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The Relationship between English Language Learners' Mathematics Problem Solving Strategies and the Mathematics Register

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The Relationship between English Language Learners' Mathematics Problem Solving Strategies and the Mathematics Register

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by

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Abstract

This case study followed three Hispanic and three Marshallese students' participation in a seventh grade pre-algebra unit over the course of four weeks. Pre-assessment items indicated that the students had difficulty with aspects of the mathematics register. Their teacher employed an interpretation of the gradual release model of instruction in which direct strategies for translating word problems into equations with unknowns were modeled prior to students doing similar problems on their own. In these situations, students showed some success in solving similar problems. However, post-assessment results indicated that none of the six students were able to successfully solve open response problems similar to the problems that were covered during the instructional unit. The main difference between the post-assessment problems and the pre- and during instruction problems was that the students were not directed to set up an equation prior to trying to solve the problem. Students were not able to apply procedures related to the mathematics register to solve contextualized problems when they were not given specific methods on how to set up the equations in advance of trying to solve the word problems. They struggled to set up an appropriate equation to represent the situation and appeared bound by the equation used to set up the situation in lieu of another strategy that might have produced a correct answer, underscoring the complexities involved in making sense of algebraic content.

Keywords: mathematics; English language learners; ELLs; language of mathematics; translation activities; middle school mathematics

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Dedication

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CHAPTER 1: INTRODUCTION

"The problem of the achievement gap in education is [often] described in terms of ethnicity—ethnic relations, issues of oppression and equity—while ironically the solutions for closing students' learning gaps in the classroom lied in tapping into their culture" (Hammond, 2015, p. 21). The connotations associated with the phrase "achievement gap" are sometimes associated with limitations in student learning based on these factors. Hammond said, "Deficit thinking defines students and their families by their weaknesses rather than their strengths, suggesting that these weaknesses stem from low intelligence, poor moral character, or inadequate social skills" (2015, p. 33). This deficiency is usually associated with students of ethnicities other than Caucasian and/or low socioeconomic status.

While there are arguably multiple groups of students unintentionally or unfairly sidelined in their pursuit of a quality education, the focus of this research was on English language learners (ELLs). This study explored the learning experiences of six middle school ELLs in their seventh grade mathematics classroom. The cultural and linguistic influences these students carried to the classroom affected how their brains processed information (Park & Huang, 2010). These six students, who were of Marshallese and Hispanic heritage, were simultaneously learning English and new mathematical content in English.

Background of the Study

Who are the Marshallese students? The Marshallese are a people on a once tropical paradise island with panoramic views of ocean waves at nearly every step a person could take. The Republic of the Marshall Islands, often referred to as the Marshall Islands, is an island country located near the equator in the Pacific Ocean. The Marshall Islands were a part of Oceania, which was comprised of two archipelagic island chains of 29 atolls, each made up of many small islands and five larger single islands in the North Pacific Ocean, about halfway between Hawaii and Australia (Central Intelligence Agency, 2018). The total land mass was 112 square miles of land with a population of just more than 53,000 people living in the Republic of the Marshall Islands. The average elevation above sea level was just over two feet with the highest point being just under thirty-three feet (U.S. Department of the Interior, 2019). Living close to water was a part of everyday life.

Regrettably, a portion of the Marshall Islands was obliterated. At the end of World War II, the U.S. was given control over the Marshall Islands as part of an agreement with the United Nations Trust Territory of the Pacific Islands. From 1945-1986, the Marshall Islands were under U.S. administration (World Atlas, 2016). During that time, the U.S. tested 67 atomic bombs on the Marshall Islands (Robbins, 2005). By doing so, the U.S. ruined entire communities and the people within those communities were forced into exile on other, less livable, islands (Langlois, 2015). To compensate for those environmental atrocities, the U.S. offered the Compact of Free Association. It was a legal document that stated Marshallese citizens were allowed to work and live in the U.S. without a visa or green card for an indefinite amount of time (Miller, 2016). A mass exodus of the Marshallese people from the Marshall Islands took place after the creation of the Compact of Free Association. Portions of land being permanently destroyed, rising sea levels, and the cost to try to repair and protect the rest of the Islands was overwhelming. The Marshallese sought better opportunities in the U.S. As of 2015, almost 25,000 Marshallese had left the Marshall Islands to live in the U.S. (Langlois, 2015). Nearly a third of the total Marshallese population resided in the U.S. in pursuit of better education, economy, and healthcare (Carpenter, 2011). The largest concentration of the Marshallese population in the U.S. was believed to be located in Springdale, Arkansas (AETN, 2018).

The Marshallese traditionally took a laissez-faire approach to formal education. That may have caused an abrasive transition to the typical American culture in classroom expectation and procedural requirements. “For example, while compulsory education to age 14 is often the case in the Republic of the Marshall Islands, the laws are not strictly enforced and daily school attendance is not given much attention” (Heine, 2002). Because of this, Marshallese families were surprised to learn in the U.S., school attendance was taken seriously and could even lead to the involvement of law enforcement agencies in some cases. Methods of instruction within Marshallese classrooms were also different from that of American classrooms. “Where students may be expected to problem solve and make decisions independently in [most] American classrooms, island students may be reluctant at first to step outside of normal family practices in which problem solving and decision making [were] shared” (Heine, 2002). An unintentional culture clash between the Marshallese students and families and the school system may have been the result.

Culturally, outsiders (Central Intelligence Agency, 2018) view the Marshallese as friendly and peaceful people. It was important in Marshallese society to have consideration for others so strangers were welcomed warmly. The Marshallese concept of family is a fluid one. Marshallese culture, family includes extended members, and many households include multiple generations (McElfish et al., 2015). The whole family actively participates in the raising of a child. A nurse in Springdale had the following experience:

> A matrilineal system wherein all related members of a generation is considered the joint parents of a child. “[Kids] will show up [to school] one day with someone and say, ‘This is my mom,’” said Sandy Hainline--Williams, an American nurse who has become a cultural liaison for Springdale’s Marshallese. “And the next day, a different woman: ‘This is my mom.’” These attitudes, anthropologists believe, were born of the ethos of extreme generosity necessary for crowded island life. “There’s a general idea that things belong to everyone, as opposed to specific people. (Joyce, 2015)

The Marshallese culture and their concept of a fluid family bound families together through grandparents, parents, grandchildren, aunts, uncles, and cousins and thereby made family groups and gatherings big events (Ratliffe, 2011).

Who are the Hispanic students? “Latino youth are extraordinarily diverse, and their experiences resist facile generalizations” (Suarez-Orozco, Suarez-Orozco, & Doucet, 2004). Latinos immigrants come from dozens of countries in Central America, with a varied range of cultural traditions. Latino families came from a collectivist culture and often had large families. What traditional U.S. American culture would consider extended family members are often critical sources of tangible instrumental and emotional support in Latino culture (Suárez-Orozco, Suárez-Orozco, & Doucet, 2004)? As far as parental education levels, again, there is a wide variance. Some children come from highly educated professionals and others had parents who are illiterate, low skilled, and economically struggling (Suarez-Orozco, 2000).

Latino families immigrate for a variety of reasons, legally or illegally, including escaping political and religious persecution, or promise of better jobs and the hope for a better education. There is also an element of fluidity in Latino youth when it came to settling for residency. “Some immigrant origin youth [came] to settle permanently, over time losing their ties to their homelands; others [followed] their parents from one migrant camp to another” (Suarez-Orozco, Suarez-Orozco, & Doucet, 2004). Then, there were those in the middle that engaged in a hybrid system of transitional living, living both “here and there”- or shuttling between their country of birth and their country of choice (Suárez-Orozco & Suárez-Orozco, 2001).

There are Latino students across the spectrum when it comes to preparedness for U. S. American-style schooling. There are youths from middle-class, upper-status urban backgrounds. These students are usually well versed in literacy and have proficient study skills. Parents that are more educated are better able to guide their children in various aspects of school such as how to study, structure an essay, access information for school projects, and provide necessary resources, including additional books, a home computer, and even tutors (Suárez-Orozco, Suárez-Orozco, & Doucet, 2004). These parents are more likely to be educationally involved with their children and educational system savvy. On the other hand, children who have parents with limited education are at a disadvantage in academics. There are students from countries riddled with humanitarian issues or poverty-stricken countries with little or no formal schooling. Students from those struggling countries have missed critical years of classroom experience and often could not read and write in their native language of Spanish at all or at a proficient level for their age (Paez, 2001).

Stanton-Salazar (2001) found that low-income Mexican immigrant parents generally highly valued educational success for their children and had high educational aspirations for them. However, few of those parents actually understood the school experience of their children in American schools and thus could not empathize with them. The parents were often unable to support their children in ways that were aligned with U.S. American schooling expectations in concrete ways, such as helping them with homework or attending parent-teacher conferences. This impaired the role the parents could have in facilitating their children's access to postsecondary education because they did not perceive to have access to as many resources as their peers.

In a further cultural divide, Latino families often came from traditions that encouraged a high power distance between families and the school system (Livermore, 2013). Put another way, school authorities were highly respected by Hispanic families and parents were expected to stay out of the daily tasks of their child’s education. The traditional U.S. educational system had the total opposite approach when it came to parental involvement. “Good” parents were expected to be highly immersed in their children’s school activities with everything from volunteering in the classroom, actively helping their children with homework, and being a staunch advocate for their children’s education (Blair, 2014). Most Latino parents would consider it impolite to impose their expectations on teachers. Erroneously, American teachers often interpreted this distance and respect as lack of caring on the Latino parent’s part (Lopez, 2003).

General Phenomenon of Interest. Students who are English language learners (ELLs) or considered culturally and linguistically diverse (CLD) (for the purposes of this research, the terms ELL and CLD are used interchangeably) are one of the fastest growing student populations in the United States and underserved academically (Crotts, 2013). ELLs are those who truly have a language other than English present from birth to an extent that potentially impacts a student’s English language development (Arkansas Department of Education, 2019, May 8). An aspect of the increasing presence of ELLs in U.S. schools is these students are at high risk for academic failure (Ruiz-de-Velasco & Fix, 2000). For example, for the 2017-2018 academic year, the high school graduation rate for limited English proficient students in Arkansas was 83 percent compared to an overall high school graduation rate of 89 percent (University of Arkansas, 2019).

A study in California explored how curriculum and instruction, teacher education, and policy intersected to shape the classroom experiences of multilingual students in K–12 schools. Six school districts were utilized to explore what blocked the gate to enrollment in and successful completion of secondary mathematics courses for students classified as ever English learners (ever ELs). The initial quantitative findings indicated that half of all students in those districts repeated a mathematics course between 8th and 10th grades, with limited evidence of additional learning during the students’ second time in the course (Thompson, 2017). The study concluded that interactions between course placement policies, ways of knowing, and student motivational factors shaped students' mathematics course-taking trajectories (Thompson, 2017). In other words, the opportunity to learn was necessary but not sufficient for success in education. The findings held true not just for students that were currently classified as ELLs, but also for students that had previously been classified as an ELL and had reached English proficiency (Thompson, 2017). How ELLs were initially served in mathematics courses had long-term effects on their mathematical education.

Statement of the Problem

A holistic view of the mathematics discipline's achievement gap between native speakers and ELLs was well documented through research (Zehr, 2010). Figure 1 detailed the need to focus on ELLs in mathematics for Arkansas using data from the 2016-2017 school year for the state.

Share of Arkansas ELs and All Students Meeting or Exceeding Standards in Math (%), by Grade, SY 2016–17

	Grade 3 (%)	Grade 4 (%)	Grade 5 (%)	Grade 6 (%)	Grade 7 (%)	Grade 8 (%)	Grade 9 (%)	Grade 10 (%)
Share of ELs who met or exceeded standards	43.2	38.8	38.5	50.9	24.0	22.6	10.1	7.0
Share of all students who met or exceeded standards	59.5	56.3	53.3	62.9	49.4	45.2	31.5	26.2

EL = English Learner; SY = School Year.
Note: To generate data report used to create this table, visit the ADE Data Center and select "2016-2017" under "School Year Range" and "Add State of Arkansas" under "Districts." Then, in the "Student Assessment" category, select "ACT Aspire Achievement (2016-2018)," and when asked to "copy selections to all grades and subjects listed below" select "English Learners" and "Combined Population."
Source: ADE, "ADE Data Center—Generate Custom Report."

Figure 1. Proficiency of ELs for the 2016-2017 SY

A portion of the published material in the field of mathematics education on ELLs focused on teaching students of Latino/Latina descent. While these students were an important part of the ELL demographics, they were not the only category of ELLs. Few studied the mathematical needs of ELLs whose native language and culture was not Spanish in nature. That left out a portion of ELLs, including the Marshallese. In addition, since the emphasis had been on documenting the disparity between ELLs and their peers rather than the specifics of why the divergence existed beyond ethnicity, it was sometimes difficult to separate linguistic, cognitive, sociocultural, and pedagogical strands that could be influencing why there was a mathematical achievement gap (Janzen, 2008). Research in mathematics education increasingly recognized the role of mathematics content language in the education of ELLs as well as the role of culture in the classroom (Babaci-Wilhite, 2016).

Classroom studies documented the fact that underserved English learners, poor students, and students of color routinely received less instruction in higher order skills development than other students (Allington and McGill-Franzen, 1989; Darling-Hammond, 2001; Oakes, 2005; Gandara and Contreras, 2010). There were no differences in intellectual capabilities between a typical student, English learners, poor students, or students of color based on IQ (Nisbett et al., 2012). Therefore, academic achievement should follow a normal bell curve regardless of outside labels placed on students. However, students labeled atypical struggled because educators did not offer them sufficient opportunities in the classroom to develop the cognitive skills and habits of mind that would prepare them to take on more advanced academic tasks (Jackson, 2011; Boykin and Noguera, 2011). Kozol (2005) and Oakes (2005) argued the reason they were not offered more opportunities for rigor was rooted in the education system's legacy of "separate and unequal." This may have been done consciously or unconsciously due to lack of understanding of differences in cultural interpretation or linguistic needs.

Current Efforts. "Education either functions as an instrument which is used to facilitate integration of the younger generation into the logic of the present system and bring about conformity or it becomes the practice of freedom, the means by which men and women deal critically and creatively with reality and discover how to participate in the transformation of their world" (Freire, 1993). Educators engaged in discourse about ELL mathematical perspectives and their linguistic needs within the content, synchronous implementation of mathematical instructional strategies and linguistics would benefit ELLs. To better understand the struggle of the ELL population with mathematics, an analysis of their perspectives was conducted through this research. The goal was to illuminate details on how ELLs in a seventh grade mathematics classroom learn algebraic content simultaneously with a second language.

Purpose of the Study

The purpose of this research was to describe perspectives of ELLs from one seventh-grade mathematics classroom while attending to their linguistic identity. In association with linguistic needs, researchers have demonstrated that culturally responsive education could strengthen student connectedness with school and enhance learning (Kalyanpur & Harry, 2012). When used effectively, culturally responsive pedagogy had the ability to help students build intellectual and competence (Gordon, 2001; Hammond, 2015). Part of attending to culturally responsive pedagogy was factoring in the linguistic needs of the students and instigating supports for students to fully access the content.

Significance of the Study

For Arkansas, 8 percent of all students enrolled for the 2018-2019 school year were considered ELLs, 14 percent of the student population in Northwest Arkansas, and 45 percent of the students in the school district studied were considered ELLs. Those statistics had the attention of the students, parents, teachers, and communities those schools serve because that was a significant part of the student population. Culture and language guided how people, in this case students, processed information. To ensure retention of learning, it must have been determined what students already knew and understood how the students had organized the knowledge in their schema. From there, educators had to construct culturally based connections or “scaffolds” between the existing schema and the new content (Hammond, 2015). Unsuccessful realizations of students’ backgrounds and their educational heritage led to misunderstandings and overrepresentation in outlier mathematics courses. For example, the researcher experienced a disproportionately higher representation of ELLs in her local remedial mathematics courses. Her fellow teachers echoed this same unbalanced depiction in their classrooms. Through this research, teachers and schools will have had a more comprehensive awareness of their students’ mathematical linguistic privations.

Research Question

The research question used to guide the study was: What perspectives do English language learners bring when engaged with algebraic expressions and equations content? Specifically, this study examined how Hispanic and Marshallese students experienced mathematical content based on three interrelated factors: (a) the role of teacher, (b) the language of mathematics, and (c) the approaches to problem solving variations in didactic versus inquiry situations.

Operational Definition of Terms

Algebraic Language - Mathematical equation, mathematical symbol, algebraic equation, or algebraic symbol

Complex Language - Mathematical word problems that may use more complex vocabulary, difficult to understand, and requires references in order to translate the words into a mathematical equation correctly

Didactic Instruction- Considered traditional and teacher-centered teaching methodology with modeling, reinforcement, feedback, and successive approximations considered key components (Schmidt, 2001)

Hispanic or Latino/a - A Spanish-speaking person of Latin American descent, usually from a Central American country

Inquiry-Based Instruction - Teaching methodology considered constructivist in nature and student centered with adaptability to students' cognitive development considered a key component (Schmidt, 2001)

Language Barriers -Context or words that a person does not know and understand based on language learner status

Limited of Marshallese/Spanish Word -No English equivalent of the Marshallese/Spanish word due to the Marshallese/Spanish language's fewer words compared to the English language

Marshallese - A native or inhabitant of the Marshall Islands or the Micronesian language of the Marshall Islands

Mathematics Register (Halliday & Martin, 1993) - The academic language of mathematics through which mathematicians or students communicate through consisting of technical terms and highly specialized symbolic notations

Mathematical Word Problems, Word Problems, Story, or Application Problem -Algebraic equations that are written in words or presented as a story

Simple Language Problem - Mathematical word problem using simple vocabularies, read from left to right, and does not require any references

Standard English -English that is widely recognized as acceptable in spelling, grammar, pronunciation, and vocabulary wherever English is spoken and understood

Uncommon Terms -Words that do not exist in the other language (i.e. Marshallese)

Vocabulary - Mathematical vocabularies that mean the same when defined from English to Marshallese/Spanish

Assumptions and Limitations

Assumptions. For the purposes of this research, some assumptions had to be made. One, the students of this case study could proficiently converse with the researcher in English if the students were given appropriate language supports. Two, the students could decode and encode the mathematical content if they were given appropriate language supports. Three, the students were given appropriate language supports for the mathematics content by their teacher as far as the teacher was able based on her knowledge of the students and linguistic difficulty of the mathematical content. Finally, ELLs had a different perspective of the mathematical content than their native English-speaking peers. Every effort was made by the researcher to insure these assumptions held true throughout the duration of the case study unless proven false.

Limitations. Due to time constraints and limited personnel, there were some limitations to this case study. Within this sample, Hispanic and Marshallese groups were represented. These included the use of a local small convenience sample, which affected the ability to generalize the results to other populations across the U.S. Second, the students interviewed were in a blocked math class meaning they received double the amount of mathematics instructional time compared to non-blocked students. Because these students had a history of struggling with the mathematics content, mathematics could induce some anxiety for the students. On track students and advanced placement students were not a part of this study. In addition, the researcher for this study was also an ESL instructional coach for the teacher of the focal students, which may have unintentionally influenced the presence of mathematics comprehension interventions or usage of comprehension strategies in the general education classroom.

In addition, adolescent students who may not have been motivated to put forth their best effort were the participants of this research. Since all students were tested at multiple points during the unit, willingness and motivation to complete the tests accurately and with best effort may have been a limitation. Furthermore, the students selected to interview and to participate in this project may or may not have been born in the United States. While there were multiple education perspectives being studied here, the researcher was most familiar with the educational idiosyncrasies of the United States. Because the students may not have been consistently educated in the U.S., they may have had different educational experiences including interrupted schooling. This could have caused the students to have various degrees of academic proficiency in their native languages. To participate, the students had to have ELL status based on limited English proficiency based on an English language proficiency assessment conducted by the school. The mathematics problems presented to the students were in English. In addition, attendance of the students for the academic mathematics lessons could not be predicted or measured and may have influenced the results of their mathematical perspectives of content.

CHAPTER 2: LITERATURE REVIEW

Various theories attempted to explain the differences between English language learners (ELLs) and their native speaking peers in mathematical achievement. Although the research covered a variety of such theories, this review was organized around three major themes. The first describes research studies that have explored the impact of the teacher on the mathematics classroom environment through the lens of ELLs. The second area of research reviewed the studies on the impact of language of mathematics in general. The studies that explored the relationship between the intended mathematics content and what was learned by ELLs is the third area reviewed. Although the literature presented these themes in a number of contexts, this review of the literature primarily focused on their application to ELL understanding of mathematics through English.

Related Studies: Role of the Teacher

The research on the role of the teacher in determining student success of ELLs is mixed. For example, in one study, teacher impact on achievement gains had a larger effect on mathematics achievement than on reading achievement (Nye, Konstantopoulos, & Hedges, 2004). This study determined that there was a positive effect for the lowest ELLs enrolled in English as a second language (ESL) supported courses on college preparatory mathematics coursework as well as on mathematics test scores and null effects in other academic areas (Callahan, Wilkinson, & Muller, 2010). These results also indicated that the quality of the teacher was a more significant contributor in the trajectories of student outcomes in mathematics than in reading. In addition, another study found that representing and communicating mathematics are interwoven and therefore teaching mathematics via repetition, acquisition, and transmission of vocabulary does not show improvements in students' learning (Warren, Harris, & Miller, 2014).

Teacher Knowledge. In the fall of 2015, the latest for which these statistics were available, the percentage of public school students in the United States classified as ELLs was 9.5 percent, or 4.8 million students (U.S. Department of Education, n.d.). In Arkansas for the 2018-2019 school year, 8 percent of the student population was identified as being limited English proficient, or 38, 265 students (University of Arkansas, 2019). During the 2013-2014 school year, ESL endorsements on teaching licenses was listed as a teacher shortage area for Arkansas.

Researchers concerned with the assessment of ELLs have asserted states should place a substantial focus on increasing teacher knowledge of current ELL issues such as including ELL pedagogy in pre-service teacher education and continuing teacher education (Wolf, Herman, & Dietel, 2010). Learning about linguistic supports in mathematics should also be included in pre-service teacher mathematics education programs and professional development for in-service teachers. This additional training of content for educators in textbooks and professional development materials equips teachers for the work of teaching mathematics to ELLs (Wilson, 2016).

Sleeter (2001) furthered these ideas with research on pre-service teacher education programs for diverse schools. A review of 80 different studies was conducted on the effects of numerous pre-service teacher education strategies, including student selection, cross-cultural immersion practices, multicultural coursework, and program modifications. Sleeter argued that there exists a quantity of research about multicultural education, but few of the studies examined what strategies prepare pre-service teachers to be quality multicultural educators. Furthermore, Sleeter (2001) stated that “Most of the research focuses on addressing the attitudes and lack of knowledge of White pre-service students” (p. 94). Although that was an element of the issue, stating weaknesses was not the solution to the problem.

Delpit (2006) also examined the impact of classroom routine and assessment on students. There were multiple ideas on what an effective classroom looked like (Stone, 2005), especially based on cultural differences (Rudiak-Gould, 2009). “When we are able to recognize and name a student’s learning moves and not mistake culturally different ways of learning and making meaning for intellectual deficits, we are better able to match those moves with a powerful teaching response” (Hammond, 2015, p. 5). Even in mathematics, there were stylistic subtleties to how problems were written and solved depending on the mathematician (Fitzgerald & James, 2007). Linguistic barriers compound these differences.

Instructional strategies and curriculum seemed to have an impact on ELL learning. Swanson (2016) found (a) context matters in sustaining innovation, (b) powerful curriculum and instruction could transform teaching and teachers, and (c) student impacts continue after the initial implementation. ELLs' achievement gains in mathematics could be from the challenging mathematics content and instructional and language scaffolding strategies (Cho, Yang, & Mandracchia, 2015). Pedagogical content instructional strategies like those found in Constructing Meaning© and the eight standards of mathematical practice as put forth by the Common Core State Standards Initiative (2019) influenced how the mathematical content was taught and what was expected of the ELLs to submit as evidence of learning.

Mathematics Knowledge. Teacher knowledge of mathematics has been delineated in a variety of ways: pure content, pedagogical content, knowledge of students, to name a few. Ball and colleagues (2004; 2005; 2008; 2011; 2018) described multiple over-lapping areas of knowledge and skills related to success in teaching mathematics. They argued that teaching mathematics requires specialized pedagogical content knowledge the focuses on how well teachers are able to connect mathematical concepts and procedures to students' ways of thinking. Teachers facilitate mathematical classroom discussion by leading students to compare and contrast multiple representations and solutions to problems.

Keyword Strategies in Word Problems. The use of keywords as an instructional strategy in mathematics is considered controversial amongst mathematics educators. For example, the use of keyword strategies is not consistent with specialized content knowledge previously described. It is intended to aid in the interpretation of algebraic reasoning by lowering the amount of content and thus simplifying the problem (Karp, Bush, & Dougherty, 2015).

