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Inventory Management and Control "For a Cause"

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Inventory Management and Control "For a Cause"

Inventory Management and Control "For a Cause"

A thesis submitted in partial fulfillment
of the requirements for the degree of
Master of Science in Industrial Engineering

by

John Miller
University of Arkansas
Bachelor of Science in Industrial Engineering, 2009

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This thesis is approved for recommendation to the Graduate Council.

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Dean John R. English
Thesis Director

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Dr. Ed Pohl   Dr. Kelly Sullivan
Committee Member   Committee Member

Abstract

Companies today are changing the culture of business and beginning to involve more than just labor and money in overall decisions. Just as sustainability issues and humanitarian logistics are gaining popularity, so is the idea of using business to make a difference on society in addition to making a profit. As companies position themselves across the globe to make an impact, they employ people in third-world environments that create uncertainties on both the supply and demand sides. Also, the idea of strategically planning work with the goal of minimizing costs has been replaced by companies wanting to give more work and operate with "planned inefficiencies" so that they can guarantee workers that they will be able to earn an income and feed their family. The primary objective of this research is to develop an optimal or desirable inventory control policy for companies operating "for a cause." We investigate lot-sizing rules, material requirements planning (MRP) systems, and mathematical models to determine the inventory order quantity that will minimize costs while guaranteeing a steady amount of labor at a consistent interval. This study will help companies avoid cutting costs at the expense of sacrificing a person's livelihood.

Acknowledgments

I would to first of all thank Dean John R. English for every role he has played in my life, from an advisor to a mentor to a second father. Thank you for inspiring me and giving me the chance to excel and follow in your footsteps. Next, thank you to my wife of almost three years who is the best decision I have ever made next to following Jesus. I love you with all of my heart and am thankful for how you have endured my long days and nights of projects and studying. Thank you also to my parents who have always supported me and challenged me to do my best and have been by my side through the ups and the downs. Thank you Dr. Kelly Sullivan for the friendship we've built and your quick thinking and creative ideas that always helped me along in optimization. And finally, thank you to my Lord and Savior who has taught me that faith is more than a religious title, but it is a way of life. Thank you God for allowing me to see my need for Jesus and to become more like Him every day. May I hear the words one day, "Well done my good and faithful servant."

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Chapter 1:   Introduction

Inventory management and control is crucial for companies as they attempt to reduce costs and operate efficiently within an uncertain supply chain environment. The main objectives of inventory management are how much of a specific product to order and when to place that order (Yin, Yin, & Guo, 2008). Industry leaders and academics have approached this problem from many different angles while considering multiple environments, but the real world environment continues to change. Models were previously developed within static conditions as "normal" scenarios and baseline conditions in the original research, but models have expanded to multiple types of uncertainty. Much research has focused on determining the proper inventory policy and subsequent order quantities within production, manufacturing, and retail companies in worldwide supply chains.

Although significant research has been performed concerning supply chain management, production planning, inventory control, and even uncertainties and their effects on the entire process, there are more opportunities for research advances involving more realities in real-life systems. There is a need for an intensive study of the relationships between the different sources of uncertainty in the supply chain operation process (Li & Liu, 2013).

1.1 Research Motivation

Today, not all companies operate within the same environments and with the same goals. Companies are changing the culture of business and beginning to involve more than just labor and money in overall decisions. Just as sustainability issues and humanitarian logistics are gaining popularity, so is the idea of using business to make a difference on society in addition to making a profit. Certain companies are planning their locations, operations, business practices, and even products around the idea of changing lives by providing people jobs and ultimately

hope and a future. Today, business involves changing lives, one person at a time, and that leads to the motivation behind this research.

As companies position themselves across the globe to make an impact, they employ people in third-world environments that bring new uncertainties along with them. Researchers have seen that demand is not the only source of uncertainty, but supply is as well. This is seen when external suppliers cannot guarantee quantities or lead times because of cultural, societal, or even environmental issues. Also, the idea of strategically planning work with the goal of minimizing costs has been replaced, in some cases, by companies wanting to give more work and operate with "planned inefficiencies" so that they can guarantee workers that they will be able to earn an income and feed their family. Yes, the day has come when minimizing costs and maximizing profits must be balanced with ensuring work and payment to support the needs of the workforce and a global society. There are now more variables involved than setup costs, holding costs, and order quantities. Today, people's lives, futures, and hope have entered the equation.

1.2 Research Objectives

The primary objective of this research is to develop an optimal or desirable inventory control policy for companies operating "for a cause." We investigate lot-sizing rules, MRP systems, and mathematical models to determine the inventory order quantity that minimizes costs while guaranteeing a steady amount of labor at a consistent interval. We hope this study will help companies to not cut costs at the expense of sacrificing a person's livelihood.

Our research objective is achieved through exploring mathematical modeling and simulation. We present a reproducible and scalable method for other companies with the same goals to apply within their industry.

1.3 Task Plan

To accomplish this objective, the following tasks have been completed:

Task 1.   Investigate classic lot-sizing models and attempt to adapt to the "For Cause" domains as observed by the researcher

    1.1 Develop a generalized supply chain for "For Cause" domain found in third world countries

    1.2 Provide baseline process parameters for the supply and demand sides of the supply chain

    1.3 Develop a new "desirable" inventory policy/model

    1.4 Determine optimal inventory management and control parameters for the new model

Task 2.   Develop and validate a dynamic program for the generalized supply chain representing "For Cause" industries in third world countries

    2.1 Write the dynamic program for the generalized supply chain for the "For Cause" domain in third world countries

    2.2 Run the dynamic program for a short-term planning horizon and validate the results

    2.3 Apply uncertainty to the demand using the expected value of a stochastic process

    2.4 Run the dynamic program for a medium-term planning horizon and validate the results

    2.5 Run the dynamic program for a long-term planning horizon and validate the results

Task 3.   Perform experiments and analyze the effects and interactions

Task 4.   Research conclusions and possible extensions

Task 5.   Finish writing and editing thesis

1.4 Research Contributions

  This research presents a method of combatting uncertainties from both the supply and demand sides involving quantity and timing issues as commerce has moved into a global environment (and increased the amount of uncertainties involved in the process) with a focus of advancing "a cause." Gupta and Brennan's (1995) simulation study showed that lead time (supply timing) uncertainty creates a harsh environment for MRP. When making a decision about which lot-sizing rule to use, one needs to take into account the source and level of this uncertainty and also the product structure. We take into account the source and levels of uncertainties that apply within this environment in order to find the optimal approach to inventory management and control to be used within this business model.

Chapter 2:   Literature Review

There has been significant research in the areas of inventory control, lot-sizing

techniques, manufacturing resource planning, and uncertainties. This chapter reviews the current

research in these areas.

2.1   Inventory Control

A supply chain is a network system that generally consists of suppliers, manufacturers,

distributors, and retailers that perform all activities necessary to transform raw materials into

finished products in the consumers' hands (Li & Liu, 2013). Production planning is an important

component of the supply chain that relates to managing the productive resources required to

perform this transformation from raw materials to final products, and inventory control is an

important element of all companies' production systems. An inappropriate policy of inventory

control can lead to overages and/or shortages, which can both generate extra expenses, either in

increased inventory holding costs, setup costs, or transportation costs, or decreased capital assets.

"Inventory control in a supply chain is crucial for companies to satisfy their customer demands

as well as controlling costs" (Dolgui, Ben Ammar, Hnaien, & Louly, 2013). Companies actively

employ different techniques and methods to attempt to control their inventories within their

production systems while minimizing their overall costs.

2.2   Lot-Sizing Techniques

Traditional methods of inventory control use differing techniques to determine optimal

order quantities and reorder points, the most basic rule being the lot-for-lot (LFL). In the LFL

rule, the number of units scheduled for production each period is the same as the net

requirements for that period (Nahmias, 2009). However, it is often better to group orders

together using lot-sizing techniques because the LFL does not take into consideration economical

aspects and organizational constraints (Dolgui, Louly, & Prodhon, 2005). Some of the most

common lot-sizing techniques beyond the LFL are the Economic Order Quantity (EOQ), the

Periodic Order Quantity (POQ), and the Wagner-Whitin algorithm (WW).

2.2.1   Economic Order Quantity.

The Economic Order Quantity (EOQ) technique is a basic change from the LFL

involving a constant order quantity placed at differing order intervals in order to minimize cost,

rather than just ordering based upon the net requirements of the period. Once the order quantity

has been calculated, the reorder point will then change based upon demand and inventory levels.

The EOQ formula is based on four inputs: the average demand rate, λ; the holding cost, h; the

proportional order cost per unit, c; and the setup cost, K (Nahmias, 2009).

If the order cost per unit is c and the setup cost per order placed is K, then the total fixed

cost plus order cost per cycle is $C(Q) = K + cQ$, and dividing by T gives the order cost per unit

time. Also, since Q units are consumed each cycle at rate λ, $T = Q/\lambda$, or $\lambda = Q/T$. Therefore, the

average annual setup and purchase cost is $(K + cQ)/T$.

The average inventory level, considering no safety stock, is $Q/2$ because the inventory

level decreases linearly from Q to 0 each cycle, so the average annual holding cost is simply

$h(Q/2)$. Therefore, the average annual cost, here termed $G(Q)$, is given by

$$G(Q) = \frac{K+cQ}{T} + \frac{hQ}{2} = \frac{K+cQ}{Q/\lambda} + \frac{hQ}{2}$$

$$= \frac{K\lambda}{Q} + \lambda c + \frac{hQ}{2}$$

To find the Q that will minimize $G(Q)$, consider the shape of the curve of $G(Q)$.

$$G'(Q) = \frac{-K\lambda}{Q^2} + \frac{h}{2}$$

$$G''(Q) = \frac{2K\lambda}{Q^3} > 0 \text{ for } Q > 0$$

Since $G''(Q) > 0$, $G(Q)$ is a convex function, and therefore the optimal value of Q occurs where $G'(Q) = 0$.

$$G'(Q) = \frac{-K\lambda}{Q^2} + \frac{h}{2} = 0$$

$$Q^2 = \frac{2K\lambda}{h}$$

Therefore, the economic order quantity, Q^*, can be calculated as (Nahmias, 2009)

$$Q^* = \sqrt{\frac{2K\lambda}{h}} \,.$$

2.2.2 Periodic Order Quantity.

The Periodic Order Quantity (POQ), in contrast, seeks to find a constant order interval to place orders but with changing order quantities based on the period's requirements. This method chooses a fixed reorder point but will then order the amount needed for the coming order interval. The POQ formula simply takes the average number of periods that the EOQ accounts for and then divides the EOQ by the average demand per period, rounded to the nearest integer (Molinder & Olhager, 1998). The POQ can be calculated as (van den Heuvel & Wagelmans, 2005)

$$Q = \frac{EOQ}{\lambda} = \sqrt{\frac{2K}{\lambda h}} \,.$$

2.2.3 Wagner-Whitin algorithm.

The Wagner-Whitin (WW) algorithm is a procedure that determines the minimal order cost for a dynamic deterministic demand without capacity constraint (Dolgui et al., 2005). It considers the existing inventory at the beginning of a period carried over from the last period, the demand for the current period, and then the amount ordered for this period to ensure that the current demand will be met. The WW succeeds in generating an optimal solution to the single level lot-sizing problem (Wagner & Whitin, 1958), but it is time-consuming and therefore has

led to multiple heuristics (e.g. Silver-Meal heuristic) that are easier to solve and offer approximate solutions, all of which are seeking to minimize costs. Hu, Munson, and Silver (2004) compare a modified Silver-Meal heuristic with two other lot-sizing models under time-varying, but known, demand and found that when the time horizon for a current lot size is small, an exact WW procedure will probably give the best solution. Considering an additional input, p_t (marginal production cost in period t), for a T-period lot-sizing problem, the total cost, $c_{s,t}$, to satisfy the demand in periods $s,\ldots,t$ by production in period s with $1 \leq s \leq t \leq T$ (van den Heuvel & Wagelmans, 2005) can be found by

$$c_{s,t} = K_s + \sum_{i=s}^{t}(p_s\lambda_i + \sum_{j=s+1}^{i} h_{j-1}\lambda_i).$$

Furthermore, if $f(t)$ represents the minimal cost for t periods and we fix $f(0) = 0$, then one can find the optimal production plan recursively by (van den Heuvel & Wagelmans, 2005)

$$f(t) = \min_{i=1,\ldots,t}\{f(i-1) + c_{i,t}\}.$$

2.2.3.1 Silver-Meal heuristic.

The Silver-Meal (SM) heuristic is a simpler variation of the WW shown to yield easier and almost identically optimal results. Silver and Meal (1973) actually found this heuristic to yield carrying and replenishments costs just 0.4% higher than the actual WW algorithm, which is known to yield the minimal values of these costs. Using the above notation and also $T=1,2,3\ldots$ as the time duration the current replenishment quantity is to last, R and $G(t)$ as changing quantities within the algorithm, and $M = \frac{K}{p_t h}$, the algorithm (Silver & Meal, 1973) is adapted as follows:

$T \leftarrow 1$
$R \leftarrow \lambda(1)$
$G(1) \leftarrow M$
while $T^2 * \lambda(T + 1) \leq G(T)$
    $T \leftarrow T + 1$
    $R \leftarrow R + \lambda(T)$
    $G(T) \leftarrow G(T\text{-}1) + (T\text{-}1) \lambda(T)$
end while
$Q \leftarrow R$
return Q

*where Q is final value of R, and R is the cumulative demand through period T ($\sum_{t=1}^{T} \lambda(t)$).

