertices are fixed and specified by their x and y coordinates, and the edges connecting pairs of vertices are straight lines[35]. Similarity (distance) measures in geometric graphs generally can be divided into two approaches: syntactic, where the graph is divided up into geometric “features” whose relative positions are then compared, or earth-movers distance methods (transformation or edit methods) which measure how much one graph needs to be changed in order to be transformed into the other graph [36]

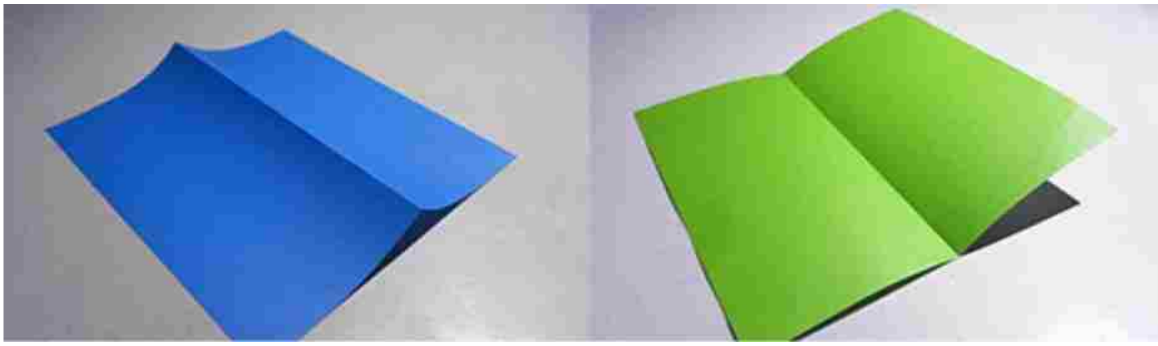


Figure 22 Examples of folds. A fold can be either a mountain (upward protruding) or a valley (downward protruding) depending on the orientation of the paper.

The origami pattern is based on two parameters: the folding angle, which is the angle measured between two parallelograms of the fold, from flat surface to a three dimensional fold, and the crease angle, which is the angle between two subsequent rows of parallelograms on the flat surface before folding.

In folding a flat sheet of material into a three-dimensional folded structure, it is important to ensure the following conditions exist, in order to guarantee that the sheet can indeed be folded in the specified pattern:

1. Tessellations (normally polygons) made of identical polygons, as shown in Figure 16, are considered a set of 2D polygons lying on a common plane (the flat sheet). These polygons must be convex. Every polygon in the 3D folded structure has a corresponding face in the 2D polygons. The shape of each face is the parallel projection of the

corresponding polygon onto a plane perpendicular to that polygon. No face may overlap with any other [37].

2. Each vertex in a folded structure must have the angles meeting there from the adjacent faces totaling 2π a limiting condition for the vertex to unfold flat [29].

Therefore, not every set of 2D polygons on a flat sheet is foldable. Indeed, Bern and Hayes [38] show that it is NP-hard (Non-deterministic Polynomial-time hardness) to determine whether a crease pattern (2D polygons) is flat foldable. Hull [39] focuses on necessary and sufficient conditions on overlap orders, resulting in a characterization of flat foldability for general crease patterns. Lang [40] describes an algorithm to construct "uniaxial" bases, which can be folded into arbitrarily complex models.

Folded patterns also have one or more elementary flat surface, each of which has a specific geometrical shape which forms the basic building elements of the folded pattern. Additionally, a combination or multiplication of these elementary flat surfaces of a specific geometrical shape constitutes the basic building cell of a folded pattern, as it is repeated in two dimensions, creating the three-dimensional folded shape.

The basic building cell of the Miura-ori pattern consists of four identical polyhedron [A, B, C, and D] each of length a, width b and an included angle 60 degree. The cell lays flat prior to folding. The repetition of the cell 'm' times in the direction of the arrow along the X axis provides the unfolded length of the tessellated sheet, while the cell repetition 'n' times in the direction of the arrow along the Y axis provides the unfolded width of the tessellated sheet.

3.2.2 Bottom-Up

A bottom-up approach is the piecing together of systems to give rise to more complex systems, thus making the original systems sub-systems of the emergent system. This strategy often resembles a "seed" model, whereby the beginnings are small but eventually grow in complexity and completeness.

The *bottom-up* approach shows the derivation of a simple set of parametric equations that describe the kinematics for the folding of a certain MIO(Modular Isometric Origami) foldcores unit cell type. All assumptions for the derivation are based on zero-thickness surfaces [41]. Once derived, these equations deliver a complete understanding on geometric and other (e.g., physical) properties of the resulting structure.

A single cell of the Miura-ori pattern is taken and parameters are named on it to derive geometric relations as shown in table 3 and figure 23. The parameters in case of flat state of the cell is assigned with the subscript 0 to indicate unfolded state as shown in table 4 and figure 24.

Table 3 The parameters for a single cell are assumed to derive the geometric relations.

L	Length of one tessellation of the folded cell with respect to the flat surface
S	Cross Sectional Side of folded cell

H	Height of the folded cell
V	Distance between vertex and side
l	Length of crease to edge of tessellation

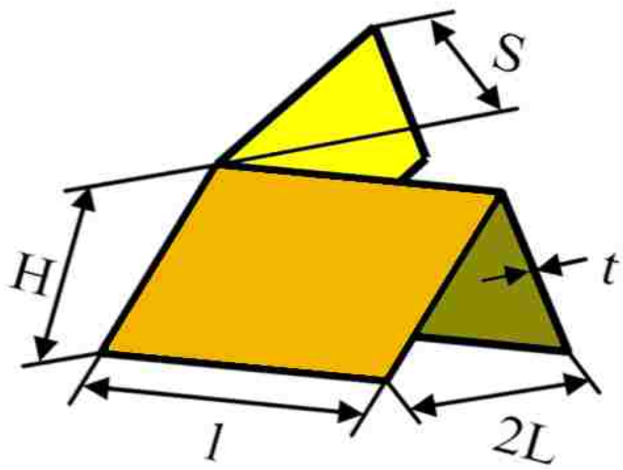


Figure 23 Parametric unit cell model used to derive geometric relations

Table 4 Parameters of the cell with index 0 for unfolded state

L_0	Length of one tessellation of the cell with respect to the flat surface in unfolded state
S_0	Cross Sectional Side of flat cell
V_0	Distance between vertex and side in flat state
H_0	Height of the folded cell in flat state

Using the cell and dimension parameters from Figure 25, the following relations between the flat parameters (with index 0) and the 3D cell can be derived and allow easy design of the folding pattern for any set of 3D parameters[41, p. 5]. The value of H^0 is assumed to be constant.

