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Grouping Techniques to Manage Large-Scale Multi-Item Multi-Echelon Inventory Systems

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Grouping Techniques to Manage Large-Scale Multi-Item Multi-Echelon Inventory Systems

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ABSTRACT

Large retail companies operate large-scale systems which may consist of thousands of stores. These retail stores and their suppliers, such as warehouses and manufacturers, form a large-scale multi-item multi-echelon inventory supply network. Operations of this kind of inventory system require a large number of human resources, computing capacity, etc.

In this research, three kinds of grouping techniques are investigated to make the large-scale inventory system "easier" to manage. The first grouping technique is a network based ABC classification method. A new classification criterion is developed so that the inventory network characteristics are included in the classification process, and this criterion is shown to be better than the traditional annual dollar usage criterion. The second grouping technique is "NIT" classification, which takes into consideration the supply structure of the inventory item types. In order to have similar operations-related attributes for items within the same group, a network based K-Means clustering methodology is developed to cluster items based on distance measures. It is believed that there is no single best model or approach to solve the problems of the complex multi-item multi-echelon inventory systems of interest. Therefore, some combinations of different grouping techniques are suggested to handle these problems.

The performance of the grouping techniques are evaluated based on effectiveness (grouping penalty cost and Sum of Squared Error) and efficiency (grouping time). Extensive experiments based on 1,024 different inventory system scenarios are carried out to evaluate the performance of the ABC classification, NIT classification, and the K-Means clustering techniques. Based on these experimental results, the characteristics of the 3 individual grouping techniques are summarized, and their performance compared. Based on the characteristics and

performance of these grouping techniques, suggestions are made to select an appropriate grouping method.

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1 Introduction

Large-scale retail systems usually consist of thousands of retail stores. These retail stores and their suppliers, such as warehouses and manufacturers, form a large-scale inventory supply network. They can be deemed as including several echelons, such as the retailer echelon, warehouse echelon, and manufacturer echelon, etc. To satisfy end customer demand, each store keeps a wide variety of items.

The inventory system of interest in this research is motivated by some real world business situations that can be commonly found in some large-scale retail systems. The structure of this system can be abstracted as shown in Figure 1.

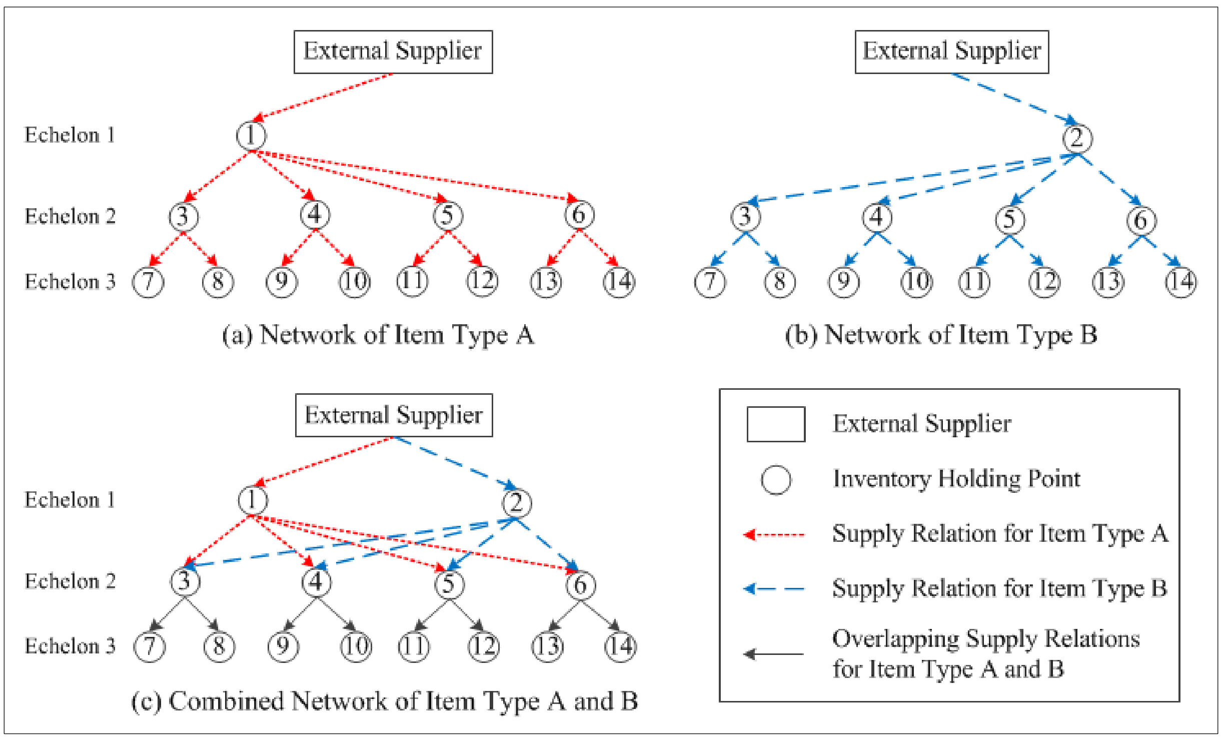

Figure 1: Multi-Echelon Inventory System

Suppose a company based in the US owns a large-scale supply chain system and resources from two different foreign countries for different items. It is also assumed that the company utilizes multiple suppliers. All the suppliers are abstracted as a single external supplier, for the following reasons: 1) the suppliers' inventories are not controlled by the company, therefore, for modeling convenience it is assumed that the inventory at the external supplier does not need to be represented in the system, 2) the characteristics of the external supplier are not significant in the problem solution process, and 3) external suppliers are assumed to have infinite supply of items; which means that the orders made to external suppliers can be shipped after the lead time for the corresponding item type. In other words, the lead time at the external supplier level includes any production or waiting delays to meet the demand.

An inventory holding point (IHP) is a location that stores inventories. Since the inventory at the external supplier is not controlled by the company, the external supplier is not considered as an IHP. A group of IHPs that share the same supply functions can be deemed as located at the same echelon of the supply network. The customer location is supposed to be located at a lower echelon than its supplier location. The echelon number for an IHP is the supply location's echelon number plus one. The external supplier is treated as located at echelon zero. In this research, when referring to N-echelon inventory system, N means the number of echelons excluding the external supplier echelon. As shown in Figure 1, the IHPs can be separated into three echelons. In the aforementioned scenario, to leverage consolidation practices for the reasons of high transportation costs, etc., the company builds two warehouses, one in each foreign country. These warehouses are represented as IHP 1 and 2 respectively in Figure 1, which are located at echelon 1. These warehouses supply different items to the regional distribution centers (DCs) in the US. These DCs are located at echelon 2, and each of them supplies a number of retail stores located geographically close to the DC. The retail stores are located at echelon 3. Some specific customer-supplier relations that may be found, such as the direct supplying from the External Supplier to a retail store are not considered in this research.

Each item type is stored at multiple locations. Based on the supply-customer relations, the locations holding the same item are connected to form a supply network, which is called Network of Item Type (NIT). Figure 1 shows two examples of NITs, Network of Item Type A and Network of Item Type B. It is supposed that the warehouses store different item types since the company does not resource the same item from different foreign countries. Based on this assumption, it can be seen from (c) in Figure 1 that NIT A and B do not share the same IHPs at echelon 1, and may share the same IHPs at echelon 2 and 3. This is consistent with the fact that the company may use the same domestic supply networks for different items. Also, in a specific NIT, each IHP has only one supply location and may have multiple customer locations. All the NITs combined form the network of inventory system (NIS); this means that each NIT is a sub-network of the NIS.

If this kind of multi-item multi-echelon supply network includes thousands of item types, it forms a large scale multi-item multi-echelon supply network that may have thousands of stock keeping units (SKUs). A SKU is an item type stocked at a particular location within the supply chain. For large scale multi-item multi-echelon supply networks, it may not be practical to determine the optimal inventory policy for each individual SKU due to several reasons: (1) it is too time consuming to calculate the optimal policy for each SKU; and (2) the implementation of the resulting optimal inventory control policies may require a large amount of management and other inventory control related resources.

From the large scale inventory systems management perspective, the management of inventory via classification/clustering can be categorized into two directions: (1) importance-based classification, and (2) operation-based clustering. The importance-based classification methods prioritize the item types and then put more effort into controlling important items, and less effort into controlling less important items. The 1st research direction tries to alleviate the large-scale inventory control problem by spending less time and energy on less important items, but typically it does not consider grouping from the inventory cost perspective. The operation-based clustering methods cluster the items with similar characteristics and implement the same group inventory control policy for items in the same group. Research predicated on operation-based clustering methods groups the items from the inventory cost perspective, i.e., implementing the same group inventory control policy for items in the same group assumes that grouping will not unduly increase the inventory cost. This second research direction does not identify the important items; therefore, it treats each item as equally important. To the best of our knowledge, only Zhang et al. (2001) and Teunter et al (2009) have tried to cluster the items from both importance and cost perspectives. However, both of these approaches are conducted based on a single location rather than grouping the item types from an inventory network perspective.

It should be noted that, in the literature, there is a lack of articles on clustering the item types from an inventory network perspective. The goal of this research is to effectively and efficiently group the item types from the network perspective so that the important items are identified, and the system size is reduced to a manageable scale without unduly sacrificing the quality of performance calculations and policy setting decisions.

The following system characteristics and relationships are assumed throughout this research.

1. The external supplier has an infinite supply of items, the inventory at the external supplier is not controlled by the company, and the external supplier is treated as located at echelon zero.

2. Each IHP is supplied by an IHP that is located at the immediate higher echelon, except those IHPs located at the first echelon, which are supplied by the external suppler.
3. The echelon number of a location is 1 plus the echelon number of its immediate supply location.
4. A location has only one supplier location, and one or multiple customer locations.
5. The time between demand arrivals are non-negative random variables.
6. The lead time at the External Supplier is a non-negative random variable.
7. The transportation time is a non-negative random variable.
8. The order handling time at an IHP can be neglected.
9. When an order is not filled, it is lost; the back order case is not considered.
10. End customer demands are satisfied by the retail locations which are the lowest echelon IHPs.
11. The retail stores are independent and non-identical.

The assumptions 1 to 4 are structure related assumptions, assumptions 5 to 7 are related to random variables, and assumptions 8 to 11 are relevant to ordering processes.

Since the large-scale datasets for the problem of interest are not available from the literature and cannot be conveniently obtained from industry, and real data does not permit experimental control of problem characteristics, an efficient data generation procedure is developed in this research that satisfies aforementioned assumptions to provide data for experimentation purposes.

As it can be seen from above discussions, this research assumes that a large-scale multi-item multi-echelon inventory system can be effectively and efficiently managed/controlled by reducing its size relying on appropriate grouping methodologies. This research studies three different grouping methodologies. The first one relates to ABC classification, which is widely used in industry. A new ABC classification criterion is developed and shown to be better than the annual dollar usage approach. The second one is an innovative grouping methodology based on NIT to reduce the size of the large-scale problem. In order to have similar operation related attributes for items within the same group, K-Means clustering is studied in this research to cluster items based on distance measures.

The general research questions in this research can be summarized as follows:

Q1: What is the best way to represent the system in a mathematical and computer data structure format to facilitate analysis of the grouping methods?

Q2: What is the most appropriate method to generate large-scale datasets that represent inventory systems and will facilitate the testing of grouping methodologies?

Q3: What system characteristics should be used during the application of grouping and clustering methods? How should these characteristics be represented mathematically?

Q4: What is the best method for importance-based classification from the network perspective?

Q5: What is the best method for operation-based clustering from the network perspective?

Accordingly, this dissertation is organized as follows: Chapter 2 reviews the literatures related to the problem and the problem solving processes; Chapter 3 discusses the research methodology; Chapter 4 discusses the modeling and quantification of the large scale multi-item multi-echelon inventory network system of interest; Chapter 5 investigates the research factors and their levels, and discusses the experimental design for this research; Chapter 6 analyzes the experimental results of the ABC classification and the K-Means clustering, and compares the three individual grouping techniques developed; and Chapter 7 is the conclusions, suggestions, and future work.

2 Literature Review

For grouping the items in a large scale inventory system, it is imperative to identify the characteristics of the system that holds the large scale inventory items. Further, to appropriately group the items according to the inventory management goals, the system and the item characteristics should be categorized so that a systematic grouping methodology can be applied. This indicates that selecting a set of grouping attributes that impact the effectiveness and efficiency of the grouping procedure is the first step of the grouping process. The next step is to select appropriate grouping techniques. As part of the grouping technique selection process, the evaluation of the grouping techniques according to the quality of resulted groups should be carried out. Ernst and Cohen (1990) point out that "clusters obtained from different data samples may exhibit large differences in attribute centroids". Thus, it should also be noted that for grouping the items in this large scale and complex inventory system properly, the system characteristics and the item attributes need to be quantified so that the quantitative grouping techniques can be applied. Therefore, the system quantifying tools need to be carefully selected. The following presents the literature review on the system characteristics and item grouping attributes, grouping techniques, evaluation of grouping techniques, and the data modeling and data generation.

2.1 System Characteristics and Grouping Attributes

A system characteristic is an evaluation criterion that can be used to categorize systems. Cohen et al. (1986) summarizes the characteristics related to the multi-echelon inventory system as: 1) number of products, (2) number of echelons, (3) network structure (series, arborescence, general), (4) reparable versus non-reparable items, (5) product family relations (multi-indentured assemblies, market groups), (6) periodic versus continuous review, (7) cost/service tradeoff measures, (8) demand process class, and (9) lead time and distribution mechanisms. This indicates that large-scale multi-item multi-echelon supply chain networks require large amounts of data to thoroughly describe the system. It also means that the system characteristics need to be carefully taken into consideration in the modeling process, and reflect the characteristics and the relations between these characteristics quantitatively. The system characteristics plus the attributes of items in the system need to be categorized according to the grouping goals and the grouping techniques applied. This research examines the system characteristics summarized by Cohen et al. (1986) and extends the system characteristics considered according to grouping needs that can be applied to networks of item types.

2.1.1 Structural and Non-Structural Attributes

An attribute representing the supply structure of an item type is deemed as a structural attribute of that item. Item attributes that do not participate in defining an item type’s supply structure are considered as non-structural attributes in this research. The structural and non-structural attributes should be thoroughly investigated to be able to select an appropriate set of grouping attributes. The method of grouping item types based on traditionally used attributes is not sufficient to support the network level inventory management practice since network structure related attributes are not considered.

Lenard and Roy (1995) criticize the existing inventory models since they are, to a large extent, disconnected to the existing inventory practice; therefore, they try to facilitate the decision making process in inventory control using a multi-criteria approach. They firstly apply the mono-item inventory control model to determine the inventory policies based on efficient policy surfaces and then extend this model to multi-item model by grouping items into functional groups using a structure of attributes. They categorize three different levels of attributes, which are (1) attributes on which differences between items prevent the grouping of these items; (2)

attributes on which differences between items weaken the grouping; and (3) attributes which are particularly useful for the inventory manager. They discuss two attributes that prevent grouping: (1) the storage structure; and (2) the strategic importance of the items. The storage structure prevents items to be grouped together since the function of warehouses is different at each echelon. In addition, the decision would be different for strategic and non-strategic items; thus, the strategic importance of the items prevents items to be grouped together. The authors point out that there are attributes, such as demand dispersion and lead time of the item types, on which differences between items weaken the grouping, and there are three attributes useful to the practitioner, i.e., the nature of the items, the supplier and the existence of functional groups. The authors build the families of items using the first five attributes. For each item family, an aggregate item is built, the parameters of which are the synthesis of the main characteristics of the items in the family. Every item in the same family applies the same inventory policy as the aggregate item.

The attribute categorization proposed by Lenard and Roy (1995) provides guidance to choose a combination of grouping attributes in the grouping framework suggested in this research. It should be noted that the NIT concept introduced in the previous section is regarded as a structural attribute, since it defines the structure of the supply network for the item, and since the supply-customer relations between the locations defined by the NIT correspond to the functions between the warehouses and their supplier locations, and functions between the warehouses and their customer locations described by Lenard and Roy (1995). The number of locations is an attribute of NIT since it is used to define NIT; therefore, it is not independently considered as a structural grouping attribute in the grouping process in this research. In the literature, the NIT as a characteristic of an item type is not considered in item type grouping processes. One of the suggestions of this dissertation is the NIT classification which is the first item grouping technique based on a structural attribute (NIT). In this research, all the grouping attributes other than NIT are deemed as non-structural attributes, which are discussed in Section 2.2, together with the grouping techniques since they are selected based on the grouping techniques applied. These attributes are categorized as non-structural attributes because they are not decided by the supply network structure.

2.2 Grouping Techniques

The grouping techniques can be classified into two main categories: grouping techniques based on importance related attributes, and grouping techniques based on operations related attributes. The former one is to identify importance of item types so that the items can be prioritized in the management process, and the latter one is to group item types with similar operational significance together to support inventory control practice.

2.2.1 Grouping using Importance Related Attributes

In inventory management practice, management is interested in identifying the most important items that have the most significant impact on the inventory cost, so that the management resources can be used optimally. In this process the grouping attributes need to be selected according to the grouping goals.

The ABC analysis is the most widely applied technique to identify important item types. The detailed illustration of the ABC technique can be found in Silver et al. (1998). From the number of classification criteria perspective, the ABC classification can be classified into three categories: (1) traditional single criterion; (2) multiple criteria; and (3) single criterion considering optimization models. The traditional single criteria ABC analysis considers the annual dollar usage, which is the multiplication of average unit cost and annual demand, as the only clustering criteria (Cohen and Ernst 1988). Criticality is another widely used attribute that relates to the importance of the item type. Criticality reflects the consequences incurred by not being able to deliver a spare part on time (Van Kampen et al, 2012). The failure of delivering a critical item will have significant impacts, such as endangering the safety of personnel, etc. The traditional single criteria ABC analysis has several drawbacks, such as over-emphasizing the importance of the item types that have high annual cost but are not important from the operational perspective and under-emphasizing the important items that have low annual cost (Flores et al. 1992). In addition, the traditional single criterion ABC analysis does not consider optimizing the inventory policy parameters for item groups (Zhang et al 2003).

Flores and Whybark (1986) suggest that more than one criterion should be considered in the ABC classification, such as lead time, criticality, commonality, obsolescence, substitutability, and reparability. Besides criticality, Ramanathan (2006) also summarizes the importance related attributes used in ABC classification as inventory cost, lead time, commonality, obsolescence, substitutability, number of requests for the item in a year, scarcity, durability, reparability, order size requirement, stockability, demand distribution, and stock-out penalty cost. The multiple criteria ABC analysis is carried out using different techniques, such as analytic hierarchy process (AHP) (Flores et al. 1992) and meta-heuristics (Guvenir and Erel 1998).

For the third type of ABC analysis, the classification criterion is related to optimization models. Zhang et al. (2001) develop a procedure to combine the processes of classifying items into groups and optimizing the inventory policy parameters for groups. They formulate the inventory control problem as minimizing inventory investment subject to constraints on average service level and replenishment frequency. They derive an expression for reorder points, through which suggest a categorization scheme and a classification criterion. The classification criterion is an expression composed of unit cost, replenishment lead time and demand. The higher values of the classification criterion indicate the higher service levels. The authors use the classification criterion to divide items based on an ABC classification technique. Each item group applies the same service constraint and order frequency, and various approximations are implemented to calculate stocking policies. Through several numerical examples, the authors verify the proposed clustering scheme does not have large errors, i.e., within 15% of the lower bound on the optimal average inventory investment.

The disadvantage of the method applied by Zhang et al. (2001) is that the importance related attributes are not considered during the ABC classification. To fill this gap, Teunter et al. (2010) develop a cost criterion based on a cost minimization approach to minimize total inventory cost while satisfying the constraint on average fill rate over all SKUs. Their cost criterion involves both an importance related attribute, i.e. shortage cost (criticality), and operations related attributes, i.e., demand rate, inventory holding cost and order quantity. The intuition of choosing the cost criterion, which comes from the approximate newsboy-type optimality condition for each SKU, to minimize the total cost is that the service level for an SKU is increasing in the ratio of the cost criterion. The advantage of this kind of ABC classification is that several related parameters are organized in a single classification criterion so that complex multi-criteria ABC classification methods are avoided. After classifying the SKUs into SKU groups, the authors use the Solver tool in Excel to find the cycle service levels for each group that minimize the total inventory cost for all SKUs while satisfying the target fill rate. Through a numerical experiment using three real life datasets, Teunter et al. (2010) verify that the cost criterion consistently performs better than other methods, i.e. the method of Zhang et al. (2001) and the traditional ABC classification, across the datasets.

Zhang et al. (2001) and Teunter et al. (2010) develop the classification criterion to cluster SKUs at one location. Inspired by their approaches, this dissertation develops a new network-based cost criterion to identify important item types through ABC classification for multi-echelon problems.

It can be noted that the traditional single location ABC classification techniques have some disadvantages. They focus on prioritizing items, but it does not guarantee the items in the same group to have similar operation related characteristics. They "may provide unacceptable performance when evaluated with respect to cost and service measures in complex inventory environments" (Ernst and Cohen, 1990); in other words, they cannot guarantee that applying the group reorder policy for SKUs in the same group will not unduly sacrifice the quality of performance calculations. In addition, the maximum number of clusters in ABC classification is usually limited to six (Silver et al. 1998).

2.2.2 Grouping Using Operations Related Attributes

Ernst and Cohen (1990) believe that beyond the traditional cost and volume attributes used in ABC analysis, all product characteristics which have a significant impact on the particular operations management problem of interest should be taken into consideration to satisfy the objective of supporting strategic planning for the operations function. The authors point out that deciding inventory policies based on individual SKUs is both computationally and conceptually impractical since it is difficult to monitor and control system performance from a strategic perspective. These indicate that item types in an inventory system should be clustered into similar groups considering operations related attributes. The operations related attributes are those attributes that are used to determine the inventory control policies. Van Kampen et al (2012) classify these characteristics into four categories: (1) volume, (2) product, (3) customer and (4) timing. The volume category includes the demand volume and the demand values. The demand value refers to the multiplication of demand and unit cost. The unit cost and lead time are commonly attributed to the product category. The third category, considering the importance of customer, is not frequently used in the clustering. According to Van Kampen et al. (2012), the fourth category has received little research attention, and the most notable attribute in this category that is used in clustering is inter-demand interval. This dissertation examines the operations related attributes in volume and product categories. The importance and operations related attributes are not exclusive to each other, rather they are the grouping attributes that are selected according to the grouping goal; this means that some of the attributes from the both categories can be applied in a certain grouping process together. Also, some attributes, such as unit cost and demand volume, can be categorized as either an importance related attribute or an operations related attribute according to the grouping objective.

In order to provide better operational performances after the grouping process, Ernst and Cohen (1990) develop an ORG (Operations Related Groups) methodology to cluster items. Taking into account all item attributes that significantly affect the operational goals, the ORG methodology can be summarized into two stages. At the first stage, the number of the groups is determined by an optimization model that minimizes the total number of groups subjecting to a constraint on the maximum operational penalty. After the number of groups is determined at the first stage, the second stage is to partition the SKUs into groups. This stage includes two steps: (1) use discriminant analysis of original variables to select the clustering variables that significantly affect determining the final groups; and (2) based on the selected clustering variables, apply the membership selection rules to group SKUs. The basic idea of membership selection rules is to reproduce the classification by minimizing the generalized squared distance between the new observation and the mean of the group centroid. The experiments conducted for the inventory system of an automobile manufacturer have shown that applying ORG methodology has superior SKU performances than implementing traditional ABC method.

Similar to Ernst and Cohen (1990), Rossetti and Achlerkar (2011) also apply statistical clustering to group items, but they attempt to solve the clustering problem and the policy-setting problem at the same time. The inventory control model in Rossetti and Achlerkar (2011) is to minimize the total inventory holding cost subjecting to expected annual order frequency and expected number of backorder constraints. The methodology presented in Hopp and Spearman (2001) to set the inventory policies, an iterative procedure that first satisfies the average order frequency constraint and then the backorder level constraint, is applied to determine the optimal reorder point and reorder quantity for SKUs. Considering the inventory control model during the clustering process, Rossetti and Achlerkar (2011) develop two clustering methodologies: Multi-Item Group Policies (MIGP) inventory segmentation and Grouped Multi-Item Individual Policies (GMIIP) inventory segmentation. The MIGP inventory segmentation method groups inventory items and determines an inventory policy for each group by applying the optimization model to the groups. The parameter of the group is determined by the mean attribute values of items in the group. Each item within the same group uses the same group policy determined for that group. Compared to MIGP, the GMIIP inventory segmentation method calculates individual inventory policies for every item within the groups. The main clustering method used in the paper is the Unweighted Pair Group Method Using Arithmetic Averages or the UPGMA clustering method described in Romesburg (1984), and the K-means clustering algorithm is also examined during the experimentation. The experimental results show that the MIGP procedure reduces the computation time to set the policies significantly, but causes a lack of identity for the items and incurs a large penalty cost compared to individual policy setting procedures. The GMIIP procedure results in closer inventory policy parameters compared to individual policy setting procedure, but more computation time is required. Both segmentation procedures developed perform better than ABC method from the perspectives of costs and service, but they need more computation time.

The operations related attributes, which are non-structural attributes, can be expressed as decimal values and their similarity is usually measured using Euclidean distance. The available clustering methods for grouping item types based on these attributes are K-Means (KM) algorithm, genetic algorithm (GA), simulated annealing algorithm (SA), tabu search (TA) algorithm, etc. Al-Sultan and Khan (1996) compare the computational performance of KM, GA, SA and TA for the clustering problem. They test these algorithms on several datasets and conclude that KM is faster than the other three algorithms by a factor that ranges from 400-5000. In addition, Maimon and Rokach (2005) summarize that only the KM and its equivalent have been applied to grouping large scale datasets. Maimon and Rokach (2005) summarize three main reasons for the popularity of K-Means algorithm: 1) the time complexity of K-Means algorithm is $O(mkl)$, where m is the number of instances; k is the number of clusters; and l is the number of iterations used by the algorithm to converge; 2) the space of K-Means algorithm is $O(k+m)$; and 3) the K-Means algorithm is order-independent. Since only K-Means is recommended for grouping large scale datasets, this dissertation applies K-Means techniques to cluster items.

Similar to Ernst and Cohen (1990) and Rossetti and Achlerkar (2011), the clustering method suggested in this dissertation also examines the effects of clustering attributes on the system performance related to the inventory management goal (such as clustering penalty cost and clustering time) and uses statistical clustering to group item types. The major difference is that this dissertation not only considers the non-structural attributes, but also structural attribute, i.e. NIT, during the clustering process. Further, this research focuses on the multi-echelon inventory system rather than the single location case.

2.2.3 Grouping of NIT

To define the similarity of NITs so that they can be grouped together accordingly, it is necessary to first model (represent) the NITs, which are decided by the structure of the inventory supply network. While the values of non-structural attributes can be represented using a number, data structures may be needed to describe the structural attributes. It should be noted that, different representations of the non-structural attributes may need different grouping techniques according to different grouping goals. Also, the different representation of NIT itself may lead to different NIT grouping techniques.

The NIT can be modeled using graph theory or mathematical expression. From the graph theory perspective, NIT can be represented using a tree, where the nodes represent locations and arcs represent the supply relation. Graph clustering has received a lot of attention lately. It is used to partition vertices in a graph into separate clusters based on measures such as vertex connectivity or neighborhood similarity. Zhou et al. (2009) point out that “a major difference between graph clustering and traditional relational data clustering is that, graph clustering measures vertex closeness based on connectivity (i.e., the number of possible paths between two vertices) and structural similarity (i.e., the number of common neighbors of two vertices); while relational data clustering measures distance mainly based on attribute similarity (i.e., Euclidian distance between two attribute vectors)”. The algorithms for attributed graph clustering can be generally categorized into two types, distance-based and model-based (Xu et al. 2012).

Zhou et al. (2009) suggest a graph clustering algorithm that uses both structural and attribute similarities through a unified distance measure. They try to combine the structural and attribute similarities into a unified framework through graph augmentation; which is implemented by inserting a set of attribute vertices, each of which holds the attributes that appear in the graph, into the original graph, and then connecting each of these inserted attribute vertices to the other vertices if they have the same attribute value. Based on the augmented graph, the vertex closeness is estimated based on their proposed model, and then graph clustering based on the random walk distance is performed.

Rather than relying on artificial design of a distance measure, Xu et al. (2012) suggest a model based approach to attributed graph clustering. In this model, which is a Bayesian probabilistic model, a principled and natural framework is proposed for capturing both structural and attribute aspects of a graph. The authors point out that clustering with the proposed model can be converted into a probabilistic inference problem, for which the variational algorithm they suggest is efficient and the proposed method significantly outperforms the state-of-art distance-based attributed graph clustering method.

Both of the attribute graph clustering methodologies discussed above are not directly applicable to the specific inventory system management problem this research is dealing with due to the following reason: the structural attribute in this research is NIT, which is a network that connects locations based on the supplier-customer relations. In this kind of graph, the vertices are locations, and the arcs are the supplier-customer relations. The graph clustering methods proposed in Xu et al. (2012) and Zhou et al. (2009) partition the vertices. Applying their methods means the locations in an NIT will be partitioned rather than the item types, which are the clustering objects in this research. This means that the supplier-customer relation between the locations regarding an item type will be disconnected if their methods are applied for NIT clustering. To overcome these disadvantages, this research suggests a new classification method based on NIT, i.e., NIT classification method.

2.3 Evaluation of the Grouping Techniques

The resulted item type groups should be evaluated from both the statistical and system optimization perspectives, so that the corresponding clustering techniques are assessed. From a statistical perspective, Tan et al. (2006) summarize five issues for cluster evaluation: (1) determining the clustering tendency of a set of data, i.e., distinguishing whether non-random structure actually exists in the data; (2) determining the correct number of clusters; (3) evaluating how well the results of a cluster analysis fit the data without reference to external information; (4) comparing the results of a cluster analysis to externally known results; and (5) comparing two sets of clusters to determine which is better. Since there are no externally known results to be compared, the 4th evaluation is not performed. All the other cluster evaluations are applied to examine the grouping techniques studied in this dissertation.

From an optimization perspective, Ernst and Cohen (1990) propose the evaluation criteria to measure the clustering effectiveness. They indicate two types of costs which are non-decreasing in the number of groups: (1) the cost penalty for using policies based on groups; and (2) the loss of discrimination (i.e., all items in a group should be similar with respect to their SKU attributes and items in different groups should be different). Also, the cost of using a small number of groups must be balanced against the computational and conceptual benefits of small group numbers. In this dissertation, the loss of discrimination is measured by sum of squared error (SSE). Both SSE and the penalty cost incurred by using policies based on groups are used in this dissertation to evaluate the effectiveness of different clustering techniques.

2.4 Data Modeling and Data Generation

The system characteristics and the item attributes selected based on previous discussions need to be quantified, so that the item types can be clustered using quantitative tools. It should be noted that considering the interactions between the system characteristics and between item types in the large scale multi-item multi-echelon inventory system of interest, not only the system and item attributes need to be quantified but also the relations between these characteristics should be quantified for item grouping purposes. This quantification is implemented using data modeling and data generation in this research.

A data model is an abstract model that is used to show the data created in the business practices. The goal of data modeling is to define the attribute values of data models and relationships between them. It facilitates communication between management and functional departments. Since data models support data and computer systems by providing the definition and format of data, a set of efficient and effective data models obtained by carefully implementing data modeling process is the foundation of a well-organized and well-functioning information system (West 2011). Data modeling is a fundamental task in this research, since it organizes the characteristics of the large-scale multi-item multi-echelon inventory system and the characteristics of the item types, so that the organized system and item characteristics can be used in the data generation and grouping procedures. The rest of this sub-section reviews the literature related to data modeling and data generation.

Rossetti and Chen (2012) develop a Cloud Computing Architecture for Supply Chain Network Simulation (CCAFSCNS) with 10 components to expedite the distributed simulation of large-scale multi-echelon supply chains. The Input Data is one of CCAFSCNS's components and it provides the information for the simulation requirements and the characteristics of a supply chain network. The characteristics considered in their paper are probability distributions, item type, location, shipment, SKU and demand generator. The authors apply the rules of relational database to design the relational tables for each system characteristic to avoid the problems related to redundancy, multiple themes and modification. In their research, they focus on the system characteristics that affect the simulation results. Similar to Rossetti and Chen (2012), this research also applies the rules of relational databases to design the relational tables for each system characteristic. The differences in designing the relational tables between Rossetti and Chen (2012) and this research are that (1) this research takes into consideration the network of item type (NIT) as one of the system characteristics, which means the inventory system of interest is considered as an inventory network formed by multiple IHPs; (2) this research not only considers the system characteristics affecting the system performance, but also the characteristics related to the importance of item types.

This research requires a large amount of data, based on the selected system characteristics, to represent the large-scale multi-item multi-echelon supply network of interest. This kind of data is unavailable in the literature and is not conveniently available from industry. To make the generated data set closely reflect the real world situation and to make it reusable in different research processes using different tools in future work, some data modeling techniques that are generally used in the industry for building information systems are implemented in this research.

Silverston et al. (1997) suggest that two modeling methodologies, top-down and bottom-up, are prominent among the many ways to create data models. In some cases these two methods are used together according to the data characteristics. These two methodologies are summarized as follows:

- Bottom-up models are often the result of a reengineering effort. They usually start with existing data structures forms, fields on application screens, or reports. These models are usually physical, application-specific, and incomplete from an enterprise perspective. They may not promote data sharing, especially if they are built without reference to other parts of the organization (Silverston et al. 1997).
- Top-down logical data models, on the other hand, are created in an abstract way by getting information from people who know the subject area. A system may not implement all the entities in a logical model, but the model serves as a reference point or template (Silverston et al. 1997).

The top-down approach is selected in this research, in which, the real world scenario is constructed first, and then the entities and associations are identified. The diagram that illustrates entity and relationship is called the E-R Diagram. During the data modeling process, the E-R diagram is used to draw the entities and associations.

Based on the E-R diagram, relational theory is used to design relational tables (models) that store attributes of the entities and the relations between the entities. A relational model is a database model based on first-order predicate logic, and it is the most frequently applied technique for the design of data models. The advantages of the relational view of data modeling are summarized by Codd (1970) as follows:

- It provides a means of describing data with its natural structure only -- that is, without superimposing any additional structure for machine representation purposes.
- It provides a basis for a high level data language which will yield maximal independence between programs on the one hand and machine representation and organization of data on the other.

- It forms a sound basis for treating derivability, redundancy, and consistency of relations.
- It permits a clearer evaluation of the scope and logical limitations of present formatted data systems, and also the relative merits (from a logical standpoint) of competing representations of data within a single system.

Based on the designed data models, an efficient data generation procedure is developed in this research to provide the datasets for grouping. Also, the designed data models can be used in practice as a blueprint for inventory system databases.

It should be noted that the relationships between system attributes should be explicitly investigated in the data modeling process, since the changes in the values of some of the attributes may bring changes in values of some other attributes, and these different attribute values could affect the system performance.

Deshpande et al. (2003) investigates how to effectively manage items with heterogeneous attributes and different service requirements. The authors conduct their study on the logistics system used to control the consumable service parts for weapon systems. Through interviews and rigorous analysis of part attribute and performance data, the authors find the service level of an item is negatively affected by its cost and less affected by priority code. Based on the analysis of data sets collected from DLA (Defense Logistics Agency), the authors identify some relationships between the values of the system attributes, such as the negative relationship between item cost and service performance, and the positive relationship between essentiality and criticality, etc. To validate their conclusion, the authors test the significance of the relationships among essentiality, weapon criticality, unit price, production lead time, administrative lead time and demand frequency through regression analysis.

Rossetti and Achlerkar (2011) develop two segmentation methodologies to cluster items in a large-scale multi-item inventory system. In order to evaluate different clustering methods through relative comparisons, they develop a data generation procedure to generate large-scale datasets. According to the authors, there is no such data generation procedure available from the literature. Based on the experience and the findings from Deshpande et al. (2003), Rossetti and Achlerkar (2011) consider the relationships between the attribute values of data models in their data generation process. Their assumptions focus on the direct or inverse proportional relationships between a pair of attributes. To make the data generation more applicable, the authors summarize the relations between average annual demand and other attributes, such as the average annual demand and the unit cost of an item are inversely proportional, and the average annual demand and the desired fill rate of an item are direct proportional, etc. In order to satisfy the assumptions about the relations between the generated values, they use a sequence of conditional probability distributions to randomly generate the attribute values. The authors develop an example specification for generating attributes. They stratify the demand into 3 strata (low demand, medium demand and high demand) with the probability of 33% for each of the strata. When the stratum (one of low demand, medium demand or high demand) is chosen, the value of the demand is generated through a uniform distribution over the stratum's range. The generation process for each of the other attributes follows the same process. The probability of choosing the stratum for other attributes is specified by their relation with annual demand (direct or inverse proportional relation between the attribute and the annual demand).

The assumptions about the relations of attributes mentioned in Deshpande et al. (2003) and Rossetti and Achlerkar (2011) are considered in this research. The main difference between the data generation method in Rossetti and Achlerkar (2011) and the implementation in this dissertation is that this dissertation generates NITs for items and use NITs to facilitate data generation of SKUs and demands.

In Table 1, key findings from the literature are summarized. These key findings correspond to the research questions that were discussed in Chapter 1. The following Chapter 3 through Chapter 6 deals with these questions based on the key findings listed in Table 1. Chapter 7 summarizes the research results according to the research questions.

Table 1: Summary of Key Findings in Literature Review

Section	Research Questions	Key Findings
2.1	Q1	• large-scale multi-item multi-echelon supply chain networks require large amounts of data to thoroughly describe the system • the system characteristics need to be carefully taken into consideration in the modeling process, and reflect the characteristics and the relations between these characteristics quantitatively • the NIT as a characteristic of an item type is not considered in item type grouping processes
2.2	Q3	See following 2.2.1, 2.2.2, and 2.2.3
2.2.1	Q4	• Inspired by Zhang et al. (2001) and Teunter et al. (2010)'s approaches, this research develops a new network-based cost criterion to identify important item types through ABC classification for multi-echelon problems • ABC classification "may provide unacceptable performance when evaluated with respect to cost and service measures in complex inventory environments" (Ernst and Cohen, 1990) • The maximum number of clusters in ABC classification is usually limited to six (Silver et al. 1998)
2.2.2	Q5	• Item types in an inventory system should be clustered into similar groups considering operations related attributes which are used to determine the inventory control policies • Maimon and Rokach (2005) summarize that only the KM and its equivalent have been applied to grouping large scale datasets; therefore, this research applies K-Means techniques to cluster items • Similar to Ernst and Cohen (1990) and Rossetti and Achlerkar (2011), the clustering method suggested in this dissertation also examines the effects of clustering attributes on the system performance related to the inventory management goal (such as clustering penalty cost and clustering time) and uses statistical clustering to group item types. The major difference is that this dissertation not only considers the non-structural attributes, but also structural attribute, i.e. NIT, during the clustering process. Further, this research focuses on the multi-echelon inventory system rather than single location case
2.2.3		• Different representations of the non-structural attributes may need different grouping techniques according to different grouping goals, and the different representation of NIT itself may lead to different NIT grouping techniques • This research suggests a new classification method based on NIT, i.e., NIT classification method
2.3	Q3, Q4, Q5	• Both SSE, which measures the loss of discrimination, and the penalty cost incurred by using policies based on groups are used in this dissertation to evaluate the effectiveness of different clustering techniques
2.4	Q1, Q2	• This research requires a large amount of data, based on the selected system characteristics, to represent the large-scale multi-item multi-echelon supply network of interest. This kind of data is unavailable in the literature and is not conveniently available from industry • There is no such data generation procedure available from the literature • The top-down approach is selected in this research, in which, the real world scenario is constructed first, and then the entities and associations are identified • The assumptions about the relations of attributes mentioned in Deshpande et al. (2003) and Rossetti and Achlerkar (2011) are considered in this research. The main difference between the data generation method in Rossetti and Achlerkar (2011) and the implementation in this dissertation is that this dissertation generates NITs for items and use NITs to facilitate data generation of SKUs and demands

3 Research Methodology

The goal of this research is to effectively and efficiently group the item types from the network perspective so that the item type groups matching the managerial goals are obtained, and meanwhile the system size is reduced to a manageable scale without unduly sacrificing the quality of performance calculations and policy setting decisions. Ratliff and Nulty (1997) point out that "there is no single best approach, best representation, best model, or best algorithm for optimizing logistics decisions". Based on the discussions in Section 2, this point of view applies to the item type grouping problem in the large scale multi-item multi-echelon inventory system of interest. It means that a set of techniques need to be applied to fulfill the goal of this research. These techniques mainly fall into four categories: (1) grouping; (2) inventory control policy optimization; (3) data modeling; and (4) data generation.

From the discussions so far, it can be seen that the total cost after grouping item types in a large scale system is decided by the item types being clustered, the inventory control policy applied, the cost model used to calculate the inventory related costs, the grouping algorithms used to group the items, and the number of groups that the items are divided into. In this research, other than the inventory policy for which the continuous reorder point reorder quantity policy is selected, each of the aspects include a set of options which are compared/tested in the corresponding sections; i.e. a set of different item types, which can be denoted as $N = \{1,2,\dots,n\}$, a set of different cost models, which can be denoted as $M = \{m_1, m_2, \dots\}$, a set of different grouping algorithms, which can be denoted as $A = \{A_1, A_2, \dots\}$, and a set of different number of groups obtained, after grouping, are compared/tested to investigate their impacts on the total costs after grouping. It should be also noted that all these elements affect the grouping time. Using these elements, the goal of this research is summarized using following mathematical formulation on Exhibit 1 to show the different aspects related to this research and these aspects are addressed in the rest of this section.

Minimize: $IC(N, p_i, m_j, A_q, k) - IC(N, p_i, m_j)$

s.t.: $k \leq \bar{k}$

$GT \leq \overline{GT}$

where

$\bar{k}$ ≡the maximum number of groups

GT ≡ grouping time

$\overline{GT}$ ≡ the maximum grouping time

IC ≡ inventory cost

Exhibit 1: Grouping Goal

As shown in Exhibit 1, the goal is to minimize the cost difference between before and after clustering while satisfying the number of groups and computational time constraints based on the elements mentioned above. The increased cost caused by grouping is usually called clustering penalty cost. In practice, the criterion $\bar{k}$ and $\overline{GT}$ could be decided by the inventory control manager according to the company's requirements. It can be seen from Exhibit 1 that this research involves several aspects: (1) modeling item type; (2) selecting inventory control policy p_i; (3) based on the inventory control policy p_i, selecting the model m_j to calculate the inventory related costs; and (4) selecting/developing the clustering algorithm A_q to partition the item types. Also, the way of determining the group policy for the item types within the group needs to be considered.

Since there is no readily available datasets with controllable characteristics that can be used for the grouping process, an effective and efficient data generation procedure is critical to provide the data for grouping. Before designing the data generation procedure, how the system can be represented as data that can be clustered should be determined through data modeling techniques.

Further, the quality of the grouping results need to be tested from statistical and optimization perspectives. Since there is no exact control policy optimization technique available for the multi-echelon inventory system described in this research, the heuristic procedure for setting the optimal inventory control parameters at single location developed in Hadley and Whitin (1963) is extended to the multi-echelon case. This approach is called Extended Hadley and Whitin Solution (EHWS). The main purpose of this optimization technique is to provide a way to relatively compare different grouping methods developed in this dissertation.

It can be seen from the discussions above that the whole research process is an integration of data modeling, data generation, grouping, and optimization procedure as shown in Figure 2.