2.2.4   Lot-sizing discussion.

De Bodt and Van Wassenhove (1983) surprisingly find that the EOQ technique was the

most effective when compared to several others that are all used within a material requirements

planning system even though the other lot-sizing techniques had been developed specifically for

that type of system. That observation indicates the opportunity for further research considering

these lot-sizing techniques in a material requirements planning environment involving

uncertainty rather than the static conditions under which they were originally tested and

developed. Wagner and Whitin (1958; 2004) agree that when demand is not steady state, even if

it is known but varies by period, the EOQ formula actually does not assure an overall minimum

cost. However, Gupta and Brennan (1995) use simulation to test multiple scenarios and find

EOQ to yield the best cost when uncertainty exists at all levels of the product structure, but they

also find a high correlation between lead time uncertainty and lot sizing rules, and therefore the

level of uncertainty present affects the lot sizing rule decision. Simpson (2001) proves via

simulation that all of the nine lot-sizing techniques considered perform similarly when demand

was positive in each planning period with low variability, but once variability is introduced

under rolling horizon conditions, WW gave the best results; however, another method, maximum

part period gain algorithm (MPG), produced near WW results with significantly fewer calculations even though it had been conspicuously absent in academic research.

A rolling horizon indicates a planning horizon in which the forecasted demand for a new period enters the forecast window every time a period ends and is dropped from the current window (Gupta & Brennan, 1994). Gupta and Brennan (1994) use simulation to test ten different lot-sizing rules considering a rolling horizon and allowing for back orders and find that the LFL performs the worst under uncertainty because it operates as a just-in-time philosophy. They found that EQS, a variant of EOQ allowing for back orders, performs the best when considering lead time uncertainty(Gupta & Brennan, 1994). Brennan and Gupta (1996) attribute this success to the built-in buffer quantities that the EQS and EOQ both contain that prove costly in the event of no uncertainty but then help significantly when uncertainty is present. They justify their use of simulation, saying that "simulation experiments allow researchers to consider variables common to the MRP system (e.g. lot-sizing rules) as well as those common to the manufacturing environment (e.g. product structure) simultaneously, thus providing relatively quick insights into MRP performance" (Gupta & Brennan, 1994).

2.3   Material Requirements Planning

Material requirements planning (MRP) is another common approach used within production systems to manage inventory control. MRP is a typical push-based inventory system in which central decisions determine the flow of materials and "push" them throughout the system rather than demand "pulling" them from level to level. "In MRP, appropriate production amounts for all levels of the production hierarchy are computed all at once based on forecasts of end-product demand and the relationship between components and end items" (Nahmias, 2009). MRP seeks to provide manufacturing, delivery, and purchasing schedules and ensure the

availability of components, materials, and products while also maintaining the lowest possible inventory (Grasso & Taylor III, 1984). Therefore, MRP pursues inventory control-related objectives well with accurate forecasts of supply and demand quantities, but uncertainty in either area can cause instability for the production system as it attempts to determine the flow of components from lower levels all the way to the end item. Further disruption comes from timing uncertainty on either the supply or demand side because planned lead times are very important to effective operation of an MRP system (Melnyk & Piper, 1985). Melnyk and Piper also found that lot-sizing can potentially be a major source of lead-time error within the system.

Many researchers attempt to evaluate the differences and possible techniques to approach these uncertainties. "Dolgui and Prodhon (2007) have focused on the development of MRP software for an uncertain environment and have shown that various techniques such as safety stock, safety lead time, and lot-sizing rules can be used to control the supply variability in order to lead the better anticipation of uncertainties" (Dolgui et al., 2013). Shorter lead times generally accompany reduced lead-time variability (Molinder & Olhager, 1998), and therefore greater variability could naturally lead to an increased lead time to account for this uncertainty. Also, the higher the level of the significant uncertainties, the higher the level of parts and finished products that are delivered late, proving that companies that use MRP must diagnose the major uncertainties so that buffer or slack can be used most effectively (Koh & Saad, 2003).   Most either approach only uncertainties related to supply or demand, or compare these different variations, but research on the combined effect of both types of uncertainty is lacking. According to Murthy and Ma (1991), no paper considers MRP with all of the potential types of uncertainties in an integrated fashion, and few papers actually deal with real case studies and actual application of MRP under uncertainty in the real world. Concerning safety stock,

unresolved issues such as what level to hold this inventory buffer (finished product, component level one, etc.) alarms the authors (Murthy & Ma, 1991). Nevertheless, forecasting, lot-sizing, and safety stock policies are of great importance for any MRP system that functions within an uncertain environment (De Bodt & van Wassenhove, 1983). "Very few works have been conducted within the MRP theory framework during the study period. Most of the reviewed papers that deal with parts and raw materials planning correspond to multilevel lot-sizing modeling approaches" (Diaz-Madronero, Mula, & Peidro, 2014). Diaz-Madronero et al. (2014) show that most studies have modeled uncertain demand with stochastic models but that this may not be the best approach because of unreliable statistical data. They suggest that fuzzy mathematical programming may be a suitable alternative modeling approach. These authors also propose that no current research considers the impact that procurement transport may have on accomplishing production plans, specifically involving near and offshore suppliers. This "can confirm the need for optimization models and tools for the production and procurement transport planning processes which contemplate different forms of long- and short-distance transport (railway, air, full truck load, grouping, milk round, routes, etc.) and different characteristics (legal or environmental restrictions)" (Diaz-Madronero et al., 2014).

2.4   Uncertainties

The two biggest types of uncertainty are quantity uncertainty and timing uncertainty, and they can both approach from either the supply side or the demand side. According to Gupta and Brennan (1995), "uncertainty is due to two major factors, demand and supply." Researchers have considered various options and techniques to study these uncertainties and their effects on the supply chain.

2.4.1   Supply uncertainty.

Supply uncertainty generally stems from issues within the production system or quantity and/or timing issues related to suppliers. Grasso and Taylor examined the impact of supply uncertainty on the performance of an MRP system due to timing issues, specifically variability in the lead time of purchased parts (1984). They found that managing the lead time does reduce variability but that safety stock has a much greater impact when shielding against supply/timing uncertainty. Buzacott and Shanthikumar (1994) furthered this research and concluded that the decision between safety lead time and safety stock is dependent upon the forecast of future required shipments: if the forecast is good, then safety lead time is preferable, but if the timing of forecasts is not known and only the mean demand can be predicted, then either safety stock or safety lead time will be effective. They decided that the accuracy of the master schedule determines the value of using MRP. Wang, Liu, and Ding (2007) then focused on the influence of the fluctuation of delivery times on the systems' upstream production planning and showed that when delivery time in the current period increases faster than in the last period (more fluctuation), the producer should schedule the production with an increasing amount of batch size.

2.4.2   Demand uncertainty.

Many models have been proposed considering only demand uncertainty because of the numerous applications within industry. However, "no analytical solutions to safety stock are given when the stochastic demand distribution is unknown" (Zhang, Xu, & Zhang, 2013). Zhang et al. (2013) study a distribution-free stochastic inventory control problem by assuming a non-stationary stochastic process whose mean and variance are not constant. Using a reinforcement technique (the action-valued method) and a BP (backpropagation) neural network, Zhang et al.

(2013) design the control parameters of the model to adaptively change automatically with demand fluctuation. The authors find that adding a correction value to the safety lead time yields a better performance when the demand follows either a known distribution or an unknown one. Danne and Dangelmaier (2009) propose that the use of a planning buffer (safety lead time) at each stage of production increases safety stocks on the considered production or subsequent stages, but they found that it could lead to an overall minimum cost when applied to a scenario considering fast-moving consumer goods. With their results, they showed that the determined planning buffer does not have to rely only upon experience and intuition but can be supported by a quantitative model.

Kazan, Nagi, and Rump (2000) consider the effect of uncertain demand on production schedules and production quantities. The authors compare the results of the Wagner Whitin algorithm, the Silver-Meal heuristic, and also a new mixed integer linear program (MILP) over different rolling planning horizons rather than just a single period. They find their MILP to be more successful than the other methods because it focuses more on certain components of the problem and therefore is the preferable tool to generate new production schedules. Jeunet and Jonard (2000) use simulation to study nine different lot-sizing techniques under uncertain demand. Their goal is twofold: to find the most cost-effective, but also to find the most robust (they defined robustness as the stability of the technique during demand fluctuations). They find a negative correlation between cost-effectiveness and robustness and conclude that "a trade-off exists between static optimality and a somewhat more dynamic efficiency that takes into account the cost of changing delivery dates" (Jeunet & Jonard, 2000). Johansen (1999) finds that Deterministic Dynamic Programming (DDP) gives the best lot size if demand uncertainty is low, but as demand uncertainty increases, a (*s, S*) policy, determined by Stochastic Dynamic

Programming (SDP), gives better sizes. Pujawan and Silver (2008) test certain heuristics (using criterion from the Silver-Meal heuristic) against a (*s, S*) policy and found the heuristics to perform better, specifically for greater levels of demand uncertainty; they recommend future research to include a non-zero replenishment lead time and non-normal demand variability.

2.4.3 Supply or demand uncertainty.

Whybark and Williams (1976) study the effects of both supply and demand uncertainties on a MRP system. They compare two techniques, safety stock and safety lead time, considering timing and quantity uncertainty in both demand and supply individually, and from these four situations, they conclude that "under conditions of uncertainty in timing, safety lead time is the preferred technique, while safety stock is preferred under conditions of quantity uncertainty" (Whybark & Williams, 1976). Wacker (1985) then studies each uncertainty and proposes that proper statistical estimates of safety stock are the best remedy for all uncertainty. They also suggest that much more research needed to be done in the area. The main difficulty of lead time uncertainties generally stems from the inter-dependence of components inventories, and Dolgui, Louly, and Prodhon (2005) attempt to analyze lead time uncertainties and random demand separately but suggest that much more research needs to be done considering both simultaneously. This is of considerable interest in the industrial sector, but they claim that it is the most complex problem. It appears that current research continues to alternate between safety stock and safety lead time as the best solution to these uncertainties with considerable focus also given to improvements in lot-sizing rules.

2.4.4 Supply and demand uncertainty.

MRP has proven effective for manufacturing planning and control and can be used within a materials planning model, and some have attempted to combine and test both supply and

demand uncertainties in relation to an MRP system. One such model that applied this to the

industry of refrigerating drinks revealed that "safety stock can provide fewer shortages with a

smaller inventory investment; therefore, the results show a distinct preference for using only

safety stock" (Alves, Machado, & Cruz Machado, 2004). The writers also suggested that the

application of this model and these findings to other industries requires the use of materials that

have high consumption rates along with high demand variability. The major conclusion of their

work is "safety stock is more robust in coping with changes in production plans over the lead

time plus order interval." Molinder (1997) performed a simulation study on an MRP system

influenced by both stochastic demand and stochastic lead times and studied the amount of lead

time variability, the amount of demand variability, and the influence of stock-out cost/inventory

holding cost ratio. Simulation is often the chosen method for difficult analytical problems, and

via simulation Molinder (1997) found that for cases with a high demand variability and low lead

time variability, the lowest cost is obtained by using safety stocks, but for cases with a high

variability in both demand and lead time, utilizing safety lead times achieves the lowest cost

(1997). He also found that the safety stock principle is always best for a low stockout/inventory

holding cost ratio.

Yin, Yin, and Guo (2008) investigate the methods and techniques for exploring the joint

effect of stochastic demand and lead times on inventory levels and present a heuristic Dynamic

Response Particle Swarm Optimization (DRPSO) for the Optimal Inventory Control Policy

(OICP). The authors formulate this as a dynamic nonlinear optimization problem considering

both demand and lead time uncertainties applied to order point and order quantity. They

conclude that the optimal inventory policy with uncertain demand and uncertain lead time is

computed easily using the improved DRPSO algorithm. They also find that as demand variance

significantly increases as the lead time increases, the proposed technique becomes more critical.

Brennan and Gupta (1996) again used simulation to test multiple lot-sizing rules and algorithms

but now under both lead time and demand uncertainty and found that a reduction in lead time

uncertainty should be of significance because the degree of lead time uncertainty dominates

demand uncertainty.