Some studies have shown positive results of ELLs performance in mathematics with the use of certain types of keyword strategies. These studies found that pre-teaching key words increased fluency in connected text written above the typical reader's reading level, particularly in expository texts. Using the keyword method with phonological keywords and direct native language keyword-translation links in the classroom led to better second language vocabulary learning at early stages of acquisition (Sagarra & Alba, 2006; Coulter & Lambert, 2015). O'Donnell, Weber, and McLaughlin (2003) corroborated this when their findings indicated that the students read more words correctly and answered more comprehension questions accurately after the material was previewed and the keywords were discussed, especially with ELLs. Metacomprehension, the ability to monitor the understanding of texts, accuracy was greater when generating keywords (de Bruin et al., 2011).

In contrast, keyword strategies, historically in mathematics instruction, included identifying words that would indicate which operation should be used to solve different types of word problems. For instance a problem like, "Ruby has 10 smiley face stamps. She gets some more stamps and now has 16 stamps altogether. How many more stamps did she get?" includes the word altogether. If this word has been pre-taught to students as a "keyword" strategy to indicate finding the sum, young students are likely to add 10 and 16 and get the incorrect answer of 16. Older students learning pre-algebra may set up the equation as 10 + 6 = x which would also likely lead to an incorrect answer (Clement and Bernhard, 2005).

Many of the revised standardized mathematics tests have incorporated more word problems into their assessment sets (Hipwell & Klenowski, 2011). These tests highlighted the importance of taking into consideration the intangible language heritage that students bring to the school environment while they were taught in the use of Standard English (Okoye-Johnson, 2011). Gerace and Mestre (1981) stated that the most difficult steps in solving problems using conventional written language were the steps of translating a mathematical word problem to a mathematical equation, and the process of translating written language into algebraic language was considered important for students' success in learning and mastering mathematics. This issue was reiterated by Cummins, Kintsch, Reusser, and Weimer (2004) when they argued the issue with word problems for students likely stemmed from the misreading and miss translating between mathematical word problems and mathematical equations to complexity in language.

Fuchs et al. (2012) found that for students who struggle with word problems in mathematics, the difficulty lies in understanding what should be done with the numbers deeply embedded in a narrative. Halladay and Neumann (2012) noted that mathematical word problems often contain extraneous information in their efforts to depict real world applications for mathematical reasoning. ELLs found it difficult to dissect the problem to distinguish pertinent information from superfluous information and process the language itself. Research suggested that teachers themselves often reverted to a “superficial approach to problem-solving” when working with word problems, and failed to make the most of teachable moments in mathematical language and concepts within the word problem (Rosales, Vicente, Chamoso, Múñez, & Orrantia, 2012). Mathematics teachers developed over time the ability to intrinsically know the extraneous information in a word problem and tended to go straight for what was needed to answer the question. A teaching opportunity was missed when the teacher did not explain to the students why a portion of the problem was relevant and why a portion was irrelevant (Karp, Bush, & Dougherty, 2015).

The keyword strategies to determine the operation or which number should be multiplied by a coefficient in a linear contextualized situation has been shown to impact all mathematics learners (Clement & Bernhard, 2005). Furthermore, it has been found that keywords became more of a hindrance to mathematical reasoning when students began to explore multistep word problems because students had to decide how and if to group keywords (Karp, Bush, and Dougherty, 2019). Key word strategies do not consider the role of the keyword within the overall meaning of the word problem situation and therefore are more limiting for successful problem solving than strategies, such as direct modeling, that emphasize understanding the entirety of the structure and quantities represented in the word problem (Carpenter et al., 2015). In a mathematics text, if one word is misunderstood, the entire sentence could be misconstrued. Take for example the following question, “Which data set shows the greatest change in range?” Every word in the question is needed to arrive at the right answer. Leith, Rose, and King (2016) found that students had to be able to decode every word correctly to make sense of this type of problem. If experience with decoding all the words and making sense of the overall meaning of the problem is limited, it has been found to be additionally confounding for ELLs. Walkington, Clinton, and Shivraj (2018) found that ELLs who did not receive opportunities with language supports such as structured student talk activities struggled more with interpreting and translating word problem situations that they were required to translate into algebraic equations that would facilitate a correct answer than solving equations that were already written with numbers, operation symbols, and variables.

Teacher Social and Emotional Competence. Jennings and Greenberg (2009) highlighted the importance of a teacher's social and emotional competence. Their proposed prosocial model contributed to the well-being of the development and maintenance of student relationships, effective classroom management, and successful social and emotional learning program implementation. Prosocial classrooms were more conducive to learning and promoted positive developmental outcomes for the students. This model seemed favorable to the collectivistic culture of the Marshallese and Hispanic people in comparison to the individualistic American culture.

> An educator's ability to recognize students' cultural displays of learning and meaning making and respond positively and constructively with teaching moves that use cultural knowledge as a scaffold to connect what the student knows to new concepts and content in order to promote effective information processing was the difference in culturally responsive teaching (Hammond, 2015, p. 15).

Mathematics teachers that were culturally and linguistically sensitive accounted for the affective filter in their students.

> It is not enough to have a classroom free of psychological and social threats. The brain needs to be part of a caring social community to maximize its sense of well-being. Marginalized students need to feel affirmed and included as valued members of a learning community (Hammond, 2015, p. 47).

Multilevel mediation analyses appeared to show a positive relationship between classroom emotional climate and grades that were mediated by engagement (Quezada et al., 2012). Teachers with more positive perceptions of the learning environment had lower initial ratings of concentration problems, disruptive behavior, and internalizing symptoms, and higher ratings of prosocial behaviors and family involvement of their students (Debnam et al., 2015). Students with refugee backgrounds, like the Marshallese, or students with limited or interrupted schooling had challenges in post-resettlement such as acquiring a new language, culture, and system while dealing with post-traumatic stress (Servan-Schreiber, Le lin, & Birmaher, 1998). A student with post-traumatic stress may have seemed aggressive, withdrawn, and unable to concentrate, or present anxiety (Coelho, 1998; Thabet, Abed & Vostanis, 2004). Deficient views made the assumption that some children, due to their genetic, cultural, experimental, or linguistic differences had faults that they needed to overcome if they were to learn (Cho, Wang, & Christ, 2019). The issue with that perspective was student's perceived failure was attributed to their home life and families, not to schools or society (Nieto, 2017). A teacher's deficit view of an ELL affected his or her instructional decisions (e.g., providing less advanced instruction or fewer opportunities to collaborate with peers), marginalizing ELLs and creating missed opportunities for learning (Roy & Roxas, 2011).

Delpit (2006) was resolute that prejudice was not the main issue for this phenomenon of mathematical achievement gap, although it did play a role. Educators tended "to perceive those different from themselves except through their own culturally clouded vision" (p. xiv). These ELLs were seen as "other" or exceptionalities rather than simply as students. Often this was associated with a negative viewpoint. Furthermore, there was a concern about schools that place curriculum and standardized testing above relationships with students. Delpit (2006) made it clear there was a difference between allowing and supporting diversity.

The cultural differences continually have implications for ELLs. While there is an increasing amount of minority students in classrooms generally, the teaching workforce remains predominantly white female. Delpit (2006) asserted,

> Teachers can… acknowledge the unfair "discourse-stacking" that our society engages in. They can discuss openly the injustices of allowing certain people to succeed, based not upon merit but upon which family they were born into, upon which discourse they had access to as children… Only after acknowledging the inequity of the system can the teacher's stance then be "Let me show you how to cheat!" In addition, of course, to cheat is to learn the discourse, which would otherwise be used to exclude them from participating in and transforming the mainstream. (p. 165)

Parker, Bartell, and Novak (2017) examined the cultural responsiveness of thirteen mathematics teachers after the teachers completed a course about culture in a mathematics classroom. They concluded teachers seemed to have expanded their cultural awareness and increased their self-efficacy about teaching in a culturally responsive manner. However, teachers did not seem to develop their knowledge related to power and privilege in society.

The Inventory of Situationally and Culturally Responsive Teaching (ISCRT). ISCRT, formerly known as Biography-Driven Performance Rubric (BDP), takes a comprehensive look at the ecology of a classroom from the standpoint of cultural and linguistic responsiveness (Murry, Herrera, Kavimandan, & Perez, 2011). Furthermore, the creation of ISCRT was based on the five Standards for Effective Pedagogy and Learning (Center for Research on Education, Diversity, & Excellence [CREDE], n. d.) and the Standards Performance Continuum (Tharp and Dalton, 2007; Murry, Herrera, Kavimandan, & Perez, 2011). The classroom observation instrument breaks down components of ELL instructional best practices into twenty-two indicators grouped into five standards (Murry, Herrera, Kavimandan, & Perez, 2011). It provides an avenue to quantify the qualitative enactment of pedagogy (Herrera et al., 2011, Murry et al., 2015). The indicators are scored by a trained observer and measured on a Likert type scale from 0 to 4 with each increase in score representing an increased level of teacher implementation. The most desirable score is a 4 for each indicator. Analysis of internal reliability of the ISCRT demonstrated acceptable internal consistency for this type of instrument (Herrera et al., 2011). See Appendix M for a copy of the full instrument.

Related Studies: Language of Mathematics

The "Language of Mathematics" includes mathematics symbols, conventions, colloquial terms like "coefficient" and other global vocabulary that have specific denotations and meanings in mathematics, such as "acute" or "ray". "The linguistic challenges include the multi-semiotic formations of mathematics, its dense noun phrases that participate in relational processes, and the precise meanings of conjunctions and implicit logical relationships that link elements in mathematics discourse" (Schleppegrell, 2007, p. 139). Mathematics content is delivered through language and mathematics and teachers are therefore teachers of the language of mathematics (Leith, Rose, & King, 2016). Despite agreement that language was crucial to mathematical achievement, mathematics textbooks and curricula did not make the language demands of their tasks evident to mathematics teachers, and teachers were often unaware of the linguistic complexity of the mathematical tasks they presented to learners (Lucero, 2012).

The language of mathematics becomes intuitive to mathematics teachers with awareness and experience. Gough (2007) observed that when teachers were not conscientious of the language ambiguities and challenges in mathematics, they failed in their teaching responsibilities and instead laid the blame on the students, quoting "'learning difficulties', cognitive confusion, and attention deficits". Fuchs, et al., 2012, concluded that when students were left to struggle with their challenges misunderstood, their achievement levels in mathematics continued to drop along with their opportunities for positive post-school outcomes.

Another study found that a correlation existed between language proficiency and achievement in mathematics (Riordain & O'Donoghue, 2009). Some teachers operated under the fallacy ELLs did not need any linguistic interventions or supports in mathematics since the content was thought of as a universal language (Adoniou, 2014). The difficulties associated with mathematics were seen as coming from the cognitive demands of the content itself. These perceptions, not formally studied, could potentially apply to mathematics presented in symbols with common meanings such as the "+" symbol. However, with word problems, language and mathematics are inextricably connected.

Further confounding this issue is that the meanings of some mathematical terms or symbols are not the same across languages (Robertson, 2009). For example, a dot is used to represent a decimal point in most dialects of English while a comma is used in some Spanish speaking countries. Commas are used in the U.S. as digit separators while other countries use spaces or superscript commas as digit separators. Delpit (2006) discussed how teachers needed to examine how they affect minority and low-income students' levels of understanding of "Standard" English. In other words, teachers unintentionally projected their culture and linguistic heritage onto their students in ways that disregarded the different meanings and cultural connotations associated with certain vocabulary words. Turner (2011) noted that if a shift were to come that focuses more on the language of mathematics, two competencies would need to be included: communication, and using symbolic as well as formal and technical language. Furthermore, "The more an individual processes these competencies, the more able he or she will be able to make effective use of his or her mathematical knowledge to solve contextual problems" (p. 26). ELLs required support to develop these linguistically bound competencies through the English language and ample processing time.

The studies of language-based mathematics problems demonstrated that words and vocabulary affected how mathematics information was processed (Tzeng & Wang, 1983). Research indicated that ELLs experienced a disadvantage of up to 15 percent in mathematics because of issues with language (Barton, Chan, King, Neville-Barton, & Sneddon, 2005; Riordain & O'Donoghue, 2009). These results indicate that while language misunderstandings can inhibit all students, ELLs are more profoundly impacted because they are grappling with second language acquisition in addition to grappling with language intensive mathematics content as opposed to only grappling with only language intensive mathematics and extraneous terms that are familiar to English first learners.

Mathematics Register. Linguistically, mathematics became associated with what some educators have termed "a register". The mathematics register is the connection between symbols that stand for ideas and concepts and the language used to describe the meanings of those symbols. The English language, along with all other languages, has evolved over hundreds of years of the development of the discipline of mathematics (Halliday & Martin, 1993). O'Halloran's (2005) described the mathematics register as processes, representations, and symbolism associated with mathematical concepts and procedures. O'Halloran focused on how this range of meaning systems influences the construction of mathematical meaning. Grammatical patterns used to interpret the mathematics register, such as mathematical vocabulary and terms may take on different meanings across cultures and different languages in contexts and vocabulary situations that are not connected to mathematics. (Schleppegrell, 2007).

For example, in the Marshall Islands, geometric navigational charts are based on ocean wave swell interactions and the direction the wind is blowing. Western geometric navigational charts rely on compasses, charts and rulers (Ascher, 1995).

Gough (2007) described mathematics teachers as multilinguals who must consciously pass the linguistic mastery of the discipline to their students. Unfortunately, researchers observed a disconnect or an unparalleled discourse style between the language of instruction in the classroom and the language of mathematics required to make sense of tasks in the textbook (Vries, Young, & Warren, 2007). Along those lines, Slavit and Ernst-Slavit (2007, p. 4) found "conversation in mathematics classrooms can be a barrier to understanding for ELLs." There were multiple expressions in mathematics to convey similar algorithmic functions; e.g., subtract, take away, minus, less, and difference (Carter & Quinnell, 2012). Galvan Carlan (as cited in Adoniou & Yi, 2014, p. 5) found, "fluency in interpersonal conversation does not equate to fluency in concepts and the discipline-specific language of mathematics." Not only did a student need to be proficient in conversational language, but also simultaneous knowledge of the register of mathematics was necessary for success in mathematics classrooms.

Translation activities from word problems written in English to equations with the goal of using those to solve the equations for the unknown have been shown to be cumbersome for ELLs. In one early study of ELLs, Hispanic ELLs took more time translating mathematical word problems into mathematical symbols and required more time to solve mathematical equations than non-minority students (Mestre & Robinson, 1983). The tasks required students to first decode the English words of the problem, with English not being their first language, and then interpret the mathematical structure and use that information to set up an appropriate equation that could be solved to get the correct answer. The ELLs tended to guess and check strategies to avoid the translation process altogether (Mestre, 1988). While guess and check methods can be helpful in finding correct answers for many word problems, equations eventually enhance students' performance with more complicated number sets, multi-step situations, and for more advanced level thinking, conjecturing, and generalized principles of mathematics.

Sentence Structure. Syntax of mathematics was shown to play a role in underachievement of the content for students if there they were already considered behind in mathematics and they experienced language difficulties (Ríordáin & O'donoghue, 2011). Because of proper English grammar, a word order nuance or syntax shift in mathematical sentences potentially alters the mathematical meaning or interpretation in dramatic ways. For example, students may have read a sentence sequentially from left to right, but the order in which they responded to the sentence was from right to left. The following is an example of a problem in which translating to an equation from left to right could provide additional challenges to ELLs: "Julie has fifteen pens. If Julie has seven more pens than Lio, how many pens does Lio have?" A left to right translation might be $15 + 7 = x$, because of the "seven more". This translation does not lead to the correct answer. In order to get the correct answer students have to shift the addition sign to the other side of the equation (i.e., $15 = 7 + x$) or use flexibility with the operations and subtract (i.e., $15-7 = x$) to get the correct answer.

Sentences in mathematics tend to convey complex relationships and abstract ideas (Geeganage et al., 2016). When dependent clauses were used with linguistically complex sentences to convey mathematical thought these structures were found to be the most difficult for ELLs to comprehend (Martiniello, 2008). For example, complex sentence structures are required for hypothesis testing or justifying an answer (Martiniello, 2008). Examples of complex sentence structures are conditional sentence types, such as if-then statements.

Sense-Making in Mathematics. Riordain & O'Donoghue (2008) found that when ELLs spent additional time familiarizing themselves with the sentence structures and colloquial mathematical terms, they communicated more at a top tier level than those students who only tried to complete direct translation activities from words to symbols. These results are consistent with advocacy documents that emphasize "making sense of problems and persevere in solving them" (Standard for Mathematical Practice One, Common Core State Standards for Mathematics, 2010). This standard was listed first in the set of practice standards as it is considered a pivotal disposition for all students to develop for success in secondary mathematics. In other words, whether situated in story contexts or presented in symbolic form, all students need the ability to make sense of the words, quantities and symbols in order to carry out logical mathematical procedures. For example, in a study about mathematical reasoning, a native Spanish speaking ELL was asked to compare the perimeter and area of various rectangles and look for patterns. The ELL did not know the correct term for rectangle in Spanish or English. However, she was able to communicate that a rectangle with longer sides has a higher perimeter. If an observer had focused on the ELL's inability to produce the right mathematical vocabulary, he would have missed the ELL's correct mathematical reasoning. Moschkovich (2011) found it was necessary to focus on the student's mathematical reasoning and not only on her proficiency in English.

Didactic vs. Inquiry Mathematics and ELLs

The literature on best curricular approaches for ELL's learning of mathematics is mixed at best. Prior to recommendations from mathematics education leaders that disputed the overemphasis on low level skills and symbolic procedures, mathematics textbooks were organized around mastery of procedures before engagement with word problems. Word problems were perceived as ways to practice numeric procedures couched in words and situations. Didactic teaching approaches along with worked examples in most textbooks fed this perception and as a result students in the U.S. underperformed in international comparative studies (Stephens et al., 2016). National documents reflected the urgency to shift the foci of mathematics curriculum materials and situate problem solving at the earliest stages of learning mathematics and not just as an extension way to an already learned procedure (NCTM, 1989; 1991; 2001; 2012; etc.).

These types of standards documents reflected the growing body of cognition on how students make sense of early mathematics content. Carpenter and Moser (1984) added detailed knowledge of young children's early learning of number concepts by interviewing and assessing their approaches to solving word problems prior to their formal exposure to these problems in classroom settings. Carpenter et al. further expanded this knowledge base by presenting the knowledge gained to teachers to determine if it would enhance their mathematics instruction in the early elementary grades. One of their seminal studies showed that kindergarten students were capable of solving word problems involving all four basic operations without first having a method demonstrated (1993).

Successful examples of students solving problems using their own intuitive strategies as opposed to following methods taught by the teacher provided impetus for educators and curriculum developers to focus on inquiry methods of teaching mathematics. The Common Core Standards for Mathematical Practice of (1) make sense of problems and persevere in solving them, (2) reason abstractly and quantitatively, (3) construct and critique the reasoning of others, (4) model with mathematics, (5) use appropriate tools strategically, (6) attend to precision, (7) look for and make use of structure, and (8) look for and express regularity in repeated reasoning, encapsulated decades of research that demonstrated the benefits of shifting lessons from didactic to inquiry-based by using the definitions as cited in Schmidt (2001). The idea of presenting problems to students and letting them use strategies that they had constructed on their own or adapted from other students began to replace more traditional methods. This change in format for mathematics lessons opened the door for promoting mathematical discourse among students and between students and the teacher (Moschkovich, 2007).

Mathematical discourse in classrooms began to become a focus of classroom based studies. The goal of mathematical discourse is to clarify logical mathematical thinking, elaborate verbal descriptions, and analyze written work (Knudsen, Lara-Meloy, Stevens, & Rutstein, 2014). While the eight mathematics practice standards were written to encompass all grades and levels of mathematics learning, algebraic reasoning, because of the content including generalizing from rules of arithmetic, numerical patterns, and modeling contexts through various methods of representation can be enhanced by a focus on discourse. However, Lager (2006) cautioned that efforts to oversimplify the expository context of mathematical discourse to improve ELL's English language acquisition can lead to incorrect problem solutions because of misunderstandings of the intent of the mathematical content.

Student Language Proficiency. Proficiency in the English language affected all facets of an ELL's ability to communicate with speakers of that language (Nadri, Baghaei, & Zohoorian, 2019). Students who were identified as ELLs had access to programs to help them simultaneously attain proficiency in English as well as meet academic standards. The English Language Proficiency Assessment for the 21st Century (ELPA21) (Cook and MacDonald, 2013) was used by the participating school district to establish the proficiency level of its ELLs. The purpose of the ELPA21 was to provide online assessments that best measure ELLs' mastery of the communication demands of the states' rigorous academic standards. The assessments were designed around the four language domains: reading, writing, speaking, and listening (ELPA21 state consortium receives additional grant funding, 2014). The marginal reliability coefficients for the total scaled score range from .92 to .94. For each of the four subcategories of listening, reading, writing and speaking the coefficients range from .63 in the listening subscale to .89 in the writing subscale (American Institutes for Research, 2014, September 15). The students were assigned a language proficiency level number constructed from the aggregate results of their four assessments.

Based on the ELPA21 level of the student, educators were given skill sets the ELL would be able to do linguistically for teachers to consider in the classroom. New content standards in mathematics (Kendall, 2011) expected all students, including ELLs, to use multiple modes to express student understanding of concepts. Multimodality refers to the use of several signs or symbols to design a symbolic product or event (Kress and van Leeuwen, 2017). Traditionally, conceptualizations of multimodality in ELL education and the content areas differed considerably (Grapin, 2019). To solve mathematical content, students engaged in arguments as they made claims, provided relevant and sufficient evidence to support their claims, and offered sound reasoning all of which involve language use (Mercer & Sams, 2006). If the student did not have the language proficiency of a native speaker of English, it affected the student's ability to fully communicate their thinking in English. A study found children who were fluent in both Spanish and English performed better than less proficient bilingual children on measures of math calculation, fluid intelligence, and reading (Swanson, Kong, & Petcu, 2018). Arkansas was an English only state by law for instruction at the time of this study (Hanna, 2017). This placed the ELLs at an educational disadvantage in mathematics classrooms as it made it more difficult for them to acquire the mathematical content and express what they had learned (Brown, 2005). The average mathematics score for non-ELLs in Grades 4 and 8 has been higher than the scores of ELLs whose first language is Spanish for several years (Bumgarner, Martin, & Brooks-Gunn, 2013).

To solve algebraic tasks, like those found in a seventh grade curriculum associated with a unit on rational expression and equations, students needed to be able to interpret a given scenario, construct one or more models of the scenario, select algebraic methods to be used to find solutions for the scenario, and determine outcomes (NCTM, 2000; Schoenfeld, 2004). Such tasks generally had inherent complexity and ambiguity that makes them cognitively demanding (Stein, Grover, & Henningsen, 1996), in addition to the linguistic challenges associated with processing the material in a non-native language context. A study furthered this idea by showing "that greater linguistic complexity increases the difficulty of English-language math items for ELLs compared to non-ELLs of equivalent math proficiency" (Martiniello, 2008, p. 333).

In a study that investigated how native Spanish speaking ELLs interacted with word problems through the lens of language it was found

> few children understood the words identical (4%) and certain (33%). These are … Spanish-English cognates, but, unlike impossible, they are infrequently encountered in conversation because more-colloquial synonyms, such as igual (equal) and seguro (sure), are available. About half of the children either ignored or confused the meanings of the words likely and unlikely (Martiniello, 2008, p. 346).

While there were different discourse patterns within mathematics, it cannot be ignored that these patterns were specific to mathematical contexts.

Translation Activities. The linguistic interpretive process further convoluted conceptual understanding and procedural fluency in mathematics. Relationships in mathematical content are tied to the ways in which different mathematical terms may be grouped together, which in turn tends to be developed by many students through operations (Mason, 1996). In mathematics classrooms, efforts to include the ability to decode and encode mathematical problems, use appropriate mathematical vocabulary, and simultaneously use correct mathematical symbols benefitted ELLs (Turner, 2011). Furthermore, writing and/or adapting word problem situations that attend to students' cultural and age related interests limit the burden of sense making when students are translating from contextualized situations to mathematical symbols (Sigley & Wilkinson, 2015).

Literal translation appeared to arise in the interplay between language difficulties and mathematical discussion (Planas, 2014). Although it may not have been deliberate, translation activities, such as the use of keywords, were given the function of reducing communication by giving more emphasis to language than to mathematics (Planas, 2014). Word-by-word translations sometimes negated the intention of a mathematical phrase by ungrouping words. For example, a problem such as, "Albert is four years less than twice the age of Eunice. If Albert is 28, how old is Eunice?" can be misrepresented in a word-by-word translation such as $4<2x=28$.

Over emphasis on translation activities can overshadow sense making with respect to the mathematics (Planas, 2014). This type of overemphasis led to alternative interpretations of the items that resulted in incorrect answers (Noble, Rosebery, Suarez, Warren, & O'Connor, 2014). Cognitive and linguistic challenges influenced what mathematical features of a problem a student, particularly an ELL, noticed when in the process of problem solving. Students were required to reverse thinking and consider several possibilities in their problem solving. What students comprehended from a problem statement correlated to their subsequent steps in algebraic reasoning (Lobato, Hohensee, & Rhodehamel, 2013).