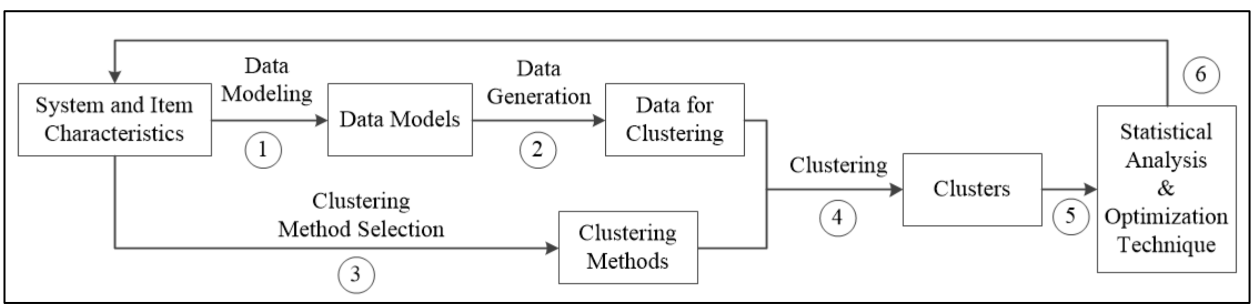

Figure 2: Research Process

The relationships among the elements in Figure 3 are summarized as follows:

1) The system and item characteristics and their relationships are quantified using data modeling; as a result, data models are developed.

2) Data generation methods are developed based on the data models to provide large scale datasets for grouping.

3) The grouping methods are selected for different system and item characteristics.

4) The data that represents the system and item is clustered using grouping methods accordingly.

5) The clusters resulting from the clustering process are evaluated from statistical analysis and optimization perspectives.

6) The grouping evaluation process identifies the attributes that have significant impact on the clustering results. These attributes should be included in the system characteristics.

The key issues regarding the inventory control policy, inventory control models, grouping techniques applied in this research are discussed in Section 3.1, 3.2, and 3.3 respectively. The data modeling and data generation are discussed in Chapter 4.

3.1 Inventory Control Policy

The main policies that are typically used in inventory control practice are (1) order point, order quantity policy (R, Q); (2) order point, order up to level policy (R, S); (3) periodic review, order up to level policy (s, S); and (4) (R, s, S) policy which is a combination of (R, S) and (s, S) policies.

The reorder point reorder quantity policy ((r, Q) policy) is selected in this research for the following reasons: (1) it is easy to implemented and widely used in the industry; (2) it is a common practice to compute order quantity Q and reorder point r separately (Hopp et al. 1997), and (3) calculating reorder points and order quantities separately does not result in large errors (Zhang, Hopp and Supatgiat 2001; Zheng 1992).

3.2 Inventory Control Model

In this sub-section, the four inventory cost models from the literature are compared and the most appropriate one is selected and extended to solve the multi-echelon problem of interest in this research. Some of these models are also used to derive the rules to build the classification criterion in section 3.3.1. For the comparison and review convenience, they are listed as follows:

Model 1: Zhang, Hopp and Supatgiat's Model

Zhang et al. (2001) formulate the inventory control problem as minimizing inventory investment subjecting to constraints on average service level and replenishment frequency. The model is as follows:

Minimize: $\sum_{i=1}^{N} c_i \left(r_i - \theta_i + \frac{Q_i}{2} + \frac{1}{2} + B_i(r_i, Q_i) \right)$

s.t. $\frac{1}{N} \sum_{i=1}^{N} \frac{D_i}{Q_i} \leq F$

$\sum_{i=1}^{N} \frac{D_i}{D_{tot}} (1 - A_i(r_i, Q_i)) \geq \mathrm{S}$

$r_i \geq \underline{r_i}$

$Q_i \geq 1$

$r_i, Q_i: integers$

Where

$N \equiv$ number of items

$c_i \equiv$ unit cost for item i

$D_i \equiv$ expected demand for item i per year

$D_{tot} \equiv \sum_{i=1}^{N} D_i$

$l_i \equiv$ replenishment leadtime for item i

$\theta_i \equiv D_i l_i$, expected demand for item i $during\ lead\ time\ l_i$

$\sigma_i \equiv$ standard deviation of demand during lead time for item i

$Q_i \equiv$ order quantity for item i

$r_i \equiv$ reorder point for item i

$\underline{r_i} \equiv$ preset value for reorder point of the item i

$A_i(r_i, Q_i) \equiv$ probability of stockout for item i

$B_i(r_i, Q_i) \equiv$ expected number of backorders for item i at any time

Exhibit 2: Zhang, Hopp and Supatgiat's Detailed Model

Model 2: Teunter, Babai and Syntetos's Model

Teunter et al. (2010) develop a model to minimize total inventory cost while satisfying the constraint on average fill rate over all SKUs. The model is as follows:

Minimize: $\sum_{i=1}^{N}\left(h_i SS_i - h_i \frac{Q_i}{2} + b_i D_i (1 - FR_i)\right)$

Where

$N \equiv$ number of SKUs

$b_i \equiv$ penalty cost for SKU i

$CSL_i \equiv 1 - \frac{h_i Q_i}{b_i D_i}$, cycle service level for SKU i

$D_i \equiv$ demand per unit time for SKU i

$FR_i \equiv$ fill rate for SKU i

$h_i \equiv$ inventory holding cost for SKU i

$L_i \equiv$ lead time for SKU i

$Q_i \equiv$ (average) order quantity for SKU i

$SS_i \equiv$ safety stock for SKU i

Exhibit 3: Teunter, Babai and Syntetos's Model

Model 3: Hopp and Spearman's Model

Hopp and Spearman (2001) develop a model to minimize the inventory related cost subjecting to order frequency and back order constraints. The model is as follows:

Minimize: $C = \sum_{i=1}^{N} h_i \bar{I}_i(R_i, Q_i)$

s.t. $\frac{1}{N}\sum_{i=1}^{N} \frac{\lambda_i}{Q_i} \leq F$

$\sum_{i=1}^{N} \bar{B}_i(R_i, Q_i) \leq B$

$R_i \geq -Q_i, i = 1,2, \dots, N$

$Q_i \geq 1, i = 1,2, \dots, N$

$Q_i \& R_i: Integers, i = 1,2, \dots, N$

Where

$i \equiv$ Item index

$N \equiv$ number of items

$F \equiv$ Target order frequency

$B \equiv$ Target number of backorders

$\lambda_i \equiv$ Demand rate for Item i

$C \equiv$ Total inventory investment

$h_i \equiv$ Holding cost for Item i

$Q_i \equiv$ Reorder quantity for Item i

$R_i \equiv$ Reorder point for Item i

$\bar{I}_i(R_i, Q_i) \equiv$ Average on hand inventory for Item i

$\bar{B}_i(R_i, Q_i) \equiv$ Expectednumber of backorders for Item i

Exhibit 4: Hopp and Spearman’s Model

Model 4: Hadley and Whitin’s Model

Ernst and Cohen (1990) apply the backorders case inventory model of Hadley and Whitin (1963). In this dissertation, it is assumed that the unfilled demand will be lost; thus, the lost sales case is listed as follows:

$$\text{Minimize: } C(r,Q) = \frac{\lambda}{Q}A + IC\left(\frac{Q}{2} + r - \mu_{LT}\right) + \left(IC + \frac{b\lambda}{Q}\right)\left[\int_r^{\infty}(x-r)h(x)dx\right] \quad (1)$$

Where

$A \equiv$ ordering cost

$b \equiv$ lost sales cost

$C \equiv$ unit cost

$\lambda \equiv$ demand rate

$I \equiv$ inventory holding charge

$\mathrm{h(x)} \equiv$ probability distribution of lead time demand assumed to be normal with parameters (μ_{LT}, σ_{LT})

Exhibit 5: Hadley and Whitin's Model

All of these four inventory cost models are applied to evaluate performance of the grouping algorithms by the authors. The 4th model, Hadley and Whitin's Model, is used to evaluate the performance of the grouping algorithms in this research for the following reasons: (1) this research considers the importance-related attributes, i.e. shortage cost, during the ABC classification process. Since Model 1 and Model 3 do not have importance-related attributes, they are not suitable for this research; (2) for Model 2, the order quantity Q_i is derived based on the historical data, and it is not optimized through the formulations. However, this dissertation needs a model that can optimize reorder point and reorder quantity without referring to historic data. Therefore, Model 4 is selected.

In Model 4, the shortage cost considered in this research is used to measure the criticality of the items. The goal of Model 4 is to find optimal r^* and Q^* to minimize the total cost. If $0 < Q^* < \infty$, $0 < r^* < \infty$, then Q^* and r^* satisfy $\partial C(r,Q)/\partial Q = 0$ and $\partial C(r,Q)/\partial r = 0$. Thus, following two equations can be derived.

$$Q = \sqrt{\frac{2\lambda[A+b*\eta(r)]}{IC}} \qquad (2)$$

$$H(r) = \frac{QIC}{\lambda b+QIC} \qquad (3)$$

Where

$\eta(r) \equiv \int_r^\infty (x-r)h(x)dx$

$H(r) \equiv$ complementary cumulative of $h(x)$

If h(x) is a normal distribution, then the equivalent of (1) can be expressed as:

$$C(r,Q) = \frac{\lambda}{Q}A + IC\left(\frac{Q}{2}+r-\mu_{LT}\right) + \left(IC+\frac{b\lambda}{Q}\right)\left[(\mu_{LT}-r)\Phi\left(\frac{r-\mu_{LT}}{\sigma_{LT}}\right) + \sigma_{LT}\phi\left(\frac{r-\mu_{LT}}{\sigma_{LT}}\right)\right] \qquad (4)$$

Where

$\Phi(x) \equiv$ cumulative distribution fucntion for x

$\phi(x) \equiv$ probability density function for x

The approximate solution is obtained by applying the heuristic iterative procedure developed in Section 4.4 of Hadley and Whitin (1963). The heuristic procedure can be summarized as:

Step 1: Initialize $Q_1 = \sqrt{\frac{2A\lambda}{IC}}$

Step 2: Calculate r_1 using equation (3) and Q_1

Step 3: Calculate Q_2 using equation (2) and r_1

Step 4: Calculate r_2 using equation (3) and Q_2

Step 5: If r_2 is close to r_1, then stop, $r^* = r_2$ and $Q^* = Q_2$; otherwise, let $Q_1 = Q_2$ and $r_1 = r_2$ and go back to Step 3.

Exhibit 6: Heuristic Procedure for Optimization

In order to apply Model 4 to the multi-echelon case in this research, several issues need to be addressed: (a) determining of the mean and variance of lead time demand; (b) determining of the optimal policy for the SKUs at upper echelons; and (c) determining of the group policy. In the following, these issues are discussed.

(a) Determining the mean and variance of lead time demand

In this research, the mean and variance of replenishment lead time will be generated during the data generation procedure. The lead time at the first echelon locations is the lead time to get items from the external supplier. The lead time at the lower echelon locations can be modeled as the transportation time from the upper echelon supplier (for the lost sales case). It is practical to model the lead time demand as a normal distribution (Axsäter 2006). In this case, according to Axsäter (2006) the mean and standard deviation of the lead time demand, which are denoted as μ_{LTD} and σ_{LTD} respectively, can be calculated as follows:

$$\mu_{LTD} = \mu_D E(L) \tag{5}$$

$$\sigma_{LTD} = \sqrt{\sigma_D{}^2 E(L) + \mu_D{}^2 Var(L)} \tag{6}$$

Where

μ_D ≡mean of demand

σ_D ≡standard deviation of demand

$E(L)$ ≡mean of lead time

$Var(L)$ ≡variance of lead time

(b) Determining the optimal policy for the SKUs at upper echelons

The demand at the upper echelon is the aggregate demand of its customer locations. In this dissertation, the customer demand is supposed to be normally distributed for several reasons: (1) it is common to use the normal distribution to model the demand since in many circumstances the demand comes from several independent customers and according to the central limit theorem that a sum of many independent random variables tend to be a normally distributed variable (Axsäter 2006); (2) unlike exponential distribution, normal distribution allows the user to determine the mean and variance of the customer demand respectively; and (3) the sum of two or more mutually independent normal random variables is still a normal random variable. The property of normal random variables mentioned in the 3rd reason is one of the key assumptions for the cost model in this research. Therefore, the mean and variance of the demand at immediate upper echelon location can be expressed as in equation (7) and (8).

$$\mu_D^u = \sum_{i=1}^{q} \mu_D^{(i)} \tag{7}$$

where

$\mu_D^u \equiv$ mean aggregate demand at the upper location

$\mu_D^{(i)} \equiv$ mean demand at the customer location *i*

$q \equiv$ the number of customer locations

$$\sigma_D^u = \sqrt{\sum_{i=1}^{q} (\sigma_D^{(i)})^2} \tag{8}$$

Where

$\sigma_D^u \equiv$ standard deviation of aggregate demand at the upper location

$\sigma_D^{(i)} \equiv$ standard deviation of demand at the customer location *i*

$q \equiv$ the number of customer locations

After the parameters of demand and lead time at the upper location are determined using equations (7) and (8), the same heuristic procedure (Exhibit 6) described in Hadley and Whitin (1963) is applied to calculate r^* and Q^*. A simple case is illustrated in Appendix 1, which describes the procedures to set approximately optimal values of r^* and Q^* for a single-item two-echelon inventory system.

(c) Determining the group policy

For each item group, one inventory control policy is applied to items within the group. For each item group, an aggregate item is built to represent the item group (Lenard and Roy 1995). The two main parameters of the aggregate item are the demand and lead time. Assume an item family has *p* items, the mean and standard deviation of the aggregate demand can be calculated as follows:

$$\mu_D^A = \frac{1}{p}\sum_{i=1}^{p} \mu_D^{(i)} \tag{9}$$

Where

μ_D^A ≡mean of aggregate demand

$\mu_D^{(i)}$ ≡mean demand of item *i* within the group

$$\sigma_D^A = \sqrt{\frac{1}{p}\sum_{i=1}^{p}(\sigma_D^{(i)})^2} \tag{10}$$

Where

σ_D^A ≡standard deviation of aggregate demand

$\sigma_D^{(i)}$ ≡standard deviation of the demand of item *i* within the group

Similarly, for the mean and standard deviation of the aggregate lead time can be calculated as follows:

$$\mu_{LT}^A = \frac{1}{p}\sum_{i=1}^{p} \mu_{LT}^{(i)} \tag{11}$$

Where

μ_{LT}^A ≡mean lead time of aggregate item

$\mu_{LT}^{(i)}$ ≡mean lead time of item *i*

$$\sigma_{LT}^A = \sqrt{\frac{1}{p}\sum_{i=1}^{p}(\sigma_{LT}^{(i)})^2} \tag{12}$$

Where

σ_{LT}^{A} ≡standard deviation of the lead time of aggregate item

$\sigma_{LT}^{(i)}$ ≡standard deviation of the lead time of item *i*

After the parameters of demand and lead time are determined using equations (9)-(12), the same procedure described in Hadley and Whitin (1963) is applied to calculate r^* and Q^*.

3.3 Grouping Techniques

This sub-section discusses the importance-based classification, the NIT classification, and the operations-based clustering suggested in this research.

3.3.1 The ABC Classification

This research develops and evaluates an optimization model based on a single criterion ABC classification technique, which groups items based on their importance. This single criterion ABC classification technique is compared with other grouping techniques. This kind of ABC classification technique avoids complex multi-criteria clustering methods and is proven to be effective by Zhang et al. (2001) and Teunter et al. (2010). This section discusses the intuitions of selecting appropriate ABC classification criterion by reviewing the two papers: Zhang et al. (2001) and Teunter et al. (2010), summarizes the rule of selecting/developing classification criterion, and develops the classification criterion for the network based ABC classification technique.

The classification criterion for Zhang et al. (2001)

The inventory cost model in Zhang et al. (2001) is listed in Exhibit 2 (Model 1) in Section 3.2. This model is to minimize inventory investment subject to constraints on average service level and replenishment frequency. The authors suggest a categorization scheme based on steps as follows:

a) Based on some approaches to the (r, Q) policy optimization problem in the literature, the reorder point is expressed as:

$$r_i = \theta_i + k_i \sigma_i$$

Where

$\theta_i \equiv D_i l_i$, expected demand for item i during lead time l_i

$\sigma_i \equiv$ standard deviation of demand during lead time for item *i*

$r_i \equiv$ reorder point for item *i*

b) Based on a single item model, given by the probability that there is no stockout during lead time (Nahmias 1997), and using a service-constrained approach with Type I service S_i, the k_i is decided as following:

$$k_i = z_{S_i}$$

Where

$z_{S_i} \equiv$ the standard normal ordinate such that $\Phi(z_{S_i}) = S_i$

c) Based on the assumption that average inventory can be approximated by $r_i - \theta_i + \frac{Q_i}{2}$ and using Type I formula to compute average service level, the following expression for the reorder point is derived (Hopp, Spearman and Zhang (1997)):

$$k_i = \sqrt{-2ln\left(\sqrt{2\pi}\frac{\sqrt{l_i c_i D_{tot}}}{\sqrt{D_i}\mu}\right)}$$

Where

$c_i \equiv$ unit cost for item *i*

$l_i \equiv$ replenishment lead time for item *i*

$\mu \equiv$ the Lagrange multiplier corresponding to the average service constraint

$D_i \equiv$ expected demand for item *i* per year

$D_{tot} \equiv \sum_{i=1}^{N} D_i$

d) The expression of k_i suggests that items with higher values of $\frac{D_i}{l_i c_i^2}$ result in higher k_i values; therefore, higher service level is obtained for the given values of D_i, l_i, and c_i. Thus, $\frac{D_i}{l_i c_i^2}$ was used by Zhang et al. (2001) as a classification criterion.

<u>The classification criterion for Teunter et al. (2010)</u>

The inventory cost model in Teunter et al. (2010) is listed in Exhibit 3 (Model 2) in Section 3.2. Model 2 is to minimize total inventory cost with the constraint on average fill rate over all SKUs. The authors propose a classification criterion based on the observation of cycle service level for SKU (CSL_i). Minimizing the total cost results in the following approximate newsboy-type optimality condition:

$$CSL_i \equiv 1 - \frac{h_i Q_i}{b_i D_i}$$

Where

b_i ≡ penalty cost for SKU *i*

h_i ≡inventory holding cost for SKU *i*

D_i ≡demand per unit time for SKU *i*

Q_i ≡(average) order quantity for SKU *i*

This condition indicates that the service level for a SKU is increasing with the increment of the ratio $\frac{b_i D_i}{h_i Q_i}$. Therefore, the ratio $\frac{b_i D_i}{h_i Q_i}$ is chosen as the classification criterion and the SKUs are ranked based on the ratio in descending order.

From the analysis of development of classification criterion in Zhang et al. (2001) and Teunter et al. (2010), it can be seen that the selection of classification criterion follows several rules:

1) The selection of classification criterion is model specific.

2) The classification criterion reflects the inventory management goal.

3) The item with higher value of the classification criterion is more important.

Extending rule 2) to the management of multi-item multi-echelon inventory system of interest leads to following rule:

4) The classification criterion should reflect the inventory network management goal.

These classification criterion selection rules are applied in this research. As discussed in Section 3.1, Hadley and Whitin's Model (Model 4 listed in Exhibit 5) is selected in this research, and a network inventory cost criterion that affects the system performance is developed based on this model. For an inventory system with more than one location, the cost function for the entire network of one item is the sum of the cost related to each location j. For one location, the inventory related cost is the sum of ordering cost (OC_j), average annual inventory holding cost (IHC_j) and the lost sale cost (LSC_j). The expressions of aforementioned three components are illustrated in equations (13) to (15).

$$OC_j = \frac{\lambda_j}{Q_{Lj}} A_j \tag{13}$$

Where

OC_j ≡ordering cost at location *j*

λ_j ≡ demand rate at location *j*

Q_{Lj} ≡ reorder quantity at location *j*

A_j ≡ ordering cost per time at location *j*

$$IHC_j = I_j C\left(\frac{Q_{Lj}}{2} + r_{Lj} - \mu_{LT\_j}\right) + I_j C\left[\int_{r_{Lj}}^{\infty} x h(x) dx - r_{Lj} H(r_{Lj})\right] \tag{14}$$

Where

IHC_j ≡average annual inventory holding cost at location *j*

I_j ≡ inventory holding charge at location *j*

C ≡ unit cost

Q_{Lj} ≡ reorder quantity at location *j*

r_{Lj} ≡ reorder point at location *j*

μ_{LT_j} ≡ mean lead time demand at location *j*

$h(x)$ ≡ probability distribution of lead time demand assumed to be normal with

parameters (μ_{LT_j}, σ_{LT_j})

$H(x)$ ≡the complementary cumulative of *h*(x)

$$LSC_j = \frac{b\lambda_j}{Q_{Lj}}\left[\int_{r_{Lj}}^{\infty} xh(x)dx - r_{Lj}H(r_{Lj})\right] \qquad (15)$$

Where

LSC_j ≡lost sale cost at location *j*

b ≡ lost sale cost

λ_j ≡ demand rate at location *j*

Q_{Lj} ≡ reorder quantity at location *j*

r_{Lj} ≡ reorder point at location *j*

$h(x)$ ≡ probability distribution of lead time demand assumed to be normal with

parameters (μ_{LT_j}, σ_{LT_j})

$H(x)$ ≡the complementary cumulative of *h*(x)

By adding the equations from (13) to (15), the cost function for the entire network of one item can be expressed as following:

$$\sum_{j=1}^{NL}\left\{\frac{\lambda_j}{Q_{Lj}}A_j + I_jC\left(\frac{Q_{Lj}}{2}+r_{Lj}-\mu_{LT\_j}\right)+\left(I_jC+\frac{b\lambda_j}{Q_{Lj}}\right)\left[\int_{r_{Lj}}^{\infty}xh(x)dx - r_{Lj}H(r_{Lj})\right]\right\} \quad (16)$$

Where

NL ≡ number of locations that hold inventory

C ≡ unit cost

b ≡ lost sales cost

A_j ≡ ordering cost per time at location *j*

I_j ≡ inventory holding charge at location *j*

λ_j ≡ demand rate at location *j*

μ_{LT_j} ≡ mean lead time demand at location *j*

σ_{LT_j} ≡ standard deviation of lead time demand at location *j*

r_{Lj} ≡ reorder point at location *j*

Q_{Lj} ≡ reorder quantity at location *j*

$h(x)$ ≡ probability distribution of lead time demand assumed to be normal with

parameters (μ_{LT_j}, σ_{LT_j})

$H(x)$ ≡the complementary cumulative of *h*(x)

Since $h(x)$ is assumed to be a normal distribution, the cost function (16) for the entire network can be further expressed as following:

$$\sum_{j=1}^{NL}\left\{\begin{array}{c}\frac{\lambda_j}{Q_{Lj}}A_j + I_jC\left(\frac{Q_{Lj}}{2}+r_{Lj}-\mu_{LT\_j}\right)\\ +\left(I_jC+\frac{b\lambda_j}{Q_{Lj}}\right)\left[\left(\mu_{LT\_j}-r_{Lj}\right)\Phi\left(\frac{r_{Lj}-\mu_{LT\_j}}{\sigma_{LT\_j}}\right)+\sigma_{LT\_j}\phi\left(\frac{r_{Lj}-\mu_{LT\_j}}{\sigma_{LT\_j}}\right)\right]\end{array}\right\} \quad (17)$$

Where

NL ≡ number of locations that hold inventory

C ≡ unit cost

$b \equiv$ lost sales cost

$A_j \equiv$ ordering cost per time at location *j*

$I_j \equiv$ inventory holding charge at location *j*

$\lambda_j \equiv$ demand rate at location *j*

$\mu_{LT\_j} \equiv$ mean lead time demand at location *j*

$\sigma_{LT\_j} \equiv$ standard deviation of lead time demand at location *j*

$r_{Lj} \equiv$ reorder point at location *j*

$Q_{Lj} \equiv$ reorder quantity at location *j*

$\Phi(x) \equiv$ cumulative distribution function for *x*

$\phi(x) \equiv$ probability density function for *x*

The total network inventory cost (NIC) in Equation (17) is selected as the network classification criterion since: 1) it is directly from the cost model; 2) it reflects the inventory cost management goal; 3) the higher value of NIC means the item is more important; and 4) it is the total network cost which represents the network management goal.

The procedure to implement the importance-based classification from the network perspective is summarized as follows:

Step 1: For each item i, calculate the network cost NIC_i.

Step 2: Sort items according to NIC_i. in descending order.

Step 3: Partition the items into three classes A, B, and C. The A class contain about 20% of the top items, the B class contain about next 30% items and the C class contain the rest of 50% items.

Exhibit 7: The Procedures for Importance-Based Classification using 3 groups

3.3.2 The NIT Classification

As discussed in Section 2.2.3, the NIT classification depends on the representation of NIT. It should be noted that the NIT representation is part of the interest of exploring the best ways to represent the system in a mathematical and computer data structure format to facilitate the analysis of the system and implementing the statistical grouping techniques. This section first discusses two NIT expressions, from graph theory and binary perspectives, and then selects the most appropriate NIT expression to implement NIT classification.

The NIT modeling is based on the following NIT related assumptions:

---

1) Each customer location has only one supplier in an NIT.

2) Each IHP is supplied by an IHP that is located at the immediate higher echelon, except those IHPs located at the first echelon, which are supplied by the external suppler.

3) Each NIT has at least one retail store.

4) End customer demands are satisfied by the retail locations which are the lowest echelon IHPs.

5) Each location is located at only one echelon.

Exhibit 8: The characteristics of NIT

In the following, the NIT modeling is discussed from graph theory expression and binary expression perspectives respectively.

<u>Graph Theory Expression</u>

The inventory network system (and the structural attribute NIT) can be expressed as a tree using graph theory as in Figure 3. A tree is a graph that does not include any simple circuits, which means it has no loops and no more than one edge between any two different vertices. In Figure 3, the external supplier is represented as the root of the tree, and the retail stores are represented as the leaves of the tree. The supplier locations are deemed as the parent nodes of their customer locations, which are deemed as the child nodes of their parent nodes. Further, the supplier-customer relations are represented by the edge that connecting the supplier and customer locations.

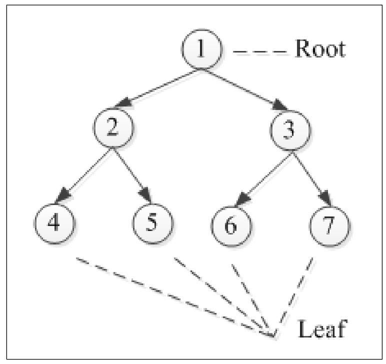

Figure 3: The representation the location network

Based on the graph representation of the inventory system, a NIT can be represented as Tree (V, E) or T (V, E), where the V is a set of vertices representing the corresponding nodes, and the E represents a set of edges that connecting two vectors. For example, the tree in Figure 3 can be represented as T(V, E), where V={1,2,3,4,5,6,7}, and E = {(1,2),(1,3),(2,4),(2,5),(3,6),(3,7)}.

The corresponding relations between the system components discussed in Section 1 and the components in the Tree are summarized as in Table 2.

Table 2: The Components of the Inventory system and the Tree

Components of the Inventory system	External Supplier	Retailer Stores	Supplier Location	Customer Location	Supplier-Customer Relation
Components of the Tree	Root	Leaf	Parent node	Child node	Edge

The tree representing a NIT has the characteristics such as: (1) it is a connected graph with n vertices and n−1 edges, where n is any positive integer; (2) it has no loops and no more than one edge between any two different vertices; in other words, there is a unique simple path between any two of its vertices; (3) it is a rooted tree in which the vertex representing the External Supplier is designated as the root and every edge is directed away from the root. In the rest of this research, the term NIT is equivalent to Tree.

It can be noted that these characteristics are consistent with the inventory location network structure related assumptions described in Section 1. In the inventory system of interest, for each item type, each IHP is supplied by an IHP that is located at the immediate higher echelon, except those IHPs located at the first echelon, which are supplied by the external suppler. This is consistent with the characteristic (2) mentioned above. The items for the inventory system are supplied by the External Supplier, which is represented as the root of the NIT tree, and the supplying direction of these items are directed away from this root. This corresponds to the NIT tree characteristic (3).

Based on the NITs represented in graphs, the relations between the NITs can be specified. Suppose there are two trees $NIT_1(V_1, E_1)$ and $NIT_2(V_2, E_2)$. The following relations can be specified:

- **Definition 1:** NIT_1 is a sub-graph of NIT_2, if $V_1 \subseteq V_2$ and $E_1 \subseteq E_2$
- **Definition 2:** The NIT_1 is equal to NIT_2, if NIT_1 is a sub-graph of NIT_2 and NIT_2 is a sub-graph of NIT_1.
- **Definition 3:** NIT_1 and NIT_2 are partly same if NIT_1 is not a sub-graph of NIT_2, NIT_2 is not a sub-graph of NIT_1, and the union of NIT_1 and NIT_2 ($NIT_1 \cup NIT_2$) is a Tree.
- **Definition 4:** NIT_1 and NIT_2 are different if the union of NIT_1 and NIT_2 ($NIT_1 \cup NIT_2$) is not a Tree.

Based on these four definitions, the relations (similarities) of the NITs can be summarized as: (1) equal; (2) sub graph; (3) partly same; (4) different.

In order to compare two NITs base on the graph representation, all vertices and edges of two NITs should be compared.

Binary Expression

The existence of each location can be expressed by a binary value where "1" indicates the existence of the corresponding location for the item and "0" means the non-existence. This means that the NIT can be expressed as a number of binary values, each of which represents the existence of a location. Based on this idea, the NIT illustrated in Figure 3 can be represented using Table 3:

Table 3: Binary Expression of NIT Based on All Locations

L1	L2	L3	L4	L5	L6	L7
1	1	1	1	1	1	1

The characteristics of the NIT listed in Exhibit 8 indicate that the structure of a NIT can be determined by the retail stores located at the lowest echelon. This means that the binary expression of the NIT in Table 3 can be represented using only the binary values for retail stores. Therefore, the ultimate resulted binary expression for NIT can be expressed as in Table 4. From now on in this dissertation, the binary expression for NIT refers to binary expression for NIT based on retail stores.

Table 4: Binary Expression for NIT Based on Retail Stores

L1	L2	L3	L4
1	1	1	1

Based on the binary expression for NIT, the comparison between two NITs can be reduced to comparing the structure of lowest echelon. The process of determining whether two $NITs$, for example NIT_1 and NIT_2, are the same can be realized by checking whether each retail store in NIT_1 exists in NIT_2 and whether each retail store in NIT_2 exists in NIT_1. If there is any retail store in NIT_1 that does not exist in NIT_2 or any retail store in NIT_2 that does not exist in NIT_1, then NIT_1 and NIT_2 are different; otherwise, NIT_1 and NIT_2 are the same.

The binary expression of NIT is selected to classify the NITs in this research because of following two reasons: 1) Lenard and Roy (1995) suggest that the storage structure is the attribute on which differences between items prevent the grouping of the items. This is because the function of the warehouse is different at different echelons, i.e., different warehouse has different customers. This means that items with different NITs should be separated; 2) compared to the graph theory expression, the binary expression of NIT is more efficient to determine whether two NITs are the same.

The process to group items into NIT groups satisfying the condition that items within the same NIT group have the same NIT structure is called NIT classification.

The NIT classification procedure can be summarized as follows:

---

1) Assign the first NIT to a new NIT group.

2) Iterate NITs from the second to the last. Compare each NIT with the first NIT in the existing NIT group(s), if the current NIT is equal to the selected NIT, add it to the corresponding group; otherwise create a new NIT group, and add the current NIT to it as the first NIT.

Exhibit 9: NIT Classification Procedure

3.3.3 The K-Means Clustering

It can be seen from the discussions that the clusters formed after classifying the item types based on NITs would have two characteristics: (1) still include very large number of item types; (2) the item characteristics values of some item types in a group are too different from most of the remaining item types so that applying the same inventory control group policy for these items is not reasonable. Thus, the item groups resulting only based on the NIT classification may not satisfy the commonly held objective of grouping, which is "minimum within-group variability and maximum between-group variability", from the other item characteristics perspectives, such as unit cost, demand rate, ordering cost, etc. This motivates the further clustering these resulted groups into smaller groups based on the other most related attributes.

This section discusses the K-Means Clustering from following perspectives: 1) selecting clustering attributes based on whether they have an impact on the inventory management goal, 2) specifying the distance measurement for the selected attributes, 3) based on the distance measurement, introducing the K-Means clustering algorithm, 4) the technique to optimize the number of clusters, and 5) a K-Means clustering method to cluster both structural and non-structural attributes.

Since the Hadley and Whitin's Model is selected in this research to calculate the inventory cost, the variables in the model are used to cluster the items. These variables are unit cost of item *i* (C_i), lost sales cost of item *i* (b_i), ordering cost of item *i* at location *j* (A_{ij}), inventory holding charge of item *i* at location *j* (I_{ij}), demand rate of item *i* at retail store j^R (λ_{ij^R}), standard deviation of demand of item *i* at retail store j^R ($\sigma_{D\_ij^R}$), mean lead time at location *j* for item *i* (MLT_{ij}), variance of lead time at location *j* for item *i* (VLT_{ij}). In the rest of this dissertation, the number of retail stores is denoted as *NR*, and the number of locations as *NL*. The distance measures for the non-structural attributes are ($2NR + 4NL$) dimensional squared Euclidean distance for the following reasons:

1) One item has one unit cost and one lost sales penalty cost.

2) The mean and variance of demand at upper echelon location other than retail store echelon can be derived based on λ_{ij^R} and $\sigma_{D\_ij^R}$ at retail stores. Therefore, only the mean and variance of demand at retail stores are considered as the clustering attributes.

3) The MLT_{ij} and VLT_{ij} may be different at different locations.

4) Except for the External Supplier, each location has an ordering cost and inventory holding cost rate.

5) In sum, 2+2NR+2NL+2($NL - 2$)= $2NR + 4NL$ clustering attributes are obtained based on the four reasons above.

Assume the location index is numbered continuously from 0 to ($NL - 1$) with the External Supplier as 0, and assume the retail stores are numbered through ($NL - NR - 1$) to ($NL - 1$). The ($2NR + 4NL$) dimensional space for the clustering attributes can be organized as follows:

$$Attribute\ List = \ (C_i, b_i, \lambda_{i(NL-NR-1)}, \dots, \lambda_{i(NL-1)}, \sigma_{D_{i(NL-NR-1)}}, \dots, \sigma_{D_{(NL-1)}},$$
$$A_{i1} \dots, A_{i(NL-1)}, I_{i1}, \dots, I_{i(NL-1)}, MLT_{i0}, \dots, MLT_{i(NL-1)}, VLT_{i0}, \dots,\ VLT_{i(NL-1)})$$

Exhibit 10: Attribute List for Non-Structural Attributes

Using the attributes in Exhibit 10, the K-Means algorithm is selected in this research to cluster the items since it is commonly used in practice to cluster large datasets. The commonly used distance measure in K-Means algorithm is the Euclidean distance. The distance between two items, X_A and X_B, can be expressed as:

$$d(X_A, X_B) = d_{AB} = \left| (C'_A - C'_B)^2 + (b'_A - b'_B)^2 + \sum_{j=NL-NR-1}^{NL-1} \left[\left(\lambda'_{Aj} - \lambda'_{Bj}\right)^2 + \left(\sigma'_{D_{Aj}} - \sigma'_{D_{Bj}}\right)^2 \right] + \sum_{j=0}^{NL-1} \left[\left(MLT'_{Aj} - MLT'_{Bj}\right)^2 + \left(VLT'_{Aj} - VLT'_{Bj}\right)^2 \right] + \sum_{j=1}^{NL-1} [(A'_A - A'_B)^2 + (I'_A - I'_B)^2] \right|^{0.5}$$

Where

$C'_i, b'_i, \lambda'_{ij}, \sigma'_{D_{ij}}, MLT'_{ij}, VLT'_{ij}, A'_i, I'_i$ are the normalized values.

Exhibit 11: Euclidean Distance

The normalized values for continuous-valued are obtained based on following formula:

$$X_i'^{(t)} = \frac{X_i^{(t)} - \min\left(X_i^{(t)}\right)}{\max\left(X_i^{(t)}\right) - \min\left(X_i^{(t)}\right))}$$

Where

$X_i^{(t)} \equiv$ *t*-th attribute of item *i*

$X_i'^{(t)} \equiv$ the normalized value of *t*-th attribute of item *i*

Exhibit 12: Normalization

Since all the clustering attributes in Exhibit 10 are continuous-valued, putting normalization formulations into Exhibit 11, the Euclidean distance between two items can be expressed as:

$$d(X_A, X_B) = \left| \left(\frac{C_A - C_B}{max(C_i) - min(C_i)}\right)^2 + \left(\frac{b_A - b_B}{max(b_i) - min(b_i)}\right)^2 \right.$$

$$+ \sum_{j=NL-NR-1}^{NL-1} \left[\left(\frac{\lambda_{Aj} - \lambda_{Bj}}{max(\lambda_{ij}; j \in (NL - NR - 1, NL - 1)) - min(\lambda_{ij}; j \in (NL - NR - 1, NL - 1))}\right)^2 \right.$$

$$\left. + \left(\frac{\sigma_{D_{Aj}} - \sigma_{D_{Bj}}}{max(\sigma_{D_{ij}}; j \in (NL - NR - 1, NL - 1)) - min(\sigma_{D_{ij}}; j \in (NL - NR - 1, NL - 1))}\right)^2 \right]$$

$$+ \sum_{j=0}^{NL-1} \left[\left(\frac{MLT_{Aj} - MLT_{Bj}}{max(MLT_{ij}) - min(MLT_{ij})}\right)^2 + \left(\frac{VLT_{Aj} - VLT_{Bj}}{max(VLT_{ij}) - min(VLT_{ij})}\right)^2 \right]$$

$$\left. + \sum_{j=1}^{NL-1} \left[\left(\frac{A_A - A_B}{max(A_i) - min(A_i)}\right)^2 + \left(\frac{I_A - I_B}{max(I_i) - min(I_i)}\right)^2 \right] \right|^{0.5}$$

Exhibit 13: Euclidean Distance- Version 2

Based on the Euclidean Distance showed in Exhibit 13, a commonly used iterative refinement algorithm, which is introduced in Exhibit 14, adopted from MacKay (2003) to solve the K-Means problem is applied in this research.

Initialization Step Set *K* means $\{m^{(k)}\}$ to random values.

Assignment Step Each data point *n* is assigned to the nearest mean.

Denote the guess for the cluster $k^{(n)}$ that the point $\mathrm{x}^{(n)}$ belongs to by $\hat{k}^{(n)}$.

$$\hat{k}^{(n)} = \underset{k}{\operatorname{argmin}}\{d(m^{(k)}, \mathrm{x}^{(n)})\}$$

An alternative, equivalent representation of this assignment of points to clusters is given by 'responsibilities', which are indicator variables $r_k^{(n)}$. In the assignment step, set $r_k^{(n)}$ to one if mean *k* is the closest mean to data point $\mathrm{x}^{(n)}$; otherwise $r_k^{(n)}$ is zero.

$$r_k^{(n)} = \begin{cases} 1 & \text{if } \hat{k}^{(n)} = k \\ 0 & \text{if } \hat{k}^{(n)} \neq k \end{cases}$$

if a tie happens, $\hat{k}^{(n)}$ is set to the smallest of the winning $\{k\}$.

Update Step The model parameters, the means, are adjusted to match the sample means of the data points that they are responsible for.

$$\mathrm{m}^{(k)} = \frac{\sum_n r_k^{(n)}}{R^{(k)}}$$

where $R^{(k)}$ is the total responsibility of mean k,

$$R^{(k)} = \sum_n r_k^{(n)}$$

If $R^{(k)} = 0$, keep $\mathrm{m}^{(k)}$.

Repeat the assignment step and update step until the assignments do not change.

Exhibit 14: The K-Means Algorithm

The K-Means clustering algorithm requires the number of clusters K as an input parameter. This parameter affects the performance of the clustering results significantly; thus, it should be determined carefully. Tibshirani et al. (2001) develop a gap statistic approach to find the optimal number of clusters based on the within-cluster scatter. Supposing the data $\{x_{ij}\}$, i=1, 2,…, n, j=1, 2,…, p, is composed of p dimensional space on n observations, supposing the observations are partitioned in k clusters, C_1, C_2,…, C_k, and denoting C_r as the indices of observations in cluster r, Tibshirani et al. (2001) propose an error measure W_k, as following:

$$W_k = \sum\nolimits_{r=1}^{k} \left(\frac{1}{2n_r} \sum_{i,i' \in C_r} d_{ii'} \right)$$

Where

$d_{ii'} \equiv$ The Euclidean distance between two observations i and i'

$n_r \equiv$ The number of observations in k-th cluster

Exhibit 15: Error Measure

Their estimate of the optimal number of clusters is the value of k satisfying following condition:

$$Maximize\ Gap_n(k) = E_n^*\{\log(W_k)\} - \log(W_k)$$

Where

$E_n^* \equiv$ The expectation under a sample of size n based on the reference distribution

Exhibit 16: Gap Statistic

The authors conclude that the uniform distribution is the most appropriate distribution to perform a gap test. Considering partitioning n uniform data points in p dimensions with k centers, and assuming that the centers are equally aligned, the expectation of $\log(W_k)$ can be approximately expressed as (Tibshirani et al. 2001):

$$\log(pn/12) - (2/p)\log(k) + constant$$

Thus, using the uniform distribution as the reference distribution, the gap statistic can be expressed as:

$$Gap_n(k) = \log(pn/12) - (2/p)\log(k) - \log(W_k) + constant$$

Exhibit 17: Gap Statistic for Uniform Distribution

As shown, the last part of the expression in Exhibit 17 is a constant, which is independent of n, p and k; thus, the constant can be ignored when comparing the value of Gap Statistic for different k.

As discussed previously, the intuition behind the NIT classification is that structural attributes (NIT in this research) prevents the grouping of items (Lenard and Roy 1995). The relaxation of this assumption leads to another clustering method, i.e., clustering the items based on both structural and non-structural attributes. The measurement for both structural and non-structural attributes is the distance metrics for mixed-type attributes. Maimon and Rokach (2005) suggest a way to calculate distance metrics for mixed-type attributes between two instances using the Euclidean distance metric where the difference between binary attributes is calculated as 0 or 1, and the difference between numeric attributes is calculated as the distance between their normalized values. In this research, the non-structural attributes are numeric attributes and the NIT attribute can be represented as a set of binary attributes, each of which represents the existence of a location with the value 1 meaning "exist" and the value 0 meaning "not-exist". Therefore, the number of the clustering attributes for an item increases from $(2NR + 4NL)$ to $(3NR + 4NL)$ where NR is the number of retail stores and NL is the number of locations. Among the $(3NR + 4NL)$ clustering attributes, $(2NR + 4NL)$ clustering attributes are non-structural attributes with numerical values, and NR clustering attributes are structural attributes with binary values. The attribute list with both structural and non-structural attributes can be organized as follows:

$$
\begin{aligned}
&Attribute\ List \\
&= (E_{i(NL-NR-1)}, \dots, E_{i(NL-1)}, C_i, b_i, \lambda_{i(NL-NR-1)}, \dots, \lambda_{i(NL-1)}, \sigma_{D_{i(NL-NR-1)}}, \dots, \sigma_{D_{(NL-1)}}, \\
&A_{i1} \dots, A_{i(NL-1)}, I_{i1}, \dots, I_{i(NL-1)}, MLT_{i0}, \dots, MLT_{i(NL-1)}, VLT_{i0}, \dots, VLT_{i(NL-1)})
\end{aligned}
$$

Where

E_{ij} ≡ the existence of location *j* for item *i*

C_i ≡ unit cost of item *i*

b_i ≡ lost sales cost of item *i*

λ_{ij} ≡ demand rate of item *i* at retail store *j*

$\sigma_{D_{(ij)}}$ ≡ standard deviation of demand of item *i* at retail store *j*

A_{ij} ≡ ordering cost of item *i* at location *j*

I_{ij} ≡ inventory holding charge item *i* at location *j*

MLT_{ij} ≡ mean lead time at location *j* for item *i*

VLT_{ij} ≡ variance of lead time at location *j* for item *i*

Exhibit 18: Attribute List for both Structural and Non-Structural Attributes

Maimon and Rokach (2005) define the dissimilarity $d(x_i, x_j)$ between two instances consisting of *p* attributes of mixed type as:

$$d(x_i, x_j) = \frac{\sum_{n=1}^{p} \delta_{ij}^{(n)} d_{ij}^{(n)}}{\sum_{n=1}^{p} \delta_{ij}^{(n)}}$$

Where

$\delta_{ij}^{(n)} = 0$ if one of the values is missing

If the attribute is binary, $d^{(n)}(x_i, x_j) = 0$ if $x_{in} = x_{jn}$; otherwise $d^{(n)}(x_i, x_j) = 1$

If the attribute is continuous-valued, $d_{ij}^{(n)} = \frac{|x_{in} - x_{jn}|}{max_h x_{hn} - min_h x_{hn}}$, where *h* runs over all non-missing instances for *n*-th attribute.