Fuzzy logic has also been introduced to model the customer's demand variability and the

unreliability of external suppliers, and the results showed that the model is well sensitive to

changes in parameters (Mahnam, Yadollahpour, Famil-Dardashti, & Hejazi, 2009). Li and Liu

(2013) focused on several uncertainties: demand, production process, supply chain structure,

inventory policy implementation, and vendor order placement lead time delays. They found that

the larger the vendor order placement lead time delay is, the larger the inventory deviation

amplitude is and the longer the bullwhip phenomenon exists. They suggested that upstream

activities can respond to downstream inaccuracies by extending their reaction times with certain

delays and also that supply chain managers can use their findings to find the weak links in their

supply chains, design a better inventory control strategy, and improve their supply chain's

performance. They propose that more research should be done, such as a detailed study of the

relationship between different uncertainty sources in the supply chain operation process.

Thangham and Uthayakumar (2009) also propose that fuzzy numbers could bring more realistic

results because of the unreliability of data dealing with uncertainties in the supply chain. They

use a "possibilistic" decision model to determine the supply chain configuration and optimal

policies and then converted the problem into a multi-objective optimization problem.

Guide and Srivastava (2000) perform an intensive literature review considering the

different types of uncertainty and even the combination of supply and demand quantity and

timing uncertainties, and their conclusion was that the previous work had not included real data.

Their recommendation for future research was to focus on models that actually use the real

environment. Dolgui and Prodhon (2007) agree and say that simultaneously taking into account

the uncertainties will be very practical to industry because it will give a more realistic

assessment.

2.5   Aggregate Production Planning

Aggregate production planning (APP) can be defined as medium-term capacity planning,

generally over a 3-18 month planning horizon, determining optimal production, workforce, and

inventory levels for each time period within the planning horizon to meet aggregate demands at

the product type level (Fung, Tang, & Wang, 2003; Mirzapour Al-e-hashem, Malekly, &

Aryanezhad, 2011; Tang, Wang, & Fung, 2000). Wang and Liang (2004) simply define it as

"matching capacity to demand of forecasted, varying customer orders over the medium term."

Master production schedule, capacity plan, and material requirements planning all depend on

APP in a hierarchical manner (Baykasoglu & Gocken, 2010). Figure 1 is an adaptation of Lee's

(1990) depiction of this hierarchical flow:

Figure 1: Simplified Production Planning and Inventory Control Hierarchy
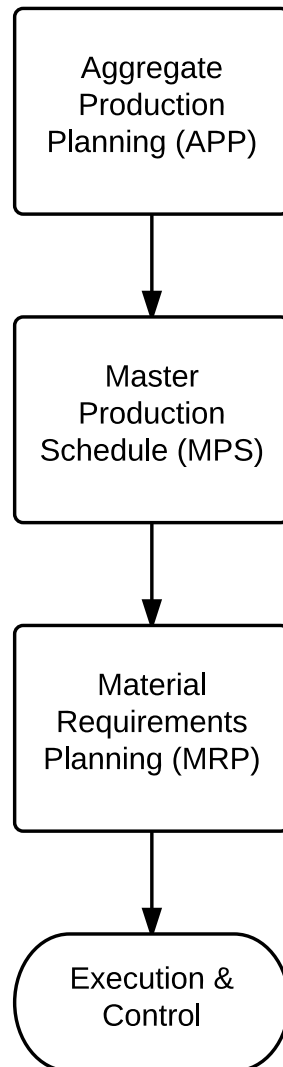
This planning is usually applied to one type of product or a group of very similar products so that the aggregate approach is warranted. However, the supply chain's robustness determines how well this planning performs within an uncertain environment (Mirzapour Al-e-hashem et al., 2011). Sahinidis and Mirzapour Al-e-hashem et al. (2004; 2011) classify the research that considers uncertainty into four categories:

(1) stochastic programming approach – some parameters are modeled as random variables with known probability distributions; (2) fuzzy programming approach – considers some variables as fuzzy numbers; (3) stochastic dynamic programming approach – includes applications of random variables in dynamic programming in all areas of multi-stage decision making; (4) robust optimization approach – includes uncertainty by setting up different scenarios that demonstrate realizations of uncertain parameters.

Holt, Modigliani, and Simon (1955) propose the HMMS rule and model whose linear decision rules try to find the optimum production rate and labor levels that minimize overall costs. The HMMS is one of the best-known classical models within aggregate production and inventory planning (Fung et al., 2003). Mirzapour Al-e-hashem et al. (2011) develop a new robust multi-objective aggregate production planning (RMAPP) model which first models the problem as a multi-objective mixed integer nonlinear program, then transforms it into a linear one reformulated as a robust multi-objective linear program, and finally solves it as a single-objective problem using the LP-metrics method.

Wang and Liang (2005) and Saad (1982) catalog all deterministic decision models considering APP into six categories: (1) linear programming (LP); (2) linear decision rule; (3) transportation method; (4) management coefficient approach; (5) search decision rule; (6) simulation. However, real-world problems generally involve uncertainties, especially when considering costs, demands, and capacities. Zimmerman (1976) first introduces fuzzy set theory into traditional LP problems. Fung, Tang, and Wang (2003) then propose a fuzzy multi-product aggregate production planning (FMAPP) model to accommodate different scenarios and differing decision-maker preferences and attempt to model fuzzy demand and fuzzy capacity using triangular fuzzy numbers. Wang and Liang (Wang & Liang, 2005) also incorporate fuzzy

numbers and develop a possibilistic linear programming (PLP) approach for solving multi-product APP decision problems with uncertain demand, operating costs, and capacity while attempting to minimize total cost. Others try to use an algebraic targeting approach to locate an optimal production rate for aggregate planning in a supply chain (Foo, Ooi, Tan, & Tan, 2008). LP models are the most accepted among the optimization models and can be categorized as: (1) deterministic optimization models; (2) stochastic programming models; and (3) fuzzy optimization models (Fung et al., 2003; Tang et al., 2000).

Zimmerman (1983) shows the difference between probability theory and possibility theory and how heuristic algorithms are generally more efficient when using fuzzy sets and that the human element of the decision maker can often be modeled better by fuzzy set theory rather than traditional mathematics. Tang et al. (2000) also show that uncertainties within production capacities, production times, demands, etc. cannot be adequately modeled by frequency-based probability distribution and therefore fuzzy set theory is needed to model APP. Baykasoglu and Gocken (2010) agree that representing uncertain parameters with fuzzy numbers is more applicable to real world settings. Fung et al. (2003) use triangular fuzzy numbers and propose a fuzzy formulation and simulation approach to multi-product aggregate production planning (MAPP) problems under financial constraints. Baykasoglu and Gocken (2010) also use triangular fuzzy numbers to define the parameters within their multi-product fuzzy multi-objective APP problem. They (Baykasoglu & Gocken, 2010) apply Zimmerman's (1976) max-min approach that has been widely adopted to transform fuzzy linear programs into crisp equivalents. Lee (1990) investigates fuzzy APP problems for a single product type with a fuzzy objective, fuzzy workforce levels, and fuzzy demands in each period. Tang et al. (2000) use a fuzzy approach to determine production, inventory, and workforce levels for a product in each

time period with the goal of minimizing the total production and inventory costs or maximizing profit. Fuzziness is fundamental in the future demand of an MRP environment, and Lee, Kramer, and Hwang (1990) use part-period balancing with uncertainty by modeling fuzzy demand as triangular fuzzy numbers to determine lot sizes and arrive at associated membership functions. These authors (Lee, Kramer, & Hwang, 1991) also compare three different lot-sizing methods altered to include fuzzy set theory and suggest that part-period balancing may be the best choice. Baykasoglu (2001) formulates the APP problem as a pre-emptive goal-programming model and then adapts the multiple-objective tabu search algorithm as a solution mechanism for this formulation.

Vörös (1999) confirms that MRP has difficulties within certain industries, such as fashion/retail, because of unpredictability and the role it plays in the sequencing of production. He (Vörös, 1999) shows that products should be produced in the order of increasing unpredictability. Migliorelli and Swan (1988) find a bridge between MRP and aggregate planning; when expectations are beyond the MRP system's capability, they find solutions by supplementing it with aggregate planning. Lee (1990) agrees that fuzziness is intrinsic in the future demand within an MRP system.

Chapter 3: Model

We consider sample demand data from an actual company making textile products in a third-world country and employing impoverished people that were previously unemployed. We realize that the state of the uncertainty means that we must react once each uncertainty occurs, and therefore we model the optimization problem as a dynamic program so that the decision maker can make decisions and then respond and react according to what occurs each period. We use what Sahinidis (2004) and Mirzapour Al-e-hashem et al. (2011) consider a "Stochastic Dynamic Programming Approach" and apply a random variable within a dynamic program.

3.1   Assumptions

We consider the supply chain of companies operating in third-world environments for the purpose of creating jobs. We specifically study one company producing textile products that all use the same component fabric called "umber." This company produces all of its products, whether purses, bags, scarves, or numerous other items, using this same component material. Therefore we design a method for determining the production quantity or amount, a_p, that a company should begin producing of its main component material in time period one in order to minimize the total cost, v, of the operation while never allowing the production quantity to decrease.

We model this policy as a dynamic program that first determines the minimum total costs and production quantities of the last time period, T, and then recursively solves backward in time to find the minimum total costs and production quantities for the preceding time periods. Each total cost represents the cost from that time period forward, so the model solves the cost for just that current period and then uses the expected value of the total cost for the future periods to yield the total cost from that point on until the end of the planning horizon. Therefore, the

minimum total costs found in period one corresponding to the initial inventory level and the previous maximum amount produced in one time period represent the minimum possible total costs considering the entire planning horizon.

Most "for cause" companies generally operate and employ workers in third-world environments, and the company we specifically consider is a retail company producing textile products in one such environment. Therefore, there is uncertainty on both the supply and demand side, but for this problem we choose to approach the demand uncertainty and leave the supply uncertainty as a future extension.

3.1.1 Triangularly-distributed demand.

Much of the former research on uncertain demand suggests using the triangular distribution, and therefore we also assume that our demands are triangular random variables and model them using the triangular distribution. However, as the triangular distribution is continuous but demands are discrete, the model assumes discretized values.

The probability distribution function (PDF) of the triangular distribution, where a is the minimum, b represents the mode, and c is the maximum value within the possible range of values, is as follows:

$$f(x|a,b,c) = \begin{cases} \frac{2(x-a)}{(c-a)(b-a)}, a \leq x \leq b \\ \frac{2(c-x)}{(c-a)(c-b)}, b < x \leq c \\ 0, x < a, x > c \end{cases} \qquad (1)$$

3.1.2 Expected value.

Rather than simulating and using triangular random numbers to generate a different demand each repetition, we assume all of the values within each triangular distribution need to be considered and therefore use the probabilities of each triangular random number to find the minimum expected value of the total cost for each period. We assume that the actual demands are not known ahead of time but instead use demand forecasts to calculate these expected values systematically for each period and then allow the model to react to the observed demand in a manner that minimizes expected costs over the remainder of the time horizon.

The model uses the cumulative distribution function (CDF) of the triangular distribution to determine the expected value of the cost for each time period. After integration of the PDF, the CDF is given as:

$$F(x|a,b,c) = \begin{cases} 0, x < a \\ \frac{(x-a)^2}{(c-a)(b-a)}, a \leq x \leq b \\ 1 - \frac{(c-x)^2}{(c-a)(c-b)}, b < x \leq c \\ 1, x > c \end{cases} \tag{2}$$

However, the model assumes a discretized triangular distribution. Therefore, the CDF is computed from x_1 to x_2, where $\frac{x_2-x_1}{2}$ is an integer, for the range $b-5$ to $b+5$ for every demand (assuming a, b, and c are integers and $a = b-5$ and $c = b+5$). These probabilities are used within the model to determine the expected value of the cost for each time period using:

$$\sum_{x \in X} \left(p_x^t * (c_h * (i - x)^+ + c_l * (x - i)^+ + v(i, j, t + 1)) \right)$$

(3)

*all parameters and variables in (3) are listed in Table 1 and Table 2 below in 3.2; b^+ in (3) denotes the maximum value within $(0, b)$.

3.1.3   Static or increasing production quantity.

"For cause" companies want to minimize cost and maximize profit while not sacrificing the amount of jobs they provide to workers. The model thus assumes that the production amount can remain the same or increase, but it employs a constraint that does not allow the amount to decrease from the previous maximum. So once the production quantity for a time period increases from the previous period, the amount in every successive period must be greater than or equal to the new maximum. The model stores this quantity as y and then uses y as the minimum possible production quantity for the next time period. If the minimum cost in the next period results in a larger production quantity, y is then updated and becomes the new "previous maximum," or the minimum possible production quantity for the successive periods.