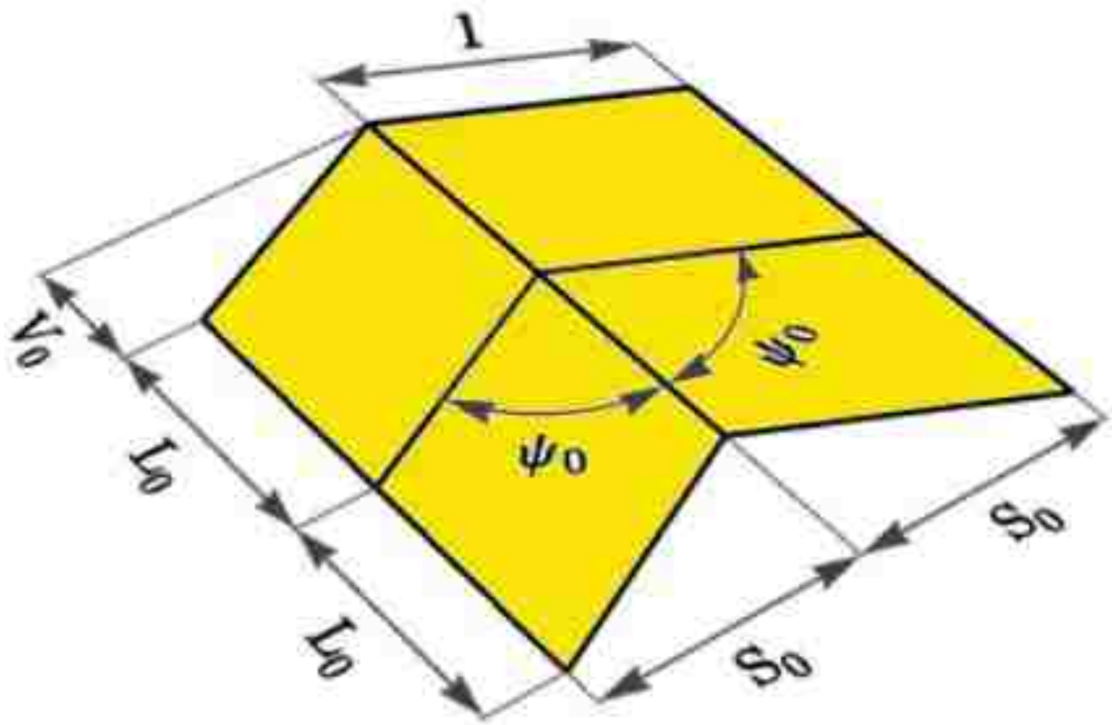


Figure 24 Parameterizations of 3D cell in the flat state

A relation is derived between the flat state and folded state based on Fig. 25

with $L > 0$, $S > 0$, $V > 0$ and $H > 0$

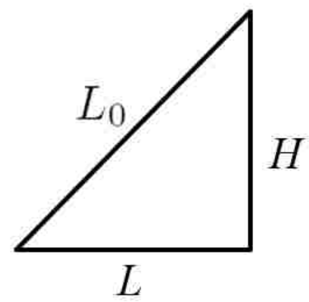


Figure 25 Correlation of parameters between flat and folded states

$$L_0 = \sqrt{H^2 + L^2}, \tag{3.2}$$

$$S_0 = \sqrt{\frac{H^2V^2}{H^2+L^2} + S^2} \text{ and} \tag{3.3}$$

$$V_0 = \frac{LV}{\sqrt{H^2+L^2}}. \tag{3.4}$$

The kinematic equations are formulated when one 3D parameter is taken as a control variable, thereby shaping the functions of other 3D parameters with given constant paper parameters. In the case of H as control with $0 \leq \mathrm{H} \leq L_0S_0/\sqrt{S_0^2 + V_0^2}$, from the flat state to the flat-folded or block state, this leads to

$$V(H) = \frac{L_0V_0}{\sqrt{L_0^2 - H^2}}, \tag{3.5}$$

$$S(H) = \frac{\sqrt{L_0^2S_0^2 - H^2(S_0^2 + V_0^2)}}{\sqrt{L_0^2 - H^2}} \text{ and} \tag{3.6}$$

$$L(H) = \sqrt{L_0^2 - H^2}. \tag{3.7}$$

When the unit model cell is transformed between different states, which alter the parametric representations between flat state and various other parametrizations, the relation can be obtained by substitution,

$$L = H\tan(\alpha)\csc(\psi),\ \ S = l\sin(\psi)\ and\ \ V = l\cos(\psi)\,. \tag{3.8}$$

Relation between angle of the 3D cell model and parameters is shown in figure 26.

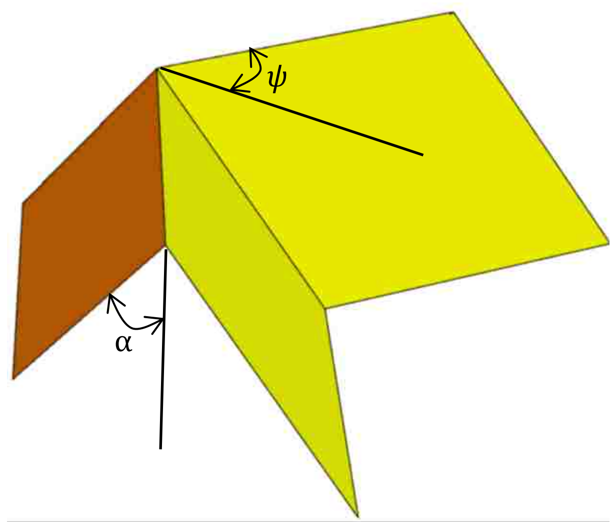


Figure 26 Different angular parameters

The derived equations can be used to design patterns and simulate the folding process.

3.3 Conceptual Design of Continuous Folding Machine

Figure 27 shows the layout of the setup in Solidworks. All the components were designed around keeping the width of the roll 45 cm. The setup consists of the XY configuration laser etching setup, the forming section and the funnel type conveyor section.