Exhibit 19: Distance Metrics for Mixed-Type Attributes

Using the distance measures in Exhibit 19 and the aforementioned K-Means algorithm, the items can be partitioned using both structural and non-structural attributes.

3.3.4 The categories of grouping techniques

The terms “Classification” and “Clustering” have similar meanings. Both of them refer to cluster observations into smaller groups. However, the processes of classification and clustering have some subtle differences. Their differences can be summarized into three perspectives: (1) classification has determined labels before the grouping process, but clustering does not; (2) while classification uses a clearly declared rule to group observations, clustering clusters observations based on the distance measurement; (3) classification is one kind of supervised process, but clustering is unsupervised process.

The ABC analysis and NIT grouping methods are categorized as classification techniques and K-Means is a clustering technique. A summary of the comparison of ABC classification, NIT classification, and K-Means clustering is illustrated in Table 5.

Table 5: Grouping Categories

	Classification		Clustering
	ABC	NIT	K-Means
Label Pre-determined	A, B, C	NIT1, NIT2,…	None
Grouping Method	Pareto principle	Same NIT structure	Based on Euclidean distance

3.3.5 The Grouping Technique Evaluation

Bonner (1964) first argued that "there is no universal definition for what is a good clustering"; the qualities of the clustering are determined based on the beholders experience. In this dissertation, the clustering results are tested to quantitatively compare the clustering techniques applied from both effectiveness (statistical and optimization) and efficiency (grouping time) perspectives.

3.3.5.1 SSE as an Evaluation Criteria

From the statistical perspective, the compactness of the clusters is measured using Sum of Squared Error (SSE). The SSE is chosen as the compactness measurement since it is the simplest and most widely used criterion measure for clustering (Maimon and Rokach 2005). SSE can be calculated as follows:

$$SSE = \sum_{r=1}^{k} \sum_{i \in C_r} \sum_{j=1}^{m} \left(x_{ij} - \bar{x}_j^{(r)} \right)^2$$

Where k ≡ the number of clusters

$m \equiv$ the number of dimensions of an observation

$C_r \equiv r$-th centroid

$\bar{x}_j^{(r)} \equiv$ the value of *j*-th dimension of *r*-th centroid

$x_{ij} \equiv$ the value of *j*-th dimension of *i*-th observation

$i \in C_i \equiv$ *i*-th observation in the group determined by *i*-th centroid

Exhibit 20: Sum of Squared Error

3.3.5.2 Penalty Cost as an Evaluation Criteria

From the optimization perspective, the Hadley and Whitin's Model discussed in Section 3.2 is used to calculate the total inventory cost. The clustering penalty cost (CPC), which is the increased cost caused by clustering, is used to measure the clustering effectiveness. The clustering penalty cost can be expressed as following:

$$CPC = IC(N, p, m, A_q, k) - IC(N, p, m)$$

Where $IC \equiv$ total inventory cost

$N \equiv$ the set of item types

$p \equiv$ inventory control policy, which is continuous (r, Q) policy

$m \equiv$ inventory control model, which is Hadley and Whitin's Model

$A_q \equiv$ clustering algorithms q

$k \equiv$ the number of clusters

Exhibit 21: Clustering Penalty Cost

It can be seen from Exhibit 21 that CPC is calculated by total cost after grouping, i.e. $IC(N, p, m, A_q, k)$, minus total cost before grouping, i.e. $IC(N, p, m)$. The total cost before grouping is calculated based on following steps:

Step 1: for each item type in the system, optimize r and Q for each items as shown on Exhibit 6. And then, using formula (17), which is defined based on p and m, the inventory cost before grouping is calculated. Appendix 1 shows an example of calculating a single item's total cost.

Step 2: summing all item's inventory cost before grouping results the total inventory cost before grouping.

The difference between the calculation of before and after grouping total cost is that after grouping total cost is calculated based on the group inventory policy. Based on selected A_q and k, items are grouped into different groups, and each group is treated as an aggregate item. The aggregate item inventory policy related parameters are calculated based on formulas (9)-(12). The total cost after grouping is calculated based on following steps:

Step 1: for each aggregate item in the system, optimize r and Q for each items as shown on Exhibit 6.

Step 2: for each item, apply optimized r and Q from the corresponding aggregate item (group) to calculate the total inventory cost using formula (17).

Step 3: sum all item's inventory cost using group policy to obtain the total inventory cost after grouping.

The %CPC measures the percent deviation with respect to the optimal individual inventory policy. The value of %CPC shows the percent increase of total inventory cost when the group policy is applied.

The %CPC is calculated using following equation:

$$\%\mathrm{CPC} = \frac{Total\ Cost\ after\ clustering - Total\ Cost\ before\ clustering}{Total\ Cost\ before\ clustering}$$

$$= \frac{Total\ Cost\ using\ Group\ Policy - Total\ Cost\ using\ \ Individual\ Policy}{Total\ Cost\ using\ \ Individual\ Policy}$$

The %CPC is used to measure the effectiveness from the optimization perspective in the rest of this dissertation.

3.3.5.3 The Grouping Time

From the efficiency perspective, the grouping time is used to assess the quality of different grouping techniques. The lower the grouping time, the more efficient the grouping technique. As discussed in Maimon and Rokach (2005), the time complexity of K-Means algorithm relates to three attributes: 1) the number of instances (items); 2) the number of clusters; and 3) the number of iterations used by the algorithm to converge. Besides these three attributes, the number of clustering attributes is also an important factor affecting the clustering time, since the more clustering attributes, the more computational memory is required to execute the clustering. The effects of the aforementioned four factors on the grouping time of K-Means are investigated in section 6.3. In addition, the comparisons of grouping time between different grouping techniques are discussed in section 6.4.

4 Data Modeling and Generation

This Chapter first represents the system, item characteristics and their relations using data modeling. Based on the data models created during the data modeling process, the data generation procedures are discussed. The goal of data modeling and data generation discussed in this chapter is to provide data for the research of grouping techniques.

4.1 Data Modeling

The characteristics of the inventory system of interest including the item characteristics need to be represented in mathematical and computer data structure format to facilitate analysis of the grouping methods. The data modeling process deals with the 1st research question mentioned in the Introduction Chapter. The way to mathematically model structural attribute NIT is discussed in section 3.3.2. This section explains the data modeling process applied in this research from the computer data perspective.

The system of interest is made of large number of system elements that interact with each other system wide. The item types in the inventory system are also the system elements, and they interact with each other and with the other system elements as well. This means that quantifying item type characteristics requires the quantification of other interacting elements with item types. In brief, the quantification of the inventory system is to quantify the inventory system elements and their relations. There are a variety of system element interactions, such as an item type can be stored at specific locations, or an item type can be sold at certain locations, etc. To understand the inventory system better, careful study of the interactions among system characteristics is essential.

In the data modeling research, a top-down methodology is used to conduct the modeling process, in which, the real world scenario is constructed first, and then the entities and associations are identified. The steps of the data modeling process are as follows:

a) Summarizing the system characteristics

b) Building the E-R diagram

c) Mapping the E-R diagram to the relational model

Following, each step is summarized briefly.

4.1.1 Summarizing System Characteristics

The system characteristics involve a set of system element characteristics. The system elements are identified based on the system description in Chapter 1, the optimization model discussed in section 3.2, and the clustering attributes discussed in section 3.3. The identifying system elements process basically is to identify the entities that independently exist in the system and can be uniquely identified. These entities are item type, location, inventory policy, and probability distribution. Based on these entities, the following section builds the related E-R diagram.

4.1.2 Building the E-R Diagram

The E-R diagram is built through following steps: 1) identifying entities and drawing the entity diagram; 2) identifying associations and drawing the association diagrams; and 3) specifying the domain for each attribute. To illustrate the quantifiable aspects of the inventory system of interest, such as relationships, behavior, structure etc., the UML diagram is employed to draw the E-R diagram. A class is used to describe a group of objects with similar attributes, common operations, common relationships to other objects, and common semantics (Rumbaugh 1991). In UML diagram, the notation for class and association are represented as shown in

Figure 4 and Figure 5 respectively. The multiplicity in Figure 5 specifies “the number of instances of one class that may relate to a single instance of an associated class” (Rumbaugh 1991). The values for the multiplicity can be zero (0), one (1), or many (*). The multiplicity is specified as [lower limit .. upper limit], where lower limit corresponds to the minimum multiplicity and upper limit corresponds to maximum multiplicity. The notations in Figure 4 and Figure 5 are used to draw the UML diagram for the entire inventory system.

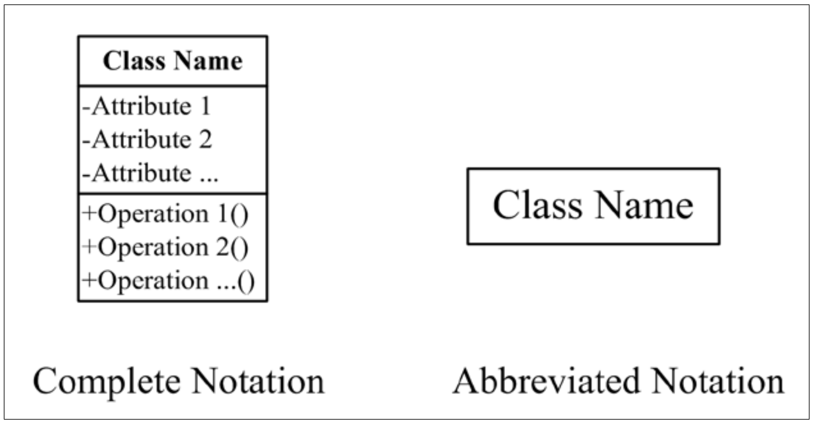

Figure 4: UML Diagram Notation for Class

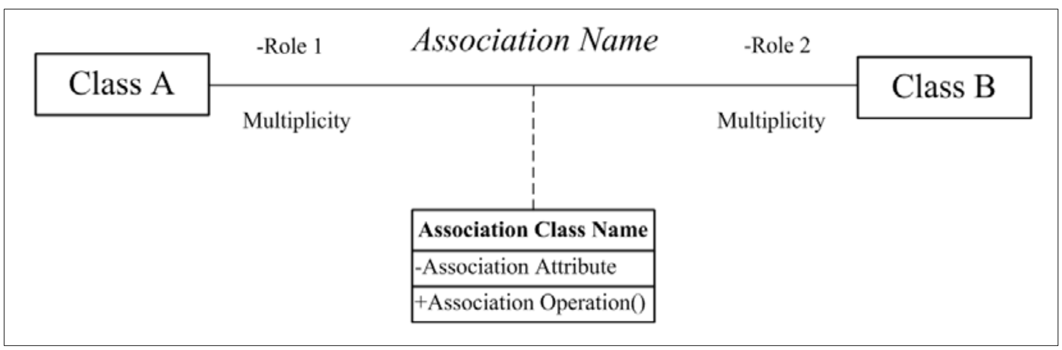

Figure 5: UML Diagram Notation for Association

The complete UML diagram of the inventory system is illustrated in Figure 6. The detailed data modeling procedures are documented in Appendix 2, which includes the E-R diagram, as shown on Figure 6, building process in details.

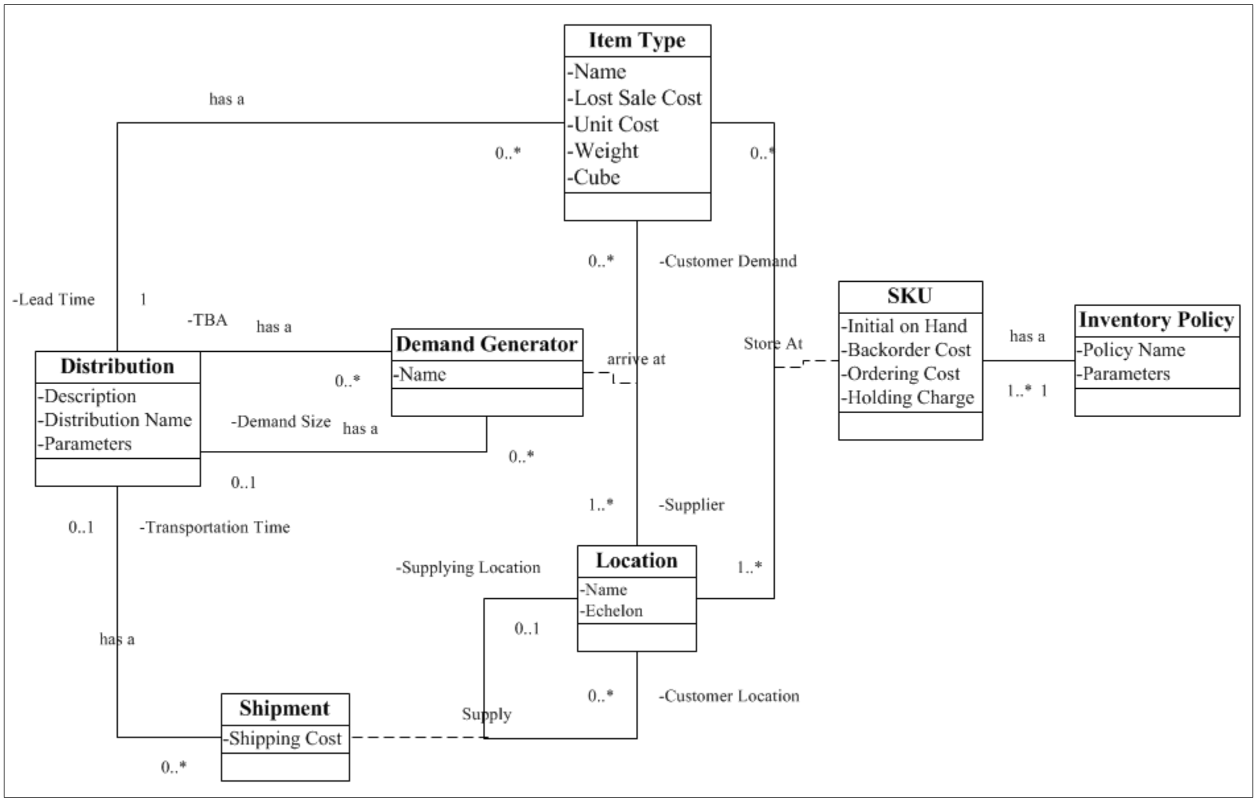

Figure 6: The E-R Diagram of the Inventory System

4.1.3 Mapping the E-R Diagram to the Relational Model

Based on the E-R diagram derived in the previous section, this section discusses how to design the relational tables corresponding to the entity and association classes. The schema of the tables is shown in the following format:

Table Name (<u>Primary Key(s)</u>, Attribute 1, Attribute 2, … , Attribute N)

Normal forms are rules used to provide the guidance of designing tables. The tables corresponding to the E-R diagram are built based on the third normal form. The tables are in third normal form if they satisfy all the following criteria:

1) For each row, attributes must be atomic with only one value

2) Each non-primary key field is fully functional dependent on every key of the table

3) Each non-primary key field is non-transitively dependent on every key of the table

Exhibit 22: The Criteria for Third Normal Form

After mapping the UML diagram in Figure 6 to the relational models, there are six tables resulted: Distribution, Location, Item Type, Demand Generator, SKU, and Shipment. The detailed mapping process is described in Appendix 2.

4.1.4 Two Special Data Models

Based on the relational data models developed in the previous sections, this section discusses two special data models: (1) Network of Item Type (NIT) and (2) Inventory System (IS).

NIT data model contains two attributes: locations and the supply relation between locations. The NIT for a specific item type can be derived using SKU table and Shipment table, and an example is given in Appendix 2. When the ID of the item type is available, all the locations of the corresponding item can be obtained from SKU table. Given the location IDs, the related supply relation can be found in the Shipment table.

All the system elements are part of the Inventory System; thus, IS data model is an aggregate of all the aforementioned data models.

The implementation of the data models in Java is recorded in Appendix 3.The next section discusses the data generation procedure based on the data models developed in this section.

4.2 Data Generation

This section discusses a method to generate large-scale datasets that represent the inventory system and facilitate the testing of grouping methodologies. Based on the data models designed in previous section, a data generation procedure was developed and used to generate large-scale datasets. The basic goal of the data generation procedure is to generate a large scale dataset including information of item types, locations, and SKUs, etc. To generate such a large dataset, some automated procedure using programming tools such as Java, was needed to conveniently and efficiently generate objects of SKUs holding both item type information and location information.

This section is organized into four parts: 1) the relationships between data modeling and data generation; 2) data generation procedure; 3) data quantification for inputs; and 4) data generation evaluation.

4.2.1 Relationships between Data Modeling and Data Generation

The data modeling process determines the attributes of the objects contained in the inventory system of interest and the relationships among the system objects. The connections between data modeling and data generation can be summarized as follows:

1) The attributes of data models resulting from the data modeling process determine what needs to be generated during the data generation process. The data generation process generates new values and assigns the generated values to the attributes of the data models.
2) The data modeling process determines the relations among the data models and some relations are used to facilitate the data generation process. For example, each Item Type model contains one Location Network data model. The SKUs, which are determined by item type and locations, can be generated by iterating the locations in the location network of the corresponding item type.
3) The data modeling process determines the structure of the storage files. After data generation, the generated inventory system needs to be stored in files (such as CSV

files). During the clustering process, the information of the inventory system is read from the files. The structures of the files are specified by the data models.

4.2.2 Data Generation Procedure

This section first discusses the generation of NIS and NIT, and then discusses the entire data generation process.

4.2.2.1 NIS and NIT Generation

As discussed in Chapter 1, all the NITs combined form the network of inventory system (NIS); this means that each NIT is a sub-network of the NIS. Both NIS and NIT are location networks. During the data generation process, the NIS is first built; then, NIT for each item type is built based on NIS and stored in the corresponding item type object.

Two kinds of inputs are needed to build the NIS: (1) the probability distribution for number of echelons, and (2) the probability distribution(s) for number of customers for a supplier location. The NIS building process follows the assumption that each customer location has only one supplier location for item type. Figure 7 illustrates the process of building a three echelon inventory system, in which there is 1 external supplier at echelon 0, 1 location at echelon 1, 2 locations at echelon 2, and 5 locations at echelon 3. The first step is to create a single External Supplier. The following steps are to add locations from the highest echelon to the lowest. NC_l represents the number of customer locations for a supplier location l. As shown in Figure 7 (A), where NC_0 = 1 means there is 1 customer location for the external supplier (with location ID 0), adding locations starts from echelon 1 by putting 1 location at this echelon; on (B), where NC_1 = 2 means that there are 2 customer locations for location 1, 2 customer locations are assigned for the location 1; on (C), NC_2=2 and NC_3=3 indicates there are 2 customer locations for location 2, and 3 customer locations for location 2. Two customer locations are assigned to location 2 and three customer locations are assigned to location 3 and this finishes the building of the entire supply network.

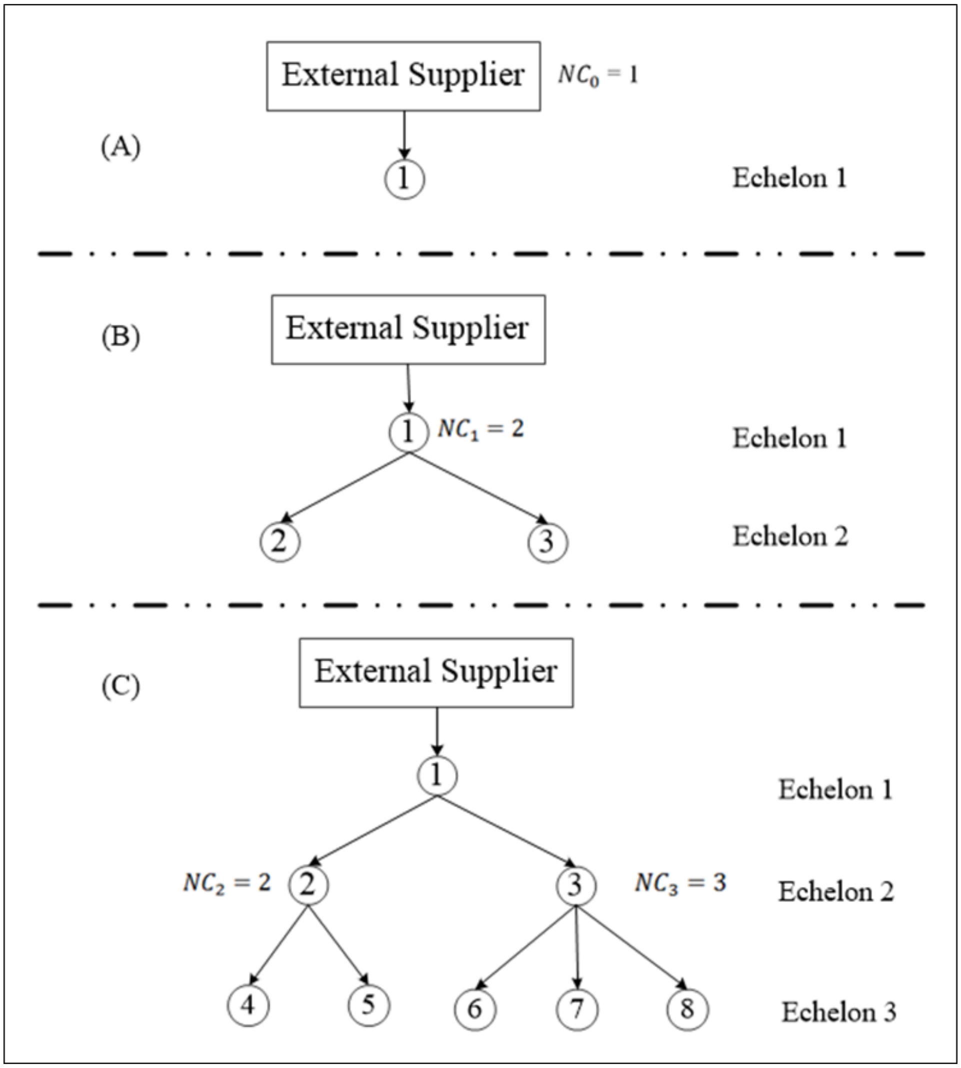

Figure 7: Building the Supply Network

In reality, if an item type exists at a number of retail stores, which is located at the lowest echelon, its entire supply network locations (or NIT) can be determined based on the assumption that a customer location has only one supplier location for an item type. In the data generation process, it is assumed that the item type's existence at a retail store follows a probability distribution. Based on this assumption, the lowest echelon IHPs are randomly determined (generated). Once the retail stores having the item type are found, the item type's supply network can be decided; therefore the NIT is fixed as aforementioned.

Based on the supply network (NIS) built in Figure 7, Figure 8 illustrates the NIT building process. Suppose an item type is stored at location 4, 5, 6, and 7, which are located at echelon 3 of the supply network as shown in Figure 8 (A). Since location 4 and 5 have single supplier location 2, and location 6 and 7 has supplier location 3 as shown in Figure 7, the location 2 and 3 are decided as located at echelon 2 of the NIT as in Figure 8 (B). Further, since both locations 2 and 3 have single supplier 1, location 1 is determined as located at echelon 1 of the NIT as in Figure 8 (C). Finally, since the External Supplier supplies the IHPs at echelon 1 and there is only location 1 at the echelon, External Supplier is connected to location 1, and this finishes building of the entire NIT as in Figure 8 (D). In sum, besides the two inputs needed to build the NIS (the number of echelons, and number of customer locations for each supplier location), the NIT building process requires one more input, i.e., the probability of item type's existence at a location located at the lowest echelon.

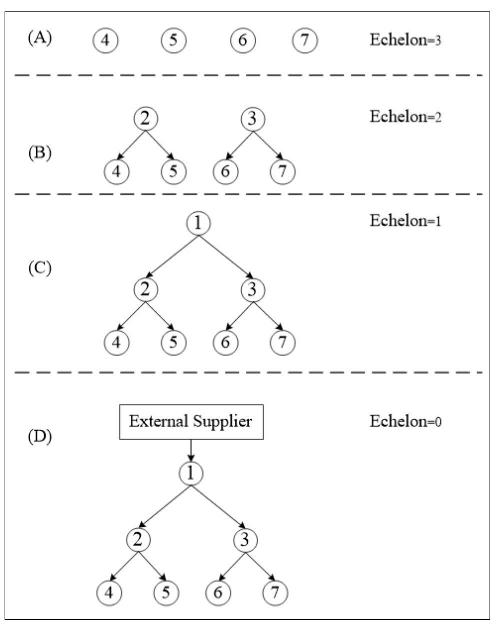

Figure 8: Building the Network of Item Type (NIT)

In reality, the number of retail stores is usually larger than the number of warehouses, this indicates the number of customers at lower echelons is larger than the number of customers at upper echelons. This characteristic of location networks can be achieved by applying a set of uniform distributions satisfying the condition as following:

$$UB_{e-1} \leq LB_e$$

Where

UB_{e-1} ≡ upper bound of the number of customers for locations at echelon e-1

LB_e ≡ lower bound of the number of customers for locations at echelon e

An example implementation of the mechanism above is illustrated in Table 6.

Table 6：Implementation of Uniform Distributions

Echelon	Distribution for number of customers for a supplier location l (NC_l)
0	~ Uniform(1, 2)
1	~ Uniform(2, 3)
2	~ Uniform(3, 5)
…	…

Another consideration about NIT is the lead time at IHPs. Usually, the lead time at the lower echelons is shorter than that of upper echelons. This condition is satisfied using the mechanism illustrated as follows:

(1) Generate the lead time at the External Supplier (LTatES).

(2) For the IHP other than the External Supplier, the lead time is set as the product of the lead time of its supplier and a random variable from a Uniform (0.5, 1).

4.2.2.2 Summary of the Data Generation Procedure

This section presents the inputs and overview of the data generation procedure to create a large-scale dataset that can quantitatively represent the multi-item multi-echelon inventory system of interest.

The inputs of the data generation can be summarized as follows:

D_{NE}: The probability distribution for number of echelons

D_{NC}: The probability distribution for number of customers for a supplier location

PI: Probability of Item Type's existence at a retail store (across any retailer location)

NI: Number of Item Types

D_{UC}: The probability distribution of item's Unit Cost

D_{LSC}: The probability distribution of item's Lost-Sale-Cost-to-Unit-Cost Ratio

D_{OC}: The probability distribution of ordering cost

D_{HC}: The probability distribution of holding cost rate

D_{DR}: The probability distribution for demand rate

D_{DV}: The probability distribution for Demand Variance-to-Mean Ratio

D_{LTES}: Lead Time distribution at External Supplier (day)

D_{LTIHP}: Lead Time distribution at IHP (day)

D_{LTV}: Lead Time Variance-to-Mean Ratio

The overview of data generation procedure can be summarized as in Figure 9. As shown, the first step is to generate the physical network of inventory system (NIS), which holds the entire inventory system. The second step is to generate the item types in the inventory system. The third step deals with generating SKUs. The last step is to generate demands at the retailer level. The rest of this section discusses the data generation procedure based on the inputs and the aforementioned steps.

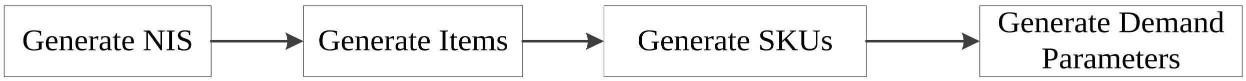

Figure 9：Overview of data generation procedure

Step 1: Generating NIS

The number of echelons of the inventory system is randomly determined based on the probability distribution D_{NE}, which can be any discrete distribution; the discrete uniform distributions is used in this research.

Once the number of echelons is determined, the physical inventory network system is generated from top to bottom as discussed in 4.2.2.1. An example of a generated NIS is shown in Figure 7 (C).

Step 2: Generating Items

This step first generates the structural attribute (NIT), and then generates the remaining item attribute values for the corresponding items. The generation process of NIT is discussed in 4.2.2.1, and an example of generated NIT is given in Figure 8 (D). And then, the non-structural attributes such as unit cost, lost sale cost, mean and variance of lead time at external supplier are generated using distributions D_{UC}, D_{LSC}, D_{LTES}, and D_{LTV}. All the items are generated by repeating these two steps.

Step 3：Generating SKUs

Generation of SKUs for an item follows two steps. First, for an item type, iterate the location information (location ID) sequentially in an NIT and combine this information with the item type information (item type ID) stored at the corresponding location. This method forms SKUs by combining item type ID and location ID automatically. This process is illustrated in

Figure 10. Second, for each SKU, ordering cost, inventory holding cost rate, and the mean and variance of the Lead Time distribution between IHPs are generated using D_{OC}, D_{HC}, D_{LTIHP} and D_{LTV}. These two steps are repeated for all items.

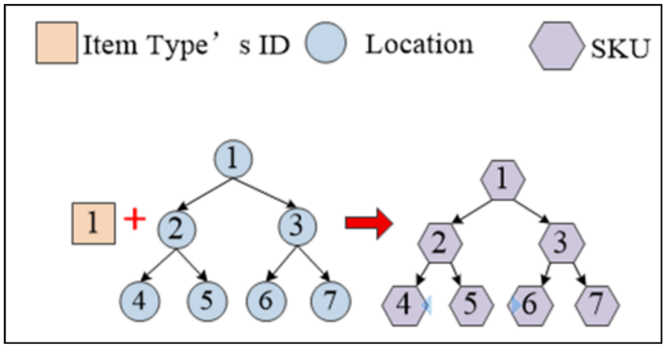

Figure 10: SKU Generation Process

Step 4: Generating Demand Characteristics

Generation of demand for an item type also follows two steps. First, iterate all the SKUs associated with the item, and create a demand generator for each SKU at the lowest echelon. This process is illustrated on Figure 11. Second, for each Demand Generator, generate mean and variance of demand rate using D_{DR} and D_{DV}. These two steps are repeated for all item types.

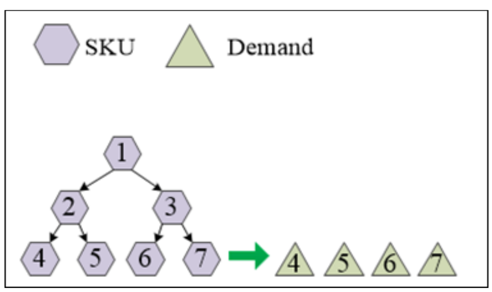

Figure 11: Demand Generation Process

The pseudo code of the data generation procedure is recorded in Appendix 3.

4.2.3 Data Quantification for Inputs

Before the implementation of data generation, the values for input attributes and the relationship between the attributes should be quantified. This section discusses these two issues.

4.2.3.1 The Range of Input Values

The input attributes are the ones that affect the system performance, for instance, total inventory cost in this research. Based on the study of Chaharsooghi and Heydari (2010), Deshpande et al. (2003), Ehrhardt (1984), Lee et al. (1997), Metters (1997), REM Associates, and a case study from Tmall.com, the range of the input values are summarized in Table 7. The detailed discussion about the data in Table 7 is in Appendix 4.

Table 7: Summary of Input Range

Attribute	Range	Reference
Unit Cost ($)	[1, 200,000]	Deshpande et al. (2003) ,data from Tmall
Lost-Sale-Cost-to-Unit-Cost Ratio	[0.1,1]	Metters (1997)
Ordering Cost ($)	[100,10000]	Lee et al.(1997)
Inventory Holding Charge ($/$/year)	[12%,35%]	REM Associates
Demand Rate (yearly)	[1,2000000]	Deshpande et al. (2003), data from Tmall
Demand Variance-to-Mean Ratio	[0.1,4]	Metters (1997), Lee et al.(1997)
Mean lead time at ES (day)	[10,250]	Deshpande et al. (2003)
Mean Lead Time at an IHP (day)	[10,55]	Deshpande et al. (2003)
Lead Time Variance-to-Mean Ratio	[0.01, 2]	Chaharsooghi and Heydari (2010), Ehrhardt (1984)

4.2.3.2 The Relationship between the Attributes

As mentioned in section 2.4, the motivation of generating data in this research is to provide large-scale controllable datasets that closely reflect real inventory systems. This means that, on the one hand, the large scale inventory dataset is not conveniently available from the industry; on the other hand the real data from the industry cannot be manipulated to satisfy the experimental needs in the research.

One of the important system characteristics in an inventory system is that some of the system attributes interact with each other system wide, therefore their relations need to be closely modeled to reflect the real world scenarios. Generally speaking, the generated data in this research should have the characteristics such as 1) large scale; 2) the inputs can be controlled so that different inventory systems can be generated according to the research objectives; 3) the interactions (relationships) between the system characteristics should be closely modeled. Keeping these perspective in mind, in the following data generation steps, first a set of data was generated based on Rossetti and Achlerkar (2011), and then a regression analysis was implemented to build three models reflecting the relationships between unit cost and lead time at external supplier, ordering cost, and demand respectively.

Based on the literature and experience, some of the attribute relations in an inventory system can be summarized as follows:

---

1) Average annual demand of an item is inversely proportional to its unit cost.

2) Average annual demand of an item is inversely proportional to the mean of its replenishment lead time from External Supplier.

3) The ordering cost of an item is directly proportional to its unit cost.

4) The replenishment lead time of an item from External Supplier is directly proportional to its unit cost.

Exhibit 23: The Assumptions about the Relationships between Attributes

The first two assumptions can also be found in Deshpande et al. (2003) and Rossetti and Achlerkar (2011). The 3rd assumption is summarized based on experience. One of the reasons for the high ordering cost for the items with high unit cost is that the shipping cost is more expensive due to the higher insurance cost for more expensive items (the insurance cost is proportional to the value of items in practice). In this research, the shipping cost is included in the ordering cost. The 4th assumption is assumed based on the 1st and 2nd assumptions. In the real business world, the fast-moving consumer goods have relatively low cost and high demand. The production efficiency for these products is relatively high to meet the customer demands; thus, their replenishment lead time is relatively short. The 1st and 3rd assumptions are also verified by the regression analysis discussed in Appendix 4.

The mechanism to deal with relationships between attributes is developed based on Rossetti and Achlerkar (2011). They use a sequence of conditional probability distributions to randomly generate the attribute values. Their mechanism is illustrated using Table 8 (Adopted from Rossetti and Achlerkar, (2011)).

Table 8: Attribute Values

% of Total	Average Annual Demand	Unit Cost	Mean Lead Time
		[1000-10000]M-30%	[5-20]M-30%
		[1-1000]L-60%	[1-5]L-60%
60%	[100-500]H	[10000-100000]H-10%	[20-200]H-10%
		[10000-100000]H-30%	[1-5]L-30%
		[1000-10000]M-60%	[5-20]M-60%
30%	[10-100]M	[1-1000]L-10%	[20-200]H-10%
		[1-1000]L-30%	[1-5]L-30%
		[10000-100000]H-60%	[20-200]H-60%
10%	[0.5-10]L	[1000-10000]M-10%	[5-20]M-10%

The mechanism developed by Rossetti and Achlerkar (2011) follows several rules:

- Each attribute is associated with three uniform distributions: one for generating low range values, one for generating medium range values, and one for generating high range values. The ranges of these three distributions are continuous but not overlapped.

- The values of average annual demand are generated first, then, the values of other attributes, such as unit cost and mean lead time, are generated based on their relations to average annual demand.
- The generation of the value of average annual demand follows two steps: first, a discrete probability distribution function is used to select the high, medium or low demand category based on the discrete probabilities specified for each demand category. For example, according to the "% of Total" column, the average annual demand has 60% chance to be high category, 30% chance to be medium category, and 10% chance to be low category; the second step is to generate the average annual demand value according to the distribution specified for that category.
- The generation of attribute values other than average annual demand also follows two steps. The generation of Mean Lead Time is described here as an example. The Mean Lead Time also has three value distribution category, i.e., high, medium and low. First, the chance of value category is determined by the direct or inverse relationship between average annual demand and Mean Lead Time and by the value category of the average annual demand. Suppose the values generated for average annual demand belongs to the high category. Since the average annual demand of an item is inversely proportional to the mean of its replenishment lead time, the low category value for Mean Lead Time has more chance and high category has less chance. As a result, because the average annual demand belongs to the high category, the value category for Mean Lead Time is determined by the discrete probability distribution function with 10% for high category, 30% for medium category and 60% for low category; second, the values of the Mean Lead Time is generated according to the distribution selected in the first step.

As mentioned previously, these data generation rules used in Rossetti and Achlerkar (2011) are applied in this dissertation to deal with the relations between attributes mentioned in Exhibit 23. Table 9 and Table 10 are the results of attribute values assignment according to the mechanism proposed by Rossetti and Achlerkar (2011); these results satisfy the attribute relationships in Exhibit 23. The corresponding range for High (H), Medium (M) and Low (L) is specified in Table 11. The difference between their method and the method implemented in this dissertation is that instead of generating average annual demand, this dissertation first generates the values for unit cost, and then determine the attribute values for the other related attributes.

Table 9: Attribute Values for Assumption 1, 3 and 4

Unit Cost	Ordering Cost	Average Annual Demand	Replenishment Lead Time
H	H-60%	H-10%	H-60%
	M-30%	M-30%	M-30%
	L-10%	L-60%	L-10%
M	H-20%	H-20%	H-20%
	M-60%	M-60%	M-60%
	L-20%	L-20%	L-20%
L	H-10%	H-60%	H-10%
	M-30%	M-30%	M-30%
	L-60%	L-10%	L-60%

Table 10: Attribute Values for Assumption 2

Average Annual Demand	Replenishment Lead Time
H	H-10%
	M-30%
	L-60%
M	H-30%
	M-60%
	L-10%
L	H-60%
	M-30%
	L-10%

Table 11: Attribute Ranges

Item Attributes	Category Values					
	Low (L)		Medium (M)		High (H)	
	Range Low	Range High	Range Low	Range High	Range Low	Range High
Average annual demand (unit)	1	50	50	25,000	25,000	2,000,000
Unit cost ($)	0.2	150	150	1,000	1,000	200,000
Ordering cost ($)	100	500	500	2,500	2,500	10,000
Mean replenishment lead time from ES(day)	10	50	50	100	100	250

Regression Models reflecting the attribute relations

Setting the ranges of the attributes as in Table 7 and Table 11, and the probability for High (H), Medium (M), and Low (L) range of the Unit Cost to 10%, 30% and 60% respectively, a set of data is generated. A regression analysis was implemented based on the generated data to test whether the pre-assumed relationships remain between unit cost and lead time at external supplier, unit cost and ordering cost, and unit cost and demand in the resulting data set. The regression models obtained can be summarized as follows:

(1) LTatESmean = 68.6 + 0.000349 unit cost

(2) orderingCost = 2126 + 0.0112 unit cost

(3) demandMean = 330114 - 0.164 unit cost

In the regression models above, plus sign indicates the directly proportional relation, and the minus sign indicates the inversely proportional relation between the corresponding attributes. It can be seen from the regression equations that the relations pre-assumed in Exhibit 23 remain in the generated dataset. The regression analysis results are listed in Appendix 4.

4.2.4 Data Generation Evaluation

In this dissertation, the effectiveness of the data generation methodology is measured by the diversity of the data that represent the system of interest. A data generation process that provides wide variety of inventory systems will facilitate the investigation of the large scale multi item multi echelon inventory system characteristics. As discussed in the previous sections, the multi-item multi-echelon inventory systems include structural and non-structural attributes. To measure the system diversity quantitatively, these attributes should be mathematically represented, so that the statistic diversity measurements can be applied in the process.

In the following section, first the representation of structural attributes, NIS and NIT is discussed, and then the calculation process of the diversity measurement (SSE) is presented accordingly. Since the non-structural attributes are represented using decimal numbers, and their diversity can be calculated directly based on these values using statistical diversity measurement such as SSE, the non-structural attribute diversity is not specifically discussed. Based on the evaluation criterion SSE, the diversities of the NIS and NIT are discussed in the following sections. In addition, a 20-item two-echelon inventory system is generated to visualize the distances of the generated data and the grouping results.

4.2.4.1 The Representation of the Structural Attributes and the Diversity measurement

As previously indicated, SSE can be used as the diversity measurement for both structural and non-structural attributes. In one set of generated data, the diversity means the differences between the system elements in that dataset. Since the calculation of SSE for NIS and NIT is the same, an instance involving two NISes is used to discuss the structural attribute diversity (difference) in this section.

Traditionally in a binary system, 1 represents the existence and 0 the non-existence. Therefore, the structural attribute NIS (or NIT) can be represented as a list of 0 and 1, each of which represents the status of existence of a location in the NIS. Figure 12 shows two NISes, and they can be represented using a list of 0 and 1 as in Table 12.

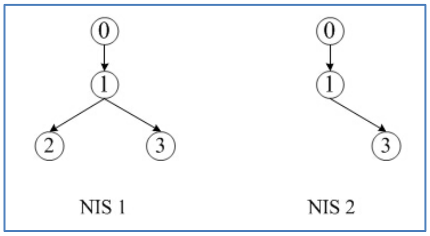

Figure 12: Two NISes

Table 12: Binary representation of NIS

NIS1	Location (j)	0	1	2	3
	x_{1j}	1	1	1	1
NIS2	Location (j)	0	1	2	3
	x_{2j}	1	1	0	1

Based on the binary representation of the NISes, their corresponding diversity measurement, SSE can be calculated using following formula:

$$\sum_{i=1}^{m} \sum_{j=0}^{n-1} \left(x_{ij} - \bar{x_j}\right)^2$$

where $x_{ij} \equiv$ jth dimention (location)of ith observation(NIS i)

$\bar{x_j} \equiv$ the mean of jth dimention

$n \equiv$ number of locations

$m \equiv$number of NISes/NITs

The SSE calculation process for the above example is summarized on Table 13, in which the difference between the two NISes is measured using SSE=0.5.

Table 13: the Calculation of SSE for the NISes

$\bar{x}_j$		1	1	0.5	1
$x_{1j} - \bar{x}_j$	NIS1	0	0	0.5	0
$x_{2j} - \bar{x}_j$	NIS2	0	0	-0.5	0
$(x_{1j} - \bar{x}_j)^2$	NIS1	0	0	0.25	0
$(x_{2j} - \bar{x}_j)^2$	NIS2	0	0	0.25	0
$\sum_{i=1}^{2}\sum_{j=0}^{3}(x_{ij} - \bar{x}_j)^2$	0.5				

Based on the calculation process above, it can be seen that SSE is a positive value, which could result in larger values when applied to the larger networks, and smaller values for smaller networks comparison. Therefore, the relative comparison measurement, adjusted SSE is applied for comparing the NISes in this research. The adjusted SSE can be calculated using following formula:

$$SSE_{adj} = SSE/NL_{max}$$

Where $NL_{max} \equiv the\ maximum\ number\ of\ locations\ among\ the\ NISes\ generated$

It can be seen from the discussion above that the NIS shape decides the difference between the NISes. Also, it can be seen from the discussion in section 4.2.2.1, that the shape of a NIS is decided by the number of echelon, and the probability distribution used to generate the number of customers locations; this means that by changing number of echelon, and the parameters of the probability distribution, the diversity (shape difference) of the generated NISes can be changed. To control the NIS diversity in a set of generated data, finding out which input parameters affect the diversity most has practical value. In this dissertation, Discrete Uniform (DU) distribution is selected to generate the number of customer locations.

4.2.4.2 Diversity of NIS

Based on two kinds of experiments, by controlling the input parameters, mean and variance, of selected DU, the sensitivity of the diversity as output to these input parameters is discussed. The two sets of experiments can be summarized as 1) varying the mean of DU while controlling variance of it; 2) varying the variance of the DU, while controlling it's mean. Simply put, the goal is to investigate how the diversity of the NIS is affected by the mean and variance of distribution of number of customer locations. All the test cases are two echelon location networks, and 1000 NISes are generated to perform the experiments. The experimental input parameters and the results are organized into two scenarios as follows.

Scenario 1: Control Variance of the DU, and Vary the Mean of DU

This experiment is carried out based on three cases. In all cases, the variance is set to 0.67. The mean for these cases are set to 2, 3, and 4 respectively. The settings are listed on Table 14.