This allows our model to never decrease the production quantity from one period to the next, which translates to sustained, and potentially increased, labor. This assumption allows the model to operate in the "for cause" domain. Our model also solves recursively, so it must consider all possible values of y and offer minimum total costs for each since the value of y affects the production quantity possibilities. This allows our model to present options within each time period based upon different possibilities of y so that the decision maker can choose depending upon their current level of inventory and also their current value of y.

3.1.4   No back orders.

Due to the retail industry and fashion styles changing for every season, and also considering the small size of most "for cause" companies, we assume that any demand that the company does not meet becomes a lost sale and incurs a penalty considering the amount of potential revenue lost minus the production cost to produce the product. Because of the small size of these companies and changing styles within the retail industry, the model does not allow for any backordered items.

3.1.5   Low holding cost.

Low property prices in third-world environments translate to extremely low holding costs, so we consider a holding cost of $1 per unit of inventory per time period and keep this constant. This is the only inventory cost beyond production.

3.1.6   Component inventory.

We assume that the company produces all products using the same component material, so we use component inventory, not final product inventory, as our unit for demand, current inventory level, and production quantity. This allows for the aggregate planning approach and employs risk pooling to account for the overall demand of component material. Since all products use the same component material, rather than estimating the demand and uncertainty of every product individually, this approach allows all of the products to be aggregated, and the model only assumes one demand and one uncertainty per time period. Whenever one product's forecast is higher than the actual demand and another product's forecast is lower than it's actual demand, this approach allows for some of these to offset each other and therefore is more accurate and requires less safety stock to protect against uncertainty.

3.1.7 Period *t* production can serve period *t+1* demand.

The inventory that we are modeling is at the component level, so any quantity ordered this time period will not be completed until the end of the time period. Then when the actual demand of a product is known for the next month, this recently produced component material will now be ready to satisfy the demand of that next month. Any production quantity the model assumes in period *t* will not be able to satisfy product demand in period *t*. Therefore, only initial inventory entering period *t* can serve demand in period *t*, and component material produced in period *t* will be complete at the end of period *t* and able to satisfy demand in period *t+1*.

3.2   Parameters and Variables

The model contains six input parameters. The first parameter is the total forecasted sales for each time period, *t*, translated to demand of component material, d_t. The numbers vary seasonally because of the nature of the retail industry, and these numbers are modeled as triangular random variables so that uncertainty is inherent. The second parameter is the production cost per unit (feet, meters, etc.) of component material, c_p. This cost includes all labor and material costs necessary to produce one unit of the component material. The third input is the holding cost per unit per one time period, c_h, which allows for the overall holding cost to be calculated dependent on the number of time periods that a unit is held at the warehouse. The fourth parameter is the lost sales cost per unit of component material, c_l, which accounts for the amount of revenue that could be generated from one unit of component minus the cost required to produce that unit, or the profit that would have been accumulated from the sale of a product. Table 1 below lists the notation of the parameters:

Table 1: Input Parameters

d_t	Forecasted demand as units of component material per time period
c_p	Production cost per unit of component material
c_h	Holding cost per unit of component material per one time period
c_l	Lost sales cost per unit of component material
I	Maximum inventory (inventory capacity)
J	Maximum production (production capacity)
T	Number of time periods in the planning horizon
p^t_x	Probability of demand x in period t from triangular distribution

The model uses one decision variable and three state variables, or a total of four variables. The model helps the decision maker determine the amount of component material to produce each time period, j. The model also monitors the maximum value of production in previous periods at a state variable, y, so that it can help the decision maker determine the correct amount of production in order to minimize cost but also to not allow production to decrease. The other two state variables are the current inventory level, i, which is the cumulative total of initial inventory and production minus demand, and the time period, t, within the planning horizon.

Table 2: Variables

j	Production quantity (units)
y	Maximum production thus far (units)
i	Current inventory level (units)
t	Time period

The model reports the production quantity, j, in time period t that results in the minimum expected cost for time periods t, $t+1$, …, T, considering a level of inventory, i, and a maximum

previous production quantity, y, as the production amount decision $a_p(i, y, t)$. The minimum total cost associated with this production quantity is $v(i, y, t)$.

3.3   Building the Model

The model developed in this thesis uses the input parameters defined previously and determines the production amount yielding the minimum expected cost from time period t on for the rest of the planning horizon considering several constraints. First of all, as stated before, production is not allowed to decrease. The model continuously monitors the current maximum amount of component produced in any previous time period, y, and only allows the current period's production to be the greater than or equal to y. Also, inventory balance is maintained in that the inventory at the beginning of month t equals the amount of inventory entering the time period $t–1$ minus the demand in $t–1$ plus the production in $t–1$. This means that any production in time period t cannot be used to fill demand until time period $t+1$. Thus, $i_{t-1} - d_{t-1}$ is the remaining inventory in period $t–1$ after satisfying demand in $t–1$ (if $i_{t-1} - d_{t-1} < 0$, remaining inventory is 0). In this case, $i_{t-1} - d_{t-1} + j_{t-1}$ is the amount of inventory on hand in period $t–1$ after production, which is then carried into period t; however, we impose a maximum inventory level of I based upon our inventory capacity (the implication is that any inventory above this level is lost each time period):

$$
i_t = \begin{cases}
j_{t-1} & , \ i_{t-1} - d_{t-1} < 0 \\
i_{t-1} - d_{t-1} + j_{t-1} & , \ 0 \leq i_{t-1} - d_{t-1} \leq I - j_{t-1} \\
I & , \ i_{t-1} - d_{t-1} > I - j_{t-1}
\end{cases}
\tag{4}
$$

The model then calculates the cost for each time period t from 1 to T, each possible level of inventory i from 0 to I, and each possible maximum production amount thus far, y, from 0 to J

by finding the minimum cost of every possible level of production, j, from y to J using the cost equation (5). As it calculates all of these costs, the model chooses the minimum cost within each combination of time period t, inventory level i, and maximum production amount y. The model represents the holding amount of inventory (amount not being used to satisfy demand) in a time period as the maximum value of $i - x$ (inventory minus demand) and zero and the amount of lost sales (amount of demand not being satisfied because of an inventory shortage) as the maximum value of $x - i$ (demand minus inventory) and zero; v represents the minimum total cost of the time period specified. All other parameters and variables are listed in Table 1 and Table 2 above:

$$v(i, y, t) = \min_{y \le j \le J} \left(c_p * j + \sum_{x \in X} \left(p_x^t * (c_h * (i - x)^+ + c_l * (x - i)^+ + v(i, j, t + 1)) \right) \right)$$

(5)

*In (5) above, b^+ denotes max $\{0, b\}$.

The model calculates every combination using all levels of j between y and J and chooses that minimum to recommend the lowest total cost from time period t on considering that this production amount j will become the previous maximum amount y for period $t + 1$. It begins in time period T and recursively solves for all time periods backward and accumulates the total cost so that the decision in period t will account for the rest of the planning horizon. Figure 2 below shows the pseudocode used by the model:

Figure 2: Total Cost Algorithm in Pseudocode

```
function total_cost(cₚ, cₕ, cₗ, I, J, T, d[1],…, d[T])
    for t = T to 1 do
        for I = 0 to I do
            for y = 0 to J do
                v(i, y, t) ← M
                for j = y to J do
                    k ← 0
                    for x = dₜ – 5 to dₜ + 5 do
                        if (x - i) > 0 then
                            aₗ ← x - i
                        else
                            aₗ ← 0
                        end if
                        if (i - x) > 0 then
                            aₕ ← i – x
                        else
                            aₕ ← 0
                        end if
                        if aₕ + j ≤ I
                            k ← k + pₓᵗ * (aₗ * cₗ + aₕ * cₕ + v(aₕ + j, j, t + 1))
                        else if
                            k ← k + pₓᵗ * (aₗ * cₗ + aₕ * cₕ + v(I + 1, j, t + 1))
                        end if
                    end for
                    if (cₚ * (j – 1) + k) ≤ v(i, y, t)
                        v(i, y, t) ← cₚ * (j – 1) + k
                        aₚ(i, y, t) ← (j – 1)
                    end if
                end for
            end for
        end for
    end for
    return v(i, y, t), aₚ(i, y, t)
```

This algorithm solves for the minimum cost of production from time period t to the end of the planning horizon, $v(i, y, t)$, and also reports the associated production amount, $a_p(i, y, t)$. This gives the decision maker the ability to choose the production amount based upon initial inventory and initial maximum production amount thus far and guarantees the minimum for that production scenario.

3.4 Validation

3.4.1 Triangular distribution.

The model assumes that the demands follow a triangular distribution. This has become a common assumption for much research modeling uncertain demands, and this model applies the triangular distribution within the Stochastic Dynamic Programming Approach (Sahinidis, 2004; Mirzapour Al-e-hashem et al., 2011).

The probability distribution function (PDF) of the triangular distribution, where a is the minimum, b represents the mode, and c is the maximum value within the possible range of values, is as follows:

$$f(x|a,b,c) = \begin{cases} \frac{2(x-a)}{(c-a)(b-a)}, a \leq x \leq b \\ \frac{2(c-x)}{(c-a)(c-b)}, b < x \leq c \\ 0, x < a, x > c \end{cases} \tag{1}$$

The triangular distribution can be used to generate random numbers that represent the demands within each time period. The histogram below shows 100,000 sample random numbers generated using the triangular distribution where $a = 0$, $b = 5$, and $c = 10$:

Figure 3: Histogram of 100,000 Triangular Random Numbers
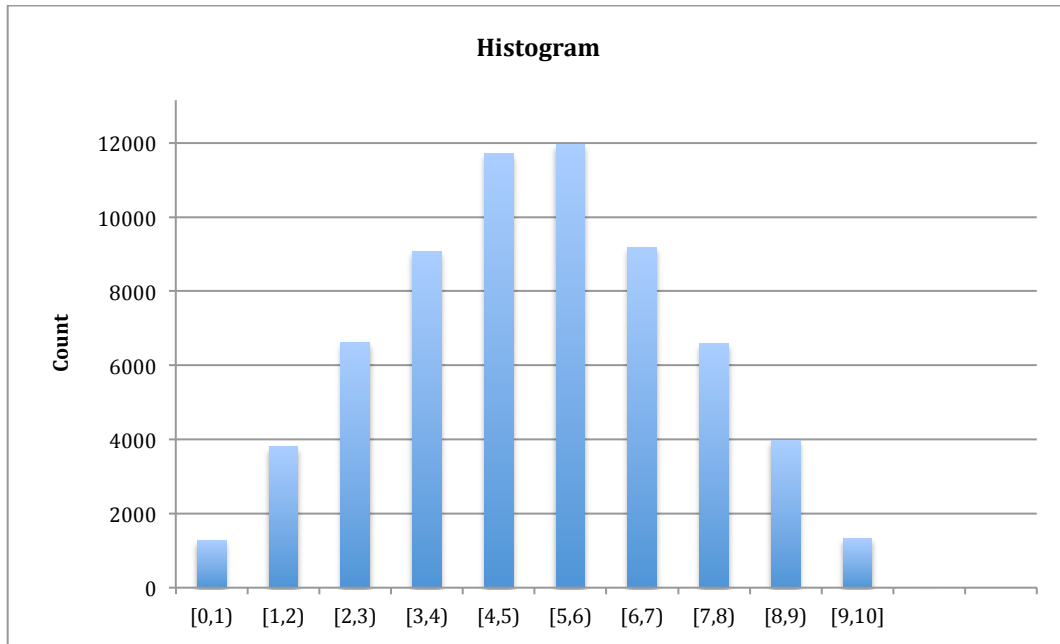
The model uses the cumulative distribution function (CDF) of the triangular distribution to determine the expected value of the cost for each time period. After integration of the PDF, the CDF is given as:

$$F(x|a,b,c) = \begin{cases} 0, x < a \\ \frac{(x-a)^2}{(c-a)(b-a)}, a \leq x \leq b \\ 1 - \frac{(c-x)^2}{(c-a)(c-b)}, b < x \leq c \\ 1, x > c \end{cases} \tag{2}$$

However, the model assumes a discretized triangular distribution. Therefore, the CDF is computed from x_1 to x_2, where $\frac{x_2-x_1}{2}$ is an integer, for the range $b-5$ to $b+5$ for every demand (this assumes $a = b - 5$ and $c = b + 5$). These probabilities are used within the model to determine the expected value of the cost for each time period using:

$$\sum_{x \in X} \left(p_x^t * (c_h * (i - x)^+ + c_l * (x - i)^+ + v(i, j, t + 1)) \right)$$

(3)