Sense-making and inquiry learning approaches are consistent in that the goal is to provide opportunities for students to apply their current understandings to novel situations productively. Productive struggle is an idea that has shown promise for improving students' inquiry approaches (Boaler, 2016). Moser and colleagues (2011) showed that when students make mistakes in mathematics, certain brain activity happens that does not happen when students work a problem correctly. When students were struggling with the material, they were developing brain synapses and pathways needed for learning to take place. Without the mistakes, the brain pathways were not developed. For teachers and students with a growth mindset, the act of making a mistake resulted in particularly significant brain growth. Boaler (n.d.) concluded, "the importance of mistakes both suggest strongly that we need math environments in which students are given open tasks and challenging work that causes them to struggle, experience cognitive conflict, and make mistakes."

Summary of Prior Research

Research into culturally and linguistically diverse students' mathematics experiences came from a variety of perspectives. Some studies described teaching strategies that enhance ELLs' learning in mathematics classrooms. Other studies emphasized instructional methods that were found to limit students' learning of mathematics content. Language supports and holistic decoding strategies were found to be successful with ELLs. Inherently, these strategies are consistent with the eight mathematical practice standards (*Common Core State Standards in Mathematics, 2010*). In contrast, methods focused on mastery of specific methods taught by the teacher, word by word translations and lack of connections to broad mathematics standards were found to be less effective for improving ELLs' learning of the content (Cho, Yang, & Mandracchia, 2015).

The current research base is limited by several factors. While some studies have measured student achievement in mathematics based on strategies designed to improve the learning experiences of ELLs, most research has focused on quantitative research and evaluation methods that can determine whether or not students' test performance improved as a result of these strategies. Most of these types of studies do not account for specific types of strategies that students use to get correct answers. For students who do not improve during an instructional unit, multiple-choice items on standardized tests do not provide evidence of the types of interpretations and meanings that would determine both productive mathematical processes or unproductive errors and misconceptions.

CHAPTER 3: METHODOLOGY

The purpose of this case study was to describe perspectives and detail experiences of linguistically diverse students in a seventh grade mathematics classroom. Six students, three Hispanic and three Marshallese, from a class of 21 students were selected for specific data collection related to how ELLs process pre-algebra content.

Research Design

The design for this research was a qualitative case study. Case studies reflect a naturalistic approach and tend to be attuned to the complexities and interactions in a particular context (Stake, 1995). Relationships and the in-depth process of how they were woven together in a given situation was the focus (Denscombe, 2003). Patterns in the Hispanic and Marshallese mathematical ideas were expressed through language or artifacts, such as how they behaved within the classroom as expressed through their actions observed by the researcher (Fetterman, 2010) were documented. In order to answer the research question, the researcher had to view mathematics from the subjects' perspective and interpret the findings taking into account the influence of culture and language on responses.

The qualitative case study approach was ideal to answer the research question as the researcher gathered information on the perspectives of the bounded group of Hispanic and Marshallese students in mathematics. The research design was appropriate because linguistic influences are not generally quantifiable and tend to be highly individualized based on experiences. For those reasons, the researcher sought to develop an in-depth understanding of the case by collecting multiple forms of data (Creswell & Poth, 2018).

Target Population and Sample

Population. The location of the research study was in Northwest Arkansas and the participants had been identified as current ELLs by their school district. ELL enrollment in Arkansas was on the rise with 8.1 percent of all student enrollment for the state or 38,265 students in the 2018-2019 academic year. Furthermore, 14 percent of all student enrollment in Northwest Arkansas or 24,388 students were classified at ELLs (University of Arkansas, 2019). Figure 2 detailed the breakdown of what languages were spoken in Arkansas schools for 2017-2018 academic year, the latest year those statistics were available.

Home Languages Other Than English Spoken by Arkansas Students, SY 2017–18

	Number of Students	Share of Students with a Home Language Other Than English (%)
Spanish	44,628	82.7
Marshallese	3,381	6.3
Vietnamese	852	1.6
Arabic	555	1.0
Laotian	523	1.0
Other (95 languages)	4,033	7.5

SY = School Year

Notes: Counts include but are not limited to English Learners. This table uses data from the October 1, 2017 student count.

Source: Arkansas Department of Education (ADE), "ADE Data Center—Student Home Language by District," accessed July 10, 2018, https://adedata.arkansas.gov/statewide/Districts/StudentHomeLanguage.aspx.

Figure 2. Home Languages in Arkansas 2017-2018 SY

There was a large ELL population in Arkansas and they tended to live in concentrated groups rather than spread evenly throughout the state. The majority of ELLs attended school in Northwest Arkansas. The region included the leading school district with ELLs accounting for 45.3 percent of its total student enrollment for the 2018-2019 school year and comprising 25 percent of all ELLs in the state. In 2015, the State of Arkansas Bureau of Legislative Research indicated that despite overall gains in academic achievement for ELLs since 2008-2009, these students were still performing well below their non-ELL peers academically in mathematics.

District Setting. Table 1 detailed the Springdale School District demographics for the 2017-2018 school year.

Table 1. School district demographics for the 2017-2018 SY

Ethnicity	Number of Students	Percent of Students
Hispanic	10,289	46.78%
Asian	371	1.69%
American Indian	115	0.52%
Black	536	2.44%
White	7,571	34.42%
Hawaii/Pacific Islander	2,816	12.80%
Two or More	297	1.35%
TOTAL	21,995	100.00%

School Setting. The latest breakdown of demographics for the specific school where the research took place can be found in Figure 3. These statistics were from 2016.

Figure 3: School level demographics

Classroom setting. The sample for this research consisted of a seventh grade blocked mathematics class. If students were in a blocked class, the students historically struggled with the content for a variety of reasons. The students within the blocked class received double the amount of instructional time as their peers in a non-blocked class. The reasoning behind the extended time was to help build content skills. The students built content skills by being given more processing time and increased frequency in instructional interventions.

The class for this research consisted of twenty-one students. There were nine Hispanic students, five Marshallese students, and seven Caucasian students. Thirteen of the students were female and nine were male. Although there were twenty-one students in the class, this case study focused on six of those students. The six students had to have been categorized as ELLs by the school district.

As part of their classwork, students were placed in intentional small groups of four that rotated depending on what was being asked of the class to do. A few of the reasons students were grouped together were language level, mathematics proficiency level, ability to work together, students with different or same problem-solving strategies, or any combination of those reasons. While problem solving, students routinely worked in collaboration to talk about and work through the mathematics content. Mutual respect was expressly taught by the teacher and enforced within the classroom and between the students. When students were asked to talk through their thinking aloud to the class, the students had to defend their answers. Their teacher would then call on other students to express their agreement or disagreement with what the student said as well as defend their answer. Again, this was done in a respectful manner. No one student or group of students dominated the conversations in the classroom. Responses came from every student every day based on how the teacher asked the students questions. There were no indications of a student feeling unengaged or unvalued in class based on verbal and nonverbal communication, especially from the viewpoint of an ELL. Based on the observations of the researcher and conversations held by the students, the students had relational equity between each other and between themselves and the teacher.

Sample. For this study, a sample of the classroom population was studied in depth. Six students from the seventh grade mathematics class were the focus of the research. These students were all ELLs with varying degrees of mathematical performance achievement. Four of the students were considered to have an English language proficiency level of 2 by ELPA21 standards. One student had an English language proficiency level categorized as a long term 3 and one had a classification of a long term 5. The phrase “long term” in front of the language level means the student had been classified at that language level longer than typically normed language acquisition rates. The school district, school, classroom, and sample all had high instances of low socioeconomic status for the students and their families. The native languages of the sample students were Spanish and Marshallese.

Procedures

Confidentiality. Permission to conduct this study was granted from the University of Arkansas Institutional Review Board (see Appendix A), as well as the administration of the Springdale School District where the study was conducted (see Appendix B). Permission of individual parties to participate in this study was obtained prior to commencement of this project. A letter to parents/guardians (see Appendix C) along with an Informed Consent (see Appendix D) was sent home with each student in the appropriate language, and a signature from the parent or guardian was obtained before data for that child was reported. The Informed Consent explained the purpose and procedures of the study. It also explained that participation was completely voluntary and that there were no rewards for participating nor penalties for not participating. It explained that the child could withdraw from the study at any time without penalty. The teacher was also given an informed consent form, (see Appendix F) detailing her participation was completely voluntary and that there were no rewards for participating. All information was kept confidential to the extent allowed by applicable State and Federal law and University policy. Confidentiality was assured and maintained by the researcher through the establishment of a code. Each student and the teacher were assigned pseudonyms to establish the code. All data were recorded and reported anonymously using the code. Only the researcher had access to the code, and all data were kept in a secure location with the researcher on a computer that was password protected or in a locked file cabinet in a locked University office. Once the study was successfully defended, the code was destroyed.

Target Students for Case Study. Purposeful recruitment took place for this research. Each student chosen for this study was done so based on documented ELL status from the school district and membership in a particular class. First, the researcher conducted classroom observations and collected student artifacts from the students' normal classwork. Second, six ELLs were selected based on their variability of performance in class. Their teacher indicated two high performing, two mid performing, and two low performing students based on their grades and ELL status. These students were observed regularly during class for the duration of the unit with detailed notes taken about what they were doing to make sense of the content. To further the depth of data collection and clarify what the researcher saw in class, interviews were conducted with each of the selected students. These interviews were on school grounds with a teacher present as a follow-up into their thought processes for solving mathematical problems. At the conclusion of the unit, a summative post assessment was collected from the students' normal classwork to add to the body of data for analysis.

Data Collection. Given the nature of the research question and the qualitative case study research approach, the researcher used multiple data sources and collection techniques in this study. The researcher determined patterns of the language-sharing groups, Hispanic and

Marshallese, through engagement in extensive fieldwork and collection of data primarily through interviews, observations, and student artifacts (Atkinson, 2015; Fetterman, 2010). This deeper exploration of the issues facilitated simultaneous data triangulation, authentication, and validation of findings.

The class was given a pretest with a three part problem to solve during scheduled class time before the instruction of the unit began with the researcher present. For the duration of the unit, the researcher conducted face-to-face observations three times a week for ninety minutes each. During those observations, field notes and student artifacts through embedded activities were collected. After the conclusion of the unit, the students were given three, grade and unit level appropriate, word problems to solve with the researcher present. As the students were working, the researcher asked the students to talk aloud through their thinking while solving. Purposeful questions to discover the student's mathematical thought processes were asked by the researcher for clarification. These interviews were recorded using audio/visual technology and field notes for further analysis.

Instruments. During the course of this research, instruments were used for consistency and quality of data collection. Student artifacts from regular classroom work were collected throughout the duration of the unit to monitor student progress. The researcher kept observation notes via the Classroom Observation Notes template seen in figure 4. This template was filled out in real-time by the researcher during each observation.

Classroom Observations Template	
Name of Teacher:	Name of Observer:
Date of Observation:	Length of Observation:
Topic of Lesson:	
Comments of Observer:	
Comments by Students:	
Comments by Teacher:	

Figure 4: Classroom Observations Template.

Furthermore, a mid-unit student interview was conducted using a predetermined set of problems for the students of the case study as shown in Figure 5. The students of this case study were given the following instrument to fill out while being recorded with the researcher.

Name ______________________________Date ___________________ Block __________

Student Interview Problems

QUESTION	ANSWER
Ricky's dad called an electrician who charges $50 to travel to a customer's home and $75 per hour of labor completed. The electrician traveled to Ricky's house and completed 3 hours of labor. What was the total amount of money the electrician charged Ricky's dad?	
Kai is 4 years younger than half Abigail's age. Kai is 12 years old. How old is Abigail?	
A couple of friends wanted to get something to eat while at a festival. They bought two pieces of fried chicken and three orange sodas for $7. Later, another group of friends bought four pieces of fried chicken and three orange sodas for $11. How much did each piece of fried chicken and each orange soda cost?	

Figure 5: Student Interview Problems.

The researcher asked a selection of investigative questions as the students solved the above problems to gain insight into their mathematical thought processes from a pool of questions shown in Figure 6. The researcher selected the questions based on what seemed most pertinent to each student's problem solving progression or to clarify what the researcher was witnessing the student doing.

Semi-Structured Student Interview Questions

1. Make sense of problems and persevere in solving them.
    - How would you describe the problem in your own words?
    - How would you describe what you are trying to find?
    - Talk me through the steps you have used to solve it.
2. Reason abstractly and quantitatively.
    - What do the numbers in the problem represent?
    - What is the relationship between the quantities?
    - What does _____ mean to you? (e.g. symbol, quantity, diagram)
    - How did you decide in this task that you needed to use…..? Could you have used another operation or property to solve the task? Why or why not?

3. Construct viable arguments and critique the reasoning of others.
    - What mathematical evidence supports your solution?
    - How could you prove…..?
    - How did you decide to try that strategy?
    - Did you try a method that did not work out? Why didn't it work?
4. Model with mathematics.
    - What number model could you construct to represent the problem?
    - Would it help to create a diagram, graph, table, …?
    - What are some ways you can visually represent….?
5. Use appropriate tools strategically.
    - What mathematical tools could we use to visualize and represent the situation?
    - What do you know that is not stated in the problem?
    - What estimate did you make for your solution?
    - In this situation would it be helpful to use
        - A graph?
        - A number line?
        - A ruler?
        - A diagram?
        - A calculator?
        - A manipulative?
6. Attend to precision.
    - What mathematical terms apply in this situation?
    - How did you know your solution was reasonable?

- How are you showing the meaning of the quantities?
- What symbols or mathematical notations are important in this problem?
- What mathematical language, definition, or properties can you use to explain…?
- How could you test your solution to see if it answers the problem?

7. Look for and make use of structure.
   - What observation do you make about…?
   - What parts of the problem might you eliminate? Simplify?
   - What patterns do you find in…?
   - What ideas have we learned before that were useful in solving this problem?
   - How does this relate to…?

8. Look for and express regularity in repeating reasoning.
   - Will the same strategy work in other situations?
   - What is going to happen in this situation?
   - What would happen if…?
   - What predictions or generalizations can this pattern support?

Figure 6: Semi-Structured Student Interview Questions (Institute for Advanced Study/Park City Mathematics Institute/Created by Learning Services, Modified by Melisa Hancock, 2013 as cited in Arkansas Department of Education, n.d.)

Each interview was recorded and transcribed for qualitative analysis.

Another instrument used during the course of this study was the ISCRT (Herrera et al., 2010). A licensing fee was paid to utilize the instrument. The teacher had her cultural and linguistic pedagogical responsiveness observed and scored using the ISCRT rubric. A portion of the ISCRT, all that was used to score the teacher, can be seen in Appendix J.

Data Analysis

For this research, a qualitative data analysis technique was utilized. The qualitative data was examined using a compare and contrast analysis method (Creswell, 2018). For the analysis of data, verbatim quotes from participants were synthesized using in vivo codes for cultural and linguistic interpretation of mathematics content. A single class was studied through classroom observations, analysis of student artifacts, and focused interviews of six ELLs. The process of analyzing the qualitative data included meticulously going through the data sets numerous times to ensure thorough familiarity with the context. Data coding simultaneously took place on

Google Docs through font coloring. Data sorting and re-analysis followed to make sure all determined clusters were appropriate. The in vivo codes created clusters according to similarities and differences. During this process and in reflective sessions, the researcher made connections within the records and was able to compile the qualitative data. Once the data collection and sorting was complete, the researcher employed deep analysis of the data for interpretation.

Summary

In summary, this research was conducted through a qualitative case study. It focused on the mathematical perspectives and thought processes of six culturally and linguistically diverse students in a seventh grade middle school mathematics class. With observations, classroom artifacts, field notes, and recorded interviews, the researcher compiled data about the linguistic perspectives of ELLs in mathematics. The following section details the findings of the research.

CHAPTER 4: FINDINGS

This study traced the learning of six students in a seventh-grade mathematics class over the course of a six-week period. The school district assigned each of the six students a level of English language proficiency based on the results of ELPA21 since they were not native English speakers. The data collected came from their classwork, observations, teacher input, and student pre and post interviews. The research and the data highlighted three interrelated categories of instructional methods that impacted the six ELLs' strategy use, sense making, and learning of algebraic concepts over four -week period. The three interrelated categories of results detailed in this chapter include (a) the impact of the teacher's interpretation and implementation of the gradual release method on how mathematics content was perceived by these six students, (b) the impact of the mathematics register on how the content was interpreted, and (c) the disconnect between students' strategies resulting from didactic teaching methods and their strategies used within inquiry contexts.

Description of the Sample

This seventh grade mathematics class was located at a middle school in the Springdale School District, an urban area of the United States. Sixth and seventh grade students attended the middle school. Teachers were expected to teach students using only English. Depending on the classroom, students may or may not have been permitted to converse with each other in their native languages if their first language was not English. In Ms. Roswell's classroom, students could converse with each other in their language of choice. However, all academic correspondence with her had to be in English as Ms. Roswell was monolingual. The following table provides a breakdown of the class demographics based on native language.

Table 2. Number of Students in Sample Class Demographics

Class Demographics, $\boldsymbol{N = 21}$

Language	Student Native Language	Female	Male	Limited English Proficient
Spanish	9	7	2	5
Marshallese	5	0	5	5
English	7	6	2	0

Based on predetermined factors, six focal students from the class of twenty-one were chosen. Table 3 provides individual student information for the class.

Table 3. Individual Demographics for the Class

Pseudonym	Native Language	Gender	ELPA21 Level	Class Achievement Level
Bryan*	Marshallese	Male	2	Average
Robert*	Marshallese	Male	2	High
John*	Marshallese	Male	2	Low
Gina*	Spanish	Female	2	Low
Jacqueline*	Spanish	Female	Long Term 5	High
Josie*	Spanish	Female	Long Term 3	Average
Cassie	Spanish	Female	2	Average
Landon[+]	Marshallese	Male	2	Low
Jared[+]	Marshallese	Male	2	Low
Trudi	Spanish	Female	2	High
Deidra**	Spanish	Female	___	Average
Rosalind**	Spanish	Female	___	Low
Randy**	Spanish	Male	___	High
Dean**	Spanish	Male	___	Low
Cain	English	Male	___	Average
Trent	English	Male	___	Low
Ali	English	Female	___	Average
Mackenzie	English	Female	___	Average
Glenda	English	Female	___	Average
Desi	English	Female	___	Low
Carissa	English	Female	___	High

*Denotes a student of focus. +Denotes students that moved schools during the unit. **Denotes a student was not a native speaker of English and was no longer considered limited English proficient.

To be a student of focus, the student had to have documented ELL status from the school district at the time of the study. These students were identified by the school district as being ELLs through ELPA21, a state standardized assessment on language proficiency level. Eleven of the 21 students were native English speakers or had reached proficient English speaker status and therefore were not considered for in depth case study analysis. All 10 ELLs were pre-assessed through the use of semi-structured interviews involving algebraic equations/expressions word problems (see Appendix H for a complete list of questions). One of these 10 was eliminated because approval to participate was denied. Of the remaining nine students, two additional students were eliminated for further study because they moved during data collection. Of the seven remaining, one additional student showed early hesitancy to verbalize her thinking during the pre-assessment interview and was eliminated from further data analysis. The teacher communicated the mathematical achievement level of each of the students. She divided the achievement levels into the three categories of high, average, and low. This information was based on her assessment of the student's understanding of the material and performance in class. An ELL of each language and of each mathematical achievement level comprised the six students of focus.

The "Sadie and Eric" problem with subsequent questions in Table 4 was utilized for the pre-assessment interview. Ms. Roswell gave the whole class the problem set as part of a pre-assessment for the unit. Each of the focal student's responses was also detailed in Table 4. The strategies of six students: Bryan, Robert, John, Gina, Jacqueline, and Josie were documented in a unit on rational numbers and equations. Appendix K detailed the expectations of the unit. Both paper and pencil and online periodic assessments were given throughout the instructional unit.

The students' strategies/answers on the pre-assessment question were recopied in Table 4 for comparison and analysis. A copy of the actual student work is in Appendix L.

Table 4. Sadie and Eric Problem

Sadie computes the perimeter of a rectangle by adding the length, l, and width, and doubling this sum. Eric computes the perimeter of a rectangle by doubling the length, l, doubling the width, w, and adding the doubled amounts.

1. Write an equation for Sadie's way of calculating the perimeter. Write an equation for Eric's way as well.
2. Use both equations to find the perimeter of a rectangle with width 30.2 cm and length 75.7 cm.
3. Explain why Sadie and Eric always get the same answer, no matter what the length and width of the rectangle are.

Name	Problem 1	Problem 2	Problem 3
Bryan	Sadie: $l + w = x(2)$ Eric: $(l + 2) + (w * 2) = x$	Sadie: $(75.7 + 30.2)2 = 211.8cm$ Eric: $(75.7 * 2) + (30.2 * 2) = 211.8cm$	<u>Because I did backward…..</u>
Robert	S: $p = l + w\ l + w$ E: $l + l + w + w$	Sadie: $75.7 + 30.2 = 105.9 * 2 = 211.8cm$ Eric: $2 * 30.2 = 60.4$ $2 * 75.7 = 151.4$ $60.4 + 151.4 = 211.8cm$	Is because we used 30.2 and 75.7 on both sides and times them by 2.
John	Eric 4 Sadie 4	S 30.2cm+75.7cm=105.4cm*2=211.8 E $75.7 + 30.2 = 105.9 * 2 = 211.8$	Because Sadie and Eric doubled amounts the answer
Gina	Sadie: $P = 2(l + w)$ Eric: $P = (l * 2) + (w * 2)$	Sadie: $30.2 + 75.2)2 = 211.8cm$ Eric: $(30.2 * 2) + (75.7 * 2) = 211.8cm$	Saddie and Eric always get the same answer because the equations your just doubling the length and width

Table 4. Sadie and Eric Problem (Cont.)

Name	Problem 1	Problem 2	Problem 3
Jacqueline	Sadie: $l + w = ? * 2$ Eric: $(l * 2) + (w * 2) =$	$= (30.2 + 75.7) * 2 = 211.8cm$ $= (75.7 * 2) + (30.2 * 2) = 211.8cm$	They got the same answer because in both of the equations you're just doubling the length and width.
Josie	Sadie: $(l + w)2$ Eric: $(l * 2) + (w * 2) =$	Eric: $(75.7 * 2) + (30.2 * 2) = 211.8 + 211.8 = 423.6$ Sadie: $(75.7 + 30.2)2 = 211.8$	They get the same answer because both times by 2 and its tecnecly the thing. Even though Sadie multiply the 2 in the end.

For problem one, the students were asked to write a model to represent Sadie and Eric's thinking. There was not consistency in mathematical symbol usage among the student responses. John did not write down a linear equation with an unknown. Bryan and Jacqueline had similar sense-making tendencies for Sadie and Eric. For Sadie, the two made improper use of the equal sign and used it as a step marker in a sequence rather than in a sense of equality. Robert had a unique representation of Sadie and Eric's thinking. He knew how to calculate perimeter and wrote his understanding of the calculation rather than Sadie and Eric's. Gina and Josie had similar thoughts in modeling Sadie and Eric's thinking and were the most correct for problem one. For problem two, one of the six students made a mistake on numeric fluency for the Eric component of the problem, but did the Sadie component correctly. Five of the six students arrived at the correct answer for problem two although their methodologies for doing so did not match the equations they created in problem one. In problem three, the students were asked to explain their thinking. While not the intention of the exercise, it did highlight some of the students' struggles to communicate their thinking in English. Three of the six students were able to correctly identify through written communication why Sadie and Eric will always get the same answer by using their models. Based on the pre-test, there were some discrepancies between mathematics modeling and sense making for the students.

Background Information on Six Students

In addition to the pre-unit interviews with the six selected students, Ms. Roswell provided additional narrative information on each child both in writing and verbally for questions given to her. She noted that she had ongoing discipline problems with Bryan. He was seated next to and relied heavily on Robert for translations. By translations in this case, Robert would take the English phrase and replace it with other, usually lower level, English phrases with similar meaning. Robert came from a prominent family in the Marshallese community and his parents had assimilated more to typical American social norms in education than the other Marshallese parents had per Ms. Roswell. John sat across from Robert. John had a laissez-faire attitude toward school consistent with the stereotypically easy-going island lifestyle of the Marshallese. In class, it was observed he waited for his peers to work through a problem before he would attempt it. The three boys all had the same English proficiency level of two. However, their comfort level in speaking was observed to be noticeably different. Bryan would speak to his Marshallese peers regularly, but not the other students unless placed in groups with them. Robert was observed to be the most social of the three and the most willing to take risk in verbal transactions. He did not hesitate to try to pronounce words in class and would repeat words when corrected. When Bryan, Robert, and John would converse, it was observed to be in lower level, conversational English, with Marshallese words periodically dispersed within the dialogue.

The three Hispanic female students of focus sat together. Ms. Roswell shared that Gina had a language proficiency level of two, Jacqueline was a long-term level five, and Josie was a long-term level three. When the three girls worked together, their discourse was almost exclusively in Spanish. Ms. Roswell noted that Jacqueline would speak on behalf of the other girls in English to the teacher or the rest of the class during mathematical discussions. Ms. Roswell was unaware Jacqueline was considered an ELL until she pulled up her student file. Jacqueline was proficient in spoken English language. Reading and writing in English was what kept her from achieving native speaker proficiency. Josie had plateaued in her acquisition of English at a level three for some time and this seemed to be correlated to her consistent placement in lower level core courses. Gina had the lowest English proficiency of the three girls and she spoke the least voluntarily.