Table 14: Parameters for Scenario 1

	LB	UB	Mean	Var
case 1	1	3	2	0.67
case 2	2	4	3	0.67
case 3	3	5	4	0.67

Figure 13 shows the resulting different NIS frequency. Each bar on the figure represents the number of the same NISes generated. It can be seen that the NIS with 5 locations shown on Figure 14 is generated most with 168 occurrences. Figure 15 and Figure 16 show the results of case 2, in which the network with 9 locations is generated the most with 116 occurrences. Figure 17 and Figure 18 illustrates the results of case 3, from which it can be seen that a16 location network is generated 92 times with the highest frequency.

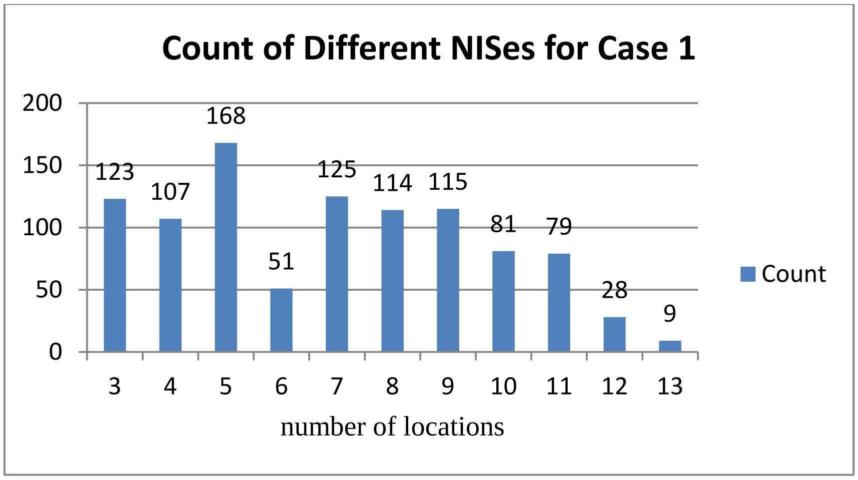

Figure 13: Plot for count of different NISes for case 1

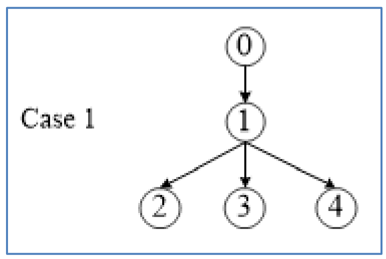

Figure 14: The NIS with 5 Locations

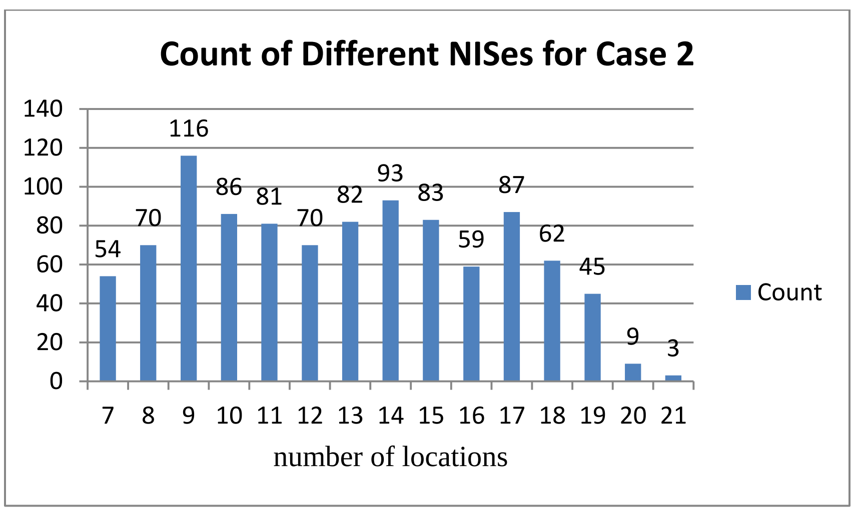

Figure 15: Plot for count of different NISes for case 2

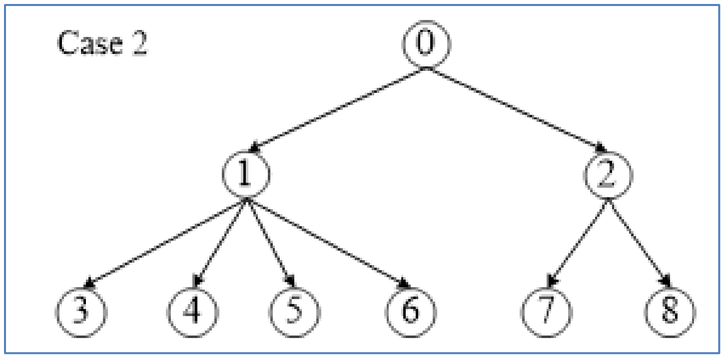

Figure 16: The NIS with 9 Locations

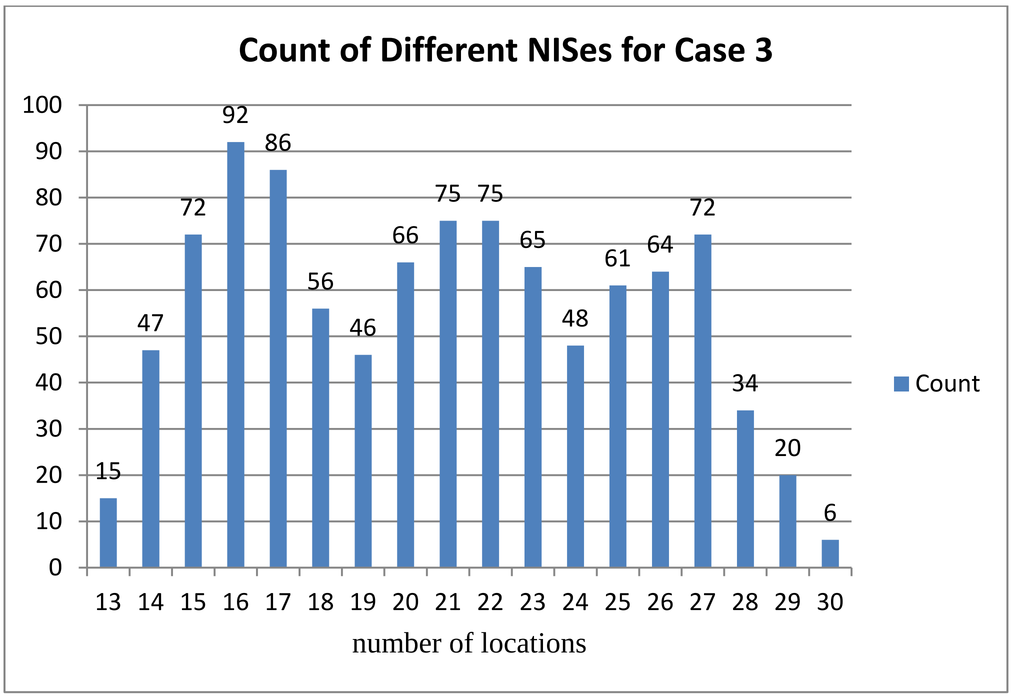

Figure 17: Plot for count of different NISes for case 3

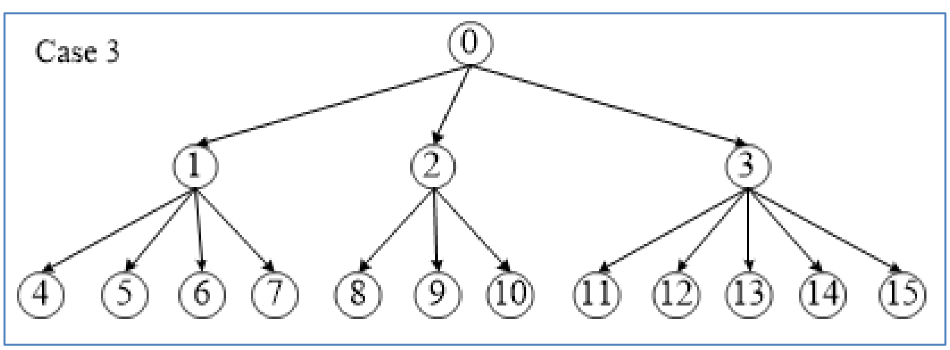

Figure 18: The NIS with 16 Locations

Scenario 2: control mean of the DU, and vary the variance of DU

This experiment is also implemented based on three cases. In all cases, the mean is set to 3.5. The variances for these cases are set to 0.25, 1.25, and 2.92 respectively. The settings are listed on Table 15.

Table 15: Parameters for Scenario 2

	LB	UB	Mean	Var
case 4	3	4	3.5	0.25
case 5	2	5	3.5	1.25
case 6	1	6	3.5	2.92

Figure 19 shows the resulting different NIS frequency. Each bar on the figure represents the number of same NISes generated. It can be seen that the NIS with 14 locations shown on Figure 20 is generated most with 208 occurrences. Figure 21 and Figure 22 show the results of case 5, in which the network with 10 locations is generated the most with 72 occurrences. Figure 23 and Figure 24 illustrates the results of case 6, from which it can be seen that an 8 location network is generated 59 times with the highest frequency.

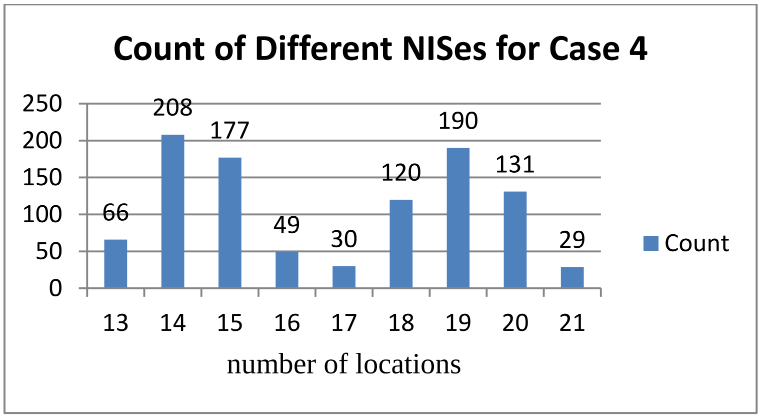

Figure 19: Plot for count of different NISes for case 4

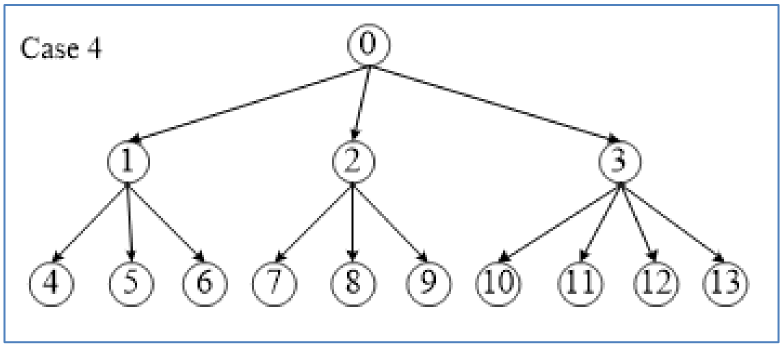

Figure 20: The NIS with 14 Locations

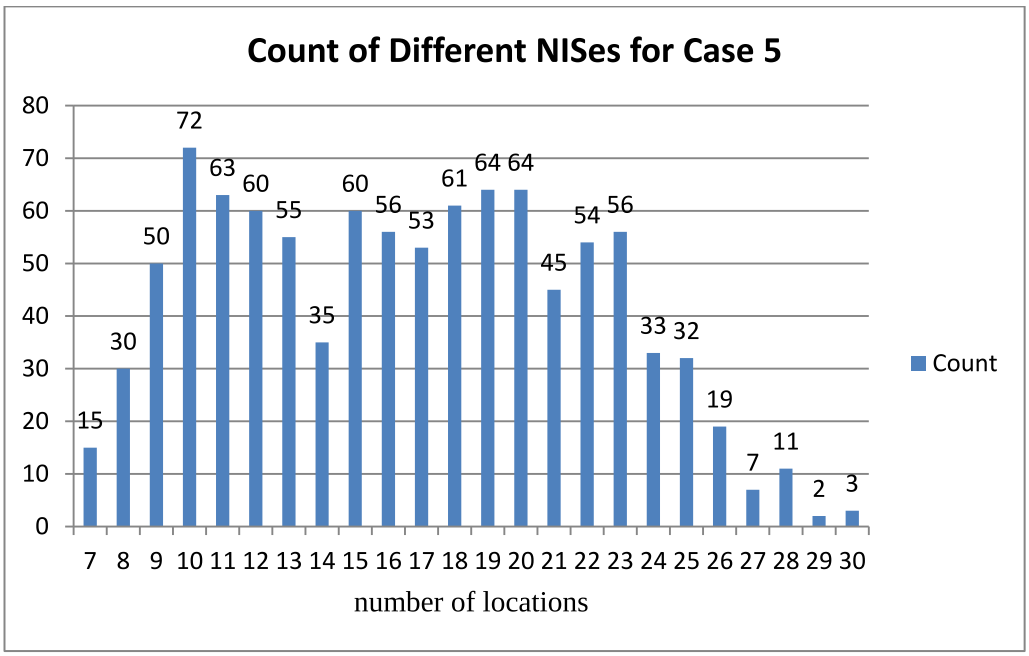

Figure 21: Plot for count of different NISes for case 5

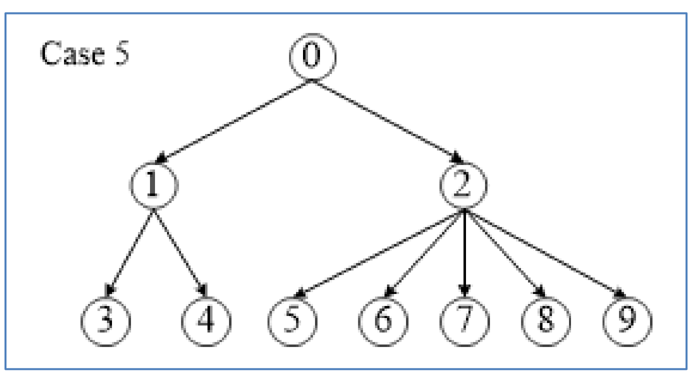

Figure 22: The NIS With 10 Locations

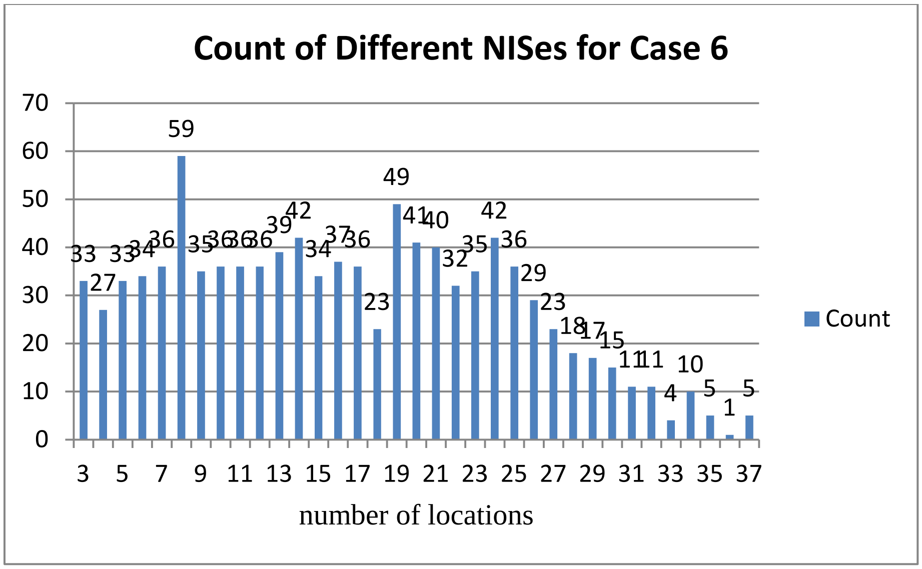

Figure 23: Plot for count of different NISes for case 6

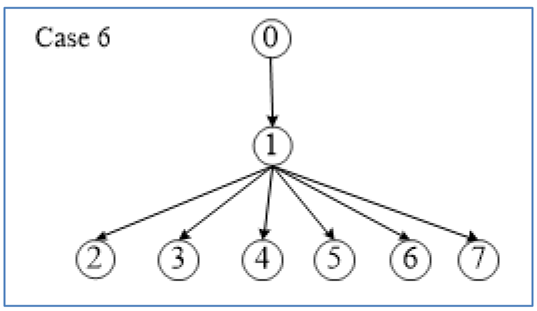

Figure 24: The NIS with 8 Locations

The experimental results of the two scenarios are summarized on following Table 16 and Table 17.

Table 16: Results for Scenario 1

Scenario 1	case 1	case 2	case 3
Mean of DU	2	3	4
Var of DU	0.67	0.67	0.67
SSE of NISes	1536.43	2057.84	2546.18
SSE_{adj}	118.19	97.99	82.13
# of locations of the most frequent location network	5	9	16
# of different networks	11	15	18

Table 17: Results for Scenario 2

Scenario 2	case 4	case 5	case 6
Mean of DU	3.5	3.5	3.5
Var of DU	0.25	1.25	2.92
SSE of NISes	1397.32	3079.21	4703.04
SSE_{adj}	66.54	99.33	109.37
# of locations of the most frequent location network	14	10	8
# of different networks	9	24	35

From the SSE perspective, as shown in Table 16, when the variance of DU is controlled, the SSE increases with the increase of mean of DU; and, when the mean of DU is controlled, as shown in Table 17, the SSE increases with the increase of variance of DU.

From the adjusted SSE perspective, when the variance of DU is controlled, the SSE_{adj} decreases with the increase of mean of DU; and, when the mean of DU is controlled, the SSE_{adj} increases with the increase of variance of DU.

From the number of different networks generated, the number of different networks generated is increased considerably following the increase of the mean while the variance is kept unchanged (Table 16); and when the mean is kept unchanged, the number of different networks generated is also increased considerably following the increase of the variance (Table 17).

From the number of locations of the most frequently generated perspective, in scenario 1, the number of locations of the most frequently generated increases following increase of mean; and, in scenario 2 it decreases following increase of variance. This helps to diversify generation of NIS, and since NITs are generated based on NISes, therefore it decides the diversity of NITs indirectly.

4.2.4.3 Diversity of NIT

The diversity of NIT is affected by two parameters: number of retailers at lowest echelon (*NR*) and probability of existence of an item at a retail store (*PE*). Two sets of experiments are implemented to investigate the impact of these two parameters on the diversity of NITs generated. The experiments can be summarized as: (1) control the *NR* and investigate how *PE* affects the adjusted SSE; and (2) investigate how *NR* affects the adjusted SSE.

<u>Experiment 1</u>

This experiment is carried out based on the NIS, which has four retailers. The *PE*s used to generate the NIT are 0.1, 0.3, 0.5, 0.7, and 0.9. Using these *PE*s, 1000 NITs are generated and the resulting adjusted SSEs are plotted in Figure 25.

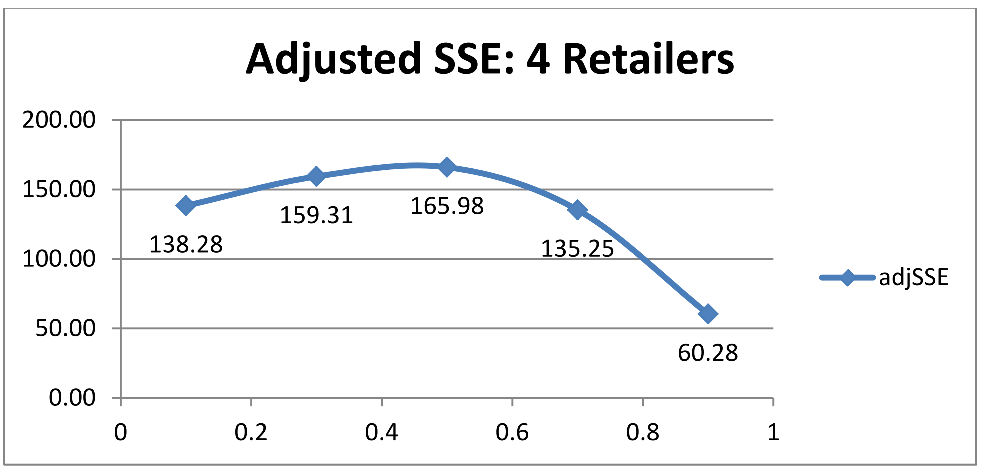

Figure 25: Adjusted SSE of 4 Retailers

For each *PE*, the distributions of the generated NITs are plotted as shown in Figure 26 to Figure 30. In the figures from Figure 26 to Figure 30, 1 to 4 on the horizontal axis are the NITs with 1 retailer; 5-10 are the NITs with 2 retailers; 11-14 are the NITs with 3 retailers; and 15 is the NITs with 4 retailers.

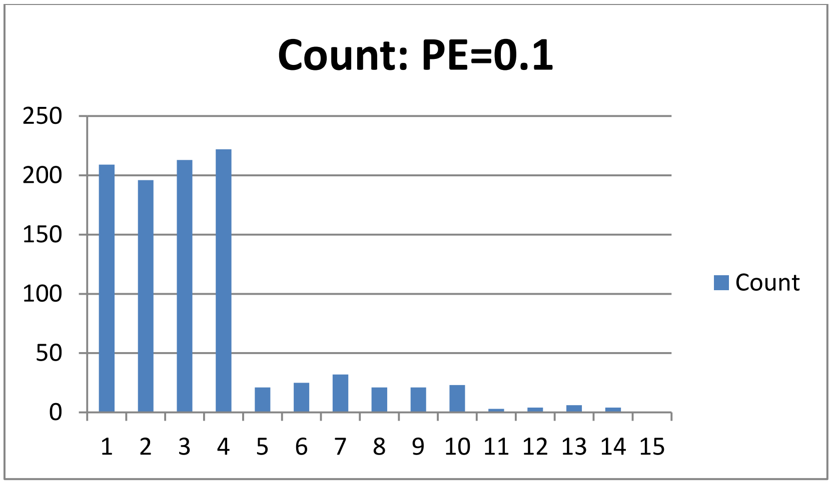

Figure 26: Plot for count of different NITs when *PE*=0.1

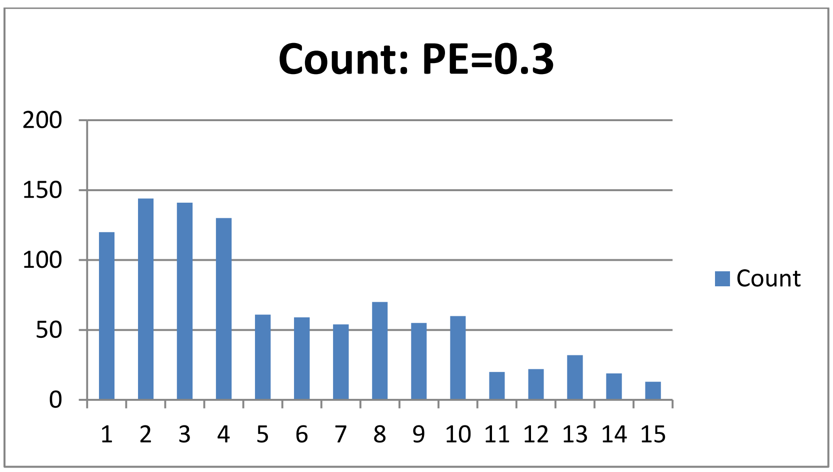

Figure 27: Plot for count of different NITs when *PE*=0.3

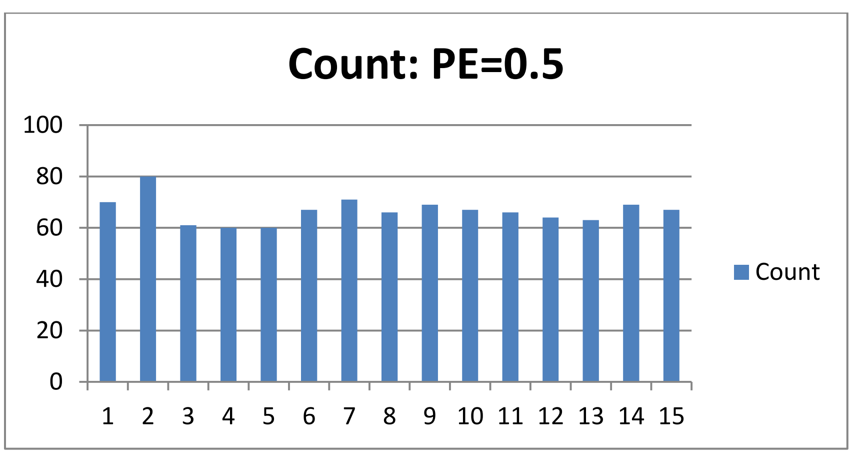

Figure 28: Plot for count of different NITs when *PE*=0.5

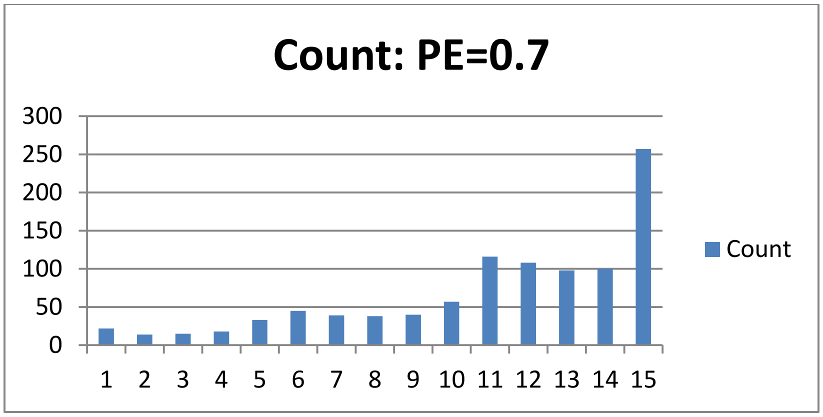

Figure 29: Plot for count of different NITs when *PE*=0.7

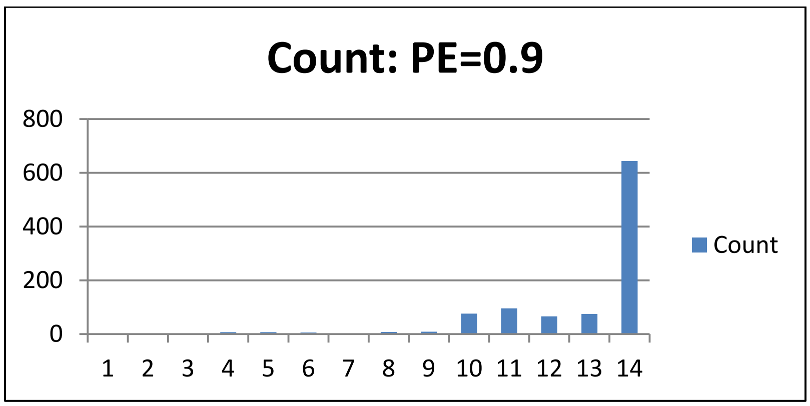

Figure 30: Plot for count of different NITs when *PE*=0.9

Based on the results of experiment 1, the following conclusions can be made:

(1) The adjusted SSEs of NITs increase when PE has range 0.1 to 0.5 and decrease when it has range 0.5 to 1.

(2) When the values of *PE* increase, the generated NITs tend to have larger number of locations.

Experiment 2

In order to investigate how *NR* affects the adjusted SSE of the generated NITs, the values of *NR* are set to 3, 4 and 5. Since *PE* also affects the SSE of the generated NIT, five values of *PE*, i.e., 0.1, 0.3, 0.5, 0.7, and 0.9, are used to generate NITs for each value of *NR*. For experiment 2, 1000 NITs are generated to calculate SSE, and the experiment results are summarized in Table

18. For 3 retailer and 5 retailer cases, the changing patterns of SSE according to different *PE*s are illustrated in Figure 31 and Figure 32.

Table 18: Experiment Results for Experiment 2

PE	Adjusted SSE for 3 NR	Adjusted SSE for 4 NR	Adjusted SSE for 5 NR
0.1	140.06	138.28	131.95
0.3	148.65	159.31	164.35
0.5	147.50	165.98	178.39
0.7	121.45	135.25	152.35
0.9	49.63	60.28	62.85

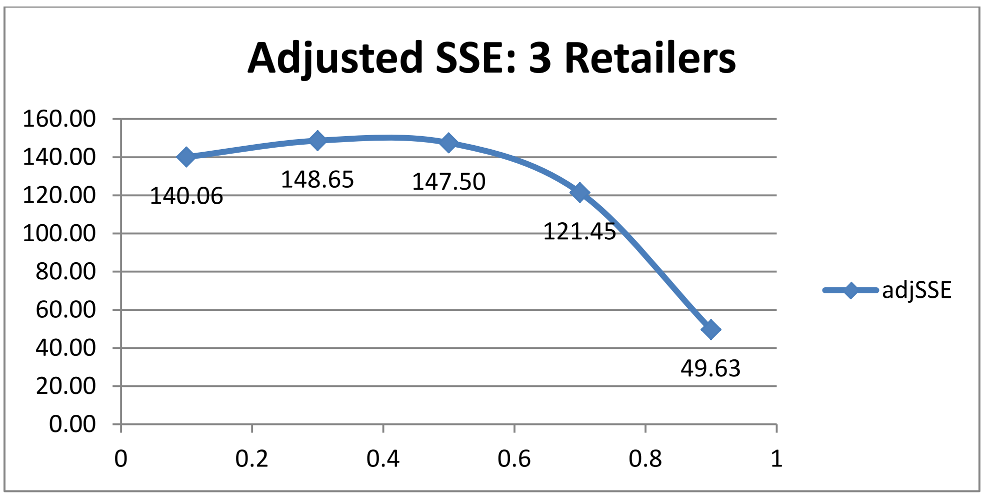

Figure 31: Adjusted SSE of 3 Retailers

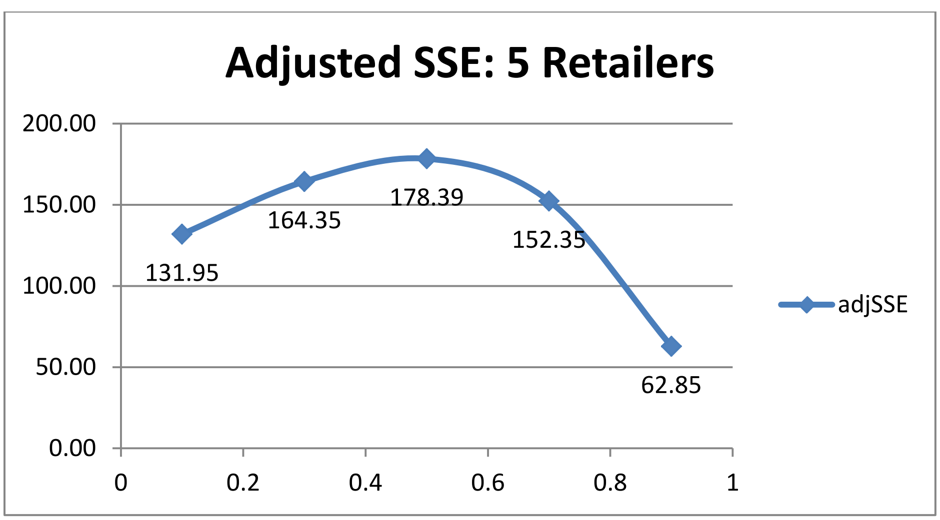

Figure 32: Adjusted SSE of 5 Retailers

Based on the results of experiment 2, the following can be concluded:

(1) Adjusted SSEs of NITs increase when *PE* ranges from 0.1 to 0.5 and decrease when it ranges from 0.5 to 1 for different values of *NR*.

(2) It can be seen from Table 18 that when PE equals 0.1, the adjusted SSE decreases with the increasing of *NR*. When the value of PE is larger than 0.1, the adjusted SSE increases with the increasing of *NR*.

These conclusion help to set PE values when generating NITs, for example, to get higher SSE, which is the diversity measurement of diversity of NITs, for NITs generated, one can set PE value closer to 0.5.

4.2.4.4 Data Generation and Grouping of the data

As part of the evaluation process of the generated data, this section first illustrates the generated data using normalized Euclidean Distance between items and the Mean Item. The Mean Item is obtained by averaging attribute values of all items generated. And then, the generated items are tentatively grouped using K-Means clustering to see whether the generated data is "good" for grouping.

In this instance of data, the structure of the NIS is a two echelon inventory system with one location on echelon 1 and two locations on echelon 2. The figure of the NIS can be found in Figure 7(B). Using the data generation process discussed in section 4.2.2, and the mechanism discussed in section 4.2.3.2, which generates the values of the attributes keeping the relations, 20 items were generated. The attributes listed in section 3.3.3, i.e. the attribute list for non-structural attributes (attribute list 1) and the attribute list for both structural and non-structural attributes (attribute list 2) are discussed respectively. For each item, normalized Euclidean distances between its attributes and the Mean Item attributes based on both attribute lists 1 and attribute lists 2 are calculated and stored in Table 19.

Table 19: The Euclidean Distance between Items and the Mean Item

Item#	Distance of Non-Structural Attributes	Distance of Non-Structural and Structural Attributes
1	3.58	4.29
2	0.79	0.89
3	2.01	2.12
4	2.87	3.57
5	0.95	1.05
6	2.75	3.35
7	3.03	3.63
8	1.69	1.79
9	4.2	4.9
10	2.62	3.23
11	1.84	1.94
12	2.53	2.63
13	2.9	3.6
14	2.08	2.19
15	2.15	2.25
16	1.42	1.52
17	3.57	4.17
18	1.89	2
19	2.9	3.5
20	1.28	1.38

In order to visualize the distances between items, the normalized Euclidean distances between items and the Mean Item are drawn in Figure 33 for both attribute list 1 and attribute list 2. It can be seen from Figure 33 that, there are three items, which are item 4, 13 and 19, having different relative distances (or the relative sequence based on the distance) to the Mean Item; this means that when applying attributes in list 1 and list 2 separately, the items show different grouping tendency when the grouping is based on the Euclidean Distance.

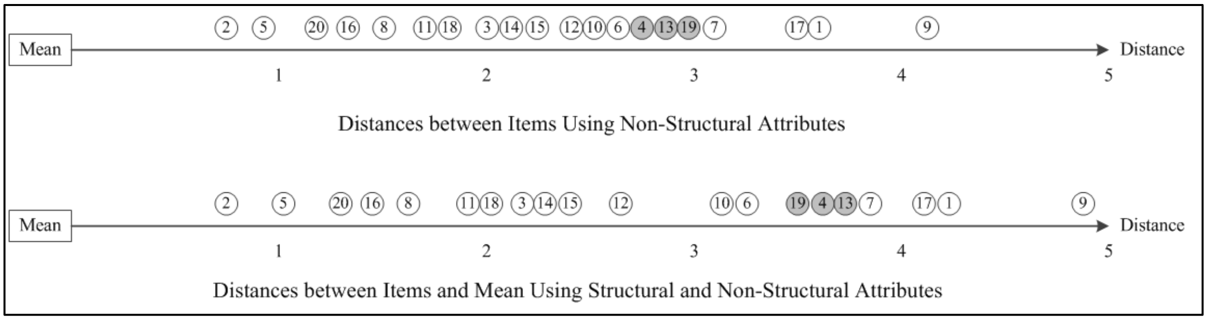

Figure 33: The Plot of Euclidean Distances between items and the Mean Item

The K-Means clustering discussed in section 3.3.3 is applied to group the 20 items into 3 groups. The grouping results based on attribute list 1 and attribute list 2 are illustrated in Figure 34.

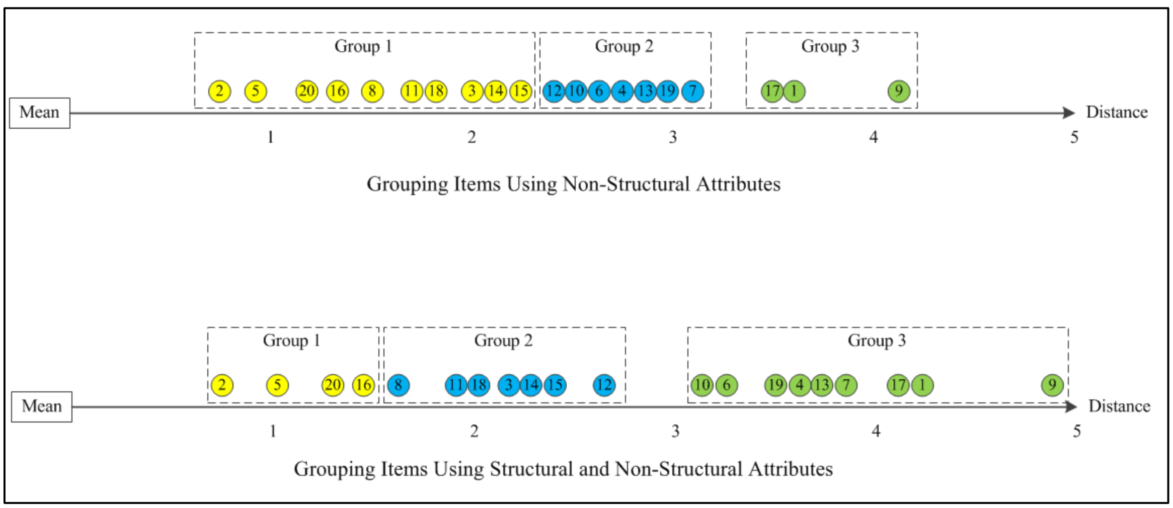

Figure 34: Grouping Results for 20 items

Based on the distances in Table 19 and grouping results in Figure 34, three observations are obtained from this tentative grouping process as follows:

(1) The distribution (based on normalized Euclidean distance) of the items using non-structural attributes and both non-structural and structural attributes are different; this can be seen from the observation that relative positions of the three items (4, 13 and 19) in this instance are different.

(2) The generated items show some data patterns that can be grouped, which means that items “visually near” based on normalized Euclidean distance tend to be grouped together.

(3) For the same dataset, the grouping results could be different when using different grouping attributes.

The generated data in this research will be systematically evaluated further in the following chapters based on more specific and larger data instances.

5 Experimental Design

Based on the guidelines for designing an experiment suggested by Montgomery (2001), this chapter identifies the research problems; selects the factors, levels to answer the research questions; the corresponding response variables for the questions; and, the experimental designs accordingly. This chapter first investigates the research factors and their levels, and then discusses the experimental design for this research.

5.1 Research Factors Analysis

As illustrated in Exhibit 1, the research factors considered are categorized into 5 main factor categories. The inventory control policy studied in this dissertation is continuous reorder point reorder quantity policy; this factor is fixed to one level. Also, the cost model is fixed to the cost model discussed in 3.2. Except for these two factors, the impacts of other three main factor categories are studied in this research. This section investigates these main research factor categories and the levels of the factors.

5.1.1 The Factor Category N

The factor Category N (characteristics of the inventory system) can be classified into three sub-categories. The 1st category is non-structural attributes. There are 9 factors belonging to this category, and they can be found in Table 7. The 2nd category is structural attributes; number of locations, number of echelons and NIT is considered for this category. The 3rd category is the number of items. The 1st and 2nd categories are the characteristics of one single item, and the 3rd category describes the scale of the items involved.

5.1.2 The factor category A_q and k

The factor category A_q is the grouping methods. Three grouping techniques were discussed in section 3.3. The study of this factor is a focus of this dissertation. This sub-section discusses the research questions that are specific for ABC classification and K-Means clustering.

5.1.2.1 Research Questions regarding ABC Classification

The research questions are considered from two perspectives: 1) classification criterion and 2) the number of groups.

1) The classification criterion

In order to evaluate the effectiveness of the clustering criterion NIC, the traditional ABC classification criterion, i.e., annual dollar usage is also implemented. The formula used to calculate the network annual dollar usage ($NADU$) is as following:

$$NADU = \sum_{i=1}^{n} \sum_{j=1}^{r} \lambda_{ij} \times C_i$$

Where

$n \equiv$ number of item types

$r \equiv$ number of retail stores

$C_i \equiv$ unit cost of item *i*

$\lambda_{ij} \equiv$ annual demand of item *i* at location *j*

2) The number of groups

As discussed in Teunter et al. (2010), typical A, B, and C classes contain around 20%, 30%, and 50% of all SKUs respectively. Teunter et al. (2010) proposes an extension of 3 classes to 6 classes ABC classification with 4% for A class, 7% for B class, 10% for C class, 16 % for D class, 25% for E class, and 38% for F class. In this research, the typical three classes ABC classification, and a seven classes ABC classification extended from Teunter et al. (2010)'s 6 classes ABC classification are implemented. The details about the extension from 6 classes to 7 classes are introduced in section 6.2.

In sum, there are two specific research questions related to ABC Classification:

A1: Whether the classification criterion NIC is better than NADU?

A2: Whether 7-group is better than 3-group for ABC classification?

5.1.2.2 Research Questions regarding K-Means Clustering

Six K-Means Clustering related research questions are discussed in this section.

K1: Which non-structural attributes are significant?

The selection of clustering attributes is critical for K-Means clustering. From the efficiency perspective, the more clustering attributes, the more clustering time will be taken. Too many clustering attributes may make clustering time infeasible. From the effectiveness perspective, adding some not significant clustering attributes may reduce the system performance. Thus, this research question is to identify the significant non-structural attributes.

K2: Whether the item types having the same NIT structure tend to be clustered into the same group?

After grouping the items using the K-Means clustering, a study of main NIT is performed to identify the dominating structure in each group. An example of main NIT analysis is illustrated in Table 33. If in each group, a majority of items have the same NIT structure, then it can be concluded as a trend that to group items with same NIT together exists.

K3: Whether the structural attributes affect the clustering results?

The structural attribute considered in this research is NIT. The binary expression for NIT (illustrated in Table 4) is used as the structural attributes values for K-Means. In other words, the structural attributes is a set of binary variables, each of which represent the existence of a retail store. Experiments are designed to test the changes of effectiveness and efficiency when structural attributes are involved.

K4: Which factors affect the K-Means clustering time?

The clustering time (in seconds) is crucial for large scale grouping. Maimon and Rokach (2005) summarize three factors affecting the K-Means clustering time: the number of instances, the number of clusters, and the number of iterations used by the algorithm to converge. Besides these three factors, the number of clustering attributes is also considered for the study of clustering time.

K5: How does the number of clusters *k* affect the clustering results?

As mentioned previously, the number of clusters affects clustering performance which are measured based on %CPC (Percent Grouping Penalty Cost), SSE, and GT (Grouping Time) in this research. Two levels of k, i.e., 3 and 7 groups are used to investigate the changes of the selected performance measures.

5.1.2.3 Research Questions for comparing different grouping techniques

The three grouping techniques, ABC, NIT, and K-means need to be compared based on the same number of groups. Since NIT classification can have 7 groups when the number of retailer locations is 3, and the number of groups for ABC classification can be legitimately extended to 7 groups, while K-means clustering has the flexibility of setting the number of groups K, the number of groups for comparison of the grouping techniques is set to 7 groups. Basing the comparison of the grouping techniques on 7 groups rather than 3 groups is also because 7-group case results better performance measures for both ABC and K-means grouping techniques.

Based on Figure 35, it should be noted that when NIT structure has 2 retailer locations as on NIS_1, there are 3 possible different NITs groups, which is calculated as $C_{(2,1)} + C_{(2,2)} = \frac{2!}{1!(2-1)!} + \frac{2!}{2!(2-2)!} = 3$, and in the same way when there are 3 retailer locations as on NIS_2, the number of different NITs is $C_{(3,1)} + C_{(3,2)} + C_{(3,3)} = \frac{3!}{1!(3-1)!} + \frac{3!}{2!(3-2)!} + \frac{3!}{3!(3-3)!} = 7$, this inflexibility of number of groups in NIT's case is also a reason for setting the groups number to 7.

The research question this section deals with is which grouping technique performs better than the other ones with respect to the grouping performance measures, i.e. %CPC, SSE, and GT.