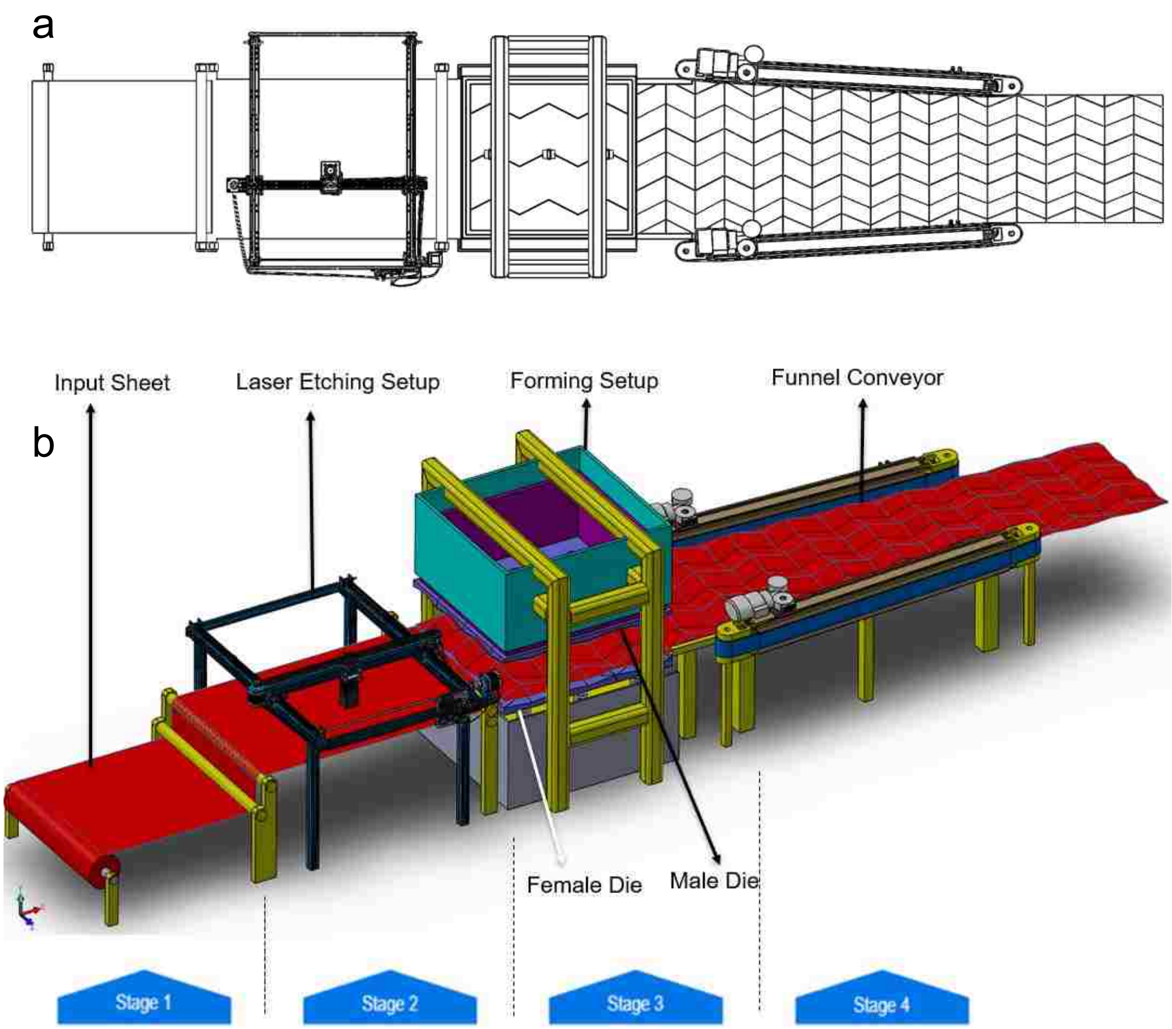


Figure 27 Model of the layout a)2D Line Drawing b)CAD drawing

The entire layout has 4 major processes from the input of the material to be processed to the final forming stage. The various aspects

3.4 Pre-processing of the Sheet

Two methods for processing the sheet before the forming process are by using either laser etching which serves as a reference for the dies by reducing the resistance on the crease lines, or by heat treating the sheet to a pliable temperature before carrying out the forming process.

3.4.1 Using Laser Etching

The polycarbonate sheet roll that has been chosen as the material for the forming process goes through initial processing by means of a laser. A 2D diagram of the pattern to be etched is fed to a laser. This is done to etch the miura-ori pattern to be folded. The laser is fed with the pattern based on the dimension of the sheet.

A simple laser setup having a X-Y configuration as shown in Figure 28 is utilized. It is made up of a solid H- shaped anodized aluminum alloy structure to etch the pattern. A high-power solid-state laser is utilized for the engraving process.

Figure 28 Laser setup with XY plane movement configuration

The laser setup shown in Figure 28 is fed with the desired CAD model of the pattern to be etched (Figure 29). The desired spacing of the pattern is determined based on the above-mentioned calculations.

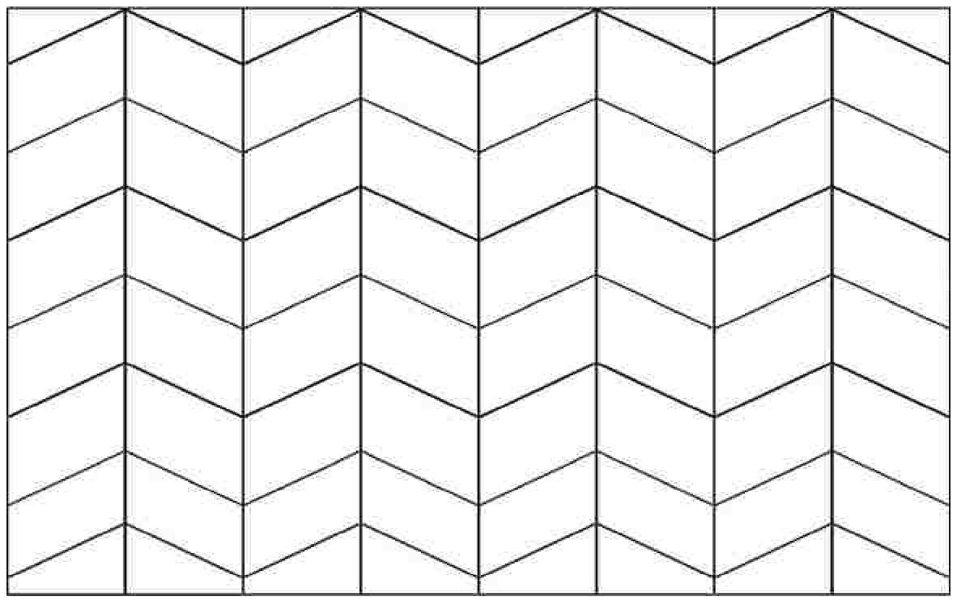


Figure 29 2D sketch of the pattern to be laser etched

3.4.1.1 Sequence of Etching

The pattern shown in Figure 30 is etched with importance given to the completion of the first column before the laser moves on to the next column. This sequence of operation is achieved by altering the G-code fed into the laser setup or by altering the .STL file of the 2D pattern.

Rollers are used to move the Polycarbonate sheet after the completion of a single etching process of the 2D pattern. Stepper motors are utilized for precise control of the rollers. A

3-Phase stepper motor having a phase change angle of 0.2 is to be used to maintain tight control.

3.5 Forming of the Etched Sheet

The etched sheet is fed to the forming section which has the male and female dies. The dies consist of 3 pairs of dies that have indents for the Miura-ori pattern to be formed.

The dimensions of the die are based on the width of the Polycarbonate roll. The Miura-ori pattern to be etched is based on the dimension from Chapter 4.2.

The forming process has the following steps:

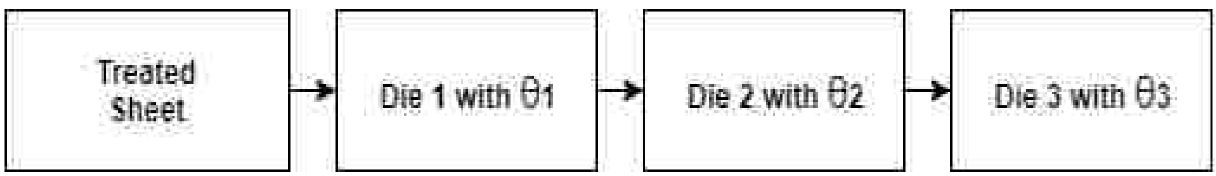


Figure 30 Sequence of actions in Forming

3.5.1 Parameters of the Die

The forming process has three different pairs of dies which have variations in the depth of the Miura-ori pattern by order of increasing magnitude. This makes it a step by step forming process.

Figure 31 shows the visual representation of the die having indents for the Miura-ori pattern.