Theme One: Socio-mathematical Norms and Gradual Release Instructional Strategy

The teacher typically began the class with a bell ringer or warm up activity in which the class sat on the floor close to the board to discuss the problem. This activity usually took five to ten minutes. Following that activity, the students would return to their desks for approximately twenty to thirty minutes of instruction. For the remainder of the class, the students would practice what was presented to them in small groups as an assignment to be turned in before the completion of the class period. Nearly every day for the six-week observation followed that instructional classroom model.

Students were expected to collaborate with each other and reach a consensus on the mathematics. Ms. Roswell would walk from group to group to check in on the students and make corrections as necessary. Before answering any student questions, she would ask the student if s/he had conferred with at least two other peers before asking her a question. If the student's answer was no, Ms. Roswell would direct the student back to the group at hand.

Teacher Knowledge. Ms. Roswell was a licensed educator in the areas of middle school mathematics, science, language arts, social studies, and elementary education at the time of data collection. She had been a teacher for seven years at the time of this research, all within the same school and district. During this study, Ms. Roswell was working towards earning her English as a second language (ESL) endorsement on her teaching license and furthering her ELL educational knowledge. She had completed two of the four required graduate level courses in Arkansas towards an ESL endorsement. She was halfway through the third course and was set to enroll in the fourth. The fourth course would conclude the required courses for the series per state standards. Ms. Roswell took her education a step further and volunteered to participate in an optional fifth course about ELL education. The district Ms. Roswell worked within has had a high ELL population for several years. Because of that, the district also provided extensive professional development for all of its teachers geared toward classroom best practices for ELLs. Ms. Roswell participated in that professional development.

Mrs. Roswell utilized Constructing Meaning© questioning systems with her students during the unit to elicit student responses. Constructing Meaning© provided teachers with a process and tools for weaving explicit language instruction into teaching, such as through the use of language frames, and was based on backward design and a gradual release of responsibility model (E.L. Achieve, 2013). Ms. Roswell gave the students language frames to help them communicate in an academic style. An example of language frames Ms. Roswell gave to the students can be seen in the following figure. Evidence of the students' attempts to use these language frames were seen in their answer to the third item on their pre-test and in their submitted work.

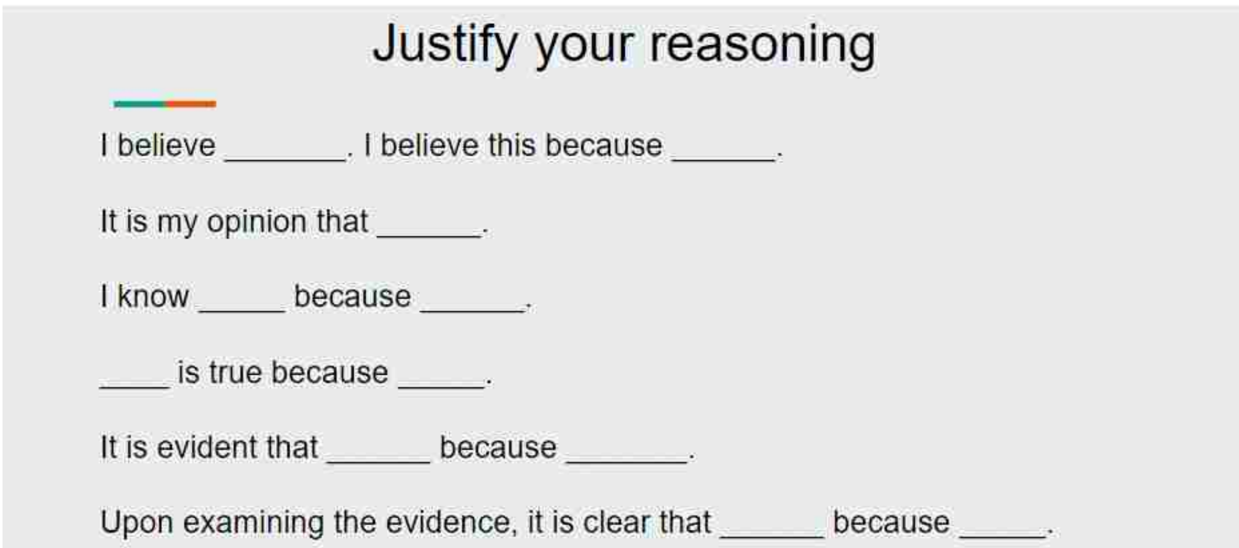


Figure 7. Language Frames

Ms. Roswell also implemented a form of the “gradual release” model of instruction. Ms. Roswell would model her thinking of how to solve the problems for the students during whole class instruction. Next, the students would work in pairs or small groups on a parallel problem while Ms. Roswell walked around the room to answer questions the students had. Then, students had to complete an individual assignment with similar problems to what they had been shown. Ms. Roswell attempted to follow the eight standards of mathematical practices as put forth by the National Council for Teachers of Mathematics. She exposed her students to the standards by the having them posted around the room for students to reference. She expected her students to look for and make use of structure in their problem solving strategies. Ms. Roswell had established socio-mathematical (Tatsis & Koleza, 2008) norms for her classroom by following those two methodologies. Socio-mathematical norms were ‘normative aspects of mathematics discussions specific to students’ mathematical activity’ (Yackel & Cobb, 1996). Examples of socio-mathematical norms were the understandings of what counts as mathematically different, sophisticated, efficient and elegant (Yackel, 2001) in her student's work. The mathematical methodologies modeled by Ms. Roswell were transferred and utilized by the students as evidenced in their submitted work. They students' submitted work were copies of the equation formats shown to them.

Keyword Strategies in Solving Word Problems. To practice word problem literacy, Ms. Roswell gave the students a set of three or so word problems to solve daily. She had the students read through the problems first. During their reading, the students had to underline or circle words or phrases they or a classmate may not know. The students would then share out to Ms. Roswell and the class the vocabulary they chose. These were often a mix of mathematics register words and Standard English vocabulary. Ms. Roswell had the students discuss in their small groups what they thought the vocabulary meant and the groups came to a consensus. The groups then shared out what they thought and Ms. Roswell made corrections or additions as needed. After that round of teaching vocabulary for the word problems, Ms. Roswell then taught the mathematical content. She placed emphasis on mathematical register specific vocabulary and expressly taught this using direct instruction to the students. The students could not properly set up an algebraic representation of the word problem without being able to fully comprehend the English process or comparison words in the scenario.

In addition to pre-teaching vocabulary, Ms. Roswell taught the students to pick out keywords from the word problems. For example, she told the students specific words equated to specific operations. The six students tended to read the problems at least twice, usually more than that if there were unfamiliar words, before deciding which parts of the problem were needed to answer the question. Underlining or circling keywords in a problem and making notes in the margins about what the words meant was taught to be normal classroom practice. This strategy was seen consistently in Gina, Jaqueline and Josie's submitted work. All of the students were successful in identifying relevant information. However, numeric distractors or phrases were not utilized in the word problems at that point in time. This eventually caused tension between the didactic nature of the equations unit and the ELLs' mathematical sense making. As demonstrated by the posttest responses in the following figure, if the keywords the students were accustomed to in class were removed from the problem or used in a different way from what they had been shown, the students were unable to perform the mathematical task.

Pre and during instruction the tasks were presented with intermediary steps that focused on setting up equations with an unknown. Over emphasis on translation activities in favor of problem solving caused additional challenges for ELLs in mathematics. For example, their linguistic understanding of what was happening in the problem influenced how ELLs processed the mathematical problems. The students were interviewed as part of this research. During the interviews, the six students were given mathematics problems to work. The researcher requested the students talk aloud as they solved the problems. The questions given to the ELLs during the posttest did not follow the approach presented to the students in class and the students struggled to make sense of the problem. One of the questions given to the students was: *Kai is 4 years younger than half Abigail's age. Kai is 12 years old. How old is Abigail?* The correct answer was meant to be 32 years old.

Table 5: Kai and Abigail Problem

Kai is 4 years younger than half Abigail's age. Kai is 12 years old. How old is Abigail?

Pseudonym	ELPA21 Level	Dialogue
Robert	2	Robert: Kyle, right? Kyle is four years younger than Abigail's age. Kyle is 12. How old is Abigail? Twelve…so eight, nine, ten, eleven, twelve. She's eight. Researcher: Is that what you think? All right, show me how you got that. Robert: 'Cause…well I did eight plus four is twelve. And it said that Kyle's twelve, Abigail is four years younger than…half Abigail's age. Oh.
Josie	Long Term 3	Josie: Kyle is four years younger than Abigail's age. He is 12 years old…so there's 12 divided by two is six, then you minus six from four, that equals two…two? Researcher: Who is two? Josie: Abigail?
Gina	2	Gina: Kai is four years younger than half Abigail's age. Kai is 12 years old. How old is Abigail? Researcher: So, what are the important parts of that? Gina: That Kai is four years younger than half of Abigail's age. Researcher: Right. Gina: And, Kai is…Kai is 12 years old. So, I will do…four subtract 12. Wait. Wait. Four minus…Wait, four plus six. Researcher: How'd you get that? Gina: 'Cause I don't know, it says younger than half. So, in my head, I did 12 divided by two and got six. Researcher: Very good. Gina: So I thought I'd do four plus six equals 12. Then, I will do 12 minus six… Researcher: Yeah? Gina: I get six. Researcher: Good. Gina: Then, I'd do six divided by four, and I get 1.5. But Abigail can't be 1.5 years old, so it's closer to two, so she would be two years old?

For the “Kai and Abigail” problem, the students understood the individual words of the mathematical question, but had difficulty translating the grouping of the words or the phrases due to the syntax of the content in English. It must be considered whether and to what extent mathematical assessment questions were evaluating mathematical knowledge versus knowledge of English.

Teacher Social and Emotional Competence. Ms. Roswell took conscious strides to account for the affective filter of her students using various methods. For example, Bryan was homeless during the course of this study. Ms. Roswell kept school supplies, snacks, and small toiletries in her classroom for him to use discreetly if he needed them. Ms. Roswell greeted students by name as they walked into the door every day, celebrated their birthdays monthly, and took the time to learn and speak some phrases and words in the students’ native languages. The Marshallese and Hispanic cultures were collectivistic and this was complementary for the norm of students working in groups. Ms. Roswell usually did intentional grouping in her classroom with students being placed heterogeneously or homogeneously based on language ability, mathematical problem solving style, mathematical ability, cooperativeness, and/or work style. On occasion, students were instructed to choose with whom they wanted to work with that day. The grouping strategy varied depending on the type of activity being implemented and the type of product the students were expected to be able to produce. By the end of the class session, every student was expected to speak at least once. Ms. Roswell set a mutually respectful tone in her classroom through leading by example and verbalizing her expected social norms to the students. She consistently enforced those social norms to the students and the students adhered to them during the unit.

ISCRT. As part of the research process, an attempt to quantify the qualitative interactions between cultures and implementations of cultural responsiveness was assembled. Ms. Roswell was scored on her cultural responsiveness in the classroom using the ISCRT scoring rubric© as developed by Socorro Herrera et al. (2010) Kansas State University, CIMA. Ms. Roswell's indicated scores were located in Appendix J. A score of four was the most desirable in each of the twenty-two indicators.

Based on her ISCRT scores (See Appendix J) during one observation, Ms. Roswell had some indicators of a culturally responsive teacher for her students. ISCRT was broken down into twenty-two indicators that were grouped into five standards. The five standards were Joint Productive Activity, Language and Literacy Development, Contextualization, Challenging Activities, and Instructional Conversation. Ms. Roswell had an average score of 3.00 in Joint Productive Activity, 2.75 for Language and Literacy Development, 2.67 in Contextualization, 3.40 in Challenging Activities, and a 4.00 in Instructional Conversation. This was a scheduled observation with a coach that had worked with her on using EL strategies. Her composite ISCRT score was 3.16 putting her in the favorable category of a five point Likert scale with four being the highest possible score.

Summary of Theme One. Ms. Roswell, a seventh grade mathematics teacher, had an influence on the classroom environment. She employed a form of gradual release for her instruction style in which she modeled for the students how to solve the problems using an equation or inequality. While solving the problem, Ms. Roswell would point out what she deemed as keywords and equate them to operations or positioning of variables within an equation or inequality. Ms. Roswell would then have the students work in pairs or small groups to try to solve a problem similar to the one they had been shown. She gave the students language frames to help the ELLs communicate during discourse. It was evident she cared for her students by the way she interacted with them. She facilitated a welcoming environment for the six ELLs.

Theme Two: Language of Mathematics

Galileo said, "The great book of nature can be read only by those who know the language in which it was written. And this language is mathematics." The language of mathematics for this study was broken down into three components. Those components were the mathematics register, sentence structure, and mathematical symbols. Each idea affected the mathematical discourse of Bryan, Robert, John, Gina, Jacqueline, and Josie.

Mathematics Register. The mathematics register consisted of the technical language specific to the discipline. The unit involved several Kahoot© quizzes in which students had to read a mathematical expression and translate it into English. All six students struggled with these tasks. For example, a problem like $8y^3$ required students to write it as, “The product of eight and y cubed”. The six students would typically write, “Three times eight times y”.

Another example of the influence of the technical language aspect occurred early on in the instructional unit on the Hamster problem. This problem required students to interpret, write and, solve an inequality problem. Five of six students set up an inequality that was modeled for them by the substitute teacher they had on that particular day as shown in Jacqueline’s work in figure 9. However, Bryan only wrote an expression. He also rewrote other inequalities on that page as equations with numbers only, highlighting the challenge of ELLs to make sense of the purpose of variables.

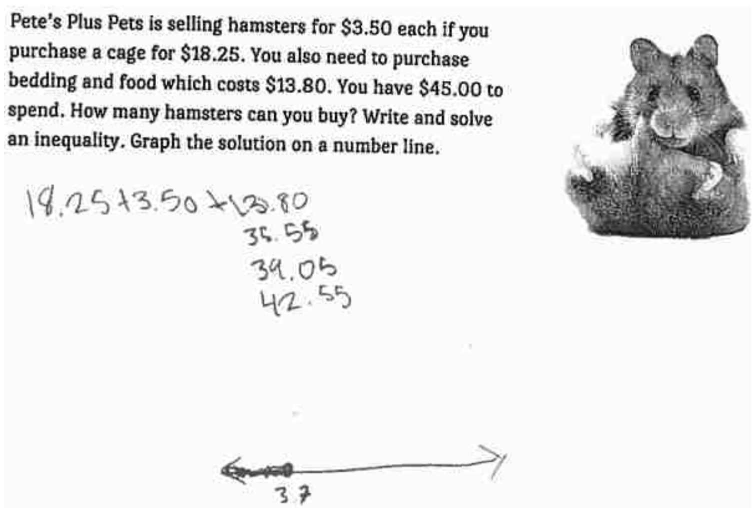


Figure 8. *Bryan's (ELPA21 Level 2) work sample discussing the thinking behind the mathematics*

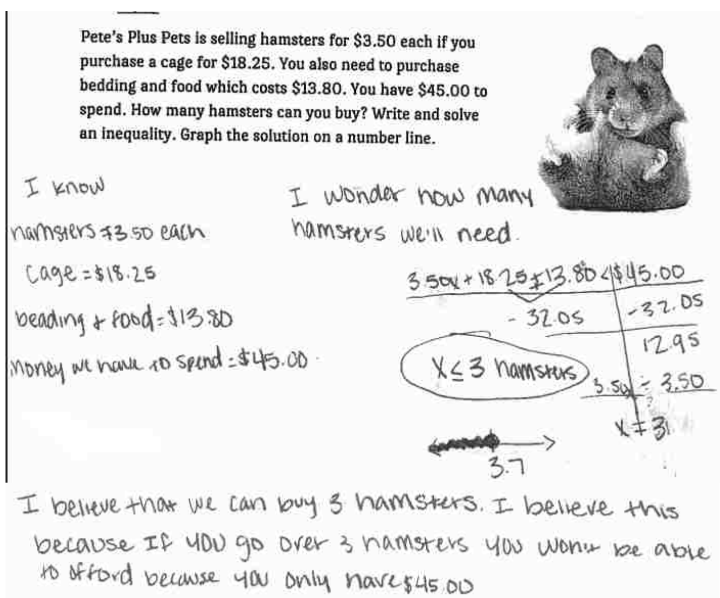


Figure 9. *Jacqueline's (ELPA21 Level Long Term 5) work sample discussing the thinking behind the mathematics*

Some of their confounding issues with the mathematics register were masked throughout the instructional unit because Ms. Roswell typically went through a gradual release model example first of how to set up equations and inequalities, and then they did a similar example on their own. For example, Jacqueline and Josie's work looked nearly identical (Figure 10). Five of the six ELLs had similar work. All of the students were told to mark out the first problem in that problem set because they had not been shown an example of how to do a problem exactly like problem one.

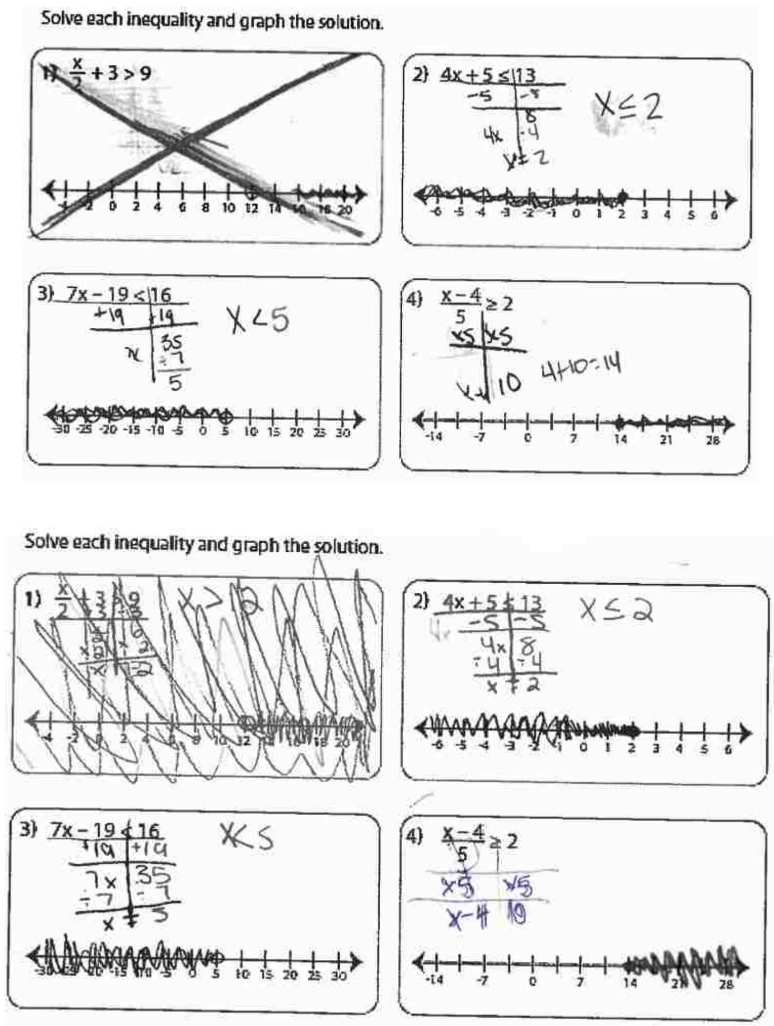


Figure 10. Comparison between Jacqueline and Josie's work

Sentence Structure. The idiosyncrasies in the proper syntax of the English language provided an irregular translation pattern into mathematical sentences. Mathematics was more than just a collection of numbers. The meaning behind the numbers and how the numbers, symbols, and text were ordered communicated specific meaning through mathematical language. There were signs the syntactic structure of English convoluted the translation of the mathematics between the native speakers of the Marshallese language Bryan, Robert and John and the native speakers of Spanish Gina, Jacqueline, and Josie. Each of the native languages had grammatical distinctions from English.

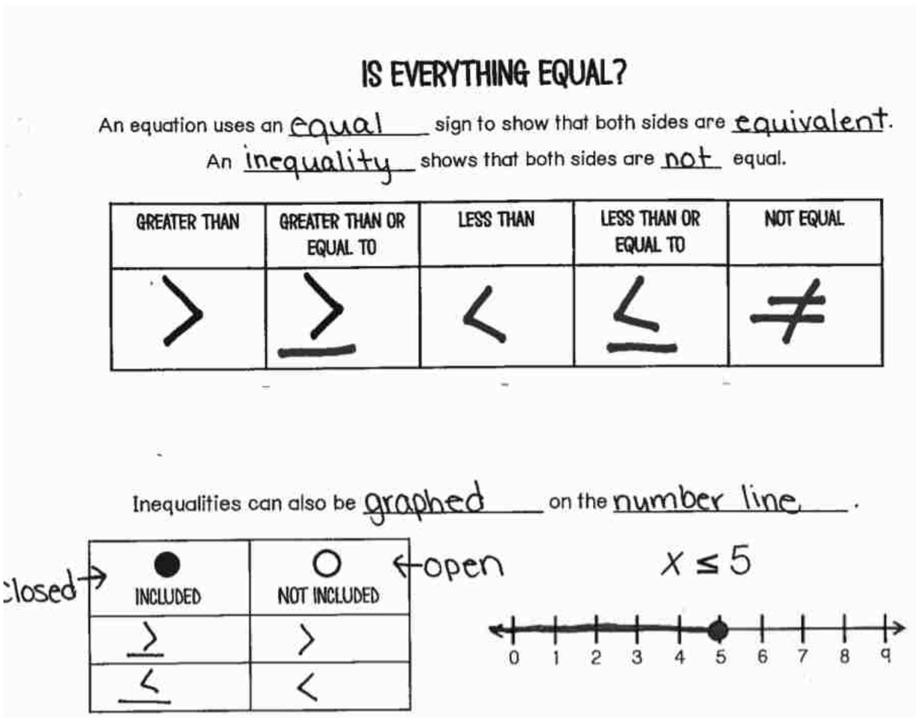
Ms. Roswell practiced a spiraling technique in her educational pedagogy. Spiraling was a term used to illustrate the resurgence of previous content periodically to help with content retention in students. The warm-up activity or bell ringer task was generally a review of the previous day's work. One of the warm-up activities given to students during the observed unit was to translate the following phrases into algebraic expressions or equations as shown in the list below.

- A number divided by six is thirty-two
- The sum of twelve and a number is twenty
- Five more than ten times a number
- Four less than twice a number is ten
- A number squared minus eight
- The quotient of twenty and a number is ten
- The product of negative three and a number is twelve
- A number squared minus ten
- A number increased by eight is negative three
- Nine less than a number is fifteen

Through an educational technology tool called Kahoot.it©, the students had a quick formative assessment over those phrases. The students and teacher had an immediate visual representation of the content the class as a whole retained from the previous material in the way of a summary of responses after each question. The students had an 80-90 percent correct response rate on most of the items. On the day this activity took place, the mathematics register vocabulary was not the issue as the students correctly chose the operation taking place (i.e. quotient, product, sum, etc.) consistently. The ELLs had an issue when the order of the mathematical operation-taking place was different from the word order given. In other words, a word-by-word translation did not work due to the structure of the sentence. For example, "four less than twice a number is ten" caused some debate amongst the ELLs. The students wanted to write down $4 - 2x = 10$ or $4 < 2x = 10$ rather than the intended $2x - 4 = 10$ because an attempt to do word-by-word translations. All six students quickly ruled out the $4 < 2x = 10$ because they had not seen those symbols put together in that manner previously and this was meant to be a review. The students were debating whether $4 - 2x = 10$ or $2x - 4 = 10$ was correct. John, Bryan, and Gina thought $4 - 2x = 10$ was right while Robert, Jacqueline and Jose argued for $2x - 4 = 10$. When the mathematical sentence was structured in chronological order in both English and mathematically, the ELLs did correctly and consistently perform a word-by-word translation. In other words, when the word order followed the order of operations the students were successful.

Mathematics Symbols. Students were pre-taught vocabulary and symbols using direct instruction. As part of the instructional strategy, students were given a mathematical symbols notes page as seen in the following figure. This notes page contributed to some mathematical misunderstandings by perpetuating some misconceptions about inequalities. For example, the statement "An inequality shows that both sides are not equal" was contradicted by the table of symbols directly below it showing a possibility of equality.

Ms. Roswell deliberated through each symbol and collaborated with the students a list of potential vocabulary words that could equate to the math symbol. This list was not an exhaustive list, but a representation of the most common words the students would see when translating symbols to words or decoding words to symbols in class that day and during the unit. The students frequently referenced this notes page for the rest of the unit and carried it around the classroom with them to various stations to solve mathematical inequalities. The problem with this occurred when the ELLs assumed that if a word or phrase was listed under a particular mathematics symbol, that word or phrase always went with that particular symbol. For example, the students equated the phrase "less than" with the mathematical symbol $<$ when it could have also been associated with subtraction for that unit depending on the context.