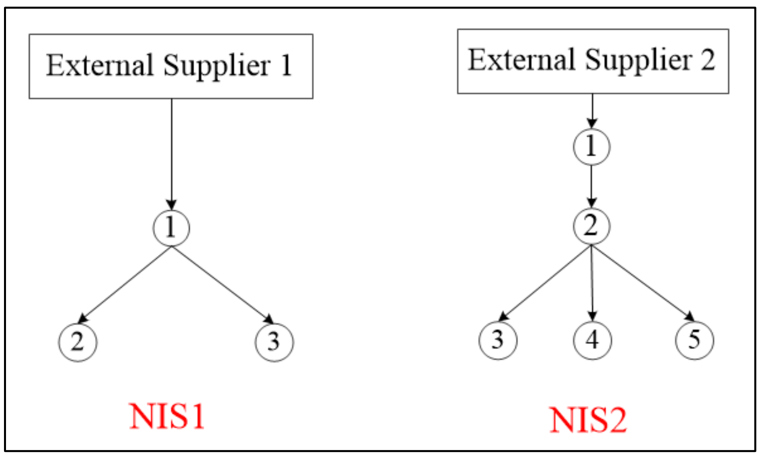

Figure 35: Two NISes

5.1.3 The factor category m_j

From the study of classification criterion in section 3.3.1, it can be seen that the performance of the grouping techniques should depend on the selected cost model if they are derived using cost models. For example, Zhang et al. (2001) develop a classification criterion $\frac{D_i}{l_i c_i^2}$ (CC1) based on the cost model (Model 1) to minimize inventory investment and prove that this classification criterion is appropriate for the model. Teunter et al. (2010) develop another classification criterion $\frac{b_i D_i}{h_i Q_i}$ (CC2) based on the cost model (Model 2) to minimize total inventory cost and compare the CC2 and CC1 based on Model 2, and prove that CC2 is better than CC1. This seems right since the CC1 is constructed based on Model 1, and the conditions derived in Model 1 for CC1 (higher values of $\frac{D_i}{l_i c_i^2}$ results in better system performance) may not hold for Model 2.

In this dissertation, the system performance measures selected are calculated using EHWS (Extended Hedley and Whitin Solution) discussed in Section 3.2.

5.2 The Design

The experimental designs are based on experimental factors and levels selected. This section discusses the factors, levels, and experimental designs based on the research questions in the previous section.

5.2.1 Experimental Design for ABC Grouping

This section deals with questions A1 and A2 mentioned in Section 5.1.2.1. The factor and the factor levels are summarized on Table 20.

Table 20: Factors and Levels for ABC grouping

Research Question	Factor	Level 1	Level 2
A1	Classification Criterion	NADU	NIC
A2	Number of Groups	3	7

One replicate randomized complete block design is used to analyze A1 and A2. Each block in this case represents one scenario (an inventory system) that contains all the system characteristics listed in Table 21. There are 1024 (2^{10}=1024) system scenarios considered on each comparison experiment by each system attribute taking 2 levels. The resulting values for %CPC and GT are the observations observed for each of these 1024 system scenarios. For

CPC and %CPC calculation process refer to Section 3.3.5.2. A complete block is an experimental setup that tests the two levels of the factors by putting them in one scenario. For example, for A1 use both NIC and NADU to implement ABC classification; for A2 use both 3 groups and 7 groups to implement ABC classification. In an experiment, randomization is used to control unknown and uncontrolled nuisance factors (Montgomery (2001)). For example, setup time is one of the uncontrolled nuisance factors. When running a 1024-scenario experiment, at the beginning the computer start allocating resources such as RAM for the calculation, therefore the scenarios implemented during this period run relatively slowly than the scenarios implemented after the setting up; this results in the variances on GT. To control the effect of the setting up process to the experimental results, the randomization is implemented by randomizing the sequence of the levels within each block experiment.

Table 21: The Factor Index for Item Characteristics

Factor Index	Attribute	Low	High
A	Unit Cost ($)	[1,10]	[1,20]
B	Lost-Sale-Cost-to-Unit-Cost Ratio	[0.1,0.2]	[0.1, 0.5]
C	Ordering Cost ($)	[10,20]	[10,50]
D	Inventory Holding Charge ($/$/year)	[1%,10%]	[1%,20%]
E	Demand Rate (yearly)	[1,100]	[1,200]
F	Demand Variance-to-Mean Ratio	[0.1,0.2]	[0.1, 0.5]
G	Mean lead time at ES (day)	[10,20]	[10,50]
J	Lead Time Variance-to-Mean Ratio	[0.1,0.2]	[0.1, 0.5]
K	Number of Items	1100	2200
L	Number of IHPs	3	5

Since there are 1 factor 2 level in dealing with A1 and A2, Paired t-test is used to analyze the performance measures %CPC and GT.

5.2.2 Experimental Design for K-Means Clustering

The research question K1 is to identify the most significant non-structural attributes for the grouping process. As shown in Table 21, 8 (A to J) factors (attributes) are investigated to decide whether they are significant on the performance measure %CPC. Montgomery (2001) points out that “because resources are usually limited, the number of replicates that the experimenter can employ may be restricted. Frequently, available resources only allow a single replicate of the design to be run, unless the experimenter is willing to omit some of the original factors”. Sine there are 8 factors that are important for this research to investigate and any one of them cannot be omitted in this section of experiment, and since the large-scale character when it comes to the data scale this section of experiments need to handle, a single replicate 2^k factorial design is implemented to investigate the item characteristics.

To deal with the research question K1, 256（2^8）design points are considered. One design point represents one inventory system scenario that the values of system characters (factor) are set to specific values. An example of design points is given on Table 22, on which -1 represents low level, and 1 represents high level of corresponding factor. The factors’ values within a design point are the input for the data generation mechanism discussed in Section 4.2. Based on the generated data, the K-Means clustering technique discussed in Section 3.3.3 is used to grouping the system. The grouping penalty cost (%CPC) is used as the response variable to evaluate the significance of the on-structural attributes. For the experimental results, the stepwise regression is used to identify the significant non-structural attributes.

Table 22: An example of design points for K1

Design Point /Scenario	A	B	C	D	E	F	G	J
1	-1	-1	-1	-1	-1	-1	-1	-1
2	1	-1	-1	-1	-1	-1	-1	-1
…								
256	1	1	1	1	1	1	1	1

5.2.3 Experimental Design for Comparing the Three Grouping Methods

The goal of the experiment is to compare the three grouping methods. The research factor is the grouping method, and there are three factor levels, i.e., ABC classification, NIT classification, and K-Means clustering implemented in this research.

The experiments implement one replicate randomized complete block design. Each block in this case represents one scenario (inventory system). A complete block is an experimental setup that tests the three methods by putting them in one scenario. The randomization is implemented by randomizing the sequence of the experiments corresponding to each of the grouping methods within each block. To compare the grouping methods based on 7 groups, the number of IHP is seleceted as 5 (3 echelon 3-retailer case). The 9 attributes with indices from A to K on Table 21 are considered on each experiment. There are 512 (2^9=512) system scenarios (blocks) considered on each comparison experiment. In this three grouping method comparison experiment, since the grouping method factor has 3 levels, therefore there are three design points as shown on Table 23. The resulted block design is shown on Table 24. Each scenario (block) is generated using the data generation mechanism discussed in Section 4.2.

Table 23: Design Points for the Three Grouping Method Comparison

Design Point	Grouping Method
1	ABC classification
2	NIT classification
3	K-Means clustering

Table 24: Block Design for the Three Grouping Method Comparison

Grouping Method	Block			
	1	2	…	512
(1)ABC classification	y11	y12	…	y1_512
(2)NIT classification	y21	y22	…	y2_512
(3)K-Means clustering	y31	y32	…	y3_512

The response variables for the experiments are %CPC, SSE, and GT. ANOVA is used to analyze the experimental results. If the null hypotheses are rejected, the Fisher's LSD method is used to conduct multiple comparisons.

6 Experimental Results and Analysis

The datasets generated for the experiments in this chapter are based on the analysis of the system characteristics of large scale multi-item, multi-echelon inventory systems in the previous sections. The non-structural and structural attributes selected for the grouping are listed on Table 21. As discussed in the previous sections these attributes are chosen based on the inventory system characteristics summarized in Cohen et al. (1986) and the cost model used in this research. The data generation process is summarized on Section 4.2.2.2. The relationships between the attributes generated are introduced in Section 4.2.3.2.

Based on the data generation mechanism discussed on chapter 4, values of the system characteristics for the experiments in this chapter are generated. To introduce the generated attribute values and their characteristics, a pilot experiment is presented. Using this pilot experiment the three grouping methods are implemented based on three attributes, i.e. Unit Cost, Ordering Cost (on a specific location), and Demand Rate (on a specific location) to illustrate the relationships between clusters and the performance measures, i.e. %CPC and SSE, visually. Also, in this pilot experiment, the consistency between these performance measures is visually presented. After clarifying these relationships, the following sections implement more specific experiments regarding the grouping methods of interest based on the same data generation mechanism.

Following the pilot experiment, rest of this chapter analyzes the experimental results of ABC classification and K-Means clustering, and then the comparison of the three individual grouping techniques is conducted.

6.1 Pilot Experiment

This section presents some pilot experiments to help better understanding of the following experiments carried out in rest of this chapter using visual tools, such as plots, charts, etc.

6.1.1 Data for Pilot Experiment

The NIS selected (the NIS2 as shown in Figure 35) for this part of the experiment is a three-echelon structure, which has one location on echelon 1, one location on echelon 2, and three locations on echelon 3. Since the goal is to show visually the experimental results in this section, only one scenario is considered. The high level ranges of the non-structural attributes as shown on Table 21 are selected as the attribute value ranges in this part of experiment, except the value of number of items, which is set as 10000.

For overall generated data, we need more spread out data with some data points compacted in certain areas so that we can check whether the grouping methods are grouping the compacted data together. To present better the grouping mechanism visually, the following steps are taken to generate data that shows clear pattern:

1) Use data generation mechanism mentioned in Chapter 4 to generate 10,000 items. The generated data is plotted in Figure 36 to visualize the data points. In Section 6.3.1, the significance of non-structural attributes is studied using stepwise regression analysis. The results show that ordering cost, demand and unit cost are the most significant non-structural attributes which affect the grouping penalty cost; thus, these three attributes are selected to plot the items. In addition, the items located at location 3 are selected to plot the items. The values of the selected attributes are normalized to plot the data points as shown in Figure 36.

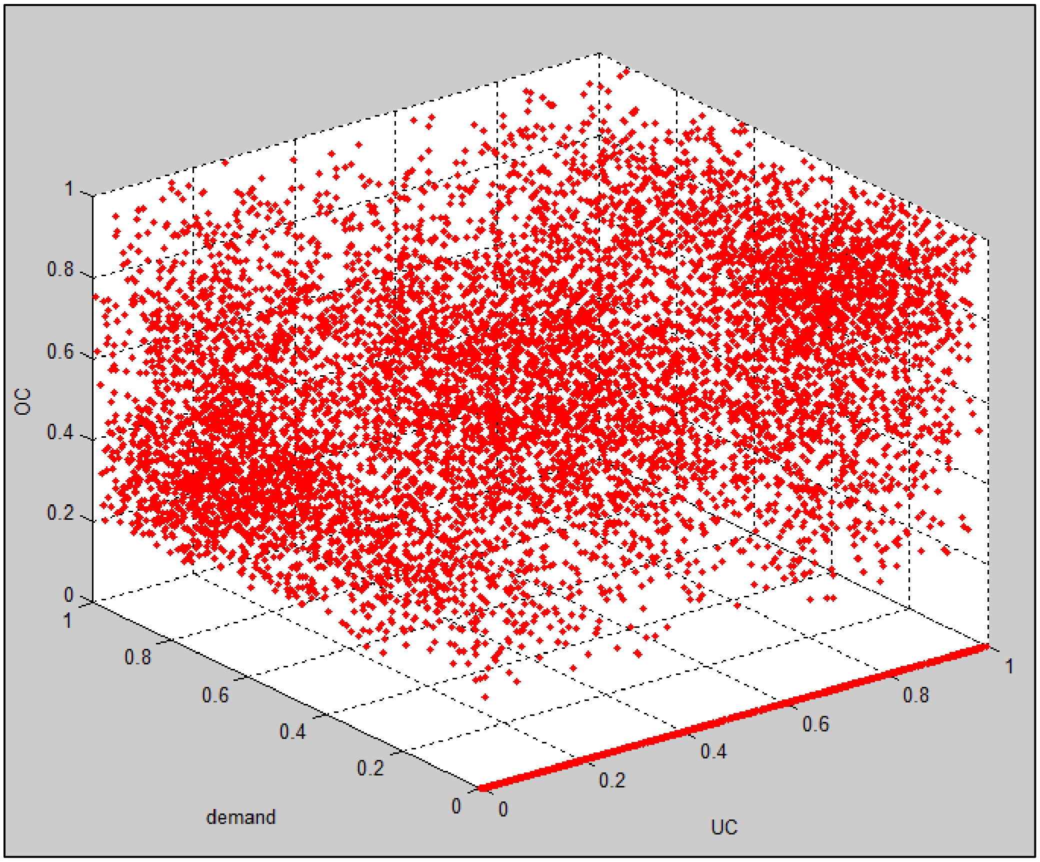

Figure 36: 10,000 Items

2) It should be noted that the data points on Unit Cost axis represent the item types that are not stored at location 3, and this can be seen based on the NIT structures on Figure 37. On Figure 37, it can be seen that the item types stored at location 3 have NITs as shown on part (a), and the item types not stored at location 3 have NITs as shown on part (b). It can also be seen from the plot that the NIT structure, which represents an item type's storage structure in an inventory network, affects the data pattern as shown on Figure 36.

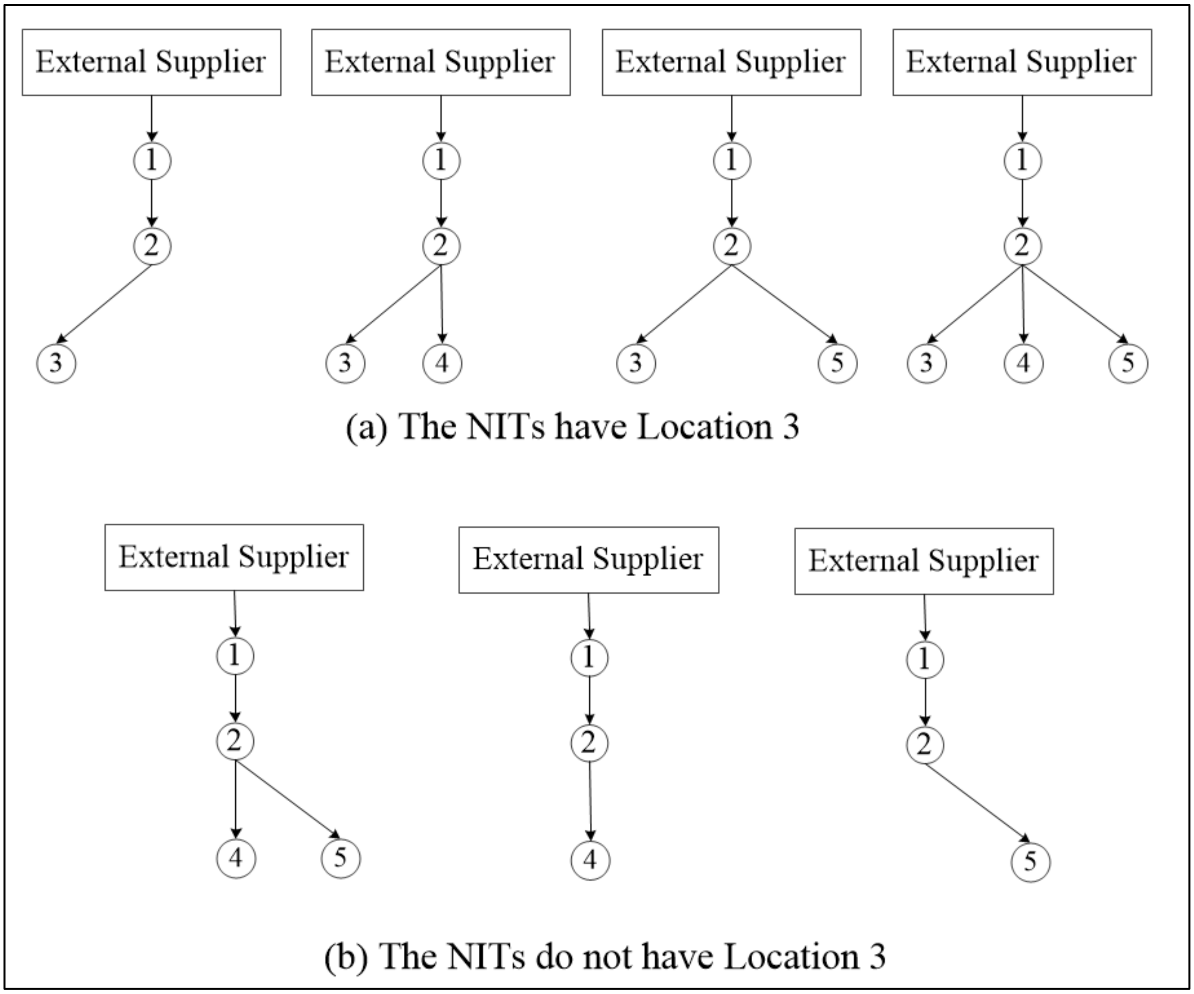

Figure 37: NIT structures

As mentioned in Lenard and Roy (1995), the storage structure is the attribute that prevent items grouping together. NIT represents the storage structure of a specific item type in an inventory network, and it represents the supplier and customer relationships between the locations for the item type. Therefore, it can be seen that the data points on the Unit Cost axis tend to be clustered separate to the other data points that not falling on to the Unit Cost axis.

The goal of the data generation in this pilot experiment section is to obtain data points with clear cluster patterns visually (in 3D plot). To obtain this kind of data points, the entire space shown on Figure 36 is divided into 27 cubic spaces based on Table 25, on which LB and UB represent Lower Bound and Upper Bound respectively.

Table 25: 27 Modules

Module	Unit Cost		Demand		Ordering Cost	
	LB	UB	LB	UB	LB	UB
1	0.15	0.4	0.15	0.4	0.15	0.4
2	0.15	0.4	0.15	0.4	0.45	0.7
3	0.15	0.4	0.15	0.4	0.75	1
4	0.15	0.4	0.45	0.7	0.15	0.4
5	0.15	0.4	0.45	0.7	0.45	0.7
6	0.15	0.4	0.45	0.7	0.75	1
7	0.15	0.4	0.75	1	0.15	0.4
8	0.15	0.4	0.75	1	0.45	0.7
9	0.15	0.4	0.75	1	0.75	1
10	0.45	0.7	0.15	0.4	0.15	0.4
11	0.45	0.7	0.15	0.4	0.45	0.7
12	0.45	0.7	0.15	0.4	0.75	1
13	0.45	0.7	0.45	0.7	0.15	0.4
14	0.45	0.7	0.45	0.7	0.45	0.7
15	0.45	0.7	0.45	0.7	0.75	1
16	0.45	0.7	0.75	1	0.15	0.4
17	0.45	0.7	0.75	1	0.45	0.7
18	0.45	0.7	0.75	1	0.75	1
19	0.75	1	0.15	0.4	0.15	0.4
20	0.75	1	0.15	0.4	0.45	0.7
21	0.75	1	0.15	0.4	0.75	1
22	0.75	1	0.45	0.7	0.15	0.4
23	0.75	1	0.45	0.7	0.45	0.7
24	0.75	1	0.45	0.7	0.75	1
25	0.75	1	0.75	1	0.15	0.4
26	0.75	1	0.75	1	0.45	0.7
27	0.75	1	0.75	1	0.75	1

To obtain 7 groups of data showing clustering patterns that this section of experiments require, 6 spaces with the largest number of data points and the Unit Cost axis data points are selected (this is to keep the location related attributes in the clustering patterns). To obtain clusters that having enough between cluster distances, the spaces of data points that close to other selected clusters are replaced by spaces that having enough distance to the selected spaces. This way, the data clusters with total number of 2572 items shown on Figure 38 are obtained.

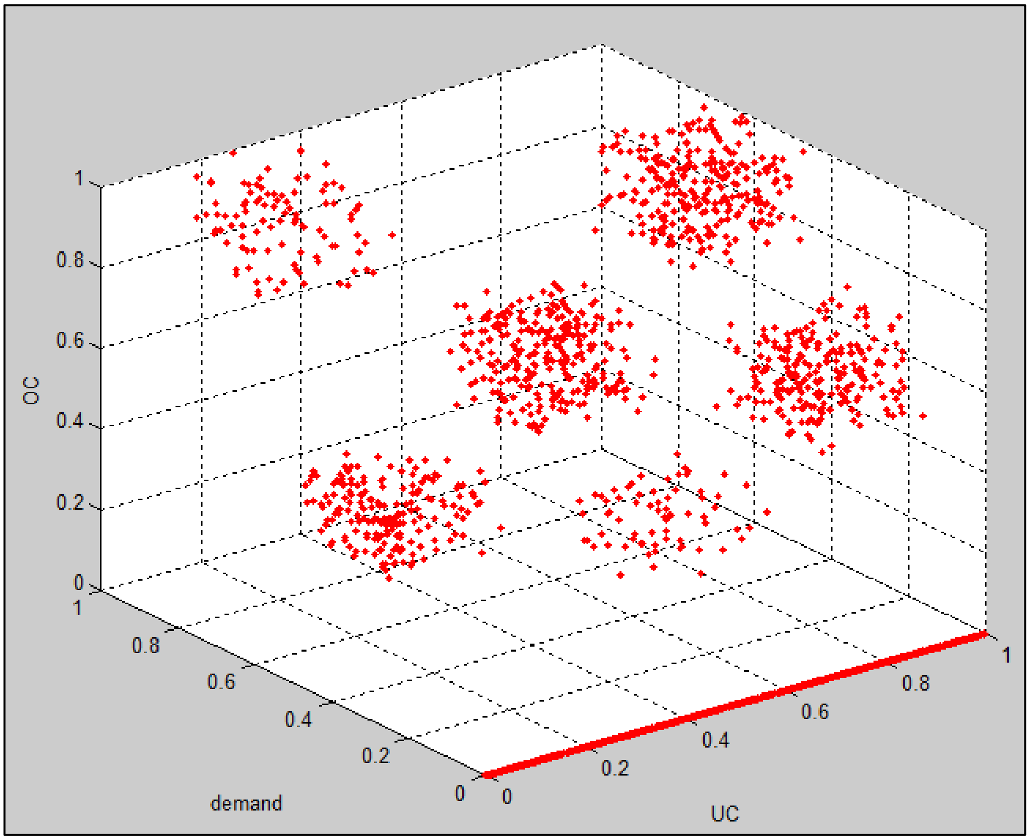

Figure 38: 2572 Items before Grouping

6.1.2 Pilot Experiment and Result

Using the dataset generated in the previous section, this section implements ABC, NIT, and K-means grouping methods to obtain corresponding item groups. And then, the grouping results are evaluated based on cluster plots, SSE, and CPC%. This is to examine the grouping methods' effectiveness. Since the NIT with 3 retailer stores will result in 7 NIT grouping, to keep the consistency of the experiment regarding the number of groups, 7-group grouping is implemented for ABC, NIT, and K-means grouping methods. For ABC grouping, the classification criterion NIC is selected. The details about ABC grouping with 7 groups and NIC as classification criterion are discussed in Section 6.2.

The grouping results of K-means, NIT, and ABC are plotted as on Figure 39, Figure 40, and Figure 41. The resulted SSE and CPC% values are summarized on Table 26. In Figure 39 to Figure 41, each color represents items which are grouped in the same group. It can be seen from Figure 39 that K-means results the completely separated clusters with unique color for each cluster. For the convenience in the rest of this research, the clusters grouped by K-means clustering are referred as K-Means cluster. Figure 40 and Figure 41 show the NIT and ABC grouping results with mixed colors compared to corresponding K-Means clusters. This indicates that items which are close to each other based on the distance of ordering cost, demand and unit cost are grouped together when K-means clustering is used, but are separated to different groups when NIT and ABC classification are applied. It should be noted that, on Figure 40, the clusters not on Unit Cost axis are mixed with 4 colors, while Unit Cost axis clusters are mixed with 3 colors. This is due to the structures shown on Figure 37, i.e., the items on Unit Cost Axis belongs to one of the three structures in part (b) of the figure, and remaining ones belong to one of the structures in part (a) of the figure. And, on Figure 41, all the clusters are mixed with 7 colors. From the degree of mixture of the colors, it can be seen that NIT clusters are better grouped than ABC's, for items which are close to each other based on the distance of ordering cost, demand and unit cost are separated to more different groups in ABC than in NIT classification. This is consistent with the corresponding SSE and CPC% results in Table 26. On Table 26, both SSE and CPC% values of K-means clustering are lower than NIT and ABC's; while NIT's SSE and CPC% values are lower than ABC's.

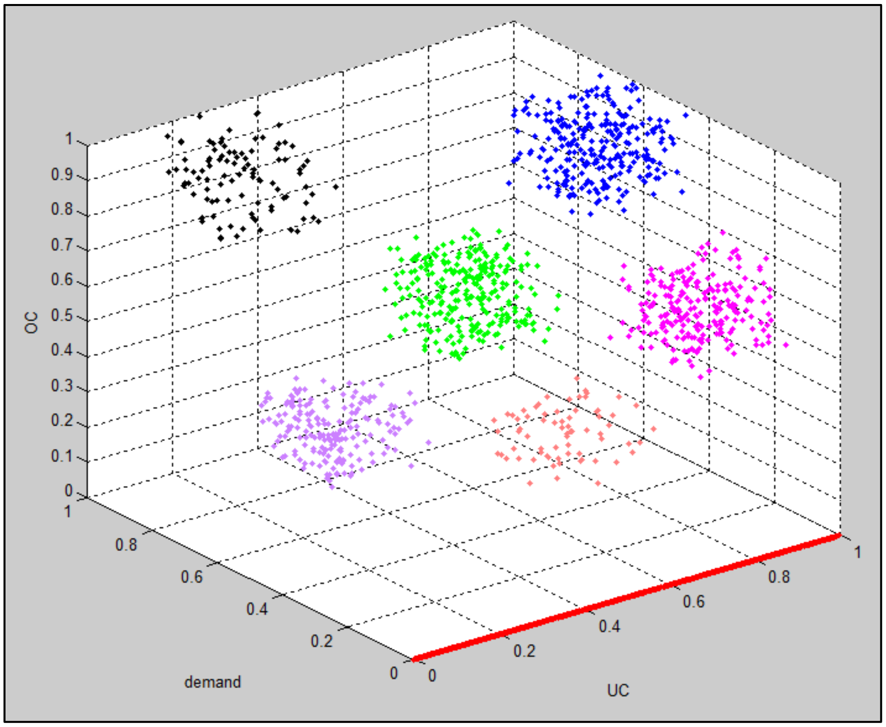

Figure 39: Visualization of K-Means Clustering Results

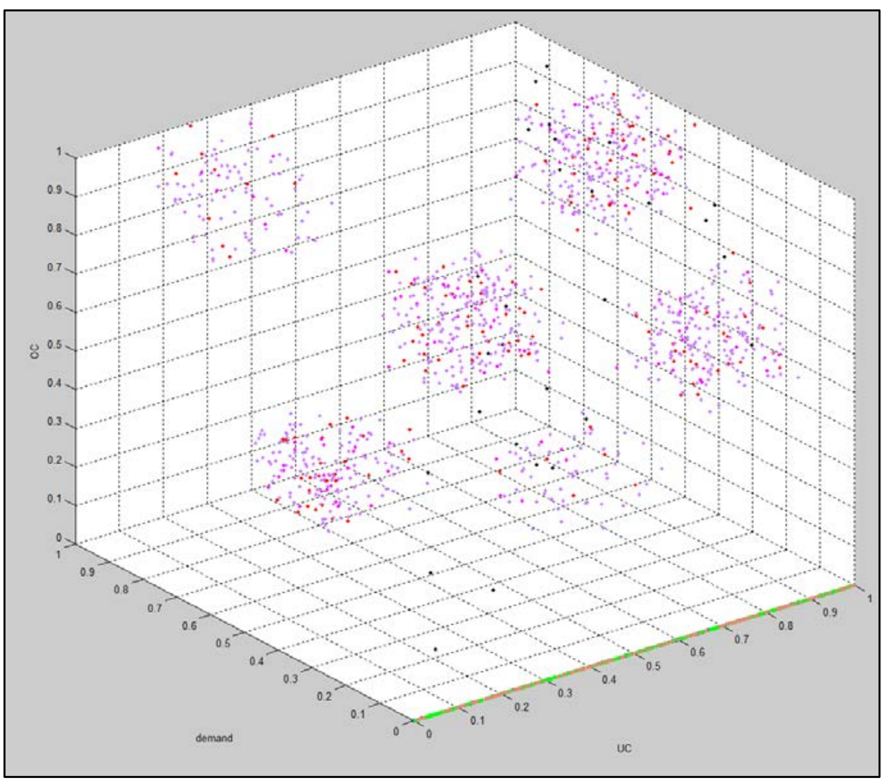

Figure 40: Visualization of NIT Grouping Results

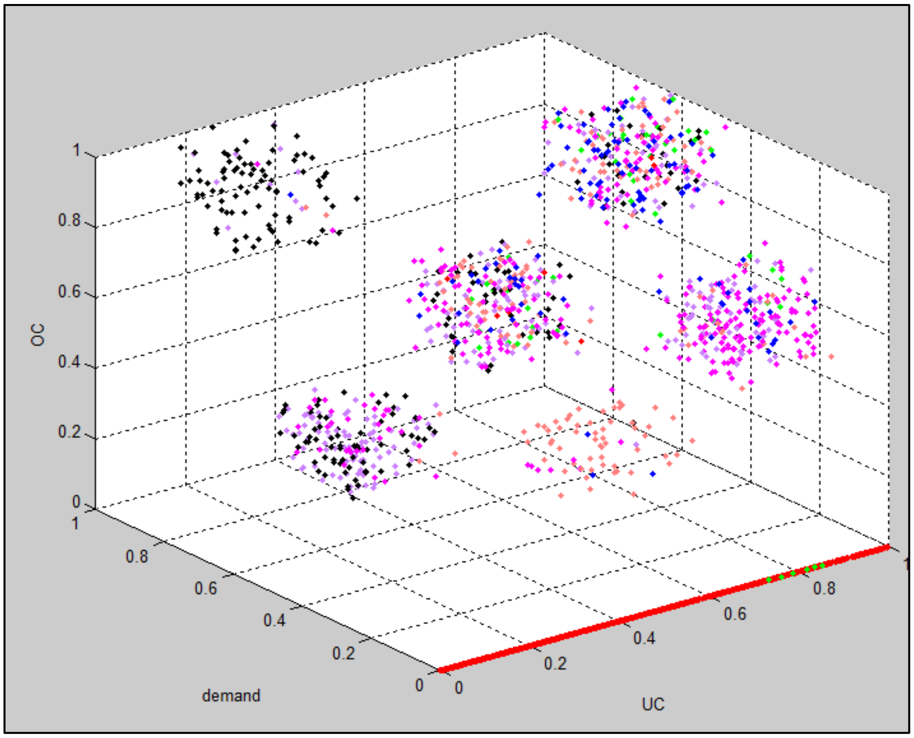

Figure 41: Visualization of ABC Grouping Results

Table 26: Grouping Result of Three Method

	%CPC	SSE
K-Means	9.2	10.78
NIT	10.4	3350.27
ABC	12.3	4342.32

It can be seen from the results that the visual results (plots), the SSE and CPC% are consistent. This consistency can be found between SSE and CPC% values; this is because CPC% as SSE also reflects the distance (difference) between items that are grouped.

The following two tables, Table 27 and Table 28, show the percentage of items that are in common between K-means groups and ABC groups and between K-Means groups and NIT groups respectively. Since K-means performs the best in grouping, it is taken as benchmark in these comparisons. In the tables, the decimal numbers represent the percentage of the items that common in the grouping results for both of the grouping methods compared. In Table 27, not including the last row, each column on the table is the comparison between each ABC resulted group against all the K-means resulted groups. The resulting maximum percentage from the comparison is put on the last row. This percentage is taken as the maximum similarity percentage between one ABC group and all the K-means groups. It can be seen from Table 27 that maximum percentage similarity ranges from 0.13 to 0.53 for the comparison of K-means and ABC methods. Similarly, it can be seen from Table 28 that the maximum percentage similarity ranges from 0.06 to 0.72 for the comparison of K-means and NIT methods. When comparing ABC and NIT groups to K-means groups, since the average maximum similarity percentage in ABC case is 28.71% and it's 30.29% in NIT case, it can be concluded that relative to ABC method, NIT results more similar groups to K-means method.

Table 27: Common items within groups between K-means and ABC

K-means Group/ABC Group	1	2	3	4	5	6	7
1	0.53	0.01	0.02	0.04	0.06	0.11	0.24
2	0.02	0.06	0.09	0.18	0.24	0.27	0.15
3	0.01	0.13	0.21	0.18	0.16	0.21	0.1
4	0.1	0.01	0.06	0.07	0.17	0.33	0.26
5	0.38	0	0	0.02	0.07	0.17	0.36
6	0.11	0.08	0.08	0.08	0.22	0.21	0.23
7	0.08	0	0.02	0.08	0.16	0.26	0.39
Max%	0.53	0.13	0.21	0.18	0.24	0.33	0.39

Table 28: Common items within groups between K-means and NIT

K-means Group/NIT Group	1	2	3	4	5	6	7
1	0	0.74	0.13	0.13	0	0	0
2	0.15	0	0	0	0.72	0.12	0.02
3	0.14	0	0	0	0.73	0.11	0.02
4	0.09	0	0	0	0.77	0.09	0.06
5	0.15	0	0	0	0.7	0.12	0.02
6	0.11	0	0	0	0.76	0.1	0.04
7	0.1	0	0	0	0.79	0.09	0.01
Max%	0.15	0.74	0.13	0.13	0.79	0.12	0.06

In this section, the three grouping methods are compared using the three attributes selected, and the results are visually presented using figures. The results show that K-means has lowest CPC% and SSE. More comprehensive investigation and comparison of these methods are carried out in the following sections.

6.2 Analysis of ABC Classification results

This part of experiment is to answer questions A1 and A2 mentioned in Section 5.1.2.1. 12 factors are used in this ABC classification experiment, including all the characteristics of item types (10 factors listed in Table 21) plus number of groups (NG) and classification criterion. The same scenarios (2^{10}=1024) are used for both levels of A1 and A2, and the sequence of the levels are randomly decided; therefore, the randomized complete block designs are implemented for dealing with questions A1 and A2 respectively.

A1：whether the classification criterion NIC is better than NADU?

The two classification criteria are compared based on two response variables, the percentage of clustering penalty cost (%CPC) and the grouping time (GT). The %CPC indicates the effectiveness of the grouping technique, and GT represents the efficiency of the grouping technique.

The comparisons of the across scenario means of %CPC and GT for NADU and NIC are shown in Figure 42. The Appendix 5 illustrates the organization of the experimental results and the calculation of the across scenario means.

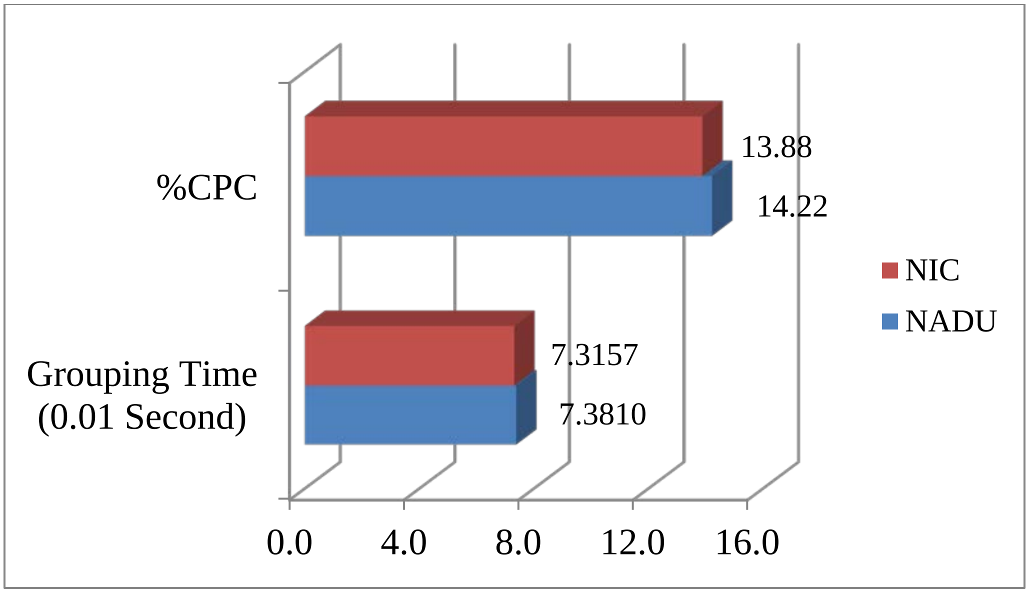

Figure 42: Comparisons between NADU and NIC

Figure 42 shows that NIC has lower %CPC and shorter GT. Paired t-test is further implemented to investigate whether there are significant differences between NIC and NADU for the two response variables. Paired t-test is the most appropriate analysis in this case because (1) the NIC and NADU are calculated based on the same scenarios (subjects), and their calculations involve the same inputs such as unit cost and annual demand rate, which satisfy the "related paired observations" assumption of t-test; and (2) a paired t-test does not require both samples to have equal variance. The test is based on the mean value of % CPC (percent-clustering penalty cost) and the mean value of the ABC classification time (GT). Minitab is used for the following tests.

The NIC and NADU test based on % CPC uses the following hypothesis:

H_0: $\mu_1 - \mu_2 = 0$ (% CPC does not change using NIC and NADU)

H_1: $\mu_1 - \mu_2 < 0$ (NIC results smaller % CPC than NADU)

(μ_1: mean of %CPC for NIC; μ_2: mean of %CPC for NADU; α=0.05)

The test results of NIC and NADU testing based on % CPC are shown in Exhibit 24. The resulted P-Value (0.000) indicates that the mean difference within pairs is statistically significant, therefore it can be concluded that NIC performs better as an ABC classification criterion than NADU for the case of interest. This is consistent with the trend shown in Figure 42. It can be seen that NIC results 0.34% less CPC in average.

Paired T-Test and CI: %CPC_NIC, %CPC_NADU

```
Paired T for %CPC_NIC - %CPC_NADU
               N      Mean    StDev  SE Mean
%CPC_NIC    2048   13.8830   1.3361   0.0295
%CPC_NADU   2048   14.2228   1.3390   0.0296
Difference  2048  -0.33979  0.18106  0.00400

95% upper bound for mean difference: -0.33321
T-Test of mean difference = 0 (vs < 0): T-Value = -84.93  P-Value = 0.000
```

Exhibit 24: Paired T-Test for Penalty Cost (A1)

The NIC and NADU test based on GT uses the following hypothesis:

H_0: $\mu_1 - \mu_2 = 0$ (GT does not change using NIC and NADU)

H_1: $\mu_1 - \mu_2 < 0$ (NIC results smaller GT than NADU)

(μ_1: mean of GT for NIC; μ_2: mean of GT for NADU; α=0.05)

Exhibit 25 shows the test results of NIC and NADU testing based on GT. The resulted P-Value (0.000) indicates that the mean difference within pairs is statistically significant, therefore it can be concluded that NIC based ABC classification uses considerably less time than NADU.

Paired T-Test and CI: GT_NIC, GT_NADU

```
Paired T for GT_NIC - GT_NADU
               N       Mean     StDev   SE Mean
GT_NIC      2049   0.073157  0.039922  0.000882
GT_NADU     2049   0.073810  0.040363  0.000892
Difference  2049  -0.000653  0.005250  0.000116

95% upper bound for mean difference: -0.000462
```

```
T-Test of mean difference = 0 (vs < 0): T-Value = -5.63  P-Value = 0.000
```

Exhibit 25: Paired T-Test for Grouping Time (A1)

A2：whether 7 groups are better than 3 groups for ABC classification?

In answering this question, 7 groups are considered due to: 1) to investigate how the number of groups affects the grouping results for ABC grouping, a group number more than 3 should be selected; 2) the comparison between ABC, NIT and K-Means grouping methods should be carried out based on same number of groups, and the selected NIT structure allows 7 group classification for the NIT method; 3) Teunter et al. (2010)'s suggestion on the percentage of number of items for each group for a 6-group ABC classification can be extended conveniently to the 7-group case.

Teunter et al. (2010) suggests 4%, 7%, 10%, 16%, 25% and 38% for a 6-group ABC classification case. The ratios between the adjacent two groups in this case can be summarized as on Table 29; it can be seen that the average of this ratio is 1.57. By lowering this ratio to 1.5, this research extends the 6-group case to 7-group case as on Table 30.

Table 29: ABC Setup in Teunter et al. (2010)

	A	B	C	D	E	F
%	4	7	10	16	25	38
Ratio		1.75	1.43	1.6	1.56	1.52

Table 30: ABC Setup for 7 Groups

A	B	C	D	E	F	G
3	5	7	11	16	23	35

The comparisons of the across scenario means of %CPC and GT for 3 and 7-group cases are shown in Figure 43.

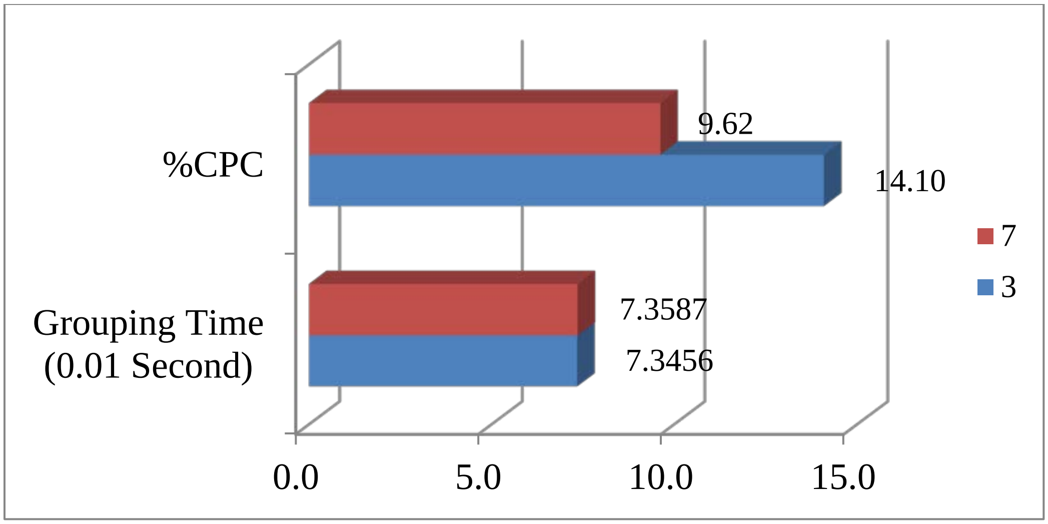

Figure 43: Comparisons between 3 and 7 groups for ABC

Figure 43 shows that 7-group has lower %CPC and slightly longer GT. Paired t-test is further implemented to investigate whether there are significant differences between 3 and 7-group cases for the two response variables.

Paired t-test is used for the analysis in this case for the same reasons as mentioned previously (in A1).

The 3-group and 7-group test based on % CPC uses the following hypothesis:

H_0: $\mu_1 - \mu_2 = 0$ (% CPC does not change in 3-group and 7-group cases)

H_1: $\mu_1 - \mu_2 < 0$ (7-group case have smaller % CPC than 3-group case)

(μ_1: mean of %CPC for 7-group case; μ_2: mean of %CPC for 3-group case; α=0.05)

The test results of 3-group and 7-group testing based on % CPC are shown in Exhibit 26. The resulted P-Value (0.000) indicates that the mean difference within pairs is statistically significant, therefore it can be concluded that 7-group is better than 3-group grouping for ABC classification, and the average of %CPC of 7 group is 4.48% lower than that of 3 group case. This is consistent with the trend shown in Figure 43. This is consistent with the commonly held view that the more groups the items are divided into the less the resulting penalty cost will be.