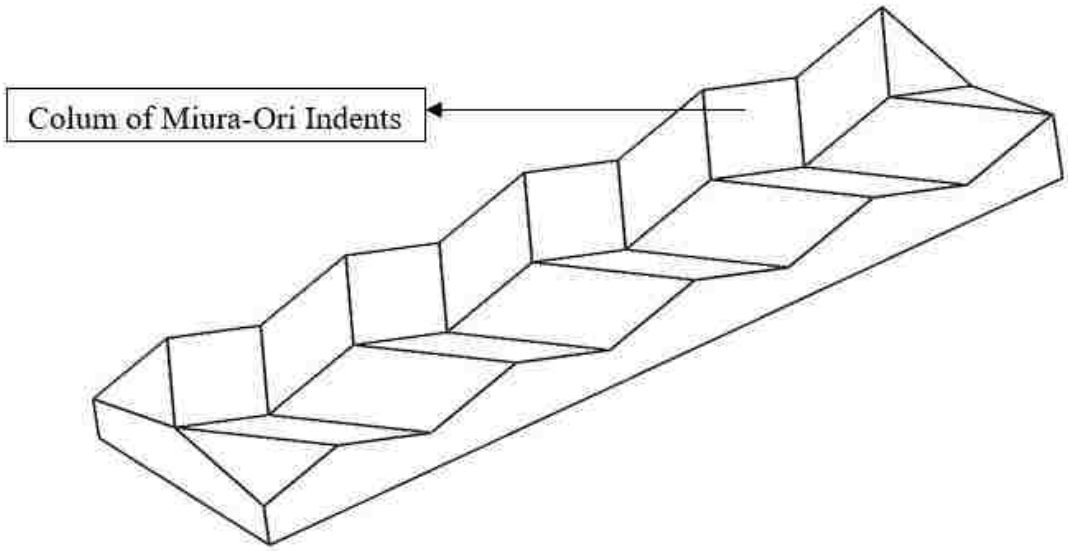


Figure 31 Cast Iron Die

The Angle 'ψ' for the first set of male and female die is 70^o. It increases in increments of 5^o to each successive die.

The etched PC sheet goes through the series of dies and is gradually formed as shown in Figure 32. The line at the side represents the rate of bending with respect to a single miura cell.

The sequence of operations is as follows:

- The etched sheet goes through an initial degree of forming with crease angle at 70^o. This leads to a depth of 2 mm.
- In the second stage, the crease angle is further increased to 75^o with a corresponding increase in height to 4 mm.

- In the final stage of forming, the crease angle is raised to 80^o with the peak at

6 mm.

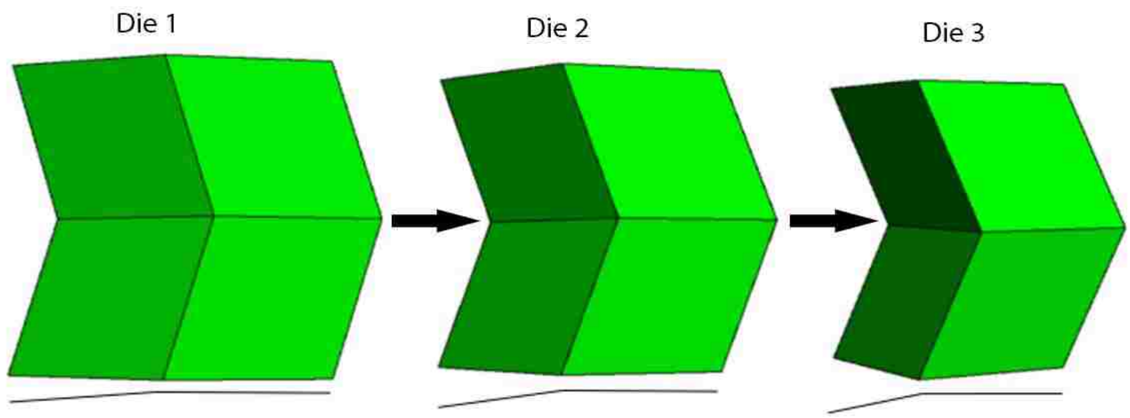


Figure 32 Change in unit cell dimension based on die

It is noted that the forming of the sheet is partially completed in this state. This is shown as the formation of creases on the sheet which serve as the guidelines until the origami pattern is formed to the required dimensions.

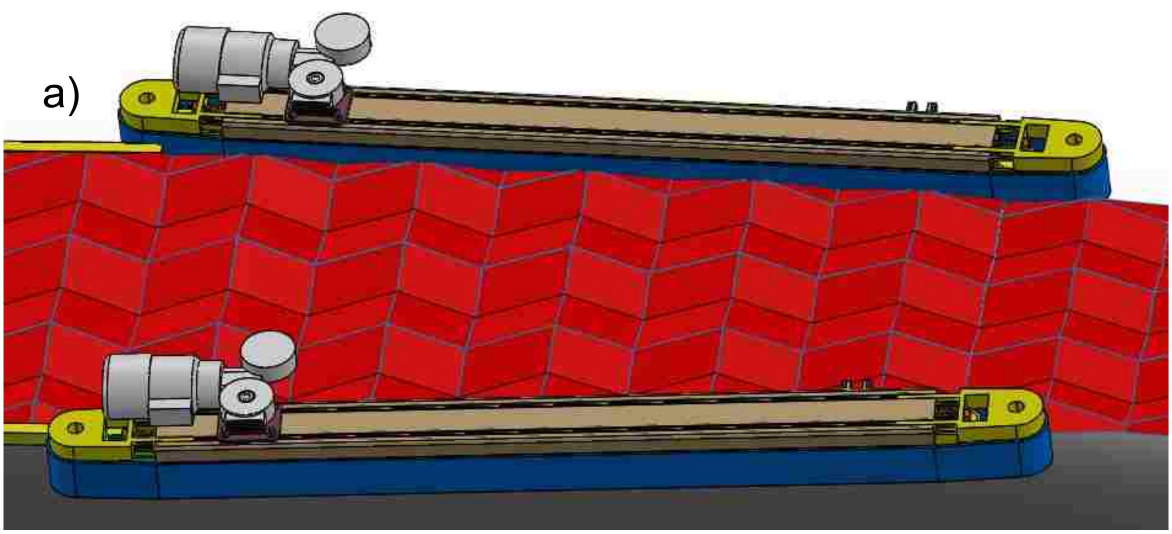
3.6 Funnel-type conveyor system

A conveyor system is a common piece of mechanical handling equipment that moves materials from one location to another. Conveyors are especially useful in applications involving the transportation of heavy or bulky materials. Conveyor systems allow quick and efficient transportation for a wide variety of materials, which make them very popular in the material handling and packaging industries.

The conveyor system used here has two different uses:

1. The major purpose is to ensure that the formed sheet, which is in its semi-rigid state, is further compressed by the conveyor system until the pattern is formed to its required state.
2. The secondary purpose is to push the formed sheet to the output stage.

The formed sheet is fed through a conveyor which is tapered at an angle of 3 degrees as shown in Figure 33 with respect to its central axis. The conveyor is in place for a length of 88 cm. The conveyor setup is a fully enclosed aluminum shell with conveyor belts inclined at the sides.



b)