*all parameters and variables listed in Table 1 and Table 2 above

3.4.2 Model.

We code the model in MatLab and then make initial assumptions to test the validity of the model. First of all, the decision in period 1 is the most insightful and will yield the lowest total cost for the entire planning horizon, so we focus on the output from period 1. Also, the initial inventory appears to have more effect on production amounts and costs than the maximum previous production quantity, so we plot the changing production amounts and costs for one maximum previous production amount, y, and show the impact that an increasing initial inventory has on production and costs. We examine the costs for $t=1$ and $y=0$, or $v(i,0,1)$. Using the assumptions from above and deciding to alter the initial demand, d, for three different scenarios (d, $2 * d$, and $3 * d$), the model runs and reports the following results for the costs below in Figure 4 ("1,1,1" represents "1 * d, 1 * c_p, 1 * c_l," "2,1,1" represents "2 * d, 1 * c_p, 1 * c_l," and so forth):

Figure 4: Total Cost $v(i,0,1)$ at $y=0$, $t=1$
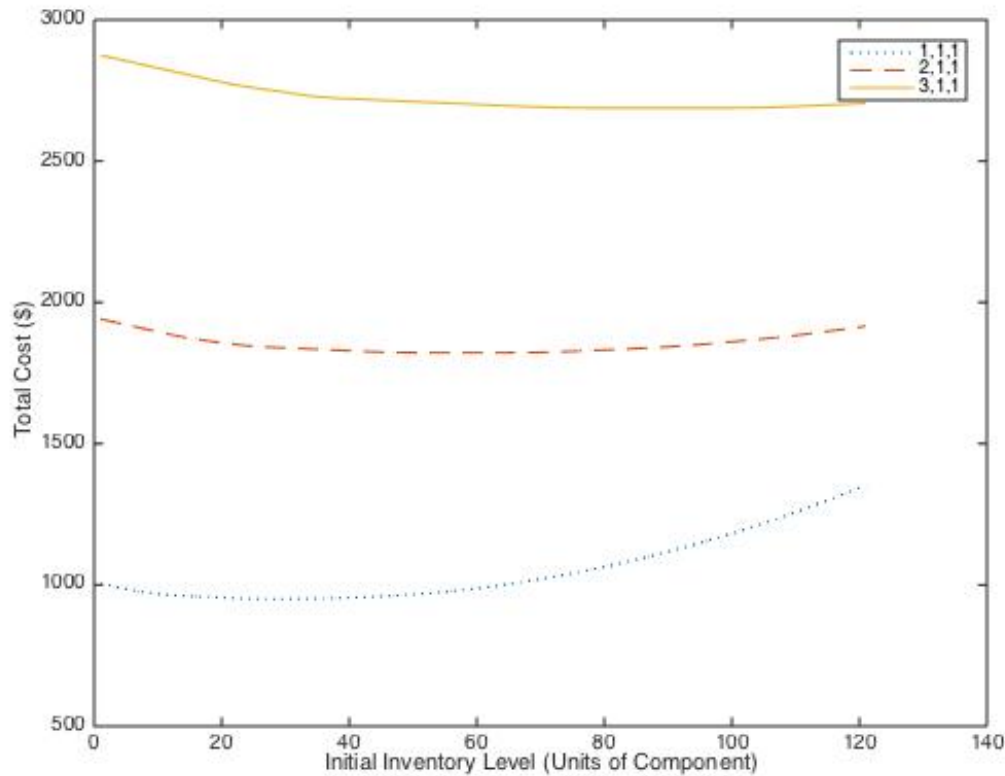
This data in Figure 4 fits perfectly with intuition. Considering the function with the lowest demand ("1,1,1"), the total cost along the y-axis initially decreases as the inventory level, i, increases along the x-axis. If the previous maximum production amount, y, is 0, then the total cost will decrease as inventory increases because less production will be needed per month. However, as the initial inventory level grows, the total cost proves to be convex and begins to increase after its optimal minimum value because the inventory's impact surpasses that of the demand, and holding costs begin to play a larger role. The model also presents the production quantities in Figure 5 below recommended to yield these minimum costs:

Figure 5: Production Quantity (*j*) for *y*=0, *t*=1 – $a_p(i,0,1)$
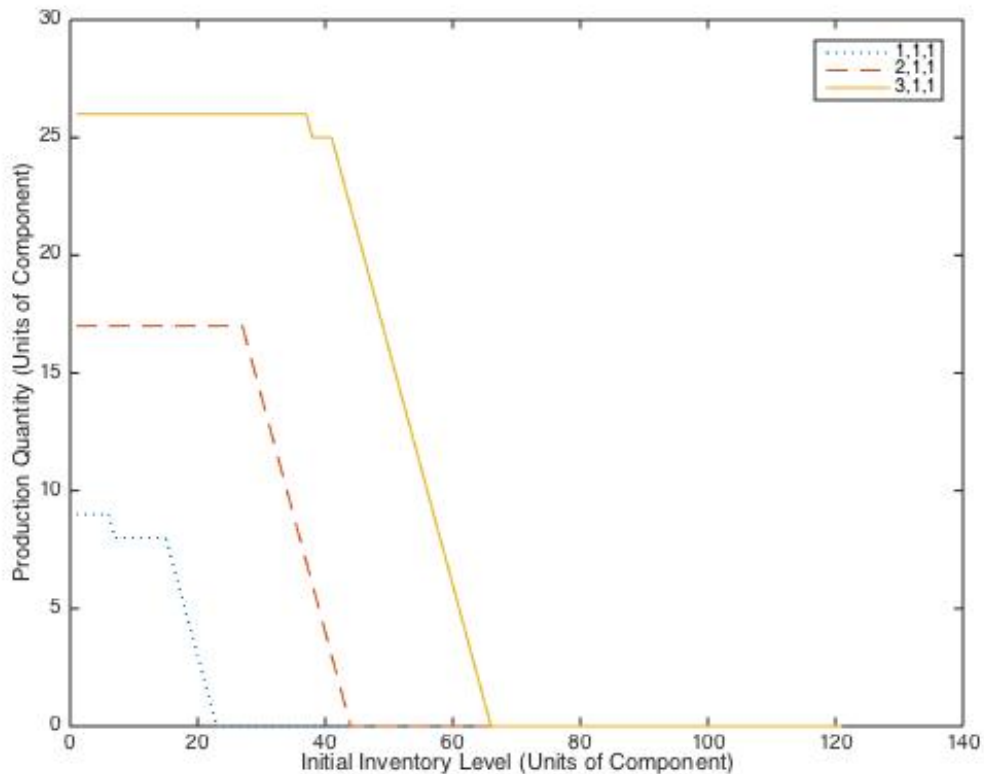
Again, these production amounts, *j*, in Figure 5 above match expectations. As the level of inventory increases, the amount recommended to produce this month decreases. These levels all increase as expected along the y-axis as demands increase ("1,1,1" to "2,1,1" to "3,1,1") and decrease as inventory level increases (x-axis). The model appears to be performing as intended.

Chapter 4: Results

From our validation, the initial inventory level clearly appears to be important in the model,

so we decide to test the impact of demand, production cost, and lost sales' cost upon the

minimum total cost and associated production amount as the level of inventory increases. The

impact of demand seems to be intuitive, but we also want to see the correlated impact when other

parameters change alongside demand. Table 3 below is a matrix of the 27 different scenarios we

use in the experiments. The values of d^*, $c_p{}^*$, and $c_l{}^*$ represent the coefficients of the parameters

for that scenario. For example, d, c_p, and c_l represent the baseline values of each parameter, so

1,1,1 denotes 1 * d, 1 * c_p, and 1 * c_l. Therefore, scenario 3 with d^*, $c_p{}^*$, and $c_l{}^*$ values of 3, 1,

and 1 means that demand for that scenario is 3 * d, production cost is 1 * c_p, and lost sales cost is

1 * c_l. See Table 3 below for all of the scenarios:

Table 3: Scenarios 1-27 with Corresponding Values of d^*, $c_p{}^*$ $c_l{}^*$

	$d^*, c_p{}^*, c_l{}^*$			$d^*, c_p{}^*, c_l{}^*$			$d^*, c_p{}^*, c_l{}^*$
1	1,1,1	|	10	1,1,2	|	19	1,1,3
2	2,1,1	|	11	2,1,2	|	20	2,1,3
3	3,1,1	|	12	3,1,2	|	21	3,1,3
4	1,2,1	|	13	1,2,2	|	22	1,2,3
5	2,2,1	|	14	2,2,2	|	23	2,2,3
6	3,2,1	|	15	3,2,2	|	24	3,2,3
7	1,3,1	|	16	1,3,2	|	25	1,3,3
8	2,3,1	|	17	2,3,2	|	26	2,3,3
9	3,3,1	|	18	3,3,2	|	27	3,3,3

4.1   One Parameter Change

We first look at the impact of either demand, production cost, and lost sales cost changing in

isolation with the other two remaining stationary.

4.1.1   Demand.

Production cost and lost sales cost initialize at \$2 and \$5, respectively. Demand starts at the lowest amount and then increases to two- and three-times the initial amounts (see Figure 4 and Figure 5 in 3.4.2 above for this experiment). Figure 4 above shows that, as demand increases, the total cost, v, also increases. The associated production amounts increase as well in Figure 5. Even though the amounts each still reduce to zero once this becomes cost-effective, this decline occurs later as the demands increase because more component material is needed to meet the larger demands.

4.1.2   Production cost.

Next we consider the impact of changing production cost alone. As the price per unit of component material increases, the total cost should also increase, and the production amount may decrease due to the impact of production costs versus lost sales costs. Figure 6 below shows the effect of a changing production cost when demands and lost sales cost both remain stationary:

Figure 6: Impact of Changing c_p – Total Cost, $v(i,0,1)$
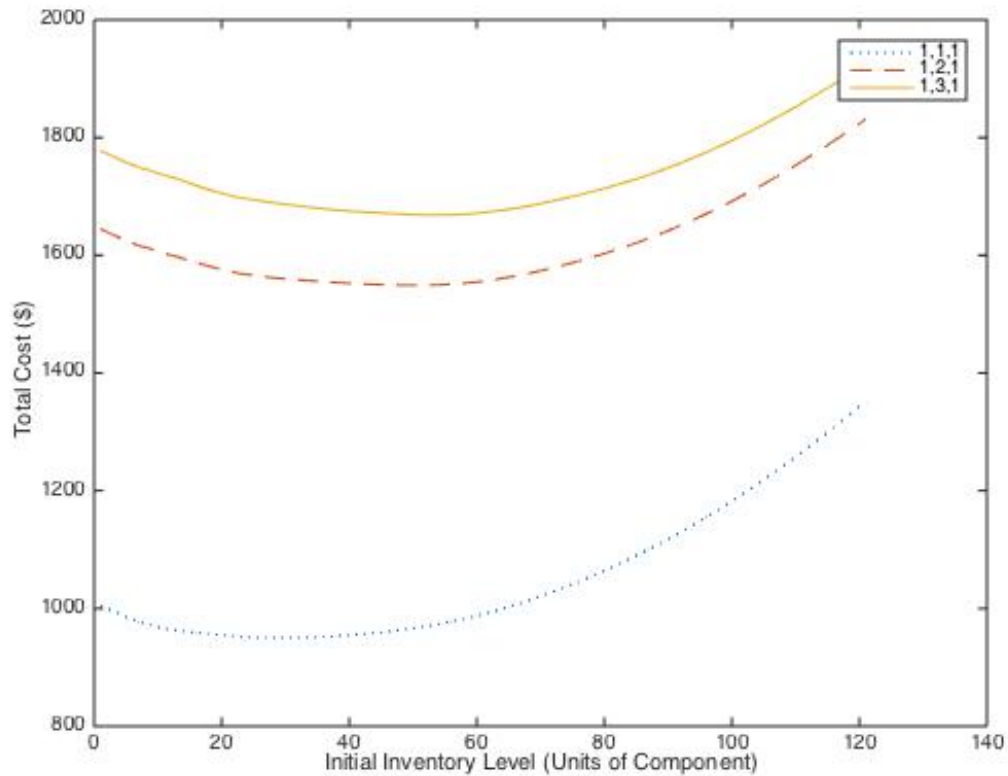
The total cost naturally increases in Figure 6 as the production cost increases ("1,1,1" to "1,2,1" to "1,3,1"). Each of these total cost functions is still convex and follows the same pattern as before, but the highest two functions actually result in lower total costs than the highest two functions from Figure 5, suggesting less production because of the increased c_p. Now we look at the associated production amounts that yield these minimum costs for each inventory level in Figure 7 below:

**Figure 7: Impact of Changing c_p - Production Quantity, $a_p(i,0,1)$**
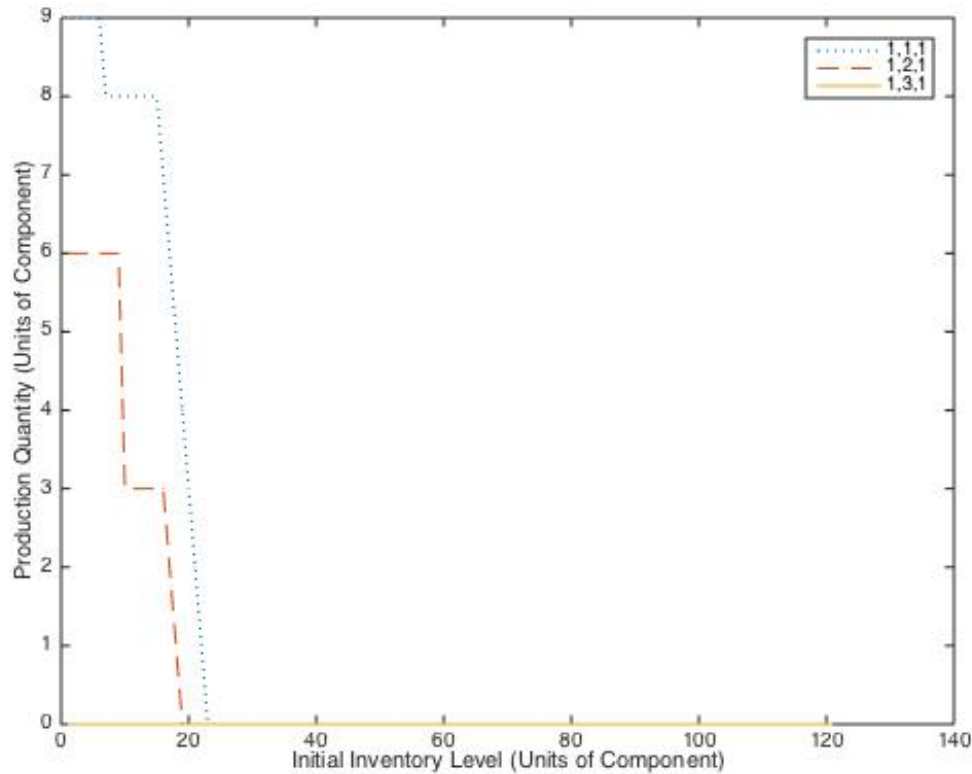
The lowest total costs on Figure 6 correspond with the lowest production cost, but the associated production amount is actually higher than the two options with higher production costs. The function with the highest production cost is "1,3,1," and it actually recommends to produce 0 units of component material for all levels of initial inventory. This actually makes sense because when the production cost reaches a significantly higher amount than the stationary lost sales cost, it becomes advantageous to not produce any product at all and just suffer the price of lost sales. The two scenarios with lower production costs still recommend a decrease to zero because of the effect of the production cost. Production cost does have an impact upon the minimum possible total costs and the recommended production amounts, and all recommended production quantities have decreased from the original scenarios.

4.1.3 Lost sales cost.

Now we consider the impact of the lost sales cost. An initial assumption is that this will have an opposite effect than that of the production cost because the higher the cost of lost sales, the more the company should want to produce in order to meet demands and avoid these costs. The three original scenarios are used except now with lost sales costs increasing to two- and three- times the original amounts below in Figure 8:

Figure 8: Impact of Changing c_l - Total Cost, $v(i,0,1)$
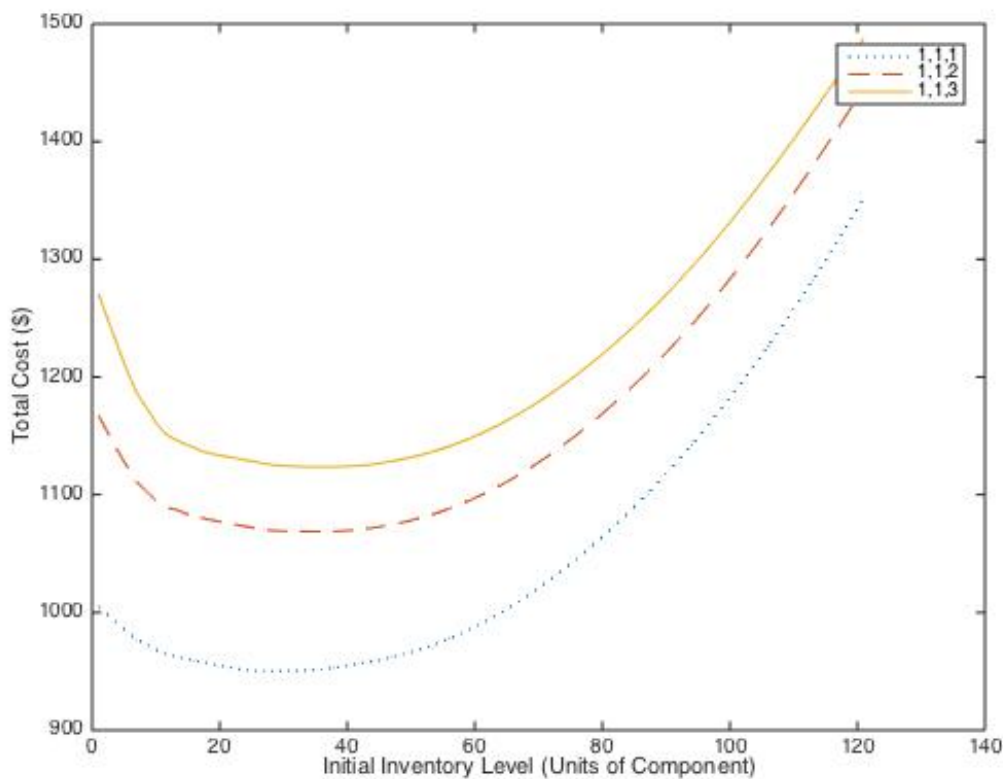
The total cost functions still follow a similar pattern of increasing in Figure 8 as the cost of lost sales increases ("1,1,1" to "1,1,2" to "1,1,3"). The functions are still convex but the highest

two are lower than their original counterparts. Now look at the associated production amounts in Figure 9:

Figure 9: Impact of Changing c_l - Production Quantity, $a_p(i,0,1)$
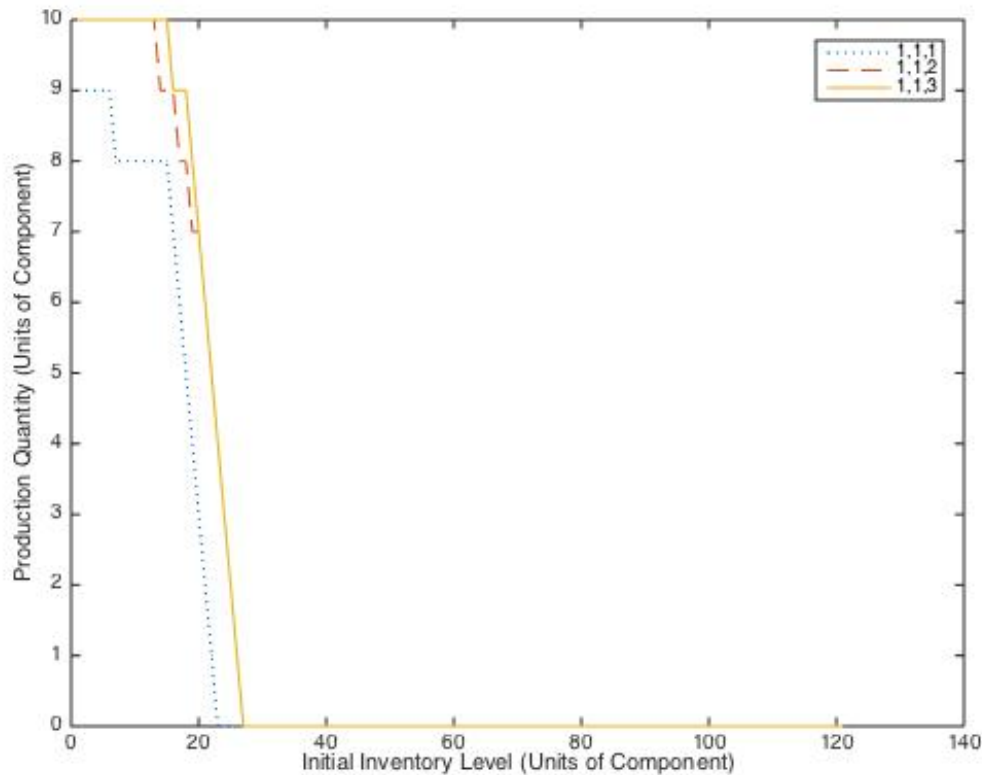
The lowest lost sales cost function ("1,1,1") shows the lowest recommended production, opposite of the effect of the changing production cost (as expected). All three scenarios still decrease to 0 once it becomes cost effective to do so. The two functions with the highest lost sales costs ("1,1,3" and "1,1,2") almost mirror each other, suggesting that any more increase in the lost sales cost will follow the exact same production recommendations. These are also lower production quantities than their original counterparts.

4.2   Two Parameters Change

Now we examine the effect of two parameters changing concurrently within the model. We test to see the interactions between demand, production cost, and lost sales cost with one of those remaining stationary while the other two change.

4.2.1   Demand and production cost.

The original scenarios are again the baselines for this experiment, but then the demand increases to two- and three-times the low amount as the production cost also increases to two- and three-time its original for each value of the demand. The minimum total costs for each scenario are shown below in Figure 10:

Figure 10: Impact of Changing *d* and *c_p* - Total Cost, *v(i,0,1)*
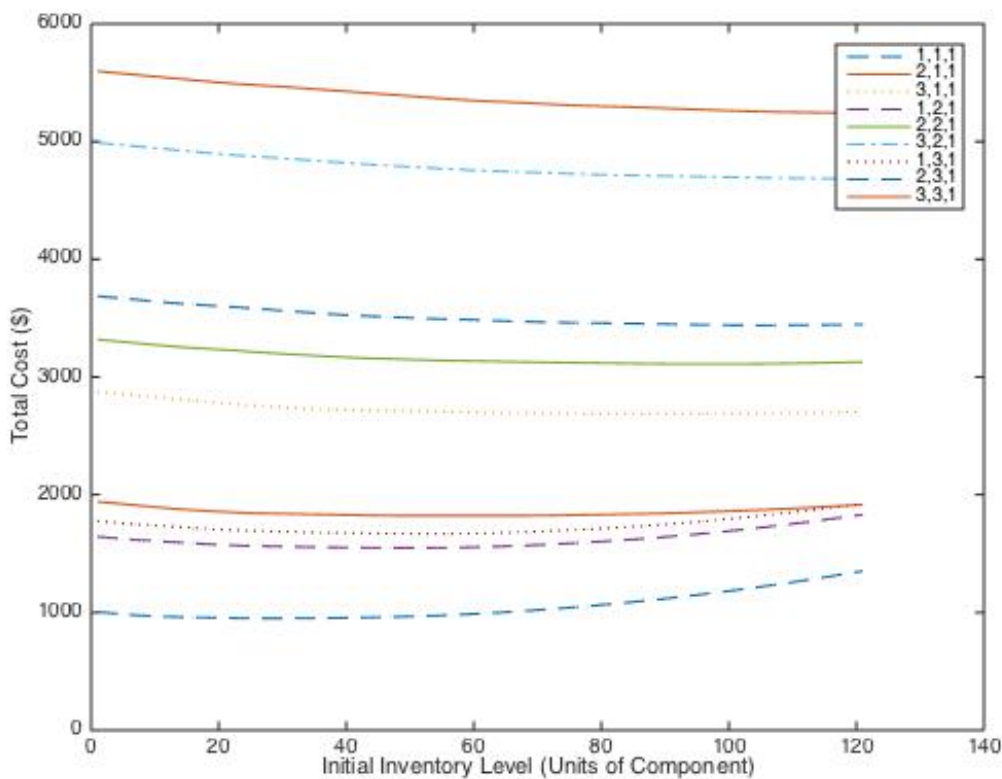
The lowest total cost function in Figure 10 corresponds with the original scenario employing the lowest values of d, c_p, and c_l ("1,1,1"). The functions yielding the highest 2 total costs beginning at \$4,989.72 and \$5,597.25 are the ones using the highest d but also the highest 2 values of c_p ("3,3,1" and "3,2,1"), showing that the interaction with c_p amplifies the effect of d. The corresponding production quantities are listed below in Figure 11:

Figure 11: Impact of Changing d and c_p - Production Quantity, $a_p(i,0,1)$
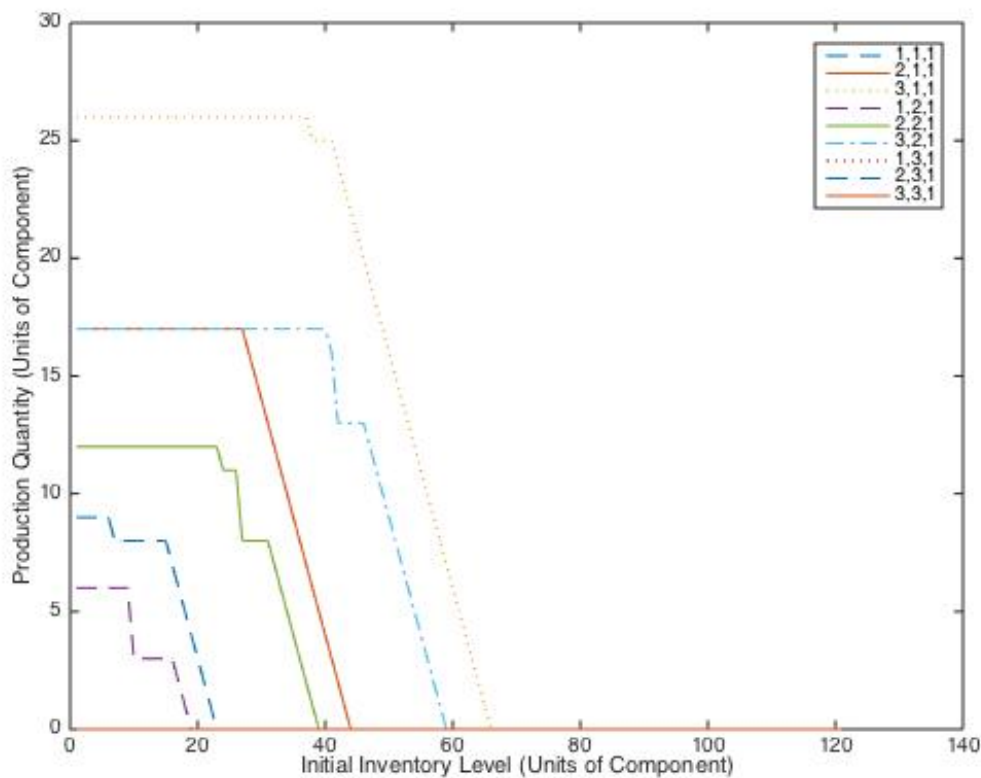
The lowest total cost at an inventory level of zero from Figure 10 corresponds with an initial production amount of 9 on Figure 11. The highest total costs from Figure 10 correspond with production amounts of 0 and 17 in Figure 11 ("3,3,1" and "3,2,1". When c_p is the lowest value, the recommended production amount is actually the highest at 26 ("3,1,1") but then is

lowers to 17 and then to 0 as c_p increases, further displaying the impact of c_p. The larger the value of c_p is, the less the model recommends to produce.

4.2.2 Demand and lost sales cost.

Now the production cost remains constant while the demands and lost sales cost each change from the original values to two- and three-times those values. Notice the impact of the interactions in Figure 12:

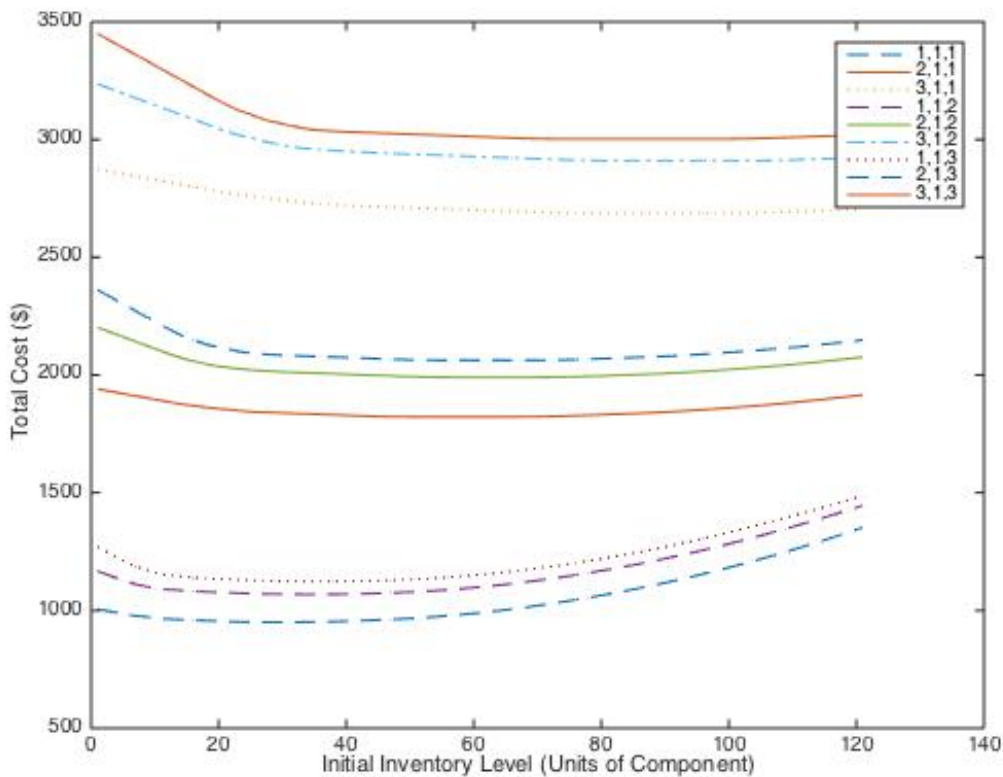**Figure 12: Impact of Changing d and c_l - Total Cost, $v(i,0,1)$**

Figure 12 shows that lost sales cost has a minimal impact. The lowest three total costs in Figure 12 correspond to the functions with the lowest d but all three values of c_l ("1,1,1",

"1,1,2", "1,1,3"). The middle three total costs all use the second value of d along with all three values of c_l. The highest total costs all correspond with the highest values of d, showing that the interaction of c_l with d only slightly alters its impact on the minimum total cost for each scenario ("3,1,3", "3,1,2", "3,1,1"). Figure 13 below shows the corresponding production quantities:

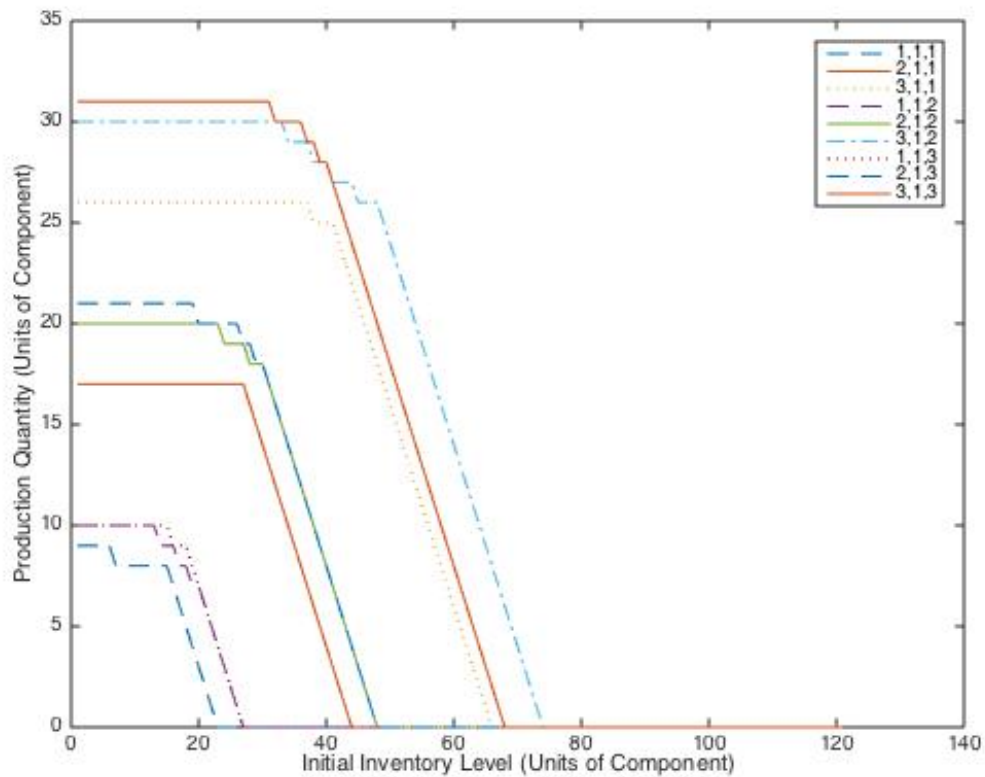**Figure 13: Impact of Changing d and c_l - Production Quantity, $a_p(i,0,1)$**

Figure 13 shows that the recommended production amounts all follow the same pattern as the total costs. Lost sales cost only slightly impacts the effect of changing demands upon the minimum total costs, but the production amounts do in fact increase slightly over the original amounts.

4.2.3   Production cost and lost sales cost.

The demands remain stationary and allow us to observe the interaction between production

cost and lost sales cost. Figure 14 shows the minimum total costs for each scenario:

Figure 14: Impact of Changing c_p and c_l - Total Cost, $v(i,0,1)$
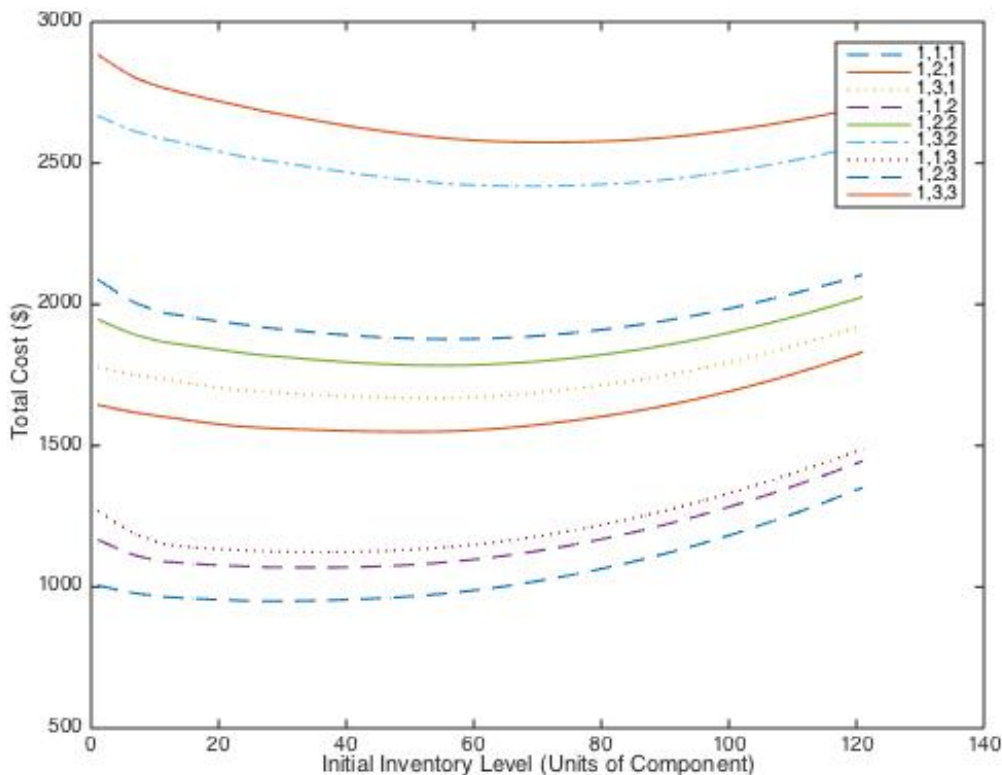
Much like the previous experiment, the lowest costs in Figure 14 correspond with all three

values of lost sales cost but with the lowest value of production cost ("1,1,1", "1,1,2", "1,1,3").

The highest total costs accompany the highest value of c_p and the highest two values of c_l

("1,3,3" and "1,3,2"). These were significantly higher, though, than the function containing the

highest c_p and the lowest c_l, showing that c_l adds some significance to the interaction, though it is

less than c_p. Figure 15 below shows the corresponding production quantities that yield the costs in Figure 14:

Figure 15: Impact of Changing c_p and c_l - Production Quantity, $a_p(i,0,1)$
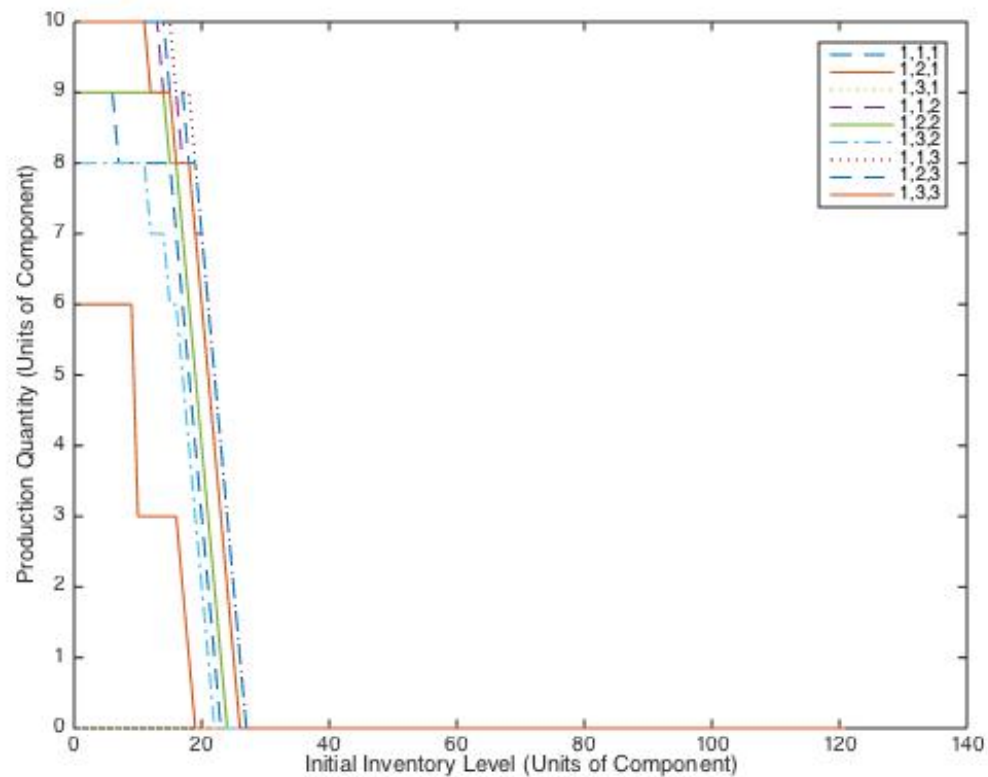
Although the interaction between c_p and c_l did not affect the minimum total costs substantially, Figure 15 shows that it did in fact affect the recommended production amounts to achieve these minimum costs. The initial production amounts all reduce to 10 or less in this situation, showing that this interaction does indeed affect the recommended amount produced per time period for the horizon.