Paired T-Test and CI: %CPC_7, %CPC_3

```
Paired T for %CPC_7 - %CPC_3
                 N      Mean    StDev  SE Mean
%CPC_7        2048   14.0091   1.3466   0.0298
%CPC_3        2048   14.0967   1.3486   0.0298
Difference    2048  -0.08755  0.13773  0.00304

95% upper bound for mean difference: -0.08254
T-Test of mean difference = 0 (vs < 0): T-Value = -28.77  P-Value = 0.000
```

Exhibit 26: Paired T-Test for Penalty Cost (A2)

The 3-group and 7-group test based on GT uses the following hypothesis:

H_0: $\mu_1 - \mu_2 = 0$ (GT does not change for 3-group and 7-group cases)

H_1: $\mu_1 - \mu_2 < 0$ (7-group results smaller GT than 3-group)

(μ_1: mean of GT for 7-group; μ_2: mean of GT for 3-group; $\alpha=0.05$)

Exhibit 27 shows the test results of 3-group and 7-group testing based on GT. The resulted P-Value (0.283) indicates that the mean difference within pairs is statistically insignificant, therefore it can be concluded that 7-group based ABC classification uses no less time than 3-group based ones.

Paired T-Test and CI: GT_7, GT_3

```
Paired T for GT_7 - GT_3
                 N      Mean      StDev   SE Mean
GT_7          2048  0.073587  0.040332  0.000891
GT_3          2048  0.073456  0.040234  0.000889
Difference    2048  0.000131  0.005538  0.000122

95% upper bound for mean difference: 0.000333
T-Test of mean difference = 0 (vs < 0): T-Value = 1.07  P-Value = 0.858
```

Exhibit 27: Paired T-Test for Grouping Time (A2)

6.3 Analysis of K-Means Clustering Results

The five K-Means clustering related questions mentioned in Section 5.1.2.2 are dealt with in this section.

6.3.1 The Significant Non-Structural Attributes

This section deals with Question **K1** (which non-structural attributes are significant?) in Section 5.1.2.2. As illustrated in Table 21, the non-structural attributes are indexed with A to J. The stepwise regression, the most widely used variable selection technique, is applied to find significant variables. A detailed introduction of stepwise regression can be found in (Montgomery and Runger 2003). Based on the response variable percentage of clustering penalty cost (%CPC), Exhibit 28 illustrates the Minitab stepwise regression output for K-Means clustering with 7 groups for the scenario of 2200 items and 5 IHPs.

Results for: k7-2200items-Ihp5
Regression Analysis: %CPC versus A, B, C, D, E, F, G, J

```
Stepwise Selection of Terms
Candidate terms: A, B, C, D, E, F, G, J

            ------Step 1-----    ------Step 2------    ------Step 3------
               Coef         P       Coef          P       Coef          P
Constant      4.994                 5.825                 6.396
C           0.11651     0.000     0.11651     0.000     0.11651     0.000
E                                -0.01102     0.000    -0.01102     0.000
A                                                      -0.03172     0.000
B
D
J
G

S                    0.657049              0.597227              0.548275
R-sq                   64.06%                70.43%                75.17%
R-sq(adj)              63.92%                70.19%                74.88%
R-sq(pred)             63.49%                69.72%                74.38%
Mallows' Cp           298.25                202.83                132.13

            ------Step 4------    ------Step 5------    ------Step 6------
               Coef          P       Coef          P       Coef          P
Constant      7.080                 6.451                 6.722
C           0.11651      0.000    0.11651      0.000    0.11651      0.000
```

```
E              -0.01102     0.000    -0.01102     0.000    -0.01102     0.000
A              -0.03172     0.000    -0.03172     0.000    -0.03172     0.000
B                -3.036     0.000      -3.036     0.000      -3.036     0.000
D                                        7.86     0.000        7.86     0.000
J                                                            -1.203     0.002
G

S                        0.498906              0.458661              0.450382
R-sq                       79.52%                82.76%                83.45%
R-sq(adj)                  79.20%                82.42%                83.05%
R-sq(pred)                 78.70%                81.93%                82.50%
Mallows' Cp                 67.50                 19.91                 11.45

             ------Step 7------
                   Coef         P
Constant          6.891
C               0.11651     0.000
E              -0.01102     0.000
A              -0.03172     0.000
B                -3.036     0.000
D                  7.86     0.000
J                -1.203     0.001
G              -0.00755     0.044

S                        0.447605
R-sq                       83.72%
R-sq(adj)                  83.26%
R-sq(pred)                 82.65%
Mallows' Cp                  9.33

α to enter = 0.05, α to remove = 0.05
```

Exhibit 28: Minitab Stepwise Regression Output

At the level of significance (Alpha) of 0.05, 7 of the 8 factors have significant effect on the %CPC. Figure 44 is the main effects plot for %CPC. The magnitude of the vertical displacement indicates the strength of the main effect of the corresponding factor. As shown in Figure 44, factor with index “C” (Ordering Cost) has significantly stronger effect than any other factors; therefore, it is the most significant factor. The main effects plot also shows the direction of the main effects. In addition, from the analysis of the stepwise regression results (in Exhibit 28), it can be seen that the main effect plots of factor “F” (Demand Variance-to-Mean Ratio) is very flat; this means that only factor “F” is not significant when Alpha equals to 0.05. The residual plot for the analysis of main effects for %CPC is shown in Figure 45. The residual plot supports the normality assumption of the residuals.

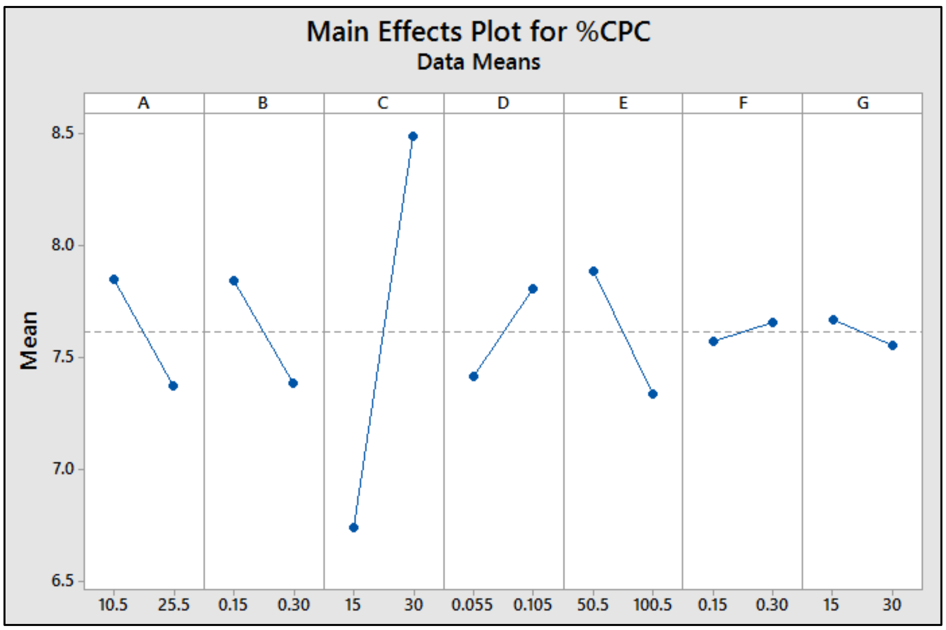

Figure 44: Main Effects Plot for %CPC

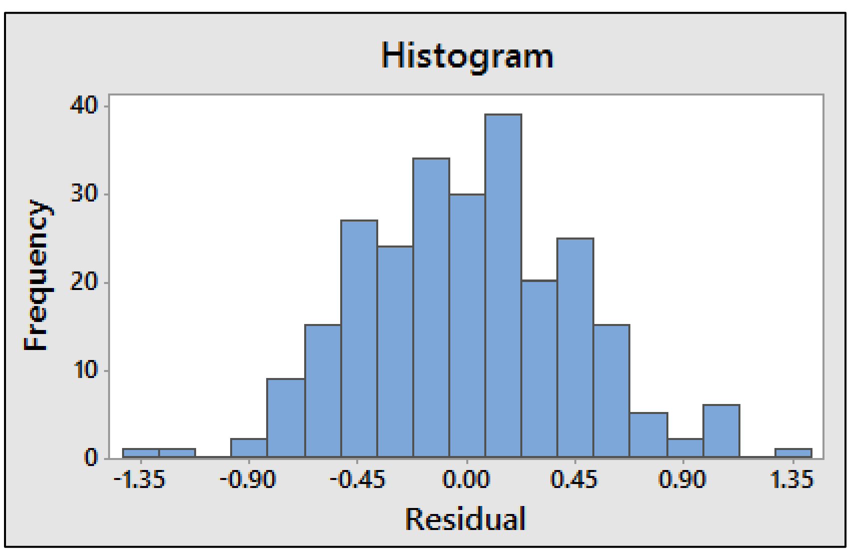

Figure 45: Residual Plot for Significant Factor Analysis

The same stepwise regression procedure to analyze K-Means clustering with 7 groups, 2200 items and 5 IHPs is carried out to analyze 8 different inventory system setups, and each setup includes 256 (2^8=256) scenarios and the results are listed in Table 31.

Table 31: Regression Analysis on 8 Non-Structural Attributes

# of items	# of IHPs	# of k	A	B	C	D	E	F	G	J	R-sq
1100	3	3	4	3	1	5	2	N	7	6	60.58%
		7	2	4	1	5	3	N	7	6	79.64%
	5	3	4	3	1	6	2	5	7	N	67.59%
		7	3	4	1	5	2	N	6	7	80.30%
2200	3	3	5	3	1	4	2	N	7	6	57.37%
		7	4	5	1	3	2	N	N	6	80.56%
	5	3	5	3	1	6	2	N	N	4	53.14%
		7	3	4	1	5	2	N	7	6	83.72%

In Table 31, the first three columns define the scenarios investigated, columns A-J record the parameters of the 8 non-structural attributes, and the last column records the R-square for all the significant attributes. The numbers in columns A to J reflect the significance of the corresponding attributes; “1” means the corresponding attributes is the most significant attribute, and the higher the number, the less significant the attribute is. In Table 31, the symbol “N” means the attribute is not significant.

It can be seen from Table 31 that F (Demand Variance-to-Mean Ratio), G (Mean lead time at ES), and J (Lead Time Variance-to-Mean Ratio) are not significant in some scenarios. And, their significance rankings are mostly low. To further evaluate the significance of the factors based on the R-square values, the R-square values and their cross scenario average values are listed on Table 32. It can be seen from the average R-square values that factor F, G, and J are the least significant among the 8 factors with 0.1%, 0.5%, and1.3%. Since the insignificance in certain scenarios and the very low corresponding R-square values of these three factors, this research rules out them from the grouping processes as insignificant attributes.

Table 32: Regression Analysis on 9 Non-Structural Attributes – R-square

# of items	# of locations	# of k	A	B	C	D	E	F	G	J	R-sq
1100	3	3	3.0%	5.2%	39.7%	2.3%	7.0%	0.0%	1.3%	2.1%	60.6%
		7	5.6%	3.2%	61.9%	3.0%	4.6%	0.0%	0.5%	0.8%	79.6%
	5	3	2.6%	4.8%	46.8%	0.8%	10.9%	1.0%	0.7%	0.0%	67.6%
		7	4.9%	3.7%	60.6%	1.2%	9.1%	0.0%	0.6%	0.4%	80.3%
2200	3	3	2.7%	4.0%	33.9%	3.7%	10.5%	0.0%	0.8%	1.8%	57.4%
		7	4.3%	3.9%	60.8%	5.2%	5.6%	0.0%	0.0%	0.8%	80.6%
	5	3	2.6%	5.9%	29.1%	0.9%	10.7%	0.0%	0.0%	3.9%	52.0%
		7	4.7%	4.4%	64.1%	3.2%	6.4%	0.0%	0.3%	0.7%	83.7%
Average			3.8%	4.4%	49.6%	2.5%	8.1%	0.1%	0.5%	1.3%	70.2%

6.3.2 The Tendency of Clustering Same NIT Structures Together

This section deals with Question **K2** (whether the item types having the same NIT structure tend to be clustered into the same group when using Non-Structural attributes?) in Section 5.1.2.2.

The non-structural attributes are listed in Table 21 with Factor Indices from A to J, among which C (Ordering Cost), D (Inventory Holding Charge), E (Demand Rate), F (Demand Variance-to-Mean Ratio) are location related attributes. Location related attributes means when an item is not stored on a specific location, the corresponding location related attribute value should not be numerically involved in the grouping process. However, it should be noted that location related attributes are considered on the locations where the specific item is stored. The non-structural attributes for K-means clustering method are organized as shown in Exhibit 10, where all the non-structural attributes on the corresponding locations of the inventory system are listed (represented). This means that the location related attributes should be involved in the grouping process to reflect the storage status of an item in the inventory network system. As mentioned in 3.3.2, an item type's existence at a specific location can be represented using binary system, i.e. 0 represents non-existence, and 1 existence. Considering the numeric characteristic of the non-structural attributes involved in the K-means clustering process, the non-existing non-structural attribute on a specific location needs to be numerically represented. Therefore, the 0's representing the non-existence of a specific item on a specific location is treated as numeric 0 rather than binary 0 in this part of the research. Further, to avoid the confusion, these numeric 0s are substituted by 0.0001, which is a sufficiently small value that does not affect the calculation of the grouping performance measures. With all these adjustments, it should be noted that, following the input structure (representation) of the non-structural attributes on Exhibit 10, the non-structural attributes all together actually carries storage structural information by reflecting whether certain items exist on certain locations and by reflecting the storage networks as whole for the item types. This means that NIT structural information is put into the clustering process by this way of representation (organization) of the clustering attribute input values.

From the discussion so far, there are three ways to take the storage structure into consideration in the clustering process; first is to have binary system represent the storage status of the item types in the inventory network; second is to use non-structural attributes organized in the way described in the immediately previous paragraph; and third is to use both first and second ways together. Considering this section investigates K-means clustering method characteristics, and the first way is actually the NIT classification process, in this section of the research the second way is investigated to find out whether the involvement of the storage structural information in the clustering process using aforementioned non-structural clustering attributes representation evidently affects the clustering results. The third way is the research topic of next section.

As shown in Exhibit 10, 8 non-structural attributes (factors A to J in the Table 21) are used in the clustering. And then, statistical analysis is conducted for each resulted cluster to see whether the items with certain storage structure dominate the entire group. Dominate here means whether majority of the item types in the same clusters have the same NITs. This is measured by calculating the percentage of the same NITs in a resulting cluster. Table 33 is an illustration of the main (dominating) NIT analysis. For group 1, 22 out of total 26 items, which means 85% of the items, have the storage structure NIT A. Thus, the main NIT for group 1 is NIT A. Similarly, the main NIT for group 2 is NIT B dominating by 81% as the main structure, and the main NIT for group 3 is NIT C and 79% of the items in this group have this structure. In this case, the average percentage of the main structure across the clusters is the average of 85%, 81% and 79%, which is 81.7%.

Table 33: An Example of Main NIT Analysis

	NIT A	NIT B	NIT C	Main Structure	% of Main Structure
Group 1	22	3	1	NIT A	85
Group 2	3	22	2	NIT B	81
Group 3	3	3	23	NIT C	79

Considering the 5 significant non-structural attributes selected in the previous section and inventory system setup shown in Table 31, 3-group and 7-group K-means clustering was implemented to answer whether the item types having the same NIT structure tend to be clustered into the same group when using significant Non-Structural attributes. The results show that in 3-group case the across scenario average of % of Main Structure means is 81.3 (variance 69.6), 7-group case the across scenario average of % of Main Structure means is 90.6 (variance

56.0). With more than 80% of NITs having the same structure, the results show that the same NIT tends to be grouped together during the K-Means clustering process. It should also be noted from the results that when the grouping number increases, the tendency of clustering the same NIT structures together is more significant.

6.3.3 The Effect of Structural Attributes on the Clustering Results

This section deals with Question **K3** (whether the structural attributes affect the clustering results?) in Section 5.1.2.2 by adding structural attributes into the K-means attribute list in the clustering process.

The binary expression (illustrated in section 3.3.2) is used to represent structural attribute NIT, i.e., one NIT is represented as a set of binary values, each of which represent the existence of a retail store at a location. Table 34 records the results of 4 response variables, SSE, %CPC (the percentage of clustering penalty cost), average percentage of main NIT, and GT (Grouping Time). Each response in Table 34 is the average value of 1024 (2^{10}=1024) scenarios involving the 10 factors in Table 21.

Table 34: Results for Structural and Non-Structural Attributes

	# of Groups	SSE(10^2)	%CPC	Avg % of Main NIT	GT (0.01secs)
Significant Non-Structural Attributes	3	13.46	10.00	81.27	7.07
Structural and Significant Non-Structural Attributes	3	15.72	11.55	87.19	12.27
Significant Non-Structural Attributes	7	10.43	8.98	90.55	9.80
Structural and Significant Non-Structural Attributes	7	10.64	9.20	96.66	21.77

Figure 46 and Figure 47 show the changes of the four response variables in this experiment.

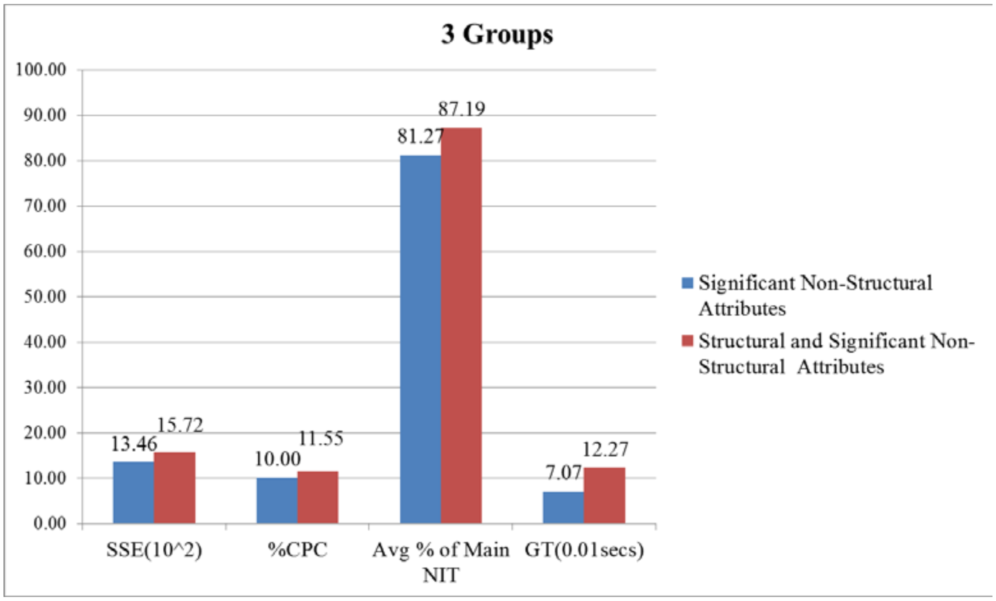

Figure 46: Comparisons between Significant Non-Structural Attributes and Structural and Significant Non-Structural Attributes (3-group Case)

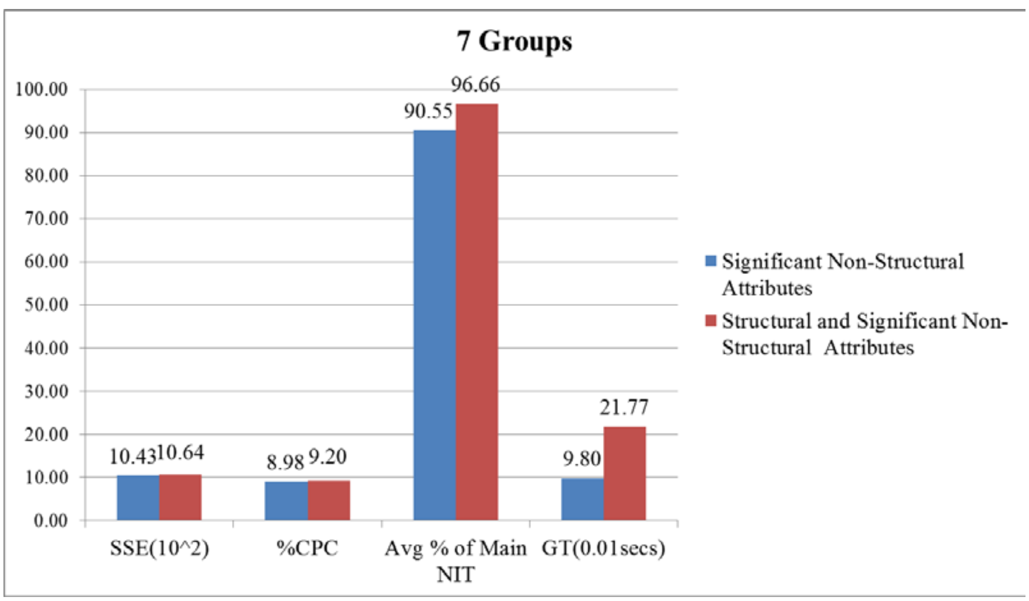

Figure 47: Comparisons between Significant Non-Structural Attributes and Structural and Significant Non-Structural Attributes (7-group Case)

Figure 46 is the comparisons between K-Means clustering using significant non-structural attributes and clustering using structural and significant non-structural attributes in the

3-group case. From Figure 46, it can be seen that adding the structural attributes increase SSE, %CPC, Avg % of Main NIT, and GT. In addition, Figure 47 is the comparisons between K-Means clustering using significant non-structural attributes and clustering using structural and significant non-structural attributes in the 7-group case. This figure also indicates that adding the structural attributes increases the values of the four responses.

The results show that values of SSE, %CPC, Avg % of Main NIT increase 16.8%, 15.5%, and 7.3% respectively in 3-group case while the increase on GT is 73.5%, and 2.0%, 2.5%, and 6.7%respectively in 7-group case, while the increase on GT is 122.2%. The increase on %CPC and SEE shows that the repeated involvement of structural attributes with 0 and 1 values, while the important non-structural attributes already are carrying the structural information (refer to Section 6.3.2), causes the unfavorable increase on these performance measures. In other words, the repeated involvement of the structural attributes in distance calculation in grouping procedure results larger distance between items, and this affects the grouping results, therefore causes higher %CPC and SSE. Since the K-means clustering using the significant non-structural attributes includes the structural information as previously discussed, and the consistent increases on SSE, %CPC, and Avg % of Main NIT are trivial compared to the increase of GT, for the reason of experimental efficiency and unfavorable increase on %CPC, SSE, the following experiments are implemented based on significant non-structural attributes.

6.3.4 The Factors Affecting the K-Means Clustering Time

This section deals with Question **K4** (which factors affect the K-Means clustering time?) in Section 5.1.2.2.

As discussed in Maimon and Rokach (2005), the time complexity of K-Means algorithm relates to three factors: 1) the number of instances; 2) the number of clusters; and 3) the number of iterations used by the algorithm to converge. Besides these three factors, the number of clustering attributes is also an important factor affecting the clustering time.

This section investigates the impact of these four factors. The experimental setup is as on Table 35. The 15 attributes on Table 35 correspond to the significant non-structural attributes for NIS2 on Figure 35, and the 30 attributes correspond to all the non-structural attributes for NIS2.

Table 35: Experimental Setup for Four Factor Analysis of K-Means Clustering Time

Factor	Low	High
# of iterations to converge	3	7
# of clusters	3	7
# of instance (items)	1100	2200
# of clustering attributes	15	30

Table 36 lists the GT (Grouping Time) for the four factors and the stepwise regression results are shown in Exhibit 29. The stepwise regression results show that number of items, number of attributes, number of clusters, and number of iterations significantly affect the clustering time, and these four contribute around 90.55% of the variance of the GT.

Table 36: Results for 4 factors on Grouping Time

# of Iterations	# of Clusters	# of Items	# of Attributes	GT
3	3	1100	15	0.056
3	3	1100	30	0.106
3	3	2200	15	0.110
3	3	2200	30	0.213
3	7	1100	15	0.078
3	7	1100	30	0.155
3	7	2200	15	0.156
3	7	2200	30	0.300
7	3	1100	15	0.072
7	3	1100	30	0.136
7	3	2200	15	0.144
7	3	2200	30	0.272
7	7	1100	15	0.113
7	7	1100	30	0.230
7	7	2200	15	0.223
7	7	2200	30	0.433

Stepwise Regression: CT versus iterations, clusters, items, attributes

Stepwise Selection of Terms

Candidate terms: Iterations, Clusters, Items, Attributes

	-------Step 1------		-------Step 2------		-------Step 3------	
	Coef	P	Coef	P	Coef	P
Constant	0.0050		-0.1624		-0.2529	
Items	0.000103	0.017	0.000103	0.002	0.000103	0.000
Attributes			0.00744	0.003	0.00744	0.000
Clusters					0.01811	0.010
Iterations						
S		0.0834502		0.0605775		0.0471744
R-sq		34.45%		67.93%		82.05%
R-sq(adj)		29.77%		62.99%		77.56%
R-sq(pred)		14.38%		51.41%		68.08%
Mallows' Cp		64.30		27.33		12.90

	-------Step 4------	
	Coef	P
Constant	-0.3232	
Items	0.000103	0.000
Attributes	0.00744	0.000
Clusters	0.01811	0.002
Iterations	0.01406	0.009
S		0.0357464
R-sq		90.55%
R-sq(adj)		87.11%
R-sq(pred)		80.01%
Mallows' Cp		5.00

α to enter = 0.05, α to remove = 0.05

Exhibit 29: Stepwise Regression Analysis on GT based on 4 factors

The R-Squares for the significant factors affecting the grouping time is plotted in Figure 48. The results show that the number of items is the most significant factor which contributes to 34.45% of the R-Squares, the number of clustering attributes is the second significant factor which explains 33.48% of the R-Squares, the number of clusters is the third significant factor which explains 14.12% of the R-Squares, and the number of iteration for K-Means to converge is the fourth significant factor which contributes to 8.5% of the R-Squares.

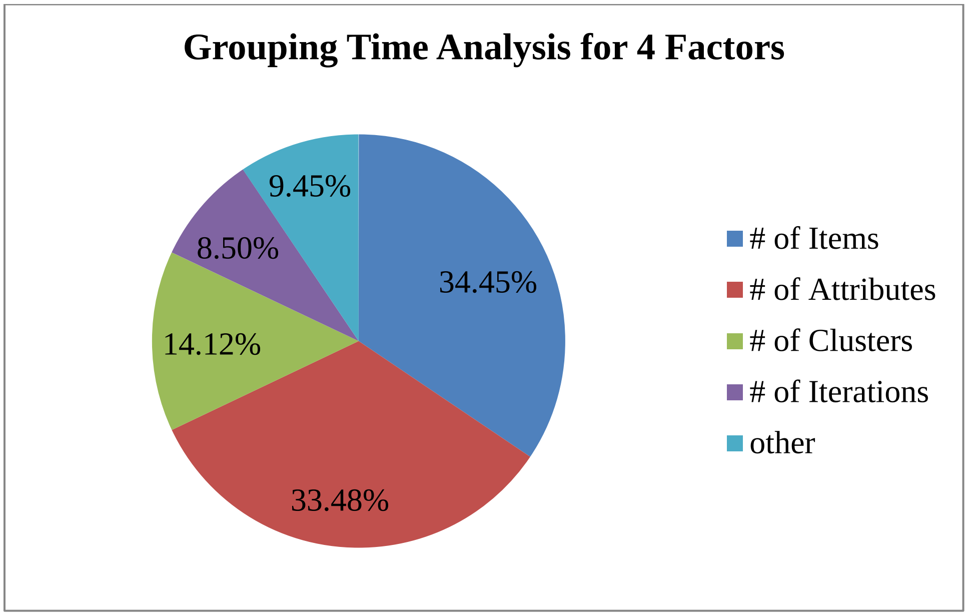

Figure 48: Grouping Time Analysis for Four Factors

6.3.5 The Effect of the Number of Clusters K on the Clustering Results

This section deals with Question **K5** (how the number of clusters k affects the clustering results?) in Section 5.1.2.2.

This experiment is conducted based on two scenarios by comparing the trend on both gap statistic mentioned in section 3.3.3 and the SSE values for different number of groups. In both scenarios, the 10 attributes (listed on Table 21) including the structural attributes, non-structural attributes, and the number of items are used to generate data for the K-Means clustering. For the 1st scenario, all the 10 factors are chosen at their low levels. And for the 2nd scenario, all the 10 factors are chosen at their high levels.

The maximum number of groups is calculated based on the simple rule of sum ($k \approx \sqrt{n/2}$). In the 2nd scenario, there are 2200 items. Thus, the maximum number of groups is set to be 33.

The results of the adjusted SSE and gap statistics (refer to Section 3.3.3) are illustrated in Figure 49 and Figure 50. In these figures, the horizontal axis is number of groups, and the vertical axis is the % of change of Gap_adj (Adjusted Gap) and SSE_adj (Adjusted SSE). The original values of SSE are adjusted to the percent ratio of the original value to the maximum SSE, and the original values of gap statistics are adjusted to the percentage ratio of the original value to the maximum gap statistic. The purpose of adjusting values of SSE and gap statistics is to see the trends of SSE and gap statistic in the same figure.

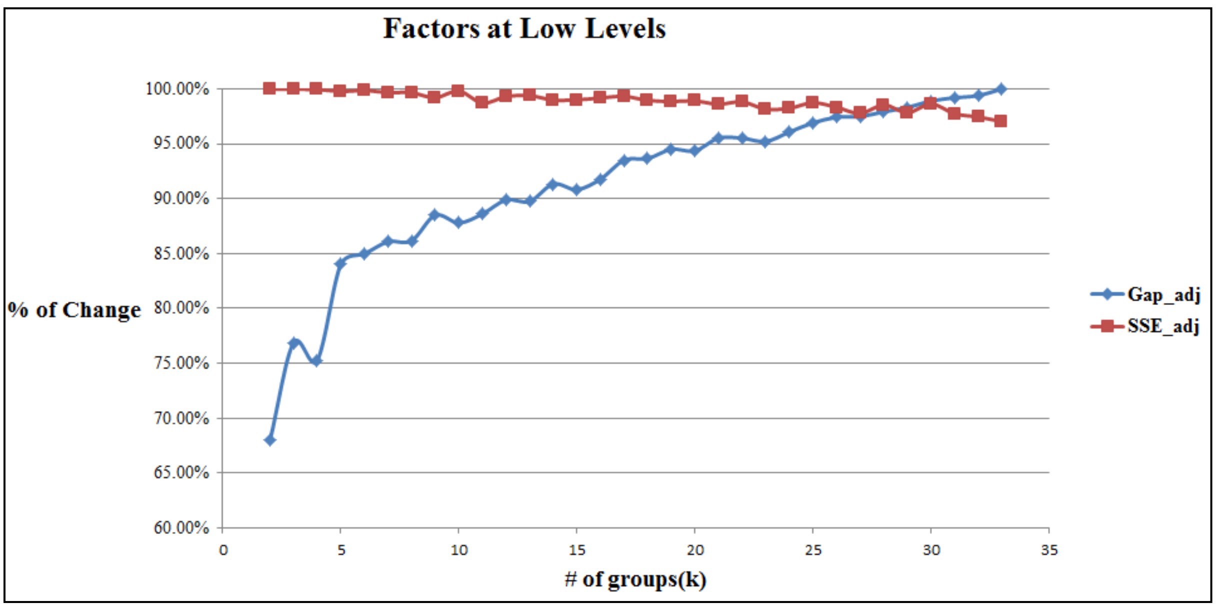

Figure 49: Trend of SSE and Gap Statistics for Scenario 1

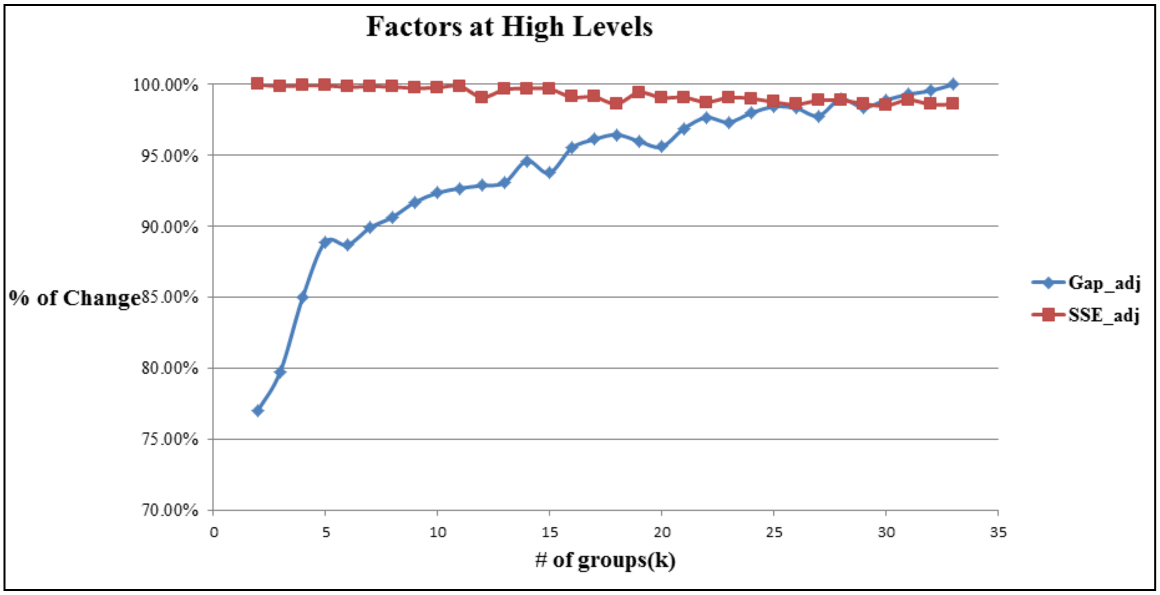

Figure 50: Trend of SSE and Gap Statistics for Scenario 2

Figure 49 shows that SSE is not monotonically decreasing with k, this may be due to the randomness of choosing initial seeds; however, its overall trend is decreasing as shown. Also, the gap statistic is not monotonically increasing; however, its overall trend is increasing. The same conclusions can be derived based on Figure 50. These trends indicate that in general, the larger the *k* value is, the better the clustering results are. This means that in practice, a simple way to determine *k* is to fix its values to the largest acceptable value.

6.4 Comparison between Grouping Methods

This section compares the ABC classification, NIT classification, and K-Means clustering. The comparisons are based on following experimental setups, which are derived based on previous individual grouping methods related experiments, for the three grouping methods respectively:

ABC classification:

- NIC is selected as the classification criteria

- 7-group classification is selected to implement the comparison with the other grouping techniques

K-means clustering:

- significant non-structural attributes are selected as the clustering attributes
- 7-group clustering is selected to implement the comparison with the other grouping techniques
- Number of iteration is set to 3

NIT classification:

- Based on an inventory system with 3 echelon 3 retailers, which allows comparing NIT with the other grouping methods based on 7 groups

The ABC classification, NIT classification, and K-means clustering methods are compared based on the following experimental design:

- Number of replication is 1
- Randomized Complete Block Design

The system characteristics used in the experiments are listed on Table 21. To compare the grouping methods based on 7 groups, the number of IHP is seleceted as 5 (3 echelon 3-retailer case). The other 9 attributes listed on Table 21 takes 2 levels as shown on the table. This way, there are 512 (2^9=512) system scenarios considered on each comparison experiment. The experiments implement Randomized Complete Block Design. Each block in this case represents one scenario. A complete block is an experimental setup that tests the three methods by putting them in one scenario. The randomization is implemented by randomizing the sequence of the experiments corresponding to each of the grouping methods within each block. Following are the hypothesis and the corresponding statistical analysis.

Experiment based on %CPC

The hypothesis of comparing the three grouping methods:

H_0: $\mu_1 = \mu_2 = \mu_3$ (% CPC does not change for the three grouping methods)

H_1: $\mu_1 \neq \mu_2 \neq \mu_3$ (% CPC for the three different grouping methods are different)

(μ_1: mean of %CPC for ABC; μ_2: mean of %CPC for NIT; mean of %CPC for K-means; α=0.05)

ANOVA is used to compare the means of %CPC for the three grouping methods, and the results are on Exhibit 30. It can be seen from Exhibit 30 that the Null hypothesis is rejected (P-value = 0,000), which means that %CPC changes for the three grouping methods. Since the null hypothesis is rejected in the ANOVA, it can be concluded that some of the factor level means are different. Considering ANOVA does not identify which means are different, Fisher's least significant difference (LSD) method as the multiple comparisons method is implemented to compare the three methods.

The result of Fisher's LSD is on Figure 51. The comparison results show that K-means, NIT, and ABC grouping methods have significant differences respect to %CPC. It can be seen that K-means clustering out performs both NIT and ABC classifications, while NIT grouping performs better than ABC classification.

One-way ANOVA: %CPC versus Grouping Method

Method

Null hypothesis	All means are equal
Alternative hypothesis	At least one mean is different
Significance level	α = 0.05

Equal variances were assumed for the analysis.

Factor Information

Factor	Levels	Values
Grouping Method	3	ABC, K-Means, NIT

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Grouping Method	2	5426	2712.76	2307.90	0.000
Error	1533	1802	1.18		
Total	1535	7227			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
1.08417	75.07%	75.04%	74.97%

Means

Grouping Method	N	Mean	StDev	95% CI
ABC	512	12.9424	1.0840	(12.8484, 13.0364)
K-Means	512	8.7984	1.0283	(8.7045, 8.8924)
NIT	512	12.6070	1.1375	(12.5130, 12.7010)

Pooled StDev = 1.08417

Fisher Pairwise Comparisons

Grouping Information Using the Fisher LSD Method and 95% Confidence

Grouping Method	N	Mean	Grouping
ABC	512	12.9424	A
NIT	512	12.6070	B
K-Means	512	8.7984	C

Means that do not share a letter are significantly different.

Fisher Individual Tests for Differences of Means

Difference of Levels	Difference of Means	SE of Difference	95% CI	T-Value	Adjusted P-Value
K-Means - ABC	-4.1439	0.0678	(-4.2769, -4.0110)	-61.16	0.000
NIT - ABC	-0.3354	0.0678	(-0.4683, -0.2024)	-4.95	0.000
NIT - K-Means	3.8086	0.0678	(3.6757, 3.9415)	56.21	0.000

Simultaneous confidence level = 87.81%

Exhibit 30: Minitab Analysis of Variance Output for %CPC

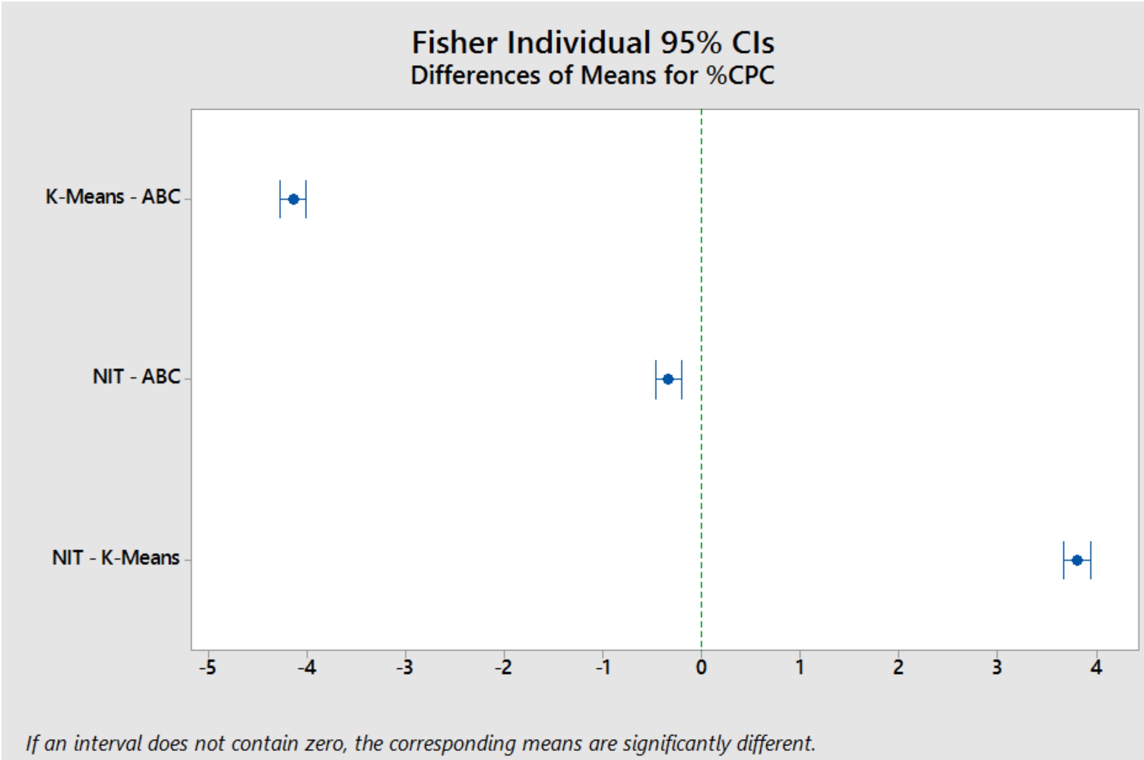

Figure 51: Fisher's Pairwise Comparisons for % CPC

<u>Experiment based on SSE</u>

The hypothesis of comparing the three grouping methods:

H_0: $\mu_1 = \mu_2 = \mu_3$ (SSE does not change for the three grouping methods)

H_1: $\mu_1 \neq \mu_2 \neq \mu_3$ (SSE for the three different grouping methods are different)

(μ_1: mean of SSE for ABC; μ_2: mean of SSE for NIT; mean of SSE for K-means; α=0.05)

It can be seen from Exhibit 31 that the Null hypothesis is rejected (P-value = 0,000), which means that SSE changes for the three grouping methods. Since the null hypothesis is rejected in the ANOVA, it can be concluded that some of the factor level means are different.

The result of Fisher's LSD is on Figure 52. The comparison results show that K-means, NIT, and ABC grouping methods have significant differences respect to SSE. It can be seen that

K-means clustering out performs both NIT and ABC classifications, while NIT grouping performs better than ABC classification.

One-way ANOVA: SSE versus Grouping Method

Method

Null hypothesis	All means are equal
Alternative hypothesis	At least one mean is different
Significance level	$\alpha = 0.05$

Equal variances were assumed for the analysis.