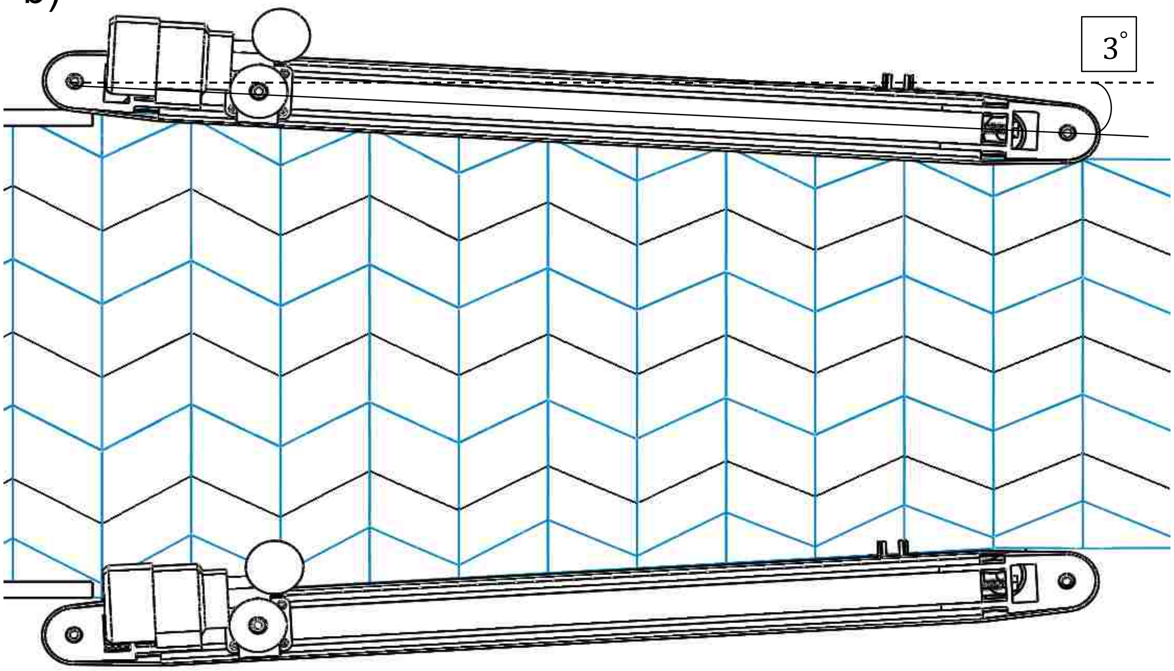


Figure 33 Funnel-type conveyor a)inclined view b)Top view shows the inclination of the conveyors along the central axis

The width of the conveyor at the start is 45 cm as shown in Figure 34. Due to the inclination produced by convergence, the width at the end of the conveyor is reduced to 37 cm. This leads to a reduction in overall width of the formed sheet by 8 cm. This gives a corresponding increase in height of the formed sheet, which converges along the lines of the crease pattern which were impinged in the forming stages.

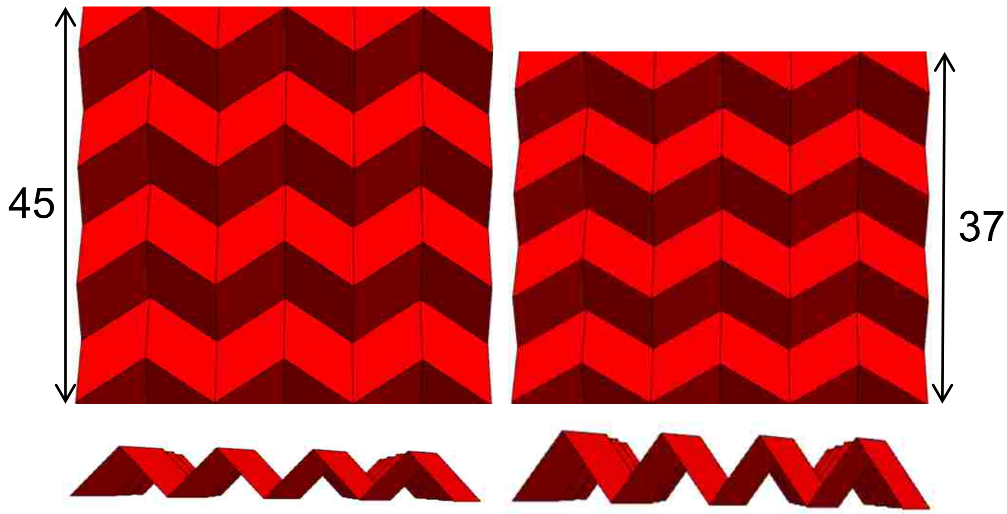


Figure 34 Shows the change in height of the formed sheet.

The conveyor funnels the sheet and is crucial for forming the final stage of the process. The path is enclosed on the top and it controls the compression of the sheet and ensures that it does not go beyond the boundaries.

The motors and rollers of the conveyor system are all synchronized and they move at a constant, slow speed.

4 SIMULATION FOR THE FORMING OF THE SHEET ALONG THE PATTERN

This chapter begins by describing the simulation method used to validate the forming process. The geometry is first described, followed by specification of the boundary and initial conditions. The simulation is carried out with various boundary conditions to validate the results that were obtained.

4.1 Finite Element Analysis

Experimental testing of innovative materials and structures like foldcores can be time-consuming and expensive. When it comes to the manufacturing of toolings, prototypes and test specimens as well as the testing and damage assessment, numerical simulations are an established efficient tool in the development process of engineering structures. Therefore, finite element (FE) models of the composite foldcore sandwich structures were developed in the commercial FE software Abaqus and Solidworks Simulation. They were both employed to study the deformation of specimens under the different conditions. The model was drawn in Solidworks and fed into Abaqus for the simulation. Details of deformation from the finite element analysis are reported.

4.1.1 Boundary Conditions

The patterned sheet model was first built in SolidWorks by tiling identical Miura-ori units shown in Fig.35.

1. Such a patterned sheet was then meshed and modeled with shell elements of type S4R in Abaqus. A material constitutive relation was used.
2. The simulation is carried out for the final stage of the forming process in the funnel type conveyor region.
3. The top and bottom contact points of the patterned sheet were both constrained along the Z-axis while allowing free movement in the other two axes. (see Fig. 36).
4. One of the sides parallel to the creases in the model is fully fixed while the other two sides perpendicular to the fixed side are used as load points.
5. A load of 30 N is applied.
6. Self-contact was defined, which took into account hard contact and friction between the surfaces and the patterned sheet.

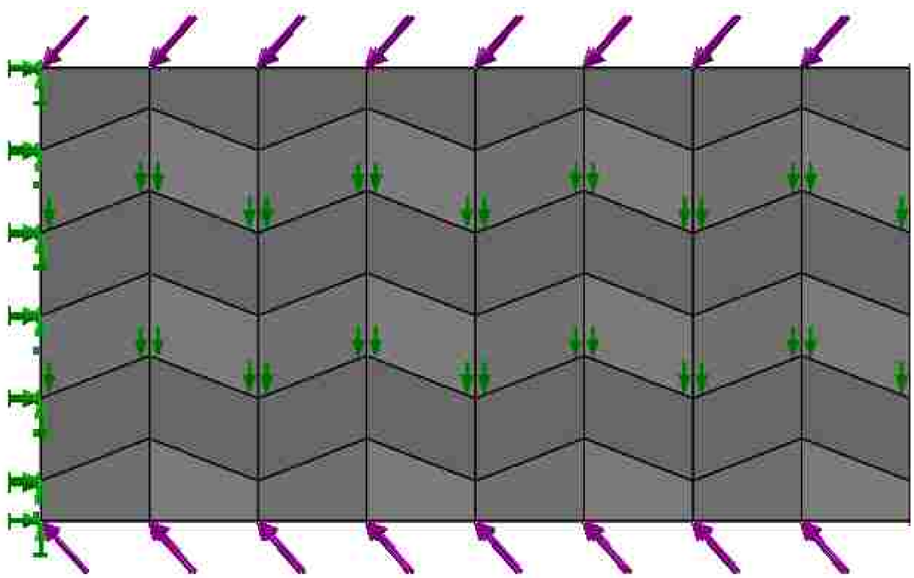


Figure 35 Boundary conditions as show in Solidworks Simulations

4.2 Element size

Different mesh sizes were used as shown in Fig.36 plots the mesh for 1 mm element size. A smaller element size of 1.0 mm resulted in a reduced force compared with that for 1.2 mm mesh size, but the CPU time increased considerably. Further refinement to 0.8 mm did not seem to improve the accuracy significantly. Hence an element size of 1mm was selected as the default element size for all the subsequent simulations.