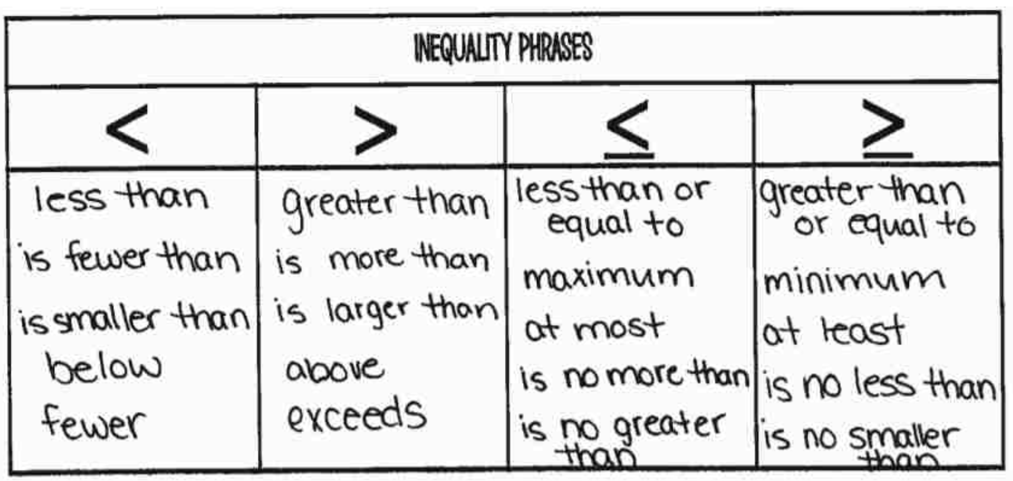
IS EVERYTHING EQUAL?

An equation uses an equal sign to show that both sides are equivalent.
An inequality shows that both sides are not equal.

GREATER THAN	GREATER THAN OR EQUAL TO	LESS THAN	LESS THAN OR EQUAL TO	NOT EQUAL
$>$	$\geq$	$<$	$\leq$	$\neq$

Inequalities can also be graphed on the number line.

closed → ● INCLUDED	○ NOT INCLUDED ← open
$\geq$	$>$
$\leq$	$<$



INEQUALITY PHRASES			
$<$	$>$	$\leq$	$\geq$
less than	greater than	less than or equal to	greater than or equal to
is fewer than	is more than	maximum	minimum
is smaller than	is larger than	at most	at least
below	above	is no more than	is no less than
fewer	exceeds	is no greater than	is no smaller than

Figure 11. *Teacher copy of notes pages students and the teacher collaboratively filled out*

During this collaboration process, the students had to read the words, draw the symbols, write down the English phrases said to them, and say the words and symbols aloud. The students were using all four modalities (reading, writing, speaking, and listening) to acquire understanding of the mathematics symbols.

Summary of Theme Two. The discourse for the classroom predominately consisted of the language of mathematics. The mathematics register or the technical language associated with the discipline, along with the sentence structure and mathematical symbolism communicated through English proved to be problematic for the ELLs. The six students were prone to errors when translating word problems to algebraic equations when word-by-word translations were used as opposed to grouping based on mathematical structure.

Theme Three: ELLs' Responses to Didactic and Inquiry approaches

Novel Situations. While masked during traditional instructional episodes, the six students' problems with translation activities were revealed during the post unit interview utilizing three prompts. Even though two of the tasks were similar to the pretest and problems presented during instruction, none of the six students solved the posttest problems correctly. The third item was meant to be an investigation of what the students would do with a type of problem they had not seen before nor given any additional supports. Table 5 details each of the student's responses to the first two posttest items.

Table 6. Question 1 and 2 of the Posttest

Student Posttest Problems

1. Ricky's dad called an electrician who charges $50 to travel to a customer's home and $75 per hour of labor completed. The electrician traveled to Ricky's house and completed 3 hours of labor. What was the total amount of money the electrician charged Ricky's dad?
2. Kai is 4 years younger than half Abigail's age. Kai is 12 years old. How old is Abigail.

Name	Problem 1	Problem 2
Bryan	75x÷50=(1.5)	12−4=8÷4=(2)
Robert	3x+75=50 −75 −75 3x −25 ÷3 ÷3 x= 8.25 amount of money 75 −50 25	1/2x+4=12 −4 −4 1/2 · 8 ÷1/2 x= [illegible] abigail is 4 years old 8÷2 =4 8/2 =4
John	3x+75=50 −75 −75 3x 25 ÷3 ÷3 x=8.3 total	K 4 year now Kai 12 A 12+2=14 year
Gina	3x+50=75 −50 −50 25 25 ÷3 ÷3 x=$8.33	4+6=12 −6 −6 6 6 x=1.5 nearest 2
Jacqueline	3x+50=75 −50 −50 25 3x ÷3 ÷3 x=$8.3 x=$8.3	Chose to Answer Through Dialogue
Josie	3x+50=75 −50 −50 3x 25 ÷3 ÷3 x=8.30	12÷2=6−4=2

For problem one, students were asked to determine how much money Ricky's dad was charged for the electrical work that he wanted done. All six students set up a linear equation. However, none of them correctly modeled the scenario with their equation. Through the gradual release model, the students had learned the smallest number goes next to the variable and the total was not shown to them as a possible variable. In their verbalized dialogue, the students expressed they knew the answer was not supposed to be negative so they set the equation equal to 75 instead of 50. In question two, the linguistics challenged the mathematics modeling as supported by the variety in problem solving strategies. Five of the six students calculated Abigail's age to be younger than Kai's and did not attend to precision by questioning that outcome based on the initial question. None of the students calculated Abigail's age correctly.

Student Language Proficiency. Ms. Roswell made efforts to accommodate for assorted language proficiency levels in her classroom. She regularly used and had posted around the room language frames to help students communicate in an academic style. Periodically during the unit, the students were presented with word problems and asked to underline or circle words they did not fully understand or words their friends might not understand. Ms. Roswell would then list out and discuss the words or phrases the students indicated they struggled with to the whole class. Table 6 listed out the words from the Sadie and Eric problem the students marked. Their markings indicated what words they did not understand or words their friends might not understand. Their language level according to ELPA21 and native language was also indicated. Josie and Jacqueline seemed to have indicated the most words even though they have the highest two English proficiency levels.

Sadie computes the perimeter of a rectangle by adding the length, l, and width, and doubling this sum. Eric computes the perimeter of a rectangle by doubling the length, l, doubling the width, w, and adding the doubled amounts.

Table 7. Student Indicated Vocabulary Difficulties

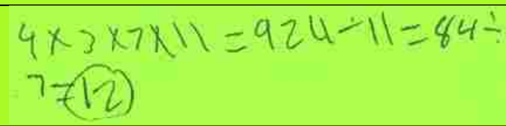
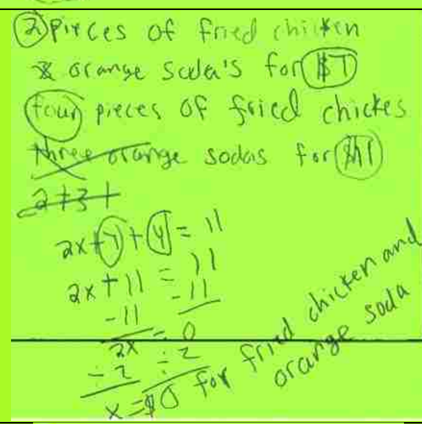
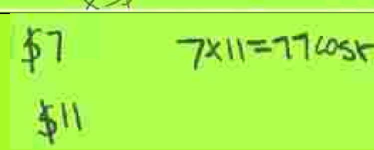
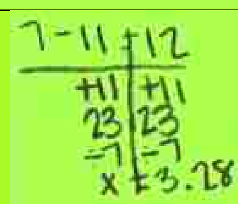
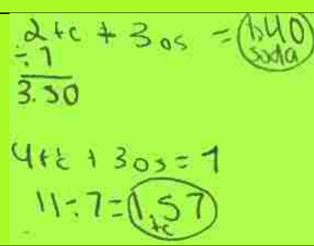
Pseudonym	ELPA21 Level	Vocabulary Words/Phrases
John	2	Perimeter, doubling, adding the doubled amounts
Robert	2	Computes
Bryan	2	Computes, sum
Gina	2	Adding the length, width, doubling, doubling the length, doubling the width, adding the doubled amounts
Josie	Long Term 3	Adding the length, width, doubling the sum, perimeter, rectangle, doubling the length, doubling the width, adding the doubled amounts
Jacqueline	Long Term 5	Adding the length, width, doubling the sum, doubling the length, doubling the width, adding the doubled amounts

Problem Three. Problem three was given to the students during the posttest unit interview to investigate what the students would do with a problem that had not been modeled for them. The problem was posed with an inquiry approach. In other words, no pre-teaching of any additional mathematical vocabulary took place nor had the students encountered a system of equations at that point. The following figures outlined the six students' responses to problem three and detailed Jacqueline's dialogue as she reasoned through the problem.

Table 8: Posttest Question 3 Results

Student Posttest Problem 3

A couple of friends wanted to get something to each while at a festival. They bought two pieces of fried chicken and three orange sodas for $7. Later, another group of friends bought four pieces of fried chicken and three orange sodas for $11. How much did each piece of fried chicken and each orange soda cost?

Name	Problem 3
Bryan	 4x3x7x11=924÷11=84÷7=(12)
Robert	 (2) pieces of fried chicken 3 orange soda's for ($7) (four) pieces of fried chicken three orange sodas for ($11) 2+3+ 2x+(7)+(4)=11 2x+11=11 -11 -11 2x=0 ÷2 ÷2 x=$0 for fried chicken and orange soda
John	 $7 7x11=77 cost $11
Gina	 7-11 = 12 +11 +11 23 23 -7 -7 x = 3.28
Jacqueline	Answered Through Dialogue
Josie	 2fc + 3os = (6.40 soda) ÷7 3.30 4fc + 3os = 7 11÷7=(1.57 fc)

Jacqueline: A couple friends wanted to get something to eat while at a festival. They bought two pieces of fried chicken and three orange sodas for $7. Later, another group of friends bought four pieces of fried chicken and three orange sodas for $11. How much did each piece of fried chicken and each soda cost?

Maybe do seven divided by two to get the orange sodas, 'cause there was ... or three, 'cause there was three orange sodas. And then I would get a decimal, so maybe if you do that

Wait. Yeah, 2.3, so that must be 2.3, and maybe also divided by two because of the two pieces of fried chicken. That would get you another decimal. Then you add them up and it would be $7. Then for this one you would do the same. So the two pieces of fried chicken would be 3.5.

Then the three soda would be 2.3 and then you could do how much the three of them costed divided by seven to get the answer for each one of them. Wait. For this one you would round, right?

It would just be 3.4. $3.4.

Then for the 11, and then the four fried pieces of chicken it'd be $2.75, and for the sodas it would be a decimal. That's too many numbers so I think you would have to round.

It would be $3.07.

Then for how much did each piece of fried chicken and I think ... Does it say individual, or does it say all together?

All together. For both of them? I feel like you should add these two just so that you can know the price. It would be easier.

And then add the two and the four?
So it would be $3. And for the sodas it would be 1.2.

Figure 12. Jacqueline's Dialogue for Problem 3

Despite having no experience with a system of equations, three of the six students intrinsically knew there were two different equations simultaneously occurring within the problem and two of those three attempted to solve the system as an algebraic system. The students had not had that type of problem modeled for them in advance of them having the opportunity to make sense of it. It also involved a mathematical structure not experienced during the instructional unit (system of two linear equations instead of one linear equation). The fact that three of the six students recognized that the situation involved two unknowns demonstrated some evidence of sense-making and attempted to deal with the two unknowns simultaneously which is required for solving systems of equations.

Gradual Release to Didactic. Theme three outlined the differences in how the six ELLs approached solving word problems based on whether a didactic method had been used versus an inquiry method. Gradual release has been characterized by practicing teachers as "I do, we do, you do". In content areas outside of mathematics, this interpretation may in fact be successful for ELLs (Daniel and Pray, 2017). However, using this interpretation in mathematics was, for Mrs. Roswell, interchangeable with long held traditional didactic methods that emphasize showing students a mathematics strategy, having them practice it with the teacher's support, and then having students practice the same method independently. Mrs. Roswell used an interpretation of gradual release that was interchangeable with traditional didactic methods to teach mathematics.

Overall Impact on the Six ELLs

Bryan, Robert, John, Josie, Gina, and Jacqueline were ELLs in a seventh grade mathematics classroom. Their trajectory through a unit on expressions and inequalities was complicated by three different factors. First, Ms. Roswell created a classroom that was flexible and comfortable for all learners. The implementation of tasks was direct with specific instructions on how to represent keywords and phrases and expressions and equations from word problems. When six students were given tasks to solve immediately following the modeling of similar tasks by Ms. Roswell, their responses were mostly correct. However, when students were asked to solve a word problem on the post assessment which was not directly modeled for them, all six students failed to translate the word problem into an appropriate equation and therefore could not correctly solve the problem. Acquiring the language of mathematics in synchronicity with the English language was complicated due to the intricate nature of both. For example, on the first problem in the posttest about the electrician, the students appeared to set up an equation in which they were solving for the hourly rate rather than the total amount the electrician charged for services. For these six ELLs, didactic instruction through modeling first and direct translation activities did not support "making sense of problems and persevere in solving them" as the Common Core State Standards intended (2010).

CHAPTER 5: DISCUSSION, IMPLICATIONS, RECOMMENDATIONS

The research question used to guide the analysis was: What perspectives do English language learners (ELLs) bring when engaged with algebraic expressions and equations content? Analysis of data indicated that the six target students had some success with translation activities and solving word problems when supported by direct instructional strategies involving how to set up equations/inequalities, but struggled to apply those strategies during the post unit interviews when similar problems were posed without directions to set up intermediary steps. These results suggest that gradual release methods that require students to set up algebraic equations based on keywords and previous translation activities did not appear to support success in problem solving. These methods appeared to overshadow the potential for sense making activities related to the context and quantities in the problems. The findings from this case study provided insight and furthered previous findings by specifically examining Hispanic and Marshallese students. Implications for each theme are described in the following paragraphs.

Theme One: Structured Student talk is limited in Didactic Environments

Teacher Knowledge. Ms. Roswell had the necessary qualifications to teach a middle school mathematics class in Arkansas, she was also actively seeking additional training to better serve her culturally and linguistically diverse students. Ms. Roswell utilized her previous teaching experience to inform her instructional strategies. She knew where the students were likely to struggle and spent additional instructional time over those topics. The Constructing Meaning© instructional methodology and Ms. Roswell's demeanor helped these students feel competent enough to try the problem sets, question each other's procedures, and try again if necessary. The way Ms. Roswell structured the class was conducive to students debating with each other about the mathematical solving strategy used when solving problems.

It was problematic to determine ELL difficulties based on student work during the unit due to the nature of how the work was submitted. The didactic teaching approach meant learners sat passively while the teacher demonstrated how problems were solved (Ledibane, Kaiser, & van der Walt, 2018). Each problem type was modeled for students before they had a chance at attempting to make sense of it on their own. In addition, the teacher had a built in check system in which the students could check their answers for correctness before the work was turned in. In other words, mathematical misunderstandings were masked through parroting or copying. For example, consider Jacqueline's work during the unit and posttest.

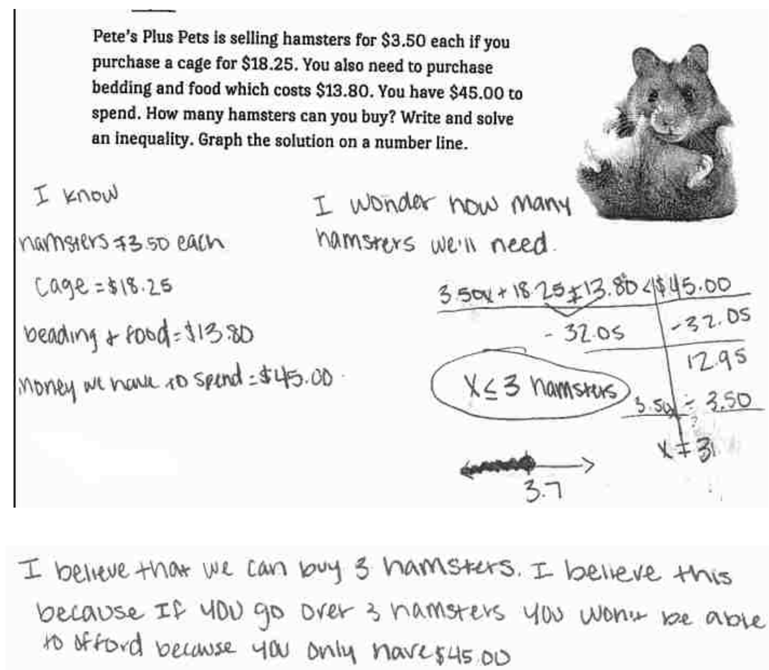


Figure 13. Jacqueline's hamster problem

Jacqueline was considered a high performing mathematics student by Ms. Roswell and Jacqueline performed well on the hamster problem. An example like the hamster problem had been modeled for her before she attempted to solve this one. Also in the directions of the hamster problem, Jacqueline was told or in this case reminded how it was intended she solve the problem by using an inequality and graphing the solution on the number line. When those supports were not utilized like in the posttest, Jacqueline did not make sense of the problem correctly.

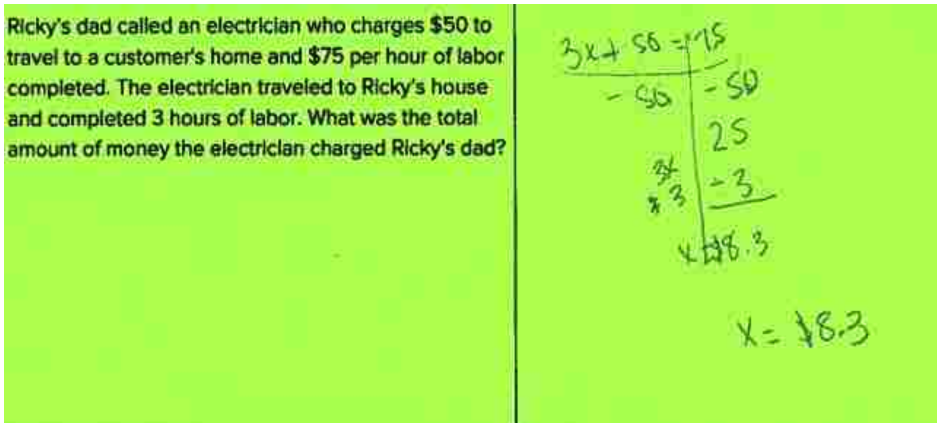


Figure 14. Jacqueline's posttest electrician problem

The idea that Jacqueline understood how to solve a problem similar to the hamster problem was challenged in the posttest. Jacqueline incorrectly set up her equation to model the scenario and instead repeated what she had been shown to do. In this instance as well as for the other five students in this case study, posttest items were solved incorrectly perhaps due to the tension between how the gradual release model was implemented and making sense of problems.

The quality of teacher knowledge had implications in the classroom. A qualified, experienced mathematics teacher seemed available to the ELLs. Ms. Roswell asked the students questions about their mathematical procedures to help the students get to the intended mathematical outcome and she provided linguistic supports for the students. These students had a history of struggling in mathematics and appeared to be closer to being on par with their peers during this unit. However, she focused heavily on language acquisition and mathematical procedures perhaps to the detriment of mathematical understanding.

Keyword Strategies in Solving Word Problems. This particular unit was heavy on mathematical content in written context rather than simply expressions or equations to be solved by a standard algorithm. The strategy of using keywords was used to highlight the essential vocabulary for the unit and lower the amount of vocabulary within the mathematical content. The teacher, Ms. Roswell, practiced the pedagogical strategy of pre-teaching vocabulary, or in this case keywords when problem solving. The students as a result would look for those particular words in their problem sets in order to figure out what was expected of them to answer the questions and essentially disregarded the rest of the information. Ms. Roswell used explicit instruction on what the mathematical register phrases meant in terms of an expression or equation. Issues did arise for both the Hispanic and Marshallese students if a word or phrase had not been explicitly taught or if the word or phrase could have more than one meaning depending on the context. The students were successful in identifying relevant information. However, distractor information was not utilized. The students did not consistently interpret the relevant information correctly.

Based on the findings, it appeared the students were more focused on trying to set up an equation/inequality than solving the problem. They appeared to be reading to find the numbers and variables needed to set up the equations rather than seeking actual meaning from the word problem. This could imply the need for more instructional focus on sense-making related to the context of the problem. Moreover, traditional didactic methods that focus on modeling a particular method provide limitations to the usefulness of student discourse. While students were given opportunities to discuss their strategies, the substance of the mathematical conversations was focused on the level of correctness of the work based on what was modeled for them rather than comparing/contrasting different approaches to solving the same problem.

Teacher Social and Emotional Competence. As observed through the students' engagement and body language, this classroom was an emotionally safe place to intellectually grow. It was observed the students worked through the problem sets, asked questions if they thought they were needed, and implemented modeled problem solving strategies. Another study's findings furthered the theoretical assumptions in the literature that supportive social relationships influence achievement through motivational and affective pathways (Ahmed, Minnaert, van der Werf, & Kuyper, 2010). The classroom atmosphere encouraged the students to want to perform well as evidenced by the students submitting their work in the way it was modeled for them. No particular student dominated the classroom conversation and by the end of the class, every student had a chance to share their ideas about applying the mathematics content to the tasks. The majority of the classes during the observations were spent with students in collaboration with each other talking through the procedures of how to solve the modeled problem sets.

The groups were chosen with intention for a variety of reasons ranging from English language proficiency level, mathematical proficiency, and ability to work productively together. Other times, the students had to choose whom they wanted to work with or to work individually. Through student on task behavior, collaboration, and body language, it was apparent the students had respect for each other. The researcher thought the explicit instruction by the teacher to the students of how to appropriately interact with each other and consistent enforcement of norms contributed to the students understanding of appropriate social interaction. The collaborative atmosphere was also consistent with and supportive of the Hispanic and Marshallese cultures being collectivist. Additionally, the Marshallese culture was traditionally an oral one and being able to talk through the mathematical content was in line with their cultural methods of processing information.

There were implications based on the conclusions drawn for social and emotional competence. If students were explicitly taught appropriate social interaction and those norms were enforced, students learned and applied appropriate social behavior. In other words, student achievement was not influenced by some students being marginalized over others in this case study. Second, Danielak, Gupta, and Elby (2014) suggest that students' ways of knowing and beliefs about what counts as knowing and learning, their personal epistemologies, contributes to their sense of self as knowers and learners. The six students did not question the mathematical models being shown to them nor the methodology of instruction possibly because it did not occur to them to do so. The students repeated what was modeled for them. For student understanding of content, providing an adequate learning atmosphere did not appear to be sufficient for the six students.

ISCRT. Ms. Roswell was scored on her cultural responsiveness in the classroom using the ISCRT scoring rubric© as developed by Socorro Herrera et al. (2010) Kansas State University, CIMA. Her scores were adequate for each of the five pillars of (1) joint productive activity, (2) language and literacy development, (3) contextualization, (4), challenging activities, (5) instructional conversation indicating she had an adequately culturally responsive classroom for her diverse students. However, it could be argued that she was consciously implementing pedagogical best practices for her ELLs since she was observed for a class on ELL instructional methods.

Implications of Theme One. The role of the teacher had an impact on how the mathematical content was perceived. Ms. Roswell had most of the necessary credentials to be a quality teacher for her students and was working on what she had left to attain, an ESL endorsement. Because of her training in cultural responsiveness and natural demeanor, the ELLs did not appear intimidated by the mathematical content. A variety of combined factors could explain students' lack of success on the post assessment items as a result of instruction. While Mrs. Roswell was considered a culturally responsive teacher, the tasks were not contextualized for either Marshallese or Hispanic students. Instead they were more typical Anglo type of contexts. It's possible that word problems situated in contexts more familiar and more relevant to the ELL's might have superseded their tendency to mimic the strategy modeled for them by the teacher. Furthermore, the implementation of the gradual release model that focused on presenting the equation first could have interfered with the students' sense-making strategies. Many of the problems posed did not require an equation to get a correct answer. Translating to equations after students have gotten a correct answer may have enhanced their ability to connect a variety of ways of thinking about the word problems to symbols.

Theme Two: Sense Making with Translation Activities

The nature of the discipline and content in a mathematics classroom was communicated through a mathematics specific register, word problems, and syntax of the language used to communicate thought processes. Each area of study had an impact over ELL perspectives in mathematics. Whether intended or not, the six ELLs used word by word translation techniques, often to the detriment of successful problem solving. Encouraging them to try to make sense of what is happening with the quantities and operations in the problem prior to completing the translation process from words to symbols could improve their success on this task. Also, like problem solving itself, there are expressions and equations in word form that have multiple correct equivalent symbolic representations. Having students compare and contrast differences engages them in higher level thinking about the purpose of equations for problem solving and encourages their own flexibility with the content and the language.