Factor Information

Factor	Levels	Values
Grouping Method	3	ABC, K-Means, NIT

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Grouping Method	2	649503647	324751824	559.15	0.000
Error	1533	890366386	580800		
Total	1535	1539870033			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
762.102	42.18%	42.10%	41.95%

Means

Grouping Method	N	Mean	StDev	95% CI
ABC	512	2954.9	969.4	(2888.8, 3020.9)
K-Means	512	1372.5	466.3	(1306.4, 1438.6)
NIT	512	2321.5	764.9	(2255.4, 2387.5)

Pooled StDev = 762.102

Fisher Pairwise Comparisons

Grouping Information Using the Fisher LSD Method and 95% Confidence

Grouping Method	N	Mean	Grouping
ABC	512	2954.9	A

```
NIT        512   2321.5     B
K-Means    512   1372.5       C
Means that do not share a letter are significantly different.

Fisher Individual Tests for Differences of Means
Difference of  Difference       SE of                                  Adjusted
Levels           of Means  Difference          95% CI          T-Value   P-Value
K-Means - ABC     -1582.4        47.6  (-1675.8, -1488.9)       -33.22     0.000
NIT - ABC          -633.4        47.6  ( -726.8,   -540.0)      -13.30     0.000
NIT - K-Means       949.0        47.6  (  855.6,   1042.4)       19.92     0.000
Simultaneous confidence level = 87.81%
```

Exhibit 31: Minitab Analysis of Variance Output for SSE

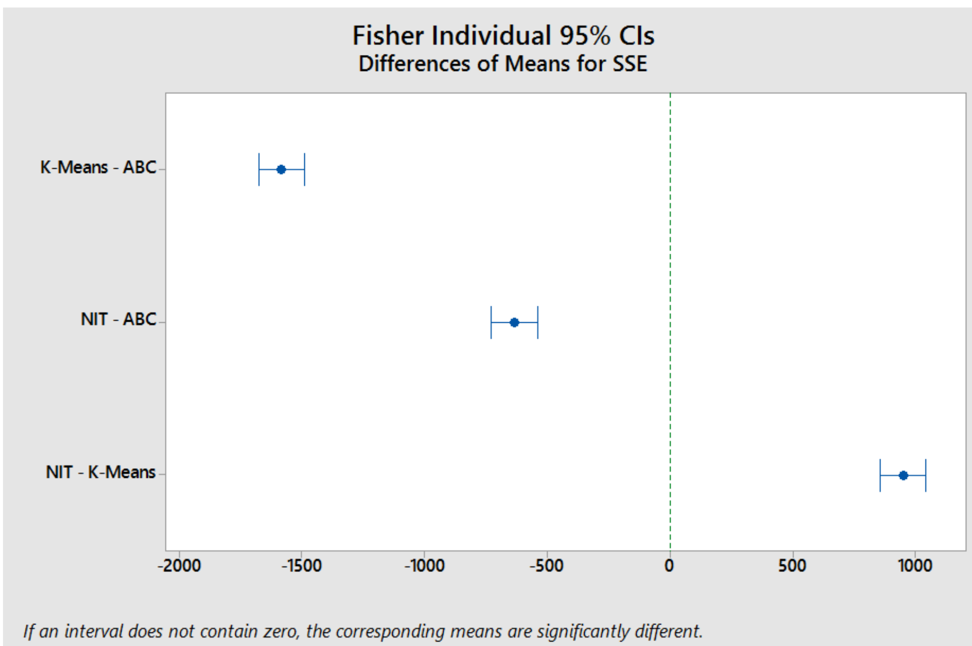

Figure 52: Fisher's Pairwise Comparisons for SSE

<u>Experiment based on GT</u>

The hypothesis of comparing the three grouping methods:

H_0: $\mu_1 = \mu_2 = \mu_3$ (GT does not change for the three grouping methods)

H_1: $\mu_1 \neq \mu_2 \neq \mu_3$ (GT for the three different grouping methods are different)

(μ_1: mean of GT for ABC; μ_2: mean of GT for NIT; mean of GT for K-means; α=0.05)

It can be seen from Exhibit 32 that the Null hypothesis is rejected (P-value = 0,000), which means that GT changes for the three grouping methods. Since the null hypothesis is rejected in the ANOVA, it can be concluded that some of the factor level means are different.

The result of Fisher's LSD is on Figure 53. The comparison results show that K-means have significant difference with NIT and ABC grouping methods respect to GT, while NIT and ABC do not have significant difference. It can be seen that K-means takes longer clustering time than both NIT and ABC classifications.

One-way ANOVA: GT versus Grouping Method

Method

Null hypothesis	All means are equal
Alternative hypothesis	At least one mean is different
Significance level	$\alpha = 0.05$

Equal variances were assumed for the analysis.

Factor Information

Factor	Levels	Values
Grouping Method	3	ABC, K-Means, NIT

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Grouping Method	2	0.2185	0.109264	68.49	0.000
Error	1533	2.4455	0.001595		
Total	1535	2.6640			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.0399405	8.20%	8.08%	7.84%

Means

Grouping Method	N	Mean	StDev	95% CI
ABC	512	0.09046	0.04480	(0.08699, 0.09392)
K-Means	512	0.11520	0.03902	(0.11174, 0.11866)
NIT	512	0.08937	0.03545	(0.08591, 0.09284)

Pooled StDev = 0.0399405

Fisher Pairwise Comparisons

Grouping Information Using the Fisher LSD Method and 95% Confidence

Grouping

Method	N	Mean	Grouping
K-Means	512	0.11520	A
ABC	512	0.09046	B
NIT	512	0.08937	B

Means that do not share a letter are significantly different.

Fisher Individual Tests for Differences of Means

Difference of Levels	Difference of Means	SE of Difference	95% CI	T-Value	Adjusted P-Value
K-Means - ABC	0.02474	0.00250	(0.01985, 0.02964)	9.91	0.000
NIT - ABC	-0.00108	0.00250	(-0.00598, 0.00381)	-0.43	0.665
NIT - K-Means	-0.02583	0.00250	(-0.03072, -0.02093)	-10.35	0.000

Simultaneous confidence level = 87.81%

Exhibit 32: Minitab Analysis of Variance Output for GT

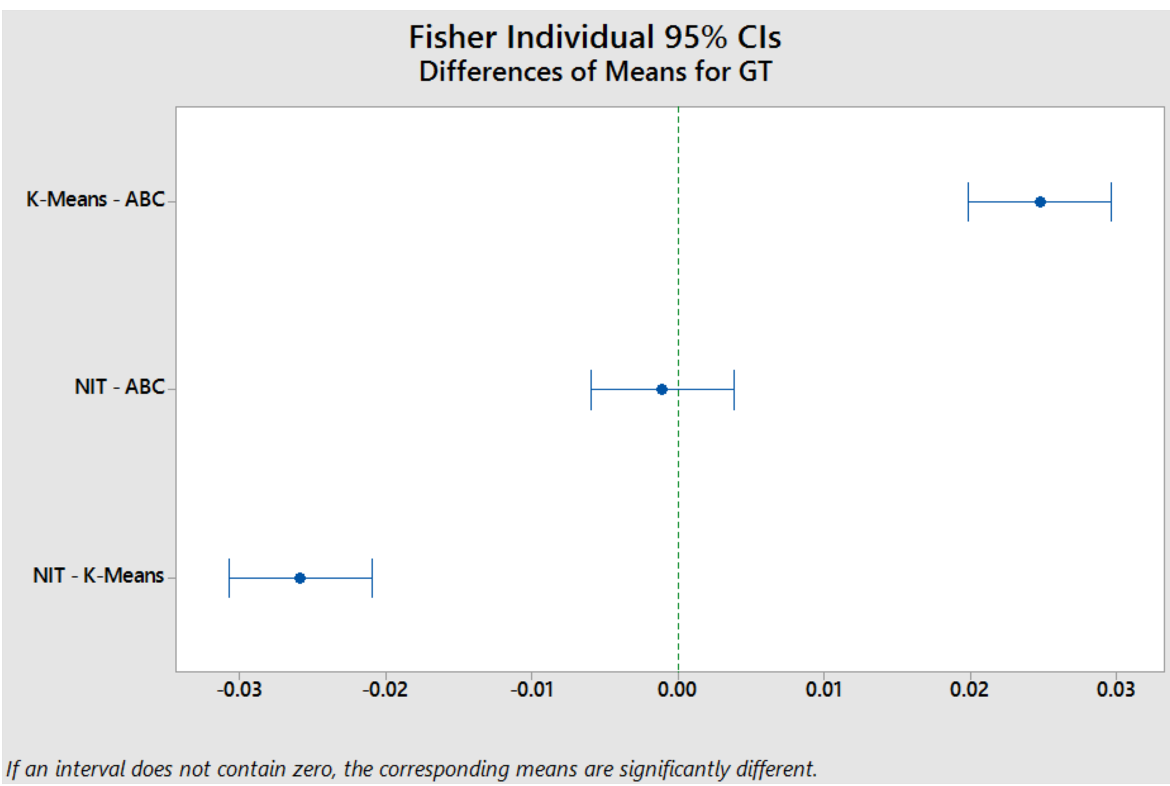

Figure 53: Fisher's Pairwise Comparisons for GT

6.5 Conclusions on the Grouping Methods

The experimental results regarding the grouping methods can be summarized as following:

ABC classification

- NIC as the classification criteria out performs NADU regarding to both %CPC and GT
- 7-group classification is significantly better than 3-group case according to %CPC values, and the resulted GT shows there is no significant difference between 7-group and 3-group cases

K-means Clustering

- The significant non-structural attributes identified in this research are Unit Cost, Lost-Sale-Cost-to-Unit-Cost Ratio, Ordering Cost, Inventory Holding Charge, Demand Rate
- Structural information can be put in the K-means algorithm by identifying and representing location related attribute values to form clusters that having structural within group similarity
- After considering location related attributes in the K-means clustering, there is no need to add structural attributes represented in binary values
- Among the four factors that affect the GT, the number of items is the most significant factor, the number of clustering attributes is the second significant factor, the number of clusters is the third significant factor, and the number of iteration for K-Means to converge is the fourth significant factor
- In general, the larger the *k* value is, the better the clustering results are

Comparison between the Three Grouping Techniques

- Based on %CPC and SSE, K-means clustering out performs both NIT and ABC classifications, while NIT grouping performs better than ABC classification
- Based on GT, K-means takes longer clustering time than both NIT and ABC classifications, while there is no significant difference between NIT and ABC

7 Summary

This chapter discusses the conclusions, suggestions, and future work.

7.1 Conclusions

The main focus of this research is to investigate the grouping techniques for large scale multi-item multi-echelon inventory system. The whole research is driven by six main research questions (Q1 to Q6) discussed in the introduction chapter.

The 1st research question is about the representation of the inventory system of interest and the inventory items to facilitate analysis and implementation of grouping methods. In order to deal with this question, the characteristics of the inventory items are categorized as structural and non-structural attributes. The most important structural attribute is the network of item type (NIT) which demonstrates the storage structure of an item. This attribute can be including in the grouping process either by represented using binary system (in NIT classification and K-means clustering), or in the way of organizing the grouping attributes listed in Exhibit 10. Involving the NIT by representing it using binary system in K-means clustering is not suggested based on the corresponding experimental results. This research is the first study that models the structural attribute of an item using its supply network, or put it in other words, using the network of item type (NIT). In section 3.3.2, the NIT is modeled using graph theory representation (binary expression). Compared to ABC classification and K-Means clustering, the experimental results show that the NIT classification using binary expression uses the shortest time to group large number of items. The non-structural attributes are modeled as continuous decimal values to calculate the Euclidean distance between items.

The 2nd research question is about the data generation method. A multi-item multi-echelon (MIME) data generation procedure is developed to generate different datasets representing a variety of large scale inventory systems. The inputs of the data generation are user configurable, so that controllable large scale MIME inventory system datasets are generated. This helps the experiments and facilitates the testing of the grouping methods. The data generation procedure is designed using the Java Classes developed in the process of dealing with 1st research question. It is believed that this dissertation is the first research that uses the NIT for data generation. As illustrated in Section 4.2.2.2, the structural attribute NIT facilitates the generation of SKUs and demands in the data generation process for the large scale inventory system of interest. To make the generated dataset more realistic, six directly related papers are studied to determine the range of data generation inputs, and a procedure involving a sequence of conditional probability distributions suggested by Rossetti and Achlerkar (2011) is used to satisfy the direct or inverse proportional relationships between a pair of input attributes.

The 3rd research question is about which system and item characteristics should be used in the grouping process. The answer to this question depends on the grouping method. For ABC classification, the characteristics that are involved in the classification criterion participate in the grouping process. For NIT classification, the structural attribute NIT is used for separating the items. For K-Means clustering, the characteristics that involved in the cost model are used as the clustering attributes. As discussed in Section 6.3, not all system and item characteristics that are involved in the cost model significantly affect the clustering results. Also, the experimental results show that including structural attributes in the K-Means clustering attributes increase clustering penalty cost and consumes more clustering time. Thus, structural attributes are not suggested to be directly involved in the K-Means clustering.

The 4th research question related investigation deals with the importance-based classification from the network perspective. This research question also relates to the question

A1 and A2 discussed in Section 4.2.4.4 and 6.2. This dissertation discusses the process of selecting appropriate ABC classification criterion by reviewing Zhang et al. (2001) and Teunter et al. (2010), and summarizes the rules of selecting/developing classification criterion. Based on the summarized rules, two network-based classification criteria are developed. The 1st classification criterion, the network inventory cost (NIC), which is shown in Equation (13) of Section 3.3.1, is developed based on Hadley and Whitin (1963)'s Model to calculate the inventory cost of the entire network. The 2nd classification criterion, the network annual dollar usage (NADU), is the sum of traditionally applied annual dollar usage of an item in the entire inventory network. The purpose of the 2nd classification criterion is to evaluate the new classification criterion NIC. Two paired-t tests are implemented to compare the effectiveness and efficiency of NIC and NADU. The measurement for effectiveness is the clustering penalty cost and the measurement for efficiency is the grouping time. The results show that NIC is significantly better than NADU from both effectiveness and efficiency perspectives. Also, the experimental results show that compared to 3 groups, using 7 groups results in smaller clustering penalty cost.

The 5th research question relates to K-Means clustering. According to Maimon and Rokach (2005), only K-Means clustering and its equivalent have been successfully applied to grouping large scale datasets. In order to understand the performance of the K-Means clustering for the large scale inventory system of interest, five K-Means related research questions (K1 to K5) are investigated in Section 6.3. The experimental results lead to following suggestions: 1) structural attributes should not be involved as the clustering attributes in K-Means clustering; 2) the simplest way to determine *k* is to set its value to the maximum allowed value; 3) number of items, number of clustering attributes, number of clusters, and number of iterations significantly affect the clustering time; and 4) items having the same NIT structure tend to be clustered into the same group.

7.2 Suggestions on Combining the Grouping Techniques in Practice

Individually, ABC classification, NIT classification, and K-Means clustering have their own unique advantages and disadvantages. This validates Ratliff and Nulty (1997)'s view of "there is no single best approach, best representation, best model, or best algorithm for optimizing logistics decisions". This indicates that it is reasonable to explore the ways to combine these techniques so that they can be used to their best advantages while avoiding their disadvantages in practice. It should be also noted from the analysis in the previous section that the sequence of applying these importance-based classifications and operation-based clustering may affect the grouping results and practical meaning of the groups. This could lead to different inventory management strategies in practice. In the following, the possible ways of integrating classification and clustering techniques are discussed in the context of two stage grouping and three stage grouping.

<u>ABC Classification + K-Means Clustering</u>

When individually used, ABC classification as an importance-based grouping technique identifies the important item types in the entire inventory system. It helps management to prioritize the items according to their importance for the management and financial resources allocation, so that the inventory items can be managed more effectively. When individually implemented, K-means clustering as an operational-attributes-based clustering technique groups together items with similar (close) characteristics from the operational perspective; this facilitates the implementation of inventory control policy so that lower penalty cost and higher service level can be achieved effectively. These techniques can be combined sequentially according to the management goal.

If the grouping is carried out in the sequence of the ABC classification first and then the K-Means clustering, the items will be grouped into 3 or 7 groups first that are prioritized according to the importance of the item types; in this way, each item is labeled as A item, B item, or C item indicating their importance. Then, using K-Means clustering, each resulting group from first step is further divided into item groups based on the operational-attributes; this facilitates determining the optimal group policy for the items while making the importance of the items identifiable. This combination of network-based ABC classification and K-Means clustering extends the traditional single-location-based ABC classification to the network-based and cost model involved grouping level.

If the grouping is carried out in the sequence of the K-Means clustering first and then the ABC classification, the items will firstly be grouped into operational groups, each of which holds item types with similar operational characteristics. In this way, the number of item groups is not restricted to 3 or 7 at the first stage. Further, classifying each of these operational groups using the ABC classification developed in this research labels the items in each group according to their importance to the management. This combination of network-based K-Means clustering and ABC classification extends the K-Means clustering to item importance involved grouping level.

NIT Classification + K-Means Clustering

As mentioned previously, as a classification technique based on the storage structure attribute, NIT classification makes it possible to group the items in such a way that the items in the same group share exactly the same storage structure. This corresponds to Lenard and Roy

(1995)'s suggestion that the differences between the storage structure of items prevent the items being grouped together for the function of the warehouse is different at different echelons.

If the grouping is implemented in the sequence of the NIT classification first and then the K-Means clustering, the items with identical storage structures are grouped together first, and then the resulting item groups are further divided into groups based on the operation-based attributes. This facilitates the group inventory policy calculation for the items with the same structural attributes. The ultimate groups will strictly have their own identical structures.

If the grouping is implemented in the sequence of the K-Means clustering first and then the NIT Classification, the resulted ultimate groups will also strictly have their own identical structures.

NIT Classification + ABC Classification

The items grouped together using NIT classification and ABC classification sequentially share the same storage structure and have the importance labels such as A, B, or C. Grouping first using NIT classification and then ABC classification helps prioritizing the items having the same storage structure by determining the importance of the items. On the other hand, grouping first using ABC classification and then NIT classification helps with identifying the storage structure of items with the same importance (priority); in other words, it helps to group the items with the same importance and same structure together.

ABC Classification +NIT Classification + K-Means Clustering

A three stage grouping that combine all the three grouping techniques introduced so far in this research sequentially can result in groups with more complete involvement of the system characteristics, which may be very favorable in some cases in practice. There are 6 ways of combination for these methods, and they would result different final groups. It should be noted that no matter in which sequences the grouping is implemented using these three grouping techniques together, the final resulted groups will have their own unique storage structure due to the involvement of NIT in the grouping process; they will have importance labels for each item type included; and they will have operational-attributes with similar (close) values. The diversity of the resulting groups from applying different sequences in the grouping process provides a means of investigating a wide variety of clustering scenarios.

The different grouping techniques have their own advantages according to the management goal. From the three aspects of resulted groups, whether importance of items are identified, whether have same structure, whether have similar operations-related attributes, the 7 grouping techniques can be compared as in Table 37.

Table 37: Characteristics of the 7 Grouping Techniques

Grouping Method	Identifying Important Items	Having Same Structure	Having Similar Operations-Related Attributes
K-Means			Yes
ABC	Yes		
NIT		Yes	
ABC+ K-Means	Yes		Yes
NIT+K-Means		Yes	Yes
NIT+ABC	Yes	Yes	
ABC+NIT+K-Means	Yes	Yes	Yes

7.3 Future Work

For grouping techniques, there are still many aspects need to be investigated. This section recommends three aspects to further investigate.

One of the five main factor categories affecting the grouping results is the cost model. The future research can use the cost model applied by the companies to test the performance of the network based ABC classification, NIT classification and K-Means clustering, and select the most appropriate grouping method or a combination of grouping methods. The cost model applied by the companies can be further investigated based on the methodologies developed in this research.

From the NIT perspective, the NIT modeling can be further investigated and applied to more realistic scenarios when industrial data is available. Currently, there are several assumptions about NIT modeling, such as each customer location has only one supply location. These assumptions may be relaxed after analyzing the characteristics of the storage structures in the industry. In this research, NIT is studied based on inventory control related activities. NIT can be investigated to integrate the inventory control, transportation and warehousing activities.

For K-Means clustering technique, the quality of the clustering results relates to the selection of initial seeds. This research did not investigate this field. The knowledge gained from the industry could help set up some rules for the selection of initial seeds, which are the centroids of each group setup in the Initialization Step in Exhibit 14. In addition, industrial instances could provide some insights into the clustering attributes selection, choosing optimal number of groups, etc.

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Appendix 1: A Case Study of the Multi-Echelon Inventory Cost Model

In this appendix, using a small instance, the cost model is illustrated by setting the optimal policies for a single-item two-echelon inventory system. As shown in Figure 54, consider a warehouse located at echelon 1 that supplies two retail stores at echelon 2.

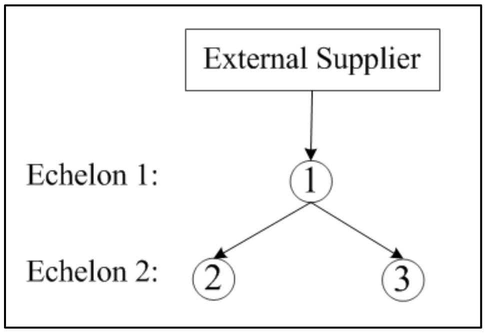

Figure 54: Multi-Echelon Inventory Cost Model Example

The continuous reorder point reorder quantity policy is used for this two echelon inventory system. The data used to calculate the cost are listed as follows:

$C \equiv \text{unit cost} = 100 \text{ dollars}$

$b = \text{lost sales cost} = 50 \text{ dollars}$

$A_1 \equiv \text{ordering cost at location } 3 = 500 \text{ dollars}$

$A_2 \equiv \text{ordering cost at location } 2 = 200 \text{ dollars}$

$A_3 \equiv \text{ordering cost at location } 3 = 300 \text{ dollars}$

$I_1 \equiv \text{inventory holding charge at location } 1 = 0.15 \text{ \$/\$/unit/year}$

$I_2 \equiv \text{inventory holding charge at location } 2 = 0.2 \text{ \$/\$/unit/year}$

$I_3 \equiv \text{inventory holding charge at location } 2 = 0.25 \text{ \$/\$/unit/year}$

$\lambda_2 \equiv \text{demand rate at location } 2 = 1500\ units/year$

$\sigma_{D\_2} \equiv \text{standard deviation of demand at location } 2 = 20 \text{ units/year}$

$\lambda_3 \equiv \text{demand rate at location } 3 = 2000 \text{ units/year}$

$\sigma_{D\_3} \equiv$ standard deviation of demand at location 3 $= 30$ units/year

$E(LT_1) \equiv$ mean lead time at external supplier $= 0.3$ year

$E(LT_2) \equiv$ mean lead time from location 1 to location 2 $= 0.1$ year

$E(LT_3) \equiv$ mean lead time from location 1 to location 3 $= 0.15$ year

$Var(LT_1) \equiv$ variance of lead time from external supplier $= 0.03$ year

$Var(LT_2) \equiv$ variance of lead time from location 1 to location 2 $= 0.01$ year

$Var(LT_3) \equiv$ variance of lead time from location 1 to location 3 $= 0.02$ year

The goal is to minimize the total inventory related cost for the whole network:

$$\text{Minimize } Total\ Cost = \sum_{i=1}^{3} \left\{ \begin{array}{c} \dfrac{\lambda_i}{Q_{Li}} A_i + I_i C \left(\dfrac{Q_{Li}}{2} + r_{Li} - \mu_{LT\_i} \right) \\ + \left(I_i C + \dfrac{b\lambda_i}{Q_{Li}} \right) \left[(\mu_{LT\_i} - r_{Li}) \Phi \left(\dfrac{r_{Li} - \mu_{LT\_i}}{\sigma_{LT\_i}} \right) + \sigma_{LT\_i} \phi \left(\dfrac{r_{Li} - \mu_{LT\_i}}{\sigma_{LT\_i}} \right) \right] \end{array} \right\}$$

Where

$\mu_{LT\_i} \equiv mean\ lead\ time\ demand\ at\ location\ i$

$\sigma_{LT\_i} \equiv standard\ deviation\ of\ lead\ time\ demand\ at\ location\ i$

$r_{Li} \equiv reorder\ point\ at\ location\ i$

$Q_{Li} \equiv reorder\ quantity\ at\ location\ i$

<u>For location 2</u>

$\mu_{LT\_2} = \lambda_2 E(LT_2) = 1500 \times 0.1 = 150$

$\sigma_{LT\_2} = \sqrt{\sigma_{D\_2}^2 E(LT_2) + \lambda_2^2 Var(LT_2)} = \sqrt{20^2 \times 0.1 + 1500^2 \times 0.01} = 150.13$

$Q_1 = Q_w = \sqrt{\frac{2\lambda_2 \mathrm{A}_2}{I_2 C}} = \sqrt{\frac{2 \times 1500 \times 200}{0.2 \times 100}} = 173.2$

$\Phi\left(\frac{r_1 - \mu_{LT\_2}}{\sigma_{LT\_2}}\right) = \frac{Q_1 I_2 C}{\lambda_2 b + Q_1 I_2 C} = \frac{173.2 \times 0.2 \times 100}{1500 \times 50 + 173.2 \times 0.2 \times 100} = 0.0441$

From the normal tables, $\frac{r_1 - \mu_{LT\_2}}{\sigma_{LT\_2}}$=1.704

$r_1 = 1.704 \times \sigma_{LT\_2} + \mu_{LT\_2} = 1.704 \times 150.13 + 150 = 405.82$

$\phi\left(\frac{r_1 - \mu_{LT\_2}}{\sigma_{LT\_2}}\right) = \phi(1.704) = 0.0933$

$\eta(r_1) = (\mu_{LT\_2} - r_1)\Phi\left(\frac{r_1 - \mu_{LT\_2}}{\sigma_{LT\_2}}\right) + \sigma_{LT_2}\phi\left(\frac{r_1 - \mu_{LT\_2}}{\sigma_{LT\_2}}\right)$

$= (150 - 405.82) * 0.0441 + 150.13 * 0.0933 =$2.7254

$Q_2 = \sqrt{\frac{2\lambda_2[A_2 + b * \eta(r_1)]}{I_2 C}} = \sqrt{\frac{2\times1500\times[200+50\times2.7254]}{0.2\times100}} = 224.59$

$\Phi\left(\frac{r_2 - \mu_{LT\_2}}{\sigma_{LT\_2}}\right) = \frac{Q_2 I_2 C}{\lambda_2 b + Q_2 I_2 C} = \frac{224.59\times0.2\times100}{1500\times50+224.59\times0.2\times100} = 0.0565$

From the normal tables, $\frac{r_2 - \mu_{LT\_2}}{\sigma_{LT\_2}}$=1.579

$r_2 = 1.579 \times \sigma_{LT\_2} + \mu_{LT\_2} = 1.579 \times 150.13 + 150 = 387.05$

Then, the stop criterion is checked. In this example, the stop criterion is $\frac{r_1 - r_2}{r_1} \leq 0.02$.

Since $\frac{r_1 - r_2}{r_1} = \frac{405.82-387.05}{405.82} = 0.046 > 0.02$, continue to search the optimal solution.

$\phi\left(\frac{r_2 - \mu_{LT\_2}}{\sigma_{LT\_2}}\right) = \phi(1.579) = 0.1147$

$\eta(r_2) = (\mu_{LT\_2} - r_2)\Phi\left(\frac{r_2 - \mu_{LT\_2}}{\sigma_{LT\_2}}\right) + \sigma_{LT\_2}\phi\left(\frac{r_2 - \mu_{LT\_2}}{\sigma_{LT\_2}}\right)$

$= (150 - 387.05) * 0.0565 + 150.13 * 0.1147 = 3.8265$

$Q_3 = \sqrt{\frac{2\lambda_2[A_2 + b * \eta(r_2)]}{I_2 C}} = \sqrt{\frac{2\times1500\times[200+50\times3.8265]}{0.2\times100}} = 242.28$

$\Phi\left(\frac{r_3 - \mu_{LT\_2}}{\sigma_{LT\_2}}\right) = \frac{Q_3 I_2 C}{\lambda_2 b + Q_3 I_2 C} = \frac{242.28\times0.2\times100}{1500\times50+ 242.28\times0.2\times100} = 0.0607$

From the normal tables, $\frac{r_2 - \mu_{LT\_2}}{\sigma_{LT\_2}}$=1.552

$r_3 = 1.552 \times \sigma_{LT\_2} + \mu_{LT\_2} = 1.552 \times 150.13 + 150 = 383$

Since $\frac{r_2 - r_3}{r_2} = \frac{387.05-383}{387.05} = 0.01 < 0.02$, the optimal solution is obtained.

The optimal reorder point for location 2: $r_{L2}^{*} = r_3 = 383$

The optimal reorder quantity for location 2: $Q_{L2}^{*} = Q_3 = 242.28 \approx 242$

The total cost for location 2:

$$\frac{\lambda_2}{Q_{L2}^{*}} A_2 + I_2 C\left(\frac{Q_{L2}^{*}}{2} + r_{L2}^{*} - \mu_{LT\_2}\right) + \left(I_2 C + \frac{b\lambda_2}{Q_{L2}^{*}}\right)\left[\left(\mu_{LT\_2} - r_{L2}^{*}\right)\Phi\left(\frac{r_{L2}^{*} - \mu_{LT\_2}}{\sigma_{LT\_2}}\right) + \sigma_{LT\_2}\phi\left(\frac{r_{L2}^{*} - \mu_{LT\_2}}{\sigma_{LT\_2}}\right)\right]$$

$$= \frac{1500}{242} \times 200 + 0.2 \times 100 \times \left(\frac{242}{2} + 383 - 150\right) + \left(0.2 \times 100 + \frac{50 \times 1500}{242}\right)$$

$$\times \left[(150 - 383)\Phi\left(\frac{383 - 150}{150.13}\right) + 150.13 \times \phi\left(\frac{383 - 150}{150.13}\right)\right] = 9334.76$$

<u>For location 3</u>

$\mu_{LT\_3} = \lambda_3 E(LT_3) = 2000 \times 0.15 = 300$

$\sigma_{LT\_3} = \sqrt{\sigma_{D\_3}^2 E(LT_3) + \lambda_3^2 Var(LT_3)} = \sqrt{30^2 \times 0.15 + 2000^2 \times 0.02} = 283.08$

$Q_1 = Q_w = \sqrt{\frac{2\lambda_3 \mathrm{A}_3}{I_3 C}} = \sqrt{\frac{2\times2000\times300}{0.25\times100}} = 219.09$

$\Phi\left(\frac{r_1 - \mu_{LT\_3}}{\sigma_{LT\_3}}\right) = \frac{Q_1 I_3 C}{\lambda_3 b + Q_1 I_3 C} = \frac{219.09\times0.25\times100}{2000\times50+219.09\times0.25\times100} = 0.0519$

From the normal tables, $\frac{r_1 - \mu_{LT\_2}}{\sigma_{LT\_2}}$=1.626

$r_1 = 1.626 \times \sigma_{LT\_2} + \mu_{LT\_2} = 1.626 \times 283.08 + 300 = 760.29$

$\phi\left(\frac{r_1 - \mu_{LT\_3}}{\sigma_{LT\_3}}\right) = \phi(1.626) = 0.1063$

$\eta(r_1) = \left(\mu_{LT\_3} - r_1\right)\Phi\left(\frac{r_1 - \mu_{LT\_3}}{\sigma_{LT\_3}}\right) + \sigma_{LT\_3}\phi\left(\frac{r_1 - \mu_{LT\_3}}{\sigma_{LT\_3}}\right)$

$= (300 - 760.29) * 0.0519 + 283.08 * 0.1063$ =6.202

$Q_2 = \sqrt{\frac{2\lambda_3[A_3 + b*\eta(r_1)]}{I_3 C}} = \sqrt{\frac{2\times2000\times[300+50\times6.202]}{0.25\times100}} = 312.43$

$\Phi\left(\frac{r_2 - \mu_{LT\_3}}{\sigma_{LT\_3}}\right) = \frac{Q_2 I_3 C}{\lambda_3 b + Q_2 I_3 C} = \frac{312.43\times0.25\times100}{2000\times50+312.43\times0.25\times100} = 0.0724$

From the normal tables, $\frac{r_2-\mu_{LT\_3}}{\sigma_{LT\_3}}$=1.447

$r_2 = 1.447 \times \sigma_{LT\_3} + \mu_{LT\_3} = 1.447 \times 283.08 + 300 = 709.62$

Since $\frac{r_1-r_2}{r_1} = \frac{760.29-709.62}{760.29} = 0.067 > 0.02$, continue to search the optimal solution.

$\phi\left(\frac{r_2-\mu_{LT\_3}}{\sigma_{LT\_3}}\right) = \phi(1.447) = 0.1401$

$\eta(r_2) = (\mu_{LT\_3} - r_2)\Phi\left(\frac{r_2-\mu_{LT\_3}}{\sigma_{LT\_3}}\right) + \sigma_{LT_2}\phi\left(\frac{r_2-\mu_{LT\_3}}{\sigma_{LT\_3}}\right)$

$$= (300 - 709.62) * 0.0724 + 283.08 * 0.1401 = 10.003$$

$Q_3 = \sqrt{\frac{2\lambda_3[A_3+b*\eta(r_2)]}{I_3C}} = \sqrt{\frac{2\times2000\times[300+50\times10.003]}{0.25\times100}} = 357.8$

$\Phi\left(\frac{r_3-\mu_{LT\_3}}{\sigma_{LT\_3}}\right) = \frac{Q_3I_3C}{\lambda_3b+Q_3I_3C} = \frac{357.8\ \times0.25\times100}{2000\times50+\ 357.8\ \times0.25\times100} = 0.0821$

From the normal tables, $\frac{r_2-\mu_{LT\_3}}{\sigma_{LT\_3}}$=1.396

$r_3 = 1.396 \times \sigma_{LT\_3} + \mu_{LT\_3} = 1.396 \times 283.08 + 300 = 695.18$

Since $\frac{r_2-r_3}{r_2} = \frac{709.62-695.18}{709.62} = 0.02 \leq 0.02$, the optimal solution is obtained.

The optimal reorder point for location 3: $r_{L3}^* = r_3 = 695.18 \approx 695$

The optimal reorder quantity for location 3: $Q_{L2}^* = Q_3 = 357.8 \approx 358$

The total cost for location 3:

$\frac{\lambda_3}{Q_{L3}^*}A_3 + I_3C\left(\frac{Q_{L3}^*}{2} + r_{L3}^* - \mu_{LT\_3}\right) + \left(I_3C + \frac{b\lambda_3}{Q_{L3}^*}\right)\left[(\mu_{LT\_3} - r_{L3}^*)\Phi\left(\frac{r_{L3}^*-\mu_{LT\_3}}{\sigma_{LT\_3}}\right) + \sigma_{LT\_3}\phi\left(\frac{r_{L3}^*-\mu_{LT\_3}}{\sigma_{LT\_3}}\right)\right]$

$$= \frac{2000}{358} \times 300 + 0.25 \times 100 \times \left(\frac{358}{2} + 695 - 300\right) + \left(0.25 \times 100 + \frac{50 \times 2000}{358}\right)$$

$$\times \left[(300 - 695)\Phi\left(\frac{695 - 300}{283.08}\right) + 283.08 \times \phi\left(\frac{695 - 300}{283.08}\right)\right] = 18142.14$$

<u>For location 1</u>

The mean demand rate at location 1: $\lambda_1 = \lambda_2 + \lambda_3 = 1500 + 2000 = 3500$

The standard deviation of demand at location 1: $\sigma_{D\_1} = \sqrt{\sigma_{D\_2}^2 + \sigma_{D\_3}^2} = \sqrt{20^2 + 30^2} = 36$

$\mu_{LT\_1} = \lambda_1 E(LT_1) = 3500 \times 0.3 = 1050$

$\sigma_{LT\_1} = \sqrt{\sigma_{D\_1}^2 E(LT_1) + \lambda_1^2 Var(LT_1)} = \sqrt{36^2 \times 0.3 + 3500^2 \times 0.03} = 606.54$

$Q_1 = Q_w = \sqrt{\frac{2\lambda_1 \mathrm{A}_1}{I_1 C}} = \sqrt{\frac{2\times 3500\times 500}{0.15\times 100}} = 483.05$

$\Phi\left(\frac{r_1 - \mu_{LT\_1}}{\sigma_{LT\_1}}\right) = \frac{Q_1 I_1 C}{\lambda_1 b + Q_1 I_1 C} = \frac{483.05\times 0.15\times 100}{3500\times 50 + 483.05\times 0.15\times 100} = 0.0398$

From the normal tables, $\frac{r_1 - \mu_{LT\_1}}{\sigma_{LT\_1}}$=1.7535

$r_1 = 1.7535 \times \sigma_{LT\_1} + \mu_{LT\_1} = 1.7535 \times 606.54 + 1050 = 2113.57$

$\phi\left(\frac{r_1 - \mu_{LT\_1}}{\sigma_{LT\_1}}\right) = \phi(1.7535) = 0.0857$

$\eta(r_1) = (\mu_{LT\_1} - r_1)\Phi\left(\frac{r_1 - \mu_{LT\_1}}{\sigma_{LT\_1}}\right) + \sigma_{LT\_1}\phi\left(\frac{r_1 - \mu_{LT\_1}}{\sigma_{LT\_1}}\right)$

$= (1050 - 2113.57) * 0.0398 + 606.54 * 0.0857 =$9.6504

$Q_2 = \sqrt{\frac{2\lambda_1 [A_1 + b * \eta(r_1)]}{I_1 C}} = \sqrt{\frac{2\times 3500\times[500+50\times 9.6504]}{0.15\times 100}} = 677.13$

$\Phi\left(\frac{r_2 - \mu_{LT\_1}}{\sigma_{LT\_1}}\right) = \frac{Q_2 I_1 C}{\lambda_1 b + Q_2 I_1 C} = \frac{677.13\times 0.15\times 100}{3500\times 50 + 677.13\times 0.15\times 100} = 0.0549$

From the normal tables, $\frac{r_2 - \mu_{LT\_1}}{\sigma_{LT\_1}}$=1.59

$r_2 = 1.59 \times \sigma_{LT\_3} + \mu_{LT\_3} = 1.59 \times 606.54 + 1050 = 2014.4$

Since $\frac{r_1 - r_2}{r_1} = \frac{2113.57 - 2014.4}{2113.57} = 0.047 > 0.02$, continue to search the optimal solution.

$\phi\left(\frac{r_2 - \mu_{LT\_1}}{\sigma_{LT\_1}}\right) = \phi(1.59) = 0.1126$

$\eta(r_2) = (\mu_{LT\_1} - r_2)\Phi\left(\frac{r_2 - \mu_{LT\_1}}{\sigma_{LT\_1}}\right) + \sigma_{LT_2}\phi\left(\frac{r_2 - \mu_{LT\_1}}{\sigma_{LT\_1}}\right)$

$= (1050 - 2014.4) * 0.0549 + 606.54 * 0.1126 = 15.35$

$$Q_3 = \sqrt{\frac{2\lambda_1[A_1+b*\eta(r_2)]}{I_1 C}} = \sqrt{\frac{2\times3500\times[500+50\times15.35]}{0.15\times100}} = 769$$

$$\Phi\left(\frac{r_3-\mu_{LT\_1}}{\sigma_{LT\_1}}\right) = \frac{Q_3 I_1 C}{\lambda_1 b+Q_3 I_1 C} = \frac{769\times0.15\times100}{3500\times50+769\times0.15\times100} = 0.0618$$

From the normal tables, $\frac{r_2-\mu_{LT\_3}}{\sigma_{LT\_3}}$=1.545

$$r_3 = 1.545\times\sigma_{LT\_3}+\mu_{LT\_3} = 1.545\times606.54+1050 = 1987.1$$

Since $\frac{r_2-r_3}{r_2} = \frac{2014.4-1987.1}{2014.4} = 0.013 \leq 0.02$, the optimal solution is obtained.

The optimal reorder point for location 1: $r_{L1}^* = r_3 = 1987.1 \approx 1987$

The optimal reorder quantity for location 1: $Q_{L1}^* = Q_3 = 769$

The total cost for location 1:

$$\frac{\lambda_1}{Q_{L1}^*}A_1 + I_1 C\left(\frac{Q_{L1}^*}{2}+r_{L1}^*-\mu_{LT\_1}\right)+\left(I_1 C+\frac{b\lambda_1}{Q_{L1}^*}\right)\left[\left(\mu_{LT\_1}-r_{L1}^*\right)\Phi\left(\frac{r_{L1}^*-\mu_{LT\_1}}{\sigma_{LT\_1}}\right)+\sigma_{LT\_1}\phi\left(\frac{r_{L1}^*-\mu_{LT\_1}}{\sigma_{LT\_1}}\right)\right]$$

$$= \frac{3500}{769}\times500+0.15\times100\times\left(\frac{769}{2}+1987-1050\right)+\left(0.15\times100+\frac{50\times3500}{769}\right)$$

$$\times\left[(1050-1987)\Phi\left(\frac{1987-1050}{606.54}\right)+606.54\times\phi\left(\frac{1987-1050}{606.54}\right)\right]$$

$$= 24618.42$$

In sum, the total cost for the whole network= total cost for location 1+ total cost for location 2+ total cost for location 3=$24618.42 + 9334.76 + 18142.14 = 52095.32$. The summary of final policies and costs are shown in following table.

Table 38: The Summary of the Final Policies and Costs

	r	Q	cost
Location 1	1987	769	24618.42
Location 2	383	242	9334.76
Location 3	695	358	18142.14

Appendix 2: Data Modeling

This Appendix implements the steps mentioned in Section 4.1.2 and 4.1.3 to build the data models. The E-R diagram is built and then mapped to relational tables. Based on the system characteristics discussed in Section 1, this appendix first discusses the process to build the E-R diagram, and then implements the mapping process from E-R diagram to relational models. At the end of this Appendix, an example is given to illustrate the deriving process of the NIT for a specific item based on the SKU table and the Shipment table.

a) E-R Diagram Building Process

This section models the inventory system through following steps: (1) identifying entities and drawing the entity diagram; (2) identifying associations and drawing the association diagrams; and (3) specifying the domain for each attribute. It should be noted that the third step "specifying the domain for each attribute" is completed during the step 1 and step 2.

Step 1: Identifying the Entities

Based on the inventory system characteristics, there are four entities: probability distribution, item type, location, and inventory policy. The entities and their attributes are summarized and represented using UML diagram notation as in Figure 55.

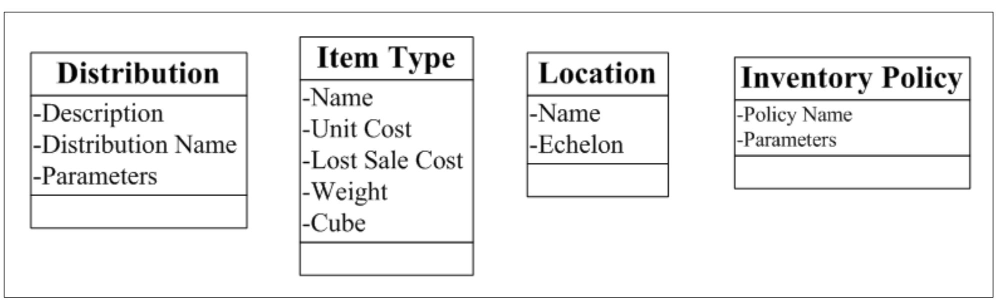

Figure 55: Entity Classes

As it is shown in Figure 55, the Distribution entity has three attributes: description, distribution name, and parameters. The description attribute is a string that describes the usage of the probability distribution. The distribution name attribute is a string that represents the distribution type such as normal, exponential, uniform etc. The parameters attribute is a string that stores the parameters of the distribution. If there are more than one parameter values, they are separated by a comma.

The Item Type entity has five attributes: name, unit cost, lost sales cost, weight, and cube. The name attribute is a string that describes the item type. The unit cost attribute is a positive decimal that records the cost of a single unit of the item. The lost sales cost attribute is a positive decimal that indicates the penalty cost for lost sales. The weight attribute is a positive decimal that represents the weight of the item. The cube attribute is also a positive decimal that shows the size of the item.

The Location entity represents an IHP or an external supplier. The Location entity has two attributes: name and echelon. The name attribute is a string that represents the location. The echelon attribute is an integer value that represents the echelon at which the location is located. The default echelon value for the external supplier is zero.

The Inventory Policy entity has two attributes: policy name and parameters. The policy name attribute is a string that describes the ordering policy. The parameter attribute is a string that stores the parameters of the policy. If there are more than one parameter values, they are separated by a comma.

<u>Step 2: Identifying the Association</u>

Based on the entities described in previous section, this section discusses each association in detail.

Item Type – Distribution Association

Association description: each item type has an associated probability distribution that represents the lead time at external supplier.

Multiplicity: One item type has one and only one lead time distribution. One distribution belongs to zero or many item types. Distributions can be used to represent lead time at the external suppler, demand rate, transportation time etc. The distributions that are not used to represent the lead time have no relationship with item type, thus the minimum cardinality from distribution to item type is zero. A distribution can be used to represent the lead time for more than one item type, thus the maximum cardinality is many. The Item Type – Distribution Association is illustrated as in Figure 56.

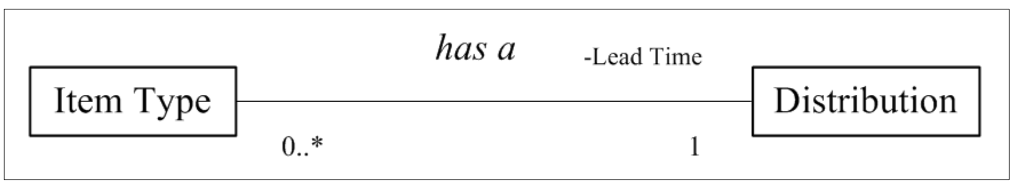

Figure 56: Item Type – Distribution Association

Arrive at Association

Association description: end customer demands arrive at retail stores. It is assumed that each customer demand only associates with a single item type; therefore the customer demand can be modeled as the item type. In reality a customer demand may contain multiple item types; in this case the customer demand is separated into different item types. The “arrive at” association has two related probability distributions, one represents the time between arrivals (TBA), and the other represents the demand size. The attributes of the “arrive at” association are stored in the Demand Generator association class. The name attribute is a string which represents the descriptive name for the generator.