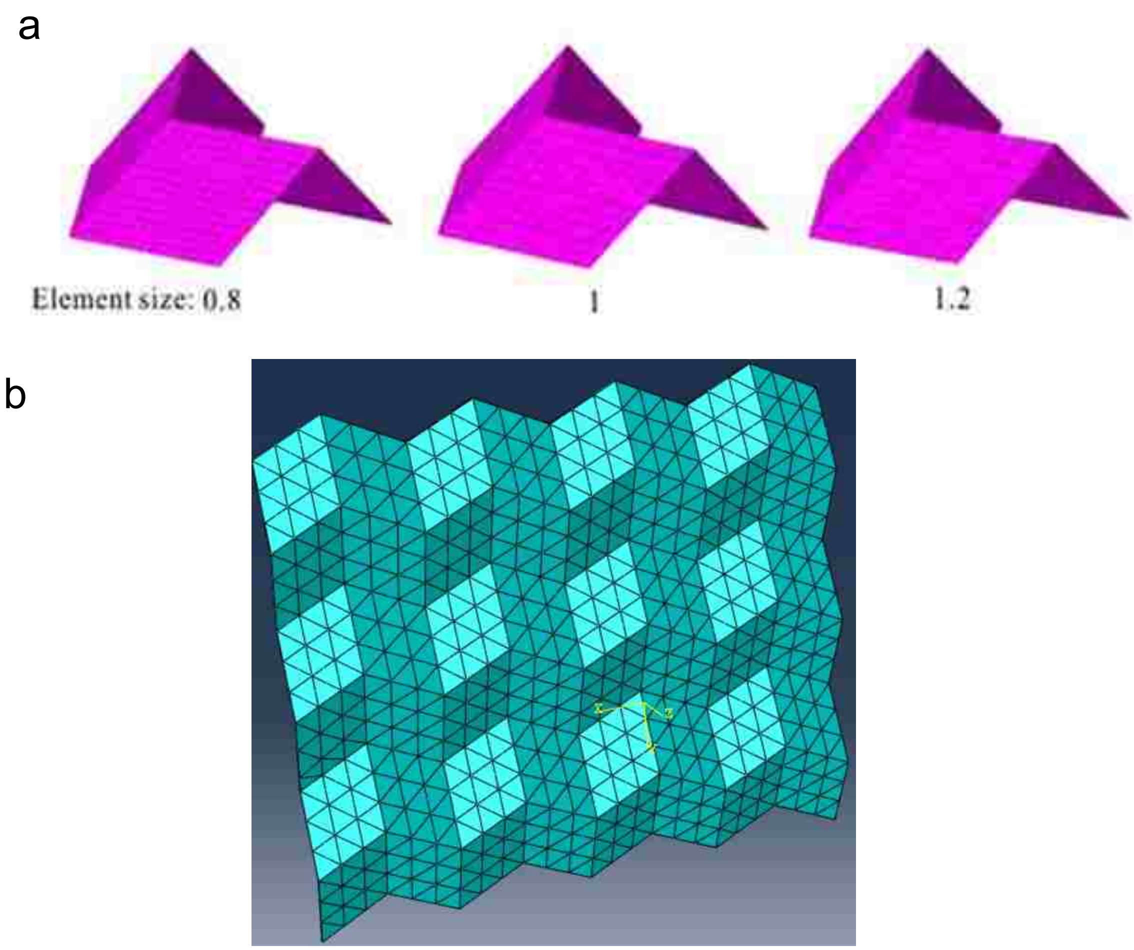


Figure 36 (a) FE models with different element size ,(b) Final meshed model

4.3 Loading rate

The loading rate in the simulation is given in increasing steps of 5N. Therefore, a compression force of 20, 25, 30 N was applied, respectively. The force value seems insensitive to the loading rates over a range of displacement, though a load of 20 N gave the lowest value of the deformation. Figure 37 shows the deformation of the model for 20 N load.

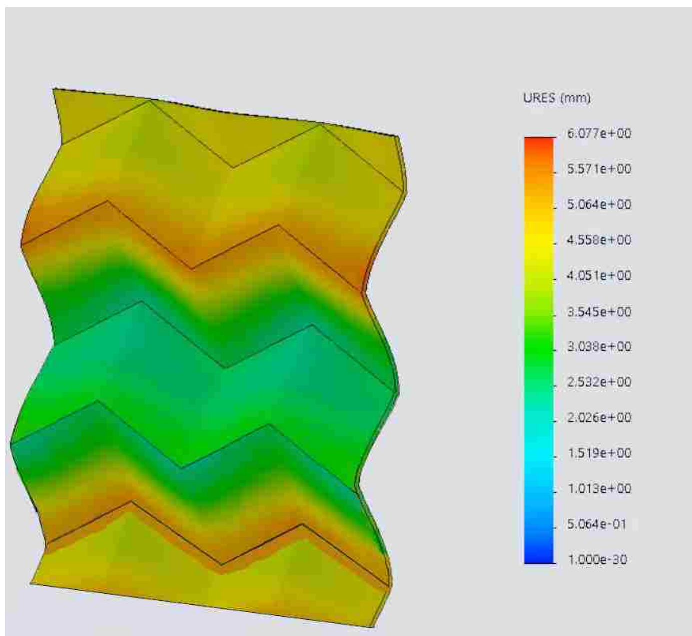


Figure 37 Deformation of the model on loading

5 RESULTS

The various results obtained from the simulation evaluate the feasibility of the final stage of the forming process.

5.1 Evaluation of Results

The PC sheet is subjected to a load of 3 kg along its horizontal sides. The deformation that occurs is found to be in line with the desired stress-strain values for the chosen material. A compression of 6 mm in overall width of the formed pattern is observed with a corresponding increase in height of the pattern.

The static displacement of the model is simulated in Abaqus and the result is shown in Figure 38.