Mathematics Register. The mathematics register focused on the content specific technical language. A unit on rational expressions and equations was content vocabulary heavy. A working knowledge of the mathematical technical language was required for the students to be able to perform the requested tasks. Without it, it was difficult to decipher what was being described in the scenario or being asked of the students to do within the scenario. The students seemed to perform well when there was only one mathematical operation going on. The ELLs had an issue when there was more than one operation happening or when exponential syntax was used. For example, m^3 , $8y^2$, product of twice a and b, and cubed of $a + b$ were problems the ELLs needed to review.

Possible implications of those findings are as follows. One, the linguistic demand of the mathematical register when more than one operation was happening was perhaps too much to process at the students' current English proficiency level with the language. More practice or another educational strategy to describe how the mathematical language is woven together when more than one operation was occurring may have been beneficial. Grouping of operations and intentions of groups needed more attention for these students. Vocabulary specific to the register of mathematics would likely not be encountered outside of the classroom context at that point for the ELLs. Therefore, the intentional exposure of the vocabulary within context would help with content retention and understanding. Next, the exponential language consistently presented issues for the students and the vocabulary associated with exponents was not expressly taught to the students during the unit. The mathematical language associated with exponents did not have similar usage outside of the discipline. Again, it was unlikely ELLs would come across exponents outside of a mathematical classroom context. The brevity of exposure to exponential language devoid of applicable context could have made it difficult to internalize and retain the proper usage of the words for students, particularly for ELLs simultaneously learning the language though which the mathematics was communicated. Those findings reiterated the need to expressly expose the students to mathematical registry items in context so they could grapple with it for the appropriate schema to develop.

Sentence Structure. As part of this unit, students were asked to translate and decode mathematics register content between words to symbols and symbols to words. Because of the sentence structure in English, a literal word for word translation may have made the mathematics symbols incorrect with the intent of the phrase. For example, the phrase four less than twice a number is out of chronological order if translated word for word into mathematical symbols. Some of the ELLs wrote $4 - 2x$ or $4 < 2x$, instead of the intended $2x - 4$. This showed the students understood the vocabulary in isolation but not necessarily grouped together in a sentence.

Using the problems given as an example in the findings, it was apparent a working knowledge of English syntax and mathematics register vocabulary were needed in order to make sense of the problem. There were not context clues given via redundancy in text or a pictorial representation of the scenario. The questions used about the problem were also sequentially written meaning the first question had to be answered correctly in order to be able to get the following questions correct.

Based on student outcomes, had the teacher utilized an instructional strategy for students to wrestle with sentence structure in English in addition to the vocabulary, the ELLs might have had different results in the quality of mathematics register translations. Chow and Ekholm (2019) concluded syntax usage is a stronger predictor of mathematical performance than vocabulary. There were some sentence structures unique to English that may not easily translate to the Spanish and Marshallese languages. Mathematical sentences written in English may not be in chronological order for a literal word for word translation. This linguistic difference awareness and the need to discuss the intention of grouping in mathematical sentences became apparent for ELLs.

Mathematics Symbols. Mathematics symbols were used to convey mathematical ideas. This unit made heavy use of the mathematics symbols: ≤, ≥, <, >, +, -, =. The students had to understand the mathematical intent of those symbols and be able to translate or decode the symbols into the mathematical register phrases and back again at a proficient level to be successful. Comparative language and symbols were challenging linguistically even for native English speakers, especially when comparing abstract quantities. The mathematical symbols had a host of mathematical vocabulary that could be attached to them. For example, the concept of subtraction could have been conveyed using the words or phrases like *minus*, *difference*, *less than, decrease*, or *loss* among other words or phrases. Because of this, the teacher and students talked through and collectively created a notes page of possible vocabulary words associated with each symbol in the mathematical register. The students frequently referenced this notes page while doing their problem sets and with no prompting, took the page with them when moving about the room to solve other problem sets.

A symbol was arguably not a word; however, that did not imply there were not linguistic challenges associated with the symbol. Tang et al. (2006, p. 10775) suggested

> results further indicate that the different biological encoding of numbers [a type of symbol] may be shaped by visual reading experience during language acquisition and other cultural factors such as mathematics learning strategies and education systems, which cannot be explained completely by the differences in languages *per se.*

The meaning of the symbol was communicated through language, in this case, English. A notes page was given to the students with the intention of helping them remember vocabulary. However, the students associated the symbols on the notes page exclusively with the words on the notes page and saw the words as indicators for a computation to be completed. The words associated with the symbols on the notes page may have had different meanings depending on the context and if they were grouped together with other words or symbols.

Theme Three: Potential Benefits of Inquiry Methods to Improve ELLs' Learning of Mathematics

The student artifacts provide a glimmer of the possibilities and potential if ELLs are positioned as inquirers that are capable of making sense of mathematics, whether it is presented in word problem situations or symbols only. These six students had a limited set of opportunities to approach problem solving by using what they knew about the numbers and context to solve the problems. Inquiry methods provide opportunities for all students to try to make sense of the mathematics prior to being shown a particular strategy ahead of their own steps. Didactic methods, the predominate instructional strategy utilized during this unit, limited opportunities for students to interpret problem situations in ways that might have been different from, yet mathematically productive, for these six learners. Students discourse opportunities were limited to comparing/contrasting correct versus incorrect replication of the approach modeled to them by the teacher. Therefore, students did not have the opportunity to engage in conversations that might have broadened their understandings of the multitude of different, yet mathematically correct, possible representations of linear word problem situations.

Recommendations to Further ELL Mathematical Sense Making

Based on this research, there were recommendations to further ELL mathematical sense making. One, the six students in this study were told how to solve the problem first rather than allowed to attempt to make sense of the problem for themselves. As a result, it appeared the students were more focused on trying to make the problem fit the equation model shown to them rather than making sense of the problem using their own thinking. Two, after Ms. Roswell answered the students' questions about vocabulary, it may have behooved the students to put the vocabulary back in the context of the word problem and then consider the mathematical implications of the problem based on their new understanding. Three, the ELLs attempted to do word by word translations in word problems in an effort to make sense of the problems. In English, some words were grouped together to convey a singular idea in mathematics. A word by word translation could break apart the intended grouping in mathematics and produce an altered translation. The altered translation could then lead to incorrect problem solving by the ELLs. A lesson on mathematical grouping of words to convey meaning in English may be beneficial to help these six ELLs grow in mathematical linguistics.

Inquiry approaches that emphasize sense making strategies with equations has shown to improve students' relational understandings and achievement in algebra (Knuth et al., 2006). Just as understanding the entirety of a word problem structure enhances students' success opportunities with problem solving, flexible interpretations of the equations used represent word problems may also enhance ELL students' success in pre-algebra content. This type of activity allows them to examine different forms of equations to represent contextualized situations and explore how the correct answer is found in each of the various forms.

Limitations

Limitations in this study included the use of a local, small sample, which affected the ability to generalize the results to other populations across the U.S. In Arkansas during the time of this study, Hispanic students made up approximately 12 percent of the student population and Marshallese (Pacific Islander) students made up 2 percent (NCES, n.d.). The focus on those two groups was pertinent for that area. There were numerous other subpopulations of culturally and linguistically diverse students across the United States that were not the focus of this research. Second, all students interviewed were part of a blocked math class meaning they received double the amount of mathematics instructional time compared to non-blocked students. In other words, this study's particular group of students had a history of mathematical underachievement and did not include students that have demonstrated mathematical success previously. Third, the researcher of the study was an ELL instructional coach for the teacher of these students, which may have unintentionally influenced the presence of mathematics comprehension interventions or usage of comprehension strategies in the general education classroom. The mathematical instructional strategies may have been different from a teacher that did not have training in how to teach ELLs so the students may have responded differently.

Certain variables were not the focus of this study and therefore not controlled. These variables were language levels of the participants beyond English learner status, country of birth, stability of home life, and migrant status. Within this sample, only Hispanic and Marshallese ELL were represented. The participants consisted of adolescents in seventh grade. Attendance of the students for the academic lessons could not be predicted and may have influenced the results of their mathematical achievement and perspectives of content.

Recommendations for Further Research

This research added to the body of knowledge about mathematical education and helped to fill a void in ELL mathematical education. The ELLs in this case study provided a representation of their seventh grade mathematics class. To further fill the gaps in mathematics education knowledge, it could be useful to examine other grade levels as the specific mathematical content changes as well as the student's mental development by age may play a role. The students in this study were all in a blocked mathematics class with a trained mathematics teacher who also had training in ELL instruction. It may be beneficial to gain additional perspectives from students of varying mathematical ability and from students without an ELL trained mathematics teacher. In addition, a larger sample could make the results more able to be generalized. Future research could also study ELL in a learning environment that focuses on sense making and developing multiple problem solving strategies. Even with limited opportunities for inquiry on the part of the students, they demonstrated the capacity to do it. In a class in which inquiry is a regular and consistent expectation, ELLs may overcome limitations of the language and mathematical syntax and narrow or overcome completely the acknowledged gap in achievement between many of them and English first students.

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APPENDICES

Appendix A: University of Arkansas Internal Review Board Approval

UNIVERSITY OF ARKANSAS

To:	Sarah Reeves Frederickson
From:	Douglas James Adams, Chair IRB Committee
Date:	02/25/2019
Action:	**Expedited Approval**
Action Date:	02/25/2019
Protocol #:	1810150656
Study Title:	Ell Perspectives in Mathematics
Expiration Date:	02/03/2020
Last Approval Date:	

The above-referenced protocol has been approved following expedited review by the IRB Committee that oversees research with human subjects.

If the research involves collaboration with another institution then the research cannot commence until the Committee receives written notification of approval from the collaborating institution's IRB.

It is the Principal Investigator's responsibility to obtain review and continued approval before the expiration date.

Protocols are approved for a maximum period of one year. You may not continue any research activity beyond the expiration date without Committee approval. Please submit continuation requests early enough to allow sufficient time for review. Failure to receive approval for continuation before the expiration date will result in the automatic suspension of the approval of this protocol. Information collected following suspension is unapproved research and cannot be reported or published as research data. If you do not wish continued approval, please notify the Committee of the study closure.

Adverse Events: Any serious or unexpected adverse event must be reported to the IRB Committee within 48 hours. All other adverse events should be reported within 10 working days.

Amendments: If you wish to change any aspect of this study, such as the procedures, the consent forms, study personnel, or number of participants, please submit an amendment to the IRB. All changes must be approved by the IRB Committee before they can be initiated.

You must maintain a research file for at least 3 years after completion of the study. This file should include all correspondence with the IRB Committee, original signed consent forms, and study data.

cc: Laura B Kent, Investigator

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Appendix B: School District Internal Review Board Approval

SPRINGDALE
SCHOOL DISTRICT

Director of Accountability
Melody Morgan

TO: Sarah Frederickson

FROM: Melody Morgan

DATE: December 20, 2018

SUBJECT: Research Request

Dear Ms. Frederickson,

Thank you for your submission to the Springdale School District to conduct research in our district. We appreciate your interest and desire to include Springdale in this important work.

The committee has reviewed your proposal to study the ***Perspectives of English Language Learners in Mathematics***. The committee has conditionally approved your research proposal pending IRB approval. I have also provided Dr. Dodson a copy of your proposal and encourage you to work with her and keep her informed as you begin your research project at Sonora Middle School. I assured Dr. Dodson this would not interfere with the school day and that your research would protect the identity of students, staff, school and district. Please forward a copy of your IRB approval once it is final.

Last, Dr. Bridgforth has staff that can assist with the Marshallese translation of your parent letter and informed consent forms. Please work directly with her to get this completed. As we discussed by phone, you will need to obtain parent consent for all students in the classroom you will be observing even though a select few will chosen as target participants.

Sincerely,

Melody Morgan

Melody Morgan
Director of Accountability and Assessment
Springdale School District

cc: Dr. Marcia Smith
Dr. Kathy Morledge
Dr. Mary Bridgforth
Ms. Tara Harshaw
Dr. Martha Dodson

Appendix C: Parent Letter

Dear Parent/Guardian,

My name is Sarah Frederickson. I am a doctoral student in the Department of Curriculum and Instruction at the University of Arkansas with a focus on mathematics teaching and learning. During the instructional unit Rational Numbers and Equations in your child's mathematics class, I will be observing and documenting strategies students use to solve a variety of mathematics problems. A small selection of students will be asked to do a video-recorded interview where they will talk about their thought process when solving mathematics problems.

All of the information I obtain from your child's class and your child will be kept confidential. Your child's name will not be used on any of the forms they complete, and no information about your child will ever leave school premises with a name attached. The short survey that your child completes will be marked with a number I select but no one who works in the school will ever know this number or the responses of your child.

The Springdale School District and the University of Arkansas have approved the survey. However, your child does not have to participate in the survey. Participation or non-participation will not affect your child's grades. Your child's teacher will be present in the classroom during the survey. She will not be involved in the student survey process and will not be told who does and does not participate.

The information from the survey and selection of interviews will help me learn more about the factors that contribute to students' success in middle school mathematics. There are no known risks associated with participation in this study.

If you and your child agree that your child may take part in the research, please return a signed copy of the attached permission form. You may keep the other copy for future reference.

Thank you for your consideration,

Sarah Frederickson
Ph.D. Candidate
Department of Curriculum and Instruction
College of Education and Health Professions
University of Arkansas
Fayetteville, AR 72701
479-575-7198
sar008@uark.edu

Spanish Parent Letter

Queridos Padres/Guardianes,

Mi nombre es Sarah Frederickson. Soy una estudiante de doctorado en el Departamento de Currículo e Instrucción en la Universidad de Arkansas con enfoque en enseñanza y aprendizaje de matemáticas. Durante la unidad instructiva de Números Racionales y Ecuaciones, estaré documentando estrategias que los estudiantes usan para resolver una variedad de problemas matemáticos.

Toda la información que obtenga de su hijo(a) se mantendrá confidencial. El nombre de su hijo(a) no se usará en ninguno de los formularios que llenen, y ninguna información acerca de su hijo(a) saldrá de la propiedad de la escuela con un nombre adscrito. La encuesta que su hijo(a) complete será marcada con un número que yo seleccione pero nadie que trabaja en la escuela sabrá este número ni las repuestas de su hijo(a).

El distrito escolar de Springdale y la Universidad de Arkansas han aprobado la encuesta. Sin embargo, su hijo(a) no tiene que participar en la encuesta y la participación o falta de participación no afectará las calificaciones de su hijo(a). El maestro(a) de su hijo(a) estará presente en el salón de clases durante la encuesta. Sin embargo, el maestro(a) no participará en el proceso de la encuesta y no se le informará quien participará o no.

La información de la encuesta nos ayudará a aprender acerca de los factores que contribuyen al éxito de los estudiantes en matemáticas de la escuela intermedia. No hay ningún riesgo asociado con la participación en este estudio.

Si usted y su hijo(a) están de acuerdo con que él o ella tomen parte en el estudio, por favor regrese una copia firmada del formulario de permiso en el sobre incluído. Puede quedarse con la otra copia para futura referencia.

Gracias por su consideración,

Sarah Frederickson
Candidata Ph.D.
Departamento de Currículo e Instrucción
Colegio de Educación y Profesiones de la Salud
Universidad de Arkansas
Fayetteville, AR 72701
479-575-7198
sar008@uark.edu

Marshallese Parent Letter

Dear Jinen im Jemen/Rikejbarok rijikuul ro,

Eta in Sarah Frederickson. Ña ij juon doctoral rijikuul ilo Department eo an Curriculum im Instruction ilo University of Arkansas im ij focus on katakin im ekatak kin mathematics ak bwinbwin. Ilo instructional unit eo Rational Numbers and Equations ilo kilaaj in math eo an ajiri eo najim, inaaj etale im jei strategy ko rijikuul ro rej kojerbali nan aer solve ak lo uwaak ñan elõñ buraablom ko ilo mathematics ak bwinbwin. Juon kurub jidikdik in rijikuul renaaj kajitok ippaer ñan aer kommane juon video im rej rekoot e aer interview im renaaj konono kin lomnak ko aer elañe rej kajion solve e problem in mathematics ko.

Aolepen melele ko inaaj buki jen kilaaj eo an im jen e reban kwaloki ñan jabdrewot. Reban kojerbale etan ajiri eo najim ilo jabdrewot form ko renaaj kanni, im ejjelok melele ikijien ajiri eo najim renaaj kadiwojlok jen mon jikuul ko im rebn likit etan ie. Survey kadudu eo im ajiri eo najim enaaj kadedelok renaaj kokaleiki kin juon nomba inaaj kelete im ejjelok juon ilo jikuul eo enaaj jela kin nomba in ak uwaak ko an ajiri in ilo survey in.

Springdale jikuul tijtirik im University of Arkansas emoj aer komalimi survey in. Ijo wot ke, ajri eo najim ejjab aikuj bok konaan ilo survey in. Bok konaan ak jab bok konaan eban jelet kireet eo an ajiri eo najim. Enaaj bed rikaki eo an ajiri eo najim ilo kilaajruum eo ilo ien eo ej kanne survey eo. Ijo wot ke, eban bok konaan ilo survey process in an rijikuul in im reban ba nane won ej bok konaan im won ejjab bok konaan.

Melele ko jen survey im selection in interview ko enaaj jiban eo bwe in ekatak elaplok kin factor koi m rej komman bwe rijikuul eo en tobrak ilo kilaaj in mathematics ak bwinbwin eo ilo middle jikuul. Ejjelok menin uwota koi m rej ekejel ilo an bok konaan ilo ekatak in.

Elane kwe im ajri eo najim komro ej errā bwe ajiri eo en bok konaan ilo research in, jouj im koroltok form in malim in ilo envelope in im jaini. Komaroñ boke wot copy ne juon ñan am reference tokalik.

Kommol kin am kommane waween in,

Sarah Frederickson
Ph.D. Candidate
Department of Curriculum and Instruction
College of Education and Health Professions
University of Arkansas
Fayetteville, AR 72701
479-575-7198
sar008@uark.edu

Appendix D: Informed Consent of Students

INFORMED CONSENT
For Children Under 18

Title: Diverse Student Perceptions in Middle School Mathematics

Investigators		
Sarah Frederickson Ph.D. Candidate Department of CIED College of Ed. and Health Prof. University of Arkansas Fayetteville, AR 72701 479-575-7198 sar008@uark.edu	Laura B. Kent, Ph.D. Associate Professor Department of CIED College of Ed. and Health Prof. University of Arkansas Fayetteville, AR 72701 479-575-8762 lkent@uark.edu	Ro Windwalker Research Compliance 109 MLKG University of Arkansas Fayetteville, AR 72701 479-575-2208 irb@uark.edu

Description: The present study will investigate English language learner perspectives in mathematics. Specifically, it will examine English language learner impact on the teacher's classroom practice. The observation of your child's classroom will last for an entire unit and will take approximately 30-60 minutes per day for the duration of the unit. Videotaping, audio recording, and/or photographs of some observations and interviews will be done with permission of teachers, parents and students.
Risks and Benefits: The benefits include improved teaching and learning practices for English Language Learners and students whose first language is English. There are no anticipated risks to participating in the study. Your child's grades are in no way affected by your participation or non-participation in this study.
Voluntary Participation: Your child's participation in the observation is voluntary.
Confidentiality: Student artifacts such as drawings, writings, photographs, audio recordings, and/or videos will be assigned a subject number and will be used to match student artifacts with classroom observations/videos. All information will be coded. Your child's name will not be used in any publication resulting from this research. All information will be kept confidential to the extent allowed by law and University policy.
Right to Withdraw: Your child is free to refuse to participate in this study and to withdraw from this study at any time. Your decision to withdraw will bring no negative consequences—no penalty to you.
Informed Consent: I have read the description, including the purpose of the study, the procedures to be used, the potential risks, the confidentiality as well as the option to withdraw from the study at any time. I give my consent for my child to be part of the classroom observation.

____________________________________ ____________ yes/no
yes/no

Student's Name Date Observation
Video/Photos

______________________________ ___________
Parent's Signature Date

I have discussed this study with my parent/guardian and agree to be part of the classroom observation.

______________________________ ___________
Student's Signature Date

Student Informed Consent Spanish

PERMISO INFORMADO

Para Menores de 18

Título: Percepciones de Estudiantes Diversos en Matemáticas de Escuela Intermedia

Investigators		
Sarah Frederickson Candidata Ph.D. Departamento de Currículo e Instrucción. Colegio de Educación y Prof. de la Salud Universidad de Arkansas Fayetteville, AR 72701 479-575-7198 sar008@uark.edu	Laura B. Kent, Ph.D. Profesora Ascoiada Departamento de Currículo e Instrucción. Colegio de Educación y Prof. de la Salud Universidad de Arkansas Fayetteville, AR 72701 479-575-8762 lkent@uark.edu	Ro Windwalker Adhesión de Estudios 109 MLKG Universidad de Arkansas Fayetteville, AR 72701 479-575-2208 irb@uark.edu

Descripción: El estudio presente investigará las perspectivas en matemáticas del estudiante aprendiendo inglés. Específicamente, se estará examinando el impacto que tiene el estudiante aprendiendo Inglés en la practica docente del maestro(a). La observación del salón de clases de su hijo(a) durará una unidad entera y tomará aproximadamente 30-60 minutos al día durante el plazo de la unidad. Grabaciones de video, audio, y/o fotografías de algunas de las observaciones y entrevistas se realizarán con la autorización de los maestros, padres y estudiantes.

Riesgos y Beneficios: Los beneficios incluyen practicas mejoradas de enseñanza y aprendizaje para estudiantes aprendiendo inglés y los estudiantes que tienen inglés como su lengua natal. No hay ningún riesgo anticipado al participar en el estudio. Las calificaciones de su hijo(a) no serán afectadas en ninguna manera al participar o no participar en el estudio.

Participación Voluntaria: La participación de su hijo(a) en esta observación es voluntaria.

Confidencialidad: Artefactos del estudiante como dibujos, escritos, fotografías, grabaciones de audio y/o videos serán asignados un número y será usado para asociar artefactos del estudiante con las observaciones/videos del salón. Toda la información será codificada. El nombre de su hijo(a) no se usará en ninguna publicación que resulte del estudio. Toda la información se mantendrá confidencial hasta el grado permitido por la ley y las pólizas de la Universidad.

Derecho a Retiro: Su hijo(a) tiene derecho a rechazar la participación en el estudio y de retirarse del estudio en cualquier momento. Su decisión de retirarse no tendrá ninguna consecuencia negativa—ninguna penalidad para usted.

Permiso Informado: He leído la descripción, incluyendo el propósito del estudio, los procedimientos que se usarán, los posibles riesgos, la confidencialidad al igual que la opción de retirarse del estudio en cualquier momento. Doy mi permiso para que mi hijo(a) tome parte en esta observación.

______________________________	____________	si/no	si/no
Nombre del Estudiante	Fecha	Observación	Video/Fotos

_________________________________ ______________
Firma del Padre/Madre/Guardián Fecha

He discutido el estudio con mi padre/madre/guardián y estoy de acuerdo en tomar parte en esta observación.

_________________________________ ______________
Firma del Estudiante Fecha

Student Informed Consent Marshallese
MALIM IN KAROÑ
Ñan ajiri ro edik jen 18 aer iiõ

Title: Waween ak kajojo Rijikuul ro ilo Middle School Kolmenlokjen Ikijien Mathematics ak Bwinbwin

Investigators		
Sarah Frederickson Ph.D. Candidate Department of CIED College of Ed. and Health Prof. University of Arkansas Fayetteville, AR 72701 479-575-7198 sar008@uark.edu	Laura B. Kent, Ph.D. Associate Professor Department of CIED College of Ed. and Health Prof. University of Arkansas Fayetteville, AR 72701 479-575-8762 lkent@uark.edu	Ro Windwalker Research Compliance 109 MLKG University of Arkansas Fayetteville, AR 72701 479-575-2208 irb@uark.edu

Kemlele: Ekatak eo kio enaaj etale perspective ko an eo ej ekatak kajin English ikijien mathematics ak bwinbwin. Elaptata, enaaj etale impact eo an eo ej ekatak kajin English ion waween an rikaki eo ilo kilaajruum eo katakin. Observation eo kin an ajiri eo bed ilo kilaajruum enaaj bok aolepen juon unit im enaaj aetokan 30-60 minute ko ilo juon raan nan jemlokin unit eo. Aer videotape, rekoot aer konon, im/ak pijaik jet iaan observation ko im interview ko renaaj kommani elañe rikaki eo enaaj komalimi, jinen im jemen im rijikuul ro.
Kauwotata ko im Emman ko: Emman ko ekoba an emmanlok aer katakin im ekatak ñan English Language Learner ro ak ro rej ekatak kajin English im rijikuul ro im kajin eo aer jinoin tata ej English. Ejjelok kauwotatata ko ilo aer bok konaer ilo ekatak in. Eban jelet kireet ko an ajiri eo najim ilo am bok konaam ak jab bok konaam ilo ekatak in.
Balontier in bok Konaam: An ajiri eo najim bok konaan ilo ekatak in ej an wot pepe.
Waween kejbarok melele ko: Melele ko ikijien rijikuul eo ainwot jiña ko an, bwebwenato ko an, pija ko an ainikien aer rekoot e an konono, im/ak video ko renaaj lelok juon jabjek nomba eo renaaj kojerbale ñan match e artifact ko an rijikuul eo ippan kilaajruum observation ko/video ko. Aolep melele renaaj code i. Reban kojerbale etan ajiri eo najim ilo jabdrewot publication ko renaaj jebar jen ekatak in. Aolep melele renaaj lukkun kejbaroki ekkar ñan kakien ko an University eo.
Jimwe ñan Bojrak: Ajiri eo najim emaroñ kwalok an jab konaan im bojrak jen an bed ilo ekatak in jabdrewot ien. Lomnak eo am ñan am bojrak eban jelet jabdrewot jorāān – eban wor penalty ak kaje ñan eok.
Mālim in Karoñ: Ededelok ao riti kemlele kein, ekoba wunleplep in ekatak in, waween ko renaaj kojerbali, kauwotata ko, waween kejbarok melele ko im barainwot kelet ne ñan bojrak jen ekatak in ilo jabdrewot ien. Ij lewaj malim ñan ajiri in nāju bwe en mottan kilaajruum ekatak in.