Multiplicity: a customer demand arrives at one or more retail stores, and one location may have many customer demands or may have no customer demand arrival. This is because only the retail stores directly face the end customer demands; the IHPs at higher echelons do not have customer demand. A demand generator has one and only one TBA distribution. A demand generator has zero or one demand size distribution. When a demand generator has no demand size distribution, it is assumed that the demand size has constant value 1. A TBA distribution may associate with zero or many demand generators. The distribution can represent TBA distribution, demand size distribution, etc. If a distribution is not used to model a TBA, its multiplicity is zero; and if it is used to model many TBAs, its multiplicity is many. The demand size distribution has the same multiplicity as TBA distribution. Figure 57 illustrates the "arrive at" association.

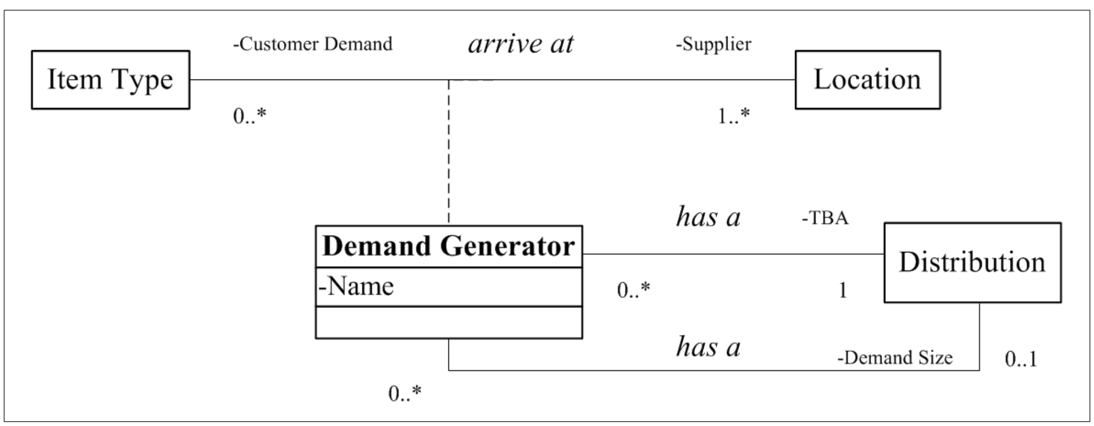

Figure 57: Arrive at Association

Store at Association

Association description: items are stored at the IHPs. Attributes of the "store at" association are stored in the SKU association class. SKU association class has four attributes: initial on hand, backorder cost, ordering cost, and holding charge. The initial on hand attribute is a non-negative integer, and it is the initial amount of inventory on hand. The backorder cost attribute is a positive decimal, and it is the backorder cost in $/unit/time. The ordering cost is a positive decimal, and it is the cost of an order in $/order. The holding charge cost is a positive decimal, and it is the cost of holding inventory in $/$/time. Each SKU has an inventory policy.

Multiplicity: an item type is stored at one or many locations. A location may have zero or many item types. The zero multiplicity corresponds to the assumption that the external supplier does not hold any inventory. Each SKU has one and only one inventory policy, and one inventory policy can be applied to one or many SKUs. Figure 58 illustrates the “store at” association.

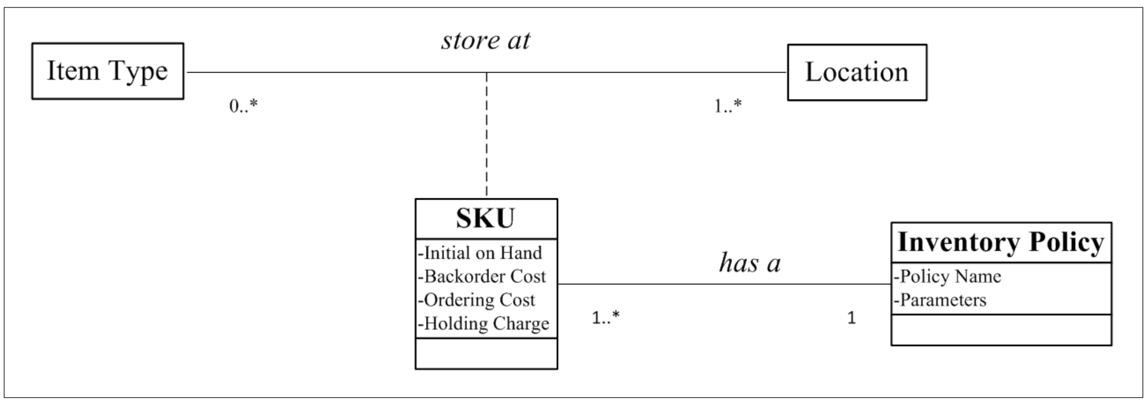

Figure 58: Store at Association

Supply association

Association description: as mentioned in Section 1, each IHP is supplied by an IHP that located at the immediate higher echelon, except those IHPs located at the first echelon, which are supplied by the external suppler. Supply association has one attribute, which is the shipping cost. The shipping cost is a positive decimal, and it is the cost of shipping from supplier to the customer in $/shipment. The attribute of the “supply” association are stored in the shipment association class. The shipment association class has a transportation time distribution.

Multiplicity: as it is discussed in Section 1, there are three different customer-supplier relations: End Customer Demand-IHP, IHP-IHP, and IHP-External Supplier. The End Customer

Demand-IHP relation reflects the demand arrival process from the end customer to the retail store, which is the location at the lowest echelon. In this case, the location at the lowest echelon has no customer locations, which means the minimum multiplicity at the Customer Location side is zero. For a specific NIT, each IHP has only one supply location and may have multiple customer locations. This indicates that the maximum multiplicity at the Supplier Location side is one, and the maximum multiplicity at the Customer Location side is many. The external supplier is located at the highest echelon and do not have a supplier location; thus the minimum multiplicity at the supplier location side is zero. Figure 59 illustrates the "supply" association.

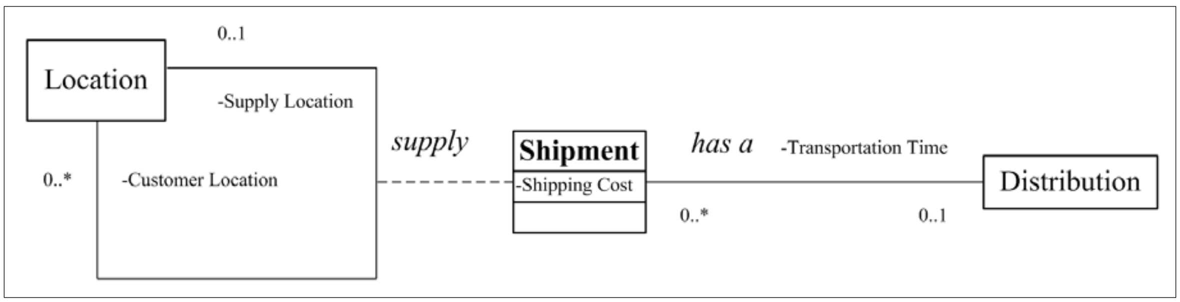

Figure 59: Supply Association

The complete UML diagram for the inventory system resulted by putting all the entities and associations is shown in Figure 6.

b) Mapping the E-R Diagram to the Relational Model

Based on the E-R diagram derived in the previous section, this section designs the relational tables corresponding to the entity and association classes. The schema of the tables is shown in the following format:

Table Name (<u>Primary Key(s)</u>, Attribute 1, Attribute 2… Attribute N)

The rest of this section illustrates the mapping process from E-R diagram to tables.

Distribution

(<u>ID</u>, Description, Distribution Name, Parameters)

The Distribution entity is mapped to Distribution Table. An ID field is added as the primary key of the Distribution Table.

Location

(<u>ID</u>, Name, Echelon)

The Location entity is mapped to Location Table. An ID field is added as the primary key of the Location Table.

Item Type

(<u>ID</u>, Name, Unit Cost, Lost sales Cost, Weight, Cube, LeadTimeID(references Distribution Table), LeadTimeMean, LeadTimeVar)

The Item Type entity is mapped to Item Type Table. An ID field is added as the primary key of the Item Type Table. Item Type Table has a buried attribute LeadTimeID that implements Item Type – Distribution association in Figure 56. The lead time mean(LeadTimeMean) and lead time variance(LeadTimeVar) for the lead time distribution are also stored in the Item Type table.

Demand Generator

(<u>ItemTypeID</u>(reference Item Type Table), <u>LocationID</u>(reference Location Table), Name, TBAID(reference Distribution Table), DemandSizeID(reference Distribution Table), AnnualDemandMean, AnnualDemandVar)

The “arrive at” association in Figure 57 is mapped to Demand Generator Table. The association attribute, name, is turned into an attribute column in the Demand Generator Table. The TBAID is buried into Demand Generator Table to implement the “has a TBA” association. The DemandSizeID is buried into Demand Generator Table to implement the “has a Demand Size” association. The mean of annual demand (AnnualDemandMean) and the variance of annual demand (AnnualDemandVar) are also stored in the Demand Generator table.

SKU

(ItemTypeID(reference Item Type Table), LocationID(reference Location Table), Initial on Hand, Backorder Cost, Ordering Cost, Holding Charge, Policy Name, ReorderPoint, ReorderQuantity, Min, Max, ReviewPeriod, LeadTimeMean, LeadTimeVar)

The "store at" association in Figure 58 is mapped to SKU Table. The association attributes, initial on hand, backorder cost, ordering cost, holding charge, are turned into attribute columns in the SKU Table. The lead time mean (LeadTimeMean) and lead time variance (LeadTimeVar) of the SKU reordering time are also stored in the SKU table.

In order to satisfy the requirements of third normal form, the Inventory Policy entity should be mapped to an Inventory Policy table, and the primary key of Inventory Policy Table should be buried into SKU Table. This would increase the complexity of the table structure and the data input process. Two revisions are made to solve this problem: (1) the attributes of the Inventory Policy entity are stored in the SKU Table, and (2) the parameters of the inventory policy are separated into five columns. As a result, the SKU Table violates the second normal form since some of the attributes, such as ReorderPt, ReorderQty, Min, Max, and ReviewPeriod, are not dependent on the primary keys. These columns are determined by the non-primary key column Policy Name. As aforementioned, the purpose of denormalizing the SKU table is to reduce the complexity of the data structure. Compared to storing the information of Inventory Policy in a separate table, putting it in the SKU table makes it easier to input all the SKU information in one table.

Shipment

(<u>SupplyLocationID</u>(reference Location Table), <u>CustomerLocationID</u>(reference Location Table), Shipping Cost, TransportationTimeID(reference Distribution Table), ShippingTimeMean, ShippingTimeVar)

The "supply" association in Figure 59 is mapped to Shipment Table. The association attribute, shipping cost, is turned into an attribute column in the Shipment Table. The TransportationTimeID is buried into Shipment Table to implement the "has a transportation time" association between Shipment class and Distribution class. The mean of transportation time (ShippingTimeMean) and variance of transportation time (ShippingTimeVar) are also stored in the Shipment table.

c) An Example: Deriving the NIT from SKU Table and Shipment Table

Following is an illustration of the deriving process of the NIT of a specific item type based on SKU table and Shipment table. Figure 60 is the SKU table of item type 1 (as shown on Column B/ItemTypeID), on which, Column A/LocationID shows there are 5 locations on its location network. Figure 61 is the Shipment table of inventory network. The Column A (SupplyingLocationID) and Column B (CustomerLocationID) reflects the supplier and customer relationships in the network. It can be seen from Figure 61 that Location 1 gets items from Location 0; Location 2 is supplied by Location 1, and there are 3 customer locations, e.g. Location 3, 4, 5, for Location 2. Based on these supplier and customer relations, the NIT of item type 1 can be derived as shown on Figure 62.

SKU

	A	B	C	D	E	F	G	H	I	J	K	L
1	LocationID	ItemTypeID	InitialOnHand	BackOrderingCost	OrderingCost	HoldingCharge	StockOutCost	LTmean	LTvar	Policy	ReorderPt	ReorderQty
2	1	1	0	1	37.53	0.15432	0.29	19.63973	8.260146165	R_Q	304	2128
3	2	1	0	1	36.35	0.19588	0.29	13.25357	3.923012533	R_Q	304	2128
4	3	1	0	1	30.71	0.14018	0.29	9.88692	1.21869283	R_Q	144	1008
5	4	1	0	1	31.65	0.06306	0.29	11.32601	1.762231663	R_Q	136	952
6	5	1	0	1	32.13	0.02889	0.29	8.35557	3.656322807	R_Q	24	168

Figure 60: An Instance of SKU Table

SHIPMENT

	A	B	C	D
1	SupplyingLocationID	CustomerLocationID	ShippingTimeMean	ShippingTimeVar
2	0	1	52.23	29.12
3	1	2	24.21	9.5
4	2	3	52.15	2.74
5	2	4	19.13	2.98
6	2	5	21.96	16.71

Figure 61: An Instance of Shipment Table

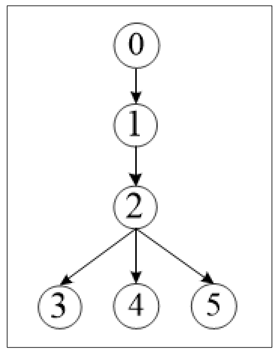

Figure 62: NIT of Item Type 1

Appendix 3: Data Representation in Java Classes and the Data Generation Algorithm

This appendix contains two parts. The first part illustrates the implementation of data models (discussed in Section 4.1 and Appendix 2) using Java classes. Based on the developed Java classes, the second part demonstrates the data generation processes (discussed in Section 4.2.2.2) using several algorithms.

Data Representation in Java Classes

Each table designed in the Data Modeling process is implemented as a Java class. The attributes and data types of these Java classes are summarized in the data modeling appendix. In this table, the first column is the Java Class name, the second column is the corresponding table name, the third column is the attribute name in the tables, the fourth column is the corresponding attributes represented in Java, and fifth column is the Java data type for the fourth column. The data types presented in the fifth column could be the original Java data types or the data types (classes) created in this research. Except for the foreign keys, the Java data type is selected based on the attribute description in the table design. The data types of the foreign keys are implemented by the data type where the foreign keys reference to. For example, in Item Type table, the attribute LeadTimeID is used to reference Distribution Table; in the corresponding Java class ItemTypeDM, the LeadTimeID attribute is implemented by a class attribute myLeadTime, whose data type is CDFInfo class (since the Distribution Table is implemented as CDFInfo class). The attribute myLocationNetwork and the data type LocationNetwork in ItemTypeDM class will be discussed later. The domain of the data type CDFName is {Normal, Beta, Lognormal, Gamma, Weibull, PearsonType5, PearsonType6, Triangular, Bernoulli, Geometric, ShiftedGeometric, Poisson,VConstant, Exponential, DUniform, Uniform, JohnsonB, LogLogistic, NegativeBinomial, Binomial, DEmpirical}.

Table 39: Java Classes Based on Tables

Java Class Name	Table Name	Attributes in Table	Attributes in Java	Data Type
CDFInfo	Distribution	ID	myID	int
		Description	myDescription	String
		Distribution Name	myType	CDF name
		Parameters	myParams	String
LocationDM	Location	ID	myID	int
		Name	myName	String
		Echelon	myLevel	int
ItemTypeDM	Item Type	ID	myID	int
		Name	myName	String
		Unit Cost	myCost	double
		Lost sales Cost	myLostSaleCost	double
		Weight	myWeight	double
		Cube	myCube	double
		LeadTimeID	myLeadTime	CDFInfo
		LeadTimeMean	myLeadTimeATESmean	double
		LeadTimeVar	myLeadTimeATESvar	double
			myLocationNetwork	LocationNetwork
SKUDM	SKU	LocationID	myLocation	LocationDM
		ItemTypeID	myItemType	ItemTypeDM
		Initial on Hand	myInitialOnHand	int
		Ordering Cost	myOrderingCost	double
		Holding Charge	myHoldingCharge	double
		Stockout Cost	myStockOutCost	double
		Policy Name	myPolicy	String
		ReorderPoint	myReorderPt	int
		ReorderQuantity	myReorderQty	int
		ReviewPeriod	myReviewPeriod	double
		Min	myMin	int
		Max	myMax	int
		LeadTimeMean	myLeadTimeMean	double
		LeadTimeVar	myLeadTimeVar	double
ShipmentDM	Shipment	SupplyLocationID	mySupplyingLocation	LocationDM
		CustomerLocationID	myCustomerLocation	LocationDM
		TransportationTimeID	myShippingTime	CDFInfo
		ShippingTimeMean	myShippingTimeMean	double
		ShippingTimeVar	myShippingTimeVar	double
		Shipping Cost	myShippingCost	double
DemandGeneratorDM	Demand Generator	LocationID	mySupplier	LocationDM
		ItemTypeID	myItemType	ItemTypeDM
		TBAID	myTimeBtwEvents	CDFInfo
		DemandSizeID	myAmt	CDFInfo
		Name	myName	String
		AnnualDemandMean	myAnnualDemandMean	double
		AnnualDemandVar	myAnnualDemandVar	double

From the object oriented programming perspective, a Java class LocationNetwork is created to represent the supply network (the NIT data model described in Section 4.1.4). This class has five attributes: myLocations, myRelation, myLevelLocations, myItemLocationAtRetailers, and myCustomers. The data type and description of the attributes are summarized in Table 39. It should be noted that the ItemTypeDM class in Table 39 has an attribute named myLocationNetwork whose data type is LocationNetwork (class), and this attribute stores the information of Supply Network for the corresponding item type.

Table 40: LocationNetwork Class

Attribute	Data Type	Description
myLocations	Set<Integer>	a set of integer that records all the location IDs within a supply network
myRelation	Map<Integer, Integer>	a map that stores all the customer-supplier relations; the key is the customer ID, and the value is the supplier ID; myRelation attribute is the implementation of CSmap
myLevelLocations	Map<Integer, Set<Integer>>	make it easier to find the locations at a certain echelon; the key is the echelon number, and the value is a Set of location IDs
myItemLocationAtRetailers	Set<Integer>	a set that stores the location IDs at the retail echelon (the lowest echelon in the supply network)
myCustomers	Map<Integer, Set<Integer>>	help find the customer location IDs; the key is the supplier location ID, and the value is the customer location ID

NetworkDM class is developed to store all the characteristics of the inventory system (The IS data model described in Section 4.1.4). As aforementioned, the NIS is formed by combining all the NITs in the inventory system; this means that the entire supply network of inventory system information can be represented by modeling the NIS. The NIS is implemented as an attribute myLocationNetwork in the NetworkDM whose data type is LocationNetwork . As mentioned in Section 2.1 and repeated here as a reminder, Cohen et al. (1986) point out that the characteristics of an multi-echelon multi-item inventory system includes: 1) number of products, (2) number of echelons, (3) network structure (series, arborescence, general), (4) periodic versus continuous review, (5) cost/service tradeoff measures, (6) demand process class, and (7) lead time and distribution mechanisms, etc. The attributes of NetworkDM class will store all of these characteristics. The data type and description of the attributes are summarized in Table 41.

Table 41: NetworkDM Class

Attribute	Data Type	Description
myCDFs	Map<Integer, CDFInfo>	Store the CDFInfo objects (distributions) that are used in inventory systems; the Integer stores the ID of the CDFInfo object
myItemTypes	Set<ItemTypeDM>	Store the item types in the inventory systems
myLocations	Map<Integer, LocationDM>	Store the locations in the inventory systems
myLevelLocations	Map<Integer, LocationDM>	Permits easy look up of locations by echelon.
myShipments	Map<LocationDM, ShipmentDM>	Store the shipping information for each customer location.
mySKUs	Set<SKUDM>	Store the SKUs in the inventory systems.
myDemandGeneratorDMs	Set<DemandGeneratorDM>	Store the DemandGeneratorDM objects in the inventory systems
myLocationNetwork	LocationNetwork	Store the Supply Network of the inventory systems

<u>Data Generation Algorithms</u>

The rest of this appendix uses several algorithms to illustrate the data generation processes. The inputs of the data generation and four main data generation steps are summarized in Section 4.2.2.2. The step 1 corresponds to Algorithm 2. Step 2, 3 and 4 correspond to the three for loops in Algorithm 1.

Algorithm 1: The data generation procedure

```
generate a NetworkDM object networkdm;

generate an integer value NE using D_NE;
create a LocationNetwork object LN and initialize its attributes; (refer to Algorithm 2)
set networkdm.myLocationNetwork= LN;

create and add the LocationDM objects to networkdm.myLocations using LN ;(refer to
Algorithm 4)

create and add the ShipmentDM objects to networkdm.myShipments using LN (refer to
Algorithm 5)

for loopCounter = 1 to NI
        create an ItemTypeDM object itemtypedm;
        set itemtypedm.myID = loopCounter;
        set itemtypedm.myName = “Item Type loopCounter”;
        generate a double value dblUnitCost using D_UC;
        set itemtypedm.myCost = dblUnitCost;
        generate a double value dblLSCR using D_LSC;
        set itemtypedm. myLostSaleCost= dblUnitCost* dblLSCR;
        generate a double value dblLeadTime using D_LTES;
        generate a double value dblLTVTMR Value using D_LTV;
        set dblLeadTimeVar = dblLeadTime * doubleLTVTMR;
        set itemtypedm. myLeadTimeATESmean= dblLeadTime;
        set itemtypedm. myLeadTimeATESvar= dblLeadTimeVar;
        add itemtypedm to networkdm.myItemTypes;
        increase loopCounter by 1;
end for

for each itemtypedmElement in networkdm.myItemTypes
        create a LocationNetwork object locationNetwork;
        set the attributes of locationNetwork using buildNewNIT method; (refer to Algorithm 6)
        for each intLocaitonID in locationNetwork.myLocations
                get locationdm from networkdm.myLocations using intLocaitonID;
                set intEchelonValue = the locationdm.myLevel;
                if (!intEchelonValue == 0) then
                        create a SKUDM object skudm;
                        set skudm.myLocation = locationdm;
                        set skudm.myItemType = itemtypedmElement;
                        generate a double value dblHoldingCharge by D_HC;
```

```
                    set skudm.myHoldingCharge = dblHoldingCharge;
                    generate a double value dblOrderingCost by $D_{OC}$;
                    set skudm.myOrderingCost = dblOrderingCost;
                    set skudm. myStockOutCost= itemtypedmElement.myLostSaleCost;
                    generate a double value dblLTMean using $D_{LTIHP}$;
                    set skudm. myLeadTimeMean= dblLTMean;
                    generate a double value dblLTVTMR Value using $D_{LTV}$;
                    set dblLTV= dblLTMean * dblLTVTMR;
                    set skudm. myLeadTimeVar= dblLTV;

                    add skudm to networkdm.mySKUs;
               end if
          end for each
          set itemtypedmElement.myLocationNetwork = locationNetwork;
end for each

for each skudmElement in networkdm.mySKUs
          set locationdm = skudmElement.myLocation;
          set intEchelonValue = locationdm.myLevel;
          if (intEchelonValue == NE)
               set skudmElement.myPolicy =" R_Q";
               set skudmElement.myReorderPt = 1;
               set skudmElement.myReorderQty = 1;
               set intLocationID = locationdm.myID ;
               set itemtypedm =skudmElement.myItemType;
               set intItemTypeID = itemtypedm.myID;
               generate a double value dblDM using $D_{DR}$;
               generate a double value dblDVTMR using $D_{DV}$;
               set dblDV= dblDM * dblDVTMR;

               create a DemandGeneratorDM object dgdm;
               set dgdm.myName = "DG for Item Type intItemTypeID Location
               intLocationID" ;
               set dgdm.mySupplier = locationdm;
               set dgdm.myItemType = itemtypedm;
               set dgdm. myAnnualDemandMean = dblDM;
               set dgdm. myAnnualDemandVar = dblDV;
               add dgdm to networkdm.myDemandGeneratorDMs;
          end if
end for each
```

Algorithm 2: generateNetwork() in LocationNetwork class

```
addLocation(0, 0) ;(refer to Algorithm 3)
set intCumulativeNum = 0;
set intCurrentLevel = 1;
define Set<Integer> locationSet;
```

while (*intCurrentLevel* <= *NE*)
    set *locationSet*= get the value of this.myLevelLocations using the key (*intCurrentLevel*-1) ;
    while (*locationSet* has a *intElement*)
        generate *intNumOfCustomer* by D_{NC};
        for *loopCounter* = 1 to *intNumOfCustomer*
            intCumulativeNum =*intCumulativeNum* +1;
            addLocation(*intCumulativeNum* , *intCurrentLevel*); (refer to Algorithm 3)
            add a map entry to this.myRelation using *intCumulativeNum* as key and *intElement* as value;
        end for
    end while
    intCurrentLevel=*intCurrentLevel*+1;
end while

Algorithm 3: addLocation(Integer *LocationID*, Integer *LevelID*) in LocationNetwork class

if (!this.myLevelLocations.containsKey(*LevelID*))
    create a LinkedHashSet<Integer> object *set1*;
    add a map entry to this.myLevelLocations using *LevelID* as key and *set1* as value;
end If
add *LocationID* to this.myLocations;
locationSet= get the value from this.myLevelLocations using *LevelID*;
add *LocationID* to *locationSet*;

Algorithm 4: create and add the LocationDM objects to *networkdm*.myLocations using *LN*

define Set<Integer> *locationSet*;
for *loopCounter* = 0 to *NE*
    set *locationSet*=*networkdm*.myLocationNetwork.myLevelLocations.get(*loopCounter*);
    for each *intElement* in *locationSet*
        if (*intElement* == 0) then
            create the LocationDM object *locationdmES*;
            set *locationdmES*.myID=*intElement*;
            set *locationdmES*.myName="External Supplier";
            set *locationdmES*. myLevel =*loopCounter*;
            add *locationdmES* to *networkdm*.myLocations using *intElement* as key and *locationdmES* as value;
        else
            create a LocationDM object *locationdmIHP*;
            set *locationdmIHP*.myID=*intElement*;
            set *locationdmIHP*.myName="Location *intElement*";
            set *locationdmIHP*. myLevel=*loopCounter*;
            add *locationdmIHP* to *networkdm*.myLocations using *intElement* as key and *locationdmIHP* as value;

end if
end for each
increase *loopCounter* by 1;
end for

Algorithm 5: create and add the ShipmentDM objects to *networkdm*.myShipments using *LN*

for each *mapEntry* in *networkdm.* myRelation.entrySet()
set *intCustomerID* = the key of the *mapEntry*;
set *intSupplierID* = the value of the *mapEntry*;
generate a double value *dblShippingTime* using D_{LTIHP};
generate a double value *dblLTVTMR* value using D_{LTV};
set *dblLTV*= *dblShippingTime* * *dblLTVTMR*;

create a ShipmentDM object *shipmentdm*;
set *shipmentdm*.mySupplyingLocation.myID= *intSupplierID*;
set *shipmentdm*.myCustomerLocation.myID= *intCustomerID*;
set *shipmentdm*.myShippingTimeMean= *dblShippingTime*;
set *shipmentdm*.myShippingTimeVar= *dblLTV*;
end for each

Algorithm 6: buildNewNIT method in LocationNetwork class

For each *itemtypedmElement* in *networkdm*.myItemTypes
create a new LocationNetwork object *locationNetwork*;
set *locationNetwork* =*networkdm*.myLocationNetwork;
set *intLocationIDSet*=get the value of *locationNetwork*.myLevelLocations using the key *NE*;
for each *intElement* in *intLocationIDSet*
generate a double value *dblProb*~Uniform(0,1);
if (*dblProb* <= PI) then
add *intElement* to *locationNetwork*.myItemLocationAtRetailers;
end if
end for each

for *loopCounter*= *NE* to 1
set *intLevelLocationsSet* = get the value of *locationNetwork.* myLevelLocations using the key *loopCounter*;
if (*loopCounter* == *NE*)
for each *intElement* in *intLevelLocationsSet*
if (*locationNetwork.* myItemLocationAtRetailers does not contain the element *intElement*)
delete the element *intElement* from *locationNetwork.* myLocations;
delete the element *intElement* from *intLevelLocationsSet*;
delete the entry from *locationNetwork*.myRelation using the key *intElement*;
end if

```
                end for each
            else
                for each intElement in intLevelLocationsSet
                    set intCustomerIDSet=get the value of
                    locationNetwork.myLevelLocations using the key
                    (loopCounter+1);
                    set existCustomer=False;
                    for each intElement2 in intCustomerIDSet
                        if ( exist an entry using intElement2 as the key and
                        intElement as value in locationNetwork.myRelation)
                            set existCustomer=True;
                        end if
                    end for each
                    if (existCustomer=False)
                        delete the element intElement from
                        locationNetwork.myLocations;
                        delete the element intElement from intLevelLocationsSet;
                        delete the entry from locationNetwork.myRelation using
                        the key intElement;
                    end if
                end for each
            end if
            decrease loopCounter by 1;
        end for
end for each
```

Appendix 4: Data Analysis for Inputs

This appendix shows the details about the data analysis processes. The purpose of data analysis is to generate data that represents real world inventory system. The data analysis serves two main goals: 1) determine the data range for the attributes, and 2) investigate the relationships between attributes.

The attributes studied in this appendix are as follows:

$C \equiv$ unit cost of an item

$b \equiv$ lost sales cost of an item

$\lambda \equiv$ demand rate at retail store

$\sigma_R \equiv$ standard deviation of demand at retail store R

$A \equiv$ ordering cost

$I \equiv$ inventory holding charge

$E(LT_{ES}) \equiv$ mean lead time at external supplier

$E(LT_{IHP}) \equiv$ mean lead time at an inventory holding point

$Var(LT_{ES}) \equiv$ variance of lead time at external supplier

$Var(LT_{IHP}) \equiv$ variance of lead time at an inventory holding point

Exhibit 33: Attributes List for the Study

The rest of this appendix discusses the quantification of these attributes.

The Range of Input Values

The inventory holding charge is calculated by dividing the inventory holding cost using the number of on hand inventory. The inventory holding charge consists of capital costs, inventory service costs, storage space costs, and inventory risk cost (REM Associates). REM Associates collect the estimate of carrying costs as a percentage of inventory value from 13 textbooks. According to the values from these textbook, the range of the inventory holding charge is from 12%-35%.

The ordering cost is the expense spent on placing an order. This cost includes the activities related to ordering process, such as making invoices, billing, arranging shipping, etc. P&G estimated that the cost of each invoicing is between $35 to $75 (Lee et al. 1997). In this dissertation, the shipping cost is also included as part of the ordering cost. In the scenario discussed in Section 1, the inventory system considered is an international business that sourcing the products from different countries; thus, the cost of international shipping should be considered. Considering shipping a 40 foot container from China to the US as an example, a total of $7,000 shipping cost may occur with the $4000 for ocean freight and $3000 for inland trucking. It should be noted that this international shipping cost only occurs from external supplier to the warehouses located at echelon 1. Thus, the ordering cost range for inventory holding points (IHPs) located at 1st echelon and other echelons should be different. Based on above observations, in this dissertation the range of ordering cost is determined as [3,000, 10,000] for IHPs located at 1st echelon, and [100, 3000] for other IHPs. It's assumed here that each order triggers a shipment, and the consolidation of different items in one shipment is not considered.

The research conducted by Deshpande et al. (2003) provides some insights about attribute values, such as unit cost, demand rate and lead time. The authors investigate the data from the U.S. military weapon system. They select a representative 21 weapon system containing 200,000 service parts and conduct a series of data analysis. Table 42 and Table 43 summarize some statistics that directly related to this dissertation. This inventory system has characteristics of low demand, high item cost and long production lead time. In Table 42, the production lead time is related to the lead time at external supplier. Also, the LRT in Table 43 corresponds to the lead time at an inventory holding point. LRT is the abbreviation for Logistics Response Time which means the lead time needed to fulfill a customer order.

Table 42: Part Attributes for Weapon System

	Mean	Std
Production Lead Time ($)	173	102
Demand Frequency (yearly)	16	86
Unit Price ($)	242.5	314

Table 43: Average Response Time (LRT) by Cost Categories

Unit Price ($)	Very Low(VL) [$0,$150]	Low (L) [$150,$500]	Medium (M) [$500,$1000]	High (H) [$10,000,$200,000]	Very High (VH) [$200,000+]
Average LRT (days)	26.3	29	33.7	54.8	97.4
No. of Parts	19.80%	14.70%	40.80%	22.70%	2%

The inventory system described in Section 1 has some differences with the system studied in Deshpande et al. (2003). The products sold in the inventory system described in Section 1 are categorized as Consumer Packaged Goods (CPG) which has characteristics of high demand, low cost and short production lead time. In order to get some insights in the CPG, data from Tmall.com was collected, the largest B2C (business-to-consumer) online retail platform in Asia.

The data collection from Tmall.com serves the following two purposes: 1) investigate the values range for unit cost, annual demand and shipping cost; and 2) investigate the direct or inverse proportional relationship between unit cost, annual demand and shipping cost. Sixty observations that covering 15 major categories and 60 sub-categories were collected. Each observation is the top seller in that sub-category. Data for three attributes are collected: 1) the unit price in Chinese Yuan (CNY), 2) monthly demand in units, and 3) shipping cost in CNY.

The collected data is shown in Table 44. It should be noted that: 1) the estimated annual sales for each item is calculated by the monthly sales times twelve, and 2) in the shipping cost column, the asterisk before some values means that the shipping cost is not available for that category, and the values are found from similar items.

Table 44: Data from Tmall

	Category	Sub-Category	Price (CNY)	Estimated Annual Sales	Shipping Cost (CNY)
1	Appliances	Air Conditioners	3499	17,580	*180
		Humidifiers	89	486,648	20
		Laundry	1299	21,924	*380
		Microwave Ovens	699	22,392	300
		Refrigerators	1399	46,596	*100
		Vacuums	2035	101,148	200
2	Automotive	Interior Accessories	26	139,932	20
		Tools & Equipment	248	90,312	500
		Wheels & Tires	460	2,628	500
3	Baby	Baby Food	182.4	80,748	37
		Baby Toys	1.5	1,884,336	15
4	Beaty	Bath	129	214,536	30
		fragrance	19.9	129,840	20
		Hair Care	68	209,724	15
		Makeup	9.98	337,140	5
		skin care	9.9	986,352	8
5	Books	Biographies	46.8	49,452	30
		Children's Books	69	118,500	15
		Education & Reference	36.8	134,472	15
6	clothing	men's clothing	19.8	947,268	20
		women's clothing	23.98	1,304,376	30
		underwear	6.5	1,193,004	20
		accessories	1.68	2,292,780	18
7	Electronics	Camera	559	27,552	50
		Cell Phone	75	166,368	15
		Desktops	3940	2,700	300
		Kitchen	65	167,328	20
		Laptops	5146.5	13,944	50
		Office Electronics	58	66,456	20
8	Health & Personal Care	House Supplies	4.9	276,000	*15
		Personal Care	29.9	1,079,424	*20
		Medical Supplies	299	120,360	*20
		Nutrition	11	625,596	*15
9	Home	Bedding	889	106,680	20
		Cleaning Supplies	65	323,028	20
		Decorating	226	33,660	20
		Furniture	1698	14,772	*320
		Storage	329	31,296	*120
10	Jewelry	Bracelets	46.8	93,984	20
		Earrings	218	7,788	35
		Necklaces	179	59,628	20
		Rings	65	37,944	20
		watch	399	254,940	20
11	Luggage & Bag	Backpacks	320	66,012	30
		Briefcases	198	293,520	20
		Luggage	239	311,652	50
		wallet	69	236,352	20
12	Office Products	office furniture	299	38,016	*175
		office supplies	4.7	90,540	15
13	shoe	men's shoe	98.82	212,232	60
		women's shoe	138	108,792	20
14	Software	Accounting	480	2,316	*20
		Business & Office	328	2,688	*20
		Education & Reference	13	368,196	*15
15	Sports & Outdoors	Bikes	1399	16,668	*100
		Water Sports	59	642,864	22
		Fitness	228.58	39,816	150
		Golf	29	26,436	15
		Fishing	129.92	24,876	60
		Clothing	59	93,924	20

A summary of the major statistics for Tmall data is listed in Table 45. The unit price and the shipping cost are converted from CNY to USD (US dollar) based on the exchange rate of 6.2:1.

Table 45: The Main Statistics for Tmall Data

	Min	20th percentile	80th percentile	Max	Mean	Std
Unit Price ($)	0.24	3.87	74.19	830.08	77.35	157.51
Annual Demand	2,316	24,876	323,028	2,292,780	281,600	457,710
Shipping Cost($)	0.81	2.42	16.13	80.65	11.91	18.69

Even though the research conducted by Deshpande et al. (2003) and the data collected from Tmall.com provide the insights into the mean demand values, the variance of an item demand is not available. The study of demand variance can be found in Lee et al. (1997) and Metters (1997). Lee et al. (1997) investigate the product data from two specific corporate examples and conclude that the ranges of demand variance-to-mean ratio are [0.23, 4.7] and [0.49, 3.37] respectively. Based on this observation, Metters (1997) conducts an experimental design to test the bullwhip effect using the variance-to-mean ratios as 0.5, 2 and 4. Based on the information from aforementioned two articles, in this dissertation, the range of demand variance-to-mean ratio is considered as [0.1, 4].

The lost-sale penalty cost occurs when the demand is unsatisfied. In the experimental design conducted by Metters (1997), the lost-sale penalty cost is set to be 0, 50% or 100% of the production cost. Based on these values, the range of the lost-sale-cost-to-unit-cost ratio is set to be [0.1,1] in this dissertation.

Ehrhardt (1984) and Chaharsooghi and Heydari (2010) study the impact of lead time on SC performances. Both of these two studies investigate the effects of variance of lead time on the supply chain performance such as inventory holding cost. In the experiments, Ehrhardt (1984) vary the lead time variance-to-mean ratio from 0 to 100%, and Chaharsooghi and Heydari (2010) select the lead time variance-to-mean ratio from 1.7% to 200%. Based on these two studies, the range of lead time variance-to-mean ratio is selected as [0.01, 2] in this dissertation.

Based on the aforementioned studies, the range of input values are summarized in Table 7.

The Relationship between Attributes

The purpose in this part is to test the relationships between three attributes (unit price, mean annual demand and shipping cost) collected from Tmall (Table 44), especially to test the 1st and 3rd assumptions mentioned in Exhibit 23. The simple linear regression considering a single repressor *x* and response variable *Y* is used to analyze the two assumptions.

Assumption 1: Average annual demand of an item is inversely proportional to its unit cost.

Selecting the Price as *x* and the Estimated Annual Sales as *Y*, the regression analysis from Minitab is shown in Exhibit 34. From the ANOVA table, it can be seen that the regression model is significant at p=0.05. The regression coefficient is -119, which means that the annual demand is inversely proportional to price.

Regression Analysis: Annual Demand versus Price

```
The regression equation is
Annual Demand = 338856 - 119 Price


Predictor     Coef  SE Coef      T      P
Constant    338856    64311   5.27  0.000
Price      -119.39    59.51  -2.01  0.050


S = 446412   R-Sq = 6.5%   R-Sq(adj) = 4.9%


Analysis of Variance
```

Source	DF	SS	MS	F	P
Regression	1	8.02006E+11	8.02006E+11	4.02	0.050
Residual Error	58	1.15584E+13	1.99283E+11		
Total	59	1.23604E+13			

Exhibit 34: Regression Analysis: Annual Demand versus Price

Assumption 3: The ordering cost is directly proportional to its unit cost.

Selecting the Price as *x* and the Shipping cost as *Y*, the regression analysis from Minitab is shown in Exhibit 35. From the ANOVA table, it can be noted that the regression is significant at p=0.003. The regression coefficient is 0.0448, which means that the shipping cost is directly proportional to price.

Regression Analysis: Shipping Cost versus Price

```
The regression equation is
Shipping Cost = 52.3 + 0.0448 Price
```

Predictor	Coef	SE Coef	T	P
Constant	52.33	15.59	3.36	0.001
Price	0.04484	0.01443	3.11	0.003

S = 108.226   R-Sq = 14.3%   R-Sq(adj) = 12.8%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	113136	113136	9.66	0.003
Residual Error	58	679348	11713		
Total	59	792484			

Exhibit 35: Regression Analysis: Shipping Cost Versus Price

Regression Analysis between Unit Cost and Lead Time at External Supplier, Ordering Cost, and Demand

a) Regression Analysis: LTatESmean versus unit cost

```
The regression equation is
LTatESmean = 68.6 + 0.000349 unit cost

Predictor         Coef     SE Coef      T      P
Constant        68.630       1.792  38.30  0.000
unit cost   0.00034919  0.00004570   7.64  0.000

S = 54.3526   R-Sq = 5.5%   R-Sq(adj) = 5.4%

Analysis of Variance
Source           DF       SS      MS      F      P
Regression        1   172496  172496  58.39  0.000
Residual Error  998  2948295    2954
Total           999  3120791
```

b) Regression Analysis: orderingCost versus unit cost

```
The regression equation is
orderingCost = 2126 + 0.0112 unit cost

Predictor       Coef   SE Coef      T      P
Constant     2126.41     86.45  24.60  0.000
unit cost   0.011224  0.002205   5.09  0.000

S = 2622.45   R-Sq = 2.5%   R-Sq(adj) = 2.4%

Analysis of Variance
Source           DF          SS          MS      F      P
Regression        1   178223993   178223993  25.91  0.000
Residual Error  998  6863511878     6877266
Total           999  7041735871
```

c) Regression Analysis: demandMean versus unit cost

```
The regression equation is
demandMean = 330114 - 0.164 unit cost

Predictor     Coef  SE Coef      T      P
Constant    330114    18588  17.76  0.000
```

unit cost -0.1643   0.4741 -0.35 0.729

S = 563857   R-Sq = 0.0%   R-Sq(adj) = 0.0%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	38181514795	38181514795	0.12	0.729
Residual Error	998	3.17299E+14	3.17935E+11		
Total	999	3.17337E+14			

Appendix 5: The Experimental Data for ABC Classification

This appendix illustrates the organization of the experimental results and the calculation of the across scenario means. The experimental data for ABC Classification is introduced as an example in this appendix, and the experimental results for NIT classification and K-Means clustering follow the same organization. After implementing the ABC classification, the performances of the ABC classification are listed in Table 46. The levels for the experiment parameters (factor A to L are introduced in Table 21) are recorded in the columns from A to L. The low level is represented using "-1" and the high level is represented as "1". In addition, the column "NG" records the levels for number of groups and the column "S1" records the levels for classification criteria. The last three columns record the performance measures, i.e., percent increase of clustering penalty cost (%CPC), SSE and grouping time (GT).

A total of 1024 scenarios are used to compare the classification criteria NIC and NADU.

Table 46: The Performance of ABC Classification

No.	A	B	C	D	E	F	G	H	J	K	L	NG	S1	%CPC	SSE	GT
1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	19.5	1.26E+09	0.169
2	-1	-1	-1	-1	-1	-1	-1	1	-1	-1	1	-1	-1	16.6	2.44E+09	0.089
3	-1	-1	-1	-1	1	1	-1	-1	-1	1	1	-1	-1	15.2	2.24E+10	0.228
4	-1	1	1	1	1	1	-1	-1	1	1	1	-1	-1	16.2	2.90E+10	0.132
5	-1	1	1	1	1	1	-1	1	-1	-1	-1	-1	-1	19.2	6.83E+09	0.033
…	…	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	17.5	1.23E+09	0.024
1023	-1	1	1	1	1	1	1	1	1	1	1	-1	-1	15.7	2.91E+10	0.123
1024	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	-1	-1	15.5	2.39E+09	0.042

The across scenario mean for a response variable is the average of the values of all scenarios. For example, the across scenario mean for grouping time is shown as following:

$$GT_{across-scenario} = \sum_{i=1}^{1024} GT_i$$

The across scenario mean for %CPC and SSE follow the same formulation.