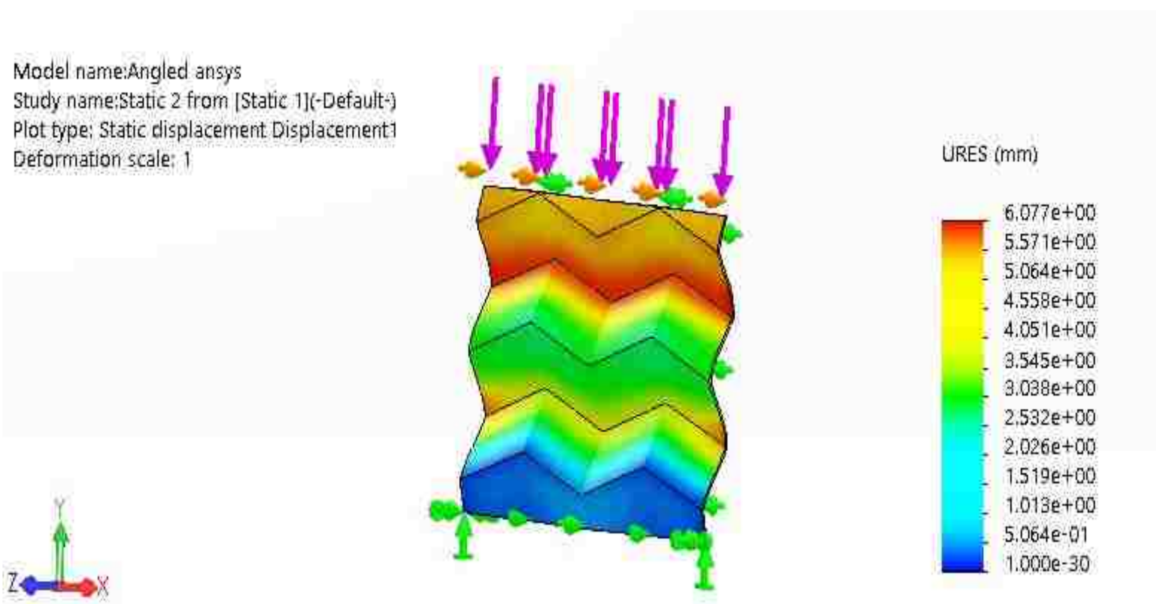


Figure 38 Static displacement of the sheet in the final forming stage

When the element size is kept at 1 mm and a triangle mesh is generated for the tessellated sheet, the FE analysis is carried out for two opposing loads as shown in figure 39.

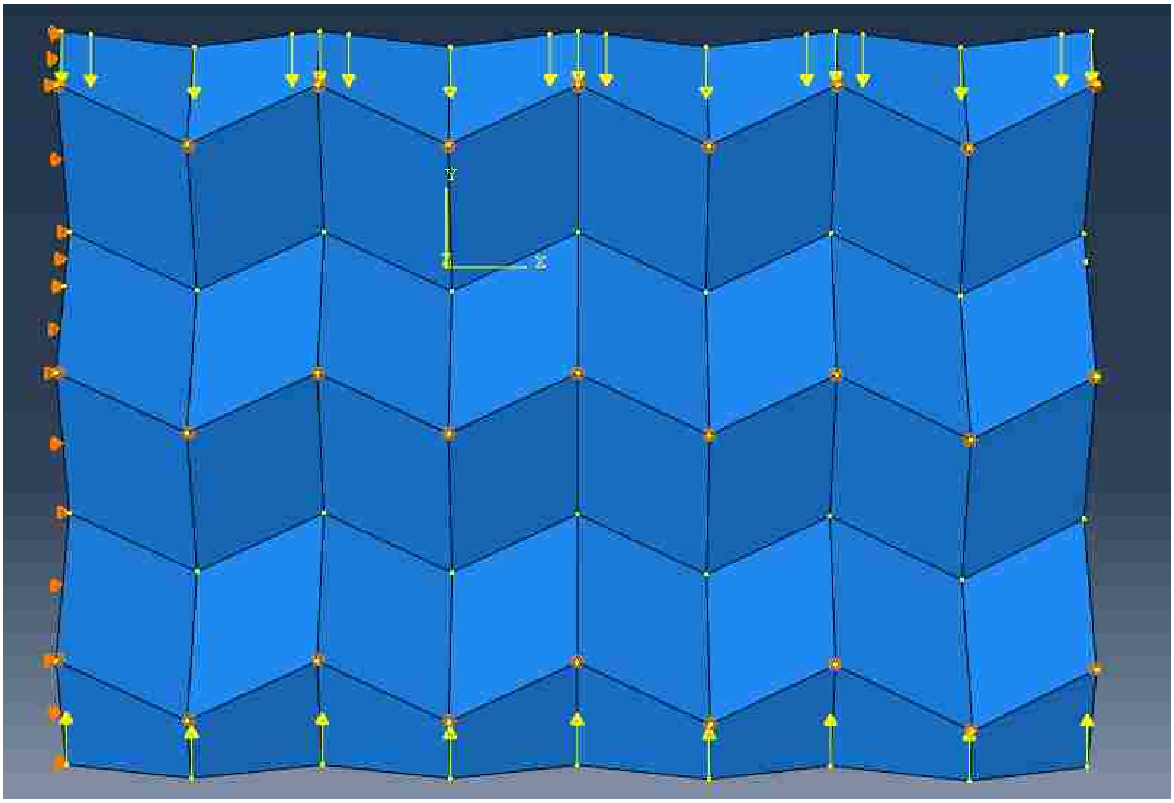


Figure 39 Boundary condition which simulate funnel-type conveyor stage

A load of 2.5 kg is applied on the horizontal side with the focus on compression of the tessellated sheet (figure 41). The side of the sheet in contact with the roller is fixed. The points of the creases on both sides of the sheet are given free range of motion with respect to the central axis of the sheet. This allows compression along the crease patterns and forming of the required textured sheets. The Von Mises stress is used to determine if a given material will yield or fracture. It is mostly used for ductile materials.

It is seen that the value of stress is within safe limits of 1.98 e4 N/mm and there is no deformation to the point of shearing for this load value, as shown in Figure 40.

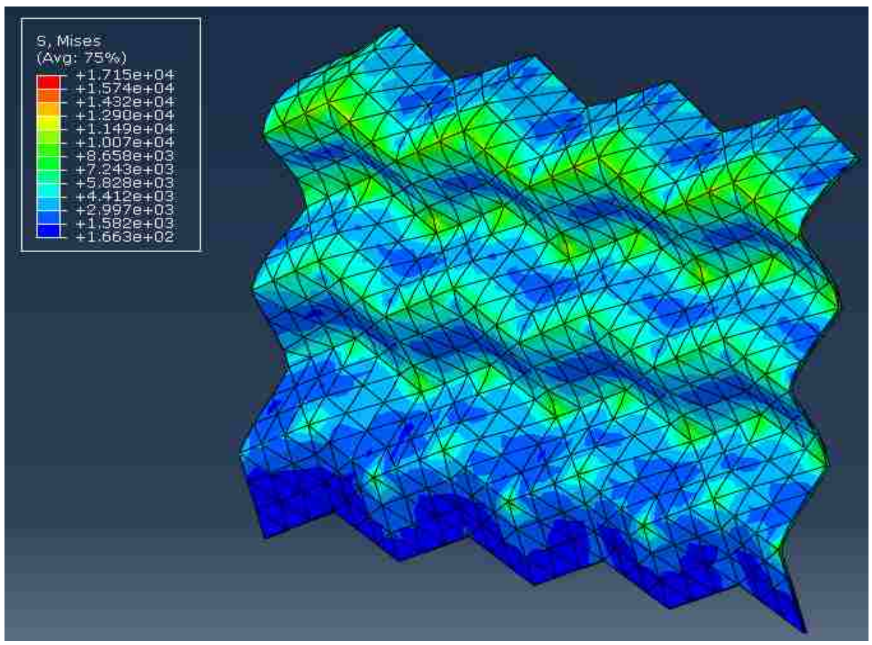


Figure 40 Von Mises stress of the tessellated sheet

The maximum principal stress, shown in figure 41, indicates the point of failure is along the crease lines in the upper sections of the tessellated sheet.