______________________________ __________ aet/jaab

aet/jaab

Etan Rijikuul eo Rainin Observation

video/pija

______________________________ ___________

Jaini etan Jinen im Jemen Rainin

Ededelok ao kenaan kin ekatak in ippan jino im jema/rikejbarok ro ao im ij erra in mottan kilaajruum observation in.

____________________________ ___________

Jaini etan Rijikuul eo Rainin

Appendix E: Informed Teacher Consent

INFORMED CONSENT
For Teacher Participant

Title: Diverse Student Perceptions in Middle School Mathematics

Investigators		
Sarah Frederickson Ph.D. Candidate Department of CIED College of Ed. and Health Prof. University of Arkansas Fayetteville, AR 72701 479-575-7198 sar008@uark.edu	Laura B. Kent, Ph.D. Associate Professor Department of CIED College of Ed. and Health Prof. University of Arkansas Fayetteville, AR 72701 479-575-8762 lkent@uark.edu	Ro Windwalker Research Compliance 109 MLKG University of Arkansas Fayetteville, AR 72701 479-575-2208 irb@uark.edu

Description: The present study will investigate English language learner perspectives in mathematics. Specifically, it will examine English language learner impact on the teacher's classroom practice. The observation of the teacher's classroom will last for an entire unit and will take approximately 30-60 minutes per day for the duration of the unit. Videotaping, audio recording, and/or photographs of some observations and interviews will be done with permission of teachers, parents and students.
Risks and Benefits: The benefits include improved teaching and learning practices for English Language Learners and students whose first language is English. There are no anticipated risks to participating in the study. Your evaluation is in no way affected by your participation or non-participation in this study.
Voluntary Participation: Your participation in the observation is voluntary.
Confidentiality: Student artifacts such as drawings, writings, photographs, audio recordings, and/or videos will be assigned a subject number and will be used to match student artifacts with classroom observations/videos. All information will be coded. The students' names nor the teacher's name will be used in any publication resulting from this research. All information will be kept confidential to the extent allowed by law and University policy.
Right to Withdraw: You are free to refuse to participate in this study and to withdraw from this study at any time. Your decision to withdraw will bring no negative consequences—no penalty to you.
Informed Consent: I have read the description, including the purpose of the study, the procedures to be used, the potential risks, the confidentiality as well as the option to withdraw from the study at any time. I give my consent to be part of the classroom observation.

____________________________________ ____________ yes/no
yes/no

Teacher's Name Date Observation
Video/Photos

______________________________ ____________
Teacher's Signature Date

Appendix F: Classroom Observation Notes

Classroom Observation Notes	
Name of Teacher:	**Name of Observer:**
Date of Observation:	**Length of Observation:**
Topic of Lesson:	
Comments of Observer:	
Comments by Teacher:	
Comments by Students:	

Appendix G: Student Interview Problems

Name ____________________________Date _________________ Block __________

Student Interview Problems

QUESTION	ANSWER
Ricky's dad called an electrician who charges $50 to travel to a customer's home and $75 per hour of labor completed. The electrician traveled to Ricky's house and completed 3 hours of labor. What was the total amount of money the electrician charged Ricky's dad?	
Kai is 4 years younger than half Abigail's age. Kai is 12 years old. How old is Abigail?	
A couple of friends wanted to get something to eat while at a festival. They bought two pieces of fried chicken and three orange sodas for $7. Later, another group of friends bought four pieces of fried chicken and three orange sodas for $11. How much did each piece of fried chicken and each orange soda cost?	

Appendix H: Semi-Structured Student Interview Questions

1. Make sense of problems and persevere in solving them.

- How would you describe the problem in your own words?
- How would you describe what you are trying to find?
- Talk me through the steps you've used to solve it.

2. Reason abstractly and quantitatively.

- What do the numbers in the problem represent?
- What is the relationship between the quantities?
- What does _____ mean to you? (e.g. symbol, quantity, diagram)
- How did you decide in this task that you needed to use…..? Could you have used another operation or property to solve the task? Why or why not?

3. Construct viable arguments and critique the reasoning of others.

- What mathematical evidence supports your solution?
- How could you prove…..?
- How did you decide to try that strategy?
- Did you try a method that did not work out? Why didn't it work?

4. Model with mathematics.

- What number model could you construct to represent the problem?
- Would it help to create a diagram, graph, table, …?
- What are some ways you can visually represent….?

5. Use appropriate tools strategically.

- What mathematical tools could we use to visualize and represent the situation?
- What do you know that is not stated in the problem?
- What estimate did you make for your solution?
- In this situation would it be helpful to use

    - A graph?
    - A number line?
    - A ruler?
    - A diagram?
    - A calculator?
    - A manipulative?

6. Attend to precision.

- What mathematical terms apply in this situation?
- How did you know your solution was reasonable?
- How are you showing the meaning of the quantities?
- What symbols or mathematical notations are important in this problem?
- What mathematical language, definition, or properties can you use to explain…?
- How could you test your solution to see if it answers the problem?

7. Look for and make use of structure.

- What observation do you make about…?
- What parts of the problem might you eliminate? Simplify?
- What patterns do you find in…?
- What ideas have we learned before that were useful in solving this problem?
- How does this relate to…?

8. Look for and express regularity in repeating reasoning.

- Will the same strategy work in other situations?
- What is going to happen in this situation?
- What would happen if…?
- What predictions or generalizations can this pattern support?

Appendix I: ELPA21 Achievement Level Descriptors Grade Band 6-8

Reading Level 2 Early Intermediate	Reading Level 3 Intermediate	Reading Level 5 Advanced
7th Score Range: 486-533	7th Score Range: 534-608	7th Score Range: 642 or above
When reading grade appropriate text, the student at Level 2 is working on:	When reading grade appropriate text, the student at Level 3 is working on:	When reading grade appropriate text, the student at Level 5 is working on:
Identifying the main topic and a few key details in simple written texts; identifying key words and phrases; responding to simple comments and questions on a variety of topics as well as some wh-questions; gathering and recording information.	Determining the central idea or theme and supporting details; responding to others' comments and answering questions on familiar topics; gathering information from a few sources; using context clues to determine the meanings of general academic and content-specific words and phrases; explaining an author's argument; analyzing the arguments and claims made in text, distinguishing between those that are supported by reasons or evidence and those that are not.	Determining central ideas or themes and how they are supported by specific details; summarizing key ideas in text; responding to others' comments and answering questions on a variety of topics, adding relevant and specific evidence; gathering information from sources, evaluating its credibility, and paraphrasing the data; determining whether reasoning is sound and evidence is sufficient to support claims; determine the meaning of figurative and connotative language.
Writing Level 2 Early Intermediate	Writing Level 3 Intermediate	Writing Level 5 Advanced
7th Score Range: 474-519	7th Score Range: 520-596	7th Score Range: 625 or above
When writing, the student at level 2 is working on:	When writing, the student at level 3 is working on:	When writing, the student at level 5 is working on:
Participating in short written exchanges; composing claims, narratives, or informational texts about familiar topics, providing a reason or fact to support the claim; responding to simple and wh- questions; recounting a brief sequences of events in order; using frequently occurring general academic and content-specific words and phrases.	Participating in written exchanges with some details; constructing a claim about a topic, introducing the topic, and providing reasons and facts in logical order; providing a concluding statement; asking and answering questions, adding relevant information; expressing own ideas in writing; recounting a short sequence of events in order with beginning, middle, and end; using common transitional words and phrases.	Participating in extended written exchanges on a variety of tipics and texts; adding evidence and summarizing ideas; composing narrative and informational texts with relevant details about a variety of topics; constructing a claim, introducing the topic and providing compelling, ordered reasons to support the claim, recounting a complex sequence of event with a beginning, middle, and end; adapting language choices and style to the purpose and audience; precisely expressing ideas while maintaining a consistent style and tone.

Listening Level 2 Early Intermediate	Listening Level 3 Intermediate	Listening Level 5 Advanced
7th Score Range: 430-472	7th Score Range: 473-552	7th Score Range: 597 or above
When listening, the student at level 2 is working on:	When listening, the student at level 3 is working on:	When listening, the student at level 5 is working on:
Recognizing the main topic and retelling a few key points; responding to simple questions and wh- questions; determining the meaning of frequently occurring words, phrases and expressions.	Determining the main idea and a few supporting details; paraphrasing the main idea; participating in discussions, building on the ideas of others and answering questions; determining the meaning of general education and content specific words.	Determining main idea or ideas and how each idea is supported with evidence; gathering information from multiple oral sources and evaluating the credibility of the information; quoting or citing examples while paraphrasing data and conclusions; determining the meaning of general academic, context specific, figurative and idiomatic phrases,.
Speaking Level 2 Early Intermediate	Speaking Level 3 Intermediate	Speaking Level 5 Advanced
7th Score Range: 475-526	7th Score Range: 527-581	7th Score Range: 611 or Above
When speaking, the student at level 2 is working on:	When speaking, the student at level 3 is working on:	When speaking, the student at level 5 is working on:
Offering an opinion or prediction using simple grammatical structures and vocabulary; responding to questions with words relevant to the topic; interpreting the information in a picture or graph about a familiar topic, constructing a claim and providing a supporting reason; producing simple and compound sentences.	Describing a picture or a graph using general academic and content-specific vocabulary, and compound as well as complex sentences; constructing a claim and providing several supporting reasons or facts in a logical order; adapting language choices to audience; delivering a short oral presentation, or recounting a brief sequence of events in order using linking words.	Making predictions and drawing conclusions from a variety of sources; asking and answering questions, and stating opinions with appropriate grammatical structures and vocabulary; recounting a complex sequence of events; making a claim with simple, compound, and complex sentences.

Adapted from the Oregon Department of Education (2017

Appendix J: Ms. Roswell's ISCRT Scores

I. Joint Productive Activity

	Not Observed 0	Emerging 1	Developing 2	Enacting 3	Integrating 4
		The teacher:	*The teacher:*	*The teacher:*	*The teacher:*
LE	A. No evidence of a respectful learning environment	A. Creates an environment that respects students as individual learners	A. Creates a culturally and linguistically respectful learning environment	A. Creates a low-risk learning environment that values diverse perspectives	A. Orchestrates conditions and situations to ensure that students collaborate as equal members in a low-risk learning community
TC	B. No collaboration between teacher and students	B. Collaborates with students but no evidence of a joint product	B. Collaborates with whole class to create a joint product or students collaborate on a joint product in pairs or small groups	B. Collaboratively guides small groups of students, especially those that need higher levels of support, to create joint products	B. Collaborates with students to create joint products that integrate language and content standards
TPSI	C. Students work independently of one another	C. Provides minimal opportunities for student interaction	C. Provides occasional structured opportunities for student interaction	C. Provides frequent structured opportunities for purposeful student interaction	C. Provides consistent structured opportunities for purposeful student interaction that promote development of the CLD student biography
PGD	D. Pair or group students based on random grouping or student self-selection	D. Pair or group students based on one dimensions of the CLD student biography	D. Pair or group students based on two or three dimensions of the CLD student biography	D. Pair or group students based on two or three dimensions of the CLD student biography as appropriate for the task/activity	D. Pair or group students based on all four dimensions of the CLD student biography as appropriate for the task/activity
AC	E. No connections between the activity and the lesson	E. Makes minimal connections between the strategy/activity and the lesson	E. Makes occasional relevant connections between the strategy/activity and the lesson	E. Frequently uses insights from the strategy/activity to make connections affirm learning, or modify instruction as needed	E. Consistently uses insights from the strategy/activity to make connections, affirm learning, and modify instruction as needed
LE= Learning Environment TC= Teacher Collaboration TPSI= Total Group, Partner, Small Group, Individual PGD= Partner/Grouping Determination; AC= Activity Connections					

II. Language & Literacy Development

	Not Observed 0	Emerging 1	Developing 2	Enacting 3	Integrating 4
		The teacher provides:	*The teacher provides:*	*The teacher provides:*	*The teacher provides:*
LSRW	A. Instruction is dominated by teacher talk and students are passive listeners	A. Listening, speaking, reading, & writing (LSRW) activities with minimal opportunities for students' academic language development	A. L, S, R, & W activities with occasional opportunities for students' academic language development	A. Frequent opportunities for student expression and academic language development in activities that integrate L, S, R, & W	A. Consistent opportunities for student expression and academic language development in higher-order thinking activities that integrate L, S, R, & W
QRM	B. No use of questioning (Q), rephrasing (R), or modeling (M) to assist language and literacy development	B. Minimal use of Q, R, or M to assist language and literacy development	B. Occasional use of Q, R, or M to assist language and literacy development	B. Frequent use of purposeful Q, R, and M to assist language and literacy development	B. Consistent use of purposeful Q, R, and M to assist academic language and literacy development and to build students' capacities to pose questions about their own thinking
L1	C. No evidence of native language in environment or instruction	C. Minimal evidence of native language in environment and/or instruction	C. Occasional opportunities for students to use their native language during the lesson	C. Frequent, explicit, purposeful opportunities for students to use their native language during the lesson in ways that support academic learning	C. Consistent, systematic opportunities for students to use their native language during the lesson in ways that support academic language and literacy development
LBK	D. No references to students' prior knowledge and background experiences related to language and literacy development*	D. Minimal references to prior knowledge and background experiences related to language and literacy development*	D. Occasional references to prior knowledge and background experiences related to language and literacy development*	D. Frequent references to prior knowledge and background experiences related to academic language and literacy development*	D. Consistent use of students' culture-bound ways of comprehending, communicating, and expressing themselves as a springboard for academic language and literacy development*

Notes:
*PA = Phonemic Awareness; P = Phonics; V = Vocabulary; F = Fluency; C = Comprehension

LSRW = Listening, Speaking, Reading, Writing QRM = Questioning, Rephrasing, Modeling L1 = Native Language LBK = Background Knowledge of Language/Literacy

III. Contextualization

	Not Observed 0	Emerging 1	Developing 2	Enacting 3	Integrating 4
		The teacher:	*The teacher:*	*The teacher:*	*The teacher:*
BK3	A. No pre-assessment of students' academic knowledge about the topic	A. Conducts pre-assessment of only students' academic knowledge about the topic	A. Conducts pre-assessment of students' funds of knowledge, prior knowledge, and academic knowledge about the topic or key content vocabulary	A. Conducts pre-assessment that provides all students the opportunity to share/document their funds of knowledge, prior knowledge, and academic knowledge about the topic or key content vocabulary	A. Conducts pre-assessment that provides all students the opportunity to share/document their funds of knowledge, prior knowledge, and academic knowledge about the topic and key content vocabulary; teacher documents students' background knowledge for use throughout the lesson
A/CL	B. Focus is solely on content delivery	B. Provides minimal opportunities for students to share with peers content-related connections to their background knowledge	B. Provides occasional opportunities for students to share with peers content-related connections to their background knowledge	B. Provides frequent opportunities for students to share/document their content-related connections to their background knowledge and purposefully listens/observes as students share/document	B. Provides consistent opportunities for students to share/document their content-related connections to their background knowledge and uses insights gleaned to highlight student assets, support academic learning, and maximize the community of learners
BIO	C. New information is presented in an abstract, disconnected manner	C. Makes minimal connections between students' sociocultural, linguistic, cognitive, and academic dimensions and new academic concepts	C. Makes occasional connections between students' sociocultural, linguistic, cognitive, and academic dimensions and the new academic concepts	C. Makes frequent and purposeful connections between students' individual biographies, including what was learned about their knowledge and experiences from home, community, and school, and the new academic concepts	C. Systematically makes consistent and purposeful connections between students' individual biographies, including what was learned about their knowledge and experiences from home, community, and school, and the new academic concepts, with applications to the real world

BK3 = Funds of Knowledge (family), Prior Knowledge (community), Academic Knowledge (school) A/CL = Assets/Community of Learners BIO = CLD Biography Connections

IV. Challenging Activities

	Not Observed 0	Emerging 1	Developing 2	Enacting 3	Integrating 4
		Teacher instruction and strategy use:	*Teacher instruction and strategy use:*	*Teacher instruction and strategy use:*	*Teacher instruction and strategy use:*
ACOM	A. No accommodations for linguistic or academic levels	A. Provides minimal accommodations based on students' linguistic and academic levels	A. Provides occasional, structured accommodations based on students' linguistic and academic levels	A. Provides frequent, structured accommodations based on students' linguistic and academic levels that build upon culture-bound patterns of knowing, learning, and applying	A. Provides consistent, systematic, structured accommodations based on students' linguistic and academic levels that build upon culture-bound patterns of knowing, learning, and applying
CO/LO	B. Makes no reference to lesson objectives	B. Includes verbally stated or posted lesson objectives that reflect content standards	B. Includes verbally stated and posted content and language objectives that reflect content standards	B. Includes content and language objectives that (1) are verbally stated and posted, (2) reflect content and language standards, and (3) are revisited during the lesson	B. Includes content and language objectives that (1) are verbally stated and posted, (2) reflect content and language standards, and (3) are interwoven throughout the lesson
S/E	C. Strategies/activities are not aligned to standards and do not reflect expectations	C. Includes strategies/ activities that are aligned to standards and that reflect vague expectations	C. Includes strategies/ activities that are aligned to standards and that reflect clear expectations	C. Includes challenging strategies/ activities that are aligned to standards and that reflect clear expectations	C. Includes challenging strategies/ activities that reflect skillful integration of multiple standards, clear expectations, and higher-order thinking skills
AF	D. Does not consider students' states of mind/affective filter	D. Minimally attends to students' states of mind/affective filter	D. Occasionally monitors students' states of mind/affective filter and adjusts instruction accordingly	D. Frequently monitors students' states of mind/affective filter and adjusts instructional conditions accordingly	D. Consistently monitors the states of mind/affective filter of individual students and of the whole group and adjusts instructional conditions and situations accordingly
FB	E. Provides no feedback on student performance	E. Provides minimal feedback on student performance	E. Provides occasional feedback on student performance to confirm/disconfirm learning	E. Provides frequent feedback on student performance to confirm/disconfirm learning and to advance student learning	E. Uses systematic formative assessment to provide consistent feedback on student performance to confirm/disconfirm learning and to advance student learning

V. Instructional Conversation

	Not Observed 0	Emerging 1	Developing 2	Enacting 3	Integrating 4
		With individuals and small groups of students, the teacher:	*With individuals and small groups of students, the teacher:*	*With individuals and small groups of students, the teacher:*	*With individuals and small groups of students, the teacher:*
ESTK	A. Lecture predominates	A. Uses questioning to elicit student talk	A. Elicits student talk with questioning, listening, and rephrasing	A. Elicits student talk with questioning, listening, rephrasing, and explicit modeling of turn-taking and questioning structures	A. Elicits student talk about the content through student-led discussion and questioning
KTU	B. Teacher responds in ways that validate students	B. Responds in ways that minimally promote higher-order thinking and individual connections from the known to the unknown	B. Responds in ways that occasionally promote higher-order thinking and individual connections from the known to the unknown	B. Responds in ways that frequently promote higher-order thinking and individual connections from the known to the unknown	B. Responds in ways that consistently promote higher-order thinking, elaboration of connections from the known to the unknown, and application beyond the classroom
BICS/ CALP	C. Teacher conversation is not on topic	C. Uses BICS and/or CALP to discuss the content/topic; provides minimal opportunities for academic talk among students	C. Uses CALP to discuss the content/topic and provides occasional opportunities for academic talk, including use of key content vocabulary, among students	C. Provides frequent opportunities for academic talk, including use of key content vocabulary, in which the teacher bridges between student talk and academic language	C. Facilitates consistent opportunities for student-led academic conversations using key content vocabulary
REV	D. Incorporates no revoicing of students' learning	D. Includes minimal revoicing of learning, limited to repeating students' words	D. Includes occasional revoicing of learning, limited to repeating and/or rephrasing	D. Includes frequent revoicing of learning that challenges students to solidify or expand upon connections to the academic content and vocabulary	D. Includes consistent revoicing of learning that challenges students to solidify, expand upon, and make deeper connections to the academic content and vocabulary
SAV	E. Does not invite students to articulate their views/judgments/ processes	E. Provides minimal opportunities for students to articulate their views/ judgments/processes	E. Provides occasional opportunities for students to articulate their views/ judgments/processes and provide rationales	E. Provides frequent, purposeful opportunities for students to articulate their views/judgments/processes and provide rationales	E. Provides consistent, structured opportunities for students to articulate their views/judgments/processes and provide rationales

ESTK = Eliciting Student Talk KTU = Known to Unknown BICS/CALP = Basic Interpersonal Communication Skills/Cognitive Academic Language Proficiency
REV = Revoicing SAV = Students Articulate Views

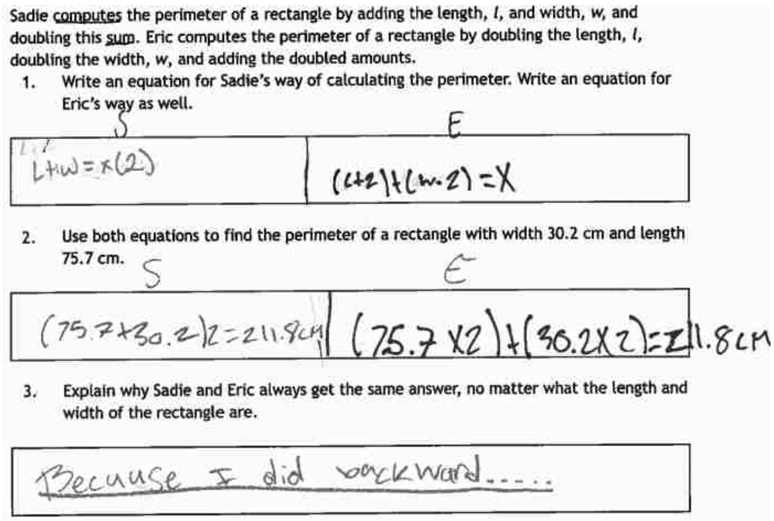
Appendix K: Arkansas Mathematics Content Standards for the Unit

Ratios and Proportional Relationships	Analyze proportional relationships and use them to solve real-world and mathematical problems
AR.Math.Content.7.RP.A.1	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units For example: If a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the *complex fraction* $^{1/2}/_{1/4}$ miles per hour, equivalently 2 miles per hour.
AR.Math.Content.7.RP.A.2	Recognize and represent proportional relationships between quantities: • Decide whether two quantities are in a proportional relationship (e.g., by testing for equivalent ratios in a table or graphing on a *coordinate plane* and observing whether the graph is a straight line through the origin) • Identify unit rate (also known as the constant of proportionality) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships • Represent proportional relationships by equations (e.g., if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$) • Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1, r)$ where r is the unit rate Note: Unit rate connects to slope concept in 8th grade.
The Number System	Apply and extend previous understandings of operations with fractions
AR.Math.Content.7.NS.A.1	Apply and extend previous understandings of addition and subtraction to add and subtract *rational numbers* Represent addition and subtraction on a horizontal or vertical number line diagram: • Describe situations in which opposite quantities combine to make 0 and show that a number and its opposite have a sum of 0 (*additive inverses*) (e.g., A hydrogen atom has 0 charge because its two constituents are oppositely charged.) • Understand $p + q$ as a number where p is the starting point and q represents a distance from p in the positive or negative direction depending on whether q is positive or negative • Interpret sums of *rational numbers* by describing real-world contexts (e.g., 3 + 2 means beginning at 3, move 2 units to the right and end at the sum of 5; 3 + (-2) means beginning at 3, move 2 units to the left and end at the sum of 1; 70 + (-30) = 40 could mean after earning \$70, \$30 was spent on a new video game, leaving a balance of \$40) • Understand subtraction of *rational numbers* as adding the *additive inverse*, $p - q = p + (-q)$ • Show that the distance between two *rational numbers* on the number line is the *absolute value* of their difference and apply this principle in real-world contexts (e.g., the distance between -5 and 6 is 11. -5 and 6 are 11 units apart on the number line) Fluently add and subtract *rational numbers* by applying properties of operations as strategies

AR.Math.Content.7.RP.A.3	Use proportional relationships to solve multi-step ratio and percent problems Note: Examples include but are not limited to simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease.
AR.Math.Content.7.NS.A.2	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide *rational numbers:* • Understand that multiplication is extended from fractions to all *rational numbers* by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, and the rules for multiplying signed numbers • Interpret products of *rational numbers* by describing real-world contexts • Understand that *integers* can be divided, provided that the divisor is not zero, and every quotient of *integers* (with non-zero divisor) is a rational number (e.g., if *p* and *q* are *integers*, then $-(p/q) = (-p)/q = p/(-q)$) • Interpret quotients of *rational numbers* by describing real-world contexts • Fluently multiply and divide *rational numbers* by applying properties of operations as strategies • Convert a fraction to a decimal using long division • Know that the decimal form of a fraction terminates in 0s or eventually repeats
AR.Math.Content.7.NS.A.3	Solve real-world and mathematical problems involving the four operations with *rational numbers*, including but not limited to *complex fractions*
Expressions and Equations	Use properties of operations to generate equivalent expressions
AR.Math.Content.7.EE.A.1	Apply properties of operations as strategies to add, subtract, expand, and factor linear expressions with rational coefficients
AR.Math.Content.7.EE.A.2	Understand how the quantities in a problem are related by rewriting an expression in different forms For example: $a + 0.05a = 1.05a$ means that 'increase by 5%' is the same as 'multiply by 1.05' or the perimeter of a square with side length *s* can be written as $s+s+s+s$ or $4s$.
Expressions and Equations	Solve real-life and mathematical problems using numerical and algebraic expressions and equations
AR.Math.Content.7.EE.B.3	Solve multi-step, real-life, and mathematical problems posed with positive and negative rational numbers in any form using tools strategically: • Apply properties of operations to calculate with numbers in any form (e.g., -(1/4)(n-4)) • Convert between forms as appropriate (e.g., if a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50) Assess the reasonableness of answers using mental computation and estimation strategies (e.g., if you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation)

AR.Math.Content.7.EE.B.4	• Use variables to represent quantities in a real-world or mathematical problem • Construct simple equations and inequalities to solve problems by reasoning about the quantities • Solve word problems leading to equations of these forms $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently • Write an algebraic solution identifying the sequence of the operations used to mirror the arithmetic solution (e.g., The perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width? Subtract 2*6 from 54 and divide by 2; (2*6) + 2w = 54) • Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers Graph the solution set of the inequality and interpret it in the context of the problem (e.g., As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.)