4.3   Three Parameters Change

Finally, we observe the interactions of all three parameters. All three range from the initial

low amounts, two-times those amounts, and three-times the initial amounts (the scenarios are

explained in Table 3 in Chapter 4). The interactions are set up so that each parameter possibility

is paired with all possibilities of the other two so that 27 different scenarios result. Figure 16

displays the minimum total costs of these scenarios:

Figure 16: Impact of Changing d, c_p, and c_l - Total Cost, $v(i,0,1)$
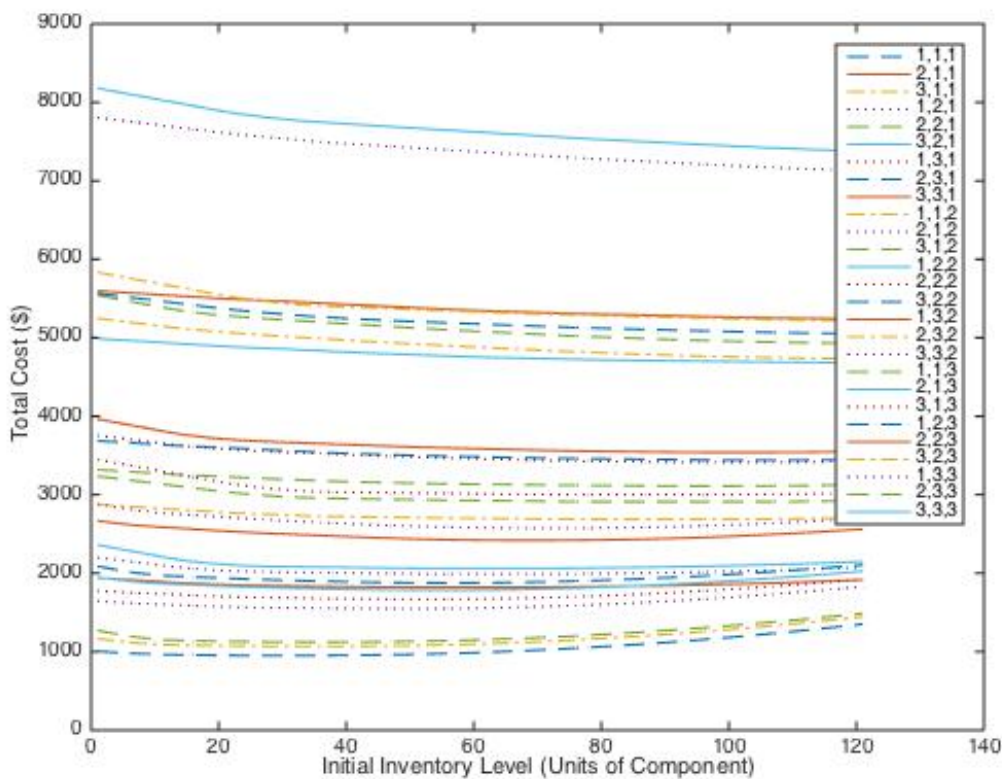

The experimental conditions in Figure 16 raise some very interesting interactions. First of

all, the three lowest total costs, all beginning between $1,000 and $1,500, belong to functions

with d and c_p both equaling the lowest values but c_l equaling all three of it's possible values ("1,1,1", "1,1,2", "1,1,3"). This proves that the lost sales cost has the least effect of all of these parameters.

Second, the two highest total costs ($8,179.30 and $7,806.30) belong to the scenarios with d and c_p both equaling their highest values and c_l equaling its two highest values ("3,3,3" and "3,3,2"). So the interaction of all three parameters has the largest impact, and even the two most impactful parameters, d and c_p, produce a greater impact when in conjunction with c_l. When c_l remains it's minimum ("3,3,1"), the total cost is $5,597.25, which is substantially lower. Therefore, d and c_p have the greatest impact, but c_l does indeed have an impact as well when it is paired along with the other two.

Finally, another notable grouping of total costs begins between $5,000 and $6,000. Upon inspection, none of these scenarios have d and c_p at their lowest values, and only one of these uses c_l as its lowest value ("3,3,1"). This again proves that d and c_p have the greatest impression on the total cost but that c_l serves a purpose as well, dramatically more so when paired with high values of the other two parameters. Now we examine the production amounts in Figure 17 that yield the costs in Figure 16:

Figure 17: Impact of Changing *d*, c_p, and c_l - Production Quantity, $a_p(i,0,1)$
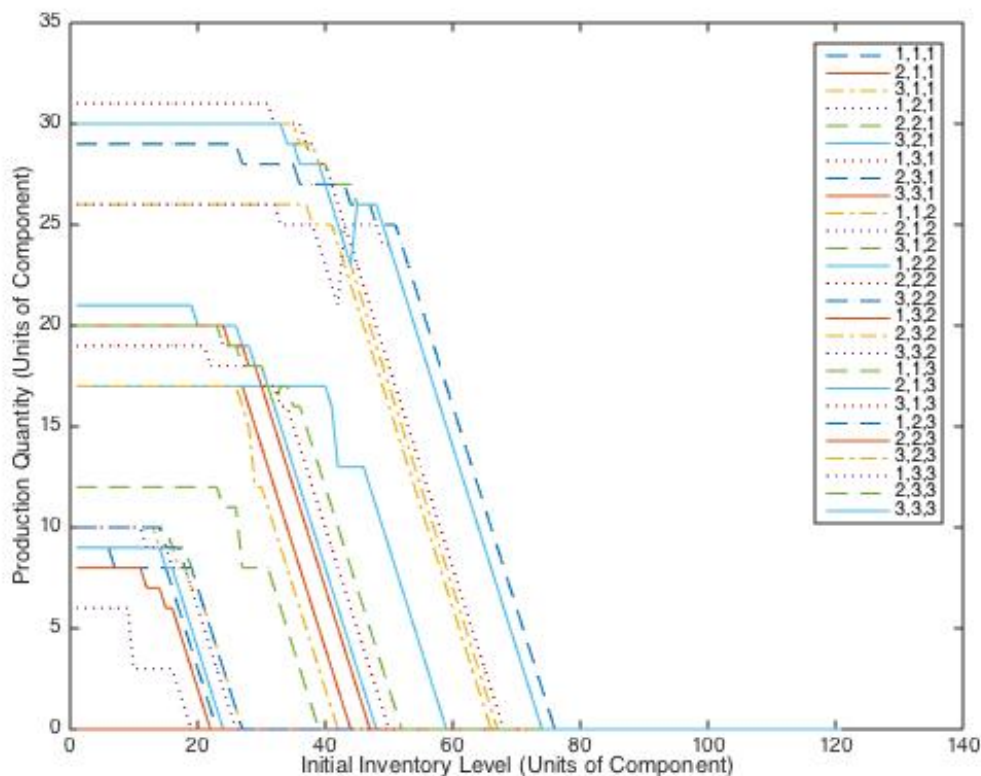
The three lowest total costs from Figure 16 all correspond with some of the lowest initial

productions in Figure 17 ("1,1,1", "1,1,2", "1,1,3"). They are not the absolute lowest, but all

three recommend producing 9 or 10 units of component material when the initial inventory

equals 0. The highest costs from Figure 16 correspond with 2 of the highest recommended

production amounts, 26 and 30, from Figure 17 ("3,3,3" and "3,3,2"). And finally, the third

notable grouping from Figure 16 corresponds with diverse production amounts ranging from 0 to

30 on Figure 17. The lower production values in this grouping all correspond to high c_p (i.e.

"3,2,3") and the higher production values correspond to low c_p (i.e. "3,3,1"). The highest

production amount, a_p, on Figure 17 (31) actually corresponds with a low c_p ("3,1,3"), proving

that c_p has the greatest effect on the recommended production amount when seeking to minimize

total cost.

Chapter 5: Conclusion and Extensions

We discuss the impact of the research and the model and offer extensions for future research. This research makes great insights into inventory control and production of "for cause" companies and opens doors for much more study in this area. Inventory control in the "for cause" domain has many facets yet to be explored.

5.1   Conclusion

The model performs well for inventory horizons up to three years and potentially beyond. The magnitude of changes in minimum costs and production amounts amidst changes in demand, production cost, and lost sales cost show the model to be fairly robust and adaptable. Demand proves to be the biggest factor of the three parameters studied affecting the minimum total costs and associated production amounts, which again should be intuitive. But its interactions with production cost and lost sales cost show the effect to be amplified when paired with other changes. All parameters prove to impact the total costs and recommended production amounts, and production cost proves to actually have the greatest impact on the recommended production amount if the decision maker seeks to minimize the amount produced, but demand far outweighs the two costs when the decision maker seeks the lowest total cost.

However, an even more prominent influence emerges with the amount of initial inventory. Inventory plays a significant role in determining the amount of production that leads to the lowest cost from that time period t throughout the rest of the planning horizon. A low initial level of component material inventory leads to a high production amount recommended per time period, but a higher inventory drastically decreases the production quantity until it reaches the lowest it can possibly become (in the example studied, this is zero because we assume the maximum previous production amount, y, to be zero).

Therefore, the magnitude of demand, initial inventory, and production cost all prove to be very important factors affecting the model's results. This model accurately uses a "for cause" company's demand, production cost, lost sales cost, holding cost, inventory capacity, and production capacity to generate the amount of component to produce beginning in the opening time period that will lead to the minimum cost for the planning horizon. This model also honors the desire to never produce less in one month than has been produced in any previous month, therefore only allowing production to remain constant or increase, which translates to sustained jobs in third world countries and even produces more possible work for the impoverished. We are confident that our model performs as intended and produces unique and applicable results, especially when compared to one such company's current strategy of inventory control that consists of "ordering more material when inventory is running low." This model will ensure lower costs and while sustaining or even increasing labor.

5.2   Extensions

Although the model performs at its intended capacity, there are many directions for future research to expand this topic. The first extension is to directly include supply uncertainty into the model. This model did not explicitly include uncertainty within the production. The model uses a dynamic programming approach that allows the decision maker to respond to and adjust according to uncertainty, but it does not allow for uncertainty within the model yet as it does on the demand side. This will be a very applicable and value-adding extension.

The next extension is to use simulation with randomly generated numbers from the demand distributions to show how the model performs over time against real possibilities of demand. Rather than assuming the expected value, this would test actual demand scenarios that occur and then allow the dynamic program to react and adjust.

Another future area of research is to include sales within the model. This model minimizes costs, but adding sales allows the model to maximize profit. This may result in higher costs but better overall profit for the company and therefore more labor for workers.

This model uses a real scenario, but using actual, larger numbers for demand could show how the model can react in different, real-life situations. The model should be tested with larger numbers because these companies are growing and will require larger possibilities of supply and demand.

Another possible extension is to include more possible values in the triangular distributions. This model assumes a minimum (a) of the mode minus five ($b - 5$) and a maximum (c) of the mode plus five ($b + 5$) for all possible demands, regardless of magnitude. An extension of the research would be to include larger forecasted demands.

The final extension is to examine the impact of the maximum previous production quantity, y, by choosing one inventory level, i, per scenario and plotting the total costs and recommended production amounts as y changes. Choose one initial inventory level, such as $i = 0$, and then examine v and a_p as the maximum previous production, y, increases over the x-axis. This will not show any of the impact of the initial inventory that we see, but it will better show the impact of the maximum previous production quantity.

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