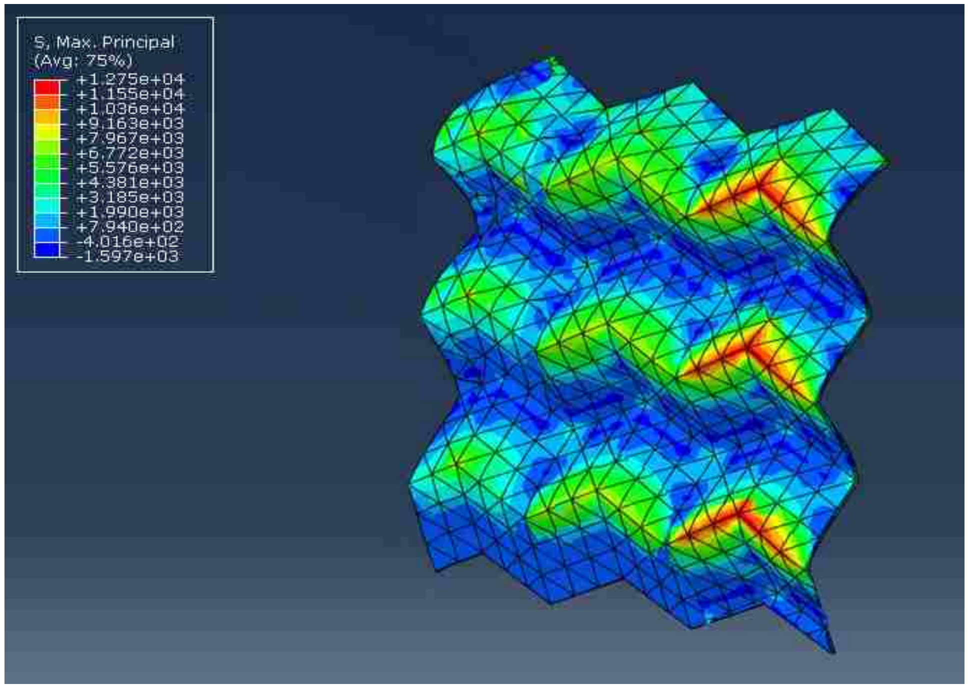


Figure 41 Maximum Principal Stress of the tessellated sheet.

The forces at the primary forming stage are found to be increase progressively with each die as shown in figure 42. This is in accordance with the depth of deformation of the model.

5.2 Failure Modes of the Model

Various limitations of the layout have been identified and are presented as follows:

- The prime mover of the PC sheet is rollers throughout the layout, hence there is a chance of wrinkling in the sheet.

- The etching of the crease pattern by the laser is the most time-consuming section of the layout.
- During initial setup of the machine, the die does not start forming until the sheet is in contact with the conveyor so that there is sufficient pressure holding the sheet along both directions.
- All the motors are to be perfectly synchronized with the flow of the sheet so that the forming process is accurately carried out on the etched surface.
- In case of error in forming, it should be identified early so that the subsequent patterns are formed appropriately.
- Friction or lubrication error may occur depending on the material of the sheet. Hence proper lubrication may be needed to be provided based on practical results.

6 CONCLUSIONS

The main objective of this simulation is a demonstration to show a new approach for the production of textured sheets. The various parameters that govern the folding process are clearly defined. The results from the simulation show the working processes of the proposed method. The various modes of failures are elaborated for the proposed method. A plausible means to eliminate such limitations is provided.

7 FUTURE WORK

The limitation of this research being fully numerical can be addressed by building a model in the future. Further work would need to validate this model with additional data or a physical prototype. The approach used is framed upon ideal work conditions. The scope of future work is specified below:

- A scaled down prototype can be built to evaluate crucial aspects of the model.
- The production time taken an be calculated with accuracy using the prototype.
- A variety of different materials can be tested for feasibility of the production process.

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APPENDICES

Appendix A. **Mathematica Code**

Miura Map Code was generated with reference from Thomas C. Hull’s code.

```
unit1    [{x_, y_}, a_] := {{x, y, 0}, {-1 + x, y, 0}, {-1 + x - Cot[a], -1 + y,
           0}, {-Cot[a] + x, -1 + y, 0}
                                  };
unit2    [{x_, y_},  a_] := {{x, y, 0}, {-Cot[a] + x, 1 + y, 0}, {-1 + x - Cot[a], 1 + y,
            0}, {-1 + x, y, 0}
                                  };
D2       [t_, a1_] :=   ArcCos[Cos[     t] - ((Sin[t])^2*Sin[a1]*        Sin[\[Pi] - a1])/(1 -
         (Cos[a1]*Cos[\[Pi] - a1] +  Sin[a1]*Sin[\[Pi] - a1]*Cos[t]))
                                  ];
FoldRecur [poly_, {m_, n_}, a_, t_] :=
 Which [m == 0 && n == 0, poly,   m == 0 && OddQ[n],
  FoldRecur [
         Transpose [ RotationMatrix[-\[Pi] + D2[\[Pi] - t, a], {-1, 0, 0}].
```

```
        Transpose [poly - Table [{m, n - 1, 0}, {4}]]] + Table [{m, n - 1, 0}, {4}],

        {m,  n - 1}, a, t

                ],

        m == 0 && EvenQ[n],

FoldRecur [

        Transpose [RotationMatrix[\[Pi] - D2[\[Pi] - t, a], {-1, 0, 0}].

        Transpose [poly - Table [{m, n - 1, 0}, {4}]]] + Table [{m, n - 1, 0}, {4}], {m,

         n - 1}, a, t

                ],

OddQ[m] && EvenQ[n],

FoldRecur [

        Transpose [ RotationMatrix[-t, {-Cot[a], -1, 0}].

        Transpose [poly - Table [{m - 1, n, 0}, {4}]]] + Table [{m - 1, n, 0}, {4}], {m - 1,

        n}, a, t

                ],

EvenQ[m] && EvenQ[n],

FoldRecur [
```

```
        Transpose [ RotationMatrix[t, {-Cot[a], -1, 0}].
        Transpose [poly - Table [{m - 1, n, 0}, {4}]]] + Table [{m - 1, n, 0}, {4}], {m - 1,
        n}, a, t
                ],
  OddQ[m] && OddQ[n],
  FoldRecur [
        Transpose [RotationMatrix[t, {-Cot[a], 1, 0}].
        Transpose [poly - Table [{m - 1, n - 1, 0}, {4}]]] + Table [{m - 1, n - 1, 0}, {4}],
        {m - 1, n}, a, t
                ],
  EvenQ[m] && OddQ[n],
  FoldRecur [Transpose[RotationMatrix[-t, {-Cot[a], 1, 0}].
        Transpose[poly - Table[{m - 1, n - 1, 0}, {4}]]] +     Table[{m - 1, n - 1, 0}, {4}],
        {m - 1, n}, a, t]
        ]
Manipulate [
        Graphics3D [
```

```
Flatten[Table[{Magenta,     Polygon[FoldRecur[unit1[{m, n},
Angle], {m, n}, Angle, t]],     Polygon[FoldRecur[unit2[{m, n},
Angle], {m, n + 1}, Angle, t]

        ]},

 {m, 0, length - 1}, {n, 0, width - 2, 2}]

        ],

Boxed -> False,   PlotRange -> All, ImageSize -> {500, 300}

],

{{t, 0.001, "folding angle: "}, 0.001, \[Pi], Appearance -> "Labeled"},

{{Angle, 1.3, "crease angle: "}, 1.57, 0.01, Appearance -> "Labeled"},

{{length, 4}, {2, 3, 4, 5, 6, 7, 8, 9, 10}, SetterBar},

{{width, 4}, {2, 4, 6, 8, 10}}, SaveDefinitions -> True]
```

VITA AUCTORIS

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