Bryan's Pretest. He had an ELPA21 level of 2.



Sadie computes the perimeter of a rectangle by adding the length, *l*, and width, *w*, and doubling this sum. Eric computes the perimeter of a rectangle by doubling the length, *l*, doubling the width, *w*, and adding the doubled amounts.

1. Write an equation for Sadie's way of calculating the perimeter. Write an equation for Eric's way as well.

S	E
L+W=x(2)	(L+2)+(w.2)=X

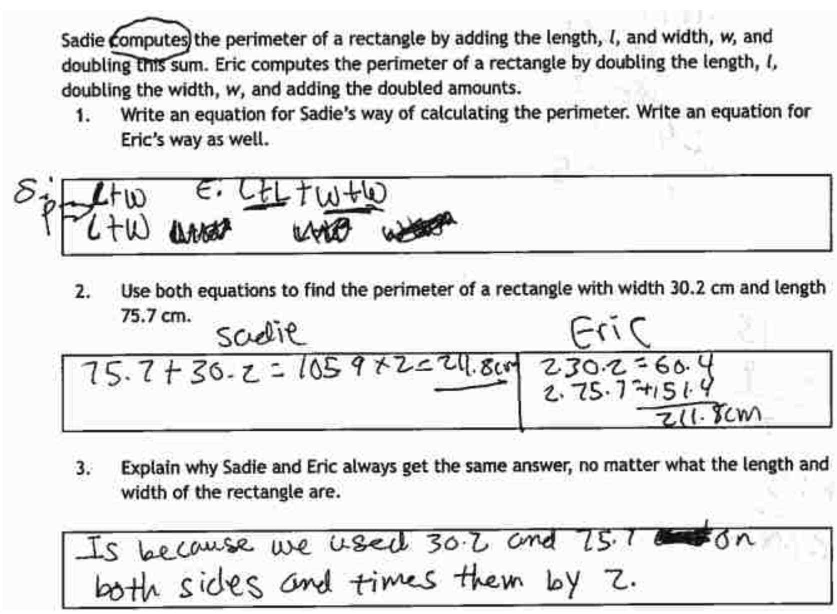
2. Use both equations to find the perimeter of a rectangle with width 30.2 cm and length 75.7 cm.

S	E
(75.7+30.2)2=211.8cm	(75.7 X2)+(30.2X2)=211.8cm

3. Explain why Sadie and Eric always get the same answer, no matter what the length and width of the rectangle are.

Because I did backward.....

Robert's Pretest. He had an ELPA21 level of 2.



Sadie computes the perimeter of a rectangle by adding the length, *l*, and width, *w*, and doubling this sum. Eric computes the perimeter of a rectangle by doubling the length, *l*, doubling the width, *w*, and adding the doubled amounts.

1. Write an equation for Sadie's way of calculating the perimeter. Write an equation for Eric's way as well.

S: L+W E: L+L+W+W
P: L+W

2. Use both equations to find the perimeter of a rectangle with width 30.2 cm and length 75.7 cm.

Sadie	Eric
75.7+30.2=105.9×2=211.8cm	2 30.2=60.4 2.75.7=+151.4 211.8cm

3. Explain why Sadie and Eric always get the same answer, no matter what the length and width of the rectangle are.

Is because we used 30.2 and 75.7 on both sides and times them by 2.

John's Pretest. He had an ELPA21 level of 2.

Sadie computes the perimeter of a rectangle by adding the length, *l*, and width, *w*, and doubling this sum. Eric computes the perimeter of a rectangle by doubling the length, *l*, doubling the width, *w*, and adding the doubled amounts.

1. Write an equation for Sadie's way of calculating the perimeter. Write an equation for Eric's way as well.

Eric 4 Sadie 4

2. Use both equations to find the perimeter of a rectangle with width 30.2 cm and length 75.7 cm.

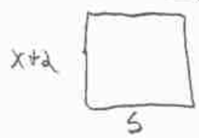
S 30.2cm =75.7cm = 105.9 x 2= 211.8

E 75.7 +30.2 =105.9x2 =211.8

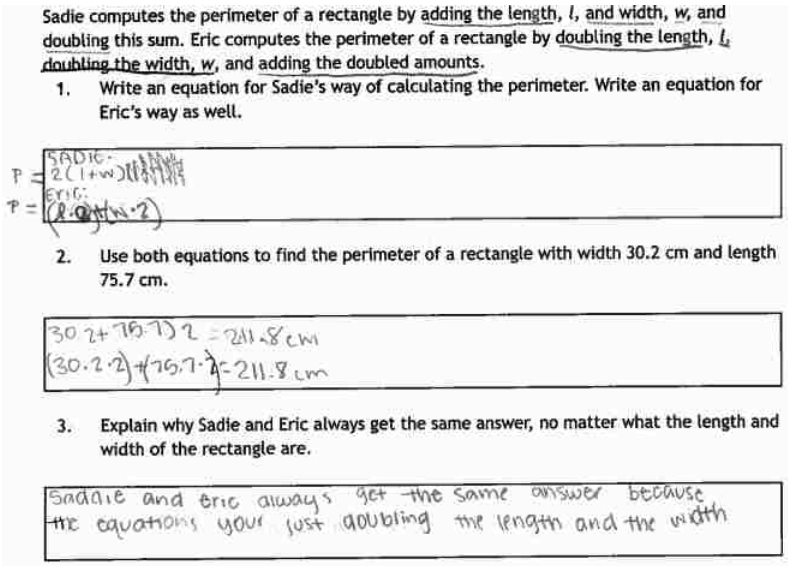
3. Explain why Sadie and Eric always get the same answer, no matter what the length and width of the rectangle are.

Because Sadie and Eric doubled amounts the answer

24+ 5= 29+58 58



Gina's Pretest. She had an ELPA21 level of 2.



Sadie computes the perimeter of a rectangle by adding the length, *l*, and width, *w*, and doubling this sum. Eric computes the perimeter of a rectangle by doubling the length, *l*, doubling the width, *w*, and adding the doubled amounts.

1. Write an equation for Sadie's way of calculating the perimeter. Write an equation for Eric's way as well.

P = SADIE: 2(l+w)
P = ERIC: (l·2)+(w·2)

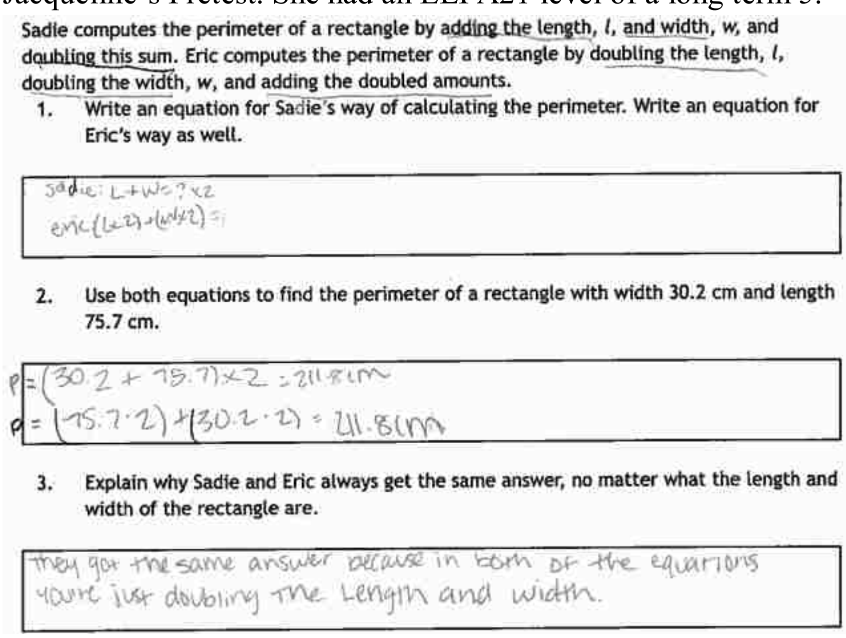
2. Use both equations to find the perimeter of a rectangle with width 30.2 cm and length 75.7 cm.

(30.2+75.7)2 = 211.8cm
(30.2·2)+(75.7·2)=211.8 cm

3. Explain why Sadie and Eric always get the same answer, no matter what the length and width of the rectangle are.

Saddie and Eric always get the same answer because the equations your just doubling the length and the width

Jacqueline's Pretest. She had an ELPA21 level of a long term 5.



Sadie computes the perimeter of a rectangle by adding the length, *l*, and width, *w*, and doubling this sum. Eric computes the perimeter of a rectangle by doubling the length, *l*, doubling the width, *w*, and adding the doubled amounts.

1. Write an equation for Sadie's way of calculating the perimeter. Write an equation for Eric's way as well.

sadie: L+W=?x2
eric(Lx2)+(Wx2)=

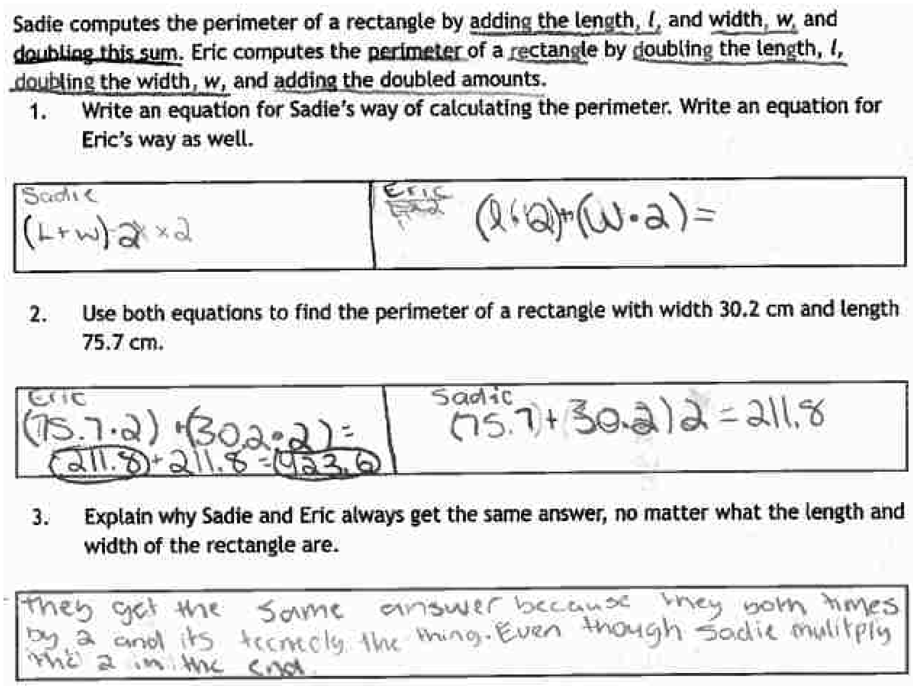
2. Use both equations to find the perimeter of a rectangle with width 30.2 cm and length 75.7 cm.

P=(30.2 + 75.7)x2 =211.8cm
P= (75.7·2)+(30.2·2) = 211.8cm

3. Explain why Sadie and Eric always get the same answer, no matter what the length and width of the rectangle are.

They got the same answer because in both of the equations you're just doubling the Length and width.

Josie's Pretest. She had an ELPA21 level of a long term 3.



Sadie computes the perimeter of a rectangle by adding the length, *l*, and width, *w*, and doubling this sum. Eric computes the perimeter of a rectangle by doubling the length, *l*, doubling the width, *w*, and adding the doubled amounts.

1. Write an equation for Sadie's way of calculating the perimeter. Write an equation for Eric's way as well.

Sadie (L+w)·2 ×2	Eric (l·2)+(W·2)=

2. Use both equations to find the perimeter of a rectangle with width 30.2 cm and length 75.7 cm.

Eric (75.7·2) +(30.2·2)= 211.8+211.8=423.6	Sadie (75.7+30.2)2=211.8

3. Explain why Sadie and Eric always get the same answer, no matter what the length and width of the rectangle are.

They get the same answer because they both times by 2 and its tecnecly the thing. Even though Sadie mulitply the 2 in the end.

Appendix M: ISCRT

I. Joint Productive Activity

	Not Observed 0	Emerging 1	Developing 2	Enacting 3	Integrating 4
		The teacher:	*The teacher:*	*The teacher:*	*The teacher:*
LE	F. No evidence of a respectful learning environment	F. Creates an environment that respects students as individual learners	F. Creates a culturally and linguistically respectful learning environment	F. Creates a low-risk learning environment that values diverse perspectives	F. Orchestrates conditions and situations to ensure that students collaborate as equal members in a low-risk learning community
TC	G. No collaboration between teacher and students	G. Collaborates with students but no evidence of a joint product	G. Collaborates with whole class to create a joint product or students collaborate on a joint product in pairs or small groups	G. Collaboratively guides small groups of students, especially those that need higher levels of support, to create joint products	G. Collaborates with students to create joint products that integrate language and content standards
TPSI	H. Students work independently of one another	H. Provides minimal opportunities for student interaction	H. Provides occasional structured opportunities for student interaction	H. Provides frequent structured opportunities for purposeful student interaction	H. Provides consistent structured opportunities for purposeful student interaction that promote development of the CLD student biography
PGD	I. Pair or group students based on random grouping or student self-selection	I. Pair or group students based on one dimensions of the CLD student biography	I. Pair or group students based on two or three dimensions of the CLD student biography	I. Pair or group students based on two or three dimensions of the CLD student biography as appropriate for the task/activity	I. Pair or group students based on all four dimensions of the CLD student biography as appropriate for the task/activity
AC	J. No connections between the activity and the lesson	J. Makes minimal connections between the strategy/activity and the lesson	J. Makes occasional relevant connections between the strategy/activity and the lesson	J. Frequently uses insights from the strategy/activity to make connections affirm learning, or modify instruction as needed	J. Consistently uses insights from the strategy/activity to make connections, affirm learning, and modify instruction as needed
LE= Learning Environment TC= Teacher Collaboration TPSI= Total Group, Partner, Small Group, Individual PGD= Partner/Grouping Determination; AC= Activity Connections					

II. Language & Literacy Development

	Not Observed 0	Emerging 1	Developing 2	Enacting 3	Integrating 4
		The teacher provides:	*The teacher provides:*	*The teacher provides:*	*The teacher provides:*
LSRW	E. Instruction is dominated by teacher talk and students are passive listeners	E. Listening, speaking, reading, & writing (LSRW) activities with minimal opportunities for students' academic language development	E. L, S, R, & W activities with occasional opportunities for students' academic language development	E. Frequent opportunities for student expression and academic language development in activities that integrate L, S, R, & W	E. Consistent opportunities for student expression and academic language development in higher-order thinking activities that integrate L, S, R, & W
QRM	F. No use of questioning (Q), rephrasing (R), or modeling (M) to assist language and literacy development	F. Minimal use of Q, R, or M to assist language and literacy development	F. Occasional use of Q, R, or M to assist language and literacy development	F. Frequent use of purposeful Q, R, and M to assist language and literacy development	F. Consistent use of purposeful Q, R, and M to assist academic language and literacy development and to build students' capacities to pose questions about their own thinking
L1	G. No evidence of native language in environment or instruction	G. Minimal evidence of native language in environment and/or instruction	G. Occasional opportunities for students to use their native language during the lesson	G. Frequent, explicit, purposeful opportunities for students to use their native language during the lesson in ways that support academic learning	G. Consistent, systematic opportunities for students to use their native language during the lesson in ways that support academic language and literacy development
LBK	H. No references to students' prior knowledge and background experiences related to language and literacy development*	H. Minimal references to prior knowledge and background experiences related to language and literacy development*	H. Occasional references to prior knowledge and background experiences related to language and literacy development*	H. Frequent references to prior knowledge and background experiences related to academic language and literacy development*	H. Consistent use of students' culture-bound ways of comprehending, communicating, and expressing themselves as a springboard for academic language and literacy development*

Notes:
*PA = Phonemic Awareness; P = Phonics; V = Vocabulary; F = Fluency; C = Comprehension

LSRW = Listening, Speaking, Reading, Writing QRM = Questioning, Rephrasing, Modeling L1 = Native Language LBK = Background Knowledge of Language/Literacy

III. Contextualization

	Not Observed 0	Emerging 1	Developing 2	Enacting 3	Integrating 4
		The teacher:	*The teacher:*	*The teacher:*	*The teacher:*
BK3	D. No pre-assessment of students' academic knowledge about the topic	D. Conducts pre-assessment of only students' academic knowledge about the topic	D. Conducts pre-assessment of students' funds of knowledge, prior knowledge, and academic knowledge about the topic or key content vocabulary	D. Conducts pre-assessment that provides all students the opportunity to share/document their funds of knowledge, prior knowledge, and academic knowledge about the topic or key content vocabulary	D. Conducts pre-assessment that provides all students the opportunity to share/document their funds of knowledge, prior knowledge, and academic knowledge about the topic and key content vocabulary; teacher documents students' background knowledge for use throughout the lesson
A/CL	E. Focus is solely on content delivery	E. Provides minimal opportunities for students to share with peers content-related connections to their background knowledge	E. Provides occasional opportunities for students to share with peers content-related connections to their background knowledge	E. Provides frequent opportunities for students to share/document their content-related connections to their background knowledge and purposefully listens/observes as students share/document	E. Provides consistent opportunities for students to share/document their content-related connections to their background knowledge and uses insights gleaned to highlight student assets, support academic learning, and maximize the community of learners
BIO	F. New information is presented in an abstract, disconnected manner	F. Makes minimal connections between students' sociocultural, linguistic, cognitive, and academic dimensions and new academic concepts	F. Makes occasional connections between students' sociocultural, linguistic, cognitive, and academic dimensions and the new academic concepts	F. Makes frequent and purposeful connections between students' individual biographies, including what was learned about their knowledge and experiences from home, community, and school, and the new academic concepts	F. Systematically makes consistent and purposeful connections between students' individual biographies, including what was learned about their knowledge and experiences from home, community, and school, and the new academic concepts, with applications to the real world

BK3 = Funds of Knowledge (family), Prior Knowledge (community), Academic Knowledge (school) A/CL = Assets/Community of Learners BIO = CLD Biography Connections

IV. Challenging Activities

	Not Observed 0	Emerging 1	Developing 2	Enacting 3	Integrating 4
		Teacher instruction and strategy use:	*Teacher instruction and strategy use:*	*Teacher instruction and strategy use:*	*Teacher instruction and strategy use:*
ACOM	F. No accommodations for linguistic or academic levels	F. Provides minimal accommodations based on students' linguistic and academic levels	F. Provides occasional, structured accommodations based on students' linguistic and academic levels	F. Provides frequent, structured accommodations based on students' linguistic and academic levels that build upon culture-bound patterns of knowing, learning, and applying	F. Provides consistent, systematic, structured accommodations based on students' linguistic and academic levels that build upon culture-bound patterns of knowing, learning, and applying
CO/LO	G. Makes no reference to lesson objectives	G. Includes verbally stated or posted lesson objectives that reflect content standards	G. Includes verbally stated and posted content and language objectives that reflect content standards	G. Includes content and language objectives that (1) are verbally stated and posted, (2) reflect content and language standards, and (3) are revisited during the lesson	G. Includes content and language objectives that (1) are verbally stated and posted, (2) reflect content and language standards, and (3) are interwoven throughout the lesson
S/E	H. Strategies/activities are not aligned to standards and do not reflect expectations	H. Includes strategies/ activities that are aligned to standards and that reflect vague expectations	H. Includes strategies/ activities that are aligned to standards and that reflect clear expectations	H. Includes challenging strategies/ activities that are aligned to standards and that reflect clear expectations	H. Includes challenging strategies/ activities that reflect skillful integration of multiple standards, clear expectations, and higher-order thinking skills
AF	I. Does not consider students' states of mind/affective filter	I. Minimally attends to students' states of mind/affective filter	I. Occasionally monitors students' states of mind/affective filter and adjusts instruction accordingly	I. Frequently monitors students' states of mind/affective filter and adjusts instructional conditions accordingly	I. Consistently monitors the states of mind/affective filter of individual students and of the whole group and adjusts instructional conditions and situations accordingly
FB	J. Provides no feedback on student performance	J. Provides minimal feedback on student performance	J. Provides occasional feedback on student performance to confirm/disconfirm learning	J. Provides frequent feedback on student performance to confirm/disconfirm learning and to advance student learning	J. Uses systematic formative assessment to provide consistent feedback on student performance to confirm/disconfirm learning and to advance student learning

ACOM = Accommodations CO/LO = Content Objectives & Language Objectives S/E = Standards/Expectations AF = Affective Filter FB = Feedback (formative assessment)

Adapted from CREDE (1999) Standards for Effective Pedagogy and Learning

V. Instructional Conversation

	Not Observed 0	Emerging 1	Developing 2	Enacting 3	Integrating 4
		With individuals and small groups of students, the teacher:	*With individuals and small groups of students, the teacher:*	*With individuals and small groups of students, the teacher:*	*With individuals and small groups of students, the teacher:*
ESTK	F. Lecture predominates	F. Uses questioning to elicit student talk	F. Elicits student talk with questioning, listening, and rephrasing	F. Elicits student talk with questioning, listening, rephrasing, and explicit modeling of turn-taking and questioning structures	F. Elicits student talk about the content through student-led discussion and questioning
KTU	G. Teacher responds in ways that validate students	G. Responds in ways that minimally promote higher-order thinking and individual connections from the known to the unknown	G. Responds in ways that occasionally promote higher-order thinking and individual connections from the known to the unknown	G. Responds in ways that frequently promote higher-order thinking and individual connections from the known to the unknown	G. Responds in ways that consistently promote higher-order thinking, elaboration of connections from the known to the unknown, and application beyond the classroom
BICS/ CALP	H. Teacher conversation is not on topic	H. Uses BICS and/or CALP to discuss the content/topic; provides minimal opportunities for academic talk among students	H. Uses CALP to discuss the content/topic and provides occasional opportunities for academic talk, including use of key content vocabulary, among students	H. Provides frequent opportunities for academic talk, including use of key content vocabulary, in which the teacher bridges between student talk and academic language	H. Facilitates consistent opportunities for student-led academic conversations using key content vocabulary
REV	I. Incorporates no revoicing of students' learning	I. Includes minimal revoicing of learning, limited to repeating students' words	I. Includes occasional revoicing of learning, limited to repeating and/or rephrasing	I. Includes frequent revoicing of learning that challenges students to solidify or expand upon connections to the academic content and vocabulary	I. Includes consistent revoicing of learning that challenges students to solidify, expand upon, and make deeper connections to the academic content and vocabulary
SAV	J. Does not invite students to articulate their views/judgments/ processes	J. Provides minimal opportunities for students to articulate their views/ judgments/processes	J. Provides occasional opportunities for students to articulate their views/ judgments/processes and provide rationales	J. Provides frequent, purposeful opportunities for students to articulate their views/judgments/processes and provide rationales	J. Provides consistent, structured opportunities for students to articulate their views/judgments/processes and provide rationales

ESTK = Eliciting Student Talk KTU = Known to Unknown BICS/CALP = Basic Interpersonal Communication Skills/Cognitive Academic Language Proficiency
REV = Revoicing SAV = Students Articulate Views